Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.4-Inverse-hyperbolic-cotangent/7.4.1-Inverse-hyperbolic-cotangent-functions

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3.246 $\int \coth^{-1}(1+id-d\tan(a+bx))\,dx$ . . . 937
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3.248 $\int (e+fx)^3\coth^{-1}(\cot(a+bx))\,dx$ . . . 943
3.249 $\int (e+fx)^2\coth^{-1}(\cot(a+bx))\,dx$ . . . 948
3.250 $\int (e+fx)\coth^{-1}(\cot(a+bx))\,dx$ . . . 952
3.251 $\int \coth^{-1}(\cot(a+bx))\,dx$ . . . 957
3.252 $\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx}\,dx$ . . . 960
3.253 $\int x^2\coth^{-1}(c+d\cot(a+bx))\,dx$ . . . 962
3.254 $\int x\coth^{-1}(c+d\cot(a+bx))\,dx$ . . . 967
3.255 $\int \coth^{-1}(c+d\cot(a+bx))\,dx$ . . . 971
3.256 $\int \frac{\coth^{-1}(c+d\cot(a+bx))}{x}\,dx$ . . . 977
3.257 $\int x^2\coth^{-1}(1+id+d\cot(a+bx))\,dx$ . . . 979
3.258 $\int x\coth^{-1}(1+id+d\cot(a+bx))\,dx$ . . . 984

3.259 $\int \coth^{-1}(1+id+d\cot(a+bx))\,dx$ . . . 988
3.260 $\int \frac{\coth^{-1}(1+id+d\cot(a+bx))}{x}\,dx$ . . . 992
3.261 $\int x^2\coth^{-1}(1-id-d\cot(a+bx))\,dx$ . . . 994
3.262 $\int x\coth^{-1}(1-id-d\cot(a+bx))\,dx$ . . . 999
3.263 $\int \coth^{-1}(1-id-d\cot(a+bx))\,dx$ . . . 1003
3.264 $\int \frac{\coth^{-1}(1-id-d\cot(a+bx))}{x}\,dx$ . . . 1007
3.265 $\int \frac{\left(a+b\coth^{-1}(cx^n)\right)\left(d+e\log\left(fx^m\right)\right)}{x}\,dx$ . . . 1009
3.266 $\int x^5\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$ . . . 1013
3.267 $\int x^3\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$ . . . 1019
3.268 $\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$ . . . 1025
3.269 $\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x}\,dx$ . . . 1030
3.270 $\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^3}\,dx$ . . . 1035
3.271 $\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^5}\,dx$ . . . 1040
3.272 $\int x^4\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$ . . . 1045
3.273 $\int x^2\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$ . . . 1052
3.274 $\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$ . . . 1058
3.275 $\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^2}\,dx$ . . . 1062
3.276 $\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^4}\,dx$ . . . 1066
3.277 $\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^6}\,dx$ . . . 1071
3.278 $\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx$ . . . 1076
3.279 $\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx$ . . . 1082
3.280 $\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx$ . . . 1088
3.281 $\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^2}\,dx$ . . . 1091
3.282 $\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^3}\,dx$ . . . 1098
3.283 $\int \coth^{-1}(e^x)\,dx$ . . . 1104
3.284 $\int x\coth^{-1}(e^x)\,dx$ . . . 1107
3.285 $\int x^2\coth^{-1}(e^x)\,dx$ . . . 1110
3.286 $\int \coth^{-1}\left(e^{a+bx}\right)\,dx$ . . . 1113
3.287 $\int x\coth^{-1}\left(e^{a+bx}\right)\,dx$ . . . 1116
3.288 $\int x^2\coth^{-1}\left(e^{a+bx}\right)\,dx$ . . . 1119
3.289 $\int \coth^{-1}\left(a+bf^{c+dx}\right)\,dx$ . . . 1122
3.290 $\int x\coth^{-1}\left(a+bf^{c+dx}\right)\,dx$ . . . 1126
3.291 $\int x^2\coth^{-1}\left(a+bf^{c+dx}\right)\,dx$ . . . 1130
3.292 $\int \frac{1}{\left(a-ax^2\right)\left(b-2b\coth^{-1}(x)\right)}\,dx$ . . . 1135
3.293 $\int x^3\coth^{-1}\left(a+bx^4\right)\,dx$ . . . 1137
3.294 $\int x^{-1+n}\coth^{-1}(a+bx^n)\,dx$ . . . 1140
3.295 $\int e^{c(a+bx)}\coth^{-1}(\sinh(ac+bcx))\,dx$ . . . 1143
3.296 $\int e^{c(a+bx)}\coth^{-1}(\cosh(ac+bcx))\,dx$ . . . 1147
3.297 $\int e^{c(a+bx)}\coth^{-1}(\tanh(ac+bcx))\,dx$ . . . 1150
3.298 $\int e^{c(a+bx)}\coth^{-1}(\coth(ac+bcx))\,dx$ . . . 1153
3.299 $\int e^{c(a+bx)}\coth^{-1}(\mathrm{sech}(ac+bcx))\,dx$ . . . 1156
3.300 $\int e^{c(a+bx)}\coth^{-1}(\mathrm{csch}(ac+bcx))\,dx$ . . . 1159

4 Listing of Grading functions **1163**

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [300]. This is test number [198].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (300)	% 0. (0)
Mathematica	% 98. (294)	% 2. (6)
Maple	% 91. (273)	% 9. (27)
Maxima	% 75.67 (227)	% 24.33 (73)
Fricas	% 74.33 (223)	% 25.67 (77)
Sympy	% 31.67 (95)	% 68.33 (205)
Giac	% 9.67 (29)	% 90.33 (271)

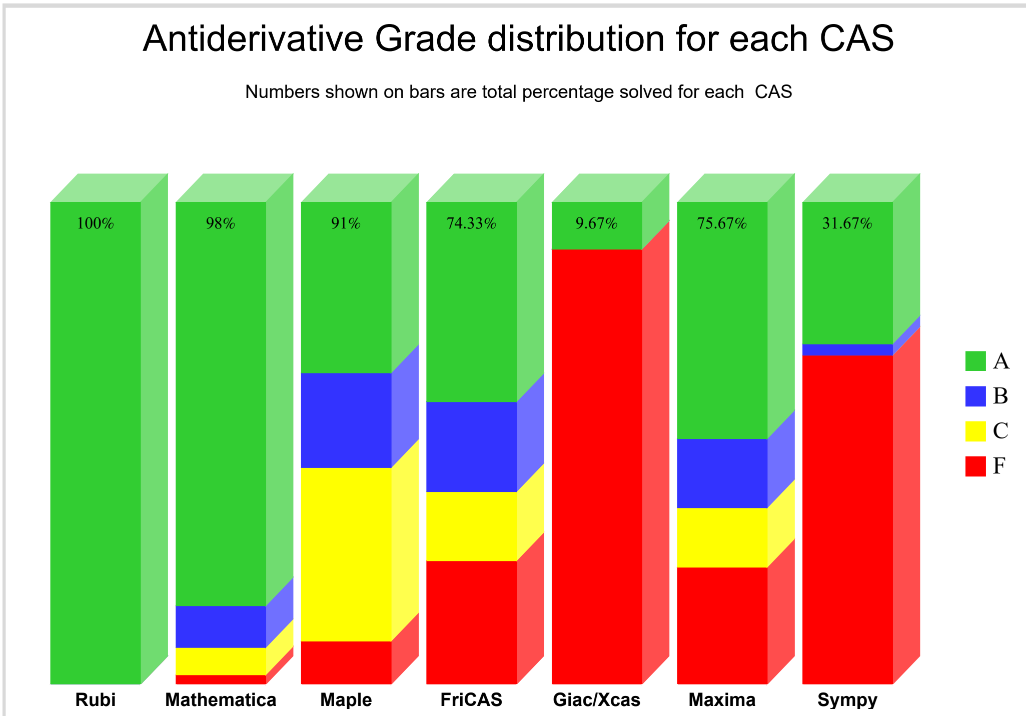
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

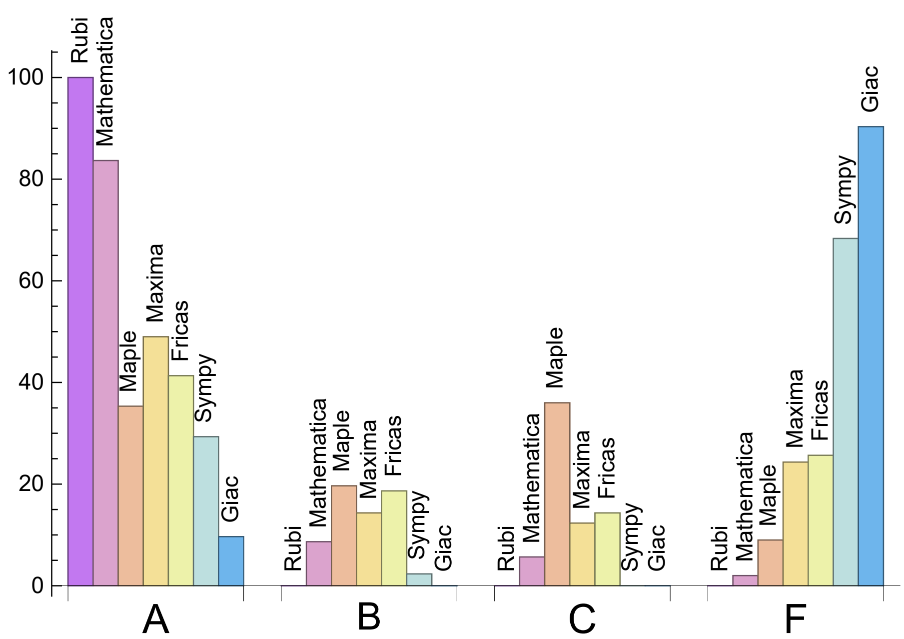
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	83.67	8.67	5.67	2.
Maple	35.33	19.67	36.	9.
Maxima	49.	14.33	12.33	24.33
Fricas	41.33	18.67	14.33	25.67
Sympy	29.33	2.33	0.	68.33
Giac	9.67	0.	0.	90.33

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.23	125.8	0.93	79.	1.
Mathematica	1.01	221.68	1.43	78.	0.93
Maple	3.42	18855.4	149.72	306.	2.61
Maxima	1.42	157.59	1.76	105.	1.51
Fricas	1.64	630.2	5.32	223.	3.56
Sympy	8.34	76.19	1.34	46.	1.15
Giac	0.28	4.83	0.19	0.	0.

1.4 list of integrals that has no closed form antiderivative

{42, 43, 120, 121, 122, 126, 127, 206, 211, 216, 220, 225, 230, 235, 239, 243, 247, 252, 256, 260, 264, 280}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {76, 78}

Mathematica {13, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 40, 41, 48, 49, 56, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 82, 98, 100, 101, 106, 109, 110, 111, 112, 113, 114, 115, 116, 118, 125, 210, 215, 217, 218, 224, 229, 233, 238, 242, 246, 250, 255, 259, 263, 270, 271, 278, 281, 282, 295, 296, 297, 298, 299, 300}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

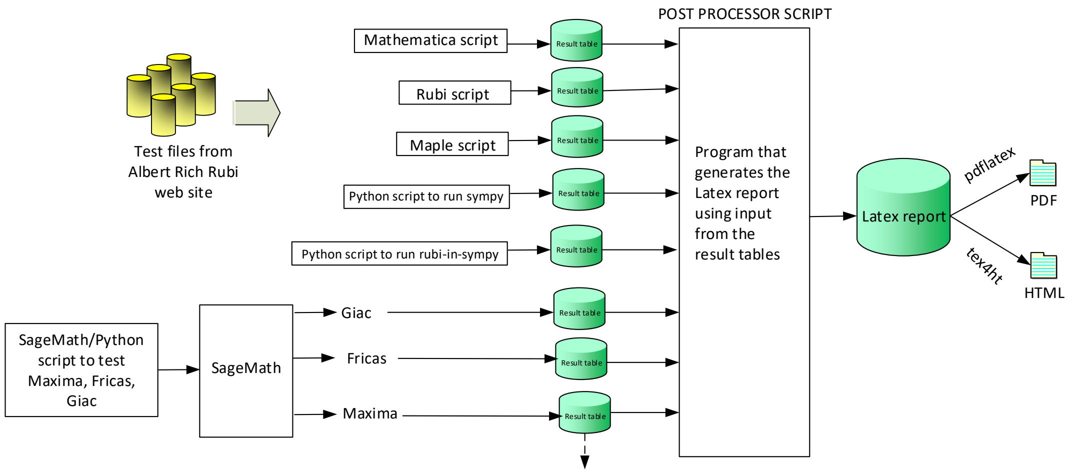
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 71, 72, 76, 77, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 96, 97, 99, 102, 103, 104, 105, 106, 107, 108, 110, 111, 120, 121, 122, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 211, 212, 213, 214, 216, 217, 218, 219, 220, 221, 222, 223, 225, 226, 227, 228, 230, 232, 233, 234, 235, 236, 237, 239, 240, 241, 243, 244, 245, 247, 249, 250, 251, 252, 253, 254, 256, 257, 258, 260, 261, 262, 264, 266, 267, 268, 270, 271, 272, 273, 274, 278, 280, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300 }

B grade: { 41, 70, 95, 98, 100, 101, 109, 139, 150, 210, 215, 224, 229, 231, 238, 242, 246, 248, 255, 259, 263, 275, 276, 279, 281, 283 }

C grade: { 24, 26, 34, 66, 73, 74, 75, 78, 79, 112, 113, 114, 115, 116, 118, 265, 282 }

F grade: { 117, 119, 123, 124, 269, 277}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 15, 35, 36, 37, 38, 42, 43, 48, 49, 50, 51, 52, 53, 54, 57, 58, 60, 64, 65, 66, 67, 68, 72, 74, 75, 77, 78, 80, 81, 83, 84, 85, 87, 88, 89, 90, 91, 92, 94, 95, 97, 98, 99, 100, 101, 105, 106, 107, 108, 113, 120, 121, 122, 125, 126, 127, 133, 134, 135, 140, 151, 162, 171, 180, 189, 197, 198, 199, 200, 206, 211, 216, 220, 225, 230, 235, 239, 243, 247, 252, 256, 260, 264, 280, 282, 283, 284, 285, 286, 288, 289, 292, 293, 298 }

B grade: { 12, 14, 16, 17, 19, 20, 21, 22, 28, 39, 40, 41, 56, 59, 61, 62, 63, 69, 70, 71, 76, 82, 86, 96, 102, 103, 104, 109, 110, 111, 116, 123, 124, 129, 130, 131, 185, 186, 194, 195, 196, 205, 210, 215, 219, 224, 229, 234, 238, 242, 246, 251, 255, 259, 263, 287, 290, 291, 294 }

C grade: { 18, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 55, 73, 79, 93, 112, 114, 115, 117, 118, 128, 132, 136, 137, 138, 139, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 152, 153, 154, 155, 156, 157, 159, 160, 161, 163, 167, 168, 169, 170, 176, 177, 178, 179, 187, 188, 193, 201, 202, 203, 204, 207, 208, 209, 212, 213, 214, 217, 218, 221, 222, 223, 226, 227, 228, 231, 232, 233, 236, 237, 240, 241, 244, 245, 248, 249, 250, 253, 254, 257, 258, 261, 262, 265, 266, 267, 268, 269, 272, 273, 274, 278, 295, 296, 297, 299, 300 }

F grade: { 44, 45, 46, 47, 119, 158, 164, 165, 166, 172, 173, 174, 175, 181, 182, 183, 184, 190, 191, 192, 270, 271, 275, 276, 277, 279, 281 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 20, 21, 23, 35, 36, 37, 38, 51, 52, 53, 57, 58, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 77, 78, 83, 84, 85, 87, 88, 89, 90, 91, 92, 94, 96, 97, 99, 103, 104, 105, 107, 108, 110, 122, 126, 127, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 142, 143, 144, 145, 147, 148, 149, 150, 154, 155, 156, 157, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 235, 239, 243, 247, 252, 256, 260, 264, 268, 280, 284, 285, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 299

B grade: { 7, 16, 17, 19, 22, 25, 27, 31, 33, 55, 56, 59, 61, 86, 93, 95, 98, 100, 101, 102, 109, 140, 151, 234, 238, 240, 241, 242, 244, 245, 246, 251, 255, 257, 258, 259, 261, 262, 263, 283, 286, 295, 300 }

C grade: { 141, 152, 153, 159, 160, 161, 162, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 185, 186, 187, 188, 189, 266, 267, 269, 272, 273, 274 }

F grade: { 18, 24, 26, 28, 29, 30, 32, 34, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 54, 73, 76, 79, 80, 81, 82, 106, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 123, 124, 125, 128, 136, 146, 158, 166, 175, 184, 190, 191, 192, 193, 231, 232, 233, 236, 237, 248, 249, 250, 253, 254, 265, 270, 271, 275, 276, 277, 278, 279, 281, 282 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 16, 20, 22, 35, 36, 37, 38, 42, 43, 50, 51, 52, 55, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 83, 84, 85, 87, 88, 89, 90, 91, 92, 94, 96, 97, 99, 104, 105, 120, 121, 122, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 152, 153, 154, 155, 156, 157, 159, 160, 163, 193, 194, 195, 196, 197, 198, 199, 206, 211, 216, 220, 225, 230, 235, 239, 243, 247, 252, 256, 259, 260, 263, 264, 266, 267, 268, 272, 273, 274, 280, 289, 292, 293, 296, 297, 298, 299 }

B grade: { 44, 45, 46, 47, 53, 54, 57, 95, 102, 103, 107, 108, 151, 161, 162, 164, 165, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 185, 186, 187, 188, 189, 200, 205, 210, 215, 219, 224, 229, 234, 238, 242, 246, 251, 255, 283, 286, 294, 295, 300 }

C grade: { 201, 202, 203, 204, 207, 208, 209, 212, 213, 214, 217, 218, 221, 222, 223, 226, 227, 228, 231, 232, 233, 236, 237, 240, 241, 244, 245, 248, 249, 250, 253, 254, 257, 258, 261, 262, 265, 284, 285, 287, 288, 290, 291 }

F grade: { 7, 13, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 39, 40, 41, 48, 49, 56, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 86, 93, 98, 100, 101, 106, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 123, 124, 125, 158, 166, 175, 184, 190, 191, 192, 269, 270, 271, 275, 276, 277, 278, 279, 281, 282 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 16, 20, 22, 35, 36, 37, 38, 42, 43, 53, 54, 57, 58, 63, 64, 65, 67, 68, 90, 94, 96, 97, 99, 103, 104, 105, 122, 126, 127, 129, 130, 131, 133, 134, 135, 137, 138, 139, 140, 143, 144, 145, 147, 148, 149, 150, 151, 155, 157, 162, 170, 171, 178, 179, 180, 189, 194, 195, 196, 198, 199, 211, 216, 225, 230, 235, 252, 266, 267, 268, 272, 273, 274, 292, 293, 297 }

B grade: { 60, 87, 88, 89, 91, 92, 156 }

C grade: { }

F grade: { 7, 13, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 39, 40, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 55, 56, 59, 61, 62, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 93, 95, 98, 100, 101, 102, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 123, 124, 125, 128, 132, 136, 141, 142, 146, 152, 153, 154, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 172, 173, 174, 175, 176, 177, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 197, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 224, 226, 227, 228, 229, 231, 232, 233, 234, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 269, 270, 271, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 294, 295, 296, 298, 299, 300 }

2.1.7 Giac

A grade: { 42, 43, 120, 121, 122, 126, 127, 194, 195, 196, 197, 198, 199, 206, 211, 216, 220, 225, 230, 235, 239, 243, 247, 252, 256, 260, 264, 280, 298 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 123, 124, 125, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 200, 201, 202, 203, 204, 205, 207, 208, 209, 210, 212, 213, 214, 215, 217, 218, 219, 221, 222, 223, 224, 226, 227, 228, 229, 231, 232, 233, 234, 236, 237, 238, 240, 241, 242, 244, 245, 246, 248, 249, 250, 251, 253, 254, 255, 257, 258, 259, 261, 262, 263, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 299, 300 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	67	55	82	120	49	0
normalized size	1	1.	1.31	1.08	1.61	2.35	0.96	0.
time (sec)	N/A	0.028	0.009	0.035	0.965	1.54	2.07	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	62	122	54	0
normalized size	1	1.	1.	0.98	1.24	2.44	1.08	0.
time (sec)	N/A	0.037	0.008	0.035	0.958	1.576	3.798	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	57	47	70	99	41	0
normalized size	1	1.	1.39	1.15	1.71	2.41	1.	0.
time (sec)	N/A	0.024	0.008	0.031	0.966	1.599	1.284	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	41	47	99	46	0
normalized size	1	1.	1.	1.02	1.18	2.48	1.15	0.
time (sec)	N/A	0.03	0.008	0.032	0.968	1.573	2.373	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	47	39	55	78	32	0
normalized size	1	1.	1.52	1.26	1.77	2.52	1.03	0.
time (sec)	N/A	0.013	0.007	0.033	0.973	1.618	0.919	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	23	34	77	27	0
normalized size	1	1.	1.	0.92	1.36	3.08	1.08	0.
time (sec)	N/A	0.007	0.003	0.043	0.987	1.68	0.609	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	37	116	0	0	0
normalized size	1	1.	0.93	1.32	4.14	0.	0.	0.
time (sec)	N/A	0.01	0.008	0.042	0.997	0.	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	35	41	99	26	0
normalized size	1	1.	1.	1.17	1.37	3.3	0.87	0.
time (sec)	N/A	0.021	0.008	0.036	0.974	1.7	0.611	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	47	39	49	80	24	0
normalized size	1	1.	1.52	1.26	1.58	2.58	0.77	0.
time (sec)	N/A	0.016	0.008	0.039	0.973	1.519	2.571	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	48	54	120	46	0
normalized size	1	1.	1.	1.02	1.15	2.55	0.98	0.
time (sec)	N/A	0.03	0.01	0.039	0.945	1.57	4.121	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	57	47	69	100	32	0
normalized size	1	1.	1.39	1.15	1.68	2.44	0.78	0.
time (sec)	N/A	0.022	0.012	0.036	0.944	1.573	1.96	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	80	196	182	224	114	0
normalized size	1	1.	0.76	1.87	1.73	2.13	1.09	0.
time (sec)	N/A	0.246	0.022	0.052	0.971	1.577	7.505	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	87	196	209	0	0	0
normalized size	1	1.	0.69	1.54	1.65	0.	0.	0.
time (sec)	N/A	0.226	0.443	0.053	0.961	0.	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	62	176	159	184	90	0
normalized size	1	1.	0.77	2.17	1.96	2.27	1.11	0.
time (sec)	N/A	0.163	0.017	0.052	0.955	1.597	5.101	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	66	176	181	0	0	0
normalized size	1	1.	0.64	1.71	1.76	0.	0.	0.
time (sec)	N/A	0.153	0.238	0.052	0.974	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	43	155	131	143	60	0
normalized size	1	1.	0.8	2.87	2.43	2.65	1.11	0.
time (sec)	N/A	0.078	0.012	0.056	0.986	1.506	4.527	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	58	58	46	122	182	0	0	0
normalized size	1	1.	0.79	2.1	3.14	0.	0.	0.
time (sec)	N/A	0.078	0.084	0.119	0.987	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	114	487	0	0	0	0
normalized size	1	1.	1.18	5.02	0.	0.	0.	0.
time (sec)	N/A	0.232	0.063	0.381	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	55	55	49	159	197	0	0	0
normalized size	1	1.	0.89	2.89	3.58	0.	0.	0.
time (sec)	N/A	0.108	0.1	0.056	0.982	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	57	164	130	181	56	0
normalized size	1	1.	0.93	2.69	2.13	2.97	0.92	0.
time (sec)	N/A	0.099	0.017	0.055	0.972	1.899	1.01	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	87	224	238	0	0	0
normalized size	1	1.	0.84	2.17	2.31	0.	0.	0.
time (sec)	N/A	0.171	0.246	0.06	0.989	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	82	185	208	223	90	0
normalized size	1	1.	0.91	2.06	2.31	2.48	1.	0.
time (sec)	N/A	0.172	0.022	0.06	0.983	1.838	6.423	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	186	186	117	1141	390	0	0	0
normalized size	1	1.	0.63	6.13	2.1	0.	0.	0.
time (sec)	N/A	0.717	0.526	1.901	1.012	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	196	196	175	806	0	0	0	0
normalized size	1	1.	0.89	4.11	0.	0.	0.	0.
time (sec)	N/A	0.58	0.567	1.106	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	139	88	684	354	0	0	0
normalized size	1	1.	0.63	4.92	2.55	0.	0.	0.
time (sec)	N/A	0.417	0.3	0.619	0.995	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	149	149	140	765	0	0	0	0
normalized size	1	1.	0.94	5.13	0.	0.	0.	0.
time (sec)	N/A	0.333	0.353	0.57	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	95	95	68	3070	290	0	0	0
normalized size	1	1.	0.72	32.32	3.05	0.	0.	0.
time (sec)	N/A	0.183	0.141	0.404	0.999	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	79	180	0	0	0	0
normalized size	1	1.	0.93	2.12	0.	0.	0.	0.
time (sec)	N/A	0.165	0.098	0.115	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	150	150	156	564	0	0	0	0
normalized size	1	1.	1.04	3.76	0.	0.	0.	0.
time (sec)	N/A	0.35	0.076	0.253	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	72	796	0	0	0	0
normalized size	1	1.	0.91	10.08	0.	0.	0.	0.
time (sec)	N/A	0.199	0.132	0.342	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	95	95	79	3673	340	0	0	0
normalized size	1	1.	0.83	38.66	3.58	0.	0.	0.
time (sec)	N/A	0.219	0.167	0.422	0.992	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	154	154	142	895	0	0	0	0
normalized size	1	1.	0.92	5.81	0.	0.	0.	0.
time (sec)	N/A	0.369	0.213	0.657	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	141	141	118	661	462	0	0	0
normalized size	1	1.	0.84	4.69	3.28	0.	0.	0.
time (sec)	N/A	0.464	0.23	0.536	1.01	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	164	164	741	926	0	0	0	0
normalized size	1	1.	4.52	5.65	0.	0.	0.	0.
time (sec)	N/A	0.034	7.684	0.625	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	213	334	373	558	427	0
normalized size	1	1.	0.87	1.36	1.52	2.28	1.74	0.
time (sec)	N/A	0.181	0.116	0.04	0.972	1.584	25.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	150	233	267	382	282	0
normalized size	1	1.	0.89	1.38	1.58	2.26	1.67	0.
time (sec)	N/A	0.127	0.079	0.037	0.974	1.627	7.312	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	98	148	177	258	182	0
normalized size	1	1.	0.89	1.35	1.61	2.35	1.65	0.
time (sec)	N/A	0.133	0.054	0.036	0.973	1.549	3.323	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	69	76	88	142	87	0
normalized size	1	1.	1.21	1.33	1.54	2.49	1.53	0.
time (sec)	N/A	0.066	0.012	0.038	0.974	1.515	1.373	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	390	390	671	785	0	0	0	0
normalized size	1	1.	1.72	2.01	0.	0.	0.	0.
time (sec)	N/A	0.947	1.344	0.218	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	590	590	753	2218	0	0	0	0
normalized size	1	1.	1.28	3.76	0.	0.	0.	0.
time (sec)	N/A	0.87	7.815	0.41	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	657	657	1838	4128	0	0	0	0
normalized size	1	1.	2.8	6.28	0.	0.	0.	0.
time (sec)	N/A	0.951	12.88	0.596	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	24.127	0.713	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	4.359	0.724	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	119	0	0	760	0	0
normalized size	1	1.	1.92	0.	0.	12.26	0.	0.
time (sec)	N/A	0.113	0.118	0.457	0.	1.747	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	226	0	0	1496	0	0
normalized size	1	1.	1.77	0.	0.	11.69	0.	0.
time (sec)	N/A	0.34	0.321	0.456	0.	1.985	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	329	0	0	2610	0	0
normalized size	1	1.	1.64	0.	0.	13.05	0.	0.
time (sec)	N/A	1.047	0.636	0.455	0.	2.245	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	431	0	0	4140	0	0
normalized size	1	1.	1.52	0.	0.	14.63	0.	0.
time (sec)	N/A	1.343	0.961	0.461	0.	3.193	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	186	186	125	199	0	0	0	0
normalized size	1	1.	0.67	1.07	0.	0.	0.	0.
time (sec)	N/A	0.083	1.039	0.448	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	77	190	0	0	0	0
normalized size	1	1.	0.53	1.32	0.	0.	0.	0.
time (sec)	N/A	0.05	0.112	0.368	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	30	52	0	92	0	0
normalized size	1	1.	0.81	1.41	0.	2.49	0.	0.
time (sec)	N/A	0.025	0.044	0.226	0.	1.536	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	45	112	90	140	0	0
normalized size	1	1.	0.54	1.35	1.08	1.69	0.	0.
time (sec)	N/A	0.053	0.05	0.238	1.017	1.584	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	55	176	134	189	0	0
normalized size	1	1.	0.44	1.42	1.08	1.52	0.	0.
time (sec)	N/A	0.082	0.061	0.261	1.026	1.679	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	36	3	0
normalized size	1	1.	1.	1.33	1.33	12.	1.	0.
time (sec)	N/A	0.023	0.025	0.043	0.976	1.507	0.411	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	182	15	0
normalized size	1	1.	1.	1.08	0.	15.17	1.25	0.
time (sec)	N/A	0.026	0.009	0.059	0.	1.614	4.822	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	61	707	231	165	0	0
normalized size	1	1.	0.98	11.4	3.73	2.66	0.	0.
time (sec)	N/A	0.051	0.083	0.494	1.025	1.569	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	37	37	34	75	103	0	0	0
normalized size	1	1.	0.92	2.03	2.78	0.	0.	0.
time (sec)	N/A	0.059	0.049	0.035	0.963	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	13	8	38	5	0
normalized size	1	1.	1.	1.62	1.	4.75	0.62	0.
time (sec)	N/A	0.014	0.004	0.029	0.954	1.558	0.817	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	44	39	46	74	31	0
normalized size	1	1.	1.22	1.08	1.28	2.06	0.86	0.
time (sec)	N/A	0.031	0.027	0.027	0.946	1.588	0.819	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	28	99	103	111	0	0
normalized size	1	1.	0.74	2.61	2.71	2.92	0.	0.
time (sec)	N/A	0.017	0.032	0.043	0.967	1.584	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	53	63	111	88	0
normalized size	1	1.	1.	1.06	1.26	2.22	1.76	0.
time (sec)	N/A	0.034	0.051	0.028	0.949	1.593	1.329	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	43	131	159	165	0	0
normalized size	1	1.	0.64	1.96	2.37	2.46	0.	0.
time (sec)	N/A	0.035	0.052	0.049	0.962	1.582	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	81	199	143	279	0	0
normalized size	1	1.	0.8	1.97	1.42	2.76	0.	0.
time (sec)	N/A	0.126	0.039	0.035	0.96	1.551	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	92	146	107	213	117	0
normalized size	1	1.	1.18	1.87	1.37	2.73	1.5	0.
time (sec)	N/A	0.102	0.023	0.036	0.958	1.611	3.197	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	56	89	82	176	76	0
normalized size	1	1.	0.86	1.37	1.26	2.71	1.17	0.
time (sec)	N/A	0.072	0.022	0.033	0.972	1.6	1.609	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	43	36	42	135	41	0
normalized size	1	1.	1.23	1.03	1.2	3.86	1.17	0.
time (sec)	N/A	0.016	0.015	0.03	0.974	1.544	0.819	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	92	92	259	81	173	0	0	0
normalized size	1	1.	2.82	0.88	1.88	0.	0.	0.
time (sec)	N/A	0.097	0.168	0.076	0.972	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	55	63	73	192	144	0
normalized size	1	1.	0.86	0.98	1.14	3.	2.25	0.
time (sec)	N/A	0.051	0.051	0.037	0.959	1.676	2.885	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	76	82	115	279	410	0
normalized size	1	1.	0.84	0.91	1.28	3.1	4.56	0.
time (sec)	N/A	0.1	0.107	0.043	0.96	1.759	3.456	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	263	263	203	967	432	0	0	0
normalized size	1	1.	0.77	3.68	1.64	0.	0.	0.
time (sec)	N/A	0.349	1.564	0.056	0.988	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	204	204	607	729	350	0	0	0
normalized size	1	1.	2.98	3.57	1.72	0.	0.	0.
time (sec)	N/A	0.278	4.527	0.055	1.002	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	136	136	106	365	273	0	0	0
normalized size	1	1.	0.78	2.68	2.01	0.	0.	0.
time (sec)	N/A	0.209	0.247	0.053	0.985	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	55	151	188	0	0	0
normalized size	1	1.	0.68	1.86	2.32	0.	0.	0.
time (sec)	N/A	0.09	0.07	0.102	0.978	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	714	985	0	0	0	0
normalized size	1	1.	4.82	6.66	0.	0.	0.	0.
time (sec)	N/A	0.091	3.205	0.543	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	251	251	206	342	329	0	0	0
normalized size	1	1.	0.82	1.36	1.31	0.	0.	0.
time (sec)	N/A	0.719	1.04	0.096	1.028	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	370	370	291	467	486	0	0	0
normalized size	1	1.	0.79	1.26	1.31	0.	0.	0.
time (sec)	N/A	0.83	2.184	0.099	1.024	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	673	597	529	1230	0	0	0	0
normalized size	1	0.89	0.79	1.83	0.	0.	0.	0.
time (sec)	N/A	1.099	0.586	0.49	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	185	176	259	0	0	0
normalized size	1	1.	1.54	1.47	2.16	0.	0.	0.
time (sec)	N/A	0.127	0.067	0.137	0.987	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	292	360	502	297	259	0	0	0
normalized size	1	1.23	1.72	1.02	0.89	0.	0.	0.
time (sec)	N/A	0.5	4.258	0.142	0.996	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	738	738	5552	19686	0	0	0	0
normalized size	1	1.	7.52	26.67	0.	0.	0.	0.
time (sec)	N/A	1.53	35.665	1.251	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	619	619	575	738	0	0	0	0
normalized size	1	1.	0.93	1.19	0.	0.	0.	0.
time (sec)	N/A	2.199	0.666	0.191	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	738	738	719	970	0	0	0	0
normalized size	1	1.	0.97	1.31	0.	0.	0.	0.
time (sec)	N/A	2.371	0.694	0.161	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	335	335	596	2098	0	0	0	0
normalized size	1	1.	1.78	6.26	0.	0.	0.	0.
time (sec)	N/A	0.749	0.898	0.586	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	59	42	55	111	0	0
normalized size	1	1.	1.16	0.82	1.08	2.18	0.	0.
time (sec)	N/A	0.015	0.018	0.036	0.961	1.606	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	52	37	49	95	0	0
normalized size	1	1.	1.24	0.88	1.17	2.26	0.	0.
time (sec)	N/A	0.011	0.015	0.033	0.958	1.532	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	27	35	76	0	0
normalized size	1	1.	1.	1.23	1.59	3.45	0.	0.
time (sec)	N/A	0.006	0.006	0.034	0.952	1.639	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	33	89	0	0	0
normalized size	1	1.	1.	1.74	4.68	0.	0.	0.
time (sec)	N/A	0.02	0.01	0.054	0.96	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	45	32	42	84	92	0
normalized size	1	1.	1.8	1.28	1.68	3.36	3.68	0.
time (sec)	N/A	0.014	0.024	0.038	0.969	1.571	3.663	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	58	37	49	107	160	0
normalized size	1	1.	1.38	0.88	1.17	2.55	3.81	0.
time (sec)	N/A	0.015	0.019	0.035	0.985	1.591	9.807	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	31	35	32	111	121	0
normalized size	1	1.	0.82	0.92	0.84	2.92	3.18	0.
time (sec)	N/A	0.018	0.015	0.034	0.957	1.579	9.513	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	25	30	26	96	39	0
normalized size	1	1.	0.81	0.97	0.84	3.1	1.26	0.
time (sec)	N/A	0.014	0.01	0.036	0.99	1.655	1.607	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	22	74	87	0
normalized size	1	1.	1.	0.75	1.1	3.7	4.35	0.
time (sec)	N/A	0.009	0.007	0.036	0.967	1.645	0.781	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	29	24	99	126	0
normalized size	1	1.	1.	1.21	1.	4.12	5.25	0.
time (sec)	N/A	0.01	0.016	0.04	0.986	1.64	2.157	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	85	140	0	0	0
normalized size	1	1.	0.93	3.04	5.	0.	0.	0.
time (sec)	N/A	0.022	0.013	0.146	0.991	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	30	20	65	15	0
normalized size	1	1.	1.	1.58	1.05	3.42	0.79	0.
time (sec)	N/A	0.006	0.002	0.056	0.977	1.859	0.254	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	97	61	198	421	0	0
normalized size	1	1.	2.55	1.61	5.21	11.08	0.	0.
time (sec)	N/A	0.023	0.045	0.09	1.269	2.004	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	66	70	84	108	56	0
normalized size	1	1.	1.69	1.79	2.15	2.77	1.44	0.
time (sec)	N/A	0.022	0.032	0.036	0.966	1.924	1.738	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	42	95	109	211	97	0
normalized size	1	1.	0.78	1.76	2.02	3.91	1.8	0.
time (sec)	N/A	0.048	0.035	0.034	0.956	1.872	3.133	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	35	35	286	59	151	0	0	0
normalized size	1	1.	8.17	1.69	4.31	0.	0.	0.
time (sec)	N/A	0.025	0.025	0.042	0.981	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	43	54	72	171	136	0
normalized size	1	1.	0.9	1.12	1.5	3.56	2.83	0.
time (sec)	N/A	0.045	0.026	0.04	0.976	1.912	4.826	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	25	25	117	34	78	0	0	0
normalized size	1	1.	4.68	1.36	3.12	0.	0.	0.
time (sec)	N/A	0.023	0.013	0.029	0.979	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	35	35	312	59	178	0	0	0
normalized size	1	1.	8.91	1.69	5.09	0.	0.	0.
time (sec)	N/A	0.03	0.023	0.044	0.99	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	270	786	450	856	0	0
normalized size	1	1.	1.61	4.68	2.68	5.1	0.	0.
time (sec)	N/A	0.342	0.281	0.046	0.992	2.114	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	174	477	279	548	369	0
normalized size	1	1.	1.45	3.98	2.32	4.57	3.08	0.
time (sec)	N/A	0.204	0.162	0.04	0.971	1.965	18.117	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	138	184	147	319	173	0
normalized size	1	1.	1.42	1.9	1.52	3.29	1.78	0.
time (sec)	N/A	0.171	0.043	0.04	0.961	1.892	2.626	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	48	42	49	157	46	0
normalized size	1	1.	1.2	1.05	1.22	3.92	1.15	0.
time (sec)	N/A	0.025	0.015	0.033	0.971	1.913	0.761	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	206	202	0	0	0	0
normalized size	1	1.	1.58	1.55	0.	0.	0.	0.
time (sec)	N/A	0.142	0.113	0.153	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	125	141	163	583	0	0
normalized size	1	1.	1.09	1.23	1.42	5.07	0.	0.
time (sec)	N/A	0.169	0.201	0.043	0.997	3.111	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	174	236	393	1766	0	0
normalized size	1	1.	1.04	1.41	2.35	10.57	0.	0.
time (sec)	N/A	0.234	0.346	0.052	1.045	9.186	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	374	374	1054	2694	1068	0	0	0
normalized size	1	1.	2.82	7.2	2.86	0.	0.	0.
time (sec)	N/A	0.637	7.236	0.069	1.972	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	221	220	295	857	540	0	0	0
normalized size	1	1.	1.33	3.88	2.44	0.	0.	0.
time (sec)	N/A	0.443	0.594	0.062	1.961	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	111	226	0	0	0	0
normalized size	1	1.	1.14	2.33	0.	0.	0.	0.
time (sec)	N/A	0.116	0.148	0.111	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	214	214	1721	1845	0	0	0	0
normalized size	1	1.	8.04	8.62	0.	0.	0.	0.
time (sec)	N/A	0.153	16.684	0.855	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	480	485	470	783	0	0	0	0
normalized size	1	1.01	0.98	1.63	0.	0.	0.	0.
time (sec)	N/A	1.743	8.636	0.158	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	546	546	2594	10477	0	0	0	0
normalized size	1	1.	4.75	19.19	0.	0.	0.	0.
time (sec)	N/A	1.043	10.423	3.685	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	326	325	600	12285	0	0	0	0
normalized size	1	1.	1.84	37.68	0.	0.	0.	0.
time (sec)	N/A	0.721	1.314	1.107	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	132	132	208	485	0	0	0	0
normalized size	1	1.	1.58	3.67	0.	0.	0.	0.
time (sec)	N/A	0.227	0.32	0.14	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	308	308	0	3796	0	0	0	0
normalized size	1	1.	0.	12.32	0.	0.	0.	0.
time (sec)	N/A	0.187	22.435	0.71	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1089	1094	1899	4619	0	0	0	0
normalized size	1	1.	1.74	4.24	0.	0.	0.	0.
time (sec)	N/A	2.782	17.417	0.844	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	162	162	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.246	2.354	1.493	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	2.517	1.368	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	0.343	1.286	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.098	1.043	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	460	460	0	1492	0	0	0	0
normalized size	1	1.	0.	3.24	0.	0.	0.	0.
time (sec)	N/A	0.598	0.283	2.439	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	302	302	0	696	0	0	0	0
normalized size	1	1.	0.	2.3	0.	0.	0.	0.
time (sec)	N/A	0.346	0.51	1.202	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	89	98	119	0	0	0	0
normalized size	1	1.	1.1	1.34	0.	0.	0.	0.
time (sec)	N/A	0.059	0.415	0.667	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.09	0.983	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.752	0.976	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	34	676	0	72	0	0
normalized size	1	1.	0.92	18.27	0.	1.95	0.	0.
time (sec)	N/A	0.027	0.059	0.164	0.	1.64	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	59	26	31	19	0
normalized size	1	1.	0.87	2.57	1.13	1.35	0.83	0.
time (sec)	N/A	0.009	0.017	0.076	1.177	1.632	0.557	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	48	26	31	19	0
normalized size	1	1.	0.87	2.09	1.13	1.35	0.83	0.
time (sec)	N/A	0.007	0.016	0.075	1.172	1.652	0.292	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	32	22	23	19	0
normalized size	1	1.	1.12	2.	1.38	1.44	1.19	0.
time (sec)	N/A	0.003	0.008	0.06	1.152	1.574	0.195	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	354	46	22	0	0
normalized size	1	1.	0.9	16.86	2.19	1.05	0.	0.
time (sec)	N/A	0.044	0.016	0.189	0.978	1.621	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	18	20	23	27	14	0
normalized size	1	1.	1.06	1.18	1.35	1.59	0.82	0.
time (sec)	N/A	0.009	0.016	0.069	1.175	1.723	0.314	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	18	20	26	30	19	0
normalized size	1	1.	0.78	0.87	1.13	1.3	0.83	0.
time (sec)	N/A	0.01	0.014	0.074	1.171	1.594	0.773	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	26	32	20	0
normalized size	1	1.	0.87	0.87	1.13	1.39	0.87	0.
time (sec)	N/A	0.009	0.015	0.075	1.179	1.668	1.277	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	62	9175	0	223	0	0
normalized size	1	1.	0.87	129.23	0.	3.14	0.	0.
time (sec)	N/A	0.035	0.118	0.773	0.	1.744	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	37	3418	49	72	78	0
normalized size	1	1.	0.88	81.38	1.17	1.71	1.86	0.
time (sec)	N/A	0.026	0.034	0.372	1.359	1.571	3.678	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	37	3418	49	72	60	0
normalized size	1	1.	0.88	81.38	1.17	1.71	1.43	0.
time (sec)	N/A	0.025	0.056	0.372	1.37	1.632	1.886	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	74	3418	49	70	37	0
normalized size	1	1.	2.18	100.53	1.44	2.06	1.09	0.
time (sec)	N/A	0.025	0.082	0.368	1.368	1.525	0.586	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	45	62	20	0
normalized size	1	1.	1.	0.94	2.81	3.88	1.25	0.
time (sec)	N/A	0.005	0.006	0.098	1.373	1.628	0.414	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	53	3774	51	69	0	0
normalized size	1	1.	1.08	77.02	1.04	1.41	0.	0.
time (sec)	N/A	0.026	0.043	0.284	2.473	1.654	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	37	1095	73	69	0	0
normalized size	1	1.	0.95	28.08	1.87	1.77	0.	0.
time (sec)	N/A	0.026	0.049	0.21	1.196	1.675	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	42	3213	46	73	32	0
normalized size	1	1.	1.17	89.25	1.28	2.03	0.89	0.
time (sec)	N/A	0.023	0.034	0.375	1.402	1.609	0.753	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	34	3217	49	68	37	0
normalized size	1	1.	1.1	103.77	1.58	2.19	1.19	0.
time (sec)	N/A	0.014	0.047	0.357	1.4	1.581	1.182	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	37	3217	49	72	39	0
normalized size	1	1.	0.58	50.27	0.77	1.12	0.61	0.
time (sec)	N/A	0.035	0.031	0.362	1.407	1.611	1.925	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	97	63382	0	468	0	0
normalized size	1	1.	0.88	576.2	0.	4.25	0.	0.
time (sec)	N/A	0.059	0.114	6.384	0.	1.717	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	54	18111	73	119	56	0
normalized size	1	1.	0.89	296.9	1.2	1.95	0.92	0.
time (sec)	N/A	0.041	0.031	1.102	1.545	1.592	6.056	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	54	18111	73	120	58	0
normalized size	1	1.	0.89	296.9	1.2	1.97	0.95	0.
time (sec)	N/A	0.041	0.027	1.096	1.561	1.568	3.559	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	54	18111	73	120	60	0
normalized size	1	1.	1.02	341.72	1.38	2.26	1.13	0.
time (sec)	N/A	0.031	0.022	1.081	1.546	1.557	3.39	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	99	18111	73	117	41	0
normalized size	1	1.	2.91	532.68	2.15	3.44	1.21	0.
time (sec)	N/A	0.015	0.074	1.083	1.559	1.502	1.732	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	69	109	20	0
normalized size	1	1.	1.	0.94	4.31	6.81	1.25	0.
time (sec)	N/A	0.005	0.006	0.106	1.556	1.596	0.804	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	104	21848	101	119	0	0
normalized size	1	1.	1.35	283.74	1.31	1.55	0.	0.
time (sec)	N/A	0.096	0.058	0.847	2.51	1.7	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	62	7683	166	115	0	0
normalized size	1	1.	0.91	112.99	2.44	1.69	0.	0.
time (sec)	N/A	0.044	0.039	0.415	2.082	1.725	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	66	7366	97	117	0	0
normalized size	1	1.	1.1	122.77	1.62	1.95	0.	0.
time (sec)	N/A	0.041	0.036	0.426	1.403	1.619	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	60	17237	70	120	51	0
normalized size	1	1.	1.09	313.4	1.27	2.18	0.93	0.
time (sec)	N/A	0.038	0.024	1.385	1.597	1.659	1.212	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	50	17235	72	112	56	0
normalized size	1	1.	1.61	555.97	2.32	3.61	1.81	0.
time (sec)	N/A	0.014	0.022	1.347	1.597	1.602	2.145	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	54	17234	73	116	60	0
normalized size	1	1.	0.84	269.28	1.14	1.81	0.94	0.
time (sec)	N/A	0.034	0.034	1.273	1.599	1.698	3.612	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	51	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.084	0.346	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	79	130774	116	293	0	0
normalized size	1	1.	0.98	1614.49	1.43	3.62	0.	0.
time (sec)	N/A	0.059	0.048	4.901	1.807	1.552	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	55	28786	69	225	0	0
normalized size	1	1.	0.98	514.04	1.23	4.02	0.	0.
time (sec)	N/A	0.035	0.038	1.134	1.824	1.66	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	4303	41	185	0	0
normalized size	1	1.	1.	138.81	1.32	5.97	0.	0.
time (sec)	N/A	0.015	0.022	0.314	1.83	1.593	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	22	63	17	0
normalized size	1	1.	1.	1.08	1.83	5.25	1.42	0.
time (sec)	N/A	0.004	0.05	0.076	1.457	1.63	8.224	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	29	972	50	205	0	0
normalized size	1	1.	0.66	22.09	1.14	4.66	0.	0.
time (sec)	N/A	0.028	0.018	11.445	1.812	1.696	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	45	0	88	315	0	0
normalized size	1	1.	0.69	0.	1.35	4.85	0.	0.
time (sec)	N/A	0.038	0.024	180.	1.793	1.727	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	66	0	146	448	0	0
normalized size	1	1.	0.72	0.	1.59	4.87	0.	0.
time (sec)	N/A	0.063	0.027	180.	1.796	1.713	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	51	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.521	1.83	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	106	131085	240	729	0	0
normalized size	1	1.	1.08	1337.6	2.45	7.44	0.	0.
time (sec)	N/A	0.081	0.097	6.12	2.487	1.666	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	83	29109	167	544	0	0
normalized size	1	1.	1.11	388.12	2.23	7.25	0.	0.
time (sec)	N/A	0.054	0.062	1.408	2.438	1.761	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	56	4626	108	416	0	0
normalized size	1	1.	1.12	92.52	2.16	8.32	0.	0.
time (sec)	N/A	0.031	0.068	0.366	2.443	1.635	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	625	63	213	36	0
normalized size	1	1.	0.96	22.32	2.25	7.61	1.29	0.
time (sec)	N/A	0.013	0.065	0.187	2.44	1.681	16.298	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	24	77	20	0
normalized size	1	1.	1.	1.07	1.71	5.5	1.43	0.
time (sec)	N/A	0.005	0.006	0.078	1.48	1.614	16.134	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	53	0	104	691	0	0
normalized size	1	1.	0.76	0.	1.49	9.87	0.	0.
time (sec)	N/A	0.048	0.071	180.	2.453	1.739	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	70	0	182	1065	0	0
normalized size	1	1.	0.69	0.	1.78	10.44	0.	0.
time (sec)	N/A	0.063	0.062	180.	2.451	1.815	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	92	0	258	1439	0	0
normalized size	1	1.	0.64	0.	1.8	10.06	0.	0.
time (sec)	N/A	0.09	0.046	180.	2.461	1.742	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	51	0	0	0	0	0
normalized size	1	1.	0.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.507	1.891	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	114	29456	266	1100	0	0
normalized size	1	1.	1.24	320.17	2.89	11.96	0.	0.
time (sec)	N/A	0.071	0.042	1.396	3.544	1.794	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	86	4977	194	936	0	0
normalized size	1	1.	1.21	70.1	2.73	13.18	0.	0.
time (sec)	N/A	0.049	0.045	0.379	3.545	1.813	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	49	952	127	548	54	0
normalized size	1	1.	1.04	20.26	2.7	11.66	1.15	0.
time (sec)	N/A	0.029	0.031	0.195	3.57	1.782	24.514	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	27	634	82	262	42	0
normalized size	1	1.	0.79	18.65	2.41	7.71	1.24	0.
time (sec)	N/A	0.014	0.049	0.171	3.531	1.648	24.167	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	41	227	24	0
normalized size	1	1.	1.	0.94	2.56	14.19	1.5	0.
time (sec)	N/A	0.005	0.006	0.074	1.471	1.61	16.46	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	74	0	234	1841	0	0
normalized size	1	1.	0.76	0.	2.41	18.98	0.	0.
time (sec)	N/A	0.066	0.106	180.	3.551	1.935	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	93	0	328	2475	0	0
normalized size	1	1.	0.71	0.	2.5	18.89	0.	0.
time (sec)	N/A	0.087	0.05	180.	3.566	2.12	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	107	0	448	3069	0	0
normalized size	1	1.	0.63	0.	2.64	18.05	0.	0.
time (sec)	N/A	0.127	0.043	180.	3.54	2.24	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	71	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.132	2.566	0.	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	146	2984280	513	1288	0	0
normalized size	1	1.	0.88	18086.6	3.11	7.81	0.	0.
time (sec)	N/A	0.133	0.103	87.445	1.819	1.721	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	106	953037	344	917	0	0
normalized size	1	1.	0.88	7876.34	2.84	7.58	0.	0.
time (sec)	N/A	0.08	0.077	36.951	1.799	1.769	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	71	252344	223	649	0	0
normalized size	1	1.	0.87	3077.37	2.72	7.91	0.	0.
time (sec)	N/A	0.046	0.065	18.674	1.791	1.728	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	41	71611	138	489	0	0
normalized size	1	1.	0.85	1491.9	2.88	10.19	0.	0.
time (sec)	N/A	0.02	0.043	15.611	1.792	1.604	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	88	389	51	0
normalized size	1	1.	1.	1.05	4.4	19.45	2.55	0.
time (sec)	N/A	0.007	0.014	0.087	1.739	1.703	1.047	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	60	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.081	0.855	0.	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	67	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	0.04	1.964	0.	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	67	0	0	0	0	0
normalized size	1	1.	0.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	0.041	2.029	0.	0.	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	34	676	0	72	0	0
normalized size	1	1.	0.92	18.27	0.	1.95	0.	0.
time (sec)	N/A	0.01	0.033	0.159	0.	1.639	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	59	18	31	39	18
normalized size	1	1.	0.87	2.57	0.78	1.35	1.7	0.78
time (sec)	N/A	0.014	0.021	0.092	0.946	1.344	91.849	1.173

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	48	18	31	60	18
normalized size	1	1.	0.87	2.09	0.78	1.35	2.61	0.78
time (sec)	N/A	0.008	0.015	0.075	0.934	1.268	37.983	1.139

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	32	14	23	36	14
normalized size	1	1.	1.12	2.	0.88	1.44	2.25	0.88
time (sec)	N/A	0.003	0.006	0.064	0.934	1.284	3.99	1.124

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	27	11	22	0	12
normalized size	1	1.	0.9	1.29	0.52	1.05	0.	0.57
time (sec)	N/A	0.031	0.017	0.073	0.945	1.49	0.	1.137

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	18	20	15	27	34	16
normalized size	1	1.	1.06	1.18	0.88	1.59	2.	0.94
time (sec)	N/A	0.009	0.016	0.074	0.942	1.618	12.187	1.162

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	18	20	15	30	39	15
normalized size	1	1.	0.78	0.87	0.65	1.3	1.7	0.65
time (sec)	N/A	0.009	0.014	0.076	0.944	1.471	77.289	1.14

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	47	21	45	213	0	0
normalized size	1	1.	1.74	0.78	1.67	7.89	0.	0.
time (sec)	N/A	0.034	0.02	0.052	1.153	1.667	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	81	449	76	325	0	0
normalized size	1	1.	1.59	8.8	1.49	6.37	0.	0.
time (sec)	N/A	0.066	0.014	0.154	1.184	1.618	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	109	471	105	435	0	0
normalized size	1	1.	1.42	6.12	1.36	5.65	0.	0.
time (sec)	N/A	0.09	0.029	0.141	1.178	1.771	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	345	5294	379	2587	0	0
normalized size	1	1.	1.12	17.24	1.23	8.43	0.	0.
time (sec)	N/A	0.465	0.452	5.085	2.289	2.064	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	259	4990	290	2114	0	0
normalized size	1	1.	1.12	21.6	1.26	9.15	0.	0.
time (sec)	N/A	0.379	0.163	4.741	2.157	1.894	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	131	306	192	1601	0	0
normalized size	1	1.	0.87	2.04	1.28	10.67	0.	0.
time (sec)	N/A	0.232	1.246	0.164	2.111	1.851	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	5.265	0.331	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	144	1726	201	1377	0	0
normalized size	1	1.	0.93	11.14	1.3	8.88	0.	0.
time (sec)	N/A	0.307	0.191	34.221	3.355	1.886	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	118	1667	169	1160	0	0
normalized size	1	1.	0.92	13.02	1.32	9.06	0.	0.
time (sec)	N/A	0.269	0.114	37.152	3.467	1.765	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	91	1584	136	953	0	0
normalized size	1	1.	0.9	15.68	1.35	9.44	0.	0.
time (sec)	N/A	0.231	0.096	10.194	3.486	1.743	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	69	69	201	247	97	707	0	0
normalized size	1	1.	2.91	3.58	1.41	10.25	0.	0.
time (sec)	N/A	0.14	0.858	0.156	3.449	2.139	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	2.972	0.364	0.	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	144	1802	197	1280	0	0
normalized size	1	1.	0.86	10.73	1.17	7.62	0.	0.
time (sec)	N/A	0.303	0.185	39.194	3.472	2.099	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	119	1745	166	1081	0	0
normalized size	1	1.	0.86	12.55	1.19	7.78	0.	0.
time (sec)	N/A	0.268	0.103	33.599	3.506	2.138	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	93	1664	135	894	0	0
normalized size	1	1.	0.85	15.13	1.23	8.13	0.	0.
time (sec)	N/A	0.238	0.091	16.123	3.501	2.064	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	76	76	200	271	99	667	0	0
normalized size	1	1.	2.63	3.57	1.3	8.78	0.	0.
time (sec)	N/A	0.15	0.89	0.158	3.901	1.96	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	3.114	0.372	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	303	303	353	5222	374	2560	0	0
normalized size	1	1.	1.17	17.23	1.23	8.45	0.	0.
time (sec)	N/A	0.465	0.397	5.121	2.281	2.327	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	229	229	267	4918	288	2093	0	0
normalized size	1	1.	1.17	21.48	1.26	9.14	0.	0.
time (sec)	N/A	0.382	0.158	4.785	2.226	2.238	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	131	306	192	1585	0	0
normalized size	1	1.	0.87	2.04	1.28	10.57	0.	0.
time (sec)	N/A	0.235	1.126	0.171	2.27	2.167	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.147	5.48	0.337	0.	0.	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	141	1698	197	1280	0	0
normalized size	1	1.	0.93	11.17	1.3	8.42	0.	0.
time (sec)	N/A	0.308	0.174	32.161	3.429	2.26	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	116	1641	166	1081	0	0
normalized size	1	1.	0.92	13.02	1.32	8.58	0.	0.
time (sec)	N/A	0.269	0.102	35.69	3.52	2.042	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	90	1560	135	894	0	0
normalized size	1	1.	0.9	15.6	1.35	8.94	0.	0.
time (sec)	N/A	0.24	0.093	15.664	3.714	1.971	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	69	69	197	247	97	667	0	0
normalized size	1	1.	2.86	3.58	1.41	9.67	0.	0.
time (sec)	N/A	0.145	0.787	0.161	3.597	2.053	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	3.149	0.372	0.	0.	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	147	1830	201	1377	0	0
normalized size	1	1.	0.89	11.09	1.22	8.35	0.	0.
time (sec)	N/A	0.31	0.188	37.705	3.718	2.15	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	121	1771	169	1160	0	0
normalized size	1	1.	0.88	12.93	1.23	8.47	0.	0.
time (sec)	N/A	0.27	0.111	37.721	3.564	2.058	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	94	1688	136	953	0	0
normalized size	1	1.	0.86	15.49	1.25	8.74	0.	0.
time (sec)	N/A	0.238	0.113	19.147	3.538	1.841	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	76	76	208	271	99	707	0	0
normalized size	1	1.	2.74	3.57	1.3	9.3	0.	0.
time (sec)	N/A	0.144	0.75	0.175	3.528	1.747	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	3.299	0.392	0.	0.	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	302	654	7429	0	4514	0	0
normalized size	1	1.	2.17	24.6	0.	14.95	0.	0.
time (sec)	N/A	0.242	0.314	8.453	0.	2.596	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	409	5543	0	3357	0	0
normalized size	1	1.	1.75	23.69	0.	14.35	0.	0.
time (sec)	N/A	0.172	0.188	13.508	0.	2.209	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	162	162	263	2543	0	2313	0	0
normalized size	1	1.	1.62	15.7	0.	14.28	0.	0.
time (sec)	N/A	0.111	0.132	10.639	0.	2.28	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	78	178	246	1493	0	0
normalized size	1	1.	0.99	2.25	3.11	18.9	0.	0.
time (sec)	N/A	0.048	0.023	0.158	1.815	1.896	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	5.064	1.157	0.	0.	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	395	346	6820	0	5871	0	0
normalized size	1	1.	0.88	17.27	0.	14.86	0.	0.
time (sec)	N/A	0.5	0.315	6.798	0.	2.656	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	257	6482	0	4545	0	0
normalized size	1	1.	0.87	21.97	0.	15.41	0.	0.
time (sec)	N/A	0.397	0.14	7.796	0.	2.491	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	4654	612	502	3210	0	0
normalized size	1	1.	23.99	3.15	2.59	16.55	0.	0.
time (sec)	N/A	0.251	13.224	0.131	2.13	2.434	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.146	0.355	0.382	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	155	2339	460	980	0	0
normalized size	1	1.	0.91	13.76	2.71	5.76	0.	0.
time (sec)	N/A	0.304	0.188	30.058	1.229	1.837	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	119	2249	333	811	0	0
normalized size	1	1.	0.89	16.91	2.5	6.1	0.	0.
time (sec)	N/A	0.253	0.106	17.08	1.144	1.731	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	766	292	358	603	0	0
normalized size	1	1.	8.24	3.14	3.85	6.48	0.	0.
time (sec)	N/A	0.155	2.858	0.175	1.554	1.71	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	0.727	0.444	0.	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	156	2449	459	992	0	0
normalized size	1	1.	0.91	14.32	2.68	5.8	0.	0.
time (sec)	N/A	0.298	0.177	29.632	1.22	1.89	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	120	2351	332	821	0	0
normalized size	1	1.	0.9	17.54	2.48	6.13	0.	0.
time (sec)	N/A	0.245	0.109	13.712	1.151	1.778	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	94	94	723	297	355	610	0	0
normalized size	1	1.	7.69	3.16	3.78	6.49	0.	0.
time (sec)	N/A	0.156	2.825	0.178	1.596	1.667	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.085	0.72	0.44	0.	0.	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	302	654	7429	0	3677	0	0
normalized size	1	1.	2.17	24.6	0.	12.18	0.	0.
time (sec)	N/A	0.235	0.287	6.052	0.	2.684	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	409	5543	0	2646	0	0
normalized size	1	1.	1.75	23.69	0.	11.31	0.	0.
time (sec)	N/A	0.171	0.175	13.674	0.	2.438	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	162	162	263	2543	0	1724	0	0
normalized size	1	1.	1.62	15.7	0.	10.64	0.	0.
time (sec)	N/A	0.11	0.121	10.737	0.	2.138	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	78	265	248	1029	0	0
normalized size	1	1.	0.99	3.35	3.14	13.03	0.	0.
time (sec)	N/A	0.048	0.032	0.161	1.69	1.832	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.107	1.173	0.	0.	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	339	6662	0	4748	0	0
normalized size	1	1.	0.87	17.04	0.	12.14	0.	0.
time (sec)	N/A	0.503	0.306	6.855	0.	2.993	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	253	6312	0	3838	0	0
normalized size	1	1.	0.86	21.54	0.	13.1	0.	0.
time (sec)	N/A	0.403	0.125	8.394	0.	2.834	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	4463	629	529	2919	0	0
normalized size	1	1.	23.01	3.24	2.73	15.05	0.	0.
time (sec)	N/A	0.249	12.993	0.139	1.891	2.785	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.093	0.348	0.391	0.	0.	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	155	2449	462	505	0	0
normalized size	1	1.	0.92	14.58	2.75	3.01	0.	0.
time (sec)	N/A	0.306	0.177	29.797	1.174	1.773	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	119	2351	335	429	0	0
normalized size	1	1.	0.9	17.81	2.54	3.25	0.	0.
time (sec)	N/A	0.257	0.109	17.385	1.139	1.691	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	709	299	386	339	0	0
normalized size	1	1.	7.62	3.22	4.15	3.65	0.	0.
time (sec)	N/A	0.159	3.364	0.181	1.545	1.774	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	0.673	0.444	0.	0.	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	155	2339	463	504	0	0
normalized size	1	1.	0.92	13.84	2.74	2.98	0.	0.
time (sec)	N/A	0.295	0.197	29.139	1.378	1.689	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	119	2249	336	427	0	0
normalized size	1	1.	0.89	16.91	2.53	3.21	0.	0.
time (sec)	N/A	0.248	0.104	20.552	1.281	1.772	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	94	94	605	304	389	335	0	0
normalized size	1	1.	6.44	3.23	4.14	3.56	0.	0.
time (sec)	N/A	0.15	2.638	0.193	1.758	1.745	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	0.75	0.457	0.	0.	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	131	920	0	979	0	0
normalized size	1	1.	0.82	5.75	0.	6.12	0.	0.
time (sec)	N/A	0.574	0.315	0.47	0.	1.928	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	236	4034	447	579	362	0
normalized size	1	1.	0.79	13.58	1.51	1.95	1.22	0.
time (sec)	N/A	0.386	0.171	10.293	1.113	1.696	113.193	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	192	3320	366	440	286	0
normalized size	1	1.	0.85	14.76	1.63	1.96	1.27	0.
time (sec)	N/A	0.256	0.145	6.22	1.076	1.681	33.845	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	129	2616	231	297	209	0
normalized size	1	1.	0.92	18.69	1.65	2.12	1.49	0.
time (sec)	N/A	0.119	0.106	0.602	1.028	1.625	12.663	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	C	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	381	381	0	864	225	0	0	0
normalized size	1	1.	0.	2.27	0.59	0.	0.	0.
time (sec)	N/A	0.439	0.213	1.104	1.779	0.	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	247	247	161	0	0	0	0	0
normalized size	1	1.	0.65	0.	0.	0.	0.	0.
time (sec)	N/A	0.488	0.15	2.582	0.	0.	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	339	339	307	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.717	0.145	6.231	0.	0.	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	236	4194	428	587	345	0
normalized size	1	1.	0.75	13.31	1.36	1.86	1.1	0.
time (sec)	N/A	0.749	0.149	1.72	1.11	1.676	43.404	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	183	3515	340	452	265	0
normalized size	1	1.	0.74	14.23	1.38	1.83	1.07	0.
time (sec)	N/A	0.621	0.123	1.083	1.193	1.728	15.704	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	144	2210	240	304	155	0
normalized size	1	1.	1.38	21.25	2.31	2.92	1.49	0.
time (sec)	N/A	0.198	0.017	0.566	1.141	1.699	9.844	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	94	332	0	0	0	0	0
normalized size	1	0.9	3.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.269	0.204	1.642	0.	0.	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	191	457	0	0	0	0	0
normalized size	1	0.97	2.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.463	0.383	4.322	0.	0.	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	256	250	0	0	0	0	0	0
normalized size	1	0.98	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.67	0.28	4.318	0.	0.	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	512	512	677	8491	0	0	0	0
normalized size	1	1.	1.32	16.58	0.	0.	0.	0.
time (sec)	N/A	0.767	4.252	1.996	0.	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	546	546	1287	0	0	0	0	0
normalized size	1	1.	2.36	0.	0.	0.	0.	0.
time (sec)	N/A	1.387	3.212	1.994	0.	0.	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	100	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.252	0.273	0.74	0.	0.	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	560	560	1236	0	0	0	0	0
normalized size	1	1.	2.21	0.	0.	0.	0.	0.
time (sec)	N/A	1.261	3.562	0.832	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	712	712	1318	937	0	0	0	0
normalized size	1	1.	1.85	1.32	0.	0.	0.	0.
time (sec)	N/A	1.113	5.571	3.198	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	51	31	78	262	0	0
normalized size	1	1.	2.04	1.24	3.12	10.48	0.	0.
time (sec)	N/A	0.012	0.034	0.052	1.119	1.776	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	71	62	80	374	0	0
normalized size	1	1.	1.39	1.22	1.57	7.33	0.	0.
time (sec)	N/A	0.047	0.022	0.049	1.123	1.661	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	93	79	103	462	0	0
normalized size	1	1.	1.33	1.13	1.47	6.6	0.	0.
time (sec)	N/A	0.073	0.019	0.046	1.109	1.778	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	68	67	144	417	0	0
normalized size	1	1.	1.66	1.63	3.51	10.17	0.	0.
time (sec)	N/A	0.016	0.073	0.072	1.072	1.679	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	113	153	146	583	0	0
normalized size	1	1.	1.36	1.84	1.76	7.02	0.	0.
time (sec)	N/A	0.064	0.031	0.07	1.135	1.612	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	149	185	192	730	0	0
normalized size	1	1.	1.25	1.55	1.61	6.13	0.	0.
time (sec)	N/A	0.099	0.03	0.076	1.125	1.755	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	108	164	284	899	0	0
normalized size	1	1.	0.64	0.98	1.69	5.35	0.	0.
time (sec)	N/A	0.132	0.075	0.199	1.089	1.601	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	177	590	262	1193	0	0
normalized size	1	1.	0.82	2.73	1.21	5.52	0.	0.
time (sec)	N/A	2.714	0.104	0.174	1.284	1.62	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	235	666	338	1453	0	0
normalized size	1	1.	0.87	2.48	1.26	5.4	0.	0.
time (sec)	N/A	2.586	0.07	0.175	1.273	1.914	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	19	28	57	14	0
normalized size	1	1.	1.	1.12	1.65	3.35	0.82	0.
time (sec)	N/A	0.044	0.052	0.072	1.127	1.648	1.315	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	39	46	50	149	60	0
normalized size	1	1.	0.89	1.05	1.14	3.39	1.36	0.
time (sec)	N/A	0.052	0.016	0.078	1.047	1.822	9.037	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	42	118	54	359	0	0
normalized size	1	1.	0.89	2.51	1.15	7.64	0.	0.
time (sec)	N/A	0.055	0.036	0.119	1.065	1.859	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	153	794	248	624	0	0
normalized size	1	1.	1.43	7.42	2.32	5.83	0.	0.
time (sec)	N/A	0.155	0.164	0.329	1.614	1.744	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	49	49	60	824	86	236	0	0
normalized size	1	1.	1.22	16.82	1.76	4.82	0.	0.
time (sec)	N/A	0.079	0.086	0.281	1.118	1.654	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	45	45	46	351	58	55	63	0
normalized size	1	1.	1.02	7.8	1.29	1.22	1.4	0.
time (sec)	N/A	0.06	0.075	0.25	1.058	1.707	25.793	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	45	45	46	68	57	55	0	47
normalized size	1	1.	1.02	1.51	1.27	1.22	0.	1.04
time (sec)	N/A	0.059	0.08	0.069	1.181	1.512	0.	1.183

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	49	49	59	939	86	238	0	0
normalized size	1	1.	1.2	19.16	1.76	4.86	0.	0.
time (sec)	N/A	0.071	0.083	0.292	1.039	1.756	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	150	920	248	625	0	0
normalized size	1	1.	1.4	8.6	2.32	5.84	0.	0.
time (sec)	N/A	0.148	0.157	0.333	1.748	1.79	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [75] had the largest ratio of [1.333]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.	8	0.375
2	A	4	3	1.	8	0.375
3	A	4	3	1.	8	0.375
4	A	4	3	1.	8	0.375
5	A	3	3	1.	6	0.5
6	A	2	2	1.	4	0.5
7	A	1	1	1.	8	0.125
8	A	5	5	1.	8	0.625
9	A	3	3	1.	8	0.375
10	A	4	3	1.	8	0.375
11	A	4	3	1.	8	0.375
12	A	15	7	1.	10	0.7
13	A	14	9	1.	10	0.9
14	A	10	7	1.	10	0.7
15	A	9	8	1.	10	0.8
16	A	5	5	1.	8	0.625
17	A	5	5	1.	6	0.833
18	A	6	5	1.	10	0.5
19	A	4	4	1.	10	0.4
20	A	8	7	1.	10	0.7
21	A	8	7	1.	10	0.7
22	A	13	8	1.	10	0.8
23	A	33	11	1.	10	1.1
24	A	22	11	1.	10	1.1
25	A	18	10	1.	10	1.
26	A	11	9	1.	10	0.9
27	A	8	8	1.	8	1.
28	A	5	6	1.	6	1.
29	A	8	6	1.	10	0.6
30	A	5	6	1.	10	0.6
31	A	7	6	1.	10	0.6
32	A	14	11	1.	10	1.1
33	A	16	8	1.	10	0.8
34	A	1	1	1.	14	0.071
35	A	4	4	1.	14	0.286
36	A	4	4	1.	14	0.286
37	A	5	5	1.	14	0.357
38	A	5	4	1.	12	0.333
39	A	27	13	1.	14	0.929
40	A	25	13	1.	14	0.929
41	A	23	11	1.	14	0.786

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
42	A	0	0	0.	0	0.
43	A	0	0	0.	0	0.
44	A	5	6	1.	16	0.375
45	A	7	9	1.	16	0.562
46	A	8	9	1.	16	0.562
47	A	8	9	1.	16	0.562
48	A	3	3	1.	15	0.2
49	A	2	2	1.	15	0.133
50	A	1	1	1.	15	0.067
51	A	2	2	1.	15	0.133
52	A	3	2	1.	15	0.133
53	A	1	1	1.	14	0.071
54	A	1	1	1.	14	0.071
55	A	4	4	1.	14	0.286
56	A	4	4	1.	13	0.308
57	A	1	1	1.	12	0.083
58	A	3	3	1.	13	0.231
59	A	2	2	1.	12	0.167
60	A	4	3	1.	13	0.231
61	A	3	3	1.	12	0.25
62	A	7	5	1.	10	0.5
63	A	7	5	1.	10	0.5
64	A	7	5	1.	8	0.625
65	A	3	3	1.	6	0.5
66	A	5	5	1.	10	0.5
67	A	7	5	1.	10	0.5
68	A	5	4	1.	10	0.4
69	A	19	15	1.	12	1.25
70	A	15	13	1.	12	1.083
71	A	12	10	1.	10	1.
72	A	6	6	1.	8	0.75
73	A	2	2	1.	12	0.167
74	A	17	15	1.	12	1.25
75	A	21	16	1.	12	1.333
76	A	37	7	0.89	16	0.438
77	A	5	5	1.	14	0.357
78	A	37	10	1.23	16	0.625
79	A	57	11	1.	16	0.688
80	A	55	16	1.	18	0.889
81	A	65	19	1.	18	1.056
82	A	12	8	1.	19	0.421
83	A	6	4	1.	10	0.4
84	A	5	4	1.	8	0.5
85	A	4	4	1.	6	0.667

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
86	A	2	2	1.	10	0.2
87	A	4	4	1.	10	0.4
88	A	5	4	1.	10	0.4
89	A	3	2	1.	12	0.167
90	A	3	2	1.	12	0.167
91	A	2	2	1.	12	0.167
92	A	4	4	1.	12	0.333
93	A	2	2	1.	10	0.2
94	A	3	3	1.	4	0.75
95	A	2	2	1.	10	0.2
96	A	4	4	1.	12	0.333
97	A	5	4	1.	14	0.286
98	A	2	2	1.	14	0.143
99	A	6	6	1.	14	0.429
100	A	3	3	1.	12	0.25
101	A	3	3	1.	19	0.158
102	A	7	5	1.	18	0.278
103	A	7	5	1.	18	0.278
104	A	7	5	1.	16	0.312
105	A	4	3	1.	10	0.3
106	A	5	5	1.	18	0.278
107	A	7	5	1.	18	0.278
108	A	5	4	1.	18	0.222
109	A	16	13	1.	20	0.65
110	A	13	10	1.	18	0.556
111	A	6	6	1.	12	0.5
112	A	2	2	1.	20	0.1
113	A	21	19	1.01	20	0.95
114	A	21	14	1.	20	0.7
115	A	15	11	1.	18	0.611
116	A	6	7	1.	12	0.583
117	A	2	2	1.	20	0.1
118	A	30	18	1.	20	0.9
119	A	6	4	1.	18	0.222
120	A	0	0	0.	0	0.
121	A	0	0	0.	0	0.
122	A	0	0	0.	0	0.
123	A	9	7	1.	40	0.175
124	A	7	6	1.	40	0.15
125	A	2	3	1.	38	0.079
126	A	0	0	0.	0	0.
127	A	0	0	0.	0	0.
128	A	2	2	1.	11	0.182
129	A	2	2	1.	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	2	2	1.	9	0.222
131	A	2	2	1.	7	0.286
132	A	2	2	1.	11	0.182
133	A	2	2	1.	11	0.182
134	A	2	2	1.	11	0.182
135	A	2	2	1.	11	0.182
136	A	3	2	1.	13	0.154
137	A	3	2	1.	13	0.154
138	A	3	2	1.	13	0.154
139	A	3	3	1.	11	0.273
140	A	2	2	1.	9	0.222
141	A	3	3	1.	13	0.231
142	A	3	3	1.	13	0.231
143	A	3	2	1.	13	0.154
144	A	1	1	1.	13	0.077
145	A	2	2	1.	13	0.154
146	A	4	2	1.	13	0.154
147	A	4	2	1.	13	0.154
148	A	4	2	1.	13	0.154
149	A	4	3	1.	13	0.231
150	A	3	3	1.	11	0.273
151	A	2	2	1.	9	0.222
152	A	4	3	1.	13	0.231
153	A	4	4	1.	13	0.308
154	A	4	3	1.	13	0.231
155	A	4	2	1.	13	0.154
156	A	1	1	1.	13	0.077
157	A	2	2	1.	13	0.154
158	A	1	1	1.	13	0.077
159	A	5	4	1.	13	0.308
160	A	4	4	1.	13	0.308
161	A	3	3	1.	11	0.273
162	A	2	2	1.	9	0.222
163	A	4	3	1.	13	0.231
164	A	5	4	1.	13	0.308
165	A	6	4	1.	13	0.308
166	A	2	2	1.	13	0.154
167	A	6	5	1.	13	0.385
168	A	5	5	1.	13	0.385
169	A	4	4	1.	13	0.308
170	A	3	3	1.	11	0.273
171	A	2	2	1.	9	0.222
172	A	5	4	1.	13	0.308
173	A	6	5	1.	13	0.385

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
174	A	7	5	1.	13	0.385
175	A	3	2	1.	13	0.154
176	A	6	5	1.	13	0.385
177	A	5	4	1.	13	0.308
178	A	4	3	1.	13	0.231
179	A	3	3	1.	11	0.273
180	A	2	2	1.	9	0.222
181	A	6	4	1.	13	0.308
182	A	7	5	1.	13	0.385
183	A	8	5	1.	13	0.385
184	A	1	1	1.	13	0.077
185	A	6	3	1.	13	0.231
186	A	5	3	1.	13	0.231
187	A	4	3	1.	13	0.231
188	A	3	3	1.	11	0.273
189	A	2	2	1.	9	0.222
190	A	1	1	1.	13	0.077
191	A	2	2	1.	13	0.154
192	A	3	2	1.	13	0.154
193	A	2	2	1.	11	0.182
194	A	2	2	1.	11	0.182
195	A	2	2	1.	9	0.222
196	A	2	2	1.	7	0.286
197	A	2	2	1.	11	0.182
198	A	2	2	1.	11	0.182
199	A	2	2	1.	11	0.182
200	A	6	4	1.	3	1.333
201	A	8	5	1.	5	1.
202	A	10	6	1.	7	0.857
203	A	11	6	1.	15	0.4
204	A	9	5	1.	13	0.385
205	A	7	4	1.	11	0.364
206	A	0	0	0.	0	0.
207	A	8	7	1.	16	0.438
208	A	7	7	1.	16	0.438
209	A	6	6	1.	14	0.429
210	A	5	5	1.	12	0.417
211	A	0	0	0.	0	0.
212	A	8	7	1.	19	0.368
213	A	7	7	1.	19	0.368
214	A	6	6	1.	17	0.353
215	A	5	5	1.	15	0.333
216	A	0	0	0.	0	0.
217	A	11	6	1.	15	0.4

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
218	A	9	5	1.	13	0.385
219	A	7	4	1.	11	0.364
220	A	0	0	0.	0	0.
221	A	8	7	1.	16	0.438
222	A	7	7	1.	16	0.438
223	A	6	6	1.	14	0.429
224	A	5	5	1.	12	0.417
225	A	0	0	0.	0	0.
226	A	8	7	1.	19	0.368
227	A	7	7	1.	19	0.368
228	A	6	6	1.	17	0.353
229	A	5	5	1.	15	0.333
230	A	0	0	0.	0	0.
231	A	12	6	1.	15	0.4
232	A	10	6	1.	15	0.4
233	A	8	5	1.	13	0.385
234	A	6	4	1.	7	0.571
235	A	0	0	0.	0	0.
236	A	11	6	1.	15	0.4
237	A	9	5	1.	13	0.385
238	A	7	4	1.	11	0.364
239	A	0	0	0.	0	0.
240	A	7	7	1.	20	0.35
241	A	6	6	1.	18	0.333
242	A	5	5	1.	16	0.312
243	A	0	0	0.	0	0.
244	A	7	7	1.	21	0.333
245	A	6	6	1.	19	0.316
246	A	5	5	1.	17	0.294
247	A	0	0	0.	0	0.
248	A	12	6	1.	15	0.4
249	A	10	6	1.	15	0.4
250	A	8	5	1.	13	0.385
251	A	6	4	1.	7	0.571
252	A	0	0	0.	0	0.
253	A	11	6	1.	15	0.4
254	A	9	5	1.	13	0.385
255	A	7	4	1.	11	0.364
256	A	0	0	0.	0	0.
257	A	7	7	1.	20	0.35
258	A	6	6	1.	18	0.333
259	A	5	5	1.	16	0.312
260	A	0	0	0.	0	0.
261	A	7	7	1.	21	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
262	A	6	6	1.	19	0.316
263	A	5	5	1.	17	0.294
264	A	0	0	0.	0	0.
265	A	11	8	1.	24	0.333
266	A	23	11	1.	27	0.407
267	A	14	11	1.	27	0.407
268	A	7	8	1.	25	0.32
269	A	21	11	1.	27	0.407
270	A	13	13	1.	27	0.482
271	A	17	14	1.	27	0.518
272	A	26	15	1.	27	0.556
273	A	21	15	1.	27	0.556
274	A	9	8	1.	24	0.333
275	A	8	8	0.9	27	0.296
276	A	17	16	0.97	27	0.593
277	A	26	18	0.98	27	0.667
278	A	22	17	1.	22	0.773
279	A	38	20	1.	21	0.952
280	A	0	0	0.	0	0.
281	A	38	22	1.	24	0.917
282	A	32	17	1.	24	0.708
283	A	2	2	1.	4	0.5
284	A	7	4	1.	6	0.667
285	A	9	5	1.	8	0.625
286	A	2	2	1.	8	0.25
287	A	7	4	1.	10	0.4
288	A	9	5	1.	12	0.417
289	A	6	6	1.	12	0.5
290	A	25	8	1.	14	0.571
291	A	29	9	1.	16	0.562
292	A	1	1	1.	20	0.05
293	A	4	4	1.	12	0.333
294	A	4	4	1.	14	0.286
295	A	8	7	1.	20	0.35
296	A	5	5	1.	20	0.25
297	A	3	2	1.	20	0.1
298	A	3	2	1.	20	0.1
299	A	5	5	1.	20	0.25
300	A	8	7	1.	20	0.35

Chapter 3

Listing of integrals

3.1 $\int x^5 \coth^{-1}(ax)\, dx$

Optimal. Leaf size=51

$$\frac{x^3}{18a^3} + \frac{x}{6a^5} - \frac{\tanh^{-1}(ax)}{6a^6} + \frac{x^5}{30a} + \frac{1}{6}x^6 \coth^{-1}(ax)$$

```
[Out] x/(6*a^5) + x^3/(18*a^3) + x^5/(30*a) + (x^6*ArcCoth[a*x])/6 - ArcTanh[a*x]
/(6*a^6)
```

Rubi [A] time = 0.0279481, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5917, 302, 206}

$$\frac{x^3}{18a^3} + \frac{x}{6a^5} - \frac{\tanh^{-1}(ax)}{6a^6} + \frac{x^5}{30a} + \frac{1}{6}x^6 \coth^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*ArcCoth[a*x],x]
```

```
[Out] x/(6*a^5) + x^3/(18*a^3) + x^5/(30*a) + (x^6*ArcCoth[a*x])/6 - ArcTanh[a*x]
/(6*a^6)
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^5 \coth^{-1}(ax)\,dx &= \frac{1}{6}x^6\coth^{-1}(ax)-\frac{1}{6}a\int \frac{x^6}{1-a^2x^2}\,dx\\
&= \frac{1}{6}x^6\coth^{-1}(ax)-\frac{1}{6}a\int \left(-\frac{1}{a^6}-\frac{x^2}{a^4}-\frac{x^4}{a^2}+\frac{1}{a^6\left(1-a^2x^2\right)}\right)dx\\
&= \frac{x}{6a^5}+\frac{x^3}{18a^3}+\frac{x^5}{30a}+\frac{1}{6}x^6\coth^{-1}(ax)-\frac{\int \frac{1}{1-a^2x^2}\,dx}{6a^5}\\
&= \frac{x}{6a^5}+\frac{x^3}{18a^3}+\frac{x^5}{30a}+\frac{1}{6}x^6\coth^{-1}(ax)-\frac{\tanh^{-1}(ax)}{6a^6}
\end{aligned}
$$

Mathematica [A] time = 0.0089957, size = 67, normalized size = 1.31

$$\frac{x^3}{18a^3}+\frac{x}{6a^5}+\frac{\log(1-ax)}{12a^6}-\frac{\log(ax+1)}{12a^6}+\frac{x^5}{30a}+\frac{1}{6}x^6\coth^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*ArcCoth[a*x],x]

[Out] x/(6*a^5) + x^3/(18*a^3) + x^5/(30*a) + (x^6*ArcCoth[a*x])/6 + Log[1 - a*x]
/(12*a^6) - Log[1 + a*x]/(12*a^6)
```

Maple [A] time = 0.035, size = 55, normalized size = 1.1

$$\frac{x^6\operatorname{arccoth}(ax)}{6}+\frac{x^5}{30\,a}+\frac{x^3}{18\,a^3}+\frac{x}{6\,a^5}+\frac{\ln(ax-1)}{12\,a^6}-\frac{\ln(ax+1)}{12\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arccoth(a*x),x)

[Out] 1/6*x^6*arccoth(a*x)+1/30*x^5/a+1/18*x^3/a^3+1/6*x/a^5+1/12/a^6*ln(a*x-1)-1
/12/a^6*ln(a*x+1)
```

Maxima [A] time = 0.964772, size = 82, normalized size = 1.61

$$\frac{1}{6}\,x^6\operatorname{arcoth}(ax)+\frac{1}{180}\,a\left(\frac{2\left(3\,a^4x^5+5\,a^2x^3+15\,x\right)}{a^6}-\frac{15\,\log(ax+1)}{a^7}+\frac{15\,\log(ax-1)}{a^7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccoth(a*x),x, algorithm="maxima")

[Out] 1/6*x^6*arccoth(a*x) + 1/180*a*(2*(3*a^4*x^5 + 5*a^2*x^3 + 15*x)/a^6 - 15*l
og(a*x + 1)/a^7 + 15*log(a*x - 1)/a^7)
```

Fricas [A] time = 1.54015, size = 120, normalized size = 2.35

$$\frac{6\,a^5x^5+10\,a^3x^3+30\,ax+15\left(a^6x^6-1\right)\log\left(\frac{ax+1}{ax-1}\right)}{180\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccoth(a*x),x, algorithm="fricas")
```

```
[Out] 1/180*(6*a^5*x^5 + 10*a^3*x^3 + 30*a*x + 15*(a^6*x^6 - 1)*log((a*x + 1)/(a*
x - 1)))/a^6
```

Sympy [A] time = 2.07, size = 49, normalized size = 0.96

$$\begin{cases}\frac{x^6\operatorname{acoth}(ax)}{6}+\frac{x^5}{30a}+\frac{x^3}{18a^3}+\frac{x}{6a^5}-\frac{\operatorname{acoth}(ax)}{6a^6} & \text{for } a\neq 0\\ \frac{i\pi x^6}{12} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*acoth(a*x),x)
```

```
[Out] Piecewise((x**6*acoth(a*x)/6 + x**5/(30*a) + x**3/(18*a**3) + x/(6*a**5) -
acoth(a*x)/(6*a**6), Ne(a, 0)), (I*pi*x**6/12, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\operatorname{arcoth}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccoth(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^5*arccoth(a*x), x)
```

3.2 $\int x^4 \coth^{-1}(ax)\,dx$

Optimal. Leaf size=50

$$\frac{x^2}{10a^3}+\frac{\log\left(1-a^2x^2\right)}{10a^5}+\frac{x^4}{20a}+\frac{1}{5}x^5\coth^{-1}(ax)$$

```
[Out] x^2/(10*a^3) + x^4/(20*a) + (x^5*ArcCoth[a*x])/5 + Log[1 - a^2*x^2]/(10*a^5
)
```

Rubi [A] time = 0.0371068, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5917, 266, 43}

$$\frac{x^2}{10a^3}+\frac{\log\left(1-a^2x^2\right)}{10a^5}+\frac{x^4}{20a}+\frac{1}{5}x^5\coth^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcCoth[a*x],x]
```

```
[Out] x^2/(10*a^3) + x^4/(20*a) + (x^5*ArcCoth[a*x])/5 + Log[1 - a^2*x^2]/(10*a^5
)
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4\coth^{-1}(ax)\,dx &= \frac{1}{5}x^5\coth^{-1}(ax)-\frac{1}{5}a\int\frac{x^5}{1-a^2x^2}\,dx\\ &= \frac{1}{5}x^5\coth^{-1}(ax)-\frac{1}{10}a\,\text{Subst}\left(\int\frac{x^2}{1-a^2x}\,dx,x,x^2\right)\\ &= \frac{1}{5}x^5\coth^{-1}(ax)-\frac{1}{10}a\,\text{Subst}\left(\int\left(-\frac{1}{a^4}-\frac{x}{a^2}-\frac{1}{a^4\left(-1+a^2x\right)}\right)dx,x,x^2\right)\\ &= \frac{x^2}{10a^3}+\frac{x^4}{20a}+\frac{1}{5}x^5\coth^{-1}(ax)+\frac{\log\left(1-a^2x^2\right)}{10a^5}\end{aligned}$$

Mathematica [A] time = 0.0081977, size = 50, normalized size = 1.

$$\frac{x^2}{10a^3}+\frac{\log\left(1-a^2x^2\right)}{10a^5}+\frac{x^4}{20a}+\frac{1}{5}x^5\coth^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcCoth[a*x],x]
```

```
[Out] x^2/(10*a^3) + x^4/(20*a) + (x^5*ArcCoth[a*x])/5 + Log[1 - a^2*x^2]/(10*a^5
)
```

Maple [A] time = 0.035, size = 49, normalized size = 1.

$$\frac{x^5\text{arccoth}\,(ax)}{5}+\frac{x^4}{20\,a}+\frac{x^2}{10\,a^3}+\frac{\ln\,(ax-1)}{10\,a^5}+\frac{\ln\,(ax+1)}{10\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccoth(a*x),x)
```

```
[Out] 1/5*x^5*arccoth(a*x)+1/20*x^4/a+1/10*x^2/a^3+1/10/a^5*ln(a*x-1)+1/10/a^5*ln
(a*x+1)
```

Maxima [A] time = 0.958241, size = 62, normalized size = 1.24

$$\frac{1}{5}\,x^5\,\text{arcoth}\,(ax)+\frac{1}{20}\,a\left(\frac{a^2x^4+2\,x^2}{a^4}+\frac{2\,\log\left(a^2x^2-1\right)}{a^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(a*x),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arccoth(a*x) + 1/20*a*((a^2*x^4 + 2*x^2)/a^4 + 2*log(a^2*x^2 - 1)/a
^6)
```

Fricas [A] time = 1.57557, size = 122, normalized size = 2.44

$$\frac{2\,a^5x^5\log\left(\frac{ax+1}{ax-1}\right)+a^4x^4+2\,a^2x^2+2\,\log\left(a^2x^2-1\right)}{20\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(a*x),x, algorithm="fricas")
```

```
[Out] 1/20*(2*a^5*x^5*log((a*x + 1)/(a*x - 1)) + a^4*x^4 + 2*a^2*x^2 + 2*log(a^2*
x^2 - 1))/a^5
```

Sympy [A] time = 3.79828, size = 54, normalized size = 1.08

$$\begin{cases} \frac{x^5 \operatorname{acoth}(ax)}{5} + \frac{x^4}{20a} + \frac{x^2}{10a^3} + \frac{\log(ax+1)}{5a^5} - \frac{\operatorname{acoth}(ax)}{5a^5} & \text{for } a \neq 0 \\ \frac{i\pi x^5}{10} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acoth(a*x),x)
```

```
[Out] Piecewise((x**5*acoth(a*x)/5 + x**4/(20*a) + x**2/(10*a**3) + log(a*x + 1)/
(5*a**5) - acoth(a*x)/(5*a**5), Ne(a, 0)), (I*pi*x**5/10, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{arcoth}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^4*arccoth(a*x), x)
```

3.3 $\int x^3 \coth^{-1}(ax)\,dx$

Optimal. Leaf size=41

$$\frac{x}{4a^3}-\frac{\tanh^{-1}(ax)}{4a^4}+\frac{x^3}{12a}+\frac{1}{4}x^4\coth^{-1}(ax)$$

```
[Out] x/(4*a^3) + x^3/(12*a) + (x^4*ArcCoth[a*x])/4 - ArcTanh[a*x]/(4*a^4)
```

Rubi [A] time = 0.0237262, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5917, 302, 206}

$$\frac{x}{4a^3}-\frac{\tanh^{-1}(ax)}{4a^4}+\frac{x^3}{12a}+\frac{1}{4}x^4\coth^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCoth[a*x],x]
```

```
[Out] x/(4*a^3) + x^3/(12*a) + (x^4*ArcCoth[a*x])/4 - ArcTanh[a*x]/(4*a^4)
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3\coth^{-1}(ax)\,dx &= \frac{1}{4}x^4\coth^{-1}(ax)-\frac{1}{4}a\int\frac{x^4}{1-a^2x^2}\,dx\\
&= \frac{1}{4}x^4\coth^{-1}(ax)-\frac{1}{4}a\int\left(-\frac{1}{a^4}-\frac{x^2}{a^2}+\frac{1}{a^4\left(1-a^2x^2\right)}\right)dx\\
&= \frac{x}{4a^3}+\frac{x^3}{12a}+\frac{1}{4}x^4\coth^{-1}(ax)-\frac{\int\frac{1}{1-a^2x^2}\,dx}{4a^3}\\
&= \frac{x}{4a^3}+\frac{x^3}{12a}+\frac{1}{4}x^4\coth^{-1}(ax)-\frac{\tanh^{-1}(ax)}{4a^4}
\end{aligned}$$

Mathematica [A] time = 0.0082089, size = 57, normalized size = 1.39

$$\frac{x}{4a^3}+\frac{\log(1-ax)}{8a^4}-\frac{\log(ax+1)}{8a^4}+\frac{x^3}{12a}+\frac{1}{4}x^4\coth^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCoth[a*x],x]
```

```
[Out] x/(4*a^3) + x^3/(12*a) + (x^4*ArcCoth[a*x])/4 + Log[1 - a*x]/(8*a^4) - Log[
1 + a*x]/(8*a^4)
```

Maple [A] time = 0.031, size = 47, normalized size = 1.2

$$\frac{x^4\mathrm{arccoth}\,(ax)}{4}+\frac{x^3}{12\,a}+\frac{x}{4\,a^3}+\frac{\ln(ax-1)}{8\,a^4}-\frac{\ln(ax+1)}{8\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccoth(a*x),x)
```

```
[Out] 1/4*x^4*arccoth(a*x)+1/12*x^3/a+1/4*x/a^3+1/8/a^4*ln(a*x-1)-1/8/a^4*ln(a*x+
1)
```

Maxima [A] time = 0.966109, size = 70, normalized size = 1.71

$$\frac{1}{4}\,x^4\,\mathrm{arcoth}\,(ax)+\frac{1}{24}\,a\left(\frac{2\left(a^2x^3+3\,x\right)}{a^4}-\frac{3\,\log(ax+1)}{a^5}+\frac{3\,\log(ax-1)}{a^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(a*x),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccoth(a*x) + 1/24*a*(2*(a^2*x^3 + 3*x)/a^4 - 3*log(a*x + 1)/a^5 +
 3*log(a*x - 1)/a^5)
```

Fricas [A] time = 1.59866, size = 99, normalized size = 2.41

$$\frac{2\,a^3x^3+6\,ax+3\left(a^4x^4-1\right)\log\left(\frac{ax+1}{ax-1}\right)}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(a*x),x, algorithm="fricas")
```

```
[Out] 1/24*(2*a^3*x^3 + 6*a*x + 3*(a^4*x^4 - 1)*log((a*x + 1)/(a*x - 1)))/a^4
```

Sympy [A] time = 1.28445, size = 41, normalized size = 1.

$$\begin{cases} \frac{x^4 \operatorname{acoth}(ax)}{4} + \frac{x^3}{12a} + \frac{x}{4a^3} - \frac{\operatorname{acoth}(ax)}{4a^4} & \text{for } a \neq 0 \\ \frac{i\pi x^4}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acoth(a*x),x)
```

```
[Out] Piecewise((x**4*acoth(a*x)/4 + x**3/(12*a) + x/(4*a**3) - acoth(a*x)/(4*a**
4), Ne(a, 0)), (I*pi*x**4/8, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arcoth}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*arccoth(a*x), x)
```

3.4 $\int x^2 \coth^{-1}(ax)\,dx$

Optimal. Leaf size=40

$$\frac{\log\left(1-a^2x^2\right)}{6a^3}+\frac{x^2}{6a}+\frac{1}{3}x^3\coth^{-1}(ax)$$

```
[Out] x^2/(6*a) + (x^3*ArcCoth[a*x])/3 + Log[1 - a^2*x^2]/(6*a^3)
```

Rubi [A] time = 0.0298628, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5917, 266, 43}

$$\frac{\log\left(1-a^2x^2\right)}{6a^3}+\frac{x^2}{6a}+\frac{1}{3}x^3\coth^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[a*x],x]
```

```
[Out] x^2/(6*a) + (x^3*ArcCoth[a*x])/3 + Log[1 - a^2*x^2]/(6*a^3)
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2\coth^{-1}(ax)\,dx &= \frac{1}{3}x^3\coth^{-1}(ax)-\frac{1}{3}a\int\frac{x^3}{1-a^2x^2}\,dx\\
&= \frac{1}{3}x^3\coth^{-1}(ax)-\frac{1}{6}a\,\text{Subst}\left(\int\frac{x}{1-a^2x}\,dx,x,x^2\right)\\
&= \frac{1}{3}x^3\coth^{-1}(ax)-\frac{1}{6}a\,\text{Subst}\left(\int\left(-\frac{1}{a^2}-\frac{1}{a^2\left(-1+a^2x\right)}\right)dx,x,x^2\right)\\
&= \frac{x^2}{6a}+\frac{1}{3}x^3\coth^{-1}(ax)+\frac{\log\left(1-a^2x^2\right)}{6a^3}
\end{aligned}$$

Mathematica [A] time = 0.0078073, size = 40, normalized size = 1.

$$\frac{\log\left(1-a^2x^2\right)}{6a^3}+\frac{x^2}{6a}+\frac{1}{3}x^3\coth^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[a*x],x]
```

```
[Out] x^2/(6*a) + (x^3*ArcCoth[a*x])/3 + Log[1 - a^2*x^2]/(6*a^3)
```

Maple [A] time = 0.032, size = 41, normalized size = 1.

$$\frac{x^3\mathrm{arccoth}\,(ax)}{3}+\frac{x^2}{6\,a}+\frac{\ln\,(ax-1)}{6\,a^3}+\frac{\ln\,(ax+1)}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(a*x),x)
```

```
[Out] 1/3*x^3*arccoth(a*x)+1/6*x^2/a+1/6/a^3*ln(a*x-1)+1/6/a^3*ln(a*x+1)
```

Maxima [A] time = 0.968443, size = 47, normalized size = 1.18

$$\frac{1}{3}\,x^3\,\mathrm{arcoth}\,(ax)+\frac{1}{6}\,a\left(\frac{x^2}{a^2}+\frac{\log\left(a^2x^2-1\right)}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(a*x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(a*x) + 1/6*a*(x^2/a^2 + log(a^2*x^2 - 1)/a^4)
```

Fricas [A] time = 1.57336, size = 99, normalized size = 2.48

$$\frac{a^3x^3\log\left(\frac{ax+1}{ax-1}\right)+a^2x^2+\log\left(a^2x^2-1\right)}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(a*x),x, algorithm="fricas")
```

```
[Out] 1/6*(a^3*x^3*log((a*x + 1)/(a*x - 1)) + a^2*x^2 + log(a^2*x^2 - 1))/a^3
```

Sympy [A] time = 2.3732, size = 46, normalized size = 1.15

$$\begin{cases} \frac{x^3\,\mathrm{acoth}\,(ax)}{3}+\frac{x^2}{6a}+\frac{\log\,(ax+1)}{3a^3}-\frac{\mathrm{acoth}\,(ax)}{3a^3} & \text{for } a\neq 0 \\ \frac{i\pi x^3}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(a*x),x)
```

```
[Out] Piecewise((x**3*acoth(a*x)/3 + x**2/(6*a) + log(a*x + 1)/(3*a**3) - acoth(a
*x)/(3*a**3), Ne(a, 0)), (I*pi*x**3/6, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arcoth}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(a*x), x)
```

3.5 $\int x\coth^{-1}(ax)\,dx$

Optimal. Leaf size=31

$$-\frac{\tanh^{-1}(ax)}{2a^2}+\frac{1}{2}x^2\coth^{-1}(ax)+\frac{x}{2a}$$

```
[Out] x/(2*a) + (x^2*ArcCoth[a*x])/2 - ArcTanh[a*x]/(2*a^2)
```

Rubi [A] time = 0.0132059, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5917, 321, 206}

$$-\frac{\tanh^{-1}(ax)}{2a^2}+\frac{1}{2}x^2\coth^{-1}(ax)+\frac{x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[a*x],x]
```

```
[Out] x/(2*a) + (x^2*ArcCoth[a*x])/2 - ArcTanh[a*x]/(2*a^2)
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x\coth^{-1}(ax)\,dx &= \frac{1}{2}x^2\coth^{-1}(ax)-\frac{1}{2}a\int\frac{x^2}{1-a^2x^2}\,dx\\ &= \frac{x}{2a}+\frac{1}{2}x^2\coth^{-1}(ax)-\frac{\int\frac{1}{1-a^2x^2}\,dx}{2a}\\ &= \frac{x}{2a}+\frac{1}{2}x^2\coth^{-1}(ax)-\frac{\tanh^{-1}(ax)}{2a^2}\end{aligned}$$

Mathematica [A] time = 0.0069993, size = 47, normalized size = 1.52

$$\frac{\log(1-ax)}{4a^2}-\frac{\log(ax+1)}{4a^2}+\frac{1}{2}x^2\coth^{-1}(ax)+\frac{x}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[a*x],x]
```

```
[Out] x/(2*a) + (x^2*ArcCoth[a*x])/2 + Log[1 - a*x]/(4*a^2) - Log[1 + a*x]/(4*a^2
)
```

Maple [A] time = 0.033, size = 39, normalized size = 1.3

$$\frac{x^2\operatorname{arccoth}(ax)}{2}+\frac{x}{2a}+\frac{\ln(ax-1)}{4a^2}-\frac{\ln(ax+1)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(a*x),x)
```

```
[Out] 1/2*x^2*arccoth(a*x)+1/2*x/a+1/4/a^2*ln(a*x-1)-1/4/a^2*ln(a*x+1)
```

Maxima [A] time = 0.973175, size = 55, normalized size = 1.77

$$\frac{1}{2}x^2\operatorname{arcoth}(ax)+\frac{1}{4}a\left(\frac{2x}{a^2}-\frac{\log(ax+1)}{a^3}+\frac{\log(ax-1)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(a*x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccoth(a*x) + 1/4*a*(2*x/a^2 - log(a*x + 1)/a^3 + log(a*x - 1)/a^3
)
```

Fricas [A] time = 1.61763, size = 78, normalized size = 2.52

$$\frac{2ax+\left(a^2x^2-1\right)\log\left(\frac{ax+1}{ax-1}\right)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(a*x),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*x + (a^2*x^2 - 1)*log((a*x + 1)/(a*x - 1)))/a^2
```

Sympy [A] time = 0.918862, size = 32, normalized size = 1.03

$$\begin{cases}\frac{x^2\operatorname{acoth}(ax)}{2}+\frac{x}{2a}-\frac{\operatorname{acoth}(ax)}{2a^2} & \text{for } a\neq 0\\ \frac{i\pi x^2}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(a*x),x)
```

```
[Out] Piecewise((x**2*acoth(a*x)/2 + x/(2*a) - acoth(a*x)/(2*a**2), Ne(a, 0)), (I
*pi*x**2/4, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arcoth}(ax) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(a*x),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(a*x), x)
```

3.6 $\int \coth^{-1}(ax)\,dx$

Optimal. Leaf size=25

$$\frac{\log(1-a^2x^2)}{2a}+x\coth^{-1}(ax)$$

```
[Out] x*ArcCoth[a*x] + Log[1 - a^2*x^2]/(2*a)
```

Rubi [A] time = 0.0068101, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5911, 260}

$$\frac{\log(1-a^2x^2)}{2a}+x\coth^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x],x]
```

```
[Out] x*ArcCoth[a*x] + Log[1 - a^2*x^2]/(2*a)
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \coth^{-1}(ax)\,dx &= x\coth^{-1}(ax)-a\int \frac{x}{1-a^2x^2}\,dx\\ &= x\coth^{-1}(ax)+\frac{\log(1-a^2x^2)}{2a}\end{aligned}$$

Mathematica [A] time = 0.0030681, size = 25, normalized size = 1.

$$\frac{\log(1-a^2x^2)}{2a}+x\coth^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x],x]
```

```
[Out] x*ArcCoth[a*x] + Log[1 - a^2*x^2]/(2*a)
```

Maple [A] time = 0.043, size = 23, normalized size = 0.9

$$x\text{arccoth}\left(ax\right)+\frac{\ln\left(a^{2}x^{2}-1\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x),x)
```

```
[Out] x*arccoth(a*x)+1/2/a*ln(a^2*x^2-1)
```

Maxima [A] time = 0.986804, size = 34, normalized size = 1.36

$$\frac{2\,ax\,\text{arcoth}\left(ax\right)+\log\left(-a^{2}x^{2}+1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*x*arccoth(a*x) + log(-a^2*x^2 + 1))/a
```

Fricas [A] time = 1.68022, size = 77, normalized size = 3.08

$$\frac{ax\log\left(\frac{ax+1}{ax-1}\right)+\log\left(a^{2}x^{2}-1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x),x, algorithm="fricas")
```

```
[Out] 1/2*(a*x*log((a*x + 1)/(a*x - 1)) + log(a^2*x^2 - 1))/a
```

Sympy [A] time = 0.609134, size = 27, normalized size = 1.08

$$\begin{cases} x\,\text{acoth}\left(ax\right)+\frac{\log\left(ax+1\right)}{a}-\frac{\text{acoth}\left(ax\right)}{a} & \text{for}\ a\neq 0\\ \frac{i\pi x}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x),x)
```

```
[Out] Piecewise((x*acoth(a*x) + log(a*x + 1)/a - acoth(a*x)/a, Ne(a, 0)), (I*pi*x
/2, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\text{arcoth}\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x),x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x), x)
```

3.7 $\int \frac{\coth^{-1}(ax)}{x}\,dx$

Optimal. Leaf size=28

$$\frac{1}{2}\text{PolyLog}\left(2,-\frac{1}{ax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{1}{ax}\right)$$

```
[Out] PolyLog[2, -(1/(a*x))]/2 - PolyLog[2, 1/(a*x)]/2
```

Rubi [A] time = 0.0097539, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5913}

$$\frac{1}{2}\text{PolyLog}\left(2,-\frac{1}{ax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{1}{ax}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]/x,x]
```

```
[Out] PolyLog[2, -(1/(a*x))]/2 - PolyLog[2, 1/(a*x)]/2
```

Rule 5913

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b*PolyLog[2, -(c*x)^(-1)])/2, x] - Simp[(b*PolyLog[2, 1/(c*x)])/
2, x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{\coth^{-1}(ax)}{x}\,dx=\frac{1}{2}\text{Li}_2\left(-\frac{1}{ax}\right)-\frac{1}{2}\text{Li}_2\left(\frac{1}{ax}\right)$$

Mathematica [A] time = 0.0081032, size = 26, normalized size = 0.93

$$\frac{1}{2}\left(\text{PolyLog}\left(2,-\frac{1}{ax}\right)-\text{PolyLog}\left(2,\frac{1}{ax}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]/x,x]
```

```
[Out] (PolyLog[2, -(1/(a*x))] - PolyLog[2, 1/(a*x)])/2
```

Maple [A] time = 0.042, size = 37, normalized size = 1.3

$$\ln(ax)\,\text{arccoth}(ax)-\frac{\text{dilog}(ax)}{2}-\frac{\text{dilog}(ax+1)}{2}-\frac{\ln(ax)\ln(ax+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)/x,x)
```

```
[Out] ln(a*x)*arccoth(a*x)-1/2*dilog(a*x)-1/2*dilog(a*x+1)-1/2*ln(a*x)*ln(a*x+1)
```

Maxima [B] time = 0.997304, size = 116, normalized size = 4.14

$$-\frac{1}{2}a\left(\frac{\log(ax+1)}{a}-\frac{\log(ax-1)}{a}\right)\log(x)-\frac{1}{2}a\left(\frac{\log(ax-1)\log(ax)+\mathrm{Li}_2(-ax+1)}{a}-\frac{\log(ax+1)\log(-ax)+\mathrm{Li}_2(ax+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*(log(a*x + 1)/a - log(a*x - 1)/a)*log(x) - 1/2*a*((log(a*x - 1)*log(
a*x) + dilog(-a*x + 1))/a - (log(a*x + 1)*log(-a*x) + dilog(a*x + 1))/a) +
arccoth(a*x)*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{arcoth}(ax)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\mathrm{acoth}(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)/x,x)
```

```
[Out] Integral(acoth(a*x)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\mathrm{arcoth}(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)/x, x)
```

3.8 $\int \frac{\coth^{-1}(ax)}{x^2} dx$

Optimal. Leaf size=30

$$-\frac{1}{2}a\log\left(1-a^2x^2\right)+a\log(x)-\frac{\coth^{-1}(ax)}{x}$$

```
[Out] -(ArcCoth[a*x]/x) + a*Log[x] - (a*Log[1 - a^2*x^2])/2
```

Rubi [A] time = 0.020794, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5917, 266, 36, 29, 31}

$$-\frac{1}{2}a\log\left(1-a^2x^2\right)+a\log(x)-\frac{\coth^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]/x^2,x]

[Out] -(ArcCoth[a*x]/x) + a*Log[x] - (a*Log[1 - a^2*x^2])/2
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^{-1}(ax)}{x^2}\,dx &= -\frac{\coth^{-1}(ax)}{x} + a\int \frac{1}{x\left(1-a^2x^2\right)}\,dx \\
&= -\frac{\coth^{-1}(ax)}{x} + \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\left(1-a^2x\right)}\,dx, x, x^2\right) \\
&= -\frac{\coth^{-1}(ax)}{x} + \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) + \frac{1}{2}a^3\,\text{Subst}\left(\int \frac{1}{1-a^2x}\,dx, x, x^2\right) \\
&= -\frac{\coth^{-1}(ax)}{x} + a\log(x) - \frac{1}{2}a\log\left(1-a^2x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0075953, size = 30, normalized size = 1.

$$
-\frac{1}{2}a\log\left(1-a^2x^2\right) + a\log(x) - \frac{\coth^{-1}(ax)}{x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]/x^2,x]
```

```
[Out] -(ArcCoth[a*x]/x) + a*Log[x] - (a*Log[1 - a^2*x^2])/2
```

Maple [A] time = 0.036, size = 35, normalized size = 1.2

$$
-\frac{\text{arccoth}\,(ax)}{x} - \frac{a\ln\,(ax-1)}{2} + a\ln\,(ax) - \frac{a\ln\,(ax+1)}{2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)/x^2,x)
```

```
[Out] -arccoth(a*x)/x-1/2*a*ln(a*x-1)+a*ln(a*x)-1/2*a*ln(a*x+1)
```

Maxima [A] time = 0.974439, size = 41, normalized size = 1.37

$$
-\frac{1}{2}a\left(\log\left(a^2x^2-1\right) - \log\left(x^2\right)\right) - \frac{\text{arcoth}\,(ax)}{x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*(log(a^2*x^2 - 1) - log(x^2)) - arccoth(a*x)/x
```

Fricas [A] time = 1.70012, size = 99, normalized size = 3.3

$$
-\frac{ax\log\left(a^2x^2-1\right) - 2\,ax\log\,(x) + \log\left(\frac{ax+1}{ax-1}\right)}{2\,x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a*x*log(a^2*x^2 - 1) - 2*a*x*log(x) + log((a*x + 1)/(a*x - 1)))/x
```

Sympy [A] time = 0.610749, size = 26, normalized size = 0.87

$$a\log(x) - a\log(ax+1) + a\,\mathrm{acoth}(ax) - \frac{\mathrm{acoth}(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)/x**2,x)
```

```
[Out] a*log(x) - a*log(a*x + 1) + a*acoth(a*x) - acoth(a*x)/x
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{arcoth}(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)/x^2, x)
```

3.9 $\int \frac{\coth^{-1}(ax)}{x^3} \, dx$

Optimal. Leaf size=31

$$\frac{1}{2}a^2\tanh^{-1}(ax) - \frac{\coth^{-1}(ax)}{2x^2} - \frac{a}{2x}$$

```
[Out] -a/(2*x) - ArcCoth[a*x]/(2*x^2) + (a^2*ArcTanh[a*x])/2
```

Rubi [A] time = 0.0163453, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5917, 325, 206}

$$\frac{1}{2}a^2\tanh^{-1}(ax) - \frac{\coth^{-1}(ax)}{2x^2} - \frac{a}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]/x^3,x]
```

```
[Out] -a/(2*x) - ArcCoth[a*x]/(2*x^2) + (a^2*ArcTanh[a*x])/2
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)}{x^3} \, dx &= -\frac{\coth^{-1}(ax)}{2x^2} + \frac{1}{2}a \int \frac{1}{x^2\left(1-a^2x^2\right)} \, dx \\
&= -\frac{a}{2x} - \frac{\coth^{-1}(ax)}{2x^2} + \frac{1}{2}a^3 \int \frac{1}{1-a^2x^2} \, dx \\
&= -\frac{a}{2x} - \frac{\coth^{-1}(ax)}{2x^2} + \frac{1}{2}a^2\tanh^{-1}(ax)
\end{aligned}$$

Mathematica [A] time = 0.0082258, size = 47, normalized size = 1.52

$$-\frac{1}{4}a^2\log(1-ax)+\frac{1}{4}a^2\log(ax+1)-\frac{\coth^{-1}(ax)}{2x^2}-\frac{a}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]/x^3,x]
```

```
[Out] -a/(2*x) - ArcCoth[a*x]/(2*x^2) - (a^2*Log[1 - a*x])/4 + (a^2*Log[1 + a*x])
/4
```

Maple [A] time = 0.039, size = 39, normalized size = 1.3

$$-\frac{\operatorname{arccoth}(ax)}{2x^2}-\frac{a}{2x}-\frac{a^2\ln(ax-1)}{4}+\frac{a^2\ln(ax+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)/x^3,x)
```

```
[Out] -1/2*arccoth(a*x)/x^2-1/2*a/x-1/4*a^2*ln(a*x-1)+1/4*a^2*ln(a*x+1)
```

Maxima [A] time = 0.972809, size = 49, normalized size = 1.58

$$\frac{1}{4}\left(a\log(ax+1)-a\log(ax-1)-\frac{2}{x}\right)a-\frac{\operatorname{arcoth}(ax)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(a*log(a*x + 1) - a*log(a*x - 1) - 2/x)*a - 1/2*arccoth(a*x)/x^2
```

Fricas [A] time = 1.51869, size = 80, normalized size = 2.58

$$-\frac{2ax-\left(a^2x^2-1\right)\log\left(\frac{ax+1}{ax-1}\right)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*x - (a^2*x^2 - 1)*log((a*x + 1)/(a*x - 1)))/x^2
```

Sympy [A] time = 2.57081, size = 24, normalized size = 0.77

$$\frac{a^2\operatorname{acoth}(ax)}{2}-\frac{a}{2x}-\frac{\operatorname{acoth}(ax)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)/x**3,x)
```

```
[Out] a**2*acoth(a*x)/2 - a/(2*x) - acoth(a*x)/(2*x**2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(ax)}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)/x^3, x)
```

3.10 $\int \frac{\coth^{-1}(ax)}{x^4} \, dx$

Optimal. Leaf size=47

$$-\frac{1}{6}a^3\log\left(1-a^2x^2\right)+\frac{1}{3}a^3\log(x)-\frac{a}{6x^2}-\frac{\coth^{-1}(ax)}{3x^3}$$

```
[Out] -a/(6*x^2) - ArcCoth[a*x]/(3*x^3) + (a^3*Log[x])/3 - (a^3*Log[1 - a^2*x^2])
/6
```

Rubi [A] time = 0.0302444, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5917, 266, 44}

$$-\frac{1}{6}a^3\log\left(1-a^2x^2\right)+\frac{1}{3}a^3\log(x)-\frac{a}{6x^2}-\frac{\coth^{-1}(ax)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]/x^4,x]
```

```
[Out] -a/(6*x^2) - ArcCoth[a*x]/(3*x^3) + (a^3*Log[x])/3 - (a^3*Log[1 - a^2*x^2])
/6
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\coth^{-1}(ax)}{x^4}\,dx = -\frac{\coth^{-1}(ax)}{3x^3} + \frac{1}{3}a\int \frac{1}{x^3\left(1-a^2x^2\right)}\,dx$$
$$= -\frac{\coth^{-1}(ax)}{3x^3} + \frac{1}{6}a\,\text{Subst}\left(\int \frac{1}{x^2\left(1-a^2x\right)}\,dx, x, x^2\right)$$
$$= -\frac{\coth^{-1}(ax)}{3x^3} + \frac{1}{6}a\,\text{Subst}\left(\int \left(\frac{1}{x^2} + \frac{a^2}{x} - \frac{a^4}{-1+a^2x}\right)dx, x, x^2\right)$$
$$= -\frac{a}{6x^2} - \frac{\coth^{-1}(ax)}{3x^3} + \frac{1}{3}a^3\log(x) - \frac{1}{6}a^3\log\left(1-a^2x^2\right)$$

Mathematica [A] time = 0.0095672, size = 47, normalized size = 1.

$$-\frac{1}{6}a^3\log\left(1-a^2x^2\right) + \frac{1}{3}a^3\log(x) - \frac{a}{6x^2} - \frac{\coth^{-1}(ax)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]/x^4,x]
```

```
[Out] -a/(6*x^2) - ArcCoth[a*x]/(3*x^3) + (a^3*Log[x])/3 - (a^3*Log[1 - a^2*x^2])
/6
```

Maple [A] time = 0.039, size = 48, normalized size = 1.

$$-\frac{\text{arccoth}\,(ax)}{3\,x^3} - \frac{a^3\ln(ax-1)}{6} - \frac{a}{6\,x^2} + \frac{a^3\ln(ax)}{3} - \frac{a^3\ln(ax+1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)/x^4,x)
```

```
[Out] -1/3*arccoth(a*x)/x^3-1/6*a^3*ln(a*x-1)-1/6*a/x^2+1/3*a^3*ln(a*x)-1/6*a^3*l
n(a*x+1)
```

Maxima [A] time = 0.945418, size = 54, normalized size = 1.15

$$-\frac{1}{6}\left(a^2\log\left(a^2x^2-1\right) - a^2\log\left(x^2\right) + \frac{1}{x^2}\right)a - \frac{\text{arcoth}\,(ax)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(a^2*log(a^2*x^2 - 1) - a^2*log(x^2) + 1/x^2)*a - 1/3*arccoth(a*x)/x^3
```

Fricas [A] time = 1.5701, size = 120, normalized size = 2.55

$$-\frac{a^3x^3\log\left(a^2x^2-1\right) - 2\,a^3x^3\log(x) + ax + \log\left(\frac{ax+1}{ax-1}\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(a^3*x^3*log(a^2*x^2 - 1) - 2*a^3*x^3*log(x) + a*x + log((a*x + 1)/(a*
x - 1)))/x^3
```

Sympy [A] time = 4.12088, size = 46, normalized size = 0.98

$$\frac{a^3 \log(x)}{3} - \frac{a^3 \log(ax+1)}{3} + \frac{a^3 \operatorname{acoth}(ax)}{3} - \frac{a}{6x^2} - \frac{\operatorname{acoth}(ax)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)/x**4,x)
```

```
[Out] a**3*log(x)/3 - a**3*log(a*x + 1)/3 + a**3*acoth(a*x)/3 - a/(6*x**2) - acot
h(a*x)/(3*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(ax)}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)/x^4, x)
```

3.11 $\int \frac{\coth^{-1}(ax)}{x^5}\,dx$

Optimal. Leaf size=41

$$-\frac{a^3}{4x}+\frac{1}{4}a^4\tanh^{-1}(ax)-\frac{a}{12x^3}-\frac{\coth^{-1}(ax)}{4x^4}$$

```
[Out] -a/(12*x^3) - a^3/(4*x) - ArcCoth[a*x]/(4*x^4) + (a^4*ArcTanh[a*x])/4
```

Rubi [A] time = 0.0218977, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5917, 325, 206}

$$-\frac{a^3}{4x}+\frac{1}{4}a^4\tanh^{-1}(ax)-\frac{a}{12x^3}-\frac{\coth^{-1}(ax)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]/x^5,x]
```

```
[Out] -a/(12*x^3) - a^3/(4*x) - ArcCoth[a*x]/(4*x^4) + (a^4*ArcTanh[a*x])/4
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)}{x^5}\,dx &= -\frac{\coth^{-1}(ax)}{4x^4}+\frac{1}{4}a\int \frac{1}{x^4\left(1-a^2x^2\right)}\,dx\\
&= -\frac{a}{12x^3}-\frac{\coth^{-1}(ax)}{4x^4}+\frac{1}{4}a^3\int \frac{1}{x^2\left(1-a^2x^2\right)}\,dx\\
&= -\frac{a}{12x^3}-\frac{a^3}{4x}-\frac{\coth^{-1}(ax)}{4x^4}+\frac{1}{4}a^5\int \frac{1}{1-a^2x^2}\,dx\\
&= -\frac{a}{12x^3}-\frac{a^3}{4x}-\frac{\coth^{-1}(ax)}{4x^4}+\frac{1}{4}a^4\tanh^{-1}(ax)
\end{aligned}$$

Mathematica [A] time = 0.0118316, size = 57, normalized size = 1.39

$$-\frac{a^3}{4x}-\frac{1}{8}a^4\log(1-ax)+\frac{1}{8}a^4\log(ax+1)-\frac{a}{12x^3}-\frac{\coth^{-1}(ax)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]/x^5,x]
```

```
[Out] -a/(12*x^3) - a^3/(4*x) - ArcCoth[a*x]/(4*x^4) - (a^4*Log[1 - a*x])/8 + (a^
4*Log[1 + a*x])/8
```

Maple [A] time = 0.036, size = 47, normalized size = 1.2

$$-\frac{\operatorname{arccoth}(ax)}{4\,x^4}-\frac{a^4\ln(ax-1)}{8}-\frac{a}{12\,x^3}-\frac{a^3}{4\,x}+\frac{a^4\ln(ax+1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)/x^5,x)
```

```
[Out] -1/4*arccoth(a*x)/x^4-1/8*a^4*ln(a*x-1)-1/12*a/x^3-1/4*a^3/x+1/8*a^4*ln(a*x
+1)
```

Maxima [A] time = 0.94443, size = 69, normalized size = 1.68

$$\frac{1}{24}\left(3\,a^3\log(ax+1)-3\,a^3\log(ax-1)-\frac{2\left(3\,a^2x^2+1\right)}{x^3}\right)a-\frac{\operatorname{arcoth}(ax)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x^5,x, algorithm="maxima")
```

```
[Out] 1/24*(3*a^3*log(a*x + 1) - 3*a^3*log(a*x - 1) - 2*(3*a^2*x^2 + 1)/x^3)*a -
1/4*arccoth(a*x)/x^4
```

Fricas [A] time = 1.5728, size = 100, normalized size = 2.44

$$-\frac{6\,a^3x^3+2\,ax-3\left(a^4x^4-1\right)\log\left(\frac{ax+1}{ax-1}\right)}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x^5,x, algorithm="fricas")
```

```
[Out] -1/24*(6*a^3*x^3 + 2*a*x - 3*(a^4*x^4 - 1)*log((a*x + 1)/(a*x - 1)))/x^4
```

Sympy [A] time = 1.9597, size = 32, normalized size = 0.78

$$\frac{a^4 \operatorname{acoth}(ax)}{4} - \frac{a^3}{4x} - \frac{a}{12x^3} - \frac{\operatorname{acoth}(ax)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)/x**5,x)
```

```
[Out] a**4*acoth(a*x)/4 - a**3/(4*x) - a/(12*x**3) - acoth(a*x)/(4*x**4)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(ax)}{x^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)/x^5, x)
```

3.12 $\int x^5 \coth^{-1}(ax)^2 \, dx$

Optimal. Leaf size=105

$$\frac{x^4}{60a^2}+\frac{4x^2}{45a^4}+\frac{23\log\left(1-a^2x^2\right)}{90a^6}+\frac{x^3\coth^{-1}(ax)}{9a^3}+\frac{x\coth^{-1}(ax)}{3a^5}-\frac{\coth^{-1}(ax)^2}{6a^6}+\frac{1}{6}x^6\coth^{-1}(ax)^2+\frac{x^5\coth^{-1}(ax}{15a}$$

```
[Out] (4*x^2)/(45*a^4) + x^4/(60*a^2) + (x*ArcCoth[a*x])/(3*a^5) + (x^3*ArcCoth[a
*x])/(9*a^3) + (x^5*ArcCoth[a*x])/(15*a) - ArcCoth[a*x]^2/(6*a^6) + (x^6*Ar
cCoth[a*x]^2)/6 + (23*Log[1 - a^2*x^2])/(90*a^6)
```

Rubi [A] time = 0.245691, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5917, 5981, 266, 43, 5911, 260, 5949}

$$\frac{x^4}{60a^2}+\frac{4x^2}{45a^4}+\frac{23\log\left(1-a^2x^2\right)}{90a^6}+\frac{x^3\coth^{-1}(ax)}{9a^3}+\frac{x\coth^{-1}(ax)}{3a^5}-\frac{\coth^{-1}(ax)^2}{6a^6}+\frac{1}{6}x^6\coth^{-1}(ax)^2+\frac{x^5\coth^{-1}(ax}{15a}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*ArcCoth[a*x]^2,x]
```

```
[Out] (4*x^2)/(45*a^4) + x^4/(60*a^2) + (x*ArcCoth[a*x])/(3*a^5) + (x^3*ArcCoth[a
*x])/(9*a^3) + (x^5*ArcCoth[a*x])/(15*a) - ArcCoth[a*x]^2/(6*a^6) + (x^6*Ar
cCoth[a*x]^2)/6 + (23*Log[1 - a^2*x^2])/(90*a^6)
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5981

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x^5 \coth^{-1}(ax)^2\,dx = \frac{1}{6}x^6\coth^{-1}(ax)^2 - \frac{1}{3}a\int \frac{x^6\coth^{-1}(ax)}{1-a^2x^2}\,dx$$
$$= \frac{1}{6}x^6\coth^{-1}(ax)^2 + \frac{\int x^4\coth^{-1}(ax)\,dx}{3a} - \frac{\int \frac{x^4\coth^{-1}(ax)}{1-a^2x^2}\,dx}{3a}$$
$$= \frac{x^5\coth^{-1}(ax)}{15a} + \frac{1}{6}x^6\coth^{-1}(ax)^2 - \frac{1}{15}\int \frac{x^5}{1-a^2x^2}\,dx + \frac{\int x^2\coth^{-1}(ax)\,dx}{3a^3} - \frac{\int \frac{x^2\coth^{-1}(ax)}{1-a^2x^2}\,dx}{3a^3}$$
$$= \frac{x^3\coth^{-1}(ax)}{9a^3} + \frac{x^5\coth^{-1}(ax)}{15a} + \frac{1}{6}x^6\coth^{-1}(ax)^2 - \frac{1}{30}\text{Subst}\left(\int \frac{x^2}{1-a^2x}\,dx, x, x^2\right) + \frac{\int \coth^{-1}}{3a}$$
$$= \frac{x\coth^{-1}(ax)}{3a^5} + \frac{x^3\coth^{-1}(ax)}{9a^3} + \frac{x^5\coth^{-1}(ax)}{15a} - \frac{\coth^{-1}(ax)^2}{6a^6} + \frac{1}{6}x^6\coth^{-1}(ax)^2 - \frac{1}{30}\text{Subst}\Big($$
$$= \frac{x^2}{30a^4} + \frac{x^4}{60a^2} + \frac{x\coth^{-1}(ax)}{3a^5} + \frac{x^3\coth^{-1}(ax)}{9a^3} + \frac{x^5\coth^{-1}(ax)}{15a} - \frac{\coth^{-1}(ax)^2}{6a^6} + \frac{1}{6}x^6\coth^{-1}(ax)$$
$$= \frac{4x^2}{45a^4} + \frac{x^4}{60a^2} + \frac{x\coth^{-1}(ax)}{3a^5} + \frac{x^3\coth^{-1}(ax)}{9a^3} + \frac{x^5\coth^{-1}(ax)}{15a} - \frac{\coth^{-1}(ax)^2}{6a^6} + \frac{1}{6}x^6\coth^{-1}(ax)$$

Mathematica [A] time = 0.0218823, size = 80, normalized size = 0.76

$$\frac{3a^4x^4 + 16a^2x^2 + 46\log\left(1-a^2x^2\right) + 4ax\left(3a^4x^4 + 5a^2x^2 + 15\right)\coth^{-1}(ax) + 30\left(a^6x^6-1\right)\coth^{-1}(ax)^2}{180a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*ArcCoth[a*x]^2,x]
```

```
[Out] (16*a^2*x^2 + 3*a^4*x^4 + 4*a*x*(15 + 5*a^2*x^2 + 3*a^4*x^4)*ArcCoth[a*x] +
 30*(-1 + a^6*x^6)*ArcCoth[a*x]^2 + 46*Log[1 - a^2*x^2])/(180*a^6)
```

Maple [B] time = 0.052, size = 196, normalized size = 1.9

$$\frac{x^6\left(\operatorname{arccoth}(ax)\right)^2}{6} + \frac{x^5\operatorname{arccoth}(ax)}{15\,a} + \frac{x^3\operatorname{arccoth}(ax)}{9\,a^3} + \frac{x\operatorname{arccoth}(ax)}{3\,a^5} + \frac{\operatorname{arccoth}(ax)\ln(ax-1)}{6\,a^6} - \frac{\operatorname{arccoth}(ax)\ln(a}{6\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arccoth(a*x)^2,x)
```

```
[Out] 1/6*x^6*arccoth(a*x)^2+1/15*x^5*arccoth(a*x)/a+1/9*x^3*arccoth(a*x)/a^3+1/3
*x*arccoth(a*x)/a^5+1/6/a^6*arccoth(a*x)*ln(a*x-1)-1/6/a^6*arccoth(a*x)*ln(
a*x+1)+1/24/a^6*ln(a*x-1)^2-1/12/a^6*ln(a*x-1)*ln(1/2+1/2*a*x)+1/12/a^6*ln(
-1/2*a*x+1/2)*ln(1/2+1/2*a*x)-1/12/a^6*ln(-1/2*a*x+1/2)*ln(a*x+1)+1/24/a^6*
ln(a*x+1)^2+1/60*x^4/a^2+4/45*x^2/a^4+23/90/a^6*ln(a*x-1)+23/90/a^6*ln(a*x+
1)
```

Maxima [A] time = 0.970857, size = 182, normalized size = 1.73

$$\frac{1}{6}x^6\operatorname{arcoth}(ax)^2+\frac{1}{90}a\left(\frac{2\left(3a^4x^5+5a^2x^3+15x\right)}{a^6}-\frac{15\log(ax+1)}{a^7}+\frac{15\log(ax-1)}{a^7}\right)\operatorname{arcoth}(ax)+\frac{6a^4x^4+32a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccoth(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*x^6*arccoth(a*x)^2 + 1/90*a*(2*(3*a^4*x^5 + 5*a^2*x^3 + 15*x)/a^6 - 15*
log(a*x + 1)/a^7 + 15*log(a*x - 1)/a^7)*arccoth(a*x) + 1/360*(6*a^4*x^4 + 3
2*a^2*x^2 - 2*(15*log(a*x - 1) - 46)*log(a*x + 1) + 15*log(a*x + 1)^2 + 15*
log(a*x - 1)^2 + 92*log(a*x - 1))/a^6
```

Fricas [A] time = 1.57716, size = 224, normalized size = 2.13

$$\frac{6a^4x^4+32a^2x^2+15\left(a^6x^6-1\right)\log\left(\frac{ax+1}{ax-1}\right)^2+4\left(3a^5x^5+5a^3x^3+15ax\right)\log\left(\frac{ax+1}{ax-1}\right)+92\log\left(a^2x^2-1\right)}{360a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccoth(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/360*(6*a^4*x^4 + 32*a^2*x^2 + 15*(a^6*x^6 - 1)*log((a*x + 1)/(a*x - 1))^2
 + 4*(3*a^5*x^5 + 5*a^3*x^3 + 15*a*x)*log((a*x + 1)/(a*x - 1)) + 92*log(a^2
*x^2 - 1))/a^6
```

Sympy [A] time = 7.50491, size = 114, normalized size = 1.09

$$\begin{cases}\frac{x^6\operatorname{acoth}^2(ax)}{6}+\frac{x^5\operatorname{acoth}(ax)}{15a}+\frac{x^4}{60a^2}+\frac{x^3\operatorname{acoth}(ax)}{9a^3}+\frac{4x^2}{45a^4}+\frac{x\operatorname{acoth}(ax)}{3a^5}+\frac{23\log(ax+1)}{45a^6}-\frac{\operatorname{acoth}^2(ax)}{6a^6}-\frac{23\operatorname{acoth}(ax)}{45a^6} & \text{for } a\neq 0\\ -\frac{\pi^2x^6}{24} & \text{otherwis}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*acoth(a*x)**2,x)
```

```
[Out] Piecewise((x**6*acoth(a*x)**2/6 + x**5*acoth(a*x)/(15*a) + x**4/(60*a**2) +
 x**3*acoth(a*x)/(9*a**3) + 4*x**2/(45*a**4) + x*acoth(a*x)/(3*a**5) + 23*l
og(a*x + 1)/(45*a**6) - acoth(a*x)**2/(6*a**6) - 23*acoth(a*x)/(45*a**6), N
e(a, 0)), (-pi**2*x**6/24, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \operatorname{arcoth}(ax)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccoth(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^5*arccoth(a*x)^2, x)
```

3.13 $\int x^4 \coth^{-1}(ax)^2 \, dx$

Optimal. Leaf size=127

$$-\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{5a^5}+\frac{x^3}{30a^2}+\frac{x^2\coth^{-1}(ax)}{5a^3}+\frac{3x}{10a^4}-\frac{3\tanh^{-1}(ax)}{10a^5}+\frac{\coth^{-1}(ax)^2}{5a^5}-\frac{2\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{5a^5}+$$

```
[Out] (3*x)/(10*a^4) + x^3/(30*a^2) + (x^2*ArcCoth[a*x])/(5*a^3) + (x^4*ArcCoth[a
*x])/(10*a) + ArcCoth[a*x]^2/(5*a^5) + (x^5*ArcCoth[a*x]^2)/5 - (3*ArcTanh[
a*x])/(10*a^5) - (2*ArcCoth[a*x]*Log[2/(1 - a*x)])/(5*a^5) - PolyLog[2, 1 -
 2/(1 - a*x)]/(5*a^5)
```

Rubi [A] time = 0.225659, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.9, Rules used = {5917, 5981, 302, 206, 321, 5985, 5919, 2402, 2315}

$$-\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{5a^5}+\frac{x^3}{30a^2}+\frac{x^2\coth^{-1}(ax)}{5a^3}+\frac{3x}{10a^4}-\frac{3\tanh^{-1}(ax)}{10a^5}+\frac{\coth^{-1}(ax)^2}{5a^5}-\frac{2\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{5a^5}+$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcCoth[a*x]^2,x]
```


```
[Out] (3*x)/(10*a^4) + x^3/(30*a^2) + (x^2*ArcCoth[a*x])/(5*a^3) + (x^4*ArcCoth[a
*x])/(10*a) + ArcCoth[a*x]^2/(5*a^5) + (x^5*ArcCoth[a*x]^2)/5 - (3*ArcTanh[
a*x])/(10*a^5) - (2*ArcCoth[a*x]*Log[2/(1 - a*x)])/(5*a^5) - PolyLog[2, 1 -
 2/(1 - a*x)]/(5*a^5)
```


Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5981

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5985

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int x^4 \coth^{-1}(ax)^2\,dx = \frac{1}{5}x^5\coth^{-1}(ax)^2 - \frac{1}{5}(2a)\int \frac{x^5\coth^{-1}(ax)}{1-a^2x^2}\,dx$$

$$= \frac{1}{5}x^5\coth^{-1}(ax)^2 + \frac{2\int x^3\coth^{-1}(ax)\,dx}{5a} - \frac{2\int \frac{x^3\coth^{-1}(ax)}{1-a^2x^2}\,dx}{5a}$$

$$= \frac{x^4\coth^{-1}(ax)}{10a} + \frac{1}{5}x^5\coth^{-1}(ax)^2 - \frac{1}{10}\int \frac{x^4}{1-a^2x^2}\,dx + \frac{2\int x\coth^{-1}(ax)\,dx}{5a^3} - \frac{2\int \frac{x\coth^{-1}(ax)}{1-a^2x^2}\,dx}{5a^3}$$

$$= \frac{x^2\coth^{-1}(ax)}{5a^3} + \frac{x^4\coth^{-1}(ax)}{10a} + \frac{\coth^{-1}(ax)^2}{5a^5} + \frac{1}{5}x^5\coth^{-1}(ax)^2 - \frac{1}{10}\int\left(-\frac{1}{a^4} - \frac{x^2}{a^2} + \frac{1}{a^4\left(1-a\right.}\right.$$

$$= \frac{3x}{10a^4} + \frac{x^3}{30a^2} + \frac{x^2\coth^{-1}(ax)}{5a^3} + \frac{x^4\coth^{-1}(ax)}{10a} + \frac{\coth^{-1}(ax)^2}{5a^5} + \frac{1}{5}x^5\coth^{-1}(ax)^2 - \frac{2\coth^{-1}(ax}{5a}$$

$$= \frac{3x}{10a^4} + \frac{x^3}{30a^2} + \frac{x^2\coth^{-1}(ax)}{5a^3} + \frac{x^4\coth^{-1}(ax)}{10a} + \frac{\coth^{-1}(ax)^2}{5a^5} + \frac{1}{5}x^5\coth^{-1}(ax)^2 - \frac{3\tanh^{-1}(ax}{10a^5}$$

$$= \frac{3x}{10a^4} + \frac{x^3}{30a^2} + \frac{x^2\coth^{-1}(ax)}{5a^3} + \frac{x^4\coth^{-1}(ax)}{10a} + \frac{\coth^{-1}(ax)^2}{5a^5} + \frac{1}{5}x^5\coth^{-1}(ax)^2 - \frac{3\tanh^{-1}(ax}{10a^5}$$

Mathematica [A] time = 0.443444, size = 87, normalized size = 0.69

$$\frac{6\text{PolyLog}\left(2,e^{-2\coth^{-1}(ax)}\right)+ax\left(a^2x^2+9\right)+6\left(a^5x^5-1\right)\coth^{-1}(ax)^2+3\coth^{-1}(ax)\left(a^4x^4+2a^2x^2-4\log\left(1-e^{-2c\cdots}\right.\right.}{30a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*ArcCoth[a*x]^2,x]
```

```
[Out] (a*x*(9 + a^2*x^2) + 6*(-1 + a^5*x^5)*ArcCoth[a*x]^2 + 3*ArcCoth[a*x]*(-3 +
 2*a^2*x^2 + a^4*x^4 - 4*Log[1 - E^(-2*ArcCoth[a*x])]) + 6*PolyLog[2, E^(-2
*ArcCoth[a*x])])/(30*a^5)
```

Maple [A] time = 0.053, size = 196, normalized size = 1.5

$$\frac{x^5(\text{arccoth}(ax))^2}{5}+\frac{x^4\text{arccoth}(ax)}{10a}+\frac{x^2\text{arccoth}(ax)}{5a^3}+\frac{\text{arccoth}(ax)\ln(ax-1)}{5a^5}+\frac{\text{arccoth}(ax)\ln(ax+1)}{5a^5}+\frac{x^3}{30a^{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccoth(a*x)^2,x)
```

```
[Out] 1/5*x^5*arccoth(a*x)^2+1/10*x^4*arccoth(a*x)/a+1/5*x^2*arccoth(a*x)/a^3+1/5
/a^5*arccoth(a*x)*ln(a*x-1)+1/5/a^5*arccoth(a*x)*ln(a*x+1)+1/30*x^3/a^2+3/1
0*x/a^4+3/20/a^5*ln(a*x-1)-3/20/a^5*ln(a*x+1)+1/20/a^5*ln(a*x-1)^2-1/5/a^5*
dilog(1/2+1/2*a*x)-1/10/a^5*ln(a*x-1)*ln(1/2+1/2*a*x)-1/10/a^5*ln(-1/2*a*x+
1/2)*ln(1/2+1/2*a*x)+1/10/a^5*ln(-1/2*a*x+1/2)*ln(a*x+1)-1/20/a^5*ln(a*x+1)
^2
```

Maxima [A] time = 0.960673, size = 209, normalized size = 1.65

$$\frac{1}{5}x^5\,\text{arcoth}(ax)^2+\frac{1}{60}a^2\left(\frac{2a^3x^3+18ax-3\log(ax+1)^2+6\log(ax+1)\log(ax-1)+3\log(ax-1)^2+9\log(ax\cdots}{a^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arccoth(a*x)^2 + 1/60*a^2*((2*a^3*x^3 + 18*a*x - 3*log(a*x + 1)^2 +
 6*log(a*x + 1)*log(a*x - 1) + 3*log(a*x - 1)^2 + 9*log(a*x - 1))/a^7 - 12*
(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a^7 - 9*log(a*x +
 1)/a^7) + 1/10*a*((a^2*x^4 + 2*x^2)/a^4 + 2*log(a^2*x^2 - 1)/a^6)*arccoth(
a*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^4\,\text{arcoth}(ax)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^4*arccoth(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{acoth}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acoth(a*x)**2,x)
```

```
[Out] Integral(x**4*acoth(a*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{arcoth}(ax)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4*arccoth(a*x)^2, x)
```

3.14 $\int x^3 \coth^{-1}(ax)^2\,dx$

Optimal. Leaf size=81

$$\frac{x^2}{12a^2}+\frac{\log\left(1-a^2x^2\right)}{3a^4}+\frac{x\coth^{-1}(ax)}{2a^3}-\frac{\coth^{-1}(ax)^2}{4a^4}+\frac{1}{4}x^4\coth^{-1}(ax)^2+\frac{x^3\coth^{-1}(ax)}{6a}$$

```
[Out] x^2/(12*a^2) + (x*ArcCoth[a*x])/(2*a^3) + (x^3*ArcCoth[a*x])/(6*a) - ArcCot
h[a*x]^2/(4*a^4) + (x^4*ArcCoth[a*x]^2)/4 + Log[1 - a^2*x^2]/(3*a^4)
```

Rubi [A] time = 0.162949, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5917, 5981, 266, 43, 5911, 260, 5949}

$$\frac{x^2}{12a^2}+\frac{\log\left(1-a^2x^2\right)}{3a^4}+\frac{x\coth^{-1}(ax)}{2a^3}-\frac{\coth^{-1}(ax)^2}{4a^4}+\frac{1}{4}x^4\coth^{-1}(ax)^2+\frac{x^3\coth^{-1}(ax)}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCoth[a*x]^2,x]
```

```
[Out] x^2/(12*a^2) + (x*ArcCoth[a*x])/(2*a^3) + (x^3*ArcCoth[a*x])/(6*a) - ArcCot
h[a*x]^2/(4*a^4) + (x^4*ArcCoth[a*x]^2)/4 + Log[1 - a^2*x^2]/(3*a^4)
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5981

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
```

```
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int x^3 \coth^{-1}(ax)^2\,dx &= \frac{1}{4}x^4\coth^{-1}(ax)^2 - \frac{1}{2}a\int \frac{x^4\coth^{-1}(ax)}{1-a^2x^2}\,dx \\
&= \frac{1}{4}x^4\coth^{-1}(ax)^2 + \frac{\int x^2\coth^{-1}(ax)\,dx}{2a} - \frac{\int \frac{x^2\coth^{-1}(ax)}{1-a^2x^2}\,dx}{2a} \\
&= \frac{x^3\coth^{-1}(ax)}{6a} + \frac{1}{4}x^4\coth^{-1}(ax)^2 - \frac{1}{6}\int \frac{x^3}{1-a^2x^2}\,dx + \frac{\int \coth^{-1}(ax)\,dx}{2a^3} - \frac{\int \frac{\coth^{-1}(ax)}{1-a^2x^2}\,dx}{2a^3} \\
&= \frac{x\coth^{-1}(ax)}{2a^3} + \frac{x^3\coth^{-1}(ax)}{6a} - \frac{\coth^{-1}(ax)^2}{4a^4} + \frac{1}{4}x^4\coth^{-1}(ax)^2 - \frac{1}{12}\operatorname{Subst}\left(\int \frac{x}{1-a^2x}\,dx, x, x\right. \\
&= \frac{x\coth^{-1}(ax)}{2a^3} + \frac{x^3\coth^{-1}(ax)}{6a} - \frac{\coth^{-1}(ax)^2}{4a^4} + \frac{1}{4}x^4\coth^{-1}(ax)^2 + \frac{\log\left(1-a^2x^2\right)}{4a^4} - \frac{1}{12}\operatorname{Subst}\left(\right. \\
&= \frac{x^2}{12a^2} + \frac{x\coth^{-1}(ax)}{2a^3} + \frac{x^3\coth^{-1}(ax)}{6a} - \frac{\coth^{-1}(ax)^2}{4a^4} + \frac{1}{4}x^4\coth^{-1}(ax)^2 + \frac{\log\left(1-a^2x^2\right)}{3a^4}
\end{aligned}$$

Mathematica [A] time = 0.0167572, size = 62, normalized size = 0.77

$$\frac{a^2x^2 + 4\log\left(1-a^2x^2\right) + 2ax\left(a^2x^2+3\right)\coth^{-1}(ax) + 3\left(a^4x^4-1\right)\coth^{-1}(ax)^2}{12a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCoth[a*x]^2,x]
```

```
[Out] (a^2*x^2 + 2*a*x*(3 + a^2*x^2)*ArcCoth[a*x] + 3*(-1 + a^4*x^4)*ArcCoth[a*x]
^2 + 4*Log[1 - a^2*x^2])/(12*a^4)
```

Maple [B] time = 0.052, size = 176, normalized size = 2.2

$$\frac{x^4(\operatorname{arccoth}(ax))^2}{4} + \frac{x^3\operatorname{arccoth}(ax)}{6a} + \frac{x\operatorname{arccoth}(ax)}{2a^3} + \frac{\operatorname{arccoth}(ax)\ln(ax-1)}{4a^4} - \frac{\operatorname{arccoth}(ax)\ln(ax+1)}{4a^4} + \frac{(\ln(ax-}{16a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccoth(a*x)^2,x)
```

```
[Out] 1/4*x^4*arccoth(a*x)^2+1/6*x^3*arccoth(a*x)/a+1/2*x*arccoth(a*x)/a^3+1/4/a^
4*arccoth(a*x)*ln(a*x-1)-1/4/a^4*arccoth(a*x)*ln(a*x+1)+1/16/a^4*ln(a*x-1)^
```

```
2-1/8/a^4*ln(a*x-1)*ln(1/2+1/2*a*x)-1/8/a^4*ln(-1/2*a*x+1/2)*ln(a*x+1)+1/8/
a^4*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+1/16/a^4*ln(a*x+1)^2+1/12*x^2/a^2+1/3/
a^4*ln(a*x-1)+1/3/a^4*ln(a*x+1)
```

Maxima [A] time = 0.955027, size = 159, normalized size = 1.96

$$\frac{1}{4}x^4\,\mathrm{arcoth}\,(ax)^2 + \frac{1}{12}a\left(\frac{2\left(a^2x^3+3x\right)}{a^4} - \frac{3\log(ax+1)}{a^5} + \frac{3\log(ax-1)}{a^5}\right)\mathrm{arcoth}\,(ax) + \frac{4a^2x^2 - 2(3\log(ax-1) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccoth(a*x)^2 + 1/12*a*(2*(a^2*x^3 + 3*x)/a^4 - 3*log(a*x + 1)/a^5
 + 3*log(a*x - 1)/a^5)*arccoth(a*x) + 1/48*(4*a^2*x^2 - 2*(3*log(a*x - 1) -
 8)*log(a*x + 1) + 3*log(a*x + 1)^2 + 3*log(a*x - 1)^2 + 16*log(a*x - 1))/a
^4
```

Fricas [A] time = 1.59719, size = 184, normalized size = 2.27

$$\frac{4a^2x^2 + 3\left(a^4x^4-1\right)\log\left(\frac{ax+1}{ax-1}\right)^2 + 4\left(a^3x^3+3ax\right)\log\left(\frac{ax+1}{ax-1}\right) + 16\log\left(a^2x^2-1\right)}{48a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(4*a^2*x^2 + 3*(a^4*x^4 - 1)*log((a*x + 1)/(a*x - 1))^2 + 4*(a^3*x^3 +
 3*a*x)*log((a*x + 1)/(a*x - 1)) + 16*log(a^2*x^2 - 1))/a^4
```

Sympy [A] time = 5.10093, size = 90, normalized size = 1.11

$$\begin{cases} \frac{x^4\,\mathrm{acoth}^2(ax)}{4} + \frac{x^3\,\mathrm{acoth}(ax)}{6a} + \frac{x^2}{12a^2} + \frac{x\,\mathrm{acoth}(ax)}{2a^3} + \frac{2\log(ax+1)}{3a^4} - \frac{\mathrm{acoth}^2(ax)}{4a^4} - \frac{2\,\mathrm{acoth}(ax)}{3a^4} & \text{for } a \neq 0 \\ -\frac{\pi^2x^4}{16} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acoth(a*x)**2,x)
```

```
[Out] Piecewise((x**4*acoth(a*x)**2/4 + x**3*acoth(a*x)/(6*a) + x**2/(12*a**2) +
x*acoth(a*x)/(2*a**3) + 2*log(a*x + 1)/(3*a**4) - acoth(a*x)**2/(4*a**4) -
2*acoth(a*x)/(3*a**4), Ne(a, 0)), (-pi**2*x**4/16, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\,\mathrm{arcoth}\,(ax)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*arccoth(a*x)^2, x)
```

3.15 $\int x^2 \coth^{-1}(ax)^2\,dx$

Optimal. Leaf size=103

$$-\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{3a^3}+\frac{x}{3a^2}-\frac{\tanh^{-1}(ax)}{3a^3}+\frac{\coth^{-1}(ax)^2}{3a^3}-\frac{2\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{3a^3}+\frac{1}{3}x^3\coth^{-1}(ax)^2+\frac{x^2\coth}{3a}$$

```
[Out] x/(3*a^2) + (x^2*ArcCoth[a*x])/(3*a) + ArcCoth[a*x]^2/(3*a^3) + (x^3*ArcCot
h[a*x]^2)/3 - ArcTanh[a*x]/(3*a^3) - (2*ArcCoth[a*x]*Log[2/(1 - a*x)])/(3*a
^3) - PolyLog[2, 1 - 2/(1 - a*x)]/(3*a^3)
```

Rubi [A] time = 0.153214, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.8, Rules used = {5917, 5981, 321, 206, 5985, 5919, 2402, 2315}

$$-\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{3a^3}+\frac{x}{3a^2}-\frac{\tanh^{-1}(ax)}{3a^3}+\frac{\coth^{-1}(ax)^2}{3a^3}-\frac{2\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{3a^3}+\frac{1}{3}x^3\coth^{-1}(ax)^2+\frac{x^2\coth}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[a*x]^2,x]
```

```
[Out] x/(3*a^2) + (x^2*ArcCoth[a*x])/(3*a) + ArcCoth[a*x]^2/(3*a^3) + (x^3*ArcCot
h[a*x]^2)/3 - ArcTanh[a*x]/(3*a^3) - (2*ArcCoth[a*x]*Log[2/(1 - a*x)])/(3*a
^3) - PolyLog[2, 1 - 2/(1 - a*x)]/(3*a^3)
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5981

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5985

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(ax)^2\,dx &= \frac{1}{3}x^3\coth^{-1}(ax)^2 - \frac{1}{3}(2a)\int \frac{x^3\coth^{-1}(ax)}{1-a^2x^2}\,dx \\
&= \frac{1}{3}x^3\coth^{-1}(ax)^2 + \frac{2\int x\coth^{-1}(ax)\,dx}{3a} - \frac{2\int \frac{x\coth^{-1}(ax)}{1-a^2x^2}\,dx}{3a} \\
&= \frac{x^2\coth^{-1}(ax)}{3a} + \frac{\coth^{-1}(ax)^2}{3a^3} + \frac{1}{3}x^3\coth^{-1}(ax)^2 - \frac{1}{3}\int \frac{x^2}{1-a^2x^2}\,dx - \frac{2\int \frac{\coth^{-1}(ax)}{1-ax}\,dx}{3a^2} \\
&= \frac{x}{3a^2} + \frac{x^2\coth^{-1}(ax)}{3a} + \frac{\coth^{-1}(ax)^2}{3a^3} + \frac{1}{3}x^3\coth^{-1}(ax)^2 - \frac{2\coth^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{3a^3} - \frac{\int \frac{1}{1-a^2x^2}\,dx}{3a^2} \\
&= \frac{x}{3a^2} + \frac{x^2\coth^{-1}(ax)}{3a} + \frac{\coth^{-1}(ax)^2}{3a^3} + \frac{1}{3}x^3\coth^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)}{3a^3} - \frac{2\coth^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{3a^3} \\
&= \frac{x}{3a^2} + \frac{x^2\coth^{-1}(ax)}{3a} + \frac{\coth^{-1}(ax)^2}{3a^3} + \frac{1}{3}x^3\coth^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)}{3a^3} - \frac{2\coth^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{3a^3}
\end{aligned}$$

Mathematica [A] time = 0.23807, size = 66, normalized size = 0.64

$$\frac{\text{PolyLog}\left(2,e^{-2\coth^{-1}(ax)}\right)+\left(a^3x^3-1\right)\coth^{-1}(ax)^2+\coth^{-1}(ax)\left(a^2x^2-2\log\left(1-e^{-2\coth^{-1}(ax)}\right)-1\right)+ax}{3a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*ArcCoth[a*x]^2,x]
```

```
[Out] (a*x + (-1 + a^3*x^3)*ArcCoth[a*x]^2 + ArcCoth[a*x]*(-1 + a^2*x^2 - 2*Log[1
 - E^(-2*ArcCoth[a*x])]) + PolyLog[2, E^(-2*ArcCoth[a*x])])/(3*a^3)
```

Maple [A] time = 0.052, size = 176, normalized size = 1.7

$$\frac{x^3 (\operatorname{arccoth}(ax))^2}{3} + \frac{x^2 \operatorname{arccoth}(ax)}{3a} + \frac{\operatorname{arccoth}(ax)\ln(ax-1)}{3a^3} + \frac{\operatorname{arccoth}(ax)\ln(ax+1)}{3a^3} + \frac{x}{3a^2} + \frac{\ln(ax-1)}{6a^3} - \frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(a*x)^2,x)
```

```
[Out] 1/3*x^3*arccoth(a*x)^2+1/3*x^2*arccoth(a*x)/a+1/3/a^3*arccoth(a*x)*ln(a*x-1
)+1/3/a^3*arccoth(a*x)*ln(a*x+1)+1/3*x/a^2+1/6/a^3*ln(a*x-1)-1/6/a^3*ln(a*x
+1)+1/12/a^3*ln(a*x-1)^2-1/3/a^3*dilog(1/2+1/2*a*x)-1/6/a^3*ln(a*x-1)*ln(1/
2+1/2*a*x)+1/6/a^3*ln(-1/2*a*x+1/2)*ln(a*x+1)-1/6/a^3*ln(-1/2*a*x+1/2)*ln(1
/2+1/2*a*x)-1/12/a^3*ln(a*x+1)^2
```

Maxima [A] time = 0.974019, size = 181, normalized size = 1.76

$$\frac{1}{3}x^3 \operatorname{arcoth}(ax)^2 + \frac{1}{12}a^2\left(\frac{4\,ax - \log(ax+1)^2 + 2\,\log(ax+1)\log(ax-1) + \log(ax-1)^2 + 2\,\log(ax-1)}{a^5} - \frac{4\left(\log(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(a*x)^2 + 1/12*a^2*((4*a*x - log(a*x + 1)^2 + 2*log(a*x + 1)
*log(a*x - 1) + log(a*x - 1)^2 + 2*log(a*x - 1))/a^5 - 4*(log(a*x - 1)*log(
1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a^5 - 2*log(a*x + 1)/a^5) + 1/3*a*(
x^2/a^2 + log(a^2*x^2 - 1)/a^4)*arccoth(a*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(x^2 \operatorname{arcoth}(ax)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*arccoth(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{acoth}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(a*x)**2,x)
```

```
[Out] Integral(x**2*acoth(a*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arcoth}(ax)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(a*x)^2, x)
```

3.16 $\int x \coth^{-1}(ax)^2 \, dx$

Optimal. Leaf size=54

$$\frac{\log\left(1-a^2x^2\right)}{2a^2}-\frac{\coth^{-1}(ax)^2}{2a^2}+\frac{1}{2}x^2\coth^{-1}(ax)^2+\frac{x\coth^{-1}(ax)}{a}$$

```
[Out] (x*ArcCoth[a*x])/a - ArcCoth[a*x]^2/(2*a^2) + (x^2*ArcCoth[a*x]^2)/2 + Log[
1 - a^2*x^2]/(2*a^2)
```

Rubi [A] time = 0.0783636, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5917, 5981, 5911, 260, 5949}

$$\frac{\log\left(1-a^2x^2\right)}{2a^2}-\frac{\coth^{-1}(ax)^2}{2a^2}+\frac{1}{2}x^2\coth^{-1}(ax)^2+\frac{x\coth^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[a*x]^2,x]
```

```
[Out] (x*ArcCoth[a*x])/a - ArcCoth[a*x]^2/(2*a^2) + (x^2*ArcCoth[a*x]^2)/2 + Log[
1 - a^2*x^2]/(2*a^2)
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5981

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int x\coth^{-1}(ax)^2\,dx &= \frac{1}{2}x^2\coth^{-1}(ax)^2 - a\int\frac{x^2\coth^{-1}(ax)}{1-a^2x^2}\,dx\\
&= \frac{1}{2}x^2\coth^{-1}(ax)^2 + \frac{\int\coth^{-1}(ax)\,dx}{a} - \frac{\int\frac{\coth^{-1}(ax)}{1-a^2x^2}\,dx}{a}\\
&= \frac{x\coth^{-1}(ax)}{a} - \frac{\coth^{-1}(ax)^2}{2a^2} + \frac{1}{2}x^2\coth^{-1}(ax)^2 - \int\frac{x}{1-a^2x^2}\,dx\\
&= \frac{x\coth^{-1}(ax)}{a} - \frac{\coth^{-1}(ax)^2}{2a^2} + \frac{1}{2}x^2\coth^{-1}(ax)^2 + \frac{\log\left(1-a^2x^2\right)}{2a^2}
\end{aligned}
$$

Mathematica [A] time = 0.012074, size = 43, normalized size = 0.8

$$
\frac{\log\left(1-a^2x^2\right)+\left(a^2x^2-1\right)\coth^{-1}(ax)^2+2ax\coth^{-1}(ax)}{2a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[a*x]^2,x]
```

```
[Out] (2*a*x*ArcCoth[a*x] + (-1 + a^2*x^2)*ArcCoth[a*x]^2 + Log[1 - a^2*x^2])/(2*
a^2)
```

Maple [B] time = 0.056, size = 155, normalized size = 2.9

$$
\frac{x^2\left(\operatorname{arccoth}(ax)\right)^2}{2}+\frac{x\operatorname{arccoth}(ax)}{a}+\frac{\operatorname{arccoth}(ax)\ln(ax-1)}{2a^2}-\frac{\operatorname{arccoth}(ax)\ln(ax+1)}{2a^2}+\frac{\left(\ln(ax-1)\right)^2}{8a^2}-\frac{\ln(ax-1)}{4a^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(a*x)^2,x)
```

```
[Out] 1/2*x^2*arccoth(a*x)^2+x*arccoth(a*x)/a+1/2/a^2*arccoth(a*x)*ln(a*x-1)-1/2/
a^2*arccoth(a*x)*ln(a*x+1)+1/8/a^2*ln(a*x-1)^2-1/4/a^2*ln(a*x-1)*ln(1/2+1/2
*a*x)+1/2/a^2*ln(a*x-1)+1/2/a^2*ln(a*x+1)-1/4/a^2*ln(-1/2*a*x+1/2)*ln(a*x+1
)+1/4/a^2*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+1/8/a^2*ln(a*x+1)^2
```

Maxima [B] time = 0.986487, size = 131, normalized size = 2.43

$$
\frac{1}{2}x^2\operatorname{arcoth}(ax)^2+\frac{1}{2}a\left(\frac{2x}{a^2}-\frac{\log(ax+1)}{a^3}+\frac{\log(ax-1)}{a^3}\right)\operatorname{arcoth}(ax)-\frac{2\left(\log(ax-1)-2\right)\log(ax+1)-\log(ax+1)^2}{8a^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccoth(a*x)^2 + 1/2*a*(2*x/a^2 - log(a*x + 1)/a^3 + log(a*x - 1)/a
^3)*arccoth(a*x) - 1/8*(2*(log(a*x - 1) - 2)*log(a*x + 1) - log(a*x + 1)^2
- log(a*x - 1)^2 - 4*log(a*x - 1))/a^2
```

Fricas [A] time = 1.50581, size = 143, normalized size = 2.65

$$\frac{4\,ax\log\left(\frac{ax+1}{ax-1}\right)+\left(a^{2}x^{2}-1\right)\log\left(\frac{ax+1}{ax-1}\right)^{2}+4\,\log\left(a^{2}x^{2}-1\right)}{8\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*a*x*log((a*x + 1)/(a*x - 1)) + (a^2*x^2 - 1)*log((a*x + 1)/(a*x - 1)
)^2 + 4*log(a^2*x^2 - 1))/a^2
```

Sympy [A] time = 4.52711, size = 60, normalized size = 1.11

$$\begin{cases} \frac{x^{2}\operatorname{acoth}^{2}(ax)}{2}+\frac{x\operatorname{acoth}(ax)}{a}+\frac{\log(ax+1)}{a^{2}}-\frac{\operatorname{acoth}^{2}(ax)}{2a^{2}}-\frac{\operatorname{acoth}(ax)}{a^{2}} & \text{for } a\neq 0 \\ -\frac{\pi^{2}x^{2}}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(a*x)**2,x)
```

```
[Out] Piecewise((x**2*acoth(a*x)**2/2 + x*acoth(a*x)/a + log(a*x + 1)/a**2 - acot
h(a*x)**2/(2*a**2) - acoth(a*x)/a**2, Ne(a, 0)), (-pi**2*x**2/8, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\operatorname{arcoth}(ax)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(a*x)^2, x)
```

3.17 $\int \coth^{-1}(ax)^2\,dx$

Optimal. Leaf size=58

$$-\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a}+x\coth^{-1}(ax)^2+\frac{\coth^{-1}(ax)^2}{a}-\frac{2\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a}$$

```
[Out] ArcCoth[a*x]^2/a + x*ArcCoth[a*x]^2 - (2*ArcCoth[a*x]*Log[2/(1 - a*x)])/a -
 PolyLog[2, 1 - 2/(1 - a*x)]/a
```

Rubi [A] time = 0.0777673, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5911, 5985, 5919, 2402, 2315}

$$-\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a}+x\coth^{-1}(ax)^2+\frac{\coth^{-1}(ax)^2}{a}-\frac{2\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]^2,x]
```

```
[Out] ArcCoth[a*x]^2/a + x*ArcCoth[a*x]^2 - (2*ArcCoth[a*x]*Log[2/(1 - a*x)])/a -
 PolyLog[2, 1 - 2/(1 - a*x)]/a
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5985

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \coth^{-1}(ax)^2\,dx &= x\coth^{-1}(ax)^2-(2a)\int \frac{x\coth^{-1}(ax)}{1-a^2x^2}\,dx\\
&= \frac{\coth^{-1}(ax)^2}{a}+x\coth^{-1}(ax)^2-2\int \frac{\coth^{-1}(ax)}{1-ax}\,dx\\
&= \frac{\coth^{-1}(ax)^2}{a}+x\coth^{-1}(ax)^2-\frac{2\coth^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a}+2\int \frac{\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx\\
&= \frac{\coth^{-1}(ax)^2}{a}+x\coth^{-1}(ax)^2-\frac{2\coth^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a}-\frac{2\,\mathrm{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1-ax}\right)}{a}\\
&= \frac{\coth^{-1}(ax)^2}{a}+x\coth^{-1}(ax)^2-\frac{2\coth^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a}-\frac{\mathrm{Li}_2\left(1-\frac{2}{1-ax}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0836344, size = 46, normalized size = 0.79

$$\frac{\mathrm{PolyLog}\left(2,e^{-2\coth^{-1}(ax)}\right)+\coth^{-1}(ax)\left((ax-1)\coth^{-1}(ax)-2\log\left(1-e^{-2\coth^{-1}(ax)}\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[a*x]^2,x]
```

```
[Out] (ArcCoth[a*x]*((-1 + a*x)*ArcCoth[a*x] - 2*Log[1 - E^(-2*ArcCoth[a*x])]) +
PolyLog[2, E^(-2*ArcCoth[a*x])])/a
```

Maple [B] time = 0.119, size = 122, normalized size = 2.1

$$x\left(\mathrm{arccoth}\left(ax\right)\right)^2-2\,\frac{\mathrm{arccoth}\left(ax\right)}{a}\ln\left(1+\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)-2\,\frac{\mathrm{arccoth}\left(ax\right)}{a}\ln\left(1-\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)+\frac{\left(\mathrm{arccoth}\left(ax\right)\right)^2}{a}-2\,\frac{1}{a}\mathrm{polylog}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)^2,x)
```

```
[Out] x*arccoth(a*x)^2-2/a*arccoth(a*x)*ln(1+1/((a*x-1)/(a*x+1))^(1/2))-2/a*arcco
th(a*x)*ln(1-1/((a*x-1)/(a*x+1))^(1/2))+arccoth(a*x)^2/a-2/a*polylog(2,-1/(
(a*x-1)/(a*x+1))^(1/2))-2/a*polylog(2,1/((a*x-1)/(a*x+1))^(1/2))
```

Maxima [B] time = 0.986916, size = 182, normalized size = 3.14

$$x\,\mathrm{arcoth}\left(ax\right)^2+\frac{1}{4}\left(a\left(\frac{\log\left(ax+1\right)^2+2\,\log\left(ax+1\right)\log\left(ax-1\right)-\log\left(ax-1\right)^2}{a^3}-\frac{4\left(\log\left(ax-1\right)\log\left(\frac{1}{2}\,ax+\frac{1}{2}\right)+\mathrm{Li}_2\right.}{a^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2,x, algorithm="maxima")
```

```
[Out] x*arccoth(a*x)^2 + 1/4*(a*((log(a*x + 1)^2 + 2*log(a*x + 1)*log(a*x - 1) -
log(a*x - 1)^2)/a^3 - 4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x +
 1/2))/a^3) - 2*(log(a*x + 1)/a - log(a*x - 1)/a)*log(a^2*x^2 - 1)/a)*a + a
rccoth(a*x)*log(a^2*x^2 - 1)/a
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{arcoth}\,(ax)^2\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \text{acoth}^2\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)**2,x)
```

```
[Out] Integral(acoth(a*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{arcoth}\,(ax)^2\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)^2, x)
```

3.18 $\int \frac{\coth^{-1}(ax)^2}{x} dx$

Optimal. Leaf size=97

$$\frac{1}{2}\text{PolyLog}\left(3,1-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(3,1-\frac{2ax}{ax+1}\right)+\coth^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{ax+1}\right)-\coth^{-1}(ax)\text{PolyLog}$$

```
[Out] 2*ArcCoth[a*x]^2*ArcCoth[1 - 2/(1 - a*x)] + ArcCoth[a*x]*PolyLog[2, 1 - 2/(
1 + a*x)] - ArcCoth[a*x]*PolyLog[2, 1 - (2*a*x)/(1 + a*x)] + PolyLog[3, 1 -
 2/(1 + a*x)]/2 - PolyLog[3, 1 - (2*a*x)/(1 + a*x)]/2
```

Rubi [A] time = 0.232009, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5915, 6053, 5949, 6057, 6610}

$$\frac{1}{2}\text{PolyLog}\left(3,1-\frac{2}{ax+1}\right)-\frac{1}{2}\text{PolyLog}\left(3,1-\frac{2ax}{ax+1}\right)+\coth^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{ax+1}\right)-\coth^{-1}(ax)\text{PolyLog}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]^2/x,x]
```

```
[Out] 2*ArcCoth[a*x]^2*ArcCoth[1 - 2/(1 - a*x)] + ArcCoth[a*x]*PolyLog[2, 1 - 2/(
1 + a*x)] - ArcCoth[a*x]*PolyLog[2, 1 - (2*a*x)/(1 + a*x)] + PolyLog[3, 1 -
 2/(1 + a*x)]/2 - PolyLog[3, 1 - (2*a*x)/(1 + a*x)]/2
```

Rule 5915

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcCoth[c*x])^p*ArcCoth[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcCoth[c*x])^(p - 1)*ArcCoth[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6053

```
Int[(ArcCoth[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[SimplifyIntegrand[1 + 1/u, x]]*(a +
 b*ArcCoth[c*x])^p)/(d + e*x^2), x], x] - Dist[1/2, Int[(Log[SimplifyIntegr
and[1 - 1/u, x]]*(a + b*ArcCoth[c*x])^p)/(d + e*x^2), x], x] /; FreeQ[{a, b
, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 -
c*x))^2, 0]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6057

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcCoth[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\coth^{-1}(ax)^2}{x}\,dx = 2\coth^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1-ax}\right)-(4a)\int \frac{\coth^{-1}(ax)\coth^{-1}\left(1-\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx$$

$$= 2\coth^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1-ax}\right)+(2a)\int \frac{\coth^{-1}(ax)\log\left(\frac{2}{1+ax}\right)}{1-a^2x^2}\,dx-(2a)\int \frac{\coth^{-1}(ax)\log\left(\frac{2a}{1+a}\right.}{1-a^2x^2}$$

$$= 2\coth^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1-ax}\right)+\coth^{-1}(ax)\mathrm{Li}_2\left(1-\frac{2}{1+ax}\right)-\coth^{-1}(ax)\mathrm{Li}_2\left(1-\frac{2ax}{1+ax}\right)-a$$

$$= 2\coth^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1-ax}\right)+\coth^{-1}(ax)\mathrm{Li}_2\left(1-\frac{2}{1+ax}\right)-\coth^{-1}(ax)\mathrm{Li}_2\left(1-\frac{2ax}{1+ax}\right)+\frac{1}{2}$$

Mathematica [A] time = 0.0634424, size = 114, normalized size = 1.18

$$-\coth^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{-2\coth^{-1}(ax)}\right)-\coth^{-1}(ax)\mathrm{PolyLog}\left(2,e^{2\coth^{-1}(ax)}\right)-\frac{1}{2}\mathrm{PolyLog}\left(3,-e^{-2\coth^{-1}(ax)}\right)+\frac{1}{2}\mathrm{Poly}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[a*x]^2/x,x]
```

```
[Out] (2*ArcCoth[a*x]^3)/3 + ArcCoth[a*x]^2*Log[1 + E^(-2*ArcCoth[a*x])] - ArcCot
h[a*x]^2*Log[1 - E^(2*ArcCoth[a*x])] - ArcCoth[a*x]*PolyLog[2, -E^(-2*ArcCo
th[a*x])] - ArcCoth[a*x]*PolyLog[2, E^(2*ArcCoth[a*x])] - PolyLog[3, -E^(-2
*ArcCoth[a*x])]/2 + PolyLog[3, E^(2*ArcCoth[a*x])]/2
```

Maple [C] time = 0.381, size = 487, normalized size = 5.

$$\ln(ax)(\mathrm{arccoth}(ax))^2+\frac{i}{2}\pi\,\mathrm{csgn}\left(i\left(\frac{ax+1}{ax-1}+1\right)\right)\mathrm{csgn}\left(i\left(\frac{ax+1}{ax-1}-1\right)^{-1}\right)\mathrm{csgn}\left(i\left(\frac{ax+1}{ax-1}+1\right)\left(\frac{ax+1}{ax-1}-1\right)^{-1}\right)(\mathrm{arccoth}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)^2/x,x)
```

```
[Out] ln(a*x)*arccoth(a*x)^2+1/2*I*Pi*csgn(I*((a*x+1)/(a*x-1)+1))*csgn(I/((a*x+1)
/(a*x-1)-1))*csgn(I/((a*x+1)/(a*x-1)-1)*((a*x+1)/(a*x-1)+1))*arccoth(a*x)^2
-1/2*I*Pi*csgn(I*((a*x+1)/(a*x-1)+1))*csgn(I/((a*x+1)/(a*x-1)-1)*((a*x+1)/(
a*x-1)+1))^2*arccoth(a*x)^2-1/2*I*Pi*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I/((a
*x+1)/(a*x-1)-1)*((a*x+1)/(a*x-1)+1))^2*arccoth(a*x)^2+1/2*I*Pi*csgn(I/((a*
x+1)/(a*x-1)-1)*((a*x+1)/(a*x-1)+1))^3*arccoth(a*x)^2+arccoth(a*x)^2*ln((a*
x+1)/(a*x-1)-1)-arccoth(a*x)^2*ln(1-1/((a*x-1)/(a*x+1))^(1/2))-2*arccoth(a*
x)*polylog(2,1/((a*x-1)/(a*x+1))^(1/2))+2*polylog(3,1/((a*x-1)/(a*x+1))^(1/
2))-arccoth(a*x)^2*ln(1+1/((a*x-1)/(a*x+1))^(1/2))-2*arccoth(a*x)*polylog(2
,-1/((a*x-1)/(a*x+1))^(1/2))+2*polylog(3,-1/((a*x-1)/(a*x+1))^(1/2))+arccot
h(a*x)*polylog(2,-(a*x+1)/(a*x-1))-1/2*polylog(3,-(a*x+1)/(a*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(ax)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(arccoth(a*x)^2/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arcoth}(ax)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)^2/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^2(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)**2/x,x)
```

```
[Out] Integral(acoth(a*x)**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(ax)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)^2/x, x)
```

3.19 $\int \frac{\coth^{-1}(ax)^2}{x^2} dx$

Optimal. Leaf size=55

$$-a\text{PolyLog}\left(2, \frac{2}{ax+1}-1\right)+a\coth^{-1}(ax)^2-\frac{\coth^{-1}(ax)^2}{x}+2a\log\left(2-\frac{2}{ax+1}\right)\coth^{-1}(ax)$$

```
[Out] a*ArcCoth[a*x]^2 - ArcCoth[a*x]^2/x + 2*a*ArcCoth[a*x]*Log[2 - 2/(1 + a*x)]
 - a*PolyLog[2, -1 + 2/(1 + a*x)]
```

Rubi [A] time = 0.108329, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5917, 5989, 5933, 2447}

$$-a\text{PolyLog}\left(2, \frac{2}{ax+1}-1\right)+a\coth^{-1}(ax)^2-\frac{\coth^{-1}(ax)^2}{x}+2a\log\left(2-\frac{2}{ax+1}\right)\coth^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]^2/x^2,x]
```

```
[Out] a*ArcCoth[a*x]^2 - ArcCoth[a*x]^2/x + 2*a*ArcCoth[a*x]*Log[2 - 2/(1 + a*x)]
 - a*PolyLog[2, -1 + 2/(1 + a*x)]
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5989

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcCoth[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5933

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcCoth[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\coth^{-1}(ax)^2}{x^2}\,dx = -\frac{\coth^{-1}(ax)^2}{x} + (2a)\int \frac{\coth^{-1}(ax)}{x\left(1-a^2x^2\right)}\,dx$$
$$= a\coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{x} + (2a)\int \frac{\coth^{-1}(ax)}{x(1+ax)}\,dx$$
$$= a\coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{x} + 2a\coth^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right) - \left(2a^2\right)\int \frac{\log\left(2-\frac{2}{1+ax}\right)}{1-a^2x^2}\,dx$$
$$= a\coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{x} + 2a\coth^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right) - a\text{Li}_2\left(-1+\frac{2}{1+ax}\right)$$

Mathematica [A] time = 0.100025, size = 49, normalized size = 0.89

$$-a\text{PolyLog}\left(2,-e^{-2\coth^{-1}(ax)}\right) + \frac{(ax-1)\coth^{-1}(ax)^2}{x} + 2a\coth^{-1}(ax)\log\left(e^{-2\coth^{-1}(ax)}+1\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[a*x]^2/x^2,x]
```

```
[Out] ((-1 + a*x)*ArcCoth[a*x]^2)/x + 2*a*ArcCoth[a*x]*Log[1 + E^(-2*ArcCoth[a*x]
)] - a*PolyLog[2, -E^(-2*ArcCoth[a*x])]
```

Maple [B] time = 0.056, size = 159, normalized size = 2.9

$$-\frac{(\text{arccoth}\,(ax))^2}{x} - a\text{arccoth}\,(ax)\ln(ax-1) + 2\,a\text{arccoth}\,(ax)\ln(ax) - a\text{arccoth}\,(ax)\ln(ax+1) - \frac{a\,(\ln(ax-1))^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)^2/x^2,x)
```

```
[Out] -arccoth(a*x)^2/x-a*arccoth(a*x)*ln(a*x-1)+2*a*arccoth(a*x)*ln(a*x)-a*arcco
th(a*x)*ln(a*x+1)-1/4*a*ln(a*x-1)^2+a*dilog(1/2+1/2*a*x)+1/2*a*ln(a*x-1)*ln
(1/2+1/2*a*x)+1/2*a*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)-1/2*a*ln(-1/2*a*x+1/2)
*ln(a*x+1)+1/4*a*ln(a*x+1)^2-a*dilog(a*x)-a*dilog(a*x+1)-a*ln(a*x)*ln(a*x+1
)
```

Maxima [B] time = 0.981925, size = 197, normalized size = 3.58

$$\frac{1}{4}a^2\left(\frac{\log(ax+1)^2 - 2\,\log(ax+1)\log(ax-1) - \log(ax-1)^2}{a} + \frac{4\left(\log(ax-1)\log\left(\frac{1}{2}\,ax+\frac{1}{2}\right) + \text{Li}_2\left(-\frac{1}{2}\,ax+\frac{1}{2}\right)\right)}{a} - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*((log(a*x + 1)^2 - 2*log(a*x + 1)*log(a*x - 1) - log(a*x - 1)^2)/a
 + 4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a - 4*(log(a*
```

```
x + 1)*log(x) + dilog(-a*x))/a + 4*(log(-a*x + 1)*log(x) + dilog(a*x))/a) -
 a*(log(a^2*x^2 - 1) - log(x^2))*arccoth(a*x) - arccoth(a*x)^2/x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arcoth}\left(ax\right)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)^2/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^2\left(ax\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)**2/x**2,x)
```

```
[Out] Integral(acoth(a*x)**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}\left(ax\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)^2/x^2, x)
```

3.20 $\int \frac{\coth^{-1}(ax)^2}{x^3}\,dx$

Optimal. Leaf size=61

$$-\frac{1}{2}a^2\log\left(1-a^2x^2\right)+a^2\log(x)+\frac{1}{2}a^2\coth^{-1}(ax)^2-\frac{\coth^{-1}(ax)^2}{2x^2}-\frac{a\coth^{-1}(ax)}{x}$$

```
[Out] -((a*ArcCoth[a*x])/x) + (a^2*ArcCoth[a*x]^2)/2 - ArcCoth[a*x]^2/(2*x^2) + a
^2*Log[x] - (a^2*Log[1 - a^2*x^2])/2
```

Rubi [A] time = 0.0985716, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5917, 5983, 266, 36, 29, 31, 5949}

$$-\frac{1}{2}a^2\log\left(1-a^2x^2\right)+a^2\log(x)+\frac{1}{2}a^2\coth^{-1}(ax)^2-\frac{\coth^{-1}(ax)^2}{2x^2}-\frac{a\coth^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]^2/x^3,x]
```

```
[Out] -((a*ArcCoth[a*x])/x) + (a^2*ArcCoth[a*x]^2)/2 - ArcCoth[a*x]^2/(2*x^2) + a
^2*Log[x] - (a^2*Log[1 - a^2*x^2])/2
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5983

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcCoth[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)^2}{x^3}\,dx &= -\frac{\coth^{-1}(ax)^2}{2x^2} + a\int \frac{\coth^{-1}(ax)}{x^2\left(1-a^2x^2\right)}\,dx \\
&= -\frac{\coth^{-1}(ax)^2}{2x^2} + a\int \frac{\coth^{-1}(ax)}{x^2}\,dx + a^3\int \frac{\coth^{-1}(ax)}{1-a^2x^2}\,dx \\
&= -\frac{a\coth^{-1}(ax)}{x} + \frac{1}{2}a^2\coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{2x^2} + a^2\int \frac{1}{x\left(1-a^2x^2\right)}\,dx \\
&= -\frac{a\coth^{-1}(ax)}{x} + \frac{1}{2}a^2\coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{x\left(1-a^2x\right)}\,dx, x, x^2\right) \\
&= -\frac{a\coth^{-1}(ax)}{x} + \frac{1}{2}a^2\coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) + \frac{1}{2}a^4\,\text{Subst}\left(\int \frac{}{1-}\right. \\
&= -\frac{a\coth^{-1}(ax)}{x} + \frac{1}{2}a^2\coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{2x^2} + a^2\log(x) - \frac{1}{2}a^2\log\left(1-a^2x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0170421, size = 57, normalized size = 0.93

$$-\frac{1}{2}a^2\log\left(1-a^2x^2\right) + \frac{\left(a^2x^2-1\right)\coth^{-1}(ax)^2}{2x^2} + a^2\log(x) - \frac{a\coth^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]^2/x^3,x]
```

```
[Out] -((a*ArcCoth[a*x])/x) + ((-1 + a^2*x^2)*ArcCoth[a*x]^2)/(2*x^2) + a^2*Log[x
] - (a^2*Log[1 - a^2*x^2])/2
```

Maple [B] time = 0.055, size = 164, normalized size = 2.7

$$-\frac{(\text{arccoth}\,(ax))^2}{2\,x^2} - \frac{a\text{arccoth}\,(ax)}{x} - \frac{a^2\text{arccoth}\,(ax)\ln(ax-1)}{2} + \frac{a^2\text{arccoth}\,(ax)\ln(ax+1)}{2} - \frac{a^2\left(\ln(ax-1)\right)^2}{8} + \frac{a^2\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)^2/x^3,x)
```

```
[Out] -1/2*arccoth(a*x)^2/x^2-a*arccoth(a*x)/x-1/2*a^2*arccoth(a*x)*ln(a*x-1)+1/2
*a^2*arccoth(a*x)*ln(a*x+1)-1/8*a^2*ln(a*x-1)^2+1/4*a^2*ln(a*x-1)*ln(1/2+1/
2*a*x)-1/2*a^2*ln(a*x-1)+a^2*ln(a*x)-1/2*a^2*ln(a*x+1)+1/4*a^2*ln(-1/2*a*x+
1/2)*ln(a*x+1)-1/4*a^2*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)-1/8*a^2*ln(a*x+1)^2
```

Maxima [A] time = 0.971504, size = 130, normalized size = 2.13

$$\frac{1}{8}\left(2\left(\log\left(ax-1\right)-2\right)\log\left(ax+1\right)-\log\left(ax+1\right)^2-\log\left(ax-1\right)^2-4\,\log\left(ax-1\right)+8\,\log\left(x\right)\right)a^2+\frac{1}{2}\left(a\log\left(ax+1\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*(2*(log(a*x - 1) - 2)*log(a*x + 1) - log(a*x + 1)^2 - log(a*x - 1)^2 -
4*log(a*x - 1) + 8*log(x))*a^2 + 1/2*(a*log(a*x + 1) - a*log(a*x - 1) - 2/x
)*a*arccoth(a*x) - 1/2*arccoth(a*x)^2/x^2
```

Fricas [A] time = 1.89944, size = 181, normalized size = 2.97

$$-\frac{4\,a^2x^2\log\left(a^2x^2-1\right)-8\,a^2x^2\log\left(x\right)+4\,ax\log\left(\frac{ax+1}{ax-1}\right)-\left(a^2x^2-1\right)\log\left(\frac{ax+1}{ax-1}\right)^2}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(4*a^2*x^2*log(a^2*x^2 - 1) - 8*a^2*x^2*log(x) + 4*a*x*log((a*x + 1)/(
a*x - 1)) - (a^2*x^2 - 1)*log((a*x + 1)/(a*x - 1))^2)/x^2
```

Sympy [A] time = 1.0102, size = 56, normalized size = 0.92

$$a^2\log\left(x\right)-a^2\log\left(ax+1\right)+\frac{a^2\operatorname{acoth}^2\left(ax\right)}{2}+a^2\operatorname{acoth}\left(ax\right)-\frac{a\operatorname{acoth}\left(ax\right)}{x}-\frac{\operatorname{acoth}^2\left(ax\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)**2/x**3,x)
```

```
[Out] a**2*log(x) - a**2*log(a*x + 1) + a**2*acoth(a*x)**2/2 + a**2*acoth(a*x) -
a*acoth(a*x)/x - acoth(a*x)**2/(2*x**2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{arcoth}\left(ax\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)^2/x^3, x)
```

3.21 $\int \frac{\coth^{-1}(ax)^2}{x^4} \, dx$

Optimal. Leaf size=103

$$-\frac{1}{3}a^3\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{a^2}{3x}+\frac{1}{3}a^3\tanh^{-1}(ax)+\frac{1}{3}a^3\coth^{-1}(ax)^2+\frac{2}{3}a^3\log\left(2-\frac{2}{ax+1}\right)\coth^{-1}(ax)-\frac{a\coth^{-1}}{3x^2}$$

```
[Out] -a^2/(3*x) - (a*ArcCoth[a*x])/(3*x^2) + (a^3*ArcCoth[a*x]^2)/3 - ArcCoth[a*
x]^2/(3*x^3) + (a^3*ArcTanh[a*x])/3 + (2*a^3*ArcCoth[a*x]*Log[2 - 2/(1 + a*
x)])/3 - (a^3*PolyLog[2, -1 + 2/(1 + a*x)])/3
```

Rubi [A] time = 0.170784, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5917, 5983, 325, 206, 5989, 5933, 2447}

$$-\frac{1}{3}a^3\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{a^2}{3x}+\frac{1}{3}a^3\tanh^{-1}(ax)+\frac{1}{3}a^3\coth^{-1}(ax)^2+\frac{2}{3}a^3\log\left(2-\frac{2}{ax+1}\right)\coth^{-1}(ax)-\frac{a\coth^{-1}}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]^2/x^4,x]
```

```
[Out] -a^2/(3*x) - (a*ArcCoth[a*x])/(3*x^2) + (a^3*ArcCoth[a*x]^2)/3 - ArcCoth[a*
x]^2/(3*x^3) + (a^3*ArcTanh[a*x])/3 + (2*a^3*ArcCoth[a*x]*Log[2 - 2/(1 + a*
x)])/3 - (a^3*PolyLog[2, -1 + 2/(1 + a*x)])/3
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5983

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcCoth[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5989

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcCoth[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5933

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcCoth[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)^2}{x^4}\,dx &= -\frac{\coth^{-1}(ax)^2}{3x^3}+\frac{1}{3}(2a)\int \frac{\coth^{-1}(ax)}{x^3\left(1-a^2x^2\right)}\,dx\\
&= -\frac{\coth^{-1}(ax)^2}{3x^3}+\frac{1}{3}(2a)\int \frac{\coth^{-1}(ax)}{x^3}\,dx+\frac{1}{3}\left(2a^3\right)\int \frac{\coth^{-1}(ax)}{x\left(1-a^2x^2\right)}\,dx\\
&= -\frac{a\coth^{-1}(ax)}{3x^2}+\frac{1}{3}a^3\coth^{-1}(ax)^2-\frac{\coth^{-1}(ax)^2}{3x^3}+\frac{1}{3}a^2\int \frac{1}{x^2\left(1-a^2x^2\right)}\,dx+\frac{1}{3}\left(2a^3\right)\int \frac{\coth^{-1}}{x(1+}\\
&= -\frac{a^2}{3x}-\frac{a\coth^{-1}(ax)}{3x^2}+\frac{1}{3}a^3\coth^{-1}(ax)^2-\frac{\coth^{-1}(ax)^2}{3x^3}+\frac{2}{3}a^3\coth^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)+\frac{1}{3}a\\
&= -\frac{a^2}{3x}-\frac{a\coth^{-1}(ax)}{3x^2}+\frac{1}{3}a^3\coth^{-1}(ax)^2-\frac{\coth^{-1}(ax)^2}{3x^3}+\frac{1}{3}a^3\tanh^{-1}(ax)+\frac{2}{3}a^3\coth^{-1}(ax)\log
\end{aligned}$$

Mathematica [A] time = 0.246084, size = 87, normalized size = 0.84

$$\frac{-a^3x^3\text{PolyLog}\left(2,-e^{-2\coth^{-1}(ax)}\right)-a^2x^2+\left(a^3x^3-1\right)\coth^{-1}(ax)^2+ax\coth^{-1}(ax)\left(a^2x^2+2a^2x^2\log\left(e^{-2\coth^{-1}(ax)}+\right.\right.}{3x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[a*x]^2/x^4,x]
```

```
[Out] (-(a^2*x^2) + (-1 + a^3*x^3)*ArcCoth[a*x]^2 + a*x*ArcCoth[a*x]*(-1 + a^2*x^
2 + 2*a^2*x^2*Log[1 + E^(-2*ArcCoth[a*x])]) - a^3*x^3*PolyLog[2, -E^(-2*Arc
Coth[a*x])])/(3*x^3)
```

Maple [B] time = 0.06, size = 224, normalized size = 2.2

$$-\frac{(\operatorname{arccoth}(ax))^2}{3x^3}-\frac{a^3\operatorname{arccoth}(ax)\ln(ax-1)}{3}-\frac{a\operatorname{arccoth}(ax)}{3x^2}+\frac{2a^3\operatorname{arccoth}(ax)\ln(ax)}{3}-\frac{a^3\operatorname{arccoth}(ax)\ln(ax+}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)^2/x^4,x)
```

```
[Out] -1/3*arccoth(a*x)^2/x^3-1/3*a^3*arccoth(a*x)*ln(a*x-1)-1/3*a*arccoth(a*x)/x
^2+2/3*a^3*arccoth(a*x)*ln(a*x)-1/3*a^3*arccoth(a*x)*ln(a*x+1)-1/3*a^2/x-1/
6*a^3*ln(a*x-1)+1/6*a^3*ln(a*x+1)-1/12*a^3*ln(a*x-1)^2+1/3*a^3*dilog(1/2+1/
2*a*x)+1/6*a^3*ln(a*x-1)*ln(1/2+1/2*a*x)+1/6*a^3*ln(-1/2*a*x+1/2)*ln(1/2+1/
2*a*x)-1/6*a^3*ln(-1/2*a*x+1/2)*ln(a*x+1)+1/12*a^3*ln(a*x+1)^2-1/3*a^3*dilo
g(a*x)-1/3*a^3*dilog(a*x+1)-1/3*a^3*ln(a*x)*ln(a*x+1)
```

Maxima [A] time = 0.989151, size = 238, normalized size = 2.31

$$\frac{1}{12}\left(4\left(\log\left(ax-1\right)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)a-4\left(\log\left(ax+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-ax\right)\right)a+4\left(\log\left(-ax+1\right)\log\left(x\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))*a - 4*(lo
g(a*x + 1)*log(x) + dilog(-a*x))*a + 4*(log(-a*x + 1)*log(x) + dilog(a*x))*
a + 2*a*log(a*x + 1) - 2*a*log(a*x - 1) + (a*x*log(a*x + 1)^2 - 2*a*x*log(a
*x + 1)*log(a*x - 1) - a*x*log(a*x - 1)^2 - 4)/x)*a^2 - 1/3*(a^2*log(a^2*x^
2 - 1) - a^2*log(x^2) + 1/x^2)*a*arccoth(a*x) - 1/3*arccoth(a*x)^2/x^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{arcoth}\left(ax\right)^2}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)^2/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{acoth}^2\left(ax\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)**2/x**4,x)
```

```
[Out] Integral(acoth(a*x)**2/x**4, x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(ax)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)^2/x^4, x)
```

3.22 $\int \frac{\coth^{-1}(ax)^2}{x^5} dx$

Optimal. Leaf size=90

$$-\frac{a^2}{12x^2} - \frac{1}{3}a^4 \log\left(1-a^2x^2\right) + \frac{2}{3}a^4 \log(x) + \frac{1}{4}a^4 \coth^{-1}(ax)^2 - \frac{a^3 \coth^{-1}(ax)}{2x} - \frac{a \coth^{-1}(ax)}{6x^3} - \frac{\coth^{-1}(ax)^2}{4x^4}$$

```
[Out] -a^2/(12*x^2) - (a*ArcCoth[a*x])/(6*x^3) - (a^3*ArcCoth[a*x])/(2*x) + (a^4*
ArcCoth[a*x]^2)/4 - ArcCoth[a*x]^2/(4*x^4) + (2*a^4*Log[x])/3 - (a^4*Log[1
- a^2*x^2])/3
```

Rubi [A] time = 0.17234, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.8, Rules used = {5917, 5983, 266, 44, 36, 29, 31, 5949}

$$-\frac{a^2}{12x^2} - \frac{1}{3}a^4 \log\left(1-a^2x^2\right) + \frac{2}{3}a^4 \log(x) + \frac{1}{4}a^4 \coth^{-1}(ax)^2 - \frac{a^3 \coth^{-1}(ax)}{2x} - \frac{a \coth^{-1}(ax)}{6x^3} - \frac{\coth^{-1}(ax)^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]^2/x^5,x]
```

```
[Out] -a^2/(12*x^2) - (a*ArcCoth[a*x])/(6*x^3) - (a^3*ArcCoth[a*x])/(2*x) + (a^4*
ArcCoth[a*x]^2)/4 - ArcCoth[a*x]^2/(4*x^4) + (2*a^4*Log[x])/3 - (a^4*Log[1
- a^2*x^2])/3
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5983

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcCoth[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)^2}{x^5}\,dx &= -\frac{\coth^{-1}(ax)^2}{4x^4}+\frac{1}{2}a\int \frac{\coth^{-1}(ax)}{x^4\left(1-a^2x^2\right)}\,dx\\
&= -\frac{\coth^{-1}(ax)^2}{4x^4}+\frac{1}{2}a\int \frac{\coth^{-1}(ax)}{x^4}\,dx+\frac{1}{2}a^3\int \frac{\coth^{-1}(ax)}{x^2\left(1-a^2x^2\right)}\,dx\\
&= -\frac{a\coth^{-1}(ax)}{6x^3}-\frac{\coth^{-1}(ax)^2}{4x^4}+\frac{1}{6}a^2\int \frac{1}{x^3\left(1-a^2x^2\right)}\,dx+\frac{1}{2}a^3\int \frac{\coth^{-1}(ax)}{x^2}\,dx+\frac{1}{2}a^5\int \frac{\coth}{1-}\\
&= -\frac{a\coth^{-1}(ax)}{6x^3}-\frac{a^3\coth^{-1}(ax)}{2x}+\frac{1}{4}a^4\coth^{-1}(ax)^2-\frac{\coth^{-1}(ax)^2}{4x^4}+\frac{1}{12}a^2\,\mathrm{Subst}\left(\int \frac{1}{x^2\left(1-a^2x\right.}\right.\\
&= -\frac{a\coth^{-1}(ax)}{6x^3}-\frac{a^3\coth^{-1}(ax)}{2x}+\frac{1}{4}a^4\coth^{-1}(ax)^2-\frac{\coth^{-1}(ax)^2}{4x^4}+\frac{1}{12}a^2\,\mathrm{Subst}\left(\int\left(\frac{1}{x^2}+\frac{a^2}{x}\right.\right.\\
&= -\frac{a^2}{12x^2}-\frac{a\coth^{-1}(ax)}{6x^3}-\frac{a^3\coth^{-1}(ax)}{2x}+\frac{1}{4}a^4\coth^{-1}(ax)^2-\frac{\coth^{-1}(ax)^2}{4x^4}+\frac{1}{6}a^4\log(x)-\frac{1}{12}a^4\\
&= -\frac{a^2}{12x^2}-\frac{a\coth^{-1}(ax)}{6x^3}-\frac{a^3\coth^{-1}(ax)}{2x}+\frac{1}{4}a^4\coth^{-1}(ax)^2-\frac{\coth^{-1}(ax)^2}{4x^4}+\frac{2}{3}a^4\log(x)-\frac{1}{3}a^4\,\mathrm{lo}
\end{aligned}$$

Mathematica [A] time = 0.0216711, size = 82, normalized size = 0.91

$$-\frac{a^2}{12x^2}-\frac{1}{3}a^4\log\left(1-a^2x^2\right)-\frac{a\left(3a^2x^2+1\right)\coth^{-1}(ax)}{6x^3}+\frac{\left(a^4x^4-1\right)\coth^{-1}(ax)^2}{4x^4}+\frac{2}{3}a^4\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]^2/x^5,x]
```

```
[Out] -a^2/(12*x^2) - (a*(1 + 3*a^2*x^2)*ArcCoth[a*x])/(6*x^3) + ((-1 + a^4*x^4)*
ArcCoth[a*x]^2)/(4*x^4) + (2*a^4*Log[x])/3 - (a^4*Log[1 - a^2*x^2])/3
```

Maple [B] time = 0.06, size = 185, normalized size = 2.1

$$-\frac{(\mathrm{arccoth}\,(ax))^2}{4\,x^4}-\frac{a^4\mathrm{arccoth}\,(ax)\ln(ax-1)}{4}-\frac{a\,\mathrm{arccoth}\,(ax)}{6\,x^3}-\frac{a^3\mathrm{arccoth}\,(ax)}{2\,x}+\frac{a^4\mathrm{arccoth}\,(ax)\ln(ax+1)}{4}-\frac{a^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)^2/x^5,x)
```

```
[Out] -1/4*arccoth(a*x)^2/x^4-1/4*a^4*arccoth(a*x)*ln(a*x-1)-1/6*a*arccoth(a*x)/x
^3-1/2*a^3*arccoth(a*x)/x+1/4*a^4*arccoth(a*x)*ln(a*x+1)-1/16*a^4*ln(a*x-1)
^2+1/8*a^4*ln(a*x-1)*ln(1/2+1/2*a*x)-1/8*a^4*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*
x)+1/8*a^4*ln(-1/2*a*x+1/2)*ln(a*x+1)-1/16*a^4*ln(a*x+1)^2-1/3*a^4*ln(a*x-1
)-1/12*a^2/x^2+2/3*a^4*ln(a*x)-1/3*a^4*ln(a*x+1)
```

Maxima [B] time = 0.98254, size = 208, normalized size = 2.31

$$\frac{1}{48}\left(32\,a^2\log\left(x\right)-\frac{3\,a^2x^2\log\left(ax+1\right)^2+3\,a^2x^2\log\left(ax-1\right)^2+16\,a^2x^2\log\left(ax-1\right)-2\left(3\,a^2x^2\log\left(ax-1\right)-8\,a^2x^2\right)\log\left(\right.}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/48*(32*a^2*log(x) - (3*a^2*x^2*log(a*x + 1)^2 + 3*a^2*x^2*log(a*x - 1)^2
+ 16*a^2*x^2*log(a*x - 1) - 2*(3*a^2*x^2*log(a*x - 1) - 8*a^2*x^2)*log(a*x
+ 1) + 4)/x^2)*a^2 + 1/12*(3*a^3*log(a*x + 1) - 3*a^3*log(a*x - 1) - 2*(3*a
^2*x^2 + 1)/x^3)*a*arccoth(a*x) - 1/4*arccoth(a*x)^2/x^4
```

Fricas [A] time = 1.83772, size = 223, normalized size = 2.48

$$-\frac{16\,a^4x^4\log\left(a^2x^2-1\right)-32\,a^4x^4\log\left(x\right)+4\,a^2x^2-3\left(a^4x^4-1\right)\log\left(\frac{ax+1}{ax-1}\right)^2+4\left(3\,a^3x^3+ax\right)\log\left(\frac{ax+1}{ax-1}\right)}{48\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/48*(16*a^4*x^4*log(a^2*x^2 - 1) - 32*a^4*x^4*log(x) + 4*a^2*x^2 - 3*(a^4
*x^4 - 1)*log((a*x + 1)/(a*x - 1))^2 + 4*(3*a^3*x^3 + a*x)*log((a*x + 1)/(a
*x - 1)))/x^4
```

Sympy [A] time = 6.42276, size = 90, normalized size = 1.

$$\frac{2a^4\log\left(x\right)}{3}-\frac{2a^4\log\left(ax+1\right)}{3}+\frac{a^4\operatorname{acoth}^2\left(ax\right)}{4}+\frac{2a^4\operatorname{acoth}\left(ax\right)}{3}-\frac{a^3\operatorname{acoth}\left(ax\right)}{2x}-\frac{a^2}{12x^2}-\frac{a\operatorname{acoth}\left(ax\right)}{6x^3}-\frac{\operatorname{acoth}^2\left(ax\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)**2/x**5,x)
```

```
[Out] 2*a**4*log(x)/3 - 2*a**4*log(a*x + 1)/3 + a**4*acoth(a*x)**2/4 + 2*a**4*aco
th(a*x)/3 - a**3*acoth(a*x)/(2*x) - a**2/(12*x**2) - a*acoth(a*x)/(6*x**3)
- acoth(a*x)**2/(4*x**4)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(ax)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)^2/x^5, x)
```

3.23 $\int x^5 \coth^{-1}(ax)^3 \, dx$

Optimal. Leaf size=186

$$-\frac{23\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{30a^6}+\frac{x^3}{60a^3}+\frac{x^4\coth^{-1}(ax)}{20a^2}+\frac{x^3\coth^{-1}(ax)^2}{6a^3}+\frac{4x^2\coth^{-1}(ax)}{15a^4}+\frac{19x}{60a^5}-\frac{19\tanh^{-1}(ax)}{60a^6}+\frac{x\,\text{co}}{}$$

```
[Out] (19*x)/(60*a^5) + x^3/(60*a^3) + (4*x^2*ArcCoth[a*x])/(15*a^4) + (x^4*ArcCo
th[a*x])/(20*a^2) + (23*ArcCoth[a*x]^2)/(30*a^6) + (x*ArcCoth[a*x]^2)/(2*a^
5) + (x^3*ArcCoth[a*x]^2)/(6*a^3) + (x^5*ArcCoth[a*x]^2)/(10*a) - ArcCoth[a
*x]^3/(6*a^6) + (x^6*ArcCoth[a*x]^3)/6 - (19*ArcTanh[a*x])/(60*a^6) - (23*A
rcCoth[a*x]*Log[2/(1 - a*x)])/(15*a^6) - (23*PolyLog[2, 1 - 2/(1 - a*x)])/(
30*a^6)
```

Rubi [A] time = 0.717193, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 33, number of rules used = 11, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.1, Rules used = {5917, 5981, 302, 206, 321, 5985, 5919, 2402, 2315, 5911, 5949}

$$-\frac{23\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{30a^6}+\frac{x^3}{60a^3}+\frac{x^4\coth^{-1}(ax)}{20a^2}+\frac{x^3\coth^{-1}(ax)^2}{6a^3}+\frac{4x^2\coth^{-1}(ax)}{15a^4}+\frac{19x}{60a^5}-\frac{19\tanh^{-1}(ax)}{60a^6}+\frac{x\,\text{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*ArcCoth[a*x]^3,x]
```


```
[Out] (19*x)/(60*a^5) + x^3/(60*a^3) + (4*x^2*ArcCoth[a*x])/(15*a^4) + (x^4*ArcCo
th[a*x])/(20*a^2) + (23*ArcCoth[a*x]^2)/(30*a^6) + (x*ArcCoth[a*x]^2)/(2*a^
5) + (x^3*ArcCoth[a*x]^2)/(6*a^3) + (x^5*ArcCoth[a*x]^2)/(10*a) - ArcCoth[a
*x]^3/(6*a^6) + (x^6*ArcCoth[a*x]^3)/6 - (19*ArcTanh[a*x])/(60*a^6) - (23*A
rcCoth[a*x]*Log[2/(1 - a*x)])/(15*a^6) - (23*PolyLog[2, 1 - 2/(1 - a*x)])/(
30*a^6)
```


Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5981

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5985

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x^5 \coth^{-1}(ax)^3\,dx = \frac{1}{6}x^6\coth^{-1}(ax)^3 - \frac{1}{2}a\int \frac{x^6\coth^{-1}(ax)^2}{1-a^2x^2}\,dx$$

$$= \frac{1}{6}x^6\coth^{-1}(ax)^3 + \frac{\int x^4\coth^{-1}(ax)^2\,dx}{2a} - \frac{\int \frac{x^4\coth^{-1}(ax)^2}{1-a^2x^2}\,dx}{2a}$$

$$= \frac{x^5\coth^{-1}(ax)^2}{10a} + \frac{1}{6}x^6\coth^{-1}(ax)^3 - \frac{1}{5}\int \frac{x^5\coth^{-1}(ax)}{1-a^2x^2}\,dx + \frac{\int x^2\coth^{-1}(ax)^2\,dx}{2a^3} - \frac{\int \frac{x^2\coth^{-1}(a}{1-a^2x^2}}{2a^3}$$

$$= \frac{x^3\coth^{-1}(ax)^2}{6a^3} + \frac{x^5\coth^{-1}(ax)^2}{10a} + \frac{1}{6}x^6\coth^{-1}(ax)^3 + \frac{\int \coth^{-1}(ax)^2\,dx}{2a^5} - \frac{\int \frac{\coth^{-1}(ax)^2}{1-a^2x^2}\,dx}{2a^5} + \int$$

$$= \frac{x^4\coth^{-1}(ax)}{20a^2} + \frac{x\coth^{-1}(ax)^2}{2a^5} + \frac{x^3\coth^{-1}(ax)^2}{6a^3} + \frac{x^5\coth^{-1}(ax)^2}{10a} - \frac{\coth^{-1}(ax)^3}{6a^6} + \frac{1}{6}x^6\coth^{-1}$$

$$= \frac{4x^2\coth^{-1}(ax)}{15a^4} + \frac{x^4\coth^{-1}(ax)}{20a^2} + \frac{23\coth^{-1}(ax)^2}{30a^6} + \frac{x\coth^{-1}(ax)^2}{2a^5} + \frac{x^3\coth^{-1}(ax)^2}{6a^3} + \frac{x^5\coth}{10}$$

$$= \frac{19x}{60a^5} + \frac{x^3}{60a^3} + \frac{4x^2\coth^{-1}(ax)}{15a^4} + \frac{x^4\coth^{-1}(ax)}{20a^2} + \frac{23\coth^{-1}(ax)^2}{30a^6} + \frac{x\coth^{-1}(ax)^2}{2a^5} + \frac{x^3\coth^{-1}}{6a^3}$$

$$= \frac{19x}{60a^5} + \frac{x^3}{60a^3} + \frac{4x^2\coth^{-1}(ax)}{15a^4} + \frac{x^4\coth^{-1}(ax)}{20a^2} + \frac{23\coth^{-1}(ax)^2}{30a^6} + \frac{x\coth^{-1}(ax)^2}{2a^5} + \frac{x^3\coth^{-1}}{6a^3}$$

$$= \frac{19x}{60a^5} + \frac{x^3}{60a^3} + \frac{4x^2\coth^{-1}(ax)}{15a^4} + \frac{x^4\coth^{-1}(ax)}{20a^2} + \frac{23\coth^{-1}(ax)^2}{30a^6} + \frac{x\coth^{-1}(ax)^2}{2a^5} + \frac{x^3\coth^{-1}}{6a^3}$$

Mathematica [A] time = 0.525733, size = 117, normalized size = 0.63

$$\frac{46\text{PolyLog}\left(2, e^{-2\coth^{-1}(ax)}\right) + ax\left(a^2x^2+19\right) + 10\left(a^6x^6-1\right)\coth^{-1}(ax)^3 + 2\left(3a^5x^5+5a^3x^3+15ax-23\right)\coth^{-1}(ax)^2}{60a^6}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5*ArcCoth[a*x]^3,x]
```

```
[Out] (a*x*(19 + a^2*x^2) + 2*(-23 + 15*a*x + 5*a^3*x^3 + 3*a^5*x^5)*ArcCoth[a*x]
^2 + 10*(-1 + a^6*x^6)*ArcCoth[a*x]^3 + ArcCoth[a*x]*(-19 + 16*a^2*x^2 + 3*
a^4*x^4 - 92*Log[1 - E^(-2*ArcCoth[a*x])]) + 46*PolyLog[2, E^(-2*ArcCoth[a*
x])])/(60*a^6)
```

Maple [C] time = 1.901, size = 1141, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arccoth(a*x)^3,x)
```

```
[Out] 23/30*arccoth(a*x)^2/a^6+1/4/a^6*arccoth(a*x)^2*ln(a*x-1)-1/4/a^6*arccoth(a
*x)^2*ln(a*x+1)-23/15/a^6*arccoth(a*x)*ln(1+1/((a*x-1)/(a*x+1))^(1/2))-1/4/
a^6*arccoth(a*x)^2*ln((a*x-1)/(a*x+1))-41/120/a^6/(-1+((a*x-1)/(a*x+1))^(1/
2))*((a*x-1)/(a*x+1))^(1/2)-41/120/a^6/(((a*x-1)/(a*x+1))^(1/2)+1)*((a*x-1)
/(a*x+1))^(1/2)+1/80/a^5/(((a*x-1)/(a*x+1))^(1/2)*a*x+((a*x-1)/(a*x+1))^(1/
2)+a*x)*x-1/80/a^5/(((a*x-1)/(a*x+1))^(1/2)*a*x+((a*x-1)/(a*x+1))^(1/2)-a*x
```

```
)*x+1/120/a^6*((a*x-1)/(a*x+1))^(1/2)/(2*((a*x-1)/(a*x+1))^(1/2)*a*x+((a*x-
1)/(a*x+1))^(1/2)-2*a*x+1)+1/120/a^6*((a*x-1)/(a*x+1))^(1/2)/(2*((a*x-1)/(a
*x+1))^(1/2)*a*x+((a*x-1)/(a*x+1))^(1/2)+2*a*x-1)-1/6*arccoth(a*x)^3/a^6+1/
6*x^6*arccoth(a*x)^3+4/15*x^2*arccoth(a*x)/a^4+1/10*x^5*arccoth(a*x)^2/a+1/
20*x^4*arccoth(a*x)/a^2+1/2*x*arccoth(a*x)^2/a^5+1/6*x^3*arccoth(a*x)^2/a^3
-1/80/a^6/(((a*x-1)/(a*x+1))^(1/2)*a*x+((a*x-1)/(a*x+1))^(1/2)+a*x)+1/80/a^
6/(((a*x-1)/(a*x+1))^(1/2)*a*x+((a*x-1)/(a*x+1))^(1/2)-a*x)-19/60/a^6*arcco
th(a*x)+23/15/a^6*dilog(1/((a*x-1)/(a*x+1))^(1/2))-23/15/a^6*dilog(1+1/((a*
x-1)/(a*x+1))^(1/2))-1/8*I/a^6*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1
))*csgn(I*(a*x+1)/(a*x-1))*csgn(I/((a*x+1)/(a*x-1)-1))*arccoth(a*x)^2+1/4*I
/a^6*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/(a*x-1))^2*arccoth(a
*x)^2+1/8*I/a^6*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I*(a*
x+1)/(a*x-1))*arccoth(a*x)^2-1/8*I/a^6*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2
*csgn(I*(a*x+1)/(a*x-1))*arccoth(a*x)^2+1/8*I/a^6*Pi*csgn(I*(a*x+1)/(a*x-1)
/((a*x+1)/(a*x-1)-1))^2*csgn(I/((a*x+1)/(a*x-1)-1))*arccoth(a*x)^2-1/8*I/a^
6*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^3*arccoth(a*x)^2-1/8*I/a^6
*Pi*csgn(I*(a*x+1)/(a*x-1))^3*arccoth(a*x)^2-1/120/a^5*((a*x-1)/(a*x+1))^(1
/2)/(2*((a*x-1)/(a*x+1))^(1/2)*a*x+((a*x-1)/(a*x+1))^(1/2)-2*a*x+1)*x-1/120
/a^5*((a*x-1)/(a*x+1))^(1/2)/(2*((a*x-1)/(a*x+1))^(1/2)*a*x+((a*x-1)/(a*x+1
))^(1/2)+2*a*x-1)*x
```

Maxima [A] time = 1.01173, size = 390, normalized size = 2.1

$$\frac{1}{6}x^6\operatorname{arcoth}(ax)^3+\frac{1}{60}a\left(\frac{2\left(3a^4x^5+5a^2x^3+15x\right)}{a^6}-\frac{15\log(ax+1)}{a^7}+\frac{15\log(ax-1)}{a^7}\right)\operatorname{arcoth}(ax)^2+\frac{1}{240}a\left(\frac{4a^3x^3\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccoth(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/6*x^6*arccoth(a*x)^3 + 1/60*a*(2*(3*a^4*x^5 + 5*a^2*x^3 + 15*x)/a^6 - 15*
log(a*x + 1)/a^7 + 15*log(a*x - 1)/a^7)*arccoth(a*x)^2 + 1/240*a*(((4*a^3*x
^3 + (15*log(a*x - 1) - 46)*log(a*x + 1)^2 - 5*log(a*x + 1)^3 + 5*log(a*x -
 1)^3 + 76*a*x - (15*log(a*x - 1)^2 - 92*log(a*x - 1))*log(a*x + 1) + 46*lo
g(a*x - 1)^2 + 38*log(a*x - 1))/a - 184*(log(a*x - 1)*log(1/2*a*x + 1/2) +
dilog(-1/2*a*x + 1/2))/a - 38*log(a*x + 1)/a)/a^6 + 2*(6*a^4*x^4 + 32*a^2*x
^2 - 2*(15*log(a*x - 1) - 46)*log(a*x + 1) + 15*log(a*x + 1)^2 + 15*log(a*x
 - 1)^2 + 92*log(a*x - 1))*arccoth(a*x)/a^7)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(x^5\operatorname{arcoth}(ax)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccoth(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^5*arccoth(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\operatorname{acoth}^3(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*acoth(a*x)**3,x)
```

```
[Out] Integral(x**5*acoth(a*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \operatorname{arcoth}(ax)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccoth(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^5*arccoth(a*x)^3, x)
```

3.24 $\int x^4 \coth^{-1}(ax)^3 \, dx$

Optimal. Leaf size=196

$$\frac{3\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{10a^5}-\frac{3\coth^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{5a^5}+\frac{x^2}{20a^3}+\frac{\log\left(1-a^2x^2\right)}{2a^5}+\frac{x^3\coth^{-1}(ax)}{10a^2}+\frac{3x^2\coth^{-}}{10a^3}$$

```
[Out] x^2/(20*a^3) + (9*x*ArcCoth[a*x])/(10*a^4) + (x^3*ArcCoth[a*x])/(10*a^2) -
(9*ArcCoth[a*x]^2)/(20*a^5) + (3*x^2*ArcCoth[a*x]^2)/(10*a^3) + (3*x^4*ArcC
oth[a*x]^2)/(20*a) + ArcCoth[a*x]^3/(5*a^5) + (x^5*ArcCoth[a*x]^3)/5 - (3*A
rcCoth[a*x]^2*Log[2/(1 - a*x)])/(5*a^5) + Log[1 - a^2*x^2]/(2*a^5) - (3*Arc
Coth[a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/(5*a^5) + (3*PolyLog[3, 1 - 2/(1 - a
*x)])/(10*a^5)
```

Rubi [A] time = 0.580282, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 11, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.1, Rules used = {5917, 5981, 266, 43, 5911, 260, 5949, 5985, 5919, 6059, 6610}

$$\frac{3\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{10a^5}-\frac{3\coth^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{5a^5}+\frac{x^2}{20a^3}+\frac{\log\left(1-a^2x^2\right)}{2a^5}+\frac{x^3\coth^{-1}(ax)}{10a^2}+\frac{3x^2\coth^{-}}{10a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcCoth[a*x]^3,x]
```

```
[Out] x^2/(20*a^3) + (9*x*ArcCoth[a*x])/(10*a^4) + (x^3*ArcCoth[a*x])/(10*a^2) -
(9*ArcCoth[a*x]^2)/(20*a^5) + (3*x^2*ArcCoth[a*x]^2)/(10*a^3) + (3*x^4*ArcC
oth[a*x]^2)/(20*a) + ArcCoth[a*x]^3/(5*a^5) + (x^5*ArcCoth[a*x]^3)/5 - (3*A
rcCoth[a*x]^2*Log[2/(1 - a*x)])/(5*a^5) + Log[1 - a^2*x^2]/(2*a^5) - (3*Arc
Coth[a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/(5*a^5) + (3*PolyLog[3, 1 - 2/(1 - a
*x)])/(10*a^5)
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5981

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5985

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 6059

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcCoth[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int x^4 \coth^{-1}(ax)^3\,dx = \frac{1}{5}x^5\coth^{-1}(ax)^3 - \frac{1}{5}(3a)\int \frac{x^5\coth^{-1}(ax)^2}{1-a^2x^2}\,dx$$

$$= \frac{1}{5}x^5\coth^{-1}(ax)^3 + \frac{3\int x^3\coth^{-1}(ax)^2\,dx}{5a} - \frac{3\int \frac{x^3\coth^{-1}(ax)^2}{1-a^2x^2}\,dx}{5a}$$

$$= \frac{3x^4\coth^{-1}(ax)^2}{20a} + \frac{1}{5}x^5\coth^{-1}(ax)^3 - \frac{3}{10}\int \frac{x^4\coth^{-1}(ax)}{1-a^2x^2}\,dx + \frac{3\int x\coth^{-1}(ax)^2\,dx}{5a^3} - \frac{3\int x}{}$$

$$= \frac{3x^2\coth^{-1}(ax)^2}{10a^3} + \frac{3x^4\coth^{-1}(ax)^2}{20a} + \frac{\coth^{-1}(ax)^3}{5a^5} + \frac{1}{5}x^5\coth^{-1}(ax)^3 - \frac{3\int \frac{\coth^{-1}(ax)^2}{1-ax}\,dx}{5a^4} +$$

$$= \frac{x^3\coth^{-1}(ax)}{10a^2} + \frac{3x^2\coth^{-1}(ax)^2}{10a^3} + \frac{3x^4\coth^{-1}(ax)^2}{20a} + \frac{\coth^{-1}(ax)^3}{5a^5} + \frac{1}{5}x^5\coth^{-1}(ax)^3 - \frac{3\,\mathrm{c}}{}$$

$$= \frac{9x\coth^{-1}(ax)}{10a^4} + \frac{x^3\coth^{-1}(ax)}{10a^2} - \frac{9\coth^{-1}(ax)^2}{20a^5} + \frac{3x^2\coth^{-1}(ax)^2}{10a^3} + \frac{3x^4\coth^{-1}(ax)^2}{20a} + \mathrm{cot}$$

$$= \frac{9x\coth^{-1}(ax)}{10a^4} + \frac{x^3\coth^{-1}(ax)}{10a^2} - \frac{9\coth^{-1}(ax)^2}{20a^5} + \frac{3x^2\coth^{-1}(ax)^2}{10a^3} + \frac{3x^4\coth^{-1}(ax)^2}{20a} + \mathrm{cot}$$

$$= \frac{x^2}{20a^3} + \frac{9x\coth^{-1}(ax)}{10a^4} + \frac{x^3\coth^{-1}(ax)}{10a^2} - \frac{9\coth^{-1}(ax)^2}{20a^5} + \frac{3x^2\coth^{-1}(ax)^2}{10a^3} + \frac{3x^4\coth^{-1}(ax)}{20a}$$

Mathematica [C] time = 0.566502, size = 175, normalized size = 0.89

$$\frac{-24\coth^{-1}(ax)\mathrm{PolyLog}\left(2,e^{2\coth^{-1}(ax)}\right) + 12\mathrm{PolyLog}\left(3,e^{2\coth^{-1}(ax)}\right) + 2a^2x^2 - 40\log\left(\frac{1}{ax\sqrt{1-\frac{1}{a^2x^2}}}\right) + 8a^5x^5\coth^{-1}(}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*ArcCoth[a*x]^3,x]
```

```
[Out] (-2 - I*Pi^3 + 2*a^2*x^2 + 36*a*x*ArcCoth[a*x] + 4*a^3*x^3*ArcCoth[a*x] - 1
8*ArcCoth[a*x]^2 + 12*a^2*x^2*ArcCoth[a*x]^2 + 6*a^4*x^4*ArcCoth[a*x]^2 + 8
*ArcCoth[a*x]^3 + 8*a^5*x^5*ArcCoth[a*x]^3 - 24*ArcCoth[a*x]^2*Log[1 - E^(2
*ArcCoth[a*x])] - 40*Log[1/(a*Sqrt[1 - 1/(a^2*x^2)]*x)] - 24*ArcCoth[a*x]*P
olyLog[2, E^(2*ArcCoth[a*x])] + 12*PolyLog[3, E^(2*ArcCoth[a*x])])/(40*a^5)
```

Maple [C] time = 1.106, size = 806, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccoth(a*x)^3,x)
```

```
[Out] 1/5*arccoth(a*x)^3/a^5-1/20/a^5-9/20*arccoth(a*x)^2/a^5+1/a^5*arccoth(a*x)+
6/5/a^5*polylog(3,1/((a*x-1)/(a*x+1))^(1/2))-1/a^5*ln(1+1/((a*x-1)/(a*x+1))
^(1/2))-1/a^5*ln(1/((a*x-1)/(a*x+1))^(1/2)-1)+6/5/a^5*polylog(3,-1/((a*x-1)
/(a*x+1))^(1/2))+1/20*x^2/a^3+1/5*x^5*arccoth(a*x)^3+3/10*x^2*arccoth(a*x)^
2/a^3+9/10*x*arccoth(a*x)/a^4+1/10*x^3*arccoth(a*x)/a^2+3/20*x^4*arccoth(a*
x)^2/a+3/20*I/a^5*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))*csgn(I*(a*
```

```
x+1)/(a*x-1))*csgn(I/((a*x+1)/(a*x-1)-1))*arccoth(a*x)^2-3/5/a^5*arccoth(a*
x)^2*ln(2)+3/10/a^5*arccoth(a*x)^2*ln(a*x-1)+3/10/a^5*arccoth(a*x)^2*ln(a*x
+1)+3/10/a^5*arccoth(a*x)^2*ln((a*x-1)/(a*x+1))+3/5/a^5*arccoth(a*x)^2*ln((
a*x+1)/(a*x-1)-1)-3/5/a^5*arccoth(a*x)^2*ln(1-1/((a*x-1)/(a*x+1))^(1/2))-6/
5/a^5*arccoth(a*x)*polylog(2,1/((a*x-1)/(a*x+1))^(1/2))-3/5/a^5*arccoth(a*x
)^2*ln(1+1/((a*x-1)/(a*x+1))^(1/2))-6/5/a^5*arccoth(a*x)*polylog(2,-1/((a*x
-1)/(a*x+1))^(1/2))-3/20*I/a^5*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1
))^2*csgn(I/((a*x+1)/(a*x-1)-1))*arccoth(a*x)^2-3/20*I/a^5*Pi*csgn(I*(a*x+1
)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I*(a*x+1)/(a*x-1))*arccoth(a*x)^2-3/1
0*I/a^5*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/(a*x-1))^2*arccot
h(a*x)^2+3/20*I/a^5*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I*(a*x+1)/(a*
x-1))*arccoth(a*x)^2+3/20*I/a^5*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-
1))^3*arccoth(a*x)^2+3/20*I/a^5*Pi*csgn(I*(a*x+1)/(a*x-1))^3*arccoth(a*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(a^5x^5+1\right)\log\left(ax+1\right)^3+3\left(a^4x^4+2\,a^2x^2-2\left(a^5x^5-1\right)\log\left(ax-1\right)\right)\log\left(ax+1\right)^2}{80\,a^5}+\frac{1}{8}\int-\frac{5\left(a^5x^5+a^4x^4\right)\log\left(ax-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/80*(2*(a^5*x^5 + 1)*log(a*x + 1)^3 + 3*(a^4*x^4 + 2*a^2*x^2 - 2*(a^5*x^5
- 1)*log(a*x - 1))*log(a*x + 1)^2)/a^5 + 1/8*integrate(-1/5*(5*(a^5*x^5 + a
^4*x^4)*log(a*x - 1)^3 + 3*(a^4*x^4 + 2*a^2*x^2 - 5*(a^5*x^5 + a^4*x^4)*log
(a*x - 1)^2 - 2*(a^5*x^5 - 1)*log(a*x - 1))*log(a*x + 1))/(a^5*x + a^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^4\,\mathrm{arcoth}\left(ax\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^4*arccoth(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\,\mathrm{acoth}^3\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acoth(a*x)**3,x)
```

```
[Out] Integral(x**4*acoth(a*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{arcoth}(ax)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^4*arccoth(a*x)^3, x)
```

3.25 $\int x^3 \coth^{-1}(ax)^3 \, dx$

Optimal. Leaf size=139

$$-\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^4}+\frac{x^2\coth^{-1}(ax)}{4a^2}+\frac{x}{4a^3}-\frac{\tanh^{-1}(ax)}{4a^4}+\frac{3x\coth^{-1}(ax)^2}{4a^3}-\frac{\coth^{-1}(ax)^3}{4a^4}+\frac{\coth^{-1}(ax)^2}{a^4}-\frac{2\log\left(\right.}{}$$

```
[Out] x/(4*a^3) + (x^2*ArcCoth[a*x])/(4*a^2) + ArcCoth[a*x]^2/a^4 + (3*x*ArcCoth[
a*x]^2)/(4*a^3) + (x^3*ArcCoth[a*x]^2)/(4*a) - ArcCoth[a*x]^3/(4*a^4) + (x^
4*ArcCoth[a*x]^3)/4 - ArcTanh[a*x]/(4*a^4) - (2*ArcCoth[a*x]*Log[2/(1 - a*x
)])/a^4 - PolyLog[2, 1 - 2/(1 - a*x)]/a^4
```

Rubi [A] time = 0.41668, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {5917, 5981, 321, 206, 5985, 5919, 2402, 2315, 5911, 5949}

$$-\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^4}+\frac{x^2\coth^{-1}(ax)}{4a^2}+\frac{x}{4a^3}-\frac{\tanh^{-1}(ax)}{4a^4}+\frac{3x\coth^{-1}(ax)^2}{4a^3}-\frac{\coth^{-1}(ax)^3}{4a^4}+\frac{\coth^{-1}(ax)^2}{a^4}-\frac{2\log\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCoth[a*x]^3,x]
```

```
[Out] x/(4*a^3) + (x^2*ArcCoth[a*x])/(4*a^2) + ArcCoth[a*x]^2/a^4 + (3*x*ArcCoth[
a*x]^2)/(4*a^3) + (x^3*ArcCoth[a*x]^2)/(4*a) - ArcCoth[a*x]^3/(4*a^4) + (x^
4*ArcCoth[a*x]^3)/4 - ArcTanh[a*x]/(4*a^4) - (2*ArcCoth[a*x]*Log[2/(1 - a*x
)])/a^4 - PolyLog[2, 1 - 2/(1 - a*x)]/a^4
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5981

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 5985

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x^3 \coth^{-1}(ax)^3\,dx = \frac{1}{4}x^4\coth^{-1}(ax)^3 - \frac{1}{4}(3a)\int \frac{x^4\coth^{-1}(ax)^2}{1-a^2x^2}\,dx$$

$$= \frac{1}{4}x^4\coth^{-1}(ax)^3 + \frac{3\int x^2\coth^{-1}(ax)^2\,dx}{4a} - \frac{3\int \frac{x^2\coth^{-1}(ax)^2}{1-a^2x^2}\,dx}{4a}$$

$$= \frac{x^3\coth^{-1}(ax)^2}{4a} + \frac{1}{4}x^4\coth^{-1}(ax)^3 - \frac{1}{2}\int \frac{x^3\coth^{-1}(ax)}{1-a^2x^2}\,dx + \frac{3\int \coth^{-1}(ax)^2\,dx}{4a^3} - \frac{3\int \frac{\coth^{-1}(ax}{1-a^2x^2}}{4a^3}$$

$$= \frac{3x\coth^{-1}(ax)^2}{4a^3} + \frac{x^3\coth^{-1}(ax)^2}{4a} - \frac{\coth^{-1}(ax)^3}{4a^4} + \frac{1}{4}x^4\coth^{-1}(ax)^3 + \frac{\int x\coth^{-1}(ax)\,dx}{2a^2} - \int \frac{x\,c}{}$$

$$= \frac{x^2\coth^{-1}(ax)}{4a^2} + \frac{\coth^{-1}(ax)^2}{a^4} + \frac{3x\coth^{-1}(ax)^2}{4a^3} + \frac{x^3\coth^{-1}(ax)^2}{4a} - \frac{\coth^{-1}(ax)^3}{4a^4} + \frac{1}{4}x^4\coth^{-1}($$

$$= \frac{x}{4a^3} + \frac{x^2\coth^{-1}(ax)}{4a^2} + \frac{\coth^{-1}(ax)^2}{a^4} + \frac{3x\coth^{-1}(ax)^2}{4a^3} + \frac{x^3\coth^{-1}(ax)^2}{4a} - \frac{\coth^{-1}(ax)^3}{4a^4} + \frac{1}{4}x^4\,c$$

$$= \frac{x}{4a^3} + \frac{x^2\coth^{-1}(ax)}{4a^2} + \frac{\coth^{-1}(ax)^2}{a^4} + \frac{3x\coth^{-1}(ax)^2}{4a^3} + \frac{x^3\coth^{-1}(ax)^2}{4a} - \frac{\coth^{-1}(ax)^3}{4a^4} + \frac{1}{4}x^4\,c$$

$$= \frac{x}{4a^3} + \frac{x^2\coth^{-1}(ax)}{4a^2} + \frac{\coth^{-1}(ax)^2}{a^4} + \frac{3x\coth^{-1}(ax)^2}{4a^3} + \frac{x^3\coth^{-1}(ax)^2}{4a} - \frac{\coth^{-1}(ax)^3}{4a^4} + \frac{1}{4}x^4\,c$$

Mathematica [A] time = 0.299733, size = 88, normalized size = 0.63

$$\frac{4\text{PolyLog}\left(2, e^{-2\coth^{-1}(ax)}\right) + \left(a^4x^4-1\right)\coth^{-1}(ax)^3 + \left(a^3x^3+3ax-4\right)\coth^{-1}(ax)^2 + \coth^{-1}(ax)\left(a^2x^2 - 8\log\left(1-e^{-2}\right.\right.}{4a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*ArcCoth[a*x]^3,x]
```

```
[Out] (a*x + (-4 + 3*a*x + a^3*x^3)*ArcCoth[a*x]^2 + (-1 + a^4*x^4)*ArcCoth[a*x]^
3 + ArcCoth[a*x]*(-1 + a^2*x^2 - 8*Log[1 - E^(-2*ArcCoth[a*x])]) + 4*PolyLo
g[2, E^(-2*ArcCoth[a*x])])/(4*a^4)
```

Maple [C] time = 0.619, size = 684, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccoth(a*x)^3,x)
```

```
[Out] arccoth(a*x)^2/a^4+3/8/a^4*arccoth(a*x)^2*ln(a*x-1)-3/8/a^4*arccoth(a*x)^2*
ln(a*x+1)-2/a^4*arccoth(a*x)*ln(1+1/((a*x-1)/(a*x+1))^(1/2))-3/8/a^4*arccot
h(a*x)^2*ln((a*x-1)/(a*x+1))-1/4/a^4/(-1+((a*x-1)/(a*x+1))^(1/2))*((a*x-1)/
(a*x+1))^(1/2)-1/4/a^4/(((a*x-1)/(a*x+1))^(1/2)+1)*((a*x-1)/(a*x+1))^(1/2)-
1/4/a^4*arccoth(a*x)+1/4*x^4*arccoth(a*x)^3+3/4*x*arccoth(a*x)^2/a^3+1/4*x^
3*arccoth(a*x)^2/a+1/4*x^2*arccoth(a*x)/a^2+2/a^4*dilog(1/((a*x-1)/(a*x+1))
^(1/2))-2/a^4*dilog(1+1/((a*x-1)/(a*x+1))^(1/2))-3/16*I/a^4*Pi*csgn(I*(a*x+
1)/(a*x-1))^3*arccoth(a*x)^2-3/16*I/a^4*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/
(a*x-1)-1))^3*arccoth(a*x)^2+3/16*I/a^4*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/
(a*x-1)-1))^2*csgn(I/((a*x+1)/(a*x-1)-1))*arccoth(a*x)^2-1/4*arccoth(a*x)^3
```

```
/a^4-3/16*I/a^4*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I*(a*x+1)/(a*x-1)
)*arccoth(a*x)^2+3/8*I/a^4*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1
)/(a*x-1))^2*arccoth(a*x)^2+3/16*I/a^4*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(
a*x-1)-1))^2*csgn(I*(a*x+1)/(a*x-1))*arccoth(a*x)^2-3/16*I/a^4*Pi*csgn(I*(a
*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1))*csgn(I/((a*x+1)/
(a*x-1)-1))*arccoth(a*x)^2
```

Maxima [B] time = 0.994781, size = 354, normalized size = 2.55

$$\frac{1}{4}x^4\operatorname{arcoth}(ax)^3+\frac{1}{8}a\left(\frac{2\left(a^2x^3+3x\right)}{a^4}-\frac{3\log(ax+1)}{a^5}+\frac{3\log(ax-1)}{a^5}\right)\operatorname{arcoth}(ax)^2+\frac{1}{32}a\left(\frac{(3\log(ax-1)-8)\log(ax+1)^2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccoth(a*x)^3 + 1/8*a*(2*(a^2*x^3 + 3*x)/a^4 - 3*log(a*x + 1)/a^5
+ 3*log(a*x - 1)/a^5)*arccoth(a*x)^2 + 1/32*a*((((3*log(a*x - 1) - 8)*log(a
*x + 1)^2 - log(a*x + 1)^3 + log(a*x - 1)^3 + 8*a*x - (3*log(a*x - 1)^2 - 1
6*log(a*x - 1))*log(a*x + 1) + 8*log(a*x - 1)^2 + 4*log(a*x - 1))/a - 32*(l
og(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a - 4*log(a*x + 1)/
a)/a^4 + 2*(4*a^2*x^2 - 2*(3*log(a*x - 1) - 8)*log(a*x + 1) + 3*log(a*x + 1
)^2 + 3*log(a*x - 1)^2 + 16*log(a*x - 1))*arccoth(a*x)/a^5)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(x^3\operatorname{arcoth}(ax)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^3*arccoth(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\operatorname{acoth}^3(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acoth(a*x)**3,x)
```

```
[Out] Integral(x**3*acoth(a*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\operatorname{arcoth}(ax)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*arccoth(a*x)^3, x)
```

3.26 $\int x^2 \coth^{-1}(ax)^3 \, dx$

Optimal. Leaf size=149

$$\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^3}-\frac{\coth^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^3}+\frac{\log\left(1-a^2x^2\right)}{2a^3}+\frac{\coth^{-1}(ax)^3}{3a^3}-\frac{\coth^{-1}(ax)^2}{2a^3}+\frac{x\coth^{-1}}{a^2}$$

```
[Out] (x*ArcCoth[a*x])/a^2 - ArcCoth[a*x]^2/(2*a^3) + (x^2*ArcCoth[a*x]^2)/(2*a)
+ ArcCoth[a*x]^3/(3*a^3) + (x^3*ArcCoth[a*x]^3)/3 - (ArcCoth[a*x]^2*Log[2/(
1 - a*x)])/a^3 + Log[1 - a^2*x^2]/(2*a^3) - (ArcCoth[a*x]*PolyLog[2, 1 - 2/
(1 - a*x)])/a^3 + PolyLog[3, 1 - 2/(1 - a*x)]/(2*a^3)
```

Rubi [A] time = 0.332522, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.9, Rules used = {5917, 5981, 5911, 260, 5949, 5985, 5919, 6059, 6610}

$$\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^3}-\frac{\coth^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^3}+\frac{\log\left(1-a^2x^2\right)}{2a^3}+\frac{\coth^{-1}(ax)^3}{3a^3}-\frac{\coth^{-1}(ax)^2}{2a^3}+\frac{x\coth^{-1}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[a*x]^3,x]
```

```
[Out] (x*ArcCoth[a*x])/a^2 - ArcCoth[a*x]^2/(2*a^3) + (x^2*ArcCoth[a*x]^2)/(2*a)
+ ArcCoth[a*x]^3/(3*a^3) + (x^3*ArcCoth[a*x]^3)/3 - (ArcCoth[a*x]^2*Log[2/(
1 - a*x)])/a^3 + Log[1 - a^2*x^2]/(2*a^3) - (ArcCoth[a*x]*PolyLog[2, 1 - 2/
(1 - a*x)])/a^3 + PolyLog[3, 1 - 2/(1 - a*x)]/(2*a^3)
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5981

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5985

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 6059

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcCoth[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int x^2\coth^{-1}(ax)^3\,dx = \frac{1}{3}x^3\coth^{-1}(ax)^3 - a\int \frac{x^3\coth^{-1}(ax)^2}{1-a^2x^2}\,dx$$

$$= \frac{1}{3}x^3\coth^{-1}(ax)^3 + \frac{\int x\coth^{-1}(ax)^2\,dx}{a} - \frac{\int \frac{x\coth^{-1}(ax)^2}{1-a^2x^2}\,dx}{a}$$

$$= \frac{x^2\coth^{-1}(ax)^2}{2a} + \frac{\coth^{-1}(ax)^3}{3a^3} + \frac{1}{3}x^3\coth^{-1}(ax)^3 - \frac{\int \frac{\coth^{-1}(ax)^2}{1-ax}\,dx}{a^2} - \int \frac{x^2\coth^{-1}(ax)}{1-a^2x^2}\,dx$$

$$= \frac{x^2\coth^{-1}(ax)^2}{2a} + \frac{\coth^{-1}(ax)^3}{3a^3} + \frac{1}{3}x^3\coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^3} + \frac{\int \coth^{-1}(ax)\,dx}{a^2} -$$

$$= \frac{x\coth^{-1}(ax)}{a^2} - \frac{\coth^{-1}(ax)^2}{2a^3} + \frac{x^2\coth^{-1}(ax)^2}{2a} + \frac{\coth^{-1}(ax)^3}{3a^3} + \frac{1}{3}x^3\coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^2}{a^3}$$

$$= \frac{x\coth^{-1}(ax)}{a^2} - \frac{\coth^{-1}(ax)^2}{2a^3} + \frac{x^2\coth^{-1}(ax)^2}{2a} + \frac{\coth^{-1}(ax)^3}{3a^3} + \frac{1}{3}x^3\coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^2}{a^3}$$

Mathematica [C] time = 0.352743, size = 140, normalized size = 0.94

$$\frac{-24\coth^{-1}(ax)\text{PolyLog}\left(2,e^{2\coth^{-1}(ax)}\right)+12\text{PolyLog}\left(3,e^{2\coth^{-1}(ax)}\right)-24\log\left(\frac{1}{ax\sqrt{1-\frac{1}{a^2x^2}}}\right)+8a^3x^3\coth^{-1}(ax)^3+12}{24a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*ArcCoth[a*x]^3,x]
```

```
[Out] ((-I)*Pi^3 + 24*a*x*ArcCoth[a*x] - 12*ArcCoth[a*x]^2 + 12*a^2*x^2*ArcCoth[a
*x]^2 + 8*ArcCoth[a*x]^3 + 8*a^3*x^3*ArcCoth[a*x]^3 - 24*ArcCoth[a*x]^2*Log
[1 - E^(2*ArcCoth[a*x])] - 24*Log[1/(a*Sqrt[1 - 1/(a^2*x^2)]*x)] - 24*ArcCo
th[a*x]*PolyLog[2, E^(2*ArcCoth[a*x])] + 12*PolyLog[3, E^(2*ArcCoth[a*x])])
/(24*a^3)
```

Maple [C] time = 0.57, size = 765, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(a*x)^3,x)
```

```
[Out] 1/4*I/a^3*csgn(I*(a*x+1)/(a*x-1))*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1
))*csgn(I/((a*x+1)/(a*x-1)-1))*arccoth(a*x)^2*Pi-1/2*arccoth(a*x)^2/a^3-1/4
*I/a^3*csgn(I*(a*x+1)/(a*x-1))*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^
2*arccoth(a*x)^2*Pi+1/4*I/a^3*csgn(I*(a*x+1)/(a*x-1))*arccoth(a*x)^2*Pi*csg
n(I/((a*x-1)/(a*x+1))^(1/2))^2-1/4*I/a^3*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a
*x-1)-1))^2*csgn(I/((a*x+1)/(a*x-1)-1))*arccoth(a*x)^2*Pi+x*arccoth(a*x)/a^
2+2/a^3*polylog(3,1/((a*x-1)/(a*x+1))^(1/2))+2/a^3*polylog(3,-1/((a*x-1)/(a
*x+1))^(1/2))-1/a^3*ln(1/((a*x-1)/(a*x+1))^(1/2)-1)-1/2*I/a^3*csgn(I*(a*x+1
)/(a*x-1))^2*arccoth(a*x)^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))+1/a^3*arccot
h(a*x)-1/a^3*ln(1+1/((a*x-1)/(a*x+1))^(1/2))+1/3*arccoth(a*x)^3/a^3+1/3*x^3
*arccoth(a*x)^3+1/2/a^3*arccoth(a*x)^2*ln(a*x-1)+1/2/a^3*arccoth(a*x)^2*ln(
a*x+1)+1/2/a^3*arccoth(a*x)^2*ln((a*x-1)/(a*x+1))+1/a^3*arccoth(a*x)^2*ln((
a*x+1)/(a*x-1)-1)-1/a^3*arccoth(a*x)^2*ln(1-1/((a*x-1)/(a*x+1))^(1/2))-2/a^
3*arccoth(a*x)*polylog(2,1/((a*x-1)/(a*x+1))^(1/2))-1/a^3*arccoth(a*x)^2*ln
(1+1/((a*x-1)/(a*x+1))^(1/2))-2/a^3*arccoth(a*x)*polylog(2,-1/((a*x-1)/(a*x
+1))^(1/2))-1/a^3*arccoth(a*x)^2*ln(2)+1/2*x^2*arccoth(a*x)^2/a+1/4*I/a^3*c
sgn(I*(a*x+1)/(a*x-1))^3*arccoth(a*x)^2*Pi+1/4*I/a^3*csgn(I*(a*x+1)/(a*x-1)
/((a*x+1)/(a*x-1)-1))^3*arccoth(a*x)^2*Pi
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(a^3x^3+1\right)\log\left(ax+1\right)^3+3\left(a^2x^2-\left(a^3x^3-1\right)\log\left(ax-1\right)\right)\log\left(ax+1\right)^2}{24\,a^3}+\frac{1}{8}\int-\frac{\left(a^3x^3+a^2x^2\right)\log\left(ax-1\right)^3+\left(2\,a^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/24*((a^3*x^3 + 1)*log(a*x + 1)^3 + 3*(a^2*x^2 - (a^3*x^3 - 1)*log(a*x - 1
))*log(a*x + 1)^2)/a^3 + 1/8*integrate(-((a^3*x^3 + a^2*x^2)*log(a*x - 1)^3
```

```
 + (2*a^2*x^2 - 3*(a^3*x^3 + a^2*x^2)*log(a*x - 1)^2 - 2*(a^3*x^3 - 1)*log(
a*x - 1))*log(a*x + 1))/(a^3*x + a^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2\,\text{arcoth}\,(ax)^3\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^2*arccoth(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\,\text{acoth}^3\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(a*x)**3,x)
```

```
[Out] Integral(x**2*acoth(a*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\,\text{arcoth}\,(ax)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(a*x)^3, x)
```

3.27 $\int x\coth^{-1}(ax)^3\,dx$

Optimal. Leaf size=95

$$-\frac{3\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^2}-\frac{\coth^{-1}(ax)^3}{2a^2}+\frac{3\coth^{-1}(ax)^2}{2a^2}-\frac{3\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a^2}+\frac{1}{2}x^2\coth^{-1}(ax)^3+\frac{3x\coth^{-1}(}{2a}$$

```
[Out] (3*ArcCoth[a*x]^2)/(2*a^2) + (3*x*ArcCoth[a*x]^2)/(2*a) - ArcCoth[a*x]^3/(2
*a^2) + (x^2*ArcCoth[a*x]^3)/2 - (3*ArcCoth[a*x]*Log[2/(1 - a*x)])/a^2 - (3
*PolyLog[2, 1 - 2/(1 - a*x)])/(2*a^2)
```

Rubi [A] time = 0.183365, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {5917, 5981, 5911, 5985, 5919, 2402, 2315, 5949}

$$-\frac{3\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^2}-\frac{\coth^{-1}(ax)^3}{2a^2}+\frac{3\coth^{-1}(ax)^2}{2a^2}-\frac{3\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a^2}+\frac{1}{2}x^2\coth^{-1}(ax)^3+\frac{3x\coth^{-1}(}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[a*x]^3,x]
```

```
[Out] (3*ArcCoth[a*x]^2)/(2*a^2) + (3*x*ArcCoth[a*x]^2)/(2*a) - ArcCoth[a*x]^3/(2
*a^2) + (x^2*ArcCoth[a*x]^3)/2 - (3*ArcCoth[a*x]*Log[2/(1 - a*x)])/a^2 - (3
*PolyLog[2, 1 - 2/(1 - a*x)])/(2*a^2)
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5981

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5985

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int x\coth^{-1}(ax)^3\,dx &= \frac{1}{2}x^2\coth^{-1}(ax)^3 - \frac{1}{2}(3a)\int \frac{x^2\coth^{-1}(ax)^2}{1-a^2x^2}\,dx\\
&= \frac{1}{2}x^2\coth^{-1}(ax)^3 + \frac{3\int \coth^{-1}(ax)^2\,dx}{2a} - \frac{3\int \frac{\coth^{-1}(ax)^2}{1-a^2x^2}\,dx}{2a}\\
&= \frac{3x\coth^{-1}(ax)^2}{2a} - \frac{\coth^{-1}(ax)^3}{2a^2} + \frac{1}{2}x^2\coth^{-1}(ax)^3 - 3\int \frac{x\coth^{-1}(ax)}{1-a^2x^2}\,dx\\
&= \frac{3\coth^{-1}(ax)^2}{2a^2} + \frac{3x\coth^{-1}(ax)^2}{2a} - \frac{\coth^{-1}(ax)^3}{2a^2} + \frac{1}{2}x^2\coth^{-1}(ax)^3 - \frac{3\int \frac{\coth^{-1}(ax)}{1-ax}\,dx}{a}\\
&= \frac{3\coth^{-1}(ax)^2}{2a^2} + \frac{3x\coth^{-1}(ax)^2}{2a} - \frac{\coth^{-1}(ax)^3}{2a^2} + \frac{1}{2}x^2\coth^{-1}(ax)^3 - \frac{3\coth^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^2} + \frac{3}{-}\\
&= \frac{3\coth^{-1}(ax)^2}{2a^2} + \frac{3x\coth^{-1}(ax)^2}{2a} - \frac{\coth^{-1}(ax)^3}{2a^2} + \frac{1}{2}x^2\coth^{-1}(ax)^3 - \frac{3\coth^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^2} - \frac{3}{-}\\
&= \frac{3\coth^{-1}(ax)^2}{2a^2} + \frac{3x\coth^{-1}(ax)^2}{2a} - \frac{\coth^{-1}(ax)^3}{2a^2} + \frac{1}{2}x^2\coth^{-1}(ax)^3 - \frac{3\coth^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^2} - \frac{3}{-}
\end{aligned}$$

Mathematica [A] time = 0.140715, size = 68, normalized size = 0.72

$$\frac{3\text{PolyLog}\left(2,e^{-2\coth^{-1}(ax)}\right)+\coth^{-1}(ax)\left(\left(a^2x^2-1\right)\coth^{-1}(ax)^2+3(ax-1)\coth^{-1}(ax)-6\log\left(1-e^{-2\coth^{-1}(ax)}\right)\right)}{2a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*ArcCoth[a*x]^3,x]
```

```
[Out] (ArcCoth[a*x]*(3*(-1 + a*x)*ArcCoth[a*x] + (-1 + a^2*x^2)*ArcCoth[a*x]^2 -
6*Log[1 - E^(-2*ArcCoth[a*x])]) + 3*PolyLog[2, E^(-2*ArcCoth[a*x])])/(2*a^2
```

)

Maple [C] time = 0.404, size = 3070, normalized size = 32.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(a*x)^3,x)
```

```
[Out] 3/2*arccoth(a*x)^2/a^2+3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)
-1))^3*dilog(1/((a*x-1)/(a*x+1))^(1/2))-3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1)
/((a*x+1)/(a*x-1)-1))^3*dilog(1+1/((a*x-1)/(a*x+1))^(1/2))+3/8*I/a^2*Pi*csg
n(I*(a*x+1)/(a*x-1))^3*dilog(1/((a*x-1)/(a*x+1))^(1/2))-3/8*I/a^2*Pi*csgn(I
*(a*x+1)/(a*x-1))^3*dilog(1+1/((a*x-1)/(a*x+1))^(1/2))+3/8*I/a^2*Pi*csgn(I*
(a*x+1)/(a*x-1))^3*polylog(2,1/((a*x-1)/(a*x+1))^(1/2))+3/8*I/a^2*Pi*csgn(I
*(a*x+1)/(a*x-1))^3*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))-3/8*I/a^2*Pi*csgn
(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^3*arccoth(a*x)^2-3/8*I/a^2*Pi*csgn(
I*(a*x+1)/(a*x-1))^3*arccoth(a*x)^2+3/4/a^2*arccoth(a*x)^2*ln(a*x-1)-3/4/a^
2*arccoth(a*x)^2*ln(a*x+1)-3/4/a^2*arccoth(a*x)^2*ln((a*x-1)/(a*x+1))+3/8*I
/a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I*(a*x+1)/(a*x-1))*polylog(2
,1/((a*x-1)/(a*x+1))^(1/2))-3/4*I/a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*cs
gn(I*(a*x+1)/(a*x-1))^2*polylog(2,1/((a*x-1)/(a*x+1))^(1/2))+3/4*I/a^2*Pi*c
sgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/(a*x-1))^2*arccoth(a*x)^2+3/8
*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I*(a*x+1)/(a*x
-1))*arccoth(a*x)^2-3/8*I/a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I*(
a*x+1)/(a*x-1))*arccoth(a*x)^2+3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)
/(a*x-1)-1))^2*csgn(I/((a*x+1)/(a*x-1)-1))*arccoth(a*x)^2-3/4*I/a^2*Pi*csgn
(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/(a*x-1))^2*polylog(2,-1/((a*x-1)
/(a*x+1))^(1/2))-3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2
*csgn(I/((a*x+1)/(a*x-1)-1))*dilog(1/((a*x-1)/(a*x+1))^(1/2))+3/8*I/a^2*Pi*
csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I/((a*x+1)/(a*x-1)-1))*d
ilog(1+1/((a*x-1)/(a*x+1))^(1/2))-3/4*I/a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/
2))*csgn(I*(a*x+1)/(a*x-1))^2*dilog(1/((a*x-1)/(a*x+1))^(1/2))+3/4*I/a^2*Pi
*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/(a*x-1))^2*dilog(1+1/((a*x-
1)/(a*x+1))^(1/2))-3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))
^2*csgn(I*(a*x+1)/(a*x-1))*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))-3/8*I/a^2*
Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I*(a*x+1)/(a*x-1))*po
lylog(2,1/((a*x-1)/(a*x+1))^(1/2))-3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*
x+1)/(a*x-1)-1))^2*csgn(I/((a*x+1)/(a*x-1)-1))*polylog(2,1/((a*x-1)/(a*x+1)
)^(1/2))+3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^3*arccoth
(a*x)*ln(1-1/((a*x-1)/(a*x+1))^(1/2))+3/8*I/a^2*Pi*csgn(I/((a*x-1)/(a*x+1))
^(1/2))^2*csgn(I*(a*x+1)/(a*x-1))*dilog(1/((a*x-1)/(a*x+1))^(1/2))-3/8*I/a^
2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I*(a*x+1)/(a*x-1))*
dilog(1/((a*x-1)/(a*x+1))^(1/2))+3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+
1)/(a*x-1)-1))^2*csgn(I*(a*x+1)/(a*x-1))*dilog(1+1/((a*x-1)/(a*x+1))^(1/2))
-3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I/((a*x+1)
/(a*x-1)-1))*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))-3/8*I/a^2*Pi*csgn(I/((a*
x-1)/(a*x+1))^(1/2))^2*csgn(I*(a*x+1)/(a*x-1))*dilog(1+1/((a*x-1)/(a*x+1))^
(1/2))+3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1))^3*arccoth(a*x)*ln(1-1/((a*x-1)/
(a*x+1))^(1/2))+3/8*I/a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I*(a*x+
1)/(a*x-1))*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))+3/8*I/a^2*Pi*csgn(I*(a*x+
1)/(a*x-1)/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1))*csgn(I/((a*x+1)/(a*
x-1)-1))*arccoth(a*x)*ln(1-1/((a*x-1)/(a*x+1))^(1/2))-3/a^2*arccoth(a*x)*ln
(1+1/((a*x-1)/(a*x+1))^(1/2))-3/2/a^2*arccoth(a*x)*ln(1-1/((a*x-1)/(a*x+1))
^(1/2))-1/2*arccoth(a*x)^3/a^2+1/2*x^2*arccoth(a*x)^3+3/8*I/a^2*Pi*csgn(I*(
a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^3*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))
```

```
+3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^3*polylog(2,1/((a
*x-1)/(a*x+1))^(1/2))-3/4*I/a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(
a*x+1)/(a*x-1))^2*arccoth(a*x)*ln(1-1/((a*x-1)/(a*x+1))^(1/2))-3/8*I/a^2*Pi
*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I/((a*x+1)/(a*x-1)-1))*
arccoth(a*x)*ln(1-1/((a*x-1)/(a*x+1))^(1/2))-3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a
*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I*(a*x+1)/(a*x-1))*arccoth(a*x)*ln(1-1/((
a*x-1)/(a*x+1))^(1/2))-3/2/a^2*polylog(2,1/((a*x-1)/(a*x+1))^(1/2))-3/2/a^2
*dilog(1+1/((a*x-1)/(a*x+1))^(1/2))-3/2/a^2*polylog(2,-1/((a*x-1)/(a*x+1))^
(1/2))+3/2/a^2*dilog(1/((a*x-1)/(a*x+1))^(1/2))+3/8*I/a^2*Pi*csgn(I*(a*x+1)
/(a*x-1)/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1))*csgn(I/((a*x+1)/(a*x-
1)-1))*dilog(1/((a*x-1)/(a*x+1))^(1/2))-3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1)
/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1))*csgn(I/((a*x+1)/(a*x-1)-1))*a
rccoth(a*x)^2-3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))*csgn
(I*(a*x+1)/(a*x-1))*csgn(I/((a*x+1)/(a*x-1)-1))*dilog(1+1/((a*x-1)/(a*x+1))
^(1/2))+3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))*csgn(I*(a*
x+1)/(a*x-1))*csgn(I/((a*x+1)/(a*x-1)-1))*polylog(2,1/((a*x-1)/(a*x+1))^(1/
2))+3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)
/(a*x-1))*csgn(I/((a*x+1)/(a*x-1)-1))*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))
+3/2*x*arccoth(a*x)^2/a+3/8*I/a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn
(I*(a*x+1)/(a*x-1))*arccoth(a*x)*ln(1-1/((a*x-1)/(a*x+1))^(1/2))
```

Maxima [B] time = 0.999338, size = 290, normalized size = 3.05

$$\frac{1}{2}x^2\operatorname{arcoth}(ax)^3+\frac{3}{4}a\left(\frac{2x}{a^2}-\frac{\log(ax+1)}{a^3}+\frac{\log(ax-1)}{a^3}\right)\operatorname{arcoth}(ax)^2+\frac{1}{16}a\left(\frac{3(\log(ax-1)-2)\log(ax+1)^2-\log(ax+1)^3+\log(ax-1)^3}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccoth(a*x)^3 + 3/4*a*(2*x/a^2 - log(a*x + 1)/a^3 + log(a*x - 1)/a
^3)*arccoth(a*x)^2 + 1/16*a*(((3*(log(a*x - 1) - 2)*log(a*x + 1)^2 - log(a*
x + 1)^3 + log(a*x - 1)^3 - 3*(log(a*x - 1)^2 - 4*log(a*x - 1))*log(a*x + 1
) + 6*log(a*x - 1)^2)/a - 24*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*
a*x + 1/2))/a)/a^2 - 6*(2*(log(a*x - 1) - 2)*log(a*x + 1) - log(a*x + 1)^2
- log(a*x - 1)^2 - 4*log(a*x - 1))*arccoth(a*x)/a^3)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(x\operatorname{arcoth}(ax)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x*arccoth(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\operatorname{acoth}^3(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(a*x)**3,x)
```

```
[Out] Integral(x*acoth(a*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arcoth}(ax)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(a*x)^3, x)
```

3.28 $\int \coth^{-1}(ax)^3 \, dx$

Optimal. Leaf size=85

$$\frac{3\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a}-\frac{3\coth^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a}+x\coth^{-1}(ax)^3+\frac{\coth^{-1}(ax)^3}{a}-\frac{3\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)^2}{a}$$

```
[Out] ArcCoth[a*x]^3/a + x*ArcCoth[a*x]^3 - (3*ArcCoth[a*x]^2*Log[2/(1 - a*x)])/a
 - (3*ArcCoth[a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/a + (3*PolyLog[3, 1 - 2/(1
- a*x)])/(2*a)
```

Rubi [A] time = 0.165484, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {5911, 5985, 5919, 5949, 6059, 6610}

$$\frac{3\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a}-\frac{3\coth^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a}+x\coth^{-1}(ax)^3+\frac{\coth^{-1}(ax)^3}{a}-\frac{3\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)^2}{a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]^3,x]
```

```
[Out] ArcCoth[a*x]^3/a + x*ArcCoth[a*x]^3 - (3*ArcCoth[a*x]^2*Log[2/(1 - a*x)])/a
 - (3*ArcCoth[a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/a + (3*PolyLog[3, 1 - 2/(1
- a*x)])/(2*a)
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5985

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6059

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcCoth[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \coth^{-1}(ax)^3\,dx = x\coth^{-1}(ax)^3 - (3a)\int \frac{x\coth^{-1}(ax)^2}{1-a^2x^2}\,dx$$
$$= \frac{\coth^{-1}(ax)^3}{a} + x\coth^{-1}(ax)^3 - 3\int \frac{\coth^{-1}(ax)^2}{1-ax}\,dx$$
$$= \frac{\coth^{-1}(ax)^3}{a} + x\coth^{-1}(ax)^3 - \frac{3\coth^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a} + 6\int \frac{\coth^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx$$
$$= \frac{\coth^{-1}(ax)^3}{a} + x\coth^{-1}(ax)^3 - \frac{3\coth^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a} - \frac{3\coth^{-1}(ax)\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{a} + 3\int \frac{\text{Li}}{}$$
$$= \frac{\coth^{-1}(ax)^3}{a} + x\coth^{-1}(ax)^3 - \frac{3\coth^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a} - \frac{3\coth^{-1}(ax)\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{a} + \frac{3\text{Li}_3(1}{2}$$

Mathematica [A] time = 0.0976196, size = 79, normalized size = 0.93

$$-\frac{3\coth^{-1}(ax)\text{PolyLog}\left(2,e^{2\coth^{-1}(ax)}\right)}{a} + \frac{3\text{PolyLog}\left(3,e^{2\coth^{-1}(ax)}\right)}{2a} + x\coth^{-1}(ax)^3 + \frac{\coth^{-1}(ax)^3}{a} - \frac{3\coth^{-1}(ax)^2}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[a*x]^3,x]
```

```
[Out] ArcCoth[a*x]^3/a + x*ArcCoth[a*x]^3 - (3*ArcCoth[a*x]^2*Log[1 - E^(2*ArcCot
h[a*x])])/a - (3*ArcCoth[a*x]*PolyLog[2, E^(2*ArcCoth[a*x])])/a + (3*PolyLo
g[3, E^(2*ArcCoth[a*x])])/(2*a)
```

Maple [B] time = 0.115, size = 180, normalized size = 2.1

$$x(\text{arccoth}(ax))^3 - 3\frac{(\text{arccoth}(ax))^2}{a}\ln\left(1+\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right) - 3\frac{(\text{arccoth}(ax))^2}{a}\ln\left(1-\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right) + \frac{(\text{arccoth}(ax))^3}{a} - 6\,\text{arcc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)^3,x)
```

```
[Out] x*arccoth(a*x)^3-3/a*arccoth(a*x)^2*ln(1+1/((a*x-1)/(a*x+1))^(1/2))-3/a*arc
coth(a*x)^2*ln(1-1/((a*x-1)/(a*x+1))^(1/2))+arccoth(a*x)^3/a-6/a*arccoth(a*
x)*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))-6/a*arccoth(a*x)*polylog(2,1/((a*x
```

```
-1)/(a*x+1))^(1/2))+6/a*polylog(3,-1/((a*x-1)/(a*x+1))^(1/2))+6/a*polylog(3
,1/((a*x-1)/(a*x+1))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(ax+1)\log(ax+1)^3-3(ax-1)\log(ax+1)^2\log(ax-1)}{8a}+\frac{1}{8}\int-\frac{(ax+1)\log(ax-1)^3-3\left((ax+1)\log(ax-1)^2+2\right.}{ax+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((a*x + 1)*log(a*x + 1)^3 - 3*(a*x - 1)*log(a*x + 1)^2*log(a*x - 1))/a
+ 1/8*integrate(-((a*x + 1)*log(a*x - 1)^3 - 3*((a*x + 1)*log(a*x - 1)^2 +
2*(a*x - 1)*log(a*x - 1))*log(a*x + 1))/(a*x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\mathrm{arcoth}\,(ax)^3\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{acoth}^3\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)**3,x)
```

```
[Out] Integral(acoth(a*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{arcoth}\,(ax)^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)^3, x)
```

3.29 $\int \frac{\coth^{-1}(ax)^3}{x}\,dx$

Optimal. Leaf size=150

$$\frac{3}{4}\mathrm{PolyLog}\left(4,1-\frac{2}{ax+1}\right)-\frac{3}{4}\mathrm{PolyLog}\left(4,1-\frac{2ax}{ax+1}\right)+\frac{3}{2}\coth^{-1}(ax)^2\mathrm{PolyLog}\left(2,1-\frac{2}{ax+1}\right)-\frac{3}{2}\coth^{-1}(ax)^2\mathrm{Po}$$

```
[Out] 2*ArcCoth[a*x]^3*ArcCoth[1 - 2/(1 - a*x)] + (3*ArcCoth[a*x]^2*PolyLog[2, 1
- 2/(1 + a*x)])/2 - (3*ArcCoth[a*x]^2*PolyLog[2, 1 - (2*a*x)/(1 + a*x)])/2
+ (3*ArcCoth[a*x]*PolyLog[3, 1 - 2/(1 + a*x)])/2 - (3*ArcCoth[a*x]*PolyLog[
3, 1 - (2*a*x)/(1 + a*x)])/2 + (3*PolyLog[4, 1 - 2/(1 + a*x)])/4 - (3*PolyL
og[4, 1 - (2*a*x)/(1 + a*x)])/4
```

Rubi [A] time = 0.350385, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5915, 6053, 5949, 6057, 6061, 6610}

$$\frac{3}{4}\mathrm{PolyLog}\left(4,1-\frac{2}{ax+1}\right)-\frac{3}{4}\mathrm{PolyLog}\left(4,1-\frac{2ax}{ax+1}\right)+\frac{3}{2}\coth^{-1}(ax)^2\mathrm{PolyLog}\left(2,1-\frac{2}{ax+1}\right)-\frac{3}{2}\coth^{-1}(ax)^2\mathrm{Po}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]^3/x,x]
```


```
[Out] 2*ArcCoth[a*x]^3*ArcCoth[1 - 2/(1 - a*x)] + (3*ArcCoth[a*x]^2*PolyLog[2, 1
- 2/(1 + a*x)])/2 - (3*ArcCoth[a*x]^2*PolyLog[2, 1 - (2*a*x)/(1 + a*x)])/2
+ (3*ArcCoth[a*x]*PolyLog[3, 1 - 2/(1 + a*x)])/2 - (3*ArcCoth[a*x]*PolyLog[
3, 1 - (2*a*x)/(1 + a*x)])/2 + (3*PolyLog[4, 1 - 2/(1 + a*x)])/4 - (3*PolyL
og[4, 1 - (2*a*x)/(1 + a*x)])/4
```


Rule 5915

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcCoth[c*x])^p*ArcCoth[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcCoth[c*x])^(p - 1)*ArcCoth[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6053

```
Int[(ArcCoth[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[SimplifyIntegrand[1 + 1/u, x]]*(a +
 b*ArcCoth[c*x])^p)/(d + e*x^2), x], x] - Dist[1/2, Int[(Log[SimplifyIntegr
and[1 - 1/u, x]]*(a + b*ArcCoth[c*x])^p)/(d + e*x^2), x], x] /; FreeQ[{a, b
, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 -
c*x))^2, 0]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6057

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcCoth[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
```

```
] - Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6061

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcCoth[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\coth^{-1}(ax)^3}{x}\,dx = 2\coth^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1-ax}\right)-(6a)\int \frac{\coth^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx$$

$$= 2\coth^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1-ax}\right)+(3a)\int \frac{\coth^{-1}(ax)^2\log\left(\frac{2}{1+ax}\right)}{1-a^2x^2}\,dx-(3a)\int \frac{\coth^{-1}(ax)^2\log\left(\right.}{1-a^2x^2}$$

$$= 2\coth^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1-ax}\right)+\frac{3}{2}\coth^{-1}(ax)^2\text{Li}_2\left(1-\frac{2}{1+ax}\right)-\frac{3}{2}\coth^{-1}(ax)^2\text{Li}_2\left(1-\frac{2ax}{1+a}\right.$$

$$= 2\coth^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1-ax}\right)+\frac{3}{2}\coth^{-1}(ax)^2\text{Li}_2\left(1-\frac{2}{1+ax}\right)-\frac{3}{2}\coth^{-1}(ax)^2\text{Li}_2\left(1-\frac{2ax}{1+a}\right.$$

$$= 2\coth^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1-ax}\right)+\frac{3}{2}\coth^{-1}(ax)^2\text{Li}_2\left(1-\frac{2}{1+ax}\right)-\frac{3}{2}\coth^{-1}(ax)^2\text{Li}_2\left(1-\frac{2ax}{1+a}\right.$$

Mathematica [A] time = 0.0758937, size = 156, normalized size = 1.04

$$\frac{1}{64}\left(-96\coth^{-1}(ax)^2\text{PolyLog}\left(2,-e^{-2\coth^{-1}(ax)}\right)-96\coth^{-1}(ax)^2\text{PolyLog}\left(2,e^{2\coth^{-1}(ax)}\right)-96\coth^{-1}(ax)\text{PolyLog}\left(3,\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[a*x]^3/x,x]
```

```
[Out] (-Pi^4 + 32*ArcCoth[a*x]^4 + 64*ArcCoth[a*x]^3*Log[1 + E^(-2*ArcCoth[a*x])]
 - 64*ArcCoth[a*x]^3*Log[1 - E^(2*ArcCoth[a*x])] - 96*ArcCoth[a*x]^2*PolyLo
g[2, -E^(-2*ArcCoth[a*x])] - 96*ArcCoth[a*x]^2*PolyLog[2, E^(2*ArcCoth[a*x]
)] - 96*ArcCoth[a*x]*PolyLog[3, -E^(-2*ArcCoth[a*x])] + 96*ArcCoth[a*x]*Pol
yLog[3, E^(2*ArcCoth[a*x])] - 48*PolyLog[4, -E^(-2*ArcCoth[a*x])] - 48*Poly
Log[4, E^(2*ArcCoth[a*x])])/64
```

Maple [C] time = 0.253, size = 564, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)^3/x,x)
```

```
[Out] ln(a*x)*arccoth(a*x)^3+arccoth(a*x)^3*ln((a*x+1)/(a*x-1)-1)+3/2*arccoth(a*x
)^2*polylog(2,-(a*x+1)/(a*x-1))-3/2*arccoth(a*x)*polylog(3,-(a*x+1)/(a*x-1)
)+3/4*polylog(4,-(a*x+1)/(a*x-1))+1/2*I*Pi*csgn(I/((a*x+1)/(a*x-1)-1)*((a*x
+1)/(a*x-1)+1))^3*arccoth(a*x)^3+1/2*I*Pi*csgn(I*((a*x+1)/(a*x-1)+1))*csgn(
I/((a*x+1)/(a*x-1)-1))*csgn(I/((a*x+1)/(a*x-1)-1)*((a*x+1)/(a*x-1)+1))*arcc
oth(a*x)^3-1/2*I*Pi*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I/((a*x+1)/(a*x-1)-1)*
((a*x+1)/(a*x-1)+1))^2*arccoth(a*x)^3-1/2*I*Pi*csgn(I*((a*x+1)/(a*x-1)+1))*
csgn(I/((a*x+1)/(a*x-1)-1)*((a*x+1)/(a*x-1)+1))^2*arccoth(a*x)^3-arccoth(a*
x)^3*ln(1-1/((a*x-1)/(a*x+1))^(1/2))-3*arccoth(a*x)^2*polylog(2,1/((a*x-1)/
(a*x+1))^(1/2))+6*arccoth(a*x)*polylog(3,1/((a*x-1)/(a*x+1))^(1/2))-6*polyl
og(4,1/((a*x-1)/(a*x+1))^(1/2))-arccoth(a*x)^3*ln(1+1/((a*x-1)/(a*x+1))^(1/
2))-3*arccoth(a*x)^2*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))+6*arccoth(a*x)*p
olylog(3,-1/((a*x-1)/(a*x+1))^(1/2))-6*polylog(4,-1/((a*x-1)/(a*x+1))^(1/2)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(ax)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x,x, algorithm="maxima")
```

```
[Out] integrate(arccoth(a*x)^3/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arcoth}(ax)^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)^3/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^3(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)**3/x,x)
```

```
[Out] Integral(acoth(a*x)**3/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(ax)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)^3/x, x)
```

3.30 $\int \frac{\coth^{-1}(ax)^3}{x^2}\,dx$

Optimal. Leaf size=79

$$-\frac{3}{2}a\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-3a\coth^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+a\coth^{-1}(ax)^3-\frac{\coth^{-1}(ax)^3}{x}+3a\log\left(2-\right.$$

```
[Out] a*ArcCoth[a*x]^3 - ArcCoth[a*x]^3/x + 3*a*ArcCoth[a*x]^2*Log[2 - 2/(1 + a*x
)] - 3*a*ArcCoth[a*x]*PolyLog[2, -1 + 2/(1 + a*x)] - (3*a*PolyLog[3, -1 + 2
/(1 + a*x)])/2
```

Rubi [A] time = 0.199122, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5917, 5989, 5933, 5949, 6057, 6610}

$$-\frac{3}{2}a\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-3a\coth^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+a\coth^{-1}(ax)^3-\frac{\coth^{-1}(ax)^3}{x}+3a\log\left(2-\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]^3/x^2,x]
```

```
[Out] a*ArcCoth[a*x]^3 - ArcCoth[a*x]^3/x + 3*a*ArcCoth[a*x]^2*Log[2 - 2/(1 + a*x
)] - 3*a*ArcCoth[a*x]*PolyLog[2, -1 + 2/(1 + a*x)] - (3*a*PolyLog[3, -1 + 2
/(1 + a*x)])/2
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5989

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcCoth[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5933

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcCoth[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6057

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcCoth[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\coth^{-1}(ax)^3}{x^2}\,dx = -\frac{\coth^{-1}(ax)^3}{x} + (3a)\int \frac{\coth^{-1}(ax)^2}{x\left(1-a^2x^2\right)}\,dx$$
$$= a\coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{x} + (3a)\int \frac{\coth^{-1}(ax)^2}{x(1+ax)}\,dx$$
$$= a\coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{x} + 3a\coth^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right) - \left(6a^2\right)\int \frac{\coth^{-1}(ax)\log\left(2-\frac{2}{1+a\ldots}\right.}{1-a^2x^2}$$
$$= a\coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{x} + 3a\coth^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right) - 3a\coth^{-1}(ax)\mathrm{Li}_2\left(-1+\frac{2}{1+ax}\right)$$
$$= a\coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{x} + 3a\coth^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right) - 3a\coth^{-1}(ax)\mathrm{Li}_2\left(-1+\frac{2}{1+ax}\right)$$

Mathematica [A] time = 0.131787, size = 72, normalized size = 0.91

$$-3a\coth^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{-2\coth^{-1}(ax)}\right) - \frac{3}{2}a\mathrm{PolyLog}\left(3,-e^{-2\coth^{-1}(ax)}\right) + \frac{(ax-1)\coth^{-1}(ax)^3}{x} + 3a\coth^{-1}(ax)^2\log$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[a*x]^3/x^2,x]
```

```
[Out] ((-1 + a*x)*ArcCoth[a*x]^3)/x + 3*a*ArcCoth[a*x]^2*Log[1 + E^(-2*ArcCoth[a*
x])] - 3*a*ArcCoth[a*x]*PolyLog[2, -E^(-2*ArcCoth[a*x])] - (3*a*PolyLog[3,
-E^(-2*ArcCoth[a*x])])/2
```

Maple [C] time = 0.342, size = 796, normalized size = 10.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)^3/x^2,x)
```

```
[Out] -arccoth(a*x)^3/x-3/2*a*arccoth(a*x)^2*ln(a*x-1)+3*a*arccoth(a*x)^2*ln(a*x)
-3/2*a*arccoth(a*x)^2*ln(a*x+1)-3/2*a*arccoth(a*x)^2*ln((a*x-1)/(a*x+1))-a*
arccoth(a*x)^3+3*a*arccoth(a*x)*polylog(2,-(a*x+1)/(a*x-1))-3/2*a*polylog(3
,-(a*x+1)/(a*x-1))+3/2*I*a*arccoth(a*x)^2*csgn(I/((a*x+1)/(a*x-1)-1)*((a*x+
```

```
1)/(a*x-1)+1))*csgn(I*((a*x+1)/(a*x-1)+1))*csgn(I/((a*x+1)/(a*x-1)-1))*Pi-3
/2*I*a*arccoth(a*x)^2*csgn(I/((a*x+1)/(a*x-1)-1)*((a*x+1)/(a*x-1)+1))^2*csg
n(I*((a*x+1)/(a*x-1)+1))*Pi-3/2*I*a*arccoth(a*x)^2*csgn(I/((a*x+1)/(a*x-1)-
1)*((a*x+1)/(a*x-1)+1))^2*csgn(I/((a*x+1)/(a*x-1)-1))*Pi+3/2*I*a*arccoth(a*
x)^2*csgn(I/((a*x+1)/(a*x-1)-1)*((a*x+1)/(a*x-1)+1))^3*Pi+3/2*I*a*arccoth(a
*x)^2*csgn(I*(a*x+1)/(a*x-1))^2*csgn(I/((a*x-1)/(a*x+1))^(1/2))*Pi-3/4*I*a*
arccoth(a*x)^2*csgn(I*(a*x+1)/(a*x-1))^3*Pi-3/4*I*a*arccoth(a*x)^2*csgn(I*(
a*x+1)/(a*x-1))*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*Pi-3/4*I*a*arccoth(a*x)^2
*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^3*Pi+3/4*I*a*arccoth(a*x)^2*cs
gn(I*(a*x+1)/(a*x-1))*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*Pi+3/4*
I*a*arccoth(a*x)^2*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1)/((a*x
+1)/(a*x-1)-1))^2*Pi-3/4*I*a*arccoth(a*x)^2*csgn(I*(a*x+1)/(a*x-1))*csgn(I/
((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))*Pi+3*a*arc
coth(a*x)^2*ln(2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/8*a*(log(a*x + 1) - log(x))*log(a)^3 + 3/8*a*integrate(x*log(a*x - 1)/(a*
x^3 + x^2), x)*log(a)^2 - 3/8*a*integrate(x*log(x)/(a*x^3 + x^2), x)*log(a)
^2 - 1/8*(a*log(a*x + 1) - a*log(x) - 1/x)*log(a)^3 + 3/4*a^2*integrate(x^2
*log(a*x + 1)*log(a*x - 1)/(a*x^3 + x^2), x) - 3/2*a^2*integrate(x^2*log(a*
x + 1)*log(x)/(a*x^3 + x^2), x) + 3/4*a*integrate(x*log(a*x - 1)*log(x)/(a*
x^3 + x^2), x)*log(a) - 3/8*a*integrate(x*log(x)^2/(a*x^3 + x^2), x)*log(a)
 + 3/8*integrate(log(a*x - 1)/(a*x^3 + x^2), x)*log(a)^2 - 3/8*integrate(lo
g(x)/(a*x^3 + x^2), x)*log(a)^2 + 3/8*a*integrate(x*log(a*x + 1)*log(a*x -
1)^2/(a*x^3 + x^2), x) - 3/8*a*integrate(x*log(a*x - 1)^2*log(x)/(a*x^3 + x
^2), x) + 3/8*a*integrate(x*log(a*x - 1)*log(x)^2/(a*x^3 + x^2), x) - 1/8*a
*integrate(x*log(x)^3/(a*x^3 + x^2), x) - 3/4*a*integrate(x*log(a*x + 1)*lo
g(a*x - 1)/(a*x^3 + x^2), x) - 3/8*integrate(a*x*log(a*x - 1)^2/(a*x^3 + x^
2), x)*log(a) - 3/8*integrate(log(a*x - 1)^2/(a*x^3 + x^2), x)*log(a) + 3/4
*integrate(log(a*x - 1)*log(x)/(a*x^3 + x^2), x)*log(a) - 3/8*integrate(log
(x)^2/(a*x^3 + x^2), x)*log(a) - 3/8*(a^2*log(a*x - 1) - a^2*log(x) + a/x)*
log(-1/(a*x) + 1)^2/a + 1/8*log(-1/(a*x) + 1)^3/x - 1/8*((a*x + 1)*log(a*x
+ 1)^3 - 3*(2*a*x*log(x) - (a*x - 1)*log(a*x - 1))*log(a*x + 1)^2)/x + 1/8*
(3*(a^3*x*log(a*x - 1)^2 + a^3*x*log(x)^2 - 2*a^3*x*log(x) + 2*a^2 - 2*(a^3
*x*log(x) - a^3*x)*log(a*x - 1))*log(-1/(a*x) + 1)/(a*x) - (a^4*x*log(a*x -
 1)^3 - a^4*x*log(x)^3 + 3*a^4*x*log(x)^2 - 6*a^4*x*log(x) + 6*a^3 - 3*(a^4
*x*log(x) - a^4*x)*log(a*x - 1)^2 + 3*(a^4*x*log(x)^2 - 2*a^4*x*log(x) + 2*
a^4*x)*log(a*x - 1))/(a^2*x))/a + 3/8*integrate(log(a*x + 1)*log(a*x - 1)^2
/(a*x^3 + x^2), x) - 3/8*integrate(log(a*x - 1)^2*log(x)/(a*x^3 + x^2), x)
+ 3/8*integrate(log(a*x - 1)*log(x)^2/(a*x^3 + x^2), x) - 1/8*integrate(log
(x)^3/(a*x^3 + x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arcoth}(ax)^3}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)^3/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^3(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)**3/x**2,x)
```

```
[Out] Integral(acoth(a*x)**3/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(ax)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)^3/x^2, x)
```

3.31 $\int \frac{\coth^{-1}(ax)^3}{x^3}\,dx$

Optimal. Leaf size=95

$$-\frac{3}{2}a^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{1}{2}a^2\coth^{-1}(ax)^3+\frac{3}{2}a^2\coth^{-1}(ax)^2+3a^2\log\left(2-\frac{2}{ax+1}\right)\coth^{-1}(ax)-\frac{\coth^{-1}(ax)}{2x^2}$$

```
[Out] (3*a^2*ArcCoth[a*x]^2)/2 - (3*a*ArcCoth[a*x]^2)/(2*x) + (a^2*ArcCoth[a*x]^3
)/2 - ArcCoth[a*x]^3/(2*x^2) + 3*a^2*ArcCoth[a*x]*Log[2 - 2/(1 + a*x)] - (3
*a^2*PolyLog[2, -1 + 2/(1 + a*x)])/2
```

Rubi [A] time = 0.219236, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5917, 5983, 5989, 5933, 2447, 5949}

$$-\frac{3}{2}a^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{1}{2}a^2\coth^{-1}(ax)^3+\frac{3}{2}a^2\coth^{-1}(ax)^2+3a^2\log\left(2-\frac{2}{ax+1}\right)\coth^{-1}(ax)-\frac{\coth^{-1}(ax)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]^3/x^3,x]
```


```
[Out] (3*a^2*ArcCoth[a*x]^2)/2 - (3*a*ArcCoth[a*x]^2)/(2*x) + (a^2*ArcCoth[a*x]^3
)/2 - ArcCoth[a*x]^3/(2*x^2) + 3*a^2*ArcCoth[a*x]*Log[2 - 2/(1 + a*x)] - (3
*a^2*PolyLog[2, -1 + 2/(1 + a*x)])/2
```


Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5983

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcCoth[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5989

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcCoth[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5933

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcCoth[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^{-1}(ax)^3}{x^3}\,dx &= -\frac{\coth^{-1}(ax)^3}{2x^2}+\frac{1}{2}(3a)\int \frac{\coth^{-1}(ax)^2}{x^2\left(1-a^2x^2\right)}\,dx\\
&= -\frac{\coth^{-1}(ax)^3}{2x^2}+\frac{1}{2}(3a)\int \frac{\coth^{-1}(ax)^2}{x^2}\,dx+\frac{1}{2}\left(3a^3\right)\int \frac{\coth^{-1}(ax)^2}{1-a^2x^2}\,dx\\
&= -\frac{3a\coth^{-1}(ax)^2}{2x}+\frac{1}{2}a^2\coth^{-1}(ax)^3-\frac{\coth^{-1}(ax)^3}{2x^2}+\left(3a^2\right)\int \frac{\coth^{-1}(ax)}{x\left(1-a^2x^2\right)}\,dx\\
&= \frac{3}{2}a^2\coth^{-1}(ax)^2-\frac{3a\coth^{-1}(ax)^2}{2x}+\frac{1}{2}a^2\coth^{-1}(ax)^3-\frac{\coth^{-1}(ax)^3}{2x^2}+\left(3a^2\right)\int \frac{\coth^{-1}(ax)}{x(1+ax)}\,dx\\
&= \frac{3}{2}a^2\coth^{-1}(ax)^2-\frac{3a\coth^{-1}(ax)^2}{2x}+\frac{1}{2}a^2\coth^{-1}(ax)^3-\frac{\coth^{-1}(ax)^3}{2x^2}+3a^2\coth^{-1}(ax)\log\left(2-\frac{}{1+}\right.\\
&= \frac{3}{2}a^2\coth^{-1}(ax)^2-\frac{3a\coth^{-1}(ax)^2}{2x}+\frac{1}{2}a^2\coth^{-1}(ax)^3-\frac{\coth^{-1}(ax)^3}{2x^2}+3a^2\coth^{-1}(ax)\log\left(2-\frac{}{1+}\right.
\end{aligned}
$$

Mathematica [A] time = 0.166635, size = 79, normalized size = 0.83

$$
\frac{1}{2}\left(\frac{\coth^{-1}(ax)\left(\left(a^2x^2-1\right)\coth^{-1}(ax)^2+6a^2x^2\log\left(e^{-2\coth^{-1}(ax)}+1\right)+3ax(ax-1)\coth^{-1}(ax)\right)}{x^2}-3a^2\text{PolyLog}\left(2,-e^{-2}\right.\right.
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[a*x]^3/x^3,x]
```

```
[Out] ((ArcCoth[a*x]*(3*a*x*(-1 + a*x)*ArcCoth[a*x] + (-1 + a^2*x^2)*ArcCoth[a*x]
^2 + 6*a^2*x^2*Log[1 + E^(-2*ArcCoth[a*x])]))/x^2 - 3*a^2*PolyLog[2, -E^(-2
*ArcCoth[a*x])])/2
```

Maple [C] time = 0.422, size = 3673, normalized size = 38.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)^3/x^3,x)
```

```
[Out] -3/4*I*a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/(a*x-1))^2*arc
coth(a*x)*ln(1+I/((a*x-1)/(a*x+1))^(1/2))+3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-
1))^3*arccoth(a*x)^2-3/16*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-
1))^3*polylog(2,-(a*x+1)/(a*x-1))+3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x
+1)/(a*x-1)-1))^3*arccoth(a*x)^2+3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1))^3*dil
og(1-I/((a*x-1)/(a*x+1))^(1/2))-3/16*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1))^3*pol
ylog(2,-(a*x+1)/(a*x-1))+3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-
1)-1))^3*dilog(1+I/((a*x-1)/(a*x+1))^(1/2))-3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*
x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I*(a*x+1)/(a*x-1))*arccoth(a*x)*ln(1+I/((a
*x-1)/(a*x+1))^(1/2))-3/4*a^2*arccoth(a*x)^2*ln(a*x-1)+3/4*a^2*arccoth(a*x)
^2*ln(a*x+1)+3/4*a^2*arccoth(a*x)^2*ln((a*x-1)/(a*x+1))+3/2*a^2*arccoth(a*x
)*ln((a*x+1)/(a*x-1)+1)+3/2*a^2*arccoth(a*x)*ln(1+I/((a*x-1)/(a*x+1))^(1/2)
)+3/2*a^2*arccoth(a*x)*ln(1-I/((a*x-1)/(a*x+1))^(1/2))-1/2*arccoth(a*x)^3/x
^2+1/2*a^2*arccoth(a*x)^3+3/2*a^2*dilog(1-I/((a*x-1)/(a*x+1))^(1/2))+3/2*a^
2*dilog(1+I/((a*x-1)/(a*x+1))^(1/2))+3/4*a^2*polylog(2,-(a*x+1)/(a*x-1))+3/
8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-
1))*csgn(I/((a*x+1)/(a*x-1)-1))*arccoth(a*x)*ln(1-I/((a*x-1)/(a*x+1))^(1/2)
)-3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(
a*x-1))*csgn(I/((a*x+1)/(a*x-1)-1))*arccoth(a*x)*ln((a*x+1)/(a*x-1)+1)+3/8*
I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1)
)*csgn(I/((a*x+1)/(a*x-1)-1))*arccoth(a*x)*ln(1+I/((a*x-1)/(a*x+1))^(1/2))-
3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I/((a*x+1)/
(a*x-1)-1))*arccoth(a*x)*ln(1-I/((a*x-1)/(a*x+1))^(1/2))+3/8*I*a^2*Pi*csgn(
I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I*(a*x+1)/(a*x-1))*arccoth(a*
x)*ln((a*x+1)/(a*x-1)+1)+3/8*I*a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csg
n(I*(a*x+1)/(a*x-1))*arccoth(a*x)*ln(1-I/((a*x-1)/(a*x+1))^(1/2))-3/8*I*a^2
*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^3*arccoth(a*x)*ln((a*x+1)/(
a*x-1)+1)+3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^3*arccot
h(a*x)*ln(1+I/((a*x-1)/(a*x+1))^(1/2))+3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/
((a*x+1)/(a*x-1)-1))^3*arccoth(a*x)*ln(1-I/((a*x-1)/(a*x+1))^(1/2))+3/8*I*a
^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/(a*x-1))^2*polylog(2,-
(a*x+1)/(a*x-1))+3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1))^3*arccoth(a*x)*ln(1-I
/((a*x-1)/(a*x+1))^(1/2))+3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1))^3*arccoth(a*
x)*ln(1+I/((a*x-1)/(a*x+1))^(1/2))-3/16*I*a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(
1/2))^2*csgn(I*(a*x+1)/(a*x-1))*polylog(2,-(a*x+1)/(a*x-1))-3/8*I*a^2*Pi*cs
gn(I*(a*x+1)/(a*x-1))^3*arccoth(a*x)*ln((a*x+1)/(a*x-1)+1)-3/4*I*a^2*Pi*csg
n(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/(a*x-1))^2*dilog(1-I/((a*x-1)/(
a*x+1))^(1/2))-3/4*I*a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/
(a*x-1))^2*dilog(1+I/((a*x-1)/(a*x+1))^(1/2))+3/8*I*a^2*Pi*csgn(I/((a*x-1)/
(a*x+1))^(1/2))^2*csgn(I*(a*x+1)/(a*x-1))*dilog(1+I/((a*x-1)/(a*x+1))^(1/2)
)+3/8*I*a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I*(a*x+1)/(a*x-1))*di
log(1-I/((a*x-1)/(a*x+1))^(1/2))-3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+
1)/(a*x-1)-1))^2*csgn(I*(a*x+1)/(a*x-1))*dilog(1-I/((a*x-1)/(a*x+1))^(1/2))
-3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I/((a*x+1)
/(a*x-1)-1))*dilog(1+I/((a*x-1)/(a*x+1))^(1/2))-3/8*I*a^2*Pi*csgn(I*(a*x+1)
/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I/((a*x+1)/(a*x-1)-1))*dilog(1-I/((a*x
-1)/(a*x+1))^(1/2))+3/16*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1
))^2*csgn(I*(a*x+1)/(a*x-1))*polylog(2,-(a*x+1)/(a*x-1))+3/16*I*a^2*Pi*csgn
(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I/((a*x+1)/(a*x-1)-1))*polyl
og(2,-(a*x+1)/(a*x-1))-3/4*I*a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*
(a*x+1)/(a*x-1))^2*arccoth(a*x)^2-3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x
+1)/(a*x-1)-1))^2*csgn(I*(a*x+1)/(a*x-1))*arccoth(a*x)^2+3/8*I*a^2*Pi*csgn(
I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I*(a*x+1)/(a*x-1))*arccoth(a*x)^2+3/8*I*a
^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^3*dilog(1-I/((a*x-1)/(a*x
+1))^(1/2))+3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1))^3*dilog(1+I/((a*x-1)/(a*x+
1))^(1/2))-3/2*a^2*arccoth(a*x)^2-3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x
+1)/(a*x-1)-1))^2*csgn(I/((a*x+1)/(a*x-1)-1))*arccoth(a*x)^2-3/8*I*a^2*Pi*c
sgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I*(a*x+1)/(a*x-1))*dilog(
1+I/((a*x-1)/(a*x+1))^(1/2))-3/4*I*a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*c
```

```
sgn(I*(a*x+1)/(a*x-1))^2*arccoth(a*x)*ln(1-I/((a*x-1)/(a*x+1))^(1/2))-3/2*a
*arccoth(a*x)^2/x+3/4*I*a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+
1)/(a*x-1))^2*arccoth(a*x)*ln((a*x+1)/(a*x-1)+1)+3/8*I*a^2*Pi*csgn(I/((a*x-
1)/(a*x+1))^(1/2))^2*csgn(I*(a*x+1)/(a*x-1))*arccoth(a*x)*ln(1+I/((a*x-1)/(
a*x+1))^(1/2))-3/16*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))*cs
gn(I*(a*x+1)/(a*x-1))*csgn(I/((a*x+1)/(a*x-1)-1))*polylog(2,-(a*x+1)/(a*x-1
))+3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/
(a*x-1))*csgn(I/((a*x+1)/(a*x-1)-1))*dilog(1+I/((a*x-1)/(a*x+1))^(1/2))+3/8
*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1
))*csgn(I/((a*x+1)/(a*x-1)-1))*dilog(1-I/((a*x-1)/(a*x+1))^(1/2))+3/8*I*a^2
*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1))*csg
n(I/((a*x+1)/(a*x-1)-1))*arccoth(a*x)^2-3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)
/((a*x+1)/(a*x-1)-1))^2*csgn(I*(a*x+1)/(a*x-1))*arccoth(a*x)*ln(1-I/((a*x-1
)/(a*x+1))^(1/2))-3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^
2*csgn(I/((a*x+1)/(a*x-1)-1))*arccoth(a*x)*ln(1+I/((a*x-1)/(a*x+1))^(1/2))-
3/8*I*a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I*(a*x+1)/(a*x-1))*arcc
oth(a*x)*ln((a*x+1)/(a*x-1)+1)+3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)
/(a*x-1)-1))^2*csgn(I/((a*x+1)/(a*x-1)-1))*arccoth(a*x)*ln((a*x+1)/(a*x-1)+
1)
```

Maxima [B] time = 0.991751, size = 340, normalized size = 3.58

$$\frac{3}{4}\left(a\log\left(ax+1\right)-a\log\left(ax-1\right)-\frac{2}{x}\right)a\operatorname{arcoth}\left(ax\right)^{2}-\frac{1}{16}\left(a^{2}\left(\frac{3\left(\log\left(ax-1\right)-2\right)\log\left(ax+1\right)^{2}-\log\left(ax+1\right)^{3}+\log\left(ax\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x^3,x, algorithm="maxima")
```

```
[Out] 3/4*(a*log(a*x + 1) - a*log(a*x - 1) - 2/x)*a*arccoth(a*x)^2 - 1/16*(a^2*((
3*(log(a*x - 1) - 2)*log(a*x + 1)^2 - log(a*x + 1)^3 + log(a*x - 1)^3 - 3*(
log(a*x - 1)^2 - 4*log(a*x - 1))*log(a*x + 1) + 6*log(a*x - 1)^2)/a - 24*(l
og(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a + 24*(log(a*x + 1
)*log(x) + dilog(-a*x))/a - 24*(log(-a*x + 1)*log(x) + dilog(a*x))/a) - 6*(
2*(log(a*x - 1) - 2)*log(a*x + 1) - log(a*x + 1)^2 - log(a*x - 1)^2 - 4*log
(a*x - 1) + 8*log(x))*a*arccoth(a*x))*a - 1/2*arccoth(a*x)^3/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arcoth}\left(ax\right)^{3}}{x^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x^3,x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)^3/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^{3}\left(ax\right)}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)**3/x**3,x)
```

```
[Out] Integral(acoth(a*x)**3/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(ax)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x^3,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)^3/x^3, x)
```

3.32 $\int \frac{\coth^{-1}(ax)^3}{x^4} \, dx$

Optimal. Leaf size=154

$$-\frac{1}{2}a^3\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-a^3\coth^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{1}{2}a^3\log\left(1-a^2x^2\right)+a^3\log(x)+\frac{1}{3}a^3\coth^{-1}(a$$

```
[Out] -((a^2*ArcCoth[a*x])/x) + (a^3*ArcCoth[a*x]^2)/2 - (a*ArcCoth[a*x]^2)/(2*x^
2) + (a^3*ArcCoth[a*x]^3)/3 - ArcCoth[a*x]^3/(3*x^3) + a^3*Log[x] - (a^3*Lo
g[1 - a^2*x^2])/2 + a^3*ArcCoth[a*x]^2*Log[2 - 2/(1 + a*x)] - a^3*ArcCoth[a
*x]*PolyLog[2, -1 + 2/(1 + a*x)] - (a^3*PolyLog[3, -1 + 2/(1 + a*x)])/2
```

Rubi [A] time = 0.368827, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.1, Rules used = {5917, 5983, 266, 36, 29, 31, 5949, 5989, 5933, 6057, 6610}

$$-\frac{1}{2}a^3\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-a^3\coth^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{1}{2}a^3\log\left(1-a^2x^2\right)+a^3\log(x)+\frac{1}{3}a^3\coth^{-1}(a$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]^3/x^4,x]
```


```
[Out] -((a^2*ArcCoth[a*x])/x) + (a^3*ArcCoth[a*x]^2)/2 - (a*ArcCoth[a*x]^2)/(2*x^
2) + (a^3*ArcCoth[a*x]^3)/3 - ArcCoth[a*x]^3/(3*x^3) + a^3*Log[x] - (a^3*Lo
g[1 - a^2*x^2])/2 + a^3*ArcCoth[a*x]^2*Log[2 - 2/(1 + a*x)] - a^3*ArcCoth[a
*x]*PolyLog[2, -1 + 2/(1 + a*x)] - (a^3*PolyLog[3, -1 + 2/(1 + a*x)])/2
```


Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5983

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcCoth[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5989

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcCoth[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5933

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcCoth[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6057

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcCoth[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\coth^{-1}(ax)^3}{x^4}dx = -\frac{\coth^{-1}(ax)^3}{3x^3} + a\int \frac{\coth^{-1}(ax)^2}{x^3(1-a^2x^2)}dx$$
$$= -\frac{\coth^{-1}(ax)^3}{3x^3} + a\int \frac{\coth^{-1}(ax)^2}{x^3}dx + a^3\int \frac{\coth^{-1}(ax)^2}{x(1-a^2x^2)}dx$$
$$= -\frac{a\coth^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^3\coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{3x^3} + a^2\int \frac{\coth^{-1}(ax)}{x^2(1-a^2x^2)}dx + a^3\int \frac{\coth^{-1}(ax)^2}{x(1+ax)}dx$$
$$= -\frac{a\coth^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^3\coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{3x^3} + a^3\coth^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right) + a^2\int \frac{\coth^{-1}}{x}$$
$$= -\frac{a^2\coth^{-1}(ax)}{x} + \frac{1}{2}a^3\coth^{-1}(ax)^2 - \frac{a\coth^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^3\coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{3x^3} + a^3\coth^{-1}$$
$$= -\frac{a^2\coth^{-1}(ax)}{x} + \frac{1}{2}a^3\coth^{-1}(ax)^2 - \frac{a\coth^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^3\coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{3x^3} + a^3\coth^{-1}$$
$$= -\frac{a^2\coth^{-1}(ax)}{x} + \frac{1}{2}a^3\coth^{-1}(ax)^2 - \frac{a\coth^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^3\coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{3x^3} + a^3\coth^{-1}$$
$$= -\frac{a^2\coth^{-1}(ax)}{x} + \frac{1}{2}a^3\coth^{-1}(ax)^2 - \frac{a\coth^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^3\coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{3x^3} + a^3\log(x)$$

Mathematica [A] time = 0.21288, size = 142, normalized size = 0.92

$$\frac{1}{6}\left(-6a^3\coth^{-1}(ax)\text{PolyLog}\left(2,-e^{-2\coth^{-1}(ax)}\right) - 3a^3\text{PolyLog}\left(3,-e^{-2\coth^{-1}(ax)}\right) + 6a^3\log\left(\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}\right) + 2a^3\coth^{-1}(ax\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[a*x]^3/x^4,x]
```

```
[Out] ((-6*a^2*ArcCoth[a*x])/x + 3*a^3*ArcCoth[a*x]^2 - (3*a*ArcCoth[a*x]^2)/x^2
+ 2*a^3*ArcCoth[a*x]^3 - (2*ArcCoth[a*x]^3)/x^3 + 6*a^3*ArcCoth[a*x]^2*Log[
1 + E^(-2*ArcCoth[a*x])] + 6*a^3*Log[1/Sqrt[1 - 1/(a^2*x^2)]] - 6*a^3*ArcCo
th[a*x]*PolyLog[2, -E^(-2*ArcCoth[a*x])] - 3*a^3*PolyLog[3, -E^(-2*ArcCoth[
a*x])])/6
```

Maple [C] time = 0.657, size = 895, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)^3/x^4,x)
```

```
[Out] -a^3*arccoth(a*x)-1/2*a^3*polylog(3,-(a*x+1)/(a*x-1))+a^3*ln((a*x+1)/(a*x-1
)+1)-1/3*arccoth(a*x)^3/x^3-a^2*arccoth(a*x)/x-1/2*a^3*arccoth(a*x)^2*ln(a*
x-1)+a^3*arccoth(a*x)^2*ln(a*x)+a^3*arccoth(a*x)*polylog(2,-(a*x+1)/(a*x-1)
)-1/2*a^3*arccoth(a*x)^2*ln(a*x+1)-1/2*a^3*arccoth(a*x)^2*ln((a*x-1)/(a*x+1
))+a^3*arccoth(a*x)^2*ln(2)+1/2*a^3*arccoth(a*x)^2-1/4*I*a^3*arccoth(a*x)^2
*Pi*csgn(I*(a*x+1)/(a*x-1))^3+1/2*I*a^3*arccoth(a*x)^2*Pi*csgn(I/((a*x+1)/(
a*x-1)-1)*((a*x+1)/(a*x-1)+1))^3-1/2*a*arccoth(a*x)^2/x^2-1/3*a^3*arccoth(a
```

```
*x)^3-1/4*I*a^3*arccoth(a*x)^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1
))^3+1/2*I*a^3*arccoth(a*x)^2*Pi*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I/((a*x+1
)/(a*x-1)-1)*((a*x+1)/(a*x-1)+1))*csgn(I*((a*x+1)/(a*x-1)+1))-1/4*I*a^3*arc
coth(a*x)^2*Pi*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/
(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1))+1/4*I*a^3*arccoth(a*x)^2*Pi*csgn(I*(a*x
+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I*(a*x+1)/(a*x-1))+1/4*I*a^3*arccot
h(a*x)^2*Pi*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*
x-1)-1))^2+1/2*I*a^3*arccoth(a*x)^2*Pi*csgn(I*(a*x+1)/(a*x-1))^2*csgn(I/((a
*x-1)/(a*x+1))^(1/2))-1/4*I*a^3*arccoth(a*x)^2*Pi*csgn(I*(a*x+1)/(a*x-1))*c
sgn(I/((a*x-1)/(a*x+1))^(1/2))^2-1/2*I*a^3*arccoth(a*x)^2*Pi*csgn(I/((a*x+1
)/(a*x-1)-1))*csgn(I/((a*x+1)/(a*x-1)-1)*((a*x+1)/(a*x-1)+1))^2-1/2*I*a^3*a
rccoth(a*x)^2*Pi*csgn(I/((a*x+1)/(a*x-1)-1)*((a*x+1)/(a*x-1)+1))^2*csgn(I*(
(a*x+1)/(a*x-1)+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/4*a^4*integrate(x^4*log(a*x + 1)*log(a*x - 1)/(a*x^5 + x^4), x) - 1/2*a^4
*integrate(x^4*log(a*x + 1)*log(x)/(a*x^5 + x^4), x) + 1/16*(2*a^2*log(a*x
+ 1) - 2*a^2*log(x) - (2*a*x - 1)/x^2)*a*log(a)^3 + 3/8*a*integrate(x*log(a
*x - 1)/(a*x^5 + x^4), x)*log(a)^2 - 3/8*a*integrate(x*log(x)/(a*x^5 + x^4)
, x)*log(a)^2 - 1/48*(6*a^3*log(a*x + 1) - 6*a^3*log(x) - (6*a^2*x^2 - 3*a*
x + 2)/x^3)*log(a)^3 + 1/4*a^2*integrate(x^2*log(a*x + 1)/(a*x^5 + x^4), x)
 + 3/4*a*integrate(x*log(a*x - 1)*log(x)/(a*x^5 + x^4), x)*log(a) - 3/8*a*i
ntegrate(x*log(x)^2/(a*x^5 + x^4), x)*log(a) + 3/8*integrate(log(a*x - 1)/(
a*x^5 + x^4), x)*log(a)^2 - 3/8*integrate(log(x)/(a*x^5 + x^4), x)*log(a)^2
 + 3/8*a*integrate(x*log(a*x + 1)*log(a*x - 1)^2/(a*x^5 + x^4), x) - 3/8*a*
integrate(x*log(a*x - 1)^2*log(x)/(a*x^5 + x^4), x) + 3/8*a*integrate(x*log
(a*x - 1)*log(x)^2/(a*x^5 + x^4), x) - 1/8*a*integrate(x*log(x)^3/(a*x^5 +
x^4), x) - 1/4*a*integrate(x*log(a*x + 1)*log(a*x - 1)/(a*x^5 + x^4), x) -
3/8*integrate(a*x*log(a*x - 1)^2/(a*x^5 + x^4), x)*log(a) - 3/8*integrate(l
og(a*x - 1)^2/(a*x^5 + x^4), x)*log(a) + 3/4*integrate(log(a*x - 1)*log(x)/
(a*x^5 + x^4), x)*log(a) - 3/8*integrate(log(x)^2/(a*x^5 + x^4), x)*log(a)
- 1/48*(6*a^4*log(a*x - 1) - 6*a^4*log(x) + (6*a^3*x^2 + 3*a^2*x + 2*a)/x^3
)*log(-1/(a*x) + 1)^2/a + 1/864*(6*(18*a^5*x^3*log(a*x - 1)^2 + 18*a^5*x^3*
log(x)^2 - 66*a^5*x^3*log(x) + 66*a^4*x^2 + 15*a^3*x + 4*a^2 - 6*(6*a^5*x^3
*log(x) - 11*a^5*x^3)*log(a*x - 1))*log(-1/(a*x) + 1)/(a*x^3) - (36*a^6*x^3
*log(a*x - 1)^3 - 36*a^6*x^3*log(x)^3 + 198*a^6*x^3*log(x)^2 - 510*a^6*x^3*
log(x) + 510*a^5*x^2 + 57*a^4*x + 8*a^3 - 18*(6*a^6*x^3*log(x) - 11*a^6*x^3
)*log(a*x - 1)^2 + 6*(18*a^6*x^3*log(x)^2 - 66*a^6*x^3*log(x) + 85*a^6*x^3)
*log(a*x - 1))/(a^2*x^3))/a + 1/24*log(-1/(a*x) + 1)^3/x^3 - 1/24*((a^3*x^3
 + 1)*log(a*x + 1)^3 - 3*(2*a^3*x^3*log(x) - a*x - (a^3*x^3 - 1)*log(a*x -
1))*log(a*x + 1)^2)/x^3 + 3/8*integrate(log(a*x + 1)*log(a*x - 1)^2/(a*x^5
+ x^4), x) - 3/8*integrate(log(a*x - 1)^2*log(x)/(a*x^5 + x^4), x) + 3/8*in
tegrate(log(a*x - 1)*log(x)^2/(a*x^5 + x^4), x) - 1/8*integrate(log(x)^3/(a
*x^5 + x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arcoth}(ax)^3}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x^4,x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)^3/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^3(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)**3/x**4,x)
```

```
[Out] Integral(acoth(a*x)**3/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(ax)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x^4,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)^3/x^4, x)
```

3.33 $\int \frac{\coth^{-1}(ax)^3}{x^5} dx$

Optimal. Leaf size=141

$$-a^4\text{PolyLog}\left(2, \frac{2}{ax+1} - 1\right) - \frac{a^2\coth^{-1}(ax)}{4x^2} - \frac{a^3}{4x} + \frac{1}{4}a^4\tanh^{-1}(ax) + \frac{1}{4}a^4\coth^{-1}(ax)^3 + a^4\coth^{-1}(ax)^2 - \frac{3a^3\coth}{4}$$

```
[Out] -a^3/(4*x) - (a^2*ArcCoth[a*x])/(4*x^2) + a^4*ArcCoth[a*x]^2 - (a*ArcCoth[a
*x]^2)/(4*x^3) - (3*a^3*ArcCoth[a*x]^2)/(4*x) + (a^4*ArcCoth[a*x]^3)/4 - Ar
cCoth[a*x]^3/(4*x^4) + (a^4*ArcTanh[a*x])/4 + 2*a^4*ArcCoth[a*x]*Log[2 - 2/
(1 + a*x)] - a^4*PolyLog[2, -1 + 2/(1 + a*x)]
```

Rubi [A] time = 0.463885, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.8, Rules used = {5917, 5983, 325, 206, 5989, 5933, 2447, 5949}

$$-a^4\text{PolyLog}\left(2, \frac{2}{ax+1} - 1\right) - \frac{a^2\coth^{-1}(ax)}{4x^2} - \frac{a^3}{4x} + \frac{1}{4}a^4\tanh^{-1}(ax) + \frac{1}{4}a^4\coth^{-1}(ax)^3 + a^4\coth^{-1}(ax)^2 - \frac{3a^3\coth}{4}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]^3/x^5,x]
```


```
[Out] -a^3/(4*x) - (a^2*ArcCoth[a*x])/(4*x^2) + a^4*ArcCoth[a*x]^2 - (a*ArcCoth[a
*x]^2)/(4*x^3) - (3*a^3*ArcCoth[a*x]^2)/(4*x) + (a^4*ArcCoth[a*x]^3)/4 - Ar
cCoth[a*x]^3/(4*x^4) + (a^4*ArcTanh[a*x])/4 + 2*a^4*ArcCoth[a*x]*Log[2 - 2/
(1 + a*x)] - a^4*PolyLog[2, -1 + 2/(1 + a*x)]
```


Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5983

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcCoth[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 5989

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcCoth[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5933

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcCoth[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\coth^{-1}(ax)^3}{x^5}\,dx = -\frac{\coth^{-1}(ax)^3}{4x^4} + \frac{1}{4}(3a)\int \frac{\coth^{-1}(ax)^2}{x^4\left(1-a^2x^2\right)}\,dx$$

$$= -\frac{\coth^{-1}(ax)^3}{4x^4} + \frac{1}{4}(3a)\int \frac{\coth^{-1}(ax)^2}{x^4}\,dx + \frac{1}{4}\left(3a^3\right)\int \frac{\coth^{-1}(ax)^2}{x^2\left(1-a^2x^2\right)}\,dx$$

$$= -\frac{a\coth^{-1}(ax)^2}{4x^3} - \frac{\coth^{-1}(ax)^3}{4x^4} + \frac{1}{2}a^2\int \frac{\coth^{-1}(ax)}{x^3\left(1-a^2x^2\right)}\,dx + \frac{1}{4}\left(3a^3\right)\int \frac{\coth^{-1}(ax)^2}{x^2}\,dx + \frac{1}{4}\left(3a^5\right)\int$$

$$= -\frac{a\coth^{-1}(ax)^2}{4x^3} - \frac{3a^3\coth^{-1}(ax)^2}{4x} + \frac{1}{4}a^4\coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{4x^4} + \frac{1}{2}a^2\int \frac{\coth^{-1}(ax)}{x^3}\,dx + \frac{1}{2}$$

$$= -\frac{a^2\coth^{-1}(ax)}{4x^2} + a^4\coth^{-1}(ax)^2 - \frac{a\coth^{-1}(ax)^2}{4x^3} - \frac{3a^3\coth^{-1}(ax)^2}{4x} + \frac{1}{4}a^4\coth^{-1}(ax)^3 - \frac{\coth^{-1}(}{4x^4}$$

$$= -\frac{a^3}{4x} - \frac{a^2\coth^{-1}(ax)}{4x^2} + a^4\coth^{-1}(ax)^2 - \frac{a\coth^{-1}(ax)^2}{4x^3} - \frac{3a^3\coth^{-1}(ax)^2}{4x} + \frac{1}{4}a^4\coth^{-1}(ax)^3 -$$

$$= -\frac{a^3}{4x} - \frac{a^2\coth^{-1}(ax)}{4x^2} + a^4\coth^{-1}(ax)^2 - \frac{a\coth^{-1}(ax)^2}{4x^3} - \frac{3a^3\coth^{-1}(ax)^2}{4x} + \frac{1}{4}a^4\coth^{-1}(ax)^3 -$$

Mathematica [A] time = 0.230209, size = 118, normalized size = 0.84

$$\frac{-4a^4x^4\mathrm{PolyLog}\left(2,-e^{-2\coth^{-1}(ax)}\right) - a^3x^3 + ax\left(4a^3x^3-3a^2x^2-1\right)\coth^{-1}(ax)^2 + \left(a^4x^4-1\right)\coth^{-1}(ax)^3 + a^2x^2\coth^{-1}(}{4x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[a*x]^3/x^5,x]
```

```
[Out] (-(a^3*x^3) + a*x*(-1 - 3*a^2*x^2 + 4*a^3*x^3)*ArcCoth[a*x]^2 + (-1 + a^4*x
^4)*ArcCoth[a*x]^3 + a^2*x^2*ArcCoth[a*x]*(-1 + a^2*x^2 + 8*a^2*x^2*Log[1 +
 E^(-2*ArcCoth[a*x])]) - 4*a^4*x^4*PolyLog[2, -E^(-2*ArcCoth[a*x])])/(4*x^4
)
```

Maple [C] time = 0.536, size = 661, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)^3/x^5,x)
```

```
[Out] 1/4*a^4-3/8*a^4*arccoth(a*x)^2*ln(a*x-1)+3/8*a^4*arccoth(a*x)^2*ln(a*x+1)+2
*a^4*arccoth(a*x)*ln(1+I/((a*x-1)/(a*x+1))^(1/2))+3/8*a^4*arccoth(a*x)^2*ln
((a*x-1)/(a*x+1))+2*a^4*arccoth(a*x)*ln(1-I/((a*x-1)/(a*x+1))^(1/2))+1/4*a^
4*arccoth(a*x)+1/4*a^4*arccoth(a*x)^3+3/16*I*a^4*Pi*csgn(I/((a*x-1)/(a*x+1)
)^(1/2))^2*csgn(I*(a*x+1)/(a*x-1))*arccoth(a*x)^2+3/16*I*a^4*Pi*csgn(I*(a*x
+1)/(a*x-1))^3*arccoth(a*x)^2-1/4*a^2*arccoth(a*x)/x^2-3/8*I*a^4*Pi*csgn(I/
((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/(a*x-1))^2*arccoth(a*x)^2+3/16*I*a^
4*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1))*cs
gn(I/((a*x+1)/(a*x-1)-1))*arccoth(a*x)^2+3/16*I*a^4*Pi*csgn(I*(a*x+1)/(a*x-
1)/((a*x+1)/(a*x-1)-1))^3*arccoth(a*x)^2-3/16*I*a^4*Pi*csgn(I*(a*x+1)/(a*x-
1)/((a*x+1)/(a*x-1)-1))^2*csgn(I/((a*x+1)/(a*x-1)-1))*arccoth(a*x)^2-1/4*ar
ccoth(a*x)^3/x^4-1/4*a*arccoth(a*x)^2/x^3-3/4*a^3*arccoth(a*x)^2/x-1/4*a^3/
x-a^4*arccoth(a*x)^2-3/16*I*a^4*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-
1))^2*csgn(I*(a*x+1)/(a*x-1))*arccoth(a*x)^2+2*a^4*dilog(1+I/((a*x-1)/(a*x+
1))^(1/2))+2*a^4*dilog(1-I/((a*x-1)/(a*x+1))^(1/2))
```

Maxima [B] time = 1.01002, size = 462, normalized size = 3.28

$$\frac{1}{8}\left(3\,a^3\log\left(ax+1\right)-3\,a^3\log\left(ax-1\right)-\frac{2\left(3\,a^2x^2+1\right)}{x^3}\right)a\,\mathrm{arcoth}\left(ax\right)^2+\frac{1}{32}\left(\left(32\left(\log\left(ax-1\right)\log\left(\frac{1}{2}\,ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*(3*a^3*log(a*x + 1) - 3*a^3*log(a*x - 1) - 2*(3*a^2*x^2 + 1)/x^3)*a*arc
coth(a*x)^2 + 1/32*((32*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x +
 1/2))*a - 32*(log(a*x + 1)*log(x) + dilog(-a*x))*a + 32*(log(-a*x + 1)*log
(x) + dilog(a*x))*a + 4*a*log(a*x + 1) - 4*a*log(a*x - 1) + (a*x*log(a*x +
1)^3 - a*x*log(a*x - 1)^3 - 8*a*x*log(a*x - 1)^2 - (3*a*x*log(a*x - 1) - 8*
a*x)*log(a*x + 1)^2 + (3*a*x*log(a*x - 1)^2 - 16*a*x*log(a*x - 1))*log(a*x
+ 1) - 8)/x)*a^2 + 2*(32*a^2*log(x) - (3*a^2*x^2*log(a*x + 1)^2 + 3*a^2*x^2
*log(a*x - 1)^2 + 16*a^2*x^2*log(a*x - 1) - 2*(3*a^2*x^2*log(a*x - 1) - 8*a
^2*x^2)*log(a*x + 1) + 4)/x^2)*a*arccoth(a*x))*a - 1/4*arccoth(a*x)^3/x^4
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arcoth}\,(ax)^3}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x^5,x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)^3/x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{acoth}^3\,(ax)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)**3/x**5,x)
```

```
[Out] Integral(acoth(a*x)**3/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arcoth}\,(ax)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x^5,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)^3/x^5, x)
```

3.34 $\int \frac{\coth^{-1}(cx)^2}{d+ex}\,dx$

Optimal. Leaf size=164

$$-\frac{\text{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{2e}-\frac{\coth^{-1}(cx)\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e}+\frac{\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2e}+\frac{\coth^{-1}(cx)\text{PolyL}}{e}$$

```
[Out] -((ArcCoth[c*x]^2*Log[2/(1 + c*x)])/e) + (ArcCoth[c*x]^2*Log[(2*c*(d + e*x)
)/((c*d + e)*(1 + c*x))])/e + (ArcCoth[c*x]*PolyLog[2, 1 - 2/(1 + c*x)])/e
- (ArcCoth[c*x]*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e +
PolyLog[3, 1 - 2/(1 + c*x)]/(2*e) - PolyLog[3, 1 - (2*c*(d + e*x))/((c*d +
e)*(1 + c*x))]/(2*e)
```

Rubi [A] time = 0.0337196, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5923}

$$-\frac{\text{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{2e}-\frac{\coth^{-1}(cx)\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e}+\frac{\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2e}+\frac{\coth^{-1}(cx)\text{PolyL}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[c*x]^2/(d + e*x),x]
```

```
[Out] -((ArcCoth[c*x]^2*Log[2/(1 + c*x)])/e) + (ArcCoth[c*x]^2*Log[(2*c*(d + e*x)
)/((c*d + e)*(1 + c*x))])/e + (ArcCoth[c*x]*PolyLog[2, 1 - 2/(1 + c*x)])/e
- (ArcCoth[c*x]*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e +
PolyLog[3, 1 - 2/(1 + c*x)]/(2*e) - PolyLog[3, 1 - (2*c*(d + e*x))/((c*d +
e)*(1 + c*x))]/(2*e)
```

Rule 5923

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 -Simp[((a + b*ArcCoth[c*x])^2*Log[2/(1 + c*x)])/e, x] + (Simp[((a + b*ArcC
oth[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Simp[(b*(a
+ b*ArcCoth[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e, x] - Simp[(b*(a + b*ArcCo
th[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Sim
p[(b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e), x] - Simp[(b^2*PolyLog[3, 1 - (2
*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e), x]) /; FreeQ[{a, b, c, d, e},
x] && NeQ[c^2*d^2 - e^2, 0]
```

Rubi steps

$$\int \frac{\coth^{-1}(cx)^2}{d+ex}\,dx = -\frac{\coth^{-1}(cx)^2\log\left(\frac{2}{1+cx}\right)}{e}+\frac{\coth^{-1}(cx)^2\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e}+\frac{\coth^{-1}(cx)\text{Li}_2\left(1-\frac{2}{1+cx}\right)}{e}-\frac{\coth^{-1}(c}{}$$

Mathematica [C] time = 7.68431, size = 741, normalized size = 4.52

$$24(e-cd)(cd+e)\left(6e\coth^{-1}(cx)\text{PolyLog}\left(2,\frac{(cd+e)e^{2\coth^{-1}(cx)}}{cd-e}\right)-3e\text{PolyLog}\left(3,\frac{(cd+e)e^{2\coth^{-1}(cx)}}{cd-e}\right)-12e\coth^{-1}(cx)\text{PolyLog}\left(2,-e^{\tanh^{-1}\left(\frac{e}{cd}\right)+\coth^{-1}(cx)}\right)-12e\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[c*x]^2/(d + e*x),x]
```

```
[Out] ((-I)*e*Pi^3 + 8*c*d*ArcCoth[c*x]^3 + 8*e*ArcCoth[c*x]^3 - 24*e*ArcCoth[c*x
]^2*Log[1 - E^(2*ArcCoth[c*x])] - 24*e*ArcCoth[c*x]*PolyLog[2, E^(2*ArcCoth
[c*x])] + 12*e*PolyLog[3, E^(2*ArcCoth[c*x])] + (24*(-(c*d) + e)*(c*d + e)*
(-2*c*d*ArcCoth[c*x]^3 + 6*e*ArcCoth[c*x]^3 + (4*c*d*Sqrt[1 - e^2/(c^2*d^2)
]*ArcCoth[c*x]^3)/E^ArcTanh[e/(c*d)] + (6*I)*e*Pi*ArcCoth[c*x]*Log[(E^(-Arc
Coth[c*x]) + E^ArcCoth[c*x])/2] + 6*e*ArcCoth[c*x]^2*Log[1 + ((c*d + e)*E^(
2*ArcCoth[c*x]))/(-(c*d) + e)] - 6*e*ArcCoth[c*x]^2*Log[1 - E^(ArcCoth[c*x]
 + ArcTanh[e/(c*d)])] - 6*e*ArcCoth[c*x]^2*Log[1 + E^(ArcCoth[c*x] + ArcTan
h[e/(c*d)])] - 6*e*ArcCoth[c*x]^2*Log[1 - E^(2*(ArcCoth[c*x] + ArcTanh[e/(c
*d)]))] - 12*e*ArcCoth[c*x]*ArcTanh[e/(c*d)]*Log[(I/2)*E^(-ArcCoth[c*x] - A
rcTanh[e/(c*d)])*(-1 + E^(2*(ArcCoth[c*x] + ArcTanh[e/(c*d)])))] - 6*e*ArcC
oth[c*x]^2*Log[(c*d*(-1 + E^(2*ArcCoth[c*x])) + e*(1 + E^(2*ArcCoth[c*x])))
/(2*E^ArcCoth[c*x])] - (6*I)*e*Pi*ArcCoth[c*x]*Log[1/Sqrt[1 - 1/(c^2*x^2)]]
 + 6*e*ArcCoth[c*x]^2*Log[(d + e*x)/(Sqrt[1 - 1/(c^2*x^2)]*x)] + 12*e*ArcCo
th[c*x]*ArcTanh[e/(c*d)]*Log[I*Sinh[ArcCoth[c*x] + ArcTanh[e/(c*d)]]] + 6*e
*ArcCoth[c*x]*PolyLog[2, ((c*d + e)*E^(2*ArcCoth[c*x]))/(c*d - e)] - 12*e*A
rcCoth[c*x]*PolyLog[2, -E^(ArcCoth[c*x] + ArcTanh[e/(c*d)])] - 12*e*ArcCoth
[c*x]*PolyLog[2, E^(ArcCoth[c*x] + ArcTanh[e/(c*d)])] - 6*e*ArcCoth[c*x]*Po
lyLog[2, E^(2*(ArcCoth[c*x] + ArcTanh[e/(c*d)]))] - 3*e*PolyLog[3, ((c*d +
e)*E^(2*ArcCoth[c*x]))/(c*d - e)] + 12*e*PolyLog[3, -E^(ArcCoth[c*x] + ArcT
anh[e/(c*d)])] + 12*e*PolyLog[3, E^(ArcCoth[c*x] + ArcTanh[e/(c*d)])] + 3*e
*PolyLog[3, E^(2*(ArcCoth[c*x] + ArcTanh[e/(c*d)]))]))/(6*c^2*d^2 - 6*e^2))
/(24*e^2)
```

Maple [C] time = 0.625, size = 926, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(c*x)^2/(e*x+d),x)
```

```
[Out] ln(c*e*x+c*d)/e*arccoth(c*x)^2-1/e*arccoth(c*x)^2*ln(c*d*((c*x+1)/(c*x-1)-1
)+((c*x+1)/(c*x-1)+1)*e)+1/e*arccoth(c*x)^2*ln((c*x+1)/(c*x-1)-1)-1/e*arcco
th(c*x)^2*ln(1-1/((c*x-1)/(c*x+1))^(1/2))-2/e*arccoth(c*x)*polylog(2,1/((c*
x-1)/(c*x+1))^(1/2))+2/e*polylog(3,1/((c*x-1)/(c*x+1))^(1/2))-1/e*arccoth(c
*x)^2*ln(1+1/((c*x-1)/(c*x+1))^(1/2))-2/e*arccoth(c*x)*polylog(2,-1/((c*x-1
)/(c*x+1))^(1/2))+2/e*polylog(3,-1/((c*x-1)/(c*x+1))^(1/2))-1/2*I/e*Pi*arcc
oth(c*x)^2*csgn(I*(c*d*((c*x+1)/(c*x-1)-1)+((c*x+1)/(c*x-1)+1)*e)/((c*x+1)/
(c*x-1)-1))^2*csgn(I/((c*x+1)/(c*x-1)-1))+1/2*I/e*Pi*arccoth(c*x)^2*csgn(I*
(c*d*((c*x+1)/(c*x-1)-1)+((c*x+1)/(c*x-1)+1)*e)/((c*x+1)/(c*x-1)-1))^3+1/2*
I/e*Pi*arccoth(c*x)^2*csgn(I*(c*d*((c*x+1)/(c*x-1)-1)+((c*x+1)/(c*x-1)+1)*e
))*csgn(I*(c*d*((c*x+1)/(c*x-1)-1)+((c*x+1)/(c*x-1)+1)*e)/((c*x+1)/(c*x-1)-
1))*csgn(I/((c*x+1)/(c*x-1)-1))-1/2*I/e*Pi*arccoth(c*x)^2*csgn(I*(c*d*((c*x
+1)/(c*x-1)-1)+((c*x+1)/(c*x-1)+1)*e))*csgn(I*(c*d*((c*x+1)/(c*x-1)-1)+((c*
x+1)/(c*x-1)+1)*e)/((c*x+1)/(c*x-1)-1))^2+1/(c*d+e)*arccoth(c*x)^2*ln(1-(c*
d+e)/(c*d-e)*(c*x+1)/(c*x-1))+1/(c*d+e)*arccoth(c*x)*polylog(2,(c*d+e)/(c*d
-e)*(c*x+1)/(c*x-1))-1/2/(c*d+e)*polylog(3,(c*d+e)/(c*d-e)*(c*x+1)/(c*x-1))
+c/e*d/(c*d+e)*arccoth(c*x)^2*ln(1-(c*d+e)/(c*d-e)*(c*x+1)/(c*x-1))+c/e*d/(
c*d+e)*arccoth(c*x)*polylog(2,(c*d+e)/(c*d-e)*(c*x+1)/(c*x-1))-1/2*c/e*d/(c
*d+e)*polylog(3,(c*d+e)/(c*d-e)*(c*x+1)/(c*x-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(cx)^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c*x)^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(arccoth(c*x)^2/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arcoth}(cx)^2}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c*x)^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(arccoth(c*x)^2/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^2(cx)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(c*x)**2/(e*x+d),x)
```

```
[Out] Integral(acoth(c*x)**2/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(cx)^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c*x)^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(arccoth(c*x)^2/(e*x + d), x)
```

3.35 $\int (c + dx^2)^4 \coth^{-1}(ax)\,dx$

Optimal. Leaf size=245

$$\frac{d^2x^4(378a^4c^2+180a^2cd+35d^2)}{1260a^5}+\frac{dx^2(378a^4c^2d+420a^6c^3+180a^2cd^2+35d^3)}{630a^7}+\frac{(378a^4c^2d^2+420a^6c^3d+315a^8c^4+\cdots}{630a\cdots}$$

```
[Out] (d*(420*a^6*c^3 + 378*a^4*c^2*d + 180*a^2*c*d^2 + 35*d^3)*x^2)/(630*a^7) +
(d^2*(378*a^4*c^2 + 180*a^2*c*d + 35*d^2)*x^4)/(1260*a^5) + (d^3*(36*a^2*c
+ 7*d)*x^6)/(378*a^3) + (d^4*x^8)/(72*a) + c^4*x*ArcCoth[a*x] + (4*c^3*d*x^
3*ArcCoth[a*x])/3 + (6*c^2*d^2*x^5*ArcCoth[a*x])/5 + (4*c*d^3*x^7*ArcCoth[a
*x])/7 + (d^4*x^9*ArcCoth[a*x])/9 + ((315*a^8*c^4 + 420*a^6*c^3*d + 378*a^4
*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*Log[1 - a^2*x^2])/(630*a^9)
```

Rubi [A] time = 0.180539, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {194, 5977, 1810, 260}

$$\frac{d^2x^4(378a^4c^2+180a^2cd+35d^2)}{1260a^5}+\frac{dx^2(378a^4c^2d+420a^6c^3+180a^2cd^2+35d^3)}{630a^7}+\frac{(378a^4c^2d^2+420a^6c^3d+315a^8c^4+\cdots}{630a\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^4*ArcCoth[a*x],x]
```


```
[Out] (d*(420*a^6*c^3 + 378*a^4*c^2*d + 180*a^2*c*d^2 + 35*d^3)*x^2)/(630*a^7) +
(d^2*(378*a^4*c^2 + 180*a^2*c*d + 35*d^2)*x^4)/(1260*a^5) + (d^3*(36*a^2*c
+ 7*d)*x^6)/(378*a^3) + (d^4*x^8)/(72*a) + c^4*x*ArcCoth[a*x] + (4*c^3*d*x^
3*ArcCoth[a*x])/3 + (6*c^2*d^2*x^5*ArcCoth[a*x])/5 + (4*c*d^3*x^7*ArcCoth[a
*x])/7 + (d^4*x^9*ArcCoth[a*x])/9 + ((315*a^8*c^4 + 420*a^6*c^3*d + 378*a^4
*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*Log[1 - a^2*x^2])/(630*a^9)
```


Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5977

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \left(c+dx^2\right)^4 \coth^{-1}(ax)\,dx = c^4x\coth^{-1}(ax)+\frac{4}{3}c^3dx^3\coth^{-1}(ax)+\frac{6}{5}c^2d^2x^5\coth^{-1}(ax)+\frac{4}{7}cd^3x^7\coth^{-1}(ax)+\frac{1}{9}d^{\cdot}$$

$$= c^4x\coth^{-1}(ax)+\frac{4}{3}c^3dx^3\coth^{-1}(ax)+\frac{6}{5}c^2d^2x^5\coth^{-1}(ax)+\frac{4}{7}cd^3x^7\coth^{-1}(ax)+\frac{1}{9}d^{\cdot}$$

$$= \frac{d\left(420a^6c^3+378a^4c^2d+180a^2cd^2+35d^3\right)x^2}{630a^7}+\frac{d^2\left(378a^4c^2+180a^2cd+35d^2\right)x^4}{1260a^5}+\cdots$$

$$= \frac{d\left(420a^6c^3+378a^4c^2d+180a^2cd^2+35d^3\right)x^2}{630a^7}+\frac{d^2\left(378a^4c^2+180a^2cd+35d^2\right)x^4}{1260a^5}+\cdots$$

Mathematica [A] time = 0.116038, size = 213, normalized size = 0.87

$$\frac{a^2dx^2\left(3a^6\left(756c^2dx^2+1680c^3+240cd^2x^4+35d^3x^6\right)+4a^4d\left(1134c^2+270cdx^2+35d^2x^4\right)+30a^2d^2\left(72c+7dx^2\right)+\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^4*ArcCoth[a*x],x]
```

```
[Out] (a^2*d*x^2*(420*d^3 + 30*a^2*d^2*(72*c + 7*d*x^2) + 4*a^4*d*(1134*c^2 + 270
*c*d*x^2 + 35*d^2*x^4) + 3*a^6*(1680*c^3 + 756*c^2*d*x^2 + 240*c*d^2*x^4 +
35*d^3*x^6)) + 24*a^9*x*(315*c^4 + 420*c^3*d*x^2 + 378*c^2*d^2*x^4 + 180*c*
d^3*x^6 + 35*d^4*x^8)*ArcCoth[a*x] + 12*(315*a^8*c^4 + 420*a^6*c^3*d + 378*
a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*Log[1 - a^2*x^2])/(7560*a^9)
```

Maple [A] time = 0.04, size = 334, normalized size = 1.4

$$\frac{2c^3dx^2}{3a}+\frac{2\ln(ax+1)cd^3}{7a^7}+\frac{\ln(ax-1)c^4}{2a}+\frac{\ln(ax-1)d^4}{18a^9}+\frac{\ln(ax+1)c^4}{2a}+\frac{\ln(ax+1)d^4}{18a^9}+\frac{x^6d^4}{54a^3}+\frac{x^2d^4}{18a^7}+\frac{x^4d^4}{36a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^4*arccoth(a*x),x)
```

```
[Out] 2/3/a*c^3*d*x^2+2/7/a^7*ln(a*x+1)*c*d^3+1/2/a*ln(a*x-1)*c^4+1/18/a^9*ln(a*x
-1)*d^4+1/2/a*ln(a*x+1)*c^4+1/18/a^9*ln(a*x+1)*d^4+1/54/a^3*x^6*d^4+1/18/a^
7*x^2*d^4+1/36/a^5*x^4*d^4+2/7/a^7*ln(a*x-1)*c*d^3+2/3/a^3*ln(a*x+1)*c^3*d+
2/3/a^3*ln(a*x-1)*c^3*d+3/5/a^5*ln(a*x-1)*c^2*d^2+3/5/a^5*ln(a*x+1)*c^2*d^2
+1/7/a^3*x^4*c*d^3+2/7/a^5*x^2*c*d^3+3/5/a^3*c^2*d^2*x^2+2/21/a*c*d^3*x^6+3
/10/a*c^2*d^2*x^4+c^4*x*arccoth(a*x)+4/3*c^3*d*x^3*arccoth(a*x)+6/5*c^2*d^2
*x^5*arccoth(a*x)+4/7*c*d^3*x^7*arccoth(a*x)+1/72*d^4*x^8/a+1/9*d^4*x^9*arc
coth(a*x)
```

Maxima [A] time = 0.972144, size = 373, normalized size = 1.52

$$\frac{1}{7560}a\left(\frac{105a^6d^4x^8+20\left(36a^6cd^3+7a^4d^4\right)x^6+6\left(378a^6c^2d^2+180a^4cd^3+35a^2d^4\right)x^4+12\left(420a^6c^3d+378a^4c^2d^2\cdots\right.}{a^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arccoth(a*x),x, algorithm="maxima")
```

```
[Out] 1/7560*a*((105*a^6*d^4*x^8 + 20*(36*a^6*c*d^3 + 7*a^4*d^4)*x^6 + 6*(378*a^6
*c^2*d^2 + 180*a^4*c*d^3 + 35*a^2*d^4)*x^4 + 12*(420*a^6*c^3*d + 378*a^4*c^
2*d^2 + 180*a^2*c*d^3 + 35*d^4)*x^2)/a^8 + 12*(315*a^8*c^4 + 420*a^6*c^3*d
+ 378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*log(a*x + 1)/a^10 + 12*(315*a^8
*c^4 + 420*a^6*c^3*d + 378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*log(a*x -
1)/a^10) + 1/315*(35*d^4*x^9 + 180*c*d^3*x^7 + 378*c^2*d^2*x^5 + 420*c^3*d*
x^3 + 315*c^4*x)*arccoth(a*x)
```

Fricas [A] time = 1.58354, size = 558, normalized size = 2.28

$$\frac{105\,a^{8}d^{4}x^{8} + 20\left(36\,a^{8}cd^{3} + 7\,a^{6}d^{4}\right)x^{6} + 6\left(378\,a^{8}c^{2}d^{2} + 180\,a^{6}cd^{3} + 35\,a^{4}d^{4}\right)x^{4} + 12\left(420\,a^{8}c^{3}d + 378\,a^{6}c^{2}d^{2} + 180\,a^{4}cd}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arccoth(a*x),x, algorithm="fricas")
```

```
[Out] 1/7560*(105*a^8*d^4*x^8 + 20*(36*a^8*c*d^3 + 7*a^6*d^4)*x^6 + 6*(378*a^8*c^
2*d^2 + 180*a^6*c*d^3 + 35*a^4*d^4)*x^4 + 12*(420*a^8*c^3*d + 378*a^6*c^2*d
^2 + 180*a^4*c*d^3 + 35*a^2*d^4)*x^2 + 12*(315*a^8*c^4 + 420*a^6*c^3*d + 37
8*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*log(a^2*x^2 - 1) + 12*(35*a^9*d^4*x
^9 + 180*a^9*c*d^3*x^7 + 378*a^9*c^2*d^2*x^5 + 420*a^9*c^3*d*x^3 + 315*a^9*
c^4*x)*log((a*x + 1)/(a*x - 1)))/a^9
```

Sympy [A] time = 25.0004, size = 427, normalized size = 1.74

$$\begin{cases} c^{4}x\operatorname{acoth}(ax) + \frac{4c^{3}dx^{3}\operatorname{acoth}(ax)}{3} + \frac{6c^{2}d^{2}x^{5}\operatorname{acoth}(ax)}{5} + \frac{4cd^{3}x^{7}\operatorname{acoth}(ax)}{7} + \frac{d^{4}x^{9}\operatorname{acoth}(ax)}{9} + \frac{c^{4}\log\left(x-\frac{1}{a}\right)}{a} + \frac{c^{4}\operatorname{acoth}(ax)}{a} + \frac{2c^{3}dx^{2}}{3a} + \cdots \\ \frac{i\pi\left(c^{4}x+\frac{4c^{3}dx^{3}}{3}+\frac{6c^{2}d^{2}x^{5}}{5}+\frac{4cd^{3}x^{7}}{7}+\frac{d^{4}x^{9}}{9}\right)}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**4*acoth(a*x),x)
```

```
[Out] Piecewise((c**4*x*acoth(a*x) + 4*c**3*d*x**3*acoth(a*x)/3 + 6*c**2*d**2*x**
5*acoth(a*x)/5 + 4*c*d**3*x**7*acoth(a*x)/7 + d**4*x**9*acoth(a*x)/9 + c**4
*log(x - 1/a)/a + c**4*acoth(a*x)/a + 2*c**3*d*x**2/(3*a) + 3*c**2*d**2*x**
4/(10*a) + 2*c*d**3*x**6/(21*a) + d**4*x**8/(72*a) + 4*c**3*d*log(x - 1/a)/
(3*a**3) + 4*c**3*d*acoth(a*x)/(3*a**3) + 3*c**2*d**2*x**2/(5*a**3) + c*d**
3*x**4/(7*a**3) + d**4*x**6/(54*a**3) + 6*c**2*d**2*log(x - 1/a)/(5*a**5) +
 6*c**2*d**2*acoth(a*x)/(5*a**5) + 2*c*d**3*x**2/(7*a**5) + d**4*x**4/(36*a
**5) + 4*c*d**3*log(x - 1/a)/(7*a**7) + 4*c*d**3*acoth(a*x)/(7*a**7) + d**4
*x**2/(18*a**7) + d**4*log(x - 1/a)/(9*a**9) + d**4*acoth(a*x)/(9*a**9), Ne
(a, 0)), (I*pi*(c**4*x + 4*c**3*d*x**3/3 + 6*c**2*d**2*x**5/5 + 4*c*d**3*x*
*7/7 + d**4*x**9/9)/2, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(dx^{2} + c\right)^{4} \operatorname{arcoth}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arccoth(a*x),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^4*arccoth(a*x), x)
```

3.36 $\int (c + dx^2)^3 \coth^{-1}(ax)\, dx$

Optimal. Leaf size=169

$$\frac{dx^2\left(35a^4c^2+21a^2cd+5d^2\right)}{70a^5}+\frac{\left(35a^4c^2d+35a^6c^3+21a^2cd^2+5d^3\right)\log\left(1-a^2x^2\right)}{70a^7}+\frac{d^2x^4\left(21a^2c+5d\right)}{140a^3}+c^2dx^3\coth^{-1}($$

```
[Out] (d*(35*a^4*c^2 + 21*a^2*c*d + 5*d^2)*x^2)/(70*a^5) + (d^2*(21*a^2*c + 5*d)*
x^4)/(140*a^3) + (d^3*x^6)/(42*a) + c^3*x*ArcCoth[a*x] + c^2*d*x^3*ArcCoth[
a*x] + (3*c*d^2*x^5*ArcCoth[a*x])/5 + (d^3*x^7*ArcCoth[a*x])/7 + ((35*a^6*c
^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*Log[1 - a^2*x^2])/(70*a^7)
```

Rubi [A] time = 0.1268, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {194, 5977, 1810, 260}

$$\frac{dx^2\left(35a^4c^2+21a^2cd+5d^2\right)}{70a^5}+\frac{\left(35a^4c^2d+35a^6c^3+21a^2cd^2+5d^3\right)\log\left(1-a^2x^2\right)}{70a^7}+\frac{d^2x^4\left(21a^2c+5d\right)}{140a^3}+c^2dx^3\coth^{-1}($$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3*ArcCoth[a*x],x]
```

```
[Out] (d*(35*a^4*c^2 + 21*a^2*c*d + 5*d^2)*x^2)/(70*a^5) + (d^2*(21*a^2*c + 5*d)*
x^4)/(140*a^3) + (d^3*x^6)/(42*a) + c^3*x*ArcCoth[a*x] + c^2*d*x^3*ArcCoth[
a*x] + (3*c*d^2*x^5*ArcCoth[a*x])/5 + (d^3*x^7*ArcCoth[a*x])/7 + ((35*a^6*c
^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*Log[1 - a^2*x^2])/(70*a^7)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5977

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \left(c+dx^2\right)^3 \coth^{-1}(ax)\,dx = c^3x\coth^{-1}(ax) + c^2dx^3\coth^{-1}(ax) + \frac{3}{5}cd^2x^5\coth^{-1}(ax) + \frac{1}{7}d^3x^7\coth^{-1}(ax) - a\int \frac{c^3}{}$$

$$= c^3x\coth^{-1}(ax) + c^2dx^3\coth^{-1}(ax) + \frac{3}{5}cd^2x^5\coth^{-1}(ax) + \frac{1}{7}d^3x^7\coth^{-1}(ax) - a\int \Big(-$$

$$= \frac{d\left(35a^4c^2+21a^2cd+5d^2\right)x^2}{70a^5} + \frac{d^2\left(21a^2c+5d\right)x^4}{140a^3} + \frac{d^3x^6}{42a} + c^3x\coth^{-1}(ax) + c^2dx^3\,\mathrm{co}$$

$$= \frac{d\left(35a^4c^2+21a^2cd+5d^2\right)x^2}{70a^5} + \frac{d^2\left(21a^2c+5d\right)x^4}{140a^3} + \frac{d^3x^6}{42a} + c^3x\coth^{-1}(ax) + c^2dx^3\,\mathrm{co}$$

Mathematica [A] time = 0.0786468, size = 150, normalized size = 0.89

$$\frac{a^2dx^2\left(a^4\left(210c^2+63cdx^2+10d^2x^4\right)+3a^2d\left(42c+5dx^2\right)+30d^2\right)+6\left(35a^4c^2d+35a^6c^3+21a^2cd^2+5d^3\right)\log\left(1-a\right.}{420a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3*ArcCoth[a*x],x]
```

```
[Out] (a^2*d*x^2*(30*d^2 + 3*a^2*d*(42*c + 5*d*x^2) + a^4*(210*c^2 + 63*c*d*x^2 +
 10*d^2*x^4)) + 12*a^7*x*(35*c^3 + 35*c^2*d*x^2 + 21*c*d^2*x^4 + 5*d^3*x^6)
*ArcCoth[a*x] + 6*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*Log[1
- a^2*x^2])/(420*a^7)
```

Maple [A] time = 0.037, size = 233, normalized size = 1.4

$$\frac{d^3x^7\mathrm{arccoth}\,(ax)}{7} + \frac{3\,cd^2x^5\mathrm{arccoth}\,(ax)}{5} + c^2dx^3\mathrm{arccoth}\,(ax) + c^3x\mathrm{arccoth}\,(ax) + \frac{3\,cd^2x^2}{10\,a^3} + \frac{3\,cd^2x^4}{20\,a} + \frac{c^2x^2d}{2\,a} + \frac{d^3x}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3*arccoth(a*x),x)
```

```
[Out] 1/7*d^3*x^7*arccoth(a*x)+3/5*c*d^2*x^5*arccoth(a*x)+c^2*d*x^3*arccoth(a*x)+
c^3*x*arccoth(a*x)+3/10/a^3*c*d^2*x^2+3/20/a*c*d^2*x^4+1/2/a*x^2*c^2*d+1/42
*d^3*x^6/a+1/14/a^5*d^3*x^2+1/2/a*ln(a*x-1)*c^3+1/2/a^3*ln(a*x-1)*c^2*d+3/1
0/a^5*ln(a*x-1)*c*d^2+1/14/a^7*ln(a*x-1)*d^3+1/2/a*ln(a*x+1)*c^3+1/2/a^3*ln
(a*x+1)*c^2*d+3/10/a^5*ln(a*x+1)*c*d^2+1/14/a^7*ln(a*x+1)*d^3+1/28/a^3*x^4*
d^3
```

Maxima [A] time = 0.974128, size = 267, normalized size = 1.58

$$\frac{1}{420}\,a\left(\frac{10\,a^4d^3x^6+3\left(21\,a^4cd^2+5\,a^2d^3\right)x^4+6\left(35\,a^4c^2d+21\,a^2cd^2+5\,d^3\right)x^2}{a^6} + \frac{6\left(35\,a^6c^3+35\,a^4c^2d+21\,a^2cd^2+5\,d\right.}{a^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*arccoth(a*x),x, algorithm="maxima")
```

```
[Out] 1/420*a*((10*a^4*d^3*x^6 + 3*(21*a^4*c*d^2 + 5*a^2*d^3)*x^4 + 6*(35*a^4*c^2
*d + 21*a^2*c*d^2 + 5*d^3)*x^2)/a^6 + 6*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2
```

```
*c*d^2 + 5*d^3)*log(a*x + 1)/a^8 + 6*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*
d^2 + 5*d^3)*log(a*x - 1)/a^8) + 1/35*(5*d^3*x^7 + 21*c*d^2*x^5 + 35*c^2*d*
x^3 + 35*c^3*x)*arccoth(a*x)
```

Fricas [A] time = 1.62707, size = 382, normalized size = 2.26

$$\frac{10\,a^6d^3x^6+3\left(21\,a^6cd^2+5\,a^4d^3\right)x^4+6\left(35\,a^6c^2d+21\,a^4cd^2+5\,a^2d^3\right)x^2+6\left(35\,a^6c^3+35\,a^4c^2d+21\,a^2cd^2+5\,d^3\right)\log\left(a\right.}{420\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*arccoth(a*x),x, algorithm="fricas")
```

```
[Out] 1/420*(10*a^6*d^3*x^6 + 3*(21*a^6*c*d^2 + 5*a^4*d^3)*x^4 + 6*(35*a^6*c^2*d
+ 21*a^4*c*d^2 + 5*a^2*d^3)*x^2 + 6*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d
^2 + 5*d^3)*log(a^2*x^2 - 1) + 6*(5*a^7*d^3*x^7 + 21*a^7*c*d^2*x^5 + 35*a^7
*c^2*d*x^3 + 35*a^7*c^3*x)*log((a*x + 1)/(a*x - 1)))/a^7
```

Sympy [A] time = 7.31205, size = 282, normalized size = 1.67

$$\begin{cases} c^3x\operatorname{acoth}(ax)+c^2dx^3\operatorname{acoth}(ax)+\frac{3cd^2x^5\operatorname{acoth}(ax)}{5}+\frac{d^3x^7\operatorname{acoth}(ax)}{7}+\frac{c^3\log\left(x-\frac{1}{a}\right)}{a}+\frac{c^3\operatorname{acoth}(ax)}{a}+\frac{c^2dx^2}{2a}+\frac{3cd^2x^4}{20a}+\frac{d^3x^6}{42a}+\frac{c^2}{} \\ \frac{i\pi\left(c^3x+c^2dx^3+\frac{3cd^2x^5}{5}+\frac{d^3x^7}{7}\right)}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3*acoth(a*x),x)
```

```
[Out] Piecewise((c**3*x*acoth(a*x) + c**2*d*x**3*acoth(a*x) + 3*c*d**2*x**5*acoth
(a*x)/5 + d**3*x**7*acoth(a*x)/7 + c**3*log(x - 1/a)/a + c**3*acoth(a*x)/a
+ c**2*d*x**2/(2*a) + 3*c*d**2*x**4/(20*a) + d**3*x**6/(42*a) + c**2*d*log(
x - 1/a)/a**3 + c**2*d*acoth(a*x)/a**3 + 3*c*d**2*x**2/(10*a**3) + d**3*x**
4/(28*a**3) + 3*c*d**2*log(x - 1/a)/(5*a**5) + 3*c*d**2*acoth(a*x)/(5*a**5)
 + d**3*x**2/(14*a**5) + d**3*log(x - 1/a)/(7*a**7) + d**3*acoth(a*x)/(7*a*
*7), Ne(a, 0)), (I*pi*(c**3*x + c**2*d*x**3 + 3*c*d**2*x**5/5 + d**3*x**7/7
)/2, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)^3\operatorname{arcoth}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*arccoth(a*x),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^3*arccoth(a*x), x)
```

3.37 $\int (c+dx^2)^2 \coth^{-1}(ax)\,dx$

Optimal. Leaf size=110

$$\frac{(15a^4c^2+10a^2cd+3d^2)\log(1-a^2x^2)}{30a^5}+\frac{dx^2(10a^2c+3d)}{30a^3}+c^2x\coth^{-1}(ax)+\frac{2}{3}cdx^3\coth^{-1}(ax)+\frac{d^2x^4}{20a}+\frac{1}{5}d^2x^5\text{co}$$

```
[Out] (d*(10*a^2*c + 3*d)*x^2)/(30*a^3) + (d^2*x^4)/(20*a) + c^2*x*ArcCoth[a*x] +
 (2*c*d*x^3*ArcCoth[a*x])/3 + (d^2*x^5*ArcCoth[a*x])/5 + ((15*a^4*c^2 + 10*
a^2*c*d + 3*d^2)*Log[1 - a^2*x^2])/(30*a^5)
```

Rubi [A] time = 0.133164, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {194, 5977, 1594, 1247, 698}

$$\frac{(15a^4c^2+10a^2cd+3d^2)\log(1-a^2x^2)}{30a^5}+\frac{dx^2(10a^2c+3d)}{30a^3}+c^2x\coth^{-1}(ax)+\frac{2}{3}cdx^3\coth^{-1}(ax)+\frac{d^2x^4}{20a}+\frac{1}{5}d^2x^5\text{co}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2*ArcCoth[a*x],x]
```

```
[Out] (d*(10*a^2*c + 3*d)*x^2)/(30*a^3) + (d^2*x^4)/(20*a) + c^2*x*ArcCoth[a*x] +
 (2*c*d*x^3*ArcCoth[a*x])/3 + (d^2*x^5*ArcCoth[a*x])/5 + ((15*a^4*c^2 + 10*
a^2*c*d + 3*d^2)*Log[1 - a^2*x^2])/(30*a^5)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5977

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
```

```
&& IntegerQ[m]))
```

Rubi steps

$$\int \left(c+dx^2\right)^2 \coth^{-1}(ax)\,dx = c^2x\coth^{-1}(ax)+\frac{2}{3}cdx^3\coth^{-1}(ax)+\frac{1}{5}d^2x^5\coth^{-1}(ax)-a\int \frac{c^2x+\frac{2}{3}cdx^3+\frac{d^2x^5}{5}}{1-a^2x^2}\,dx$$

$$= c^2x\coth^{-1}(ax)+\frac{2}{3}cdx^3\coth^{-1}(ax)+\frac{1}{5}d^2x^5\coth^{-1}(ax)-a\int \frac{x\left(c^2+\frac{2}{3}cdx^2+\frac{d^2x^4}{5}\right)}{1-a^2x^2}\,dx$$

$$= c^2x\coth^{-1}(ax)+\frac{2}{3}cdx^3\coth^{-1}(ax)+\frac{1}{5}d^2x^5\coth^{-1}(ax)-\frac{1}{2}a\,\text{Subst}\left(\int \frac{c^2+\frac{2cdx}{3}+\frac{d^2x^2}{5}}{1-a^2x}\,d\right.$$

$$= c^2x\coth^{-1}(ax)+\frac{2}{3}cdx^3\coth^{-1}(ax)+\frac{1}{5}d^2x^5\coth^{-1}(ax)-\frac{1}{2}a\,\text{Subst}\left(\int\left(-\frac{d\left(10a^2c+3d\right)}{15a^4}\right.\right.$$

$$= \frac{d\left(10a^2c+3d\right)x^2}{30a^3}+\frac{d^2x^4}{20a}+c^2x\coth^{-1}(ax)+\frac{2}{3}cdx^3\coth^{-1}(ax)+\frac{1}{5}d^2x^5\coth^{-1}(ax)+\frac{(15}{}$$

Mathematica [A] time = 0.0538446, size = 98, normalized size = 0.89

$$\frac{\left(30a^4c^2+20a^2cd+6d^2\right)\log\left(1-a^2x^2\right)+4a^5x\coth^{-1}(ax)\left(15c^2+10cdx^2+3d^2x^4\right)+a^2dx^2\left(a^2\left(20c+3dx^2\right)+6d\right)}{60a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2*ArcCoth[a*x],x]
```

```
[Out] (a^2*d*x^2*(6*d + a^2*(20*c + 3*d*x^2)) + 4*a^5*x*(15*c^2 + 10*c*d*x^2 + 3*
d^2*x^4)*ArcCoth[a*x] + (30*a^4*c^2 + 20*a^2*c*d + 6*d^2)*Log[1 - a^2*x^2])
/(60*a^5)
```

Maple [A] time = 0.036, size = 148, normalized size = 1.4

$$\frac{d^2x^5\text{arccoth}\,(ax)}{5}+\frac{2\,cdx^3\text{arccoth}\,(ax)}{3}+c^2x\text{arccoth}\,(ax)+\frac{d^2x^4}{20\,a}+\frac{cdx^2}{3\,a}+\frac{d^2x^2}{10\,a^3}+\frac{\ln(ax-1)\,c^2}{2\,a}+\frac{\ln(ax-1)\,cd}{3\,a^3}+\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2*arccoth(a*x),x)
```

```
[Out] 1/5*d^2*x^5*arccoth(a*x)+2/3*c*d*x^3*arccoth(a*x)+c^2*x*arccoth(a*x)+1/20*d
^2*x^4/a+1/3/a*c*d*x^2+1/10/a^3*x^2*d^2+1/2/a*ln(a*x-1)*c^2+1/3/a^3*ln(a*x-
1)*c*d+1/10/a^5*ln(a*x-1)*d^2+1/2/a*ln(a*x+1)*c^2+1/3/a^3*ln(a*x+1)*c*d+1/1
0/a^5*ln(a*x+1)*d^2
```

Maxima [A] time = 0.973441, size = 177, normalized size = 1.61

$$\frac{1}{60}\,a\left(\frac{3\,a^2d^2x^4+2\left(10\,a^2cd+3\,d^2\right)x^2}{a^4}+\frac{2\left(15\,a^4c^2+10\,a^2cd+3\,d^2\right)\log(ax+1)}{a^6}+\frac{2\left(15\,a^4c^2+10\,a^2cd+3\,d^2\right)\log(ax-\right.}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2*arccoth(a*x),x, algorithm="maxima")
```

```
[Out] 1/60*a*((3*a^2*d^2*x^4 + 2*(10*a^2*c*d + 3*d^2)*x^2)/a^4 + 2*(15*a^4*c^2 +
10*a^2*c*d + 3*d^2)*log(a*x + 1)/a^6 + 2*(15*a^4*c^2 + 10*a^2*c*d + 3*d^2)*
log(a*x - 1)/a^6) + 1/15*(3*d^2*x^5 + 10*c*d*x^3 + 15*c^2*x)*arccoth(a*x)
```

Fricas [A] time = 1.5485, size = 258, normalized size = 2.35

$$\frac{3\,a^4d^2x^4 + 2\left(10\,a^4cd + 3\,a^2d^2\right)x^2 + 2\left(15\,a^4c^2 + 10\,a^2cd + 3\,d^2\right)\log\left(a^2x^2 - 1\right) + 2\left(3\,a^5d^2x^5 + 10\,a^5cdx^3 + 15\,a^5c^2x\right)\mathrm{l}}{60\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2*arccoth(a*x),x, algorithm="fricas")
```

```
[Out] 1/60*(3*a^4*d^2*x^4 + 2*(10*a^4*c*d + 3*a^2*d^2)*x^2 + 2*(15*a^4*c^2 + 10*a
^2*c*d + 3*d^2)*log(a^2*x^2 - 1) + 2*(3*a^5*d^2*x^5 + 10*a^5*c*d*x^3 + 15*a
^5*c^2*x)*log((a*x + 1)/(a*x - 1)))/a^5
```

Sympy [A] time = 3.32318, size = 182, normalized size = 1.65

$$\begin{cases} c^2x\operatorname{acoth}(ax) + \frac{2cdx^3\operatorname{acoth}(ax)}{3} + \frac{d^2x^5\operatorname{acoth}(ax)}{5} + \frac{c^2\log\left(x-\frac{1}{a}\right)}{a} + \frac{c^2\operatorname{acoth}(ax)}{a} + \frac{cdx^2}{3a} + \frac{d^2x^4}{20a} + \frac{2cd\log\left(x-\frac{1}{a}\right)}{3a^3} + \frac{2cd\operatorname{acoth}(ax)}{3a^3} + \\ \frac{i\pi\left(c^2x+\frac{2cdx^3}{3}+\frac{d^2x^5}{5}\right)}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2*acoth(a*x),x)
```

```
[Out] Piecewise((c**2*x*acoth(a*x) + 2*c*d*x**3*acoth(a*x)/3 + d**2*x**5*acoth(a*
x)/5 + c**2*log(x - 1/a)/a + c**2*acoth(a*x)/a + c*d*x**2/(3*a) + d**2*x**4
/(20*a) + 2*c*d*log(x - 1/a)/(3*a**3) + 2*c*d*acoth(a*x)/(3*a**3) + d**2*x*
*2/(10*a**3) + d**2*log(x - 1/a)/(5*a**5) + d**2*acoth(a*x)/(5*a**5), Ne(a,
 0)), (I*pi*(c**2*x + 2*c*d*x**3/3 + d**2*x**5/5)/2, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(dx^2 + c\right)^2 \operatorname{arcoth}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2*arccoth(a*x),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^2*arccoth(a*x), x)
```

3.38 $\int \left(c+dx^2\right)\coth^{-1}(ax)\,dx$

Optimal. Leaf size=57

$$\frac{\left(3a^2c+d\right)\log\left(1-a^2x^2\right)}{6a^3}+cx\coth^{-1}(ax)+\frac{dx^2}{6a}+\frac{1}{3}dx^3\coth^{-1}(ax)$$

```
[Out] (d*x^2)/(6*a) + c*x*ArcCoth[a*x] + (d*x^3*ArcCoth[a*x])/3 + ((3*a^2*c + d)*
Log[1 - a^2*x^2])/(6*a^3)
```

Rubi [A] time = 0.0659503, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5977, 1593, 444, 43}

$$\frac{\left(3a^2c+d\right)\log\left(1-a^2x^2\right)}{6a^3}+cx\coth^{-1}(ax)+\frac{dx^2}{6a}+\frac{1}{3}dx^3\coth^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)*ArcCoth[a*x],x]
```

```
[Out] (d*x^2)/(6*a) + c*x*ArcCoth[a*x] + (d*x^3*ArcCoth[a*x])/3 + ((3*a^2*c + d)*
Log[1 - a^2*x^2])/(6*a^3)
```

Rule 5977

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int\left(c+dx^2\right)\coth^{-1}(ax)\,dx &= cx\coth^{-1}(ax)+\frac{1}{3}dx^3\coth^{-1}(ax)-a\int\frac{cx+\frac{dx^3}{3}}{1-a^2x^2}\,dx\\ &= cx\coth^{-1}(ax)+\frac{1}{3}dx^3\coth^{-1}(ax)-a\int\frac{x\left(c+\frac{dx^2}{3}\right)}{1-a^2x^2}\,dx\\ &= cx\coth^{-1}(ax)+\frac{1}{3}dx^3\coth^{-1}(ax)-\frac{1}{2}a\,\text{Subst}\left(\int\frac{c+\frac{dx}{3}}{1-a^2x}\,dx,x,x^2\right)\\ &= cx\coth^{-1}(ax)+\frac{1}{3}dx^3\coth^{-1}(ax)-\frac{1}{2}a\,\text{Subst}\left(\int\left(-\frac{d}{3a^2}+\frac{-3a^2c-d}{3a^2\left(-1+a^2x\right)}\right)dx,x,x^2\right)\\ &= \frac{dx^2}{6a}+cx\coth^{-1}(ax)+\frac{1}{3}dx^3\coth^{-1}(ax)+\frac{\left(3a^2c+d\right)\log\left(1-a^2x^2\right)}{6a^3}\end{aligned}$$

Mathematica [A] time = 0.0116903, size = 69, normalized size = 1.21

$$\frac{c\log\left(1-a^2x^2\right)}{2a}+\frac{d\log\left(1-a^2x^2\right)}{6a^3}+cx\coth^{-1}(ax)+\frac{dx^2}{6a}+\frac{1}{3}dx^3\coth^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)*ArcCoth[a*x],x]
```

```
[Out] (d*x^2)/(6*a) + c*x*ArcCoth[a*x] + (d*x^3*ArcCoth[a*x])/3 + (c*Log[1 - a^2*
x^2])/(2*a) + (d*Log[1 - a^2*x^2])/(6*a^3)
```

Maple [A] time = 0.038, size = 76, normalized size = 1.3

$$\frac{dx^3\text{arccoth}\,(ax)}{3}+cx\text{arccoth}\,(ax)+\frac{dx^2}{6\,a}+\frac{\ln(ax-1)\,c}{2\,a}+\frac{\ln(ax-1)\,d}{6\,a^3}+\frac{\ln(ax+1)\,c}{2\,a}+\frac{\ln(ax+1)\,d}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)*arccoth(a*x),x)
```

```
[Out] 1/3*d*x^3*arccoth(a*x)+c*x*arccoth(a*x)+1/6*d*x^2/a+1/2/a*ln(a*x-1)*c+1/6/a
^3*ln(a*x-1)*d+1/2/a*ln(a*x+1)*c+1/6/a^3*ln(a*x+1)*d
```

Maxima [A] time = 0.974406, size = 88, normalized size = 1.54

$$\frac{1}{6}a\left(\frac{dx^2}{a^2}+\frac{\left(3\,a^2c+d\right)\log(ax+1)}{a^4}+\frac{\left(3\,a^2c+d\right)\log(ax-1)}{a^4}\right)+\frac{1}{3}\left(dx^3+3\,cx\right)\text{arcoth}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)*arccoth(a*x),x, algorithm="maxima")
```

```
[Out] 1/6*a*(d*x^2/a^2 + (3*a^2*c + d)*log(a*x + 1)/a^4 + (3*a^2*c + d)*log(a*x -
 1)/a^4) + 1/3*(d*x^3 + 3*c*x)*arccoth(a*x)
```

Fricas [A] time = 1.51526, size = 142, normalized size = 2.49

$$\frac{a^2dx^2 + \left(3\,a^2c + d\right)\log\left(a^2x^2 - 1\right) + \left(a^3dx^3 + 3\,a^3cx\right)\log\left(\frac{ax+1}{ax-1}\right)}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)*arccoth(a*x),x, algorithm="fricas")
```

```
[Out] 1/6*(a^2*d*x^2 + (3*a^2*c + d)*log(a^2*x^2 - 1) + (a^3*d*x^3 + 3*a^3*c*x)*l
og((a*x + 1)/(a*x - 1)))/a^3
```

Sympy [A] time = 1.37328, size = 87, normalized size = 1.53

$$\begin{cases} cx\operatorname{acoth}(ax) + \frac{dx^3\operatorname{acoth}(ax)}{3} + \frac{c\log\left(x-\frac{1}{a}\right)}{a} + \frac{c\operatorname{acoth}(ax)}{a} + \frac{dx^2}{6a} + \frac{d\log\left(x-\frac{1}{a}\right)}{3a^3} + \frac{d\operatorname{acoth}(ax)}{3a^3} & \text{for } a \neq 0 \\ \frac{i\pi\left(cx+\frac{dx^3}{3}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)*acoth(a*x),x)
```

```
[Out] Piecewise((c*x*acoth(a*x) + d*x**3*acoth(a*x)/3 + c*log(x - 1/a)/a + c*acot
h(a*x)/a + d*x**2/(6*a) + d*log(x - 1/a)/(3*a**3) + d*acoth(a*x)/(3*a**3),
Ne(a, 0)), (I*pi*(c*x + d*x**3/3)/2, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(dx^2 + c\right)\operatorname{arcoth}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)*arccoth(a*x),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*arccoth(a*x), x)
```

3.39 $\int \frac{\coth^{-1}(ax)}{c+dx^2} \, dx$

Optimal. Leaf size=390

$$-\frac{i\text{PolyLog}\left(2,1+\frac{2\sqrt{c}\sqrt{d}(1-ax)}{\left(-\sqrt{d}+ia\sqrt{c}\right)\left(\sqrt{c}-i\sqrt{d}x\right)}\right)}{4\sqrt{c}\sqrt{d}}+\frac{i\text{PolyLog}\left(2,1-\frac{2\sqrt{c}\sqrt{d}(ax+1)}{\left(\sqrt{d}+ia\sqrt{c}\right)\left(\sqrt{c}-i\sqrt{d}x\right)}\right)}{4\sqrt{c}\sqrt{d}}-\frac{\log\left(1-\frac{1}{ax}\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}\sqrt{d}}+\frac{\log\left(\frac{1}{ax}\right.}{}$$

```
[Out] -(ArcTan[(Sqrt[d]*x)/Sqrt[c]]*Log[1 - 1/(a*x)])/(2*Sqrt[c]*Sqrt[d]) + (ArcT
an[(Sqrt[d]*x)/Sqrt[c]]*Log[1 + 1/(a*x)])/(2*Sqrt[c]*Sqrt[d]) + (ArcTan[(Sq
rt[d]*x)/Sqrt[c]]*Log[(-2*Sqrt[c]*Sqrt[d]*(1 - a*x))/((I*a*Sqrt[c] - Sqrt[d
])*(Sqrt[c] - I*Sqrt[d]*x))])/(2*Sqrt[c]*Sqrt[d]) - (ArcTan[(Sqrt[d]*x)/Sqr
t[c]]*Log[(2*Sqrt[c]*Sqrt[d]*(1 + a*x))/((I*a*Sqrt[c] + Sqrt[d])*(Sqrt[c] -
 I*Sqrt[d]*x))])/(2*Sqrt[c]*Sqrt[d]) - ((I/4)*PolyLog[2, 1 + (2*Sqrt[c]*Sqr
t[d]*(1 - a*x))/((I*a*Sqrt[c] - Sqrt[d])*(Sqrt[c] - I*Sqrt[d]*x))])/(Sqrt[c
]*Sqrt[d]) + ((I/4)*PolyLog[2, 1 - (2*Sqrt[c]*Sqrt[d]*(1 + a*x))/((I*a*Sqrt
[c] + Sqrt[d])*(Sqrt[c] - I*Sqrt[d]*x))])/(Sqrt[c]*Sqrt[d])
```

Rubi [A] time = 0.946685, antiderivative size = 390, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 13, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {5973, 205, 2470, 12, 260, 6688, 4876, 4848, 2391, 4856, 2402, 2315, 2447}

$$-\frac{i\text{PolyLog}\left(2,1+\frac{2\sqrt{c}\sqrt{d}(1-ax)}{\left(-\sqrt{d}+ia\sqrt{c}\right)\left(\sqrt{c}-i\sqrt{d}x\right)}\right)}{4\sqrt{c}\sqrt{d}}+\frac{i\text{PolyLog}\left(2,1-\frac{2\sqrt{c}\sqrt{d}(ax+1)}{\left(\sqrt{d}+ia\sqrt{c}\right)\left(\sqrt{c}-i\sqrt{d}x\right)}\right)}{4\sqrt{c}\sqrt{d}}-\frac{\log\left(1-\frac{1}{ax}\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}\sqrt{d}}+\frac{\log\left(\frac{1}{ax}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]/(c + d*x^2),x]
```

```
[Out] -(ArcTan[(Sqrt[d]*x)/Sqrt[c]]*Log[1 - 1/(a*x)])/(2*Sqrt[c]*Sqrt[d]) + (ArcT
an[(Sqrt[d]*x)/Sqrt[c]]*Log[1 + 1/(a*x)])/(2*Sqrt[c]*Sqrt[d]) + (ArcTan[(Sq
rt[d]*x)/Sqrt[c]]*Log[(-2*Sqrt[c]*Sqrt[d]*(1 - a*x))/((I*a*Sqrt[c] - Sqrt[d
])*(Sqrt[c] - I*Sqrt[d]*x))])/(2*Sqrt[c]*Sqrt[d]) - (ArcTan[(Sqrt[d]*x)/Sqr
t[c]]*Log[(2*Sqrt[c]*Sqrt[d]*(1 + a*x))/((I*a*Sqrt[c] + Sqrt[d])*(Sqrt[c] -
 I*Sqrt[d]*x))])/(2*Sqrt[c]*Sqrt[d]) - ((I/4)*PolyLog[2, 1 + (2*Sqrt[c]*Sqr
t[d]*(1 - a*x))/((I*a*Sqrt[c] - Sqrt[d])*(Sqrt[c] - I*Sqrt[d]*x))])/(Sqrt[c
]*Sqrt[d]) + ((I/4)*PolyLog[2, 1 - (2*Sqrt[c]*Sqrt[d]*(1 + a*x))/((I*a*Sqrt
[c] + Sqrt[d])*(Sqrt[c] - I*Sqrt[d]*x))])/(Sqrt[c]*Sqrt[d])
```

Rule 5973

```
Int[ArcCoth[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[1/2, Int[
Log[1 + 1/(c*x)]/(d + e*x^2), x], x] - Dist[1/2, Int[Log[1 - 1/(c*x)]/(d +
e*x^2), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
```

```
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\coth^{-1}(ax)}{c+dx^2}\,dx = -\left(\frac{1}{2}\int \frac{\log\left(1-\frac{1}{ax}\right)}{c+dx^2}\,dx\right)+\frac{1}{2}\int \frac{\log\left(1+\frac{1}{ax}\right)}{c+dx^2}\,dx$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1-\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}}+\frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1+\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}}+\frac{\int \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}\left(1-\frac{1}{ax}\right)x^2}\,dx}{2a}+\frac{\int \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}\left(1+\frac{1}{ax}\right)x^2}\,dx}{2a}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1-\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}}+\frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1+\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}}+\frac{\int \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\left(1-\frac{1}{ax}\right)x^2}\,dx}{2a\sqrt{c}\sqrt{d}}+\frac{\int \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\left(1+\frac{1}{ax}\right)x^2}\,dx}{2a\sqrt{c}\sqrt{d}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1-\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}}+\frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1+\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}}+\frac{\int \frac{a\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{x(-1+ax)}\,dx}{2a\sqrt{c}\sqrt{d}}+\frac{\int \frac{a\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{x(1+ax)}\,dx}{2a\sqrt{c}\sqrt{d}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1-\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}}+\frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1+\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}}+\frac{\int \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{x(-1+ax)}\,dx}{2\sqrt{c}\sqrt{d}}+\frac{\int \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{x(1+ax)}\,dx}{2\sqrt{c}\sqrt{d}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1-\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}}+\frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1+\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}}+\frac{\int \left(-\frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{x}+\frac{a\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{-1+ax}\right)dx}{2\sqrt{c}\sqrt{d}}+\frac{\int \Big(}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1-\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}}+\frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1+\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}}+\frac{a\int \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{-1+ax}\,dx}{2\sqrt{c}\sqrt{d}}-\frac{a\int \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{1+ax}\,dx}{2\sqrt{c}\sqrt{d}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1-\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}}+\frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1+\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}}+\frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(-\frac{2\sqrt{c}\sqrt{d}(1-ax)}{\left(ia\sqrt{c}-\sqrt{d}\right)\left(\sqrt{c}-i\sqrt{d}x\right)}\right)}{2\sqrt{c}\sqrt{d}}-$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1-\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}}+\frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(1+\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}}+\frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(-\frac{2\sqrt{c}\sqrt{d}(1-ax)}{\left(ia\sqrt{c}-\sqrt{d}\right)\left(\sqrt{c}-i\sqrt{d}x\right)}\right)}{2\sqrt{c}\sqrt{d}}-$$

Mathematica [A] time = 1.34434, size = 671, normalized size = 1.72

$$\frac{a\left(i\left(\text{PolyLog}\left(2,\frac{\left(2i\sqrt{a^2cd}+a^2c-d\right)\left(\sqrt{a^2cd}+iadx\right)}{\left(a^2c+d\right)\left(\sqrt{a^2cd}-iadx\right)}\right)-\text{PolyLog}\left(2,\frac{\left(-2i\sqrt{a^2cd}+a^2c-d\right)\left(\sqrt{a^2cd}+iadx\right)}{\left(a^2c+d\right)\left(\sqrt{a^2cd}-iadx\right)}\right)\right)-2i\cos^{-1}\left(\frac{a^2c-d}{a^2c+d}\right)\tan^{-1}\left(\frac{ac}{x\sqrt{a^2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]/(c + d*x^2),x]
```

```
[Out] (a*((-2*I)*ArcCos[(a^2*c - d)/(a^2*c + d)]*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)]
+ 4*ArcCoth[a*x]*ArcTan[(a*d*x)/Sqrt[a^2*c*d]] - (ArcCos[(a^2*c - d)/(a^2*c
 + d)] + 2*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)])*Log[(2*d*(a^2*c - I*Sqrt[a^2*c*
d])*(-1 + a*x))/((a^2*c + d)*(I*Sqrt[a^2*c*d] + a*d*x))] - (ArcCos[(a^2*c -
 d)/(a^2*c + d)] - 2*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)])*Log[(2*d*(a^2*c + I*S
qrt[a^2*c*d])*(1 + a*x))/((a^2*c + d)*(I*Sqrt[a^2*c*d] + a*d*x))] + (ArcCos
[(a^2*c - d)/(a^2*c + d)] + 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] + ArcTan[(a*
d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d])/(Sqrt[a^2*c + d]*E^ArcCot
h[a*x]*Sqrt[-(a^2*c) + d + (a^2*c + d)*Cosh[2*ArcCoth[a*x]]])] + (ArcCos[(a
^2*c - d)/(a^2*c + d)] - 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] + ArcTan[(a*d*x
)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d]*E^ArcCoth[a*x])/(Sqrt[a^2*c +
 d]*Sqrt[-(a^2*c) + d + (a^2*c + d)*Cosh[2*ArcCoth[a*x]]])] + I*(-PolyLog[2
, ((a^2*c - d - (2*I)*Sqrt[a^2*c*d])*(Sqrt[a^2*c*d] + I*a*d*x))/((a^2*c + d
)*(Sqrt[a^2*c*d] - I*a*d*x))] + PolyLog[2, ((a^2*c - d + (2*I)*Sqrt[a^2*c*d
])*(Sqrt[a^2*c*d] + I*a*d*x))/((a^2*c + d)*(Sqrt[a^2*c*d] - I*a*d*x))])))/(
4*Sqrt[a^2*c*d])
```

Maple [B] time = 0.218, size = 785, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)/(d*x^2+c),x)
```

```
[Out] -1/2*a^3/d/(a^4*c^2+2*a^2*c*d+d^2)*ln(1-(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*
(-a^2*c*d)^(1/2)-d))*arccoth(a*x)*(-a^2*c*d)^(1/2)*c-a/(a^4*c^2+2*a^2*c*d+d
^2)*ln(1-(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))*arccoth(a*
x)*(-a^2*c*d)^(1/2)+1/2*a^3/d/(a^4*c^2+2*a^2*c*d+d^2)*arccoth(a*x)^2*(-a^2*
c*d)^(1/2)*c+a/(a^4*c^2+2*a^2*c*d+d^2)*arccoth(a*x)^2*(-a^2*c*d)^(1/2)-1/4*
a^3/d/(a^4*c^2+2*a^2*c*d+d^2)*polylog(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*
(-a^2*c*d)^(1/2)-d))*(-a^2*c*d)^(1/2)*c-1/2*a/(a^4*c^2+2*a^2*c*d+d^2)*polyl
og(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))*(-a^2*c*d)^(1/
2)-1/2/a/c/(a^4*c^2+2*a^2*c*d+d^2)*ln(1-(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*
(-a^2*c*d)^(1/2)-d))*arccoth(a*x)*(-a^2*c*d)^(1/2)*d+1/2/a/c/(a^4*c^2+2*a^2
*c*d+d^2)*arccoth(a*x)^2*(-a^2*c*d)^(1/2)*d-1/4/a/c/(a^4*c^2+2*a^2*c*d+d^2)
*polylog(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))*(-a^2*c*
d)^(1/2)*d+1/2/a*(-a^2*c*d)^(1/2)/c/d*arccoth(a*x)*ln(1-(a^2*c+d)*(a*x+1)/(
a*x-1)/(a^2*c+2*(-a^2*c*d)^(1/2)-d))-1/2/a*(-a^2*c*d)^(1/2)/c/d*arccoth(a*x
)^2+1/4/a*(-a^2*c*d)^(1/2)/c/d*polylog(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c+2
*(-a^2*c*d)^(1/2)-d))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arcoth}(ax)}{dx^2+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)/(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}(ax)}{c+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)/(d*x**2+c),x)
```

```
[Out] Integral(acoth(a*x)/(c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(ax)}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)/(d*x^2 + c), x)
```

3.40 $\int \frac{\coth^{-1}(ax)}{\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=590

$$\frac{i\,\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}-\frac{i\,\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{a\sqrt{c}+i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}+\frac{i\,\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}+i\sqrt{d}x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}-\frac{i\,\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}+i\sqrt{d}x\right)}{a\sqrt{c}+i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}+\frac{a\log\left(1-\right.}{4c\left(a^2c\right.}$$

```
[Out] (x*ArcCoth[a*x])/(2*c*(c + d*x^2)) + (ArcCoth[a*x]*ArcTan[(Sqrt[d]*x)/Sqrt[
c]])/(2*c^(3/2)*Sqrt[d]) + ((I/8)*Log[(Sqrt[d]*(1 - a*x))/(I*a*Sqrt[c] + Sq
rt[d])]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*Sqrt[d]) - ((I/8)*Log[-((S
qrt[d]*(1 + a*x))/(I*a*Sqrt[c] - Sqrt[d]))]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]])
/(c^(3/2)*Sqrt[d]) - ((I/8)*Log[-((Sqrt[d]*(1 - a*x))/(I*a*Sqrt[c] - Sqrt[d
]))]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*Sqrt[d]) + ((I/8)*Log[(Sqrt[d
]*(1 + a*x))/(I*a*Sqrt[c] + Sqrt[d])]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(c^(3
/2)*Sqrt[d]) + (a*Log[1 - a^2*x^2])/(4*c*(a^2*c + d)) - (a*Log[c + d*x^2])/
(4*c*(a^2*c + d)) + ((I/8)*PolyLog[2, (a*(Sqrt[c] - I*Sqrt[d]*x))/(a*Sqrt[c
] - I*Sqrt[d])]])/(c^(3/2)*Sqrt[d]) - ((I/8)*PolyLog[2, (a*(Sqrt[c] - I*Sqrt
[d]*x))/(a*Sqrt[c] + I*Sqrt[d])]])/(c^(3/2)*Sqrt[d]) + ((I/8)*PolyLog[2, (a*
(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] - I*Sqrt[d])]])/(c^(3/2)*Sqrt[d]) - ((I/
8)*PolyLog[2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] + I*Sqrt[d])]])/(c^(3/2
)*Sqrt[d])
```

Rubi [A] time = 0.869703, antiderivative size = 590, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 13, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {199, 205, 5977, 6725, 517, 444, 36, 31, 4908, 2409, 2394, 2393, 2391}

$$\frac{i\,\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}-\frac{i\,\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{a\sqrt{c}+i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}+\frac{i\,\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}+i\sqrt{d}x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}-\frac{i\,\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}+i\sqrt{d}x\right)}{a\sqrt{c}+i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}+\frac{a\log\left(1-\right.}{4c\left(a^2c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]/(c + d*x^2)^2,x]
```


```
[Out] (x*ArcCoth[a*x])/(2*c*(c + d*x^2)) + (ArcCoth[a*x]*ArcTan[(Sqrt[d]*x)/Sqrt[
c]])/(2*c^(3/2)*Sqrt[d]) + ((I/8)*Log[(Sqrt[d]*(1 - a*x))/(I*a*Sqrt[c] + Sq
rt[d])]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*Sqrt[d]) - ((I/8)*Log[-((S
qrt[d]*(1 + a*x))/(I*a*Sqrt[c] - Sqrt[d]))]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]])
/(c^(3/2)*Sqrt[d]) - ((I/8)*Log[-((Sqrt[d]*(1 - a*x))/(I*a*Sqrt[c] - Sqrt[d
]))]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*Sqrt[d]) + ((I/8)*Log[(Sqrt[d
]*(1 + a*x))/(I*a*Sqrt[c] + Sqrt[d])]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(c^(3
/2)*Sqrt[d]) + (a*Log[1 - a^2*x^2])/(4*c*(a^2*c + d)) - (a*Log[c + d*x^2])/
(4*c*(a^2*c + d)) + ((I/8)*PolyLog[2, (a*(Sqrt[c] - I*Sqrt[d]*x))/(a*Sqrt[c
] - I*Sqrt[d])]])/(c^(3/2)*Sqrt[d]) - ((I/8)*PolyLog[2, (a*(Sqrt[c] - I*Sqrt
[d]*x))/(a*Sqrt[c] + I*Sqrt[d])]])/(c^(3/2)*Sqrt[d]) + ((I/8)*PolyLog[2, (a*
(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] - I*Sqrt[d])]])/(c^(3/2)*Sqrt[d]) - ((I/
8)*PolyLog[2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] + I*Sqrt[d])]])/(c^(3/2
)*Sqrt[d])
```


Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
```

```
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5977

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4908

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
```

```
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\coth^{-1}(ax)}{\left(c+dx^2\right)^2}\,dx = \frac{x\coth^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} - a\int \frac{\frac{x}{2c(c+dx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}}{1-a^2x^2}\,dx$$

$$= \frac{x\coth^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} - a\int\left(-\frac{x}{2c(-1+ax)(1+ax)\left(c+dx^2\right)} - \frac{\tan^{-1}\left(\frac{\sqrt{d}}{\sqrt{c}}\right.}{2c^{3/2}\sqrt{d}\left(-1+\right.}\right.$$

$$= \frac{x\coth^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} + \frac{a\int\frac{x}{(-1+ax)(1+ax)(c+dx^2)}\,dx}{2c} + \frac{a\int\frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{-1+a^2x^2}\,dx}{2c^{3/2}\sqrt{d}}$$

$$= \frac{x\coth^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} + \frac{a\int\frac{x}{(-1+a^2x^2)(c+dx^2)}\,dx}{2c} + \frac{(ia)\int\frac{\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{-1+a^2x^2}\,dx}{4c^{3/2}\sqrt{d}} - \frac{(ia)\int}{4}$$

$$= \frac{x\coth^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} + \frac{a\,\mathrm{Subst}\left(\int\frac{1}{(-1+a^2x)(c+dx)}\,dx, x, x^2\right)}{4c} + \frac{(ia)\int\left(-\frac{\log\left(1-\frac{i\sqrt{}}{}\right.}{2(1-ax}\right.}{4c}$$

$$= \frac{x\coth^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} - \frac{(ia)\int\frac{\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{1-ax}\,dx}{8c^{3/2}\sqrt{d}} - \frac{(ia)\int\frac{\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{1+ax}\,dx}{8c^{3/2}\sqrt{d}} + \frac{(ia)\int\frac{\log}{}}{8c^3}$$

$$= \frac{x\coth^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} + \frac{i\log\left(\frac{\sqrt{d}(1-ax)}{ia\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} - \frac{i\log\left(-\frac{\sqrt{d}(1+ax)}{ia\sqrt{c}-\sqrt{d}}\right)\log\left(\right.}{8c^{3/2}\sqrt{d}}$$

$$= \frac{x\coth^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} + \frac{i\log\left(\frac{\sqrt{d}(1-ax)}{ia\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} - \frac{i\log\left(-\frac{\sqrt{d}(1+ax)}{ia\sqrt{c}-\sqrt{d}}\right)\log\left(\right.}{8c^{3/2}\sqrt{d}}$$

$$= \frac{x\coth^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} + \frac{i\log\left(\frac{\sqrt{d}(1-ax)}{ia\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} - \frac{i\log\left(-\frac{\sqrt{d}(1+ax)}{ia\sqrt{c}-\sqrt{d}}\right)\log\left(\right.}{8c^{3/2}\sqrt{d}}$$

Mathematica [A] time = 7.81475, size = 753, normalized size = 1.28

$$-\frac{a\left(i\left(\mathrm{PolyLog}\left(2,\frac{\left(-2i\sqrt{a^2cd}+a^2c-d\right)\left(\sqrt{a^2cd}+iadx\right)}{\left(a^2c+d\right)\left(\sqrt{a^2cd}-iadx\right)}\right)-\mathrm{PolyLog}\left(2,\frac{\left(2i\sqrt{a^2cd}+a^2c-d\right)\left(\sqrt{a^2cd}+iadx\right)}{\left(a^2c+d\right)\left(\sqrt{a^2cd}-iadx\right)}\right)\right)+2i\cos^{-1}\left(\frac{a^2c-d}{a^2c+d}\right)\tan^{-1}\left(\frac{ac}{x\sqrt{a^2cd}}\right)-4\coth^{-1}(ax)\tan^{-1}\left(\frac{adx}{\sqrt{a^2cd}}\right)+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[a*x]/(c + d*x^2)^2,x]
```

```
[Out] -(a*((2*Log[1 + ((a^2*c + d)*Cosh[2*ArcCoth[a*x]])/(-(a^2*c) + d)])/(a^2*c
+ d) + ((2*I)*ArcCos[(a^2*c - d)/(a^2*c + d)]*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x
```

```
)] - 4*ArcCoth[a*x]*ArcTan[(a*d*x)/Sqrt[a^2*c*d]] + (ArcCos[(a^2*c - d)/(a^
2*c + d)] + 2*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)])*Log[(2*d*(a^2*c - I*Sqrt[a^2
*c*d])*(-1 + a*x))/((a^2*c + d)*(I*Sqrt[a^2*c*d] + a*d*x))] + (ArcCos[(a^2*
c - d)/(a^2*c + d)] - 2*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)])*Log[(2*d*(a^2*c +
I*Sqrt[a^2*c*d])*(1 + a*x))/((a^2*c + d)*(I*Sqrt[a^2*c*d] + a*d*x))] - (Arc
Cos[(a^2*c - d)/(a^2*c + d)] + 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] + ArcTan[
(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d])/(Sqrt[a^2*c + d]*E^Arc
Coth[a*x]*Sqrt[-(a^2*c) + d + (a^2*c + d)*Cosh[2*ArcCoth[a*x]]])] - (ArcCos
[(a^2*c - d)/(a^2*c + d)] - 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] + ArcTan[(a*
d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d]*E^ArcCoth[a*x])/(Sqrt[a^2*
c + d]*Sqrt[-(a^2*c) + d + (a^2*c + d)*Cosh[2*ArcCoth[a*x]]])] + I*(PolyLog
[2, ((a^2*c - d - (2*I)*Sqrt[a^2*c*d])*(Sqrt[a^2*c*d] + I*a*d*x))/((a^2*c +
 d)*(Sqrt[a^2*c*d] - I*a*d*x))] - PolyLog[2, ((a^2*c - d + (2*I)*Sqrt[a^2*c
*d])*(Sqrt[a^2*c*d] + I*a*d*x))/((a^2*c + d)*(Sqrt[a^2*c*d] - I*a*d*x))]))/
Sqrt[a^2*c*d] - (4*ArcCoth[a*x]*Sinh[2*ArcCoth[a*x]])/(-(a^2*c) + d + (a^2*
c + d)*Cosh[2*ArcCoth[a*x]])))/(8*c)
```

Maple [B] time = 0.41, size = 2218, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)/(d*x^2+c)^2,x)
```

```
[Out] -1/4*a^3/(a^2*c+d)^2*ln(a^2*c/(a*x-1)^2*(a*x+1)^2-2*a^2*c*(a*x+1)/(a*x-1)+d
/(a*x-1)^2*(a*x+1)^2+a^2*c+2*(a*x+1)/(a*x-1)*d+d)-1/2*a^3/(a^2*c+d)^2*ln((a
*x-1)/(a*x+1))-1/4*a^4*(c*d)^(1/2)/d/(a^2*c+d)^2*arctan(a/d*(c*d)^(1/2))+1/
4*(c*d)^(1/2)/c^2*d/(a^2*c+d)^2*arctan(a/d*(c*d)^(1/2))-3/4*a*d*ln(1-(a^2*c
+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))*arccoth(a*x)/(a^2*c+d)/c/
(a^4*c^2+2*a^2*c*d+d^2)*(-a^2*c*d)^(1/2)-1/4*a^5/d/(a^2*c+d)/(a^4*c^2+2*a^2
*c*d+d^2)*ln(1-(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))*arcc
oth(a*x)*(-a^2*c*d)^(1/2)*c-1/4/a/c^2*d^2/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)
*ln(1-(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))*arccoth(a*x)*
(-a^2*c*d)^(1/2)+1/4*a^2*(c*d)^(1/2)/c/d/(a^2*c+d)*arctan(a/d*(c*d)^(1/2))-
1/4*a*(-a^2*c*d)^(1/2)/c/d/(a^2*c+d)*arccoth(a*x)^2+1/8*a*(-a^2*c*d)^(1/2)/
c/d/(a^2*c+d)*polylog(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c+2*(-a^2*c*d)^(1/2)
-d))+1/4/a*(-a^2*c*d)^(1/2)/c^2/(a^2*c+d)*arccoth(a*x)*ln(1-(a^2*c+d)*(a*x+
1)/(a*x-1)/(a^2*c+2*(-a^2*c*d)^(1/2)-d))-3/4*a^3/(a^2*c+d)/(a^4*c^2+2*a^2*c
*d+d^2)*ln(1-(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))*arccot
h(a*x)*(-a^2*c*d)^(1/2)+1/4*a^2*(c*d)^(1/2)/c/d/(a^2*c+d)*arctan(1/(a^2*c+d
)*d^2/(c*d)^(1/2)*x+1/(a^2*c+d)*d/(c*d)^(1/2)*a*c+a^2/(a^2*c+d)*(c*d)^(1/2)
*x-a/(a^2*c+d)*(c*d)^(1/2))-1/2*a/c/(a^2*c+d)^2*d*ln((a*x-1)/(a*x+1))+1/2*a
^4*arccoth(a*x)/(a^2*c+d)/(a^2*d*x^2+a^2*c)*x+1/8/a*(-a^2*c*d)^(1/2)/c^2/(a
^2*c+d)*polylog(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c+2*(-a^2*c*d)^(1/2)-d))-1
/4/a*(-a^2*c*d)^(1/2)/c^2/(a^2*c+d)*arccoth(a*x)^2-1/4*a/c/(a^2*c+d)^2*d*ln
(a^2*c/(a*x-1)^2*(a*x+1)^2-2*a^2*c*(a*x+1)/(a*x-1)+d/(a*x-1)^2*(a*x+1)^2+a^
2*c+2*(a*x+1)/(a*x-1)*d+d)+3/4*a^3/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*arccot
h(a*x)^2*(-a^2*c*d)^(1/2)-3/8*a^3/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*polylog
(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))*(-a^2*c*d)^(1/2)
+1/4*(c*d)^(1/2)/c^2*d/(a^2*c+d)^2*arctan(1/(a^2*c+d)*d^2/(c*d)^(1/2)*x+1/(
a^2*c+d)*d/(c*d)^(1/2)*a*c+a^2/(a^2*c+d)*(c*d)^(1/2)*x-a/(a^2*c+d)*(c*d)^(1
/2))-1/4*(c*d)^(1/2)/c^2/(a^2*c+d)*arctan(1/(a^2*c+d)*d^2/(c*d)^(1/2)*x+1/(
a^2*c+d)*d/(c*d)^(1/2)*a*c+a^2/(a^2*c+d)*(c*d)^(1/2)*x-a/(a^2*c+d)*(c*d)^(1
/2))-1/2*a^3*arccoth(a*x)/(a^2*c+d)/(a^2*d*x^2+a^2*c)-1/4*(c*d)^(1/2)/c^2/(
a^2*c+d)*arctan(a/d*(c*d)^(1/2))-1/4*a^4*(c*d)^(1/2)/d/(a^2*c+d)^2*arctan(1
/(a^2*c+d)*d^2/(c*d)^(1/2)*x+1/(a^2*c+d)*d/(c*d)^(1/2)*a*c+a^2/(a^2*c+d)*(c
```

```
*d)^(1/2)*x-a/(a^2*c+d)*(c*d)^(1/2))-3/8*a/c*d/(a^2*c+d)/(a^4*c^2+2*a^2*c*d
+d^2)*polylog(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))*(-a
^2*c*d)^(1/2)-1/8/a/c^2*d^2/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*polylog(2,(a^
2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))*(-a^2*c*d)^(1/2)+1/4/a
/c^2*d^2/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*arccoth(a*x)^2*(-a^2*c*d)^(1/2)+
3/4*a/c*d/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*arccoth(a*x)^2*(-a^2*c*d)^(1/2)
-1/2*a^3*arccoth(a*x)/c/(a^2*c+d)/(a^2*d*x^2+a^2*c)*d*x^2+1/2*a^2*arccoth(a
*x)/c/(a^2*c+d)/(a^2*d*x^2+a^2*c)*x*d+1/4*a*(-a^2*c*d)^(1/2)/c/d/(a^2*c+d)*
arccoth(a*x)*ln(1-(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c+2*(-a^2*c*d)^(1/2)-d))-1
/8*a^5/d/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*polylog(2,(a^2*c+d)*(a*x+1)/(a*x
-1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))*(-a^2*c*d)^(1/2)*c+1/4*a^5/d/(a^2*c+d)/(a
^4*c^2+2*a^2*c*d+d^2)*arccoth(a*x)^2*(-a^2*c*d)^(1/2)*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arcoth}\,(ax)}{d^2x^4 + 2\,cdx^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arcoth}\,(ax)}{\left(dx^2 + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)/(d*x^2 + c)^2, x)
```

3.41 $\int \frac{\coth^{-1}(ax)}{\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=657

$$\frac{3i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}-\frac{3i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{a\sqrt{c}+i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}+\frac{3i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}+i\sqrt{d}x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}-\frac{3i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}+i\sqrt{d}x\right)}{a\sqrt{c}+i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}+\cdots$$

```
[Out] a/(8*c*(a^2*c + d)*(c + d*x^2)) + (x*ArcCoth[a*x])/(4*c*(c + d*x^2)^2) + (3
*x*ArcCoth[a*x])/(8*c^2*(c + d*x^2)) + (3*ArcCoth[a*x]*ArcTan[(Sqrt[d]*x)/S
qrt[c]])/(8*c^(5/2)*Sqrt[d]) + (((3*I)/32)*Log[(Sqrt[d]*(1 - a*x))/(I*a*Sqr
t[c] + Sqrt[d])]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d]) - (((3*I
)/32)*Log[-((Sqrt[d]*(1 + a*x))/(I*a*Sqrt[c] - Sqrt[d]))]*Log[1 - (I*Sqrt[d
]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d]) - (((3*I)/32)*Log[-((Sqrt[d]*(1 - a*x))/(I
*a*Sqrt[c] - Sqrt[d]))]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d]) +
 (((3*I)/32)*Log[(Sqrt[d]*(1 + a*x))/(I*a*Sqrt[c] + Sqrt[d])]*Log[1 + (I*Sq
rt[d]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d]) + (a*(5*a^2*c + 3*d)*Log[1 - a^2*x^2])
/(16*c^2*(a^2*c + d)^2) - (a*(5*a^2*c + 3*d)*Log[c + d*x^2])/(16*c^2*(a^2*c
 + d)^2) + (((3*I)/32)*PolyLog[2, (a*(Sqrt[c] - I*Sqrt[d]*x))/(a*Sqrt[c] -
I*Sqrt[d])])/(c^(5/2)*Sqrt[d]) - (((3*I)/32)*PolyLog[2, (a*(Sqrt[c] - I*Sqr
t[d]*x))/(a*Sqrt[c] + I*Sqrt[d])])/(c^(5/2)*Sqrt[d]) + (((3*I)/32)*PolyLog[
2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] - I*Sqrt[d])])/(c^(5/2)*Sqrt[d])
- (((3*I)/32)*PolyLog[2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] + I*Sqrt[d]
)])/(c^(5/2)*Sqrt[d])
```

Rubi [A] time = 0.950909, antiderivative size = 657, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {199, 205, 5977, 6725, 571, 77, 4908, 2409, 2394, 2393, 2391}

$$\frac{3i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}-\frac{3i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{a\sqrt{c}+i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}+\frac{3i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}+i\sqrt{d}x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}-\frac{3i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}+i\sqrt{d}x\right)}{a\sqrt{c}+i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]/(c + d*x^2)^3,x]
```

```
[Out] a/(8*c*(a^2*c + d)*(c + d*x^2)) + (x*ArcCoth[a*x])/(4*c*(c + d*x^2)^2) + (3
*x*ArcCoth[a*x])/(8*c^2*(c + d*x^2)) + (3*ArcCoth[a*x]*ArcTan[(Sqrt[d]*x)/S
qrt[c]])/(8*c^(5/2)*Sqrt[d]) + (((3*I)/32)*Log[(Sqrt[d]*(1 - a*x))/(I*a*Sqr
t[c] + Sqrt[d])]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d]) - (((3*I
)/32)*Log[-((Sqrt[d]*(1 + a*x))/(I*a*Sqrt[c] - Sqrt[d]))]*Log[1 - (I*Sqrt[d
]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d]) - (((3*I)/32)*Log[-((Sqrt[d]*(1 - a*x))/(I
*a*Sqrt[c] - Sqrt[d]))]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d]) +
 (((3*I)/32)*Log[(Sqrt[d]*(1 + a*x))/(I*a*Sqrt[c] + Sqrt[d])]*Log[1 + (I*Sq
rt[d]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d]) + (a*(5*a^2*c + 3*d)*Log[1 - a^2*x^2])
/(16*c^2*(a^2*c + d)^2) - (a*(5*a^2*c + 3*d)*Log[c + d*x^2])/(16*c^2*(a^2*c
 + d)^2) + (((3*I)/32)*PolyLog[2, (a*(Sqrt[c] - I*Sqrt[d]*x))/(a*Sqrt[c] -
I*Sqrt[d])])/(c^(5/2)*Sqrt[d]) - (((3*I)/32)*PolyLog[2, (a*(Sqrt[c] - I*Sqr
t[d]*x))/(a*Sqrt[c] + I*Sqrt[d])])/(c^(5/2)*Sqrt[d]) + (((3*I)/32)*PolyLog[
2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] - I*Sqrt[d])])/(c^(5/2)*Sqrt[d])
- (((3*I)/32)*PolyLog[2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] + I*Sqrt[d]
)])/(c^(5/2)*Sqrt[d])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5977

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 4908

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\coth^{-1}(ax)}{\left(c+dx^2\right)^3}\,dx = \frac{x\coth^{-1}(ax)}{4c\left(c+dx^2\right)^2}+\frac{3x\coth^{-1}(ax)}{8c^2\left(c+dx^2\right)}+\frac{3\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}}-a\int\frac{\frac{x}{4c\left(c+dx^2\right)^2}+\frac{3x}{8c^2\left(c+dx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}}}{1-a^2x^2}$$

$$=\frac{x\coth^{-1}(ax)}{4c\left(c+dx^2\right)^2}+\frac{3x\coth^{-1}(ax)}{8c^2\left(c+dx^2\right)}+\frac{3\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}}-a\int\left(-\frac{x\left(5c+3dx^2\right)}{8c^2\left(-1+a^2x^2\right)\left(c+dx^2\right)^2}-\ldots\right)$$

$$=\frac{x\coth^{-1}(ax)}{4c\left(c+dx^2\right)^2}+\frac{3x\coth^{-1}(ax)}{8c^2\left(c+dx^2\right)}+\frac{3\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}}+\frac{a\int\frac{x\left(5c+3dx^2\right)}{\left(-1+a^2x^2\right)\left(c+dx^2\right)^2}dx}{8c^2}+\frac{(3a)\int\frac{\tan^{-1}(\ldots)}{-1+a\ldots}}{8c^{5/2}\sqrt{\ldots}}$$

$$=\frac{x\coth^{-1}(ax)}{4c\left(c+dx^2\right)^2}+\frac{3x\coth^{-1}(ax)}{8c^2\left(c+dx^2\right)}+\frac{3\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}}+\frac{a\,\text{Subst}\left(\int\frac{5c+3dx}{\left(-1+a^2x\right)(c+dx)^2}dx,x,x^2\right)}{16c^2}+\ldots$$

$$=\frac{x\coth^{-1}(ax)}{4c\left(c+dx^2\right)^2}+\frac{3x\coth^{-1}(ax)}{8c^2\left(c+dx^2\right)}+\frac{3\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}}+\frac{a\,\text{Subst}\left(\int\left(\frac{a^2\left(5a^2c+3d\right)}{\left(a^2c+d\right)^2\left(-1+a^2x\right)}-\frac{2cd}{\left(a^2c+d\right)(c\ldots}\ldots\right)\right)}{16c^2}$$

$$=\frac{a}{8c\left(a^2c+d\right)\left(c+dx^2\right)}+\frac{x\coth^{-1}(ax)}{4c\left(c+dx^2\right)^2}+\frac{3x\coth^{-1}(ax)}{8c^2\left(c+dx^2\right)}+\frac{3\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}}+\frac{a\left(5a^2c+3d\ldots\right)}{16c^2\left(\ldots\right.}$$

$$=\frac{a}{8c\left(a^2c+d\right)\left(c+dx^2\right)}+\frac{x\coth^{-1}(ax)}{4c\left(c+dx^2\right)^2}+\frac{3x\coth^{-1}(ax)}{8c^2\left(c+dx^2\right)}+\frac{3\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}}+\frac{3i\log\left(\frac{\sqrt{d}(1-\ldots}{ia\sqrt{c}+\ldots}\right)}{32\ldots}$$

$$=\frac{a}{8c\left(a^2c+d\right)\left(c+dx^2\right)}+\frac{x\coth^{-1}(ax)}{4c\left(c+dx^2\right)^2}+\frac{3x\coth^{-1}(ax)}{8c^2\left(c+dx^2\right)}+\frac{3\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}}+\frac{3i\log\left(\frac{\sqrt{d}(1-\ldots}{ia\sqrt{c}+\ldots}\right)}{32\ldots}$$

$$=\frac{a}{8c\left(a^2c+d\right)\left(c+dx^2\right)}+\frac{x\coth^{-1}(ax)}{4c\left(c+dx^2\right)^2}+\frac{3x\coth^{-1}(ax)}{8c^2\left(c+dx^2\right)}+\frac{3\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}}+\frac{3i\log\left(\frac{\sqrt{d}(1-\ldots}{ia\sqrt{c}+\ldots}\right)}{32\ldots}$$

Mathematica [B] time = 12.88, size = 1838, normalized size = 2.8

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[a*x]/(c + d*x^2)^3,x]
```

```
[Out] a^5*((-5*Log[1 + ((a^2*c + d)*Cosh[2*ArcCoth[a*x]])/(-(a^2*c) + d)])/(16*a^
2*c*(a^2*c + d)^2) - (3*d*Log[1 + ((a^2*c + d)*Cosh[2*ArcCoth[a*x]])/(-(a^2
*c) + d)])/(16*a^4*c^2*(a^2*c + d)^2) + (3*((-2*I)*ArcCos[-((-(a^2*c) + d)/
(a^2*c + d))]*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] + 4*ArcCoth[a*x]*ArcTan[(a*d*
x)/Sqrt[a^2*c*d]] - (ArcCos[-((-(a^2*c) + d)/(a^2*c + d))] - 2*ArcTan[(a*c)
```

```
/(Sqrt[a^2*c*d]*x)])*Log[1 - ((-(a^2*c) + d - (2*I)*Sqrt[a^2*c*d])*(2*d - (
(2*I)*Sqrt[a^2*c*d])/(a*x)))/((a^2*c + d)*(2*d + ((2*I)*Sqrt[a^2*c*d])/(a*x
)))] + (-ArcCos[-((-(a^2*c) + d)/(a^2*c + d))] - 2*ArcTan[(a*c)/(Sqrt[a^2*c
*d]*x)])*Log[1 - ((-(a^2*c) + d + (2*I)*Sqrt[a^2*c*d])*(2*d - ((2*I)*Sqrt[a
^2*c*d])/(a*x)))/((a^2*c + d)*(2*d + ((2*I)*Sqrt[a^2*c*d])/(a*x)))] + (ArcC
os[-((-(a^2*c) + d)/(a^2*c + d))] + (2*I)*((-I)*ArcTan[(a*c)/(Sqrt[a^2*c*d]
*x)] - I*ArcTan[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d])/(Sqrt[
a^2*c + d]*E^ArcCoth[a*x]*Sqrt[-(a^2*c) + d + (a^2*c + d)*Cosh[2*ArcCoth[a*
x]]])] + (ArcCos[-((-(a^2*c) + d)/(a^2*c + d))] - (2*I)*((-I)*ArcTan[(a*c)/
(Sqrt[a^2*c*d]*x)] - I*ArcTan[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^
2*c*d]*E^ArcCoth[a*x])/(Sqrt[a^2*c + d]*Sqrt[-(a^2*c) + d + (a^2*c + d)*Cos
h[2*ArcCoth[a*x]]])] + I*(PolyLog[2, ((-(a^2*c) + d - (2*I)*Sqrt[a^2*c*d])*
(2*d - ((2*I)*Sqrt[a^2*c*d])/(a*x)))/((a^2*c + d)*(2*d + ((2*I)*Sqrt[a^2*c*
d])/(a*x)))] - PolyLog[2, ((-(a^2*c) + d + (2*I)*Sqrt[a^2*c*d])*(2*d - ((2*
I)*Sqrt[a^2*c*d])/(a*x)))/((a^2*c + d)*(2*d + ((2*I)*Sqrt[a^2*c*d])/(a*x)))
])))/(32*a^2*c*Sqrt[a^2*c*d]*(a^2*c + d)) + (3*d*((-2*I)*ArcCos[-((-(a^2*c)
 + d)/(a^2*c + d))]*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] + 4*ArcCoth[a*x]*ArcTan
[(a*d*x)/Sqrt[a^2*c*d]] - (ArcCos[-((-(a^2*c) + d)/(a^2*c + d))] - 2*ArcTan
[(a*c)/(Sqrt[a^2*c*d]*x)])*Log[1 - ((-(a^2*c) + d - (2*I)*Sqrt[a^2*c*d])*(2
*d - ((2*I)*Sqrt[a^2*c*d])/(a*x)))/((a^2*c + d)*(2*d + ((2*I)*Sqrt[a^2*c*d]
)/(a*x)))] + (-ArcCos[-((-(a^2*c) + d)/(a^2*c + d))] - 2*ArcTan[(a*c)/(Sqrt
[a^2*c*d]*x)])*Log[1 - ((-(a^2*c) + d + (2*I)*Sqrt[a^2*c*d])*(2*d - ((2*I)*
Sqrt[a^2*c*d])/(a*x)))/((a^2*c + d)*(2*d + ((2*I)*Sqrt[a^2*c*d])/(a*x)))] +
 (ArcCos[-((-(a^2*c) + d)/(a^2*c + d))] + (2*I)*((-I)*ArcTan[(a*c)/(Sqrt[a^
2*c*d]*x)] - I*ArcTan[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d])/
(Sqrt[a^2*c + d]*E^ArcCoth[a*x]*Sqrt[-(a^2*c) + d + (a^2*c + d)*Cosh[2*ArcC
oth[a*x]]])] + (ArcCos[-((-(a^2*c) + d)/(a^2*c + d))] - (2*I)*((-I)*ArcTan[
(a*c)/(Sqrt[a^2*c*d]*x)] - I*ArcTan[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*S
qrt[a^2*c*d]*E^ArcCoth[a*x])/(Sqrt[a^2*c + d]*Sqrt[-(a^2*c) + d + (a^2*c +
d)*Cosh[2*ArcCoth[a*x]]])] + I*(PolyLog[2, ((-(a^2*c) + d - (2*I)*Sqrt[a^2*
c*d])*(2*d - ((2*I)*Sqrt[a^2*c*d])/(a*x)))/((a^2*c + d)*(2*d + ((2*I)*Sqrt[
a^2*c*d])/(a*x)))] - PolyLog[2, ((-(a^2*c) + d + (2*I)*Sqrt[a^2*c*d])*(2*d
- ((2*I)*Sqrt[a^2*c*d])/(a*x)))/((a^2*c + d)*(2*d + ((2*I)*Sqrt[a^2*c*d])/(
a*x)))])))/(32*a^4*c^2*Sqrt[a^2*c*d]*(a^2*c + d)) - (d*ArcCoth[a*x]*Sinh[2*
ArcCoth[a*x]])/(2*a^2*c*(a^2*c + d)*(-(a^2*c) + d + a^2*c*Cosh[2*ArcCoth[a*
x]] + d*Cosh[2*ArcCoth[a*x]])^2) - (2*a^2*c*d - 5*a^4*c^2*ArcCoth[a*x]*Sinh
[2*ArcCoth[a*x]] - 8*a^2*c*d*ArcCoth[a*x]*Sinh[2*ArcCoth[a*x]] - 3*d^2*ArcC
oth[a*x]*Sinh[2*ArcCoth[a*x]])/(8*a^4*c^2*(a^2*c + d)^2*(-(a^2*c) + d + a^2
*c*Cosh[2*ArcCoth[a*x]] + d*Cosh[2*ArcCoth[a*x]])))
```

Maple [B] time = 0.596, size = 4128, normalized size = 6.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)/(d*x^2+c)^3,x)
```

```
[Out] 1/8*a^5/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2*d-5/8*a^5/(a^4*c^2+2*a^
2*c*d+d^2)/(a^2*c+d)*ln((a*x-1)/(a*x+1))-5/16*a^5/(a^4*c^2+2*a^2*c*d+d^2)/(
a^2*c+d)*ln(a^2*c/(a*x-1)^2*(a*x+1)^2-2*a^2*c*(a*x+1)/(a*x-1)+d/(a*x-1)^2*(
a*x+1)^2+a^2*c+2*(a*x+1)/(a*x-1)*d+d)+3/4*a^5*arccoth(a*x)^2/(a^4*c^2+2*a^2
*c*d+d^2)^2*(-a^2*c*d)^(1/2)-3/8*a^5*polylog(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a
^2*c-2*(-a^2*c*d)^(1/2)-d))/(a^4*c^2+2*a^2*c*d+d^2)^2*(-a^2*c*d)^(1/2)-1/8*
a^2*(c*d)^(1/2)/c^2/(a^4*c^2+2*a^2*c*d+d^2)*arctan(a/d*(c*d)^(1/2))-3/16*(c
*d)^(1/2)/c^3*d/(a^4*c^2+2*a^2*c*d+d^2)*arctan(a/d*(c*d)^(1/2))-5/8*a^7/(a^
4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2*c*arccoth(a*x)-9/8*a^3/c/(a^4*c^2+
```

```
2*a^2*c*d+d^2)^2*ln(1-(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d
))*arccoth(a*x)*(-a^2*c*d)^(1/2)*d-1/8*a^2*(c*d)^(1/2)/c^2/(a^4*c^2+2*a^2*c
*d+d^2)*arctan(1/(a^2*c+d)*d^2/(c*d)^(1/2)*x+1/(a^2*c+d)*d/(c*d)^(1/2)*a*c+
a^2/(a^2*c+d)*(c*d)^(1/2)*x-a/(a^2*c+d)*(c*d)^(1/2))-3/8*a*(-a^2*c*d)^(1/2)
/c^2/(a^4*c^2+2*a^2*c*d+d^2)*arccoth(a*x)^2-3/8*a^5/(a^4*c^2+2*a^2*c*d+d^2)
/(a^2*d*x^2+a^2*c)^2*d*arccoth(a*x)+3/16*a*(-a^2*c*d)^(1/2)/c^2/(a^4*c^2+2*
a^2*c*d+d^2)*polylog(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c+2*(-a^2*c*d)^(1/2)-
d))-3/4*a^5*ln(1-(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))*ar
ccoth(a*x)/(a^4*c^2+2*a^2*c*d+d^2)^2*(-a^2*c*d)^(1/2)-3/16*(c*d)^(1/2)/c^3*
d/(a^4*c^2+2*a^2*c*d+d^2)*arctan(1/(a^2*c+d)*d^2/(c*d)^(1/2)*x+1/(a^2*c+d)*
d/(c*d)^(1/2)*a*c+a^2/(a^2*c+d)*(c*d)^(1/2)*x-a/(a^2*c+d)*(c*d)^(1/2))-1/8*
a^7/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2*d*x^2-5/8*a^7/(a^4*c^2+2*a^
2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2/c*arccoth(a*x)*x^4*d^2-3/4*a/c^2*d^2/(a^4*c^
2+2*a^2*c*d+d^2)^2*ln(1-(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^(1/2)
-d))*arccoth(a*x)*(-a^2*c*d)^(1/2)-3/16*a^7/d/(a^4*c^2+2*a^2*c*d+d^2)^2*ln(
1-(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))*arccoth(a*x)*(-a^
2*c*d)^(1/2)*c+3/8*a^8/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2*arccoth(
a*x)*x^3*d-5/4*a^7/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2*arccoth(a*x)
*x^2*d+5/4*a^6/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2*arccoth(a*x)*x*d
+3/16*(c*d)^(1/2)/c^3*d^2/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*arctan(1/(a^2*c
+d)*d^2/(c*d)^(1/2)*x+1/(a^2*c+d)*d/(c*d)^(1/2)*a*c+a^2/(a^2*c+d)*(c*d)^(1/
2)*x-a/(a^2*c+d)*(c*d)^(1/2))-5/16*a^6*(c*d)^(1/2)/d/(a^2*c+d)/(a^4*c^2+2*a
^2*c*d+d^2)*arctan(a/d*(c*d)^(1/2))+5/16*a^4*(c*d)^(1/2)/c/d/(a^4*c^2+2*a^2
*c*d+d^2)*arctan(a/d*(c*d)^(1/2))-3/16*a^4*(c*d)^(1/2)/c/(a^2*c+d)/(a^4*c^2
+2*a^2*c*d+d^2)*arctan(a/d*(c*d)^(1/2))+3/16*(c*d)^(1/2)/c^3*d^2/(a^2*c+d)/
(a^4*c^2+2*a^2*c*d+d^2)*arctan(a/d*(c*d)^(1/2))-1/8*a^7/(a^4*c^2+2*a^2*c*d+
d^2)/(a^2*d*x^2+a^2*c)^2/c*d^2*x^4+1/8*a^5/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x
^2+a^2*c)^2/c*d^2*x^2+9/8*a^3/c*d/(a^4*c^2+2*a^2*c*d+d^2)^2*arccoth(a*x)^2*
(-a^2*c*d)^(1/2)-9/16*a^3/c*d/(a^4*c^2+2*a^2*c*d+d^2)^2*polylog(2,(a^2*c+d)
*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))*(-a^2*c*d)^(1/2)-3/8*a/c^2*d
^2/(a^4*c^2+2*a^2*c*d+d^2)^2*polylog(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(
-a^2*c*d)^(1/2)-d))*(-a^2*c*d)^(1/2)-3/16/a*(-a^2*c*d)^(1/2)/c^3*d/(a^4*c^2
+2*a^2*c*d+d^2)*arccoth(a*x)^2-a^3/(a^4*c^2+2*a^2*c*d+d^2)/c*d/(a^2*c+d)*ln
((a*x-1)/(a*x+1))+3/32*a^3*(-a^2*c*d)^(1/2)/c/d/(a^4*c^2+2*a^2*c*d+d^2)*pol
ylog(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c+2*(-a^2*c*d)^(1/2)-d))-3/32*a^7/d/(
a^4*c^2+2*a^2*c*d+d^2)^2*polylog(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2
*c*d)^(1/2)-d))*(-a^2*c*d)^(1/2)*c+3/16/a/c^3*d^3/(a^4*c^2+2*a^2*c*d+d^2)^2
*arccoth(a*x)^2*(-a^2*c*d)^(1/2)-5/16*a^6*(c*d)^(1/2)/d/(a^2*c+d)/(a^4*c^2+
2*a^2*c*d+d^2)*arctan(1/(a^2*c+d)*d^2/(c*d)^(1/2)*x+1/(a^2*c+d)*d/(c*d)^(1/
2)*a*c+a^2/(a^2*c+d)*(c*d)^(1/2)*x-a/(a^2*c+d)*(c*d)^(1/2))+3/8*a*(-a^2*c*d
)^(1/2)/c^2/(a^4*c^2+2*a^2*c*d+d^2)*arccoth(a*x)*ln(1-(a^2*c+d)*(a*x+1)/(a*
x-1)/(a^2*c+2*(-a^2*c*d)^(1/2)-d))-3/32/a/c^3*d^3/(a^4*c^2+2*a^2*c*d+d^2)^2
*polylog(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))*(-a^2*c*
d)^(1/2)-3/16*a^3*(-a^2*c*d)^(1/2)/c/d/(a^4*c^2+2*a^2*c*d+d^2)*arccoth(a*x)
^2-3/8*a/(a^4*c^2+2*a^2*c*d+d^2)/c^2*d^2/(a^2*c+d)*ln((a*x-1)/(a*x+1))-1/2*
a^3/(a^4*c^2+2*a^2*c*d+d^2)/c*d/(a^2*c+d)*ln(a^2*c/(a*x-1)^2*(a*x+1)^2-2*a^
2*c*(a*x+1)/(a*x-1)+d/(a*x-1)^2*(a*x+1)^2+a^2*c+2*(a*x+1)/(a*x-1)*d+d)-3/16
*a/(a^4*c^2+2*a^2*c*d+d^2)/c^2*d^2/(a^2*c+d)*ln(a^2*c/(a*x-1)^2*(a*x+1)^2-2
*a^2*c*(a*x+1)/(a*x-1)+d/(a*x-1)^2*(a*x+1)^2+a^2*c+2*(a*x+1)/(a*x-1)*d+d)+5
/8*a^8/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2*c*arccoth(a*x)*x+3/32/a*
(-a^2*c*d)^(1/2)/c^3*d/(a^4*c^2+2*a^2*c*d+d^2)*polylog(2,(a^2*c+d)*(a*x+1)/
(a*x-1)/(a^2*c+2*(-a^2*c*d)^(1/2)-d))+3/4*a*d^2*arccoth(a*x)^2/c^2/(a^4*c^2
+2*a^2*c*d+d^2)^2*(-a^2*c*d)^(1/2)+3/16*a^7/d/(a^4*c^2+2*a^2*c*d+d^2)^2*arc
coth(a*x)^2*(-a^2*c*d)^(1/2)*c-3/16*a^4*(c*d)^(1/2)/c/(a^2*c+d)/(a^4*c^2+2*
a^2*c*d+d^2)*arctan(1/(a^2*c+d)*d^2/(c*d)^(1/2)*x+1/(a^2*c+d)*d/(c*d)^(1/2)
*a*c+a^2/(a^2*c+d)*(c*d)^(1/2)*x-a/(a^2*c+d)*(c*d)^(1/2))+5/16*a^4*(c*d)^(1
/2)/c/d/(a^4*c^2+2*a^2*c*d+d^2)*arctan(1/(a^2*c+d)*d^2/(c*d)^(1/2)*x+1/(a^2
*c+d)*d/(c*d)^(1/2)*a*c+a^2/(a^2*c+d)*(c*d)^(1/2)*x-a/(a^2*c+d)*(c*d)^(1/2)
)+3/4*a^6/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2/c*arccoth(a*x)*x^3*d^
```

```
2+3/16/a*(-a^2*c*d)^(1/2)/c^3*d/(a^4*c^2+2*a^2*c*d+d^2)*arccoth(a*x)*ln(1-(
a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c+2*(-a^2*c*d)^(1/2)-d))-3/16/a/c^3*d^3/(a^4*
c^2+2*a^2*c*d+d^2)^2*ln(1-(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^(1/
2)-d))*arccoth(a*x)*(-a^2*c*d)^(1/2)+3/8*a^4/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d
*x^2+a^2*c)^2/c^2*arccoth(a*x)*x^3*d^3+3/16*a^3*(-a^2*c*d)^(1/2)/c/d/(a^4*c
^2+2*a^2*c*d+d^2)*arccoth(a*x)*ln(1-(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c+2*(-a^
2*c*d)^(1/2)-d))-3/4*a^5/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2/c*arcc
oth(a*x)*x^2*d^2+5/8*a^4/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2/c*arcc
oth(a*x)*x*d^2+5/16*a^2*(c*d)^(1/2)/c^2*d/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)
*arctan(a/d*(c*d)^(1/2))-3/8*a^5/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^
2/c^2*arccoth(a*x)*x^4*d^3+5/16*a^2*(c*d)^(1/2)/c^2*d/(a^2*c+d)/(a^4*c^2+2*
a^2*c*d+d^2)*arctan(1/(a^2*c+d)*d^2/(c*d)^(1/2)*x+1/(a^2*c+d)*d/(c*d)^(1/2)
*a*c+a^2/(a^2*c+d)*(c*d)^(1/2)*x-a/(a^2*c+d)*(c*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arcoth}(ax)}{d^3x^6 + 3\,cd^2x^4 + 3\,c^2dx^2 + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)/(d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(ax)}{\left(dx^2 + c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)/(d*x^2 + c)^3, x)
```

3.42 $\int \sqrt{c+dx^2}\coth^{-1}(ax)\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\coth^{-1}(ax)\sqrt{c+dx^2},x\right)$$

```
[Out] Unintegrable[Sqrt[c + d*x^2]*ArcCoth[a*x], x]
```

Rubi [A] time = 0.0208016, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt{c+dx^2}\coth^{-1}(ax)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c + d*x^2]*ArcCoth[a*x],x]
```

```
[Out] Defer[Int][Sqrt[c + d*x^2]*ArcCoth[a*x], x]
```

Rubi steps

$$\int \sqrt{c+dx^2}\coth^{-1}(ax)\,dx=\int \sqrt{c+dx^2}\coth^{-1}(ax)\,dx$$

Mathematica [A] time = 24.1269, size = 0, normalized size = 0.

$$\int \sqrt{c+dx^2}\coth^{-1}(ax)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c + d*x^2]*ArcCoth[a*x],x]
```

```
[Out] Integrate[Sqrt[c + d*x^2]*ArcCoth[a*x], x]
```

Maple [A] time = 0.713, size = 0, normalized size = 0.

$$\int \sqrt{dx^2+c}\,\text{arccoth}\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*arccoth(a*x),x)
```

```
[Out] int((d*x^2+c)^(1/2)*arccoth(a*x),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*arccoth(a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{dx^2+c}\,\text{arcoth}\,(ax)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*arccoth(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^2 + c)*arccoth(a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{c+dx^2}\,\text{acoth}\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*acoth(a*x),x)
```

```
[Out] Integral(sqrt(c + d*x**2)*acoth(a*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{dx^2+c}\,\text{arcoth}\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*arccoth(a*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)*arccoth(a*x), x)
```

3.43 $\int \frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}},x\right)$$

```
[Out] Unintegrable[ArcCoth[a*x]/Sqrt[c + d*x^2], x]
```

Rubi [A] time = 0.0234482, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCoth[a*x]/Sqrt[c + d*x^2],x]
```

```
[Out] Defer[Int][ArcCoth[a*x]/Sqrt[c + d*x^2], x]
```

Rubi steps

$$\int \frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}}\,dx=\int \frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}}\,dx$$

Mathematica [A] time = 4.35927, size = 0, normalized size = 0.

$$\int \frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCoth[a*x]/Sqrt[c + d*x^2],x]
```

```
[Out] Integrate[ArcCoth[a*x]/Sqrt[c + d*x^2], x]
```

Maple [A] time = 0.724, size = 0, normalized size = 0.

$$\int \text{arccoth}(ax)\frac{1}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)/(d*x^2+c)^(1/2),x)
```

```
[Out] int(arccoth(a*x)/(d*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arcoth}\,(ax)}{\sqrt{dx^2+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)/sqrt(d*x^2 + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{acoth}\,(ax)}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(acoth(a*x)/sqrt(c + d*x**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arcoth}\,(ax)}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)/sqrt(d*x^2 + c), x)
```

3.44 $\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=62

$$\frac{x\coth^{-1}(ax)}{c\sqrt{c+dx^2}}-\frac{\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{c\sqrt{a^2c+d}}$$

```
[Out] (x*ArcCoth[a*x])/(c*Sqrt[c + d*x^2]) - ArcTanh[(a*Sqrt[c + d*x^2])/Sqrt[a^2
*c + d]]/(c*Sqrt[a^2*c + d])
```

Rubi [A] time = 0.112951, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {191, 5977, 12, 444, 63, 208}

$$\frac{x\coth^{-1}(ax)}{c\sqrt{c+dx^2}}-\frac{\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{c\sqrt{a^2c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]/(c + d*x^2)^(3/2),x]
```

```
[Out] (x*ArcCoth[a*x])/(c*Sqrt[c + d*x^2]) - ArcTanh[(a*Sqrt[c + d*x^2])/Sqrt[a^2
*c + d]]/(c*Sqrt[a^2*c + d])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5977

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(ax)}{\left(c+dx^2\right)^{3/2}}\,dx &= \frac{x\coth^{-1}(ax)}{c\sqrt{c+dx^2}} - a\int \frac{x}{c\left(1-a^2x^2\right)\sqrt{c+dx^2}}\,dx\\ &= \frac{x\coth^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{a\int \frac{x}{\left(1-a^2x^2\right)\sqrt{c+dx^2}}\,dx}{c}\\ &= \frac{x\coth^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{\left(1-a^2x\right)\sqrt{c+dx}}\,dx, x, x^2\right)}{2c}\\ &= \frac{x\coth^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{a^2c}{d}-\frac{a^2x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{cd}\\ &= \frac{x\coth^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{c\sqrt{a^2c+d}}\end{aligned}$$

Mathematica [A] time = 0.117746, size = 119, normalized size = 1.92

$$\frac{\frac{-\log\left(\sqrt{a^2c+d}\sqrt{c+dx^2}+ac-dx\right)-\log\left(\sqrt{a^2c+d}\sqrt{c+dx^2}+ac+dx\right)+\log(1-ax)+\log(ax+1)}{\sqrt{a^2c+d}}+\frac{2x\coth^{-1}(ax)}{\sqrt{c+dx^2}}}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]/(c + d*x^2)^(3/2),x]
```

```
[Out] ((2*x*ArcCoth[a*x])/Sqrt[c + d*x^2] + (Log[1 - a*x] + Log[1 + a*x] - Log[a*
c - d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]] - Log[a*c + d*x + Sqrt[a^2*c + d
]*Sqrt[c + d*x^2]])/Sqrt[a^2*c + d])/(2*c)
```

Maple [F] time = 0.457, size = 0, normalized size = 0.

$$\int \mathrm{arccoth}\,(ax)\left(dx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)/(d*x^2+c)^(3/2),x)
```

```
[Out] int(arccoth(a*x)/(d*x^2+c)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.74724, size = 760, normalized size = 12.26

$$\left[\frac{2\left(a^2c+d\right)\sqrt{dx^2+c}\,x\log\left(\frac{ax+1}{ax-1}\right)+\sqrt{a^2c+d}\left(dx^2+c\right)\log\left(\frac{a^4d^2x^4+8\,a^4c^2+8\,a^2cd+2\left(4\,a^4cd+3\,a^2d^2\right)x^2-4\left(a^3dx^2+2\,a^3c+ad\right)\sqrt{a^2c+d}\sqrt{dx}}{a^4x^4-2\,a^2x^2+1}\right.}{4\left(a^2c^3+c^2d+\left(a^2c^2d+cd^2\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2*c + d)*sqrt(d*x^2 + c)*x*log((a*x + 1)/(a*x - 1)) + sqrt(a^2*c
 + d)*(d*x^2 + c)*log((a^4*d^2*x^4 + 8*a^4*c^2 + 8*a^2*c*d + 2*(4*a^4*c*d +
 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2 + 2*a^3*c + a*d)*sqrt(a^2*c + d)*sqrt(d*x^2
+ c) + d^2)/(a^4*x^4 - 2*a^2*x^2 + 1)))/(a^2*c^3 + c^2*d + (a^2*c^2*d + c*d
^2)*x^2), 1/2*((a^2*c + d)*sqrt(d*x^2 + c)*x*log((a*x + 1)/(a*x - 1)) + sqr
t(-a^2*c - d)*(d*x^2 + c)*arctan(1/2*(a^2*d*x^2 + 2*a^2*c + d)*sqrt(-a^2*c
- d)*sqrt(d*x^2 + c)/(a^3*c^2 + a*c*d + (a^3*c*d + a*d^2)*x^2)))/(a^2*c^3 +
 c^2*d + (a^2*c^2*d + c*d^2)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}(ax)}{\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(acoth(a*x)/(c + d*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(ax)}{\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)/(d*x^2 + c)^(3/2), x)
```

3.45 $\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=128

$$-\frac{(3a^2c+2d)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{3c^2(a^2c+d)^{3/2}}+\frac{a}{3c(a^2c+d)\sqrt{c+dx^2}}+\frac{2x\coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\coth^{-1}(ax)}{3c(c+dx^2)^{3/2}}$$

```
[Out] a/(3*c*(a^2*c + d)*Sqrt[c + d*x^2]) + (x*ArcCoth[a*x])/(3*c*(c + d*x^2)^(3/
2)) + (2*x*ArcCoth[a*x])/(3*c^2*Sqrt[c + d*x^2]) - ((3*a^2*c + 2*d)*ArcTanh
[(a*Sqrt[c + d*x^2])/Sqrt[a^2*c + d]])/(3*c^2*(a^2*c + d)^(3/2))
```

Rubi [A] time = 0.339828, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {192, 191, 5977, 6688, 12, 571, 78, 63, 208}

$$-\frac{(3a^2c+2d)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{3c^2(a^2c+d)^{3/2}}+\frac{a}{3c(a^2c+d)\sqrt{c+dx^2}}+\frac{2x\coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\coth^{-1}(ax)}{3c(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]/(c + d*x^2)^(5/2),x]
```

```
[Out] a/(3*c*(a^2*c + d)*Sqrt[c + d*x^2]) + (x*ArcCoth[a*x])/(3*c*(c + d*x^2)^(3/
2)) + (2*x*ArcCoth[a*x])/(3*c^2*Sqrt[c + d*x^2]) - ((3*a^2*c + 2*d)*ArcTanh
[(a*Sqrt[c + d*x^2])/Sqrt[a^2*c + d]])/(3*c^2*(a^2*c + d)^(3/2))
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5977

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\coth^{-1}(ax)}{\left(c+dx^2\right)^{5/2}}\,dx = \frac{x\coth^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - a\int \frac{\frac{x}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x}{3c^2\sqrt{c+dx^2}}}{1-a^2x^2}\,dx$$

$$= \frac{x\coth^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - a\int \frac{x\left(3c+2dx^2\right)}{3c^2\left(1-a^2x^2\right)\left(c+dx^2\right)^{3/2}}\,dx$$

$$= \frac{x\coth^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{a\int \frac{x\left(3c+2dx^2\right)}{\left(1-a^2x^2\right)\left(c+dx^2\right)^{3/2}}\,dx}{3c^2}$$

$$= \frac{x\coth^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{a\,\mathrm{Subst}\left(\int \frac{3c+2dx}{\left(1-a^2x\right)(c+dx)^{3/2}}\,dx, x, x^2\right)}{6c^2}$$

$$= \frac{a}{3c\left(a^2c+d\right)\sqrt{c+dx^2}} + \frac{x\coth^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{\left(a\left(3a^2c+2d\right)\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-a^2x\right)\sqrt{c+dx}}\,d\right.}{6c^2\left(a^2c+d\right)}$$

$$= \frac{a}{3c\left(a^2c+d\right)\sqrt{c+dx^2}} + \frac{x\coth^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{\left(a\left(3a^2c+2d\right)\right)\mathrm{Subst}\left(\int \frac{1}{1+\frac{a^2c}{d}-\frac{a^2x^2}{d}}\,dx,\right.}{3c^2d\left(a^2c+d\right)}$$

$$= \frac{a}{3c\left(a^2c+d\right)\sqrt{c+dx^2}} + \frac{x\coth^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{\left(3a^2c+2d\right)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{3c^2\left(a^2c+d\right)^{3/2}}$$

Mathematica [A] time = 0.321201, size = 226, normalized size = 1.77

$$\frac{\frac{2ac}{\left(a^2c+d\right)\sqrt{c+dx^2}} - \frac{\left(3a^2c+2d\right)\log\left(\sqrt{a^2c+d}\sqrt{c+dx^2}+ac-dx\right)}{\left(a^2c+d\right)^{3/2}} - \frac{\left(3a^2c+2d\right)\log\left(\sqrt{a^2c+d}\sqrt{c+dx^2}+ac+dx\right)}{\left(a^2c+d\right)^{3/2}} + \frac{\left(3a^2c+2d\right)\log(1-ax)}{\left(a^2c+d\right)^{3/2}} + \frac{\left(3a^2c+2d\right)\log(ax+1)}{\left(a^2c+d\right)^{3/2}} + \ldots}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]/(c + d*x^2)^(5/2),x]
```

```
[Out] ((2*a*c)/((a^2*c + d)*Sqrt[c + d*x^2]) + (2*x*(3*c + 2*d*x^2)*ArcCoth[a*x])
/(c + d*x^2)^(3/2) + ((3*a^2*c + 2*d)*Log[1 - a*x])/(a^2*c + d)^(3/2) + ((3
*a^2*c + 2*d)*Log[1 + a*x])/(a^2*c + d)^(3/2) - ((3*a^2*c + 2*d)*Log[a*c -
d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]])/(a^2*c + d)^(3/2) - ((3*a^2*c + 2*d
)*Log[a*c + d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]])/(a^2*c + d)^(3/2))/(6*c
^2)
```

Maple [F] time = 0.456, size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(ax)\left(dx^2+c\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)/(d*x^2+c)^(5/2),x)
```

```
[Out] int(arccoth(a*x)/(d*x^2+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.98491, size = 1496, normalized size = 11.69

$$\left[\frac{\left(3\,a^2c^3+\left(3\,a^2cd^2+2\,d^3\right)x^4+2\,c^2d+2\left(3\,a^2c^2d+2\,cd^2\right)x^2\right)\sqrt{a^2c+d}\log\left(\frac{a^4d^2x^4+8\,a^4c^2+8\,a^2cd+2\left(4\,a^4cd+3\,a^2d^2\right)x^2-4\left(a^3dx^2+2\right.}{a^4x^4-2\,a^2x^2+1}\right.}{12\left(a^4c^6+2\,a^2c^5d+c^4d^2+\left(a^4\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*((3*a^2*c^3 + (3*a^2*c*d^2 + 2*d^3)*x^4 + 2*c^2*d + 2*(3*a^2*c^2*d +
2*c*d^2)*x^2)*sqrt(a^2*c + d)*log((a^4*d^2*x^4 + 8*a^4*c^2 + 8*a^2*c*d + 2*
(4*a^4*c*d + 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2 + 2*a^3*c + a*d)*sqrt(a^2*c + d)
*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 - 2*a^2*x^2 + 1)) + 2*(2*a^3*c^3 + 2*a*c^2
*d + 2*(a^3*c^2*d + a*c*d^2)*x^2 + (2*(a^4*c^2*d + 2*a^2*c*d^2 + d^3)*x^3 +
 3*(a^4*c^3 + 2*a^2*c^2*d + c*d^2)*x)*log((a*x + 1)/(a*x - 1)))*sqrt(d*x^2
+ c))/(a^4*c^6 + 2*a^2*c^5*d + c^4*d^2 + (a^4*c^4*d^2 + 2*a^2*c^3*d^3 + c^2
*d^4)*x^4 + 2*(a^4*c^5*d + 2*a^2*c^4*d^2 + c^3*d^3)*x^2), 1/6*((3*a^2*c^3 +
 (3*a^2*c*d^2 + 2*d^3)*x^4 + 2*c^2*d + 2*(3*a^2*c^2*d + 2*c*d^2)*x^2)*sqrt(
-a^2*c - d)*arctan(1/2*(a^2*d*x^2 + 2*a^2*c + d)*sqrt(-a^2*c - d)*sqrt(d*x^
2 + c)/(a^3*c^2 + a*c*d + (a^3*c*d + a*d^2)*x^2)) + (2*a^3*c^3 + 2*a*c^2*d
+ 2*(a^3*c^2*d + a*c*d^2)*x^2 + (2*(a^4*c^2*d + 2*a^2*c*d^2 + d^3)*x^3 + 3*
(a^4*c^3 + 2*a^2*c^2*d + c*d^2)*x)*log((a*x + 1)/(a*x - 1)))*sqrt(d*x^2 + c
))/(a^4*c^6 + 2*a^2*c^5*d + c^4*d^2 + (a^4*c^4*d^2 + 2*a^2*c^3*d^3 + c^2*d^
4)*x^4 + 2*(a^4*c^5*d + 2*a^2*c^4*d^2 + c^3*d^3)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}{\left(ax \right)}}{\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(acoth(a*x)/(c + d*x**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(ax)}{\left(dx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)/(d*x^2 + c)^(5/2), x)
```

3.46 $\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx$

Optimal. Leaf size=200

$$-\frac{\left(15a^4c^2+20a^2cd+8d^2\right)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{15c^3\left(a^2c+d\right)^{5/2}}+\frac{a\left(7a^2c+4d\right)}{15c^2\left(a^2c+d\right)^2\sqrt{c+dx^2}}+\frac{a}{15c\left(a^2c+d\right)\left(c+dx^2\right)^{3/2}}+\frac{8x\coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}}$$

```
[Out] a/(15*c*(a^2*c + d)*(c + d*x^2)^(3/2)) + (a*(7*a^2*c + 4*d))/(15*c^2*(a^2*c
 + d)^2*Sqrt[c + d*x^2]) + (x*ArcCoth[a*x])/(5*c*(c + d*x^2)^(5/2)) + (4*x*
ArcCoth[a*x])/(15*c^2*(c + d*x^2)^(3/2)) + (8*x*ArcCoth[a*x])/(15*c^3*Sqrt[
c + d*x^2]) - ((15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*ArcTanh[(a*Sqrt[c + d*x^2]
)/Sqrt[a^2*c + d]])/(15*c^3*(a^2*c + d)^(5/2))
```

Rubi [A] time = 1.04748, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {192, 191, 5977, 6688, 12, 6715, 897, 1261, 208}

$$-\frac{\left(15a^4c^2+20a^2cd+8d^2\right)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{15c^3\left(a^2c+d\right)^{5/2}}+\frac{a\left(7a^2c+4d\right)}{15c^2\left(a^2c+d\right)^2\sqrt{c+dx^2}}+\frac{a}{15c\left(a^2c+d\right)\left(c+dx^2\right)^{3/2}}+\frac{8x\coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]/(c + d*x^2)^(7/2),x]
```


```
[Out] a/(15*c*(a^2*c + d)*(c + d*x^2)^(3/2)) + (a*(7*a^2*c + 4*d))/(15*c^2*(a^2*c
 + d)^2*Sqrt[c + d*x^2]) + (x*ArcCoth[a*x])/(5*c*(c + d*x^2)^(5/2)) + (4*x*
ArcCoth[a*x])/(15*c^2*(c + d*x^2)^(3/2)) + (8*x*ArcCoth[a*x])/(15*c^3*Sqrt[
c + d*x^2]) - ((15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*ArcTanh[(a*Sqrt[c + d*x^2]
)/Sqrt[a^2*c + d]])/(15*c^3*(a^2*c + d)^(5/2))
```


Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5977

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\coth^{-1}(ax)}{\left(c+dx^2\right)^{7/2}}\,dx = \frac{x\coth^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}} + \frac{4x\coth^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}} + \frac{8x\coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - a\int \frac{\frac{x}{5c\left(c+dx^2\right)^{5/2}} + \frac{4x}{15c^2\left(c+dx^2\right)^{3/2}} + \frac{8x}{15c^3\sqrt{c+dx^2}}}{1-a^2x^2}$$

$$= \frac{x\coth^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}} + \frac{4x\coth^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}} + \frac{8x\coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - a\int \frac{x\left(15c^2+20cdx^2+8d^2x^4\right)}{15c^3\left(1-a^2x^2\right)\left(c+dx^2\right)^{5/2}}\,dx$$

$$= \frac{x\coth^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}} + \frac{4x\coth^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}} + \frac{8x\coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a\int \frac{x\left(15c^2+20cdx^2+8d^2x^4\right)}{\left(1-a^2x^2\right)\left(c+dx^2\right)^{5/2}}\,dx}{15c^3}$$

$$= \frac{x\coth^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}} + \frac{4x\coth^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}} + \frac{8x\coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a\,\mathrm{Subst}\left(\int \frac{15c^2+20cdx+8d^2x^2}{\left(1-a^2x\right)(c+dx)^{5/2}}\,dx, x, x^2\right)}{30c^3}$$

$$= \frac{x\coth^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}} + \frac{4x\coth^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}} + \frac{8x\coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a\,\mathrm{Subst}\left(\int \frac{3c^2+4cx^2+8x^4}{x^4\left(\frac{a^2c+d}{d}-\frac{a^2x^2}{d}\right)}\,dx, x, \sqrt{c+dx^2}\right)}{15c^3d}$$

$$= \frac{x\coth^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}} + \frac{4x\coth^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}} + \frac{8x\coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a\,\mathrm{Subst}\left(\int\left(\frac{3c^2d}{\left(a^2c+d\right)x^4} + \frac{cd\left(7a^2c+4d\right)}{\left(a^2c+d\right)^2x^2} + \frac{d(15a^4}{(a^2c+}\right.\right.}{15c^3d}$$

$$= \frac{a}{15c\left(a^2c+d\right)\left(c+dx^2\right)^{3/2}} + \frac{a\left(7a^2c+4d\right)}{15c^2\left(a^2c+d\right)^2\sqrt{c+dx^2}} + \frac{x\coth^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}} + \frac{4x\coth^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}} + \frac{8x}{15}$$

$$= \frac{a}{15c\left(a^2c+d\right)\left(c+dx^2\right)^{3/2}} + \frac{a\left(7a^2c+4d\right)}{15c^2\left(a^2c+d\right)^2\sqrt{c+dx^2}} + \frac{x\coth^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}} + \frac{4x\coth^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}} + \frac{8x}{15}$$

Mathematica [A] time = 0.636018, size = 329, normalized size = 1.64

$$\left(15a^4c^2+20a^2cd+8d^2\right)\log(1-ax)\left(c+dx^2\right)^{5/2} + \left(15a^4c^2+20a^2cd+8d^2\right)\log(ax+1)\left(c+dx^2\right)^{5/2} - \left(15a^4c^2+20a^2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]/(c + d*x^2)^(7/2),x]
```

```
[Out] (2*a*c*Sqrt[a^2*c + d]*(c + d*x^2)*(d*(5*c + 4*d*x^2) + a^2*c*(8*c + 7*d*x^
2)) + 2*(a^2*c + d)^(5/2)*x*(15*c^2 + 20*c*d*x^2 + 8*d^2*x^4)*ArcCoth[a*x]
+ (15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*(c + d*x^2)^(5/2)*Log[1 - a*x] + (15*a^
4*c^2 + 20*a^2*c*d + 8*d^2)*(c + d*x^2)^(5/2)*Log[1 + a*x] - (15*a^4*c^2 +
20*a^2*c*d + 8*d^2)*(c + d*x^2)^(5/2)*Log[a*c - d*x + Sqrt[a^2*c + d]*Sqrt[
c + d*x^2]] - (15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*(c + d*x^2)^(5/2)*Log[a*c +
 d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]])/(30*c^3*(a^2*c + d)^(5/2)*(c + d*x
^2)^(5/2))
```

Maple [F] time = 0.455, size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(ax)\left(dx^2+c\right)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)/(d*x^2+c)^(7/2),x)
```

```
[Out] int(arccoth(a*x)/(d*x^2+c)^(7/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.24513, size = 2610, normalized size = 13.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/60*((15*a^4*c^5 + 20*a^2*c^4*d + (15*a^4*c^2*d^3 + 20*a^2*c*d^4 + 8*d^5)
*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 + 20*a^2*c^2*d^3 + 8*c*d^4)*x^4 + 3*(1
5*a^4*c^4*d + 20*a^2*c^3*d^2 + 8*c^2*d^3)*x^2)*sqrt(a^2*c + d)*log((a^4*d^2
*x^4 + 8*a^4*c^2 + 8*a^2*c*d + 2*(4*a^4*c*d + 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2
 + 2*a^3*c + a*d)*sqrt(a^2*c + d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 - 2*a^2*x
^2 + 1)) + 2*(16*a^5*c^5 + 26*a^3*c^4*d + 10*a*c^3*d^2 + 2*(7*a^5*c^3*d^2 +
 11*a^3*c^2*d^3 + 4*a*c*d^4)*x^4 + 6*(5*a^5*c^4*d + 8*a^3*c^3*d^2 + 3*a*c^2
*d^3)*x^2 + (8*(a^6*c^3*d^2 + 3*a^4*c^2*d^3 + 3*a^2*c*d^4 + d^5)*x^5 + 20*(
a^6*c^4*d + 3*a^4*c^3*d^2 + 3*a^2*c^2*d^3 + c*d^4)*x^3 + 15*(a^6*c^5 + 3*a^
4*c^4*d + 3*a^2*c^3*d^2 + c^2*d^3)*x)*log((a*x + 1)/(a*x - 1)))*sqrt(d*x^2
+ c))/(a^6*c^9 + 3*a^4*c^8*d + 3*a^2*c^7*d^2 + c^6*d^3 + (a^6*c^6*d^3 + 3*a
^4*c^5*d^4 + 3*a^2*c^4*d^5 + c^3*d^6)*x^6 + 3*(a^6*c^7*d^2 + 3*a^4*c^6*d^3
+ 3*a^2*c^5*d^4 + c^4*d^5)*x^4 + 3*(a^6*c^8*d + 3*a^4*c^7*d^2 + 3*a^2*c^6*d
^3 + c^5*d^4)*x^2), 1/30*((15*a^4*c^5 + 20*a^2*c^4*d + (15*a^4*c^2*d^3 + 20
*a^2*c*d^4 + 8*d^5)*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 + 20*a^2*c^2*d^3 +
8*c*d^4)*x^4 + 3*(15*a^4*c^4*d + 20*a^2*c^3*d^2 + 8*c^2*d^3)*x^2)*sqrt(-a^2
*c - d)*arctan(1/2*(a^2*d*x^2 + 2*a^2*c + d)*sqrt(-a^2*c - d)*sqrt(d*x^2 +
c)/(a^3*c^2 + a*c*d + (a^3*c*d + a*d^2)*x^2)) + (16*a^5*c^5 + 26*a^3*c^4*d
+ 10*a*c^3*d^2 + 2*(7*a^5*c^3*d^2 + 11*a^3*c^2*d^3 + 4*a*c*d^4)*x^4 + 6*(5*
a^5*c^4*d + 8*a^3*c^3*d^2 + 3*a*c^2*d^3)*x^2 + (8*(a^6*c^3*d^2 + 3*a^4*c^2*
d^3 + 3*a^2*c*d^4 + d^5)*x^5 + 20*(a^6*c^4*d + 3*a^4*c^3*d^2 + 3*a^2*c^2*d^
3 + c*d^4)*x^3 + 15*(a^6*c^5 + 3*a^4*c^4*d + 3*a^2*c^3*d^2 + c^2*d^3)*x)*lo
g((a*x + 1)/(a*x - 1)))*sqrt(d*x^2 + c))/(a^6*c^9 + 3*a^4*c^8*d + 3*a^2*c^7
*d^2 + c^6*d^3 + (a^6*c^6*d^3 + 3*a^4*c^5*d^4 + 3*a^2*c^4*d^5 + c^3*d^6)*x^
6 + 3*(a^6*c^7*d^2 + 3*a^4*c^6*d^3 + 3*a^2*c^5*d^4 + c^4*d^5)*x^4 + 3*(a^6*
c^8*d + 3*a^4*c^7*d^2 + 3*a^2*c^6*d^3 + c^5*d^4)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)/(d*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(ax)}{(dx^2 + c)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)/(d*x^2 + c)^(7/2), x)
```

3.47 $\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{9/2}}\, dx$

Optimal. Leaf size=283

$$\frac{a\left(19a^4c^2+22a^2cd+8d^2\right)}{35c^3\left(a^2c+d\right)^3\sqrt{c+dx^2}}-\frac{\left(70a^4c^2d+35a^6c^3+56a^2cd^2+16d^3\right)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{35c^4\left(a^2c+d\right)^{7/2}}+\frac{a\left(11a^2c+6d\right)}{105c^2\left(a^2c+d\right)^2\left(c+dx^2\right)^{3/2}}+$$

```
[Out] a/(35*c*(a^2*c + d)*(c + d*x^2)^(5/2)) + (a*(11*a^2*c + 6*d))/(105*c^2*(a^2
*c + d)^2*(c + d*x^2)^(3/2)) + (a*(19*a^4*c^2 + 22*a^2*c*d + 8*d^2))/(35*c^
3*(a^2*c + d)^3*Sqrt[c + d*x^2]) + (x*ArcCoth[a*x])/(7*c*(c + d*x^2)^(7/2))
 + (6*x*ArcCoth[a*x])/(35*c^2*(c + d*x^2)^(5/2)) + (8*x*ArcCoth[a*x])/(35*c
^3*(c + d*x^2)^(3/2)) + (16*x*ArcCoth[a*x])/(35*c^4*Sqrt[c + d*x^2]) - ((35
*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2 + 16*d^3)*ArcTanh[(a*Sqrt[c + d*x^2]
)/Sqrt[a^2*c + d]])/(35*c^4*(a^2*c + d)^(7/2))
```

Rubi [A] time = 1.34323, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {192, 191, 5977, 6688, 12, 6715, 1619, 63, 208}

$$\frac{a\left(19a^4c^2+22a^2cd+8d^2\right)}{35c^3\left(a^2c+d\right)^3\sqrt{c+dx^2}}-\frac{\left(70a^4c^2d+35a^6c^3+56a^2cd^2+16d^3\right)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{35c^4\left(a^2c+d\right)^{7/2}}+\frac{a\left(11a^2c+6d\right)}{105c^2\left(a^2c+d\right)^2\left(c+dx^2\right)^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]/(c + d*x^2)^(9/2),x]
```



```
[Out] a/(35*c*(a^2*c + d)*(c + d*x^2)^(5/2)) + (a*(11*a^2*c + 6*d))/(105*c^2*(a^2
*c + d)^2*(c + d*x^2)^(3/2)) + (a*(19*a^4*c^2 + 22*a^2*c*d + 8*d^2))/(35*c^
3*(a^2*c + d)^3*Sqrt[c + d*x^2]) + (x*ArcCoth[a*x])/(7*c*(c + d*x^2)^(7/2))
 + (6*x*ArcCoth[a*x])/(35*c^2*(c + d*x^2)^(5/2)) + (8*x*ArcCoth[a*x])/(35*c
^3*(c + d*x^2)^(3/2)) + (16*x*ArcCoth[a*x])/(35*c^4*Sqrt[c + d*x^2]) - ((35
*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2 + 16*d^3)*ArcTanh[(a*Sqrt[c + d*x^2]
)/Sqrt[a^2*c + d]])/(35*c^4*(a^2*c + d)^(7/2))
```


Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5977

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
```

```
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 1619

```
Int[((Px_)*((c_.) + (d_.)*(x_))^(n_.))/((a_.) + (b_.)*(x_)), x_Symbol] :> I
nt[ExpandIntegrand[1/Sqrt[c + d*x], (Px*(c + d*x)^(n + 1/2))/(a + b*x), x],
 x] /; FreeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[n + 1/2, 0] && GtQ
[Expon[Px, x], 2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx = \frac{x\coth^{-1}(ax)}{7c(c+dx^2)^{7/2}} + \frac{6x\coth^{-1}(ax)}{35c^2(c+dx^2)^{5/2}} + \frac{8x\coth^{-1}(ax)}{35c^3(c+dx^2)^{3/2}} + \frac{16x\coth^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - a\int \frac{\frac{x}{7c(c+dx^2)^{7/2}} + \frac{6}{35c^2(c+}}{}$$

$$= \frac{x\coth^{-1}(ax)}{7c(c+dx^2)^{7/2}} + \frac{6x\coth^{-1}(ax)}{35c^2(c+dx^2)^{5/2}} + \frac{8x\coth^{-1}(ax)}{35c^3(c+dx^2)^{3/2}} + \frac{16x\coth^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - a\int \frac{x(35c^3+70c^2dx^2}{35c^4(1-a^2x}$$

$$= \frac{x\coth^{-1}(ax)}{7c(c+dx^2)^{7/2}} + \frac{6x\coth^{-1}(ax)}{35c^2(c+dx^2)^{5/2}} + \frac{8x\coth^{-1}(ax)}{35c^3(c+dx^2)^{3/2}} + \frac{16x\coth^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{a\int \frac{x(35c^3+70c^2dx^2+56cd^2x^4}{(1-a^2x^2)(c+dx^2)^7}}{35c^4}$$

$$= \frac{x\coth^{-1}(ax)}{7c(c+dx^2)^{7/2}} + \frac{6x\coth^{-1}(ax)}{35c^2(c+dx^2)^{5/2}} + \frac{8x\coth^{-1}(ax)}{35c^3(c+dx^2)^{3/2}} + \frac{16x\coth^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{a\,\mathrm{Subst}\left(\int \frac{35c^3+70c^2dx+5}{(1-a^2x)}\right.}{70c}$$

$$= \frac{x\coth^{-1}(ax)}{7c(c+dx^2)^{7/2}} + \frac{6x\coth^{-1}(ax)}{35c^2(c+dx^2)^{5/2}} + \frac{8x\coth^{-1}(ax)}{35c^3(c+dx^2)^{3/2}} + \frac{16x\coth^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{a\,\mathrm{Subst}\left(\int\left(\frac{5c^3d}{(a^2c+d)(c+dx}\right.\right.}{}$$

$$= \frac{a}{35c(a^2c+d)(c+dx^2)^{5/2}} + \frac{a(11a^2c+6d)}{105c^2(a^2c+d)^2(c+dx^2)^{3/2}} + \frac{a(19a^4c^2+22a^2cd+8d^2)}{35c^3(a^2c+d)^3\sqrt{c+dx^2}} + \frac{x\coth^{-1}}{7c(c+dx}$$

$$= \frac{a}{35c(a^2c+d)(c+dx^2)^{5/2}} + \frac{a(11a^2c+6d)}{105c^2(a^2c+d)^2(c+dx^2)^{3/2}} + \frac{a(19a^4c^2+22a^2cd+8d^2)}{35c^3(a^2c+d)^3\sqrt{c+dx^2}} + \frac{x\coth^{-1}}{7c(c+dx}$$

$$= \frac{a}{35c(a^2c+d)(c+dx^2)^{5/2}} + \frac{a(11a^2c+6d)}{105c^2(a^2c+d)^2(c+dx^2)^{3/2}} + \frac{a(19a^4c^2+22a^2cd+8d^2)}{35c^3(a^2c+d)^3\sqrt{c+dx^2}} + \frac{x\coth^{-1}}{7c(c+dx}$$

Mathematica [A] time = 0.961486, size = 431, normalized size = 1.52

$$\frac{2ac\sqrt{a^2c+d}(c+dx^2)\left(3(19a^4c^2+22a^2cd+8d^2)(c+dx^2)^2+3c^2(a^2c+d)^2+c(11a^2c+6d)(a^2c+d)(c+dx^2)\right)+3(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]/(c + d*x^2)^(9/2),x]
```

```
[Out] (2*a*c*Sqrt[a^2*c + d]*(c + d*x^2)*(3*c^2*(a^2*c + d)^2 + c*(a^2*c + d)*(11
*a^2*c + 6*d)*(c + d*x^2) + 3*(19*a^4*c^2 + 22*a^2*c*d + 8*d^2)*(c + d*x^2)
^2) + 6*(a^2*c + d)^(7/2)*x*(35*c^3 + 70*c^2*d*x^2 + 56*c*d^2*x^4 + 16*d^3*
x^6)*ArcCoth[a*x] + 3*(35*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2 + 16*d^3)*(
c + d*x^2)^(7/2)*Log[1 - a*x] + 3*(35*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2
 + 16*d^3)*(c + d*x^2)^(7/2)*Log[1 + a*x] - 3*(35*a^6*c^3 + 70*a^4*c^2*d +
56*a^2*c*d^2 + 16*d^3)*(c + d*x^2)^(7/2)*Log[a*c - d*x + Sqrt[a^2*c + d]*Sq
rt[c + d*x^2]] - 3*(35*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2 + 16*d^3)*(c +
 d*x^2)^(7/2)*Log[a*c + d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]])/(210*c^4*(a
^2*c + d)^(7/2)*(c + d*x^2)^(7/2))
```

Maple [F] time = 0.461, size = 0, normalized size = 0.

$$\int \mathrm{arccoth}(ax)(dx^2+c)^{-\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)/(d*x^2+c)^(9/2),x)
```

```
[Out] int(arccoth(a*x)/(d*x^2+c)^(9/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.19344, size = 4140, normalized size = 14.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/420*(3*(35*a^6*c^7 + 70*a^4*c^6*d + 56*a^2*c^5*d^2 + (35*a^6*c^3*d^4 + 7
0*a^4*c^2*d^5 + 56*a^2*c*d^6 + 16*d^7)*x^8 + 16*c^4*d^3 + 4*(35*a^6*c^4*d^3
 + 70*a^4*c^3*d^4 + 56*a^2*c^2*d^5 + 16*c*d^6)*x^6 + 6*(35*a^6*c^5*d^2 + 70
*a^4*c^4*d^3 + 56*a^2*c^3*d^4 + 16*c^2*d^5)*x^4 + 4*(35*a^6*c^6*d + 70*a^4*
c^5*d^2 + 56*a^2*c^4*d^3 + 16*c^3*d^4)*x^2)*sqrt(a^2*c + d)*log((a^4*d^2*x^
4 + 8*a^4*c^2 + 8*a^2*c*d + 2*(4*a^4*c*d + 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2 +
2*a^3*c + a*d)*sqrt(a^2*c + d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 - 2*a^2*x^2
+ 1)) + 2*(142*a^7*c^7 + 320*a^5*c^6*d + 244*a^3*c^5*d^2 + 66*a*c^4*d^3 + 6
*(19*a^7*c^4*d^3 + 41*a^5*c^3*d^4 + 30*a^3*c^2*d^5 + 8*a*c*d^6)*x^6 + 2*(18
2*a^7*c^5*d^2 + 397*a^5*c^4*d^3 + 293*a^3*c^3*d^4 + 78*a*c^2*d^5)*x^4 + 2*(
196*a^7*c^6*d + 434*a^5*c^5*d^2 + 325*a^3*c^4*d^3 + 87*a*c^3*d^4)*x^2 + 3*(
16*(a^8*c^4*d^3 + 4*a^6*c^3*d^4 + 6*a^4*c^2*d^5 + 4*a^2*c*d^6 + d^7)*x^7 +
56*(a^8*c^5*d^2 + 4*a^6*c^4*d^3 + 6*a^4*c^3*d^4 + 4*a^2*c^2*d^5 + c*d^6)*x^
5 + 70*(a^8*c^6*d + 4*a^6*c^5*d^2 + 6*a^4*c^4*d^3 + 4*a^2*c^3*d^4 + c^2*d^5
)*x^3 + 35*(a^8*c^7 + 4*a^6*c^6*d + 6*a^4*c^5*d^2 + 4*a^2*c^4*d^3 + c^3*d^4
)*x)*log((a*x + 1)/(a*x - 1)))*sqrt(d*x^2 + c))/(a^8*c^12 + 4*a^6*c^11*d +
6*a^4*c^10*d^2 + 4*a^2*c^9*d^3 + c^8*d^4 + (a^8*c^8*d^4 + 4*a^6*c^7*d^5 + 6
*a^4*c^6*d^6 + 4*a^2*c^5*d^7 + c^4*d^8)*x^8 + 4*(a^8*c^9*d^3 + 4*a^6*c^8*d^
4 + 6*a^4*c^7*d^5 + 4*a^2*c^6*d^6 + c^5*d^7)*x^6 + 6*(a^8*c^10*d^2 + 4*a^6*
c^9*d^3 + 6*a^4*c^8*d^4 + 4*a^2*c^7*d^5 + c^6*d^6)*x^4 + 4*(a^8*c^11*d + 4*
a^6*c^10*d^2 + 6*a^4*c^9*d^3 + 4*a^2*c^8*d^4 + c^7*d^5)*x^2), 1/210*(3*(35*
a^6*c^7 + 70*a^4*c^6*d + 56*a^2*c^5*d^2 + (35*a^6*c^3*d^4 + 70*a^4*c^2*d^5
+ 56*a^2*c*d^6 + 16*d^7)*x^8 + 16*c^4*d^3 + 4*(35*a^6*c^4*d^3 + 70*a^4*c^3*
d^4 + 56*a^2*c^2*d^5 + 16*c*d^6)*x^6 + 6*(35*a^6*c^5*d^2 + 70*a^4*c^4*d^3 +
 56*a^2*c^3*d^4 + 16*c^2*d^5)*x^4 + 4*(35*a^6*c^6*d + 70*a^4*c^5*d^2 + 56*a
^2*c^4*d^3 + 16*c^3*d^4)*x^2)*sqrt(-a^2*c - d)*arctan(1/2*(a^2*d*x^2 + 2*a^
2*c + d)*sqrt(-a^2*c - d)*sqrt(d*x^2 + c)/(a^3*c^2 + a*c*d + (a^3*c*d + a*d
^2)*x^2)) + (142*a^7*c^7 + 320*a^5*c^6*d + 244*a^3*c^5*d^2 + 66*a*c^4*d^3 +
 6*(19*a^7*c^4*d^3 + 41*a^5*c^3*d^4 + 30*a^3*c^2*d^5 + 8*a*c*d^6)*x^6 + 2*(
182*a^7*c^5*d^2 + 397*a^5*c^4*d^3 + 293*a^3*c^3*d^4 + 78*a*c^2*d^5)*x^4 + 2
```

```
*(196*a^7*c^6*d + 434*a^5*c^5*d^2 + 325*a^3*c^4*d^3 + 87*a*c^3*d^4)*x^2 + 3
*(16*(a^8*c^4*d^3 + 4*a^6*c^3*d^4 + 6*a^4*c^2*d^5 + 4*a^2*c*d^6 + d^7)*x^7
+ 56*(a^8*c^5*d^2 + 4*a^6*c^4*d^3 + 6*a^4*c^3*d^4 + 4*a^2*c^2*d^5 + c*d^6)*
x^5 + 70*(a^8*c^6*d + 4*a^6*c^5*d^2 + 6*a^4*c^4*d^3 + 4*a^2*c^3*d^4 + c^2*d
^5)*x^3 + 35*(a^8*c^7 + 4*a^6*c^6*d + 6*a^4*c^5*d^2 + 4*a^2*c^4*d^3 + c^3*d
^4)*x)*log((a*x + 1)/(a*x - 1)))*sqrt(d*x^2 + c))/(a^8*c^12 + 4*a^6*c^11*d
+ 6*a^4*c^10*d^2 + 4*a^2*c^9*d^3 + c^8*d^4 + (a^8*c^8*d^4 + 4*a^6*c^7*d^5 +
 6*a^4*c^6*d^6 + 4*a^2*c^5*d^7 + c^4*d^8)*x^8 + 4*(a^8*c^9*d^3 + 4*a^6*c^8*
d^4 + 6*a^4*c^7*d^5 + 4*a^2*c^6*d^6 + c^5*d^7)*x^6 + 6*(a^8*c^10*d^2 + 4*a^
6*c^9*d^3 + 6*a^4*c^8*d^4 + 4*a^2*c^7*d^5 + c^6*d^6)*x^4 + 4*(a^8*c^11*d +
4*a^6*c^10*d^2 + 6*a^4*c^9*d^3 + 4*a^2*c^8*d^4 + c^7*d^5)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)/(d*x**2+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(ax)}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)/(d*x^2 + c)^(9/2), x)
```

3.48 $\int \sqrt{a-ax^2}\coth^{-1}(x)\,dx$

Optimal. Leaf size=186

$$-\frac{ia\sqrt{1-x^2}\text{PolyLog}\left(2,-\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{2\sqrt{a-ax^2}}+\frac{ia\sqrt{1-x^2}\text{PolyLog}\left(2,\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{2\sqrt{a-ax^2}}+\frac{1}{2}\sqrt{a-ax^2}+\frac{1}{2}x\sqrt{a-ax^2}\coth^{-1}(x)-\frac{a\sqrt{1-x^2}\text{ t}}{}$$

```
[Out] Sqrt[a - a*x^2]/2 + (x*Sqrt[a - a*x^2]*ArcCoth[x])/2 - (a*Sqrt[1 - x^2]*Arc
Coth[x]*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/Sqrt[a - a*x^2] - ((I/2)*a*Sqrt[1
- x^2]*PolyLog[2, ((-I)*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a - a*x^2] + ((I/2)
*a*Sqrt[1 - x^2]*PolyLog[2, (I*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a - a*x^2]
```

Rubi [A] time = 0.0831836, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5943, 5955, 5951}

$$-\frac{ia\sqrt{1-x^2}\text{PolyLog}\left(2,-\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{2\sqrt{a-ax^2}}+\frac{ia\sqrt{1-x^2}\text{PolyLog}\left(2,\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{2\sqrt{a-ax^2}}+\frac{1}{2}\sqrt{a-ax^2}+\frac{1}{2}x\sqrt{a-ax^2}\coth^{-1}(x)-\frac{a\sqrt{1-x^2}\text{ t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*x^2]*ArcCoth[x],x]
```


```
[Out] Sqrt[a - a*x^2]/2 + (x*Sqrt[a - a*x^2]*ArcCoth[x])/2 - (a*Sqrt[1 - x^2]*Arc
Coth[x]*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/Sqrt[a - a*x^2] - ((I/2)*a*Sqrt[1
- x^2]*PolyLog[2, ((-I)*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a - a*x^2] + ((I/2)
*a*Sqrt[1 - x^2]*PolyLog[2, (I*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a - a*x^2]
```


Rule 5943

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcCoth[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcCoth[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 5955

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcCoth[c*x])
^p/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5951

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcCoth[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\int \sqrt{a-ax^2}\coth^{-1}(x)\,dx = \frac{1}{2}\sqrt{a-ax^2}+\frac{1}{2}x\sqrt{a-ax^2}\coth^{-1}(x)+\frac{1}{2}a\int\frac{\coth^{-1}(x)}{\sqrt{a-ax^2}}\,dx$$
$$= \frac{1}{2}\sqrt{a-ax^2}+\frac{1}{2}x\sqrt{a-ax^2}\coth^{-1}(x)+\frac{\left(a\sqrt{1-x^2}\right)\int\frac{\coth^{-1}(x)}{\sqrt{1-x^2}}\,dx}{2\sqrt{a-ax^2}}$$
$$= \frac{1}{2}\sqrt{a-ax^2}+\frac{1}{2}x\sqrt{a-ax^2}\coth^{-1}(x)-\frac{a\sqrt{1-x^2}\coth^{-1}(x)\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)}{\sqrt{a-ax^2}}-\frac{ia\sqrt{1-x^2}\mathrm{Li}_2\left(-\right.}{2\sqrt{a-ax^2}}$$

Mathematica [A] time = 1.03914, size = 125, normalized size = 0.67

$$-\frac{\sqrt{a-ax^2}\left(-4\mathrm{PolyLog}\left(2,-e^{-\coth^{-1}(x)}\right)+4\mathrm{PolyLog}\left(2,e^{-\coth^{-1}(x)}\right)-2\coth\left(\frac{1}{2}\coth^{-1}(x)\right)-4\coth^{-1}(x)\log\left(1-e^{-\coth}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a - a*x^2]*ArcCoth[x],x]
```

```
[Out] -(Sqrt[a - a*x^2]*(-2*Coth[ArcCoth[x]/2] - ArcCoth[x]*Csch[ArcCoth[x]/2]^2
- 4*ArcCoth[x]*Log[1 - E^(-ArcCoth[x])] + 4*ArcCoth[x]*Log[1 + E^(-ArcCoth[
x])] - 4*PolyLog[2, -E^(-ArcCoth[x])] + 4*PolyLog[2, E^(-ArcCoth[x])] - Arc
Coth[x]*Sech[ArcCoth[x]/2]^2 + 2*Tanh[ArcCoth[x]/2]))/(8*Sqrt[1 - x^(-2)]*x
)
```

Maple [A] time = 0.448, size = 199, normalized size = 1.1

$$\frac{\mathrm{arccoth}(x)\,x+1}{2}\sqrt{-(-1+x)(1+x)a}-\frac{\mathrm{arccoth}(x)}{-2+2x}\sqrt{-(-1+x)(1+x)a}\sqrt{\frac{-1+x}{1+x}}\ln\left(1+\frac{1}{\sqrt{\frac{-1+x}{1+x}}}\right)-\frac{1}{-2+2x}\sqrt{-(-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*x^2+a)^(1/2)*arccoth(x),x)
```

```
[Out] 1/2*(arccoth(x)*x+1)*(-(-1+x)*(1+x)*a)^(1/2)-1/2*(-(-1+x)*(1+x)*a)^(1/2)*((
-1+x)/(1+x))^(1/2)/(-1+x)*arccoth(x)*ln(1+1/((-1+x)/(1+x))^(1/2))-1/2*(-(-1
+x)*(1+x)*a)^(1/2)*((-1+x)/(1+x))^(1/2)/(-1+x)*polylog(2,-1/((-1+x)/(1+x))^
(1/2))+1/2*(-(-1+x)*(1+x)*a)^(1/2)*((-1+x)/(1+x))^(1/2)/(-1+x)*arccoth(x)*l
n(1-1/((-1+x)/(1+x))^(1/2))+1/2*(-(-1+x)*(1+x)*a)^(1/2)*((-1+x)/(1+x))^(1/2
)/(-1+x)*polylog(2,1/((-1+x)/(1+x))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^2+a)^(1/2)*arccoth(x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-ax^2 + a}\,\text{arcoth}\left(x\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^2+a)^(1/2)*arccoth(x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a*x^2 + a)*arccoth(x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a\left(x - 1\right)\left(x + 1\right)}\,\text{acoth}\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x**2+a)**(1/2)*acoth(x),x)
```

```
[Out] Integral(sqrt(-a*(x - 1)*(x + 1))*acoth(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-ax^2 + a}\,\text{arcoth}\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^2+a)^(1/2)*arccoth(x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*x^2 + a)*arccoth(x), x)
```

3.49 $\int \frac{\coth^{-1}(x)}{\sqrt{a-ax^2}}\,dx$

Optimal. Leaf size=144

$$-\frac{i\sqrt{1-x^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{\sqrt{a-ax^2}}+\frac{i\sqrt{1-x^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{\sqrt{a-ax^2}}-\frac{2\sqrt{1-x^2}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)\coth^{-1}(x)}{\sqrt{a-ax^2}}$$

```
[Out] (-2*Sqrt[1 - x^2]*ArcCoth[x]*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/Sqrt[a - a*x^
2] - (I*Sqrt[1 - x^2]*PolyLog[2, ((-I)*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a -
a*x^2] + (I*Sqrt[1 - x^2]*PolyLog[2, (I*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a -
 a*x^2]
```

Rubi [A] time = 0.0495992, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5955, 5951}

$$-\frac{i\sqrt{1-x^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{\sqrt{a-ax^2}}+\frac{i\sqrt{1-x^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{\sqrt{a-ax^2}}-\frac{2\sqrt{1-x^2}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)\coth^{-1}(x)}{\sqrt{a-ax^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[x]/Sqrt[a - a*x^2],x]
```

```
[Out] (-2*Sqrt[1 - x^2]*ArcCoth[x]*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/Sqrt[a - a*x^
2] - (I*Sqrt[1 - x^2]*PolyLog[2, ((-I)*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a -
a*x^2] + (I*Sqrt[1 - x^2]*PolyLog[2, (I*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a -
 a*x^2]
```

Rule 5955

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcCoth[c*x])
^p/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5951

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcCoth[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])]])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(x)}{\sqrt{a-ax^2}}\,dx &= \frac{\sqrt{1-x^2}\int \frac{\coth^{-1}(x)}{\sqrt{1-x^2}}\,dx}{\sqrt{a-ax^2}}\\ &= -\frac{2\sqrt{1-x^2}\coth^{-1}(x)\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)}{\sqrt{a-ax^2}}-\frac{i\sqrt{1-x^2}\,\mathrm{Li}_2\left(-\frac{i\sqrt{1-x}}{\sqrt{1+x}}\right)}{\sqrt{a-ax^2}}+\frac{i\sqrt{1-x^2}\,\mathrm{Li}_2\left(\frac{i\sqrt{1-x}}{\sqrt{1+x}}\right)}{\sqrt{a-ax^2}}\end{aligned}$$

Mathematica [A] time = 0.112466, size = 77, normalized size = 0.53

$$\frac{\sqrt{a-ax^2}\left(\text{PolyLog}\left(2,-e^{-\coth^{-1}(x)}\right)-\text{PolyLog}\left(2,e^{-\coth^{-1}(x)}\right)+\coth^{-1}(x)\left(\log\left(1-e^{-\coth^{-1}(x)}\right)-\log\left(e^{-\coth^{-1}(x)}+\right.\right.\right.}{a\sqrt{1-\frac{1}{x^2}}x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[x]/Sqrt[a - a*x^2],x]
```

```
[Out] (Sqrt[a - a*x^2]*(ArcCoth[x]*(Log[1 - E^(-ArcCoth[x])] - Log[1 + E^(-ArcCot
h[x])]) + PolyLog[2, -E^(-ArcCoth[x])] - PolyLog[2, E^(-ArcCoth[x])]))/(a*S
qrt[1 - x^(-2)]*x)
```

Maple [A] time = 0.368, size = 190, normalized size = 1.3

$$-\frac{\operatorname{arccoth}(x)}{(-1+x)\,a}\ln\left(1+\frac{1}{\sqrt{\frac{-1+x}{1+x}}}\right)\sqrt{\frac{-1+x}{1+x}}\sqrt{-(-1+x)(1+x)\,a}-\frac{1}{(-1+x)\,a}\operatorname{polylog}\left(2,-\frac{1}{\sqrt{\frac{-1+x}{1+x}}}\right)\sqrt{\frac{-1+x}{1+x}}\sqrt{-(-1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x)/(-a*x^2+a)^(1/2),x)
```

```
[Out] -ln(1+1/((-1+x)/(1+x))^(1/2))*arccoth(x)*((-1+x)/(1+x))^(1/2)*(-(-1+x)*(1+x
)*a)^(1/2)/(-1+x)/a-polylog(2,-1/((-1+x)/(1+x))^(1/2))*((-1+x)/(1+x))^(1/2)
*(-(-1+x)*(1+x)*a)^(1/2)/(-1+x)/a+ln(1-1/((-1+x)/(1+x))^(1/2))*arccoth(x)*(
(-1+x)/(1+x))^(1/2)*(-(-1+x)*(1+x)*a)^(1/2)/(-1+x)/a+polylog(2,1/((-1+x)/(1
+x))^(1/2))*((-1+x)/(1+x))^(1/2)*(-(-1+x)*(1+x)*a)^(1/2)/(-1+x)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-a*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-ax^2+a}\,\operatorname{arcoth}(x)}{ax^2-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-a*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a*x^2 + a)*arccoth(x)/(a*x^2 - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}(x)}{\sqrt{-a(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x)/(-a*x**2+a)**(1/2),x)
```

```
[Out] Integral(acoth(x)/sqrt(-a*(x - 1)*(x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(x)}{\sqrt{-ax^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-a*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccoth(x)/sqrt(-a*x^2 + a), x)
```

3.50 $\int \frac{\coth^{-1}(x)}{(a-ax^2)^{3/2}}\,dx$

Optimal. Leaf size=37

$$\frac{x\coth^{-1}(x)}{a\sqrt{a-ax^2}}-\frac{1}{a\sqrt{a-ax^2}}$$

```
[Out] -(1/(a*Sqrt[a - a*x^2])) + (x*ArcCoth[x])/(a*Sqrt[a - a*x^2])
```

Rubi [A] time = 0.0254153, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {5959}

$$\frac{x\coth^{-1}(x)}{a\sqrt{a-ax^2}}-\frac{1}{a\sqrt{a-ax^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[x]/(a - a*x^2)^(3/2),x]
```

```
[Out] -(1/(a*Sqrt[a - a*x^2])) + (x*ArcCoth[x])/(a*Sqrt[a - a*x^2])
```

Rule 5959

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcCoth[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{3/2}}\,dx=-\frac{1}{a\sqrt{a-ax^2}}+\frac{x\coth^{-1}(x)}{a\sqrt{a-ax^2}}$$

Mathematica [A] time = 0.0438355, size = 30, normalized size = 0.81

$$\frac{\sqrt{a-ax^2}\left(1-x\coth^{-1}(x)\right)}{a^2\left(x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[x]/(a - a*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a - a*x^2]*(1 - x*ArcCoth[x]))/(a^2*(-1 + x^2))
```

Maple [A] time = 0.226, size = 52, normalized size = 1.4

$$-\frac{\operatorname{arccoth}(x)-1}{(-2+2x)a^2}\sqrt{-(-1+x)(1+x)a}-\frac{\operatorname{arccoth}(x)+1}{(2+2x)a^2}\sqrt{-(-1+x)(1+x)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x)/(-a*x^2+a)^(3/2),x)
```

```
[Out] -1/2*(arccoth(x)-1)*(-(-1+x)*(1+x)*a)^(1/2)/(-1+x)/a^2-1/2*(arccoth(x)+1)*(
-(-1+x)*(1+x)*a)^(1/2)/(1+x)/a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-a*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5362, size = 92, normalized size = 2.49

$$-\frac{\sqrt{-ax^2+a}\left(x\log\left(\frac{x+1}{x-1}\right)-2\right)}{2\left(a^2x^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-a*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-a*x^2 + a)*(x*log((x + 1)/(x - 1)) - 2)/(a^2*x^2 - a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}(x)}{\left(-a(x-1)(x+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x)/(-a*x**2+a)**(3/2),x)
```

```
[Out] Integral(acoth(x)/(-a*(x - 1)*(x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(x)}{\left(-ax^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-a*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arccoth(x)/(-a*x^2 + a)^(3/2), x)
```

3.51 $\int \frac{\coth^{-1}(x)}{(a-ax^2)^{5/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{2}{3a^2\sqrt{a-ax^2}}+\frac{2x\coth^{-1}(x)}{3a^2\sqrt{a-ax^2}}-\frac{1}{9a\left(a-ax^2\right)^{3/2}}+\frac{x\coth^{-1}(x)}{3a\left(a-ax^2\right)^{3/2}}$$

```
[Out] -1/(9*a*(a - a*x^2)^(3/2)) - 2/(3*a^2*Sqrt[a - a*x^2]) + (x*ArcCoth[x])/(3*
a*(a - a*x^2)^(3/2)) + (2*x*ArcCoth[x])/(3*a^2*Sqrt[a - a*x^2])
```

Rubi [A] time = 0.0534298, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5961, 5959}

$$-\frac{2}{3a^2\sqrt{a-ax^2}}+\frac{2x\coth^{-1}(x)}{3a^2\sqrt{a-ax^2}}-\frac{1}{9a\left(a-ax^2\right)^{3/2}}+\frac{x\coth^{-1}(x)}{3a\left(a-ax^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[x]/(a - a*x^2)^(5/2),x]
```

```
[Out] -1/(9*a*(a - a*x^2)^(3/2)) - 2/(3*a^2*Sqrt[a - a*x^2]) + (x*ArcCoth[x])/(3*
a*(a - a*x^2)^(3/2)) + (2*x*ArcCoth[x])/(3*a^2*Sqrt[a - a*x^2])
```

Rule 5961

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcCoth[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcCoth[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5959

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcCoth[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(x)}{\left(a-ax^2\right)^{5/2}}\,dx &= -\frac{1}{9a\left(a-ax^2\right)^{3/2}}+\frac{x\coth^{-1}(x)}{3a\left(a-ax^2\right)^{3/2}}+\frac{2\int \frac{\coth^{-1}(x)}{(a-ax^2)^{3/2}}\,dx}{3a}\\ &= -\frac{1}{9a\left(a-ax^2\right)^{3/2}}-\frac{2}{3a^2\sqrt{a-ax^2}}+\frac{x\coth^{-1}(x)}{3a\left(a-ax^2\right)^{3/2}}+\frac{2x\coth^{-1}(x)}{3a^2\sqrt{a-ax^2}}\end{aligned}$$

Mathematica [A] time = 0.0499682, size = 45, normalized size = 0.54

$$-\frac{\sqrt{a-ax^2}\left(-6x^2+\left(6x^3-9x\right)\coth^{-1}(x)+7\right)}{9a^3\left(x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[x]/(a - a*x^2)^(5/2),x]
```

```
[Out] -(Sqrt[a - a*x^2]*(7 - 6*x^2 + (-9*x + 6*x^3)*ArcCoth[x]))/(9*a^3*(-1 + x^2
)^2)
```

Maple [A] time = 0.238, size = 112, normalized size = 1.4

$$\frac{(1+x)\,(-1+3\,\mathrm{arccoth}\,(x))}{72\,(-1+x)^{2}\,a^{3}}\sqrt{-(-1+x)\,(1+x)\,a}-\frac{3\,\mathrm{arccoth}\,(x)-3}{(-8+8\,x)\,a^{3}}\sqrt{-(-1+x)\,(1+x)\,a}-\frac{3\,\mathrm{arccoth}\,(x)+3}{(8+8\,x)\,a^{3}}\sqrt{-(-1+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x)/(-a*x^2+a)^(5/2),x)
```

```
[Out] 1/72*(1+x)*(-1+3*arccoth(x))*(-(-1+x)*(1+x)*a)^(1/2)/(-1+x)^2/a^3-3/8*(arcc
oth(x)-1)*(-(-1+x)*(1+x)*a)^(1/2)/(-1+x)/a^3-3/8*(arccoth(x)+1)*(-(-1+x)*(1
+x)*a)^(1/2)/(1+x)/a^3+1/72*(1+3*arccoth(x))*(-1+x)*(-(-1+x)*(1+x)*a)^(1/2)
/(1+x)^2/a^3
```

Maxima [A] time = 1.01709, size = 90, normalized size = 1.08

$$\frac{1}{3}\left(\frac{2\,x}{\sqrt{-ax^{2}+a}a^{2}}+\frac{x}{\left(-ax^{2}+a\right)^{\frac{3}{2}}a}\right)\mathrm{arcoth}\,(x)-\frac{2}{3\,\sqrt{-ax^{2}+a}a^{2}}-\frac{1}{9\left(-ax^{2}+a\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-a*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*x/(sqrt(-a*x^2 + a)*a^2) + x/((-a*x^2 + a)^(3/2)*a))*arccoth(x) - 2/
3/(sqrt(-a*x^2 + a)*a^2) - 1/9/((-a*x^2 + a)^(3/2)*a)
```

Fricas [A] time = 1.58402, size = 140, normalized size = 1.69

$$\frac{\sqrt{-ax^{2}+a}\left(12\,x^{2}-3\left(2\,x^{3}-3\,x\right)\log\left(\frac{x+1}{x-1}\right)-14\right)}{18\left(a^{3}x^{4}-2\,a^{3}x^{2}+a^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-a*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(-a*x^2 + a)*(12*x^2 - 3*(2*x^3 - 3*x)*log((x + 1)/(x - 1)) - 14)/
(a^3*x^4 - 2*a^3*x^2 + a^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}(x)}{(-a(x-1)(x+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x)/(-a*x**2+a)**(5/2),x)
```

```
[Out] Integral(acoth(x)/(-a*(x - 1)*(x + 1))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(x)}{\left(-ax^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-a*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(arccoth(x)/(-a*x^2 + a)^(5/2), x)
```

3.52 $\int \frac{\coth^{-1}(x)}{\left(a-ax^2\right)^{7/2}}\,dx$

Optimal. Leaf size=124

$$-\frac{8}{15a^3\sqrt{a-ax^2}}-\frac{4}{45a^2\left(a-ax^2\right)^{3/2}}+\frac{8x\coth^{-1}(x)}{15a^3\sqrt{a-ax^2}}+\frac{4x\coth^{-1}(x)}{15a^2\left(a-ax^2\right)^{3/2}}-\frac{1}{25a\left(a-ax^2\right)^{5/2}}+\frac{x\coth^{-1}(x)}{5a\left(a-ax^2\right)^{5/2}}$$

```
[Out] -1/(25*a*(a - a*x^2)^(5/2)) - 4/(45*a^2*(a - a*x^2)^(3/2)) - 8/(15*a^3*Sqrt
[a - a*x^2]) + (x*ArcCoth[x])/(5*a*(a - a*x^2)^(5/2)) + (4*x*ArcCoth[x])/(1
5*a^2*(a - a*x^2)^(3/2)) + (8*x*ArcCoth[x])/(15*a^3*Sqrt[a - a*x^2])
```

Rubi [A] time = 0.0820825, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5961, 5959}

$$-\frac{8}{15a^3\sqrt{a-ax^2}}-\frac{4}{45a^2\left(a-ax^2\right)^{3/2}}+\frac{8x\coth^{-1}(x)}{15a^3\sqrt{a-ax^2}}+\frac{4x\coth^{-1}(x)}{15a^2\left(a-ax^2\right)^{3/2}}-\frac{1}{25a\left(a-ax^2\right)^{5/2}}+\frac{x\coth^{-1}(x)}{5a\left(a-ax^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[x]/(a - a*x^2)^(7/2),x]
```

```
[Out] -1/(25*a*(a - a*x^2)^(5/2)) - 4/(45*a^2*(a - a*x^2)^(3/2)) - 8/(15*a^3*Sqrt
[a - a*x^2]) + (x*ArcCoth[x])/(5*a*(a - a*x^2)^(5/2)) + (4*x*ArcCoth[x])/(1
5*a^2*(a - a*x^2)^(3/2)) + (8*x*ArcCoth[x])/(15*a^3*Sqrt[a - a*x^2])
```

Rule 5961

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcCoth[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcCoth[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5959

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcCoth[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(x)}{\left(a-ax^2\right)^{7/2}}\,dx &= -\frac{1}{25a\left(a-ax^2\right)^{5/2}}+\frac{x\coth^{-1}(x)}{5a\left(a-ax^2\right)^{5/2}}+\frac{4\int \frac{\coth^{-1}(x)}{\left(a-ax^2\right)^{5/2}}\,dx}{5a}\\
&= -\frac{1}{25a\left(a-ax^2\right)^{5/2}}-\frac{4}{45a^2\left(a-ax^2\right)^{3/2}}+\frac{x\coth^{-1}(x)}{5a\left(a-ax^2\right)^{5/2}}+\frac{4x\coth^{-1}(x)}{15a^2\left(a-ax^2\right)^{3/2}}+\frac{8\int \frac{\coth^{-1}(x)}{\left(a-ax^2\right)^{3/2}}\,dx}{15a^2}\\
&= -\frac{1}{25a\left(a-ax^2\right)^{5/2}}-\frac{4}{45a^2\left(a-ax^2\right)^{3/2}}-\frac{8}{15a^3\sqrt{a-ax^2}}+\frac{x\coth^{-1}(x)}{5a\left(a-ax^2\right)^{5/2}}+\frac{4x\coth^{-1}(x)}{15a^2\left(a-ax^2\right)^{3/2}}+\frac{8x\,c}{15a^3}\end{aligned}$$

Mathematica [A] time = 0.0605613, size = 55, normalized size = 0.44

$$\frac{\sqrt{a-ax^2}\left(120x^4-260x^2-15\left(8x^4-20x^2+15\right)x\coth^{-1}(x)+149\right)}{225a^4\left(x^2-1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[x]/(a - a*x^2)^(7/2),x]
```

```
[Out] (Sqrt[a - a*x^2]*(149 - 260*x^2 + 120*x^4 - 15*x*(15 - 20*x^2 + 8*x^4)*ArcC
oth[x]))/(225*a^4*(-1 + x^2)^3)
```

Maple [A] time = 0.261, size = 176, normalized size = 1.4

$$-\frac{(1+x)^2(-1+5\,\mathrm{arccoth}\,(x))}{800\,(-1+x)^3a^4}\sqrt{-(-1+x)(1+x)a}+\frac{(5+5x)(-1+3\,\mathrm{arccoth}\,(x))}{288\,(-1+x)^2a^4}\sqrt{-(-1+x)(1+x)a}-\frac{5\,\mathrm{arccoth}}{(-16+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x)/(-a*x^2+a)^(7/2),x)
```

```
[Out] -1/800*(1+x)^2*(-1+5*arccoth(x))*(-(-1+x)*(1+x)*a)^(1/2)/(-1+x)^3/a^4+5/288
*(1+x)*(-1+3*arccoth(x))*(-(-1+x)*(1+x)*a)^(1/2)/(-1+x)^2/a^4-5/16*(arccoth
(x)-1)*(-(-1+x)*(1+x)*a)^(1/2)/(-1+x)/a^4-5/16*(arccoth(x)+1)*(-(-1+x)*(1+x
)*a)^(1/2)/(1+x)/a^4+5/288*(1+3*arccoth(x))*(-1+x)*(-(-1+x)*(1+x)*a)^(1/2)/
(1+x)^2/a^4-1/800*(1+5*arccoth(x))*(-1+x)^2*(-(-1+x)*(1+x)*a)^(1/2)/(1+x)^3
/a^4
```

Maxima [A] time = 1.0259, size = 134, normalized size = 1.08

$$\frac{1}{15}\left(\frac{8x}{\sqrt{-ax^2+a}a^3}+\frac{4x}{\left(-ax^2+a\right)^{\frac{3}{2}}a^2}+\frac{3x}{\left(-ax^2+a\right)^{\frac{5}{2}}a}\right)\mathrm{arcoth}\,(x)-\frac{8}{15\sqrt{-ax^2+a}a^3}-\frac{4}{45\left(-ax^2+a\right)^{\frac{3}{2}}a^2}-\frac{1}{25\left(-ax^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-a*x^2+a)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*(8*x/(sqrt(-a*x^2 + a)*a^3) + 4*x/((-a*x^2 + a)^(3/2)*a^2) + 3*x/((-a*
x^2 + a)^(5/2)*a))*arccoth(x) - 8/15/(sqrt(-a*x^2 + a)*a^3) - 4/45/((-a*x^2
 + a)^(3/2)*a^2) - 1/25/((-a*x^2 + a)^(5/2)*a)
```

Fricas [A] time = 1.67926, size = 189, normalized size = 1.52

$$\frac{\left(240\,x^4-520\,x^2-15\left(8\,x^5-20\,x^3+15\,x\right)\log\left(\frac{x+1}{x-1}\right)+298\right)\sqrt{-ax^2+a}}{450\left(a^4x^6-3\,a^4x^4+3\,a^4x^2-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-a*x^2+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/450*(240*x^4 - 520*x^2 - 15*(8*x^5 - 20*x^3 + 15*x)*log((x + 1)/(x - 1))
+ 298)*sqrt(-a*x^2 + a)/(a^4*x^6 - 3*a^4*x^4 + 3*a^4*x^2 - a^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x)/(-a*x**2+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(x)}{\left(-ax^2 + a\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-a*x^2+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(arccoth(x)/(-a*x^2 + a)^(7/2), x)
```

3.53 $\int \frac{1}{(1-x^2)\coth^{-1}(x)}\,dx$

Optimal. Leaf size=3

$$\log\left(\coth^{-1}(x)\right)$$

```
[Out] Log[ArcCoth[x]]
```

Rubi [A] time = 0.0233692, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5947}

$$\log\left(\coth^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^2)*ArcCoth[x]),x]
```

```
[Out] Log[ArcCoth[x]]
```

Rule 5947

```
Int[1/(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol
] :> Simp[Log[RemoveContent[a + b*ArcCoth[c*x], x]]/(b*c*d), x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\int \frac{1}{(1-x^2)\coth^{-1}(x)}\,dx = \log\left(\coth^{-1}(x)\right)$$

Mathematica [A] time = 0.0250802, size = 3, normalized size = 1.

$$\log\left(\coth^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x^2)*ArcCoth[x]),x]
```

```
[Out] Log[ArcCoth[x]]
```

Maple [A] time = 0.043, size = 4, normalized size = 1.3

$$\ln\left(\operatorname{arccoth}(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)/arccoth(x),x)
```

```
[Out] ln(arccoth(x))
```

Maxima [A] time = 0.975901, size = 4, normalized size = 1.33

$$\log(\operatorname{arcoth}(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)/arccoth(x),x, algorithm="maxima")
```

```
[Out] log(arccoth(x))
```

Fricas [B] time = 1.50668, size = 36, normalized size = 12.

$$\log\left(\log\left(\frac{x+1}{x-1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)/arccoth(x),x, algorithm="fricas")
```

```
[Out] log(log((x + 1)/(x - 1)))
```

Sympy [A] time = 0.411158, size = 3, normalized size = 1.

$$\log(\operatorname{acoth}(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)/acoth(x),x)
```

```
[Out] log(acoth(x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(x^2-1)\operatorname{arcoth}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)/arccoth(x),x, algorithm="giac")
```

```
[Out] integrate(-1/((x^2 - 1)*arccoth(x)), x)
```

3.54 $\int \frac{\coth^{-1}(x)^n}{1-x^2}\,dx$

Optimal. Leaf size=12

$$\frac{\coth^{-1}(x)^{n+1}}{n+1}$$

```
[Out] ArcCoth[x]^(1 + n)/(1 + n)
```

Rubi [A] time = 0.0263976, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5949}

$$\frac{\coth^{-1}(x)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[x]^n/(1 - x^2),x]

[Out] ArcCoth[x]^(1 + n)/(1 + n)
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\coth^{-1}(x)^n}{1-x^2}\,dx = \frac{\coth^{-1}(x)^{1+n}}{1+n}$$

Mathematica [A] time = 0.0089221, size = 12, normalized size = 1.

$$\frac{\coth^{-1}(x)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[x]^n/(1 - x^2),x]

[Out] ArcCoth[x]^(1 + n)/(1 + n)
```

Maple [A] time = 0.059, size = 13, normalized size = 1.1

$$\frac{(\operatorname{arccoth}(x))^{1+n}}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x)^n/(-x^2+1),x)
```

```
[Out] arccoth(x)^(1+n)/(1+n)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)^n/(-x^2+1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.61418, size = 182, normalized size = 15.17

$$\frac{\cosh\left(n\log\left(\frac{1}{2}\,\log\left(\frac{x+1}{x-1}\right)\right)\right)\log\left(\frac{x+1}{x-1}\right)+\log\left(\frac{x+1}{x-1}\right)\sinh\left(n\log\left(\frac{1}{2}\,\log\left(\frac{x+1}{x-1}\right)\right)\right)}{2\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)^n/(-x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(n*log(1/2*log((x + 1)/(x - 1))))*log((x + 1)/(x - 1)) + log((x +
1)/(x - 1))*sinh(n*log(1/2*log((x + 1)/(x - 1)))))/(n + 1)
```

Sympy [A] time = 4.82176, size = 15, normalized size = 1.25

$$\begin{cases}\frac{\operatorname{acoth}^{n+1}(x)}{n+1} & \text{for}\ n\neq-1\\ \log(\operatorname{acoth}(x)) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x)**n/(-x**2+1),x)
```

```
[Out] Piecewise((acoth(x)**(n + 1)/(n + 1), Ne(n, -1)), (log(acoth(x)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\operatorname{arcoth}(x)^{n}}{x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)^n/(-x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-arccoth(x)^n/(x^2 - 1), x)
```

3.55 $\int \frac{\coth^{-1}(x)^2}{\left(1-x^2\right)^2}\,dx$

Optimal. Leaf size=62

$$\frac{x}{4\left(1-x^2\right)}+\frac{x\coth^{-1}(x)^2}{2\left(1-x^2\right)}-\frac{\coth^{-1}(x)}{2\left(1-x^2\right)}+\frac{1}{4}\tanh^{-1}(x)+\frac{1}{6}\coth^{-1}(x)^3$$

```
[Out] x/(4*(1 - x^2)) - ArcCoth[x]/(2*(1 - x^2)) + (x*ArcCoth[x]^2)/(2*(1 - x^2))
 + ArcCoth[x]^3/6 + ArcTanh[x]/4
```

Rubi [A] time = 0.0506488, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5957, 5995, 199, 206}

$$\frac{x}{4\left(1-x^2\right)}+\frac{x\coth^{-1}(x)^2}{2\left(1-x^2\right)}-\frac{\coth^{-1}(x)}{2\left(1-x^2\right)}+\frac{1}{4}\tanh^{-1}(x)+\frac{1}{6}\coth^{-1}(x)^3$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[x]^2/(1 - x^2)^2,x]
```

```
[Out] x/(4*(1 - x^2)) - ArcCoth[x]/(2*(1 - x^2)) + (x*ArcCoth[x]^2)/(2*(1 - x^2))
 + ArcCoth[x]^3/6 + ArcTanh[x]/4
```

Rule 5957

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcCoth[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcCoth[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5995

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcCoth[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcCoth[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^{-1}(x)^2}{\left(1-x^2\right)^2}\,dx &= \frac{x\coth^{-1}(x)^2}{2\left(1-x^2\right)} + \frac{1}{6}\coth^{-1}(x)^3 - \int \frac{x\coth^{-1}(x)}{\left(1-x^2\right)^2}\,dx \\
&= -\frac{\coth^{-1}(x)}{2\left(1-x^2\right)} + \frac{x\coth^{-1}(x)^2}{2\left(1-x^2\right)} + \frac{1}{6}\coth^{-1}(x)^3 + \frac{1}{2}\int \frac{1}{\left(1-x^2\right)^2}\,dx \\
&= \frac{x}{4\left(1-x^2\right)} - \frac{\coth^{-1}(x)}{2\left(1-x^2\right)} + \frac{x\coth^{-1}(x)^2}{2\left(1-x^2\right)} + \frac{1}{6}\coth^{-1}(x)^3 + \frac{1}{4}\int \frac{1}{1-x^2}\,dx \\
&= \frac{x}{4\left(1-x^2\right)} - \frac{\coth^{-1}(x)}{2\left(1-x^2\right)} + \frac{x\coth^{-1}(x)^2}{2\left(1-x^2\right)} + \frac{1}{6}\coth^{-1}(x)^3 + \frac{1}{4}\tanh^{-1}(x)
\end{aligned}
$$

Mathematica [A] time = 0.0831476, size = 61, normalized size = 0.98

$$
\frac{-3\left(x^2-1\right)\log(1-x)+3\left(x^2-1\right)\log(x+1)+4\left(x^2-1\right)\coth^{-1}(x)^3-6x-12x\coth^{-1}(x)^2+12\coth^{-1}(x)}{24\left(x^2-1\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[x]^2/(1 - x^2)^2,x]
```

```
[Out] (-6*x + 12*ArcCoth[x] - 12*x*ArcCoth[x]^2 + 4*(-1 + x^2)*ArcCoth[x]^3 - 3*(
-1 + x^2)*Log[1 - x] + 3*(-1 + x^2)*Log[1 + x])/(24*(-1 + x^2))
```

Maple [C] time = 0.494, size = 707, normalized size = 11.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x)^2/(-x^2+1)^2,x)
```

```
[Out] -1/4*arccoth(x)^2/(-1+x)-1/4*arccoth(x)^2*ln(-1+x)-1/4*arccoth(x)^2/(1+x)+1
/4*arccoth(x)^2*ln(1+x)+1/4*arccoth(x)^2*ln((-1+x)/(1+x))+1/24*(6*I*csgn(I*
(1+x)/(-1+x))^2*csgn(I/((-1+x)/(1+x))^(1/2))*arccoth(x)^2*Pi-6*I*arccoth(x)
^2*Pi*csgn(I/((-1+x)/(1+x))^(1/2))*csgn(I*(1+x)/(-1+x))^2*x^2+3*I*arccoth(x
)^2*Pi*csgn(I*(1+x)/(-1+x))^3*x^2+3*I*csgn(I/((1+x)/(-1+x)-1))*csgn(I*(1+x)
/(-1+x)/((1+x)/(-1+x)-1))^2*arccoth(x)^2*Pi+3*I*arccoth(x)^2*Pi*csgn(I*(1+x
)/(-1+x)/((1+x)/(-1+x)-1))*csgn(I*(1+x)/(-1+x))*csgn(I/((1+x)/(-1+x)-1))*x^
2-3*I*arccoth(x)^2*Pi*csgn(I*(1+x)/(-1+x)/((1+x)/(-1+x)-1))^2*csgn(I*(1+x)/
(-1+x))*x^2+3*I*arccoth(x)^2*Pi*csgn(I/((-1+x)/(1+x))^(1/2))^2*csgn(I*(1+x)
/(-1+x))*x^2-3*I*csgn(I*(1+x)/(-1+x))^3*arccoth(x)^2*Pi-3*I*arccoth(x)^2*Pi
*csgn(I*(1+x)/(-1+x)/((1+x)/(-1+x)-1))^2*csgn(I/((1+x)/(-1+x)-1))*x^2-3*I*c
sgn(I*(1+x)/(-1+x)/((1+x)/(-1+x)-1))^3*arccoth(x)^2*Pi+3*I*csgn(I*(1+x)/(-1
+x))*csgn(I*(1+x)/(-1+x)/((1+x)/(-1+x)-1))^2*arccoth(x)^2*Pi+3*I*arccoth(x)
^2*Pi*csgn(I*(1+x)/(-1+x)/((1+x)/(-1+x)-1))^3*x^2-3*I*csgn(I*(1+x)/(-1+x))*
csgn(I/((-1+x)/(1+x))^(1/2))^2*arccoth(x)^2*Pi-3*I*csgn(I/((1+x)/(-1+x)-1))
*csgn(I*(1+x)/(-1+x))*csgn(I*(1+x)/(-1+x)/((1+x)/(-1+x)-1))*arccoth(x)^2*Pi
+4*x^2*arccoth(x)^3-4*arccoth(x)^3+6*arccoth(x)*x^2+6*arccoth(x)-6*x)/(-1+x
)/(1+x)
```

Maxima [B] time = 1.02543, size = 231, normalized size = 3.73

$$-\frac{1}{4}\left(\frac{2x}{x^2-1}-\log(x+1)+\log(x-1)\right)\operatorname{arcoth}(x)^2-\frac{\left(\left(x^2-1\right)\log(x+1)^2-2\left(x^2-1\right)\log(x+1)\log(x-1)+\left(x^2-\right.\right.}{8\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)^2/(-x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*x/(x^2 - 1) - log(x + 1) + log(x - 1))*arccoth(x)^2 - 1/8*((x^2 - 1
)*log(x + 1)^2 - 2*(x^2 - 1)*log(x + 1)*log(x - 1) + (x^2 - 1)*log(x - 1)^2
 - 4)*arccoth(x)/(x^2 - 1) + 1/48*((x^2 - 1)*log(x + 1)^3 - 3*(x^2 - 1)*log
(x + 1)^2*log(x - 1) - (x^2 - 1)*log(x - 1)^3 + 3*((x^2 - 1)*log(x - 1)^2 +
 2*x^2 - 2)*log(x + 1) - 6*(x^2 - 1)*log(x - 1) - 12*x)/(x^2 - 1)
```

Fricas [A] time = 1.56907, size = 165, normalized size = 2.66

$$\frac{\left(x^2-1\right)\log\left(\frac{x+1}{x-1}\right)^3-6\,x\log\left(\frac{x+1}{x-1}\right)^2+6\left(x^2+1\right)\log\left(\frac{x+1}{x-1}\right)-12\,x}{48\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)^2/(-x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/48*((x^2 - 1)*log((x + 1)/(x - 1))^3 - 6*x*log((x + 1)/(x - 1))^2 + 6*(x^
2 + 1)*log((x + 1)/(x - 1)) - 12*x)/(x^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{acoth}^2(x)}{(x-1)^2(x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x)**2/(-x**2+1)**2,x)
```

```
[Out] Integral(acoth(x)**2/((x - 1)**2*(x + 1)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{arcoth}(x)^2}{\left(x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)^2/(-x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(x)^2/(x^2 - 1)^2, x)
```

3.56 $\int \frac{x \coth^{-1}(x)}{1-x^2} \, dx$

Optimal. Leaf size=37

$$\frac{1}{2}\text{PolyLog}\left(2,\frac{x+1}{x-1}\right)-\frac{1}{2}\coth^{-1}(x)^2+\log\left(\frac{2}{1-x}\right)\coth^{-1}(x)$$

```
[Out] -ArcCoth[x]^2/2 + ArcCoth[x]*Log[2/(1 - x)] + PolyLog[2, (1 + x)/(-1 + x)]/
2
```

Rubi [A] time = 0.0592361, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5985, 5919, 2402, 2315}

$$\frac{1}{2}\text{PolyLog}\left(2,\frac{x+1}{x-1}\right)-\frac{1}{2}\coth^{-1}(x)^2+\log\left(\frac{2}{1-x}\right)\coth^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcCoth[x])/(1 - x^2),x]
```

```
[Out] -ArcCoth[x]^2/2 + ArcCoth[x]*Log[2/(1 - x)] + PolyLog[2, (1 + x)/(-1 + x)]/
2
```

Rule 5985

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{x\coth^{-1}(x)}{1-x^2}\,dx = -\frac{1}{2}\coth^{-1}(x)^2 + \int \frac{\coth^{-1}(x)}{1-x}\,dx$$
$$= -\frac{1}{2}\coth^{-1}(x)^2 + \coth^{-1}(x)\log\left(\frac{2}{1-x}\right) - \int \frac{\log\left(\frac{2}{1-x}\right)}{1-x^2}\,dx$$
$$= -\frac{1}{2}\coth^{-1}(x)^2 + \coth^{-1}(x)\log\left(\frac{2}{1-x}\right) + \text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-x}\right)$$
$$= -\frac{1}{2}\coth^{-1}(x)^2 + \coth^{-1}(x)\log\left(\frac{2}{1-x}\right) + \frac{1}{2}\text{Li}_2\left(\frac{1+x}{-1+x}\right)$$

Mathematica [A] time = 0.0491352, size = 34, normalized size = 0.92

$$\frac{1}{2}\left(\coth^{-1}(x)\left(\coth^{-1}(x) + 2\log\left(1 - e^{-2\coth^{-1}(x)}\right)\right) - \text{PolyLog}\left(2, e^{-2\coth^{-1}(x)}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*ArcCoth[x])/(1 - x^2),x]
```

```
[Out] (ArcCoth[x]*(ArcCoth[x] + 2*Log[1 - E^(-2*ArcCoth[x])]) - PolyLog[2, E^(-2*
ArcCoth[x])])/2
```

Maple [B] time = 0.035, size = 75, normalized size = 2.

$$-\frac{\text{arccoth}(x)\ln(-1+x)}{2} - \frac{\text{arccoth}(x)\ln(1+x)}{2} - \frac{(\ln(-1+x))^2}{8} + \frac{1}{2}\text{dilog}\left(\frac{1}{2}+\frac{x}{2}\right) + \frac{\ln(-1+x)}{4}\ln\left(\frac{1}{2}+\frac{x}{2}\right) - \frac{1}{4}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(x)/(-x^2+1),x)
```

```
[Out] -1/2*arccoth(x)*ln(-1+x)-1/2*arccoth(x)*ln(1+x)-1/8*ln(-1+x)^2+1/2*dilog(1/
2+1/2*x)+1/4*ln(-1+x)*ln(1/2+1/2*x)-1/4*(ln(1+x)-ln(1/2+1/2*x))*ln(-1/2*x+1
/2)+1/8*ln(1+x)^2
```

Maxima [B] time = 0.96325, size = 103, normalized size = 2.78

$$\frac{1}{4}(\log(x+1) - \log(x-1))\log\left(x^2-1\right) - \frac{1}{2}\,\text{arcoth}(x)\log\left(x^2-1\right) - \frac{1}{8}\,\log(x+1)^2 - \frac{1}{4}\,\log(x+1)\log(x-1) + \frac{1}{8}\,\text{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(x)/(-x^2+1),x, algorithm="maxima")
```

```
[Out] 1/4*(log(x + 1) - log(x - 1))*log(x^2 - 1) - 1/2*arccoth(x)*log(x^2 - 1) -
1/8*log(x + 1)^2 - 1/4*log(x + 1)*log(x - 1) + 1/8*log(x - 1)^2 + 1/2*log(x
 - 1)*log(1/2*x + 1/2) + 1/2*dilog(-1/2*x + 1/2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x\,\text{arcoth}\,(x)}{x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(x)/(-x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-x*arccoth(x)/(x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x\,\text{acoth}\,(x)}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(x)/(-x**2+1),x)
```

```
[Out] -Integral(x*acoth(x)/(x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{x\,\text{arcoth}\,(x)}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(x)/(-x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-x*arccoth(x)/(x^2 - 1), x)
```

3.57 $\int \frac{\coth^{-1}(x)}{1-x^2} \, dx$

Optimal. Leaf size=8

$$\frac{1}{2}\coth^{-1}(x)^2$$

```
[Out] ArcCoth[x]^2/2
```

Rubi [A] time = 0.0138965, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5949}

$$\frac{1}{2}\coth^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[x]/(1 - x^2),x]
```

```
[Out] ArcCoth[x]^2/2
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\coth^{-1}(x)}{1-x^2} \, dx = \frac{1}{2}\coth^{-1}(x)^2$$

Mathematica [A] time = 0.0039697, size = 8, normalized size = 1.

$$\frac{1}{2}\coth^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[x]/(1 - x^2),x]
```

```
[Out] ArcCoth[x]^2/2
```

Maple [A] time = 0.029, size = 13, normalized size = 1.6

$$\operatorname{Artanh}(x)\operatorname{arccoth}(x) - \frac{(\operatorname{Artanh}(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x)/(-x^2+1),x)
```

```
[Out] arctanh(x)*arccoth(x)-1/2*arctanh(x)^2
```

Maxima [A] time = 0.953711, size = 8, normalized size = 1.

$$\frac{1}{2}\,\operatorname{arcoth}(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*arccoth(x)^2
```

Fricas [B] time = 1.55847, size = 38, normalized size = 4.75

$$\frac{1}{8}\,\log\left(\frac{x+1}{x-1}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-x^2+1),x, algorithm="fricas")
```

```
[Out] 1/8*log((x + 1)/(x - 1))^2
```

Sympy [A] time = 0.817232, size = 5, normalized size = 0.62

$$\frac{\operatorname{acoth}^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x)/(-x**2+1),x)
```

```
[Out] acoth(x)**2/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\operatorname{arcoth}(x)}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-arccoth(x)/(x^2 - 1), x)
```

3.58 $\int \frac{x \coth^{-1}(x)}{(1-x^2)^2} \, dx$

Optimal. Leaf size=36

$$-\frac{x}{4(1-x^2)}+\frac{\coth^{-1}(x)}{2(1-x^2)}-\frac{1}{4}\tanh^{-1}(x)$$

```
[Out] -x/(4*(1 - x^2)) + ArcCoth[x]/(2*(1 - x^2)) - ArcTanh[x]/4
```

Rubi [A] time = 0.0307194, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5995, 199, 206}

$$-\frac{x}{4(1-x^2)}+\frac{\coth^{-1}(x)}{2(1-x^2)}-\frac{1}{4}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcCoth[x])/(1 - x^2)^2,x]
```

```
[Out] -x/(4*(1 - x^2)) + ArcCoth[x]/(2*(1 - x^2)) - ArcTanh[x]/4
```

Rule 5995

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcCoth[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcCoth[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\coth^{-1}(x)}{\left(1-x^2\right)^2}\,dx &= \frac{\coth^{-1}(x)}{2\left(1-x^2\right)} - \frac{1}{2}\int \frac{1}{\left(1-x^2\right)^2}\,dx \\
&= -\frac{x}{4\left(1-x^2\right)} + \frac{\coth^{-1}(x)}{2\left(1-x^2\right)} - \frac{1}{4}\int \frac{1}{1-x^2}\,dx \\
&= -\frac{x}{4\left(1-x^2\right)} + \frac{\coth^{-1}(x)}{2\left(1-x^2\right)} - \frac{1}{4}\tanh^{-1}(x)
\end{aligned}
$$

Mathematica [A] time = 0.027431, size = 44, normalized size = 1.22

$$
\frac{x}{4\left(x^2-1\right)} - \frac{\coth^{-1}(x)}{2\left(x^2-1\right)} + \frac{1}{8}\log(1-x) - \frac{1}{8}\log(x+1)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcCoth[x])/(1 - x^2)^2,x]
```

```
[Out] x/(4*(-1 + x^2)) - ArcCoth[x]/(2*(-1 + x^2)) + Log[1 - x]/8 - Log[1 + x]/8
```

Maple [A] time = 0.027, size = 39, normalized size = 1.1

$$
-\frac{\operatorname{arccoth}(x)}{2x^2-2} + \frac{1}{-8+8x} + \frac{\ln(-1+x)}{8} + \frac{1}{8+8x} - \frac{\ln(1+x)}{8}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(x)/(-x^2+1)^2,x)
```

```
[Out] -1/2/(x^2-1)*arccoth(x)+1/8/(-1+x)+1/8*ln(-1+x)+1/8/(1+x)-1/8*ln(1+x)
```

Maxima [A] time = 0.946376, size = 46, normalized size = 1.28

$$
\frac{x}{4\left(x^2-1\right)} - \frac{\operatorname{arcoth}(x)}{2\left(x^2-1\right)} - \frac{1}{8}\log(x+1) + \frac{1}{8}\log(x-1)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(x)/(-x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x/(x^2 - 1) - 1/2*arccoth(x)/(x^2 - 1) - 1/8*log(x + 1) + 1/8*log(x - 1
)
```

Fricas [A] time = 1.58823, size = 74, normalized size = 2.06

$$
-\frac{\left(x^2+1\right)\log\left(\frac{x+1}{x-1}\right) - 2x}{8\left(x^2-1\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(x)/(-x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((x^2 + 1)*log((x + 1)/(x - 1)) - 2*x)/(x^2 - 1)
```

Sympy [A] time = 0.819492, size = 31, normalized size = 0.86

$$-\frac{x^2 \operatorname{acoth}(x)}{4x^2-4}+\frac{x}{4x^2-4}-\frac{\operatorname{acoth}(x)}{4x^2-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(x)/(-x**2+1)**2,x)
```

```
[Out] -x**2*acoth(x)/(4*x**2 - 4) + x/(4*x**2 - 4) - acoth(x)/(4*x**2 - 4)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \operatorname{arcoth}(x)}{\left(x^2-1\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(x)/(-x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(x)/(x^2 - 1)^2, x)
```

3.59 $\int \frac{\coth^{-1}(x)}{(1-x^2)^2} \, dx$

Optimal. Leaf size=38

$$-\frac{1}{4(1-x^2)}+\frac{x\coth^{-1}(x)}{2(1-x^2)}+\frac{1}{4}\coth^{-1}(x)^2$$

```
[Out] -1/(4*(1 - x^2)) + (x*ArcCoth[x])/(2*(1 - x^2)) + ArcCoth[x]^2/4
```

Rubi [A] time = 0.0174931, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5957, 261}

$$-\frac{1}{4(1-x^2)}+\frac{x\coth^{-1}(x)}{2(1-x^2)}+\frac{1}{4}\coth^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[x]/(1 - x^2)^2,x]
```

```
[Out] -1/(4*(1 - x^2)) + (x*ArcCoth[x])/(2*(1 - x^2)) + ArcCoth[x]^2/4
```

Rule 5957

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcCoth[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcCoth[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(x)}{(1-x^2)^2}\,dx &= \frac{x\coth^{-1}(x)}{2(1-x^2)}+\frac{1}{4}\coth^{-1}(x)^2-\frac{1}{2}\int \frac{x}{(1-x^2)^2}\,dx\\ &= -\frac{1}{4(1-x^2)}+\frac{x\coth^{-1}(x)}{2(1-x^2)}+\frac{1}{4}\coth^{-1}(x)^2\end{aligned}$$

Mathematica [A] time = 0.0323973, size = 28, normalized size = 0.74

$$\frac{(x^2-1)\coth^{-1}(x)^2-2x\coth^{-1}(x)+1}{4(x^2-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[x]/(1 - x^2)^2,x]
```

```
[Out] (1 - 2*x*ArcCoth[x] + (-1 + x^2)*ArcCoth[x]^2)/(4*(-1 + x^2))
```

Maple [B] time = 0.043, size = 99, normalized size = 2.6

$$-\frac{\operatorname{arccoth}(x)}{-4+4x}-\frac{\operatorname{arccoth}(x)\ln(-1+x)}{4}-\frac{\operatorname{arccoth}(x)}{4+4x}+\frac{\operatorname{arccoth}(x)\ln(1+x)}{4}+\frac{1}{8}\left(\ln(1+x)-\ln\left(\frac{1}{2}+\frac{x}{2}\right)\right)\ln\left(-\frac{x}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x)/(-x^2+1)^2,x)
```

```
[Out] -1/4*arccoth(x)/(-1+x)-1/4*arccoth(x)*ln(-1+x)-1/4*arccoth(x)/(1+x)+1/4*arc
coth(x)*ln(1+x)+1/8*(ln(1+x)-ln(1/2+1/2*x))*ln(-1/2*x+1/2)-1/16*ln(1+x)^2-1
/16*ln(-1+x)^2+1/8*ln(-1+x)*ln(1/2+1/2*x)+1/8/(-1+x)-1/8/(1+x)
```

Maxima [B] time = 0.967018, size = 103, normalized size = 2.71

$$-\frac{1}{4}\left(\frac{2x}{x^2-1}-\log(x+1)+\log(x-1)\right)\operatorname{arccoth}(x)-\frac{(x^2-1)\log(x+1)^2-2(x^2-1)\log(x+1)\log(x-1)+(x^2-1)}{16(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*x/(x^2 - 1) - log(x + 1) + log(x - 1))*arccoth(x) - 1/16*((x^2 - 1)
*log(x + 1)^2 - 2*(x^2 - 1)*log(x + 1)*log(x - 1) + (x^2 - 1)*log(x - 1)^2
- 4)/(x^2 - 1)
```

Fricas [A] time = 1.58369, size = 111, normalized size = 2.92

$$\frac{(x^2-1)\log\left(\frac{x+1}{x-1}\right)^2-4x\log\left(\frac{x+1}{x-1}\right)+4}{16(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/16*((x^2 - 1)*log((x + 1)/(x - 1))^2 - 4*x*log((x + 1)/(x - 1)) + 4)/(x^2
 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{acoth}(x)}{(x-1)^2(x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x)/(-x**2+1)**2,x)
```

```
[Out] Integral(acoth(x)/((x - 1)**2*(x + 1)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(x)}{(x^2-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(x)/(x^2 - 1)^2, x)
```

3.60 $\int \frac{x \coth^{-1}(x)}{(1-x^2)^3} dx$

Optimal. Leaf size=50

$$-\frac{3x}{32(1-x^2)}-\frac{x}{16(1-x^2)^2}+\frac{\coth^{-1}(x)}{4(1-x^2)^2}-\frac{3}{32}\tanh^{-1}(x)$$

```
[Out] -x/(16*(1 - x^2)^2) - (3*x)/(32*(1 - x^2)) + ArcCoth[x]/(4*(1 - x^2)^2) - (
3*ArcTanh[x])/32
```

Rubi [A] time = 0.0341809, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5995, 199, 206}

$$-\frac{3x}{32(1-x^2)}-\frac{x}{16(1-x^2)^2}+\frac{\coth^{-1}(x)}{4(1-x^2)^2}-\frac{3}{32}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcCoth[x])/(1 - x^2)^3,x]
```

```
[Out] -x/(16*(1 - x^2)^2) - (3*x)/(32*(1 - x^2)) + ArcCoth[x]/(4*(1 - x^2)^2) - (
3*ArcTanh[x])/32
```

Rule 5995

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcCoth[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcCoth[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x\coth^{-1}(x)}{\left(1-x^2\right)^3}\,dx = \frac{\coth^{-1}(x)}{4\left(1-x^2\right)^2} - \frac{1}{4}\int \frac{1}{\left(1-x^2\right)^3}\,dx$$
$$= -\frac{x}{16\left(1-x^2\right)^2} + \frac{\coth^{-1}(x)}{4\left(1-x^2\right)^2} - \frac{3}{16}\int \frac{1}{\left(1-x^2\right)^2}\,dx$$
$$= -\frac{x}{16\left(1-x^2\right)^2} - \frac{3x}{32\left(1-x^2\right)} + \frac{\coth^{-1}(x)}{4\left(1-x^2\right)^2} - \frac{3}{32}\int \frac{1}{1-x^2}\,dx$$
$$= -\frac{x}{16\left(1-x^2\right)^2} - \frac{3x}{32\left(1-x^2\right)} + \frac{\coth^{-1}(x)}{4\left(1-x^2\right)^2} - \frac{3}{32}\tanh^{-1}(x)$$

Mathematica [A] time = 0.0505672, size = 50, normalized size = 1.

$$\frac{1}{64}\left(\frac{6x}{x^2-1} - \frac{4x}{\left(x^2-1\right)^2} + \frac{16\coth^{-1}(x)}{\left(x^2-1\right)^2} + 3\log(1-x) - 3\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcCoth[x])/(1 - x^2)^3,x]
```

```
[Out] ((-4*x)/(-1 + x^2)^2 + (6*x)/(-1 + x^2) + (16*ArcCoth[x])/(-1 + x^2)^2 + 3*
Log[1 - x] - 3*Log[1 + x])/64
```

Maple [A] time = 0.028, size = 53, normalized size = 1.1

$$\frac{\text{arccoth}\,(x)}{4\left(x^2-1\right)^2} - \frac{1}{64\,(-1+x)^2} + \frac{3}{-64+64\,x} + \frac{3\,\ln(-1+x)}{64} + \frac{1}{64\,(1+x)^2} + \frac{3}{64+64\,x} - \frac{3\,\ln(1+x)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(x)/(-x^2+1)^3,x)
```

```
[Out] 1/4/(x^2-1)^2*arccoth(x)-1/64/(-1+x)^2+3/64/(-1+x)+3/64*ln(-1+x)+1/64/(1+x)
^2+3/64/(1+x)-3/64*ln(1+x)
```

Maxima [A] time = 0.949001, size = 63, normalized size = 1.26

$$\frac{3\,x^3-5\,x}{32\left(x^4-2\,x^2+1\right)} + \frac{\text{arcoth}\,(x)}{4\left(x^2-1\right)^2} - \frac{3}{64}\,\log(x+1) + \frac{3}{64}\,\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(x)/(-x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(3*x^3 - 5*x)/(x^4 - 2*x^2 + 1) + 1/4*arccoth(x)/(x^2 - 1)^2 - 3/64*lo
g(x + 1) + 3/64*log(x - 1)
```

Fricas [A] time = 1.59255, size = 111, normalized size = 2.22

$$\frac{6\,x^3 - \left(3\,x^4 - 6\,x^2 - 5\right)\log\left(\frac{x+1}{x-1}\right) - 10\,x}{64\,\left(x^4 - 2\,x^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(x)/(-x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(6*x^3 - (3*x^4 - 6*x^2 - 5)*log((x + 1)/(x - 1)) - 10*x)/(x^4 - 2*x^2
 + 1)
```

Sympy [B] time = 1.32896, size = 88, normalized size = 1.76

$$-\frac{3x^4\operatorname{acoth}(x)}{32x^4 - 64x^2 + 32} + \frac{3x^3}{32x^4 - 64x^2 + 32} + \frac{6x^2\operatorname{acoth}(x)}{32x^4 - 64x^2 + 32} - \frac{5x}{32x^4 - 64x^2 + 32} + \frac{5\operatorname{acoth}(x)}{32x^4 - 64x^2 + 32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(x)/(-x**2+1)**3,x)
```

```
[Out] -3*x**4*acoth(x)/(32*x**4 - 64*x**2 + 32) + 3*x**3/(32*x**4 - 64*x**2 + 32)
 + 6*x**2*acoth(x)/(32*x**4 - 64*x**2 + 32) - 5*x/(32*x**4 - 64*x**2 + 32)
+ 5*acoth(x)/(32*x**4 - 64*x**2 + 32)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x\operatorname{arcoth}(x)}{\left(x^2 - 1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(x)/(-x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-x*arccoth(x)/(x^2 - 1)^3, x)
```

3.61 $\int \frac{\coth^{-1}(x)}{(1-x^2)^3}\,dx$

Optimal. Leaf size=67

$$-\frac{3}{16\left(1-x^2\right)}-\frac{1}{16\left(1-x^2\right)^2}+\frac{3x\coth^{-1}(x)}{8\left(1-x^2\right)}+\frac{x\coth^{-1}(x)}{4\left(1-x^2\right)^2}+\frac{3}{16}\coth^{-1}(x)^2$$

```
[Out] -1/(16*(1 - x^2)^2) - 3/(16*(1 - x^2)) + (x*ArcCoth[x])/(4*(1 - x^2)^2) + (
3*x*ArcCoth[x])/(8*(1 - x^2)) + (3*ArcCoth[x]^2)/16
```

Rubi [A] time = 0.0352498, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5961, 5957, 261}

$$-\frac{3}{16\left(1-x^2\right)}-\frac{1}{16\left(1-x^2\right)^2}+\frac{3x\coth^{-1}(x)}{8\left(1-x^2\right)}+\frac{x\coth^{-1}(x)}{4\left(1-x^2\right)^2}+\frac{3}{16}\coth^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[x]/(1 - x^2)^3,x]
```

```
[Out] -1/(16*(1 - x^2)^2) - 3/(16*(1 - x^2)) + (x*ArcCoth[x])/(4*(1 - x^2)^2) + (
3*x*ArcCoth[x])/(8*(1 - x^2)) + (3*ArcCoth[x]^2)/16
```

Rule 5961

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcCoth[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcCoth[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5957

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcCoth[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcCoth[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\coth^{-1}(x)}{(1-x^2)^3}\,dx = -\frac{1}{16(1-x^2)^2} + \frac{x\coth^{-1}(x)}{4(1-x^2)^2} + \frac{3}{4}\int \frac{\coth^{-1}(x)}{(1-x^2)^2}\,dx$$
$$= -\frac{1}{16(1-x^2)^2} + \frac{x\coth^{-1}(x)}{4(1-x^2)^2} + \frac{3x\coth^{-1}(x)}{8(1-x^2)} + \frac{3}{16}\coth^{-1}(x)^2 - \frac{3}{8}\int \frac{x}{(1-x^2)^2}\,dx$$
$$= -\frac{1}{16(1-x^2)^2} - \frac{3}{16(1-x^2)} + \frac{x\coth^{-1}(x)}{4(1-x^2)^2} + \frac{3x\coth^{-1}(x)}{8(1-x^2)} + \frac{3}{16}\coth^{-1}(x)^2$$

Mathematica [A] time = 0.0523017, size = 43, normalized size = 0.64

$$-\frac{-3x^2 + 2(3x^2-5)x\coth^{-1}(x) - 3(x^2-1)^2\coth^{-1}(x)^2 + 4}{16(x^2-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[x]/(1 - x^2)^3,x]
```

```
[Out] -(4 - 3*x^2 + 2*x*(-5 + 3*x^2)*ArcCoth[x] - 3*(-1 + x^2)^2*ArcCoth[x]^2)/(1
6*(-1 + x^2)^2)
```

Maple [B] time = 0.049, size = 131, normalized size = 2.

$$\frac{\operatorname{arccoth}(x)}{16(-1+x)^2} - \frac{3\operatorname{arccoth}(x)}{-16+16x} - \frac{3\operatorname{arccoth}(x)\ln(-1+x)}{16} - \frac{\operatorname{arccoth}(x)}{16(1+x)^2} - \frac{3\operatorname{arccoth}(x)}{16+16x} + \frac{3\operatorname{arccoth}(x)\ln(1+x)}{16} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x)/(-x^2+1)^3,x)
```

```
[Out] 1/16*arccoth(x)/(-1+x)^2-3/16*arccoth(x)/(-1+x)-3/16*arccoth(x)*ln(-1+x)-1/
16*arccoth(x)/(1+x)^2-3/16*arccoth(x)/(1+x)+3/16*arccoth(x)*ln(1+x)-3/64*ln
(-1+x)^2+3/32*ln(-1+x)*ln(1/2+1/2*x)+3/32*(ln(1+x)-ln(1/2+1/2*x))*ln(-1/2*x
+1/2)-3/64*ln(1+x)^2-1/64/(-1+x)^2+7/64/(-1+x)-1/64/(1+x)^2-7/64/(1+x)
```

Maxima [B] time = 0.961914, size = 159, normalized size = 2.37

$$-\frac{1}{16}\left(\frac{2(3x^3-5x)}{x^4-2x^2+1} - 3\log(x+1) + 3\log(x-1)\right)\operatorname{arcoth}(x) - \frac{3(x^4-2x^2+1)\log(x+1)^2 - 6(x^4-2x^2+1)\log(}{64(x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(2*(3*x^3 - 5*x)/(x^4 - 2*x^2 + 1) - 3*log(x + 1) + 3*log(x - 1))*arc
coth(x) - 1/64*(3*(x^4 - 2*x^2 + 1)*log(x + 1)^2 - 6*(x^4 - 2*x^2 + 1)*log(
x + 1)*log(x - 1) + 3*(x^4 - 2*x^2 + 1)*log(x - 1)^2 - 12*x^2 + 16)/(x^4 -
2*x^2 + 1)
```

Fricas [A] time = 1.58159, size = 165, normalized size = 2.46

$$\frac{3\left(x^{4}-2\,x^{2}+1\right)\log\left(\frac{x+1}{x-1}\right)^{2}+12\,x^{2}-4\left(3\,x^{3}-5\,x\right)\log\left(\frac{x+1}{x-1}\right)-16}{64\left(x^{4}-2\,x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(3*(x^4 - 2*x^2 + 1)*log((x + 1)/(x - 1))^2 + 12*x^2 - 4*(3*x^3 - 5*x)
*log((x + 1)/(x - 1)) - 16)/(x^4 - 2*x^2 + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\operatorname{acoth}(x)}{x^{6}-3x^{4}+3x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x)/(-x**2+1)**3,x)
```

```
[Out] -Integral(acoth(x)/(x**6 - 3*x**4 + 3*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{\operatorname{arcoth}(x)}{\left(x^{2}-1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-arccoth(x)/(x^2 - 1)^3, x)
```

3.62 $\int x^3 \coth^{-1}(a+bx)\,dx$

Optimal. Leaf size=101

$$\frac{(6a^2+1)x}{4b^3}+\frac{(a+bx)^3}{12b^4}-\frac{a(a+bx)^2}{2b^4}+\frac{(1-a)^4\log(-a-bx+1)}{8b^4}-\frac{(a+1)^4\log(a+bx+1)}{8b^4}+\frac{1}{4}x^4\coth^{-1}(a+bx)$$

```
[Out] ((1 + 6*a^2)*x)/(4*b^3) - (a*(a + b*x)^2)/(2*b^4) + (a + b*x)^3/(12*b^4) +
(x^4*ArcCoth[a + b*x])/4 + ((1 - a)^4*Log[1 - a - b*x])/(8*b^4) - ((1 + a)^
4*Log[1 + a + b*x])/(8*b^4)
```

Rubi [A] time = 0.126333, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6112, 5927, 702, 633, 31}

$$\frac{(6a^2+1)x}{4b^3}+\frac{(a+bx)^3}{12b^4}-\frac{a(a+bx)^2}{2b^4}+\frac{(1-a)^4\log(-a-bx+1)}{8b^4}-\frac{(a+1)^4\log(a+bx+1)}{8b^4}+\frac{1}{4}x^4\coth^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCoth[a + b*x],x]
```

```
[Out] ((1 + 6*a^2)*x)/(4*b^3) - (a*(a + b*x)^2)/(2*b^4) + (a + b*x)^3/(12*b^4) +
(x^4*ArcCoth[a + b*x])/4 + ((1 - a)^4*Log[1 - a - b*x])/(8*b^4) - ((1 + a)^
4*Log[1 + a + b*x])/(8*b^4)
```

Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rule 5927

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcCoth[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int x^3 \coth^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^3 \coth^{-1}(x)\,dx, x, a+bx\right)}{b}$$

$$= \frac{1}{4}x^4 \coth^{-1}(a+bx) - \frac{1}{4}\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^4}{1-x^2}\,dx, x, a+bx\right)$$

$$= \frac{1}{4}x^4 \coth^{-1}(a+bx) - \frac{1}{4}\text{Subst}\left(\int \left(-\frac{1+6a^2}{b^4}+\frac{4ax}{b^4}-\frac{x^2}{b^4}+\frac{1+6a^2+a^4-4a\left(1+a^2\right)x}{b^4\left(1-x^2\right)}\right) dx,$$

$$= \frac{\left(1+6a^2\right)x}{4b^3} - \frac{a(a+bx)^2}{2b^4} + \frac{(a+bx)^3}{12b^4} + \frac{1}{4}x^4 \coth^{-1}(a+bx) - \frac{\text{Subst}\left(\int \frac{1+6a^2+a^4-4a\left(1+a^2\right)x}{1-x^2}\,dx, x\right.}{4b^4}$$

$$= \frac{\left(1+6a^2\right)x}{4b^3} - \frac{a(a+bx)^2}{2b^4} + \frac{(a+bx)^3}{12b^4} + \frac{1}{4}x^4 \coth^{-1}(a+bx) - \frac{(1-a)^4\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, a+\right.}{8b^4}$$

$$= \frac{\left(1+6a^2\right)x}{4b^3} - \frac{a(a+bx)^2}{2b^4} + \frac{(a+bx)^3}{12b^4} + \frac{1}{4}x^4 \coth^{-1}(a+bx) + \frac{(1-a)^4\log(1-a-bx)}{8b^4} - \frac{(1+a}{}$$

Mathematica [A] time = 0.038819, size = 81, normalized size = 0.8

$$\frac{6\left(3a^2+1\right)bx-6ab^2x^2+6b^4x^4\coth^{-1}(a+bx)+3(a-1)^4\log(-a-bx+1)-3(a+1)^4\log(a+bx+1)+2b^3x^3}{24b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCoth[a + b*x],x]
```

```
[Out] (6*(1 + 3*a^2)*b*x - 6*a*b^2*x^2 + 2*b^3*x^3 + 6*b^4*x^4*ArcCoth[a + b*x] +
 3*(-1 + a)^4*Log[1 - a - b*x] - 3*(1 + a)^4*Log[1 + a + b*x])/(24*b^4)
```

Maple [B] time = 0.035, size = 199, normalized size = 2.

$$\frac{a}{4\,b^4}+\frac{x}{4\,b^3}+\frac{x^4 \text{arccoth}\,(bx+a)}{4}-\frac{\ln(bx+a+1)}{8\,b^4}+\frac{\ln(bx+a-1)}{8\,b^4}+\frac{x^3}{12\,b}-\frac{ax^2}{4\,b^2}+\frac{3\,a^2x}{4\,b^3}+\frac{13\,a^3}{12\,b^4}+\frac{\ln(bx+a-1)\,a^4}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccoth(b*x+a),x)
```

```
[Out] 1/4/b^4*a+1/4*x/b^3+1/4*x^4*arccoth(b*x+a)-1/8/b^4*ln(b*x+a+1)+1/8/b^4*ln(b
*x+a-1)+1/12*x^3/b-1/4/b^2*x^2*a+3/4/b^3*x*a^2+13/12/b^4*a^3+1/8/b^4*ln(b*x
+a-1)*a^4-1/2/b^4*ln(b*x+a-1)*a^3+3/4/b^4*ln(b*x+a-1)*a^2-1/2/b^4*ln(b*x+a-
1)*a-1/8/b^4*ln(b*x+a+1)*a^4-1/2/b^4*ln(b*x+a+1)*a^3-3/4/b^4*ln(b*x+a+1)*a^
2-1/2/b^4*ln(b*x+a+1)*a
```

Maxima [A] time = 0.959818, size = 143, normalized size = 1.42

$$\frac{1}{4}x^4 \text{arcoth}\,(bx+a)+\frac{1}{24}b\left(\frac{2\left(b^2x^3-3\,abx^2+3\left(3\,a^2+1\right)x\right)}{b^4}-\frac{3\left(a^4+4\,a^3+6\,a^2+4\,a+1\right)\log(bx+a+1)}{b^5}+\frac{3\left(a^4-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccoth(b*x + a) + 1/24*b*(2*(b^2*x^3 - 3*a*b*x^2 + 3*(3*a^2 + 1)*x
)/b^4 - 3*(a^4 + 4*a^3 + 6*a^2 + 4*a + 1)*log(b*x + a + 1)/b^5 + 3*(a^4 - 4
*a^3 + 6*a^2 - 4*a + 1)*log(b*x + a - 1)/b^5)
```

Fricas [A] time = 1.55089, size = 279, normalized size = 2.76

$$\frac{3\,b^4x^4\log\left(\frac{bx+a+1}{bx+a-1}\right)+2\,b^3x^3-6\,ab^2x^2+6\left(3\,a^2+1\right)bx-3\left(a^4+4\,a^3+6\,a^2+4\,a+1\right)\log\left(bx+a+1\right)+3\left(a^4-4\,a^3+\right.}{24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^4*x^4*log((b*x + a + 1)/(b*x + a - 1)) + 2*b^3*x^3 - 6*a*b^2*x^2
+ 6*(3*a^2 + 1)*b*x - 3*(a^4 + 4*a^3 + 6*a^2 + 4*a + 1)*log(b*x + a + 1) +
3*(a^4 - 4*a^3 + 6*a^2 - 4*a + 1)*log(b*x + a - 1))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acoth(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arcoth}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*arccoth(b*x + a), x)
```

3.63 $\int x^2 \coth^{-1}(a+bx)\,dx$

Optimal. Leaf size=78

$$\frac{(a+bx)^2}{6b^3}-\frac{ax}{b^2}+\frac{(1-a)^3\log(-a-bx+1)}{6b^3}+\frac{(a+1)^3\log(a+bx+1)}{6b^3}+\frac{1}{3}x^3\coth^{-1}(a+bx)$$

```
[Out] -((a*x)/b^2) + (a + b*x)^2/(6*b^3) + (x^3*ArcCoth[a + b*x])/3 + ((1 - a)^3*
Log[1 - a - b*x])/(6*b^3) + ((1 + a)^3*Log[1 + a + b*x])/(6*b^3)
```

Rubi [A] time = 0.101837, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6112, 5927, 702, 633, 31}

$$\frac{(a+bx)^2}{6b^3}-\frac{ax}{b^2}+\frac{(1-a)^3\log(-a-bx+1)}{6b^3}+\frac{(a+1)^3\log(a+bx+1)}{6b^3}+\frac{1}{3}x^3\coth^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[a + b*x],x]
```

```
[Out] -((a*x)/b^2) + (a + b*x)^2/(6*b^3) + (x^3*ArcCoth[a + b*x])/3 + ((1 - a)^3*
Log[1 - a - b*x])/(6*b^3) + ((1 + a)^3*Log[1 + a + b*x])/(6*b^3)
```

Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rule 5927

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcCoth[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int x^2\coth^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)^2\coth^{-1}(x)\,dx,x,a+bx\right)}{b}$$
$$=\frac{1}{3}x^3\coth^{-1}(a+bx)-\frac{1}{3}\text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}{1-x^2}dx,x,a+bx\right)$$
$$=\frac{1}{3}x^3\coth^{-1}(a+bx)-\frac{1}{3}\text{Subst}\left(\int\left(\frac{3a}{b^3}-\frac{x}{b^3}-\frac{a\left(3+a^2\right)-\left(1+3a^2\right)x}{b^3\left(1-x^2\right)}\right)dx,x,a+bx\right)$$
$$=-\frac{ax}{b^2}+\frac{(a+bx)^2}{6b^3}+\frac{1}{3}x^3\coth^{-1}(a+bx)+\frac{\text{Subst}\left(\int\frac{a\left(3+a^2\right)-\left(1+3a^2\right)x}{1-x^2}dx,x,a+bx\right)}{3b^3}$$
$$=-\frac{ax}{b^2}+\frac{(a+bx)^2}{6b^3}+\frac{1}{3}x^3\coth^{-1}(a+bx)-\frac{(1-a)^3\,\text{Subst}\left(\int\frac{1}{1-x}dx,x,a+bx\right)}{6b^3}-\frac{(1+a)^3\,\text{Su}}{}$$
$$=-\frac{ax}{b^2}+\frac{(a+bx)^2}{6b^3}+\frac{1}{3}x^3\coth^{-1}(a+bx)+\frac{(1-a)^3\log(1-a-bx)}{6b^3}+\frac{(1+a)^3\log(1+a+bx}{6b^3}$$

Mathematica [A] time = 0.0233845, size = 92, normalized size = 1.18

$$\frac{\left(-a^3+3a^2-3a+1\right)\log(-a-bx+1)}{6b^3}+\frac{\left(a^3+3a^2+3a+1\right)\log(a+bx+1)}{6b^3}-\frac{2ax}{3b^2}+\frac{1}{3}x^3\coth^{-1}(a+bx)+\frac{x^2}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[a + b*x],x]
```

```
[Out] (-2*a*x)/(3*b^2) + x^2/(6*b) + (x^3*ArcCoth[a + b*x])/3 + ((1 - 3*a + 3*a^2
 - a^3)*Log[1 - a - b*x])/(6*b^3) + ((1 + 3*a + 3*a^2 + a^3)*Log[1 + a + b*
x])/(6*b^3)
```

Maple [B] time = 0.036, size = 146, normalized size = 1.9

$$\frac{x^3\text{arccoth}\,(bx+a)}{3}+\frac{x^2}{6\,b}-\frac{2\,ax}{3\,b^2}-\frac{5\,a^2}{6\,b^3}-\frac{\ln(bx+a-1)\,a^3}{6\,b^3}+\frac{\ln(bx+a-1)\,a^2}{2\,b^3}-\frac{\ln(bx+a-1)\,a}{2\,b^3}+\frac{\ln(bx+a-1)}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(b*x+a),x)
```

```
[Out] 1/3*x^3*arccoth(b*x+a)+1/6*x^2/b-2/3*a*x/b^2-5/6/b^3*a^2-1/6/b^3*ln(b*x+a-1
)*a^3+1/2/b^3*ln(b*x+a-1)*a^2-1/2/b^3*ln(b*x+a-1)*a+1/6/b^3*ln(b*x+a-1)+1/6
/b^3*ln(b*x+a+1)*a^3+1/2/b^3*ln(b*x+a+1)*a^2+1/2/b^3*ln(b*x+a+1)*a+1/6/b^3*
ln(b*x+a+1)
```

Maxima [A] time = 0.958009, size = 107, normalized size = 1.37

$$\frac{1}{3}\,x^3\,\text{arcoth}\,(bx+a)+\frac{1}{6}\,b\left(\frac{bx^2-4\,ax}{b^3}+\frac{\left(a^3+3\,a^2+3\,a+1\right)\log(bx+a+1)}{b^4}-\frac{\left(a^3-3\,a^2+3\,a-1\right)\log(bx+a-1)}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(b*x + a) + 1/6*b*((b*x^2 - 4*a*x)/b^3 + (a^3 + 3*a^2 + 3*a
+ 1)*log(b*x + a + 1)/b^4 - (a^3 - 3*a^2 + 3*a - 1)*log(b*x + a - 1)/b^4)
```

Fricas [A] time = 1.61077, size = 213, normalized size = 2.73

$$\frac{b^3x^3\log\left(\frac{bx+a+1}{bx+a-1}\right)+b^2x^2-4\,abx+\left(a^3+3\,a^2+3\,a+1\right)\log\left(bx+a+1\right)-\left(a^3-3\,a^2+3\,a-1\right)\log\left(bx+a-1\right)}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(b^3*x^3*log((b*x + a + 1)/(b*x + a - 1)) + b^2*x^2 - 4*a*b*x + (a^3 +
3*a^2 + 3*a + 1)*log(b*x + a + 1) - (a^3 - 3*a^2 + 3*a - 1)*log(b*x + a - 1
))/b^3
```

Sympy [A] time = 3.19662, size = 117, normalized size = 1.5

$$\begin{cases} \frac{a^3\operatorname{acoth}(a+bx)}{3b^3}+\frac{a^2\log(a+bx+1)}{b^3}-\frac{a^2\operatorname{acoth}(a+bx)}{b^3}-\frac{2ax}{3b^2}+\frac{a\operatorname{acoth}(a+bx)}{b^3}+\frac{x^3\operatorname{acoth}(a+bx)}{3}+\frac{x^2}{6b}+\frac{\log(a+bx+1)}{3b^3}-\frac{\operatorname{acoth}(a+bx)}{3b^3} & \text{for } b \\ \frac{x^3\operatorname{acoth}(a)}{3} & \text{other} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(b*x+a),x)
```

```
[Out] Piecewise((a**3*acoth(a + b*x)/(3*b**3) + a**2*log(a + b*x + 1)/b**3 - a**2
*acoth(a + b*x)/b**3 - 2*a*x/(3*b**2) + a*acoth(a + b*x)/b**3 + x**3*acoth(
a + b*x)/3 + x**2/(6*b) + log(a + b*x + 1)/(3*b**3) - acoth(a + b*x)/(3*b**
3), Ne(b, 0)), (x**3*acoth(a)/3, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arcoth}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(b*x + a), x)
```

3.64 $\int x \coth^{-1}(a + bx)\, dx$

Optimal. Leaf size=65

$$\frac{(1-a)^2 \log(-a-bx+1)}{4b^2} - \frac{(a+1)^2 \log(a+bx+1)}{4b^2} + \frac{1}{2} x^2 \coth^{-1}(a+bx) + \frac{x}{2b}$$

```
[Out] x/(2*b) + (x^2*ArcCoth[a + b*x])/2 + ((1 - a)^2*Log[1 - a - b*x])/(4*b^2) -
 ((1 + a)^2*Log[1 + a + b*x])/(4*b^2)
```

Rubi [A] time = 0.0716315, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6112, 5927, 702, 633, 31}

$$\frac{(1-a)^2 \log(-a-bx+1)}{4b^2} - \frac{(a+1)^2 \log(a+bx+1)}{4b^2} + \frac{1}{2} x^2 \coth^{-1}(a+bx) + \frac{x}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[a + b*x],x]
```

```
[Out] x/(2*b) + (x^2*ArcCoth[a + b*x])/2 + ((1 - a)^2*Log[1 - a - b*x])/(4*b^2) -
 ((1 + a)^2*Log[1 + a + b*x])/(4*b^2)
```

Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rule 5927

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcCoth[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int x\coth^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)\coth^{-1}(x)\,dx,x,a+bx\right)}{b}$$
$$= \frac{1}{2}x^2\coth^{-1}(a+bx)-\frac{1}{2}\text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{1-x^2}dx,x,a+bx\right)$$
$$= \frac{1}{2}x^2\coth^{-1}(a+bx)-\frac{1}{2}\text{Subst}\left(\int\left(-\frac{1}{b^2}+\frac{1+a^2-2ax}{b^2\left(1-x^2\right)}\right)dx,x,a+bx\right)$$
$$= \frac{x}{2b}+\frac{1}{2}x^2\coth^{-1}(a+bx)-\frac{\text{Subst}\left(\int\frac{1+a^2-2ax}{1-x^2}dx,x,a+bx\right)}{2b^2}$$
$$= \frac{x}{2b}+\frac{1}{2}x^2\coth^{-1}(a+bx)-\frac{(1-a)^2\,\text{Subst}\left(\int\frac{1}{1-x}dx,x,a+bx\right)}{4b^2}+\frac{(1+a)^2\,\text{Subst}\left(\int\frac{1}{-1-x}dx,x,\right.}{4b^2}$$
$$= \frac{x}{2b}+\frac{1}{2}x^2\coth^{-1}(a+bx)+\frac{(1-a)^2\log(1-a-bx)}{4b^2}-\frac{(1+a)^2\log(1+a+bx)}{4b^2}$$

Mathematica [A] time = 0.0219382, size = 56, normalized size = 0.86

$$\frac{2b^2x^2\coth^{-1}(a+bx)+(a-1)^2\log(-a-bx+1)-(a+1)^2\log(a+bx+1)+2bx}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[a + b*x],x]
```

```
[Out] (2*b*x + 2*b^2*x^2*ArcCoth[a + b*x] + (-1 + a)^2*Log[1 - a - b*x] - (1 + a)
^2*Log[1 + a + b*x])/(4*b^2)
```

Maple [A] time = 0.033, size = 89, normalized size = 1.4

$$\frac{x^2\text{arccoth}\,(bx+a)}{2}-\frac{\text{arccoth}\,(bx+a)\,a^2}{2\,b^2}+\frac{x}{2\,b}+\frac{a}{2\,b^2}-\frac{\ln(bx+a-1)\,a}{2\,b^2}+\frac{\ln(bx+a-1)}{4\,b^2}-\frac{\ln(bx+a+1)\,a}{2\,b^2}-\frac{\ln(bx}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(b*x+a),x)
```

```
[Out] 1/2*x^2*arccoth(b*x+a)-1/2/b^2*arccoth(b*x+a)*a^2+1/2*x/b+1/2/b^2*a-1/2/b^2
*ln(b*x+a-1)*a+1/4/b^2*ln(b*x+a-1)-1/2/b^2*ln(b*x+a+1)*a-1/4/b^2*ln(b*x+a+1
)
```

Maxima [A] time = 0.971898, size = 82, normalized size = 1.26

$$\frac{1}{2}x^2\,\text{arcoth}\,(bx+a)+\frac{1}{4}b\left(\frac{2\,x}{b^2}-\frac{\left(a^2+2\,a+1\right)\log(bx+a+1)}{b^3}+\frac{\left(a^2-2\,a+1\right)\log(bx+a-1)}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccoth(b*x + a) + 1/4*b*(2*x/b^2 - (a^2 + 2*a + 1)*log(b*x + a + 1
)/b^3 + (a^2 - 2*a + 1)*log(b*x + a - 1)/b^3)
```

Fricas [A] time = 1.59959, size = 176, normalized size = 2.71

$$\frac{b^2x^2 \log\left(\frac{bx+a+1}{bx+a-1}\right) + 2\,bx - \left(a^2 + 2\,a + 1\right)\log\left(bx + a + 1\right) + \left(a^2 - 2\,a + 1\right)\log\left(bx + a - 1\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^2*log((b*x + a + 1)/(b*x + a - 1)) + 2*b*x - (a^2 + 2*a + 1)*log
(b*x + a + 1) + (a^2 - 2*a + 1)*log(b*x + a - 1))/b^2
```

Sympy [A] time = 1.60945, size = 76, normalized size = 1.17

$$\begin{cases} -\frac{a^2 \operatorname{acoth}(a+bx)}{2b^2} - \frac{a\log(a+bx+1)}{b^2} + \frac{a\operatorname{acoth}(a+bx)}{b^2} + \frac{x^2\operatorname{acoth}(a+bx)}{2} + \frac{x}{2b} - \frac{\operatorname{acoth}(a+bx)}{2b^2} & \text{for } b \neq 0 \\ \frac{x^2 \operatorname{acoth}(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(b*x+a),x)
```

```
[Out] Piecewise((-a**2*acoth(a + b*x)/(2*b**2) - a*log(a + b*x + 1)/b**2 + a*acot
h(a + b*x)/b**2 + x**2*acoth(a + b*x)/2 + x/(2*b) - acoth(a + b*x)/(2*b**2)
, Ne(b, 0)), (x**2*acoth(a)/2, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arcoth}(bx + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(b*x + a), x)
```

3.65 $\int \coth^{-1}(a+bx)\,dx$

Optimal. Leaf size=35

$$\frac{\log\left(1-(a+bx)^2\right)}{2b}+\frac{(a+bx)\coth^{-1}(a+bx)}{b}$$

```
[Out] ((a + b*x)*ArcCoth[a + b*x])/b + Log[1 - (a + b*x)^2]/(2*b)
```

Rubi [A] time = 0.0159401, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6104, 5911, 260}

$$\frac{\log\left(1-(a+bx)^2\right)}{2b}+\frac{(a+bx)\coth^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x],x]
```

```
[Out] ((a + b*x)*ArcCoth[a + b*x])/b + Log[1 - (a + b*x)^2]/(2*b)
```

Rule 6104

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}
, x] && IGtQ[p, 0]
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \coth^{-1}(a+bx)\,dx &= \frac{\text{Subst}\left(\int \coth^{-1}(x)\,dx,x,a+bx\right)}{b}\\ &= \frac{(a+bx)\coth^{-1}(a+bx)}{b}-\frac{\text{Subst}\left(\int \frac{x}{1-x^2}\,dx,x,a+bx\right)}{b}\\ &= \frac{(a+bx)\coth^{-1}(a+bx)}{b}+\frac{\log\left(1-(a+bx)^2\right)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0145124, size = 43, normalized size = 1.23

$$\frac{(a+1)\log(a+bx+1)-(a-1)\log(-a-bx+1)}{2b}+x\coth^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a + b*x],x]
```

```
[Out] x*ArcCoth[a + b*x] + (-((-1 + a)*Log[1 - a - b*x]) + (1 + a)*Log[1 + a + b*
x])/(2*b)
```

Maple [A] time = 0.03, size = 36, normalized size = 1.

$$x\mathrm{arccoth}\left(bx+a\right)+\frac{\mathrm{arccoth}\left(bx+a\right)a}{b}+\frac{\ln\left(\left(bx+a\right)^{2}-1\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a),x)
```

```
[Out] x*arccoth(b*x+a)+1/b*arccoth(b*x+a)*a+1/2/b*ln((b*x+a)^2-1)
```

Maxima [A] time = 0.973925, size = 42, normalized size = 1.2

$$\frac{2\left(bx+a\right)\mathrm{arcoth}\left(bx+a\right)+\log\left(-\left(bx+a\right)^{2}+1\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*x + a)*arccoth(b*x + a) + log(-(b*x + a)^2 + 1))/b
```

Fricas [A] time = 1.54404, size = 135, normalized size = 3.86

$$\frac{bx\log\left(\frac{bx+a+1}{bx+a-1}\right)+\left(a+1\right)\log\left(bx+a+1\right)-\left(a-1\right)\log\left(bx+a-1\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x*log((b*x + a + 1)/(b*x + a - 1)) + (a + 1)*log(b*x + a + 1) - (a -
 1)*log(b*x + a - 1))/b
```

Sympy [A] time = 0.8191, size = 41, normalized size = 1.17

$$\begin{cases}\frac{a\,\mathrm{acoth}\left(a+bx\right)}{b}+x\,\mathrm{acoth}\left(a+bx\right)+\frac{\log\left(a+bx+1\right)}{b}-\frac{\mathrm{acoth}\left(a+bx\right)}{b} & \text{for}\ b\neq 0\\ x\,\mathrm{acoth}\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a),x)
```

```
[Out] Piecewise((a*acoth(a + b*x)/b + x*acoth(a + b*x) + log(a + b*x + 1)/b - aco
th(a + b*x)/b, Ne(b, 0)), (x*acoth(a), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}(bx + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a), x)
```

3.66 $\int \frac{\coth^{-1}(a+bx)}{x}\,dx$

Optimal. Leaf size=92

$$\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2}{a+bx+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2bx}{(1-a)(a+bx+1)}\right)+\log\left(\frac{2}{a+bx+1}\right)\left(-\coth^{-1}(a+bx)\right)+\log$$

```
[Out] -(ArcCoth[a + b*x]*Log[2/(1 + a + b*x)]) + ArcCoth[a + b*x]*Log[(2*b*x)/((1
 - a)*(1 + a + b*x))] + PolyLog[2, 1 - 2/(1 + a + b*x)]/2 - PolyLog[2, 1 -
(2*b*x)/((1 - a)*(1 + a + b*x))]/2
```

Rubi [A] time = 0.0973951, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6112, 5921, 2402, 2315, 2447}

$$\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2}{a+bx+1}\right)-\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2bx}{(1-a)(a+bx+1)}\right)+\log\left(\frac{2}{a+bx+1}\right)\left(-\coth^{-1}(a+bx)\right)+\log$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x]/x,x]
```

```
[Out] -(ArcCoth[a + b*x]*Log[2/(1 + a + b*x)]) + ArcCoth[a + b*x]*Log[(2*b*x)/((1
 - a)*(1 + a + b*x))] + PolyLog[2, 1 - 2/(1 + a + b*x)]/2 - PolyLog[2, 1 -
(2*b*x)/((1 - a)*(1 + a + b*x))]/2
```

Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rule 5921

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcCoth[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcCoth[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
```

```
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\coth^{-1}(a+bx)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\coth^{-1}(x)}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bx\right)}{b}$$

$$= -\coth^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right) + \coth^{-1}(a+bx)\log\left(\frac{2bx}{(1-a)(1+a+bx)}\right) + \text{Subst}\left(\int \frac{\log\left(\right.}{1-}\right.$$

$$= -\coth^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right) + \coth^{-1}(a+bx)\log\left(\frac{2bx}{(1-a)(1+a+bx)}\right) - \frac{1}{2}\text{Li}_2\left(1-\frac{}{(1-a}\right.$$

$$= -\coth^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right) + \coth^{-1}(a+bx)\log\left(\frac{2bx}{(1-a)(1+a+bx)}\right) + \frac{1}{2}\text{Li}_2\left(1-\frac{2}{1+a}\right.$$

Mathematica [C] time = 0.167891, size = 259, normalized size = 2.82

$$\frac{1}{8}\left(-4\text{PolyLog}\left(2, e^{2\tanh^{-1}(a)-2\tanh^{-1}(a+bx)}\right) - 4\text{PolyLog}\left(2, -e^{2\tanh^{-1}(a+bx)}\right) + 4\left(\tanh^{-1}(a)-\tanh^{-1}(a+bx)\right)^2 - (\pi-2\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[a + b*x]/x,x]
```

```
[Out] (ArcCoth[a + b*x] - ArcTanh[a + b*x])*Log[x] + ArcTanh[a + b*x]*(-Log[1/Sqr
t[1 - (a + b*x)^2]] + Log[(-I)*Sinh[ArcTanh[a] - ArcTanh[a + b*x]]]) + (4*(
ArcTanh[a] - ArcTanh[a + b*x])^2 - (Pi - (2*I)*ArcTanh[a + b*x])^2 - 8*(Arc
Tanh[a] - ArcTanh[a + b*x])*Log[1 - E^(2*ArcTanh[a] - 2*ArcTanh[a + b*x])]
- (4*I)*(Pi - (2*I)*ArcTanh[a + b*x])*Log[1 + E^(2*ArcTanh[a + b*x])] + 4*(
I*Pi + 2*ArcTanh[a + b*x])*Log[2/Sqrt[1 - (a + b*x)^2]] + 8*(ArcTanh[a] - A
rcTanh[a + b*x])*Log[(-2*I)*Sinh[ArcTanh[a] - ArcTanh[a + b*x]]] - 4*PolyLo
g[2, E^(2*ArcTanh[a] - 2*ArcTanh[a + b*x])] - 4*PolyLog[2, -E^(2*ArcTanh[a
+ b*x])])/8
```

Maple [A] time = 0.076, size = 81, normalized size = 0.9

$$\ln(bx)\,\text{arccoth}(bx+a) - \frac{1}{2}\text{dilog}\left(\frac{bx+a+1}{1+a}\right) - \frac{\ln(bx)}{2}\ln\left(\frac{bx+a+1}{1+a}\right) + \frac{1}{2}\text{dilog}\left(\frac{bx+a-1}{a-1}\right) + \frac{\ln(bx)}{2}\ln\left(\frac{bx+a-1}{a-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)/x,x)
```

```
[Out] ln(b*x)*arccoth(b*x+a)-1/2*dilog((b*x+a+1)/(1+a))-1/2*ln(b*x)*ln((b*x+a+1)/
(1+a))+1/2*dilog((b*x+a-1)/(a-1))+1/2*ln(b*x)*ln((b*x+a-1)/(a-1))
```

Maxima [A] time = 0.971844, size = 173, normalized size = 1.88

$$-\frac{1}{2}b\left(\frac{\log(bx+a+1)}{b}-\frac{\log(bx+a-1)}{b}\right)\log(x)+\frac{1}{2}b\left(\frac{\log(bx+a+1)\log\left(-\frac{bx+a+1}{a+1}+1\right)+\mathrm{Li}_2\left(\frac{bx+a+1}{a+1}\right)}{b}-\frac{\log(bx+\dots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] -1/2*b*(log(b*x + a + 1)/b - log(b*x + a - 1)/b)*log(x) + 1/2*b*((log(b*x +
 a + 1)*log(-(b*x + a + 1)/(a + 1) + 1) + dilog((b*x + a + 1)/(a + 1)))/b -
 (log(b*x + a - 1)*log(-(b*x + a - 1)/(a - 1) + 1) + dilog((b*x + a - 1)/(a
 - 1)))/b) + arccoth(b*x + a)*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arcoth}(bx+a)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(b*x + a)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{acoth}(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)/x,x)
```

```
[Out] Integral(acoth(a + b*x)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arcoth}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a)/x, x)
```

3.67 $\int \frac{\coth^{-1}(a+bx)}{x^2} \, dx$

Optimal. Leaf size=64

$$\frac{b\log(x)}{1-a^2} - \frac{b\log(-a-bx+1)}{2(1-a)} - \frac{b\log(a+bx+1)}{2(a+1)} - \frac{\coth^{-1}(a+bx)}{x}$$

```
[Out] -(ArcCoth[a + b*x]/x) + (b*Log[x])/(1 - a^2) - (b*Log[1 - a - b*x])/(2*(1 -
 a)) - (b*Log[1 + a + b*x])/(2*(1 + a))
```

Rubi [A] time = 0.0510754, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6110, 371, 706, 31, 633}

$$\frac{b\log(x)}{1-a^2} - \frac{b\log(-a-bx+1)}{2(1-a)} - \frac{b\log(a+bx+1)}{2(a+1)} - \frac{\coth^{-1}(a+bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x]/x^2,x]
```

```
[Out] -(ArcCoth[a + b*x]/x) + (b*Log[x])/(1 - a^2) - (b*Log[1 - a - b*x])/(2*(1 -
 a)) - (b*Log[1 + a + b*x])/(2*(1 + a))
```

Rule 6110

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[((e + f*x)^(m + 1)*(a + b*ArcCoth[c + d*x])^p)/(f*(m
 + 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcCot
h[c + d*x])^(p - 1))/(1 - (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
```

```
-(a*c)]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(a+bx)}{x^2}\,dx &= -\frac{\coth^{-1}(a+bx)}{x}+b\int \frac{1}{x\left(1-(a+bx)^2\right)}\,dx\\ &= -\frac{\coth^{-1}(a+bx)}{x}+b\,\text{Subst}\left(\int \frac{1}{(-a+x)\left(1-x^2\right)}\,dx,x,a+bx\right)\\ &= -\frac{\coth^{-1}(a+bx)}{x}+\frac{b\,\text{Subst}\left(\int \frac{1}{-a+x}\,dx,x,a+bx\right)}{1-a^2}+\frac{b\,\text{Subst}\left(\int \frac{a+x}{1-x^2}\,dx,x,a+bx\right)}{1-a^2}\\ &= -\frac{\coth^{-1}(a+bx)}{x}+\frac{b\log(x)}{1-a^2}+\frac{b\,\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,a+bx\right)}{2(1-a)}+\frac{b\,\text{Subst}\left(\int \frac{1}{-1-x}\,dx,x,a+bx\right)}{2(1+a)}\\ &= -\frac{\coth^{-1}(a+bx)}{x}+\frac{b\log(x)}{1-a^2}-\frac{b\log(1-a-bx)}{2(1-a)}-\frac{b\log(1+a+bx)}{2(1+a)}\end{aligned}$$

Mathematica [A] time = 0.0505785, size = 55, normalized size = 0.86

$$\frac{b((a+1)\log(-a-bx+1)-(a-1)\log(a+bx+1)-2\log(x))}{2\left(a^2-1\right)}-\frac{\coth^{-1}(a+bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a + b*x]/x^2,x]
```

```
[Out] -(ArcCoth[a + b*x]/x) + (b*(-2*Log[x] + (1 + a)*Log[1 - a - b*x] - (-1 + a)
*Log[1 + a + b*x]))/(2*(-1 + a^2))
```

Maple [A] time = 0.037, size = 63, normalized size = 1.

$$-\frac{\operatorname{arccoth}(bx+a)}{x}+\frac{b\ln(bx+a-1)}{2a-2}-\frac{b\ln(bx)}{(1+a)(a-1)}-\frac{b\ln(bx+a+1)}{2+2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)/x^2,x)
```

```
[Out] -arccoth(b*x+a)/x+b/(2*a-2)*ln(b*x+a-1)-b/(a-1)/(1+a)*ln(b*x)-b/(2+2*a)*ln(
b*x+a+1)
```

Maxima [A] time = 0.959147, size = 73, normalized size = 1.14

$$-\frac{1}{2}b\left(\frac{\log(bx+a+1)}{a+1}-\frac{\log(bx+a-1)}{a-1}+\frac{2\log(x)}{a^2-1}\right)-\frac{\operatorname{arcoth}(bx+a)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*(log(b*x + a + 1)/(a + 1) - log(b*x + a - 1)/(a - 1) + 2*log(x)/(a^2
 - 1)) - arccoth(b*x + a)/x
```

Fricas [A] time = 1.67592, size = 192, normalized size = 3.

$$-\frac{(a-1)bx\log(bx+a+1)-(a+1)bx\log(bx+a-1)+2\,bx\log(x)+\left(a^2-1\right)\log\left(\frac{bx+a+1}{bx+a-1}\right)}{2\left(a^2-1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*((a - 1)*b*x*log(b*x + a + 1) - (a + 1)*b*x*log(b*x + a - 1) + 2*b*x*l
og(x) + (a^2 - 1)*log((b*x + a + 1)/(b*x + a - 1)))/((a^2 - 1)*x)
```

Sympy [A] time = 2.8849, size = 144, normalized size = 2.25

$$\begin{cases} \frac{b\operatorname{acoth}(bx-1)}{2}-\frac{\operatorname{acoth}(bx-1)}{x}-\frac{1}{2x} & \text{for}\ a=-1\\ -\frac{b\operatorname{acoth}(bx+1)}{2}-\frac{\operatorname{acoth}(bx+1)}{x}+\frac{1}{2x} & \text{for}\ a=1\\ -\frac{a^2\operatorname{acoth}(a+bx)}{a^2x-x}-\frac{abx\operatorname{acoth}(a+bx)}{a^2x-x}-\frac{bx\log(x)}{a^2x-x}+\frac{bx\log(a+bx+1)}{a^2x-x}-\frac{bx\operatorname{acoth}(a+bx)}{a^2x-x}+\frac{\operatorname{acoth}(a+bx)}{a^2x-x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)/x**2,x)
```

```
[Out] Piecewise((b*acoth(b*x - 1)/2 - acoth(b*x - 1)/x - 1/(2*x), Eq(a, -1)), (-b
*acoth(b*x + 1)/2 - acoth(b*x + 1)/x + 1/(2*x), Eq(a, 1)), (-a**2*acoth(a +
 b*x)/(a**2*x - x) - a*b*x*acoth(a + b*x)/(a**2*x - x) - b*x*log(x)/(a**2*x
 - x) + b*x*log(a + b*x + 1)/(a**2*x - x) - b*x*acoth(a + b*x)/(a**2*x - x)
 + acoth(a + b*x)/(a**2*x - x), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a)/x^2, x)
```

3.68 $\int \frac{\coth^{-1}(a+bx)}{x^3}\,dx$

Optimal. Leaf size=90

$$\frac{ab^2\log(x)}{\left(1-a^2\right)^2}-\frac{b}{2\left(1-a^2\right)x}-\frac{b^2\log(-a-bx+1)}{4(1-a)^2}+\frac{b^2\log(a+bx+1)}{4(a+1)^2}-\frac{\coth^{-1}(a+bx)}{2x^2}$$

```
[Out] -b/(2*(1 - a^2)*x) - ArcCoth[a + b*x]/(2*x^2) + (a*b^2*Log[x])/(1 - a^2)^2
- (b^2*Log[1 - a - b*x])/(4*(1 - a)^2) + (b^2*Log[1 + a + b*x])/(4*(1 + a)^
2)
```

Rubi [A] time = 0.0997384, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6110, 371, 710, 801}

$$\frac{ab^2\log(x)}{\left(1-a^2\right)^2}-\frac{b}{2\left(1-a^2\right)x}-\frac{b^2\log(-a-bx+1)}{4(1-a)^2}+\frac{b^2\log(a+bx+1)}{4(a+1)^2}-\frac{\coth^{-1}(a+bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x]/x^3,x]
```

```
[Out] -b/(2*(1 - a^2)*x) - ArcCoth[a + b*x]/(2*x^2) + (a*b^2*Log[x])/(1 - a^2)^2
- (b^2*Log[1 - a - b*x])/(4*(1 - a)^2) + (b^2*Log[1 + a + b*x])/(4*(1 + a)^
2)
```

Rule 6110

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[((e + f*x)^(m + 1)*(a + b*ArcCoth[c + d*x])^p)/(f*(m
 + 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcCot
h[c + d*x])^(p - 1))/(1 - (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(a+bx)}{x^3}\,dx &= -\frac{\coth^{-1}(a+bx)}{2x^2}+\frac{1}{2}b\int \frac{1}{x^2\left(1-(a+bx)^2\right)}\,dx\\ &= -\frac{\coth^{-1}(a+bx)}{2x^2}+\frac{1}{2}b^2\,\text{Subst}\left(\int \frac{1}{(-a+x)^2\left(1-x^2\right)}\,dx,x,a+bx\right)\\ &= -\frac{b}{2\left(1-a^2\right)x}-\frac{\coth^{-1}(a+bx)}{2x^2}-\frac{b^2\,\text{Subst}\left(\int \frac{-a-x}{(-a+x)\left(1-x^2\right)}\,dx,x,a+bx\right)}{2\left(1-a^2\right)}\\ &= -\frac{b}{2\left(1-a^2\right)x}-\frac{\coth^{-1}(a+bx)}{2x^2}-\frac{b^2\,\text{Subst}\left(\int \left(-\frac{2a}{\left(-1+a^2\right)(a-x)}+\frac{-1-a}{2(-1+a)(-1+x)}+\frac{-1+a}{2(1+a)(1+x)}\right)dx,x,a+bx\right)}{2\left(1-a^2\right)}\\ &= -\frac{b}{2\left(1-a^2\right)x}-\frac{\coth^{-1}(a+bx)}{2x^2}+\frac{ab^2\log(x)}{\left(1-a^2\right)^2}-\frac{b^2\log(1-a-bx)}{4(1-a)^2}+\frac{b^2\log(1+a+bx)}{4(1+a)^2}\end{aligned}$$

Mathematica [A] time = 0.106819, size = 76, normalized size = 0.84

$$\frac{1}{4}\left(b\left(\frac{4ab\log(x)}{\left(a^2-1\right)^2}+\frac{2}{\left(a^2-1\right)x}-\frac{b\log(-a-bx+1)}{(a-1)^2}+\frac{b\log(a+bx+1)}{(a+1)^2}\right)-\frac{2\coth^{-1}(a+bx)}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a + b*x]/x^3,x]
```

```
[Out] ((-2*ArcCoth[a + b*x])/x^2 + b*(2/((-1 + a^2)*x) + (4*a*b*Log[x])/(-1 + a^2
)^2 - (b*Log[1 - a - b*x])/(-1 + a)^2 + (b*Log[1 + a + b*x])/(1 + a)^2))/4
```

Maple [A] time = 0.043, size = 82, normalized size = 0.9

$$-\frac{\text{arccoth}\,(bx+a)}{2\,x^2}-\frac{b^2\ln(bx+a-1)}{4\,(a-1)^2}+\frac{b}{(2\,a-2)\,(1+a)\,x}+\frac{ab^2\ln(bx)}{(a-1)^2\,(1+a)^2}+\frac{b^2\ln(bx+a+1)}{4\,(1+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)/x^3,x)
```

```
[Out] -1/2*arccoth(b*x+a)/x^2-1/4*b^2/(a-1)^2*ln(b*x+a-1)+1/2*b/(a-1)/(1+a)/x+b^2
*a/(a-1)^2/(1+a)^2*ln(b*x)+1/4*b^2*ln(b*x+a+1)/(1+a)^2
```

Maxima [A] time = 0.959889, size = 115, normalized size = 1.28

$$\frac{1}{4}\left(\frac{4\,ab\log(x)}{a^4-2\,a^2+1}+\frac{b\log(bx+a+1)}{a^2+2\,a+1}-\frac{b\log(bx+a-1)}{a^2-2\,a+1}+\frac{2}{\left(a^2-1\right)x}\right)b-\frac{\text{arcoth}\,(bx+a)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*a*b*log(x)/(a^4 - 2*a^2 + 1) + b*log(b*x + a + 1)/(a^2 + 2*a + 1) -
b*log(b*x + a - 1)/(a^2 - 2*a + 1) + 2/((a^2 - 1)*x))*b - 1/2*arccoth(b*x +
 a)/x^2
```

Fricas [A] time = 1.75893, size = 279, normalized size = 3.1

$$\frac{\left(a^2-2a+1\right)b^2x^2\log\left(bx+a+1\right)-\left(a^2+2a+1\right)b^2x^2\log\left(bx+a-1\right)+4ab^2x^2\log\left(x\right)+2\left(a^2-1\right)bx-\left(a^4-2a^2+\right.}{4\left(a^4-2a^2+1\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*((a^2 - 2*a + 1)*b^2*x^2*log(b*x + a + 1) - (a^2 + 2*a + 1)*b^2*x^2*log
(b*x + a - 1) + 4*a*b^2*x^2*log(x) + 2*(a^2 - 1)*b*x - (a^4 - 2*a^2 + 1)*lo
g((b*x + a + 1)/(b*x + a - 1)))/((a^4 - 2*a^2 + 1)*x^2)
```

Sympy [A] time = 3.45566, size = 410, normalized size = 4.56

$$\begin{cases}\frac{b^2\operatorname{acoth}(bx-1)}{8}-\frac{b}{8x}-\frac{\operatorname{acoth}(bx-1)}{2x^2}-\frac{1}{8x^2}\\ \frac{b^2\operatorname{acoth}(bx+1)}{8}-\frac{b}{8x}-\frac{\operatorname{acoth}(bx+1)}{2x^2}+\frac{1}{8x^2}\\ -\frac{a^4\operatorname{acoth}(a+bx)}{2a^4x^2-4a^2x^2+2x^2}+\frac{a^2b^2x^2\operatorname{acoth}(a+bx)}{2a^4x^2-4a^2x^2+2x^2}+\frac{a^2bx}{2a^4x^2-4a^2x^2+2x^2}+\frac{2a^2\operatorname{acoth}(a+bx)}{2a^4x^2-4a^2x^2+2x^2}+\frac{2ab^2x^2\log(x)}{2a^4x^2-4a^2x^2+2x^2}-\frac{2ab^2x^2\log(a+bx+1)}{2a^4x^2-4a^2x^2+2x^2}+\frac{2ab^2x^2\operatorname{acot}}{2a^4x^2-4a^2}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)/x**3,x)
```

```
[Out] Piecewise((b**2*acoth(b*x - 1)/8 - b/(8*x) - acoth(b*x - 1)/(2*x**2) - 1/(8
*x**2), Eq(a, -1)), (b**2*acoth(b*x + 1)/8 - b/(8*x) - acoth(b*x + 1)/(2*x*
*2) + 1/(8*x**2), Eq(a, 1)), (-a**4*acoth(a + b*x)/(2*a**4*x**2 - 4*a**2*x*
*2 + 2*x**2) + a**2*b**2*x**2*acoth(a + b*x)/(2*a**4*x**2 - 4*a**2*x**2 + 2
*x**2) + a**2*b*x/(2*a**4*x**2 - 4*a**2*x**2 + 2*x**2) + 2*a**2*acoth(a + b
*x)/(2*a**4*x**2 - 4*a**2*x**2 + 2*x**2) + 2*a*b**2*x**2*log(x)/(2*a**4*x**
2 - 4*a**2*x**2 + 2*x**2) - 2*a*b**2*x**2*log(a + b*x + 1)/(2*a**4*x**2 - 4
*a**2*x**2 + 2*x**2) + 2*a*b**2*x**2*acoth(a + b*x)/(2*a**4*x**2 - 4*a**2*x
**2 + 2*x**2) + b**2*x**2*acoth(a + b*x)/(2*a**4*x**2 - 4*a**2*x**2 + 2*x**
2) - b*x/(2*a**4*x**2 - 4*a**2*x**2 + 2*x**2) - acoth(a + b*x)/(2*a**4*x**2
 - 4*a**2*x**2 + 2*x**2), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(bx+a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a)/x^3, x)
```

3.69 $\int x^3 \coth^{-1}(a+bx)^2 \, dx$

Optimal. Leaf size=263

$$\frac{a\left(a^2+1\right)\text{PolyLog}\left(2,-\frac{a+bx+1}{-a-bx+1}\right)}{b^4}+\frac{\left(6a^2+1\right)\log\left(1-(a+bx)^2\right)}{4b^4}-\frac{a\left(a^2+1\right)\coth^{-1}(a+bx)^2}{b^4}-\frac{\left(a^4+6a^2+1\right)\coth^{-1}(a}{4b^4}$$

```
[Out] -((a*x)/b^3) + (a + b*x)^2/(12*b^4) + ((1 + 6*a^2)*(a + b*x)*ArcCoth[a + b*
x])/(2*b^4) - (a*(a + b*x)^2*ArcCoth[a + b*x])/b^4 + ((a + b*x)^3*ArcCoth[a
 + b*x])/(6*b^4) - (a*(1 + a^2)*ArcCoth[a + b*x]^2)/b^4 - ((1 + 6*a^2 + a^4
)*ArcCoth[a + b*x]^2)/(4*b^4) + (x^4*ArcCoth[a + b*x]^2)/4 + (a*ArcTanh[a +
 b*x])/b^4 + (2*a*(1 + a^2)*ArcCoth[a + b*x]*Log[2/(1 - a - b*x)])/b^4 + Lo
g[1 - (a + b*x)^2]/(12*b^4) + ((1 + 6*a^2)*Log[1 - (a + b*x)^2])/(4*b^4) +
(a*(1 + a^2)*PolyLog[2, -((1 + a + b*x)/(1 - a - b*x))])/b^4
```

Rubi [A] time = 0.348779, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 15, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.25, Rules used = {6112, 5929, 5911, 260, 5917, 321, 206, 266, 43, 6049, 5949, 5985, 5919, 2402, 2315}

$$\frac{a\left(a^2+1\right)\text{PolyLog}\left(2,-\frac{a+bx+1}{-a-bx+1}\right)}{b^4}+\frac{\left(6a^2+1\right)\log\left(1-(a+bx)^2\right)}{4b^4}-\frac{a\left(a^2+1\right)\coth^{-1}(a+bx)^2}{b^4}-\frac{\left(a^4+6a^2+1\right)\coth^{-1}(a}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCoth[a + b*x]^2,x]
```

```
[Out] -((a*x)/b^3) + (a + b*x)^2/(12*b^4) + ((1 + 6*a^2)*(a + b*x)*ArcCoth[a + b*
x])/(2*b^4) - (a*(a + b*x)^2*ArcCoth[a + b*x])/b^4 + ((a + b*x)^3*ArcCoth[a
 + b*x])/(6*b^4) - (a*(1 + a^2)*ArcCoth[a + b*x]^2)/b^4 - ((1 + 6*a^2 + a^4
)*ArcCoth[a + b*x]^2)/(4*b^4) + (x^4*ArcCoth[a + b*x]^2)/4 + (a*ArcTanh[a +
 b*x])/b^4 + (2*a*(1 + a^2)*ArcCoth[a + b*x]*Log[2/(1 - a - b*x)])/b^4 + Lo
g[1 - (a + b*x)^2]/(12*b^4) + ((1 + 6*a^2)*Log[1 - (a + b*x)^2])/(4*b^4) +
(a*(1 + a^2)*PolyLog[2, -((1 + a + b*x)/(1 - a - b*x))])/b^4
```

Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rule 5929

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcCoth[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcCoth[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6049

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCoth[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5985

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int x^3 \coth^{-1}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^3 \coth^{-1}(x)^2\,dx, x, a+bx\right)}{b}$$

$$= \frac{1}{4}x^4\coth^{-1}(a+bx)^2 - \frac{1}{2}\text{Subst}\left(\int \left(-\frac{\left(1+6a^2\right)\coth^{-1}(x)}{b^4} + \frac{4ax\coth^{-1}(x)}{b^4} - \frac{x^2\coth^{-1}(x)}{b^4}\right.\right.$$

$$= \frac{1}{4}x^4\coth^{-1}(a+bx)^2 + \frac{\text{Subst}\left(\int x^2\coth^{-1}(x)\,dx, x, a+bx\right)}{2b^4} - \frac{\text{Subst}\left(\int \frac{\left(1+6a^2+a^4-4a\left(1+a^2\right)x\right)\text{co}}{1-x^2}\right.}{2b^4}$$

$$= \frac{\left(1+6a^2\right)(a+bx)\coth^{-1}(a+bx)}{2b^4} - \frac{a(a+bx)^2\coth^{-1}(a+bx)}{b^4} + \frac{(a+bx)^3\coth^{-1}(a+bx)}{6b^4} +$$

$$= -\frac{ax}{b^3} + \frac{\left(1+6a^2\right)(a+bx)\coth^{-1}(a+bx)}{2b^4} - \frac{a(a+bx)^2\coth^{-1}(a+bx)}{b^4} + \frac{(a+bx)^3\coth^{-1}(a+}{6b^4}$$

$$= -\frac{ax}{b^3} + \frac{\left(1+6a^2\right)(a+bx)\coth^{-1}(a+bx)}{2b^4} - \frac{a(a+bx)^2\coth^{-1}(a+bx)}{b^4} + \frac{(a+bx)^3\coth^{-1}(a+}{6b^4}$$

$$= -\frac{ax}{b^3} + \frac{(a+bx)^2}{12b^4} + \frac{\left(1+6a^2\right)(a+bx)\coth^{-1}(a+bx)}{2b^4} - \frac{a(a+bx)^2\coth^{-1}(a+bx)}{b^4} + \frac{(a+bx)}{}$$

$$= -\frac{ax}{b^3} + \frac{(a+bx)^2}{12b^4} + \frac{\left(1+6a^2\right)(a+bx)\coth^{-1}(a+bx)}{2b^4} - \frac{a(a+bx)^2\coth^{-1}(a+bx)}{b^4} + \frac{(a+bx)}{}$$

$$= -\frac{ax}{b^3} + \frac{(a+bx)^2}{12b^4} + \frac{\left(1+6a^2\right)(a+bx)\coth^{-1}(a+bx)}{2b^4} - \frac{a(a+bx)^2\coth^{-1}(a+bx)}{b^4} + \frac{(a+bx)}{}$$

Mathematica [A] time = 1.56441, size = 203, normalized size = 0.77

$$-\frac{12\left(a^3+a\right)\text{PolyLog}\left(2, e^{-2\coth^{-1}(a+bx)}\right) + 3\left(a^4-4a^3+6a^2-4a-b^4x^4+1\right)\coth^{-1}(a+bx)^2 - 2\coth^{-1}(a+bx)\left(9a^2bx\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*ArcCoth[a + b*x]^2,x]
```

```
[Out] -(1 + 11*a^2 + 10*a*b*x - b^2*x^2 + 3*(1 - 4*a + 6*a^2 - 4*a^3 + a^4 - b^4*
x^4)*ArcCoth[a + b*x]^2 - 2*ArcCoth[a + b*x]*(9*a + 13*a^3 + 3*b*x + 9*a^2*
b*x - 3*a*b^2*x^2 + b^3*x^3 + 12*(a + a^3)*Log[1 - E^(-2*ArcCoth[a + b*x])]
) + 8*Log[1/((a + b*x)*Sqrt[1 - (a + b*x)^(-2)])] + 36*a^2*Log[1/((a + b*x)
*Sqrt[1 - (a + b*x)^(-2)])] + 12*(a + a^3)*PolyLog[2, E^(-2*ArcCoth[a + b*x
])])/(12*b^4)
```

Maple [B] time = 0.056, size = 967, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccoth(b*x+a)^2,x)
```

```
[Out] 1/4/b^4*arccoth(b*x+a)*ln(b*x+a-1)*a^4+1/8/b^4*ln(-1/2*b*x-1/2*a+1/2)*ln(1/
2+1/2*b*x+1/2*a)*a^4+1/2/b^4*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2+1/2*b*x+1/2*a)*a
^3-1/b^4*arccoth(b*x+a)*ln(b*x+a+1)*a-1/b^4*arccoth(b*x+a)*ln(b*x+a+1)*a^3-
1/4/b^4*arccoth(b*x+a)*ln(b*x+a+1)*a^4-11/12/b^4*a^2-1/2/b^2*arccoth(b*x+a)
*x^2*a+1/2/b^4*ln(1/2+1/2*b*x+1/2*a)*ln(b*x+a-1)*a^3+3/2/b^4*arccoth(b*x+a)
*ln(b*x+a-1)*a^2-1/8/b^4*ln(1/2+1/2*b*x+1/2*a)*ln(b*x+a-1)*a^4+3/2/b^3*arcc
oth(b*x+a)*x*a^2-1/b^4*arccoth(b*x+a)*ln(b*x+a-1)*a-1/2/b^4*ln(-1/2*b*x-1/2
*a+1/2)*ln(b*x+a+1)*a-1/b^4*arccoth(b*x+a)*ln(b*x+a-1)*a^3-3/2/b^4*arccoth(
b*x+a)*ln(b*x+a+1)*a^2-3/4/b^4*ln(1/2+1/2*b*x+1/2*a)*ln(b*x+a-1)*a^2-1/2/b^
4*ln(-1/2*b*x-1/2*a+1/2)*ln(b*x+a+1)*a^3+3/2/b^4*ln(b*x+a-1)*a^2-1/2/b^4*ln
(b*x+a-1)*a+3/2/b^4*ln(b*x+a+1)*a^2+1/2/b^4*ln(b*x+a+1)*a-5/6*a*x/b^3+1/2/b
^4*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2+1/2*b*x+1/2*a)*a-1/8/b^4*ln(-1/2*b*x-1/2*a
+1/2)*ln(b*x+a+1)*a^4+3/4/b^4*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2+1/2*b*x+1/2*a)*
a^2+1/2/b^4*ln(1/2+1/2*b*x+1/2*a)*ln(b*x+a-1)*a-3/4/b^4*ln(-1/2*b*x-1/2*a+1
/2)*ln(b*x+a+1)*a^2+1/4*x^4*arccoth(b*x+a)^2+1/b^4*dilog(1/2+1/2*b*x+1/2*a)
*a+1/2/b^4*arccoth(b*x+a)*a+1/4/b^4*arccoth(b*x+a)*ln(b*x+a-1)+1/2/b^3*arcc
oth(b*x+a)*x+3/8/b^4*ln(b*x+a-1)^2*a^2-1/4/b^4*ln(b*x+a-1)^2*a+1/16/b^4*ln(
b*x+a+1)^2*a^4+3/8/b^4*ln(b*x+a+1)^2*a^2+1/4/b^4*ln(b*x+a+1)^2*a+1/16/b^4*l
n(b*x+a-1)^2*a^4-1/4/b^4*ln(b*x+a-1)^2*a^3+1/b^4*dilog(1/2+1/2*b*x+1/2*a)*a
^3-1/8/b^4*ln(-1/2*b*x-1/2*a+1/2)*ln(b*x+a+1)-1/4/b^4*arccoth(b*x+a)*ln(b*x
+a+1)-1/8/b^4*ln(b*x+a-1)*ln(1/2+1/2*b*x+1/2*a)+13/6/b^4*arccoth(b*x+a)*a^3
+1/6/b*arccoth(b*x+a)*x^3+1/4/b^4*ln(b*x+a+1)^2*a^3+1/8/b^4*ln(-1/2*b*x-1/2
*a+1/2)*ln(1/2+1/2*b*x+1/2*a)+1/16/b^4*ln(b*x+a+1)^2+1/16/b^4*ln(b*x+a-1)^2
+1/3/b^4*ln(b*x+a+1)+1/3/b^4*ln(b*x+a-1)+1/12*x^2/b^2
```

Maxima [A] time = 0.988167, size = 432, normalized size = 1.64

$$\frac{1}{4} x^4 \operatorname{arcoth}(bx+a)^2 + \frac{1}{48} b^2 \left(\frac{48\left(a^3+a\right)\left(\log(bx+a-1)\log\left(\frac{1}{2} bx+\frac{1}{2} a+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2} bx-\frac{1}{2} a+\frac{1}{2}\right)\right)}{b^6} + \frac{4\left(13 a^3 + \right.}{} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccoth(b*x + a)^2 + 1/48*b^2*(48*(a^3 + a)*(log(b*x + a - 1)*log(1
/2*b*x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))/b^6 + 4*(13*a^3 + 18
```

```
*a^2 + 9*a + 4)*log(b*x + a + 1)/b^6 + (4*b^2*x^2 - 40*a*b*x + 3*(a^4 + 4*a
^3 + 6*a^2 + 4*a + 1)*log(b*x + a + 1)^2 - 6*(a^4 + 4*a^3 + 6*a^2 + 4*a + 1
)*log(b*x + a + 1)*log(b*x + a - 1) + 3*(a^4 - 4*a^3 + 6*a^2 - 4*a + 1)*log
(b*x + a - 1)^2 - 4*(13*a^3 - 18*a^2 + 9*a - 4)*log(b*x + a - 1))/b^6) + 1/
12*b*(2*(b^2*x^3 - 3*a*b*x^2 + 3*(3*a^2 + 1)*x)/b^4 - 3*(a^4 + 4*a^3 + 6*a^
2 + 4*a + 1)*log(b*x + a + 1)/b^5 + 3*(a^4 - 4*a^3 + 6*a^2 - 4*a + 1)*log(b
*x + a - 1)/b^5)*arccoth(b*x + a)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^3\,\text{arcoth}\,(bx+a)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*arccoth(b*x + a)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\,\text{acoth}^2\,(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acoth(b*x+a)**2,x)
```

```
[Out] Integral(x**3*acoth(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\,\text{arcoth}\,(bx+a)^2\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*arccoth(b*x + a)^2, x)
```

3.70 $\int x^2 \coth^{-1}(a+bx)^2 \, dx$

Optimal. Leaf size=204

$$-\frac{(3a^2+1)\,\text{PolyLog}\left(2,-\frac{a+bx+1}{-a-bx+1}\right)}{3b^3}+\frac{a\left(a^2+3\right)\coth^{-1}(a+bx)^2}{3b^3}+\frac{(3a^2+1)\coth^{-1}(a+bx)^2}{3b^3}-\frac{2\left(3a^2+1\right)\log\left(\frac{2}{-a-bx}\right.}{3b}$$

```
[Out] x/(3*b^2) - (2*a*(a + b*x)*ArcCoth[a + b*x])/b^3 + ((a + b*x)^2*ArcCoth[a +
 b*x])/(3*b^3) + (a*(3 + a^2)*ArcCoth[a + b*x]^2)/(3*b^3) + ((1 + 3*a^2)*Ar
cCoth[a + b*x]^2)/(3*b^3) + (x^3*ArcCoth[a + b*x]^2)/3 - ArcTanh[a + b*x]/(
3*b^3) - (2*(1 + 3*a^2)*ArcCoth[a + b*x]*Log[2/(1 - a - b*x)])/(3*b^3) - (a
*Log[1 - (a + b*x)^2])/b^3 - ((1 + 3*a^2)*PolyLog[2, -((1 + a + b*x)/(1 - a
 - b*x))])/(3*b^3)
```

Rubi [A] time = 0.278272, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.083, Rules used = {6112, 5929, 5911, 260, 5917, 321, 206, 6049, 5949, 5985, 5919, 2402, 2315}

$$-\frac{(3a^2+1)\,\text{PolyLog}\left(2,-\frac{a+bx+1}{-a-bx+1}\right)}{3b^3}+\frac{a\left(a^2+3\right)\coth^{-1}(a+bx)^2}{3b^3}+\frac{(3a^2+1)\coth^{-1}(a+bx)^2}{3b^3}-\frac{2\left(3a^2+1\right)\log\left(\frac{2}{-a-bx}\right.}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[a + b*x]^2,x]
```


```
[Out] x/(3*b^2) - (2*a*(a + b*x)*ArcCoth[a + b*x])/b^3 + ((a + b*x)^2*ArcCoth[a +
 b*x])/(3*b^3) + (a*(3 + a^2)*ArcCoth[a + b*x]^2)/(3*b^3) + ((1 + 3*a^2)*Ar
cCoth[a + b*x]^2)/(3*b^3) + (x^3*ArcCoth[a + b*x]^2)/3 - ArcTanh[a + b*x]/(
3*b^3) - (2*(1 + 3*a^2)*ArcCoth[a + b*x]*Log[2/(1 - a - b*x)])/(3*b^3) - (a
*Log[1 - (a + b*x)^2])/b^3 - ((1 + 3*a^2)*PolyLog[2, -((1 + a + b*x)/(1 - a
 - b*x))])/(3*b^3)
```


Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rule 5929

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcCoth[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcCoth[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6049

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCoth[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5985

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int x^2 \coth^{-1}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^2 \coth^{-1}(x)^2\,dx, x, a+bx\right)}{b}$$

$$= \frac{1}{3}x^3\coth^{-1}(a+bx)^2 - \frac{2}{3}\text{Subst}\left(\int \left(\frac{3a\coth^{-1}(x)}{b^3} - \frac{x\coth^{-1}(x)}{b^3} - \frac{\left(a\left(3+a^2\right)-\left(1+3a^2\right.}{b^3\left(1-x^2\right.}\right.\right.$$

$$= \frac{1}{3}x^3\coth^{-1}(a+bx)^2 + \frac{2\,\text{Subst}\left(\int x\coth^{-1}(x)\,dx, x, a+bx\right)}{3b^3} + \frac{2\,\text{Subst}\left(\int \frac{\left(a\left(3+a^2\right)-\left(1+3a^2\right)\right.}{1-x^2}\right.}{3b^3}$$

$$= -\frac{2a(a+bx)\coth^{-1}(a+bx)}{b^3} + \frac{(a+bx)^2\coth^{-1}(a+bx)}{3b^3} + \frac{1}{3}x^3\coth^{-1}(a+bx)^2 - \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{x}{3b^2} - \frac{2a(a+bx)\coth^{-1}(a+bx)}{b^3} + \frac{(a+bx)^2\coth^{-1}(a+bx)}{3b^3} + \frac{1}{3}x^3\coth^{-1}(a+bx)^2 - \frac{a\,\mathrm{lo}}{}$$

$$= \frac{x}{3b^2} - \frac{2a(a+bx)\coth^{-1}(a+bx)}{b^3} + \frac{(a+bx)^2\coth^{-1}(a+bx)}{3b^3} + \frac{a\left(3+a^2\right)\coth^{-1}(a+bx)^2}{3b^3}$$

$$= \frac{x}{3b^2} - \frac{2a(a+bx)\coth^{-1}(a+bx)}{b^3} + \frac{(a+bx)^2\coth^{-1}(a+bx)}{3b^3} + \frac{a\left(3+a^2\right)\coth^{-1}(a+bx)^2}{3b^3}$$

$$= \frac{x}{3b^2} - \frac{2a(a+bx)\coth^{-1}(a+bx)}{b^3} + \frac{(a+bx)^2\coth^{-1}(a+bx)}{3b^3} + \frac{a\left(3+a^2\right)\coth^{-1}(a+bx)^2}{3b^3}$$

$$= \frac{x}{3b^2} - \frac{2a(a+bx)\coth^{-1}(a+bx)}{b^3} + \frac{(a+bx)^2\coth^{-1}(a+bx)}{3b^3} + \frac{a\left(3+a^2\right)\coth^{-1}(a+bx)^2}{3b^3}$$

Mathematica [B] time = 4.52669, size = 607, normalized size = 2.98

$$-\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\left(1-(a+bx)^2\right)\left(\frac{4\left(3a^2+1\right)\text{PolyLog}\left(2,e^{-2\coth^{-1}(a+bx)}\right)}{(a+bx)^3\left(1-\frac{1}{(a+bx)^2}\right)^{3/2}} + \frac{9a^2\coth^{-1}(a+bx)^2}{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}} + \frac{-3\left(a^2-1\right)\coth^{-1}(a+bx)^2+6a\coth^{-1}(a}{\sqrt{1-\frac{1}{(a+bx)^2}}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*ArcCoth[a + b*x]^2,x]
```

```
[Out] -((a + b*x)*Sqrt[1 - (a + b*x)^(-2)]*(1 - (a + b*x)^2)*((4*ArcCoth[a + b*x]
)/((a + b*x)*Sqrt[1 - (a + b*x)^(-2)]) + (3*ArcCoth[a + b*x]^2)/((a + b*x)*
Sqrt[1 - (a + b*x)^(-2)]) - (12*a*ArcCoth[a + b*x]^2)/((a + b*x)*Sqrt[1 - (
a + b*x)^(-2)]) + (9*a^2*ArcCoth[a + b*x]^2)/((a + b*x)*Sqrt[1 - (a + b*x)^
(-2)]) + (-1 + 6*a*ArcCoth[a + b*x] - 3*(-1 + a^2)*ArcCoth[a + b*x]^2)/Sqrt
[1 - (a + b*x)^(-2)] + Cosh[3*ArcCoth[a + b*x]] - 6*a*ArcCoth[a + b*x]*Cosh
[3*ArcCoth[a + b*x]] + ArcCoth[a + b*x]^2*Cosh[3*ArcCoth[a + b*x]] + 3*a^2*
ArcCoth[a + b*x]^2*Cosh[3*ArcCoth[a + b*x]] + (6*ArcCoth[a + b*x]*Log[1 - E
^(-2*ArcCoth[a + b*x])])/((a + b*x)*Sqrt[1 - (a + b*x)^(-2)]) + (18*a^2*Arc
```

```
Coth[a + b*x]*Log[1 - E^(-2*ArcCoth[a + b*x])])/((a + b*x)*Sqrt[1 - (a + b*
x)^(-2)]) - (18*a*Log[1/((a + b*x)*Sqrt[1 - (a + b*x)^(-2)])])/((a + b*x)*S
qrt[1 - (a + b*x)^(-2)]) + (4*(1 + 3*a^2)*PolyLog[2, E^(-2*ArcCoth[a + b*x]
)])/((a + b*x)^3*(1 - (a + b*x)^(-2))^(3/2)) - ArcCoth[a + b*x]^2*Sinh[3*Ar
cCoth[a + b*x]] - 3*a^2*ArcCoth[a + b*x]^2*Sinh[3*ArcCoth[a + b*x]] - 2*Arc
Coth[a + b*x]*Log[1 - E^(-2*ArcCoth[a + b*x])]*Sinh[3*ArcCoth[a + b*x]] - 6
*a^2*ArcCoth[a + b*x]*Log[1 - E^(-2*ArcCoth[a + b*x])]*Sinh[3*ArcCoth[a + b
*x]] + 6*a*Log[1/((a + b*x)*Sqrt[1 - (a + b*x)^(-2)])]*Sinh[3*ArcCoth[a + b
*x]]))/(12*b^3)
```

Maple [B] time = 0.055, size = 729, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(b*x+a)^2,x)
```

```
[Out] -1/b^3*ln(b*x+a-1)*a-1/b^3*ln(b*x+a+1)*a+1/3*x^3*arccoth(b*x+a)^2+1/6/b^3*l
n(-1/2*b*x-1/2*a+1/2)*ln(b*x+a+1)+1/3/b^3*arccoth(b*x+a)*ln(b*x+a+1)-1/6/b^
3*ln(b*x+a-1)*ln(1/2+1/2*b*x+1/2*a)-1/b^3*dilog(1/2+1/2*b*x+1/2*a)*a^2-1/12
/b^3*ln(b*x+a+1)^2*a^3-1/4/b^3*ln(b*x+a+1)^2*a^2-1/4/b^3*ln(b*x+a+1)^2*a-1/
12/b^3*ln(b*x+a-1)^2*a^3+1/4/b^3*ln(b*x+a-1)^2*a^2-1/4/b^3*ln(b*x+a-1)^2*a-
5/3/b^3*arccoth(b*x+a)*a^2+1/3/b*arccoth(b*x+a)*x^2+1/3/b^3*arccoth(b*x+a)*
ln(b*x+a-1)-1/6/b^3*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2+1/2*b*x+1/2*a)-1/2/b^3*ln
(1/2+1/2*b*x+1/2*a)*ln(b*x+a-1)*a^2+1/6/b^3*ln(1/2+1/2*b*x+1/2*a)*ln(b*x+a-
1)*a^3+1/3/b^3*arccoth(b*x+a)*ln(b*x+a+1)*a^3-1/6/b^3*ln(-1/2*b*x-1/2*a+1/2
)*ln(1/2+1/2*b*x+1/2*a)*a^3-1/2/b^3*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2+1/2*b*x+1
/2*a)*a^2-1/b^3*arccoth(b*x+a)*ln(b*x+a-1)*a-4/3/b^2*arccoth(b*x+a)*x*a-1/2
/b^3*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2+1/2*b*x+1/2*a)*a+1/6/b^3*ln(-1/2*b*x-1/2
*a+1/2)*ln(b*x+a+1)*a^3+1/2/b^3*ln(1/2+1/2*b*x+1/2*a)*ln(b*x+a-1)*a+1/2/b^3
*ln(-1/2*b*x-1/2*a+1/2)*ln(b*x+a+1)*a^2+1/2/b^3*ln(-1/2*b*x-1/2*a+1/2)*ln(b
*x+a+1)*a+1/b^3*arccoth(b*x+a)*ln(b*x+a+1)*a^2+1/b^3*arccoth(b*x+a)*ln(b*x+
a+1)*a-1/3/b^3*arccoth(b*x+a)*ln(b*x+a-1)*a^3+1/b^3*arccoth(b*x+a)*ln(b*x+a
-1)*a^2+1/6/b^3*ln(b*x+a-1)-1/6/b^3*ln(b*x+a+1)-1/12/b^3*ln(b*x+a+1)^2-1/3/
b^3*dilog(1/2+1/2*b*x+1/2*a)+1/3/b^3*a+1/12/b^3*ln(b*x+a-1)^2+1/3*x/b^2
```

Maxima [A] time = 1.00212, size = 350, normalized size = 1.72

$$\frac{1}{3}x^3\operatorname{arcoth}(bx+a)^2 - \frac{1}{12}b^2\left(\frac{4\left(3a^2+1\right)\left(\log(bx+a-1)\log\left(\frac{1}{2}bx+\frac{1}{2}a+\frac{1}{2}\right)+\operatorname{Li}_2\left(-\frac{1}{2}bx-\frac{1}{2}a+\frac{1}{2}\right)\right)}{b^5}+\frac{2\left(5a^2+6a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(b*x + a)^2 - 1/12*b^2*(4*(3*a^2 + 1)*(log(b*x + a - 1)*log(
1/2*b*x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))/b^5 + 2*(5*a^2 + 6*
a + 1)*log(b*x + a + 1)/b^5 + ((a^3 + 3*a^2 + 3*a + 1)*log(b*x + a + 1)^2 -
 2*(a^3 + 3*a^2 + 3*a + 1)*log(b*x + a + 1)*log(b*x + a - 1) + (a^3 - 3*a^2
 + 3*a - 1)*log(b*x + a - 1)^2 - 4*b*x - 2*(5*a^2 - 6*a + 1)*log(b*x + a -
1))/b^5) + 1/3*b*((b*x^2 - 4*a*x)/b^3 + (a^3 + 3*a^2 + 3*a + 1)*log(b*x + a
 + 1)/b^4 - (a^3 - 3*a^2 + 3*a - 1)*log(b*x + a - 1)/b^4)*arccoth(b*x + a)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2 \operatorname{arcoth}\left(bx+a\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*arccoth(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arcoth}\left(bx+a\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(b*x + a)^2, x)
```

3.71 $\int x \coth^{-1}(a+bx)^2 \, dx$

Optimal. Leaf size=136

$$\frac{a\text{PolyLog}\left(2,-\frac{a+bx+1}{-a-bx+1}\right)}{b^2}-\frac{\left(a^2+1\right)\coth^{-1}(a+bx)^2}{2b^2}+\frac{\log\left(1-(a+bx)^2\right)}{2b^2}-\frac{a\coth^{-1}(a+bx)^2}{b^2}+\frac{(a+bx)\coth^{-1}(a+bx}{b^2}$$

```
[Out] ((a + b*x)*ArcCoth[a + b*x])/b^2 - (a*ArcCoth[a + b*x]^2)/b^2 - ((1 + a^2)*
ArcCoth[a + b*x]^2)/(2*b^2) + (x^2*ArcCoth[a + b*x]^2)/2 + (2*a*ArcCoth[a +
 b*x]*Log[2/(1 - a - b*x)])/b^2 + Log[1 - (a + b*x)^2]/(2*b^2) + (a*PolyLog
[2, -((1 + a + b*x)/(1 - a - b*x))])/b^2
```

Rubi [A] time = 0.209095, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6112, 5929, 5911, 260, 6049, 5949, 5985, 5919, 2402, 2315}

$$\frac{a\text{PolyLog}\left(2,-\frac{a+bx+1}{-a-bx+1}\right)}{b^2}-\frac{\left(a^2+1\right)\coth^{-1}(a+bx)^2}{2b^2}+\frac{\log\left(1-(a+bx)^2\right)}{2b^2}-\frac{a\coth^{-1}(a+bx)^2}{b^2}+\frac{(a+bx)\coth^{-1}(a+bx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[a + b*x]^2,x]
```

```
[Out] ((a + b*x)*ArcCoth[a + b*x])/b^2 - (a*ArcCoth[a + b*x]^2)/b^2 - ((1 + a^2)*
ArcCoth[a + b*x]^2)/(2*b^2) + (x^2*ArcCoth[a + b*x]^2)/2 + (2*a*ArcCoth[a +
 b*x]*Log[2/(1 - a - b*x)])/b^2 + Log[1 - (a + b*x)^2]/(2*b^2) + (a*PolyLog
[2, -((1 + a + b*x)/(1 - a - b*x))])/b^2
```

Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rule 5929

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcCoth[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcCoth[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6049

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCoth[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5985

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int x \coth^{-1}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)\coth^{-1}(x)^2\,dx,x,a+bx\right)}{b}$$
$$= \frac{1}{2}x^2\coth^{-1}(a+bx)^2 - \text{Subst}\left(\int\left(-\frac{\coth^{-1}(x)}{b^2}+\frac{\left(1+a^2-2ax\right)\coth^{-1}(x)}{b^2\left(1-x^2\right)}\right)dx,x,a+bx\right)$$
$$= \frac{1}{2}x^2\coth^{-1}(a+bx)^2 + \frac{\text{Subst}\left(\int\coth^{-1}(x)\,dx,x,a+bx\right)}{b^2} - \frac{\text{Subst}\left(\int\frac{\left(1+a^2-2ax\right)\coth^{-1}(x)}{1-x^2}\,dx,x,\right.}{b^2}$$
$$= \frac{(a+bx)\coth^{-1}(a+bx)}{b^2} + \frac{1}{2}x^2\coth^{-1}(a+bx)^2 - \frac{\text{Subst}\left(\int\frac{x}{1-x^2}\,dx,x,a+bx\right)}{b^2} - \frac{\text{Subst}\left(\int\left(\frac{(1}{}\right.\right.}{}$$
$$= \frac{(a+bx)\coth^{-1}(a+bx)}{b^2} + \frac{1}{2}x^2\coth^{-1}(a+bx)^2 + \frac{\log\left(1-(a+bx)^2\right)}{2b^2} + \frac{(2a)\,\text{Subst}\left(\int\frac{x\coth^{-1}(}{1-x^2}\right.}{b^2}$$
$$= \frac{(a+bx)\coth^{-1}(a+bx)}{b^2} - \frac{a\coth^{-1}(a+bx)^2}{b^2} - \frac{\left(1+a^2\right)\coth^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\coth^{-1}(a+bx$$
$$= \frac{(a+bx)\coth^{-1}(a+bx)}{b^2} - \frac{a\coth^{-1}(a+bx)^2}{b^2} - \frac{\left(1+a^2\right)\coth^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\coth^{-1}(a+bx$$
$$= \frac{(a+bx)\coth^{-1}(a+bx)}{b^2} - \frac{a\coth^{-1}(a+bx)^2}{b^2} - \frac{\left(1+a^2\right)\coth^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\coth^{-1}(a+bx$$
$$= \frac{(a+bx)\coth^{-1}(a+bx)}{b^2} - \frac{a\coth^{-1}(a+bx)^2}{b^2} - \frac{\left(1+a^2\right)\coth^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\coth^{-1}(a+bx$$

Mathematica [A] time = 0.246845, size = 106, normalized size = 0.78

$$\frac{-2a\text{PolyLog}\left(2,e^{-2\coth^{-1}(a+bx)}\right)+\left(-a^2+2a+b^2x^2-1\right)\coth^{-1}(a+bx)^2-2\log\left(\frac{1}{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}\right)+2\coth^{-1}(a+bx)\left(2\right.}{2b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*ArcCoth[a + b*x]^2,x]
```

```
[Out] ((-1 + 2*a - a^2 + b^2*x^2)*ArcCoth[a + b*x]^2 + 2*ArcCoth[a + b*x]*(a + b*
x + 2*a*Log[1 - E^(-2*ArcCoth[a + b*x])]) - 2*Log[1/((a + b*x)*Sqrt[1 - (a
+ b*x)^(-2)])] - 2*a*PolyLog[2, E^(-2*ArcCoth[a + b*x])])/(2*b^2)
```

Maple [B] time = 0.053, size = 365, normalized size = 2.7

$$\frac{x^2\left(\text{arccoth}\,(bx+a)\right)^2}{2} - \frac{\left(\text{arccoth}\,(bx+a)\right)^2a^2}{2\,b^2} + \frac{x\text{arccoth}\,(bx+a)}{b} + \frac{\text{arccoth}\,(bx+a)\,a}{b^2} - \frac{\text{arccoth}\,(bx+a)\ln(bx+a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(b*x+a)^2,x)
```

```
[Out] 1/2*x^2*arccoth(b*x+a)^2-1/2/b^2*arccoth(b*x+a)^2*a^2+1/b*arccoth(b*x+a)*x+
1/b^2*arccoth(b*x+a)*a-1/b^2*arccoth(b*x+a)*ln(b*x+a-1)*a+1/2/b^2*arccoth(b
```

```
*x+a)*ln(b*x+a-1)-1/b^2*arccoth(b*x+a)*ln(b*x+a+1)*a-1/2/b^2*arccoth(b*x+a)
*ln(b*x+a+1)+1/2/b^2*ln(b*x+a-1)+1/2/b^2*ln(b*x+a+1)-1/4/b^2*ln(b*x+a-1)^2*
a+1/b^2*dilog(1/2+1/2*b*x+1/2*a)*a+1/2/b^2*ln(1/2+1/2*b*x+1/2*a)*ln(b*x+a-1
)*a+1/8/b^2*ln(b*x+a-1)^2-1/4/b^2*ln(b*x+a-1)*ln(1/2+1/2*b*x+1/2*a)+1/2/b^2
*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2+1/2*b*x+1/2*a)*a-1/2/b^2*ln(-1/2*b*x-1/2*a+1
/2)*ln(b*x+a+1)*a+1/4/b^2*ln(b*x+a+1)^2*a+1/4/b^2*ln(-1/2*b*x-1/2*a+1/2)*ln
(1/2+1/2*b*x+1/2*a)-1/4/b^2*ln(-1/2*b*x-1/2*a+1/2)*ln(b*x+a+1)+1/8/b^2*ln(b
*x+a+1)^2
```

Maxima [A] time = 0.984672, size = 273, normalized size = 2.01

$$\frac{1}{2}\,x^2\,\mathrm{arcoth}\,(bx+a)^2+\frac{1}{8}\,b^2\left(\frac{8\left(\log(bx+a-1)\log\left(\frac{1}{2}\,bx+\frac{1}{2}\,a+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a+\frac{1}{2}\right)\right)a}{b^4}+\frac{4\,(a+1)\log(bx+\dots}{b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccoth(b*x + a)^2 + 1/8*b^2*(8*(log(b*x + a - 1)*log(1/2*b*x + 1/2
*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))*a/b^4 + 4*(a + 1)*log(b*x + a +
1)/b^4 + ((a^2 + 2*a + 1)*log(b*x + a + 1)^2 - 2*(a^2 + 2*a + 1)*log(b*x +
a + 1)*log(b*x + a - 1) + (a^2 - 2*a + 1)*log(b*x + a - 1)^2 - 4*(a - 1)*lo
g(b*x + a - 1))/b^4) + 1/2*b*(2*x/b^2 - (a^2 + 2*a + 1)*log(b*x + a + 1)/b^
3 + (a^2 - 2*a + 1)*log(b*x + a - 1)/b^3)*arccoth(b*x + a)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x\,\mathrm{arcoth}\,(bx+a)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x*arccoth(b*x + a)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\mathrm{acoth}^2\,(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(b*x+a)**2,x)
```

```
[Out] Integral(x*acoth(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\mathrm{arcoth}\,(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(b*x + a)^2, x)
```

3.72 $\int \coth^{-1}(a+bx)^2\,dx$

Optimal. Leaf size=81

$$-\frac{\text{PolyLog}\left(2,-\frac{a+bx+1}{-a-bx+1}\right)}{b}+\frac{(a+bx)\coth^{-1}(a+bx)^2}{b}+\frac{\coth^{-1}(a+bx)^2}{b}-\frac{2\log\left(\frac{2}{-a-bx+1}\right)\coth^{-1}(a+bx)}{b}$$

```
[Out] ArcCoth[a + b*x]^2/b + ((a + b*x)*ArcCoth[a + b*x]^2)/b - (2*ArcCoth[a + b*
x]*Log[2/(1 - a - b*x)])/b - PolyLog[2, -((1 + a + b*x)/(1 - a - b*x))]/b
```

Rubi [A] time = 0.0902758, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {6104, 5911, 5985, 5919, 2402, 2315}

$$-\frac{\text{PolyLog}\left(2,-\frac{a+bx+1}{-a-bx+1}\right)}{b}+\frac{(a+bx)\coth^{-1}(a+bx)^2}{b}+\frac{\coth^{-1}(a+bx)^2}{b}-\frac{2\log\left(\frac{2}{-a-bx+1}\right)\coth^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x]^2,x]
```

```
[Out] ArcCoth[a + b*x]^2/b + ((a + b*x)*ArcCoth[a + b*x]^2)/b - (2*ArcCoth[a + b*
x]*Log[2/(1 - a - b*x)])/b - PolyLog[2, -((1 + a + b*x)/(1 - a - b*x))]/b
```

Rule 6104

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}
, x] && IGtQ[p, 0]
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5985

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \coth^{-1}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \coth^{-1}(x)^2\,dx, x, a+bx\right)}{b}$$
$$= \frac{(a+bx)\coth^{-1}(a+bx)^2}{b} - \frac{2\,\text{Subst}\left(\int \frac{x\coth^{-1}(x)}{1-x^2}\,dx, x, a+bx\right)}{b}$$
$$= \frac{\coth^{-1}(a+bx)^2}{b} + \frac{(a+bx)\coth^{-1}(a+bx)^2}{b} - \frac{2\,\text{Subst}\left(\int \frac{\coth^{-1}(x)}{1-x}\,dx, x, a+bx\right)}{b}$$
$$= \frac{\coth^{-1}(a+bx)^2}{b} + \frac{(a+bx)\coth^{-1}(a+bx)^2}{b} - \frac{2\coth^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{b} + \frac{2\,\text{Subst}\left(\int \frac{\log(\ldots}{1-\ldots}\right.}{\ldots}$$
$$= \frac{\coth^{-1}(a+bx)^2}{b} + \frac{(a+bx)\coth^{-1}(a+bx)^2}{b} - \frac{2\coth^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{b} - \frac{2\,\text{Subst}\left(\int \frac{\log(2\ldots}{1-2\ldots}\right.}{b}$$
$$= \frac{\coth^{-1}(a+bx)^2}{b} + \frac{(a+bx)\coth^{-1}(a+bx)^2}{b} - \frac{2\coth^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{b} - \frac{\text{Li}_2\left(1-\frac{2}{1-a-bx}\right)}{b}$$

Mathematica [A] time = 0.0696864, size = 55, normalized size = 0.68

$$\frac{\text{PolyLog}\left(2, e^{-2\coth^{-1}(a+bx)}\right) + \coth^{-1}(a+bx)\left((a+bx-1)\coth^{-1}(a+bx) - 2\log\left(1-e^{-2\coth^{-1}(a+bx)}\right)\right)}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[a + b*x]^2,x]
```

```
[Out] (ArcCoth[a + b*x]*((-1 + a + b*x)*ArcCoth[a + b*x] - 2*Log[1 - E^(-2*ArcCot
h[a + b*x])]) + PolyLog[2, E^(-2*ArcCoth[a + b*x])])/b
```

Maple [A] time = 0.102, size = 151, normalized size = 1.9

$$x(\text{arccoth}(bx+a))^2 + \frac{(\text{arccoth}(bx+a))^2 a}{b} - 2\frac{\text{arccoth}(bx+a)}{b}\ln\left(1+\frac{1}{\sqrt{\frac{bx+a-1}{bx+a+1}}}\right) - 2\frac{\text{arccoth}(bx+a)}{b}\ln\left(1-\frac{1}{\sqrt{\frac{bx+\ldots}{bx+\ldots}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)^2,x)
```

```
[Out] x*arccoth(b*x+a)^2+1/b*arccoth(b*x+a)^2*a-2/b*arccoth(b*x+a)*ln(1+1/((b*x+a
-1)/(b*x+a+1))^(1/2))-2/b*arccoth(b*x+a)*ln(1-1/((b*x+a-1)/(b*x+a+1))^(1/2)
)+arccoth(b*x+a)^2/b-2/b*polylog(2,-1/((b*x+a-1)/(b*x+a+1))^(1/2))-2/b*poly
```

```
log(2,1/((b*x+a-1)/(b*x+a+1))^(1/2))
```

Maxima [A] time = 0.97802, size = 188, normalized size = 2.32

$$-\frac{1}{4}b^2\left(\frac{(a+1)\log(bx+a+1)^2-2(a+1)\log(bx+a+1)\log(bx+a-1)+(a-1)\log(bx+a-1)^2}{b^3}+\frac{4\left(\log(bx+a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*(((a + 1)*log(b*x + a + 1)^2 - 2*(a + 1)*log(b*x + a + 1)*log(b*x
+ a - 1) + (a - 1)*log(b*x + a - 1)^2)/b^3 + 4*(log(b*x + a - 1)*log(1/2*b*
x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))/b^3) + b*((a + 1)*log(b*x
 + a + 1)/b^2 - (a - 1)*log(b*x + a - 1)/b^2)*arccoth(b*x + a) + x*arccoth(
b*x + a)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\operatorname{arcoth}(bx+a)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(arccoth(b*x + a)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acoth}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)**2,x)
```

```
[Out] Integral(acoth(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a)^2, x)
```

3.73 $\int \frac{\coth^{-1}(a+bx)^2}{x} dx$

Optimal. Leaf size=148

$$\frac{1}{2}\text{PolyLog}\left(3,1-\frac{2}{a+bx+1}\right)-\frac{1}{2}\text{PolyLog}\left(3,1-\frac{2bx}{(1-a)(a+bx+1)}\right)+\coth^{-1}(a+bx)\text{PolyLog}\left(2,1-\frac{2}{a+bx+1}\right)-$$

```
[Out] -(ArcCoth[a + b*x]^2*Log[2/(1 + a + b*x)]) + ArcCoth[a + b*x]^2*Log[(2*b*x)
/((1 - a)*(1 + a + b*x))] + ArcCoth[a + b*x]*PolyLog[2, 1 - 2/(1 + a + b*x)
] - ArcCoth[a + b*x]*PolyLog[2, 1 - (2*b*x)/((1 - a)*(1 + a + b*x))] + Poly
Log[3, 1 - 2/(1 + a + b*x)]/2 - PolyLog[3, 1 - (2*b*x)/((1 - a)*(1 + a + b*
x))]/2
```

Rubi [A] time = 0.0913941, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6112, 5923}

$$\frac{1}{2}\text{PolyLog}\left(3,1-\frac{2}{a+bx+1}\right)-\frac{1}{2}\text{PolyLog}\left(3,1-\frac{2bx}{(1-a)(a+bx+1)}\right)+\coth^{-1}(a+bx)\text{PolyLog}\left(2,1-\frac{2}{a+bx+1}\right)-$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x]^2/x,x]
```

```
[Out] -(ArcCoth[a + b*x]^2*Log[2/(1 + a + b*x)]) + ArcCoth[a + b*x]^2*Log[(2*b*x)
/((1 - a)*(1 + a + b*x))] + ArcCoth[a + b*x]*PolyLog[2, 1 - 2/(1 + a + b*x)
] - ArcCoth[a + b*x]*PolyLog[2, 1 - (2*b*x)/((1 - a)*(1 + a + b*x))] + Poly
Log[3, 1 - 2/(1 + a + b*x)]/2 - PolyLog[3, 1 - (2*b*x)/((1 - a)*(1 + a + b*
x))]/2
```

Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rule 5923

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 -Simp[((a + b*ArcCoth[c*x])^2*Log[2/(1 + c*x)])/e, x] + (Simp[((a + b*ArcC
oth[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Simp[(b*(a
+ b*ArcCoth[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e, x] - Simp[(b*(a + b*ArcCo
th[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Sim
p[(b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e), x] - Simp[(b^2*PolyLog[3, 1 - (2
*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e), x]) /; FreeQ[{a, b, c, d, e},
x] && NeQ[c^2*d^2 - e^2, 0]
```

Rubi steps

$$\int \frac{\coth^{-1}(a+bx)^2}{x} dx = \frac{\text{Subst}\left(\int \frac{\coth^{-1}(x)^2}{-\frac{a}{b}+\frac{x}{b}} dx, x, a+bx\right)}{b}$$

$$= -\coth^{-1}(a+bx)^2 \log\left(\frac{2}{1+a+bx}\right)+\coth^{-1}(a+bx)^2 \log\left(\frac{2bx}{(1-a)(1+a+bx)}\right)+\coth^{-1}(a+b$$

Mathematica [C] time = 3.20549, size = 714, normalized size = 4.82

$$2\coth^{-1}(a+bx)\text{PolyLog}\left(2,-\sqrt{\frac{a-1}{a+1}}e^{\coth^{-1}(a+bx)}\right)+2\coth^{-1}(a+bx)\text{PolyLog}\left(2,\sqrt{\frac{a-1}{a+1}}e^{\coth^{-1}(a+bx)}\right)-\coth^{-1}(a$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[a + b*x]^2/x,x]
```

```
[Out] (-I/24)*Pi^3 - (2*ArcCoth[a + b*x]^3)/3 - (2*a*ArcCoth[a + b*x]^3)/3 + (2*S
qrt[1 - a^(-2)]*a*E^ArcTanh[a^(-1)]*ArcCoth[a + b*x]^3)/3 - I*Pi*ArcCoth[a
+ b*x]*Log[(E^(-ArcCoth[a + b*x]) + E^ArcCoth[a + b*x])/2] + ArcCoth[a + b*
x]^2*Log[1 - Sqrt[(-1 + a)/(1 + a)]*E^ArcCoth[a + b*x]] + ArcCoth[a + b*x]^
2*Log[1 + Sqrt[(-1 + a)/(1 + a)]*E^ArcCoth[a + b*x]] - ArcCoth[a + b*x]^2*L
og[1 - E^(2*ArcCoth[a + b*x])] - ArcCoth[a + b*x]^2*Log[1 - ((-1 + a)*E^(2*
ArcCoth[a + b*x]))/(1 + a)] + ArcCoth[a + b*x]^2*Log[1 - E^(2*ArcCoth[a + b
*x] - 2*ArcTanh[a^(-1)])] - 2*ArcCoth[a + b*x]*ArcTanh[a^(-1)]*Log[(I/2)*(E
^(ArcCoth[a + b*x] - ArcTanh[a^(-1)]) - E^(-ArcCoth[a + b*x] + ArcTanh[a^(-
1)]))] + ArcCoth[a + b*x]^2*Log[(-1 - E^(2*ArcCoth[a + b*x]) + a*(-1 + E^(2
*ArcCoth[a + b*x])))/(2*E^ArcCoth[a + b*x])] + I*Pi*ArcCoth[a + b*x]*Log[1/
Sqrt[1 - (a + b*x)^(-2)]] - ArcCoth[a + b*x]^2*Log[-((b*x)/((a + b*x)*Sqrt[
1 - (a + b*x)^(-2)]))] + 2*ArcCoth[a + b*x]*ArcTanh[a^(-1)]*Log[I*Sinh[ArcC
oth[a + b*x] - ArcTanh[a^(-1)]]] + 2*ArcCoth[a + b*x]*PolyLog[2, -(Sqrt[(-1
 + a)/(1 + a)]*E^ArcCoth[a + b*x])] + 2*ArcCoth[a + b*x]*PolyLog[2, Sqrt[(-
1 + a)/(1 + a)]*E^ArcCoth[a + b*x]] - ArcCoth[a + b*x]*PolyLog[2, E^(2*ArcC
oth[a + b*x])] - ArcCoth[a + b*x]*PolyLog[2, ((-1 + a)*E^(2*ArcCoth[a + b*x
]))/(1 + a)] + ArcCoth[a + b*x]*PolyLog[2, E^(2*ArcCoth[a + b*x] - 2*ArcTan
h[a^(-1)])] - 2*PolyLog[3, -(Sqrt[(-1 + a)/(1 + a)]*E^ArcCoth[a + b*x])] -
2*PolyLog[3, Sqrt[(-1 + a)/(1 + a)]*E^ArcCoth[a + b*x]] + PolyLog[3, E^(2*A
rcCoth[a + b*x])]/2 + PolyLog[3, ((-1 + a)*E^(2*ArcCoth[a + b*x]))/(1 + a)]
/2 - PolyLog[3, E^(2*ArcCoth[a + b*x] - 2*ArcTanh[a^(-1)])]/2
```

Maple [C] time = 0.543, size = 985, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)^2/x,x)
```

```
[Out] ln(b*x)*arccoth(b*x+a)^2-arccoth(b*x+a)^2*ln(a*((b*x+a+1)/(b*x+a-1)-1)-(b*x
+a+1)/(b*x+a-1)-1)+arccoth(b*x+a)^2*ln((b*x+a+1)/(b*x+a-1)-1)-arccoth(b*x+a
)^2*ln(1-1/((b*x+a-1)/(b*x+a+1))^(1/2))-2*arccoth(b*x+a)*polylog(2,1/((b*x+
a-1)/(b*x+a+1))^(1/2))+2*polylog(3,1/((b*x+a-1)/(b*x+a+1))^(1/2))-arccoth(b
*x+a)^2*ln(1+1/((b*x+a-1)/(b*x+a+1))^(1/2))-2*arccoth(b*x+a)*polylog(2,-1/(
(b*x+a-1)/(b*x+a+1))^(1/2))+2*polylog(3,-1/((b*x+a-1)/(b*x+a+1))^(1/2))-1/2
*I*Pi*arccoth(b*x+a)^2*csgn(I*(a*((b*x+a+1)/(b*x+a-1)-1)-(b*x+a+1)/(b*x+a-1
)-1))*csgn(I*(a*((b*x+a+1)/(b*x+a-1)-1)-(b*x+a+1)/(b*x+a-1)-1)/((b*x+a+1)/(
b*x+a-1)-1))^2+I*Pi*arccoth(b*x+a)^2*csgn(I*(a*((b*x+a+1)/(b*x+a-1)-1)-(b*x
+a+1)/(b*x+a-1)-1)/((b*x+a+1)/(b*x+a-1)-1))^2+1/2*I*Pi*arccoth(b*x+a)^2*csg
n(I*(a*((b*x+a+1)/(b*x+a-1)-1)-(b*x+a+1)/(b*x+a-1)-1))*csgn(I*(a*((b*x+a+1)
/(b*x+a-1)-1)-(b*x+a+1)/(b*x+a-1)-1)/((b*x+a+1)/(b*x+a-1)-1))*csgn(I/((b*x+
a+1)/(b*x+a-1)-1))-I*Pi*arccoth(b*x+a)^2-1/2*I*Pi*arccoth(b*x+a)^2*csgn(I*(
a*((b*x+a+1)/(b*x+a-1)-1)-(b*x+a+1)/(b*x+a-1)-1)/((b*x+a+1)/(b*x+a-1)-1))^2
*csgn(I/((b*x+a+1)/(b*x+a-1)-1))-1/2*I*Pi*arccoth(b*x+a)^2*csgn(I*(a*((b*x+
```

```
a+1)/(b*x+a-1)-1)-(b*x+a+1)/(b*x+a-1)-1)/((b*x+a+1)/(b*x+a-1)-1))^3+a/(a-1)
*arccoth(b*x+a)^2*ln(1-(a-1)*(b*x+a+1)/(b*x+a-1)/(1+a))+a/(a-1)*arccoth(b*x
+a)*polylog(2,(a-1)*(b*x+a+1)/(b*x+a-1)/(1+a))-1/2*a/(a-1)*polylog(3,(a-1)*
(b*x+a+1)/(b*x+a-1)/(1+a))-1/(a-1)*arccoth(b*x+a)^2*ln(1-(a-1)*(b*x+a+1)/(b
*x+a-1)/(1+a))-1/(a-1)*arccoth(b*x+a)*polylog(2,(a-1)*(b*x+a+1)/(b*x+a-1)/(
1+a))+1/2/(a-1)*polylog(3,(a-1)*(b*x+a+1)/(b*x+a-1)/(1+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(arccoth(b*x + a)^2/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arcoth}(bx+a)^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(b*x + a)^2/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^2(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)**2/x,x)
```

```
[Out] Integral(acoth(a + b*x)**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a)^2/x, x)
```

3.74 $\int \frac{\coth^{-1}(a+bx)^2}{x^2} dx$

Optimal. Leaf size=251

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{a+bx+1}\right)}{1-a^2}-\frac{b\text{PolyLog}\left(2,1-\frac{2bx}{(1-a)(a+bx+1)}\right)}{1-a^2}+\frac{b\text{PolyLog}\left(2,-\frac{a+bx+1}{-a-bx+1}\right)}{2(1-a)}-\frac{b\text{PolyLog}\left(2,1-\frac{2}{a+bx+1}\right)}{2(a+1)}-$$

```
[Out] -(ArcCoth[a + b*x]^2/x) + (b*ArcCoth[a + b*x]*Log[2/(1 - a - b*x)])/(1 - a)
 + (b*ArcCoth[a + b*x]*Log[2/(1 + a + b*x)])/(1 + a) - (2*b*ArcCoth[a + b*x
]*Log[2/(1 + a + b*x)])/(1 - a^2) + (2*b*ArcCoth[a + b*x]*Log[(2*b*x)/((1 -
 a)*(1 + a + b*x))])/(1 - a^2) + (b*PolyLog[2, -((1 + a + b*x)/(1 - a - b*x
))])/(2*(1 - a)) - (b*PolyLog[2, 1 - 2/(1 + a + b*x)])/(2*(1 + a)) + (b*Pol
yLog[2, 1 - 2/(1 + a + b*x)])/(1 - a^2) - (b*PolyLog[2, 1 - (2*b*x)/((1 - a
)*(1 + a + b*x))])/(1 - a^2)
```

Rubi [A] time = 0.719278, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 15, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.25, Rules used = {6110, 371, 706, 31, 633, 6741, 6122, 6688, 12, 6725, 5921, 2402, 2315, 2447, 5919}

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{a+bx+1}\right)}{1-a^2}-\frac{b\text{PolyLog}\left(2,1-\frac{2bx}{(1-a)(a+bx+1)}\right)}{1-a^2}+\frac{b\text{PolyLog}\left(2,-\frac{a+bx+1}{-a-bx+1}\right)}{2(1-a)}-\frac{b\text{PolyLog}\left(2,1-\frac{2}{a+bx+1}\right)}{2(a+1)}-$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x]^2/x^2,x]
```


```
[Out] -(ArcCoth[a + b*x]^2/x) + (b*ArcCoth[a + b*x]*Log[2/(1 - a - b*x)])/(1 - a)
 + (b*ArcCoth[a + b*x]*Log[2/(1 + a + b*x)])/(1 + a) - (2*b*ArcCoth[a + b*x
]*Log[2/(1 + a + b*x)])/(1 - a^2) + (2*b*ArcCoth[a + b*x]*Log[(2*b*x)/((1 -
 a)*(1 + a + b*x))])/(1 - a^2) + (b*PolyLog[2, -((1 + a + b*x)/(1 - a - b*x
))])/(2*(1 - a)) - (b*PolyLog[2, 1 - 2/(1 + a + b*x)])/(2*(1 + a)) + (b*Pol
yLog[2, 1 - 2/(1 + a + b*x)])/(1 - a^2) - (b*PolyLog[2, 1 - (2*b*x)/((1 - a
)*(1 + a + b*x))])/(1 - a^2)
```


Rule 6110

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[((e + f*x)^(m + 1)*(a + b*ArcCoth[c + d*x])^p)/(f*(m
 + 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcCot
h[c + d*x])^(p - 1))/(1 - (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
```

```
0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6122

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subs
t[Int[((d*e - c*f)/d + (f*x)/d)^m*(-(C/d^2) + (C*x^2)/d^2)^q*(a + b*ArcCoth
[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q},
x] && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 5921

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcCoth[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcCoth[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rubi steps

$$\int \frac{\coth^{-1}(a+bx)^2}{x^2}\,dx = -\frac{\coth^{-1}(a+bx)^2}{x}+(2b)\int \frac{\coth^{-1}(a+bx)}{x\left(1-(a+bx)^2\right)}\,dx$$

$$= -\frac{\coth^{-1}(a+bx)^2}{x}+(2b)\int \frac{\coth^{-1}(a+bx)}{x\left(1-a^2-2abx-b^2x^2\right)}\,dx$$

$$= -\frac{\coth^{-1}(a+bx)^2}{x}+2\,\text{Subst}\left(\int \frac{\coth^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)\left(1-x^2\right)}\,dx,x,a+bx\right)$$

$$= -\frac{\coth^{-1}(a+bx)^2}{x}+2\,\text{Subst}\left(\int \frac{b\coth^{-1}(x)}{(-a+x)\left(1-x^2\right)}\,dx,x,a+bx\right)$$

$$= -\frac{\coth^{-1}(a+bx)^2}{x}+(2b)\,\text{Subst}\left(\int \frac{\coth^{-1}(x)}{(-a+x)\left(1-x^2\right)}\,dx,x,a+bx\right)$$

$$= -\frac{\coth^{-1}(a+bx)^2}{x}+(2b)\,\text{Subst}\left(\int\left(\frac{\coth^{-1}(x)}{\left(-1+a^2\right)(a-x)}+\frac{\coth^{-1}(x)}{2(-1+a)(-1+x)}-\frac{\coth^{-1}(x)}{2(1+a)(1+x}\right.\right.$$

$$= -\frac{\coth^{-1}(a+bx)^2}{x}-\frac{b\,\text{Subst}\left(\int \frac{\coth^{-1}(x)}{-1+x}\,dx,x,a+bx\right)}{1-a}-\frac{b\,\text{Subst}\left(\int \frac{\coth^{-1}(x)}{1+x}\,dx,x,a+bx\right)}{1+a}$$

$$= -\frac{\coth^{-1}(a+bx)^2}{x}+\frac{b\coth^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{1-a}+\frac{b\coth^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right)}{1+a}-\frac{2b\cot}{}$$

$$= -\frac{\coth^{-1}(a+bx)^2}{x}+\frac{b\coth^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{1-a}+\frac{b\coth^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right)}{1+a}-\frac{2b\cot}{}$$

$$= -\frac{\coth^{-1}(a+bx)^2}{x}+\frac{b\coth^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{1-a}+\frac{b\coth^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right)}{1+a}-\frac{2b\cot}{}$$

Mathematica [C] time = 1.04006, size = 206, normalized size = 0.82

$$bx\text{PolyLog}\left(2,e^{2\tanh^{-1}\left(\frac{1}{a}\right)-2\coth^{-1}(a+bx)}\right)-\left(\sqrt{1-\frac{1}{a^2}}abxe^{\tanh^{-1}\left(\frac{1}{a}\right)}+a^2-1\right)\coth^{-1}(a+bx)^2+bx\coth^{-1}(a+bx)\left(-2\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[a + b*x]^2/x^2,x]
```

```
[Out] (-((-1 + a^2 + Sqrt[1 - a^(-2)]*a*b*E^ArcTanh[a^(-1)]*x)*ArcCoth[a + b*x]^2
) + b*x*ArcCoth[a + b*x]*((-I)*Pi + 2*ArcTanh[a^(-1)] - 2*Log[1 - E^(-2*Arc
Coth[a + b*x] + 2*ArcTanh[a^(-1)])]) + b*x*(I*Pi*(Log[1 + E^(2*ArcCoth[a +
b*x])] - Log[1/Sqrt[1 - (a + b*x)^(-2)]]) + 2*ArcTanh[a^(-1)]*(Log[1 - E^(-
2*ArcCoth[a + b*x] + 2*ArcTanh[a^(-1)])] - Log[I*Sinh[ArcCoth[a + b*x] - Ar
cTanh[a^(-1)]]])) + b*x*PolyLog[2, E^(-2*ArcCoth[a + b*x] + 2*ArcTanh[a^(-1
)])])/((-1 + a^2)*x)
```

Maple [A] time = 0.096, size = 342, normalized size = 1.4

$$-\frac{(\mathrm{arccoth}(bx+a))^2}{x}+2\,\frac{b\mathrm{arccoth}(bx+a)\ln(bx+a-1)}{2a-2}-2\,\frac{b\mathrm{arccoth}(bx+a)\ln(bx)}{(1+a)(a-1)}-2\,\frac{b\mathrm{arccoth}(bx+a)\ln(bx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)^2/x^2,x)
```

```
[Out] -arccoth(b*x+a)^2/x+2*b*arccoth(b*x+a)/(2*a-2)*ln(b*x+a-1)-2*b*arccoth(b*x+
a)/(a-1)/(1+a)*ln(b*x)-2*b*arccoth(b*x+a)/(2+2*a)*ln(b*x+a+1)+b/(a-1)/(1+a)
*dilog((b*x+a+1)/(1+a))+b/(a-1)/(1+a)*ln(b*x)*ln((b*x+a+1)/(1+a))-b/(a-1)/(
1+a)*dilog((b*x+a-1)/(a-1))-b/(a-1)/(1+a)*ln(b*x)*ln((b*x+a-1)/(a-1))+1/2*b
/(1+a)*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2+1/2*b*x+1/2*a)-1/2*b/(1+a)*ln(-1/2*b*x
-1/2*a+1/2)*ln(b*x+a+1)+1/2*b/(1+a)*dilog(1/2+1/2*b*x+1/2*a)+1/4*b/(1+a)*ln
(b*x+a+1)^2+1/4*b/(a-1)*ln(b*x+a-1)^2-1/2*b/(a-1)*dilog(1/2+1/2*b*x+1/2*a)-
1/2*b/(a-1)*ln(b*x+a-1)*ln(1/2+1/2*b*x+1/2*a)
```

Maxima [A] time = 1.02835, size = 329, normalized size = 1.31

$$\frac{1}{4}\,b^2\left(\frac{(a-1)\log(bx+a+1)^2-2\,(a-1)\log(bx+a+1)\log(bx+a-1)+(a+1)\log(bx+a-1)^2}{a^2b-b}-\frac{4\left(\log(bx+a-1)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*(((a - 1)*log(b*x + a + 1)^2 - 2*(a - 1)*log(b*x + a + 1)*log(b*x +
 a - 1) + (a + 1)*log(b*x + a - 1)^2)/(a^2*b - b) - 4*(log(b*x + a - 1)*log
(1/2*b*x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))/(a^2*b - b) + 4*(l
og(b*x/(a + 1) + 1)*log(x) + dilog(-b*x/(a + 1)))/(a^2*b - b) - 4*(log(b*x/
(a - 1) + 1)*log(x) + dilog(-b*x/(a - 1)))/(a^2*b - b)) - b*(log(b*x + a +
1)/(a + 1) - log(b*x + a - 1)/(a - 1) + 2*log(x)/(a^2 - 1))*arccoth(b*x + a
) - arccoth(b*x + a)^2/x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{arcoth}(bx+a)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(arccoth(b*x + a)^2/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^2(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)**2/x**2,x)
```

```
[Out] Integral(acoth(a + b*x)**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(bx+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a)^2/x^2, x)
```

3.75 $\int \frac{\coth^{-1}(a+bx)^2}{x^3}\,dx$

Optimal. Leaf size=370

$$\frac{ab^2\text{PolyLog}\left(2,1-\frac{2}{a+bx+1}\right)}{\left(1-a^2\right)^2}-\frac{ab^2\text{PolyLog}\left(2,1-\frac{2bx}{(1-a)(a+bx+1)}\right)}{\left(1-a^2\right)^2}+\frac{b^2\text{PolyLog}\left(2,-\frac{a+bx+1}{-a-bx+1}\right)}{4(1-a)^2}+\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{a+bx+1}\right)}{4(a+1)^2}\ldots$$

```
[Out] -((b*ArcCoth[a + b*x])/((1 - a^2)*x)) - ArcCoth[a + b*x]^2/(2*x^2) + (b^2*L
og[x])/(1 - a^2)^2 + (b^2*ArcCoth[a + b*x]*Log[2/(1 - a - b*x)])/(2*(1 - a)
^2) - (b^2*Log[1 - a - b*x])/(2*(1 - a)^2*(1 + a)) - (b^2*ArcCoth[a + b*x]*
Log[2/(1 + a + b*x)])/(2*(1 + a)^2) - (2*a*b^2*ArcCoth[a + b*x]*Log[2/(1 +
a + b*x)])/(1 - a^2)^2 + (2*a*b^2*ArcCoth[a + b*x]*Log[(2*b*x)/((1 - a)*(1
+ a + b*x))])/(1 - a^2)^2 - (b^2*Log[1 + a + b*x])/(2*(1 - a)*(1 + a)^2) +
(b^2*PolyLog[2, -((1 + a + b*x)/(1 - a - b*x))])/(4*(1 - a)^2) + (b^2*PolyL
og[2, 1 - 2/(1 + a + b*x)])/(4*(1 + a)^2) + (a*b^2*PolyLog[2, 1 - 2/(1 + a
+ b*x)])/(1 - a^2)^2 - (a*b^2*PolyLog[2, 1 - (2*b*x)/((1 - a)*(1 + a + b*x)
)])/(1 - a^2)^2
```

Rubi [A] time = 0.829986, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 16, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {6110, 371, 710, 801, 6741, 6122, 6725, 5927, 706, 31, 633, 5921, 2402, 2315, 2447, 5919}

$$\frac{ab^2\text{PolyLog}\left(2,1-\frac{2}{a+bx+1}\right)}{\left(1-a^2\right)^2}-\frac{ab^2\text{PolyLog}\left(2,1-\frac{2bx}{(1-a)(a+bx+1)}\right)}{\left(1-a^2\right)^2}+\frac{b^2\text{PolyLog}\left(2,-\frac{a+bx+1}{-a-bx+1}\right)}{4(1-a)^2}+\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{a+bx+1}\right)}{4(a+1)^2}\ldots$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x]^2/x^3,x]
```


```
[Out] -((b*ArcCoth[a + b*x])/((1 - a^2)*x)) - ArcCoth[a + b*x]^2/(2*x^2) + (b^2*L
og[x])/(1 - a^2)^2 + (b^2*ArcCoth[a + b*x]*Log[2/(1 - a - b*x)])/(2*(1 - a)
^2) - (b^2*Log[1 - a - b*x])/(2*(1 - a)^2*(1 + a)) - (b^2*ArcCoth[a + b*x]*
Log[2/(1 + a + b*x)])/(2*(1 + a)^2) - (2*a*b^2*ArcCoth[a + b*x]*Log[2/(1 +
a + b*x)])/(1 - a^2)^2 + (2*a*b^2*ArcCoth[a + b*x]*Log[(2*b*x)/((1 - a)*(1
+ a + b*x))])/(1 - a^2)^2 - (b^2*Log[1 + a + b*x])/(2*(1 - a)*(1 + a)^2) +
(b^2*PolyLog[2, -((1 + a + b*x)/(1 - a - b*x))])/(4*(1 - a)^2) + (b^2*PolyL
og[2, 1 - 2/(1 + a + b*x)])/(4*(1 + a)^2) + (a*b^2*PolyLog[2, 1 - 2/(1 + a
+ b*x)])/(1 - a^2)^2 - (a*b^2*PolyLog[2, 1 - (2*b*x)/((1 - a)*(1 + a + b*x)
)])/(1 - a^2)^2
```


Rule 6110

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[((e + f*x)^(m + 1)*(a + b*ArcCoth[c + d*x])^p)/(f*(m
 + 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcCot
h[c + d*x])^(p - 1))/(1 - (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
```

```
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6122

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subs
t[Int[((d*e - c*f)/d + (f*x)/d)^m*(-(C/d^2) + (C*x^2)/d^2)^q*(a + b*ArcCoth
[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q},
x] && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 5927

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcCoth[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
```

```
-(a*c)]
```

Rule 5921

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcCoth[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcCoth[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rubi steps

$$\int \frac{\coth^{-1}(a+bx)^2}{x^3}\,dx = -\frac{\coth^{-1}(a+bx)^2}{2x^2} + b\int \frac{\coth^{-1}(a+bx)}{x^2\left(1-(a+bx)^2\right)}\,dx$$

$$= -\frac{\coth^{-1}(a+bx)^2}{2x^2} + b\int \frac{\coth^{-1}(a+bx)}{x^2\left(1-a^2-2abx-b^2x^2\right)}\,dx$$

$$= -\frac{\coth^{-1}(a+bx)^2}{2x^2} + \text{Subst}\left(\int \frac{\coth^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\left(1-x^2\right)}\,dx, x, a+bx\right)$$

$$= -\frac{\coth^{-1}(a+bx)^2}{2x^2} + \text{Subst}\left(\int \left(-\frac{b^2\coth^{-1}(x)}{\left(-1+a^2\right)(a-x)^2} - \frac{2ab^2\coth^{-1}(x)}{\left(-1+a^2\right)^2(a-x)} - \frac{b^2\coth^{-1}(x)}{2(-1+a)^2(-1+\right.\right.$$

$$= -\frac{\coth^{-1}(a+bx)^2}{2x^2} - \frac{b^2\,\text{Subst}\left(\int \frac{\coth^{-1}(x)}{-1+x}\,dx, x, a+bx\right)}{2(1-a)^2} + \frac{b^2\,\text{Subst}\left(\int \frac{\coth^{-1}(x)}{1+x}\,dx, x, a+bx\right.}{2(1+a)^2}$$

$$= -\frac{b\coth^{-1}(a+bx)}{\left(1-a^2\right)x} - \frac{\coth^{-1}(a+bx)^2}{2x^2} + \frac{b^2\coth^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{2(1-a)^2} - \frac{b^2\coth^{-1}(a+bx)}{2(1+a}$$

$$= -\frac{b\coth^{-1}(a+bx)}{\left(1-a^2\right)x} - \frac{\coth^{-1}(a+bx)^2}{2x^2} + \frac{b^2\coth^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{2(1-a)^2} - \frac{b^2\coth^{-1}(a+bx)}{2(1+a}$$

$$= -\frac{b\coth^{-1}(a+bx)}{\left(1-a^2\right)x} - \frac{\coth^{-1}(a+bx)^2}{2x^2} + \frac{b^2\log(x)}{\left(1-a^2\right)^2} + \frac{b^2\coth^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{2(1-a)^2} - \frac{b^2\,\text{co}}{}$$

$$= -\frac{b\coth^{-1}(a+bx)}{\left(1-a^2\right)x} - \frac{\coth^{-1}(a+bx)^2}{2x^2} + \frac{b^2\log(x)}{\left(1-a^2\right)^2} + \frac{b^2\coth^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{2(1-a)^2} - \frac{b^2\log}{2(1-}$$

Mathematica [C] time = 2.18402, size = 291, normalized size = 0.79

$$\frac{-2ab^2x^2\text{PolyLog}\left(2, e^{2\tanh^{-1}\left(\frac{1}{a}\right)-2\coth^{-1}(a+bx)}\right) + \left(a^2\left(b^2x^2\left(2\sqrt{1-\frac{1}{a^2}}e^{\tanh^{-1}\left(\frac{1}{a}\right)}-1\right)+2\right)-a^4+b^2x^2-1\right)\coth^{-1}(a+}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[a + b*x]^2/x^3,x]
```

```
[Out] ((-1 - a^4 + b^2*x^2 + a^2*(2 + b^2*(-1 + 2*Sqrt[1 - a^(-2)]*E^ArcTanh[a^(-
1)])*x^2))*ArcCoth[a + b*x]^2 + 2*b*x*ArcCoth[a + b*x]*(-1 + a^2 + a*b*x +
I*a*b*Pi*x - 2*a*b*x*ArcTanh[a^(-1)] + 2*a*b*x*Log[1 - E^(-2*ArcCoth[a + b*
x] + 2*ArcTanh[a^(-1)])]) + 2*b^2*x^2*((-I)*a*Pi*Log[1 + E^(2*ArcCoth[a + b
*x])] + I*a*Pi*Log[1/Sqrt[1 - (a + b*x)^(-2)]] + Log[-((b*x)/((a + b*x)*Sqr
t[1 - (a + b*x)^(-2)]))] - 2*a*ArcTanh[a^(-1)]*(Log[1 - E^(-2*ArcCoth[a + b
*x] + 2*ArcTanh[a^(-1)])] - Log[I*Sinh[ArcCoth[a + b*x] - ArcTanh[a^(-1)]]]
)) - 2*a*b^2*x^2*PolyLog[2, E^(-2*ArcCoth[a + b*x] + 2*ArcTanh[a^(-1)])])/(
2*(-1 + a^2)^2*x^2)
```

Maple [A] time = 0.099, size = 467, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)^2/x^3,x)
```

```
[Out] -1/2*arccoth(b*x+a)^2/x^2-1/2*b^2*arccoth(b*x+a)/(a-1)^2*ln(b*x+a-1)+b*arcc
oth(b*x+a)/(a-1)/(1+a)/x+2*b^2*arccoth(b*x+a)*a/(a-1)^2/(1+a)^2*ln(b*x)+1/2
*b^2*arccoth(b*x+a)/(1+a)^2*ln(b*x+a+1)-1/8*b^2/(a-1)^2*ln(b*x+a-1)^2+1/4*b
^2/(a-1)^2*dilog(1/2+1/2*b*x+1/2*a)+1/4*b^2/(a-1)^2*ln(b*x+a-1)*ln(1/2+1/2*
b*x+1/2*a)-1/4*b^2/(1+a)^2*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2+1/2*b*x+1/2*a)+1/4
*b^2/(1+a)^2*ln(-1/2*b*x-1/2*a+1/2)*ln(b*x+a+1)-1/4*b^2/(1+a)^2*dilog(1/2+1
/2*b*x+1/2*a)-1/8*b^2/(1+a)^2*ln(b*x+a+1)^2-b^2/(a-1)/(1+a)/(2*a-2)*ln(b*x+
a-1)+b^2/(a-1)^2/(1+a)^2*ln(b*x)+b^2/(a-1)/(1+a)/(2+2*a)*ln(b*x+a+1)-b^2*a/
(a-1)^2/(1+a)^2*dilog((b*x+a+1)/(1+a))-b^2*a/(a-1)^2/(1+a)^2*ln(b*x)*ln((b*
x+a+1)/(1+a))+b^2*a/(a-1)^2/(1+a)^2*dilog((b*x+a-1)/(a-1))+b^2*a/(a-1)^2/(1
+a)^2*ln(b*x)*ln((b*x+a-1)/(a-1))
```

Maxima [A] time = 1.02374, size = 486, normalized size = 1.31

$$\frac{1}{8}\left(\frac{8\left(\log\left(bx+a-1\right)\log\left(\frac{1}{2}\,bx+\frac{1}{2}\,a+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a+\frac{1}{2}\right)\right)a}{a^4-2\,a^2+1}-\frac{8\left(\log\left(\frac{bx}{a+1}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{bx}{a+1}\right)\right)a}{a^4-2\,a^2+1}+\frac{8\left(\log\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*(8*(log(b*x + a - 1)*log(1/2*b*x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2*
a + 1/2))*a/(a^4 - 2*a^2 + 1) - 8*(log(b*x/(a + 1) + 1)*log(x) + dilog(-b*x
/(a + 1)))*a/(a^4 - 2*a^2 + 1) + 8*(log(b*x/(a - 1) + 1)*log(x) + dilog(-b*
x/(a - 1)))*a/(a^4 - 2*a^2 + 1) - ((a^2 - 2*a + 1)*log(b*x + a + 1)^2 - 2*(
a^2 - 2*a + 1)*log(b*x + a + 1)*log(b*x + a - 1) + (a^2 + 2*a + 1)*log(b*x
+ a - 1)^2)/(a^4 - 2*a^2 + 1) + 4*log(b*x + a + 1)/(a^3 + a^2 - a - 1) - 4*
log(b*x + a - 1)/(a^3 - a^2 - a + 1) + 8*log(x)/(a^4 - 2*a^2 + 1))*b^2 + 1/
2*(4*a*b*log(x)/(a^4 - 2*a^2 + 1) + b*log(b*x + a + 1)/(a^2 + 2*a + 1) - b*
log(b*x + a - 1)/(a^2 - 2*a + 1) + 2/((a^2 - 1)*x))*b*arccoth(b*x + a) - 1/
2*arccoth(b*x + a)^2/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{arcoth}\left(bx+a\right)^2}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(arccoth(b*x + a)^2/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{acoth}^2\left(a+bx\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)**2/x**3,x)
```

```
[Out] Integral(acoth(a + b*x)**2/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(bx+a)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a)^2/x^3, x)
```

3.76 $\int \frac{\coth^{-1}(a+bx)}{c+dx^2}\,dx$

Optimal. Leaf size=673

$$\frac{\text{PolyLog}\left(2,-\frac{(-a-bx+1)\left(a^2d+b^2c\right)}{(a+bx)\left(-(1-a)ad+b^2c-b\sqrt{-c}\sqrt{d}\right)}\right)}{4\sqrt{-c}\sqrt{d}}-\frac{\text{PolyLog}\left(2,-\frac{(-a-bx+1)\left(a^2d+b^2c\right)}{(a+bx)\left(-(1-a)ad+b^2c+b\sqrt{-c}\sqrt{d}\right)}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{\text{PolyLog}\left(2,\frac{(a+bx+1)\left(a^2d+b^2c\right)}{(a+bx)\left(a(a+1)d+b^2c-b\sqrt{\cdot}\right.}\right.}{4\sqrt{-c}\sqrt{d}}$$

```
[Out] (Log[-((1 - a - b*x)/(a + b*x))]*Log[1 + ((b^2*c + a^2*d)*(1 - a - b*x))/((
b^2*c - b*Sqrt[-c]*Sqrt[d] - (1 - a)*a*d)*(a + b*x))])/(4*Sqrt[-c]*Sqrt[d])
 - (Log[-((1 - a - b*x)/(a + b*x))]*Log[1 + ((b^2*c + a^2*d)*(1 - a - b*x))
/((b^2*c + b*Sqrt[-c]*Sqrt[d] - (1 - a)*a*d)*(a + b*x))])/(4*Sqrt[-c]*Sqrt[
d]) + (Log[(1 + a + b*x)/(a + b*x)]*Log[1 - ((b^2*c + a^2*d)*(1 + a + b*x))
/((b^2*c - b*Sqrt[-c]*Sqrt[d] + a*(1 + a)*d)*(a + b*x))])/(4*Sqrt[-c]*Sqrt[
d]) - (Log[(1 + a + b*x)/(a + b*x)]*Log[1 - ((b^2*c + a^2*d)*(1 + a + b*x))
/((b^2*c + b*Sqrt[-c]*Sqrt[d] + a*(1 + a)*d)*(a + b*x))])/(4*Sqrt[-c]*Sqrt[
d]) + PolyLog[2, -(((b^2*c + a^2*d)*(1 - a - b*x))/((b^2*c - b*Sqrt[-c]*Sqr
t[d] - (1 - a)*a*d)*(a + b*x)))]/(4*Sqrt[-c]*Sqrt[d]) - PolyLog[2, -(((b^2*
c + a^2*d)*(1 - a - b*x))/((b^2*c + b*Sqrt[-c]*Sqrt[d] - (1 - a)*a*d)*(a +
b*x)))]/(4*Sqrt[-c]*Sqrt[d]) + PolyLog[2, ((b^2*c + a^2*d)*(1 + a + b*x))/(
(b^2*c - b*Sqrt[-c]*Sqrt[d] + a*(1 + a)*d)*(a + b*x))]/(4*Sqrt[-c]*Sqrt[d])
 - PolyLog[2, ((b^2*c + a^2*d)*(1 + a + b*x))/((b^2*c + b*Sqrt[-c]*Sqrt[d]
+ a*(1 + a)*d)*(a + b*x))]/(4*Sqrt[-c]*Sqrt[d])
```

Rubi [A] time = 1.09947, antiderivative size = 597, normalized size of antiderivative = 0.89, number of steps used = 37, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6116, 2513, 2409, 2394, 2393, 2391, 205}

$$-\frac{\text{PolyLog}\left(2,-\frac{\sqrt{d}(-a-bx+1)}{b\sqrt{-c}-(1-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{d}(-a-bx+1)}{(1-a)\sqrt{d}+b\sqrt{-c}}\right)}{4\sqrt{-c}\sqrt{d}}-\frac{\text{PolyLog}\left(2,-\frac{\sqrt{d}(a+bx+1)}{b\sqrt{-c}-(a+1)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{d}(a+bx+1)}{(a+1)\sqrt{d}+b\sqrt{-c}}\right)}{4\sqrt{-c}\sqrt{d}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[ArcCoth[a + b*x]/(c + d*x^2),x]
```

```
[Out] (ArcTan[(Sqrt[d]*x)/Sqrt[c]]*(Log[-1 + a + b*x] - Log[-((1 - a - b*x)/(a +
b*x))] - Log[a + b*x]))/(2*Sqrt[c]*Sqrt[d]) + (ArcTan[(Sqrt[d]*x)/Sqrt[c]]*
(Log[a + b*x] - Log[1 + a + b*x] + Log[(1 + a + b*x)/(a + b*x)]))/(2*Sqrt[c
]*Sqrt[d]) - (Log[-1 + a + b*x]*Log[(b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c]
- (1 - a)*Sqrt[d])])/(4*Sqrt[-c]*Sqrt[d]) + (Log[1 + a + b*x]*Log[(b*(Sqrt[
-c] - Sqrt[d]*x))/(b*Sqrt[-c] + (1 + a)*Sqrt[d])])/(4*Sqrt[-c]*Sqrt[d]) + (
Log[-1 + a + b*x]*Log[(b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] + (1 - a)*Sqrt
[d])])/(4*Sqrt[-c]*Sqrt[d]) - (Log[1 + a + b*x]*Log[(b*(Sqrt[-c] + Sqrt[d]*
x))/(b*Sqrt[-c] - (1 + a)*Sqrt[d])])/(4*Sqrt[-c]*Sqrt[d]) - PolyLog[2, -((S
qrt[d]*(1 - a - b*x))/(b*Sqrt[-c] - (1 - a)*Sqrt[d]))]/(4*Sqrt[-c]*Sqrt[d])
 + PolyLog[2, (Sqrt[d]*(1 - a - b*x))/(b*Sqrt[-c] + (1 - a)*Sqrt[d])]/(4*Sq
rt[-c]*Sqrt[d]) - PolyLog[2, -((Sqrt[d]*(1 + a + b*x))/(b*Sqrt[-c] - (1 + a
)*Sqrt[d]))]/(4*Sqrt[-c]*Sqrt[d]) + PolyLog[2, (Sqrt[d]*(1 + a + b*x))/(b*S
qrt[-c] + (1 + a)*Sqrt[d])]/(4*Sqrt[-c]*Sqrt[d])
```

Rule 6116

```
Int[ArcCoth[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Dist
[1/2, Int[Log[(1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] - Dist[1/2, Int[
Log[(-1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x]
```

```
&& RationalQ[n]
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\coth^{-1}(a+bx)}{c+dx^2}\,dx = -\left(\frac{1}{2}\int \frac{\log\left(\frac{-1+a+bx}{a+bx}\right)}{c+dx^2}\,dx\right) + \frac{1}{2}\int \frac{\log\left(\frac{1+a+bx}{a+bx}\right)}{c+dx^2}\,dx$$

$$= -\left(\frac{1}{2}\int \frac{\log(-1+a+bx)}{c+dx^2}\,dx\right) + \frac{1}{2}\int \frac{\log(1+a+bx)}{c+dx^2}\,dx - \frac{1}{2}\left(-\log(-1+a+bx) + \log\left(\frac{-1+a}{a+b}\right.\right.$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2\sqrt{c}\sqrt{d}} + \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\left(\log(a+bx) - \log\right.}{2\sqrt{c}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2\sqrt{c}\sqrt{d}} + \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\left(\log(a+bx) - \log\right.}{2\sqrt{c}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2\sqrt{c}\sqrt{d}} + \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\left(\log(a+bx) - \log\right.}{2\sqrt{c}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2\sqrt{c}\sqrt{d}} + \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\left(\log(a+bx) - \log\right.}{2\sqrt{c}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2\sqrt{c}\sqrt{d}} + \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\left(\log(a+bx) - \log\right.}{2\sqrt{c}}$$

Mathematica [A] time = 0.58637, size = 529, normalized size = 0.79

$$\frac{\text{PolyLog}\left(2, \frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{(a-1)\sqrt{d}+b\sqrt{-c}}\right) - \text{PolyLog}\left(2, \frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{(a+1)\sqrt{d}+b\sqrt{-c}}\right) - \text{PolyLog}\left(2, \frac{b\left(\sqrt{-c}+\sqrt{d}x\right)}{b\sqrt{-c}-(a-1)\sqrt{d}}\right) + \text{PolyLog}\left(2, \frac{b\left(\sqrt{-c}+\sqrt{d}x\right)}{b\sqrt{-c}-(a+1)\sqrt{d}}\right) + \log(}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[a + b*x]/(c + d*x^2),x]
```

```
[Out] (Log[(Sqrt[d]*(-1 + a + b*x))/(b*Sqrt[-c] + (-1 + a)*Sqrt[d])]*Log[Sqrt[-c]
 - Sqrt[d]*x] - Log[(-1 + a + b*x)/(a + b*x)]*Log[Sqrt[-c] - Sqrt[d]*x] - L
og[(Sqrt[d]*(1 + a + b*x))/(b*Sqrt[-c] + (1 + a)*Sqrt[d])]*Log[Sqrt[-c] - S
qrt[d]*x] + Log[(1 + a + b*x)/(a + b*x)]*Log[Sqrt[-c] - Sqrt[d]*x] - Log[-(
(Sqrt[d]*(-1 + a + b*x))/(b*Sqrt[-c] - (-1 + a)*Sqrt[d]))]*Log[Sqrt[-c] + S
qrt[d]*x] + Log[(-1 + a + b*x)/(a + b*x)]*Log[Sqrt[-c] + Sqrt[d]*x] + Log[-
((Sqrt[d]*(1 + a + b*x))/(b*Sqrt[-c] - (1 + a)*Sqrt[d]))]*Log[Sqrt[-c] + Sq
rt[d]*x] - Log[(1 + a + b*x)/(a + b*x)]*Log[Sqrt[-c] + Sqrt[d]*x] + PolyLog
[2, (b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] + (-1 + a)*Sqrt[d])] - PolyLog[2
, (b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] + (1 + a)*Sqrt[d])] - PolyLog[2, (
b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] - (-1 + a)*Sqrt[d])] + PolyLog[2, (b*
(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] - (1 + a)*Sqrt[d])])/(4*Sqrt[-c]*Sqrt[d
])
```

Maple [B] time = 0.49, size = 1230, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)/(d*x^2+c),x)
```

```
[Out] 1/2/b*(-b^2*c*d)^(1/2)/c/d*arccoth(b*x+a)*ln(1-(a^2*d+b^2*c-2*a*d+d)*(b*x+a
+1)/(b*x+a-1)/(a^2*d+b^2*c+2*(-b^2*c*d)^(1/2)-d))-1/2/b*(-b^2*c*d)^(1/2)/c/
d*arccoth(b*x+a)^2+1/4/b*(-b^2*c*d)^(1/2)/c/d*polylog(2,(a^2*d+b^2*c-2*a*d+
d)*(b*x+a+1)/(b*x+a-1)/(a^2*d+b^2*c+2*(-b^2*c*d)^(1/2)-d))-1/2/b*(-b^2*c*d)
^(1/2)/c*ln(1-(a^2*d+b^2*c-2*a*d+d)*(b*x+a+1)/(b*x+a-1)/(a^2*d+b^2*c-2*(-b^
2*c*d)^(1/2)-d))*arccoth(b*x+a)*a^2/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d)-1/2*
b*(-b^2*c*d)^(1/2)/d*ln(1-(a^2*d+b^2*c-2*a*d+d)*(b*x+a+1)/(b*x+a-1)/(a^2*d+
b^2*c-2*(-b^2*c*d)^(1/2)-d))*arccoth(b*x+a)/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)
-d)+1/2/b*(-b^2*c*d)^(1/2)/c*arccoth(b*x+a)^2*a^2/(a^2*d+b^2*c-2*(-b^2*c*d)
^(1/2)-d)+1/2*b*(-b^2*c*d)^(1/2)/d*arccoth(b*x+a)^2/(a^2*d+b^2*c-2*(-b^2*c*
d)^(1/2)-d)-1/4/b*(-b^2*c*d)^(1/2)/c*polylog(2,(a^2*d+b^2*c-2*a*d+d)*(b*x+a
+1)/(b*x+a-1)/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d))*a^2/(a^2*d+b^2*c-2*(-b^2*
c*d)^(1/2)-d)-1/4*b*(-b^2*c*d)^(1/2)/d*polylog(2,(a^2*d+b^2*c-2*a*d+d)*(b*x
+a+1)/(b*x+a-1)/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d))/(a^2*d+b^2*c-2*(-b^2*c*
d)^(1/2)-d)-b/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d)*ln(1-(a^2*d+b^2*c-2*a*d+d)
*(b*x+a+1)/(b*x+a-1)/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d))*arccoth(b*x+a)+b/(
a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d)*arccoth(b*x+a)^2+1/2/b*(-b^2*c*d)^(1/2)/c
*ln(1-(a^2*d+b^2*c-2*a*d+d)*(b*x+a+1)/(b*x+a-1)/(a^2*d+b^2*c-2*(-b^2*c*d)^(
1/2)-d))*arccoth(b*x+a)/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d)-1/2/b*(-b^2*c*d)
^(1/2)/c*arccoth(b*x+a)^2/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d)-1/2*b/(a^2*d+b
^2*c-2*(-b^2*c*d)^(1/2)-d)*polylog(2,(a^2*d+b^2*c-2*a*d+d)*(b*x+a+1)/(b*x+a
-1)/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d))+1/4/b*(-b^2*c*d)^(1/2)/c*polylog(2,
(a^2*d+b^2*c-2*a*d+d)*(b*x+a+1)/(b*x+a-1)/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d
))/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arcoth}(bx+a)}{dx^2+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arccoth(b*x + a)/(d*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(bx+a)}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a)/(d*x^2 + c), x)
```

3.77 $\int \frac{\coth^{-1}(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=120

$$-\frac{\text{PolyLog}\left(2,1-\frac{2b(c+dx)}{(a+bx+1)(-ad+bc+d)}\right)}{2d}+\frac{\text{PolyLog}\left(2,1-\frac{2}{a+bx+1}\right)}{2d}+\frac{\coth^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(a+bx+1)(-ad+bc+d)}\right)}{d}-\frac{\log\left(\frac{2}{a+bx}\right.}{}$$

```
[Out] -((ArcCoth[a + b*x]*Log[2/(1 + a + b*x)])/d) + (ArcCoth[a + b*x]*Log[(2*b*(
c + d*x))/((b*c + d - a*d)*(1 + a + b*x))])/d + PolyLog[2, 1 - 2/(1 + a + b
*x)]/(2*d) - PolyLog[2, 1 - (2*b*(c + d*x))/((b*c + d - a*d)*(1 + a + b*x))
]/(2*d)
```

Rubi [A] time = 0.126501, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6112, 5921, 2402, 2315, 2447}

$$-\frac{\text{PolyLog}\left(2,1-\frac{2b(c+dx)}{(a+bx+1)(-ad+bc+d)}\right)}{2d}+\frac{\text{PolyLog}\left(2,1-\frac{2}{a+bx+1}\right)}{2d}+\frac{\coth^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(a+bx+1)(-ad+bc+d)}\right)}{d}-\frac{\log\left(\frac{2}{a+bx}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x]/(c + d*x),x]
```

```
[Out] -((ArcCoth[a + b*x]*Log[2/(1 + a + b*x)])/d) + (ArcCoth[a + b*x]*Log[(2*b*(
c + d*x))/((b*c + d - a*d)*(1 + a + b*x))])/d + PolyLog[2, 1 - 2/(1 + a + b
*x)]/(2*d) - PolyLog[2, 1 - (2*b*(c + d*x))/((b*c + d - a*d)*(1 + a + b*x))
]/(2*d)
```

Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rule 5921

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcCoth[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcCoth[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\coth^{-1}(a+bx)}{c+dx}\,dx = \frac{\text{Subst}\left(\int \frac{\coth^{-1}(x)}{\frac{bc-ad}{b}+\frac{dx}{b}}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\coth^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right)}{d} + \frac{\coth^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+d-ad)(1+a+bx)}\right)}{d} + \frac{\text{Subst}\left(\int \frac{\log\left(\frac{2}{1+x}\right)}{1-x^2}\,dx, x,\right.}{d}$$

$$= -\frac{\coth^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right)}{d} + \frac{\coth^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+d-ad)(1+a+bx)}\right)}{d} - \frac{\text{Li}_2\left(1-\frac{2b(c+dx)}{(bc+d-ad)(1+a+bx}\right.}{2d}$$

$$= -\frac{\coth^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right)}{d} + \frac{\coth^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+d-ad)(1+a+bx)}\right)}{d} + \frac{\text{Li}_2\left(1-\frac{2}{1+a+bx}\right)}{2d} - \frac{\text{Li}_2}{}$$

Mathematica [A] time = 0.0665651, size = 185, normalized size = 1.54

$$-\frac{\text{PolyLog}\left(2,\frac{b(c+dx)}{-ad+bc-d}\right)}{2d} + \frac{\text{PolyLog}\left(2,\frac{b(c+dx)}{-ad+bc+d}\right)}{2d} + \frac{\log(c+dx)\log\left(\frac{d(-a-bx+1)}{-ad+bc+d}\right)}{2d} - \frac{\log\left(\frac{a+bx-1}{a+bx}\right)\log(c+dx)}{2d} - \frac{\log(c+dx}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[a + b*x]/(c + d*x),x]
```

```
[Out] (Log[(d*(1 - a - b*x))/(b*c + d - a*d)]*Log[c + d*x])/(2*d) - (Log[(-1 + a
+ b*x)/(a + b*x)]*Log[c + d*x])/(2*d) - (Log[-((d*(1 + a + b*x))/(b*c - d -
 a*d))]*Log[c + d*x])/(2*d) + (Log[(1 + a + b*x)/(a + b*x)]*Log[c + d*x])/(
2*d) - PolyLog[2, (b*(c + d*x))/(b*c - d - a*d)]/(2*d) + PolyLog[2, (b*(c +
 d*x))/(b*c + d - a*d)]/(2*d)
```

Maple [A] time = 0.137, size = 176, normalized size = 1.5

$$\frac{\ln(d(bx+a)-ad+bc)\,\text{arccoth}\,(bx+a)}{d} - \frac{\ln(d(bx+a)-ad+bc)}{2d}\ln\left(\frac{d(bx+a)+d}{ad-bc+d}\right) - \frac{1}{2d}\text{dilog}\left(\frac{d(bx+a)+d}{ad-bc+d}\right) + \frac{\text{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)/(d*x+c),x)
```

```
[Out] ln(d*(b*x+a)-a*d+b*c)/d*arccoth(b*x+a)-1/2/d*ln(d*(b*x+a)-a*d+b*c)*ln((d*(b
*x+a)+d)/(a*d-b*c+d))-1/2/d*dilog((d*(b*x+a)+d)/(a*d-b*c+d))+1/2/d*ln((d*(b
*x+a)-d)/(a*d-b*c-d))*ln(d*(b*x+a)-a*d+b*c)+1/2/d*dilog((d*(b*x+a)-d)/(a*d-
b*c-d))
```

Maxima [A] time = 0.987377, size = 259, normalized size = 2.16

$$-\frac{1}{2}b\left(\frac{\log(bx+a-1)\log\left(\frac{bdx+ad-d}{bc-ad+d}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad-d}{bc-ad+d}\right)}{bd}-\frac{\log(bx+a+1)\log\left(\frac{bdx+ad+d}{bc-ad-d}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad+d}{bc-ad-d}\right)}{bd}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*b*((log(b*x + a - 1)*log((b*d*x + a*d - d)/(b*c - a*d + d) + 1) + dilo
g(-(b*d*x + a*d - d)/(b*c - a*d + d)))/(b*d) - (log(b*x + a + 1)*log((b*d*x
 + a*d + d)/(b*c - a*d - d) + 1) + dilog(-(b*d*x + a*d + d)/(b*c - a*d - d)
))/(b*d)) - 1/2*b*(log(b*x + a + 1)/b - log(b*x + a - 1)/b)*log(d*x + c)/d
+ arccoth(b*x + a)*log(d*x + c)/d
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arcoth}(bx+a)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(arccoth(b*x + a)/(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a)/(d*x + c), x)
```

3.78 $\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x}}\,dx$

Optimal. Leaf size=292

$$\frac{d\text{PolyLog}\left(2,-\frac{b(cx+d)}{ac-bd+c}\right)}{2c^2}-\frac{d\text{PolyLog}\left(2,\frac{b(cx+d)}{-ac+bd+c}\right)}{2c^2}+\frac{d\log\left(-\frac{-a-bx+1}{a+bx}\right)\log(cx+d)}{2c^2}-\frac{d\log(cx+d)\log\left(\frac{c(-a-bx+1)}{-ac+bd+c}\right)}{2c^2}+\ldots$$

```
[Out] ((1 - a - b*x)*Log[-((1 - a - b*x)/(a + b*x))])/(2*b*c) + Log[a + b*x]/(2*b
*c) + Log[1 + a + b*x]/(2*b*c) + ((a + b*x)*Log[(1 + a + b*x)/(a + b*x)])/(
2*b*c) - (d*Log[(c*(1 - a - b*x))/(c - a*c + b*d)]*Log[d + c*x])/(2*c^2) +
(d*Log[-((1 - a - b*x)/(a + b*x))]*Log[d + c*x])/(2*c^2) + (d*Log[(c*(1 + a
 + b*x))/(c + a*c - b*d)]*Log[d + c*x])/(2*c^2) - (d*Log[(1 + a + b*x)/(a +
 b*x)]*Log[d + c*x])/(2*c^2) + (d*PolyLog[2, -((b*(d + c*x))/(c + a*c - b*d
))])/(2*c^2) - (d*PolyLog[2, (b*(d + c*x))/(c - a*c + b*d)])/(2*c^2)
```

Rubi [A] time = 0.499929, antiderivative size = 360, normalized size of antiderivative = 1.23, number of steps used = 37, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6116, 2513, 2409, 2389, 2295, 2394, 2393, 2391, 193, 43}

$$\frac{d\text{PolyLog}\left(2,\frac{c(-a-bx+1)}{-ac+bd+c}\right)}{2c^2}-\frac{d\text{PolyLog}\left(2,\frac{c(a+bx+1)}{ac-bd+c}\right)}{2c^2}+\frac{d\log(a+bx-1)\log\left(\frac{b(cx+d)}{-ac+bd+c}\right)}{2c^2}-\frac{d\left(\log(a+bx-1)-\log\left(-\frac{-a-b\ldots}{a+\ldots}\right.\right.}{2\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Int[ArcCoth[a + b*x]/(c + d/x),x]
```

```
[Out] ((1 - a - b*x)*Log[-1 + a + b*x])/(2*b*c) + (x*(Log[-1 + a + b*x] - Log[-((
1 - a - b*x)/(a + b*x))] - Log[a + b*x]))/(2*c) + ((1 + a + b*x)*Log[1 + a
+ b*x])/(2*b*c) + (x*(Log[a + b*x] - Log[1 + a + b*x] + Log[(1 + a + b*x)/(
a + b*x)]))/(2*c) - (d*(Log[-1 + a + b*x] - Log[-((1 - a - b*x)/(a + b*x))]
 - Log[a + b*x])*Log[d + c*x])/(2*c^2) - (d*(Log[a + b*x] - Log[1 + a + b*x
] + Log[(1 + a + b*x)/(a + b*x)])*Log[d + c*x])/(2*c^2) - (d*Log[1 + a + b*
x]*Log[-((b*(d + c*x))/(c + a*c - b*d))])/(2*c^2) + (d*Log[-1 + a + b*x]*Lo
g[(b*(d + c*x))/(c - a*c + b*d)])/(2*c^2) + (d*PolyLog[2, (c*(1 - a - b*x))
/(c - a*c + b*d)])/(2*c^2) - (d*PolyLog[2, (c*(1 + a + b*x))/(c + a*c - b*d
)])/(2*c^2)
```

Rule 6116

```
Int[ArcCoth[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Dist
[1/2, Int[Log[(1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] - Dist[1/2, Int[
Log[(-1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x]
 && RationalQ[n]
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x}}\,dx = -\left(\frac{1}{2}\int \frac{\log\left(\frac{-1+a+bx}{a+bx}\right)}{c+\frac{d}{x}}\,dx\right)+\frac{1}{2}\int \frac{\log\left(\frac{1+a+bx}{a+bx}\right)}{c+\frac{d}{x}}\,dx$$

$$= -\left(\frac{1}{2}\int \frac{\log(-1+a+bx)}{c+\frac{d}{x}}\,dx\right)+\frac{1}{2}\int \frac{\log(1+a+bx)}{c+\frac{d}{x}}\,dx-\frac{1}{2}\left(-\log(-1+a+bx)+\log\left(\frac{-1+a+}{a+b}\right.\right.$$

$$= -\left(\frac{1}{2}\int \left(\frac{\log(-1+a+bx)}{c}-\frac{d\log(-1+a+bx)}{c(d+cx)}\right)dx\right)+\frac{1}{2}\int \left(\frac{\log(1+a+bx)}{c}-\frac{d\log(1+a+b}{c(d+cx)}\right.$$

$$= -\frac{\int \log(-1+a+bx)\,dx}{2c}+\frac{\int \log(1+a+bx)\,dx}{2c}+\frac{d\int \frac{\log(-1+a+bx)}{d+cx}\,dx}{2c}-\frac{d\int \frac{\log(1+a+bx)}{d+cx}\,dx}{2c}-\frac{1}{2}\left(-\right.$$

$$= \frac{x\left(\log(-1+a+bx)-\log\left(-\frac{1-a-bx}{a+bx}\right)-\log(a+bx)\right)}{2c}+\frac{x\left(\log(a+bx)-\log(1+a+bx)+\log\left(\frac{1+}{a}\right.\right.}{2c}$$

$$= \frac{(1-a-bx)\log(-1+a+bx)}{2bc}+\frac{x\left(\log(-1+a+bx)-\log\left(-\frac{1-a-bx}{a+bx}\right)-\log(a+bx)\right)}{2c}+\frac{(1+a+b}{}$$

$$= \frac{(1-a-bx)\log(-1+a+bx)}{2bc}+\frac{x\left(\log(-1+a+bx)-\log\left(-\frac{1-a-bx}{a+bx}\right)-\log(a+bx)\right)}{2c}+\frac{(1+a+b}{}$$

Mathematica [C] time = 4.25848, size = 502, normalized size = 1.72

$$\frac{bcd\text{PolyLog}\left(2,\exp\left(2\tanh^{-1}\left(\frac{c}{ac-bd}\right)-2\coth^{-1}(a+bx)\right)\right)-bcd\text{PolyLog}\left(2,e^{-2\coth^{-1}(a+bx)}\right)+b^2d^2\sqrt{1-\frac{c^2}{(ac-bd)^2}}\coth^{-}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[a + b*x]/(c + d/x),x]
```

```
[Out] (2*a*c^2*ArcCoth[a + b*x] - I*b*c*d*Pi*ArcCoth[a + b*x] + 2*b*c^2*x*ArcCoth
[a + b*x] + b*c*d*ArcCoth[a + b*x]^2 + a*b*c*d*ArcCoth[a + b*x]^2 - b^2*d^2
*ArcCoth[a + b*x]^2 - a*b*c*d*Sqrt[1 - c^2/(a*c - b*d)^2]*E^ArcTanh[c/(a*c
- b*d)]*ArcCoth[a + b*x]^2 + b^2*d^2*Sqrt[1 - c^2/(a*c - b*d)^2]*E^ArcTanh[
c/(a*c - b*d)]*ArcCoth[a + b*x]^2 + 2*b*c*d*ArcCoth[a + b*x]*ArcTanh[c/(a*c
 - b*d)] + 2*b*c*d*ArcCoth[a + b*x]*Log[1 - E^(-2*ArcCoth[a + b*x])] + I*b*
c*d*Pi*Log[1 + E^(2*ArcCoth[a + b*x])] - 2*b*c*d*ArcCoth[a + b*x]*Log[1 - E
^(-2*ArcCoth[a + b*x] + 2*ArcTanh[c/(a*c - b*d)])] + 2*b*c*d*ArcTanh[c/(a*c
 - b*d)]*Log[1 - E^(-2*ArcCoth[a + b*x] + 2*ArcTanh[c/(a*c - b*d)])] - I*b*
c*d*Pi*Log[1/Sqrt[1 - (a + b*x)^(-2)]] - 2*c^2*Log[1/((a + b*x)*Sqrt[1 - (a
 + b*x)^(-2)])] - 2*b*c*d*ArcTanh[c/(a*c - b*d)]*Log[I*Sinh[ArcCoth[a + b*x
] - ArcTanh[c/(a*c - b*d)]]] - b*c*d*PolyLog[2, E^(-2*ArcCoth[a + b*x])] +
b*c*d*PolyLog[2, E^(-2*ArcCoth[a + b*x] + 2*ArcTanh[c/(a*c - b*d)])])/(2*b*
c^3)
```

Maple [A] time = 0.142, size = 297, normalized size = 1.

$$\frac{x\text{arccoth}(bx+a)}{c}+\frac{\text{arccoth}(bx+a)\,a}{bc}-\frac{\text{arccoth}(bx+a)\,d\ln(c(bx+a)-ac+bd)}{c^2}+\frac{\ln\left(a^2c^2-2\,abcd+b^2d^2+2\,(c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)/(c+d/x),x)
```

```
[Out] arccoth(b*x+a)/c*x+1/b*arccoth(b*x+a)/c*a-arccoth(b*x+a)*d/c^2*ln(c*(b*x+a)
-a*c+b*d)+1/2/b/c*ln(a^2*c^2-2*a*b*c*d+b^2*d^2+2*(c*(b*x+a)-a*c+b*d)*a*c-2*
(c*(b*x+a)-a*c+b*d)*b*d+(c*(b*x+a)-a*c+b*d)^2-c^2)-1/2/c^2*d*ln(c*(b*x+a)-a
*c+b*d)*ln((c*(b*x+a)-c)/(a*c-b*d-c))-1/2/c^2*d*dilog((c*(b*x+a)-c)/(a*c-b*
d-c))+1/2/c^2*d*ln(c*(b*x+a)-a*c+b*d)*ln((c*(b*x+a)+c)/(a*c-b*d+c))+1/2/c^2
*d*dilog((c*(b*x+a)+c)/(a*c-b*d+c))
```

Maxima [A] time = 0.996411, size = 259, normalized size = 0.89

$$\frac{1}{2}b\left(\frac{\left(\log\left(cx+d\right)\log\left(\frac{bcx+bd}{ac-bd+c}+1\right)+\mathrm{Li}_2\left(-\frac{bcx+bd}{ac-bd+c}\right)\right)d}{bc^2}-\frac{\left(\log\left(cx+d\right)\log\left(\frac{bcx+bd}{ac-bd-c}+1\right)+\mathrm{Li}_2\left(-\frac{bcx+bd}{ac-bd-c}\right)\right)d}{bc^2}+\frac{(a+1)\mathrm{l}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(c+d/x),x, algorithm="maxima")
```

```
[Out] 1/2*b*((log(c*x + d)*log((b*c*x + b*d)/(a*c - b*d + c) + 1) + dilog(-(b*c*x
 + b*d)/(a*c - b*d + c)))*d/(b*c^2) - (log(c*x + d)*log((b*c*x + b*d)/(a*c
- b*d - c) + 1) + dilog(-(b*c*x + b*d)/(a*c - b*d - c)))*d/(b*c^2) + (a + 1
)*log(b*x + a + 1)/(b^2*c) - (a - 1)*log(b*x + a - 1)/(b^2*c)) + (x/c - d*l
og(c*x + d)/c^2)*arccoth(b*x + a)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x\,\mathrm{arcoth}\left(bx+a\right)}{cx+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(c+d/x),x, algorithm="fricas")
```

```
[Out] integral(x*arccoth(b*x + a)/(c*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)/(c+d/x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\mathrm{arcoth}\left(bx+a\right)}{c+\frac{d}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(c+d/x),x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a)/(c + d/x), x)
```

3.79 $\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx$

Optimal. Leaf size=738

$$\frac{\sqrt{d}\text{PolyLog}\left(2,\frac{\sqrt{-c}(-a-bx+1)}{(1-a)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}}-\frac{\sqrt{d}\text{PolyLog}\left(2,\frac{\sqrt{-c}(-a-bx+1)}{(1-a)\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}}+\frac{\sqrt{d}\text{PolyLog}\left(2,\frac{\sqrt{-c}(a+bx+1)}{(a+1)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}}-\frac{\sqrt{d}\text{PolyLog}\left(2,\frac{\sqrt{-}}{(a+}\right.}{4(-c)^{3/2}}$$

```
[Out] ((1 - a - b*x)*Log[-1 + a + b*x])/(2*b*c) + (x*(Log[-1 + a + b*x] - Log[-((
1 - a - b*x)/(a + b*x))] - Log[a + b*x]))/(2*c) - (Sqrt[d]*ArcTan[(Sqrt[c]*
x)/Sqrt[d]]*(Log[-1 + a + b*x] - Log[-((1 - a - b*x)/(a + b*x))] - Log[a +
b*x]))/(2*c^(3/2)) + ((1 + a + b*x)*Log[1 + a + b*x])/(2*b*c) + (x*(Log[a +
 b*x] - Log[1 + a + b*x] + Log[(1 + a + b*x)/(a + b*x)]))/(2*c) - (Sqrt[d]*
ArcTan[(Sqrt[c]*x)/Sqrt[d]]*(Log[a + b*x] - Log[1 + a + b*x] + Log[(1 + a +
 b*x)/(a + b*x)]))/(2*c^(3/2)) + (Sqrt[d]*Log[-1 + a + b*x]*Log[-((b*(Sqrt[
d] - Sqrt[-c]*x))/((1 - a)*Sqrt[-c] - b*Sqrt[d]))])/(4*(-c)^(3/2)) - (Sqrt[
d]*Log[1 + a + b*x]*Log[(b*(Sqrt[d] - Sqrt[-c]*x))/((1 + a)*Sqrt[-c] + b*Sq
rt[d])])/(4*(-c)^(3/2)) + (Sqrt[d]*Log[1 + a + b*x]*Log[-((b*(Sqrt[d] + Sqr
t[-c]*x))/((1 + a)*Sqrt[-c] - b*Sqrt[d]))])/(4*(-c)^(3/2)) - (Sqrt[d]*Log[-
1 + a + b*x]*Log[(b*(Sqrt[d] + Sqrt[-c]*x))/((1 - a)*Sqrt[-c] + b*Sqrt[d])]
)/(4*(-c)^(3/2)) + (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(1 - a - b*x))/((1 - a)*Sq
rt[-c] - b*Sqrt[d])])/(4*(-c)^(3/2)) - (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(1 - a
 - b*x))/((1 - a)*Sqrt[-c] + b*Sqrt[d])])/(4*(-c)^(3/2)) + (Sqrt[d]*PolyLog
[2, (Sqrt[-c]*(1 + a + b*x))/((1 + a)*Sqrt[-c] - b*Sqrt[d])])/(4*(-c)^(3/2)
) - (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(1 + a + b*x))/((1 + a)*Sqrt[-c] + b*Sqrt
[d])])/(4*(-c)^(3/2))
```

Rubi [A] time = 1.53032, antiderivative size = 738, normalized size of antiderivative = 1., number of steps used = 57, number of rules used = 11, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {6116, 2513, 2409, 2389, 2295, 2394, 2393, 2391, 193, 321, 205}

$$\frac{\sqrt{d}\text{PolyLog}\left(2,\frac{\sqrt{-c}(-a-bx+1)}{(1-a)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}}-\frac{\sqrt{d}\text{PolyLog}\left(2,\frac{\sqrt{-c}(-a-bx+1)}{(1-a)\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}}+\frac{\sqrt{d}\text{PolyLog}\left(2,\frac{\sqrt{-c}(a+bx+1)}{(a+1)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}}-\frac{\sqrt{d}\text{PolyLog}\left(2,\frac{\sqrt{-}}{(a+}\right.}{4(-c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x]/(c + d/x^2),x]
```


```
[Out] ((1 - a - b*x)*Log[-1 + a + b*x])/(2*b*c) + (x*(Log[-1 + a + b*x] - Log[-((
1 - a - b*x)/(a + b*x))] - Log[a + b*x]))/(2*c) - (Sqrt[d]*ArcTan[(Sqrt[c]*
x)/Sqrt[d]]*(Log[-1 + a + b*x] - Log[-((1 - a - b*x)/(a + b*x))] - Log[a +
b*x]))/(2*c^(3/2)) + ((1 + a + b*x)*Log[1 + a + b*x])/(2*b*c) + (x*(Log[a +
 b*x] - Log[1 + a + b*x] + Log[(1 + a + b*x)/(a + b*x)]))/(2*c) - (Sqrt[d]*
ArcTan[(Sqrt[c]*x)/Sqrt[d]]*(Log[a + b*x] - Log[1 + a + b*x] + Log[(1 + a +
 b*x)/(a + b*x)]))/(2*c^(3/2)) + (Sqrt[d]*Log[-1 + a + b*x]*Log[-((b*(Sqrt[
d] - Sqrt[-c]*x))/((1 - a)*Sqrt[-c] - b*Sqrt[d]))])/(4*(-c)^(3/2)) - (Sqrt[
d]*Log[1 + a + b*x]*Log[(b*(Sqrt[d] - Sqrt[-c]*x))/((1 + a)*Sqrt[-c] + b*Sq
rt[d])])/(4*(-c)^(3/2)) + (Sqrt[d]*Log[1 + a + b*x]*Log[-((b*(Sqrt[d] + Sqr
t[-c]*x))/((1 + a)*Sqrt[-c] - b*Sqrt[d]))])/(4*(-c)^(3/2)) - (Sqrt[d]*Log[-
1 + a + b*x]*Log[(b*(Sqrt[d] + Sqrt[-c]*x))/((1 - a)*Sqrt[-c] + b*Sqrt[d])]
)/(4*(-c)^(3/2)) + (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(1 - a - b*x))/((1 - a)*Sq
rt[-c] - b*Sqrt[d])])/(4*(-c)^(3/2)) - (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(1 - a
 - b*x))/((1 - a)*Sqrt[-c] + b*Sqrt[d])])/(4*(-c)^(3/2)) + (Sqrt[d]*PolyLog
[2, (Sqrt[-c]*(1 + a + b*x))/((1 + a)*Sqrt[-c] - b*Sqrt[d])])/(4*(-c)^(3/2)
```

```
) - (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(1 + a + b*x))/((1 + a)*Sqrt[-c] + b*Sqrt
[d])])/(4*(-c)^(3/2))
```

Rule 6116

```
Int[ArcCoth[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Dist
[1/2, Int[Log[(1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] - Dist[1/2, Int[
Log[(-1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x]
 && RationalQ[n]
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx = -\left(\frac{1}{2}\int \frac{\log\left(\frac{-1+a+bx}{a+bx}\right)}{c+\frac{d}{x^2}}\,dx\right)+\frac{1}{2}\int \frac{\log\left(\frac{1+a+bx}{a+bx}\right)}{c+\frac{d}{x^2}}\,dx$$

$$= -\left(\frac{1}{2}\int \frac{\log(-1+a+bx)}{c+\frac{d}{x^2}}\,dx\right)+\frac{1}{2}\int \frac{\log(1+a+bx)}{c+\frac{d}{x^2}}\,dx-\frac{1}{2}\left(-\log(-1+a+bx)+\log\left(\frac{-1+}{a}\right.\right.$$

$$= -\left(\frac{1}{2}\int\left(\frac{\log(-1+a+bx)}{c}-\frac{d\log(-1+a+bx)}{c\left(d+cx^2\right)}\right)dx\right)+\frac{1}{2}\int\left(\frac{\log(1+a+bx)}{c}-\frac{d\log(1+a}{c\left(d+cx^2\right.}\right.$$

$$= \frac{x\left(\log(-1+a+bx)-\log\left(-\frac{1-a-bx}{a+bx}\right)-\log(a+bx)\right)}{2c}+\frac{x\left(\log(a+bx)-\log(1+a+bx)+\log\right.}{2c}$$

$$= \frac{x\left(\log(-1+a+bx)-\log\left(-\frac{1-a-bx}{a+bx}\right)-\log(a+bx)\right)}{2c}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\left(\log(-1+a+bx)-\log\right.}{2c^{3/2}}$$

$$= \frac{(1-a-bx)\log(-1+a+bx)}{2bc}+\frac{x\left(\log(-1+a+bx)-\log\left(-\frac{1-a-bx}{a+bx}\right)-\log(a+bx)\right)}{2c}-\frac{\sqrt{d}\tan}{}$$

$$= \frac{(1-a-bx)\log(-1+a+bx)}{2bc}+\frac{x\left(\log(-1+a+bx)-\log\left(-\frac{1-a-bx}{a+bx}\right)-\log(a+bx)\right)}{2c}-\frac{\sqrt{d}\tan}{}$$

$$= \frac{(1-a-bx)\log(-1+a+bx)}{2bc}+\frac{x\left(\log(-1+a+bx)-\log\left(-\frac{1-a-bx}{a+bx}\right)-\log(a+bx)\right)}{2c}-\frac{\sqrt{d}\tan}{}$$

$$= \frac{(1-a-bx)\log(-1+a+bx)}{2bc}+\frac{x\left(\log(-1+a+bx)-\log\left(-\frac{1-a-bx}{a+bx}\right)-\log(a+bx)\right)}{2c}-\frac{\sqrt{d}\tan}{}$$

Mathematica [C] time = 35.6651, size = 5552, normalized size = 7.52

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[a + b*x]/(c + d/x^2),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 1.251, size = 19686, normalized size = 26.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)/(c+d/x^2),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(c+d/x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2 \operatorname{arcoth}(bx+a)}{cx^2+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(c+d/x^2),x, algorithm="fricas")
```

```
[Out] integral(x^2*arccoth(b*x + a)/(c*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)/(c+d/x**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(bx+a)}{c+\frac{d}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(c+d/x^2),x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a)/(c + d/x^2), x)
```

3.80 $\int \frac{\coth^{-1}(a+bx)}{c+d\sqrt{x}}\,dx$

Optimal. Leaf size=619

$$\frac{c\text{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}c-\sqrt{-a-1}d}\right)}{d^2}+\frac{c\text{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{-a-1}d+\sqrt{b}c}\right)}{d^2}-\frac{c\text{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}c-\sqrt{1-a}d}\right)}{d^2}-\frac{c\text{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{1-a}d+\sqrt{b}c}\right)}{d^2}+\frac{c\log}{}$$

```
[Out] (2*Sqrt[1 + a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[1 + a]])/(Sqrt[b]*d) - (2*Sqrt
[1 - a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[1 - a]])/(Sqrt[b]*d) + (c*Log[(d*(Sq
rt[-1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-1 - a]*d)]*Log[c + d*Sqrt
[x]])/d^2 - (c*Log[(d*(Sqrt[1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[1
- a]*d)]*Log[c + d*Sqrt[x]])/d^2 + (c*Log[-((d*(Sqrt[-1 - a] + Sqrt[b]*Sqrt
[x]))/(Sqrt[b]*c - Sqrt[-1 - a]*d))]*Log[c + d*Sqrt[x]])/d^2 - (c*Log[-((d*
(Sqrt[1 - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c - Sqrt[1 - a]*d))]*Log[c + d*Sq
rt[x]])/d^2 - (Sqrt[x]*Log[-((1 - a - b*x)/(a + b*x))])/d + (c*Log[c + d*Sq
rt[x]]*Log[-((1 - a - b*x)/(a + b*x))])/d^2 + (Sqrt[x]*Log[(1 + a + b*x)/(a
 + b*x)])/d - (c*Log[c + d*Sqrt[x]]*Log[(1 + a + b*x)/(a + b*x)])/d^2 + (c*
PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[-1 - a]*d)])/d^2 + (
c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-1 - a]*d)])/d^2 -
 (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[1 - a]*d)])/d^2
- (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[1 - a]*d)])/d^2
```

Rubi [A] time = 2.19896, antiderivative size = 619, normalized size of antiderivative = 1., number of steps used = 55, number of rules used = 16, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {6116, 190, 43, 2528, 2523, 12, 481, 205, 2524, 2418, 260, 2416, 2394, 2393, 2391, 208}

$$\frac{c\text{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}c-\sqrt{-a-1}d}\right)}{d^2}+\frac{c\text{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{-a-1}d+\sqrt{b}c}\right)}{d^2}-\frac{c\text{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}c-\sqrt{1-a}d}\right)}{d^2}-\frac{c\text{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{1-a}d+\sqrt{b}c}\right)}{d^2}+\frac{c\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x]/(c + d*Sqrt[x]),x]
```


```
[Out] (2*Sqrt[1 + a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[1 + a]])/(Sqrt[b]*d) - (2*Sqrt
[1 - a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[1 - a]])/(Sqrt[b]*d) + (c*Log[(d*(Sq
rt[-1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-1 - a]*d)]*Log[c + d*Sqrt
[x]])/d^2 - (c*Log[(d*(Sqrt[1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[1
- a]*d)]*Log[c + d*Sqrt[x]])/d^2 + (c*Log[-((d*(Sqrt[-1 - a] + Sqrt[b]*Sqrt
[x]))/(Sqrt[b]*c - Sqrt[-1 - a]*d))]*Log[c + d*Sqrt[x]])/d^2 - (c*Log[-((d*
(Sqrt[1 - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c - Sqrt[1 - a]*d))]*Log[c + d*Sq
rt[x]])/d^2 - (Sqrt[x]*Log[-((1 - a - b*x)/(a + b*x))])/d + (c*Log[c + d*Sq
rt[x]]*Log[-((1 - a - b*x)/(a + b*x))])/d^2 + (Sqrt[x]*Log[(1 + a + b*x)/(a
 + b*x)])/d - (c*Log[c + d*Sqrt[x]]*Log[(1 + a + b*x)/(a + b*x)])/d^2 + (c*
PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[-1 - a]*d)])/d^2 + (
c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-1 - a]*d)])/d^2 -
 (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[1 - a]*d)])/d^2
- (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[1 - a]*d)])/d^2
```


Rule 6116

```
Int[ArcCoth[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Dist
[1/2, Int[Log[(1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] - Dist[1/2, Int[
Log[(-1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x]
```

```
 && RationalQ[n]
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 2523

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[(x*(a + b*Log[c*
RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
```

```
RFx, x] && IntegerQ[p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\coth^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = -\left(\frac{1}{2}\int \frac{\log\left(\frac{-1+a+bx}{a+bx}\right)}{c+d\sqrt{x}}\,dx\right) + \frac{1}{2}\int \frac{\log\left(\frac{1+a+bx}{a+bx}\right)}{c+d\sqrt{x}}\,dx$$

$$= -\text{Subst}\left(\int \frac{x\log\left(\frac{-1+a+bx^2}{a+bx^2}\right)}{c+dx}\,dx, x, \sqrt{x}\right) + \text{Subst}\left(\int \frac{x\log\left(\frac{1+a+bx^2}{a+bx^2}\right)}{c+dx}\,dx, x, \sqrt{x}\right)$$

$$= -\text{Subst}\left(\int\left(\frac{\log\left(\frac{-1+a+bx^2}{a+bx^2}\right)}{d} - \frac{c\log\left(\frac{-1+a+bx^2}{a+bx^2}\right)}{d(c+dx)}\right)dx, x, \sqrt{x}\right) + \text{Subst}\left(\int\left(\frac{\log\left(\frac{1+a+bx^2}{a+bx^2}\right)}{d} - \frac{c\,\mathrm{l}\ldots}{\ldots}\right.\right.$$

$$= -\frac{\text{Subst}\left(\int \log\left(\frac{-1+a+bx^2}{a+bx^2}\right)dx, x, \sqrt{x}\right)}{d} + \frac{\text{Subst}\left(\int \log\left(\frac{1+a+bx^2}{a+bx^2}\right)dx, x, \sqrt{x}\right)}{d} + \frac{c\,\text{Subst}\left(\int \frac{\log(\ldots}{\ldots}\right.}{\ldots}$$

$$= -\frac{\sqrt{x}\log\left(-\frac{1-a-bx}{a+bx}\right)}{d} + \frac{c\log\left(c+d\sqrt{x}\right)\log\left(-\frac{1-a-bx}{a+bx}\right)}{d^2} + \frac{\sqrt{x}\log\left(\frac{1+a+bx}{a+bx}\right)}{d} - \frac{c\log\left(c+d\sqrt{x}\right)\log\ldots}{d^2}$$

$$= -\frac{\sqrt{x}\log\left(-\frac{1-a-bx}{a+bx}\right)}{d} + \frac{c\log\left(c+d\sqrt{x}\right)\log\left(-\frac{1-a-bx}{a+bx}\right)}{d^2} + \frac{\sqrt{x}\log\left(\frac{1+a+bx}{a+bx}\right)}{d} - \frac{c\log\left(c+d\sqrt{x}\right)\log\ldots}{d^2}$$

$$= -\frac{\sqrt{x}\log\left(-\frac{1-a-bx}{a+bx}\right)}{d} + \frac{c\log\left(c+d\sqrt{x}\right)\log\left(-\frac{1-a-bx}{a+bx}\right)}{d^2} + \frac{\sqrt{x}\log\left(\frac{1+a+bx}{a+bx}\right)}{d} - \frac{c\log\left(c+d\sqrt{x}\right)\log\ldots}{d^2}$$

$$= \frac{2\sqrt{1+a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}d} - \frac{2\sqrt{1-a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}d} - \frac{\sqrt{x}\log\left(-\frac{1-a-bx}{a+bx}\right)}{d} + \frac{c\log\left(c+d\sqrt{x}\right)\log\left(\ldots\right.}{d^2}$$

$$= \frac{2\sqrt{1+a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}d} - \frac{2\sqrt{1-a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}d} - \frac{\sqrt{x}\log\left(-\frac{1-a-bx}{a+bx}\right)}{d} + \frac{c\log\left(c+d\sqrt{x}\right)\log\left(\ldots\right.}{d^2}$$

$$= \frac{2\sqrt{1+a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}d} - \frac{2\sqrt{1-a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}d} + \frac{c\log\left(\frac{d\left(\sqrt{-1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-1-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2} - \frac{c\,\mathrm{l}\ldots}{\ldots}$$

$$= \frac{2\sqrt{1+a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}d} - \frac{2\sqrt{1-a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}d} + \frac{c\log\left(\frac{d\left(\sqrt{-1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-1-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2} - \frac{c\,\mathrm{l}\ldots}{\ldots}$$

$$= \frac{2\sqrt{1+a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}d} - \frac{2\sqrt{1-a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}d} + \frac{c\log\left(\frac{d\left(\sqrt{-1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-1-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2} - \frac{c\,\mathrm{l}\ldots}{\ldots}$$

<u>Mathematica</u> [A] time = 0.666141, size = 575, normalized size = 0.93

$$\frac{c\text{PolyLog}\left(2, \frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}c-\sqrt{-a-1}d}\right) + c\text{PolyLog}\left(2, \frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{-a-1}d+\sqrt{b}c}\right) - c\text{PolyLog}\left(2, \frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}c-\sqrt{1-a}d}\right) - c\text{PolyLog}\left(2, \frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{1-a}d+\sqrt{b}c}\right) + c\log\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a + b*x]/(c + d*Sqrt[x]),x]
```

```
[Out] ((2*Sqrt[1 + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[1 + a]])/Sqrt[b] - (2*Sqrt[
1 - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[1 - a]])/Sqrt[b] + c*Log[(d*(Sqrt[-
1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-1 - a]*d)]*Log[c + d*Sqrt[x]]
 - c*Log[(d*(Sqrt[1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[1 - a]*d)]*L
og[c + d*Sqrt[x]] + c*Log[(d*(Sqrt[-1 - a] + Sqrt[b]*Sqrt[x]))/(-(Sqrt[b]*c
) + Sqrt[-1 - a]*d)]*Log[c + d*Sqrt[x]] - c*Log[(d*(Sqrt[1 - a] + Sqrt[b]*S
qrt[x]))/(-(Sqrt[b]*c) + Sqrt[1 - a]*d)]*Log[c + d*Sqrt[x]] - d*Sqrt[x]*Log
[(-1 + a + b*x)/(a + b*x)] + c*Log[c + d*Sqrt[x]]*Log[(-1 + a + b*x)/(a + b
*x)] + d*Sqrt[x]*Log[(1 + a + b*x)/(a + b*x)] - c*Log[c + d*Sqrt[x]]*Log[(1
 + a + b*x)/(a + b*x)] + c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c
- Sqrt[-1 - a]*d)] + c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sq
rt[-1 - a]*d)] - c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[1
 - a]*d)] - c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[1 - a]
*d)])/d^2
```

Maple [A] time = 0.191, size = 738, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)/(c+d*x^(1/2)),x)
```

```
[Out] 2*arccoth(b*x+a)*x^(1/2)/d-2*arccoth(b*x+a)*c/d^2*ln(c+d*x^(1/2))+2/(a*b*d^
2+b*d^2)^(1/2)*arctan(1/2*(2*b*(c+d*x^(1/2))-2*b*c)/(a*b*d^2+b*d^2)^(1/2))+
2/(a*b*d^2+b*d^2)^(1/2)*arctan(1/2*(2*b*(c+d*x^(1/2))-2*b*c)/(a*b*d^2+b*d^2
)^(1/2))*a+2/(a*b*d^2-b*d^2)^(1/2)*arctan(1/2*(2*b*(c+d*x^(1/2))-2*b*c)/(a*
b*d^2-b*d^2)^(1/2))-2/(a*b*d^2-b*d^2)^(1/2)*arctan(1/2*(2*b*(c+d*x^(1/2))-2
*b*c)/(a*b*d^2-b*d^2)^(1/2))*a+1/d^2*c*ln(c+d*x^(1/2))*ln((-b*(c+d*x^(1/2))
+b*c+(-a*b*d^2-b*d^2)^(1/2))/(b*c+(-a*b*d^2-b*d^2)^(1/2)))+1/d^2*c*ln(c+d*x
^(1/2))*ln((b*(c+d*x^(1/2))-b*c+(-a*b*d^2-b*d^2)^(1/2))/(-b*c+(-a*b*d^2-b*d
^2)^(1/2)))+1/d^2*c*dilog((-b*(c+d*x^(1/2))+b*c+(-a*b*d^2-b*d^2)^(1/2))/(b*
c+(-a*b*d^2-b*d^2)^(1/2)))+1/d^2*c*dilog((b*(c+d*x^(1/2))-b*c+(-a*b*d^2-b*d
^2)^(1/2))/(-b*c+(-a*b*d^2-b*d^2)^(1/2)))-1/d^2*c*ln(c+d*x^(1/2))*ln((-b*(c
+d*x^(1/2))+b*c+(-a*b*d^2+b*d^2)^(1/2))/(b*c+(-a*b*d^2+b*d^2)^(1/2)))-1/d^2
*c*ln(c+d*x^(1/2))*ln((b*(c+d*x^(1/2))-b*c+(-a*b*d^2+b*d^2)^(1/2))/(-b*c+(-
a*b*d^2+b*d^2)^(1/2)))-1/d^2*c*dilog((-b*(c+d*x^(1/2))+b*c+(-a*b*d^2+b*d^2)
^(1/2))/(b*c+(-a*b*d^2+b*d^2)^(1/2)))-1/d^2*c*dilog((b*(c+d*x^(1/2))-b*c+(-
a*b*d^2+b*d^2)^(1/2))/(-b*c+(-a*b*d^2+b*d^2)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(bx+a)}{d\sqrt{x}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(c+d*x^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(arccoth(b*x + a)/(d*sqrt(x) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d\sqrt{x}\operatorname{arcoth}(bx+a) - c\operatorname{arcoth}(bx+a)}{d^2x - c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(c+d*x^(1/2)),x, algorithm="fricas")
```

```
[Out] integral((d*sqrt(x)*arccoth(b*x + a) - c*arccoth(b*x + a))/(d^2*x - c^2), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)/(c+d*x**(1/2)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(bx+a)}{d\sqrt{x}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(c+d*x^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a)/(d*sqrt(x) + c), x)
```

3.81 $\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx$

Optimal. Leaf size=738

$$-\frac{d^2\text{PolyLog}\left(2,-\frac{\sqrt{b}\left(c\sqrt{x}+d\right)}{\sqrt{-a-1}c-\sqrt{b}d}\right)}{c^3}+\frac{d^2\text{PolyLog}\left(2,-\frac{\sqrt{b}\left(c\sqrt{x}+d\right)}{\sqrt{1-a}c-\sqrt{b}d}\right)}{c^3}-\frac{d^2\text{PolyLog}\left(2,\frac{\sqrt{b}\left(c\sqrt{x}+d\right)}{\sqrt{-a-1}c+\sqrt{b}d}\right)}{c^3}+\frac{d^2\text{PolyLog}\left(2,\frac{\sqrt{b}\left(c\sqrt{x}+d\right)}{\sqrt{1-a}c+\sqrt{b}d}\right)}{c^3}$$

```
[Out] (-2*Sqrt[1 + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[1 + a]])/(Sqrt[b]*c^2) + (2
*Sqrt[1 - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[1 - a]])/(Sqrt[b]*c^2) - (d^2
*Log[(c*(Sqrt[-1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-1 - a]*c + Sqrt[b]*d)]*Log
[d + c*Sqrt[x]])/c^3 + (d^2*Log[(c*(Sqrt[1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[1
 - a]*c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 - (d^2*Log[(c*(Sqrt[-1 - a] +
 Sqrt[b]*Sqrt[x]))/(Sqrt[-1 - a]*c - Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 +
(d^2*Log[(c*(Sqrt[1 - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[1 - a]*c - Sqrt[b]*d)]*L
og[d + c*Sqrt[x]])/c^3 + ((1 - a)*Log[1 - a - b*x])/(2*b*c) + (d*Sqrt[x]*Lo
g[-((1 - a - b*x)/(a + b*x))])/c^2 - (x*Log[-((1 - a - b*x)/(a + b*x))])/(2
*c) - (d^2*Log[d + c*Sqrt[x]]*Log[-((1 - a - b*x)/(a + b*x))])/c^3 + ((1 +
a)*Log[1 + a + b*x])/(2*b*c) - (d*Sqrt[x]*Log[(1 + a + b*x)/(a + b*x)])/c^2
 + (x*Log[(1 + a + b*x)/(a + b*x)])/(2*c) + (d^2*Log[d + c*Sqrt[x]]*Log[(1
+ a + b*x)/(a + b*x)])/c^3 - (d^2*PolyLog[2, -((Sqrt[b]*(d + c*Sqrt[x]))/(S
qrt[-1 - a]*c - Sqrt[b]*d))])/c^3 + (d^2*PolyLog[2, -((Sqrt[b]*(d + c*Sqrt[
x]))/(Sqrt[1 - a]*c - Sqrt[b]*d))])/c^3 - (d^2*PolyLog[2, (Sqrt[b]*(d + c*S
qrt[x]))/(Sqrt[-1 - a]*c + Sqrt[b]*d)])/c^3 + (d^2*PolyLog[2, (Sqrt[b]*(d +
 c*Sqrt[x]))/(Sqrt[1 - a]*c + Sqrt[b]*d)])/c^3
```

Rubi [A] time = 2.37149, antiderivative size = 738, normalized size of antiderivative = 1., number of steps used = 65, number of rules used = 19, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.056, Rules used = {6116, 190, 44, 2528, 2523, 12, 481, 205, 2525, 446, 72, 2524, 2418, 260, 2416, 2394, 2393, 2391, 208}

$$-\frac{d^2\text{PolyLog}\left(2,-\frac{\sqrt{b}\left(c\sqrt{x}+d\right)}{\sqrt{-a-1}c-\sqrt{b}d}\right)}{c^3}+\frac{d^2\text{PolyLog}\left(2,-\frac{\sqrt{b}\left(c\sqrt{x}+d\right)}{\sqrt{1-a}c-\sqrt{b}d}\right)}{c^3}-\frac{d^2\text{PolyLog}\left(2,\frac{\sqrt{b}\left(c\sqrt{x}+d\right)}{\sqrt{-a-1}c+\sqrt{b}d}\right)}{c^3}+\frac{d^2\text{PolyLog}\left(2,\frac{\sqrt{b}\left(c\sqrt{x}+d\right)}{\sqrt{1-a}c+\sqrt{b}d}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x]/(c + d/Sqrt[x]),x]
```

```
[Out] (-2*Sqrt[1 + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[1 + a]])/(Sqrt[b]*c^2) + (2
*Sqrt[1 - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[1 - a]])/(Sqrt[b]*c^2) - (d^2
*Log[(c*(Sqrt[-1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-1 - a]*c + Sqrt[b]*d)]*Log
[d + c*Sqrt[x]])/c^3 + (d^2*Log[(c*(Sqrt[1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[1
 - a]*c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 - (d^2*Log[(c*(Sqrt[-1 - a] +
 Sqrt[b]*Sqrt[x]))/(Sqrt[-1 - a]*c - Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 +
(d^2*Log[(c*(Sqrt[1 - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[1 - a]*c - Sqrt[b]*d)]*L
og[d + c*Sqrt[x]])/c^3 + ((1 - a)*Log[1 - a - b*x])/(2*b*c) + (d*Sqrt[x]*Lo
g[-((1 - a - b*x)/(a + b*x))])/c^2 - (x*Log[-((1 - a - b*x)/(a + b*x))])/(2
*c) - (d^2*Log[d + c*Sqrt[x]]*Log[-((1 - a - b*x)/(a + b*x))])/c^3 + ((1 +
a)*Log[1 + a + b*x])/(2*b*c) - (d*Sqrt[x]*Log[(1 + a + b*x)/(a + b*x)])/c^2
 + (x*Log[(1 + a + b*x)/(a + b*x)])/(2*c) + (d^2*Log[d + c*Sqrt[x]]*Log[(1
+ a + b*x)/(a + b*x)])/c^3 - (d^2*PolyLog[2, -((Sqrt[b]*(d + c*Sqrt[x]))/(S
qrt[-1 - a]*c - Sqrt[b]*d))])/c^3 + (d^2*PolyLog[2, -((Sqrt[b]*(d + c*Sqrt[
x]))/(Sqrt[1 - a]*c - Sqrt[b]*d))])/c^3 - (d^2*PolyLog[2, (Sqrt[b]*(d + c*S
qrt[x]))/(Sqrt[-1 - a]*c + Sqrt[b]*d)])/c^3 + (d^2*PolyLog[2, (Sqrt[b]*(d +
```

```
 c*Sqrt[x]))/(Sqrt[1 - a]*c + Sqrt[b]*d)])/c^3
```

Rule 6116

```
Int[ArcCoth[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Dist
[1/2, Int[Log[(1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] - Dist[1/2, Int[
Log[(-1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x]
 && RationalQ[n]
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 2523

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[(x*(a + b*Log[c*
RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
```

```
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = -\left(\frac{1}{2}\int \frac{\log\left(\frac{-1+a+bx}{a+bx}\right)}{c+\frac{d}{\sqrt{x}}}\,dx\right)+\frac{1}{2}\int \frac{\log\left(\frac{1+a+bx}{a+bx}\right)}{c+\frac{d}{\sqrt{x}}}\,dx$$

$$= -\text{Subst}\left(\int \frac{x^2\log\left(\frac{-1+a+bx^2}{a+bx^2}\right)}{d+cx}\,dx, x, \sqrt{x}\right)+\text{Subst}\left(\int \frac{x^2\log\left(\frac{1+a+bx^2}{a+bx^2}\right)}{d+cx}\,dx, x, \sqrt{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{d\log\left(\frac{-1+a+bx^2}{a+bx^2}\right)}{c^2}+\frac{x\log\left(\frac{-1+a+bx^2}{a+bx^2}\right)}{c}+\frac{d^2\log\left(\frac{-1+a+bx^2}{a+bx^2}\right)}{c^2(d+cx)}\right)dx, x, \sqrt{x}\right)+\text{Subst}\Bigg($$

$$= -\frac{\text{Subst}\left(\int x\log\left(\frac{-1+a+bx^2}{a+bx^2}\right)dx, x, \sqrt{x}\right)}{c}+\frac{\text{Subst}\left(\int x\log\left(\frac{1+a+bx^2}{a+bx^2}\right)dx, x, \sqrt{x}\right)}{c}+\frac{d\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{d\sqrt{x}\log\left(-\frac{1-a-bx}{a+bx}\right)}{c^2}-\frac{x\log\left(-\frac{1-a-bx}{a+bx}\right)}{2c}-\frac{d^2\log\left(d+c\sqrt{x}\right)\log\left(-\frac{1-a-bx}{a+bx}\right)}{c^3}-\frac{d\sqrt{x}\log\left(\frac{1+a+bx}{a+bx}\right)}{c^2}+$$

$$= \frac{d\sqrt{x}\log\left(-\frac{1-a-bx}{a+bx}\right)}{c^2}-\frac{x\log\left(-\frac{1-a-bx}{a+bx}\right)}{2c}-\frac{d^2\log\left(d+c\sqrt{x}\right)\log\left(-\frac{1-a-bx}{a+bx}\right)}{c^3}-\frac{d\sqrt{x}\log\left(\frac{1+a+bx}{a+bx}\right)}{c^2}+$$

$$= \frac{d\sqrt{x}\log\left(-\frac{1-a-bx}{a+bx}\right)}{c^2}-\frac{x\log\left(-\frac{1-a-bx}{a+bx}\right)}{2c}-\frac{d^2\log\left(d+c\sqrt{x}\right)\log\left(-\frac{1-a-bx}{a+bx}\right)}{c^3}-\frac{d\sqrt{x}\log\left(\frac{1+a+bx}{a+bx}\right)}{c^2}+$$

$$= -\frac{2\sqrt{1+a}\,d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}c^2}+\frac{2\sqrt{1-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}c^2}+\frac{d\sqrt{x}\log\left(-\frac{1-a-bx}{a+bx}\right)}{c^2}-\frac{x\log\left(-\frac{1-a-bx}{a+bx}\right)}{2c}$$

$$= -\frac{2\sqrt{1+a}\,d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}c^2}+\frac{2\sqrt{1-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}c^2}+\frac{(1-a)\log(1-a-bx)}{2bc}+\frac{d\sqrt{x}\log\left(-\right.}{c^2}$$

$$= -\frac{2\sqrt{1+a}\,d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}c^2}+\frac{2\sqrt{1-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}c^2}-\frac{d^2\log\left(\frac{c\left(\sqrt{-1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-1-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3}$$

$$= -\frac{2\sqrt{1+a}\,d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}c^2}+\frac{2\sqrt{1-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}c^2}-\frac{d^2\log\left(\frac{c\left(\sqrt{-1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-1-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3}$$

$$= -\frac{2\sqrt{1+a}\,d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}c^2}+\frac{2\sqrt{1-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}c^2}-\frac{d^2\log\left(\frac{c\left(\sqrt{-1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-1-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3}$$

Mathematica [A] time = 0.693836, size = 719, normalized size = 0.97

$$-2bd^2\text{PolyLog}\left(2,\frac{\sqrt{b}(c\sqrt{x}+d)}{\sqrt{b}d-\sqrt{-a-1}c}\right)-2bd^2\text{PolyLog}\left(2,\frac{\sqrt{b}(c\sqrt{x}+d)}{\sqrt{-a-1}c+\sqrt{b}d}\right)+2bd^2\text{PolyLog}\left(2,\frac{\sqrt{b}(c\sqrt{x}+d)}{\sqrt{b}d-\sqrt{1-a}c}\right)+2bd^2\text{PolyLog}\left(2,\frac{\sqrt{b}(c\sqrt{x}+d)}{\sqrt{1-a}c+\sqrt{b}d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a + b*x]/(c + d/Sqrt[x]),x]
```

```
[Out] (-4*Sqrt[1 + a]*Sqrt[b]*c*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[1 + a]] + 4*Sqrt[
1 - a]*Sqrt[b]*c*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[1 - a]] - 2*b*d^2*Log[(c*
(Sqrt[-1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-1 - a]*c + Sqrt[b]*d)]*Log[d + c*S
qrt[x]] + 2*b*d^2*Log[(c*(Sqrt[1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[1 - a]*c +
Sqrt[b]*d)]*Log[d + c*Sqrt[x]] - 2*b*d^2*Log[(c*(Sqrt[-1 - a] + Sqrt[b]*Sqr
t[x]))/(Sqrt[-1 - a]*c - Sqrt[b]*d)]*Log[d + c*Sqrt[x]] + 2*b*d^2*Log[(c*(S
qrt[1 - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[1 - a]*c - Sqrt[b]*d)]*Log[d + c*Sqrt[
x]] + c^2*Log[1 - a - b*x] - a*c^2*Log[1 - a - b*x] + 2*b*c*d*Sqrt[x]*Log[(
-1 + a + b*x)/(a + b*x)] - b*c^2*x*Log[(-1 + a + b*x)/(a + b*x)] - 2*b*d^2*
Log[d + c*Sqrt[x]]*Log[(-1 + a + b*x)/(a + b*x)] + c^2*Log[1 + a + b*x] + a
*c^2*Log[1 + a + b*x] - 2*b*c*d*Sqrt[x]*Log[(1 + a + b*x)/(a + b*x)] + b*c^
2*x*Log[(1 + a + b*x)/(a + b*x)] + 2*b*d^2*Log[d + c*Sqrt[x]]*Log[(1 + a +
b*x)/(a + b*x)] - 2*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(-(Sqrt[-1 -
 a]*c) + Sqrt[b]*d)] - 2*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-
1 - a]*c + Sqrt[b]*d)] + 2*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(-(Sq
rt[1 - a]*c) + Sqrt[b]*d)] + 2*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(
Sqrt[1 - a]*c + Sqrt[b]*d)])/(2*b*c^3)
```

Maple [A] time = 0.161, size = 970, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)/(c+d/x^(1/2)),x)
```

```
[Out] arccoth(b*x+a)/c*x-2*arccoth(b*x+a)/c^2*d*x^(1/2)+2*arccoth(b*x+a)*d^2/c^3*
ln(d+c*x^(1/2))+1/c^3*d^2*ln(d+c*x^(1/2))*ln((-b*(d+c*x^(1/2))+b*d+(-a*b*c^
2+b*c^2)^(1/2))/(b*d+(-a*b*c^2+b*c^2)^(1/2)))+1/c^3*d^2*ln(d+c*x^(1/2))*ln(
(b*(d+c*x^(1/2))-b*d+(-a*b*c^2+b*c^2)^(1/2))/(-b*d+(-a*b*c^2+b*c^2)^(1/2)))
+1/c^3*d^2*dilog((-b*(d+c*x^(1/2))+b*d+(-a*b*c^2+b*c^2)^(1/2))/(b*d+(-a*b*c
^2+b*c^2)^(1/2)))+1/c^3*d^2*dilog((b*(d+c*x^(1/2))-b*d+(-a*b*c^2+b*c^2)^(1/
2))/(-b*d+(-a*b*c^2+b*c^2)^(1/2)))-1/c^3*d^2*ln(d+c*x^(1/2))*ln((-b*(d+c*x^
(1/2))+b*d+(-a*b*c^2-b*c^2)^(1/2))/(b*d+(-a*b*c^2-b*c^2)^(1/2)))-1/c^3*d^2*
ln(d+c*x^(1/2))*ln((b*(d+c*x^(1/2))-b*d+(-a*b*c^2-b*c^2)^(1/2))/(-b*d+(-a*b
*c^2-b*c^2)^(1/2)))-1/c^3*d^2*dilog((-b*(d+c*x^(1/2))+b*d+(-a*b*c^2-b*c^2)^
(1/2))/(b*d+(-a*b*c^2-b*c^2)^(1/2)))-1/c^3*d^2*dilog((b*(d+c*x^(1/2))-b*d+(
-a*b*c^2-b*c^2)^(1/2))/(-b*d+(-a*b*c^2-b*c^2)^(1/2)))-1/2/b/c*a*ln(b*(d+c*x
^(1/2))^2-2*(d+c*x^(1/2))*b*d+a*c^2+b*d^2-c^2)+2/c*a*d/(a*b*c^2-b*c^2)^(1/2
)*arctan(1/2*(2*b*(d+c*x^(1/2))-2*b*d)/(a*b*c^2-b*c^2)^(1/2))+1/2/b/c*ln(b*
(d+c*x^(1/2))^2-2*(d+c*x^(1/2))*b*d+a*c^2+b*d^2-c^2)-2/c*d/(a*b*c^2-b*c^2)^
(1/2)*arctan(1/2*(2*b*(d+c*x^(1/2))-2*b*d)/(a*b*c^2-b*c^2)^(1/2))+1/2/b/c*l
n(b*(d+c*x^(1/2))^2-2*(d+c*x^(1/2))*b*d+a*c^2+b*d^2+c^2)-2/c*d/(a*b*c^2+b*c
^2)^(1/2)*arctan(1/2*(2*b*(d+c*x^(1/2))-2*b*d)/(a*b*c^2+b*c^2)^(1/2))+1/2/b
/c*a*ln(b*(d+c*x^(1/2))^2-2*(d+c*x^(1/2))*b*d+a*c^2+b*d^2+c^2)-2/c*a*d/(a*b
*c^2+b*c^2)^(1/2)*arctan(1/2*(2*b*(d+c*x^(1/2))-2*b*d)/(a*b*c^2+b*c^2)^(1/2
```

```
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(bx+a+1)\log(bx+a+1)-(bx+a-1)\log(bx+a-1)}{2\,bc}-\frac{1}{2}\int\frac{d\log(bx+a+1)-d\log(bx+a-1)}{c^2\sqrt{x}+cd}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(c+d/x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*((b*x + a + 1)*log(b*x + a + 1) - (b*x + a - 1)*log(b*x + a - 1))/(b*c)
 - 1/2*integrate((d*log(b*x + a + 1) - d*log(b*x + a - 1))/(c^2*sqrt(x) + c
*d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{cx\operatorname{arcoth}(bx+a)-d\sqrt{x}\operatorname{arcoth}(bx+a)}{c^2x-d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(c+d/x^(1/2)),x, algorithm="fricas")
```

```
[Out] integral((c*x*arccoth(b*x + a) - d*sqrt(x)*arccoth(b*x + a))/(c^2*x - d^2),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)/(c+d/x**(1/2)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{arcoth}(bx+a)}{c+\frac{d}{\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(c+d/x^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a)/(c + d/sqrt(x)), x)
```

3.82 $\int \frac{\coth^{-1}(d+ex)}{a+bx+cx^2}\,dx$

Optimal. Leaf size=335

$$-\frac{\text{PolyLog}\left(2,\frac{2\left(-e\left(b-\sqrt{b^2-4ac}\right)-2c(d+ex)+2cd\right)}{(d+ex+1)\left(-e\sqrt{b^2-4ac}+be-2cd+2c\right)}+1\right)}{2\sqrt{b^2-4ac}}+\frac{\text{PolyLog}\left(2,\frac{2\left(-e\left(\sqrt{b^2-4ac}+b\right)-2c(d+ex)+2cd\right)}{(d+ex+1)\left(e\left(\sqrt{b^2-4ac}+b\right)+2c(1-d)\right)}+1\right)}{2\sqrt{b^2-4ac}}+\frac{\coth^{-1}(d+ex)\log\left(\cdots\right)}{\sqrt{b^{\cdots}}}$$

```
[Out] (ArcCoth[d + e*x]*Log[(2*e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*(1 - d) +
 (b - Sqrt[b^2 - 4*a*c])*e)*(1 + d + e*x))])/Sqrt[b^2 - 4*a*c] - (ArcCoth[d
 + e*x]*Log[(2*e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*(1 - d) + (b + Sqrt
[b^2 - 4*a*c])*e)*(1 + d + e*x))])/Sqrt[b^2 - 4*a*c] - PolyLog[2, 1 + (2*(2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*x)))/((2*c - 2*c*d + b*e - Sq
rt[b^2 - 4*a*c]*e)*(1 + d + e*x))]/(2*Sqrt[b^2 - 4*a*c]) + PolyLog[2, 1 + (
2*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*x)))/((2*c*(1 - d) + (b +
 Sqrt[b^2 - 4*a*c])*e)*(1 + d + e*x))]/(2*Sqrt[b^2 - 4*a*c])
```

Rubi [A] time = 0.748577, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {618, 206, 6728, 6112, 5921, 2402, 2315, 2447}

$$-\frac{\text{PolyLog}\left(2,\frac{2\left(-e\left(b-\sqrt{b^2-4ac}\right)-2c(d+ex)+2cd\right)}{(d+ex+1)\left(-e\sqrt{b^2-4ac}+be-2cd+2c\right)}+1\right)}{2\sqrt{b^2-4ac}}+\frac{\text{PolyLog}\left(2,\frac{2\left(-e\left(\sqrt{b^2-4ac}+b\right)-2c(d+ex)+2cd\right)}{(d+ex+1)\left(e\left(\sqrt{b^2-4ac}+b\right)+2c(1-d)\right)}+1\right)}{2\sqrt{b^2-4ac}}+\frac{\coth^{-1}(d+ex)\log\left(\cdots\right)}{\sqrt{b^{\cdots}}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] (ArcCoth[d + e*x]*Log[(2*e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*(1 - d) +
 (b - Sqrt[b^2 - 4*a*c])*e)*(1 + d + e*x))])/Sqrt[b^2 - 4*a*c] - (ArcCoth[d
 + e*x]*Log[(2*e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*(1 - d) + (b + Sqrt
[b^2 - 4*a*c])*e)*(1 + d + e*x))])/Sqrt[b^2 - 4*a*c] - PolyLog[2, 1 + (2*(2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*x)))/((2*c - 2*c*d + b*e - Sq
rt[b^2 - 4*a*c]*e)*(1 + d + e*x))]/(2*Sqrt[b^2 - 4*a*c]) + PolyLog[2, 1 + (
2*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*x)))/((2*c*(1 - d) + (b +
 Sqrt[b^2 - 4*a*c])*e)*(1 + d + e*x))]/(2*Sqrt[b^2 - 4*a*c])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
```

```
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rule 5921

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcCoth[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcCoth[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\coth^{-1}(d+ex)}{a+bx+cx^2}\,dx = \int \left(\frac{2c\coth^{-1}(d+ex)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)} - \frac{2c\coth^{-1}(d+ex)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}+2cx\right)}\right)dx$$

$$= \frac{(2c)\int \frac{\coth^{-1}(d+ex)}{b-\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{\coth^{-1}(d+ex)}{b+\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{(2c)\,\text{Subst}\left(\int \frac{\coth^{-1}(x)}{\frac{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}{e}+\frac{2cx}{e}}\,dx, x, d+ex\right)}{\sqrt{b^2-4ac}e} - \frac{(2c)\,\text{Subst}\left(\int \frac{\coth^{-1}(x)}{\frac{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}{e}+\frac{2cx}{e}}\,dx, x, d+ex\right)}{\sqrt{b^2-4ac}e}$$

$$= \frac{\coth^{-1}(d+ex)\log\left(\frac{2e\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(2c-2cd+be-\sqrt{b^2-4ac}e\right)(1+d+ex)}\right)}{\sqrt{b^2-4ac}} - \frac{\coth^{-1}(d+ex)\log\left(\frac{2e\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2c(1-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(1+d+ex}\right.}{\sqrt{b^2-4ac}}$$

$$= \frac{\coth^{-1}(d+ex)\log\left(\frac{2e\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(2c-2cd+be-\sqrt{b^2-4ac}e\right)(1+d+ex)}\right)}{\sqrt{b^2-4ac}} - \frac{\coth^{-1}(d+ex)\log\left(\frac{2e\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2c(1-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(1+d+ex}\right.}{\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.898204, size = 596, normalized size = 1.78

$$-\text{PolyLog}\left(2, \frac{e\left(\sqrt{b^2-4ac}-b-2cx\right)}{e\left(\sqrt{b^2-4ac}-b\right)+2c(d+1)}\right) + \text{PolyLog}\left(2, \frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{-e\sqrt{b^2-4ac}+be-2cd+2c}\right) - \text{PolyLog}\left(2, \frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{e\left(\sqrt{b^2-4ac}+b\right)-2c(d-1)}\right) + \text{PolyLog}\left(2,\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] (Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[(2*c*(-1 + d + e*x))/(2*c*(-1 + d)
+ (-b + Sqrt[b^2 - 4*a*c])*e)] - Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[(2*
c*(-1 + d + e*x))/(2*c*(-1 + d) - (b + Sqrt[b^2 - 4*a*c])*e)] - Log[b - Sqr
t[b^2 - 4*a*c] + 2*c*x]*Log[(-1 + d + e*x)/(d + e*x)] + Log[b + Sqrt[b^2 -
4*a*c] + 2*c*x]*Log[(-1 + d + e*x)/(d + e*x)] - Log[b - Sqrt[b^2 - 4*a*c] +
 2*c*x]*Log[(2*c*(1 + d + e*x))/(2*c*(1 + d) + (-b + Sqrt[b^2 - 4*a*c])*e)]
 + Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[(2*c*(1 + d + e*x))/(2*c*(1 + d)
- (b + Sqrt[b^2 - 4*a*c])*e)] + Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[(1 +
 d + e*x)/(d + e*x)] - Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[(1 + d + e*x)
/(d + e*x)] - PolyLog[2, (e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*(1 + d)
+ (-b + Sqrt[b^2 - 4*a*c])*e)] + PolyLog[2, (e*(b - Sqrt[b^2 - 4*a*c] + 2*c
*x))/(2*c - 2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e)] - PolyLog[2, (e*(b + Sqrt[b
^2 - 4*a*c] + 2*c*x))/(-2*c*(-1 + d) + (b + Sqrt[b^2 - 4*a*c])*e)] + PolyLo
g[2, (e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*(1 + d) + (b + Sqrt[b^2 - 4*
a*c])*e)])/(2*Sqrt[b^2 - 4*a*c])
```

Maple [B] time = 0.586, size = 2098, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(e*x+d)/(c*x^2+b*x+a),x)
```

```
[Out] 1/e*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*arccoth(e*x+d)*ln(1-(a*e^2-b*d*e+c
*d^2+b*e-2*c*d+c)*(e*x+d+1)/(e*x+d-1)/(a*e^2-b*e*d+c*d^2+(-e^2*(4*a*c-b^2))
^(1/2)-c))-1/e*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*arccoth(e*x+d)^2+1/2/e*
(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*polylog(2,(a*e^2-b*d*e+c*d^2+b*e-2*c*d
+c)*(e*x+d+1)/(e*x+d-1)/(a*e^2-b*e*d+c*d^2+(-e^2*(4*a*c-b^2))^(1/2)-c))-e*(
e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*ln(1-(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)*(e
*x+d+1)/(e*x+d-1)/(a*e^2-b*e*d+c*d^2-(-e^2*(4*a*c-b^2))^(1/2)-c))*arccoth(e
*x+d)*a/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)+(e^2*(-4*a*c+b^2))^(
1/2)/(4*a*c-b^2)*ln(1-(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)*(e*x+d+1)/(e*x+d-1)/(
a*e^2-b*e*d+c*d^2-(-e^2*(4*a*c-b^2))^(1/2)-c))*arccoth(e*x+d)*b*d/(a*e^2-b*
e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)-1/e*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b
^2)*ln(1-(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)*(e*x+d+1)/(e*x+d-1)/(a*e^2-b*e*d+c
*d^2-(-e^2*(4*a*c-b^2))^(1/2)-c))*arccoth(e*x+d)*c*d^2/(a*e^2-b*e*d+c*d^2-(
e^2*(-4*a*c+b^2))^(1/2)-c)+e*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*arccoth(e
*x+d)^2*a/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)-(e^2*(-4*a*c+b^2))
^(1/2)/(4*a*c-b^2)*arccoth(e*x+d)^2*b*d/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2
))^(1/2)-c)+1/e*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*arccoth(e*x+d)^2*c*d^2
/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)-1/2*e*(e^2*(-4*a*c+b^2))^(1
/2)/(4*a*c-b^2)*polylog(2,(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)*(e*x+d+1)/(e*x+d-
1)/(a*e^2-b*e*d+c*d^2-(-e^2*(4*a*c-b^2))^(1/2)-c))*a/(a*e^2-b*e*d+c*d^2-(e^
2*(-4*a*c+b^2))^(1/2)-c)+1/2*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*polylog(2
,(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)*(e*x+d+1)/(e*x+d-1)/(a*e^2-b*e*d+c*d^2-(-e
^2*(4*a*c-b^2))^(1/2)-c))*b*d/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c
)-1/2/e*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*polylog(2,(a*e^2-b*d*e+c*d^2+b
*e-2*c*d+c)*(e*x+d+1)/(e*x+d-1)/(a*e^2-b*e*d+c*d^2-(-e^2*(4*a*c-b^2))^(1/2)
-c))*c*d^2/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)-e/(a*e^2-b*e*d+c*
d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)*ln(1-(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)*(e*x+d
+1)/(e*x+d-1)/(a*e^2-b*e*d+c*d^2-(-e^2*(4*a*c-b^2))^(1/2)-c))*arccoth(e*x+d
)+1/e*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*ln(1-(a*e^2-b*d*e+c*d^2+b*e-2*c*
d+c)*(e*x+d+1)/(e*x+d-1)/(a*e^2-b*e*d+c*d^2-(-e^2*(4*a*c-b^2))^(1/2)-c))*ar
ccoth(e*x+d)*c/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)+e/(a*e^2-b*e*
d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)*arccoth(e*x+d)^2-1/e*(e^2*(-4*a*c+b^2))
^(1/2)/(4*a*c-b^2)*arccoth(e*x+d)^2*c/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))
^(1/2)-c)-1/2*e/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)*polylog(2,(a
*e^2-b*d*e+c*d^2+b*e-2*c*d+c)*(e*x+d+1)/(e*x+d-1)/(a*e^2-b*e*d+c*d^2-(-e^2*
(4*a*c-b^2))^(1/2)-c))+1/2/e*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*polylog(2
,(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)*(e*x+d+1)/(e*x+d-1)/(a*e^2-b*e*d+c*d^2-(-e
^2*(4*a*c-b^2))^(1/2)-c))*c/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(e*x+d)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arcoth}(ex+d)}{cx^2+bx+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral(arccoth(e*x + d)/(c*x^2 + b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(e*x+d)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(ex+d)}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(e*x+d)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate(arccoth(e*x + d)/(c*x^2 + b*x + a), x)
```

3.83 $\int x^2 \coth^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=51

$$\frac{x^{5/2}}{15} + \frac{x^{3/2}}{9} + \frac{1}{3}x^3 \coth^{-1}\left(\sqrt{x}\right) + \frac{\sqrt{x}}{3} - \frac{1}{3}\tanh^{-1}\left(\sqrt{x}\right)$$

```
[Out] Sqrt[x]/3 + x^(3/2)/9 + x^(5/2)/15 + (x^3*ArcCoth[Sqrt[x]])/3 - ArcTanh[Sqr
t[x]]/3
```

Rubi [A] time = 0.0147834, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6098, 50, 63, 206}

$$\frac{x^{5/2}}{15} + \frac{x^{3/2}}{9} + \frac{1}{3}x^3 \coth^{-1}\left(\sqrt{x}\right) + \frac{\sqrt{x}}{3} - \frac{1}{3}\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[Sqrt[x]],x]
```

```
[Out] Sqrt[x]/3 + x^(3/2)/9 + x^(5/2)/15 + (x^3*ArcCoth[Sqrt[x]])/3 - ArcTanh[Sqr
t[x]]/3
```

Rule 6098

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^2 \coth^{-1}\left(\sqrt{x}\right) dx &= \frac{1}{3}x^3 \coth^{-1}\left(\sqrt{x}\right) - \frac{1}{6}\int \frac{x^{5/2}}{1-x}\, dx \\
&= \frac{x^{5/2}}{15} + \frac{1}{3}x^3 \coth^{-1}\left(\sqrt{x}\right) - \frac{1}{6}\int \frac{x^{3/2}}{1-x}\, dx \\
&= \frac{x^{3/2}}{9} + \frac{x^{5/2}}{15} + \frac{1}{3}x^3 \coth^{-1}\left(\sqrt{x}\right) - \frac{1}{6}\int \frac{\sqrt{x}}{1-x}\, dx \\
&= \frac{\sqrt{x}}{3} + \frac{x^{3/2}}{9} + \frac{x^{5/2}}{15} + \frac{1}{3}x^3 \coth^{-1}\left(\sqrt{x}\right) - \frac{1}{6}\int \frac{1}{(1-x)\sqrt{x}}\, dx \\
&= \frac{\sqrt{x}}{3} + \frac{x^{3/2}}{9} + \frac{x^{5/2}}{15} + \frac{1}{3}x^3 \coth^{-1}\left(\sqrt{x}\right) - \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1-x^2}\, dx, x, \sqrt{x}\right) \\
&= \frac{\sqrt{x}}{3} + \frac{x^{3/2}}{9} + \frac{x^{5/2}}{15} + \frac{1}{3}x^3 \coth^{-1}\left(\sqrt{x}\right) - \frac{1}{3}\tanh^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0175619, size = 59, normalized size = 1.16

$$
\frac{1}{90}\left(6x^{5/2} + 10x^{3/2} + 30x^3 \coth^{-1}\left(\sqrt{x}\right) + 30\sqrt{x} + 15\log\left(1-\sqrt{x}\right) - 15\log\left(\sqrt{x}+1\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[Sqrt[x]],x]
```

```
[Out] (30*Sqrt[x] + 10*x^(3/2) + 6*x^(5/2) + 30*x^3*ArcCoth[Sqrt[x]] + 15*Log[1 -
 Sqrt[x]] - 15*Log[1 + Sqrt[x]])/90
```

Maple [A] time = 0.036, size = 42, normalized size = 0.8

$$
\frac{x^3}{3}\text{arccoth}\left(\sqrt{x}\right) + \frac{1}{15}x^{\frac{5}{2}} + \frac{1}{9}x^{\frac{3}{2}} + \frac{1}{3}\sqrt{x} + \frac{1}{6}\ln\left(-1+\sqrt{x}\right) - \frac{1}{6}\ln\left(1+\sqrt{x}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(x^(1/2)),x)
```

```
[Out] 1/3*x^3*arccoth(x^(1/2))+1/15*x^(5/2)+1/9*x^(3/2)+1/3*x^(1/2)+1/6*ln(-1+x^(
1/2))-1/6*ln(1+x^(1/2))
```

Maxima [A] time = 0.961345, size = 55, normalized size = 1.08

$$
\frac{1}{3}x^3\,\text{arcoth}\left(\sqrt{x}\right) + \frac{1}{15}x^{\frac{5}{2}} + \frac{1}{9}x^{\frac{3}{2}} + \frac{1}{3}\sqrt{x} - \frac{1}{6}\log\left(\sqrt{x}+1\right) + \frac{1}{6}\log\left(\sqrt{x}-1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(sqrt(x)) + 1/15*x^(5/2) + 1/9*x^(3/2) + 1/3*sqrt(x) - 1/6*l
og(sqrt(x) + 1) + 1/6*log(sqrt(x) - 1)
```

Fricas [A] time = 1.60583, size = 111, normalized size = 2.18

$$\frac{1}{6}\left(x^{3}-1\right)\log\left(\frac{x+2\sqrt{x}+1}{x-1}\right)+\frac{1}{45}\left(3x^{2}+5x+15\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/6*(x^3 - 1)*log((x + 2*sqrt(x) + 1)/(x - 1)) + 1/45*(3*x^2 + 5*x + 15)*sq
rt(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{2}\operatorname{acoth}\left(\sqrt{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(x**(1/2)),x)
```

```
[Out] Integral(x**2*acoth(sqrt(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{2}\operatorname{arcoth}\left(\sqrt{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(x^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(sqrt(x)), x)
```

3.84 $\int x \coth^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=42

$$\frac{x^{3/2}}{6}+\frac{1}{2}x^2\coth^{-1}\left(\sqrt{x}\right)+\frac{\sqrt{x}}{2}-\frac{1}{2}\tanh^{-1}\left(\sqrt{x}\right)$$

```
[Out] Sqrt[x]/2 + x^(3/2)/6 + (x^2*ArcCoth[Sqrt[x]])/2 - ArcTanh[Sqrt[x]]/2
```

Rubi [A] time = 0.0107724, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6098, 50, 63, 206}

$$\frac{x^{3/2}}{6}+\frac{1}{2}x^2\coth^{-1}\left(\sqrt{x}\right)+\frac{\sqrt{x}}{2}-\frac{1}{2}\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[Sqrt[x]],x]

[Out] Sqrt[x]/2 + x^(3/2)/6 + (x^2*ArcCoth[Sqrt[x]])/2 - ArcTanh[Sqrt[x]]/2
```

Rule 6098

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int x\coth^{-1}\left(\sqrt{x}\right)dx &= \frac{1}{2}x^2\coth^{-1}\left(\sqrt{x}\right)-\frac{1}{4}\int \frac{x^{3/2}}{1-x}dx\\
&= \frac{x^{3/2}}{6}+\frac{1}{2}x^2\coth^{-1}\left(\sqrt{x}\right)-\frac{1}{4}\int \frac{\sqrt{x}}{1-x}dx\\
&= \frac{\sqrt{x}}{2}+\frac{x^{3/2}}{6}+\frac{1}{2}x^2\coth^{-1}\left(\sqrt{x}\right)-\frac{1}{4}\int \frac{1}{(1-x)\sqrt{x}}dx\\
&= \frac{\sqrt{x}}{2}+\frac{x^{3/2}}{6}+\frac{1}{2}x^2\coth^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}dx,x,\sqrt{x}\right)\\
&= \frac{\sqrt{x}}{2}+\frac{x^{3/2}}{6}+\frac{1}{2}x^2\coth^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\tanh^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0145469, size = 52, normalized size = 1.24

$$\frac{1}{12}\left(2x^{3/2}+6x^2\coth^{-1}\left(\sqrt{x}\right)+6\sqrt{x}+3\log\left(1-\sqrt{x}\right)-3\log\left(\sqrt{x}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[Sqrt[x]],x]
```

```
[Out] (6*Sqrt[x] + 2*x^(3/2) + 6*x^2*ArcCoth[Sqrt[x]] + 3*Log[1 - Sqrt[x]] - 3*Lo
g[1 + Sqrt[x]])/12
```

Maple [A] time = 0.033, size = 37, normalized size = 0.9

$$\frac{x^2}{2}\text{arccoth}\left(\sqrt{x}\right)+\frac{1}{6}x^{\frac{3}{2}}+\frac{1}{2}\sqrt{x}+\frac{1}{4}\ln\left(-1+\sqrt{x}\right)-\frac{1}{4}\ln\left(1+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(x^(1/2)),x)
```

```
[Out] 1/2*x^2*arccoth(x^(1/2))+1/6*x^(3/2)+1/2*x^(1/2)+1/4*ln(-1+x^(1/2))-1/4*ln(
1+x^(1/2))
```

Maxima [A] time = 0.957906, size = 49, normalized size = 1.17

$$\frac{1}{2}x^2\,\text{arcoth}\left(\sqrt{x}\right)+\frac{1}{6}x^{\frac{3}{2}}+\frac{1}{2}\sqrt{x}-\frac{1}{4}\log\left(\sqrt{x}+1\right)+\frac{1}{4}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccoth(sqrt(x)) + 1/6*x^(3/2) + 1/2*sqrt(x) - 1/4*log(sqrt(x) + 1)
 + 1/4*log(sqrt(x) - 1)
```

Fricas [A] time = 1.53167, size = 95, normalized size = 2.26

$$\frac{1}{4}\left(x^2-1\right)\log\left(\frac{x+2\sqrt{x}+1}{x-1}\right)+\frac{1}{6}(x+3)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(x^2 - 1)*log((x + 2*sqrt(x) + 1)/(x - 1)) + 1/6*(x + 3)*sqrt(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{acoth}\left(\sqrt{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(x**(1/2)),x)
```

```
[Out] Integral(x*acoth(sqrt(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arcoth}\left(\sqrt{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(x^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(sqrt(x)), x)
```

3.85 $\int \coth^{-1}\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=22

$$\sqrt{x}-\tanh^{-1}\left(\sqrt{x}\right)+x\coth^{-1}\left(\sqrt{x}\right)$$

```
[Out] Sqrt[x] + x*ArcCoth[Sqrt[x]] - ArcTanh[Sqrt[x]]
```

Rubi [A] time = 0.006253, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6092, 50, 63, 206}

$$\sqrt{x}-\tanh^{-1}\left(\sqrt{x}\right)+x\coth^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Sqrt[x]],x]
```

```
[Out] Sqrt[x] + x*ArcCoth[Sqrt[x]] - ArcTanh[Sqrt[x]]
```

Rule 6092

```
Int[ArcCoth[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcCoth[c*x^n], x] - Dist
[c*n, Int[x^n/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \coth^{-1}\left(\sqrt{x}\right)dx &= x\coth^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\int \frac{\sqrt{x}}{1-x}\,dx\\ &= \sqrt{x}+x\coth^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\int \frac{1}{(1-x)\sqrt{x}}\,dx\\ &= \sqrt{x}+x\coth^{-1}\left(\sqrt{x}\right)-\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{x}\right)\\ &= \sqrt{x}+x\coth^{-1}\left(\sqrt{x}\right)-\tanh^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.0059219, size = 22, normalized size = 1.

$$\sqrt{x}-\tanh^{-1}\left(\sqrt{x}\right)+x\coth^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Sqrt[x]],x]
```

```
[Out] Sqrt[x] + x*ArcCoth[Sqrt[x]] - ArcTanh[Sqrt[x]]
```

Maple [A] time = 0.034, size = 27, normalized size = 1.2

$$x\text{arccoth}\left(\sqrt{x}\right)+\sqrt{x}+\frac{1}{2}\ln\left(-1+\sqrt{x}\right)-\frac{1}{2}\ln\left(1+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x^(1/2)),x)
```

```
[Out] x*arccoth(x^(1/2))+x^(1/2)+1/2*ln(-1+x^(1/2))-1/2*ln(1+x^(1/2))
```

Maxima [A] time = 0.951836, size = 35, normalized size = 1.59

$$x\,\text{arcoth}\left(\sqrt{x}\right)+\sqrt{x}-\frac{1}{2}\log\left(\sqrt{x}+1\right)+\frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2)),x, algorithm="maxima")
```

```
[Out] x*arccoth(sqrt(x)) + sqrt(x) - 1/2*log(sqrt(x) + 1) + 1/2*log(sqrt(x) - 1)
```

Fricas [A] time = 1.63877, size = 76, normalized size = 3.45

$$\frac{1}{2}(x-1)\log\left(\frac{x+2\sqrt{x}+1}{x-1}\right)+\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*(x - 1)*log((x + 2*sqrt(x) + 1)/(x - 1)) + sqrt(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acoth}\left(\sqrt{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x**(1/2)),x)
```

```
[Out] Integral(acoth(sqrt(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}\left(\sqrt{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(sqrt(x)), x)
```

3.86 $\int \frac{\coth^{-1}(\sqrt{x})}{x} \, dx$

Optimal. Leaf size=19

$$\text{PolyLog}\left(2,-\frac{1}{\sqrt{x}}\right)-\text{PolyLog}\left(2,\frac{1}{\sqrt{x}}\right)$$

```
[Out] PolyLog[2, -(1/Sqrt[x])] - PolyLog[2, 1/Sqrt[x]]
```

Rubi [A] time = 0.0197034, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6096, 5913}

$$\text{PolyLog}\left(2,-\frac{1}{\sqrt{x}}\right)-\text{PolyLog}\left(2,\frac{1}{\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Sqrt[x]]/x,x]
```

```
[Out] PolyLog[2, -(1/Sqrt[x])] - PolyLog[2, 1/Sqrt[x]]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcCoth[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 5913

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b*PolyLog[2, -(c*x)^(-1)])/2, x] - Simp[(b*PolyLog[2, 1/(c*x)])/
2, x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x} \, dx &= 2\,\text{Subst}\left(\int \frac{\coth^{-1}(x)}{x} \, dx, x, \sqrt{x}\right)\\ &= \text{Li}_2\left(-\frac{1}{\sqrt{x}}\right)-\text{Li}_2\left(\frac{1}{\sqrt{x}}\right)\end{aligned}$$

Mathematica [A] time = 0.0101691, size = 19, normalized size = 1.

$$\text{PolyLog}\left(2,-\frac{1}{\sqrt{x}}\right)-\text{PolyLog}\left(2,\frac{1}{\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Sqrt[x]]/x,x]
```

```
[Out] PolyLog[2, -(1/Sqrt[x])] - PolyLog[2, 1/Sqrt[x]]
```

Maple [B] time = 0.054, size = 33, normalized size = 1.7

$$\ln(x)\operatorname{arccoth}\left(\sqrt{x}\right)-\operatorname{dilog}\left(\sqrt{x}\right)-\operatorname{dilog}\left(1+\sqrt{x}\right)-\frac{\ln(x)}{2}\ln\left(1+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x^(1/2))/x,x)
```

```
[Out] ln(x)*arccoth(x^(1/2))-dilog(x^(1/2))-dilog(1+x^(1/2))-1/2*ln(x)*ln(1+x^(1/
2))
```

Maxima [B] time = 0.959881, size = 89, normalized size = 4.68

$$-\frac{1}{2}\left(\log\left(\sqrt{x}+1\right)-\log\left(\sqrt{x}-1\right)\right)\log(x)+\operatorname{arcoth}\left(\sqrt{x}\right)\log(x)+\log\left(-\sqrt{x}\right)\log\left(\sqrt{x}+1\right)-\frac{1}{2}\log(x)\log\left(\sqrt{x}-1\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x,x, algorithm="maxima")
```

```
[Out] -1/2*(log(sqrt(x) + 1) - log(sqrt(x) - 1))*log(x) + arccoth(sqrt(x))*log(x)
 + log(-sqrt(x))*log(sqrt(x) + 1) - 1/2*log(x)*log(sqrt(x) - 1) + dilog(sqr
t(x) + 1) - dilog(-sqrt(x) + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arcoth}\left(\sqrt{x}\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(sqrt(x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{acoth}\left(\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x**(1/2))/x,x)
```

```
[Out] Integral(acoth(sqrt(x))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}\left(\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(sqrt(x))/x, x)
```

3.87 $\int \frac{\coth^{-1}(\sqrt{x})}{x^2} \, dx$

Optimal. Leaf size=25

$$-\frac{1}{\sqrt{x}}+\tanh^{-1}\left(\sqrt{x}\right)-\frac{\coth^{-1}\left(\sqrt{x}\right)}{x}$$

```
[Out] -(1/Sqrt[x]) - ArcCoth[Sqrt[x]]/x + ArcTanh[Sqrt[x]]
```

Rubi [A] time = 0.013518, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6098, 51, 63, 206}

$$-\frac{1}{\sqrt{x}}+\tanh^{-1}\left(\sqrt{x}\right)-\frac{\coth^{-1}\left(\sqrt{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Sqrt[x]]/x^2,x]

[Out] -(1/Sqrt[x]) - ArcCoth[Sqrt[x]]/x + ArcTanh[Sqrt[x]]
```

Rule 6098

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^2}\,dx &= -\frac{\coth^{-1}\left(\sqrt{x}\right)}{x} + \frac{1}{2}\int \frac{1}{(1-x)x^{3/2}}\,dx \\
&= -\frac{1}{\sqrt{x}} - \frac{\coth^{-1}\left(\sqrt{x}\right)}{x} + \frac{1}{2}\int \frac{1}{(1-x)\sqrt{x}}\,dx \\
&= -\frac{1}{\sqrt{x}} - \frac{\coth^{-1}\left(\sqrt{x}\right)}{x} + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{x}\right) \\
&= -\frac{1}{\sqrt{x}} - \frac{\coth^{-1}\left(\sqrt{x}\right)}{x} + \tanh^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0240027, size = 45, normalized size = 1.8

$$-\frac{1}{\sqrt{x}} - \frac{1}{2}\log\left(1-\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{\coth^{-1}\left(\sqrt{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Sqrt[x]]/x^2,x]
```

```
[Out] -(1/Sqrt[x]) - ArcCoth[Sqrt[x]]/x - Log[1 - Sqrt[x]]/2 + Log[1 + Sqrt[x]]/2
```

Maple [A] time = 0.038, size = 32, normalized size = 1.3

$$-\frac{1}{x}\text{arccoth}\left(\sqrt{x}\right) - \frac{1}{\sqrt{x}} - \frac{1}{2}\ln\left(-1+\sqrt{x}\right) + \frac{1}{2}\ln\left(1+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x^(1/2))/x^2,x)
```

```
[Out] -arccoth(x^(1/2))/x-1/x^(1/2)-1/2*ln(-1+x^(1/2))+1/2*ln(1+x^(1/2))
```

Maxima [A] time = 0.969192, size = 42, normalized size = 1.68

$$-\frac{\text{arcoth}\left(\sqrt{x}\right)}{x} - \frac{1}{\sqrt{x}} + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] -arccoth(sqrt(x))/x - 1/sqrt(x) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) -
1)
```

Fricas [A] time = 1.57107, size = 84, normalized size = 3.36

$$\frac{(x-1)\log\left(\frac{x+2\sqrt{x}+1}{x-1}\right) - 2\sqrt{x}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*((x - 1)*log((x + 2*sqrt(x) + 1)/(x - 1)) - 2*sqrt(x))/x
```

Sympy [B] time = 3.66259, size = 92, normalized size = 3.68

$$\frac{x^{\frac{5}{2}}\operatorname{acoth}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}-x^{\frac{3}{2}}}-\frac{2x^{\frac{3}{2}}\operatorname{acoth}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}-x^{\frac{3}{2}}}+\frac{\sqrt{x}\operatorname{acoth}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}-x^{\frac{3}{2}}}-\frac{x^2}{x^{\frac{5}{2}}-x^{\frac{3}{2}}}+\frac{x}{x^{\frac{5}{2}}-x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x**(1/2))/x**2,x)
```

```
[Out] x**(5/2)*acoth(sqrt(x))/(x**(5/2) - x**(3/2)) - 2*x**(3/2)*acoth(sqrt(x))/(
x**(5/2) - x**(3/2)) + sqrt(x)*acoth(sqrt(x))/(x**(5/2) - x**(3/2)) - x**2/
(x**(5/2) - x**(3/2)) + x/(x**(5/2) - x**(3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}\left(\sqrt{x}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(sqrt(x))/x^2, x)
```

3.88 $\int \frac{\coth^{-1}(\sqrt{x})}{x^3}\,dx$

Optimal. Leaf size=42

$$-\frac{1}{6x^{3/2}}-\frac{\coth^{-1}(\sqrt{x})}{2x^2}-\frac{1}{2\sqrt{x}}+\frac{1}{2}\tanh^{-1}(\sqrt{x})$$

```
[Out] -1/(6*x^(3/2)) - 1/(2*Sqrt[x]) - ArcCoth[Sqrt[x]]/(2*x^2) + ArcTanh[Sqrt[x]
]/2
```

Rubi [A] time = 0.0147932, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6098, 51, 63, 206}

$$-\frac{1}{6x^{3/2}}-\frac{\coth^{-1}(\sqrt{x})}{2x^2}-\frac{1}{2\sqrt{x}}+\frac{1}{2}\tanh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Sqrt[x]]/x^3,x]
```

```
[Out] -1/(6*x^(3/2)) - 1/(2*Sqrt[x]) - ArcCoth[Sqrt[x]]/(2*x^2) + ArcTanh[Sqrt[x]
]/2
```

Rule 6098

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = -\frac{\coth^{-1}\left(\sqrt{x}\right)}{2x^2} + \frac{1}{4}\int \frac{1}{(1-x)x^{5/2}}\,dx$$
$$= -\frac{1}{6x^{3/2}} - \frac{\coth^{-1}\left(\sqrt{x}\right)}{2x^2} + \frac{1}{4}\int \frac{1}{(1-x)x^{3/2}}\,dx$$
$$= -\frac{1}{6x^{3/2}} - \frac{1}{2\sqrt{x}} - \frac{\coth^{-1}\left(\sqrt{x}\right)}{2x^2} + \frac{1}{4}\int \frac{1}{(1-x)\sqrt{x}}\,dx$$
$$= -\frac{1}{6x^{3/2}} - \frac{1}{2\sqrt{x}} - \frac{\coth^{-1}\left(\sqrt{x}\right)}{2x^2} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{1}{6x^{3/2}} - \frac{1}{2\sqrt{x}} - \frac{\coth^{-1}\left(\sqrt{x}\right)}{2x^2} + \frac{1}{2}\tanh^{-1}\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.0191363, size = 58, normalized size = 1.38

$$-\frac{1}{6x^{3/2}} - \frac{\coth^{-1}\left(\sqrt{x}\right)}{2x^2} - \frac{1}{2\sqrt{x}} - \frac{1}{4}\log\left(1-\sqrt{x}\right) + \frac{1}{4}\log\left(\sqrt{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Sqrt[x]]/x^3,x]
```

```
[Out] -1/(6*x^(3/2)) - 1/(2*Sqrt[x]) - ArcCoth[Sqrt[x]]/(2*x^2) - Log[1 - Sqrt[x]
]/4 + Log[1 + Sqrt[x]]/4
```

Maple [A] time = 0.035, size = 37, normalized size = 0.9

$$-\frac{1}{2\,x^2}\text{arccoth}\left(\sqrt{x}\right) - \frac{1}{4}\ln\left(-1+\sqrt{x}\right) - \frac{1}{6}x^{-\frac{3}{2}} - \frac{1}{2}\frac{1}{\sqrt{x}} + \frac{1}{4}\ln\left(1+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x^(1/2))/x^3,x)
```

```
[Out] -1/2*arccoth(x^(1/2))/x^2-1/4*ln(-1+x^(1/2))-1/6/x^(3/2)-1/2/x^(1/2)+1/4*ln
(1+x^(1/2))
```

Maxima [A] time = 0.985373, size = 49, normalized size = 1.17

$$-\frac{3\,x+1}{6\,x^{\frac{3}{2}}} - \frac{\text{arcoth}\left(\sqrt{x}\right)}{2\,x^2} + \frac{1}{4}\,\log\left(\sqrt{x}+1\right) - \frac{1}{4}\,\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] -1/6*(3*x + 1)/x^(3/2) - 1/2*arccoth(sqrt(x))/x^2 + 1/4*log(sqrt(x) + 1) -
1/4*log(sqrt(x) - 1)
```

Fricas [A] time = 1.59093, size = 107, normalized size = 2.55

$$\frac{3\left(x^2-1\right)\log\left(\frac{x+2\sqrt{x}+1}{x-1}\right)-2\left(3x+1\right)\sqrt{x}}{12x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*(x^2 - 1)*log((x + 2*sqrt(x) + 1)/(x - 1)) - 2*(3*x + 1)*sqrt(x))/x
^2
```

Sympy [B] time = 9.8067, size = 160, normalized size = 3.81

$$\frac{3x^{\frac{7}{2}}\operatorname{acoth}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}-6x^{\frac{5}{2}}}-\frac{3x^{\frac{5}{2}}\operatorname{acoth}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}-6x^{\frac{5}{2}}}-\frac{3x^{\frac{3}{2}}\operatorname{acoth}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}-6x^{\frac{5}{2}}}+\frac{3\sqrt{x}\operatorname{acoth}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}-6x^{\frac{5}{2}}}-\frac{3x^3}{6x^{\frac{7}{2}}-6x^{\frac{5}{2}}}+\frac{2x^2}{6x^{\frac{7}{2}}-6x^{\frac{5}{2}}}+\frac{x}{6x^{\frac{7}{2}}-6x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x**(1/2))/x**3,x)
```

```
[Out] 3*x**(7/2)*acoth(sqrt(x))/(6*x**(7/2) - 6*x**(5/2)) - 3*x**(5/2)*acoth(sqrt
(x))/(6*x**(7/2) - 6*x**(5/2)) - 3*x**(3/2)*acoth(sqrt(x))/(6*x**(7/2) - 6*
x**(5/2)) + 3*sqrt(x)*acoth(sqrt(x))/(6*x**(7/2) - 6*x**(5/2)) - 3*x**3/(6*
x**(7/2) - 6*x**(5/2)) + 2*x**2/(6*x**(7/2) - 6*x**(5/2)) + x/(6*x**(7/2) -
 6*x**(5/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{arcoth}\left(\sqrt{x}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] integrate(arccoth(sqrt(x))/x^3, x)
```

3.89 $\int x^{3/2} \coth^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=38

$$\frac{x^2}{10}+\frac{2}{5}x^{5/2}\coth^{-1}\left(\sqrt{x}\right)+\frac{x}{5}+\frac{1}{5}\log(1-x)$$

```
[Out] x/5 + x^2/10 + (2*x^(5/2)*ArcCoth[Sqrt[x]])/5 + Log[1 - x]/5
```

Rubi [A] time = 0.0177347, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6098, 43}

$$\frac{x^2}{10}+\frac{2}{5}x^{5/2}\coth^{-1}\left(\sqrt{x}\right)+\frac{x}{5}+\frac{1}{5}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*ArcCoth[Sqrt[x]],x]
```

```
[Out] x/5 + x^2/10 + (2*x^(5/2)*ArcCoth[Sqrt[x]])/5 + Log[1 - x]/5
```

Rule 6098

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\coth^{-1}\left(\sqrt{x}\right) dx &= \frac{2}{5}x^{5/2}\coth^{-1}\left(\sqrt{x}\right)-\frac{1}{5}\int \frac{x^2}{1-x}\,dx\\ &= \frac{2}{5}x^{5/2}\coth^{-1}\left(\sqrt{x}\right)-\frac{1}{5}\int \left(-1+\frac{1}{1-x}-x\right) dx\\ &= \frac{x}{5}+\frac{x^2}{10}+\frac{2}{5}x^{5/2}\coth^{-1}\left(\sqrt{x}\right)+\frac{1}{5}\log(1-x)\end{aligned}$$

Mathematica [A] time = 0.0154204, size = 31, normalized size = 0.82

$$\frac{1}{10}\left(4x^{5/2}\coth^{-1}\left(\sqrt{x}\right)+(x+2)x+2\log(1-x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*ArcCoth[Sqrt[x]],x]
```

```
[Out] (x*(2 + x) + 4*x^(5/2)*ArcCoth[Sqrt[x]] + 2*Log[1 - x])/10
```

Maple [A] time = 0.034, size = 35, normalized size = 0.9

$$\frac{2}{5}x^{\frac{5}{2}}\operatorname{arccoth}\left(\sqrt{x}\right)+\frac{x^2}{10}+\frac{x}{5}+\frac{1}{5}\ln\left(-1+\sqrt{x}\right)+\frac{1}{5}\ln\left(1+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*arccoth(x^(1/2)),x)
```

```
[Out] 2/5*x^(5/2)*arccoth(x^(1/2))+1/10*x^2+1/5*x+1/5*ln(-1+x^(1/2))+1/5*ln(1+x^(
1/2))
```

Maxima [A] time = 0.956905, size = 32, normalized size = 0.84

$$\frac{2}{5}x^{\frac{5}{2}}\operatorname{arcoth}\left(\sqrt{x}\right)+\frac{1}{10}x^2+\frac{1}{5}x+\frac{1}{5}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arccoth(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2)*arccoth(sqrt(x)) + 1/10*x^2 + 1/5*x + 1/5*log(x - 1)
```

Fricas [A] time = 1.57908, size = 111, normalized size = 2.92

$$\frac{1}{5}x^{\frac{5}{2}}\log\left(\frac{x+2\sqrt{x}+1}{x-1}\right)+\frac{1}{10}x^2+\frac{1}{5}x+\frac{1}{5}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arccoth(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/5*x^(5/2)*log((x + 2*sqrt(x) + 1)/(x - 1)) + 1/10*x^2 + 1/5*x + 1/5*log(x
 - 1)
```

Sympy [B] time = 9.51341, size = 121, normalized size = 3.18

$$\frac{4x^{\frac{7}{2}}\operatorname{acoth}\left(\sqrt{x}\right)}{10x-10}-\frac{4x^{\frac{5}{2}}\operatorname{acoth}\left(\sqrt{x}\right)}{10x-10}+\frac{x^3}{10x-10}+\frac{x^2}{10x-10}+\frac{4x\log\left(\sqrt{x}+1\right)}{10x-10}-\frac{4x\operatorname{acoth}\left(\sqrt{x}\right)}{10x-10}-\frac{4\log\left(\sqrt{x}+1\right)}{10x-10}+\frac{4\,\mathrm{ac}}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*acoth(x**(1/2)),x)
```

```
[Out] 4*x**(7/2)*acoth(sqrt(x))/(10*x - 10) - 4*x**(5/2)*acoth(sqrt(x))/(10*x - 1
0) + x**3/(10*x - 10) + x**2/(10*x - 10) + 4*x*log(sqrt(x) + 1)/(10*x - 10)
 - 4*x*acoth(sqrt(x))/(10*x - 10) - 4*log(sqrt(x) + 1)/(10*x - 10) + 4*acot
```

```
h(sqrt(x))/(10*x - 10) - 2/(10*x - 10)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{\frac{3}{2}} \operatorname{arcoth}\left(\sqrt{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arccoth(x^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)*arccoth(sqrt(x)), x)
```

3.90 $\int \sqrt{x} \coth^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=31

$$\frac{2}{3}x^{3/2}\coth^{-1}\left(\sqrt{x}\right)+\frac{x}{3}+\frac{1}{3}\log(1-x)$$

```
[Out] x/3 + (2*x^(3/2)*ArcCoth[Sqrt[x]])/3 + Log[1 - x]/3
```

Rubi [A] time = 0.0135038, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6098, 43}

$$\frac{2}{3}x^{3/2}\coth^{-1}\left(\sqrt{x}\right)+\frac{x}{3}+\frac{1}{3}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcCoth[Sqrt[x]],x]
```

```
[Out] x/3 + (2*x^(3/2)*ArcCoth[Sqrt[x]])/3 + Log[1 - x]/3
```

Rule 6098

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\coth^{-1}\left(\sqrt{x}\right) dx &= \frac{2}{3}x^{3/2}\coth^{-1}\left(\sqrt{x}\right)-\frac{1}{3}\int \frac{x}{1-x}\,dx\\ &= \frac{2}{3}x^{3/2}\coth^{-1}\left(\sqrt{x}\right)-\frac{1}{3}\int \left(-1+\frac{1}{1-x}\right)dx\\ &= \frac{x}{3}+\frac{2}{3}x^{3/2}\coth^{-1}\left(\sqrt{x}\right)+\frac{1}{3}\log(1-x)\end{aligned}$$

Mathematica [A] time = 0.0101959, size = 25, normalized size = 0.81

$$\frac{1}{3}\left(2x^{3/2}\coth^{-1}\left(\sqrt{x}\right)+x+\log(1-x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcCoth[Sqrt[x]],x]
```

```
[Out] (x + 2*x^(3/2)*ArcCoth[Sqrt[x]] + Log[1 - x])/3
```

Maple [A] time = 0.036, size = 30, normalized size = 1.

$$\frac{2}{3}x^{\frac{3}{2}}\operatorname{arccoth}\left(\sqrt{x}\right)+\frac{x}{3}+\frac{1}{3}\ln\left(-1+\sqrt{x}\right)+\frac{1}{3}\ln\left(1+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x^(1/2))*x^(1/2),x)
```

```
[Out] 2/3*x^(3/2)*arccoth(x^(1/2))+1/3*x+1/3*ln(-1+x^(1/2))+1/3*ln(1+x^(1/2))
```

Maxima [A] time = 0.990299, size = 26, normalized size = 0.84

$$\frac{2}{3}\,x^{\frac{3}{2}}\operatorname{arcoth}\left(\sqrt{x}\right)+\frac{1}{3}\,x+\frac{1}{3}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)*arccoth(sqrt(x)) + 1/3*x + 1/3*log(x - 1)
```

Fricas [A] time = 1.65532, size = 96, normalized size = 3.1

$$\frac{1}{3}\,x^{\frac{3}{2}}\log\left(\frac{x+2\,\sqrt{x}+1}{x-1}\right)+\frac{1}{3}\,x+\frac{1}{3}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))*x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*x^(3/2)*log((x + 2*sqrt(x) + 1)/(x - 1)) + 1/3*x + 1/3*log(x - 1)
```

Sympy [A] time = 1.60655, size = 39, normalized size = 1.26

$$\frac{2x^{\frac{3}{2}}\operatorname{acoth}\left(\sqrt{x}\right)}{3}+\frac{x}{3}+\frac{2\log\left(\sqrt{x}+1\right)}{3}-\frac{2\operatorname{acoth}\left(\sqrt{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x**(1/2))*x**(1/2),x)
```

```
[Out] 2*x**(3/2)*acoth(sqrt(x))/3 + x/3 + 2*log(sqrt(x) + 1)/3 - 2*acoth(sqrt(x))
/3
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x}\operatorname{arcoth}\left(\sqrt{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))*x^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)*arccoth(sqrt(x)), x)
```

3.91 $\int \frac{\coth^{-1}(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal. Leaf size=20

$$\log(1-x)+2\sqrt{x}\coth^{-1}\left(\sqrt{x}\right)$$

[Out] 2*Sqrt[x]*ArcCoth[Sqrt[x]] + Log[1 - x]

Rubi [A] time = 0.0092563, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6098, 31}

$$\log(1-x)+2\sqrt{x}\coth^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Sqrt[x]]/Sqrt[x],x]

[Out] 2*Sqrt[x]*ArcCoth[Sqrt[x]] + Log[1 - x]

Rule 6098

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx&=2\sqrt{x}\coth^{-1}\left(\sqrt{x}\right)-\int \frac{1}{1-x}\,dx\\&=2\sqrt{x}\coth^{-1}\left(\sqrt{x}\right)+\log(1-x)\end{aligned}$$

Mathematica [A] time = 0.007357, size = 20, normalized size = 1.

$$\log(1-x)+2\sqrt{x}\coth^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Sqrt[x]]/Sqrt[x],x]

[Out] 2*Sqrt[x]*ArcCoth[Sqrt[x]] + Log[1 - x]

Maple [A] time = 0.036, size = 15, normalized size = 0.8

$$2\,\mathrm{arccoth}\left(\sqrt{x}\right)\sqrt{x}+\ln\left(-1+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x^(1/2))/x^(1/2),x)
```

```
[Out] 2*arccoth(x^(1/2))*x^(1/2)+ln(-1+x)
```

Maxima [A] time = 0.967133, size = 22, normalized size = 1.1

$$2\sqrt{x}\,\mathrm{arcoth}\left(\sqrt{x}\right)+\log\left(-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)*arccoth(sqrt(x)) + log(-x + 1)
```

Fricas [A] time = 1.64507, size = 74, normalized size = 3.7

$$\sqrt{x}\log\left(\frac{x+2\sqrt{x}+1}{x-1}\right)+\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x)*log((x + 2*sqrt(x) + 1)/(x - 1)) + log(x - 1)
```

Sympy [B] time = 0.780662, size = 87, normalized size = 4.35

$$\frac{2x^{\frac{3}{2}}\,\mathrm{acoth}\left(\sqrt{x}\right)}{x-1}-\frac{2\sqrt{x}\,\mathrm{acoth}\left(\sqrt{x}\right)}{x-1}+\frac{2x\log\left(\sqrt{x}+1\right)}{x-1}-\frac{2x\,\mathrm{acoth}\left(\sqrt{x}\right)}{x-1}-\frac{2\log\left(\sqrt{x}+1\right)}{x-1}+\frac{2\,\mathrm{acoth}\left(\sqrt{x}\right)}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x**(1/2))/x**(1/2),x)
```

```
[Out] 2*x**(3/2)*acoth(sqrt(x))/(x - 1) - 2*sqrt(x)*acoth(sqrt(x))/(x - 1) + 2*x*
log(sqrt(x) + 1)/(x - 1) - 2*x*acoth(sqrt(x))/(x - 1) - 2*log(sqrt(x) + 1)/
(x - 1) + 2*acoth(sqrt(x))/(x - 1)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\mathrm{arcoth}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccoth(sqrt(x))/sqrt(x), x)
```

3.92 $\int \frac{\coth^{-1}(\sqrt{x})}{x^{3/2}}\,dx$

Optimal. Leaf size=24

$$-\log(1-x)+\log(x)-\frac{2\coth^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

```
[Out] (-2*ArcCoth[Sqrt[x]])/Sqrt[x] - Log[1 - x] + Log[x]
```

Rubi [A] time = 0.0104366, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6098, 36, 31, 29}

$$-\log(1-x)+\log(x)-\frac{2\coth^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Sqrt[x]]/x^(3/2),x]
```

```
[Out] (-2*ArcCoth[Sqrt[x]])/Sqrt[x] - Log[1 - x] + Log[x]
```

Rule 6098

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx &= -\frac{2\coth^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}+\int \frac{1}{(1-x)x}\,dx\\ &= -\frac{2\coth^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}+\int \frac{1}{1-x}\,dx+\int \frac{1}{x}\,dx\\ &= -\frac{2\coth^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}-\log(1-x)+\log(x)\end{aligned}$$

Mathematica [A] time = 0.0164811, size = 24, normalized size = 1.

$$-\log(1-x)+\log(x)-\frac{2\coth^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Sqrt[x]]/x^(3/2),x]
```

```
[Out] (-2*ArcCoth[Sqrt[x]])/Sqrt[x] - Log[1 - x] + Log[x]
```

Maple [A] time = 0.04, size = 29, normalized size = 1.2

$$-2\frac{\mathrm{arccoth}\left(\sqrt{x}\right)}{\sqrt{x}}-\ln\left(-1+\sqrt{x}\right)+\ln(x)-\ln\left(1+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x^(1/2))/x^(3/2),x)
```

```
[Out] -2*arccoth(x^(1/2))/x^(1/2)-ln(-1+x^(1/2))+ln(x)-ln(1+x^(1/2))
```

Maxima [A] time = 0.986294, size = 24, normalized size = 1.

$$-\frac{2\,\mathrm{arcoth}\left(\sqrt{x}\right)}{\sqrt{x}}-\log(x-1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x^(3/2),x, algorithm="maxima")
```

```
[Out] -2*arccoth(sqrt(x))/sqrt(x) - log(x - 1) + log(x)
```

Fricas [A] time = 1.63988, size = 99, normalized size = 4.12

$$-\frac{x\log(x-1)-x\log(x)+\sqrt{x}\log\left(\frac{x+2\sqrt{x}+1}{x-1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x^(3/2),x, algorithm="fricas")
```

```
[Out] -(x*log(x - 1) - x*log(x) + sqrt(x)*log((x + 2*sqrt(x) + 1)/(x - 1)))/x
```

Sympy [B] time = 2.15684, size = 126, normalized size = 5.25

$$-\frac{2x^{\frac{3}{2}}\,\mathrm{acoth}\left(\sqrt{x}\right)}{x^2-x}+\frac{2\sqrt{x}\,\mathrm{acoth}\left(\sqrt{x}\right)}{x^2-x}+\frac{x^2\log(x)}{x^2-x}-\frac{2x^2\log\left(\sqrt{x}+1\right)}{x^2-x}+\frac{2x^2\,\mathrm{acoth}\left(\sqrt{x}\right)}{x^2-x}-\frac{x\log(x)}{x^2-x}+\frac{2x\log\left(\sqrt{x}+\right.}{x^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x**(1/2))/x**(3/2),x)
```

```
[Out] -2*x**(3/2)*acoth(sqrt(x))/(x**2 - x) + 2*sqrt(x)*acoth(sqrt(x))/(x**2 - x)
 + x**2*log(x)/(x**2 - x) - 2*x**2*log(sqrt(x) + 1)/(x**2 - x) + 2*x**2*aco
th(sqrt(x))/(x**2 - x) - x*log(x)/(x**2 - x) + 2*x*log(sqrt(x) + 1)/(x**2 -
 x) - 2*x*acoth(sqrt(x))/(x**2 - x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}\left(\sqrt{x}\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arccoth(sqrt(x))/x^(3/2), x)
```

3.93 $\int \frac{\coth^{-1}(ax^5)}{x} dx$

Optimal. Leaf size=28

$$\frac{1}{10}\text{PolyLog}\left(2,-\frac{1}{ax^5}\right)-\frac{1}{10}\text{PolyLog}\left(2,\frac{1}{ax^5}\right)$$

```
[Out] PolyLog[2, -(1/(a*x^5))]/10 - PolyLog[2, 1/(a*x^5)]/10
```

Rubi [A] time = 0.0217677, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6096, 5913}

$$\frac{1}{10}\text{PolyLog}\left(2,-\frac{1}{ax^5}\right)-\frac{1}{10}\text{PolyLog}\left(2,\frac{1}{ax^5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x^5]/x,x]
```

```
[Out] PolyLog[2, -(1/(a*x^5))]/10 - PolyLog[2, 1/(a*x^5)]/10
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcCoth[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 5913

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b*PolyLog[2, -(c*x)^(-1)])/2, x] - Simp[(b*PolyLog[2, 1/(c*x)])/
2, x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(ax^5)}{x} dx &= \frac{1}{5}\text{Subst}\left(\int \frac{\coth^{-1}(ax)}{x} dx, x, x^5\right)\\ &= \frac{1}{10}\text{Li}_2\left(-\frac{1}{ax^5}\right)-\frac{1}{10}\text{Li}_2\left(\frac{1}{ax^5}\right)\end{aligned}$$

Mathematica [A] time = 0.0134413, size = 26, normalized size = 0.93

$$\frac{1}{10}\left(\text{PolyLog}\left(2,-\frac{1}{ax^5}\right)-\text{PolyLog}\left(2,\frac{1}{ax^5}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x^5]/x,x]
```

```
[Out] (PolyLog[2, -(1/(a*x^5))] - PolyLog[2, 1/(a*x^5)])/10
```

Maple [C] time = 0.146, size = 85, normalized size = 3.

$$\ln(x)\,\mathrm{arccoth}\left(a x^5\right)+\frac{1}{2}\sum_{\_R1=\mathrm{RootOf}\left(a\_Z^5-1\right)}\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)-\frac{1}{2}\sum_{\_R1=\mathrm{RootOf}\left(a\_Z^5+1\right)}\ln(x)\ln\left(\frac{\_R1}{\_R1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x^5)/x,x)
```

```
[Out] ln(x)*arccoth(a*x^5)+1/2*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1),_R1=R
ootOf(_Z^5*a-1))-1/2*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1),_R1=RootO
f(_Z^5*a+1))
```

Maxima [B] time = 0.990651, size = 140, normalized size = 5.

$$-\frac{1}{2}a\left(\frac{\log\left(ax^5+1\right)}{a}-\frac{\log\left(ax^5-1\right)}{a}\right)\log(x)-\frac{1}{10}a\left(\frac{\log\left(ax^5-1\right)\log\left(ax^5\right)+\mathrm{Li}_2\left(-ax^5+1\right)}{a}-\frac{\log\left(ax^5+1\right)\log\left(-ax^5\right)}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x^5)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*(log(a*x^5 + 1)/a - log(a*x^5 - 1)/a)*log(x) - 1/10*a*((log(a*x^5 -
1)*log(a*x^5) + dilog(-a*x^5 + 1))/a - (log(a*x^5 + 1)*log(-a*x^5) + dilog(
a*x^5 + 1))/a) + arccoth(a*x^5)*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{arcoth}\left(ax^5\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x^5)/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x^5)/x, x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: KeyError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x**5)/x,x)
```

```
[Out] Exception raised: KeyError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}\left(ax^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x^5)/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x^5)/x, x)
```

3.94 $\int \coth^{-1}\left(\frac{1}{x}\right) dx$

Optimal. Leaf size=19

$$\frac{1}{2}\log\left(1-x^2\right)+x\coth^{-1}\left(\frac{1}{x}\right)$$

```
[Out] x*ArcCoth[x^(-1)] + Log[1 - x^2]/2
```

Rubi [A] time = 0.0060927, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {6092, 263, 260}

$$\frac{1}{2}\log\left(1-x^2\right)+x\coth^{-1}\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[x^(-1)],x]
```

```
[Out] x*ArcCoth[x^(-1)] + Log[1 - x^2]/2
```

Rule 6092

```
Int[ArcCoth[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcCoth[c*x^n], x] - Dist
[c*n, Int[x^n/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \coth^{-1}\left(\frac{1}{x}\right) dx &= x\coth^{-1}\left(\frac{1}{x}\right)+\int \frac{1}{\left(1-\frac{1}{x^2}\right)x}\,dx\\ &= x\coth^{-1}\left(\frac{1}{x}\right)+\int \frac{x}{-1+x^2}\,dx\\ &= x\coth^{-1}\left(\frac{1}{x}\right)+\frac{1}{2}\log\left(1-x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0018144, size = 19, normalized size = 1.

$$\frac{1}{2}\log\left(1-x^2\right)+x\coth^{-1}\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[x^(-1)],x]

[Out] x*ArcCoth[x^(-1)] + Log[1 - x^2]/2
```

Maple [A] time = 0.056, size = 30, normalized size = 1.6

$$x\operatorname{arccoth}\left(x^{-1}\right)+\frac{\ln\left(x^{-1}-1\right)}{2}-\ln\left(x^{-1}\right)+\frac{\ln\left(x^{-1}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1/x),x)

[Out] x*arccoth(1/x)+1/2*ln(1/x-1)-ln(1/x)+1/2*ln(1/x+1)
```

Maxima [A] time = 0.976526, size = 20, normalized size = 1.05

$$x\operatorname{arcoth}\left(\frac{1}{x}\right)+\frac{1}{2}\log\left(x^{2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1/x),x, algorithm="maxima")

[Out] x*arccoth(1/x) + 1/2*log(x^2 - 1)
```

Fricas [A] time = 1.85874, size = 65, normalized size = 3.42

$$\frac{1}{2}x\log\left(-\frac{x+1}{x-1}\right)+\frac{1}{2}\log\left(x^{2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1/x),x, algorithm="fricas")

[Out] 1/2*x*log(-(x + 1)/(x - 1)) + 1/2*log(x^2 - 1)
```

Sympy [A] time = 0.254185, size = 15, normalized size = 0.79

$$x\operatorname{acoth}\left(\frac{1}{x}\right)+\log\left(x+1\right)-\operatorname{acoth}\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1/x),x)

[Out] x*acoth(1/x) + log(x + 1) - acoth(1/x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}\left(\frac{1}{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1/x),x, algorithm="giac")
```

```
[Out] integrate(arccoth(1/x), x)
```

3.95 $\int \frac{\coth^{-1}(ax^n)}{x} \, dx$

Optimal. Leaf size=38

$$\frac{\text{PolyLog}\left(2,-\frac{x^{-n}}{a}\right)}{2n}-\frac{\text{PolyLog}\left(2,\frac{x^{-n}}{a}\right)}{2n}$$

```
[Out] PolyLog[2, -(1/(a*x^n))]/(2*n) - PolyLog[2, 1/(a*x^n)]/(2*n)
```

Rubi [A] time = 0.023247, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6096, 5913}

$$\frac{\text{PolyLog}\left(2,-\frac{x^{-n}}{a}\right)}{2n}-\frac{\text{PolyLog}\left(2,\frac{x^{-n}}{a}\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x^n]/x,x]
```

```
[Out] PolyLog[2, -(1/(a*x^n))]/(2*n) - PolyLog[2, 1/(a*x^n)]/(2*n)
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcCoth[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 5913

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b*PolyLog[2, -(c*x)^(-1)])/2, x] - Simp[(b*PolyLog[2, 1/(c*x)])/
2, x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(ax^n)}{x} \, dx &= \frac{\text{Subst}\left(\int \frac{\coth^{-1}(ax)}{x} \, dx, x, x^n\right)}{n}\\ &= \frac{\text{Li}_2\left(-\frac{x^{-n}}{a}\right)}{2n}-\frac{\text{Li}_2\left(\frac{x^{-n}}{a}\right)}{2n}\end{aligned}$$

Mathematica [B] time = 0.0454459, size = 97, normalized size = 2.55

$$\frac{-\text{PolyLog}(2,1-ax^n)+\text{PolyLog}(2,ax^n+1)+n\log(x)\log(ax^n-1)-n\log(x)\log(ax^n+1)-\log(ax^n)\log(ax^n-}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x^n]/x,x]
```

```
[Out] (2*n*ArcCoth[a*x^n]*Log[x] + n*Log[x]*Log[-1 + a*x^n] - Log[a*x^n]*Log[-1 +
 a*x^n] - n*Log[x]*Log[1 + a*x^n] + Log[-(a*x^n)]*Log[1 + a*x^n] - PolyLog[
2, 1 - a*x^n] + PolyLog[2, 1 + a*x^n])/(2*n)
```

Maple [A] time = 0.09, size = 61, normalized size = 1.6

$$\frac{\ln(ax^n)\operatorname{arccoth}(ax^n)}{n} - \frac{\operatorname{dilog}(ax^n)}{2n} - \frac{\operatorname{dilog}(ax^n+1)}{2n} - \frac{\ln(ax^n)\ln(ax^n+1)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x^n)/x,x)
```

```
[Out] 1/n*ln(a*x^n)*arccoth(a*x^n)-1/2/n*dilog(a*x^n)-1/2/n*dilog(a*x^n+1)-1/2/n*
ln(a*x^n)*ln(a*x^n+1)
```

Maxima [B] time = 1.26882, size = 198, normalized size = 5.21

$$-\frac{1}{2}an\left(\frac{\log\left(\frac{ax^n+1}{a}\right)}{an} - \frac{\log\left(\frac{ax^n-1}{a}\right)}{an}\right)\log(x) + \frac{1}{2}an\left(\frac{\log(ax^n+1)\log(x) - \log(ax^n-1)\log(x)}{an} - \frac{n\log(ax^n+1)\log(x) + \cdots}{an^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x^n)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*n*(log((a*x^n + 1)/a)/(a*n) - log((a*x^n - 1)/a)/(a*n))*log(x) + 1/2
*a*n*((log(a*x^n + 1)*log(x) - log(a*x^n - 1)*log(x))/(a*n) - (n*log(a*x^n
+ 1)*log(x) + dilog(-a*x^n))/(a*n^2) + (n*log(-a*x^n + 1)*log(x) + dilog(a*
x^n))/(a*n^2)) + arccoth(a*x^n)*log(x)
```

Fricas [B] time = 2.00394, size = 421, normalized size = 11.08

$$-\frac{n\log(a\cosh(n\log(x)) + a\sinh(n\log(x)) + 1)\log(x) - n\log(-a\cosh(n\log(x)) - a\sinh(n\log(x)) + 1)\log(x) - n\,l\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x^n)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(n*log(a*cosh(n*log(x)) + a*sinh(n*log(x)) + 1)*log(x) - n*log(-a*cosh
(n*log(x)) - a*sinh(n*log(x)) + 1)*log(x) - n*log(x)*log((a*cosh(n*log(x))
+ a*sinh(n*log(x)) + 1)/(a*cosh(n*log(x)) + a*sinh(n*log(x)) - 1)) - dilog(
a*cosh(n*log(x)) + a*sinh(n*log(x))) + dilog(-a*cosh(n*log(x)) - a*sinh(n*l
og(x))))/n
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}(ax^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x**n)/x,x)
```

```
[Out] Integral(acoth(a*x**n)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(ax^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x^n)/x, x)
```

3.96 $\int (a+bx)\coth^{-1}(a+bx)\,dx$

Optimal. Leaf size=39

$$-\frac{\tanh^{-1}(a+bx)}{2b}+\frac{(a+bx)^2\coth^{-1}(a+bx)}{2b}+\frac{x}{2}$$

```
[Out] x/2 + ((a + b*x)^2*ArcCoth[a + b*x])/(2*b) - ArcTanh[a + b*x]/(2*b)
```

Rubi [A] time = 0.0224787, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6108, 5917, 321, 206}

$$-\frac{\tanh^{-1}(a+bx)}{2b}+\frac{(a+bx)^2\coth^{-1}(a+bx)}{2b}+\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*ArcCoth[a + b*x],x]
```

```
[Out] x/2 + ((a + b*x)^2*ArcCoth[a + b*x])/(2*b) - ArcTanh[a + b*x]/(2*b)
```

Rule 6108

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcCoth[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+bx)\coth^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int x\coth^{-1}(x)\,dx, x, a+bx\right)}{b}$$
$$= \frac{(a+bx)^2\coth^{-1}(a+bx)}{2b} - \frac{\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, a+bx\right)}{2b}$$
$$= \frac{x}{2} + \frac{(a+bx)^2\coth^{-1}(a+bx)}{2b} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, a+bx\right)}{2b}$$
$$= \frac{x}{2} + \frac{(a+bx)^2\coth^{-1}(a+bx)}{2b} - \frac{\tanh^{-1}(a+bx)}{2b}$$

Mathematica [A] time = 0.0320044, size = 66, normalized size = 1.69

$$\frac{a^2\log(a+bx+1) - \left(a^2-1\right)\log(-a-bx+1) - \log(a+bx+1) + 2bx(2a+bx)\coth^{-1}(a+bx) + 2bx}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*ArcCoth[a + b*x],x]
```

```
[Out] (2*b*x + 2*b*x*(2*a + b*x)*ArcCoth[a + b*x] - (-1 + a^2)*Log[1 - a - b*x] -
 Log[1 + a + b*x] + a^2*Log[1 + a + b*x])/(4*b)
```

Maple [B] time = 0.036, size = 70, normalized size = 1.8

$$\frac{b\,\text{arccoth}(bx+a)\,x^2}{2} + \text{arccoth}(bx+a)\,xa + \frac{\text{arccoth}(bx+a)\,a^2}{2b} + \frac{x}{2} + \frac{a}{2b} + \frac{\ln(bx+a-1)}{4b} - \frac{\ln(bx+a+1)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*arccoth(b*x+a),x)
```

```
[Out] 1/2*b*arccoth(b*x+a)*x^2+arccoth(b*x+a)*x*a+1/2/b*arccoth(b*x+a)*a^2+1/2*x+
1/2*a/b+1/4/b*ln(b*x+a-1)-1/4/b*ln(b*x+a+1)
```

Maxima [A] time = 0.966219, size = 84, normalized size = 2.15

$$\frac{1}{4}b\left(\frac{2x}{b} + \frac{\left(a^2-1\right)\log(bx+a+1)}{b^2} - \frac{\left(a^2-1\right)\log(bx+a-1)}{b^2}\right) + \frac{1}{2}\left(bx^2+2ax\right)\text{arccoth}(bx+a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*arccoth(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*b*(2*x/b + (a^2 - 1)*log(b*x + a + 1)/b^2 - (a^2 - 1)*log(b*x + a - 1)/
b^2) + 1/2*(b*x^2 + 2*a*x)*arccoth(b*x + a)
```

Fricas [A] time = 1.92371, size = 108, normalized size = 2.77

$$\frac{2\,bx + \left(b^2x^2 + 2\,abx + a^2 - 1\right)\log\left(\frac{bx+a+1}{bx+a-1}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*arccoth(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*x + (b^2*x^2 + 2*a*b*x + a^2 - 1)*log((b*x + a + 1)/(b*x + a - 1))
)/b
```

Sympy [A] time = 1.73783, size = 56, normalized size = 1.44

$$\begin{cases} \frac{a^2 \operatorname{acoth}(a+bx)}{2b} + ax\operatorname{acoth}(a+bx) + \frac{bx^2 \operatorname{acoth}(a+bx)}{2} + \frac{x}{2} - \frac{\operatorname{acoth}(a+bx)}{2b} & \text{for } b \neq 0 \\ ax\operatorname{acoth}(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*acoth(b*x+a),x)
```

```
[Out] Piecewise((a**2*acoth(a + b*x)/(2*b) + a*x*acoth(a + b*x) + b*x**2*acoth(a
+ b*x)/2 + x/2 - acoth(a + b*x)/(2*b), Ne(b, 0)), (a*x*acoth(a), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx + a)\operatorname{arcoth}(bx + a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*arccoth(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*arccoth(b*x + a), x)
```

3.97 $\int (a+bx)^2 \coth^{-1}(a+bx)\,dx$

Optimal. Leaf size=54

$$\frac{(a+bx)^2}{6b}+\frac{\log\left(1-(a+bx)^2\right)}{6b}+\frac{(a+bx)^3\coth^{-1}(a+bx)}{3b}$$

```
[Out] (a + b*x)^2/(6*b) + ((a + b*x)^3*ArcCoth[a + b*x])/(3*b) + Log[1 - (a + b*x
)^2]/(6*b)
```

Rubi [A] time = 0.0479171, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6108, 5917, 266, 43}

$$\frac{(a+bx)^2}{6b}+\frac{\log\left(1-(a+bx)^2\right)}{6b}+\frac{(a+bx)^3\coth^{-1}(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*ArcCoth[a + b*x],x]
```

```
[Out] (a + b*x)^2/(6*b) + ((a + b*x)^3*ArcCoth[a + b*x])/(3*b) + Log[1 - (a + b*x
)^2]/(6*b)
```

Rule 6108

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcCoth[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^2 \coth^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int x^2 \coth^{-1}(x)\,dx, x, a+bx\right)}{b}$$
$$= \frac{(a+bx)^3 \coth^{-1}(a+bx)}{3b} - \frac{\text{Subst}\left(\int \frac{x^3}{1-x^2}\,dx, x, a+bx\right)}{3b}$$
$$= \frac{(a+bx)^3 \coth^{-1}(a+bx)}{3b} - \frac{\text{Subst}\left(\int \frac{x}{1-x}\,dx, x, (a+bx)^2\right)}{6b}$$
$$= \frac{(a+bx)^3 \coth^{-1}(a+bx)}{3b} - \frac{\text{Subst}\left(\int \left(-1+\frac{1}{1-x}\right)dx, x, (a+bx)^2\right)}{6b}$$
$$= \frac{(a+bx)^2}{6b} + \frac{(a+bx)^3 \coth^{-1}(a+bx)}{3b} + \frac{\log\left(1-(a+bx)^2\right)}{6b}$$

Mathematica [A] time = 0.0345896, size = 42, normalized size = 0.78

$$\frac{(a+bx)^2 + \log\left(1-(a+bx)^2\right) + 2(a+bx)^3 \coth^{-1}(a+bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*ArcCoth[a + b*x],x]
```

```
[Out] ((a + b*x)^2 + 2*(a + b*x)^3*ArcCoth[a + b*x] + Log[1 - (a + b*x)^2])/(6*b)
```

Maple [A] time = 0.034, size = 95, normalized size = 1.8

$$\frac{b^2 \operatorname{arccoth}(bx+a)\,x^3}{3} + b\operatorname{arccoth}(bx+a)\,x^2 a + \operatorname{arccoth}(bx+a)\,xa^2 + \frac{\operatorname{arccoth}(bx+a)\,a^3}{3b} + \frac{bx^2}{6} + \frac{ax}{3} + \frac{a^2}{6b} + \frac{\ln(bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*arccoth(b*x+a),x)
```

```
[Out] 1/3*b^2*arccoth(b*x+a)*x^3+b*arccoth(b*x+a)*x^2*a+arccoth(b*x+a)*x*a^2+1/3/
b*arccoth(b*x+a)*a^3+1/6*b*x^2+1/3*a*x+1/6/b*a^2+1/6/b*ln(b*x+a-1)+1/6/b*ln
(b*x+a+1)
```

Maxima [A] time = 0.95623, size = 109, normalized size = 2.02

$$\frac{1}{6}\,b\left(\frac{bx^2+2\,ax}{b} + \frac{\left(a^3+1\right)\log(bx+a+1)}{b^2} - \frac{\left(a^3-1\right)\log(bx+a-1)}{b^2}\right) + \frac{1}{3}\left(b^2x^3+3\,abx^2+3\,a^2x\right)\operatorname{arcoth}(bx+a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arccoth(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/6*b*((b*x^2 + 2*a*x)/b + (a^3 + 1)*log(b*x + a + 1)/b^2 - (a^3 - 1)*log(b
*x + a - 1)/b^2) + 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)*arccoth(b*x + a)
```

Fricas [A] time = 1.87158, size = 211, normalized size = 3.91

$$\frac{b^2x^2 + 2\,abx + \left(a^3 + 1\right)\log\left(bx + a + 1\right) - \left(a^3 - 1\right)\log\left(bx + a - 1\right) + \left(b^3x^3 + 3\,ab^2x^2 + 3\,a^2bx\right)\log\left(\frac{bx+a+1}{bx+a-1}\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arccoth(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(b^2*x^2 + 2*a*b*x + (a^3 + 1)*log(b*x + a + 1) - (a^3 - 1)*log(b*x + a
 - 1) + (b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)*log((b*x + a + 1)/(b*x + a - 1)
))/b
```

Sympy [A] time = 3.13341, size = 97, normalized size = 1.8

$$\begin{cases} \frac{a^3\operatorname{acoth}(a+bx)}{3b} + a^2x\operatorname{acoth}(a+bx) + abx^2\operatorname{acoth}(a+bx) + \frac{ax}{3} + \frac{b^2x^3\operatorname{acoth}(a+bx)}{3} + \frac{bx^2}{6} + \frac{\log\left(\frac{a}{b}+x+\frac{1}{b}\right)}{3b} - \frac{\operatorname{acoth}(a+bx)}{3b} & \text{fo} \\ a^2x\operatorname{acoth}(a) & \text{ot} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*acoth(b*x+a),x)
```

```
[Out] Piecewise((a**3*acoth(a + b*x)/(3*b) + a**2*x*acoth(a + b*x) + a*b*x**2*aco
th(a + b*x) + a*x/3 + b**2*x**3*acoth(a + b*x)/3 + b*x**2/6 + log(a/b + x +
 1/b)/(3*b) - acoth(a + b*x)/(3*b), Ne(b, 0)), (a**2*x*acoth(a), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx + a)^2 \operatorname{arcoth}(bx + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arccoth(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^2*arccoth(b*x + a), x)
```

3.98 $\int \frac{\coth^{-1}(a+bx)}{a+bx}\,dx$

Optimal. Leaf size=35

$$\frac{\text{PolyLog}\left(2,-\frac{1}{a+bx}\right)}{2b}-\frac{\text{PolyLog}\left(2,\frac{1}{a+bx}\right)}{2b}$$

```
[Out] PolyLog[2, -(a + b*x)^(-1)]/(2*b) - PolyLog[2, (a + b*x)^(-1)]/(2*b)
```

Rubi [A] time = 0.0254893, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6108, 5913}

$$\frac{\text{PolyLog}\left(2,-\frac{1}{a+bx}\right)}{2b}-\frac{\text{PolyLog}\left(2,\frac{1}{a+bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x]/(a + b*x),x]
```

```
[Out] PolyLog[2, -(a + b*x)^(-1)]/(2*b) - PolyLog[2, (a + b*x)^(-1)]/(2*b)
```

Rule 6108

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcCoth[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rule 5913

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b*PolyLog[2, -(c*x)^(-1)])/2, x] - Simp[(b*PolyLog[2, 1/(c*x)])/
2, x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(a+bx)}{a+bx}\,dx &= \frac{\text{Subst}\left(\int \frac{\coth^{-1}(x)}{x}\,dx,x,a+bx\right)}{b}\\ &= \frac{\text{Li}_2\left(-\frac{1}{a+bx}\right)}{2b}-\frac{\text{Li}_2\left(\frac{1}{a+bx}\right)}{2b}\end{aligned}$$

Mathematica [B] time = 0.0254113, size = 286, normalized size = 8.17

$$-\frac{\text{PolyLog}(2,-a-bx)}{2b}+\frac{\text{PolyLog}(2,a+bx)}{2b}-\frac{\log^2\left(\frac{ab-(a-1)b}{b(a+bx)}\right)}{4b}+\frac{\log^2\left(\frac{ab-(a+1)b}{b(a+bx)}\right)}{4b}-\frac{\log\left(\frac{b(a+bx-1)}{(a-1)b-ab}\right)\log\left(\frac{ab-(a-1)b}{b(a+bx)}\right)}{2b}+\frac{\log}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[a + b*x]/(a + b*x),x]
```

```
[Out] -(Log[(b*(-1 + a + b*x))/((-1 + a)*b - a*b)]*Log[(-((-1 + a)*b) + a*b)/(b*(
a + b*x))])/(2*b) - Log[(-((-1 + a)*b) + a*b)/(b*(a + b*x))]^2/(4*b) + (Log
[(b*(-1 - a - b*x))/((-1 - a)*b + a*b)]*Log[(a*b - (1 + a)*b)/(b*(a + b*x))
])/(2*b) + Log[(a*b - (1 + a)*b)/(b*(a + b*x))]^2/(4*b) + (Log[(-((-1 + a)*
b) + a*b)/(b*(a + b*x))]*Log[(-1 + a + b*x)/(a + b*x)])/(2*b) - (Log[(a*b -
 (1 + a)*b)/(b*(a + b*x))]*Log[(1 + a + b*x)/(a + b*x)])/(2*b) - PolyLog[2,
 -a - b*x]/(2*b) + PolyLog[2, a + b*x]/(2*b)
```

Maple [A] time = 0.042, size = 59, normalized size = 1.7

$$\frac{\ln(bx+a)\operatorname{arccoth}(bx+a)}{b}-\frac{\operatorname{dilog}(bx+a)}{2b}-\frac{\operatorname{dilog}(bx+a+1)}{2b}-\frac{\ln(bx+a)\ln(bx+a+1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)/(b*x+a),x)
```

```
[Out] 1/b*ln(b*x+a)*arccoth(b*x+a)-1/2/b*dilog(b*x+a)-1/2/b*dilog(b*x+a+1)-1/2/b*
ln(b*x+a)*ln(b*x+a+1)
```

Maxima [B] time = 0.981154, size = 151, normalized size = 4.31

$$-\frac{1}{2}b\left(\frac{\log(bx+a)\log(bx+a-1)+\operatorname{Li}_2(-bx-a+1)}{b^2}-\frac{\log(bx+a+1)\log(-bx-a)+\operatorname{Li}_2(bx+a+1)}{b^2}\right)-\frac{1}{2}\left(\log(b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*b*((log(b*x + a)*log(b*x + a - 1) + dilog(-b*x - a + 1))/b^2 - (log(b*
x + a + 1)*log(-b*x - a) + dilog(b*x + a + 1))/b^2) - 1/2*(log(b*x + a + 1)
/b - log(b*x + a - 1)/b)*log(b*x + a) + arccoth(b*x + a)*log(b*x + a)/b
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arcoth}(bx+a)}{bx+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(arccoth(b*x + a)/(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{acoth}(a+bx)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)/(b*x+a),x)
```

```
[Out] Integral(acoth(a + b*x)/(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(bx+a)}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a)/(b*x + a), x)
```

3.99 $\int \frac{\coth^{-1}(a+bx)}{(a+bx)^2}\,dx$

Optimal. Leaf size=48

$$\frac{\log(a+bx)}{b}-\frac{\log\left(1-(a+bx)^2\right)}{2b}-\frac{\coth^{-1}(a+bx)}{b(a+bx)}$$

```
[Out] -(ArcCoth[a + b*x]/(b*(a + b*x))) + Log[a + b*x]/b - Log[1 - (a + b*x)^2]/(
2*b)
```

Rubi [A] time = 0.0451322, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6108, 5917, 266, 36, 31, 29}

$$\frac{\log(a+bx)}{b}-\frac{\log\left(1-(a+bx)^2\right)}{2b}-\frac{\coth^{-1}(a+bx)}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x]/(a + b*x)^2,x]
```

```
[Out] -(ArcCoth[a + b*x]/(b*(a + b*x))) + Log[a + b*x]/b - Log[1 - (a + b*x)^2]/(
2*b)
```

Rule 6108

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcCoth[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(a+bx)}{(a+bx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\coth^{-1}(x)}{x^2}\,dx, x, a+bx\right)}{b}\\
&= -\frac{\coth^{-1}(a+bx)}{b(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{x(1-x^2)}\,dx, x, a+bx\right)}{b}\\
&= -\frac{\coth^{-1}(a+bx)}{b(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{(1-x)x}\,dx, x, (a+bx)^2\right)}{2b}\\
&= -\frac{\coth^{-1}(a+bx)}{b(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, (a+bx)^2\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, (a+bx)^2\right)}{2b}\\
&= -\frac{\coth^{-1}(a+bx)}{b(a+bx)} + \frac{\log(a+bx)}{b} - \frac{\log\left(1-(a+bx)^2\right)}{2b}
\end{aligned}$$

Mathematica [A] time = 0.0257019, size = 43, normalized size = 0.9

$$-\frac{-2\log(a+bx)+\log\left(1-(a+bx)^2\right)+\frac{2\coth^{-1}(a+bx)}{a+bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a + b*x]/(a + b*x)^2,x]
```

```
[Out] -((2*ArcCoth[a + b*x])/(a + b*x) - 2*Log[a + b*x] + Log[1 - (a + b*x)^2])/(
2*b)
```

Maple [A] time = 0.04, size = 54, normalized size = 1.1

$$-\frac{\text{arccoth}\,(bx+a)}{b\,(bx+a)} - \frac{\ln\,(bx+a-1)}{2\,b} + \frac{\ln\,(bx+a)}{b} - \frac{\ln\,(bx+a+1)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)/(b*x+a)^2,x)
```

```
[Out] -arccoth(b*x+a)/b/(b*x+a)-1/2/b*ln(b*x+a-1)+ln(b*x+a)/b-1/2/b*ln(b*x+a+1)
```

Maxima [A] time = 0.97605, size = 72, normalized size = 1.5

$$-\frac{\log\,(bx+a+1)}{2\,b} + \frac{\log\,(bx+a)}{b} - \frac{\log\,(bx+a-1)}{2\,b} - \frac{\text{arccoth}\,(bx+a)}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*log(b*x + a + 1)/b + log(b*x + a)/b - 1/2*log(b*x + a - 1)/b - arccoth
(b*x + a)/((b*x + a)*b)
```

Fricas [A] time = 1.91232, size = 171, normalized size = 3.56

$$-\frac{(bx+a)\log\left(b^2x^2+2\,abx+a^2-1\right)-2\,(bx+a)\log\left(bx+a\right)+\log\left(\frac{bx+a+1}{bx+a-1}\right)}{2\left(b^2x+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((b*x + a)*log(b^2*x^2 + 2*a*b*x + a^2 - 1) - 2*(b*x + a)*log(b*x + a)
 + log((b*x + a + 1)/(b*x + a - 1)))/(b^2*x + a*b)
```

Sympy [A] time = 4.82632, size = 136, normalized size = 2.83

$$\begin{cases} \frac{a\log\left(\frac{a}{b}+x\right)}{ab+b^2x}-\frac{a\log\left(\frac{a}{b}+x+\frac{1}{b}\right)}{ab+b^2x}+\frac{a\operatorname{acoth}\left(a+bx\right)}{ab+b^2x}+\frac{bx\log\left(\frac{a}{b}+x\right)}{ab+b^2x}-\frac{bx\log\left(\frac{a}{b}+x+\frac{1}{b}\right)}{ab+b^2x}+\frac{bx\operatorname{acoth}\left(a+bx\right)}{ab+b^2x}-\frac{\operatorname{acoth}\left(a+bx\right)}{ab+b^2x} & \text{for}\ b\neq 0\\ \frac{x\operatorname{acoth}\left(a\right)}{a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)/(b*x+a)**2,x)
```

```
[Out] Piecewise((a*log(a/b + x)/(a*b + b**2*x) - a*log(a/b + x + 1/b)/(a*b + b**2
*x) + a*acoth(a + b*x)/(a*b + b**2*x) + b*x*log(a/b + x)/(a*b + b**2*x) - b
*x*log(a/b + x + 1/b)/(a*b + b**2*x) + b*x*acoth(a + b*x)/(a*b + b**2*x) -
acoth(a + b*x)/(a*b + b**2*x), Ne(b, 0)), (x*acoth(a)/a**2, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}\left(bx+a\right)}{\left(bx+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a)/(b*x + a)^2, x)
```

3.100 $\int \frac{\coth^{-1}(1+x)}{2+2x} \, dx$

Optimal. Leaf size=25

$$\frac{1}{4}\text{PolyLog}\left(2,-\frac{1}{x+1}\right)-\frac{1}{4}\text{PolyLog}\left(2,\frac{1}{x+1}\right)$$

```
[Out] PolyLog[2, -(1 + x)^(-1)]/4 - PolyLog[2, (1 + x)^(-1)]/4
```

Rubi [A] time = 0.0227001, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6108, 12, 5913}

$$\frac{1}{4}\text{PolyLog}\left(2,-\frac{1}{x+1}\right)-\frac{1}{4}\text{PolyLog}\left(2,\frac{1}{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[1 + x]/(2 + 2*x),x]
```

```
[Out] PolyLog[2, -(1 + x)^(-1)]/4 - PolyLog[2, (1 + x)^(-1)]/4
```

Rule 6108

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcCoth[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5913

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b*PolyLog[2, -(c*x)^(-1)])/2, x] - Simp[(b*PolyLog[2, 1/(c*x)])/
2, x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(1+x)}{2+2x} \, dx &= \text{Subst}\left(\int \frac{\coth^{-1}(x)}{2x} \, dx, x, 1+x\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{\coth^{-1}(x)}{x} \, dx, x, 1+x\right)\\ &= \frac{1}{4}\text{Li}_2\left(-\frac{1}{1+x}\right)-\frac{1}{4}\text{Li}_2\left(\frac{1}{1+x}\right)\end{aligned}$$

Mathematica [B] time = 0.0131804, size = 117, normalized size = 4.68

$$-\frac{1}{4}\text{PolyLog}(2,-x-1)+\frac{1}{4}\text{PolyLog}(2,x+1)+\frac{1}{8}\log^2\left(-\frac{1}{x+1}\right)-\frac{1}{8}\log^2\left(\frac{1}{x+1}\right)+\frac{1}{4}\log(x+2)\log\left(-\frac{1}{x+1}\right)-\frac{1}{4}\log\Big($$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[1 + x]/(2 + 2*x),x]
```

```
[Out] Log[-(1 + x)^(-1)]^2/8 - (Log[-x]*Log[(1 + x)^(-1)])/4 - Log[(1 + x)^(-1)]^
2/8 + (Log[(1 + x)^(-1)]*Log[x/(1 + x)])/4 + (Log[-(1 + x)^(-1)]*Log[2 + x]
)/4 - (Log[-(1 + x)^(-1)]*Log[(2 + x)/(1 + x)])/4 - PolyLog[2, -1 - x]/4 +
PolyLog[2, 1 + x]/4
```

Maple [A] time = 0.029, size = 34, normalized size = 1.4

$$\frac{\ln(1+x)\,\mathrm{arccoth}(1+x)}{2}-\frac{\mathrm{dilog}(1+x)}{4}-\frac{\mathrm{dilog}(x+2)}{4}-\frac{\ln(1+x)\ln(x+2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1+x)/(2+2*x),x)
```

```
[Out] 1/2*ln(1+x)*arccoth(1+x)-1/4*dilog(1+x)-1/4*dilog(x+2)-1/4*ln(1+x)*ln(x+2)
```

Maxima [B] time = 0.978667, size = 78, normalized size = 3.12

$$-\frac{1}{4}(\log(x+2)-\log(x))\log(x+1)+\frac{1}{2}\,\mathrm{arcoth}(x+1)\log(x+1)-\frac{1}{4}\log(x+1)\log(x)+\frac{1}{4}\log(x+2)\log(-x-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+x)/(2+2*x),x, algorithm="maxima")
```

```
[Out] -1/4*(log(x + 2) - log(x))*log(x + 1) + 1/2*arccoth(x + 1)*log(x + 1) - 1/4
*log(x + 1)*log(x) + 1/4*log(x + 2)*log(-x - 1) - 1/4*dilog(-x) + 1/4*dilog
(x + 2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{arcoth}(x+1)}{2(x+1)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+x)/(2+2*x),x, algorithm="fricas")
```

```
[Out] integral(1/2*arccoth(x + 1)/(x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\mathrm{acoth}(x+1)}{x+1}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1+x)/(2+2*x),x)

[Out] Integral(acoth(x + 1)/(x + 1), x)/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(x+1)}{2(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+x)/(2+2*x),x, algorithm="giac")

[Out] integrate(1/2*arccoth(x + 1)/(x + 1), x)
```

3.101 $\int \frac{\coth^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx$

Optimal. Leaf size=35

$$\frac{\text{PolyLog}\left(2,-\frac{1}{a+bx}\right)}{2d}-\frac{\text{PolyLog}\left(2,\frac{1}{a+bx}\right)}{2d}$$

```
[Out] PolyLog[2, -(a + b*x)^(-1)]/(2*d) - PolyLog[2, (a + b*x)^(-1)]/(2*d)
```

Rubi [A] time = 0.0301393, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6108, 12, 5913}

$$\frac{\text{PolyLog}\left(2,-\frac{1}{a+bx}\right)}{2d}-\frac{\text{PolyLog}\left(2,\frac{1}{a+bx}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] PolyLog[2, -(a + b*x)^(-1)]/(2*d) - PolyLog[2, (a + b*x)^(-1)]/(2*d)
```

Rule 6108

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcCoth[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5913

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b*PolyLog[2, -(c*x)^(-1)])/2, x] - Simp[(b*PolyLog[2, 1/(c*x)])/
2, x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx &= \frac{\text{Subst}\left(\int \frac{b\coth^{-1}(x)}{dx}\,dx,x,a+bx\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{\coth^{-1}(x)}{x}\,dx,x,a+bx\right)}{d}\\ &= \frac{\text{Li}_2\left(-\frac{1}{a+bx}\right)}{2d}-\frac{\text{Li}_2\left(\frac{1}{a+bx}\right)}{2d}\end{aligned}$$

Mathematica [B] time = 0.023135, size = 312, normalized size = 8.91

$$b\left(-\frac{\text{PolyLog}(2,-a-bx)}{2bd}+\frac{\text{PolyLog}(2,a+bx)}{2bd}-\frac{\log^2\left(\frac{ab-(a-1)b}{b(a+bx)}\right)}{4bd}+\frac{\log^2\left(\frac{ab-(a+1)b}{b(a+bx)}\right)}{4bd}-\frac{\log\left(\frac{b(a+bx-1)}{(a-1)b-ab}\right)\log\left(\frac{ab-(a-1)b}{b(a+bx)}\right)}{2bd}+\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] b*(-(Log[(b*(-1 + a + b*x))/((-1 + a)*b - a*b)]*Log[(-((-1 + a)*b) + a*b)/(
b*(a + b*x))])/(2*b*d) - Log[(-((-1 + a)*b) + a*b)/(b*(a + b*x))]^2/(4*b*d)
 + (Log[(b*(-1 - a - b*x))/((-1 - a)*b + a*b)]*Log[(a*b - (1 + a)*b)/(b*(a
+ b*x))])/(2*b*d) + Log[(a*b - (1 + a)*b)/(b*(a + b*x))]^2/(4*b*d) + (Log[(
-((-1 + a)*b) + a*b)/(b*(a + b*x))]*Log[(-1 + a + b*x)/(a + b*x)])/(2*b*d)
- (Log[(a*b - (1 + a)*b)/(b*(a + b*x))]*Log[(1 + a + b*x)/(a + b*x)])/(2*b*
d) - PolyLog[2, -a - b*x]/(2*b*d) + PolyLog[2, a + b*x]/(2*b*d))
```

Maple [A] time = 0.044, size = 59, normalized size = 1.7

$$\frac{\ln(bx+a)\,\text{arccoth}(bx+a)}{d}-\frac{\text{dilog}(bx+a)}{2d}-\frac{\text{dilog}(bx+a+1)}{2d}-\frac{\ln(bx+a)\ln(bx+a+1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)/(a*d/b+d*x),x)
```

```
[Out] 1/d*ln(b*x+a)*arccoth(b*x+a)-1/2/d*dilog(b*x+a)-1/2/d*dilog(b*x+a+1)-1/2/d*
ln(b*x+a)*ln(b*x+a+1)
```

Maxima [B] time = 0.98962, size = 178, normalized size = 5.09

$$-\frac{1}{2}b\left(\frac{\log(bx+a)\log(bx+a-1)+\text{Li}_2(-bx-a+1)}{bd}-\frac{\log(bx+a+1)\log(-bx-a)+\text{Li}_2(bx+a+1)}{bd}\right)-\frac{b\left(\frac{\log(bx+a+1)}{b}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

```
[Out] -1/2*b*((log(b*x + a)*log(b*x + a - 1) + dilog(-b*x - a + 1))/(b*d) - (log(
b*x + a + 1)*log(-b*x - a) + dilog(b*x + a + 1))/(b*d)) - 1/2*b*(log(b*x +
a + 1)/b - log(b*x + a - 1)/b)*log(d*x + a*d/b)/d + arccoth(b*x + a)*log(d*
x + a*d/b)/d
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\text{arcoth}(bx+a)}{bdx+ad},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

```
[Out] integral(b*arccoth(b*x + a)/(b*d*x + a*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b\int \frac{\operatorname{acoth}(a+bx)}{a+bx}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)/(a*d/b+d*x),x)
```

```
[Out] b*Integral(acoth(a + b*x)/(a + b*x), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(bx+a)}{dx+\frac{ad}{b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a)/(d*x + a*d/b), x)
```

3.102 $\int (e+fx)^3 \left(a+b\coth^{-1}(c+dx)\right) dx$

Optimal. Leaf size=168

$$\frac{(e+fx)^4\left(a+b\coth^{-1}(c+dx)\right)}{4f}+\frac{bfx\left(\left(6c^2+1\right)f^2-12cdef+6d^2e^2\right)}{4d^3}+\frac{bf^2(c+dx)^2(de-cf)}{2d^4}-\frac{b(-cf+de-f)^4\log(}{8d^4f}$$

```
[Out] (b*f*(6*d^2*e^2 - 12*c*d*e*f + (1 + 6*c^2)*f^2)*x)/(4*d^3) + (b*f^2*(d*e -
c*f)*(c + d*x)^2)/(2*d^4) + (b*f^3*(c + d*x)^3)/(12*d^4) + ((e + f*x)^4*(a
+ b*ArcCoth[c + d*x]))/(4*f) + (b*(d*e + f - c*f)^4*Log[1 - c - d*x])/(8*d^
4*f) - (b*(d*e - f - c*f)^4*Log[1 + c + d*x])/(8*d^4*f)
```

Rubi [A] time = 0.341981, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6112, 5927, 702, 633, 31}

$$\frac{(e+fx)^4\left(a+b\coth^{-1}(c+dx)\right)}{4f}+\frac{bfx\left(\left(6c^2+1\right)f^2-12cdef+6d^2e^2\right)}{4d^3}+\frac{bf^2(c+dx)^2(de-cf)}{2d^4}-\frac{b(-cf+de-f)^4\log(}{8d^4f}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*(a + b*ArcCoth[c + d*x]),x]
```

```
[Out] (b*f*(6*d^2*e^2 - 12*c*d*e*f + (1 + 6*c^2)*f^2)*x)/(4*d^3) + (b*f^2*(d*e -
c*f)*(c + d*x)^2)/(2*d^4) + (b*f^3*(c + d*x)^3)/(12*d^4) + ((e + f*x)^4*(a
+ b*ArcCoth[c + d*x]))/(4*f) + (b*(d*e + f - c*f)^4*Log[1 - c - d*x])/(8*d^
4*f) - (b*(d*e - f - c*f)^4*Log[1 + c + d*x])/(8*d^4*f)
```

Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rule 5927

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcCoth[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int (e+fx)^3\left(a+b\coth^{-1}(c+dx)\right)dx=\frac{\text{Subst}\left(\int\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^3\left(a+b\coth^{-1}(x)\right)dx,x,c+dx\right)}{d}$$

$$=\frac{(e+fx)^4\left(a+b\coth^{-1}(c+dx)\right)}{4f}-\frac{b\,\text{Subst}\left(\int\frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^4}{1-x^2}dx,x,c+dx\right)}{4f}$$

$$=\frac{(e+fx)^4\left(a+b\coth^{-1}(c+dx)\right)}{4f}-\frac{b\,\text{Subst}\left(\int\left(-\frac{f^2\left(6d^2e^2-12cdef+\left(1+6c^2\right)f^2\right)}{d^4}-\ldots\right.\right.}{}$$

$$=\frac{bf\left(6d^2e^2-12cdef+\left(1+6c^2\right)f^2\right)x}{4d^3}+\frac{bf^2(de-cf)(c+dx)^2}{2d^4}+\frac{bf^3(c+dx)^3}{12d^4}\ldots$$

$$=\frac{bf\left(6d^2e^2-12cdef+\left(1+6c^2\right)f^2\right)x}{4d^3}+\frac{bf^2(de-cf)(c+dx)^2}{2d^4}+\frac{bf^3(c+dx)^3}{12d^4}\ldots$$

$$=\frac{bf\left(6d^2e^2-12cdef+\left(1+6c^2\right)f^2\right)x}{4d^3}+\frac{bf^2(de-cf)(c+dx)^2}{2d^4}+\frac{bf^3(c+dx)^3}{12d^4}\ldots$$

Mathematica [A] time = 0.281329, size = 270, normalized size = 1.61

$$\frac{6dx\left(4ad^3e^3+bf\left(\left(3c^2+1\right)f^2-8cdef+6d^2e^2\right)\right)+6d^2fx^2\left(6ad^2e^2+bf(2de-cf)\right)+2d^3f^2x^3(12ade+bf)+6ad^4f^3\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*(a + b*ArcCoth[c + d*x]),x]
```

```
[Out] (6*d*(4*a*d^3*e^3 + b*f*(6*d^2*e^2 - 8*c*d*e*f + (1 + 3*c^2)*f^2))*x + 6*d^
2*f*(6*a*d^2*e^2 + b*f*(2*d*e - c*f))*x^2 + 2*d^3*f^2*(12*a*d*e + b*f)*x^3
+ 6*a*d^4*f^3*x^4 + 6*b*d^4*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*A
rcCoth[c + d*x] - 3*b*(-1 + c)*(4*d^3*e^3 - 6*(-1 + c)*d^2*e^2*f + 4*(-1 +
c)^2*d*e*f^2 - (-1 + c)^3*f^3)*Log[1 - c - d*x] - 3*b*(1 + c)*(-4*d^3*e^3 +
 6*(1 + c)*d^2*e^2*f - 4*(1 + c)^2*d*e*f^2 + (1 + c)^3*f^3)*Log[1 + c + d*x
])/(24*d^4)
```

Maple [B] time = 0.046, size = 786, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*(a+b*arccoth(d*x+c)),x)
```

```
[Out] 1/4*a/f*e^4+13/12/d^4*b*f^3*c^3+1/4/d^4*b*f^3*c+3/2/d^2*b*f*c*e^2-5/2/d^3*b
*f^2*c^2*e-1/2/d*b*ln(d*x+c-1)*c*e^3+1/2/d*b*ln(d*x+c+1)*c*e^3-1/2/d^4*b*f^
3*ln(d*x+c-1)*c^3-1/8/d^4*b*f^3*ln(d*x+c+1)*c^4-3/4/d^2*b*f*ln(d*x+c+1)*e^2
```

```
+1/2/d^3*b*f^2*ln(d*x+c-1)*e+1/4*a*f^3*x^4+a*x*e^3+1/12/d*b*f^3*x^3+1/2/d*b
*ln(d*x+c-1)*e^3+1/2/d*b*ln(d*x+c+1)*e^3-1/8/d^4*b*f^3*ln(d*x+c+1)+1/8/d^4*
b*f^3*ln(d*x+c-1)+1/4*b*f^3*arccoth(d*x+c)*x^4+arccoth(d*x+c)*x*b*e^3-1/8*b
/f*ln(d*x+c+1)*e^4+1/4*b/f*arccoth(d*x+c)*e^4+1/4*b/d^3*f^3*x+1/8*b/f*ln(d*
x+c-1)*e^4+a*f^2*x^3*e+3/2*a*f*x^2*e^2+3/4*b/d^3*f^3*c^2*x+1/2/d^3*b*f^2*ln
(d*x+c+1)*e+3/4/d^2*b*f*ln(d*x+c-1)*e^2+1/8/d^4*b*f^3*ln(d*x+c-1)*c^4-1/2/d
^4*b*f^3*ln(d*x+c+1)*c-3/4/d^4*b*f^3*ln(d*x+c+1)*c^2-1/2/d^4*b*f^3*ln(d*x+c
+1)*c^3-1/2/d^4*b*f^3*ln(d*x+c-1)*c+3/4/d^4*b*f^3*ln(d*x+c-1)*c^2+b*f^2*arc
coth(d*x+c)*e*x^3+3/2*b*f*arccoth(d*x+c)*e^2*x^2-1/4/d^2*b*f^3*x^2*c+1/2/d*
b*f^2*e*x^2+3/2*b/d*f*e^2*x+3/2/d^3*b*f^2*ln(d*x+c+1)*c*e+3/2/d^3*b*f^2*ln(
d*x+c-1)*c^2*e+1/2/d^3*b*f^2*ln(d*x+c+1)*c^3*e-3/2/d^2*b*f*ln(d*x+c+1)*c*e^
2-3/4/d^2*b*f*ln(d*x+c+1)*c^2*e^2+3/2/d^3*b*f^2*ln(d*x+c+1)*c^2*e+3/4/d^2*b
*f*ln(d*x+c-1)*c^2*e^2-2*b/d^2*f^2*c*e*x-3/2/d^2*b*f*ln(d*x+c-1)*c*e^2-1/2/
d^3*b*f^2*ln(d*x+c-1)*c^3*e-3/2/d^3*b*f^2*ln(d*x+c-1)*c*e
```

Maxima [B] time = 0.992158, size = 450, normalized size = 2.68

$$\frac{1}{4}af^3x^4 + aef^2x^3 + \frac{3}{2}ae^2fx^2 + \frac{3}{4}\left(2x^2\operatorname{arcoth}(dx+c) + d\left(\frac{2x}{d^2} - \frac{(c^2+2c+1)\log(dx+c+1)}{d^3} + \frac{(c^2-2c+1)\log(dx}{d^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(a+b*arccoth(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*a*f^3*x^4 + a*e*f^2*x^3 + 3/2*a*e^2*f*x^2 + 3/4*(2*x^2*arccoth(d*x + c)
 + d*(2*x/d^2 - (c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(
d*x + c - 1)/d^3))*b*e^2*f + 1/2*(2*x^3*arccoth(d*x + c) + d*((d*x^2 - 4*c*
x)/d^3 + (c^3 + 3*c^2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3*c
- 1)*log(d*x + c - 1)/d^4))*b*e*f^2 + 1/24*(6*x^4*arccoth(d*x + c) + d*(2*(
d^2*x^3 - 3*c*d*x^2 + 3*(3*c^2 + 1)*x)/d^4 - 3*(c^4 + 4*c^3 + 6*c^2 + 4*c +
 1)*log(d*x + c + 1)/d^5 + 3*(c^4 - 4*c^3 + 6*c^2 - 4*c + 1)*log(d*x + c -
1)/d^5))*b*f^3 + a*e^3*x + 1/2*(2*(d*x + c)*arccoth(d*x + c) + log(-(d*x +
c)^2 + 1))*b*e^3/d
```

Fricas [B] time = 2.11405, size = 856, normalized size = 5.1

$$\frac{6\,ad^4f^3x^4 + 2\left(12\,ad^4ef^2 + bd^3f^3\right)x^3 + 6\left(6\,ad^4e^2f + 2\,bd^3ef^2 - bcd^2f^3\right)x^2 + 6\left(4\,ad^4e^3 + 6\,bd^3e^2f - 8\,bcd^2ef^2 + \left(3\,bc^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(a+b*arccoth(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(6*a*d^4*f^3*x^4 + 2*(12*a*d^4*e*f^2 + b*d^3*f^3)*x^3 + 6*(6*a*d^4*e^2
*f + 2*b*d^3*e*f^2 - b*c*d^2*f^3)*x^2 + 6*(4*a*d^4*e^3 + 6*b*d^3*e^2*f - 8*
b*c*d^2*e*f^2 + (3*b*c^2 + b)*d*f^3)*x + 3*(4*(b*c + b)*d^3*e^3 - 6*(b*c^2
+ 2*b*c + b)*d^2*e^2*f + 4*(b*c^3 + 3*b*c^2 + 3*b*c + b)*d*e*f^2 - (b*c^4 +
 4*b*c^3 + 6*b*c^2 + 4*b*c + b)*f^3)*log(d*x + c + 1) - 3*(4*(b*c - b)*d^3*
e^3 - 6*(b*c^2 - 2*b*c + b)*d^2*e^2*f + 4*(b*c^3 - 3*b*c^2 + 3*b*c - b)*d*e
*f^2 - (b*c^4 - 4*b*c^3 + 6*b*c^2 - 4*b*c + b)*f^3)*log(d*x + c - 1) + 3*(b
*d^4*f^3*x^4 + 4*b*d^4*e*f^2*x^3 + 6*b*d^4*e^2*f*x^2 + 4*b*d^4*e^3*x)*log((
d*x + c + 1)/(d*x + c - 1)))/d^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*(a+b*acoth(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)^3 (b \operatorname{arcoth}(dx+c)+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(a+b*arccoth(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*(b*arccoth(d*x + c) + a), x)
```

3.103 $\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right) dx$

Optimal. Leaf size=120

$$\frac{(e+fx)^3\left(a+b\coth^{-1}(c+dx)\right)}{3f}+\frac{bfx(de-cf)}{d^2}+\frac{b(-cf+de+f)^3\log(-c-dx+1)}{6d^3f}-\frac{b(de-(c+1)f)^3\log(c+dx+1)}{6d^3f}$$

```
[Out] (b*f*(d*e - c*f)*x)/d^2 + (b*f^2*(c + d*x)^2)/(6*d^3) + ((e + f*x)^3*(a + b
*ArcCoth[c + d*x]))/(3*f) + (b*(d*e + f - c*f)^3*Log[1 - c - d*x])/(6*d^3*f
) - (b*(d*e - (1 + c)*f)^3*Log[1 + c + d*x])/(6*d^3*f)
```

Rubi [A] time = 0.203979, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6112, 5927, 702, 633, 31}

$$\frac{(e+fx)^3\left(a+b\coth^{-1}(c+dx)\right)}{3f}+\frac{bfx(de-cf)}{d^2}+\frac{b(-cf+de+f)^3\log(-c-dx+1)}{6d^3f}-\frac{b(de-(c+1)f)^3\log(c+dx+1)}{6d^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*(a + b*ArcCoth[c + d*x]),x]
```

```
[Out] (b*f*(d*e - c*f)*x)/d^2 + (b*f^2*(c + d*x)^2)/(6*d^3) + ((e + f*x)^3*(a + b
*ArcCoth[c + d*x]))/(3*f) + (b*(d*e + f - c*f)^3*Log[1 - c - d*x])/(6*d^3*f
) - (b*(d*e - (1 + c)*f)^3*Log[1 + c + d*x])/(6*d^3*f)
```

Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rule 5927

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcCoth[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right) dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2 \left(a+b\coth^{-1}(x)\right) dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^3\left(a+b\coth^{-1}(c+dx)\right)}{3f} - \frac{b\,\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^3}{1-x^2} dx, x, c+dx\right)}{3f}$$

$$= \frac{(e+fx)^3\left(a+b\coth^{-1}(c+dx)\right)}{3f} - \frac{b\,\text{Subst}\left(\int \left(-\frac{3f^2(de-cf)}{d^3} - \frac{f^3x}{d^3} + \frac{(de-cf)\left(d^2e\right.}{}\right.\right.}{}$$

$$= \frac{bf(de-cf)x}{d^2} + \frac{bf^2(c+dx)^2}{6d^3} + \frac{(e+fx)^3\left(a+b\coth^{-1}(c+dx)\right)}{3f} - \frac{b\,\text{Subst}\left(\right.}{}$$

$$= \frac{bf(de-cf)x}{d^2} + \frac{bf^2(c+dx)^2}{6d^3} + \frac{(e+fx)^3\left(a+b\coth^{-1}(c+dx)\right)}{3f} - \frac{\left(b(de+f\right.}{}$$

$$= \frac{bf(de-cf)x}{d^2} + \frac{bf^2(c+dx)^2}{6d^3} + \frac{(e+fx)^3\left(a+b\coth^{-1}(c+dx)\right)}{3f} + \frac{b(de+f}{}$$

Mathematica [A] time = 0.161644, size = 174, normalized size = 1.45

$$\frac{2dx\left(3ad^2e^2+bf(3de-2cf)\right)+d^2fx^2(6ade+bf)+2ad^3f^2x^3+2bd^3x\left(3e^2+3efx+f^2x^2\right)\coth^{-1}(c+dx)-b(c-1)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*(a + b*ArcCoth[c + d*x]),x]
```

```
[Out] (2*d*(3*a*d^2*e^2 + b*f*(3*d*e - 2*c*f))*x + d^2*f*(6*a*d*e + b*f)*x^2 + 2*
a*d^3*f^2*x^3 + 2*b*d^3*x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcCoth[c + d*x] - b*
(-1 + c)*(3*d^2*e^2 - 3*(-1 + c)*d*e*f + (-1 + c)^2*f^2)*Log[1 - c - d*x] +
 b*(1 + c)*(3*d^2*e^2 - 3*(1 + c)*d*e*f + (1 + c)^2*f^2)*Log[1 + c + d*x])/
(6*d^3)
```

Maple [B] time = 0.04, size = 477, normalized size = 4.

$$\frac{bf\ln(dx+c-1)c^2e}{2d^2} + \frac{af^2x^3}{3} + axe^2 - \frac{2bf^2cx}{3d^2} + \frac{bf^2\ln(dx+c+1)}{6d^3} + \frac{b\ln(dx+c-1)e^2}{2d} + afx^2e - \frac{b\ln(dx+c+1}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(a+b*arccoth(d*x+c)),x)
```

```
[Out] 1/2/d^2*b*f*ln(d*x+c-1)*c^2*e+1/3*a*f^2*x^3+a*x*e^2-2/3*b/d^2*f^2*c*x+1/6/d
^3*b*f^2*ln(d*x+c+1)+1/2/d*b*ln(d*x+c-1)*e^2+a*f*x^2*e-1/6*b/f*ln(d*x+c+1)*
e^3+1/6*b/f*ln(d*x+c-1)*e^3+1/3*b/f*arccoth(d*x+c)*e^3+arccoth(d*x+c)*x*b*e
^2+1/3*b*f^2*arccoth(d*x+c)*x^3+1/2/d*b*ln(d*x+c+1)*e^2+1/6/d^3*b*f^2*ln(d*
```

```
x+c-1)+1/6/d*b*f^2*x^2-1/2/d^2*b*f*ln(d*x+c+1)*c^2*e-1/d^2*b*f*ln(d*x+c+1)*
c*e-1/d^2*b*f*ln(d*x+c-1)*c*e+1/3*a/f*e^3-5/6/d^3*b*f^2*c^2+1/d^2*b*f*c*e+b
/d*f*e*x+1/6/d^3*b*f^2*ln(d*x+c+1)*c^3+1/2/d^3*b*f^2*ln(d*x+c+1)*c^2-1/2/d^
3*b*f^2*ln(d*x+c-1)*c-1/6/d^3*b*f^2*ln(d*x+c-1)*c^3+1/2/d^3*b*f^2*ln(d*x+c+
1)*c-1/2/d^2*b*f*ln(d*x+c+1)*e+1/2/d^2*b*f*ln(d*x+c-1)*e+1/2/d^3*b*f^2*ln(d
*x+c-1)*c^2+b*f*arccoth(d*x+c)*e*x^2-1/2/d*b*ln(d*x+c-1)*c*e^2+1/2/d*b*ln(d
*x+c+1)*c*e^2
```

Maxima [A] time = 0.970765, size = 279, normalized size = 2.32

$$\frac{1}{3}af^2x^3 + aefx^2 + \frac{1}{2}\left(2x^2\operatorname{arcoth}(dx+c) + d\left(\frac{2x}{d^2} - \frac{(c^2+2c+1)\log(dx+c+1)}{d^3} + \frac{(c^2-2c+1)\log(dx+c-1)}{d^3}\right)\right)be$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccoth(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*a*f^2*x^3 + a*e*f*x^2 + 1/2*(2*x^2*arccoth(d*x + c) + d*(2*x/d^2 - (c^2
 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*b
*e*f + 1/6*(2*x^3*arccoth(d*x + c) + d*((d*x^2 - 4*c*x)/d^3 + (c^3 + 3*c^2
+ 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3*c - 1)*log(d*x + c - 1)/
d^4))*b*f^2 + a*e^2*x + 1/2*(2*(d*x + c)*arccoth(d*x + c) + log(-(d*x + c)^
2 + 1))*b*e^2/d
```

Fricas [B] time = 1.96481, size = 548, normalized size = 4.57

$$\frac{2ad^3f^2x^3 + (6ad^3ef + bd^2f^2)x^2 + 2(3ad^3e^2 + 3bd^2ef - 2bcdf^2)x + (3(bc+b)d^2e^2 - 3(bc^2+2bc+b)def + (bc^3+3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccoth(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*d^3*f^2*x^3 + (6*a*d^3*e*f + b*d^2*f^2)*x^2 + 2*(3*a*d^3*e^2 + 3*b
*d^2*e*f - 2*b*c*d*f^2)*x + (3*(b*c + b)*d^2*e^2 - 3*(b*c^2 + 2*b*c + b)*d*
e*f + (b*c^3 + 3*b*c^2 + 3*b*c + b)*f^2)*log(d*x + c + 1) - (3*(b*c - b)*d^
2*e^2 - 3*(b*c^2 - 2*b*c + b)*d*e*f + (b*c^3 - 3*b*c^2 + 3*b*c - b)*f^2)*lo
g(d*x + c - 1) + (b*d^3*f^2*x^3 + 3*b*d^3*e*f*x^2 + 3*b*d^3*e^2*x)*log((d*x
 + c + 1)/(d*x + c - 1)))/d^3
```

Sympy [A] time = 18.1172, size = 369, normalized size = 3.08

$$\begin{cases} ae^2x + aefx^2 + \frac{af^2x^3}{3} + \frac{bc^3f^2\operatorname{acoth}(c+dx)}{3d^3} - \frac{bc^2ef\operatorname{acoth}(c+dx)}{d^2} + \frac{bc^2f^2\log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{d^3} - \frac{bc^2f^2\operatorname{acoth}(c+dx)}{d^3} + \frac{bce^2\operatorname{acoth}(c+dx)}{d} - \frac{2bcef\log}{d} \\ (a+b\operatorname{acoth}(c))\left(e^2x + efx^2 + \frac{f^2x^3}{3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(a+b*acoth(d*x+c)),x)
```

```
[Out] Piecewise((a*e**2*x + a*e*f*x**2 + a*f**2*x**3/3 + b*c**3*f**2*acoth(c + d*
x)/(3*d**3) - b*c**2*e*f*acoth(c + d*x)/d**2 + b*c**2*f**2*log(c/d + x + 1/
d)/d**3 - b*c**2*f**2*acoth(c + d*x)/d**3 + b*c*e**2*acoth(c + d*x)/d - 2*b
*c*e*f*log(c/d + x + 1/d)/d**2 + 2*b*c*e*f*acoth(c + d*x)/d**2 - 2*b*c*f**2
*x/(3*d**2) + b*c*f**2*acoth(c + d*x)/d**3 + b*e**2*x*acoth(c + d*x) + b*e*
f*x**2*acoth(c + d*x) + b*f**2*x**3*acoth(c + d*x)/3 + b*e**2*log(c/d + x +
 1/d)/d - b*e**2*acoth(c + d*x)/d + b*e*f*x/d + b*f**2*x**2/(6*d) - b*e*f*a
coth(c + d*x)/d**2 + b*f**2*log(c/d + x + 1/d)/(3*d**3) - b*f**2*acoth(c +
d*x)/(3*d**3), Ne(d, 0)), ((a + b*acoth(c))*(e**2*x + e*f*x**2 + f**2*x**3/
3), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)^2 (b\operatorname{arcoth}(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccoth(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*(b*arccoth(d*x + c) + a), x)
```

3.104 $\int (e+fx)\left(a+b\coth^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=97

$$\frac{(e+fx)^2\left(a+b\coth^{-1}(c+dx)\right)}{2f}+\frac{b(-cf+de+f)^2\log(-c-dx+1)}{4d^2f}-\frac{b(de-(c+1)f)^2\log(c+dx+1)}{4d^2f}+\frac{bfx}{2d}$$

```
[Out] (b*f*x)/(2*d) + ((e + f*x)^2*(a + b*ArcCoth[c + d*x]))/(2*f) + (b*(d*e + f
- c*f)^2*Log[1 - c - d*x])/(4*d^2*f) - (b*(d*e - (1 + c)*f)^2*Log[1 + c + d
*x])/(4*d^2*f)
```

Rubi [A] time = 0.171195, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6112, 5927, 702, 633, 31}

$$\frac{(e+fx)^2\left(a+b\coth^{-1}(c+dx)\right)}{2f}+\frac{b(-cf+de+f)^2\log(-c-dx+1)}{4d^2f}-\frac{b(de-(c+1)f)^2\log(c+dx+1)}{4d^2f}+\frac{bfx}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*(a + b*ArcCoth[c + d*x]),x]
```

```
[Out] (b*f*x)/(2*d) + ((e + f*x)^2*(a + b*ArcCoth[c + d*x]))/(2*f) + (b*(d*e + f
- c*f)^2*Log[1 - c - d*x])/(4*d^2*f) - (b*(d*e - (1 + c)*f)^2*Log[1 + c + d
*x])/(4*d^2*f)
```

Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rule 5927

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcCoth[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int (e+fx)\left(a+b\coth^{-1}(c+dx)\right)dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)\left(a+b\coth^{-1}(x)\right)dx,x,c+dx\right)}{d}$$
$$= \frac{(e+fx)^2\left(a+b\coth^{-1}(c+dx)\right)}{2f} - \frac{b\,\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2}{1-x^2}dx,x,c+dx\right)}{2f}$$
$$= \frac{(e+fx)^2\left(a+b\coth^{-1}(c+dx)\right)}{2f} - \frac{b\,\text{Subst}\left(\int \left(-\frac{f^2}{d^2}+\frac{d^2e^2-2cdef+\left(1+c^2\right)f^2+2f(de\cdots}{d^2\left(1-x^2\right)}\right.\right.}{2f}$$
$$= \frac{bfx}{2d} + \frac{(e+fx)^2\left(a+b\coth^{-1}(c+dx)\right)}{2f} - \frac{b\,\text{Subst}\left(\int \frac{d^2e^2-2cdef+\left(1+c^2\right)f^2+2f(de-\cdots}{1-x^2}\right.}{2d^2f}$$
$$= \frac{bfx}{2d} + \frac{(e+fx)^2\left(a+b\coth^{-1}(c+dx)\right)}{2f} - \frac{\left(b(de+f-cf)^2\right)\text{Subst}\left(\int \frac{1}{1-x}dx,\cdots\right.}{4d^2f}$$
$$= \frac{bfx}{2d} + \frac{(e+fx)^2\left(a+b\coth^{-1}(c+dx)\right)}{2f} + \frac{b(de+f-cf)^2\log(1-c-dx)}{4d^2f} - \frac{b\cdots}{}$$

Mathematica [A] time = 0.0427247, size = 138, normalized size = 1.42

$$aex+\frac{1}{2}afx^2+\frac{b\left(c^2-2c+1\right)f\log(-c-dx+1)}{4d^2}+\frac{b\left(-c^2-2c-1\right)f\log(c+dx+1)}{4d^2}+\frac{be((c+1)\log(c+dx+1)-(\cdots}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*(a + b*ArcCoth[c + d*x]),x]
```

```
[Out] a*e*x + (b*f*x)/(2*d) + (a*f*x^2)/2 + b*e*x*ArcCoth[c + d*x] + (b*f*x^2*Arc
Coth[c + d*x])/2 + (b*(1 - 2*c + c^2)*f*Log[1 - c - d*x])/(4*d^2) + (b*(-1
- 2*c - c^2)*f*Log[1 + c + d*x])/(4*d^2) + (b*e*(-((-1 + c)*Log[1 - c - d*x
]) + (1 + c)*Log[1 + c + d*x]))/(2*d)
```

Maple [B] time = 0.04, size = 184, normalized size = 1.9

$$\frac{ax^2f}{2}-\frac{ac^2f}{2d^2}+axe+\frac{ace}{d}+\frac{b\,\text{arccoth}\,(dx+c)\,fx^2}{2}-\frac{\text{arccoth}\,(dx+c)\,bc^2f}{2d^2}+\text{arccoth}\,(dx+c)\,xbe+\frac{b\,\text{arccoth}\,(dx+\cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(a+b*arccoth(d*x+c)),x)
```

```
[Out] 1/2*a*x^2*f-1/2/d^2*a*c^2*f+a*x*e+1/d*a*c*e+1/2*b*arccoth(d*x+c)*f*x^2-1/2/
d^2*b*arccoth(d*x+c)*f*c^2+arccoth(d*x+c)*x*b*e+1/d*arccoth(d*x+c)*b*c*e+1/
2*b*f*x/d+1/2/d^2*b*c*f-1/2/d^2*b*ln(d*x+c-1)*c*f+1/2/d*b*ln(d*x+c-1)*e+1/4
/d^2*b*ln(d*x+c-1)*f-1/2/d^2*b*ln(d*x+c+1)*c*f+1/2/d*b*ln(d*x+c+1)*e-1/4/d^
2*b*ln(d*x+c+1)*f
```

Maxima [A] time = 0.960865, size = 147, normalized size = 1.52

$$\frac{1}{2}afx^2+\frac{1}{4}\left(2x^2\operatorname{arcoth}(dx+c)+d\left(\frac{2x}{d^2}-\frac{(c^2+2c+1)\log(dx+c+1)}{d^3}+\frac{(c^2-2c+1)\log(dx+c-1)}{d^3}\right)\right)bf+aex+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccoth(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*a*f*x^2 + 1/4*(2*x^2*arccoth(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1)*lo
g(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*b*f + a*e*x + 1
/2*(2*(d*x + c)*arccoth(d*x + c) + log(-(d*x + c)^2 + 1))*b*e/d
```

Fricas [A] time = 1.8918, size = 319, normalized size = 3.29

$$\frac{2ad^2fx^2+2(2ad^2e+bdf)x+(2(bc+b)de-(bc^2+2bc+b)f)\log(dx+c+1)-(2(bc-b)de-(bc^2-2bc+b)f)\log}{4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccoth(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*d^2*f*x^2 + 2*(2*a*d^2*e + b*d*f)*x + (2*(b*c + b)*d*e - (b*c^2 +
2*b*c + b)*f)*log(d*x + c + 1) - (2*(b*c - b)*d*e - (b*c^2 - 2*b*c + b)*f)*
log(d*x + c - 1) + (b*d^2*f*x^2 + 2*b*d^2*e*x)*log((d*x + c + 1)/(d*x + c -
 1)))/d^2
```

Sympy [A] time = 2.6259, size = 173, normalized size = 1.78

$$\begin{cases} aex+\frac{afx^2}{2}-\frac{bc^2f\operatorname{acoth}(c+dx)}{2d^2}+\frac{bce\operatorname{acoth}(c+dx)}{d}-\frac{bcf\log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{d^2}+\frac{bcf\operatorname{acoth}(c+dx)}{d^2}+bex\operatorname{acoth}(c+dx)+\frac{bfx^2\operatorname{acoth}(c+dx)}{2}+\frac{be}{} \\ (a+b\operatorname{acoth}(c))\left(ex+\frac{fx^2}{2}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*acoth(d*x+c)),x)
```

```
[Out] Piecewise((a*e*x + a*f*x**2/2 - b*c**2*f*acoth(c + d*x)/(2*d**2) + b*c*e*ac
oth(c + d*x)/d - b*c*f*log(c/d + x + 1/d)/d**2 + b*c*f*acoth(c + d*x)/d**2
+ b*e*x*acoth(c + d*x) + b*f*x**2*acoth(c + d*x)/2 + b*e*log(c/d + x + 1/d)
/d - b*e*acoth(c + d*x)/d + b*f*x/(2*d) - b*f*acoth(c + d*x)/(2*d**2), Ne(d
, 0)), ((a + b*acoth(c))*(e*x + f*x**2/2), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int(fx+e)(b\operatorname{arcoth}(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccoth(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(b*arccoth(d*x + c) + a), x)
```

3.105 $\int \left(a+b\coth^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=40

$$ax+\frac{b\log\left(1-(c+dx)^2\right)}{2d}+\frac{b(c+dx)\coth^{-1}(c+dx)}{d}$$

```
[Out] a*x + (b*(c + d*x)*ArcCoth[c + d*x])/d + (b*Log[1 - (c + d*x)^2])/(2*d)
```

Rubi [A] time = 0.0245296, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6104, 5911, 260}

$$ax+\frac{b\log\left(1-(c+dx)^2\right)}{2d}+\frac{b(c+dx)\coth^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcCoth[c + d*x],x]
```

```
[Out] a*x + (b*(c + d*x)*ArcCoth[c + d*x])/d + (b*Log[1 - (c + d*x)^2])/(2*d)
```

Rule 6104

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}
, x] && IGtQ[p, 0]
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\coth^{-1}(c+dx)\right)\,dx &= ax+b\int \coth^{-1}(c+dx)\,dx\\ &= ax+\frac{b\,\text{Subst}\left(\int \coth^{-1}(x)\,dx,x,c+dx\right)}{d}\\ &= ax+\frac{b(c+dx)\coth^{-1}(c+dx)}{d}-\frac{b\,\text{Subst}\left(\int \frac{x}{1-x^2}\,dx,x,c+dx\right)}{d}\\ &= ax+\frac{b(c+dx)\coth^{-1}(c+dx)}{d}+\frac{b\log\left(1-(c+dx)^2\right)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0149388, size = 48, normalized size = 1.2

$$ax+\frac{b((c+1)\log(c+dx+1)-(c-1)\log(-c-dx+1))}{2d}+bx\coth^{-1}(c+dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcCoth[c + d*x],x]
```

```
[Out] a*x + b*x*ArcCoth[c + d*x] + (b*(-((-1 + c)*Log[1 - c - d*x]) + (1 + c)*Log
[1 + c + d*x]))/(2*d)
```

Maple [A] time = 0.033, size = 42, normalized size = 1.1

$$ax + b\operatorname{arccoth}(dx+c)\,x + \frac{b\operatorname{arccoth}(dx+c)\,c}{d} + \frac{b\ln\left((dx+c)^2-1\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arccoth(d*x+c),x)
```

```
[Out] a*x+b*arccoth(d*x+c)*x+b/d*arccoth(d*x+c)*c+1/2*b/d*ln((d*x+c)^2-1)
```

Maxima [A] time = 0.970909, size = 49, normalized size = 1.22

$$ax + \frac{\left(2\,(dx+c)\operatorname{arcoth}(dx+c) + \log\left(-(dx+c)^2+1\right)\right)b}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccoth(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + 1/2*(2*(d*x + c)*arccoth(d*x + c) + log(-(d*x + c)^2 + 1))*b/d
```

Fricas [A] time = 1.91274, size = 157, normalized size = 3.92

$$\frac{bdx\log\left(\frac{dx+c+1}{dx+c-1}\right) + 2\,adx + (bc+b)\log(dx+c+1) - (bc-b)\log(dx+c-1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccoth(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(b*d*x*log((d*x + c + 1)/(d*x + c - 1)) + 2*a*d*x + (b*c + b)*log(d*x +
 c + 1) - (b*c - b)*log(d*x + c - 1))/d
```

Sympy [A] time = 0.761472, size = 46, normalized size = 1.15

$$ax + b\left(\begin{cases} \frac{c\operatorname{acoth}(c+dx)}{d} + x\operatorname{acoth}(c+dx) + \frac{\log(c+dx+1)}{d} - \frac{\operatorname{acoth}(c+dx)}{d} & \text{for}\ d \neq 0 \\ x\operatorname{acoth}(c) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*acoth(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((c*acoth(c + d*x)/d + x*acoth(c + d*x) + log(c + d*x + 1)
/d - acoth(c + d*x)/d, Ne(d, 0)), (x*acoth(c), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int b \operatorname{arcoth}(dx + c) + a \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccoth(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(b*arccoth(d*x + c) + a, x)
```

3.106 $\int \frac{a+b\coth^{-1}(c+dx)}{e+fx}\,dx$

Optimal. Leaf size=130

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{2f}+\frac{b\text{PolyLog}\left(2,1-\frac{2}{c+dx+1}\right)}{2f}+\frac{\left(a+b\coth^{-1}(c+dx)\right)\log\left(\frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{f}-$$

```
[Out] -(((a + b*ArcCoth[c + d*x])*Log[2/(1 + c + d*x)])/f) + ((a + b*ArcCoth[c +
d*x])*Log[(2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/f + (b*PolyLog[
2, 1 - 2/(1 + c + d*x)])/(2*f) - (b*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e +
f - c*f)*(1 + c + d*x))])/(2*f)
```

Rubi [A] time = 0.14231, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6112, 5921, 2402, 2315, 2447}

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{2f}+\frac{b\text{PolyLog}\left(2,1-\frac{2}{c+dx+1}\right)}{2f}+\frac{\left(a+b\coth^{-1}(c+dx)\right)\log\left(\frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{f}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[c + d*x])/(e + f*x),x]
```

```
[Out] -(((a + b*ArcCoth[c + d*x])*Log[2/(1 + c + d*x)])/f) + ((a + b*ArcCoth[c +
d*x])*Log[(2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/f + (b*PolyLog[
2, 1 - 2/(1 + c + d*x)])/(2*f) - (b*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e +
f - c*f)*(1 + c + d*x))])/(2*f)
```

Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rule 5921

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcCoth[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcCoth[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{a+b\coth^{-1}(c+dx)}{e+fx}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\coth^{-1}(x)}{\frac{de-cf}{d}+\frac{fx}{d}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)\log\left(\frac{2}{1+c+dx}\right)}{f} + \frac{\left(a+b\coth^{-1}(c+dx)\right)\log\left(\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{f} + \frac{b\,\mathrm{S}}{}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)\log\left(\frac{2}{1+c+dx}\right)}{f} + \frac{\left(a+b\coth^{-1}(c+dx)\right)\log\left(\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{f} - \frac{b\mathrm{L}}{}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)\log\left(\frac{2}{1+c+dx}\right)}{f} + \frac{\left(a+b\coth^{-1}(c+dx)\right)\log\left(\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{f} + \frac{b\mathrm{L}}{}$$

Mathematica [A] time = 0.11257, size = 206, normalized size = 1.58

$$-\frac{b\text{PolyLog}\left(2,\frac{d(e+fx)}{-cf+de-f}\right)}{2f} + \frac{b\text{PolyLog}\left(2,\frac{d(e+fx)}{-cf+de+f}\right)}{2f} + \frac{a\log(e+fx)}{f} + \frac{b\log(e+fx)\log\left(\frac{f(-c-dx+1)}{-cf+de+f}\right)}{2f} - \frac{b\log\left(-\frac{-c-dx+1}{c+dx}\right)}{2f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCoth[c + d*x])/(e + f*x),x]
```

```
[Out] (a*Log[e + f*x])/f + (b*Log[(f*(1 - c - d*x))/(d*e + f - c*f)]*Log[e + f*x]
)/(2*f) - (b*Log[-((1 - c - d*x)/(c + d*x))]*Log[e + f*x])/(2*f) - (b*Log[-
((f*(1 + c + d*x))/(d*e - f - c*f))]*Log[e + f*x])/(2*f) + (b*Log[(1 + c +
d*x)/(c + d*x)]*Log[e + f*x])/(2*f) - (b*PolyLog[2, (d*(e + f*x))/(d*e - f
- c*f)])/(2*f) + (b*PolyLog[2, (d*(e + f*x))/(d*e + f - c*f)])/(2*f)
```

Maple [A] time = 0.153, size = 202, normalized size = 1.6

$$\frac{a\ln\left((dx+c)f-cf+de\right)}{f} + \frac{b\ln\left((dx+c)f-cf+de\right)\operatorname{arccoth}(dx+c)}{f} - \frac{b\ln\left((dx+c)f-cf+de\right)}{2f}\ln\left(\frac{(dx+c)f+f}{cf-de+f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(d*x+c))/(f*x+e),x)
```

```
[Out] a*ln((d*x+c)*f-c*f+d*e)/f+b*ln((d*x+c)*f-c*f+d*e)/f*arccoth(d*x+c)-1/2*b/f*
ln((d*x+c)*f-c*f+d*e)*ln(((d*x+c)*f+f)/(c*f-d*e+f))-1/2*b/f*dilog(((d*x+c)*
f+f)/(c*f-d*e+f))+1/2*b/f*ln((d*x+c)*f-c*f+d*e)*ln(((d*x+c)*f-f)/(c*f-d*e-f
```

```
))+1/2*b/f*dilog(((d*x+c)*f-f)/(c*f-d*e-f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}b\int\frac{\log\left(\frac{1}{dx+c}+1\right)-\log\left(-\frac{1}{dx+c}+1\right)}{fx+e}\,dx+\frac{a\log\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))/(f*x+e),x, algorithm="maxima")
```

```
[Out] 1/2*b*integrate((log(1/(d*x + c) + 1) - log(-1/(d*x + c) + 1))/(f*x + e), x
) + a*log(f*x + e)/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\operatorname{arcoth}\left(dx+c\right)+a}{fx+e},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((b*arccoth(d*x + c) + a)/(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(d*x+c))/(f*x+e),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\operatorname{arcoth}\left(dx+c\right)+a}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(d*x + c) + a)/(f*x + e), x)
```

3.107 $\int \frac{a+b\coth^{-1}(c+dx)}{(e+fx)^2}\,dx$

Optimal. Leaf size=115

$$-\frac{a+b\coth^{-1}(c+dx)}{f(e+fx)}-\frac{bd\log(-c-dx+1)}{2f(-cf+de+f)}+\frac{bd\log(c+dx+1)}{2f(-cf+de-f)}-\frac{bd\log(e+fx)}{(-cf+de+f)(de-(c+1)f)}$$

```
[Out] -((a + b*ArcCoth[c + d*x])/(f*(e + f*x))) - (b*d*Log[1 - c - d*x])/(2*f*(d*
e + f - c*f)) + (b*d*Log[1 + c + d*x])/(2*f*(d*e - f - c*f)) - (b*d*Log[e +
 f*x])/((d*e + f - c*f)*(d*e - (1 + c)*f))
```

Rubi [A] time = 0.168724, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6110, 1982, 705, 31, 632}

$$-\frac{a+b\coth^{-1}(c+dx)}{f(e+fx)}-\frac{bd\log(-c-dx+1)}{2f(-cf+de+f)}+\frac{bd\log(c+dx+1)}{2f(-cf+de-f)}-\frac{bd\log(e+fx)}{(-cf+de+f)(de-(c+1)f)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[c + d*x])/(e + f*x)^2,x]
```

```
[Out] -((a + b*ArcCoth[c + d*x])/(f*(e + f*x))) - (b*d*Log[1 - c - d*x])/(2*f*(d*
e + f - c*f)) + (b*d*Log[1 + c + d*x])/(2*f*(d*e - f - c*f)) - (b*d*Log[e +
 f*x])/((d*e + f - c*f)*(d*e - (1 + c)*f))
```

Rule 6110

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[((e + f*x)^(m + 1)*(a + b*ArcCoth[c + d*x])^p)/(f*(m
 + 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcCot
h[c + d*x])^(p - 1))/(1 - (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 1982

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{a+b\coth^{-1}(c+dx)}{(e+fx)^2}\,dx = -\frac{a+b\coth^{-1}(c+dx)}{f(e+fx)}+\frac{(bd)\int \frac{1}{(e+fx)\left(1-(c+dx)^2\right)}\,dx}{f}$$

$$= -\frac{a+b\coth^{-1}(c+dx)}{f(e+fx)}+\frac{(bd)\int \frac{1}{(e+fx)\left(1-c^2-2cdx-d^2x^2\right)}\,dx}{f}$$

$$= -\frac{a+b\coth^{-1}(c+dx)}{f(e+fx)}+\frac{(bd)\int \frac{-d^2e+2cdf+d^2fx}{1-c^2-2cdx-d^2x^2}\,dx}{f\left(-d^2e^2+2cdef+\left(1-c^2\right)f^2\right)}+\frac{(bdf)\int \frac{1}{e+fx}\,dx}{-d^2e^2+2cdef+\left(1-c^2\right)f^2}$$

$$= -\frac{a+b\coth^{-1}(c+dx)}{f(e+fx)}-\frac{bd\log(e+fx)}{(de-f-cf)(de+f-cf)}-\frac{\left(bd^3\right)\int \frac{1}{-d-cd-d^2x}\,dx}{2f(de-f-cf)}+\frac{\left(bd^3\right)\int \frac{1}{d-\cdots}}{2f(de+\cdots}$$

$$= -\frac{a+b\coth^{-1}(c+dx)}{f(e+fx)}-\frac{bd\log(1-c-dx)}{2f(de+f-cf)}+\frac{bd\log(1+c+dx)}{2f(de-f-cf)}-\frac{bd\log(e+fx)}{(de-f-cf)(de+f\cdots}$$

Mathematica [A] time = 0.200933, size = 125, normalized size = 1.09

$$\frac{1}{2}\left(-\frac{2a}{f(e+fx)}-\frac{2bd\log(e+fx)}{\left(c^2-1\right)f^2-2cdef+d^2e^2}+\frac{bd\log(-c-dx+1)}{f((c-1)f-de)}-\frac{bd\log(c+dx+1)}{f(cf-de+f)}-\frac{2b\coth^{-1}(c+dx)}{f(e+fx)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCoth[c + d*x])/(e + f*x)^2,x]
```

```
[Out] ((-2*a)/(f*(e + f*x)) - (2*b*ArcCoth[c + d*x])/(f*(e + f*x)) + (b*d*Log[1 -
 c - d*x])/(f*(-(d*e) + (-1 + c)*f)) - (b*d*Log[1 + c + d*x])/(f*(-(d*e) +
f + c*f)) - (2*b*d*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2))/2
```

Maple [A] time = 0.043, size = 141, normalized size = 1.2

$$-\frac{ad}{\left(dfx+de\right)f}-\frac{bd\text{arccoth}\,(dx+c)}{\left(dfx+de\right)f}+\frac{bd\ln(dx+c-1)}{f\left(2cf-2de-2f\right)}-\frac{bd\ln(dx+c+1)}{f\left(2cf-2de+2f\right)}-\frac{bd\ln\left((dx+c)f-cf+de\right)}{\left(cf-de-f\right)\left(cf-de+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(d*x+c))/(f*x+e)^2,x)
```

```
[Out] -d*a/(d*f*x+d*e)/f-d*b/(d*f*x+d*e)/f*arccoth(d*x+c)+d*b/f/(2*c*f-2*d*e-2*f)
*ln(d*x+c-1)-d*b/f/(2*c*f-2*d*e+2*f)*ln(d*x+c+1)-d*b/(c*f-d*e-f)/(c*f-d*e+f
)*ln((d*x+c)*f-c*f+d*e)
```

Maxima [A] time = 0.996587, size = 163, normalized size = 1.42

$$\frac{1}{2}\left(d\left(\frac{\log(dx+c+1)}{def-(c+1)f^2}-\frac{\log(dx+c-1)}{def-(c-1)f^2}-\frac{2\log\left(fx+e\right)}{d^2e^2-2\,cdef+\left(c^2-1\right)f^2}\right)-\frac{2\,\mathrm{arcoth}\,(dx+c)}{f^2x+ef}\right)b-\frac{a}{f^2x+ef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(d*(log(d*x + c + 1)/(d*e*f - (c + 1)*f^2) - log(d*x + c - 1)/(d*e*f -
(c - 1)*f^2) - 2*log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 - 1)*f^2)) - 2*ar
ccoth(d*x + c)/(f^2*x + e*f))*b - a/(f^2*x + e*f)
```

Fricas [B] time = 3.11053, size = 583, normalized size = 5.07

$$-\frac{2\,ad^2e^2-4\,acdef+2\left(ac^2-a\right)f^2-\left(bd^2e^2-(bc-b)def+\left(bd^2ef-(bc-b)df^2\right)x\right)\log(dx+c+1)+\left(bd^2e^2-(bc+b)d\right.}{2\left(d^2e^3f-2\,cde^2f^2+\left(c^2-1\right)ef\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d^2*e^2 - 4*a*c*d*e*f + 2*(a*c^2 - a)*f^2 - (b*d^2*e^2 - (b*c - b
)*d*e*f + (b*d^2*e*f - (b*c - b)*d*f^2)*x)*log(d*x + c + 1) + (b*d^2*e^2 -
(b*c + b)*d*e*f + (b*d^2*e*f - (b*c + b)*d*f^2)*x)*log(d*x + c - 1) + 2*(b*
d*f^2*x + b*d*e*f)*log(f*x + e) + (b*d^2*e^2 - 2*b*c*d*e*f + (b*c^2 - b)*f^
2)*log((d*x + c + 1)/(d*x + c - 1)))/(d^2*e^3*f - 2*c*d*e^2*f^2 + (c^2 - 1)
*e*f^3 + (d^2*e^2*f^2 - 2*c*d*e*f^3 + (c^2 - 1)*f^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(d*x+c))/(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{arcoth}\,(dx+c)+a}{\left(fx+e\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(d*x + c) + a)/(f*x + e)^2, x)
```

3.108 $\int \frac{a+b\coth^{-1}(c+dx)}{(e+fx)^3}\,dx$

Optimal. Leaf size=167

$$-\frac{a+b\coth^{-1}(c+dx)}{2f(e+fx)^2}-\frac{bd^2\log(-c-dx+1)}{4f(-cf+de+f)^2}+\frac{bd^2\log(c+dx+1)}{4f(-cf+de-f)^2}-\frac{bd^2(de-cf)\log(e+fx)}{(-cf+de+f)^2(de-(c+1)f)^2}+\frac{}{2(e+fx)(-}$$

```
[Out] (b*d)/(2*(d*e + f - c*f)*(d*e - (1 + c)*f)*(e + f*x)) - (a + b*ArcCoth[c +
d*x])/(2*f*(e + f*x)^2) - (b*d^2*Log[1 - c - d*x])/(4*f*(d*e + f - c*f)^2)
+ (b*d^2*Log[1 + c + d*x])/(4*f*(d*e - f - c*f)^2) - (b*d^2*(d*e - c*f)*Log
[e + f*x])/((d*e + f - c*f)^2*(d*e - (1 + c)*f)^2)
```

Rubi [A] time = 0.234421, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6110, 1982, 709, 800}

$$-\frac{a+b\coth^{-1}(c+dx)}{2f(e+fx)^2}-\frac{bd^2\log(-c-dx+1)}{4f(-cf+de+f)^2}+\frac{bd^2\log(c+dx+1)}{4f(-cf+de-f)^2}-\frac{bd^2(de-cf)\log(e+fx)}{(-cf+de+f)^2(de-(c+1)f)^2}+\frac{}{2(e+fx)(-}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[c + d*x])/(e + f*x)^3,x]
```

```
[Out] (b*d)/(2*(d*e + f - c*f)*(d*e - (1 + c)*f)*(e + f*x)) - (a + b*ArcCoth[c +
d*x])/(2*f*(e + f*x)^2) - (b*d^2*Log[1 - c - d*x])/(4*f*(d*e + f - c*f)^2)
+ (b*d^2*Log[1 + c + d*x])/(4*f*(d*e - f - c*f)^2) - (b*d^2*(d*e - c*f)*Log
[e + f*x])/((d*e + f - c*f)^2*(d*e - (1 + c)*f)^2)
```

Rule 6110

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[((e + f*x)^(m + 1)*(a + b*ArcCoth[c + d*x])^p)/(f*(m
 + 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcCot
h[c + d*x])^(p - 1))/(1 - (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 1982

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
```

```
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+b\coth^{-1}(c+dx)}{(e+fx)^3}\,dx = -\frac{a+b\coth^{-1}(c+dx)}{2f(e+fx)^2}+\frac{(bd)\int \frac{1}{(e+fx)^2\left(1-(c+dx)^2\right)}\,dx}{2f}$$

$$= -\frac{a+b\coth^{-1}(c+dx)}{2f(e+fx)^2}+\frac{(bd)\int \frac{1}{(e+fx)^2\left(1-c^2-2cdx-d^2x^2\right)}\,dx}{2f}$$

$$= \frac{bd}{2(de+f-cf)(de-(1+c)f)(e+fx)}-\frac{a+b\coth^{-1}(c+dx)}{2f(e+fx)^2}+\frac{(bd)\int \frac{-d(de-2cf)+d^2fx}{(e+fx)\left(1-c^2-2cdx-d^2x^2\right.}}{2f\left(-d^2e^2+2cdef+\left(1-c\right.\right.}$$

$$= \frac{bd}{2(de+f-cf)(de-(1+c)f)(e+fx)}-\frac{a+b\coth^{-1}(c+dx)}{2f(e+fx)^2}+\frac{(bd)\int \left(\frac{d^2(-de+(1+c)f)}{2(de+f-cf)(1-c-dx)}+\right.}{2f\left(-\right.}$$

$$= \frac{bd}{2(de+f-cf)(de-(1+c)f)(e+fx)}-\frac{a+b\coth^{-1}(c+dx)}{2f(e+fx)^2}-\frac{bd^2\log(1-c-dx)}{4f(de+f-cf)^2}+\frac{bd^2\,\mathrm{l}}{4f(}$$

Mathematica [A] time = 0.346272, size = 174, normalized size = 1.04

$$\frac{1}{4}\left(-\frac{2a}{f(e+fx)^2}+\frac{2bd}{(e+fx)\left(\left(c^2-1\right)f^2-2cdef+d^2e^2\right)}-\frac{4bd^2(de-cf)\log(e+fx)}{\left(\left(c^2-1\right)f^2-2cdef+d^2e^2\right)^2}-\frac{bd^2\log(-c-dx+1)}{f(-cf+de+f)^2}+\frac{bd^2\,\mathrm{log}}{f(cf}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCoth[c + d*x])/(e + f*x)^3,x]
```

```
[Out] ((-2*a)/(f*(e + f*x)^2) + (2*b*d)/((d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)*(
e + f*x)) - (2*b*ArcCoth[c + d*x])/(f*(e + f*x)^2) - (b*d^2*Log[1 - c - d*x
])/(f*(d*e + f - c*f)^2) + (b*d^2*Log[1 + c + d*x])/(f*(-(d*e) + f + c*f)^2
) - (4*b*d^2*(d*e - c*f)*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^
2)^2)/4
```

Maple [A] time = 0.052, size = 236, normalized size = 1.4

$$-\frac{ad^2}{2\left(dfx+de\right)^2 f}-\frac{bd^2\operatorname{arccoth}(dx+c)}{2\left(dfx+de\right)^2 f}-\frac{bd^2\ln(dx+c-1)}{4f\left(cf-de-f\right)^2}+\frac{bd^2\ln(dx+c+1)}{4f\left(cf-de+f\right)^2}+\frac{bd^2}{\left(2cf-2de-2f\right)\left(cf-de+f\right)\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(d*x+c))/(f*x+e)^3,x)
```

```
[Out] -1/2*d^2*a/(d*f*x+d*e)^2/f-1/2*d^2*b/(d*f*x+d*e)^2/f*arccoth(d*x+c)-1/4*d^2
*b/f/(c*f-d*e-f)^2*ln(d*x+c-1)+1/4*d^2*b/f/(c*f-d*e+f)^2*ln(d*x+c+1)+1/2*d^
2*b/(c*f-d*e-f)/(c*f-d*e+f)/(d*f*x+d*e)+d^2*b*f/(c*f-d*e-f)^2/(c*f-d*e+f)^2
*ln((d*x+c)*f-c*f+d*e)*c-d^3*b/(c*f-d*e-f)^2/(c*f-d*e+f)^2*ln((d*x+c)*f-c*f
+d*e)*e
```

Maxima [A] time = 1.04528, size = 393, normalized size = 2.35

$$\frac{1}{4}\left(d\left(\frac{d\log(dx+c+1)}{d^2e^2f-2(c+1)def^2+\left(c^2+2c+1\right)f^3}-\frac{d\log(dx+c-1)}{d^2e^2f-2(c-1)def^2+\left(c^2-2c+1\right)f^3}-\frac{4(c}{d^4e^4-4cd^3e^3f+2\left(3c^2-\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))/(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(d*(d*log(d*x + c + 1)/(d^2*e^2*f - 2*(c + 1)*d*e*f^2 + (c^2 + 2*c + 1)
*f^3) - d*log(d*x + c - 1)/(d^2*e^2*f - 2*(c - 1)*d*e*f^2 + (c^2 - 2*c + 1)
*f^3) - 4*(d^2*e - c*d*f)*log(f*x + e)/(d^4*e^4 - 4*c*d^3*e^3*f + 2*(3*c^2
- 1)*d^2*e^2*f^2 - 4*(c^3 - c)*d*e*f^3 + (c^4 - 2*c^2 + 1)*f^4) + 2/(d^2*e^
3 - 2*c*d*e^2*f + (c^2 - 1)*e*f^2 + (d^2*e^2*f - 2*c*d*e*f^2 + (c^2 - 1)*f^
3)*x)) - 2*arccoth(d*x + c)/(f^3*x^2 + 2*e*f^2*x + e^2*f))*b - 1/2*a/(f^3*x
^2 + 2*e*f^2*x + e^2*f)
```

Fricas [B] time = 9.18629, size = 1766, normalized size = 10.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))/(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*d^4*e^4 - 2*(4*a*c + b)*d^3*e^3*f + 4*(3*a*c^2 + b*c - a)*d^2*e^2
*f^2 - 2*(4*a*c^3 + b*c^2 - 4*a*c - b)*d*e*f^3 + 2*(a*c^4 - 2*a*c^2 + a)*f^
4 - 2*(b*d^3*e^2*f^2 - 2*b*c*d^2*e*f^3 + (b*c^2 - b)*d*f^4)*x - (b*d^4*e^4
- 2*(b*c - b)*d^3*e^3*f + (b*c^2 - 2*b*c + b)*d^2*e^2*f^2 + (b*d^4*e^2*f^2
- 2*(b*c - b)*d^3*e*f^3 + (b*c^2 - 2*b*c + b)*d^2*f^4)*x^2 + 2*(b*d^4*e^3*f
 - 2*(b*c - b)*d^3*e^2*f^2 + (b*c^2 - 2*b*c + b)*d^2*e*f^3)*x)*log(d*x + c
+ 1) + (b*d^4*e^4 - 2*(b*c + b)*d^3*e^3*f + (b*c^2 + 2*b*c + b)*d^2*e^2*f^2
 + (b*d^4*e^2*f^2 - 2*(b*c + b)*d^3*e*f^3 + (b*c^2 + 2*b*c + b)*d^2*f^4)*x^
2 + 2*(b*d^4*e^3*f - 2*(b*c + b)*d^3*e^2*f^2 + (b*c^2 + 2*b*c + b)*d^2*e*f^
3)*x)*log(d*x + c - 1) + 4*(b*d^3*e^3*f - b*c*d^2*e^2*f^2 + (b*d^3*e*f^3 -
b*c*d^2*f^4)*x^2 + 2*(b*d^3*e^2*f^2 - b*c*d^2*e*f^3)*x)*log(f*x + e) + (b*d
^4*e^4 - 4*b*c*d^3*e^3*f + 2*(3*b*c^2 - b)*d^2*e^2*f^2 - 4*(b*c^3 - b*c)*d*
e*f^3 + (b*c^4 - 2*b*c^2 + b)*f^4)*log((d*x + c + 1)/(d*x + c - 1)))/(d^4*e
^6*f - 4*c*d^3*e^5*f^2 + 2*(3*c^2 - 1)*d^2*e^4*f^3 - 4*(c^3 - c)*d*e^3*f^4
+ (c^4 - 2*c^2 + 1)*e^2*f^5 + (d^4*e^4*f^3 - 4*c*d^3*e^3*f^4 + 2*(3*c^2 - 1
)*d^2*e^2*f^5 - 4*(c^3 - c)*d*e*f^6 + (c^4 - 2*c^2 + 1)*f^7)*x^2 + 2*(d^4*e
^5*f^2 - 4*c*d^3*e^4*f^3 + 2*(3*c^2 - 1)*d^2*e^3*f^4 - 4*(c^3 - c)*d*e^2*f^
5 + (c^4 - 2*c^2 + 1)*e*f^6)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(d*x+c))/(f*x+e)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arcoth}(dx + c) + a}{(fx + e)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))/(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(d*x + c) + a)/(f*x + e)^3, x)
```

3.109 $\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=374

$$-\frac{b^2\left(\left(3c^2+1\right)f^2-6cdef+3d^2e^2\right)\text{PolyLog}\left(2,-\frac{c+dx+1}{-c-dx+1}\right)}{3d^3}-\frac{(de-cf)\left(\left(c^2+3\right)f^2-2cdef+d^2e^2\right)\left(a+b\coth^{-1}(c+\right.}{3d^3f}$$

```
[Out] (b^2*f^2*x)/(3*d^2) + (2*a*b*f*(d*e - c*f)*x)/d^2 + (2*b^2*f*(d*e - c*f)*(c
 + d*x)*ArcCoth[c + d*x])/d^3 + (b*f^2*(c + d*x)^2*(a + b*ArcCoth[c + d*x])
)/(3*d^3) - ((d*e - c*f)*(d^2*e^2 - 2*c*d*e*f + (3 + c^2)*f^2)*(a + b*ArcCo
th[c + d*x])^2)/(3*d^3*f) + ((3*d^2*e^2 - 6*c*d*e*f + (1 + 3*c^2)*f^2)*(a +
 b*ArcCoth[c + d*x])^2)/(3*d^3) + ((e + f*x)^3*(a + b*ArcCoth[c + d*x])^2)/
(3*f) - (b^2*f^2*ArcTanh[c + d*x])/(3*d^3) - (2*b*(3*d^2*e^2 - 6*c*d*e*f +
(1 + 3*c^2)*f^2)*(a + b*ArcCoth[c + d*x])*Log[2/(1 - c - d*x)])/(3*d^3) + (
b^2*f*(d*e - c*f)*Log[1 - (c + d*x)^2])/d^3 - (b^2*(3*d^2*e^2 - 6*c*d*e*f +
 (1 + 3*c^2)*f^2)*PolyLog[2, -((1 + c + d*x)/(1 - c - d*x))])/(3*d^3)
```

Rubi [A] time = 0.636536, antiderivative size = 374, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.65, Rules used = {6112, 5929, 5911, 260, 5917, 321, 206, 6049, 5949, 5985, 5919, 2402, 2315}

$$-\frac{b^2\left(\left(3c^2+1\right)f^2-6cdef+3d^2e^2\right)\text{PolyLog}\left(2,-\frac{c+dx+1}{-c-dx+1}\right)}{3d^3}-\frac{(de-cf)\left(\left(c^2+3\right)f^2-2cdef+d^2e^2\right)\left(a+b\coth^{-1}(c+\right.}{3d^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*(a + b*ArcCoth[c + d*x])^2,x]
```


```
[Out] (b^2*f^2*x)/(3*d^2) + (2*a*b*f*(d*e - c*f)*x)/d^2 + (2*b^2*f*(d*e - c*f)*(c
 + d*x)*ArcCoth[c + d*x])/d^3 + (b*f^2*(c + d*x)^2*(a + b*ArcCoth[c + d*x])
)/(3*d^3) - ((d*e - c*f)*(d^2*e^2 - 2*c*d*e*f + (3 + c^2)*f^2)*(a + b*ArcCo
th[c + d*x])^2)/(3*d^3*f) + ((3*d^2*e^2 - 6*c*d*e*f + (1 + 3*c^2)*f^2)*(a +
 b*ArcCoth[c + d*x])^2)/(3*d^3) + ((e + f*x)^3*(a + b*ArcCoth[c + d*x])^2)/
(3*f) - (b^2*f^2*ArcTanh[c + d*x])/(3*d^3) - (2*b*(3*d^2*e^2 - 6*c*d*e*f +
(1 + 3*c^2)*f^2)*(a + b*ArcCoth[c + d*x])*Log[2/(1 - c - d*x)])/(3*d^3) + (
b^2*f*(d*e - c*f)*Log[1 - (c + d*x)^2])/d^3 - (b^2*(3*d^2*e^2 - 6*c*d*e*f +
 (1 + 3*c^2)*f^2)*PolyLog[2, -((1 + c + d*x)/(1 - c - d*x))])/(3*d^3)
```


Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rule 5929

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcCoth[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcCoth[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6049

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCoth[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5985

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int (e+fx)^2\left(a+b\coth^{-1}(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2\left(a+b\coth^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$=\frac{(e+fx)^3\left(a+b\coth^{-1}(c+dx)\right)^2}{3f}-\frac{(2b)\,\text{Subst}\left(\int\left(-\frac{3f^2(de-cf)\left(a+b\coth^{-1}(x)\right)}{d^3}\right.\right.}{}$$

$$=\frac{(e+fx)^3\left(a+b\coth^{-1}(c+dx)\right)^2}{3f}-\frac{(2b)\,\text{Subst}\left(\int\frac{\left((de-cf)\left(d^2e^2-2cdef+3f^2+c^2f^2\right.\right.}{}\right.}{}$$

$$=\frac{2abf(de-cf)x}{d^2}+\frac{bf^2(c+dx)^2\left(a+b\coth^{-1}(c+dx)\right)}{3d^3}+\frac{(e+fx)^3\left(a+b\,\text{cc}\right.}{3f}$$

$$=\frac{b^2f^2x}{3d^2}+\frac{2abf(de-cf)x}{d^2}+\frac{2b^2f(de-cf)(c+dx)\coth^{-1}(c+dx)}{d^3}+\frac{bf^2(c+}{}$$

$$=\frac{b^2f^2x}{3d^2}+\frac{2abf(de-cf)x}{d^2}+\frac{2b^2f(de-cf)(c+dx)\coth^{-1}(c+dx)}{d^3}+\frac{bf^2(c+}{}$$

$$=\frac{b^2f^2x}{3d^2}+\frac{2abf(de-cf)x}{d^2}+\frac{2b^2f(de-cf)(c+dx)\coth^{-1}(c+dx)}{d^3}+\frac{bf^2(c+}{}$$

$$=\frac{b^2f^2x}{3d^2}+\frac{2abf(de-cf)x}{d^2}+\frac{2b^2f(de-cf)(c+dx)\coth^{-1}(c+dx)}{d^3}+\frac{bf^2(c+}{}$$

$$=\frac{b^2f^2x}{3d^2}+\frac{2abf(de-cf)x}{d^2}+\frac{2b^2f(de-cf)(c+dx)\coth^{-1}(c+dx)}{d^3}+\frac{bf^2(c+}{}$$

Mathematica [B] time = 7.23623, size = 1054, normalized size = 2.82

$$\frac{1}{3}a^2f^2x^3+a^2efx^2+a^2e^2x+\frac{1}{3}ab\left(2x\left(3e^2+3fxe+f^2x^2\right)\coth^{-1}(c+dx)+\frac{dfx(6de-4cf+dfx)-(c-1)\left(3d^2e^2-\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)^2*(a + b*ArcCoth[c + d*x])^2,x]
```

```
[Out] a^2*e^2*x + a^2*e*f*x^2 + (a^2*f^2*x^3)/3 + (a*b*(2*x*(3*e^2 + 3*e*f*x + f^
2*x^2)*ArcCoth[c + d*x] + (d*f*x*(6*d*e - 4*c*f + d*f*x) - (-1 + c)*(3*d^2*
e^2 - 3*(-1 + c)*d*e*f + (-1 + c)^2*f^2)*Log[1 - c - d*x] + (1 + c)*(3*d^2*
e^2 - 3*(1 + c)*d*e*f + (1 + c)^2*f^2)*Log[1 + c + d*x])/d^3))/3 + (b^2*e^2
*(1 - (c + d*x)^2)*(ArcCoth[c + d*x]*(ArcCoth[c + d*x] - (c + d*x)*ArcCoth[
c + d*x] + 2*Log[1 - E^(-2*ArcCoth[c + d*x])]) - PolyLog[2, E^(-2*ArcCoth[c
 + d*x])]))/(d*(c + d*x)^2*(1 - (c + d*x)^(-2))) - (b^2*e*f*(1 - (c + d*x)^
2)*(2*c*ArcCoth[c + d*x]^2 + (c + d*x)^2*(1 - (c + d*x)^(-2))*ArcCoth[c + d
*x]^2 - 2*(c + d*x)*ArcCoth[c + d*x]*(-1 + c*ArcCoth[c + d*x]) + 4*c*ArcCot
h[c + d*x]*Log[1 - E^(-2*ArcCoth[c + d*x])] - 2*Log[1/((c + d*x)*Sqrt[1 - (
c + d*x)^(-2)])] - 2*c*PolyLog[2, E^(-2*ArcCoth[c + d*x])]))/(d^2*(c + d*x)
^2*(1 - (c + d*x)^(-2))) - (b^2*f^2*(c + d*x)*Sqrt[1 - (c + d*x)^(-2)]*(1 -
 (c + d*x)^2)*((4*ArcCoth[c + d*x])/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) +
(3*ArcCoth[c + d*x]^2)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) - (12*c*ArcCoth
[c + d*x]^2)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (9*c^2*ArcCoth[c + d*x]
^2)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (-1 + 6*c*ArcCoth[c + d*x] + 3*A
rcCoth[c + d*x]^2 - 3*c^2*ArcCoth[c + d*x]^2)/Sqrt[1 - (c + d*x)^(-2)] + Co
sh[3*ArcCoth[c + d*x]] - 6*c*ArcCoth[c + d*x]*Cosh[3*ArcCoth[c + d*x]] + Ar
cCoth[c + d*x]^2*Cosh[3*ArcCoth[c + d*x]] + 3*c^2*ArcCoth[c + d*x]^2*Cosh[3
*ArcCoth[c + d*x]] + (6*ArcCoth[c + d*x]*Log[1 - E^(-2*ArcCoth[c + d*x])])/
((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (18*c^2*ArcCoth[c + d*x]*Log[1 - E^(
-2*ArcCoth[c + d*x])])/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) - (18*c*Log[1/(
(c + d*x)*Sqrt[1 - (c + d*x)^(-2)])])/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)])
+ (4*(1 + 3*c^2)*PolyLog[2, E^(-2*ArcCoth[c + d*x])])/((c + d*x)^3*(1 - (c
+ d*x)^(-2))^(3/2)) - ArcCoth[c + d*x]^2*Sinh[3*ArcCoth[c + d*x]] - 3*c^2*A
rcCoth[c + d*x]^2*Sinh[3*ArcCoth[c + d*x]] - 2*ArcCoth[c + d*x]*Log[1 - E^(
-2*ArcCoth[c + d*x])]*Sinh[3*ArcCoth[c + d*x]] - 6*c^2*ArcCoth[c + d*x]*Log
[1 - E^(-2*ArcCoth[c + d*x])]*Sinh[3*ArcCoth[c + d*x]] + 6*c*Log[1/((c + d*
x)*Sqrt[1 - (c + d*x)^(-2)])]*Sinh[3*ArcCoth[c + d*x]]))/(12*d^3)
```

Maple [B] time = 0.069, size = 2694, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(a+b*arccoth(d*x+c))^2,x)
```

```
[Out] -1/d^2*b^2*f*arccoth(d*x+c)*ln(d*x+c+1)*e+1/6/d^3*b^2*f^2*ln(d*x+c+1)*ln(-1
/2*d*x-1/2*c+1/2)*c^3-1/2/d^2*b^2*f*ln(d*x+c+1)*ln(-1/2*d*x-1/2*c+1/2)*e+1/
2/d^2*b^2*f*ln(1/2+1/2*d*x+1/2*c)*ln(-1/2*d*x-1/2*c+1/2)*e+1/3*a^2*f^2*x^3+
a^2*x*e^2+1/3*b^2*f^2*x/d^2+1/3/d*a*b*f^2*x^2+1/3*a^2/f*e^3+1/3/d^3*b^2*f^2
*c-5/3/d^3*a*b*f^2*c^2+1/2/d^2*b^2*f*ln(1/2+1/2*d*x+1/2*c)*ln(-1/2*d*x-1/2*
c+1/2)*c^2*e-1/2/d^2*b^2*f*ln(d*x+c-1)*ln(1/2+1/2*d*x+1/2*c)*c^2*e-1/d^2*b^
2*f*arccoth(d*x+c)*ln(d*x+c+1)*c^2*e-1/2/d^2*b^2*f*ln(d*x+c+1)*ln(-1/2*d*x-
1/2*c+1/2)*c^2*e-2/d^2*a*b*f*ln(d*x+c-1)*c*e-1/d^2*a*b*f*ln(d*x+c+1)*c^2*e-
2/d^2*a*b*f*ln(d*x+c+1)*c*e+1/d^2*a*b*f*ln(d*x+c-1)*c^2*e-1/d^2*b^2*f*ln(d*
x+c+1)*ln(-1/2*d*x-1/2*c+1/2)*c*e-2/d^2*b^2*f*arccoth(d*x+c)*ln(d*x+c-1)*c*
e+1/d^2*b^2*f*arccoth(d*x+c)*ln(d*x+c-1)*c^2*e+1/d^2*b^2*f*ln(1/2+1/2*d*x+1
/2*c)*ln(-1/2*d*x-1/2*c+1/2)*c*e-2/d^2*b^2*f*arccoth(d*x+c)*ln(d*x+c+1)*c*e
+1/d^2*b^2*f*ln(d*x+c-1)*ln(1/2+1/2*d*x+1/2*c)*c*e-1/12/d^3*b^2*f^2*ln(d*x+
c+1)^2-1/3/d^3*b^2*f^2*dilog(1/2+1/2*d*x+1/2*c)+1/6/d^3*b^2*f^2*ln(d*x+c-1)
-1/4/d*b^2*ln(d*x+c+1)^2*e^2-1/d*b^2*dilog(1/2+1/2*d*x+1/2*c)*e^2+1/3*b^2/f
*arccoth(d*x+c)^2*e^3+1/12*b^2/f*ln(d*x+c-1)^2*e^3+1/12*b^2/f*ln(d*x+c+1)^2
*e^3+1/3*b^2*f^2*arccoth(d*x+c)^2*x^3+arccoth(d*x+c)^2*x*b^2*e^2+a^2*f*x^2*
e+1/4/d*b^2*ln(d*x+c-1)^2*e^2-1/6/d^3*b^2*f^2*ln(d*x+c+1)+1/12/d^3*b^2*f^2*
ln(d*x+c-1)^2-5/3/d^3*b^2*f^2*arccoth(d*x+c)*c^2+1/3/d*b^2*f^2*arccoth(d*x+
```

```
c)*x^2-1/6/d^3*b^2*f^2*ln(d*x+c-1)*ln(1/2+1/2*d*x+1/2*c)+1/3/d^3*a*b*f^2*ln
(d*x+c+1)+1/6/d^3*b^2*f^2*ln(d*x+c+1)*ln(-1/2*d*x-1/2*c+1/2)-1/6/d^3*b^2*f^
2*ln(1/2+1/2*d*x+1/2*c)*ln(-1/2*d*x-1/2*c+1/2)+1/3/d^3*b^2*f^2*arccoth(d*x+
c)*ln(d*x+c+1)+1/3/d^3*b^2*f^2*arccoth(d*x+c)*ln(d*x+c-1)+1/3/d^3*a*b*f^2*l
n(d*x+c-1)+1/d*a*b*ln(d*x+c-1)*e^2+1/d*a*b*ln(d*x+c+1)*e^2-1/d^3*b^2*f^2*ln
(d*x+c+1)*c-1/12/d^3*b^2*f^2*ln(d*x+c-1)^2*c^3-1/12/d^3*b^2*f^2*ln(d*x+c+1)
^2*c^3-1/4/d^3*b^2*f^2*ln(d*x+c+1)^2*c^2-1/4/d^3*b^2*f^2*ln(d*x+c+1)^2*c+1/
4/d^2*b^2*f*ln(d*x+c-1)^2*e-1/d^3*b^2*f^2*dilog(1/2+1/2*d*x+1/2*c)*c^2+2/3*
a*b*f^2*arccoth(d*x+c)*x^3+b^2*f*arccoth(d*x+c)^2*e*x^2+1/3*b^2/f*arccoth(d
*x+c)*ln(d*x+c-1)*e^3-1/6*b^2/f*ln(d*x+c+1)*ln(-1/2*d*x-1/2*c+1/2)*e^3+1/6*
b^2/f*ln(1/2+1/2*d*x+1/2*c)*ln(-1/2*d*x-1/2*c+1/2)*e^3-1/3*b^2/f*arccoth(d*
x+c)*ln(d*x+c+1)*e^3-1/6*b^2/f*ln(d*x+c-1)*ln(1/2+1/2*d*x+1/2*c)*e^3+1/3*a*
b/f*ln(d*x+c-1)*e^3-1/3*a*b/f*ln(d*x+c+1)*e^3+2/3*a*b/f*arccoth(d*x+c)*e^3+
2*arccoth(d*x+c)*x*a*b*e^2+1/d^2*b^2*f*ln(d*x+c-1)*e+1/d^2*b^2*f*ln(d*x+c+1
)*e+1/4/d^3*b^2*f^2*ln(d*x+c-1)^2*c^2-1/4/d^3*b^2*f^2*ln(d*x+c-1)^2*c+1/4/d
^2*b^2*f*ln(d*x+c+1)^2*e-1/d^3*b^2*f^2*ln(d*x+c-1)*c-1/4/d*b^2*ln(d*x+c+1)^
2*c*e^2-1/4/d*b^2*ln(d*x+c-1)^2*c*e^2+1/d*b^2*arccoth(d*x+c)*ln(d*x+c-1)*e^
2+1/2/d*b^2*ln(d*x+c+1)*ln(-1/2*d*x-1/2*c+1/2)*e^2-1/2/d*b^2*ln(1/2+1/2*d*x
+1/2*c)*ln(-1/2*d*x-1/2*c+1/2)*e^2-1/2/d*b^2*ln(d*x+c-1)*ln(1/2+1/2*d*x+1/2
*c)*e^2+1/d*b^2*arccoth(d*x+c)*ln(d*x+c+1)*e^2-4/3*a*b/d^2*f^2*c*x+1/4/d^2*
b^2*f*ln(d*x+c+1)^2*c^2*e+1/4/d^2*b^2*f*ln(d*x+c-1)^2*c^2*e-1/2/d^2*b^2*f*l
n(d*x+c-1)^2*c*e+1/d*a*b*ln(d*x+c+1)*c*e^2-1/d*a*b*ln(d*x+c-1)*c*e^2+1/d^2*
a*b*f*ln(d*x+c-1)*e-4/3/d^2*b^2*f^2*arccoth(d*x+c)*c*x+2/d^2*a*b*f*e*c-1/d^
3*b^2*f^2*arccoth(d*x+c)*ln(d*x+c-1)*c+1/3/d^3*b^2*f^2*arccoth(d*x+c)*ln(d*
x+c+1)*c^3+1/d^3*b^2*f^2*arccoth(d*x+c)*ln(d*x+c+1)*c^2+1/d^3*b^2*f^2*arcco
th(d*x+c)*ln(d*x+c+1)*c+2*a*b*f*arccoth(d*x+c)*e*x^2+2/d*b^2*f*arccoth(d*x+
c)*e*x+2/d^2*b^2*f*arccoth(d*x+c)*e*c+2*a*b/d*f*e*x-1/2/d^3*b^2*f^2*ln(d*x+
c-1)*ln(1/2+1/2*d*x+1/2*c)*c^2-1/3/d^3*b^2*f^2*arccoth(d*x+c)*ln(d*x+c-1)*c
^3+1/2/d^3*b^2*f^2*ln(d*x+c-1)*ln(1/2+1/2*d*x+1/2*c)*c-1/2/d^2*b^2*f*ln(d*x
+c-1)*ln(1/2+1/2*d*x+1/2*c)*e+1/2/d*b^2*ln(d*x+c-1)*ln(1/2+1/2*d*x+1/2*c)*c
*e^2+1/d*b^2*arccoth(d*x+c)*ln(d*x+c+1)*c*e^2+1/2/d^2*b^2*f*ln(d*x+c+1)^2*c
*e+2/d^2*b^2*f*dilog(1/2+1/2*d*x+1/2*c)*c*e+1/d^3*a*b*f^2*ln(d*x+c-1)*c^2-1
/d*b^2*arccoth(d*x+c)*ln(d*x+c-1)*c*e^2+1/2/d*b^2*ln(d*x+c+1)*ln(-1/2*d*x-1
/2*c+1/2)*c*e^2-1/2/d*b^2*ln(1/2+1/2*d*x+1/2*c)*ln(-1/2*d*x-1/2*c+1/2)*c*e^
2+1/d^2*b^2*f*arccoth(d*x+c)*ln(d*x+c-1)*e+1/6/d^3*b^2*f^2*ln(d*x+c-1)*ln(1
/2+1/2*d*x+1/2*c)*c^3+1/d^3*a*b*f^2*ln(d*x+c+1)*c^2+1/3/d^3*a*b*f^2*ln(d*x+
c+1)*c^3-1/d^2*a*b*f*ln(d*x+c+1)*e-1/3/d^3*a*b*f^2*ln(d*x+c-1)*c^3+1/d^3*a*
b*f^2*ln(d*x+c+1)*c-1/d^3*a*b*f^2*ln(d*x+c-1)*c+1/2/d^3*b^2*f^2*ln(d*x+c+1)
*ln(-1/2*d*x-1/2*c+1/2)*c^2+1/2/d^3*b^2*f^2*ln(d*x+c+1)*ln(-1/2*d*x-1/2*c+1
/2)*c-1/6/d^3*b^2*f^2*ln(1/2+1/2*d*x+1/2*c)*ln(-1/2*d*x-1/2*c+1/2)*c^3-1/2/
d^3*b^2*f^2*ln(1/2+1/2*d*x+1/2*c)*ln(-1/2*d*x-1/2*c+1/2)*c^2-1/2/d^3*b^2*f^
2*ln(1/2+1/2*d*x+1/2*c)*ln(-1/2*d*x-1/2*c+1/2)*c+1/d^3*b^2*f^2*arccoth(d*x+
c)*ln(d*x+c-1)*c^2
```

Maxima [B] time = 1.97234, size = 1068, normalized size = 2.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccoth(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*f^2*x^3 + a^2*e*f*x^2 + (2*x^2*arccoth(d*x + c) + d*(2*x/d^2 - (c^2
 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*a
*b*e*f + 1/3*(2*x^3*arccoth(d*x + c) + d*((d*x^2 - 4*c*x)/d^3 + (c^3 + 3*c^
2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3*c - 1)*log(d*x + c - 1
)/d^4))*a*b*f^2 + a^2*e^2*x + (2*(d*x + c)*arccoth(d*x + c) + log(-(d*x + c
```

```
)^2 + 1))*a*b*e^2/d - 1/3*(3*d^2*e^2 - 6*c*d*e*f + 3*c^2*f^2 + f^2)*(log(d*
x + c - 1)*log(1/2*d*x + 1/2*c + 1/2) + dilog(-1/2*d*x - 1/2*c + 1/2))*b^2/
d^3 - 1/6*(5*c^2*f^2 - 6*d*e*f - 6*(d*e*f - f^2)*c + f^2)*b^2*log(d*x + c +
 1)/d^3 + 1/12*(4*b^2*d*f^2*x + (b^2*d^3*f^2*x^3 + 3*b^2*d^3*e*f*x^2 + 3*b^
2*d^3*e^2*x + (c^3*f^2 + 3*d^2*e^2 - 3*(d*e*f - f^2)*c^2 - 3*d*e*f + 3*(d^2
*e^2 - 2*d*e*f + f^2)*c + f^2)*b^2)*log(d*x + c + 1)^2 + (b^2*d^3*f^2*x^3 +
 3*b^2*d^3*e*f*x^2 + 3*b^2*d^3*e^2*x + (c^3*f^2 - 3*d^2*e^2 - 3*(d*e*f + f^
2)*c^2 - 3*d*e*f + 3*(d^2*e^2 + 2*d*e*f + f^2)*c - f^2)*b^2)*log(d*x + c -
1)^2 + 2*(b^2*d^2*f^2*x^2 + 2*(3*d^2*e*f - 2*c*d*f^2)*b^2*x - (b^2*d^3*f^2*
x^3 + 3*b^2*d^3*e*f*x^2 + 3*b^2*d^3*e^2*x + (c^3*f^2 - 3*d^2*e^2 - 3*(d*e*f
 + f^2)*c^2 - 3*d*e*f + 3*(d^2*e^2 + 2*d*e*f + f^2)*c - f^2)*b^2)*log(d*x +
 c - 1))*log(d*x + c + 1) - 2*(b^2*d^2*f^2*x^2 + 2*(3*d^2*e*f - 2*c*d*f^2)*
b^2*x - (5*c^2*f^2 + 6*d*e*f - 6*(d*e*f + f^2)*c + f^2)*b^2)*log(d*x + c -
1))/d^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(a^2f^2x^2 + 2\,a^2efx + a^2e^2 + \left(b^2f^2x^2 + 2\,b^2efx + b^2e^2\right)\mathrm{arcoth}\left(dx+c\right)^2 + 2\left(abf^2x^2 + 2\,abefx + abe^2\right)\mathrm{arcoth}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccoth(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*f^2*x^2 + 2*a^2*e*f*x + a^2*e^2 + (b^2*f^2*x^2 + 2*b^2*e*f*x +
 b^2*e^2)*arccoth(d*x + c)^2 + 2*(a*b*f^2*x^2 + 2*a*b*e*f*x + a*b*e^2)*arcc
oth(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(a+b*acoth(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx+e\right)^2\left(b\,\mathrm{arcoth}\left(dx+c\right)+a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccoth(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*(b*arccoth(d*x + c) + a)^2, x)
```

3.110 $\int (e+fx)\left(a+b\coth^{-1}(c+dx)\right)^2\,dx$

Optimal. Leaf size=221

$$-\frac{b^2(de-cf)\text{PolyLog}\left(2,-\frac{c+dx+1}{-c-dx+1}\right)}{d^2}-\frac{\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^2f}+\frac{(de-cf)\left(a+b\coth^{-1}\right.}{d^2}$$

```
[Out] (a*b*f*x)/d + (b^2*f*(c + d*x)*ArcCoth[c + d*x])/d^2 + ((d*e - c*f)*(a + b*
ArcCoth[c + d*x])^2)/d^2 - ((d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)*(a + b*Ar
cCoth[c + d*x])^2)/(2*d^2*f) + ((e + f*x)^2*(a + b*ArcCoth[c + d*x])^2)/(2*
f) - (2*b*(d*e - c*f)*(a + b*ArcCoth[c + d*x])*Log[2/(1 - c - d*x)])/d^2 +
(b^2*f*Log[1 - (c + d*x)^2])/(2*d^2) - (b^2*(d*e - c*f)*PolyLog[2, -((1 + c
 + d*x)/(1 - c - d*x))])/d^2
```

Rubi [A] time = 0.442664, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6112, 5929, 5911, 260, 6049, 5949, 5985, 5919, 2402, 2315}

$$-\frac{b^2(de-cf)\text{PolyLog}\left(2,-\frac{c+dx+1}{-c-dx+1}\right)}{d^2}+\frac{\left(-\frac{\left(c^2+1\right)f}{d}+2ce-\frac{de^2}{f}\right)\left(a+b\coth^{-1}(c+dx)\right)^2}{2d}+\frac{(de-cf)\left(a+b\coth^{-1}(c+d\right.}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*(a + b*ArcCoth[c + d*x])^2,x]
```

```
[Out] (a*b*f*x)/d + (b^2*f*(c + d*x)*ArcCoth[c + d*x])/d^2 + ((d*e - c*f)*(a + b*
ArcCoth[c + d*x])^2)/d^2 + ((2*c*e - (d*e^2)/f - ((1 + c^2)*f)/d)*(a + b*Ar
cCoth[c + d*x])^2)/(2*d) + ((e + f*x)^2*(a + b*ArcCoth[c + d*x])^2)/(2*f) -
 (2*b*(d*e - c*f)*(a + b*ArcCoth[c + d*x])*Log[2/(1 - c - d*x)])/d^2 + (b^2
*f*Log[1 - (c + d*x)^2])/(2*d^2) - (b^2*(d*e - c*f)*PolyLog[2, -((1 + c + d
*x)/(1 - c - d*x))])/d^2
```

Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rule 5929

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcCoth[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcCoth[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6049

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCoth[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5985

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int (e+fx)\left(a+b\coth^{-1}(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)\left(a+b\coth^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^2\left(a+b\coth^{-1}(c+dx)\right)^2}{2f} - \frac{b\,\text{Subst}\left(\int\left(-\frac{f^2\left(a+b\coth^{-1}(x)\right)}{d^2}+\frac{\left(d^2e^2-2cde\right.}{}\right.\right.}{}$$

$$= \frac{(e+fx)^2\left(a+b\coth^{-1}(c+dx)\right)^2}{2f} - \frac{b\,\text{Subst}\left(\int \frac{\left(d^2e^2-2cdef+\left(1+c^2\right)f^2+2f(de-cf)x\right)}{1-x^2}\right.}{d^2 f}$$

$$= \frac{abfx}{d} + \frac{(e+fx)^2\left(a+b\coth^{-1}(c+dx)\right)^2}{2f} - \frac{b\,\text{Subst}\left(\int\left(\frac{d^2e^2\left(1+\frac{f\left(-2cde+f+c^2f\right)}{d^2e^2}\right)(}{1-x^2}\right.\right.}{}$$

$$= \frac{abfx}{d} + \frac{b^2f(c+dx)\coth^{-1}(c+dx)}{d^2} + \frac{(e+fx)^2\left(a+b\coth^{-1}(c+dx)\right)^2}{2f} - \frac{(b}{}$$

$$= \frac{abfx}{d} + \frac{b^2f(c+dx)\coth^{-1}(c+dx)}{d^2} + \frac{(de-cf)\left(a+b\coth^{-1}(c+dx)\right)^2}{d^2} - \frac{(a}{}$$

$$= \frac{abfx}{d} + \frac{b^2f(c+dx)\coth^{-1}(c+dx)}{d^2} + \frac{(de-cf)\left(a+b\coth^{-1}(c+dx)\right)^2}{d^2} - \frac{(a}{}$$

$$= \frac{abfx}{d} + \frac{b^2f(c+dx)\coth^{-1}(c+dx)}{d^2} + \frac{(de-cf)\left(a+b\coth^{-1}(c+dx)\right)^2}{d^2} - \frac{(a}{}$$

$$= \frac{abfx}{d} + \frac{b^2f(c+dx)\coth^{-1}(c+dx)}{d^2} + \frac{(de-cf)\left(a+b\coth^{-1}(c+dx)\right)^2}{d^2} - \frac{(a}{}$$

Mathematica [A] time = 0.593743, size = 295, normalized size = 1.33

$$\frac{2b^2(de-cf)\text{PolyLog}\left(2,e^{-2\coth^{-1}(c+dx)}\right) - a^2c^2f + 2a^2cde + 2a^2d^2ex + a^2d^2fx^2 + 2b\coth^{-1}(c+dx)\left(-(c+dx)(acf-\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)*(a + b*ArcCoth[c + d*x])^2,x]
```

```
[Out] (2*a^2*c*d*e + 2*a*b*c*f - a^2*c^2*f + 2*a^2*d^2*e*x + 2*a*b*d*f*x + a^2*d^
2*f*x^2 + b^2*(-1 + c + d*x)*(2*d*e + f - c*f + d*f*x)*ArcCoth[c + d*x]^2 +
 2*b*ArcCoth[c + d*x]*(-((c + d*x)*(-(b*f) + a*c*f - a*d*(2*e + f*x))) - 2*
b*(d*e - c*f)*Log[1 - E^(-2*ArcCoth[c + d*x])]) + a*b*f*Log[1 - c - d*x] -
a*b*f*Log[1 + c + d*x] - 4*a*b*d*e*Log[1/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)
])] - 2*b^2*f*Log[1/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)])] + 4*a*b*c*f*Log[1
/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)])] + 2*b^2*(d*e - c*f)*PolyLog[2, E^(-2
*ArcCoth[c + d*x])])/(2*d^2)
```

Maple [B] time = 0.062, size = 857, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(a+b*arccoth(d*x+c))^2,x)
```

```
[Out] -1/4/d^2*b^2*ln(1/2+1/2*d*x+1/2*c)*ln(d*x+c-1)*f+1/d*b^2*arccoth(d*x+c)*f*x
+1/d^2*b^2*arccoth(d*x+c)*f*c-1/2/d^2*b^2*arccoth(d*x+c)^2*f*c^2+1/2/d^2*b^
2*arccoth(d*x+c)*ln(d*x+c-1)*f+1/d^2*a*b*c*f+1/2/d^2*b^2*ln(-1/2*d*x-1/2*c+
1/2)*ln(1/2+1/2*d*x+1/2*c)*c*f-1/d^2*b^2*arccoth(d*x+c)*ln(d*x+c+1)*c*f+1/2
*a^2*x^2*f+a^2*x*e-1/d*b^2*dilog(1/2+1/2*d*x+1/2*c)*e-1/4/d*b^2*ln(d*x+c+1)
^2*e+1/2/d^2*b^2*ln(d*x+c+1)*f+1/2/d^2*b^2*ln(d*x+c-1)*f+1/8/d^2*b^2*ln(d*x
+c+1)^2*f+1/8/d^2*b^2*ln(d*x+c-1)^2*f+1/4/d*b^2*ln(d*x+c-1)^2*e-1/2/d^2*b^2
*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)*c*f-1/d^2*a*b*arccoth(d*x+c)*c^2*f+2/d*
arccoth(d*x+c)*a*b*c*e-1/d^2*a*b*ln(d*x+c-1)*c*f-1/d^2*a*b*ln(d*x+c+1)*c*f-
1/d^2*b^2*arccoth(d*x+c)*ln(d*x+c-1)*c*f+1/2/d^2*b^2*ln(1/2+1/2*d*x+1/2*c)*
ln(d*x+c-1)*c*f-1/2/d^2*a^2*c^2*f+1/d*a^2*c*e+1/2*b^2*arccoth(d*x+c)^2*f*x^
2+arccoth(d*x+c)^2*x*b^2*e+1/4/d^2*b^2*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2+1/2*d*
x+1/2*c)*f-1/4/d^2*b^2*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)*f+a*b*arccoth(d*x
+c)*f*x^2+2*arccoth(d*x+c)*x*a*b*e-1/2/d*b^2*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2+
1/2*d*x+1/2*c)*e+1/d*b^2*arccoth(d*x+c)*ln(d*x+c-1)*e+1/2/d*b^2*ln(-1/2*d*x
-1/2*c+1/2)*ln(d*x+c+1)*e-1/2/d*b^2*ln(1/2+1/2*d*x+1/2*c)*ln(d*x+c-1)*e+1/d
*b^2*arccoth(d*x+c)*ln(d*x+c+1)*e+1/d*arccoth(d*x+c)^2*b^2*c*e+1/d*a*b*ln(d
*x+c-1)*e+1/d*a*b*ln(d*x+c+1)*e-1/2/d^2*b^2*arccoth(d*x+c)*ln(d*x+c+1)*f-1/
4/d^2*b^2*ln(d*x+c-1)^2*c*f+1/d^2*b^2*dilog(1/2+1/2*d*x+1/2*c)*c*f-1/2/d^2*
a*b*ln(d*x+c+1)*f+1/4/d^2*b^2*ln(d*x+c+1)^2*c*f+1/2/d^2*a*b*ln(d*x+c-1)*f+a
*b*f*x/d
```

Maxima [A] time = 1.96123, size = 540, normalized size = 2.44

$$\frac{1}{2}\,a^2 f x^2 + \frac{1}{2}\left(2\,x^2 \operatorname{arcoth}\left(dx+c\right) + d\left(\frac{2\,x}{d^2} - \frac{\left(c^2+2\,c+1\right)\log\left(dx+c+1\right)}{d^3} + \frac{\left(c^2-2\,c+1\right)\log\left(dx+c-1\right)}{d^3}\right)\right)abf + a^2ex$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccoth(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*f*x^2 + 1/2*(2*x^2*arccoth(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1)*
log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*a*b*f + a^2*e
*x + (2*(d*x + c)*arccoth(d*x + c) + log(-(d*x + c)^2 + 1))*a*b*e/d - (d*e
- c*f)*(log(d*x + c - 1)*log(1/2*d*x + 1/2*c + 1/2) + dilog(-1/2*d*x - 1/2*
c + 1/2))*b^2/d^2 + 1/2*(c*f + f)*b^2*log(d*x + c + 1)/d^2 + 1/8*((b^2*d^2*
f*x^2 + 2*b^2*d^2*e*x - (c^2*f - 2*(d*e - f)*c - 2*d*e + f)*b^2)*log(d*x +
c + 1)^2 + (b^2*d^2*f*x^2 + 2*b^2*d^2*e*x - (c^2*f - 2*(d*e + f)*c + 2*d*e
+ f)*b^2)*log(d*x + c - 1)^2 + 2*(2*b^2*d*f*x - (b^2*d^2*f*x^2 + 2*b^2*d^2*
e*x - (c^2*f - 2*(d*e + f)*c + 2*d*e + f)*b^2)*log(d*x + c - 1))*log(d*x +
c + 1) - 4*(b^2*d*f*x + (c*f - f)*b^2)*log(d*x + c - 1))/d^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(a^2 f x + a^2 e + \left(b^2 f x + b^2 e\right)\operatorname{arcoth}\left(dx+c\right)^2 + 2\left(abfx + abe\right)\operatorname{arcoth}\left(dx+c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccoth(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*f*x + a^2*e + (b^2*f*x + b^2*e)*arccoth(d*x + c)^2 + 2*(a*b*f*
x + a*b*e)*arccoth(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\operatorname{acoth}(c + dx))^2 (e + fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*acoth(d*x+c))**2,x)
```

```
[Out] Integral((a + b*acoth(c + d*x))**2*(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx + e)(b\operatorname{arcoth}(dx + c) + a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccoth(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(b*arccoth(d*x + c) + a)^2, x)
```

3.111 $\int \left(a+b\coth^{-1}(c+dx)\right)^2\,dx$

Optimal. Leaf size=97

$$-\frac{b^2\text{PolyLog}\left(2,-\frac{c+dx+1}{-c-dx+1}\right)}{d}+\frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2}{d}+\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{d}-\frac{2b\log\left(\frac{2}{-c-dx+1}\right)\left(a+b\coth^{-1}\right.}{d}$$

```
[Out] (a + b*ArcCoth[c + d*x])^2/d + ((c + d*x)*(a + b*ArcCoth[c + d*x])^2)/d - (
2*b*(a + b*ArcCoth[c + d*x])*Log[2/(1 - c - d*x)])/d - (b^2*PolyLog[2, -((1
 + c + d*x)/(1 - c - d*x))])/d
```

Rubi [A] time = 0.11621, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6104, 5911, 5985, 5919, 2402, 2315}

$$-\frac{b^2\text{PolyLog}\left(2,-\frac{c+dx+1}{-c-dx+1}\right)}{d}+\frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2}{d}+\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{d}-\frac{2b\log\left(\frac{2}{-c-dx+1}\right)\left(a+b\coth^{-1}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[c + d*x])^2,x]
```

```
[Out] (a + b*ArcCoth[c + d*x])^2/d + ((c + d*x)*(a + b*ArcCoth[c + d*x])^2)/d - (
2*b*(a + b*ArcCoth[c + d*x])*Log[2/(1 - c - d*x)])/d - (b^2*PolyLog[2, -((1
 + c + d*x)/(1 - c - d*x))])/d
```

Rule 6104

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}
, x] && IGtQ[p, 0]
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5985

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \left(a+b\coth^{-1}(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \left(a+b\coth^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$
$$= \frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2}{d} - \frac{(2b)\,\text{Subst}\left(\int \frac{x\left(a+b\coth^{-1}(x)\right)}{1-x^2} dx, x, c+dx\right)}{d}$$
$$= \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2}{d} - \frac{(2b)\,\text{Subst}\left(\int \frac{a+b\coth^{-}}{1-x}\right.}{d}$$
$$= \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2}{d} - \frac{2b\left(a+b\coth^{-1}(c+d\right.}{d}$$
$$= \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2}{d} - \frac{2b\left(a+b\coth^{-1}(c+d\right.}{d}$$
$$= \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2}{d} - \frac{2b\left(a+b\coth^{-1}(c+d\right.}{d}$$

Mathematica [A] time = 0.148008, size = 111, normalized size = 1.14

$$\frac{b^2\text{PolyLog}\left(2, e^{-2\coth^{-1}(c+dx)}\right) + a\left(ac+adx-2b\log\left(\frac{1}{(c+dx)\sqrt{1-\frac{1}{(c+dx)^2}}}\right)\right) + 2b\coth^{-1}(c+dx)\left(ac+adx-b\log\left(1-e^{-2}\right.\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCoth[c + d*x])^2,x]
```

```
[Out] (b^2*(-1 + c + d*x)*ArcCoth[c + d*x]^2 + 2*b*ArcCoth[c + d*x]*(a*c + a*d*x
- b*Log[1 - E^(-2*ArcCoth[c + d*x])]) + a*(a*c + a*d*x - 2*b*Log[1/((c + d*
x)*Sqrt[1 - (c + d*x)^(-2)])]) + b^2*PolyLog[2, E^(-2*ArcCoth[c + d*x])])/d
```

Maple [B] time = 0.111, size = 226, normalized size = 2.3

$$(\text{arccoth}\,(dx+c))^2 xb^2 + \frac{(\text{arccoth}\,(dx+c))^2 b^2 c}{d} + 2\,\text{arccoth}\,(dx+c)\,xab - 2\,\frac{\text{arccoth}\,(dx+c)\,b^2}{d}\ln\left(1-\frac{1}{\sqrt{\frac{dx+c-1}{dx+c+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(d*x+c))^2,x)
```

```
[Out] arccoth(d*x+c)^2*x*b^2+1/d*arccoth(d*x+c)^2*b^2*c+2*arccoth(d*x+c)*x*a*b-2/
d*ln(1-1/((d*x+c-1)/(d*x+c+1))^(1/2))*arccoth(d*x+c)*b^2-2/d*ln(1+1/((d*x+c
-1)/(d*x+c+1))^(1/2))*arccoth(d*x+c)*b^2+1/d*b^2*arccoth(d*x+c)^2+2/d*arcco
th(d*x+c)*a*b*c+a^2*x-2/d*polylog(2,1/((d*x+c-1)/(d*x+c+1))^(1/2))*b^2-2/d*
polylog(2,-1/((d*x+c-1)/(d*x+c+1))^(1/2))*b^2+1/d*a*b*ln((d*x+c)^2-1)+a^2*c
/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^2x + \frac{1}{4}b^2\left(\frac{dx\log(dx+c-1)^2 + (dx+c+1)\log(dx+c+1)^2 - 2(dx+c-1)\log(dx+c+1)\log(dx+c-1)}{d} + \int \frac{2(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/4*b^2*((d*x*log(d*x + c - 1)^2 + (d*x + c + 1)*log(d*x + c + 1)^2
 - 2*(d*x + c - 1)*log(d*x + c + 1)*log(d*x + c - 1))/d + integrate(2*(c^2
+ (c*d - 3*d)*x - 2*c + 1)*log(d*x + c - 1)/(d^2*x^2 + 2*c*d*x + c^2 - 1),
x)) + (2*(d*x + c)*arccoth(d*x + c) + log(-(d*x + c)^2 + 1))*a*b/d
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^2 \operatorname{arcoth}(dx+c)^2 + 2\,ab \operatorname{arcoth}(dx+c) + a^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arccoth(d*x + c)^2 + 2*a*b*arccoth(d*x + c) + a^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{acoth}(c+dx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(d*x+c))**2,x)
```

```
[Out] Integral((a + b*acoth(c + d*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arcoth}(dx+c) + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(d*x + c) + a)^2, x)
```

3.112 $\int \frac{(a+b\coth^{-1}(c+dx))^2}{e+fx}\,dx$

Optimal. Leaf size=214

$$-\frac{b\left(a+b\coth^{-1}(c+dx)\right)\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{f}+\frac{b\text{PolyLog}\left(2,1-\frac{2}{c+dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)}{f}-\ldots$$

```
[Out] -(((a + b*ArcCoth[c + d*x])^2*Log[2/(1 + c + d*x)])/f) + ((a + b*ArcCoth[c
+ d*x])^2*Log[(2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/f + (b*(a +
 b*ArcCoth[c + d*x])*PolyLog[2, 1 - 2/(1 + c + d*x)])/f - (b*(a + b*ArcCoth
[c + d*x])*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])
/f + (b^2*PolyLog[3, 1 - 2/(1 + c + d*x)])/(2*f) - (b^2*PolyLog[3, 1 - (2*d
*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/(2*f)
```

Rubi [A] time = 0.153347, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {6112, 5923}

$$-\frac{b\left(a+b\coth^{-1}(c+dx)\right)\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{f}+\frac{b\text{PolyLog}\left(2,1-\frac{2}{c+dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)}{f}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[c + d*x])^2/(e + f*x),x]
```

```
[Out] -(((a + b*ArcCoth[c + d*x])^2*Log[2/(1 + c + d*x)])/f) + ((a + b*ArcCoth[c
+ d*x])^2*Log[(2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/f + (b*(a +
 b*ArcCoth[c + d*x])*PolyLog[2, 1 - 2/(1 + c + d*x)])/f - (b*(a + b*ArcCoth
[c + d*x])*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])
/f + (b^2*PolyLog[3, 1 - 2/(1 + c + d*x)])/(2*f) - (b^2*PolyLog[3, 1 - (2*d
*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/(2*f)
```

Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rule 5923

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 -Simp[((a + b*ArcCoth[c*x])^2*Log[2/(1 + c*x)])/e, x] + (Simp[((a + b*ArcC
oth[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Simp[(b*(a
+ b*ArcCoth[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e, x] - Simp[(b*(a + b*ArcCo
th[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Sim
p[(b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e), x] - Simp[(b^2*PolyLog[3, 1 - (2
*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e), x]) /; FreeQ[{a, b, c, d, e},
x] && NeQ[c^2*d^2 - e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{e+fx}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\coth^{-1}(x)\right)^2}{\frac{de-cf}{d}+\frac{fx}{d}}\,dx, x, c+dx\right)}{d}$$
$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^2\log\left(\frac{2}{1+c+dx}\right)}{f} + \frac{\left(a+b\coth^{-1}(c+dx)\right)^2\log\left(\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{f}$$

Mathematica [C] time = 16.6836, size = 1721, normalized size = 8.04

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCoth[c + d*x])^2/(e + f*x),x]
```

```
[Out] (a^2*Log[e + f*x])/f + 2*a*b*(((ArcCoth[c + d*x] - ArcTanh[c + d*x])*Log[e
+ f*x])/f - (I*(I*ArcTanh[c + d*x]*(-Log[1/Sqrt[1 - (c + d*x)^2]] + Log[I*S
inh[ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]]]) + ((-I)*(I*ArcTanh[(d*e -
c*f)/f] + I*ArcTanh[c + d*x])^2 - (I/4)*(Pi - (2*I)*ArcTanh[c + d*x])^2 + 2
*(I*ArcTanh[(d*e - c*f)/f] + I*ArcTanh[c + d*x])*Log[1 - E^((2*I)*(I*ArcTan
h[(d*e - c*f)/f] + I*ArcTanh[c + d*x]))] + (Pi - (2*I)*ArcTanh[c + d*x])*Lo
g[1 - E^(I*(Pi - (2*I)*ArcTanh[c + d*x]))] - (Pi - (2*I)*ArcTanh[c + d*x])*
Log[2*Sin[(Pi - (2*I)*ArcTanh[c + d*x])/2]] - 2*(I*ArcTanh[(d*e - c*f)/f] +
 I*ArcTanh[c + d*x])*Log[(2*I)*Sinh[ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*
x]]] - I*PolyLog[2, E^((2*I)*(I*ArcTanh[(d*e - c*f)/f] + I*ArcTanh[c + d*x]
))] - I*PolyLog[2, E^(I*(Pi - (2*I)*ArcTanh[c + d*x]))])/2))/f) - (b^2*(d*e
 - c*f + f*(c + d*x))*(1 - (c + d*x)^2)*(-(I*f*Pi^3 - 8*d*e*ArcCoth[c + d*x
]^3 - 8*f*ArcCoth[c + d*x]^3 + 8*c*f*ArcCoth[c + d*x]^3 + 24*f*ArcCoth[c +
d*x]^2*Log[1 - E^(2*ArcCoth[c + d*x])] + 24*f*ArcCoth[c + d*x]*PolyLog[2, E
^(2*ArcCoth[c + d*x])] - 12*f*PolyLog[3, E^(2*ArcCoth[c + d*x])]])/(24*f^2)
+ ((-(d*e) - f + c*f)*(-(d*e) + f + c*f)*(2*d*e*ArcCoth[c + d*x]^3 - 6*f*Ar
cCoth[c + d*x]^3 - 2*c*f*ArcCoth[c + d*x]^3 - (4*d*e*Sqrt[(d^2*e^2 - 2*c*d*
e*f + (-1 + c^2)*f^2)/(d*e - c*f)^2]*ArcCoth[c + d*x]^3)/E^ArcTanh[f/(d*e -
 c*f)] + (4*c*f*Sqrt[(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)/(d*e - c*f)^2]*
ArcCoth[c + d*x]^3)/E^ArcTanh[f/(d*e - c*f)] + (6*I)*f*Pi*ArcCoth[c + d*x]*
Log[2] - f*ArcCoth[c + d*x]^2*Log[64] - (6*I)*f*Pi*ArcCoth[c + d*x]*Log[E^(
-ArcCoth[c + d*x]) + E^ArcCoth[c + d*x]] + 6*f*ArcCoth[c + d*x]^2*Log[1 - E
^(2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]))] + 12*f*ArcCoth[c + d*x]*A
rcTanh[f/(d*e - c*f)]*Log[(I/2)*E^(-ArcCoth[c + d*x] - ArcTanh[f/(d*e - c*f
)])*(-1 + E^(2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)])))] + 6*f*ArcCoth
[c + d*x]^2*Log[-((d*e*(-1 + E^(2*ArcCoth[c + d*x])) + (1 + c + E^(2*ArcCot
h[c + d*x]) - c*E^(2*ArcCoth[c + d*x]))*f)/E^ArcCoth[c + d*x])] - 6*f*ArcCo
th[c + d*x]^2*Log[(-(d*e*(-1 + E^(2*ArcCoth[c + d*x]))) + (-1 - E^(2*ArcCot
h[c + d*x]) + c*(-1 + E^(2*ArcCoth[c + d*x])))*f)/(d*e - (1 + c)*f)] + 6*f*
ArcCoth[c + d*x]^2*Log[1 - (E^ArcCoth[c + d*x]*Sqrt[d*e + f - c*f])/Sqrt[d*
e - (1 + c)*f]] + 6*f*ArcCoth[c + d*x]^2*Log[1 + (E^ArcCoth[c + d*x]*Sqrt[d
*e + f - c*f])/Sqrt[d*e - (1 + c)*f]] + (6*I)*f*Pi*ArcCoth[c + d*x]*Log[1/S
qrt[1 - (c + d*x)^(-2)]] - 6*f*ArcCoth[c + d*x]^2*Log[-(f/Sqrt[1 - (c + d*x
)^(-2)]) - (d*e)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (c*f)/((c + d*x)*Sq
rt[1 - (c + d*x)^(-2)])] - 12*f*ArcCoth[c + d*x]*ArcTanh[f/(d*e - c*f)]*Log
[I*Sinh[ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]]] + 6*f*ArcCoth[c + d*x]*
PolyLog[2, E^(2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]))] - 6*f*ArcCoth
[c + d*x]*PolyLog[2, (E^(2*ArcCoth[c + d*x])*(d*e + f - c*f))/(d*e - (1 + c
)*f)] + 12*f*ArcCoth[c + d*x]*PolyLog[2, -((E^ArcCoth[c + d*x]*Sqrt[d*e + f
 - c*f])/Sqrt[d*e - (1 + c)*f])] + 12*f*ArcCoth[c + d*x]*PolyLog[2, (E^ArcC
oth[c + d*x]*Sqrt[d*e + f - c*f])/Sqrt[d*e - (1 + c)*f]] - 3*f*PolyLog[3, E
```

```
^(2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]))] + 3*f*PolyLog[3, (E^(2*Ar
cCoth[c + d*x])*(d*e + f - c*f))/(d*e - (1 + c)*f)] - 12*f*PolyLog[3, -((E^
ArcCoth[c + d*x]*Sqrt[d*e + f - c*f])/Sqrt[d*e - (1 + c)*f])] - 12*f*PolyLo
g[3, (E^ArcCoth[c + d*x]*Sqrt[d*e + f - c*f])/Sqrt[d*e - (1 + c)*f]]))/(6*f
^2*(d*e + f - c*f)*(d*e - (1 + c)*f))))/(d*(c + d*x)^2*(e + f*x)*(1 - (c +
d*x)^(-2)))
```

Maple [C] time = 0.855, size = 1845, normalized size = 8.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(d*x+c))^2/(f*x+e),x)
```

```
[Out] -1/2*I*b^2/f*Pi*arccoth(d*x+c)^2*csgn(I*(((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*
x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^
2*csgn(I/((d*x+c+1)/(d*x+c-1)-1))-b^2/f*arccoth(d*x+c)^2*ln(1+1/((d*x+c-1)/
(d*x+c+1))^(1/2))-2*b^2/f*arccoth(d*x+c)*polylog(2,-1/((d*x+c-1)/(d*x+c+1))
^(1/2))-b^2/f*arccoth(d*x+c)^2*ln(1-1/((d*x+c-1)/(d*x+c+1))^(1/2))-2*b^2/f*
arccoth(d*x+c)*polylog(2,1/((d*x+c-1)/(d*x+c+1))^(1/2))-b^2/(c*f-d*e-f)*arc
coth(d*x+c)^2*ln(1-(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))-b^2/(c*f-d*
e-f)*arccoth(d*x+c)*polylog(2,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))+
b^2/f*arccoth(d*x+c)^2*ln((d*x+c+1)/(d*x+c-1)-1)-b^2/f*arccoth(d*x+c)^2*ln(
((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-
1)-1)*f)+b^2*ln((d*x+c)*f-c*f+d*e)/f*arccoth(d*x+c)^2-1/2*b^2*c/(c*f-d*e-f)
*polylog(3,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))-a*b/f*dilog(((d*x+c
)*f+f)/(c*f-d*e+f))+a*b/f*dilog(((d*x+c)*f-f)/(c*f-d*e-f))+1/2*I*b^2/f*Pi*a
rccoth(d*x+c)^2*csgn(I*(((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))
*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f))*csgn(I*(((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d
*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))
*csgn(I/((d*x+c+1)/(d*x+c-1)-1))-a*b/f*ln((d*x+c)*f-c*f+d*e)*ln(((d*x+c)*f+
f)/(c*f-d*e+f))+2*a*b*ln((d*x+c)*f-c*f+d*e)/f*arccoth(d*x+c)+a*b/f*ln((d*x+
c)*f-c*f+d*e)*ln(((d*x+c)*f-f)/(c*f-d*e-f))+b^2*c/(c*f-d*e-f)*arccoth(d*x+c
)^2*ln(1-(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))+b^2*c/(c*f-d*e-f)*arc
coth(d*x+c)*polylog(2,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))-I*b^2/f*
Pi*arccoth(d*x+c)^2+a^2*ln((d*x+c)*f-c*f+d*e)/f+2*b^2/f*polylog(3,1/((d*x+c
-1)/(d*x+c+1))^(1/2))+2*b^2/f*polylog(3,-1/((d*x+c-1)/(d*x+c+1))^(1/2))+1/2
*b^2/(c*f-d*e-f)*polylog(3,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))+1/2
*d*b^2/f*e/(c*f-d*e-f)*polylog(3,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f
))+I*b^2/f*Pi*arccoth(d*x+c)^2*csgn(I*(((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+
c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^2-
1/2*I*b^2/f*Pi*arccoth(d*x+c)^2*csgn(I*(((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x
+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^3
-d*b^2/f*e/(c*f-d*e-f)*arccoth(d*x+c)^2*ln(1-(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1
)/(c*f-d*e+f))-d*b^2/f*e/(c*f-d*e-f)*arccoth(d*x+c)*polylog(2,(c*f-d*e-f)*(
d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))-1/2*I*b^2/f*Pi*arccoth(d*x+c)^2*csgn(I*(((d
*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-
1)*f))*csgn(I*(((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d
*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2\log\left(fx+e\right)}{f}+\int\frac{b^2\left(\log\left(\frac{1}{dx+c}+1\right)-\log\left(-\frac{1}{dx+c}+1\right)\right)^2}{4\left(fx+e\right)}+\frac{ab\left(\log\left(\frac{1}{dx+c}+1\right)-\log\left(-\frac{1}{dx+c}+1\right)\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^2/(f*x+e),x, algorithm="maxima")
```

```
[Out] a^2*log(f*x + e)/f + integrate(1/4*b^2*(log(1/(d*x + c) + 1) - log(-1/(d*x
+ c) + 1))^2/(f*x + e) + a*b*(log(1/(d*x + c) + 1) - log(-1/(d*x + c) + 1))
/(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \operatorname{arcoth}(dx+c)^2 + 2\,ab \operatorname{arcoth}(dx+c) + a^2}{fx+e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^2/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccoth(d*x + c)^2 + 2*a*b*arccoth(d*x + c) + a^2)/(f*x + e),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(d*x+c))**2/(f*x+e),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcoth}(dx+c)+a)^2}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^2/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(d*x + c) + a)^2/(f*x + e), x)
```

3.113 $\int \frac{(a+b\coth^{-1}(c+dx))^2}{(e+fx)^2}\,dx$

Optimal. Leaf size=480

$$\frac{b^2d\mathrm{PolyLog}\left(2,-\frac{c+dx+1}{-c-dx+1}\right)}{2f(-cf+de+f)}+\frac{b^2d\mathrm{PolyLog}\left(2,1-\frac{2}{c+dx+1}\right)}{2f(-cf+de-f)}-\frac{b^2d\mathrm{PolyLog}\left(2,1-\frac{2}{c+dx+1}\right)}{(-cf+de+f)(de-(c+1)f)}+\frac{b^2d\mathrm{PolyLog}\left(2,1-\frac{\ldots}{(c+d\ldots}\right)}{(-cf+de+f)(de-\ldots}$$

```
[Out] -((a + b*ArcCoth[c + d*x])^2/(f*(e + f*x))) + (b^2*d*ArcCoth[c + d*x]*Log[2
/(1 - c - d*x)])/(f*(d*e + f - c*f)) - (a*b*d*Log[1 - c - d*x])/(f*(d*e + f
 - c*f)) - (b^2*d*ArcCoth[c + d*x]*Log[2/(1 + c + d*x)])/(f*(d*e - f - c*f)
) + (2*b^2*d*ArcCoth[c + d*x]*Log[2/(1 + c + d*x)])/((d*e + f - c*f)*(d*e -
 (1 + c)*f)) + (a*b*d*Log[1 + c + d*x])/(f*(d*e - f - c*f)) + (2*a*b*d*Log[
e + f*x])/(f^2 - (d*e - c*f)^2) - (2*b^2*d*ArcCoth[c + d*x]*Log[(2*d*(e + f
*x))/((d*e + f - c*f)*(1 + c + d*x))])/((d*e + f - c*f)*(d*e - (1 + c)*f))
+ (b^2*d*PolyLog[2, -((1 + c + d*x)/(1 - c - d*x))])/(2*f*(d*e + f - c*f))
+ (b^2*d*PolyLog[2, 1 - 2/(1 + c + d*x)])/(2*f*(d*e - f - c*f)) - (b^2*d*Po
lyLog[2, 1 - 2/(1 + c + d*x)])/((d*e + f - c*f)*(d*e - (1 + c)*f)) + (b^2*d
*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/((d*e + f
 - c*f)*(d*e - (1 + c)*f))
```

Rubi [A] time = 1.74301, antiderivative size = 485, normalized size of antiderivative = 1.01, number of steps used = 21, number of rules used = 19, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.95, Rules used = {6110, 1982, 705, 31, 632, 6741, 6122, 706, 633, 6688, 12, 6725, 72, 6742, 5919, 2402, 2315, 5921, 2447}

$$\frac{b^2d\mathrm{PolyLog}\left(2,-\frac{c+dx+1}{-c-dx+1}\right)}{2f(-cf+de+f)}+\frac{b^2d\mathrm{PolyLog}\left(2,1-\frac{2}{c+dx+1}\right)}{2f(-cf+de-f)}-\frac{b^2d\mathrm{PolyLog}\left(2,1-\frac{2}{c+dx+1}\right)}{(-cf+de+f)(de-(c+1)f)}+\frac{b^2d\mathrm{PolyLog}\left(2,1-\frac{\ldots}{(c+d\ldots}\right)}{(-cf+de+f)(de-\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[c + d*x])^2/(e + f*x)^2,x]
```

```
[Out] -((a + b*ArcCoth[c + d*x])^2/(f*(e + f*x))) + (b^2*d*ArcCoth[c + d*x]*Log[2
/(1 - c - d*x)])/(f*(d*e + f - c*f)) - (a*b*d*Log[1 - c - d*x])/(f*(d*e + f
 - c*f)) - (b^2*d*ArcCoth[c + d*x]*Log[2/(1 + c + d*x)])/(f*(d*e - f - c*f)
) + (2*b^2*d*ArcCoth[c + d*x]*Log[2/(1 + c + d*x)])/((d*e + f - c*f)*(d*e -
 (1 + c)*f)) + (a*b*d*Log[1 + c + d*x])/(f*(d*e - f - c*f)) - (2*a*b*d*Log[
e + f*x])/((d*e + f - c*f)*(d*e - (1 + c)*f)) - (2*b^2*d*ArcCoth[c + d*x]*L
og[(2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/((d*e + f - c*f)*(d*e
- (1 + c)*f)) + (b^2*d*PolyLog[2, -((1 + c + d*x)/(1 - c - d*x))])/(2*f*(d*
e + f - c*f)) + (b^2*d*PolyLog[2, 1 - 2/(1 + c + d*x)])/(2*f*(d*e - f - c*f
)) - (b^2*d*PolyLog[2, 1 - 2/(1 + c + d*x)])/((d*e + f - c*f)*(d*e - (1 + c
)*f)) + (b^2*d*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x
))])/((d*e + f - c*f)*(d*e - (1 + c)*f))
```

Rule 6110

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[((e + f*x)^(m + 1)*(a + b*ArcCoth[c + d*x])^p)/(f*(m
 + 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcCot
h[c + d*x])^(p - 1))/(1 - (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 1982

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6122

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subs
t[Int[((d*e - c*f)/d + (f*x)/d)^m*(-(C/d^2) + (C*x^2)/d^2)^q*(a + b*ArcCoth
[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q},
x] && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5921

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcCoth[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcCoth[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{(e+fx)^2}\,dx = -\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2bd)\int \frac{a+b\coth^{-1}(c+dx)}{(e+fx)\left(1-(c+dx)^2\right)}\,dx}{f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2bd)\int \frac{a+b\coth^{-1}(c+dx)}{(e+fx)\left(1-c^2-2cdx-d^2x^2\right)}\,dx}{f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2b)\,\mathrm{Subst}\left(\int \frac{a+b\coth^{-1}(x)}{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)(1-x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2b)\,\mathrm{Subst}\left(\int \frac{d\left(a+b\coth^{-1}(x)\right)}{(de-cf+fx)(1-x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2bd)\,\mathrm{Subst}\left(\int \frac{a+b\coth^{-1}(x)}{(de-cf+fx)(1-x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2bd)\,\mathrm{Subst}\left(\int \left(-\frac{a}{(-1+x)(1+x)(de-cf+fx)} - \frac{b\coth^{-1}(x)}{(-1+x)(1+x)(de-cf+fx)}\right.\right.}{f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{(2abd)\,\mathrm{Subst}\left(\int \frac{1}{(-1+x)(1+x)(de-cf+fx)}\,dx, x, c+dx\right)}{f} - \frac{(2b^2d)}{}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{(2abd)\,\mathrm{Subst}\left(\int \left(\frac{1}{2(de+f-cf)(-1+x)} + \frac{1}{2(-de+(1+c)f)(1+x)} + \frac{}{(de+(1}\right.\right.}{f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{abd\log(1-c-dx)}{f(de+f-cf)} + \frac{abd\log(1+c+dx)}{f(de-f-cf)} - \frac{2abd\log(}{(de+f-cf)(d}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{b^2d\coth^{-1}(c+dx)\log\left(\frac{2}{1-c-dx}\right)}{f(de+f-cf)} - \frac{abd\log(1-c-dx)}{f(de+f-cf)} - \frac{b^2d}{}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{b^2d\coth^{-1}(c+dx)\log\left(\frac{2}{1-c-dx}\right)}{f(de+f-cf)} - \frac{abd\log(1-c-dx)}{f(de+f-cf)} - \frac{b^2d}{}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{b^2d\coth^{-1}(c+dx)\log\left(\frac{2}{1-c-dx}\right)}{f(de+f-cf)} - \frac{abd\log(1-c-dx)}{f(de+f-cf)} - \frac{b^2d}{}$$

Mathematica [C] time = 8.63578, size = 470, normalized size = 0.98

$$b^2d\left(1-(c+dx)^2\right)(e+fx)\left(\frac{f\left(-\mathrm{PolyLog}\left(2,\exp\left(-2\left(\tanh^{-1}\left(\frac{f}{de-cf}\right)+\coth^{-1}(c+dx)\right)\right)\right)-2\tanh^{-1}\left(\frac{f}{cf-de}\right)\log\left(1-\exp\left(-2\left(\tanh^{-1}\left(\frac{f}{de-cf}\right)+\coth^{-1}(c+dx)\right)\right)\right)+\coth^{-1}(c+dx)\left(2\log\left(1-e\right.\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCoth[c + d*x])^2/(e + f*x)^2,x]
```

```
[Out] (-(a^2/f) + (2*a*b*((f - c^2*f + d^2*e*x + c*d*(e - f*x))*ArcCoth[c + d*x]
- d*(e + f*x)*Log[-((d*(e + f*x))/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]))]))/
```

```
((d*e + f - c*f)*(d*e - (1 + c)*f)) + (b^2*d*(e + f*x)*(1 - (c + d*x)^2)*((
E^ArcTanh[f/(-(d*e) + c*f)]*ArcCoth[c + d*x]^2)/((-(d*e) + c*f)*Sqrt[1 - f^
2/(d*e - c*f)^2]) + ArcCoth[c + d*x]^2/(d*e + d*f*x) + (f*((-I)*Pi*Log[1 +
E^(2*ArcCoth[c + d*x])] - 2*ArcTanh[f/(-(d*e) + c*f)]*Log[1 - E^(-2*(ArcCot
h[c + d*x] + ArcTanh[f/(d*e - c*f)]))] + ArcCoth[c + d*x]*(I*Pi + 2*ArcTanh
[f/(d*e - c*f)] + 2*Log[1 - E^(-2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)
]))]) + I*Pi*Log[1/Sqrt[1 - (c + d*x)^(-2)]] + 2*ArcTanh[f/(-(d*e) + c*f)]*
Log[I*Sinh[ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]]] - PolyLog[2, E^(-2*(
ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]))]))/(d^2*e^2 - 2*c*d*e*f + (-1 +
 c^2)*f^2)))/((c + d*x)^2*(f - f/(c + d*x)^2)))/(e + f*x)
```

Maple [A] time = 0.158, size = 783, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(d*x+c))^2/(f*x+e)^2,x)
```

```
[Out] -d*a^2/(d*f*x+d*e)/f-d*b^2/(d*f*x+d*e)/f*arccoth(d*x+c)^2+2*d*b^2/f*arccoth
(d*x+c)/(2*c*f-2*d*e-2*f)*ln(d*x+c-1)-2*d*b^2/f*arccoth(d*x+c)/(2*c*f-2*d*e
+2*f)*ln(d*x+c+1)-2*d*b^2*arccoth(d*x+c)/(c*f-d*e-f)/(c*f-d*e+f)*ln((d*x+c)
*f-c*f+d*e)+d*b^2/(c*f-d*e-f)/(c*f-d*e+f)*ln(((d*x+c)*f+f)/(c*f-d*e+f))*ln(
(d*x+c)*f-c*f+d*e)+d*b^2/(c*f-d*e-f)/(c*f-d*e+f)*dilog(((d*x+c)*f+f)/(c*f-d
*e+f))-d*b^2/(c*f-d*e-f)/(c*f-d*e+f)*ln(((d*x+c)*f-f)/(c*f-d*e-f))*ln((d*x+
c)*f-c*f+d*e)-d*b^2/(c*f-d*e-f)/(c*f-d*e+f)*dilog(((d*x+c)*f-f)/(c*f-d*e-f)
)+1/4*d*b^2/f/(c*f-d*e-f)*ln(d*x+c-1)^2-1/2*d*b^2/f/(c*f-d*e-f)*dilog(1/2+1
/2*d*x+1/2*c)-1/2*d*b^2/f/(c*f-d*e-f)*ln(d*x+c-1)*ln(1/2+1/2*d*x+1/2*c)-1/2
*d*b^2/f/(c*f-d*e+f)*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)+1/2*d*b^2/f/(c*f-d*
e+f)*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2+1/2*d*x+1/2*c)+1/2*d*b^2/f/(c*f-d*e+f)*d
ilog(1/2+1/2*d*x+1/2*c)+1/4*d*b^2/f/(c*f-d*e+f)*ln(d*x+c+1)^2-2*d*a*b/(d*f*
x+d*e)/f*arccoth(d*x+c)+2*d*a*b/f/(2*c*f-2*d*e-2*f)*ln(d*x+c-1)-2*d*a*b/f/(
2*c*f-2*d*e+2*f)*ln(d*x+c+1)-2*d*a*b/(c*f-d*e-f)/(c*f-d*e+f)*ln((d*x+c)*f-c
*f+d*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(d\left(\frac{\log\left(dx+c+1\right)}{def-(c+1)f^2}-\frac{\log\left(dx+c-1\right)}{def-(c-1)f^2}-\frac{2\,\log\left(fx+e\right)}{d^2e^2-2\,cdef+\left(c^2-1\right)f^2}\right)-\frac{2\,\mathrm{arcoth}\left(dx+c\right)}{f^2x+ef}\right)ab-\frac{1}{4}\,b^2\left(\frac{\log\left(dx+c+1\right)^2}{f^2x+ef}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^2/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] (d*(log(d*x + c + 1)/(d*e*f - (c + 1)*f^2) - log(d*x + c - 1)/(d*e*f - (c -
 1)*f^2) - 2*log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 - 1)*f^2)) - 2*arccot
h(d*x + c)/(f^2*x + e*f))*a*b - 1/4*b^2*(log(d*x + c + 1)^2/(f^2*x + e*f) +
 integrate(-((d*f*x + c*f + f)*log(d*x + c - 1)^2 + 2*(d*f*x + d*e - (d*f*x
 + c*f + f)*log(d*x + c - 1))*log(d*x + c + 1))/(d*f^3*x^3 + c*e^2*f + e^2*
f + (2*d*e*f^2 + c*f^3 + f^3)*x^2 + (d*e^2*f + 2*c*e*f^2 + 2*e*f^2)*x), x))
 - a^2/(f^2*x + e*f)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \operatorname{arcoth}(dx+c)^2 + 2\,ab \operatorname{arcoth}(dx+c) + a^2}{f^2x^2 + 2\,efx + e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^2/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccoth(d*x + c)^2 + 2*a*b*arccoth(d*x + c) + a^2)/(f^2*x^2 +
 2*e*f*x + e^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(d*x+c))**2/(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcoth}(dx+c) + a)^2}{(fx+e)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^2/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(d*x + c) + a)^2/(f*x + e)^2, x)
```

3.114 $\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=546

$$-\frac{b^2\left(\left(3c^2+1\right)f^2-6cdef+3d^2e^2\right)\text{PolyLog}\left(2,1-\frac{2}{-c-dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)}{d^3}+\frac{b^3\left(\left(3c^2+1\right)f^2-6cdef+3d^2\right.}{2d}$$

```
[Out] (a*b^2*f^2*x)/d^2 + (b^3*f^2*(c + d*x)*ArcCoth[c + d*x])/d^3 - (b*f^2*(a +
b*ArcCoth[c + d*x])^2)/(2*d^3) + (3*b*f*(d*e - c*f)*(a + b*ArcCoth[c + d*x]
)^2)/d^3 + (3*b*f*(d*e - c*f)*(c + d*x)*(a + b*ArcCoth[c + d*x])^2)/d^3 + (
b*f^2*(c + d*x)^2*(a + b*ArcCoth[c + d*x])^2)/(2*d^3) - ((d*e - c*f)*(d^2*e
^2 - 2*c*d*e*f + (3 + c^2)*f^2)*(a + b*ArcCoth[c + d*x])^3)/(3*d^3*f) + ((3
*d^2*e^2 - 6*c*d*e*f + (1 + 3*c^2)*f^2)*(a + b*ArcCoth[c + d*x])^3)/(3*d^3)
 + ((e + f*x)^3*(a + b*ArcCoth[c + d*x])^3)/(3*f) - (6*b^2*f*(d*e - c*f)*(a
 + b*ArcCoth[c + d*x])*Log[2/(1 - c - d*x)])/d^3 - (b*(3*d^2*e^2 - 6*c*d*e*
f + (1 + 3*c^2)*f^2)*(a + b*ArcCoth[c + d*x])^2*Log[2/(1 - c - d*x)])/d^3 +
 (b^3*f^2*Log[1 - (c + d*x)^2])/(2*d^3) - (3*b^3*f*(d*e - c*f)*PolyLog[2, -
((1 + c + d*x)/(1 - c - d*x))])/d^3 - (b^2*(3*d^2*e^2 - 6*c*d*e*f + (1 + 3*
c^2)*f^2)*(a + b*ArcCoth[c + d*x])*PolyLog[2, 1 - 2/(1 - c - d*x)])/d^3 + (
b^3*(3*d^2*e^2 - 6*c*d*e*f + (1 + 3*c^2)*f^2)*PolyLog[3, 1 - 2/(1 - c - d*x
)])/(2*d^3)
```

Rubi [A] time = 1.04283, antiderivative size = 546, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {6112, 5929, 5911, 5985, 5919, 2402, 2315, 5917, 5981, 260, 5949, 6049, 6059, 6610}

$$-\frac{b^2\left(\left(3c^2+1\right)f^2-6cdef+3d^2e^2\right)\text{PolyLog}\left(2,1-\frac{2}{-c-dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)}{d^3}+\frac{b^3\left(\left(3c^2+1\right)f^2-6cdef+3d^2\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*(a + b*ArcCoth[c + d*x])^3,x]
```

```
[Out] (a*b^2*f^2*x)/d^2 + (b^3*f^2*(c + d*x)*ArcCoth[c + d*x])/d^3 - (b*f^2*(a +
b*ArcCoth[c + d*x])^2)/(2*d^3) + (3*b*f*(d*e - c*f)*(a + b*ArcCoth[c + d*x]
)^2)/d^3 + (3*b*f*(d*e - c*f)*(c + d*x)*(a + b*ArcCoth[c + d*x])^2)/d^3 + (
b*f^2*(c + d*x)^2*(a + b*ArcCoth[c + d*x])^2)/(2*d^3) - ((d*e - c*f)*(d^2*e
^2 - 2*c*d*e*f + (3 + c^2)*f^2)*(a + b*ArcCoth[c + d*x])^3)/(3*d^3*f) + ((3
*d^2*e^2 - 6*c*d*e*f + (1 + 3*c^2)*f^2)*(a + b*ArcCoth[c + d*x])^3)/(3*d^3)
 + ((e + f*x)^3*(a + b*ArcCoth[c + d*x])^3)/(3*f) - (6*b^2*f*(d*e - c*f)*(a
 + b*ArcCoth[c + d*x])*Log[2/(1 - c - d*x)])/d^3 - (b*(3*d^2*e^2 - 6*c*d*e*
f + (1 + 3*c^2)*f^2)*(a + b*ArcCoth[c + d*x])^2*Log[2/(1 - c - d*x)])/d^3 +
 (b^3*f^2*Log[1 - (c + d*x)^2])/(2*d^3) - (3*b^3*f*(d*e - c*f)*PolyLog[2, -
((1 + c + d*x)/(1 - c - d*x))])/d^3 - (b^2*(3*d^2*e^2 - 6*c*d*e*f + (1 + 3*
c^2)*f^2)*(a + b*ArcCoth[c + d*x])*PolyLog[2, 1 - 2/(1 - c - d*x)])/d^3 + (
b^3*(3*d^2*e^2 - 6*c*d*e*f + (1 + 3*c^2)*f^2)*PolyLog[3, 1 - 2/(1 - c - d*x
)])/(2*d^3)
```

Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
```

```
Q[p, 0]
```

Rule 5929

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcCoth[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcCoth[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5985

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5981

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6049

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCoth[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 6059

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcCoth[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2 \left(a+b\coth^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^3 \left(a+b\coth^{-1}(c+dx)\right)^3}{3f} - \frac{b\,\text{Subst}\left(\int \left(-\frac{3f^2(de-cf)\left(a+b\coth^{-1}(x)\right)^2}{d^3} - \frac{f^3x(\cdots}{\cdots}\right.\right.}{\cdots}$$

$$= \frac{(e+fx)^3 \left(a+b\coth^{-1}(c+dx)\right)^3}{3f} - \frac{b\,\text{Subst}\left(\int \frac{\left((de-cf)\left(d^2e^2-2cdef+3f^2+c^2f^2\right)+f\left(3d^2\cdots\right.\right.}{1\cdots}\right.}{\cdots}$$

$$= \frac{3bf(de-cf)(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2}{d^3} + \frac{bf^2(c+dx)^2\left(a+b\coth^{-1}(c+\cdots\right.}{2d^3}$$

$$= \frac{3bf(de-cf)\left(a+b\coth^{-1}(c+dx)\right)^2}{d^3} + \frac{3bf(de-cf)(c+dx)\left(a+b\coth^{-1}(c+\cdots\right.}{d^3}$$

$$= \frac{ab^2f^2x}{d^2} - \frac{bf^2\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^3} + \frac{3bf(de-cf)\left(a+b\coth^{-1}(c+dx)\right)^2}{d^3}$$

$$= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c+dx)\coth^{-1}(c+dx)}{d^3} - \frac{bf^2\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^3} + \frac{3bf(d\cdots}{\cdots}$$

$$= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c+dx)\coth^{-1}(c+dx)}{d^3} - \frac{bf^2\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^3} + \frac{3bf(d\cdots}{\cdots}$$

$$= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c+dx)\coth^{-1}(c+dx)}{d^3} - \frac{bf^2\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^3} + \frac{3bf(d\cdots}{\cdots}$$

Mathematica [C] time = 10.423, size = 2594, normalized size = 4.75

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)^2*(a + b*ArcCoth[c + d*x])^3,x]
```

```
[Out] (a^2*(a*d^2*e^2 + 3*b*d*e*f - 2*b*c*f^2)*x)/d^2 + (a^2*f*(2*a*d*e + b*f)*x^
2)/(2*d) + (a^3*f^2*x^3)/3 + a^2*b*x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcCoth[c
+ d*x] + ((3*a^2*b*d^2*e^2 - 3*a^2*b*c*d^2*e^2 + 3*a^2*b*d*e*f - 6*a^2*b*c*
d*e*f + 3*a^2*b*c^2*d*e*f + a^2*b*f^2 - 3*a^2*b*c*f^2 + 3*a^2*b*c^2*f^2 - a
^2*b*c^3*f^2)*Log[1 - c - d*x])/(2*d^3) + ((3*a^2*b*d^2*e^2 + 3*a^2*b*c*d^2
*e^2 - 3*a^2*b*d*e*f - 6*a^2*b*c*d*e*f - 3*a^2*b*c^2*d*e*f + a^2*b*f^2 + 3*
a^2*b*c*f^2 + 3*a^2*b*c^2*f^2 + a^2*b*c^3*f^2)*Log[1 + c + d*x])/(2*d^3) +
(3*a*b^2*e^2*(1 - (c + d*x)^2)*(ArcCoth[c + d*x]*(ArcCoth[c + d*x] - (c + d
*x)*ArcCoth[c + d*x] + 2*Log[1 - E^(-2*ArcCoth[c + d*x])]) - PolyLog[2, E^(
-2*ArcCoth[c + d*x])]))/(d*(c + d*x)^2*(1 - (c + d*x)^(-2))) - (3*a*b^2*e*f
*(1 - (c + d*x)^2)*(2*c*ArcCoth[c + d*x]^2 + (c + d*x)^2*(1 - (c + d*x)^(-2
))*ArcCoth[c + d*x]^2 - 2*(c + d*x)*ArcCoth[c + d*x]*(-1 + c*ArcCoth[c + d*
x]) + 4*c*ArcCoth[c + d*x]*Log[1 - E^(-2*ArcCoth[c + d*x])] - 2*Log[1/((c +
 d*x)*Sqrt[1 - (c + d*x)^(-2)])] - 2*c*PolyLog[2, E^(-2*ArcCoth[c + d*x])])
)/(d^2*(c + d*x)^2*(1 - (c + d*x)^(-2))) + (b^3*e^2*(1 - (c + d*x)^2)*((I/8
```

```
)*Pi^3 - ArcCoth[c + d*x]^3 - (c + d*x)*ArcCoth[c + d*x]^3 + 3*ArcCoth[c +
d*x]^2*Log[1 - E^(2*ArcCoth[c + d*x])] + 3*ArcCoth[c + d*x]*PolyLog[2, E^(2
*ArcCoth[c + d*x])] - (3*PolyLog[3, E^(2*ArcCoth[c + d*x])])/2))/(d*(c + d*
x)^2*(1 - (c + d*x)^(-2))) - (b^3*e*f*(1 - (c + d*x)^2)*(I*c*Pi^3 - 12*ArcC
oth[c + d*x]^2 + 12*(c + d*x)*ArcCoth[c + d*x]^2 - 8*c*ArcCoth[c + d*x]^3 -
 8*c*(c + d*x)*ArcCoth[c + d*x]^3 + 4*(c + d*x)^2*(1 - (c + d*x)^(-2))*ArcC
oth[c + d*x]^3 - 24*ArcCoth[c + d*x]*Log[1 - E^(-2*ArcCoth[c + d*x])] + 24*
c*ArcCoth[c + d*x]^2*Log[1 - E^(2*ArcCoth[c + d*x])] + 12*PolyLog[2, E^(-2*
ArcCoth[c + d*x])] + 24*c*ArcCoth[c + d*x]*PolyLog[2, E^(2*ArcCoth[c + d*x]
)] - 12*c*PolyLog[3, E^(2*ArcCoth[c + d*x])]))/(4*d^2*(c + d*x)^2*(1 - (c +
 d*x)^(-2))) - (a*b^2*f^2*(c + d*x)*Sqrt[1 - (c + d*x)^(-2)]*(1 - (c + d*x)
^2)*((4*ArcCoth[c + d*x])/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (3*ArcCoth
[c + d*x]^2)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) - (12*c*ArcCoth[c + d*x]^
2)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (9*c^2*ArcCoth[c + d*x]^2)/((c +
d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (-1 + 6*c*ArcCoth[c + d*x] + 3*ArcCoth[c +
 d*x]^2 - 3*c^2*ArcCoth[c + d*x]^2)/Sqrt[1 - (c + d*x)^(-2)] + Cosh[3*ArcCo
th[c + d*x]] - 6*c*ArcCoth[c + d*x]*Cosh[3*ArcCoth[c + d*x]] + ArcCoth[c +
d*x]^2*Cosh[3*ArcCoth[c + d*x]] + 3*c^2*ArcCoth[c + d*x]^2*Cosh[3*ArcCoth[c
 + d*x]] + (6*ArcCoth[c + d*x]*Log[1 - E^(-2*ArcCoth[c + d*x])])/((c + d*x)
*Sqrt[1 - (c + d*x)^(-2)]) + (18*c^2*ArcCoth[c + d*x]*Log[1 - E^(-2*ArcCoth
[c + d*x])])/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) - (18*c*Log[1/((c + d*x)*
Sqrt[1 - (c + d*x)^(-2)])])/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (4*(1 +
3*c^2)*PolyLog[2, E^(-2*ArcCoth[c + d*x])])/((c + d*x)^3*(1 - (c + d*x)^(-2
))^(3/2)) - ArcCoth[c + d*x]^2*Sinh[3*ArcCoth[c + d*x]] - 3*c^2*ArcCoth[c +
 d*x]^2*Sinh[3*ArcCoth[c + d*x]] - 2*ArcCoth[c + d*x]*Log[1 - E^(-2*ArcCoth
[c + d*x])]*Sinh[3*ArcCoth[c + d*x]] - 6*c^2*ArcCoth[c + d*x]*Log[1 - E^(-2
*ArcCoth[c + d*x])]*Sinh[3*ArcCoth[c + d*x]] + 6*c*Log[1/((c + d*x)*Sqrt[1
- (c + d*x)^(-2)])]*Sinh[3*ArcCoth[c + d*x]]))/(4*d^3) + (b^3*f^2*(1 - (c +
 d*x)^2)*(3*c*PolyLog[2, E^(-2*ArcCoth[c + d*x])] + ((c + d*x)^3*(1 - (c +
d*x)^(-2))^(3/2)*(((-3*I)*Pi^3)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) - ((9*
I)*c^2*Pi^3)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (24*ArcCoth[c + d*x])/S
qrt[1 - (c + d*x)^(-2)] - (72*c*ArcCoth[c + d*x]^2)/Sqrt[1 - (c + d*x)^(-2)
] - (48*ArcCoth[c + d*x]^2)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (216*c*A
rcCoth[c + d*x]^2)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) - (24*ArcCoth[c + d
*x]^3)/Sqrt[1 - (c + d*x)^(-2)] + (24*c^2*ArcCoth[c + d*x]^3)/Sqrt[1 - (c +
 d*x)^(-2)] + (24*ArcCoth[c + d*x]^3)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)])
+ (96*c*ArcCoth[c + d*x]^3)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (72*c^2*
ArcCoth[c + d*x]^3)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) - 24*ArcCoth[c + d
*x]*Cosh[3*ArcCoth[c + d*x]] + 72*c*ArcCoth[c + d*x]^2*Cosh[3*ArcCoth[c + d
*x]] - 8*ArcCoth[c + d*x]^3*Cosh[3*ArcCoth[c + d*x]] - 24*c^2*ArcCoth[c + d
*x]^3*Cosh[3*ArcCoth[c + d*x]] + (432*c*ArcCoth[c + d*x]*Log[1 - E^(-2*ArcC
oth[c + d*x])])/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) - (72*ArcCoth[c + d*x]
^2*Log[1 - E^(2*ArcCoth[c + d*x])])/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) -
(216*c^2*ArcCoth[c + d*x]^2*Log[1 - E^(2*ArcCoth[c + d*x])])/((c + d*x)*Sqr
t[1 - (c + d*x)^(-2)]) - (72*Log[1/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)])])/(
(c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (96*(1 + 3*c^2)*ArcCoth[c + d*x]*Poly
Log[2, E^(2*ArcCoth[c + d*x])])/((c + d*x)^3*(1 - (c + d*x)^(-2))^(3/2)) -
(48*(1 + 3*c^2)*PolyLog[3, E^(2*ArcCoth[c + d*x])])/((c + d*x)^3*(1 - (c +
d*x)^(-2))^(3/2)) + I*Pi^3*Sinh[3*ArcCoth[c + d*x]] + (3*I)*c^2*Pi^3*Sinh[3
*ArcCoth[c + d*x]] - 72*c*ArcCoth[c + d*x]^2*Sinh[3*ArcCoth[c + d*x]] - 8*A
rcCoth[c + d*x]^3*Sinh[3*ArcCoth[c + d*x]] - 24*c^2*ArcCoth[c + d*x]^3*Sinh
[3*ArcCoth[c + d*x]] - 144*c*ArcCoth[c + d*x]*Log[1 - E^(-2*ArcCoth[c + d*x
])]*Sinh[3*ArcCoth[c + d*x]] + 24*ArcCoth[c + d*x]^2*Log[1 - E^(2*ArcCoth[c
 + d*x])]*Sinh[3*ArcCoth[c + d*x]] + 72*c^2*ArcCoth[c + d*x]^2*Log[1 - E^(2
*ArcCoth[c + d*x])]*Sinh[3*ArcCoth[c + d*x]] + 24*Log[1/((c + d*x)*Sqrt[1 -
 (c + d*x)^(-2)])]*Sinh[3*ArcCoth[c + d*x]]))/96))/(d^3*(c + d*x)^2*(1 - (c
 + d*x)^(-2)))
```

Maple [C] time = 3.685, size = 10477, normalized size = 19.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(a+b*arccoth(d*x+c))^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccoth(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*f^2*x^3 + a^3*e*f*x^2 + 3/2*(2*x^2*arccoth(d*x + c) + d*(2*x/d^2 -
(c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3
))*a^2*b*e*f + 1/2*(2*x^3*arccoth(d*x + c) + d*((d*x^2 - 4*c*x)/d^3 + (c^3
+ 3*c^2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3*c - 1)*log(d*x +
 c - 1)/d^4))*a^2*b*f^2 + a^3*e^2*x + 3/2*(2*(d*x + c)*arccoth(d*x + c) + l
og(-(d*x + c)^2 + 1))*a^2*b*e^2/d + 1/24*((b^3*d^3*f^2*x^3 + 3*b^3*d^3*e*f*
x^2 + 3*b^3*d^3*e^2*x + (c^3*f^2 + 3*d^2*e^2 - 3*(d*e*f - f^2)*c^2 - 3*d*e*
f + 3*(d^2*e^2 - 2*d*e*f + f^2)*c + f^2)*b^3)*log(d*x + c + 1)^3 + 3*(2*a*b
^2*d^3*f^2*x^3 + (6*a*b^2*d^3*e*f + b^3*d^2*f^2)*x^2 + 2*(3*a*b^2*d^3*e^2 +
 (3*d^2*e*f - 2*c*d*f^2)*b^3)*x - (b^3*d^3*f^2*x^3 + 3*b^3*d^3*e*f*x^2 + 3*
b^3*d^3*e^2*x + (c^3*f^2 - 3*d^2*e^2 - 3*(d*e*f + f^2)*c^2 - 3*d*e*f + 3*(d
^2*e^2 + 2*d*e*f + f^2)*c - f^2)*b^3)*log(d*x + c - 1))*log(d*x + c + 1)^2)
/d^3 + integrate(-1/8*((b^3*d^3*f^2*x^3 + (2*d^3*e*f + c*d^2*f^2 + d^2*f^2)
*b^3*x^2 + (d^3*e^2 + 2*c*d^2*e*f + 2*d^2*e*f)*b^3*x + (c*d^2*e^2 + d^2*e^2
)*b^3)*log(d*x + c - 1)^3 - 6*(a*b^2*d^3*f^2*x^3 + (2*d^3*e*f + c*d^2*f^2 +
 d^2*f^2)*a*b^2*x^2 + (d^3*e^2 + 2*c*d^2*e*f + 2*d^2*e*f)*a*b^2*x + (c*d^2*
e^2 + d^2*e^2)*a*b^2)*log(d*x + c - 1)^2 + (4*a*b^2*d^3*f^2*x^3 + 2*(6*a*b^
2*d^3*e*f + b^3*d^2*f^2)*x^2 - 3*(b^3*d^3*f^2*x^3 + (2*d^3*e*f + c*d^2*f^2
+ d^2*f^2)*b^3*x^2 + (d^3*e^2 + 2*c*d^2*e*f + 2*d^2*e*f)*b^3*x + (c*d^2*e^2
 + d^2*e^2)*b^3)*log(d*x + c - 1)^2 + 4*(3*a*b^2*d^3*e^2 + (3*d^2*e*f - 2*c
*d*f^2)*b^3)*x + 2*(6*(c*d^2*e^2 + d^2*e^2)*a*b^2 - (c^3*f^2 - 3*d^2*e^2 -
3*(d*e*f + f^2)*c^2 - 3*d*e*f + 3*(d^2*e^2 + 2*d*e*f + f^2)*c - f^2)*b^3 +
(6*a*b^2*d^3*f^2 - b^3*d^3*f^2)*x^3 - 3*(b^3*d^3*e*f - 2*(2*d^3*e*f + c*d^2
*f^2 + d^2*f^2)*a*b^2)*x^2 - 3*(b^3*d^3*e^2 - 2*(d^3*e^2 + 2*c*d^2*e*f + 2*
d^2*e*f)*a*b^2)*x)*log(d*x + c - 1))*log(d*x + c + 1))/(d^3*x + c*d^2 + d^2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(a^3f^2x^2 + 2\,a^3efx + a^3e^2 + \left(b^3f^2x^2 + 2\,b^3efx + b^3e^2\right)\text{arcoth}\,(dx+c)^3 + 3\left(ab^2f^2x^2 + 2\,ab^2efx + ab^2e^2\right)\text{arcot}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccoth(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*f^2*x^2 + 2*a^3*e*f*x + a^3*e^2 + (b^3*f^2*x^2 + 2*b^3*e*f*x +
 b^3*e^2)*arccoth(d*x + c)^3 + 3*(a*b^2*f^2*x^2 + 2*a*b^2*e*f*x + a*b^2*e^2
)*arccoth(d*x + c)^2 + 3*(a^2*b*f^2*x^2 + 2*a^2*b*e*f*x + a^2*b*e^2)*arccot
h(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(a+b*acoth(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)^2 (b \operatorname{arcoth}(dx+c)+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccoth(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*(b*arccoth(d*x + c) + a)^3, x)
```

3.115 $\int (e+fx)\left(a+b\coth^{-1}(c+dx)\right)^3\,dx$

Optimal. Leaf size=326

$$-\frac{3b^2(de-cf)\text{PolyLog}\left(2,1-\frac{2}{-c-dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)}{d^2}+\frac{3b^3(de-cf)\text{PolyLog}\left(3,1-\frac{2}{-c-dx+1}\right)}{2d^2}-\frac{3b^3f\text{PolyLog}}{2d}$$

```
[Out] (3*b*f*(a + b*ArcCoth[c + d*x])^2)/(2*d^2) + (3*b*f*(c + d*x)*(a + b*ArcCot
h[c + d*x])^2)/(2*d^2) + ((d*e - c*f)*(a + b*ArcCoth[c + d*x])^3)/d^2 - ((d
^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)*(a + b*ArcCoth[c + d*x])^3)/(2*d^2*f) +
 ((e + f*x)^2*(a + b*ArcCoth[c + d*x])^3)/(2*f) - (3*b^2*f*(a + b*ArcCoth[c
 + d*x])*Log[2/(1 - c - d*x)])/d^2 - (3*b*(d*e - c*f)*(a + b*ArcCoth[c + d*
x])^2*Log[2/(1 - c - d*x)])/d^2 - (3*b^3*f*PolyLog[2, -((1 + c + d*x)/(1 -
c - d*x))])/(2*d^2) - (3*b^2*(d*e - c*f)*(a + b*ArcCoth[c + d*x])*PolyLog[2
, 1 - 2/(1 - c - d*x)])/d^2 + (3*b^3*(d*e - c*f)*PolyLog[3, 1 - 2/(1 - c -
d*x)])/(2*d^2)
```

Rubi [A] time = 0.720792, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {6112, 5929, 5911, 5985, 5919, 2402, 2315, 6049, 5949, 6059, 6610}

$$-\frac{3b^2(de-cf)\text{PolyLog}\left(2,1-\frac{2}{-c-dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)}{d^2}+\frac{3b^3(de-cf)\text{PolyLog}\left(3,1-\frac{2}{-c-dx+1}\right)}{2d^2}-\frac{3b^3f\text{PolyLog}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*(a + b*ArcCoth[c + d*x])^3,x]
```

```
[Out] (3*b*f*(a + b*ArcCoth[c + d*x])^2)/(2*d^2) + (3*b*f*(c + d*x)*(a + b*ArcCot
h[c + d*x])^2)/(2*d^2) + ((d*e - c*f)*(a + b*ArcCoth[c + d*x])^3)/d^2 + ((2
*c*e - (d*e^2)/f - ((1 + c^2)*f)/d)*(a + b*ArcCoth[c + d*x])^3)/(2*d) + ((e
 + f*x)^2*(a + b*ArcCoth[c + d*x])^3)/(2*f) - (3*b^2*f*(a + b*ArcCoth[c + d
*x])*Log[2/(1 - c - d*x)])/d^2 - (3*b*(d*e - c*f)*(a + b*ArcCoth[c + d*x])^
2*Log[2/(1 - c - d*x)])/d^2 - (3*b^3*f*PolyLog[2, -((1 + c + d*x)/(1 - c -
d*x))])/(2*d^2) - (3*b^2*(d*e - c*f)*(a + b*ArcCoth[c + d*x])*PolyLog[2, 1
- 2/(1 - c - d*x)])/d^2 + (3*b^3*(d*e - c*f)*PolyLog[3, 1 - 2/(1 - c - d*x)
])/(2*d^2)
```

Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rule 5929

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcCoth[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcCoth[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5985

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 6049

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCoth[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6059

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcCoth[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (e+fx)\left(a+b\coth^{-1}(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int\left(\frac{de-cf}{d}+\frac{fx}{d}\right)\left(a+b\coth^{-1}(x)\right)^3\,dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^2\left(a+b\coth^{-1}(c+dx)\right)^3}{2f} - \frac{(3b)\,\text{Subst}\left(\int\left(-\frac{f^2\left(a+b\coth^{-1}(x)\right)^2}{d^2}+\frac{\left(d^2e^2-2cd\right.}{}\right.\right.}{}$$

$$= \frac{(e+fx)^2\left(a+b\coth^{-1}(c+dx)\right)^3}{2f} - \frac{(3b)\,\text{Subst}\left(\int\frac{\left(d^2e^2-2cdef+\left(1+c^2\right)f^2+2f(de-cf)x\right)(}{1-x^2}\right.}{2d^2f}$$

$$= \frac{3bf(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^2} + \frac{(e+fx)^2\left(a+b\coth^{-1}(c+dx)\right)^3}{2f} - \frac{(3b)}{}$$

$$= \frac{3bf\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^2} + \frac{3bf(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^2} + \frac{(e+fx)^2}{}$$

$$= \frac{3bf\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^2} + \frac{3bf(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^2} + \frac{(de-cf)\,(}{}$$

$$= \frac{3bf\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^2} + \frac{3bf(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^2} + \frac{(de-cf)\,(}{}$$

$$= \frac{3bf\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^2} + \frac{3bf(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^2} + \frac{(de-cf)\,(}{}$$

$$= \frac{3bf\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^2} + \frac{3bf(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^2} + \frac{(de-cf)\,(}{}$$

Mathematica [C] time = 1.31426, size = 600, normalized size = 1.84

$$\frac{12ab^2de\left(\text{PolyLog}\left(2,e^{-2\coth^{-1}(c+dx)}\right)+\coth^{-1}(c+dx)\left((c+dx-1)\coth^{-1}(c+dx)-2\log\left(1-e^{-2\coth^{-1}(c+dx)}\right)\right)\right)-12ab}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)*(a + b*ArcCoth[c + d*x])^3,x]
```

```
[Out] (2*a^2*(2*a*d*e + 3*b*f - 2*a*c*f)*(c + d*x) + 2*a^3*f*(c + d*x)^2 - 6*a^2*
b*(c + d*x)*(c*f - d*(2*e + f*x))*ArcCoth[c + d*x] + 3*a^2*b*(2*d*e + f - 2
*c*f)*Log[1 - c - d*x] + 3*a^2*b*(2*d*e - (1 + 2*c)*f)*Log[1 + c + d*x] + 1
2*a*b^2*f*((c + d*x)*ArcCoth[c + d*x] + ((-1 + (c + d*x)^2)*ArcCoth[c + d*x
]^2)/2 - Log[1/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)])]) + 12*a*b^2*d*e*(ArcCo
th[c + d*x]*((-1 + c + d*x)*ArcCoth[c + d*x] - 2*Log[1 - E^(-2*ArcCoth[c +
d*x])]) + PolyLog[2, E^(-2*ArcCoth[c + d*x])]) - 12*a*b^2*c*f*(ArcCoth[c +
d*x]*((-1 + c + d*x)*ArcCoth[c + d*x] - 2*Log[1 - E^(-2*ArcCoth[c + d*x])])
 + PolyLog[2, E^(-2*ArcCoth[c + d*x])]) + 2*b^3*f*(ArcCoth[c + d*x]*(3*(-1
+ c + d*x)*ArcCoth[c + d*x] + (-1 + c^2 + 2*c*d*x + d^2*x^2)*ArcCoth[c + d*
x]^2 - 6*Log[1 - E^(-2*ArcCoth[c + d*x])]) + 3*PolyLog[2, E^(-2*ArcCoth[c +
 d*x])]) + 4*b^3*d*e*((-I/8)*Pi^3 + ArcCoth[c + d*x]^3 + (c + d*x)*ArcCoth[
```

```
c + d*x]^3 - 3*ArcCoth[c + d*x]^2*Log[1 - E^(2*ArcCoth[c + d*x])] - 3*ArcCo
th[c + d*x]*PolyLog[2, E^(2*ArcCoth[c + d*x])] + (3*PolyLog[3, E^(2*ArcCoth
[c + d*x])])/2) - 4*b^3*c*f*((-I/8)*Pi^3 + ArcCoth[c + d*x]^3 + (c + d*x)*A
rcCoth[c + d*x]^3 - 3*ArcCoth[c + d*x]^2*Log[1 - E^(2*ArcCoth[c + d*x])] -
3*ArcCoth[c + d*x]*PolyLog[2, E^(2*ArcCoth[c + d*x])] + (3*PolyLog[3, E^(2*
ArcCoth[c + d*x])])/2))/(4*d^2)
```

Maple [C] time = 1.107, size = 12285, normalized size = 37.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(a+b*arccoth(d*x+c))^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a^3fx^2 + \frac{3}{4}\left(2x^2\operatorname{arcoth}(dx+c) + d\left(\frac{2x}{d^2} - \frac{(c^2+2c+1)\log(dx+c+1)}{d^3} + \frac{(c^2-2c+1)\log(dx+c-1)}{d^3}\right)\right)a^2bf + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccoth(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*f*x^2 + 3/4*(2*x^2*arccoth(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1)*
log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*a^2*b*f + a^3
*e*x + 3/2*(2*(d*x + c)*arccoth(d*x + c) + log(-(d*x + c)^2 + 1))*a^2*b*e/d
 + 1/16*((b^3*d^2*f*x^2 + 2*b^3*d^2*e*x - (c^2*f - 2*(d*e - f)*c - 2*d*e +
f)*b^3)*log(d*x + c + 1)^3 + 3*(2*a*b^2*d^2*f*x^2 + 2*(2*a*b^2*d^2*e + b^3*
d*f)*x - (b^3*d^2*f*x^2 + 2*b^3*d^2*e*x - (c^2*f - 2*(d*e + f)*c + 2*d*e +
f)*b^3)*log(d*x + c - 1))*log(d*x + c + 1)^2)/d^2 + integrate(-1/8*((b^3*d^
2*f*x^2 + (d^2*e + c*d*f + d*f)*b^3*x + (c*d*e + d*e)*b^3)*log(d*x + c - 1)
^3 - 6*(a*b^2*d^2*f*x^2 + (d^2*e + c*d*f + d*f)*a*b^2*x + (c*d*e + d*e)*a*b
^2)*log(d*x + c - 1)^2 + 3*(2*a*b^2*d^2*f*x^2 - (b^3*d^2*f*x^2 + (d^2*e + c
*d*f + d*f)*b^3*x + (c*d*e + d*e)*b^3)*log(d*x + c - 1)^2 + 2*(2*a*b^2*d^2*
e + b^3*d*f)*x + (4*(c*d*e + d*e)*a*b^2 + (c^2*f - 2*(d*e + f)*c + 2*d*e +
f)*b^3 + (4*a*b^2*d^2*f - b^3*d^2*f)*x^2 - 2*(b^3*d^2*e - 2*(d^2*e + c*d*f
+ d*f)*a*b^2)*x)*log(d*x + c - 1))*log(d*x + c + 1))/(d^2*x + c*d + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(a^3fx + a^3e + \left(b^3fx + b^3e\right)\operatorname{arcoth}(dx+c)^3 + 3\left(ab^2fx + ab^2e\right)\operatorname{arcoth}(dx+c)^2 + 3\left(a^2bfx + a^2be\right)\operatorname{arcoth}\cdots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccoth(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*f*x + a^3*e + (b^3*f*x + b^3*e)*arccoth(d*x + c)^3 + 3*(a*b^2*
f*x + a*b^2*e)*arccoth(d*x + c)^2 + 3*(a^2*b*f*x + a^2*b*e)*arccoth(d*x + c
```

```
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*acoth(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)(b\operatorname{arcoth}(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccoth(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(b*arccoth(d*x + c) + a)^3, x)
```

3.116 $\int \left(a+b\coth^{-1}(c+dx)\right)^3\,dx$

Optimal. Leaf size=132

$$-\frac{3b^2\text{PolyLog}\left(2,1-\frac{2}{-c-dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)}{d}+\frac{3b^3\text{PolyLog}\left(3,1-\frac{2}{-c-dx+1}\right)}{2d}+\frac{(c+dx)\left(a+b\coth^{-1}(c+dx\right.}{d}$$

```
[Out] (a + b*ArcCoth[c + d*x])^3/d + ((c + d*x)*(a + b*ArcCoth[c + d*x])^3)/d - (
3*b*(a + b*ArcCoth[c + d*x])^2*Log[2/(1 - c - d*x)])/d - (3*b^2*(a + b*ArcC
oth[c + d*x])*PolyLog[2, 1 - 2/(1 - c - d*x)])/d + (3*b^3*PolyLog[3, 1 - 2/
(1 - c - d*x)])/(2*d)
```

Rubi [A] time = 0.227334, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6104, 5911, 5985, 5919, 5949, 6059, 6610}

$$-\frac{3b^2\text{PolyLog}\left(2,1-\frac{2}{-c-dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)}{d}+\frac{3b^3\text{PolyLog}\left(3,1-\frac{2}{-c-dx+1}\right)}{2d}+\frac{(c+dx)\left(a+b\coth^{-1}(c+dx\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[c + d*x])^3,x]
```

```
[Out] (a + b*ArcCoth[c + d*x])^3/d + ((c + d*x)*(a + b*ArcCoth[c + d*x])^3)/d - (
3*b*(a + b*ArcCoth[c + d*x])^2*Log[2/(1 - c - d*x)])/d - (3*b^2*(a + b*ArcC
oth[c + d*x])*PolyLog[2, 1 - 2/(1 - c - d*x)])/d + (3*b^3*PolyLog[3, 1 - 2/
(1 - c - d*x)])/(2*d)
```

Rule 6104

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}
, x] && IGtQ[p, 0]
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5985

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6059

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcCoth[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \left(a+b\coth^{-1}(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int \left(a+b\coth^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$

$$=\frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^3}{d}-\frac{(3b)\,\text{Subst}\left(\int \frac{x\left(a+b\coth^{-1}(x)\right)^2}{1-x^2}\,dx,x,c+dx\right)}{d}$$

$$=\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{d}+\frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^3}{d}-\frac{(3b)\,\text{Subst}\left(\int \frac{\left(a+b\coth^{-1}(x\right.}{1-x}\right.}{d}$$

$$=\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{d}+\frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^3}{d}-\frac{3b\left(a+b\coth^{-1}(c+dx)\right)^2}{d}$$

$$=\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{d}+\frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^3}{d}-\frac{3b\left(a+b\coth^{-1}(c+dx)\right)^2}{d}$$

$$=\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{d}+\frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^3}{d}-\frac{3b\left(a+b\coth^{-1}(c+dx)\right)^2}{d}$$

Mathematica [C] time = 0.319898, size = 208, normalized size = 1.58

$$6ab^2\left(\text{PolyLog}\left(2,e^{-2\coth^{-1}(c+dx)}\right)+\coth^{-1}(c+dx)\left((c+dx-1)\coth^{-1}(c+dx)-2\log\left(1-e^{-2\coth^{-1}(c+dx)}\right)\right)\right)+2b^3\left(-3\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCoth[c + d*x])^3,x]
```

```
[Out] (2*a^3*(c + d*x) + 6*a^2*b*(c + d*x)*ArcCoth[c + d*x] + 3*a^2*b*Log[1 - (c
+ d*x)^2] + 6*a*b^2*(ArcCoth[c + d*x]*((-1 + c + d*x)*ArcCoth[c + d*x] - 2*
Log[1 - E^(-2*ArcCoth[c + d*x])]) + PolyLog[2, E^(-2*ArcCoth[c + d*x])]) +
2*b^3*((-I/8)*Pi^3 + ArcCoth[c + d*x]^3 + (c + d*x)*ArcCoth[c + d*x]^3 - 3*
ArcCoth[c + d*x]^2*Log[1 - E^(2*ArcCoth[c + d*x])] - 3*ArcCoth[c + d*x]*Pol
yLog[2, E^(2*ArcCoth[c + d*x])] + (3*PolyLog[3, E^(2*ArcCoth[c + d*x])])/2)
```

```
)/(2*d)
```

Maple [B] time = 0.14, size = 485, normalized size = 3.7

$$xa^3 + \frac{a^3c}{d} + (\operatorname{arccoth}(dx+c))^3 xb^3 + \frac{(\operatorname{arccoth}(dx+c))^3 b^3c}{d} - 3\frac{(\operatorname{arccoth}(dx+c))^2 b^3}{d}\ln\left(1+\frac{1}{\sqrt{\frac{dx+c-1}{dx+c+1}}}\right) - 3\,\frac{(\operatorname{arcc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(d*x+c))^3,x)
```

```
[Out] x*a^3+1/d*a^3*c+arccoth(d*x+c)^3*x*b^3+1/d*arccoth(d*x+c)^3*b^3*c-3/d*ln(1+
1/((d*x+c-1)/(d*x+c+1))^(1/2))*arccoth(d*x+c)^2*b^3-3/d*ln(1-1/((d*x+c-1)/(
d*x+c+1))^(1/2))*arccoth(d*x+c)^2*b^3+1/d*b^3*arccoth(d*x+c)^3-6/d*polylog(
2,1/((d*x+c-1)/(d*x+c+1))^(1/2))*arccoth(d*x+c)*b^3-6/d*polylog(2,-1/((d*x+
c-1)/(d*x+c+1))^(1/2))*arccoth(d*x+c)*b^3+6/d*polylog(3,-1/((d*x+c-1)/(d*x+
c+1))^(1/2))*b^3+6/d*polylog(3,1/((d*x+c-1)/(d*x+c+1))^(1/2))*b^3+3*arccoth
(d*x+c)^2*x*a*b^2+3/d*arccoth(d*x+c)^2*a*b^2*c-6/d*ln(1+1/((d*x+c-1)/(d*x+c
+1))^(1/2))*arccoth(d*x+c)*a*b^2-6/d*ln(1-1/((d*x+c-1)/(d*x+c+1))^(1/2))*ar
ccoth(d*x+c)*a*b^2+3/d*a*b^2*arccoth(d*x+c)^2-6/d*polylog(2,1/((d*x+c-1)/(d
*x+c+1))^(1/2))*a*b^2-6/d*polylog(2,-1/((d*x+c-1)/(d*x+c+1))^(1/2))*a*b^2+3
*arccoth(d*x+c)*x*a^2*b+3/d*arccoth(d*x+c)*a^2*b*c+3/2/d*a^2*b*ln((d*x+c)^2
-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^3x + \frac{3\left(2\,(dx+c)\operatorname{arcoth}(dx+c) + \log\left(-(dx+c)^2+1\right)\right)a^2b}{2\,d} + \frac{\left(b^3dx + b^3(c+1)\right)\log(dx+c+1)^3 + 3\left(2\,ab^2dx - (b\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 3/2*(2*(d*x + c)*arccoth(d*x + c) + log(-(d*x + c)^2 + 1))*a^2*b/d
+ 1/8*((b^3*d*x + b^3*(c + 1))*log(d*x + c + 1)^3 + 3*(2*a*b^2*d*x - (b^3*d
*x + b^3*(c - 1))*log(d*x + c - 1))*log(d*x + c + 1)^2)/d + integrate(-1/8*
((b^3*d*x + b^3*(c + 1))*log(d*x + c - 1)^3 - 6*(a*b^2*d*x + a*b^2*(c + 1))
*log(d*x + c - 1)^2 + 3*(4*a*b^2*d*x - (b^3*d*x + b^3*(c + 1))*log(d*x + c
- 1)^2 + 2*(2*a*b^2*(c + 1) - b^3*(c - 1) + (2*a*b^2*d - b^3*d)*x)*log(d*x
+ c - 1))*log(d*x + c + 1))/(d*x + c + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(b^3\operatorname{arcoth}(dx+c)^3 + 3\,ab^2\operatorname{arcoth}(dx+c)^2 + 3\,a^2b\operatorname{arcoth}(dx+c) + a^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arccoth(d*x + c)^3 + 3*a*b^2*arccoth(d*x + c)^2 + 3*a^2*b*arcc
oth(d*x + c) + a^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{acoth}(c + dx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(d*x+c))**3,x)
```

```
[Out] Integral((a + b*acoth(c + d*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arcoth}(dx + c) + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(d*x + c) + a)^3, x)
```

3.117 $\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{e+fx}\,dx$

Optimal. Leaf size=308

$$-\frac{3b^2\left(a+b\coth^{-1}(c+dx)\right)\text{PolyLog}\left(3,1-\frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{2f}+\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{c+dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)}{2f}$$

```
[Out] -(((a + b*ArcCoth[c + d*x])^3*Log[2/(1 + c + d*x)])/f) + ((a + b*ArcCoth[c
+ d*x])^3*Log[(2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/f + (3*b*(a
 + b*ArcCoth[c + d*x])^2*PolyLog[2, 1 - 2/(1 + c + d*x)])/(2*f) - (3*b*(a +
 b*ArcCoth[c + d*x])^2*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 +
 c + d*x))])/(2*f) + (3*b^2*(a + b*ArcCoth[c + d*x])*PolyLog[3, 1 - 2/(1 +
c + d*x)])/(2*f) - (3*b^2*(a + b*ArcCoth[c + d*x])*PolyLog[3, 1 - (2*d*(e +
 f*x))/((d*e + f - c*f)*(1 + c + d*x))])/(2*f) + (3*b^3*PolyLog[4, 1 - 2/(1
 + c + d*x)])/(4*f) - (3*b^3*PolyLog[4, 1 - (2*d*(e + f*x))/((d*e + f - c*f
)*(1 + c + d*x))])/(4*f)
```

Rubi [A] time = 0.187135, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {6112, 5925}

$$-\frac{3b^2\left(a+b\coth^{-1}(c+dx)\right)\text{PolyLog}\left(3,1-\frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{2f}+\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{c+dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[c + d*x])^3/(e + f*x),x]
```

```
[Out] -(((a + b*ArcCoth[c + d*x])^3*Log[2/(1 + c + d*x)])/f) + ((a + b*ArcCoth[c
+ d*x])^3*Log[(2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/f + (3*b*(a
 + b*ArcCoth[c + d*x])^2*PolyLog[2, 1 - 2/(1 + c + d*x)])/(2*f) - (3*b*(a +
 b*ArcCoth[c + d*x])^2*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 +
 c + d*x))])/(2*f) + (3*b^2*(a + b*ArcCoth[c + d*x])*PolyLog[3, 1 - 2/(1 +
c + d*x)])/(2*f) - (3*b^2*(a + b*ArcCoth[c + d*x])*PolyLog[3, 1 - (2*d*(e +
 f*x))/((d*e + f - c*f)*(1 + c + d*x))])/(2*f) + (3*b^3*PolyLog[4, 1 - 2/(1
 + c + d*x)])/(4*f) - (3*b^3*PolyLog[4, 1 - (2*d*(e + f*x))/((d*e + f - c*f
)*(1 + c + d*x))])/(4*f)
```

Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rule 5925

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^3/((d_) + (e_.)*(x_)), x_Symbol] :>
 -Simp[((a + b*ArcCoth[c*x])^3*Log[2/(1 + c*x)])/e, x] + (Simp[((a + b*ArcC
oth[c*x])^3*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Simp[(3*b*(
a + b*ArcCoth[c*x])^2*PolyLog[2, 1 - 2/(1 + c*x)])/(2*e), x] - Simp[(3*b*(a
 + b*ArcCoth[c*x])^2*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])
/(2*e), x] + Simp[(3*b^2*(a + b*ArcCoth[c*x])*PolyLog[3, 1 - 2/(1 + c*x)])/
```

```
(2*e), x] - Simp[(3*b^2*(a + b*ArcCoth[c*x])*PolyLog[3, 1 - (2*c*(d + e*x))
/((c*d + e)*(1 + c*x))])/(2*e), x] + Simp[(3*b^3*PolyLog[4, 1 - 2/(1 + c*x)
])/(4*e), x] - Simp[(3*b^3*PolyLog[4, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c
*x))])/(4*e), x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{e+fx}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\coth^{-1}(x)\right)^3}{\frac{de-cf}{d}+\frac{fx}{d}}\,dx, x, c+dx\right)}{d}$$
$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^3\log\left(\frac{2}{1+c+dx}\right)}{f} + \frac{\left(a+b\coth^{-1}(c+dx)\right)^3\log\left(\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{f}$$

Mathematica [F] time = 22.4348, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCoth[c + d*x])^3/(e + f*x),x]
```

```
[Out] Integrate[(a + b*ArcCoth[c + d*x])^3/(e + f*x), x]
```

Maple [C] time = 0.71, size = 3796, normalized size = 12.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(d*x+c))^3/(f*x+e),x)
```

```
[Out] 3/2*a^2*b/f*dilog(((d*x+c)*f-f)/(c*f-d*e-f))-b^3/(c*f-d*e-f)*arccoth(d*x+c)
^3*ln(1-(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))+3/2*b^3/(c*f-d*e-f)*ar
ccoth(d*x+c)*polylog(3,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))-3*b^3/f
*arccoth(d*x+c)^2*polylog(2,1/((d*x+c-1)/(d*x+c+1))^(1/2))-b^3/f*arccoth(d*
x+c)^3*ln(1-1/((d*x+c-1)/(d*x+c+1))^(1/2))-3*b^3/f*arccoth(d*x+c)^2*polylog
(2,-1/((d*x+c-1)/(d*x+c+1))^(1/2))+6*b^3/f*arccoth(d*x+c)*polylog(3,-1/((d*
x+c-1)/(d*x+c+1))^(1/2))+b^3*ln((d*x+c)*f-c*f+d*e)/f*arccoth(d*x+c)^3+b^3/f
*arccoth(d*x+c)^3*ln((d*x+c+1)/(d*x+c-1)-1)+6*b^3/f*arccoth(d*x+c)*polylog(
3,1/((d*x+c-1)/(d*x+c+1))^(1/2))-b^3/f*arccoth(d*x+c)^3*ln(1+1/((d*x+c-1)/(
d*x+c+1))^(1/2))-b^3/f*arccoth(d*x+c)^3*ln(((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(
d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)-3/2*b^3/(c*f-d*e-f)*arc
coth(d*x+c)^2*polylog(2,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))+3/4*b^
3*c/(c*f-d*e-f)*polylog(4,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))+6*a*
b^2/f*polylog(3,1/((d*x+c-1)/(d*x+c+1))^(1/2))+6*a*b^2/f*polylog(3,-1/((d*x
+c-1)/(d*x+c+1))^(1/2))+3/2*a*b^2/(c*f-d*e-f)*polylog(3,(c*f-d*e-f)*(d*x+c+
1)/(d*x+c-1)/(c*f-d*e+f))-3/2*a^2*b/f*dilog(((d*x+c)*f+f)/(c*f-d*e+f))+3/2*
b^3*c/(c*f-d*e-f)*arccoth(d*x+c)^2*polylog(2,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1
)/(c*f-d*e+f))-6*a*b^2/f*arccoth(d*x+c)*polylog(2,-1/((d*x+c-1)/(d*x+c+1))^
(1/2))-3*a*b^2/f*arccoth(d*x+c)^2*ln(1-1/((d*x+c-1)/(d*x+c+1))^(1/2))-6*a*b
```

```
^2/f*arccoth(d*x+c)*polylog(2,1/((d*x+c-1)/(d*x+c+1))^(1/2))-3*a*b^2/(c*f-d
*e-f)*arccoth(d*x+c)^2*ln(1-(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))-3*
a*b^2/(c*f-d*e-f)*arccoth(d*x+c)*polylog(2,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/
(c*f-d*e+f))+3*a*b^2/f*arccoth(d*x+c)^2*ln((d*x+c+1)/(d*x+c-1)-1)-3*a*b^2/f
*arccoth(d*x+c)^2*ln(((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*
d+(-(d*x+c+1)/(d*x+c-1)-1)*f)+b^3*c/(c*f-d*e-f)*arccoth(d*x+c)^3*ln(1-(c*f-
d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))-3/2*b^3*c/(c*f-d*e-f)*arccoth(d*x+c
)*polylog(3,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))+3*a^2*b*ln((d*x+c)
*f-c*f+d*e)/f*arccoth(d*x+c)-I*b^3/f*Pi*arccoth(d*x+c)^3-3/2*a^2*b/f*ln(((d
*x+c)*f+f)/(c*f-d*e+f))*ln((d*x+c)*f-c*f+d*e)+3/2*a^2*b/f*ln(((d*x+c)*f-f)/
(c*f-d*e-f))*ln((d*x+c)*f-c*f+d*e)+3*a*b^2*ln((d*x+c)*f-c*f+d*e)/f*arccoth(
d*x+c)^2-3*a*b^2/f*arccoth(d*x+c)^2*ln(1+1/((d*x+c-1)/(d*x+c+1))^(1/2))-3/2
*a*b^2*c/(c*f-d*e-f)*polylog(3,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))
-3*d*a*b^2/f*e/(c*f-d*e-f)*arccoth(d*x+c)^2*ln(1-(c*f-d*e-f)*(d*x+c+1)/(d*x
+c-1)/(c*f-d*e+f))-3*d*a*b^2/f*e/(c*f-d*e-f)*arccoth(d*x+c)*polylog(2,(c*f-
d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))-3/2*I*a*b^2/f*Pi*arccoth(d*x+c)^2*c
sgn(I*(((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/
(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^2*csgn(I/((d*x+c+1)/(d*x+c-1)-1))-
3/2*I*a*b^2/f*Pi*arccoth(d*x+c)^2*csgn(I*(((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d
*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f))*csgn(I*(((d*x+c+1)/(d*x
+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+
c+1)/(d*x+c-1)-1))^2+1/2*I*b^3/f*Pi*arccoth(d*x+c)^3*csgn(I*(((d*x+c+1)/(d*
x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f))*csgn
(I*(((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*
x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))*csgn(I/((d*x+c+1)/(d*x+c-1)-1))-6*b^3
/f*polylog(4,1/((d*x+c-1)/(d*x+c+1))^(1/2))+a^3*ln((d*x+c)*f-c*f+d*e)/f-3/4
*b^3/(c*f-d*e-f)*polylog(4,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))-6*b
^3/f*polylog(4,-1/((d*x+c-1)/(d*x+c+1))^(1/2))+3*a*b^2*c/(c*f-d*e-f)*arccot
h(d*x+c)^2*ln(1-(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))+3*a*b^2*c/(c*f
-d*e-f)*arccoth(d*x+c)*polylog(2,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f
))+I*b^3/f*Pi*arccoth(d*x+c)^3*csgn(I*(((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+
c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^2-
3/4*d*b^3/f*e/(c*f-d*e-f)*polylog(4,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*
e+f))-3*I*a*b^2/f*Pi*arccoth(d*x+c)^2-1/2*I*b^3/f*Pi*arccoth(d*x+c)^3*csgn(
I*(((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x
+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^3-3/2*I*a*b^2/f*Pi*arccoth(d*x+c)^2*cs
gn(I*(((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(
d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^3-1/2*I*b^3/f*Pi*arccoth(d*x+c)^3*c
sgn(I*(((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/
(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^2*csgn(I/((d*x+c+1)/(d*x+c-1)-1))+
3/2*d*a*b^2/f*e/(c*f-d*e-f)*polylog(3,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-
d*e+f))-3/2*d*b^3/f*e/(c*f-d*e-f)*arccoth(d*x+c)^2*polylog(2,(c*f-d*e-f)*(d
*x+c+1)/(d*x+c-1)/(c*f-d*e+f))-d*b^3/f*e/(c*f-d*e-f)*arccoth(d*x+c)^3*ln(1-
(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))+3/2*d*b^3/f*e/(c*f-d*e-f)*arcc
oth(d*x+c)*polylog(3,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))-1/2*I*b^3
/f*Pi*arccoth(d*x+c)^3*csgn(I*(((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*
x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f))*csgn(I*(((d*x+c+1)/(d*x+c-1)-1)*c*
f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c
-1)-1))^2+3*I*a*b^2/f*Pi*arccoth(d*x+c)^2*csgn(I*(((d*x+c+1)/(d*x+c-1)-1)*c
*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+
c-1)-1))^2+3/2*I*a*b^2/f*Pi*arccoth(d*x+c)^2*csgn(I*(((d*x+c+1)/(d*x+c-1)-1
)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f))*csgn(I*(((d*
x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1
)*f)/((d*x+c+1)/(d*x+c-1)-1))*csgn(I/((d*x+c+1)/(d*x+c-1)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^3\log\left(fx+e\right)}{f}+\int\frac{b^3\left(\log\left(\frac{1}{dx+c}+1\right)-\log\left(-\frac{1}{dx+c}+1\right)\right)^3}{8\left(fx+e\right)}+\frac{3\,ab^2\left(\log\left(\frac{1}{dx+c}+1\right)-\log\left(-\frac{1}{dx+c}+1\right)\right)^2}{4\left(fx+e\right)}+\frac{3\,a^2b\left(\log\left(\frac{1}{dx+c}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^3/(f*x+e),x, algorithm="maxima")
```

```
[Out] a^3*log(f*x + e)/f + integrate(1/8*b^3*(log(1/(d*x + c) + 1) - log(-1/(d*x
+ c) + 1))^3/(f*x + e) + 3/4*a*b^2*(log(1/(d*x + c) + 1) - log(-1/(d*x + c)
 + 1))^2/(f*x + e) + 3/2*a^2*b*(log(1/(d*x + c) + 1) - log(-1/(d*x + c) + 1
))/(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\,\text{arcoth}\,(dx+c)^3+3\,ab^2\,\text{arcoth}\,(dx+c)^2+3\,a^2b\,\text{arcoth}\,(dx+c)+a^3}{fx+e},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^3/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((b^3*arccoth(d*x + c)^3 + 3*a*b^2*arccoth(d*x + c)^2 + 3*a^2*b*arc
coth(d*x + c) + a^3)/(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(d*x+c))**3/(f*x+e),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\,\text{arcoth}\,(dx+c)+a)^3}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^3/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(d*x + c) + a)^3/(f*x + e), x)
```

3.118 $\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx$

Optimal. Leaf size=1089

result too large to display

```
[Out] -((a + b*ArcCoth[c + d*x])^3/(f*(e + f*x))) + (3*a*b^2*d*ArcCoth[c + d*x]*L
og[2/(1 - c - d*x)])/(f*(d*e + f - c*f)) + (3*b^3*d*ArcCoth[c + d*x]^2*Log[
2/(1 - c - d*x)])/(2*f*(d*e + f - c*f)) - (3*a^2*b*d*Log[1 - c - d*x])/(2*f
*(d*e + f - c*f)) - (3*a*b^2*d*ArcCoth[c + d*x]*Log[2/(1 + c + d*x)])/(f*(d
*e - f - c*f)) + (6*a*b^2*d*ArcCoth[c + d*x]*Log[2/(1 + c + d*x)])/((d*e +
f - c*f)*(d*e - (1 + c)*f)) - (3*b^3*d*ArcCoth[c + d*x]^2*Log[2/(1 + c + d*
x)])/(2*f*(d*e - f - c*f)) + (3*b^3*d*ArcCoth[c + d*x]^2*Log[2/(1 + c + d*x
)])/((d*e + f - c*f)*(d*e - (1 + c)*f)) + (3*a^2*b*d*Log[1 + c + d*x])/(2*f
*(d*e - f - c*f)) + (3*a^2*b*d*Log[e + f*x])/(f^2 - (d*e - c*f)^2) - (6*a*b
^2*d*ArcCoth[c + d*x]*Log[(2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])
/((d*e + f - c*f)*(d*e - (1 + c)*f)) - (3*b^3*d*ArcCoth[c + d*x]^2*Log[(2*d
*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/((d*e + f - c*f)*(d*e - (1 +
c)*f)) + (3*a*b^2*d*PolyLog[2, -((1 + c + d*x)/(1 - c - d*x))])/(2*f*(d*e +
 f - c*f)) + (3*b^3*d*ArcCoth[c + d*x]*PolyLog[2, 1 - 2/(1 - c - d*x)])/(2*
f*(d*e + f - c*f)) + (3*a*b^2*d*PolyLog[2, 1 - 2/(1 + c + d*x)])/(2*f*(d*e
- f - c*f)) - (3*a*b^2*d*PolyLog[2, 1 - 2/(1 + c + d*x)])/((d*e + f - c*f)*
(d*e - (1 + c)*f)) + (3*b^3*d*ArcCoth[c + d*x]*PolyLog[2, 1 - 2/(1 + c + d*
x)])/(2*f*(d*e - f - c*f)) - (3*b^3*d*ArcCoth[c + d*x]*PolyLog[2, 1 - 2/(1
+ c + d*x)])/((d*e + f - c*f)*(d*e - (1 + c)*f)) + (3*a*b^2*d*PolyLog[2, 1
- (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/((d*e + f - c*f)*(d*e -
 (1 + c)*f)) + (3*b^3*d*ArcCoth[c + d*x]*PolyLog[2, 1 - (2*d*(e + f*x))/((d
*e + f - c*f)*(1 + c + d*x))])/((d*e + f - c*f)*(d*e - (1 + c)*f)) - (3*b^3
*d*PolyLog[3, 1 - 2/(1 - c - d*x)])/(4*f*(d*e + f - c*f)) + (3*b^3*d*PolyLo
g[3, 1 - 2/(1 + c + d*x)])/(4*f*(d*e - f - c*f)) - (3*b^3*d*PolyLog[3, 1 -
2/(1 + c + d*x)])/(2*(d*e + f - c*f)*(d*e - (1 + c)*f)) + (3*b^3*d*PolyLog[
3, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/(2*(d*e + f - c*f)
*(d*e - (1 + c)*f))
```

Rubi [A] time = 2.78214, antiderivative size = 1094, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 18, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.9, Rules used = {6110, 6741, 6122, 6688, 12, 6725, 72, 6742, 5919, 2402, 2315, 5921, 2447, 5949, 6059, 6610, 6057, 5923}

$$\frac{3d\coth^{-1}(c+dx)^2\log\left(\frac{2}{-c-dx+1}\right)b^3}{2f(de-cf+f)}-\frac{3d\coth^{-1}(c+dx)^2\log\left(\frac{2}{c+dx+1}\right)b^3}{2f(de-cf-f)}+\frac{3d\coth^{-1}(c+dx)^2\log\left(\frac{2}{c+dx+1}\right)b^3}{(de-cf+f)(de-(c+1)f)}-\frac{3d\,\mathrm{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[c + d*x])^3/(e + f*x)^2,x]
```


```
[Out] -((a + b*ArcCoth[c + d*x])^3/(f*(e + f*x))) + (3*a*b^2*d*ArcCoth[c + d*x]*L
og[2/(1 - c - d*x)])/(f*(d*e + f - c*f)) + (3*b^3*d*ArcCoth[c + d*x]^2*Log[
2/(1 - c - d*x)])/(2*f*(d*e + f - c*f)) - (3*a^2*b*d*Log[1 - c - d*x])/(2*f
*(d*e + f - c*f)) - (3*a*b^2*d*ArcCoth[c + d*x]*Log[2/(1 + c + d*x)])/(f*(d
*e - f - c*f)) + (6*a*b^2*d*ArcCoth[c + d*x]*Log[2/(1 + c + d*x)])/((d*e +
f - c*f)*(d*e - (1 + c)*f)) - (3*b^3*d*ArcCoth[c + d*x]^2*Log[2/(1 + c + d*
x)])/(2*f*(d*e - f - c*f)) + (3*b^3*d*ArcCoth[c + d*x]^2*Log[2/(1 + c + d*x
)])/((d*e + f - c*f)*(d*e - (1 + c)*f)) + (3*a^2*b*d*Log[1 + c + d*x])/(2*f
```

```
*(d*e - f - c*f)) - (3*a^2*b*d*Log[e + f*x])/((d*e + f - c*f)*(d*e - (1 + c
)*f)) - (6*a*b^2*d*ArcCoth[c + d*x]*Log[(2*d*(e + f*x))/((d*e + f - c*f)*(1
 + c + d*x))])/((d*e + f - c*f)*(d*e - (1 + c)*f)) - (3*b^3*d*ArcCoth[c + d
*x]^2*Log[(2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/((d*e + f - c*f
)*(d*e - (1 + c)*f)) + (3*a*b^2*d*PolyLog[2, -((1 + c + d*x)/(1 - c - d*x))
])/(2*f*(d*e + f - c*f)) + (3*b^3*d*ArcCoth[c + d*x]*PolyLog[2, 1 - 2/(1 -
c - d*x)])/(2*f*(d*e + f - c*f)) + (3*a*b^2*d*PolyLog[2, 1 - 2/(1 + c + d*x
)])/(2*f*(d*e - f - c*f)) - (3*a*b^2*d*PolyLog[2, 1 - 2/(1 + c + d*x)])/((d
*e + f - c*f)*(d*e - (1 + c)*f)) + (3*b^3*d*ArcCoth[c + d*x]*PolyLog[2, 1 -
 2/(1 + c + d*x)])/(2*f*(d*e - f - c*f)) - (3*b^3*d*ArcCoth[c + d*x]*PolyLo
g[2, 1 - 2/(1 + c + d*x)])/((d*e + f - c*f)*(d*e - (1 + c)*f)) + (3*a*b^2*d
*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/((d*e + f
 - c*f)*(d*e - (1 + c)*f)) + (3*b^3*d*ArcCoth[c + d*x]*PolyLog[2, 1 - (2*d*
(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/((d*e + f - c*f)*(d*e - (1 + c
)*f)) - (3*b^3*d*PolyLog[3, 1 - 2/(1 - c - d*x)])/(4*f*(d*e + f - c*f)) + (
3*b^3*d*PolyLog[3, 1 - 2/(1 + c + d*x)])/(4*f*(d*e - f - c*f)) - (3*b^3*d*P
olyLog[3, 1 - 2/(1 + c + d*x)])/(2*(d*e + f - c*f)*(d*e - (1 + c)*f)) + (3*
b^3*d*PolyLog[3, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/(2*(
d*e + f - c*f)*(d*e - (1 + c)*f))
```

Rule 6110

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[((e + f*x)^(m + 1)*(a + b*ArcCoth[c + d*x])^p)/(f*(m
 + 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcCot
h[c + d*x])^(p - 1))/(1 - (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6122

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subs
t[Int[((d*e - c*f)/d + (f*x)/d)^m*(-(C/d^2) + (C*x^2)/d^2)^q*(a + b*ArcCoth
[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q},
x] && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5921

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcCoth[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcCoth[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6059

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcCoth[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6057

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcCoth[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 5923

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 -Simp[((a + b*ArcCoth[c*x])^2*Log[2/(1 + c*x)])/e, x] + (Simp[((a + b*ArcC
oth[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Simp[(b*(a
+ b*ArcCoth[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e, x] - Simp[(b*(a + b*ArcCo
th[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Sim
p[(b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e), x] - Simp[(b^2*PolyLog[3, 1 - (2
*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e), x]) /; FreeQ[{a, b, c, d, e},
x] && NeQ[c^2*d^2 - e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx = -\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{(3bd)\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{(e+fx)\left(1-(c+dx)^2\right)}\,dx}{f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{(3bd)\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{(e+fx)\left(1-c^2-2cdx-d^2x^2\right)}\,dx}{f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{(3b)\,\mathrm{Subst}\left(\int \frac{\left(a+b\coth^{-1}(x)\right)^2}{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)(1-x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{(3b)\,\mathrm{Subst}\left(\int \frac{d\left(a+b\coth^{-1}(x)\right)^2}{(de-cf+fx)(1-x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{(3bd)\,\mathrm{Subst}\left(\int \frac{\left(a+b\coth^{-1}(x)\right)^2}{(de-cf+fx)(1-x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{(3bd)\,\mathrm{Subst}\left(\int \left(-\frac{a^2}{(-1+x)(1+x)(de-cf+fx)} - \frac{2ab\coth^{-1}(x)}{(-1+x)(1+x)(de-cf+}\right.\right.}{f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(e+fx)} - \frac{\left(3a^2bd\right)\mathrm{Subst}\left(\int \frac{1}{(-1+x)(1+x)(de-cf+fx)}\,dx, x, c+dx\right)}{f} - \frac{(6}{}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(e+fx)} - \frac{\left(3a^2bd\right)\mathrm{Subst}\left(\int \left(\frac{1}{2(de+f-cf)(-1+x)} + \frac{1}{2(-de+(1+c)f)(1+x)} + \right.\right.}{f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(e+fx)} - \frac{3a^2bd\log(1-c-dx)}{2f(de+f-cf)} + \frac{3a^2bd\log(1+c+dx)}{2f(de-f-cf)} - \frac{3a}{(de+f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{3ab^2d\coth^{-1}(c+dx)\log\left(\frac{2}{1-c-dx}\right)}{f(de+f-cf)} + \frac{3b^3d\coth^{-1}(c+dx}{2f(de+f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{3ab^2d\coth^{-1}(c+dx)\log\left(\frac{2}{1-c-dx}\right)}{f(de+f-cf)} + \frac{3b^3d\coth^{-1}(c+dx}{2f(de+f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{3ab^2d\coth^{-1}(c+dx)\log\left(\frac{2}{1-c-dx}\right)}{f(de+f-cf)} + \frac{3b^3d\coth^{-1}(c+dx}{2f(de+f}$$

<u>Mathematica</u> [C] time = 17.4167, size = 1899, normalized size = 1.74

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCoth[c + d*x])^3/(e + f*x)^2,x]
```

```
[Out] -(a^3/(f*(e + f*x))) - (3*a^2*b*ArcCoth[c + d*x])/(f*(e + f*x)) + (3*a^2*b*
d*Log[1 - c - d*x])/(2*f*(-(d*e) - f + c*f)) - (3*a^2*b*d*Log[1 + c + d*x])
/(2*f*(-(d*e) + f + c*f)) - (3*a^2*b*d*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f -
 f^2 + c^2*f^2) + (3*a*b^2*(1 - (c + d*x)^2)*(f/Sqrt[1 - (c + d*x)^(-2)] +
(d*e - c*f)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]))^2*((E^ArcTanh[f/(-(d*e) +
 c*f)]*ArcCoth[c + d*x]^2)/((-(d*e) + c*f)*Sqrt[1 - f^2/(d*e - c*f)^2]) + A
```

```
rcCoth[c + d*x]^2/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]*(f/Sqrt[1 - (c + d*x)
^(-2)] + (d*e - c*f)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]))) + (f*(I*Pi*ArcC
oth[c + d*x] + 2*ArcCoth[c + d*x]*ArcTanh[f/(d*e - c*f)] - I*Pi*Log[1 + E^(
2*ArcCoth[c + d*x])] + 2*ArcCoth[c + d*x]*Log[1 - E^(-2*(ArcCoth[c + d*x] +
 ArcTanh[f/(d*e - c*f)]))] - 2*ArcTanh[f/(-(d*e) + c*f)]*Log[1 - E^(-2*(Arc
Coth[c + d*x] + ArcTanh[f/(d*e - c*f)]))] + I*Pi*Log[1/Sqrt[1 - (c + d*x)^(
-2)]] + 2*ArcTanh[f/(-(d*e) + c*f)]*Log[I*Sinh[ArcCoth[c + d*x] + ArcTanh[f
/(d*e - c*f)]]] - PolyLog[2, E^(-2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f
)]))]))/(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)))/(d*f*(e + f*x)^2) - (b^3*(
1 - (c + d*x)^2)*(f/Sqrt[1 - (c + d*x)^(-2)] + (d*e - c*f)/((c + d*x)*Sqrt[
1 - (c + d*x)^(-2)]))^2*((d*ArcCoth[c + d*x]^3)/(f*(c + d*x)*Sqrt[1 - (c +
d*x)^(-2)]*(-(f/Sqrt[1 - (c + d*x)^(-2)]) - (d*e)/((c + d*x)*Sqrt[1 - (c +
d*x)^(-2)]) + (c*f)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]))) - (d*(2*d*e*ArcC
oth[c + d*x]^3 - 6*f*ArcCoth[c + d*x]^3 - 2*c*f*ArcCoth[c + d*x]^3 - (4*d*e
*Sqrt[(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)/(d*e - c*f)^2]*ArcCoth[c + d*x
]^3)/E^ArcTanh[f/(d*e - c*f)] + (4*c*f*Sqrt[(d^2*e^2 - 2*c*d*e*f + (-1 + c^
2)*f^2)/(d*e - c*f)^2]*ArcCoth[c + d*x]^3)/E^ArcTanh[f/(d*e - c*f)] + (6*I)
*f*Pi*ArcCoth[c + d*x]*Log[2] - f*ArcCoth[c + d*x]^2*Log[64] - (6*I)*f*Pi*A
rcCoth[c + d*x]*Log[E^(-ArcCoth[c + d*x]) + E^ArcCoth[c + d*x]] + 6*f*ArcCo
th[c + d*x]^2*Log[1 - E^(2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]))] +
12*f*ArcCoth[c + d*x]*ArcTanh[f/(d*e - c*f)]*Log[(I/2)*E^(-ArcCoth[c + d*x]
 - ArcTanh[f/(d*e - c*f)])*(-1 + E^(2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e -
c*f)])))] + 6*f*ArcCoth[c + d*x]^2*Log[-((d*e*(-1 + E^(2*ArcCoth[c + d*x]))
 + (1 + c + E^(2*ArcCoth[c + d*x]) - c*E^(2*ArcCoth[c + d*x]))*f)/E^ArcCoth
[c + d*x])] - 6*f*ArcCoth[c + d*x]^2*Log[(-(d*e*(-1 + E^(2*ArcCoth[c + d*x]
))) + (-1 - E^(2*ArcCoth[c + d*x]) + c*(-1 + E^(2*ArcCoth[c + d*x])))*f)/(d
*e - (1 + c)*f)] + 6*f*ArcCoth[c + d*x]^2*Log[1 - (E^ArcCoth[c + d*x]*Sqrt[
d*e + f - c*f])/Sqrt[d*e - (1 + c)*f]] + 6*f*ArcCoth[c + d*x]^2*Log[1 + (E^
ArcCoth[c + d*x]*Sqrt[d*e + f - c*f])/Sqrt[d*e - (1 + c)*f]] + (6*I)*f*Pi*A
rcCoth[c + d*x]*Log[1/Sqrt[1 - (c + d*x)^(-2)]] - 6*f*ArcCoth[c + d*x]^2*Lo
g[-(f/Sqrt[1 - (c + d*x)^(-2)]) - (d*e)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]
) + (c*f)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)])] - 12*f*ArcCoth[c + d*x]*Arc
Tanh[f/(d*e - c*f)]*Log[I*Sinh[ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]]]
+ 6*f*ArcCoth[c + d*x]*PolyLog[2, E^(2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e -
 c*f)]))] - 6*f*ArcCoth[c + d*x]*PolyLog[2, (E^(2*ArcCoth[c + d*x])*(d*e +
f - c*f))/(d*e - (1 + c)*f)] + 12*f*ArcCoth[c + d*x]*PolyLog[2, -((E^ArcCot
h[c + d*x]*Sqrt[d*e + f - c*f])/Sqrt[d*e - (1 + c)*f])] + 12*f*ArcCoth[c +
d*x]*PolyLog[2, (E^ArcCoth[c + d*x]*Sqrt[d*e + f - c*f])/Sqrt[d*e - (1 + c)
*f]] - 3*f*PolyLog[3, E^(2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]))] +
3*f*PolyLog[3, (E^(2*ArcCoth[c + d*x])*(d*e + f - c*f))/(d*e - (1 + c)*f)]
- 12*f*PolyLog[3, -((E^ArcCoth[c + d*x]*Sqrt[d*e + f - c*f])/Sqrt[d*e - (1
+ c)*f])] - 12*f*PolyLog[3, (E^ArcCoth[c + d*x]*Sqrt[d*e + f - c*f])/Sqrt[d
*e - (1 + c)*f]]))/(2*f*(d*e + f - c*f)*(d*e - (1 + c)*f))))/(d^2*(e + f*x)
^2)
```

Maple [C] time = 0.844, size = 4619, normalized size = 4.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(d*x+c))^3/(f*x+e)^2,x)
```

```
[Out] 3/4*I*d^2*b^3/f/(c*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*e*csgn(I*(d*x+c
+1)/(d*x+c-1))^3+3/4*I*d*b^3/(c*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*cs
gn(I*(d*x+c+1)/(d*x+c-1)/((d*x+c+1)/(d*x+c-1)-1))*csgn(I*(d*x+c+1)/(d*x+c-1
))*csgn(I/((d*x+c+1)/(d*x+c-1)-1))+3/4*I*d^2*b^3/f/(c*f-d*e-f)/(c*f-d*e+f)*
```

```
arccoth(d*x+c)^2*Pi*e*csgn(I*(d*x+c+1)/(d*x+c-1)/((d*x+c+1)/(d*x+c-1)-1))^3
+3/2*I*d*b^3/(c*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*c*csgn(I/((d*x+c-1
)/(d*x+c+1))^(1/2))*csgn(I*(d*x+c+1)/(d*x+c-1))^2-3/2*I*d*b^3/(c*f-d*e-f)/(
c*f-d*e+f)*arccoth(d*x+c)^2*Pi*csgn(I*(((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+
c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f))*csgn(I*(((d*x+c+1)/(d*x+c-
1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1
)/(d*x+c-1)-1))*csgn(I/((d*x+c+1)/(d*x+c-1)-1))+3/4*I*d*b^3/(c*f-d*e-f)/(c*
f-d*e+f)*arccoth(d*x+c)^2*Pi*c*csgn(I*(d*x+c+1)/(d*x+c-1)/((d*x+c+1)/(d*x+c
-1)-1))^2*csgn(I/((d*x+c+1)/(d*x+c-1)-1))+3/4*I*d*b^3/(c*f-d*e-f)/(c*f-d*e+
f)*arccoth(d*x+c)^2*Pi*c*csgn(I*(d*x+c+1)/(d*x+c-1)/((d*x+c+1)/(d*x+c-1)-1)
)^2*csgn(I*(d*x+c+1)/(d*x+c-1))-3/4*I*d*b^3/(c*f-d*e-f)/(c*f-d*e+f)*arccoth
(d*x+c)^2*Pi*c*csgn(I/((d*x+c-1)/(d*x+c+1))^(1/2))^2*csgn(I*(d*x+c+1)/(d*x+
c-1))-d*b^3/(d*f*x+d*e)/f*arccoth(d*x+c)^3-d*a^3/(d*f*x+d*e)/f+3*d^2*b^3/(c
*f-d*e-f)^2/(c*f-d*e+f)*e*arccoth(d*x+c)^2*ln(1-(c*f-d*e-f)*(d*x+c+1)/(d*x+
c-1)/(c*f-d*e+f))+3*d^2*b^3/(c*f-d*e-f)^2/(c*f-d*e+f)*e*arccoth(d*x+c)*poly
log(2,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))+3*I*d*b^3/(c*f-d*e-f)/(c
*f-d*e+f)*arccoth(d*x+c)^2*Pi+3/4*I*d^2*b^3/f/(c*f-d*e-f)/(c*f-d*e+f)*arcco
th(d*x+c)^2*Pi*e*csgn(I/((d*x+c-1)/(d*x+c+1))^(1/2))^2*csgn(I*(d*x+c+1)/(d*
x+c-1))-3/4*I*d^2*b^3/f/(c*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*e*csgn(
I*(d*x+c+1)/(d*x+c-1)/((d*x+c+1)/(d*x+c-1)-1))^2*csgn(I/((d*x+c+1)/(d*x+c-1
)-1))-d*b^3/f*arccoth(d*x+c)^3/(c*f-d*e+f)-3/4*I*d^2*b^3/f/(c*f-d*e-f)/(c*f
-d*e+f)*arccoth(d*x+c)^2*Pi*e*csgn(I*(d*x+c+1)/(d*x+c-1)/((d*x+c+1)/(d*x+c-
1)-1))^2*csgn(I*(d*x+c+1)/(d*x+c-1))-3/2*I*d^2*b^3/f/(c*f-d*e-f)/(c*f-d*e+f
)*arccoth(d*x+c)^2*Pi*e*csgn(I/((d*x+c-1)/(d*x+c+1))^(1/2))*csgn(I*(d*x+c+1
)/(d*x+c-1))^2-3*d*b^3*f/(c*f-d*e-f)^2/(c*f-d*e+f)*c*arccoth(d*x+c)*polylog
(2,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))-3*d*b^3*f/(c*f-d*e-f)^2/(c*
f-d*e+f)*c*arccoth(d*x+c)^2*ln(1-(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f
))-3*I*d*b^3/(c*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*csgn(I*(((d*x+c+1)
/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/(
(d*x+c+1)/(d*x+c-1)-1))^2+3/2*I*d*b^3/(c*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c
)^2*Pi*csgn(I*(((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d
*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^3+3/4*I*d*b^3/(c*f-d*e-f)/
(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*csgn(I*(d*x+c+1)/(d*x+c-1))^3+3/4*I*d*b^3/(
c*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*csgn(I*(d*x+c+1)/(d*x+c-1)/((d*x
+c+1)/(d*x+c-1)-1))^3+3*d*a*b^2/(c*f-d*e-f)/(c*f-d*e+f)*dilog(((d*x+c)*f+f)
/(c*f-d*e+f))-3*d*a*b^2/(c*f-d*e-f)/(c*f-d*e+f)*dilog(((d*x+c)*f-f)/(c*f-d*
e-f))+3/4*d*a*b^2/f/(c*f-d*e-f)*ln(d*x+c-1)^2-3/2*d*a*b^2/f/(c*f-d*e-f)*dil
og(1/2+1/2*d*x+1/2*c)+3/2*d*a*b^2/f/(c*f-d*e+f)*dilog(1/2+1/2*d*x+1/2*c)+3/
4*d*a*b^2/f/(c*f-d*e+f)*ln(d*x+c+1)^2+3*d*a^2*b/f/(2*c*f-2*d*e-2*f)*ln(d*x+
c-1)-3/2*d*b^3/f*arccoth(d*x+c)^2/(c*f-d*e+f)*ln((d*x+c-1)/(d*x+c+1))-3*d*a
^2*b/(d*f*x+d*e)/f*arccoth(d*x+c)-3*d*b^3/f*arccoth(d*x+c)^2/(2*c*f-2*d*e+2
*f)*ln(d*x+c+1)+3*d*b^3/f*arccoth(d*x+c)^2/(2*c*f-2*d*e-2*f)*ln(d*x+c-1)-3*
d*b^3*arccoth(d*x+c)^2/(c*f-d*e-f)/(c*f-d*e+f)*ln((d*x+c)*f-c*f+d*e)+3*d*b^
3*arccoth(d*x+c)^2/(c*f-d*e-f)/(c*f-d*e+f)*ln(((d*x+c+1)/(d*x+c-1)-1)*c*f+(
1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)-3*d*a*b^2/(d*f*x+d*e
)/f*arccoth(d*x+c)^2-3*d*a^2*b/f/(2*c*f-2*d*e+2*f)*ln(d*x+c+1)-3*d*a^2*b/(c
*f-d*e-f)/(c*f-d*e+f)*ln((d*x+c)*f-c*f+d*e)-3/2*d*b^3*f/(c*f-d*e-f)^2/(c*f-
d*e+f)*polylog(3,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))-3*d*b^3/(c*f-
d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*ln(2)-3/2*d^2*b^3/(c*f-d*e-f)^2/(c*f-d*
e+f)*e*polylog(3,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))+3/2*I*d*b^3/(
c*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*csgn(I*(((d*x+c+1)/(d*x+c-1)-1)*
c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x
+c-1)-1))^2*csgn(I/((d*x+c+1)/(d*x+c-1)-1))-3/4*I*d*b^3/(c*f-d*e-f)/(c*f-d*
e+f)*arccoth(d*x+c)^2*Pi*c*csgn(I*(d*x+c+1)/(d*x+c-1))^3+3/2*I*d*b^3/(c*f-d
*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*csgn(I*(((d*x+c+1)/(d*x+c-1)-1)*c*f+(
1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f))*csgn(I*(((d*x+c+1)/
(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((
d*x+c+1)/(d*x+c-1)-1))^2-3/2*I*d*b^3/(c*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)
^2*Pi*csgn(I/((d*x+c-1)/(d*x+c+1))^(1/2))*csgn(I*(d*x+c+1)/(d*x+c-1))^2+3/4
```

```
*I*d*b^3/(c*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*csgn(I/((d*x+c-1)/(d*x
+c+1))^(1/2))^2*csgn(I*(d*x+c+1)/(d*x+c-1))-3/4*I*d*b^3/(c*f-d*e-f)/(c*f-d*
e+f)*arccoth(d*x+c)^2*Pi*csgn(I*(d*x+c+1)/(d*x+c-1)/((d*x+c+1)/(d*x+c-1)-1)
)^2*csgn(I/((d*x+c+1)/(d*x+c-1)-1))-3/4*I*d*b^3/(c*f-d*e-f)/(c*f-d*e+f)*arc
coth(d*x+c)^2*Pi*csgn(I*(d*x+c+1)/(d*x+c-1)/((d*x+c+1)/(d*x+c-1)-1))^2*csgn
(I*(d*x+c+1)/(d*x+c-1))-3/4*I*d*b^3/(c*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^
2*Pi*c*csgn(I*(d*x+c+1)/(d*x+c-1)/((d*x+c+1)/(d*x+c-1)-1))^3-3/4*I*d*b^3/(c
*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*c*csgn(I*(d*x+c+1)/(d*x+c-1)/((d*
x+c+1)/(d*x+c-1)-1))*csgn(I*(d*x+c+1)/(d*x+c-1))*csgn(I/((d*x+c+1)/(d*x+c-1
)-1))+3/4*I*d^2*b^3/f/(c*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*e*csgn(I*
(d*x+c+1)/(d*x+c-1)/((d*x+c+1)/(d*x+c-1)-1))*csgn(I*(d*x+c+1)/(d*x+c-1))*cs
gn(I/((d*x+c+1)/(d*x+c-1)-1))+3/2*d*b^3*f/(c*f-d*e-f)^2/(c*f-d*e+f)*c*polyl
og(3,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))+6*d*a*b^2/f*arccoth(d*x+c
)/(2*c*f-2*d*e-2*f)*ln(d*x+c-1)-6*d*a*b^2/f*arccoth(d*x+c)/(2*c*f-2*d*e+2*f
)*ln(d*x+c+1)-6*d*a*b^2*arccoth(d*x+c)/(c*f-d*e-f)/(c*f-d*e+f)*ln((d*x+c)*f
-c*f+d*e)+3*d*a*b^2/(c*f-d*e-f)/(c*f-d*e+f)*ln(((d*x+c)*f+f)/(c*f-d*e+f))*l
n((d*x+c)*f-c*f+d*e)-3*d*a*b^2/(c*f-d*e-f)/(c*f-d*e+f)*ln(((d*x+c)*f-f)/(c*
f-d*e-f))*ln((d*x+c)*f-c*f+d*e)-3/2*d*a*b^2/f/(c*f-d*e-f)*ln(d*x+c-1)*ln(1/
2+1/2*d*x+1/2*c)-3/2*d*a*b^2/f/(c*f-d*e+f)*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c+
1)+3/2*d*a*b^2/f/(c*f-d*e+f)*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2+1/2*d*x+1/2*c)+3
*d*b^3*f/(c*f-d*e-f)^2/(c*f-d*e+f)*arccoth(d*x+c)^2*ln(1-(c*f-d*e-f)*(d*x+c
+1)/(d*x+c-1)/(c*f-d*e+f))+3*d*b^3*f/(c*f-d*e-f)^2/(c*f-d*e+f)*arccoth(d*x+
c)*polylog(2,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^3/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] 3/2*(d*(log(d*x + c + 1)/(d*e*f - (c + 1)*f^2) - log(d*x + c - 1)/(d*e*f -
(c - 1)*f^2) - 2*log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 - 1)*f^2)) - 2*ar
ccoth(d*x + c)/(f^2*x + e*f))*a^2*b - a^3/(f^2*x + e*f) + 1/8*(((d^2*e*f -
c*d*f^2 + d*f^2)*b^3*x + (c*d*e*f - c^2*f^2 + d*e*f + f^2)*b^3)*log(d*x + c
 + 1)^3 - 3*(2*(d^2*e^2 - 2*c*d*e*f + c^2*f^2 - f^2)*a*b^2 + ((d^2*e*f - c*
d*f^2 - d*f^2)*b^3*x + (c*d*e*f - c^2*f^2 - d*e*f + f^2)*b^3)*log(d*x + c -
 1))*log(d*x + c + 1)^2)/(d^2*e^3*f - 2*c*d*e^2*f^2 + c^2*e*f^3 - e*f^3 + (
d^2*e^2*f^2 - 2*c*d*e*f^3 + c^2*f^4 - f^4)*x) + integrate(-1/8*(((d^2*e*f -
 c*d*f^2 + d*f^2)*b^3*x + (c*d*e*f - c^2*f^2 + d*e*f + f^2)*b^3)*log(d*x +
c - 1)^3 - 6*((d^2*e*f - c*d*f^2 + d*f^2)*a*b^2*x + (c*d*e*f - c^2*f^2 + d*
e*f + f^2)*a*b^2)*log(d*x + c - 1)^2 - 3*(4*(d^2*e*f - c*d*f^2 + d*f^2)*a*b
^2*x + 4*(d^2*e^2 - c*d*e*f + d*e*f)*a*b^2 + ((d^2*e*f - c*d*f^2 + d*f^2)*b
^3*x + (c*d*e*f - c^2*f^2 + d*e*f + f^2)*b^3)*log(d*x + c - 1)^2 + 2*(b^3*d
^2*f^2*x^2 - 2*(c*d*e*f - c^2*f^2 + d*e*f + f^2)*a*b^2 + (c*d*e*f - d*e*f)*
b^3 - (2*(d^2*e*f - c*d*f^2 + d*f^2)*a*b^2 - (d^2*e*f + c*d*f^2 - d*f^2)*b^
3)*x)*log(d*x + c - 1))*log(d*x + c + 1))/(c*d*e^3*f - c^2*e^2*f^2 + d*e^3*
f + e^2*f^2 + (d^2*e*f^3 - c*d*f^4 + d*f^4)*x^3 + (2*d^2*e^2*f^2 - c*d*e*f^
3 - c^2*f^4 + 3*d*e*f^3 + f^4)*x^2 + (d^2*e^3*f + c*d*e^2*f^2 - 2*c^2*e*f^3
 + 3*d*e^2*f^2 + 2*e*f^3)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\,\text{arcoth}\left(dx+c\right)^3+3\,ab^2\,\text{arcoth}\left(dx+c\right)^2+3\,a^2b\,\text{arcoth}\left(dx+c\right)+a^3}{f^2x^2+2\,efx+e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^3/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccoth(d*x + c)^3 + 3*a*b^2*arccoth(d*x + c)^2 + 3*a^2*b*arc
coth(d*x + c) + a^3)/(f^2*x^2 + 2*e*f*x + e^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(d*x+c))**3/(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arcoth}(dx+c)+a)^3}{(fx+e)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^3/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(d*x + c) + a)^3/(f*x + e)^2, x)
```

3.119 $\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right) dx$

Optimal. Leaf size=162

$$\frac{bd(e+fx)^{m+2}\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{d(e+fx)}{-cf+de-f}\right)}{2f(m+1)(m+2)(de-(c+1)f)}-\frac{bd(e+fx)^{m+2}\text{Hypergeometric2F1}\left(1,m+2,m+3,\right.}{2f(m+1)(m+2)(-cf+de+f)}$$

```
[Out] ((e + f*x)^(1 + m)*(a + b*ArcCoth[c + d*x]))/(f*(1 + m)) + (b*d*(e + f*x)^(
2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e - f - c*f)])/(
2*f*(d*e - (1 + c)*f)*(1 + m)*(2 + m)) - (b*d*(e + f*x)^(2 + m)*Hypergeomet
ric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e + f - c*f)])/(2*f*(d*e + f - c*f
)*(1 + m)*(2 + m))
```

Rubi [A] time = 0.246004, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6112, 5927, 712, 68}

$$\frac{(e+fx)^{m+1}\left(a+b\coth^{-1}(c+dx)\right)}{f(m+1)}+\frac{bd(e+fx)^{m+2}\,_2F_1\left(1,m+2;m+3;\frac{d(e+fx)}{de-cf-f}\right)}{2f(m+1)(m+2)(de-(c+1)f)}-\frac{bd(e+fx)^{m+2}\,_2F_1\left(1,m+2;m+3;\right.}{2f(m+1)(m+2)(-cf+de+}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^m*(a + b*ArcCoth[c + d*x]),x]
```

```
[Out] ((e + f*x)^(1 + m)*(a + b*ArcCoth[c + d*x]))/(f*(1 + m)) + (b*d*(e + f*x)^(
2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e - f - c*f)])/(
2*f*(d*e - (1 + c)*f)*(1 + m)*(2 + m)) - (b*d*(e + f*x)^(2 + m)*Hypergeomet
ric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e + f - c*f)])/(2*f*(d*e + f - c*f
)*(1 + m)*(2 + m))
```

Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rule 5927

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcCoth[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 712

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
```

```
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^m \left(a+b\coth^{-1}(x)\right)\,dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^{1+m}\left(a+b\coth^{-1}(c+dx)\right)}{f(1+m)} - \frac{b\,\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+m}}{1-x^2}\,dx, x, c+dx\right)}{f(1+m)}$$

$$= \frac{(e+fx)^{1+m}\left(a+b\coth^{-1}(c+dx)\right)}{f(1+m)} - \frac{b\,\text{Subst}\left(\int \left(\frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+m}}{2(1-x)} + \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1}}{2(1+x)} \right.\right.}{f(1+m)}$$

$$= \frac{(e+fx)^{1+m}\left(a+b\coth^{-1}(c+dx)\right)}{f(1+m)} - \frac{b\,\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+m}}{1-x}\,dx, x, c+dx\right)}{2f(1+m)}$$

$$= \frac{(e+fx)^{1+m}\left(a+b\coth^{-1}(c+dx)\right)}{f(1+m)} + \frac{bd(e+fx)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{d(}{de-}\right.}{2f(de-(1+c)f)(1+m)(2+m)}$$

Mathematica [F] time = 2.35383, size = 0, normalized size = 0.

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e + f*x)^m*(a + b*ArcCoth[c + d*x]),x]

[Out] Integrate[(e + f*x)^m*(a + b*ArcCoth[c + d*x]), x]
```

Maple [F] time = 1.493, size = 0, normalized size = 0.

$$\int \left(fx+e\right)^m \left(a+b\,\text{arccoth}\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*(a+b*arccoth(d*x+c)),x)

[Out] int((f*x+e)^m*(a+b*arccoth(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arccoth(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \operatorname{arcoth}(dx + c) + a)(fx + e)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arccoth(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*arccoth(d*x + c) + a)*(f*x + e)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*(a+b*acoth(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arcoth}(dx + c) + a)(fx + e)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arccoth(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(d*x + c) + a)*(f*x + e)^m, x)
```

3.120 $\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^2 dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left((e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^2,x\right)$$

```
[Out] Unintegrable[(e + f*x)^m*(a + b*ArcCoth[c + d*x])^2, x]
```

Rubi [A] time = 0.0665056, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Int[(e + f*x)^m*(a + b*ArcCoth[c + d*x])^2,x]
```

```
[Out] Defer[Subst][Defer[Int][((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcCoth[x])^2, x
], x, c + d*x]/d
```

Rubi steps

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^m \left(a+b\coth^{-1}(x)\right)^2 dx,x,c+dx\right)}{d}$$

Mathematica [A] time = 2.51712, size = 0, normalized size = 0.

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e + f*x)^m*(a + b*ArcCoth[c + d*x])^2,x]
```

```
[Out] Integrate[(e + f*x)^m*(a + b*ArcCoth[c + d*x])^2, x]
```

Maple [A] time = 1.368, size = 0, normalized size = 0.

$$\int \left(fx+e\right)^m (a+b\,\mathrm{arccoth}\,(dx+c))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*(a+b*arccoth(d*x+c))^2,x)
```

```
[Out] int((f*x+e)^m*(a+b*arccoth(d*x+c))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arccoth(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2 \operatorname{arcoth}(dx+c)^2 + 2\,ab \operatorname{arcoth}(dx+c) + a^2\right)\left(fx+e\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arccoth(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccoth(d*x + c)^2 + 2*a*b*arccoth(d*x + c) + a^2)*(f*x + e)^
m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*(a+b*acoth(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arcoth}(dx+c)+a)^2 \left(fx+e\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arccoth(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(d*x + c) + a)^2*(f*x + e)^m, x)
```

3.121 $\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left((e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^3, x\right)$$

```
[Out] Unintegrable[(e + f*x)^m*(a + b*ArcCoth[c + d*x])^3, x]
```

Rubi [A] time = 0.0654614, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(e + f*x)^m*(a + b*ArcCoth[c + d*x])^3,x]
```

```
[Out] Defer[Subst][Defer[Int][((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcCoth[x])^3, x
], x, c + d*x]/d
```

Rubi steps

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^3 \, dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^m \left(a+b\coth^{-1}(x)\right)^3 \, dx, x, c+dx\right)}{d}$$

Mathematica [A] time = 0.342944, size = 0, normalized size = 0.

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e + f*x)^m*(a + b*ArcCoth[c + d*x])^3,x]
```

```
[Out] Integrate[(e + f*x)^m*(a + b*ArcCoth[c + d*x])^3, x]
```

Maple [A] time = 1.286, size = 0, normalized size = 0.

$$\int \left(fx+e\right)^m \left(a+b\,\mathrm{arccoth}\left(dx+c\right)\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*(a+b*arccoth(d*x+c))^3,x)
```

```
[Out] int((f*x+e)^m*(a+b*arccoth(d*x+c))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arccoth(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^3\,\mathrm{arcoth}\left(dx+c\right)^3+3\,ab^2\,\mathrm{arcoth}\left(dx+c\right)^2+3\,a^2b\,\mathrm{arcoth}\left(dx+c\right)+a^3\right)\left(fx+e\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arccoth(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccoth(d*x + c)^3 + 3*a*b^2*arccoth(d*x + c)^2 + 3*a^2*b*arc
coth(d*x + c) + a^3)*(f*x + e)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*(a+b*acoth(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\,\mathrm{arcoth}\left(dx+c\right)+a\right)^3\left(fx+e\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arccoth(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(d*x + c) + a)^3*(f*x + e)^m, x)
```

3.122 $\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

Optimal. Leaf size=42

$$\text{Unintegrable}\left(\frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^n}{1-c^2x^2},x\right)$$

```
[Out] Unintegrable[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),
x]
```

Rubi [A] time = 0.047806, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

```
[Out] Defer[Int][(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Rubi steps

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Mathematica [A] time = 0.0983633, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

```
[Out] Integrate[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Maple [A] time = 1.043, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2+1}\left(a+b\,\mathrm{arccoth}\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

```
[Out] int((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(b \operatorname{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] -integrate((b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\left(b \operatorname{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="fricas")
```

```
[Out] integral(-(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a+b \operatorname{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^n}{c^2x^2-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
```

```
[Out] -Integral((a + b*acoth(sqrt(-c*x + 1)/sqrt(c*x + 1)))**n/(c**2*x**2 - 1), x
)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(b \operatorname{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] integrate(-(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

3.123 $\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

Optimal. Leaf size=460

$$-\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}+\frac{3b^2\text{PolyLog}\left(3,1-\frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1\right)}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}-\frac{3b}{}$$

```
[Out] (-2*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*ArcCoth[1 - 2/(1 - Sqrt[
1 - c*x]/Sqrt[1 + c*x])])/c - (3*b*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])^2*PolyLog[2, 1 - 2/(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x])])/(2*c) + (3*b*(a
 + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2, 1 - (2*Sqrt[1 - c*x
])/(Sqrt[1 + c*x]*(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x]))])/(2*c) - (3*b^2*(a +
b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, 1 - 2/(1 + Sqrt[1 - c*x]
/Sqrt[1 + c*x])])/(2*c) + (3*b^2*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]
])*PolyLog[3, 1 - (2*Sqrt[1 - c*x])/(Sqrt[1 + c*x]*(1 + Sqrt[1 - c*x]/Sqrt[
1 + c*x]))])/(2*c) - (3*b^3*PolyLog[4, 1 - 2/(1 + Sqrt[1 - c*x]/Sqrt[1 + c*
x])])/(4*c) + (3*b^3*PolyLog[4, 1 - (2*Sqrt[1 - c*x])/(Sqrt[1 + c*x]*(1 + S
qrt[1 - c*x]/Sqrt[1 + c*x]))])/(4*c)
```

Rubi [A] time = 0.598234, antiderivative size = 460, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {6681, 5915, 6053, 5949, 6057, 6061, 6610}

$$-\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}+\frac{3b^2\text{PolyLog}\left(3,1-\frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1\right)}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}-\frac{3b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```


```
[Out] (-2*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*ArcCoth[1 - 2/(1 - Sqrt[
1 - c*x]/Sqrt[1 + c*x])])/c - (3*b*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])^2*PolyLog[2, 1 - 2/(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x])])/(2*c) + (3*b*(a
 + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2, 1 - (2*Sqrt[1 - c*x
])/(Sqrt[1 + c*x]*(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x]))])/(2*c) - (3*b^2*(a +
b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, 1 - 2/(1 + Sqrt[1 - c*x]
/Sqrt[1 + c*x])])/(2*c) + (3*b^2*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]
])*PolyLog[3, 1 - (2*Sqrt[1 - c*x])/(Sqrt[1 + c*x]*(1 + Sqrt[1 - c*x]/Sqrt[
1 + c*x]))])/(2*c) - (3*b^3*PolyLog[4, 1 - 2/(1 + Sqrt[1 - c*x]/Sqrt[1 + c*
x])])/(4*c) + (3*b^3*PolyLog[4, 1 - (2*Sqrt[1 - c*x])/(Sqrt[1 + c*x]*(1 + S
qrt[1 - c*x]/Sqrt[1 + c*x]))])/(4*c)
```


Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rule 5915

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcCoth[c*x])^p*ArcCoth[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcCoth[c*x])^(p - 1)*ArcCoth[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6053

```
Int[(ArcCoth[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[SimplifyIntegrand[1 + 1/u, x]]*(a +
 b*ArcCoth[c*x])^p)/(d + e*x^2), x], x] - Dist[1/2, Int[(Log[SimplifyIntegr
and[1 - 1/u, x]]*(a + b*ArcCoth[c*x])^p)/(d + e*x^2), x], x] /; FreeQ[{a, b
, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 -
c*x))^2, 0]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6057

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcCoth[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6061

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcCoth[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a+b\coth^{-1}(x))^3}{x}dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{2\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(6b)\,\text{Subst}\left(\int \frac{\coth^{-1}\left(1-\frac{2}{1-x}\right)(a+b\coth^{-1}}{1-x^2}\right.}{c}$$

$$= -\frac{2\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{(3b)\,\text{Subst}\left(\int \frac{(a+b\coth^{-1}(x))^2\log\left(\frac{2}{1+x}\right)}{1-x^2}\right.}{c}$$

$$= -\frac{2\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{3b\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\text{Li}_2\left(1-\right.}{2c}$$

$$= -\frac{2\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{3b\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\text{Li}_2\left(1-\right.}{2c}$$

$$= -\frac{2\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{3b\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\text{Li}_2\left(1-\right.}{2c}$$

Mathematica [F] time = 0.283333, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] Integrate[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2), x]
```

Maple [B] time = 2.439, size = 1492, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x)
```

```
[Out] -3*a*b^2/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1/((-c*x+1)^(1/2)/(c*
x+1)^(1/2)-1)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)+1)-3*a*b^2/c*arccoth((-c*x+1
)^(1/2)/(c*x+1)^(1/2))*polylog(2,-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*((-c*x
+1)^(1/2)/(c*x+1)^(1/2)+1))+3*a*b^2/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))
^2*ln(1-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1
))^(1/2))+6*a*b^2/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,1/(((-c
*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))+3*a*b
^2/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+1/(((-c*x+1)^(1/2)/(c*x+1
)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))+6*a*b^2/c*arccoth((-c*x
```

```
+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-
c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))-1/2*a^3/c*ln(c*x-1)+1/2*a^3/c*ln(c*x+
1)+6*b^3/c*polylog(4,-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(
c*x+1)^(1/2)+1))^(1/2))+6*b^3/c*polylog(4,1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-
1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))-3/4*b^3/c*polylog(4,-1/((-c*x+1
)^(1/2)/(c*x+1)^(1/2)-1)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))+b^3/c*arccoth((-
c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1+1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c
*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))+3*b^3/c*arccoth((-c*x+1)^(1/2)/(c*x+1)
^(1/2))^2*polylog(2,-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c
*x+1)^(1/2)+1))^(1/2))-6*b^3/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylo
g(3,-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^
(1/2))+b^3/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1-1/(((-c*x+1)^(1/2
)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))+3*b^3/c*arccoth
((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)
-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))-6*b^3/c*arccoth((-c*x+1)^(1/2)
/(c*x+1)^(1/2))*polylog(3,1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/
2)/(c*x+1)^(1/2)+1))^(1/2))-b^3/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*l
n(1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)+1)-3/
2*b^3/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,-1/((-c*x+1)^(1/2
)/(c*x+1)^(1/2)-1)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))+3/2*b^3/c*arccoth((-c*
x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*((-
c*x+1)^(1/2)/(c*x+1)^(1/2)+1))-6*a*b^2/c*polylog(3,-1/(((-c*x+1)^(1/2)/(c*x
+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))+3/2*a*b^2/c*polylog(3
,-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))-6*a*
b^2/c*polylog(3,1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)
^(1/2)+1))^(1/2))+3*a^2*b/c*dilog(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1
)^(1/2)/(c*x+1)^(1/2)+1))-3/4*a^2*b/c*dilog(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1
)^2/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a^3\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)-\frac{\left(b^3\log(cx+1)-b^3\log(-cx+1)\right)\log\left(-\sqrt{cx+1}+\sqrt{-cx+1}\right)^3}{16c}-\int\frac{4\left(\sqrt{cx+1}b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] 1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) - 1/16*(b^3*log(c*x + 1) - b^3*lo
g(-c*x + 1))*log(-sqrt(c*x + 1) + sqrt(-c*x + 1))^3/c - integrate(1/32*(4*(
sqrt(c*x + 1)*b^3 - sqrt(-c*x + 1)*b^3)*log(sqrt(c*x + 1) + sqrt(-c*x + 1))
^3 + 24*(sqrt(c*x + 1)*a*b^2 - sqrt(-c*x + 1)*a*b^2)*log(sqrt(c*x + 1) + sq
rt(-c*x + 1))^2 + 3*(4*(sqrt(c*x + 1)*b^3 - sqrt(-c*x + 1)*b^3)*log(sqrt(c*
x + 1) + sqrt(-c*x + 1)) + (8*a*b^2 - (b^3*c*x - b^3)*log(c*x + 1) + (b^3*c
*x - b^3)*log(-c*x + 1))*sqrt(c*x + 1) - (8*a*b^2 - (b^3*c*x + b^3)*log(c*x
 + 1) + (b^3*c*x + b^3)*log(-c*x + 1))*sqrt(-c*x + 1))*log(-sqrt(c*x + 1) +
 sqrt(-c*x + 1))^2 + 48*(sqrt(c*x + 1)*a^2*b - sqrt(-c*x + 1)*a^2*b)*log(sq
rt(c*x + 1) + sqrt(-c*x + 1)) - 12*(4*sqrt(c*x + 1)*a^2*b - 4*sqrt(-c*x + 1
)*a^2*b + (sqrt(c*x + 1)*b^3 - sqrt(-c*x + 1)*b^3)*log(sqrt(c*x + 1) + sqrt
(-c*x + 1))^2 + 4*(sqrt(c*x + 1)*a*b^2 - sqrt(-c*x + 1)*a*b^2)*log(sqrt(c*x
 + 1) + sqrt(-c*x + 1)))*log(-sqrt(c*x + 1) + sqrt(-c*x + 1)))/((c^2*x^2 -
1)*sqrt(c*x + 1) - (c^2*x^2 - 1)*sqrt(-c*x + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b^3\,\text{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3+3\,ab^2\,\text{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2+3\,a^2b\,\text{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a^3}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="fricas")
```

```
[Out] integral(-(b^3*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b^2*arccoth(sq
rt(-c*x + 1)/sqrt(c*x + 1))^2 + 3*a^2*b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1
)) + a^3)/(c^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{a^3}{c^2x^2-1}\,dx-\int\frac{b^3\,\text{acoth}^3\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx-\int\frac{3ab^2\,\text{acoth}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx-\int\frac{3a^2b\,\text{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a**3/(c**2*x**2 - 1), x) - Integral(b**3*acoth(sqrt(-c*x + 1)/sqr
t(c*x + 1))**3/(c**2*x**2 - 1), x) - Integral(3*a*b**2*acoth(sqrt(-c*x + 1)
/sqrt(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(3*a**2*b*acoth(sqrt(-c*x
+ 1)/sqrt(c*x + 1))/(c**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{\left(b\,\text{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] integrate(-(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^3/(c^2*x^2 - 1), x
)
```

3.124 $\int \frac{\left(a+b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

Optimal. Leaf size=302

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}+\frac{b\text{PolyLog}\left(2,1-\frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1\right)}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}-\frac{b^2\text{I}}{}$$

```
[Out] (-2*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*ArcCoth[1 - 2/(1 - Sqrt[
1 - c*x]/Sqrt[1 + c*x])])/c - (b*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]
])*PolyLog[2, 1 - 2/(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x])])/c + (b*(a + b*ArcCo
th[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, 1 - (2*Sqrt[1 - c*x])/(Sqrt[1 +
 c*x]*(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x]))])/c - (b^2*PolyLog[3, 1 - 2/(1 + S
qrt[1 - c*x]/Sqrt[1 + c*x])])/(2*c) + (b^2*PolyLog[3, 1 - (2*Sqrt[1 - c*x])
/(Sqrt[1 + c*x]*(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x]))])/(2*c)
```

Rubi [A] time = 0.346365, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6681, 5915, 6053, 5949, 6057, 6610}

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}+\frac{b\text{PolyLog}\left(2,1-\frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1\right)}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}-\frac{b^2\text{I}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```



```
[Out] (-2*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*ArcCoth[1 - 2/(1 - Sqrt[
1 - c*x]/Sqrt[1 + c*x])])/c - (b*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]
])*PolyLog[2, 1 - 2/(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x])])/c + (b*(a + b*ArcCo
th[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, 1 - (2*Sqrt[1 - c*x])/(Sqrt[1 +
 c*x]*(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x]))])/c - (b^2*PolyLog[3, 1 - 2/(1 + S
qrt[1 - c*x]/Sqrt[1 + c*x])])/(2*c) + (b^2*PolyLog[3, 1 - (2*Sqrt[1 - c*x])
/(Sqrt[1 + c*x]*(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x]))])/(2*c)
```


Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rule 5915

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcCoth[c*x])^p*ArcCoth[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcCoth[c*x])^(p - 1)*ArcCoth[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6053

```
Int[(ArcCoth[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[SimplifyIntegrand[1 + 1/u, x]]*(a +
 b*ArcCoth[c*x])^p)/(d + e*x^2), x], x] - Dist[1/2, Int[(Log[SimplifyIntegr
and[1 - 1/u, x]]*(a + b*ArcCoth[c*x])^p)/(d + e*x^2), x], x] /; FreeQ[{a, b
, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 -
c*x))^2, 0]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6057

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcCoth[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(a+b\coth^{-1}(x)\right)^2}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{2\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(4b)\,\text{Subst}\left(\int \frac{\coth^{-1}\left(1-\frac{2}{1-x}\right)\left(a+b\coth^{-1}\right.}{1-x^2}\right.}{c}$$

$$= -\frac{2\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{(2b)\,\text{Subst}\left(\int \frac{\left(a+b\coth^{-1}(x)\right)\log\left(\frac{2}{1+x}\right)}{1-x^2}\right.}{c}$$

$$= -\frac{2\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{b\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{Li}_2\left(1-\frac{}{1+}\right.}{c}$$

$$= -\frac{2\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{b\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{Li}_2\left(1-\frac{}{1+}\right.}{c}$$

Mathematica [F] time = 0.510229, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] Integrate[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2), x]
```

Maple [B] time = 1.202, size = 696, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x)
```

```
[Out] -1/2*a^2/c*ln(c*x-1)+1/2*a^2/c*ln(c*x+1)+b^2/c*arccoth((-c*x+1)^(1/2)/(c*x+
1)^(1/2))^2*ln(1+1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1
)^(1/2)+1))^(1/2))+2*b^2/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,
-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2
))-2*b^2/c*polylog(3,-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(
c*x+1)^(1/2)+1))^(1/2))-b^2/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1/
((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)+1)-b^2/c*
arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-1/((-c*x+1)^(1/2)/(c*x+1)^
(1/2)-1)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))+1/2*b^2/c*polylog(3,-1/((-c*x+1)
^(1/2)/(c*x+1)^(1/2)-1)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))+b^2/c*arccoth((-c
*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*
x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))+2*b^2/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^
(1/2))*polylog(2,1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1
)^(1/2)+1))^(1/2))-2*b^2/c*polylog(3,1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((
-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))+2*a*b/c*dilog(((-c*x+1)^(1/2)/(c*x+1
)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))-1/2*a*b/c*dilog(((-c*x+1)^(1/2
)/(c*x+1)^(1/2)-1)^2/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a^2\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)+\frac{\left(b^2\log(cx+1)-b^2\log(-cx+1)\right)\log\left(-\sqrt{cx+1}+\sqrt{-cx+1}\right)^2}{8c}+\int -\frac{2\left(\sqrt{cx+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] 1/2*a^2*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/8*(b^2*log(c*x + 1) - b^2*log
(-c*x + 1))*log(-sqrt(c*x + 1) + sqrt(-c*x + 1))^2/c + integrate(-1/8*(2*(s
qrt(c*x + 1)*b^2 - sqrt(-c*x + 1)*b^2)*log(sqrt(c*x + 1) + sqrt(-c*x + 1))^
2 + 8*(sqrt(c*x + 1)*a*b - sqrt(-c*x + 1)*a*b)*log(sqrt(c*x + 1) + sqrt(-c*
x + 1)) - (4*(sqrt(c*x + 1)*b^2 - sqrt(-c*x + 1)*b^2)*log(sqrt(c*x + 1) + s
qrt(-c*x + 1)) + (8*a*b - (b^2*c*x - b^2)*log(c*x + 1) + (b^2*c*x - b^2)*lo
g(-c*x + 1))*sqrt(c*x + 1) - (8*a*b - (b^2*c*x + b^2)*log(c*x + 1) + (b^2*c
*x + b^2)*log(-c*x + 1))*sqrt(-c*x + 1))*log(-sqrt(c*x + 1) + sqrt(-c*x + 1
)))/((c^2*x^2 - 1)*sqrt(c*x + 1) - (c^2*x^2 - 1)*sqrt(-c*x + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b^2\operatorname{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2+2\,ab\operatorname{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a^2}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="fricas")
```

```
[Out] integral(-(b^2*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b*arccoth(sqrt
(-c*x + 1)/sqrt(c*x + 1)) + a^2)/(c^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{a^2}{c^2x^2-1}\,dx-\int\frac{b^2\operatorname{acoth}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx-\int\frac{2ab\operatorname{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a**2/(c**2*x**2 - 1), x) - Integral(b**2*acoth(sqrt(-c*x + 1)/sqr
t(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(2*a*b*acoth(sqrt(-c*x + 1)/sq
rt(c*x + 1))/(c**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{\left(b\operatorname{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] integrate(-(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2/(c^2*x^2 - 1), x
)
```

3.125 $\int \frac{a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

Optimal. Leaf size=89

$$-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{cx+1}}{\sqrt{1-cx}}\right)}{2c}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{cx+1}}{\sqrt{1-cx}}\right)}{2c}-\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}$$

```
[Out] -((a*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/c) - (b*PolyLog[2, -(Sqrt[1 + c*x]/S
qrt[1 - c*x])])/(2*c) + (b*PolyLog[2, Sqrt[1 + c*x]/Sqrt[1 - c*x]])/(2*c)
```

Rubi [A] time = 0.0586061, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {206, 6681, 5913}

$$-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{cx+1}}{\sqrt{1-cx}}\right)}{2c}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{cx+1}}{\sqrt{1-cx}}\right)}{2c}-\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] -((a*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/c) - (b*PolyLog[2, -(Sqrt[1 + c*x]/S
qrt[1 - c*x])])/(2*c) + (b*PolyLog[2, Sqrt[1 + c*x]/Sqrt[1 - c*x]])/(2*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rule 5913

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b*PolyLog[2, -(c*x)^(-1)])/2, x] - Simp[(b*PolyLog[2, 1/(c*x)])/
2, x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{a+b\coth^{-1}(x)}{x}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}\\ &= -\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}-\frac{b\text{Li}_2\left(-\frac{\sqrt{1+cx}}{\sqrt{1-cx}}\right)}{2c}+\frac{b\text{Li}_2\left(\frac{\sqrt{1+cx}}{\sqrt{1-cx}}\right)}{2c}\end{aligned}$$

Mathematica [A] time = 0.414956, size = 98, normalized size = 1.1

$$\frac{b\left(\text{PolyLog}\left(2,-e^{-\tanh^{-1}(cx)}\right)-\text{PolyLog}\left(2,e^{-\tanh^{-1}(cx)}\right)+\tanh^{-1}(cx)\left(2\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)+\log\left(1-e^{-\tanh^{-1}(cx)}\right)-\log\left(\right.\right.\right.}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] (a*ArcTanh[c*x])/c + (b*(ArcTanh[c*x]*(2*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x
]] + Log[1 - E^(-ArcTanh[c*x])] - Log[1 + E^(-ArcTanh[c*x])]) + PolyLog[2,
-E^(-ArcTanh[c*x])] - PolyLog[2, E^(-ArcTanh[c*x])]))/(2*c)
```

Maple [A] time = 0.667, size = 119, normalized size = 1.3

$$-\frac{a\ln(cx-1)}{2c}+\frac{a\ln(cx+1)}{2c}+\frac{b}{c}\text{dilog}\left(\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}-1\right)\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}+1\right)^{-1}\right)-\frac{b}{4c}\text{dilog}\left(\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x)
```

```
[Out] -1/2*a/c*ln(c*x-1)+1/2*a/c*ln(c*x+1)+b/c*dilog(((-c*x+1)^(1/2)/(c*x+1)^(1/2
)-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))-1/4*b/c*dilog(((-c*x+1)^(1/2)/(c*x+1
)^(1/2)-1)^2/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}b\left(\frac{(\log(cx+1)-\log(-cx+1))\log\left(\sqrt{cx+1}+\sqrt{-cx+1}\right)-(\log(cx+1)-\log(-cx+1))\log\left(-\sqrt{cx+1}+\sqrt{-cx+1}\right)}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="maxima")
```

```
[Out] 1/4*b*(((log(c*x + 1) - log(-c*x + 1))*log(sqrt(c*x + 1) + sqrt(-c*x + 1))
- (log(c*x + 1) - log(-c*x + 1))*log(-sqrt(c*x + 1) + sqrt(-c*x + 1)))/c -
2*integrate(-1/2*sqrt(c*x + 1)*(log(c*x + 1) - log(-c*x + 1))/((c^2*x^2 - 1
)*sqrt(c*x + 1) + (c^2*x^2 - 1)*sqrt(-c*x + 1)), x) - 2*integrate(1/2*sqrt(
c*x + 1)*(log(c*x + 1) - log(-c*x + 1))/((c^2*x^2 - 1)*sqrt(c*x + 1) - (c^2
*x^2 - 1)*sqrt(-c*x + 1)), x)) + 1/2*a*(log(c*x + 1)/c - log(c*x - 1)/c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b\,\text{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="fricas")
```

```
[Out] integral(-(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a}{c^2x^2-1}\,dx - \int \frac{b\,\operatorname{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a/(c**2*x**2 - 1), x) - Integral(b*acoth(sqrt(-c*x + 1)/sqrt(c*x
+ 1))/(c**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{b\,\operatorname{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="giac")
```

```
[Out] integrate(-(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

3.126 $\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

Optimal. Leaf size=42

$$\text{Unintegrable}\left(\frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)},x\right)$$

```
[Out] Unintegrable[1/((1 - c^2*x^2)*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]]))
, x]
```

Rubi [A] time = 0.0447068, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),
x]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Mathematica [A] time = 0.0901088, size = 0, normalized size = 0.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x
]
```

Maple [A] time = 0.983, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2+1}\left(a+b\,\mathrm{arccoth}\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(c^2x^2-1\right)\left(b\operatorname{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="maxima")
```

```
[Out] -integrate(1/((c^2*x^2 - 1)*(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
 x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{1}{ac^2x^2+\left(bc^2x^2-b\right)\operatorname{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="fricas")
```

```
[Out] integral(-1/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccoth(sqrt(-c*x + 1)/sqrt(c*x +
1)) - a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{ac^2x^2-a+bc^2x^2\operatorname{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-b\operatorname{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*acoth((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

```
[Out] -Integral(1/(a*c**2*x**2 - a + b*c**2*x**2*acoth(sqrt(-c*x + 1)/sqrt(c*x +
1)) - b*acoth(sqrt(-c*x + 1)/sqrt(c*x + 1))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(c^2x^2-1\right)\left(b\operatorname{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
 x)
```

3.127 $\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$

Optimal. Leaf size=42

$$\text{Unintegrable}\left(\frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2},x\right)$$

```
[Out] Unintegrable[1/((1 - c^2*x^2)*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^
2), x]
```

Rubi [A] time = 0.0426659, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
```

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
, x]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Mathematica [A] time = 0.751716, size = 0, normalized size = 0.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
 x]
```

Maple [A] time = 0.976, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2+1}\left(a+b\,\mathrm{arccoth}\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)^{-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{4\,cx}{\sqrt{cx+1}\sqrt{-cx+1}b^2c\log\left(\sqrt{cx+1}+\sqrt{-cx+1}\right)-\sqrt{cx+1}\sqrt{-cx+1}b^2c\log\left(-\sqrt{cx+1}+\sqrt{-cx+1}\right)+2\sqrt{cx+1}\sqrt{-cx+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="maxima")
```

```
[Out] 4*c*x/(sqrt(c*x + 1)*sqrt(-c*x + 1)*b^2*c*log(sqrt(c*x + 1) + sqrt(-c*x + 1
)) - sqrt(c*x + 1)*sqrt(-c*x + 1)*b^2*c*log(-sqrt(c*x + 1) + sqrt(-c*x + 1)
) + 2*sqrt(c*x + 1)*sqrt(-c*x + 1)*a*b*c) - integrate(-4/((b^2*c^2*x^2 - b^
2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*log(sqrt(c*x + 1) + sqrt(-c*x + 1)) - (b^2*
c^2*x^2 - b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*log(-sqrt(c*x + 1) + sqrt(-c*x
+ 1)) + 2*(a*b*c^2*x^2 - a*b)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{1}{a^2c^2x^2+\left(b^2c^2x^2-b^2\right)\mathrm{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2-a^2+2\left(abc^2x^2-ab\right)\mathrm{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="fricas")
```

```
[Out] integral(-1/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccoth(sqrt(-c*x + 1)/sqrt(
c*x + 1))^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccoth(sqrt(-c*x + 1)/sqrt(c*x +
 1))), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int\frac{1}{a^2c^2x^2-a^2+2abc^2x^2\,\mathrm{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-2ab\,\mathrm{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+b^2c^2x^2\,\mathrm{acoth}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-b^2\,\mathrm{acoth}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*acoth((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,x
)
```

```
[Out] -Integral(1/(a**2*c**2*x**2 - a**2 + 2*a*b*c**2*x**2*acoth(sqrt(-c*x + 1)/s
qrt(c*x + 1)) - 2*a*b*acoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + b**2*c**2*x**2*
```

```
acoth(sqrt(-c*x + 1)/sqrt(c*x + 1))**2 - b**2*acoth(sqrt(-c*x + 1)/sqrt(c*x
 + 1))**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(c^{2}x^{2}-1\right)\left(b \operatorname{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2
), x)
```

3.128 $\int x^m \coth^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=37

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))}{m+1}-\frac{bx^{m+2}}{m^2+3m+2}$$

```
[Out] -((b*x^(2 + m))/(2 + 3*m + m^2)) + (x^(1 + m)*ArcCoth[Tanh[a + b*x]])/(1 +
m)
```

Rubi [A] time = 0.027411, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 30}

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))}{m+1}-\frac{bx^{m+2}}{m^2+3m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] -((b*x^(2 + m))/(2 + 3*m + m^2)) + (x^(1 + m)*ArcCoth[Tanh[a + b*x]])/(1 +
m)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^m \coth^{-1}(\tanh(a+bx))\,dx &= \frac{x^{1+m}\coth^{-1}(\tanh(a+bx))}{1+m}-\frac{b\int x^{1+m}\,dx}{1+m}\\ &= -\frac{bx^{2+m}}{2+3m+m^2}+\frac{x^{1+m}\coth^{-1}(\tanh(a+bx))}{1+m}\end{aligned}$$

Mathematica [A] time = 0.0591708, size = 34, normalized size = 0.92

$$x^m\left(\frac{x\left(\coth^{-1}(\tanh(a+bx))-bx\right)}{m+1}+\frac{bx^2}{m+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] x^m*((b*x^2)/(2 + m) + (x*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/(1 + m))
```

Maple [C] time = 0.164, size = 676, normalized size = 18.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arccoth(tanh(b*x+a)),x)
```

```
[Out] 1/(1+m)*x*x^m*ln(exp(b*x+a))-1/4*x*(4*b*x+2*I*Pi*csgn(I*exp(2*b*x+2*a))^3+I
*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*m-I*Pi*csgn(I*exp(2*b*x+2*a
))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2*m+I*Pi*csgn(I*exp(2*b*x+2*a)
)^3*m+I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b
*x+2*a)/(exp(2*b*x+2*a)+1))*m+4*I*Pi+2*I*Pi*m-4*I*Pi*csgn(I/(exp(2*b*x+2*a)
+1))^2-2*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2*m-4*I*Pi*csgn(I*exp(b*x+a))*csgn
(I*exp(2*b*x+2*a))^2+I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3*m-2*I
*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+4*I*
Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+2*I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a
)+1))^3-2*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+
2*a)+1))^2+2*I*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+2*I*Pi*csgn(I
/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*
x+2*a)+1))-2*I*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*m-I*Pi*csgn(I
/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2*m+2*I*Pi*c
sgn(I/(exp(2*b*x+2*a)+1))^3*m)/(1+m)/(2+m)*x^m
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63962, size = 72, normalized size = 1.95

$$\frac{\left((bm+b)x^2+(am+2\,a)x\right)x^m}{m^2+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x^2 + (a*m + 2*a)*x)*x^m/(m^2 + 3*m + 2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*acoth(tanh(b*x+a)),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{arcoth}(\tanh(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^m*arccoth(tanh(b*x + a)), x)
```

3.129 $\int x^2 \coth^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=23

$$\frac{1}{3}x^3 \coth^{-1}(\tanh(a+bx)) - \frac{bx^4}{12}$$

```
[Out] -(b*x^4)/12 + (x^3*ArcCoth[Tanh[a + b*x]])/3
```

Rubi [A] time = 0.0090215, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 30}

$$\frac{1}{3}x^3 \coth^{-1}(\tanh(a+bx)) - \frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[Tanh[a + b*x]],x]

[Out] -(b*x^4)/12 + (x^3*ArcCoth[Tanh[a + b*x]])/3
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2 \coth^{-1}(\tanh(a+bx))\,dx &= \frac{1}{3}x^3 \coth^{-1}(\tanh(a+bx)) - \frac{1}{3}b\int x^3\,dx\\ &= -\frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(\tanh(a+bx))\end{aligned}$$

Mathematica [A] time = 0.0171361, size = 20, normalized size = 0.87

$$-\frac{1}{12}x^3 \left(bx - 4\coth^{-1}(\tanh(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[Tanh[a + b*x]],x]

[Out] -(x^3*(b*x - 4*ArcCoth[Tanh[a + b*x]]))/12
```

Maple [B] time = 0.076, size = 59, normalized size = 2.6

$$\frac{x^3 \mathrm{arccoth}\left(\tanh\left(bx+a\right)\right)}{3}+\frac{1}{3b^3}\left(-\frac{\left(bx+a\right)^4}{4}+\left(bx+a\right)^3 a-\frac{3a^2\left(bx+a\right)^2}{2}+\left(bx+a\right)a^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(tanh(b*x+a)),x)
```

```
[Out] 1/3*x^3*arccoth(tanh(b*x+a))+1/3/b^3*(-1/4*(b*x+a)^4+(b*x+a)^3*a-3/2*a^2*(b
*x+a)^2+(b*x+a)*a^3)
```

Maxima [A] time = 1.17738, size = 26, normalized size = 1.13

$$-\frac{1}{12}bx^4+\frac{1}{3}x^3\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/12*b*x^4 + 1/3*x^3*arccoth(tanh(b*x + a))
```

Fricas [A] time = 1.63204, size = 31, normalized size = 1.35

$$\frac{1}{4}bx^4+\frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

Sympy [A] time = 0.556857, size = 19, normalized size = 0.83

$$-\frac{bx^4}{12}+\frac{x^3\,\mathrm{acoth}\left(\tanh\left(a+bx\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(tanh(b*x+a)),x)
```

```
[Out] -b*x**4/12 + x**3*acoth(tanh(a + b*x))/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(tanh(b*x + a)), x)
```

3.130 $\int x \coth^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=23

$$\frac{1}{2}x^2\coth^{-1}(\tanh(a+bx))-\frac{bx^3}{6}$$

```
[Out] -(b*x^3)/6 + (x^2*ArcCoth[Tanh[a + b*x]])/2
```

Rubi [A] time = 0.0074933, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6240, 30}

$$\frac{1}{2}x^2\coth^{-1}(\tanh(a+bx))-\frac{bx^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] -(b*x^3)/6 + (x^2*ArcCoth[Tanh[a + b*x]])/2
```

Rule 6240

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Tanh[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\coth^{-1}(\tanh(a+bx))\,dx &= \frac{1}{2}x^2\coth^{-1}(\tanh(a+bx))-\frac{1}{2}b\int x^2\,dx\\ &= -\frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(\tanh(a+bx))\end{aligned}$$

Mathematica [A] time = 0.0157631, size = 20, normalized size = 0.87

$$-\frac{1}{6}x^2\left(bx-3\coth^{-1}(\tanh(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] -(x^2*(b*x - 3*ArcCoth[Tanh[a + b*x]]))/6
```

Maple [B] time = 0.075, size = 48, normalized size = 2.1

$$\frac{x^2 \operatorname{arccoth}(\tanh(bx+a))}{2} + \frac{1}{2b^2}\left(-\frac{(bx+a)^3}{3} + (bx+a)^2 a - a^2(bx+a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(tanh(b*x+a)),x)
```

```
[Out] 1/2*x^2*arccoth(tanh(b*x+a))+1/2/b^2*(-1/3*(b*x+a)^3+(b*x+a)^2*a-a^2*(b*x+a
))
```

Maxima [A] time = 1.17243, size = 26, normalized size = 1.13

$$-\frac{1}{6} bx^3 + \frac{1}{2} x^2 \operatorname{arcoth}(\tanh(bx+a))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/6*b*x^3 + 1/2*x^2*arccoth(tanh(b*x + a))
```

Fricas [A] time = 1.65176, size = 31, normalized size = 1.35

$$\frac{1}{3} bx^3 + \frac{1}{2} ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

Sympy [A] time = 0.291645, size = 19, normalized size = 0.83

$$-\frac{bx^3}{6} + \frac{x^2 \operatorname{acoth}(\tanh(a+bx))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(tanh(b*x+a)),x)
```

```
[Out] -b*x**3/6 + x**2*acoth(tanh(a + b*x))/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arcoth}(\tanh(bx+a))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(tanh(b*x + a)), x)
```

3.131 $\int \coth^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=16

$$\frac{\coth^{-1}(\tanh(a+bx))^2}{2b}$$

```
[Out] ArcCoth[Tanh[a + b*x]]^2/(2*b)
```

Rubi [A] time = 0.003068, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2157, 30}

$$\frac{\coth^{-1}(\tanh(a+bx))^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]],x]

[Out] ArcCoth[Tanh[a + b*x]]^2/(2*b)
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \coth^{-1}(\tanh(a+bx))\,dx &= \frac{\text{Subst}\left(\int x\,dx, x, \coth^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{\coth^{-1}(\tanh(a+bx))^2}{2b}\end{aligned}$$

Mathematica [A] time = 0.0083259, size = 18, normalized size = 1.12

$$x\coth^{-1}(\tanh(a+bx))-\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]],x]

[Out] -(b*x^2)/2 + x*ArcCoth[Tanh[a + b*x]]
```

Maple [B] time = 0.06, size = 32, normalized size = 2.

$$\frac{1}{b}\left(\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)\operatorname{arccoth}\left(\tanh\left(bx+a\right)\right)-\frac{\left(\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a)),x)
```

```
[Out] 1/b*(arctanh(tanh(b*x+a))*arccoth(tanh(b*x+a))-1/2*arctanh(tanh(b*x+a))^2)
```

Maxima [A] time = 1.15234, size = 22, normalized size = 1.38

$$-\frac{1}{2}\,bx^{2}+x\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/2*b*x^2 + x*arccoth(tanh(b*x + a))
```

Fricas [A] time = 1.57382, size = 23, normalized size = 1.44

$$\frac{1}{2}\,bx^{2}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*x
```

Sympy [A] time = 0.194614, size = 19, normalized size = 1.19

$$\begin{cases}\frac{\operatorname{acoth}^{2}\left(\tanh\left(a+bx\right)\right)}{2b} & \text{for } b\neq 0\\ x\operatorname{acoth}\left(\tanh\left(a\right)\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a)),x)
```

```
[Out] Piecewise((acoth(tanh(a + b*x))**2/(2*b), Ne(b, 0)), (x*acoth(tanh(a)), Tru
e))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a)), x)
```

3.132 $\int \frac{\coth^{-1}(\tanh(a+bx))}{x}\,dx$

Optimal. Leaf size=21

$$bx - \log(x)\left(bx - \coth^{-1}(\tanh(a+bx))\right)$$

```
[Out] b*x - (b*x - ArcCoth[Tanh[a + b*x]])*Log[x]
```

Rubi [A] time = 0.0439774, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2158, 29}

$$bx - \log(x)\left(bx - \coth^{-1}(\tanh(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]/x,x]
```

```
[Out] b*x - (b*x - ArcCoth[Tanh[a + b*x]])*Log[x]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(\tanh(a+bx))}{x}\,dx &= bx - \left(bx - \coth^{-1}(\tanh(a+bx))\right)\int \frac{1}{x}\,dx\\ &= bx - \left(bx - \coth^{-1}(\tanh(a+bx))\right)\log(x)\end{aligned}$$

Mathematica [A] time = 0.0161214, size = 19, normalized size = 0.9

$$\log(x)\left(\coth^{-1}(\tanh(a+bx)) - bx\right) + bx$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]/x,x]
```

```
[Out] b*x + (-(b*x) + ArcCoth[Tanh[a + b*x]])*Log[x]
```

Maple [C] time = 0.189, size = 354, normalized size = 16.9

$$\ln(x)\ln\left(e^{bx+a}\right) - \ln(x)xb + bx + \frac{i}{2}\pi\ln(x)\,\mathrm{csgn}\left(ie^{bx+a}\right)\left(\mathrm{csgn}\left(ie^{2bx+2a}\right)\right)^2 - \frac{i}{2}\pi\ln(x) + \frac{i}{4}\pi\ln(x)\,\mathrm{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))/x,x)
```

```
[Out] ln(x)*ln(exp(b*x+a))-ln(x)*x*b+b*x+1/2*I*Pi*ln(x)*csgn(I*exp(b*x+a))*csgn(I
*exp(2*b*x+2*a))^2-1/2*I*Pi*ln(x)+1/4*I*Pi*ln(x)*csgn(I/(exp(2*b*x+2*a)+1))
*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/4*I*Pi*ln(x)*csgn(I*exp(2*b*
x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/4*I*Pi*ln(x)*csgn(I*e
xp(2*b*x+2*a))^3-1/4*I*Pi*ln(x)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3
+1/2*I*Pi*ln(x)*csgn(I/(exp(2*b*x+2*a)+1))^2-1/2*I*Pi*ln(x)*csgn(I/(exp(2*b
*x+2*a)+1))^3-1/4*I*Pi*ln(x)*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*
a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/4*I*Pi*ln(x)*csgn(I*exp(b*x
+a))^2*csgn(I*exp(2*b*x+2*a))
```

Maxima [A] time = 0.977923, size = 46, normalized size = 2.19

$$-b\left(x+\frac{a}{b}\right)\log\left(x\right)+b\left(x+\frac{a\log\left(x\right)}{b}\right)+\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -b*(x + a/b)*log(x) + b*(x + a*log(x)/b) + arccoth(tanh(b*x + a))*log(x)
```

Fricas [A] time = 1.62091, size = 22, normalized size = 1.05

$$bx+a\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] b*x + a*log(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))/x,x)
```

```
[Out] Integral(acoth(tanh(a + b*x))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a))/x, x)
```

3.133 $\int \frac{\coth^{-1}(\tanh(a+bx))}{x^2} dx$

Optimal. Leaf size=17

$$b\log(x) - \frac{\coth^{-1}(\tanh(a+bx))}{x}$$

```
[Out] -(ArcCoth[Tanh[a + b*x]]/x) + b*Log[x]
```

Rubi [A] time = 0.0092769, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 29}

$$b\log(x) - \frac{\coth^{-1}(\tanh(a+bx))}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]/x^2,x]
```

```
[Out] -(ArcCoth[Tanh[a + b*x]]/x) + b*Log[x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(\tanh(a+bx))}{x^2} dx &= -\frac{\coth^{-1}(\tanh(a+bx))}{x} + b\int \frac{1}{x} dx \\ &= -\frac{\coth^{-1}(\tanh(a+bx))}{x} + b\log(x) \end{aligned}$$

Mathematica [A] time = 0.016392, size = 18, normalized size = 1.06

$$-\frac{\coth^{-1}(\tanh(a+bx))}{x} + b\log(x) + b$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]/x^2,x]
```

```
[Out] b - ArcCoth[Tanh[a + b*x]]/x + b*Log[x]
```

Maple [A] time = 0.069, size = 20, normalized size = 1.2

$$-\frac{\operatorname{arccoth}(\tanh(bx+a))}{x}+b\ln(bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))/x^2,x)
```

```
[Out] -arccoth(tanh(b*x+a))/x+b*ln(b*x)
```

Maxima [A] time = 1.17533, size = 23, normalized size = 1.35

$$b\log(x)-\frac{\operatorname{arcoth}(\tanh(bx+a))}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))/x^2,x, algorithm="maxima")
```

```
[Out] b*log(x) - arccoth(tanh(b*x + a))/x
```

Fricas [A] time = 1.72301, size = 27, normalized size = 1.59

$$\frac{bx\log(x)-a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))/x^2,x, algorithm="fricas")
```

```
[Out] (b*x*log(x) - a)/x
```

Sympy [A] time = 0.314397, size = 14, normalized size = 0.82

$$b\log(x)-\frac{\operatorname{acoth}(\tanh(a+bx))}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))/x**2,x)
```

```
[Out] b*log(x) - acoth(tanh(a + b*x))/x
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{arcoth}(\tanh(bx+a))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a))/x^2, x)
```

3.134 $\int \frac{\coth^{-1}(\tanh(a+bx))}{x^3}\,dx$

Optimal. Leaf size=23

$$-\frac{\coth^{-1}(\tanh(a+bx))}{2x^2}-\frac{b}{2x}$$

```
[Out] -b/(2*x) - ArcCoth[Tanh[a + b*x]]/(2*x^2)
```

Rubi [A] time = 0.0095948, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 30}

$$-\frac{\coth^{-1}(\tanh(a+bx))}{2x^2}-\frac{b}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]/x^3,x]
```

```
[Out] -b/(2*x) - ArcCoth[Tanh[a + b*x]]/(2*x^2)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(\tanh(a+bx))}{x^3}\,dx &= -\frac{\coth^{-1}(\tanh(a+bx))}{2x^2}+\frac{1}{2}b\int \frac{1}{x^2}\,dx\\ &= -\frac{b}{2x}-\frac{\coth^{-1}(\tanh(a+bx))}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.0135521, size = 18, normalized size = 0.78

$$-\frac{\coth^{-1}(\tanh(a+bx))+bx}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]/x^3,x]
```

```
[Out] -(b*x + ArcCoth[Tanh[a + b*x]])/(2*x^2)
```

Maple [A] time = 0.074, size = 20, normalized size = 0.9

$$-\frac{b}{2x}-\frac{\operatorname{arccoth}\left(\tanh\left(bx+a\right)\right)}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))/x^3,x)
```

```
[Out] -1/2*b/x-1/2*arccoth(tanh(b*x+a))/x^2
```

Maxima [A] time = 1.17109, size = 26, normalized size = 1.13

$$-\frac{b}{2x}-\frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b/x - 1/2*arccoth(tanh(b*x + a))/x^2
```

Fricas [A] time = 1.59421, size = 30, normalized size = 1.3

$$-\frac{2bx+a}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x + a)/x^2
```

Sympy [A] time = 0.773247, size = 19, normalized size = 0.83

$$-\frac{b}{2x}-\frac{\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))/x**3,x)
```

```
[Out] -b/(2*x) - acoth(tanh(a + b*x))/(2*x**2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))/x^3,x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a))/x^3, x)
```

3.135 $\int \frac{\coth^{-1}(\tanh(a+bx))}{x^4} dx$

Optimal. Leaf size=23

$$-\frac{\coth^{-1}(\tanh(a+bx))}{3x^3}-\frac{b}{6x^2}$$

```
[Out] -b/(6*x^2) - ArcCoth[Tanh[a + b*x]]/(3*x^3)
```

Rubi [A] time = 0.0088794, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 30}

$$-\frac{\coth^{-1}(\tanh(a+bx))}{3x^3}-\frac{b}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]/x^4,x]
```

```
[Out] -b/(6*x^2) - ArcCoth[Tanh[a + b*x]]/(3*x^3)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(\tanh(a+bx))}{x^4}\,dx &= -\frac{\coth^{-1}(\tanh(a+bx))}{3x^3}+\frac{1}{3}b\int \frac{1}{x^3}\,dx\\ &= -\frac{b}{6x^2}-\frac{\coth^{-1}(\tanh(a+bx))}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.0154072, size = 20, normalized size = 0.87

$$-\frac{2\coth^{-1}(\tanh(a+bx))+bx}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]/x^4,x]
```

```
[Out] -(b*x + 2*ArcCoth[Tanh[a + b*x]])/(6*x^3)
```

Maple [A] time = 0.075, size = 20, normalized size = 0.9

$$-\frac{b}{6x^2}-\frac{\operatorname{arccoth}(\tanh(bx+a))}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))/x^4,x)
```

```
[Out] -1/6*b/x^2-1/3*arccoth(tanh(b*x+a))/x^3
```

Maxima [A] time = 1.17902, size = 26, normalized size = 1.13

$$-\frac{b}{6x^2}-\frac{\operatorname{arcoth}(\tanh(bx+a))}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*b/x^2 - 1/3*arccoth(tanh(b*x + a))/x^3
```

Fricas [A] time = 1.668, size = 32, normalized size = 1.39

$$-\frac{3bx+2a}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b*x + 2*a)/x^3
```

Sympy [A] time = 1.27728, size = 20, normalized size = 0.87

$$-\frac{b}{6x^2}-\frac{\operatorname{acoth}(\tanh(a+bx))}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))/x**4,x)
```

```
[Out] -b/(6*x**2) - acoth(tanh(a + b*x))/(3*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(\tanh(bx+a))}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))/x^4,x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a))/x^4, x)
```

3.136 $\int x^m \coth^{-1}(\tanh(a+bx))^2 \, dx$

Optimal. Leaf size=71

$$-\frac{2bx^{m+2}\coth^{-1}(\tanh(a+bx))}{m^2+3m+2}+\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))^2}{m+1}+\frac{2b^2x^{m+3}}{m^3+6m^2+11m+6}$$

```
[Out] (2*b^2*x^(3 + m))/(6 + 11*m + 6*m^2 + m^3) - (2*b*x^(2 + m)*ArcCoth[Tanh[a
+ b*x]])/(2 + 3*m + m^2) + (x^(1 + m)*ArcCoth[Tanh[a + b*x]]^2)/(1 + m)
```

Rubi [A] time = 0.0352614, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$-\frac{2bx^{m+2}\coth^{-1}(\tanh(a+bx))}{m^2+3m+2}+\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))^2}{m+1}+\frac{2b^2x^{m+3}}{m^3+6m^2+11m+6}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] (2*b^2*x^(3 + m))/(6 + 11*m + 6*m^2 + m^3) - (2*b*x^(2 + m)*ArcCoth[Tanh[a
+ b*x]])/(2 + 3*m + m^2) + (x^(1 + m)*ArcCoth[Tanh[a + b*x]]^2)/(1 + m)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^m \coth^{-1}(\tanh(a+bx))^2 \, dx &= \frac{x^{1+m}\coth^{-1}(\tanh(a+bx))^2}{1+m}-\frac{(2b)\int x^{1+m}\coth^{-1}(\tanh(a+bx)) \, dx}{1+m}\\ &= -\frac{2bx^{2+m}\coth^{-1}(\tanh(a+bx))}{2+3m+m^2}+\frac{x^{1+m}\coth^{-1}(\tanh(a+bx))^2}{1+m}+\frac{\left(2b^2\right)\int x^{2+m} \, dx}{2+3m+m^2}\\ &= \frac{2b^2x^{3+m}}{6+11m+6m^2+m^3}-\frac{2bx^{2+m}\coth^{-1}(\tanh(a+bx))}{2+3m+m^2}+\frac{x^{1+m}\coth^{-1}(\tanh(a+bx))^2}{1+m}\end{aligned}$$

Mathematica [A] time = 0.11794, size = 62, normalized size = 0.87

$$\frac{x^{m+1}\left(\left(m^2+5m+6\right)\coth^{-1}(\tanh(a+bx))^2-2b(m+3)x\coth^{-1}(\tanh(a+bx))+2b^2x^2\right)}{(m+1)(m+2)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] (x^(1 + m)*(2*b^2*x^2 - 2*b*(3 + m)*x*ArcCoth[Tanh[a + b*x]] + (6 + 5*m + m
^2)*ArcCoth[Tanh[a + b*x]]^2))/((1 + m)*(2 + m)*(3 + m))
```

Maple [C] time = 0.773, size = 9175, normalized size = 129.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arccoth(tanh(b*x+a))^2,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74427, size = 223, normalized size = 3.14

$$\frac{\left(4\left(b^2m^2+3\,b^2m+2\,b^2\right)x^3+8\left(abm^2+4\,abm+3\,ab\right)x^2+\left(4\,a^2m^2-\pi^2\left(m^2+5\,m+6\right)+20\,a^2m+24\,a^2\right)x\right)x^m}{4\left(m^3+6\,m^2+11\,m+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(b^2*m^2 + 3*b^2*m + 2*b^2)*x^3 + 8*(a*b*m^2 + 4*a*b*m + 3*a*b)*x^2
+ (4*a^2*m^2 - pi^2*(m^2 + 5*m + 6) + 20*a^2*m + 24*a^2)*x)*x^m/(m^3 + 6*m^
2 + 11*m + 6)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*acoth(tanh(b*x+a))**2,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{arcoth}(\tanh(bx + a))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*arccoth(tanh(b*x + a))^2, x)
```

3.137 $\int x^3 \coth^{-1}(\tanh(a+bx))^2\,dx$

Optimal. Leaf size=42

$$-\frac{1}{10}bx^5\coth^{-1}(\tanh(a+bx))+\frac{1}{4}x^4\coth^{-1}(\tanh(a+bx))^2+\frac{b^2x^6}{60}$$

```
[Out] (b^2*x^6)/60 - (b*x^5*ArcCoth[Tanh[a + b*x]])/10 + (x^4*ArcCoth[Tanh[a + b*
x]]^2)/4
```

Rubi [A] time = 0.0260149, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$-\frac{1}{10}bx^5\coth^{-1}(\tanh(a+bx))+\frac{1}{4}x^4\coth^{-1}(\tanh(a+bx))^2+\frac{b^2x^6}{60}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] (b^2*x^6)/60 - (b*x^5*ArcCoth[Tanh[a + b*x]])/10 + (x^4*ArcCoth[Tanh[a + b*
x]]^2)/4
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^3\coth^{-1}(\tanh(a+bx))^2\,dx&=\frac{1}{4}x^4\coth^{-1}(\tanh(a+bx))^2-\frac{1}{2}b\int x^4\coth^{-1}(\tanh(a+bx))\,dx\\&=-\frac{1}{10}bx^5\coth^{-1}(\tanh(a+bx))+\frac{1}{4}x^4\coth^{-1}(\tanh(a+bx))^2+\frac{1}{10}b^2\int x^5\,dx\\&=\frac{b^2x^6}{60}-\frac{1}{10}bx^5\coth^{-1}(\tanh(a+bx))+\frac{1}{4}x^4\coth^{-1}(\tanh(a+bx))^2\end{aligned}$$

Mathematica [A] time = 0.0336754, size = 37, normalized size = 0.88

$$\frac{1}{60}x^4\left(-6bx\coth^{-1}(\tanh(a+bx))+15\coth^{-1}(\tanh(a+bx))^2+b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] (x^4*(b^2*x^2 - 6*b*x*ArcCoth[Tanh[a + b*x]] + 15*ArcCoth[Tanh[a + b*x]]^2)
)/60
```

Maple [C] time = 0.372, size = 3418, normalized size = 81.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccoth(tanh(b*x+a))^2,x)
```

```
[Out] 1/20*I*Pi*b*x^5-1/16*Pi^2*x^4*csgn(I/(exp(2*b*x+2*a)+1))^6-1/16*Pi^2*x^4*cs
gn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(
2*b*x+2*a)+1))-1/20*I*Pi*b*x^5*csgn(I/(exp(2*b*x+2*a)+1))^2+1/8*Pi^2*x^4*cs
gn(I/(exp(2*b*x+2*a)+1))^5-1/16*Pi^2*x^4+1/4*x^4*ln(exp(b*x+a))^2+1/16*Pi^2
*x^4*csgn(I/(exp(2*b*x+2*a)+1))^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))
^2-1/16*Pi^2*x^4*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(2*b*x+2*a))^3+1/60
*x^6*b^2-1/16*Pi^2*x^4*csgn(I/(exp(2*b*x+2*a)+1))^4-1/64*Pi^2*x^4*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^6-1/16*Pi^2*x^4*csgn(I/(exp(2*b*x+2*a)+1))
^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/16*Pi^2*x^4*csgn(I/(exp(2*
b*x+2*a)+1))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/32*Pi^2*x^4*cs
gn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/64*Pi^
2*x^4*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4+
1/32*Pi^2*x^4*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+
1))^5-3/32*Pi^2*x^4*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))^4+1/16*Pi^2
*x^4*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^5+1/32*Pi^2*x^4*csgn(I*exp(2
*b*x+2*a))^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/16*Pi^2*x^4*csgn
(I*exp(2*b*x+2*a))^3*csgn(I/(exp(2*b*x+2*a)+1))^2+1/16*Pi^2*x^4*csgn(I*exp(
2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3*csgn(I/(exp(2*b*x+2*a)+1))^2+1/20*I*Pi*b*x
^5*csgn(I/(exp(2*b*x+2*a)+1))^3-1/16*Pi^2*x^4*csgn(I/(exp(2*b*x+2*a)+1))^4*
csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/16*Pi^2*
x^4*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a)
)-1/8*Pi^2*x^4*csgn(I/(exp(2*b*x+2*a)+1))^3+1/8*Pi^2*x^4*csgn(I/(exp(2*b*x+
2*a)+1))^2+1/8*Pi^2*x^4*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/16*Pi
^2*x^4*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1
/64*Pi^2*x^4*csgn(I*exp(2*b*x+2*a))^6-1/16*Pi^2*x^4*csgn(I*exp(b*x+a))^2*cs
gn(I*exp(2*b*x+2*a))-1/16*Pi^2*x^4*csgn(I*exp(2*b*x+2*a))^3+1/16*Pi^2*x^4*c
sgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/64*
Pi^2*x^4*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)
+1))^4+1/32*Pi^2*x^4*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(
2*b*x+2*a)+1))^5-1/64*Pi^2*x^4*csgn(I*exp(b*x+a))^4*csgn(I*exp(2*b*x+2*a))^
2+1/16*Pi^2*x^4*csgn(I*exp(b*x+a))^3*csgn(I*exp(2*b*x+2*a))^3-1/16*Pi^2*x^4
*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/40*I*Pi*b*x^5*csgn(I/(exp(2*
b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/40*I*Pi*b*x^5*cs
gn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/20*I*Pi*b*x^5*csgn(I*exp(b*x+a)
)*csgn(I*exp(2*b*x+2*a))^2-1/40*I*Pi*b*x^5*csgn(I*exp(2*b*x+2*a))*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/40*I*Pi*b*x^5*csgn(I/(exp(2*b*x+2*a)+1
))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/8*Pi^
2*x^4*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a)
)^2+1/16*Pi^2*x^4*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(2*b*x+2*a))*csgn(
I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/16*Pi^2*x^4*csgn(I*exp(b*x+a))^2*c
sgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)+1))^2-1/8*Pi^2*x^4*csgn(I*exp(
b*x+a))*csgn(I*exp(2*b*x+2*a))^2*csgn(I/(exp(2*b*x+2*a)+1))^2-1/16*Pi^2*x^4
*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2*csgn(I/
(exp(2*b*x+2*a)+1))^2-1/32*Pi^2*x^4*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b
```

```
*x+a))^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))
+1/32*Pi^2*x^4*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))^2*csgn(I*exp(2
*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/16*Pi^2*x^4*csgn(I
/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp
(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/16*Pi^2*x^4*csgn(I/(exp(2*b*x+2*a)+1))*cs
gn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+
2*a)+1))^2-1/16*Pi^2*x^4*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*
csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4-1/32*Pi^2*x^4*csgn(I*exp(b*x+a)
)^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/16
*Pi^2*x^4*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)
/(exp(2*b*x+2*a)+1))^3-1/32*Pi^2*x^4*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(
2*b*x+2*a))^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/32*Pi^2*x^4*csgn(
I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2
*b*x+2*a)+1))^2-1/64*Pi^2*x^4*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x
+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/32*Pi^2*x^4*csgn(I/(
exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*
x+2*a)+1))^3+1/32*Pi^2*x^4*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)
)^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/16*Pi^2*x^4*csgn(I/(exp(2
*b*x+2*a)+1))^3*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a
)+1))+(-1/10*b*x^5-1/8*I*Pi*x^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3
-1/8*I*Pi*x^4*csgn(I*exp(2*b*x+2*a))^3+1/4*I*Pi*x^4*csgn(I*exp(b*x+a))*csgn
(I*exp(2*b*x+2*a))^2+1/8*I*Pi*x^4*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2
*a)/(exp(2*b*x+2*a)+1))^2+1/8*I*Pi*x^4*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/8*I*Pi*x^4*csgn(I/(exp(2*b*x+2*a)+1))*
csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/8*I*Pi*x
^4*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/4*I*Pi*x^4*csgn(I/(exp(2*b
*x+2*a)+1))^3-1/4*I*Pi*x^4+1/4*I*Pi*x^4*csgn(I/(exp(2*b*x+2*a)+1))^2)*ln(ex
p(b*x+a))+1/40*I*Pi*b*x^5*csgn(I*exp(2*b*x+2*a))^3+1/40*I*Pi*b*x^5*csgn(I*e
xp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/32*Pi^2*x^4*csgn(I*exp(b*x+a))^2*csgn
(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/16*Pi^2*
x^4*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(
2*b*x+2*a)+1))^2
```

Maxima [A] time = 1.3594, size = 49, normalized size = 1.17

$$\frac{1}{60}\,b^2x^6 - \frac{1}{10}\,bx^5\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right) + \frac{1}{4}\,x^4\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(tanh(b*x+a))^2,x, algorithm="maxima")

[Out] 1/60*b^2*x^6 - 1/10*b*x^5*arccoth(tanh(b*x + a)) + 1/4*x^4*arccoth(tanh(b*x
 + a))^2
```

Fricas [A] time = 1.57062, size = 72, normalized size = 1.71

$$\frac{1}{6}\,b^2x^6 + \frac{2}{5}\,abx^5 - \frac{1}{16}\left(\pi^2 - 4\,a^2\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(tanh(b*x+a))^2,x, algorithm="fricas")

[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 - 1/16*(pi^2 - 4*a^2)*x^4
```

Sympy [A] time = 3.67829, size = 78, normalized size = 1.86

$$\begin{cases} \frac{x^3 \operatorname{acoth}^3 (\tanh (a+bx))}{3b} - \frac{x^2 \operatorname{acoth}^4 (\tanh (a+bx))}{4b^2} + \frac{x \operatorname{acoth}^5 (\tanh (a+bx))}{10b^3} - \frac{\operatorname{acoth}^6 (\tanh (a+bx))}{60b^4} & \text{for } b \neq 0 \\ \frac{x^4 \operatorname{acoth}^2 (\tanh (a))}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acoth(tanh(b*x+a))**2,x)
```

```
[Out] Piecewise((x**3*acoth(tanh(a + b*x))**3/(3*b) - x**2*acoth(tanh(a + b*x))**
4/(4*b**2) + x*acoth(tanh(a + b*x))**5/(10*b**3) - acoth(tanh(a + b*x))**6/
(60*b**4), Ne(b, 0)), (x**4*acoth(tanh(a))**2/4, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arcoth}(\tanh (bx + a))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*arccoth(tanh(b*x + a))^2, x)
```

3.138 $\int x^2 \coth^{-1}(\tanh(a+bx))^2 \, dx$

Optimal. Leaf size=42

$$-\frac{1}{6}bx^4\coth^{-1}(\tanh(a+bx))+\frac{1}{3}x^3\coth^{-1}(\tanh(a+bx))^2+\frac{b^2x^5}{30}$$

```
[Out] (b^2*x^5)/30 - (b*x^4*ArcCoth[Tanh[a + b*x]])/6 + (x^3*ArcCoth[Tanh[a + b*x
]]^2)/3
```

Rubi [A] time = 0.02533, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$-\frac{1}{6}bx^4\coth^{-1}(\tanh(a+bx))+\frac{1}{3}x^3\coth^{-1}(\tanh(a+bx))^2+\frac{b^2x^5}{30}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] (b^2*x^5)/30 - (b*x^4*ArcCoth[Tanh[a + b*x]])/6 + (x^3*ArcCoth[Tanh[a + b*x
]]^2)/3
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^2\coth^{-1}(\tanh(a+bx))^2\,dx &= \frac{1}{3}x^3\coth^{-1}(\tanh(a+bx))^2-\frac{1}{3}(2b)\int x^3\coth^{-1}(\tanh(a+bx))\,dx\\
&= -\frac{1}{6}bx^4\coth^{-1}(\tanh(a+bx))+\frac{1}{3}x^3\coth^{-1}(\tanh(a+bx))^2+\frac{1}{6}b^2\int x^4\,dx\\
&= \frac{b^2x^5}{30}-\frac{1}{6}bx^4\coth^{-1}(\tanh(a+bx))+\frac{1}{3}x^3\coth^{-1}(\tanh(a+bx))^2
\end{aligned}$$

Mathematica [A] time = 0.0561201, size = 37, normalized size = 0.88

$$\frac{1}{30}x^3\left(-5bx\coth^{-1}(\tanh(a+bx))+10\coth^{-1}(\tanh(a+bx))^2+b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] (x^3*(b^2*x^2 - 5*b*x*ArcCoth[Tanh[a + b*x]] + 10*ArcCoth[Tanh[a + b*x]]^2)
)/30
```

Maple [C] time = 0.372, size = 3418, normalized size = 81.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(tanh(b*x+a))^2,x)
```

```
[Out] -1/12*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a))*csgn(I*e
xp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/6*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))
^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/6*Pi^2*x^3*csgn(I/(exp(2*b
*x+2*a)+1))^5-1/12*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))^6+1/12*Pi^2*x^3*csgn
(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*
a)+1))^3+1/6*Pi^2*x^3*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*csgn(I/(e
xp(2*b*x+2*a)+1))^3-1/12*Pi^2*x^3+1/3*x^3*ln(exp(b*x+a))^2-1/12*Pi^2*x^3*cs
gn(I/(exp(2*b*x+2*a)+1))^4*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(ex
p(2*b*x+2*a)+1))-1/12*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+
2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/48*Pi^2*x^3*csgn(I/(exp(2
*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2
*a)+1))^2+(-1/6*b*x^4-1/3*I*Pi*x^3*csgn(I/(exp(2*b*x+2*a)+1))^3-1/6*I*Pi*x^
3*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/6*I*Pi*x^3*csgn(I*exp(2*b*x
+2*a)/(exp(2*b*x+2*a)+1))^3+1/3*I*Pi*x^3*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*
x+2*a))^2-1/6*I*Pi*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*cs
gn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/6*I*Pi*x^3*csgn(I*exp(2*b*x+2*a))
*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/6*I*Pi*x^3*csgn(I*exp(2*b*x+
2*a))^3-1/3*I*Pi*x^3+1/6*I*Pi*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b
*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/3*I*Pi*x^3*csgn(I/(exp(2*b*x+2*a)+1))^2)*ln
(exp(b*x+a))-1/12*I*Pi*b*x^4*csgn(I/(exp(2*b*x+2*a)+1))^2+1/12*I*Pi*b*x^4*c
sgn(I/(exp(2*b*x+2*a)+1))^3+1/12*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))^4*csgn
(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/24*Pi^2*x^3*csgn(I*exp(2*b*x+2*a)
)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^5-1/12*Pi^2*x^3*csgn(I*exp(2*b*
x+2*a)/(exp(2*b*x+2*a)+1))^3*csgn(I/(exp(2*b*x+2*a)+1))^3+1/12*Pi^2*x^3*csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3*csgn(I/(exp(2*b*x+2*a)+1))^2-1/24*
Pi^2*x^3*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))
^3-1/12*Pi^2*x^3*csgn(I*exp(2*b*x+2*a))^3*csgn(I/(exp(2*b*x+2*a)+1))^3-1/48
*Pi^2*x^3*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)
)^4+1/12*Pi^2*x^3*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^5+1/24*Pi^2*x^3
*csgn(I*exp(2*b*x+2*a))^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/48*
Pi^2*x^3*csgn(I*exp(b*x+a))^4*csgn(I*exp(2*b*x+2*a))^2+1/12*Pi^2*x^3*csgn(I
*exp(b*x+a))^3*csgn(I*exp(2*b*x+2*a))^3-1/8*Pi^2*x^3*csgn(I*exp(b*x+a))^2*c
sgn(I*exp(2*b*x+2*a))^4-1/48*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*e
xp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4+1/24*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1)
)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^5+1/12*Pi^2*x^3*csgn(I*exp(2*b*
x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/12*Pi^2*x^3*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2*csgn(I/(exp(2*b*x+2*a)+1))^3+1/12*Pi^2*x
^3*csgn(I*exp(2*b*x+2*a))^3*csgn(I/(exp(2*b*x+2*a)+1))^2-1/6*Pi^2*x^3*csgn(
I/(exp(2*b*x+2*a)+1))^3-1/24*I*Pi*b*x^4*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2
*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/12*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))*cs
gn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4-1/24*Pi^2*
x^3*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(
2*b*x+2*a)+1))^3-1/12*Pi^2*x^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-
1/12*Pi^2*x^3*csgn(I*exp(2*b*x+2*a))^3+1/6*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+
```

```
1))^2-1/12*Pi^2*x^3*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp
(2*b*x+2*a)+1))^3-1/12*Pi^2*x^3*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^3
*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/12*Pi^2*x^3*csgn(I/(exp(2*b*
x+2*a)+1))^3*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1
))-1/24*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))^2*csgn(I*exp
(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/24*Pi^2*x^3*csgn
(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn(I*e
xp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/12*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1)
)*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*
b*x+2*a)+1))-1/12*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))*cs
gn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/48*Pi^
2*x^3*csgn(I*exp(2*b*x+2*a))^6-1/48*Pi^2*x^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b
*x+2*a)+1))^6+1/30*x^5*b^2+1/12*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/12*Pi^2*x^3*csgn(I*exp(b*x+a))^2*csg
n(I*exp(2*b*x+2*a))+1/6*Pi^2*x^3*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^
2+1/24*I*Pi*b*x^4*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/12*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))
^4-1/24*I*Pi*b*x^4*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*
b*x+2*a)+1))^2+1/24*I*Pi*b*x^4*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-
1/12*I*Pi*b*x^4*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/12*Pi^2*x^3*c
sgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+1/2
4*I*Pi*b*x^4*csgn(I*exp(2*b*x+2*a))^3+1/24*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+
1))^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/
24*Pi^2*x^3*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+
2*a)/(exp(2*b*x+2*a)+1))^2+1/24*I*Pi*b*x^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x
+2*a)+1))^3+1/12*I*Pi*b*x^4-1/24*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I
*exp(2*b*x+2*a))^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/24*Pi^2*x^3*
csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(
exp(2*b*x+2*a)+1))^2+1/24*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*
b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/12*Pi^2*x^3*csgn(
I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2
*b*x+2*a)+1))^2
```

Maxima [A] time = 1.37034, size = 49, normalized size = 1.17

$$\frac{1}{30}\,b^2x^5 - \frac{1}{6}\,bx^4 \operatorname{arcoth}\left(\tanh\left(bx+a\right)\right) + \frac{1}{3}\,x^3 \operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/30*b^2*x^5 - 1/6*b*x^4*arccoth(tanh(b*x + a)) + 1/3*x^3*arccoth(tanh(b*x
+ a))^2
```

Fricas [A] time = 1.63241, size = 72, normalized size = 1.71

$$\frac{1}{5}\,b^2x^5 + \frac{1}{2}\,abx^4 - \frac{1}{12}\left(\pi^2 - 4\,a^2\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 - 1/12*(pi^2 - 4*a^2)*x^3
```

Sympy [A] time = 1.88637, size = 60, normalized size = 1.43

$$\begin{cases} \frac{x^2 \operatorname{acoth}^3(\tanh(a+bx))}{3b} - \frac{x \operatorname{acoth}^4(\tanh(a+bx))}{6b^2} + \frac{\operatorname{acoth}^5(\tanh(a+bx))}{30b^3} & \text{for } b \neq 0 \\ \frac{x^3 \operatorname{acoth}^2(\tanh(a))}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(tanh(b*x+a))**2,x)
```

```
[Out] Piecewise((x**2*acoth(tanh(a + b*x))**3/(3*b) - x*acoth(tanh(a + b*x))**4/(
6*b**2) + acoth(tanh(a + b*x))**5/(30*b**3), Ne(b, 0)), (x**3*acoth(tanh(a)
)**2/3, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arcoth}(\tanh(bx+a))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(tanh(b*x + a))^2, x)
```

3.139 $\int x \coth^{-1}(\tanh(a+bx))^2 \, dx$

Optimal. Leaf size=34

$$\frac{x \coth^{-1}(\tanh(a+bx))^3}{3b} - \frac{\coth^{-1}(\tanh(a+bx))^4}{12b^2}$$

```
[Out] (x*ArcCoth[Tanh[a + b*x]]^3)/(3*b) - ArcCoth[Tanh[a + b*x]]^4/(12*b^2)
```

Rubi [A] time = 0.0253549, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2168, 2157, 30}

$$\frac{x \coth^{-1}(\tanh(a+bx))^3}{3b} - \frac{\coth^{-1}(\tanh(a+bx))^4}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] (x*ArcCoth[Tanh[a + b*x]]^3)/(3*b) - ArcCoth[Tanh[a + b*x]]^4/(12*b^2)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x \coth^{-1}(\tanh(a+bx))^2 \, dx &= \frac{x \coth^{-1}(\tanh(a+bx))^3}{3b} - \frac{\int \coth^{-1}(\tanh(a+bx))^3 \, dx}{3b} \\
&= \frac{x \coth^{-1}(\tanh(a+bx))^3}{3b} - \frac{\operatorname{Subst}\left(\int x^3 \, dx, x, \coth^{-1}(\tanh(a+bx))\right)}{3b^2} \\
&= \frac{x \coth^{-1}(\tanh(a+bx))^3}{3b} - \frac{\coth^{-1}(\tanh(a+bx))^4}{12b^2}
\end{aligned}$$

Mathematica [B] time = 0.0822737, size = 74, normalized size = 2.18

$$\frac{(a+bx)\left(4\left(2a^2+abx-b^2x^2\right)\coth^{-1}(\tanh(a+bx))-(3a-bx)(a+bx)^2-6(a-bx)\coth^{-1}(\tanh(a+bx))^2\right)}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] ((a + b*x)*(-((3*a - b*x)*(a + b*x)^2) + 4*(2*a^2 + a*b*x - b^2*x^2)*ArcCot
h[Tanh[a + b*x]] - 6*(a - b*x)*ArcCoth[Tanh[a + b*x]]^2))/(12*b^2)
```

Maple [C] time = 0.368, size = 3418, normalized size = 100.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(tanh(b*x+a))^2,x)
```

```
[Out] -1/4*Pi^2*x^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*
b*x+2*a))^2-1/8*Pi^2*x^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a)
)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/4*Pi^2*x^2*csgn(I/(exp(2*b*
x+2*a)+1))^5+1/16*Pi^2*x^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))^2*cs
gn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/16*Pi^2*x^2*csgn(I/(exp(2*b*x+2
*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^5-1/8*Pi^2*x^2+1/2*x^2*ln
(exp(b*x+a))^2+1/16*Pi^2*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*
a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/16*Pi^2*x^2*csgn(I/(exp
(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2
*a)+1))^3-1/8*Pi^2*x^2*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(2*b*x+2*a)/(
exp(2*b*x+2*a)+1))^3-1/8*Pi^2*x^2*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(2
*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/8*Pi^2*x^2*csgn(I/(exp(2*b*x+2*a)+1))^2*c
sgn(I*exp(2*b*x+2*a))^3+1/8*Pi^2*x^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/8*Pi^2*x^2*csgn(I*exp(b*x+a))^2*csgn(I
*exp(2*b*x+2*a))+1/4*Pi^2*x^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1
/8*Pi^2*x^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)
)^2-1/32*Pi^2*x^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a)/(exp(2
*b*x+2*a)+1))^4-1/32*Pi^2*x^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a
)/(exp(2*b*x+2*a)+1))^4+1/16*Pi^2*x^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b
*x+2*a)/(exp(2*b*x+2*a)+1))^5+1/8*Pi^2*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(
I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-3/16*Pi^2*x^2*csgn(I*exp(b*x+a))^2*c
sgn(I*exp(2*b*x+2*a))^4+1/8*Pi^2*x^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*
a))^5+1/16*Pi^2*x^2*csgn(I*exp(2*b*x+2*a))^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b
*x+2*a)+1))^2-1/16*Pi^2*x^2*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/
(exp(2*b*x+2*a)+1))^3-1/8*Pi^2*x^2*csgn(I*exp(2*b*x+2*a))^3*csgn(I/(exp(2*b
*x+2*a)+1))^3+1/8*Pi^2*x^2*csgn(I/(exp(2*b*x+2*a)+1))^4*csgn(I*exp(2*b*x+2*
a)/(exp(2*b*x+2*a)+1))^2-1/32*Pi^2*x^2*csgn(I*exp(b*x+a))^4*csgn(I*exp(2*b*
x+2*a))^2+1/8*Pi^2*x^2*csgn(I*exp(b*x+a))^3*csgn(I*exp(2*b*x+2*a))^3-1/32*P
i^2*x^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*
b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/16*Pi^2*x^2*csgn(I/(exp(2*b*x+2*a)+1))^2*c
sgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/16*Pi^2
*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))^4*csgn(I*exp(2*b*x+2
*a)/(exp(2*b*x+2*a)+1))-1/4*Pi^2*x^2*csgn(I/(exp(2*b*x+2*a)+1))^3-1/8*Pi^2*
x^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/32*Pi^2*x^2*csgn(I*exp(2*
b*x+2*a)/(exp(2*b*x+2*a)+1))^6-1/16*Pi^2*x^2*csgn(I*exp(b*x+a))^2*csgn(I*ex
p(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/8*Pi^2*x^2*csgn
(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)+1))^3+1/12*I
*Pi*b*x^3*csgn(I*exp(2*b*x+2*a))^3-1/16*Pi^2*x^2*csgn(I/(exp(2*b*x+2*a)+1))
*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2
*b*x+2*a)+1))+1/16*Pi^2*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))^2
*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/8*Pi^
2*x^2*csgn(I/(exp(2*b*x+2*a)+1))^6-1/8*Pi^2*x^2*csgn(I*exp(2*b*x+2*a))^3-1/
```

```
32*Pi^2*x^2*csgn(I*exp(2*b*x+2*a))^6-1/8*Pi^2*x^2*csgn(I/(exp(2*b*x+2*a)+1)
)^4+1/4*Pi^2*x^2*csgn(I/(exp(2*b*x+2*a)+1))^2+(-1/3*b*x^3+1/4*I*Pi*x^2*csgn
(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/4*I*Pi
*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a
)/(exp(2*b*x+2*a)+1))+1/4*I*Pi*x^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+
2*a)/(exp(2*b*x+2*a)+1))^2-1/2*I*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))^3-1/4*I*
Pi*x^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/2*I*Pi*x^2*csgn(I/(exp
(2*b*x+2*a)+1))^2+1/2*I*Pi*x^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-
1/4*I*Pi*x^2*csgn(I*exp(2*b*x+2*a))^3-1/2*I*Pi*x^2-1/4*I*Pi*x^2*csgn(I*exp(
b*x+a))^2*csgn(I*exp(2*b*x+2*a)))*ln(exp(b*x+a))+1/12*I*Pi*b*x^3*csgn(I*exp
(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/8*Pi^2*x^2*csgn(I/(exp(2*b*x+2*a)+1))^2
*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+1/12*x^4*b^2+1/8*Pi^2*x^2*csgn
(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(
2*b*x+2*a)+1))^2+1/8*Pi^2*x^2*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(2*b*x
+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/6*I*Pi*b*x^3*csgn(I/(exp
(2*b*x+2*a)+1))^3+1/4*Pi^2*x^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*
csgn(I/(exp(2*b*x+2*a)+1))^3-1/8*Pi^2*x^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b
*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/8*Pi^2*x^2*csgn(I/
(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x
+2*a)+1))+1/6*I*Pi*b*x^3+1/12*I*Pi*b*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*
exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/8*Pi^2*x^2*csgn
(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*
b*x+2*a)+1))^4+1/8*Pi^2*x^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/6*I*Pi*b*x^3*csgn(I/(exp(2*b*x+2
*a)+1))^2-1/8*Pi^2*x^2*csgn(I/(exp(2*b*x+2*a)+1))^4*csgn(I*exp(2*b*x+2*a))*
csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/6*I*Pi*b*x^3*csgn(I*exp(b*x+a))
*csgn(I*exp(2*b*x+2*a))^2-1/12*I*Pi*b*x^3*csgn(I*exp(2*b*x+2*a))*csgn(I*exp
(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/8*Pi^2*x^2*csgn(I/(exp(2*b*x+2*a)+1))*c
sgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x
+2*a)+1))-1/8*Pi^2*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))*csgn(I
*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/12*I*Pi*b*
x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+
1/12*I*Pi*b*x^3*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))
```

Maxima [A] time = 1.36817, size = 49, normalized size = 1.44

$$\frac{1}{12}\,b^2x^4 - \frac{1}{3}\,bx^3\,\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right) + \frac{1}{2}\,x^2\,\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/12*b^2*x^4 - 1/3*b*x^3*arccoth(tanh(b*x + a)) + 1/2*x^2*arccoth(tanh(b*x
+ a))^2
```

Fricas [A] time = 1.52462, size = 70, normalized size = 2.06

$$\frac{1}{4}\,b^2x^4 + \frac{2}{3}\,abx^3 - \frac{1}{8}\left(\pi^2 - 4\,a^2\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/4*b^2*x^4 + 2/3*a*b*x^3 - 1/8*(pi^2 - 4*a^2)*x^2
```

Sympy [A] time = 0.586048, size = 37, normalized size = 1.09

$$\frac{b^2x^4}{12} - \frac{bx^3 \operatorname{acoth}(\tanh(a+bx))}{3} + \frac{x^2 \operatorname{acoth}^2(\tanh(a+bx))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(tanh(b*x+a))**2,x)
```

```
[Out] b**2*x**4/12 - b*x**3*acoth(tanh(a + b*x))/3 + x**2*acoth(tanh(a + b*x))**2
/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arcoth}(\tanh(bx+a))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(tanh(b*x + a))^2, x)
```

3.140 $\int \coth^{-1}(\tanh(a+bx))^2\,dx$

Optimal. Leaf size=16

$$\frac{\coth^{-1}(\tanh(a+bx))^3}{3b}$$

```
[Out] ArcCoth[Tanh[a + b*x]]^3/(3*b)
```

Rubi [A] time = 0.0053532, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2157, 30}

$$\frac{\coth^{-1}(\tanh(a+bx))^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] ArcCoth[Tanh[a + b*x]]^3/(3*b)
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \coth^{-1}(\tanh(a+bx))^2\,dx &= \frac{\text{Subst}\left(\int x^2\,dx, x, \coth^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{\coth^{-1}(\tanh(a+bx))^3}{3b}\end{aligned}$$

Mathematica [A] time = 0.00554, size = 16, normalized size = 1.

$$\frac{\coth^{-1}(\tanh(a+bx))^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] ArcCoth[Tanh[a + b*x]]^3/(3*b)
```

Maple [A] time = 0.098, size = 15, normalized size = 0.9

$$\frac{(\operatorname{arccoth}(\tanh(bx+a)))^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^2,x)
```

```
[Out] 1/3*arccoth(tanh(b*x+a))^3/b
```

Maxima [B] time = 1.37255, size = 45, normalized size = 2.81

$$\frac{1}{3}b^2x^3 - bx^2 \operatorname{arccoth}(\tanh(bx+a)) + x \operatorname{arccoth}(\tanh(bx+a))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3 - b*x^2*arccoth(tanh(b*x + a)) + x*arccoth(tanh(b*x + a))^2
```

Fricas [A] time = 1.62762, size = 62, normalized size = 3.88

$$\frac{1}{3}b^2x^3 + abx^2 - \frac{1}{4}\left(\pi^2 - 4a^2\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/3*b^2*x^3 + a*b*x^2 - 1/4*(pi^2 - 4*a^2)*x
```

Sympy [A] time = 0.414211, size = 20, normalized size = 1.25

$$\begin{cases} \frac{\operatorname{acoth}^3(\tanh(a+bx))}{3b} & \text{for } b \neq 0 \\ x \operatorname{acoth}^2(\tanh(a)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**2,x)
```

```
[Out] Piecewise((acoth(tanh(a + b*x))**3/(3*b), Ne(b, 0)), (x*acoth(tanh(a))**2,
True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(\tanh(bx+a))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^2, x)
```

3.141 $\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x} dx$

Optimal. Leaf size=49

$$-bx\left(bx-\coth^{-1}(\tanh(a+bx))\right)+\frac{1}{2}\coth^{-1}(\tanh(a+bx))^2+\log(x)\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2$$

```
[Out] -(b*x*(b*x - ArcCoth[Tanh[a + b*x]])) + ArcCoth[Tanh[a + b*x]]^2/2 + (b*x -
 ArcCoth[Tanh[a + b*x]])^2*Log[x]
```

Rubi [A] time = 0.0259427, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2159, 2158, 29}

$$-bx\left(bx-\coth^{-1}(\tanh(a+bx))\right)+\frac{1}{2}\coth^{-1}(\tanh(a+bx))^2+\log(x)\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^2/x,x]
```

```
[Out] -(b*x*(b*x - ArcCoth[Tanh[a + b*x]])) + ArcCoth[Tanh[a + b*x]]^2/2 + (b*x -
 ArcCoth[Tanh[a + b*x]])^2*Log[x]
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x}\,dx &= \frac{1}{2}\coth^{-1}(\tanh(a+bx))^2-\left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{\coth^{-1}(\tanh(a+bx))}{x}\,dx\\ &= -bx\left(bx-\coth^{-1}(\tanh(a+bx))\right)+\frac{1}{2}\coth^{-1}(\tanh(a+bx))^2-\left(\left(bx-\coth^{-1}(\tanh(a+\right.\right.\\ &= -bx\left(bx-\coth^{-1}(\tanh(a+bx))\right)+\frac{1}{2}\coth^{-1}(\tanh(a+bx))^2+\left(bx-\coth^{-1}(\tanh(a+b\right.\end{aligned}$$

Mathematica [A] time = 0.0427117, size = 53, normalized size = 1.08

$$\frac{1}{2}(a+bx)^2-(a+bx)\left(-2\coth^{-1}(\tanh(a+bx))+a+2bx\right)+\log(bx)\left(\coth^{-1}(\tanh(a+bx))-bx\right)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^2/x,x]
```

```
[Out] (a + b*x)^2/2 - (a + b*x)*(a + 2*b*x - 2*ArcCoth[Tanh[a + b*x]]) + (-(b*x)
+ ArcCoth[Tanh[a + b*x]])^2*Log[b*x]
```

Maple [C] time = 0.284, size = 3774, normalized size = 77.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^2/x,x)
```

```
[Out] -1/8*ln(x)*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))^2*csgn(I*exp(
2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+I*Pi*b*x*csgn(I*exp
(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/2*I*ln(exp(b*x+a))*ln(x)*Pi*csgn(I*exp(
2*b*x+2*a))^3-1/2*I*Pi*x*b*csgn(I*exp(2*b*x+2*a))^3-1/2*I*Pi*x*b*csgn(I*exp
(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/8*ln(x)*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))
*csgn(I*exp(2*b*x+2*a))^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/2*I*l
n(exp(b*x+a))*ln(x)*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/8*ln(x
)*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x
+2*a)/(exp(2*b*x+2*a)+1))^3+1/2*I*ln(x)*Pi*x*b*csgn(I*exp(2*b*x+2*a)/(exp(2
*b*x+2*a)+1))^3+1/2*I*Pi*x*b*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(
exp(2*b*x+2*a)+1))^2-1/2*I*ln(x)*Pi*x*b*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2
*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/2*I*ln(exp(b*x+a))*ln(x)*Pi*csgn(I/(exp(2
*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+
1))+b^2*x^2*ln(x)+2*b*ln(exp(b*x+a))*x-1/2*Pi^2*ln(x)*csgn(I/(exp(2*b*x+2*a
)+1))^3+1/2*Pi^2*ln(x)*csgn(I/(exp(2*b*x+2*a)+1))^2-1/2*I*Pi*x*b*csgn(I/(ex
p(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*
a)+1))+1/2*ln(x)*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(b*x+a))*csgn(
I*exp(2*b*x+2*a))^2-1/16*ln(x)*Pi^2*csgn(I*exp(b*x+a))^4*csgn(I*exp(2*b*x+2
*a))^2+1/4*ln(x)*Pi^2*csgn(I*exp(b*x+a))^3*csgn(I*exp(2*b*x+2*a))^3-1/4*ln(
x)*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+
2*a))+1/4*ln(x)*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(2*b*x+2*a))*cs
gn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/16*ln(x)*Pi^2*csgn(I*exp(2*b*x+
2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4+1/8*ln(x)*Pi^2*csgn(I*e
xp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^5-1/4*ln(x)*Pi^2*c
sgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3*csgn(I/(exp(2*b*x+2*a)+1))^3+1/4
*ln(x)*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a
)+1))^2-I*ln(exp(b*x+a))*ln(x)*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+I*ln(exp(b*x
+a))*ln(x)*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2-I*Pi*x*b*csgn(I/(exp(2*b*x+2*a)+
1))^3+1/8*ln(x)*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))^3*cs
gn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/16*ln(x)*Pi^2*csgn(I/(exp(2*b*x
+2*a)+1))^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+
1))^2+I*Pi*x*b*csgn(I/(exp(2*b*x+2*a)+1))^2-1/4*ln(x)*Pi^2*csgn(I*exp(b*x+a
))*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-3/2
*b^2*x^2+1/8*ln(x)*Pi^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))^2*csgn(
I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/8*ln(x)*Pi^2*csgn(I/(exp(2*b*x+2*a
)+1))*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-
1/4*ln(x)*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp
(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/4*ln(x)*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^4
*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/4*ln(x)
*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+
2*a)/(exp(2*b*x+2*a)+1))-1/2*ln(x)*Pi^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x
+2*a))^2*csgn(I/(exp(2*b*x+2*a)+1))^2-1/4*ln(x)*Pi^2*csgn(I*exp(2*b*x+2*a))
```

```
*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2*csgn(I/(exp(2*b*x+2*a)+1))^2+1
/4*ln(x)*Pi^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x
+2*a)+1))^2+1/2*ln(x)*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^5-1/4*ln(x)*Pi^2*csgn
(I/(exp(2*b*x+2*a)+1))^4-1/4*ln(x)*Pi^2*csgn(I*exp(2*b*x+2*a))^3-1/16*ln(x)
*Pi^2*csgn(I*exp(2*b*x+2*a))^6-1/4*ln(x)*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^6-
1/4*ln(x)*Pi^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-I*ln(exp(b*x+a))
*ln(x)*Pi-I*Pi*x*b+1/4*ln(x)*Pi^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))
^3*csgn(I/(exp(2*b*x+2*a)+1))^2-1/16*ln(x)*Pi^2*csgn(I*exp(2*b*x+2*a)/(exp(
2*b*x+2*a)+1))^6-3/8*ln(x)*Pi^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))
^4+1/4*ln(x)*Pi^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^5+1/8*ln(x)*Pi^
2*csgn(I*exp(2*b*x+2*a))^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/8*
ln(x)*Pi^2*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1
))^3-1/16*ln(x)*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a)/(ex
p(2*b*x+2*a)+1))^4+1/4*ln(x)*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^4*csgn(I*exp(2
*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/4*ln(x)*Pi^2*csgn(I*exp(2*b*x+2*a))^3*csg
n(I/(exp(2*b*x+2*a)+1))^3-1/4*Pi^2*ln(x)*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*
b*x+2*a))+1/2*Pi^2*ln(x)*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/4*Pi
^2*ln(x)*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2
-1/4*ln(x)*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b
*x+2*a)+1))^2+1/4*ln(x)*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+
2*a))^3+1/2*I*ln(x)*Pi*x*b*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)
)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-2*b*ln(exp(b*x+a))*ln(x)*x+I*ln
(x)*Pi*x*b+ln(x)*ln(exp(b*x+a))^2+1/8*ln(x)*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))
*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^5-1/8*ln(x)*Pi^2*csgn(I*exp(b*x+
a))^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/
4*ln(x)*Pi^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2
*a)/(exp(2*b*x+2*a)+1))^3-1/4*ln(x)*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*
exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4-I*ln(x)*Pi*x*b*
csgn(I/(exp(2*b*x+2*a)+1))^2-1/4*Pi^2*ln(x)+I*ln(x)*Pi*x*b*csgn(I/(exp(2*b*
x+2*a)+1))^3+1/4*ln(x)*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))*c
sgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/4*ln(x)
*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^
2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/2*I*ln(x)*Pi*x*b*csgn(I*exp
(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-I*ln(x)*Pi*x*b*csgn(I*exp(b*x+a))*csgn(I*
exp(2*b*x+2*a))^2+1/2*I*ln(exp(b*x+a))*ln(x)*Pi*csgn(I/(exp(2*b*x+2*a)+1))*
csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/2*I*ln(exp(b*x+a))*ln(x)*Pi*c
sgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+1/2*I*ln(x)*Pi*x*b*csgn(I*exp(2*
b*x+2*a))^3+1/2*I*ln(exp(b*x+a))*ln(x)*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp
(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/2*I*Pi*x*b*csgn(I*exp(b*x+a))^2*csgn(I*
exp(2*b*x+2*a))+1/8*ln(x)*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a)
)^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+I*ln
(exp(b*x+a))*ln(x)*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/2*I*Pi*
b*x*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-
1/2*I*ln(x)*Pi*x*b*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*
b*x+2*a)+1))^2
```

Maxima [C] time = 2.47337, size = 51, normalized size = 1.04

$$\frac{1}{2}b^2x^2 + \frac{1}{8}(-8i\pi b + 16ab)x - \frac{1}{4}\left(\pi^2 + 4i\pi a - 4a^2\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2/x,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2 + 1/8*(-8*I*pi*b + 16*a*b)*x - 1/4*(pi^2 + 4*I*pi*a - 4*a^2)*lo
g(x)
```

Fricas [A] time = 1.65362, size = 69, normalized size = 1.41

$$\frac{1}{2}\,b^2x^2 + 2\,abx - \frac{1}{4}\left(\pi^2 - 4\,a^2\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2/x,x, algorithm="fricas")
```

```
[Out] 1/2*b^2*x^2 + 2*a*b*x - 1/4*(pi^2 - 4*a^2)*log(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^2\left(\tanh\left(a + bx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**2/x,x)
```

```
[Out] Integral(acoth(tanh(a + b*x))**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}\left(\tanh\left(bx + a\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^2/x, x)
```

3.142 $\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2}\,dx$

Optimal. Leaf size=39

$$-\frac{\coth^{-1}(\tanh(a+bx))^2}{x}-2b\log(x)\left(bx-\coth^{-1}(\tanh(a+bx))\right)+2b^2x$$

```
[Out] 2*b^2*x - ArcCoth[Tanh[a + b*x]]^2/x - 2*b*(b*x - ArcCoth[Tanh[a + b*x]])*L
og[x]
```

Rubi [A] time = 0.0258467, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2158, 29}

$$-\frac{\coth^{-1}(\tanh(a+bx))^2}{x}-2b\log(x)\left(bx-\coth^{-1}(\tanh(a+bx))\right)+2b^2x$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^2/x^2,x]
```

```
[Out] 2*b^2*x - ArcCoth[Tanh[a + b*x]]^2/x - 2*b*(b*x - ArcCoth[Tanh[a + b*x]])*L
og[x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2}\,dx &= -\frac{\coth^{-1}(\tanh(a+bx))^2}{x}+(2b)\int \frac{\coth^{-1}(\tanh(a+bx))}{x}\,dx\\ &= 2b^2x-\frac{\coth^{-1}(\tanh(a+bx))^2}{x}-\left(2b\left(bx-\coth^{-1}(\tanh(a+bx))\right)\right)\int \frac{1}{x}\,dx\\ &= 2b^2x-\frac{\coth^{-1}(\tanh(a+bx))^2}{x}-2b\left(bx-\coth^{-1}(\tanh(a+bx))\right)\log(x)\end{aligned}$$

Mathematica [A] time = 0.0489712, size = 37, normalized size = 0.95

$$-\frac{\coth^{-1}(\tanh(a+bx))^2}{x}+2b(\log(x)+1)\coth^{-1}(\tanh(a+bx))-2b^2x\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^2/x^2,x]
```

```
[Out] -(ArcCoth[Tanh[a + b*x]]^2/x) - 2*b^2*x*Log[x] + 2*b*ArcCoth[Tanh[a + b*x]]
*(1 + Log[x])
```

Maple [C] time = 0.21, size = 1095, normalized size = 28.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^2/x^2,x)
```

```
[Out] -I*Pi*ln(exp(b*x+a))/x*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/2*I*Pi
*ln(exp(b*x+a))/x*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2
*a)+1))^2+1/2*I*Pi*ln(x)*b*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)
/(exp(2*b*x+2*a)+1))^2+I*Pi*ln(x)*b*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a
))^2+2*b^2*x-1/2*I*Pi*ln(x)*b*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2
*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/2*I*Pi*ln(exp(b*x+a))/x*cs
gn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(
2*b*x+2*a)+1))-2*ln(x)*x*b^2+2*ln(x)*ln(exp(b*x+a))*b-I*Pi*ln(x)*b*csgn(I/(
exp(2*b*x+2*a)+1))^3-I*Pi*ln(exp(b*x+a))/x*csgn(I/(exp(2*b*x+2*a)+1))^2+I*P
i*ln(x)*b*csgn(I/(exp(2*b*x+2*a)+1))^2+1/2*I*Pi*ln(exp(b*x+a))/x*csgn(I*exp
(2*b*x+2*a))^3+1/16*Pi^2*(-2*csgn(I/(exp(2*b*x+2*a)+1))^2+csgn(I/(exp(2*b*x
+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-
csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*cs
gn(I/(exp(2*b*x+2*a)+1))^3+csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*cs
gn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+csgn(I*exp(2*b*x+2*a))^3-csgn(I*e
xp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+csgn(I*exp(2*b*x
+2*a)/(exp(2*b*x+2*a)+1))^3+2)^2/x+1/2*I*Pi*ln(exp(b*x+a))/x*csgn(I*exp(b*x
+a))^2*csgn(I*exp(2*b*x+2*a))+1/2*I*Pi*ln(x)*b*csgn(I*exp(2*b*x+2*a))*csgn(
I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/2*I*Pi*ln(x)*b*csgn(I*exp(b*x+a))^
2*csgn(I*exp(2*b*x+2*a))-1/2*I*Pi*ln(exp(b*x+a))/x*csgn(I/(exp(2*b*x+2*a)+1
))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/2*I*Pi*ln(x)*b*csgn(I*exp(
2*b*x+2*a))^3-1/2*I*Pi*ln(x)*b*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+
1/2*I*Pi*ln(exp(b*x+a))/x*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+I*Pi*
ln(exp(b*x+a))/x-I*Pi*ln(x)*b+I*Pi*ln(exp(b*x+a))/x*csgn(I/(exp(2*b*x+2*a)+
1))^3-1/x*ln(exp(b*x+a))^2
```

Maxima [A] time = 1.19632, size = 73, normalized size = 1.87

$$2\,b\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)\log\left(x\right)-2\left(b\left(x+\frac{a}{b}\right)\log\left(x\right)-b\left(x+\frac{a\log\left(x\right)}{b}\right)\right)b-\frac{\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2/x^2,x, algorithm="maxima")
```

```
[Out] 2*b*arccoth(tanh(b*x + a))*log(x) - 2*(b*(x + a/b)*log(x) - b*(x + a*log(x)
/b))*b - arccoth(tanh(b*x + a))^2/x
```

Fricas [A] time = 1.67534, size = 69, normalized size = 1.77

$$\frac{4\,b^2x^2 + 8\,abx\log\left(x\right) + \pi^2 - 4\,a^2}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^2*x^2 + 8*a*b*x*log(x) + pi^2 - 4*a^2)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^2\left(\tanh\left(a + bx\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**2/x**2,x)
```

```
[Out] Integral(acoth(tanh(a + b*x))**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}\left(\tanh\left(bx + a\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^2/x^2, x)
```

3.143 $\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^3}\,dx$

Optimal. Leaf size=36

$$-\frac{\coth^{-1}(\tanh(a+bx))^2}{2x^2}-\frac{b\coth^{-1}(\tanh(a+bx))}{x}+b^2\log(x)$$

```
[Out] -((b*ArcCoth[Tanh[a + b*x]])/x) - ArcCoth[Tanh[a + b*x]]^2/(2*x^2) + b^2*Lo
g[x]
```

Rubi [A] time = 0.0228216, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 29}

$$-\frac{\coth^{-1}(\tanh(a+bx))^2}{2x^2}-\frac{b\coth^{-1}(\tanh(a+bx))}{x}+b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^2/x^3,x]
```

```
[Out] -((b*ArcCoth[Tanh[a + b*x]])/x) - ArcCoth[Tanh[a + b*x]]^2/(2*x^2) + b^2*Lo
g[x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^3}\,dx &= -\frac{\coth^{-1}(\tanh(a+bx))^2}{2x^2}+b\int \frac{\coth^{-1}(\tanh(a+bx))}{x^2}\,dx\\ &= -\frac{b\coth^{-1}(\tanh(a+bx))}{x}-\frac{\coth^{-1}(\tanh(a+bx))^2}{2x^2}+b^2\int \frac{1}{x}\,dx\\ &= -\frac{b\coth^{-1}(\tanh(a+bx))}{x}-\frac{\coth^{-1}(\tanh(a+bx))^2}{2x^2}+b^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.0337682, size = 42, normalized size = 1.17

$$-\frac{2bx\coth^{-1}(\tanh(a+bx))+\coth^{-1}(\tanh(a+bx))^2-b^2x^2(2\log(x)+3)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^2/x^3,x]
```

```
[Out] -(2*b*x*ArcCoth[Tanh[a + b*x]] + ArcCoth[Tanh[a + b*x]]^2 - b^2*x^2*(3 + 2*
Log[x]))/(2*x^2)
```

Maple [C] time = 0.375, size = 3213, normalized size = 89.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^2/x^3,x)
```

```
[Out] -1/2/x^2*ln(exp(b*x+a))^2-1/4*(4*b*x-2*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+I*
Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-I*Pi*
csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+2*I*Pi*csgn(I/(exp(2*b*x+2*a)+1
))^2-2*I*Pi-I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-I*Pi*csgn(I/(e
xp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2
*a)+1))+2*I*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-I*Pi*csgn(I*exp(
2*b*x+2*a))^3+I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*
b*x+2*a)+1))^2)/x^2*ln(exp(b*x+a))+1/32*(6*Pi^2*csgn(I*exp(b*x+a))^2*csgn(I
*exp(2*b*x+2*a))^4+8*I*Pi*b*x*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2
*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+4*Pi^2+4*Pi^2*csgn(I*exp(2*b
*x+2*a)/(exp(2*b*x+2*a)+1))^3+Pi^2*csgn(I*exp(2*b*x+2*a))^6+4*Pi^2*csgn(I*e
xp(2*b*x+2*a))^3-4*Pi^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2
*b*x+2*a)+1))^2+8*I*Pi*b*x*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+8*I*
Pi*b*x*csgn(I*exp(2*b*x+2*a))^3-4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^4*csgn(I*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^3*cs
gn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*Pi^2*csgn(I*exp(2*b*x+2*a))^3*c
sgn(I/(exp(2*b*x+2*a)+1))^3+Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*
b*x+2*a)/(exp(2*b*x+2*a)+1))^4+32*b^2*x^2*ln(x)+4*Pi^2*csgn(I/(exp(2*b*x+2*
a)+1))^6-2*Pi^2*csgn(I*exp(2*b*x+2*a))^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2
*a)+1))^2+2*Pi^2*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+
2*a)+1))^3-4*Pi^2*csgn(I*exp(2*b*x+2*a))^3*csgn(I/(exp(2*b*x+2*a)+1))^2+4*P
i^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2*csgn
(I/(exp(2*b*x+2*a)+1))^2-8*Pi^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2
-8*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2+8*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^3-4*
Pi^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^5-2*Pi^2*csgn(I*exp(2*b*x+2*
a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^5+Pi^2*csgn(I*exp(2*b*x+2*a))
^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4-2*Pi^2*csgn(I/(exp(2*b*x+2*a
)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^5+16*I*Pi*x*b+4*Pi^2*csgn(I
/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-4*Pi^2*c
sgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3*csgn(I/(exp(2*b*x+2*a)+1))^2+4*P
i^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-16*I*Pi*x*b*csgn(I/(exp(2*b
*x+2*a)+1))^2+16*I*Pi*x*b*csgn(I/(exp(2*b*x+2*a)+1))^3-4*Pi^2*csgn(I*exp(b*
x+a))^3*csgn(I*exp(2*b*x+2*a))^3-4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*e
xp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^3*csg
n(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+Pi^2*csgn(I*e
xp(b*x+a))^4*csgn(I*exp(2*b*x+2*a))^2-8*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^5+4
*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^4-2*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn
(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+2*Pi^2*csgn(
I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))^4*csgn(I*exp(2*b*x+2*a)/(exp(2
*b*x+2*a)+1))-2*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))^3*cs
gn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-2*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))
*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*Pi^
2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*
b*x+2*a)+1))^2+8*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(b*x+a))*csgn(
```

```
I*exp(2*b*x+2*a))^2+4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^4*csgn(I*exp(2*b*x+2*
a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+4*Pi^2*csgn(I/(exp(2*b*x+2*a)
+1))^3*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-8*Pi^2*csgn(I/(exp(2*b*x
+2*a)+1))^3*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-4*Pi^2*csgn(I/(exp(
2*b*x+2*a)+1))^3*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*
a)+1))^2+Pi^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^6+4*Pi^2*csgn(I/(ex
p(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*
a)+1))-2*Pi^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*
x+2*a)/(exp(2*b*x+2*a)+1))^2-16*I*Pi*b*x*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*
x+2*a))^2-8*I*Pi*b*x*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*
x+2*a)+1))^2+8*I*Pi*x*b*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-4*Pi^2*
csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*
x+2*a)+1))^3+2*Pi^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(
2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-8*I*Pi*b*x*csgn(I/(exp(2*b*x+2*a)+1))*csgn
(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2
*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+4*Pi^2*csgn(I/(exp(2*b*x+2*a)+
1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4+Pi^2
*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a
)/(exp(2*b*x+2*a)+1))^2-2*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a)
)^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+4*Pi
^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*c
sgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1)
)*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*
b*x+2*a)+1))+2*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))^2*csgn(I*
exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)))/x^2
```

Maxima [A] time = 1.40233, size = 46, normalized size = 1.28

$$b^2 \log(x) - \frac{b \operatorname{arcoth}(\tanh(bx+a))}{x} - \frac{\operatorname{arcoth}(\tanh(bx+a))^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2/x^3,x, algorithm="maxima")
```

```
[Out] b^2*log(x) - b*arccoth(tanh(b*x + a))/x - 1/2*arccoth(tanh(b*x + a))^2/x^2
```

Fricas [A] time = 1.6086, size = 73, normalized size = 2.03

$$\frac{8\,b^2x^2\log(x) - 16\,abx + \pi^2 - 4\,a^2}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(8*b^2*x^2*log(x) - 16*a*b*x + pi^2 - 4*a^2)/x^2
```

Sympy [A] time = 0.752501, size = 32, normalized size = 0.89

$$b^2 \log(x) - \frac{b \operatorname{acoth}(\tanh(a+bx))}{x} - \frac{\operatorname{acoth}^2(\tanh(a+bx))}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**2/x**3,x)
```

```
[Out] b**2*log(x) - b*acoth(tanh(a + b*x))/x - acoth(tanh(a + b*x))**2/(2*x**2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^{2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^2/x^3, x)
```

3.144 $\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4} dx$

Optimal. Leaf size=31

$$\frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

```
[Out] ArcCoth[Tanh[a + b*x]]^3/(3*x^3*(b*x - ArcCoth[Tanh[a + b*x]]))
```

Rubi [A] time = 0.0135441, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2167}

$$\frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^2/x^4,x]
```

```
[Out] ArcCoth[Tanh[a + b*x]]^3/(3*x^3*(b*x - ArcCoth[Tanh[a + b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4} dx = \frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A] time = 0.0474149, size = 34, normalized size = 1.1

$$-\frac{bx\coth^{-1}(\tanh(a+bx))+\coth^{-1}(\tanh(a+bx))^2+b^2x^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^2/x^4,x]
```

```
[Out] -(b^2*x^2 + b*x*ArcCoth[Tanh[a + b*x]] + ArcCoth[Tanh[a + b*x]]^2)/(3*x^3)
```

Maple [C] time = 0.357, size = 3217, normalized size = 103.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b*x+a))^2/x^4,x)

[Out] -1/3/x^3*ln(exp(b*x+a))^2-1/6*(2*b*x+I*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-I*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*I*Pi+2*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2-I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-I*Pi*csgn(I*exp(2*b*x+2*a))^3+2*I*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-2*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3)/x^3*ln(exp(b*x+a))-1/48*(-4*I*Pi*x*b*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-6*Pi^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))^4-4*Pi^2-4*Pi^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-Pi^2*csgn(I*exp(2*b*x+2*a))^6-4*Pi^2*csgn(I*exp(2*b*x+2*a))^3+4*Pi^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-4*Pi^2*csgn(I*exp(2*b*x+2*a))^3*csgn(I/(exp(2*b*x+2*a)+1))^3-Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4-4*I*Pi*b*x*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+4*I*Pi*b*x*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+8*I*Pi*b*x*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+4*I*Pi*b*x*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-4*I*Pi*x*b*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-4*I*Pi*b*x*csgn(I*exp(2*b*x+2*a))^3-4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^6+2*Pi^2*csgn(I*exp(2*b*x+2*a))^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-2*Pi^2*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+16*b^2*x^2+4*Pi^2*csgn(I*exp(2*b*x+2*a))^3*csgn(I/(exp(2*b*x+2*a)+1))^2-4*Pi^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2*csgn(I/(exp(2*b*x+2*a)+1))^2+8*I*Pi*x*b*csgn(I/(exp(2*b*x+2*a)+1))^2-8*I*Pi*x*b*csgn(I/(exp(2*b*x+2*a)+1))^3+8*Pi^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+8*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2-8*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^3+4*Pi^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^5+2*Pi^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^5-Pi^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4+2*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^5-4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+4*Pi^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3*csgn(I/(exp(2*b*x+2*a)+1))^2-4*Pi^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-8*I*Pi*x*b+4*Pi^2*csgn(I*exp(b*x+a))^3*csgn(I*exp(2*b*x+2*a))^3+4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi^2*csgn(I*exp(b*x+a))^4*csgn(I*exp(2*b*x+2*a))^2+8*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^5-4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^4+2*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-2*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+2*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-4*Pi^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-8*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^4*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+8*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-Pi^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^6-4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+2*Pi^2*csgn(I*exp(b*x+a))^2*csg

```
n(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+4*Pi^2*cs
gn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+
2*a)+1))^3-2*Pi^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*
b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*e
xp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(
I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4-Pi^2*csgn(I/(
exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*
b*x+2*a)+1))^2+2*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))^2*csgn(
I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-4*Pi^2*csgn(I
/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp
(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*
exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+
1))-2*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x
+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)))/x^3
```

Maxima [A] time = 1.40024, size = 49, normalized size = 1.58

$$-\frac{b^2}{3\,x}-\frac{b\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)}{3\,x^2}-\frac{\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^2/x - 1/3*b*arccoth(tanh(b*x + a))/x^2 - 1/3*arccoth(tanh(b*x + a))^
2/x^3
```

Fricas [A] time = 1.58129, size = 68, normalized size = 2.19

$$-\frac{12\,b^2x^2+12\,abx-\pi^2+4\,a^2}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(12*b^2*x^2 + 12*a*b*x - pi^2 + 4*a^2)/x^3
```

Sympy [A] time = 1.18208, size = 37, normalized size = 1.19

$$-\frac{b^2}{3x}-\frac{b\,\mathrm{acoth}\left(\tanh\left(a+bx\right)\right)}{3x^2}-\frac{\mathrm{acoth}^2\left(\tanh\left(a+bx\right)\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**2/x**4,x)
```

```
[Out] -b**2/(3*x) - b*acoth(tanh(a + b*x))/(3*x**2) - acoth(tanh(a + b*x))**2/(3*
x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(\tanh(bx+a))^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^2/x^4, x)
```

3.145 $\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^5}\,dx$

Optimal. Leaf size=64

$$\frac{\coth^{-1}(\tanh(a+bx))^3}{4x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)}+\frac{b\coth^{-1}(\tanh(a+bx))^3}{12x^3\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}$$

```
[Out] (b*ArcCoth[Tanh[a + b*x]]^3)/(12*x^3*(b*x - ArcCoth[Tanh[a + b*x]])^2) + Ar
cCoth[Tanh[a + b*x]]^3/(4*x^4*(b*x - ArcCoth[Tanh[a + b*x]]))
```

Rubi [A] time = 0.0351377, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2171, 2167}

$$\frac{\coth^{-1}(\tanh(a+bx))^3}{4x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)}+\frac{b\coth^{-1}(\tanh(a+bx))^3}{12x^3\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^2/x^5,x]
```

```
[Out] (b*ArcCoth[Tanh[a + b*x]]^3)/(12*x^3*(b*x - ArcCoth[Tanh[a + b*x]])^2) + Ar
cCoth[Tanh[a + b*x]]^3/(4*x^4*(b*x - ArcCoth[Tanh[a + b*x]]))
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^5}\,dx &= \frac{\coth^{-1}(\tanh(a+bx))^3}{4x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)}+\frac{b\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4}\,dx}{4\left(bx-\coth^{-1}(\tanh(a+bx))\right)}\\ &= \frac{b\coth^{-1}(\tanh(a+bx))^3}{12x^3\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}+\frac{\coth^{-1}(\tanh(a+bx))^3}{4x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)}\end{aligned}$$

Mathematica [A] time = 0.0306651, size = 37, normalized size = 0.58

$$-\frac{2bx\coth^{-1}(\tanh(a+bx))+3\coth^{-1}(\tanh(a+bx))^2+b^2x^2}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^2/x^5,x]
```

```
[Out] -(b^2*x^2 + 2*b*x*ArcCoth[Tanh[a + b*x]] + 3*ArcCoth[Tanh[a + b*x]]^2)/(12*
x^4)
```

Maple [C] time = 0.362, size = 3217, normalized size = 50.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^2/x^5,x)
```

```
[Out] -1/4/x^4*ln(exp(b*x+a))^2-1/24*(4*b*x+3*I*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-3*I*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp
(2*b*x+2*a))-6*I*Pi-3*I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-3*I*
Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/
(exp(2*b*x+2*a)+1))+6*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2-3*I*Pi*csgn(I*exp(2
*b*x+2*a))^3+6*I*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+3*I*Pi*csgn
(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-6*I*Pi*c
sgn(I/(exp(2*b*x+2*a)+1))^3)/x^4*ln(exp(b*x+a))-1/192*(-18*Pi^2*csgn(I*exp(
b*x+a))^2*csgn(I*exp(2*b*x+2*a))^4-12*Pi^2-12*Pi^2*csgn(I*exp(2*b*x+2*a)/(e
xp(2*b*x+2*a)+1))^3-16*I*Pi*x*b*csgn(I/(exp(2*b*x+2*a)+1))^3-3*Pi^2*csgn(I*
exp(2*b*x+2*a))^6-8*I*Pi*b*x*csgn(I*exp(2*b*x+2*a))^3-8*I*Pi*b*x*csgn(I*exp
(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+16*I*Pi*x*b*csgn(I/(exp(2*b*x+2*a)+1))^2-
12*Pi^2*csgn(I*exp(2*b*x+2*a))^3+12*Pi^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(
2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+12*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^4*csgn(
I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-12*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^3
*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-12*Pi^2*csgn(I*exp(2*b*x+2*a))
^3*csgn(I/(exp(2*b*x+2*a)+1))^3-3*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4-8*I*Pi*b*x*csgn(I/(exp(2*b*x+2*a)+1))*
csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-12*Pi^2*cs
gn(I/(exp(2*b*x+2*a)+1))^6+6*Pi^2*csgn(I*exp(2*b*x+2*a))^4*csgn(I*exp(2*b*x
+2*a)/(exp(2*b*x+2*a)+1))^2-6*Pi^2*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*
x+2*a)/(exp(2*b*x+2*a)+1))^3+16*b^2*x^2+8*I*Pi*b*x*csgn(I/(exp(2*b*x+2*a)+1
))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+12*Pi^2*csgn(I*exp(2*b*x+2*a
))^3*csgn(I/(exp(2*b*x+2*a)+1))^2-12*Pi^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp
(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2*csgn(I/(exp(2*b*x+2*a)+1))^2+16*I*Pi*b*x*
csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+8*I*Pi*b*x*csgn(I*exp(2*b*x+2*a
))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-8*I*Pi*x*b*csgn(I*exp(b*x+a)
)^2*csgn(I*exp(2*b*x+2*a))+24*Pi^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a)
)^2+24*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2-24*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))
^3+12*Pi^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^5+6*Pi^2*csgn(I*exp(2*
b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^5-3*Pi^2*csgn(I*exp(2*b
*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4+6*Pi^2*csgn(I/(exp(2
*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^5-12*Pi^2*csgn(I/(e
xp(2*b*x+2*a)+1))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+12*Pi^2*csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3*csgn(I/(exp(2*b*x+2*a)+1))^2-12*Pi
^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-16*I*Pi*x*b+12*Pi^2*csgn(I*e
xp(b*x+a))^3*csgn(I*exp(2*b*x+2*a))^3+12*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*cs
gn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+12*Pi^2*csgn(I/(exp(2*b*x+2*a)+1)
)^3*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-3*Pi^2
*csgn(I*exp(b*x+a))^4*csgn(I*exp(2*b*x+2*a))^2+24*Pi^2*csgn(I/(exp(2*b*x+2*
a)+1))^5-12*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^4+6*Pi^2*csgn(I/(exp(2*b*x+2*a)
+1))^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-6
```

```
*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))^4*csgn(I*exp(2*b*x+
2*a)/(exp(2*b*x+2*a)+1))+6*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x
+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+6*Pi^2*csgn(I/(exp(2*b
*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+
1))^3-12*Pi^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+
2*a)/(exp(2*b*x+2*a)+1))^2-24*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(
b*x+a))*csgn(I*exp(2*b*x+2*a))^2-12*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^4*csgn(
I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-12*Pi^2*csgn(I/
(exp(2*b*x+2*a)+1))^3*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+24*Pi^2*c
sgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+12*
Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2
*a)/(exp(2*b*x+2*a)+1))^2-3*Pi^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^
6-12*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*
x+2*a)/(exp(2*b*x+2*a)+1))+6*Pi^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a
))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+12*Pi^2*csgn(I*exp(b*x+a))
*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-6*Pi^
2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*
b*x+2*a)+1))^3+12*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(b*x+a))^2*cs
gn(I*exp(2*b*x+2*a))-12*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*
a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4-3*Pi^2*csgn(I/(exp(2*b*x+2*
a)+1))^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))
^2+6*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+
2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-12*Pi^2*csgn(I/(exp(2*b*x
+2*a)+1))*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)
/(exp(2*b*x+2*a)+1))^2+12*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a)
)*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-6*Pi^2
*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))^2*c
sgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)))/x^4
```

Maxima [A] time = 1.40713, size = 49, normalized size = 0.77

$$-\frac{b^2}{12\,x^2}-\frac{b\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)}{6\,x^3}-\frac{\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/12*b^2/x^2 - 1/6*b*arccoth(tanh(b*x + a))/x^3 - 1/4*arccoth(tanh(b*x + a
))^2/x^4
```

Fricas [A] time = 1.61147, size = 72, normalized size = 1.12

$$-\frac{24\,b^2x^2+32\,abx-3\,\pi^2+12\,a^2}{48\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/48*(24*b^2*x^2 + 32*a*b*x - 3*pi^2 + 12*a^2)/x^4
```

Sympy [A] time = 1.92498, size = 39, normalized size = 0.61

$$-\frac{b^2}{12x^2}-\frac{b\,\mathrm{acoth}\left(\tanh\left(a+bx\right)\right)}{6x^3}-\frac{\mathrm{acoth}^2\left(\tanh\left(a+bx\right)\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**2/x**5,x)
```

```
[Out] -b**2/(12*x**2) - b*acoth(tanh(a + b*x))/(6*x**3) - acoth(tanh(a + b*x))**2
/(4*x**4)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^2/x^5, x)
```

3.146 $\int x^m \coth^{-1}(\tanh(a+bx))^3\,dx$

Optimal. Leaf size=110

$$\frac{6b^2x^{m+3}\coth^{-1}(\tanh(a+bx))}{m^3+6m^2+11m+6}-\frac{3bx^{m+2}\coth^{-1}(\tanh(a+bx))^2}{m^2+3m+2}+\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))^3}{m+1}-\frac{6b^3x^{m}}{(m+1)\left(m^3+9m\right.}$$

```
[Out] (-6*b^3*x^(4 + m))/((1 + m)*(24 + 26*m + 9*m^2 + m^3)) + (6*b^2*x^(3 + m)*A
rcCoth[Tanh[a + b*x]])/(6 + 11*m + 6*m^2 + m^3) - (3*b*x^(2 + m)*ArcCoth[Ta
nh[a + b*x]]^2)/(2 + 3*m + m^2) + (x^(1 + m)*ArcCoth[Tanh[a + b*x]]^3)/(1 +
 m)
```

Rubi [A] time = 0.0593837, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$\frac{6b^2x^{m+3}\coth^{-1}(\tanh(a+bx))}{m^3+6m^2+11m+6}-\frac{3bx^{m+2}\coth^{-1}(\tanh(a+bx))^2}{m^2+3m+2}+\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))^3}{m+1}-\frac{6b^3x^{m}}{(m+1)\left(m^3+9m\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] (-6*b^3*x^(4 + m))/((1 + m)*(24 + 26*m + 9*m^2 + m^3)) + (6*b^2*x^(3 + m)*A
rcCoth[Tanh[a + b*x]])/(6 + 11*m + 6*m^2 + m^3) - (3*b*x^(2 + m)*ArcCoth[Ta
nh[a + b*x]]^2)/(2 + 3*m + m^2) + (x^(1 + m)*ArcCoth[Tanh[a + b*x]]^3)/(1 +
 m)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^m \coth^{-1}(\tanh(a+bx))^3\,dx &= \frac{x^{1+m}\coth^{-1}(\tanh(a+bx))^3}{1+m}-\frac{(3b)\int x^{1+m}\coth^{-1}(\tanh(a+bx))^2\,dx}{1+m}\\
&= -\frac{3bx^{2+m}\coth^{-1}(\tanh(a+bx))^2}{2+3m+m^2}+\frac{x^{1+m}\coth^{-1}(\tanh(a+bx))^3}{1+m}+\frac{\left(6b^2\right)\int x^{2+m}\text{co}}{2-}\\
&= \frac{6b^2x^{3+m}\coth^{-1}(\tanh(a+bx))}{6+11m+6m^2+m^3}-\frac{3bx^{2+m}\coth^{-1}(\tanh(a+bx))^2}{2+3m+m^2}+\frac{x^{1+m}\coth^{-1}(\text{t}}{1+}\\
&= -\frac{6b^3x^{4+m}}{(4+m)\left(6+11m+6m^2+m^3\right)}+\frac{6b^2x^{3+m}\coth^{-1}(\tanh(a+bx))}{6+11m+6m^2+m^3}-\frac{3bx^{2+m}\text{coth}}{2+}
\end{aligned}$$

Mathematica [A] time = 0.113509, size = 97, normalized size = 0.88

$$\frac{x^{m+1}\left(6b^2(m+4)x^2\coth^{-1}(\tanh(a+bx))-3b\left(m^2+7m+12\right)x\coth^{-1}(\tanh(a+bx))^2+\left(m^3+9m^2+26m+24\right)\coth$$
$$(m+1)(m+2)(m+3)(m+4)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] (x^(1 + m)*(-6*b^3*x^3 + 6*b^2*(4 + m)*x^2*ArcCoth[Tanh[a + b*x]] - 3*b*(12
 + 7*m + m^2)*x*ArcCoth[Tanh[a + b*x]]^2 + (24 + 26*m + 9*m^2 + m^3)*ArcCot
h[Tanh[a + b*x]]^3))/((1 + m)*(2 + m)*(3 + m)*(4 + m))
```

Maple [C] time = 6.384, size = 63382, normalized size = 576.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arccoth(tanh(b*x+a))^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71674, size = 468, normalized size = 4.25

$$\left(4\left(b^3m^3+6\,b^3m^2+11\,b^3m+6\,b^3\right)x^4+12\left(ab^2m^3+7\,ab^2m^2+14\,ab^2m+8\,ab^2\right)x^3+3\left(4\,a^2bm^3+32\,a^2bm^2+76\,a^2bm\right.$$
$$4\left(m^4+10\,m^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(b^3*m^3 + 6*b^3*m^2 + 11*b^3*m + 6*b^3)*x^4 + 12*(a*b^2*m^3 + 7*a*b
^2*m^2 + 14*a*b^2*m + 8*a*b^2)*x^3 + 3*(4*a^2*b*m^3 + 32*a^2*b*m^2 + 76*a^2
*b*m - pi^2*(b*m^3 + 8*b*m^2 + 19*b*m + 12*b) + 48*a^2*b)*x^2 + (4*a^3*m^3
+ 36*a^3*m^2 + 104*a^3*m - 3*pi^2*(a*m^3 + 9*a*m^2 + 26*a*m + 24*a) + 96*a^
3)*x)*x^m/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*acoth(tanh(b*x+a))**3,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{arcoth}(\tanh(bx+a))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*arccoth(tanh(b*x + a))^3, x)
```

3.147 $\int x^4 \coth^{-1}(\tanh(a+bx))^3 \, dx$

Optimal. Leaf size=61

$$\frac{1}{35}b^2x^7\coth^{-1}(\tanh(a+bx))-\frac{1}{10}bx^6\coth^{-1}(\tanh(a+bx))^2+\frac{1}{5}x^5\coth^{-1}(\tanh(a+bx))^3-\frac{1}{280}b^3x^8$$

```
[Out] -(b^3*x^8)/280 + (b^2*x^7*ArcCoth[Tanh[a + b*x]])/35 - (b*x^6*ArcCoth[Tanh[
a + b*x]]^2)/10 + (x^5*ArcCoth[Tanh[a + b*x]]^3)/5
```

Rubi [A] time = 0.0411641, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$\frac{1}{35}b^2x^7\coth^{-1}(\tanh(a+bx))-\frac{1}{10}bx^6\coth^{-1}(\tanh(a+bx))^2+\frac{1}{5}x^5\coth^{-1}(\tanh(a+bx))^3-\frac{1}{280}b^3x^8$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] -(b^3*x^8)/280 + (b^2*x^7*ArcCoth[Tanh[a + b*x]])/35 - (b*x^6*ArcCoth[Tanh[
a + b*x]]^2)/10 + (x^5*ArcCoth[Tanh[a + b*x]]^3)/5
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^4\coth^{-1}(\tanh(a+bx))^3\,dx &= \frac{1}{5}x^5\coth^{-1}(\tanh(a+bx))^3-\frac{1}{5}(3b)\int x^5\coth^{-1}(\tanh(a+bx))^2\,dx\\ &= -\frac{1}{10}bx^6\coth^{-1}(\tanh(a+bx))^2+\frac{1}{5}x^5\coth^{-1}(\tanh(a+bx))^3+\frac{1}{5}b^2\int x^6\coth^{-1}(\tan\\ &= \frac{1}{35}b^2x^7\coth^{-1}(\tanh(a+bx))-\frac{1}{10}bx^6\coth^{-1}(\tanh(a+bx))^2+\frac{1}{5}x^5\coth^{-1}(\tanh(a+\\ &= -\frac{1}{280}b^3x^8+\frac{1}{35}b^2x^7\coth^{-1}(\tanh(a+bx))-\frac{1}{10}bx^6\coth^{-1}(\tanh(a+bx))^2+\frac{1}{5}x^5\cot\end{aligned}$$

Mathematica [A] time = 0.030511, size = 54, normalized size = 0.89

$$-\frac{1}{280}x^5\left(-8b^2x^2\coth^{-1}(\tanh(a+bx))+28bx\coth^{-1}(\tanh(a+bx))^2-56\coth^{-1}(\tanh(a+bx))^3+b^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] -(x^5*(b^3*x^3 - 8*b^2*x^2*ArcCoth[Tanh[a + b*x]] + 28*b*x*ArcCoth[Tanh[a +
 b*x]]^2 - 56*ArcCoth[Tanh[a + b*x]]^3))/280
```

Maple [C] time = 1.102, size = 18111, normalized size = 296.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccoth(tanh(b*x+a))^3,x)
```

```
[Out] result too large to display
```

Maxima [A] time = 1.54533, size = 73, normalized size = 1.2

$$-\frac{1}{10}\,bx^{6}\,\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^{2}+\frac{1}{5}\,x^{5}\,\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^{3}-\frac{1}{280}\,\left(b^{2}x^{8}-8\,bx^{7}\,\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -1/10*b*x^6*arccoth(tanh(b*x + a))^2 + 1/5*x^5*arccoth(tanh(b*x + a))^3 - 1
/280*(b^2*x^8 - 8*b*x^7*arccoth(tanh(b*x + a)))*b
```

Fricas [A] time = 1.59236, size = 119, normalized size = 1.95

$$\frac{1}{8}\,b^{3}x^{8}+\frac{3}{7}\,ab^{2}x^{7}-\frac{1}{8}\,\left(\pi^{2}b-4\,a^{2}b\right)x^{6}-\frac{1}{20}\,\left(3\,\pi^{2}a-4\,a^{3}\right)x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/8*b^3*x^8 + 3/7*a*b^2*x^7 - 1/8*(pi^2*b - 4*a^2*b)*x^6 - 1/20*(3*pi^2*a -
 4*a^3)*x^5
```

Sympy [A] time = 6.05556, size = 56, normalized size = 0.92

$$-\frac{b^{3}x^{8}}{280}+\frac{b^{2}x^{7}\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}{35}-\frac{bx^{6}\operatorname{acoth}^{2}\left(\tanh\left(a+bx\right)\right)}{10}+\frac{x^{5}\operatorname{acoth}^{3}\left(\tanh\left(a+bx\right)\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acoth(tanh(b*x+a))**3,x)
```

```
[Out] -b**3*x**8/280 + b**2*x**7*acoth(tanh(a + b*x))/35 - b*x**6*acoth(tanh(a +
b*x))**2/10 + x**5*acoth(tanh(a + b*x))**3/5
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{arcoth}(\tanh(bx+a))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(x^4*arccoth(tanh(b*x + a))^3, x)
```

3.148 $\int x^3 \coth^{-1}(\tanh(a+bx))^3\,dx$

Optimal. Leaf size=61

$$\frac{1}{20}b^2x^6\coth^{-1}(\tanh(a+bx))-\frac{3}{20}bx^5\coth^{-1}(\tanh(a+bx))^2+\frac{1}{4}x^4\coth^{-1}(\tanh(a+bx))^3-\frac{1}{140}b^3x^7$$

```
[Out] -(b^3*x^7)/140 + (b^2*x^6*ArcCoth[Tanh[a + b*x]])/20 - (3*b*x^5*ArcCoth[Tan
h[a + b*x]]^2)/20 + (x^4*ArcCoth[Tanh[a + b*x]]^3)/4
```

Rubi [A] time = 0.0407912, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$\frac{1}{20}b^2x^6\coth^{-1}(\tanh(a+bx))-\frac{3}{20}bx^5\coth^{-1}(\tanh(a+bx))^2+\frac{1}{4}x^4\coth^{-1}(\tanh(a+bx))^3-\frac{1}{140}b^3x^7$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] -(b^3*x^7)/140 + (b^2*x^6*ArcCoth[Tanh[a + b*x]])/20 - (3*b*x^5*ArcCoth[Tan
h[a + b*x]]^2)/20 + (x^4*ArcCoth[Tanh[a + b*x]]^3)/4
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^3\coth^{-1}(\tanh(a+bx))^3\,dx &= \frac{1}{4}x^4\coth^{-1}(\tanh(a+bx))^3-\frac{1}{4}(3b)\int x^4\coth^{-1}(\tanh(a+bx))^2\,dx\\ &= -\frac{3}{20}bx^5\coth^{-1}(\tanh(a+bx))^2+\frac{1}{4}x^4\coth^{-1}(\tanh(a+bx))^3+\frac{1}{10}\left(3b^2\right)\int x^5\,\text{cot}\\ &= \frac{1}{20}b^2x^6\coth^{-1}(\tanh(a+bx))-\frac{3}{20}bx^5\coth^{-1}(\tanh(a+bx))^2+\frac{1}{4}x^4\coth^{-1}(\tanh(\\ &= -\frac{1}{140}b^3x^7+\frac{1}{20}b^2x^6\coth^{-1}(\tanh(a+bx))-\frac{3}{20}bx^5\coth^{-1}(\tanh(a+bx))^2+\frac{1}{4}x^4\end{aligned}$$

Mathematica [A] time = 0.0271736, size = 54, normalized size = 0.89

$$-\frac{1}{140}x^4\left(-7b^2x^2\coth^{-1}(\tanh(a+bx))+21bx\coth^{-1}(\tanh(a+bx))^2-35\coth^{-1}(\tanh(a+bx))^3+b^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] -(x^4*(b^3*x^3 - 7*b^2*x^2*ArcCoth[Tanh[a + b*x]] + 21*b*x*ArcCoth[Tanh[a +
 b*x]]^2 - 35*ArcCoth[Tanh[a + b*x]]^3))/140
```

Maple [C] time = 1.096, size = 18111, normalized size = 296.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccoth(tanh(b*x+a))^3,x)
```

```
[Out] result too large to display
```

Maxima [A] time = 1.56117, size = 73, normalized size = 1.2

$$-\frac{3}{20}\,bx^5\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2+\frac{1}{4}\,x^4\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^3-\frac{1}{140}\left(b^2x^7-7\,bx^6\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -3/20*b*x^5*arccoth(tanh(b*x + a))^2 + 1/4*x^4*arccoth(tanh(b*x + a))^3 - 1
/140*(b^2*x^7 - 7*b*x^6*arccoth(tanh(b*x + a)))*b
```

Fricas [A] time = 1.5684, size = 120, normalized size = 1.97

$$\frac{1}{7}\,b^3x^7+\frac{1}{2}\,ab^2x^6-\frac{3}{20}\left(\pi^2b-4\,a^2b\right)x^5-\frac{1}{16}\left(3\,\pi^2a-4\,a^3\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/7*b^3*x^7 + 1/2*a*b^2*x^6 - 3/20*(pi^2*b - 4*a^2*b)*x^5 - 1/16*(3*pi^2*a
- 4*a^3)*x^4
```

Sympy [A] time = 3.559, size = 58, normalized size = 0.95

$$-\frac{b^3x^7}{140}+\frac{b^2x^6\,\mathrm{acoth}\left(\tanh\left(a+bx\right)\right)}{20}-\frac{3bx^5\,\mathrm{acoth}^2\left(\tanh\left(a+bx\right)\right)}{20}+\frac{x^4\,\mathrm{acoth}^3\left(\tanh\left(a+bx\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acoth(tanh(b*x+a))**3,x)
```

```
[Out] -b**3*x**7/140 + b**2*x**6*acoth(tanh(a + b*x))/20 - 3*b*x**5*acoth(tanh(a
+ b*x))**2/20 + x**4*acoth(tanh(a + b*x))**3/4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arcoth}(\tanh(bx+a))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*arccoth(tanh(b*x + a))^3, x)
```

3.149 $\int x^2 \coth^{-1}(\tanh(a+bx))^3 \, dx$

Optimal. Leaf size=53

$$\frac{\coth^{-1}(\tanh(a+bx))^6}{60b^3}-\frac{x\coth^{-1}(\tanh(a+bx))^5}{10b^2}+\frac{x^2\coth^{-1}(\tanh(a+bx))^4}{4b}$$

```
[Out] (x^2*ArcCoth[Tanh[a + b*x]]^4)/(4*b) - (x*ArcCoth[Tanh[a + b*x]]^5)/(10*b^2
) + ArcCoth[Tanh[a + b*x]]^6/(60*b^3)
```

Rubi [A] time = 0.0312969, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2157, 30}

$$\frac{\coth^{-1}(\tanh(a+bx))^6}{60b^3}-\frac{x\coth^{-1}(\tanh(a+bx))^5}{10b^2}+\frac{x^2\coth^{-1}(\tanh(a+bx))^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] (x^2*ArcCoth[Tanh[a + b*x]]^4)/(4*b) - (x*ArcCoth[Tanh[a + b*x]]^5)/(10*b^2
) + ArcCoth[Tanh[a + b*x]]^6/(60*b^3)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\coth^{-1}(\tanh(a+bx))^3\,dx &= \frac{x^2\coth^{-1}(\tanh(a+bx))^4}{4b}-\frac{\int x\coth^{-1}(\tanh(a+bx))^4\,dx}{2b}\\ &= \frac{x^2\coth^{-1}(\tanh(a+bx))^4}{4b}-\frac{x\coth^{-1}(\tanh(a+bx))^5}{10b^2}+\frac{\int \coth^{-1}(\tanh(a+bx))^5\,dx}{10b^2}\\ &= \frac{x^2\coth^{-1}(\tanh(a+bx))^4}{4b}-\frac{x\coth^{-1}(\tanh(a+bx))^5}{10b^2}+\frac{\text{Subst}\left(\int x^5\,dx,x,\coth^{-1}(\text{tan}\right.}{10b^3}\\ &= \frac{x^2\coth^{-1}(\tanh(a+bx))^4}{4b}-\frac{x\coth^{-1}(\tanh(a+bx))^5}{10b^2}+\frac{\coth^{-1}(\tanh(a+bx))^6}{60b^3}\end{aligned}$$

Mathematica [A] time = 0.0222424, size = 54, normalized size = 1.02

$$-\frac{1}{60}x^3\left(-6b^2x^2\coth^{-1}(\tanh(a+bx))+15bx\coth^{-1}(\tanh(a+bx))^2-20\coth^{-1}(\tanh(a+bx))^3+b^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] -(x^3*(b^3*x^3 - 6*b^2*x^2*ArcCoth[Tanh[a + b*x]] + 15*b*x*ArcCoth[Tanh[a +
 b*x]]^2 - 20*ArcCoth[Tanh[a + b*x]]^3))/60
```

Maple [C] time = 1.081, size = 18111, normalized size = 341.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(tanh(b*x+a))^3,x)
```

```
[Out] result too large to display
```

Maxima [A] time = 1.54597, size = 73, normalized size = 1.38

$$-\frac{1}{4}bx^4\operatorname{arcoth}(\tanh(bx+a))^2+\frac{1}{3}x^3\operatorname{arcoth}(\tanh(bx+a))^3-\frac{1}{60}\left(b^2x^6-6bx^5\operatorname{arcoth}(\tanh(bx+a))\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -1/4*b*x^4*arccoth(tanh(b*x + a))^2 + 1/3*x^3*arccoth(tanh(b*x + a))^3 - 1/
60*(b^2*x^6 - 6*b*x^5*arccoth(tanh(b*x + a)))*b
```

Fricas [A] time = 1.55735, size = 120, normalized size = 2.26

$$\frac{1}{6}b^3x^6+\frac{3}{5}ab^2x^5-\frac{3}{16}\left(\pi^2b-4a^2b\right)x^4-\frac{1}{12}\left(3\pi^2a-4a^3\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/6*b^3*x^6 + 3/5*a*b^2*x^5 - 3/16*(pi^2*b - 4*a^2*b)*x^4 - 1/12*(3*pi^2*a
- 4*a^3)*x^3
```

Sympy [A] time = 3.3897, size = 60, normalized size = 1.13

$$\begin{cases}\frac{x^2\operatorname{acoth}^4(\tanh(a+bx))}{4b}-\frac{x\operatorname{acoth}^5(\tanh(a+bx))}{10b^2}+\frac{\operatorname{acoth}^6(\tanh(a+bx))}{60b^3} & \text{for } b\neq 0\\ \frac{x^3\operatorname{acoth}^3(\tanh(a))}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(tanh(b*x+a))**3,x)
```

```
[Out] Piecewise((x**2*acoth(tanh(a + b*x))**4/(4*b) - x*acoth(tanh(a + b*x))**5/(
10*b**2) + acoth(tanh(a + b*x))**6/(60*b**3), Ne(b, 0)), (x**3*acoth(tanh(a
))**3/3, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arcoth}(\tanh(bx+a))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(tanh(b*x + a))^3, x)
```

3.150 $\int x \coth^{-1}(\tanh(a+bx))^3 \, dx$

Optimal. Leaf size=34

$$\frac{x \coth^{-1}(\tanh(a+bx))^4}{4b} - \frac{\coth^{-1}(\tanh(a+bx))^5}{20b^2}$$

```
[Out] (x*ArcCoth[Tanh[a + b*x]]^4)/(4*b) - ArcCoth[Tanh[a + b*x]]^5/(20*b^2)
```

Rubi [A] time = 0.0149233, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2168, 2157, 30}

$$\frac{x \coth^{-1}(\tanh(a+bx))^4}{4b} - \frac{\coth^{-1}(\tanh(a+bx))^5}{20b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] (x*ArcCoth[Tanh[a + b*x]]^4)/(4*b) - ArcCoth[Tanh[a + b*x]]^5/(20*b^2)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x \coth^{-1}(\tanh(a+bx))^3 \, dx &= \frac{x \coth^{-1}(\tanh(a+bx))^4}{4b} - \frac{\int \coth^{-1}(\tanh(a+bx))^4 \, dx}{4b} \\
&= \frac{x \coth^{-1}(\tanh(a+bx))^4}{4b} - \frac{\text{Subst}\left(\int x^4 \, dx, x, \coth^{-1}(\tanh(a+bx))\right)}{4b^2} \\
&= \frac{x \coth^{-1}(\tanh(a+bx))^4}{4b} - \frac{\coth^{-1}(\tanh(a+bx))^5}{20b^2}
\end{aligned}$$

Mathematica [B] time = 0.0736339, size = 99, normalized size = 2.91

$$\frac{(a+bx)\left(10\left(2a^2+abx-b^2x^2\right)\coth^{-1}(\tanh(a+bx))^2+(4a-bx)(a+bx)^3-5(3a-bx)(a+bx)^2\coth^{-1}(\tanh(a+b\cdots}{20b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] ((a + b*x)*((4*a - b*x)*(a + b*x)^3 - 5*(3*a - b*x)*(a + b*x)^2*ArcCoth[Tan
h[a + b*x]] + 10*(2*a^2 + a*b*x - b^2*x^2)*ArcCoth[Tanh[a + b*x]]^2 - 10*(a
 - b*x)*ArcCoth[Tanh[a + b*x]]^3))/(20*b^2)
```

Maple [C] time = 1.083, size = 18111, normalized size = 532.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(tanh(b*x+a))^3,x)
```

```
[Out] result too large to display
```

Maxima [A] time = 1.55891, size = 73, normalized size = 2.15

$$-\frac{1}{2}\,bx^3\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2+\frac{1}{2}\,x^2\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^3-\frac{1}{20}\,\left(b^2x^5-5\,bx^4\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -1/2*b*x^3*arccoth(tanh(b*x + a))^2 + 1/2*x^2*arccoth(tanh(b*x + a))^3 - 1/
20*(b^2*x^5 - 5*b*x^4*arccoth(tanh(b*x + a)))*b
```

Fricas [A] time = 1.50232, size = 117, normalized size = 3.44

$$\frac{1}{5}\,b^3x^5+\frac{3}{4}\,ab^2x^4-\frac{1}{4}\,\left(\pi^2b-4\,a^2b\right)x^3-\frac{1}{8}\,\left(3\,\pi^2a-4\,a^3\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/5*b^3*x^5 + 3/4*a*b^2*x^4 - 1/4*(pi^2*b - 4*a^2*b)*x^3 - 1/8*(3*pi^2*a -
4*a^3)*x^2
```

Sympy [A] time = 1.73176, size = 41, normalized size = 1.21

$$\begin{cases}\frac{x\,\mathrm{acoth}^4\left(\tanh\left(a+bx\right)\right)}{4b}-\frac{\mathrm{acoth}^5\left(\tanh\left(a+bx\right)\right)}{20b^2} & \text{for } b\neq 0\\ \frac{x^2\,\mathrm{acoth}^3\left(\tanh\left(a\right)\right)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(tanh(b*x+a))**3,x)
```

```
[Out] Piecewise((x*acoth(tanh(a + b*x))**4/(4*b) - acoth(tanh(a + b*x))**5/(20*b*
*2), Ne(b, 0)), (x**2*acoth(tanh(a))**3/2, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arcoth}(\tanh(bx+a))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(tanh(b*x + a))^3, x)
```

3.151 $\int \coth^{-1}(\tanh(a+bx))^3\,dx$

Optimal. Leaf size=16

$$\frac{\coth^{-1}(\tanh(a+bx))^4}{4b}$$

```
[Out] ArcCoth[Tanh[a + b*x]]^4/(4*b)
```

Rubi [A] time = 0.0051923, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2157, 30}

$$\frac{\coth^{-1}(\tanh(a+bx))^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] ArcCoth[Tanh[a + b*x]]^4/(4*b)
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \coth^{-1}(\tanh(a+bx))^3\,dx &= \frac{\text{Subst}\left(\int x^3\,dx, x, \coth^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{\coth^{-1}(\tanh(a+bx))^4}{4b}\end{aligned}$$

Mathematica [A] time = 0.0056073, size = 16, normalized size = 1.

$$\frac{\coth^{-1}(\tanh(a+bx))^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] ArcCoth[Tanh[a + b*x]]^4/(4*b)
```

Maple [A] time = 0.106, size = 15, normalized size = 0.9

$$\frac{(\operatorname{arccoth}(\tanh(bx+a)))^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^3,x)
```

```
[Out] 1/4*arccoth(tanh(b*x+a))^4/b
```

Maxima [B] time = 1.55637, size = 69, normalized size = 4.31

$$-\frac{3}{2}bx^2\operatorname{arcoth}(\tanh(bx+a))^2 + x\operatorname{arcoth}(\tanh(bx+a))^3 - \frac{1}{4}\left(b^2x^4 - 4bx^3\operatorname{arcoth}(\tanh(bx+a))\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -3/2*b*x^2*arccoth(tanh(b*x + a))^2 + x*arccoth(tanh(b*x + a))^3 - 1/4*(b^2
*x^4 - 4*b*x^3*arccoth(tanh(b*x + a)))*b
```

Fricas [B] time = 1.59564, size = 109, normalized size = 6.81

$$\frac{1}{4}b^3x^4 + ab^2x^3 - \frac{3}{8}\left(\pi^2 b - 4a^2 b\right)x^2 - \frac{1}{4}\left(3\pi^2 a - 4a^3\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/4*b^3*x^4 + a*b^2*x^3 - 3/8*(pi^2*b - 4*a^2*b)*x^2 - 1/4*(3*pi^2*a - 4*a^
3)*x
```

Sympy [A] time = 0.803763, size = 20, normalized size = 1.25

$$\begin{cases} \frac{\operatorname{acoth}^4(\tanh(a+bx))}{4b} & \text{for } b \neq 0 \\ x\operatorname{acoth}^3(\tanh(a)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**3,x)
```

```
[Out] Piecewise((acoth(tanh(a + b*x))**4/(4*b), Ne(b, 0)), (x*acoth(tanh(a))**3,
True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}(\tanh(bx+a))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^3, x)
```

3.152 $\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x}\,dx$

Optimal. Leaf size=77

$$bx\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2-\frac{1}{2}\coth^{-1}(\tanh(a+bx))^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)+\frac{1}{3}\coth^{-1}(\tanh(a+bx))^3$$

```
[Out] b*x*(b*x - ArcCoth[Tanh[a + b*x]])^2 - ((b*x - ArcCoth[Tanh[a + b*x]])*ArcC
oth[Tanh[a + b*x]]^2)/2 + ArcCoth[Tanh[a + b*x]]^3/3 - (b*x - ArcCoth[Tanh[
a + b*x]])^3*Log[x]
```

Rubi [A] time = 0.0962767, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2159, 2158, 29}

$$bx\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2-\frac{1}{2}\coth^{-1}(\tanh(a+bx))^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)+\frac{1}{3}\coth^{-1}(\tanh(a+bx))^3$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^3/x,x]
```

```
[Out] b*x*(b*x - ArcCoth[Tanh[a + b*x]])^2 - ((b*x - ArcCoth[Tanh[a + b*x]])*ArcC
oth[Tanh[a + b*x]]^2)/2 + ArcCoth[Tanh[a + b*x]]^3/3 - (b*x - ArcCoth[Tanh[
a + b*x]])^3*Log[x]
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x}\,dx &= \frac{1}{3}\coth^{-1}(\tanh(a+bx))^3-\left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x}\,d\\ &= -\frac{1}{2}\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2+\frac{1}{3}\coth^{-1}(\tanh(a+bx))^3\\ &= bx\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2-\frac{1}{2}\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+\\ &= bx\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2-\frac{1}{2}\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+\end{aligned}$$

Mathematica [A] time = 0.0580561, size = 104, normalized size = 1.35

$$(a+bx)\left(a^2-3a\left(-\coth^{-1}(\tanh(a+bx))+a+bx\right)+3\left(-\coth^{-1}(\tanh(a+bx))+a+bx\right)^2\right)+\frac{1}{3}(a+bx)^3-\frac{1}{2}(a+bx)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^3/x,x]
```

```
[Out] (a + b*x)^3/3 + (a + b*x)*(a^2 - 3*a*(a + b*x - ArcCoth[Tanh[a + b*x]]) + 3
*(a + b*x - ArcCoth[Tanh[a + b*x]])^2) - ((a + b*x)^2*(2*a + 3*b*x - 3*ArcC
oth[Tanh[a + b*x]]))/2 + (-(b*x) + ArcCoth[Tanh[a + b*x]])^3*Log[b*x]
```

Maple [C] time = 0.847, size = 21848, normalized size = 283.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^3/x,x)
```

```
[Out] result too large to display
```

Maxima [C] time = 2.51028, size = 101, normalized size = 1.31

$$\frac{1}{3}b^3x^3+\frac{1}{24}\left(-18i\pi b^2+36ab^2\right)x^2-\frac{1}{24}\left(18\pi^2b+72i\pi ab-72a^2b\right)x+\frac{1}{8}\left(i\pi^3-6\pi^2a-12i\pi a^2+8a^3\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3 + 1/24*(-18*I*pi*b^2 + 36*a*b^2)*x^2 - 1/24*(18*pi^2*b + 72*I*p
i*a*b - 72*a^2*b)*x + 1/8*(I*pi^3 - 6*pi^2*a - 12*I*pi*a^2 + 8*a^3)*log(x)
```

Fricas [A] time = 1.69985, size = 119, normalized size = 1.55

$$\frac{1}{3}b^3x^3+\frac{3}{2}ab^2x^2-\frac{3}{4}\left(\pi^2b-4a^2b\right)x-\frac{1}{4}\left(3\pi^2a-4a^3\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x,x, algorithm="fricas")
```

```
[Out] 1/3*b^3*x^3 + 3/2*a*b^2*x^2 - 3/4*(pi^2*b - 4*a^2*b)*x - 1/4*(3*pi^2*a - 4*
a^3)*log(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{acoth}^3(\tanh(a+bx))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**3/x,x)
```

```
[Out] Integral(acoth(tanh(a + b*x))**3/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^{3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^3/x, x)
```

3.153 $\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^2}\,dx$

Optimal. Leaf size=68

$$-3b^2x\left(bx-\coth^{-1}(\tanh(a+bx))\right)-\frac{\coth^{-1}(\tanh(a+bx))^3}{x}+\frac{3}{2}b\coth^{-1}(\tanh(a+bx))^2+3b\log(x)\left(bx-\coth^{-1}(\tan\right.$$

```
[Out] -3*b^2*x*(b*x - ArcCoth[Tanh[a + b*x]]) + (3*b*ArcCoth[Tanh[a + b*x]]^2)/2
- ArcCoth[Tanh[a + b*x]]^3/x + 3*b*(b*x - ArcCoth[Tanh[a + b*x]])^2*Log[x]
```

Rubi [A] time = 0.0435156, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2168, 2159, 2158, 29}

$$-3b^2x\left(bx-\coth^{-1}(\tanh(a+bx))\right)-\frac{\coth^{-1}(\tanh(a+bx))^3}{x}+\frac{3}{2}b\coth^{-1}(\tanh(a+bx))^2+3b\log(x)\left(bx-\coth^{-1}(\tan\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^3/x^2,x]
```

```
[Out] -3*b^2*x*(b*x - ArcCoth[Tanh[a + b*x]]) + (3*b*ArcCoth[Tanh[a + b*x]]^2)/2
- ArcCoth[Tanh[a + b*x]]^3/x + 3*b*(b*x - ArcCoth[Tanh[a + b*x]])^2*Log[x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^2}\,dx = -\frac{\coth^{-1}(\tanh(a+bx))^3}{x} + (3b)\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x}\,dx$$
$$= \frac{3}{2}b\coth^{-1}(\tanh(a+bx))^2 - \frac{\coth^{-1}(\tanh(a+bx))^3}{x} - \left(3b\left(bx - \coth^{-1}(\tanh(a+bx\right.\right.$$
$$= -3b^2x\left(bx - \coth^{-1}(\tanh(a+bx))\right) + \frac{3}{2}b\coth^{-1}(\tanh(a+bx))^2 - \frac{\coth^{-1}(\tanh(a+}{x}$$
$$= -3b^2x\left(bx - \coth^{-1}(\tanh(a+bx))\right) + \frac{3}{2}b\coth^{-1}(\tanh(a+bx))^2 - \frac{\coth^{-1}(\tanh(a+}{x}$$

Mathematica [A] time = 0.039466, size = 62, normalized size = 0.91

$$-6b^2x\log(x)\coth^{-1}(\tanh(a+bx)) - \frac{\coth^{-1}(\tanh(a+bx))^3}{x} + 3b(\log(x)+1)\coth^{-1}(\tanh(a+bx))^2 + \frac{3}{2}b^3x^2(2\log($$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^3/x^2,x]
```

```
[Out] -(ArcCoth[Tanh[a + b*x]]^3/x) - 6*b^2*x*ArcCoth[Tanh[a + b*x]]*Log[x] + 3*b
*ArcCoth[Tanh[a + b*x]]^2*(1 + Log[x]) + (3*b^3*x^2*(-1 + 2*Log[x]))/2
```

Maple [C] time = 0.415, size = 7683, normalized size = 113.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^3/x^2,x)
```

```
[Out] result too large to display
```

Maxima [C] time = 2.08232, size = 166, normalized size = 2.44

$$3\,b\,\mathrm{arcoth}\,(\tanh\,(bx+a))^2\log\,(x) - \frac{3}{2}\left(2\,\mathrm{arcoth}\,(\tanh\,(bx+a))^2\log\,(x) - \left(bx^2 + (2i\,\pi + 4\,a)x + 2\left(-\frac{i\,\pi(bx+a)}{b} - \right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^2,x, algorithm="maxima")
```

```
[Out] 3*b*arccoth(tanh(b*x + a))^2*log(x) - 3/2*(2*arccoth(tanh(b*x + a))^2*log(x
) - (b*x^2 + (2*I*pi + 4*a)*x + 2*(-I*pi*(b*x + a)/b - (b*x + a)^2/b)*log(x
) + 2*arccoth(tanh(b*x + a))^2*log(x)/b + 2*(I*pi*a + a^2)*log(x)/b)*b)*b -
 arccoth(tanh(b*x + a))^3/x
```

Fricas [A] time = 1.72542, size = 115, normalized size = 1.69

$$\frac{2\,b^3x^3 + 12\,ab^2x^2 + 3\,\pi^2a - 4\,a^3 - 3\left(\pi^2b - 4\,a^2b\right)x\log\,(x)}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^3*x^3 + 12*a*b^2*x^2 + 3*pi^2*a - 4*a^3 - 3*(pi^2*b - 4*a^2*b)*x*l
og(x))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^3(\tanh(a+bx))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**3/x**2,x)
```

```
[Out] Integral(acoth(tanh(a + b*x))**3/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(\tanh(bx+a))^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^3/x^2, x)
```

3.154 $\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^3}\,dx$

Optimal. Leaf size=60

$$-3b^2\log(x)\left(bx-\coth^{-1}(\tanh(a+bx))\right)-\frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2}-\frac{3b\coth^{-1}(\tanh(a+bx))^2}{2x}+3b^3x$$

```
[Out] 3*b^3*x - (3*b*ArcCoth[Tanh[a + b*x]]^2)/(2*x) - ArcCoth[Tanh[a + b*x]]^3/(
2*x^2) - 3*b^2*(b*x - ArcCoth[Tanh[a + b*x]])*Log[x]
```

Rubi [A] time = 0.0411344, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2158, 29}

$$-3b^2\log(x)\left(bx-\coth^{-1}(\tanh(a+bx))\right)-\frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2}-\frac{3b\coth^{-1}(\tanh(a+bx))^2}{2x}+3b^3x$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^3/x^3,x]
```

```
[Out] 3*b^3*x - (3*b*ArcCoth[Tanh[a + b*x]]^2)/(2*x) - ArcCoth[Tanh[a + b*x]]^3/(
2*x^2) - 3*b^2*(b*x - ArcCoth[Tanh[a + b*x]])*Log[x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^3}\,dx &= -\frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2}+\frac{1}{2}(3b)\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2}\,dx\\
&= -\frac{3b\coth^{-1}(\tanh(a+bx))^2}{2x}-\frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2}+\left(3b^2\right)\int \frac{\coth^{-1}(\tanh(a+b
}{x}\\
&= 3b^3x-\frac{3b\coth^{-1}(\tanh(a+bx))^2}{2x}-\frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2}-\left(3b^2\left(bx-\coth^{-1}(\tanh\right.\right.\\
&= 3b^3x-\frac{3b\coth^{-1}(\tanh(a+bx))^2}{2x}-\frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2}-3b^2\left(bx-\coth^{-1}(\tanh\right.
\end{aligned}$$

Mathematica [A] time = 0.0363074, size = 66, normalized size = 1.1

$$3b^2\log(x)\left(\coth^{-1}(\tanh(a+bx))-bx\right)-\frac{\left(\coth^{-1}(\tanh(a+bx))-bx\right)^3}{2x^2}-\frac{3b\left(\coth^{-1}(\tanh(a+bx))-bx\right)^2}{x}+b^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^3/x^3,x]
```

```
[Out] b^3*x - (3*b*(-(b*x) + ArcCoth[Tanh[a + b*x]])^2)/x - (-(b*x) + ArcCoth[Tan
h[a + b*x]])^3/(2*x^2) + 3*b^2*(-(b*x) + ArcCoth[Tanh[a + b*x]])*Log[x]
```

Maple [C] time = 0.426, size = 7366, normalized size = 122.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^3/x^3,x)
```

```
[Out] result too large to display
```

Maxima [A] time = 1.40292, size = 97, normalized size = 1.62

$$3\left(b\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)\log\left(x\right)-\left(b\left(x+\frac{a}{b}\right)\log\left(x\right)-b\left(x+\frac{a\log\left(x\right)}{b}\right)\right)b\right)b-\frac{3\,b\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2}{2\,x}-\mathrm{arcoth}\left(\mathrm{ta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^3,x, algorithm="maxima")
```

```
[Out] 3*(b*arccoth(tanh(b*x + a))*log(x) - (b*(x + a/b)*log(x) - b*(x + a*log(x)/
b))*b)*b - 3/2*b*arccoth(tanh(b*x + a))^2/x - 1/2*arccoth(tanh(b*x + a))^3/
x^2
```

Fricas [A] time = 1.61919, size = 117, normalized size = 1.95

$$\frac{8\,b^3x^3+24\,ab^2x^2\log\left(x\right)+3\,\pi^2a-4\,a^3+6\left(\pi^2b-4\,a^2b\right)x}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(8*b^3*x^3 + 24*a*b^2*x^2*log(x) + 3*pi^2*a - 4*a^3 + 6*(pi^2*b - 4*a^2
*b)*x)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^3(\tanh(a+bx))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**3/x**3,x)
```

```
[Out] Integral(acoth(tanh(a + b*x))**3/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(\tanh(bx+a))^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^3/x^3, x)
```

3.155 $\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^4} dx$

Optimal. Leaf size=55

$$-\frac{b^2 \coth^{-1}(\tanh(a+bx))}{x}-\frac{b \coth^{-1}(\tanh(a+bx))^2}{2x^2}-\frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3}+b^3\log(x)$$

```
[Out] -((b^2*ArcCoth[Tanh[a + b*x]])/x) - (b*ArcCoth[Tanh[a + b*x]]^2)/(2*x^2) -
ArcCoth[Tanh[a + b*x]]^3/(3*x^3) + b^3*Log[x]
```

Rubi [A] time = 0.0381152, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 29}

$$-\frac{b^2 \coth^{-1}(\tanh(a+bx))}{x}-\frac{b \coth^{-1}(\tanh(a+bx))^2}{2x^2}-\frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3}+b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^3/x^4,x]
```

```
[Out] -((b^2*ArcCoth[Tanh[a + b*x]])/x) - (b*ArcCoth[Tanh[a + b*x]]^2)/(2*x^2) -
ArcCoth[Tanh[a + b*x]]^3/(3*x^3) + b^3*Log[x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^4} dx &= -\frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3}+b\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^3} dx\\
&= -\frac{b \coth^{-1}(\tanh(a+bx))^2}{2x^2}-\frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3}+b^2\int \frac{\coth^{-1}(\tanh(a+bx))}{x^2} dx\\
&= -\frac{b^2 \coth^{-1}(\tanh(a+bx))}{x}-\frac{b \coth^{-1}(\tanh(a+bx))^2}{2x^2}-\frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3}+b^3\int\\
&= -\frac{b^2 \coth^{-1}(\tanh(a+bx))}{x}-\frac{b \coth^{-1}(\tanh(a+bx))^2}{2x^2}-\frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3}+b^3\log
\end{aligned}$$

Mathematica [A] time = 0.0243607, size = 60, normalized size = 1.09

$$\frac{-6b^2x^2\coth^{-1}(\tanh(a+bx))-3bx\coth^{-1}(\tanh(a+bx))^2-2\coth^{-1}(\tanh(a+bx))^3+b^3x^3(6\log(x)+11)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^3/x^4,x]
```

```
[Out] (-6*b^2*x^2*ArcCoth[Tanh[a + b*x]] - 3*b*x*ArcCoth[Tanh[a + b*x]]^2 - 2*Arc
Coth[Tanh[a + b*x]]^3 + b^3*x^3*(11 + 6*Log[x]))/(6*x^3)
```

Maple [C] time = 1.385, size = 17237, normalized size = 313.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^3/x^4,x)
```

```
[Out] result too large to display
```

Maxima [A] time = 1.59677, size = 70, normalized size = 1.27

$$\left(b^2\log\left(x\right)-\frac{b\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)}{x}\right)b-\frac{b\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2}{2\,x^2}-\frac{\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^4,x, algorithm="maxima")
```

```
[Out] (b^2*log(x) - b*arccoth(tanh(b*x + a))/x)*b - 1/2*b*arccoth(tanh(b*x + a))^
2/x^2 - 1/3*arccoth(tanh(b*x + a))^3/x^3
```

Fricas [A] time = 1.65861, size = 120, normalized size = 2.18

$$\frac{24\,b^3x^3\log\left(x\right)-72\,ab^2x^2+6\,\pi^2a-8\,a^3+9\left(\pi^2b-4\,a^2b\right)x}{24\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/24*(24*b^3*x^3*log(x) - 72*a*b^2*x^2 + 6*pi^2*a - 8*a^3 + 9*(pi^2*b - 4*a
^2*b)*x)/x^3
```

Sympy [A] time = 1.21213, size = 51, normalized size = 0.93

$$b^3\log\left(x\right)-\frac{b^2\,\mathrm{acoth}\left(\tanh\left(a+bx\right)\right)}{x}-\frac{b\,\mathrm{acoth}^2\left(\tanh\left(a+bx\right)\right)}{2x^2}-\frac{\mathrm{acoth}^3\left(\tanh\left(a+bx\right)\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**3/x**4,x)
```

```
[Out] b**3*log(x) - b**2*acoth(tanh(a + b*x))/x - b*acoth(tanh(a + b*x))**2/(2*x*
*2) - acoth(tanh(a + b*x))**3/(3*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(\tanh(bx+a))^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^4,x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^3/x^4, x)
```

3.156 $\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5} dx$

Optimal. Leaf size=31

$$\frac{\coth^{-1}(\tanh(a+bx))^4}{4x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

```
[Out] ArcCoth[Tanh[a + b*x]]^4/(4*x^4*(b*x - ArcCoth[Tanh[a + b*x]]))
```

Rubi [A] time = 0.0137358, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2167}

$$\frac{\coth^{-1}(\tanh(a+bx))^4}{4x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^3/x^5,x]
```

```
[Out] ArcCoth[Tanh[a + b*x]]^4/(4*x^4*(b*x - ArcCoth[Tanh[a + b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5} dx = \frac{\coth^{-1}(\tanh(a+bx))^4}{4x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A] time = 0.0222898, size = 50, normalized size = 1.61

$$-\frac{b^2x^2\coth^{-1}(\tanh(a+bx))+bx\coth^{-1}(\tanh(a+bx))^2+\coth^{-1}(\tanh(a+bx))^3+b^3x^3}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^3/x^5,x]
```

```
[Out] -(b^3*x^3 + b^2*x^2*ArcCoth[Tanh[a + b*x]] + b*x*ArcCoth[Tanh[a + b*x]]^2 +
 ArcCoth[Tanh[a + b*x]]^3)/(4*x^4)
```

Maple [C] time = 1.347, size = 17235, normalized size = 556.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^3/x^5,x)
```

```
[Out] result too large to display
```

Maxima [A] time = 1.59718, size = 72, normalized size = 2.32

$$-\frac{1}{4}\,b\left(\frac{b^2}{x}+\frac{b\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)}{x^2}\right)-\frac{b\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2}{4\,x^3}-\frac{\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*b*(b^2/x + b*arccoth(tanh(b*x + a))/x^2) - 1/4*b*arccoth(tanh(b*x + a)
)^2/x^3 - 1/4*arccoth(tanh(b*x + a))^3/x^4
```

Fricas [A] time = 1.60225, size = 112, normalized size = 3.61

$$-\frac{16\,b^3x^3+24\,ab^2x^2-3\,\pi^2a+4\,a^3-4\left(\pi^2b-4\,a^2b\right)x}{16\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^5,x, algorithm="fricas")
```

```
[Out] -1/16*(16*b^3*x^3 + 24*a*b^2*x^2 - 3*pi^2*a + 4*a^3 - 4*(pi^2*b - 4*a^2*b)*
x)/x^4
```

Sympy [B] time = 2.14484, size = 56, normalized size = 1.81

$$-\frac{b^3}{4x}-\frac{b^2\,\mathrm{acoth}\left(\tanh\left(a+bx\right)\right)}{4x^2}-\frac{b\,\mathrm{acoth}^2\left(\tanh\left(a+bx\right)\right)}{4x^3}-\frac{\mathrm{acoth}^3\left(\tanh\left(a+bx\right)\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**3/x**5,x)
```

```
[Out] -b**3/(4*x) - b**2*acoth(tanh(a + b*x))/(4*x**2) - b*acoth(tanh(a + b*x))**
2/(4*x**3) - acoth(tanh(a + b*x))**3/(4*x**4)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^5,x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^3/x^5, x)
```

3.157 $\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^6}\,dx$

Optimal. Leaf size=64

$$\frac{\coth^{-1}(\tanh(a+bx))^4}{5x^5\left(bx-\coth^{-1}(\tanh(a+bx))\right)}+\frac{b\coth^{-1}(\tanh(a+bx))^4}{20x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}$$

```
[Out] (b*ArcCoth[Tanh[a + b*x]]^4)/(20*x^4*(b*x - ArcCoth[Tanh[a + b*x]])^2) + Ar
cCoth[Tanh[a + b*x]]^4/(5*x^5*(b*x - ArcCoth[Tanh[a + b*x]]))
```

Rubi [A] time = 0.0335485, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2171, 2167}

$$\frac{\coth^{-1}(\tanh(a+bx))^4}{5x^5\left(bx-\coth^{-1}(\tanh(a+bx))\right)}+\frac{b\coth^{-1}(\tanh(a+bx))^4}{20x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^3/x^6,x]
```

```
[Out] (b*ArcCoth[Tanh[a + b*x]]^4)/(20*x^4*(b*x - ArcCoth[Tanh[a + b*x]])^2) + Ar
cCoth[Tanh[a + b*x]]^4/(5*x^5*(b*x - ArcCoth[Tanh[a + b*x]]))
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^6}\,dx &= \frac{\coth^{-1}(\tanh(a+bx))^4}{5x^5\left(bx-\coth^{-1}(\tanh(a+bx))\right)}+\frac{b\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5}\,dx}{5\left(bx-\coth^{-1}(\tanh(a+bx))\right)}\\ &= \frac{b\coth^{-1}(\tanh(a+bx))^4}{20x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}+\frac{\coth^{-1}(\tanh(a+bx))^4}{5x^5\left(bx-\coth^{-1}(\tanh(a+bx))\right)}\end{aligned}$$

Mathematica [A] time = 0.0339501, size = 54, normalized size = 0.84

$$-\frac{2b^2x^2\coth^{-1}(\tanh(a+bx))+3bx\coth^{-1}(\tanh(a+bx))^2+4\coth^{-1}(\tanh(a+bx))^3+b^3x^3}{20x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^3/x^6,x]
```

```
[Out] -(b^3*x^3 + 2*b^2*x^2*ArcCoth[Tanh[a + b*x]] + 3*b*x*ArcCoth[Tanh[a + b*x]]
^2 + 4*ArcCoth[Tanh[a + b*x]]^3)/(20*x^5)
```

Maple [C] time = 1.273, size = 17234, normalized size = 269.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^3/x^6,x)
```

```
[Out] result too large to display
```

Maxima [A] time = 1.59881, size = 73, normalized size = 1.14

$$-\frac{1}{20}\,b\left(\frac{b^2}{x^2}+\frac{2\,b\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)}{x^3}\right)-\frac{3\,b\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2}{20\,x^4}-\frac{\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^6,x, algorithm="maxima")
```

```
[Out] -1/20*b*(b^2/x^2 + 2*b*arccoth(tanh(b*x + a))/x^3) - 3/20*b*arccoth(tanh(b*
x + a))^2/x^4 - 1/5*arccoth(tanh(b*x + a))^3/x^5
```

Fricas [A] time = 1.69803, size = 116, normalized size = 1.81

$$-\frac{40\,b^3x^3+80\,ab^2x^2-12\,\pi^2a+16\,a^3-15\left(\pi^2b-4\,a^2b\right)x}{80\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^6,x, algorithm="fricas")
```

```
[Out] -1/80*(40*b^3*x^3 + 80*a*b^2*x^2 - 12*pi^2*a + 16*a^3 - 15*(pi^2*b - 4*a^2*
b)*x)/x^5
```

Sympy [A] time = 3.61206, size = 60, normalized size = 0.94

$$-\frac{b^3}{20x^2}-\frac{b^2\,\mathrm{acoth}\left(\tanh\left(a+bx\right)\right)}{10x^3}-\frac{3b\,\mathrm{acoth}^2\left(\tanh\left(a+bx\right)\right)}{20x^4}-\frac{\mathrm{acoth}^3\left(\tanh\left(a+bx\right)\right)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**3/x**6,x)
```

```
[Out] -b**3/(20*x**2) - b**2*acoth(tanh(a + b*x))/(10*x**3) - 3*b*acoth(tanh(a +
b*x))**2/(20*x**4) - acoth(tanh(a + b*x))**3/(5*x**5)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(\tanh(bx+a))^3}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^6,x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^3/x^6, x)
```

3.158 $\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=53

$$-\frac{x^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{(m+1)\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

```
[Out] -((x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (b*x)/(b*x - ArcCoth[Tanh[a
 + b*x]])])/((1 + m)*(b*x - ArcCoth[Tanh[a + b*x]])))
```

Rubi [A] time = 0.0284971, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2164}

$$-\frac{x^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{(m+1)\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] -((x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (b*x)/(b*x - ArcCoth[Tanh[a
 + b*x]])])/((1 + m)*(b*x - ArcCoth[Tanh[a + b*x]])))
```

Rule 2164

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)*Hypergeometric2F1[1, n + 1, n + 2, -((a*v)/(b*u
- a*v))])/((n + 1)*(b*u - a*v)), x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))}\,dx=-\frac{x^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{(1+m)\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A] time = 0.0842978, size = 51, normalized size = 0.96

$$\frac{x^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{bx}{\coth^{-1}(\tanh(a+bx))-bx}\right)}{(m+1)\left(\coth^{-1}(\tanh(a+bx))-bx\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((b*x)/(-(b*x) + ArcCoth[Tan
h[a + b*x]]))])/((1 + m)*(-(b*x) + ArcCoth[Tanh[a + b*x]]))
```

Maple [F] time = 0.346, size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{arccoth}(\tanh(bx+a))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arccoth(tanh(b*x+a)),x)
```

```
[Out] int(x^m/arccoth(tanh(b*x+a)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{arcoth}(\tanh(bx+a))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] integrate(x^m/arccoth(tanh(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{\operatorname{arcoth}(\tanh(bx+a))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] integral(x^m/arccoth(tanh(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{acoth}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/acoth(tanh(b*x+a)),x)
```

```
[Out] Integral(x**m/acoth(tanh(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{arcoth}(\tanh(bx+a))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^m/arccoth(tanh(b*x + a)), x)
```

3.159 $\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))} dx$

Optimal. Leaf size=81

$$\frac{x^2 \left(bx - \coth^{-1}(\tanh(a+bx))\right)}{2b^2} + \frac{x \left(bx - \coth^{-1}(\tanh(a+bx))\right)^2}{b^3} + \frac{\left(bx - \coth^{-1}(\tanh(a+bx))\right)^3 \log\left(\coth^{-1}(\tanh(a\right.}{b^4}$$

```
[Out] x^3/(3*b) + (x^2*(b*x - ArcCoth[Tanh[a + b*x]]))/(2*b^2) + (x*(b*x - ArcCot
h[Tanh[a + b*x]])^2)/b^3 + ((b*x - ArcCoth[Tanh[a + b*x]])^3*Log[ArcCoth[Ta
nh[a + b*x]]])/b^4
```

Rubi [A] time = 0.0588508, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2159, 2158, 2157, 29}

$$\frac{x^2 \left(bx - \coth^{-1}(\tanh(a+bx))\right)}{2b^2} + \frac{x \left(bx - \coth^{-1}(\tanh(a+bx))\right)^2}{b^3} + \frac{\left(bx - \coth^{-1}(\tanh(a+bx))\right)^3 \log\left(\coth^{-1}(\tanh(a\right.}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] x^3/(3*b) + (x^2*(b*x - ArcCoth[Tanh[a + b*x]]))/(2*b^2) + (x*(b*x - ArcCot
h[Tanh[a + b*x]])^2)/b^3 + ((b*x - ArcCoth[Tanh[a + b*x]])^3*Log[ArcCoth[Ta
nh[a + b*x]]])/b^4
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{x^3}{3b} - \frac{\left(-bx+\coth^{-1}(\tanh(a+bx))\right)\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))}\,dx}{b}$$

$$= \frac{x^3}{3b} + \frac{x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{2b^2} + \frac{\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2 \int \frac{x}{\coth^{-1}(\tanh(}$$

$$= \frac{x^3}{3b} + \frac{x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{2b^2} + \frac{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}{b^3} - \frac{(-bx+\mathrm{co}}{}$$

$$= \frac{x^3}{3b} + \frac{x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{2b^2} + \frac{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}{b^3} - \frac{(-bx+\mathrm{co}}{}$$

$$= \frac{x^3}{3b} + \frac{x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{2b^2} + \frac{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}{b^3} + \frac{(bx-\mathrm{cotl}}{}$$

Mathematica [A] time = 0.0481871, size = 79, normalized size = 0.98

$$-\frac{x^2\left(\coth^{-1}(\tanh(a+bx))-bx\right)}{2b^2} + \frac{x\left(\coth^{-1}(\tanh(a+bx))-bx\right)^2}{b^3} - \frac{\left(\coth^{-1}(\tanh(a+bx))-bx\right)^3 \log\left(\coth^{-1}(\mathrm{tan}\right.}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] x^3/(3*b) - (x^2*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/(2*b^2) + (x*(-(b*x) +
ArcCoth[Tanh[a + b*x]])^2)/b^3 - ((-(b*x) + ArcCoth[Tanh[a + b*x]])^3*Log[A
rcCoth[Tanh[a + b*x]]])/b^4
```

Maple [C] time = 4.901, size = 130774, normalized size = 1614.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arccoth(tanh(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [C] time = 1.80687, size = 116, normalized size = 1.43

$$\frac{4\,b^2x^3 + (3i\,\pi b - 6\,ab)x^2 - \left(3\,\pi^2 + 12i\,\pi a - 12\,a^2\right)x}{12\,b^3} - \frac{\left(i\,\pi^3 - 6\,\pi^2 a - 12i\,\pi a^2 + 8\,a^3\right)\log\left(-i\,\pi + 2\,bx + 2\,a\right)}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*b^2*x^3 + (3*I*pi*b - 6*a*b)*x^2 - (3*pi^2 + 12*I*pi*a - 12*a^2)*x)
/b^3 - 1/8*(I*pi^3 - 6*pi^2*a - 12*I*pi*a^2 + 8*a^3)*log(-I*pi + 2*b*x + 2*
a)/b^4
```

Fricas [A] time = 1.5517, size = 293, normalized size = 3.62

$$\frac{8\,b^3x^3 - 12\,ab^2x^2 - 6\left(\pi^2b - 4\,a^2b\right)x - 6\left(\pi^3 - 12\,\pi a^2\right)\arctan\left(-\frac{2\,bx+2\,a-\sqrt{4\,b^2x^2+8\,abx+\pi^2+4\,a^2}}{\pi}\right) + 3\left(3\,\pi^2a - 4\,a^3\right)\log\left(4\,b^2x\right.}{24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/24*(8*b^3*x^3 - 12*a*b^2*x^2 - 6*(pi^2*b - 4*a^2*b)*x - 6*(pi^3 - 12*pi*a
^2)*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/pi) +
3*(3*pi^2*a - 4*a^3)*log(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/b^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/acoth(tanh(b*x+a)),x)
```

```
[Out] Integral(x**3/acoth(tanh(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^3/arccoth(tanh(b*x + a)), x)
```

3.160 $\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))} dx$

Optimal. Leaf size=56

$$\frac{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^2}+\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^3}+\frac{x^2}{2b}$$

```
[Out] x^2/(2*b) + (x*(b*x - ArcCoth[Tanh[a + b*x]]))/b^2 + ((b*x - ArcCoth[Tanh[a
 + b*x]])^2*Log[ArcCoth[Tanh[a + b*x]]])/b^3
```

Rubi [A] time = 0.0346625, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2159, 2158, 2157, 29}

$$\frac{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^2}+\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^3}+\frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcCoth[Tanh[a + b*x]],x]
```


```
[Out] x^2/(2*b) + (x*(b*x - ArcCoth[Tanh[a + b*x]]))/b^2 + ((b*x - ArcCoth[Tanh[a
 + b*x]])^2*Log[ArcCoth[Tanh[a + b*x]]])/b^3
```


Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{x^2}{2b} - \frac{\left(-bx+\coth^{-1}(\tanh(a+bx))\right)\int \frac{x}{\coth^{-1}(\tanh(a+bx))}\,dx}{b}$$
$$= \frac{x^2}{2b} + \frac{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^2} + \frac{\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2 \int \frac{1}{\coth^{-1}(\tanh(a+bx)}}{b^2}$$
$$= \frac{x^2}{2b} + \frac{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^2} + \frac{\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2 \text{Subst}\left(\int \frac{1}{x}\,dx, x,\right.}{b^3}$$
$$= \frac{x^2}{2b} + \frac{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^2} + \frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2 \log\left(\coth^{-1}(\tanh(\right.}{b^3}$$

Mathematica [A] time = 0.0383789, size = 55, normalized size = 0.98

$$-\frac{x\left(\coth^{-1}(\tanh(a+bx))-bx\right)}{b^2} + \frac{\left(\coth^{-1}(\tanh(a+bx))-bx\right)^2 \log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^3} + \frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] x^2/(2*b) - (x*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/b^2 + ((-(b*x) + ArcCoth[
Tanh[a + b*x]])^2*Log[ArcCoth[Tanh[a + b*x]]])/b^3
```

Maple [C] time = 1.134, size = 28786, normalized size = 514.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arccoth(tanh(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [C] time = 1.82435, size = 69, normalized size = 1.23

$$\frac{bx^2 + (i\pi - 2a)x}{2b^2} - \frac{\left(\pi^2 + 4i\pi a - 4a^2\right)\log\left(-i\pi + 2bx + 2a\right)}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + (I*pi - 2*a)*x)/b^2 - 1/4*(pi^2 + 4*I*pi*a - 4*a^2)*log(-I*pi
+ 2*b*x + 2*a)/b^3
```

Fricas [A] time = 1.66004, size = 225, normalized size = 4.02

$$\frac{4b^2x^2 - 8abx - 16\pi a \arctan\left(-\frac{2bx+2a-\sqrt{4b^2x^2+8abx+\pi^2+4a^2}}{\pi}\right) - \left(\pi^2-4a^2\right)\log\left(4b^2x^2+8abx+\pi^2+4a^2\right)}{8b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b^2*x^2 - 8*a*b*x - 16*pi*a*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 +
8*a*b*x + pi^2 + 4*a^2))/pi) - (pi^2 - 4*a^2)*log(4*b^2*x^2 + 8*a*b*x + pi^
2 + 4*a^2))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{acoth}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/acoth(tanh(b*x+a)),x)
```

```
[Out] Integral(x**2/acoth(tanh(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{arcoth}(\tanh(bx+a))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2/arccoth(tanh(b*x + a)), x)
```

3.161 $\int \frac{x}{\coth^{-1}(\tanh(a+bx))} dx$

Optimal. Leaf size=31

$$\frac{(bx - \coth^{-1}(\tanh(a + bx))) \log(\coth^{-1}(\tanh(a + bx)))}{b^2} + \frac{x}{b}$$

```
[Out] x/b + ((b*x - ArcCoth[Tanh[a + b*x]])*Log[ArcCoth[Tanh[a + b*x]]])/b^2
```

Rubi [A] time = 0.0150948, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2158, 2157, 29}

$$\frac{(bx - \coth^{-1}(\tanh(a + bx))) \log(\coth^{-1}(\tanh(a + bx)))}{b^2} + \frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] x/b + ((b*x - ArcCoth[Tanh[a + b*x]])*Log[ArcCoth[Tanh[a + b*x]]])/b^2
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\coth^{-1}(\tanh(a + bx))} dx &= \frac{x}{b} - \frac{(-bx + \coth^{-1}(\tanh(a + bx))) \int \frac{1}{\coth^{-1}(\tanh(a+bx))} dx}{b} \\
&= \frac{x}{b} - \frac{(-bx + \coth^{-1}(\tanh(a + bx))) \operatorname{Subst}\left(\int \frac{1}{x} dx, x, \coth^{-1}(\tanh(a + bx))\right)}{b^2} \\
&= \frac{x}{b} + \frac{(bx - \coth^{-1}(\tanh(a + bx))) \log(\coth^{-1}(\tanh(a + bx)))}{b^2}
\end{aligned}$$

Mathematica [A] time = 0.0221102, size = 31, normalized size = 1.

$$\frac{x}{b} - \frac{(\coth^{-1}(\tanh(a + bx)) - bx) \log(\coth^{-1}(\tanh(a + bx)))}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] x/b - ((-(b*x) + ArcCoth[Tanh[a + b*x]])*Log[ArcCoth[Tanh[a + b*x]]])/b^2
```

Maple [C] time = 0.314, size = 4303, normalized size = 138.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arccoth(tanh(b*x+a)),x)
```

```
[Out] 1/b*ln(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csg
n(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(ex
p(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(e
xp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csg
n(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csg
n(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*e
xp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*
a+2*Pi)*x-1/b^2*ln(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+
2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-P
i*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*
Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*
a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*
a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2
+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))
-b*x-a)+4*I*a+2*Pi)*ln(exp(b*x+a))-1/2*I/b^2*ln(-2*Pi*csgn(I/(exp(2*b*x+2*a
)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b
*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+
2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(
b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2
*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b
*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^
3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)*Pi*csgn(I/(exp(2*b*x+2*a)+
1))^2+1/4*I/b^2*ln(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+
2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-P
i*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*
Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*
a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*
a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2
+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))
-b*x-a)+4*I*a+2*Pi)*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*cs
gn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/4*I/b^2*ln(-2*Pi*csgn(I/(exp(2*b*
x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2
*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I
*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*
b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)
+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)*Pi*csgn(I/(exp(2*b*x+
2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/2*I/b^2*ln(-2*Pi*csg
n(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*
a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))
*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1)
)^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*
csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*
a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(e
```

```
xp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)*Pi*csgn(
I/(exp(2*b*x+2*a)+1))^3+1/4*I/b^2*ln(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*
csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(ex
p(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2
*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*c
sgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*c
sgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(ex
p(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4
*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*
b*x+2*a))-1/2*I/b^2*ln(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*
b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1
))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^
2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*
x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*
x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1
))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x
+a))-b*x-a)+4*I*a+2*Pi)*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/4*
I/b^2*ln(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*c
sgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(
exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/
(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*c
sgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*c
sgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I
*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*
I*a+2*Pi)*Pi*csgn(I*exp(2*b*x+2*a))^3-1/4*I/b^2*ln(-2*Pi*csgn(I/(exp(2*b*x+
2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(
2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b
*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*e
xp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*
x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(
2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1
))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)*Pi*csgn(I*exp(2*b*x+2*a
))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/4*I/b^2*ln(-2*Pi*csgn(I/(e
xp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*cs
gn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(
I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi
*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I
*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*cs
gn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b
*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)*Pi*csgn(I*exp(
2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/2*I/b^2*ln(-2*Pi*csgn(I/(exp(2*b*x+2*a)+
1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x
+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*
a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*
x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a
))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x
+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+
4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)*Pi+x/b
```

Maxima [C] time = 1.83026, size = 41, normalized size = 1.32

$$\frac{x}{b} - \frac{(-i\pi + 2a)\log(-i\pi + 2bx + 2a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] x/b - 1/2*(-I*pi + 2*a)*log(-I*pi + 2*b*x + 2*a)/b^2
```

Fricas [B] time = 1.59347, size = 185, normalized size = 5.97

$$\frac{2\,bx + 2\,\pi \arctan\left(-\frac{2\,bx+2\,a-\sqrt{4\,b^2x^2+8\,abx+\pi^2+4\,a^2}}{\pi}\right) - a\log\left(4\,b^2x^2 + 8\,abx + \pi^2 + 4\,a^2\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x + 2*pi*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 +
4*a^2))/pi) - a*log(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{acoth}(\tanh(a + bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acoth(tanh(b*x+a)),x)
```

```
[Out] Integral(x/acoth(tanh(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{arcoth}(\tanh(bx + a))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x/arccoth(tanh(b*x + a)), x)
```

3.162 $\int \frac{1}{\coth^{-1}(\tanh(a+bx))} \, dx$

Optimal. Leaf size=12

$$\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b}$$

```
[Out] Log[ArcCoth[Tanh[a + b*x]]]/b
```

Rubi [A] time = 0.0040205, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2157, 29}

$$\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^(-1),x]
```

```
[Out] Log[ArcCoth[Tanh[a + b*x]]]/b
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\coth^{-1}(\tanh(a+bx))} \, dx &= \frac{\text{Subst}\left(\int \frac{1}{x} \, dx, x, \coth^{-1}(\tanh(a+bx))\right)}{b} \\
&= \frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.04961, size = 12, normalized size = 1.

$$\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^(-1),x]
```

```
[Out] Log[ArcCoth[Tanh[a + b*x]]]/b
```

Maple [A] time = 0.076, size = 13, normalized size = 1.1

$$\frac{\ln\left(\operatorname{arccoth}\left(\tanh\left(bx+a\right)\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccoth(tanh(b*x+a)),x)
```

```
[Out] ln(arccoth(tanh(b*x+a)))/b
```

Maxima [C] time = 1.45703, size = 22, normalized size = 1.83

$$\frac{\log\left(-\frac{1}{2}i\pi-bx-a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] log(-1/2*I*pi - b*x - a)/b
```

Fricas [B] time = 1.63011, size = 63, normalized size = 5.25

$$\frac{\log\left(4\,b^2x^2+8\,abx+\pi^2+4\,a^2\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*log(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2)/b
```

Sympy [A] time = 8.22448, size = 17, normalized size = 1.42

$$\begin{cases}\frac{\log\left(\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)\right)}{b} & \text{for } b\neq 0\\ \frac{x}{\operatorname{acoth}\left(\tanh\left(a\right)\right)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acoth(tanh(b*x+a)),x)
```

```
[Out] Piecewise((log(acoth(tanh(a + b*x)))/b, Ne(b, 0)), (x/acoth(tanh(a)), True)
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(1/arccoth(tanh(b*x + a)), x)
```

3.163 $\int \frac{1}{x \coth^{-1}(\tanh(a+bx))} dx$

Optimal. Leaf size=44

$$\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}$$

```
[Out] -(Log[x]/(b*x - ArcCoth[Tanh[a + b*x]])) + Log[ArcCoth[Tanh[a + b*x]]]/(b*x
 - ArcCoth[Tanh[a + b*x]])
```

Rubi [A] time = 0.0284736, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2160, 2157, 29}

$$\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*ArcCoth[Tanh[a + b*x]]),x]
```

```
[Out] -(Log[x]/(b*x - ArcCoth[Tanh[a + b*x]])) + Log[ArcCoth[Tanh[a + b*x]]]/(b*x
 - ArcCoth[Tanh[a + b*x]])
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x \coth^{-1}(\tanh(a+bx))} dx &= -\frac{\int \frac{1}{x} dx}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{b \int \frac{1}{\coth^{-1}(\tanh(a+bx))} dx}{bx-\coth^{-1}(\tanh(a+bx))}\\
&= -\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{\text{Subst}\left(\int \frac{1}{x} dx, x, \coth^{-1}(\tanh(a+bx))\right)}{bx-\coth^{-1}(\tanh(a+bx))}\\
&= -\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{bx-\coth^{-1}(\tanh(a+bx))}
\end{aligned}$$

Mathematica [A] time = 0.0183468, size = 29, normalized size = 0.66

$$\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)-\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*ArcCoth[Tanh[a + b*x]]),x]
```

```
[Out] (-Log[x] + Log[ArcCoth[Tanh[a + b*x]]])/(b*x - ArcCoth[Tanh[a + b*x]])
```

Maple [C] time = 11.445, size = 972, normalized size = 22.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arccoth(tanh(b*x+a)),x)
```

```
[Out] -4*I/(2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(
I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(
2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*
b*x+2*a))^3-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-Pi*csgn(I*exp(
2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(b*x+a
))^2*csgn(I*exp(2*b*x+2*a))+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-
4*I*b*x-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+4*I*ln(exp(b*x+a))+2*Pi)*ln(-2*Pi
*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*
x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)
+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a
)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+
a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*
x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a
)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)+4*I/
(2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp
(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x
+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2
*a))^3-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-Pi*csgn(I*exp(2*b*x
+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(b*x+a))^2*
csgn(I*exp(2*b*x+2*a))+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-4*I*b
*x-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+4*I*ln(exp(b*x+a))+2*Pi)*ln(x)
```

Maxima [C] time = 1.81207, size = 50, normalized size = 1.14

$$\frac{2\,\log\left(-i\,\pi+2\,bx+2\,a\right)}{i\,\pi-2\,a}-\frac{2\,\log\left(x\right)}{i\,\pi-2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 2*log(-I*pi + 2*b*x + 2*a)/(I*pi - 2*a) - 2*log(x)/(I*pi - 2*a)
```

Fricas [A] time = 1.69563, size = 205, normalized size = 4.66

$$-\frac{2\left(2\,\pi\arctan\left(-\frac{2\,bx+2\,a-\sqrt{4\,b^2x^2+8\,abx+\pi^2+4\,a^2}}{\pi}\right)+a\log\left(4\,b^2x^2+8\,abx+\pi^2+4\,a^2\right)-2\,a\log\left(x\right)\right)}{\pi^2+4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] -2*(2*pi*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/p
i) + a*log(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2) - 2*a*log(x))/(pi^2 + 4*a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{acoth}(\tanh(a+bx))} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/acoth(tanh(b*x+a)),x)
```

```
[Out] Integral(1/(x*acoth(tanh(a + b*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{arcoth}(\tanh(bx+a))} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(1/(x*arccoth(tanh(b*x + a))), x)
```

3.164 $\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))} \, dx$

Optimal. Leaf size=65

$$\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}-\frac{b\log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}+\frac{b\log\left(\coth^{-1}(\tanh(a+bx))\right)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}$$

```
[Out] 1/(x*(b*x - ArcCoth[Tanh[a + b*x]])) - (b*Log[x])/(b*x - ArcCoth[Tanh[a + b
*x]])^2 + (b*Log[ArcCoth[Tanh[a + b*x]]])/(b*x - ArcCoth[Tanh[a + b*x]])^2
```

Rubi [A] time = 0.0378546, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2163, 2160, 2157, 29}

$$\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}-\frac{b\log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}+\frac{b\log\left(\coth^{-1}(\tanh(a+bx))\right)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*ArcCoth[Tanh[a + b*x]]),x]
```

```
[Out] 1/(x*(b*x - ArcCoth[Tanh[a + b*x]])) - (b*Log[x])/(b*x - ArcCoth[Tanh[a + b
*x]])^2 + (b*Log[ArcCoth[Tanh[a + b*x]]])/(b*x - ArcCoth[Tanh[a + b*x]])^2
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))} dx = \frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)} + \frac{b\int \frac{1}{x\coth^{-1}(\tanh(a+bx))} dx}{bx-\coth^{-1}(\tanh(a+bx))}$$

$$= \frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)} - \frac{b\int \frac{1}{x} dx}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2} + \frac{b^2\int \frac{1}{\coth^{-1}(\text{ta}}}{\left(bx-\coth^{-1}(\text{ta}\right.}$$

$$= \frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)} - \frac{b\log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2} + \frac{b\,\text{Subst}\left(\int \frac{1}{x} dx\right.}{\left(bx-\text{cotl}\right.}$$

$$= \frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)} - \frac{b\log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2} + \frac{b\log\left(\coth^{-1}(\text{ta}\right.}{\left(bx-\coth^{-1}(\text{ta}\right.}$$

Mathematica [A] time = 0.024122, size = 45, normalized size = 0.69

$$\frac{bx\left(\log\left(\coth^{-1}(\tanh(a+bx))\right)-\log(x)+1\right)-\coth^{-1}(\tanh(a+bx))}{x\left(\coth^{-1}(\tanh(a+bx))-bx\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*ArcCoth[Tanh[a + b*x]]),x]
```

```
[Out] (-ArcCoth[Tanh[a + b*x]] + b*x*(1 - Log[x] + Log[ArcCoth[Tanh[a + b*x]]]))/
(x*(-(b*x) + ArcCoth[Tanh[a + b*x]])^2)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \operatorname{arccoth}(\tanh(bx+a))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arccoth(tanh(b*x+a)),x)
```

```
[Out] int(1/x^2/arccoth(tanh(b*x+a)),x)
```

Maxima [C] time = 1.7929, size = 88, normalized size = 1.35

$$-\frac{4\,b\log(-i\,\pi+2\,bx+2\,a)}{\pi^2+4i\,\pi a-4\,a^2} + \frac{4\,b\log(x)}{\pi^2+4i\,\pi a-4\,a^2} + \frac{2}{(i\,\pi-2\,a)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -4*b*log(-I*pi + 2*b*x + 2*a)/(pi^2 + 4*I*pi*a - 4*a^2) + 4*b*log(x)/(pi^2
+ 4*I*pi*a - 4*a^2) + 2/((I*pi - 2*a)*x)
```

Fricas [B] time = 1.72745, size = 315, normalized size = 4.85

$$\frac{2\left(16\,\pi abx\arctan\left(-\frac{2\,bx+2\,a-\sqrt{4\,b^2x^2+8\,abx+\pi^2+4\,a^2}}{\pi}\right)-2\,\pi^2a-8\,a^3-\left(\pi^2b-4\,a^2b\right)x\log\left(4\,b^2x^2+8\,abx+\pi^2+4\,a^2\right)+2\left(\pi^2\right.\right.}{\left(\pi^4+8\,\pi^2a^2+16\,a^4\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 2*(16*pi*a*b*x*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a
^2))/pi) - 2*pi^2*a - 8*a^3 - (pi^2*b - 4*a^2*b)*x*log(4*b^2*x^2 + 8*a*b*x
+ pi^2 + 4*a^2) + 2*(pi^2*b - 4*a^2*b)*x*log(x))/((pi^4 + 8*pi^2*a^2 + 16*a
^4)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/acoth(tanh(b*x+a)),x)
```

```
[Out] Integral(1/(x**2*acoth(tanh(a + b*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*arccoth(tanh(b*x + a))), x)
```

3.165 $\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))} dx$

Optimal. Leaf size=92

$$-\frac{b^2 \log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3}+\frac{b^2 \log\left(\coth^{-1}(\tanh(a+bx))\right)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3}+\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}+\frac{}{x\left(bx-\coth\right.}$$

```
[Out] b/(x*(b*x - ArcCoth[Tanh[a + b*x]])^2) + 1/(2*x^2*(b*x - ArcCoth[Tanh[a + b
*x]])) - (b^2*Log[x])/(b*x - ArcCoth[Tanh[a + b*x]])^3 + (b^2*Log[ArcCoth[T
anh[a + b*x]]])/(b*x - ArcCoth[Tanh[a + b*x]])^3
```

Rubi [A] time = 0.0628528, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2163, 2160, 2157, 29}

$$-\frac{b^2 \log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3}+\frac{b^2 \log\left(\coth^{-1}(\tanh(a+bx))\right)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3}+\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}+\frac{}{x\left(bx-\coth\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*ArcCoth[Tanh[a + b*x]]),x]
```

```
[Out] b/(x*(b*x - ArcCoth[Tanh[a + b*x]])^2) + 1/(2*x^2*(b*x - ArcCoth[Tanh[a + b
*x]])) - (b^2*Log[x])/(b*x - ArcCoth[Tanh[a + b*x]])^3 + (b^2*Log[ArcCoth[T
anh[a + b*x]]])/(b*x - ArcCoth[Tanh[a + b*x]])^3
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))} dx = \frac{1}{2x^2 \left(bx - \coth^{-1}(\tanh(a+bx))\right)} - \frac{b \int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))} dx}{-bx + \coth^{-1}(\tanh(a+bx))}$$

$$= \frac{b}{x \left(bx - \coth^{-1}(\tanh(a+bx))\right)^2} + \frac{1}{2x^2 \left(bx - \coth^{-1}(\tanh(a+bx))\right)} - \frac{}{\left(bx - \coth^{-1}(\mathrm{ta}\right.}$$

$$= \frac{b}{x \left(bx - \coth^{-1}(\tanh(a+bx))\right)^2} + \frac{1}{2x^2 \left(bx - \coth^{-1}(\tanh(a+bx))\right)} + \frac{}{\left(bx - \coth^{-1}(\mathrm{ta}\right.}$$

$$= \frac{b}{x \left(bx - \coth^{-1}(\tanh(a+bx))\right)^2} + \frac{1}{2x^2 \left(bx - \coth^{-1}(\tanh(a+bx))\right)} - \frac{b^2 \mathrm{lo}}{\left(bx - \coth^{-1}(\mathrm{ta}\right.}$$

$$= \frac{b}{x \left(bx - \coth^{-1}(\tanh(a+bx))\right)^2} + \frac{1}{2x^2 \left(bx - \coth^{-1}(\tanh(a+bx))\right)} - \frac{b^2 \mathrm{lo}}{\left(bx - \coth^{-1}(\mathrm{ta}\right.}$$

Mathematica [A] time = 0.0270553, size = 66, normalized size = 0.72

$$\frac{b^2x^2 \left(2\log\left(\coth^{-1}(\tanh(a+bx))\right) - 2\log(x) + 3\right) - 4bx\coth^{-1}(\tanh(a+bx)) + \coth^{-1}(\tanh(a+bx))^2}{2x^2 \left(bx - \coth^{-1}(\tanh(a+bx))\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*ArcCoth[Tanh[a + b*x]]),x]
```

```
[Out] (-4*b*x*ArcCoth[Tanh[a + b*x]] + ArcCoth[Tanh[a + b*x]]^2 + b^2*x^2*(3 - 2*
Log[x] + 2*Log[ArcCoth[Tanh[a + b*x]]]))/(2*x^2*(b*x - ArcCoth[Tanh[a + b*x
]])^3)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \operatorname{arccoth}(\tanh(bx+a))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/arccoth(tanh(b*x+a)),x)
```

```
[Out] int(1/x^3/arccoth(tanh(b*x+a)),x)
```

Maxima [C] time = 1.79593, size = 146, normalized size = 1.59

$$\frac{8\,b^2 \log(-i\pi + 2\,bx + 2\,a)}{-i\pi^3 + 6\,\pi^2 a + 12i\,\pi a^2 - 8\,a^3} - \frac{8\,b^2 \log(x)}{-i\pi^3 + 6\,\pi^2 a + 12i\,\pi a^2 - 8\,a^3} - \frac{2\,(i\pi + 4\,bx - 2\,a)}{\left(2\,\pi^2 + 8i\,\pi a - 8\,a^2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 8*b^2*log(-I*pi + 2*b*x + 2*a)/(-I*pi^3 + 6*pi^2*a + 12*I*pi*a^2 - 8*a^3) -
 8*b^2*log(x)/(-I*pi^3 + 6*pi^2*a + 12*I*pi*a^2 - 8*a^3) - 2*(I*pi + 4*b*x
- 2*a)/((2*pi^2 + 8*I*pi*a - 8*a^2)*x^2)
```

Fricas [B] time = 1.71336, size = 448, normalized size = 4.87

$$-\frac{2\left(\pi^4 a + 8\,\pi^2 a^3 + 16\,a^5 - 8\left(\pi^3 b^2 - 12\,\pi a^2 b^2\right)x^2 \arctan\left(-\frac{2\,bx+2\,a-\sqrt{4\,b^2x^2+8\,abx+\pi^2+4\,a^2}}{\pi}\right) - 4\left(3\,\pi^2 ab^2 - 4\,a^3b^2\right)x^2 \log(}{\left(\pi^6 + 12\,\pi^4 a^2 + 48\,\pi^2 a^4 + 64\,a^6\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] -2*(pi^4*a + 8*pi^2*a^3 + 16*a^5 - 8*(pi^3*b^2 - 12*pi*a^2*b^2)*x^2*arctan(
-(2*b*x + 2*a - sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/pi) - 4*(3*pi^2*a
*b^2 - 4*a^3*b^2)*x^2*log(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2) + 8*(3*pi^2*a
*b^2 - 4*a^3*b^2)*x^2*log(x) + 2*(pi^4*b - 16*a^4*b)*x)/((pi^6 + 12*pi^4*a^
2 + 48*pi^2*a^4 + 64*a^6)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \operatorname{acoth}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/acoth(tanh(b*x+a)),x)
```

```
[Out] Integral(1/(x**3*acoth(tanh(a + b*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \operatorname{arcoth}(\tanh(bx+a))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(1/(x^3*arccoth(tanh(b*x + a))), x)
```

3.166 $\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^2}\,dx$

Optimal. Leaf size=65

$$-\frac{x^m\text{Hypergeometric2F1}\left(1,m,m+1,\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{b\left(bx-\coth^{-1}(\tanh(a+bx))\right)}-\frac{x^m}{b\coth^{-1}(\tanh(a+bx))}$$

```
[Out] -(x^m/(b*ArcCoth[Tanh[a + b*x]])) - (x^m*Hypergeometric2F1[1, m, 1 + m, (b*
x)/(b*x - ArcCoth[Tanh[a + b*x]])])/(b*(b*x - ArcCoth[Tanh[a + b*x]]))
```

Rubi [A] time = 0.0390504, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 2164}

$$-\frac{x^m\,{}_2F_1\left(1,m;m+1;\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{b\left(bx-\coth^{-1}(\tanh(a+bx))\right)}-\frac{x^m}{b\coth^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] -(x^m/(b*ArcCoth[Tanh[a + b*x]])) - (x^m*Hypergeometric2F1[1, m, 1 + m, (b*
x)/(b*x - ArcCoth[Tanh[a + b*x]])])/(b*(b*x - ArcCoth[Tanh[a + b*x]]))
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2164

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)*Hypergeometric2F1[1, n + 1, n + 2, -((a*v)/(b*u
- a*v))])/((n + 1)*(b*u - a*v)), x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^2}\,dx &= -\frac{x^m}{b\coth^{-1}(\tanh(a+bx))}+\frac{m\int \frac{x^{-1+m}}{\coth^{-1}(\tanh(a+bx))}\,dx}{b}\\ &= -\frac{x^m}{b\coth^{-1}(\tanh(a+bx))}-\frac{x^m\,{}_2F_1\left(1,m;1+m;\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{b\left(bx-\coth^{-1}(\tanh(a+bx))\right)}\end{aligned}$$

Mathematica [A] time = 0.521421, size = 51, normalized size = 0.78

$$\frac{x^{m+1}\text{Hypergeometric2F1}\left(2,m+1,m+2,-\frac{bx}{\coth^{-1}(\tanh(a+bx))-bx}\right)}{(m+1)\left(\coth^{-1}(\tanh(a+bx))-bx\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((b*x)/(-(b*x) + ArcCoth[Tan
h[a + b*x]]))])/((1 + m)*(-(b*x) + ArcCoth[Tanh[a + b*x]])^2)
```

Maple [F] time = 1.83, size = 0, normalized size = 0.

$$\int \frac{x^m}{(\text{arccoth}(\tanh(bx+a)))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arccoth(tanh(b*x+a))^2,x)
```

```
[Out] int(x^m/arccoth(tanh(b*x+a))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\text{arcoth}(\tanh(bx+a))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m/arccoth(tanh(b*x + a))^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{\text{arcoth}(\tanh(bx+a))^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] integral(x^m/arccoth(tanh(b*x + a))^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\text{acoth}^2(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/acoth(tanh(b*x+a))**2,x)
```

```
[Out] Integral(x**m/acoth(tanh(a + b*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{arcoth}(\tanh(bx+a))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/arccoth(tanh(b*x + a))^2, x)
```

3.167 $\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^2} dx$

Optimal. Leaf size=98

$$\frac{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^3}+\frac{4x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}{b^4}+\frac{4\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3\log\left(\coth^{-1}(\right.}{b^5}$$

```
[Out] (4*x^3)/(3*b^2) + (2*x^2*(b*x - ArcCoth[Tanh[a + b*x]]))/b^3 + (4*x*(b*x -
ArcCoth[Tanh[a + b*x]])^2)/b^4 - x^4/(b*ArcCoth[Tanh[a + b*x]]) + (4*(b*x -
 ArcCoth[Tanh[a + b*x]])^3*Log[ArcCoth[Tanh[a + b*x]]])/b^5
```

Rubi [A] time = 0.0808205, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2168, 2159, 2158, 2157, 29}

$$\frac{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^3}+\frac{4x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}{b^4}+\frac{4\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3\log\left(\coth^{-1}(\right.}{b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] (4*x^3)/(3*b^2) + (2*x^2*(b*x - ArcCoth[Tanh[a + b*x]]))/b^3 + (4*x*(b*x -
ArcCoth[Tanh[a + b*x]])^2)/b^4 - x^4/(b*ArcCoth[Tanh[a + b*x]]) + (4*(b*x -
 ArcCoth[Tanh[a + b*x]])^3*Log[ArcCoth[Tanh[a + b*x]]])/b^5
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^2}\,dx = -\frac{x^4}{b\coth^{-1}(\tanh(a+bx))} + \frac{4\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))}\,dx}{b}$$

$$= \frac{4x^3}{3b^2} - \frac{x^4}{b\coth^{-1}(\tanh(a+bx))} - \frac{\left(4\left(-bx+\coth^{-1}(\tanh(a+bx))\right)\right)\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))}\,d}{b^2}$$

$$= \frac{4x^3}{3b^2} + \frac{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^3} - \frac{x^4}{b\coth^{-1}(\tanh(a+bx))} + \frac{\left(4\left(-bx+\coth^{-1}(\right.\right.}{}$$

$$= \frac{4x^3}{3b^2} + \frac{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^3} + \frac{4x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}{b^4} - \frac{}{b\coth^{-1}(}$$

$$= \frac{4x^3}{3b^2} + \frac{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^3} + \frac{4x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}{b^4} - \frac{}{b\coth^{-1}(}$$

$$= \frac{4x^3}{3b^2} + \frac{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^3} + \frac{4x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}{b^4} - \frac{}{b\coth^{-1}(}$$

Mathematica [A] time = 0.0974712, size = 106, normalized size = 1.08

$$-\frac{x^2\left(\coth^{-1}(\tanh(a+bx))-bx\right)}{b^3} - \frac{\left(\coth^{-1}(\tanh(a+bx))-bx\right)^4}{b^5\coth^{-1}(\tanh(a+bx))} + \frac{3x\left(\coth^{-1}(\tanh(a+bx))-bx\right)^2}{b^4} - \frac{4\left(\coth^{-1}(\tanh\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] x^3/(3*b^2) - (x^2*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/b^3 + (3*x*(-(b*x) +
ArcCoth[Tanh[a + b*x]])^2)/b^4 - (-(b*x) + ArcCoth[Tanh[a + b*x]])^4/(b^5*A
rcCoth[Tanh[a + b*x]]) - (4*(-(b*x) + ArcCoth[Tanh[a + b*x]])^3*Log[ArcCoth
[Tanh[a + b*x]]])/b^5
```

Maple [C] time = 6.12, size = 131085, normalized size = 1337.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arccoth(tanh(b*x+a))^2,x)
```

```
[Out] result too large to display
```

Maxima [C] time = 2.48721, size = 240, normalized size = 2.45

$$\frac{4\left(16\,b^4x^4 - 3\,\pi^4 - 24i\,\pi^3a + 72\,\pi^2a^2 + 96i\,\pi a^3 - 48\,a^4 + \left(16i\,\pi b^3 - 32\,ab^3\right)x^3 - \left(24\,\pi^2b^2 + 96i\,\pi ab^2 - 96\,a^2b^2\right)x^2 + (18\right.}{192\,b^6x - 96i\,\pi b^5 + 192\,ab^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 4*(16*b^4*x^4 - 3*pi^4 - 24*I*pi^3*a + 72*pi^2*a^2 + 96*I*pi*a^3 - 48*a^4 +
 (16*I*pi*b^3 - 32*a*b^3)*x^3 - (24*pi^2*b^2 + 96*I*pi*a*b^2 - 96*a^2*b^2)*
x^2 + (18*I*pi^3*b - 108*pi^2*a*b - 216*I*pi*a^2*b + 144*a^3*b)*x)/(192*b^6
*x - 96*I*pi*b^5 + 192*a*b^5) - 1/2*(I*pi^3 - 6*pi^2*a - 12*I*pi*a^2 + 8*a^
3)*log(-I*pi + 2*b*x + 2*a)/b^5
```

Fricas [B] time = 1.66643, size = 729, normalized size = 7.44

$$16\,b^5x^5 - 16\,ab^4x^4 + 9\,\pi^4a + 24\,\pi^2a^3 - 48\,a^5 - 32\left(\pi^2b^3 - 2\,a^2b^3\right)x^3 - 12\left(7\,\pi^2ab^2 - 20\,a^3b^2\right)x^2 - 12\left(\pi^4b - 6\,\pi^2a^2b - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(16*b^5*x^5 - 16*a*b^4*x^4 + 9*pi^4*a + 24*pi^2*a^3 - 48*a^5 - 32*(pi^
2*b^3 - 2*a^2*b^3)*x^3 - 12*(7*pi^2*a*b^2 - 20*a^3*b^2)*x^2 - 12*(pi^4*b -
6*pi^2*a^2*b - 8*a^4*b)*x - 12*(pi^5 - 8*pi^3*a^2 - 48*pi*a^4 + 4*(pi^3*b^2
 - 12*pi*a^2*b^2)*x^2 + 8*(pi^3*a*b - 12*pi*a^3*b)*x)*arctan(-(2*b*x + 2*a
- sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/pi) + 6*(3*pi^4*a + 8*pi^2*a^3
- 16*a^5 + 4*(3*pi^2*a*b^2 - 4*a^3*b^2)*x^2 + 8*(3*pi^2*a^2*b - 4*a^4*b)*x)
*log(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/(4*b^7*x^2 + 8*a*b^6*x + pi^2*b^5
 + 4*a^2*b^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{acoth}^2\left(\tanh\left(a + bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/acoth(tanh(b*x+a))**2,x)
```

```
[Out] Integral(x**4/acoth(tanh(a + b*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{arcoth}\left(\tanh\left(bx + a\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(x^4/arccoth(tanh(b*x + a))^2, x)
```

3.168 $\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2} dx$

Optimal. Leaf size=75

$$\frac{3x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^3}+\frac{3\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^4}-\frac{x^3}{b\coth^{-1}(\tanh(a+bx)}$$

```
[Out] (3*x^2)/(2*b^2) + (3*x*(b*x - ArcCoth[Tanh[a + b*x]]))/b^3 - x^3/(b*ArcCoth
[Tanh[a + b*x]]) + (3*(b*x - ArcCoth[Tanh[a + b*x]])^2*Log[ArcCoth[Tanh[a +
 b*x]]])/b^4
```

Rubi [A] time = 0.0537226, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2168, 2159, 2158, 2157, 29}

$$\frac{3x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^3}+\frac{3\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^4}-\frac{x^3}{b\coth^{-1}(\tanh(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcCoth[Tanh[a + b*x]]^2,x]
```


```
[Out] (3*x^2)/(2*b^2) + (3*x*(b*x - ArcCoth[Tanh[a + b*x]]))/b^3 - x^3/(b*ArcCoth
[Tanh[a + b*x]]) + (3*(b*x - ArcCoth[Tanh[a + b*x]])^2*Log[ArcCoth[Tanh[a +
 b*x]]])/b^4
```


Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2}\,dx = -\frac{x^3}{b\coth^{-1}(\tanh(a+bx))} + \frac{3\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))}\,dx}{b}$$
$$= \frac{3x^2}{2b^2} - \frac{x^3}{b\coth^{-1}(\tanh(a+bx))} - \frac{\left(3\left(-bx+\coth^{-1}(\tanh(a+bx))\right)\right)\int \frac{x}{\coth^{-1}(\tanh(a+bx}$$
$$= \frac{3x^2}{2b^2} + \frac{3x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^3} - \frac{x^3}{b\coth^{-1}(\tanh(a+bx))} + \frac{\left(3\left(-bx+\coth^{-1}\right.\right.}{}$$
$$= \frac{3x^2}{2b^2} + \frac{3x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^3} - \frac{x^3}{b\coth^{-1}(\tanh(a+bx))} + \frac{\left(3\left(-bx+\coth^{-1}\right.\right.}{}$$
$$= \frac{3x^2}{2b^2} + \frac{3x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^3} - \frac{x^3}{b\coth^{-1}(\tanh(a+bx))} + \frac{3\left(bx-\coth^{-1}(\mathrm{t}\right.}{}$$

Mathematica [A] time = 0.0615903, size = 83, normalized size = 1.11

$$\frac{\left(\coth^{-1}(\tanh(a+bx))-bx\right)^3}{b^4\coth^{-1}(\tanh(a+bx))} - \frac{2x\left(\coth^{-1}(\tanh(a+bx))-bx\right)}{b^3} + \frac{3\left(\coth^{-1}(\tanh(a+bx))-bx\right)^2\log\left(\coth^{-1}(\tan\right.}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] x^2/(2*b^2) - (2*x*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/b^3 + (-(b*x) + ArcCo
th[Tanh[a + b*x]])^3/(b^4*ArcCoth[Tanh[a + b*x]]) + (3*(-(b*x) + ArcCoth[Ta
nh[a + b*x]])^2*Log[ArcCoth[Tanh[a + b*x]]])/b^4
```

Maple [C] time = 1.408, size = 29109, normalized size = 388.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arccoth(tanh(b*x+a))^2,x)
```

```
[Out] result too large to display
```

Maxima [C] time = 2.43762, size = 167, normalized size = 2.23

$$\frac{4\left(4b^3x^3+i\pi^3-6\pi^2a-12i\pi a^2+8a^3+\left(6i\pi b^2-12ab^2\right)x^2+\left(4\pi^2b+16i\pi ab-16a^2b\right)x\right)}{32b^5x-16i\pi b^4+32ab^4} - \frac{\left(3\pi^2+12i\pi a-12a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 4*(4*b^3*x^3 + I*pi^3 - 6*pi^2*a - 12*I*pi*a^2 + 8*a^3 + (6*I*pi*b^2 - 12*a
*b^2)*x^2 + (4*pi^2*b + 16*I*pi*a*b - 16*a^2*b)*x)/(32*b^5*x - 16*I*pi*b^4
+ 32*a*b^4) - 1/4*(3*pi^2 + 12*I*pi*a - 12*a^2)*log(-I*pi + 2*b*x + 2*a)/b^
4
```

Fricas [B] time = 1.76149, size = 544, normalized size = 7.25

$$\frac{16\,b^4x^4 - 32\,ab^3x^3 - 2\,\pi^4 + 32\,a^4 + 4\left(\pi^2b^2 - 28\,a^2b^2\right)x^2 - 8\left(5\,\pi^2ab + 4\,a^3b\right)x - 48\left(4\,\pi ab^2x^2 + 8\,\pi a^2bx + \pi^3a + 4\,\pi a^3\right)}{8\left(4\,b^6x^2 + 8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(16*b^4*x^4 - 32*a*b^3*x^3 - 2*pi^4 + 32*a^4 + 4*(pi^2*b^2 - 28*a^2*b^2
)*x^2 - 8*(5*pi^2*a*b + 4*a^3*b)*x - 48*(4*pi*a*b^2*x^2 + 8*pi*a^2*b*x + pi
^3*a + 4*pi*a^3)*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4
*a^2))/pi) - 3*(pi^4 - 16*a^4 + 4*(pi^2*b^2 - 4*a^2*b^2)*x^2 + 8*(pi^2*a*b
- 4*a^3*b)*x)*log(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/(4*b^6*x^2 + 8*a*b^5
*x + pi^2*b^4 + 4*a^2*b^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{acoth}^2\left(\tanh\left(a + bx\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/acoth(tanh(b*x+a))**2,x)
```

```
[Out] Integral(x**3/acoth(tanh(a + b*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{arcoth}\left(\tanh\left(bx + a\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(x^3/arccoth(tanh(b*x + a))^2, x)
```

3.169 $\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2} dx$

Optimal. Leaf size=50

$$\frac{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{x^2}{b\coth^{-1}(\tanh(a+bx))}+\frac{2x}{b^2}$$

```
[Out] (2*x)/b^2 - x^2/(b*ArcCoth[Tanh[a + b*x]]) + (2*(b*x - ArcCoth[Tanh[a + b*x
]])*Log[ArcCoth[Tanh[a + b*x]]])/b^3
```

Rubi [A] time = 0.031317, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2168, 2158, 2157, 29}

$$\frac{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{x^2}{b\coth^{-1}(\tanh(a+bx))}+\frac{2x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] (2*x)/b^2 - x^2/(b*ArcCoth[Tanh[a + b*x]]) + (2*(b*x - ArcCoth[Tanh[a + b*x
]])*Log[ArcCoth[Tanh[a + b*x]]])/b^3
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2}\,dx = -\frac{x^2}{b\coth^{-1}(\tanh(a+bx))} + \frac{2\int \frac{x}{\coth^{-1}(\tanh(a+bx))}\,dx}{b}$$
$$= \frac{2x}{b^2} - \frac{x^2}{b\coth^{-1}(\tanh(a+bx))} - \frac{\left(2\left(-bx+\coth^{-1}(\tanh(a+bx))\right)\right)\int \frac{1}{\coth^{-1}(\tanh(a+bx))}\,dx}{b^2}$$
$$= \frac{2x}{b^2} - \frac{x^2}{b\coth^{-1}(\tanh(a+bx))} - \frac{\left(2\left(-bx+\coth^{-1}(\tanh(a+bx))\right)\right)\text{Subst}\left(\int \frac{1}{x}\,dx, x, \text{cot}\right.}{b^3}$$
$$= \frac{2x}{b^2} - \frac{x^2}{b\coth^{-1}(\tanh(a+bx))} + \frac{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\log\left(\coth^{-1}(\tanh(a+bx\right.}{b^3}$$

Mathematica [A] time = 0.0680071, size = 56, normalized size = 1.12

$$\frac{-\frac{\left(\coth^{-1}(\tanh(a+bx))-bx\right)^2}{\coth^{-1}(\tanh(a+bx))} + 2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\log\left(\coth^{-1}(\tanh(a+bx))\right) + bx}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] (b*x - (-(b*x) + ArcCoth[Tanh[a + b*x]])^2/ArcCoth[Tanh[a + b*x]] + 2*(b*x
- ArcCoth[Tanh[a + b*x]])*Log[ArcCoth[Tanh[a + b*x]]])/b^3
```

Maple [C] time = 0.366, size = 4626, normalized size = 92.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arccoth(tanh(b*x+a))^2,x)
```

```
[Out] -4*I*x^2/b/(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1)
)*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(
I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn
(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*P
i*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-P
i*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+2*Pi+4*I*ln(exp(b*x+a)))+2/b^2*ln(
-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp
(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x
+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*
x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp
(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp
(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*
x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)
*x-2/b^3*ln(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1)
)*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(
I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn
(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*P
i*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-P
i*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csg
```

```
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)
+4*I*a+2*Pi)*ln(exp(b*x+a))-I/b^3*ln(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*
csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(ex
p(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2
*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*c
sgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*c
sgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(ex
p(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4
*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+1/2*I
/b^3*ln(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*cs
gn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(e
xp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(
exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*cs
gn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*cs
gn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I
*a+2*Pi)*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*
b*x+2*a)/(exp(2*b*x+2*a)+1))-1/2*I/b^3*ln(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^
2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a
)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(
exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a)
)^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2
+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a
)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*
b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)*Pi*csgn(I/(exp(2*b*x+2*a)+1))*cs
gn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+I/b^3*ln(-2*Pi*csgn(I/(exp(2*b*x+
2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(
2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b
*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*e
xp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*
x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(
2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1
))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)*Pi*csgn(I/(exp(2*b*x+2*
a)+1))^3+1/2*I/b^3*ln(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b
*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)
)-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2
+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x
+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x
+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)
)^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+
a))-b*x-a)+4*I*a+2*Pi)*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-I/b^3
*ln(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I
*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2
*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(
2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I
*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I
*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(
2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2
*Pi)*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/2*I/b^3*ln(-2*Pi*csgn
(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a
))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*
csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))
^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*c
sgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a
))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(ex
p(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)*Pi*csgn(I
*exp(2*b*x+2*a))^3-1/2*I/b^3*ln(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(
I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b
*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+
```

```
2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I
*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I
*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b
*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(l
n(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x
+2*a)/(exp(2*b*x+2*a)+1))^2+1/2*I/b^3*ln(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2
+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)
/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(e
xp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))
^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+
Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)
/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b
*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b
*x+2*a)+1))^3+I/b^3*ln(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*
b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1
))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^
2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*
x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*
x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1
))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x
+a))-b*x-a)+4*I*a+2*Pi)*Pi+2*x/b^2
```

Maxima [C] time = 2.44275, size = 108, normalized size = 2.16

$$\frac{4\left(4\,b^{2}x^{2}+\pi^{2}+4i\,\pi a-4\,a^{2}+\left(-2i\,\pi b+4\,ab\right)x\right)}{16\,b^{4}x-8i\,\pi b^{3}+16\,ab^{3}}-\frac{\left(-i\,\pi+2\,a\right)\log\left(-i\,\pi+2\,bx+2\,a\right)}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 4*(4*b^2*x^2 + pi^2 + 4*I*pi*a - 4*a^2 + (-2*I*pi*b + 4*a*b)*x)/(16*b^4*x -
 8*I*pi*b^3 + 16*a*b^3) - (-I*pi + 2*a)*log(-I*pi + 2*b*x + 2*a)/b^3
```

Fricas [B] time = 1.63523, size = 416, normalized size = 8.32

$$\frac{4\,b^{3}x^{3}+8\,ab^{2}x^{2}+2\,\pi^{2}bx-\pi^{2}a-4\,a^{3}+2\left(4\,\pi b^{2}x^{2}+8\,\pi abx+\pi^{3}+4\,\pi a^{2}\right)\arctan\left(-\frac{2\,bx+2\,a-\sqrt{4\,b^{2}x^{2}+8\,abx+\pi^{2}+4\,a^{2}}}{\pi}\right)-\left(4\,a\right.}{4\,b^{5}x^{2}+8\,ab^{4}x+\pi^{2}b^{3}+4\,a^{2}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] (4*b^3*x^3 + 8*a*b^2*x^2 + 2*pi^2*b*x - pi^2*a - 4*a^3 + 2*(4*pi*b^2*x^2 +
8*pi*a*b*x + pi^3 + 4*pi*a^2)*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 + 8*a*b
*x + pi^2 + 4*a^2))/pi) - (4*a*b^2*x^2 + 8*a^2*b*x + pi^2*a + 4*a^3)*log(4*
b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/(4*b^5*x^2 + 8*a*b^4*x + pi^2*b^3 + 4*a^
2*b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{2}}{\operatorname{acoth}^{2}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/acoth(tanh(b*x+a))**2,x)
```

```
[Out] Integral(x**2/acoth(tanh(a + b*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/arccoth(tanh(b*x + a))^2, x)
```

3.170 $\int \frac{x}{\coth^{-1}(\tanh(a+bx))^2}\,dx$

Optimal. Leaf size=28

$$\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^2}-\frac{x}{b\coth^{-1}(\tanh(a+bx))}$$

```
[Out] -(x/(b*ArcCoth[Tanh[a + b*x]])) + Log[ArcCoth[Tanh[a + b*x]]]/b^2
```

Rubi [A] time = 0.013374, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2168, 2157, 29}

$$\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^2}-\frac{x}{b\coth^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] -(x/(b*ArcCoth[Tanh[a + b*x]])) + Log[ArcCoth[Tanh[a + b*x]]]/b^2
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\coth^{-1}(\tanh(a+bx))^2}\,dx &= -\frac{x}{b\coth^{-1}(\tanh(a+bx))}+\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}\,dx}{b}\\
&= -\frac{x}{b\coth^{-1}(\tanh(a+bx))}+\frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \coth^{-1}(\tanh(a+bx))\right)}{b^2}\\
&= -\frac{x}{b\coth^{-1}(\tanh(a+bx))}+\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^2}
\end{aligned}$$

Mathematica [A] time = 0.0654313, size = 27, normalized size = 0.96

$$\frac{-\frac{bx}{\coth^{-1}(\tanh(a+bx))}+\log\left(\coth^{-1}(\tanh(a+bx))\right)+1}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] (1 - (b*x)/ArcCoth[Tanh[a + b*x]] + Log[ArcCoth[Tanh[a + b*x]]])/b^2
```

Maple [C] time = 0.187, size = 625, normalized size = 22.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arccoth(tanh(b*x+a))^2,x)
```

```
[Out] -4*I*x/b/(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*
csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/
(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I
/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*
csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*
csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(
I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+2*Pi+4*I*ln(exp(b*x+a)))+1/b^2*ln(ln
(exp(b*x+a))-1/4*I*Pi*(-2*csgn(I/(exp(2*b*x+2*a)+1))^2+csgn(I/(exp(2*b*x+2*
a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-csg
n(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*csgn(
I/(exp(2*b*x+2*a)+1))^3+csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*csgn(
I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+csgn(I*exp(2*b*x+2*a))^3-csgn(I*exp(
2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+csgn(I*exp(2*b*x+2*
a)/(exp(2*b*x+2*a)+1))^3+2))
```

Maxima [C] time = 2.4401, size = 63, normalized size = 2.25

$$\frac{4\,(-i\,\pi+2\,a)}{8\,b^3x-4i\,\pi b^2+8\,ab^2}+\frac{\log\left(-i\,\pi+2\,bx+2\,a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 4*(-I*pi + 2*a)/(8*b^3*x - 4*I*pi*b^2 + 8*a*b^2) + log(-I*pi + 2*b*x + 2*a)
/b^2
```

Fricas [B] time = 1.68068, size = 213, normalized size = 7.61

$$\frac{8\,abx+2\,\pi^2+8\,a^2+\left(4\,b^2x^2+8\,abx+\pi^2+4\,a^2\right)\log\left(4\,b^2x^2+8\,abx+\pi^2+4\,a^2\right)}{2\left(4\,b^4x^2+8\,ab^3x+\pi^2b^2+4\,a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(8*a*b*x + 2*pi^2 + 8*a^2 + (4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2)*log(4*
b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/(4*b^4*x^2 + 8*a*b^3*x + pi^2*b^2 + 4*a^
2*b^2)
```

Sympy [A] time = 16.2979, size = 36, normalized size = 1.29

$$\begin{cases} -\frac{x}{b\operatorname{acoth}(\tanh(a+bx))} + \frac{\log(\operatorname{acoth}(\tanh(a+bx)))}{b^2} & \text{for } b \neq 0 \\ \frac{x^2}{2\operatorname{acoth}^2(\tanh(a))} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acoth(tanh(b*x+a))**2,x)
```

```
[Out] Piecewise((-x/(b*acoth(tanh(a + b*x))) + log(acoth(tanh(a + b*x)))/b**2, Ne
(b, 0)), (x**2/(2*acoth(tanh(a))**2), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{arcoth}(\tanh(bx+a))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(x/arccoth(tanh(b*x + a))^2, x)
```

3.171 $\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2} \, dx$

Optimal. Leaf size=14

$$-\frac{1}{b\coth^{-1}(\tanh(a+bx))}$$

[Out] -(1/(b*ArcCoth[Tanh[a + b*x]]))

Rubi [A] time = 0.0048201, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2157, 30}

$$-\frac{1}{b\coth^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b*x]]^(-2),x]

[Out] -(1/(b*ArcCoth[Tanh[a + b*x]]))

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2} \, dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2} \, dx, x, \coth^{-1}(\tanh(a+bx))\right)}{b}\\ &= -\frac{1}{b\coth^{-1}(\tanh(a+bx))}\end{aligned}$$

Mathematica [A] time = 0.0058844, size = 14, normalized size = 1.

$$-\frac{1}{b\coth^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b*x]]^(-2),x]

[Out] -(1/(b*ArcCoth[Tanh[a + b*x]]))

Maple [A] time = 0.078, size = 15, normalized size = 1.1

$$-\frac{1}{b\mathrm{arccoth}\left(\tanh\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccoth(tanh(b*x+a))^2,x)
```

```
[Out] -1/b/arccoth(tanh(b*x+a))
```

Maxima [C] time = 1.47959, size = 24, normalized size = 1.71

$$\frac{4}{\left(-2i\,\pi-4\,bx-4\,a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 4/((-2*I*pi - 4*b*x - 4*a)*b)
```

Fricas [B] time = 1.61425, size = 77, normalized size = 5.5

$$-\frac{4\left(bx+a\right)}{4\,b^3x^2+8\,ab^2x+\pi^2b+4\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] -4*(b*x + a)/(4*b^3*x^2 + 8*a*b^2*x + pi^2*b + 4*a^2*b)
```

Sympy [A] time = 16.1339, size = 20, normalized size = 1.43

$$\begin{cases} -\frac{1}{b\,\mathrm{acoth}\left(\tanh\left(a+bx\right)\right)} & \text{for}\ b\neq 0\\ \frac{x}{\mathrm{acoth}^2\left(\tanh\left(a\right)\right)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acoth(tanh(b*x+a))**2,x)
```

```
[Out] Piecewise((-1/(b*acoth(tanh(a + b*x))), Ne(b, 0)), (x/acoth(tanh(a))**2, Tr
ue))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^(-2), x)
```

3.172 $\int \frac{1}{x \coth^{-1}(\tanh(a+bx))^2} dx$

Optimal. Leaf size=70

$$-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}+\frac{\log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}-\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}$$

```
[Out] -(1/((b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]])) + Log[x]/(b*x
- ArcCoth[Tanh[a + b*x]])^2 - Log[ArcCoth[Tanh[a + b*x]]]/(b*x - ArcCoth[Ta
nh[a + b*x]])^2
```

Rubi [A] time = 0.0479788, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2163, 2160, 2157, 29}

$$-\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}+\frac{\log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}-\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*ArcCoth[Tanh[a + b*x]]^2),x]
```

```
[Out] -(1/((b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]])) + Log[x]/(b*x
- ArcCoth[Tanh[a + b*x]])^2 - Log[ArcCoth[Tanh[a + b*x]]]/(b*x - ArcCoth[Ta
nh[a + b*x]])^2
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^2}\,dx = -\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))} + \frac{\int \frac{1}{x\coth^{-1}(\tanh(a+bx))}\,dx}{-bx+\coth^{-1}(\tanh(a+bx}$$

$$= -\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))} - \frac{}{\left(bx-\coth^{-1}(\tanh(a+bx)\right.}$$

$$= -\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))} + \frac{\log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx)\right.}$$

$$= -\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))} + \frac{\log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx)\right.}$$

Mathematica [A] time = 0.0713631, size = 53, normalized size = 0.76

$$\frac{\coth^{-1}(\tanh(a+bx))\left(-\log\left(\coth^{-1}(\tanh(a+bx))\right)+\log(bx)+1\right)-bx}{\coth^{-1}(\tanh(a+bx))\left(\coth^{-1}(\tanh(a+bx))-bx\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*ArcCoth[Tanh[a + b*x]]^2),x]
```

```
[Out] (-(b*x) + ArcCoth[Tanh[a + b*x]]*(1 + Log[b*x] - Log[ArcCoth[Tanh[a + b*x]]
]))/(ArcCoth[Tanh[a + b*x]]*(-(b*x) + ArcCoth[Tanh[a + b*x]])^2)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\mathrm{arccoth}\left(\tanh\left(bx+a\right)\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arccoth(tanh(b*x+a))^2,x)
```

```
[Out] int(1/x/arccoth(tanh(b*x+a))^2,x)
```

Maxima [C] time = 2.45303, size = 104, normalized size = 1.49

$$\frac{4\,\log\left(-i\,\pi+2\,bx+2\,a\right)}{\pi^2+4i\,\pi a-4\,a^2} - \frac{4\,\log\left(x\right)}{\pi^2+4i\,\pi a-4\,a^2} - \frac{4}{\pi^2+4i\,\pi a-4\,a^2+(2i\,\pi b-4\,ab)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 4*log(-I*pi + 2*b*x + 2*a)/(pi^2 + 4*I*pi*a - 4*a^2) - 4*log(x)/(pi^2 + 4*I
*pi*a - 4*a^2) - 4/(pi^2 + 4*I*pi*a - 4*a^2 + (2*I*pi*b - 4*a*b)*x)
```

Fricas [B] time = 1.73918, size = 691, normalized size = 9.87

$$-\frac{2\left(2\,\pi^4 - 32\,a^4 - 8\left(\pi^2 ab + 4\,a^3 b\right)x + 16\left(4\,\pi ab^2x^2 + 8\,\pi a^2bx + \pi^3 a + 4\,\pi a^3\right)\arctan\left(-\frac{2\,bx+2\,a-\sqrt{4\,b^2x^2+8\,abx+\pi^2+4\,a^2}}{\pi}\right) - \left(\cdots\right.\right.}{\pi^6 + 12\,\pi^4a^2 + 48\,\pi^2a^4 + 64\,a^6 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] -2*(2*pi^4 - 32*a^4 - 8*(pi^2*a*b + 4*a^3*b)*x + 16*(4*pi*a*b^2*x^2 + 8*pi*
a^2*b*x + pi^3*a + 4*pi*a^3)*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 + 8*a*b*
x + pi^2 + 4*a^2))/pi) - (pi^4 - 16*a^4 + 4*(pi^2*b^2 - 4*a^2*b^2)*x^2 + 8*
(pi^2*a*b - 4*a^3*b)*x)*log(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2) + 2*(pi^4 -
 16*a^4 + 4*(pi^2*b^2 - 4*a^2*b^2)*x^2 + 8*(pi^2*a*b - 4*a^3*b)*x)*log(x))/
(pi^6 + 12*pi^4*a^2 + 48*pi^2*a^4 + 64*a^6 + 4*(pi^4*b^2 + 8*pi^2*a^2*b^2 +
 16*a^4*b^2)*x^2 + 8*(pi^4*a*b + 8*pi^2*a^3*b + 16*a^5*b)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,\mathrm{acoth}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/acoth(tanh(b*x+a))**2,x)
```

```
[Out] Integral(1/(x*acoth(tanh(a + b*x))**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x*arccoth(tanh(b*x + a))^2), x)
```

3.173 $\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^2}\,dx$

Optimal. Leaf size=102

$$-\frac{2b}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}+\frac{\cdots}{\left(bx\cdots\right.}$$

```
[Out] (-2*b)/((b*x - ArcCoth[Tanh[a + b*x]])^2*ArcCoth[Tanh[a + b*x]]) + 1/(x*(b*
x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]) + (2*b*Log[x])/(b*x - A
rcCoth[Tanh[a + b*x]])^3 - (2*b*Log[ArcCoth[Tanh[a + b*x]]])/(b*x - ArcCoth
[Tanh[a + b*x]])^3
```

Rubi [A] time = 0.0626622, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2171, 2163, 2160, 2157, 29}

$$-\frac{2b}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\coth^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}+\frac{\cdots}{\left(bx\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*ArcCoth[Tanh[a + b*x]]^2),x]
```

```
[Out] (-2*b)/((b*x - ArcCoth[Tanh[a + b*x]])^2*ArcCoth[Tanh[a + b*x]]) + 1/(x*(b*
x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]) + (2*b*Log[x])/(b*x - A
rcCoth[Tanh[a + b*x]])^3 - (2*b*Log[ArcCoth[Tanh[a + b*x]]])/(b*x - ArcCoth
[Tanh[a + b*x]])^3
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))} - \frac{(2b)\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^2}\,dx}{-bx+\coth^{-1}(\tanh(a+bx))}$$

$$= -\frac{2b}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\coth^{-1}(\tanh(a+bx))} + \frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx\right.}$$

$$= -\frac{2b}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\coth^{-1}(\tanh(a+bx))} + \frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx\right.}$$

$$= -\frac{2b}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\coth^{-1}(\tanh(a+bx))} + \frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx\right.}$$

$$= -\frac{2b}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\coth^{-1}(\tanh(a+bx))} + \frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx\right.}$$

Mathematica [A] time = 0.0619438, size = 70, normalized size = 0.69

$$\frac{\coth^{-1}(\tanh(a+bx))^2 + 2bx\coth^{-1}(\tanh(a+bx))\left(\log(x)-\log\left(\coth^{-1}(\tanh(a+bx))\right)\right) - b^2x^2}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3\coth^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*ArcCoth[Tanh[a + b*x]]^2),x]
```

```
[Out] (-(b^2*x^2) + ArcCoth[Tanh[a + b*x]]^2 + 2*b*x*ArcCoth[Tanh[a + b*x]]*(Log[
x] - Log[ArcCoth[Tanh[a + b*x]]]))/(x*(b*x - ArcCoth[Tanh[a + b*x]])^3*ArcC
oth[Tanh[a + b*x]])
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\operatorname{arccoth}\left(\tanh\left(bx+a\right)\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arccoth(tanh(b*x+a))^2,x)
```

```
[Out] int(1/x^2/arccoth(tanh(b*x+a))^2,x)
```

Maxima [C] time = 2.45069, size = 182, normalized size = 1.78

$$-\frac{16\,b\log\left(-i\,\pi+2\,bx+2\,a\right)}{-i\,\pi^3+6\,\pi^2 a+12 i\,\pi a^2-8\,a^3} + \frac{16\,b\log\left(x\right)}{-i\,\pi^3+6\,\pi^2 a+12 i\,\pi a^2-8\,a^3} - \frac{4\left(i\,\pi-4\,bx-2\,a\right)}{\left(2\,\pi^2 b+8 i\,\pi ab-8\,a^2 b\right)x^2-\left(i\,\pi^3-6\,\pi^2 a-12 i\,\pi a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] -16*b*log(-I*pi + 2*b*x + 2*a)/(-I*pi^3 + 6*pi^2*a + 12*I*pi*a^2 - 8*a^3) +
 16*b*log(x)/(-I*pi^3 + 6*pi^2*a + 12*I*pi*a^2 - 8*a^3) - 4*(I*pi - 4*b*x -
 2*a)/((2*pi^2*b + 8*I*pi*a*b - 8*a^2*b)*x^2 - (I*pi^3 - 6*pi^2*a - 12*I*pi
*a^2 + 8*a^3)*x)
```

Fricas [B] time = 1.81465, size = 1065, normalized size = 10.44

$$4\left(\pi^6 + 4\pi^4a^2 - 16\pi^2a^4 - 64a^6 + 8\left(\pi^4b^2 - 16a^4b^2\right)x^2 + 4\left(5\pi^4ab + 8\pi^2a^3b - 48a^5b\right)x - 8\left(4\left(\pi^3b^3 - 12\pi a^2b^3\right)x^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 4*(pi^6 + 4*pi^4*a^2 - 16*pi^2*a^4 - 64*a^6 + 8*(pi^4*b^2 - 16*a^4*b^2)*x^2
 + 4*(5*pi^4*a*b + 8*pi^2*a^3*b - 48*a^5*b)*x - 8*(4*(pi^3*b^3 - 12*pi*a^2*
b^3)*x^3 + 8*(pi^3*a*b^2 - 12*pi*a^3*b^2)*x^2 + (pi^5*b - 8*pi^3*a^2*b - 48
*pi*a^4*b)*x)*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^
2))/pi) - 4*(4*(3*pi^2*a*b^3 - 4*a^3*b^3)*x^3 + 8*(3*pi^2*a^2*b^2 - 4*a^4*b
^2)*x^2 + (3*pi^4*a*b + 8*pi^2*a^3*b - 16*a^5*b)*x)*log(4*b^2*x^2 + 8*a*b*x
 + pi^2 + 4*a^2) + 8*(4*(3*pi^2*a*b^3 - 4*a^3*b^3)*x^3 + 8*(3*pi^2*a^2*b^2
- 4*a^4*b^2)*x^2 + (3*pi^4*a*b + 8*pi^2*a^3*b - 16*a^5*b)*x)*log(x))/(4*(pi
^6*b^2 + 12*pi^4*a^2*b^2 + 48*pi^2*a^4*b^2 + 64*a^6*b^2)*x^3 + 8*(pi^6*a*b
+ 12*pi^4*a^3*b + 48*pi^2*a^5*b + 64*a^7*b)*x^2 + (pi^8 + 16*pi^6*a^2 + 96*
pi^4*a^4 + 256*pi^2*a^6 + 256*a^8)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \operatorname{acoth}^2(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/acoth(tanh(b*x+a))**2,x)
```

```
[Out] Integral(1/(x**2*acoth(tanh(a + b*x))**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \operatorname{arcoth}(\tanh(bx+a))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*arccoth(tanh(b*x + a))^2), x)
```

3.174 $\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^2} \, dx$

Optimal. Leaf size=143

$$-\frac{3b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3\coth^{-1}(\tanh(a+bx))}+\frac{3b^2\log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^4}-\frac{3b^2\log\left(\coth^{-1}(\tanh(a+bx))\right)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^4}$$

```
[Out] (-3*b^2)/((b*x - ArcCoth[Tanh[a + b*x]])^3*ArcCoth[Tanh[a + b*x]]) + (3*b)/
(2*x*(b*x - ArcCoth[Tanh[a + b*x]])^2*ArcCoth[Tanh[a + b*x]]) + 1/(2*x^2*(b
*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]) + (3*b^2*Log[x])/(b*x
- ArcCoth[Tanh[a + b*x]])^4 - (3*b^2*Log[ArcCoth[Tanh[a + b*x]]])/(b*x - Ar
cCoth[Tanh[a + b*x]])^4
```

Rubi [A] time = 0.0896868, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2171, 2163, 2160, 2157, 29}

$$-\frac{3b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3\coth^{-1}(\tanh(a+bx))}+\frac{3b^2\log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^4}-\frac{3b^2\log\left(\coth^{-1}(\tanh(a+bx))\right)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*ArcCoth[Tanh[a + b*x]]^2),x]
```

```
[Out] (-3*b^2)/((b*x - ArcCoth[Tanh[a + b*x]])^3*ArcCoth[Tanh[a + b*x]]) + (3*b)/
(2*x*(b*x - ArcCoth[Tanh[a + b*x]])^2*ArcCoth[Tanh[a + b*x]]) + 1/(2*x^2*(b
*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]) + (3*b^2*Log[x])/(b*x
- ArcCoth[Tanh[a + b*x]])^4 - (3*b^2*Log[ArcCoth[Tanh[a + b*x]]])/(b*x - Ar
cCoth[Tanh[a + b*x]])^4
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))} + \frac{(3b)\int \frac{1}{x^2\coth^{-1}(\tanh(a+b}}{2\left(bx-\coth^{-1}(\tanh(a\right.}$$

$$= \frac{3b}{2x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\coth^{-1}(\tanh(a+bx))} + \frac{}{2x^2\left(bx-\coth^{-1}(\tanh(\right.}$$

$$= -\frac{3b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3\coth^{-1}(\tanh(a+bx))} + \frac{}{2x\left(bx-\coth^{-1}(\tanh(a\right.}$$

$$= -\frac{3b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3\coth^{-1}(\tanh(a+bx))} + \frac{}{2x\left(bx-\coth^{-1}(\tanh(a\right.}$$

$$= -\frac{3b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3\coth^{-1}(\tanh(a+bx))} + \frac{}{2x\left(bx-\coth^{-1}(\tanh(a\right.}$$

$$= -\frac{3b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3\coth^{-1}(\tanh(a+bx))} + \frac{}{2x\left(bx-\coth^{-1}(\tanh(a\right.}$$

Mathematica [A] time = 0.0456956, size = 92, normalized size = 0.64

$$-\frac{-3b^2x^2\coth^{-1}(\tanh(a+bx))\left(-2\log\left(\coth^{-1}(\tanh(a+bx))\right)+2\log(x)-1\right)-6bx\coth^{-1}(\tanh(a+bx))^2+\coth^{-}}{2x^2\coth^{-1}(\tanh(a+bx))\left(\coth^{-1}(\tanh(a+bx))-bx\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*ArcCoth[Tanh[a + b*x]]^2),x]
```

```
[Out] -(2*b^3*x^3 - 6*b*x*ArcCoth[Tanh[a + b*x]]^2 + ArcCoth[Tanh[a + b*x]]^3 - 3
*b^2*x^2*ArcCoth[Tanh[a + b*x]]*(-1 + 2*Log[x] - 2*Log[ArcCoth[Tanh[a + b*x
]]]))/(2*x^2*ArcCoth[Tanh[a + b*x]]*(-(b*x) + ArcCoth[Tanh[a + b*x]])^4)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(\operatorname{arccoth}\left(\tanh\left(bx+a\right)\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/arccoth(tanh(b*x+a))^2,x)
```

```
[Out] int(1/x^3/arccoth(tanh(b*x+a))^2,x)
```

Maxima [C] time = 2.46131, size = 258, normalized size = 1.8

$$-\frac{48\,b^2\log\left(-i\,\pi+2\,bx+2\,a\right)}{\pi^4+8i\,\pi^3a-24\,\pi^2a^2-32i\,\pi a^3+16\,a^4}+\frac{48\,b^2\log\left(x\right)}{\pi^4+8i\,\pi^3a-24\,\pi^2a^2-32i\,\pi a^3+16\,a^4}-\frac{4\left(24\,b^2x^2+\right.}{\left(-4i\,\pi^3b+24\,\pi^2ab+48i\,\pi a^2b-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] -48*b^2*log(-I*pi + 2*b*x + 2*a)/(pi^4 + 8*I*pi^3*a - 24*pi^2*a^2 - 32*I*pi
*a^3 + 16*a^4) + 48*b^2*log(x)/(pi^4 + 8*I*pi^3*a - 24*pi^2*a^2 - 32*I*pi*a
^3 + 16*a^4) - 4*(24*b^2*x^2 + pi^2 + 4*I*pi*a - 4*a^2 + (-6*I*pi*b + 12*a*
b)*x)/((-4*I*pi^3*b + 24*pi^2*a*b + 48*I*pi*a^2*b - 32*a^3*b)*x^3 - (2*pi^4
 + 16*I*pi^3*a - 48*pi^2*a^2 - 64*I*pi*a^3 + 32*a^4)*x^2)
```

Fricas [B] time = 1.74182, size = 1439, normalized size = 10.06

$$\frac{2\left(\pi^8+8\,\pi^6a^2-128\,\pi^2a^6-256\,a^8-96\left(3\,\pi^4ab^3+8\,\pi^2a^3b^3-16\,a^5b^3\right)x^3+12\left(\pi^6b^2-44\,\pi^4a^2b^2-144\,\pi^2a^4b^2+192\,a^6\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 2*(pi^8 + 8*pi^6*a^2 - 128*pi^2*a^6 - 256*a^8 - 96*(3*pi^4*a*b^3 + 8*pi^2*a
^3*b^3 - 16*a^5*b^3)*x^3 + 12*(pi^6*b^2 - 44*pi^4*a^2*b^2 - 144*pi^2*a^4*b^
2 + 192*a^6*b^2)*x^2 - 8*(5*pi^6*a*b + 36*pi^4*a^3*b + 48*pi^2*a^5*b - 64*a
^7*b)*x + 384*(4*(pi^3*a*b^4 - 4*pi*a^3*b^4)*x^4 + 8*(pi^3*a^2*b^3 - 4*pi*a
^4*b^3)*x^3 + (pi^5*a*b^2 - 16*pi*a^5*b^2)*x^2)*arctan(-(2*b*x + 2*a - sqrt
(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/pi) - 12*(4*(pi^4*b^4 - 24*pi^2*a^2*b
^4 + 16*a^4*b^4)*x^4 + 8*(pi^4*a*b^3 - 24*pi^2*a^3*b^3 + 16*a^5*b^3)*x^3 +
(pi^6*b^2 - 20*pi^4*a^2*b^2 - 80*pi^2*a^4*b^2 + 64*a^6*b^2)*x^2)*log(4*b^2*
x^2 + 8*a*b*x + pi^2 + 4*a^2) + 24*(4*(pi^4*b^4 - 24*pi^2*a^2*b^4 + 16*a^4*
b^4)*x^4 + 8*(pi^4*a*b^3 - 24*pi^2*a^3*b^3 + 16*a^5*b^3)*x^3 + (pi^6*b^2 -
20*pi^4*a^2*b^2 - 80*pi^2*a^4*b^2 + 64*a^6*b^2)*x^2)*log(x))/(4*(pi^8*b^2 +
 16*pi^6*a^2*b^2 + 96*pi^4*a^4*b^2 + 256*pi^2*a^6*b^2 + 256*a^8*b^2)*x^4 +
8*(pi^8*a*b + 16*pi^6*a^3*b + 96*pi^4*a^5*b + 256*pi^2*a^7*b + 256*a^9*b)*x
^3 + (pi^10 + 20*pi^8*a^2 + 160*pi^6*a^4 + 640*pi^4*a^6 + 1280*pi^2*a^8 + 1
024*a^10)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^3\operatorname{acoth}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/acoth(tanh(b*x+a))**2,x)
```

```
[Out] Integral(1/(x**3*acoth(tanh(a + b*x))**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \operatorname{arcoth}(\tanh(bx+a))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^3*arccoth(tanh(b*x + a))^2), x)
```

3.175 $\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^3} \, dx$

Optimal. Leaf size=94

$$-\frac{mx^{m-1}\text{Hypergeometric2F1}\left(1,m-1,m,\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{2b^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}-\frac{mx^{m-1}}{2b^2\coth^{-1}(\tanh(a+bx))}-\frac{x^m}{2b\coth^{-1}(\tanh(a+bx)}$$

```
[Out] -x^m/(2*b*ArcCoth[Tanh[a + b*x]]^2) - (m*x^(-1 + m))/(2*b^2*ArcCoth[Tanh[a
+ b*x]]) - (m*x^(-1 + m)*Hypergeometric2F1[1, -1 + m, m, (b*x)/(b*x - ArcCo
th[Tanh[a + b*x]])])/(2*b^2*(b*x - ArcCoth[Tanh[a + b*x]]))
```

Rubi [A] time = 0.0599338, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 2164}

$$-\frac{mx^{m-1}\,{}_2F_1\left(1,m-1;m;\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{2b^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}-\frac{mx^{m-1}}{2b^2\coth^{-1}(\tanh(a+bx))}-\frac{x^m}{2b\coth^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] -x^m/(2*b*ArcCoth[Tanh[a + b*x]]^2) - (m*x^(-1 + m))/(2*b^2*ArcCoth[Tanh[a
+ b*x]]) - (m*x^(-1 + m)*Hypergeometric2F1[1, -1 + m, m, (b*x)/(b*x - ArcCo
th[Tanh[a + b*x]])])/(2*b^2*(b*x - ArcCoth[Tanh[a + b*x]]))
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2164

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)*Hypergeometric2F1[1, n + 1, n + 2, -((a*v)/(b*u
- a*v))])/((n + 1)*(b*u - a*v)), x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] && !IntegerQ[n]
```

Rubi steps

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{x^m}{2b\coth^{-1}(\tanh(a+bx))^2} + \frac{m\int \frac{x^{-1+m}}{\coth^{-1}(\tanh(a+bx))^2}\,dx}{2b}$$

$$= -\frac{x^m}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{mx^{-1+m}}{2b^2\coth^{-1}(\tanh(a+bx))} - \frac{((1-m)m)\int \frac{x^{-2+m}}{\coth^{-1}(\tanh(a}{2b^2}$$

$$= -\frac{x^m}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{mx^{-1+m}}{2b^2\coth^{-1}(\tanh(a+bx))} - \frac{mx^{-1+m}\,{}_2F_1\left(1,-1+m;m\right.}{2b^2\left(bx-\coth^{-1}(t\right.}$$

Mathematica [A] time = 0.50711, size = 51, normalized size = 0.54

$$\frac{x^{m+1}\text{Hypergeometric2F1}\left(3,m+1,m+2,-\frac{bx}{\coth^{-1}(\tanh(a+bx))-bx}\right)}{(m+1)\left(\coth^{-1}(\tanh(a+bx))-bx\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[3, 1 + m, 2 + m, -((b*x)/(-(b*x) + ArcCoth[Tan
h[a + b*x]]))])/((1 + m)*(-(b*x) + ArcCoth[Tanh[a + b*x]])^3)
```

Maple [F] time = 1.891, size = 0, normalized size = 0.

$$\int \frac{x^m}{(\text{arccoth}(\tanh(bx+a)))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arccoth(tanh(b*x+a))^3,x)
```

```
[Out] int(x^m/arccoth(tanh(b*x+a))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\text{arcoth}(\tanh(bx+a))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] integrate(x^m/arccoth(tanh(b*x + a))^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{\text{arcoth}(\tanh(bx+a))^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] integral(x^m/arccoth(tanh(b*x + a))^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{acoth}^3(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/acoth(tanh(b*x+a))**3,x)
```

```
[Out] Integral(x**m/acoth(tanh(a + b*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{arcoth}(\tanh(bx+a))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(x^m/arccoth(tanh(b*x + a))^3, x)
```

3.176 $\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^3} \, dx$

Optimal. Leaf size=92

$$-\frac{2x^3}{b^2 \coth^{-1}(\tanh(a+bx))} + \frac{6x\left(bx - \coth^{-1}(\tanh(a+bx))\right)}{b^4} + \frac{6\left(bx - \coth^{-1}(\tanh(a+bx))\right)^2 \log\left(\coth^{-1}(\tanh(a\right.}{b^5}$$

```
[Out] (3*x^2)/b^3 + (6*x*(b*x - ArcCoth[Tanh[a + b*x]]))/b^4 - x^4/(2*b*ArcCoth[T
anh[a + b*x]]^2) - (2*x^3)/(b^2*ArcCoth[Tanh[a + b*x]]) + (6*(b*x - ArcCoth
[Tanh[a + b*x]])^2*Log[ArcCoth[Tanh[a + b*x]]])/b^5
```

Rubi [A] time = 0.0712986, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2168, 2159, 2158, 2157, 29}

$$-\frac{2x^3}{b^2 \coth^{-1}(\tanh(a+bx))} + \frac{6x\left(bx - \coth^{-1}(\tanh(a+bx))\right)}{b^4} + \frac{6\left(bx - \coth^{-1}(\tanh(a+bx))\right)^2 \log\left(\coth^{-1}(\tanh(a\right.}{b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcCoth[Tanh[a + b*x]]^3,x]
```


```
[Out] (3*x^2)/b^3 + (6*x*(b*x - ArcCoth[Tanh[a + b*x]]))/b^4 - x^4/(2*b*ArcCoth[T
anh[a + b*x]]^2) - (2*x^3)/(b^2*ArcCoth[Tanh[a + b*x]]) + (6*(b*x - ArcCoth
[Tanh[a + b*x]])^2*Log[ArcCoth[Tanh[a + b*x]]])/b^5
```


Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{x^4}{2b\coth^{-1}(\tanh(a+bx))^2} + \frac{2\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2}\,dx}{b}$$

$$= -\frac{x^4}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{2x^3}{b^2\coth^{-1}(\tanh(a+bx))} + \frac{6\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))}\,dx}{b^2}$$

$$= \frac{3x^2}{b^3} - \frac{x^4}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{2x^3}{b^2\coth^{-1}(\tanh(a+bx))} - \frac{\left(6\left(-bx+\coth^{-1}(\tanh(a\right.\right.}{}$$

$$= \frac{3x^2}{b^3} + \frac{6x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^4} - \frac{x^4}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{2x^3}{b^2\coth^{-1}(\tanh(a}$$

$$= \frac{3x^2}{b^3} + \frac{6x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^4} - \frac{x^4}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{2x^3}{b^2\coth^{-1}(\tanh(a}$$

$$= \frac{3x^2}{b^3} + \frac{6x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^4} - \frac{x^4}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{2x^3}{b^2\coth^{-1}(\tanh(a}$$

Mathematica [A] time = 0.0418133, size = 114, normalized size = 1.24

$$-\frac{\left(\coth^{-1}(\tanh(a+bx))-bx\right)^4}{2b^5\coth^{-1}(\tanh(a+bx))^2} + \frac{4\left(\coth^{-1}(\tanh(a+bx))-bx\right)^3}{b^5\coth^{-1}(\tanh(a+bx))} - \frac{3x\left(\coth^{-1}(\tanh(a+bx))-bx\right)}{b^4} + \frac{6\left(\coth^{-1}(\tanh\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] x^2/(2*b^3) - (3*x*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/b^4 + (4*(-(b*x) + Ar
cCoth[Tanh[a + b*x]])^3)/(b^5*ArcCoth[Tanh[a + b*x]]) - (-(b*x) + ArcCoth[T
anh[a + b*x]])^4/(2*b^5*ArcCoth[Tanh[a + b*x]]^2) + (6*(-(b*x) + ArcCoth[Ta
nh[a + b*x]])^2*Log[ArcCoth[Tanh[a + b*x]]])/b^5
```

Maple [C] time = 1.396, size = 29456, normalized size = 320.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arccoth(tanh(b*x+a))^3,x)
```

```
[Out] result too large to display
```

Maxima [C] time = 3.54373, size = 266, normalized size = 2.89

$$\frac{8\left(16b^4x^4+7\pi^4+56i\pi^3a-168\pi^2a^2-224i\pi a^3+112a^4+\left(32i\pi b^3-64ab^3\right)x^3+\left(44\pi^2b^2+176i\pi ab^2-176a^2b^2\right)x^2}{256b^7x^2-64\pi^2b^5-256i\pi ab^5+256a^2b^5+\left(-256i\pi b^6+512ab^6\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 8*(16*b^4*x^4 + 7*pi^4 + 56*I*pi^3*a - 168*pi^2*a^2 - 224*I*pi*a^3 + 112*a^
4 + (32*I*pi*b^3 - 64*a*b^3)*x^3 + (44*pi^2*b^2 + 176*I*pi*a*b^2 - 176*a^2*
b^2)*x^2 + (4*I*pi^3*b - 24*pi^2*a*b - 48*I*pi*a^2*b + 32*a^3*b)*x)/(256*b^
7*x^2 - 64*pi^2*b^5 - 256*I*pi*a*b^5 + 256*a^2*b^5 + (-256*I*pi*b^6 + 512*a
*b^6)*x) - 1/2*(3*pi^2 + 12*I*pi*a - 12*a^2)*log(-I*pi + 2*b*x + 2*a)/b^5
```

Fricas [B] time = 1.79415, size = 1100, normalized size = 11.96

$$\frac{64\,b^6x^6 - 128\,ab^5x^5 - 7\,\pi^6 - 28\,\pi^4a^2 + 112\,\pi^2a^4 + 448\,a^6 + 32\left(\pi^2b^4 - 36\,a^2b^4\right)x^4 - 512\left(\pi^2ab^3 + 3\,a^3b^3\right)x^3 - 32\left(\pi^4 b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(64*b^6*x^6 - 128*a*b^5*x^5 - 7*pi^6 - 28*pi^4*a^2 + 112*pi^2*a^4 + 448
*a^6 + 32*(pi^2*b^4 - 36*a^2*b^4)*x^4 - 512*(pi^2*a*b^3 + 3*a^3*b^3)*x^3 -
32*(pi^4*b^2 + 32*pi^2*a^2*b^2)*x^2 - 32*(5*pi^4*a*b + 12*pi^2*a^3*b - 32*a
^5*b)*x - 96*(16*pi*a*b^4*x^4 + 64*pi*a^2*b^3*x^3 + pi^5*a + 8*pi^3*a^3 + 1
6*pi*a^5 + 8*(pi^3*a*b^2 + 12*pi*a^3*b^2)*x^2 + 16*(pi^3*a^2*b + 4*pi*a^4*b
)*x)*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/pi) -
 6*(pi^6 + 4*pi^4*a^2 - 16*pi^2*a^4 - 64*a^6 + 16*(pi^2*b^4 - 4*a^2*b^4)*x^
4 + 64*(pi^2*a*b^3 - 4*a^3*b^3)*x^3 + 8*(pi^4*b^2 + 8*pi^2*a^2*b^2 - 48*a^4
*b^2)*x^2 + 16*(pi^4*a*b - 16*a^5*b)*x)*log(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*
a^2))/(16*b^9*x^4 + 64*a*b^8*x^3 + pi^4*b^5 + 8*pi^2*a^2*b^5 + 16*a^4*b^5 +
 8*(pi^2*b^7 + 12*a^2*b^7)*x^2 + 16*(pi^2*a*b^6 + 4*a^3*b^6)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{acoth}^3\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/acoth(tanh(b*x+a))**3,x)
```

```
[Out] Integral(x**4/acoth(tanh(a + b*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(x^4/arccoth(tanh(b*x + a))^3, x)
```

3.177 $\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^3} \, dx$

Optimal. Leaf size=71

$$-\frac{3x^2}{2b^2\coth^{-1}(\tanh(a+bx))}+\frac{3\left(bx-\coth^{-1}(\tanh(a+bx))\right)\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^4}-\frac{x^3}{2b\coth^{-1}(\tanh(a+bx))}$$

```
[Out] (3*x)/b^3 - x^3/(2*b*ArcCoth[Tanh[a + b*x]]^2) - (3*x^2)/(2*b^2*ArcCoth[Tan
h[a + b*x]]) + (3*(b*x - ArcCoth[Tanh[a + b*x]])*Log[ArcCoth[Tanh[a + b*x]]
])/b^4
```

Rubi [A] time = 0.0491256, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2168, 2158, 2157, 29}

$$-\frac{3x^2}{2b^2\coth^{-1}(\tanh(a+bx))}+\frac{3\left(bx-\coth^{-1}(\tanh(a+bx))\right)\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^4}-\frac{x^3}{2b\coth^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] (3*x)/b^3 - x^3/(2*b*ArcCoth[Tanh[a + b*x]]^2) - (3*x^2)/(2*b^2*ArcCoth[Tan
h[a + b*x]]) + (3*(b*x - ArcCoth[Tanh[a + b*x]])*Log[ArcCoth[Tanh[a + b*x]]
])/b^4
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{x^3}{2b\coth^{-1}(\tanh(a+bx))^2} + \frac{3\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2}\,dx}{2b}$$

$$= -\frac{x^3}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{3x^2}{2b^2\coth^{-1}(\tanh(a+bx))} + \frac{3\int \frac{x}{\coth^{-1}(\tanh(a+bx))}\,dx}{b^2}$$

$$= \frac{3x}{b^3} - \frac{x^3}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{3x^2}{2b^2\coth^{-1}(\tanh(a+bx))} - \frac{\left(3\left(-bx+\coth^{-1}(\tanh(\right.\right.}{}$$

$$= \frac{3x}{b^3} - \frac{x^3}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{3x^2}{2b^2\coth^{-1}(\tanh(a+bx))} - \frac{\left(3\left(-bx+\coth^{-1}(\tanh(\right.\right.}{}$$

$$= \frac{3x}{b^3} - \frac{x^3}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{3x^2}{2b^2\coth^{-1}(\tanh(a+bx))} + \frac{3\left(bx-\coth^{-1}(\tanh(a+\right.}{}$$

Mathematica [A] time = 0.0451163, size = 86, normalized size = 1.21

$$-\frac{3b^2x^2\coth^{-1}(\tanh(a+bx)) - bx\coth^{-1}(\tanh(a+bx))^2\left(6\log\left(\coth^{-1}(\tanh(a+bx))\right)+11\right) + \coth^{-1}(\tanh(a+bx))^3}{2b^4\coth^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] -(b^3*x^3 + 3*b^2*x^2*ArcCoth[Tanh[a + b*x]] + ArcCoth[Tanh[a + b*x]]^3*(5
+ 6*Log[ArcCoth[Tanh[a + b*x]]]) - b*x*ArcCoth[Tanh[a + b*x]]^2*(11 + 6*Log
[ArcCoth[Tanh[a + b*x]]]))/(2*b^4*ArcCoth[Tanh[a + b*x]]^2)
```

Maple [C] time = 0.379, size = 4977, normalized size = 70.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arccoth(tanh(b*x+a))^3,x)
```

```
[Out] -2*I*(-6*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))^2+3*Pi*x^2*csgn(I/(exp(2*b*x+2*a
)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-3*Pi
*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2
+6*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))^3+3*Pi*x^2*csgn(I*exp(b*x+a))^2*csgn(I
*exp(2*b*x+2*a))-6*Pi*x^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+3*Pi*
x^2*csgn(I*exp(2*b*x+2*a))^3-3*Pi*x^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b
*x+2*a)/(exp(2*b*x+2*a)+1))^2+3*Pi*x^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a
)+1))^3+12*I*x^2*ln(exp(b*x+a))+6*Pi*x^2+4*I*x^3*b)/b^2/(-2*Pi*csgn(I/(exp(
2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(
I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*e
xp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*cs
gn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*ex
p(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(
I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+
2*a)+1))^3+2*Pi+4*I*ln(exp(b*x+a)))^2+3/b^3*ln(-2*Pi*csgn(I/(exp(2*b*x+2*a)
+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*
x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2
```

```
*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b
*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*
a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*
x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3
+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)*x-3/b^4*ln(-2*Pi*csgn(I/(ex
p(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I
*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*
csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*
exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*
x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)*ln(exp(b*x+a))-
3/2*I/b^4*ln(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1
))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn
(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csg
n(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*
Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-
Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*cs
gn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a
)+4*I*a+2*Pi)*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+3/4*I/b^4*ln(-2*Pi*csgn(I/(ex
p(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I
*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*
csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*
exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*
x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)*Pi*csgn(I/(exp(
2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)
+1))-3/4*I/b^4*ln(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2
*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi
*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*P
i*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a
))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a
))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+
Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-
b*x-a)+4*I*a+2*Pi)*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp
(2*b*x+2*a)+1))^2+3/2*I/b^4*ln(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I
/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*
x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2
*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*
exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*
exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*
x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln
(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+3/4*I/b^4*l
n(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*e
xp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b
*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*
b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*e
xp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*e
xp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*
b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*P
i)*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-3/2*I/b^4*ln(-2*Pi*csgn(I
/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))
*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*cs
gn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3
+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csg
n(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))
*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(
2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)*Pi*csgn(I*e
```

```
xp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+3/4*I/b^4*ln(-2*Pi*csgn(I/(exp(2*b*x+2*
a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*
b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x
+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp
(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+
2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*
b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))
^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)*Pi*csgn(I*exp(2*b*x+2*a))
^3-3/4*I/b^4*ln(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a
)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*c
sgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*
csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))
-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))
^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi
*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*
x-a)+4*I*a+2*Pi)*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x
+2*a)+1))^2+3/4*I/b^4*ln(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(
2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)
+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)
)^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*
b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*
b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)
+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b
*x+a))-b*x-a)+4*I*a+2*Pi)*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+3/
2*I/b^4*ln(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))
*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I
/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(
I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi
*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi
*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn
(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+
4*I*a+2*Pi)*Pi+3*x/b^3
```

Maxima [C] time = 3.54468, size = 194, normalized size = 2.73

$$\frac{8\left(16\,b^3x^3-5i\,\pi^3+30\,\pi^2a+60i\,\pi a^2-40\,a^3+\left(-16i\,\pi b^2+32\,ab^2\right)x^2+\left(8\,\pi^2b+32i\,\pi ab-32\,a^2b\right)x\right)}{128\,b^6x^2-32\,\pi^2b^4-128i\,\pi ab^4+128\,a^2b^4+\left(-128i\,\pi b^5+256\,ab^5\right)x}-\frac{(-3i\,\pi+6\,a)\log}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 8*(16*b^3*x^3 - 5*I*pi^3 + 30*pi^2*a + 60*I*pi*a^2 - 40*a^3 + (-16*I*pi*b^2
 + 32*a*b^2)*x^2 + (8*pi^2*b + 32*I*pi*a*b - 32*a^2*b)*x)/(128*b^6*x^2 - 32
*pi^2*b^4 - 128*I*pi*a*b^4 + 128*a^2*b^4 + (-128*I*pi*b^5 + 256*a*b^5)*x) -
 1/2*(-3*I*pi + 6*a)*log(-I*pi + 2*b*x + 2*a)/b^4
```

Fricas [B] time = 1.81278, size = 936, normalized size = 13.18

$$32\,b^5x^5+128\,ab^4x^4-5\,\pi^4a-40\,\pi^2a^3-80\,a^5+8\left(5\,\pi^2b^3+12\,a^2b^3\right)x^3+4\left(11\,\pi^2ab^2-36\,a^3b^2\right)x^2+2\left(3\,\pi^4b-16\,\pi^2a^2b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(32*b^5*x^5 + 128*a*b^4*x^4 - 5*pi^4*a - 40*pi^2*a^3 - 80*a^5 + 8*(5*pi
^2*b^3 + 12*a^2*b^3)*x^3 + 4*(11*pi^2*a*b^2 - 36*a^3*b^2)*x^2 + 2*(3*pi^4*b
 - 16*pi^2*a^2*b - 112*a^4*b)*x + 6*(16*pi*b^4*x^4 + 64*pi*a*b^3*x^3 + pi^5
 + 8*pi^3*a^2 + 16*pi*a^4 + 8*(pi^3*b^2 + 12*pi*a^2*b^2)*x^2 + 16*(pi^3*a*b
 + 4*pi*a^3*b)*x)*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 +
4*a^2))/pi) - 3*(16*a*b^4*x^4 + 64*a^2*b^3*x^3 + pi^4*a + 8*pi^2*a^3 + 16*a
^5 + 8*(pi^2*a*b^2 + 12*a^3*b^2)*x^2 + 16*(pi^2*a^2*b + 4*a^4*b)*x)*log(4*b
^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/(16*b^8*x^4 + 64*a*b^7*x^3 + pi^4*b^4 + 8
*pi^2*a^2*b^4 + 16*a^4*b^4 + 8*(pi^2*b^6 + 12*a^2*b^6)*x^2 + 16*(pi^2*a*b^5
 + 4*a^3*b^5)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{acoth}^3(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/acoth(tanh(b*x+a))**3,x)
```

```
[Out] Integral(x**3/acoth(tanh(a + b*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{arcoth}(\tanh(bx+a))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(x^3/arccoth(tanh(b*x + a))^3, x)
```

3.178 $\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^3} dx$

Optimal. Leaf size=47

$$-\frac{x}{b^2\coth^{-1}(\tanh(a+bx))}+\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{x^2}{2b\coth^{-1}(\tanh(a+bx))^2}$$

```
[Out] -x^2/(2*b*ArcCoth[Tanh[a + b*x]]^2) - x/(b^2*ArcCoth[Tanh[a + b*x]]) + Log[
ArcCoth[Tanh[a + b*x]]]/b^3
```

Rubi [A] time = 0.0286383, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2157, 29}

$$-\frac{x}{b^2\coth^{-1}(\tanh(a+bx))}+\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{x^2}{2b\coth^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] -x^2/(2*b*ArcCoth[Tanh[a + b*x]]^2) - x/(b^2*ArcCoth[Tanh[a + b*x]]) + Log[
ArcCoth[Tanh[a + b*x]]]/b^3
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{x^2}{2b\coth^{-1}(\tanh(a+bx))^2} + \frac{\int \frac{x}{\coth^{-1}(\tanh(a+bx))^2}\,dx}{b}$$
$$= -\frac{x^2}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{x}{b^2\coth^{-1}(\tanh(a+bx))} + \frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}\,dx}{b^2}$$
$$= -\frac{x^2}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{x}{b^2\coth^{-1}(\tanh(a+bx))} + \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \coth^{-1}(t$$
$$= -\frac{x^2}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{x}{b^2\coth^{-1}(\tanh(a+bx))} + \frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^3}$$

Mathematica [A] time = 0.0312588, size = 49, normalized size = 1.04

$$\frac{-\frac{b^2x^2}{\coth^{-1}(\tanh(a+bx))^2} - \frac{2bx}{\coth^{-1}(\tanh(a+bx))} + 2\log\left(\coth^{-1}(\tanh(a+bx))\right) + 3}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] (3 - (b^2*x^2)/ArcCoth[Tanh[a + b*x]]^2 - (2*b*x)/ArcCoth[Tanh[a + b*x]] +
2*Log[ArcCoth[Tanh[a + b*x]]])/(2*b^3)
```

Maple [C] time = 0.195, size = 952, normalized size = 20.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arccoth(tanh(b*x+a))^3,x)
```

```
[Out] -4*I*(-2*Pi*x*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*x*csgn(I/(exp(2*b*x+2*a)+1))*
csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*x*csgn(
I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*x*cs
gn(I/(exp(2*b*x+2*a)+1))^3+Pi*x*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))
-2*Pi*x*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*x*csgn(I*exp(2*b*x+2
*a))^3-Pi*x*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)
)^2+Pi*x*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*x*ln(exp(b*x+a))+2
*Pi*x+2*I*x^2*b)/b^2/(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b
*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)
)-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2
+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x
+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x
+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)
)^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+2*Pi+4*I*ln(exp(b*x+a)))
^2+1/b^3*ln(ln(exp(b*x+a))-1/4*I*Pi*(-2*csgn(I/(exp(2*b*x+2*a)+1))^2+csgn(I
/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*
x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)
+1))^2+2*csgn(I/(exp(2*b*x+2*a)+1))^3+csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x
+2*a))-2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+csgn(I*exp(2*b*x+2*a))
^3-csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+csgn(
```

```
I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+2))
```

Maxima [C] time = 3.57015, size = 127, normalized size = 2.7

$$-\frac{8\left(3\pi^2+12i\pi a-12a^2+(8i\pi b-16ab)x\right)}{64b^5x^2-16\pi^2b^3-64i\pi ab^3+64a^2b^3+\left(-64i\pi b^4+128ab^4\right)x}+\frac{\log\left(-i\pi+2bx+2a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -8*(3*pi^2 + 12*I*pi*a - 12*a^2 + (8*I*pi*b - 16*a*b)*x)/(64*b^5*x^2 - 16*p
i^2*b^3 - 64*I*pi*a*b^3 + 64*a^2*b^3 + (-64*I*pi*b^4 + 128*a*b^4)*x) + log(
-I*pi + 2*b*x + 2*a)/b^3
```

Fricas [B] time = 1.78249, size = 548, normalized size = 11.66

$$\frac{64ab^3x^3+3\pi^4+24\pi^2a^2+48a^4+4\left(5\pi^2b^2+44a^2b^2\right)x^2+40\left(\pi^2ab+4a^3b\right)x+\left(16b^4x^4+64ab^3x^3+\pi^4+8\pi^2a^2+\right.}{2\left(16b^7x^4+64ab^6x^3+\pi^4b^3+8\pi^2a^2b^3+16a^4b^3+8\left(\pi^2b^5+12a^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(64*a*b^3*x^3 + 3*pi^4 + 24*pi^2*a^2 + 48*a^4 + 4*(5*pi^2*b^2 + 44*a^2*
b^2)*x^2 + 40*(pi^2*a*b + 4*a^3*b)*x + (16*b^4*x^4 + 64*a*b^3*x^3 + pi^4 +
8*pi^2*a^2 + 16*a^4 + 8*(pi^2*b^2 + 12*a^2*b^2)*x^2 + 16*(pi^2*a*b + 4*a^3*
b)*x)*log(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/(16*b^7*x^4 + 64*a*b^6*x^3 +
 pi^4*b^3 + 8*pi^2*a^2*b^3 + 16*a^4*b^3 + 8*(pi^2*b^5 + 12*a^2*b^5)*x^2 + 1
6*(pi^2*a*b^4 + 4*a^3*b^4)*x)
```

Sympy [A] time = 24.5137, size = 54, normalized size = 1.15

$$\begin{cases} -\frac{x^2}{2b\operatorname{acoth}^2(\tanh(a+bx))}-\frac{x}{b^2\operatorname{acoth}(\tanh(a+bx))}+\frac{\log(\operatorname{acoth}(\tanh(a+bx)))}{b^3} & \text{for } b\neq 0 \\ \frac{x^3}{3\operatorname{acoth}^3(\tanh(a))} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/acoth(tanh(b*x+a))**3,x)
```

```
[Out] Piecewise((-x**2/(2*b*acoth(tanh(a + b*x))**2) - x/(b**2*acoth(tanh(a + b*x
))) + log(acoth(tanh(a + b*x)))/b**3, Ne(b, 0)), (x**3/(3*acoth(tanh(a))**3
), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\operatorname{arcoth}(\tanh(bx+a))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(x^2/arccoth(tanh(b*x + a))^3, x)
```

3.179 $\int \frac{x}{\coth^{-1}(\tanh(a+bx))^3} dx$

Optimal. Leaf size=34

$$-\frac{1}{2b^2\coth^{-1}(\tanh(a+bx))}-\frac{x}{2b\coth^{-1}(\tanh(a+bx))^2}$$

```
[Out] -x/(2*b*ArcCoth[Tanh[a + b*x]]^2) - 1/(2*b^2*ArcCoth[Tanh[a + b*x]])
```

Rubi [A] time = 0.0141126, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2168, 2157, 30}

$$-\frac{1}{2b^2\coth^{-1}(\tanh(a+bx))}-\frac{x}{2b\coth^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] -x/(2*b*ArcCoth[Tanh[a + b*x]]^2) - 1/(2*b^2*ArcCoth[Tanh[a + b*x]])
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\coth^{-1}(\tanh(a+bx))^3} dx &= -\frac{x}{2b\coth^{-1}(\tanh(a+bx))^2}+\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2} dx}{2b}\\
&= -\frac{x}{2b\coth^{-1}(\tanh(a+bx))^2}+\frac{\text{Subst}\left(\int \frac{1}{x^2} dx, x, \coth^{-1}(\tanh(a+bx))\right)}{2b^2}\\
&= -\frac{x}{2b\coth^{-1}(\tanh(a+bx))^2}-\frac{1}{2b^2\coth^{-1}(\tanh(a+bx))}
\end{aligned}$$

Mathematica [A] time = 0.0493936, size = 27, normalized size = 0.79

$$-\frac{\coth^{-1}(\tanh(a+bx))+bx}{2b^2\coth^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] -(b*x + ArcCoth[Tanh[a + b*x]])/(2*b^2*ArcCoth[Tanh[a + b*x]]^2)
```

Maple [C] time = 0.171, size = 634, normalized size = 18.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arccoth(tanh(b*x+a))^3,x)
```

```
[Out] -2*I*(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn
(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp
(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(ex
p(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn
(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn
(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+2*Pi+4*I*b*x+4*I*ln(exp(b*x+a)))/b^2/(-2
*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2
*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2
*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+
2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b
*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2
*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+
2*a)/(exp(2*b*x+2*a)+1))^3+2*Pi+4*I*ln(exp(b*x+a)))^2
```

Maxima [C] time = 3.53148, size = 82, normalized size = 2.41

$$\frac{8i\pi-32\,bx-16\,a}{32\,b^4x^2-8\,\pi^2b^2-32i\,\pi ab^2+32\,a^2b^2+\left(-32i\,\pi b^3+64\,ab^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] (8*I*pi - 32*b*x - 16*a)/(32*b^4*x^2 - 8*pi^2*b^2 - 32*I*pi*a*b^2 + 32*a^2*
b^2 + (-32*I*pi*b^3 + 64*a*b^3)*x)
```

Fricas [B] time = 1.64833, size = 262, normalized size = 7.71

$$-\frac{2\left(8\,b^3x^3+20\,ab^2x^2+16\,a^2bx+\pi^2a+4\,a^3\right)}{16\,b^6x^4+64\,ab^5x^3+\pi^4b^2+8\,\pi^2a^2b^2+16\,a^4b^2+8\left(\pi^2b^4+12\,a^2b^4\right)x^2+16\left(\pi^2ab^3+4\,a^3b^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] -2*(8*b^3*x^3 + 20*a*b^2*x^2 + 16*a^2*b*x + pi^2*a + 4*a^3)/(16*b^6*x^4 + 6
4*a*b^5*x^3 + pi^4*b^2 + 8*pi^2*a^2*b^2 + 16*a^4*b^2 + 8*(pi^2*b^4 + 12*a^2
*b^4)*x^2 + 16*(pi^2*a*b^3 + 4*a^3*b^3)*x)
```

Sympy [A] time = 24.1672, size = 42, normalized size = 1.24

$$\begin{cases} -\frac{x}{2b\operatorname{acoth}^{2}(\tanh(a+bx))} - \frac{1}{2b^{2}\operatorname{acoth}(\tanh(a+bx))} & \text{for } b \neq 0 \\ \frac{x^{2}}{2\operatorname{acoth}^{3}(\tanh(a))} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acoth(tanh(b*x+a))**3,x)
```

```
[Out] Piecewise((-x/(2*b*acoth(tanh(a + b*x))**2) - 1/(2*b**2*acoth(tanh(a + b*x)
)), Ne(b, 0)), (x**2/(2*acoth(tanh(a))**3), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{arcoth}(\tanh(bx+a))^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(x/arccoth(tanh(b*x + a))^3, x)
```

3.180 $\int \frac{1}{\coth^{-1}(\tanh(a+bx))^3} dx$

Optimal. Leaf size=16

$$-\frac{1}{2b\coth^{-1}(\tanh(a+bx))^2}$$

```
[Out] -1/(2*b*ArcCoth[Tanh[a + b*x]]^2)
```

Rubi [A] time = 0.0053586, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2157, 30}

$$-\frac{1}{2b\coth^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^(-3),x]
```

```
[Out] -1/(2*b*ArcCoth[Tanh[a + b*x]]^2)
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\coth^{-1}(\tanh(a+bx))^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3} dx, x, \coth^{-1}(\tanh(a+bx))\right)}{b}\\ &= -\frac{1}{2b\coth^{-1}(\tanh(a+bx))^2}\end{aligned}$$

Mathematica [A] time = 0.0055339, size = 16, normalized size = 1.

$$-\frac{1}{2b\coth^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^(-3),x]
```

```
[Out] -1/(2*b*ArcCoth[Tanh[a + b*x]]^2)
```

Maple [A] time = 0.074, size = 15, normalized size = 0.9

$$-\frac{1}{2b\left(\operatorname{arccoth}\left(\tanh\left(bx+a\right)\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccoth(tanh(b*x+a))^3,x)
```

```
[Out] -1/2/b/arccoth(tanh(b*x+a))^2
```

Maxima [C] time = 1.47068, size = 41, normalized size = 2.56

$$\frac{8}{\left(4\,\pi^{2}-16i\,\pi(bx+a)-16\,(bx+a)^{2}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 8/((4*pi^2 - 16*I*pi*(b*x + a) - 16*(b*x + a)^2)*b)
```

Fricas [B] time = 1.60991, size = 227, normalized size = 14.19

$$-\frac{2\left(4\,b^{2}x^{2}+8\,abx-\pi^{2}+4\,a^{2}\right)}{16\,b^{5}x^{4}+64\,ab^{4}x^{3}+\pi^{4}b+8\,\pi^{2}a^{2}b+16\,a^{4}b+8\left(\pi^{2}b^{3}+12\,a^{2}b^{3}\right)x^{2}+16\left(\pi^{2}ab^{2}+4\,a^{3}b^{2}\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] -2*(4*b^2*x^2 + 8*a*b*x - pi^2 + 4*a^2)/(16*b^5*x^4 + 64*a*b^4*x^3 + pi^4*b
 + 8*pi^2*a^2*b + 16*a^4*b + 8*(pi^2*b^3 + 12*a^2*b^3)*x^2 + 16*(pi^2*a*b^2
 + 4*a^3*b^2)*x)
```

Sympy [A] time = 16.4599, size = 24, normalized size = 1.5

$$\begin{cases} -\frac{1}{2b\operatorname{acoth}^{2}\left(\tanh\left(a+bx\right)\right)} & \text{for}\: b\neq 0 \\ \frac{x}{\operatorname{acoth}^{3}\left(\tanh\left(a\right)\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acoth(tanh(b*x+a))**3,x)
```

```
[Out] Piecewise((-1/(2*b*acoth(tanh(a + b*x))**2), Ne(b, 0)), (x/acoth(tanh(a))**
3, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^(-3), x)
```

3.181 $\int \frac{1}{x \coth^{-1}(\tanh(a+bx))^3} dx$

Optimal. Leaf size=97

$$\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2 \coth^{-1}(\tanh(a+bx))} - \frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right) \coth^{-1}(\tanh(a+bx))^2} - \frac{}{\left(bx-\text{c}\right.}$$

```
[Out] -1/(2*(b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]^2) + 1/((b*x -
ArcCoth[Tanh[a + b*x]])^2*ArcCoth[Tanh[a + b*x]]) - Log[x]/(b*x - ArcCoth[T
anh[a + b*x]])^3 + Log[ArcCoth[Tanh[a + b*x]]]/(b*x - ArcCoth[Tanh[a + b*x]
])^3
```

Rubi [A] time = 0.0658827, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2163, 2160, 2157, 29}

$$\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2 \coth^{-1}(\tanh(a+bx))} - \frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right) \coth^{-1}(\tanh(a+bx))^2} - \frac{}{\left(bx-\text{c}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*ArcCoth[Tanh[a + b*x]]^3),x]
```

```
[Out] -1/(2*(b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]^2) + 1/((b*x -
ArcCoth[Tanh[a + b*x]])^2*ArcCoth[Tanh[a + b*x]]) - Log[x]/(b*x - ArcCoth[T
anh[a + b*x]])^3 + Log[ArcCoth[Tanh[a + b*x]]]/(b*x - ArcCoth[Tanh[a + b*x]
])^3
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2} - \frac{\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^2}\,dx}{bx-\coth^{-1}(\tanh(a+b}$$

$$= -\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2} + \frac{}{\left(bx-\coth^{-1}(\tanh(a+\right.}$$

$$= -\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2} + \frac{}{\left(bx-\coth^{-1}(\tanh(a+\right.}$$

$$= -\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2} + \frac{}{\left(bx-\coth^{-1}(\tanh(a+\right.}$$

$$= -\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2} + \frac{}{\left(bx-\coth^{-1}(\tanh(a+\right.}$$

Mathematica [A] time = 0.106451, size = 74, normalized size = 0.76

$$\frac{-4bx\coth^{-1}(\tanh(a+bx)) + \coth^{-1}(\tanh(a+bx))^2\left(-2\log\left(\coth^{-1}(\tanh(a+bx))\right)+2\log(bx)+3\right)+b^2x^2}{2\coth^{-1}(\tanh(a+bx))^2\left(\coth^{-1}(\tanh(a+bx))-bx\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*ArcCoth[Tanh[a + b*x]]^3),x]
```

```
[Out] (b^2*x^2 - 4*b*x*ArcCoth[Tanh[a + b*x]] + ArcCoth[Tanh[a + b*x]]^2*(3 + 2*L
og[b*x] - 2*Log[ArcCoth[Tanh[a + b*x]]]))/(2*ArcCoth[Tanh[a + b*x]]^2*(-(b*
x) + ArcCoth[Tanh[a + b*x]])^3)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\mathrm{arccoth}\left(\tanh\left(bx+a\right)\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arccoth(tanh(b*x+a))^3,x)
```

```
[Out] int(1/x/arccoth(tanh(b*x+a))^3,x)
```

Maxima [C] time = 3.55119, size = 234, normalized size = 2.41

$$\frac{8\left(-3i\pi+4bx+6a\right)}{2\pi^4+16i\pi^3a-48\pi^2a^2-64i\pi a^3+32a^4-\left(8\pi^2b^2+32i\pi ab^2-32a^2b^2\right)x^2+\left(8i\pi^3b-48\pi^2ab-96i\pi a^2b+64a^3b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 8*(-3*I*pi + 4*b*x + 6*a)/(2*pi^4 + 16*I*pi^3*a - 48*pi^2*a^2 - 64*I*pi*a^3
 + 32*a^4 - (8*pi^2*b^2 + 32*I*pi*a*b^2 - 32*a^2*b^2)*x^2 + (8*I*pi^3*b - 4
8*pi^2*a*b - 96*I*pi*a^2*b + 64*a^3*b)*x) + 8*log(-I*pi + 2*b*x + 2*a)/(-I*
pi^3 + 6*pi^2*a + 12*I*pi*a^2 - 8*a^3) - 8*log(x)/(-I*pi^3 + 6*pi^2*a + 12*
I*pi*a^2 - 8*a^3)
```

Fricas [B] time = 1.93464, size = 1841, normalized size = 18.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] -8*(9*pi^6*a + 60*pi^4*a^3 + 48*pi^2*a^5 - 192*a^7 + 8*(pi^4*b^3 - 16*a^4*b
^3)*x^3 + 4*(9*pi^4*a*b^2 + 8*pi^2*a^3*b^2 - 112*a^5*b^2)*x^2 + 4*(pi^6*b +
 16*pi^4*a^2*b + 16*pi^2*a^4*b - 128*a^6*b)*x - 2*(pi^7 - 4*pi^5*a^2 - 80*p
i^3*a^4 - 192*pi*a^6 + 16*(pi^3*b^4 - 12*pi*a^2*b^4)*x^4 + 64*(pi^3*a*b^3 -
 12*pi*a^3*b^3)*x^3 + 8*(pi^5*b^2 - 144*pi*a^4*b^2)*x^2 + 16*(pi^5*a*b - 8*
pi^3*a^3*b - 48*pi*a^5*b)*x)*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 + 8*a*b*
x + pi^2 + 4*a^2))/pi) - (3*pi^6*a + 20*pi^4*a^3 + 16*pi^2*a^5 - 64*a^7 + 1
6*(3*pi^2*a*b^4 - 4*a^3*b^4)*x^4 + 64*(3*pi^2*a^2*b^3 - 4*a^4*b^3)*x^3 + 8*
(3*pi^4*a*b^2 + 32*pi^2*a^3*b^2 - 48*a^5*b^2)*x^2 + 16*(3*pi^4*a^2*b + 8*pi
^2*a^4*b - 16*a^6*b)*x)*log(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2) + 2*(3*pi^6
*a + 20*pi^4*a^3 + 16*pi^2*a^5 - 64*a^7 + 16*(3*pi^2*a*b^4 - 4*a^3*b^4)*x^4
 + 64*(3*pi^2*a^2*b^3 - 4*a^4*b^3)*x^3 + 8*(3*pi^4*a*b^2 + 32*pi^2*a^3*b^2
- 48*a^5*b^2)*x^2 + 16*(3*pi^4*a^2*b + 8*pi^2*a^4*b - 16*a^6*b)*x)*log(x))/
(pi^10 + 20*pi^8*a^2 + 160*pi^6*a^4 + 640*pi^4*a^6 + 1280*pi^2*a^8 + 1024*a
^10 + 16*(pi^6*b^4 + 12*pi^4*a^2*b^4 + 48*pi^2*a^4*b^4 + 64*a^6*b^4)*x^4 +
64*(pi^6*a*b^3 + 12*pi^4*a^3*b^3 + 48*pi^2*a^5*b^3 + 64*a^7*b^3)*x^3 + 8*(p
i^8*b^2 + 24*pi^6*a^2*b^2 + 192*pi^4*a^4*b^2 + 640*pi^2*a^6*b^2 + 768*a^8*b
^2)*x^2 + 16*(pi^8*a*b + 16*pi^6*a^3*b + 96*pi^4*a^5*b + 256*pi^2*a^7*b + 2
56*a^9*b)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{acoth}^{3}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/acoth(tanh(b*x+a))**3,x)
```

```
[Out] Integral(1/(x*acoth(tanh(a + b*x))**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x*arccoth(tanh(b*x + a))^3), x)
```

3.182 $\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^3} \, dx$

Optimal. Leaf size=131

$$\frac{3b}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3 \coth^{-1}(\tanh(a+bx))} - \frac{3b}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2 \coth^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\right.}$$

```
[Out] (-3*b)/(2*(b*x - ArcCoth[Tanh[a + b*x]])^2*ArcCoth[Tanh[a + b*x]]^2) + 1/(x
*(b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]^2) + (3*b)/((b*x - A
rcCoth[Tanh[a + b*x]])^3*ArcCoth[Tanh[a + b*x]]) - (3*b*Log[x])/(b*x - ArcC
oth[Tanh[a + b*x]])^4 + (3*b*Log[ArcCoth[Tanh[a + b*x]]])/(b*x - ArcCoth[Ta
nh[a + b*x]])^4
```

Rubi [A] time = 0.0874151, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2171, 2163, 2160, 2157, 29}

$$\frac{3b}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3 \coth^{-1}(\tanh(a+bx))} - \frac{3b}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2 \coth^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*ArcCoth[Tanh[a + b*x]]^3),x]
```


```
[Out] (-3*b)/(2*(b*x - ArcCoth[Tanh[a + b*x]])^2*ArcCoth[Tanh[a + b*x]]^2) + 1/(x
*(b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]^2) + (3*b)/((b*x - A
rcCoth[Tanh[a + b*x]])^3*ArcCoth[Tanh[a + b*x]]) - (3*b*Log[x])/(b*x - ArcC
oth[Tanh[a + b*x]])^4 + (3*b*Log[ArcCoth[Tanh[a + b*x]]])/(b*x - ArcCoth[Ta
nh[a + b*x]])^4
```


Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^3} dx = \frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2} - \frac{(3b)\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^3}}{-bx+\coth^{-1}(\tanh(a+}$$

$$= -\frac{3b}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\coth^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\coth^{-1}(\tanh(\right.}$$

$$= -\frac{3b}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\coth^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\coth^{-1}(\tanh(\right.}$$

$$= -\frac{3b}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\coth^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\coth^{-1}(\tanh(\right.}$$

$$= -\frac{3b}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\coth^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\coth^{-1}(\tanh(\right.}$$

$$= -\frac{3b}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\coth^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\coth^{-1}(\tanh(\right.}$$

Mathematica [A] time = 0.050491, size = 93, normalized size = 0.71

$$-\frac{-6b^2x^2\coth^{-1}(\tanh(a+bx)) + 2\coth^{-1}(\tanh(a+bx))^3 + 3bx\coth^{-1}(\tanh(a+bx))^2\left(-2\log\left(\coth^{-1}(\tanh(a+b\right.\right.}{2x\coth^{-1}(\tanh(a+bx))^2\left(\coth^{-1}(\tanh(a+bx))-bx\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*ArcCoth[Tanh[a + b*x]]^3),x]
```

```
[Out] -(b^3*x^3 - 6*b^2*x^2*ArcCoth[Tanh[a + b*x]] + 2*ArcCoth[Tanh[a + b*x]]^3 +
 3*b*x*ArcCoth[Tanh[a + b*x]]^2*(1 + 2*Log[x] - 2*Log[ArcCoth[Tanh[a + b*x]
]]))/(2*x*ArcCoth[Tanh[a + b*x]]^2*(-(b*x) + ArcCoth[Tanh[a + b*x]])^4)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(\operatorname{arccoth}\left(\tanh\left(bx+a\right)\right)\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arccoth(tanh(b*x+a))^3,x)
```

```
[Out] int(1/x^2/arccoth(tanh(b*x+a))^3,x)
```

Maxima [C] time = 3.5656, size = 328, normalized size = 2.5

$$\frac{48\,b\log\left(-i\,\pi+2\,bx+2\,a\right)}{\pi^4+8i\,\pi^3a-24\,\pi^2a^2-32i\,\pi a^3+16\,a^4}-\frac{48\,b\log\left(x\right)}{\pi^4+8i\,\pi^3a-24\,\pi^2a^2-32i\,\pi a^3+16\,a^4}+\frac{}{\left(-4i\,\pi^3b^2+24\,\pi^2ab^2+48i\,\pi a^2b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 48*b*log(-I*pi + 2*b*x + 2*a)/(pi^4 + 8*I*pi^3*a - 24*pi^2*a^2 - 32*I*pi*a^
3 + 16*a^4) - 48*b*log(x)/(pi^4 + 8*I*pi^3*a - 24*pi^2*a^2 - 32*I*pi*a^3 +
16*a^4) + 8*(12*b^2*x^2 - pi^2 - 4*I*pi*a + 4*a^2 + (-9*I*pi*b + 18*a*b)*x)
/((-4*I*pi^3*b^2 + 24*pi^2*a*b^2 + 48*I*pi*a^2*b^2 - 32*a^3*b^2)*x^3 - (4*p
i^4*b + 32*I*pi^3*a*b - 96*pi^2*a^2*b - 128*I*pi*a^3*b + 64*a^4*b)*x^2 + (I
*pi^5 - 10*pi^4*a - 40*I*pi^3*a^2 + 80*pi^2*a^3 + 80*I*pi*a^4 - 32*a^5)*x)
```

Fricas [B] time = 2.11983, size = 2475, normalized size = 18.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 8*(6*pi^8*a + 64*pi^6*a^3 + 192*pi^4*a^5 - 512*a^9 + 96*(3*pi^4*a*b^4 + 8*p
i^2*a^3*b^4 - 16*a^5*b^4)*x^4 - 12*(pi^6*b^3 - 92*pi^4*a^2*b^3 - 272*pi^2*a
^4*b^3 + 448*a^6*b^3)*x^3 + 8*(11*pi^6*a*b^2 + 228*pi^4*a^3*b^2 + 528*pi^2*
a^5*b^2 - 832*a^7*b^2)*x^2 - (5*pi^8*b - 176*pi^6*a^2*b - 1440*pi^4*a^4*b -
 1792*pi^2*a^6*b + 3328*a^8*b)*x - 96*(16*(pi^3*a*b^5 - 4*pi*a^3*b^5)*x^5 +
 64*(pi^3*a^2*b^4 - 4*pi*a^4*b^4)*x^4 + 8*(pi^5*a*b^3 + 8*pi^3*a^3*b^3 - 48
*pi*a^5*b^3)*x^3 + 16*(pi^5*a^2*b^2 - 16*pi*a^6*b^2)*x^2 + (pi^7*a*b + 4*pi
^5*a^3*b - 16*pi^3*a^5*b - 64*pi*a^7*b)*x)*arctan(-(2*b*x + 2*a - sqrt(4*b^
2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/pi) + 3*(16*(pi^4*b^5 - 24*pi^2*a^2*b^5 +
16*a^4*b^5)*x^5 + 64*(pi^4*a*b^4 - 24*pi^2*a^3*b^4 + 16*a^5*b^4)*x^4 + 8*(p
i^6*b^3 - 12*pi^4*a^2*b^3 - 272*pi^2*a^4*b^3 + 192*a^6*b^3)*x^3 + 16*(pi^6*
a*b^2 - 20*pi^4*a^3*b^2 - 80*pi^2*a^5*b^2 + 64*a^7*b^2)*x^2 + (pi^8*b - 16*
pi^6*a^2*b - 160*pi^4*a^4*b - 256*pi^2*a^6*b + 256*a^8*b)*x)*log(4*b^2*x^2
+ 8*a*b*x + pi^2 + 4*a^2) - 6*(16*(pi^4*b^5 - 24*pi^2*a^2*b^5 + 16*a^4*b^5)
*x^5 + 64*(pi^4*a*b^4 - 24*pi^2*a^3*b^4 + 16*a^5*b^4)*x^4 + 8*(pi^6*b^3 - 1
2*pi^4*a^2*b^3 - 272*pi^2*a^4*b^3 + 192*a^6*b^3)*x^3 + 16*(pi^6*a*b^2 - 20*
pi^4*a^3*b^2 - 80*pi^2*a^5*b^2 + 64*a^7*b^2)*x^2 + (pi^8*b - 16*pi^6*a^2*b
- 160*pi^4*a^4*b - 256*pi^2*a^6*b + 256*a^8*b)*x)*log(x))/(16*(pi^8*b^4 + 1
6*pi^6*a^2*b^4 + 96*pi^4*a^4*b^4 + 256*pi^2*a^6*b^4 + 256*a^8*b^4)*x^5 + 64
*(pi^8*a*b^3 + 16*pi^6*a^3*b^3 + 96*pi^4*a^5*b^3 + 256*pi^2*a^7*b^3 + 256*a
^9*b^3)*x^4 + 8*(pi^10*b^2 + 28*pi^8*a^2*b^2 + 288*pi^6*a^4*b^2 + 1408*pi^4
*a^6*b^2 + 3328*pi^2*a^8*b^2 + 3072*a^10*b^2)*x^3 + 16*(pi^10*a*b + 20*pi^8
*a^3*b + 160*pi^6*a^5*b + 640*pi^4*a^7*b + 1280*pi^2*a^9*b + 1024*a^11*b)*x
^2 + (pi^12 + 24*pi^10*a^2 + 240*pi^8*a^4 + 1280*pi^6*a^6 + 3840*pi^4*a^8 +
 6144*pi^2*a^10 + 4096*a^12)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\operatorname{acoth}^3\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/acoth(tanh(b*x+a))**3,x)
```

```
[Out] Integral(1/(x**2*acoth(tanh(a + b*x))**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*arccoth(tanh(b*x + a))^3), x)
```

3.183 $\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^3} dx$

Optimal. Leaf size=170

$$\frac{6b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^4 \coth^{-1}(\tanh(a+bx))} - \frac{3b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3 \coth^{-1}(\tanh(a+bx))^2} - \frac{}{\left(bx-\text{co}\right.}$$

```
[Out] (-3*b^2)/((b*x - ArcCoth[Tanh[a + b*x]])^3*ArcCoth[Tanh[a + b*x]]^2) + (2*b
)/(x*(b*x - ArcCoth[Tanh[a + b*x]])^2*ArcCoth[Tanh[a + b*x]]^2) + 1/(2*x^2*
(b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]^2) + (6*b^2)/((b*x -
ArcCoth[Tanh[a + b*x]])^4*ArcCoth[Tanh[a + b*x]]) - (6*b^2*Log[x])/(b*x - A
rcCoth[Tanh[a + b*x]])^5 + (6*b^2*Log[ArcCoth[Tanh[a + b*x]]])/(b*x - ArcCo
th[Tanh[a + b*x]])^5
```

Rubi [A] time = 0.126766, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2171, 2163, 2160, 2157, 29}

$$\frac{6b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^4 \coth^{-1}(\tanh(a+bx))} - \frac{3b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3 \coth^{-1}(\tanh(a+bx))^2} - \frac{}{\left(bx-\text{co}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*ArcCoth[Tanh[a + b*x]]^3),x]
```

```
[Out] (-3*b^2)/((b*x - ArcCoth[Tanh[a + b*x]])^3*ArcCoth[Tanh[a + b*x]]^2) + (2*b
)/(x*(b*x - ArcCoth[Tanh[a + b*x]])^2*ArcCoth[Tanh[a + b*x]]^2) + 1/(2*x^2*
(b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]^2) + (6*b^2)/((b*x -
ArcCoth[Tanh[a + b*x]])^4*ArcCoth[Tanh[a + b*x]]) - (6*b^2*Log[x])/(b*x - A
rcCoth[Tanh[a + b*x]])^5 + (6*b^2*Log[ArcCoth[Tanh[a + b*x]]])/(b*x - ArcCo
th[Tanh[a + b*x]])^5
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^3}\,dx = \frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2} + \frac{(2b)\int \frac{1}{x^2\coth^{-1}(\tanh(a+b}}{bx-\coth^{-1}(\tanh(a+}$$

$$= \frac{2b}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\coth^{-1}(\tanh(a+bx))^2} + \frac{}{2x^2\left(bx-\coth^{-1}(\tanh(\right.}$$

$$= -\frac{3b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3\coth^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\coth^{-1}(\tanh(a\right.}$$

$$= -\frac{3b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3\coth^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\coth^{-1}(\tanh(a\right.}$$

$$= -\frac{3b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3\coth^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\coth^{-1}(\tanh(a\right.}$$

$$= -\frac{3b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3\coth^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\coth^{-1}(\tanh(a\right.}$$

$$= -\frac{3b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3\coth^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\coth^{-1}(\tanh(a\right.}$$

Mathematica [A] time = 0.0433847, size = 107, normalized size = 0.63

$$\frac{8b^3x^3\coth^{-1}(\tanh(a+bx)) - 12b^2x^2\coth^{-1}(\tanh(a+bx))^2\left(\log(x)-\log\left(\coth^{-1}(\tanh(a+bx))\right)\right) - 8bx\coth^{-1}(\tan}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)^5\coth^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*ArcCoth[Tanh[a + b*x]]^3),x]
```

```
[Out] (-(b^4*x^4) + 8*b^3*x^3*ArcCoth[Tanh[a + b*x]] - 8*b*x*ArcCoth[Tanh[a + b*x
]]^3 + ArcCoth[Tanh[a + b*x]]^4 - 12*b^2*x^2*ArcCoth[Tanh[a + b*x]]^2*(Log[
x] - Log[ArcCoth[Tanh[a + b*x]]]))/(2*x^2*(b*x - ArcCoth[Tanh[a + b*x]])^5*
ArcCoth[Tanh[a + b*x]]^2)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(\operatorname{arccoth}(\tanh(bx+a))\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/arccoth(tanh(b*x+a))^3,x)
```

```
[Out] int(1/x^3/arccoth(tanh(b*x+a))^3,x)
```

Maxima [C] time = 3.5402, size = 448, normalized size = 2.64

$$\frac{192\,b^2\log\left(-i\,\pi+2\,bx+2\,a\right)}{i\,\pi^5-10\,\pi^4a-40i\,\pi^3a^2+80\,\pi^2a^3+80i\,\pi a^4-32\,a^5}-\frac{192\,b^2\log\left(x\right)}{i\,\pi^5-10\,\pi^4a-40i\,\pi^3a^2+80\,\pi^2a^3+80i\,\pi a^4-32\,a^5}+\frac{}{\left(8\,\pi^4b^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 192*b^2*log(-I*pi + 2*b*x + 2*a)/(I*pi^5 - 10*pi^4*a - 40*I*pi^3*a^2 + 80*p
i^2*a^3 + 80*I*pi*a^4 - 32*a^5) - 192*b^2*log(x)/(I*pi^5 - 10*pi^4*a - 40*I
*pi^3*a^2 + 80*pi^2*a^3 + 80*I*pi*a^4 - 32*a^5) + 8*(96*b^3*x^3 - I*pi^3 +
6*pi^2*a + 12*I*pi*a^2 - 8*a^3 + (-72*I*pi*b^2 + 144*a*b^2)*x^2 - (8*pi^2*b
 + 32*I*pi*a*b - 32*a^2*b)*x)/((8*pi^4*b^2 + 64*I*pi^3*a*b^2 - 192*pi^2*a^2
*b^2 - 256*I*pi*a^3*b^2 + 128*a^4*b^2)*x^4 + (-8*I*pi^5*b + 80*pi^4*a*b + 3
20*I*pi^3*a^2*b - 640*pi^2*a^3*b - 640*I*pi*a^4*b + 256*a^5*b)*x^3 - (2*pi^
6 + 24*I*pi^5*a - 120*pi^4*a^2 - 320*I*pi^3*a^3 + 480*pi^2*a^4 + 384*I*pi*a
^5 - 128*a^6)*x^2)
```

Fricas [B] time = 2.23993, size = 3069, normalized size = 18.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 8*(3*pi^10*a + 44*pi^8*a^3 + 224*pi^6*a^5 + 384*pi^4*a^7 - 256*pi^2*a^9 - 1
024*a^11 + 192*(pi^6*b^5 - 20*pi^4*a^2*b^5 - 80*pi^2*a^4*b^5 + 64*a^6*b^5)*
x^5 + 96*(11*pi^6*a*b^4 - 140*pi^4*a^3*b^4 - 624*pi^2*a^5*b^4 + 448*a^7*b^4
)*x^4 + 16*(5*pi^8*b^3 + 16*pi^6*a^2*b^3 - 1440*pi^4*a^4*b^3 - 4864*pi^2*a^
6*b^3 + 3328*a^8*b^3)*x^3 + 4*(65*pi^8*a*b^2 - 272*pi^6*a^3*b^2 - 4896*pi^4
*a^5*b^2 - 9472*pi^2*a^7*b^2 + 6400*a^9*b^2)*x^2 + 2*(3*pi^10*b - 12*pi^8*a
^2*b - 416*pi^6*a^4*b - 1920*pi^4*a^6*b - 2304*pi^2*a^8*b + 1024*a^10*b)*x
- 48*(16*(pi^5*b^6 - 40*pi^3*a^2*b^6 + 80*pi*a^4*b^6)*x^6 + 64*(pi^5*a*b^5
- 40*pi^3*a^3*b^5 + 80*pi*a^5*b^5)*x^5 + 8*(pi^7*b^4 - 28*pi^5*a^2*b^4 - 40
0*pi^3*a^4*b^4 + 960*pi*a^6*b^4)*x^4 + 16*(pi^7*a*b^3 - 36*pi^5*a^3*b^3 - 8
0*pi^3*a^5*b^3 + 320*pi*a^7*b^3)*x^3 + (pi^9*b^2 - 32*pi^7*a^2*b^2 - 224*pi
^5*a^4*b^2 + 1280*pi*a^8*b^2)*x^2)*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 +
8*a*b*x + pi^2 + 4*a^2))/pi) - 24*(16*(5*pi^4*a*b^6 - 40*pi^2*a^3*b^6 + 16*
a^5*b^6)*x^6 + 64*(5*pi^4*a^2*b^5 - 40*pi^2*a^4*b^5 + 16*a^6*b^5)*x^5 + 8*(
5*pi^6*a*b^4 + 20*pi^4*a^3*b^4 - 464*pi^2*a^5*b^4 + 192*a^7*b^4)*x^4 + 16*(
5*pi^6*a^2*b^3 - 20*pi^4*a^4*b^3 - 144*pi^2*a^6*b^3 + 64*a^8*b^3)*x^3 + (5*
pi^8*a*b^2 - 224*pi^4*a^5*b^2 - 512*pi^2*a^7*b^2 + 256*a^9*b^2)*x^2)*log(4*
b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2) + 48*(16*(5*pi^4*a*b^6 - 40*pi^2*a^3*b^6
+ 16*a^5*b^6)*x^6 + 64*(5*pi^4*a^2*b^5 - 40*pi^2*a^4*b^5 + 16*a^6*b^5)*x^5
+ 8*(5*pi^6*a*b^4 + 20*pi^4*a^3*b^4 - 464*pi^2*a^5*b^4 + 192*a^7*b^4)*x^4 +
 16*(5*pi^6*a^2*b^3 - 20*pi^4*a^4*b^3 - 144*pi^2*a^6*b^3 + 64*a^8*b^3)*x^3
+ (5*pi^8*a*b^2 - 224*pi^4*a^5*b^2 - 512*pi^2*a^7*b^2 + 256*a^9*b^2)*x^2)*l
og(x))/(16*(pi^10*b^4 + 20*pi^8*a^2*b^4 + 160*pi^6*a^4*b^4 + 640*pi^4*a^6*b
```

```
^4 + 1280*pi^2*a^8*b^4 + 1024*a^10*b^4)*x^6 + 64*(pi^10*a*b^3 + 20*pi^8*a^3
*b^3 + 160*pi^6*a^5*b^3 + 640*pi^4*a^7*b^3 + 1280*pi^2*a^9*b^3 + 1024*a^11*
b^3)*x^5 + 8*(pi^12*b^2 + 32*pi^10*a^2*b^2 + 400*pi^8*a^4*b^2 + 2560*pi^6*a
^6*b^2 + 8960*pi^4*a^8*b^2 + 16384*pi^2*a^10*b^2 + 12288*a^12*b^2)*x^4 + 16
*(pi^12*a*b + 24*pi^10*a^3*b + 240*pi^8*a^5*b + 1280*pi^6*a^7*b + 3840*pi^4
*a^9*b + 6144*pi^2*a^11*b + 4096*a^13*b)*x^3 + (pi^14 + 28*pi^12*a^2 + 336*
pi^10*a^4 + 2240*pi^8*a^6 + 8960*pi^6*a^8 + 21504*pi^4*a^10 + 28672*pi^2*a^
12 + 16384*a^14)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \operatorname{acoth}^3(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/acoth(tanh(b*x+a))**3,x)
```

```
[Out] Integral(1/(x**3*acoth(tanh(a + b*x))**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \operatorname{arcoth}(\tanh(bx+a))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x^3*arccoth(tanh(b*x + a))^3), x)
```

3.184 $\int x^m \coth^{-1}(\tanh(a+bx))^n \, dx$

Optimal. Leaf size=79

$$\frac{x^m \left(\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)^{-m} \coth^{-1}(\tanh(a+bx))^{n+1} \text{Hypergeometric2F1}\left(-m,n+1,n+2,-\frac{\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{b(n+1)}$$

```
[Out] (x^m*ArcCoth[Tanh[a + b*x]]^(1 + n)*Hypergeometric2F1[-m, 1 + n, 2 + n, -(A
rcCoth[Tanh[a + b*x]]/(b*x - ArcCoth[Tanh[a + b*x]]))])/(b*(1 + n)*((b*x)/(
b*x - ArcCoth[Tanh[a + b*x]]))^m)
```

Rubi [A] time = 0.0494153, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2173}

$$\frac{x^m \left(\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)^{-m} \coth^{-1}(\tanh(a+bx))^{n+1} \,_2F_1\left(-m,n+1;n+2;-\frac{\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcCoth[Tanh[a + b*x]]^n,x]
```

```
[Out] (x^m*ArcCoth[Tanh[a + b*x]]^(1 + n)*Hypergeometric2F1[-m, 1 + n, 2 + n, -(A
rcCoth[Tanh[a + b*x]]/(b*x - ArcCoth[Tanh[a + b*x]]))])/(b*(1 + n)*((b*x)/(
b*x - ArcCoth[Tanh[a + b*x]]))^m)
```

Rule 2173

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(u^m*v^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, -((
a*v)/(b*u - a*v))])/(b*(n + 1)*((b*u)/(b*u - a*v))^m), x] /; NeQ[b*u - a*v,
 0]] /; PiecewiseLinearQ[u, v, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rubi steps

$$\int x^m \coth^{-1}(\tanh(a+bx))^n \, dx = \frac{x^m \left(\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)^{-m} \coth^{-1}(\tanh(a+bx))^{1+n} \,_2F_1\left(-m,1+n;2+n;-\frac{\coth^{-1}(t}{bx-\coth^{-1}}\right.}{b(1+n)}$$

Mathematica [A] time = 0.132399, size = 71, normalized size = 0.9

$$\frac{x^{m+1} \coth^{-1}(\tanh(a+bx))^n \left(\frac{bx}{\coth^{-1}(\tanh(a+bx))-bx}+1\right)^{-n} \text{Hypergeometric2F1}\left(m+1,-n,m+2,-\frac{bx}{\coth^{-1}(\tanh(a+bx))-bx}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcCoth[Tanh[a + b*x]]^n,x]
```

```
[Out] (x^(1 + m)*ArcCoth[Tanh[a + b*x]]^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((
b*x)/(-(b*x) + ArcCoth[Tanh[a + b*x]]))])/((1 + m)*(1 + (b*x)/(-(b*x) + Arc
```

```
Coth[Tanh[a + b*x]]))^n)
```

Maple [F] time = 2.566, size = 0, normalized size = 0.

$$\int x^m (\operatorname{arccoth}(\tanh(bx+a)))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arccoth(tanh(b*x+a))^n,x)
```

```
[Out] int(x^m*arccoth(tanh(b*x+a))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{arcoth}(\tanh(bx+a))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] integrate(x^m*arccoth(tanh(b*x + a))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^m \operatorname{arcoth}(\tanh(bx+a))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] integral(x^m*arccoth(tanh(b*x + a))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{acoth}^n(\tanh(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*acoth(tanh(b*x+a))**n,x)
```

```
[Out] Integral(x**m*acoth(tanh(a + b*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{arcoth}(\tanh(bx+a))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a))^n,x, algorithm="giac")
```

```
[Out] integrate(x^m*arccoth(tanh(b*x + a))^n, x)
```

3.185 $\int x^4 \coth^{-1}(\tanh(a+bx))^n\,dx$

Optimal. Leaf size=165

$$-\frac{4x^3\coth^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}+\frac{12x^2\coth^{-1}(\tanh(a+bx))^{n+3}}{b^3(n+1)(n+2)(n+3)}-\frac{24x\coth^{-1}(\tanh(a+bx))^{n+4}}{b^4(n+1)(n+2)(n+3)(n+4)}+\frac{24\coth^{-1}(\mathrm{t}}{b^5(n+1)(n+2)}$$

```
[Out] (x^4*ArcCoth[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (4*x^3*ArcCoth[Tanh[a +
b*x]]^(2 + n))/(b^2*(1 + n)*(2 + n)) + (12*x^2*ArcCoth[Tanh[a + b*x]]^(3 +
n))/(b^3*(1 + n)*(2 + n)*(3 + n)) - (24*x*ArcCoth[Tanh[a + b*x]]^(4 + n))/(
b^4*(1 + n)*(2 + n)*(3 + n)*(4 + n)) + (24*ArcCoth[Tanh[a + b*x]]^(5 + n))/
(b^5*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n))
```

Rubi [A] time = 0.133038, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2157, 30}

$$-\frac{4x^3\coth^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}+\frac{12x^2\coth^{-1}(\tanh(a+bx))^{n+3}}{b^3(n+1)(n+2)(n+3)}-\frac{24x\coth^{-1}(\tanh(a+bx))^{n+4}}{b^4(n+1)(n+2)(n+3)(n+4)}+\frac{24\coth^{-1}(\mathrm{t}}{b^5(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcCoth[Tanh[a + b*x]]^n,x]
```

```
[Out] (x^4*ArcCoth[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (4*x^3*ArcCoth[Tanh[a +
b*x]]^(2 + n))/(b^2*(1 + n)*(2 + n)) + (12*x^2*ArcCoth[Tanh[a + b*x]]^(3 +
n))/(b^3*(1 + n)*(2 + n)*(3 + n)) - (24*x*ArcCoth[Tanh[a + b*x]]^(4 + n))/(
b^4*(1 + n)*(2 + n)*(3 + n)*(4 + n)) + (24*ArcCoth[Tanh[a + b*x]]^(5 + n))/
(b^5*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n))
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^4 \coth^{-1}(\tanh(a+bx))^n\,dx = \frac{x^4\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4\int x^3\coth^{-1}(\tanh(a+bx))^{1+n}\,dx}{b(1+n)}$$
$$= \frac{x^4\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4x^3\coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12\int x^2\coth^{-1}(\tanh(a}{b^2(1+n)(2+}$$
$$= \frac{x^4\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4x^3\coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12x^2\coth^{-1}(\tanh(a+}{b^3(1+n)(2+n)(3}$$
$$= \frac{x^4\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4x^3\coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12x^2\coth^{-1}(\tanh(a+}{b^3(1+n)(2+n)(3}$$
$$= \frac{x^4\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4x^3\coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12x^2\coth^{-1}(\tanh(a+}{b^3(1+n)(2+n)(3}$$
$$= \frac{x^4\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4x^3\coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12x^2\coth^{-1}(\tanh(a+}{b^3(1+n)(2+n)(3}$$

Mathematica [A] time = 0.103393, size = 146, normalized size = 0.88

$$\frac{\coth^{-1}(\tanh(a+bx))^{n+1}\left(-4b^3\left(n^3+12n^2+47n+60\right)x^3\coth^{-1}(\tanh(a+bx))+12b^2\left(n^2+9n+20\right)x^2\coth^{-1}(\tanh(}{b^5(n+1)(n+2)(n+}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcCoth[Tanh[a + b*x]]^n,x]
```

```
[Out] (ArcCoth[Tanh[a + b*x]]^(1 + n)*(b^4*(120 + 154*n + 71*n^2 + 14*n^3 + n^4)*
x^4 - 4*b^3*(60 + 47*n + 12*n^2 + n^3)*x^3*ArcCoth[Tanh[a + b*x]] + 12*b^2*
(20 + 9*n + n^2)*x^2*ArcCoth[Tanh[a + b*x]]^2 - 24*b*(5 + n)*x*ArcCoth[Tanh
[a + b*x]]^3 + 24*ArcCoth[Tanh[a + b*x]]^4))/(b^5*(1 + n)*(2 + n)*(3 + n)*(
4 + n)*(5 + n))
```

Maple [B] time = 87.445, size = 2984280, normalized size = 18086.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccoth(tanh(b*x+a))^n,x)
```

```
[Out] result too large to display
```

Maxima [C] time = 1.81928, size = 513, normalized size = 3.11

$$\left(4\left(n^4+10n^3+35n^2+50n+24\right)b^5x^5-3i\pi^5+30\pi^4a+120i\pi^3a^2-240\pi^2a^3-240i\pi a^4+96a^5-2\left(i\pi\left(n^4+6n^3+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(tanh(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] (4*(n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 - 3*I*pi^5 + 30*pi^4*a + 120
*I*pi^3*a^2 - 240*pi^2*a^3 - 240*I*pi*a^4 + 96*a^5 - 2*(I*pi*(n^4 + 6*n^3 +
```

```
 11*n^2 + 6*n)*b^4 - 2*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a*b^4)*x^4 + (4*pi^2*(n
^3 + 3*n^2 + 2*n)*b^3 + 16*I*pi*(n^3 + 3*n^2 + 2*n)*a*b^3 - 16*(n^3 + 3*n^2
 + 2*n)*a^2*b^3)*x^3 + (6*I*pi^3*(n^2 + n)*b^2 - 36*pi^2*(n^2 + n)*a*b^2 -
72*I*pi*(n^2 + n)*a^2*b^2 + 48*(n^2 + n)*a^3*b^2)*x^2 - (6*pi^4*b*n + 48*I*
pi^3*a*b*n - 144*pi^2*a^2*b*n - 192*I*pi*a^3*b*n + 96*a^4*b*n)*x)*(cosh(-n*
log(-I*pi + 2*b*x + 2*a)) - sinh(-n*log(-I*pi + 2*b*x + 2*a)))/((2^(n + 2)*
n^5 + 15*2^(n + 2)*n^4 + 85*2^(n + 2)*n^3 + 225*2^(n + 2)*n^2 + 137*2^(n +
3)*n + 15*2^(n + 5))*b^5)
```

Fricas [B] time = 1.72095, size = 1288, normalized size = 7.81

$$\frac{2\left(2\left(b^5n^4 + 10\,b^5n^3 + 35\,b^5n^2 + 50\,b^5n + 24\,b^5\right)x^5 + 15\,\pi^4a - 120\,\pi^2a^3 + 48\,a^5 + 2\left(ab^4n^4 + 6\,ab^4n^3 + 11\,ab^4n^2 + 6\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(tanh(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(2*(b^5*n^4 + 10*b^5*n^3 + 35*b^5*n^2 + 50*b^5*n + 24*b^5)*x^5 + 15*
pi^4*a - 120*pi^2*a^3 + 48*a^5 + 2*(a*b^4*n^4 + 6*a*b^4*n^3 + 11*a*b^4*n^2
+ 6*a*b^4*n)*x^4 - 2*(4*a^2*b^3*n^3 + 12*a^2*b^3*n^2 + 8*a^2*b^3*n - pi^2*(
b^3*n^3 + 3*b^3*n^2 + 2*b^3*n))*x^3 + 6*(4*a^3*b^2*n^2 + 4*a^3*b^2*n - 3*pi
^2*(a*b^2*n^2 + a*b^2*n))*x^2 - 3*(pi^4*b*n - 24*pi^2*a^2*b*n + 16*a^4*b*n)
*x)*(b^2*x^2 + 2*a*b*x + 1/4*pi^2 + a^2)^(1/2*n)*cos(2*n*arctan(-2*b*x/pi -
 2*a/pi + sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2)/pi)) - (2*pi*(b^4*n^4 +
6*b^4*n^3 + 11*b^4*n^2 + 6*b^4*n)*x^4 + 3*pi^5 - 120*pi^3*a^2 + 240*pi*a^4
- 16*pi*(a*b^3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 6*(pi^3*(b^2*n^2 + b^2*
n) - 12*pi*(a^2*b^2*n^2 + a^2*b^2*n))*x^2 + 48*(pi^3*a*b*n - 4*pi*a^3*b*n)*
x)*(b^2*x^2 + 2*a*b*x + 1/4*pi^2 + a^2)^(1/2*n)*sin(2*n*arctan(-2*b*x/pi -
2*a/pi + sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2)/pi)))/(b^5*n^5 + 15*b^5*n
^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acoth(tanh(b*x+a))**n,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{arcoth}\left(\tanh\left(bx + a\right)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(tanh(b*x+a))^n,x, algorithm="giac")
```

```
[Out] integrate(x^4*arccoth(tanh(b*x + a))^n, x)
```

3.186 $\int x^3 \coth^{-1}(\tanh(a+bx))^n \, dx$

Optimal. Leaf size=121

$$-\frac{3x^2\coth^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}+\frac{6x\coth^{-1}(\tanh(a+bx))^{n+3}}{b^3(n+1)(n+2)(n+3)}-\frac{6\coth^{-1}(\tanh(a+bx))^{n+4}}{b^4(n+1)(n+2)(n+3)(n+4)}+\frac{x^3\coth^{-1}(\tanh(a+b$$

```
[Out] (x^3*ArcCoth[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (3*x^2*ArcCoth[Tanh[a +
b*x]]^(2 + n))/(b^2*(1 + n)*(2 + n)) + (6*x*ArcCoth[Tanh[a + b*x]]^(3 + n))
/(b^3*(1 + n)*(2 + n)*(3 + n)) - (6*ArcCoth[Tanh[a + b*x]]^(4 + n))/(b^4*(1
 + n)*(2 + n)*(3 + n)*(4 + n))
```

Rubi [A] time = 0.0796348, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2157, 30}

$$-\frac{3x^2\coth^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}+\frac{6x\coth^{-1}(\tanh(a+bx))^{n+3}}{b^3(n+1)(n+2)(n+3)}-\frac{6\coth^{-1}(\tanh(a+bx))^{n+4}}{b^4(n+1)(n+2)(n+3)(n+4)}+\frac{x^3\coth^{-1}(\tanh(a+b$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCoth[Tanh[a + b*x]]^n,x]
```

```
[Out] (x^3*ArcCoth[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (3*x^2*ArcCoth[Tanh[a +
b*x]]^(2 + n))/(b^2*(1 + n)*(2 + n)) + (6*x*ArcCoth[Tanh[a + b*x]]^(3 + n))
/(b^3*(1 + n)*(2 + n)*(3 + n)) - (6*ArcCoth[Tanh[a + b*x]]^(4 + n))/(b^4*(1
 + n)*(2 + n)*(3 + n)*(4 + n))
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^3 \coth^{-1}(\tanh(a+bx))^n\,dx = \frac{x^3\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{3\int x^2\coth^{-1}(\tanh(a+bx))^{1+n}\,dx}{b(1+n)}$$
$$= \frac{x^3\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{3x^2\coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{6\int x\coth^{-1}(\tanh(}{b^2(1+n)(2}$$
$$= \frac{x^3\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{3x^2\coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{6x\coth^{-1}(\tanh(a}{b^3(1+n)(2+n)(}$$
$$= \frac{x^3\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{3x^2\coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{6x\coth^{-1}(\tanh(a}{b^3(1+n)(2+n)(}$$
$$= \frac{x^3\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{3x^2\coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{6x\coth^{-1}(\tanh(a}{b^3(1+n)(2+n)(}$$

Mathematica [A] time = 0.0773392, size = 106, normalized size = 0.88

$$\frac{\coth^{-1}(\tanh(a+bx))^{n+1}\left(-3b^2\left(n^2+7n+12\right)x^2\coth^{-1}(\tanh(a+bx))+6b(n+4)x\coth^{-1}(\tanh(a+bx))^2-6\coth}{b^4(n+1)(n+2)(n+3)(n+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCoth[Tanh[a + b*x]]^n,x]
```

```
[Out] (ArcCoth[Tanh[a + b*x]]^(1 + n)*(b^3*(24 + 26*n + 9*n^2 + n^3)*x^3 - 3*b^2*
(12 + 7*n + n^2)*x^2*ArcCoth[Tanh[a + b*x]] + 6*b*(4 + n)*x*ArcCoth[Tanh[a
+ b*x]]^2 - 6*ArcCoth[Tanh[a + b*x]]^3))/(b^4*(1 + n)*(2 + n)*(3 + n)*(4 +
n))
```

Maple [B] time = 36.951, size = 953037, normalized size = 7876.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccoth(tanh(b*x+a))^n,x)
```

```
[Out] result too large to display
```

Maxima [C] time = 1.79852, size = 344, normalized size = 2.84

$$\frac{\left(8\left(n^3+6n^2+11n+6\right)b^4x^4-3\pi^4-24i\pi^3a+72\pi^2a^2+96i\pi a^3-48a^4+\left(-4i\pi\left(n^3+3n^2+2n\right)b^3+8\left(n^3+3n^2\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(tanh(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] (8*(n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 - 3*pi^4 - 24*I*pi^3*a + 72*pi^2*a^2 +
96*I*pi*a^3 - 48*a^4 + (-4*I*pi*(n^3 + 3*n^2 + 2*n)*b^3 + 8*(n^3 + 3*n^2 +
2*n)*a*b^3)*x^3 + (6*pi^2*(n^2 + n)*b^2 + 24*I*pi*(n^2 + n)*a*b^2 - 24*(n^2
 + n)*a^2*b^2)*x^2 + (6*I*pi^3*b*n - 36*pi^2*a*b*n - 72*I*pi*a^2*b*n + 48*a
```

```
^3*b*n)*x)*(cosh(-n*log(-I*pi + 2*b*x + 2*a)) - sinh(-n*log(-I*pi + 2*b*x +
 2*a)))/((2^(n + 3)*n^4 + 5*2^(n + 4)*n^3 + 35*2^(n + 3)*n^2 + 25*2^(n + 4)
*n + 3*2^(n + 6))*b^4)
```

Fricas [B] time = 1.76948, size = 917, normalized size = 7.58

$$\frac{\left(8\left(b^4n^3 + 6\,b^4n^2 + 11\,b^4n + 6\,b^4\right)x^4 - 3\,\pi^4 + 72\,\pi^2a^2 - 48\,a^4 + 8\left(ab^3n^3 + 3\,ab^3n^2 + 2\,ab^3n\right)x^3 - 6\left(4\,a^2b^2n^2 + 4\,a^2b^2n - \right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(tanh(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] 1/8*((8*(b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 3*pi^4 + 72*pi^2*a^2
 - 48*a^4 + 8*(a*b^3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 6*(4*a^2*b^2*n^2
+ 4*a^2*b^2*n - pi^2*(b^2*n^2 + b^2*n))*x^2 - 12*(3*pi^2*a*b*n - 4*a^3*b*n)
*x)*(b^2*x^2 + 2*a*b*x + 1/4*pi^2 + a^2)^(1/2*n)*cos(2*n*arctan(-2*b*x/pi -
 2*a/pi + sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2)/pi)) - 2*(2*pi*(b^3*n^3
+ 3*b^3*n^2 + 2*b^3*n)*x^3 + 12*pi^3*a - 48*pi*a^3 - 12*pi*(a*b^2*n^2 + a*b
^2*n)*x^2 - 3*(pi^3*b*n - 12*pi*a^2*b*n)*x)*(b^2*x^2 + 2*a*b*x + 1/4*pi^2 +
 a^2)^(1/2*n)*sin(2*n*arctan(-2*b*x/pi - 2*a/pi + sqrt(4*b^2*x^2 + 8*a*b*x
+ pi^2 + 4*a^2)/pi)))/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^
4)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acoth(tanh(b*x+a))**n,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arcoth}\left(\tanh\left(bx + a\right)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(tanh(b*x+a))^n,x, algorithm="giac")
```

```
[Out] integrate(x^3*arccoth(tanh(b*x + a))^n, x)
```

3.187 $\int x^2 \coth^{-1}(\tanh(a+bx))^n \, dx$

Optimal. Leaf size=82

$$-\frac{2x\coth^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}+\frac{2\coth^{-1}(\tanh(a+bx))^{n+3}}{b^3(n+1)(n+2)(n+3)}+\frac{x^2\coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}$$

```
[Out] (x^2*ArcCoth[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (2*x*ArcCoth[Tanh[a + b*
x]]^(2 + n))/(b^2*(1 + n)*(2 + n)) + (2*ArcCoth[Tanh[a + b*x]]^(3 + n))/(b^
3*(1 + n)*(2 + n)*(3 + n))
```

Rubi [A] time = 0.0461923, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2157, 30}

$$-\frac{2x\coth^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}+\frac{2\coth^{-1}(\tanh(a+bx))^{n+3}}{b^3(n+1)(n+2)(n+3)}+\frac{x^2\coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[Tanh[a + b*x]]^n,x]
```

```
[Out] (x^2*ArcCoth[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (2*x*ArcCoth[Tanh[a + b*
x]]^(2 + n))/(b^2*(1 + n)*(2 + n)) + (2*ArcCoth[Tanh[a + b*x]]^(3 + n))/(b^
3*(1 + n)*(2 + n)*(3 + n))
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^2 \coth^{-1}(\tanh(a+bx))^n\,dx = \frac{x^2\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{2\int x\coth^{-1}(\tanh(a+bx))^{1+n}\,dx}{b(1+n)}$$
$$= \frac{x^2\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{2x\coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{2\int \coth^{-1}(\tanh(a+b}{b^2(1+n)(2+n}$$
$$= \frac{x^2\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{2x\coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{2\,\text{Subst}\left(\int x^{2+n}\,dx, x, c\right.}{b^3(1+n)}$$
$$= \frac{x^2\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{2x\coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{2\coth^{-1}(\tanh(a+bx))}{b^3(1+n)(2+n)(3+}$$

Mathematica [A] time = 0.0645757, size = 71, normalized size = 0.87

$$\frac{\coth^{-1}(\tanh(a+bx))^{n+1}\left(-2b(n+3)x\coth^{-1}(\tanh(a+bx))+2\coth^{-1}(\tanh(a+bx))^2+b^2\left(n^2+5n+6\right)x^2\right)}{b^3(n+1)(n+2)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[Tanh[a + b*x]]^n,x]
```

```
[Out] (ArcCoth[Tanh[a + b*x]]^(1 + n)*(b^2*(6 + 5*n + n^2)*x^2 - 2*b*(3 + n)*x*Ar
cCoth[Tanh[a + b*x]] + 2*ArcCoth[Tanh[a + b*x]]^2))/(b^3*(1 + n)*(2 + n)*(3
 + n))
```

Maple [C] time = 18.674, size = 252344, normalized size = 3077.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(tanh(b*x+a))^n,x)
```

```
[Out] result too large to display
```

Maxima [C] time = 1.79112, size = 223, normalized size = 2.72

$$\frac{\left(4\left(n^2+3n+2\right)b^3x^3+i\pi^3-6\pi^2a-12i\pi a^2+8a^3+\left(-2i\pi\left(n^2+n\right)b^2+4\left(n^2+n\right)ab^2\right)x^2+\left(2\pi^2bn+8i\pi abn-8a^2b\right.}{\left(2^{n+2}n^3+3\cdot 2^{n+3}n^2+11\cdot 2^{n+2}n+3\cdot 2^{n+3}\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(tanh(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] (4*(n^2 + 3*n + 2)*b^3*x^3 + I*pi^3 - 6*pi^2*a - 12*I*pi*a^2 + 8*a^3 + (-2*
I*pi*(n^2 + n)*b^2 + 4*(n^2 + n)*a*b^2)*x^2 + (2*pi^2*b*n + 8*I*pi*a*b*n -
8*a^2*b*n)*x)*(cosh(-n*log(-I*pi + 2*b*x + 2*a)) - sinh(-n*log(-I*pi + 2*b*
x + 2*a)))/((2^(n + 2)*n^3 + 3*2^(n + 3)*n^2 + 11*2^(n + 2)*n + 3*2^(n + 3)
)*b^3)
```

Fricas [B] time = 1.72839, size = 649, normalized size = 7.91

$$\frac{2\left(2\left(b^3n^2+3\,b^3n+2\,b^3\right)x^3-3\,\pi^2a+4\,a^3+2\left(ab^2n^2+ab^2n\right)x^2+\left(\pi^2bn-4\,a^2bn\right)x\right)\left(b^2x^2+2\,abx+\frac{1}{4}\,\pi^2+a^2\right)^{\frac{1}{2}\,n}\,\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(tanh(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(2*(b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 3*pi^2*a + 4*a^3 + 2*(a*b^2*n^2
 + a*b^2*n)*x^2 + (pi^2*b*n - 4*a^2*b*n)*x)*(b^2*x^2 + 2*a*b*x + 1/4*pi^2 +
 a^2)^(1/2*n)*cos(2*n*arctan(-2*b*x/pi - 2*a/pi + sqrt(4*b^2*x^2 + 8*a*b*x
+ pi^2 + 4*a^2)/pi)) + (8*pi*a*b*n*x - 2*pi*(b^2*n^2 + b^2*n)*x^2 + pi^3 -
12*pi*a^2)*(b^2*x^2 + 2*a*b*x + 1/4*pi^2 + a^2)^(1/2*n)*sin(2*n*arctan(-2*b
*x/pi - 2*a/pi + sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2)/pi)))/(b^3*n^3 +
6*b^3*n^2 + 11*b^3*n + 6*b^3)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(tanh(b*x+a))**n,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arcoth}(\tanh(bx+a))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(tanh(b*x+a))^n,x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(tanh(b*x + a))^n, x)
```

3.188 $\int x \coth^{-1}(\tanh(a+bx))^n \, dx$

Optimal. Leaf size=48

$$\frac{x \coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)} - \frac{\coth^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}$$

```
[Out] (x*ArcCoth[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - ArcCoth[Tanh[a + b*x]]^(2
+ n)/(b^2*(1 + n)*(2 + n))
```

Rubi [A] time = 0.0200901, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2168, 2157, 30}

$$\frac{x \coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)} - \frac{\coth^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[Tanh[a + b*x]]^n,x]
```

```
[Out] (x*ArcCoth[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - ArcCoth[Tanh[a + b*x]]^(2
+ n)/(b^2*(1 + n)*(2 + n))
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x \coth^{-1}(\tanh(a+bx))^n \, dx &= \frac{x \coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{\int \coth^{-1}(\tanh(a+bx))^{1+n} \, dx}{b(1+n)} \\
&= \frac{x \coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{\text{Subst}\left(\int x^{1+n} \, dx, x, \coth^{-1}(\tanh(a+bx))\right)}{b^2(1+n)} \\
&= \frac{x \coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{\coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)}
\end{aligned}$$

Mathematica [A] time = 0.0428894, size = 41, normalized size = 0.85

$$\frac{\left(b(n+2)x-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^{n+1}}{b^2(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[Tanh[a + b*x]]^n,x]
```

```
[Out] ((b*(2 + n)*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]^(1 + n))/(b^
2*(1 + n)*(2 + n))
```

Maple [C] time = 15.611, size = 71611, normalized size = 1491.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(tanh(b*x+a))^n,x)
```

```
[Out] result too large to display
```

Maxima [C] time = 1.79195, size = 138, normalized size = 2.88

$$\frac{\left(4\,b^2(n+1)x^2+\pi^2+4i\,\pi a-4\,a^2-2\,(i\,\pi bn-2\,abn)x\right)(\cosh\left(-n\log\left(-i\,\pi+2\,bx+2\,a\right)\right)-\sinh\left(-n\log\left(-i\,\pi+2\,bx\right.\right.}{\left(2^{n+2}n^2+3\cdot 2^{n+2}n+2^{n+3}\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(tanh(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] (4*b^2*(n + 1)*x^2 + pi^2 + 4*I*pi*a - 4*a^2 - 2*(I*pi*b*n - 2*a*b*n)*x)*(c
osh(-n*log(-I*pi + 2*b*x + 2*a)) - sinh(-n*log(-I*pi + 2*b*x + 2*a)))/((2^(
n + 2)*n^2 + 3*2^(n + 2)*n + 2^(n + 3))*b^2)
```

Fricas [B] time = 1.60396, size = 489, normalized size = 10.19

$$\frac{\left(4\,abnx+4\left(b^2n+b^2\right)x^2+\pi^2-4\,a^2\right)\left(b^2x^2+2\,abx+\frac{1}{4}\,\pi^2+a^2\right)^{\frac{1}{2}\,n}\cos\left(2\,n\arctan\left(-\frac{2\,bx}{\pi}-\frac{2\,a}{\pi}+\frac{\sqrt{4\,b^2x^2+8\,abx+\pi^2+4\,a^2}}{\pi}\right.\right.}{4\left(b^2n^2+3\,b^2n+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(tanh(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] 1/4*((4*a*b*n*x + 4*(b^2*n + b^2)*x^2 + pi^2 - 4*a^2)*(b^2*x^2 + 2*a*b*x +
1/4*pi^2 + a^2)^(1/2*n)*cos(2*n*arctan(-2*b*x/pi - 2*a/pi + sqrt(4*b^2*x^2
+ 8*a*b*x + pi^2 + 4*a^2)/pi)) - 2*(pi*b*n*x - 2*pi*a)*(b^2*x^2 + 2*a*b*x +
 1/4*pi^2 + a^2)^(1/2*n)*sin(2*n*arctan(-2*b*x/pi - 2*a/pi + sqrt(4*b^2*x^2
```

```
+ 8*a*b*x + pi^2 + 4*a^2)/pi)))/(b^2*n^2 + 3*b^2*n + 2*b^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(tanh(b*x+a))**n,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arcoth}(\tanh(bx+a))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(tanh(b*x+a))^n,x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(tanh(b*x + a))^n, x)
```

3.189 $\int \coth^{-1}(\tanh(a+bx))^n \, dx$

Optimal. Leaf size=20

$$\frac{\coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}$$

```
[Out] ArcCoth[Tanh[a + b*x]]^(1 + n)/(b*(1 + n))
```

Rubi [A] time = 0.0065514, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2157, 30}

$$\frac{\coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^n,x]
```

```
[Out] ArcCoth[Tanh[a + b*x]]^(1 + n)/(b*(1 + n))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \coth^{-1}(\tanh(a+bx))^n \, dx &= \frac{\text{Subst}\left(\int x^n \, dx, x, \coth^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0142894, size = 20, normalized size = 1.

$$\frac{\coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^n,x]
```

```
[Out] ArcCoth[Tanh[a + b*x]]^(1 + n)/(b*(1 + n))
```

Maple [A] time = 0.087, size = 21, normalized size = 1.1

$$\frac{(\operatorname{arccoth}(\tanh(bx+a)))^{1+n}}{b(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^n,x)
```

```
[Out] arccoth(tanh(b*x+a))^(1+n)/b/(1+n)
```

Maxima [C] time = 1.73917, size = 88, normalized size = 4.4

$$\frac{(-i\pi+2bx+2a)(\cosh(-n\log(-i\pi+2bx+2a))-\sinh(-n\log(-i\pi+2bx+2a)))}{\left(2^{n+1}n+2^{n+1}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] (-I*pi + 2*b*x + 2*a)*(cosh(-n*log(-I*pi + 2*b*x + 2*a)) - sinh(-n*log(-I*p
i + 2*b*x + 2*a)))/((2^(n + 1)*n + 2^(n + 1))*b)
```

Fricas [B] time = 1.7029, size = 389, normalized size = 19.45

$$\frac{2(bx+a)\left(b^2x^2+2abx+\frac{1}{4}\pi^2+a^2\right)^{\frac{1}{2}n}\cos\left(2n\arctan\left(-\frac{2bx}{\pi}-\frac{2a}{\pi}+\frac{\sqrt{4b^2x^2+8abx+\pi^2+4a^2}}{\pi}\right)\right)-\pi\left(b^2x^2+2abx+\frac{1}{4}\pi^2+a^2\right.}{2(bn+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b*x + a)*(b^2*x^2 + 2*a*b*x + 1/4*pi^2 + a^2)^(1/2*n)*cos(2*n*arcta
n(-2*b*x/pi - 2*a/pi + sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2)/pi)) - pi*(
b^2*x^2 + 2*a*b*x + 1/4*pi^2 + a^2)^(1/2*n)*sin(2*n*arctan(-2*b*x/pi - 2*a/
pi + sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2)/pi)))/(b*n + b)
```

Sympy [A] time = 1.04688, size = 51, normalized size = 2.55

$$\begin{cases} \frac{x}{\operatorname{acoth}(\tanh(a))} & \text{for } b=0 \wedge n=-1 \\ x\operatorname{acoth}^{n}(\tanh(a)) & \text{for } b=0 \\ \frac{\log(\operatorname{acoth}(\tanh(a+bx)))}{b} & \text{for } n=-1 \\ \frac{\operatorname{acoth}(\tanh(a+bx))\operatorname{acoth}^{n}(\tanh(a+bx))}{bn+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**n,x)
```

```
[Out] Piecewise((x/acoth(tanh(a)), Eq(b, 0) & Eq(n, -1)), (x*acoth(tanh(a))**n, E
q(b, 0)), (log(acoth(tanh(a + b*x)))/b, Eq(n, -1)), (acoth(tanh(a + b*x))*a
coth(tanh(a + b*x))**n/(b*n + b), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}(\tanh(bx + a))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^n,x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^n, x)
```

3.190 $\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x} \, dx$

Optimal. Leaf size=64

$$\frac{\coth^{-1}(\tanh(a+bx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,-\frac{\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{(n+1)\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

```
[Out] (ArcCoth[Tanh[a + b*x]]^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, -(ArcCot
h[Tanh[a + b*x]]/(b*x - ArcCoth[Tanh[a + b*x]]))])/((1 + n)*(b*x - ArcCoth[
Tanh[a + b*x]]))
```

Rubi [A] time = 0.0208313, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2164}

$$\frac{\coth^{-1}(\tanh(a+bx))^{n+1}\,_2F_1\left(1,n+1;n+2;-\frac{\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{(n+1)\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^n/x,x]
```

```
[Out] (ArcCoth[Tanh[a + b*x]]^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, -(ArcCot
h[Tanh[a + b*x]]/(b*x - ArcCoth[Tanh[a + b*x]]))])/((1 + n)*(b*x - ArcCoth[
Tanh[a + b*x]]))
```

Rule 2164

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)*Hypergeometric2F1[1, n + 1, n + 2, -((a*v)/(b*u
- a*v))])/((n + 1)*(b*u - a*v)), x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] && !IntegerQ[n]
```

Rubi steps

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x} \, dx = \frac{\coth^{-1}(\tanh(a+bx))^{1+n}\,_2F_1\left(1,1+n;2+n;-\frac{\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{(1+n)\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A] time = 0.0811981, size = 60, normalized size = 0.94

$$\frac{\coth^{-1}(\tanh(a+bx))^n\left(\frac{\coth^{-1}(\tanh(a+bx))}{bx}\right)^{-n}\text{Hypergeometric2F1}\left(-n,-n,1-n,1-\frac{\coth^{-1}(\tanh(a+bx))}{bx}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^n/x,x]
```

```
[Out] (ArcCoth[Tanh[a + b*x]]^n*Hypergeometric2F1[-n, -n, 1 - n, 1 - ArcCoth[Tanh
[a + b*x]]/(b*x)])/(n*(ArcCoth[Tanh[a + b*x]]/(b*x))^n)
```

Maple [F] time = 0.855, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{arccoth}(\tanh(bx+a)))^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^n/x,x)
```

```
[Out] int(arccoth(tanh(b*x+a))^n/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(\tanh(bx+a))^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^n/x,x, algorithm="maxima")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^n/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arcoth}(\tanh(bx+a))^n}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^n/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(tanh(b*x + a))^n/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^n(\tanh(a+bx))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**n/x,x)
```

```
[Out] Integral(acoth(tanh(a + b*x))**n/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(\tanh(bx+a))^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^n/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^n/x, x)
```

3.191 $\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^2}\,dx$

Optimal. Leaf size=71

$$\frac{b\coth^{-1}(\tanh(a+bx))^n \text{Hypergeometric2F1}\left(1,n,n+1,-\frac{\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\coth^{-1}(\tanh(a+bx))^n}{x}$$

```
[Out] -(ArcCoth[Tanh[a + b*x]]^n/x) + (b*ArcCoth[Tanh[a + b*x]]^n*Hypergeometric2
F1[1, n, 1 + n, -(ArcCoth[Tanh[a + b*x]]/(b*x - ArcCoth[Tanh[a + b*x]]))])/
(b*x - ArcCoth[Tanh[a + b*x]])
```

Rubi [A] time = 0.0412581, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 2164}

$$\frac{b\coth^{-1}(\tanh(a+bx))^n\,{}_2F_1\left(1,n;n+1;-\frac{\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\coth^{-1}(\tanh(a+bx))^n}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^n/x^2,x]
```

```
[Out] -(ArcCoth[Tanh[a + b*x]]^n/x) + (b*ArcCoth[Tanh[a + b*x]]^n*Hypergeometric2
F1[1, n, 1 + n, -(ArcCoth[Tanh[a + b*x]]/(b*x - ArcCoth[Tanh[a + b*x]]))])/
(b*x - ArcCoth[Tanh[a + b*x]])
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2164

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)*Hypergeometric2F1[1, n + 1, n + 2, -((a*v)/(b*u
- a*v))])/((n + 1)*(b*u - a*v)), x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^2}\,dx &= -\frac{\coth^{-1}(\tanh(a+bx))^n}{x}+(bn)\int \frac{\coth^{-1}(\tanh(a+bx))^{-1+n}}{x}\,dx\\ &= -\frac{\coth^{-1}(\tanh(a+bx))^n}{x}+\frac{b\coth^{-1}(\tanh(a+bx))^n\,{}_2F_1\left(1,n;1+n;-\frac{\coth^{-1}(\tanh(a+\right.}{bx-\coth^{-1}(\tanh(a+bx))}\end{aligned}$$

Mathematica [A] time = 0.0404399, size = 67, normalized size = 0.94

$$\frac{\coth^{-1}(\tanh(a+bx))^n\left(\frac{\coth^{-1}(\tanh(a+bx))}{bx}\right)^{-n}\text{Hypergeometric2F1}\left(1-n,-n,2-n,1-\frac{\coth^{-1}(\tanh(a+bx))}{bx}\right)}{(n-1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^n/x^2,x]
```

```
[Out] (ArcCoth[Tanh[a + b*x]]^n*Hypergeometric2F1[1 - n, -n, 2 - n, 1 - ArcCoth[T
anh[a + b*x]]/(b*x)])/((-1 + n)*x*(ArcCoth[Tanh[a + b*x]]/(b*x))^n)
```

Maple [F] time = 1.964, size = 0, normalized size = 0.

$$\int \frac{(\text{arccoth}(\tanh(bx+a)))^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^n/x^2,x)
```

```
[Out] int(arccoth(tanh(b*x+a))^n/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arcoth}(\tanh(bx+a))^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^n/x^2,x, algorithm="maxima")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^n/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arcoth}(\tanh(bx+a))^n}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^n/x^2,x, algorithm="fricas")
```

```
[Out] integral(arccoth(tanh(b*x + a))^n/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{acoth}^n(\tanh(a+bx))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**n/x**2,x)
```

```
[Out] Integral(acoth(tanh(a + b*x))**n/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^{n}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^n/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^n/x^2, x)
```

3.192 $\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^3} dx$

Optimal. Leaf size=101

$$\frac{b^2 n \coth^{-1}(\tanh(a+bx))^{n-1} \text{Hypergeometric2F1}\left(1, n-1, n, -\frac{\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)} - \frac{\coth^{-1}(\tanh(a+bx))^n}{2x^2} - \frac{bn \coth^{-1}(\tanh(a+bx))^{n-1}}{2x}$$

```
[Out] -(b*n*ArcCoth[Tanh[a + b*x]]^(-1 + n))/(2*x) - ArcCoth[Tanh[a + b*x]]^n/(2*
x^2) + (b^2*n*ArcCoth[Tanh[a + b*x]]^(-1 + n)*Hypergeometric2F1[1, -1 + n,
n, -(ArcCoth[Tanh[a + b*x]]/(b*x - ArcCoth[Tanh[a + b*x]]))])/(2*(b*x - Arc
Coth[Tanh[a + b*x]]))
```

Rubi [A] time = 0.0692208, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 2164}

$$\frac{b^2 n \coth^{-1}(\tanh(a+bx))^{n-1} {}_2F_1\left(1, n-1; n; -\frac{\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)} - \frac{\coth^{-1}(\tanh(a+bx))^n}{2x^2} - \frac{bn \coth^{-1}(\tanh(a+bx))^{n-1}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^n/x^3,x]
```

```
[Out] -(b*n*ArcCoth[Tanh[a + b*x]]^(-1 + n))/(2*x) - ArcCoth[Tanh[a + b*x]]^n/(2*
x^2) + (b^2*n*ArcCoth[Tanh[a + b*x]]^(-1 + n)*Hypergeometric2F1[1, -1 + n,
n, -(ArcCoth[Tanh[a + b*x]]/(b*x - ArcCoth[Tanh[a + b*x]]))])/(2*(b*x - Arc
Coth[Tanh[a + b*x]]))
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2164

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)*Hypergeometric2F1[1, n + 1, n + 2, -((a*v)/(b*u
- a*v))])/((n + 1)*(b*u - a*v)), x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^3}\,dx = -\frac{\coth^{-1}(\tanh(a+bx))^n}{2x^2} + \frac{1}{2}(bn)\int \frac{\coth^{-1}(\tanh(a+bx))^{-1+n}}{x^2}\,dx$$

$$= -\frac{bn\coth^{-1}(\tanh(a+bx))^{-1+n}}{2x} - \frac{\coth^{-1}(\tanh(a+bx))^n}{2x^2} - \frac{1}{2}\left(b^2(1-n)n\right)\int \frac{\coth^{-1}}{}$$

$$= -\frac{bn\coth^{-1}(\tanh(a+bx))^{-1+n}}{2x} - \frac{\coth^{-1}(\tanh(a+bx))^n}{2x^2} + \frac{b^2n\coth^{-1}(\tanh(a+bx}{2\left(bx\right.}$$

Mathematica [A] time = 0.0411354, size = 67, normalized size = 0.66

$$\frac{\coth^{-1}(\tanh(a+bx))^n\left(\frac{\coth^{-1}(\tanh(a+bx))}{bx}\right)^{-n}\text{Hypergeometric2F1}\left(2-n,-n,3-n,1-\frac{\coth^{-1}(\tanh(a+bx))}{bx}\right)}{(n-2)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^n/x^3,x]
```

```
[Out] (ArcCoth[Tanh[a + b*x]]^n*Hypergeometric2F1[2 - n, -n, 3 - n, 1 - ArcCoth[T
anh[a + b*x]]/(b*x)])/((-2 + n)*x^2*(ArcCoth[Tanh[a + b*x]]/(b*x))^n)
```

Maple [F] time = 2.029, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{arccoth}(\tanh(bx+a)))^n}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^n/x^3,x)
```

```
[Out] int(arccoth(tanh(b*x+a))^n/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(\tanh(bx+a))^n}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^n/x^3,x, algorithm="maxima")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^n/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arcoth}(\tanh(bx+a))^n}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^n/x^3,x, algorithm="fricas")
```

```
[Out] integral(arccoth(tanh(b*x + a))^n/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^n(\tanh(a+bx))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**n/x**3,x)
```

```
[Out] Integral(acoth(tanh(a + b*x))**n/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(\tanh(bx+a))^n}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^n/x^3,x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^n/x^3, x)
```

3.193 $\int x^m \coth^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=37

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))}{m+1}-\frac{bx^{m+2}}{m^2+3m+2}$$

```
[Out] -((b*x^(2 + m))/(2 + 3*m + m^2)) + (x^(1 + m)*ArcCoth[Tanh[a + b*x]])/(1 +
m)
```

Rubi [A] time = 0.0100579, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 30}

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))}{m+1}-\frac{bx^{m+2}}{m^2+3m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] -((b*x^(2 + m))/(2 + 3*m + m^2)) + (x^(1 + m)*ArcCoth[Tanh[a + b*x]])/(1 +
m)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^m \coth^{-1}(\tanh(a+bx))\,dx &= \frac{x^{1+m}\coth^{-1}(\tanh(a+bx))}{1+m}-\frac{b\int x^{1+m}\,dx}{1+m}\\ &= -\frac{bx^{2+m}}{2+3m+m^2}+\frac{x^{1+m}\coth^{-1}(\tanh(a+bx))}{1+m}\end{aligned}$$

Mathematica [A] time = 0.033024, size = 34, normalized size = 0.92

$$x^m\left(\frac{x\left(\coth^{-1}(\tanh(a+bx))-bx\right)}{m+1}+\frac{bx^2}{m+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] x^m*((b*x^2)/(2 + m) + (x*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/(1 + m))
```

Maple [C] time = 0.159, size = 676, normalized size = 18.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arccoth(tanh(b*x+a)),x)
```

```
[Out] 1/(1+m)*x*x^m*ln(exp(b*x+a))-1/4*x*(4*b*x-I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*c
sgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2*m-4*I*Pi*csgn(I*exp(b*x+a))*csgn
(I*exp(2*b*x+2*a))^2-2*I*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*m+I
*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*m+I*Pi*csgn(I*exp(2*b*x+2*a
))^3*m+4*I*Pi+2*I*Pi*m+2*I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-2
*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2*m+2*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3*m-
I*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2*m+2
*I*Pi*csgn(I*exp(2*b*x+2*a))^3+I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1
))^3*m+2*I*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-4*I*Pi*csgn(I/(ex
p(2*b*x+2*a)+1))^2-2*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/
(exp(2*b*x+2*a)+1))^2+2*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*
a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+4*I*Pi*csgn(I/(exp(2*b*x+2*a)
+1))^3-2*I*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+
1))^2+I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b
*x+2*a)/(exp(2*b*x+2*a)+1))*m)/(1+m)/(2+m)*x^m
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63888, size = 72, normalized size = 1.95

$$\frac{\left((bm + b)x^2 + (am + 2\,a)x\right)x^m}{m^2 + 3\,m + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x^2 + (a*m + 2*a)*x)*x^m/(m^2 + 3*m + 2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*acoth(tanh(b*x+a)),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{arcoth}(\tanh(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^m*arccoth(tanh(b*x + a)), x)
```

3.194 $\int x^2 \coth^{-1}(\coth(a+bx))\,dx$

Optimal. Leaf size=23

$$\frac{1}{3}x^3\coth^{-1}(\coth(a+bx))-\frac{bx^4}{12}$$

```
[Out] -(b*x^4)/12 + (x^3*ArcCoth[Coth[a + b*x]])/3
```

Rubi [A] time = 0.014254, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 30}

$$\frac{1}{3}x^3\coth^{-1}(\coth(a+bx))-\frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[Coth[a + b*x]],x]

[Out] -(b*x^4)/12 + (x^3*ArcCoth[Coth[a + b*x]])/3
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\coth^{-1}(\coth(a+bx))\,dx &= \frac{1}{3}x^3\coth^{-1}(\coth(a+bx))-\frac{1}{3}b\int x^3\,dx\\ &= -\frac{bx^4}{12}+\frac{1}{3}x^3\coth^{-1}(\coth(a+bx))\end{aligned}$$

Mathematica [A] time = 0.0213134, size = 20, normalized size = 0.87

$$-\frac{1}{12}x^3\left(bx-4\coth^{-1}(\coth(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[Coth[a + b*x]],x]

[Out] -(x^3*(b*x - 4*ArcCoth[Coth[a + b*x]]))/12
```

Maple [B] time = 0.092, size = 59, normalized size = 2.6

$$\frac{x^3 \operatorname{arccoth}(\coth(bx+a))}{3} + \frac{1}{3b^3}\left(-\frac{(bx+a)^4}{4} + (bx+a)^3 a - \frac{3a^2(bx+a)^2}{2} + (bx+a)a^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(coth(b*x+a)),x)
```

```
[Out] 1/3*x^3*arccoth(coth(b*x+a))+1/3/b^3*(-1/4*(b*x+a)^4+(b*x+a)^3*a-3/2*a^2*(b
*x+a)^2+(b*x+a)*a^3)
```

Maxima [A] time = 0.945767, size = 18, normalized size = 0.78

$$\frac{1}{4}bx^4 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

Fricas [A] time = 1.34393, size = 31, normalized size = 1.35

$$\frac{1}{4}x^4 b + \frac{1}{3}x^3 a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*b + 1/3*x^3*a
```

Sympy [A] time = 91.8494, size = 39, normalized size = 1.7

$$\begin{cases} 0 & \text{for}\: a = \log\left(-e^{-bx}\right) \vee a = \log\left(e^{-bx}\right) \\ -\frac{bx^4}{12} + \frac{x^3 \operatorname{acoth}\left(\frac{1}{\tanh(a+bx)}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(coth(b*x+a)),x)
```

```
[Out] Piecewise((0, Eq(a, log(exp(-b*x))) | Eq(a, log(-exp(-b*x)))), (-b*x**4/12
+ x**3*acoth(1/tanh(a + b*x))/3, True))
```

Giac [A] time = 1.17349, size = 18, normalized size = 0.78

$$\frac{1}{4}bx^4 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

3.195 $\int x \coth^{-1}(\coth(a + bx))\, dx$

Optimal. Leaf size=23

$$\frac{1}{2}x^2 \coth^{-1}(\coth(a + bx)) - \frac{bx^3}{6}$$

```
[Out] -(b*x^3)/6 + (x^2*ArcCoth[Coth[a + b*x]])/2
```

Rubi [A] time = 0.0077072, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6242, 30}

$$\frac{1}{2}x^2 \coth^{-1}(\coth(a + bx)) - \frac{bx^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[Coth[a + b*x]],x]
```

```
[Out] -(b*x^3)/6 + (x^2*ArcCoth[Coth[a + b*x]])/2
```

Rule 6242

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Coth[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int x \coth^{-1}(\coth(a + bx))\, dx &= \frac{1}{2}x^2 \coth^{-1}(\coth(a + bx)) - \frac{1}{2}b \int x^2\, dx \\ &= -\frac{bx^3}{6} + \frac{1}{2}x^2 \coth^{-1}(\coth(a + bx)) \end{aligned}$$

Mathematica [A] time = 0.015436, size = 20, normalized size = 0.87

$$-\frac{1}{6}x^2 \left(bx - 3\coth^{-1}(\coth(a + bx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[Coth[a + b*x]],x]
```

```
[Out] -(x^2*(b*x - 3*ArcCoth[Coth[a + b*x]]))/6
```

Maple [B] time = 0.075, size = 48, normalized size = 2.1

$$\frac{x^2 \operatorname{arccoth}(\coth(bx+a))}{2} + \frac{1}{2b^2}\left(-\frac{(bx+a)^3}{3} + (bx+a)^2 a - a^2 (bx+a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(coth(b*x+a)),x)
```

```
[Out] 1/2*x^2*arccoth(coth(b*x+a))+1/2/b^2*(-1/3*(b*x+a)^3+(b*x+a)^2*a-a^2*(b*x+a
))
```

Maxima [A] time = 0.933736, size = 18, normalized size = 0.78

$$\frac{1}{3}bx^3 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

Fricas [A] time = 1.26847, size = 31, normalized size = 1.35

$$\frac{1}{3}x^3b + \frac{1}{2}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*b + 1/2*x^2*a
```

Sympy [A] time = 37.9831, size = 60, normalized size = 2.61

$$\begin{cases} 0 & \text{for } a = \log\left(-e^{-bx}\right) \vee a = \log\left(e^{-bx}\right) \\ \frac{x^2 \operatorname{acoth}(\coth(a))}{2} & \text{for } b = 0 \\ \frac{x \operatorname{acoth}^2\left(\frac{1}{\tanh(a+bx)}\right)}{2b} - \frac{\operatorname{acoth}^3\left(\frac{1}{\tanh(a+bx)}\right)}{6b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(coth(b*x+a)),x)
```

```
[Out] Piecewise((0, Eq(a, log(exp(-b*x))) | Eq(a, log(-exp(-b*x)))), (x**2*acoth(
coth(a))/2, Eq(b, 0)), (x*acoth(1/tanh(a + b*x))**2/(2*b) - acoth(1/tanh(a
+ b*x))**3/(6*b**2), True))
```

Giac [A] time = 1.13859, size = 18, normalized size = 0.78

$$\frac{1}{3}\,bx^3 + \frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

3.196 $\int \coth^{-1}(\coth(a+bx))\,dx$

Optimal. Leaf size=16

$$\frac{\coth^{-1}(\coth(a+bx))^2}{2b}$$

```
[Out] ArcCoth[Coth[a + b*x]]^2/(2*b)
```

Rubi [A] time = 0.0031558, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2157, 30}

$$\frac{\coth^{-1}(\coth(a+bx))^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Coth[a + b*x]],x]

[Out] ArcCoth[Coth[a + b*x]]^2/(2*b)
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \coth^{-1}(\coth(a+bx))\,dx &= \frac{\text{Subst}\left(\int x\,dx, x, \coth^{-1}(\coth(a+bx))\right)}{b}\\ &= \frac{\coth^{-1}(\coth(a+bx))^2}{2b}\end{aligned}$$

Mathematica [A] time = 0.0059037, size = 18, normalized size = 1.12

$$x\coth^{-1}(\coth(a+bx))-\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Coth[a + b*x]],x]

[Out] -(b*x^2)/2 + x*ArcCoth[Coth[a + b*x]]
```

Maple [B] time = 0.064, size = 32, normalized size = 2.

$$\frac{1}{b}\left(\operatorname{Artanh}\left(\coth\left(bx+a\right)\right)\operatorname{arccoth}\left(\coth\left(bx+a\right)\right)-\frac{\left(\operatorname{Artanh}\left(\coth\left(bx+a\right)\right)\right)^{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(coth(b*x+a)),x)
```

```
[Out] 1/b*(arctanh(coth(b*x+a))*arccoth(coth(b*x+a))-1/2*arctanh(coth(b*x+a))^2)
```

Maxima [A] time = 0.934248, size = 14, normalized size = 0.88

$$\frac{1}{2}\,bx^{2}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*x
```

Fricas [A] time = 1.28358, size = 23, normalized size = 1.44

$$\frac{1}{2}x^{2}b+xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*b + x*a
```

Sympy [A] time = 3.98976, size = 36, normalized size = 2.25

$$\begin{cases} 0 & \text{for}\ a=\log\left(-e^{-bx}\right)\vee a=\log\left(e^{-bx}\right) \\ -\frac{bx^{2}}{2}+x\operatorname{acoth}\left(\frac{1}{\tanh\left(a+bx\right)}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(coth(b*x+a)),x)
```

```
[Out] Piecewise((0, Eq(a, log(exp(-b*x))) | Eq(a, log(-exp(-b*x)))), (-b*x**2/2 +
 x*acoth(1/tanh(a + b*x)), True))
```

Giac [A] time = 1.12394, size = 14, normalized size = 0.88

$$\frac{1}{2}\,bx^{2}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x
```

3.197 $\int \frac{\coth^{-1}(\coth(a+bx))}{x} \, dx$

Optimal. Leaf size=21

$$bx - \log(x)\left(bx - \coth^{-1}(\coth(a+bx))\right)$$

```
[Out] b*x - (b*x - ArcCoth[Coth[a + b*x]])*Log[x]
```

Rubi [A] time = 0.0309233, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2158, 29}

$$bx - \log(x)\left(bx - \coth^{-1}(\coth(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Coth[a + b*x]]/x,x]
```

```
[Out] b*x - (b*x - ArcCoth[Coth[a + b*x]])*Log[x]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(\coth(a+bx))}{x} \, dx &= bx - \left(bx - \coth^{-1}(\coth(a+bx))\right) \int \frac{1}{x} \, dx \\ &= bx - \left(bx - \coth^{-1}(\coth(a+bx))\right) \log(x)\end{aligned}$$

Mathematica [A] time = 0.0166107, size = 19, normalized size = 0.9

$$\log(x)\left(\coth^{-1}(\coth(a+bx)) - bx\right) + bx$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Coth[a + b*x]]/x,x]
```

```
[Out] b*x + (-(b*x) + ArcCoth[Coth[a + b*x]])*Log[x]
```

Maple [A] time = 0.073, size = 27, normalized size = 1.3

$$bx + a\ln(x) + \ln(x)(\operatorname{arccoth}(\coth(bx+a)) - bx - a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(coth(b*x+a))/x,x)
```

```
[Out] b*x+a*ln(x)+ln(x)*(arccoth(coth(b*x+a))-b*x-a)
```

Maxima [A] time = 0.944969, size = 11, normalized size = 0.52

$$bx + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(coth(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] b*x + a*log(x)
```

Fricas [A] time = 1.48964, size = 22, normalized size = 1.05

$$bx + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(coth(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] b*x + a*log(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}(\coth(a + bx))}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(coth(b*x+a))/x,x)
```

```
[Out] Integral(acoth(coth(a + b*x))/x, x)
```

Giac [A] time = 1.13719, size = 12, normalized size = 0.57

$$bx + a\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(coth(b*x+a))/x,x, algorithm="giac")
```

```
[Out] b*x + a*log(abs(x))
```

3.198 $\int \frac{\coth^{-1}(\coth(a+bx))}{x^2}\,dx$

Optimal. Leaf size=17

$$b\log(x)-\frac{\coth^{-1}(\coth(a+bx))}{x}$$

```
[Out] -(ArcCoth[Coth[a + b*x]]/x) + b*Log[x]
```

Rubi [A] time = 0.008717, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 29}

$$b\log(x)-\frac{\coth^{-1}(\coth(a+bx))}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Coth[a + b*x]]/x^2,x]

[Out] -(ArcCoth[Coth[a + b*x]]/x) + b*Log[x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(\coth(a+bx))}{x^2}\,dx &= -\frac{\coth^{-1}(\coth(a+bx))}{x}+b\int \frac{1}{x}\,dx\\ &= -\frac{\coth^{-1}(\coth(a+bx))}{x}+b\log(x)\end{aligned}$$

Mathematica [A] time = 0.0160234, size = 18, normalized size = 1.06

$$-\frac{\coth^{-1}(\coth(a+bx))}{x}+b\log(x)+b$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Coth[a + b*x]]/x^2,x]
```

```
[Out] b - ArcCoth[Coth[a + b*x]]/x + b*Log[x]
```

Maple [A] time = 0.074, size = 20, normalized size = 1.2

$$-\frac{\operatorname{arccoth}(\coth(bx+a))}{x}+b\ln(bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(coth(b*x+a))/x^2,x)
```

```
[Out] -arccoth(coth(b*x+a))/x+b*ln(b*x)
```

Maxima [A] time = 0.941743, size = 15, normalized size = 0.88

$$b\log(x)-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(coth(b*x+a))/x^2,x, algorithm="maxima")
```

```
[Out] b*log(x) - a/x
```

Fricas [A] time = 1.61768, size = 27, normalized size = 1.59

$$\frac{bx\log(x)-a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(coth(b*x+a))/x^2,x, algorithm="fricas")
```

```
[Out] (b*x*log(x) - a)/x
```

Sympy [A] time = 12.1868, size = 34, normalized size = 2.

$$\begin{cases} 0 & \text{for}\: a = \log\left(-e^{-bx}\right) \vee a = \log\left(e^{-bx}\right) \\ b\log(x) - \frac{\operatorname{acoth}\left(\frac{1}{\tanh(a+bx)}\right)}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(coth(b*x+a))/x**2,x)
```

```
[Out] Piecewise((0, Eq(a, log(exp(-b*x))) | Eq(a, log(-exp(-b*x)))), (b*log(x) -
acoth(1/tanh(a + b*x))/x, True))
```

Giac [A] time = 1.16178, size = 16, normalized size = 0.94

$$b\log(|x|) - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(coth(b*x+a))/x^2,x, algorithm="giac")
```

```
[Out] b*log(abs(x)) - a/x
```

3.199 $\int \frac{\coth^{-1}(\coth(a+bx))}{x^3} \, dx$

Optimal. Leaf size=23

$$-\frac{\coth^{-1}(\coth(a+bx))}{2x^2}-\frac{b}{2x}$$

```
[Out] -b/(2*x) - ArcCoth[Coth[a + b*x]]/(2*x^2)
```

Rubi [A] time = 0.009241, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 30}

$$-\frac{\coth^{-1}(\coth(a+bx))}{2x^2}-\frac{b}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Coth[a + b*x]]/x^3,x]
```

```
[Out] -b/(2*x) - ArcCoth[Coth[a + b*x]]/(2*x^2)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(\coth(a+bx))}{x^3}\,dx &= -\frac{\coth^{-1}(\coth(a+bx))}{2x^2}+\frac{1}{2}b\int \frac{1}{x^2}\,dx\\ &= -\frac{b}{2x}-\frac{\coth^{-1}(\coth(a+bx))}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.0141821, size = 18, normalized size = 0.78

$$-\frac{\coth^{-1}(\coth(a+bx))+bx}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Coth[a + b*x]]/x^3,x]
```

```
[Out] -(b*x + ArcCoth[Coth[a + b*x]])/(2*x^2)
```

Maple [A] time = 0.076, size = 20, normalized size = 0.9

$$-\frac{b}{2x}-\frac{\operatorname{arccoth}\left(\coth\left(bx+a\right)\right)}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(coth(b*x+a))/x^3,x)
```

```
[Out] -1/2*b/x-1/2*arccoth(coth(b*x+a))/x^2
```

Maxima [A] time = 0.944455, size = 15, normalized size = 0.65

$$-\frac{2\,bx+a}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(coth(b*x+a))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x + a)/x^2
```

Fricas [A] time = 1.47088, size = 30, normalized size = 1.3

$$-\frac{2\,bx+a}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(coth(b*x+a))/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x + a)/x^2
```

Sympy [A] time = 77.2892, size = 39, normalized size = 1.7

$$\begin{cases} 0 & \text{for}\: a = \log{\left(-e^{-bx} \right)} \vee a = \log{\left(e^{-bx} \right)} \\ -\frac{b}{2x} - \frac{\operatorname{acoth}{\left(\frac{1}{\tanh{\left(a+bx \right)}} \right)}}{2x^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(coth(b*x+a))/x**3,x)
```

```
[Out] Piecewise((0, Eq(a, log(exp(-b*x))) | Eq(a, log(-exp(-b*x)))), (-b/(2*x) -
acoth(1/tanh(a + b*x))/(2*x**2), True))
```

Giac [A] time = 1.1404, size = 15, normalized size = 0.65

$$-\frac{2\,bx + a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(coth(b*x+a))/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x + a)/x^2
```

3.200 $\int \coth^{-1}(\cosh(x))\,dx$

Optimal. Leaf size=27

$$-\text{PolyLog}(2,-e^x)+\text{PolyLog}(2,e^x)-2x\tanh^{-1}(e^x)+x\coth^{-1}(\cosh(x))$$

```
[Out] x*ArcCoth[Cosh[x]] - 2*x*ArcTanh[E^x] - PolyLog[2, -E^x] + PolyLog[2, E^x]
```

Rubi [A] time = 0.0342236, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {6272, 4182, 2279, 2391}

$$-\text{PolyLog}(2,-e^x)+\text{PolyLog}(2,e^x)-2x\tanh^{-1}(e^x)+x\coth^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Cosh[x]],x]
```

```
[Out] x*ArcCoth[Cosh[x]] - 2*x*ArcTanh[E^x] - PolyLog[2, -E^x] + PolyLog[2, E^x]
```

Rule 6272

```
Int[ArcCoth[u_], x_Symbol] :> Simp[x*ArcCoth[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \coth^{-1}(\cosh(x))\,dx &= x\coth^{-1}(\cosh(x))+\int x\text{csch}(x)\,dx\\
&= x\coth^{-1}(\cosh(x))-2x\tanh^{-1}(e^x)-\int \log(1-e^x)\,dx+\int \log(1+e^x)\,dx\\
&= x\coth^{-1}(\cosh(x))-2x\tanh^{-1}(e^x)-\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^x\right)+\text{Subst}\left(\int \frac{\log(1+x)}{x}\right.\\
&= x\coth^{-1}(\cosh(x))-2x\tanh^{-1}(e^x)-\text{Li}_2(-e^x)+\text{Li}_2(e^x)
\end{aligned}$$

Mathematica [A] time = 0.0196667, size = 47, normalized size = 1.74

$$\text{PolyLog}(2,-e^{-x})-\text{PolyLog}(2,e^{-x})+x(\log(1-e^{-x})-\log(e^{-x}+1))+x\coth^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Cosh[x]],x]
```

```
[Out] x*ArcCoth[Cosh[x]] + x*(Log[1 - E^(-x)] - Log[1 + E^(-x)]) + PolyLog[2, -E^
(-x)] - PolyLog[2, E^(-x)]
```

Maple [A] time = 0.052, size = 21, normalized size = 0.8

$$x\text{arccoth}(\cosh(x))+2\,\text{dilog}(\mathrm{e}^{-x})-\frac{\text{dilog}\left(\mathrm{e}^{-2x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(cosh(x)),x)
```

```
[Out] x*arccoth(cosh(x))+2*dilog(exp(-x))-1/2*dilog(exp(-2*x))
```

Maxima [A] time = 1.15314, size = 45, normalized size = 1.67

$$x\,\text{arcoth}(\cosh(x))-x\log(e^x+1)+x\log(-e^x+1)-\text{Li}_2(-e^x)+\text{Li}_2(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(cosh(x)),x, algorithm="maxima")
```

```
[Out] x*arccoth(cosh(x)) - x*log(e^x + 1) + x*log(-e^x + 1) - dilog(-e^x) + dilog
(e^x)
```

Fricas [B] time = 1.66712, size = 213, normalized size = 7.89

$$\frac{1}{2}x\log\left(\frac{\cosh(x)+1}{\cosh(x)-1}\right)-x\log(\cosh(x)+\sinh(x)+1)+x\log(-\cosh(x)-\sinh(x)+1)+\text{Li}_2(\cosh(x)+\sinh(x))-\text{L}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x*log((cosh(x) + 1)/(cosh(x) - 1)) - x*log(cosh(x) + sinh(x) + 1) + x*l
og(-cosh(x) - sinh(x) + 1) + dilog(cosh(x) + sinh(x)) - dilog(-cosh(x) - si
nh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \text{acoth}(\cosh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(cosh(x)),x)
```

```
[Out] Integral(acoth(cosh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}(\cosh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(cosh(x)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(cosh(x)), x)
```

3.201 $\int x \coth^{-1}(\cosh(x))\,dx$

Optimal. Leaf size=51

$$-x\text{PolyLog}(2,-e^x)+x\text{PolyLog}(2,e^x)+\text{PolyLog}(3,-e^x)-\text{PolyLog}(3,e^x)-x^2\tanh^{-1}(e^x)+\frac{1}{2}x^2\coth^{-1}(\cosh(x))$$

```
[Out] (x^2*ArcCoth[Cosh[x]])/2 - x^2*ArcTanh[E^x] - x*PolyLog[2, -E^x] + x*PolyLo
g[2, E^x] + PolyLog[3, -E^x] - PolyLog[3, E^x]
```

Rubi [A] time = 0.0664171, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6274, 4182, 2531, 2282, 6589}

$$-x\text{PolyLog}(2,-e^x)+x\text{PolyLog}(2,e^x)+\text{PolyLog}(3,-e^x)-\text{PolyLog}(3,e^x)-x^2\tanh^{-1}(e^x)+\frac{1}{2}x^2\coth^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[Cosh[x]],x]
```

```
[Out] (x^2*ArcCoth[Cosh[x]])/2 - x^2*ArcTanh[E^x] - x*PolyLog[2, -E^x] + x*PolyLo
g[2, E^x] + PolyLog[3, -E^x] - PolyLog[3, E^x]
```

Rule 6274

```
Int[((a_.) + ArcCoth[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcCoth[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x\coth^{-1}(\cosh(x))\,dx &= \frac{1}{2}x^2\coth^{-1}(\cosh(x))+\frac{1}{2}\int x^2\mathrm{csch}(x)\,dx\\
&= \frac{1}{2}x^2\coth^{-1}(\cosh(x))-x^2\tanh^{-1}(e^x)-\int x\log(1-e^x)\,dx+\int x\log(1+e^x)\,dx\\
&= \frac{1}{2}x^2\coth^{-1}(\cosh(x))-x^2\tanh^{-1}(e^x)-x\mathrm{Li}_2(-e^x)+x\mathrm{Li}_2(e^x)+\int \mathrm{Li}_2(-e^x)\,dx-\int \mathrm{Li}_2(e
\\
&= \frac{1}{2}x^2\coth^{-1}(\cosh(x))-x^2\tanh^{-1}(e^x)-x\mathrm{Li}_2(-e^x)+x\mathrm{Li}_2(e^x)+\mathrm{Subst}\left(\int\frac{\mathrm{Li}_2(-x)}{x}\,dx,x,e
\\
&= \frac{1}{2}x^2\coth^{-1}(\cosh(x))-x^2\tanh^{-1}(e^x)-x\mathrm{Li}_2(-e^x)+x\mathrm{Li}_2(e^x)+\mathrm{Li}_3(-e^x)-\mathrm{Li}_3(e^x)
\end{aligned}$$

Mathematica [A] time = 0.0135662, size = 81, normalized size = 1.59

$$\frac{1}{2}\left(2x\mathrm{PolyLog}(2,-e^{-x})-2x\mathrm{PolyLog}(2,e^{-x})+2\mathrm{PolyLog}(3,-e^{-x})-2\mathrm{PolyLog}(3,e^{-x})+x^2\log(1-e^{-x})-x^2\log(e$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[Cosh[x]],x]
```

```
[Out] (x^2*ArcCoth[Cosh[x]] + x^2*Log[1 - E^(-x)] - x^2*Log[1 + E^(-x)] + 2*x*Pol
yLog[2, -E^(-x)] - 2*x*PolyLog[2, E^(-x)] + 2*PolyLog[3, -E^(-x)] - 2*PolyL
og[3, E^(-x)])/2
```

Maple [C] time = 0.154, size = 449, normalized size = 8.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(cosh(x)),x)
```

```
[Out] x*polylog(2,exp(x))-x*polylog(2,-exp(x))+1/8*I*Pi*csgn(I*(exp(x)-1)^2)^3*x^
2-1/8*I*Pi*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-1)^2)^2*x^2+1/8*I*Pi*csgn
(I*(exp(x)-1)^2)*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-1)^2)*x^2-1/8*I*Pi*
csgn(I*(exp(x)-1)^2)*csgn(I*exp(-x)*(exp(x)-1)^2)^2*x^2-1/8*I*Pi*csgn(I*(ex
p(x)+1)^2)*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)+1)^2)*x^2+1/8*I*Pi*csgn(I
*exp(-x)*(exp(x)-1)^2)^3*x^2+1/8*I*Pi*csgn(I*(exp(x)+1)^2)*csgn(I*exp(-x)*(
exp(x)+1)^2)^2*x^2+1/4*I*Pi*csgn(I*(exp(x)+1))*csgn(I*(exp(x)+1)^2)^2*x^2-1
/8*I*Pi*csgn(I*(exp(x)+1))^2*csgn(I*(exp(x)+1)^2)*x^2-1/8*I*Pi*csgn(I*exp(-
x)*(exp(x)+1)^2)^3*x^2+1/8*I*Pi*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)+1)^2
)^2*x^2+polylog(3,-exp(x))-polylog(3,exp(x))-1/8*I*Pi*csgn(I*(exp(x)+1)^2)^
3*x^2+1/2*x^2*ln(1-exp(x))-1/2*x^2*ln(exp(x)-1)-1/4*I*Pi*csgn(I*(exp(x)-1))
*csgn(I*(exp(x)-1)^2)^2*x^2+1/8*I*Pi*csgn(I*(exp(x)-1))^2*csgn(I*(exp(x)-1)
^2)*x^2
```

Maxima [A] time = 1.18363, size = 76, normalized size = 1.49

$$\frac{1}{2}x^2\operatorname{arcoth}(\cosh(x)) - \frac{1}{2}x^2\log(e^x+1) + \frac{1}{2}x^2\log(-e^x+1) - x\operatorname{Li}_2(-e^x) + x\operatorname{Li}_2(e^x) + \operatorname{Li}_3(-e^x) - \operatorname{Li}_3(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccoth(cosh(x)) - 1/2*x^2*log(e^x + 1) + 1/2*x^2*log(-e^x + 1) - x
*dilog(-e^x) + x*dilog(e^x) + polylog(3, -e^x) - polylog(3, e^x)
```

Fricas [C] time = 1.61809, size = 325, normalized size = 6.37

$$\frac{1}{4}x^2\log\left(\frac{\cosh(x)+1}{\cosh(x)-1}\right) - \frac{1}{2}x^2\log(\cosh(x)+\sinh(x)+1) + \frac{1}{2}x^2\log(-\cosh(x)-\sinh(x)+1) + x\operatorname{Li}_2(\cosh(x)+\sinh$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/4*x^2*log((cosh(x) + 1)/(cosh(x) - 1)) - 1/2*x^2*log(cosh(x) + sinh(x) +
1) + 1/2*x^2*log(-cosh(x) - sinh(x) + 1) + x*dilog(cosh(x) + sinh(x)) - x*d
ilog(-cosh(x) - sinh(x)) - polylog(3, cosh(x) + sinh(x)) + polylog(3, -cosh
(x) - sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\operatorname{acoth}(\cosh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(cosh(x)),x)
```

```
[Out] Integral(x*acoth(cosh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\operatorname{arcoth}(\cosh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(cosh(x)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(cosh(x)), x)
```

3.202 $\int x^2 \coth^{-1}(\cosh(x))\,dx$

Optimal. Leaf size=77

$-x^2\text{PolyLog}(2,-e^x)+x^2\text{PolyLog}(2,e^x)+2x\text{PolyLog}(3,-e^x)-2x\text{PolyLog}(3,e^x)-2\text{PolyLog}(4,-e^x)+2\text{PolyLo}$

```
[Out] (x^3*ArcCoth[Cosh[x]])/3 - (2*x^3*ArcTanh[E^x])/3 - x^2*PolyLog[2, -E^x] +
x^2*PolyLog[2, E^x] + 2*x*PolyLog[3, -E^x] - 2*x*PolyLog[3, E^x] - 2*PolyLo
g[4, -E^x] + 2*PolyLog[4, E^x]
```

Rubi [A] time = 0.0900436, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6274, 4182, 2531, 6609, 2282, 6589}

$-x^2\text{PolyLog}(2,-e^x)+x^2\text{PolyLog}(2,e^x)+2x\text{PolyLog}(3,-e^x)-2x\text{PolyLog}(3,e^x)-2\text{PolyLog}(4,-e^x)+2\text{PolyLo}$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[Cosh[x]],x]
```

```
[Out] (x^3*ArcCoth[Cosh[x]])/3 - (2*x^3*ArcTanh[E^x])/3 - x^2*PolyLog[2, -E^x] +
x^2*PolyLog[2, E^x] + 2*x*PolyLog[3, -E^x] - 2*x*PolyLog[3, E^x] - 2*PolyLo
g[4, -E^x] + 2*PolyLog[4, E^x]
```

Rule 6274

```
Int[((a_.) + ArcCoth[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcCoth[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
```

```
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(\cosh(x))\,dx &= \frac{1}{3}x^3 \coth^{-1}(\cosh(x)) + \frac{1}{3}\int x^3 \operatorname{csch}(x)\,dx \\
&= \frac{1}{3}x^3 \coth^{-1}(\cosh(x)) - \frac{2}{3}x^3 \tanh^{-1}(e^x) - \int x^2 \log(1-e^x)\,dx + \int x^2 \log(1+e^x)\,dx \\
&= \frac{1}{3}x^3 \coth^{-1}(\cosh(x)) - \frac{2}{3}x^3 \tanh^{-1}(e^x) - x^2\mathrm{Li}_2(-e^x) + x^2\mathrm{Li}_2(e^x) + 2\int x\mathrm{Li}_2(-e^x)\,dx - 2\int \\
&= \frac{1}{3}x^3 \coth^{-1}(\cosh(x)) - \frac{2}{3}x^3 \tanh^{-1}(e^x) - x^2\mathrm{Li}_2(-e^x) + x^2\mathrm{Li}_2(e^x) + 2x\mathrm{Li}_3(-e^x) - 2x\mathrm{Li}_3(e^x) \\
&= \frac{1}{3}x^3 \coth^{-1}(\cosh(x)) - \frac{2}{3}x^3 \tanh^{-1}(e^x) - x^2\mathrm{Li}_2(-e^x) + x^2\mathrm{Li}_2(e^x) + 2x\mathrm{Li}_3(-e^x) - 2x\mathrm{Li}_3(e^x) \\
&= \frac{1}{3}x^3 \coth^{-1}(\cosh(x)) - \frac{2}{3}x^3 \tanh^{-1}(e^x) - x^2\mathrm{Li}_2(-e^x) + x^2\mathrm{Li}_2(e^x) + 2x\mathrm{Li}_3(-e^x) - 2x\mathrm{Li}_3(e^x)
\end{aligned}$$

Mathematica [A] time = 0.0286605, size = 109, normalized size = 1.42

$$\frac{1}{24}\left(24x^2\mathrm{PolyLog}(2,-e^{-x}) + 24x^2\mathrm{PolyLog}(2,e^x) + 48x\mathrm{PolyLog}(3,-e^{-x}) - 48x\mathrm{PolyLog}(3,e^x) + 48\mathrm{PolyLog}(4,-e^{-x})\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[Cosh[x]],x]
```

```
[Out] (Pi^4 - 2*x^4 + 8*x^3*ArcCoth[Cosh[x]] - 8*x^3*Log[1 + E^(-x)] + 8*x^3*Log[
1 - E^x] + 24*x^2*PolyLog[2, -E^(-x)] + 24*x^2*PolyLog[2, E^x] + 48*x*PolyL
og[3, -E^(-x)] - 48*x*PolyLog[3, E^x] + 48*PolyLog[4, -E^(-x)] + 48*PolyLog
[4, E^x])/24
```

Maple [C] time = 0.141, size = 471, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(cosh(x)),x)
```

```
[Out] x^2*polylog(2,exp(x))-x^2*polylog(2,-exp(x))-2*polylog(4,-exp(x))+2*polylog
(4,exp(x))-1/12*I*Pi*csgn(I*(exp(x)-1)^2)*csgn(I*exp(-x)*(exp(x)-1)^2)^2*x^
```

```
3+1/12*I*Pi*csgn(I*(exp(x)-1))^2*csgn(I*(exp(x)-1)^2)*x^3-1/12*I*Pi*csgn(I*
(exp(x)+1))^2*csgn(I*(exp(x)+1)^2)*x^3+1/6*I*Pi*csgn(I*(exp(x)+1))*csgn(I*(
exp(x)+1)^2)^2*x^3+1/12*I*Pi*csgn(I*(exp(x)-1)^2)*csgn(I*exp(-x))*csgn(I*ex
p(-x)*(exp(x)-1)^2)*x^3+1/3*x^3*ln(1-exp(x))-1/3*x^3*ln(exp(x)-1)+2*x*polyl
og(3,-exp(x))-2*x*polylog(3,exp(x))+1/12*I*Pi*csgn(I*(exp(x)-1)^2)^3*x^3-1/
12*I*Pi*csgn(I*exp(-x)*(exp(x)+1)^2)^3*x^3-1/6*I*Pi*csgn(I*(exp(x)-1))*csgn
(I*(exp(x)-1)^2)^2*x^3+1/12*I*Pi*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)+1)^
2)^2*x^3-1/12*I*Pi*csgn(I*(exp(x)+1)^2)^3*x^3-1/12*I*Pi*csgn(I*(exp(x)+1)^2
)*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)+1)^2)*x^3+1/12*I*Pi*csgn(I*exp(-x)
*(exp(x)-1)^2)^3*x^3+1/12*I*Pi*csgn(I*(exp(x)+1)^2)*csgn(I*exp(-x)*(exp(x)+
1)^2)^2*x^3-1/12*I*Pi*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-1)^2)^2*x^3
```

Maxima [A] time = 1.17831, size = 105, normalized size = 1.36

$$\frac{1}{3}x^3 \operatorname{arcoth}(\cosh(x)) - \frac{1}{3}x^3 \log(e^x + 1) + \frac{1}{3}x^3 \log(-e^x + 1) - x^2 \operatorname{Li}_2(-e^x) + x^2 \operatorname{Li}_2(e^x) + 2x\operatorname{Li}_3(-e^x) - 2x\operatorname{Li}_3(e^x) - $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(cosh(x)) - 1/3*x^3*log(e^x + 1) + 1/3*x^3*log(-e^x + 1) - x
^2*dilog(-e^x) + x^2*dilog(e^x) + 2*x*polylog(3, -e^x) - 2*x*polylog(3, e^x
) - 2*polylog(4, -e^x) + 2*polylog(4, e^x)
```

Fricas [C] time = 1.77078, size = 435, normalized size = 5.65

$$\frac{1}{6}x^3 \log\left(\frac{\cosh(x)+1}{\cosh(x)-1}\right) - \frac{1}{3}x^3 \log(\cosh(x) + \sinh(x) + 1) + \frac{1}{3}x^3 \log(-\cosh(x) - \sinh(x) + 1) + x^2 \operatorname{Li}_2(\cosh(x) + $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/6*x^3*log((cosh(x) + 1)/(cosh(x) - 1)) - 1/3*x^3*log(cosh(x) + sinh(x) +
1) + 1/3*x^3*log(-cosh(x) - sinh(x) + 1) + x^2*dilog(cosh(x) + sinh(x)) - x
^2*dilog(-cosh(x) - sinh(x)) - 2*x*polylog(3, cosh(x) + sinh(x)) + 2*x*poly
log(3, -cosh(x) - sinh(x)) + 2*polylog(4, cosh(x) + sinh(x)) - 2*polylog(4,
 -cosh(x) - sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{acoth}(\cosh(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(cosh(x)),x)
```

```
[Out] Integral(x**2*acoth(cosh(x)), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arcoth}(\cosh(x)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(cosh(x)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(cosh(x)), x)
```

3.203 $\int x^2 \coth^{-1}(c + d \tanh(a + bx))\, dx$

Optimal. Leaf size=307

$$-\frac{x\text{PolyLog}\left(3,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2}+\frac{x\text{PolyLog}\left(3,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2}+\frac{\text{PolyLog}\left(4,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^3}-\frac{\text{PolyLog}\left(4,-\frac{(c+d}{}\right.}{8b^3}$$

```
[Out] (x^3*ArcCoth[c + d*Tanh[a + b*x]])/3 + (x^3*Log[1 + ((1 - c - d)*E^(2*a + 2
*b*x))/(1 - c + d)])/6 - (x^3*Log[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c
- d)])/6 + (x^2*PolyLog[2, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))])/(
4*b) - (x^2*PolyLog[2, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))])/(4*b)
 - (x*PolyLog[3, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))])/(4*b^2) + (
x*PolyLog[3, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))])/(4*b^2) + PolyL
og[4, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))]/(8*b^3) - PolyLog[4, -(
((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))]/(8*b^3)
```

Rubi [A] time = 0.464891, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6244, 2190, 2531, 6609, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(3,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2}+\frac{x\text{PolyLog}\left(3,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2}+\frac{\text{PolyLog}\left(4,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^3}-\frac{\text{PolyLog}\left(4,-\frac{(c+d}{}\right.}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[c + d*Tanh[a + b*x]],x]
```


```
[Out] (x^3*ArcCoth[c + d*Tanh[a + b*x]])/3 + (x^3*Log[1 + ((1 - c - d)*E^(2*a + 2
*b*x))/(1 - c + d)])/6 - (x^3*Log[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c
- d)])/6 + (x^2*PolyLog[2, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))])/(
4*b) - (x^2*PolyLog[2, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))])/(4*b)
 - (x*PolyLog[3, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))])/(4*b^2) + (
x*PolyLog[3, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))])/(4*b^2) + PolyL
og[4, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))]/(8*b^3) - PolyLog[4, -(
((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))]/(8*b^3)
```


Rule 6244

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Tanh[a + b*x]])/(f*
(m + 1)), x] + (Dist[(b*(1 - c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^
(2*a + 2*b*x))/(1 - c + d + (1 - c - d)*E^(2*a + 2*b*x)), x], x] - Dist[(b*
(1 + c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*a + 2*b*x))/(1 + c -
d + (1 + c + d)*E^(2*a + 2*b*x)), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &
& IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\coth^{-1}(c+d\tanh(a+bx))\,dx=\frac{1}{3}x^3\coth^{-1}(c+d\tanh(a+bx))+\frac{1}{3}(b(1-c-d))\int\frac{e^{2a+2bx}x^3}{1-c+d+(1-c-d)e^{2a+2bx}}$$

$$=\frac{1}{3}x^3\coth^{-1}(c+d\tanh(a+bx))+\frac{1}{6}x^3\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-\frac{1}{6}x^3\log\left(1+\right.$$

$$=\frac{1}{3}x^3\coth^{-1}(c+d\tanh(a+bx))+\frac{1}{6}x^3\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-\frac{1}{6}x^3\log\left(1+\right.$$

$$=\frac{1}{3}x^3\coth^{-1}(c+d\tanh(a+bx))+\frac{1}{6}x^3\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-\frac{1}{6}x^3\log\left(1+\right.$$

$$=\frac{1}{3}x^3\coth^{-1}(c+d\tanh(a+bx))+\frac{1}{6}x^3\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-\frac{1}{6}x^3\log\left(1+\right.$$

$$=\frac{1}{3}x^3\coth^{-1}(c+d\tanh(a+bx))+\frac{1}{6}x^3\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-\frac{1}{6}x^3\log\left(1+\right.$$

Mathematica [A] time = 0.452205, size = 345, normalized size = 1.12

$$\frac{-6b^2x^2\text{PolyLog}\left(2,\frac{(c-d-1)(\sinh(2(a+bx))-\cosh(2(a+bx)))}{c+d-1}\right)+6b^2x^2\text{PolyLog}\left(2,\frac{(c-d+1)(\sinh(2(a+bx))-\cosh(2(a+bx)))}{c+d+1}\right)-6bx\text{PolyLog}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^3*ArcCoth[c + d*Tanh[a + b*x]])/3 + (4*b^3*x^3*Log[1 + ((-1 + c - d)*(Co
sh[2*(a + b*x)] - Sinh[2*(a + b*x)]))/(-1 + c + d)] - 4*b^3*x^3*Log[1 + ((1
 + c - d)*(Cosh[2*(a + b*x)] - Sinh[2*(a + b*x)]))/(1 + c + d)] - 6*b^2*x^2
*PolyLog[2, ((-1 + c - d)*(-Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)]))/(-1 + c
 + d)] + 6*b^2*x^2*PolyLog[2, ((1 + c - d)*(-Cosh[2*(a + b*x)] + Sinh[2*(a
+ b*x)]))/(1 + c + d)] - 6*b*x*PolyLog[3, ((-1 + c - d)*(-Cosh[2*(a + b*x)]
 + Sinh[2*(a + b*x)]))/(-1 + c + d)] + 6*b*x*PolyLog[3, ((1 + c - d)*(-Cosh
[2*(a + b*x)] + Sinh[2*(a + b*x)]))/(1 + c + d)] - 3*PolyLog[4, ((-1 + c -
d)*(-Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)]))/(-1 + c + d)] + 3*PolyLog[4, (
(1 + c - d)*(-Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)]))/(1 + c + d)])/(24*b^3
)
```

Maple [C] time = 5.085, size = 5294, normalized size = 17.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(c+d*tanh(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [A] time = 2.28858, size = 379, normalized size = 1.23

$$\frac{1}{3}x^3 \operatorname{arcoth}(d\tanh(bx+a)+c) - \frac{1}{18}bd\left(\frac{4b^3x^3\log\left(\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}+1\right)+6b^2x^2\mathrm{Li}_2\left(-\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right)-6bx\mathrm{Li}_3(-\frac{(c+}{}}{b^4d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(d*tanh(b*x + a) + c) - 1/18*b*d*((4*b^3*x^3*log((c + d + 1)
*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 6*b^2*x^2*dilog(-(c + d + 1)*e^(2*b*x +
 2*a)/(c - d + 1)) - 6*b*x*polylog(3, -(c + d + 1)*e^(2*b*x + 2*a)/(c - d +
 1)) + 3*polylog(4, -(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^4*d) - (4
*b^3*x^3*log((c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1) + 1) + 6*b^2*x^2*dilog
(-(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)) - 6*b*x*polylog(3, -(c + d - 1)*
e^(2*b*x + 2*a)/(c - d - 1)) + 3*polylog(4, -(c + d - 1)*e^(2*b*x + 2*a)/(c
 - d - 1)))/(b^4*d))
```

Fricas [C] time = 2.06443, size = 2587, normalized size = 8.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(b^3*x^3*log(((c + 1)*cosh(b*x + a) + d*sinh(b*x + a))/((c - 1)*cosh(b*
x + a) + d*sinh(b*x + a))) - 3*b^2*x^2*dilog(sqrt(-(c + d + 1)/(c - d + 1))
```

```
*(cosh(b*x + a) + sinh(b*x + a))) - 3*b^2*x^2*dilog(-sqrt(-(c + d + 1)/(c -
 d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(sqrt(-(c + d -
1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(-sqrt(-(
c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + a^3*log(2*(c + d
 + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt(-(c
+ d + 1)/(c - d + 1))) + a^3*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1
)*sinh(b*x + a) - 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d + 1))) - a^3*log(2
*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sq
rt(-(c + d - 1)/(c - d - 1))) - a^3*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c
+ d - 1)*sinh(b*x + a) - 2*(c - d - 1)*sqrt(-(c + d - 1)/(c - d - 1))) + 6*
b*x*polylog(3, sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a
))) + 6*b*x*polylog(3, -sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sin
h(b*x + a))) - 6*b*x*polylog(3, sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x +
a) + sinh(b*x + a))) - 6*b*x*polylog(3, -sqrt(-(c + d - 1)/(c - d - 1))*(co
sh(b*x + a) + sinh(b*x + a))) - (b^3*x^3 + a^3)*log(sqrt(-(c + d + 1)/(c -
d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b^3*x^3 + a^3)*log(-sqrt(-(
c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b^3*x^3 + a
^3)*log(sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1)
 + (b^3*x^3 + a^3)*log(-sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sin
h(b*x + a)) + 1) - 6*polylog(4, sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x +
a) + sinh(b*x + a))) - 6*polylog(4, -sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b
*x + a) + sinh(b*x + a))) + 6*polylog(4, sqrt(-(c + d - 1)/(c - d - 1))*(co
sh(b*x + a) + sinh(b*x + a))) + 6*polylog(4, -sqrt(-(c + d - 1)/(c - d - 1)
)*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(c+d*tanh(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arcoth}(d \tanh(bx + a) + c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(c+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(d*tanh(b*x + a) + c), x)
```

3.204 $\int x \coth^{-1}(c + d\tanh(a + bx))\, dx$

Optimal. Leaf size=231

$$-\frac{\text{PolyLog}\left(3,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^2}+\frac{\text{PolyLog}\left(3,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^2}+\frac{x\text{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{x\text{PolyLog}\left(2,-\frac{(c+a}{}\right.}{4b}$$

```
[Out] (x^2*ArcCoth[c + d*Tanh[a + b*x]])/2 + (x^2*Log[1 + ((1 - c - d)*E^(2*a + 2
*b*x))/(1 - c + d)])/4 - (x^2*Log[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c
- d)])/4 + (x*PolyLog[2, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))])/(4*
b) - (x*PolyLog[2, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))])/(4*b) - P
olyLog[3, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))]/(8*b^2) + PolyLog[3
, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))]/(8*b^2)
```

Rubi [A] time = 0.379466, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6244, 2190, 2531, 2282, 6589}

$$-\frac{\text{PolyLog}\left(3,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^2}+\frac{\text{PolyLog}\left(3,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^2}+\frac{x\text{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{x\text{PolyLog}\left(2,-\frac{(c+a}{}\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^2*ArcCoth[c + d*Tanh[a + b*x]])/2 + (x^2*Log[1 + ((1 - c - d)*E^(2*a + 2
*b*x))/(1 - c + d)])/4 - (x^2*Log[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c
- d)])/4 + (x*PolyLog[2, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))])/(4*
b) - (x*PolyLog[2, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))])/(4*b) - P
olyLog[3, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))]/(8*b^2) + PolyLog[3
, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))]/(8*b^2)
```

Rule 6244

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Tanh[a + b*x]])/(f*
(m + 1)), x] + (Dist[(b*(1 - c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^
(2*a + 2*b*x))/(1 - c + d + (1 - c - d)*E^(2*a + 2*b*x)), x], x] - Dist[(b*
(1 + c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*a + 2*b*x))/(1 + c -
d + (1 + c + d)*E^(2*a + 2*b*x)), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &
& IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\coth^{-1}(c+d\tanh(a+bx))\,dx = \frac{1}{2}x^2\coth^{-1}(c+d\tanh(a+bx)) + \frac{1}{2}(b(1-c-d))\int \frac{e^{2a+2bx}x^2}{1-c+d+(1-c-d)e^{2a+2bx}}$$

$$= \frac{1}{2}x^2\coth^{-1}(c+d\tanh(a+bx)) + \frac{1}{4}x^2\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{4}x^2\log\left(1+\right.$$

$$= \frac{1}{2}x^2\coth^{-1}(c+d\tanh(a+bx)) + \frac{1}{4}x^2\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{4}x^2\log\left(1+\right.$$

$$= \frac{1}{2}x^2\coth^{-1}(c+d\tanh(a+bx)) + \frac{1}{4}x^2\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{4}x^2\log\left(1+\right.$$

$$= \frac{1}{2}x^2\coth^{-1}(c+d\tanh(a+bx)) + \frac{1}{4}x^2\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{4}x^2\log\left(1+\right.$$

Mathematica [A] time = 0.163279, size = 259, normalized size = 1.12

$$-2bx\text{PolyLog}\left(2,\frac{(c-d-1)(\sinh(2(a+bx))-\cosh(2(a+bx)))}{c+d-1}\right)+2bx\text{PolyLog}\left(2,\frac{(c-d+1)(\sinh(2(a+bx))-\cosh(2(a+bx)))}{c+d+1}\right)-\text{PolyLog}\left(3,\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^2*ArcCoth[c + d*Tanh[a + b*x]])/2 + (2*b^2*x^2*Log[1 + ((-1 + c - d)*(Co
sh[2*(a + b*x)] - Sinh[2*(a + b*x)]))/(-1 + c + d)] - 2*b^2*x^2*Log[1 + ((1
 + c - d)*(Cosh[2*(a + b*x)] - Sinh[2*(a + b*x)]))/(1 + c + d)] - 2*b*x*Pol
yLog[2, ((-1 + c - d)*(-Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)]))/(-1 + c + d
)] + 2*b*x*PolyLog[2, ((1 + c - d)*(-Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)])
)/(1 + c + d)] - PolyLog[3, ((-1 + c - d)*(-Cosh[2*(a + b*x)] + Sinh[2*(a +
 b*x)]))/(-1 + c + d)] + PolyLog[3, ((1 + c - d)*(-Cosh[2*(a + b*x)] + Sinh
[2*(a + b*x)]))/(1 + c + d)])/(8*b^2)
```

Maple [C] time = 4.741, size = 4990, normalized size = 21.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(c+d*tanh(b*x+a)),x)
```

```
[Out] -1/8*I*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*((exp(2*b*x+2*a)+1)*c+(exp(
2*b*x+2*a)-1)*d-exp(2*b*x+2*a)-1)/(exp(2*b*x+2*a)+1))^2+1/8*I*Pi*x^2*csgn(I
/(exp(2*b*x+2*a)+1))*csgn(I*((exp(2*b*x+2*a)+1)*c+(exp(2*b*x+2*a)-1)*d+exp(
2*b*x+2*a)+1)/(exp(2*b*x+2*a)+1))^2-1/4*x^2*ln((exp(2*b*x+2*a)+1)*c+(exp(2*
b*x+2*a)-1)*d-exp(2*b*x+2*a)-1)+1/8*I*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csg
n(I*((exp(2*b*x+2*a)+1)*c+(exp(2*b*x+2*a)-1)*d-exp(2*b*x+2*a)-1))*csgn(I*((
exp(2*b*x+2*a)+1)*c+(exp(2*b*x+2*a)-1)*d-exp(2*b*x+2*a)-1)/(exp(2*b*x+2*a)+
1))+1/8*I*Pi*x^2*csgn(I*((exp(2*b*x+2*a)+1)*c+(exp(2*b*x+2*a)-1)*d-exp(2*b*
x+2*a)-1)/(exp(2*b*x+2*a)+1))^3-1/4/b^2*a^2/(1+c+d)*ln(exp(2*b*x+2*a)*c+exp
(2*b*x+2*a)*d+exp(2*b*x+2*a)+c-d+1)-1/8*I*Pi*x^2*csgn(I*((exp(2*b*x+2*a)+1)
*c+(exp(2*b*x+2*a)-1)*d+exp(2*b*x+2*a)+1)/(exp(2*b*x+2*a)+1))^3+1/2/b*a*c/(
1+c+d)*ln((c*exp(b*x+a)+exp(b*x+a)*d+(-(1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/(
-(1+c-d)*(1+c+d))^(1/2))*x-1/2/b*d/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(-1-
c+d))*x*a-1/2/b*c/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(-1-c+d))*x*a+1/4*x^2
*ln((exp(2*b*x+2*a)+1)*c+(exp(2*b*x+2*a)-1)*d+exp(2*b*x+2*a)+1)+1/2/b*a*d/(
1+c+d)*ln((-c*exp(b*x+a)-exp(b*x+a)*d+(-(1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/
(-(1+c-d)*(1+c+d))^(1/2))*x+1/2/b*a*d/(1+c+d)*ln((c*exp(b*x+a)+exp(b*x+a)*d
+(-(1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/(-(1+c-d)*(1+c+d))^(1/2))*x+1/2/b*a*c
/(1+c+d)*ln((-c*exp(b*x+a)-exp(b*x+a)*d+(-(1+c-d)*(1+c+d))^(1/2)-exp(b*x+a)
)/(-(1+c-d)*(1+c+d))^(1/2))*x-1/4/b/(1+c+d)*polylog(2,(1+c+d)*exp(2*b*x+2*a
)/(-1-c+d))*x+1/2/b^2*a^2/(1+c+d)*ln((-c*exp(b*x+a)-exp(b*x+a)*d+(-(1+c-d)*
(1+c+d))^(1/2)-exp(b*x+a))/(-(1+c-d)*(1+c+d))^(1/2))+1/2/b^2*a^2/(1+c+d)*ln
((c*exp(b*x+a)+exp(b*x+a)*d+(-(1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/(-(1+c-d)*
(1+c+d))^(1/2))-1/4/b^2/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(-1-c+d))*a^2-1
/4/b^2/(1+c+d)*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(-1-c+d))*a+1/8/b^2*c/(1+c+
d)*polylog(3,(1+c+d)*exp(2*b*x+2*a)/(-1-c+d))+1/2/b^2*a/(1+c+d)*dilog((-c*e
xp(b*x+a)-exp(b*x+a)*d+(-(1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/(-(1+c-d)*(1+c+
d))^(1/2))-1/8/b^2*c/(c+d-1)*polylog(3,(c+d-1)*exp(2*b*x+2*a)/(1-c+d))-1/8/
b^2*d/(c+d-1)*polylog(3,(c+d-1)*exp(2*b*x+2*a)/(1-c+d))+1/2/b^2*a/(c+d-1)*d
ilog((-c*exp(b*x+a)-exp(b*x+a)*d+(-(c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/(-(c-
d-1)*(c+d-1))^(1/2))+1/2/b^2*a/(c+d-1)*dilog((c*exp(b*x+a)+exp(b*x+a)*d+(-(
c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/(-(c-d-1)*(c+d-1))^(1/2))-1/4/b^2/(c+d-1)
*ln(1-(c+d-1)*exp(2*b*x+2*a)/(1-c+d))*a^2-1/4/b/(c+d-1)*polylog(2,(c+d-1)*e
xp(2*b*x+2*a)/(1-c+d))*x-1/4/b^2/(c+d-1)*polylog(2,(c+d-1)*exp(2*b*x+2*a)/(
1-c+d))*a+1/2/b^2*a^2/(c+d-1)*ln((-c*exp(b*x+a)-exp(b*x+a)*d+(-(c-d-1)*(c+d
-1))^(1/2)+exp(b*x+a))/(-(c-d-1)*(c+d-1))^(1/2))+1/2/b^2*a^2/(c+d-1)*ln((c*
exp(b*x+a)+exp(b*x+a)*d+(-(c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/(-(c-d-1)*(c+d
-1))^(1/2))+1/4*c/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(1-c+d))*x^2+1/4*d/(c
+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(1-c+d))*x^2+1/8/b^2/(c+d-1)*polylog(3,(c
+d-1)*exp(2*b*x+2*a)/(1-c+d))-1/4/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(1-c+
d))*x^2+1/2/b^2*a/(1+c+d)*dilog((c*exp(b*x+a)+exp(b*x+a)*d+(-(1+c-d)*(1+c+d
))^(1/2)+exp(b*x+a))/(-(1+c-d)*(1+c+d))^(1/2))+1/8/b^2*d/(1+c+d)*polylog(3,
(1+c+d)*exp(2*b*x+2*a)/(-1-c+d))-1/4*d/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/
(-1-c+d))*x^2-1/4*c/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(-1-c+d))*x^2-1/2/b
*d*a/(c+d-1)*ln((-c*exp(b*x+a)-exp(b*x+a)*d+(-(c-d-1)*(c+d-1))^(1/2)+exp(b*
x+a))/(-(c-d-1)*(c+d-1))^(1/2))*x-1/2/b*d*a/(c+d-1)*ln((c*exp(b*x+a)+exp(b*
x+a)*d+(-(c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/(-(c-d-1)*(c+d-1))^(1/2))*x+1/2
/b*c/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(1-c+d))*x*a+1/2/b*d/(c+d-1)*ln(1-
(c+d-1)*exp(2*b*x+2*a)/(1-c+d))*x*a-1/2/b*c*a/(c+d-1)*ln((-c*exp(b*x+a)-exp
(b*x+a)*d+(-(c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/(-(c-d-1)*(c+d-1))^(1/2))*x-
1/2/b*c*a/(c+d-1)*ln((c*exp(b*x+a)+exp(b*x+a)*d+(-(c-d-1)*(c+d-1))^(1/2)-ex
p(b*x+a))/(-(c-d-1)*(c+d-1))^(1/2))*x-1/4/b^2*a^2/(c+d-1)*ln(exp(2*b*x+2*a)
*c+exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+c-d-1)+1/2/b^2*a^2*d/(1+c+d)*ln((-c*exp(
b*x+a)-exp(b*x+a)*d+(-(1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/(-(1+c-d)*(1+c+d))
^(1/2))+1/2/b^2*a^2*d/(1+c+d)*ln((c*exp(b*x+a)+exp(b*x+a)*d+(-(1+c-d)*(1+c+
```

```
d))^(1/2)+exp(b*x+a))/(-(1+c-d)*(1+c+d))^(1/2))+1/2/b^2*a^2*c/(1+c+d)*ln((-
c*exp(b*x+a)-exp(b*x+a)*d+(-(1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/(-(1+c-d)*(1
+c+d))^(1/2))+1/2/b^2*a^2*c/(1+c+d)*ln((c*exp(b*x+a)+exp(b*x+a)*d+(-(1+c-d)
*(1+c+d))^(1/2)+exp(b*x+a))/(-(1+c-d)*(1+c+d))^(1/2))-1/4/b^2*d/(1+c+d)*ln(
1-(1+c+d)*exp(2*b*x+2*a)/(-1-c+d))*a^2-1/4/b^2*d/(1+c+d)*polylog(2,(1+c+d)*
exp(2*b*x+2*a)/(-1-c+d))*a-1/4/b^2*c/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(-
1-c+d))*a^2-1/4/b^2*c/(1+c+d)*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(-1-c+d))*a+
1/2/b^2*a*c/(1+c+d)*dilog((-c*exp(b*x+a)-exp(b*x+a)*d+(-(1+c-d)*(1+c+d))^(1
/2)-exp(b*x+a))/(-(1+c-d)*(1+c+d))^(1/2))+1/2/b^2*a*c/(1+c+d)*dilog((c*exp(
b*x+a)+exp(b*x+a)*d+(-(1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/(-(1+c-d)*(1+c+d))
^(1/2))+1/2/b^2*a*d/(1+c+d)*dilog((-c*exp(b*x+a)-exp(b*x+a)*d+(-(1+c-d)*(1+
c+d))^(1/2)-exp(b*x+a))/(-(1+c-d)*(1+c+d))^(1/2))+1/2/b^2*a*d/(1+c+d)*dilog
((c*exp(b*x+a)+exp(b*x+a)*d+(-(1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/(-(1+c-d)*
(1+c+d))^(1/2))+1/2/b*a/(1+c+d)*ln((-c*exp(b*x+a)-exp(b*x+a)*d+(-(1+c-d)*(1
+c+d))^(1/2)-exp(b*x+a))/(-(1+c-d)*(1+c+d))^(1/2))*x+1/2/b*a/(1+c+d)*ln((c*
exp(b*x+a)+exp(b*x+a)*d+(-(1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/(-(1+c-d)*(1+c
+d))^(1/2))*x-1/4/b*d/(1+c+d)*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(-1-c+d))*x-
1/4/b*c/(1+c+d)*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(-1-c+d))*x-1/2/b/(1+c+d)*
ln(1-(1+c+d)*exp(2*b*x+2*a)/(-1-c+d))*x*a-1/4/b^2*a^2*c/(1+c+d)*ln(exp(2*b*
x+2*a)*c+exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+c-d+1)-1/4/b^2*a^2*d/(1+c+d)*ln(ex
p(2*b*x+2*a)*c+exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+c-d+1)-1/4/(1+c+d)*ln(1-(1+c
+d)*exp(2*b*x+2*a)/(-1-c+d))*x^2+1/8/b^2/(1+c+d)*polylog(3,(1+c+d)*exp(2*b*
x+2*a)/(-1-c+d))-1/8*I*Pi*x^2*csgn(I*((exp(2*b*x+2*a)+1)*c+(exp(2*b*x+2*a)-
1)*d-exp(2*b*x+2*a)-1))*csgn(I*((exp(2*b*x+2*a)+1)*c+(exp(2*b*x+2*a)-1)*d-e
xp(2*b*x+2*a)-1)/(exp(2*b*x+2*a)+1))^2+1/8*I*Pi*x^2*csgn(I*((exp(2*b*x+2*a)
+1)*c+(exp(2*b*x+2*a)-1)*d+exp(2*b*x+2*a)+1))*csgn(I*((exp(2*b*x+2*a)+1)*c+
(exp(2*b*x+2*a)-1)*d+exp(2*b*x+2*a)+1)/(exp(2*b*x+2*a)+1))^2-1/2/b^2*d*a/(c
+d-1)*dilog((-c*exp(b*x+a)-exp(b*x+a)*d+(-(c-d-1)*(c+d-1))^(1/2)+exp(b*x+a)
)/(-(c-d-1)*(c+d-1))^(1/2))-1/2/b^2*d*a/(c+d-1)*dilog((c*exp(b*x+a)+exp(b*x
+a)*d+(-(c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/(-(c-d-1)*(c+d-1))^(1/2))+1/4/b^
2*c/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(1-c+d))*a^2-1/2/b^2*c*a/(c+d-1)*di
log((c*exp(b*x+a)+exp(b*x+a)*d+(-(c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/(-(c-d-
1)*(c+d-1))^(1/2))+1/4/b^2*a^2*c/(c+d-1)*ln(exp(2*b*x+2*a)*c+exp(2*b*x+2*a)
*d-exp(2*b*x+2*a)+c-d-1)+1/4/b^2*d*a^2/(c+d-1)*ln(exp(2*b*x+2*a)*c+exp(2*b*
x+2*a)*d-exp(2*b*x+2*a)+c-d-1)+1/4/b*c/(c+d-1)*polylog(2,(c+d-1)*exp(2*b*x+
2*a)/(1-c+d))*x+1/4/b^2*c/(c+d-1)*polylog(2,(c+d-1)*exp(2*b*x+2*a)/(1-c+d))
*a+1/4/b^2*d/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(1-c+d))*a^2+1/4/b*d/(c+d-
1)*polylog(2,(c+d-1)*exp(2*b*x+2*a)/(1-c+d))*x+1/4/b^2*d/(c+d-1)*polylog(2,
(c+d-1)*exp(2*b*x+2*a)/(1-c+d))*a-1/2/b/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)
/(1-c+d))*x*a+1/2/b*a/(c+d-1)*ln((-c*exp(b*x+a)-exp(b*x+a)*d+(-(c-d-1)*(c+d
-1))^(1/2)+exp(b*x+a))/(-(c-d-1)*(c+d-1))^(1/2))*x+1/2/b*a/(c+d-1)*ln((c*ex
p(b*x+a)+exp(b*x+a)*d+(-(c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/(-(c-d-1)*(c+d-1
))^(1/2))*x-1/2/b^2*c*a^2/(c+d-1)*ln((-c*exp(b*x+a)-exp(b*x+a)*d+(-(c-d-1)*
(c+d-1))^(1/2)+exp(b*x+a))/(-(c-d-1)*(c+d-1))^(1/2))-1/2/b^2*c*a^2/(c+d-1)*
ln((c*exp(b*x+a)+exp(b*x+a)*d+(-(c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/(-(c-d-1
)*(c+d-1))^(1/2))-1/2/b^2*d*a^2/(c+d-1)*ln((-c*exp(b*x+a)-exp(b*x+a)*d+(-(c
-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/(-(c-d-1)*(c+d-1))^(1/2))-1/2/b^2*d*a^2/(c
+d-1)*ln((c*exp(b*x+a)+exp(b*x+a)*d+(-(c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/(-
(c-d-1)*(c+d-1))^(1/2))-1/2/b^2*c*a/(c+d-1)*dilog((-c*exp(b*x+a)-exp(b*x+a)
*d+(-(c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/(-(c-d-1)*(c+d-1))^(1/2))-1/8*I*Pi*
x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*((exp(2*b*x+2*a)+1)*c+(exp(2*b*x+2*a)
-1)*d+exp(2*b*x+2*a)+1))*csgn(I*((exp(2*b*x+2*a)+1)*c+(exp(2*b*x+2*a)-1)*d+
exp(2*b*x+2*a)+1)/(exp(2*b*x+2*a)+1))
```

Maxima [A] time = 2.15735, size = 290, normalized size = 1.26

$$-\frac{1}{8}\,bd\left(\frac{2\,b^2x^2\log\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}+1\right)+2\,bx\mathrm{Li}_2\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)-\mathrm{Li}_3(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1})}{b^3d}-\frac{2\,b^2x^2\log\left(\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*b*d*((2*b^2*x^2*log((c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 2*b
*x*dilog(-(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)) - polylog(3, -(c + d + 1
)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^3*d) - (2*b^2*x^2*log((c + d - 1)*e^(2*b
*x + 2*a)/(c - d - 1) + 1) + 2*b*x*dilog(-(c + d - 1)*e^(2*b*x + 2*a)/(c -
d - 1)) - polylog(3, -(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)))/(b^3*d)) +
1/2*x^2*arccoth(d*tanh(b*x + a) + c)
```

Fricas [C] time = 1.89422, size = 2114, normalized size = 9.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^2*log(((c + 1)*cosh(b*x + a) + d*sinh(b*x + a))/((c - 1)*cosh(b*
x + a) + d*sinh(b*x + a))) - 2*b*x*dilog(sqrt(-(c + d + 1)/(c - d + 1))*(co
sh(b*x + a) + sinh(b*x + a))) - 2*b*x*dilog(-sqrt(-(c + d + 1)/(c - d + 1))
*(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(sqrt(-(c + d - 1)/(c - d -
1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(-sqrt(-(c + d - 1)/(c -
d - 1))*(cosh(b*x + a) + sinh(b*x + a))) - a^2*log(2*(c + d + 1)*cosh(b*x +
 a) + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d
+ 1))) - a^2*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a)
- 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d + 1))) + a^2*log(2*(c + d - 1)*cos
h(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt(-(c + d - 1)/
(c - d - 1))) + a^2*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*
x + a) - 2*(c - d - 1)*sqrt(-(c + d - 1)/(c - d - 1))) - (b^2*x^2 - a^2)*lo
g(sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b^
2*x^2 - a^2)*log(-sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x
+ a)) + 1) + (b^2*x^2 - a^2)*log(sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x +
 a) + sinh(b*x + a)) + 1) + (b^2*x^2 - a^2)*log(-sqrt(-(c + d - 1)/(c - d -
 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 2*polylog(3, sqrt(-(c + d + 1)/
(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*polylog(3, -sqrt(-(c + d
+ 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*polylog(3, sqrt(-(c
+ d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*polylog(3, -sqrt
(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(c+d*tanh(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arcoth}(d \tanh(bx + a) + c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(c+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(d*tanh(b*x + a) + c), x)
```

3.205 $\int \coth^{-1}(c + d\tanh(a + bx))\,dx$

Optimal. Leaf size=150

$$\frac{\text{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{\text{PolyLog}\left(2,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}+\frac{1}{2}x\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)-\frac{1}{2}x\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right.$$

```
[Out] x*ArcCoth[c + d*Tanh[a + b*x]] + (x*Log[1 + ((1 - c - d)*E^(2*a + 2*b*x))/(
1 - c + d)])/2 - (x*Log[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/2 +
 PolyLog[2, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))]/(4*b) - PolyLog[2
, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))]/(4*b)
```

Rubi [A] time = 0.231966, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6236, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{\text{PolyLog}\left(2,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}+\frac{1}{2}x\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)-\frac{1}{2}x\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[c + d*Tanh[a + b*x]],x]
```


```
[Out] x*ArcCoth[c + d*Tanh[a + b*x]] + (x*Log[1 + ((1 - c - d)*E^(2*a + 2*b*x))/(
1 - c + d)])/2 - (x*Log[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/2 +
 PolyLog[2, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))]/(4*b) - PolyLog[2
, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))]/(4*b)
```


Rule 6236

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Ar
cCoth[c + d*Tanh[a + b*x]], x] + (Dist[b*(1 - c - d), Int[(x*E^(2*a + 2*b*x
))/(1 - c + d + (1 - c - d)*E^(2*a + 2*b*x)), x], x] - Dist[b*(1 + c + d),
Int[(x*E^(2*a + 2*b*x))/(1 + c - d + (1 + c + d)*E^(2*a + 2*b*x)), x], x])
/; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, 1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \coth^{-1}(c+d\tanh(a+bx))\,dx = x\coth^{-1}(c+d\tanh(a+bx)) + (b(1-c-d))\int \frac{e^{2a+2bx}x}{1-c+d+(1-c-d)e^{2a+2bx}}\,dx - (b$$

$$= x\coth^{-1}(c+d\tanh(a+bx)) + \frac{1}{2}x\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{2}x\log\left(1+\frac{(1+c-}{1+}\right.$$

$$= x\coth^{-1}(c+d\tanh(a+bx)) + \frac{1}{2}x\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{2}x\log\left(1+\frac{(1+c-}{1+}\right.$$

$$= x\coth^{-1}(c+d\tanh(a+bx)) + \frac{1}{2}x\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{2}x\log\left(1+\frac{(1+c-}{1+}\right.$$

Mathematica [A] time = 1.24638, size = 131, normalized size = 0.87

$$\frac{\text{PolyLog}\left(2,-\frac{(c+d-1)e^{2(a+bx)}}{c-d-1}\right)-\text{PolyLog}\left(2,-\frac{(c+d+1)e^{2(a+bx)}}{c-d+1}\right)+2bx\left(\log\left(\frac{(c+d-1)e^{2(a+bx)}}{c-d-1}+1\right)-\log\left(\frac{(c+d+1)e^{2(a+bx)}}{c-d+1}+1\right)\right)}{4b}+x\,c$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[c + d*Tanh[a + b*x]],x]
```

```
[Out] x*ArcCoth[c + d*Tanh[a + b*x]] + (2*b*x*(Log[1 + ((-1 + c + d)*E^(2*(a + b*
x)))/(-1 + c - d)] - Log[1 + ((1 + c + d)*E^(2*(a + b*x)))/(1 + c - d)]) +
PolyLog[2, -(((-1 + c + d)*E^(2*(a + b*x)))/(-1 + c - d))] - PolyLog[2, -((
(1 + c + d)*E^(2*(a + b*x)))/(1 + c - d))])/(4*b)
```

Maple [B] time = 0.164, size = 306, normalized size = 2.

$$-\frac{\text{arccoth}(c+d\tanh(bx+a))\ln(d\tanh(bx+a)-d)}{2b}+\frac{\text{arccoth}(c+d\tanh(bx+a))\ln(d\tanh(bx+a)+d)}{2b}+\frac{1}{4b}\text{di}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(c+d*tanh(b*x+a)),x)
```

```
[Out] -1/2/b*arccoth(c+d*tanh(b*x+a))*ln(d*tanh(b*x+a)-d)+1/2/b*arccoth(c+d*tanh(
b*x+a))*ln(d*tanh(b*x+a)+d)+1/4/b*dilog((d*tanh(b*x+a)+c+1)/(1+c+d))+1/4/b*
ln(d*tanh(b*x+a)-d)*ln((d*tanh(b*x+a)+c+1)/(1+c+d))-1/4/b*dilog((d*tanh(b*x
+a)+c-1)/(c+d-1))-1/4/b*ln(d*tanh(b*x+a)-d)*ln((d*tanh(b*x+a)+c-1)/(c+d-1))
-1/4/b*dilog((d*tanh(b*x+a)+c+1)/(1+c-d))-1/4/b*ln(d*tanh(b*x+a)+d)*ln((d*t
anh(b*x+a)+c+1)/(1+c-d))+1/4/b*dilog((d*tanh(b*x+a)+c-1)/(c-d-1))+1/4/b*ln(
d*tanh(b*x+a)+d)*ln((d*tanh(b*x+a)+c-1)/(c-d-1))
```

Maxima [A] time = 2.1111, size = 192, normalized size = 1.28

$$-\frac{1}{4}bd\left(\frac{2bx\log\left(\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}+1\right)+\text{Li}_2\left(-\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right)}{b^2d}-\frac{2bx\log\left(\frac{(c+d-1)e^{(2bx+2a)}}{c-d-1}+1\right)+\text{Li}_2\left(-\frac{(c+d-1)e^{(2bx+2a)}}{c-d-1}\right)}{b^2d}\right)+x\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/4*b*d*((2*b*x*log((c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + dilog(-
(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^2*d) - (2*b*x*log((c + d - 1)*
e^(2*b*x + 2*a)/(c - d - 1) + 1) + dilog(-(c + d - 1)*e^(2*b*x + 2*a)/(c -
d - 1)))/(b^2*d)) + x*arccoth(d*tanh(b*x + a) + c)
```

Fricas [B] time = 1.85118, size = 1601, normalized size = 10.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x*log(((c + 1)*cosh(b*x + a) + d*sinh(b*x + a))/((c - 1)*cosh(b*x +
a) + d*sinh(b*x + a))) + a*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*
sinh(b*x + a) + 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d + 1))) + a*log(2*(c
+ d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) - 2*(c - d + 1)*sqrt(-
(c + d + 1)/(c - d + 1))) - a*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d -
1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt(-(c + d - 1)/(c - d - 1))) - a*log(2*
(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) - 2*(c - d - 1)*sqr
t(-(c + d - 1)/(c - d - 1))) - (b*x + a)*log(sqrt(-(c + d + 1)/(c - d + 1))
*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-sqrt(-(c + d + 1)/(c
 - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x + a)*log(sqrt(-(c +
d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x + a)*log(-s
qrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - dilog(
sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) - dilog(-sq
rt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + dilog(sqrt(
-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + dilog(-sqrt(-(
c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(c+d*tanh(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}\left(d\tanh\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*tanh(b*x + a) + c), x)
```

3.206 $\int \frac{\coth^{-1}(c+d\tanh(a+bx))}{x} \, dx$

Optimal. Leaf size=17

$$\text{CannotIntegrate}\left(\frac{\coth^{-1}(d\tanh(a+bx)+c)}{x},x\right)$$

```
[Out] CannotIntegrate[ArcCoth[c + d*Tanh[a + b*x]]/x, x]
```

Rubi [A] time = 0.0662613, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth^{-1}(c+d\tanh(a+bx))}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCoth[c + d*Tanh[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcCoth[c + d*Tanh[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\coth^{-1}(c+d\tanh(a+bx))}{x} \, dx = \int \frac{\coth^{-1}(c+d\tanh(a+bx))}{x} \, dx$$

Mathematica [A] time = 5.26462, size = 0, normalized size = 0.

$$\int \frac{\coth^{-1}(c+d\tanh(a+bx))}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCoth[c + d*Tanh[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcCoth[c + d*Tanh[a + b*x]]/x, x]
```

Maple [A] time = 0.331, size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(c+d\tanh(bx+a))}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(c+d*tanh(b*x+a))/x,x)
```

```
[Out] int(arccoth(c+d*tanh(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*tanh(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arccoth(d*tanh(b*x + a) + c)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+c\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*tanh(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(d*tanh(b*x + a) + c)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(c+d*tanh(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*tanh(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*tanh(b*x + a) + c)/x, x)
```

3.207 $\int x^3 \coth^{-1}(1+d+d\tanh(a+bx))\,dx$

Optimal. Leaf size=155

$$\frac{3x^2\text{PolyLog}\left(3,-(d+1)e^{2a+2bx}\right)}{8b^2}-\frac{3x\text{PolyLog}\left(4,-(d+1)e^{2a+2bx}\right)}{8b^3}+\frac{3\text{PolyLog}\left(5,-(d+1)e^{2a+2bx}\right)}{16b^4}-\frac{x^3\text{PolyLog}\left(2,-\right.}{4b}$$

```
[Out] (b*x^5)/20 + (x^4*ArcCoth[1 + d + d*Tanh[a + b*x]])/4 - (x^4*Log[1 + (1 + d
)*E^(2*a + 2*b*x)])/8 - (x^3*PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))])/(4*b)
+ (3*x^2*PolyLog[3, -((1 + d)*E^(2*a + 2*b*x))])/(8*b^2) - (3*x*PolyLog[4,
-((1 + d)*E^(2*a + 2*b*x))])/(8*b^3) + (3*PolyLog[5, -((1 + d)*E^(2*a + 2*b
*x))])/(16*b^4)
```

Rubi [A] time = 0.30719, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6240, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(3,-(d+1)e^{2a+2bx}\right)}{8b^2}-\frac{3x\text{PolyLog}\left(4,-(d+1)e^{2a+2bx}\right)}{8b^3}+\frac{3\text{PolyLog}\left(5,-(d+1)e^{2a+2bx}\right)}{16b^4}-\frac{x^3\text{PolyLog}\left(2,-\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^5)/20 + (x^4*ArcCoth[1 + d + d*Tanh[a + b*x]])/4 - (x^4*Log[1 + (1 + d
)*E^(2*a + 2*b*x)])/8 - (x^3*PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))])/(4*b)
+ (3*x^2*PolyLog[3, -((1 + d)*E^(2*a + 2*b*x))])/(8*b^2) - (3*x*PolyLog[4,
-((1 + d)*E^(2*a + 2*b*x))])/(8*b^3) + (3*PolyLog[5, -((1 + d)*E^(2*a + 2*b
*x))])/(16*b^4)
```

Rule 6240

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Tanh[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3 \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \frac{1}{4}x^4\coth^{-1}(1+d+d\tanh(a+bx)) + \frac{1}{4}b\int \frac{x^4}{1+(1+d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{4}(b(1+d))\int \frac{e^{2a+2bx}x^4}{1+(1+d)e^{2a+}$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1+d)e^{2a+2bx}\right) +$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1+d)e^{2a+2bx}\right) -$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1+d)e^{2a+2bx}\right) -$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1+d)e^{2a+2bx}\right) -$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1+d)e^{2a+2bx}\right) -$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1+d)e^{2a+2bx}\right) -$$

Mathematica [A] time = 0.191028, size = 144, normalized size = 0.93

$$\frac{1}{16}\left(\frac{6x^2\text{PolyLog}\left(3,-\frac{e^{-2(a+bx)}}{d+1}\right)}{b^2} + \frac{6x\text{PolyLog}\left(4,-\frac{e^{-2(a+bx)}}{d+1}\right)}{b^3} + \frac{3\text{PolyLog}\left(5,-\frac{e^{-2(a+bx)}}{d+1}\right)}{b^4} + \frac{4x^3\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{d+1}\right)}{b}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (4*x^4*ArcCoth[1 + d + d*Tanh[a + b*x]] - 2*x^4*Log[1 + 1/((1 + d)*E^(2*(a
+ b*x)))] + (4*x^3*PolyLog[2, -(1/((1 + d)*E^(2*(a + b*x))))])/b + (6*x^2*P
olyLog[3, -(1/((1 + d)*E^(2*(a + b*x))))])/b^2 + (6*x*PolyLog[4, -(1/((1 +
d)*E^(2*(a + b*x))))])/b^3 + (3*PolyLog[5, -(1/((1 + d)*E^(2*(a + b*x))))])
/b^4)/16
```

Maple [C] time = 34.221, size = 1726, normalized size = 11.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccoth(1+d+d*tanh(b*x+a)),x)
```

```
[Out] 1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(
2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/8/b^4*a^4/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*
b*x+2*a)+1)+1/2/b^4*d*a^3/(1+d)*dilog(1+exp(b*x+a)*(-d-1)^(1/2))+1/2/b^4*d*
a^3/(1+d)*dilog(1-exp(b*x+a)*(-d-1)^(1/2))-3/8/b^4*d*a^4/(1+d)*ln(1+(1+d)*e
xp(2*b*x+2*a))-1/4/b^4*d*a^3/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))+1/2/b^4
*d*a^4/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))+1/2/b^4*d*a^4/(1+d)*ln(1-exp(b*x
+a)*(-d-1)^(1/2))+1/2/b^3*a^3/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))*x-1/4/b*d
/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*x^3+3/8/b^2*d/(1+d)*polylog(3,-(1+d
)*exp(2*b*x+2*a))*x^2+1/2/b^3*a^3/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))*x-1/2
/b^3*a^3/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x-3/8/b^3*d/(1+d)*polylog(4,-(1+d
)*exp(2*b*x+2*a))*x-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)
*d+exp(2*b*x+2*a)+1))^3+1/16*I*x^4*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x
+2*a))-1/8*I*x^4*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/16*I*x^4*
Pi*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^3-1/2/b^3*d*a^3/(1+d)*ln(1+(
1+d)*exp(2*b*x+2*a))*x+1/2/b^3*d*a^3/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))*x+
1/2/b^3*d*a^3/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))*x+1/16*I*x^4*Pi*csgn(I*ex
p(2*b*x+2*a))^3+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1
/20*b*x^5-1/16*I*x^4*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a
))^2-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*
b*x+2*a)+1))^2-1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(
exp(2*b*x+2*a)+1))^2+1/16*I*x^4*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b
*x+2*a)+1))*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))+1/8*x^4*ln(exp(2*b*
x+2*a)*d+exp(2*b*x+2*a)+1)-1/8*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x^4+1/2/b
^4*a^4/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))-3/8/b^4*a^4/(1+d)*ln(1+(1+d)*exp
(2*b*x+2*a))-1/4/b/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*x^3-1/4/b^4/(1+d)
*polylog(2,-(1+d)*exp(2*b*x+2*a))*a^3+3/8/b^2/(1+d)*polylog(3,-(1+d)*exp(2*
b*x+2*a))*x^2-3/8/b^3/(1+d)*polylog(4,-(1+d)*exp(2*b*x+2*a))*x+1/2/b^4*a^4/
(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))+3/16/b^4*d/(1+d)*polylog(5,-(1+d)*exp(2
*b*x+2*a))+1/2/b^4*a^3/(1+d)*dilog(1+exp(b*x+a)*(-d-1)^(1/2))+1/2/b^4*a^3/(
1+d)*dilog(1-exp(b*x+a)*(-d-1)^(1/2))-1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(
exp(2*b*x+2*a)+1))*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2+1/16*I*x^4
*Pi*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(
exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))^2-1/8/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*
x^4+3/16/b^4/(1+d)*polylog(5,-(1+d)*exp(2*b*x+2*a))-1/4*x^4*ln(exp(b*x+a))-
1/8*x^4*ln(d)+1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*
a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))^2-1/16*I*x^4*Pi*csgn(I/(exp(2*b*
x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*
a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))-1/8/b^4*d*a^4/(1+d)*ln(exp(2*b*x
+2*a)*d+exp(2*b*x+2*a)+1)
```

Maxima [A] time = 3.35491, size = 201, normalized size = 1.3

$$\frac{1}{4}x^4 \operatorname{arcoth}(d\tanh(bx+a)+d+1)+\frac{1}{40}\left(\frac{2x^5}{d}-\frac{5\left(2b^4x^4\log\left((d+1)e^{(2bx+2a)}+1\right)+4b^3x^3\operatorname{Li}_2\left(-(d+1)e^{(2bx+2a)}\right)\right.}{\quad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccoth(d*tanh(b*x + a) + d + 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*log
((d + 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog(-(d + 1)*e^(2*b*x + 2*a)) -
 6*b^2*x^2*polylog(3, -(d + 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, -(d + 1)
*e^(2*b*x + 2*a)) - 3*polylog(5, -(d + 1)*e^(2*b*x + 2*a)))/(b^5*d))*b*d
```

Fricas [C] time = 1.88589, size = 1377, normalized size = 8.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/40*(2*b^5*x^5 + 5*b^4*x^4*log(((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/(
d*cosh(b*x + a) + d*sinh(b*x + a))) - 20*b^3*x^3*dilog(1/2*sqrt(-4*d - 4)*(
cosh(b*x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-1/2*sqrt(-4*d - 4)*(cos
h(b*x + a) + sinh(b*x + a))) - 5*a^4*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1
)*sinh(b*x + a) + sqrt(-4*d - 4)) - 5*a^4*log(2*(d + 1)*cosh(b*x + a) + 2*(
d + 1)*sinh(b*x + a) - sqrt(-4*d - 4)) + 60*b^2*x^2*polylog(3, 1/2*sqrt(-4*
d - 4)*(cosh(b*x + a) + sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -1/2*sqrt(-
4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, 1/2*sqrt(-4*
d - 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, -1/2*sqrt(-4*d
 - 4)*(cosh(b*x + a) + sinh(b*x + a))) - 5*(b^4*x^4 - a^4)*log(1/2*sqrt(-4*
d - 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 5*(b^4*x^4 - a^4)*log(-1/2*sq
rt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 120*polylog(5, 1/2*sqrt
(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) + 120*polylog(5, -1/2*sqrt(-4*d
 - 4)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acoth(1+d+d*tanh(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arcoth}(d\tanh(bx+a)+d+1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^3*arccoth(d*tanh(b*x + a) + d + 1), x)
```

3.208 $\int x^2 \coth^{-1}(1+d+d\tanh(a+bx))\,dx$

Optimal. Leaf size=128

$$\frac{x\text{PolyLog}\left(3,-(d+1)e^{2a+2bx}\right)}{4b^2}-\frac{\text{PolyLog}\left(4,-(d+1)e^{2a+2bx}\right)}{8b^3}-\frac{x^2\text{PolyLog}\left(2,-(d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{6}x^3\log\left((d+1)e^{2\right.$$

```
[Out] (b*x^4)/12 + (x^3*ArcCoth[1 + d + d*Tanh[a + b*x]])/3 - (x^3*Log[1 + (1 + d
)*E^(2*a + 2*b*x)])/6 - (x^2*PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))])/(4*b)
+ (x*PolyLog[3, -((1 + d)*E^(2*a + 2*b*x))])/(4*b^2) - PolyLog[4, -((1 + d)
*E^(2*a + 2*b*x))]/(8*b^3)
```

Rubi [A] time = 0.269003, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6240, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{x\text{PolyLog}\left(3,-(d+1)e^{2a+2bx}\right)}{4b^2}-\frac{\text{PolyLog}\left(4,-(d+1)e^{2a+2bx}\right)}{8b^3}-\frac{x^2\text{PolyLog}\left(2,-(d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{6}x^3\log\left((d+1)e^{2\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^4)/12 + (x^3*ArcCoth[1 + d + d*Tanh[a + b*x]])/3 - (x^3*Log[1 + (1 + d
)*E^(2*a + 2*b*x)])/6 - (x^2*PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))])/(4*b)
+ (x*PolyLog[3, -((1 + d)*E^(2*a + 2*b*x))])/(4*b^2) - PolyLog[4, -((1 + d)
*E^(2*a + 2*b*x))]/(8*b^3)
```

Rule 6240

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Tanh[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \frac{1}{3}x^3\coth^{-1}(1+d+d\tanh(a+bx)) + \frac{1}{3}b\int \frac{x^3}{1+(1+d)e^{2a+2bx}}\,dx$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{3}(b(1+d))\int \frac{e^{2a+2bx}x^3}{1+(1+d)e^{2a+2bx}}$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+d)e^{2a+2bx}\right) + \frac{1}{2}$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+d)e^{2a+2bx}\right) - \frac{x^2}{}$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+d)e^{2a+2bx}\right) - \frac{x^2}{}$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+d)e^{2a+2bx}\right) - \frac{x^2}{}$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+d)e^{2a+2bx}\right) - \frac{x^2}{}$$

Mathematica [A] time = 0.113944, size = 118, normalized size = 0.92

$$\frac{1}{24}\left(\frac{6x\text{PolyLog}\left(3,-\frac{e^{-2(a+bx)}}{d+1}\right)}{b^2} + \frac{3\text{PolyLog}\left(4,-\frac{e^{-2(a+bx)}}{d+1}\right)}{b^3} + \frac{6x^2\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{d+1}\right)}{b} - 4x^3\log\left(\frac{e^{-2(a+bx)}}{d+1}+1\right) + 8x^3\,\text{co}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (8*x^3*ArcCoth[1 + d + d*Tanh[a + b*x]] - 4*x^3*Log[1 + 1/((1 + d)*E^(2*(a
+ b*x)))] + (6*x^2*PolyLog[2, -(1/((1 + d)*E^(2*(a + b*x))))])/b + (6*x*Pol
yLog[3, -(1/((1 + d)*E^(2*(a + b*x))))])/b^2 + (3*PolyLog[4, -(1/((1 + d)*E
```

```
^(2*(a + b*x))))])/b^3)/24
```

Maple [C] time = 37.152, size = 1667, normalized size = 13.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(1+d+d*tanh(b*x+a)),x)
```

```
[Out] -1/3*x^3*ln(exp(b*x+a))-1/6*x^3*ln(d)-1/4/b*d/(1+d)*polylog(2,-(1+d)*exp(2*
b*x+2*a))*x^2-1/6*I*x^3*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/12
*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+
1))^2-1/2/b^3*a^3/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))-1/8/b^3*d/(1+d)*polyl
og(4,-(1+d)*exp(2*b*x+2*a))-1/6*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x^3-1/2/
b^3*a^2/(1+d)*dilog(1+exp(b*x+a)*(-d-1)^(1/2))-1/2/b^3*a^2/(1+d)*dilog(1-ex
p(b*x+a)*(-d-1)^(1/2))+1/3/b^3/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*a^3-1/4/b/(
1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*x^2+1/4/b^3/(1+d)*polylog(2,-(1+d)*ex
p(2*b*x+2*a))*a^2+1/4/b^2/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2*a))*x-1/2/b^3*
a^3/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))+1/4/b^3*d/(1+d)*polylog(2,-(1+d)*ex
p(2*b*x+2*a))*a^2+1/4/b^2*d/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2*a))*x-1/2/b^
2*a^2/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))*x-1/2/b^2*a^2/(1+d)*ln(1-exp(b*x+
a)*(-d-1)^(1/2))*x-1/2/b^3*d*a^3/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))-1/2/b^
3*d*a^3/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))-1/2/b^3*d*a^2/(1+d)*dilog(1+exp
(b*x+a)*(-d-1)^(1/2))-1/2/b^3*d*a^2/(1+d)*dilog(1-exp(b*x+a)*(-d-1)^(1/2))+
1/2/b^2/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x*a^2+1/3/b^3*d/(1+d)*ln(1+(1+d)*e
xp(2*b*x+2*a))*a^3+1/12*b*x^4+1/6/b^3*a^3/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b
*x+2*a)+1)+1/6/b^3*d*a^3/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1)-1/12*I
*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d/(exp(2*b*x+2*a)+
1)*exp(2*b*x+2*a))^2-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+
2*a)/(exp(2*b*x+2*a)+1))^2+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I/
(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))^2-1/2/b^2*d*a^2/(1+
d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))*x+1/2/b^2*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a
))*x*a^2-1/2/b^2*d*a^2/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))*x-1/8/b^3/(1+d)*
polylog(4,-(1+d)*exp(2*b*x+2*a))-1/6/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x^3+1
/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))^3+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(
exp(2*b*x+2*a)+1))^3-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a
)*d+exp(2*b*x+2*a)+1))^3+1/12*I*x^3*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*
x+2*a))+1/12*I*x^3*Pi*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))*csgn(I/(e
xp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))^2-1/12*I*x^3*Pi*csgn(
I*d)*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2+1/6*x^3*ln(exp(2*b*x+2*a
)*d+exp(2*b*x+2*a)+1)+1/12*I*x^3*Pi*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2
*a))^3-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)*d+ex
p(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)
+1))+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I
*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/12*I*x^3*Pi*csgn(I*d)*csgn(I*exp(2*b*
x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))
```

Maxima [A] time = 3.46662, size = 169, normalized size = 1.32

$$\frac{1}{3}\,x^3\,\mathrm{arcoth}\left(d\tanh\left(bx+a\right)+d+1\right)+\frac{1}{36}\left(\frac{3\,x^4}{d}-\frac{2\left(4\,b^3x^3\log\left(\left(d+1\right)e^{\left(2\,bx+2\,a\right)}+1\right)+6\,b^2x^2\mathrm{Li}_2\left(-\left(d+1\right)e^{\left(2\,bx+2\,a\right)}\right)}{b^4d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(d*tanh(b*x + a) + d + 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*log
((d + 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-(d + 1)*e^(2*b*x + 2*a)) -
 6*b*x*polylog(3, -(d + 1)*e^(2*b*x + 2*a)) + 3*polylog(4, -(d + 1)*e^(2*b*
x + 2*a)))/(b^4*d))*b*d
```

Fricas [C] time = 1.76501, size = 1160, normalized size = 9.06

$$\frac{b^4x^4 + 2\,b^3x^3\log\left(\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 6\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{-4\,d-4}(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))\right) - 6\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(b^4*x^4 + 2*b^3*x^3*log(((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/(d*
cosh(b*x + a) + d*sinh(b*x + a))) - 6*b^2*x^2*dilog(1/2*sqrt(-4*d - 4)*(cos
h(b*x + a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-1/2*sqrt(-4*d - 4)*(cosh(b*
x + a) + sinh(b*x + a))) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*si
nh(b*x + a) + sqrt(-4*d - 4)) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d +
1)*sinh(b*x + a) - sqrt(-4*d - 4)) + 12*b*x*polylog(3, 1/2*sqrt(-4*d - 4)*(
cosh(b*x + a) + sinh(b*x + a))) + 12*b*x*polylog(3, -1/2*sqrt(-4*d - 4)*(co
sh(b*x + a) + sinh(b*x + a))) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(-4*d - 4)*(c
osh(b*x + a) + sinh(b*x + a)) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sqrt(-4*d -
 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 12*polylog(4, 1/2*sqrt(-4*d - 4)
*(cosh(b*x + a) + sinh(b*x + a))) - 12*polylog(4, -1/2*sqrt(-4*d - 4)*(cosh
(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(1+d+d*tanh(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arcoth}\left(d\tanh\left(bx+a\right)+d+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(d*tanh(b*x + a) + d + 1), x)
```

3.209 $\int x \coth^{-1}(1 + d + d\tanh(a + bx))\,dx$

Optimal. Leaf size=101

$$\frac{\text{PolyLog}\left(3,-(d+1)e^{2a+2bx}\right)}{8b^2}-\frac{x\text{PolyLog}\left(2,-(d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{4}x^2\log\left((d+1)e^{2a+2bx}+1\right)+\frac{1}{2}x^2\coth^{-1}(d\tanh(a$$

```
[Out] (b*x^3)/6 + (x^2*ArcCoth[1 + d + d*Tanh[a + b*x]])/2 - (x^2*Log[1 + (1 + d)
*E^(2*a + 2*b*x)])/4 - (x*PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))])/(4*b) + P
olyLog[3, -((1 + d)*E^(2*a + 2*b*x))]/(8*b^2)
```

Rubi [A] time = 0.230903, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6240, 2184, 2190, 2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3,-(d+1)e^{2a+2bx}\right)}{8b^2}-\frac{x\text{PolyLog}\left(2,-(d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{4}x^2\log\left((d+1)e^{2a+2bx}+1\right)+\frac{1}{2}x^2\coth^{-1}(d\tanh(a$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcCoth[1 + d + d*Tanh[a + b*x]])/2 - (x^2*Log[1 + (1 + d)
*E^(2*a + 2*b*x)])/4 - (x*PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))])/(4*b) + P
olyLog[3, -((1 + d)*E^(2*a + 2*b*x))]/(8*b^2)
```

Rule 6240

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Tanh[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \frac{1}{2}x^2\coth^{-1}(1+d+d\tanh(a+bx)) + \frac{1}{2}b\int \frac{x^2}{1+(1+d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^3}{6} + \frac{1}{2}x^2\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{2}(b(1+d))\int \frac{e^{2a+2bx}x^2}{1+(1+d)e^{2a+2bx}}$$
$$= \frac{bx^3}{6} + \frac{1}{2}x^2\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{4}x^2\log\left(1+(1+d)e^{2a+2bx}\right) + \frac{1}{2}$$
$$= \frac{bx^3}{6} + \frac{1}{2}x^2\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{4}x^2\log\left(1+(1+d)e^{2a+2bx}\right) - \frac{xL}{}$$
$$= \frac{bx^3}{6} + \frac{1}{2}x^2\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{4}x^2\log\left(1+(1+d)e^{2a+2bx}\right) - \frac{xL}{}$$
$$= \frac{bx^3}{6} + \frac{1}{2}x^2\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{4}x^2\log\left(1+(1+d)e^{2a+2bx}\right) - \frac{xL}{}$$

Mathematica [A] time = 0.0958579, size = 91, normalized size = 0.9

$$\frac{2bx\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{d+1}\right)+\text{PolyLog}\left(3,-\frac{e^{-2(a+bx)}}{d+1}\right)+2b^2x^2\left(2\coth^{-1}(d\tanh(a+bx)+d+1)-\log\left(\frac{e^{-2(a+bx)}}{d+1}+1\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (2*b^2*x^2*(2*ArcCoth[1 + d + d*Tanh[a + b*x]] - Log[1 + 1/((1 + d)*E^(2*(a
 + b*x)))]) + 2*b*x*PolyLog[2, -(1/((1 + d)*E^(2*(a + b*x))))] + PolyLog[3,
 -(1/((1 + d)*E^(2*(a + b*x))))])/(8*b^2)
```

Maple [C] time = 10.194, size = 1584, normalized size = 15.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(1+d+d*tanh(b*x+a)),x)
```

```
[Out] -1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+
1))^2+1/6*b*x^3-1/4/b^2*a^2/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1)+1/2
```

```
/b^2*a/(1+d)*dilog(1-exp(b*x+a)*(-d-1)^(1/2))-1/4/b^2/(1+d)*ln(1+(1+d)*exp(
2*b*x+2*a))*a^2-1/4/b/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*x-1/4/b^2/(1+d
)*polylog(2,-(1+d)*exp(2*b*x+2*a))*a+1/2/b^2*a^2/(1+d)*ln(1+exp(b*x+a)*(-d-
1)^(1/2))+1/2/b^2*a^2/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))+1/8/b^2*d/(1+d)*p
olylog(3,-(1+d)*exp(2*b*x+2*a))+1/2/b^2*a/(1+d)*dilog(1+exp(b*x+a)*(-d-1)^(
1/2))-1/4*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x^2+1/4*x^2*ln(exp(2*b*x+2*a)*
d+exp(2*b*x+2*a)+1)-1/4/b^2*d*a^2/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+
1)+1/8*I*x^2*Pi*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b
*x+2*a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))^2-1/8*I*x^2*Pi*csgn(I*d)*cs
gn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2-1/2*x^2*ln(exp(b*x+a))-1/4*x^2*
ln(d)+1/8*I*x^2*Pi*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^3-1/8*I*x^2*
Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d/(exp(2*b*x+2*a)+1)*ex
p(2*b*x+2*a))^2-1/4/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x^2+1/8/b^2/(1+d)*poly
log(3,-(1+d)*exp(2*b*x+2*a))+1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I
/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))^2+1/8*I*x^2*Pi*csg
n(I*exp(2*b*x+2*a))^3+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)
)^3-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)
+1))^3+1/2/b^2*d*a/(1+d)*dilog(1+exp(b*x+a)*(-d-1)^(1/2))+1/2/b^2*d*a/(1+d)
*dilog(1-exp(b*x+a)*(-d-1)^(1/2))-1/2/b/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x*
a-1/4/b^2*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*a^2-1/4/b*d/(1+d)*polylog(2,-(
1+d)*exp(2*b*x+2*a))*x-1/4/b^2*d/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*a+1
/2/b*a/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))*x+1/2/b*a/(1+d)*ln(1-exp(b*x+a)*
(-d-1)^(1/2))*x+1/2/b^2*d*a^2/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))+1/2/b^2*d
*a^2/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a
)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/8*I*x^2*Pi*csgn(I*exp(b
*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/4*I*x^2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(
2*b*x+2*a))^2+1/8*I*x^2*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+
1))*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))+1/8*I*x^2*Pi*csgn(I/(exp(2*
b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1
))-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b
*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))-
1/2/b*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x*a+1/2/b*d*a/(1+d)*ln(1+exp(b*x+a
)*(-d-1)^(1/2))*x+1/2/b*d*a/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))*x
```

Maxima [A] time = 3.48612, size = 136, normalized size = 1.35

$$\frac{1}{24}\left(\frac{4x^3}{d}-\frac{3\left(2b^2x^2\log\left((d+1)e^{(2bx+2a)}+1\right)+2bx\text{Li}_2\left(-(d+1)e^{(2bx+2a)}\right)-\text{Li}_3(-(d+1)e^{(2bx+2a)})\right)}{b^3d}\right)bd+\frac{1}{2}x^2\,\text{arcot}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/24*(4*x^3/d - 3*(2*b^2*x^2*log((d + 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dilog
(-(d + 1)*e^(2*b*x + 2*a)) - polylog(3, -(d + 1)*e^(2*b*x + 2*a)))/(b^3*d))
*b*d + 1/2*x^2*arccoth(d*tanh(b*x + a) + d + 1)
```

Fricas [C] time = 1.74289, size = 953, normalized size = 9.44

$$2b^3x^3+3b^2x^2\log\left(\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right)-6bx\text{Li}_2\left(\frac{1}{2}\sqrt{-4d-4}(\cosh(bx+a)+\sinh(bx+a))\right)-6bx\text{Li}_2\left(-\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 + 3*b^2*x^2*log(((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/(
d*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b*x*dilog(1/2*sqrt(-4*d - 4)*(cosh(
b*x + a) + sinh(b*x + a))) - 6*b*x*dilog(-1/2*sqrt(-4*d - 4)*(cosh(b*x + a)
 + sinh(b*x + a))) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x
 + a) + sqrt(-4*d - 4)) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sin
h(b*x + a) - sqrt(-4*d - 4)) - 3*(b^2*x^2 - a^2)*log(1/2*sqrt(-4*d - 4)*(co
sh(b*x + a) + sinh(b*x + a)) + 1) - 3*(b^2*x^2 - a^2)*log(-1/2*sqrt(-4*d -
4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 6*polylog(3, 1/2*sqrt(-4*d - 4)*(
cosh(b*x + a) + sinh(b*x + a))) + 6*polylog(3, -1/2*sqrt(-4*d - 4)*(cosh(b*
x + a) + sinh(b*x + a))))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(1+d+d*tanh(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arcoth}\left(d \tanh\left(bx+a\right)+d+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(d*tanh(b*x + a) + d + 1), x)
```

3.210 $\int \coth^{-1}(1+d+d\tanh(a+bx))\,dx$

Optimal. Leaf size=69

$$-\frac{\text{PolyLog}\left(2,-(d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{2}x\log\left((d+1)e^{2a+2bx}+1\right)+x\coth^{-1}(d\tanh(a+bx)+d+1)+\frac{bx^2}{2}$$

```
[Out] (b*x^2)/2 + x*ArcCoth[1 + d + d*Tanh[a + b*x]] - (x*Log[1 + (1 + d)*E^(2*a
+ 2*b*x)])/2 - PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))]/(4*b)
```

Rubi [A] time = 0.140006, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6232, 2184, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,-(d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{2}x\log\left((d+1)e^{2a+2bx}+1\right)+x\coth^{-1}(d\tanh(a+bx)+d+1)+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcCoth[1 + d + d*Tanh[a + b*x]] - (x*Log[1 + (1 + d)*E^(2*a
+ 2*b*x)])/2 - PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))]/(4*b)
```

Rule 6232

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Ar
cCoth[c + d*Tanh[a + b*x]], x] + Dist[b, Int[x/(c - d + c*E^(2*a + 2*b*x)),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \coth^{-1}(1+d+d\tanh(a+bx))\,dx = x\coth^{-1}(1+d+d\tanh(a+bx)) + b\int \frac{x}{1+(1+d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^2}{2} + x\coth^{-1}(1+d+d\tanh(a+bx)) - (b(1+d))\int \frac{e^{2a+2bx}x}{1+(1+d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^2}{2} + x\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{2}x\log\left(1+(1+d)e^{2a+2bx}\right) + \frac{1}{2}\int \log$$
$$= \frac{bx^2}{2} + x\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{2}x\log\left(1+(1+d)e^{2a+2bx}\right) + \frac{\text{Subst}\left(\right.}{}$$
$$= \frac{bx^2}{2} + x\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{2}x\log\left(1+(1+d)e^{2a+2bx}\right) - \frac{\text{Li}_2\left(-(1\right.}{}$$

Mathematica [B] time = 0.858181, size = 201, normalized size = 2.91

$$\frac{-2\text{PolyLog}\left(2,-\sqrt{-d-1}e^{a+bx}\right) - 2\text{PolyLog}\left(2,\sqrt{-d-1}e^{a+bx}\right) - 2\log\left(e^{a+bx}\right)\log\left(1-\sqrt{-d-1}e^{a+bx}\right) - 2\log\left(e^{a+bx}\right)\log\left(\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] x*ArcCoth[1 + d + d*Tanh[a + b*x]] + (b^2*x^2 + Log[E^(a + b*x)]^2 - 2*Log[
E^(a + b*x)]*Log[1 - Sqrt[-1 - d]*E^(a + b*x)] - 2*Log[E^(a + b*x)]*Log[1 +
 Sqrt[-1 - d]*E^(a + b*x)] + 2*Log[E^(a + b*x)]*Log[E^(-a - b*x) + (1 + d)*
E^(a + b*x)] - 2*b*x*Log[(2 + d)*Cosh[a + b*x] + d*Sinh[a + b*x]] - 2*PolyL
og[2, -(Sqrt[-1 - d]*E^(a + b*x))] - 2*PolyLog[2, Sqrt[-1 - d]*E^(a + b*x)]
)/(4*b)
```

Maple [B] time = 0.156, size = 247, normalized size = 3.6

$$-\frac{\text{arccoth}\left(1+d+d\tanh\left(bx+a\right)\right)\ln\left(d\tanh\left(bx+a\right)-d\right)}{2b} + \frac{\text{arccoth}\left(1+d+d\tanh\left(bx+a\right)\right)\ln\left(d\tanh\left(bx+a\right)+d\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1+d+d*tanh(b*x+a)),x)
```

```
[Out] -1/2/b*arccoth(1+d+d*tanh(b*x+a))*ln(d*tanh(b*x+a)-d)+1/2/b*arccoth(1+d+d*t
anh(b*x+a))*ln(d*tanh(b*x+a)+d)+1/4/b*dilog((d*tanh(b*x+a)+d+2)/(2*d+2))+1/
4/b*ln(d*tanh(b*x+a)-d)*ln((d*tanh(b*x+a)+d+2)/(2*d+2))-1/4/b*dilog(1/2*(d*
tanh(b*x+a)+d)/d)-1/4/b*ln(d*tanh(b*x+a)-d)*ln(1/2*(d*tanh(b*x+a)+d)/d)+1/8
/b*ln(d*tanh(b*x+a)+d)^2-1/4/b*dilog(1+1/2*d*tanh(b*x+a)+1/2*d)-1/4/b*ln(d*
tanh(b*x+a)+d)*ln(1+1/2*d*tanh(b*x+a)+1/2*d)
```

Maxima [A] time = 3.44926, size = 97, normalized size = 1.41

$$\frac{1}{4}bd\left(\frac{2x^2}{d} - \frac{2bx\log\left((d+1)e^{(2bx+2a)}+1\right) + \text{Li}_2\left(-(d+1)e^{(2bx+2a)}\right)}{b^2d}\right) + x\,\text{arccoth}\left(d\tanh\left(bx+a\right)+d+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*b*d*(2*x^2/d - (2*b*x*log((d + 1)*e^(2*b*x + 2*a) + 1) + dilog(-(d + 1)
*e^(2*b*x + 2*a)))/(b^2*d)) + x*arccoth(d*tanh(b*x + a) + d + 1)
```

Fricas [B] time = 2.13949, size = 707, normalized size = 10.25

$$b^2x^2 + bx\log\left(\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) + a\log\left(2\,(d+1)\cosh\,(bx+a) + 2\,(d+1)\sinh\,(bx+a) + \sqrt{-4\,d-4}\right) + a\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + b*x*log(((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/(d*cosh(b*
x + a) + d*sinh(b*x + a))) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh
(b*x + a) + sqrt(-4*d - 4)) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sin
h(b*x + a) - sqrt(-4*d - 4)) - (b*x + a)*log(1/2*sqrt(-4*d - 4)*(cosh(b*x +
 a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-1/2*sqrt(-4*d - 4)*(cosh(b*x + a
) + sinh(b*x + a)) + 1) - dilog(1/2*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*
x + a))) - dilog(-1/2*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acoth}\left(d\tanh\left(a+bx\right)+d+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1+d+d*tanh(b*x+a)),x)
```

```
[Out] Integral(acoth(d*tanh(a + b*x) + d + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arccoth}\left(d\tanh\left(bx+a\right)+d+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*tanh(b*x + a) + d + 1), x)
```

3.211 $\int \frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x}\,dx$

Optimal. Leaf size=18

$$\text{CannotIntegrate}\left(\frac{\coth^{-1}(d\tanh(a+bx)+d+1)}{x},x\right)$$

```
[Out] CannotIntegrate[ArcCoth[1 + d + d*Tanh[a + b*x]]/x, x]
```

Rubi [A] time = 0.0772992, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCoth[1 + d + d*Tanh[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcCoth[1 + d + d*Tanh[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x}\,dx=\int \frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x}\,dx$$

Mathematica [A] time = 2.97231, size = 0, normalized size = 0.

$$\int \frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCoth[1 + d + d*Tanh[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcCoth[1 + d + d*Tanh[a + b*x]]/x, x]
```

Maple [A] time = 0.364, size = 0, normalized size = 0.

$$\int \frac{\mathrm{arccoth}\left(1+d+d\tanh\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1+d+d*tanh(b*x+a))/x,x)
```

```
[Out] int(arccoth(1+d+d*tanh(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(d\tanh(bx+a)+d+1)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+d+d*tanh(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arccoth(d*tanh(b*x + a) + d + 1)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arcoth}(d\tanh(bx+a)+d+1)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+d+d*tanh(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(d*tanh(b*x + a) + d + 1)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}(d\tanh(a+bx)+d+1)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1+d+d*tanh(b*x+a))/x,x)
```

```
[Out] Integral(acoth(d*tanh(a + b*x) + d + 1)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(d\tanh(bx+a)+d+1)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+d+d*tanh(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*tanh(b*x + a) + d + 1)/x, x)
```

3.212 $\int x^3 \coth^{-1}(1-d-d\tanh(a+bx))\,dx$

Optimal. Leaf size=168

$$\frac{3x^2\text{PolyLog}\left(3,-(1-d)e^{2a+2bx}\right)}{8b^2}-\frac{3x\text{PolyLog}\left(4,-(1-d)e^{2a+2bx}\right)}{8b^3}+\frac{3\text{PolyLog}\left(5,-(1-d)e^{2a+2bx}\right)}{16b^4}-\frac{x^3\text{PolyLog}\left(2,-(\right.}{4b}$$

```
[Out] (b*x^5)/20 + (x^4*ArcCoth[1 - d - d*Tanh[a + b*x]])/4 - (x^4*Log[1 + (1 - d
)*E^(2*a + 2*b*x)])/8 - (x^3*PolyLog[2, -((1 - d)*E^(2*a + 2*b*x))])/(4*b)
+ (3*x^2*PolyLog[3, -((1 - d)*E^(2*a + 2*b*x))])/(8*b^2) - (3*x*PolyLog[4,
-((1 - d)*E^(2*a + 2*b*x))])/(8*b^3) + (3*PolyLog[5, -((1 - d)*E^(2*a + 2*b
*x))])/(16*b^4)
```

Rubi [A] time = 0.30328, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6240, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(3,-(1-d)e^{2a+2bx}\right)}{8b^2}-\frac{3x\text{PolyLog}\left(4,-(1-d)e^{2a+2bx}\right)}{8b^3}+\frac{3\text{PolyLog}\left(5,-(1-d)e^{2a+2bx}\right)}{16b^4}-\frac{x^3\text{PolyLog}\left(2,-(\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^5)/20 + (x^4*ArcCoth[1 - d - d*Tanh[a + b*x]])/4 - (x^4*Log[1 + (1 - d
)*E^(2*a + 2*b*x)])/8 - (x^3*PolyLog[2, -((1 - d)*E^(2*a + 2*b*x))])/(4*b)
+ (3*x^2*PolyLog[3, -((1 - d)*E^(2*a + 2*b*x))])/(8*b^2) - (3*x*PolyLog[4,
-((1 - d)*E^(2*a + 2*b*x))])/(8*b^3) + (3*PolyLog[5, -((1 - d)*E^(2*a + 2*b
*x))])/(16*b^4)
```

Rule 6240

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Tanh[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3\coth^{-1}(1-d-d\tanh(a+bx))\,dx = \frac{1}{4}x^4\coth^{-1}(1-d-d\tanh(a+bx))+\frac{1}{4}b\int\frac{x^4}{1+(1-d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^5}{20}+\frac{1}{4}x^4\coth^{-1}(1-d-d\tanh(a+bx))-\frac{1}{4}(b(1-d))\int\frac{e^{2a+2bx}x^4}{1+(1-d)e^{2a+2}}$$
$$= \frac{bx^5}{20}+\frac{1}{4}x^4\coth^{-1}(1-d-d\tanh(a+bx))-\frac{1}{8}x^4\log\left(1+(1-d)e^{2a+2bx}\right)+$$
$$= \frac{bx^5}{20}+\frac{1}{4}x^4\coth^{-1}(1-d-d\tanh(a+bx))-\frac{1}{8}x^4\log\left(1+(1-d)e^{2a+2bx}\right)-$$
$$= \frac{bx^5}{20}+\frac{1}{4}x^4\coth^{-1}(1-d-d\tanh(a+bx))-\frac{1}{8}x^4\log\left(1+(1-d)e^{2a+2bx}\right)-$$
$$= \frac{bx^5}{20}+\frac{1}{4}x^4\coth^{-1}(1-d-d\tanh(a+bx))-\frac{1}{8}x^4\log\left(1+(1-d)e^{2a+2bx}\right)-$$
$$= \frac{bx^5}{20}+\frac{1}{4}x^4\coth^{-1}(1-d-d\tanh(a+bx))-\frac{1}{8}x^4\log\left(1+(1-d)e^{2a+2bx}\right)-$$
$$= \frac{bx^5}{20}+\frac{1}{4}x^4\coth^{-1}(1-d-d\tanh(a+bx))-\frac{1}{8}x^4\log\left(1+(1-d)e^{2a+2bx}\right)-$$

Mathematica [A] time = 0.185126, size = 144, normalized size = 0.86

$$\frac{1}{16}\left(\frac{6x^2\text{PolyLog}\left(3,\frac{e^{-2(a+bx)}}{d-1}\right)}{b^2}+\frac{6x\text{PolyLog}\left(4,\frac{e^{-2(a+bx)}}{d-1}\right)}{b^3}+\frac{3\text{PolyLog}\left(5,\frac{e^{-2(a+bx)}}{d-1}\right)}{b^4}+\frac{4x^3\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{d-1}\right)}{b}-2x^4\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (4*x^4*ArcCoth[1 - d - d*Tanh[a + b*x]] - 2*x^4*Log[1 - 1/((-1 + d)*E^(2*(a
 + b*x)))] + (4*x^3*PolyLog[2, 1/((-1 + d)*E^(2*(a + b*x)))])/b + (6*x^2*Po
lyLog[3, 1/((-1 + d)*E^(2*(a + b*x)))])/b^2 + (6*x*PolyLog[4, 1/((-1 + d)*E
^(2*(a + b*x)))])/b^3 + (3*PolyLog[5, 1/((-1 + d)*E^(2*(a + b*x)))])/b^4)/1
6
```

Maple [C] time = 39.194, size = 1802, normalized size = 10.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccoth(1-d-d*tanh(b*x+a)),x)
```

```
[Out] -1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*
x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))+1
/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2
*b*x+2*a)/(exp(2*b*x+2*a)+1))+3/16/b^4*d/(d-1)*polylog(5,(d-1)*exp(2*b*x+2*
a))-1/2/b^4*a^3/(d-1)*dilog(1-exp(b*x+a)*(d-1)^(1/2))+1/2/b^4*d*a^3/(d-1)*d
ilog(1-exp(b*x+a)*(d-1)^(1/2))-3/16/b^4/(d-1)*polylog(5,(d-1)*exp(2*b*x+2*a
))+1/8/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x^4+3/8/b^4/(d-1)*ln(1-(d-1)*exp(2*
b*x+2*a))*a^4+1/4/b/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*x^3+1/4/b^4/(d-1)
*polylog(2,(d-1)*exp(2*b*x+2*a))*a^3-1/2/b^4*a^4/(d-1)*ln(1-exp(b*x+a)*(d-1
)^(1/2))+1/2/b^4*d*a^3/(d-1)*dilog(1+exp(b*x+a)*(d-1)^(1/2))-1/2/b^3*a^3/(d
-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))*x-1/2/b^3*a^3/(d-1)*ln(1+exp(b*x+a)*(d-1)^
(1/2))*x-3/8/b^4*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*a^4-1/4/b*d/(d-1)*polyl
og(2,(d-1)*exp(2*b*x+2*a))*x^3-1/4/b^4*d/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*
a))*a^3+3/8/b^2*d/(d-1)*polylog(3,(d-1)*exp(2*b*x+2*a))*x^2-3/8/b^3*d/(d-1)
*polylog(4,(d-1)*exp(2*b*x+2*a))*x+1/2/b^4*d*a^4/(d-1)*ln(1-exp(b*x+a)*(d-1
)^(1/2))+1/2/b^4*d*a^4/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))+1/2/b^3/(d-1)*ln(
1-(d-1)*exp(2*b*x+2*a))*x*a^3+1/16*I*x^4*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp
(2*b*x+2*a))-1/8*I*x^4*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/16*
I*x^4*Pi*csgn(I*exp(2*b*x+2*a))^3+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(
2*b*x+2*a)+1))^3+1/20*b*x^5-1/16*I*x^4*Pi*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2
*b*x+2*a))^3-1/16*I*x^4*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+
2*a))^2+1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*
(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))^2+1/16*I*x^4*Pi*csgn(I*(exp(2*b*x+2*a)
*d-exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x
+2*a)-1))^2-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/
(exp(2*b*x+2*a)+1))^2-1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x
+2*a)/(exp(2*b*x+2*a)+1))^2+1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*
b*x+2*a)*d-exp(2*b*x+2*a)-1))^3-1/2/b^4*a^3/(d-1)*dilog(1+exp(b*x+a)*(d-1)^
(1/2))+1/8*x^4*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1)-1/2/b^4*a^4/(d-1)*ln(1
+exp(b*x+a)*(d-1)^(1/2))-1/8*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x^4+1/16*I*
x^4*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d/(exp(2*
b*x+2*a)+1)*exp(2*b*x+2*a))-1/8*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b
*x+2*a)*d-exp(2*b*x+2*a)-1))^2+1/8*I*x^4*Pi*csgn(I*d/(exp(2*b*x+2*a)+1)*exp
(2*b*x+2*a))^2-1/8/b^4*d*a^4/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1)-1/
16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d/(exp(2*b*x+2
*a)+1)*exp(2*b*x+2*a))^2+1/8/b^4*a^4/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*
a)-1)-1/4*x^4*ln(exp(b*x+a))-1/8*x^4*ln(d)-3/8/b^2/(d-1)*polylog(3,(d-1)*ex
p(2*b*x+2*a))*x^2+3/8/b^3/(d-1)*polylog(4,(d-1)*exp(2*b*x+2*a))*x-1/2/b^3*d
/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x*a^3+1/2/b^3*d*a^3/(d-1)*ln(1-exp(b*x+a)
*(d-1)^(1/2))*x+1/2/b^3*d*a^3/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))*x
```

Maxima [A] time = 3.4718, size = 197, normalized size = 1.17

$$-\frac{1}{4}\,x^4\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+d-1\right)+\frac{1}{40}\left(\frac{2\,x^5}{d}-\frac{5\left(2\,b^4x^4\log\left(-(d-1)e^{(2\,bx+2\,a)}+1\right)+4\,b^3x^3\mathrm{Li}_2\left((d-1)e^{(2\,bx+2\,a)}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/4*x^4*arccoth(d*tanh(b*x + a) + d - 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*lo
g(-(d - 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog((d - 1)*e^(2*b*x + 2*a))
- 6*b^2*x^2*polylog(3, (d - 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, (d - 1)*
e^(2*b*x + 2*a)) - 3*polylog(5, (d - 1)*e^(2*b*x + 2*a)))/(b^5*d))*b*d
```

Fricas [C] time = 2.09938, size = 1280, normalized size = 7.62

$$\frac{2\,b^5x^5-5\,b^4x^4\log\left(\frac{d\cosh(bx+a)+d\sinh(bx+a)}{(d-2)\cosh(bx+a)+d\sinh(bx+a)}\right)-20\,b^3x^3\mathrm{Li}_2\left(\sqrt{d-1}(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))\right)-20\,b^3x^3\mathrm{Li}_2\left(-\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/40*(2*b^5*x^5 - 5*b^4*x^4*log((d*cosh(b*x + a) + d*sinh(b*x + a))/((d - 2
)*cosh(b*x + a) + d*sinh(b*x + a))) - 20*b^3*x^3*dilog(sqrt(d - 1)*(cosh(b*
x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-sqrt(d - 1)*(cosh(b*x + a) + s
inh(b*x + a))) - 5*a^4*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a
) + 2*sqrt(d - 1)) - 5*a^4*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x
 + a) - 2*sqrt(d - 1)) + 60*b^2*x^2*polylog(3, sqrt(d - 1)*(cosh(b*x + a) +
 sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -sqrt(d - 1)*(cosh(b*x + a) + sinh
(b*x + a))) - 120*b*x*polylog(4, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)
)) - 120*b*x*polylog(4, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) - 5*(
b^4*x^4 - a^4)*log(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 5*(b^
4*x^4 - a^4)*log(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 120*po
lylog(5, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) + 120*polylog(5, -sqr
t(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acoth(1-d-d*tanh(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\operatorname{arcoth}\left(-d\tanh\left(bx+a\right)-d+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^3*arccoth(-d*tanh(b*x + a) - d + 1), x)
```

3.213 $\int x^2 \coth^{-1}(1-d-d\tanh(a+bx))\,dx$

Optimal. Leaf size=139

$$\frac{x\text{PolyLog}\left(3,-(1-d)e^{2a+2bx}\right)}{4b^2}-\frac{\text{PolyLog}\left(4,-(1-d)e^{2a+2bx}\right)}{8b^3}-\frac{x^2\text{PolyLog}\left(2,-(1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{6}x^3\log\left((1-d)e^{2a}\right.$$

```
[Out] (b*x^4)/12 + (x^3*ArcCoth[1 - d - d*Tanh[a + b*x]])/3 - (x^3*Log[1 + (1 - d
)*E^(2*a + 2*b*x)])/6 - (x^2*PolyLog[2, -((1 - d)*E^(2*a + 2*b*x))])/(4*b)
+ (x*PolyLog[3, -((1 - d)*E^(2*a + 2*b*x))])/(4*b^2) - PolyLog[4, -((1 - d)
*E^(2*a + 2*b*x))]/(8*b^3)
```

Rubi [A] time = 0.268458, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6240, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{x\text{PolyLog}\left(3,-(1-d)e^{2a+2bx}\right)}{4b^2}-\frac{\text{PolyLog}\left(4,-(1-d)e^{2a+2bx}\right)}{8b^3}-\frac{x^2\text{PolyLog}\left(2,-(1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{6}x^3\log\left((1-d)e^{2a}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^4)/12 + (x^3*ArcCoth[1 - d - d*Tanh[a + b*x]])/3 - (x^3*Log[1 + (1 - d
)*E^(2*a + 2*b*x)])/6 - (x^2*PolyLog[2, -((1 - d)*E^(2*a + 2*b*x))])/(4*b)
+ (x*PolyLog[3, -((1 - d)*E^(2*a + 2*b*x))])/(4*b^2) - PolyLog[4, -((1 - d)
*E^(2*a + 2*b*x))]/(8*b^3)
```

Rule 6240

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Tanh[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \coth^{-1}(1-d-d\tanh(a+bx))\,dx = \frac{1}{3}x^3\coth^{-1}(1-d-d\tanh(a+bx)) + \frac{1}{3}b\int \frac{x^3}{1+(1-d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{3}(b(1-d))\int \frac{e^{2a+2bx}x^3}{1+(1-d)e^{2a+2bx}}$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-d)e^{2a+2bx}\right) + \frac{1}{2}$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-d)e^{2a+2bx}\right) - \frac{x^2}{}$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-d)e^{2a+2bx}\right) - \frac{x^2}{}$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-d)e^{2a+2bx}\right) - \frac{x^2}{}$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-d)e^{2a+2bx}\right) - \frac{x^2}{}$$

Mathematica [A] time = 0.102574, size = 119, normalized size = 0.86

$$\frac{1}{24}\left(\frac{6x\text{PolyLog}\left(3,\frac{e^{-2(a+bx)}}{d-1}\right)}{b^2} + \frac{3\text{PolyLog}\left(4,\frac{e^{-2(a+bx)}}{d-1}\right)}{b^3} + \frac{6x^2\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{d-1}\right)}{b} - 4x^3\log\left(1-\frac{e^{-2(a+bx)}}{d-1}\right) + 8x^3\coth^{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (8*x^3*ArcCoth[1 - d - d*Tanh[a + b*x]] - 4*x^3*Log[1 - 1/((-1 + d)*E^(2*(a
 + b*x)))] + (6*x^2*PolyLog[2, 1/((-1 + d)*E^(2*(a + b*x)))])/b + (6*x*Poly
Log[3, 1/((-1 + d)*E^(2*(a + b*x)))])/b^2 + (3*PolyLog[4, 1/((-1 + d)*E^(2*
```

```
(a + b*x)))])/b^3)/24
```

Maple [C] time = 33.599, size = 1745, normalized size = 12.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(1-d-d*tanh(b*x+a)),x)
```

```
[Out] -1/6*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1)
)^2-1/2/b^3*d*a^2/(d-1)*dilog(1-exp(b*x+a)*(d-1)^(1/2))-1/2/b^3*d*a^2/(d-1)
*dilog(1+exp(b*x+a)*(d-1)^(1/2))+1/3/b^3*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))
*a^3-1/4/b*d/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*x^2+1/4/b^3*d/(d-1)*poly
log(2,(d-1)*exp(2*b*x+2*a))*a^2+1/4/b^2*d/(d-1)*polylog(3,(d-1)*exp(2*b*x+2
*a))*x-1/2/b^2/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x*a^2+1/2/b^2*a^2/(d-1)*ln(
1-exp(b*x+a)*(d-1)^(1/2))*x-1/2/b^3*d*a^3/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2)
)-1/2/b^3*d*a^3/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))-1/3*x^3*ln(exp(b*x+a))-1
/6*x^3*ln(d)-1/6*I*x^3*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/12*
I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1
))^2+1/12*b*x^4+1/8/b^3/(d-1)*polylog(4,(d-1)*exp(2*b*x+2*a))+1/6/(d-1)*ln(
1-(d-1)*exp(2*b*x+2*a))*x^3+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I
/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))^2+1/12*I*x^3*Pi*cs
gn(I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*
b*x+2*a)*d-exp(2*b*x+2*a)-1))^2-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*
b*x+2*a)+1))*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2+1/2/b^2*a^2/(d-1
)*ln(1+exp(b*x+a)*(d-1)^(1/2))*x+1/6*x^3*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)
-1)+1/6/b^3*d*a^3/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1)-1/12*I*x^3*Pi
*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/12*I*
x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1
))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))-1/6/b^3*a
^3/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1)+1/12*I*x^3*Pi*csgn(I/(exp(2*
b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))^3+1/12*I*x^3*Pi*csgn(I*exp
(2*b*x+2*a))^3+1/6*I*x^3*Pi*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2-1
/12*I*x^3*Pi*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^3+1/12*I*x^3*Pi*cs
gn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/6*d/(d-1)*ln(1-(d-1)*exp(2*b*x+
2*a))*x^3-1/3/b^3/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*a^3+1/4/b/(d-1)*polylog(
2,(d-1)*exp(2*b*x+2*a))*x^2-1/4/b^3/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*a
^2-1/4/b^2/(d-1)*polylog(3,(d-1)*exp(2*b*x+2*a))*x+1/2/b^3*a^3/(d-1)*ln(1-e
xp(b*x+a)*(d-1)^(1/2))+1/2/b^3*a^3/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))-1/8/b
^3*d/(d-1)*polylog(4,(d-1)*exp(2*b*x+2*a))+1/2/b^3*a^2/(d-1)*dilog(1-exp(b*
x+a)*(d-1)^(1/2))+1/2/b^3*a^2/(d-1)*dilog(1+exp(b*x+a)*(d-1)^(1/2))+1/12*I*
x^3*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/12*I*x^3*Pi*csgn(I*d)*
csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2+1/12*I*x^3*Pi*csgn(I/(exp(2*b
*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)
)+1/12*I*x^3*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*
d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))+1/2/b^2*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2
*a))*x*a^2-1/2/b^2*d*a^2/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))*x-1/2/b^2*d*a^2
/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))*x
```

Maxima [A] time = 3.50631, size = 166, normalized size = 1.19

$$-\frac{1}{3}x^3 \operatorname{arcoth}\left(d\tanh\left(bx+a\right)+d-1\right)+\frac{1}{36}\left(\frac{3x^4}{d}-\frac{2\left(4b^3x^3\log\left(-(d-1)e^{(2bx+2a)}+1\right)+6b^2x^2\mathrm{Li}_2\left((d-1)e^{(2bx+2a)}\right.\right.}{b^4d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/3*x^3*arccoth(d*tanh(b*x + a) + d - 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*lo
g(-(d - 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog((d - 1)*e^(2*b*x + 2*a))
- 6*b*x*polylog(3, (d - 1)*e^(2*b*x + 2*a)) + 3*polylog(4, (d - 1)*e^(2*b*x
 + 2*a)))/(b^4*d))*b*d
```

Fricas [C] time = 2.13774, size = 1081, normalized size = 7.78

$$b^4x^4 - 2\,b^3x^3\log\left(\frac{d\cosh(bx+a)+d\sinh(bx+a)}{(d-2)\cosh(bx+a)+d\sinh(bx+a)}\right) - 6\,b^2x^2\mathrm{Li}_2\left(\sqrt{d-1}(\cosh(bx+a)+\sinh(bx+a))\right) - 6\,b^2x^2\mathrm{Li}_2\left(-\sqrt{d-1}(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(b^4*x^4 - 2*b^3*x^3*log((d*cosh(b*x + a) + d*sinh(b*x + a))/((d - 2)*
cosh(b*x + a) + d*sinh(b*x + a))) - 6*b^2*x^2*dilog(sqrt(d - 1)*(cosh(b*x +
 a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-sqrt(d - 1)*(cosh(b*x + a) + sinh(
b*x + a))) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a) +
2*sqrt(d - 1)) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a
) - 2*sqrt(d - 1)) + 12*b*x*polylog(3, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*
x + a))) + 12*b*x*polylog(3, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)))
- 2*(b^3*x^3 + a^3)*log(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) -
2*(b^3*x^3 + a^3)*log(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 1
2*polylog(4, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) - 12*polylog(4, -
sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(1-d-d*tanh(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arcoth}(-d\tanh(bx+a) - d + 1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(-d*tanh(b*x + a) - d + 1), x)
```

3.214 $\int x \coth^{-1}(1-d-d\tanh(a+bx))\,dx$

Optimal. Leaf size=110

$$\frac{\text{PolyLog}\left(3,-(1-d)e^{2a+2bx}\right)}{8b^2}-\frac{x\text{PolyLog}\left(2,-(1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{4}x^2\log\left((1-d)e^{2a+2bx}+1\right)+\frac{1}{2}x^2\coth^{-1}(d(-\tanh($$

```
[Out] (b*x^3)/6 + (x^2*ArcCoth[1 - d - d*Tanh[a + b*x]])/2 - (x^2*Log[1 + (1 - d)
*E^(2*a + 2*b*x)])/4 - (x*PolyLog[2, -((1 - d)*E^(2*a + 2*b*x))])/(4*b) + P
olyLog[3, -((1 - d)*E^(2*a + 2*b*x))]/(8*b^2)
```

Rubi [A] time = 0.238174, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6240, 2184, 2190, 2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3,-(1-d)e^{2a+2bx}\right)}{8b^2}-\frac{x\text{PolyLog}\left(2,-(1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{4}x^2\log\left((1-d)e^{2a+2bx}+1\right)+\frac{1}{2}x^2\coth^{-1}(d(-\tanh($$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcCoth[1 - d - d*Tanh[a + b*x]])/2 - (x^2*Log[1 + (1 - d)
*E^(2*a + 2*b*x)])/4 - (x*PolyLog[2, -((1 - d)*E^(2*a + 2*b*x))])/(4*b) + P
olyLog[3, -((1 - d)*E^(2*a + 2*b*x))]/(8*b^2)
```

Rule 6240

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Tanh[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\coth^{-1}(1-d-d\tanh(a+bx))\,dx = \frac{1}{2}x^2\coth^{-1}(1-d-d\tanh(a+bx))+\frac{1}{2}b\int\frac{x^2}{1+(1-d)e^{2a+2bx}}\,dx$$
$$=\frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1-d-d\tanh(a+bx))-\frac{1}{2}(b(1-d))\int\frac{e^{2a+2bx}x^2}{1+(1-d)e^{2a+2bx}}\,d$$
$$=\frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1-d-d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1-d)e^{2a+2bx}\right)+\frac{1}{2}\int$$
$$=\frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1-d-d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1-d)e^{2a+2bx}\right)-\frac{x\mathrm{Li}}{}$$
$$=\frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1-d-d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1-d)e^{2a+2bx}\right)-\frac{x\mathrm{Li}}{}$$
$$=\frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1-d-d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1-d)e^{2a+2bx}\right)-\frac{x\mathrm{Li}}{}$$

Mathematica [A] time = 0.0907819, size = 93, normalized size = 0.85

$$\frac{2bx\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{d-1}\right)+\text{PolyLog}\left(3,\frac{e^{-2(a+bx)}}{d-1}\right)+2b^2x^2\left(2\coth^{-1}(d(-\tanh(a+bx))-d+1)-\log\left(1-\frac{e^{-2(a+bx)}}{d-1}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (2*b^2*x^2*(2*ArcCoth[1 - d - d*Tanh[a + b*x]] - Log[1 - 1/((-1 + d)*E^(2*(
a + b*x)))]) + 2*b*x*PolyLog[2, 1/((-1 + d)*E^(2*(a + b*x)))] + PolyLog[3,
1/((-1 + d)*E^(2*(a + b*x)))])/(8*b^2)
```

Maple [C] time = 16.123, size = 1664, normalized size = 15.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(1-d-d*tanh(b*x+a)),x)
```

```
[Out] 1/2/b^2*d*a^2/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))+1/2/b^2*d*a/(d-1)*dilog(1-
exp(b*x+a)*(d-1)^(1/2))+1/2/b^2*d*a/(d-1)*dilog(1+exp(b*x+a)*(d-1)^(1/2))-1
```

```
/4/b^2*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*a^2-1/4/b*d/(d-1)*polylog(2,(d-1)
*exp(2*b*x+2*a))*x-1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a
)/(exp(2*b*x+2*a)+1))^2+1/6*b*x^3+1/8*I*x^2*Pi*csgn(I*(exp(2*b*x+2*a)*d-exp
(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-
1))^2+1/4/b^2/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*a^2+1/4/b/(d-1)*polylog(2,(d
-1)*exp(2*b*x+2*a))*x+1/4/b^2/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*a-1/2/b
^2*a^2/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))-1/2/b^2*a^2/(d-1)*ln(1+exp(b*x+a)
*(d-1)^(1/2))+1/8/b^2*d/(d-1)*polylog(3,(d-1)*exp(2*b*x+2*a))-1/2/b^2*a/(d-
1)*dilog(1-exp(b*x+a)*(d-1)^(1/2))-1/2/b^2*a/(d-1)*dilog(1+exp(b*x+a)*(d-1)
^(1/2))-1/4*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x^2+1/4/b^2*a^2/(d-1)*ln(exp
(2*b*x+2*a)*d-exp(2*b*x+2*a)-1)+1/4*I*x^2*Pi*csgn(I*d/(exp(2*b*x+2*a)+1)*ex
p(2*b*x+2*a))^2+1/4/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x^2-1/8/b^2/(d-1)*poly
log(3,(d-1)*exp(2*b*x+2*a))-1/8*I*x^2*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)
+1)*exp(2*b*x+2*a))^2-1/2*x^2*ln(exp(b*x+a))-1/4*x^2*ln(d)+1/8*I*x^2*Pi*csg
n(I/(exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b
*x+2*a)-1))^2-1/4/b^2*d/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*a+1/2/b/(d-1)
*ln(1-(d-1)*exp(2*b*x+2*a))*x*a-1/2/b*a/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))*
x-1/2/b*a/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))*x+1/2/b^2*d*a^2/(d-1)*ln(1-exp
(b*x+a)*(d-1)^(1/2))-1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))
*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2+1/4*x^2*ln(exp(2*b*x+2*a)*d-
exp(2*b*x+2*a)-1)-1/4/b^2*d*a^2/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1)
+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a))^3+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(
exp(2*b*x+2*a)+1))^3+1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)
*d-exp(2*b*x+2*a)-1))^3-1/4*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2
*a)*d-exp(2*b*x+2*a)-1))^2-1/8*I*x^2*Pi*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b
*x+2*a))^3-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(e
xp(2*b*x+2*a)+1))^2+1/8*I*x^2*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a)
)-1/4*I*x^2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/8*I*x^2*Pi*csg
n(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))*csgn(I/
(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))+1/8*I*x^2*Pi*csgn(I
*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d/(exp(2*b*x+2*a)+1)*e
xp(2*b*x+2*a))+1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a
))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/2/b*d/(d-1)*ln(1-(d-1)*exp(2
*b*x+2*a))*x*a+1/2/b*d*a/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))*x+1/2/b*d*a/(d-
1)*ln(1+exp(b*x+a)*(d-1)^(1/2))*x
```

Maxima [A] time = 3.50123, size = 135, normalized size = 1.23

$$\frac{1}{24}\left(\frac{4\,x^3}{d}-\frac{3\left(2\,b^2x^2\log\left(-(d-1)e^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left((d-1)e^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3((d-1)e^{(2\,bx+2\,a)})\right)}{b^3d}\right)bd-\frac{1}{2}\,x^2\,\mathrm{arcoth}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/24*(4*x^3/d - 3*(2*b^2*x^2*log(-(d - 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dilo
g((d - 1)*e^(2*b*x + 2*a)) - polylog(3, (d - 1)*e^(2*b*x + 2*a)))/(b^3*d))*
b*d - 1/2*x^2*arccoth(d*tanh(b*x + a) + d - 1)
```

Fricas [C] time = 2.06429, size = 894, normalized size = 8.13

$$2\,b^3x^3-3\,b^2x^2\log\left(\frac{d\cosh(bx+a)+d\sinh(bx+a)}{(d-2)\cosh(bx+a)+d\sinh(bx+a)}\right)-6\,bx\mathrm{Li}_2\left(\sqrt{d-1}(\cosh(bx+a)+\sinh(bx+a))\right)-6\,bx\mathrm{Li}_2\left(-\sqrt{d-1}(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 - 3*b^2*x^2*log((d*cosh(b*x + a) + d*sinh(b*x + a))/((d - 2
)*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b*x*dilog(sqrt(d - 1)*(cosh(b*x + a
) + sinh(b*x + a))) - 6*b*x*dilog(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x +
a))) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a) + 2*sqrt
(d - 1)) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a) - 2*
sqrt(d - 1)) - 3*(b^2*x^2 - a^2)*log(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x
+ a)) + 1) - 3*(b^2*x^2 - a^2)*log(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x +
 a)) + 1) + 6*polylog(3, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) + 6*p
olylog(3, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(1-d-d*tanh(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arcoth}(-d \tanh(bx + a) - d + 1) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(-d*tanh(b*x + a) - d + 1), x)
```

3.215 $\int \coth^{-1}(1-d-d\tanh(a+bx))\,dx$

Optimal. Leaf size=76

$$-\frac{\text{PolyLog}\left(2,-(1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{2}x\log\left((1-d)e^{2a+2bx}+1\right)+x\coth^{-1}(d(-\tanh(a+bx))-d+1)+\frac{bx^2}{2}$$

```
[Out] (b*x^2)/2 + x*ArcCoth[1 - d - d*Tanh[a + b*x]] - (x*Log[1 + (1 - d)*E^(2*a
+ 2*b*x)])/2 - PolyLog[2, -((1 - d)*E^(2*a + 2*b*x))]/(4*b)
```

Rubi [A] time = 0.149527, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6232, 2184, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,-(1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{2}x\log\left((1-d)e^{2a+2bx}+1\right)+x\coth^{-1}(d(-\tanh(a+bx))-d+1)+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcCoth[1 - d - d*Tanh[a + b*x]] - (x*Log[1 + (1 - d)*E^(2*a
+ 2*b*x)])/2 - PolyLog[2, -((1 - d)*E^(2*a + 2*b*x))]/(4*b)
```

Rule 6232

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Ar
cCoth[c + d*Tanh[a + b*x]], x] + Dist[b, Int[x/(c - d + c*E^(2*a + 2*b*x)),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \coth^{-1}(1-d-d\tanh(a+bx))\,dx = x\coth^{-1}(1-d-d\tanh(a+bx)) + b\int \frac{x}{1+(1-d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^2}{2} + x\coth^{-1}(1-d-d\tanh(a+bx)) - (b(1-d))\int \frac{e^{2a+2bx}x}{1+(1-d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^2}{2} + x\coth^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{2}x\log\left(1+(1-d)e^{2a+2bx}\right) + \frac{1}{2}\int \log($$
$$= \frac{bx^2}{2} + x\coth^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{2}x\log\left(1+(1-d)e^{2a+2bx}\right) + \frac{\text{Subst}\left(\int\right.}{}$$
$$= \frac{bx^2}{2} + x\coth^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{2}x\log\left(1+(1-d)e^{2a+2bx}\right) - \frac{\text{Li}_2\left(-(1\right.}{}$$

Mathematica [B] time = 0.890278, size = 200, normalized size = 2.63

$$\frac{-2\text{PolyLog}\left(2,-\sqrt{d-1}e^{a+bx}\right) - 2\text{PolyLog}\left(2,\sqrt{d-1}e^{a+bx}\right) - 2\log\left(e^{a+bx}\right)\log\left(1-\sqrt{d-1}e^{a+bx}\right) - 2\log\left(e^{a+bx}\right)\log\left(\sqrt{d}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] x*ArcCoth[1 - d - d*Tanh[a + b*x]] + (b^2*x^2 + Log[E^(a + b*x)]^2 - 2*Log[
E^(a + b*x)]*Log[1 - Sqrt[-1 + d]*E^(a + b*x)] - 2*Log[E^(a + b*x)]*Log[1 +
 Sqrt[-1 + d]*E^(a + b*x)] + 2*Log[E^(a + b*x)]*Log[E^(-a - b*x)*(-1 + (-1
+ d)*E^(2*(a + b*x)))] - 2*b*x*Log[(-2 + d)*Cosh[a + b*x] + d*Sinh[a + b*x]
] - 2*PolyLog[2, -(Sqrt[-1 + d]*E^(a + b*x))] - 2*PolyLog[2, Sqrt[-1 + d]*E
^(a + b*x)])/(4*b)
```

Maple [B] time = 0.158, size = 271, normalized size = 3.6

$$\frac{\text{arccoth}\,(1-d-d\tanh(bx+a))\ln(-d\tanh(bx+a)-d)}{2b} - \frac{\text{arccoth}\,(1-d-d\tanh(bx+a))\ln(-d\tanh(bx+a)+d)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1-d-d*tanh(b*x+a)),x)
```

```
[Out] 1/2/b*arccoth(1-d-d*tanh(b*x+a))*ln(-d*tanh(b*x+a)-d)-1/2/b*arccoth(1-d-d*t
anh(b*x+a))*ln(-d*tanh(b*x+a)+d)+1/8/b*ln(-d*tanh(b*x+a)-d)^2-1/4/b*dilog(1
-1/2*d*tanh(b*x+a)-1/2*d)-1/4/b*ln(-d*tanh(b*x+a)-d)*ln(1-1/2*d*tanh(b*x+a)
-1/2*d)-1/4/b*dilog(-1/2*(-d*tanh(b*x+a)-d)/d)-1/4/b*ln(-d*tanh(b*x+a)+d)*l
n(-1/2*(-d*tanh(b*x+a)-d)/d)+1/4/b*dilog((-d*tanh(b*x+a)-d+2)/(-2*d+2))+1/4
/b*ln(-d*tanh(b*x+a)+d)*ln((-d*tanh(b*x+a)-d+2)/(-2*d+2))
```

Maxima [A] time = 3.90124, size = 99, normalized size = 1.3

$$\frac{1}{4}bd\left(\frac{2x^2}{d} - \frac{2bx\log\left(-(d-1)e^{(2bx+2a)}+1\right) + \text{Li}_2\left((d-1)e^{(2bx+2a)}\right)}{b^2d}\right) - x\,\text{arcoth}\,(d\tanh(bx+a)+d-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-d-d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*b*d*(2*x^2/d - (2*b*x*log(-(d - 1)*e^(2*b*x + 2*a) + 1) + dilog((d - 1)
*e^(2*b*x + 2*a)))/(b^2*d)) - x*arccoth(d*tanh(b*x + a) + d - 1)
```

Fricas [B] time = 1.96049, size = 667, normalized size = 8.78

$$b^2x^2 - bx\log\left(\frac{d\cosh(bx+a)+d\sinh(bx+a)}{(d-2)\cosh(bx+a)+d\sinh(bx+a)}\right) + a\log\left(2(d-1)\cosh(bx+a) + 2(d-1)\sinh(bx+a) + 2\sqrt{d-1}\right) + a\log($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-d-d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - b*x*log((d*cosh(b*x + a) + d*sinh(b*x + a))/((d - 2)*cosh(b*
x + a) + d*sinh(b*x + a))) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh
(b*x + a) + 2*sqrt(d - 1)) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh
(b*x + a) - 2*sqrt(d - 1)) - (b*x + a)*log(sqrt(d - 1)*(cosh(b*x + a) + sin
h(b*x + a)) + 1) - (b*x + a)*log(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a
)) + 1) - dilog(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) - dilog(-sqrt(
d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acoth}(-d\tanh(a+bx) - d + 1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1-d-d*tanh(b*x+a)),x)
```

```
[Out] Integral(acoth(-d*tanh(a + b*x) - d + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}(-d\tanh(bx+a) - d + 1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-d-d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(-d*tanh(b*x + a) - d + 1), x)
```

3.216 $\int \frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x}\,dx$

Optimal. Leaf size=21

$$\text{CannotIntegrate}\left(\frac{\coth^{-1}(d(-\tanh(a+bx))-d+1)}{x},x\right)$$

```
[Out] CannotIntegrate[ArcCoth[1 - d - d*Tanh[a + b*x]]/x, x]
```

Rubi [A] time = 0.0645733, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCoth[1 - d - d*Tanh[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcCoth[1 - d - d*Tanh[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x}\,dx = \int \frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x}\,dx$$

Mathematica [A] time = 3.11442, size = 0, normalized size = 0.

$$\int \frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCoth[1 - d - d*Tanh[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcCoth[1 - d - d*Tanh[a + b*x]]/x, x]
```

Maple [A] time = 0.372, size = 0, normalized size = 0.

$$\int \frac{\mathrm{arccoth}\left(1-d-d\tanh\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1-d-d*tanh(b*x+a))/x,x)
```

```
[Out] int(arccoth(1-d-d*tanh(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+d-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-d-d*tanh(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -integrate(arccoth(d*tanh(b*x + a) + d - 1)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+d-1\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-d-d*tanh(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(-arccoth(d*tanh(b*x + a) + d - 1)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}\left(-d\tanh\left(a+bx\right)-d+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1-d-d*tanh(b*x+a))/x,x)
```

```
[Out] Integral(acoth(-d*tanh(a + b*x) - d + 1)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}\left(-d\tanh\left(bx+a\right)-d+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-d-d*tanh(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(-d*tanh(b*x + a) - d + 1)/x, x)
```

3.217 $\int x^2 \coth^{-1}(c + d\coth(a + bx))\, dx$

Optimal. Leaf size=303

$$-\frac{x\text{PolyLog}\left(3,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2}+\frac{x\text{PolyLog}\left(3,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2}+\frac{\text{PolyLog}\left(4,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^3}-\frac{\text{PolyLog}\left(4,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^3}$$

```
[Out] (x^3*ArcCoth[c + d*Coth[a + b*x]])/3 + (x^3*Log[1 - ((1 - c - d)*E^(2*a + 2
*b*x))/(1 - c + d)])/6 - (x^3*Log[1 - ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c
- d)])/6 + (x^2*PolyLog[2, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(4*b
) - (x^2*PolyLog[2, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(4*b) - (x*
PolyLog[3, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(4*b^2) + (x*PolyLog
[3, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(4*b^2) + PolyLog[4, ((1 -
c - d)*E^(2*a + 2*b*x))/(1 - c + d)]/(8*b^3) - PolyLog[4, ((1 + c + d)*E^(2
*a + 2*b*x))/(1 + c - d)]/(8*b^3)
```

Rubi [A] time = 0.465445, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6246, 2190, 2531, 6609, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(3,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2}+\frac{x\text{PolyLog}\left(3,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2}+\frac{\text{PolyLog}\left(4,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^3}-\frac{\text{PolyLog}\left(4,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^3*ArcCoth[c + d*Coth[a + b*x]])/3 + (x^3*Log[1 - ((1 - c - d)*E^(2*a + 2
*b*x))/(1 - c + d)])/6 - (x^3*Log[1 - ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c
- d)])/6 + (x^2*PolyLog[2, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(4*b
) - (x^2*PolyLog[2, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(4*b) - (x*
PolyLog[3, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(4*b^2) + (x*PolyLog
[3, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(4*b^2) + PolyLog[4, ((1 -
c - d)*E^(2*a + 2*b*x))/(1 - c + d)]/(8*b^3) - PolyLog[4, ((1 + c + d)*E^(2
*a + 2*b*x))/(1 + c - d)]/(8*b^3)
```

Rule 6246

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Coth[a + b*x]])/(f*
(m + 1)), x] + (-Dist[(b*(1 - c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E
^(2*a + 2*b*x))/(1 - c + d - (1 - c - d)*E^(2*a + 2*b*x)), x], x] + Dist[(b
*(1 + c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*a + 2*b*x))/(1 + c -
 d - (1 + c + d)*E^(2*a + 2*b*x)), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \coth^{-1}(c+d\coth(a+bx))\,dx = \frac{1}{3}x^3\coth^{-1}(c+d\coth(a+bx)) - \frac{1}{3}(b(1-c-d))\int \frac{e^{2a+2bx}x^3}{1-c+d+(-1+c+d)e^{2a}$$

$$= \frac{1}{3}x^3\coth^{-1}(c+d\coth(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{6}x^3\log\Big(1$$

$$= \frac{1}{3}x^3\coth^{-1}(c+d\coth(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{6}x^3\log\Big(1$$

$$= \frac{1}{3}x^3\coth^{-1}(c+d\coth(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{6}x^3\log\Big(1$$

$$= \frac{1}{3}x^3\coth^{-1}(c+d\coth(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{6}x^3\log\Big(1$$

$$= \frac{1}{3}x^3\coth^{-1}(c+d\coth(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{6}x^3\log\Big(1$$

Mathematica [A] time = 0.39741, size = 353, normalized size = 1.17

$$-6b^2x^2\text{PolyLog}\left(2,\frac{(c-d-1)(\cosh(2(a+bx))-\sinh(2(a+bx)))}{c+d-1}\right)+6b^2x^2\text{PolyLog}\left(2,\frac{(c-d+1)(\cosh(2(a+bx))-\sinh(2(a+bx)))}{c+d+1}\right)-6bx\text{PolyL}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*ArcCoth[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^3*ArcCoth[c + d*Coth[a + b*x]])/3 + (4*b^3*x^3*Log[(2*(Cosh[a + b*x] - S
inh[a + b*x])*(d*Cosh[a + b*x] + (-1 + c)*Sinh[a + b*x]))/(-1 + c + d)] - 4
*b^3*x^3*Log[1 + ((1 + c - d)*(-Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)]))/(1
+ c + d)] - 6*b^2*x^2*PolyLog[2, ((-1 + c - d)*(Cosh[2*(a + b*x)] - Sinh[2*
(a + b*x)]))/(-1 + c + d)] + 6*b^2*x^2*PolyLog[2, ((1 + c - d)*(Cosh[2*(a +
 b*x)] - Sinh[2*(a + b*x)]))/(1 + c + d)] - 6*b*x*PolyLog[3, ((-1 + c - d)*
(Cosh[2*(a + b*x)] - Sinh[2*(a + b*x)]))/(-1 + c + d)] + 6*b*x*PolyLog[3, (
(1 + c - d)*(Cosh[2*(a + b*x)] - Sinh[2*(a + b*x)]))/(1 + c + d)] - 3*PolyL
og[4, ((-1 + c - d)*(Cosh[2*(a + b*x)] - Sinh[2*(a + b*x)]))/(-1 + c + d)]
+ 3*PolyLog[4, ((1 + c - d)*(Cosh[2*(a + b*x)] - Sinh[2*(a + b*x)]))/(1 + c
 + d)])/(24*b^3)
```

Maple [C] time = 5.121, size = 5222, normalized size = 17.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(c+d*coth(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [A] time = 2.28051, size = 374, normalized size = 1.23

$$\frac{1}{3}x^3\,\mathrm{arcoth}\left(d\coth\left(bx+a\right)+c\right)-\frac{1}{18}bd\left(\frac{4\,b^3x^3\log\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}+1\right)+6\,b^2x^2\mathrm{Li}_2\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)-6\,bx\mathrm{Li}_3\left(\frac{(c+d+1)e^{(}}{c-d}\right.}{b^4d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(c+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(d*coth(b*x + a) + c) - 1/18*b*d*((4*b^3*x^3*log(-(c + d + 1
)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 6*b^2*x^2*dilog((c + d + 1)*e^(2*b*x +
 2*a)/(c - d + 1)) - 6*b*x*polylog(3, (c + d + 1)*e^(2*b*x + 2*a)/(c - d +
1)) + 3*polylog(4, (c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^4*d) - (4*b
^3*x^3*log(-(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1) + 1) + 6*b^2*x^2*dilog(
(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)) - 6*b*x*polylog(3, (c + d - 1)*e^(
2*b*x + 2*a)/(c - d - 1)) + 3*polylog(4, (c + d - 1)*e^(2*b*x + 2*a)/(c - d
 - 1)))/(b^4*d))
```

Fricas [C] time = 2.32668, size = 2560, normalized size = 8.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(b^3*x^3*log((d*cosh(b*x + a) + (c + 1)*sinh(b*x + a))/(d*cosh(b*x + a)
 + (c - 1)*sinh(b*x + a))) - 3*b^2*x^2*dilog(sqrt((c + d + 1)/(c - d + 1))*
```

```
(cosh(b*x + a) + sinh(b*x + a))) - 3*b^2*x^2*dilog(-sqrt((c + d + 1)/(c - d
 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(sqrt((c + d - 1)/
(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(-sqrt((c +
d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + a^3*log(2*(c + d + 1
)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt((c + d +
 1)/(c - d + 1))) + a^3*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sin
h(b*x + a) - 2*(c - d + 1)*sqrt((c + d + 1)/(c - d + 1))) - a^3*log(2*(c +
d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt((c
+ d - 1)/(c - d - 1))) - a^3*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1
)*sinh(b*x + a) - 2*(c - d - 1)*sqrt((c + d - 1)/(c - d - 1))) + 6*b*x*poly
log(3, sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 6*b
*x*polylog(3, -sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)
)) - 6*b*x*polylog(3, sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b
*x + a))) - 6*b*x*polylog(3, -sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a)
+ sinh(b*x + a))) - (b^3*x^3 + a^3)*log(sqrt((c + d + 1)/(c - d + 1))*(cosh
(b*x + a) + sinh(b*x + a)) + 1) - (b^3*x^3 + a^3)*log(-sqrt((c + d + 1)/(c
- d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b^3*x^3 + a^3)*log(sqrt((
c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b^3*x^3 + a
^3)*log(-sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1)
 - 6*polylog(4, sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a
))) - 6*polylog(4, -sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x
 + a))) + 6*polylog(4, sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(
b*x + a))) + 6*polylog(4, -sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + s
inh(b*x + a))))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(c+d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arcoth}(d \coth(bx + a) + c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(c+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(d*coth(b*x + a) + c), x)
```

3.218 $\int x \coth^{-1}(c + d \coth(a + bx)) \, dx$

Optimal. Leaf size=229

$$-\frac{\text{PolyLog}\left(3, \frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^2} + \frac{\text{PolyLog}\left(3, \frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^2} + \frac{x\text{PolyLog}\left(2, \frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b} - \frac{x\text{PolyLog}\left(2, \frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}$$

```
[Out] (x^2*ArcCoth[c + d*Coth[a + b*x]])/2 + (x^2*Log[1 - ((1 - c - d)*E^(2*a + 2
*b*x))/(1 - c + d)])/4 - (x^2*Log[1 - ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c
- d)])/4 + (x*PolyLog[2, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(4*b)
- (x*PolyLog[2, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(4*b) - PolyLog
[3, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)]/(8*b^2) + PolyLog[3, ((1 + c
 + d)*E^(2*a + 2*b*x))/(1 + c - d)]/(8*b^2)
```

Rubi [A] time = 0.382299, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6246, 2190, 2531, 2282, 6589}

$$-\frac{\text{PolyLog}\left(3, \frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^2} + \frac{\text{PolyLog}\left(3, \frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^2} + \frac{x\text{PolyLog}\left(2, \frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b} - \frac{x\text{PolyLog}\left(2, \frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^2*ArcCoth[c + d*Coth[a + b*x]])/2 + (x^2*Log[1 - ((1 - c - d)*E^(2*a + 2
*b*x))/(1 - c + d)])/4 - (x^2*Log[1 - ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c
- d)])/4 + (x*PolyLog[2, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(4*b)
- (x*PolyLog[2, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(4*b) - PolyLog
[3, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)]/(8*b^2) + PolyLog[3, ((1 + c
 + d)*E^(2*a + 2*b*x))/(1 + c - d)]/(8*b^2)
```

Rule 6246

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Coth[a + b*x]])/(f*
(m + 1)), x] + (-Dist[(b*(1 - c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E
^(2*a + 2*b*x))/(1 - c + d - (1 - c - d)*E^(2*a + 2*b*x)), x], x] + Dist[(b
*(1 + c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*a + 2*b*x))/(1 + c -
 d - (1 + c + d)*E^(2*a + 2*b*x)), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\coth^{-1}(c+d\coth(a+bx))\,dx=\frac{1}{2}x^2\coth^{-1}(c+d\coth(a+bx))-\frac{1}{2}(b(1-c-d))\int\frac{e^{2a+2bx}x^2}{1-c+d+(-1+c+d)e^{2a+}$$

$$=\frac{1}{2}x^2\coth^{-1}(c+d\coth(a+bx))+\frac{1}{4}x^2\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-\frac{1}{4}x^2\log\left(1\right.$$

$$=\frac{1}{2}x^2\coth^{-1}(c+d\coth(a+bx))+\frac{1}{4}x^2\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-\frac{1}{4}x^2\log\left(1\right.$$

$$=\frac{1}{2}x^2\coth^{-1}(c+d\coth(a+bx))+\frac{1}{4}x^2\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-\frac{1}{4}x^2\log\left(1\right.$$

$$=\frac{1}{2}x^2\coth^{-1}(c+d\coth(a+bx))+\frac{1}{4}x^2\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-\frac{1}{4}x^2\log\left(1\right.$$

Mathematica [A] time = 0.158329, size = 267, normalized size = 1.17

$$-2bx\text{PolyLog}\left(2,\frac{(c-d-1)(\cosh(2(a+bx))-\sinh(2(a+bx)))}{c+d-1}\right)+2bx\text{PolyLog}\left(2,\frac{(c-d+1)(\cosh(2(a+bx))-\sinh(2(a+bx)))}{c+d+1}\right)-\text{PolyLog}\left(3,\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*ArcCoth[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^2*ArcCoth[c + d*Coth[a + b*x]])/2 + (2*b^2*x^2*Log[(2*(Cosh[a + b*x] - S
inh[a + b*x])*(d*Cosh[a + b*x] + (-1 + c)*Sinh[a + b*x]))/(-1 + c + d)] - 2
*b^2*x^2*Log[1 + ((1 + c - d)*(-Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)]))/(1
+ c + d)] - 2*b*x*PolyLog[2, ((-1 + c - d)*(Cosh[2*(a + b*x)] - Sinh[2*(a +
 b*x)]))/(-1 + c + d)] + 2*b*x*PolyLog[2, ((1 + c - d)*(Cosh[2*(a + b*x)] -
 Sinh[2*(a + b*x)]))/(1 + c + d)] - PolyLog[3, ((-1 + c - d)*(Cosh[2*(a + b
*x)] - Sinh[2*(a + b*x)]))/(-1 + c + d)] + PolyLog[3, ((1 + c - d)*(Cosh[2*
(a + b*x)] - Sinh[2*(a + b*x)]))/(1 + c + d)])/(8*b^2)
```

Maple [C] time = 4.785, size = 4918, normalized size = 21.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(c+d*coth(b*x+a)),x)
```

```
[Out] 1/2/b^2*a^2*c/(1+c+d)*ln((c*exp(b*x+a)+exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)
+exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))+1/2/b^2*a^2*d/(1+c+d)*ln((-c*exp(b*x+
a)-exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2)
)+1/2/b^2*a^2*d/(1+c+d)*ln((c*exp(b*x+a)+exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/
2)+exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))+1/2/b*a/(1+c+d)*ln((-c*exp(b*x+a)-e
xp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))*x+
1/4*x^2*ln((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d+exp(2*b*x+2*a)-1)-1/8*
I*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x
+2*a)+1)*d-exp(2*b*x+2*a)+1)/(exp(2*b*x+2*a)-1))^2-1/2/b*d/(1+c+d)*ln(1-(1+
c+d)*exp(2*b*x+2*a)/(1+c-d))*x*a+1/2/b^2*a^2/(c+d-1)*ln((c*exp(b*x+a)+exp(b
*x+a)*d+((c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))+1/4*c/
(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*x^2+1/4*d/(c+d-1)*ln(1-(c+d-1)
*exp(2*b*x+2*a)/(c-d-1))*x^2-1/8/b^2*c/(c+d-1)*polylog(3,(c+d-1)*exp(2*b*x+
2*a)/(c-d-1))-1/8/b^2*d/(c+d-1)*polylog(3,(c+d-1)*exp(2*b*x+2*a)/(c-d-1))+1
/2/b^2*a/(c+d-1)*dilog((-c*exp(b*x+a)-exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)+
exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))+1/2/b^2*a/(c+d-1)*dilog((c*exp(b*x+a)+
exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))-1
/4/b^2/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*a^2-1/4/b/(c+d-1)*polyl
og(2,(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*x-1/4/b^2/(c+d-1)*polylog(2,(c+d-1)*ex
p(2*b*x+2*a)/(c-d-1))*a+1/2/b^2*a^2/(c+d-1)*ln((-c*exp(b*x+a)-exp(b*x+a)*d+
((c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))-1/4/(c+d-1)*ln
(1-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*x^2+1/8/b^2/(c+d-1)*polylog(3,(c+d-1)*ex
p(2*b*x+2*a)/(c-d-1))+1/8*I*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*((exp(
2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d+exp(2*b*x+2*a)-1)/(exp(2*b*x+2*a)-1))^
2-1/4/b^2*a^2/(1+c+d)*ln(exp(2*b*x+2*a)*c+exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-c
+d-1)+1/2/b*a/(1+c+d)*ln((c*exp(b*x+a)+exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)
+exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))*x-1/4/b*d/(1+c+d)*polylog(2,(1+c+d)*e
xp(2*b*x+2*a)/(1+c-d))*x-1/4/b*c/(1+c+d)*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(
1+c-d))*x-1/2/b/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*x*a+1/2/b^2*a*
c/(1+c+d)*dilog((-c*exp(b*x+a)-exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)-exp(b*x
+a))/((1+c-d)*(1+c+d))^(1/2))+1/2/b^2*a*c/(1+c+d)*dilog((c*exp(b*x+a)+exp(b
*x+a)*d+((1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))+1/2/b^
2*a*d/(1+c+d)*dilog((-c*exp(b*x+a)-exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)-exp
(b*x+a))/((1+c-d)*(1+c+d))^(1/2))+1/2/b^2*a*d/(1+c+d)*dilog((c*exp(b*x+a)+e
xp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))-1/
4/b^2*d/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*a^2-1/4/b^2*d/(1+c+d)*
polylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*a-1/4/b^2*c/(1+c+d)*ln(1-(1+c+d)*
exp(2*b*x+2*a)/(1+c-d))*a^2-1/4/b^2*c/(1+c+d)*polylog(2,(1+c+d)*exp(2*b*x+2
*a)/(1+c-d))*a+1/2/b^2*a^2*c/(1+c+d)*ln((-c*exp(b*x+a)-exp(b*x+a)*d+((1+c-d
)*(1+c+d))^(1/2)-exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))-1/4*d/(1+c+d)*ln(1-(1
+c+d)*exp(2*b*x+2*a)/(1+c-d))*x^2-1/4*c/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)
/(1+c-d))*x^2-1/4/b/(1+c+d)*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*x+1/2
/b^2*a^2/(1+c+d)*ln((-c*exp(b*x+a)-exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)-exp
(b*x+a))/((1+c-d)*(1+c+d))^(1/2))+1/2/b^2*a^2/(1+c+d)*ln((c*exp(b*x+a)+exp(
b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))-1/4/b
^2/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*a^2-1/4/b^2/(1+c+d)*polylog
(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*a+1/2/b^2*a/(1+c+d)*dilog((c*exp(b*x+a)+
exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))+1
/8/b^2*c/(1+c+d)*polylog(3,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/8/b^2*d/(1+c+d
)*polylog(3,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/2/b^2*a/(1+c+d)*dilog((-c*exp
(b*x+a)-exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1+c-d)*(1+c+d))^
(1/2))-1/4*x^2*ln((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d-exp(2*b*x+2*a)+
1)-1/2/b*c/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*x*a+1/2/b*a*c/(1+c+
d)*ln((-c*exp(b*x+a)-exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1+c
-d)*(1+c+d))^(1/2))*x+1/2/b*a*c/(1+c+d)*ln((c*exp(b*x+a)+exp(b*x+a)*d+((1+c
```

```
-d)*(1+c+d))^(1/2)+exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))*x+1/2/b*a*d/(1+c+d)
*ln((-c*exp(b*x+a)-exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1+c-d
)*(1+c+d))^(1/2))*x+1/2/b*a*d/(1+c+d)*ln((c*exp(b*x+a)+exp(b*x+a)*d+((1+c-d
)*(1+c+d))^(1/2)+exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))*x+1/4/b^2*a^2*c/(c+d-
1)*ln(exp(2*b*x+2*a)*c+exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-c+d+1)+1/4/b^2*d*a^2
/(c+d-1)*ln(exp(2*b*x+2*a)*c+exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-c+d+1)+1/8*I*P
i*x^2*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d+exp(2*b*x+2*a)-1))*
csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d+exp(2*b*x+2*a)-1)/(exp(2*
b*x+2*a)-1))^2+1/2/b*c/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*x*a+1/2
/b*d/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*x*a-1/2/b*c*a/(c+d-1)*ln(
(-c*exp(b*x+a)-exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/((c-d-1)*(c
+d-1))^(1/2))*x-1/2/b*c*a/(c+d-1)*ln((c*exp(b*x+a)+exp(b*x+a)*d+((c-d-1)*(c
+d-1))^(1/2)-exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))*x-1/2/b*d*a/(c+d-1)*ln((-
c*exp(b*x+a)-exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/((c-d-1)*(c+d
-1))^(1/2))*x-1/2/b*d*a/(c+d-1)*ln((c*exp(b*x+a)+exp(b*x+a)*d+((c-d-1)*(c+d
-1))^(1/2)-exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))*x-1/4/b^2*a^2/(c+d-1)*ln(ex
p(2*b*x+2*a)*c+exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-c+d+1)-1/4/b^2*a^2*c/(1+c+d)
*ln(exp(2*b*x+2*a)*c+exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-c+d-1)-1/4/b^2*a^2*d/(
1+c+d)*ln(exp(2*b*x+2*a)*c+exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-c+d-1)-1/8*I*Pi*
x^2*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d+exp(2*b*x+2*a)-1)/(ex
p(2*b*x+2*a)-1))^3+1/8*I*Pi*x^2*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a
)+1)*d-exp(2*b*x+2*a)+1)/(exp(2*b*x+2*a)-1))^3-1/4/(1+c+d)*ln(1-(1+c+d)*exp
(2*b*x+2*a)/(1+c-d))*x^2+1/8/b^2/(1+c+d)*polylog(3,(1+c+d)*exp(2*b*x+2*a)/(
1+c-d))-1/8*I*Pi*x^2*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d-exp(
2*b*x+2*a)+1))*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d-exp(2*b*x+
2*a)+1)/(exp(2*b*x+2*a)-1))^2-1/2/b^2*c*a^2/(c+d-1)*ln((-c*exp(b*x+a)-exp(b
*x+a)*d+((c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))-1/2/b^
2*c*a^2/(c+d-1)*ln((c*exp(b*x+a)+exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)-exp(b
*x+a))/((c-d-1)*(c+d-1))^(1/2))-1/2/b^2*d*a^2/(c+d-1)*ln((-c*exp(b*x+a)-exp
(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))-1/2/
b^2*d*a^2/(c+d-1)*ln((c*exp(b*x+a)+exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)-exp
(b*x+a))/((c-d-1)*(c+d-1))^(1/2))-1/2/b^2*c*a/(c+d-1)*dilog((-c*exp(b*x+a)-
exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))-1
/2/b^2*c*a/(c+d-1)*dilog((c*exp(b*x+a)+exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)
-exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))-1/2/b^2*d*a/(c+d-1)*dilog((-c*exp(b*x
+a)-exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2
))-1/2/b^2*d*a/(c+d-1)*dilog((c*exp(b*x+a)+exp(b*x+a)*d+((c-d-1)*(c+d-1))^(
1/2)-exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))+1/4/b^2*c/(c+d-1)*ln(1-(c+d-1)*ex
p(2*b*x+2*a)/(c-d-1))*a^2+1/4/b*c/(c+d-1)*polylog(2,(c+d-1)*exp(2*b*x+2*a)/
(c-d-1))*x+1/4/b^2*c/(c+d-1)*polylog(2,(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*a+1/
4/b^2*d/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*a^2+1/4/b*d/(c+d-1)*po
lylog(2,(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*x+1/4/b^2*d/(c+d-1)*polylog(2,(c+d-
1)*exp(2*b*x+2*a)/(c-d-1))*a-1/2/b/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(c-d
-1))*x*a+1/2/b*a/(c+d-1)*ln((-c*exp(b*x+a)-exp(b*x+a)*d+((c-d-1)*(c+d-1))^(
1/2)+exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))*x+1/2/b*a/(c+d-1)*ln((c*exp(b*x+a
)+exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))
*x+1/8*I*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*((exp(2*b*x+2*a)-1)*c+(ex
p(2*b*x+2*a)+1)*d-exp(2*b*x+2*a)+1))*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*
x+2*a)+1)*d-exp(2*b*x+2*a)+1)/(exp(2*b*x+2*a)-1))-1/8*I*Pi*x^2*csgn(I/(exp(
2*b*x+2*a)-1))*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d+exp(2*b*x+
2*a)-1))*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d+exp(2*b*x+2*a)-1
)/(exp(2*b*x+2*a)-1))
```

Maxima [A] time = 2.22562, size = 288, normalized size = 1.26

$$-\frac{1}{8}\,bd\left(\frac{2\,b^2x^2\log\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}+1\right)+2\,bx\mathrm{Li}_2\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)-\mathrm{Li}_3\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)}{b^3d}-\frac{2\,b^2x^2\log\left(-\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(c+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*b*d*((2*b^2*x^2*log(-(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 2*
b*x*dilog((c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)) - polylog(3, (c + d + 1)
*e^(2*b*x + 2*a)/(c - d + 1)))/(b^3*d) - (2*b^2*x^2*log(-(c + d - 1)*e^(2*b
*x + 2*a)/(c - d - 1) + 1) + 2*b*x*dilog((c + d - 1)*e^(2*b*x + 2*a)/(c - d
 - 1)) - polylog(3, (c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)))/(b^3*d)) + 1/
2*x^2*arccoth(d*coth(b*x + a) + c)
```

Fricas [C] time = 2.23779, size = 2093, normalized size = 9.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^2*log((d*cosh(b*x + a) + (c + 1)*sinh(b*x + a))/(d*cosh(b*x + a)
 + (c - 1)*sinh(b*x + a))) - 2*b*x*dilog(sqrt((c + d + 1)/(c - d + 1))*(cos
h(b*x + a) + sinh(b*x + a))) - 2*b*x*dilog(-sqrt((c + d + 1)/(c - d + 1))*(
cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(sqrt((c + d - 1)/(c - d - 1))
*(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(-sqrt((c + d - 1)/(c - d -
1))*(cosh(b*x + a) + sinh(b*x + a))) - a^2*log(2*(c + d + 1)*cosh(b*x + a)
+ 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt((c + d + 1)/(c - d + 1))
) - a^2*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) - 2*(
c - d + 1)*sqrt((c + d + 1)/(c - d + 1))) + a^2*log(2*(c + d - 1)*cosh(b*x
+ a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt((c + d - 1)/(c - d
- 1))) + a^2*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a)
- 2*(c - d - 1)*sqrt((c + d - 1)/(c - d - 1))) - (b^2*x^2 - a^2)*log(sqrt((
c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b^2*x^2 - a
^2)*log(-sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1)
 + (b^2*x^2 - a^2)*log(sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(
b*x + a)) + 1) + (b^2*x^2 - a^2)*log(-sqrt((c + d - 1)/(c - d - 1))*(cosh(b
*x + a) + sinh(b*x + a)) + 1) + 2*polylog(3, sqrt((c + d + 1)/(c - d + 1))*
(cosh(b*x + a) + sinh(b*x + a))) + 2*polylog(3, -sqrt((c + d + 1)/(c - d +
1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*polylog(3, sqrt((c + d - 1)/(c - d
 - 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*polylog(3, -sqrt((c + d - 1)/(c
 - d - 1))*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(c+d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arcoth}(d \coth(bx + a) + c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(c+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(d*coth(b*x + a) + c), x)
```

3.219 $\int \coth^{-1}(c + d\coth(a + bx))\,dx$

Optimal. Leaf size=150

$$\frac{\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}+\frac{1}{2}x\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)-\frac{1}{2}x\log\left(1-\frac{(c+d+1)e^{2a}}{c-d+1}\right.$$

```
[Out] x*ArcCoth[c + d*Coth[a + b*x]] + (x*Log[1 - ((1 - c - d)*E^(2*a + 2*b*x))/(
1 - c + d)])/2 - (x*Log[1 - ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/2 +
 PolyLog[2, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)]/(4*b) - PolyLog[2, (
(1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)]/(4*b)
```

Rubi [A] time = 0.234753, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6238, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}+\frac{1}{2}x\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)-\frac{1}{2}x\log\left(1-\frac{(c+d+1)e^{2a}}{c-d+1}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[c + d*Coth[a + b*x]],x]
```

```
[Out] x*ArcCoth[c + d*Coth[a + b*x]] + (x*Log[1 - ((1 - c - d)*E^(2*a + 2*b*x))/(
1 - c + d)])/2 - (x*Log[1 - ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/2 +
 PolyLog[2, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)]/(4*b) - PolyLog[2, (
(1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)]/(4*b)
```

Rule 6238

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Ar
cCoth[c + d*Coth[a + b*x]], x] + (-Dist[b*(1 - c - d), Int[(x*E^(2*a + 2*b*
x))/(1 - c + d - (1 - c - d)*E^(2*a + 2*b*x)), x], x] + Dist[b*(1 + c + d),
 Int[(x*E^(2*a + 2*b*x))/(1 + c - d - (1 + c + d)*E^(2*a + 2*b*x)), x], x])
 /; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, 1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \coth^{-1}(c+d\coth(a+bx))\,dx = x\coth^{-1}(c+d\coth(a+bx)) - (b(1-c-d))\int \frac{e^{2a+2bx}x}{1-c+d+(-1+c+d)e^{2a+2bx}}\,dx$$
$$= x\coth^{-1}(c+d\coth(a+bx)) + \frac{1}{2}x\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{2}x\log\left(1-\frac{(1+$$
$$= x\coth^{-1}(c+d\coth(a+bx)) + \frac{1}{2}x\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{2}x\log\left(1-\frac{(1+$$
$$= x\coth^{-1}(c+d\coth(a+bx)) + \frac{1}{2}x\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{2}x\log\left(1-\frac{(1+$$

Mathematica [A] time = 1.12623, size = 131, normalized size = 0.87

$$x\coth^{-1}(d\coth(a+bx)+c) - \frac{-\text{PolyLog}\left(2,\frac{(c+d-1)e^{2(a+bx)}}{c-d-1}\right)+\text{PolyLog}\left(2,\frac{(c+d+1)e^{2(a+bx)}}{c-d+1}\right)-2bx\left(\log\left(1-\frac{(c+d-1)e^{2(a+bx)}}{c-d-1}\right.\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[c + d*Coth[a + b*x]],x]
```

```
[Out] x*ArcCoth[c + d*Coth[a + b*x]] - (-2*b*x*(Log[1 - ((-1 + c + d)*E^(2*(a + b
*x)))/(-1 + c - d)] - Log[1 - ((1 + c + d)*E^(2*(a + b*x)))/(1 + c - d)]) -
 PolyLog[2, ((-1 + c + d)*E^(2*(a + b*x)))/(-1 + c - d)] + PolyLog[2, ((1 +
 c + d)*E^(2*(a + b*x)))/(1 + c - d)])/(4*b)
```

Maple [B] time = 0.171, size = 306, normalized size = 2.

$$-\frac{\operatorname{arccoth}(c+d\coth(bx+a))\ln(d\coth(bx+a)-d)}{2b}+\frac{\operatorname{arccoth}(c+d\coth(bx+a))\ln(d\coth(bx+a)+d)}{2b}+\frac{1}{4b}\mathrm{di}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(c+d*coth(b*x+a)),x)
```

```
[Out] -1/2/b*arccoth(c+d*coth(b*x+a))*ln(d*coth(b*x+a)-d)+1/2/b*arccoth(c+d*coth(
b*x+a))*ln(d*coth(b*x+a)+d)+1/4/b*dilog((d*coth(b*x+a)+c+1)/(1+c+d))+1/4/b*
ln(d*coth(b*x+a)-d)*ln((d*coth(b*x+a)+c+1)/(1+c+d))-1/4/b*dilog((d*coth(b*x
+a)+c-1)/(c+d-1))-1/4/b*ln(d*coth(b*x+a)-d)*ln((d*coth(b*x+a)+c-1)/(c+d-1))
+1/4/b*dilog((d*coth(b*x+a)+c-1)/(c-d-1))+1/4/b*ln(d*coth(b*x+a)+d)*ln((d*c
oth(b*x+a)+c-1)/(c-d-1))-1/4/b*dilog((d*coth(b*x+a)+c+1)/(1+c-d))-1/4/b*ln(
d*coth(b*x+a)+d)*ln((d*coth(b*x+a)+c+1)/(1+c-d))
```

Maxima [A] time = 2.27015, size = 192, normalized size = 1.28

$$-\frac{1}{4}bd\left(\frac{2bx\log\left(-\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}+1\right)+\text{Li}_2\left(\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right)}{b^2d}-\frac{2bx\log\left(-\frac{(c+d-1)e^{(2bx+2a)}}{c-d-1}+1\right)+\text{Li}_2\left(\frac{(c+d-1)e^{(2bx+2a)}}{c-d-1}\right)}{b^2d}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/4*b*d*((2*b*x*log(-(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + dilog(
(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^2*d) - (2*b*x*log(-(c + d - 1)
*e^(2*b*x + 2*a)/(c - d - 1) + 1) + dilog((c + d - 1)*e^(2*b*x + 2*a)/(c -
d - 1)))/(b^2*d)) + x*arccoth(d*coth(b*x + a) + c)
```

Fricas [B] time = 2.16709, size = 1585, normalized size = 10.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x*log((d*cosh(b*x + a) + (c + 1)*sinh(b*x + a))/(d*cosh(b*x + a) + (
c - 1)*sinh(b*x + a))) + a*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*
sinh(b*x + a) + 2*(c - d + 1)*sqrt((c + d + 1)/(c - d + 1))) + a*log(2*(c +
 d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) - 2*(c - d + 1)*sqrt((c
 + d + 1)/(c - d + 1))) - a*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)
*sinh(b*x + a) + 2*(c - d - 1)*sqrt((c + d - 1)/(c - d - 1))) - a*log(2*(c
+ d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) - 2*(c - d - 1)*sqrt((
c + d - 1)/(c - d - 1))) - (b*x + a)*log(sqrt((c + d + 1)/(c - d + 1))*(cos
h(b*x + a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-sqrt((c + d + 1)/(c - d +
 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x + a)*log(sqrt((c + d - 1)/
(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x + a)*log(-sqrt((c
+ d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - dilog(sqrt((c
+ d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) - dilog(-sqrt((c + d
 + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + dilog(sqrt((c + d - 1
)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + dilog(-sqrt((c + d - 1)/(
c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(c+d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}\left(d\coth\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*coth(b*x + a) + c), x)
```

3.220 $\int \frac{\coth^{-1}(c+d\coth(a+bx))}{x}\,dx$

Optimal. Leaf size=17

$$\text{CannotIntegrate}\left(\frac{\coth^{-1}(d\coth(a+bx)+c)}{x},x\right)$$

```
[Out] CannotIntegrate[ArcCoth[c + d*Coth[a + b*x]]/x, x]
```

Rubi [A] time = 0.147025, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth^{-1}(c+d\coth(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCoth[c + d*Coth[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcCoth[c + d*Coth[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\coth^{-1}(c+d\coth(a+bx))}{x}\,dx=\int \frac{\coth^{-1}(c+d\coth(a+bx))}{x}\,dx$$

Mathematica [A] time = 5.47975, size = 0, normalized size = 0.

$$\int \frac{\coth^{-1}(c+d\coth(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCoth[c + d*Coth[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcCoth[c + d*Coth[a + b*x]]/x, x]
```

Maple [A] time = 0.337, size = 0, normalized size = 0.

$$\int \frac{\mathrm{arccoth}\left(c+d\coth\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(c+d*coth(b*x+a))/x,x)
```

```
[Out] int(arccoth(c+d*coth(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(d \coth(bx+a)+c)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*coth(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arccoth(d*coth(b*x + a) + c)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arcoth}(d \coth(bx+a)+c)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*coth(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(d*coth(b*x + a) + c)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(c+d*coth(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(d \coth(bx+a)+c)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*coth(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*coth(b*x + a) + c)/x, x)
```

3.221 $\int x^3 \coth^{-1}(1+d+d\coth(a+bx))\,dx$

Optimal. Leaf size=152

$$\frac{3x^2\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{8b^2}-\frac{3x\text{PolyLog}\left(4,(d+1)e^{2a+2bx}\right)}{8b^3}+\frac{3\text{PolyLog}\left(5,(d+1)e^{2a+2bx}\right)}{16b^4}-\frac{x^3\text{PolyLog}\left(2,(d\right.}{4b}$$

```
[Out] (b*x^5)/20 + (x^4*ArcCoth[1 + d + d*Coth[a + b*x]])/4 - (x^4*Log[1 - (1 + d
)*E^(2*a + 2*b*x)])/8 - (x^3*PolyLog[2, (1 + d)*E^(2*a + 2*b*x)])/(4*b) + (
3*x^2*PolyLog[3, (1 + d)*E^(2*a + 2*b*x)])/(8*b^2) - (3*x*PolyLog[4, (1 + d
)*E^(2*a + 2*b*x)])/(8*b^3) + (3*PolyLog[5, (1 + d)*E^(2*a + 2*b*x)])/(16*b
^4)
```

Rubi [A] time = 0.30751, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6242, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{8b^2}-\frac{3x\text{PolyLog}\left(4,(d+1)e^{2a+2bx}\right)}{8b^3}+\frac{3\text{PolyLog}\left(5,(d+1)e^{2a+2bx}\right)}{16b^4}-\frac{x^3\text{PolyLog}\left(2,(d\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCoth[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] (b*x^5)/20 + (x^4*ArcCoth[1 + d + d*Coth[a + b*x]])/4 - (x^4*Log[1 - (1 + d
)*E^(2*a + 2*b*x)])/8 - (x^3*PolyLog[2, (1 + d)*E^(2*a + 2*b*x)])/(4*b) + (
3*x^2*PolyLog[3, (1 + d)*E^(2*a + 2*b*x)])/(8*b^2) - (3*x*PolyLog[4, (1 + d
)*E^(2*a + 2*b*x)])/(8*b^3) + (3*PolyLog[5, (1 + d)*E^(2*a + 2*b*x)])/(16*b
^4)
```

Rule 6242

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Coth[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3 \coth^{-1}(1+d+d\coth(a+bx))\,dx = \frac{1}{4}x^4\coth^{-1}(1+d+d\coth(a+bx)) + \frac{1}{4}b\int \frac{x^4}{1+(-1-d)e^{2a+2bx}}\,dx$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1+d+d\coth(a+bx)) + \frac{1}{4}(b(1+d))\int \frac{e^{2a+2bx}x^4}{1+(-1-d)e^{2a+2b\ldots}}$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1+d+d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1+d)e^{2a+2bx}\right) + \frac{1}{2}\ldots$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1+d+d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1+d)e^{2a+2bx}\right) - \frac{x^3\ldots}{\ldots}$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1+d+d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1+d)e^{2a+2bx}\right) - \frac{x^3\ldots}{\ldots}$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1+d+d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1+d)e^{2a+2bx}\right) - \frac{x^3\ldots}{\ldots}$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1+d+d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1+d)e^{2a+2bx}\right) - \frac{x^3\ldots}{\ldots}$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1+d+d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1+d)e^{2a+2bx}\right) - \frac{x^3\ldots}{\ldots}$$

Mathematica [A] time = 0.174493, size = 141, normalized size = 0.93

$$\frac{1}{16}\left(\frac{6x^2\text{PolyLog}\left(3,\frac{e^{-2(a+bx)}}{d+1}\right)}{b^2} + \frac{6x\text{PolyLog}\left(4,\frac{e^{-2(a+bx)}}{d+1}\right)}{b^3} + \frac{3\text{PolyLog}\left(5,\frac{e^{-2(a+bx)}}{d+1}\right)}{b^4} + \frac{4x^3\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{d+1}\right)}{b} - 2x^4\log\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCoth[1 + d + d*Coth[a + b*x]],x]
```

[Out] (4*x^4*ArcCoth[1 + d + d*Coth[a + b*x]] - 2*x^4*Log[1 - 1/((1 + d)*E^(2*(a + b*x)))] + (4*x^3*PolyLog[2, 1/((1 + d)*E^(2*(a + b*x)))])/b + (6*x^2*PolyLog[3, 1/((1 + d)*E^(2*(a + b*x)))])/b^2 + (6*x*PolyLog[4, 1/((1 + d)*E^(2*(a + b*x)))])/b^3 + (3*PolyLog[5, 1/((1 + d)*E^(2*(a + b*x)))])/b^4)/16

Maple [C] time = 32.161, size = 1698, normalized size = 11.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*arccoth(1+d+d*coth(b*x+a)),x)

[Out] -1/8/b^4*d*a^4/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1)+1/2/b^3*a^3/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))*x+1/2/b^3*a^3/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))*x-1/2/b^3*a^3/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x-1/4/b*d/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*x^3+3/8/b^2*d/(1+d)*polylog(3,(1+d)*exp(2*b*x+2*a))*x^2-3/8/b^3*d/(1+d)*polylog(4,(1+d)*exp(2*b*x+2*a))*x+1/2/b^4*d*a^3/(1+d)*dilog(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b^4*d*a^3/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))+1/2/b^4*d*a^4/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))-3/8/b^4*d*a^4/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))-1/4/b^4*d*a^3/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2+1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^2+1/16*I*x^4*Pi*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^2-1/16*I*x^4*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/2/b^3*d*a^3/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))*x-1/2/b^3*d*a^3/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x+1/2/b^3*d*a^3/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))*x+1/2/b^4*d*a^4/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^3+1/16*I*x^4*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/8*I*x^4*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a))^3+1/20*b*x^5+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^3+1/8*x^4*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1)-1/8/b^4*a^4/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1)+3/16/b^4/(1+d)*polylog(5,(1+d)*exp(2*b*x+2*a))-1/8/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x^4-1/8*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x^4-1/4/b^4*a^3/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))-1/4/b/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*x^3+3/8/b^2/(1+d)*polylog(3,(1+d)*exp(2*b*x+2*a))*x^2-3/8/b^3/(1+d)*polylog(4,(1+d)*exp(2*b*x+2*a))*x+1/2/b^4*a^3/(1+d)*dilog(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b^4*a^3/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))+3/16/b^4*d/(1+d)*polylog(5,(1+d)*exp(2*b*x+2*a))+1/2/b^4*a^4/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b^4*a^4/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))-3/8/b^4*a^4/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))+1/16*I*x^4*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^3-1/4*x^4*ln(exp(b*x+a))-1/8*x^4*ln(d)-1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))+1/16*I*x^4*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))

Maxima [A] time = 3.42852, size = 197, normalized size = 1.3

$$\frac{1}{4}x^4 \operatorname{arcoth}(d\coth(bx+a)+d+1) + \frac{1}{40}\left(\frac{2x^5}{d} - \frac{5\left(2b^4x^4\log\left(-(d+1)e^{(2bx+2a)}+1\right) + 4b^3x^3\mathrm{Li}_2\left((d+1)e^{(2bx+2a)}\right) - 6\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccoth(d*coth(b*x + a) + d + 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*log
(-(d + 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog((d + 1)*e^(2*b*x + 2*a)) -
 6*b^2*x^2*polylog(3, (d + 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, (d + 1)*e
^(2*b*x + 2*a)) - 3*polylog(5, (d + 1)*e^(2*b*x + 2*a)))/(b^5*d))*b*d
```

Fricas [C] time = 2.26028, size = 1280, normalized size = 8.42

$$\frac{2b^5x^5 + 5b^4x^4\log\left(\frac{d\cosh(bx+a)+(d+2)\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 20b^3x^3\mathrm{Li}_2\left(\sqrt{d+1}(\cosh(bx+a)+\sinh(bx+a))\right) - 20b^3x^3\mathrm{Li}_2\left(-\sqrt{d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/40*(2*b^5*x^5 + 5*b^4*x^4*log((d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/(
d*cosh(b*x + a) + d*sinh(b*x + a))) - 20*b^3*x^3*dilog(sqrt(d + 1)*(cosh(b*
x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-sqrt(d + 1)*(cosh(b*x + a) + s
inh(b*x + a))) - 5*a^4*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a
) + 2*sqrt(d + 1)) - 5*a^4*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x
 + a) - 2*sqrt(d + 1)) + 60*b^2*x^2*polylog(3, sqrt(d + 1)*(cosh(b*x + a) +
 sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -sqrt(d + 1)*(cosh(b*x + a) + sinh
(b*x + a))) - 120*b*x*polylog(4, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)
)) - 120*b*x*polylog(4, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) - 5*(
b^4*x^4 - a^4)*log(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 5*(b^
4*x^4 - a^4)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 120*po
lylog(5, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) + 120*polylog(5, -sqr
t(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acoth(1+d+d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arcoth}(d\coth(bx+a)+d+1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(1+d+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^3*arccoth(d*coth(b*x + a) + d + 1), x)
```

3.222 $\int x^2 \coth^{-1}(1+d+d\coth(a+bx))\,dx$

Optimal. Leaf size=126

$$\frac{x\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{4b^2}-\frac{\text{PolyLog}\left(4,(d+1)e^{2a+2bx}\right)}{8b^3}-\frac{x^2\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{6}x^3\log\left(1-(d+1)e^{2a+2b}\right.$$

```
[Out] (b*x^4)/12 + (x^3*ArcCoth[1 + d + d*Coth[a + b*x]])/3 - (x^3*Log[1 - (1 + d
)*E^(2*a + 2*b*x)])/6 - (x^2*PolyLog[2, (1 + d)*E^(2*a + 2*b*x)])/(4*b) + (
x*PolyLog[3, (1 + d)*E^(2*a + 2*b*x)])/(4*b^2) - PolyLog[4, (1 + d)*E^(2*a
+ 2*b*x)]/(8*b^3)
```

Rubi [A] time = 0.26909, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6242, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{x\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{4b^2}-\frac{\text{PolyLog}\left(4,(d+1)e^{2a+2bx}\right)}{8b^3}-\frac{x^2\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{6}x^3\log\left(1-(d+1)e^{2a+2b}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] (b*x^4)/12 + (x^3*ArcCoth[1 + d + d*Coth[a + b*x]])/3 - (x^3*Log[1 - (1 + d
)*E^(2*a + 2*b*x)])/6 - (x^2*PolyLog[2, (1 + d)*E^(2*a + 2*b*x)])/(4*b) + (
x*PolyLog[3, (1 + d)*E^(2*a + 2*b*x)])/(4*b^2) - PolyLog[4, (1 + d)*E^(2*a
+ 2*b*x)]/(8*b^3)
```

Rule 6242

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Coth[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x^2\coth^{-1}(1+d+d\coth(a+bx))\,dx &= \frac{1}{3}x^3\coth^{-1}(1+d+d\coth(a+bx))+\frac{1}{3}b\int\frac{x^3}{1+(-1-d)e^{2a+2bx}}\,dx\\ &= \frac{bx^4}{12}+\frac{1}{3}x^3\coth^{-1}(1+d+d\coth(a+bx))+\frac{1}{3}(b(1+d))\int\frac{e^{2a+2bx}x^3}{1+(-1-d)e^{2a}}\\ &= \frac{bx^4}{12}+\frac{1}{3}x^3\coth^{-1}(1+d+d\coth(a+bx))-\frac{1}{6}x^3\log\left(1-(1+d)e^{2a+2bx}\right)+\\ &= \frac{bx^4}{12}+\frac{1}{3}x^3\coth^{-1}(1+d+d\coth(a+bx))-\frac{1}{6}x^3\log\left(1-(1+d)e^{2a+2bx}\right)-\\ &= \frac{bx^4}{12}+\frac{1}{3}x^3\coth^{-1}(1+d+d\coth(a+bx))-\frac{1}{6}x^3\log\left(1-(1+d)e^{2a+2bx}\right)-\\ &= \frac{bx^4}{12}+\frac{1}{3}x^3\coth^{-1}(1+d+d\coth(a+bx))-\frac{1}{6}x^3\log\left(1-(1+d)e^{2a+2bx}\right)-\\ &= \frac{bx^4}{12}+\frac{1}{3}x^3\coth^{-1}(1+d+d\coth(a+bx))-\frac{1}{6}x^3\log\left(1-(1+d)e^{2a+2bx}\right)-\end{aligned}$$

Mathematica [A] time = 0.102156, size = 116, normalized size = 0.92

$$\frac{1}{24}\left(\frac{6x\text{PolyLog}\left(3,\frac{e^{-2(a+bx)}}{d+1}\right)}{b^2}+\frac{3\text{PolyLog}\left(4,\frac{e^{-2(a+bx)}}{d+1}\right)}{b^3}+\frac{6x^2\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{d+1}\right)}{b}-4x^3\log\left(1-\frac{e^{-2(a+bx)}}{d+1}\right)+8x^3\cot\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] (8*x^3*ArcCoth[1 + d + d*Coth[a + b*x]] - 4*x^3*Log[1 - 1/((1 + d)*E^(2*(a
+ b*x)))] + (6*x^2*PolyLog[2, 1/((1 + d)*E^(2*(a + b*x)))])/b + (6*x*PolyLo
g[3, 1/((1 + d)*E^(2*(a + b*x)))])/b^2 + (3*PolyLog[4, 1/((1 + d)*E^(2*(a +
```

```
b*x)))])/b^3)/24
```

Maple [C] time = 35.69, size = 1641, normalized size = 13.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(1+d+d*coth(b*x+a)),x)
```

```
[Out] -1/4/b*d/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*x^2-1/12*I*x^3*Pi*csgn(I/(ex
p(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^3+1/12*I*x^3*Pi*csgn(I
*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^3-1/3*x^3*ln(exp(b*x+a))-1/6*x^3*ln(d)-
1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2
*a)-1))^2-1/12*I*x^3*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a
))^2+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(ex
p(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^2-1/6*I*x^3*Pi*csgn(I*exp(b*x+a))*csgn(I*
exp(2*b*x+2*a))^2+1/12*b*x^4+1/6/b^3*a^3/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*
x+2*a)-1)+1/6*x^3*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1)+1/12*I*x^3*Pi*csgn(
I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x
+2*a)*d+exp(2*b*x+2*a)-1))^2-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x
+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/6/b^3*d*a^3/(1+d)
*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1)+1/12*I*x^3*Pi*csgn(I*d/(exp(2*b*x+2*
a)-1)*exp(2*b*x+2*a))^3-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(ex
p(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)
*d+exp(2*b*x+2*a)-1))-1/2/b^3*d*a^2/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))+1
/2/b^2/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x*a^2+1/3/b^3*d/(1+d)*ln(1-(1+d)*ex
p(2*b*x+2*a))*a^3-1/2/b^3*d*a^3/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))-1/2/b^3*
d*a^3/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a
))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))+1/1
2*I*x^3*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d/(ex
p(2*b*x+2*a)-1)*exp(2*b*x+2*a))+1/4/b^3*d/(1+d)*polylog(2,(1+d)*exp(2*b*x+2
*a))*a^2+1/4/b^2*d/(1+d)*polylog(3,(1+d)*exp(2*b*x+2*a))*x-1/2/b^2*a^2/(1+d
)*ln(1-exp(b*x+a)*(1+d)^(1/2))*x-1/2/b^2*a^2/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1
/2))*x-1/2/b^3*d*a^2/(1+d)*dilog(1-exp(b*x+a)*(1+d)^(1/2))+1/12*I*x^3*Pi*cs
gn(I*exp(2*b*x+2*a))^3+1/4/b^2/(1+d)*polylog(3,(1+d)*exp(2*b*x+2*a))*x-1/2/
b^3*a^3/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))-1/2/b^3*a^3/(1+d)*ln(1+exp(b*x+a
)*(1+d)^(1/2))-1/6*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x^3-1/8/b^3*d/(1+d)*p
olylog(4,(1+d)*exp(2*b*x+2*a))-1/2/b^3*a^2/(1+d)*dilog(1-exp(b*x+a)*(1+d)^(
1/2))-1/2/b^3*a^2/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))+1/3/b^3/(1+d)*ln(1-
(1+d)*exp(2*b*x+2*a))*a^3-1/4/b/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*x^2+1
/4/b^3/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*a^2+1/12*I*x^3*Pi*csgn(I*exp(b
*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/6/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x^3-1/
8/b^3/(1+d)*polylog(4,(1+d)*exp(2*b*x+2*a))-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+
2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2+1/2/b^2*d/(1+d)*ln(1-(1+d
)*exp(2*b*x+2*a))*x*a^2-1/2/b^2*d*a^2/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))*x-
1/2/b^2*d*a^2/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))*x
```

Maxima [A] time = 3.5198, size = 166, normalized size = 1.32

$$\frac{1}{3}\,x^3\,\mathrm{arcoth}\left(d\coth\left(bx+a\right)+d+1\right)+\frac{1}{36}\left(\frac{3\,x^4}{d}-\frac{2\left(4\,b^3x^3\log\left(-(d+1)e^{(2\,bx+2\,a)}+1\right)+6\,b^2x^2\mathrm{Li}_2\left((d+1)e^{(2\,bx+2\,a)}\right)-6\right.}{b^4d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(d*coth(b*x + a) + d + 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*log
(-(d + 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog((d + 1)*e^(2*b*x + 2*a)) -
 6*b*x*polylog(3, (d + 1)*e^(2*b*x + 2*a)) + 3*polylog(4, (d + 1)*e^(2*b*x
+ 2*a)))/(b^4*d))*b*d
```

Fricas [C] time = 2.04236, size = 1081, normalized size = 8.58

$$\frac{b^4x^4 + 2\,b^3x^3\log\left(\frac{d\cosh(bx+a)+(d+2)\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 6\,b^2x^2\mathrm{Li}_2\left(\sqrt{d+1}(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))\right) - 6\,b^2x^2\mathrm{Li}_2\left(-\sqrt{d+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(b^4*x^4 + 2*b^3*x^3*log((d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/(d*
cosh(b*x + a) + d*sinh(b*x + a))) - 6*b^2*x^2*dilog(sqrt(d + 1)*(cosh(b*x +
 a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-sqrt(d + 1)*(cosh(b*x + a) + sinh(
b*x + a))) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a) +
2*sqrt(d + 1)) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a
) - 2*sqrt(d + 1)) + 12*b*x*polylog(3, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*
x + a))) + 12*b*x*polylog(3, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)))
- 2*(b^3*x^3 + a^3)*log(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) -
2*(b^3*x^3 + a^3)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 1
2*polylog(4, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) - 12*polylog(4, -
sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(1+d+d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arcoth}\left(d\coth\left(bx+a\right)+d+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+d+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(d*coth(b*x + a) + d + 1), x)
```

3.223 $\int x\coth^{-1}(1+d+d\coth(a+bx))\,dx$

Optimal. Leaf size=100

$$\frac{\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{8b^2}-\frac{x\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(d+1)e^{2a+2bx}\right)+\frac{1}{2}x^2\coth^{-1}(d\coth(a+bx)\cdot$$

```
[Out] (b*x^3)/6 + (x^2*ArcCoth[1 + d + d*Coth[a + b*x]])/2 - (x^2*Log[1 - (1 + d)
*E^(2*a + 2*b*x)])/4 - (x*PolyLog[2, (1 + d)*E^(2*a + 2*b*x)])/(4*b) + Poly
Log[3, (1 + d)*E^(2*a + 2*b*x)]/(8*b^2)
```

Rubi [A] time = 0.24039, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6242, 2184, 2190, 2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{8b^2}-\frac{x\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(d+1)e^{2a+2bx}\right)+\frac{1}{2}x^2\coth^{-1}(d\coth(a+bx)\cdot$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcCoth[1 + d + d*Coth[a + b*x]])/2 - (x^2*Log[1 - (1 + d)
*E^(2*a + 2*b*x)])/4 - (x*PolyLog[2, (1 + d)*E^(2*a + 2*b*x)])/(4*b) + Poly
Log[3, (1 + d)*E^(2*a + 2*b*x)]/(8*b^2)
```

Rule 6242

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Coth[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\coth^{-1}(1+d+d\coth(a+bx))\,dx=\frac{1}{2}x^2\coth^{-1}(1+d+d\coth(a+bx))+\frac{1}{2}b\int\frac{x^2}{1+(-1-d)e^{2a+2bx}}\,dx$$
$$=\frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1+d+d\coth(a+bx))+\frac{1}{2}(b(1+d))\int\frac{e^{2a+2bx}x^2}{1+(-1-d)e^{2a+}}$$
$$=\frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1+d+d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1+d)e^{2a+2bx}\right)+$$
$$=\frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1+d+d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1+d)e^{2a+2bx}\right)-$$
$$=\frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1+d+d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1+d)e^{2a+2bx}\right)-$$
$$=\frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1+d+d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1+d)e^{2a+2bx}\right)-$$

Mathematica [A] time = 0.092782, size = 90, normalized size = 0.9

$$\frac{2bx\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{d+1}\right)+\text{PolyLog}\left(3,\frac{e^{-2(a+bx)}}{d+1}\right)+2b^2x^2\left(2\coth^{-1}(d\coth(a+bx)+d+1)-\log\left(1-\frac{e^{-2(a+bx)}}{d+1}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] (2*b^2*x^2*(2*ArcCoth[1 + d + d*Coth[a + b*x]] - Log[1 - 1/((1 + d)*E^(2*(a
 + b*x)))]) + 2*b*x*PolyLog[2, 1/((1 + d)*E^(2*(a + b*x)))] + PolyLog[3, 1/
((1 + d)*E^(2*(a + b*x)))])/(8*b^2)
```

Maple [C] time = 15.664, size = 1560, normalized size = 15.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(1+d+d*coth(b*x+a)),x)
```

```
[Out] 1/6*b*x^3+1/8/b^2/(1+d)*polylog(3,(1+d)*exp(2*b*x+2*a))-1/4/(1+d)*ln(1-(1+d
)*exp(2*b*x+2*a))*x^2-1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1)
```

```
)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/8*I*x^2*Pi*csgn(I/(exp(2*
b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))
^2+1/8*I*x^2*Pi*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b
*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^2-1/8*I*x^2*Pi*csgn(I*exp(2
*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2-1/2*x^2*ln(exp(b*x+a
))-1/4*x^2*ln(d)-1/2/b/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x*a+1/4*x^2*ln(exp(
2*b*x+2*a)*d+exp(2*b*x+2*a)-1)-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn
(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2-1/8*I*x^2*Pi*csgn(I*d)*csgn(I*d/(ex
p(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/4*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x^
2+1/8/b^2*d/(1+d)*polylog(3,(1+d)*exp(2*b*x+2*a))+1/2/b^2*a/(1+d)*dilog(1-e
xp(b*x+a)*(1+d)^(1/2))+1/2/b^2*a/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))-1/4/
b^2/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a^2-1/4/b/(1+d)*polylog(2,(1+d)*exp(2*
b*x+2*a))*x-1/4/b^2/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*a+1/2/b^2*a^2/(1+
d)*ln(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b^2*a^2/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/
2))-1/4/b^2*a^2/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1)-1/4/b^2*d/(1+d)
*ln(1-(1+d)*exp(2*b*x+2*a))*a^2-1/4/b*d/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a
))*x-1/4/b^2*d/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*a+1/2/b*a/(1+d)*ln(1-e
xp(b*x+a)*(1+d)^(1/2))*x+1/2/b*a/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))*x+1/2/b
^2*d*a^2/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b^2*d*a^2/(1+d)*ln(1+exp(b*
x+a)*(1+d)^(1/2))+1/2/b^2*d*a/(1+d)*dilog(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b^2
*d*a/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))-1/4/b^2*d*a^2/(1+d)*ln(exp(2*b*x
+2*a)*d+exp(2*b*x+2*a)-1)+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a))^3-1/8*I*x^2*P
i*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^3+1/8*I*x^
2*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^3+1/8*I*x^2*Pi*csgn(I*exp(
2*b*x+2*a)/(exp(2*b*x+2*a)-1))^3+1/8*I*x^2*Pi*csgn(I*exp(b*x+a))^2*csgn(I*e
xp(2*b*x+2*a))-1/4*I*x^2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/8
*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a
)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))+1/8*I*
x^2*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2
*a)/(exp(2*b*x+2*a)-1))+1/8*I*x^2*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2
*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))-1/2/b*d/(1+d)*ln(
1-(1+d)*exp(2*b*x+2*a))*x*a+1/2/b*d*a/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))*x+
1/2/b*d*a/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))*x
```

Maxima [A] time = 3.7136, size = 135, normalized size = 1.35

$$\frac{1}{24}\left(\frac{4\,x^3}{d}-\frac{3\left(2\,b^2x^2\log\left(-(d+1)e^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left((d+1)e^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3((d+1)e^{(2\,bx+2\,a)})\right)}{b^3d}\right)bd+\frac{1}{2}\,x^2\,\mathrm{arcoth}\,(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/24*(4*x^3/d - 3*(2*b^2*x^2*log(-(d + 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dilo
g((d + 1)*e^(2*b*x + 2*a)) - polylog(3, (d + 1)*e^(2*b*x + 2*a)))/(b^3*d))*
b*d + 1/2*x^2*arccoth(d*coth(b*x + a) + d + 1)
```

Fricas [C] time = 1.97112, size = 894, normalized size = 8.94

$$\frac{2\,b^3x^3+3\,b^2x^2\log\left(\frac{d\cosh(bx+a)+(d+2)\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right)-6\,bx\mathrm{Li}_2\left(\sqrt{d+1}(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))\right)-6\,bx\mathrm{Li}_2\left(-\sqrt{d+1}(\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 + 3*b^2*x^2*log((d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/(
d*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b*x*dilog(sqrt(d + 1)*(cosh(b*x + a
) + sinh(b*x + a))) - 6*b*x*dilog(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x +
a))) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a) + 2*sqrt
(d + 1)) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a) - 2*
sqrt(d + 1)) - 3*(b^2*x^2 - a^2)*log(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x
+ a)) + 1) - 3*(b^2*x^2 - a^2)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x +
 a)) + 1) + 6*polylog(3, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) + 6*p
olylog(3, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(1+d+d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arcoth}\left(d \coth\left(bx + a\right) + d + 1\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1+d+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(d*coth(b*x + a) + d + 1), x)
```

3.224 $\int \coth^{-1}(1+d+d\coth(a+bx))\,dx$

Optimal. Leaf size=69

$$-\frac{\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{2}x\log\left(1-(d+1)e^{2a+2bx}\right)+x\coth^{-1}(d\coth(a+bx)+d+1)+\frac{bx^2}{2}$$

```
[Out] (b*x^2)/2 + x*ArcCoth[1 + d + d*Coth[a + b*x]] - (x*Log[1 - (1 + d)*E^(2*a
+ 2*b*x)])/2 - PolyLog[2, (1 + d)*E^(2*a + 2*b*x)]/(4*b)
```

Rubi [A] time = 0.144776, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6234, 2184, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{2}x\log\left(1-(d+1)e^{2a+2bx}\right)+x\coth^{-1}(d\coth(a+bx)+d+1)+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcCoth[1 + d + d*Coth[a + b*x]] - (x*Log[1 - (1 + d)*E^(2*a
+ 2*b*x)])/2 - PolyLog[2, (1 + d)*E^(2*a + 2*b*x)]/(4*b)
```

Rule 6234

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Ar
cCoth[c + d*Coth[a + b*x]], x] + Dist[b, Int[x/(c - d - c*E^(2*a + 2*b*x)),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \coth^{-1}(1+d+d\coth(a+bx))\,dx = x\coth^{-1}(1+d+d\coth(a+bx)) + b\int \frac{x}{1+(-1-d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^2}{2} + x\coth^{-1}(1+d+d\coth(a+bx)) + (b(1+d))\int \frac{e^{2a+2bx}x}{1+(-1-d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^2}{2} + x\coth^{-1}(1+d+d\coth(a+bx)) - \frac{1}{2}x\log\left(1-(1+d)e^{2a+2bx}\right) + \frac{1}{2}\int \mathrm{l}$$
$$= \frac{bx^2}{2} + x\coth^{-1}(1+d+d\coth(a+bx)) - \frac{1}{2}x\log\left(1-(1+d)e^{2a+2bx}\right) + \frac{\mathrm{Subst}}{}$$
$$= \frac{bx^2}{2} + x\coth^{-1}(1+d+d\coth(a+bx)) - \frac{1}{2}x\log\left(1-(1+d)e^{2a+2bx}\right) - \frac{\mathrm{Li}_2\left((\right.}{}$$

Mathematica [B] time = 0.786928, size = 197, normalized size = 2.86

$$\frac{-2\mathrm{PolyLog}\left(2,-\sqrt{d+1}e^{a+bx}\right) - 2\mathrm{PolyLog}\left(2,\sqrt{d+1}e^{a+bx}\right) - 2\log\left(e^{a+bx}\right)\log\left(1-\sqrt{d+1}e^{a+bx}\right) - 2\log\left(e^{a+bx}\right)\log\left(\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] x*ArcCoth[1 + d + d*Coth[a + b*x]] + (b^2*x^2 + Log[E^(a + b*x)]^2 - 2*Log[
E^(a + b*x)]*Log[1 - Sqrt[1 + d]*E^(a + b*x)] - 2*Log[E^(a + b*x)]*Log[1 +
Sqrt[1 + d]*E^(a + b*x)] + 2*Log[E^(a + b*x)]*Log[E^(-a - b*x)*(-1 + (1 + d
)*E^(2*(a + b*x)))] - 2*b*x*Log[d*Cosh[a + b*x] + (2 + d)*Sinh[a + b*x]] -
2*PolyLog[2, -(Sqrt[1 + d]*E^(a + b*x))] - 2*PolyLog[2, Sqrt[1 + d]*E^(a +
b*x)])/(4*b)
```

Maple [B] time = 0.161, size = 247, normalized size = 3.6

$$\frac{\mathrm{arccoth}\left(1+d+d\coth\left(bx+a\right)\right)\ln\left(d\coth\left(bx+a\right)+d\right)}{2b} - \frac{\mathrm{arccoth}\left(1+d+d\coth\left(bx+a\right)\right)\ln\left(d\coth\left(bx+a\right)-d\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1+d+d*coth(b*x+a)),x)
```

```
[Out] 1/2/b*arccoth(1+d+d*coth(b*x+a))*ln(d*coth(b*x+a)+d)-1/2/b*arccoth(1+d+d*co
th(b*x+a))*ln(d*coth(b*x+a)-d)+1/4/b*dilog((d*coth(b*x+a)+d+2)/(2*d+2))+1/4
/b*ln(d*coth(b*x+a)-d)*ln((d*coth(b*x+a)+d+2)/(2*d+2))-1/4/b*dilog(1/2*(d*c
oth(b*x+a)+d)/d)-1/4/b*ln(d*coth(b*x+a)-d)*ln(1/2*(d*coth(b*x+a)+d)/d)+1/8/
b*ln(d*coth(b*x+a)+d)^2-1/4/b*dilog(1+1/2*d*coth(b*x+a)+1/2*d)-1/4/b*ln(d*c
oth(b*x+a)+d)*ln(1+1/2*d*coth(b*x+a)+1/2*d)
```

Maxima [A] time = 3.59745, size = 97, normalized size = 1.41

$$\frac{1}{4}bd\left(\frac{2x^2}{d} - \frac{2bx\log\left(-(d+1)e^{(2bx+2a)}+1\right) + \mathrm{Li}_2\left((d+1)e^{(2bx+2a)}\right)}{b^2d}\right) + x\,\mathrm{arcoth}\left(d\coth\left(bx+a\right)+d+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*b*d*(2*x^2/d - (2*b*x*log(-(d + 1)*e^(2*b*x + 2*a) + 1) + dilog((d + 1)
*e^(2*b*x + 2*a)))/(b^2*d)) + x*arccoth(d*coth(b*x + a) + d + 1)
```

Fricas [B] time = 2.05268, size = 667, normalized size = 9.67

$$\frac{b^2x^2 + bx\log\left(\frac{d\cosh(bx+a)+(d+2)\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) + a\log\left(2(d+1)\cosh(bx+a) + 2(d+1)\sinh(bx+a) + 2\sqrt{d+1}\right) + a\log\left(2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + b*x*log((d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/(d*cosh(b*
x + a) + d*sinh(b*x + a))) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh
(b*x + a) + 2*sqrt(d + 1)) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh
(b*x + a) - 2*sqrt(d + 1)) - (b*x + a)*log(sqrt(d + 1)*(cosh(b*x + a) + sin
h(b*x + a)) + 1) - (b*x + a)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a
)) + 1) - dilog(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) - dilog(-sqrt(
d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acoth}\left(d\coth\left(a+bx\right)+d+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1+d+d*coth(b*x+a)),x)
```

```
[Out] Integral(acoth(d*coth(a + b*x) + d + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}\left(d\coth\left(bx+a\right)+d+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+d+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*coth(b*x + a) + d + 1), x)
```

3.225 $\int \frac{\coth^{-1}(1+d+d\coth(a+bx))}{x}\,dx$

Optimal. Leaf size=18

$$\text{CannotIntegrate}\left(\frac{\coth^{-1}(d\coth(a+bx)+d+1)}{x},x\right)$$

```
[Out] CannotIntegrate[ArcCoth[1 + d + d*Coth[a + b*x]]/x, x]
```

Rubi [A] time = 0.0609226, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth^{-1}(1+d+d\coth(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCoth[1 + d + d*Coth[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcCoth[1 + d + d*Coth[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\coth^{-1}(1+d+d\coth(a+bx))}{x}\,dx=\int \frac{\coth^{-1}(1+d+d\coth(a+bx))}{x}\,dx$$

Mathematica [A] time = 3.14883, size = 0, normalized size = 0.

$$\int \frac{\coth^{-1}(1+d+d\coth(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCoth[1 + d + d*Coth[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcCoth[1 + d + d*Coth[a + b*x]]/x, x]
```

Maple [A] time = 0.372, size = 0, normalized size = 0.

$$\int \frac{\text{arccoth}\,(1+d+d\coth\,(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1+d+d*coth(b*x+a))/x,x)
```

```
[Out] int(arccoth(1+d+d*coth(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}\left(d \coth\left(bx + a\right) + d + 1\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+d+d*coth(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arccoth(d*coth(b*x + a) + d + 1)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arcoth}\left(d \coth\left(bx + a\right) + d + 1\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+d+d*coth(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(d*coth(b*x + a) + d + 1)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}\left(d \coth\left(a + bx\right) + d + 1\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1+d+d*coth(b*x+a))/x,x)
```

```
[Out] Integral(acoth(d*coth(a + b*x) + d + 1)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}\left(d \coth\left(bx + a\right) + d + 1\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+d+d*coth(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*coth(b*x + a) + d + 1)/x, x)
```

3.226 $\int x^3 \coth^{-1}(1-d-d\coth(a+bx))\,dx$

Optimal. Leaf size=165

$$\frac{3x^2\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{8b^2}-\frac{3x\text{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)}{8b^3}+\frac{3\text{PolyLog}\left(5,(1-d)e^{2a+2bx}\right)}{16b^4}-\frac{x^3\text{PolyLog}\left(2,(1-\right.}{4b}$$

```
[Out] (b*x^5)/20 + (x^4*ArcCoth[1 - d - d*Coth[a + b*x]])/4 - (x^4*Log[1 - (1 - d
)*E^(2*a + 2*b*x)])/8 - (x^3*PolyLog[2, (1 - d)*E^(2*a + 2*b*x)])/(4*b) + (
3*x^2*PolyLog[3, (1 - d)*E^(2*a + 2*b*x)])/(8*b^2) - (3*x*PolyLog[4, (1 - d
)*E^(2*a + 2*b*x)])/(8*b^3) + (3*PolyLog[5, (1 - d)*E^(2*a + 2*b*x)])/(16*b
^4)
```

Rubi [A] time = 0.309702, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6242, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{8b^2}-\frac{3x\text{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)}{8b^3}+\frac{3\text{PolyLog}\left(5,(1-d)e^{2a+2bx}\right)}{16b^4}-\frac{x^3\text{PolyLog}\left(2,(1-\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCoth[1 - d - d*Coth[a + b*x]],x]
```


```
[Out] (b*x^5)/20 + (x^4*ArcCoth[1 - d - d*Coth[a + b*x]])/4 - (x^4*Log[1 - (1 - d
)*E^(2*a + 2*b*x)])/8 - (x^3*PolyLog[2, (1 - d)*E^(2*a + 2*b*x)])/(4*b) + (
3*x^2*PolyLog[3, (1 - d)*E^(2*a + 2*b*x)])/(8*b^2) - (3*x*PolyLog[4, (1 - d
)*E^(2*a + 2*b*x)])/(8*b^3) + (3*PolyLog[5, (1 - d)*E^(2*a + 2*b*x)])/(16*b
^4)
```


Rule 6242

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Coth[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3 \coth^{-1}(1-d-d\coth(a+bx))\,dx = \frac{1}{4}x^4\coth^{-1}(1-d-d\coth(a+bx)) + \frac{1}{4}b\int \frac{x^4}{1+(-1+d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1-d-d\coth(a+bx)) + \frac{1}{4}(b(1-d))\int \frac{e^{2a+2bx}x^4}{1+(-1+d)e^{2a+2bx}}$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1-d)e^{2a+2bx}\right) + \frac{1}{2}\int$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1-d)e^{2a+2bx}\right) - \frac{x^3\mathrm{L}}{}$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1-d)e^{2a+2bx}\right) - \frac{x^3\mathrm{L}}{}$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1-d)e^{2a+2bx}\right) - \frac{x^3\mathrm{L}}{}$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1-d)e^{2a+2bx}\right) - \frac{x^3\mathrm{L}}{}$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1-d)e^{2a+2bx}\right) - \frac{x^3\mathrm{L}}{}$$

Mathematica [A] time = 0.187995, size = 147, normalized size = 0.89

$$\frac{1}{16}\left(\frac{6x^2\text{PolyLog}\left(3,-\frac{e^{-2(a+bx)}}{d-1}\right)}{b^2} + \frac{6x\text{PolyLog}\left(4,-\frac{e^{-2(a+bx)}}{d-1}\right)}{b^3} + \frac{3\text{PolyLog}\left(5,-\frac{e^{-2(a+bx)}}{d-1}\right)}{b^4} + \frac{4x^3\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{d-1}\right)}{b} - 2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCoth[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (4*x^4*ArcCoth[1 - d - d*Coth[a + b*x]] - 2*x^4*Log[1 + 1/((-1 + d)*E^(2*(a
 + b*x)))] + (4*x^3*PolyLog[2, -(1/((-1 + d)*E^(2*(a + b*x))))])/b + (6*x^2
*PolyLog[3, -(1/((-1 + d)*E^(2*(a + b*x))))])/b^2 + (6*x*PolyLog[4, -(1/((-
1 + d)*E^(2*(a + b*x))))])/b^3 + (3*PolyLog[5, -(1/((-1 + d)*E^(2*(a + b*x)
)))])/b^4)/16
```

Maple [C] time = 37.705, size = 1830, normalized size = 11.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccoth(1-d-d*coth(b*x+a)),x)
```

```
[Out] -1/16*I*x^4*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/16
*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a
)-1)*exp(2*b*x+2*a))^2+1/2/b^4*d*a^3/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))-
3/8/b^4*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a^4-1/4/b*d/(d-1)*polylog(2,-(d-
1)*exp(2*b*x+2*a))*x^3-1/4/b^4*d/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*a^3
+3/8/b^2*d/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))*x^2-3/8/b^3*d/(d-1)*polyl
og(4,-(d-1)*exp(2*b*x+2*a))*x-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn
(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2-1/16*I*x^4*Pi*csgn(I*d/(exp(2*b*x+2
*a)-1)*exp(2*b*x+2*a))^3+1/16*I*x^4*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*
x+2*a))-1/8*I*x^4*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/16*I*x^4
*Pi*csgn(I*exp(2*b*x+2*a))^3+1/20*b*x^5+1/8/b^4*a^4/(d-1)*ln(exp(2*b*x+2*a)
*d-exp(2*b*x+2*a)+1)-1/2/b^4*a^3/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))-1/2/
b^4*a^3/(d-1)*dilog(1-exp(b*x+a)*(1-d)^(1/2))+3/16/b^4*d/(d-1)*polylog(5,-(
d-1)*exp(2*b*x+2*a))-1/2/b^4*a^4/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))+3/8/b^4
*a^4/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))+1/4/b^4*a^3/(d-1)*polylog(2,-(d-1)*ex
p(2*b*x+2*a))-1/2/b^4*a^4/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))+1/4/b/(d-1)*po
lylog(2,-(d-1)*exp(2*b*x+2*a))*x^3-3/8/b^2/(d-1)*polylog(3,-(d-1)*exp(2*b*x
+2*a))*x^2+3/8/b^3/(d-1)*polylog(4,-(d-1)*exp(2*b*x+2*a))*x-1/8*d/(d-1)*ln(
1+(d-1)*exp(2*b*x+2*a))*x^4+1/8/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^4-3/16/b
^4/(d-1)*polylog(5,-(d-1)*exp(2*b*x+2*a))+1/2/b^4*d*a^3/(d-1)*dilog(1-exp(b
*x+a)*(1-d)^(1/2))-1/2/b^3*a^3/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))*x+1/2/b^4
*d*a^4/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^4*d*a^4/(d-1)*ln(1-exp(b*x+
a)*(1-d)^(1/2))+1/2/b^3*a^3/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x-1/2/b^3*a^3/
(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))*x-1/8*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-1)
*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))^2+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)
/(exp(2*b*x+2*a)-1))^3+1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2
*a)*d-exp(2*b*x+2*a)+1))^3+1/8*I*x^4*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b
*x+2*a))^2-1/4*x^4*ln(exp(b*x+a))-1/8*x^4*ln(d)-1/8/b^4*d*a^4/(d-1)*ln(exp(
2*b*x+2*a)*d-exp(2*b*x+2*a)+1)-1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2+1/16*I*x^4*Pi*csgn(I*(exp(2*b*x+2*a)*d
-exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2
*a)+1))^2+1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1
)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))^2+1/8*x^4*ln(exp(2*b*x+2*a)*d-exp(2*
b*x+2*a)+1)+1/2/b^3*d*a^3/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))*x+1/2/b^3*d*a^
3/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))*x-1/2/b^3*d/(d-1)*ln(1+(d-1)*exp(2*b*x
+2*a))*x*a^3-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a
)*d-exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*
x+2*a)+1))+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)-1))*
csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))+1/16*I*x^4*Pi*csgn(I*d)*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a)
)
```

Maxima [A] time = 3.71832, size = 201, normalized size = 1.22

$$-\frac{1}{4}x^4\operatorname{arcoth}\left(d\coth\left(bx+a\right)+d-1\right)+\frac{1}{40}\left(\frac{2x^5}{d}-\frac{5\left(2b^4x^4\log\left(\left(d-1\right)e^{\left(2bx+2a\right)}+1\right)+4b^3x^3\operatorname{Li}_2\left(-\left(d-1\right)e^{\left(2bx+2a\right)}\right)-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(1-d-d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/4*x^4*arccoth(d*coth(b*x + a) + d - 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*lo
g((d - 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog(-(d - 1)*e^(2*b*x + 2*a))
- 6*b^2*x^2*polylog(3, -(d - 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, -(d - 1
)*e^(2*b*x + 2*a)) - 3*polylog(5, -(d - 1)*e^(2*b*x + 2*a)))/(b^5*d))*b*d
```

Fricas [C] time = 2.14981, size = 1377, normalized size = 8.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(1-d-d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/40*(2*b^5*x^5 - 5*b^4*x^4*log((d*cosh(b*x + a) + d*sinh(b*x + a))/(d*cosh
(b*x + a) + (d - 2)*sinh(b*x + a))) - 20*b^3*x^3*dilog(1/2*sqrt(-4*d + 4)*(
cosh(b*x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-1/2*sqrt(-4*d + 4)*(cos
h(b*x + a) + sinh(b*x + a))) - 5*a^4*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1
)*sinh(b*x + a) + sqrt(-4*d + 4)) - 5*a^4*log(2*(d - 1)*cosh(b*x + a) + 2*(
d - 1)*sinh(b*x + a) - sqrt(-4*d + 4)) + 60*b^2*x^2*polylog(3, 1/2*sqrt(-4*
d + 4)*(cosh(b*x + a) + sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -1/2*sqrt(-
4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, 1/2*sqrt(-4*
d + 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, -1/2*sqrt(-4*d
 + 4)*(cosh(b*x + a) + sinh(b*x + a))) - 5*(b^4*x^4 - a^4)*log(1/2*sqrt(-4*
d + 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 5*(b^4*x^4 - a^4)*log(-1/2*sq
rt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 120*polylog(5, 1/2*sqrt
(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) + 120*polylog(5, -1/2*sqrt(-4*d
 + 4)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acoth(1-d-d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\operatorname{arcoth}\left(-d\coth\left(bx+a\right)-d+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(1-d-d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^3*arccoth(-d*coth(b*x + a) - d + 1), x)
```

3.227 $\int x^2 \coth^{-1}(1 - d - d \coth(a + bx)) \, dx$

Optimal. Leaf size=137

$$\frac{x\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{4b^2}-\frac{\text{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)}{8b^3}-\frac{x^2\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{6}x^3\log\left(1-(1-d)e^{2a+2b}\right.$$

```
[Out] (b*x^4)/12 + (x^3*ArcCoth[1 - d - d*Coth[a + b*x]])/3 - (x^3*Log[1 - (1 - d
)*E^(2*a + 2*b*x)])/6 - (x^2*PolyLog[2, (1 - d)*E^(2*a + 2*b*x)])/(4*b) + (
x*PolyLog[3, (1 - d)*E^(2*a + 2*b*x)])/(4*b^2) - PolyLog[4, (1 - d)*E^(2*a
+ 2*b*x)]/(8*b^3)
```

Rubi [A] time = 0.270101, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6242, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{x\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{4b^2}-\frac{\text{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)}{8b^3}-\frac{x^2\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{6}x^3\log\left(1-(1-d)e^{2a+2b}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (b*x^4)/12 + (x^3*ArcCoth[1 - d - d*Coth[a + b*x]])/3 - (x^3*Log[1 - (1 - d
)*E^(2*a + 2*b*x)])/6 - (x^2*PolyLog[2, (1 - d)*E^(2*a + 2*b*x)])/(4*b) + (
x*PolyLog[3, (1 - d)*E^(2*a + 2*b*x)])/(4*b^2) - PolyLog[4, (1 - d)*E^(2*a
+ 2*b*x)]/(8*b^3)
```

Rule 6242

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Coth[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \coth^{-1}(1-d-d\coth(a+bx))\,dx = \frac{1}{3}x^3\coth^{-1}(1-d-d\coth(a+bx)) + \frac{1}{3}b\int \frac{x^3}{1+(-1+d)e^{2a+2bx}}\,dx$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1-d-d\coth(a+bx)) + \frac{1}{3}(b(1-d))\int \frac{e^{2a+2bx}x^3}{1+(-1+d)e^{2a-}}$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1-d-d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-d)e^{2a+2bx}\right) +$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1-d-d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-d)e^{2a+2bx}\right) -$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1-d-d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-d)e^{2a+2bx}\right) -$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1-d-d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-d)e^{2a+2bx}\right) -$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1-d-d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-d)e^{2a+2bx}\right) -$$

Mathematica [A] time = 0.11101, size = 121, normalized size = 0.88

$$\frac{1}{24}\left(\frac{6x\text{PolyLog}\left(3,-\frac{e^{-2(a+bx)}}{d-1}\right)}{b^2} + \frac{3\text{PolyLog}\left(4,-\frac{e^{-2(a+bx)}}{d-1}\right)}{b^3} + \frac{6x^2\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{d-1}\right)}{b} - 4x^3\log\left(\frac{e^{-2(a+bx)}}{d-1}+1\right) + 8x\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (8*x^3*ArcCoth[1 - d - d*Coth[a + b*x]] - 4*x^3*Log[1 + 1/((-1 + d)*E^(2*(a
 + b*x)))] + (6*x^2*PolyLog[2, -(1/((-1 + d)*E^(2*(a + b*x))))])/b + (6*x*P
olyLog[3, -(1/((-1 + d)*E^(2*(a + b*x))))])/b^2 + (3*PolyLog[4, -(1/((-1 +
```

```
d)*E^(2*(a + b*x))))])/b^3)/24
```

Maple [C] time = 37.721, size = 1771, normalized size = 12.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(1-d-d*coth(b*x+a)),x)
```

```
[Out] 1/12*I*x^3*Pi*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x
+2*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))^2+1/6/b^3*d*a^3/(d-1)*ln(exp(
2*b*x+2*a)*d-exp(2*b*x+2*a)+1)+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csg
n(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))^2-1/12*I*x^3*Pi
*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^3+1/12*I*x^3*Pi*csgn(I*exp(2*b
*x+2*a)/(exp(2*b*x+2*a)-1))^3-1/3*x^3*ln(exp(b*x+a))-1/6*x^3*ln(d)+1/6*I*x^
3*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/12*I*x^3*Pi*csgn(I/(ex
p(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2-1/12*I*x^3*Pi*
csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/6*I*x^3*Pi*csgn(I
*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/12*b*x^4+1/6*x^3*ln(exp(2*b*x+2*a)*
d-exp(2*b*x+2*a)+1)-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))
*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/2/b^3*d*a^2/(d-1)*dilog(1+
exp(b*x+a)*(1-d)^(1/2))-1/2/b^3*d*a^2/(d-1)*dilog(1-exp(b*x+a)*(1-d)^(1/2))
-1/2/b^3*d*a^3/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))-1/2/b^3*d*a^3/(d-1)*ln(1-
exp(b*x+a)*(1-d)^(1/2))+1/3/b^3*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a^3-1/4/
b*d/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*x^2+1/4/b^3*d/(d-1)*polylog(2,-(
d-1)*exp(2*b*x+2*a))*a^2+1/4/b^2*d/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))*x
-1/2/b^2/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x*a^2+1/2/b^2*a^2/(d-1)*ln(1+exp(
b*x+a)*(1-d)^(1/2))*x+1/2/b^2*a^2/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))*x-1/6/
b^3*a^3/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1)+1/12*I*x^3*Pi*csgn(I/(e
xp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))^3-1/6*d/(d-1)*ln(1+(d
-1)*exp(2*b*x+2*a))*x^3-1/8/b^3*d/(d-1)*polylog(4,-(d-1)*exp(2*b*x+2*a))+1/
2/b^3*a^2/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^3*a^2/(d-1)*dilog(1-e
xp(b*x+a)*(1-d)^(1/2))-1/3/b^3/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a^3+1/4/b/(
d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*x^2-1/4/b^3/(d-1)*polylog(2,-(d-1)*ex
p(2*b*x+2*a))*a^2-1/4/b^2/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))*x+1/2/b^3*
a^3/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^3*a^3/(d-1)*ln(1-exp(b*x+a)*(1
-d)^(1/2))+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)-1))*
csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))+1/12*I*x^3*Pi*csgn(I*d)*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a)
)-1/6*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1
))^2+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))^3+1/12*I*x^3*Pi*csgn(I*exp(b*x+a)
)^2*csgn(I*exp(2*b*x+2*a))+1/6/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^3+1/8/b^3
/(d-1)*polylog(4,-(d-1)*exp(2*b*x+2*a))-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)
)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2-1/12*I*x^3*Pi*csgn(I/(exp(2*b
*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2
*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))+1/2/b^2*d/(d-1)*ln(1+(d-1)*exp(
2*b*x+2*a))*x*a^2-1/2/b^2*d*a^2/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))*x-1/2/b^
2*d*a^2/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))*x
```

Maxima [A] time = 3.56393, size = 169, normalized size = 1.23

$$-\frac{1}{3}x^3\,\mathrm{arcoth}\left(d\coth\left(bx+a\right)+d-1\right)+\frac{1}{36}\left(\frac{3x^4}{d}-\frac{2\left(4b^3x^3\log\left((d-1)e^{(2bx+2a)}+1\right)+6b^2x^2\mathrm{Li}_2\left(-(d-1)e^{(2bx+2a)}\right)-\right.}{b^4d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1-d-d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/3*x^3*arccoth(d*coth(b*x + a) + d - 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*lo
g((d - 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-(d - 1)*e^(2*b*x + 2*a))
- 6*b*x*polylog(3, -(d - 1)*e^(2*b*x + 2*a)) + 3*polylog(4, -(d - 1)*e^(2*b
*x + 2*a)))/(b^4*d))*b*d
```

Fricas [C] time = 2.05805, size = 1160, normalized size = 8.47

$$\frac{b^4x^4 - 2\,b^3x^3\log\left(\frac{d\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+(d-2)\sinh(bx+a)}\right) - 6\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{-4\,d+4}(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))\right) - 6\,b^2x^2\mathrm{Li}_2\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1-d-d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(b^4*x^4 - 2*b^3*x^3*log((d*cosh(b*x + a) + d*sinh(b*x + a))/(d*cosh(b
*x + a) + (d - 2)*sinh(b*x + a))) - 6*b^2*x^2*dilog(1/2*sqrt(-4*d + 4)*(cos
h(b*x + a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-1/2*sqrt(-4*d + 4)*(cosh(b*
x + a) + sinh(b*x + a))) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*si
nh(b*x + a) + sqrt(-4*d + 4)) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d -
1)*sinh(b*x + a) - sqrt(-4*d + 4)) + 12*b*x*polylog(3, 1/2*sqrt(-4*d + 4)*(
cosh(b*x + a) + sinh(b*x + a))) + 12*b*x*polylog(3, -1/2*sqrt(-4*d + 4)*(co
sh(b*x + a) + sinh(b*x + a))) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(-4*d + 4)*(c
osh(b*x + a) + sinh(b*x + a)) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sqrt(-4*d +
 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 12*polylog(4, 1/2*sqrt(-4*d + 4)
*(cosh(b*x + a) + sinh(b*x + a))) - 12*polylog(4, -1/2*sqrt(-4*d + 4)*(cosh
(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(1-d-d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\,\mathrm{arcoth}\left(-d\coth\left(bx+a\right)-d+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1-d-d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(-d*coth(b*x + a) - d + 1), x)
```

3.228 $\int x \coth^{-1}(1-d-d\coth(a+bx))\,dx$

Optimal. Leaf size=109

$$\frac{\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{8b^2}-\frac{x\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(1-d)e^{2a+2bx}\right)+\frac{1}{2}x^2\coth^{-1}(d(-\coth(a+bx$$

```
[Out] (b*x^3)/6 + (x^2*ArcCoth[1 - d - d*Coth[a + b*x]])/2 - (x^2*Log[1 - (1 - d)
*E^(2*a + 2*b*x)])/4 - (x*PolyLog[2, (1 - d)*E^(2*a + 2*b*x)])/(4*b) + Poly
Log[3, (1 - d)*E^(2*a + 2*b*x)]/(8*b^2)
```

Rubi [A] time = 0.237728, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6242, 2184, 2190, 2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{8b^2}-\frac{x\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(1-d)e^{2a+2bx}\right)+\frac{1}{2}x^2\coth^{-1}(d(-\coth(a+bx$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcCoth[1 - d - d*Coth[a + b*x]])/2 - (x^2*Log[1 - (1 - d)
*E^(2*a + 2*b*x)])/4 - (x*PolyLog[2, (1 - d)*E^(2*a + 2*b*x)])/(4*b) + Poly
Log[3, (1 - d)*E^(2*a + 2*b*x)]/(8*b^2)
```

Rule 6242

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Coth[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\coth^{-1}(1-d-d\coth(a+bx))\,dx = \frac{1}{2}x^2\coth^{-1}(1-d-d\coth(a+bx))+\frac{1}{2}b\int\frac{x^2}{1+(-1+d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1-d-d\coth(a+bx))+\frac{1}{2}(b(1-d))\int\frac{e^{2a+2bx}x^2}{1+(-1+d)e^{2a+2}}$$
$$= \frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1-d-d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1-d)e^{2a+2bx}\right)+\frac{1}{2}$$
$$= \frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1-d-d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1-d)e^{2a+2bx}\right)-\frac{x}{}$$
$$= \frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1-d-d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1-d)e^{2a+2bx}\right)-\frac{x}{}$$
$$= \frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1-d-d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1-d)e^{2a+2bx}\right)-\frac{x}{}$$

Mathematica [A] time = 0.112547, size = 94, normalized size = 0.86

$$\frac{2bx\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{d-1}\right)+\text{PolyLog}\left(3,-\frac{e^{-2(a+bx)}}{d-1}\right)+2b^2x^2\left(2\coth^{-1}(d(-\coth(a+bx))-d+1)-\log\left(\frac{e^{-2(a+bx)}}{d-1}+1\right)\right.}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (2*b^2*x^2*(2*ArcCoth[1 - d - d*Coth[a + b*x]] - Log[1 + 1/((-1 + d)*E^(2*(
a + b*x)))]) + 2*b*x*PolyLog[2, -(1/((-1 + d)*E^(2*(a + b*x))))] + PolyLog[
3, -(1/((-1 + d)*E^(2*(a + b*x))))])/(8*b^2)
```

Maple [C] time = 19.147, size = 1688, normalized size = 15.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(1-d-d*coth(b*x+a)),x)
```

```
[Out] -1/4/b^2*d*a^2/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1)-1/8*I*x^2*Pi*csg
n(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^3+1/6*b*x^3+1/8*I*x^2*Pi*csgn(I*(e
```

```
xp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a
)*d-exp(2*b*x+2*a)+1))^2-1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)
-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/8*I*x^2*Pi*csgn(I*exp(
2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2-1/2*x^2*ln(exp(b*x+
a))-1/4*x^2*ln(d)+1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x
+2*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))^2+1/4*x^2*ln(exp(2*b*x+2*a)*d
-exp(2*b*x+2*a)+1)-1/2/b^2*a/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))-1/4*d/(d
-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^2-1/2/b^2*a/(d-1)*dilog(1-exp(b*x+a)*(1-d)
^(1/2))+1/4/b^2/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a^2+1/4/b/(d-1)*polylog(2,
-(d-1)*exp(2*b*x+2*a))*x+1/4/b^2/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*a-1
/2/b^2*a^2/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))-1/2/b^2*a^2/(d-1)*ln(1-exp(b*
x+a)*(1-d)^(1/2))+1/8/b^2*d/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))-1/8*I*x^
2*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2
-1/8*I*x^2*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/4*I
*x^2*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/4/(d-1)*ln(1+(d-1)*
exp(2*b*x+2*a))*x^2-1/8/b^2/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))+1/8*I*x^
2*Pi*csgn(I*exp(2*b*x+2*a))^3-1/4*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2
*b*x+2*a)*d-exp(2*b*x+2*a)+1))^2+1/4/b^2*a^2/(d-1)*ln(exp(2*b*x+2*a)*d-exp(
2*b*x+2*a)+1)+1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d-exp(
2*b*x+2*a)+1))^3+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^3+1
/2/b^2*d*a/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^2*d*a/(d-1)*dilog(1-
exp(b*x+a)*(1-d)^(1/2))-1/4/b^2*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a^2-1/4/
b*d/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*x-1/4/b^2*d/(d-1)*polylog(2,-(d-
1)*exp(2*b*x+2*a))*a+1/2/b/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x*a-1/2/b*a/(d-
1)*ln(1+exp(b*x+a)*(1-d)^(1/2))*x-1/2/b*a/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2)
)*x+1/2/b^2*d*a^2/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^2*d*a^2/(d-1)*ln
(1-exp(b*x+a)*(1-d)^(1/2))+1/8*I*x^2*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b
*x+2*a))-1/4*I*x^2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/8*I*x^2
*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)
/(exp(2*b*x+2*a)-1))+1/8*I*x^2*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*
x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))-1/2/b*d/(d-1)*ln(1+(
d-1)*exp(2*b*x+2*a))*x*a+1/2/b*d*a/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))*x+1/2
/b*d*a/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))*x-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+
2*a)-1))*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)
-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))
```

Maxima [A] time = 3.53762, size = 136, normalized size = 1.25

$$\frac{1}{24}\left(\frac{4x^3}{d} - \frac{3\left(2b^2x^2\log\left((d-1)e^{(2bx+2a)}+1\right) + 2bx\mathrm{Li}_2\left(-(d-1)e^{(2bx+2a)}\right) - \mathrm{Li}_3(-(d-1)e^{(2bx+2a)})\right)}{b^3d}\right)bd - \frac{1}{2}x^2\,\mathrm{arcoth}\,(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1-d-d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/24*(4*x^3/d - 3*(2*b^2*x^2*log((d - 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dilog
(-(d - 1)*e^(2*b*x + 2*a)) - polylog(3, -(d - 1)*e^(2*b*x + 2*a)))/(b^3*d))
*b*d - 1/2*x^2*arccoth(d*coth(b*x + a) + d - 1)
```

Fricas [C] time = 1.84068, size = 953, normalized size = 8.74

$$2b^3x^3 - 3b^2x^2\log\left(\frac{d\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+(d-2)\sinh(bx+a)}\right) - 6bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4d+4}(\cosh(bx+a)+\sinh(bx+a))\right) - 6bx\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1-d-d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 - 3*b^2*x^2*log((d*cosh(b*x + a) + d*sinh(b*x + a))/(d*cosh
(b*x + a) + (d - 2)*sinh(b*x + a))) - 6*b*x*dilog(1/2*sqrt(-4*d + 4)*(cosh(
b*x + a) + sinh(b*x + a))) - 6*b*x*dilog(-1/2*sqrt(-4*d + 4)*(cosh(b*x + a)
 + sinh(b*x + a))) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x
 + a) + sqrt(-4*d + 4)) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sin
h(b*x + a) - sqrt(-4*d + 4)) - 3*(b^2*x^2 - a^2)*log(1/2*sqrt(-4*d + 4)*(co
sh(b*x + a) + sinh(b*x + a)) + 1) - 3*(b^2*x^2 - a^2)*log(-1/2*sqrt(-4*d +
4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 6*polylog(3, 1/2*sqrt(-4*d + 4)*(
cosh(b*x + a) + sinh(b*x + a))) + 6*polylog(3, -1/2*sqrt(-4*d + 4)*(cosh(b*
x + a) + sinh(b*x + a))))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(1-d-d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arcoth}\left(-d \coth\left(bx+a\right)-d+1\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1-d-d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(-d*coth(b*x + a) - d + 1), x)
```

3.229 $\int \coth^{-1}(1-d-d\coth(a+bx))\,dx$

Optimal. Leaf size=76

$$-\frac{\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{2}x\log\left(1-(1-d)e^{2a+2bx}\right)+x\coth^{-1}(d(-\coth(a+bx))-d+1)+\frac{bx^2}{2}$$

```
[Out] (b*x^2)/2 + x*ArcCoth[1 - d - d*Coth[a + b*x]] - (x*Log[1 - (1 - d)*E^(2*a
+ 2*b*x)])/2 - PolyLog[2, (1 - d)*E^(2*a + 2*b*x)]/(4*b)
```

Rubi [A] time = 0.144469, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6234, 2184, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{2}x\log\left(1-(1-d)e^{2a+2bx}\right)+x\coth^{-1}(d(-\coth(a+bx))-d+1)+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcCoth[1 - d - d*Coth[a + b*x]] - (x*Log[1 - (1 - d)*E^(2*a
+ 2*b*x)])/2 - PolyLog[2, (1 - d)*E^(2*a + 2*b*x)]/(4*b)
```

Rule 6234

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Ar
cCoth[c + d*Coth[a + b*x]], x] + Dist[b, Int[x/(c - d - c*E^(2*a + 2*b*x)),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \coth^{-1}(1-d-d\coth(a+bx))\,dx = x\coth^{-1}(1-d-d\coth(a+bx)) + b\int \frac{x}{1+(-1+d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^2}{2} + x\coth^{-1}(1-d-d\coth(a+bx)) + (b(1-d))\int \frac{e^{2a+2bx}x}{1+(-1+d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^2}{2} + x\coth^{-1}(1-d-d\coth(a+bx)) - \frac{1}{2}x\log\left(1-(1-d)e^{2a+2bx}\right) + \frac{1}{2}\int \mathrm{lo}$$
$$= \frac{bx^2}{2} + x\coth^{-1}(1-d-d\coth(a+bx)) - \frac{1}{2}x\log\left(1-(1-d)e^{2a+2bx}\right) + \frac{\mathrm{Subst}}{}$$
$$= \frac{bx^2}{2} + x\coth^{-1}(1-d-d\coth(a+bx)) - \frac{1}{2}x\log\left(1-(1-d)e^{2a+2bx}\right) - \frac{\mathrm{Li}_2\left((1\right.}{}$$

Mathematica [B] time = 0.749857, size = 208, normalized size = 2.74

$$\frac{-2\mathrm{PolyLog}\left(2,-\sqrt{1-d}e^{a+bx}\right) - 2\mathrm{PolyLog}\left(2,\sqrt{1-d}e^{a+bx}\right) - 2\log\left(e^{a+bx}\right)\log\left(1-\sqrt{1-d}e^{a+bx}\right) - 2\log\left(e^{a+bx}\right)\log\left(\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] x*ArcCoth[1 - d - d*Coth[a + b*x]] + (b^2*x^2 + Log[E^(a + b*x)]^2 - 2*Log[
E^(a + b*x)]*Log[1 - Sqrt[1 - d]*E^(a + b*x)] - 2*Log[E^(a + b*x)]*Log[1 +
Sqrt[1 - d]*E^(a + b*x)] + 2*Log[E^(a + b*x)]*Log[E^(-a - b*x)*(1 + (-1 + d
)*E^(2*(a + b*x)))] - 2*b*x*Log[d*Cosh[a + b*x] + (-2 + d)*Sinh[a + b*x]] -
 2*PolyLog[2, -(Sqrt[1 - d]*E^(a + b*x))] - 2*PolyLog[2, Sqrt[1 - d]*E^(a +
 b*x)])/(4*b)
```

Maple [B] time = 0.175, size = 271, normalized size = 3.6

$$\frac{\mathrm{arccoth}\,(1-d-d\coth(bx+a))\ln(-d\coth(bx+a)-d)}{2b} - \frac{\mathrm{arccoth}\,(1-d-d\coth(bx+a))\ln(-d\coth(bx+a)+d}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1-d-d*coth(b*x+a)),x)
```

```
[Out] 1/2/b*arccoth(1-d-d*coth(b*x+a))*ln(-d*coth(b*x+a)-d)-1/2/b*arccoth(1-d-d*c
oth(b*x+a))*ln(-d*coth(b*x+a)+d)+1/8/b*ln(-d*coth(b*x+a)-d)^2-1/4/b*dilog(1
-1/2*d*coth(b*x+a)-1/2*d)-1/4/b*ln(-d*coth(b*x+a)-d)*ln(1-1/2*d*coth(b*x+a)
-1/2*d)-1/4/b*dilog(-1/2*(-d*coth(b*x+a)-d)/d)-1/4/b*ln(-d*coth(b*x+a)+d)*l
n(-1/2*(-d*coth(b*x+a)-d)/d)+1/4/b*dilog((-d*coth(b*x+a)-d+2)/(-2*d+2))+1/4
/b*ln(-d*coth(b*x+a)+d)*ln((-d*coth(b*x+a)-d+2)/(-2*d+2))
```

Maxima [A] time = 3.52798, size = 99, normalized size = 1.3

$$\frac{1}{4}bd\left(\frac{2x^2}{d} - \frac{2bx\log\left((d-1)e^{(2bx+2a)}+1\right) + \mathrm{Li}_2\left(-(d-1)e^{(2bx+2a)}\right)}{b^2d}\right) - x\,\mathrm{arcoth}\,(d\coth(bx+a)+d-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-d-d*coth(b*x+a)),x, algorithm="maxima")

[Out] 1/4*b*d*(2*x^2/d - (2*b*x*log((d - 1)*e^(2*b*x + 2*a) + 1) + dilog(-(d - 1)*e^(2*b*x + 2*a)))/(b^2*d)) - x*arccoth(d*coth(b*x + a) + d - 1)

Fricas [B] time = 1.74687, size = 707, normalized size = 9.3

$$\frac{b^2x^2 - bx\log\left(\frac{d\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+(d-2)\sinh(bx+a)}\right) + a\log\left(2\,(d-1)\cosh\,(bx+a) + 2\,(d-1)\sinh\,(bx+a) + \sqrt{-4\,d+4}\right) + a\log\left(2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-d-d*coth(b*x+a)),x, algorithm="fricas")

[Out] 1/2*(b^2*x^2 - b*x*log((d*cosh(b*x + a) + d*sinh(b*x + a))/(d*cosh(b*x + a) + (d - 2)*sinh(b*x + a))) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a) + sqrt(-4*d + 4)) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a) - sqrt(-4*d + 4)) - (b*x + a)*log(1/2*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-1/2*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - dilog(1/2*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) - dilog(-1/2*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))))/b

---

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acoth}\left(-d\coth\left(a+bx\right)-d+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(1-d-d*coth(b*x+a)),x)

[Out] Integral(acoth(-d*coth(a + b*x) - d + 1), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}\left(-d\coth\left(bx+a\right)-d+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-d-d*coth(b*x+a)),x, algorithm="giac")

[Out] integrate(arccoth(-d*coth(b*x + a) - d + 1), x)

3.230 $\int \frac{\coth^{-1}(1-d-d\coth(a+bx))}{x}\,dx$

Optimal. Leaf size=21

$$\text{CannotIntegrate}\left(\frac{\coth^{-1}(d(-\coth(a+bx))-d+1)}{x},x\right)$$

```
[Out] CannotIntegrate[ArcCoth[1 - d - d*Coth[a + b*x]]/x, x]
```

Rubi [A] time = 0.0834828, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth^{-1}(1-d-d\coth(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCoth[1 - d - d*Coth[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcCoth[1 - d - d*Coth[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\coth^{-1}(1-d-d\coth(a+bx))}{x}\,dx=\int \frac{\coth^{-1}(1-d-d\coth(a+bx))}{x}\,dx$$

Mathematica [A] time = 3.29888, size = 0, normalized size = 0.

$$\int \frac{\coth^{-1}(1-d-d\coth(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCoth[1 - d - d*Coth[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcCoth[1 - d - d*Coth[a + b*x]]/x, x]
```

Maple [A] time = 0.392, size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(1-d-d\coth(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1-d-d*coth(b*x+a))/x,x)
```

```
[Out] int(arccoth(1-d-d*coth(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\operatorname{arcoth}\left(d\coth\left(bx+a\right)+d-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-d-d*coth(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -integrate(arccoth(d*coth(b*x + a) + d - 1)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\operatorname{arcoth}\left(d\coth\left(bx+a\right)+d-1\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-d-d*coth(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(-arccoth(d*coth(b*x + a) + d - 1)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}\left(-d\coth\left(a+bx\right)-d+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1-d-d*coth(b*x+a))/x,x)
```

```
[Out] Integral(acoth(-d*coth(a + b*x) - d + 1)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}\left(-d\coth\left(bx+a\right)-d+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-d-d*coth(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(-d*coth(b*x + a) - d + 1)/x, x)
```

3.231 $\int (e+fx)^3 \coth^{-1}(\tan(a+bx))\,dx$

Optimal. Leaf size=302

$$\frac{3if^2(e+fx)\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3}-\frac{3if^2(e+fx)\text{PolyLog}\left(4,ie^{2i(a+bx)}\right)}{8b^3}+\frac{3f(e+fx)^2\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{8b^2}-$$

```
[Out] ((e + f*x)^4*ArcCoth[Tan[a + b*x]])/(4*f) + ((I/4)*(e + f*x)^4*ArcTan[E^((2
*I)*(a + b*x))])/f - ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (3*f*(e +
f*x)^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (3*f*(e + f*x)^2*Pol
yLog[3, I*E^((2*I)*(a + b*x))])/(8*b^2) + (((3*I)/8)*f^2*(e + f*x)*PolyLog[
4, (-I)*E^((2*I)*(a + b*x))])/b^3 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E
^((2*I)*(a + b*x))])/b^3 - (3*f^3*PolyLog[5, (-I)*E^((2*I)*(a + b*x))])/(16
*b^4) + (3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```

Rubi [A] time = 0.24248, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6252, 4181, 2531, 6609, 2282, 6589}

$$\frac{3if^2(e+fx)\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3}-\frac{3if^2(e+fx)\text{PolyLog}\left(4,ie^{2i(a+bx)}\right)}{8b^3}+\frac{3f(e+fx)^2\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{8b^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*ArcCoth[Tan[a + b*x]],x]
```

```
[Out] ((e + f*x)^4*ArcCoth[Tan[a + b*x]])/(4*f) + ((I/4)*(e + f*x)^4*ArcTan[E^((2
*I)*(a + b*x))])/f - ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (3*f*(e +
f*x)^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (3*f*(e + f*x)^2*Pol
yLog[3, I*E^((2*I)*(a + b*x))])/(8*b^2) + (((3*I)/8)*f^2*(e + f*x)*PolyLog[
4, (-I)*E^((2*I)*(a + b*x))])/b^3 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E
^((2*I)*(a + b*x))])/b^3 - (3*f^3*PolyLog[5, (-I)*E^((2*I)*(a + b*x))])/(16
*b^4) + (3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```

Rule 6252

```
Int[ArcCoth[Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcCoth[Tan[a + b*x]])/(f*(m + 1)), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
 e, f}, x] && IGtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (e+fx)^3\coth^{-1}(\tan(a+bx))\,dx = \frac{(e+fx)^4\coth^{-1}(\tan(a+bx))}{4f} - \frac{b\int (e+fx)^4\sec(2a+2bx)\,dx}{4f}$$

$$= \frac{(e+fx)^4\coth^{-1}(\tan(a+bx))}{4f} + \frac{i(e+fx)^4\tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{1}{2}\int (e+fx)^3\log\left(1\right.$$

$$= \frac{(e+fx)^4\coth^{-1}(\tan(a+bx))}{4f} + \frac{i(e+fx)^4\tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} - \frac{i(e+fx)^3\text{Li}_2\left(-ie^{2i(\right.}}{4b}$$

$$= \frac{(e+fx)^4\coth^{-1}(\tan(a+bx))}{4f} + \frac{i(e+fx)^4\tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} - \frac{i(e+fx)^3\text{Li}_2\left(-ie^{2i(\right.}}{4b}$$

$$= \frac{(e+fx)^4\coth^{-1}(\tan(a+bx))}{4f} + \frac{i(e+fx)^4\tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} - \frac{i(e+fx)^3\text{Li}_2\left(-ie^{2i(\right.}}{4b}$$

$$= \frac{(e+fx)^4\coth^{-1}(\tan(a+bx))}{4f} + \frac{i(e+fx)^4\tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} - \frac{i(e+fx)^3\text{Li}_2\left(-ie^{2i(\right.}}{4b}$$

$$= \frac{(e+fx)^4\coth^{-1}(\tan(a+bx))}{4f} + \frac{i(e+fx)^4\tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} - \frac{i(e+fx)^3\text{Li}_2\left(-ie^{2i(\right.}}{4b}$$

Mathematica [B] time = 0.314003, size = 654, normalized size = 2.17

$$\frac{1}{4}x\left(6e^2fx+4e^3+4ef^2x^2+f^3x^3\right)\coth^{-1}(\tan(a+bx)) + \frac{6b^2e^2f\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)-6b^2e^2f\text{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*ArcCoth[Tan[a + b*x]],x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcCoth[Tan[a + b*x]])/4 + (
-8*b^4*e^3*x*Log[1 - I*E^((2*I)*(a + b*x))] - 12*b^4*e^2*f*x^2*Log[1 - I*E^
```

```
((2*I)*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] - 2*b^4
*f^3*x^4*Log[1 - I*E^((2*I)*(a + b*x))] + 8*b^4*e^3*x*Log[1 + I*E^((2*I)*(a
 + b*x))] + 12*b^4*e^2*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 8*b^4*e*f^2*x
^3*Log[1 + I*E^((2*I)*(a + b*x))] + 2*b^4*f^3*x^4*Log[1 + I*E^((2*I)*(a + b
*x))] - (4*I)*b^3*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))] + (4*I)*
b^3*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))] + 6*b^2*e^2*f*PolyLog[3,
(-I)*E^((2*I)*(a + b*x))] + 12*b^2*e*f^2*x*PolyLog[3, (-I)*E^((2*I)*(a + b*
x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] - 6*b^2*e^2*f*Pol
yLog[3, I*E^((2*I)*(a + b*x))] - 12*b^2*e*f^2*x*PolyLog[3, I*E^((2*I)*(a +
b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^((2*I)*(a + b*x))] + (6*I)*b*e*f^2*Po
lyLog[4, (-I)*E^((2*I)*(a + b*x))] + (6*I)*b*f^3*x*PolyLog[4, (-I)*E^((2*I)
*(a + b*x))] - (6*I)*b*e*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))] - (6*I)*b*f^
3*x*PolyLog[4, I*E^((2*I)*(a + b*x))] - 3*f^3*PolyLog[5, (-I)*E^((2*I)*(a +
 b*x))] + 3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```

Maple [C] time = 8.453, size = 7429, normalized size = 24.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*arccoth(tan(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{16}\left(f^3x^4+4\,ef^2x^3+6\,e^2fx^2+4\,e^3x\right)\log\left(2\,\cos\left(2\,bx+2\,a\right)^2+2\,\sin\left(2\,bx+2\,a\right)^2+4\,\sin\left(2\,bx+2\,a\right)+2\right)-\frac{1}{16}\left(f^3x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arccoth(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/16*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*log(2*cos(2*b*x + 2*a)
^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/16*(f^3*x^4 + 4*e*f
^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*
a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(1/2*((b*f^3*x^4 + 4*b*e*f^2*x^3
+ 6*b*e^2*f*x^2 + 4*b*e^3*x)*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f^3*x^4
 + 4*b*e*f^2*x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*sin(4*b*x + 4*a)*sin(2*b*x +
2*a) + (b*f^3*x^4 + 4*b*e*f^2*x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*cos(2*b*x +
2*a))/(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x
)
```

Fricas [C] time = 2.5959, size = 4514, normalized size = 14.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arccoth(tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/32*(3*f^3*polylog(5, (I*tan(b*x + a)^2 + 2*tan(b*x + a) - I)/(tan(b*x +
a)^2 + 1)) - 3*f^3*polylog(5, (I*tan(b*x + a)^2 - 2*tan(b*x + a) - I)/(tan(
b*x + a)^2 + 1)) + 3*f^3*polylog(5, (-I*tan(b*x + a)^2 + 2*tan(b*x + a) + I
)/(tan(b*x + a)^2 + 1)) - 3*f^3*polylog(5, (-I*tan(b*x + a)^2 - 2*tan(b*x +
 a) + I)/(tan(b*x + a)^2 + 1)) - (4*I*b^3*f^3*x^3 + 12*I*b^3*e*f^2*x^2 + 12
*I*b^3*e^2*f*x + 4*I*b^3*e^3)*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x +
a) - I + 1)/(tan(b*x + a)^2 + 1) + 1) - (4*I*b^3*f^3*x^3 + 12*I*b^3*e*f^2*x
^2 + 12*I*b^3*e^2*f*x + 4*I*b^3*e^3)*dilog(-((I + 1)*tan(b*x + a)^2 - 2*tan
(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1) + 1) - (-4*I*b^3*f^3*x^3 - 12*I*b^3
*e*f^2*x^2 - 12*I*b^3*e^2*f*x - 4*I*b^3*e^3)*dilog(-(-(I - 1)*tan(b*x + a)^
2 + 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) - (-4*I*b^3*f^3*x^3 -
 12*I*b^3*e*f^2*x^2 - 12*I*b^3*e^2*f*x - 4*I*b^3*e^3)*dilog(-(-(I - 1)*tan(
b*x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) + 2*(b^4*f^3
*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^
2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(((I + 1)*tan(b*x + a)^2 + 2*tan(
b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) - 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f
 + 4*a^3*b*e*f^2 - a^4*f^3)*log(((I + 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) +
 I - 1)/(tan(b*x + a)^2 + 1)) + 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*
e*f^2 - a^4*f^3)*log(((I + 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I - 1)/(t
an(b*x + a)^2 + 1)) - 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 +
4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(
((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) + 2
*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e
^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log((-(I - 1)*tan(b*x + a)^
2 + 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) - 2*(b^4*f^3*x^4 + 4*b^4*
e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f +
 4*a^3*b*e*f^2 - a^4*f^3)*log((-(I - 1)*tan(b*x + a)^2 - 2*tan(b*x + a) + I
 + 1)/(tan(b*x + a)^2 + 1)) - 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*
f^2 - a^4*f^3)*log(((I - 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) + I + 1)/(tan
(b*x + a)^2 + 1)) + 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*
f^3)*log(((I - 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I + 1)/(tan(b*x + a)^
2 + 1)) - 4*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x)
*log((tan(b*x + a) + 1)/(tan(b*x + a) - 1)) - (6*I*b*f^3*x + 6*I*b*e*f^2)*p
olylog(4, (I*tan(b*x + a)^2 + 2*tan(b*x + a) - I)/(tan(b*x + a)^2 + 1)) - (
6*I*b*f^3*x + 6*I*b*e*f^2)*polylog(4, (I*tan(b*x + a)^2 - 2*tan(b*x + a) -
I)/(tan(b*x + a)^2 + 1)) - (-6*I*b*f^3*x - 6*I*b*e*f^2)*polylog(4, (-I*tan(
b*x + a)^2 + 2*tan(b*x + a) + I)/(tan(b*x + a)^2 + 1)) - (-6*I*b*f^3*x - 6*
I*b*e*f^2)*polylog(4, (-I*tan(b*x + a)^2 - 2*tan(b*x + a) + I)/(tan(b*x + a
)^2 + 1)) - 6*(b^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*polylog(3, (I*tan(b
*x + a)^2 + 2*tan(b*x + a) - I)/(tan(b*x + a)^2 + 1)) + 6*(b^2*f^3*x^2 + 2*
b^2*e*f^2*x + b^2*e^2*f)*polylog(3, (I*tan(b*x + a)^2 - 2*tan(b*x + a) - I)
/(tan(b*x + a)^2 + 1)) - 6*(b^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*polylo
g(3, (-I*tan(b*x + a)^2 + 2*tan(b*x + a) + I)/(tan(b*x + a)^2 + 1)) + 6*(b^
2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*polylog(3, (-I*tan(b*x + a)^2 - 2*ta
n(b*x + a) + I)/(tan(b*x + a)^2 + 1)))/b^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e+fx\right)^{3} \operatorname{acoth}\left(\tan\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*acoth(tan(b*x+a)),x)
```

```
[Out] Integral((e + f*x)**3*acoth(tan(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)^3 \operatorname{arcoth}(\tan(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arccoth(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*arccoth(tan(b*x + a)), x)
```

3.232 $\int (e+fx)^2 \coth^{-1}(\tan(a+bx))\,dx$

Optimal. Leaf size=234

$$\frac{f(e+fx)\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{4b^2}-\frac{f(e+fx)\text{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{4b^2}+\frac{if^2\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3}-\frac{if^2\text{PolyLog}\left(4,ie^{2i(a+bx)}\right)}{8b^3}$$

```
[Out] ((e + f*x)^3*ArcCoth[Tan[a + b*x]])/(3*f) + ((I/3)*(e + f*x)^3*ArcTan[E^((2
*I)*(a + b*x))])/f - ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*(e + f*
x)*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(4*b^2) - (f*(e + f*x)*PolyLog[3,
I*E^((2*I)*(a + b*x))])/(4*b^2) + ((I/8)*f^2*PolyLog[4, (-I)*E^((2*I)*(a +
b*x))])/b^3 - ((I/8)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])/b^3
```

Rubi [A] time = 0.17167, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6252, 4181, 2531, 6609, 2282, 6589}

$$\frac{f(e+fx)\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{4b^2}-\frac{f(e+fx)\text{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{4b^2}+\frac{if^2\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3}-\frac{if^2\text{PolyLog}\left(4,ie^{2i(a+bx)}\right)}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*ArcCoth[Tan[a + b*x]],x]
```

```
[Out] ((e + f*x)^3*ArcCoth[Tan[a + b*x]])/(3*f) + ((I/3)*(e + f*x)^3*ArcTan[E^((2
*I)*(a + b*x))])/f - ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*(e + f*
x)*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(4*b^2) - (f*(e + f*x)*PolyLog[3,
I*E^((2*I)*(a + b*x))])/(4*b^2) + ((I/8)*f^2*PolyLog[4, (-I)*E^((2*I)*(a +
b*x))])/b^3 - ((I/8)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])/b^3
```

Rule 6252

```
Int[ArcCoth[Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcCoth[Tan[a + b*x]])/(f*(m + 1)), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
 e, f}, x] && IGtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (e+fx)^2\coth^{-1}(\tan(a+bx))\,dx = \frac{(e+fx)^3\coth^{-1}(\tan(a+bx))}{3f} - \frac{b\int (e+fx)^3\sec(2a+2bx)\,dx}{3f}$$
$$= \frac{(e+fx)^3\coth^{-1}(\tan(a+bx))}{3f} + \frac{i(e+fx)^3\tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{1}{2}\int (e+fx)^2\text{lo}$$
$$= \frac{(e+fx)^3\coth^{-1}(\tan(a+bx))}{3f} + \frac{i(e+fx)^3\tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} - \frac{i(e+fx)^2\text{Li}_2\left(-\right.}{4b}$$
$$= \frac{(e+fx)^3\coth^{-1}(\tan(a+bx))}{3f} + \frac{i(e+fx)^3\tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} - \frac{i(e+fx)^2\text{Li}_2\left(-\right.}{4b}$$
$$= \frac{(e+fx)^3\coth^{-1}(\tan(a+bx))}{3f} + \frac{i(e+fx)^3\tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} - \frac{i(e+fx)^2\text{Li}_2\left(-\right.}{4b}$$
$$= \frac{(e+fx)^3\coth^{-1}(\tan(a+bx))}{3f} + \frac{i(e+fx)^3\tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} - \frac{i(e+fx)^2\text{Li}_2\left(-\right.}{4b}$$

Mathematica [A] time = 0.188441, size = 409, normalized size = 1.75

$$\frac{1}{3}x\left(3e^2+3efx+f^2x^2\right)\coth^{-1}(\tan(a+bx)) + \frac{-6ib^2(e+fx)^2\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)+6ib^2(e+fx)^2\text{PolyLog}\left(2,ie^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*ArcCoth[Tan[a + b*x]],x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcCoth[Tan[a + b*x]])/3 + (-12*b^3*e^2*x*Lo
g[1 - I*E^((2*I)*(a + b*x))] - 12*b^3*e*f*x^2*Log[1 - I*E^((2*I)*(a + b*x))
] - 4*b^3*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] + 12*b^3*e^2*x*Log[1 + I*E
^((2*I)*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 4*b^3
*f^2*x^3*Log[1 + I*E^((2*I)*(a + b*x))] - (6*I)*b^2*(e + f*x)^2*PolyLog[2,
(-I)*E^((2*I)*(a + b*x))] + (6*I)*b^2*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a
+ b*x))] + 6*b*e*f*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] + 6*b*f^2*x*PolyLog
[3, (-I)*E^((2*I)*(a + b*x))] - 6*b*e*f*PolyLog[3, I*E^((2*I)*(a + b*x))] -
 6*b*f^2*x*PolyLog[3, I*E^((2*I)*(a + b*x))] + (3*I)*f^2*PolyLog[4, (-I)*E^
```

```
((2*I)*(a + b*x))] - (3*I)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])/(24*b^3)
```

Maple [C] time = 13.508, size = 5543, normalized size = 23.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*arccoth(tan(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{12}\left(f^2x^3+3\,efx^2+3\,e^2x\right)\log\left(2\,\cos\left(2\,bx+2\,a\right)^2+2\,\sin\left(2\,bx+2\,a\right)^2+4\,\sin\left(2\,bx+2\,a\right)+2\right)-\frac{1}{12}\left(f^2x^3+3\,efx^2+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arccoth(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x
 + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*
log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) -
 integrate(2/3*((b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(4*b*x + 4*a)*cos(
2*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*sin(4*b*x + 4*a)*sin(2
*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(2*b*x + 2*a))/(cos(
4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

Fricas [C] time = 2.20852, size = 3357, normalized size = 14.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arccoth(tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/48*(3*I*f^2*polylog(4, (I*tan(b*x + a)^2 + 2*tan(b*x + a) - I)/(tan(b*x +
 a)^2 + 1)) + 3*I*f^2*polylog(4, (I*tan(b*x + a)^2 - 2*tan(b*x + a) - I)/(t
an(b*x + a)^2 + 1)) - 3*I*f^2*polylog(4, (-I*tan(b*x + a)^2 + 2*tan(b*x + a
) + I)/(tan(b*x + a)^2 + 1)) - 3*I*f^2*polylog(4, (-I*tan(b*x + a)^2 - 2*ta
n(b*x + a) + I)/(tan(b*x + a)^2 + 1)) + (6*I*b^2*f^2*x^2 + 12*I*b^2*e*f*x +
 6*I*b^2*e^2)*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(tan
(b*x + a)^2 + 1) + 1) + (6*I*b^2*f^2*x^2 + 12*I*b^2*e*f*x + 6*I*b^2*e^2)*di
log(-((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)
 + 1) + (-6*I*b^2*f^2*x^2 - 12*I*b^2*e*f*x - 6*I*b^2*e^2)*dilog(-(-(I - 1)*
tan(b*x + a)^2 + 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) + (-6*I*
b^2*f^2*x^2 - 12*I*b^2*e*f*x - 6*I*b^2*e^2)*dilog(-(-(I - 1)*tan(b*x + a)^2
 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) - 4*(b^3*f^2*x^3 + 3*b
^3*e*f*x^2 + 3*b^3*e^2*x + 3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(((I + 1
)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) + 4*(3*a*b
```

```
^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(((I + 1)*tan(b*x + a)^2 + 2*I*tan(b*x +
 a) + I - 1)/(tan(b*x + a)^2 + 1)) - 4*(3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2
)*log(((I + 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I - 1)/(tan(b*x + a)^2 +
 1)) + 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x + 3*a*b^2*e^2 - 3*a^2*b
*e*f + a^3*f^2)*log(((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1)/(tan(
b*x + a)^2 + 1)) - 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x + 3*a*b^2*e
^2 - 3*a^2*b*e*f + a^3*f^2)*log((-(I - 1)*tan(b*x + a)^2 + 2*tan(b*x + a) +
 I + 1)/(tan(b*x + a)^2 + 1)) + 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*
x + 3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log((-(I - 1)*tan(b*x + a)^2 - 2*t
an(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + 4*(3*a*b^2*e^2 - 3*a^2*b*e*f +
 a^3*f^2)*log(((I - 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) + I + 1)/(tan(b*x
+ a)^2 + 1)) - 4*(3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(((I - 1)*tan(b*x
 + a)^2 - 2*I*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + 8*(b^3*f^2*x^3
+ 3*b^3*e*f*x^2 + 3*b^3*e^2*x)*log((tan(b*x + a) + 1)/(tan(b*x + a) - 1)) +
 6*(b*f^2*x + b*e*f)*polylog(3, (I*tan(b*x + a)^2 + 2*tan(b*x + a) - I)/(ta
n(b*x + a)^2 + 1)) - 6*(b*f^2*x + b*e*f)*polylog(3, (I*tan(b*x + a)^2 - 2*t
an(b*x + a) - I)/(tan(b*x + a)^2 + 1)) + 6*(b*f^2*x + b*e*f)*polylog(3, (-I
*tan(b*x + a)^2 + 2*tan(b*x + a) + I)/(tan(b*x + a)^2 + 1)) - 6*(b*f^2*x +
b*e*f)*polylog(3, (-I*tan(b*x + a)^2 - 2*tan(b*x + a) + I)/(tan(b*x + a)^2
+ 1)))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (e+fx)^2 \operatorname{acoth}(\tan(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*acoth(tan(b*x+a)),x)
```

```
[Out] Integral((e + f*x)**2*acoth(tan(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)^2 \operatorname{arcoth}(\tan(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arccoth(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*arccoth(tan(b*x + a)), x)
```

3.233 $\int (e + fx) \coth^{-1}(\tan(a + bx)) \, dx$

Optimal. Leaf size=162

$$\frac{f\text{PolyLog}\left(3, -ie^{2i(a+bx)}\right)}{8b^2} - \frac{f\text{PolyLog}\left(3, ie^{2i(a+bx)}\right)}{8b^2} - \frac{i(e+fx)\text{PolyLog}\left(2, -ie^{2i(a+bx)}\right)}{4b} + \frac{i(e+fx)\text{PolyLog}\left(2, ie^{2i(a+bx)}\right)}{4b}$$

```
[Out] ((e + f*x)^2*ArcCoth[Tan[a + b*x]])/(2*f) + ((I/2)*(e + f*x)^2*ArcTan[E^((2
*I)*(a + b*x))])/f - ((I/4)*(e + f*x)*PolyLog[2, (-I)*E^((2*I)*(a + b*x))])
/b + ((I/4)*(e + f*x)*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*PolyLog[3,
(-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (f*PolyLog[3, I*E^((2*I)*(a + b*x))])/
(8*b^2)
```

Rubi [A] time = 0.111402, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6252, 4181, 2531, 2282, 6589}

$$\frac{f\text{PolyLog}\left(3, -ie^{2i(a+bx)}\right)}{8b^2} - \frac{f\text{PolyLog}\left(3, ie^{2i(a+bx)}\right)}{8b^2} - \frac{i(e+fx)\text{PolyLog}\left(2, -ie^{2i(a+bx)}\right)}{4b} + \frac{i(e+fx)\text{PolyLog}\left(2, ie^{2i(a+bx)}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*ArcCoth[Tan[a + b*x]],x]
```

```
[Out] ((e + f*x)^2*ArcCoth[Tan[a + b*x]])/(2*f) + ((I/2)*(e + f*x)^2*ArcTan[E^((2
*I)*(a + b*x))])/f - ((I/4)*(e + f*x)*PolyLog[2, (-I)*E^((2*I)*(a + b*x))])
/b + ((I/4)*(e + f*x)*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*PolyLog[3,
(-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (f*PolyLog[3, I*E^((2*I)*(a + b*x))])/
(8*b^2)
```

Rule 6252

```
Int[ArcCoth[Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcCoth[Tan[a + b*x]])/(f*(m + 1)), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
 e, f}, x] && IGtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (e+fx)\coth^{-1}(\tan(a+bx))\,dx = \frac{(e+fx)^2\coth^{-1}(\tan(a+bx))}{2f} - \frac{b\int (e+fx)^2\sec(2a+2bx)\,dx}{2f}$$

$$= \frac{(e+fx)^2\coth^{-1}(\tan(a+bx))}{2f} + \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} + \frac{1}{2}\int (e+fx)\log\left(\right.$$

$$= \frac{(e+fx)^2\coth^{-1}(\tan(a+bx))}{2f} + \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} - \frac{i(e+fx)\text{Li}_2\left(-ie^{2}\right.}{4b}$$

$$= \frac{(e+fx)^2\coth^{-1}(\tan(a+bx))}{2f} + \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} - \frac{i(e+fx)\text{Li}_2\left(-ie^{2}\right.}{4b}$$

$$= \frac{(e+fx)^2\coth^{-1}(\tan(a+bx))}{2f} + \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} - \frac{i(e+fx)\text{Li}_2\left(-ie^{2}\right.}{4b}$$

Mathematica [A] time = 0.132029, size = 263, normalized size = 1.62

$$-be\left(\frac{i\text{PolyLog}\left(2,-ie^{i(2a+2bx)}\right)}{4b^2} - \frac{i\text{PolyLog}\left(2,ie^{i(2a+2bx)}\right)}{4b^2} - \frac{ix\tan^{-1}\left(e^{2ia+2ibx}\right)}{b}\right) + \frac{f\left(2ibx\text{PolyLog}(2,-\sin(2(a+bx\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)*ArcCoth[Tan[a + b*x]],x]
```

```
[Out] e*x*ArcCoth[Tan[a + b*x]] + (f*x^2*ArcCoth[Tan[a + b*x]])/2 - b*e*(((-I)*x*
ArcTan[E^((2*I)*a + (2*I)*b*x)])/b + ((I/4)*PolyLog[2, (-I)*E^(I*(2*a + 2*b
*x))])/b^2 - ((I/4)*PolyLog[2, I*E^(I*(2*a + 2*b*x))])/b^2) + (f*((4*I)*b^2
*x^2*ArcTan[Cos[2*(a + b*x)] + I*Sin[2*(a + b*x)]] + (2*I)*b*x*PolyLog[2, I
*Cos[2*(a + b*x)] - Sin[2*(a + b*x)]] - (2*I)*b*x*PolyLog[2, (-I)*Cos[2*(a
+ b*x)] + Sin[2*(a + b*x)]] - PolyLog[3, I*Cos[2*(a + b*x)] - Sin[2*(a + b*
x)]] + PolyLog[3, (-I)*Cos[2*(a + b*x)] + Sin[2*(a + b*x)]]))/(8*b^2)
```

Maple [C] time = 10.639, size = 2543, normalized size = 15.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*arccoth(tan(b*x+a)),x)
```

```
[Out] -1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^3*x^2-1/4*I*P
i*x*e*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^3-1/4*ln(exp(2*I*(b
```

```
*x+a))-I)*x^2*f-1/2*ln(exp(2*I*(b*x+a))-I)*x*e-1/2*I/b^2*f*a*dilog(((-I)^(1
/2)-exp(I*(b*x+a)))/(-I)^(1/2))-1/2*I/b^2*f*a*dilog(((-I)^(1/2)+exp(I*(b*x+
a)))/(-I)^(1/2))+1/2*I/b*e*(I*b*x+I*a)*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^
(1/2))-1/2*I/b*e*dilog(1+exp(I*(b*x+a))*(-1)^(3/4))-1/2*I/b*e*dilog(1-exp(I
*(b*x+a))*(-1)^(3/4))+1/2*I/b*e*dilog(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2
))+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3-1/4*I*P
i*x*e*csgn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^3+1/8*f*polylog
(3,-I*exp(2*I*(b*x+a)))/b^2-1/8*f*polylog(3,I*exp(2*I*(b*x+a)))/b^2+1/4/b^2
*f*(I*b*x+I*a)^2*ln(1-I*exp(2*I*(b*x+a)))+1/4/b^2*f*(I*b*x+I*a)*polylog(2,I
*exp(2*I*(b*x+a)))-1/4/b^2*f*(I*b*x+I*a)^2*ln(1+I*exp(2*I*(b*x+a)))-1/4/b^2
*f*(I*b*x+I*a)*polylog(2,-I*exp(2*I*(b*x+a)))+1/4*I*Pi*x*e+1/8*I*Pi*f*x^2+1
/4*I*Pi*x*e*csgn((1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3+1/2*I/b
*e*dilog(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))+1/4/b^2*f*a^2*ln(-exp(2*I*
(b*x+a))+I)-1/2/b*e*a*ln(-exp(2*I*(b*x+a))+I)+1/2/b*a*e*ln(exp(2*I*(b*x+a))
+I)-1/4/b^2*f*a^2*ln(exp(2*I*(b*x+a))+I)+1/8*I*Pi*f*csgn((1+I)*(exp(2*I*(b*
x+a))+I)/(exp(2*I*(b*x+a))+1))^2*x^2-1/4*I*Pi*x*e*csgn((1-I)*(exp(2*I*(b*x+
a))-I)/(exp(2*I*(b*x+a))+1))^2-1/8*I*Pi*f*csgn((1-I)*(exp(2*I*(b*x+a))-I)/(
exp(2*I*(b*x+a))+1))^2*x^2+1/2*I/b*e*(I*b*x+I*a)*ln(((-I)^(1/2)+exp(I*(b*x+
a)))/(-I)^(1/2))-1/2*I/b*e*(I*b*x+I*a)*ln(1+exp(I*(b*x+a))*(-1)^(3/4))-1/2*
I/b*e*(I*b*x+I*a)*ln(1-exp(I*(b*x+a))*(-1)^(3/4))+1/2*I/b^2*f*a*dilog(1+exp
(I*(b*x+a))*(-1)^(3/4))+(1/4*f*x^2+1/2*e*x)*ln(exp(2*I*(b*x+a))+I)+1/2*I/b^
2*f*a*dilog(1-exp(I*(b*x+a))*(-1)^(3/4))+1/8*I*Pi*f*csgn((1-I)*(exp(2*I*(b*
x+a))-I)/(exp(2*I*(b*x+a))+1))^3*x^2-1/8*I*Pi*f*csgn(I/(exp(2*I*(b*x+a))+1)
)*csgn(I*(exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a)
)+1))*x^2-1/4*I*Pi*x*e*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a)
)+I))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))+1/4*I*Pi*x*e*csgn((
1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2-1/4*I*Pi*x*e*csgn(I*(exp(
2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2-1/4*I*
Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*csgn((1-I)*(exp(2*
I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2-1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))-I
)/(exp(2*I*(b*x+a))+1))*csgn((1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1
))^2*x^2+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*csg
n((1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))-1/8*I*Pi*f*csgn(I*(exp(2
*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2*x^2-1/8
*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))*csgn((1+I)*(exp(2
*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))*x^2+1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a)
)-I)/(exp(2*I*(b*x+a))+1))*csgn((1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a)
)+1))*x^2-1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))*cs
gn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))+1/8*I*Pi*f*csgn(I/(exp(
2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2*x^2+1/
8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*
(b*x+a))+1))^2*x^2+1/4*I*Pi*x*e*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*
I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))
+I))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2+1/4*I*Pi*x*e*csgn(
I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))*csgn((1+I)*(exp(2*I*(b*x+a))+I
)/(exp(2*I*(b*x+a))+1))^2+1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(
b*x+a))+1))*csgn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2*x^2-1/2
*I/b^2*f*a*(I*b*x+I*a)*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))-1/2*I/b^2
*f*a*(I*b*x+I*a)*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))-1/8*I*Pi*f*csgn
(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^
2*x^2-1/4*I*Pi*x*e*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))-I)
/(exp(2*I*(b*x+a))+1))^2+1/2*I/b^2*f*a*(I*b*x+I*a)*ln(1+exp(I*(b*x+a))*(-1)
^(3/4))+1/2*I/b^2*f*a*(I*b*x+I*a)*ln(1-exp(I*(b*x+a))*(-1)^(3/4))+1/8*I*Pi*
f*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3*x^2-1/8*I*Pi*f*csgn((
1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^3*x^2+1/8*I*Pi*f*csgn(I/(ex
p(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))-I)
/(exp(2*I*(b*x+a))+1))*x^2+1/4*I*Pi*x*e*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I
*(exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{8}\left(fx^2+2\,ex\right)\log\left(2\,\cos\left(2\,bx+2\,a\right)^2+2\,\sin\left(2\,bx+2\,a\right)^2+4\,\sin\left(2\,bx+2\,a\right)+2\right)-\frac{1}{8}\left(fx^2+2\,ex\right)\log\left(2\,\cos\left(2\,bx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arccoth(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/8*(f*x^2 + 2*e*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin
(2*b*x + 2*a) + 2) - 1/8*(f*x^2 + 2*e*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2
*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(((b*f*x^2 + 2*b*e*x)*co
s(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f*x^2 + 2*b*e*x)*sin(4*b*x + 4*a)*sin(
2*b*x + 2*a) + (b*f*x^2 + 2*b*e*x)*cos(2*b*x + 2*a))/(cos(4*b*x + 4*a)^2 +
sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

Fricas [C] time = 2.27982, size = 2313, normalized size = 14.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arccoth(tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/16*((2*I*b*f*x + 2*I*b*e)*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a)
 - I + 1)/(tan(b*x + a)^2 + 1) + 1) + (2*I*b*f*x + 2*I*b*e)*dilog(-((I + 1)
*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1) + 1) + (-2*I
*b*f*x - 2*I*b*e)*dilog(-(-(I - 1)*tan(b*x + a)^2 + 2*tan(b*x + a) + I + 1)
/(tan(b*x + a)^2 + 1) + 1) + (-2*I*b*f*x - 2*I*b*e)*dilog(-(-(I - 1)*tan(b*
x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) - 2*(b^2*f*x^2
 + 2*b^2*e*x + 2*a*b*e - a^2*f)*log(((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a
) - I + 1)/(tan(b*x + a)^2 + 1)) + 2*(2*a*b*e - a^2*f)*log(((I + 1)*tan(b*x
 + a)^2 + 2*I*tan(b*x + a) + I - 1)/(tan(b*x + a)^2 + 1)) - 2*(2*a*b*e - a^
2*f)*log(((I + 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I - 1)/(tan(b*x + a)^
2 + 1)) + 2*(b^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a^2*f)*log(((I + 1)*tan(b*x
+ a)^2 - 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) - 2*(b^2*f*x^2 + 2*b
^2*e*x + 2*a*b*e - a^2*f)*log((-(I - 1)*tan(b*x + a)^2 + 2*tan(b*x + a) + I
 + 1)/(tan(b*x + a)^2 + 1)) + 2*(b^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a^2*f)*l
og((-(I - 1)*tan(b*x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1))
 + 2*(2*a*b*e - a^2*f)*log(((I - 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) + I +
 1)/(tan(b*x + a)^2 + 1)) - 2*(2*a*b*e - a^2*f)*log(((I - 1)*tan(b*x + a)^2
 - 2*I*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + 4*(b^2*f*x^2 + 2*b^2*e
*x)*log((tan(b*x + a) + 1)/(tan(b*x + a) - 1)) + f*polylog(3, (I*tan(b*x +
a)^2 + 2*tan(b*x + a) - I)/(tan(b*x + a)^2 + 1)) - f*polylog(3, (I*tan(b*x
+ a)^2 - 2*tan(b*x + a) - I)/(tan(b*x + a)^2 + 1)) + f*polylog(3, (-I*tan(b
*x + a)^2 + 2*tan(b*x + a) + I)/(tan(b*x + a)^2 + 1)) - f*polylog(3, (-I*ta
n(b*x + a)^2 - 2*tan(b*x + a) + I)/(tan(b*x + a)^2 + 1)))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(e+fx\right)\operatorname{acoth}\left(\tan\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*acoth(tan(b*x+a)),x)
```

```
[Out] Integral((e + f*x)*acoth(tan(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)\operatorname{arcoth}(\tan(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arccoth(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*arccoth(tan(b*x + a)), x)
```

3.234 $\int \coth^{-1}(\tan(a+bx))\,dx$

Optimal. Leaf size=79

$$-\frac{i\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b}+ix\tan^{-1}\left(e^{2i(a+bx)}\right)+x\coth^{-1}(\tan(a+bx))$$

```
[Out] x*ArcCoth[Tan[a + b*x]] + I*x*ArcTan[E^((2*I)*(a + b*x))] - ((I/4)*PolyLog[
2, (-I)*E^((2*I)*(a + b*x))])/b + ((I/4)*PolyLog[2, I*E^((2*I)*(a + b*x))])
/b
```

Rubi [A] time = 0.047899, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6248, 4181, 2279, 2391}

$$-\frac{i\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b}+ix\tan^{-1}\left(e^{2i(a+bx)}\right)+x\coth^{-1}(\tan(a+bx))$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tan[a + b*x]],x]
```

```
[Out] x*ArcCoth[Tan[a + b*x]] + I*x*ArcTan[E^((2*I)*(a + b*x))] - ((I/4)*PolyLog[
2, (-I)*E^((2*I)*(a + b*x))])/b + ((I/4)*PolyLog[2, I*E^((2*I)*(a + b*x))])
/b
```

Rule 6248

```
Int[ArcCoth[Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcCoth[Tan[a + b
*x]], x] - Dist[b, Int[x*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \coth^{-1}(\tan(a+bx))\,dx = x\coth^{-1}(\tan(a+bx)) - b\int x\sec(2a+2bx)\,dx$$
$$= x\coth^{-1}(\tan(a+bx)) + ix\tan^{-1}\left(e^{2i(a+bx)}\right) + \frac{1}{2}\int \log\left(1-ie^{i(2a+2bx)}\right)dx - \frac{1}{2}\int \log\left(1+ie^{i\ldots}\right.$$
$$= x\coth^{-1}(\tan(a+bx)) + ix\tan^{-1}\left(e^{2i(a+bx)}\right) - \frac{i\,\mathrm{Subst}\left(\int \frac{\log(1-ix)}{x}dx, x, e^{i(2a+2bx)}\right)}{4b} + \frac{i\,\mathrm{Subst}\ldots}{}$$
$$= x\coth^{-1}(\tan(a+bx)) + ix\tan^{-1}\left(e^{2i(a+bx)}\right) - \frac{i\mathrm{Li}_2\left(-ie^{2i(a+bx)}\right)}{4b} + \frac{i\mathrm{Li}_2\left(ie^{2i(a+bx)}\right)}{4b}$$

Mathematica [A] time = 0.0230426, size = 78, normalized size = 0.99

$$\frac{-i\mathrm{PolyLog}\left(2,-ie^{2i(a+bx)}\right) + i\mathrm{PolyLog}\left(2,ie^{2i(a+bx)}\right) + 4bx\left(\coth^{-1}(\tan(a+bx)) + i\tan^{-1}\left(e^{2i(a+bx)}\right)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tan[a + b*x]],x]
```

```
[Out] (4*b*x*(ArcCoth[Tan[a + b*x]] + I*ArcTan[E^((2*I)*(a + b*x))]) - I*PolyLog[
2, (-I)*E^((2*I)*(a + b*x))] + I*PolyLog[2, I*E^((2*I)*(a + b*x))])/(4*b)
```

Maple [B] time = 0.158, size = 178, normalized size = 2.3

$$\frac{\arctan(\tan(bx+a))\,\mathrm{arccoth}(\tan(bx+a))}{b} + \frac{\arctan(\tan(bx+a))}{2b}\ln\left(1+\frac{i(1+i\tan(bx+a))^2}{1+(\tan(bx+a))^2}\right) - \frac{\frac{i}{4}}{b}\mathrm{polylog}\left(2,\frac{-i(\ldots}{1\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tan(b*x+a)),x)
```

```
[Out] 1/b*arctan(tan(b*x+a))*arccoth(tan(b*x+a))+1/2/b*arctan(tan(b*x+a))*ln(1+I*
(1+I*tan(b*x+a))^2/(1+tan(b*x+a)^2))-1/4*I/b*polylog(2,-I*(1+I*tan(b*x+a))^
2/(1+tan(b*x+a)^2))-1/2/b*arctan(tan(b*x+a))*ln(1-I*(1+I*tan(b*x+a))^2/(1+t
an(b*x+a)^2))+1/4*I/b*polylog(2,I*(1+I*tan(b*x+a))^2/(1+tan(b*x+a)^2))
```

Maxima [B] time = 1.8146, size = 246, normalized size = 3.11

$$\frac{4(bx+a)\,\mathrm{arcoth}(\tan(bx+a)) + \left(\arctan\left(\frac{1}{2}\tan(bx+a)+\frac{1}{2},\frac{1}{2}\tan(bx+a)+\frac{1}{2}\right) - \arctan\left(\frac{1}{2}\tan(bx+a)-\frac{1}{2},-\frac{1}{2}\mathrm{ta}\ldots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(b*x + a)*arccoth(tan(b*x + a)) + (arctan2(1/2*tan(b*x + a) + 1/2, 1
/2*tan(b*x + a) + 1/2) - arctan2(1/2*tan(b*x + a) - 1/2, -1/2*tan(b*x + a)
+ 1/2))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log(1/2*tan(b*x + a)^2 + tan(b*
x + a) + 1/2) + (b*x + a)*log(1/2*tan(b*x + a)^2 - tan(b*x + a) + 1/2) - I*
dilog((1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2) + I*dilog(-(1/2*I - 1/2)*ta
```

```
n(b*x + a) + 1/2*I + 1/2) + I*dilog((1/2*I - 1/2)*tan(b*x + a) + 1/2*I + 1/
2) - I*dilog(-(1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2))/b
```

Fricas [B] time = 1.896, size = 1493, normalized size = 18.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*log((tan(b*x + a) + 1)/(tan(b*x + a) - 1)) - 2*(b*x + a)*log(((I
 + 1)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) + 2*a*
log(((I + 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) + I - 1)/(tan(b*x + a)^2 + 1
)) - 2*a*log(((I + 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I - 1)/(tan(b*x +
 a)^2 + 1)) + 2*(b*x + a)*log(((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I
+ 1)/(tan(b*x + a)^2 + 1)) - 2*(b*x + a)*log((-(I - 1)*tan(b*x + a)^2 + 2*t
an(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + 2*(b*x + a)*log((-(I - 1)*tan(
b*x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + 2*a*log(((I -
1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) - 2*a*l
og(((I - 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)
) + I*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(tan(b*x + a
)^2 + 1) + 1) + I*dilog(-((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1)/
(tan(b*x + a)^2 + 1) + 1) - I*dilog(-(-(I - 1)*tan(b*x + a)^2 + 2*tan(b*x +
 a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) - I*dilog(-(-(I - 1)*tan(b*x + a)^2
- 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acoth}(\tan(a + bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tan(b*x+a)),x)
```

```
[Out] Integral(acoth(tan(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}(\tan(bx + a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(tan(b*x + a)), x)
```

3.235 $\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx}\,dx$

Optimal. Leaf size=17

$$\text{CannotIntegrate}\left(\frac{\coth^{-1}(\tan(a+bx))}{e+fx},x\right)$$

```
[Out] CannotIntegrate[ArcCoth[Tan[a + b*x]]/(e + f*x), x]
```

Rubi [A] time = 0.0410895, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCoth[Tan[a + b*x]]/(e + f*x),x]
```

```
[Out] Defer[Int][ArcCoth[Tan[a + b*x]]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx}\,dx=\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx}\,dx$$

Mathematica [A] time = 5.06394, size = 0, normalized size = 0.

$$\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCoth[Tan[a + b*x]]/(e + f*x),x]
```

```
[Out] Integrate[ArcCoth[Tan[a + b*x]]/(e + f*x), x]
```

Maple [A] time = 1.157, size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(\tan(bx+a))}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tan(b*x+a))/(f*x+e),x)
```

```
[Out] int(arccoth(tan(b*x+a))/(f*x+e),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(\tan(bx+a))}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tan(b*x+a))/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(arccoth(tan(b*x + a))/(f*x + e), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arcoth}(\tan(bx+a))}{fx+e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tan(b*x+a))/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral(arccoth(tan(b*x + a))/(f*x + e), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}(\tan(a+bx))}{e+fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tan(b*x+a))/(f*x+e),x)
```

```
[Out] Integral(acoth(tan(a + b*x))/(e + f*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(\tan(bx+a))}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tan(b*x+a))/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(arccoth(tan(b*x + a))/(f*x + e), x)
```

3.236 $\int x^2 \coth^{-1}(c + d\tan(a + bx))\,dx$

Optimal. Leaf size=395

$$\frac{x\text{PolyLog}\left(3,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b^2}-\frac{x\text{PolyLog}\left(3,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b^2}+\frac{i\text{PolyLog}\left(4,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{8b^3}-\frac{i\text{PolyLog}\left(4,-\frac{(c-}{}\right.}{8b^3}$$

```
[Out] (x^3*ArcCoth[c + d*Tan[a + b*x]])/3 + (x^3*Log[1 + ((1 - c + I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 - c - I*d)])/6 - (x^3*Log[1 + ((1 + c - I*d)*E^((2*I)*a
+ (2*I)*b*x))/(1 + c + I*d)])/6 - ((I/4)*x^2*PolyLog[2, -(((1 - c + I*d)*E^
((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/b + ((I/4)*x^2*PolyLog[2, -(((1 + c
 - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/b + (x*PolyLog[3, -(((1 -
 c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/(4*b^2) - (x*PolyLog[3,
 -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/(4*b^2) + ((I/8
)*PolyLog[4, -(((1 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/b^3
 - ((I/8)*PolyLog[4, -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d
))])/b^3
```

Rubi [A] time = 0.500079, antiderivative size = 395, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6268, 2190, 2531, 6609, 2282, 6589}

$$\frac{x\text{PolyLog}\left(3,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b^2}-\frac{x\text{PolyLog}\left(3,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b^2}+\frac{i\text{PolyLog}\left(4,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{8b^3}-\frac{i\text{PolyLog}\left(4,-\frac{(c-}{}\right.}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcCoth[c + d*Tan[a + b*x]])/3 + (x^3*Log[1 + ((1 - c + I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 - c - I*d)])/6 - (x^3*Log[1 + ((1 + c - I*d)*E^((2*I)*a
+ (2*I)*b*x))/(1 + c + I*d)])/6 - ((I/4)*x^2*PolyLog[2, -(((1 - c + I*d)*E^
((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/b + ((I/4)*x^2*PolyLog[2, -(((1 + c
 - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/b + (x*PolyLog[3, -(((1 -
 c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/(4*b^2) - (x*PolyLog[3,
 -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/(4*b^2) + ((I/8
)*PolyLog[4, -(((1 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/b^3
 - ((I/8)*PolyLog[4, -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d
))])/b^3
```

Rule 6268

```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Tan[a + b*x]])/(f*(m
 + 1)), x] + (-Dist[(I*b*(1 + c - I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)
*E^(2*I*a + 2*I*b*x))/(1 + c + I*d + (1 + c - I*d)*E^(2*I*a + 2*I*b*x)), x]
, x] + Dist[(I*b*(1 - c + I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*I*
a + 2*I*b*x))/(1 - c - I*d + (1 - c + I*d)*E^(2*I*a + 2*I*b*x)), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, 1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\coth^{-1}(c+d\tan(a+bx))\,dx=\frac{1}{3}x^3\coth^{-1}(c+d\tan(a+bx))+\frac{1}{3}(b(i(1-c)-d))\int\frac{e^{2ia+2ibx}x^3}{1-c-id+(1-c+id)e^{2ia}\cdots}$$

$$=\frac{1}{3}x^3\coth^{-1}(c+d\tan(a+bx))+\frac{1}{6}x^3\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)-\frac{1}{6}x^3\log\Big(1\cdots$$

$$=\frac{1}{3}x^3\coth^{-1}(c+d\tan(a+bx))+\frac{1}{6}x^3\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)-\frac{1}{6}x^3\log\Big(1\cdots$$

$$=\frac{1}{3}x^3\coth^{-1}(c+d\tan(a+bx))+\frac{1}{6}x^3\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)-\frac{1}{6}x^3\log\Big(1\cdots$$

$$=\frac{1}{3}x^3\coth^{-1}(c+d\tan(a+bx))+\frac{1}{6}x^3\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)-\frac{1}{6}x^3\log\Big(1\cdots$$

$$=\frac{1}{3}x^3\coth^{-1}(c+d\tan(a+bx))+\frac{1}{6}x^3\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)-\frac{1}{6}x^3\log\Big(1\cdots$$

Mathematica [A] time = 0.315059, size = 346, normalized size = 0.88

$$\frac{1}{3}x^3\coth^{-1}(d\tan(a+bx)+c)+\frac{-6ib^2x^2\text{PolyLog}\left(2,\frac{(-c+id+1)e^{2i(a+bx)}}{c+id-1}\right)+6ib^2x^2\text{PolyLog}\left(2,-\frac{(c-id+1)e^{2i(a+bx)}}{c+id+1}\right)+6bx\text{Pol}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcCoth[c + d*Tan[a + b*x]])/3 + (4*b^3*x^3*Log[1 + ((-1 + c - I*d)*E^
((2*I)*(a + b*x)))/(-1 + c + I*d)] - 4*b^3*x^3*Log[1 + ((1 + c - I*d)*E^((2
*I)*(a + b*x)))/(1 + c + I*d)] - (6*I)*b^2*x^2*PolyLog[2, ((1 - c + I*d)*E^
((2*I)*(a + b*x)))/(-1 + c + I*d)] + (6*I)*b^2*x^2*PolyLog[2, -(((1 + c - I
*d)*E^((2*I)*(a + b*x)))/(1 + c + I*d))] + 6*b*x*PolyLog[3, ((1 - c + I*d)*
E^((2*I)*(a + b*x)))/(-1 + c + I*d)] - 6*b*x*PolyLog[3, -(((1 + c - I*d)*E^
((2*I)*(a + b*x)))/(1 + c + I*d))] + (3*I)*PolyLog[4, ((1 - c + I*d)*E^((2*
I)*(a + b*x)))/(-1 + c + I*d)] - (3*I)*PolyLog[4, -(((1 + c - I*d)*E^((2*I)
*(a + b*x)))/(1 + c + I*d))])/(24*b^3)
```

Maple [C] time = 6.798, size = 6820, normalized size = 17.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(c+d*tan(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(c+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*x^3*log((c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*b
*x + 2*a) + (c^2 + d^2 + 2*c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 + 2*(c^2 -
 d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 2*c + 1) - 1/12*x^3*log((c^2 + d^2 - 2*c
 + 1)*cos(2*b*x + 2*a)^2 + 4*(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 - 2*c
+ 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 + 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*x + 2
*a) - 2*c + 1) - 4*b*d*integrate(-1/3*(2*(c^2 + d^2 - 1)*x^3*cos(2*b*x + 2*
a)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^3*sin(2*b*x + 2*a)^
2 + (c^2 - d^2 - 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2*b*x + 2*a) - (c
^2 - d^2 - 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^3*cos(2*b*x
 + 2*a) + (c^2 - d^2 - 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 + d^
4 + 2*(c^2 + 1)*d^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*cos(4*b*x +
 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x + 2*a)^2 +
(c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 + d^4
 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 + 2*(c^4 + d^4 -
 2*(3*c^2 - 1)*d^2 - 2*c^2 + 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4
*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*(c^4 - d^
4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3 - c)*d - 2*(c*d^3 + (
c^3 - c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*sin(2*b*x + 2*a))*si
n(4*b*x + 4*a) + 8*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1), x)
```

Fricas [C] time = 2.65594, size = 5871, normalized size = 14.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/48*(8*b^3*x^3*log((d*tan(b*x + a) + c + 1)/(d*tan(b*x + a) + c - 1)) + 6*
I*b^2*x^2*dilog(-((2*I*(c + 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 - 2*I*(c +
 1)*d + (2*I*c^2 + 4*(c + 1)*d - 2*I*d^2 + 4*I*c + 2*I)*tan(b*x + a) + 4*c
+ 2)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) - 6*
I*b^2*x^2*dilog(-((-2*I*(c + 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 + 2*I*(c
+ 1)*d + (-2*I*c^2 + 4*(c + 1)*d + 2*I*d^2 - 4*I*c - 2*I)*tan(b*x + a) + 4*
c + 2)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) -
6*I*b^2*x^2*dilog(-((2*I*(c - 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 - 2*I*(c
 - 1)*d + (2*I*c^2 + 4*(c - 1)*d - 2*I*d^2 - 4*I*c + 2*I)*tan(b*x + a) - 4*
c + 2)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1) + 1) +
6*I*b^2*x^2*dilog(-((-2*I*(c - 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 + 2*I*(
c - 1)*d + (-2*I*c^2 + 4*(c - 1)*d + 2*I*d^2 + 4*I*c - 2*I)*tan(b*x + a) -
4*c + 2)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1) + 1)
+ 4*a^3*log(((I*(c + 1)*d + d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)*d + (I*c^
2 + I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/(tan(b*x + a)^2 + 1)) + 4*a^
3*log(((I*(c + 1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*(c + 1)*d + (I*c^2 + I*
d^2 + 2*I*c + I)*tan(b*x + a) + 2*c + 1)/(tan(b*x + a)^2 + 1)) - 4*a^3*log(
((I*(c - 1)*d + d^2)*tan(b*x + a)^2 - c^2 + I*(c - 1)*d + (I*c^2 + I*d^2 -
2*I*c + I)*tan(b*x + a) + 2*c - 1)/(tan(b*x + a)^2 + 1)) - 4*a^3*log(((I*(c
 - 1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*(c - 1)*d + (I*c^2 + I*d^2 - 2*I*c
+ I)*tan(b*x + a) - 2*c + 1)/(tan(b*x + a)^2 + 1)) - 6*b*x*polylog(3, ((c^2
 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*tan(b*x + a)^2 - c^2 - 2*I*(c + 1)*d + d^
2 + (2*I*c^2 - 4*(c + 1)*d - 2*I*d^2 + 4*I*c + 2*I)*tan(b*x + a) - 2*c - 1)
/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) - 6*b*x*poly
log(3, ((c^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*tan(b*x + a)^2 - c^2 + 2*I*(c
 + 1)*d + d^2 + (-2*I*c^2 - 4*(c + 1)*d + 2*I*d^2 - 4*I*c - 2*I)*tan(b*x +
a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1))
 + 6*b*x*polylog(3, ((c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*tan(b*x + a)^2 -
 c^2 - 2*I*(c - 1)*d + d^2 + (2*I*c^2 - 4*(c - 1)*d - 2*I*d^2 - 4*I*c + 2*I
)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2
 - 2*c + 1)) + 6*b*x*polylog(3, ((c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*tan(
b*x + a)^2 - c^2 + 2*I*(c - 1)*d + d^2 + (-2*I*c^2 - 4*(c - 1)*d + 2*I*d^2
+ 4*I*c - 2*I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^
2 + c^2 + d^2 - 2*c + 1)) - 4*(b^3*x^3 + a^3)*log(((2*I*(c + 1)*d + 2*d^2)*
tan(b*x + a)^2 + 2*c^2 - 2*I*(c + 1)*d + (2*I*c^2 + 4*(c + 1)*d - 2*I*d^2 +
 4*I*c + 2*I)*tan(b*x + a) + 4*c + 2)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2
 + c^2 + d^2 + 2*c + 1)) - 4*(b^3*x^3 + a^3)*log(((-2*I*(c + 1)*d + 2*d^2)*
tan(b*x + a)^2 + 2*c^2 + 2*I*(c + 1)*d + (-2*I*c^2 + 4*(c + 1)*d + 2*I*d^2
- 4*I*c - 2*I)*tan(b*x + a) + 4*c + 2)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^
2 + c^2 + d^2 + 2*c + 1)) + 4*(b^3*x^3 + a^3)*log(((2*I*(c - 1)*d + 2*d^2)*
tan(b*x + a)^2 + 2*c^2 - 2*I*(c - 1)*d + (2*I*c^2 + 4*(c - 1)*d - 2*I*d^2 -
 4*I*c + 2*I)*tan(b*x + a) - 4*c + 2)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2
 + c^2 + d^2 - 2*c + 1)) + 4*(b^3*x^3 + a^3)*log(((-2*I*(c - 1)*d + 2*d^2)*
tan(b*x + a)^2 + 2*c^2 + 2*I*(c - 1)*d + (-2*I*c^2 + 4*(c - 1)*d + 2*I*d^2
+ 4*I*c - 2*I)*tan(b*x + a) - 4*c + 2)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^
2 + c^2 + d^2 - 2*c + 1)) + 3*I*polylog(4, ((c^2 + 2*I*(c + 1)*d - d^2 + 2*
c + 1)*tan(b*x + a)^2 - c^2 - 2*I*(c + 1)*d + d^2 + (2*I*c^2 - 4*(c + 1)*d
- 2*I*d^2 + 4*I*c + 2*I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*tan
(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) - 3*I*polylog(4, ((c^2 - 2*I*(c + 1)*d
- d^2 + 2*c + 1)*tan(b*x + a)^2 - c^2 + 2*I*(c + 1)*d + d^2 + (-2*I*c^2 - 4
*(c + 1)*d + 2*I*d^2 - 4*I*c - 2*I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2
```

```
*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) - 3*I*polylog(4, ((c^2 + 2*I
*(c - 1)*d - d^2 - 2*c + 1)*tan(b*x + a)^2 - c^2 - 2*I*(c - 1)*d + d^2 + (2
*I*c^2 - 4*(c - 1)*d - 2*I*d^2 - 4*I*c + 2*I)*tan(b*x + a) + 2*c - 1)/((c^2
 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1)) + 3*I*polylog(4, (
(c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*tan(b*x + a)^2 - c^2 + 2*I*(c - 1)*d
+ d^2 + (-2*I*c^2 - 4*(c - 1)*d + 2*I*d^2 + 4*I*c - 2*I)*tan(b*x + a) + 2*c
 - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1)))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(c+d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arcoth}\left(d \tan\left(bx + a\right) + c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(c+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(d*tan(b*x + a) + c), x)
```

3.237 $\int x\coth^{-1}(c+d\tan(a+bx))\,dx$

Optimal. Leaf size=295

$$\frac{\text{PolyLog}\left(3,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{8b^2}-\frac{\text{PolyLog}\left(3,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{8b^2}-\frac{ix\text{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b}+\frac{ix\text{PolyLog}\left(2,-\right.}{4b}$$

```
[Out] (x^2*ArcCoth[c + d*Tan[a + b*x]])/2 + (x^2*Log[1 + ((1 - c + I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 - c - I*d)])/4 - (x^2*Log[1 + ((1 + c - I*d)*E^((2*I)*a
+ (2*I)*b*x))/(1 + c + I*d)])/4 - ((I/4)*x*PolyLog[2, -(((1 - c + I*d)*E^((
2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/b + ((I/4)*x*PolyLog[2, -(((1 + c - I
*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/b + PolyLog[3, -(((1 - c + I*
d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))]/(8*b^2) - PolyLog[3, -(((1 + c
- I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))]/(8*b^2)
```

Rubi [A] time = 0.396999, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6268, 2190, 2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{8b^2}-\frac{\text{PolyLog}\left(3,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{8b^2}-\frac{ix\text{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b}+\frac{ix\text{PolyLog}\left(2,-\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcCoth[c + d*Tan[a + b*x]])/2 + (x^2*Log[1 + ((1 - c + I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 - c - I*d)])/4 - (x^2*Log[1 + ((1 + c - I*d)*E^((2*I)*a
+ (2*I)*b*x))/(1 + c + I*d)])/4 - ((I/4)*x*PolyLog[2, -(((1 - c + I*d)*E^((
2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/b + ((I/4)*x*PolyLog[2, -(((1 + c - I
*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/b + PolyLog[3, -(((1 - c + I*
d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))]/(8*b^2) - PolyLog[3, -(((1 + c
- I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))]/(8*b^2)
```

Rule 6268

```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Tan[a + b*x]])/(f*(m
 + 1)), x] + (-Dist[(I*b*(1 + c - I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)
*E^(2*I*a + 2*I*b*x))/(1 + c + I*d + (1 + c - I*d)*E^(2*I*a + 2*I*b*x)), x]
, x] + Dist[(I*b*(1 - c + I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*I*
a + 2*I*b*x))/(1 - c - I*d + (1 - c + I*d)*E^(2*I*a + 2*I*b*x)), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, 1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\coth^{-1}(c+d\tan(a+bx))\,dx = \frac{1}{2}x^2\coth^{-1}(c+d\tan(a+bx)) + \frac{1}{2}(b(i(1-c)-d))\int \frac{e^{2ia+2ibx}x^2}{1-c-id+(1-c+id)e^{2ia+2ib\cdots}}$$

$$= \frac{1}{2}x^2\coth^{-1}(c+d\tan(a+bx)) + \frac{1}{4}x^2\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) - \frac{1}{4}x^2\log\left(1+\cdots\right.$$

$$= \frac{1}{2}x^2\coth^{-1}(c+d\tan(a+bx)) + \frac{1}{4}x^2\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) - \frac{1}{4}x^2\log\left(1+\cdots\right.$$

$$= \frac{1}{2}x^2\coth^{-1}(c+d\tan(a+bx)) + \frac{1}{4}x^2\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) - \frac{1}{4}x^2\log\left(1+\cdots\right.$$

$$= \frac{1}{2}x^2\coth^{-1}(c+d\tan(a+bx)) + \frac{1}{4}x^2\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) - \frac{1}{4}x^2\log\left(1+\cdots\right.$$

Mathematica [A] time = 0.140406, size = 257, normalized size = 0.87

$$\frac{1}{2}x^2\coth^{-1}(d\tan(a+bx)+c) + \frac{-2ibx\text{PolyLog}\left(2,\frac{(-c+id+1)e^{2i(a+bx)}}{c+id-1}\right)+2ibx\text{PolyLog}\left(2,-\frac{(c-id+1)e^{2i(a+bx)}}{c+id+1}\right)+\text{PolyLog}\left(3,\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcCoth[c + d*Tan[a + b*x]])/2 + (2*b^2*x^2*Log[1 + ((-1 + c - I*d)*E^
((2*I)*(a + b*x)))/(-1 + c + I*d)] - 2*b^2*x^2*Log[1 + ((1 + c - I*d)*E^((2
*I)*(a + b*x)))/(1 + c + I*d)] - (2*I)*b*x*PolyLog[2, ((1 - c + I*d)*E^((2*
I)*(a + b*x)))/(-1 + c + I*d)] + (2*I)*b*x*PolyLog[2, -(((1 + c - I*d)*E^((
2*I)*(a + b*x)))/(1 + c + I*d))] + PolyLog[3, ((1 - c + I*d)*E^((2*I)*(a +
b*x)))/(-1 + c + I*d)] - PolyLog[3, -(((1 + c - I*d)*E^((2*I)*(a + b*x)))/(
1 + c + I*d))])/(8*b^2)
```

Maple [C] time = 7.796, size = 6482, normalized size = 22.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(c+d*tan(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(c+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -2*b*d*integrate(-(2*(c^2 + d^2 - 1)*x^2*cos(2*b*x + 2*a)^2 + 2*c*d*x^2*sin
(2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^2*sin(2*b*x + 2*a)^2 + (c^2 - d^2 - 1)*
x^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x + 2*a) - (c^2 - d^2 - 1)*x^2*co
s(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^2*cos(2*b*x + 2*a) + (c^2 - d^2
 - 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 + d^4 + 2*(c^2 + 1)*d^2
+ (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*cos(4*b*x + 4*a)^2 + 4*(c^4 + d
^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x + 2*a)^2 + (c^4 + d^4 + 2*(c^2
+ 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 -
 2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 + 2*(c^4 + d^4 - 2*(3*c^2 - 1)*d^2 -
 2*c^2 + 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(c*d^3 + (c^3 - c)*
d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*(c^4 - d^4 - 2*c^2 + 1)*cos(2
*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3 - c)*d - 2*(c*d^3 + (c^3 - c)*d)*cos(2*b*
x + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*sin(2*b*x + 2*a))*sin(4*b*x + 4*a) + 8*(
c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1), x) + 1/8*x^2*log((c^2 + d^2 + 2
*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 + 2*
c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 + 2*(c^2 - d^2 + 2*c + 1)*cos(2*b*x +
 2*a) + 2*c + 1) - 1/8*x^2*log((c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a)^2 + 4
*(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 - 2*c + 1)*sin(2*b*x + 2*a)^2 + c^
2 + d^2 + 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*x + 2*a) - 2*c + 1)
```

Fricas [C] time = 2.49072, size = 4545, normalized size = 15.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/16*(4*b^2*x^2*log((d*tan(b*x + a) + c + 1)/(d*tan(b*x + a) + c - 1)) + 2*
I*b*x*dilog(-((2*I*(c + 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 - 2*I*(c + 1)*
d + (2*I*c^2 + 4*(c + 1)*d - 2*I*d^2 + 4*I*c + 2*I)*tan(b*x + a) + 4*c + 2)
/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) - 2*I*b*
x*dilog(-((-2*I*(c + 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 + 2*I*(c + 1)*d +
 (-2*I*c^2 + 4*(c + 1)*d + 2*I*d^2 - 4*I*c - 2*I)*tan(b*x + a) + 4*c + 2)/(
(c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) - 2*I*b*x*
dilog(-((2*I*(c - 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 - 2*I*(c - 1)*d + (2
*I*c^2 + 4*(c - 1)*d - 2*I*d^2 - 4*I*c + 2*I)*tan(b*x + a) - 4*c + 2)/((c^2
 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1) + 1) + 2*I*b*x*dilo
g(-((-2*I*(c - 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 + 2*I*(c - 1)*d + (-2*I
*c^2 + 4*(c - 1)*d + 2*I*d^2 + 4*I*c - 2*I)*tan(b*x + a) - 4*c + 2)/((c^2 +
```

```
d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1) + 1) - 2*a^2*log(((I*
(c + 1)*d + d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)*d + (I*c^2 + I*d^2 + 2*I*
c + I)*tan(b*x + a) - 2*c - 1)/(tan(b*x + a)^2 + 1)) - 2*a^2*log(((I*(c + 1
)*d - d^2)*tan(b*x + a)^2 + c^2 + I*(c + 1)*d + (I*c^2 + I*d^2 + 2*I*c + I)
*tan(b*x + a) + 2*c + 1)/(tan(b*x + a)^2 + 1)) + 2*a^2*log(((I*(c - 1)*d +
d^2)*tan(b*x + a)^2 - c^2 + I*(c - 1)*d + (I*c^2 + I*d^2 - 2*I*c + I)*tan(b
*x + a) + 2*c - 1)/(tan(b*x + a)^2 + 1)) + 2*a^2*log(((I*(c - 1)*d - d^2)*t
an(b*x + a)^2 + c^2 + I*(c - 1)*d + (I*c^2 + I*d^2 - 2*I*c + I)*tan(b*x + a
) - 2*c + 1)/(tan(b*x + a)^2 + 1)) - 2*(b^2*x^2 - a^2)*log(((2*I*(c + 1)*d
+ 2*d^2)*tan(b*x + a)^2 + 2*c^2 - 2*I*(c + 1)*d + (2*I*c^2 + 4*(c + 1)*d -
2*I*d^2 + 4*I*c + 2*I)*tan(b*x + a) + 4*c + 2)/((c^2 + d^2 + 2*c + 1)*tan(b
*x + a)^2 + c^2 + d^2 + 2*c + 1)) - 2*(b^2*x^2 - a^2)*log(((-2*I*(c + 1)*d
+ 2*d^2)*tan(b*x + a)^2 + 2*c^2 + 2*I*(c + 1)*d + (-2*I*c^2 + 4*(c + 1)*d +
 2*I*d^2 - 4*I*c - 2*I)*tan(b*x + a) + 4*c + 2)/((c^2 + d^2 + 2*c + 1)*tan(
b*x + a)^2 + c^2 + d^2 + 2*c + 1)) + 2*(b^2*x^2 - a^2)*log(((2*I*(c - 1)*d
+ 2*d^2)*tan(b*x + a)^2 + 2*c^2 - 2*I*(c - 1)*d + (2*I*c^2 + 4*(c - 1)*d -
2*I*d^2 - 4*I*c + 2*I)*tan(b*x + a) - 4*c + 2)/((c^2 + d^2 - 2*c + 1)*tan(b
*x + a)^2 + c^2 + d^2 - 2*c + 1)) + 2*(b^2*x^2 - a^2)*log(((-2*I*(c - 1)*d
+ 2*d^2)*tan(b*x + a)^2 + 2*c^2 + 2*I*(c - 1)*d + (-2*I*c^2 + 4*(c - 1)*d +
 2*I*d^2 + 4*I*c - 2*I)*tan(b*x + a) - 4*c + 2)/((c^2 + d^2 - 2*c + 1)*tan(
b*x + a)^2 + c^2 + d^2 - 2*c + 1)) - polylog(3, ((c^2 + 2*I*(c + 1)*d - d^2
 + 2*c + 1)*tan(b*x + a)^2 - c^2 - 2*I*(c + 1)*d + d^2 + (2*I*c^2 - 4*(c +
1)*d - 2*I*d^2 + 4*I*c + 2*I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1
)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) - polylog(3, ((c^2 - 2*I*(c + 1)*d
 - d^2 + 2*c + 1)*tan(b*x + a)^2 - c^2 + 2*I*(c + 1)*d + d^2 + (-2*I*c^2 -
4*(c + 1)*d + 2*I*d^2 - 4*I*c - 2*I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 +
2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) + polylog(3, ((c^2 + 2*I*(c
 - 1)*d - d^2 - 2*c + 1)*tan(b*x + a)^2 - c^2 - 2*I*(c - 1)*d + d^2 + (2*I*
c^2 - 4*(c - 1)*d - 2*I*d^2 - 4*I*c + 2*I)*tan(b*x + a) + 2*c - 1)/((c^2 +
d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1)) + polylog(3, ((c^2 -
2*I*(c - 1)*d - d^2 - 2*c + 1)*tan(b*x + a)^2 - c^2 + 2*I*(c - 1)*d + d^2 +
 (-2*I*c^2 - 4*(c - 1)*d + 2*I*d^2 + 4*I*c - 2*I)*tan(b*x + a) + 2*c - 1)/(
(c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1)))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(c+d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arcoth}(d \tan(bx + a) + c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(c+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(d*tan(b*x + a) + c), x)
```

3.238 $\int \coth^{-1}(c + d\tan(a + bx))\,dx$

Optimal. Leaf size=194

$$-\frac{i\text{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b}+\frac{i\text{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b}+\frac{1}{2}x\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)-\frac{1}{2}x\log\left(1+\right.$$

```
[Out] x*ArcCoth[c + d*Tan[a + b*x]] + (x*Log[1 + ((1 - c + I*d)*E^((2*I)*a + (2*I
)*b*x))/(1 - c - I*d)])/2 - (x*Log[1 + ((1 + c - I*d)*E^((2*I)*a + (2*I)*b*
x))/(1 + c + I*d)])/2 - ((I/4)*PolyLog[2, -(((1 - c + I*d)*E^((2*I)*a + (2*
I)*b*x))/(1 - c - I*d))])/b + ((I/4)*PolyLog[2, -(((1 + c - I*d)*E^((2*I)*a
 + (2*I)*b*x))/(1 + c + I*d))])/b
```

Rubi [A] time = 0.250733, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6260, 2190, 2279, 2391}

$$-\frac{i\text{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b}+\frac{i\text{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b}+\frac{1}{2}x\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)-\frac{1}{2}x\log\left(1+\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[c + d*Tan[a + b*x]],x]
```


```
[Out] x*ArcCoth[c + d*Tan[a + b*x]] + (x*Log[1 + ((1 - c + I*d)*E^((2*I)*a + (2*I
)*b*x))/(1 - c - I*d)])/2 - (x*Log[1 + ((1 + c - I*d)*E^((2*I)*a + (2*I)*b*
x))/(1 + c + I*d)])/2 - ((I/4)*PolyLog[2, -(((1 - c + I*d)*E^((2*I)*a + (2*
I)*b*x))/(1 - c - I*d))])/b + ((I/4)*PolyLog[2, -(((1 + c - I*d)*E^((2*I)*a
 + (2*I)*b*x))/(1 + c + I*d))])/b
```


Rule 6260

```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Coth[c + d*Tan[a + b*x]], x] + (-Dist[I*b*(1 + c - I*d), Int[(x*E^(2*I*a +
2*I*b*x))/(1 + c + I*d + (1 + c - I*d)*E^(2*I*a + 2*I*b*x)), x], x] + Dist[
I*b*(1 - c + I*d), Int[(x*E^(2*I*a + 2*I*b*x))/(1 - c - I*d + (1 - c + I*d)
*E^(2*I*a + 2*I*b*x)), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c + I*d)^2
, 1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \coth^{-1}(c+d\tan(a+bx))\,dx = x\coth^{-1}(c+d\tan(a+bx)) + (b(i(1-c)-d))\int \frac{e^{2ia+2ibx}x}{1-c-id+(1-c+id)e^{2ia+2ibx}}\,dx -$$

$$= x\coth^{-1}(c+d\tan(a+bx)) + \frac{1}{2}x\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) - \frac{1}{2}x\log\left(1+\frac{(1+c-}{1+}\right.$$

$$= x\coth^{-1}(c+d\tan(a+bx)) + \frac{1}{2}x\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) - \frac{1}{2}x\log\left(1+\frac{(1+c-}{1+}\right.$$

$$= x\coth^{-1}(c+d\tan(a+bx)) + \frac{1}{2}x\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) - \frac{1}{2}x\log\left(1+\frac{(1+c-}{1+}\right.$$

Mathematica [B] time = 13.2235, size = 4654, normalized size = 23.99

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[c + d*Tan[a + b*x]],x]
```

```
[Out] x*ArcCoth[c + d*Tan[a + b*x]] + (d*(-(a*Log[-(Sec[(a + b*x)/2]^2*((-1 + c)*
Cos[a + b*x] + d*Sin[a + b*x]))]) + a*Log[Sec[(a + b*x)/2]^2*(Cos[a + b*x]
+ c*Cos[a + b*x] + d*Sin[a + b*x])] + (a + b*x)*Log[(-d + Sqrt[1 - 2*c + c^
2 + d^2])/(-1 + c) + Tan[(a + b*x)/2]] + I*Log[((-1 + c)*(1 + I*Tan[(a + b*
x)/2]))/(-1 + c + I*d - I*Sqrt[1 - 2*c + c^2 + d^2])]*Log[(-d + Sqrt[1 - 2*
c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/2]] - I*Log[-(((-1 + c)*(I + Tan[(
a + b*x)/2]))/(I - I*c - d + Sqrt[1 - 2*c + c^2 + d^2]))]*Log[(-d + Sqrt[1
- 2*c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/2]] + (a + b*x)*Log[(d + Sqrt[
1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]] + I*Log[((-1 + c)*(-I + T
an[(a + b*x)/2]))/(I - I*c + d + Sqrt[1 - 2*c + c^2 + d^2])]*Log[(d + Sqrt[
1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]] - I*Log[((-1 + c)*(I + Ta
n[(a + b*x)/2]))/(-I + I*c + d + Sqrt[1 - 2*c + c^2 + d^2])]*Log[(d + Sqrt[
1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]] - (a + b*x)*Log[-((d + Sq
rt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2]] - I*Log[((1 + c)*(-I
+ Tan[(a + b*x)/2]))/(-I - I*c + d + Sqrt[1 + 2*c + c^2 + d^2])]*Log[-((d +
 Sqrt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2]] + I*Log[((1 + c)*(
I + Tan[(a + b*x)/2]))/(I + I*c + d + Sqrt[1 + 2*c + c^2 + d^2])]*Log[-((d
+ Sqrt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2]] - (a + b*x)*Log[(
-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(1 + c)] + I*Log
[((1 + c)*(1 - I*Tan[(a + b*x)/2]))/(1 + c - I*d + I*Sqrt[1 + 2*c + c^2 + d
^2])]*Log[(-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(1 +
c)] - I*Log[((1 + c)*(1 + I*Tan[(a + b*x)/2]))/(1 + c + I*d - I*Sqrt[1 + 2*
c + c^2 + d^2])]*Log[(-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x
)/2])/(1 + c)] + I*PolyLog[2, (d + Sqrt[1 - 2*c + c^2 + d^2] - (-1 + c)*Tan
[(a + b*x)/2])/(I - I*c + d + Sqrt[1 - 2*c + c^2 + d^2])] - I*PolyLog[2, (d
 + Sqrt[1 - 2*c + c^2 + d^2] - (-1 + c)*Tan[(a + b*x)/2])/(-I + I*c + d + S
qrt[1 - 2*c + c^2 + d^2])] - I*PolyLog[2, (-d + Sqrt[1 - 2*c + c^2 + d^2] +
 (-1 + c)*Tan[(a + b*x)/2])/(I - I*c - d + Sqrt[1 - 2*c + c^2 + d^2])] + I*
PolyLog[2, (-d + Sqrt[1 - 2*c + c^2 + d^2] + (-1 + c)*Tan[(a + b*x)/2])/(-I
 + I*c - d + Sqrt[1 - 2*c + c^2 + d^2])] - I*PolyLog[2, (d + Sqrt[1 + 2*c +
 c^2 + d^2] - (1 + c)*Tan[(a + b*x)/2])/(-I - I*c + d + Sqrt[1 + 2*c + c^2
```

+ d^2])] + I*PolyLog[2, (d + Sqrt[1 + 2*c + c^2 + d^2] - (1 + c)*Tan[(a + b*x)/2])/(I + I*c + d + Sqrt[1 + 2*c + c^2 + d^2])] + I*PolyLog[2, (-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(-I - I*c - d + Sqrt[1 + 2*c + c^2 + d^2])] - I*PolyLog[2, (-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(I + I*c - d + Sqrt[1 + 2*c + c^2 + d^2])])*((-2*a)/(b*(-1 + c^2 + d^2 - Cos[2*(a + b*x)] + c^2*Cos[2*(a + b*x)] - d^2*Cos[2*(a + b*x)] + 2*c*d*Sin[2*(a + b*x)])) + (2*(a + b*x))/(b*(-1 + c^2 + d^2 - Cos[2*(a + b*x)] + c^2*Cos[2*(a + b*x)] - d^2*Cos[2*(a + b*x)] + 2*c*d*Sin[2*(a + b*x)]))))/(Log[(-d + Sqrt[1 - 2*c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/2]] + Log[(d + Sqrt[1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]] - Log[-((d + Sqrt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2]] - Log[(-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(1 + c)] + (Log[(-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(1 + c)]*Sec[(a + b*x)/2]^2)/(2*(1 - I*Tan[(a + b*x)/2])) - (Log[(-d + Sqrt[1 - 2*c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(2*(1 + I*Tan[(a + b*x)/2])) + (Log[(-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(1 + c)]*Sec[(a + b*x)/2]^2)/(2*(1 + I*Tan[(a + b*x)/2])) + ((I/2)*Log[(d + Sqrt[1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(-I + Tan[(a + b*x)/2]) - ((I/2)*Log[-((d + Sqrt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(-I + Tan[(a + b*x)/2]) - ((I/2)*Log[(-d + Sqrt[1 - 2*c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(I + Tan[(a + b*x)/2]) - ((I/2)*Log[(d + Sqrt[1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(I + Tan[(a + b*x)/2]) + ((I/2)*Log[-((d + Sqrt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(I + Tan[(a + b*x)/2]) + ((a + b*x)*Sec[(a + b*x)/2]^2)/(2*((-d + Sqrt[1 - 2*c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/2])) + ((I/2)*Log[((-1 + c)*(1 + I*Tan[(a + b*x)/2]))/(-1 + c + I*d - I*Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/((-d + Sqrt[1 - 2*c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/2]) - ((I/2)*Log[-(((-1 + c)*(I + Tan[(a + b*x)/2]))/(I - I*c - d + Sqrt[1 - 2*c + c^2 + d^2]))]*Sec[(a + b*x)/2]^2)/((-d + Sqrt[1 - 2*c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/2]) + ((a + b*x)*Sec[(a + b*x)/2]^2)/(2*((d + Sqrt[1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2])) + ((I/2)*Log[((-1 + c)*(-I + Tan[(a + b*x)/2]))/(I - I*c + d + Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/((d + Sqrt[1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]) - ((I/2)*Log[((-1 + c)*(I + Tan[(a + b*x)/2]))/(-I + I*c + d + Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/((d + Sqrt[1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]) - ((a + b*x)*Sec[(a + b*x)/2]^2)/(2*(-((d + Sqrt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2])) - ((I/2)*Log[((1 + c)*(-I + Tan[(a + b*x)/2]))/(-I - I*c + d + Sqrt[1 + 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-((d + Sqrt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2]) + ((I/2)*Log[((1 + c)*(I + Tan[(a + b*x)/2]))/(I + I*c + d + Sqrt[1 + 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-((d + Sqrt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2]) + ((I/2)*(-1 + c)*Log[1 - (d + Sqrt[1 - 2*c + c^2 + d^2] - (-1 + c)*Tan[(a + b*x)/2])/(I - I*c + d + Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(d + Sqrt[1 - 2*c + c^2 + d^2] - (-1 + c)*Tan[(a + b*x)/2]) - ((I/2)*(-1 + c)*Log[1 - (d + Sqrt[1 - 2*c + c^2 + d^2] - (-1 + c)*Tan[(a + b*x)/2])/(-I + I*c + d + Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(d + Sqrt[1 - 2*c + c^2 + d^2] - (-1 + c)*Tan[(a + b*x)/2]) + ((I/2)*(-1 + c)*Log[1 - (-d + Sqrt[1 - 2*c + c^2 + d^2] + (-1 + c)*Tan[(a + b*x)/2])/(I - I*c - d + Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-d + Sqrt[1 - 2*c + c^2 + d^2] + (-1 + c)*Tan[(a + b*x)/2]) - ((I/2)*(-1 + c)*Log[1 - (-d + Sqrt[1 - 2*c + c^2 + d^2] + (-1 + c)*Tan[(a + b*x)/2])/(-I + I*c - d + Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-d + Sqrt[1 - 2*c + c^2 + d^2] + (-1 + c)*Tan[(a + b*x)/2]) - ((I/2)*(1 + c)*Log[1 - (d + Sqrt[1 + 2*c + c^2 + d^2] - (1 + c)*Tan[(a + b*x)/2])/(-I - I*c + d + Sqrt[1 + 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(d + Sqrt[1 + 2*c + c^2 + d^2] - (1 + c)*Tan[(a + b*x)/2]) + ((I/2)*(1 + c)*Log[1 - (d + Sqrt[1 + 2*c + c^2 + d^2] - (1 + c)*Tan[(a + b*x)/2])/(I + I*c + d + Sqrt[1 + 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(d + Sqrt[1 + 2*c + c^2 + d^2] - (1 +

```
 c)*Tan[(a + b*x)/2]) - ((1 + c)*(a + b*x)*Sec[(a + b*x)/2]^2)/(2*(-d + Sqr
t[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])) + ((I/2)*(1 + c)*Log[((
1 + c)*(1 - I*Tan[(a + b*x)/2]))/(1 + c - I*d + I*Sqrt[1 + 2*c + c^2 + d^2]
)]*Sec[(a + b*x)/2]^2)/(-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b
*x)/2]) - ((I/2)*(1 + c)*Log[((1 + c)*(1 + I*Tan[(a + b*x)/2]))/(1 + c + I*
d - I*Sqrt[1 + 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-d + Sqrt[1 + 2*c +
c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2]) - ((I/2)*(1 + c)*Log[1 - (-d + Sqrt[
1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(-I - I*c - d + Sqrt[1 + 2
*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-d + Sqrt[1 + 2*c + c^2 + d^2] + (1
+ c)*Tan[(a + b*x)/2]) + ((I/2)*(1 + c)*Log[1 - (-d + Sqrt[1 + 2*c + c^2 +
d^2] + (1 + c)*Tan[(a + b*x)/2])/(I + I*c - d + Sqrt[1 + 2*c + c^2 + d^2])]
*Sec[(a + b*x)/2]^2)/(-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x
)/2]) + (a*Cos[(a + b*x)/2]^2*(-(Sec[(a + b*x)/2]^2*(d*Cos[a + b*x] - (-1 +
 c)*Sin[a + b*x])) - Sec[(a + b*x)/2]^2*((-1 + c)*Cos[a + b*x] + d*Sin[a +
b*x])*Tan[(a + b*x)/2]))/((-1 + c)*Cos[a + b*x] + d*Sin[a + b*x]) + (a*Cos[
(a + b*x)/2]^2*(Sec[(a + b*x)/2]^2*(d*Cos[a + b*x] - Sin[a + b*x] - c*Sin[a
 + b*x]) + Sec[(a + b*x)/2]^2*(Cos[a + b*x] + c*Cos[a + b*x] + d*Sin[a + b*
x])*Tan[(a + b*x)/2]))/(Cos[a + b*x] + c*Cos[a + b*x] + d*Sin[a + b*x]))
```

Maple [B] time = 0.131, size = 612, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(c+d*tan(b*x+a)),x)
```

```
[Out] 1/b*arctan(tan(b*x+a))*arccoth(c+d*tan(b*x+a))-1/2/b*arctan((c+d*tan(b*x+a)
)/d-c/d)*ln(d*((c+d*tan(b*x+a))/d-c/d)+c+1)+1/2/b*arctan((c+d*tan(b*x+a))/d
-c/d)*ln(d*((c+d*tan(b*x+a))/d-c/d)+c-1)+1/4*I/b*ln(d*((c+d*tan(b*x+a))/d-c
/d)+c-1)*ln((I*d-d*((c+d*tan(b*x+a))/d-c/d))/(I*d+c-1))-1/4*I/b*ln(d*((c+d*
tan(b*x+a))/d-c/d)+c-1)*ln((I*d+d*((c+d*tan(b*x+a))/d-c/d))/(1-c+I*d))+1/4*
I/b*dilog((I*d-d*((c+d*tan(b*x+a))/d-c/d))/(I*d+c-1))-1/4*I/b*dilog((I*d+d*
((c+d*tan(b*x+a))/d-c/d))/(1-c+I*d))-1/4*I/b*ln(d*((c+d*tan(b*x+a))/d-c/d)+
c+1)*ln((I*d-d*((c+d*tan(b*x+a))/d-c/d))/(1+c+I*d))+1/4*I/b*ln(d*((c+d*tan(
b*x+a))/d-c/d)+c+1)*ln((I*d+d*((c+d*tan(b*x+a))/d-c/d))/(I*d-c-1))-1/4*I/b*
dilog((I*d-d*((c+d*tan(b*x+a))/d-c/d))/(1+c+I*d))+1/4*I/b*dilog((I*d+d*((c+
d*tan(b*x+a))/d-c/d))/(I*d-c-1))
```

Maxima [B] time = 2.12998, size = 502, normalized size = 2.59

$$4\,(bx+a)\operatorname{arcoth}\left(d\tan\left(bx+a\right)+c\right)+\left(\arctan\left(\frac{d^2\tan(bx+a)+(c+1)d}{c^2+d^2+2\,c+1},\frac{(c+1)d\tan(bx+a)+c^2+2\,c+1}{c^2+d^2+2\,c+1}\right)-\arctan\left(\frac{d^2\tan(bx+a)+(c-1)d}{c^2+d^2-2\,c+1},\ \cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(b*x + a)*arccoth(d*tan(b*x + a) + c) + (arctan2((d^2*tan(b*x + a) +
 (c + 1)*d)/(c^2 + d^2 + 2*c + 1), ((c + 1)*d*tan(b*x + a) + c^2 + 2*c + 1)
/(c^2 + d^2 + 2*c + 1)) - arctan2((d^2*tan(b*x + a) + (c - 1)*d)/(c^2 + d^2
 - 2*c + 1), ((c - 1)*d*tan(b*x + a) + c^2 - 2*c + 1)/(c^2 + d^2 - 2*c + 1)
))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log((d^2*tan(b*x + a)^2 + 2*(c + 1)*
d*tan(b*x + a) + c^2 + 2*c + 1)/(c^2 + d^2 + 2*c + 1)) + (b*x + a)*log((d^2
```

```
*tan(b*x + a)^2 + 2*(c - 1)*d*tan(b*x + a) + c^2 - 2*c + 1)/(c^2 + d^2 - 2*
c + 1)) - I*dilog(-(I*d*tan(b*x + a) - d)/(I*c + d + I)) + I*dilog(-(I*d*ta
n(b*x + a) - d)/(I*c + d - I)) - I*dilog((I*d*tan(b*x + a) + d)/(-I*c + d +
 I)) + I*dilog((I*d*tan(b*x + a) + d)/(-I*c + d - I)))/b
```

Fricas [B] time = 2.43372, size = 3210, normalized size = 16.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*log((d*tan(b*x + a) + c + 1)/(d*tan(b*x + a) + c - 1)) - 2*(b*x
+ a)*log(((2*I*(c + 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 - 2*I*(c + 1)*d +
(2*I*c^2 + 4*(c + 1)*d - 2*I*d^2 + 4*I*c + 2*I)*tan(b*x + a) + 4*c + 2)/((c
^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) - 2*(b*x + a)*lo
g(((-2*I*(c + 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 + 2*I*(c + 1)*d + (-2*I*
c^2 + 4*(c + 1)*d + 2*I*d^2 - 4*I*c - 2*I)*tan(b*x + a) + 4*c + 2)/((c^2 +
d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) + 2*(b*x + a)*log(((2
*I*(c - 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 - 2*I*(c - 1)*d + (2*I*c^2 + 4
*(c - 1)*d - 2*I*d^2 - 4*I*c + 2*I)*tan(b*x + a) - 4*c + 2)/((c^2 + d^2 - 2
*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1)) + 2*(b*x + a)*log(((-2*I*(c
- 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 + 2*I*(c - 1)*d + (-2*I*c^2 + 4*(c -
 1)*d + 2*I*d^2 + 4*I*c - 2*I)*tan(b*x + a) - 4*c + 2)/((c^2 + d^2 - 2*c +
1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1)) + 2*a*log(((I*(c + 1)*d + d^2)*ta
n(b*x + a)^2 - c^2 + I*(c + 1)*d + (I*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a)
 - 2*c - 1)/(tan(b*x + a)^2 + 1)) + 2*a*log(((I*(c + 1)*d - d^2)*tan(b*x +
a)^2 + c^2 + I*(c + 1)*d + (I*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a) + 2*c +
 1)/(tan(b*x + a)^2 + 1)) - 2*a*log(((I*(c - 1)*d + d^2)*tan(b*x + a)^2 - c
^2 + I*(c - 1)*d + (I*c^2 + I*d^2 - 2*I*c + I)*tan(b*x + a) + 2*c - 1)/(tan
(b*x + a)^2 + 1)) - 2*a*log(((I*(c - 1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*(
c - 1)*d + (I*c^2 + I*d^2 - 2*I*c + I)*tan(b*x + a) - 2*c + 1)/(tan(b*x + a
)^2 + 1)) + I*dilog(-((2*I*(c + 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 - 2*I*
(c + 1)*d + (2*I*c^2 + 4*(c + 1)*d - 2*I*d^2 + 4*I*c + 2*I)*tan(b*x + a) +
4*c + 2)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1)
- I*dilog(-((-2*I*(c + 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 + 2*I*(c + 1)*d
 + (-2*I*c^2 + 4*(c + 1)*d + 2*I*d^2 - 4*I*c - 2*I)*tan(b*x + a) + 4*c + 2)
/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) - I*dilo
g(-((2*I*(c - 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 - 2*I*(c - 1)*d + (2*I*c
^2 + 4*(c - 1)*d - 2*I*d^2 - 4*I*c + 2*I)*tan(b*x + a) - 4*c + 2)/((c^2 + d
^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1) + 1) + I*dilog(-((-2*I*
(c - 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 + 2*I*(c - 1)*d + (-2*I*c^2 + 4*(
c - 1)*d + 2*I*d^2 + 4*I*c - 2*I)*tan(b*x + a) - 4*c + 2)/((c^2 + d^2 - 2*c
 + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1) + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(c+d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}(d\tan(bx+a)+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*tan(b*x + a) + c), x)
```

3.239 $\int \frac{\coth^{-1}(c+d\tan(a+bx))}{x}\,dx$

Optimal. Leaf size=17

$$\text{CannotIntegrate}\left(\frac{\coth^{-1}(d\tan(a+bx)+c)}{x},x\right)$$

```
[Out] CannotIntegrate[ArcCoth[c + d*Tan[a + b*x]]/x, x]
```

Rubi [A] time = 0.145621, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth^{-1}(c+d\tan(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCoth[c + d*Tan[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcCoth[c + d*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\coth^{-1}(c+d\tan(a+bx))}{x}\,dx = \int \frac{\coth^{-1}(c+d\tan(a+bx))}{x}\,dx$$

Mathematica [A] time = 0.355154, size = 0, normalized size = 0.

$$\int \frac{\coth^{-1}(c+d\tan(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCoth[c + d*Tan[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcCoth[c + d*Tan[a + b*x]]/x, x]
```

Maple [A] time = 0.382, size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(c+d\tan(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(c+d*tan(b*x+a))/x,x)
```

```
[Out] int(arccoth(c+d*tan(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(d\tan(bx+a)+c)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*tan(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arccoth(d*tan(b*x + a) + c)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arcoth}(d\tan(bx+a)+c)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*tan(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(d*tan(b*x + a) + c)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(c+d*tan(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(d\tan(bx+a)+c)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*tan(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*tan(b*x + a) + c)/x, x)
```

3.240 $\int x^2 \coth^{-1}(1 - id + d\tan(a + bx))\,dx$

Optimal. Leaf size=170

$$-\frac{x\text{PolyLog}\left(3, -(1-id)e^{2ia+2ibx}\right)}{4b^2} - \frac{i\text{PolyLog}\left(4, -(1-id)e^{2ia+2ibx}\right)}{8b^3} + \frac{ix^2\text{PolyLog}\left(2, -(1-id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{6}x^3\log\left(1\right.$$

```
[Out] (I/12)*b*x^4 + (x^3*ArcCoth[1 - I*d + d*Tan[a + b*x]])/3 - (x^3*Log[1 + (1
- I*d)*E^((2*I)*a + (2*I)*b*x)])/6 + ((I/4)*x^2*PolyLog[2, -((1 - I*d)*E^((
2*I)*a + (2*I)*b*x))])/b - (x*PolyLog[3, -((1 - I*d)*E^((2*I)*a + (2*I)*b*x
))])/(4*b^2) - ((I/8)*PolyLog[4, -((1 - I*d)*E^((2*I)*a + (2*I)*b*x))])/b^3
```

Rubi [A] time = 0.304345, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {6264, 2184, 2190, 2531, 6609, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(3, -(1-id)e^{2ia+2ibx}\right)}{4b^2} - \frac{i\text{PolyLog}\left(4, -(1-id)e^{2ia+2ibx}\right)}{8b^3} + \frac{ix^2\text{PolyLog}\left(2, -(1-id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{6}x^3\log\left(1\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[1 - I*d + d*Tan[a + b*x]],x]
```

```
[Out] (I/12)*b*x^4 + (x^3*ArcCoth[1 - I*d + d*Tan[a + b*x]])/3 - (x^3*Log[1 + (1
- I*d)*E^((2*I)*a + (2*I)*b*x)])/6 + ((I/4)*x^2*PolyLog[2, -((1 - I*d)*E^((
2*I)*a + (2*I)*b*x))])/b - (x*PolyLog[3, -((1 - I*d)*E^((2*I)*a + (2*I)*b*x
))])/(4*b^2) - ((I/8)*PolyLog[4, -((1 - I*d)*E^((2*I)*a + (2*I)*b*x))])/b^3
```

Rule 6264

```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Tan[a + b*x]])/(f*(m
 + 1)), x] + Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c + I*d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \coth^{-1}(1-id+d\tan(a+bx))\,dx = \frac{1}{3}x^3\coth^{-1}(1-id+d\tan(a+bx)) + \frac{1}{3}(ib)\int \frac{x^3}{1+(1-id)e^{2ia+2ibx}}\,dx$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\coth^{-1}(1-id+d\tan(a+bx)) - \frac{1}{3}(b(i+d))\int \frac{e^{2ia+2ibx}x^3}{1+(1-id)e^{2ia+2}}$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\coth^{-1}(1-id+d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-id)e^{2ia+2ibx}\right) +$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\coth^{-1}(1-id+d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-id)e^{2ia+2ibx}\right) +$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\coth^{-1}(1-id+d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-id)e^{2ia+2ibx}\right) +$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\coth^{-1}(1-id+d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-id)e^{2ia+2ibx}\right) +$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\coth^{-1}(1-id+d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-id)e^{2ia+2ibx}\right) +$$

Mathematica [A] time = 0.187783, size = 155, normalized size = 0.91

$$\frac{1}{3}x^3\coth^{-1}(d\tan(a+bx)-id+1) - \frac{6ib^2x^2\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{d+i}\right) + 6bx\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{d+i}\right) - 3i\text{PolyLog}\left(4,-\frac{ie^{-2i}}{d}\right.}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[1 - I*d + d*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcCoth[1 - I*d + d*Tan[a + b*x]])/3 - (4*b^3*x^3*Log[1 + I/((I + d)*E
^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, (-I)/((I + d)*E^((2*I)*(a +
 b*x)))] + 6*b*x*PolyLog[3, (-I)/((I + d)*E^((2*I)*(a + b*x)))] - (3*I)*Pol
```

```
yLog[4, (-I)/((I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

Maple [C] time = 30.058, size = 2339, normalized size = 13.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(1-I*d+d*tan(b*x+a)),x)
```

```
[Out] -1/2/b^2*a^2*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x+1/2/b^2*d/(I
+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*x*a^2-1/2/b^2*a^2*d/(I+d)*ln(1-I*exp(I*(
b*x+a))*(-I*(I+d))^(1/2))*x-1/6*x^3*ln(d)-1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(
b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^
2+1/12*I*b*x^4-1/8*I/b^3*d/(I+d)*polylog(4,I*(I+d)*exp(2*I*(b*x+a)))+1/6*I/
b^3*a^3/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)+1/3*I/b^3/(I+d)*l
n(1-I*(I+d)*exp(2*I*(b*x+a)))*a^3-1/4*I/b^2/(I+d)*polylog(3,I*(I+d)*exp(2*I
*(b*x+a)))*x-1/2*I/b^3*a^3/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/
6*I/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*x^3-1/2/b^3*a^2/(I+d)*dilog(1-I*ex
p(I*(b*x+a))*(-I*(I+d))^(1/2))-1/2/b^3*a^2/(I+d)*dilog(1+I*exp(I*(b*x+a))*(
-I*(I+d))^(1/2))-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x
+a))/(exp(2*I*(b*x+a))+1))^2+1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csg
n(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))^2+1/8/b
^3/(I+d)*polylog(4,I*(I+d)*exp(2*I*(b*x+a)))-1/3*x^3*ln(exp(I*(b*x+a)))+1/6
*x^3*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)+1/4*I/b*d/(I+d)*polylog(2,
I*(I+d)*exp(2*I*(b*x+a)))*x^2+1/12*I*x^3*Pi*csgn(I*d)*csgn(I*exp(2*I*(b*x+a
))/(exp(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))+1/
12*I*x^3*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))-1/6*I*x^3*Pi*
csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2-1/12*I*x^3*Pi*csgn(I*d)*c
sgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2-1/12*I*x^3*Pi*csgn(I*exp(2
*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*
x+a)))^2-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))
/(exp(2*I*(b*x+a))+1))^2+1/2*I/b^3*a^2*d/(I+d)*dilog(1-I*exp(I*(b*x+a))*(-I
*(I+d))^(1/2))+1/2*I/b^3*a^2*d/(I+d)*dilog(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1
/2))+1/2*I/b^2/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*x*a^2-1/4/b^2*d/(I+d)*p
olylog(3,I*(I+d)*exp(2*I*(b*x+a)))*x-1/2/b^3*a^3*d/(I+d)*ln(1-I*exp(I*(b*x+
a))*(-I*(I+d))^(1/2))-1/2/b^3*a^3*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^
(1/2))+1/6/b^3*a^3*d/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)+1/3/
b^3*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a^3+1/12*I*x^3*Pi*csgn(I*d/(exp(
2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^3-1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a)
)+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))^3-1/12*I*x^3*Pi*csgn(I*(I*exp
(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))*csgn((I*exp(2*I*(
b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))+1/12*I*x^3*Pi*csgn(I*d/
(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*
(b*x+a)))-1/4*I/b^3*d/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*a^2-1/2*I/b
^2*a^2/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x-1/2*I/b^3*a^3/(I+d)*
ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))+1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*
x+a))+exp(2*I*(b*x+a))*d+I))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+
I)/(exp(2*I*(b*x+a))+1))^2+1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I
*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+
a))*d+I)/(exp(2*I*(b*x+a))+1))^2-1/2*I/b^2*a^2/(I+d)*ln(1+I*exp(I*(b*x+a))*
(-I*(I+d))^(1/2))*x-1/4/b/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*x^2+1/4
/b^3/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*a^2-1/6*d/(I+d)*ln(1-I*(I+d)
*exp(2*I*(b*x+a)))*x^3+1/12*I*x^3*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+
a))*d+I)/(exp(2*I*(b*x+a))+1))^2-1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*
exp(2*I*(b*x+a)))^2+1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+
a)))^3-1/12*I*x^3*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*
```

```
I*(b*x+a))+1))^3+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))^3+1/12*I*x^3*Pi*csg
n(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^3-1/12*I*x^3*Pi*csgn(I/(exp(2*I*
(b*x+a))+1))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I))*csgn(I*(I*ex
p(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))+1/12*I*x^3*Pi*cs
gn(I/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))
/(exp(2*I*(b*x+a))+1))
```

Maxima [B] time = 1.2286, size = 460, normalized size = 2.71

$$\frac{\frac{12\left((bx+a)^3-3(bx+a)^2a+3(bx+a)a^2\right)\operatorname{arcoth}(d\tan(bx+a)-id+1)}{b^2}-\frac{-3i(bx+a)^4+12i(bx+a)^3a-18i(bx+a)^2a^2+\left(8i(bx+a)^3-18i(bx+a)^2a+18i(bx+a)a^2\right)\arctan(}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arccoth(d*tan(b*
x + a) - I*d + 1)/b^2 - (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
+ a)^2*a^2 + (8*I*(b*x + a)^3 - 18*I*(b*x + a)^2*a + 18*I*(b*x + a)*a^2)*ar
ctan2(-d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) + cos(2*b*
x + 2*a) + 1) + (-12*I*(b*x + a)^2 + 18*I*(b*x + a)*a - 9*I*a^2)*dilog((I*d
 - 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(b*x + a)
*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^2 + 2*d
*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, (I*d
 - 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (I*d - 1)*e^(2*I*b*x + 2*I*a)))
/b^2)/b
```

Fricas [C] time = 1.83746, size = 980, normalized size = 5.76

$$\frac{ib^4x^4+2b^3x^3\log\left(\frac{\left((d+i)e^{(2ibx+2ia)}+i\right)e^{(-2ibx-2ia)}}{d}\right)+6ib^2x^2\operatorname{Li}_2\left(\frac{1}{2}\sqrt{4id-4}e^{(ibx+ia)}\right)+6ib^2x^2\operatorname{Li}_2\left(-\frac{1}{2}\sqrt{4id-4}e^{(ibx+ia)}\right)-ia^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(I*b^4*x^4 + 2*b^3*x^3*log(((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b
*x - 2*I*a)/d) + 6*I*b^2*x^2*dilog(1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) + 6
*I*b^2*x^2*dilog(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - I*a^4 + 2*a^3*log(
((2*d + 2*I)*e^(I*b*x + I*a) + I*sqrt(4*I*d - 4))/(2*d + 2*I)) + 2*a^3*log(
((2*d + 2*I)*e^(I*b*x + I*a) - I*sqrt(4*I*d - 4))/(2*d + 2*I)) - 12*b*x*pol
ylog(3, 1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - 12*b*x*polylog(3, -1/2*sqrt(
4*I*d - 4)*e^(I*b*x + I*a)) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(4*I*d - 4)*e^(
I*b*x + I*a) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I
*a) + 1) - 12*I*polylog(4, 1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - 12*I*poly
log(4, -1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(1-I*d+d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arcoth}(d\tan(bx+a) - id + 1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(d*tan(b*x + a) - I*d + 1), x)
```

3.241 $\int x \coth^{-1}(1 - id + d\tan(a + bx))\,dx$

Optimal. Leaf size=133

$$-\frac{\text{PolyLog}\left(3, -(1-id)e^{2ia+2ibx}\right)}{8b^2} + \frac{ix\text{PolyLog}\left(2, -(1-id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{4}x^2\log\left(1 + (1-id)e^{2ia+2ibx}\right) + \frac{1}{2}x^2\coth^{-1}(d\tan$$

```
[Out] (I/6)*b*x^3 + (x^2*ArcCoth[1 - I*d + d*Tan[a + b*x]])/2 - (x^2*Log[1 + (1 -
 I*d)*E^((2*I)*a + (2*I)*b*x)])/4 + ((I/4)*x*PolyLog[2, -((1 - I*d)*E^((2*I
)*a + (2*I)*b*x))])/b - PolyLog[3, -((1 - I*d)*E^((2*I)*a + (2*I)*b*x))]/(8
*b^2)
```

Rubi [A] time = 0.253474, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6264, 2184, 2190, 2531, 2282, 6589}

$$-\frac{\text{PolyLog}\left(3, -(1-id)e^{2ia+2ibx}\right)}{8b^2} + \frac{ix\text{PolyLog}\left(2, -(1-id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{4}x^2\log\left(1 + (1-id)e^{2ia+2ibx}\right) + \frac{1}{2}x^2\coth^{-1}(d\tan$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[1 - I*d + d*Tan[a + b*x]],x]
```

```
[Out] (I/6)*b*x^3 + (x^2*ArcCoth[1 - I*d + d*Tan[a + b*x]])/2 - (x^2*Log[1 + (1 -
 I*d)*E^((2*I)*a + (2*I)*b*x)])/4 + ((I/4)*x*PolyLog[2, -((1 - I*d)*E^((2*I
)*a + (2*I)*b*x))])/b - PolyLog[3, -((1 - I*d)*E^((2*I)*a + (2*I)*b*x))]/(8
*b^2)
```

Rule 6264

```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Tan[a + b*x]])/(f*(m
 + 1)), x] + Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c + I*d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\coth^{-1}(1-id+d\tan(a+bx))\,dx=\frac{1}{2}x^2\coth^{-1}(1-id+d\tan(a+bx))+\frac{1}{2}(ib)\int\frac{x^2}{1+(1-id)e^{2ia+2ibx}}\,dx$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1-id+d\tan(a+bx))-\frac{1}{2}(b(i+d))\int\frac{e^{2ia+2ibx}x^2}{1+(1-id)e^{2ia+}}$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1-id+d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1-id)e^{2ia+2ibx}\right)\cdot$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1-id+d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1-id)e^{2ia+2ibx}\right)\cdot$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1-id+d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1-id)e^{2ia+2ibx}\right)\cdot$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1-id+d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1-id)e^{2ia+2ibx}\right)\cdot$$

Mathematica [A] time = 0.10564, size = 119, normalized size = 0.89

$$\frac{1}{2}x^2\coth^{-1}(d\tan(a+bx)-id+1)-\frac{2ibx\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{d+i}\right)+\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{d+i}\right)+2b^2x^2\log\left(1+\frac{ie^{-2i(a+bx)}}{d+i}\right.}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[1 - I*d + d*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcCoth[1 - I*d + d*Tan[a + b*x]])/2 - (2*b^2*x^2*Log[1 + I/((I + d)*E
^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/((I + d)*E^((2*I)*(a + b*x
)))] + PolyLog[3, (-I)/((I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

Maple [C] time = 17.08, size = 2249, normalized size = 16.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(1-I*d+d*tan(b*x+a)),x)
```

```
[Out] 1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^3-1/8*I*x^2*Pi*c
sgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))^3+1/8*I
*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^3+1/4*x^2*ln(I*exp(
2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)-1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))+1)
*exp(2*I*(b*x+a)))^2+1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+
a)))^3-1/8*I/b^2/(I+d)*polylog(3,I*(I+d)*exp(2*I*(b*x+a)))-1/4*d/(I+d)*ln(1
-I*(I+d)*exp(2*I*(b*x+a)))*x^2+1/2/b^2*a/(I+d)*dilog(1-I*exp(I*(b*x+a))*(-I
*(I+d))^(1/2))+1/2/b^2*a/(I+d)*dilog(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))+1
/6*I*b*x^3-1/4*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2-1
/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+
a))+1))*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1)
)-1/4/b/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*x-1/4/b^2/(I+d)*polylog(2
,I*(I+d)*exp(2*I*(b*x+a)))*a-1/8/b^2*d/(I+d)*polylog(3,I*(I+d)*exp(2*I*(b*x
+a)))-1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*
I*(b*x+a))+1))^3+1/8*I*x^2*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I
)/(exp(2*I*(b*x+a))+1))^2-1/4*x^2*ln(d)+1/8*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))
^2*csgn(I*exp(2*I*(b*x+a)))+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))^3-1/4*I/(
I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*x^2-1/2*x^2*ln(exp(I*(b*x+a)))+1/2*I/b*
a/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x+1/2*I/b*a/(I+d)*ln(1+I*ex
p(I*(b*x+a))*(-I*(I+d))^(1/2))*x-1/2/b*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)
))*x*a+1/2/b*a*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x+1/2/b*a*d/
(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x-1/8*I*x^2*Pi*csgn(I*d/(exp(
2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+
a)))^2+1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2
*I*(b*x+a))+1))*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*
x+a))+1))^2+1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I))*
csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))^2-1/
2*I/b^2*a*d/(I+d)*dilog(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/2*I/b^2*a*d/
(I+d)*dilog(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/8*I*x^2*Pi*csgn(I*exp(2*
I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^2-1/8*I*x^2*Pi*cs
gn(I*d)*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2+1/8*I*x^2*Pi*csgn
(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))+1)*exp
(2*I*(b*x+a)))-1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*
x+a))/(exp(2*I*(b*x+a))+1))^2-1/4/b^2*a^2*d/(I+d)*ln(I*exp(2*I*(b*x+a))+exp
(2*I*(b*x+a))*d+I)-1/4/b^2*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a^2+1/2/b
^2*a^2*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))+1/2/b^2*a^2*d/(I+d)*
ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/4*I/b^2*a^2/(I+d)*ln(I*exp(2*I*(b
*x+a))+exp(2*I*(b*x+a))*d+I)-1/2*I/b/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*x
*a+1/4*I/b*d/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*x+1/4*I/b^2*d/(I+d)*
polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*a+1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))
+1))*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))
+1/8*I*x^2*Pi*csgn(I*d)*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(
I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))-1/4*I/b^2/(I+d)*ln(1-I*(I+d)*exp
(2*I*(b*x+a)))*a^2+1/2*I/b^2*a^2/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/
2))-1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I*d/(ex
p(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2+1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))
+1))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))
^2+1/2*I/b^2*a^2/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/8*I*x^2*Pi
*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d
+I))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))
```

Maxima [B] time = 1.1443, size = 333, normalized size = 2.5

$$\frac{\frac{12\left((bx+a)^2-2\,(bx+a)a\right)\operatorname{arcoth}(d\tan(bx+a)-i\,d+1)}{b}-\frac{-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\operatorname{Li}_2\left((i\,d-1)e^{(2i\,bx+2i\,a)}\right)+\left(6i\,(bx+a)^2-12i\,(bx+a)a\right)\arctan(-d\cos(2\,bx+2\,\ldots}{\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arccoth(d*tan(b*x + a) - I*d + 1)/b
- (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((I*d - 1)*e^(2*I*b
*x + 2*I*a)) + (6*I*(b*x + a)^2 - 12*I*(b*x + a)*a)*arctan2(-d*cos(2*b*x +
2*a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) + cos(2*b*x + 2*a) + 1) + 3*((b
*x + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin
(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) + 3*polylo
g(3, (I*d - 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

Fricas [C] time = 1.73063, size = 811, normalized size = 6.1

$$\frac{2i\,b^3x^3 + 3\,b^2x^2\log\left(\frac{\left((d+i)e^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{d}\right) + 2i\,a^3 + 6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{4i\,d-4}e^{(i\,bx+i\,a)}\right) + 6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{4i\,d-4}e^{(i\,bx+i\,a)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*I*b^3*x^3 + 3*b^2*x^2*log(((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I
*b*x - 2*I*a)/d) + 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(4*I*d - 4)*e^(I*b*x + I
*a)) + 6*I*b*x*dilog(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - 3*a^2*log(((2*
d + 2*I)*e^(I*b*x + I*a) + I*sqrt(4*I*d - 4))/(2*d + 2*I)) - 3*a^2*log(((2*
d + 2*I)*e^(I*b*x + I*a) - I*sqrt(4*I*d - 4))/(2*d + 2*I)) - 3*(b^2*x^2 - a
^2)*log(1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) - 3*(b^2*x^2 - a^2)*log(-1
/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) - 6*polylog(3, 1/2*sqrt(4*I*d - 4)*
e^(I*b*x + I*a)) - 6*polylog(3, -1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(1-I*d+d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\mathrm{arcoth}\left(d\tan\left(bx+a\right)-i\,d+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(d*tan(b*x + a) - I*d + 1), x)
```

3.242 $\int \coth^{-1}(1 - id + d\tan(a + bx))\,dx$

Optimal. Leaf size=93

$$\frac{i\text{PolyLog}\left(2, -(1-id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{2}x\log\left(1+(1-id)e^{2ia+2ibx}\right) + x\coth^{-1}(d\tan(a+bx) - id + 1) + \frac{1}{2}ibx^2$$

```
[Out] (I/2)*b*x^2 + x*ArcCoth[1 - I*d + d*Tan[a + b*x]] - (x*Log[1 + (1 - I*d)*E^
((2*I)*a + (2*I)*b*x)])/2 + ((I/4)*PolyLog[2, -((1 - I*d)*E^((2*I)*a + (2*I
)*b*x))])/b
```

Rubi [A] time = 0.154592, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6256, 2184, 2190, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2, -(1-id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{2}x\log\left(1+(1-id)e^{2ia+2ibx}\right) + x\coth^{-1}(d\tan(a+bx) - id + 1) + \frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[1 - I*d + d*Tan[a + b*x]],x]
```


```
[Out] (I/2)*b*x^2 + x*ArcCoth[1 - I*d + d*Tan[a + b*x]] - (x*Log[1 + (1 - I*d)*E^
((2*I)*a + (2*I)*b*x)])/2 + ((I/4)*PolyLog[2, -((1 - I*d)*E^((2*I)*a + (2*I
)*b*x))])/b
```


Rule 6256

```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Coth[c + d*Tan[a + b*x]], x] + Dist[I*b, Int[x/(c + I*d + c*E^(2*I*a + 2*I*
b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \coth^{-1}(1-id+d\tan(a+bx))\,dx = x\coth^{-1}(1-id+d\tan(a+bx))+(ib)\int \frac{x}{1+(1-id)e^{2ia+2ibx}}\,dx$$
$$=\frac{1}{2}ibx^2+x\coth^{-1}(1-id+d\tan(a+bx))-(b(i+d))\int \frac{e^{2ia+2ibx}x}{1+(1-id)e^{2ia+2ibx}}\,dx$$
$$=\frac{1}{2}ibx^2+x\coth^{-1}(1-id+d\tan(a+bx))-\frac{1}{2}x\log\left(1+(1-id)e^{2ia+2ibx}\right)+\frac{1}{2}\int$$
$$=\frac{1}{2}ibx^2+x\coth^{-1}(1-id+d\tan(a+bx))-\frac{1}{2}x\log\left(1+(1-id)e^{2ia+2ibx}\right)-\frac{i\,\mathrm{Su}}{}$$
$$=\frac{1}{2}ibx^2+x\coth^{-1}(1-id+d\tan(a+bx))-\frac{1}{2}x\log\left(1+(1-id)e^{2ia+2ibx}\right)+\frac{i\mathrm{Li}}{}$$

Mathematica [B] time = 2.85825, size = 766, normalized size = 8.24

$$\frac{x\sec^2(a+bx)(\cos(bx)+i\sin(bx))(\sin(bx)+i\cos(bx))(d\sin(a+bx)+(2-id)\cos(a+bx))\left(\mathrm{PolyLog}\left(2,-\frac{1}{2}(\cos(a)\right.\right.}{(\tan(a+bx)-i)(d\tan(a+bx)-id+2)(id\sin(a+bx)+(d+2i)\cos(a+bx))\left(\frac{\sec^2(bx)\log\left(\frac{\sec(bx)}{}\right)}{\mathrm{t}}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[1 - I*d + d*Tan[a + b*x]],x]
```

```
[Out] x*ArcCoth[1 - I*d + d*Tan[a + b*x]] + (x*((2*I)*b*x*Log[2*Cos[b*x]*(Cos[b*x
] - I*Sin[b*x])] - Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((2*I + d)*Cos[a + b*x
] + I*d*Sin[a + b*x]))/(2*(I + d))]*Log[1 - I*Tan[b*x]] + Log[(Sec[b*x]*((2
 - I*d)*Cos[a + b*x] + d*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*Log[1 +
I*Tan[b*x]] - PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] - PolyLog[2, (Sec[b*x]
*(d*Cos[a] + I*(2*I + d)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*(I + d
))] + PolyLog[2, -((Cos[a] + I*Sin[a])*(d*Cos[a] + I*(2*I + d)*Sin[a])*(-I
+ Tan[b*x]))/2]*Sec[a + b*x]^2*(Cos[b*x] + I*Sin[b*x])*(I*Cos[b*x] + Sin[b
*x])*((2 - I*d)*Cos[a + b*x] + d*Sin[a + b*x]))/(((2*I + d)*Cos[a + b*x] +
I*d*Sin[a + b*x])*((I*Log[1 + I*Tan[b*x]]*Sec[b*x]*(d*Cos[a] + I*(2*I + d)*
Sin[a]))/((2*I + d)*Cos[a + b*x] + I*d*Sin[a + b*x]) + (Log[1 - I*Tan[b*x]]
*Sec[b*x]*((-I)*d*Cos[a] + (2*I + d)*Sin[a]))/((2*I + d)*Cos[a + b*x] + I*d
*Sin[a + b*x]) + 2*b*x*(1 - I*Tan[b*x]) + (Log[(Sec[b*x]*((2 - I*d)*Cos[a +
 b*x] + d*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*Sec[b*x]^2)/(-I + Tan[b
*x]) - (Log[1 + ((Cos[a] + I*Sin[a])*(d*Cos[a] + I*(2*I + d)*Sin[a])*(-I +
Tan[b*x]))/2]*Sec[b*x]^2)/(-I + Tan[b*x]) + Log[(Sec[b*x]*(Cos[a] - I*Sin[a
])*((2*I + d)*Cos[a + b*x] + I*d*Sin[a + b*x]))/(2*(I + d))]*(-I + Tan[b*x]
) - (Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((2*I + d)*Cos[a + b*x] + I*d*Sin[a
+ b*x]))/(2*(I + d))]*Sec[b*x]^2)/(I + Tan[b*x]))*(-I + Tan[a + b*x])*(2 -
I*d + d*Tan[a + b*x]))
```

Maple [B] time = 0.175, size = 292, normalized size = 3.1

$$\frac{\frac{i}{2}\operatorname{arccoth}(1-id+d\tan(bx+a))\ln(id+d\tan(bx+a))}{b}-\frac{\frac{i}{2}\operatorname{arccoth}(1-id+d\tan(bx+a))\ln(-id+d\tan(bx+a}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1-I*d+d*tan(b*x+a)),x)
```

```
[Out] 1/2*I/b*arccoth(1-I*d+d*tan(b*x+a))*ln(I*d+d*tan(b*x+a))-1/2*I/b*arccoth(1-
I*d+d*tan(b*x+a))*ln(-I*d+d*tan(b*x+a))-1/8*I/b*ln(-I*d+d*tan(b*x+a))^2+1/4
*I/b*dilog(1-1/2*I*d+1/2*d*tan(b*x+a))+1/4*I/b*ln(-I*d+d*tan(b*x+a))*ln(1-1
/2*I*d+1/2*d*tan(b*x+a))+1/4*I/b*dilog(1/2*I*(-I*d+d*tan(b*x+a))/d)+1/4*I/b
*ln(I*d+d*tan(b*x+a))*ln(1/2*I*(-I*d+d*tan(b*x+a))/d)-1/4*I/b*dilog((2-I*d+
d*tan(b*x+a))/(-2*I*d+2))-1/4*I/b*ln(I*d+d*tan(b*x+a))*ln((2-I*d+d*tan(b*x+
a))/(-2*I*d+2))
```

Maxima [B] time = 1.55359, size = 358, normalized size = 3.85

$$-\frac{4(bx+a)d\left(\frac{\log(d\tan(bx+a)-id+2)}{d}-\frac{\log(\tan(bx+a)-i)}{d}\right)-d\left(\frac{2i\left(\log(d\tan(bx+a)-id+2)\log\left(-\frac{id\tan(bx+a)+d+2i}{2d+2i}+1\right)+\mathrm{Li}_2\left(\frac{id\tan(bx+a)+d+2i}{2d+2i}\right)\right)}{d}-2i\,l\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*(b*x + a)*d*(log(d*tan(b*x + a) - I*d + 2)/d - log(tan(b*x + a) - I
)/d) - d*(2*I*(log(d*tan(b*x + a) - I*d + 2)*log(-(I*d*tan(b*x + a) + d + 2
*I)/(2*d + 2*I) + 1) + dilog((I*d*tan(b*x + a) + d + 2*I)/(2*d + 2*I)))/d -
 (2*I*log(d*tan(b*x + a) - I*d + 2)*log(tan(b*x + a) - I) - I*log(tan(b*x +
 a) - I)^2)/d + 2*I*(log(1/2*d*tan(b*x + a) - 1/2*I*d + 1)*log(tan(b*x + a)
 - I) + dilog(-1/2*d*tan(b*x + a) + 1/2*I*d))/d - 2*I*(log(tan(b*x + a) - I
)*log(-1/2*I*tan(b*x + a) + 1/2) + dilog(1/2*I*tan(b*x + a) + 1/2))/d) - 8*
(b*x + a)*arccoth(d*tan(b*x + a) - I*d + 1))/b
```

Fricas [B] time = 1.71042, size = 603, normalized size = 6.48

$$\frac{ib^2x^2+bx\log\left(\frac{\left((d+i)e^{(2ibx+2ia)}+i\right)e^{(-2ibx-2ia)}}{d}\right)-ia^2-(bx+a)\log\left(\frac{1}{2}\sqrt{4id-4}e^{(ibx+ia)}+1\right)-(bx+a)\log\left(-\frac{1}{2}\sqrt{4id-4}e^{(ibx+}\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*b^2*x^2 + b*x*log(((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*
I*a)/d) - I*a^2 - (b*x + a)*log(1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) -
(b*x + a)*log(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) + a*log(((2*d + 2*I
)*e^(I*b*x + I*a) + I*sqrt(4*I*d - 4))/(2*d + 2*I)) + a*log(((2*d + 2*I)*e^
(I*b*x + I*a) - I*sqrt(4*I*d - 4))/(2*d + 2*I)) + I*dilog(1/2*sqrt(4*I*d -
4)*e^(I*b*x + I*a)) + I*dilog(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1-I*d+d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}\left(d\tan\left(bx+a\right)-i\,d+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*tan(b*x + a) - I*d + 1), x)
```

3.243 $\int \frac{\coth^{-1}(1-id+d\tan(a+bx))}{x}\,dx$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(\frac{\coth^{-1}(d\tan(a+bx)-id+1)}{x},x\right)$$

```
[Out] CannotIntegrate[ArcCoth[1 - I*d + d*Tan[a + b*x]]/x, x]
```

Rubi [A] time = 0.0885803, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth^{-1}(1-id+d\tan(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCoth[1 - I*d + d*Tan[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcCoth[1 - I*d + d*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\coth^{-1}(1-id+d\tan(a+bx))}{x}\,dx=\int \frac{\coth^{-1}(1-id+d\tan(a+bx))}{x}\,dx$$

Mathematica [A] time = 0.727147, size = 0, normalized size = 0.

$$\int \frac{\coth^{-1}(1-id+d\tan(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCoth[1 - I*d + d*Tan[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcCoth[1 - I*d + d*Tan[a + b*x]]/x, x]
```

Maple [A] time = 0.444, size = 0, normalized size = 0.

$$\int \frac{\text{arccoth}\,(1-id+d\tan(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1-I*d+d*tan(b*x+a))/x,x)
```

```
[Out] int(arccoth(1-I*d+d*tan(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-i\,bx + \frac{1}{4}\left(-i\,\pi - 4i\,a - 2\,\log\left(d\right)\right)\log\left(x\right) - \frac{1}{2}i\int \frac{\arctan\left(-d\cos\left(2\,bx + 2\,a\right) + \sin\left(2\,bx + 2\,a\right), -d\sin\left(2\,bx + 2\,a\right) - \mathrm{c}\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-I*d+d*tan(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -I*b*x + 1/4*(-I*pi - 4*I*a - 2*log(d))*log(x) - 1/2*I*integrate(arctan2(-d
*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - cos(2*b*x + 2*a
) - 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*s
in(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\frac{\left((d+i)e^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{d}\right)}{2\,x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-I*d+d*tan(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*log(((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)/d)/
x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1-I*d+d*tan(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}\left(d\tan\left(bx + a\right) - i\,d + 1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-I*d+d*tan(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*tan(b*x + a) - I*d + 1)/x, x)
```

3.244 $\int x^2 \coth^{-1}(1+id-d\tan(a+bx))\,dx$

Optimal. Leaf size=171

$$-\frac{x\text{PolyLog}\left(3,-(1+id)e^{2ia+2ibx}\right)}{4b^2}-\frac{i\text{PolyLog}\left(4,-(1+id)e^{2ia+2ibx}\right)}{8b^3}+\frac{ix^2\text{PolyLog}\left(2,-(1+id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{6}x^3\log\left(1+\right.$$

```
[Out] (I/12)*b*x^4 + (x^3*ArcCoth[1 + I*d - d*Tan[a + b*x]])/3 - (x^3*Log[1 + (1
+ I*d)*E^((2*I)*a + (2*I)*b*x)])/6 + ((I/4)*x^2*PolyLog[2, -((1 + I*d)*E^((
2*I)*a + (2*I)*b*x))])/b - (x*PolyLog[3, -((1 + I*d)*E^((2*I)*a + (2*I)*b*x
))])/(4*b^2) - ((I/8)*PolyLog[4, -((1 + I*d)*E^((2*I)*a + (2*I)*b*x))])/b^3
```

Rubi [A] time = 0.297572, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6264, 2184, 2190, 2531, 6609, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(3,-(1+id)e^{2ia+2ibx}\right)}{4b^2}-\frac{i\text{PolyLog}\left(4,-(1+id)e^{2ia+2ibx}\right)}{8b^3}+\frac{ix^2\text{PolyLog}\left(2,-(1+id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{6}x^3\log\left(1+\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[1 + I*d - d*Tan[a + b*x]],x]
```

```
[Out] (I/12)*b*x^4 + (x^3*ArcCoth[1 + I*d - d*Tan[a + b*x]])/3 - (x^3*Log[1 + (1
+ I*d)*E^((2*I)*a + (2*I)*b*x)])/6 + ((I/4)*x^2*PolyLog[2, -((1 + I*d)*E^((
2*I)*a + (2*I)*b*x))])/b - (x*PolyLog[3, -((1 + I*d)*E^((2*I)*a + (2*I)*b*x
))])/(4*b^2) - ((I/8)*PolyLog[4, -((1 + I*d)*E^((2*I)*a + (2*I)*b*x))])/b^3
```

Rule 6264

```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Tan[a + b*x]])/(f*(m
 + 1)), x] + Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c + I*d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\coth^{-1}(1+id-d\tan(a+bx))\,dx=\frac{1}{3}x^3\coth^{-1}(1+id-d\tan(a+bx))+\frac{1}{3}(ib)\int\frac{x^3}{1+(1+id)e^{2ia+2ibx}}\,dx$$

$$=\frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1+id-d\tan(a+bx))-\frac{1}{3}(b(i-d))\int\frac{e^{2ia+2ibx}x}{1+(1+id)e^{2i}}$$

$$=\frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1+id-d\tan(a+bx))-\frac{1}{6}x^3\log\left(1+(1+id)e^{2ia+2ibx}\right.$$

$$=\frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1+id-d\tan(a+bx))-\frac{1}{6}x^3\log\left(1+(1+id)e^{2ia+2ibx}\right.$$

$$=\frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1+id-d\tan(a+bx))-\frac{1}{6}x^3\log\left(1+(1+id)e^{2ia+2ibx}\right.$$

$$=\frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1+id-d\tan(a+bx))-\frac{1}{6}x^3\log\left(1+(1+id)e^{2ia+2ibx}\right.$$

$$=\frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1+id-d\tan(a+bx))-\frac{1}{6}x^3\log\left(1+(1+id)e^{2ia+2ibx}\right.$$

Mathematica [A] time = 0.176928, size = 156, normalized size = 0.91

$$\frac{1}{3}x^3\coth^{-1}(d(-\tan(a+bx))+id+1)-\frac{6ib^2x^2\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{d-i}\right)+6bx\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{d-i}\right)-3i\text{PolyLog}\left(4,\right.}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[1 + I*d - d*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcCoth[1 + I*d - d*Tan[a + b*x]])/3 - (4*b^3*x^3*Log[1 - I/((-I + d)*
E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, I/((-I + d)*E^((2*I)*(a +
b*x)))] + 6*b*x*PolyLog[3, I/((-I + d)*E^((2*I)*(a + b*x)))] - (3*I)*PolyLo
```

```
g[4, I/((-I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

Maple [C] time = 29.632, size = 2449, normalized size = 14.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(1+I*d-d*tan(b*x+a)),x)
```

```
[Out] -1/2*I/b^3*a^2*d/(-d+I)*dilog(1-I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))+1/2*I/b
^2/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b*x+a)))*x*a^2-1/6*x^3*ln(d)-1/12*I*x^3*Pi
*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))+1
)*exp(2*I*(b*x+a)))^2+1/12*I*b*x^4-1/2/b^2*d/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(
b*x+a)))*x*a^2+1/2/b^2*a^2*d/(-d+I)*ln(1+I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2)
)*x+1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I
*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+
a))+1))^2-1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(e
xp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I
*(b*x+a))+1))+1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b
*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2+1/12*I*x^3*Pi*csgn(I
*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))*d-I*ex
p(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2-1/2*I/b^2*a^2/(-d+I)*ln(1+I*exp(I
*(b*x+a))*(-I*(-d+I))^(1/2))*x+1/4*I/b^3*d/(-d+I)*polylog(2,I*(-d+I)*exp(2*
I*(b*x+a)))*a^2-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+
a))/(exp(2*I*(b*x+a))+1))^2-1/3*x^3*ln(exp(I*(b*x+a)))+1/6*x^3*ln(exp(2*I*(
b*x+a))*d-I*exp(2*I*(b*x+a))-I)-1/2*I/b^3*a^3/(-d+I)*ln(1-I*exp(I*(b*x+a))*
(-I*(-d+I))^(1/2))+1/6*I/b^3*a^3/(-d+I)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+
a))*d+I)+1/8*I/b^3*d/(-d+I)*polylog(4,I*(-d+I)*exp(2*I*(b*x+a)))+1/12*I*x^3
*Pi*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3+
1/6*d/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b*x+a)))*x^3-1/2/b^3*a^2/(-d+I)*dilog(1
+I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))-1/2/b^3*a^2/(-d+I)*dilog(1-I*exp(I*(b*
x+a))*(-I*(-d+I))^(1/2))-1/4/b/(-d+I)*polylog(2,I*(-d+I)*exp(2*I*(b*x+a)))*
x^2+1/4/b^3/(-d+I)*polylog(2,I*(-d+I)*exp(2*I*(b*x+a)))*a^2-1/12*I*x^3*Pi*c
sgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3+1/3
*I/b^3/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b*x+a)))*a^3+1/12*I*x^3*Pi*csgn(I*d)*c
sgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I*(b*x+a))+1)*
exp(2*I*(b*x+a)))+1/12*I*x^3*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*
x+a)))-1/6*I*x^3*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2-1/12*
I*x^3*Pi*csgn(I*d)*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2-1/12*I
*x^3*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I*(b*
x+a))+1)*exp(2*I*(b*x+a)))^2-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csg
n(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^2+1/8/b^3/(-d+I)*polylog(4,I*(-d
+I)*exp(2*I*(b*x+a)))+1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(
b*x+a)))^3-1/6*I/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b*x+a)))*x^3+1/2/b^2*a^2*d/(
-d+I)*ln(1-I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))*x-1/4*I/b*d/(-d+I)*polylog(2
,I*(-d+I)*exp(2*I*(b*x+a)))*x^2-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*
csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))*
d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))-1/2*I/b^2*a^2/(-d+I)*ln(1-I*e
xp(I*(b*x+a))*(-I*(-d+I))^(1/2))*x+1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))
+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))+1/2/b^3
*a^3*d/(-d+I)*ln(1+I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))+1/2/b^3*a^3*d/(-d+I)
*ln(1-I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))+1/4/b^2*d/(-d+I)*polylog(3,I*(-d+
I)*exp(2*I*(b*x+a)))*x-1/6/b^3*a^3*d/(-d+I)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(
b*x+a))*d+I)-1/3/b^3*d/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b*x+a)))*a^3-1/2*I/b^3
*a^2*d/(-d+I)*dilog(1+I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))-1/4*I/b^2/(-d+I)*
polylog(3,I*(-d+I)*exp(2*I*(b*x+a)))*x-1/2*I/b^3*a^3/(-d+I)*ln(1+I*exp(I*(b
```

```
*x+a))*(-I*(-d+I))^(1/2))-1/12*I*x^3*Pi*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*
(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2+1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+a))+
1)*exp(2*I*(b*x+a)))^2-1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b
*x+a)))^3+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))^3+1/12*I*x^3*Pi*csgn(I*exp
(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^3+1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a)
)+1))*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1)
)
```

Maxima [B] time = 1.21992, size = 459, normalized size = 2.68

$$-\frac{\frac{12\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\operatorname{arcoth}(d\tan(bx+a)-i\,d-1)}{b^2}+\frac{-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2+\left(8i\,(bx+a)^3-18i\,(bx+a)^2a+18i\,(bx+a)a^2\right)\,\mathrm{ar}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arccoth(d*tan(b
*x + a) - I*d - 1)/b^2 + (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
 + a)^2*a^2 + (8*I*(b*x + a)^3 - 18*I*(b*x + a)^2*a + 18*I*(b*x + a)*a^2)*a
rctan2(d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + cos(2*b
*x + 2*a) + 1) + (-12*I*(b*x + a)^2 + 18*I*(b*x + a)*a - 9*I*a^2)*dilog((-I
*d - 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(b*x +
a)*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^2 - 2
*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, (-
I*d - 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (-I*d - 1)*e^(2*I*b*x + 2*I*
a)))/b^2)/b
```

Fricas [C] time = 1.89045, size = 992, normalized size = 5.8

$$\frac{i\,b^4x^4-2\,b^3x^3\log\left(\frac{de^{(2i\,bx+2i\,a)}}{(d-i)e^{(2i\,bx+2i\,a)}-i}\right)+6i\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{-4i\,d-4}e^{(i\,bx+i\,a)}\right)+6i\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{-4i\,d-4}e^{(i\,bx+i\,a)}\right)-i\,a^4+2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(I*b^4*x^4 - 2*b^3*x^3*log(d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*x +
 2*I*a) - I)) + 6*I*b^2*x^2*dilog(1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) + 6
*I*b^2*x^2*dilog(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - I*a^4 + 2*a^3*log
(((2*d - 2*I)*e^(I*b*x + I*a) + I*sqrt(-4*I*d - 4))/(2*d - 2*I)) + 2*a^3*lo
g(((2*d - 2*I)*e^(I*b*x + I*a) - I*sqrt(-4*I*d - 4))/(2*d - 2*I)) - 12*b*x*
polylog(3, 1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - 12*b*x*polylog(3, -1/2*s
qrt(-4*I*d - 4)*e^(I*b*x + I*a)) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(-4*I*d -
4)*e^(I*b*x + I*a) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sqrt(-4*I*d - 4)*e^(I*
b*x + I*a) + 1) - 12*I*polylog(4, 1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - 1
2*I*polylog(4, -1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(1+I*d-d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arcoth}(-d\tan(bx+a)+id+1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(-d*tan(b*x + a) + I*d + 1), x)
```

3.245 $\int x \coth^{-1}(1 + id - d\tan(a + bx))\,dx$

Optimal. Leaf size=134

$$-\frac{\text{PolyLog}\left(3,-(1+id)e^{2ia+2ibx}\right)}{8b^2}+\frac{ix\text{PolyLog}\left(2,-(1+id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{4}x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)+\frac{1}{2}x^2\coth^{-1}(d($$

```
[Out] (I/6)*b*x^3 + (x^2*ArcCoth[1 + I*d - d*Tan[a + b*x]])/2 - (x^2*Log[1 + (1 +
 I*d)*E^((2*I)*a + (2*I)*b*x)])/4 + ((I/4)*x*PolyLog[2, -((1 + I*d)*E^((2*I
)*a + (2*I)*b*x))])/b - PolyLog[3, -((1 + I*d)*E^((2*I)*a + (2*I)*b*x))]/(8
*b^2)
```

Rubi [A] time = 0.24538, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6264, 2184, 2190, 2531, 2282, 6589}

$$-\frac{\text{PolyLog}\left(3,-(1+id)e^{2ia+2ibx}\right)}{8b^2}+\frac{ix\text{PolyLog}\left(2,-(1+id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{4}x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)+\frac{1}{2}x^2\coth^{-1}(d($$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[1 + I*d - d*Tan[a + b*x]],x]
```


```
[Out] (I/6)*b*x^3 + (x^2*ArcCoth[1 + I*d - d*Tan[a + b*x]])/2 - (x^2*Log[1 + (1 +
 I*d)*E^((2*I)*a + (2*I)*b*x)])/4 + ((I/4)*x*PolyLog[2, -((1 + I*d)*E^((2*I
)*a + (2*I)*b*x))])/b - PolyLog[3, -((1 + I*d)*E^((2*I)*a + (2*I)*b*x))]/(8
*b^2)
```


Rule 6264

```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Tan[a + b*x]])/(f*(m
 + 1)), x] + Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c + I*d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\coth^{-1}(1+id-d\tan(a+bx))\,dx=\frac{1}{2}x^2\coth^{-1}(1+id-d\tan(a+bx))+\frac{1}{2}(ib)\int\frac{x^2}{1+(1+id)e^{2ia+2ibx}}\,dx$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1+id-d\tan(a+bx))-\frac{1}{2}(b(i-d))\int\frac{e^{2ia+2ibx}x^2}{1+(1+id)e^{2ia+2ibx}}$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1+id-d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)+\frac{1}{2}$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1+id-d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)+$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1+id-d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)+$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1+id-d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)+$$

Mathematica [A] time = 0.108779, size = 120, normalized size = 0.9

$$\frac{1}{2}x^2\coth^{-1}(d(-\tan(a+bx))+id+1)-\frac{2ibx\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{d-i}\right)+\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{d-i}\right)+2b^2x^2\log\left(1-\frac{ie^{-2i(a+bx)}}{d-i}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[1 + I*d - d*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcCoth[1 + I*d - d*Tan[a + b*x]])/2 - (2*b^2*x^2*Log[1 - I/((-I + d)*
E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/((-I + d)*E^((2*I)*(a + b*x)
))] + PolyLog[3, I/((-I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

Maple [C] time = 13.712, size = 2351, normalized size = 17.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(1+I*d-d*tan(b*x+a)),x)
```

```
[Out] -1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp
(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I
*(b*x+a))+1))+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^3-
1/8*I*x^2*Pi*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a
))+1))^2+1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^3+1/8
*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2-1/8*I*x^2*Pi*csgn
(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3+1/6*I*
b*x^3-1/4*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2+1/8*I*
x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b
*x+a))-I)/(exp(2*I*(b*x+a))+1))^2-1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*
exp(2*I*(b*x+a)))^3-1/8*I/b^2/(-d+I)*polylog(3,I*(-d+I)*exp(2*I*(b*x+a)))-1
/4*I/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b*x+a)))*x^2-1/4*x^2*ln(d)+1/8*I*x^2*Pi*
csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))+1/4*x^2*ln(exp(2*I*(b*x+a
))*d-I*exp(2*I*(b*x+a))-I)+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))^3+1/8*I*x^
2*Pi*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3
+1/8/b^2*d/(-d+I)*polylog(3,I*(-d+I)*exp(2*I*(b*x+a)))+1/2/b^2*a/(-d+I)*dil
og(1+I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))+1/2/b^2*a/(-d+I)*dilog(1-I*exp(I*(
b*x+a))*(-I*(-d+I))^(1/2))-1/4/b/(-d+I)*polylog(2,I*(-d+I)*exp(2*I*(b*x+a))
)*x-1/4/b^2/(-d+I)*polylog(2,I*(-d+I)*exp(2*I*(b*x+a)))*a+1/4*d/(-d+I)*ln(1
-I*(-d+I)*exp(2*I*(b*x+a)))*x^2-1/2*x^2*ln(exp(I*(b*x+a)))+1/2*I/b*a/(-d+I)
*ln(1-I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))*x-1/2/b*a*d/(-d+I)*ln(1+I*exp(I*(
b*x+a))*(-I*(-d+I))^(1/2))*x-1/2/b*a*d/(-d+I)*ln(1-I*exp(I*(b*x+a))*(-I*(-d
+I))^(1/2))*x-1/4*I/b^2*d/(-d+I)*polylog(2,I*(-d+I)*exp(2*I*(b*x+a)))*a-1/2
*I/b/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b*x+a)))*x*a+1/2*I/b*a/(-d+I)*ln(1+I*exp
(I*(b*x+a))*(-I*(-d+I))^(1/2))*x+1/2*I/b^2*a*d/(-d+I)*dilog(1-I*exp(I*(b*x+
a))*(-I*(-d+I))^(1/2))+1/8*I*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b
*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a
))+1))^2-1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(
d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2+1/2*I/b^2*a*d/(-d+I)*dilog(1+I*e
xp(I*(b*x+a))*(-I*(-d+I))^(1/2))-1/4*I/b*d/(-d+I)*polylog(2,I*(-d+I)*exp(2*
I*(b*x+a)))*x+1/2/b*d/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b*x+a)))*x*a-1/8*I*x^2*
Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*c
sgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))-1/8*I*x
^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1)
)^2-1/8*I*x^2*Pi*csgn(I*d)*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^
2+1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(
2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))-1/4*I/b^2/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b
*x+a)))*a^2-1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a
))/(exp(2*I*(b*x+a))+1))^2+1/2*I/b^2*a^2/(-d+I)*ln(1+I*exp(I*(b*x+a))*(-I*(
-d+I))^(1/2))+1/2*I/b^2*a^2/(-d+I)*ln(1-I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))
-1/4*I/b^2*a^2/(-d+I)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d+I)+1/4/b^2*d
/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b*x+a)))*a^2-1/2/b^2*a^2*d/(-d+I)*ln(1+I*exp
(I*(b*x+a))*(-I*(-d+I))^(1/2))-1/2/b^2*a^2*d/(-d+I)*ln(1-I*exp(I*(b*x+a))*(
-I*(-d+I))^(1/2))+1/4/b^2*a^2*d/(-d+I)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a
))*d+I)+1/8*I*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(
2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b
*x+a))+1))^2+1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+
a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))+1/8*I*x^2*Pi*csgn(I*d)*c
sgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I*(b*x+a))+1)*
exp(2*I*(b*x+a)))-1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1)
)*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2
```

Maxima [B] time = 1.15077, size = 332, normalized size = 2.48

$$-\frac{\frac{12\left((bx+a)^2-2\,(bx+a)a\right)\operatorname{arcoth}(d\tan(bx+a)-i\,d-1)}{b}+\frac{-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\operatorname{Li}_2\left((-i\,d-1)e^{(2i\,bx+2i\,a)}\right)+\left(6i\,(bx+a)^2-12i\,(bx+a)a\right)\arctan(d\cos(2}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arccoth(d*tan(b*x + a) - I*d - 1)/b
 + (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((-I*d - 1)*e^(2*I
*b*x + 2*I*a)) + (6*I*(b*x + a)^2 - 12*I*(b*x + a)*a)*arctan2(d*cos(2*b*x +
 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + cos(2*b*x + 2*a) + 1) + 3*(
(b*x + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*s
in(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) + 3*poly
log(3, (-I*d - 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

Fricas [C] time = 1.778, size = 821, normalized size = 6.13

$$2i\,b^3x^3 - 3\,b^2x^2\log\left(\frac{de^{(2i\,bx+2i\,a)}}{(d-i)e^{(2i\,bx+2i\,a)}-i}\right) + 2i\,a^3 + 6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,d-4}e^{(i\,bx+i\,a)}\right) + 6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,d-4}e^{(i\,bx+i\,a)}\right) - 3\,a^2\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*I*b^3*x^3 - 3*b^2*x^2*log(d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*x
 + 2*I*a) - I)) + 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I
*a)) + 6*I*b*x*dilog(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - 3*a^2*log(((2
*d - 2*I)*e^(I*b*x + I*a) + I*sqrt(-4*I*d - 4))/(2*d - 2*I)) - 3*a^2*log(((
2*d - 2*I)*e^(I*b*x + I*a) - I*sqrt(-4*I*d - 4))/(2*d - 2*I)) - 3*(b^2*x^2
- a^2)*log(1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1) - 3*(b^2*x^2 - a^2)*lo
g(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1) - 6*polylog(3, 1/2*sqrt(-4*I*d
 - 4)*e^(I*b*x + I*a)) - 6*polylog(3, -1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)
))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(1+I*d-d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\mathrm{arcoth}\left(-d\tan\left(bx+a\right)+i\,d+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(-d*tan(b*x + a) + I*d + 1), x)
```

3.246 $\int \coth^{-1}(1+id-d\tan(a+bx))\,dx$

Optimal. Leaf size=94

$$\frac{i\mathrm{PolyLog}\left(2,-(1+id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{2}x\log\left(1+(1+id)e^{2ia+2ibx}\right)+x\coth^{-1}(d(-\tan(a+bx))+id+1)+\frac{1}{2}ibx^2$$

```
[Out] (I/2)*b*x^2 + x*ArcCoth[1 + I*d - d*Tan[a + b*x]] - (x*Log[1 + (1 + I*d)*E^
((2*I)*a + (2*I)*b*x)])/2 + ((I/4)*PolyLog[2, -((1 + I*d)*E^((2*I)*a + (2*I
)*b*x))])/b
```

Rubi [A] time = 0.156441, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6256, 2184, 2190, 2279, 2391}

$$\frac{i\mathrm{PolyLog}\left(2,-(1+id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{2}x\log\left(1+(1+id)e^{2ia+2ibx}\right)+x\coth^{-1}(d(-\tan(a+bx))+id+1)+\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[1 + I*d - d*Tan[a + b*x]],x]
```

```
[Out] (I/2)*b*x^2 + x*ArcCoth[1 + I*d - d*Tan[a + b*x]] - (x*Log[1 + (1 + I*d)*E^
((2*I)*a + (2*I)*b*x)])/2 + ((I/4)*PolyLog[2, -((1 + I*d)*E^((2*I)*a + (2*I
)*b*x))])/b
```

Rule 6256

```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Coth[c + d*Tan[a + b*x]], x] + Dist[I*b, Int[x/(c + I*d + c*E^(2*I*a + 2*I*
b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \coth^{-1}(1+id-d\tan(a+bx))\,dx = x\coth^{-1}(1+id-d\tan(a+bx)) + (ib)\int \frac{x}{1+(1+id)e^{2ia+2ibx}}\,dx$$
$$= \frac{1}{2}ibx^2 + x\coth^{-1}(1+id-d\tan(a+bx)) - (b(i-d))\int \frac{e^{2ia+2ibx}x}{1+(1+id)e^{2ia+2ibx}}\,dx$$
$$= \frac{1}{2}ibx^2 + x\coth^{-1}(1+id-d\tan(a+bx)) - \frac{1}{2}x\log\left(1+(1+id)e^{2ia+2ibx}\right) + \frac{1}{2}\int \mathrm{lo}$$
$$= \frac{1}{2}ibx^2 + x\coth^{-1}(1+id-d\tan(a+bx)) - \frac{1}{2}x\log\left(1+(1+id)e^{2ia+2ibx}\right) - \frac{i\,\mathrm{Subs}}{}$$
$$= \frac{1}{2}ibx^2 + x\coth^{-1}(1+id-d\tan(a+bx)) - \frac{1}{2}x\log\left(1+(1+id)e^{2ia+2ibx}\right) + \frac{i\mathrm{Li}_2\left(-\right.}{}$$

Mathematica [B] time = 2.82527, size = 723, normalized size = 7.69

$$x\coth^{-1}(d(-\tan(a+bx))+id+1) - \frac{x\sec(a+bx)(\cos(bx)+i\sin(bx))(\sin(bx)+i\cos(bx))\left(-\mathrm{PolyLog}\left(2,\frac{1}{2}(\cos(a)+i\right.\right.}{(\tan(a+bx)-i)(id\sin(a+bx)+(d-2i)\cos(a+bx))\left(-\frac{\sec^2(bx)\log\left(\right.}{}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[1 + I*d - d*Tan[a + b*x]],x]
```

```
[Out] x*ArcCoth[1 + I*d - d*Tan[a + b*x]] - (x*((-2*I)*b*x*Log[2*Cos[b*x]*(Cos[b*
x] - I*Sin[b*x])] + Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-2*I + d)*Cos[a + b
*x] + I*d*Sin[a + b*x]))/(2*(-I + d))]*Log[1 - I*Tan[b*x]] - Log[(Sec[b*x]*
((2 + I*d)*Cos[a + b*x] - d*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*Log[1
 + I*Tan[b*x]] + PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] + PolyLog[2, (Sec[b
*x]*(d*Cos[a] + (2 + I*d)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*(-I +
 d))] - PolyLog[2, ((Cos[a] + I*Sin[a])*(d*Cos[a] + (2 + I*d)*Sin[a])*(-I +
 Tan[b*x]))/2])*Sec[a + b*x]*(Cos[b*x] + I*Sin[b*x])*(I*Cos[b*x] + Sin[b*x]
))/(((-2*I + d)*Cos[a + b*x] + I*d*Sin[a + b*x])*((I*Log[1 - I*Tan[b*x]]*Se
c[b*x]*(d*Cos[a] + (2 + I*d)*Sin[a]))/((-2*I + d)*Cos[a + b*x] + I*d*Sin[a
+ b*x]) + (Log[1 + I*Tan[b*x]]*Sec[b*x]*((-I)*d*Cos[a] + (-2*I + d)*Sin[a])
)/((-2*I + d)*Cos[a + b*x] + I*d*Sin[a + b*x]) - (Log[(Sec[b*x]*((2 + I*d)*
Cos[a + b*x] - d*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*Sec[b*x]^2)/(-I
+ Tan[b*x]) + (Log[1 - ((Cos[a] + I*Sin[a])*(d*Cos[a] + (2 + I*d)*Sin[a])*(
-I + Tan[b*x]))/2]*Sec[b*x]^2)/(-I + Tan[b*x]) - Log[1 - (Sec[b*x]*(d*Cos[a
] + (2 + I*d)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*(-I + d))]*(-I +
Tan[b*x]) + (Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-2*I + d)*Cos[a + b*x] + I
*d*Sin[a + b*x]))/(2*(-I + d))]*Sec[b*x]^2)/(I + Tan[b*x]) + (2*I)*b*x*(I +
 Tan[b*x]))*(-I + Tan[a + b*x]))
```

Maple [B] time = 0.178, size = 297, normalized size = 3.2

$$\frac{\frac{i}{2}\operatorname{arccoth}\left(1+id-d\tan\left(bx+a\right)\right)\ln\left(id+d\tan\left(bx+a\right)\right)}{b} - \frac{\frac{i}{2}\operatorname{arccoth}\left(1+id-d\tan\left(bx+a\right)\right)\ln\left(id-d\tan\left(bx+a\right)\right)}{b} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1+I*d-d*tan(b*x+a)),x)
```

```
[Out] 1/2*I/b*arccoth(1+I*d-d*tan(b*x+a))*ln(I*d+d*tan(b*x+a))-1/2*I/b*arccoth(1+
I*d-d*tan(b*x+a))*ln(I*d-d*tan(b*x+a))-1/8*I/b*ln(I*d-d*tan(b*x+a))^2+1/4*I
/b*dilog(1+1/2*I*d-1/2*d*tan(b*x+a))+1/4*I/b*ln(I*d-d*tan(b*x+a))*ln(1+1/2*
I*d-1/2*d*tan(b*x+a))+1/4*I/b*dilog(1/2*I*(-I*d+d*tan(b*x+a))/d)+1/4*I/b*ln
(I*d+d*tan(b*x+a))*ln(1/2*I*(-I*d+d*tan(b*x+a))/d)-1/4*I/b*dilog((-2-I*d+d*
tan(b*x+a))/(-2*I*d-2))-1/4*I/b*ln(I*d+d*tan(b*x+a))*ln((-2-I*d+d*tan(b*x+a
))/(-2*I*d-2))
```

Maxima [B] time = 1.59644, size = 355, normalized size = 3.78

$$-\frac{4\,(bx+a)d\left(\frac{\log(d\tan(bx+a)-i\,d-2)}{d}-\frac{\log(\tan(bx+a)-i)}{d}\right)+d\left(-\frac{2i\left(\log(d\tan(bx+a)-i\,d-2)\log\left(-\frac{i\,d\tan(bx+a)+d-2i}{2\,d-2i}+1\right)+\mathrm{Li}_2\left(\frac{i\,d\tan(bx+a)+d-2i}{2\,d-2i}\right)\right)}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*(b*x + a)*d*(log(d*tan(b*x + a) - I*d - 2)/d - log(tan(b*x + a) - I
)/d) + d*(-2*I*(log(d*tan(b*x + a) - I*d - 2)*log(-(I*d*tan(b*x + a) + d -
2*I)/(2*d - 2*I) + 1) + dilog((I*d*tan(b*x + a) + d - 2*I)/(2*d - 2*I)))/d
+ (2*I*log(d*tan(b*x + a) - I*d - 2)*log(tan(b*x + a) - I) - I*log(tan(b*x
+ a) - I)^2)/d - 2*I*(log(-1/2*d*tan(b*x + a) + 1/2*I*d + 1)*log(tan(b*x +
a) - I) + dilog(1/2*d*tan(b*x + a) - 1/2*I*d))/d + 2*I*(log(tan(b*x + a) -
I)*log(-1/2*I*tan(b*x + a) + 1/2) + dilog(1/2*I*tan(b*x + a) + 1/2))/d) + 8
*(b*x + a)*arccoth(d*tan(b*x + a) - I*d - 1))/b
```

Fricas [B] time = 1.66666, size = 610, normalized size = 6.49

$$\frac{i\,b^2x^2-bx\log\left(\frac{de^{(2i\,bx+2i\,a)}}{(d-i)e^{(2i\,bx+2i\,a)}-i}\right)-i\,a^2-(bx+a)\log\left(\frac{1}{2}\sqrt{-4i\,d-4}e^{(i\,bx+i\,a)}+1\right)-(bx+a)\log\left(-\frac{1}{2}\sqrt{-4i\,d-4}e^{(i\,bx+i\,a)}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*b^2*x^2 - b*x*log(d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*x + 2*I*a)
 - I)) - I*a^2 - (b*x + a)*log(1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1) -
(b*x + a)*log(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1) + a*log(((2*d - 2*
I)*e^(I*b*x + I*a) + I*sqrt(-4*I*d - 4))/(2*d - 2*I)) + a*log(((2*d - 2*I)*
e^(I*b*x + I*a) - I*sqrt(-4*I*d - 4))/(2*d - 2*I)) + I*dilog(1/2*sqrt(-4*I*
d - 4)*e^(I*b*x + I*a)) + I*dilog(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1+I*d-d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arccoth}\left(-d\tan\left(bx+a\right)+i\,d+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(-d*tan(b*x + a) + I*d + 1), x)
```

3.247 $\int \frac{\coth^{-1}(1+id-d\tan(a+bx))}{x}\,dx$

Optimal. Leaf size=23

$$\text{CannotIntegrate}\left(\frac{\coth^{-1}(d(-\tan(a+bx))+id+1)}{x},x\right)$$

```
[Out] CannotIntegrate[ArcCoth[1 + I*d - d*Tan[a + b*x]]/x, x]
```

Rubi [A] time = 0.0846946, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth^{-1}(1+id-d\tan(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCoth[1 + I*d - d*Tan[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcCoth[1 + I*d - d*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\coth^{-1}(1+id-d\tan(a+bx))}{x}\,dx=\int \frac{\coth^{-1}(1+id-d\tan(a+bx))}{x}\,dx$$

Mathematica [A] time = 0.720247, size = 0, normalized size = 0.

$$\int \frac{\coth^{-1}(1+id-d\tan(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCoth[1 + I*d - d*Tan[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcCoth[1 + I*d - d*Tan[a + b*x]]/x, x]
```

Maple [A] time = 0.44, size = 0, normalized size = 0.

$$\int \frac{\mathrm{arccoth}\left(1+id-d\tan\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1+I*d-d*tan(b*x+a))/x,x)
```

```
[Out] int(arccoth(1+I*d-d*tan(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-i\,bx + \frac{1}{4}\,(-i\,\pi - 4i\,a - 2\,\log(d))\log(x) + \frac{1}{2}i\int \frac{\arctan\left(d\cos(2\,bx + 2\,a) + \sin(2\,bx + 2\,a), -d\sin(2\,bx + 2\,a) + \cos(2\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+I*d-d*tan(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -I*b*x + 1/4*(-I*pi - 4*I*a - 2*log(d))*log(x) + 1/2*I*integrate(arctan2(d*
cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + cos(2*b*x + 2*a)
 + 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*si
n(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\log\left(\frac{de^{(2i\,bx+2i\,a)}}{(d-i)e^{(2i\,bx+2i\,a)}-i}\right)}{2\,x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+I*d-d*tan(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(-1/2*log(d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*x + 2*I*a) - I))/
x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1+I*d-d*tan(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(-d\tan(bx + a) + i\,d + 1)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+I*d-d*tan(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(-d*tan(b*x + a) + I*d + 1)/x, x)
```

3.248 $\int (e+fx)^3 \coth^{-1}(\cot(a+bx))\,dx$

Optimal. Leaf size=302

$$\frac{3if^2(e+fx)\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3}-\frac{3if^2(e+fx)\text{PolyLog}\left(4,ie^{2i(a+bx)}\right)}{8b^3}+\frac{3f(e+fx)^2\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{8b^2}-\frac{3}{}$$

```
[Out] ((e + f*x)^4*ArcCoth[Cot[a + b*x]])/(4*f) + ((I/4)*(e + f*x)^4*ArcTan[E^((2
*I)*(a + b*x))])/f - ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (3*f*(e +
f*x)^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (3*f*(e + f*x)^2*Pol
yLog[3, I*E^((2*I)*(a + b*x))])/(8*b^2) + (((3*I)/8)*f^2*(e + f*x)*PolyLog[
4, (-I)*E^((2*I)*(a + b*x))])/b^3 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E
^((2*I)*(a + b*x))])/b^3 - (3*f^3*PolyLog[5, (-I)*E^((2*I)*(a + b*x))])/(16
*b^4) + (3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```

Rubi [A] time = 0.235077, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6254, 4181, 2531, 6609, 2282, 6589}

$$\frac{3if^2(e+fx)\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3}-\frac{3if^2(e+fx)\text{PolyLog}\left(4,ie^{2i(a+bx)}\right)}{8b^3}+\frac{3f(e+fx)^2\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{8b^2}-\frac{3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*ArcCoth[Cot[a + b*x]],x]
```


```
[Out] ((e + f*x)^4*ArcCoth[Cot[a + b*x]])/(4*f) + ((I/4)*(e + f*x)^4*ArcTan[E^((2
*I)*(a + b*x))])/f - ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (3*f*(e +
f*x)^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (3*f*(e + f*x)^2*Pol
yLog[3, I*E^((2*I)*(a + b*x))])/(8*b^2) + (((3*I)/8)*f^2*(e + f*x)*PolyLog[
4, (-I)*E^((2*I)*(a + b*x))])/b^3 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E
^((2*I)*(a + b*x))])/b^3 - (3*f^3*PolyLog[5, (-I)*E^((2*I)*(a + b*x))])/(16
*b^4) + (3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```


Rule 6254

```
Int[ArcCoth[Cot[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcCoth[Cot[a + b*x]])/(f*(m + 1)), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
 e, f}, x] && IGtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (e+fx)^3\coth^{-1}(\cot(a+bx))\,dx = \frac{(e+fx)^4\coth^{-1}(\cot(a+bx))}{4f} - \frac{b\int (e+fx)^4\sec(2a+2bx)\,dx}{4f}$$

$$= \frac{(e+fx)^4\coth^{-1}(\cot(a+bx))}{4f} + \frac{i(e+fx)^4\tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{1}{2}\int (e+fx)^3\log\left(1\right.$$

$$= \frac{(e+fx)^4\coth^{-1}(\cot(a+bx))}{4f} + \frac{i(e+fx)^4\tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} - \frac{i(e+fx)^3\text{Li}_2\left(-ie^{2i(\right.}}{4b}$$

$$= \frac{(e+fx)^4\coth^{-1}(\cot(a+bx))}{4f} + \frac{i(e+fx)^4\tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} - \frac{i(e+fx)^3\text{Li}_2\left(-ie^{2i(\right.}}{4b}$$

$$= \frac{(e+fx)^4\coth^{-1}(\cot(a+bx))}{4f} + \frac{i(e+fx)^4\tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} - \frac{i(e+fx)^3\text{Li}_2\left(-ie^{2i(\right.}}{4b}$$

$$= \frac{(e+fx)^4\coth^{-1}(\cot(a+bx))}{4f} + \frac{i(e+fx)^4\tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} - \frac{i(e+fx)^3\text{Li}_2\left(-ie^{2i(\right.}}{4b}$$

$$= \frac{(e+fx)^4\coth^{-1}(\cot(a+bx))}{4f} + \frac{i(e+fx)^4\tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} - \frac{i(e+fx)^3\text{Li}_2\left(-ie^{2i(\right.}}{4b}$$

Mathematica [B] time = 0.286991, size = 654, normalized size = 2.17

$$\frac{1}{4}x\left(6e^2fx+4e^3+4ef^2x^2+f^3x^3\right)\coth^{-1}(\cot(a+bx)) + \frac{6b^2e^2f\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)-6b^2e^2f\text{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*ArcCoth[Cot[a + b*x]],x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcCoth[Cot[a + b*x]])/4 + (
-8*b^4*e^3*x*Log[1 - I*E^((2*I)*(a + b*x))] - 12*b^4*e^2*f*x^2*Log[1 - I*E^
```

```
((2*I)*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] - 2*b^4
*f^3*x^4*Log[1 - I*E^((2*I)*(a + b*x))] + 8*b^4*e^3*x*Log[1 + I*E^((2*I)*(a
 + b*x))] + 12*b^4*e^2*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 8*b^4*e*f^2*x
^3*Log[1 + I*E^((2*I)*(a + b*x))] + 2*b^4*f^3*x^4*Log[1 + I*E^((2*I)*(a + b
*x))] - (4*I)*b^3*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))] + (4*I)*
b^3*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))] + 6*b^2*e^2*f*PolyLog[3,
(-I)*E^((2*I)*(a + b*x))] + 12*b^2*e*f^2*x*PolyLog[3, (-I)*E^((2*I)*(a + b*
x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] - 6*b^2*e^2*f*Pol
yLog[3, I*E^((2*I)*(a + b*x))] - 12*b^2*e*f^2*x*PolyLog[3, I*E^((2*I)*(a +
b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^((2*I)*(a + b*x))] + (6*I)*b*e*f^2*Po
lyLog[4, (-I)*E^((2*I)*(a + b*x))] + (6*I)*b*f^3*x*PolyLog[4, (-I)*E^((2*I)
*(a + b*x))] - (6*I)*b*e*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))] - (6*I)*b*f^
3*x*PolyLog[4, I*E^((2*I)*(a + b*x))] - 3*f^3*PolyLog[5, (-I)*E^((2*I)*(a +
 b*x))] + 3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```

Maple [C] time = 6.052, size = 7429, normalized size = 24.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*arccoth(cot(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{16}\left(f^3x^4+4\,ef^2x^3+6\,e^2fx^2+4\,e^3x\right)\log\left(2\,\cos\left(2\,bx+2\,a\right)^2+2\,\sin\left(2\,bx+2\,a\right)^2+4\,\sin\left(2\,bx+2\,a\right)+2\right)-\frac{1}{16}\left(f^3x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arccoth(cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/16*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*log(2*cos(2*b*x + 2*a)
^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/16*(f^3*x^4 + 4*e*f
^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*
a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(1/2*((b*f^3*x^4 + 4*b*e*f^2*x^3
+ 6*b*e^2*f*x^2 + 4*b*e^3*x)*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f^3*x^4
 + 4*b*e*f^2*x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*sin(4*b*x + 4*a)*sin(2*b*x +
2*a) + (b*f^3*x^4 + 4*b*e*f^2*x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*cos(2*b*x +
2*a))/(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x
)
```

Fricas [C] time = 2.68441, size = 3677, normalized size = 12.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arccoth(cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/32*(3*f^3*polylog(5, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - 3*f^3*poly
log(5, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + 3*f^3*polylog(5, -I*cos(2*b
*x + 2*a) + sin(2*b*x + 2*a)) - 3*f^3*polylog(5, -I*cos(2*b*x + 2*a) - sin(
2*b*x + 2*a)) - (4*I*b^3*f^3*x^3 + 12*I*b^3*e*f^2*x^2 + 12*I*b^3*e^2*f*x +
4*I*b^3*e^3)*dilog(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - (4*I*b^3*f^3*x^
3 + 12*I*b^3*e*f^2*x^2 + 12*I*b^3*e^2*f*x + 4*I*b^3*e^3)*dilog(I*cos(2*b*x
+ 2*a) - sin(2*b*x + 2*a)) - (-4*I*b^3*f^3*x^3 - 12*I*b^3*e*f^2*x^2 - 12*I*
b^3*e^2*f*x - 4*I*b^3*e^3)*dilog(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) -
(-4*I*b^3*f^3*x^3 - 12*I*b^3*e*f^2*x^2 - 12*I*b^3*e^2*f*x - 4*I*b^3*e^3)*di
log(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - 4*(b^4*f^3*x^4 + 4*b^4*e*f^2*
x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x)*log((cos(2*b*x + 2*a) + sin(2*b*x + 2*
a) + 1)/(cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1)) - 2*(4*a*b^3*e^3 - 6*a^2
*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(cos(2*b*x + 2*a) + I*sin(2*b*x +
2*a) + I) + 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log
(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I) + 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*
x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3
*b*e*f^2 - a^4*f^3)*log(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1) - 2*(b^4
*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 -
6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(I*cos(2*b*x + 2*a) - sin(2*b
*x + 2*a) + 1) + 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4
*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(-I*co
s(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1) - 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 +
 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*
f^2 - a^4*f^3)*log(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1) - 2*(4*a*b^3
*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(-cos(2*b*x + 2*a) + I
*sin(2*b*x + 2*a) + I) + 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 -
 a^4*f^3)*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I) - (-6*I*b*f^3*x -
 6*I*b*e*f^2)*polylog(4, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - (-6*I*b*f
^3*x - 6*I*b*e*f^2)*polylog(4, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - (6*
I*b*f^3*x + 6*I*b*e*f^2)*polylog(4, -I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))
 - (6*I*b*f^3*x + 6*I*b*e*f^2)*polylog(4, -I*cos(2*b*x + 2*a) - sin(2*b*x +
 2*a)) - 6*(b^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*polylog(3, I*cos(2*b*x
 + 2*a) + sin(2*b*x + 2*a)) + 6*(b^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*p
olylog(3, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - 6*(b^2*f^3*x^2 + 2*b^2*e
*f^2*x + b^2*e^2*f)*polylog(3, -I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + 6*
(b^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*polylog(3, -I*cos(2*b*x + 2*a) -
sin(2*b*x + 2*a)))/b^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e + fx\right)^{3} \operatorname{acoth}\left(\cot\left(a + bx\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*acoth(cot(b*x+a)),x)
```

```
[Out] Integral((e + f*x)**3*acoth(cot(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int {\left(f x + e\right)}^{3} \operatorname{arcoth}\left(\cot\left(b x + a\right)\right)\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arccoth(cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*arccoth(cot(b*x + a)), x)
```

3.249 $\int (e+fx)^2 \coth^{-1}(\cot(a+bx))\,dx$

Optimal. Leaf size=234

$$\frac{f(e+fx)\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{4b^2}-\frac{f(e+fx)\text{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{4b^2}+\frac{if^2\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3}-\frac{if^2\text{PolyLog}\left(4,ie^{2i(a+bx)}\right)}{8b^3}$$

```
[Out] ((e + f*x)^3*ArcCoth[Cot[a + b*x]])/(3*f) + ((I/3)*(e + f*x)^3*ArcTan[E^((2
*I)*(a + b*x))])/f - ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*(e + f*
x)*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(4*b^2) - (f*(e + f*x)*PolyLog[3,
I*E^((2*I)*(a + b*x))])/(4*b^2) + ((I/8)*f^2*PolyLog[4, (-I)*E^((2*I)*(a +
b*x))])/b^3 - ((I/8)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])/b^3
```

Rubi [A] time = 0.170911, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6254, 4181, 2531, 6609, 2282, 6589}

$$\frac{f(e+fx)\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{4b^2}-\frac{f(e+fx)\text{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{4b^2}+\frac{if^2\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3}-\frac{if^2\text{PolyLog}\left(4,ie^{2i(a+bx)}\right)}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*ArcCoth[Cot[a + b*x]],x]
```

```
[Out] ((e + f*x)^3*ArcCoth[Cot[a + b*x]])/(3*f) + ((I/3)*(e + f*x)^3*ArcTan[E^((2
*I)*(a + b*x))])/f - ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*(e + f*
x)*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(4*b^2) - (f*(e + f*x)*PolyLog[3,
I*E^((2*I)*(a + b*x))])/(4*b^2) + ((I/8)*f^2*PolyLog[4, (-I)*E^((2*I)*(a +
b*x))])/b^3 - ((I/8)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])/b^3
```

Rule 6254

```
Int[ArcCoth[Cot[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcCoth[Cot[a + b*x]])/(f*(m + 1)), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
 e, f}, x] && IGtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (e+fx)^2\coth^{-1}(\cot(a+bx))\,dx = \frac{(e+fx)^3\coth^{-1}(\cot(a+bx))}{3f} - \frac{b\int (e+fx)^3\sec(2a+2bx)\,dx}{3f}$$
$$= \frac{(e+fx)^3\coth^{-1}(\cot(a+bx))}{3f} + \frac{i(e+fx)^3\tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{1}{2}\int (e+fx)^2 \log$$
$$= \frac{(e+fx)^3\coth^{-1}(\cot(a+bx))}{3f} + \frac{i(e+fx)^3\tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} - \frac{i(e+fx)^2\text{Li}_2\left(-i\right.}{4b}$$
$$= \frac{(e+fx)^3\coth^{-1}(\cot(a+bx))}{3f} + \frac{i(e+fx)^3\tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} - \frac{i(e+fx)^2\text{Li}_2\left(-i\right.}{4b}$$
$$= \frac{(e+fx)^3\coth^{-1}(\cot(a+bx))}{3f} + \frac{i(e+fx)^3\tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} - \frac{i(e+fx)^2\text{Li}_2\left(-i\right.}{4b}$$
$$= \frac{(e+fx)^3\coth^{-1}(\cot(a+bx))}{3f} + \frac{i(e+fx)^3\tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} - \frac{i(e+fx)^2\text{Li}_2\left(-i\right.}{4b}$$

Mathematica [A] time = 0.174746, size = 409, normalized size = 1.75

$$\frac{1}{3}x\left(3e^2+3efx+f^2x^2\right)\coth^{-1}(\cot(a+bx)) + \frac{-6ib^2(e+fx)^2\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)+6ib^2(e+fx)^2\text{PolyLog}\left(2,ie^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*ArcCoth[Cot[a + b*x]],x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcCoth[Cot[a + b*x]])/3 + (-12*b^3*e^2*x*Lo
g[1 - I*E^((2*I)*(a + b*x))] - 12*b^3*e*f*x^2*Log[1 - I*E^((2*I)*(a + b*x))
] - 4*b^3*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] + 12*b^3*e^2*x*Log[1 + I*E
^((2*I)*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 4*b^3
*f^2*x^3*Log[1 + I*E^((2*I)*(a + b*x))] - (6*I)*b^2*(e + f*x)^2*PolyLog[2,
(-I)*E^((2*I)*(a + b*x))] + (6*I)*b^2*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a
+ b*x))] + 6*b*e*f*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] + 6*b*f^2*x*PolyLog
[3, (-I)*E^((2*I)*(a + b*x))] - 6*b*e*f*PolyLog[3, I*E^((2*I)*(a + b*x))] -
 6*b*f^2*x*PolyLog[3, I*E^((2*I)*(a + b*x))] + (3*I)*f^2*PolyLog[4, (-I)*E^
```

```
((2*I)*(a + b*x))] - (3*I)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])/(24*b^3)
```

Maple [C] time = 13.674, size = 5543, normalized size = 23.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*arccoth(cot(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{12}\left(f^2x^3 + 3\,efx^2 + 3\,e^2x\right)\log\left(2\,\cos\left(2\,bx + 2\,a\right)^2 + 2\,\sin\left(2\,bx + 2\,a\right)^2 + 4\,\sin\left(2\,bx + 2\,a\right) + 2\right) - \frac{1}{12}\left(f^2x^3 + 3\,efx^2 + 3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arccoth(cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x
 + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*
log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) -
 integrate(2/3*((b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(4*b*x + 4*a)*cos(
2*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*sin(4*b*x + 4*a)*sin(2
*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(2*b*x + 2*a))/(cos(
4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

Fricas [C] time = 2.43799, size = 2646, normalized size = 11.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arccoth(cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/48*(-3*I*f^2*polylog(4, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - 3*I*f^2*
polylog(4, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + 3*I*f^2*polylog(4, -I*c
os(2*b*x + 2*a) + sin(2*b*x + 2*a)) + 3*I*f^2*polylog(4, -I*cos(2*b*x + 2*a
) - sin(2*b*x + 2*a)) + (6*I*b^2*f^2*x^2 + 12*I*b^2*e*f*x + 6*I*b^2*e^2)*di
log(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + (6*I*b^2*f^2*x^2 + 12*I*b^2*e*
f*x + 6*I*b^2*e^2)*dilog(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + (-6*I*b^2
*f^2*x^2 - 12*I*b^2*e*f*x - 6*I*b^2*e^2)*dilog(-I*cos(2*b*x + 2*a) + sin(2*
b*x + 2*a)) + (-6*I*b^2*f^2*x^2 - 12*I*b^2*e*f*x - 6*I*b^2*e^2)*dilog(-I*co
s(2*b*x + 2*a) - sin(2*b*x + 2*a)) + 8*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3
*e^2*x)*log((cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1)/(cos(2*b*x + 2*a) - s
in(2*b*x + 2*a) + 1)) + 4*(3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(cos(2*b
*x + 2*a) + I*sin(2*b*x + 2*a) + I) - 4*(3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^
2)*log(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I) - 4*(b^3*f^2*x^3 + 3*b^3*
e*f*x^2 + 3*b^3*e^2*x + 3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(I*cos(2*b*
```

```
x + 2*a) + sin(2*b*x + 2*a) + 1) + 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e
^2*x + 3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(I*cos(2*b*x + 2*a) - sin(2*
b*x + 2*a) + 1) - 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x + 3*a*b^2*e^
2 - 3*a^2*b*e*f + a^3*f^2)*log(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1)
+ 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x + 3*a*b^2*e^2 - 3*a^2*b*e*f
+ a^3*f^2)*log(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1) + 4*(3*a*b^2*e^2
 - 3*a^2*b*e*f + a^3*f^2)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + I) -
 4*(3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(-cos(2*b*x + 2*a) - I*sin(2*b*
x + 2*a) + I) + 6*(b*f^2*x + b*e*f)*polylog(3, I*cos(2*b*x + 2*a) + sin(2*b
*x + 2*a)) - 6*(b*f^2*x + b*e*f)*polylog(3, I*cos(2*b*x + 2*a) - sin(2*b*x
+ 2*a)) + 6*(b*f^2*x + b*e*f)*polylog(3, -I*cos(2*b*x + 2*a) + sin(2*b*x +
2*a)) - 6*(b*f^2*x + b*e*f)*polylog(3, -I*cos(2*b*x + 2*a) - sin(2*b*x + 2*
a)))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e + fx\right)^{2} \operatorname{acoth}\left(\cot\left(a + bx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*acoth(cot(b*x+a)),x)
```

```
[Out] Integral((e + f*x)**2*acoth(cot(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx + e\right)^{2} \operatorname{arcoth}\left(\cot\left(bx + a\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arccoth(cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*arccoth(cot(b*x + a)), x)
```

3.250 $\int (e+fx)\coth^{-1}(\cot(a+bx))\,dx$

Optimal. Leaf size=162

$$\frac{f\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{8b^2}-\frac{f\text{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{8b^2}-\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx)\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b}$$

```
[Out] ((e + f*x)^2*ArcCoth[Cot[a + b*x]])/(2*f) + ((I/2)*(e + f*x)^2*ArcTan[E^((2
*I)*(a + b*x))])/f - ((I/4)*(e + f*x)*PolyLog[2, (-I)*E^((2*I)*(a + b*x))])
/b + ((I/4)*(e + f*x)*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*PolyLog[3,
(-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (f*PolyLog[3, I*E^((2*I)*(a + b*x))])/
(8*b^2)
```

Rubi [A] time = 0.110149, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6254, 4181, 2531, 2282, 6589}

$$\frac{f\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{8b^2}-\frac{f\text{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{8b^2}-\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx)\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*ArcCoth[Cot[a + b*x]],x]
```

```
[Out] ((e + f*x)^2*ArcCoth[Cot[a + b*x]])/(2*f) + ((I/2)*(e + f*x)^2*ArcTan[E^((2
*I)*(a + b*x))])/f - ((I/4)*(e + f*x)*PolyLog[2, (-I)*E^((2*I)*(a + b*x))])
/b + ((I/4)*(e + f*x)*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*PolyLog[3,
(-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (f*PolyLog[3, I*E^((2*I)*(a + b*x))])/
(8*b^2)
```

Rule 6254

```
Int[ArcCoth[Cot[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcCoth[Cot[a + b*x]])/(f*(m + 1)), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
 e, f}, x] && IGtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (e+fx)\coth^{-1}(\cot(a+bx))\,dx = \frac{(e+fx)^2\coth^{-1}(\cot(a+bx))}{2f} - \frac{b\int (e+fx)^2\sec(2a+2bx)\,dx}{2f}$$
$$= \frac{(e+fx)^2\coth^{-1}(\cot(a+bx))}{2f} + \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} + \frac{1}{2}\int (e+fx)\log\left(\right.$$
$$= \frac{(e+fx)^2\coth^{-1}(\cot(a+bx))}{2f} + \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} - \frac{i(e+fx)\mathrm{Li}_2\left(-ie^{2\cdots}\right.}{4b}$$
$$= \frac{(e+fx)^2\coth^{-1}(\cot(a+bx))}{2f} + \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} - \frac{i(e+fx)\mathrm{Li}_2\left(-ie^{2\cdots}\right.}{4b}$$
$$= \frac{(e+fx)^2\coth^{-1}(\cot(a+bx))}{2f} + \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} - \frac{i(e+fx)\mathrm{Li}_2\left(-ie^{2\cdots}\right.}{4b}$$

Mathematica [A] time = 0.120911, size = 263, normalized size = 1.62

$$-be\left(\frac{i\mathrm{PolyLog}\left(2,-ie^{i(2a+2bx)}\right)}{4b^2} - \frac{i\mathrm{PolyLog}\left(2,ie^{i(2a+2bx)}\right)}{4b^2} - \frac{ix\tan^{-1}\left(e^{2ia+2ibx}\right)}{b}\right) + \frac{f\left(2ibx\mathrm{PolyLog}(2,-\sin(2(a+bx\cdots\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)*ArcCoth[Cot[a + b*x]],x]
```

```
[Out] e*x*ArcCoth[Cot[a + b*x]] + (f*x^2*ArcCoth[Cot[a + b*x]])/2 - b*e*(((-I)*x*
ArcTan[E^((2*I)*a + (2*I)*b*x)])/b + ((I/4)*PolyLog[2, (-I)*E^(I*(2*a + 2*b
*x))])/b^2 - ((I/4)*PolyLog[2, I*E^(I*(2*a + 2*b*x))])/b^2) + (f*((4*I)*b^2
*x^2*ArcTan[Cos[2*(a + b*x)] + I*Sin[2*(a + b*x)]] + (2*I)*b*x*PolyLog[2, I
*Cos[2*(a + b*x)] - Sin[2*(a + b*x)]] - (2*I)*b*x*PolyLog[2, (-I)*Cos[2*(a
+ b*x)] + Sin[2*(a + b*x)]] - PolyLog[3, I*Cos[2*(a + b*x)] - Sin[2*(a + b*
x)]] + PolyLog[3, (-I)*Cos[2*(a + b*x)] + Sin[2*(a + b*x)]]))/(8*b^2)
```

Maple [C] time = 10.737, size = 2543, normalized size = 15.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*arccoth(cot(b*x+a)),x)
```

```
[Out] -1/4*ln(exp(2*I*(b*x+a))-I)*x^2*f-1/2*ln(exp(2*I*(b*x+a))-I)*x*e-1/2*I/b^2*
f*a*dilog(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))-1/2*I/b^2*f*a*dilog(((-I)
```

```
^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))+1/2*I/b*e*(I*b*x+I*a)*ln(((-I)^(1/2)-exp
(I*(b*x+a)))/(-I)^(1/2))-1/2*I/b*e*dilog(1+exp(I*(b*x+a))*(-1)^(3/4))-1/2*I
/b*e*dilog(1-exp(I*(b*x+a))*(-1)^(3/4))+1/2*I/b*e*dilog(((-I)^(1/2)-exp(I*(
b*x+a)))/(-I)^(1/2))+1/8*f*polylog(3,-I*exp(2*I*(b*x+a)))/b^2-1/8*f*polylog
(3,I*exp(2*I*(b*x+a)))/b^2+1/4/b^2*f*(I*b*x+I*a)^2*ln(1-I*exp(2*I*(b*x+a)))
+1/4/b^2*f*(I*b*x+I*a)*polylog(2,I*exp(2*I*(b*x+a)))-1/4/b^2*f*(I*b*x+I*a)^
2*ln(1+I*exp(2*I*(b*x+a)))-1/4/b^2*f*(I*b*x+I*a)*polylog(2,-I*exp(2*I*(b*x+
a)))+1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^3*x^2+1/4
*I*Pi*x*e*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^3+1/8*I*Pi*
f*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^3*x^2-1/4*I*Pi*x*e*
csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-
1))^2-1/4*I*Pi*x*e-1/8*I*Pi*f*x^2-1/8*I*Pi*f*csgn(I/(exp(2*I*(b*x+a))-1))*c
sgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2*x^2+1/2*I/b*e*dilog(((-I
)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))+1/4/b^2*f*a^2*ln(-exp(2*I*(b*x+a))+I)-1
/2/b*e*a*ln(-exp(2*I*(b*x+a))+I)+1/2/b*a*e*ln(exp(2*I*(b*x+a))+I)-1/4/b^2*f
*a^2*ln(exp(2*I*(b*x+a))+I)+1/2*I/b*e*(I*b*x+I*a)*ln(((-I)^(1/2)+exp(I*(b*x
+a)))/(-I)^(1/2))-1/2*I/b*e*(I*b*x+I*a)*ln(1+exp(I*(b*x+a))*(-1)^(3/4))-1/2
*I/b*e*(I*b*x+I*a)*ln(1-exp(I*(b*x+a))*(-1)^(3/4))+1/2*I/b^2*f*a*dilog(1+ex
p(I*(b*x+a))*(-1)^(3/4))+(1/4*f*x^2+1/2*e*x)*ln(exp(2*I*(b*x+a))+I)+1/8*I*P
i*f*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((1-I)*(exp(2*I*(
b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2*x^2+1/4*I*Pi*x*e*csgn(I/(exp(2*I*(b*x+a)
)-1))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2+1/4*I*Pi*x*e*csgn
(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^3+1/8*I*Pi*f*csgn(I*(exp(2*I*
(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2*x^2+1/2*I/
b^2*f*a*dilog(1-exp(I*(b*x+a))*(-1)^(3/4))+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*
x+a))-I))*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I
*(b*x+a))-1))+1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))-I))*csgn(I/(exp(2*I*(b*x+
a))-1))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))*x^2-1/2*I/b^2*f*a
*(I*b*x+I*a)*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))-1/2*I/b^2*f*a*(I*b*
x+I*a)*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))-1/4*I*Pi*x*e*csgn(I*(exp(
2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2-1/8*I*
Pi*f*csgn(I*(exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x
+a))-1))^2*x^2+1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))
*csgn((1+I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))*x^2+1/4*I*Pi*x*e*csg
n(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((1-I)*(exp(2*I*(b*x+a))
+I)/(exp(2*I*(b*x+a))-1))^2+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I))*csgn(
I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2+1/8*I*Pi*f*csgn(I/(exp(2*I*(
b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2*x^2+1/2*I/b
^2*f*a*(I*b*x+I*a)*ln(1+exp(I*(b*x+a))*(-1)^(3/4))+1/2*I/b^2*f*a*(I*b*x+I*a
)*ln(1-exp(I*(b*x+a))*(-1)^(3/4))-1/4*I*Pi*x*e*csgn((1-I)*(exp(2*I*(b*x+a))
+I)/(exp(2*I*(b*x+a))-1))^2-1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I
*(b*x+a))-1))^3*x^2-1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+
a))-1))^3+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))*cs
gn((1+I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))+1/4*I*Pi*x*e*csgn((1+I)
*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2+1/8*I*Pi*f*csgn((1+I)*(exp(2*
I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2*x^2-1/8*I*Pi*f*csgn((1-I)*(exp(2*I*(b
*x+a))+I)/(exp(2*I*(b*x+a))-1))^2*x^2-1/8*I*Pi*f*csgn((1+I)*(exp(2*I*(b*x+a
))-I)/(exp(2*I*(b*x+a))-1))^3*x^2-1/4*I*Pi*x*e*csgn((1+I)*(exp(2*I*(b*x+a))
-I)/(exp(2*I*(b*x+a))-1))^3-1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))+I))*csgn(I/
(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*x^2
-1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))*csgn((1+I)*
(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2-1/4*I*Pi*x*e*csgn(I*(exp(2*I*(
b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(
b*x+a))-1))-1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I))*csgn(I/(exp(2*I*(b*x+
a))-1))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))-1/8*I*Pi*f*csgn(I
*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))*csgn((1+I)*(exp(2*I*(b*x+a))-I)
/(exp(2*I*(b*x+a))-1))^2*x^2-1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*
I*(b*x+a))-1))*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{8}\left(fx^2+2ex\right)\log\left(2\cos\left(2bx+2a\right)^2+2\sin\left(2bx+2a\right)^2+4\sin\left(2bx+2a\right)+2\right)-\frac{1}{8}\left(fx^2+2ex\right)\log\left(2\cos\left(2bx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arccoth(cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/8*(f*x^2 + 2*e*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin
(2*b*x + 2*a) + 2) - 1/8*(f*x^2 + 2*e*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2
*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(((b*f*x^2 + 2*b*e*x)*co
s(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f*x^2 + 2*b*e*x)*sin(4*b*x + 4*a)*sin(
2*b*x + 2*a) + (b*f*x^2 + 2*b*e*x)*cos(2*b*x + 2*a))/(cos(4*b*x + 4*a)^2 +
sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

Fricas [C] time = 2.13758, size = 1724, normalized size = 10.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arccoth(cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/16*((2*I*b*f*x + 2*I*b*e)*dilog(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) +
(2*I*b*f*x + 2*I*b*e)*dilog(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + (-2*I*
b*f*x - 2*I*b*e)*dilog(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + (-2*I*b*f*
x - 2*I*b*e)*dilog(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + 4*(b^2*f*x^2 +
 2*b^2*e*x)*log((cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1)/(cos(2*b*x + 2*a)
 - sin(2*b*x + 2*a) + 1)) + 2*(2*a*b*e - a^2*f)*log(cos(2*b*x + 2*a) + I*si
n(2*b*x + 2*a) + I) - 2*(2*a*b*e - a^2*f)*log(cos(2*b*x + 2*a) - I*sin(2*b*
x + 2*a) + I) - 2*(b^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a^2*f)*log(I*cos(2*b*x
 + 2*a) + sin(2*b*x + 2*a) + 1) + 2*(b^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a^2*
f)*log(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1) - 2*(b^2*f*x^2 + 2*b^2*e*
x + 2*a*b*e - a^2*f)*log(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1) + 2*(b
^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a^2*f)*log(-I*cos(2*b*x + 2*a) - sin(2*b*x
 + 2*a) + 1) + 2*(2*a*b*e - a^2*f)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*
a) + I) - 2*(2*a*b*e - a^2*f)*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) +
I) + f*polylog(3, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - f*polylog(3, I*c
os(2*b*x + 2*a) - sin(2*b*x + 2*a)) + f*polylog(3, -I*cos(2*b*x + 2*a) + si
n(2*b*x + 2*a)) - f*polylog(3, -I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(e+fx\right)\operatorname{acoth}\left(\cot\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*acoth(cot(b*x+a)),x)
```

```
[Out] Integral((e + f*x)*acoth(cot(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx + e) \operatorname{arcoth}(\cot(bx + a))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arccoth(cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*arccoth(cot(b*x + a)), x)
```

3.251 $\int \coth^{-1}(\cot(a+bx))\,dx$

Optimal. Leaf size=79

$$-\frac{i\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b}+ix\tan^{-1}\left(e^{2i(a+bx)}\right)+x\coth^{-1}(\cot(a+bx))$$

```
[Out] x*ArcCoth[Cot[a + b*x]] + I*x*ArcTan[E^((2*I)*(a + b*x))] - ((I/4)*PolyLog[
2, (-I)*E^((2*I)*(a + b*x))])/b + ((I/4)*PolyLog[2, I*E^((2*I)*(a + b*x))])
/b
```

Rubi [A] time = 0.048343, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6250, 4181, 2279, 2391}

$$-\frac{i\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b}+ix\tan^{-1}\left(e^{2i(a+bx)}\right)+x\coth^{-1}(\cot(a+bx))$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Cot[a + b*x]],x]
```

```
[Out] x*ArcCoth[Cot[a + b*x]] + I*x*ArcTan[E^((2*I)*(a + b*x))] - ((I/4)*PolyLog[
2, (-I)*E^((2*I)*(a + b*x))])/b + ((I/4)*PolyLog[2, I*E^((2*I)*(a + b*x))])
/b
```

Rule 6250

```
Int[ArcCoth[Cot[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcCoth[Cot[a + b
*x]], x] - Dist[b, Int[x*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \coth^{-1}(\cot(a+bx))\,dx = x\coth^{-1}(\cot(a+bx)) - b\int x\sec(2a+2bx)\,dx$$
$$= x\coth^{-1}(\cot(a+bx)) + ix\tan^{-1}\left(e^{2i(a+bx)}\right) + \frac{1}{2}\int \log\left(1-ie^{i(2a+2bx)}\right)dx - \frac{1}{2}\int \log\left(1+ie^{i(\ldots}\right.$$
$$= x\coth^{-1}(\cot(a+bx)) + ix\tan^{-1}\left(e^{2i(a+bx)}\right) - \frac{i\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}dx, x, e^{i(2a+2bx)}\right)}{4b} + \frac{i\,\text{Subst}\ldots}{}$$
$$= x\coth^{-1}(\cot(a+bx)) + ix\tan^{-1}\left(e^{2i(a+bx)}\right) - \frac{i\text{Li}_2\left(-ie^{2i(a+bx)}\right)}{4b} + \frac{i\text{Li}_2\left(ie^{2i(a+bx)}\right)}{4b}$$

Mathematica [A] time = 0.0317604, size = 78, normalized size = 0.99

$$\frac{-i\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right) + i\text{PolyLog}\left(2,ie^{2i(a+bx)}\right) + 4bx\left(\coth^{-1}(\cot(a+bx)) + i\tan^{-1}\left(e^{2i(a+bx)}\right)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Cot[a + b*x]],x]
```

```
[Out] (4*b*x*(ArcCoth[Cot[a + b*x]] + I*ArcTan[E^((2*I)*(a + b*x))]) - I*PolyLog[
2, (-I)*E^((2*I)*(a + b*x))] + I*PolyLog[2, I*E^((2*I)*(a + b*x))])/(4*b)
```

Maple [B] time = 0.161, size = 265, normalized size = 3.4

$$-\frac{\text{arccoth}\left(\cot\left(bx+a\right)\right)\pi}{2b} + \frac{\text{arccoth}\left(\cot\left(bx+a\right)\right)\text{arccot}\left(\cot\left(bx+a\right)\right)}{b} - \frac{\pi}{4b}\ln\left(1+\frac{i\left(1+i\cot\left(bx+a\right)\right)^{2}}{\left(\cot\left(bx+a\right)\right)^{2}+1}\right) + \text{arccot}(\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(cot(b*x+a)),x)
```

```
[Out] -1/2/b*arccoth(cot(b*x+a))*Pi+1/b*arccoth(cot(b*x+a))*arccot(cot(b*x+a))-1/
4/b*ln(1+I*(1+I*cot(b*x+a))^2/(cot(b*x+a)^2+1))*Pi+1/2/b*ln(1+I*(1+I*cot(b*
x+a))^2/(cot(b*x+a)^2+1))*arccot(cot(b*x+a))+1/4*I/b*polylog(2,-I*(1+I*cot(
b*x+a))^2/(cot(b*x+a)^2+1))+1/4/b*ln(1-I*(1+I*cot(b*x+a))^2/(cot(b*x+a)^2+1
))*Pi-1/2/b*ln(1-I*(1+I*cot(b*x+a))^2/(cot(b*x+a)^2+1))*arccot(cot(b*x+a))-
1/4*I/b*polylog(2,I*(1+I*cot(b*x+a))^2/(cot(b*x+a)^2+1))
```

Maxima [B] time = 1.68979, size = 248, normalized size = 3.14

$$\frac{4\left(bx+a\right)\text{arcoth}\left(\frac{1}{\tan(bx+a)}\right) + \left(\arctan\left(\frac{1}{2}\tan\left(bx+a\right)+\frac{1}{2}, \frac{1}{2}\tan\left(bx+a\right)+\frac{1}{2}\right) - \arctan\left(\frac{1}{2}\tan\left(bx+a\right)-\frac{1}{2}, -\frac{1}{2}\tan(\ldots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(b*x + a)*arccoth(1/tan(b*x + a)) + (arctan2(1/2*tan(b*x + a) + 1/2,
 1/2*tan(b*x + a) + 1/2) - arctan2(1/2*tan(b*x + a) - 1/2, -1/2*tan(b*x + a
) + 1/2))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log(1/2*tan(b*x + a)^2 + tan(
```

```
b*x + a) + 1/2) + (b*x + a)*log(1/2*tan(b*x + a)^2 - tan(b*x + a) + 1/2) -
I*dilog((1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2) + I*dilog(-(1/2*I - 1/2)*
tan(b*x + a) + 1/2*I + 1/2) + I*dilog((1/2*I - 1/2)*tan(b*x + a) + 1/2*I +
1/2) - I*dilog(-(1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2))/b
```

Fricas [B] time = 1.83244, size = 1029, normalized size = 13.03

$$4\,bx\log\left(\frac{\cos(2\,bx+2\,a)+\sin(2\,bx+2\,a)+1}{\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a)+1}\right)+2\,a\log\left(\cos\left(2\,bx+2\,a\right)+i\,\sin\left(2\,bx+2\,a\right)+i\right)-2\,a\log\left(\cos\left(2\,bx+2\,a\right)-i\,\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*log((cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1)/(cos(2*b*x + 2*a)
- sin(2*b*x + 2*a) + 1)) + 2*a*log(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) +
I) - 2*a*log(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I) - 2*(b*x + a)*log(I
*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1) + 2*(b*x + a)*log(I*cos(2*b*x + 2
*a) - sin(2*b*x + 2*a) + 1) - 2*(b*x + a)*log(-I*cos(2*b*x + 2*a) + sin(2*b
*x + 2*a) + 1) + 2*(b*x + a)*log(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1
) + 2*a*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + I) - 2*a*log(-cos(2*b*
x + 2*a) - I*sin(2*b*x + 2*a) + I) + I*dilog(I*cos(2*b*x + 2*a) + sin(2*b*x
 + 2*a)) + I*dilog(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - I*dilog(-I*cos(
2*b*x + 2*a) + sin(2*b*x + 2*a)) - I*dilog(-I*cos(2*b*x + 2*a) - sin(2*b*x
+ 2*a)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acoth}\left(\cot\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(cot(b*x+a)),x)
```

```
[Out] Integral(acoth(cot(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}\left(\cot\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(cot(b*x + a)), x)
```

3.252 $\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx}\,dx$

Optimal. Leaf size=17

$$\text{CannotIntegrate}\left(\frac{\coth^{-1}(\cot(a+bx))}{e+fx},x\right)$$

```
[Out] CannotIntegrate[ArcCoth[Cot[a + b*x]]/(e + f*x), x]
```

Rubi [A] time = 0.0417984, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCoth[Cot[a + b*x]]/(e + f*x),x]
```

```
[Out] Defer[Int][ArcCoth[Cot[a + b*x]]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx}\,dx = \int \frac{\coth^{-1}(\cot(a+bx))}{e+fx}\,dx$$

Mathematica [A] time = 0.107466, size = 0, normalized size = 0.

$$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCoth[Cot[a + b*x]]/(e + f*x),x]
```

```
[Out] Integrate[ArcCoth[Cot[a + b*x]]/(e + f*x), x]
```

Maple [A] time = 1.173, size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(\cot(bx+a))}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(cot(b*x+a))/(f*x+e),x)
```

```
[Out] int(arccoth(cot(b*x+a))/(f*x+e),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(\cot(bx+a))}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(cot(b*x+a))/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(arccoth(cot(b*x + a))/(f*x + e), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arcoth}(\cot(bx+a))}{fx+e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(cot(b*x+a))/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral(arccoth(cot(b*x + a))/(f*x + e), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}(\cot(a+bx))}{e+fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(cot(b*x+a))/(f*x+e),x)
```

```
[Out] Integral(acoth(cot(a + b*x))/(e + f*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(\cot(bx+a))}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(cot(b*x+a))/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(arccoth(cot(b*x + a))/(f*x + e), x)
```

3.253 $\int x^2 \coth^{-1}(c + d\cot(a + bx))\,dx$

Optimal. Leaf size=391

$$\frac{x\text{PolyLog}\left(3,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b^2}-\frac{x\text{PolyLog}\left(3,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b^2}+\frac{i\text{PolyLog}\left(4,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{8b^3}-\frac{i\text{PolyLog}\left(4,\frac{(c+id+1)e^{2i}}{c-id+1}\right.}{8b^3}$$

```
[Out] (x^3*ArcCoth[c + d*Cot[a + b*x]])/3 + (x^3*Log[1 - ((1 - c - I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 - c + I*d)])/6 - (x^3*Log[1 - ((1 + c + I*d)*E^((2*I)*a
+ (2*I)*b*x))/(1 + c - I*d)])/6 - ((I/4)*x^2*PolyLog[2, ((1 - c - I*d)*E^((
2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/b + ((I/4)*x^2*PolyLog[2, ((1 + c + I*
d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/b + (x*PolyLog[3, ((1 - c - I*d
)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/(4*b^2) - (x*PolyLog[3, ((1 + c
+ I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/(4*b^2) + ((I/8)*PolyLog[4,
 ((1 - c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/b^3 - ((I/8)*PolyL
og[4, ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/b^3
```

Rubi [A] time = 0.503235, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6270, 2190, 2531, 6609, 2282, 6589}

$$\frac{x\text{PolyLog}\left(3,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b^2}-\frac{x\text{PolyLog}\left(3,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b^2}+\frac{i\text{PolyLog}\left(4,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{8b^3}-\frac{i\text{PolyLog}\left(4,\frac{(c+id+1)e^{2i}}{c-id+1}\right.}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[c + d*Cot[a + b*x]],x]
```


```
[Out] (x^3*ArcCoth[c + d*Cot[a + b*x]])/3 + (x^3*Log[1 - ((1 - c - I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 - c + I*d)])/6 - (x^3*Log[1 - ((1 + c + I*d)*E^((2*I)*a
+ (2*I)*b*x))/(1 + c - I*d)])/6 - ((I/4)*x^2*PolyLog[2, ((1 - c - I*d)*E^((
2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/b + ((I/4)*x^2*PolyLog[2, ((1 + c + I*
d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/b + (x*PolyLog[3, ((1 - c - I*d
)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/(4*b^2) - (x*PolyLog[3, ((1 + c
+ I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/(4*b^2) + ((I/8)*PolyLog[4,
 ((1 - c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/b^3 - ((I/8)*PolyL
og[4, ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/b^3
```


Rule 6270

```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Cot[a + b*x]])/(f*(m
 + 1)), x] + (-Dist[(I*b*(1 - c - I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)
*E^(2*I*a + 2*I*b*x))/(1 - c + I*d - (1 - c - I*d)*E^(2*I*a + 2*I*b*x)), x]
, x] + Dist[(I*b*(1 + c + I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*I*
a + 2*I*b*x))/(1 + c - I*d - (1 + c + I*d)*E^(2*I*a + 2*I*b*x)), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, 1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\coth^{-1}(c+d\cot(a+bx))\,dx=\frac{1}{3}x^3\coth^{-1}(c+d\cot(a+bx))+\frac{1}{3}(b(i(1+c)-d))\int\frac{e^{2ia+2ibx}x^3}{1+c-id+(-1-c-id)e^2\cdots}$$

$$=\frac{1}{3}x^3\coth^{-1}(c+d\cot(a+bx))+\frac{1}{6}x^3\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)-\frac{1}{6}x^3\log\Big(1\cdots$$

$$=\frac{1}{3}x^3\coth^{-1}(c+d\cot(a+bx))+\frac{1}{6}x^3\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)-\frac{1}{6}x^3\log\Big(1\cdots$$

$$=\frac{1}{3}x^3\coth^{-1}(c+d\cot(a+bx))+\frac{1}{6}x^3\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)-\frac{1}{6}x^3\log\Big(1\cdots$$

$$=\frac{1}{3}x^3\coth^{-1}(c+d\cot(a+bx))+\frac{1}{6}x^3\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)-\frac{1}{6}x^3\log\Big(1\cdots$$

$$=\frac{1}{3}x^3\coth^{-1}(c+d\cot(a+bx))+\frac{1}{6}x^3\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)-\frac{1}{6}x^3\log\Big(1\cdots$$

Mathematica [A] time = 0.306333, size = 339, normalized size = 0.87

$$\frac{1}{3}x^3\coth^{-1}(d\cot(a+bx)+c)+\frac{-6ib^2x^2\text{PolyLog}\left(2,\frac{(c+id-1)e^{2i(a+bx)}}{c-id-1}\right)+6ib^2x^2\text{PolyLog}\left(2,\frac{(c+id+1)e^{2i(a+bx)}}{c-id+1}\right)+6bx\text{PolyL}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcCoth[c + d*Cot[a + b*x]])/3 + (4*b^3*x^3*Log[1 - ((-1 + c + I*d)*E^
((2*I)*(a + b*x)))/(-1 + c - I*d)] - 4*b^3*x^3*Log[1 - ((1 + c + I*d)*E^((2
*I)*(a + b*x)))/(1 + c - I*d)] - (6*I)*b^2*x^2*PolyLog[2, ((-1 + c + I*d)*E
^((2*I)*(a + b*x)))/(-1 + c - I*d)] + (6*I)*b^2*x^2*PolyLog[2, ((1 + c + I*
d)*E^((2*I)*(a + b*x)))/(1 + c - I*d)] + 6*b*x*PolyLog[3, ((-1 + c + I*d)*E
^((2*I)*(a + b*x)))/(-1 + c - I*d)] - 6*b*x*PolyLog[3, ((1 + c + I*d)*E^((2
*I)*(a + b*x)))/(1 + c - I*d)] + (3*I)*PolyLog[4, ((-1 + c + I*d)*E^((2*I)*
(a + b*x)))/(-1 + c - I*d)] - (3*I)*PolyLog[4, ((1 + c + I*d)*E^((2*I)*(a +
 b*x)))/(1 + c - I*d)])/(24*b^3)
```

Maple [C] time = 6.855, size = 6662, normalized size = 17.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(c+d*cot(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(c+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*x^3*log((c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*b
*x + 2*a) + (c^2 + d^2 + 2*c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 - 2*(c^2 -
 d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 2*c + 1) - 1/12*x^3*log((c^2 + d^2 - 2*c
 + 1)*cos(2*b*x + 2*a)^2 + 4*(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 - 2*c
+ 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 - 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*x + 2
*a) - 2*c + 1) - 4*b*d*integrate(1/3*(2*(c^2 + d^2 - 1)*x^3*cos(2*b*x + 2*a
)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^3*sin(2*b*x + 2*a)^2
 - (c^2 - d^2 - 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2*b*x + 2*a) + (c^
2 - d^2 - 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^3*cos(2*b*x
+ 2*a) - (c^2 - d^2 - 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 + d^4
 + 2*(c^2 + 1)*d^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*cos(4*b*x +
4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x + 2*a)^2 + (
c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 + d^4
+ 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 + 2*(c^4 + d^4 -
2*(3*c^2 - 1)*d^2 - 2*c^2 - 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*
(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) - 4*(c^4 - d^4
 - 2*c^2 + 1)*cos(2*b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 - c)*d + 2*(c*d^3 + (c
^3 - c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*sin(2*b*x + 2*a))*sin
(4*b*x + 4*a) + 8*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1), x)
```

Fricas [C] time = 2.99284, size = 4748, normalized size = 12.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/48*(8*b^3*x^3*log((d*cos(2*b*x + 2*a) + (c + 1)*sin(2*b*x + 2*a) + d)/(d*
cos(2*b*x + 2*a) + (c - 1)*sin(2*b*x + 2*a) + d)) + 6*I*b^2*x^2*dilog(-(c^2
 + d^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 +
 2*(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 +
2*c + 1) + 1) - 6*I*b^2*x^2*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2
+ 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin
(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1) + 1) - 6*I*b^2*x^2*dilog(-(c
^2 + d^2 - (c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2
 + 2*(c - 1)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2
- 2*c + 1) + 1) + 6*I*b^2*x^2*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^
2 - 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*s
in(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1) + 1) + 4*a^3*log(1/2*c^2 +
 I*(c + 1)*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(
I*c^2 + I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) + c + 1/2) - 4*a^3*log(1/2*c^2
+ I*(c - 1)*d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2*
(I*c^2 + I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) - c + 1/2) + 4*a^3*log(-1/2*c^
2 + I*(c + 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/
2*(I*c^2 + I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) - c - 1/2) - 4*a^3*log(-1/2*
c^2 + I*(c - 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) +
1/2*(I*c^2 + I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) + c - 1/2) - 6*b*x*polylog
(3, ((c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*(c
 + 1)*d - I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*c + 1)) - 6*b
*x*polylog(3, ((c^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I
*c^2 - 2*(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*c
+ 1)) + 6*b*x*polylog(3, ((c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x +
 2*a) + (I*c^2 - 2*(c - 1)*d - I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a))/(c^2 +
d^2 - 2*c + 1)) + 6*b*x*polylog(3, ((c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*c
os(2*b*x + 2*a) + (-I*c^2 - 2*(c - 1)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*
a))/(c^2 + d^2 - 2*c + 1)) - 4*(b^3*x^3 + a^3)*log((c^2 + d^2 - (c^2 + 2*I*
(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c + 1)*d + I*d^2
 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1)) - 4*(b^3*x
^3 + a^3)*log((c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x
+ 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) + 2*c +
 1)/(c^2 + d^2 + 2*c + 1)) + 4*(b^3*x^3 + a^3)*log((c^2 + d^2 - (c^2 + 2*I*
(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c - 1)*d + I*d^2
 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1)) + 4*(b^3*x
^3 + a^3)*log((c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x
+ 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) - 2*c +
 1)/(c^2 + d^2 - 2*c + 1)) - 3*I*polylog(4, ((c^2 + 2*I*(c + 1)*d - d^2 + 2
*c + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin(2*
b*x + 2*a))/(c^2 + d^2 + 2*c + 1)) + 3*I*polylog(4, ((c^2 - 2*I*(c + 1)*d -
 d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*(c + 1)*d + I*d^2 - 2*I*c -
I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*c + 1)) + 3*I*polylog(4, ((c^2 + 2*I*(c
 - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*(c - 1)*d - I*d^2 -
2*I*c + I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*c + 1)) - 3*I*polylog(4, ((c^2
- 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*(c - 1)*d +
 I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*c + 1)))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(c+d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arcoth}(d\cot(bx+a)+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(c+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(d*cot(b*x + a) + c), x)
```

3.254 $\int x \coth^{-1}(c + d\cot(a + bx))\,dx$

Optimal. Leaf size=293

$$\frac{\text{PolyLog}\left(3,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{8b^2}-\frac{\text{PolyLog}\left(3,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{8b^2}-\frac{ix\text{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b}+\frac{ix\text{PolyLog}\left(2,\frac{(c+id+1)}{c-}\right.}{4b}$$

```
[Out] (x^2*ArcCoth[c + d*Cot[a + b*x]])/2 + (x^2*Log[1 - ((1 - c - I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 - c + I*d)])/4 - (x^2*Log[1 - ((1 + c + I*d)*E^((2*I)*a
+ (2*I)*b*x))/(1 + c - I*d)])/4 - ((I/4)*x*PolyLog[2, ((1 - c - I*d)*E^((2*
I)*a + (2*I)*b*x))/(1 - c + I*d)])/b + ((I/4)*x*PolyLog[2, ((1 + c + I*d)*E
^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/b + PolyLog[3, ((1 - c - I*d)*E^((2
*I)*a + (2*I)*b*x))/(1 - c + I*d)]/(8*b^2) - PolyLog[3, ((1 + c + I*d)*E^((
2*I)*a + (2*I)*b*x))/(1 + c - I*d)]/(8*b^2)
```

Rubi [A] time = 0.40347, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6270, 2190, 2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{8b^2}-\frac{\text{PolyLog}\left(3,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{8b^2}-\frac{ix\text{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b}+\frac{ix\text{PolyLog}\left(2,\frac{(c+id+1)}{c-}\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[c + d*Cot[a + b*x]],x]
```


```
[Out] (x^2*ArcCoth[c + d*Cot[a + b*x]])/2 + (x^2*Log[1 - ((1 - c - I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 - c + I*d)])/4 - (x^2*Log[1 - ((1 + c + I*d)*E^((2*I)*a
+ (2*I)*b*x))/(1 + c - I*d)])/4 - ((I/4)*x*PolyLog[2, ((1 - c - I*d)*E^((2*
I)*a + (2*I)*b*x))/(1 - c + I*d)])/b + ((I/4)*x*PolyLog[2, ((1 + c + I*d)*E
^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/b + PolyLog[3, ((1 - c - I*d)*E^((2
*I)*a + (2*I)*b*x))/(1 - c + I*d)]/(8*b^2) - PolyLog[3, ((1 + c + I*d)*E^((
2*I)*a + (2*I)*b*x))/(1 + c - I*d)]/(8*b^2)
```


Rule 6270

```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Cot[a + b*x]])/(f*(m
 + 1)), x] + (-Dist[(I*b*(1 - c - I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)
*E^(2*I*a + 2*I*b*x))/(1 - c + I*d - (1 - c - I*d)*E^(2*I*a + 2*I*b*x)), x]
, x] + Dist[(I*b*(1 + c + I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*I*
a + 2*I*b*x))/(1 + c - I*d - (1 + c + I*d)*E^(2*I*a + 2*I*b*x)), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, 1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\coth^{-1}(c+d\cot(a+bx))\,dx=\frac{1}{2}x^2\coth^{-1}(c+d\cot(a+bx))+\frac{1}{2}(b(i(1+c)-d))\int\frac{e^{2ia+2ibx}x^2}{1+c-id+(-1-c-id)e^{2ia+2i}}$$
$$=\frac{1}{2}x^2\coth^{-1}(c+d\cot(a+bx))+\frac{1}{4}x^2\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)-\frac{1}{4}x^2\log\left(1-\frac{(1}{}\right.$$
$$=\frac{1}{2}x^2\coth^{-1}(c+d\cot(a+bx))+\frac{1}{4}x^2\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)-\frac{1}{4}x^2\log\left(1-\frac{(1}{}\right.$$
$$=\frac{1}{2}x^2\coth^{-1}(c+d\cot(a+bx))+\frac{1}{4}x^2\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)-\frac{1}{4}x^2\log\left(1-\frac{(1}{}\right.$$
$$=\frac{1}{2}x^2\coth^{-1}(c+d\cot(a+bx))+\frac{1}{4}x^2\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)-\frac{1}{4}x^2\log\left(1-\frac{(1}{}\right.$$

Mathematica [A] time = 0.125174, size = 253, normalized size = 0.86

$$\frac{1}{2}x^2\coth^{-1}(d\cot(a+bx)+c)+\frac{-2ibx\text{PolyLog}\left(2,\frac{(c+id-1)e^{2i(a+bx)}}{c-id-1}\right)+2ibx\text{PolyLog}\left(2,\frac{(c+id+1)e^{2i(a+bx)}}{c-id+1}\right)+\text{PolyLog}\left(3,\frac{(c+ia}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcCoth[c + d*Cot[a + b*x]])/2 + (2*b^2*x^2*Log[1 - ((-1 + c + I*d)*E^
((2*I)*(a + b*x)))/(-1 + c - I*d)] - 2*b^2*x^2*Log[1 - ((1 + c + I*d)*E^((2
*I)*(a + b*x)))/(1 + c - I*d)] - (2*I)*b*x*PolyLog[2, ((-1 + c + I*d)*E^((2
*I)*(a + b*x)))/(-1 + c - I*d)] + (2*I)*b*x*PolyLog[2, ((1 + c + I*d)*E^((2
*I)*(a + b*x)))/(1 + c - I*d)] + PolyLog[3, ((-1 + c + I*d)*E^((2*I)*(a + b
*x)))/(-1 + c - I*d)] - PolyLog[3, ((1 + c + I*d)*E^((2*I)*(a + b*x)))/(1 +
 c - I*d)])/(8*b^2)
```

Maple [C] time = 8.394, size = 6312, normalized size = 21.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(c+d*cot(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(c+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -2*b*d*integrate((2*(c^2 + d^2 - 1)*x^2*cos(2*b*x + 2*a)^2 + 2*c*d*x^2*sin(
2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^2*sin(2*b*x + 2*a)^2 - (c^2 - d^2 - 1)*x
^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x + 2*a) + (c^2 - d^2 - 1)*x^2*cos
(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^2*cos(2*b*x + 2*a) - (c^2 - d^2
- 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 + d^4 + 2*(c^2 + 1)*d^2 +
 (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*cos(4*b*x + 4*a)^2 + 4*(c^4 + d^
4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x + 2*a)^2 + (c^4 + d^4 + 2*(c^2 +
 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 -
2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 + 2*(c^4 + d^4 - 2*(3*c^2 - 1)*d^2 -
2*c^2 - 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(c*d^3 + (c^3 - c)*d
)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) - 4*(c^4 - d^4 - 2*c^2 + 1)*cos(2*
b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 - c)*d + 2*(c*d^3 + (c^3 - c)*d)*cos(2*b*x
 + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*sin(2*b*x + 2*a))*sin(4*b*x + 4*a) + 8*(c
*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1), x) + 1/8*x^2*log((c^2 + d^2 + 2*
c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 + 2*c
 + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 - 2*(c^2 - d^2 + 2*c + 1)*cos(2*b*x +
2*a) + 2*c + 1) - 1/8*x^2*log((c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a)^2 + 4*
(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 - 2*c + 1)*sin(2*b*x + 2*a)^2 + c^2
 + d^2 - 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*x + 2*a) - 2*c + 1)
```

Fricas [C] time = 2.83357, size = 3838, normalized size = 13.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/16*(4*b^2*x^2*log((d*cos(2*b*x + 2*a) + (c + 1)*sin(2*b*x + 2*a) + d)/(d*
cos(2*b*x + 2*a) + (c - 1)*sin(2*b*x + 2*a) + d)) + 2*I*b*x*dilog(-(c^2 + d
^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(
c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c
+ 1) + 1) - 2*I*b*x*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2 + 2*c +
1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin(2*b*x +
 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1) + 1) - 2*I*b*x*dilog(-(c^2 + d^2 - (
c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c - 1)
*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1) +
 1) + 2*I*b*x*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos
(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a)
- 2*c + 1)/(c^2 + d^2 - 2*c + 1) + 1) - 2*a^2*log(1/2*c^2 + I*(c + 1)*d - 1
```

```
/2*d^2 - 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 +
2*I*c + I)*sin(2*b*x + 2*a) + c + 1/2) + 2*a^2*log(1/2*c^2 + I*(c - 1)*d -
1/2*d^2 - 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 -
 2*I*c + I)*sin(2*b*x + 2*a) - c + 1/2) - 2*a^2*log(-1/2*c^2 + I*(c + 1)*d
+ 1/2*d^2 + 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2
 + 2*I*c + I)*sin(2*b*x + 2*a) - c - 1/2) + 2*a^2*log(-1/2*c^2 + I*(c - 1)*
d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d
^2 - 2*I*c + I)*sin(2*b*x + 2*a) + c - 1/2) - 2*(b^2*x^2 - a^2)*log((c^2 +
d^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*
(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c
 + 1)) - 2*(b^2*x^2 - a^2)*log((c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2 + 2*
c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin(2*b
*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1)) + 2*(b^2*x^2 - a^2)*log((c^2 +
d^2 - (c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*
(c - 1)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c
 + 1)) + 2*(b^2*x^2 - a^2)*log((c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^2 - 2*
c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*sin(2*b
*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1)) - polylog(3, ((c^2 + 2*I*(c + 1
)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*(c + 1)*d - I*d^2 + 2*I*
c + I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*c + 1)) - polylog(3, ((c^2 - 2*I*(c
 + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*(c + 1)*d + I*d^2 -
 2*I*c - I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*c + 1)) + polylog(3, ((c^2 + 2
*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*(c - 1)*d - I*d
^2 - 2*I*c + I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*c + 1)) + polylog(3, ((c^2
 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*(c - 1)*d
+ I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*c + 1)))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(c+d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arcoth}(d \cot(bx + a) + c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(c+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(d*cot(b*x + a) + c), x)
```

3.255 $\int \coth^{-1}(c + d\cot(a + bx))\,dx$

Optimal. Leaf size=194

$$-\frac{i\text{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b}+\frac{i\text{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b}+\frac{1}{2}x\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)-\frac{1}{2}x\log\left(1-\frac{(c\cdot}{}\right.$$

```
[Out] x*ArcCoth[c + d*Cot[a + b*x]] + (x*Log[1 - ((1 - c - I*d)*E^((2*I)*a + (2*I
)*b*x))/(1 - c + I*d)])/2 - (x*Log[1 - ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*
x))/(1 + c - I*d)])/2 - ((I/4)*PolyLog[2, ((1 - c - I*d)*E^((2*I)*a + (2*I)
*b*x))/(1 - c + I*d)])/b + ((I/4)*PolyLog[2, ((1 + c + I*d)*E^((2*I)*a + (2
*I)*b*x))/(1 + c - I*d)])/b
```

Rubi [A] time = 0.249007, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6262, 2190, 2279, 2391}

$$-\frac{i\text{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b}+\frac{i\text{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b}+\frac{1}{2}x\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)-\frac{1}{2}x\log\left(1-\frac{(c\cdot}{}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[c + d*Cot[a + b*x]],x]
```

```
[Out] x*ArcCoth[c + d*Cot[a + b*x]] + (x*Log[1 - ((1 - c - I*d)*E^((2*I)*a + (2*I
)*b*x))/(1 - c + I*d)])/2 - (x*Log[1 - ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*
x))/(1 + c - I*d)])/2 - ((I/4)*PolyLog[2, ((1 - c - I*d)*E^((2*I)*a + (2*I)
*b*x))/(1 - c + I*d)])/b + ((I/4)*PolyLog[2, ((1 + c + I*d)*E^((2*I)*a + (2
*I)*b*x))/(1 + c - I*d)])/b
```

Rule 6262

```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Coth[c + d*Cot[a + b*x]], x] + (-Dist[I*b*(1 - c - I*d), Int[(x*E^(2*I*a +
2*I*b*x))/(1 - c + I*d - (1 - c - I*d)*E^(2*I*a + 2*I*b*x)), x], x] + Dist[
I*b*(1 + c + I*d), Int[(x*E^(2*I*a + 2*I*b*x))/(1 + c - I*d - (1 + c + I*d)
*E^(2*I*a + 2*I*b*x)), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - I*d)^2
, 1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \coth^{-1}(c+d\cot(a+bx))\,dx = x\coth^{-1}(c+d\cot(a+bx)) + (b(i(1+c)-d))\int \frac{e^{2ia+2ibx}x}{1+c-id+(-1-c-id)e^{2ia+2ibx}}\,dx$$
$$= x\coth^{-1}(c+d\cot(a+bx)) + \frac{1}{2}x\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) - \frac{1}{2}x\log\left(1-\frac{(1+c+}{1+}\right.$$
$$= x\coth^{-1}(c+d\cot(a+bx)) + \frac{1}{2}x\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) - \frac{1}{2}x\log\left(1-\frac{(1+c+}{1+}\right.$$
$$= x\coth^{-1}(c+d\cot(a+bx)) + \frac{1}{2}x\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) - \frac{1}{2}x\log\left(1-\frac{(1+c+}{1+}\right.$$

Mathematica [B] time = 12.9929, size = 4463, normalized size = 23.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[c + d*Cot[a + b*x]],x]
```

```
[Out] x*ArcCoth[c + d*Cot[a + b*x]] - (d*(a*Log[-(Sec[(a + b*x)/2]^2*(d*Cos[a + b
*x] + (-1 + c)*Sin[a + b*x]))] - a*Log[-(Sec[(a + b*x)/2]^2*(d*Cos[a + b*x]
 + Sin[a + b*x] + c*Sin[a + b*x]))] - (a + b*x)*Log[-((-1 + c + Sqrt[1 - 2*
c + c^2 + d^2])/d) + Tan[(a + b*x)/2]] - I*Log[(d*(-I + Tan[(a + b*x)/2]))/
(-1 + c - I*d + Sqrt[1 - 2*c + c^2 + d^2])]*Log[-((-1 + c + Sqrt[1 - 2*c +
c^2 + d^2])/d) + Tan[(a + b*x)/2]] + I*Log[(d*(I + Tan[(a + b*x)/2]))/(-1 +
 c + I*d + Sqrt[1 - 2*c + c^2 + d^2])]*Log[-((-1 + c + Sqrt[1 - 2*c + c^2 +
 d^2])/d) + Tan[(a + b*x)/2]] + (a + b*x)*Log[-((1 + c + Sqrt[1 + 2*c + c^2
 + d^2])/d) + Tan[(a + b*x)/2]] + I*Log[(d*(-I + Tan[(a + b*x)/2]))/(1 + c
- I*d + Sqrt[1 + 2*c + c^2 + d^2])]*Log[-((1 + c + Sqrt[1 + 2*c + c^2 + d^2
])/d) + Tan[(a + b*x)/2]] - I*Log[(d*(I + Tan[(a + b*x)/2]))/(1 + c + I*d +
 Sqrt[1 + 2*c + c^2 + d^2])]*Log[-((1 + c + Sqrt[1 + 2*c + c^2 + d^2])/d) +
 Tan[(a + b*x)/2]] - (a + b*x)*Log[(1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*T
an[(a + b*x)/2])/d] - I*Log[-((d*(-I + Tan[(a + b*x)/2]))/(1 - c + I*d + Sq
rt[1 - 2*c + c^2 + d^2]))]*Log[(1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(
a + b*x)/2])/d] + I*Log[-((d*(I + Tan[(a + b*x)/2]))/(1 - c - I*d + Sqrt[1
- 2*c + c^2 + d^2]))]*Log[(1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b
*x)/2])/d] + (a + b*x)*Log[(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a +
 b*x)/2])/d] + I*Log[-((d*(-I + Tan[(a + b*x)/2]))/(-1 - c + I*d + Sqrt[1 +
 2*c + c^2 + d^2]))]*Log[(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a + b
*x)/2])/d] - I*Log[-((d*(I + Tan[(a + b*x)/2]))/(-1 - c - I*d + Sqrt[1 + 2*
c + c^2 + d^2]))]*Log[(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a + b*x)
/2])/d] - I*PolyLog[2, (-1 + c + Sqrt[1 - 2*c + c^2 + d^2] - d*Tan[(a + b*x
)/2])/(-1 + c - I*d + Sqrt[1 - 2*c + c^2 + d^2])] + I*PolyLog[2, (-1 + c +
Sqrt[1 - 2*c + c^2 + d^2] - d*Tan[(a + b*x)/2])/(-1 + c + I*d + Sqrt[1 - 2*
c + c^2 + d^2])] - I*PolyLog[2, (1 + c - Sqrt[1 + 2*c + c^2 + d^2] - d*Tan[
(a + b*x)/2])/(1 + c + I*d - Sqrt[1 + 2*c + c^2 + d^2])] + I*PolyLog[2, (1
+ c + Sqrt[1 + 2*c + c^2 + d^2] - d*Tan[(a + b*x)/2])/(1 + c - I*d + Sqrt[1
 + 2*c + c^2 + d^2])] - I*PolyLog[2, (1 + c + Sqrt[1 + 2*c + c^2 + d^2] - d
*Tan[(a + b*x)/2])/(1 + c + I*d + Sqrt[1 + 2*c + c^2 + d^2])] + I*PolyLog[2
, (1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/(1 - c - I*d + S
```

qrt[1 - 2*c + c^2 + d^2])] - I*PolyLog[2, (1 - c + Sqrt[1 - 2*c + c^2 + d^2
] + d*Tan[(a + b*x)/2])/(1 - c + I*d + Sqrt[1 - 2*c + c^2 + d^2])] + I*Poly
Log[2, (-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/(-1 - c +
I*d + Sqrt[1 + 2*c + c^2 + d^2])])*((2*a)/(b*(1 - c^2 - d^2 - Cos[2*(a + b*
x)] + c^2*Cos[2*(a + b*x)] - d^2*Cos[2*(a + b*x)] - 2*c*d*Sin[2*(a + b*x)])
) - (2*(a + b*x))/(b*(1 - c^2 - d^2 - Cos[2*(a + b*x)] + c^2*Cos[2*(a + b*x
)] - d^2*Cos[2*(a + b*x)] - 2*c*d*Sin[2*(a + b*x)]))))/(-Log[-((-1 + c + Sq
rt[1 - 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]] + Log[-((1 + c + Sqrt[1 + 2
*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]] - Log[(1 - c + Sqrt[1 - 2*c + c^2 +
 d^2] + d*Tan[(a + b*x)/2])/d] + Log[(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] +
d*Tan[(a + b*x)/2])/d] - ((I/2)*Log[-((-1 + c + Sqrt[1 - 2*c + c^2 + d^2])/
d) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(-I + Tan[(a + b*x)/2]) + ((I/2)
*Log[-((1 + c + Sqrt[1 + 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]]*Sec[(a +
b*x)/2]^2)/(-I + Tan[(a + b*x)/2]) - ((I/2)*Log[(1 - c + Sqrt[1 - 2*c + c^2
 + d^2] + d*Tan[(a + b*x)/2])/d]*Sec[(a + b*x)/2]^2)/(-I + Tan[(a + b*x)/2]
) + ((I/2)*Log[(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/d]
*Sec[(a + b*x)/2]^2)/(-I + Tan[(a + b*x)/2]) + ((I/2)*Log[-((-1 + c + Sqrt[
1 - 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(I + Tan[(
a + b*x)/2]) - ((I/2)*Log[-((1 + c + Sqrt[1 + 2*c + c^2 + d^2])/d) + Tan[(a
 + b*x)/2]]*Sec[(a + b*x)/2]^2)/(I + Tan[(a + b*x)/2]) + ((I/2)*Log[(1 - c
+ Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/d]*Sec[(a + b*x)/2]^2)/(I
 + Tan[(a + b*x)/2]) - ((I/2)*Log[(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*T
an[(a + b*x)/2])/d]*Sec[(a + b*x)/2]^2)/(I + Tan[(a + b*x)/2]) - ((a + b*x)
*Sec[(a + b*x)/2]^2)/(2*(-((-1 + c + Sqrt[1 - 2*c + c^2 + d^2])/d) + Tan[(a
 + b*x)/2])) - ((I/2)*Log[(d*(-I + Tan[(a + b*x)/2]))/(-1 + c - I*d + Sqrt[
1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-((-1 + c + Sqrt[1 - 2*c + c^2
+ d^2])/d) + Tan[(a + b*x)/2]) + ((I/2)*Log[(d*(I + Tan[(a + b*x)/2]))/(-1
+ c + I*d + Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-((-1 + c + Sq
rt[1 - 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]) + ((a + b*x)*Sec[(a + b*x)/
2]^2)/(2*(-((1 + c + Sqrt[1 + 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2])) + (
(I/2)*Log[(d*(-I + Tan[(a + b*x)/2]))/(1 + c - I*d + Sqrt[1 + 2*c + c^2 + d
^2])]*Sec[(a + b*x)/2]^2)/(-((1 + c + Sqrt[1 + 2*c + c^2 + d^2])/d) + Tan[(
a + b*x)/2]) - ((I/2)*Log[(d*(I + Tan[(a + b*x)/2]))/(1 + c + I*d + Sqrt[1
+ 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-((1 + c + Sqrt[1 + 2*c + c^2 + d
^2])/d) + Tan[(a + b*x)/2]) - ((I/2)*d*Log[1 - (-1 + c + Sqrt[1 - 2*c + c^2
 + d^2] - d*Tan[(a + b*x)/2])/(-1 + c - I*d + Sqrt[1 - 2*c + c^2 + d^2])]*S
ec[(a + b*x)/2]^2)/(-1 + c + Sqrt[1 - 2*c + c^2 + d^2] - d*Tan[(a + b*x)/2]
) + ((I/2)*d*Log[1 - (-1 + c + Sqrt[1 - 2*c + c^2 + d^2] - d*Tan[(a + b*x)/
2])/(-1 + c + I*d + Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-1 + c
 + Sqrt[1 - 2*c + c^2 + d^2] - d*Tan[(a + b*x)/2]) - ((I/2)*d*Log[1 - (1 +
c - Sqrt[1 + 2*c + c^2 + d^2] - d*Tan[(a + b*x)/2])/(1 + c + I*d - Sqrt[1 +
 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(1 + c - Sqrt[1 + 2*c + c^2 + d^2]
- d*Tan[(a + b*x)/2]) + ((I/2)*d*Log[1 - (1 + c + Sqrt[1 + 2*c + c^2 + d^2]
 - d*Tan[(a + b*x)/2])/(1 + c - I*d + Sqrt[1 + 2*c + c^2 + d^2])]*Sec[(a +
b*x)/2]^2)/(1 + c + Sqrt[1 + 2*c + c^2 + d^2] - d*Tan[(a + b*x)/2]) - ((I/2
)*d*Log[1 - (1 + c + Sqrt[1 + 2*c + c^2 + d^2] - d*Tan[(a + b*x)/2])/(1 + c
 + I*d + Sqrt[1 + 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(1 + c + Sqrt[1 +
2*c + c^2 + d^2] - d*Tan[(a + b*x)/2]) - (d*(a + b*x)*Sec[(a + b*x)/2]^2)/(
2*(1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])) - ((I/2)*d*Log[
-((d*(-I + Tan[(a + b*x)/2]))/(1 - c + I*d + Sqrt[1 - 2*c + c^2 + d^2]))]*S
ec[(a + b*x)/2]^2)/(1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])
 + ((I/2)*d*Log[-((d*(I + Tan[(a + b*x)/2]))/(1 - c - I*d + Sqrt[1 - 2*c +
c^2 + d^2]))]*Sec[(a + b*x)/2]^2)/(1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Ta
n[(a + b*x)/2]) - ((I/2)*d*Log[1 - (1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*T
an[(a + b*x)/2])/(1 - c - I*d + Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2
]^2)/(1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2]) + ((I/2)*d*Lo
g[1 - (1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/(1 - c + I*d
 + Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(1 - c + Sqrt[1 - 2*c +
c^2 + d^2] + d*Tan[(a + b*x)/2]) + (d*(a + b*x)*Sec[(a + b*x)/2]^2)/(2*(-1

```
- c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])) + ((I/2)*d*Log[-((d*
(-I + Tan[(a + b*x)/2]))/(-1 - c + I*d + Sqrt[1 + 2*c + c^2 + d^2]))]*Sec[(
a + b*x)/2]^2)/(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2]) -
((I/2)*d*Log[-((d*(I + Tan[(a + b*x)/2]))/(-1 - c - I*d + Sqrt[1 + 2*c + c^
2 + d^2]))]*Sec[(a + b*x)/2]^2)/(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan
[(a + b*x)/2]) - ((I/2)*d*Log[1 - (-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*T
an[(a + b*x)/2])/(-1 - c + I*d + Sqrt[1 + 2*c + c^2 + d^2])]*Sec[(a + b*x)/
2]^2)/(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2]) - (a*Cos[(a
 + b*x)/2]^2*(-(Sec[(a + b*x)/2]^2*((-1 + c)*Cos[a + b*x] - d*Sin[a + b*x])
) - Sec[(a + b*x)/2]^2*(d*Cos[a + b*x] + (-1 + c)*Sin[a + b*x])*Tan[(a + b*
x)/2]))/(d*Cos[a + b*x] + (-1 + c)*Sin[a + b*x]) + (a*Cos[(a + b*x)/2]^2*(-
(Sec[(a + b*x)/2]^2*(Cos[a + b*x] + c*Cos[a + b*x] - d*Sin[a + b*x])) - Sec
[(a + b*x)/2]^2*(d*Cos[a + b*x] + Sin[a + b*x] + c*Sin[a + b*x])*Tan[(a + b
*x)/2]))/(d*Cos[a + b*x] + Sin[a + b*x] + c*Sin[a + b*x]))
```

Maple [B] time = 0.139, size = 629, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(c+d*cot(b*x+a)),x)
```

```
[Out] -1/2/b*arccoth(c+d*cot(b*x+a))*Pi+1/b*arccoth(c+d*cot(b*x+a))*arccot(cot(b*
x+a))+1/2/b*arctan((c+d*cot(b*x+a))/d-c/d)*ln(d*((c+d*cot(b*x+a))/d-c/d)+c+
1)-1/2/b*arctan((c+d*cot(b*x+a))/d-c/d)*ln(d*((c+d*cot(b*x+a))/d-c/d)+c-1)-
1/4*I/b*ln(d*((c+d*cot(b*x+a))/d-c/d)+c-1)*ln((I*d-d*((c+d*cot(b*x+a))/d-c/
d))/(I*d+c-1))+1/4*I/b*ln(d*((c+d*cot(b*x+a))/d-c/d)+c-1)*ln((I*d+d*((c+d*c
ot(b*x+a))/d-c/d))/(1-c+I*d))-1/4*I/b*dilog((I*d-d*((c+d*cot(b*x+a))/d-c/d)
)/(I*d+c-1))+1/4*I/b*dilog((I*d+d*((c+d*cot(b*x+a))/d-c/d))/(1-c+I*d))+1/4*
I/b*ln(d*((c+d*cot(b*x+a))/d-c/d)+c+1)*ln((I*d-d*((c+d*cot(b*x+a))/d-c/d))/
(1+c+I*d))-1/4*I/b*ln(d*((c+d*cot(b*x+a))/d-c/d)+c+1)*ln((I*d+d*((c+d*cot(b
*x+a))/d-c/d))/(I*d-c-1))+1/4*I/b*dilog((I*d-d*((c+d*cot(b*x+a))/d-c/d))/(1
+c+I*d))-1/4*I/b*dilog((I*d+d*((c+d*cot(b*x+a))/d-c/d))/(I*d-c-1))
```

Maxima [B] time = 1.89117, size = 529, normalized size = 2.73

$$4\,(bx+a)\,\mathrm{arcoth}\left(c+\frac{d}{\tan(bx+a)}\right)+\left(\arctan\left(\frac{(c+1)d+\left(c^2+2\,c+1\right)\tan(bx+a)}{c^2+d^2+2\,c+1},\frac{(c+1)d\tan(bx+a)+d^2}{c^2+d^2+2\,c+1}\right)-\arctan\left(\frac{(c-1)d+\left(c^2-2\,c+1\right)\tan(bx+a}{c^2+d^2-2\,c+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(b*x + a)*arccoth(c + d/tan(b*x + a)) + (arctan2(((c + 1)*d + (c^2 +
 2*c + 1)*tan(b*x + a))/(c^2 + d^2 + 2*c + 1), ((c + 1)*d*tan(b*x + a) + d^
2)/(c^2 + d^2 + 2*c + 1)) - arctan2(((c - 1)*d + (c^2 - 2*c + 1)*tan(b*x +
a))/(c^2 + d^2 - 2*c + 1), ((c - 1)*d*tan(b*x + a) + d^2)/(c^2 + d^2 - 2*c
+ 1)))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log((2*(c + 1)*d*tan(b*x + a) +
(c^2 + 2*c + 1)*tan(b*x + a)^2 + d^2)/(c^2 + d^2 + 2*c + 1)) + (b*x + a)*lo
g((2*(c - 1)*d*tan(b*x + a) + (c^2 - 2*c + 1)*tan(b*x + a)^2 + d^2)/(c^2 +
d^2 - 2*c + 1)) + I*dilog(-((c + 1)*tan(b*x + a) - I*c - I)/(I*c + d + I))
- I*dilog(-((c - 1)*tan(b*x + a) - I*c + I)/(I*c + d - I)) + I*dilog(-((c -
 1)*tan(b*x + a) + I*c - I)/(-I*c + d + I)) - I*dilog(-((c + 1)*tan(b*x + a
```

```
) + I*c + I)/(-I*c + d - I)))/b
```

Fricas [B] time = 2.78503, size = 2919, normalized size = 15.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*log((d*cos(2*b*x + 2*a) + (c + 1)*sin(2*b*x + 2*a) + d)/(d*cos(2
*b*x + 2*a) + (c - 1)*sin(2*b*x + 2*a) + d)) + 2*a*log(1/2*c^2 + I*(c + 1)*
d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d
^2 + 2*I*c + I)*sin(2*b*x + 2*a) + c + 1/2) - 2*a*log(1/2*c^2 + I*(c - 1)*d
 - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^
2 - 2*I*c + I)*sin(2*b*x + 2*a) - c + 1/2) + 2*a*log(-1/2*c^2 + I*(c + 1)*d
 + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^
2 + 2*I*c + I)*sin(2*b*x + 2*a) - c - 1/2) - 2*a*log(-1/2*c^2 + I*(c - 1)*d
 + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^
2 - 2*I*c + I)*sin(2*b*x + 2*a) + c - 1/2) - 2*(b*x + a)*log((c^2 + d^2 - (
c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c + 1)
*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1))
- 2*(b*x + a)*log((c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*
b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) + 2
*c + 1)/(c^2 + d^2 + 2*c + 1)) + 2*(b*x + a)*log((c^2 + d^2 - (c^2 + 2*I*(c
 - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c - 1)*d + I*d^2 +
 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1)) + 2*(b*x + a
)*log((c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) +
 (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2
 + d^2 - 2*c + 1)) + I*dilog(-(c^2 + d^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*c
 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*b
*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1) + 1) - I*dilog(-(c^2 + d^2 - (c^
2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d
- I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1) + 1)
 - I*dilog(-(c^2 + d^2 - (c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x +
2*a) + (-I*c^2 + 2*(c - 1)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c +
1)/(c^2 + d^2 - 2*c + 1) + 1) + I*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c - 1)*d
- d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c +
I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1) + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(c+d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}(d\cot(bx+a)+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*cot(b*x + a) + c), x)
```

3.256 $\int \frac{\coth^{-1}(c+d\cot(a+bx))}{x}\,dx$

Optimal. Leaf size=17

$$\text{CannotIntegrate}\left(\frac{\coth^{-1}(d\cot(a+bx)+c)}{x},x\right)$$

```
[Out] CannotIntegrate[ArcCoth[c + d*Cot[a + b*x]]/x, x]
```

Rubi [A] time = 0.0933881, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth^{-1}(c+d\cot(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCoth[c + d*Cot[a + b*x]]/x,x]

[Out] Defer[Int][ArcCoth[c + d*Cot[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\coth^{-1}(c+d\cot(a+bx))}{x}\,dx=\int \frac{\coth^{-1}(c+d\cot(a+bx))}{x}\,dx$$

Mathematica [A] time = 0.347825, size = 0, normalized size = 0.

$$\int \frac{\coth^{-1}(c+d\cot(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCoth[c + d*Cot[a + b*x]]/x,x]

[Out] Integrate[ArcCoth[c + d*Cot[a + b*x]]/x, x]
```

Maple [A] time = 0.391, size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(c+d\cot(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(c+d*cot(b*x+a))/x,x)

[Out] int(arccoth(c+d*cot(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(d\cot(bx+a)+c)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*cot(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arccoth(d*cot(b*x + a) + c)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arcoth}(d\cot(bx+a)+c)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*cot(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(d*cot(b*x + a) + c)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(c+d*cot(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(d\cot(bx+a)+c)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*cot(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*cot(b*x + a) + c)/x, x)
```

3.257 $\int x^2 \coth^{-1}(1 + id + d\cot(a + bx))\,dx$

Optimal. Leaf size=168

$$-\frac{x\text{PolyLog}\left(3,(1+id)e^{2ia+2ibx}\right)}{4b^2}-\frac{i\text{PolyLog}\left(4,(1+id)e^{2ia+2ibx}\right)}{8b^3}+\frac{ix^2\text{PolyLog}\left(2,(1+id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{6}x^3\log\left(1-(\right.$$

```
[Out] (I/12)*b*x^4 + (x^3*ArcCoth[1 + I*d + d*Cot[a + b*x]])/3 - (x^3*Log[1 - (1
+ I*d)*E^((2*I)*a + (2*I)*b*x)])/6 + ((I/4)*x^2*PolyLog[2, (1 + I*d)*E^((2*
I)*a + (2*I)*b*x)])/b - (x*PolyLog[3, (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/(
4*b^2) - ((I/8)*PolyLog[4, (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/b^3
```

Rubi [A] time = 0.305705, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {6266, 2184, 2190, 2531, 6609, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(3,(1+id)e^{2ia+2ibx}\right)}{4b^2}-\frac{i\text{PolyLog}\left(4,(1+id)e^{2ia+2ibx}\right)}{8b^3}+\frac{ix^2\text{PolyLog}\left(2,(1+id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{6}x^3\log\left(1-(\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[1 + I*d + d*Cot[a + b*x]],x]
```


```
[Out] (I/12)*b*x^4 + (x^3*ArcCoth[1 + I*d + d*Cot[a + b*x]])/3 - (x^3*Log[1 - (1
+ I*d)*E^((2*I)*a + (2*I)*b*x)])/6 + ((I/4)*x^2*PolyLog[2, (1 + I*d)*E^((2*
I)*a + (2*I)*b*x)])/b - (x*PolyLog[3, (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/(
4*b^2) - ((I/8)*PolyLog[4, (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/b^3
```


Rule 6266

```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Cot[a + b*x]])/(f*(m
 + 1)), x] + Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c - I*d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\coth^{-1}(1+id+d\cot(a+bx))\,dx=\frac{1}{3}x^3\coth^{-1}(1+id+d\cot(a+bx))+\frac{1}{3}(ib)\int\frac{x^3}{1+(-1-id)e^{2ia+2ibx}}\,dx$$

$$=\frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1+id+d\cot(a+bx))+\frac{1}{3}(b(i-d))\int\frac{e^{2ia+2ibx}x^3}{1+(-1-id)e^{2ia+}}$$

$$=\frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1+id+d\cot(a+bx))-\frac{1}{6}x^3\log\left(1-(1+id)e^{2ia+2ibx}\right)+$$

$$=\frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1+id+d\cot(a+bx))-\frac{1}{6}x^3\log\left(1-(1+id)e^{2ia+2ibx}\right)+$$

$$=\frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1+id+d\cot(a+bx))-\frac{1}{6}x^3\log\left(1-(1+id)e^{2ia+2ibx}\right)+$$

$$=\frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1+id+d\cot(a+bx))-\frac{1}{6}x^3\log\left(1-(1+id)e^{2ia+2ibx}\right)+$$

$$=\frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1+id+d\cot(a+bx))-\frac{1}{6}x^3\log\left(1-(1+id)e^{2ia+2ibx}\right)+$$

Mathematica [A] time = 0.177359, size = 155, normalized size = 0.92

$$\frac{1}{3}x^3\coth^{-1}(d\cot(a+bx)+id+1)-\frac{6ib^2x^2\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{d-i}\right)+6bx\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{d-i}\right)-3i\text{PolyLog}\left(4,-\frac{ie^{-2i}}{d}\right.}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[1 + I*d + d*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcCoth[1 + I*d + d*Cot[a + b*x]])/3 - (4*b^3*x^3*Log[1 + I/((-I + d)*
E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, (-I)/((-I + d)*E^((2*I)*(a
 + b*x)))] + 6*b*x*PolyLog[3, (-I)/((-I + d)*E^((2*I)*(a + b*x)))] - (3*I)*
```

```
PolyLog[4, (-I)/((-I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

Maple [C] time = 29.797, size = 2449, normalized size = 14.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(1+I*d+d*cot(b*x+a)),x)
```

```
[Out] -1/6*x^3*ln(d)+1/12*I*b*x^4+1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(
2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b
*x+a))+I)/(exp(2*I*(b*x+a))-1))^2-1/2*I/b^2*a^2/(-d+I)*ln(1+I*exp(I*(b*x+a)
)*(I*(-d+I))^(1/2))*x-1/2*I/b^3*a^2*d/(-d+I)*dilog(1-I*exp(I*(b*x+a))*(I*(-
d+I))^(1/2))-1/2*I/b^3*a^2*d/(-d+I)*dilog(1+I*exp(I*(b*x+a))*(I*(-d+I))^(1/
2))-1/4*I/b*d/(-d+I)*polylog(2,-I*(-d+I)*exp(2*I*(b*x+a)))*x^2+1/2/b^3*a^3*
d/(-d+I)*ln(1-I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))+1/2/b^3*a^3*d/(-d+I)*ln(1+
I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))-1/6/b^3*a^3*d/(-d+I)*ln(I*exp(2*I*(b*x+a
))-exp(2*I*(b*x+a))*d-I)+1/4/b^2*d/(-d+I)*polylog(3,-I*(-d+I)*exp(2*I*(b*x+
a)))*x+1/6*I/b^3*a^3/(-d+I)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d-I)+1/8
/b^3/(-d+I)*polylog(4,-I*(-d+I)*exp(2*I*(b*x+a)))-1/3*x^3*ln(exp(I*(b*x+a))
)-1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(exp
(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))
-1))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))
^2+1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+1/12*I*x^3
*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))-1/6*I*x^3*Pi*csgn(I*e
xp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2-1/6*I/(-d+I)*ln(1+I*(-d+I)*exp(2*
I*(b*x+a)))*x^3-1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))
^3-1/12*I*x^3*Pi*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b
*x+a))-1))^2-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*
x+a))*d-I*exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a)
)+I)/(exp(2*I*(b*x+a))-1))+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(e
xp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))+1/12*I*x^
3*Pi*csgn(I*d)*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))*csgn(I*d/(exp(
2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))-1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-
I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*d-I*exp(
2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))+1/2/b^2*a^2*d/(-d+I)*ln(1+I*exp(I*(b*
x+a))*(I*(-d+I))^(1/2))*x+1/4*I/b^3*d/(-d+I)*polylog(2,-I*(-d+I)*exp(2*I*(b
*x+a)))*a^2+1/2*I/b^2/(-d+I)*ln(1+I*(-d+I)*exp(2*I*(b*x+a)))*x*a^2-1/2*I/b^
2*a^2/(-d+I)*ln(1-I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))*x+1/8*I/b^3*d/(-d+I)*p
olylog(4,-I*(-d+I)*exp(2*I*(b*x+a)))+1/12*I*x^3*Pi*csgn((exp(2*I*(b*x+a))*d
-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^3-1/12*I*x^3*Pi*csgn(I*(exp(2*
I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^3+1/12*I*x^3*Pi*cs
gn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))-1)*e
xp(2*I*(b*x+a)))+1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a)
)+I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1)
)^2+1/6*x^3*ln(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)-1/12*I*x^3*Pi*csgn(
I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(
2*I*(b*x+a)))^2-1/3/b^3*d/(-d+I)*ln(1+I*(-d+I)*exp(2*I*(b*x+a)))*a^3-1/4*I/
b^2/(-d+I)*polylog(3,-I*(-d+I)*exp(2*I*(b*x+a)))*x-1/2*I/b^3*a^3/(-d+I)*ln(
1-I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))-1/2*I/b^3*a^3/(-d+I)*ln(1+I*exp(I*(b*x
+a))*(I*(-d+I))^(1/2))-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*
I*(b*x+a))/(exp(2*I*(b*x+a))-1))^2-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1
))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^2-1/12*I*x^3*Pi*csgn(I*d)*
csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+1/3*I/b^3/(-d+I)*ln(1+I*(
-d+I)*exp(2*I*(b*x+a)))*a^3-1/2/b^3*a^2/(-d+I)*dilog(1-I*exp(I*(b*x+a))*(I*
(-d+I))^(1/2))-1/2/b^3*a^2/(-d+I)*dilog(1+I*exp(I*(b*x+a))*(I*(-d+I))^(1/2)
```

```
)-1/4/b/(-d+I)*polylog(2,-I*(-d+I)*exp(2*I*(b*x+a)))*x^2+1/4/b^3/(-d+I)*pol
ylog(2,-I*(-d+I)*exp(2*I*(b*x+a)))*a^2+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)
))^3-1/2/b^2*d/(-d+I)*ln(1+I*(-d+I)*exp(2*I*(b*x+a)))*x*a^2+1/2/b^2*a^2*d/(
-d+I)*ln(1-I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))*x+1/6*d/(-d+I)*ln(1+I*(-d+I)*
exp(2*I*(b*x+a)))*x^3+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a
))-1))^3+1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^3
```

Maxima [B] time = 1.17363, size = 462, normalized size = 2.75

$$\frac{12\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\mathrm{arcoth}(d\cot(bx+a)+i\,d+1)}{b^2}-\frac{-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2+\left(-8i\,(bx+a)^3+18i\,(bx+a)^2a-18i\,(bx+a)a^2\right)\mathrm{arctan}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arccoth(d*cot(b*
x + a) + I*d + 1)/b^2 - (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
+ a)^2*a^2 + (-8*I*(b*x + a)^3 + 18*I*(b*x + a)^2*a - 18*I*(b*x + a)*a^2)*a
rctan2(d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) - cos(2*b*
x + 2*a) + 1) + (-12*I*(b*x + a)^2 + 18*I*(b*x + a)*a - 9*I*a^2)*dilog((I*d
 + 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(b*x + a)
*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^2 + 2*d
*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, (I*d
 + 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (I*d + 1)*e^(2*I*b*x + 2*I*a)))
/b^2)/b
```

Fricas [C] time = 1.77333, size = 505, normalized size = 3.01

$$\frac{2i\,b^4x^4+4\,b^3x^3\log\left(\frac{\left((d-i)e^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{d}\right)+6i\,b^2x^2\mathrm{Li}_2\left(-(-i\,d-1)e^{(2i\,bx+2i\,a)}\right)-2i\,a^4+4\,a^3\log\left(\frac{(d-i)e^{(2i\,bx+2i\,a)}+i}{d-i}\right)-6\,b}{24\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/24*(2*I*b^4*x^4 + 4*b^3*x^3*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I
*b*x - 2*I*a)/d) + 6*I*b^2*x^2*dilog(-(-I*d - 1)*e^(2*I*b*x + 2*I*a)) - 2*I
*a^4 + 4*a^3*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)/(d - I)) - 6*b*x*polylog
(3, (I*d + 1)*e^(2*I*b*x + 2*I*a)) - 4*(b^3*x^3 + a^3)*log((-I*d - 1)*e^(2*
I*b*x + 2*I*a) + 1) - 3*I*polylog(4, (I*d + 1)*e^(2*I*b*x + 2*I*a)))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(1+I*d+d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arcoth}\left(d\cot\left(bx+a\right)+i\,d+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(d*cot(b*x + a) + I*d + 1), x)
```

3.258 $\int x \coth^{-1}(1 + id + d \cot(a + bx))\,dx$

Optimal. Leaf size=132

$$-\frac{\text{PolyLog}\left(3,(1+id)e^{2ia+2ibx}\right)}{8b^2}+\frac{ix\text{PolyLog}\left(2,(1+id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(1+id)e^{2ia+2ibx}\right)+\frac{1}{2}x^2\coth^{-1}(d\cot(a\cdot$$

```
[Out] (I/6)*b*x^3 + (x^2*ArcCoth[1 + I*d + d*Cot[a + b*x]])/2 - (x^2*Log[1 - (1 +
 I*d)*E^((2*I)*a + (2*I)*b*x)])/4 + ((I/4)*x*PolyLog[2, (1 + I*d)*E^((2*I)*
a + (2*I)*b*x)])/b - PolyLog[3, (1 + I*d)*E^((2*I)*a + (2*I)*b*x)]/(8*b^2)
```

Rubi [A] time = 0.256505, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6266, 2184, 2190, 2531, 2282, 6589}

$$-\frac{\text{PolyLog}\left(3,(1+id)e^{2ia+2ibx}\right)}{8b^2}+\frac{ix\text{PolyLog}\left(2,(1+id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(1+id)e^{2ia+2ibx}\right)+\frac{1}{2}x^2\coth^{-1}(d\cot(a\cdot$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[1 + I*d + d*Cot[a + b*x]],x]
```

```
[Out] (I/6)*b*x^3 + (x^2*ArcCoth[1 + I*d + d*Cot[a + b*x]])/2 - (x^2*Log[1 - (1 +
 I*d)*E^((2*I)*a + (2*I)*b*x)])/4 + ((I/4)*x*PolyLog[2, (1 + I*d)*E^((2*I)*
a + (2*I)*b*x)])/b - PolyLog[3, (1 + I*d)*E^((2*I)*a + (2*I)*b*x)]/(8*b^2)
```

Rule 6266

```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Cot[a + b*x]])/(f*(m
 + 1)), x] + Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c - I*d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\coth^{-1}(1+id+d\cot(a+bx))\,dx=\frac{1}{2}x^2\coth^{-1}(1+id+d\cot(a+bx))+\frac{1}{2}(ib)\int\frac{x^2}{1+(-1-id)e^{2ia+2ibx}}\,dx$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1+id+d\cot(a+bx))+\frac{1}{2}(b(i-d))\int\frac{e^{2ia+2ibx}x^2}{1+(-1-id)e^{2ia}}$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1+id+d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1+id)e^{2ia+2ibx}\right)-$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1+id+d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1+id)e^{2ia+2ibx}\right)-$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1+id+d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1+id)e^{2ia+2ibx}\right)-$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1+id+d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1+id)e^{2ia+2ibx}\right)-$$

Mathematica [A] time = 0.108773, size = 119, normalized size = 0.9

$$\frac{1}{2}x^2\coth^{-1}(d\cot(a+bx)+id+1)-\frac{2ibx\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{d-i}\right)+\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{d-i}\right)+2b^2x^2\log\left(1+\frac{ie^{-2i(a+bx)}}{d-i}\right.}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[1 + I*d + d*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcCoth[1 + I*d + d*Cot[a + b*x]])/2 - (2*b^2*x^2*Log[1 + I/((-I + d)*
E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/((-I + d)*E^((2*I)*(a + b
*x)))] + PolyLog[3, (-I)/((-I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

Maple [C] time = 17.385, size = 2351, normalized size = 17.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(1+I*d+d*cot(b*x+a)),x)
```

```
[Out] 1/2*I/b^2*a^2/(-d+I)*ln(1+I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))+1/8*I*x^2*Pi*c
sgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I
```

```
)/(exp(2*I*(b*x+a))-1))^2-1/4*I/b^2*a^2/(-d+I)*ln(I*exp(2*I*(b*x+a))-exp(2*
I*(b*x+a))*d-I)-1/4*I/b^2/(-d+I)*ln(1+I*(-d+I)*exp(2*I*(b*x+a)))*a^2+1/2*I/
b^2*a^2/(-d+I)*ln(1-I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))+1/4/b^2*a^2*d/(-d+I)
*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d-I)-1/2/b^2*a^2*d/(-d+I)*ln(1-I*ex
p(I*(b*x+a))*(I*(-d+I))^(1/2))-1/2/b^2*a^2*d/(-d+I)*ln(1+I*exp(I*(b*x+a))*(
I*(-d+I))^(1/2))+1/4/b^2*d/(-d+I)*ln(1+I*(-d+I)*exp(2*I*(b*x+a)))*a^2-1/8*I
*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-
1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))+1/
8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^3+1/8*I*x^2*Pi*csg
n(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^3-1/8*I*x^2*Pi*csgn((exp(2*I*(
b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2+1/4*x^2*ln(exp(2*I*
(b*x+a))*d-I*exp(2*I*(b*x+a))+I)-1/4/b/(-d+I)*polylog(2,-I*(-d+I)*exp(2*I*(
b*x+a)))*x-1/4/b^2/(-d+I)*polylog(2,-I*(-d+I)*exp(2*I*(b*x+a)))*a+1/8/b^2*d
/(-d+I)*polylog(3,-I*(-d+I)*exp(2*I*(b*x+a)))-1/8*I*x^2*Pi*csgn(I*(exp(2*I*
(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^3-1/8*I/b^2/(-d+I)*p
olylog(3,-I*(-d+I)*exp(2*I*(b*x+a)))-1/4*I/(-d+I)*ln(1+I*(-d+I)*exp(2*I*(b*
x+a)))*x^2+1/6*I*b*x^3-1/4*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(
b*x+a)))^2-1/8*I*x^2*Pi*csgn(I*d)*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*
x+a)))^2-1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))*csgn(I*
d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*
x+a))-1)*exp(2*I*(b*x+a)))^3-1/4*x^2*ln(d)+1/8*I*x^2*Pi*csgn(I*exp(I*(b*x+a
)))^2*csgn(I*exp(2*I*(b*x+a)))-1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I
*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^2-1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x
+a))-1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^2-1/8*I*x^2*Pi*csgn(
I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))-1)*exp(
2*I*(b*x+a)))^2+1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a))
)*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))+1/8*I*x^2*Pi*csgn(I*exp(2*I
*(b*x+a)))^3+1/8*I*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/
(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2
*I*(b*x+a))-1))^2-1/2*x^2*ln(exp(I*(b*x+a)))+1/8*I*x^2*Pi*csgn((exp(2*I*(b*
x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^3+1/4*d/(-d+I)*ln(1+I*(
-d+I)*exp(2*I*(b*x+a)))*x^2+1/2/b^2*a/(-d+I)*dilog(1-I*exp(I*(b*x+a))*(I*(-
d+I))^(1/2))+1/2/b^2*a/(-d+I)*dilog(1+I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))+1/
8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+1/8*I*x^2*Pi*csg
n(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))*d-I
*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2+1/2/b*d/(-d+I)*ln(1+I*(-d+I)*e
xp(2*I*(b*x+a)))*x*a-1/2/b*a*d/(-d+I)*ln(1-I*exp(I*(b*x+a))*(I*(-d+I))^(1/2
))*x-1/2/b*a*d/(-d+I)*ln(1+I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))*x+1/2*I/b*a/(
-d+I)*ln(1-I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))*x+1/2*I/b*a/(-d+I)*ln(1+I*exp
(I*(b*x+a))*(I*(-d+I))^(1/2))*x-1/4*I/b*d/(-d+I)*polylog(2,-I*(-d+I)*exp(2*
I*(b*x+a)))*x-1/4*I/b^2*d/(-d+I)*polylog(2,-I*(-d+I)*exp(2*I*(b*x+a)))*a+1/
8*I*x^2*Pi*csgn(I*d)*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))*csgn(I*d
/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)
))*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1
))-1/2*I/b/(-d+I)*ln(1+I*(-d+I)*exp(2*I*(b*x+a)))*x*a+1/2*I/b^2*a*d/(-d+I)*
dilog(1-I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))+1/2*I/b^2*a*d/(-d+I)*dilog(1+I*e
xp(I*(b*x+a))*(I*(-d+I))^(1/2))-1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*c
sgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))*d
-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))
```

Maxima [B] time = 1.1394, size = 335, normalized size = 2.54

$$\frac{\frac{12\left((bx+a)^2-2\,(bx+a)a\right)\operatorname{arcoth}(d\cot(bx+a)+i\,d+1)}{b} - \frac{-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\operatorname{Li}_2\left((i\,d+1)e^{(2i\,bx+2i\,a)}\right)+\left(-6i\,(bx+a)^2+12i\,(bx+a)a\right)\arctan(d\cos(2\,bx+2\,\ldots}{\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arccoth(d*cot(b*x + a) + I*d + 1)/b
- (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((I*d + 1)*e^(2*I*b
*x + 2*I*a)) + (-6*I*(b*x + a)^2 + 12*I*(b*x + a)*a)*arctan2(d*cos(2*b*x +
2*a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1) + 3*((b
*x + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin
(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) + 3*polylo
g(3, (I*d + 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

Fricas [C] time = 1.69142, size = 429, normalized size = 3.25

$$\frac{4 i b^3 x^3 + 6 b^2 x^2 \log\left(\frac{\left((d-i) e^{(2 i b x + 2 i a)} + i\right) e^{(-2 i b x - 2 i a)}}{d}\right) + 4 i a^3 + 6 i b x \mathrm{Li}_2\left(-(-i d - 1) e^{(2 i b x + 2 i a)}\right) - 6 a^2 \log\left(\frac{(d-i) e^{(2 i b x + 2 i a)} + i}{d-i}\right) - 6}{24 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/24*(4*I*b^3*x^3 + 6*b^2*x^2*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I
*b*x - 2*I*a)/d) + 4*I*a^3 + 6*I*b*x*dilog(-(-I*d - 1)*e^(2*I*b*x + 2*I*a))
 - 6*a^2*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)/(d - I)) - 6*(b^2*x^2 - a^2)
*log((-I*d - 1)*e^(2*I*b*x + 2*I*a) + 1) - 3*polylog(3, (I*d + 1)*e^(2*I*b*
x + 2*I*a)))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(1+I*d+d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arcoth}\left(d \cot\left(b x + a\right) + i d + 1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(d*cot(b*x + a) + I*d + 1), x)
```

3.259 $\int \coth^{-1}(1 + id + d\cot(a + bx))\,dx$

Optimal. Leaf size=93

$$\frac{i\text{PolyLog}\left(2,(1+id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{2}x\log\left(1-(1+id)e^{2ia+2ibx}\right)+x\coth^{-1}(d\cot(a+bx)+id+1)+\frac{1}{2}ibx^2$$

```
[Out] (I/2)*b*x^2 + x*ArcCoth[1 + I*d + d*Cot[a + b*x]] - (x*Log[1 - (1 + I*d)*E^
((2*I)*a + (2*I)*b*x)])/2 + ((I/4)*PolyLog[2, (1 + I*d)*E^((2*I)*a + (2*I)*
b*x)])/b
```

Rubi [A] time = 0.158715, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6258, 2184, 2190, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2,(1+id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{2}x\log\left(1-(1+id)e^{2ia+2ibx}\right)+x\coth^{-1}(d\cot(a+bx)+id+1)+\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[1 + I*d + d*Cot[a + b*x]],x]
```

```
[Out] (I/2)*b*x^2 + x*ArcCoth[1 + I*d + d*Cot[a + b*x]] - (x*Log[1 - (1 + I*d)*E^
((2*I)*a + (2*I)*b*x)])/2 + ((I/4)*PolyLog[2, (1 + I*d)*E^((2*I)*a + (2*I)*
b*x)])/b
```

Rule 6258

```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Coth[c + d*Cot[a + b*x]], x] + Dist[I*b, Int[x/(c - I*d - c*E^(2*I*a + 2*I*
b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \coth^{-1}(1+id+d\cot(a+bx))\,dx = x\coth^{-1}(1+id+d\cot(a+bx))+(ib)\int \frac{x}{1+(-1-id)e^{2ia+2ibx}}\,dx$$
$$=\frac{1}{2}ibx^2+x\coth^{-1}(1+id+d\cot(a+bx))+(b(i-d))\int \frac{e^{2ia+2ibx}x}{1+(-1-id)e^{2ia+2ibx}}\,d$$
$$=\frac{1}{2}ibx^2+x\coth^{-1}(1+id+d\cot(a+bx))-\frac{1}{2}x\log\left(1-(1+id)e^{2ia+2ibx}\right)+\frac{1}{2}\int$$
$$=\frac{1}{2}ibx^2+x\coth^{-1}(1+id+d\cot(a+bx))-\frac{1}{2}x\log\left(1-(1+id)e^{2ia+2ibx}\right)-\frac{i\,\mathrm{Su}}{}$$
$$=\frac{1}{2}ibx^2+x\coth^{-1}(1+id+d\cot(a+bx))-\frac{1}{2}x\log\left(1-(1+id)e^{2ia+2ibx}\right)+\frac{i\mathrm{Li}_2}{}$$

Mathematica [B] time = 3.36414, size = 709, normalized size = 7.62

$$\frac{x\csc^2(a+bx)(\cos(bx)-i\sin(bx))(\cos(bx)+i\sin(bx))\left(i\mathrm{PolyLog}\left(2,\frac{(\cos(a}{}\right.\right.}{(\cot(a+bx)+i)(d\cot(a+bx)+id+2)\left(\frac{(d-2i)\cos(a+bx)(\log(1-i\tan(bx))-\log(1+i\tan(bx)))}{d\cos(a+bx)+(2+id)\sin(a+bx)}+\frac{d\sin(a+bx)(\log(1-i\tan(bx))-\log(1+i\tan}{(d-2i)\sin(a+bx)-id\cos(a+bx)}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[1 + I*d + d*Cot[a + b*x]],x]
```

```
[Out] x*ArcCoth[1 + I*d + d*Cot[a + b*x]] + (x*Csc[a + b*x]^2*(2*b*x*Log[2*Cos[b*
x]*(Cos[b*x] - I*Sin[b*x])] + I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*(d*Cos[a
+ b*x] + (2 + I*d)*Sin[a + b*x]))/(2*(-I + d))]*Log[1 - I*Tan[b*x]] - I*Log
[(Sec[b*x]*((-I)*Cos[a] + Sin[a])*(d*Cos[a + b*x] + (2 + I*d)*Sin[a + b*x])
)/2]*Log[1 + I*Tan[b*x]] + I*PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] - I*Pol
yLog[2, (Sec[b*x]*((2 + I*d)*Cos[a] - d*Sin[a])*(Cos[a + b*x] + I*Sin[a + b
*x]))/2] + I*PolyLog[2, ((Cos[a] - I*Sin[a])*((-2 - I*d)*Cos[a] + d*Sin[a])
*(I + Tan[b*x]))/(2*(-I + d))])*(Cos[b*x] - I*Sin[b*x])*(Cos[b*x] + I*Sin[b
*x]))/((I + Cot[a + b*x])*(2 + I*d + d*Cot[a + b*x])*((2*I)*b*x + Log[1 + (
Sec[b*x]*((-2 - I*d)*Cos[a] + d*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2]
 - Log[(Sec[b*x]*((-I)*Cos[a] + Sin[a])*(d*Cos[a + b*x] + (2 + I*d)*Sin[a +
 b*x]))/2] + ((-2*I + d)*Cos[a + b*x]*(Log[1 - I*Tan[b*x]] - Log[1 + I*Tan[
b*x]]))/(d*Cos[a + b*x] + (2 + I*d)*Sin[a + b*x]) + (d*(Log[1 - I*Tan[b*x]]
 - Log[1 + I*Tan[b*x]])*Sin[a + b*x])/((-I)*d*Cos[a + b*x] + (-2*I + d)*Sin
[a + b*x]) + 2*b*x*Tan[b*x] - I*Log[1 + (Sec[b*x]*((-2 - I*d)*Cos[a] + d*Si
n[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2]*Tan[b*x] + I*Log[(Sec[b*x]*((-I)*
Cos[a] + Sin[a])*(d*Cos[a + b*x] + (2 + I*d)*Sin[a + b*x]))/2]*Tan[b*x] - I
*Log[1 - I*Tan[b*x]]*Tan[b*x] + I*Log[1 + I*Tan[b*x]]*Tan[b*x]))
```

Maple [B] time = 0.181, size = 299, normalized size = 3.2

$$\frac{\frac{i}{2}\operatorname{arccoth}(1+id+d\cot(bx+a))\ln(id-d\cot(bx+a))}{b}-\frac{\frac{i}{2}\operatorname{arccoth}(1+id+d\cot(bx+a))\ln(id+d\cot(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1+I*d+d*cot(b*x+a)),x)
```

```
[Out] 1/2*I/b*arccoth(1+I*d+d*cot(b*x+a))*ln(I*d-d*cot(b*x+a))-1/2*I/b*arccoth(1+
I*d+d*cot(b*x+a))*ln(I*d+d*cot(b*x+a))-1/8*I/b*ln(I*d+d*cot(b*x+a))^2+1/4*I
```

```
/b*dilog(1+1/2*I*d+1/2*d*cot(b*x+a))+1/4*I/b*ln(I*d+d*cot(b*x+a))*ln(1+1/2*
I*d+1/2*d*cot(b*x+a))+1/4*I/b*dilog(1/2*I*(-I*d-d*cot(b*x+a))/d)+1/4*I/b*ln
(I*d-d*cot(b*x+a))*ln(1/2*I*(-I*d-d*cot(b*x+a))/d)-1/4*I/b*dilog((-2-I*d-d*
cot(b*x+a))/(-2*I*d-2))-1/4*I/b*ln(I*d-d*cot(b*x+a))*ln((-2-I*d-d*cot(b*x+a
))/(-2*I*d-2))
```

Maxima [B] time = 1.54505, size = 386, normalized size = 4.15

$$-\frac{4\,(bx+a)d\left(\frac{\log((i\,d+2)\tan(bx+a)+d)}{d}-\frac{\log(i\,\tan(bx+a)+1)}{d}\right)+d\left(-\frac{2i\left(\log((i\,d+2)\tan(bx+a)+d)\log\left(\frac{(d-2i)\tan(bx+a)-i\,d}{2i\,d+2}+1\right)+\mathrm{Li}_2\left(-\frac{(d-2i)\tan(bx+a)-i\,d}{2i\,d+2}\right.\right.}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*(b*x + a)*d*(log((I*d + 2)*tan(b*x + a) + d)/d - log(I*tan(b*x + a)
 + 1)/d) + d*(-2*I*(log((I*d + 2)*tan(b*x + a) + d)*log(((d - 2*I)*tan(b*x
+ a) - I*d)/(2*I*d + 2) + 1) + dilog(-((d - 2*I)*tan(b*x + a) - I*d)/(2*I*d
 + 2)))/d - 2*I*(log(1/2*(d - 2*I)*tan(b*x + a) - 1/2*I*d)*log(I*tan(b*x +
a) + 1) + dilog(-1/2*(d - 2*I)*tan(b*x + a) + 1/2*I*d + 1))/d + (2*I*log((I
*d + 2)*tan(b*x + a) + d)*log(I*tan(b*x + a) + 1) - I*log(I*tan(b*x + a) +
1)^2)/d + 2*I*(log(I*tan(b*x + a) + 1)*log(-1/2*I*tan(b*x + a) + 1/2) + dil
og(1/2*I*tan(b*x + a) + 1/2))/d) - 8*(b*x + a)*arccoth(I*d + d/tan(b*x + a)
 + 1))/b
```

Fricas [A] time = 1.77423, size = 339, normalized size = 3.65

$$\frac{2i\,b^2x^2+2\,bx\log\left(\frac{\left((d-i)e^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{d}\right)-2i\,a^2-2\,(bx+a)\log\left((-i\,d-1)e^{(2i\,bx+2i\,a)}+1\right)+2\,a\log\left(\frac{(d-i)e^{(2i\,bx+2i\,a)}+i}{d-i}\right)+}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*I*b^2*x^2 + 2*b*x*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x
- 2*I*a)/d) - 2*I*a^2 - 2*(b*x + a)*log((-I*d - 1)*e^(2*I*b*x + 2*I*a) + 1)
 + 2*a*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)/(d - I)) + I*dilog(-(-I*d - 1)
*e^(2*I*b*x + 2*I*a)))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1+I*d+d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}(d\cot(bx+a)+id+1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*cot(b*x + a) + I*d + 1), x)
```

3.260 $\int \frac{\coth^{-1}(1+id+d\cot(a+bx))}{x}\,dx$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(\frac{\coth^{-1}(d\cot(a+bx)+id+1)}{x},x\right)$$

```
[Out] CannotIntegrate[ArcCoth[1 + I*d + d*Cot[a + b*x]]/x, x]
```

Rubi [A] time = 0.0877378, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth^{-1}(1+id+d\cot(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCoth[1 + I*d + d*Cot[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcCoth[1 + I*d + d*Cot[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\coth^{-1}(1+id+d\cot(a+bx))}{x}\,dx=\int \frac{\coth^{-1}(1+id+d\cot(a+bx))}{x}\,dx$$

Mathematica [A] time = 0.673287, size = 0, normalized size = 0.

$$\int \frac{\coth^{-1}(1+id+d\cot(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCoth[1 + I*d + d*Cot[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcCoth[1 + I*d + d*Cot[a + b*x]]/x, x]
```

Maple [A] time = 0.444, size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(1+id+d\cot(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1+I*d+d*cot(b*x+a))/x,x)
```

```
[Out] int(arccoth(1+I*d+d*cot(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-i\,bx+\frac{1}{4}\,(-i\,\pi-4i\,a-2\,\log(d))\log(x)+\frac{1}{2}i\int\frac{\arctan\left(d\cos(2\,bx+2\,a)+\sin(2\,bx+2\,a),-d\sin(2\,bx+2\,a)+\mathrm{co}\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+I*d+d*cot(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -I*b*x + 1/4*(-I*pi - 4*I*a - 2*log(d))*log(x) + 1/2*I*integrate(arctan2(d*
cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + cos(2*b*x + 2*a)
 - 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*si
n(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\log\left(\frac{\left((d-i)e^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{d}\right)}{2\,x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+I*d+d*cot(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)/d)/
x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1+I*d+d*cot(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{arcoth}\left(d\cot(bx+a)+i\,d+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+I*d+d*cot(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*cot(b*x + a) + I*d + 1)/x, x)
```

3.261 $\int x^2 \coth^{-1}(1 - id - d\cot(a + bx))\,dx$

Optimal. Leaf size=169

$$-\frac{x\text{PolyLog}\left(3,(1-id)e^{2ia+2ibx}\right)}{4b^2}-\frac{i\text{PolyLog}\left(4,(1-id)e^{2ia+2ibx}\right)}{8b^3}+\frac{ix^2\text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{6}x^3\log\left(1-(1-\right.$$

```
[Out] (I/12)*b*x^4 + (x^3*ArcCoth[1 - I*d - d*Cot[a + b*x]])/3 - (x^3*Log[1 - (1
- I*d)*E^((2*I)*a + (2*I)*b*x)])/6 + ((I/4)*x^2*PolyLog[2, (1 - I*d)*E^((2*
I)*a + (2*I)*b*x)])/b - (x*PolyLog[3, (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/(
4*b^2) - ((I/8)*PolyLog[4, (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/b^3
```

Rubi [A] time = 0.294771, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6266, 2184, 2190, 2531, 6609, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(3,(1-id)e^{2ia+2ibx}\right)}{4b^2}-\frac{i\text{PolyLog}\left(4,(1-id)e^{2ia+2ibx}\right)}{8b^3}+\frac{ix^2\text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{6}x^3\log\left(1-(1-\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[1 - I*d - d*Cot[a + b*x]],x]
```


```
[Out] (I/12)*b*x^4 + (x^3*ArcCoth[1 - I*d - d*Cot[a + b*x]])/3 - (x^3*Log[1 - (1
- I*d)*E^((2*I)*a + (2*I)*b*x)])/6 + ((I/4)*x^2*PolyLog[2, (1 - I*d)*E^((2*
I)*a + (2*I)*b*x)])/b - (x*PolyLog[3, (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/(
4*b^2) - ((I/8)*PolyLog[4, (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/b^3
```


Rule 6266

```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Cot[a + b*x]])/(f*(m
 + 1)), x] + Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c - I*d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\coth^{-1}(1-id-d\cot(a+bx))\,dx = \frac{1}{3}x^3\coth^{-1}(1-id-d\cot(a+bx))+\frac{1}{3}(ib)\int\frac{x^3}{1+(-1+id)e^{2ia+2ibx}}\,dx$$
$$= \frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1-id-d\cot(a+bx))+\frac{1}{3}(b(i+d))\int\frac{e^{2ia+2ibx}x}{1+(-1+id)e^{}}$$
$$= \frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1-id-d\cot(a+bx))-\frac{1}{6}x^3\log\left(1-(1-id)e^{2ia+2ibx}\right)$$
$$= \frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1-id-d\cot(a+bx))-\frac{1}{6}x^3\log\left(1-(1-id)e^{2ia+2ibx}\right)$$
$$= \frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1-id-d\cot(a+bx))-\frac{1}{6}x^3\log\left(1-(1-id)e^{2ia+2ibx}\right)$$
$$= \frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1-id-d\cot(a+bx))-\frac{1}{6}x^3\log\left(1-(1-id)e^{2ia+2ibx}\right)$$
$$= \frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1-id-d\cot(a+bx))-\frac{1}{6}x^3\log\left(1-(1-id)e^{2ia+2ibx}\right)$$

Mathematica [A] time = 0.196546, size = 155, normalized size = 0.92

$$\frac{1}{3}x^3\coth^{-1}(d(-\cot(a+bx))-id+1)-\frac{6ib^2x^2\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{d+i}\right)+6bx\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{d+i}\right)-3i\text{PolyLog}\left(4,\frac{ie^{}}{}\right.}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[1 - I*d - d*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcCoth[1 - I*d - d*Cot[a + b*x]])/3 - (4*b^3*x^3*Log[1 + 1/((-1 + I*d
)*E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, I/((I + d)*E^((2*I)*(a +
 b*x)))] + 6*b*x*PolyLog[3, I/((I + d)*E^((2*I)*(a + b*x)))] - (3*I)*PolyLo
```

```
g[4, I/((I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

Maple [C] time = 29.139, size = 2339, normalized size = 13.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(1-I*d-d*cot(b*x+a)),x)
```

```
[Out] -1/6*x^3*ln(d)+1/12*I*b*x^4+1/4*I/b*d/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x
+a)))*x^2+1/12*I*x^3*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp
(2*I*(b*x+a))-1))^2-1/3*x^3*ln(exp(I*(b*x+a)))-1/12*I*x^3*Pi*csgn(I*(I*exp(
2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^3-1/12*I*x^3*Pi*cs
gn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))-1)*e
xp(2*I*(b*x+a)))^2-1/12*I*x^3*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*
d-I)/(exp(2*I*(b*x+a))-1))^3+1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+a))-1)*exp(
2*I*(b*x+a)))^3-1/2/b^3*a^2/(I+d)*dilog(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))
-1/2/b^3*a^2/(I+d)*dilog(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/4/b/(I+d)*po
lylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*x^2+1/4/b^3/(I+d)*polylog(2,-I*(I+d)*exp
(2*I*(b*x+a)))*a^2-1/6*d/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*x^3+1/6*x^3*l
n(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)-1/4*I/b^3*d/(I+d)*polylog(2,-I*(
I+d)*exp(2*I*(b*x+a)))*a^2+1/12*I*x^3*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*ex
p(2*I*(b*x+a)))-1/6*I*x^3*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))
)^2-1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+1/2*I/b^3
*a^2*d/(I+d)*dilog(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/2*I/b^3*a^3/(I+d)*
ln(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/8*I/b^3*d/(I+d)*polylog(4,-I*(I+d)
*exp(2*I*(b*x+a)))+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2*I*(
b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))+1/12*I*x^3*Pi*csg
n(I*d)*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))*csgn(I*d/(exp(2*I*(b*x
+a))-1)*exp(2*I*(b*x+a)))+1/6*I/b^3*a^3/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I
*(b*x+a))*d-I)+1/3*I/b^3/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*a^3-1/4*I/b^2
/(I+d)*polylog(3,-I*(I+d)*exp(2*I*(b*x+a)))*x-1/2*I/b^3*a^3/(I+d)*ln(1+I*ex
p(I*(b*x+a))*(I*(I+d))^(1/2))+1/2*I/b^3*a^2*d/(I+d)*dilog(1-I*exp(I*(b*x+a)
)*(I*(I+d))^(1/2))-1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a)
)*d-I)/(exp(2*I*(b*x+a))-1))*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)
/(exp(2*I*(b*x+a))-1))+1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*
(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))+1/12*I*x^3*Pi*csgn(
I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I))*csgn(I*(I*exp(2*I*(b*x+a))+exp
(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^2-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*
x+a))/(exp(2*I*(b*x+a))-1))*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))
^2+1/8/b^3/(I+d)*polylog(4,-I*(I+d)*exp(2*I*(b*x+a)))-1/12*I*x^3*Pi*csgn(I*
exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^2-1/12*I*x^
3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))
-1))^2-1/12*I*x^3*Pi*csgn(I*d)*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a
)))^2+1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(I*exp(2*I*(b*x+a))
+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^2-1/12*I*x^3*Pi*csgn(I/(exp(2*
I*(b*x+a))-1))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I))*csgn(I*(I*
exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))-1/6*I/(I+d)*ln
(1+I*(I+d)*exp(2*I*(b*x+a)))*x^3+1/2*I/b^2/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+
a)))*x*a^2+1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(
exp(2*I*(b*x+a))-1))*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*
I*(b*x+a))-1))^2-1/2/b^3*a^3*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))
-1/2/b^3*a^3*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))+1/6/b^3*a^3*d/(
I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)+1/3/b^3*d/(I+d)*ln(1+I*(I+
d)*exp(2*I*(b*x+a)))*a^3-1/4/b^2*d/(I+d)*polylog(3,-I*(I+d)*exp(2*I*(b*x+a)
))*x+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))^3-1/2*I/b^2*a^2/(I+d)*ln(1-I*ex
```

```
p(I*(b*x+a))*(I*(I+d))^(1/2))*x-1/2/b^2*a^2*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(
I*(I+d))^(1/2))*x-1/2/b^2*a^2*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2)
)*x+1/2/b^2*d/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*x*a^2+1/12*I*x^3*Pi*csgn
(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^3+1/12*I*x^3*Pi*csgn(I*d/(exp(2*I
*(b*x+a))-1)*exp(2*I*(b*x+a)))^3-1/2*I/b^2*a^2/(I+d)*ln(1+I*exp(I*(b*x+a))*
(I*(I+d))^(1/2))*x
```

Maxima [B] time = 1.37798, size = 463, normalized size = 2.74

$$-\frac{\frac{12\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\operatorname{arcoth}(d\cot(bx+a)+i\,d-1)}{b^2}+\frac{-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2+\left(-8i\,(bx+a)^3+18i\,(bx+a)^2a-18i\,(bx+a)a^2\right)\ldots}{\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arccoth(d*cot(b
*x + a) + I*d - 1)/b^2 + (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
 + a)^2*a^2 + (-8*I*(b*x + a)^3 + 18*I*(b*x + a)^2*a - 18*I*(b*x + a)*a^2)*
arctan2(-d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - cos(2
*b*x + 2*a) + 1) + (-12*I*(b*x + a)^2 + 18*I*(b*x + a)*a - 9*I*a^2)*dilog((
-I*d + 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(b*x
+ a)*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^2 -
 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3,
(-I*d + 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (-I*d + 1)*e^(2*I*b*x + 2*
I*a)))/b^2)/b
```

Fricas [C] time = 1.68903, size = 504, normalized size = 2.98

$$\frac{2i\,b^4x^4-4\,b^3x^3\log\left(\frac{de^{(2i\,bx+2i\,a)}}{(d+i)e^{(2i\,bx+2i\,a)}-i}\right)+6i\,b^2x^2\mathrm{Li}_2\left(-(i\,d-1)e^{(2i\,bx+2i\,a)}\right)-2i\,a^4+4\,a^3\log\left(\frac{(d+i)e^{(2i\,bx+2i\,a)}-i}{d+i}\right)-6\,bx\mathrm{polylo}\ldots}{24\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/24*(2*I*b^4*x^4 - 4*b^3*x^3*log(d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b*x
 + 2*I*a) - I)) + 6*I*b^2*x^2*dilog(-(I*d - 1)*e^(2*I*b*x + 2*I*a)) - 2*I*a
^4 + 4*a^3*log(((d + I)*e^(2*I*b*x + 2*I*a) - I)/(d + I)) - 6*b*x*polylog(3
, (-I*d + 1)*e^(2*I*b*x + 2*I*a)) - 4*(b^3*x^3 + a^3)*log((I*d - 1)*e^(2*I*
b*x + 2*I*a) + 1) - 3*I*polylog(4, (-I*d + 1)*e^(2*I*b*x + 2*I*a)))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(1-I*d-d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arcoth}(-d\cot(bx+a) - id + 1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(-d*cot(b*x + a) - I*d + 1), x)
```

3.262 $\int x \coth^{-1}(1 - id - d\cot(a + bx))\,dx$

Optimal. Leaf size=133

$$-\frac{\text{PolyLog}\left(3,(1-id)e^{2ia+2ibx}\right)}{8b^2}+\frac{ix\text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(1-id)e^{2ia+2ibx}\right)+\frac{1}{2}x^2\coth^{-1}(d(-\text{c}$$

```
[Out] (I/6)*b*x^3 + (x^2*ArcCoth[1 - I*d - d*Cot[a + b*x]])/2 - (x^2*Log[1 - (1 -
 I*d)*E^((2*I)*a + (2*I)*b*x)])/4 + ((I/4)*x*PolyLog[2, (1 - I*d)*E^((2*I)*
a + (2*I)*b*x)])/b - PolyLog[3, (1 - I*d)*E^((2*I)*a + (2*I)*b*x)]/(8*b^2)
```

Rubi [A] time = 0.247589, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6266, 2184, 2190, 2531, 2282, 6589}

$$-\frac{\text{PolyLog}\left(3,(1-id)e^{2ia+2ibx}\right)}{8b^2}+\frac{ix\text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(1-id)e^{2ia+2ibx}\right)+\frac{1}{2}x^2\coth^{-1}(d(-\text{c}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[1 - I*d - d*Cot[a + b*x]],x]
```

```
[Out] (I/6)*b*x^3 + (x^2*ArcCoth[1 - I*d - d*Cot[a + b*x]])/2 - (x^2*Log[1 - (1 -
 I*d)*E^((2*I)*a + (2*I)*b*x)])/4 + ((I/4)*x*PolyLog[2, (1 - I*d)*E^((2*I)*
a + (2*I)*b*x)])/b - PolyLog[3, (1 - I*d)*E^((2*I)*a + (2*I)*b*x)]/(8*b^2)
```

Rule 6266

```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Cot[a + b*x]])/(f*(m
 + 1)), x] + Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c - I*d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\coth^{-1}(1-id-d\cot(a+bx))\,dx=\frac{1}{2}x^2\coth^{-1}(1-id-d\cot(a+bx))+\frac{1}{2}(ib)\int\frac{x^2}{1+(-1+id)e^{2ia+2ibx}}\,dx$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1-id-d\cot(a+bx))+\frac{1}{2}(b(i+d))\int\frac{e^{2ia+2ibx}x^2}{1+(-1+id)e^{2ia+2ib}}$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1-id-d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1-id)e^{2ia+2ibx}\right)+\frac{1}{2}$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1-id-d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1-id)e^{2ia+2ibx}\right)+\frac{ix}{}$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1-id-d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1-id)e^{2ia+2ibx}\right)+\frac{ix}{}$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1-id-d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1-id)e^{2ia+2ibx}\right)+\frac{ix}{}$$

Mathematica [A] time = 0.103554, size = 119, normalized size = 0.89

$$\frac{1}{2}x^2\coth^{-1}(d(-\cot(a+bx))-id+1)-\frac{2ibx\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{d+i}\right)+\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{d+i}\right)+2b^2x^2\log\left(1+\frac{e^{-2i(a+bx)}}{-1+id}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[1 - I*d - d*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcCoth[1 - I*d - d*Cot[a + b*x]])/2 - (2*b^2*x^2*Log[1 + 1/((-1 + I*d
)*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/((I + d)*E^((2*I)*(a + b*x
)))] + PolyLog[3, I/((I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

Maple [C] time = 20.552, size = 2249, normalized size = 16.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(1-I*d-d*cot(b*x+a)),x)
```

```
[Out] 1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^3+1/8*I*x^2*Pi*c
sgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^3+1/2*I/b^2*a^2/(I+d)*ln(1+I
```

```
*exp(I*(b*x+a))*(I*(I+d))^(1/2))+1/2*I/b^2*a^2/(I+d)*ln(1-I*exp(I*(b*x+a))*
(I*(I+d))^(1/2))-1/4*I/b^2*a^2/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))
*d-I)-1/8/b^2*d/(I+d)*polylog(3,-I*(I+d)*exp(2*I*(b*x+a)))-1/4/b^2/(I+d)*po
lylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*a+1/2/b^2*a/(I+d)*dilog(1+I*exp(I*(b*x+a
))*(I*(I+d))^(1/2))-1/4*d/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*x^2+1/2/b^2*
a/(I+d)*dilog(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))+1/4*x^2*ln(I*exp(2*I*(b*x
+a))+exp(2*I*(b*x+a))*d-I)+1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*
(b*x+a))*d-I))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b
*x+a))-1))^2+1/6*I*b*x^3+1/2/b^2*a^2*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I+d)
)^(1/2))-1/4*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2-1/8
*I*x^2*Pi*csgn(I*d)*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-1/8*I
*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))*csgn(I*d/(exp(2*I*(b*
x+a))-1)*exp(2*I*(b*x+a)))^2-1/4/b^2*d/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))
*a^2+1/2/b^2*a^2*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/4*x^2*ln(
d)+1/8*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))+1/8*I*x^2
*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))*
csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^2+1/8*
I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b
*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^2-1/8*I/b^2/(I+d)*polylog(3,-I*(I+d)*exp(
2*I*(b*x+a)))-1/4*I/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*x^2-1/8*I*x^2*Pi*c
sgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^2-1/8
*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*
x+a))-1))^2-1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*cs
gn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+1/8*I*x^2*Pi*csgn(I*d/(exp(2*
I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)
))-1/4/b^2*a^2*d/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)+1/8*I*x^
2*Pi*csgn(I*exp(2*I*(b*x+a)))^3-1/4/b/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x
+a)))*x-1/8*I*x^2*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*
I*(b*x+a))-1))^3-1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d
-I)/(exp(2*I*(b*x+a))-1))^3-1/2*x^2*ln(exp(I*(b*x+a)))-1/2*I/b^2*a*d/(I+d)*
dilog(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/4*I/b^2/(I+d)*ln(1+I*(I+d)*exp(
2*I*(b*x+a)))*a^2-1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*
d-I)/(exp(2*I*(b*x+a))-1))*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(
exp(2*I*(b*x+a))-1))+1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+
a)))^3-1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+1/8*I*x
^2*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^
2+1/8*I*x^2*Pi*csgn(I*d)*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))*csgn
(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))-1/2*I/b/(I+d)*ln(1+I*(I+d)*exp(
2*I*(b*x+a)))*x*a+1/4*I/b*d/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*x+1/
4*I/b^2*d/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*a-1/2/b*d/(I+d)*ln(1+I
*(I+d)*exp(2*I*(b*x+a)))*x*a+1/2/b*a*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d)
)^(1/2))*x+1/2/b*a*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x-1/2*I/b
^2*a*d/(I+d)*dilog(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))+1/2*I/b*a/(I+d)*ln(1
+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x+1/2*I/b*a/(I+d)*ln(1-I*exp(I*(b*x+a))*
(I*(I+d))^(1/2))*x-1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(I*exp(
2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x
+a))*d-I)/(exp(2*I*(b*x+a))-1))+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(
I/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))
```

Maxima [B] time = 1.28095, size = 336, normalized size = 2.53

$$-\frac{\frac{12\left((bx+a)^2-2(bx+a)a\right)\operatorname{arcoth}(d\cot(bx+a)+i\,d-1)}{b}+\frac{-4i(bx+a)^3+12i(bx+a)^2a-6i\,bx\operatorname{Li}_2\left((-i\,d+1)e^{(2i\,bx+2i\,a)}\right)+\left(-6i(bx+a)^2+12i(bx+a)a\right)\arctan(-d\,\mathrm{co}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arccoth(d*cot(b*x + a) + I*d - 1)/b
 + (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((-I*d + 1)*e^(2*I
*b*x + 2*I*a)) + (-6*I*(b*x + a)^2 + 12*I*(b*x + a)*a)*arctan2(-d*cos(2*b*x
 + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1) + 3
*((b*x + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)
*sin(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) + 3*po
lylog(3, (-I*d + 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

Fricas [C] time = 1.77153, size = 427, normalized size = 3.21

$$\frac{4 i b^3 x^3 - 6 b^2 x^2 \log\left(\frac{d e^{(2 i bx+2 i a)}}{(d+i) e^{(2 i bx+2 i a)} - i}\right) + 4 i a^3 + 6 i bx \mathrm{Li}_2\left(-(i d - 1) e^{(2 i bx+2 i a)}\right) - 6 a^2 \log\left(\frac{(d+i) e^{(2 i bx+2 i a)} - i}{d+i}\right) - 6\left(b^2 x^2 - a^2\right) \log}{24 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/24*(4*I*b^3*x^3 - 6*b^2*x^2*log(d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b*x
 + 2*I*a) - I)) + 4*I*a^3 + 6*I*b*x*dilog(-(I*d - 1)*e^(2*I*b*x + 2*I*a)) -
 6*a^2*log(((d + I)*e^(2*I*b*x + 2*I*a) - I)/(d + I)) - 6*(b^2*x^2 - a^2)*l
og((I*d - 1)*e^(2*I*b*x + 2*I*a) + 1) - 3*polylog(3, (-I*d + 1)*e^(2*I*b*x
+ 2*I*a)))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(1-I*d-d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arcoth}(-d \cot(bx + a) - i d + 1) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(-d*cot(b*x + a) - I*d + 1), x)
```

3.263 $\int \coth^{-1}(1-id-d\cot(a+bx))\,dx$

Optimal. Leaf size=94

$$\frac{i\text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{2}x\log\left(1-(1-id)e^{2ia+2ibx}\right)+x\coth^{-1}(d(-\cot(a+bx))-id+1)+\frac{1}{2}ibx^2$$

```
[Out] (I/2)*b*x^2 + x*ArcCoth[1 - I*d - d*Cot[a + b*x]] - (x*Log[1 - (1 - I*d)*E^
((2*I)*a + (2*I)*b*x)])/2 + ((I/4)*PolyLog[2, (1 - I*d)*E^((2*I)*a + (2*I)*
b*x)])/b
```

Rubi [A] time = 0.150208, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6258, 2184, 2190, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{2}x\log\left(1-(1-id)e^{2ia+2ibx}\right)+x\coth^{-1}(d(-\cot(a+bx))-id+1)+\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[1 - I*d - d*Cot[a + b*x]],x]
```

```
[Out] (I/2)*b*x^2 + x*ArcCoth[1 - I*d - d*Cot[a + b*x]] - (x*Log[1 - (1 - I*d)*E^
((2*I)*a + (2*I)*b*x)])/2 + ((I/4)*PolyLog[2, (1 - I*d)*E^((2*I)*a + (2*I)*
b*x)])/b
```

Rule 6258

```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Coth[c + d*Cot[a + b*x]], x] + Dist[I*b, Int[x/(c - I*d - c*E^(2*I*a + 2*I*
b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \coth^{-1}(1-id-d\cot(a+bx))\,dx = x\coth^{-1}(1-id-d\cot(a+bx)) + (ib)\int \frac{x}{1+(-1+id)e^{2ia+2ibx}}\,dx$$
$$= \frac{1}{2}ibx^2 + x\coth^{-1}(1-id-d\cot(a+bx)) + (b(i+d))\int \frac{e^{2ia+2ibx}x}{1+(-1+id)e^{2ia+2ibx}}\,dx$$
$$= \frac{1}{2}ibx^2 + x\coth^{-1}(1-id-d\cot(a+bx)) - \frac{1}{2}x\log\left(1-(1-id)e^{2ia+2ibx}\right) + \frac{1}{2}\int \log$$
$$= \frac{1}{2}ibx^2 + x\coth^{-1}(1-id-d\cot(a+bx)) - \frac{1}{2}x\log\left(1-(1-id)e^{2ia+2ibx}\right) - \frac{i\,\text{Subst}}{}$$
$$= \frac{1}{2}ibx^2 + x\coth^{-1}(1-id-d\cot(a+bx)) - \frac{1}{2}x\log\left(1-(1-id)e^{2ia+2ibx}\right) + \frac{i\text{Li}_2\left((1\right.}{}$$

Mathematica [B] time = 2.63817, size = 605, normalized size = 6.44

$$\frac{x\csc^2(a+bx)(\cos(bx)-i\sin(bx))(\cos(bx)+i\sin(bx))\left(i\text{PolyLog}\left(2,\frac{(\cos(a)-i\sin(a))(\tan(bx)+i)(d\sin(a)+(2-id)\cos(a))}{2(d+i)}\right)-i\text{PolyL}\right.}{(\cot(a+bx)+i)(d\cot(a+bx)+id-2)\left(\frac{\sec^2(bx)\log\left(\frac{is}{}\right.}{}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[1 - I*d - d*Cot[a + b*x]],x]
```

```
[Out] x*ArcCoth[1 - I*d - d*Cot[a + b*x]] + (x*Csc[a + b*x]^2*(2*b*x*Log[2*Cos[b*
x]*(Cos[b*x] - I*Sin[b*x])] + I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*(d*Cos[a
+ b*x] + I*(2*I + d)*Sin[a + b*x]))/(2*(I + d))]*Log[1 - I*Tan[b*x]] - I*Lo
g[(I*Sec[b*x]*(d*Cos[a + b*x] + I*(2*I + d)*Sin[a + b*x]))/(2*Cos[a] - (2*I
)*Sin[a])]*Log[1 + I*Tan[b*x]] + I*PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] -
 I*PolyLog[2, (Sec[b*x]*((2 - I*d)*Cos[a] + d*Sin[a])*(Cos[a + b*x] + I*Sin
[a + b*x]))/2] + I*PolyLog[2, ((Cos[a] - I*Sin[a])*((2 - I*d)*Cos[a] + d*Si
n[a])*(I + Tan[b*x]))/(2*(I + d))])*(Cos[b*x] - I*Sin[b*x])*(Cos[b*x] + I*S
in[b*x]))/((I + Cot[a + b*x])*(-2 + I*d + d*Cot[a + b*x])*(-((Log[1 - I*Tan
[b*x]]*Sec[b*x]*((2*I + d)*Cos[a] + I*d*Sin[a]))/(d*Cos[a + b*x] + I*(2*I +
 d)*Sin[a + b*x])) + (Log[1 + I*Tan[b*x]]*Sec[b*x]*((2*I + d)*Cos[a] + I*d*
Sin[a]))/(d*Cos[a + b*x] + I*(2*I + d)*Sin[a + b*x]) + (Log[(I*Sec[b*x]*(d*
Cos[a + b*x] + I*(2*I + d)*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*Sec[b*
x]^2)/(1 + I*Tan[b*x]) - 2*b*x*(I + Tan[b*x]) + I*Log[1 - (Sec[b*x]*((2 - I
*d)*Cos[a] + d*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2]*(I + Tan[b*x])))
```

Maple [B] time = 0.193, size = 304, normalized size = 3.2

$$\frac{-\frac{i}{2}\text{arccoth}(1-id-d\cot(bx+a))\ln(-id-d\cot(bx+a))}{b} + \frac{\frac{i}{2}\text{arccoth}(1-id-d\cot(bx+a))\ln(id-d\cot(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1-I*d-d*cot(b*x+a)),x)
```

```
[Out] -1/2*I/b*arccoth(1-I*d-d*cot(b*x+a))*ln(-I*d-d*cot(b*x+a))+1/2*I/b*arccoth(
1-I*d-d*cot(b*x+a))*ln(I*d-d*cot(b*x+a))-1/8*I/b*ln(-I*d-d*cot(b*x+a))^2+1/
4*I/b*dilog(1-1/2*I*d-1/2*d*cot(b*x+a))+1/4*I/b*ln(-I*d-d*cot(b*x+a))*ln(1-
```

```
1/2*I*d-1/2*d*cot(b*x+a))+1/4*I/b*dilog(1/2*I*(-I*d-d*cot(b*x+a))/d)+1/4*I/
b*ln(I*d-d*cot(b*x+a))*ln(1/2*I*(-I*d-d*cot(b*x+a))/d)-1/4*I/b*dilog((2-I*d
-d*cot(b*x+a))/(-2*I*d+2))-1/4*I/b*ln(I*d-d*cot(b*x+a))*ln((2-I*d-d*cot(b*x
+a))/(-2*I*d+2))
```

Maxima [B] time = 1.75784, size = 389, normalized size = 4.14

$$-\frac{4\,(bx+a)d\left(\frac{\log((i\,d-2)\tan(bx+a)+d)}{d}-\frac{\log(i\tan(bx+a)+1)}{d}\right)-d\left(\frac{2i\left(\log((i\,d-2)\tan(bx+a)+d)\log\left(\frac{(d+2i)\tan(bx+a)-i\,d}{2i\,d-2}+1\right)+\mathrm{Li}_2\left(-\frac{(d+2i)\tan(bx+a)-}{2i\,d-2}\right.\right.}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*(b*x + a)*d*(log((I*d - 2)*tan(b*x + a) + d)/d - log(I*tan(b*x + a)
 + 1)/d) - d*(2*I*(log((I*d - 2)*tan(b*x + a) + d)*log(((d + 2*I)*tan(b*x +
 a) - I*d)/(2*I*d - 2) + 1) + dilog(-((d + 2*I)*tan(b*x + a) - I*d)/(2*I*d
- 2)))/d + 2*I*(log(-1/2*(d + 2*I)*tan(b*x + a) + 1/2*I*d)*log(I*tan(b*x +
a) + 1) + dilog(1/2*(d + 2*I)*tan(b*x + a) - 1/2*I*d + 1))/d - (2*I*log((I*
d - 2)*tan(b*x + a) + d)*log(I*tan(b*x + a) + 1) - I*log(I*tan(b*x + a) + 1
)^2)/d - 2*I*(log(I*tan(b*x + a) + 1)*log(-1/2*I*tan(b*x + a) + 1/2) + dilo
g(1/2*I*tan(b*x + a) + 1/2))/d) + 8*(b*x + a)*arccoth(I*d + d/tan(b*x + a)
- 1))/b
```

Fricas [A] time = 1.74486, size = 335, normalized size = 3.56

$$\frac{2i\,b^2x^2-2\,bx\log\left(\frac{de^{(2i\,bx+2i\,a)}}{(d+i)e^{(2i\,bx+2i\,a)}-i}\right)-2i\,a^2-2\,(bx+a)\log\left((i\,d-1)e^{(2i\,bx+2i\,a)}+1\right)+2\,a\log\left(\frac{(d+i)e^{(2i\,bx+2i\,a)}-i}{d+i}\right)+i\,\mathrm{Li}_2\left(-(\right.}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*I*b^2*x^2 - 2*b*x*log(d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b*x + 2*
I*a) - I)) - 2*I*a^2 - 2*(b*x + a)*log((I*d - 1)*e^(2*I*b*x + 2*I*a) + 1) +
 2*a*log(((d + I)*e^(2*I*b*x + 2*I*a) - I)/(d + I)) + I*dilog(-(I*d - 1)*e^
(2*I*b*x + 2*I*a)))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1-I*d-d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}\left(-d\cot\left(bx+a\right)-i\,d+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(-d*cot(b*x + a) - I*d + 1), x)
```

3.264 $\int \frac{\coth^{-1}(1-id-d\cot(a+bx))}{x}\,dx$

Optimal. Leaf size=23

$$\text{CannotIntegrate}\left(\frac{\coth^{-1}(d(-\cot(a+bx))-id+1)}{x},x\right)$$

```
[Out] CannotIntegrate[ArcCoth[1 - I*d - d*Cot[a + b*x]]/x, x]
```

Rubi [A] time = 0.0800919, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth^{-1}(1-id-d\cot(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCoth[1 - I*d - d*Cot[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcCoth[1 - I*d - d*Cot[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\coth^{-1}(1-id-d\cot(a+bx))}{x}\,dx=\int \frac{\coth^{-1}(1-id-d\cot(a+bx))}{x}\,dx$$

Mathematica [A] time = 0.749697, size = 0, normalized size = 0.

$$\int \frac{\coth^{-1}(1-id-d\cot(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCoth[1 - I*d - d*Cot[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcCoth[1 - I*d - d*Cot[a + b*x]]/x, x]
```

Maple [A] time = 0.457, size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(1-id-d\cot(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1-I*d-d*cot(b*x+a))/x,x)
```

```
[Out] int(arccoth(1-I*d-d*cot(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-i\,bx + \frac{1}{4}\left(-i\,\pi - 4i\,a - 2\,\log\left(d\right)\right)\log\left(x\right) - \frac{1}{2}i\int \frac{\arctan\left(-d\cos\left(2\,bx + 2\,a\right) + \sin\left(2\,bx + 2\,a\right), -d\sin\left(2\,bx + 2\,a\right) - \cos\left(\right.\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-I*d-d*cot(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -I*b*x + 1/4*(-I*pi - 4*I*a - 2*log(d))*log(x) - 1/2*I*integrate(arctan2(-d
*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - cos(2*b*x + 2*a
) + 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*s
in(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\log\left(\frac{de^{(2i\,bx+2i\,a)}}{(d+i)e^{(2i\,bx+2i\,a)}-i}\right)}{2\,x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-I*d-d*cot(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(-1/2*log(d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b*x + 2*I*a) - I))/
x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1-I*d-d*cot(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}\left(-d\cot\left(bx + a\right) - i\,d + 1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-I*d-d*cot(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(-d*cot(b*x + a) - I*d + 1)/x, x)
```

3.265 $\int \frac{(a+b\coth^{-1}(cx^n))(d+e\log(fx^m))}{x}\,dx$

Optimal. Leaf size=160

$$\frac{bd\text{PolyLog}\left(2,-\frac{x^{-n}}{c}\right)}{2n}-\frac{bd\text{PolyLog}\left(2,\frac{x^{-n}}{c}\right)}{2n}+\frac{be\log\left(fx^m\right)\text{PolyLog}\left(2,-\frac{x^{-n}}{c}\right)}{2n}-\frac{be\log\left(fx^m\right)\text{PolyLog}\left(2,\frac{x^{-n}}{c}\right)}{2n}+\frac{b}{}$$

```
[Out] a*d*Log[x] + (a*e*Log[f*x^m]^2)/(2*m) + (b*d*PolyLog[2, -(1/(c*x^n))])/(2*n
) + (b*e*Log[f*x^m]*PolyLog[2, -(1/(c*x^n))])/(2*n) - (b*d*PolyLog[2, 1/(c*
x^n)])/(2*n) - (b*e*Log[f*x^m]*PolyLog[2, 1/(c*x^n)])/(2*n) + (b*e*m*PolyLo
g[3, -(1/(c*x^n))])/(2*n^2) - (b*e*m*PolyLog[3, 1/(c*x^n)])/(2*n^2)
```

Rubi [A] time = 0.573596, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2301, 6742, 6096, 5913, 6072, 6070, 2374, 6589}

$$\frac{bd\text{PolyLog}\left(2,-\frac{x^{-n}}{c}\right)}{2n}-\frac{bd\text{PolyLog}\left(2,\frac{x^{-n}}{c}\right)}{2n}+\frac{be\log\left(fx^m\right)\text{PolyLog}\left(2,-\frac{x^{-n}}{c}\right)}{2n}-\frac{be\log\left(fx^m\right)\text{PolyLog}\left(2,\frac{x^{-n}}{c}\right)}{2n}+\frac{b}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcCoth[c*x^n])*(d + e*Log[f*x^m]))/x,x]
```


```
[Out] a*d*Log[x] + (a*e*Log[f*x^m]^2)/(2*m) + (b*d*PolyLog[2, -(1/(c*x^n))])/(2*n
) + (b*e*Log[f*x^m]*PolyLog[2, -(1/(c*x^n))])/(2*n) - (b*d*PolyLog[2, 1/(c*
x^n)])/(2*n) - (b*e*Log[f*x^m]*PolyLog[2, 1/(c*x^n)])/(2*n) + (b*e*m*PolyLo
g[3, -(1/(c*x^n))])/(2*n^2) - (b*e*m*PolyLog[3, 1/(c*x^n)])/(2*n^2)
```


Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcCoth[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 5913

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b*PolyLog[2, -(c*x)^(-1)])/2, x] - Simp[(b*PolyLog[2, 1/(c*x)])/
2, x]) /; FreeQ[{a, b, c}, x]
```

Rule 6072

```
Int[(Log[(d_.)*(x_)^(m_.)]*(ArcCoth[(c_.)*(x_)^(n_.)]*(b_.) + (a_)))/(x_),
x_Symbol] :> Dist[a, Int[Log[d*x^m]/x, x], x] + Dist[b, Int[(Log[d*x^m]*Arc
```

```
Coth[c*x^n])/x, x], x] /; FreeQ[{a, b, c, d, m, n}, x]
```

Rule 6070

```
Int[(ArcCoth[(c_.)*(x_)^(n_.)]*Log[(d_.)*(x_)^(m_.)])/(x_), x_Symbol] :> Di
st[1/2, Int[(Log[d*x^m]*Log[1 + 1/(c*x^n)])/x, x], x] - Dist[1/2, Int[(Log[
d*x^m]*Log[1 - 1/(c*x^n)])/x, x], x] /; FreeQ[{c, d, m, n}, x]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\coth^{-1}(cx^n)\right)\left(d+e\log\left(fx^m\right)\right)}{x}\,dx &= \int \left(\frac{d\left(a+b\coth^{-1}(cx^n)\right)}{x}+\frac{e\left(a+b\coth^{-1}(cx^n)\right)\log\left(fx^m\right)}{x}\right)dx\\
&= d\int \frac{a+b\coth^{-1}(cx^n)}{x}\,dx+e\int \frac{\left(a+b\coth^{-1}(cx^n)\right)\log\left(fx^m\right)}{x}\,dx\\
&= (ae)\int \frac{\log\left(fx^m\right)}{x}\,dx+(be)\int \frac{\coth^{-1}(cx^n)\log\left(fx^m\right)}{x}\,dx+\frac{d\,\mathrm{Subst}\left(\int \frac{a+}{}\right.}{}\\
&= ad\log(x)+\frac{ae\log^2\left(fx^m\right)}{2m}+\frac{bd\mathrm{Li}_2\left(-\frac{x^{-n}}{c}\right)}{2n}-\frac{bd\mathrm{Li}_2\left(\frac{x^{-n}}{c}\right)}{2n}-\frac{1}{2}(be)\int \frac{\log(}{}\\
&= ad\log(x)+\frac{ae\log^2\left(fx^m\right)}{2m}+\frac{bd\mathrm{Li}_2\left(-\frac{x^{-n}}{c}\right)}{2n}+\frac{be\log\left(fx^m\right)\mathrm{Li}_2\left(-\frac{x^{-n}}{c}\right)}{2n}-\frac{b}{}\\
&= ad\log(x)+\frac{ae\log^2\left(fx^m\right)}{2m}+\frac{bd\mathrm{Li}_2\left(-\frac{x^{-n}}{c}\right)}{2n}+\frac{be\log\left(fx^m\right)\mathrm{Li}_2\left(-\frac{x^{-n}}{c}\right)}{2n}-\frac{b}{}
\end{aligned}
$$

Mathematica [C] time = 0.315276, size = 131, normalized size = 0.82

$$
\frac{bcx^n\left(d+e\log\left(fx^m\right)\right)\mathrm{HypergeometricPFQ}\left(\left\{\frac{1}{2},\frac{1}{2},1\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},c^2x^{2n}\right)}{n}-\frac{bcemx^n\mathrm{HypergeometricPFQ}\left(\left\{\frac{1}{2},\frac{1}{2},\frac{1}{2},1\right\},\left\{\right.\right.}{n^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcCoth[c*x^n])*(d + e*Log[f*x^m]))/x,x]
```

```
[Out] -((b*c*e*m*x^n*HypergeometricPFQ[{1/2, 1/2, 1/2, 1}, {3/2, 3/2, 3/2}, c^2*x
^(2*n)])/n^2) + (b*c*x^n*HypergeometricPFQ[{1/2, 1/2, 1}, {3/2, 3/2}, c^2*x
^(2*n)]*(d + e*Log[f*x^m]))/n - ((a + b*ArcCoth[c*x^n] - b*ArcTanh[c*x^n])*
Log[x]*(e*m*Log[x] - 2*(d + e*Log[f*x^m])))/2
```

Maple [C] time = 0.47, size = 920, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(c*x^n))*(d+e*ln(f*x^m))/x,x)
```

```
[Out] 1/4*I/n*dilog(c*x^n)*Pi*b*e*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/2/n*dilog
(c*x^n+1)*b*d+1/n*ln(x^n)*a*d+1/2*e*a/m*ln(x^m)^2+1/4*I/n*Pi*dilog(c*x^n+1)
*b*e*csgn(I*f*x^m)^3+1/2*I/n*Pi*ln(x^n)*a*e*csgn(I*x^m)*csgn(I*f*x^m)^2-1/4
*I/n*Pi*dilog(c*x^n+1)*b*e*csgn(I*x^m)*csgn(I*f*x^m)^2+1/4*I/n*dilog(c*x^n)
*Pi*b*e*csgn(I*f*x^m)^3-1/4*I/n*ln(c*x^n)*Pi*ln(c*x^n-1)*b*e*csgn(I*f)*csgn
(I*f*x^m)^2-1/4*I/n*ln(c*x^n)*Pi*ln(c*x^n-1)*b*e*csgn(I*x^m)*csgn(I*f*x^m)^
2+1/4*I/n*Pi*dilog(c*x^n+1)*b*e*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/2*I/n
*Pi*ln(x^n)*a*e*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/2*I/n*Pi*ln(x^n)*a*e*
csgn(I*f)*csgn(I*f*x^m)^2-1/4*I/n*dilog(c*x^n)*Pi*b*e*csgn(I*x^m)*csgn(I*f*
x^m)^2-1/2*e*b/n*m*ln(x)*polylog(2,-c*x^n)+1/2*e*b/n*dilog(c*x^n+1)*m*ln(x)
+1/2*e*b/n*m*ln(x)*polylog(2,c*x^n)-1/2*e*b/n*ln(1-c*x^n)*ln(c*x^n)*ln(x^m)
+1/2*e*b/n*dilog(c*x^n)*m*ln(x)-1/4*I/n*Pi*dilog(c*x^n+1)*b*e*csgn(I*f)*csg
n(I*f*x^m)^2-1/2*e*b/n*dilog(c*x^n)*ln(x^m)-1/2/n*ln(f)*dilog(c*x^n+1)*b*e+
1/n*ln(f)*ln(x^n)*a*e+1/2*e*b*ln(x^m)*ln(1-c*x^n)*ln(x)-1/2/n*dilog(c*x^n)*
ln(f)*b*e-1/2*e*b*ln(c*x^n-1)*ln(x^m)*ln(x)-1/4*e*b*m*ln(x)^2*ln(1-c*x^n)-1
/2*e*b/n*dilog(c*x^n+1)*ln(x^m)-1/2*e*b/n^2*m*polylog(3,c*x^n)+1/4*e*b*ln(c
*x^n-1)*ln(x)^2*m-1/2/n*ln(c*x^n)*ln(c*x^n-1)*b*d+1/2*e*b/n^2*m*polylog(3,-
c*x^n)-1/2/n*ln(c*x^n)*ln(f)*ln(c*x^n-1)*b*e+1/4*I/n*ln(c*x^n)*Pi*ln(c*x^n-
1)*b*e*csgn(I*f*x^m)^3-1/4*I/n*dilog(c*x^n)*Pi*b*e*csgn(I*f)*csgn(I*f*x^m)^
2+1/2*e*b/n*ln(1-c*x^n)*ln(c*x^n)*m*ln(x)-1/2*I/n*Pi*ln(x^n)*a*e*csgn(I*f*x
^m)^3-1/2/n*dilog(c*x^n)*b*d+1/4*I/n*ln(c*x^n)*Pi*ln(c*x^n-1)*b*e*csgn(I*f)
*csgn(I*x^m)*csgn(I*f*x^m)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{ae\log\left(fx^m\right)^2}{2m} + ad\log(x) - \frac{1}{4}\left(bem\log(x)^2 - 2\,be\log(x)\log(x^m) - 2\left(e\log\left(f\right) + d\right)b\log(x)\right)\log(cx^n + 1) + \frac{1}{4}\left(bem\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*e*log(f*x^m)^2/m + a*d*log(x) - 1/4*(b*e*m*log(x)^2 - 2*b*e*log(x)*lo
g(x^m) - 2*(e*log(f) + d)*b*log(x))*log(c*x^n + 1) + 1/4*(b*e*m*log(x)^2 -
2*b*e*log(x)*log(x^m) - 2*(e*log(f) + d)*b*log(x))*log(c*x^n - 1) + integra
te(1/2*(2*b*c*e*n*x^n*log(x)*log(x^m) - (b*c*e*m*n*log(x)^2 - 2*(e*n*log(f)
 + d*n)*b*c*log(x))*x^n)/(c^2*x*x^(2*n) - x), x)
```

Fricas [C] time = 1.92843, size = 979, normalized size = 6.12

$$\frac{2\,aemn^2\log(x)^2 - 2\,bem\mathrm{polylog}\left(3, c\cosh\left(n\log(x)\right) + c\sinh\left(n\log(x)\right)\right) + 2\,bem\mathrm{polylog}\left(3, -c\cosh\left(n\log(x)\right) - c\,\mathrm{s}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*e*m*n^2*log(x)^2 - 2*b*e*m*polylog(3, c*cosh(n*log(x)) + c*sinh(n*
log(x))) + 2*b*e*m*polylog(3, -c*cosh(n*log(x)) - c*sinh(n*log(x))) + 2*(b*
e*m*n*log(x) + b*e*n*log(f) + b*d*n)*dilog(c*cosh(n*log(x)) + c*sinh(n*log(
x))) - 2*(b*e*m*n*log(x) + b*e*n*log(f) + b*d*n)*dilog(-c*cosh(n*log(x)) -
c*sinh(n*log(x))) - (b*e*m*n^2*log(x)^2 + 2*(b*e*n^2*log(f) + b*d*n^2)*log(
x))*log(c*cosh(n*log(x)) + c*sinh(n*log(x)) + 1) + (b*e*m*n^2*log(x)^2 + 2*
(b*e*n^2*log(f) + b*d*n^2)*log(x))*log(-c*cosh(n*log(x)) - c*sinh(n*log(x))
 + 1) + 4*(a*e*n^2*log(f) + a*d*n^2)*log(x) + (b*e*m*n^2*log(x)^2 + 2*(b*e*
n^2*log(f) + b*d*n^2)*log(x))*log((c*cosh(n*log(x)) + c*sinh(n*log(x)) + 1)
/(c*cosh(n*log(x)) + c*sinh(n*log(x)) - 1)))/n^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(c*x**n))*(d+e*ln(f*x**m))/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcoth}(cx^n) + a)\left(e\log\left(fx^m\right) + d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x^n) + a)*(e*log(f*x^m) + d)/x, x)
```

3.266 $\int x^5 \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$

Optimal. Leaf size=297

$$\frac{1}{6}x^6\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{ex^4\left(a+b\coth^{-1}(cx)\right)}{12c^2}-\frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{6c^4}-\frac{e\log\left(1-c^2x^2\right)\left(a+\right.}{6c^6}$$

```
[Out] (b*(3*d - e)*x)/(18*c^5) - (137*b*e*x)/(180*c^5) + (b*(3*d - e)*x^3)/(54*c^
3) - (47*b*e*x^3)/(540*c^3) + (b*(3*d - e)*x^5)/(90*c) - (b*e*x^5)/(75*c) -
 (e*x^2*(a + b*ArcCoth[c*x]))/(6*c^4) - (e*x^4*(a + b*ArcCoth[c*x]))/(12*c^
2) - (e*x^6*(a + b*ArcCoth[c*x]))/18 - (b*(3*d - e)*ArcTanh[c*x])/(18*c^6)
+ (137*b*e*ArcTanh[c*x])/(180*c^6) + (b*e*x*Log[1 - c^2*x^2])/(6*c^5) + (b*
e*x^3*Log[1 - c^2*x^2])/(18*c^3) + (b*e*x^5*Log[1 - c^2*x^2])/(30*c) - (e*(
a + b*ArcCoth[c*x])*Log[1 - c^2*x^2])/(6*c^6) + (x^6*(a + b*ArcCoth[c*x])*(
d + e*Log[1 - c^2*x^2]))/6
```

Rubi [A] time = 0.385512, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {2454, 2395, 43, 6084, 321, 207, 302, 2528, 2448, 206, 2455}

$$\frac{1}{6}x^6\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{ex^4\left(a+b\coth^{-1}(cx)\right)}{12c^2}-\frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{6c^4}-\frac{e\log\left(1-c^2x^2\right)\left(a+\right.}{6c^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```


```
[Out] (b*(3*d - e)*x)/(18*c^5) - (137*b*e*x)/(180*c^5) + (b*(3*d - e)*x^3)/(54*c^
3) - (47*b*e*x^3)/(540*c^3) + (b*(3*d - e)*x^5)/(90*c) - (b*e*x^5)/(75*c) -
 (e*x^2*(a + b*ArcCoth[c*x]))/(6*c^4) - (e*x^4*(a + b*ArcCoth[c*x]))/(12*c^
2) - (e*x^6*(a + b*ArcCoth[c*x]))/18 - (b*(3*d - e)*ArcTanh[c*x])/(18*c^6)
+ (137*b*e*ArcTanh[c*x])/(180*c^6) + (b*e*x*Log[1 - c^2*x^2])/(6*c^5) + (b*
e*x^3*Log[1 - c^2*x^2])/(18*c^3) + (b*e*x^5*Log[1 - c^2*x^2])/(30*c) - (e*(
a + b*ArcCoth[c*x])*Log[1 - c^2*x^2])/(6*c^6) + (x^6*(a + b*ArcCoth[c*x])*(
d + e*Log[1 - c^2*x^2]))/6
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6084

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2]
), x]}, Dist[a + b*ArcCoth[c*x], u, x] - Dist[b*c, Int[ExpandIntegrand[u/(1
 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)
/2, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^5\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)dx=-\frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{6c^4}-\frac{ex^4\left(a+b\coth^{-1}(cx)\right)}{12c^2}-\frac{1}{18}ex^6\left(a+b\right.$$

$$=-\frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{6c^4}-\frac{ex^4\left(a+b\coth^{-1}(cx)\right)}{12c^2}-\frac{1}{18}ex^6\left(a+b\right.$$

$$=-\frac{bex}{6c^5}-\frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{6c^4}-\frac{ex^4\left(a+b\coth^{-1}(cx)\right)}{12c^2}-\frac{1}{18}ex^6$$

$$=\frac{b(3d-e)x}{18c^5}-\frac{bex}{4c^5}+\frac{b(3d-e)x^3}{54c^3}-\frac{bex^3}{36c^3}+\frac{b(3d-e)x^5}{90c}-\frac{ex^2\left(a+\right.}{}$$

$$=\frac{b(3d-e)x}{18c^5}-\frac{bex}{4c^5}+\frac{b(3d-e)x^3}{54c^3}-\frac{bex^3}{36c^3}+\frac{b(3d-e)x^5}{90c}-\frac{ex^2\left(a+\right.}{}$$

$$=\frac{b(3d-e)x}{18c^5}-\frac{7bex}{12c^5}+\frac{b(3d-e)x^3}{54c^3}-\frac{bex^3}{36c^3}+\frac{b(3d-e)x^5}{90c}-\frac{ex^2\left(a+\right.}{}$$

$$=\frac{b(3d-e)x}{18c^5}-\frac{137bex}{180c^5}+\frac{b(3d-e)x^3}{54c^3}-\frac{47bex^3}{540c^3}+\frac{b(3d-e)x^5}{90c}-\frac{bex}{75}$$

$$=\frac{b(3d-e)x}{18c^5}-\frac{137bex}{180c^5}+\frac{b(3d-e)x^3}{54c^3}-\frac{47bex^3}{540c^3}+\frac{b(3d-e)x^5}{90c}-\frac{bex}{75}$$

Mathematica [A] time = 0.171192, size = 236, normalized size = 0.79

$$\frac{20e\log\left(1-c^2x^2\right)\left(15ac^6x^6+bcx\left(3c^4x^4+5c^2x^2+15\right)+15b\left(c^6x^6-1\right)\coth^{-1}(cx)\right)+15\log(1-cx)(-20ae+10bd-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (30*b*c*(10*d - 49*e)*x - 300*a*c^2*e*x^2 + 10*b*c^3*(10*d - 19*e)*x^3 - 15
0*a*c^4*e*x^4 + 4*b*c^5*(15*d - 11*e)*x^5 + 100*a*c^6*(3*d - e)*x^6 - 50*b*
c^2*x^2*(-6*c^4*d*x^4 + e*(6 + 3*c^2*x^2 + 2*c^4*x^4))*ArcCoth[c*x] + 15*(1
0*b*d - 20*a*e - 49*b*e)*Log[1 - c*x] - 15*(10*b*d + 20*a*e - 49*b*e)*Log[1
 + c*x] + 20*e*(15*a*c^6*x^6 + b*c*x*(15 + 5*c^2*x^2 + 3*c^4*x^4) + 15*b*(-
1 + c^6*x^6)*ArcCoth[c*x])*Log[1 - c^2*x^2])/(1800*c^6)
```

Maple [C] time = 10.293, size = 4034, normalized size = 13.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1)),x)
```

```
[Out] -1/3*b*ln((c*x+1)/(c*x-1)-1)*arccoth(c*x)*x^6*e+1/3*b*ln(2)*arccoth(c*x)*x^
6*e-1/3/c^5*b*ln((c*x+1)/(c*x-1)-1)*x*e-1/12/c^2*b*arccoth(c*x)*x^4*e-1/6/c
^4*b*arccoth(c*x)*x^2*e-1/3/c^6*b*arccoth(c*x)*ln(2)*e+1/3/c^6*b*arccoth(c*
x)*e*ln((c*x+1)/(c*x-1)-1)+1/15/c*b*ln(2)*x^5*e+1/3/c^5*b*ln(2)*x*e+1/9/c^3
*b*ln(2)*x^3*e-1/15/c*b*ln((c*x+1)/(c*x-1)-1)*x^5*e-1/9/c^3*b*ln((c*x+1)/(c
*x-1)-1)*x^3*e-23/90/c^6*b*d-49/60*b*e*x/c^5-19/180*b*e*x^3/c^3-11/450*b*e*
```

```
x^5/c-1/12*I/c^6*b*arccoth(c*x)*Pi*e*csgn(I*((c*x+1)/(c*x-1)-1)^2)*csgn(I*(
(c*x+1)/(c*x-1)-1))^2-1/60*I/c*b*csgn(I*(c*x+1)/(c*x-1))*csgn(I/((c*x-1)/(c
*x+1))^(1/2))^2*Pi*x^5*e-1/36*I/c^3*b*csgn(I*(c*x+1)/(c*x-1))*csgn(I/((c*x-
1)/(c*x+1))^(1/2))^2*Pi*x^3*e-1/12*I/c^5*b*csgn(I*(c*x+1)/(c*x-1))*csgn(I/(
(c*x-1)/(c*x+1))^(1/2))^2*Pi*x*e+1/18*I/c^3*b*csgn(I*(c*x+1)/(c*x-1))^2*csg
n(I/((c*x-1)/(c*x+1))^(1/2))*Pi*x^3*e+1/60*I/c*b*csgn(I*(c*x+1)/(c*x-1)/((c
*x+1)/(c*x-1)-1)^2)^2*csgn(I/((c*x+1)/(c*x-1)-1)^2)*Pi*x^5*e+1/36*I/c^3*b*c
sgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*csgn(I/((c*x+1)/(c*x-1)-1)^2
)*Pi*x^3*e+1/12*I/c^5*b*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*csg
n(I/((c*x+1)/(c*x-1)-1)^2)*Pi*x*e-1/30*I/c*b*csgn(I*((c*x+1)/(c*x-1)-1)^2)^
2*csgn(I*((c*x+1)/(c*x-1)-1))*Pi*x^5*e+71/75/c^6*b*e-1/6*I*b*csgn(I*((c*x+1
)/(c*x-1)-1)^2)^2*csgn(I*((c*x+1)/(c*x-1)-1))*Pi*arccoth(c*x)*x^6*e+1/6*I*b
*csgn(I*(c*x+1)/(c*x-1))^2*csgn(I/((c*x-1)/(c*x+1))^(1/2))*Pi*arccoth(c*x)*
x^6*e-1/12*I*b*csgn(I*(c*x+1)/(c*x-1))*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*Pi
*arccoth(c*x)*x^6*e+1/12*I*b*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^
2*csgn(I/((c*x+1)/(c*x-1)-1)^2)*Pi*arccoth(c*x)*x^6*e+1/12*I*b*csgn(I*(c*x+
1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*(c*x+1)/(c*x-1))*Pi*arccoth(c*x)
*x^6*e+1/12*I*b*csgn(I*((c*x+1)/(c*x-1)-1)^2)*csgn(I*((c*x+1)/(c*x-1)-1))^2
*Pi*arccoth(c*x)*x^6*e-1/12*I/c^6*b*arccoth(c*x)*Pi*e*csgn(I*(c*x+1)/(c*x-1
)/((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*(c*x+1)/(c*x-1))-1/18*I/c^3*b*csgn(I*((c*
x+1)/(c*x-1)-1)^2)^2*csgn(I*((c*x+1)/(c*x-1)-1))*Pi*x^3*e+1/30*I/c*b*csgn(I
*(c*x+1)/(c*x-1))^2*csgn(I/((c*x-1)/(c*x+1))^(1/2))*Pi*x^5*e+1/6*I/c^5*b*cs
gn(I*(c*x+1)/(c*x-1))^2*csgn(I/((c*x-1)/(c*x+1))^(1/2))*Pi*x*e+1/60*I/c*b*c
sgn(I*((c*x+1)/(c*x-1)-1)^2)*csgn(I*((c*x+1)/(c*x-1)-1))^2*Pi*x^5*e+1/36*I/
c^3*b*csgn(I*((c*x+1)/(c*x-1)-1)^2)*csgn(I*((c*x+1)/(c*x-1)-1))^2*Pi*x^3*e+
1/60*I/c*b*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*(c*x+1)/(
c*x-1))*Pi*x^5*e+1/12*I/c^5*b*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)
^2*csgn(I*(c*x+1)/(c*x-1))*Pi*x*e+1/36*I/c^3*b*csgn(I*(c*x+1)/(c*x-1)/((c*x
+1)/(c*x-1)-1)^2)^2*csgn(I*(c*x+1)/(c*x-1))*Pi*x^3*e-1/18*a*e*x^6+1/6*x^6*a
*d+23/180*I/c^6*b*e*Pi*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)
/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1))-1/12*I/c^6*b*arccoth(c*x)*e
*Pi*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1
)^2)^2-1/6*I/c^5*b*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*((c*x+1)/(c*x-1)-
1))*Pi*x*e+1/6*I/c^6*b*arccoth(c*x)*e*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2*cs
gn(I*((c*x+1)/(c*x-1)-1))-1/6*I/c^6*b*arccoth(c*x)*e*Pi*csgn(I/((c*x-1)/(c*
x+1))^(1/2))*csgn(I*(c*x+1)/(c*x-1))^2+1/12*I/c^6*b*arccoth(c*x)*Pi*e*csgn(
I/((c*x-1)/(c*x+1))^(1/2))^2*csgn(I*(c*x+1)/(c*x-1))+1/12*I/c^5*b*csgn(I*((
c*x+1)/(c*x-1)-1)^2)*csgn(I*((c*x+1)/(c*x-1)-1))^2*Pi*x*e-1/36*I/c^3*b*csgn
(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1))*csgn(I/((
c*x+1)/(c*x-1)-1)^2)*Pi*x^3*e-1/12*I/c^5*b*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/
(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1))*csgn(I/((c*x+1)/(c*x-1)-1)^2)*Pi*x*e+
1/12*I/c^6*b*arccoth(c*x)*e*Pi*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)
/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1))-1/60*I/c*b*csgn(I*(
c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1))*csgn(I/((c*x+
1)/(c*x-1)-1)^2)*Pi*x^5*e-1/12*I*b*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-
1)^2)*csgn(I*(c*x+1)/(c*x-1))*csgn(I/((c*x+1)/(c*x-1)-1)^2)*Pi*arccoth(c*x)
*x^6*e-1/12*I/c^6*b*arccoth(c*x)*e*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^3-1/12*
I/c^6*b*arccoth(c*x)*Pi*e*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3+1
/6*I/c^6*b*arccoth(c*x)*e*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^
2-23/180*I/c^6*b*e*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*csgn(
I*(c*x+1)/(c*x-1))+23/180*I/c^6*b*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*cs
gn(I*(c*x+1)/(c*x-1))-23/90*I/c^6*b*e*Pi+1/36*I/c^3*b*csgn(I*((c*x+1)/(c*x-
1)-1)^2)^3*Pi*x^3*e+1/12*I/c^6*b*arccoth(c*x)*e*Pi*csgn(I*(c*x+1)/(c*x-1))^
3+1/12*I*b*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3*Pi*arccoth(c*x)*
x^6*e-1/6*I*b*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*Pi*arccoth(c*
x)*x^6*e-1/12*I*b*csgn(I*(c*x+1)/(c*x-1))^3*Pi*arccoth(c*x)*x^6*e+1/12*I*b*
csgn(I*((c*x+1)/(c*x-1)-1)^2)^3*Pi*arccoth(c*x)*x^6*e+1/12*I/c^5*b*csgn(I*(
(c*x+1)/(c*x-1)-1)^2)^3*Pi*x*e-23/90*I/c^6*b*e*Pi*csgn(I/((c*x-1)/(c*x+1))^
(1/2))*csgn(I*(c*x+1)/(c*x-1))^2-23/180*I/c^6*b*e*Pi*csgn(I/((c*x+1)/(c*x-1
```

```
)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2-23/180*I/c^6*b*e*Pi
*csgn(I*((c*x+1)/(c*x-1)-1)^2)*csgn(I*((c*x+1)/(c*x-1)-1))^2+23/90*I/c^6*b*
e*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*((c*x+1)/(c*x-1)-1))-1/60*I/c*b
*csgn(I*(c*x+1)/(c*x-1))^3*Pi*x^5*e-1/36*I/c^3*b*csgn(I*(c*x+1)/(c*x-1))^3*
Pi*x^3*e-1/12*I/c^5*b*csgn(I*(c*x+1)/(c*x-1))^3*Pi*x*e+1/60*I/c*b*csgn(I*(c
*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3*Pi*x^5*e+1/36*I/c^3*b*csgn(I*(c*x+1)
/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3*Pi*x^3*e+1/12*I/c^5*b*csgn(I*(c*x+1)/(c*x
-1)/((c*x+1)/(c*x-1)-1)^2)^3*Pi*x*e-1/30*I/c*b*csgn(I*(c*x+1)/(c*x-1)/((c*x
+1)/(c*x-1)-1)^2)^2*Pi*x^5*e-1/18*I/c^3*b*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(
c*x-1)-1)^2)^2*Pi*x^3*e-1/6*I/c^5*b*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)
-1)^2)^2*Pi*x*e+1/60*I/c*b*csgn(I*((c*x+1)/(c*x-1)-1)^2)^3*Pi*x^5*e-23/180*
I/c^6*b*e*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^3+23/180*I/c^6*b*e*Pi*csgn(I*(c*
x+1)/(c*x-1))^3-23/180*I/c^6*b*e*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)
-1)^2)^3+23/90*I/c^6*b*e*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2
-1/6*I/c^6*b*Pi*e*arccoth(c*x)+1/30*I/c*b*Pi*x^5*e+1/18*I/c^3*b*Pi*x^3*e+1/
6*I/c^5*b*Pi*x*e+1/6*I*b*Pi*arccoth(c*x)*x^6*e-1/90/c^6*b*e*(15*arccoth(c*x
)*x^5*c^5+15*arccoth(c*x)*x^4*c^4+3*c^4*x^4+15*arccoth(c*x)*x^3*c^3+3*c^3*x
^3+15*arccoth(c*x)*x^2*c^2+8*c^2*x^2+15*arccoth(c*x)*x*c+8*c*x+15*arccoth(c
*x)+23)*(c*x-1)*ln((c*x-1)/(c*x+1))-1/12*a*e/c^2*x^4-1/6*a*e/c^4*x^2+1/6*a*
e*x^6*ln(-c^2*x^2+1)-1/6*a*e/c^6*ln(c^2*x^2-1)-1/6/c^6*b*arccoth(c*x)*d+1/1
8/c^3*b*x^3*d+1/6/c^5*b*x*d+1/30/c*b*x^5*d-1/18*b*arccoth(c*x)*x^6*e+1/6*b*
arccoth(c*x)*x^6*d+239/180/c^6*b*arccoth(c*x)*e-23/45/c^6*b*e*ln(2)
```

Maxima [C] time = 1.11322, size = 447, normalized size = 1.51

$$\frac{1}{6}adx^6+\frac{1}{36}\left(6x^6\log\left(-c^2x^2+1\right)-c^2\left(\frac{2c^4x^6+3c^2x^4+6x^2}{c^6}+\frac{6\log\left(c^2x^2-1\right)}{c^8}\right)\right)be\,\mathrm{arcoth}\,(cx)+\frac{1}{180}\left(30x^6\,\mathrm{arcoth}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima
")
```

```
[Out] 1/6*a*d*x^6 + 1/36*(6*x^6*log(-c^2*x^2 + 1) - c^2*((2*c^4*x^6 + 3*c^2*x^4 +
 6*x^2)/c^6 + 6*log(c^2*x^2 - 1)/c^8))*b*e*arccoth(c*x) + 1/180*(30*x^6*arc
coth(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 +
 15*log(c*x - 1)/c^7))*b*d + 1/36*(6*x^6*log(-c^2*x^2 + 1) - c^2*((2*c^4*x^
6 + 3*c^2*x^4 + 6*x^2)/c^6 + 6*log(c^2*x^2 - 1)/c^8))*a*e - 1/1800*(4*(-15*
I*pi*c^5 + 11*c^5)*x^5 + 10*(-10*I*pi*c^3 + 19*c^3)*x^3 + 30*(-10*I*pi*c +
49*c)*x - (-150*I*pi + 60*c^5*x^5 + 100*c^3*x^3 + 300*c*x + 735)*log(c*x +
1) - (150*I*pi + 60*c^5*x^5 + 100*c^3*x^3 + 300*c*x - 735)*log(c*x - 1))*b*
e/c^6
```

Fricas [A] time = 1.69584, size = 579, normalized size = 1.95

$$-\frac{150\,ac^4ex^4-100\left(3\,ac^6d-ac^6e\right)x^6+300\,ac^2ex^2-4\left(15\,bc^5d-11\,bc^5e\right)x^5-10\left(10\,bc^3d-19\,bc^3e\right)x^3-30\left(10\,bcd-4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas
")
```

```
[Out] -1/1800*(150*a*c^4*e*x^4 - 100*(3*a*c^6*d - a*c^6*e)*x^6 + 300*a*c^2*e*x^2
- 4*(15*b*c^5*d - 11*b*c^5*e)*x^5 - 10*(10*b*c^3*d - 19*b*c^3*e)*x^3 - 30*(
```

```
10*b*c*d - 49*b*c*e)*x - 20*(15*a*c^6*e*x^6 + 3*b*c^5*e*x^5 + 5*b*c^3*e*x^3
 + 15*b*c*e*x - 15*a*e)*log(-c^2*x^2 + 1) + 5*(15*b*c^4*e*x^4 - 10*(3*b*c^6
*d - b*c^6*e)*x^6 + 30*b*c^2*e*x^2 + 30*b*d - 147*b*e - 30*(b*c^6*e*x^6 - b
*e)*log(-c^2*x^2 + 1))*log((c*x + 1)/(c*x - 1)))/c^6
```

Sympy [A] time = 113.193, size = 362, normalized size = 1.22

$$\begin{cases} \frac{adx^6}{6} + \frac{aex^6\log\left(-c^2x^2+1\right)}{6} - \frac{aex^6}{18} - \frac{aex^4}{12c^2} - \frac{aex^2}{6c^4} - \frac{ae\log\left(-c^2x^2+1\right)}{6c^6} + \frac{bdx^6\operatorname{acoth}(cx)}{6} + \frac{bex^6\log\left(-c^2x^2+1\right)\operatorname{acoth}(cx)}{6} - \frac{bex^6\operatorname{acoth}(cx)}{18} + \frac{bdx^5}{30c} \\ \frac{dx^6\left(a+\frac{i\pi b}{2}\right)}{6} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**6/6 + a*e*x**6*log(-c**2*x**2 + 1)/6 - a*e*x**6/18 - a*e*
x**4/(12*c**2) - a*e*x**2/(6*c**4) - a*e*log(-c**2*x**2 + 1)/(6*c**6) + b*d
*x**6*acoth(c*x)/6 + b*e*x**6*log(-c**2*x**2 + 1)*acoth(c*x)/6 - b*e*x**6*a
coth(c*x)/18 + b*d*x**5/(30*c) + b*e*x**5*log(-c**2*x**2 + 1)/(30*c) - 11*b
*e*x**5/(450*c) - b*e*x**4*acoth(c*x)/(12*c**2) + b*d*x**3/(18*c**3) + b*e*
x**3*log(-c**2*x**2 + 1)/(18*c**3) - 19*b*e*x**3/(180*c**3) - b*e*x**2*acot
h(c*x)/(6*c**4) + b*d*x/(6*c**5) + b*e*x*log(-c**2*x**2 + 1)/(6*c**5) - 49*
b*e*x/(60*c**5) - b*d*acoth(c*x)/(6*c**6) - b*e*log(-c**2*x**2 + 1)*acoth(c
*x)/(6*c**6) + 49*b*e*acoth(c*x)/(60*c**6), Ne(c, 0)), (d*x**6*(a + I*pi*b/
2)/6, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\operatorname{arcoth}(cx) + a)\left(e\log\left(-c^2x^2+1\right) + d\right)x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)*x^5, x)
```

3.267 $\int x^3 \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$

Optimal. Leaf size=225

$$\frac{1}{4}x^4\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{4c^2}-\frac{e\log\left(1-c^2x^2\right)\left(a+b\coth^{-1}(cx)\right)}{4c^4}-\frac{1}{8}ex^4\left(a\right.$$

```
[Out] (b*(2*d - 3*e)*x)/(8*c^3) - (2*b*e*x)/(3*c^3) + (b*(2*d - e)*x^3)/(24*c) -
(b*e*x^3)/(18*c) - (e*x^2*(a + b*ArcCoth[c*x]))/(4*c^2) - (e*x^4*(a + b*Arc
Coth[c*x]))/8 - (b*(2*d - 3*e)*ArcTanh[c*x])/(8*c^4) + (2*b*e*ArcTanh[c*x])
/(3*c^4) + (b*e*x*Log[1 - c^2*x^2])/(4*c^3) + (b*e*x^3*Log[1 - c^2*x^2])/(1
2*c) - (e*(a + b*ArcCoth[c*x])*Log[1 - c^2*x^2])/(4*c^4) + (x^4*(a + b*ArcC
oth[c*x])*(d + e*Log[1 - c^2*x^2]))/4
```

Rubi [A] time = 0.256242, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {2454, 2395, 43, 6084, 459, 321, 206, 2471, 2448, 2455, 302}

$$\frac{1}{4}x^4\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{4c^2}-\frac{e\log\left(1-c^2x^2\right)\left(a+b\coth^{-1}(cx)\right)}{4c^4}-\frac{1}{8}ex^4\left(a\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```


```
[Out] (b*(2*d - 3*e)*x)/(8*c^3) - (2*b*e*x)/(3*c^3) + (b*(2*d - e)*x^3)/(24*c) -
(b*e*x^3)/(18*c) - (e*x^2*(a + b*ArcCoth[c*x]))/(4*c^2) - (e*x^4*(a + b*Arc
Coth[c*x]))/8 - (b*(2*d - 3*e)*ArcTanh[c*x])/(8*c^4) + (2*b*e*ArcTanh[c*x])
/(3*c^4) + (b*e*x*Log[1 - c^2*x^2])/(4*c^3) + (b*e*x^3*Log[1 - c^2*x^2])/(1
2*c) - (e*(a + b*ArcCoth[c*x])*Log[1 - c^2*x^2])/(4*c^4) + (x^4*(a + b*ArcC
oth[c*x])*(d + e*Log[1 - c^2*x^2]))/4
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6084

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2]
), x]}, Dist[a + b*ArcCoth[c*x], u, x] - Dist[b*c, Int[ExpandIntegrand[u/(1
 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)
/2, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int x^3\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)dx = -\frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{4c^2} - \frac{1}{8}ex^4\left(a+b\coth^{-1}(cx)\right) - \frac{e\left(a+b\coth\right.}{}$$

$$= -\frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{4c^2} - \frac{1}{8}ex^4\left(a+b\coth^{-1}(cx)\right) - \frac{e\left(a+b\coth\right.}{}$$

$$= \frac{b(2d-e)x^3}{24c} - \frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{4c^2} - \frac{1}{8}ex^4\left(a+b\coth^{-1}(cx)\right) -$$

$$= \frac{b(2d-3e)x}{8c^3} + \frac{b(2d-e)x^3}{24c} - \frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{4c^2} - \frac{1}{8}ex^4\left(a+b\right.$$

$$= \frac{b(2d-3e)x}{8c^3} + \frac{b(2d-e)x^3}{24c} - \frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{4c^2} - \frac{1}{8}ex^4\left(a+b\right.$$

$$= \frac{b(2d-3e)x}{8c^3} - \frac{bex}{2c^3} + \frac{b(2d-e)x^3}{24c} - \frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{4c^2} - \frac{1}{8}ex^4$$

$$= \frac{b(2d-3e)x}{8c^3} - \frac{2bex}{3c^3} + \frac{b(2d-e)x^3}{24c} - \frac{bex^3}{18c} - \frac{ex^2\left(a+b\coth^{-1}(cx)\right.}{4c^2}$$

$$= \frac{b(2d-3e)x}{8c^3} - \frac{2bex}{3c^3} + \frac{b(2d-e)x^3}{24c} - \frac{bex^3}{18c} - \frac{ex^2\left(a+b\coth^{-1}(cx)\right.}{4c^2}$$

Mathematica [A] time = 0.144613, size = 192, normalized size = 0.85

$$\frac{12e\log\left(1-c^2x^2\right)\left(3ac^4x^4+bcx\left(c^2x^2+3\right)+3b\left(c^4x^4-1\right)\coth^{-1}(cx)\right)+3\log(1-cx)(-12ae+6bd-25be)-3\log(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (6*b*c*(6*d - 25*e)*x - 36*a*c^2*e*x^2 + 2*b*c^3*(6*d - 7*e)*x^3 + 18*a*c^4
*(2*d - e)*x^4 - 18*b*c^2*x^2*(-2*c^2*d*x^2 + e*(2 + c^2*x^2))*ArcCoth[c*x]
 + 3*(6*b*d - 12*a*e - 25*b*e)*Log[1 - c*x] - 3*(6*b*d + 12*a*e - 25*b*e)*L
og[1 + c*x] + 12*e*(3*a*c^4*x^4 + b*c*x*(3 + c^2*x^2) + 3*b*(-1 + c^4*x^4)*
ArcCoth[c*x])*Log[1 - c^2*x^2])/(144*c^4)
```

Maple [C] time = 6.22, size = 3320, normalized size = 14.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1)),x)
```

```
[Out] -1/8*a*x^4*e-25/24*b*e*x/c^3-7/72*b*e*x^3/c+1/4*x^4*a*d+1/12/c*b*x^3*d+1/4/
c^3*b*x*d-1/4/c^4*b*arccoth(c*x)*d+41/24/c^4*b*arccoth(c*x)*e+1/4*b*arccoth
(c*x)*x^4*d-1/8*b*arccoth(c*x)*x^4*e+41/36/c^4*b*e-2/3/c^4*b*e*ln(2)-1/3*I/
c^4*b*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn(I*(c*x+1)/(c*x-1))^2+1/24*I
/c*b*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^3*x^3*e+1/24*I/c*b*Pi*csgn(I*(c*x+1)/
(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3*x^3*e-1/3/c^4*b*d-1/4*a*e/c^2*x^2+1/4*a*e*
x^4*ln(-c^2*x^2+1)-1/4*a*e/c^4*ln(c^2*x^2-1)+1/4*I/c^3*b*Pi*x*e-1/4*I/c^4*b
*Pi*e*arccoth(c*x)+1/6/c*b*ln(2)*x^3*e-1/6/c*b*ln((c*x+1)/(c*x-1)-1)*x^3*e-
1/4/c^2*b*arccoth(c*x)*x^2*e+1/2/c^3*b*ln(2)*x*e-1/2/c^3*b*ln((c*x+1)/(c*x-
```

1)-1)*x*e-1/2/c^4*b*arccoth(c*x)*ln(2)*e+1/2/c^4*b*arccoth(c*x)*e*ln((c*x+1
)/(c*x-1)-1)+1/2*b*ln(2)*arccoth(c*x)*x^4*e-1/2*b*ln((c*x+1)/(c*x-1)-1)*arc
coth(c*x)*x^4*e-1/3*I/c^4*b*e*Pi-1/24*I/c*b*Pi*csgn(I/((c*x+1)/(c*x-1)-1)^2
)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1))*x^3
*e-1/8*I/c^3*b*Pi*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*
x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1))*x*e+1/8*I/c^4*b*arccoth(c*x)*e*P
i*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^
2)*csgn(I*(c*x+1)/(c*x-1))-1/8*I*b*Pi*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*
(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1))*arccoth(c*x)
*x^4*e-1/12/c^4*b*e*(3*arccoth(c*x)*x^3*c^3+3*arccoth(c*x)*x^2*c^2+c^2*x^2+
3*arccoth(c*x)*x*c+c*x+3*arccoth(c*x)+4)*(c*x-1)*ln((c*x-1)/(c*x+1))+1/4*I*
b*Pi*arccoth(c*x)*x^4*e+1/12*I/c*b*Pi*x^3*e-1/6*I/c^4*b*e*Pi*csgn(I*((c*x+1
)/(c*x-1)-1)^2)^3-1/6*I/c^4*b*e*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-
1)^2)^3+1/6*I/c^4*b*e*Pi*csgn(I*(c*x+1)/(c*x-1))^3+1/3*I/c^4*b*e*Pi*csgn(I*
(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2+1/8*I*b*Pi*csgn(I*((c*x+1)/(c*x-1)
-1)^2)^3*arccoth(c*x)*x^4*e+1/8*I*b*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x
-1)-1)^2)^3*arccoth(c*x)*x^4*e-1/8*I*b*Pi*csgn(I*(c*x+1)/(c*x-1))^3*arccoth
(c*x)*x^4*e-1/4*I*b*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*arcc
oth(c*x)*x^4*e-1/4*I/c^3*b*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)
^2*x*e+1/8*I/c^4*b*arccoth(c*x)*Pi*e*csgn(I*(c*x+1)/(c*x-1))^3+1/3*I/c^4*b*
e*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*((c*x+1)/(c*x-1)-1))-1/6*I/c^4*
b*e*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)*csgn(I*((c*x+1)/(c*x-1)-1))^2-1/24*I/c
*b*Pi*csgn(I*(c*x+1)/(c*x-1))^3*x^3*e-1/12*I/c*b*Pi*csgn(I*(c*x+1)/(c*x-1)/
((c*x+1)/(c*x-1)-1)^2)^2*x^3*e+1/8*I/c^3*b*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)
^3*x*e+1/8*I/c^3*b*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3*x*e-1
/8*I/c^3*b*Pi*csgn(I*(c*x+1)/(c*x-1))^3*x*e-1/8*I/c^4*b*arccoth(c*x)*Pi*e*c
sgn(I*((c*x+1)/(c*x-1)-1)^2)^3-1/8*I/c^4*b*arccoth(c*x)*e*Pi*csgn(I*(c*x+1)
/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3-1/6*I/c^4*b*e*Pi*csgn(I/((c*x+1)/(c*x-1)-
1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2-1/6*I/c^4*b*e*Pi*csgn
(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*(c*x+1)/(c*x-1))+1/4*I/c
^4*b*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*Pi*e*arccoth(c*x)+1/6*
I/c^4*b*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*csgn(I*(c*x+1)/(c*x-1))-1/8*
I/c^4*b*arccoth(c*x)*e*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)*csgn(I*((c*x+1)/(c*
x-1)-1))^2-1/8*I/c^4*b*arccoth(c*x)*e*Pi*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn
(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2-1/4*I/c^4*b*arccoth(c*x)*e*Pi*c
sgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn(I*(c*x+1)/(c*x-1))^2+1/6*I/c^4*b*e*Pi*c
sgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*
csgn(I*(c*x+1)/(c*x-1))-1/8*I/c^4*b*arccoth(c*x)*e*Pi*csgn(I*(c*x+1)/(c*x-1
)/((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*(c*x+1)/(c*x-1))+1/8*I/c^4*b*arccoth(c*x)
*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*csgn(I*(c*x+1)/(c*x-1))+1/8*I*b*Pi*
csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)
^2*arccoth(c*x)*x^4*e+1/8*I*b*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)
^2)^2*csgn(I*(c*x+1)/(c*x-1))*arccoth(c*x)*x^4*e-1/8*I*b*Pi*csgn(I/((c*x-1)
/(c*x+1))^(1/2))^2*csgn(I*(c*x+1)/(c*x-1))*arccoth(c*x)*x^4*e+1/4*I*b*Pi*cs
gn(I/((c*x-1)/(c*x+1))^(1/2))*csgn(I*(c*x+1)/(c*x-1))^2*arccoth(c*x)*x^4*e-
1/4*I*b*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*((c*x+1)/(c*x-1)-1))*arcc
oth(c*x)*x^4*e+1/8*I*b*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)*csgn(I*((c*x+1)/(c*
x-1)-1))^2*arccoth(c*x)*x^4*e-1/12*I/c*b*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2
csgn(I((c*x+1)/(c*x-1)-1))*x^3*e+1/24*I/c*b*Pi*csgn(I*((c*x+1)/(c*x-1)-1)
^2)*csgn(I*((c*x+1)/(c*x-1)-1))^2*x^3*e+1/24*I/c*b*Pi*csgn(I/((c*x+1)/(c*x-
1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*x^3*e+1/24*I/c*b*P
i*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*(c*x+1)/(c*x-1))*x
^3*e-1/24*I/c*b*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*csgn(I*(c*x+1)/(c*x-1)
)*x^3*e+1/12*I/c*b*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn(I*(c*x+1)/(c*x-1
))^2*x^3*e-1/4*I/c^3*b*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*((c*x+1)/(
c*x-1)-1))*x*e+1/8*I/c^3*b*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)*csgn(I*((c*x+1)
/(c*x-1)-1))^2*x*e+1/8*I/c^3*b*Pi*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x
+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*x*e+1/8*I/c^3*b*Pi*csgn(I*(c*x+1)/(c*x
-1)/((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*(c*x+1)/(c*x-1))*x*e-1/8*I/c^3*b*Pi*csg

```
n(I/((c*x-1)/(c*x+1))^(1/2))^2*csgn(I*(c*x+1)/(c*x-1))*x*e+1/4*I/c^3*b*Pi*c
sgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn(I*(c*x+1)/(c*x-1))^2*x*e+1/4*I/c^4*b*ar
ccoth(c*x)*e*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*((c*x+1)/(c*x-1)-1))
```

Maxima [C] time = 1.07594, size = 366, normalized size = 1.63

$$\frac{1}{4}adx^4 + \frac{1}{8}\left(2x^4\log\left(-c^2x^2+1\right) - c^2\left(\frac{c^2x^4+2x^2}{c^4} + \frac{2\log\left(c^2x^2-1\right)}{c^6}\right)\right)be\,\mathrm{arcoth}\,(cx) + \frac{1}{24}\left(6x^4\,\mathrm{arcoth}\,(cx) + c\left(\frac{2\left(c\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima
")
```

```
[Out] 1/4*a*d*x^4 + 1/8*(2*x^4*log(-c^2*x^2 + 1) - c^2*((c^2*x^4 + 2*x^2)/c^4 + 2
*log(c^2*x^2 - 1)/c^6))*b*e*arccoth(c*x) + 1/24*(6*x^4*arccoth(c*x) + c*(2*
(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*d + 1/8*(
2*x^4*log(-c^2*x^2 + 1) - c^2*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c
^6))*a*e - 1/144*(2*(-6*I*pi*c^3 + 7*c^3)*x^3 + 6*(-6*I*pi*c + 25*c)*x - (-
18*I*pi + 12*c^3*x^3 + 36*c*x + 75)*log(c*x + 1) - (18*I*pi + 12*c^3*x^3 +
36*c*x - 75)*log(c*x - 1))*b*e/c^4
```

Fricas [A] time = 1.68059, size = 440, normalized size = 1.96

$$-\frac{36\,ac^2ex^2 - 18\left(2\,ac^4d - ac^4e\right)x^4 - 2\left(6\,bc^3d - 7\,bc^3e\right)x^3 - 6\left(6\,bcd - 25\,bce\right)x - 12\left(3\,ac^4ex^4 + bc^3ex^3 + 3\,bcex - 3\,ae\right)}{144\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas
")
```

```
[Out] -1/144*(36*a*c^2*e*x^2 - 18*(2*a*c^4*d - a*c^4*e)*x^4 - 2*(6*b*c^3*d - 7*b*
c^3*e)*x^3 - 6*(6*b*c*d - 25*b*c*e)*x - 12*(3*a*c^4*e*x^4 + b*c^3*e*x^3 + 3
*b*c*e*x - 3*a*e)*log(-c^2*x^2 + 1) + 3*(6*b*c^2*e*x^2 - 3*(2*b*c^4*d - b*c
^4*e)*x^4 + 6*b*d - 25*b*e - 6*(b*c^4*e*x^4 - b*e)*log(-c^2*x^2 + 1))*log((
c*x + 1)/(c*x - 1)))/c^4
```

Sympy [A] time = 33.8454, size = 286, normalized size = 1.27

$$\begin{cases} \frac{adx^4}{4} + \frac{aex^4\log\left(-c^2x^2+1\right)}{4} - \frac{aex^4}{8} - \frac{aex^2}{4c^2} - \frac{ae\log\left(-c^2x^2+1\right)}{4c^4} + \frac{bdx^4\,\mathrm{acoth}\,(cx)}{4} + \frac{bex^4\log\left(-c^2x^2+1\right)\mathrm{acoth}\,(cx)}{4} - \frac{bex^4\,\mathrm{acoth}\,(cx)}{8} + \frac{bdx^3}{12c} + \\ \frac{dx^4\left(a+\frac{i\pi b}{2}\right)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**4/4 + a*e*x**4*log(-c**2*x**2 + 1)/4 - a*e*x**4/8 - a*e*x
**2/(4*c**2) - a*e*log(-c**2*x**2 + 1)/(4*c**4) + b*d*x**4*acoth(c*x)/4 + b
*e*x**4*log(-c**2*x**2 + 1)*acoth(c*x)/4 - b*e*x**4*acoth(c*x)/8 + b*d*x**3
```

```
/(12*c) + b*e*x**3*log(-c**2*x**2 + 1)/(12*c) - 7*b*e*x**3/(72*c) - b*e*x**
2*acoth(c*x)/(4*c**2) + b*d*x/(4*c**3) + b*e*x*log(-c**2*x**2 + 1)/(4*c**3)
 - 25*b*e*x/(24*c**3) - b*d*acoth(c*x)/(4*c**4) - b*e*log(-c**2*x**2 + 1)*a
coth(c*x)/(4*c**4) + 25*b*e*acoth(c*x)/(24*c**4), Ne(c, 0)), (d*x**4*(a + I
*pi*b/2)/4, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arcoth}(cx) + a)\left(e \log\left(-c^2x^2 + 1\right) + d\right)x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)*x^3, x)
```

3.268 $\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)dx$

Optimal. Leaf size=140

$$-\frac{e\left(1-c^2x^2\right)\log\left(1-c^2x^2\right)\left(a+b\coth^{-1}(cx)\right)}{2c^2}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(cx)\right)-\frac{b(d-e)\tanh^{-}}{2c^2}$$

```
[Out] (b*(d - e)*x)/(2*c) - (b*e*x)/c + (d*x^2*(a + b*ArcCoth[c*x]))/2 - (e*x^2*(
a + b*ArcCoth[c*x]))/2 - (b*(d - e)*ArcTanh[c*x])/(2*c^2) + (b*e*ArcTanh[c*
x])/c^2 + (b*e*x*Log[1 - c^2*x^2])/(2*c) - (e*(1 - c^2*x^2)*(a + b*ArcCoth[
c*x])*Log[1 - c^2*x^2])/(2*c^2)
```

Rubi [A] time = 0.118628, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {2454, 2389, 2295, 6084, 321, 207, 2448, 206}

$$-\frac{e\left(1-c^2x^2\right)\log\left(1-c^2x^2\right)\left(a+b\coth^{-1}(cx)\right)}{2c^2}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(cx)\right)-\frac{b(d-e)\tanh^{-}}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (b*(d - e)*x)/(2*c) - (b*e*x)/c + (d*x^2*(a + b*ArcCoth[c*x]))/2 - (e*x^2*(
a + b*ArcCoth[c*x]))/2 - (b*(d - e)*ArcTanh[c*x])/(2*c^2) + (b*e*ArcTanh[c*
x])/c^2 + (b*e*x*Log[1 - c^2*x^2])/(2*c) - (e*(1 - c^2*x^2)*(a + b*ArcCoth[
c*x])*Log[1 - c^2*x^2])/(2*c^2)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 6084

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2]
), x]}, Dist[a + b*ArcCoth[c*x], u, x] - Dist[b*c, Int[ExpandIntegrand[u/(1
 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)
/2, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)dx=\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(cx)\right)-\frac{e\left(1-c^2x^2\right)\left(a+\right.}{}$$

$$=\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(cx)\right)-\frac{e\left(1-c^2x^2\right)\left(a+\right.}{}$$

$$=\frac{b(d-e)x}{2c}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(cx)\right)+\frac{bex}{}$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(cx)\right)$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(cx)\right)$$

Mathematica [A] time = 0.105695, size = 129, normalized size = 0.92

$$\frac{2e\log\left(1-c^2x^2\right)\left(cx(acx+b)+b\left(c^2x^2-1\right)\coth^{-1}(cx)\right)+\log(1-cx)(b(d-3e)-2ae)-\log(cx+1)(2ae+b(d-3e))+2}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (2*b*c*(d - 3*e)*x + 2*a*c^2*(d - e)*x^2 + 2*b*c^2*(d - e)*x^2*ArcCoth[c*x]
 + (b*(d - 3*e) - 2*a*e)*Log[1 - c*x] - (b*(d - 3*e) + 2*a*e)*Log[1 + c*x]
+ 2*e*(c*x*(b + a*c*x) + b*(-1 + c^2*x^2)*ArcCoth[c*x])*Log[1 - c^2*x^2])/(
4*c^2)
```

Maple [C] time = 0.602, size = 2616, normalized size = 18.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1)),x)
```

```
[Out] -1/2*a*e*x^2+1/4*I/c^2*b*arccoth(c*x)*e*Pi*csgn(I*(c*x+1)/(c*x-1))^3+1/4*I*
b*arccoth(c*x)*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3*x^2*e+1/4
*I*b*arccoth(c*x)*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^3*x^2*e-1/4*I*b*arccoth(
c*x)*Pi*csgn(I*(c*x+1)/(c*x-1))^3*x^2*e-1/2*I*b*arccoth(c*x)*Pi*csgn(I*(c*x
+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*x^2*e-1/2*I/c*b*Pi*csgn(I*(c*x+1)/(c*x
-1)/((c*x+1)/(c*x-1)-1)^2)^2*x*e+1/2*I/c^2*b*arccoth(c*x)*e*Pi*csgn(I*(c*x+
1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2-1/4*I/c^2*b*e*Pi*csgn(I*(c*x+1)/(c*x-1)
/((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*(c*x+1)/(c*x-1))-1/4*I/c^2*b*e*Pi*csgn(I/(
(c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2+1/2*I
/c^2*b*e*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*((c*x+1)/(c*x-1)-1))-1/4
*I/c^2*b*e*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)*csgn(I*((c*x+1)/(c*x-1)-1))^2+1
/2*a*e/c^2+1/2*d*a*x^2+1/2*a*e*ln(-c^2*x^2+1)*x^2-1/2*a*e/c^2*ln(-c^2*x^2+1
)+1/2/c*b*x*d+3/2/c^2*b*e-1/c^2*b*e*ln(2)-3/2*b*e*x/c-1/2/c^2*b*d-1/2/c^2*b
*arccoth(c*x)*d+5/2/c^2*b*arccoth(c*x)*e+1/2*b*arccoth(c*x)*x^2*d-1/2*b*arc
coth(c*x)*x^2*e-1/2*I*b*arccoth(c*x)*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2*csg
n(I*((c*x+1)/(c*x-1)-1))*x^2*e+1/4*I*b*arccoth(c*x)*Pi*csgn(I*((c*x+1)/(c*x
-1)-1)^2)*csgn(I*((c*x+1)/(c*x-1)-1))^2*x^2*e-1/4*I*b*arccoth(c*x)*Pi*csgn(
I/((c*x-1)/(c*x+1))^(1/2))^2*csgn(I*(c*x+1)/(c*x-1))*x^2*e+1/2*I*b*arccoth(
c*x)*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn(I*(c*x+1)/(c*x-1))^2*x^2*e+1/4
*I/c*b*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*(c*x+1)/(c
*x-1))*x*e+1/4*I/c*b*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*csg
n(I/((c*x+1)/(c*x-1)-1)^2)*x*e-1/2*I/c*b*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2
*csgn(I*((c*x+1)/(c*x-1)-1))*x*e+1/4*I/c*b*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)
*csgn(I*((c*x+1)/(c*x-1)-1))^2*x*e-1/4*I/c*b*Pi*csgn(I/((c*x-1)/(c*x+1))^(1
/2))^2*csgn(I*(c*x+1)/(c*x-1))*x*e+1/2*I/c*b*Pi*csgn(I/((c*x-1)/(c*x+1))^(1
/2))*csgn(I*(c*x+1)/(c*x-1))^2*x*e-1/4*I/c^2*b*arccoth(c*x)*e*Pi*csgn(I*(c*
x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*(c*x+1)/(c*x-1))-1/4*I/c^2*b*a
rccoth(c*x)*e*Pi*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x
+1)/(c*x-1)-1)^2)^2+1/2*I/c^2*b*arccoth(c*x)*e*Pi*csgn(I*((c*x+1)/(c*x-1)-1
)^2)^2*csgn(I*((c*x+1)/(c*x-1)-1))-1/4*I/c^2*b*arccoth(c*x)*e*Pi*csgn(I*((c
*x+1)/(c*x-1)-1)^2)*csgn(I*((c*x+1)/(c*x-1)-1))^2+1/4*I/c^2*b*arccoth(c*x)*
e*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*csgn(I*(c*x+1)/(c*x-1))-1/2*I/c^2*b*
arccoth(c*x)*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn(I*(c*x+1)/(c*x-1))^2
+1/4*I/c^2*b*e*Pi*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*
x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1))+1/4*I*b*arccoth(c*x)*Pi*csgn(I*(
c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*(c*x+1)/(c*x-1))*x^2*e+1/4*I
*b*arccoth(c*x)*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*csgn(I/(
(c*x+1)/(c*x-1)-1)^2)*x^2*e-1/2*I/c^2*b*Pi*e-1/2/c^2*b*e*(arccoth(c*x)*x*c+
arccoth(c*x)+1)*(c*x-1)*ln((c*x-1)/(c*x+1))-1/4*I/c^2*b*e*Pi*csgn(I*(c*x+1)
/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3-1/4*I/c^2*b*e*Pi*csgn(I*((c*x+1)/(c*x-1)-
1)^2)^3+1/4*I/c^2*b*Pi*e*csgn(I*(c*x+1)/(c*x-1))^3+1/2*I*b*arccoth(c*x)*Pi*
x^2*e+1/2*I/c^2*b*e*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2+1/2*
I/c*b*Pi*x*e-1/2*I/c^2*b*Pi*e*arccoth(c*x)+b*ln(2)*arccoth(c*x)*x^2*e-b*arc
coth(c*x)*ln((c*x+1)/(c*x-1)-1)*x^2*e+1/c*b*ln(2)*x*e-1/c*b*ln((c*x+1)/(c*x
-1)-1)*x*e-1/c^2*b*arccoth(c*x)*e*ln(2)+1/c^2*b*arccoth(c*x)*e*ln((c*x+1)/(
c*x-1)-1)+1/4*I/c^2*b*arccoth(c*x)*e*Pi*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(
I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1))-1/4*I*b*ar
ccoth(c*x)*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/
(c*x-1))*csgn(I/((c*x+1)/(c*x-1)-1)^2)*x^2*e-1/4*I/c*b*Pi*csgn(I*(c*x+1)/(c
*x-1)/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1))*csgn(I/((c*x+1)/(c*x-1
)-1)^2)*x*e+1/4*I/c^2*b*Pi*e*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*csgn(I*(c*x+
```

```
1)/(c*x-1))-1/2*I/c^2*b*Pi*e*csgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn(I*(c*x+1)
/(c*x-1))^2+1/4*I/c*b*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3*x*
e+1/4*I/c*b*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^3*x*e-1/4*I/c*b*Pi*csgn(I*(c*x
+1)/(c*x-1))^3*x*e-1/4*I/c^2*b*arccoth(c*x)*e*Pi*csgn(I*(c*x+1)/(c*x-1)/((c
*x+1)/(c*x-1)-1)^2)^3-1/4*I/c^2*b*arccoth(c*x)*e*Pi*csgn(I*((c*x+1)/(c*x-1)
-1)^2)^3
```

Maxima [A] time = 1.02803, size = 231, normalized size = 1.65

$$\frac{1}{2}adx^2 + \frac{1}{4}\left(2x^2\operatorname{arcoth}(cx) + c\left(\frac{2x}{c^2} - \frac{\log(cx+1)}{c^3} + \frac{\log(cx-1)}{c^3}\right)\right)bd - \frac{\left(c^2x^2 - \left(c^2x^2-1\right)\log\left(-c^2x^2+1\right) - 1\right)be\operatorname{arcot}}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*x^2 + 1/4*(2*x^2*arccoth(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log
(c*x - 1)/c^3))*b*d - 1/2*(c^2*x^2 - (c^2*x^2 - 1)*log(-c^2*x^2 + 1) - 1)*b
*e*arccoth(c*x)/c^2 - 1/2*(c^2*x^2 - (c^2*x^2 - 1)*log(-c^2*x^2 + 1) - 1)*a
*e/c^2 - 1/2*(3*c*x - (c*x + 1)*log(c*x + 1) - (c*x - 1)*log(-c*x + 1))*b*e
/c^2
```

Fricas [A] time = 1.62476, size = 297, normalized size = 2.12

$$\frac{2\left(ac^2d - ac^2e\right)x^2 + 2\left(bcd - 3bce\right)x + 2\left(ac^2ex^2 + bcex - ae\right)\log\left(-c^2x^2+1\right) + \left(\left(bc^2d - bc^2e\right)x^2 - bd + 3be + \left(bc^2ex^2 - b\right.\right.}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a*c^2*d - a*c^2*e)*x^2 + 2*(b*c*d - 3*b*c*e)*x + 2*(a*c^2*e*x^2 + b
*c*e*x - a*e)*log(-c^2*x^2 + 1) + ((b*c^2*d - b*c^2*e)*x^2 - b*d + 3*b*e +
(b*c^2*e*x^2 - b*e)*log(-c^2*x^2 + 1))*log((c*x + 1)/(c*x - 1)))/c^2
```

Sympy [A] time = 12.6635, size = 209, normalized size = 1.49

$$\begin{cases} \frac{adx^2}{2} + \frac{aex^2\log\left(-c^2x^2+1\right)}{2} - \frac{aex^2}{2} - \frac{ae\log\left(-c^2x^2+1\right)}{2c^2} + \frac{bdx^2\operatorname{acoth}(cx)}{2} + \frac{bex^2\log\left(-c^2x^2+1\right)\operatorname{acoth}(cx)}{2} - \frac{bex^2\operatorname{acoth}(cx)}{2} + \frac{bdx}{2c} + \frac{bex\log\left(-c^2x^2\right.}{2c} \\ \frac{dx^2\left(a+\frac{i\pi b}{2}\right)}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**2/2 + a*e*x**2*log(-c**2*x**2 + 1)/2 - a*e*x**2/2 - a*e*l
og(-c**2*x**2 + 1)/(2*c**2) + b*d*x**2*acoth(c*x)/2 + b*e*x**2*log(-c**2*x*
*2 + 1)*acoth(c*x)/2 - b*e*x**2*acoth(c*x)/2 + b*d*x/(2*c) + b*e*x*log(-c**
2*x**2 + 1)/(2*c) - 3*b*e*x/(2*c) - b*d*acoth(c*x)/(2*c**2) - b*e*log(-c**2
*x**2 + 1)*acoth(c*x)/(2*c**2) + 3*b*e*acoth(c*x)/(2*c**2), Ne(c, 0)), (d*x
```

```
**2*(a + I*pi*b/2)/2, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\mathrm{arcoth}\,(cx)+a)\left(e\log\left(-c^2x^2+1\right)+d\right)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)*x, x)
```

3.269 $\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x}\,dx$

Optimal. Leaf size=381

$$-\frac{1}{2}ae\text{PolyLog}\left(2,c^2x^2\right)+\frac{1}{2}be\log\left(-c^2x^2\right)\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}be\left(\log\left(-c^2x^2\right)-\log\left(1-c^2x^2\right)+\log\left(1-\frac{1}{cx}\right)+\log\left(\right.\right.$$

```
[Out] -(b*e*Log[1 + 1/(c*x)]^2*Log[-(1/(c*x))])/2 + (b*e*Log[1 - 1/(c*x)]^2*Log[1
/(c*x)])/2 + a*d*Log[x] - b*e*Log[(c + x^(-1))/c]*PolyLog[2, (c + x^(-1))/c
] + b*e*Log[1 - 1/(c*x)]*PolyLog[2, 1 - 1/(c*x)] + (b*d*PolyLog[2, -(1/(c*x
))])/2 + (b*e*Log[-(c^2*x^2)]*PolyLog[2, -(1/(c*x))])/2 - (b*e*(Log[1 - 1/(
c*x)] + Log[1 + 1/(c*x)] + Log[-(c^2*x^2)] - Log[1 - c^2*x^2])*PolyLog[2, -
(1/(c*x))])/2 - (b*d*PolyLog[2, 1/(c*x)])/2 - (b*e*Log[-(c^2*x^2)]*PolyLog[
2, 1/(c*x)])/2 + (b*e*(Log[1 - 1/(c*x)] + Log[1 + 1/(c*x)] + Log[-(c^2*x^2)
] - Log[1 - c^2*x^2])*PolyLog[2, 1/(c*x)])/2 - (a*e*PolyLog[2, c^2*x^2])/2
+ b*e*PolyLog[3, (c + x^(-1))/c] - b*e*PolyLog[3, 1 - 1/(c*x)] + b*e*PolyLo
g[3, -(1/(c*x))] - b*e*PolyLog[3, 1/(c*x)]
```

Rubi [A] time = 0.439495, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {6080, 5913, 6078, 2391, 6076, 2454, 2396, 2433, 2374, 6589, 6070}

$$-\frac{1}{2}ae\text{PolyLog}\left(2,c^2x^2\right)+\frac{1}{2}be\log\left(-c^2x^2\right)\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}be\left(\log\left(-c^2x^2\right)-\log\left(1-c^2x^2\right)+\log\left(1-\frac{1}{cx}\right)+\log\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x,x]
```


```
[Out] -(b*e*Log[1 + 1/(c*x)]^2*Log[-(1/(c*x))])/2 + (b*e*Log[1 - 1/(c*x)]^2*Log[1
/(c*x)])/2 + a*d*Log[x] - b*e*Log[(c + x^(-1))/c]*PolyLog[2, (c + x^(-1))/c
] + b*e*Log[1 - 1/(c*x)]*PolyLog[2, 1 - 1/(c*x)] + (b*d*PolyLog[2, -(1/(c*x
))])/2 + (b*e*Log[-(c^2*x^2)]*PolyLog[2, -(1/(c*x))])/2 - (b*e*(Log[1 - 1/(
c*x)] + Log[1 + 1/(c*x)] + Log[-(c^2*x^2)] - Log[1 - c^2*x^2])*PolyLog[2, -
(1/(c*x))])/2 - (b*d*PolyLog[2, 1/(c*x)])/2 - (b*e*Log[-(c^2*x^2)]*PolyLog[
2, 1/(c*x)])/2 + (b*e*(Log[1 - 1/(c*x)] + Log[1 + 1/(c*x)] + Log[-(c^2*x^2)
] - Log[1 - c^2*x^2])*PolyLog[2, 1/(c*x)])/2 - (a*e*PolyLog[2, c^2*x^2])/2
+ b*e*PolyLog[3, (c + x^(-1))/c] - b*e*PolyLog[3, 1 - 1/(c*x)] + b*e*PolyLo
g[3, -(1/(c*x))] - b*e*PolyLog[3, 1/(c*x)]
```


Rule 6080

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*(Log[(f_.) + (g_.)*(x_)^2]*(e_.) +
 (d_)))/(x_), x_Symbol] :> Dist[d, Int[(a + b*ArcCoth[c*x])/x, x], x] + Dis
t[e, Int[(Log[f + g*x^2]*(a + b*ArcCoth[c*x]))/x, x], x] /; FreeQ[{a, b, c,
 d, e, f, g}, x]
```

Rule 5913

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b*PolyLog[2, -(c*x)^(-1)])/2, x] - Simp[(b*PolyLog[2, 1/(c*x)])/
2, x]) /; FreeQ[{a, b, c}, x]
```

Rule 6078

```
Int[(Log[(f_.) + (g_.)*(x_)^2]*(ArcCoth[(c_.)*(x_)]*(b_.) + (a_)))/(x_), x_
Symbol] :> Dist[a, Int[Log[f + g*x^2]/x, x], x] + Dist[b, Int[(Log[f + g*x^
2]*ArcCoth[c*x])/x, x], x] /; FreeQ[{a, b, c, f, g}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6076

```
Int[(ArcCoth[(c_.)*(x_)]*Log[(f_.) + (g_.)*(x_)^2])/(x_), x_Symbol] :> Dist
[Log[f + g*x^2] - Log[-(c^2*x^2)] - Log[1 - 1/(c*x)] - Log[1 + 1/(c*x)], In
t[ArcCoth[c*x]/x, x], x] + (-Dist[1/2, Int[Log[1 - 1/(c*x)]^2/x, x], x] + D
ist[1/2, Int[Log[1 + 1/(c*x)]^2/x, x], x] + Int[(Log[-(c^2*x^2)]*ArcCoth[c*
x])/x, x]) /; FreeQ[{c, f, g}, x] && EqQ[c^2*f + g, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6070

```
Int[(ArcCoth[(c_.)*(x_)^(n_.)]*Log[(d_.)*(x_)^(m_.)])/(x_), x_Symbol] :> Di
st[1/2, Int[(Log[d*x^m]*Log[1 + 1/(c*x^n)])/x, x], x] - Dist[1/2, Int[(Log[
```

```
d*x^m]*Log[1 - 1/(c*x^n)])/x, x], x] /; FreeQ[{c, d, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x}\,dx = d\int \frac{a+b\coth^{-1}(cx)}{x}\,dx + e\int \frac{\left(a+b\coth^{-1}(cx)\right)\log\left(1-c^2x^2\right)}{x}\,dx$$

$$= ad\log(x)+\frac{1}{2}bd\mathrm{Li}_2\left(-\frac{1}{cx}\right)-\frac{1}{2}bd\mathrm{Li}_2\left(\frac{1}{cx}\right)+(ae)\int \frac{\log\left(1-c^2x^2\right)}{x}\,dx+$$

$$= ad\log(x)+\frac{1}{2}bd\mathrm{Li}_2\left(-\frac{1}{cx}\right)-\frac{1}{2}bd\mathrm{Li}_2\left(\frac{1}{cx}\right)-\frac{1}{2}ae\mathrm{Li}_2\left(c^2x^2\right)-\frac{1}{2}(be)\int -$$

$$= ad\log(x)+\frac{1}{2}bd\mathrm{Li}_2\left(-\frac{1}{cx}\right)-\frac{1}{2}be\left(\log\left(1-\frac{1}{cx}\right)+\log\left(1+\frac{1}{cx}\right)+\log\left(-\right.\right.$$

$$= -\frac{1}{2}be\log^2\left(1+\frac{1}{cx}\right)\log\left(-\frac{1}{cx}\right)+\frac{1}{2}be\log^2\left(1-\frac{1}{cx}\right)\log\left(\frac{1}{cx}\right)+ad\log(x$$

$$= -\frac{1}{2}be\log^2\left(1+\frac{1}{cx}\right)\log\left(-\frac{1}{cx}\right)+\frac{1}{2}be\log^2\left(1-\frac{1}{cx}\right)\log\left(\frac{1}{cx}\right)+ad\log(x$$

$$= -\frac{1}{2}be\log^2\left(1+\frac{1}{cx}\right)\log\left(-\frac{1}{cx}\right)+\frac{1}{2}be\log^2\left(1-\frac{1}{cx}\right)\log\left(\frac{1}{cx}\right)+ad\log(x$$

$$= -\frac{1}{2}be\log^2\left(1+\frac{1}{cx}\right)\log\left(-\frac{1}{cx}\right)+\frac{1}{2}be\log^2\left(1-\frac{1}{cx}\right)\log\left(\frac{1}{cx}\right)+ad\log(x$$

Mathematica [F] time = 0.212522, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x,x]
```

```
[Out] Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x, x]
```

Maple [C] time = 1.104, size = 864, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x,x)
```

```
[Out] dilog(c*x)*a*e-1/2*dilog(c*x)*b*d+ln(c*x)*a*d+1/2*b*e*ln(-c*x)*ln(c*x+1)^2+
b*e*ln(c*x+1)*polylog(2,c*x+1)-1/4*I*Pi*ln(c*x)*ln(c*x-1)*b*e*csgn(I*(c*x-1
))*csgn(I*(c*x-1)*(c*x+1))^2-1/4*I*Pi*ln(c*x)*ln(c*x-1)*b*e*csgn(I*(c*x+1))
*csgn(I*(c*x-1)*(c*x+1))^2+1/4*I*Pi*dilog(c*x)*b*e*csgn(I*(c*x-1))*csgn(I*(
c*x+1))*csgn(I*(c*x-1)*(c*x+1))+b*e*polylog(3,-c*x+1)-b*e*polylog(3,c*x+1)-
(-1/2*I*Pi*b*e*csgn(I*(c*x-1)*(c*x+1))^2-1/4*I*Pi*b*e*csgn(I*(c*x-1))*csgn(
I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))+1/4*I*Pi*b*e*csgn(I*(c*x-1))*csgn(I*(c*x
-1)*(c*x+1))^2+1/4*I*Pi*b*e*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))^2+1/4*I
```

```
*Pi*b*e*csgn(I*(c*x-1)*(c*x+1))^3+1/2*I*e*Pi*b+a*e+1/2*b*d)*dilog(c*x+1)+ln
(c*x)*ln(c*x-1)*a*e-1/2*ln(c*x)*ln(c*x-1)*b*d-1/2*ln(c*x)*ln(c*x-1)^2*b*e-l
n(c*x-1)*polylog(2,-c*x+1)*b*e-1/2*I*Pi*ln(c*x)*a*e*csgn(I*(c*x-1))*csgn(I*
(c*x+1))*csgn(I*(c*x-1)*(c*x+1))-1/4*I*Pi*ln(c*x)*ln(c*x-1)*b*e*csgn(I*(c*x
-1)*(c*x+1))^3+1/2*I*Pi*ln(c*x)*a*e*csgn(I*(c*x-1)*(c*x+1))^3-1/4*I*Pi*dilo
g(c*x)*b*e*csgn(I*(c*x-1)*(c*x+1))^3+1/4*I*Pi*ln(c*x)*ln(c*x-1)*b*e*csgn(I*
(c*x-1))*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))+1/2*I*Pi*ln(c*x)*ln(c*x-1)
*b*e*csgn(I*(c*x-1)*(c*x+1))^2-1/4*I*Pi*dilog(c*x)*b*e*csgn(I*(c*x-1))*csgn
(I*(c*x-1)*(c*x+1))^2-1/4*I*Pi*dilog(c*x)*b*e*csgn(I*(c*x+1))*csgn(I*(c*x-1
)*(c*x+1))^2+1/2*I*Pi*ln(c*x)*a*e*csgn(I*(c*x-1))*csgn(I*(c*x-1)*(c*x+1))^2
+1/2*I*Pi*ln(c*x)*a*e*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))^2+I*Pi*ln(c*x
)*a*e-I*Pi*ln(c*x)*a*e*csgn(I*(c*x-1)*(c*x+1))^2+1/2*I*Pi*dilog(c*x)*b*e*cs
gn(I*(c*x-1)*(c*x+1))^2-1/2*I*dilog(c*x)*Pi*b*e-1/2*I*ln(c*x)*Pi*ln(c*x-1)*
b*e
```

Maxima [C] time = 1.77925, size = 225, normalized size = 0.59

$$i\pi ae\log(x) - \frac{1}{2}\left(\log(cx-1)^2\log(cx) + 2\,\mathrm{Li}_2(-cx+1)\log(cx-1) - 2\,\mathrm{Li}_3(-cx+1)\right)be + \frac{1}{2}\left(\log(cx+1)^2\log(-cx) +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x,x, algorithm="maxima")
```

```
[Out] I*pi*a*e*log(x) - 1/2*(log(c*x - 1)^2*log(c*x) + 2*dilog(-c*x + 1)*log(c*x
- 1) - 2*polylog(3, -c*x + 1))*b*e + 1/2*(log(c*x + 1)^2*log(-c*x) + 2*dilo
g(c*x + 1)*log(c*x + 1) - 2*polylog(3, c*x + 1))*b*e + a*d*log(x) - 1/2*(I*
pi*b*e + b*d - 2*a*e)*(log(c*x - 1)*log(c*x) + dilog(-c*x + 1)) - 1/2*(-I*p
i*b*e - b*d - 2*a*e)*(log(c*x + 1)*log(-c*x) + dilog(c*x + 1))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bd\,\mathrm{arcoth}(cx) + ad + (be\,\mathrm{arcoth}(cx) + ae)\log\left(-c^2x^2+1\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x,x, algorithm="fricas")
```

```
[Out] integral((b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(-c^2*x^2 +
1))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{acoth}(cx))\left(d + e\log\left(-c^2x^2+1\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1))/x,x)
```

```
[Out] Integral((a + b*acoth(c*x))*(d + e*log(-c**2*x**2 + 1))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcoth}\,(cx) + a)\left(e\log\left(-c^2x^2 + 1\right) + d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x, x)
```

3.270 $\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^3}\,dx$

Optimal. Leaf size=247

$$\frac{1}{2}bc^2e\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)+\frac{1}{2}bc^2e\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{2x^2}+\frac{1}{2}c^2e(a$$

```
[Out] -(b*c^2*e*ArcCoth[c*x]^2)/2 - (b*c^2*e*ArcTanh[c*x]^2)/2 - a*c^2*e*Log[x] +
 b*c^2*e*ArcTanh[c*x]*Log[2/(1 - c*x)] + ((a + b)*c^2*e*Log[1 - c*x])/2 + (
(a - b)*c^2*e*Log[1 + c*x])/2 - (b*c*(d + e*Log[1 - c^2*x^2]))/(2*x) - ((a
+ b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/(2*x^2) + (b*c^2*ArcTanh[c*x]*(
d + e*Log[1 - c^2*x^2]))/2 - b*c^2*e*ArcCoth[c*x]*Log[2 - 2/(1 + c*x)] + (b
*c^2*e*PolyLog[2, 1 - 2/(1 - c*x)])/2 + (b*c^2*e*PolyLog[2, -1 + 2/(1 + c*x
)])/2
```

Rubi [A] time = 0.487551, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {5917, 325, 206, 6086, 6725, 801, 5989, 5933, 2447, 5984, 5918, 2402, 2315}

$$\frac{1}{2}bc^2e\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)+\frac{1}{2}bc^2e\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{2x^2}+\frac{1}{2}c^2e(a$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^3,x]
```


```
[Out] -(b*c^2*e*ArcCoth[c*x]^2)/2 - (b*c^2*e*ArcTanh[c*x]^2)/2 - a*c^2*e*Log[x] +
 b*c^2*e*ArcTanh[c*x]*Log[2/(1 - c*x)] + ((a + b)*c^2*e*Log[1 - c*x])/2 + (
(a - b)*c^2*e*Log[1 + c*x])/2 - (b*c*(d + e*Log[1 - c^2*x^2]))/(2*x) - ((a
+ b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/(2*x^2) + (b*c^2*ArcTanh[c*x]*(
d + e*Log[1 - c^2*x^2]))/2 - b*c^2*e*ArcCoth[c*x]*Log[2 - 2/(1 + c*x)] + (b
*c^2*e*PolyLog[2, 1 - 2/(1 - c*x)])/2 + (b*c^2*e*PolyLog[2, -1 + 2/(1 + c*x
)])/2
```


Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 6086

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcCoth[c*x]),
 x]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[(x
*u)/(f + g*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ
[m] && NeQ[m, -1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 5989

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcCoth[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5933

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcCoth[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^3}\,dx = -\frac{bc\left(d+e\log\left(1-c^2x^2\right)\right)}{2x} - \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{2x^2}$$

$$= -\frac{bc\left(d+e\log\left(1-c^2x^2\right)\right)}{2x} - \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{2x^2}$$

$$= -\frac{1}{2}bc^2e\tanh^{-1}(cx)^2 - \frac{bc\left(d+e\log\left(1-c^2x^2\right)\right)}{2x} - \frac{\left(a+b\coth^{-1}(cx)\right)}{}$$

$$= -\frac{1}{2}bc^2e\tanh^{-1}(cx)^2 + bc^2e\tanh^{-1}(cx)\log\left(\frac{2}{1-cx}\right) - \frac{bc\left(d+e\log\left(\right.\right.}{2x}$$

$$= -\frac{1}{2}bc^2e\coth^{-1}(cx)^2 - \frac{1}{2}bc^2e\tanh^{-1}(cx)^2 + bc^2e\tanh^{-1}(cx)\log\left(\frac{2}{1-}\right.$$

$$= -\frac{1}{2}bc^2e\coth^{-1}(cx)^2 - \frac{1}{2}bc^2e\tanh^{-1}(cx)^2 - ac^2e\log(x) + bc^2e\tanh^{-}$$

$$= -\frac{1}{2}bc^2e\coth^{-1}(cx)^2 - \frac{1}{2}bc^2e\tanh^{-1}(cx)^2 - ac^2e\log(x) + bc^2e\tanh^{-}$$

Mathematica [A] time = 0.149537, size = 161, normalized size = 0.65

$$\frac{1}{2}\left(-bc^2e\left(\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\text{PolyLog}\left(2,\frac{1}{cx}\right)\right)-\frac{e\log\left(1-c^2x^2\right)\left(a+\left(b-bc^2x^2\right)\coth^{-1}(cx)+bcx\right)}{x^2}+c^2e(a+b)\log\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^3,x]
```

```
[Out] (-((a*d)/x^2) - 2*a*c^2*e*Log[x] + (a + b)*c^2*e*Log[1 - c*x] + (a - b)*c^2
*e*Log[1 + c*x] - (b*d*(2*ArcCoth[c*x] + c*x*(2 + c*x*Log[1 - c*x] - c*x*Lo
g[1 + c*x])))/(2*x^2) - (e*(a + b*c*x + (b - b*c^2*x^2)*ArcCoth[c*x])*Log[1
 - c^2*x^2])/x^2 - b*c^2*e*(PolyLog[2, -(1/(c*x))] - PolyLog[2, 1/(c*x)]))/
2
```

Maple [F] time = 2.582, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\,\mathrm{arccoth}\left(cx\right)\right)\left(d+e\ln\left(-c^2x^2+1\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^3,x)
```

```
[Out] int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}\left(\left(c\log\left(cx+1\right)-c\log\left(cx-1\right)-\frac{2}{x}\right)c-\frac{2\,\operatorname{arcoth}\left(cx\right)}{x^{2}}\right)bd+\frac{1}{2}\left(c^{2}\left(\log\left(c^{2}x^{2}-1\right)-\log\left(x^{2}\right)\right)-\frac{\log\left(-c^{2}x^{2}+1\right)}{x^{2}}\right)ae-\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^3,x, algorithm="maxima
")
```

```
[Out] 1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arccoth(c*x)/x^2)*b*d +
1/2*(c^2*(log(c^2*x^2 - 1) - log(x^2)) - log(-c^2*x^2 + 1)/x^2)*a*e - 1/4*b
*e*(log(c*x + 1)^2/x^2 - 2*integrate(-((c*x + 1)*log(c*x - 1)^2 - (I*pi + (
I*pi*c + c)*x)*log(c*x + 1) - (-I*pi - I*pi*c*x)*log(c*x - 1))/(c*x^4 + x^3
), x)) - 1/2*a*d/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{bd\operatorname{arcoth}\left(cx\right)+ad+\left(be\operatorname{arcoth}\left(cx\right)+ae\right)\log\left(-c^{2}x^{2}+1\right)}{x^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^3,x, algorithm="fricas
")
```

```
[Out] integral((b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(-c^2*x^2 +
1))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\operatorname{acoth}\left(cx\right)\right)\left(d+e\log\left(-c^{2}x^{2}+1\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1))/x**3,x)
```

```
[Out] Integral((a + b*acoth(c*x))*(d + e*log(-c**2*x**2 + 1))/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\operatorname{arcoth}\left(cx\right)+a\right)\left(e\log\left(-c^{2}x^{2}+1\right)+d\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^3, x)
```

3.271 $\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^5}\,dx$

Optimal. Leaf size=339

$$\frac{1}{4}bc^4e\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)+\frac{1}{4}bc^4e\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{4x^4}+\frac{1}{12}c^4e(3a+$$

```
[Out] (a*c^2*e)/(4*x^2) + (5*b*c^3*e)/(12*x) + (b*c^2*e*ArcCoth[c*x])/(4*x^2) - (
b*c^4*e*ArcCoth[c*x]^2)/4 - (b*c^4*e*ArcTanh[c*x])/4 - (b*c^4*e*ArcTanh[c*x
]^2)/4 - (a*c^4*e*Log[x])/2 + (b*c^4*e*ArcTanh[c*x]*Log[2/(1 - c*x)])/2 + (
(3*a + 4*b)*c^4*e*Log[1 - c*x])/12 + ((3*a - 4*b)*c^4*e*Log[1 + c*x])/12 -
(b*c*(d + e*Log[1 - c^2*x^2]))/(12*x^3) - (b*c^3*(d + e*Log[1 - c^2*x^2]))/
(4*x) - ((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/(4*x^4) + (b*c^4*Ar
cTanh[c*x]*(d + e*Log[1 - c^2*x^2]))/4 - (b*c^4*e*ArcCoth[c*x]*Log[2 - 2/(1
 + c*x)])/2 + (b*c^4*e*PolyLog[2, 1 - 2/(1 - c*x)])/4 + (b*c^4*e*PolyLog[2,
 -1 + 2/(1 + c*x)])/4
```

Rubi [A] time = 0.717232, antiderivative size = 339, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.518, Rules used = {5917, 325, 206, 6086, 6725, 1802, 5983, 5989, 5933, 2447, 5984, 5918, 2402, 2315}

$$\frac{1}{4}bc^4e\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)+\frac{1}{4}bc^4e\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{4x^4}+\frac{1}{12}c^4e(3a+$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^5,x]
```

```
[Out] (a*c^2*e)/(4*x^2) + (5*b*c^3*e)/(12*x) + (b*c^2*e*ArcCoth[c*x])/(4*x^2) - (
b*c^4*e*ArcCoth[c*x]^2)/4 - (b*c^4*e*ArcTanh[c*x])/4 - (b*c^4*e*ArcTanh[c*x
]^2)/4 - (a*c^4*e*Log[x])/2 + (b*c^4*e*ArcTanh[c*x]*Log[2/(1 - c*x)])/2 + (
(3*a + 4*b)*c^4*e*Log[1 - c*x])/12 + ((3*a - 4*b)*c^4*e*Log[1 + c*x])/12 -
(b*c*(d + e*Log[1 - c^2*x^2]))/(12*x^3) - (b*c^3*(d + e*Log[1 - c^2*x^2]))/
(4*x) - ((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/(4*x^4) + (b*c^4*Ar
cTanh[c*x]*(d + e*Log[1 - c^2*x^2]))/4 - (b*c^4*e*ArcCoth[c*x]*Log[2 - 2/(1
 + c*x)])/2 + (b*c^4*e*PolyLog[2, 1 - 2/(1 - c*x)])/4 + (b*c^4*e*PolyLog[2,
 -1 + 2/(1 + c*x)])/4
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6086

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcCoth[c*x]),
 x]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[(x
*u)/(f + g*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ
[m] && NeQ[m, -1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 5983

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcCoth[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5989

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcCoth[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5933

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcCoth[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
```

```
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^5}\,dx = -\frac{bc\left(d+e\log\left(1-c^2x^2\right)\right)}{12x^3} - \frac{bc^3\left(d+e\log\left(1-c^2x^2\right)\right)}{4x} - \frac{\left(a+b\coth^{-1}(\right.}{}$$

$$= -\frac{bc\left(d+e\log\left(1-c^2x^2\right)\right)}{12x^3} - \frac{bc^3\left(d+e\log\left(1-c^2x^2\right)\right)}{4x} - \frac{\left(a+b\coth^{-1}(\right.}{}$$

$$= -\frac{1}{4}bc^4e\tanh^{-1}(cx)^2 - \frac{bc\left(d+e\log\left(1-c^2x^2\right)\right)}{12x^3} - \frac{bc^3\left(d+e\log\left(1-c^2x\right.\right.}{4x}$$

$$= -\frac{1}{4}bc^4e\tanh^{-1}(cx)^2 + \frac{1}{2}bc^4e\tanh^{-1}(cx)\log\left(\frac{2}{1-cx}\right) - \frac{bc\left(d+e\log\left(1\right.\right.}{12x^3}$$

$$= -\frac{1}{4}bc^4e\tanh^{-1}(cx)^2 + \frac{1}{2}bc^4e\tanh^{-1}(cx)\log\left(\frac{2}{1-cx}\right) - \frac{bc\left(d+e\log\left(1\right.\right.}{12x^3}$$

$$= \frac{ac^2e}{4x^2} + \frac{bc^3e}{6x} + \frac{bc^2e\coth^{-1}(cx)}{4x^2} - \frac{1}{4}bc^4e\coth^{-1}(cx)^2 - \frac{1}{4}bc^4e\tanh^{-1}(cx$$

$$= \frac{ac^2e}{4x^2} + \frac{5bc^3e}{12x} + \frac{bc^2e\coth^{-1}(cx)}{4x^2} - \frac{1}{4}bc^4e\coth^{-1}(cx)^2 - \frac{1}{4}bc^4e\tanh^{-1}(c$$

$$= \frac{ac^2e}{4x^2} + \frac{5bc^3e}{12x} + \frac{bc^2e\coth^{-1}(cx)}{4x^2} - \frac{1}{4}bc^4e\coth^{-1}(cx)^2 - \frac{1}{4}bc^4e\tanh^{-1}(c$$

Mathematica [A] time = 0.144746, size = 307, normalized size = 0.91

$$-\frac{1}{4}bc^4e\left(\text{PolyLog}\left(2,-\frac{1}{cx}\right) - \text{PolyLog}\left(2,\frac{1}{cx}\right)\right) + \frac{e\log\left(1-c^2x^2\right)\left(-3a-3bc^3x^3+3bc^4x^4\coth^{-1}(cx)-bcx-3b\coth^{-1}(\right.}{12x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^5,x]
```

```
[Out] -(a*d)/(4*x^4) + (a*c^2*e)/(4*x^2) + (b*c^3*e)/(6*x) - (a*c^4*e*Log[x])/2 +
 ((3*a*c^4*e + 4*b*c^4*e)*Log[1 - c*x])/12 - (b*c^4*e*(-ArcCoth[c*x]/(2*c^2
*x^2) + (-(1/(c*x)) - Log[1 - c*x]/2 + Log[1 + c*x]/2)/2))/2 + b*c^4*d*(-Ar
cCoth[c*x]/(4*c^4*x^4) + (-1/(3*c^3*x^3) - 1/(c*x) - Log[1 - c*x]/2 + Log[1
 + c*x]/2)/4) + ((3*a*c^4*e - 4*b*c^4*e)*Log[1 + c*x])/12 + (e*(-3*a - b*c*
x - 3*b*c^3*x^3 - 3*b*ArcCoth[c*x] + 3*b*c^4*x^4*ArcCoth[c*x])*Log[1 - c^2*
x^2])/(12*x^4) - (b*c^4*e*(PolyLog[2, -(1/(c*x))] - PolyLog[2, 1/(c*x)]))/4
```

Maple [F] time = 6.231, size = 0, normalized size = 0.

$$\int \frac{(a+b\,\mathrm{arccoth}\,(cx))\left(d+e\ln\left(-c^2x^2+1\right)\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^5,x)
```

```
[Out] int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{24}\left(\left(3\,c^3\log\left(cx+1\right)-3\,c^3\log\left(cx-1\right)-\frac{2\left(3\,c^2x^2+1\right)}{x^3}\right)c-\frac{6\,\mathrm{arcoth}\,(cx)}{x^4}\right)bd+\frac{1}{4}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^5,x, algorithm="maxima
")
```

```
[Out] 1/24*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c -
 6*arccoth(c*x)/x^4)*b*d + 1/4*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^
2)*c^2 - log(-c^2*x^2 + 1)/x^4)*a*e - 1/8*b*e*(log(c*x + 1)^2/x^4 - 4*integ
rate(-1/2*(2*(c*x + 1)*log(c*x - 1)^2 - (2*I*pi + (2*I*pi*c + c)*x)*log(c*x
 + 1) - (-2*I*pi - 2*I*pi*c*x)*log(c*x - 1))/(c*x^6 + x^5), x)) - 1/4*a*d/x
^4
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{bd\,\mathrm{arcoth}\,(cx)+ad+(be\,\mathrm{arcoth}\,(cx)+ae)\log\left(-c^2x^2+1\right)}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^5,x, algorithm="fricas
")
```

```
[Out] integral((b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(-c^2*x^2 +
1))/x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+b\operatorname{acoth}(cx))\left(d+e\log\left(-c^2x^2+1\right)\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1))/x**5,x)
```

```
[Out] Integral((a + b*acoth(c*x))*(d + e*log(-c**2*x**2 + 1))/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcoth}(cx)+a)\left(e\log\left(-c^2x^2+1\right)+d\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^5, x)
```

3.272 $\int x^4 \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$

Optimal. Leaf size=315

$$\frac{1}{5}x^5\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{e(4a+3b)\log(1-cx)}{20c^5}+\frac{e(4a-3b)\log(cx+1)}{20c^5}-\frac{2aex^3}{15c^2}-\frac{2aex}{5c^4}-\frac{2}{25}a$$

```
[Out] (-2*a*e*x)/(5*c^4) - (77*b*e*x^2)/(300*c^3) - (2*a*e*x^3)/(15*c^2) - (9*b*e
*x^4)/(200*c) - (2*a*e*x^5)/25 - (2*b*e*x*ArcCoth[c*x])/(5*c^4) - (2*b*e*x^
3*ArcCoth[c*x])/(15*c^2) - (2*b*e*x^5*ArcCoth[c*x])/25 + (b*e*ArcCoth[c*x]^
2)/(5*c^5) - ((4*a + 3*b)*e*Log[1 - c*x])/(20*c^5) + ((4*a - 3*b)*e*Log[1 +
 c*x])/(20*c^5) - (23*b*e*Log[1 - c^2*x^2])/(75*c^5) - (b*e*Log[1 - c^2*x^2
]^2)/(20*c^5) + (b*x^2*(d + e*Log[1 - c^2*x^2]))/(10*c^3) + (b*x^4*(d + e*L
og[1 - c^2*x^2]))/(20*c) + (x^5*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2
]))/5 + (b*Log[1 - c^2*x^2]*(d + e*Log[1 - c^2*x^2]))/(10*c^5)
```

Rubi [A] time = 0.749021, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 15, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {5917, 266, 43, 6086, 6725, 1802, 633, 31, 5981, 5911, 260, 5949, 2475, 2390, 2301}

$$\frac{1}{5}x^5\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{e(4a+3b)\log(1-cx)}{20c^5}+\frac{e(4a-3b)\log(cx+1)}{20c^5}-\frac{2aex^3}{15c^2}-\frac{2aex}{5c^4}-\frac{2}{25}a$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```



```
[Out] (-2*a*e*x)/(5*c^4) - (77*b*e*x^2)/(300*c^3) - (2*a*e*x^3)/(15*c^2) - (9*b*e
*x^4)/(200*c) - (2*a*e*x^5)/25 - (2*b*e*x*ArcCoth[c*x])/(5*c^4) - (2*b*e*x^
3*ArcCoth[c*x])/(15*c^2) - (2*b*e*x^5*ArcCoth[c*x])/25 + (b*e*ArcCoth[c*x]^
2)/(5*c^5) - ((4*a + 3*b)*e*Log[1 - c*x])/(20*c^5) + ((4*a - 3*b)*e*Log[1 +
 c*x])/(20*c^5) - (23*b*e*Log[1 - c^2*x^2])/(75*c^5) - (b*e*Log[1 - c^2*x^2
]^2)/(20*c^5) + (b*x^2*(d + e*Log[1 - c^2*x^2]))/(10*c^3) + (b*x^4*(d + e*L
og[1 - c^2*x^2]))/(20*c) + (x^5*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2
]))/5 + (b*Log[1 - c^2*x^2]*(d + e*Log[1 - c^2*x^2]))/(10*c^5)
```


Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6086

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcCoth[c*x]),
 x]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[(x
*u)/(f + g*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ
[m] && NeQ[m, -1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5981

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int x^4\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)dx=\frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{10c^3}+\frac{bx^4\left(d+e\log\left(1-c^2x^2\right)\right)}{20c}+\frac{1}{5}x^5\left(a+\right.$$

$$=\frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{10c^3}+\frac{bx^4\left(d+e\log\left(1-c^2x^2\right)\right)}{20c}+\frac{1}{5}x^5\left(a+\right.$$

$$=\frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{10c^3}+\frac{bx^4\left(d+e\log\left(1-c^2x^2\right)\right)}{20c}+\frac{1}{5}x^5\left(a+\right.$$

$$=\frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{10c^3}+\frac{bx^4\left(d+e\log\left(1-c^2x^2\right)\right)}{20c}+\frac{1}{5}x^5\left(a+\right.$$

$$=-\frac{be\log^2\left(1-c^2x^2\right)}{20c^5}+\frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{10c^3}+\frac{bx^4\left(d+e\log\left(\right.\right.}{20c}$$

$$=-\frac{2aex}{5c^4}-\frac{3bex^2}{20c^3}-\frac{2aex^3}{15c^2}-\frac{bex^4}{40c}-\frac{2}{25}aex^5-\frac{2}{25}bex^5\coth^{-1}(cx)-$$

$$=-\frac{2aex}{5c^4}-\frac{3bex^2}{20c^3}-\frac{2aex^3}{15c^2}-\frac{bex^4}{40c}-\frac{2}{25}aex^5-\frac{2bex^3\coth^{-1}(cx)}{15c^2}-$$

$$=-\frac{2aex}{5c^4}-\frac{3bex^2}{20c^3}-\frac{2aex^3}{15c^2}-\frac{bex^4}{40c}-\frac{2}{25}aex^5-\frac{2bex\coth^{-1}(cx)}{5c^4}-$$

$$=-\frac{2aex}{5c^4}-\frac{19bex^2}{100c^3}-\frac{2aex^3}{15c^2}-\frac{9bex^4}{200c}-\frac{2}{25}aex^5-\frac{2bex\coth^{-1}(cx)}{5c^4}-$$

$$=-\frac{2aex}{5c^4}-\frac{77bex^2}{300c^3}-\frac{2aex^3}{15c^2}-\frac{9bex^4}{200c}-\frac{2}{25}aex^5-\frac{2bex\coth^{-1}(cx)}{5c^4}-$$

Mathematica [A] time = 0.148721, size = 236, normalized size = 0.75

$$30c^2ex^2\log\left(1-c^2x^2\right)\left(4ac^3x^3+b\left(c^2x^2+2\right)+4bc^3x^3\coth^{-1}(cx)\right)+2\log(1-cx)(-60ae+30bd-137be)+2\log(cx$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (-240*a*c*e*x + 2*b*c^2*(30*d - 77*e)*x^2 - 80*a*c^3*e*x^3 + 3*b*c^4*(10*d
- 9*e)*x^4 + 24*a*c^5*(5*d - 2*e)*x^5 - 8*b*c*x*(-15*c^4*d*x^4 + 2*e*(15 +
5*c^2*x^2 + 3*c^4*x^4))*ArcCoth[c*x] + 120*b*e*ArcCoth[c*x]^2 + 2*(30*b*d -
 60*a*e - 137*b*e)*Log[1 - c*x] + 2*(30*b*d + 60*a*e - 137*b*e)*Log[1 + c*x
] + 30*c^2*e*x^2*(4*a*c^3*x^3 + b*(2 + c^2*x^2) + 4*b*c^3*x^3*ArcCoth[c*x])
*Log[1 - c^2*x^2] + 30*b*e*Log[1 - c^2*x^2]^2)/(600*c^5)
```

Maple [C] time = 1.72, size = 4194, normalized size = 13.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1)),x)
```

```
[Out] 1/10/c^3*b*x^2*d+1/20/c*b*x^4*d+1/5*b*arccoth(c*x)*x^5*d-46/75/c^5*b*arccot
h(c*x)*e-1/5/c^5*b*ln((c*x+1)/(c*x-1)-1)*d+1/5/c^5*b*e*ln((c*x+1)/(c*x-1)-1
)^2+137/150/c^5*b*ln((c*x+1)/(c*x-1)-1)*e+1/5/c^5*b*arccoth(c*x)*d-2/5*b*e*
x*arccoth(c*x)/c^4-2/15*b*e*x^3*arccoth(c*x)/c^2+181/600/c^5*b*e-2/5*a*e*x/
c^4-77/300*b*e*x^2/c^3-2/15*a*e*x^3/c^2-9/200*b*e*x^4/c-2/25*b*e*x^5*arccot
h(c*x)+1/20*I/c^3*b*csgn(I*(c*x+1)/(c*x-1))*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)
/(c*x-1)-1)^2)^2*Pi*x^2*e+1/40*I/c*b*csgn(I*((c*x+1)/(c*x-1)-1)^2)*csgn(I*(
(c*x+1)/(c*x-1)-1))^2*Pi*x^4*e+1/20*I/c^3*b*csgn(I*((c*x+1)/(c*x-1)-1)^2)*c
sgn(I*((c*x+1)/(c*x-1)-1))^2*Pi*x^2*e+1/20*I/c*b*csgn(I/((c*x-1)/(c*x+1))^(
1/2))*csgn(I*(c*x+1)/(c*x-1))^2*Pi*x^4*e+1/10*I/c^3*b*csgn(I/((c*x-1)/(c*x+
1))^(1/2))*csgn(I*(c*x+1)/(c*x-1))^2*Pi*x^2*e-1/40*I/c*b*csgn(I/((c*x-1)/(c
*x+1))^(1/2))^2*csgn(I*(c*x+1)/(c*x-1))*Pi*x^4*e-2/25*a*e*x^5+1/10*I/c^5*b*
Pi*ln((c*x+1)/(c*x-1)-1)*e*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*
x-1)/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1))-1/10*I*b*arccoth(c*x)*c
sgn(I*(c*x+1)/(c*x-1))*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*csgn(I
/((c*x+1)/(c*x-1)-1)^2)*Pi*x^5*e-3/20/c^5*b*d-1/10*I/c^5*b*arccoth(c*x)*e*P
i*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^
2)*csgn(I*(c*x+1)/(c*x-1))-1/40*I/c*b*csgn(I*(c*x+1)/(c*x-1))*csgn(I*(c*x+1
)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*csgn(I/((c*x+1)/(c*x-1)-1)^2)*Pi*x^4*e-1/2
0*I/c^3*b*csgn(I*(c*x+1)/(c*x-1))*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1
)^2)*csgn(I/((c*x+1)/(c*x-1)-1)^2)*Pi*x^2*e+1/5*x^5*a*d-3/40*I/c^5*b*Pi*e*c
sgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^
2-3/40*I/c^5*b*Pi*e*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*
(c*x+1)/(c*x-1))-3/40*I/c^5*b*Pi*e*csgn(I*((c*x+1)/(c*x-1)-1)^2)*csgn(I*((c
*x+1)/(c*x-1)-1))^2-1/10*I/c^5*b*arccoth(c*x)*e*Pi*csgn(I*(c*x+1)/(c*x-1))^
3-3/20*I/c^5*b*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn(I*(c*x+1)/(c*x-1))
^2-1/5*I/c^5*b*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*Pi*e*arccoth
(c*x)+1/5*I/c^5*b*Pi*ln((c*x+1)/(c*x-1)-1)*e*csgn(I*(c*x+1)/(c*x-1)/((c*x+1
)/(c*x-1)-1)^2)^2-1/40*I/c*b*csgn(I*(c*x+1)/(c*x-1))^3*Pi*x^4*e-1/20*I/c^3*
b*csgn(I*(c*x+1)/(c*x-1))^3*Pi*x^2*e+1/10*I/c^5*b*Pi*ln((c*x+1)/(c*x-1)-1)*
e*csgn(I*(c*x+1)/(c*x-1))^3+1/10*I/c^5*b*arccoth(c*x)*Pi*e*csgn(I*(c*x+1)/(
c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3-1/20*I/c*b*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/
(c*x-1)-1)^2)^2*Pi*x^4*e-1/10*I/c^3*b*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-
1)-1)^2)^2*Pi*x^2*e+1/40*I/c*b*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2
)^3*Pi*x^4*e+1/20*I/c^3*b*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3*P
i*x^2*e-1/5*I*b*arccoth(c*x)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^
2*Pi*x^5*e+1/10*I*b*arccoth(c*x)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)
^2)^3*Pi*x^5*e+1/10*I*b*arccoth(c*x)*csgn(I*((c*x+1)/(c*x-1)-1)^2)^3*Pi*x^5
*e+1/10*I/c^5*b*arccoth(c*x)*e*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^3+3/20*I/c^
5*b*Pi*e*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*((c*x+1)/(c*x-1)-1))+1/5*a*
```

```
e*x^5*ln(-c^2*x^2+1)-1/5*a*e/c^5*ln(c*x-1)+1/5*a*e/c^5*ln(c*x+1)-1/10*I*b*a
rccoth(c*x)*csgn(I*(c*x+1)/(c*x-1))^3*Pi*x^5*e-1/10/c*b*ln((c*x+1)/(c*x-1)-
1)*x^4*e-1/5/c^3*b*ln((c*x+1)/(c*x-1)-1)*x^2*e+1/10/c*b*ln(2)*x^4*e+1/5/c^3
*b*ln(2)*x^2*e+2/5/c^5*b*arccoth(c*x)*ln(2)*e-2/5/c^5*b*ln(2)*ln((c*x+1)/(c
*x-1)-1)*e-2/5*b*arccoth(c*x)*ln((c*x+1)/(c*x-1)-1)*x^5*e+2/5*b*arccoth(c*x
)*ln(2)*x^5*e-3/20*I/c^5*b*Pi*e-1/20*I/c^3*b*csgn(I/((c*x-1)/(c*x+1))^(1/2)
)^2*csgn(I*(c*x+1)/(c*x-1))*Pi*x^2*e+1/10*I/c^5*b*arccoth(c*x)*e*Pi*csgn(I*
((c*x+1)/(c*x-1)-1)^2)*csgn(I*((c*x+1)/(c*x-1)-1))^2-1/10*I/c^5*b*Pi*ln((c*
x+1)/(c*x-1)-1)*e*csgn(I*((c*x+1)/(c*x-1)-1)^2)*csgn(I*((c*x+1)/(c*x-1)-1))
^2+1/10*I/c^5*b*arccoth(c*x)*e*Pi*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x
+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2+3/40*I/c^5*b*e*Pi*csgn(I/((c*x+1)/(c*x
-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x
-1))+1/5*I/c^5*b*arccoth(c*x)*Pi*e*csgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn(I*(
c*x+1)/(c*x-1))^2-1/10*I/c^5*b*arccoth(c*x)*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(
1/2))^2*csgn(I*(c*x+1)/(c*x-1))-1/10*I/c^5*b*Pi*ln((c*x+1)/(c*x-1)-1)*e*csg
n(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2-
1/5*I/c^5*b*Pi*ln((c*x+1)/(c*x-1)-1)*e*csgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn
(I*(c*x+1)/(c*x-1))^2+1/5*I/c^5*b*Pi*ln((c*x+1)/(c*x-1)-1)*e*csgn(I*((c*x+1
)/(c*x-1)-1)^2)^2*csgn(I*((c*x+1)/(c*x-1)-1))-1/20*I/c*b*csgn(I*((c*x+1)/(c
*x-1)-1)^2)^2*csgn(I*((c*x+1)/(c*x-1)-1))*Pi*x^4*e-1/10*I/c^3*b*csgn(I*((c*
x+1)/(c*x-1)-1)^2)^2*csgn(I*((c*x+1)/(c*x-1)-1))*Pi*x^2*e-1/10*I/c^5*b*Pi*l
n((c*x+1)/(c*x-1)-1)*e*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*csgn
(I*(c*x+1)/(c*x-1))+1/10*I/c^5*b*Pi*ln((c*x+1)/(c*x-1)-1)*e*csgn(I/((c*x-1)
/(c*x+1))^(1/2))^2*csgn(I*(c*x+1)/(c*x-1))+1/10*I*b*arccoth(c*x)*csgn(I*(c*
x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*csgn(I/((c*x+1)/(c*x-1)-1)^2)*Pi*x^5*
e+1/10*I*b*arccoth(c*x)*csgn(I*((c*x+1)/(c*x-1)-1)^2)*csgn(I*((c*x+1)/(c*x-
1)-1))^2*Pi*x^5*e+1/5*I*b*arccoth(c*x)*csgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn
(I*(c*x+1)/(c*x-1))^2*Pi*x^5*e-1/10*I*b*arccoth(c*x)*csgn(I/((c*x-1)/(c*x+1
))^(1/2))^2*csgn(I*(c*x+1)/(c*x-1))*Pi*x^5*e+1/10*I*b*arccoth(c*x)*csgn(I*(
c*x+1)/(c*x-1))*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*Pi*x^5*e-1/
5*I*b*arccoth(c*x)*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*((c*x+1)/(c*x-1)-
1))*Pi*x^5*e-1/5*I/c^5*b*arccoth(c*x)*Pi*e*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2*
csgn(I*((c*x+1)/(c*x-1)-1))+1/10*I/c^5*b*arccoth(c*x)*Pi*e*csgn(I*(c*x+1)/(
c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*(c*x+1)/(c*x-1))+1/40*I/c*b*csgn(I*(
c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*csgn(I/((c*x+1)/(c*x-1)-1)^2)*Pi*x^
4*e+1/20*I/c^3*b*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*csgn(I/((c
*x+1)/(c*x-1)-1)^2)*Pi*x^2*e+1/40*I/c*b*csgn(I*(c*x+1)/(c*x-1))*csgn(I*(c*x
+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*Pi*x^4*e+1/40*I/c*b*csgn(I*((c*x+1)/(c
*x-1)-1)^2)^3*Pi*x^4*e+1/20*I/c^3*b*csgn(I*((c*x+1)/(c*x-1)-1)^2)^3*Pi*x^2*
e-1/10*I/c^5*b*Pi*ln((c*x+1)/(c*x-1)-1)*e*csgn(I*((c*x+1)/(c*x-1)-1)^2)^3-1
/10*I/c^5*b*Pi*ln((c*x+1)/(c*x-1)-1)*e*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x
-1)-1)^2)^3+3/40*I/c^5*b*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*csgn(I*(c*x
+1)/(c*x-1))-3/10/c^5*b*e*ln(2)-1/20/c^5*b*e*(4*arccoth(c*x)*x^5*c^5+c^4*x^
4+2*c^2*x^2+4*arccoth(c*x)-4*ln((c*x+1)/(c*x-1)-1)-3)*ln((c*x-1)/(c*x+1))+1
/5*I*b*arccoth(c*x)*Pi*x^5*e+1/5*I/c^5*b*arccoth(c*x)*Pi*e-1/5*I/c^5*b*Pi*l
n((c*x+1)/(c*x-1)-1)*e-3/40*I/c^5*b*Pi*e*csgn(I*((c*x+1)/(c*x-1)-1)^2)^3+3/
40*I/c^5*b*e*Pi*csgn(I*(c*x+1)/(c*x-1))^3+3/20*I/c^5*b*Pi*e*csgn(I*(c*x+1)/
(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2-3/40*I/c^5*b*Pi*e*csgn(I*(c*x+1)/(c*x-1)/(
(c*x+1)/(c*x-1)-1)^2)^3+1/20*I/c*b*Pi*x^4*e+1/10*I/c^3*b*Pi*x^2*e
```

Maxima [C] time = 1.10989, size = 428, normalized size = 1.36

$$\frac{1}{5}\,adx^5 + \frac{1}{75}\left(15\,x^5\log\left(-c^2x^2+1\right) - c^2\left(\frac{2\left(3\,c^4x^5+5\,c^2x^3+15\,x\right)}{c^6} - \frac{15\,\log\left(cx+1\right)}{c^7} + \frac{15\,\log\left(cx-1\right)}{c^7}\right)\right)be\,\text{arcoth}\,(cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima
```

```
")
```

```
[Out] 1/5*a*d*x^5 + 1/75*(15*x^5*log(-c^2*x^2 + 1) - c^2*(2*(3*c^4*x^5 + 5*c^2*x^
3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*b*e*arccoth(c*x
) + 1/20*(4*x^5*arccoth(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1
)/c^6))*b*d + 1/75*(15*x^5*log(-c^2*x^2 + 1) - c^2*(2*(3*c^4*x^5 + 5*c^2*x^
3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*a*e - 1/600*(3*
(-10*I*pi*c^4 + 9*c^4)*x^4 + 2*(-30*I*pi*c^2 + 77*c^2)*x^2 - (60*I*pi + 30*
c^4*x^4 + 60*c^2*x^2 + 120*log(c*x - 1) - 274)*log(c*x + 1) - (60*I*pi + 30
*c^4*x^4 + 60*c^2*x^2 - 274)*log(c*x - 1))*b*e/c^5
```

Fricas [A] time = 1.67589, size = 587, normalized size = 1.86

$$-\frac{80\,ac^{3}ex^{3} - 24\left(5\,ac^{5}d - 2\,ac^{5}e\right)x^{5} - 3\left(10\,bc^{4}d - 9\,bc^{4}e\right)x^{4} + 240\,acex - 30\,be\log\left(-c^{2}x^{2}+1\right)^{2} - 30\,be\log\left(\frac{cx+1}{cx-1}\right)^{2} - 2\,(3\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas
")
```

```
[Out] -1/600*(80*a*c^3*e*x^3 - 24*(5*a*c^5*d - 2*a*c^5*e)*x^5 - 3*(10*b*c^4*d - 9
*b*c^4*e)*x^4 + 240*a*c*e*x - 30*b*e*log(-c^2*x^2 + 1)^2 - 30*b*e*log((c*x
+ 1)/(c*x - 1))^2 - 2*(30*b*c^2*d - 77*b*c^2*e)*x^2 - 2*(60*a*c^5*e*x^5 + 1
5*b*c^4*e*x^4 + 30*b*c^2*e*x^2 + 30*b*d - 137*b*e)*log(-c^2*x^2 + 1) - 4*(1
5*b*c^5*e*x^5*log(-c^2*x^2 + 1) - 10*b*c^3*e*x^3 + 3*(5*b*c^5*d - 2*b*c^5*e
)*x^5 - 30*b*c*e*x + 30*a*e)*log((c*x + 1)/(c*x - 1)))/c^5
```

Sympy [A] time = 43.4042, size = 345, normalized size = 1.1

$$\begin{cases} \frac{adx^{5}}{5} + \frac{aex^{5}\log\left(-c^{2}x^{2}+1\right)}{5} - \frac{2aex^{5}}{25} - \frac{2aex^{3}}{15c^{2}} - \frac{2aex}{5c^{4}} + \frac{2ae\operatorname{acoth}(cx)}{5c^{5}} + \frac{bdx^{5}\operatorname{acoth}(cx)}{5} + \frac{bex^{5}\log\left(-c^{2}x^{2}+1\right)\operatorname{acoth}(cx)}{5} - \frac{2bex^{5}\operatorname{acoth}(cx)}{25} + \frac{bdx\cdots}{20\cdots} \\ \frac{dx^{5}\left(a+\frac{i\pi b}{2}\right)}{5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**5/5 + a*e*x**5*log(-c**2*x**2 + 1)/5 - 2*a*e*x**5/25 - 2*
a*e*x**3/(15*c**2) - 2*a*e*x/(5*c**4) + 2*a*e*acoth(c*x)/(5*c**5) + b*d*x**
5*acoth(c*x)/5 + b*e*x**5*log(-c**2*x**2 + 1)*acoth(c*x)/5 - 2*b*e*x**5*aco
th(c*x)/25 + b*d*x**4/(20*c) + b*e*x**4*log(-c**2*x**2 + 1)/(20*c) - 9*b*e*
x**4/(200*c) - 2*b*e*x**3*acoth(c*x)/(15*c**2) + b*d*x**2/(10*c**3) + b*e*x
**2*log(-c**2*x**2 + 1)/(10*c**3) - 77*b*e*x**2/(300*c**3) - 2*b*e*x*acoth(
c*x)/(5*c**4) + b*d*log(-c**2*x**2 + 1)/(10*c**5) + b*e*log(-c**2*x**2 + 1)
**2/(20*c**5) - 137*b*e*log(-c**2*x**2 + 1)/(300*c**5) + b*e*acoth(c*x)**2/
(5*c**5), Ne(c, 0)), (d*x**5*(a + I*pi*b/2)/5, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\operatorname{arcoth}(cx) + a)\left(e\log\left(-c^{2}x^{2}+1\right) + d\right)x^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)*x^4, x)
```

3.273 $\int x^2 \left(a+b \coth^{-1}(cx)\right) \left(d+e \log \left(1-c^2x^2\right)\right) dx$

Optimal. Leaf size=247

$$\frac{1}{3}x^3 \left(a+b \coth^{-1}(cx)\right) \left(e \log \left(1-c^2x^2\right)+d\right)-\frac{e(2a+b)\log(1-cx)}{6c^3}+\frac{e(2a-b)\log(cx+1)}{6c^3}-\frac{2aex}{3c^2}-\frac{2}{9}aex^3+\frac{bx^2\left(e\log\right.}{}$$

```
[Out] (-2*a*e*x)/(3*c^2) - (5*b*e*x^2)/(18*c) - (2*a*e*x^3)/9 - (2*b*e*x*ArcCoth[
c*x])/(3*c^2) - (2*b*e*x^3*ArcCoth[c*x])/9 + (b*e*ArcCoth[c*x]^2)/(3*c^3) -
 ((2*a + b)*e*Log[1 - c*x])/(6*c^3) + ((2*a - b)*e*Log[1 + c*x])/(6*c^3) -
(4*b*e*Log[1 - c^2*x^2])/(9*c^3) - (b*e*Log[1 - c^2*x^2]^2)/(12*c^3) + (b*x
^2*(d + e*Log[1 - c^2*x^2]))/(6*c) + (x^3*(a + b*ArcCoth[c*x])*(d + e*Log[1
 - c^2*x^2]))/3 + (b*Log[1 - c^2*x^2]*(d + e*Log[1 - c^2*x^2]))/(6*c^3)
```

Rubi [A] time = 0.620957, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 15, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {5917, 266, 43, 6086, 6725, 801, 633, 31, 5981, 5911, 260, 5949, 2475, 2390, 2301}

$$\frac{1}{3}x^3 \left(a+b \coth^{-1}(cx)\right) \left(e \log \left(1-c^2x^2\right)+d\right)-\frac{e(2a+b)\log(1-cx)}{6c^3}+\frac{e(2a-b)\log(cx+1)}{6c^3}-\frac{2aex}{3c^2}-\frac{2}{9}aex^3+\frac{bx^2\left(e\log\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```


```
[Out] (-2*a*e*x)/(3*c^2) - (5*b*e*x^2)/(18*c) - (2*a*e*x^3)/9 - (2*b*e*x*ArcCoth[
c*x])/(3*c^2) - (2*b*e*x^3*ArcCoth[c*x])/9 + (b*e*ArcCoth[c*x]^2)/(3*c^3) -
 ((2*a + b)*e*Log[1 - c*x])/(6*c^3) + ((2*a - b)*e*Log[1 + c*x])/(6*c^3) -
(4*b*e*Log[1 - c^2*x^2])/(9*c^3) - (b*e*Log[1 - c^2*x^2]^2)/(12*c^3) + (b*x
^2*(d + e*Log[1 - c^2*x^2]))/(6*c) + (x^3*(a + b*ArcCoth[c*x])*(d + e*Log[1
 - c^2*x^2]))/3 + (b*Log[1 - c^2*x^2]*(d + e*Log[1 - c^2*x^2]))/(6*c^3)
```


Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6086

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcCoth[c*x]),
 x]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[(x
*u)/(f + g*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ
[m] && NeQ[m, -1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5981

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
```

```
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int x^2\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)dx=\frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{6c}+\frac{1}{3}x^3\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2\right.\right.$$

$$=\frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{6c}+\frac{1}{3}x^3\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2\right.\right.$$

$$=\frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{6c}+\frac{1}{3}x^3\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2\right.\right.$$

$$=\frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{6c}+\frac{1}{3}x^3\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2\right.\right.$$

$$=-\frac{be\log^2\left(1-c^2x^2\right)}{12c^3}+\frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{6c}+\frac{1}{3}x^3\left(a+b\coth^{-1}\right.$$

$$=-\frac{2aex}{3c^2}-\frac{bex^2}{6c}-\frac{2}{9}aex^3-\frac{2}{9}bex^3\coth^{-1}(cx)-\frac{be\log^2\left(1-c^2x^2\right)}{12c^3}+\frac{bx}{}$$

$$=-\frac{2aex}{3c^2}-\frac{bex^2}{6c}-\frac{2}{9}aex^3-\frac{2bex\coth^{-1}(cx)}{3c^2}-\frac{2}{9}bex^3\coth^{-1}(cx)+\frac{be\,c}{}$$

$$=-\frac{2aex}{3c^2}-\frac{bex^2}{6c}-\frac{2}{9}aex^3-\frac{2bex\coth^{-1}(cx)}{3c^2}-\frac{2}{9}bex^3\coth^{-1}(cx)+\frac{be\,c}{}$$

$$=-\frac{2aex}{3c^2}-\frac{5bex^2}{18c}-\frac{2}{9}aex^3-\frac{2bex\coth^{-1}(cx)}{3c^2}-\frac{2}{9}bex^3\coth^{-1}(cx)+\frac{be}{}$$

Mathematica [A] time = 0.123003, size = 183, normalized size = 0.74

$$\frac{6c^2ex^2\log\left(1-c^2x^2\right)\left(2acx+2bcx\coth^{-1}(cx)+b\right)+2\log(1-cx)(-6ae+3bd-11be)+2\log(cx+1)(6ae+3bd-11be)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (-24*a*c*e*x + 2*b*c^2*(3*d - 5*e)*x^2 + 4*a*c^3*(3*d - 2*e)*x^3 + 4*b*c*x*
(3*c^2*d*x^2 - 2*e*(3 + c^2*x^2))*ArcCoth[c*x] + 12*b*e*ArcCoth[c*x]^2 + 2*
(3*b*d - 6*a*e - 11*b*e)*Log[1 - c*x] + 2*(3*b*d + 6*a*e - 11*b*e)*Log[1 +
```

```
c*x] + 6*c^2*e*x^2*(b + 2*a*c*x + 2*b*c*x*ArcCoth[c*x])*Log[1 - c^2*x^2] +
3*b*e*Log[1 - c^2*x^2]^2)/(36*c^3)
```

Maple [C] time = 1.083, size = 3515, normalized size = 14.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1)),x)
```

```
[Out] -2/3*b*e*x*arccoth(c*x)/c^2-2/3*a*e*x/c^2-5/18*b*e*x^2/c-2/9*b*e*x^3*arccot
h(c*x)+1/3*a*e*x^3*ln(-c^2*x^2+1)-1/3*a*e/c^3*ln(c*x-1)+1/3*a*e/c^3*ln(c*x+
1)+1/6/c*b*x^2*d+1/3/c^3*b*e*ln((c*x+1)/(c*x-1)-1)^2-1/3/c^3*b*ln((c*x+1)/(
c*x-1)-1)*d+11/9/c^3*b*ln((c*x+1)/(c*x-1)-1)*e+1/3/c^3*b*arccoth(c*x)*d-8/9
/c^3*b*e*arccoth(c*x)+1/3*b*arccoth(c*x)*x^3*d-1/6/c^3*b*d+5/18/c^3*b*e-2/9
*a*e*x^3+1/3*x^3*a*d-1/3/c^3*b*e*ln(2)+1/3*I/c^3*b*e*ln((c*x+1)/(c*x-1)-1)*
Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*((c*x+1)/(c*x-1)-1))-1/6*I/c^3*b*
Pi*ln((c*x+1)/(c*x-1)-1)*e*csgn(I*((c*x+1)/(c*x-1)-1)^2)*csgn(I*((c*x+1)/(c
*x-1)-1))^2-1/6*I/c^3*b*arccoth(c*x)*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2
*csgn(I*(c*x+1)/(c*x-1))+1/3*I/c^3*b*arccoth(c*x)*e*Pi*csgn(I/((c*x-1)/(c*x
+1))^(1/2))*csgn(I*(c*x+1)/(c*x-1))^2+1/6*I/c^3*b*arccoth(c*x)*e*Pi*csgn(I*
(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*(c*x+1)/(c*x-1))+1/6*I/c^3*
b*arccoth(c*x)*e*Pi*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((
c*x+1)/(c*x-1)-1)^2)^2-1/3*I/c^3*b*arccoth(c*x)*e*Pi*csgn(I*((c*x+1)/(c*x-1
)-1)^2)^2*csgn(I*((c*x+1)/(c*x-1)-1))+1/6*I/c^3*b*arccoth(c*x)*Pi*e*csgn(I*
((c*x+1)/(c*x-1)-1)^2)*csgn(I*((c*x+1)/(c*x-1)-1))^2+1/12*I/c^3*b*e*Pi*csgn
(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*csg
n(I*(c*x+1)/(c*x-1))+1/6*I*b*arccoth(c*x)*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)*
csgn(I*((c*x+1)/(c*x-1)-1))^2*x^3*e-1/12*I/c*b*Pi*csgn(I/((c*x-1)/(c*x+1))^
(1/2))^2*csgn(I*(c*x+1)/(c*x-1))*x^2*e-1/6*I/c^3*b*Pi*e-1/6*I*b*arccoth(c*x
)*Pi*csgn(I*(c*x+1)/(c*x-1))*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(
c*x-1)/((c*x+1)/(c*x-1)-1)^2)*x^3*e-1/12*I/c*b*Pi*csgn(I*(c*x+1)/(c*x-1))*c
sgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*
x^2*e+1/6*I/c^3*b*csgn(I*(c*x+1)/(c*x-1))*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(
c*x-1)-1)^2)*csgn(I/((c*x+1)/(c*x-1)-1)^2)*e*ln((c*x+1)/(c*x-1)-1)*Pi-1/6*I
/c^3*b*arccoth(c*x)*e*Pi*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-
1)/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1))-1/3/c*b*ln((c*x+1)/(c*x-1
)-1)*x^2*e+1/3/c*b*ln(2)*x^2*e-2/3/c^3*b*ln(2)*ln((c*x+1)/(c*x-1)-1)*e+2/3/
c^3*b*arccoth(c*x)*ln(2)*e-2/3*b*ln((c*x+1)/(c*x-1)-1)*arccoth(c*x)*x^3*e+2
/3*b*arccoth(c*x)*ln(2)*x^3*e+1/6/c^3*b*e*(-2*arccoth(c*x)*x^3*c^3-c^2*x^2+
2*ln((c*x+1)/(c*x-1)-1)-2*arccoth(c*x)+1)*ln((c*x-1)/(c*x+1))+1/6*I/c*b*Pi*
csgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn(I*(c*x+1)/(c*x-1))^2*x^2*e+1/12*I/c*b*
Pi*csgn(I*(c*x+1)/(c*x-1))*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*
x^2*e+1/12*I/c*b*Pi*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((
c*x+1)/(c*x-1)-1)^2)^2*x^2*e-1/6*I/c*b*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2*c
sgn(I*((c*x+1)/(c*x-1)-1))*x^2*e+1/12*I/c*b*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2
)*csgn(I*((c*x+1)/(c*x-1)-1))^2*x^2*e-1/6*I*b*arccoth(c*x)*Pi*csgn(I/((c*x-
1)/(c*x+1))^(1/2))^2*csgn(I*(c*x+1)/(c*x-1))*x^3*e+1/3*I*b*arccoth(c*x)*Pi*
csgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn(I*(c*x+1)/(c*x-1))^2*x^3*e+1/6*I*b*arc
coth(c*x)*Pi*csgn(I*(c*x+1)/(c*x-1))*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1
)-1)^2)^2*x^3*e+1/6*I*b*arccoth(c*x)*Pi*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(
I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*x^3*e-1/3*I*b*arccoth(c*x)*Pi*cs
gn(I*((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*((c*x+1)/(c*x-1)-1))*x^3*e+1/6*I/c^3*b
*e*ln((c*x+1)/(c*x-1)-1)*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*csgn(I*(c*x+1
)/(c*x-1))-1/3*I/c^3*b*e*ln((c*x+1)/(c*x-1)-1)*Pi*csgn(I/((c*x-1)/(c*x+1))^
(1/2))*csgn(I*(c*x+1)/(c*x-1))^2-1/6*I/c^3*b*e*ln((c*x+1)/(c*x-1)-1)*Pi*csg
```

```
n(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*(c*x+1)/(c*x-1))-1/6*I/
c^3*b*csgn(I/((c*x+1)/(c*x-1)-1)^2)*e*ln((c*x+1)/(c*x-1)-1)*Pi*csgn(I*(c*x+
1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2+1/6*I/c^3*b*arccoth(c*x)*e*Pi*csgn(I*(c
*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3+1/6*I/c^3*b*arccoth(c*x)*Pi*e*csgn(I
*((c*x+1)/(c*x-1)-1)^2)^3+1/3*I/c^3*b*e*ln((c*x+1)/(c*x-1)-1)*Pi*csgn(I*(c*
x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2-1/3*I/c^3*b*arccoth(c*x)*e*Pi*csgn(I*
(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2+1/12*I/c^3*b*Pi*e*csgn(I/((c*x-1)/
(c*x+1))^(1/2))^2*csgn(I*(c*x+1)/(c*x-1))-1/6*I/c^3*b*Pi*e*csgn(I/((c*x-1)/
(c*x+1))^(1/2))*csgn(I*(c*x+1)/(c*x-1))^2-1/12*I/c^3*b*Pi*e*csgn(I*(c*x+1)/
(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*(c*x+1)/(c*x-1))-1/6*I*b*arccoth(c*
x)*Pi*csgn(I*(c*x+1)/(c*x-1))^3*x^3*e+1/6*I*b*arccoth(c*x)*Pi*csgn(I*(c*x+1
)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3*x^3*e+1/6*I*b*arccoth(c*x)*Pi*csgn(I*((c
*x+1)/(c*x-1)-1)^2)^3*x^3*e-1/3*I*b*arccoth(c*x)*Pi*csgn(I*(c*x+1)/(c*x-1)/
((c*x+1)/(c*x-1)-1)^2)^2*x^3*e-1/12*I/c^3*b*e*Pi*csgn(I/((c*x+1)/(c*x-1)-1)
^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2+1/6*I/c^3*b*Pi*e*csgn(I
*((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*((c*x+1)/(c*x-1)-1))-1/12*I/c^3*b*Pi*e*csg
n(I*((c*x+1)/(c*x-1)-1)^2)*csgn(I*((c*x+1)/(c*x-1)-1))^2-1/12*I/c*b*Pi*csgn
(I*(c*x+1)/(c*x-1))^3*x^2*e+1/12*I/c*b*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(
c*x-1)-1)^2)^3*x^2*e+1/12*I/c*b*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^3*x^2*e-1/
6*I/c*b*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*x^2*e+1/6*I/c^3*
b*e*ln((c*x+1)/(c*x-1)-1)*Pi*csgn(I*(c*x+1)/(c*x-1))^3-1/6*I/c^3*b*e*ln((c*
x+1)/(c*x-1)-1)*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3-1/6*I/c^
3*b*e*ln((c*x+1)/(c*x-1)-1)*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^3-1/6*I/c^3*b*
arccoth(c*x)*e*Pi*csgn(I*(c*x+1)/(c*x-1))^3+1/3*I*b*arccoth(c*x)*Pi*x^3*e+1
/6*I/c*b*Pi*x^2*e+1/12*I/c^3*b*Pi*e*csgn(I*(c*x+1)/(c*x-1))^3-1/12*I/c^3*b*
Pi*e*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3-1/12*I/c^3*b*Pi*e*csgn
(I*((c*x+1)/(c*x-1)-1)^2)^3+1/6*I/c^3*b*Pi*e*csgn(I*(c*x+1)/(c*x-1)/((c*x+1
)/(c*x-1)-1)^2)^2-1/3*I/c^3*b*e*ln((c*x+1)/(c*x-1)-1)*Pi+1/3*I/c^3*b*arccot
h(c*x)*Pi*e
```

Maxima [C] time = 1.19273, size = 340, normalized size = 1.38

$$\frac{1}{3}\,adx^3+\frac{1}{9}\left(3\,x^3\log\left(-c^2x^2+1\right)-c^2\left(\frac{2\left(c^2x^3+3\,x\right)}{c^4}-\frac{3\,\log\left(cx+1\right)}{c^5}+\frac{3\,\log\left(cx-1\right)}{c^5}\right)\right)be\,\mathrm{arcoth}\left(cx\right)+\frac{1}{6}\left(2\,x^3\,\mathrm{arcoth}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima
")
```

```
[Out] 1/3*a*d*x^3 + 1/9*(3*x^3*log(-c^2*x^2 + 1) - c^2*(2*(c^2*x^3 + 3*x)/c^4 - 3
*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*e*arccoth(c*x) + 1/6*(2*x^3*arcc
oth(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*b*d + 1/9*(3*x^3*log(-c^2*x^
2 + 1) - c^2*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c
^5))*a*e + 1/18*((3*I*pi*c^2 - 5*c^2)*x^2 + (3*I*pi + 3*c^2*x^2 + 6*log(c*x
 - 1) - 11)*log(c*x + 1) + (3*I*pi + 3*c^2*x^2 - 11)*log(c*x - 1))*b*e/c^3
```

Fricas [A] time = 1.72752, size = 452, normalized size = 1.83

$$-\frac{24\,acex-4\left(3\,ac^3d-2\,ac^3e\right)x^3-3\,be\log\left(-c^2x^2+1\right)^2-3\,be\log\left(\frac{cx+1}{cx-1}\right)^2-2\left(3\,bc^2d-5\,bc^2e\right)x^2-2\left(6\,ac^3ex^3+3\,bc^2ex^2\right.}{36\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas
")
```

```
[Out] -1/36*(24*a*c*e*x - 4*(3*a*c^3*d - 2*a*c^3*e)*x^3 - 3*b*e*log(-c^2*x^2 + 1)
^2 - 3*b*e*log((c*x + 1)/(c*x - 1))^2 - 2*(3*b*c^2*d - 5*b*c^2*e)*x^2 - 2*(
6*a*c^3*e*x^3 + 3*b*c^2*e*x^2 + 3*b*d - 11*b*e)*log(-c^2*x^2 + 1) - 2*(3*b*
c^3*e*x^3*log(-c^2*x^2 + 1) - 6*b*c*e*x + (3*b*c^3*d - 2*b*c^3*e)*x^3 + 6*a
*e)*log((c*x + 1)/(c*x - 1)))/c^3
```

Sympy [A] time = 15.7038, size = 265, normalized size = 1.07

$$\begin{cases} \frac{adx^3}{3} + \frac{aex^3\log\left(-c^2x^2+1\right)}{3} - \frac{2aex^3}{9} - \frac{2aex}{3c^2} + \frac{2ae\operatorname{acoth}(cx)}{3c^3} + \frac{bdx^3\operatorname{acoth}(cx)}{3} + \frac{bex^3\log\left(-c^2x^2+1\right)\operatorname{acoth}(cx)}{3} - \frac{2bex^3\operatorname{acoth}(cx)}{9} + \frac{bdx^2}{6c} + \\ \frac{dx^3\left(a+\frac{i\pi b}{2}\right)}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**3/3 + a*e*x**3*log(-c**2*x**2 + 1)/3 - 2*a*e*x**3/9 - 2*a
*e*x/(3*c**2) + 2*a*e*acoth(c*x)/(3*c**3) + b*d*x**3*acoth(c*x)/3 + b*e*x**
3*log(-c**2*x**2 + 1)*acoth(c*x)/3 - 2*b*e*x**3*acoth(c*x)/9 + b*d*x**2/(6*
c) + b*e*x**2*log(-c**2*x**2 + 1)/(6*c) - 5*b*e*x**2/(18*c) - 2*b*e*x*acoth
(c*x)/(3*c**2) + b*d*log(-c**2*x**2 + 1)/(6*c**3) + b*e*log(-c**2*x**2 + 1)
**2/(12*c**3) - 11*b*e*log(-c**2*x**2 + 1)/(18*c**3) + b*e*acoth(c*x)**2/(3
*c**3), Ne(c, 0)), (d*x**3*(a + I*pi*b/2)/3, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\operatorname{arcoth}(cx) + a)\left(e\log\left(-c^2x^2+1\right) + d\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)*x^2, x)
```

3.274 $\int \left(a+b \coth^{-1}(cx)\right) \left(d+e \log \left(1-c^2x^2\right)\right) \, dx$

Optimal. Leaf size=104

$$x \left(a+b \coth^{-1}(cx)\right) \left(e \log \left(1-c^2x^2\right)+d\right)+\frac{e \left(a+b \coth^{-1}(cx)\right)^2}{bc}-2aex+\frac{b \left(e \log \left(1-c^2x^2\right)+d\right)^2}{4ce}-\frac{be \log \left(1-c^2x^2\right)}{c}$$

```
[Out] -2*a*e*x - 2*b*e*x*ArcCoth[c*x] + (e*(a + b*ArcCoth[c*x])^2)/(b*c) - (b*e*L
og[1 - c^2*x^2])/c + x*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]) + (b*(
d + e*Log[1 - c^2*x^2])^2)/(4*c*e)
```

Rubi [A] time = 0.198226, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6074, 2475, 2390, 2301, 5981, 5911, 260, 5949}

$$x \left(a+b \coth^{-1}(cx)\right) \left(e \log \left(1-c^2x^2\right)+d\right)+\frac{e \left(a+b \coth^{-1}(cx)\right)^2}{bc}-2aex+\frac{b \left(e \log \left(1-c^2x^2\right)+d\right)^2}{4ce}-\frac{be \log \left(1-c^2x^2\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] -2*a*e*x - 2*b*e*x*ArcCoth[c*x] + (e*(a + b*ArcCoth[c*x])^2)/(b*c) - (b*e*L
og[1 - c^2*x^2])/c + x*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]) + (b*(
d + e*Log[1 - c^2*x^2])^2)/(4*c*e)
```

Rule 6074

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.)), x_Symbol] :> Simp[x*(d + e*Log[f + g*x^2])*(a + b*ArcCoth[c*x]), x]
 + (-Dist[b*c, Int[(x*(d + e*Log[f + g*x^2]))/(1 - c^2*x^2), x], x] - Dist[
2*e*g, Int[(x^2*(a + b*ArcCoth[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b,
c, d, e, f, g}, x]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 5981

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx = x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)-(bc)\int \frac{x\left(d+e\log\left(1-\right.\right.}{1-c^2x^2}$$

$$= x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)-\frac{1}{2}(bc)\,\mathrm{Subst}\left(\int \frac{d+e\,\mathrm{lo}}{1}\right.$$

$$= -2aex+\frac{e\left(a+b\coth^{-1}(cx)\right)^2}{bc}+x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-\right.\right.$$

$$= -2aex-2bex\coth^{-1}(cx)+\frac{e\left(a+b\coth^{-1}(cx)\right)^2}{bc}+x\left(a+b\coth^{-1}(\right.$$

$$= -2aex-2bex\coth^{-1}(cx)+\frac{e\left(a+b\coth^{-1}(cx)\right)^2}{bc}-\frac{be\log\left(1-c^2x^2\right)}{c}$$

Mathematica [A] time = 0.0166919, size = 144, normalized size = 1.38

$$aex\log\left(1-c^2x^2\right)+\frac{2ae\tanh^{-1}(cx)}{c}+adx-2aex+\frac{bd\log\left(1-c^2x^2\right)}{2c}+\frac{be\log^2\left(1-c^2x^2\right)}{4c}-\frac{be\log\left(1-c^2x^2\right)}{c}+bex\,\mathrm{lo}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] a*d*x - 2*a*e*x + b*d*x*ArcCoth[c*x] - 2*b*e*x*ArcCoth[c*x] + (b*e*ArcCoth[
c*x]^2)/c + (2*a*e*ArcTanh[c*x])/c + (b*d*Log[1 - c^2*x^2])/(2*c) - (b*e*Lo
g[1 - c^2*x^2])/c + a*e*x*Log[1 - c^2*x^2] + b*e*x*ArcCoth[c*x]*Log[1 - c^2
*x^2] + (b*e*Log[1 - c^2*x^2]^2)/(4*c)
```

Maple [C] time = 0.566, size = 2210, normalized size = 21.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1)),x)
```

```
[Out] 1/c*b*e*ln((c*x+1)/(c*x-1)-1)^2-1/c*b*ln((c*x+1)/(c*x-1)-1)*d+2/c*b*ln((c*x
+1)/(c*x-1)-1)*e+1/c*b*arccoth(c*x)*d-2/c*b*e*arccoth(c*x)+b*arccoth(c*x)*x
*d+ln((c*x-1)/(c*x+1))*(-arccoth(c*x)*x*c+ln((c*x+1)/(c*x-1)-1)-arccoth(c*x
))*b*e/c-2/c*b*ln(2)*ln((c*x+1)/(c*x-1)-1)*e+2/c*b*arccoth(c*x)*e*ln(2)-2*b
*ln((c*x+1)/(c*x-1)-1)*arccoth(c*x)*x*e+2*b*arccoth(c*x)*ln(2)*x*e-2*a*x*e-
2*b*e*x*arccoth(c*x)+a*d*x+a*e*x*ln(-c^2*x^2+1)-a*e/c*ln(c*x-1)+a*e/c*ln(c*
x+1)+1/2*I/c*b*Pi*ln((c*x+1)/(c*x-1)-1)*e*csgn(I*(c*x+1)/(c*x-1))^3-1/2*I/c
*b*Pi*ln((c*x+1)/(c*x-1)-1)*e*csgn(I*((c*x+1)/(c*x-1)-1)^2)^3-1/2*I/c*b*Pi*
ln((c*x+1)/(c*x-1)-1)*e*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3-1/2
*I/c*b*arccoth(c*x)*e*Pi*csgn(I*(c*x+1)/(c*x-1))^3+1/2*I/c*b*arccoth(c*x)*e
*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^3+1/2*I/c*b*arccoth(c*x)*e*Pi*csgn(I*(c*x
+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3-1/2*I/c*b*csgn(I*(c*x+1)/(c*x-1)/((c*x
+1)/(c*x-1)-1)^2)^2*csgn(I/((c*x+1)/(c*x-1)-1)^2)*e*ln((c*x+1)/(c*x-1)-1)*P
i-1/2*I/c*b*arccoth(c*x)*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*csgn(I*(c*x
+1)/(c*x-1))+I/c*b*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*e*ln((c*
x+1)/(c*x-1)-1)*Pi-I/c*b*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*Pi
*e*arccoth(c*x)-1/2*I*b*Pi*arccoth(c*x)*csgn(I*(c*x+1)/(c*x-1))^3*x*e+1/2*I
*b*Pi*arccoth(c*x)*csgn(I*((c*x+1)/(c*x-1)-1)^2)^3*x*e+1/2*I*b*Pi*arccoth(c
*x)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3*x*e-I*b*Pi*arccoth(c*x)
*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*x*e+1/2*I/c*b*csgn(I*(c*x+
1)/(c*x-1))*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*csgn(I/((c*x+1)/(
c*x-1)-1)^2)*e*ln((c*x+1)/(c*x-1)-1)*Pi-1/2*I/c*b*arccoth(c*x)*e*Pi*csgn(I/
((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*csgn(I
*(c*x+1)/(c*x-1))-1/2*I*b*Pi*arccoth(c*x)*csgn(I*(c*x+1)/(c*x-1))*csgn(I/((
c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*x*e+I/c*
b*arccoth(c*x)*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn(I*(c*x+1)/(c*x-1))
^2+I*b*Pi*arccoth(c*x)*csgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn(I*(c*x+1)/(c*x-
1))^2*x*e+I/c*b*csgn(I*((c*x+1)/(c*x-1)-1))*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2
*e*ln((c*x+1)/(c*x-1)-1)*Pi+1/2*I/c*b*arccoth(c*x)*e*Pi*csgn(I*(c*x+1)/(c*x
-1)/((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*(c*x+1)/(c*x-1))-I/c*b*arccoth(c*x)*e*P
i*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*((c*x+1)/(c*x-1)-1))+1/2*I/c*b*arc
coth(c*x)*e*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)*csgn(I*((c*x+1)/(c*x-1)-1))^2+
1/2*I/c*b*arccoth(c*x)*e*Pi*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c
*x-1)/((c*x+1)/(c*x-1)-1)^2)^2-1/2*I*b*Pi*arccoth(c*x)*csgn(I/((c*x-1)/(c*x
+1))^(1/2))^2*csgn(I*(c*x+1)/(c*x-1))*x*e+1/2*I*b*Pi*arccoth(c*x)*csgn(I*(c
*x+1)/(c*x-1))*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*x*e-I*b*Pi*a
rccoth(c*x)*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*((c*x+1)/(c*x-1)-1))*x*e
+1/2*I*b*Pi*arccoth(c*x)*csgn(I*((c*x+1)/(c*x-1)-1)^2)*csgn(I*((c*x+1)/(c*x
-1)-1))^2*x*e+1/2*I*b*Pi*arccoth(c*x)*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*
(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*x*e+1/2*I/c*b*csgn(I*(c*x+1)/(c*x-
1))*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*e*ln((c*x+1)/(c*x-1)-1)*Pi-I/c*b*csgn
(I*(c*x+1)/(c*x-1))^2*csgn(I/((c*x-1)/(c*x+1))^(1/2))*e*ln((c*x+1)/(c*x-1)-
1)*Pi-1/2*I/c*b*csgn(I*(c*x+1)/(c*x-1))*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*
x-1)-1)^2)^2*e*ln((c*x+1)/(c*x-1)-1)*Pi-1/2*I/c*b*csgn(I*((c*x+1)/(c*x-1)-1
))^2*csgn(I*((c*x+1)/(c*x-1)-1)^2)*e*ln((c*x+1)/(c*x-1)-1)*Pi+I/c*b*Pi*e*ar
ccoth(c*x)+I*b*Pi*arccoth(c*x)*x*e-I/c*b*e*ln((c*x+1)/(c*x-1)-1)*Pi
```

Maxima [C] time = 1.14086, size = 240, normalized size = 2.31

$$-\left(c^2\left(\frac{2x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)-x\log\left(-c^2x^2+1\right)\right)be\operatorname{arcoth}(cx)-\left(c^2\left(\frac{2x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)-x\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima")
```

```
[Out] -(c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3) - x*log(-c^2*x^2 + 1)
)*b*e*arccoth(c*x) - (c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3) -
 x*log(-c^2*x^2 + 1))*a*e + a*d*x + 1/2*(2*c*x*arccoth(c*x) + log(-c^2*x^2
+ 1))*b*d/c + 1/2*((I*pi + 2*log(c*x - 1) - 2)*log(c*x + 1) + (I*pi - 2)*lo
g(c*x - 1))*b*e/c
```

Fricas [A] time = 1.69862, size = 304, normalized size = 2.92

$$\frac{be\log\left(-c^2x^2+1\right)^2+be\log\left(\frac{cx+1}{cx-1}\right)^2+4\,(acd-2\,ace)x+2\,(2\,acex+bd-2\,be)\log\left(-c^2x^2+1\right)+2\left(bcex\log\left(-c^2x^2+1\right.\right.}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas")
```

```
[Out] 1/4*(b*e*log(-c^2*x^2 + 1)^2 + b*e*log((c*x + 1)/(c*x - 1))^2 + 4*(a*c*d -
2*a*c*e)*x + 2*(2*a*c*e*x + b*d - 2*b*e)*log(-c^2*x^2 + 1) + 2*(b*c*e*x*log
(-c^2*x^2 + 1) + 2*a*e + (b*c*d - 2*b*c*e)*x)*log((c*x + 1)/(c*x - 1)))/c
```

Sympy [A] time = 9.84375, size = 155, normalized size = 1.49

$$\begin{cases} adx+aex\log\left(-c^2x^2+1\right)-2aex+\frac{2ae\operatorname{acoth}(cx)}{c}+bdx\operatorname{acoth}(cx)+bex\log\left(-c^2x^2+1\right)\operatorname{acoth}(cx)-2bex\operatorname{acoth}(cx) \\ dx\left(a+\frac{i\pi b}{2}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x + a*e*x*log(-c**2*x**2 + 1) - 2*a*e*x + 2*a*e*acoth(c*x)/c
 + b*d*x*acoth(c*x) + b*e*x*log(-c**2*x**2 + 1)*acoth(c*x) - 2*b*e*x*acoth(
c*x) + b*d*log(-c**2*x**2 + 1)/(2*c) + b*e*log(-c**2*x**2 + 1)**2/(4*c) - b
*e*log(-c**2*x**2 + 1)/c + b*e*acoth(c*x)**2/c, Ne(c, 0)), (d*x*(a + I*pi*b
/2), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\operatorname{arcoth}(cx)+a)\left(e\log\left(-c^2x^2+1\right)+d\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d), x)
```

3.275 $\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^2}\,dx$

Optimal. Leaf size=105

$$-\frac{1}{2}bce\text{PolyLog}\left(2,\frac{1}{1-c^2x^2}\right)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{x}-\frac{ce\left(a+b\coth^{-1}(cx)\right)^2}{b}+\frac{1}{2}bc\log\left(1-\frac{1}{1-c^2x^2}\right.$$

```
[Out] -((c*e*(a + b*ArcCoth[c*x])^2)/b) - ((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^
2*x^2]))/x + (b*c*(d + e*Log[1 - c^2*x^2])*Log[1 - (1 - c^2*x^2)^(-1)])/2 -
 (b*c*e*PolyLog[2, (1 - c^2*x^2)^(-1)])/2
```

Rubi [A] time = 0.269076, antiderivative size = 94, normalized size of antiderivative = 0.9, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6082, 2475, 2411, 2344, 2301, 2316, 2315, 5949}

$$-\frac{1}{2}bce\text{PolyLog}\left(2,c^2x^2\right)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{x}-\frac{ce\left(a+b\coth^{-1}(cx)\right)^2}{b}-\frac{bc\left(e\log\left(1-c^2x^2\right)+d\right)^2}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^2,x]
```

```
[Out] -((c*e*(a + b*ArcCoth[c*x])^2)/b) + b*c*d*Log[x] - ((a + b*ArcCoth[c*x])*(d
 + e*Log[1 - c^2*x^2]))/x - (b*c*(d + e*Log[1 - c^2*x^2])^2)/(4*e) - (b*c*e
*PolyLog[2, c^2*x^2])/2
```

Rule 6082

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*(d + e*Log[f + g*x^2])*(a +
 b*ArcCoth[c*x]))/(m + 1), x] + (-Dist[(b*c)/(m + 1), Int[(x^(m + 1)*(d + e
*Log[f + g*x^2]))/(1 - c^2*x^2), x], x] - Dist[(2*e*g)/(m + 1), Int[(x^(m +
 2)*(a + b*ArcCoth[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b, c, d, e, f,
g}, x] && ILtQ[m/2, 0]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^2}\,dx = -\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x}+(bc)\int \frac{d+e\log\left(1-c^2\right.}{x\left(1-c^2x^2\right)}$$

$$= -\frac{ce\left(a+b\coth^{-1}(cx)\right)^2}{b}-\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x}$$

$$= -\frac{ce\left(a+b\coth^{-1}(cx)\right)^2}{b}-\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x}$$

$$= -\frac{ce\left(a+b\coth^{-1}(cx)\right)^2}{b}-\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x}$$

$$= -\frac{ce\left(a+b\coth^{-1}(cx)\right)^2}{b}+bcd\log(x)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\right.}{x}$$

$$= -\frac{ce\left(a+b\coth^{-1}(cx)\right)^2}{b}+bcd\log(x)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\right.}{x}$$

Mathematica [B] time = 0.203594, size = 332, normalized size = 3.16

$$-\frac{4bcex\text{PolyLog}(2,-cx)+4bcex\text{PolyLog}(2,cx)-2bcex\text{PolyLog}\left(2,\frac{1}{2}-\frac{cx}{2}\right)-2bcex\text{PolyLog}\left(2,\frac{1}{2}(cx+1)\right)+4ae\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^2,x]
```

```
[Out] -(4*a*d + 4*b*d*ArcCoth[c*x] + 4*b*c*e*x*ArcCoth[c*x]^2 + 8*a*c*e*x*ArcTanh
[c*x] - 4*b*c*d*x*Log[x] - b*c*e*x*Log[-c^(-1) + x]^2 - b*c*e*x*Log[c^(-1)
+ x]^2 - 2*b*c*e*x*Log[c^(-1) + x]*Log[(1 - c*x)/2] + 4*b*c*e*x*Log[x]*Log[
1 - c*x] - 2*b*c*e*x*Log[-c^(-1) + x]*Log[(1 + c*x)/2] + 4*b*c*e*x*Log[x]*L
og[1 + c*x] + 4*a*e*Log[1 - c^2*x^2] + 2*b*c*d*x*Log[1 - c^2*x^2] + 4*b*e*A
rcCoth[c*x]*Log[1 - c^2*x^2] - 4*b*c*e*x*Log[x]*Log[1 - c^2*x^2] + 2*b*c*e*
x*Log[-c^(-1) + x]*Log[1 - c^2*x^2] + 2*b*c*e*x*Log[c^(-1) + x]*Log[1 - c^2
*x^2] + 4*b*c*e*x*PolyLog[2, -(c*x)] + 4*b*c*e*x*PolyLog[2, c*x] - 2*b*c*e*
x*PolyLog[2, 1/2 - (c*x)/2] - 2*b*c*e*x*PolyLog[2, (1 + c*x)/2])/(4*x)
```

Maple [F] time = 1.642, size = 0, normalized size = 0.

$$\int \frac{(a+b\,\mathrm{arccoth}\,(cx))\left(d+e\ln\left(-c^2x^2+1\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^2,x)
```

```
[Out] int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\,\mathrm{arcoth}\,(cx)}{x}\right)bd-\left(c^2\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)+\frac{\log\left(-c^2x^2+1\right)}{x}\right)ae-\frac{1}{2}\,be\left(\log \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^2,x, algorithm="maxima
")
```

```
[Out] -1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arccoth(c*x)/x)*b*d - (c^2*(log(c
*x + 1)/c - log(c*x - 1)/c) + log(-c^2*x^2 + 1)/x)*a*e - 1/2*b*e*(log(c*x +
 1)^2/x - integrate(-((c*x + 1)*log(c*x - 1)^2 - (I*pi + (I*pi*c + 2*c)*x)*
log(c*x + 1) - (-I*pi - I*pi*c*x)*log(c*x - 1))/(c*x^3 + x^2), x)) - a*d/x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{bd\,\mathrm{arcoth}\,(cx)+ad+(be\,\mathrm{arcoth}\,(cx)+ae)\log\left(-c^2x^2+1\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^2,x, algorithm="fricas
")
```

```
[Out] integral((b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(-c^2*x^2 +
1))/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+b\operatorname{acoth}(cx))\left(d+e\log\left(-c^2x^2+1\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1))/x**2,x)
```

```
[Out] Integral((a + b*acoth(c*x))*(d + e*log(-c**2*x**2 + 1))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arcoth}(cx)+a)\left(e\log\left(-c^2x^2+1\right)+d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^2, x)
```

3.276 $\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^4}\,dx$

Optimal. Leaf size=197

$$-\frac{1}{6}bc^3e\text{PolyLog}\left(2,\frac{1}{1-c^2x^2}\right)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}-\frac{c^3e\left(a+b\coth^{-1}(cx)\right)^2}{3b}+\frac{2c^2e\left(a+b\coth^{-1}\right.}{3x}$$

```
[Out] (2*c^2*e*(a + b*ArcCoth[c*x]))/(3*x) - (c^3*e*(a + b*ArcCoth[c*x])^2)/(3*b)
 - b*c^3*e*Log[x] + (b*c^3*e*Log[1 - c^2*x^2])/3 - (b*c*(1 - c^2*x^2)*(d +
e*Log[1 - c^2*x^2]))/(6*x^2) - ((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2
]))/(3*x^3) + (b*c^3*(d + e*Log[1 - c^2*x^2])*Log[1 - (1 - c^2*x^2)^(-1)])/
6 - (b*c^3*e*PolyLog[2, (1 - c^2*x^2)^(-1)])/6
```

Rubi [A] time = 0.463391, antiderivative size = 191, normalized size of antiderivative = 0.97, number of steps used = 17, number of rules used = 16, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.593, Rules used = {6082, 2475, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 5983, 5917, 266, 36, 29, 5949}

$$-\frac{1}{6}bc^3e\text{PolyLog}\left(2,c^2x^2\right)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}-\frac{c^3e\left(a+b\coth^{-1}(cx)\right)^2}{3b}+\frac{2c^2e\left(a+b\coth^{-1}(cx)\right)}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^4,x]
```

```
[Out] (2*c^2*e*(a + b*ArcCoth[c*x]))/(3*x) - (c^3*e*(a + b*ArcCoth[c*x])^2)/(3*b)
 + (b*c^3*d*Log[x])/3 - b*c^3*e*Log[x] + (b*c^3*e*Log[1 - c^2*x^2])/3 - (b*
c*(1 - c^2*x^2)*(d + e*Log[1 - c^2*x^2]))/(6*x^2) - ((a + b*ArcCoth[c*x])*(
d + e*Log[1 - c^2*x^2]))/(3*x^3) - (b*c^3*(d + e*Log[1 - c^2*x^2])^2)/(12*e
) - (b*c^3*e*PolyLog[2, c^2*x^2])/6
```

Rule 6082

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*(d + e*Log[f + g*x^2])*(a +
 b*ArcCoth[c*x]))/(m + 1), x] + (-Dist[(b*c)/(m + 1), Int[(x^(m + 1)*(d + e
*Log[f + g*x^2]))/(1 - c^2*x^2), x], x] - Dist[(2*e*g)/(m + 1), Int[(x^(m +
 2)*(a + b*ArcCoth[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b, c, d, e, f,
g}, x] && ILtQ[m/2, 0]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
```

```
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5983

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcCoth[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^4}\,dx = -\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{3x^3}+\frac{1}{3}(bc)\int\frac{d+e\log\left(1-c^2x\right.}{x^3\left(1-c^2x^2\right)}$$

$$= -\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{3x^3}+\frac{1}{6}(bc)\,\mathrm{Subst}\left(\int\frac{d+e\log}{x^2\left(1-\right.}\right.$$

$$= \frac{2c^2e\left(a+b\coth^{-1}(cx)\right)}{3x}-\frac{c^3e\left(a+b\coth^{-1}(cx)\right)^2}{3b}-\frac{\left(a+b\coth^{-1}(cx)\right)}{}$$

$$= \frac{2c^2e\left(a+b\coth^{-1}(cx)\right)}{3x}-\frac{c^3e\left(a+b\coth^{-1}(cx)\right)^2}{3b}-\frac{\left(a+b\coth^{-1}(cx)\right)}{}$$

$$= \frac{2c^2e\left(a+b\coth^{-1}(cx)\right)}{3x}-\frac{c^3e\left(a+b\coth^{-1}(cx)\right)^2}{3b}-\frac{bc\left(1-c^2x^2\right)\left(d+e\right.}{6x^2}$$

$$= \frac{2c^2e\left(a+b\coth^{-1}(cx)\right)}{3x}-\frac{c^3e\left(a+b\coth^{-1}(cx)\right)^2}{3b}+\frac{1}{3}bc^3d\log(x)-bc^3$$

$$= \frac{2c^2e\left(a+b\coth^{-1}(cx)\right)}{3x}-\frac{c^3e\left(a+b\coth^{-1}(cx)\right)^2}{3b}+\frac{1}{3}bc^3d\log(x)-bc^3$$

Mathematica [B] time = 0.382553, size = 457, normalized size = 2.32

$$\frac{1}{6}\left(-2bc^3e\mathrm{PolyLog}(2,-cx)-2bc^3e\mathrm{PolyLog}(2,cx)+bc^3e\mathrm{PolyLog}\left(2,\frac{1}{2}-\frac{cx}{2}\right)+bc^3e\mathrm{PolyLog}\left(2,\frac{1}{2}(cx+1)\right)-\frac{2ae\log\left(\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^4,x]
```

```
[Out] ((-2*a*d)/x^3 - (b*c*d)/x^2 + (4*a*c^2*e)/x - (2*b*d*ArcCoth[c*x])/x^3 + (4
*b*c^2*e*ArcCoth[c*x])/x - 2*b*c^3*e*ArcCoth[c*x]^2 - 4*a*c^3*e*ArcTanh[c*x
] - 4*b*c^3*e*Log[1/Sqrt[1 - 1/(c^2*x^2)]] + 2*b*c^3*d*Log[x] - 2*b*c^3*e*L
og[x] + (b*c^3*e*Log[-c^(-1) + x]^2)/2 + (b*c^3*e*Log[c^(-1) + x]^2)/2 + b*
c^3*e*Log[c^(-1) + x]*Log[(1 - c*x)/2] - 2*b*c^3*e*Log[x]*Log[1 - c*x] + b*
c^3*e*Log[-c^(-1) + x]*Log[(1 + c*x)/2] - 2*b*c^3*e*Log[x]*Log[1 + c*x] - b
*c^3*d*Log[1 - c^2*x^2] + b*c^3*e*Log[1 - c^2*x^2] - (2*a*e*Log[1 - c^2*x^2
])/x^3 - (b*c*e*Log[1 - c^2*x^2])/x^2 - (2*b*e*ArcCoth[c*x]*Log[1 - c^2*x^2
])/x^3 + 2*b*c^3*e*Log[x]*Log[1 - c^2*x^2] - b*c^3*e*Log[-c^(-1) + x]*Log[1
 - c^2*x^2] - b*c^3*e*Log[c^(-1) + x]*Log[1 - c^2*x^2] - 2*b*c^3*e*PolyLog[
2, -(c*x)] - 2*b*c^3*e*PolyLog[2, c*x] + b*c^3*e*PolyLog[2, 1/2 - (c*x)/2]
+ b*c^3*e*PolyLog[2, (1 + c*x)/2])/6
```

Maple [F] time = 4.322, size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{arccoth}\,(cx))\left(d + e\ln\left(-c^2x^2 + 1\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^4,x)
```

```
[Out] int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{6}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\mathrm{arcoth}\,(cx)}{x^3}\right)bd-\frac{1}{3}\left(\left(c\log\left(cx+1\right)-c\log\left(cx-1\right)-\frac{2}{x}\right)c^2+\frac{\log\left(-c^2\right.}{x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^4,x, algorithm="maxima
")
```

```
[Out] -1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arccoth(c*x)/x^3)
*b*d - 1/3*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c^2 + log(-c^2*x^2 + 1)
/x^3)*a*e - 1/6*b*e*(log(c*x + 1)^2/x^3 - 3*integrate(-1/3*(3*(c*x + 1)*log
(c*x - 1)^2 - (3*I*pi + (3*I*pi*c + 2*c)*x)*log(c*x + 1) - (-3*I*pi - 3*I*p
i*c*x)*log(c*x - 1))/(c*x^5 + x^4), x)) - 1/3*a*d/x^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{bd\,\mathrm{arcoth}\,(cx)+ad+(be\,\mathrm{arcoth}\,(cx)+ae)\log\left(-c^2x^2+1\right)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^4,x, algorithm="fricas
")
```

```
[Out] integral((b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(-c^2*x^2 +
1))/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{acoth}\,(cx))\left(d + e\log\left(-c^2x^2 + 1\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1))/x**4,x)
```

```
[Out] Integral((a + b*acoth(c*x))*(d + e*log(-c**2*x**2 + 1))/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcoth}\,(cx) + a)\left(e\log\left(-c^2x^2 + 1\right) + d\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^4, x)
```

3.277 $\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^6}\,dx$

Optimal. Leaf size=256

$$-\frac{1}{10}bc^5e\text{PolyLog}\left(2,\frac{1}{1-c^2x^2}\right)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}+\frac{2c^2e\left(a+b\coth^{-1}(cx)\right)}{15x^3}-\frac{c^5e\left(a+b\coth\right.}{5b}$$

```
[Out] (7*b*c^3*e)/(60*x^2) + (2*c^2*e*(a + b*ArcCoth[c*x]))/(15*x^3) + (2*c^4*e*(
a + b*ArcCoth[c*x]))/(5*x) - (c^5*e*(a + b*ArcCoth[c*x])^2)/(5*b) - (5*b*c^
5*e*Log[x])/6 + (19*b*c^5*e*Log[1 - c^2*x^2])/60 - (b*c*(d + e*Log[1 - c^2*
x^2]))/(20*x^4) - (b*c^3*(1 - c^2*x^2)*(d + e*Log[1 - c^2*x^2]))/(10*x^2) -
 ((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/(5*x^5) + (b*c^5*(d + e*Lo
g[1 - c^2*x^2])*Log[1 - (1 - c^2*x^2)^(-1)])/10 - (b*c^5*e*PolyLog[2, (1 -
c^2*x^2)^(-1)])/10
```

Rubi [A] time = 0.670072, antiderivative size = 250, normalized size of antiderivative = 0.98, number of steps used = 26, number of rules used = 18, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6082, 2475, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2319, 44, 5983, 5917, 266, 36, 29, 5949}

$$-\frac{1}{10}bc^5e\text{PolyLog}\left(2,c^2x^2\right)-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}+\frac{2c^2e\left(a+b\coth^{-1}(cx)\right)}{15x^3}-\frac{c^5e\left(a+b\coth^{-1}(\right.}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^6,x]
```

```
[Out] (7*b*c^3*e)/(60*x^2) + (2*c^2*e*(a + b*ArcCoth[c*x]))/(15*x^3) + (2*c^4*e*(
a + b*ArcCoth[c*x]))/(5*x) - (c^5*e*(a + b*ArcCoth[c*x])^2)/(5*b) + (b*c^5*
d*Log[x])/5 - (5*b*c^5*e*Log[x])/6 + (19*b*c^5*e*Log[1 - c^2*x^2])/60 - (b*
c*(d + e*Log[1 - c^2*x^2]))/(20*x^4) - (b*c^3*(1 - c^2*x^2)*(d + e*Log[1 -
c^2*x^2]))/(10*x^2) - ((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/(5*x^
5) - (b*c^5*(d + e*Log[1 - c^2*x^2])^2)/(20*e) - (b*c^5*e*PolyLog[2, c^2*x^
2])/10
```

Rule 6082

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*(d + e*Log[f + g*x^2])*(a +
 b*ArcCoth[c*x]))/(m + 1), x] + (-Dist[(b*c)/(m + 1), Int[(x^(m + 1)*(d + e
*Log[f + g*x^2]))/(1 - c^2*x^2), x], x] - Dist[(2*e*g)/(m + 1), Int[(x^(m +
 2)*(a + b*ArcCoth[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b, c, d, e, f,
g}, x] && ILtQ[m/2, 0]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 5983

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcCoth[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^6}dx = -\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{5x^5}+\frac{1}{5}(bc)\int \frac{d+e\log\left(1-c^2x\right.}{x^5\left(1-c^2x^2\right)}$$

$$= -\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{5x^5}+\frac{1}{10}(bc)\,\text{Subst}\left(\int \frac{d+e\log}{x^3\left(1\right.}\right.$$

$$= \frac{2c^2e\left(a+b\coth^{-1}(cx)\right)}{15x^3}-\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{5x^5}-\ldots$$

$$= \frac{2c^2e\left(a+b\coth^{-1}(cx)\right)}{15x^3}+\frac{2c^4e\left(a+b\coth^{-1}(cx)\right)}{5x}-\frac{c^5e\left(a+b\coth^{-1}(\ldots\right.}{5b}$$

$$= \frac{2c^2e\left(a+b\coth^{-1}(cx)\right)}{15x^3}+\frac{2c^4e\left(a+b\coth^{-1}(cx)\right)}{5x}-\frac{c^5e\left(a+b\coth^{-1}(\ldots\right.}{5b}$$

$$= \frac{bc^3e}{15x^2}+\frac{2c^2e\left(a+b\coth^{-1}(cx)\right)}{15x^3}+\frac{2c^4e\left(a+b\coth^{-1}(cx)\right)}{5x}-\frac{c^5e\left(a+b\ldots\right.}{\ldots}$$

$$= \frac{7bc^3e}{60x^2}+\frac{2c^2e\left(a+b\coth^{-1}(cx)\right)}{15x^3}+\frac{2c^4e\left(a+b\coth^{-1}(cx)\right)}{5x}-\frac{c^5e\left(a+b\ldots\right.}{\ldots}$$

$$= \frac{7bc^3e}{60x^2}+\frac{2c^2e\left(a+b\coth^{-1}(cx)\right)}{15x^3}+\frac{2c^4e\left(a+b\coth^{-1}(cx)\right)}{5x}-\frac{c^5e\left(a+b\ldots\right.}{\ldots}$$

Mathematica [F] time = 0.279638, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^6}dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^6,x]
```

```
[Out] Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^6, x]
```

Maple [F] time = 4.318, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\,\text{arccoth}\,(cx)\right)\left(d+e\ln\left(-c^2x^2+1\right)\right)}{x^6}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^6,x)
```

```
[Out] int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^6,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{20}\left(\left(2\,c^4\log\left(c^2x^2-1\right)-2\,c^4\log\left(x^2\right)+\frac{2\,c^2x^2+1}{x^4}\right)c+\frac{4\,\mathrm{arcoth}\left(cx\right)}{x^5}\right)bd-\frac{1}{15}\left(\left(3\,c^3\log\left(cx+1\right)-3\,c^3\log\left(cx-1\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^6,x, algorithm="maxima
")
```

```
[Out] -1/20*((2*c^4*log(c^2*x^2 - 1) - 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c +
4*arccoth(c*x)/x^5)*b*d - 1/15*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) -
2*(3*c^2*x^2 + 1)/x^3)*c^2 + 3*log(-c^2*x^2 + 1)/x^5)*a*e - 1/10*b*e*(log(c
*x + 1)^2/x^5 - 5*integrate(-1/5*(5*(c*x + 1)*log(c*x - 1)^2 - (5*I*pi + (5
*I*pi*c + 2*c)*x)*log(c*x + 1) - (-5*I*pi - 5*I*pi*c*x)*log(c*x - 1))/(c*x^
7 + x^6), x)) - 1/5*a*d/x^5
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{bd\,\mathrm{arcoth}\left(cx\right)+ad+\left(be\,\mathrm{arcoth}\left(cx\right)+ae\right)\log\left(-c^2x^2+1\right)}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^6,x, algorithm="fricas
")
```

```
[Out] integral((b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(-c^2*x^2 +
1))/x^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1))/x**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\,\mathrm{arcoth}\left(cx\right)+a\right)\left(e\log\left(-c^2x^2+1\right)+d\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^6, x)
```

3.278 $\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx$

Optimal. Leaf size=512

$$\frac{be\left(c^2f+g\right)\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c^2g}-\frac{be\left(c^2f+g\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-f}-\sqrt{g}x\right)}{(cx+1)\left(c\sqrt{-f}-\sqrt{g}\right)}\right)}{4c^2g}-\frac{be\left(c^2f+g\right)\text{PolyLog}\left(2,1-\frac{2c(\sqrt{}}{(cx+1)}\right.}{4c^2g}$$

```
[Out] (b*(d - e)*x)/(2*c) - (b*e*x)/c + (d*x^2*(a + b*ArcCoth[c*x]))/2 - (e*x^2*(
a + b*ArcCoth[c*x]))/2 + (b*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/(c*Sqrt[
g]) - (b*(d - e)*ArcTanh[c*x])/(2*c^2) - (b*e*(c^2*f + g)*ArcTanh[c*x]*Log[
2/(1 + c*x)])/(c^2*g) + (b*e*(c^2*f + g)*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] -
Sqrt[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))])/(2*c^2*g) + (b*e*(c^2*f +
g)*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1
 + c*x))])/(2*c^2*g) + (b*e*x*Log[f + g*x^2])/(2*c) + (e*(f + g*x^2)*(a + b
*ArcCoth[c*x])*Log[f + g*x^2])/(2*g) - (b*e*(c^2*f + g)*ArcTanh[c*x]*Log[f
+ g*x^2])/(2*c^2*g) + (b*e*(c^2*f + g)*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c^2*
g) - (b*e*(c^2*f + g)*PolyLog[2, 1 - (2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[
-f] - Sqrt[g])*(1 + c*x))])/(4*c^2*g) - (b*e*(c^2*f + g)*PolyLog[2, 1 - (2*
c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1 + c*x))])/(4*c^2*g)
```

Rubi [A] time = 0.76723, antiderivative size = 512, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.773, Rules used = {2454, 2389, 2295, 6084, 321, 207, 517, 2528, 2448, 205, 2470, 12, 5992, 5920, 2402, 2315, 2447}

$$\frac{be\left(c^2f+g\right)\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c^2g}-\frac{be\left(c^2f+g\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-f}-\sqrt{g}x\right)}{(cx+1)\left(c\sqrt{-f}-\sqrt{g}\right)}\right)}{4c^2g}-\frac{be\left(c^2f+g\right)\text{PolyLog}\left(2,1-\frac{2c(\sqrt{}}{(cx+1)}\right.}{4c^2g}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]),x]
```

```
[Out] (b*(d - e)*x)/(2*c) - (b*e*x)/c + (d*x^2*(a + b*ArcCoth[c*x]))/2 - (e*x^2*(
a + b*ArcCoth[c*x]))/2 + (b*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/(c*Sqrt[
g]) - (b*(d - e)*ArcTanh[c*x])/(2*c^2) - (b*e*(c^2*f + g)*ArcTanh[c*x]*Log[
2/(1 + c*x)])/(c^2*g) + (b*e*(c^2*f + g)*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] -
Sqrt[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))])/(2*c^2*g) + (b*e*(c^2*f +
g)*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1
 + c*x))])/(2*c^2*g) + (b*e*x*Log[f + g*x^2])/(2*c) + (e*(f + g*x^2)*(a + b
*ArcCoth[c*x])*Log[f + g*x^2])/(2*g) - (b*e*(c^2*f + g)*ArcTanh[c*x]*Log[f
+ g*x^2])/(2*c^2*g) + (b*e*(c^2*f + g)*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c^2*
g) - (b*e*(c^2*f + g)*PolyLog[2, 1 - (2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[
-f] - Sqrt[g])*(1 + c*x))])/(4*c^2*g) - (b*e*(c^2*f + g)*PolyLog[2, 1 - (2*
c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1 + c*x))])/(4*c^2*g)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 6084

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2]
), x]}, Dist[a + b*ArcCoth[c*x], u, x] - Dist[b*c, Int[ExpandIntegrand[u/(1
 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)
/2, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)dx=\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(cx)\right)+\frac{e\left(f+gx^2\right)\left(a\right.}{}$$

$$=\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(cx)\right)+\frac{e\left(f+gx^2\right)\left(a\right.}{}$$

$$=\frac{b(d-e)x}{2c}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(cx)\right)+\frac{e}{}$$

$$=\frac{b(d-e)x}{2c}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(cx)\right)-\frac{b}{}$$

$$=\frac{b(d-e)x}{2c}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(cx)\right)-\frac{b}{}$$

$$=\frac{b(d-e)x}{2c}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(cx)\right)-\frac{b}{}$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(c\right.$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(c\right.$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(c\right.$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(c\right.$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(c\right.$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(c\right.$$

Mathematica [A] time = 4.25167, size = 677, normalized size = 1.32

$$\frac{bc^2ef\,\mathrm{PolyLog}\left(2,\frac{\left(c^2f+g\right)e^{2\coth^{-1}(cx)}}{c^2f+2c\sqrt{-f}\sqrt{g}-g}\right)+2be\left(c^2f+g\right)\mathrm{PolyLog}\left(2,e^{-2\coth^{-1}(cx)}\right)+be\left(c^2f+g\right)\mathrm{PolyLog}\left(2,\frac{\left(c^2f+g\right)e^{2\coth}}{c^2f-2c\sqrt{-f}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]),x]
```

```
[Out] (2*b*c*d*g*x - 6*b*c*e*g*x + 2*a*c^2*d*g*x^2 - 2*a*c^2*e*g*x^2 - 2*b*d*g*Ar
cCoth[c*x] + 2*b*e*g*ArcCoth[c*x] + 2*b*c^2*d*g*x^2*ArcCoth[c*x] - 2*b*c^2*
e*g*x^2*ArcCoth[c*x] - 4*b*c^2*e*f*ArcCoth[c*x]^2 - 4*b*e*g*ArcCoth[c*x]^2
+ 4*b*c*e*Sqrt[f]*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]] - 4*b*c^2*e*f*ArcCoth
[c*x]*Log[1 - E^(-2*ArcCoth[c*x])] - 4*b*e*g*ArcCoth[c*x]*Log[1 - E^(-2*Arc
Coth[c*x])] + 2*b*c^2*e*f*ArcCoth[c*x]*Log[1 + (E^(2*ArcCoth[c*x])*(c^2*f +
 g))/(-(c^2*f) - 2*c*Sqrt[-f]*Sqrt[g] + g)] + 2*b*e*g*ArcCoth[c*x]*Log[1 +
```

```
(E^(2*ArcCoth[c*x])*(c^2*f + g))/(-(c^2*f) - 2*c*Sqrt[-f]*Sqrt[g] + g)] + 2
*b*c^2*e*f*ArcCoth[c*x]*Log[1 + (E^(2*ArcCoth[c*x])*(c^2*f + g))/(-(c^2*f)
+ 2*c*Sqrt[-f]*Sqrt[g] + g)] + 2*b*e*g*ArcCoth[c*x]*Log[1 + (E^(2*ArcCoth[c
*x])*(c^2*f + g))/(-(c^2*f) + 2*c*Sqrt[-f]*Sqrt[g] + g)] + 2*a*c^2*e*f*Log[
f + g*x^2] + 2*b*c*e*g*x*Log[f + g*x^2] + 2*a*c^2*e*g*x^2*Log[f + g*x^2] -
2*b*e*g*ArcCoth[c*x]*Log[f + g*x^2] + 2*b*c^2*e*g*x^2*ArcCoth[c*x]*Log[f +
g*x^2] + 2*b*e*(c^2*f + g)*PolyLog[2, E^(-2*ArcCoth[c*x])] + b*e*(c^2*f + g
)*PolyLog[2, (E^(2*ArcCoth[c*x])*(c^2*f + g))/(c^2*f - 2*c*Sqrt[-f]*Sqrt[g]
 - g)] + b*c^2*e*f*PolyLog[2, (E^(2*ArcCoth[c*x])*(c^2*f + g))/(c^2*f + 2*c
*Sqrt[-f]*Sqrt[g] - g)] + b*e*g*PolyLog[2, (E^(2*ArcCoth[c*x])*(c^2*f + g))
/(c^2*f + 2*c*Sqrt[-f]*Sqrt[g] - g)])/(4*c^2*g)
```

Maple [C] time = 1.996, size = 8491, normalized size = 16.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccoth(c*x))*(d+e*ln(g*x^2+f)),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccoth(c*x))*(d+e*log(g*x^2+f)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(bdx\operatorname{arcoth}(cx) + adx + (bex\operatorname{arcoth}(cx) + aex)\log\left(gx^2 + f\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccoth(c*x))*(d+e*log(g*x^2+f)),x, algorithm="fricas")
```

```
[Out] integral(b*d*x*arccoth(c*x) + a*d*x + (b*e*x*arccoth(c*x) + a*e*x)*log(g*x^
2 + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acoth(c*x))*(d+e*ln(g*x**2+f)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\mathrm{arcoth}\,(cx)+a)\Big(e\log\Big(gx^2+f\Big)+d\Big)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccoth(c*x))*(d+e*log(g*x^2+f)),x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(g*x^2 + f) + d)*x, x)
```

3.279 $\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)dx$

Optimal. Leaf size=546

$$\frac{be\text{PolyLog}\left(2,\frac{c^2(f+gx^2)}{c^2f+g}\right)}{2c}-\frac{ibe\sqrt{f}\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(1-cx)}{(-\sqrt{g}+ic\sqrt{f})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{g}}+\frac{ibe\sqrt{f}\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(cx+1)}{(\sqrt{g}+ic\sqrt{f})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{g}}+x$$

```
[Out] -2*a*e*x - 2*b*e*x*ArcCoth[c*x] + (2*a*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]
])/Sqrt[g] - (b*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[1 - 1/(c*x)])/Sqr
t[g] + (b*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[1 + 1/(c*x)])/Sqrt[g] +
 (b*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(1 - c*x)
)/((I*c*Sqrt[f] - Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[g] - (b*e*Sqrt[f
]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(1 + c*x))/((I*c*Sqrt[
f] + Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[g] - (b*e*Log[1 - c^2*x^2])/c
 + x*(a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]) + (b*Log[(g*(1 - c^2*x^2))
/(c^2*f + g)]*(d + e*Log[f + g*x^2]))/(2*c) + (b*e*PolyLog[2, (c^2*(f + g*x
^2))/(c^2*f + g)])/(2*c) - ((I/2)*b*e*Sqrt[f]*PolyLog[2, 1 + (2*Sqrt[f]*Sqr
t[g]*(1 - c*x))/((I*c*Sqrt[f] - Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[g]
 + ((I/2)*b*e*Sqrt[f]*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(1 + c*x))/((I*c*Sq
rt[f] + Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[g]
```

Rubi [A] time = 1.38741, antiderivative size = 546, normalized size of antiderivative = 1., number of steps used = 38, number of rules used = 20, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.952, Rules used = {6074, 2475, 2394, 2393, 2391, 5981, 5911, 260, 5975, 205, 5973, 2470, 12, 6688, 4876, 4848, 4856, 2402, 2315, 2447}

$$\frac{be\text{PolyLog}\left(2,\frac{c^2(f+gx^2)}{c^2f+g}\right)}{2c}-\frac{ibe\sqrt{f}\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(1-cx)}{(-\sqrt{g}+ic\sqrt{f})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{g}}+\frac{ibe\sqrt{f}\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(cx+1)}{(\sqrt{g}+ic\sqrt{f})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{g}}+x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]),x]
```

```
[Out] -2*a*e*x - 2*b*e*x*ArcCoth[c*x] + (2*a*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]
])/Sqrt[g] - (b*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[1 - 1/(c*x)])/Sqr
t[g] + (b*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[1 + 1/(c*x)])/Sqrt[g] +
 (b*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(1 - c*x)
)/((I*c*Sqrt[f] - Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[g] - (b*e*Sqrt[f
]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(1 + c*x))/((I*c*Sqrt[
f] + Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[g] - (b*e*Log[1 - c^2*x^2])/c
 + x*(a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]) + (b*Log[(g*(1 - c^2*x^2))
/(c^2*f + g)]*(d + e*Log[f + g*x^2]))/(2*c) + (b*e*PolyLog[2, (c^2*(f + g*x
^2))/(c^2*f + g)])/(2*c) - ((I/2)*b*e*Sqrt[f]*PolyLog[2, 1 + (2*Sqrt[f]*Sqr
t[g]*(1 - c*x))/((I*c*Sqrt[f] - Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[g]
 + ((I/2)*b*e*Sqrt[f]*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(1 + c*x))/((I*c*Sq
rt[f] + Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[g]
```

Rule 6074

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.)), x_Symbol] :> Simp[x*(d + e*Log[f + g*x^2])*(a + b*ArcCoth[c*x]), x]
 + (-Dist[b*c, Int[(x*(d + e*Log[f + g*x^2]))/(1 - c^2*x^2), x], x] - Dist[
2*e*g, Int[(x^2*(a + b*ArcCoth[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b,
c, d, e, f, g}, x]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5981

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5975

```
Int[(ArcCoth[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
 Dist[a, Int[1/(d + e*x^2), x], x] + Dist[b, Int[ArcCoth[c*x]/(d + e*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5973

```
Int[ArcCoth[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[1/2, Int[
Log[1 + 1/(c*x)]/(d + e*x^2), x], x] - Dist[1/2, Int[Log[1 - 1/(c*x)]/(d +
e*x^2), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
```

```
x]][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)dx = x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)-(bc)\int\frac{x\left(d+e\log\left(f+\right.\right.}{1-c^2x^2}$$

$$= x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}(bc)\,\text{Subst}\left(\int\frac{d+e\log}{1-}\right.$$

$$= -2aex+x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)+\frac{b\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right.}{}$$

$$= -2aex-2bex\coth^{-1}(cx)+\frac{2ae\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}+x\left(a+b\coth^{-1}(cx\right.$$

$$= -2aex-2bex\coth^{-1}(cx)+\frac{2ae\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}-\frac{be\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}$$

$$= -2aex-2bex\coth^{-1}(cx)+\frac{2ae\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}-\frac{be\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}$$

$$= -2aex-2bex\coth^{-1}(cx)+\frac{2ae\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}-\frac{be\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}$$

$$= -2aex-2bex\coth^{-1}(cx)+\frac{2ae\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}-\frac{be\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}$$

$$= -2aex-2bex\coth^{-1}(cx)+\frac{2ae\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}-\frac{be\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}$$

$$= -2aex-2bex\coth^{-1}(cx)+\frac{2ae\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}-\frac{be\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}$$

$$= -2aex-2bex\coth^{-1}(cx)+\frac{2ae\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}-\frac{be\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}$$

$$= -2aex-2bex\coth^{-1}(cx)+\frac{2ae\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}-\frac{be\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}$$

$$= -2aex-2bex\coth^{-1}(cx)+\frac{2ae\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}-\frac{be\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}$$

Mathematica [B] time = 3.21188, size = 1287, normalized size = 2.36

result too large to display

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]),x]

[Out] a*d*x - 2*a*e*x + b*d*x*ArcCoth[c*x] + (2*a*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[g] + (b*d*Log[1 - c^2*x^2])/(2*c) + a*e*x*Log[f + g*x^2] + b*e*(x*ArcCoth[c*x] + Log[1 - c^2*x^2]/(2*c))*Log[f + g*x^2] + (b*e*(-4*c*x*ArcCoth[c*x] + 4*Log[1/(c*Sqrt[1 - 1/(c^2*x^2)]*x)] + (Sqrt[c^2*f*g]*((-2*I)*ArcCos[(c^2*f - g)/(c^2*f + g)]*ArcTan[Sqrt[c^2*f*g]/(c*g*x)] + 4*ArcCoth[c*x]*ArcTan[(c*g*x)/Sqrt[c^2*f*g]] - (ArcCos[(c^2*f - g)/(c^2*f + g)] + 2*ArcTan[Sqrt[c^2*f*g]/(c*g*x)])*Log[((2*I)*g*(I*c^2*f + Sqrt[c^2*f*g])*(-1 + 1/(c*x)))/((c^2*f + g)*(g + (I*Sqrt[c^2*f*g])/(c*x)))] - (ArcCos[(c^2*f - g)/(c^2*f + g)] - 2*ArcTan[Sqrt[c^2*f*g]/(c*g*x)])*Log[(2*g*(c^2*f + I*Sqrt[c^2*f*g])*(1 + 1/(c*x)))/((c^2*f + g)*(g + (I*Sqrt[c^2*f*g])/(c*x)))] + (ArcCos[(c^2*f - g)/(c^2*f + g)] + 2*(ArcTan[Sqrt[c^2*f*g]/(c*g*x)] + ArcTan[(c*g*x)/Sqrt[c^2*f*g]]))*Log[(Sqrt[2]*Sqrt[c^2*f*g])/(E^ArcCoth[c*x]*Sqrt[c^2*f + g]*Sqrt[-(c^2*f) + g + (c^2*f + g)*Cosh[2*ArcCoth[c*x]]])] + (ArcCos[(c^2*f - g)/(c^2*f + g)] - 2*(ArcTan[Sqrt[c^2*f*g]/(c*g*x)] + ArcTan[(c*g*x)/Sqrt[c^2*f*g]]))*Log[(Sqrt[2]*E^ArcCoth[c*x]*Sqrt[c^2*f*g])/(Sqrt[c^2*f + g]*Sqrt[-(c^2*f) + g + (c^2*f + g)*Cosh[2*ArcCoth[c*x]]])] + I*(-PolyLog[2, ((-(c^2*f) + g + (2*I)*Sqrt[c^2*f*g])*(g - (I*Sqrt[c^2*f*g])/(c*x)))/((c^2*f + g)*(g + (I*Sqrt[c^2*f*g])/(c*x)))] + PolyLog[2, ((c^2*f - g + (2*I)*Sqrt[c^2*f*g])*(I*g + Sqrt[c^2*f*g]/(c*x)))/((c^2*f + g)*((-I)*g + Sqrt[c^2*f*g]/(c*x)))])))/g))/(2*c) - (b*e*g*(((-Log[-c^(-1) + x] - Log[c^(-1) + x] + Log[1 - c^2*x^2])*Log[f + g*x^2])/(2*g) + (Log[-c^(-1) + x]*Log[1 - (Sqrt[g]*(-c^(-1) + x))/((-I)*Sqrt[f] - Sqrt[g]/c)] + PolyLog[2, (Sqrt[g]*(-c^(-1) + x))/((-I)*Sqrt[f] - Sqrt[g]/c)])/(2*g) + (Log[-c^(-1) + x]*Log[1 - (Sqrt[g]*(-c^(-1) + x))/(I*Sqrt[f] - Sqrt[g]/c)] + PolyLog[2, (Sqrt[g]*(-c^(-1) + x))/(I*Sqrt[f] - Sqrt[g]/c)])/(2*g) + (Log[c^(-1) + x]*Log[1 - (Sqrt[g]*(c^(-1) + x))/((-I)*Sqrt[f] + Sqrt[g]/c)] + PolyLog[2, (Sqrt[g]*(c^(-1) + x))/((-I)*Sqrt[f] + Sqrt[g]/c)])/(2*g) + (Log[c^(-1) + x]*Log[1 - (Sqrt[g]*(c^(-1) + x))/(I*Sqrt[f] + Sqrt[g]/c)] + PolyLog[2, (Sqrt[g]*(c^(-1) + x))/(I*Sqrt[f] + Sqrt[g]/c)])/(2*g)))/c

---

Maple [F] time = 1.994, size = 0, normalized size = 0.

$$\int (a + b\operatorname{arccoth}(cx)) \left(d + e\ln\left(gx^2 + f\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arccoth(c*x))*(d+e*ln(g*x^2+f)),x)

[Out] int((a+b*arccoth(c*x))*(d+e*ln(g*x^2+f)),x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f)),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(bd\,\text{arcoth}\,(cx)+ad+(be\,\text{arcoth}\,(cx)+ae)\log\left(gx^2+f\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f)),x, algorithm="fricas")
```

```
[Out] integral(b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(g*x^2 + f),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(c*x))*(d+e*ln(g*x**2+f)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\text{arcoth}\,(cx)+a)\left(e\log\left(gx^2+f\right)+d\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f)),x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(g*x^2 + f) + d), x)
```

3.280 $\int \frac{\left(a+b \coth^{-1}(cx)\right)\left(d+e \log\left(f+gx^2\right)\right)}{x}\,dx$

Optimal. Leaf size=100

$$\frac{1}{2}ae\text{PolyLog}\left(2,\frac{gx^2}{f}+1\right)+be\text{CannotIntegrate}\left(\frac{\coth^{-1}(cx)\log\left(f+gx^2\right)}{x},x\right)+\frac{1}{2}bd\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}bd\text{PolyLog}$$

```
[Out] b*e*CannotIntegrate[(ArcCoth[c*x]*Log[f + g*x^2])/x, x] + a*d*Log[x] + (a*e
*Log[-((g*x^2)/f)]*Log[f + g*x^2])/2 + (b*d*PolyLog[2, -(1/(c*x))])/2 - (b*
d*PolyLog[2, 1/(c*x)])/2 + (a*e*PolyLog[2, 1 + (g*x^2)/f])/2
```

Rubi [A] time = 0.251519, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x,x]
```

```
[Out] a*d*Log[x] + (a*e*Log[-((g*x^2)/f)]*Log[f + g*x^2])/2 + (b*d*PolyLog[2, -(1
/(c*x))])/2 - (b*d*PolyLog[2, 1/(c*x)])/2 + (a*e*PolyLog[2, 1 + (g*x^2)/f])
/2 + b*e*Defer[Int][(ArcCoth[c*x]*Log[f + g*x^2])/x, x]
```

Rubi steps

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx = d\int \frac{a+b\coth^{-1}(cx)}{x}\,dx+e\int \frac{\left(a+b\coth^{-1}(cx)\right)\log\left(f+gx^2\right)}{x}\,dx$$

$$= ad\log(x)+\frac{1}{2}bd\text{Li}_2\left(-\frac{1}{cx}\right)-\frac{1}{2}bd\text{Li}_2\left(\frac{1}{cx}\right)+(ae)\int \frac{\log\left(f+gx^2\right)}{x}\,dx+$$

$$= ad\log(x)+\frac{1}{2}bd\text{Li}_2\left(-\frac{1}{cx}\right)-\frac{1}{2}bd\text{Li}_2\left(\frac{1}{cx}\right)+\frac{1}{2}(ae)\text{Subst}\left(\int \frac{\log(f+g}{x}\right.$$

$$= ad\log(x)+\frac{1}{2}ae\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right)+\frac{1}{2}bd\text{Li}_2\left(-\frac{1}{cx}\right)-\frac{1}{2}bd\text{Li}_2\left(\right.$$

$$= ad\log(x)+\frac{1}{2}ae\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right)+\frac{1}{2}bd\text{Li}_2\left(-\frac{1}{cx}\right)-\frac{1}{2}bd\text{Li}_2\left(\right.$$

Mathematica [A] time = 0.27258, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x,x]
```

```
[Out] Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x, x]
```

Maple [A] time = 0.74, size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{arccoth}\,(cx))\left(d + e\ln\left(gx^2 + f\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(c*x))*(d+e*ln(g*x^2+f))/x,x)
```

```
[Out] int((a+b*arccoth(c*x))*(d+e*ln(g*x^2+f))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$ad\log(x) + \int \frac{be\left(\log\left(\frac{1}{cx}+1\right) - \log\left(-\frac{1}{cx}+1\right)\right)\log\left(gx^2+f\right)}{2x} + \frac{bd\left(\log\left(\frac{1}{cx}+1\right) - \log\left(-\frac{1}{cx}+1\right)\right)}{2x} + \frac{ae\log\left(gx^2+f\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="maxima")
```

```
[Out] a*d*log(x) + integrate(1/2*b*e*(log(1/(c*x) + 1) - log(-1/(c*x) + 1))*log(g
*x^2 + f)/x + 1/2*b*d*(log(1/(c*x) + 1) - log(-1/(c*x) + 1))/x + a*e*log(g*
x^2 + f)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{bd\,\mathrm{arcoth}\,(cx) + ad + (be\,\mathrm{arcoth}\,(cx) + ae)\log\left(gx^2+f\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="fricas")
```

```
[Out] integral((b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(g*x^2 + f))
/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(c*x))*(d+e*ln(g*x**2+f))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcoth}\,(cx)+a)\left(e\log\left(gx^2+f\right)+d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(g*x^2 + f) + d)/x, x)
```

3.281 $\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^2}\,dx$

Optimal. Leaf size=560

$$-\frac{1}{2}bce\text{PolyLog}\left(2,\frac{c^2\left(f+gx^2\right)}{c^2f+g}\right)+\frac{1}{2}bce\text{PolyLog}\left(2,\frac{gx^2}{f}+1\right)-\frac{ibe\sqrt{g}\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(1-cx)}{\left(-\sqrt{g}+ic\sqrt{f}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{2\sqrt{f}}+\frac{ibe\sqrt{g}}{}$$

```
[Out] (2*a*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] - (b*e*Sqrt[g]*ArcTan[(
Sqrt[g]*x)/Sqrt[f]]*Log[1 - 1/(c*x)])/Sqrt[f] + (b*e*Sqrt[g]*ArcTan[(Sqrt[g
]*x)/Sqrt[f]]*Log[1 + 1/(c*x)])/Sqrt[f] + (b*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/S
qrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(1 - c*x))/((I*c*Sqrt[f] - Sqrt[g])*(Sqrt[f
] - I*Sqrt[g]*x))])/Sqrt[f] - (b*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[
(2*Sqrt[f]*Sqrt[g]*(1 + c*x))/((I*c*Sqrt[f] + Sqrt[g])*(Sqrt[f] - I*Sqrt[g]
*x))])/Sqrt[f] - ((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x + (b*c*Log
[-((g*x^2)/f)]*(d + e*Log[f + g*x^2]))/2 - (b*c*Log[(g*(1 - c^2*x^2))/(c^2*
f + g)]*(d + e*Log[f + g*x^2]))/2 - (b*c*e*PolyLog[2, (c^2*(f + g*x^2))/(c^
2*f + g)])/2 + (b*c*e*PolyLog[2, 1 + (g*x^2)/f])/2 - ((I/2)*b*e*Sqrt[g]*Pol
yLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(1 - c*x))/((I*c*Sqrt[f] - Sqrt[g])*(Sqrt[f]
 - I*Sqrt[g]*x))])/Sqrt[f] + ((I/2)*b*e*Sqrt[g]*PolyLog[2, 1 - (2*Sqrt[f]*S
qrt[g]*(1 + c*x))/((I*c*Sqrt[f] + Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[
f]
```

Rubi [A] time = 1.26144, antiderivative size = 560, normalized size of antiderivative = 1., number of steps used = 38, number of rules used = 22, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6082, 2475, 36, 29, 31, 2416, 2394, 2315, 2393, 2391, 5975, 205, 5973, 2470, 12, 260, 6688, 4876, 4848, 4856, 2402, 2447}

$$-\frac{1}{2}bce\text{PolyLog}\left(2,\frac{c^2\left(f+gx^2\right)}{c^2f+g}\right)+\frac{1}{2}bce\text{PolyLog}\left(2,\frac{gx^2}{f}+1\right)-\frac{ibe\sqrt{g}\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(1-cx)}{\left(-\sqrt{g}+ic\sqrt{f}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{2\sqrt{f}}+\frac{ibe\sqrt{g}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x^2,x]
```

```
[Out] (2*a*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] - (b*e*Sqrt[g]*ArcTan[(
Sqrt[g]*x)/Sqrt[f]]*Log[1 - 1/(c*x)])/Sqrt[f] + (b*e*Sqrt[g]*ArcTan[(Sqrt[g
]*x)/Sqrt[f]]*Log[1 + 1/(c*x)])/Sqrt[f] + (b*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/S
qrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(1 - c*x))/((I*c*Sqrt[f] - Sqrt[g])*(Sqrt[f
] - I*Sqrt[g]*x))])/Sqrt[f] - (b*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[
(2*Sqrt[f]*Sqrt[g]*(1 + c*x))/((I*c*Sqrt[f] + Sqrt[g])*(Sqrt[f] - I*Sqrt[g]
*x))])/Sqrt[f] - ((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x + (b*c*Log
[-((g*x^2)/f)]*(d + e*Log[f + g*x^2]))/2 - (b*c*Log[(g*(1 - c^2*x^2))/(c^2*
f + g)]*(d + e*Log[f + g*x^2]))/2 - (b*c*e*PolyLog[2, (c^2*(f + g*x^2))/(c^
2*f + g)])/2 + (b*c*e*PolyLog[2, 1 + (g*x^2)/f])/2 - ((I/2)*b*e*Sqrt[g]*Pol
yLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(1 - c*x))/((I*c*Sqrt[f] - Sqrt[g])*(Sqrt[f]
 - I*Sqrt[g]*x))])/Sqrt[f] + ((I/2)*b*e*Sqrt[g]*PolyLog[2, 1 - (2*Sqrt[f]*S
qrt[g]*(1 + c*x))/((I*c*Sqrt[f] + Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[
f]
```

Rule 6082

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*(d + e*Log[f + g*x^2])*(a +
```

b*ArcCoth[c*x]))/(m + 1), x] + (-Dist[(b*c)/(m + 1), Int[(x^(m + 1)*(d + e*Log[f + g*x^2]))/(1 - c^2*x^2), x], x] - Dist[(2*e*g)/(m + 1), Int[(x^(m + 2)*(a + b*ArcCoth[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b, c, d, e, f, g}, x] && ILtQ[m/2, 0]

Rule 2475

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])

Rule 36

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 31

Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2416

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

Rule 2394

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]

Rule 2315

Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]

Rule 2393

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]

Rule 2391

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 5975

```
Int[(ArcCoth[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
 Dist[a, Int[1/(d + e*x^2), x], x] + Dist[b, Int[ArcCoth[c*x]/(d + e*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5973

```
Int[ArcCoth[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[1/2, Int[
Log[1 + 1/(c*x)]/(d + e*x^2), x], x] - Dist[1/2, Int[Log[1 - 1/(c*x)]/(d +
e*x^2), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
```

```
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^2}\,dx = -\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x} + (bc)\int \frac{d+e\log\left(f+gx\right.}{x\left(1-c^2x^2\right)}$$

$$= -\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x} + \frac{1}{2}(bc)\,\mathrm{Subst}\left(\int \frac{d+e\,\mathrm{lo}}{x(1}\right.$$

$$= \frac{2ae\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x} + \frac{1}{2}(b$$

$$= \frac{2ae\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(1-\frac{1}{cx}\right)}{\sqrt{f}} + \frac{be\sqrt{g}\tan^{-1}\left(\right.}{}$$

$$= \frac{2ae\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(1-\frac{1}{cx}\right)}{\sqrt{f}} + \frac{be\sqrt{g}\tan^{-1}\left(\right.}{}$$

$$= \frac{2ae\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(1-\frac{1}{cx}\right)}{\sqrt{f}} + \frac{be\sqrt{g}\tan^{-1}\left(\right.}{}$$

$$= \frac{2ae\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(1-\frac{1}{cx}\right)}{\sqrt{f}} + \frac{be\sqrt{g}\tan^{-1}\left(\right.}{}$$

$$= \frac{2ae\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(1-\frac{1}{cx}\right)}{\sqrt{f}} + \frac{be\sqrt{g}\tan^{-1}\left(\right.}{}$$

$$= \frac{2ae\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(1-\frac{1}{cx}\right)}{\sqrt{f}} + \frac{be\sqrt{g}\tan^{-1}\left(\right.}{}$$

$$= \frac{2ae\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(1-\frac{1}{cx}\right)}{\sqrt{f}} + \frac{be\sqrt{g}\tan^{-1}\left(\right.}{}$$

$$= \frac{2ae\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(1-\frac{1}{cx}\right)}{\sqrt{f}} + \frac{be\sqrt{g}\tan^{-1}\left(\right.}{}$$

Mathematica [B] time = 3.56189, size = 1236, normalized size = 2.21

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x^2,x]
```

```
[Out] -((a*d)/x) - (b*d*ArcCoth[c*x])/x + b*c*d*Log[x] - (b*c*d*Log[1 - c^2*x^2])
/2 + a*e*((2*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] - Log[f + g*x^2]/
x) + (b*e*(-(((2*ArcCoth[c*x] + c*x*(-2*Log[x] + Log[1 - c^2*x^2]))*Log[f +
```

```
 g*x^2])/x) - 2*c*(Log[x]*(Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + Log[1 + (I*Sqrt
[g]*x)/Sqrt[f]]) + PolyLog[2, ((-I)*Sqrt[g]*x)/Sqrt[f]] + PolyLog[2, (I*Sqr
t[g]*x)/Sqrt[f]]) + c*(Log[-c^(-1) + x]*Log[(c*(Sqrt[f] - I*Sqrt[g]*x))/(c*
Sqrt[f] - I*Sqrt[g])] + Log[c^(-1) + x]*Log[(c*(Sqrt[f] - I*Sqrt[g]*x))/(c*
Sqrt[f] + I*Sqrt[g])] + Log[-c^(-1) + x]*Log[(c*(Sqrt[f] + I*Sqrt[g]*x))/(c
*Sqrt[f] + I*Sqrt[g])] - (Log[-c^(-1) + x] + Log[c^(-1) + x] - Log[1 - c^2*
x^2])*Log[f + g*x^2] + Log[c^(-1) + x]*Log[1 - (Sqrt[g]*(1 + c*x))/(I*c*Sqr
t[f] + Sqrt[g])] + PolyLog[2, (c*Sqrt[g]*(c^(-1) + x))/(I*c*Sqrt[f] + Sqrt[
g])] + PolyLog[2, (I*Sqrt[g]*(-1 + c*x))/(c*Sqrt[f] - I*Sqrt[g])] + PolyLog
[2, ((-I)*Sqrt[g]*(-1 + c*x))/(c*Sqrt[f] + I*Sqrt[g])] + PolyLog[2, (I*Sqrt
[g]*(1 + c*x))/(c*Sqrt[f] + I*Sqrt[g])]) - (c*g*((2*I)*ArcCos[(c^2*f - g)/(
c^2*f + g)]*ArcTan[(c*f)/(Sqrt[c^2*f*g]*x)] - 4*ArcCoth[c*x]*ArcTan[(c*g*x)
/Sqrt[c^2*f*g]] + (ArcCos[(c^2*f - g)/(c^2*f + g)] + 2*ArcTan[(c*f)/(Sqrt[c
^2*f*g]*x)])*Log[(2*g*(c^2*f - I*Sqrt[c^2*f*g])*(-1 + c*x))/((c^2*f + g)*(I
*Sqrt[c^2*f*g] + c*g*x))] + (ArcCos[(c^2*f - g)/(c^2*f + g)] - 2*ArcTan[(c*
f)/(Sqrt[c^2*f*g]*x)])*Log[(2*g*(c^2*f + I*Sqrt[c^2*f*g])*(1 + c*x))/((c^2*
f + g)*(I*Sqrt[c^2*f*g] + c*g*x))] - (ArcCos[(c^2*f - g)/(c^2*f + g)] + 2*(
ArcTan[(c*f)/(Sqrt[c^2*f*g]*x)] + ArcTan[(c*g*x)/Sqrt[c^2*f*g]]))*Log[(Sqrt
[2]*Sqrt[c^2*f*g])/(E^ArcCoth[c*x]*Sqrt[c^2*f + g]*Sqrt[-(c^2*f) + g + (c^2
*f + g)*Cosh[2*ArcCoth[c*x]]])] - (ArcCos[(c^2*f - g)/(c^2*f + g)] - 2*(Arc
Tan[(c*f)/(Sqrt[c^2*f*g]*x)] + ArcTan[(c*g*x)/Sqrt[c^2*f*g]]))*Log[(Sqrt[2]
*E^ArcCoth[c*x]*Sqrt[c^2*f*g])/(Sqrt[c^2*f + g]*Sqrt[-(c^2*f) + g + (c^2*f
+ g)*Cosh[2*ArcCoth[c*x]]])] + I*(PolyLog[2, ((c^2*f - g - (2*I)*Sqrt[c^2*f
*g])*(Sqrt[c^2*f*g] + I*c*g*x))/((c^2*f + g)*(Sqrt[c^2*f*g] - I*c*g*x))] -
PolyLog[2, ((c^2*f - g + (2*I)*Sqrt[c^2*f*g])*(Sqrt[c^2*f*g] + I*c*g*x))/((
c^2*f + g)*(Sqrt[c^2*f*g] - I*c*g*x))])))/Sqrt[c^2*f*g]))/2
```

Maple [F] time = 0.832, size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{arccoth}\,(cx))\left(d + e\ln\left(gx^2 + f\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(c*x))*(d+e*ln(g*x^2+f))/x^2,x)
```

```
[Out] int((a+b*arccoth(c*x))*(d+e*ln(g*x^2+f))/x^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{bd\,\mathrm{arcoth}\,(cx) + ad + (be\,\mathrm{arcoth}\,(cx) + ae)\log\left(gx^2 + f\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(g*x^2 + f))
/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(c*x))*(d+e*ln(g*x**2+f))/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arcoth}\,(cx) + a)\left(e\log\left(gx^2 + f\right) + d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(g*x^2 + f) + d)/x^2, x)
```

3.282 $\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^3}\,dx$

Optimal. Leaf size=712

$$\frac{1}{4}bc^2e\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-f}-\sqrt{g}x\right)}{(cx+1)\left(c\sqrt{-f}-\sqrt{g}\right)}\right)+\frac{1}{4}bc^2e\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-f}+\sqrt{g}x\right)}{(cx+1)\left(c\sqrt{-f}+\sqrt{g}\right)}\right)-\frac{1}{2}bc^2e\text{PolyLog}\left(2,1-\right.$$

```
[Out] (b*c*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] + (a*e*g*Log[x])/f + (b
*e*g*ArcCoth[c*x]*Log[2/(1 + c*x)])/f + b*c^2*e*ArcTanh[c*x]*Log[2/(1 + c*x
)] - (b*e*g*ArcCoth[c*x]*Log[(2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - Sq
rt[g])*(1 + c*x))])/(2*f) - (b*c^2*e*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] - Sqrt
[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))])/2 - (b*e*g*ArcCoth[c*x]*Log[(2
*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1 + c*x))])/(2*f) - (b*
c^2*e*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])
*(1 + c*x))])/2 - (a*e*g*Log[f + g*x^2])/(2*f) - (b*c*(d + e*Log[f + g*x^2]
))/(2*x) - ((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/(2*x^2) + (b*c^2*A
rcTanh[c*x]*(d + e*Log[f + g*x^2]))/2 + (b*e*g*PolyLog[2, -(1/(c*x))])/(2*f
) - (b*e*g*PolyLog[2, 1/(c*x)])/(2*f) - (b*c^2*e*PolyLog[2, 1 - 2/(1 + c*x)
])/2 - (b*e*g*PolyLog[2, 1 - 2/(1 + c*x)])/(2*f) + (b*c^2*e*PolyLog[2, 1 -
(2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))])/4 + (b*e*
g*PolyLog[2, 1 - (2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 +
c*x))])/(4*f) + (b*c^2*e*PolyLog[2, 1 - (2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sq
rt[-f] + Sqrt[g])*(1 + c*x))])/4 + (b*e*g*PolyLog[2, 1 - (2*c*(Sqrt[-f] + S
qrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1 + c*x))])/(4*f)
```

Rubi [A] time = 1.11314, antiderivative size = 712, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 17, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.708, Rules used = {5917, 325, 206, 6086, 6725, 801, 635, 205, 260, 5993, 5913, 5921, 2402, 2315, 2447, 5992, 5920}

$$\frac{1}{4}bc^2e\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-f}-\sqrt{g}x\right)}{(cx+1)\left(c\sqrt{-f}-\sqrt{g}\right)}\right)+\frac{1}{4}bc^2e\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-f}+\sqrt{g}x\right)}{(cx+1)\left(c\sqrt{-f}+\sqrt{g}\right)}\right)-\frac{1}{2}bc^2e\text{PolyLog}\left(2,1-\right.$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x^3,x]
```

```
[Out] (b*c*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] + (a*e*g*Log[x])/f + (b
*e*g*ArcCoth[c*x]*Log[2/(1 + c*x)])/f + b*c^2*e*ArcTanh[c*x]*Log[2/(1 + c*x
)] - (b*e*g*ArcCoth[c*x]*Log[(2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - Sq
rt[g])*(1 + c*x))])/(2*f) - (b*c^2*e*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] - Sqrt
[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))])/2 - (b*e*g*ArcCoth[c*x]*Log[(2
*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1 + c*x))])/(2*f) - (b*
c^2*e*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])
*(1 + c*x))])/2 - (a*e*g*Log[f + g*x^2])/(2*f) - (b*c*(d + e*Log[f + g*x^2]
))/(2*x) - ((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/(2*x^2) + (b*c^2*A
rcTanh[c*x]*(d + e*Log[f + g*x^2]))/2 + (b*e*g*PolyLog[2, -(1/(c*x))])/(2*f
) - (b*e*g*PolyLog[2, 1/(c*x)])/(2*f) - (b*c^2*e*PolyLog[2, 1 - 2/(1 + c*x)
])/2 - (b*e*g*PolyLog[2, 1 - 2/(1 + c*x)])/(2*f) + (b*c^2*e*PolyLog[2, 1 -
(2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))])/4 + (b*e*
g*PolyLog[2, 1 - (2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 +
c*x))])/(4*f) + (b*c^2*e*PolyLog[2, 1 - (2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sq
rt[-f] + Sqrt[g])*(1 + c*x))])/4 + (b*e*g*PolyLog[2, 1 - (2*c*(Sqrt[-f] + S
```

```
qrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1 + c*x))])/(4*f)
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6086

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcCoth[c*x]),
 x]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[(x
*u)/(f + g*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ
[m] && NeQ[m, -1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5993

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcCoth[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 5913

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b*PolyLog[2, -(c*x)^(-1)])/2, x] - Simp[(b*PolyLog[2, 1/(c*x)])/
2, x]) /; FreeQ[{a, b, c}, x]
```

Rule 5921

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcCoth[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcCoth[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^3}\,dx = -\frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x} - \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{2x^2}$$

$$= -\frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x} - \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{2x^2}$$

$$= -\frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x} - \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{2x^2}$$

$$= -\frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x} - \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{2x^2}$$

$$= bc^2e\tanh^{-1}(cx)\log\left(\frac{2}{1+cx}\right) - \frac{1}{2}bc^2e\tanh^{-1}(cx)\log\left(\frac{2c\left(\sqrt{-f}-\right.}{\left(c\sqrt{-f}-\sqrt{g}\right)}\right.$$

$$= \frac{aeg\log(x)}{f} + bc^2e\tanh^{-1}(cx)\log\left(\frac{2}{1+cx}\right) - \frac{1}{2}bc^2e\tanh^{-1}(cx)\log\left(\right.$$

$$= \frac{aeg\log(x)}{f} + bc^2e\tanh^{-1}(cx)\log\left(\frac{2}{1+cx}\right) - \frac{1}{2}bc^2e\tanh^{-1}(cx)\log\left(\right.$$

$$= \frac{bce\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} + \frac{aeg\log(x)}{f} + bc^2e\tanh^{-1}(cx)\log\left(\frac{2}{1+cx}\right) - \frac{1}{2}$$

$$= \frac{bce\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} + \frac{aeg\log(x)}{f} + \frac{beg\coth^{-1}(cx)\log\left(\frac{2}{1+cx}\right)}{f} + bc^2e$$

$$= \frac{bce\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} + \frac{aeg\log(x)}{f} + \frac{beg\coth^{-1}(cx)\log\left(\frac{2}{1+cx}\right)}{f} + bc^2e$$

$$= \frac{bce\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} + \frac{aeg\log(x)}{f} + \frac{beg\coth^{-1}(cx)\log\left(\frac{2}{1+cx}\right)}{f} + bc^2e$$

Mathematica [C] time = 5.57142, size = 1318, normalized size = 1.85

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x^3,x]
```

```
[Out] -(2*a*d*f - 4*b*c*e*Sqrt[f]*Sqrt[g]*x^2*ArcTan[(Sqrt[g]*x)/Sqrt[f]] - 4*a*e
*g*x^2*Log[x] + 2*a*e*g*x^2*Log[f + g*x^2] + 2*e*f*(a + b*c*x + (b - b*c^2*
x^2)*ArcCoth[c*x])*Log[f + g*x^2] + b*c^2*e*f*x^2*(-4*ArcCoth[c*x]^2 - 4*Ar
cCoth[c*x]*Log[1 - E^(-2*ArcCoth[c*x])] + 2*ArcCoth[c*x]*Log[1 + (E^(2*ArcC
oth[c*x])*(c^2*f + g))/(-(c^2*f) - 2*c*Sqrt[-f]*Sqrt[g] + g)] + 2*ArcCoth[c
*x]*Log[1 + (E^(2*ArcCoth[c*x])*(c^2*f + g))/(-(c^2*f) + 2*c*Sqrt[-f]*Sqrt[
g] + g)] + 2*PolyLog[2, E^(-2*ArcCoth[c*x])] + PolyLog[2, (E^(2*ArcCoth[c*x
])*(c^2*f + g))/(c^2*f - 2*c*Sqrt[-f]*Sqrt[g] - g)] + PolyLog[2, (E^(2*ArcC
oth[c*x])*(c^2*f + g))/(c^2*f + 2*c*Sqrt[-f]*Sqrt[g] - g)]) - b*d*(-2*(c*f*
x + g*x^2*ArcCoth[c*x]^2 + ArcCoth[c*x]*(f - c^2*f*x^2 + 2*g*x^2*Log[1 + E^
(-2*ArcCoth[c*x])]) - g*x^2*PolyLog[2, -E^(-2*ArcCoth[c*x])]) + g*x^2*(2*Ar
```

```
cCoth[c*x]*(-ArcCoth[c*x] + Log[1 + (E^(2*ArcCoth[c*x])*(c^2*f + g))/(-(c^2
*f) - 2*c*Sqrt[-f]*Sqrt[g] + g)] + Log[1 + (E^(2*ArcCoth[c*x])*(c^2*f + g))
/(-(c^2*f) + 2*c*Sqrt[-f]*Sqrt[g] + g)]) + PolyLog[2, (E^(2*ArcCoth[c*x])*(
c^2*f + g))/(c^2*f - 2*c*Sqrt[-f]*Sqrt[g] - g)] + PolyLog[2, (E^(2*ArcCoth[
c*x])*(c^2*f + g))/(c^2*f + 2*c*Sqrt[-f]*Sqrt[g] - g)])) + b*d*g*x^2*(2*Arc
Coth[c*x]^2 - (4*I)*ArcSin[Sqrt[g/(c^2*f + g)]]*ArcTanh[(c*f)/(Sqrt[-(c^2*f
*g)]*x)] - 2*ArcCoth[c*x]*(ArcCoth[c*x] + 2*Log[1 + E^(-2*ArcCoth[c*x])]) +
 2*(ArcCoth[c*x] - I*ArcSin[Sqrt[g/(c^2*f + g)]])*Log[(c^2*(-1 + E^(2*ArcCo
th[c*x]))*f + g + E^(2*ArcCoth[c*x])*g - 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcCoth[
c*x])*(c^2*f + g))] + 2*(ArcCoth[c*x] + I*ArcSin[Sqrt[g/(c^2*f + g)]])*Log[
(c^2*(-1 + E^(2*ArcCoth[c*x]))*f + g + E^(2*ArcCoth[c*x])*g + 2*Sqrt[-(c^2*
f*g)])/(E^(2*ArcCoth[c*x])*(c^2*f + g))] + 2*PolyLog[2, -E^(-2*ArcCoth[c*x]
)] - PolyLog[2, (c^2*f - g + 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcCoth[c*x])*(c^2*f
 + g))] - PolyLog[2, -((-(c^2*f) + g + 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcCoth[c*
x])*(c^2*f + g)))]) + b*e*g*x^2*(2*ArcCoth[c*x]^2 - (4*I)*ArcSin[Sqrt[g/(c^
2*f + g)]]*ArcTanh[(c*f)/(Sqrt[-(c^2*f*g)]*x)] - 2*ArcCoth[c*x]*(ArcCoth[c*
x] + 2*Log[1 + E^(-2*ArcCoth[c*x])]) + 2*(ArcCoth[c*x] - I*ArcSin[Sqrt[g/(c
^2*f + g)]])*Log[(c^2*(-1 + E^(2*ArcCoth[c*x]))*f + g + E^(2*ArcCoth[c*x])*
g - 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcCoth[c*x])*(c^2*f + g))] + 2*(ArcCoth[c*x]
 + I*ArcSin[Sqrt[g/(c^2*f + g)]])*Log[(c^2*(-1 + E^(2*ArcCoth[c*x]))*f + g
+ E^(2*ArcCoth[c*x])*g + 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcCoth[c*x])*(c^2*f + g
))] + 2*PolyLog[2, -E^(-2*ArcCoth[c*x])] - PolyLog[2, (c^2*f - g + 2*Sqrt[-
(c^2*f*g)])/(E^(2*ArcCoth[c*x])*(c^2*f + g))] - PolyLog[2, -((-(c^2*f) + g
+ 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcCoth[c*x])*(c^2*f + g)))]))/(4*f*x^2)
```

Maple [A] time = 3.198, size = 937, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(c*x))*(d+e*ln(g*x^2+f))/x^3,x)
```

```
[Out] -1/2*b*c*d/x+a*e*g*ln(x)/f-1/2*a*e*g*ln(g*x^2+f)/f-1/2/x^2*a*d-1/2*g*b*e/f*
dilog(c*x+1)-1/4*b*e*dilog((c*(-f*g)^(1/2)-(c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))
*c^2-1/4*b*e*dilog((c*(-f*g)^(1/2)+(c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))*c^2+1/4
*d*c^2*b*ln(c*x+1)-1/4*d*b*ln(c*x+1)/x^2+1/4*b*e*dilog((c*(-f*g)^(1/2)-(c*x
-1)*g-g)/(c*(-f*g)^(1/2)-g))*c^2+1/4*b*e*dilog((c*(-f*g)^(1/2)+(c*x-1)*g+g)
/(c*(-f*g)^(1/2)+g))*c^2-1/4*g*b*e/f*ln(c*x+1)*ln((c*(-f*g)^(1/2)-(c*x+1)*g
+g)/(c*(-f*g)^(1/2)+g))-1/4*g*b*e/f*ln(c*x+1)*ln((c*(-f*g)^(1/2)+(c*x+1)*g-
g)/(c*(-f*g)^(1/2)-g))+g*e*b*c/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))-1/4*d*c^
2*b*ln(c*x-1)+1/4*d*b*ln(c*x-1)/x^2-1/4*b*e*ln(c*x+1)*ln((c*(-f*g)^(1/2)-(c
*x+1)*g+g)/(c*(-f*g)^(1/2)+g))*c^2-1/4*b*e*ln(c*x+1)*ln((c*(-f*g)^(1/2)+(c*
x+1)*g-g)/(c*(-f*g)^(1/2)-g))*c^2-1/4*g*b*e/f*dilog((c*(-f*g)^(1/2)-(c*x+1)
*g+g)/(c*(-f*g)^(1/2)+g))-1/4*g*b*e/f*dilog((c*(-f*g)^(1/2)+(c*x+1)*g-g)/(c
*(-f*g)^(1/2)-g))+1/4*b*e*ln(c*x-1)*ln((c*(-f*g)^(1/2)-(c*x-1)*g-g)/(c*(-f*
g)^(1/2)-g))*c^2+1/4*b*e*ln(c*x-1)*ln((c*(-f*g)^(1/2)+(c*x-1)*g+g)/(c*(-f*g
)^(1/2)+g))*c^2+1/4*g*b*e/f*dilog((c*(-f*g)^(1/2)-(c*x-1)*g-g)/(c*(-f*g)^(1
/2)-g))+1/4*g*b*e/f*dilog((c*(-f*g)^(1/2)+(c*x-1)*g+g)/(c*(-f*g)^(1/2)+g))-
1/2*g*b*e/f*dilog(c*x)+(-1/4*b*e/x^2*ln(c*x+1)-1/4*e*(c^2*b*ln(c*x-1)*x^2-c
^2*b*ln(c*x+1)*x^2+2*x*b*c-b*ln(c*x-1)+2*a)/x^2)*ln(g*x^2+f)+1/4*g*b*e/f*ln
(c*x-1)*ln((c*(-f*g)^(1/2)-(c*x-1)*g-g)/(c*(-f*g)^(1/2)-g))+1/4*g*b*e/f*ln(
c*x-1)*ln((c*(-f*g)^(1/2)+(c*x-1)*g+g)/(c*(-f*g)^(1/2)+g))-1/2*g*b*e/f*ln(c
*x-1)*ln(c*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bd\,\text{arcoth}\,(cx)+ad+(be\,\text{arcoth}\,(cx)+ae)\log\left(gx^2+f\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(g*x^2 + f))
/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(c*x))*(d+e*ln(g*x**2+f))/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\,\text{arcoth}\,(cx)+a)\left(e\log\left(gx^2+f\right)+d\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(g*x^2 + f) + d)/x^3, x)
```

3.283 $\int \coth^{-1}(e^x)\,dx$

Optimal. Leaf size=25

$$\frac{1}{2}\text{PolyLog}\left(2,-e^{-x}\right)-\frac{1}{2}\text{PolyLog}\left(2,e^{-x}\right)$$

```
[Out] PolyLog[2, -E^(-x)]/2 - PolyLog[2, E^(-x)]/2
```

Rubi [A] time = 0.0121473, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2282, 5913}

$$\frac{1}{2}\text{PolyLog}\left(2,-e^{-x}\right)-\frac{1}{2}\text{PolyLog}\left(2,e^{-x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[E^x],x]
```

```
[Out] PolyLog[2, -E^(-x)]/2 - PolyLog[2, E^(-x)]/2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5913

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b*PolyLog[2, -(c*x)^(-1)])/2, x] - Simp[(b*PolyLog[2, 1/(c*x)])/
2, x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \coth^{-1}\left(e^x\right)\,dx &= \text{Subst}\left(\int \frac{\coth^{-1}(x)}{x}\,dx,x,e^x\right)\\ &= \frac{\text{Li}_2\left(-e^{-x}\right)}{2}-\frac{\text{Li}_2\left(e^{-x}\right)}{2}\end{aligned}$$

Mathematica [B] time = 0.0343158, size = 51, normalized size = 2.04

$$-\frac{1}{2}\text{PolyLog}\left(2,-e^x\right)+\frac{1}{2}\text{PolyLog}\left(2,e^x\right)+\frac{1}{2}x\log\left(1-e^x\right)-\frac{1}{2}x\log\left(e^x+1\right)+x\coth^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[E^x],x]
```

```
[Out] x*ArcCoth[E^x] + (x*Log[1 - E^x])/2 - (x*Log[1 + E^x])/2 - PolyLog[2, -E^x]
/2 + PolyLog[2, E^x]/2
```

Maple [A] time = 0.052, size = 31, normalized size = 1.2

$$\ln(\mathrm{e}^{x})\,\mathrm{arccoth}(\mathrm{e}^{x}) - \frac{\mathrm{dilog}(\mathrm{e}^{x})}{2} - \frac{\mathrm{dilog}(\mathrm{e}^{x}+1)}{2} - \frac{\ln(\mathrm{e}^{x})\ln(\mathrm{e}^{x}+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(exp(x)),x)
```

```
[Out] ln(exp(x))*arccoth(exp(x))-1/2*dilog(exp(x))-1/2*dilog(exp(x)+1)-1/2*ln(exp
(x))*ln(exp(x)+1)
```

Maxima [B] time = 1.11888, size = 78, normalized size = 3.12

$$-\frac{1}{2}\,x(\log(e^{x}+1) - \log(e^{x}-1)) + x\,\mathrm{arcoth}(e^{x}) + \frac{1}{2}\,\log(-e^{x})\log(e^{x}+1) - \frac{1}{2}\,x\log(e^{x}-1) + \frac{1}{2}\,\mathrm{Li}_2(e^{x}+1) - \frac{1}{2}\,\mathrm{Li}_2(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(exp(x)),x, algorithm="maxima")
```

```
[Out] -1/2*x*(log(e^x + 1) - log(e^x - 1)) + x*arccoth(e^x) + 1/2*log(-e^x)*log(e
^x + 1) - 1/2*x*log(e^x - 1) + 1/2*dilog(e^x + 1) - 1/2*dilog(-e^x + 1)
```

Fricas [B] time = 1.77649, size = 262, normalized size = 10.48

$$\frac{1}{2}\,x\log\left(\frac{\cosh(x)+\sinh(x)+1}{\cosh(x)+\sinh(x)-1}\right) - \frac{1}{2}\,x\log(\cosh(x)+\sinh(x)+1) + \frac{1}{2}\,x\log(-\cosh(x)-\sinh(x)+1) + \frac{1}{2}\,\mathrm{Li}_2(\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(exp(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x*log((cosh(x) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) - 1/2*x*log(cosh
(x) + sinh(x) + 1) + 1/2*x*log(-cosh(x) - sinh(x) + 1) + 1/2*dilog(cosh(x)
+ sinh(x)) - 1/2*dilog(-cosh(x) - sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{acoth}(e^{x})\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(exp(x)),x)
```

```
[Out] Integral(acoth(exp(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}(e^x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(exp(x)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(e^x), x)
```

3.284 $\int x \coth^{-1}(e^x)\,dx$

Optimal. Leaf size=51

$$\frac{1}{2}x\text{PolyLog}\left(2,-e^{-x}\right)-\frac{1}{2}x\text{PolyLog}\left(2,e^{-x}\right)+\frac{1}{2}\text{PolyLog}\left(3,-e^{-x}\right)-\frac{1}{2}\text{PolyLog}\left(3,e^{-x}\right)$$

```
[Out] (x*PolyLog[2, -E^(-x)])/2 - (x*PolyLog[2, E^(-x)])/2 + PolyLog[3, -E^(-x)]/
2 - PolyLog[3, E^(-x)]/2
```

Rubi [A] time = 0.0474447, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6214, 2531, 2282, 6589}

$$\frac{1}{2}x\text{PolyLog}\left(2,-e^{-x}\right)-\frac{1}{2}x\text{PolyLog}\left(2,e^{-x}\right)+\frac{1}{2}\text{PolyLog}\left(3,-e^{-x}\right)-\frac{1}{2}\text{PolyLog}\left(3,e^{-x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[E^x],x]
```

```
[Out] (x*PolyLog[2, -E^(-x)])/2 - (x*PolyLog[2, E^(-x)])/2 + PolyLog[3, -E^(-x)]/
2 - PolyLog[3, E^(-x)]/2
```

Rule 6214

```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Dist[1/2, Int
[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
 IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\coth^{-1}(e^x)\,dx = -\left(\frac{1}{2}\int x\log(1-e^{-x})\,dx\right)+\frac{1}{2}\int x\log(1+e^{-x})\,dx$$
$$=\frac{1}{2}x\text{Li}_2(-e^{-x})-\frac{1}{2}x\text{Li}_2(e^{-x})-\frac{1}{2}\int \text{Li}_2(-e^{-x})\,dx+\frac{1}{2}\int \text{Li}_2(e^{-x})\,dx$$
$$=\frac{1}{2}x\text{Li}_2(-e^{-x})-\frac{1}{2}x\text{Li}_2(e^{-x})+\frac{1}{2}\text{Subst}\left(\int\frac{\text{Li}_2(-x)}{x}\,dx,x,e^{-x}\right)-\frac{1}{2}\text{Subst}\left(\int\frac{\text{Li}_2(x)}{x}\,dx,x,e^{-x}\right)$$
$$=\frac{1}{2}x\text{Li}_2(-e^{-x})-\frac{1}{2}x\text{Li}_2(e^{-x})+\frac{\text{Li}_3(-e^{-x})}{2}-\frac{\text{Li}_3(e^{-x})}{2}$$

Mathematica [A] time = 0.0223064, size = 71, normalized size = 1.39

$$\frac{1}{4}\left(-2x\text{PolyLog}(2,-e^x)+2x\text{PolyLog}(2,e^x)+2\text{PolyLog}(3,-e^x)-2\text{PolyLog}(3,e^x)+x^2\log(1-e^x)-x^2\log(e^x+1)+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[E^x],x]
```

```
[Out] (2*x^2*ArcCoth[E^x] + x^2*Log[1 - E^x] - x^2*Log[1 + E^x] - 2*x*PolyLog[2,
-E^x] + 2*x*PolyLog[2, E^x] + 2*PolyLog[3, -E^x] - 2*PolyLog[3, E^x])/4
```

Maple [A] time = 0.049, size = 62, normalized size = 1.2

$$\frac{x^2\text{arccoth}(e^x)}{2}-\frac{x^2\ln(e^x+1)}{4}-\frac{x\text{polylog}(2,-e^x)}{2}+\frac{\text{polylog}(3,-e^x)}{2}+\frac{x^2\ln(1-e^x)}{4}+\frac{x\text{polylog}(2,e^x)}{2}-\frac{\text{polylog}(3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(exp(x)),x)
```

```
[Out] 1/2*x^2*arccoth(exp(x))-1/4*x^2*ln(exp(x)+1)-1/2*x*polylog(2,-exp(x))+1/2*p
olylog(3,-exp(x))+1/4*x^2*ln(1-exp(x))+1/2*x*polylog(2,exp(x))-1/2*polylog(
3,exp(x))
```

Maxima [A] time = 1.1231, size = 80, normalized size = 1.57

$$\frac{1}{2}x^2\,\text{arcoth}(e^x)-\frac{1}{4}x^2\log(e^x+1)+\frac{1}{4}x^2\log(-e^x+1)-\frac{1}{2}x\text{Li}_2(-e^x)+\frac{1}{2}x\text{Li}_2(e^x)+\frac{1}{2}\text{Li}_3(-e^x)-\frac{1}{2}\text{Li}_3(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(exp(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccoth(e^x) - 1/4*x^2*log(e^x + 1) + 1/4*x^2*log(-e^x + 1) - 1/2*x
*dilog(-e^x) + 1/2*x*dilog(e^x) + 1/2*polylog(3, -e^x) - 1/2*polylog(3, e^x
)
```

Fricas [C] time = 1.6609, size = 374, normalized size = 7.33

$$\frac{1}{4}x^2\log\left(\frac{\cosh(x)+\sinh(x)+1}{\cosh(x)+\sinh(x)-1}\right)-\frac{1}{4}x^2\log(\cosh(x)+\sinh(x)+1)+\frac{1}{4}x^2\log(-\cosh(x)-\sinh(x)+1)+\frac{1}{2}x\text{Li}_2(\text{co}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(exp(x)),x, algorithm="fricas")
```

```
[Out] 1/4*x^2*log((cosh(x) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) - 1/4*x^2*log(
cosh(x) + sinh(x) + 1) + 1/4*x^2*log(-cosh(x) - sinh(x) + 1) + 1/2*x*dilog(
cosh(x) + sinh(x)) - 1/2*x*dilog(-cosh(x) - sinh(x)) - 1/2*polylog(3, cosh(
x) + sinh(x)) + 1/2*polylog(3, -cosh(x) - sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{acoth}(e^x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(exp(x)),x)
```

```
[Out] Integral(x*acoth(exp(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arcoth}(e^x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(exp(x)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(e^x), x)
```

3.285 $\int x^2 \coth^{-1}(e^x)\, dx$

Optimal. Leaf size=70

$$\frac{1}{2}x^2\text{PolyLog}(2,-e^{-x})-\frac{1}{2}x^2\text{PolyLog}(2,e^{-x})+x\text{PolyLog}(3,-e^{-x})-x\text{PolyLog}(3,e^{-x})+\text{PolyLog}(4,-e^{-x})-\text{PolyLog}$$

```
[Out] (x^2*PolyLog[2, -E^(-x)])/2 - (x^2*PolyLog[2, E^(-x)])/2 + x*PolyLog[3, -E^
(-x)] - x*PolyLog[3, E^(-x)] + PolyLog[4, -E^(-x)] - PolyLog[4, E^(-x)]
```

Rubi [A] time = 0.0730378, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6214, 2531, 6609, 2282, 6589}

$$\frac{1}{2}x^2\text{PolyLog}(2,-e^{-x})-\frac{1}{2}x^2\text{PolyLog}(2,e^{-x})+x\text{PolyLog}(3,-e^{-x})-x\text{PolyLog}(3,e^{-x})+\text{PolyLog}(4,-e^{-x})-\text{PolyLog}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[E^x],x]
```

```
[Out] (x^2*PolyLog[2, -E^(-x)])/2 - (x^2*PolyLog[2, E^(-x)])/2 + x*PolyLog[3, -E^
(-x)] - x*PolyLog[3, E^(-x)] + PolyLog[4, -E^(-x)] - PolyLog[4, E^(-x)]
```

Rule 6214

```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Dist[1/2, Int
[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
 IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\coth^{-1}(e^x)\,dx = -\left(\frac{1}{2}\int x^2\log\left(1-e^{-x}\right)dx\right)+\frac{1}{2}\int x^2\log\left(1+e^{-x}\right)dx$$
$$=\frac{1}{2}x^2\text{Li}_2\left(-e^{-x}\right)-\frac{1}{2}x^2\text{Li}_2\left(e^{-x}\right)-\int x\text{Li}_2\left(-e^{-x}\right)dx+\int x\text{Li}_2\left(e^{-x}\right)dx$$
$$=\frac{1}{2}x^2\text{Li}_2\left(-e^{-x}\right)-\frac{1}{2}x^2\text{Li}_2\left(e^{-x}\right)+x\text{Li}_3\left(-e^{-x}\right)-x\text{Li}_3\left(e^{-x}\right)-\int \text{Li}_3\left(-e^{-x}\right)dx+\int \text{Li}_3\left(e^{-x}\right)dx$$
$$=\frac{1}{2}x^2\text{Li}_2\left(-e^{-x}\right)-\frac{1}{2}x^2\text{Li}_2\left(e^{-x}\right)+x\text{Li}_3\left(-e^{-x}\right)-x\text{Li}_3\left(e^{-x}\right)+\text{Subst}\left(\int\frac{\text{Li}_3(-x)}{x}dx,x,e^{-x}\right)-\text{Sul}$$
$$=\frac{1}{2}x^2\text{Li}_2\left(-e^{-x}\right)-\frac{1}{2}x^2\text{Li}_2\left(e^{-x}\right)+x\text{Li}_3\left(-e^{-x}\right)-x\text{Li}_3\left(e^{-x}\right)+\text{Li}_4\left(-e^{-x}\right)-\text{Li}_4\left(e^{-x}\right)$$

Mathematica [A] time = 0.0193149, size = 93, normalized size = 1.33

$$\frac{1}{6}\left(-3x^2\text{PolyLog}\left(2,-e^x\right)+3x^2\text{PolyLog}\left(2,e^x\right)+6x\text{PolyLog}\left(3,-e^x\right)-6x\text{PolyLog}\left(3,e^x\right)-6\text{PolyLog}\left(4,-e^x\right)+6\text{Pc}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[E^x],x]
```

```
[Out] (2*x^3*ArcCoth[E^x] + x^3*Log[1 - E^x] - x^3*Log[1 + E^x] - 3*x^2*PolyLog[2
, -E^x] + 3*x^2*PolyLog[2, E^x] + 6*x*PolyLog[3, -E^x] - 6*x*PolyLog[3, E^x
] - 6*PolyLog[4, -E^x] + 6*PolyLog[4, E^x])/6
```

Maple [A] time = 0.046, size = 79, normalized size = 1.1

$$\frac{x^3\text{arccoth}\left(e^x\right)}{3}-\frac{x^3\ln\left(e^x+1\right)}{6}-\frac{x^2\text{polylog}\left(2,-e^x\right)}{2}+x\text{polylog}\left(3,-e^x\right)-\text{polylog}\left(4,-e^x\right)+\frac{x^3\ln\left(1-e^x\right)}{6}+\frac{x^2\text{pol}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(exp(x)),x)
```

```
[Out] 1/3*x^3*arccoth(exp(x))-1/6*x^3*ln(exp(x)+1)-1/2*x^2*polylog(2,-exp(x))+x*p
olylog(3,-exp(x))-polylog(4,-exp(x))+1/6*x^3*ln(1-exp(x))+1/2*x^2*polylog(2
,exp(x))-x*polylog(3,exp(x))+polylog(4,exp(x))
```

Maxima [A] time = 1.10916, size = 103, normalized size = 1.47

$$\frac{1}{3}x^3\operatorname{arcoth}\left(e^x\right)-\frac{1}{6}x^3\log\left(e^x+1\right)+\frac{1}{6}x^3\log\left(-e^x+1\right)-\frac{1}{2}x^2\text{Li}_2\left(-e^x\right)+\frac{1}{2}x^2\text{Li}_2\left(e^x\right)+x\text{Li}_3(-e^x)-x\text{Li}_3(e^x)-\text{Li}_4(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(exp(x)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(e^x) - 1/6*x^3*log(e^x + 1) + 1/6*x^3*log(-e^x + 1) - 1/2*x
^2*dilog(-e^x) + 1/2*x^2*dilog(e^x) + x*polylog(3, -e^x) - x*polylog(3, e^x
) - polylog(4, -e^x) + polylog(4, e^x)
```

Fricas [C] time = 1.77765, size = 462, normalized size = 6.6

$$\frac{1}{6}x^3\log\left(\frac{\cosh(x)+\sinh(x)+1}{\cosh(x)+\sinh(x)-1}\right)-\frac{1}{6}x^3\log(\cosh(x)+\sinh(x)+1)+\frac{1}{6}x^3\log(-\cosh(x)-\sinh(x)+1)+\frac{1}{2}x^2\mathrm{Li}_2(\mathrm{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(exp(x)),x, algorithm="fricas")
```

```
[Out] 1/6*x^3*log((cosh(x) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) - 1/6*x^3*log(
cosh(x) + sinh(x) + 1) + 1/6*x^3*log(-cosh(x) - sinh(x) + 1) + 1/2*x^2*dilo
g(cosh(x) + sinh(x)) - 1/2*x^2*dilog(-cosh(x) - sinh(x)) - x*polylog(3, cos
h(x) + sinh(x)) + x*polylog(3, -cosh(x) - sinh(x)) + polylog(4, cosh(x) + s
inh(x)) - polylog(4, -cosh(x) - sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\operatorname{acoth}(e^x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(exp(x)),x)
```

```
[Out] Integral(x**2*acoth(exp(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\operatorname{arcoth}(e^x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(exp(x)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(e^x), x)
```

3.286 $\int \coth^{-1}\left(e^{a+bx}\right) dx$

Optimal. Leaf size=41

$$\frac{\text{PolyLog}\left(2,-e^{-a-bx}\right)}{2b}-\frac{\text{PolyLog}\left(2,e^{-a-bx}\right)}{2b}$$

```
[Out] PolyLog[2, -E^(-a - b*x)]/(2*b) - PolyLog[2, E^(-a - b*x)]/(2*b)
```

Rubi [A] time = 0.0164433, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2282, 5913}

$$\frac{\text{PolyLog}\left(2,-e^{-a-bx}\right)}{2b}-\frac{\text{PolyLog}\left(2,e^{-a-bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[E^(a + b*x)],x]
```

```
[Out] PolyLog[2, -E^(-a - b*x)]/(2*b) - PolyLog[2, E^(-a - b*x)]/(2*b)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5913

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b*PolyLog[2, -(c*x)^(-1)])/2, x] - Simp[(b*PolyLog[2, 1/(c*x)])/
2, x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \coth^{-1}\left(e^{a+bx}\right) dx &= \frac{\text{Subst}\left(\int \frac{\coth^{-1}(x)}{x} dx, x, e^{a+bx}\right)}{b}\\ &= \frac{\text{Li}_2\left(-e^{-a-bx}\right)}{2b}-\frac{\text{Li}_2\left(e^{-a-bx}\right)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0725544, size = 68, normalized size = 1.66

$$\frac{-\text{PolyLog}\left(2,-e^{a+bx}\right)+\text{PolyLog}\left(2,e^{a+bx}\right)+bx\left(\log\left(1-e^{a+bx}\right)-\log\left(e^{a+bx}+1\right)+2\coth^{-1}\left(e^{a+bx}\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[E^(a + b*x)],x]
```

```
[Out] (b*x*(2*ArcCoth[E^(a + b*x)] + Log[1 - E^(a + b*x)] - Log[1 + E^(a + b*x)])
 - PolyLog[2, -E^(a + b*x)] + PolyLog[2, E^(a + b*x)])/(2*b)
```

Maple [A] time = 0.072, size = 67, normalized size = 1.6

$$\frac{\ln\left(e^{bx+a}\right)\operatorname{arccoth}\left(e^{bx+a}\right)}{b}-\frac{\operatorname{dilog}\left(e^{bx+a}\right)}{2b}-\frac{\operatorname{dilog}\left(e^{bx+a}+1\right)}{2b}-\frac{\ln\left(e^{bx+a}\right)\ln\left(e^{bx+a}+1\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(exp(b*x+a)),x)
```

```
[Out] 1/b*ln(exp(b*x+a))*arccoth(exp(b*x+a))-1/2/b*dilog(exp(b*x+a))-1/2/b*dilog(
exp(b*x+a)+1)-1/2/b*ln(exp(b*x+a))*ln(exp(b*x+a)+1)
```

Maxima [B] time = 1.07206, size = 144, normalized size = 3.51

$$\frac{(bx+a)\operatorname{arcoth}\left(e^{(bx+a)}\right)}{b}-\frac{(bx+a)\left(\log\left(e^{(bx+a)}+1\right)-\log\left(e^{(bx+a)}-1\right)\right)-\log\left(-e^{(bx+a)}\right)\log\left(e^{(bx+a)}+1\right)+(bx+a)\log\left(e\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] (b*x + a)*arccoth(e^(b*x + a))/b - 1/2*((b*x + a)*(log(e^(b*x + a) + 1) - l
og(e^(b*x + a) - 1)) - log(-e^(b*x + a))*log(e^(b*x + a) + 1) + (b*x + a)*l
og(e^(b*x + a) - 1) - dilog(e^(b*x + a) + 1) + dilog(-e^(b*x + a) + 1))/b
```

Fricas [B] time = 1.67866, size = 417, normalized size = 10.17

$$bx\log\left(\frac{\cosh(bx+a)+\sinh(bx+a)+1}{\cosh(bx+a)+\sinh(bx+a)-1}\right)-bx\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)-a\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)-1\right)+(\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x*log((cosh(b*x + a) + sinh(b*x + a) + 1)/(cosh(b*x + a) + sinh(b*x
+ a) - 1)) - b*x*log(cosh(b*x + a) + sinh(b*x + a) + 1) - a*log(cosh(b*x +
a) + sinh(b*x + a) - 1) + (b*x + a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1)
 + dilog(cosh(b*x + a) + sinh(b*x + a)) - dilog(-cosh(b*x + a) - sinh(b*x +
 a)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acoth}\left(e^{a+bx}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(exp(b*x+a)),x)
```

```
[Out] Integral(acoth(exp(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}\left(e^{(bx+a)}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(e^(b*x + a)), x)
```

3.287 $\int x \coth^{-1}\left(e^{a+bx}\right) dx$

Optimal. Leaf size=83

$$\frac{\text{PolyLog}\left(3,-e^{-a-bx}\right)}{2b^2}-\frac{\text{PolyLog}\left(3,e^{-a-bx}\right)}{2b^2}+\frac{x\text{PolyLog}\left(2,-e^{-a-bx}\right)}{2b}-\frac{x\text{PolyLog}\left(2,e^{-a-bx}\right)}{2b}$$

```
[Out] (x*PolyLog[2, -E^(-a - b*x)])/(2*b) - (x*PolyLog[2, E^(-a - b*x)])/(2*b) +
PolyLog[3, -E^(-a - b*x)]/(2*b^2) - PolyLog[3, E^(-a - b*x)]/(2*b^2)
```

Rubi [A] time = 0.0642786, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6214, 2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3,-e^{-a-bx}\right)}{2b^2}-\frac{\text{PolyLog}\left(3,e^{-a-bx}\right)}{2b^2}+\frac{x\text{PolyLog}\left(2,-e^{-a-bx}\right)}{2b}-\frac{x\text{PolyLog}\left(2,e^{-a-bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[E^(a + b*x)],x]
```


```
[Out] (x*PolyLog[2, -E^(-a - b*x)])/(2*b) - (x*PolyLog[2, E^(-a - b*x)])/(2*b) +
PolyLog[3, -E^(-a - b*x)]/(2*b^2) - PolyLog[3, E^(-a - b*x)]/(2*b^2)
```


Rule 6214

```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Dist[1/2, Int
[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
 IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\coth^{-1}\left(e^{a+bx}\right)dx = -\left(\frac{1}{2}\int x\log\left(1-e^{-a-bx}\right)dx\right)+\frac{1}{2}\int x\log\left(1+e^{-a-bx}\right)dx$$
$$= \frac{x\text{Li}_2\left(-e^{-a-bx}\right)}{2b}-\frac{x\text{Li}_2\left(e^{-a-bx}\right)}{2b}-\frac{\int\text{Li}_2\left(-e^{-a-bx}\right)dx}{2b}+\frac{\int\text{Li}_2\left(e^{-a-bx}\right)dx}{2b}$$
$$= \frac{x\text{Li}_2\left(-e^{-a-bx}\right)}{2b}-\frac{x\text{Li}_2\left(e^{-a-bx}\right)}{2b}+\frac{\text{Subst}\left(\int\frac{\text{Li}_2(-x)}{x}dx,x,e^{-a-bx}\right)}{2b^2}-\frac{\text{Subst}\left(\int\frac{\text{Li}_2(x)}{x}dx,x,e^{-a-}\right.}{2b^2}$$
$$= \frac{x\text{Li}_2\left(-e^{-a-bx}\right)}{2b}-\frac{x\text{Li}_2\left(e^{-a-bx}\right)}{2b}+\frac{\text{Li}_3\left(-e^{-a-bx}\right)}{2b^2}-\frac{\text{Li}_3\left(e^{-a-bx}\right)}{2b^2}$$

Mathematica [A] time = 0.0309224, size = 113, normalized size = 1.36

$$\frac{-2bx\text{PolyLog}\left(2,-e^{a+bx}\right)+2bx\text{PolyLog}\left(2,e^{a+bx}\right)+2\text{PolyLog}\left(3,-e^{a+bx}\right)-2\text{PolyLog}\left(3,e^{a+bx}\right)+b^2x^2\log\left(1-e^{a+}\right.}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[E^(a + b*x)],x]
```

```
[Out] (2*b^2*x^2*ArcCoth[E^(a + b*x)] + b^2*x^2*Log[1 - E^(a + b*x)] - b^2*x^2*Lo
g[1 + E^(a + b*x)] - 2*b*x*PolyLog[2, -E^(a + b*x)] + 2*b*x*PolyLog[2, E^(a
 + b*x)] + 2*PolyLog[3, -E^(a + b*x)] - 2*PolyLog[3, E^(a + b*x)])/(4*b^2)
```

Maple [B] time = 0.07, size = 153, normalized size = 1.8

$$\frac{x^2\text{arccoth}\left(e^{bx+a}\right)}{2}-\frac{\ln\left(e^{bx+a}+1\right)x^2}{4}+\frac{\ln\left(e^{bx+a}+1\right)a^2}{4b^2}-\frac{\text{polylog}\left(2,-e^{bx+a}\right)x}{2b}+\frac{\text{polylog}\left(3,-e^{bx+a}\right)}{2b^2}+\frac{\ln\left(1-e^{b}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(exp(b*x+a)),x)
```

```
[Out] 1/2*x^2*arccoth(exp(b*x+a))-1/4*ln(exp(b*x+a)+1)*x^2+1/4/b^2*ln(exp(b*x+a)+
1)*a^2-1/2/b*polylog(2,-exp(b*x+a))*x+1/2/b^2*polylog(3,-exp(b*x+a))+1/4*ln
(1-exp(b*x+a))*x^2-1/4/b^2*ln(1-exp(b*x+a))*a^2+1/2/b*polylog(2,exp(b*x+a))
*x-1/2/b^2*polylog(3,exp(b*x+a))-1/2/b^2*a^2*arctanh(exp(b*x+a))
```

Maxima [A] time = 1.1348, size = 146, normalized size = 1.76

$$\frac{1}{2}x^2\,\text{arcoth}\left(e^{(bx+a)}\right)-\frac{1}{4}b\left(\frac{b^2x^2\log\left(e^{(bx+a)}+1\right)+2bx\text{Li}_2\left(-e^{(bx+a)}\right)-2\text{Li}_3(-e^{(bx+a)})}{b^3}-\frac{b^2x^2\log\left(-e^{(bx+a)}+1\right)+2bx}{b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccoth(e^(b*x + a)) - 1/4*b*((b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x
*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3 - (b^2*x^2*log(-e^(b
*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3)
```

Fricas [C] time = 1.61181, size = 583, normalized size = 7.02

$$\frac{b^2x^2\log\left(\frac{\cosh(bx+a)+\sinh(bx+a)+1}{\cosh(bx+a)+\sinh(bx+a)-1}\right)-b^2x^2\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)+2\,bx\mathrm{Li}_2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^2*log((cosh(b*x + a) + sinh(b*x + a) + 1)/(cosh(b*x + a) + sinh(
b*x + a) - 1)) - b^2*x^2*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 2*b*x*dil
og(cosh(b*x + a) + sinh(b*x + a)) - 2*b*x*dilog(-cosh(b*x + a) - sinh(b*x +
 a)) + a^2*log(cosh(b*x + a) + sinh(b*x + a) - 1) + (b^2*x^2 - a^2)*log(-co
sh(b*x + a) - sinh(b*x + a) + 1) - 2*polylog(3, cosh(b*x + a) + sinh(b*x +
a)) + 2*polylog(3, -cosh(b*x + a) - sinh(b*x + a)))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\mathrm{acoth}\left(e^{a}e^{bx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(exp(b*x+a)),x)
```

```
[Out] Integral(x*acoth(exp(a)*exp(b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\mathrm{arcoth}\left(e^{(bx+a)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(e^(b*x + a)), x)
```

3.288 $\int x^2 \coth^{-1}\left(e^{a+bx}\right) dx$

Optimal. Leaf size=119

$$\frac{x\text{PolyLog}\left(3,-e^{-a-bx}\right)}{b^2}-\frac{x\text{PolyLog}\left(3,e^{-a-bx}\right)}{b^2}+\frac{\text{PolyLog}\left(4,-e^{-a-bx}\right)}{b^3}-\frac{\text{PolyLog}\left(4,e^{-a-bx}\right)}{b^3}+\frac{x^2\text{PolyLog}\left(2,-e^{-a-bx}\right)}{2b}$$

```
[Out] (x^2*PolyLog[2, -E^(-a - b*x)])/(2*b) - (x^2*PolyLog[2, E^(-a - b*x)])/(2*b
) + (x*PolyLog[3, -E^(-a - b*x)])/b^2 - (x*PolyLog[3, E^(-a - b*x)])/b^2 +
PolyLog[4, -E^(-a - b*x)]/b^3 - PolyLog[4, E^(-a - b*x)]/b^3
```

Rubi [A] time = 0.0994868, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6214, 2531, 6609, 2282, 6589}

$$\frac{x\text{PolyLog}\left(3,-e^{-a-bx}\right)}{b^2}-\frac{x\text{PolyLog}\left(3,e^{-a-bx}\right)}{b^2}+\frac{\text{PolyLog}\left(4,-e^{-a-bx}\right)}{b^3}-\frac{\text{PolyLog}\left(4,e^{-a-bx}\right)}{b^3}+\frac{x^2\text{PolyLog}\left(2,-e^{-a-bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[E^(a + b*x)],x]
```

```
[Out] (x^2*PolyLog[2, -E^(-a - b*x)])/(2*b) - (x^2*PolyLog[2, E^(-a - b*x)])/(2*b
) + (x*PolyLog[3, -E^(-a - b*x)])/b^2 - (x*PolyLog[3, E^(-a - b*x)])/b^2 +
PolyLog[4, -E^(-a - b*x)]/b^3 - PolyLog[4, E^(-a - b*x)]/b^3
```

Rule 6214

```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Dist[1/2, Int
[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
 IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \coth^{-1}\left(e^{a+bx}\right) dx = -\left(\frac{1}{2}\int x^2 \log\left(1-e^{-a-bx}\right) dx\right) + \frac{1}{2}\int x^2 \log\left(1+e^{-a-bx}\right) dx$$
$$= \frac{x^2\text{Li}_2\left(-e^{-a-bx}\right)}{2b} - \frac{x^2\text{Li}_2\left(e^{-a-bx}\right)}{2b} - \frac{\int x\text{Li}_2\left(-e^{-a-bx}\right) dx}{b} + \frac{\int x\text{Li}_2\left(e^{-a-bx}\right) dx}{b}$$
$$= \frac{x^2\text{Li}_2\left(-e^{-a-bx}\right)}{2b} - \frac{x^2\text{Li}_2\left(e^{-a-bx}\right)}{2b} + \frac{x\text{Li}_3\left(-e^{-a-bx}\right)}{b^2} - \frac{x\text{Li}_3\left(e^{-a-bx}\right)}{b^2} - \frac{\int \text{Li}_3\left(-e^{-a-bx}\right) dx}{b^2} + \frac{\int \text{I}}{}$$
$$= \frac{x^2\text{Li}_2\left(-e^{-a-bx}\right)}{2b} - \frac{x^2\text{Li}_2\left(e^{-a-bx}\right)}{2b} + \frac{x\text{Li}_3\left(-e^{-a-bx}\right)}{b^2} - \frac{x\text{Li}_3\left(e^{-a-bx}\right)}{b^2} + \frac{\text{Subst}\left(\int \frac{\text{Li}_3(-x)}{x} dx, x, e^{-}\right.}{b^3}$$
$$= \frac{x^2\text{Li}_2\left(-e^{-a-bx}\right)}{2b} - \frac{x^2\text{Li}_2\left(e^{-a-bx}\right)}{2b} + \frac{x\text{Li}_3\left(-e^{-a-bx}\right)}{b^2} - \frac{x\text{Li}_3\left(e^{-a-bx}\right)}{b^2} + \frac{\text{Li}_4\left(-e^{-a-bx}\right)}{b^3} - \frac{\text{Li}_4\left(e^{-a\cdot}\right.}{b^3}$$

Mathematica [A] time = 0.0298635, size = 149, normalized size = 1.25

$$\frac{-3b^2x^2\text{PolyLog}\left(2,-e^{a+bx}\right)+3b^2x^2\text{PolyLog}\left(2,e^{a+bx}\right)+6bx\text{PolyLog}\left(3,-e^{a+bx}\right)-6bx\text{PolyLog}\left(3,e^{a+bx}\right)-6\text{PolyLog}}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[E^(a + b*x)],x]
```

```
[Out] (2*b^3*x^3*ArcCoth[E^(a + b*x)] + b^3*x^3*Log[1 - E^(a + b*x)] - b^3*x^3*Lo
g[1 + E^(a + b*x)] - 3*b^2*x^2*PolyLog[2, -E^(a + b*x)] + 3*b^2*x^2*PolyLog
[2, E^(a + b*x)] + 6*b*x*PolyLog[3, -E^(a + b*x)] - 6*b*x*PolyLog[3, E^(a +
 b*x)] - 6*PolyLog[4, -E^(a + b*x)] + 6*PolyLog[4, E^(a + b*x)])/(6*b^3)
```

Maple [A] time = 0.076, size = 185, normalized size = 1.6

$$\frac{x^3\text{arccoth}\left(e^{bx+a}\right)}{3} + \frac{\text{polylog}\left(2, e^{bx+a}\right)x^2}{2b} - \frac{\text{polylog}\left(3, e^{bx+a}\right)x}{b^2} - \frac{\ln\left(e^{bx+a}+1\right)x^3}{6} - \frac{\text{polylog}\left(2, -e^{bx+a}\right)x^2}{2b} + \text{polylog}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(exp(b*x+a)),x)
```

```
[Out] 1/3*x^3*arccoth(exp(b*x+a))+1/2/b*polylog(2,exp(b*x+a))*x^2-1/b^2*polylog(3
,exp(b*x+a))*x-1/6*ln(exp(b*x+a)+1)*x^3-1/2/b*polylog(2,-exp(b*x+a))*x^2+1/
b^2*polylog(3,-exp(b*x+a))*x+1/6*ln(1-exp(b*x+a))*x^3-1/6/b^3*ln(exp(b*x+a)
+1)*a^3+1/6/b^3*ln(1-exp(b*x+a))*a^3+1/3/b^3*a^3*arctanh(exp(b*x+a))-1/b^3*
polylog(4,-exp(b*x+a))+1/b^3*polylog(4,exp(b*x+a))
```

Maxima [A] time = 1.12452, size = 192, normalized size = 1.61

$$\frac{1}{3}x^3 \text{arcoth}\left(e^{(bx+a)}\right) - \frac{1}{6}b\left(\frac{b^3x^3\log\left(e^{(bx+a)}+1\right)+3b^2x^2\text{Li}_2\left(-e^{(bx+a)}\right)-6bx\text{Li}_3(-e^{(bx+a)})+6\text{Li}_4(-e^{(bx+a)})}{b^4} - \frac{b^3x^3\log\left(-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(e^(b*x + a)) - 1/6*b*((b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2
*x^2*dilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4, -e
^(b*x + a)))/b^4 - (b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x
+ a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(b*x + a)))/b^4)
```

Fricas [C] time = 1.75526, size = 730, normalized size = 6.13

$$b^3x^3 \log\left(\frac{\cosh(bx+a)+\sinh(bx+a)+1}{\cosh(bx+a)+\sinh(bx+a)-1}\right) - b^3x^3 \log\left(\cosh(bx+a) + \sinh(bx+a) + 1\right) + 3\,b^2x^2 \mathrm{Li}_2\left(\cosh(bx+a) + \sinh(bx -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(b^3*x^3*log((cosh(b*x + a) + sinh(b*x + a) + 1)/(cosh(b*x + a) + sinh(
b*x + a) - 1)) - b^3*x^3*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 3*b^2*x^2
*dilog(cosh(b*x + a) + sinh(b*x + a)) - 3*b^2*x^2*dilog(-cosh(b*x + a) - si
nh(b*x + a)) - a^3*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 6*b*x*polylog(3
, cosh(b*x + a) + sinh(b*x + a)) + 6*b*x*polylog(3, -cosh(b*x + a) - sinh(b
*x + a)) + (b^3*x^3 + a^3)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 6*poly
log(4, cosh(b*x + a) + sinh(b*x + a)) - 6*polylog(4, -cosh(b*x + a) - sinh(
b*x + a)))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(exp(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arcoth}\left(e^{(bx+a)}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(e^(b*x + a)), x)
```

3.289 $\int \coth^{-1}\left(a+bf^{c+dx}\right)dx$

Optimal. Leaf size=168

$$\frac{\text{PolyLog}\left(2,1-\frac{2}{a+bf^{c+dx}+1}\right)}{2d\log(f)}-\frac{\text{PolyLog}\left(2,1-\frac{2bf^{c+dx}}{(1-a)\left(a+bf^{c+dx}+1\right)}\right)}{2d\log(f)}-\frac{\log\left(\frac{2}{a+bf^{c+dx}+1}\right)\coth^{-1}\left(a+bf^{c+dx}\right)}{d\log(f)}+\frac{\log\left(\frac{2bf^{c+}}{(1-a)\left(a+bf\right.}\right.}{}$$

```
[Out] -((ArcCoth[a + b*f^(c + d*x)]*Log[2/(1 + a + b*f^(c + d*x))])/(d*Log[f])) +
 (ArcCoth[a + b*f^(c + d*x)]*Log[(2*b*f^(c + d*x))/((1 - a)*(1 + a + b*f^(c
 + d*x)))])/(d*Log[f]) + PolyLog[2, 1 - 2/(1 + a + b*f^(c + d*x))]/(2*d*Log
[f]) - PolyLog[2, 1 - (2*b*f^(c + d*x))/((1 - a)*(1 + a + b*f^(c + d*x)))]/
(2*d*Log[f])
```

Rubi [A] time = 0.132084, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2282, 6112, 5921, 2402, 2315, 2447}

$$\frac{\text{PolyLog}\left(2,1-\frac{2}{a+bf^{c+dx}+1}\right)}{2d\log(f)}-\frac{\text{PolyLog}\left(2,1-\frac{2bf^{c+dx}}{(1-a)\left(a+bf^{c+dx}+1\right)}\right)}{2d\log(f)}-\frac{\log\left(\frac{2}{a+bf^{c+dx}+1}\right)\coth^{-1}\left(a+bf^{c+dx}\right)}{d\log(f)}+\frac{\log\left(\frac{2bf^{c+}}{(1-a)\left(a+bf\right.}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*f^(c + d*x)],x]
```

```
[Out] -((ArcCoth[a + b*f^(c + d*x)]*Log[2/(1 + a + b*f^(c + d*x))])/(d*Log[f])) +
 (ArcCoth[a + b*f^(c + d*x)]*Log[(2*b*f^(c + d*x))/((1 - a)*(1 + a + b*f^(c
 + d*x)))])/(d*Log[f]) + PolyLog[2, 1 - 2/(1 + a + b*f^(c + d*x))]/(2*d*Log
[f]) - PolyLog[2, 1 - (2*b*f^(c + d*x))/((1 - a)*(1 + a + b*f^(c + d*x)))]/
(2*d*Log[f])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rule 5921

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcCoth[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcCoth[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \coth^{-1}\left(a+bf^{c+dx}\right)dx = \frac{\text{Subst}\left(\int \frac{\coth^{-1}(a+bx)}{x}\,dx, x, f^{c+dx}\right)}{d\log(f)}$$

$$= \frac{\text{Subst}\left(\int \frac{\coth^{-1}(x)}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bf^{c+dx}\right)}{bd\log(f)}$$

$$= -\frac{\coth^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1+a+bf^{c+dx}}\right)}{d\log(f)} + \frac{\coth^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(1-a)\left(1+a+bf^{c+dx}\right)}\right)}{d\log(f)} + \frac{\text{Su}}{}$$

$$= -\frac{\coth^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1+a+bf^{c+dx}}\right)}{d\log(f)} + \frac{\coth^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(1-a)\left(1+a+bf^{c+dx}\right)}\right)}{d\log(f)} - \frac{\text{Li}}{}$$

$$= -\frac{\coth^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1+a+bf^{c+dx}}\right)}{d\log(f)} + \frac{\coth^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(1-a)\left(1+a+bf^{c+dx}\right)}\right)}{d\log(f)} + \frac{\text{Li}}{}$$

Mathematica [A] time = 0.0750964, size = 108, normalized size = 0.64

$$\frac{\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a-1}\right)-\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+1}\right)+dx\log(f)\left(\log\left(\frac{a+bf^{c+dx}-1}{a-1}\right)-\log\left(\frac{a+bf^{c+dx}+1}{a+1}\right)+2\coth^{-1}\left(a+bf^{c+dx}\right)\right)}{2d\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a + b*f^(c + d*x)],x]
```

```
[Out] (d*x*Log[f]*(2*ArcCoth[a + b*f^(c + d*x)] + Log[(-1 + a + b*f^(c + d*x))/(-
1 + a)] - Log[(1 + a + b*f^(c + d*x))/(1 + a)]) + PolyLog[2, -((b*f^(c + d*
x))/(-1 + a))] - PolyLog[2, -((b*f^(c + d*x))/(1 + a))])/(2*d*Log[f])
```

Maple [A] time = 0.199, size = 164, normalized size = 1.

$$\frac{\ln\left(bf^{dx+c}\right)\operatorname{arccoth}\left(a+bf^{dx+c}\right)}{d\ln\left(f\right)}-\frac{1}{2d\ln\left(f\right)}\operatorname{dilog}\left(\frac{1+a+bf^{dx+c}}{1+a}\right)-\frac{\ln\left(bf^{dx+c}\right)}{2d\ln\left(f\right)}\ln\left(\frac{1+a+bf^{dx+c}}{1+a}\right)+\frac{1}{2d\ln\left(f\right)}\operatorname{dilog}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a+b*f^(d*x+c)),x)
```

```
[Out] 1/d/ln(f)*ln(b*f^(d*x+c))*arccoth(a+b*f^(d*x+c))-1/2/d/ln(f)*dilog((1+a+b*f
^(d*x+c))/(1+a))-1/2/d/ln(f)*ln(b*f^(d*x+c))*ln((1+a+b*f^(d*x+c))/(1+a))+1/
2/d/ln(f)*dilog((b*f^(d*x+c)+a-1)/(a-1))+1/2/d/ln(f)*ln(b*f^(d*x+c))*ln((b*
f^(d*x+c)+a-1)/(a-1))
```

Maxima [A] time = 1.08897, size = 284, normalized size = 1.69

$$\frac{\operatorname{arcoth}\left(bf^{dx+c}+a\right)\log\left(f^{dx+c}\right)}{d\log\left(f\right)}-\frac{b\left(\frac{\log\left(bf^{dx+c}+a+1\right)}{b}-\frac{\log\left(bf^{dx+c}+a-1\right)}{b}\right)\log\left(f^{dx+c}\right)-b\left(\frac{\log\left(bf^{dx+c}+a+1\right)\log\left(-\frac{bf^{dx+c}+a+1}{a+1}+1\right)+\mathrm{Li}_2\left(\frac{bf}{}\right.}{b}\right.}{2d\log\left(f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a+b*f^(d*x+c)),x, algorithm="maxima")
```

```
[Out] arccoth(b*f^(d*x + c) + a)*log(f^(d*x + c))/(d*log(f)) - 1/2*(b*(log(b*f^(d
*x + c) + a + 1)/b - log(b*f^(d*x + c) + a - 1)/b)*log(f^(d*x + c)) - b*((l
og(b*f^(d*x + c) + a + 1)*log(-(b*f^(d*x + c) + a + 1)/(a + 1) + 1) + dilog
((b*f^(d*x + c) + a + 1)/(a + 1)))/b - (log(b*f^(d*x + c) + a - 1)*log(-(b*
f^(d*x + c) + a - 1)/(a - 1) + 1) + dilog((b*f^(d*x + c) + a - 1)/(a - 1)))
/b))/(d*log(f))
```

Fricas [A] time = 1.60135, size = 899, normalized size = 5.35

$$\frac{dx\log\left(f\right)\log\left(\frac{b\cosh\left((dx+c)\log\left(f\right)\right)+b\sinh\left((dx+c)\log\left(f\right)\right)+a+1}{b\cosh\left((dx+c)\log\left(f\right)\right)+b\sinh\left((dx+c)\log\left(f\right)\right)+a-1}\right)+c\log\left(b\cosh\left((dx+c)\log\left(f\right)\right)+b\sinh\left((dx+c)\log\left(f\right)\right)+a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a+b*f^(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(d*x*log(f)*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) +
a + 1)/(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)) + c*l
og(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a + 1)*log(f) - c*
log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)*log(f) - (
d*x + c)*log(f)*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) +
a + 1)/(a + 1)) + (d*x + c)*log(f)*log((b*cosh((d*x + c)*log(f)) + b*sinh((
d*x + c)*log(f)) + a - 1)/(a - 1)) - dilog(-(b*cosh((d*x + c)*log(f)) + b*s
inh((d*x + c)*log(f)) + a + 1)/(a + 1) + 1) + dilog(-(b*cosh((d*x + c)*log(
f)) + b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1) + 1))/(d*log(f))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a+b*f**(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}\left(bf^{dx+c} + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a+b*f^(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*f^(d*x + c) + a), x)
```

3.290 $\int x \coth^{-1}\left(a + bf^{c+dx}\right) dx$

Optimal. Leaf size=216

$$-\frac{\text{PolyLog}\left(3,\frac{bf^{c+dx}}{1-a}\right)}{2d^2\log^2(f)}+\frac{\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+1}\right)}{2d^2\log^2(f)}+\frac{x\text{PolyLog}\left(2,\frac{bf^{c+dx}}{1-a}\right)}{2d\log(f)}-\frac{x\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+1}\right)}{2d\log(f)}+\frac{1}{4}x^2\log\left(1-\frac{bf^{c+dx}}{1-a}\right)$$

```
[Out] (x^2*Log[1 - (b*f^(c + d*x))/(1 - a)])/4 - (x^2*Log[1 + (b*f^(c + d*x))/(1
+ a)])/4 - (x^2*Log[1 - (a + b*f^(c + d*x))^(-1)])/4 + (x^2*Log[1 + (a + b*
f^(c + d*x))^(-1)])/4 + (x*PolyLog[2, (b*f^(c + d*x))/(1 - a)])/(2*d*Log[f]
) - (x*PolyLog[2, -((b*f^(c + d*x))/(1 + a))])/(2*d*Log[f]) - PolyLog[3, (b
*f^(c + d*x))/(1 - a)]/(2*d^2*Log[f]^2) + PolyLog[3, -((b*f^(c + d*x))/(1 +
 a))]/(2*d^2*Log[f]^2)
```

Rubi [A] time = 2.71421, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6214, 2551, 12, 6742, 2190, 2531, 2282, 6589}

$$-\frac{\text{PolyLog}\left(3,\frac{bf^{c+dx}}{1-a}\right)}{2d^2\log^2(f)}+\frac{\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+1}\right)}{2d^2\log^2(f)}+\frac{x\text{PolyLog}\left(2,\frac{bf^{c+dx}}{1-a}\right)}{2d\log(f)}-\frac{x\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+1}\right)}{2d\log(f)}+\frac{1}{4}x^2\log\left(1-\frac{bf^{c+dx}}{1-a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[a + b*f^(c + d*x)],x]
```

```
[Out] (x^2*Log[1 - (b*f^(c + d*x))/(1 - a)])/4 - (x^2*Log[1 + (b*f^(c + d*x))/(1
+ a)])/4 - (x^2*Log[1 - (a + b*f^(c + d*x))^(-1)])/4 + (x^2*Log[1 + (a + b*
f^(c + d*x))^(-1)])/4 + (x*PolyLog[2, (b*f^(c + d*x))/(1 - a)])/(2*d*Log[f]
) - (x*PolyLog[2, -((b*f^(c + d*x))/(1 + a))])/(2*d*Log[f]) - PolyLog[3, (b
*f^(c + d*x))/(1 - a)]/(2*d^2*Log[f]^2) + PolyLog[3, -((b*f^(c + d*x))/(1 +
 a))]/(2*d^2*Log[f]^2)
```

Rule 6214

```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Dist[1/2, Int
[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
 IGtQ[m, 0]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\coth^{-1}\left(a+bf^{c+dx}\right)dx = -\left(\frac{1}{2}\int x\log\left(1-\frac{1}{a+bf^{c+dx}}\right)dx\right)+\frac{1}{2}\int x\log\left(1+\frac{1}{a+bf^{c+dx}}\right)dx$$

$$= -\frac{1}{4}x^2\log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{4}x^2\log\left(1+\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{4}\int\frac{bdf^{c+dx}x^2\log(f)}{\left(-1+a+bf^{c+dx}\right)\left(a+b\right.}$$

$$= -\frac{1}{4}x^2\log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{4}x^2\log\left(1+\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{4}(bd\log(f))\int\frac{f}{\left(-1+a+bf\right.}$$

$$= -\frac{1}{4}x^2\log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{4}x^2\log\left(1+\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{4}(bd\log(f))\int\left(\frac{f^{c+dx}x^2}{-a-bf^{c+dx}}\right.$$

$$= -\frac{1}{4}x^2\log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{4}x^2\log\left(1+\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{4}(bd\log(f))\int\frac{f^{c+dx}x^2}{-1+a+bf^{c}}$$

$$= \frac{1}{4}x^2\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{4}x^2\log\left(1+\frac{bf^{c+dx}}{1+a}\right)-\frac{1}{4}x^2\log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{4}x^2\log\left(1\right.$$

$$= \frac{1}{4}x^2\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{4}x^2\log\left(1+\frac{bf^{c+dx}}{1+a}\right)-\frac{1}{4}x^2\log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{4}x^2\log\left(1\right.$$

$$= \frac{1}{4}x^2\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{4}x^2\log\left(1+\frac{bf^{c+dx}}{1+a}\right)-\frac{1}{4}x^2\log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{4}x^2\log\left(1\right.$$

$$= \frac{1}{4}x^2\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{4}x^2\log\left(1+\frac{bf^{c+dx}}{1+a}\right)-\frac{1}{4}x^2\log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{4}x^2\log\left(1\right.$$

Mathematica [A] time = 0.104145, size = 177, normalized size = 0.82

$$\frac{-2\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a-1}\right)+2\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+1}\right)+2dx\log(f)\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a-1}\right)-2dx\log(f)\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+1}\right)+}{4d^2\log^2(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[a + b*f^(c + d*x)],x]
```

```
[Out] (2*d^2*x^2*ArcCoth[a + b*f^(c + d*x)]*Log[f]^2 + d^2*x^2*Log[f]^2*Log[1 + (
b*f^(c + d*x))/(-1 + a)] - d^2*x^2*Log[f]^2*Log[1 + (b*f^(c + d*x))/(1 + a)
] + 2*d*x*Log[f]*PolyLog[2, -((b*f^(c + d*x))/(-1 + a))] - 2*d*x*Log[f]*Pol
yLog[2, -((b*f^(c + d*x))/(1 + a))] - 2*PolyLog[3, -((b*f^(c + d*x))/(-1 +
a))] + 2*PolyLog[3, -((b*f^(c + d*x))/(1 + a))])/(4*d^2*Log[f]^2)
```

Maple [B] time = 0.174, size = 590, normalized size = 2.7

$$-\frac{x^2\ln\left(bf^{dx+c}+a-1\right)}{4}+\frac{x^2\ln\left(1+a+bf^{dx+c}\right)}{4}-\frac{x^2}{4}\ln\left(1-\frac{bf^{dx}f^c}{-1-a}\right)-\frac{cx}{2d}\ln\left(1-\frac{bf^{dx}f^c}{-1-a}\right)-\frac{c^2}{4d^2}\ln\left(1-\frac{bf^{dx}f^c}{-1-a}\right)-\frac{}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(a+b*f^(d*x+c)),x)
```

```
[Out] -1/4*x^2*ln(b*f^(d*x+c)+a-1)+1/4*x^2*ln(1+a+b*f^(d*x+c))-1/4*ln(1-b*f^(d*x)
*f^c/(-1-a))*x^2-1/2/d*ln(1-b*f^(d*x)*f^c/(-1-a))*x*c-1/4/d^2*ln(1-b*f^(d*x
)*f^c/(-1-a))*c^2-1/2/ln(f)/d*polylog(2,b*f^(d*x)*f^c/(-1-a))*x-1/2/ln(f)/d
^2*polylog(2,b*f^(d*x)*f^c/(-1-a))*c+1/2/ln(f)^2/d^2*polylog(3,b*f^(d*x)*f^
c/(-1-a))-1/4/d^2*c^2*ln(1+a+b*f^(d*x)*f^c)+1/2/ln(f)/d^2*c*dilog((1+a+b*f^
(d*x)*f^c)/(1+a))+1/2/d*c*ln((1+a+b*f^(d*x)*f^c)/(1+a))*x+1/2/d^2*c^2*ln((1
+a+b*f^(d*x)*f^c)/(1+a))+1/4*ln(1-b*f^(d*x)*f^c/(1-a))*x^2+1/2/d*ln(1-b*f^(
d*x)*f^c/(1-a))*x*c+1/4/d^2*ln(1-b*f^(d*x)*f^c/(1-a))*c^2+1/2/ln(f)/d*polyl
og(2,b*f^(d*x)*f^c/(1-a))*x+1/2/ln(f)/d^2*polylog(2,b*f^(d*x)*f^c/(1-a))*c-
1/2/ln(f)^2/d^2*polylog(3,b*f^(d*x)*f^c/(1-a))+1/4/d^2*c^2*ln(b*f^(d*x)*f^c
+a-1)-1/2/ln(f)/d^2*c*dilog((b*f^(d*x)*f^c+a-1)/(a-1))-1/2/d*c*ln((b*f^(d*x
)*f^c+a-1)/(a-1))*x-1/2/d^2*c^2*ln((b*f^(d*x)*f^c+a-1)/(a-1))
```

Maxima [A] time = 1.28441, size = 262, normalized size = 1.21

$$-\frac{1}{4}bd\left(\frac{\log\left(\frac{bf^{dx}f^c}{a+1}+1\right)\log\left(f^{dx}\right)^2+2\,\text{Li}_2\left(-\frac{bf^{dx}f^c}{a+1}\right)\log\left(f^{dx}\right)-2\,\text{Li}_3(-\frac{bf^{dx}f^c}{a+1})}{bd^3\log\left(f\right)^3}-\frac{\log\left(\frac{bf^{dx}f^c}{a-1}+1\right)\log\left(f^{dx}\right)^2+2\,\text{Li}_2\left(-\frac{bf^{dx}}{a-}\right.}{bd^3\log\left(f\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(a+b*f^(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*b*d*((log(b*f^(d*x)*f^c/(a + 1) + 1)*log(f^(d*x))^2 + 2*dilog(-b*f^(d*
x)*f^c/(a + 1))*log(f^(d*x)) - 2*polylog(3, -b*f^(d*x)*f^c/(a + 1)))/(b*d^3
*log(f)^3) - (log(b*f^(d*x)*f^c/(a - 1) + 1)*log(f^(d*x))^2 + 2*dilog(-b*f^
(d*x)*f^c/(a - 1))*log(f^(d*x)) - 2*polylog(3, -b*f^(d*x)*f^c/(a - 1)))/(b*
```

```
d^3*log(f)^3))*log(f) + 1/2*x^2*arccoth(b*f^(d*x + c) + a)
```

Fricas [C] time = 1.6202, size = 1193, normalized size = 5.52

$$\frac{d^2x^2 \log(f)^2 \log\left(\frac{b\cosh\left((dx+c)\log(f)\right)+b\sinh\left((dx+c)\log(f)\right)+a+1}{b\cosh\left((dx+c)\log(f)\right)+b\sinh\left((dx+c)\log(f)\right)+a-1}\right) - c^2\log\left(b\cosh\left((dx+c)\log(f)\right) + b\sinh\left((dx+c)\log(f\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(a+b*f^(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(d^2*x^2*log(f)^2*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(
f)) + a + 1)/(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1))
 - c^2*log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a + 1)*log
(f)^2 + c^2*log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1
)*log(f)^2 - 2*d*x*dilog(-(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(
f)) + a + 1)/(a + 1) + 1)*log(f) + 2*d*x*dilog(-(b*cosh((d*x + c)*log(f)) +
 b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1) + 1)*log(f) - (d^2*x^2 - c^2)*lo
g(f)^2*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a + 1)/(a
 + 1)) + (d^2*x^2 - c^2)*log(f)^2*log((b*cosh((d*x + c)*log(f)) + b*sinh((d
*x + c)*log(f)) + a - 1)/(a - 1)) + 2*polylog(3, -(b*cosh((d*x + c)*log(f))
 + b*sinh((d*x + c)*log(f)))/(a + 1)) - 2*polylog(3, -(b*cosh((d*x + c)*log
(f)) + b*sinh((d*x + c)*log(f)))/(a - 1)))/(d^2*log(f)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(a+b*f**(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arcoth}\left(bf^{dx+c} + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(a+b*f^(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(b*f^(d*x + c) + a), x)
```

3.291 $\int x^2 \coth^{-1}\left(a+bf^{c+dx}\right)dx$

Optimal. Leaf size=269

$$-\frac{x\text{PolyLog}\left(3,\frac{bf^{c+dx}}{1-a}\right)}{d^2\log^2(f)}+\frac{x\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+1}\right)}{d^2\log^2(f)}+\frac{\text{PolyLog}\left(4,\frac{bf^{c+dx}}{1-a}\right)}{d^3\log^3(f)}-\frac{\text{PolyLog}\left(4,-\frac{bf^{c+dx}}{a+1}\right)}{d^3\log^3(f)}+\frac{x^2\text{PolyLog}\left(2,\frac{bf^{c+dx}}{1-a}\right.}{2d\log(f)}$$

```
[Out] (x^3*Log[1 - (b*f^(c + d*x))/(1 - a)])/6 - (x^3*Log[1 + (b*f^(c + d*x))/(1
+ a)])/6 - (x^3*Log[1 - (a + b*f^(c + d*x))^(-1)])/6 + (x^3*Log[1 + (a + b*
f^(c + d*x))^(-1)])/6 + (x^2*PolyLog[2, (b*f^(c + d*x))/(1 - a)])/(2*d*Log[
f]) - (x^2*PolyLog[2, -((b*f^(c + d*x))/(1 + a))])/(2*d*Log[f]) - (x*PolyLo
g[3, (b*f^(c + d*x))/(1 - a)])/(d^2*Log[f]^2) + (x*PolyLog[3, -((b*f^(c + d
*x))/(1 + a))])/(d^2*Log[f]^2) + PolyLog[4, (b*f^(c + d*x))/(1 - a)]/(d^3*L
og[f]^3) - PolyLog[4, -((b*f^(c + d*x))/(1 + a))]/(d^3*Log[f]^3)
```

Rubi [A] time = 2.58565, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {6214, 2551, 12, 6742, 2190, 2531, 6609, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(3,\frac{bf^{c+dx}}{1-a}\right)}{d^2\log^2(f)}+\frac{x\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+1}\right)}{d^2\log^2(f)}+\frac{\text{PolyLog}\left(4,\frac{bf^{c+dx}}{1-a}\right)}{d^3\log^3(f)}-\frac{\text{PolyLog}\left(4,-\frac{bf^{c+dx}}{a+1}\right)}{d^3\log^3(f)}+\frac{x^2\text{PolyLog}\left(2,\frac{bf^{c+dx}}{1-a}\right.}{2d\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[a + b*f^(c + d*x)],x]
```


```
[Out] (x^3*Log[1 - (b*f^(c + d*x))/(1 - a)])/6 - (x^3*Log[1 + (b*f^(c + d*x))/(1
+ a)])/6 - (x^3*Log[1 - (a + b*f^(c + d*x))^(-1)])/6 + (x^3*Log[1 + (a + b*
f^(c + d*x))^(-1)])/6 + (x^2*PolyLog[2, (b*f^(c + d*x))/(1 - a)])/(2*d*Log[
f]) - (x^2*PolyLog[2, -((b*f^(c + d*x))/(1 + a))])/(2*d*Log[f]) - (x*PolyLo
g[3, (b*f^(c + d*x))/(1 - a)])/(d^2*Log[f]^2) + (x*PolyLog[3, -((b*f^(c + d
*x))/(1 + a))])/(d^2*Log[f]^2) + PolyLog[4, (b*f^(c + d*x))/(1 - a)]/(d^3*L
og[f]^3) - PolyLog[4, -((b*f^(c + d*x))/(1 + a))]/(d^3*Log[f]^3)
```


Rule 6214

```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Dist[1/2, Int
[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
 IGtQ[m, 0]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\coth^{-1}\left(a+bf^{c+dx}\right)dx=-\left(\frac{1}{2}\int x^2\log\left(1-\frac{1}{a+bf^{c+dx}}\right)dx\right)+\frac{1}{2}\int x^2\log\left(1+\frac{1}{a+bf^{c+dx}}\right)dx$$
$$=-\frac{1}{6}x^3\log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{6}x^3\log\left(1+\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{6}\int\frac{bdf^{c+dx}x^3\log(f)}{\left(-1+a+bf^{c+dx}\right)\left(a+bf^{c\cdots}\right.}$$
$$=-\frac{1}{6}x^3\log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{6}x^3\log\left(1+\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{6}(bd\log(f))\int\frac{f^{c+\cdots}}{\left(-1+a+bf^{c+\cdots}\right.}$$
$$=-\frac{1}{6}x^3\log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{6}x^3\log\left(1+\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{6}(bd\log(f))\int\left(\frac{f^{c+dx}x^3}{-a-bf^{c+dx}}+\cdots\right.$$
$$=-\frac{1}{6}x^3\log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{6}x^3\log\left(1+\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{6}(bd\log(f))\int\frac{f^{c+dx}x^3}{-1+a+bf^{c+d\cdots}}$$
$$=\frac{1}{6}x^3\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{6}x^3\log\left(1+\frac{bf^{c+dx}}{1+a}\right)-\frac{1}{6}x^3\log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{6}x^3\log\left(1+\cdots\right.$$
$$=\frac{1}{6}x^3\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{6}x^3\log\left(1+\frac{bf^{c+dx}}{1+a}\right)-\frac{1}{6}x^3\log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{6}x^3\log\left(1+\cdots\right.$$
$$=\frac{1}{6}x^3\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{6}x^3\log\left(1+\frac{bf^{c+dx}}{1+a}\right)-\frac{1}{6}x^3\log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{6}x^3\log\left(1+\cdots\right.$$
$$=\frac{1}{6}x^3\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{6}x^3\log\left(1+\frac{bf^{c+dx}}{1+a}\right)-\frac{1}{6}x^3\log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{6}x^3\log\left(1+\cdots\right.$$
$$=\frac{1}{6}x^3\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{6}x^3\log\left(1+\frac{bf^{c+dx}}{1+a}\right)-\frac{1}{6}x^3\log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{6}x^3\log\left(1+\cdots\right.$$

Mathematica [A] time = 0.0697542, size = 235, normalized size = 0.87

$$\frac{3d^2x^2\log^2(f)\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a-1}\right)-3d^2x^2\log^2(f)\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+1}\right)+6\text{PolyLog}\left(4,-\frac{bf^{c+dx}}{a-1}\right)-6\text{PolyLog}\left(4,-\frac{bf^{c+d\cdots}}{a+1}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[a + b*f^(c + d*x)],x]
```

```
[Out] (2*d^3*x^3*ArcCoth[a + b*f^(c + d*x)]*Log[f]^3 + d^3*x^3*Log[f]^3*Log[1 + (
b*f^(c + d*x))/(-1 + a)] - d^3*x^3*Log[f]^3*Log[1 + (b*f^(c + d*x))/(1 + a)
] + 3*d^2*x^2*Log[f]^2*PolyLog[2, -((b*f^(c + d*x))/(-1 + a))] - 3*d^2*x^2*
Log[f]^2*PolyLog[2, -((b*f^(c + d*x))/(1 + a))] - 6*d*x*Log[f]*PolyLog[3, -
((b*f^(c + d*x))/(-1 + a))] + 6*d*x*Log[f]*PolyLog[3, -((b*f^(c + d*x))/(1
+ a))] + 6*PolyLog[4, -((b*f^(c + d*x))/(-1 + a))] - 6*PolyLog[4, -((b*f^(c
 + d*x))/(1 + a))])/(6*d^3*Log[f]^3)
```

Maple [B] time = 0.175, size = 666, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(a+b*f^(d*x+c)),x)
```

```
[Out] -1/6*x^3*ln(b*f^(d*x+c)+a-1)+1/6*x^3*ln(1+a+b*f^(d*x+c))-1/6*ln(1-b*f^(d*x)
*f^c/(-1-a))*x^3+1/2/d^2*ln(1-b*f^(d*x)*f^c/(-1-a))*x*c^2+1/3/d^3*ln(1-b*f^
(d*x)*f^c/(-1-a))*c^3-1/2/ln(f)/d*polylog(2,b*f^(d*x)*f^c/(-1-a))*x^2+1/2/l
n(f)/d^3*polylog(2,b*f^(d*x)*f^c/(-1-a))*c^2+1/ln(f)^2/d^2*polylog(3,b*f^(d
*x)*f^c/(-1-a))*x-1/ln(f)^3/d^3*polylog(4,b*f^(d*x)*f^c/(-1-a))+1/6/d^3*c^3
*ln(1+a+b*f^(d*x)*f^c)-1/2/ln(f)/d^3*c^2*dilog((1+a+b*f^(d*x)*f^c)/(1+a))-1
/2/d^2*c^2*ln((1+a+b*f^(d*x)*f^c)/(1+a))*x-1/2/d^3*c^3*ln((1+a+b*f^(d*x)*f^
c)/(1+a))+1/6*ln(1-b*f^(d*x)*f^c/(1-a))*x^3-1/2/d^2*ln(1-b*f^(d*x)*f^c/(1-a
))*x*c^2-1/3/d^3*ln(1-b*f^(d*x)*f^c/(1-a))*c^3+1/2/ln(f)/d*polylog(2,b*f^(d
*x)*f^c/(1-a))*x^2-1/2/ln(f)/d^3*polylog(2,b*f^(d*x)*f^c/(1-a))*c^2-1/ln(f)
^2/d^2*polylog(3,b*f^(d*x)*f^c/(1-a))*x+1/ln(f)^3/d^3*polylog(4,b*f^(d*x)*f
^c/(1-a))-1/6/d^3*c^3*ln(b*f^(d*x)*f^c+a-1)+1/2/ln(f)/d^3*c^2*dilog((b*f^(d
*x)*f^c+a-1)/(a-1))+1/2/d^2*c^2*ln((b*f^(d*x)*f^c+a-1)/(a-1))*x+1/2/d^3*c^3
*ln((b*f^(d*x)*f^c+a-1)/(a-1))
```

Maxima [A] time = 1.27328, size = 338, normalized size = 1.26

$$\frac{1}{3}x^3\operatorname{arcoth}\left(bf^{dx+c}+a\right)-\frac{1}{6}bd\left(\frac{\log\left(\frac{bf^{dx}f^c}{a+1}+1\right)\log\left(f^{dx}\right)^3+3\operatorname{Li}_2\left(-\frac{bf^{dx}f^c}{a+1}\right)\log\left(f^{dx}\right)^2-6\log\left(f^{dx}\right)\operatorname{Li}_3\left(-\frac{bf^{dx}f^c}{a+1}\right)+6}{bd^4\log\left(f\right)^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(a+b*f^(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(b*f^(d*x + c) + a) - 1/6*b*d*((log(b*f^(d*x)*f^c/(a + 1) +
1)*log(f^(d*x))^3 + 3*dilog(-b*f^(d*x)*f^c/(a + 1))*log(f^(d*x))^2 - 6*log(
f^(d*x))*polylog(3, -b*f^(d*x)*f^c/(a + 1)) + 6*polylog(4, -b*f^(d*x)*f^c/(
a + 1)))/(b*d^4*log(f)^4) - (log(b*f^(d*x)*f^c/(a - 1) + 1)*log(f^(d*x))^3
+ 3*dilog(-b*f^(d*x)*f^c/(a - 1))*log(f^(d*x))^2 - 6*log(f^(d*x))*polylog(3
, -b*f^(d*x)*f^c/(a - 1)) + 6*polylog(4, -b*f^(d*x)*f^c/(a - 1)))/(b*d^4*lo
g(f)^4))*log(f)
```

Fricas [C] time = 1.91391, size = 1453, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(a+b*f^(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(d^3*x^3*log(f)^3*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(
f)) + a + 1)/(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1))
 - 3*d^2*x^2*dilog(-(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) +
a + 1)/(a + 1) + 1)*log(f)^2 + 3*d^2*x^2*dilog(-(b*cosh((d*x + c)*log(f)) +
 b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1) + 1)*log(f)^2 + c^3*log(b*cosh((
d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a + 1)*log(f)^3 - c^3*log(b*c
osh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)*log(f)^3 - (d^3*x
^3 + c^3)*log(f)^3*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f))
 + a + 1)/(a + 1)) + (d^3*x^3 + c^3)*log(f)^3*log((b*cosh((d*x + c)*log(f))
 + b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1)) + 6*d*x*log(f)*polylog(3, -(b
*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)))/(a + 1)) - 6*d*x*log(f)
*polylog(3, -(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)))/(a - 1))
 - 6*polylog(4, -(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)))/(a +
```

```
 1)) + 6*polylog(4, -(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)))/
(a - 1)))/(d^3*log(f)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(a+b*f**(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arcoth}\left(bf^{dx+c} + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(a+b*f^(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(b*f^(d*x + c) + a), x)
```

3.292 $\int \frac{1}{(a-ax^2)\left(b-2b\coth^{-1}(x)\right)}\,dx$

Optimal. Leaf size=17

$$-\frac{\log\left(1-2\coth^{-1}(x)\right)}{2ab}$$

```
[Out] -Log[1 - 2*ArcCoth[x]]/(2*a*b)
```

Rubi [A] time = 0.0443519, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {5947}

$$-\frac{\log\left(1-2\coth^{-1}(x)\right)}{2ab}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - a*x^2)*(b - 2*b*ArcCoth[x])),x]
```

```
[Out] -Log[1 - 2*ArcCoth[x]]/(2*a*b)
```

Rule 5947

```
Int[1/(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol
] :> Simp[Log[RemoveContent[a + b*ArcCoth[c*x], x]]/(b*c*d), x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\int \frac{1}{\left(a-ax^2\right)\left(b-2b\coth^{-1}(x)\right)}\,dx=-\frac{\log\left(1-2\coth^{-1}(x)\right)}{2ab}$$

Mathematica [A] time = 0.0524332, size = 17, normalized size = 1.

$$-\frac{\log\left(2\coth^{-1}(x)-1\right)}{2ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - a*x^2)*(b - 2*b*ArcCoth[x])),x]
```

```
[Out] -Log[-1 + 2*ArcCoth[x]]/(2*a*b)
```

Maple [A] time = 0.072, size = 19, normalized size = 1.1

$$-\frac{\ln\left(2\,b\,\mathrm{arccoth}\left(x\right)-b\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a*x^2+a)/(b-2*b*arccoth(x)),x)
```

```
[Out] -1/2/a*ln(2*b*arccoth(x)-b)/b
```

Maxima [A] time = 1.12692, size = 28, normalized size = 1.65

$$-\frac{\log\left(\log\left(x+1\right)-\log\left(x-1\right)-1\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x^2+a)/(b-2*b*arccoth(x)),x, algorithm="maxima")
```

```
[Out] -1/2*log(log(x + 1) - log(x - 1) - 1)/(a*b)
```

Fricas [A] time = 1.64775, size = 57, normalized size = 3.35

$$-\frac{\log\left(\log\left(\frac{x+1}{x-1}\right)-1\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x^2+a)/(b-2*b*arccoth(x)),x, algorithm="fricas")
```

```
[Out] -1/2*log(log((x + 1)/(x - 1)) - 1)/(a*b)
```

Sympy [A] time = 1.31523, size = 14, normalized size = 0.82

$$-\frac{\log\left(\operatorname{acoth}\left(x\right)-\frac{1}{2}\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x**2+a)/(b-2*b*acoth(x)),x)
```

```
[Out] -log(acoth(x) - 1/2)/(2*a*b)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(ax^{2}-a\right)\left(2\,b\,\operatorname{arcoth}\left(x\right)-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x^2+a)/(b-2*b*arccoth(x)),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x^2 - a)*(2*b*arccoth(x) - b)), x)
```

3.293 $\int x^3 \coth^{-1}\left(a+bx^4\right)dx$

Optimal. Leaf size=44

$$\frac{\log\left(1-\left(a+bx^4\right)^2\right)}{8b}+\frac{\left(a+bx^4\right)\coth^{-1}\left(a+bx^4\right)}{4b}$$

```
[Out] ((a + b*x^4)*ArcCoth[a + b*x^4])/(4*b) + Log[1 - (a + b*x^4)^2]/(8*b)
```

Rubi [A] time = 0.0523872, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6715, 6104, 5911, 260}

$$\frac{\log\left(1-\left(a+bx^4\right)^2\right)}{8b}+\frac{\left(a+bx^4\right)\coth^{-1}\left(a+bx^4\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCoth[a + b*x^4],x]
```

```
[Out] ((a + b*x^4)*ArcCoth[a + b*x^4])/(4*b) + Log[1 - (a + b*x^4)^2]/(8*b)
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6104

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}
, x] && IGtQ[p, 0]
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int x^3 \coth^{-1}\left(a+bx^4\right) dx = \frac{1}{4}\operatorname{Subst}\left(\int \coth^{-1}(a+bx)\,dx, x, x^4\right)$$
$$= \frac{\operatorname{Subst}\left(\int \coth^{-1}(x)\,dx, x, a+bx^4\right)}{4b}$$
$$= \frac{\left(a+bx^4\right)\coth^{-1}\left(a+bx^4\right)}{4b} - \frac{\operatorname{Subst}\left(\int \frac{x}{1-x^2}\,dx, x, a+bx^4\right)}{4b}$$
$$= \frac{\left(a+bx^4\right)\coth^{-1}\left(a+bx^4\right)}{4b} + \frac{\log\left(1-\left(a+bx^4\right)^2\right)}{8b}$$

Mathematica [A] time = 0.016213, size = 39, normalized size = 0.89

$$\frac{\log\left(1-\left(a+bx^4\right)^2\right)+2\left(a+bx^4\right)\coth^{-1}\left(a+bx^4\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCoth[a + b*x^4],x]
```

```
[Out] (2*(a + b*x^4)*ArcCoth[a + b*x^4] + Log[1 - (a + b*x^4)^2])/(8*b)
```

Maple [A] time = 0.078, size = 46, normalized size = 1.1

$$\frac{\operatorname{arccoth}\left(bx^4+a\right)x^4}{4} + \frac{\operatorname{arccoth}\left(bx^4+a\right)a}{4b} + \frac{\ln\left(\left(bx^4+a\right)^2-1\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccoth(b*x^4+a),x)
```

```
[Out] 1/4*arccoth(b*x^4+a)*x^4+1/4/b*arccoth(b*x^4+a)*a+1/8/b*ln((b*x^4+a)^2-1)
```

Maxima [A] time = 1.04667, size = 50, normalized size = 1.14

$$\frac{2\left(bx^4+a\right)\operatorname{arcoth}\left(bx^4+a\right)+\log\left(-\left(bx^4+a\right)^2+1\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*(2*(b*x^4 + a)*arccoth(b*x^4 + a) + log(-(b*x^4 + a)^2 + 1))/b
```

Fricas [A] time = 1.82153, size = 149, normalized size = 3.39

$$\frac{bx^4\log\left(\frac{bx^4+a+1}{bx^4+a-1}\right)+(a+1)\log\left(bx^4+a+1\right)-(a-1)\log\left(bx^4+a-1\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/8*(b*x^4*log((b*x^4 + a + 1)/(b*x^4 + a - 1)) + (a + 1)*log(b*x^4 + a + 1
) - (a - 1)*log(b*x^4 + a - 1))/b
```

Sympy [A] time = 9.03658, size = 60, normalized size = 1.36

$$\begin{cases} \frac{a \operatorname{acoth}\left(a+bx^4\right)}{4b} + \frac{x^4 \operatorname{acoth}\left(a+bx^4\right)}{4} + \frac{\log\left(a+bx^4+1\right)}{4b} - \frac{\operatorname{acoth}\left(a+bx^4\right)}{4b} & \text{for } b \neq 0 \\ \frac{x^4 \operatorname{acoth}(a)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acoth(b*x**4+a),x)
```

```
[Out] Piecewise((a*acoth(a + b*x**4)/(4*b) + x**4*acoth(a + b*x**4)/4 + log(a + b
*x**4 + 1)/(4*b) - acoth(a + b*x**4)/(4*b), Ne(b, 0)), (x**4*acoth(a)/4, Tr
ue))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arcoth}\left(bx^4 + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*arccoth(b*x^4 + a), x)
```

3.294 $\int x^{-1+n} \coth^{-1}(a+bx^n)\,dx$

Optimal. Leaf size=47

$$\frac{\log\left(1-(a+bx^n)^2\right)}{2bn}+\frac{(a+bx^n)\coth^{-1}(a+bx^n)}{bn}$$

```
[Out] ((a + b*x^n)*ArcCoth[a + b*x^n])/(b*n) + Log[1 - (a + b*x^n)^2]/(2*b*n)
```

Rubi [A] time = 0.0552597, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6715, 6104, 5911, 260}

$$\frac{\log\left(1-(a+bx^n)^2\right)}{2bn}+\frac{(a+bx^n)\coth^{-1}(a+bx^n)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*ArcCoth[a + b*x^n],x]
```

```
[Out] ((a + b*x^n)*ArcCoth[a + b*x^n])/(b*n) + Log[1 - (a + b*x^n)^2]/(2*b*n)
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6104

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}
, x] && IGtQ[p, 0]
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int x^{-1+n}\coth^{-1}(a+bx^n)\,dx = \frac{\text{Subst}\left(\int \coth^{-1}(a+bx)\,dx,x,x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \coth^{-1}(x)\,dx,x,a+bx^n\right)}{bn}$$
$$= \frac{(a+bx^n)\coth^{-1}(a+bx^n)}{bn} - \frac{\text{Subst}\left(\int \frac{x}{1-x^2}\,dx,x,a+bx^n\right)}{bn}$$
$$= \frac{(a+bx^n)\coth^{-1}(a+bx^n)}{bn} + \frac{\log\left(1-(a+bx^n)^2\right)}{2bn}$$

Mathematica [A] time = 0.0357095, size = 42, normalized size = 0.89

$$\frac{\log\left(1-(a+bx^n)^2\right)+2(a+bx^n)\coth^{-1}(a+bx^n)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*ArcCoth[a + b*x^n],x]
```

```
[Out] (2*(a + b*x^n)*ArcCoth[a + b*x^n] + Log[1 - (a + b*x^n)^2])/(2*b*n)
```

Maple [B] time = 0.119, size = 118, normalized size = 2.5

$$\frac{x^n\ln(1+a+bx^n)}{2n} - \frac{x^n\ln(-1+a+bx^n)}{2n} - \frac{a}{2bn}\ln\left(x^n+\frac{a-1}{b}\right) + \frac{a}{2bn}\ln\left(x^n+\frac{1+a}{b}\right) + \frac{1}{2bn}\ln\left(x^n+\frac{a-1}{b}\right) + \frac{1}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*arccoth(a+b*x^n),x)
```

```
[Out] 1/2/n*x^n*ln(1+a+b*x^n)-1/2/n*x^n*ln(-1+a+b*x^n)-1/2/b/n*ln(x^n+(a-1)/b)*a+
1/2/b/n*ln(x^n+(1+a)/b)*a+1/2/b/n*ln(x^n+(a-1)/b)+1/2/b/n*ln(x^n+(1+a)/b)
```

Maxima [A] time = 1.06526, size = 54, normalized size = 1.15

$$\frac{2(bx^n+a)\,\text{arcoth}(bx^n+a)+\log\left(-(bx^n+a)^2+1\right)}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arccoth(a+b*x^n),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*x^n + a)*arccoth(b*x^n + a) + log(-(b*x^n + a)^2 + 1))/(b*n)
```

Fricas [B] time = 1.85904, size = 359, normalized size = 7.64

$$\frac{(a+1)\log(b\cosh(n\log(x))+b\sinh(n\log(x))+a+1)-(a-1)\log(b\cosh(n\log(x))+b\sinh(n\log(x))+a-1)}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arccoth(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*((a + 1)*log(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a + 1) - (a - 1)*log
(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a - 1) + (b*cosh(n*log(x)) + b*sinh(
n*log(x)))*log((b*cosh(n*log(x)) + b*sinh(n*log(x)) + a + 1)/(b*cosh(n*log(
x)) + b*sinh(n*log(x)) + a - 1)))/(b*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*acoth(a+b*x**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{n-1} \operatorname{arcoth}(bx^n + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arccoth(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(n - 1)*arccoth(b*x^n + a), x)
```

3.295 $\int e^{c(a+bx)}\coth^{-1}(\sinh(ac+bcx))\,dx$

Optimal. Leaf size=107

$$\frac{\left(1-\sqrt{2}\right)\log\left(-e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc}+\frac{\left(1+\sqrt{2}\right)\log\left(-e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc}+\frac{e^{ac+bcx}\coth^{-1}(\sinh(c(a+bx)))}{bc}$$

```
[Out] (E^(a*c + b*c*x)*ArcCoth[Sinh[c*(a + b*x)]])/(b*c) + ((1 - Sqrt[2])*Log[3 -
 2*Sqrt[2] - E^(2*c*(a + b*x))])/(2*b*c) + ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2]
 - E^(2*c*(a + b*x))])/(2*b*c)
```

Rubi [A] time = 0.1546, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {2194, 6276, 2282, 12, 1247, 632, 31}

$$\frac{\left(1-\sqrt{2}\right)\log\left(-e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc}+\frac{\left(1+\sqrt{2}\right)\log\left(-e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc}+\frac{e^{ac+bcx}\coth^{-1}(\sinh(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcCoth[Sinh[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcCoth[Sinh[c*(a + b*x)]])/(b*c) + ((1 - Sqrt[2])*Log[3 -
 2*Sqrt[2] - E^(2*c*(a + b*x))])/(2*b*c) + ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2]
 - E^(2*c*(a + b*x))])/(2*b*c)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6276

```
Int[((a_.) + ArcCoth[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Dist[a + b*ArcCoth[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/
(1 - u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] && !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCoth[u]), x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int e^{c(a+bx)}\coth^{-1}(\sinh(ac+bcx))\,dx &= \frac{\text{Subst}\left(\int e^x\coth^{-1}(\sinh(x))\,dx,x,ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\coth^{-1}(\sinh(c(a+bx)))}{bc}-\frac{\text{Subst}\left(\int \frac{e^x\cosh(x)}{1-\sinh^2(x)}\,dx,x,ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\coth^{-1}(\sinh(c(a+bx)))}{bc}-\frac{\text{Subst}\left(\int \frac{2x(-1-x^2)}{1-6x^2+x^4}\,dx,x,e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\coth^{-1}(\sinh(c(a+bx)))}{bc}-\frac{2\,\text{Subst}\left(\int \frac{x(-1-x^2)}{1-6x^2+x^4}\,dx,x,e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\coth^{-1}(\sinh(c(a+bx)))}{bc}-\frac{\text{Subst}\left(\int \frac{-1-x}{1-6x+x^2}\,dx,x,e^{2ac+2bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\coth^{-1}(\sinh(c(a+bx)))}{bc}+\frac{\left(1-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-3+2\sqrt{2}+x}\,dx,x,e^{2ac+2bcx}\right)}{2bc}+\\
&= \frac{e^{ac+bcx}\coth^{-1}(\sinh(c(a+bx)))}{bc}+\frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}-e^{2ac+2bcx}\right)}{2bc}+\frac{\left(1+\sqrt{2}\right)}{}
\end{aligned}
$$

Mathematica [A] time = 0.164186, size = 153, normalized size = 1.43

$$
\frac{\log\left(-2e^{c(a+bx)}-e^{2c(a+bx)}+1\right)+\log\left(2e^{c(a+bx)}-e^{2c(a+bx)}+1\right)-2\sqrt{2}\tanh^{-1}\left(\frac{e^{c(a+bx)}-1}{\sqrt{2}}\right)+2\sqrt{2}\tanh^{-1}\left(\frac{e^{c(a+bx)}+1}{\sqrt{2}}\right)-2e^{c(a+b}}{2bc}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcCoth[Sinh[a*c + b*c*x]],x]
```

```
[Out] (-2*E^(c*(a + b*x))*ArcCoth[1/(2*E^(c*(a + b*x))) - E^(c*(a + b*x))/2] - 2*
Sqrt[2]*ArcTanh[(-1 + E^(c*(a + b*x)))/Sqrt[2]] + 2*Sqrt[2]*ArcTanh[(1 + E^
(c*(a + b*x)))/Sqrt[2]] + Log[1 - 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x))] +
Log[1 + 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x))])/(2*b*c)
```

Maple [C] time = 0.329, size = 794, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arccoth(sinh(b*c*x+a*c)),x)
```

```
[Out] 1/2/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1)-1/4*I/b/c*Pi
*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))^3*exp(c*(b*
x+a))-1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(-exp(2*c
*(b*x+a))+2*exp(c*(b*x+a))+1))^2*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(-exp(2
*c*(b*x+a))+2*exp(c*(b*x+a))+1))*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))+
2*exp(c*(b*x+a))+1))^2*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(-exp(2*c*(b*x+a)
)+2*exp(c*(b*x+a))+1))*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(-exp
(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(
2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+
2*exp(c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))
+2*exp(c*(b*x+a))-1))*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(2
*c*(b*x+a))+2*exp(c*(b*x+a))-1))*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*exp(-c*
(b*x+a))*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))^3*exp(c*(b*x+a))+1/4*I/b/c*
Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+2*exp(c
*(b*x+a))-1))^2*exp(c*(b*x+a))-1/2/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))-2
*exp(c*(b*x+a))-1)+1/2/b/c*ln(exp(2*c*(b*x+a))-(1+2^(1/2))^2)*2^(1/2)-1/2/b
/c*ln(exp(2*c*(b*x+a))-(2^(1/2)-1)^2)*2^(1/2)-2*a/b+1/2/b/c*ln(exp(2*c*(b*x
+a))-(1+2^(1/2))^2)+1/2/b/c*ln(exp(2*c*(b*x+a))-(2^(1/2)-1)^2)
```

Maxima [B] time = 1.61425, size = 248, normalized size = 2.32

$$\frac{\operatorname{arcoth}\left(\sinh\left(bcx+ac\right)\right)e^{\left(\left(bx+a\right)c\right)}}{bc}+\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{\left(bcx+ac\right)}+1}{\sqrt{2}+e^{\left(bcx+ac\right)}-1}\right)}{2\,bc}-\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{\left(bcx+ac\right)}-1}{\sqrt{2}+e^{\left(bcx+ac\right)}+1}\right)}{2\,bc}+\frac{\log\left(e^{\left(2\,bcx+2\,ac\right)}+2\,e^{\left(bcx+ac\right)}-\right.}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(sinh(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arccoth(sinh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + 1/2*sqrt(2)*log(-(sqrt(2
) - e^(b*c*x + a*c) + 1)/(sqrt(2) + e^(b*c*x + a*c) - 1))/(b*c) - 1/2*sqrt(
2)*log(-(sqrt(2) - e^(b*c*x + a*c) - 1)/(sqrt(2) + e^(b*c*x + a*c) + 1))/(b
*c) + 1/2*log(e^(2*b*c*x + 2*a*c) + 2*e^(b*c*x + a*c) - 1)/(b*c) + 1/2*log(
e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x + a*c) - 1)/(b*c)
```

Fricas [B] time = 1.74446, size = 624, normalized size = 5.83

$$\frac{\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\log\left(\frac{\sinh\left(bcx+ac\right)+1}{\sinh\left(bcx+ac\right)-1}\right)+\sqrt{2}\log\left(\frac{3\left(2\sqrt{2}+3\right)\cosh\left(bcx+ac\right)^{2}-4\left(3\sqrt{2}+4\right)\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)+3}{\cosh\left(bcx+ac\right)^{2}+\sinh\left(bcx+ac\right)^{2}-3}\right.}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(sinh(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log((sinh(b*c*x + a*c) + 1)/(s
inh(b*c*x + a*c) - 1)) + sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(b*c*x + a*c)^2
 - 4*(3*sqrt(2) + 4)*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2*sqrt(2) + 3
)*sinh(b*c*x + a*c)^2 - 2*sqrt(2) - 3)/(cosh(b*c*x + a*c)^2 + sinh(b*c*x +
a*c)^2 - 3)) + log(2*(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 - 3)/(cosh(
b*c*x + a*c)^2 - 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b*c*x + a*c)^
```

```
2)))/(b*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*acoth(sinh(b*c*x+a*c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}\left(\sinh\left(bcx + ac\right)\right)e^{\left((bx+a)c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(sinh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(sinh(b*c*x + a*c))*e^((b*x + a)*c), x)
```

3.296 $\int e^{c(a+bx)} \coth^{-1}(\cosh(ac+bcx))\,dx$

Optimal. Leaf size=49

$$\frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}+\frac{e^{ac+bcx}\coth^{-1}(\cosh(c(a+bx)))}{bc}$$

```
[Out] (E^(a*c + b*c*x)*ArcCoth[Cosh[c*(a + b*x)]])/(b*c) + Log[1 - E^(2*c*(a + b*
x))]/(b*c)
```

Rubi [A] time = 0.0793503, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2194, 6276, 2282, 12, 260}

$$\frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}+\frac{e^{ac+bcx}\coth^{-1}(\cosh(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcCoth[Cosh[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcCoth[Cosh[c*(a + b*x)]])/(b*c) + Log[1 - E^(2*c*(a + b*
x))]/(b*c)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6276

```
Int[((a_.) + ArcCoth[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Dist[a + b*ArcCoth[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/
(1 - u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCoth[u]), x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int e^{c(a+bx)}\coth^{-1}(\cosh(ac+bcx))\,dx = \frac{\text{Subst}\left(\int e^x\coth^{-1}(\cosh(x))\,dx,x,ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\coth^{-1}(\cosh(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int e^x\text{csch}(x)\,dx,x,ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\coth^{-1}(\cosh(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int \frac{2x}{-1+x^2}\,dx,x,e^{ac+bcx}\right)}{bc}$$
$$= \frac{e^{ac+bcx}\coth^{-1}(\cosh(c(a+bx)))}{bc}+\frac{2\,\text{Subst}\left(\int \frac{x}{-1+x^2}\,dx,x,e^{ac+bcx}\right)}{bc}$$
$$= \frac{e^{ac+bcx}\coth^{-1}(\cosh(c(a+bx)))}{bc}+\frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}$$

Mathematica [A] time = 0.0855697, size = 60, normalized size = 1.22

$$\frac{\log\left(1-e^{2c(a+bx)}\right)+e^{c(a+bx)}\coth^{-1}\left(\frac{1}{2}e^{-c(a+bx)}\left(e^{2c(a+bx)}+1\right)\right)}{bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcCoth[Cosh[a*c + b*c*x]],x]
```

```
[Out] (E^(c*(a + b*x))*ArcCoth[(1 + E^(2*c*(a + b*x)))/(2*E^(c*(a + b*x)))] + Log
[1 - E^(2*c*(a + b*x))])/(b*c)
```

Maple [C] time = 0.281, size = 824, normalized size = 16.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arccoth(cosh(b*c*x+a*c)),x)
```

```
[Out] 1/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))+1)-1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a
))*(exp(c*(b*x+a))+1)^2)^3*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a
))+1)^2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+1)^2)^2*exp(c*(b*x+a))-1/4*
I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1)^2)^3*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*
exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+1)^2)^2*exp(c*(b*x+
a))-1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*(exp(c*(b*x+a))+1)^2)*csgn(
I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+1)^2)*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*
(exp(c*(b*x+a))-1))^2*csgn(I*(exp(c*(b*x+a))-1)^2)*exp(c*(b*x+a))+1/2*I/b/c
*Pi*csgn(I*(exp(c*(b*x+a))+1))*csgn(I*(exp(c*(b*x+a))+1)^2)^2*exp(c*(b*x+a)
)-1/2*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1))*csgn(I*(exp(c*(b*x+a))-1)^2)^2*ex
p(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*(exp(c*(b*x+a))-1)
^2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-1)^2)*exp(c*(b*x+a))-1/4*I/b/c*P
i*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-1)^2)^2*ex
p(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1)^2)*csgn(I*exp(-c*(b*x+a
))*(exp(c*(b*x+a))-1)^2)^2*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a
))-1)^2)^3*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1))^2*csgn(I*
(exp(c*(b*x+a))+1)^2)*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(e
xp(c*(b*x+a))-1)^2)^3*exp(c*(b*x+a))-1/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))
-1)-2*a/b+1/b/c*ln(exp(2*c*(b*x+a))-1)
```

Maxima [A] time = 1.1183, size = 86, normalized size = 1.76

$$\frac{\operatorname{arcoth}\left(\cosh\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc}+\frac{\log\left(e^{(bcx+ac)}+1\right)}{bc}+\frac{\log\left(e^{(bcx+ac)}-1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(cosh(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arccoth(cosh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + log(e^(b*c*x + a*c) + 1)
/(b*c) + log(e^(b*c*x + a*c) - 1)/(b*c)
```

Fricas [A] time = 1.65442, size = 236, normalized size = 4.82

$$\frac{\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\log\left(\frac{\cosh(bcx+ac)+1}{\cosh(bcx+ac)-1}\right)+2\,\log\left(\frac{2\,\sinh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(cosh(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log((cosh(b*c*x + a*c) + 1)/(c
osh(b*c*x + a*c) - 1)) + 2*log(2*sinh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sin
h(b*c*x + a*c))))/(b*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*acoth(cosh(b*c*x+a*c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}\left(\cosh\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(cosh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(cosh(b*c*x + a*c))*e^((b*x + a)*c), x)
```

3.297 $\int e^{c(a+bx)} \coth^{-1}(\tanh(ac+bcx))\,dx$

Optimal. Leaf size=45

$$\frac{e^{ac+bcx}\coth^{-1}(\tanh(c(a+bx)))}{bc}-\frac{e^{ac+bcx}}{bc}$$

```
[Out] -(E^(a*c + b*c*x)/(b*c)) + (E^(a*c + b*c*x)*ArcCoth[Tanh[c*(a + b*x)]])/(b*
c)
```

Rubi [A] time = 0.0596168, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2194, 6276}

$$\frac{e^{ac+bcx}\coth^{-1}(\tanh(c(a+bx)))}{bc}-\frac{e^{ac+bcx}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcCoth[Tanh[a*c + b*c*x]],x]
```

```
[Out] -(E^(a*c + b*c*x)/(b*c)) + (E^(a*c + b*c*x)*ArcCoth[Tanh[c*(a + b*x)]])/(b*
c)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6276

```
Int[((a_.) + ArcCoth[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Dist[a + b*ArcCoth[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/
(1 - u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCoth[u]), x]]
```

Rubi steps

$$\begin{aligned}\int e^{c(a+bx)}\coth^{-1}(\tanh(ac+bcx))\,dx &= \frac{\text{Subst}\left(\int e^x\coth^{-1}(\tanh(x))\,dx,x,ac+bcx\right)}{bc}\\ &= \frac{e^{ac+bcx}\coth^{-1}(\tanh(c(a+bx)))}{bc}-\frac{\text{Subst}\left(\int e^x\,dx,x,ac+bcx\right)}{bc}\\ &= -\frac{e^{ac+bcx}}{bc}+\frac{e^{ac+bcx}\coth^{-1}(\tanh(c(a+bx)))}{bc}\end{aligned}$$

Mathematica [A] time = 0.0752806, size = 46, normalized size = 1.02

$$\frac{e^{c(a+bx)}\left(\coth^{-1}\left(\frac{e^{2c(a+bx)}-1}{e^{2c(a+bx)}+1}\right)-1\right)}{bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcCoth[Tanh[a*c + b*c*x]],x]
```

```
[Out] (E^(c*(a + b*x))*(-1 + ArcCoth[(-1 + E^(2*c*(a + b*x)))/(1 + E^(2*c*(a + b*
x)))]))/(b*c)
```

Maple [C] time = 0.25, size = 351, normalized size = 7.8

$$\frac{e^{c(bx+a)}\ln\left(e^{c(bx+a)}\right)}{bc}+\frac{\frac{i}{4}e^{c(bx+a)}}{bc}\left(-2\pi\left(\operatorname{csgn}\left(\frac{i}{e^{2c(bx+a)}+1}\right)\right)^{3}+2\pi\left(\operatorname{csgn}\left(\frac{i}{e^{2c(bx+a)}+1}\right)\right)^{2}-\pi\left(\operatorname{csgn}\left(\frac{ie^{2c(bx+a)}}{e^{2c(bx+a)}+1}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arccoth(tanh(b*c*x+a*c)),x)
```

```
[Out] 1/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a)))+1/4*I*(-2*Pi*csgn(I/(exp(2*c*(b*x+a
))+1))^3+2*Pi*csgn(I/(exp(2*c*(b*x+a))+1))^2-Pi*csgn(I*exp(2*c*(b*x+a))/(ex
p(2*c*(b*x+a))+1))^3+Pi*csgn(I*exp(2*c*(b*x+a))/(exp(2*c*(b*x+a))+1))^2*csg
n(I*exp(2*c*(b*x+a)))+Pi*csgn(I*exp(2*c*(b*x+a))/(exp(2*c*(b*x+a))+1))^2*cs
gn(I/(exp(2*c*(b*x+a))+1))-Pi*csgn(I*exp(2*c*(b*x+a))/(exp(2*c*(b*x+a))+1))
*csgn(I*exp(2*c*(b*x+a)))*csgn(I/(exp(2*c*(b*x+a))+1))-Pi*csgn(I*exp(c*(b*x
+a)))^2*csgn(I*exp(2*c*(b*x+a)))+2*Pi*csgn(I*exp(c*(b*x+a)))*csgn(I*exp(2*c
*(b*x+a)))^2-Pi*csgn(I*exp(2*c*(b*x+a)))^3+4*I-2*Pi)/b/c*exp(c*(b*x+a))
```

Maxima [A] time = 1.05829, size = 58, normalized size = 1.29

$$\frac{\operatorname{arcoth}\left(\tanh\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc}-\frac{e^{(bcx+ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(tanh(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arccoth(tanh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - e^(b*c*x + a*c)/(b*c)
```

Fricas [A] time = 1.7068, size = 55, normalized size = 1.22

$$\frac{(bcx+ac-1)e^{(bcx+ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(tanh(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] (b*c*x + a*c - 1)*e^(b*c*x + a*c)/(b*c)
```

Sympy [A] time = 25.7929, size = 63, normalized size = 1.4

$$\begin{cases}\frac{i\pi x}{2} & \text{for } c=0\wedge(b=0\vee c=0)\\ xe^{ac}\operatorname{acoth}\left(\tanh\left(ac\right)\right) & \text{for } b=0\\ \frac{e^{ac}e^{bcx}\operatorname{acoth}\left(\tanh\left(ac+bcx\right)\right)}{bc}-\frac{e^{ac}e^{bcx}}{bc} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*acoth(tanh(b*c*x+a*c)),x)
```

```
[Out] Piecewise((I*pi*x/2, Eq(c, 0) & (Eq(b, 0) | Eq(c, 0))), (x*exp(a*c)*acoth(t
anh(a*c)), Eq(b, 0)), (exp(a*c)*exp(b*c*x)*acoth(tanh(a*c + b*c*x))/(b*c) -
 exp(a*c)*exp(b*c*x)/(b*c), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}(\tanh(bcx + ac))\, e^{((bx+a)c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(tanh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*c*x + a*c))*e^((b*x + a)*c), x)
```

3.298 $\int e^{c(a+bx)} \coth^{-1}(\coth(ac+bcx))\,dx$

Optimal. Leaf size=45

$$\frac{e^{ac+bcx}\coth^{-1}(\coth(c(a+bx)))}{bc}-\frac{e^{ac+bcx}}{bc}$$

```
[Out] -(E^(a*c + b*c*x)/(b*c)) + (E^(a*c + b*c*x)*ArcCoth[Coth[c*(a + b*x)]])/(b*
c)
```

Rubi [A] time = 0.0586381, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2194, 6276}

$$\frac{e^{ac+bcx}\coth^{-1}(\coth(c(a+bx)))}{bc}-\frac{e^{ac+bcx}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcCoth[Coth[a*c + b*c*x]],x]
```

```
[Out] -(E^(a*c + b*c*x)/(b*c)) + (E^(a*c + b*c*x)*ArcCoth[Coth[c*(a + b*x)]])/(b*
c)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6276

```
Int[((a_.) + ArcCoth[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Dist[a + b*ArcCoth[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/
(1 - u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCoth[u]), x]]
```

Rubi steps

$$\begin{aligned}\int e^{c(a+bx)}\coth^{-1}(\coth(ac+bcx))\,dx &= \frac{\text{Subst}\left(\int e^x\coth^{-1}(\coth(x))\,dx,x,ac+bcx\right)}{bc}\\ &= \frac{e^{ac+bcx}\coth^{-1}(\coth(c(a+bx)))}{bc}-\frac{\text{Subst}\left(\int e^x\,dx,x,ac+bcx\right)}{bc}\\ &= -\frac{e^{ac+bcx}}{bc}+\frac{e^{ac+bcx}\coth^{-1}(\coth(c(a+bx)))}{bc}\end{aligned}$$

Mathematica [A] time = 0.0797835, size = 46, normalized size = 1.02

$$\frac{e^{c(a+bx)}\left(\coth^{-1}\left(\frac{e^{2c(a+bx)}+1}{e^{2c(a+bx)}-1}\right)-1\right)}{bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcCoth[Coth[a*c + b*c*x]],x]
```

```
[Out] (E^(c*(a + b*x))*(-1 + ArcCoth[(1 + E^(2*c*(a + b*x)))/(-1 + E^(2*c*(a + b*
x)))]))/(b*c)
```

Maple [A] time = 0.069, size = 68, normalized size = 1.5

$$\frac{(xbc+ac)\,\mathrm{e}^{xbc+ac}-\mathrm{e}^{xbc+ac}+\mathrm{e}^{xbc+ac}\left(\operatorname{arccoth}\left(\coth\left(xbc+ac\right)\right)-xbc-ac\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arccoth(coth(b*c*x+a*c)),x)
```

```
[Out] 1/b/c*((b*c*x+a*c)*exp(b*c*x+a*c)-exp(b*c*x+a*c)+exp(b*c*x+a*c)*(arccoth(co
th(b*c*x+a*c))-x*b*c-a*c))
```

Maxima [A] time = 1.1811, size = 57, normalized size = 1.27

$$\frac{ae^{(bcx+ac)}}{b}+\frac{\left(bcxe^{(ac)}-e^{(ac)}\right)e^{(bcx)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(coth(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] a*e^(b*c*x + a*c)/b + (b*c*x*e^(a*c) - e^(a*c))*e^(b*c*x)/(b*c)
```

Fricas [A] time = 1.51155, size = 55, normalized size = 1.22

$$\frac{(bcx+ac-1)e^{(bcx+ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(coth(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] (b*c*x + a*c - 1)*e^(b*c*x + a*c)/(b*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*acoth(coth(b*c*x+a*c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18341, size = 47, normalized size = 1.04

$$\frac{\left(b^2c^2x + abc^2 - bc\right)e^{(bcx+ac)}}{b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(coth(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] (b^2*c^2*x + a*b*c^2 - b*c)*e^(b*c*x + a*c)/(b^2*c^2)
```

3.299 $\int e^{c(a+bx)} \coth^{-1}(\mathbf{sech}(ac+bcx))\,dx$

Optimal. Leaf size=49

$$\frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}+\frac{e^{ac+bcx}\coth^{-1}(\mathrm{sech}(c(a+bx)))}{bc}$$

```
[Out] (E^(a*c + b*c*x)*ArcCoth[Sech[c*(a + b*x)]])/(b*c) + Log[1 - E^(2*c*(a + b*
x))]/(b*c)
```

Rubi [A] time = 0.0706529, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2194, 6276, 2282, 12, 260}

$$\frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}+\frac{e^{ac+bcx}\coth^{-1}(\mathrm{sech}(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcCoth[Sech[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcCoth[Sech[c*(a + b*x)]])/(b*c) + Log[1 - E^(2*c*(a + b*
x))]/(b*c)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6276

```
Int[((a_.) + ArcCoth[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Dist[a + b*ArcCoth[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/
(1 - u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCoth[u]), x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int e^{c(a+bx)} \coth^{-1}(\text{sech}(ac+bcx))\,dx = \frac{\text{Subst}\left(\int e^x \coth^{-1}(\text{sech}(x))\,dx, x, ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\coth^{-1}(\text{sech}(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int e^x \text{csch}(x)\,dx, x, ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\coth^{-1}(\text{sech}(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{2x}{-1+x^2}\,dx, x, e^{ac+bcx}\right)}{bc}$$
$$= \frac{e^{ac+bcx}\coth^{-1}(\text{sech}(c(a+bx)))}{bc} + \frac{2\,\text{Subst}\left(\int \frac{x}{-1+x^2}\,dx, x, e^{ac+bcx}\right)}{bc}$$
$$= \frac{e^{ac+bcx}\coth^{-1}(\text{sech}(c(a+bx)))}{bc} + \frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}$$

Mathematica [A] time = 0.0829819, size = 59, normalized size = 1.2

$$\frac{\log\left(1-e^{2c(a+bx)}\right)+e^{c(a+bx)}\coth^{-1}\left(\frac{2e^{c(a+bx)}}{e^{2c(a+bx)}+1}\right)}{bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcCoth[Sech[a*c + b*c*x]],x]
```

```
[Out] (E^(c*(a + b*x))*ArcCoth[(2*E^(c*(a + b*x)))/(1 + E^(2*c*(a + b*x)))] + Log
[1 - E^(2*c*(a + b*x))])/(b*c)
```

Maple [C] time = 0.292, size = 939, normalized size = 19.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arccoth(sech(b*c*x+a*c)),x)
```

```
[Out] -1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1)^2)^3*exp(c*(b*x+a))+1/4*I/b/c*Pi*cs
gn(I*(exp(c*(b*x+a))+1)^2)*csgn(I*(exp(c*(b*x+a))+1)^2/(exp(2*c*(b*x+a))+1)
)^2*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))+1))*csgn(I*(exp(c*
(b*x+a))+1)^2/(exp(2*c*(b*x+a))+1))^2*exp(c*(b*x+a))+1/2*I/b/c*Pi*csgn(I*(e
xp(c*(b*x+a))+1))*csgn(I*(exp(c*(b*x+a))+1)^2)^2*exp(c*(b*x+a))-1/4*I/b/c*P
i*csgn(I*(exp(c*(b*x+a))-1)^2)*csgn(I*(exp(c*(b*x+a))-1)^2/(exp(2*c*(b*x+a)
)+1))^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1)^2)*csgn(I/(ex
p(2*c*(b*x+a))+1))*csgn(I*(exp(c*(b*x+a))+1)^2/(exp(2*c*(b*x+a))+1))*exp(c*
(b*x+a))-1/2*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1))*csgn(I*(exp(c*(b*x+a))-1)^
2)^2*exp(c*(b*x+a))+1/2*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1)^2/(exp(2*c*(b*x+
a))+1))^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1))^2*csgn(I*(
exp(c*(b*x+a))+1)^2)*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))+1
))*csgn(I*(exp(c*(b*x+a))-1)^2/(exp(2*c*(b*x+a))+1))^2*exp(c*(b*x+a))+1/4*I
/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1)^2)*csgn(I/(exp(2*c*(b*x+a))+1))*csgn(I*(e
xp(c*(b*x+a))-1)^2/(exp(2*c*(b*x+a))+1))*exp(c*(b*x+a))-1/2*I/b/c*exp(c*(b*
x+a))*Pi+1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1)^2)^3*exp(c*(b*x+a))-1/4*I/b
/c*Pi*csgn(I*(exp(c*(b*x+a))+1)^2/(exp(2*c*(b*x+a))+1))^3*exp(c*(b*x+a))-1/
4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1)^2/(exp(2*c*(b*x+a))+1))^3*exp(c*(b*x+a
))+1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1))^2*csgn(I*(exp(c*(b*x+a))-1)^2)*e
xp(c*(b*x+a))-1/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))-1)+1/b/c*exp(c*(b*x+a)
```

```
)*ln(exp(c*(b*x+a))+1)-2*a/b+1/b/c*ln(exp(2*c*(b*x+a))-1)
```

Maxima [A] time = 1.03876, size = 86, normalized size = 1.76

$$\frac{\operatorname{arcoth}\left(\operatorname{sech}\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc}+\frac{\log\left(e^{(bcx+ac)}+1\right)}{bc}+\frac{\log\left(e^{(bcx+ac)}-1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(sech(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arccoth(sech(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + log(e^(b*c*x + a*c) + 1)
/(b*c) + log(e^(b*c*x + a*c) - 1)/(b*c)
```

Fricas [A] time = 1.75632, size = 238, normalized size = 4.86

$$\frac{\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\log\left(-\frac{\cosh(bcx+ac)+1}{\cosh(bcx+ac)-1}\right)+2\,\log\left(\frac{2\,\sinh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(sech(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log(-(cosh(b*c*x + a*c) + 1)/(
cosh(b*c*x + a*c) - 1)) + 2*log(2*sinh(b*c*x + a*c)/(cosh(b*c*x + a*c) - si
nh(b*c*x + a*c))))/(b*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*acoth(sech(b*c*x+a*c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}\left(\operatorname{sech}\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(sech(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(sech(b*c*x + a*c))*e^((b*x + a)*c), x)
```

3.300 $\int e^{c(a+bx)} \coth^{-1}(\mathbf{csch}(ac+bcx))\,dx$

Optimal. Leaf size=107

$$\frac{\left(1-\sqrt{2}\right)\log\left(-e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc}+\frac{\left(1+\sqrt{2}\right)\log\left(-e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc}+\frac{e^{ac+bcx}\coth^{-1}(\operatorname{csch}(c(a+bx)))}{bc}$$

```
[Out] (E^(a*c + b*c*x)*ArcCoth[Csch[c*(a + b*x)]])/(b*c) + ((1 - Sqrt[2])*Log[3 -
 2*Sqrt[2] - E^(2*c*(a + b*x))])/(2*b*c) + ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2]
 - E^(2*c*(a + b*x))])/(2*b*c)
```

Rubi [A] time = 0.147599, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {2194, 6276, 2282, 12, 1247, 632, 31}

$$\frac{\left(1-\sqrt{2}\right)\log\left(-e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc}+\frac{\left(1+\sqrt{2}\right)\log\left(-e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc}+\frac{e^{ac+bcx}\coth^{-1}(\operatorname{csch}(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcCoth[Csch[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcCoth[Csch[c*(a + b*x)]])/(b*c) + ((1 - Sqrt[2])*Log[3 -
 2*Sqrt[2] - E^(2*c*(a + b*x))])/(2*b*c) + ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2]
 - E^(2*c*(a + b*x))])/(2*b*c)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6276

```
Int[((a_.) + ArcCoth[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Dist[a + b*ArcCoth[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/
(1 - u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCoth[u]), x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int e^{c(a+bx)}\coth^{-1}(\operatorname{csch}(ac+bcx))\,dx &= \frac{\text{Subst}\left(\int e^x\coth^{-1}(\operatorname{csch}(x))\,dx,x,ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\coth^{-1}(\operatorname{csch}(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int \frac{e^x\coth(x)\operatorname{csch}(x)}{1-\operatorname{csch}^2(x)}\,dx,x,ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\coth^{-1}(\operatorname{csch}(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int \frac{2x\left(1+x^2\right)}{1-6x^2+x^4}\,dx,x,e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\coth^{-1}(\operatorname{csch}(c(a+bx)))}{bc}+\frac{2\,\text{Subst}\left(\int \frac{x\left(1+x^2\right)}{1-6x^2+x^4}\,dx,x,e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\coth^{-1}(\operatorname{csch}(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int \frac{1+x}{1-6x+x^2}\,dx,x,e^{2ac+2bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\coth^{-1}(\operatorname{csch}(c(a+bx)))}{bc}+\frac{\left(1-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-3+2\sqrt{2}+x}\,dx,x,e^{2ac+2bcx}\right)}{2bc}+\\
&= \frac{e^{ac+bcx}\coth^{-1}(\operatorname{csch}(c(a+bx)))}{bc}+\frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}-e^{2ac+2bcx}\right)}{2bc}+\frac{\left(1+\sqrt{2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.156532, size = 150, normalized size = 1.4

$$
\frac{\log\left(-2e^{c(a+bx)}-e^{2c(a+bx)}+1\right)+\log\left(2e^{c(a+bx)}-e^{2c(a+bx)}+1\right)-2\sqrt{2}\tanh^{-1}\left(\frac{e^{c(a+bx)}-1}{\sqrt{2}}\right)+2\sqrt{2}\tanh^{-1}\left(\frac{e^{c(a+bx)}+1}{\sqrt{2}}\right)+2e^{c(a+b}}{2bc}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcCoth[Csch[a*c + b*c*x]],x]
```

```
[Out] (2*E^(c*(a + b*x))*ArcCoth[(2*E^(c*(a + b*x)))/(-1 + E^(2*c*(a + b*x)))] -
2*Sqrt[2]*ArcTanh[(-1 + E^(c*(a + b*x)))/Sqrt[2]] + 2*Sqrt[2]*ArcTanh[(1 +
E^(c*(a + b*x)))/Sqrt[2]] + Log[1 - 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x))]
+ Log[1 + 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x))])/(2*b*c)
```

Maple [C] time = 0.333, size = 920, normalized size = 8.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arccoth(csch(b*c*x+a*c)),x)
```

```
[Out] 1/2/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1)-1/4*I/b/c*Pi
*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1)
/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I/(exp(2*c*(b*x+a
))-1))*csgn(I/(exp(2*c*(b*x+a))-1)*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))^2
*exp(c*(b*x+a))-1/2*I/b/c*exp(c*(b*x+a))*Pi+1/4*I/b/c*Pi*csgn(I*(exp(2*c*(b
*x+a))+2*exp(c*(b*x+a))-1))*csgn(I/(exp(2*c*(b*x+a))-1)*(exp(2*c*(b*x+a))+2
*exp(c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+
2*exp(c*(b*x+a))-1))*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I/(exp(2*c*(b*x+a))-
1)*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(
I/(exp(2*c*(b*x+a))-1)*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))^3*exp(c*(b*x+
a))+1/2*I/b/c*Pi*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1)/(exp(2*c*(b*
x+a))-1))^2*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(
b*x+a))+1)/(exp(2*c*(b*x+a))-1))^3*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(-exp
(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a
))+1)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(-exp(2*c*
(b*x+a))+2*exp(c*(b*x+a))+1))*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(-exp(2*c
*(b*x+a))+2*exp(c*(b*x+a))+1)/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))+1/2/b/c*
ln(exp(2*c*(b*x+a))-(1+2^(1/2))^2)*2^(1/2)-1/2/b/c*ln(exp(2*c*(b*x+a))-(2^(
1/2)-1)^2)*2^(1/2)-1/2/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))-2*exp(c*(b*x+
a))-1)-2*a/b+1/2/b/c*ln(exp(2*c*(b*x+a))-(1+2^(1/2))^2)+1/2/b/c*ln(exp(2*c*
(b*x+a))-(2^(1/2)-1)^2)
```

Maxima [B] time = 1.7483, size = 248, normalized size = 2.32

$$\frac{\operatorname{arcoth}\left(\operatorname{csch}\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc}+\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(bcx+ac)}+1}{\sqrt{2}+e^{(bcx+ac)}-1}\right)}{2\,bc}-\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(bcx+ac)}-1}{\sqrt{2}+e^{(bcx+ac)}+1}\right)}{2\,bc}+\frac{\log\left(e^{(2\,bcx+2\,ac)}+2\,e^{(bcx+ac)}\right.}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(csch(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arccoth(csch(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + 1/2*sqrt(2)*log(-(sqrt(2
) - e^(b*c*x + a*c) + 1)/(sqrt(2) + e^(b*c*x + a*c) - 1))/(b*c) - 1/2*sqrt(
2)*log(-(sqrt(2) - e^(b*c*x + a*c) - 1)/(sqrt(2) + e^(b*c*x + a*c) + 1))/(b
*c) + 1/2*log(e^(2*b*c*x + 2*a*c) + 2*e^(b*c*x + a*c) - 1)/(b*c) + 1/2*log(
e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x + a*c) - 1)/(b*c)
```

Fricas [B] time = 1.79002, size = 625, normalized size = 5.84

$$\frac{\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\log\left(-\frac{\sinh(bcx+ac)+1}{\sinh(bcx+ac)-1}\right)+\sqrt{2}\log\left(\frac{3\left(2\sqrt{2}+3\right)\cosh(bcx+ac)^{2}-4\left(3\sqrt{2}+4\right)\cosh(bcx+ac)\sinh(bcx+ac)}{\cosh(bcx+ac)^{2}+\sinh(bcx+ac)^{2}-}\right.}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(csch(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log(-(sinh(b*c*x + a*c) + 1)/(
sinh(b*c*x + a*c) - 1)) + sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(b*c*x + a*c)^
2 - 4*(3*sqrt(2) + 4)*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2*sqrt(2) +
```

```
3)*sinh(b*c*x + a*c)^2 - 2*sqrt(2) - 3)/(cosh(b*c*x + a*c)^2 + sinh(b*c*x +
 a*c)^2 - 3)) + log(2*(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 - 3)/(cosh
(b*c*x + a*c)^2 - 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b*c*x + a*c)
^2)))/(b*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{ac} \int e^{bcx} \operatorname{acoth}(\operatorname{csch}(ac + bcx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*acoth(csch(b*c*x+a*c)),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*acoth(csch(a*c + b*c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}(\operatorname{csch}(bcx + ac))\,e^{((bx+a)c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(csch(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(csch(b*c*x + a*c))*e^((b*x + a)*c), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
```

```
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```