

# Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.4-Inverse-hyperbolic-cotangent/7.4.1-Inverse-hyperbolic-cotangent-functions

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3.248	$\int (e + fx)^3 \coth^{-1}(\cot(a + bx)) dx$	.1227
3.249	$\int (e + fx)^2 \coth^{-1}(\cot(a + bx)) dx$	.1233
3.250	$\int (e + fx) \coth^{-1}(\cot(a + bx)) dx$	.1238
3.251	$\int \coth^{-1}(\cot(a + bx)) dx$	.1244
3.252	$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx} dx$	.1248
3.253	$\int x^2 \coth^{-1}(c + d \cot(a + bx)) dx$	.1251
3.254	$\int x \coth^{-1}(c + d \cot(a + bx)) dx$	.1257
3.255	$\int \coth^{-1}(c + d \cot(a + bx)) dx$	.1263
3.256	$\int \frac{\coth^{-1}(c+d \cot(a+bx))}{x} dx$	.1270
3.257	$\int x^2 \coth^{-1}(1 + id + d \cot(a + bx)) dx$	.1273
3.258	$\int x \coth^{-1}(1 + id + d \cot(a + bx)) dx$	.1279
3.259	$\int \coth^{-1}(1 + id + d \cot(a + bx)) dx$	.1285
3.260	$\int \frac{\coth^{-1}(1+id+d \cot(a+bx))}{x} dx$	.1290
3.261	$\int x^2 \coth^{-1}(1 - id - d \cot(a + bx)) dx$	.1293
3.262	$\int x \coth^{-1}(1 - id - d \cot(a + bx)) dx$	.1299
3.263	$\int \coth^{-1}(1 - id - d \cot(a + bx)) dx$	.1305
3.264	$\int \frac{\coth^{-1}(1-id-d \cot(a+bx))}{x} dx$	.1310
3.265	$\int \frac{(a+b \coth^{-1}(cx^n))(d+e \log(fx^m))}{x} dx$	.1313
3.266	$\int x^5 (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2x^2)) dx$	.1318
3.267	$\int x^3 (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2x^2)) dx$	.1326
3.268	$\int x (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2x^2)) dx$	.1334
3.269	$\int \frac{(a+b \coth^{-1}(cx))(d+e \log(1-c^2x^2))}{x} dx$	.1340
3.270	$\int \frac{(a+b \coth^{-1}(cx))(d+e \log(1-c^2x^2))}{x^3} dx$	.1346
3.271	$\int \frac{(a+b \coth^{-1}(cx))(d+e \log(1-c^2x^2))}{x^5} dx$	.1352
3.272	$\int x^4 (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2x^2)) dx$	.1358
3.273	$\int x^2 (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2x^2)) dx$	.1367
3.274	$\int (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2x^2)) dx$	.1375
3.275	$\int \frac{(a+b \coth^{-1}(cx))(d+e \log(1-c^2x^2))}{x^2} dx$	.1381
3.276	$\int \frac{(a+b \coth^{-1}(cx))(d+e \log(1-c^2x^2))}{x^4} dx$	.1386
3.277	$\int \frac{(a+b \coth^{-1}(cx))(d+e \log(1-c^2x^2))}{x^6} dx$	.1392
3.278	$\int x (a + b \coth^{-1}(cx)) (d + e \log(f + gx^2)) dx$	.1399
3.279	$\int (a + b \coth^{-1}(cx)) (d + e \log(f + gx^2)) dx$	.1407
3.280	$\int \frac{(a+b \coth^{-1}(cx))(d+e \log(f+gx^2))}{x} dx$	.1416

3.281	$\int \frac{(a+b \coth^{-1}(cx))(d+e \log(f+gx^2))}{x^2} dx$	.1420
3.282	$\int \frac{(a+b \coth^{-1}(cx))(d+e \log(f+gx^2))}{x^3} dx$	.1429
3.283	$\int \coth^{-1}(e^x) dx$	.1437
3.284	$\int x \coth^{-1}(e^x) dx$	.1440
3.285	$\int x^2 \coth^{-1}(e^x) dx$	.1444
3.286	$\int \coth^{-1}(e^{a+bx}) dx$	.1448
3.287	$\int x \coth^{-1}(e^{a+bx}) dx$	.1452
3.288	$\int x^2 \coth^{-1}(e^{a+bx}) dx$	.1456
3.289	$\int \coth^{-1}(a + bf^{c+dx}) dx$	.1460
3.290	$\int x \coth^{-1}(a + bf^{c+dx}) dx$	.1465
3.291	$\int x^2 \coth^{-1}(a + bf^{c+dx}) dx$	.1471
3.292	$\int \frac{1}{(a-ax^2)(b-2b \coth^{-1}(x))} dx$	.1477
3.293	$\int x^3 \coth^{-1}(a + bx^4) dx$	.1480
3.294	$\int x^{-1+n} \coth^{-1}(a + bx^n) dx$	.1484
3.295	$\int e^{c(a+bx)} \coth^{-1}(\sinh(ac + bcx)) dx$	.1488
3.296	$\int e^{c(a+bx)} \coth^{-1}(\cosh(ac + bcx)) dx$	.1493
3.297	$\int e^{c(a+bx)} \coth^{-1}(\tanh(ac + bcx)) dx$	.1497
3.298	$\int e^{c(a+bx)} \coth^{-1}(\coth(ac + bcx)) dx$	.1501
3.299	$\int e^{c(a+bx)} \coth^{-1}(\operatorname{sech}(ac + bcx)) dx$	.1504
3.300	$\int e^{c(a+bx)} \coth^{-1}(\operatorname{csch}(ac + bcx)) dx$	.1508

#### 4 Listing of Grading functions

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# Chapter 1

## Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [ 300 ]. This is test number [ 198 ].

### 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. ( 300 )	% 0. ( 0 )
Mathematica	% 98. ( 294 )	% 2. ( 6 )
Maple	% 91. ( 273 )	% 9. ( 27 )
Maxima	% 75.67 ( 227 )	% 24.33 ( 73 )
Fricas	% 74.33 ( 223 )	% 25.67 ( 77 )
Sympy	% 31.67 ( 95 )	% 68.33 ( 205 )
Giac	% 9.67 ( 29 )	% 90.33 ( 271 )

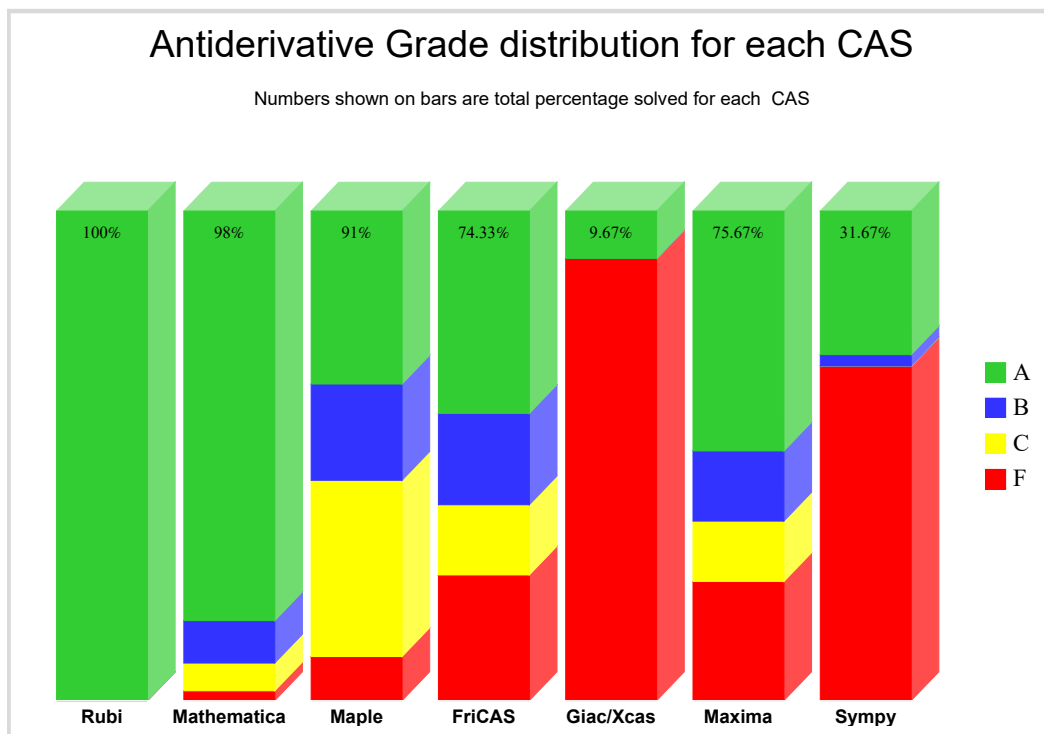
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

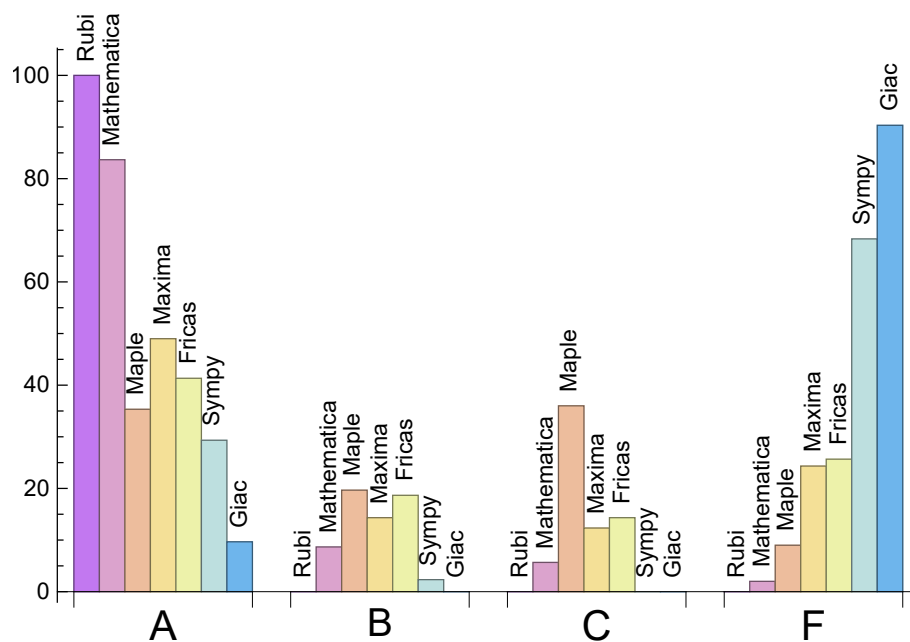
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	83.67	8.67	5.67	2.
Maple	35.33	19.67	36.	9.
Maxima	49.	14.33	12.33	24.33
Fricas	41.33	18.67	14.33	25.67
Sympy	29.33	2.33	0.	68.33
Giac	9.67	0.	0.	90.33

The following is a Bar chart illustration of the data in the above table.





The figure below compares the CAS systems for each grade level.



### 1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.23	125.8	0.93	79.	1.
Mathematica	1.01	221.68	1.43	78.	0.93
Maple	3.42	18855.4	149.72	306.	2.61
Maxima	1.42	157.59	1.76	105.	1.51
Fricas	1.64	630.2	5.32	223.	3.56
Sympy	8.34	76.19	1.34	46.	1.15
Giac	0.28	4.83	0.19	0.	0.

## 1.4 list of integrals that has no closed form antiderivative

{42, 43, 120, 121, 122, 126, 127, 206, 211, 216, 220, 225, 230, 235, 239, 243, 247, 252, 256, 260, 264, 280}

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {76, 78}

**Mathematica** {13, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 40, 41, 48, 49, 56, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 82, 98, 100, 101, 106, 109, 110, 111, 112, 113, 114, 115, 116, 118, 125, 210, 215, 217, 218, 224, 229, 233, 238, 242, 246, 250, 255, 259, 263, 270, 271, 278, 281, 282, 295, 296, 297, 298, 299, 300}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

## 1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## 1.9.2 Important note about FriCAS and Giac/X-CAS results

There are few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail with error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

## 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

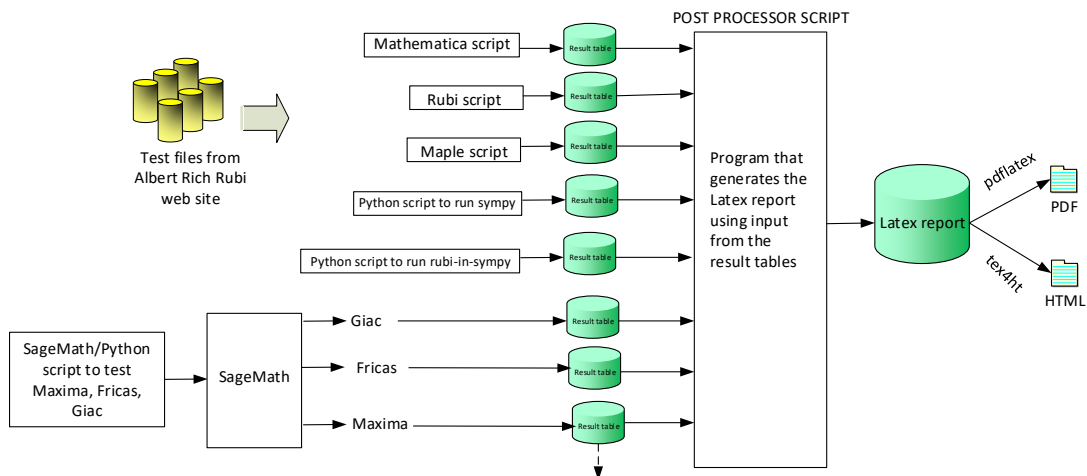
```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

    except Exception as ee:
        leafCount =1
```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

### High level overview of the CAS independent integration test build system

# Chapter 2

## detailed summary tables of results

### 2.1 List of integrals sorted by grade for each CAS

#### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300 }

B grade: { }

C grade: { }

F grade: { }

## 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 71, 72, 76, 77, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 96, 97, 99, 102, 103, 104, 105, 106, 107, 108, 110, 111, 120, 121, 122, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 211, 212, 213, 214, 216, 217, 218, 219, 220, 221, 222, 223, 225, 226, 227, 228, 230, 232, 233, 234, 235, 236, 237, 239, 240, 241, 243, 244, 245, 247, 249, 250, 251, 252, 253, 254, 256, 257, 258, 260, 261, 262, 264, 266, 267, 268, 270, 271, 272, 273, 274, 278, 280, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300 }

B grade: { 41, 70, 95, 98, 100, 101, 109, 139, 150, 210, 215, 224, 229, 231, 238, 242, 246, 248, 255, 259, 263, 275, 276, 279, 281, 283 }

C grade: { 24, 26, 34, 66, 73, 74, 75, 78, 79, 112, 113, 114, 115, 116, 118, 265, 282 }

F grade: { 117, 119, 123, 124, 269, 277 }

## 2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 15, 35, 36, 37, 38, 42, 43, 48, 49, 50, 51, 52, 53, 54, 57, 58, 60, 64, 65, 66, 67, 68, 72, 74, 75, 77, 78, 80, 81, 83, 84, 85, 87, 88, 89, 90, 91, 92, 94, 95, 97, 98, 99, 100, 101, 105, 106, 107, 108, 113, 120, 121, 122, 125, 126, 127, 133, 134, 135, 140, 151, 162, 171, 180, 189, 197, 198, 199, 200, 206, 211, 216, 220, 225, 230, 235, 239, 243, 247, 252, 256, 260, 264, 280, 282, 283, 284, 285, 286, 288, 289, 292, 293, 298 }

B grade: { 12, 14, 16, 17, 19, 20, 21, 22, 28, 39, 40, 41, 56, 59, 61, 62, 63, 69, 70, 71, 76, 82, 86, 96, 102, 103, 104, 109, 110, 111, 116, 123, 124, 129, 130, 131, 185, 186, 194, 195, 196, 205, 210, 215, 219, 224, 229, 234, 238, 242, 246, 251, 255, 259, 263, 287, 290, 291, 294 }

C grade: { 18, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 55, 73, 79, 93, 112, 114, 115, 117, 118, 128, 132, 136, 137, 138, 139, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 152, 153, 154, 155, 156, 157, 159, 160, 161, 163, 167, 168, 169, 170, 176, 177, 178, 179, 187, 188, 193, 201, 202, 203, 204, 207, 208, 209, 212, 213, 214, 217, 218, 221, 222, 223, 226, 227, 228, 231, 232, 233, 236, 237, 240, 241, 244, 245, 248, 249, 250, 253, 254, 257, 258, 261, 262, 265, 266, 267, 268, 269, 272, 273, 274, 278, 295, 296, 297, 299, 300 }

F grade: { 44, 45, 46, 47, 119, 158, 164, 165, 166, 172, 173, 174, 175, 181, 182, 183, 184, 190, 191, 192, 270, 271, 275, 276, 277, 279, 281 }



## 2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 20, 21, 23, 35, 36, 37, 38, 51, 52, 53, 57, 58, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 77, 78, 83, 84, 85, 87, 88, 89, 90, 91, 92, 94, 96, 97, 99, 103, 104, 105, 107, 108, 110, 122, 126, 127, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 142, 143, 144, 145, 147, 148, 149, 150, 154, 155, 156, 157, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 235, 239, 243, 247, 252, 256, 260, 264, 268, 280, 284, 285, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 299 }

B grade: { 7, 16, 17, 19, 22, 25, 27, 31, 33, 55, 56, 59, 61, 86, 93, 95, 98, 100, 101, 102, 109, 140, 151, 234, 238, 240, 241, 242, 244, 245, 246, 251, 255, 257, 258, 259, 261, 262, 263, 283, 286, 295, 300 }

C grade: { 141, 152, 153, 159, 160, 161, 162, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 185, 186, 187, 188, 189, 266, 267, 269, 272, 273, 274 }

F grade: { 18, 24, 26, 28, 29, 30, 32, 34, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 54, 73, 76, 79, 80, 81, 82, 106, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 123, 124, 125, 128, 136, 146, 158, 166, 175, 184, 190, 191, 192, 193, 231, 232, 233, 236, 237, 248, 249, 250, 253, 254, 265, 270, 271, 275, 276, 277, 278, 279, 281, 282 }

## 2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 16, 20, 22, 35, 36, 37, 38, 42, 43, 50, 51, 52, 55, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 83, 84, 85, 87, 88, 89, 90, 91, 92, 94, 96, 97, 99, 104, 105, 120, 121, 122, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 152, 153, 154, 155, 156, 157, 159, 160, 163, 193, 194, 195, 196, 197, 198, 199, 206, 211, 216, 220, 225, 230, 235, 239, 243, 247, 252, 256, 259, 260, 263, 264, 266, 267, 268, 272, 273, 274, 280, 289, 292, 293, 296, 297, 298, 299 }

B grade: { 44, 45, 46, 47, 53, 54, 57, 95, 102, 103, 107, 108, 151, 161, 162, 164, 165, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 185, 186, 187, 188, 189, 200, 205, 210, 215, 219, 224, 229, 234, 238, 242, 246, 251, 255, 283, 286, 294, 295, 300 }

C grade: { 201, 202, 203, 204, 207, 208, 209, 212, 213, 214, 217, 218, 221, 222, 223, 226, 227, 228, 231, 232, 233, 236, 237, 240, 241, 244, 245, 248, 249, 250, 253, 254, 257, 258, 261, 262, 265, 284, 285, 287, 288, 290, 291 }

F grade: { 7, 13, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 39, 40, 41, 48, 49, 56, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 86, 93, 98, 100, 101, 106, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 123, 124, 125, 158, 166, 175, 184, 190, 191, 192, 269, 270, 271, 275, 276, 277, 278, 279, 281, 282 }

## 2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 16, 20, 22, 35, 36, 37, 38, 42, 43, 53, 54, 57, 58, 63, 64, 65, 67, 68, 90, 94, 96, 97, 99, 103, 104, 105, 122, 126, 127, 129, 130, 131, 133, 134, 135, 137, 138, 139, 140, 143, 144, 145, 147, 148, 149, 150, 151, 155, 157, 162, 170, 171, 178, 179, 180, 189, 194, 195, 196, 198, 199, 211, 216, 225, 230, 235, 252, 266, 267, 268, 272, 273, 274, 292, 293, 297 }

B grade: { 60, 87, 88, 89, 91, 92, 156 }

C grade: { }

F grade: { 7, 13, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 39, 40, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 55, 56, 59, 61, 62, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 93, 95, 98, 100, 101, 102, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 123, 124, 125, 128, 132, 136, 141, 142, 146, 152, 153, 154, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 172, 173, 174, 175, 176, 177, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 197, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 224, 226, 227, 228, 229, 231, 232, 233, 234, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 269, 270, 271, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 294, 295, 296, 298, 299, 300 }

## 2.1.7 Giac

A grade: { 42, 43, 120, 121, 122, 126, 127, 194, 195, 196, 197, 198, 199, 206, 211, 216, 220, 225, 230, 235, 239, 243, 247, 252, 256, 260, 264, 280, 298 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 123, 124, 125, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 200, 201, 202, 203, 204, 205, 207, 208, 209, 210, 212, 213, 214, 215, 217, 218, 219, 221, 222, 223, 224, 226, 227, 228, 229, 231, 232, 233, 234, 236, 237, 238, 240, 241, 242, 244, 245, 246, 248, 249, 250, 251, 253, 254, 255, 257, 258, 259, 261, 262, 263, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 299, 300 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	67	55	82	120	49	0
normalized size	1	1.	1.31	1.08	1.61	2.35	0.96	0.
time (sec)	N/A	0.028	0.009	0.035	0.965	1.54	2.07	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	62	122	54	0
normalized size	1	1.	1.	0.98	1.24	2.44	1.08	0.
time (sec)	N/A	0.037	0.008	0.035	0.958	1.576	3.798	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	57	47	70	99	41	0
normalized size	1	1.	1.39	1.15	1.71	2.41	1.	0.
time (sec)	N/A	0.024	0.008	0.031	0.966	1.599	1.284	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	41	47	99	46	0
normalized size	1	1.	1.	1.02	1.18	2.48	1.15	0.
time (sec)	N/A	0.03	0.008	0.032	0.968	1.573	2.373	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	47	39	55	78	32	0
normalized size	1	1.	1.52	1.26	1.77	2.52	1.03	0.
time (sec)	N/A	0.013	0.007	0.033	0.973	1.618	0.919	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	23	34	77	27	0
normalized size	1	1.	1.	0.92	1.36	3.08	1.08	0.
time (sec)	N/A	0.007	0.003	0.043	0.987	1.68	0.609	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	37	116	0	0	0
normalized size	1	1.	0.93	1.32	4.14	0.	0.	0.
time (sec)	N/A	0.01	0.008	0.042	0.997	0.	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	35	41	99	26	0
normalized size	1	1.	1.	1.17	1.37	3.3	0.87	0.
time (sec)	N/A	0.021	0.008	0.036	0.974	1.7	0.611	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	47	39	49	80	24	0
normalized size	1	1.	1.52	1.26	1.58	2.58	0.77	0.
time (sec)	N/A	0.016	0.008	0.039	0.973	1.519	2.571	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	48	54	120	46	0
normalized size	1	1.	1.	1.02	1.15	2.55	0.98	0.
time (sec)	N/A	0.03	0.01	0.039	0.945	1.57	4.121	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	57	47	69	100	32	0
normalized size	1	1.	1.39	1.15	1.68	2.44	0.78	0.
time (sec)	N/A	0.022	0.012	0.036	0.944	1.573	1.96	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	80	196	182	224	114	0
normalized size	1	1.	0.76	1.87	1.73	2.13	1.09	0.
time (sec)	N/A	0.246	0.022	0.052	0.971	1.577	7.505	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	87	196	209	0	0	0
normalized size	1	1.	0.69	1.54	1.65	0.	0.	0.
time (sec)	N/A	0.226	0.443	0.053	0.961	0.	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	62	176	159	184	90	0
normalized size	1	1.	0.77	2.17	1.96	2.27	1.11	0.
time (sec)	N/A	0.163	0.017	0.052	0.955	1.597	5.101	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	66	176	181	0	0	0
normalized size	1	1.	0.64	1.71	1.76	0.	0.	0.
time (sec)	N/A	0.153	0.238	0.052	0.974	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	43	155	131	143	60	0
normalized size	1	1.	0.8	2.87	2.43	2.65	1.11	0.
time (sec)	N/A	0.078	0.012	0.056	0.986	1.506	4.527	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	58	58	46	122	182	0	0	0
normalized size	1	1.	0.79	2.1	3.14	0.	0.	0.
time (sec)	N/A	0.078	0.084	0.119	0.987	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	114	487	0	0	0	0
normalized size	1	1.	1.18	5.02	0.	0.	0.	0.
time (sec)	N/A	0.232	0.063	0.381	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	55	55	49	159	197	0	0	0
normalized size	1	1.	0.89	2.89	3.58	0.	0.	0.
time (sec)	N/A	0.108	0.1	0.056	0.982	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	57	164	130	181	56	0
normalized size	1	1.	0.93	2.69	2.13	2.97	0.92	0.
time (sec)	N/A	0.099	0.017	0.055	0.972	1.899	1.01	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	87	224	238	0	0	0
normalized size	1	1.	0.84	2.17	2.31	0.	0.	0.
time (sec)	N/A	0.171	0.246	0.06	0.989	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	82	185	208	223	90	0
normalized size	1	1.	0.91	2.06	2.31	2.48	1.	0.
time (sec)	N/A	0.172	0.022	0.06	0.983	1.838	6.423	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	186	186	117	1141	390	0	0	0
normalized size	1	1.	0.63	6.13	2.1	0.	0.	0.
time (sec)	N/A	0.717	0.526	1.901	1.012	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	196	196	175	806	0	0	0	0
normalized size	1	1.	0.89	4.11	0.	0.	0.	0.
time (sec)	N/A	0.58	0.567	1.106	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	139	88	684	354	0	0	0
normalized size	1	1.	0.63	4.92	2.55	0.	0.	0.
time (sec)	N/A	0.417	0.3	0.619	0.995	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	149	149	140	765	0	0	0	0
normalized size	1	1.	0.94	5.13	0.	0.	0.	0.
time (sec)	N/A	0.333	0.353	0.57	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	95	95	68	3070	290	0	0	0
normalized size	1	1.	0.72	32.32	3.05	0.	0.	0.
time (sec)	N/A	0.183	0.141	0.404	0.999	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	79	180	0	0	0	0
normalized size	1	1.	0.93	2.12	0.	0.	0.	0.
time (sec)	N/A	0.165	0.098	0.115	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	150	150	156	564	0	0	0	0
normalized size	1	1.	1.04	3.76	0.	0.	0.	0.
time (sec)	N/A	0.35	0.076	0.253	0.	0.	0.	0.



Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	72	796	0	0	0	0
normalized size	1	1.	0.91	10.08	0.	0.	0.	0.
time (sec)	N/A	0.199	0.132	0.342	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	95	95	79	3673	340	0	0	0
normalized size	1	1.	0.83	38.66	3.58	0.	0.	0.
time (sec)	N/A	0.219	0.167	0.422	0.992	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	154	154	142	895	0	0	0	0
normalized size	1	1.	0.92	5.81	0.	0.	0.	0.
time (sec)	N/A	0.369	0.213	0.657	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	141	141	118	661	462	0	0	0
normalized size	1	1.	0.84	4.69	3.28	0.	0.	0.
time (sec)	N/A	0.464	0.23	0.536	1.01	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	164	164	741	926	0	0	0	0
normalized size	1	1.	4.52	5.65	0.	0.	0.	0.
time (sec)	N/A	0.034	7.684	0.625	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	213	334	373	558	427	0
normalized size	1	1.	0.87	1.36	1.52	2.28	1.74	0.
time (sec)	N/A	0.181	0.116	0.04	0.972	1.584	25.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	150	233	267	382	282	0
normalized size	1	1.	0.89	1.38	1.58	2.26	1.67	0.
time (sec)	N/A	0.127	0.079	0.037	0.974	1.627	7.312	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	98	148	177	258	182	0
normalized size	1	1.	0.89	1.35	1.61	2.35	1.65	0.
time (sec)	N/A	0.133	0.054	0.036	0.973	1.549	3.323	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	69	76	88	142	87	0
normalized size	1	1.	1.21	1.33	1.54	2.49	1.53	0.
time (sec)	N/A	0.066	0.012	0.038	0.974	1.515	1.373	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	390	390	671	785	0	0	0	0
normalized size	1	1.	1.72	2.01	0.	0.	0.	0.
time (sec)	N/A	0.947	1.344	0.218	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	590	590	753	2218	0	0	0	0
normalized size	1	1.	1.28	3.76	0.	0.	0.	0.
time (sec)	N/A	0.87	7.815	0.41	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	657	657	1838	4128	0	0	0	0
normalized size	1	1.	2.8	6.28	0.	0.	0.	0.
time (sec)	N/A	0.951	12.88	0.596	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	24.127	0.713	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	4.359	0.724	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	119	0	0	760	0	0
normalized size	1	1.	1.92	0.	0.	12.26	0.	0.
time (sec)	N/A	0.113	0.118	0.457	0.	1.747	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	226	0	0	1496	0	0
normalized size	1	1.	1.77	0.	0.	11.69	0.	0.
time (sec)	N/A	0.34	0.321	0.456	0.	1.985	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	329	0	0	2610	0	0
normalized size	1	1.	1.64	0.	0.	13.05	0.	0.
time (sec)	N/A	1.047	0.636	0.455	0.	2.245	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	431	0	0	4140	0	0
normalized size	1	1.	1.52	0.	0.	14.63	0.	0.
time (sec)	N/A	1.343	0.961	0.461	0.	3.193	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	186	186	125	199	0	0	0	0
normalized size	1	1.	0.67	1.07	0.	0.	0.	0.
time (sec)	N/A	0.083	1.039	0.448	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	77	190	0	0	0	0
normalized size	1	1.	0.53	1.32	0.	0.	0.	0.
time (sec)	N/A	0.05	0.112	0.368	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	30	52	0	92	0	0
normalized size	1	1.	0.81	1.41	0.	2.49	0.	0.
time (sec)	N/A	0.025	0.044	0.226	0.	1.536	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	45	112	90	140	0	0
normalized size	1	1.	0.54	1.35	1.08	1.69	0.	0.
time (sec)	N/A	0.053	0.05	0.238	1.017	1.584	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	55	176	134	189	0	0
normalized size	1	1.	0.44	1.42	1.08	1.52	0.	0.
time (sec)	N/A	0.082	0.061	0.261	1.026	1.679	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	36	3	0
normalized size	1	1.	1.	1.33	1.33	12.	1.	0.
time (sec)	N/A	0.023	0.025	0.043	0.976	1.507	0.411	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	182	15	0
normalized size	1	1.	1.	1.08	0.	15.17	1.25	0.
time (sec)	N/A	0.026	0.009	0.059	0.	1.614	4.822	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	61	707	231	165	0	0
normalized size	1	1.	0.98	11.4	3.73	2.66	0.	0.
time (sec)	N/A	0.051	0.083	0.494	1.025	1.569	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	37	37	34	75	103	0	0	0
normalized size	1	1.	0.92	2.03	2.78	0.	0.	0.
time (sec)	N/A	0.059	0.049	0.035	0.963	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	13	8	38	5	0
normalized size	1	1.	1.	1.62	1.	4.75	0.62	0.
time (sec)	N/A	0.014	0.004	0.029	0.954	1.558	0.817	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	44	39	46	74	31	0
normalized size	1	1.	1.22	1.08	1.28	2.06	0.86	0.
time (sec)	N/A	0.031	0.027	0.027	0.946	1.588	0.819	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	28	99	103	111	0	0
normalized size	1	1.	0.74	2.61	2.71	2.92	0.	0.
time (sec)	N/A	0.017	0.032	0.043	0.967	1.584	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	53	63	111	88	0
normalized size	1	1.	1.	1.06	1.26	2.22	1.76	0.
time (sec)	N/A	0.034	0.051	0.028	0.949	1.593	1.329	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	43	131	159	165	0	0
normalized size	1	1.	0.64	1.96	2.37	2.46	0.	0.
time (sec)	N/A	0.035	0.052	0.049	0.962	1.582	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	81	199	143	279	0	0
normalized size	1	1.	0.8	1.97	1.42	2.76	0.	0.
time (sec)	N/A	0.126	0.039	0.035	0.96	1.551	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	92	146	107	213	117	0
normalized size	1	1.	1.18	1.87	1.37	2.73	1.5	0.
time (sec)	N/A	0.102	0.023	0.036	0.958	1.611	3.197	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	56	89	82	176	76	0
normalized size	1	1.	0.86	1.37	1.26	2.71	1.17	0.
time (sec)	N/A	0.072	0.022	0.033	0.972	1.6	1.609	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	43	36	42	135	41	0
normalized size	1	1.	1.23	1.03	1.2	3.86	1.17	0.
time (sec)	N/A	0.016	0.015	0.03	0.974	1.544	0.819	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	92	92	259	81	173	0	0	0
normalized size	1	1.	2.82	0.88	1.88	0.	0.	0.
time (sec)	N/A	0.097	0.168	0.076	0.972	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	55	63	73	192	144	0
normalized size	1	1.	0.86	0.98	1.14	3.	2.25	0.
time (sec)	N/A	0.051	0.051	0.037	0.959	1.676	2.885	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	76	82	115	279	410	0
normalized size	1	1.	0.84	0.91	1.28	3.1	4.56	0.
time (sec)	N/A	0.1	0.107	0.043	0.96	1.759	3.456	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	263	263	203	967	432	0	0	0
normalized size	1	1.	0.77	3.68	1.64	0.	0.	0.
time (sec)	N/A	0.349	1.564	0.056	0.988	0.	0.	0.



Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	204	204	607	729	350	0	0	0
normalized size	1	1.	2.98	3.57	1.72	0.	0.	0.
time (sec)	N/A	0.278	4.527	0.055	1.002	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	136	136	106	365	273	0	0	0
normalized size	1	1.	0.78	2.68	2.01	0.	0.	0.
time (sec)	N/A	0.209	0.247	0.053	0.985	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	55	151	188	0	0	0
normalized size	1	1.	0.68	1.86	2.32	0.	0.	0.
time (sec)	N/A	0.09	0.07	0.102	0.978	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	714	985	0	0	0	0
normalized size	1	1.	4.82	6.66	0.	0.	0.	0.
time (sec)	N/A	0.091	3.205	0.543	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	251	251	206	342	329	0	0	0
normalized size	1	1.	0.82	1.36	1.31	0.	0.	0.
time (sec)	N/A	0.719	1.04	0.096	1.028	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	370	370	291	467	486	0	0	0
normalized size	1	1.	0.79	1.26	1.31	0.	0.	0.
time (sec)	N/A	0.83	2.184	0.099	1.024	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	673	597	529	1230	0	0	0	0
normalized size	1	0.89	0.79	1.83	0.	0.	0.	0.
time (sec)	N/A	1.099	0.586	0.49	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	185	176	259	0	0	0
normalized size	1	1.	1.54	1.47	2.16	0.	0.	0.
time (sec)	N/A	0.127	0.067	0.137	0.987	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	292	360	502	297	259	0	0	0
normalized size	1	1.23	1.72	1.02	0.89	0.	0.	0.
time (sec)	N/A	0.5	4.258	0.142	0.996	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	738	738	5552	19686	0	0	0	0
normalized size	1	1.	7.52	26.67	0.	0.	0.	0.
time (sec)	N/A	1.53	35.665	1.251	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	619	619	575	738	0	0	0	0
normalized size	1	1.	0.93	1.19	0.	0.	0.	0.
time (sec)	N/A	2.199	0.666	0.191	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	738	738	719	970	0	0	0	0
normalized size	1	1.	0.97	1.31	0.	0.	0.	0.
time (sec)	N/A	2.371	0.694	0.161	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	335	335	596	2098	0	0	0	0
normalized size	1	1.	1.78	6.26	0.	0.	0.	0.
time (sec)	N/A	0.749	0.898	0.586	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	59	42	55	111	0	0
normalized size	1	1.	1.16	0.82	1.08	2.18	0.	0.
time (sec)	N/A	0.015	0.018	0.036	0.961	1.606	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	52	37	49	95	0	0
normalized size	1	1.	1.24	0.88	1.17	2.26	0.	0.
time (sec)	N/A	0.011	0.015	0.033	0.958	1.532	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	27	35	76	0	0
normalized size	1	1.	1.	1.23	1.59	3.45	0.	0.
time (sec)	N/A	0.006	0.006	0.034	0.952	1.639	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	33	89	0	0	0
normalized size	1	1.	1.	1.74	4.68	0.	0.	0.
time (sec)	N/A	0.02	0.01	0.054	0.96	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	45	32	42	84	92	0
normalized size	1	1.	1.8	1.28	1.68	3.36	3.68	0.
time (sec)	N/A	0.014	0.024	0.038	0.969	1.571	3.663	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	58	37	49	107	160	0
normalized size	1	1.	1.38	0.88	1.17	2.55	3.81	0.
time (sec)	N/A	0.015	0.019	0.035	0.985	1.591	9.807	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	31	35	32	111	121	0
normalized size	1	1.	0.82	0.92	0.84	2.92	3.18	0.
time (sec)	N/A	0.018	0.015	0.034	0.957	1.579	9.513	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	25	30	26	96	39	0
normalized size	1	1.	0.81	0.97	0.84	3.1	1.26	0.
time (sec)	N/A	0.014	0.01	0.036	0.99	1.655	1.607	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	22	74	87	0
normalized size	1	1.	1.	0.75	1.1	3.7	4.35	0.
time (sec)	N/A	0.009	0.007	0.036	0.967	1.645	0.781	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	29	24	99	126	0
normalized size	1	1.	1.	1.21	1.	4.12	5.25	0.
time (sec)	N/A	0.01	0.016	0.04	0.986	1.64	2.157	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	85	140	0	0	0
normalized size	1	1.	0.93	3.04	5.	0.	0.	0.
time (sec)	N/A	0.022	0.013	0.146	0.991	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	30	20	65	15	0
normalized size	1	1.	1.	1.58	1.05	3.42	0.79	0.
time (sec)	N/A	0.006	0.002	0.056	0.977	1.859	0.254	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	97	61	198	421	0	0
normalized size	1	1.	2.55	1.61	5.21	11.08	0.	0.
time (sec)	N/A	0.023	0.045	0.09	1.269	2.004	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	66	70	84	108	56	0
normalized size	1	1.	1.69	1.79	2.15	2.77	1.44	0.
time (sec)	N/A	0.022	0.032	0.036	0.966	1.924	1.738	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	42	95	109	211	97	0
normalized size	1	1.	0.78	1.76	2.02	3.91	1.8	0.
time (sec)	N/A	0.048	0.035	0.034	0.956	1.872	3.133	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	35	35	286	59	151	0	0	0
normalized size	1	1.	8.17	1.69	4.31	0.	0.	0.
time (sec)	N/A	0.025	0.025	0.042	0.981	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	43	54	72	171	136	0
normalized size	1	1.	0.9	1.12	1.5	3.56	2.83	0.
time (sec)	N/A	0.045	0.026	0.04	0.976	1.912	4.826	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	25	25	117	34	78	0	0	0
normalized size	1	1.	4.68	1.36	3.12	0.	0.	0.
time (sec)	N/A	0.023	0.013	0.029	0.979	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	35	35	312	59	178	0	0	0
normalized size	1	1.	8.91	1.69	5.09	0.	0.	0.
time (sec)	N/A	0.03	0.023	0.044	0.99	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	270	786	450	856	0	0
normalized size	1	1.	1.61	4.68	2.68	5.1	0.	0.
time (sec)	N/A	0.342	0.281	0.046	0.992	2.114	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	174	477	279	548	369	0
normalized size	1	1.	1.45	3.98	2.32	4.57	3.08	0.
time (sec)	N/A	0.204	0.162	0.04	0.971	1.965	18.117	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	138	184	147	319	173	0
normalized size	1	1.	1.42	1.9	1.52	3.29	1.78	0.
time (sec)	N/A	0.171	0.043	0.04	0.961	1.892	2.626	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	48	42	49	157	46	0
normalized size	1	1.	1.2	1.05	1.22	3.92	1.15	0.
time (sec)	N/A	0.025	0.015	0.033	0.971	1.913	0.761	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	206	202	0	0	0	0
normalized size	1	1.	1.58	1.55	0.	0.	0.	0.
time (sec)	N/A	0.142	0.113	0.153	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	125	141	163	583	0	0
normalized size	1	1.	1.09	1.23	1.42	5.07	0.	0.
time (sec)	N/A	0.169	0.201	0.043	0.997	3.111	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	174	236	393	1766	0	0
normalized size	1	1.	1.04	1.41	2.35	10.57	0.	0.
time (sec)	N/A	0.234	0.346	0.052	1.045	9.186	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	374	374	1054	2694	1068	0	0	0
normalized size	1	1.	2.82	7.2	2.86	0.	0.	0.
time (sec)	N/A	0.637	7.236	0.069	1.972	0.	0.	0.



Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	221	220	295	857	540	0	0	0
normalized size	1	1.	1.33	3.88	2.44	0.	0.	0.
time (sec)	N/A	0.443	0.594	0.062	1.961	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	111	226	0	0	0	0
normalized size	1	1.	1.14	2.33	0.	0.	0.	0.
time (sec)	N/A	0.116	0.148	0.111	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	214	214	1721	1845	0	0	0	0
normalized size	1	1.	8.04	8.62	0.	0.	0.	0.
time (sec)	N/A	0.153	16.684	0.855	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	480	485	470	783	0	0	0	0
normalized size	1	1.01	0.98	1.63	0.	0.	0.	0.
time (sec)	N/A	1.743	8.636	0.158	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	546	546	2594	10477	0	0	0	0
normalized size	1	1.	4.75	19.19	0.	0.	0.	0.
time (sec)	N/A	1.043	10.423	3.685	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	326	325	600	12285	0	0	0	0
normalized size	1	1.	1.84	37.68	0.	0.	0.	0.
time (sec)	N/A	0.721	1.314	1.107	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	132	132	208	485	0	0	0	0
normalized size	1	1.	1.58	3.67	0.	0.	0.	0.
time (sec)	N/A	0.227	0.32	0.14	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	308	308	0	3796	0	0	0	0
normalized size	1	1.	0.	12.32	0.	0.	0.	0.
time (sec)	N/A	0.187	22.435	0.71	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1089	1094	1899	4619	0	0	0	0
normalized size	1	1.	1.74	4.24	0.	0.	0.	0.
time (sec)	N/A	2.782	17.417	0.844	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	162	162	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.246	2.354	1.493	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	2.517	1.368	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	0.343	1.286	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.098	1.043	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	460	460	0	1492	0	0	0	0
normalized size	1	1.	0.	3.24	0.	0.	0.	0.
time (sec)	N/A	0.598	0.283	2.439	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	302	302	0	696	0	0	0	0
normalized size	1	1.	0.	2.3	0.	0.	0.	0.
time (sec)	N/A	0.346	0.51	1.202	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	89	98	119	0	0	0	0
normalized size	1	1.	1.1	1.34	0.	0.	0.	0.
time (sec)	N/A	0.059	0.415	0.667	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.09	0.983	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.752	0.976	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	34	676	0	72	0	0
normalized size	1	1.	0.92	18.27	0.	1.95	0.	0.
time (sec)	N/A	0.027	0.059	0.164	0.	1.64	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	59	26	31	19	0
normalized size	1	1.	0.87	2.57	1.13	1.35	0.83	0.
time (sec)	N/A	0.009	0.017	0.076	1.177	1.632	0.557	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	48	26	31	19	0
normalized size	1	1.	0.87	2.09	1.13	1.35	0.83	0.
time (sec)	N/A	0.007	0.016	0.075	1.172	1.652	0.292	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	32	22	23	19	0
normalized size	1	1.	1.12	2.	1.38	1.44	1.19	0.
time (sec)	N/A	0.003	0.008	0.06	1.152	1.574	0.195	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	354	46	22	0	0
normalized size	1	1.	0.9	16.86	2.19	1.05	0.	0.
time (sec)	N/A	0.044	0.016	0.189	0.978	1.621	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	18	20	23	27	14	0
normalized size	1	1.	1.06	1.18	1.35	1.59	0.82	0.
time (sec)	N/A	0.009	0.016	0.069	1.175	1.723	0.314	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	18	20	26	30	19	0
normalized size	1	1.	0.78	0.87	1.13	1.3	0.83	0.
time (sec)	N/A	0.01	0.014	0.074	1.171	1.594	0.773	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	26	32	20	0
normalized size	1	1.	0.87	0.87	1.13	1.39	0.87	0.
time (sec)	N/A	0.009	0.015	0.075	1.179	1.668	1.277	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	62	9175	0	223	0	0
normalized size	1	1.	0.87	129.23	0.	3.14	0.	0.
time (sec)	N/A	0.035	0.118	0.773	0.	1.744	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	37	3418	49	72	78	0
normalized size	1	1.	0.88	81.38	1.17	1.71	1.86	0.
time (sec)	N/A	0.026	0.034	0.372	1.359	1.571	3.678	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	37	3418	49	72	60	0
normalized size	1	1.	0.88	81.38	1.17	1.71	1.43	0.
time (sec)	N/A	0.025	0.056	0.372	1.37	1.632	1.886	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	74	3418	49	70	37	0
normalized size	1	1.	2.18	100.53	1.44	2.06	1.09	0.
time (sec)	N/A	0.025	0.082	0.368	1.368	1.525	0.586	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	45	62	20	0
normalized size	1	1.	1.	0.94	2.81	3.88	1.25	0.
time (sec)	N/A	0.005	0.006	0.098	1.373	1.628	0.414	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	53	3774	51	69	0	0
normalized size	1	1.	1.08	77.02	1.04	1.41	0.	0.
time (sec)	N/A	0.026	0.043	0.284	2.473	1.654	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	37	1095	73	69	0	0
normalized size	1	1.	0.95	28.08	1.87	1.77	0.	0.
time (sec)	N/A	0.026	0.049	0.21	1.196	1.675	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	42	3213	46	73	32	0
normalized size	1	1.	1.17	89.25	1.28	2.03	0.89	0.
time (sec)	N/A	0.023	0.034	0.375	1.402	1.609	0.753	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	34	3217	49	68	37	0
normalized size	1	1.	1.1	103.77	1.58	2.19	1.19	0.
time (sec)	N/A	0.014	0.047	0.357	1.4	1.581	1.182	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	37	3217	49	72	39	0
normalized size	1	1.	0.58	50.27	0.77	1.12	0.61	0.
time (sec)	N/A	0.035	0.031	0.362	1.407	1.611	1.925	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	97	63382	0	468	0	0
normalized size	1	1.	0.88	576.2	0.	4.25	0.	0.
time (sec)	N/A	0.059	0.114	6.384	0.	1.717	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	54	18111	73	119	56	0
normalized size	1	1.	0.89	296.9	1.2	1.95	0.92	0.
time (sec)	N/A	0.041	0.031	1.102	1.545	1.592	6.056	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	54	18111	73	120	58	0
normalized size	1	1.	0.89	296.9	1.2	1.97	0.95	0.
time (sec)	N/A	0.041	0.027	1.096	1.561	1.568	3.559	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	54	18111	73	120	60	0
normalized size	1	1.	1.02	341.72	1.38	2.26	1.13	0.
time (sec)	N/A	0.031	0.022	1.081	1.546	1.557	3.39	0.



Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	99	18111	73	117	41	0
normalized size	1	1.	2.91	532.68	2.15	3.44	1.21	0.
time (sec)	N/A	0.015	0.074	1.083	1.559	1.502	1.732	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	69	109	20	0
normalized size	1	1.	1.	0.94	4.31	6.81	1.25	0.
time (sec)	N/A	0.005	0.006	0.106	1.556	1.596	0.804	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	104	21848	101	119	0	0
normalized size	1	1.	1.35	283.74	1.31	1.55	0.	0.
time (sec)	N/A	0.096	0.058	0.847	2.51	1.7	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	62	7683	166	115	0	0
normalized size	1	1.	0.91	112.99	2.44	1.69	0.	0.
time (sec)	N/A	0.044	0.039	0.415	2.082	1.725	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	66	7366	97	117	0	0
normalized size	1	1.	1.1	122.77	1.62	1.95	0.	0.
time (sec)	N/A	0.041	0.036	0.426	1.403	1.619	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	60	17237	70	120	51	0
normalized size	1	1.	1.09	313.4	1.27	2.18	0.93	0.
time (sec)	N/A	0.038	0.024	1.385	1.597	1.659	1.212	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	50	17235	72	112	56	0
normalized size	1	1.	1.61	555.97	2.32	3.61	1.81	0.
time (sec)	N/A	0.014	0.022	1.347	1.597	1.602	2.145	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	54	17234	73	116	60	0
normalized size	1	1.	0.84	269.28	1.14	1.81	0.94	0.
time (sec)	N/A	0.034	0.034	1.273	1.599	1.698	3.612	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	51	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.084	0.346	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	79	130774	116	293	0	0
normalized size	1	1.	0.98	1614.49	1.43	3.62	0.	0.
time (sec)	N/A	0.059	0.048	4.901	1.807	1.552	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	55	28786	69	225	0	0
normalized size	1	1.	0.98	514.04	1.23	4.02	0.	0.
time (sec)	N/A	0.035	0.038	1.134	1.824	1.66	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	4303	41	185	0	0
normalized size	1	1.	1.	138.81	1.32	5.97	0.	0.
time (sec)	N/A	0.015	0.022	0.314	1.83	1.593	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	22	63	17	0
normalized size	1	1.	1.	1.08	1.83	5.25	1.42	0.
time (sec)	N/A	0.004	0.05	0.076	1.457	1.63	8.224	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	29	972	50	205	0	0
normalized size	1	1.	0.66	22.09	1.14	4.66	0.	0.
time (sec)	N/A	0.028	0.018	11.445	1.812	1.696	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	45	0	88	315	0	0
normalized size	1	1.	0.69	0.	1.35	4.85	0.	0.
time (sec)	N/A	0.038	0.024	180.	1.793	1.727	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	66	0	146	448	0	0
normalized size	1	1.	0.72	0.	1.59	4.87	0.	0.
time (sec)	N/A	0.063	0.027	180.	1.796	1.713	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	51	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.521	1.83	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	106	131085	240	729	0	0
normalized size	1	1.	1.08	1337.6	2.45	7.44	0.	0.
time (sec)	N/A	0.081	0.097	6.12	2.487	1.666	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	83	29109	167	544	0	0
normalized size	1	1.	1.11	388.12	2.23	7.25	0.	0.
time (sec)	N/A	0.054	0.062	1.408	2.438	1.761	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	56	4626	108	416	0	0
normalized size	1	1.	1.12	92.52	2.16	8.32	0.	0.
time (sec)	N/A	0.031	0.068	0.366	2.443	1.635	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	625	63	213	36	0
normalized size	1	1.	0.96	22.32	2.25	7.61	1.29	0.
time (sec)	N/A	0.013	0.065	0.187	2.44	1.681	16.298	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	24	77	20	0
normalized size	1	1.	1.	1.07	1.71	5.5	1.43	0.
time (sec)	N/A	0.005	0.006	0.078	1.48	1.614	16.134	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	53	0	104	691	0	0
normalized size	1	1.	0.76	0.	1.49	9.87	0.	0.
time (sec)	N/A	0.048	0.071	180.	2.453	1.739	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	70	0	182	1065	0	0
normalized size	1	1.	0.69	0.	1.78	10.44	0.	0.
time (sec)	N/A	0.063	0.062	180.	2.451	1.815	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	92	0	258	1439	0	0
normalized size	1	1.	0.64	0.	1.8	10.06	0.	0.
time (sec)	N/A	0.09	0.046	180.	2.461	1.742	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	51	0	0	0	0	0
normalized size	1	1.	0.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.507	1.891	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	114	29456	266	1100	0	0
normalized size	1	1.	1.24	320.17	2.89	11.96	0.	0.
time (sec)	N/A	0.071	0.042	1.396	3.544	1.794	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	86	4977	194	936	0	0
normalized size	1	1.	1.21	70.1	2.73	13.18	0.	0.
time (sec)	N/A	0.049	0.045	0.379	3.545	1.813	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	49	952	127	548	54	0
normalized size	1	1.	1.04	20.26	2.7	11.66	1.15	0.
time (sec)	N/A	0.029	0.031	0.195	3.57	1.782	24.514	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	27	634	82	262	42	0
normalized size	1	1.	0.79	18.65	2.41	7.71	1.24	0.
time (sec)	N/A	0.014	0.049	0.171	3.531	1.648	24.167	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	41	227	24	0
normalized size	1	1.	1.	0.94	2.56	14.19	1.5	0.
time (sec)	N/A	0.005	0.006	0.074	1.471	1.61	16.46	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	74	0	234	1841	0	0
normalized size	1	1.	0.76	0.	2.41	18.98	0.	0.
time (sec)	N/A	0.066	0.106	180.	3.551	1.935	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	93	0	328	2475	0	0
normalized size	1	1.	0.71	0.	2.5	18.89	0.	0.
time (sec)	N/A	0.087	0.05	180.	3.566	2.12	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	C	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	107	0	448	3069	0	0
normalized size	1	1.	0.63	0.	2.64	18.05	0.	0.
time (sec)	N/A	0.127	0.043	180.	3.54	2.24	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	71	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.132	2.566	0.	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	146	2984280	513	1288	0	0
normalized size	1	1.	0.88	18086.6	3.11	7.81	0.	0.
time (sec)	N/A	0.133	0.103	87.445	1.819	1.721	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	106	953037	344	917	0	0
normalized size	1	1.	0.88	7876.34	2.84	7.58	0.	0.
time (sec)	N/A	0.08	0.077	36.951	1.799	1.769	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	71	252344	223	649	0	0
normalized size	1	1.	0.87	3077.37	2.72	7.91	0.	0.
time (sec)	N/A	0.046	0.065	18.674	1.791	1.728	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	41	71611	138	489	0	0
normalized size	1	1.	0.85	1491.9	2.88	10.19	0.	0.
time (sec)	N/A	0.02	0.043	15.611	1.792	1.604	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	88	389	51	0
normalized size	1	1.	1.	1.05	4.4	19.45	2.55	0.
time (sec)	N/A	0.007	0.014	0.087	1.739	1.703	1.047	0.



Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	60	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.081	0.855	0.	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	67	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	0.04	1.964	0.	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	67	0	0	0	0	0
normalized size	1	1.	0.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	0.041	2.029	0.	0.	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	34	676	0	72	0	0
normalized size	1	1.	0.92	18.27	0.	1.95	0.	0.
time (sec)	N/A	0.01	0.033	0.159	0.	1.639	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	59	18	31	39	18
normalized size	1	1.	0.87	2.57	0.78	1.35	1.7	0.78
time (sec)	N/A	0.014	0.021	0.092	0.946	1.344	91.849	1.173

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	48	18	31	60	18
normalized size	1	1.	0.87	2.09	0.78	1.35	2.61	0.78
time (sec)	N/A	0.008	0.015	0.075	0.934	1.268	37.983	1.139

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	32	14	23	36	14
normalized size	1	1.	1.12	2.	0.88	1.44	2.25	0.88
time (sec)	N/A	0.003	0.006	0.064	0.934	1.284	3.99	1.124

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	27	11	22	0	12
normalized size	1	1.	0.9	1.29	0.52	1.05	0.	0.57
time (sec)	N/A	0.031	0.017	0.073	0.945	1.49	0.	1.137

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	18	20	15	27	34	16
normalized size	1	1.	1.06	1.18	0.88	1.59	2.	0.94
time (sec)	N/A	0.009	0.016	0.074	0.942	1.618	12.187	1.162

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	18	20	15	30	39	15
normalized size	1	1.	0.78	0.87	0.65	1.3	1.7	0.65
time (sec)	N/A	0.009	0.014	0.076	0.944	1.471	77.289	1.14

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	47	21	45	213	0	0
normalized size	1	1.	1.74	0.78	1.67	7.89	0.	0.
time (sec)	N/A	0.034	0.02	0.052	1.153	1.667	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	81	449	76	325	0	0
normalized size	1	1.	1.59	8.8	1.49	6.37	0.	0.
time (sec)	N/A	0.066	0.014	0.154	1.184	1.618	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	109	471	105	435	0	0
normalized size	1	1.	1.42	6.12	1.36	5.65	0.	0.
time (sec)	N/A	0.09	0.029	0.141	1.178	1.771	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	345	5294	379	2587	0	0
normalized size	1	1.	1.12	17.24	1.23	8.43	0.	0.
time (sec)	N/A	0.465	0.452	5.085	2.289	2.064	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	259	4990	290	2114	0	0
normalized size	1	1.	1.12	21.6	1.26	9.15	0.	0.
time (sec)	N/A	0.379	0.163	4.741	2.157	1.894	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	131	306	192	1601	0	0
normalized size	1	1.	0.87	2.04	1.28	10.67	0.	0.
time (sec)	N/A	0.232	1.246	0.164	2.111	1.851	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	5.265	0.331	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	144	1726	201	1377	0	0
normalized size	1	1.	0.93	11.14	1.3	8.88	0.	0.
time (sec)	N/A	0.307	0.191	34.221	3.355	1.886	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	118	1667	169	1160	0	0
normalized size	1	1.	0.92	13.02	1.32	9.06	0.	0.
time (sec)	N/A	0.269	0.114	37.152	3.467	1.765	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	91	1584	136	953	0	0
normalized size	1	1.	0.9	15.68	1.35	9.44	0.	0.
time (sec)	N/A	0.231	0.096	10.194	3.486	1.743	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	69	69	201	247	97	707	0	0
normalized size	1	1.	2.91	3.58	1.41	10.25	0.	0.
time (sec)	N/A	0.14	0.858	0.156	3.449	2.139	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	2.972	0.364	0.	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	144	1802	197	1280	0	0
normalized size	1	1.	0.86	10.73	1.17	7.62	0.	0.
time (sec)	N/A	0.303	0.185	39.194	3.472	2.099	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	119	1745	166	1081	0	0
normalized size	1	1.	0.86	12.55	1.19	7.78	0.	0.
time (sec)	N/A	0.268	0.103	33.599	3.506	2.138	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	93	1664	135	894	0	0
normalized size	1	1.	0.85	15.13	1.23	8.13	0.	0.
time (sec)	N/A	0.238	0.091	16.123	3.501	2.064	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	76	76	200	271	99	667	0	0
normalized size	1	1.	2.63	3.57	1.3	8.78	0.	0.
time (sec)	N/A	0.15	0.89	0.158	3.901	1.96	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	3.114	0.372	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	303	303	353	5222	374	2560	0	0
normalized size	1	1.	1.17	17.23	1.23	8.45	0.	0.
time (sec)	N/A	0.465	0.397	5.121	2.281	2.327	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	229	229	267	4918	288	2093	0	0
normalized size	1	1.	1.17	21.48	1.26	9.14	0.	0.
time (sec)	N/A	0.382	0.158	4.785	2.226	2.238	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	131	306	192	1585	0	0
normalized size	1	1.	0.87	2.04	1.28	10.57	0.	0.
time (sec)	N/A	0.235	1.126	0.171	2.27	2.167	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.147	5.48	0.337	0.	0.	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	141	1698	197	1280	0	0
normalized size	1	1.	0.93	11.17	1.3	8.42	0.	0.
time (sec)	N/A	0.308	0.174	32.161	3.429	2.26	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	116	1641	166	1081	0	0
normalized size	1	1.	0.92	13.02	1.32	8.58	0.	0.
time (sec)	N/A	0.269	0.102	35.69	3.52	2.042	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	90	1560	135	894	0	0
normalized size	1	1.	0.9	15.6	1.35	8.94	0.	0.
time (sec)	N/A	0.24	0.093	15.664	3.714	1.971	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	69	69	197	247	97	667	0	0
normalized size	1	1.	2.86	3.58	1.41	9.67	0.	0.
time (sec)	N/A	0.145	0.787	0.161	3.597	2.053	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	3.149	0.372	0.	0.	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	147	1830	201	1377	0	0
normalized size	1	1.	0.89	11.09	1.22	8.35	0.	0.
time (sec)	N/A	0.31	0.188	37.705	3.718	2.15	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	121	1771	169	1160	0	0
normalized size	1	1.	0.88	12.93	1.23	8.47	0.	0.
time (sec)	N/A	0.27	0.111	37.721	3.564	2.058	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	94	1688	136	953	0	0
normalized size	1	1.	0.86	15.49	1.25	8.74	0.	0.
time (sec)	N/A	0.238	0.113	19.147	3.538	1.841	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	76	76	208	271	99	707	0	0
normalized size	1	1.	2.74	3.57	1.3	9.3	0.	0.
time (sec)	N/A	0.144	0.75	0.175	3.528	1.747	0.	0.



Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	3.299	0.392	0.	0.	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	302	654	7429	0	4514	0	0
normalized size	1	1.	2.17	24.6	0.	14.95	0.	0.
time (sec)	N/A	0.242	0.314	8.453	0.	2.596	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	409	5543	0	3357	0	0
normalized size	1	1.	1.75	23.69	0.	14.35	0.	0.
time (sec)	N/A	0.172	0.188	13.508	0.	2.209	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	162	162	263	2543	0	2313	0	0
normalized size	1	1.	1.62	15.7	0.	14.28	0.	0.
time (sec)	N/A	0.111	0.132	10.639	0.	2.28	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	78	178	246	1493	0	0
normalized size	1	1.	0.99	2.25	3.11	18.9	0.	0.
time (sec)	N/A	0.048	0.023	0.158	1.815	1.896	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	5.064	1.157	0.	0.	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	395	346	6820	0	5871	0	0
normalized size	1	1.	0.88	17.27	0.	14.86	0.	0.
time (sec)	N/A	0.5	0.315	6.798	0.	2.656	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	257	6482	0	4545	0	0
normalized size	1	1.	0.87	21.97	0.	15.41	0.	0.
time (sec)	N/A	0.397	0.14	7.796	0.	2.491	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	4654	612	502	3210	0	0
normalized size	1	1.	23.99	3.15	2.59	16.55	0.	0.
time (sec)	N/A	0.251	13.224	0.131	2.13	2.434	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.146	0.355	0.382	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	155	2339	460	980	0	0
normalized size	1	1.	0.91	13.76	2.71	5.76	0.	0.
time (sec)	N/A	0.304	0.188	30.058	1.229	1.837	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	119	2249	333	811	0	0
normalized size	1	1.	0.89	16.91	2.5	6.1	0.	0.
time (sec)	N/A	0.253	0.106	17.08	1.144	1.731	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	766	292	358	603	0	0
normalized size	1	1.	8.24	3.14	3.85	6.48	0.	0.
time (sec)	N/A	0.155	2.858	0.175	1.554	1.71	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	0.727	0.444	0.	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	156	2449	459	992	0	0
normalized size	1	1.	0.91	14.32	2.68	5.8	0.	0.
time (sec)	N/A	0.298	0.177	29.632	1.22	1.89	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	120	2351	332	821	0	0
normalized size	1	1.	0.9	17.54	2.48	6.13	0.	0.
time (sec)	N/A	0.245	0.109	13.712	1.151	1.778	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	94	94	723	297	355	610	0	0
normalized size	1	1.	7.69	3.16	3.78	6.49	0.	0.
time (sec)	N/A	0.156	2.825	0.178	1.596	1.667	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.085	0.72	0.44	0.	0.	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	302	654	7429	0	3677	0	0
normalized size	1	1.	2.17	24.6	0.	12.18	0.	0.
time (sec)	N/A	0.235	0.287	6.052	0.	2.684	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	409	5543	0	2646	0	0
normalized size	1	1.	1.75	23.69	0.	11.31	0.	0.
time (sec)	N/A	0.171	0.175	13.674	0.	2.438	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	162	162	263	2543	0	1724	0	0
normalized size	1	1.	1.62	15.7	0.	10.64	0.	0.
time (sec)	N/A	0.11	0.121	10.737	0.	2.138	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	78	265	248	1029	0	0
normalized size	1	1.	0.99	3.35	3.14	13.03	0.	0.
time (sec)	N/A	0.048	0.032	0.161	1.69	1.832	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.107	1.173	0.	0.	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	339	6662	0	4748	0	0
normalized size	1	1.	0.87	17.04	0.	12.14	0.	0.
time (sec)	N/A	0.503	0.306	6.855	0.	2.993	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	253	6312	0	3838	0	0
normalized size	1	1.	0.86	21.54	0.	13.1	0.	0.
time (sec)	N/A	0.403	0.125	8.394	0.	2.834	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	4463	629	529	2919	0	0
normalized size	1	1.	23.01	3.24	2.73	15.05	0.	0.
time (sec)	N/A	0.249	12.993	0.139	1.891	2.785	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.093	0.348	0.391	0.	0.	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	155	2449	462	505	0	0
normalized size	1	1.	0.92	14.58	2.75	3.01	0.	0.
time (sec)	N/A	0.306	0.177	29.797	1.174	1.773	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	119	2351	335	429	0	0
normalized size	1	1.	0.9	17.81	2.54	3.25	0.	0.
time (sec)	N/A	0.257	0.109	17.385	1.139	1.691	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	709	299	386	339	0	0
normalized size	1	1.	7.62	3.22	4.15	3.65	0.	0.
time (sec)	N/A	0.159	3.364	0.181	1.545	1.774	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	0.673	0.444	0.	0.	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	155	2339	463	504	0	0
normalized size	1	1.	0.92	13.84	2.74	2.98	0.	0.
time (sec)	N/A	0.295	0.197	29.139	1.378	1.689	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	119	2249	336	427	0	0
normalized size	1	1.	0.89	16.91	2.53	3.21	0.	0.
time (sec)	N/A	0.248	0.104	20.552	1.281	1.772	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	94	94	605	304	389	335	0	0
normalized size	1	1.	6.44	3.23	4.14	3.56	0.	0.
time (sec)	N/A	0.15	2.638	0.193	1.758	1.745	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	0.75	0.457	0.	0.	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	131	920	0	979	0	0
normalized size	1	1.	0.82	5.75	0.	6.12	0.	0.
time (sec)	N/A	0.574	0.315	0.47	0.	1.928	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	236	4034	447	579	362	0
normalized size	1	1.	0.79	13.58	1.51	1.95	1.22	0.
time (sec)	N/A	0.386	0.171	10.293	1.113	1.696	113.193	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	192	3320	366	440	286	0
normalized size	1	1.	0.85	14.76	1.63	1.96	1.27	0.
time (sec)	N/A	0.256	0.145	6.22	1.076	1.681	33.845	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	129	2616	231	297	209	0
normalized size	1	1.	0.92	18.69	1.65	2.12	1.49	0.
time (sec)	N/A	0.119	0.106	0.602	1.028	1.625	12.663	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	C	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	381	381	0	864	225	0	0	0
normalized size	1	1.	0.	2.27	0.59	0.	0.	0.
time (sec)	N/A	0.439	0.213	1.104	1.779	0.	0.	0.



Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	247	247	161	0	0	0	0	0
normalized size	1	1.	0.65	0.	0.	0.	0.	0.
time (sec)	N/A	0.488	0.15	2.582	0.	0.	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	339	339	307	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.717	0.145	6.231	0.	0.	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	236	4194	428	587	345	0
normalized size	1	1.	0.75	13.31	1.36	1.86	1.1	0.
time (sec)	N/A	0.749	0.149	1.72	1.11	1.676	43.404	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	183	3515	340	452	265	0
normalized size	1	1.	0.74	14.23	1.38	1.83	1.07	0.
time (sec)	N/A	0.621	0.123	1.083	1.193	1.728	15.704	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	144	2210	240	304	155	0
normalized size	1	1.	1.38	21.25	2.31	2.92	1.49	0.
time (sec)	N/A	0.198	0.017	0.566	1.141	1.699	9.844	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	94	332	0	0	0	0	0
normalized size	1	0.9	3.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.269	0.204	1.642	0.	0.	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	191	457	0	0	0	0	0
normalized size	1	0.97	2.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.463	0.383	4.322	0.	0.	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	256	250	0	0	0	0	0	0
normalized size	1	0.98	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.67	0.28	4.318	0.	0.	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	512	512	677	8491	0	0	0	0
normalized size	1	1.	1.32	16.58	0.	0.	0.	0.
time (sec)	N/A	0.767	4.252	1.996	0.	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	546	546	1287	0	0	0	0	0
normalized size	1	1.	2.36	0.	0.	0.	0.	0.
time (sec)	N/A	1.387	3.212	1.994	0.	0.	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	100	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.252	0.273	0.74	0.	0.	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	560	560	1236	0	0	0	0	0
normalized size	1	1.	2.21	0.	0.	0.	0.	0.
time (sec)	N/A	1.261	3.562	0.832	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	712	712	1318	937	0	0	0	0
normalized size	1	1.	1.85	1.32	0.	0.	0.	0.
time (sec)	N/A	1.113	5.571	3.198	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	51	31	78	262	0	0
normalized size	1	1.	2.04	1.24	3.12	10.48	0.	0.
time (sec)	N/A	0.012	0.034	0.052	1.119	1.776	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	71	62	80	374	0	0
normalized size	1	1.	1.39	1.22	1.57	7.33	0.	0.
time (sec)	N/A	0.047	0.022	0.049	1.123	1.661	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	93	79	103	462	0	0
normalized size	1	1.	1.33	1.13	1.47	6.6	0.	0.
time (sec)	N/A	0.073	0.019	0.046	1.109	1.778	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	68	67	144	417	0	0
normalized size	1	1.	1.66	1.63	3.51	10.17	0.	0.
time (sec)	N/A	0.016	0.073	0.072	1.072	1.679	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	113	153	146	583	0	0
normalized size	1	1.	1.36	1.84	1.76	7.02	0.	0.
time (sec)	N/A	0.064	0.031	0.07	1.135	1.612	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	149	185	192	730	0	0
normalized size	1	1.	1.25	1.55	1.61	6.13	0.	0.
time (sec)	N/A	0.099	0.03	0.076	1.125	1.755	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	108	164	284	899	0	0
normalized size	1	1.	0.64	0.98	1.69	5.35	0.	0.
time (sec)	N/A	0.132	0.075	0.199	1.089	1.601	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	177	590	262	1193	0	0
normalized size	1	1.	0.82	2.73	1.21	5.52	0.	0.
time (sec)	N/A	2.714	0.104	0.174	1.284	1.62	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	235	666	338	1453	0	0
normalized size	1	1.	0.87	2.48	1.26	5.4	0.	0.
time (sec)	N/A	2.586	0.07	0.175	1.273	1.914	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	19	28	57	14	0
normalized size	1	1.	1.	1.12	1.65	3.35	0.82	0.
time (sec)	N/A	0.044	0.052	0.072	1.127	1.648	1.315	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	39	46	50	149	60	0
normalized size	1	1.	0.89	1.05	1.14	3.39	1.36	0.
time (sec)	N/A	0.052	0.016	0.078	1.047	1.822	9.037	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	42	118	54	359	0	0
normalized size	1	1.	0.89	2.51	1.15	7.64	0.	0.
time (sec)	N/A	0.055	0.036	0.119	1.065	1.859	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	153	794	248	624	0	0
normalized size	1	1.	1.43	7.42	2.32	5.83	0.	0.
time (sec)	N/A	0.155	0.164	0.329	1.614	1.744	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	49	49	60	824	86	236	0	0
normalized size	1	1.	1.22	16.82	1.76	4.82	0.	0.
time (sec)	N/A	0.079	0.086	0.281	1.118	1.654	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	45	45	46	351	58	55	63	0
normalized size	1	1.	1.02	7.8	1.29	1.22	1.4	0.
time (sec)	N/A	0.06	0.075	0.25	1.058	1.707	25.793	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	45	45	46	68	57	55	0	47
normalized size	1	1.	1.02	1.51	1.27	1.22	0.	1.04
time (sec)	N/A	0.059	0.08	0.069	1.181	1.512	0.	1.183

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	49	49	59	939	86	238	0	0
normalized size	1	1.	1.2	19.16	1.76	4.86	0.	0.
time (sec)	N/A	0.071	0.083	0.292	1.039	1.756	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	150	920	248	625	0	0
normalized size	1	1.	1.4	8.6	2.32	5.84	0.	0.
time (sec)	N/A	0.148	0.157	0.333	1.748	1.79	0.	0.

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [75] had the largest ratio of [ 1.333 ]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.	8	0.375
2	A	4	3	1.	8	0.375
3	A	4	3	1.	8	0.375
4	A	4	3	1.	8	0.375
5	A	3	3	1.	6	0.5
6	A	2	2	1.	4	0.5
7	A	1	1	1.	8	0.125
8	A	5	5	1.	8	0.625
9	A	3	3	1.	8	0.375
10	A	4	3	1.	8	0.375
11	A	4	3	1.	8	0.375
12	A	15	7	1.	10	0.7
13	A	14	9	1.	10	0.9
14	A	10	7	1.	10	0.7
15	A	9	8	1.	10	0.8

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	5	5	1.	8	0.625
17	A	5	5	1.	6	0.833
18	A	6	5	1.	10	0.5
19	A	4	4	1.	10	0.4
20	A	8	7	1.	10	0.7
21	A	8	7	1.	10	0.7
22	A	13	8	1.	10	0.8
23	A	33	11	1.	10	1.1
24	A	22	11	1.	10	1.1
25	A	18	10	1.	10	1.
26	A	11	9	1.	10	0.9
27	A	8	8	1.	8	1.
28	A	5	6	1.	6	1.
29	A	8	6	1.	10	0.6
30	A	5	6	1.	10	0.6
31	A	7	6	1.	10	0.6
32	A	14	11	1.	10	1.1
33	A	16	8	1.	10	0.8
34	A	1	1	1.	14	0.071
35	A	4	4	1.	14	0.286
36	A	4	4	1.	14	0.286
37	A	5	5	1.	14	0.357
38	A	5	4	1.	12	0.333
39	A	27	13	1.	14	0.929
40	A	25	13	1.	14	0.929
41	A	23	11	1.	14	0.786
42	A	0	0	0.	0	0.
43	A	0	0	0.	0	0.
44	A	5	6	1.	16	0.375
45	A	7	9	1.	16	0.562
46	A	8	9	1.	16	0.562
47	A	8	9	1.	16	0.562

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
48	A	3	3	1.	15	0.2
49	A	2	2	1.	15	0.133
50	A	1	1	1.	15	0.067
51	A	2	2	1.	15	0.133
52	A	3	2	1.	15	0.133
53	A	1	1	1.	14	0.071
54	A	1	1	1.	14	0.071
55	A	4	4	1.	14	0.286
56	A	4	4	1.	13	0.308
57	A	1	1	1.	12	0.083
58	A	3	3	1.	13	0.231
59	A	2	2	1.	12	0.167
60	A	4	3	1.	13	0.231
61	A	3	3	1.	12	0.25
62	A	7	5	1.	10	0.5
63	A	7	5	1.	10	0.5
64	A	7	5	1.	8	0.625
65	A	3	3	1.	6	0.5
66	A	5	5	1.	10	0.5
67	A	7	5	1.	10	0.5
68	A	5	4	1.	10	0.4
69	A	19	15	1.	12	1.25
70	A	15	13	1.	12	1.083
71	A	12	10	1.	10	1.
72	A	6	6	1.	8	0.75
73	A	2	2	1.	12	0.167
74	A	17	15	1.	12	1.25
75	A	21	16	1.	12	1.333
76	A	37	7	0.89	16	0.438
77	A	5	5	1.	14	0.357
78	A	37	10	1.23	16	0.625
79	A	57	11	1.	16	0.688

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	55	16	1.	18	0.889
81	A	65	19	1.	18	1.056
82	A	12	8	1.	19	0.421
83	A	6	4	1.	10	0.4
84	A	5	4	1.	8	0.5
85	A	4	4	1.	6	0.667
86	A	2	2	1.	10	0.2
87	A	4	4	1.	10	0.4
88	A	5	4	1.	10	0.4
89	A	3	2	1.	12	0.167
90	A	3	2	1.	12	0.167
91	A	2	2	1.	12	0.167
92	A	4	4	1.	12	0.333
93	A	2	2	1.	10	0.2
94	A	3	3	1.	4	0.75
95	A	2	2	1.	10	0.2
96	A	4	4	1.	12	0.333
97	A	5	4	1.	14	0.286
98	A	2	2	1.	14	0.143
99	A	6	6	1.	14	0.429
100	A	3	3	1.	12	0.25
101	A	3	3	1.	19	0.158
102	A	7	5	1.	18	0.278
103	A	7	5	1.	18	0.278
104	A	7	5	1.	16	0.312
105	A	4	3	1.	10	0.3
106	A	5	5	1.	18	0.278
107	A	7	5	1.	18	0.278
108	A	5	4	1.	18	0.222
109	A	16	13	1.	20	0.65
110	A	13	10	1.	18	0.556
111	A	6	6	1.	12	0.5

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
112	A	2	2	1.	20	0.1
113	A	21	19	1.01	20	0.95
114	A	21	14	1.	20	0.7
115	A	15	11	1.	18	0.611
116	A	6	7	1.	12	0.583
117	A	2	2	1.	20	0.1
118	A	30	18	1.	20	0.9
119	A	6	4	1.	18	0.222
120	A	0	0	0.	0	0.
121	A	0	0	0.	0	0.
122	A	0	0	0.	0	0.
123	A	9	7	1.	40	0.175
124	A	7	6	1.	40	0.15
125	A	2	3	1.	38	0.079
126	A	0	0	0.	0	0.
127	A	0	0	0.	0	0.
128	A	2	2	1.	11	0.182
129	A	2	2	1.	11	0.182
130	A	2	2	1.	9	0.222
131	A	2	2	1.	7	0.286
132	A	2	2	1.	11	0.182
133	A	2	2	1.	11	0.182
134	A	2	2	1.	11	0.182
135	A	2	2	1.	11	0.182
136	A	3	2	1.	13	0.154
137	A	3	2	1.	13	0.154
138	A	3	2	1.	13	0.154
139	A	3	3	1.	11	0.273
140	A	2	2	1.	9	0.222
141	A	3	3	1.	13	0.231
142	A	3	3	1.	13	0.231
143	A	3	2	1.	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
144	A	1	1	1.	13	0.077
145	A	2	2	1.	13	0.154
146	A	4	2	1.	13	0.154
147	A	4	2	1.	13	0.154
148	A	4	2	1.	13	0.154
149	A	4	3	1.	13	0.231
150	A	3	3	1.	11	0.273
151	A	2	2	1.	9	0.222
152	A	4	3	1.	13	0.231
153	A	4	4	1.	13	0.308
154	A	4	3	1.	13	0.231
155	A	4	2	1.	13	0.154
156	A	1	1	1.	13	0.077
157	A	2	2	1.	13	0.154
158	A	1	1	1.	13	0.077
159	A	5	4	1.	13	0.308
160	A	4	4	1.	13	0.308
161	A	3	3	1.	11	0.273
162	A	2	2	1.	9	0.222
163	A	4	3	1.	13	0.231
164	A	5	4	1.	13	0.308
165	A	6	4	1.	13	0.308
166	A	2	2	1.	13	0.154
167	A	6	5	1.	13	0.385
168	A	5	5	1.	13	0.385
169	A	4	4	1.	13	0.308
170	A	3	3	1.	11	0.273
171	A	2	2	1.	9	0.222
172	A	5	4	1.	13	0.308
173	A	6	5	1.	13	0.385
174	A	7	5	1.	13	0.385
175	A	3	2	1.	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
176	A	6	5	1.	13	0.385
177	A	5	4	1.	13	0.308
178	A	4	3	1.	13	0.231
179	A	3	3	1.	11	0.273
180	A	2	2	1.	9	0.222
181	A	6	4	1.	13	0.308
182	A	7	5	1.	13	0.385
183	A	8	5	1.	13	0.385
184	A	1	1	1.	13	0.077
185	A	6	3	1.	13	0.231
186	A	5	3	1.	13	0.231
187	A	4	3	1.	13	0.231
188	A	3	3	1.	11	0.273
189	A	2	2	1.	9	0.222
190	A	1	1	1.	13	0.077
191	A	2	2	1.	13	0.154
192	A	3	2	1.	13	0.154
193	A	2	2	1.	11	0.182
194	A	2	2	1.	11	0.182
195	A	2	2	1.	9	0.222
196	A	2	2	1.	7	0.286
197	A	2	2	1.	11	0.182
198	A	2	2	1.	11	0.182
199	A	2	2	1.	11	0.182
200	A	6	4	1.	3	1.333
201	A	8	5	1.	5	1.
202	A	10	6	1.	7	0.857
203	A	11	6	1.	15	0.4
204	A	9	5	1.	13	0.385
205	A	7	4	1.	11	0.364
206	A	0	0	0.	0	0.
207	A	8	7	1.	16	0.438

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
208	A	7	7	1.	16	0.438
209	A	6	6	1.	14	0.429
210	A	5	5	1.	12	0.417
211	A	0	0	0.	0	0.
212	A	8	7	1.	19	0.368
213	A	7	7	1.	19	0.368
214	A	6	6	1.	17	0.353
215	A	5	5	1.	15	0.333
216	A	0	0	0.	0	0.
217	A	11	6	1.	15	0.4
218	A	9	5	1.	13	0.385
219	A	7	4	1.	11	0.364
220	A	0	0	0.	0	0.
221	A	8	7	1.	16	0.438
222	A	7	7	1.	16	0.438
223	A	6	6	1.	14	0.429
224	A	5	5	1.	12	0.417
225	A	0	0	0.	0	0.
226	A	8	7	1.	19	0.368
227	A	7	7	1.	19	0.368
228	A	6	6	1.	17	0.353
229	A	5	5	1.	15	0.333
230	A	0	0	0.	0	0.
231	A	12	6	1.	15	0.4
232	A	10	6	1.	15	0.4
233	A	8	5	1.	13	0.385
234	A	6	4	1.	7	0.571
235	A	0	0	0.	0	0.
236	A	11	6	1.	15	0.4
237	A	9	5	1.	13	0.385
238	A	7	4	1.	11	0.364
239	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
240	A	7	7	1.	20	0.35
241	A	6	6	1.	18	0.333
242	A	5	5	1.	16	0.312
243	A	0	0	0.	0	0.
244	A	7	7	1.	21	0.333
245	A	6	6	1.	19	0.316
246	A	5	5	1.	17	0.294
247	A	0	0	0.	0	0.
248	A	12	6	1.	15	0.4
249	A	10	6	1.	15	0.4
250	A	8	5	1.	13	0.385
251	A	6	4	1.	7	0.571
252	A	0	0	0.	0	0.
253	A	11	6	1.	15	0.4
254	A	9	5	1.	13	0.385
255	A	7	4	1.	11	0.364
256	A	0	0	0.	0	0.
257	A	7	7	1.	20	0.35
258	A	6	6	1.	18	0.333
259	A	5	5	1.	16	0.312
260	A	0	0	0.	0	0.
261	A	7	7	1.	21	0.333
262	A	6	6	1.	19	0.316
263	A	5	5	1.	17	0.294
264	A	0	0	0.	0	0.
265	A	11	8	1.	24	0.333
266	A	23	11	1.	27	0.407
267	A	14	11	1.	27	0.407
268	A	7	8	1.	25	0.32
269	A	21	11	1.	27	0.407
270	A	13	13	1.	27	0.482
271	A	17	14	1.	27	0.518

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
272	A	26	15	1.	27	0.556
273	A	21	15	1.	27	0.556
274	A	9	8	1.	24	0.333
275	A	8	8	0.9	27	0.296
276	A	17	16	0.97	27	0.593
277	A	26	18	0.98	27	0.667
278	A	22	17	1.	22	0.773
279	A	38	20	1.	21	0.952
280	A	0	0	0.	0	0.
281	A	38	22	1.	24	0.917
282	A	32	17	1.	24	0.708
283	A	2	2	1.	4	0.5
284	A	7	4	1.	6	0.667
285	A	9	5	1.	8	0.625
286	A	2	2	1.	8	0.25
287	A	7	4	1.	10	0.4
288	A	9	5	1.	12	0.417
289	A	6	6	1.	12	0.5
290	A	25	8	1.	14	0.571
291	A	29	9	1.	16	0.562
292	A	1	1	1.	20	0.05
293	A	4	4	1.	12	0.333
294	A	4	4	1.	14	0.286
295	A	8	7	1.	20	0.35
296	A	5	5	1.	20	0.25
297	A	3	2	1.	20	0.1
298	A	3	2	1.	20	0.1
299	A	5	5	1.	20	0.25
300	A	8	7	1.	20	0.35



# Chapter 3

## Listing of integrals

### 3.1 $\int x^5 \coth^{-1}(ax) dx$

Optimal. Leaf size=51

$$\frac{x^3}{18a^3} + \frac{x}{6a^5} - \frac{\tanh^{-1}(ax)}{6a^6} + \frac{x^5}{30a} + \frac{1}{6}x^6 \coth^{-1}(ax)$$

[Out]  $x/(6*a^5) + x^3/(18*a^3) + x^5/(30*a) + (x^6*ArcCoth[a*x])/6 - ArcTanh[a*x]/(6*a^6)$

---

**Rubi [A]** time = 0.0279481, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {5917, 302, 206}

$$\frac{x^3}{18a^3} + \frac{x}{6a^5} - \frac{\tanh^{-1}(ax)}{6a^6} + \frac{x^5}{30a} + \frac{1}{6}x^6 \coth^{-1}(ax)$$

Antiderivative was successfully verified.

[In] `Int[x^5*ArcCoth[a*x], x]`

[Out]  $x/(6*a^5) + x^3/(18*a^3) + x^5/(30*a) + (x^6*ArcCoth[a*x])/6 - ArcTanh[a*x]/(6*a^6)$

#### Rule 5917

`Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]`  
`:> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c`

\*p)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 302

Int[(x\_)^(m\_)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := Int[PolynomialDivide[x^m, a + b\*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2\*n - 1]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int x^5 \coth^{-1}(ax) dx &= \frac{1}{6}x^6 \coth^{-1}(ax) - \frac{1}{6}a \int \frac{x^6}{1 - a^2x^2} dx \\
 &= \frac{1}{6}x^6 \coth^{-1}(ax) - \frac{1}{6}a \int \left( -\frac{1}{a^6} - \frac{x^2}{a^4} - \frac{x^4}{a^2} + \frac{1}{a^6(1 - a^2x^2)} \right) dx \\
 &= \frac{x}{6a^5} + \frac{x^3}{18a^3} + \frac{x^5}{30a} + \frac{1}{6}x^6 \coth^{-1}(ax) - \frac{\int \frac{1}{1 - a^2x^2} dx}{6a^5} \\
 &= \frac{x}{6a^5} + \frac{x^3}{18a^3} + \frac{x^5}{30a} + \frac{1}{6}x^6 \coth^{-1}(ax) - \frac{\tanh^{-1}(ax)}{6a^6}
 \end{aligned}$$

**Mathematica [A]** time = 0.0089957, size = 67, normalized size = 1.31

$$\frac{x^3}{18a^3} + \frac{x}{6a^5} + \frac{\log(1 - ax)}{12a^6} - \frac{\log(ax + 1)}{12a^6} + \frac{x^5}{30a} + \frac{1}{6}x^6 \coth^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*ArcCoth[a\*x], x]

[Out] x/(6\*a^5) + x^3/(18\*a^3) + x^5/(30\*a) + (x^6\*ArcCoth[a\*x])/6 + Log[1 - a\*x]/(12\*a^6) - Log[1 + a\*x]/(12\*a^6)

**Maple [A]** time = 0.035, size = 55, normalized size = 1.1

$$\frac{x^6 \operatorname{arccoth}(ax)}{6} + \frac{x^5}{30a} + \frac{x^3}{18a^3} + \frac{x}{6a^5} + \frac{\ln(ax-1)}{12a^6} - \frac{\ln(ax+1)}{12a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*arccoth(a*x),x)`

[Out] `1/6*x^6*arccoth(a*x)+1/30*x^5/a+1/18*x^3/a^3+1/6*x/a^5+1/12/a^6*ln(a*x-1)-1/12/a^6*ln(a*x+1)`

**Maxima [A]** time = 0.964772, size = 82, normalized size = 1.61

$$\frac{1}{6} x^6 \operatorname{arccoth}(ax) + \frac{1}{180} a \left( \frac{2(3a^4x^5 + 5a^2x^3 + 15x)}{a^6} - \frac{15 \log(ax+1)}{a^7} + \frac{15 \log(ax-1)}{a^7} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*arccoth(a*x),x, algorithm="maxima")`

[Out] `1/6*x^6*arccoth(a*x) + 1/180*a*(2*(3*a^4*x^5 + 5*a^2*x^3 + 15*x)/a^6 - 15*log(a*x + 1)/a^7 + 15*log(a*x - 1)/a^7)`

**Fricas [A]** time = 1.54015, size = 120, normalized size = 2.35

$$\frac{6a^5x^5 + 10a^3x^3 + 30ax + 15(a^6x^6 - 1) \log\left(\frac{ax+1}{ax-1}\right)}{180a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*arccoth(a*x),x, algorithm="fricas")`

[Out] `1/180*(6*a^5*x^5 + 10*a^3*x^3 + 30*a*x + 15*(a^6*x^6 - 1)*log((a*x + 1)/(a*x - 1)))/a^6`

**Sympy [A]** time = 2.07, size = 49, normalized size = 0.96

$$\begin{cases} \frac{x^6 \operatorname{acoth}(ax)}{6} + \frac{x^5}{30a} + \frac{x^3}{18a^3} + \frac{x}{6a^5} - \frac{\operatorname{acoth}(ax)}{6a^6} & \text{for } a \neq 0 \\ \frac{i\pi x^6}{12} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*acoth(a\*x),x)

[Out] Piecewise((x\*\*6\*acoth(a\*x)/6 + x\*\*5/(30\*a) + x\*\*3/(18\*a\*\*3) + x/(6\*a\*\*5) - acoth(a\*x)/(6\*a\*\*6), Ne(a, 0)), (I\*pi\*x\*\*6/12, True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^5 \operatorname{arccoth}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*arccoth(a\*x),x, algorithm="giac")

[Out] integrate(x^5\*arccoth(a\*x), x)

### 3.2 $\int x^4 \coth^{-1}(ax) dx$

**Optimal.** Leaf size=50

$$\frac{x^2}{10a^3} + \frac{\log(1 - a^2x^2)}{10a^5} + \frac{x^4}{20a} + \frac{1}{5}x^5 \coth^{-1}(ax)$$

[Out]  $x^2/(10*a^3) + x^4/(20*a) + (x^5*ArcCoth[a*x])/5 + \text{Log}[1 - a^2*x^2]/(10*a^5)$   
)

**Rubi [A]** time = 0.0371068, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {5917, 266, 43}

$$\frac{x^2}{10a^3} + \frac{\log(1 - a^2x^2)}{10a^5} + \frac{x^4}{20a} + \frac{1}{5}x^5 \coth^{-1}(ax)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^4*ArcCoth[a*x], x]$

[Out]  $x^2/(10*a^3) + x^4/(20*a) + (x^5*ArcCoth[a*x])/5 + \text{Log}[1 - a^2*x^2]/(10*a^5)$   
)

#### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
  := Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

### Rubi steps

$$\begin{aligned}
 \int x^4 \coth^{-1}(ax) dx &= \frac{1}{5}x^5 \coth^{-1}(ax) - \frac{1}{5}a \int \frac{x^5}{1 - a^2x^2} dx \\
 &= \frac{1}{5}x^5 \coth^{-1}(ax) - \frac{1}{10}a \operatorname{Subst}\left(\int \frac{x^2}{1 - a^2x} dx, x, x^2\right) \\
 &= \frac{1}{5}x^5 \coth^{-1}(ax) - \frac{1}{10}a \operatorname{Subst}\left(\int \left(-\frac{1}{a^4} - \frac{x}{a^2} - \frac{1}{a^4(-1 + a^2x)}\right) dx, x, x^2\right) \\
 &= \frac{x^2}{10a^3} + \frac{x^4}{20a} + \frac{1}{5}x^5 \coth^{-1}(ax) + \frac{\log(1 - a^2x^2)}{10a^5}
 \end{aligned}$$

**Mathematica [A]** time = 0.0081977, size = 50, normalized size = 1.

$$\frac{x^2}{10a^3} + \frac{\log(1 - a^2x^2)}{10a^5} + \frac{x^4}{20a} + \frac{1}{5}x^5 \coth^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*ArcCoth[a\*x], x]

[Out] x^2/(10\*a^3) + x^4/(20\*a) + (x^5\*ArcCoth[a\*x])/5 + Log[1 - a^2\*x^2]/(10\*a^5)

**Maple [A]** time = 0.035, size = 49, normalized size = 1.

$$\frac{x^5 \operatorname{arccoth}(ax)}{5} + \frac{x^4}{20a} + \frac{x^2}{10a^3} + \frac{\ln(ax - 1)}{10a^5} + \frac{\ln(ax + 1)}{10a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*arccoth(a\*x), x)

[Out] 1/5\*x^5\*arccoth(a\*x)+1/20\*x^4/a+1/10\*x^2/a^3+1/10/a^5\*ln(a\*x-1)+1/10/a^5\*ln(a\*x+1)

---

**Maxima [A]** time = 0.958241, size = 62, normalized size = 1.24

$$\frac{1}{5} x^5 \operatorname{arccoth}(ax) + \frac{1}{20} a \left( \frac{a^2 x^4 + 2x^2}{a^4} + \frac{2 \log(a^2 x^2 - 1)}{a^6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arccoth(a\*x),x, algorithm="maxima")

[Out] 1/5\*x^5\*arccoth(a\*x) + 1/20\*a\*((a^2\*x^4 + 2\*x^2)/a^4 + 2\*log(a^2\*x^2 - 1)/a^6)

---

**Fricas [A]** time = 1.57557, size = 122, normalized size = 2.44

$$\frac{2 a^5 x^5 \log\left(\frac{ax+1}{ax-1}\right) + a^4 x^4 + 2 a^2 x^2 + 2 \log(a^2 x^2 - 1)}{20 a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arccoth(a\*x),x, algorithm="fricas")

[Out] 1/20\*(2\*a^5\*x^5\*log((a\*x + 1)/(a\*x - 1)) + a^4\*x^4 + 2\*a^2\*x^2 + 2\*log(a^2\*x^2 - 1))/a^5

---

**Sympy [A]** time = 3.79828, size = 54, normalized size = 1.08

$$\begin{cases} \frac{x^5 \operatorname{acoth}(ax)}{5} + \frac{x^4}{20a} + \frac{x^2}{10a^3} + \frac{\log(ax+1)}{5a^5} - \frac{\operatorname{acoth}(ax)}{5a^5} & \text{for } a \neq 0 \\ \frac{i\pi x^5}{10} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*acoth(a\*x),x)

[Out] Piecewise((x\*\*5\*acoth(a\*x)/5 + x\*\*4/(20\*a) + x\*\*2/(10\*a\*\*3) + log(a\*x + 1)/(5\*a\*\*5) - acoth(a\*x)/(5\*a\*\*5), Ne(a, 0)), (I\*pi\*x\*\*5/10, True))

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{arccoth}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^4*arccoth(a*x), x)
```



### 3.3 $\int x^3 \coth^{-1}(ax) dx$

**Optimal.** Leaf size=41

$$\frac{x}{4a^3} - \frac{\tanh^{-1}(ax)}{4a^4} + \frac{x^3}{12a} + \frac{1}{4}x^4 \coth^{-1}(ax)$$

[Out]  $x/(4*a^3) + x^3/(12*a) + (x^4*ArcCoth[a*x])/4 - ArcTanh[a*x]/(4*a^4)$

**Rubi [A]** time = 0.0237262, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {5917, 302, 206}

$$\frac{x}{4a^3} - \frac{\tanh^{-1}(ax)}{4a^4} + \frac{x^3}{12a} + \frac{1}{4}x^4 \coth^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Int[x^3\*ArcCoth[a\*x],x]

[Out]  $x/(4*a^3) + x^3/(12*a) + (x^4*ArcCoth[a*x])/4 - ArcTanh[a*x]/(4*a^4)$

#### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.)*((d_.)*(x_.))^(m_.), x_Symbol]
  :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

#### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3 \coth^{-1}(ax) dx &= \frac{1}{4}x^4 \coth^{-1}(ax) - \frac{1}{4}a \int \frac{x^4}{1-a^2x^2} dx \\
&= \frac{1}{4}x^4 \coth^{-1}(ax) - \frac{1}{4}a \int \left( -\frac{1}{a^4} - \frac{x^2}{a^2} + \frac{1}{a^4(1-a^2x^2)} \right) dx \\
&= \frac{x}{4a^3} + \frac{x^3}{12a} + \frac{1}{4}x^4 \coth^{-1}(ax) - \frac{\int \frac{1}{1-a^2x^2} dx}{4a^3} \\
&= \frac{x}{4a^3} + \frac{x^3}{12a} + \frac{1}{4}x^4 \coth^{-1}(ax) - \frac{\tanh^{-1}(ax)}{4a^4}
\end{aligned}$$

**Mathematica [A]** time = 0.0082089, size = 57, normalized size = 1.39

$$\frac{x}{4a^3} + \frac{\log(1-ax)}{8a^4} - \frac{\log(ax+1)}{8a^4} + \frac{x^3}{12a} + \frac{1}{4}x^4 \coth^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*ArcCoth[a\*x], x]

[Out] x/(4\*a^3) + x^3/(12\*a) + (x^4\*ArcCoth[a\*x])/4 + Log[1 - a\*x]/(8\*a^4) - Log[1 + a\*x]/(8\*a^4)

**Maple [A]** time = 0.031, size = 47, normalized size = 1.2

$$\frac{x^4 \operatorname{arccoth}(ax)}{4} + \frac{x^3}{12a} + \frac{x}{4a^3} + \frac{\ln(ax-1)}{8a^4} - \frac{\ln(ax+1)}{8a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*arccoth(a\*x), x)

[Out] 1/4\*x^4\*arccoth(a\*x)+1/12\*x^3/a+1/4\*x/a^3+1/8/a^4\*ln(a\*x-1)-1/8/a^4\*ln(a\*x+1)

**Maxima [A]** time = 0.966109, size = 70, normalized size = 1.71

$$\frac{1}{4}x^4 \operatorname{arccoth}(ax) + \frac{1}{24}a \left( \frac{2(a^2x^3 + 3x)}{a^4} - \frac{3 \log(ax + 1)}{a^5} + \frac{3 \log(ax - 1)}{a^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(a\*x),x, algorithm="maxima")

[Out] 1/4\*x^4\*arccoth(a\*x) + 1/24\*a\*(2\*(a^2\*x^3 + 3\*x)/a^4 - 3\*log(a\*x + 1)/a^5 + 3\*log(a\*x - 1)/a^5)

**Fricas [A]** time = 1.59866, size = 99, normalized size = 2.41

$$\frac{2a^3x^3 + 6ax + 3(a^4x^4 - 1) \log\left(\frac{ax+1}{ax-1}\right)}{24a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(a\*x),x, algorithm="fricas")

[Out] 1/24\*(2\*a^3\*x^3 + 6\*a\*x + 3\*(a^4\*x^4 - 1)\*log((a\*x + 1)/(a\*x - 1)))/a^4

**Sympy [A]** time = 1.28445, size = 41, normalized size = 1.

$$\begin{cases} \frac{x^4 \operatorname{acoth}(ax)}{4} + \frac{x^3}{12a} + \frac{x}{4a^3} - \frac{\operatorname{acoth}(ax)}{4a^4} & \text{for } a \neq 0 \\ \frac{i\pi x^4}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*acoth(a\*x),x)

[Out] Piecewise((x\*\*4\*acoth(a\*x)/4 + x\*\*3/(12\*a) + x/(4\*a\*\*3) - acoth(a\*x)/(4\*a\*\*4), Ne(a, 0)), (I\*pi\*x\*\*4/8, True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arccoth}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*arccoth(a*x), x)
```

### 3.4 $\int x^2 \coth^{-1}(ax) dx$

**Optimal.** Leaf size=40

$$\frac{\log(1-a^2x^2)}{6a^3} + \frac{x^2}{6a} + \frac{1}{3}x^3 \coth^{-1}(ax)$$

[Out]  $x^2/(6*a) + (x^3*ArcCoth[a*x])/3 + \text{Log}[1 - a^2*x^2]/(6*a^3)$

**Rubi [A]** time = 0.0298628, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {5917, 266, 43}

$$\frac{\log(1-a^2x^2)}{6a^3} + \frac{x^2}{6a} + \frac{1}{3}x^3 \coth^{-1}(ax)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*ArcCoth[a*x], x]$

[Out]  $x^2/(6*a) + (x^3*ArcCoth[a*x])/3 + \text{Log}[1 - a^2*x^2]/(6*a^3)$

#### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
  >: Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] >: Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] >: Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
```

Q[7\*m + 4\*n + 4, 0] || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0]

### Rubi steps

$$\begin{aligned}
 \int x^2 \coth^{-1}(ax) dx &= \frac{1}{3}x^3 \coth^{-1}(ax) - \frac{1}{3}a \int \frac{x^3}{1-a^2x^2} dx \\
 &= \frac{1}{3}x^3 \coth^{-1}(ax) - \frac{1}{6}a \operatorname{Subst}\left(\int \frac{x}{1-a^2x} dx, x, x^2\right) \\
 &= \frac{1}{3}x^3 \coth^{-1}(ax) - \frac{1}{6}a \operatorname{Subst}\left(\int \left(-\frac{1}{a^2} - \frac{1}{a^2(-1+a^2x)}\right) dx, x, x^2\right) \\
 &= \frac{x^2}{6a} + \frac{1}{3}x^3 \coth^{-1}(ax) + \frac{\log(1-a^2x^2)}{6a^3}
 \end{aligned}$$

**Mathematica [A]** time = 0.0078073, size = 40, normalized size = 1.

$$\frac{\log(1-a^2x^2)}{6a^3} + \frac{x^2}{6a} + \frac{1}{3}x^3 \coth^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[a\*x], x]

[Out] x^2/(6\*a) + (x^3\*ArcCoth[a\*x])/3 + Log[1 - a^2\*x^2]/(6\*a^3)

**Maple [A]** time = 0.032, size = 41, normalized size = 1.

$$\frac{x^3 \operatorname{arccoth}(ax)}{3} + \frac{x^2}{6a} + \frac{\ln(ax-1)}{6a^3} + \frac{\ln(ax+1)}{6a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(a\*x), x)

[Out] 1/3\*x^3\*arccoth(a\*x)+1/6\*x^2/a+1/6/a^3\*ln(a\*x-1)+1/6/a^3\*ln(a\*x+1)

**Maxima [A]** time = 0.968443, size = 47, normalized size = 1.18

$$\frac{1}{3}x^3 \operatorname{arccoth}(ax) + \frac{1}{6}a \left( \frac{x^2}{a^2} + \frac{\log(a^2x^2 - 1)}{a^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(a\*x),x, algorithm="maxima")

[Out] 1/3\*x^3\*arccoth(a\*x) + 1/6\*a\*(x^2/a^2 + log(a^2\*x^2 - 1)/a^4)

**Fricas [A]** time = 1.57336, size = 99, normalized size = 2.48

$$\frac{a^3x^3 \log\left(\frac{ax+1}{ax-1}\right) + a^2x^2 + \log(a^2x^2 - 1)}{6a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(a\*x),x, algorithm="fricas")

[Out] 1/6\*(a^3\*x^3\*log((a\*x + 1)/(a\*x - 1)) + a^2\*x^2 + log(a^2\*x^2 - 1))/a^3

**Sympy [A]** time = 2.3732, size = 46, normalized size = 1.15

$$\begin{cases} \frac{x^3 \operatorname{acoth}(ax)}{3} + \frac{x^2}{6a} + \frac{\log(ax+1)}{3a^3} - \frac{\operatorname{acoth}(ax)}{3a^3} & \text{for } a \neq 0 \\ \frac{i\pi x^3}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*acoth(a\*x),x)

[Out] Piecewise((x\*\*3\*acoth(a\*x)/3 + x\*\*2/(6\*a) + log(a\*x + 1)/(3\*a\*\*3) - acoth(a\*x)/(3\*a\*\*3), Ne(a, 0)), (I\*pi\*x\*\*3/6, True))

**Giac** [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arccoth}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(a*x), x)
```



### 3.5 $\int x \coth^{-1}(ax) dx$

**Optimal.** Leaf size=31

$$-\frac{\tanh^{-1}(ax)}{2a^2} + \frac{1}{2}x^2 \coth^{-1}(ax) + \frac{x}{2a}$$

[Out]  $x/(2*a) + (x^2*ArcCoth[a*x])/2 - ArcTanh[a*x]/(2*a^2)$

**Rubi [A]** time = 0.0132059, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {5917, 321, 206}

$$-\frac{\tanh^{-1}(ax)}{2a^2} + \frac{1}{2}x^2 \coth^{-1}(ax) + \frac{x}{2a}$$

Antiderivative was successfully verified.

[In] Int[x\*ArcCoth[a\*x],x]

[Out]  $x/(2*a) + (x^2*ArcCoth[a*x])/2 - ArcTanh[a*x]/(2*a^2)$

#### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
  := Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

#### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

$Q[a, 0] \parallel \text{LtQ}[b, 0]$ )

### Rubi steps

$$\begin{aligned} \int x \coth^{-1}(ax) dx &= \frac{1}{2}x^2 \coth^{-1}(ax) - \frac{1}{2}a \int \frac{x^2}{1-a^2x^2} dx \\ &= \frac{x}{2a} + \frac{1}{2}x^2 \coth^{-1}(ax) - \frac{\int \frac{1}{1-a^2x^2} dx}{2a} \\ &= \frac{x}{2a} + \frac{1}{2}x^2 \coth^{-1}(ax) - \frac{\tanh^{-1}(ax)}{2a^2} \end{aligned}$$

**Mathematica [A]** time = 0.0069993, size = 47, normalized size = 1.52

$$\frac{\log(1-ax)}{4a^2} - \frac{\log(ax+1)}{4a^2} + \frac{1}{2}x^2 \coth^{-1}(ax) + \frac{x}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[a\*x], x]

[Out] x/(2\*a) + (x^2\*ArcCoth[a\*x])/2 + Log[1 - a\*x]/(4\*a^2) - Log[1 + a\*x]/(4\*a^2)

**Maple [A]** time = 0.033, size = 39, normalized size = 1.3

$$\frac{x^2 \operatorname{arccoth}(ax)}{2} + \frac{x}{2a} + \frac{\ln(ax-1)}{4a^2} - \frac{\ln(ax+1)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(a\*x), x)

[Out] 1/2\*x^2\*arccoth(a\*x)+1/2\*x/a+1/4/a^2\*ln(a\*x-1)-1/4/a^2\*ln(a\*x+1)

**Maxima [A]** time = 0.973175, size = 55, normalized size = 1.77

$$\frac{1}{2}x^2 \operatorname{arccoth}(ax) + \frac{1}{4}a \left( \frac{2x}{a^2} - \frac{\log(ax+1)}{a^3} + \frac{\log(ax-1)}{a^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(a\*x),x, algorithm="maxima")

[Out] 1/2\*x^2\*arccoth(a\*x) + 1/4\*a\*(2\*x/a^2 - log(a\*x + 1)/a^3 + log(a\*x - 1)/a^3 )

**Fricas [A]** time = 1.61763, size = 78, normalized size = 2.52

$$\frac{2ax + (a^2x^2 - 1)\log\left(\frac{ax+1}{ax-1}\right)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(a\*x),x, algorithm="fricas")

[Out] 1/4\*(2\*a\*x + (a^2\*x^2 - 1)\*log((a\*x + 1)/(a\*x - 1)))/a^2

**Sympy [A]** time = 0.918862, size = 32, normalized size = 1.03

$$\begin{cases} \frac{x^2 \operatorname{acoth}(ax)}{2} + \frac{x}{2a} - \frac{\operatorname{acoth}(ax)}{2a^2} & \text{for } a \neq 0 \\ \frac{i\pi x^2}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(a\*x),x)

[Out] Piecewise((x\*\*2\*acoth(a\*x)/2 + x/(2\*a) - acoth(a\*x)/(2\*a\*\*2), Ne(a, 0)), (I\*pi\*x\*\*2/4, True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arccoth}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(a*x),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(a*x), x)
```

### 3.6 $\int \coth^{-1}(ax) dx$

**Optimal.** Leaf size=25

$$\frac{\log(1 - a^2x^2)}{2a} + x \coth^{-1}(ax)$$

[Out] x\*ArcCoth[a\*x] + Log[1 - a^2\*x^2]/(2\*a)

**Rubi [A]** time = 0.0068101, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {5911, 260}

$$\frac{\log(1 - a^2x^2)}{2a} + x \coth^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x], x]

[Out] x\*ArcCoth[a\*x] + Log[1 - a^2\*x^2]/(2\*a)

#### Rule 5911

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.), x\_Symbol] :> Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c^p, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

#### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

#### Rubi steps

$$\begin{aligned} \int \coth^{-1}(ax) dx &= x \coth^{-1}(ax) - a \int \frac{x}{1 - a^2x^2} dx \\ &= x \coth^{-1}(ax) + \frac{\log(1 - a^2x^2)}{2a} \end{aligned}$$

**Mathematica [A]** time = 0.0030681, size = 25, normalized size = 1.

$$\frac{\log(1 - a^2x^2)}{2a} + x \coth^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a\*x],x]

[Out] x\*ArcCoth[a\*x] + Log[1 - a^2\*x^2]/(2\*a)

**Maple [A]** time = 0.043, size = 23, normalized size = 0.9

$$x \operatorname{arccoth}(ax) + \frac{\ln(a^2x^2 - 1)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x),x)

[Out] x\*arccoth(a\*x)+1/2/a\*ln(a^2\*x^2-1)

**Maxima [A]** time = 0.986804, size = 34, normalized size = 1.36

$$\frac{2ax \operatorname{arccoth}(ax) + \log(-a^2x^2 + 1)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x),x, algorithm="maxima")

[Out] 1/2\*(2\*a\*x\*arccoth(a\*x) + log(-a^2\*x^2 + 1))/a

**Fricas [A]** time = 1.68022, size = 77, normalized size = 3.08

$$\frac{ax \log\left(\frac{ax+1}{ax-1}\right) + \log(a^2x^2 - 1)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(a*x),x, algorithm="fricas")`

[Out]  $1/2*(a*x*\log((a*x + 1)/(a*x - 1)) + \log(a^2*x^2 - 1))/a$

**Sympy [A]** time = 0.609134, size = 27, normalized size = 1.08

$$\begin{cases} x \operatorname{acoth}(ax) + \frac{\log(ax+1)}{a} - \frac{\operatorname{acoth}(ax)}{a} & \text{for } a \neq 0 \\ \frac{i\pi x}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(a*x),x)`

[Out] `Piecewise((x*acoth(a*x) + log(a*x + 1)/a - acoth(a*x)/a, Ne(a, 0)), (I*pi*x/2, True))`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(a*x),x, algorithm="giac")`

[Out] `integrate(arccoth(a*x), x)`

$$3.7 \quad \int \frac{\coth^{-1}(ax)}{x} dx$$

**Optimal.** Leaf size=28

$$\frac{1}{2}\text{PolyLog}\left(2, -\frac{1}{ax}\right) - \frac{1}{2}\text{PolyLog}\left(2, \frac{1}{ax}\right)$$

[Out] PolyLog[2, -(1/(a\*x))]/2 - PolyLog[2, 1/(a\*x)]/2

**Rubi [A]** time = 0.0097539, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {5913}

$$\frac{1}{2}\text{PolyLog}\left(2, -\frac{1}{ax}\right) - \frac{1}{2}\text{PolyLog}\left(2, \frac{1}{ax}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]/x, x]

[Out] PolyLog[2, -(1/(a\*x))]/2 - PolyLog[2, 1/(a\*x)]/2

**Rule 5913**

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[a\*Log[x], x] + (Simp[(b\*PolyLog[2, -(c\*x)^(-1)])/2, x] - Simp[(b\*PolyLog[2, 1/(c\*x)])/2, x]) /; FreeQ[{a, b, c}, x]

**Rubi steps**

$$\int \frac{\coth^{-1}(ax)}{x} dx = \frac{1}{2}\text{Li}_2\left(-\frac{1}{ax}\right) - \frac{1}{2}\text{Li}_2\left(\frac{1}{ax}\right)$$

**Mathematica [A]** time = 0.0081032, size = 26, normalized size = 0.93

$$\frac{1}{2}\left(\text{PolyLog}\left(2, -\frac{1}{ax}\right) - \text{PolyLog}\left(2, \frac{1}{ax}\right)\right)$$



Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a\*x]/x,x]

[Out] (PolyLog[2, -(1/(a\*x))] - PolyLog[2, 1/(a\*x)])/2

**Maple [A]** time = 0.042, size = 37, normalized size = 1.3

$$\ln(ax) \operatorname{arccoth}(ax) - \frac{\operatorname{dilog}(ax)}{2} - \frac{\operatorname{dilog}(ax+1)}{2} - \frac{\ln(ax) \ln(ax+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)/x,x)

[Out] ln(a\*x)\*arccoth(a\*x)-1/2\*dilog(a\*x)-1/2\*dilog(a\*x+1)-1/2\*ln(a\*x)\*ln(a\*x+1)

**Maxima [B]** time = 0.997304, size = 116, normalized size = 4.14

$$-\frac{1}{2}a\left(\frac{\log(ax+1)}{a} - \frac{\log(ax-1)}{a}\right)\log(x) - \frac{1}{2}a\left(\frac{\log(ax-1)\log(ax) + \operatorname{Li}_2(-ax+1)}{a} - \frac{\log(ax+1)\log(-ax) + \operatorname{Li}_2(ax)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/x,x, algorithm="maxima")

[Out] -1/2\*a\*(log(a\*x + 1)/a - log(a\*x - 1)/a)\*log(x) - 1/2\*a\*((log(a\*x - 1)\*log(a\*x) + dilog(-a\*x + 1))/a - (log(a\*x + 1)\*log(-a\*x) + dilog(a\*x + 1))/a) + arccoth(a\*x)\*log(x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arccoth}(ax)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)/x, x)
```

---

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)/x,x)
```

```
[Out] Integral(acoth(a*x)/x, x)
```

---

**Giac** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)/x, x)
```

$$3.8 \quad \int \frac{\coth^{-1}(ax)}{x^2} dx$$

Optimal. Leaf size=30

$$-\frac{1}{2}a \log(1 - a^2x^2) + a \log(x) - \frac{\coth^{-1}(ax)}{x}$$

[Out]  $-(\text{ArcCoth}[a*x]/x) + a*\text{Log}[x] - (a*\text{Log}[1 - a^2*x^2])/2$

**Rubi [A]** time = 0.020794, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$ , Rules used = {5917, 266, 36, 29, 31}

$$-\frac{1}{2}a \log(1 - a^2x^2) + a \log(x) - \frac{\coth^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{ArcCoth}[a*x]/x^2, x]$

[Out]  $-(\text{ArcCoth}[a*x]/x) + a*\text{Log}[x] - (a*\text{Log}[1 - a^2*x^2])/2$

### Rule 5917

$\text{Int}[(a_.) + \text{ArcCoth}[(c_.)*(x_.)]*(b_.)]^{(p_.)}*((d_.)*(x_.))^{(m_.)}, x\_Symbol]$   
 $:\> \text{Simp}[(d*x)^{(m+1)}*(a + b*\text{ArcCoth}[c*x])^p/(d*(m+1)), x] - \text{Dist}[(b*c*p)/(d*(m+1)), \text{Int}[(d*x)^{(m+1)}*(a + b*\text{ArcCoth}[c*x])^{(p-1)}]/(1 - c^2*x^2), x], x] /;$   $\text{FreeQ}\{a, b, c, d, m\}, x] \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ (\text{EqQ}[p, 1] \ || \ \text{IntegerQ}[m]) \ \&\& \ \text{NeQ}[m, -1]$

### Rule 266

$\text{Int}[(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}, x\_Symbol] :\> \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a + b*x)^p}, x], x, x^n], x] /;$   $\text{FreeQ}\{a, b, m, n, p\}, x] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m+1)/n]]$

### Rule 36

$\text{Int}[1/(((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))), x\_Symbol] :\> \text{Dist}[b/(b*c - a*d), \text{Int}[1/(a + b*x), x], x] - \text{Dist}[d/(b*c - a*d), \text{Int}[1/(c + d*x), x], x] /;$   $\text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^-1, x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(ax)}{x^2} dx &= -\frac{\coth^{-1}(ax)}{x} + a \int \frac{1}{x(1-a^2x^2)} dx \\ &= -\frac{\coth^{-1}(ax)}{x} + \frac{1}{2}a \operatorname{Subst}\left(\int \frac{1}{x(1-a^2x)} dx, x, x^2\right) \\ &= -\frac{\coth^{-1}(ax)}{x} + \frac{1}{2}a \operatorname{Subst}\left(\int \frac{1}{x} dx, x, x^2\right) + \frac{1}{2}a^3 \operatorname{Subst}\left(\int \frac{1}{1-a^2x} dx, x, x^2\right) \\ &= -\frac{\coth^{-1}(ax)}{x} + a \log(x) - \frac{1}{2}a \log(1-a^2x^2) \end{aligned}$$

**Mathematica [A]** time = 0.0075953, size = 30, normalized size = 1.

$$-\frac{1}{2}a \log(1-a^2x^2) + a \log(x) - \frac{\coth^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]/x^2, x]
```

```
[Out] -(ArcCoth[a*x]/x) + a*Log[x] - (a*Log[1 - a^2*x^2])/2
```

**Maple [A]** time = 0.036, size = 35, normalized size = 1.2

$$-\frac{\operatorname{arccoth}(ax)}{x} - \frac{a \ln(ax-1)}{2} + a \ln(ax) - \frac{a \ln(ax+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(a*x)/x^2,x)`

[Out] `-arccoth(a*x)/x-1/2*a*ln(a*x-1)+a*ln(a*x)-1/2*a*ln(a*x+1)`

**Maxima [A]** time = 0.974439, size = 41, normalized size = 1.37

$$-\frac{1}{2}a(\log(a^2x^2 - 1) - \log(x^2)) - \frac{\operatorname{arccoth}(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(a*x)/x^2,x, algorithm="maxima")`

[Out] `-1/2*a*(log(a^2*x^2 - 1) - log(x^2)) - arccoth(a*x)/x`

**Fricas [A]** time = 1.70012, size = 99, normalized size = 3.3

$$-\frac{ax \log(a^2x^2 - 1) - 2ax \log(x) + \log\left(\frac{ax+1}{ax-1}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(a*x)/x^2,x, algorithm="fricas")`

[Out] `-1/2*(a*x*log(a^2*x^2 - 1) - 2*a*x*log(x) + log((a*x + 1)/(a*x - 1)))/x`

**Sympy [A]** time = 0.610749, size = 26, normalized size = 0.87

$$a \log(x) - a \log(ax + 1) + a \operatorname{acoth}(ax) - \frac{\operatorname{acoth}(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(a*x)/x**2,x)`

[Out] `a*log(x) - a*log(a*x + 1) + a*acoth(a*x) - acoth(a*x)/x`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(ax)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)/x^2, x)
```

### 3.9 $\int \frac{\coth^{-1}(ax)}{x^3} dx$

**Optimal.** Leaf size=31

$$\frac{1}{2}a^2 \tanh^{-1}(ax) - \frac{\coth^{-1}(ax)}{2x^2} - \frac{a}{2x}$$

[Out]  $-a/(2*x) - \text{ArcCoth}[a*x]/(2*x^2) + (a^2*\text{ArcTanh}[a*x])/2$

**Rubi [A]** time = 0.0163453, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {5917, 325, 206}

$$\frac{1}{2}a^2 \tanh^{-1}(ax) - \frac{\coth^{-1}(ax)}{2x^2} - \frac{a}{2x}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{ArcCoth}[a*x]/x^3, x]$

[Out]  $-a/(2*x) - \text{ArcCoth}[a*x]/(2*x^2) + (a^2*\text{ArcTanh}[a*x])/2$

#### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
  :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

#### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

Q[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(ax)}{x^3} dx &= -\frac{\coth^{-1}(ax)}{2x^2} + \frac{1}{2}a \int \frac{1}{x^2(1-a^2x^2)} dx \\ &= -\frac{a}{2x} - \frac{\coth^{-1}(ax)}{2x^2} + \frac{1}{2}a^3 \int \frac{1}{1-a^2x^2} dx \\ &= -\frac{a}{2x} - \frac{\coth^{-1}(ax)}{2x^2} + \frac{1}{2}a^2 \tanh^{-1}(ax)\end{aligned}$$

**Mathematica [A]** time = 0.0082258, size = 47, normalized size = 1.52

$$-\frac{1}{4}a^2 \log(1-ax) + \frac{1}{4}a^2 \log(ax+1) - \frac{\coth^{-1}(ax)}{2x^2} - \frac{a}{2x}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a\*x]/x^3,x]

[Out] -a/(2\*x) - ArcCoth[a\*x]/(2\*x^2) - (a^2\*Log[1 - a\*x])/4 + (a^2\*Log[1 + a\*x])/4

**Maple [A]** time = 0.039, size = 39, normalized size = 1.3

$$-\frac{\operatorname{arccoth}(ax)}{2x^2} - \frac{a}{2x} - \frac{a^2 \ln(ax-1)}{4} + \frac{a^2 \ln(ax+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)/x^3,x)

[Out] -1/2\*arccoth(a\*x)/x^2-1/2\*a/x-1/4\*a^2\*ln(a\*x-1)+1/4\*a^2\*ln(a\*x+1)

**Maxima [A]** time = 0.972809, size = 49, normalized size = 1.58

$$\frac{1}{4} \left( a \log(ax+1) - a \log(ax-1) - \frac{2}{x} \right) a - \frac{\operatorname{arccoth}(ax)}{2x^2}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/x^3,x, algorithm="maxima")

[Out] 1/4\*(a\*log(a\*x + 1) - a\*log(a\*x - 1) - 2/x)\*a - 1/2\*arccoth(a\*x)/x^2

**Fricas [A]** time = 1.51869, size = 80, normalized size = 2.58

$$-\frac{2ax - (a^2x^2 - 1)\log\left(\frac{ax+1}{ax-1}\right)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/x^3,x, algorithm="fricas")

[Out] -1/4\*(2\*a\*x - (a^2\*x^2 - 1)\*log((a\*x + 1)/(a\*x - 1)))/x^2

**Sympy [A]** time = 2.57081, size = 24, normalized size = 0.77

$$\frac{a^2 \operatorname{acoth}(ax)}{2} - \frac{a}{2x} - \frac{\operatorname{acoth}(ax)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(a\*x)/x\*\*3,x)

[Out] a\*\*2\*acoth(a\*x)/2 - a/(2\*x) - acoth(a\*x)/(2\*x\*\*2)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(ax)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/x^3,x, algorithm="giac")

[Out] integrate(arccoth(a\*x)/x^3, x)

### 3.10 $\int \frac{\coth^{-1}(ax)}{x^4} dx$

**Optimal.** Leaf size=47

$$-\frac{1}{6}a^3 \log(1 - a^2x^2) + \frac{1}{3}a^3 \log(x) - \frac{a}{6x^2} - \frac{\coth^{-1}(ax)}{3x^3}$$

[Out]  $-a/(6*x^2) - \text{ArcCoth}[a*x]/(3*x^3) + (a^3*\text{Log}[x])/3 - (a^3*\text{Log}[1 - a^2*x^2])/6$

**Rubi [A]** time = 0.0302444, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {5917, 266, 44}

$$-\frac{1}{6}a^3 \log(1 - a^2x^2) + \frac{1}{3}a^3 \log(x) - \frac{a}{6x^2} - \frac{\coth^{-1}(ax)}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]/x^4,x]

[Out]  $-a/(6*x^2) - \text{ArcCoth}[a*x]/(3*x^3) + (a^3*\text{Log}[x])/3 - (a^3*\text{Log}[1 - a^2*x^2])/6$

#### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^ (p_.)*((d_.)*(x_.))^ (m_.), x_Symbol]
  :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^ (p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
```

& NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{\coth^{-1}(ax)}{x^4} dx &= -\frac{\coth^{-1}(ax)}{3x^3} + \frac{1}{3}a \int \frac{1}{x^3(1-a^2x^2)} dx \\
 &= -\frac{\coth^{-1}(ax)}{3x^3} + \frac{1}{6}a \operatorname{Subst}\left(\int \frac{1}{x^2(1-a^2x)} dx, x, x^2\right) \\
 &= -\frac{\coth^{-1}(ax)}{3x^3} + \frac{1}{6}a \operatorname{Subst}\left(\int \left(\frac{1}{x^2} + \frac{a^2}{x} - \frac{a^4}{-1+a^2x}\right) dx, x, x^2\right) \\
 &= -\frac{a}{6x^2} - \frac{\coth^{-1}(ax)}{3x^3} + \frac{1}{3}a^3 \log(x) - \frac{1}{6}a^3 \log(1-a^2x^2)
 \end{aligned}$$

**Mathematica [A]** time = 0.0095672, size = 47, normalized size = 1.

$$-\frac{1}{6}a^3 \log(1-a^2x^2) + \frac{1}{3}a^3 \log(x) - \frac{a}{6x^2} - \frac{\coth^{-1}(ax)}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a\*x]/x^4,x]

[Out] -a/(6\*x^2) - ArcCoth[a\*x]/(3\*x^3) + (a^3\*Log[x])/3 - (a^3\*Log[1 - a^2\*x^2])/6

**Maple [A]** time = 0.039, size = 48, normalized size = 1.

$$-\frac{\operatorname{arccoth}(ax)}{3x^3} - \frac{a^3 \ln(ax-1)}{6} - \frac{a}{6x^2} + \frac{a^3 \ln(ax)}{3} - \frac{a^3 \ln(ax+1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)/x^4,x)

[Out] -1/3\*arccoth(a\*x)/x^3-1/6\*a^3\*ln(a\*x-1)-1/6\*a/x^2+1/3\*a^3\*ln(a\*x)-1/6\*a^3\*ln(a\*x+1)

---

**Maxima [A]** time = 0.945418, size = 54, normalized size = 1.15

$$-\frac{1}{6} \left( a^2 \log(a^2 x^2 - 1) - a^2 \log(x^2) + \frac{1}{x^2} \right) a - \frac{\operatorname{arccoth}(ax)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/x^4,x, algorithm="maxima")

[Out] -1/6\*(a^2\*log(a^2\*x^2 - 1) - a^2\*log(x^2) + 1/x^2)\*a - 1/3\*arccoth(a\*x)/x^3

---

**Fricas [A]** time = 1.5701, size = 120, normalized size = 2.55

$$-\frac{a^3 x^3 \log(a^2 x^2 - 1) - 2 a^3 x^3 \log(x) + ax + \log\left(\frac{ax+1}{ax-1}\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/x^4,x, algorithm="fricas")

[Out] -1/6\*(a^3\*x^3\*log(a^2\*x^2 - 1) - 2\*a^3\*x^3\*log(x) + a\*x + log((a\*x + 1)/(a\*x - 1)))/x^3

---

**Sympy [A]** time = 4.12088, size = 46, normalized size = 0.98

$$\frac{a^3 \log(x)}{3} - \frac{a^3 \log(ax + 1)}{3} + \frac{a^3 \operatorname{acoth}(ax)}{3} - \frac{a}{6x^2} - \frac{\operatorname{acoth}(ax)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(a\*x)/x\*\*4,x)

[Out] a\*\*3\*log(x)/3 - a\*\*3\*log(a\*x + 1)/3 + a\*\*3\*acoth(a\*x)/3 - a/(6\*x\*\*2) - acot  
h(a\*x)/(3\*x\*\*3)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(ax)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)/x^4, x)
```

### 3.11 $\int \frac{\coth^{-1}(ax)}{x^5} dx$

**Optimal.** Leaf size=41

$$-\frac{a^3}{4x} + \frac{1}{4}a^4 \tanh^{-1}(ax) - \frac{a}{12x^3} - \frac{\coth^{-1}(ax)}{4x^4}$$

[Out]  $-a/(12*x^3) - a^3/(4*x) - \text{ArcCoth}[a*x]/(4*x^4) + (a^4*\text{ArcTanh}[a*x])/4$

**Rubi [A]** time = 0.0218977, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {5917, 325, 206}

$$-\frac{a^3}{4x} + \frac{1}{4}a^4 \tanh^{-1}(ax) - \frac{a}{12x^3} - \frac{\coth^{-1}(ax)}{4x^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{ArcCoth}[a*x]/x^5, x]$

[Out]  $-a/(12*x^3) - a^3/(4*x) - \text{ArcCoth}[a*x]/(4*x^4) + (a^4*\text{ArcTanh}[a*x])/4$

#### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^ (p_.)*((d_.)*(x_.))^ (m_.), x_Symbol]
  >: Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 325

```
Int[((c_.)*(x_.))^ (m_.)*((a_.) + (b_.)*(x_)^(n_.))^ (p_.), x_Symbol] >: Simp[((c
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

#### Rule 206

```
Int[((a_.) + (b_.)*(x_)^2)^ (-1), x_Symbol] >: Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

Q[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{\coth^{-1}(ax)}{x^5} dx &= -\frac{\coth^{-1}(ax)}{4x^4} + \frac{1}{4}a \int \frac{1}{x^4(1-a^2x^2)} dx \\
 &= -\frac{a}{12x^3} - \frac{\coth^{-1}(ax)}{4x^4} + \frac{1}{4}a^3 \int \frac{1}{x^2(1-a^2x^2)} dx \\
 &= -\frac{a}{12x^3} - \frac{a^3}{4x} - \frac{\coth^{-1}(ax)}{4x^4} + \frac{1}{4}a^5 \int \frac{1}{1-a^2x^2} dx \\
 &= -\frac{a}{12x^3} - \frac{a^3}{4x} - \frac{\coth^{-1}(ax)}{4x^4} + \frac{1}{4}a^4 \tanh^{-1}(ax)
 \end{aligned}$$

**Mathematica [A]** time = 0.0118316, size = 57, normalized size = 1.39

$$-\frac{a^3}{4x} - \frac{1}{8}a^4 \log(1-ax) + \frac{1}{8}a^4 \log(ax+1) - \frac{a}{12x^3} - \frac{\coth^{-1}(ax)}{4x^4}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a\*x]/x^5,x]

[Out] -a/(12\*x^3) - a^3/(4\*x) - ArcCoth[a\*x]/(4\*x^4) - (a^4\*Log[1 - a\*x])/8 + (a^4\*Log[1 + a\*x])/8

**Maple [A]** time = 0.036, size = 47, normalized size = 1.2

$$-\frac{\operatorname{arccoth}(ax)}{4x^4} - \frac{a^4 \ln(ax-1)}{8} - \frac{a}{12x^3} - \frac{a^3}{4x} + \frac{a^4 \ln(ax+1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)/x^5,x)

[Out] -1/4\*arccoth(a\*x)/x^4-1/8\*a^4\*ln(a\*x-1)-1/12\*a/x^3-1/4\*a^3/x+1/8\*a^4\*ln(a\*x+1)

---

**Maxima [A]** time = 0.94443, size = 69, normalized size = 1.68

$$\frac{1}{24} \left( 3a^3 \log(ax+1) - 3a^3 \log(ax-1) - \frac{2(3a^2x^2+1)}{x^3} \right) a - \frac{\operatorname{arccoth}(ax)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/x^5,x, algorithm="maxima")

[Out] 1/24\*(3\*a^3\*log(a\*x + 1) - 3\*a^3\*log(a\*x - 1) - 2\*(3\*a^2\*x^2 + 1)/x^3)\*a - 1/4\*arccoth(a\*x)/x^4

---

**Fricas [A]** time = 1.5728, size = 100, normalized size = 2.44

$$\frac{6a^3x^3 + 2ax - 3(a^4x^4 - 1) \log\left(\frac{ax+1}{ax-1}\right)}{24x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/x^5,x, algorithm="fricas")

[Out] -1/24\*(6\*a^3\*x^3 + 2\*a\*x - 3\*(a^4\*x^4 - 1)\*log((a\*x + 1)/(a\*x - 1)))/x^4

---

**Sympy [A]** time = 1.9597, size = 32, normalized size = 0.78

$$\frac{a^4 \operatorname{acoth}(ax)}{4} - \frac{a^3}{4x} - \frac{a}{12x^3} - \frac{\operatorname{acoth}(ax)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(a\*x)/x\*\*5,x)

[Out] a\*\*4\*acoth(a\*x)/4 - a\*\*3/(4\*x) - a/(12\*x\*\*3) - acoth(a\*x)/(4\*x\*\*4)

---



**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(ax)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)/x^5, x)
```

### 3.12 $\int x^5 \coth^{-1}(ax)^2 dx$

**Optimal.** Leaf size=105

$$\frac{x^4}{60a^2} + \frac{4x^2}{45a^4} + \frac{23 \log(1 - a^2x^2)}{90a^6} + \frac{x^3 \coth^{-1}(ax)}{9a^3} + \frac{x \coth^{-1}(ax)}{3a^5} - \frac{\coth^{-1}(ax)^2}{6a^6} + \frac{1}{6}x^6 \coth^{-1}(ax)^2 + \frac{x^5 \coth^{-1}(ax)}{15a}$$

[Out] (4\*x^2)/(45\*a^4) + x^4/(60\*a^2) + (x\*ArcCoth[a\*x])/(3\*a^5) + (x^3\*ArcCoth[a\*x])/(9\*a^3) + (x^5\*ArcCoth[a\*x])/(15\*a) - ArcCoth[a\*x]^2/(6\*a^6) + (x^6\*ArcCoth[a\*x]^2)/6 + (23\*Log[1 - a^2\*x^2])/(90\*a^6)

**Rubi [A]** time = 0.245691, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.7$ , Rules used = {5917, 5981, 266, 43, 5911, 260, 5949}

$$\frac{x^4}{60a^2} + \frac{4x^2}{45a^4} + \frac{23 \log(1 - a^2x^2)}{90a^6} + \frac{x^3 \coth^{-1}(ax)}{9a^3} + \frac{x \coth^{-1}(ax)}{3a^5} - \frac{\coth^{-1}(ax)^2}{6a^6} + \frac{1}{6}x^6 \coth^{-1}(ax)^2 + \frac{x^5 \coth^{-1}(ax)}{15a}$$

Antiderivative was successfully verified.

[In] Int[x^5\*ArcCoth[a\*x]^2,x]

[Out] (4\*x^2)/(45\*a^4) + x^4/(60\*a^2) + (x\*ArcCoth[a\*x])/(3\*a^5) + (x^3\*ArcCoth[a\*x])/(9\*a^3) + (x^5\*ArcCoth[a\*x])/(15\*a) - ArcCoth[a\*x]^2/(6\*a^6) + (x^6\*ArcCoth[a\*x]^2)/6 + (23\*Log[1 - a^2\*x^2])/(90\*a^6)

#### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.)*((d_.)*(x_.))^(m_.), x_Symbol]
  :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 5981

```
Int[(((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.)*((f_.)*(x_.))^(m_.))/((d_.) + (
e_.)*(x_.)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.)/((d_.) + (e_.)*(x_)^2), x_Symb
ol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int x^5 \coth^{-1}(ax)^2 dx &= \frac{1}{6}x^6 \coth^{-1}(ax)^2 - \frac{1}{3}a \int \frac{x^6 \coth^{-1}(ax)}{1-a^2x^2} dx \\
&= \frac{1}{6}x^6 \coth^{-1}(ax)^2 + \frac{\int x^4 \coth^{-1}(ax) dx}{3a} - \frac{\int \frac{x^4 \coth^{-1}(ax)}{1-a^2x^2} dx}{3a} \\
&= \frac{x^5 \coth^{-1}(ax)}{15a} + \frac{1}{6}x^6 \coth^{-1}(ax)^2 - \frac{1}{15} \int \frac{x^5}{1-a^2x^2} dx + \frac{\int x^2 \coth^{-1}(ax) dx}{3a^3} - \frac{\int \frac{x^2 \coth^{-1}(ax)}{1-a^2x^2} dx}{3a^3} \\
&= \frac{x^3 \coth^{-1}(ax)}{9a^3} + \frac{x^5 \coth^{-1}(ax)}{15a} + \frac{1}{6}x^6 \coth^{-1}(ax)^2 - \frac{1}{30} \text{Subst} \left( \int \frac{x^2}{1-a^2x} dx, x, x^2 \right) + \frac{\int \coth^{-1}(ax) dx}{3a} \\
&= \frac{x \coth^{-1}(ax)}{3a^5} + \frac{x^3 \coth^{-1}(ax)}{9a^3} + \frac{x^5 \coth^{-1}(ax)}{15a} - \frac{\coth^{-1}(ax)^2}{6a^6} + \frac{1}{6}x^6 \coth^{-1}(ax)^2 - \frac{1}{30} \text{Subst} \left( \int \frac{x^2}{1-a^2x} dx, x, x^2 \right) \\
&= \frac{x^2}{30a^4} + \frac{x^4}{60a^2} + \frac{x \coth^{-1}(ax)}{3a^5} + \frac{x^3 \coth^{-1}(ax)}{9a^3} + \frac{x^5 \coth^{-1}(ax)}{15a} - \frac{\coth^{-1}(ax)^2}{6a^6} + \frac{1}{6}x^6 \coth^{-1}(ax)^2 \\
&= \frac{4x^2}{45a^4} + \frac{x^4}{60a^2} + \frac{x \coth^{-1}(ax)}{3a^5} + \frac{x^3 \coth^{-1}(ax)}{9a^3} + \frac{x^5 \coth^{-1}(ax)}{15a} - \frac{\coth^{-1}(ax)^2}{6a^6} + \frac{1}{6}x^6 \coth^{-1}(ax)^2
\end{aligned}$$

**Mathematica [A]** time = 0.0218823, size = 80, normalized size = 0.76

$$\frac{3a^4x^4 + 16a^2x^2 + 46 \log(1 - a^2x^2) + 4ax(3a^4x^4 + 5a^2x^2 + 15) \coth^{-1}(ax) + 30(a^6x^6 - 1) \coth^{-1}(ax)^2}{180a^6}$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*ArcCoth[a\*x]^2,x]

[Out] (16\*a^2\*x^2 + 3\*a^4\*x^4 + 4\*a\*x\*(15 + 5\*a^2\*x^2 + 3\*a^4\*x^4)\*ArcCoth[a\*x] + 30\*(-1 + a^6\*x^6)\*ArcCoth[a\*x]^2 + 46\*Log[1 - a^2\*x^2])/(180\*a^6)

**Maple [B]** time = 0.052, size = 196, normalized size = 1.9

$$\frac{x^6 (\operatorname{arccoth}(ax))^2}{6} + \frac{x^5 \operatorname{arccoth}(ax)}{15a} + \frac{x^3 \operatorname{arccoth}(ax)}{9a^3} + \frac{x \operatorname{arccoth}(ax)}{3a^5} + \frac{\operatorname{arccoth}(ax) \ln(ax-1)}{6a^6} - \frac{\operatorname{arccoth}(ax) \ln(ax+1)}{6a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*arccoth(a\*x)^2,x)

[Out]  $1/6*x^6*\operatorname{arccoth}(a*x)^2+1/15*x^5*\operatorname{arccoth}(a*x)/a+1/9*x^3*\operatorname{arccoth}(a*x)/a^3+1/3*x*\operatorname{arccoth}(a*x)/a^5+1/6/a^6*\operatorname{arccoth}(a*x)*\ln(a*x-1)-1/6/a^6*\operatorname{arccoth}(a*x)*\ln(a*x+1)+1/24/a^6*\ln(a*x-1)^2-1/12/a^6*\ln(a*x-1)*\ln(1/2+1/2*a*x)+1/12/a^6*\ln(-1/2*a*x+1/2)*\ln(1/2+1/2*a*x)-1/12/a^6*\ln(-1/2*a*x+1/2)*\ln(a*x+1)+1/24/a^6*\ln(a*x+1)^2+1/60*x^4/a^2+4/45*x^2/a^4+23/90/a^6*\ln(a*x-1)+23/90/a^6*\ln(a*x+1)$

**Maxima [A]** time = 0.970857, size = 182, normalized size = 1.73

$$\frac{1}{6} x^6 \operatorname{arccoth}(ax)^2 + \frac{1}{90} a \left( \frac{2(3a^4x^5 + 5a^2x^3 + 15x)}{a^6} - \frac{15 \log(ax+1)}{a^7} + \frac{15 \log(ax-1)}{a^7} \right) \operatorname{arccoth}(ax) + \frac{6a^4x^4 + 32a^2x^2 + 15(a^6x^6 - 1) \log\left(\frac{ax+1}{ax-1}\right)^2 + 4(3a^5x^5 + 5a^3x^3 + 15ax) \log\left(\frac{ax+1}{ax-1}\right) + 92 \log(a^2x^2 - 1)}{360a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*arccoth(a*x)^2,x, algorithm="maxima")`

[Out]  $1/6*x^6*\operatorname{arccoth}(a*x)^2 + 1/90*a*(2*(3*a^4*x^5 + 5*a^2*x^3 + 15*x)/a^6 - 15*\log(a*x + 1)/a^7 + 15*\log(a*x - 1)/a^7)*\operatorname{arccoth}(a*x) + 1/360*(6*a^4*x^4 + 32*a^2*x^2 - 2*(15*\log(a*x - 1) - 46)*\log(a*x + 1) + 15*\log(a*x + 1)^2 + 15*\log(a*x - 1)^2 + 92*\log(a*x - 1))/a^6$

**Fricas [A]** time = 1.57716, size = 224, normalized size = 2.13

$$\frac{6a^4x^4 + 32a^2x^2 + 15(a^6x^6 - 1) \log\left(\frac{ax+1}{ax-1}\right)^2 + 4(3a^5x^5 + 5a^3x^3 + 15ax) \log\left(\frac{ax+1}{ax-1}\right) + 92 \log(a^2x^2 - 1)}{360a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*arccoth(a*x)^2,x, algorithm="fricas")`

[Out]  $1/360*(6*a^4*x^4 + 32*a^2*x^2 + 15*(a^6*x^6 - 1)*\log((a*x + 1)/(a*x - 1))^2 + 4*(3*a^5*x^5 + 5*a^3*x^3 + 15*a*x)*\log((a*x + 1)/(a*x - 1)) + 92*\log(a^2*x^2 - 1))/a^6$

**Sympy [A]** time = 7.50491, size = 114, normalized size = 1.09

$$\begin{cases} \frac{x^6 \operatorname{acoth}^2(ax)}{\pi^2 x^6} + \frac{x^5 \operatorname{acoth}(ax)}{15a} + \frac{x^4}{60a^2} + \frac{x^3 \operatorname{acoth}(ax)}{9a^3} + \frac{4x^2}{45a^4} + \frac{x \operatorname{acoth}(ax)}{3a^5} + \frac{23 \log(ax+1)}{45a^6} - \frac{\operatorname{acoth}^2(ax)}{6a^6} - \frac{23 \operatorname{acoth}(ax)}{45a^6} & \text{for } a \neq 0 \\ -\frac{\pi^2 x^6}{24} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*acoth(a\*x)\*\*2,x)

[Out] Piecewise((x\*\*6\*acoth(a\*x)\*\*2/6 + x\*\*5\*acoth(a\*x)/(15\*a) + x\*\*4/(60\*a\*\*2) + x\*\*3\*acoth(a\*x)/(9\*a\*\*3) + 4\*x\*\*2/(45\*a\*\*4) + x\*acoth(a\*x)/(3\*a\*\*5) + 23\*log(a\*x + 1)/(45\*a\*\*6) - acoth(a\*x)\*\*2/(6\*a\*\*6) - 23\*acoth(a\*x)/(45\*a\*\*6), Ne(a, 0)), (-pi\*\*2\*x\*\*6/24, True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^5 \operatorname{arccoth}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*arccoth(a\*x)^2,x, algorithm="giac")

[Out] integrate(x^5\*arccoth(a\*x)^2, x)

### 3.13 $\int x^4 \coth^{-1}(ax)^2 dx$

**Optimal.** Leaf size=127

$$-\frac{\text{PolyLog}\left(2, 1 - \frac{2}{1-ax}\right)}{5a^5} + \frac{x^3}{30a^2} + \frac{x^2 \coth^{-1}(ax)}{5a^3} + \frac{3x}{10a^4} - \frac{3 \tanh^{-1}(ax)}{10a^5} + \frac{\coth^{-1}(ax)^2}{5a^5} - \frac{2 \log\left(\frac{2}{1-ax}\right) \coth^{-1}(ax)}{5a^5} + \frac{1}{5}$$

```
[Out] (3*x)/(10*a^4) + x^3/(30*a^2) + (x^2*ArcCoth[a*x])/(5*a^3) + (x^4*ArcCoth[a*x])/(10*a) + ArcCoth[a*x]^2/(5*a^5) + (x^5*ArcCoth[a*x]^2)/5 - (3*ArcTanh[a*x])/(10*a^5) - (2*ArcCoth[a*x]*Log[2/(1 - a*x)])/(5*a^5) - PolyLog[2, 1 - 2/(1 - a*x)]/(5*a^5)
```

**Rubi [A]** time = 0.225659, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.9$ , Rules used = {5917, 5981, 302, 206, 321, 5985, 5919, 2402, 2315}

$$-\frac{\text{PolyLog}\left(2, 1 - \frac{2}{1-ax}\right)}{5a^5} + \frac{x^3}{30a^2} + \frac{x^2 \coth^{-1}(ax)}{5a^3} + \frac{3x}{10a^4} - \frac{3 \tanh^{-1}(ax)}{10a^5} + \frac{\coth^{-1}(ax)^2}{5a^5} - \frac{2 \log\left(\frac{2}{1-ax}\right) \coth^{-1}(ax)}{5a^5} + \frac{1}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcCoth[a*x]^2,x]
```

```
[Out] (3*x)/(10*a^4) + x^3/(30*a^2) + (x^2*ArcCoth[a*x])/(5*a^3) + (x^4*ArcCoth[a*x])/(10*a) + ArcCoth[a*x]^2/(5*a^5) + (x^5*ArcCoth[a*x]^2)/5 - (3*ArcTanh[a*x])/(10*a^5) - (2*ArcCoth[a*x]*Log[2/(1 - a*x)])/(5*a^5) - PolyLog[2, 1 - 2/(1 - a*x)]/(5*a^5)
```

#### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^p_.*((d_.)*(x_))^(m_.), x_Symbol]
  :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]
```

#### Rule 5981

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^p_.*((f_.)*(x_))^(m_.))/((d_.) + (e_.)*(x_)^2), x_Symbol]
  :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(
```

$d + e*x^2$ ), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]

### Rule 302

Int[(x\_)^(m\_)/((a\_) + (b\_)\*(x\_)^(n\_)), x\_Symbol] := Int[PolynomialDivide[x^m, a + b\*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2\*n - 1]

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 321

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 5985

Int[(((a\_) + ArcCoth[(c\_)\*(x\_)]\*(b\_))^(p\_)\*(x\_))/((d\_) + (e\_)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*e\*(p + 1)), x] + Dist[1/(c\*d), Int[(a + b\*ArcCoth[c\*x])^p/(1 - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && IGtQ[p, 0]

### Rule 5919

Int[((a\_) + ArcCoth[(c\_)\*(x\_)]\*(b\_))^(p\_)/((d\_) + (e\_)\*(x\_)), x\_Symbol] := -Simp[((a + b\*ArcCoth[c\*x])^p\*Log[2/(1 + (e\*x)/d)])/e, x] + Dist[(b\*c\*p)/e, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*Log[2/(1 + (e\*x)/d)])/((1 - c^2\*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

### Rule 2402

Int[Log[(c\_)/((d\_) + (e\_)\*(x\_))]/((f\_) + (g\_)\*(x\_)^2), x\_Symbol] := -Dist[e/g, Subst[Int[Log[2\*d\*x]/(1 - 2\*d\*x), x], x, 1/(d + e\*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2\*d] && EqQ[e^2\*f + d^2\*g, 0]



Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] :> -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

Rubi steps

$$\begin{aligned}
 \int x^4 \coth^{-1}(ax)^2 dx &= \frac{1}{5}x^5 \coth^{-1}(ax)^2 - \frac{1}{5}(2a) \int \frac{x^5 \coth^{-1}(ax)}{1 - a^2x^2} dx \\
 &= \frac{1}{5}x^5 \coth^{-1}(ax)^2 + \frac{2 \int x^3 \coth^{-1}(ax) dx}{5a} - \frac{2 \int \frac{x^3 \coth^{-1}(ax)}{1 - a^2x^2} dx}{5a} \\
 &= \frac{x^4 \coth^{-1}(ax)}{10a} + \frac{1}{5}x^5 \coth^{-1}(ax)^2 - \frac{1}{10} \int \frac{x^4}{1 - a^2x^2} dx + \frac{2 \int x \coth^{-1}(ax) dx}{5a^3} - \frac{2 \int \frac{x \coth^{-1}(ax)}{1 - a^2x^2} dx}{5a^3} \\
 &= \frac{x^2 \coth^{-1}(ax)}{5a^3} + \frac{x^4 \coth^{-1}(ax)}{10a} + \frac{\coth^{-1}(ax)^2}{5a^5} + \frac{1}{5}x^5 \coth^{-1}(ax)^2 - \frac{1}{10} \int \left( -\frac{1}{a^4} - \frac{x^2}{a^2} + \frac{1}{a^4(1 - a^2x^2)} \right) dx \\
 &= \frac{3x}{10a^4} + \frac{x^3}{30a^2} + \frac{x^2 \coth^{-1}(ax)}{5a^3} + \frac{x^4 \coth^{-1}(ax)}{10a} + \frac{\coth^{-1}(ax)^2}{5a^5} + \frac{1}{5}x^5 \coth^{-1}(ax)^2 - \frac{2 \coth^{-1}(ax)}{5a^3} \\
 &= \frac{3x}{10a^4} + \frac{x^3}{30a^2} + \frac{x^2 \coth^{-1}(ax)}{5a^3} + \frac{x^4 \coth^{-1}(ax)}{10a} + \frac{\coth^{-1}(ax)^2}{5a^5} + \frac{1}{5}x^5 \coth^{-1}(ax)^2 - \frac{3 \tanh^{-1}(ax)}{10a^5} \\
 &= \frac{3x}{10a^4} + \frac{x^3}{30a^2} + \frac{x^2 \coth^{-1}(ax)}{5a^3} + \frac{x^4 \coth^{-1}(ax)}{10a} + \frac{\coth^{-1}(ax)^2}{5a^5} + \frac{1}{5}x^5 \coth^{-1}(ax)^2 - \frac{3 \tanh^{-1}(ax)}{10a^5}
 \end{aligned}$$

**Mathematica [A]** time = 0.443444, size = 87, normalized size = 0.69

$$\frac{6 \text{PolyLog}\left(2, e^{-2 \coth^{-1}(ax)}\right) + ax(a^2x^2 + 9) + 6(a^5x^5 - 1) \coth^{-1}(ax)^2 + 3 \coth^{-1}(ax) \left(a^4x^4 + 2a^2x^2 - 4 \log\left(1 - e^{-2 \coth^{-1}(ax)}\right)\right)}{30a^5}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^4\*ArcCoth[a\*x]^2, x]

[Out] (a\*x\*(9 + a^2\*x^2) + 6\*(-1 + a^5\*x^5)\*ArcCoth[a\*x]^2 + 3\*ArcCoth[a\*x]\*(-3 + 2\*a^2\*x^2 + a^4\*x^4 - 4\*Log[1 - E^(-2\*ArcCoth[a\*x])]) + 6\*PolyLog[2, E^(-2\*ArcCoth[a\*x])])/(30\*a^5)

**Maple [A]** time = 0.053, size = 196, normalized size = 1.5

$$\frac{x^5 (\operatorname{arccoth}(ax))^2}{5} + \frac{x^4 \operatorname{arccoth}(ax)}{10a} + \frac{x^2 \operatorname{arccoth}(ax)}{5a^3} + \frac{\operatorname{arccoth}(ax) \ln(ax-1)}{5a^5} + \frac{\operatorname{arccoth}(ax) \ln(ax+1)}{5a^5} + \frac{x^3}{30a^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*arccoth(a\*x)^2,x)

[Out] 1/5\*x^5\*arccoth(a\*x)^2+1/10\*x^4\*arccoth(a\*x)/a+1/5\*x^2\*arccoth(a\*x)/a^3+1/5/a^5\*arccoth(a\*x)\*ln(a\*x-1)+1/5/a^5\*arccoth(a\*x)\*ln(a\*x+1)+1/30\*x^3/a^2+3/10\*x/a^4+3/20/a^5\*ln(a\*x-1)-3/20/a^5\*ln(a\*x+1)+1/20/a^5\*ln(a\*x-1)^2-1/5/a^5\*dilog(1/2+1/2\*a\*x)-1/10/a^5\*ln(a\*x-1)\*ln(1/2+1/2\*a\*x)-1/10/a^5\*ln(-1/2\*a\*x+1/2)\*ln(1/2+1/2\*a\*x)+1/10/a^5\*ln(-1/2\*a\*x+1/2)\*ln(a\*x+1)-1/20/a^5\*ln(a\*x+1)^2

**Maxima [A]** time = 0.960673, size = 209, normalized size = 1.65

$$\frac{1}{5} x^5 \operatorname{arccoth}(ax)^2 + \frac{1}{60} a^2 \left( \frac{2a^3 x^3 + 18ax - 3 \log(ax+1)^2 + 6 \log(ax+1) \log(ax-1) + 3 \log(ax-1)^2 + 9 \log(ax-1)}{a^7} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arccoth(a\*x)^2,x, algorithm="maxima")

[Out] 1/5\*x^5\*arccoth(a\*x)^2 + 1/60\*a^2\*((2\*a^3\*x^3 + 18\*a\*x - 3\*log(a\*x + 1)^2 + 6\*log(a\*x + 1)\*log(a\*x - 1) + 3\*log(a\*x - 1)^2 + 9\*log(a\*x - 1))/a^7 - 12\*(log(a\*x - 1)\*log(1/2\*a\*x + 1/2) + dilog(-1/2\*a\*x + 1/2))/a^7 - 9\*log(a\*x + 1)/a^7) + 1/10\*a\*((a^2\*x^4 + 2\*x^2)/a^4 + 2\*log(a^2\*x^2 - 1)/a^6)\*arccoth(a\*x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(x^4 \operatorname{arccoth}(ax)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arccoth(a\*x)^2,x, algorithm="fricas")

[Out] `integral(x^4*arccoth(a*x)^2, x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{arccoth}^2(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*acoth(a*x)**2,x)`

[Out] `Integral(x**4*acoth(a*x)**2, x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{arccoth}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*arccoth(a*x)^2,x, algorithm="giac")`

[Out] `integrate(x^4*arccoth(a*x)^2, x)`

### 3.14 $\int x^3 \coth^{-1}(ax)^2 dx$

**Optimal.** Leaf size=81

$$\frac{x^2}{12a^2} + \frac{\log(1-a^2x^2)}{3a^4} + \frac{x \coth^{-1}(ax)}{2a^3} - \frac{\coth^{-1}(ax)^2}{4a^4} + \frac{1}{4}x^4 \coth^{-1}(ax)^2 + \frac{x^3 \coth^{-1}(ax)}{6a}$$

[Out]  $x^2/(12*a^2) + (x*ArcCoth[a*x])/(2*a^3) + (x^3*ArcCoth[a*x])/(6*a) - ArcCoth[a*x]^2/(4*a^4) + (x^4*ArcCoth[a*x]^2)/4 + Log[1 - a^2*x^2]/(3*a^4)$

**Rubi [A]** time = 0.162949, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.7$ , Rules used = {5917, 5981, 266, 43, 5911, 260, 5949}

$$\frac{x^2}{12a^2} + \frac{\log(1-a^2x^2)}{3a^4} + \frac{x \coth^{-1}(ax)}{2a^3} - \frac{\coth^{-1}(ax)^2}{4a^4} + \frac{1}{4}x^4 \coth^{-1}(ax)^2 + \frac{x^3 \coth^{-1}(ax)}{6a}$$

Antiderivative was successfully verified.

[In] Int[x^3\*ArcCoth[a\*x]^2,x]

[Out]  $x^2/(12*a^2) + (x*ArcCoth[a*x])/(2*a^3) + (x^3*ArcCoth[a*x])/(6*a) - ArcCoth[a*x]^2/(4*a^4) + (x^4*ArcCoth[a*x]^2)/4 + Log[1 - a^2*x^2]/(3*a^4)$

#### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^ (p_.)*((d_.)*(x_))^(m_.), x_Symbol]
  :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 5981

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^ (p_.)*((f_.)*(x_))^(m_.))/((d_.) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

#### Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

### Rule 43

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rule 5911

```
Int[((a_) + ArcCoth[(c_)*(x_)])*(b_)^(p_), x_Symbol] := Simp[x*(a + b*Arc
Coth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

### Rule 260

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

### Rule 5949

```
Int[((a_) + ArcCoth[(c_)*(x_)])*(b_)^(p_)/((d_) + (e_)*(x_)^2), x_Symb
ol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
\int x^3 \coth^{-1}(ax)^2 dx &= \frac{1}{4}x^4 \coth^{-1}(ax)^2 - \frac{1}{2}a \int \frac{x^4 \coth^{-1}(ax)}{1-a^2x^2} dx \\
&= \frac{1}{4}x^4 \coth^{-1}(ax)^2 + \frac{\int x^2 \coth^{-1}(ax) dx}{2a} - \frac{\int \frac{x^2 \coth^{-1}(ax)}{1-a^2x^2} dx}{2a} \\
&= \frac{x^3 \coth^{-1}(ax)}{6a} + \frac{1}{4}x^4 \coth^{-1}(ax)^2 - \frac{1}{6} \int \frac{x^3}{1-a^2x^2} dx + \frac{\int \coth^{-1}(ax) dx}{2a^3} - \frac{\int \frac{\coth^{-1}(ax)}{1-a^2x^2} dx}{2a^3} \\
&= \frac{x \coth^{-1}(ax)}{2a^3} + \frac{x^3 \coth^{-1}(ax)}{6a} - \frac{\coth^{-1}(ax)^2}{4a^4} + \frac{1}{4}x^4 \coth^{-1}(ax)^2 - \frac{1}{12} \text{Subst} \left( \int \frac{x}{1-a^2x} dx, x, x \right) \\
&= \frac{x \coth^{-1}(ax)}{2a^3} + \frac{x^3 \coth^{-1}(ax)}{6a} - \frac{\coth^{-1}(ax)^2}{4a^4} + \frac{1}{4}x^4 \coth^{-1}(ax)^2 + \frac{\log(1-a^2x^2)}{4a^4} - \frac{1}{12} \text{Subst} \left( \int \frac{x}{1-a^2x} dx, x, x \right) \\
&= \frac{x^2}{12a^2} + \frac{x \coth^{-1}(ax)}{2a^3} + \frac{x^3 \coth^{-1}(ax)}{6a} - \frac{\coth^{-1}(ax)^2}{4a^4} + \frac{1}{4}x^4 \coth^{-1}(ax)^2 + \frac{\log(1-a^2x^2)}{3a^4}
\end{aligned}$$

**Mathematica [A]** time = 0.0167572, size = 62, normalized size = 0.77

$$\frac{a^2x^2 + 4 \log(1 - a^2x^2) + 2ax(a^2x^2 + 3) \coth^{-1}(ax) + 3(a^4x^4 - 1) \coth^{-1}(ax)^2}{12a^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*ArcCoth[a\*x]^2,x]

[Out] (a^2\*x^2 + 2\*a\*x\*(3 + a^2\*x^2)\*ArcCoth[a\*x] + 3\*(-1 + a^4\*x^4)\*ArcCoth[a\*x]^2 + 4\*Log[1 - a^2\*x^2])/(12\*a^4)

**Maple [B]** time = 0.052, size = 176, normalized size = 2.2

$$\frac{x^4 (\operatorname{arccoth}(ax))^2}{4} + \frac{x^3 \operatorname{arccoth}(ax)}{6a} + \frac{x \operatorname{arccoth}(ax)}{2a^3} + \frac{\operatorname{arccoth}(ax) \ln(ax-1)}{4a^4} - \frac{\operatorname{arccoth}(ax) \ln(ax+1)}{4a^4} + \frac{(\ln(ax-1) - \ln(ax+1))}{16a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*arccoth(a\*x)^2,x)

[Out] 1/4\*x^4\*arccoth(a\*x)^2+1/6\*x^3\*arccoth(a\*x)/a+1/2\*x\*arccoth(a\*x)/a^3+1/4/a^4\*arccoth(a\*x)\*ln(a\*x-1)-1/4/a^4\*arccoth(a\*x)\*ln(a\*x+1)+1/16/a^4\*ln(a\*x-1)^2-1/16/a^4\*ln(a\*x+1)^2

$$2-1/8/a^4*\ln(a*x-1)*\ln(1/2+1/2*a*x)-1/8/a^4*\ln(-1/2*a*x+1/2)*\ln(a*x+1)+1/8/a^4*\ln(-1/2*a*x+1/2)*\ln(1/2+1/2*a*x)+1/16/a^4*\ln(a*x+1)^2+1/12*x^2/a^2+1/3/a^4*\ln(a*x-1)+1/3/a^4*\ln(a*x+1)$$

**Maxima [A]** time = 0.955027, size = 159, normalized size = 1.96

$$\frac{1}{4}x^4 \operatorname{arccoth}(ax)^2 + \frac{1}{12}a \left( \frac{2(a^2x^3 + 3x)}{a^4} - \frac{3 \log(ax + 1)}{a^5} + \frac{3 \log(ax - 1)}{a^5} \right) \operatorname{arccoth}(ax) + \frac{4a^2x^2 - 2(3 \log(ax - 1) - 8 \log(ax + 1) + 3 \log(ax + 1)^2 + 3 \log(ax - 1)^2 + 16 \log(ax - 1))}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(a\*x)^2,x, algorithm="maxima")

[Out] 1/4\*x^4\*arccoth(a\*x)^2 + 1/12\*a\*(2\*(a^2\*x^3 + 3\*x)/a^4 - 3\*log(a\*x + 1)/a^5 + 3\*log(a\*x - 1)/a^5)\*arccoth(a\*x) + 1/48\*(4\*a^2\*x^2 - 2\*(3\*log(a\*x - 1) - 8)\*log(a\*x + 1) + 3\*log(a\*x + 1)^2 + 3\*log(a\*x - 1)^2 + 16\*log(a\*x - 1))/a^4

**Fricas [A]** time = 1.59719, size = 184, normalized size = 2.27

$$\frac{4a^2x^2 + 3(a^4x^4 - 1)\log\left(\frac{ax+1}{ax-1}\right)^2 + 4(a^3x^3 + 3ax)\log\left(\frac{ax+1}{ax-1}\right) + 16\log(a^2x^2 - 1)}{48a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(a\*x)^2,x, algorithm="fricas")

[Out] 1/48\*(4\*a^2\*x^2 + 3\*(a^4\*x^4 - 1)\*log((a\*x + 1)/(a\*x - 1))^2 + 4\*(a^3\*x^3 + 3\*a\*x)\*log((a\*x + 1)/(a\*x - 1)) + 16\*log(a^2\*x^2 - 1))/a^4

**Sympy [A]** time = 5.10093, size = 90, normalized size = 1.11

$$\begin{cases} \frac{x^4 \operatorname{acoth}^2(ax)}{\pi^2 x^4} + \frac{x^3 \operatorname{acoth}(ax)}{6a} + \frac{x^2}{12a^2} + \frac{x \operatorname{acoth}(ax)}{2a^3} + \frac{2 \log(ax+1)}{3a^4} - \frac{\operatorname{acoth}^2(ax)}{4a^4} - \frac{2 \operatorname{acoth}(ax)}{3a^4} & \text{for } a \neq 0 \\ -\frac{\pi^2 x^4}{16} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acoth(a*x)**2,x)
```

```
[Out] Piecewise((x**4*acoth(a*x)**2/4 + x**3*acoth(a*x)/(6*a) + x**2/(12*a**2) +
x*acoth(a*x)/(2*a**3) + 2*log(a*x + 1)/(3*a**4) - acoth(a*x)**2/(4*a**4) -
2*acoth(a*x)/(3*a**4), Ne(a, 0)), (-pi**2*x**4/16, True))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arccoth}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*arccoth(a*x)^2, x)
```



### 3.15 $\int x^2 \coth^{-1}(ax)^2 dx$

**Optimal.** Leaf size=103

$$-\frac{\text{PolyLog}\left(2, 1 - \frac{2}{1-ax}\right)}{3a^3} + \frac{x}{3a^2} - \frac{\tanh^{-1}(ax)}{3a^3} + \frac{\coth^{-1}(ax)^2}{3a^3} - \frac{2 \log\left(\frac{2}{1-ax}\right) \coth^{-1}(ax)}{3a^3} + \frac{1}{3}x^3 \coth^{-1}(ax)^2 + \frac{x^2 \coth^{-1}(ax)}{3a}$$

[Out] x/(3\*a^2) + (x^2\*ArcCoth[a\*x])/(3\*a) + ArcCoth[a\*x]^2/(3\*a^3) + (x^3\*ArcCoth[a\*x]^2)/3 - ArcTanh[a\*x]/(3\*a^3) - (2\*ArcCoth[a\*x]\*Log[2/(1 - a\*x)])/(3\*a^3) - PolyLog[2, 1 - 2/(1 - a\*x)]/(3\*a^3)

**Rubi [A]** time = 0.153214, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.8$ , Rules used = {5917, 5981, 321, 206, 5985, 5919, 2402, 2315}

$$-\frac{\text{PolyLog}\left(2, 1 - \frac{2}{1-ax}\right)}{3a^3} + \frac{x}{3a^2} - \frac{\tanh^{-1}(ax)}{3a^3} + \frac{\coth^{-1}(ax)^2}{3a^3} - \frac{2 \log\left(\frac{2}{1-ax}\right) \coth^{-1}(ax)}{3a^3} + \frac{1}{3}x^3 \coth^{-1}(ax)^2 + \frac{x^2 \coth^{-1}(ax)}{3a}$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcCoth[a\*x]^2,x]

[Out] x/(3\*a^2) + (x^2\*ArcCoth[a\*x])/(3\*a) + ArcCoth[a\*x]^2/(3\*a^3) + (x^3\*ArcCoth[a\*x]^2)/3 - ArcTanh[a\*x]/(3\*a^3) - (2\*ArcCoth[a\*x]\*Log[2/(1 - a\*x)])/(3\*a^3) - PolyLog[2, 1 - 2/(1 - a\*x)]/(3\*a^3)

#### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^ (p_.)*((d_.)*(x_))^(m_.), x_Symbol]
  :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 5981

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^ (p_.)*((f_.)*(x_))^(m_.))/((d_.) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

]

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5985

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.)*(x_)/((d_) + (e_.)*(x_)^2),
x_Symbol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] := -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[(a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)]]/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)/((d_) + (e_.)*(x_))], x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(ax)^2 dx &= \frac{1}{3}x^3 \coth^{-1}(ax)^2 - \frac{1}{3}(2a) \int \frac{x^3 \coth^{-1}(ax)}{1-a^2x^2} dx \\
&= \frac{1}{3}x^3 \coth^{-1}(ax)^2 + \frac{2 \int x \coth^{-1}(ax) dx}{3a} - \frac{2 \int \frac{x \coth^{-1}(ax)}{1-a^2x^2} dx}{3a} \\
&= \frac{x^2 \coth^{-1}(ax)}{3a} + \frac{\coth^{-1}(ax)^2}{3a^3} + \frac{1}{3}x^3 \coth^{-1}(ax)^2 - \frac{1}{3} \int \frac{x^2}{1-a^2x^2} dx - \frac{2 \int \frac{\coth^{-1}(ax)}{1-ax} dx}{3a^2} \\
&= \frac{x}{3a^2} + \frac{x^2 \coth^{-1}(ax)}{3a} + \frac{\coth^{-1}(ax)^2}{3a^3} + \frac{1}{3}x^3 \coth^{-1}(ax)^2 - \frac{2 \coth^{-1}(ax) \log\left(\frac{2}{1-ax}\right)}{3a^3} - \frac{\int \frac{1}{1-a^2x^2} dx}{3a^2} \\
&= \frac{x}{3a^2} + \frac{x^2 \coth^{-1}(ax)}{3a} + \frac{\coth^{-1}(ax)^2}{3a^3} + \frac{1}{3}x^3 \coth^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)}{3a^3} - \frac{2 \coth^{-1}(ax) \log\left(\frac{2}{1-ax}\right)}{3a^3} \\
&= \frac{x}{3a^2} + \frac{x^2 \coth^{-1}(ax)}{3a} + \frac{\coth^{-1}(ax)^2}{3a^3} + \frac{1}{3}x^3 \coth^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)}{3a^3} - \frac{2 \coth^{-1}(ax) \log\left(\frac{2}{1-ax}\right)}{3a^3}
\end{aligned}$$

**Mathematica [A]** time = 0.23807, size = 66, normalized size = 0.64

$$\frac{\text{PolyLog}\left(2, e^{-2 \coth^{-1}(ax)}\right) + (a^3 x^3 - 1) \coth^{-1}(ax)^2 + \coth^{-1}(ax) \left(a^2 x^2 - 2 \log\left(1 - e^{-2 \coth^{-1}(ax)}\right) - 1\right) + ax}{3a^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2\*ArcCoth[a\*x]^2, x]

[Out] (a\*x + (-1 + a^3\*x^3)\*ArcCoth[a\*x]^2 + ArcCoth[a\*x]\*(-1 + a^2\*x^2 - 2\*Log[1 - E^(-2\*ArcCoth[a\*x])])) + PolyLog[2, E^(-2\*ArcCoth[a\*x])])/(3\*a^3)

**Maple [A]** time = 0.052, size = 176, normalized size = 1.7

$$\frac{x^3 (\operatorname{arccoth}(ax))^2}{3} + \frac{x^2 \operatorname{arccoth}(ax)}{3a} + \frac{\operatorname{arccoth}(ax) \ln(ax-1)}{3a^3} + \frac{\operatorname{arccoth}(ax) \ln(ax+1)}{3a^3} + \frac{x}{3a^2} + \frac{\ln(ax-1)}{6a^3} - \frac{\ln(ax+1)}{6a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(a\*x)^2, x)

```
[Out] 1/3*x^3*arccoth(a*x)^2+1/3*x^2*arccoth(a*x)/a+1/3/a^3*arccoth(a*x)*ln(a*x-1)
+1/3/a^3*arccoth(a*x)*ln(a*x+1)+1/3*x/a^2+1/6/a^3*ln(a*x-1)-1/6/a^3*ln(a*x
+1)+1/12/a^3*ln(a*x-1)^2-1/3/a^3*dilog(1/2+1/2*a*x)-1/6/a^3*ln(a*x-1)*ln(1/
2+1/2*a*x)+1/6/a^3*ln(-1/2*a*x+1/2)*ln(a*x+1)-1/6/a^3*ln(-1/2*a*x+1/2)*ln(1
/2+1/2*a*x)-1/12/a^3*ln(a*x+1)^2
```

**Maxima [A]** time = 0.974019, size = 181, normalized size = 1.76

$$\frac{1}{3}x^3 \operatorname{arccoth}(ax)^2 + \frac{1}{12}a^2 \left( \frac{4ax - \log(ax+1)^2 + 2 \log(ax+1) \log(ax-1) + \log(ax-1)^2 + 2 \log(ax-1)}{a^5} - \frac{4 \left( \log(ax} \right)}{a^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(a*x)^2 + 1/12*a^2*((4*a*x - log(a*x + 1)^2 + 2*log(a*x + 1)
*log(a*x - 1) + log(a*x - 1)^2 + 2*log(a*x - 1))/a^5 - 4*(log(a*x - 1)*log(
1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a^5 - 2*log(a*x + 1)/a^5) + 1/3*a*(
x^2/a^2 + log(a^2*x^2 - 1)/a^4)*arccoth(a*x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(x^2 \operatorname{arccoth}(ax)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*arccoth(a*x)^2, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{acoth}^2(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(a*x)**2,x)
```

```
[Out] Integral(x**2*acoth(a*x)**2, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arccoth}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(a*x)^2, x)
```

### 3.16 $\int x \coth^{-1}(ax)^2 dx$

**Optimal.** Leaf size=54

$$\frac{\log(1 - a^2x^2)}{2a^2} - \frac{\coth^{-1}(ax)^2}{2a^2} + \frac{1}{2}x^2 \coth^{-1}(ax)^2 + \frac{x \coth^{-1}(ax)}{a}$$

[Out] (x\*ArcCoth[a\*x])/a - ArcCoth[a\*x]^2/(2\*a^2) + (x^2\*ArcCoth[a\*x]^2)/2 + Log[1 - a^2\*x^2]/(2\*a^2)

**Rubi [A]** time = 0.0783636, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$ , Rules used = {5917, 5981, 5911, 260, 5949}

$$\frac{\log(1 - a^2x^2)}{2a^2} - \frac{\coth^{-1}(ax)^2}{2a^2} + \frac{1}{2}x^2 \coth^{-1}(ax)^2 + \frac{x \coth^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

[In] Int[x\*ArcCoth[a\*x]^2,x]

[Out] (x\*ArcCoth[a\*x])/a - ArcCoth[a\*x]^2/(2\*a^2) + (x^2\*ArcCoth[a\*x]^2)/2 + Log[1 - a^2\*x^2]/(2\*a^2)

#### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^ (p_.)*((d_.)*(x_))^(m_.), x_Symbol]
  :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 5981

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^ (p_.)*((f_.)*(x_))^(m_.))/((d_.) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

#### Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*ArcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

### Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

### Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
 \int x \coth^{-1}(ax)^2 dx &= \frac{1}{2}x^2 \coth^{-1}(ax)^2 - a \int \frac{x^2 \coth^{-1}(ax)}{1 - a^2x^2} dx \\
 &= \frac{1}{2}x^2 \coth^{-1}(ax)^2 + \frac{\int \coth^{-1}(ax) dx}{a} - \frac{\int \frac{\coth^{-1}(ax)}{1 - a^2x^2} dx}{a} \\
 &= \frac{x \coth^{-1}(ax)}{a} - \frac{\coth^{-1}(ax)^2}{2a^2} + \frac{1}{2}x^2 \coth^{-1}(ax)^2 - \int \frac{x}{1 - a^2x^2} dx \\
 &= \frac{x \coth^{-1}(ax)}{a} - \frac{\coth^{-1}(ax)^2}{2a^2} + \frac{1}{2}x^2 \coth^{-1}(ax)^2 + \frac{\log(1 - a^2x^2)}{2a^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.012074, size = 43, normalized size = 0.8

$$\frac{\log(1 - a^2x^2) + (a^2x^2 - 1) \coth^{-1}(ax)^2 + 2ax \coth^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[a*x]^2, x]
```

```
[Out] (2*a*x*ArcCoth[a*x] + (-1 + a^2*x^2)*ArcCoth[a*x]^2 + Log[1 - a^2*x^2])/(2*a^2)
```

**Maple [B]** time = 0.056, size = 155, normalized size = 2.9

$$\frac{x^2 (\operatorname{arccoth}(ax))^2}{2} + \frac{x \operatorname{arccoth}(ax)}{a} + \frac{\operatorname{arccoth}(ax) \ln(ax-1)}{2a^2} - \frac{\operatorname{arccoth}(ax) \ln(ax+1)}{2a^2} + \frac{(\ln(ax-1))^2}{8a^2} - \frac{\ln(ax-1)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(a\*x)^2,x)

[Out] 1/2\*x^2\*arccoth(a\*x)^2+x\*arccoth(a\*x)/a+1/2/a^2\*arccoth(a\*x)\*ln(a\*x-1)-1/2/a^2\*arccoth(a\*x)\*ln(a\*x+1)+1/8/a^2\*ln(a\*x-1)^2-1/4/a^2\*ln(a\*x-1)\*ln(1/2+1/2\*a\*x)+1/2/a^2\*ln(a\*x-1)+1/2/a^2\*ln(a\*x+1)-1/4/a^2\*ln(-1/2\*a\*x+1/2)\*ln(a\*x+1)+1/4/a^2\*ln(-1/2\*a\*x+1/2)\*ln(1/2+1/2\*a\*x)+1/8/a^2\*ln(a\*x+1)^2

**Maxima [B]** time = 0.986487, size = 131, normalized size = 2.43

$$\frac{1}{2} x^2 \operatorname{arccoth}(ax)^2 + \frac{1}{2} a \left( \frac{2x}{a^2} - \frac{\log(ax+1)}{a^3} + \frac{\log(ax-1)}{a^3} \right) \operatorname{arccoth}(ax) - \frac{2(\log(ax-1) - 2)\log(ax+1) - \log(ax+1)^2}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(a\*x)^2,x, algorithm="maxima")

[Out] 1/2\*x^2\*arccoth(a\*x)^2 + 1/2\*a\*(2\*x/a^2 - log(a\*x + 1)/a^3 + log(a\*x - 1)/a^3)\*arccoth(a\*x) - 1/8\*(2\*(log(a\*x - 1) - 2)\*log(a\*x + 1) - log(a\*x + 1)^2 - log(a\*x - 1)^2 - 4\*log(a\*x - 1))/a^2

**Fricas [A]** time = 1.50581, size = 143, normalized size = 2.65

$$\frac{4ax \log\left(\frac{ax+1}{ax-1}\right) + (a^2x^2 - 1) \log\left(\frac{ax+1}{ax-1}\right)^2 + 4 \log(a^2x^2 - 1)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(a\*x)^2,x, algorithm="fricas")

[Out] 1/8\*(4\*a\*x\*log((a\*x + 1)/(a\*x - 1)) + (a^2\*x^2 - 1)\*log((a\*x + 1)/(a\*x - 1))^2 + 4\*log(a^2\*x^2 - 1))/a^2



---

**Sympy [A]** time = 4.52711, size = 60, normalized size = 1.11

$$\begin{cases} \frac{x^2 \operatorname{acoth}^2(ax)}{2} + \frac{x \operatorname{acoth}(ax)}{a} + \frac{\log(ax+1)}{a^2} - \frac{\operatorname{acoth}^2(ax)}{2a^2} - \frac{\operatorname{acoth}(ax)}{a^2} & \text{for } a \neq 0 \\ -\frac{\pi^2 x^2}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(a\*x)\*\*2,x)

[Out] Piecewise((x\*\*2\*acoth(a\*x)\*\*2/2 + x\*acoth(a\*x)/a + log(a\*x + 1)/a\*\*2 - acot  
h(a\*x)\*\*2/(2\*a\*\*2) - acoth(a\*x)/a\*\*2, Ne(a, 0)), (-pi\*\*2\*x\*\*2/8, True))

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arccoth}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(a\*x)^2,x, algorithm="giac")

[Out] integrate(x\*arccoth(a\*x)^2, x)

### 3.17 $\int \coth^{-1}(ax)^2 dx$

**Optimal.** Leaf size=58

$$-\frac{\text{PolyLog}\left(2, 1 - \frac{2}{1-ax}\right)}{a} + x \coth^{-1}(ax)^2 + \frac{\coth^{-1}(ax)^2}{a} - \frac{2 \log\left(\frac{2}{1-ax}\right) \coth^{-1}(ax)}{a}$$

[Out] ArcCoth[a\*x]^2/a + x\*ArcCoth[a\*x]^2 - (2\*ArcCoth[a\*x]\*Log[2/(1 - a\*x)])/a - PolyLog[2, 1 - 2/(1 - a\*x)]/a

**Rubi [A]** time = 0.0777673, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$ , Rules used = {5911, 5985, 5919, 2402, 2315}

$$-\frac{\text{PolyLog}\left(2, 1 - \frac{2}{1-ax}\right)}{a} + x \coth^{-1}(ax)^2 + \frac{\coth^{-1}(ax)^2}{a} - \frac{2 \log\left(\frac{2}{1-ax}\right) \coth^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]^2, x]

[Out] ArcCoth[a\*x]^2/a + x\*ArcCoth[a\*x]^2 - (2\*ArcCoth[a\*x]\*Log[2/(1 - a\*x)])/a - PolyLog[2, 1 - 2/(1 - a\*x)]/a

#### Rule 5911

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.), x\_Symbol] := Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

#### Rule 5985

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*(x\_.))/((d\_.) + (e\_.)\*(x\_.)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*e\*(p + 1)), x] + Dist[1/(c\*d), Int[(a + b\*ArcCoth[c\*x])^p/(1 - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && IGtQ[p, 0]

#### Rule 5919

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] := -Simp[(a + b\*ArcCoth[c\*x])^p\*Log[2/(1 + (e\*x)/d)]/e, x] + Dist[(b\*c

p)/e, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*Log[2/(1 + (e\*x)/d)]/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

### Rule 2402

Int[Log[(c\_.)/((d\_) + (e\_.)\*(x\_))]/((f\_) + (g\_.)\*(x\_)^2), x\_Symbol] := -Dist[e/g, Subst[Int[Log[2\*d\*x]/(1 - 2\*d\*x), x], x, 1/(d + e\*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2\*d] && EqQ[e^2\*f + d^2\*g, 0]

### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

### Rubi steps

$$\begin{aligned}
 \int \coth^{-1}(ax)^2 dx &= x \coth^{-1}(ax)^2 - (2a) \int \frac{x \coth^{-1}(ax)}{1 - a^2x^2} dx \\
 &= \frac{\coth^{-1}(ax)^2}{a} + x \coth^{-1}(ax)^2 - 2 \int \frac{\coth^{-1}(ax)}{1 - ax} dx \\
 &= \frac{\coth^{-1}(ax)^2}{a} + x \coth^{-1}(ax)^2 - \frac{2 \coth^{-1}(ax) \log\left(\frac{2}{1-ax}\right)}{a} + 2 \int \frac{\log\left(\frac{2}{1-ax}\right)}{1 - a^2x^2} dx \\
 &= \frac{\coth^{-1}(ax)^2}{a} + x \coth^{-1}(ax)^2 - \frac{2 \coth^{-1}(ax) \log\left(\frac{2}{1-ax}\right)}{a} - \frac{2 \operatorname{Subst}\left(\int \frac{\log(2x)}{1-2x} dx, x, \frac{1}{1-ax}\right)}{a} \\
 &= \frac{\coth^{-1}(ax)^2}{a} + x \coth^{-1}(ax)^2 - \frac{2 \coth^{-1}(ax) \log\left(\frac{2}{1-ax}\right)}{a} - \frac{\operatorname{Li}_2\left(1 - \frac{2}{1-ax}\right)}{a}
 \end{aligned}$$

**Mathematica [A]** time = 0.0836344, size = 46, normalized size = 0.79

$$\frac{\operatorname{PolyLog}\left(2, e^{-2 \coth^{-1}(ax)}\right) + \coth^{-1}(ax) \left( (ax - 1) \coth^{-1}(ax) - 2 \log\left(1 - e^{-2 \coth^{-1}(ax)}\right) \right)}{a}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a\*x]^2, x]

[Out] (ArcCoth[a\*x]\*((-1 + a\*x)\*ArcCoth[a\*x] - 2\*Log[1 - E^(-2\*ArcCoth[a\*x])])) + PolyLog[2, E^(-2\*ArcCoth[a\*x])]/a

---

**Maple [B]** time = 0.119, size = 122, normalized size = 2.1

$$x(\operatorname{arccoth}(ax))^2 - 2 \frac{\operatorname{arccoth}(ax)}{a} \ln\left(1 + \frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right) - 2 \frac{\operatorname{arccoth}(ax)}{a} \ln\left(1 - \frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right) + \frac{(\operatorname{arccoth}(ax))^2}{a} - 2 \frac{1}{a} \operatorname{polylog}\left(2, \frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right) - 2 \frac{1}{a} \operatorname{polylog}\left(2, -\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)^2,x)

[Out] x\*arccoth(a\*x)^2-2/a\*arccoth(a\*x)\*ln(1+1/((a\*x-1)/(a\*x+1))^(1/2))-2/a\*arccoth(a\*x)\*ln(1-1/((a\*x-1)/(a\*x+1))^(1/2))+arccoth(a\*x)^2/a-2/a\*polylog(2,-1/((a\*x-1)/(a\*x+1))^(1/2))-2/a\*polylog(2,1/((a\*x-1)/(a\*x+1))^(1/2))

---

**Maxima [B]** time = 0.986916, size = 182, normalized size = 3.14

$$x \operatorname{arccoth}(ax)^2 + \frac{1}{4} \left( a \left( \frac{\log(ax+1)^2 + 2 \log(ax+1) \log(ax-1) - \log(ax-1)^2}{a^3} - \frac{4 \left( \log(ax-1) \log\left(\frac{1}{2}ax + \frac{1}{2}\right) + \operatorname{Li}_2\left(-\frac{1}{2}ax + \frac{1}{2}\right)\right)}{a^3} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^2,x, algorithm="maxima")

[Out] x\*arccoth(a\*x)^2 + 1/4\*(a\*((log(a\*x + 1)^2 + 2\*log(a\*x + 1)\*log(a\*x - 1) - log(a\*x - 1)^2)/a^3 - 4\*(log(a\*x - 1)\*log(1/2\*a\*x + 1/2) + dilog(-1/2\*a\*x + 1/2))/a^3) - 2\*(log(a\*x + 1)/a - log(a\*x - 1)/a)\*log(a^2\*x^2 - 1)/a + a\*arccoth(a\*x)\*log(a^2\*x^2 - 1)/a

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\operatorname{arccoth}(ax)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^2,x, algorithm="fricas")

[Out] `integral(arccoth(a*x)^2, x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acoth}^2(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(a*x)**2,x)`

[Out] `Integral(acoth(a*x)**2, x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(a*x)^2,x, algorithm="giac")`

[Out] `integrate(arccoth(a*x)^2, x)`

$$3.18 \quad \int \frac{\coth^{-1}(ax)^2}{x} dx$$

**Optimal.** Leaf size=97

$$\frac{1}{2}\text{PolyLog}\left(3, 1 - \frac{2}{ax+1}\right) - \frac{1}{2}\text{PolyLog}\left(3, 1 - \frac{2ax}{ax+1}\right) + \coth^{-1}(ax)\text{PolyLog}\left(2, 1 - \frac{2}{ax+1}\right) - \coth^{-1}(ax)\text{PolyLog}\left(2, 1 - \frac{2ax}{ax+1}\right)$$

[Out] 2\*ArcCoth[a\*x]^2\*ArcCoth[1 - 2/(1 - a\*x)] + ArcCoth[a\*x]\*PolyLog[2, 1 - 2/(1 + a\*x)] - ArcCoth[a\*x]\*PolyLog[2, 1 - (2\*a\*x)/(1 + a\*x)] + PolyLog[3, 1 - 2/(1 + a\*x)]/2 - PolyLog[3, 1 - (2\*a\*x)/(1 + a\*x)]/2

**Rubi [A]** time = 0.232009, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {5915, 6053, 5949, 6057, 6610}

$$\frac{1}{2}\text{PolyLog}\left(3, 1 - \frac{2}{ax+1}\right) - \frac{1}{2}\text{PolyLog}\left(3, 1 - \frac{2ax}{ax+1}\right) + \coth^{-1}(ax)\text{PolyLog}\left(2, 1 - \frac{2}{ax+1}\right) - \coth^{-1}(ax)\text{PolyLog}\left(2, 1 - \frac{2ax}{ax+1}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]^2/x, x]

[Out] 2\*ArcCoth[a\*x]^2\*ArcCoth[1 - 2/(1 - a\*x)] + ArcCoth[a\*x]\*PolyLog[2, 1 - 2/(1 + a\*x)] - ArcCoth[a\*x]\*PolyLog[2, 1 - (2\*a\*x)/(1 + a\*x)] + PolyLog[3, 1 - 2/(1 + a\*x)]/2 - PolyLog[3, 1 - (2\*a\*x)/(1 + a\*x)]/2

### Rule 5915

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^p]/(x\_), x\_Symbol] := Simp[2\*(a + b\*ArcCoth[c\*x])^p\*ArcCoth[1 - 2/(1 - c\*x)], x] - Dist[2\*b\*c\*p, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*ArcCoth[1 - 2/(1 - c\*x)])/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1]

### Rule 6053

Int[(ArcCoth[u\_] \* ((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^p) / ((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] := Dist[1/2, Int[(Log[SimplifyIntegrand[1 + 1/u, x]]\*(a + b\*ArcCoth[c\*x])^p) / (d + e\*x^2), x], x] - Dist[1/2, Int[(Log[SimplifyIntegrand[1 - 1/u, x]]\*(a + b\*ArcCoth[c\*x])^p) / (d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c\*x))^2, 0]

Rule 5949

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

Rule 6057

Int[(Log[u\_] \* ((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)) / ((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[((a + b\*ArcCoth[c\*x])^p \* PolyLog[2, 1 - u]) / (2\*c\*d), x] - Dist[(b\*p)/2, Int[((a + b\*ArcCoth[c\*x])^(p - 1) \* PolyLog[2, 1 - u]) / (d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c\*x))^2, 0]

Rule 6610

Int[(u\_)\*PolyLog[n\_, v\_], x\_Symbol] := With[{w = DerivativeDivides[v, u\*v, x]}, Simp[w\*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(ax)^2}{x} dx &= 2 \coth^{-1}(ax)^2 \coth^{-1}\left(1 - \frac{2}{1 - ax}\right) - (4a) \int \frac{\coth^{-1}(ax) \coth^{-1}\left(1 - \frac{2}{1 - ax}\right)}{1 - a^2x^2} dx \\ &= 2 \coth^{-1}(ax)^2 \coth^{-1}\left(1 - \frac{2}{1 - ax}\right) + (2a) \int \frac{\coth^{-1}(ax) \log\left(\frac{2}{1 + ax}\right)}{1 - a^2x^2} dx - (2a) \int \frac{\coth^{-1}(ax) \log\left(\frac{2}{1 - ax}\right)}{1 - a^2x^2} dx \\ &= 2 \coth^{-1}(ax)^2 \coth^{-1}\left(1 - \frac{2}{1 - ax}\right) + \coth^{-1}(ax) \text{Li}_2\left(1 - \frac{2}{1 + ax}\right) - \coth^{-1}(ax) \text{Li}_2\left(1 - \frac{2ax}{1 + ax}\right) - \\ &= 2 \coth^{-1}(ax)^2 \coth^{-1}\left(1 - \frac{2}{1 - ax}\right) + \coth^{-1}(ax) \text{Li}_2\left(1 - \frac{2}{1 + ax}\right) - \coth^{-1}(ax) \text{Li}_2\left(1 - \frac{2ax}{1 + ax}\right) + \end{aligned}$$

**Mathematica [A]** time = 0.0634424, size = 114, normalized size = 1.18

$$-\coth^{-1}(ax) \text{PolyLog}\left(2, -e^{-2\coth^{-1}(ax)}\right) - \coth^{-1}(ax) \text{PolyLog}\left(2, e^{2\coth^{-1}(ax)}\right) - \frac{1}{2} \text{PolyLog}\left(3, -e^{-2\coth^{-1}(ax)}\right) + \frac{1}{2} \text{PolyLog}\left(3, e^{2\coth^{-1}(ax)}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a\*x]^2/x, x]

```
[Out] (2*ArcCoth[a*x]^3)/3 + ArcCoth[a*x]^2*Log[1 + E^(-2*ArcCoth[a*x])] - ArcCoth[a*x]^2*Log[1 - E^(2*ArcCoth[a*x])] - ArcCoth[a*x]*PolyLog[2, -E^(-2*ArcCoth[a*x])] - ArcCoth[a*x]*PolyLog[2, E^(2*ArcCoth[a*x])] - PolyLog[3, -E^(-2*ArcCoth[a*x])]/2 + PolyLog[3, E^(2*ArcCoth[a*x])]/2
```

**Maple [C]** time = 0.381, size = 487, normalized size = 5.

$$\ln(ax) (\operatorname{arccoth}(ax))^2 + \frac{i}{2}\pi \operatorname{csgn}\left(i\left(\frac{ax+1}{ax-1} + 1\right)\right) \operatorname{csgn}\left(i\left(\frac{ax+1}{ax-1} - 1\right)^{-1}\right) \operatorname{csgn}\left(i\left(\frac{ax+1}{ax-1} + 1\right)\left(\frac{ax+1}{ax-1} - 1\right)^{-1}\right) (\operatorname{arccoth}(ax))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)^2/x, x)
```

```
[Out] ln(a*x)*arccoth(a*x)^2+1/2*I*Pi*csgn(I*((a*x+1)/(a*x-1)+1))*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I/((a*x+1)/(a*x-1)-1))*((a*x+1)/(a*x-1)+1)*arccoth(a*x)^2-1/2*I*Pi*csgn(I*((a*x+1)/(a*x-1)+1))*csgn(I/((a*x+1)/(a*x-1)-1))*((a*x+1)/(a*x-1)+1))^2*arccoth(a*x)^2-1/2*I*Pi*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I/((a*x+1)/(a*x-1)-1))*((a*x+1)/(a*x-1)+1))^2*arccoth(a*x)^2+1/2*I*Pi*csgn(I/((a*x+1)/(a*x-1)-1))*((a*x+1)/(a*x-1)+1))^3*arccoth(a*x)^2+arccoth(a*x)^2*ln((a*x+1)/(a*x-1)-1)-arccoth(a*x)^2*ln(1-1/((a*x-1)/(a*x+1))^(1/2))-2*arccoth(a*x)*polylog(2,1/((a*x-1)/(a*x+1))^(1/2))+2*polylog(3,1/((a*x-1)/(a*x+1))^(1/2))-arccoth(a*x)^2*ln(1+1/((a*x-1)/(a*x+1))^(1/2))-2*arccoth(a*x)*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))+2*polylog(3,-1/((a*x-1)/(a*x+1))^(1/2))+arccoth(a*x)*polylog(2,-(a*x+1)/(a*x-1))-1/2*polylog(3,-(a*x+1)/(a*x-1))
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(ax)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x, x, algorithm="maxima")
```

```
[Out] integrate(arccoth(a*x)^2/x, x)
```



**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arcoth}(ax)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^2/x,x, algorithm="fricas")

[Out] integral(arccoth(a\*x)^2/x, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{acoth}^2(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(a\*x)\*\*2/x,x)

[Out] Integral(acoth(a\*x)\*\*2/x, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arcoth}(ax)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^2/x,x, algorithm="giac")

[Out] integrate(arccoth(a\*x)^2/x, x)

### 3.19 $\int \frac{\coth^{-1}(ax)^2}{x^2} dx$

**Optimal.** Leaf size=55

$$-a \operatorname{PolyLog}\left(2, \frac{2}{ax+1} - 1\right) + a \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{x} + 2a \log\left(2 - \frac{2}{ax+1}\right) \coth^{-1}(ax)$$

[Out] a\*ArcCoth[a\*x]^2 - ArcCoth[a\*x]^2/x + 2\*a\*ArcCoth[a\*x]\*Log[2 - 2/(1 + a\*x)] - a\*PolyLog[2, -1 + 2/(1 + a\*x)]

**Rubi [A]** time = 0.108329, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {5917, 5989, 5933, 2447}

$$-a \operatorname{PolyLog}\left(2, \frac{2}{ax+1} - 1\right) + a \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{x} + 2a \log\left(2 - \frac{2}{ax+1}\right) \coth^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]^2/x^2,x]

[Out] a\*ArcCoth[a\*x]^2 - ArcCoth[a\*x]^2/x + 2\*a\*ArcCoth[a\*x]\*Log[2 - 2/(1 + a\*x)] - a\*PolyLog[2, -1 + 2/(1 + a\*x)]

#### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^ (p_.)*((d_.)*(x_))^(m_.), x_Symbol]
  :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 5989

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^ (p_.)/((x_)*((d_.) + (e_.)*(x_)^2)),
  x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcCoth[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

#### Rule 5933

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] := Simp[((a + b*ArcCoth[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/
(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

### Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

### Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(ax)^2}{x^2} dx &= -\frac{\coth^{-1}(ax)^2}{x} + (2a) \int \frac{\coth^{-1}(ax)}{x(1 - a^2x^2)} dx \\ &= a \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{x} + (2a) \int \frac{\coth^{-1}(ax)}{x(1 + ax)} dx \\ &= a \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{x} + 2a \coth^{-1}(ax) \log\left(2 - \frac{2}{1 + ax}\right) - (2a^2) \int \frac{\log\left(2 - \frac{2}{1 + ax}\right)}{1 - a^2x^2} dx \\ &= a \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{x} + 2a \coth^{-1}(ax) \log\left(2 - \frac{2}{1 + ax}\right) - a \operatorname{Li}_2\left(-1 + \frac{2}{1 + ax}\right) \end{aligned}$$

**Mathematica [A]** time = 0.100025, size = 49, normalized size = 0.89

$$-a \operatorname{PolyLog}\left(2, -e^{-2 \coth^{-1}(ax)}\right) + \frac{(ax - 1) \coth^{-1}(ax)^2}{x} + 2a \coth^{-1}(ax) \log\left(e^{-2 \coth^{-1}(ax)} + 1\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[a*x]^2/x^2, x]
```

```
[Out] ((-1 + a*x)*ArcCoth[a*x]^2)/x + 2*a*ArcCoth[a*x]*Log[1 + E^(-2*ArcCoth[a*x]
)] - a*PolyLog[2, -E^(-2*ArcCoth[a*x])]
```

**Maple [B]** time = 0.056, size = 159, normalized size = 2.9

$$-\frac{(\operatorname{arccoth}(ax))^2}{x} - a\operatorname{arccoth}(ax)\ln(ax-1) + 2a\operatorname{arccoth}(ax)\ln(ax) - a\operatorname{arccoth}(ax)\ln(ax+1) - \frac{a(\ln(ax-1))^2}{4} + a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)^2/x^2,x)

[Out] -arccoth(a\*x)^2/x-a\*arccoth(a\*x)\*ln(a\*x-1)+2\*a\*arccoth(a\*x)\*ln(a\*x)-a\*arccoth(a\*x)\*ln(a\*x+1)-1/4\*a\*ln(a\*x-1)^2+a\*dilog(1/2+1/2\*a\*x)+1/2\*a\*ln(a\*x-1)\*ln(1/2+1/2\*a\*x)+1/2\*a\*ln(-1/2\*a\*x+1/2)\*ln(1/2+1/2\*a\*x)-1/2\*a\*ln(-1/2\*a\*x+1/2)\*ln(a\*x+1)+1/4\*a\*ln(a\*x+1)^2-a\*dilog(a\*x)-a\*dilog(a\*x+1)-a\*ln(a\*x)\*ln(a\*x+1)

**Maxima [B]** time = 0.981925, size = 197, normalized size = 3.58

$$\frac{1}{4}a^2\left(\frac{\log(ax+1)^2 - 2\log(ax+1)\log(ax-1) - \log(ax-1)^2}{a} + \frac{4\left(\log(ax-1)\log\left(\frac{1}{2}ax + \frac{1}{2}\right) + \operatorname{Li}_2\left(-\frac{1}{2}ax + \frac{1}{2}\right)\right)}{a}\right) - 4\left(\log(ax+1)\log(x) + \operatorname{dilog}(-ax)\right)/a + 4\left(\log(-ax+1)\log(x) + \operatorname{dilog}(ax)\right)/a - a\left(\log(a^2x^2 - 1) - \log(x^2)\right)\operatorname{arccoth}(ax) - \operatorname{arccoth}(ax)^2/x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^2/x^2,x, algorithm="maxima")

[Out] 1/4\*a^2\*((log(a\*x + 1)^2 - 2\*log(a\*x + 1)\*log(a\*x - 1) - log(a\*x - 1)^2)/a + 4\*(log(a\*x - 1)\*log(1/2\*a\*x + 1/2) + dilog(-1/2\*a\*x + 1/2))/a - 4\*(log(a\*x + 1)\*log(x) + dilog(-a\*x))/a + 4\*(log(-a\*x + 1)\*log(x) + dilog(a\*x))/a) - a\*(log(a^2\*x^2 - 1) - log(x^2))\*arccoth(a\*x) - arccoth(a\*x)^2/x

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arccoth}(ax)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^2/x^2,x, algorithm="fricas")

[Out] `integral(arccoth(a*x)^2/x^2, x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^2(ax)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(a*x)**2/x**2,x)`

[Out] `Integral(acoth(a*x)**2/x**2, x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(ax)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(a*x)^2/x^2,x, algorithm="giac")`

[Out] `integrate(arccoth(a*x)^2/x^2, x)`

### 3.20 $\int \frac{\coth^{-1}(ax)^2}{x^3} dx$

**Optimal.** Leaf size=61

$$-\frac{1}{2}a^2 \log(1 - a^2x^2) + a^2 \log(x) + \frac{1}{2}a^2 \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{2x^2} - \frac{a \coth^{-1}(ax)}{x}$$

[Out]  $-\left(\frac{a \operatorname{ArcCoth}[a x]}{x}\right) + \frac{a^2 \operatorname{ArcCoth}[a x]^2}{2} - \frac{\operatorname{ArcCoth}[a x]^2}{2 x^2} + a^2 \operatorname{Log}[x] - \frac{a^2 \operatorname{Log}[1 - a^2 x^2]}{2}$

**Rubi [A]** time = 0.0985716, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.7$ , Rules used = {5917, 5983, 266, 36, 29, 31, 5949}

$$-\frac{1}{2}a^2 \log(1 - a^2x^2) + a^2 \log(x) + \frac{1}{2}a^2 \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{2x^2} - \frac{a \coth^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]^2/x^3,x]

[Out]  $-\left(\frac{a \operatorname{ArcCoth}[a x]}{x}\right) + \frac{a^2 \operatorname{ArcCoth}[a x]^2}{2} - \frac{\operatorname{ArcCoth}[a x]^2}{2 x^2} + a^2 \operatorname{Log}[x] - \frac{a^2 \operatorname{Log}[1 - a^2 x^2]}{2}$

#### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.)*((d_.)*(x_.))^(m_.), x_Symbol]
  :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 5983

```
Int[(((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.)*((f_.)*(x_.))^(m_.))/((d_.) + (
e_.)*(x_.^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcCoth[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

#### Rule 266

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[  
Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b  
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

### Rule 36

Int[1/(((a\_) + (b\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))), x\_Symbol] := Dist[b/(b\*c  
- a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x],  
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

### Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

### Rule 31

Int[((a\_) + (b\_)\*(x\_))^(p\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x,  
x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 5949

Int[((a\_) + ArcCoth[(c\_)\*(x\_)]\*(b\_))^(p\_)/((d\_) + (e\_)\*(x\_)^2), x\_Symb  
ol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b  
, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{\coth^{-1}(ax)^2}{x^3} dx &= -\frac{\coth^{-1}(ax)^2}{2x^2} + a \int \frac{\coth^{-1}(ax)}{x^2(1-a^2x^2)} dx \\
 &= -\frac{\coth^{-1}(ax)^2}{2x^2} + a \int \frac{\coth^{-1}(ax)}{x^2} dx + a^3 \int \frac{\coth^{-1}(ax)}{1-a^2x^2} dx \\
 &= -\frac{a \coth^{-1}(ax)}{x} + \frac{1}{2}a^2 \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{2x^2} + a^2 \int \frac{1}{x(1-a^2x^2)} dx \\
 &= -\frac{a \coth^{-1}(ax)}{x} + \frac{1}{2}a^2 \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^2 \text{Subst} \left( \int \frac{1}{x(1-a^2x)} dx, x, x^2 \right) \\
 &= -\frac{a \coth^{-1}(ax)}{x} + \frac{1}{2}a^2 \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^2 \text{Subst} \left( \int \frac{1}{x} dx, x, x^2 \right) + \frac{1}{2}a^4 \text{Subst} \left( \int \frac{1}{1-x^2} dx, x, x^2 \right) \\
 &= -\frac{a \coth^{-1}(ax)}{x} + \frac{1}{2}a^2 \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{2x^2} + a^2 \log(x) - \frac{1}{2}a^2 \log(1-a^2x^2)
 \end{aligned}$$

**Mathematica [A]** time = 0.0170421, size = 57, normalized size = 0.93

$$-\frac{1}{2}a^2 \log(1 - a^2x^2) + \frac{(a^2x^2 - 1) \operatorname{coth}^{-1}(ax)^2}{2x^2} + a^2 \log(x) - \frac{a \operatorname{coth}^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a\*x]^2/x^3,x]

[Out] -((a\*ArcCoth[a\*x])/x) + ((-1 + a^2\*x^2)\*ArcCoth[a\*x]^2)/(2\*x^2) + a^2\*Log[x] - (a^2\*Log[1 - a^2\*x^2])/2

**Maple [B]** time = 0.055, size = 164, normalized size = 2.7

$$-\frac{(\operatorname{arccoth}(ax))^2}{2x^2} - \frac{a \operatorname{arccoth}(ax)}{x} - \frac{a^2 \operatorname{arccoth}(ax) \ln(ax-1)}{2} + \frac{a^2 \operatorname{arccoth}(ax) \ln(ax+1)}{2} - \frac{a^2 (\ln(ax-1))^2}{8} + \frac{a^2 \ln(ax+1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)^2/x^3,x)

[Out] -1/2\*arccoth(a\*x)^2/x^2-a\*arccoth(a\*x)/x-1/2\*a^2\*arccoth(a\*x)\*ln(a\*x-1)+1/2\*a^2\*arccoth(a\*x)\*ln(a\*x+1)-1/8\*a^2\*ln(a\*x-1)^2+1/4\*a^2\*ln(a\*x-1)\*ln(1/2+1/2\*a\*x)-1/2\*a^2\*ln(a\*x-1)+a^2\*ln(a\*x)-1/2\*a^2\*ln(a\*x+1)+1/4\*a^2\*ln(-1/2\*a\*x+1/2)\*ln(a\*x+1)-1/4\*a^2\*ln(-1/2\*a\*x+1/2)\*ln(1/2+1/2\*a\*x)-1/8\*a^2\*ln(a\*x+1)^2

**Maxima [A]** time = 0.971504, size = 130, normalized size = 2.13

$$\frac{1}{8} \left( 2(\log(ax-1) - 2)\log(ax+1) - \log(ax+1)^2 - \log(ax-1)^2 - 4\log(ax-1) + 8\log(x) \right) a^2 + \frac{1}{2} \left( a\log(ax+1) - a\log(ax-1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^2/x^3,x, algorithm="maxima")

[Out] 1/8\*(2\*(log(a\*x - 1) - 2)\*log(a\*x + 1) - log(a\*x + 1)^2 - log(a\*x - 1)^2 - 4\*log(a\*x - 1) + 8\*log(x))\*a^2 + 1/2\*(a\*log(a\*x + 1) - a\*log(a\*x - 1) - 2/x)\*a\*arccoth(a\*x) - 1/2\*arccoth(a\*x)^2/x^2



---

**Fricas [A]** time = 1.89944, size = 181, normalized size = 2.97

$$\frac{4a^2x^2 \log(a^2x^2 - 1) - 8a^2x^2 \log(x) + 4ax \log\left(\frac{ax+1}{ax-1}\right) - (a^2x^2 - 1) \log\left(\frac{ax+1}{ax-1}\right)^2}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^2/x^3,x, algorithm="fricas")

[Out]  $-1/8*(4*a^2*x^2*\log(a^2*x^2 - 1) - 8*a^2*x^2*\log(x) + 4*a*x*\log((a*x + 1)/(a*x - 1)) - (a^2*x^2 - 1)*\log((a*x + 1)/(a*x - 1))^2)/x^2$

---

**Sympy [A]** time = 1.0102, size = 56, normalized size = 0.92

$$a^2 \log(x) - a^2 \log(ax + 1) + \frac{a^2 \operatorname{acoth}^2(ax)}{2} + a^2 \operatorname{acoth}(ax) - \frac{a \operatorname{acoth}(ax)}{x} - \frac{\operatorname{acoth}^2(ax)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(a\*x)\*\*2/x\*\*3,x)

[Out]  $a**2*\log(x) - a**2*\log(a*x + 1) + a**2*\operatorname{acoth}(a*x)**2/2 + a**2*\operatorname{acoth}(a*x) - a*\operatorname{acoth}(a*x)/x - \operatorname{acoth}(a*x)**2/(2*x**2)$

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(ax)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^2/x^3,x, algorithm="giac")

[Out] integrate(arccoth(a\*x)^2/x^3, x)

### 3.21 $\int \frac{\coth^{-1}(ax)^2}{x^4} dx$

**Optimal.** Leaf size=103

$$-\frac{1}{3}a^3 \text{PolyLog}\left(2, \frac{2}{ax+1} - 1\right) - \frac{a^2}{3x} + \frac{1}{3}a^3 \tanh^{-1}(ax) + \frac{1}{3}a^3 \coth^{-1}(ax)^2 + \frac{2}{3}a^3 \log\left(2 - \frac{2}{ax+1}\right) \coth^{-1}(ax) - \frac{a \coth^{-1}}{3x^2}$$

[Out]  $-a^2/(3*x) - (a*\text{ArcCoth}[a*x])/(3*x^2) + (a^3*\text{ArcCoth}[a*x]^2)/3 - \text{ArcCoth}[a*x]^2/(3*x^3) + (a^3*\text{ArcTanh}[a*x])/3 + (2*a^3*\text{ArcCoth}[a*x]*\text{Log}[2 - 2/(1 + a*x)])/3 - (a^3*\text{PolyLog}[2, -1 + 2/(1 + a*x)])/3$

**Rubi [A]** time = 0.170784, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.7$ , Rules used = {5917, 5983, 325, 206, 5989, 5933, 2447}

$$-\frac{1}{3}a^3 \text{PolyLog}\left(2, \frac{2}{ax+1} - 1\right) - \frac{a^2}{3x} + \frac{1}{3}a^3 \tanh^{-1}(ax) + \frac{1}{3}a^3 \coth^{-1}(ax)^2 + \frac{2}{3}a^3 \log\left(2 - \frac{2}{ax+1}\right) \coth^{-1}(ax) - \frac{a \coth^{-1}}{3x^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{ArcCoth}[a*x]^2/x^4, x]$

[Out]  $-a^2/(3*x) - (a*\text{ArcCoth}[a*x])/(3*x^2) + (a^3*\text{ArcCoth}[a*x]^2)/3 - \text{ArcCoth}[a*x]^2/(3*x^3) + (a^3*\text{ArcTanh}[a*x])/3 + (2*a^3*\text{ArcCoth}[a*x]*\text{Log}[2 - 2/(1 + a*x)])/3 - (a^3*\text{PolyLog}[2, -1 + 2/(1 + a*x)])/3$

#### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^ (p_.)*((d_.)*(x_.))^ (m_.), x_Symbol]
  :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 5983

```
Int[(((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^ (p_.)*((f_.)*(x_.))^ (m_.))/((d_.) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcCoth[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 5989

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)), x_Symbol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/d, Int[(a + b*ArcCoth[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5933

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_Symbol] := Simp[((a + b*ArcCoth[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Dist[(b*c*p)/d, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)^2}{x^4} dx &= -\frac{\coth^{-1}(ax)^2}{3x^3} + \frac{1}{3}(2a) \int \frac{\coth^{-1}(ax)}{x^3(1-a^2x^2)} dx \\
&= -\frac{\coth^{-1}(ax)^2}{3x^3} + \frac{1}{3}(2a) \int \frac{\coth^{-1}(ax)}{x^3} dx + \frac{1}{3}(2a^3) \int \frac{\coth^{-1}(ax)}{x(1-a^2x^2)} dx \\
&= -\frac{a \coth^{-1}(ax)}{3x^2} + \frac{1}{3}a^3 \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{3x^3} + \frac{1}{3}a^2 \int \frac{1}{x^2(1-a^2x^2)} dx + \frac{1}{3}(2a^3) \int \frac{\coth^{-1}(ax)}{x(1+ax)} dx \\
&= -\frac{a^2}{3x} - \frac{a \coth^{-1}(ax)}{3x^2} + \frac{1}{3}a^3 \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{3x^3} + \frac{2}{3}a^3 \coth^{-1}(ax) \log\left(2 - \frac{2}{1+ax}\right) + \frac{1}{3}a^4 \int \frac{1}{x(1+ax)} dx \\
&= -\frac{a^2}{3x} - \frac{a \coth^{-1}(ax)}{3x^2} + \frac{1}{3}a^3 \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{3x^3} + \frac{1}{3}a^3 \tanh^{-1}(ax) + \frac{2}{3}a^3 \coth^{-1}(ax) \log\left(2 - \frac{2}{1+ax}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.246084, size = 87, normalized size = 0.84

$$\frac{-a^3 x^3 \text{PolyLog}\left(2, -e^{-2 \coth^{-1}(ax)}\right) - a^2 x^2 + (a^3 x^3 - 1) \coth^{-1}(ax)^2 + ax \coth^{-1}(ax) \left(a^2 x^2 + 2a^2 x^2 \log\left(e^{-2 \coth^{-1}(ax)} + 1\right)\right)}{3x^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a\*x]^2/x^4, x]

[Out]  $(-(a^2 x^2) + (-1 + a^3 x^3) \text{ArcCoth}[a x]^2 + a x \text{ArcCoth}[a x] (-1 + a^2 x^2 + 2 a^2 x^2 \text{Log}[1 + E^{(-2 \text{ArcCoth}[a x])}])) - a^3 x^3 \text{PolyLog}[2, -E^{(-2 \text{ArcCoth}[a x])}]) / (3 x^3)$

**Maple [B]** time = 0.06, size = 224, normalized size = 2.2

$$-\frac{(\operatorname{arccoth}(ax))^2}{3x^3} - \frac{a^3 \operatorname{arccoth}(ax) \ln(ax-1)}{3} - \frac{a \operatorname{arccoth}(ax)}{3x^2} + \frac{2a^3 \operatorname{arccoth}(ax) \ln(ax)}{3} - \frac{a^3 \operatorname{arccoth}(ax) \ln(ax+1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)^2/x^4, x)

[Out]  $-1/3 \operatorname{arccoth}(a x)^2 / x^3 - 1/3 a^3 \operatorname{arccoth}(a x) \ln(a x - 1) - 1/3 a \operatorname{arccoth}(a x) / x^2 + 2/3 a^3 \operatorname{arccoth}(a x) \ln(a x) - 1/3 a^3 \operatorname{arccoth}(a x) \ln(a x + 1) - 1/3 a^2 / x - 1/3$

$$6a^3 \ln(ax-1) + 1/6 a^3 \ln(ax+1) - 1/12 a^3 \ln(ax-1)^2 + 1/3 a^3 \operatorname{dilog}(1/2 + 1/2ax) + 1/6 a^3 \ln(ax-1) \ln(1/2 + 1/2ax) + 1/6 a^3 \ln(-1/2ax + 1/2) \ln(1/2 + 1/2ax) - 1/6 a^3 \ln(-1/2ax + 1/2) \ln(ax+1) + 1/12 a^3 \ln(ax+1)^2 - 1/3 a^3 \operatorname{dilog}(ax) - 1/3 a^3 \operatorname{dilog}(ax+1) - 1/3 a^3 \ln(ax) \ln(ax+1)$$

**Maxima [A]** time = 0.989151, size = 238, normalized size = 2.31

$$\frac{1}{12} \left( 4 \left( \log(ax-1) \log\left(\frac{1}{2}ax + \frac{1}{2}\right) + \operatorname{Li}_2\left(-\frac{1}{2}ax + \frac{1}{2}\right) \right) a - 4(\log(ax+1) \log(x) + \operatorname{Li}_2(-ax))a + 4(\log(-ax+1) \log(x) + \operatorname{Li}_2(ax))a + 2a \log(ax+1) - 2a \log(ax-1) + (ax \log(ax+1))^2 - 2ax \log(ax+1) \log(ax-1) - ax \log(ax-1)^2 - 4/x \right) a^2 - 1/3 (a^2 \log(a^2 x^2 - 1) - a^2 \log(x^2) + 1/x^2) a \operatorname{arccoth}(ax) - 1/3 \operatorname{arccoth}(ax)^2/x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(ax)^2/x^4, x, algorithm="maxima")

[Out] 1/12\*(4\*(log(ax - 1)\*log(1/2\*ax + 1/2) + dilog(-1/2\*ax + 1/2))\*a - 4\*(log(ax + 1)\*log(x) + dilog(-ax))\*a + 4\*(log(-ax + 1)\*log(x) + dilog(ax))\*a + 2\*a\*log(ax + 1) - 2\*a\*log(ax - 1) + (ax\*log(ax + 1))^2 - 2\*ax\*log(ax + 1)\*log(ax - 1) - ax\*log(ax - 1)^2 - 4/x)\*a^2 - 1/3\*(a^2\*log(a^2\*x^2 - 1) - a^2\*log(x^2) + 1/x^2)\*a\*arccoth(ax) - 1/3\*arccoth(ax)^2/x^3

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arccoth}(ax)^2}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(ax)^2/x^4, x, algorithm="fricas")

[Out] integral(arccoth(ax)^2/x^4, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^2(ax)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)**2/x**4,x)
```

```
[Out] Integral(acoth(a*x)**2/x**4, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(ax)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)^2/x^4, x)
```

$$3.22 \quad \int \frac{\coth^{-1}(ax)^2}{x^5} dx$$

**Optimal.** Leaf size=90

$$-\frac{a^2}{12x^2} - \frac{1}{3}a^4 \log(1 - a^2x^2) + \frac{2}{3}a^4 \log(x) + \frac{1}{4}a^4 \coth^{-1}(ax)^2 - \frac{a^3 \coth^{-1}(ax)}{2x} - \frac{a \coth^{-1}(ax)}{6x^3} - \frac{\coth^{-1}(ax)^2}{4x^4}$$

[Out]  $-a^2/(12*x^2) - (a*ArcCoth[a*x])/(6*x^3) - (a^3*ArcCoth[a*x])/(2*x) + (a^4*ArcCoth[a*x]^2)/4 - ArcCoth[a*x]^2/(4*x^4) + (2*a^4*Log[x])/3 - (a^4*Log[1 - a^2*x^2])/3$

**Rubi [A]** time = 0.17234, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.8$ , Rules used = {5917, 5983, 266, 44, 36, 29, 31, 5949}

$$-\frac{a^2}{12x^2} - \frac{1}{3}a^4 \log(1 - a^2x^2) + \frac{2}{3}a^4 \log(x) + \frac{1}{4}a^4 \coth^{-1}(ax)^2 - \frac{a^3 \coth^{-1}(ax)}{2x} - \frac{a \coth^{-1}(ax)}{6x^3} - \frac{\coth^{-1}(ax)^2}{4x^4}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]^2/x^5, x]

[Out]  $-a^2/(12*x^2) - (a*ArcCoth[a*x])/(6*x^3) - (a^3*ArcCoth[a*x])/(2*x) + (a^4*ArcCoth[a*x]^2)/4 - ArcCoth[a*x]^2/(4*x^4) + (2*a^4*Log[x])/3 - (a^4*Log[1 - a^2*x^2])/3$

### Rule 5917

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcCoth[c\*x])^p)/(d\*(m + 1)), x] - Dist[(b\*c\*p)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 5983

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((f\_.)\*(x\_))^(m\_.))/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] :> Dist[1/d, Int[(f\*x)^m\*(a + b\*ArcCoth[c\*x])^p, x], x] - Dist[e/(d\*f^2), Int[((f\*x)^(m + 2)\*(a + b\*ArcCoth[c\*x])^p)/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c
- a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(p_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps



$$\begin{aligned}
\int \frac{\coth^{-1}(ax)^2}{x^5} dx &= -\frac{\coth^{-1}(ax)^2}{4x^4} + \frac{1}{2}a \int \frac{\coth^{-1}(ax)}{x^4(1-a^2x^2)} dx \\
&= -\frac{\coth^{-1}(ax)^2}{4x^4} + \frac{1}{2}a \int \frac{\coth^{-1}(ax)}{x^4} dx + \frac{1}{2}a^3 \int \frac{\coth^{-1}(ax)}{x^2(1-a^2x^2)} dx \\
&= -\frac{a \coth^{-1}(ax)}{6x^3} - \frac{\coth^{-1}(ax)^2}{4x^4} + \frac{1}{6}a^2 \int \frac{1}{x^3(1-a^2x^2)} dx + \frac{1}{2}a^3 \int \frac{\coth^{-1}(ax)}{x^2} dx + \frac{1}{2}a^5 \int \frac{\coth^{-1}(ax)}{1-a^2x^2} dx \\
&= -\frac{a \coth^{-1}(ax)}{6x^3} - \frac{a^3 \coth^{-1}(ax)}{2x} + \frac{1}{4}a^4 \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{4x^4} + \frac{1}{12}a^2 \text{Subst} \left( \int \frac{1}{x^2(1-a^2x)} dx \right) \\
&= -\frac{a \coth^{-1}(ax)}{6x^3} - \frac{a^3 \coth^{-1}(ax)}{2x} + \frac{1}{4}a^4 \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{4x^4} + \frac{1}{12}a^2 \text{Subst} \left( \int \left( \frac{1}{x^2} + \frac{a^2}{x} - \frac{a^2}{1-a^2x} \right) dx \right) \\
&= -\frac{a^2}{12x^2} - \frac{a \coth^{-1}(ax)}{6x^3} - \frac{a^3 \coth^{-1}(ax)}{2x} + \frac{1}{4}a^4 \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{4x^4} + \frac{1}{6}a^4 \log(x) - \frac{1}{12}a^4 \log(1-a^2x) \\
&= -\frac{a^2}{12x^2} - \frac{a \coth^{-1}(ax)}{6x^3} - \frac{a^3 \coth^{-1}(ax)}{2x} + \frac{1}{4}a^4 \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{4x^4} + \frac{2}{3}a^4 \log(x) - \frac{1}{3}a^4 \log(1-a^2x)
\end{aligned}$$

**Mathematica [A]** time = 0.0216711, size = 82, normalized size = 0.91

$$-\frac{a^2}{12x^2} - \frac{1}{3}a^4 \log(1-a^2x^2) - \frac{a(3a^2x^2+1)\coth^{-1}(ax)}{6x^3} + \frac{(a^4x^4-1)\coth^{-1}(ax)^2}{4x^4} + \frac{2}{3}a^4 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a\*x]^2/x^5,x]

[Out]  $-a^2/(12*x^2) - (a*(1 + 3*a^2*x^2)*\text{ArcCoth}[a*x])/(6*x^3) + ((-1 + a^4*x^4)*\text{ArcCoth}[a*x]^2)/(4*x^4) + (2*a^4*\text{Log}[x])/3 - (a^4*\text{Log}[1 - a^2*x^2])/3$

**Maple [B]** time = 0.06, size = 185, normalized size = 2.1

$$-\frac{(\operatorname{arccoth}(ax))^2}{4x^4} - \frac{a^4 \operatorname{arccoth}(ax) \ln(ax-1)}{4} - \frac{a \operatorname{arccoth}(ax)}{6x^3} - \frac{a^3 \operatorname{arccoth}(ax)}{2x} + \frac{a^4 \operatorname{arccoth}(ax) \ln(ax+1)}{4} - \frac{a^4 \ln(1-a^2x^2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)^2/x^5,x)

[Out]  $-1/4*\operatorname{arccoth}(a*x)^2/x^4-1/4*a^4*\operatorname{arccoth}(a*x)*\ln(a*x-1)-1/6*a*\operatorname{arccoth}(a*x)/x^3-1/2*a^3*\operatorname{arccoth}(a*x)/x+1/4*a^4*\operatorname{arccoth}(a*x)*\ln(a*x+1)-1/16*a^4*\ln(a*x-1)^2+1/8*a^4*\ln(a*x-1)*\ln(1/2+1/2*a*x)-1/8*a^4*\ln(-1/2*a*x+1/2)*\ln(1/2+1/2*a*x)+1/8*a^4*\ln(-1/2*a*x+1/2)*\ln(a*x+1)-1/16*a^4*\ln(a*x+1)^2-1/3*a^4*\ln(a*x-1)-1/12*a^2/x^2+2/3*a^4*\ln(a*x)-1/3*a^4*\ln(a*x+1)$

**Maxima [B]** time = 0.98254, size = 208, normalized size = 2.31

$$\frac{1}{48} \left( 32 a^2 \log(x) - \frac{3 a^2 x^2 \log(ax+1)^2 + 3 a^2 x^2 \log(ax-1)^2 + 16 a^2 x^2 \log(ax-1) - 2 (3 a^2 x^2 \log(ax-1) - 8 a^2 x^2) \log(ax+1)}{x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(a*x)^2/x^5,x, algorithm="maxima")`

[Out]  $1/48*(32*a^2*\log(x) - (3*a^2*x^2*\log(a*x + 1)^2 + 3*a^2*x^2*\log(a*x - 1)^2 + 16*a^2*x^2*\log(a*x - 1) - 2*(3*a^2*x^2*\log(a*x - 1) - 8*a^2*x^2)*\log(a*x + 1) + 4)/x^2)*a^2 + 1/12*(3*a^3*\log(a*x + 1) - 3*a^3*\log(a*x - 1) - 2*(3*a^2*x^2 + 1)/x^3)*a*\operatorname{arccoth}(a*x) - 1/4*\operatorname{arccoth}(a*x)^2/x^4$

**Fricas [A]** time = 1.83772, size = 223, normalized size = 2.48

$$\frac{16 a^4 x^4 \log(a^2 x^2 - 1) - 32 a^4 x^4 \log(x) + 4 a^2 x^2 - 3 (a^4 x^4 - 1) \log\left(\frac{ax+1}{ax-1}\right)^2 + 4 (3 a^3 x^3 + ax) \log\left(\frac{ax+1}{ax-1}\right)}{48 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(a*x)^2/x^5,x, algorithm="fricas")`

[Out]  $-1/48*(16*a^4*x^4*\log(a^2*x^2 - 1) - 32*a^4*x^4*\log(x) + 4*a^2*x^2 - 3*(a^4*x^4 - 1)*\log((a*x + 1)/(a*x - 1))^2 + 4*(3*a^3*x^3 + a*x)*\log((a*x + 1)/(a*x - 1)))/x^4$

**Sympy [A]** time = 6.42276, size = 90, normalized size = 1.

$$\frac{2a^4 \log(x)}{3} - \frac{2a^4 \log(ax+1)}{3} + \frac{a^4 \operatorname{acoth}^2(ax)}{4} + \frac{2a^4 \operatorname{acoth}(ax)}{3} - \frac{a^3 \operatorname{acoth}(ax)}{2x} - \frac{a^2}{12x^2} - \frac{a \operatorname{acoth}(ax)}{6x^3} - \frac{\operatorname{acoth}^2(ax)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(a\*x)\*\*2/x\*\*5,x)

[Out]  $2*a**4*\log(x)/3 - 2*a**4*\log(a*x + 1)/3 + a**4*acoth(a*x)**2/4 + 2*a**4*acoth(a*x)/3 - a**3*acoth(a*x)/(2*x) - a**2/(12*x**2) - a*acoth(a*x)/(6*x**3) - acoth(a*x)**2/(4*x**4)$

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(ax)^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^2/x^5,x, algorithm="giac")

[Out] integrate(arccoth(a\*x)^2/x^5, x)

### 3.23 $\int x^5 \coth^{-1}(ax)^3 dx$

**Optimal.** Leaf size=186

$$-\frac{23\text{PolyLog}\left(2, 1 - \frac{2}{1-ax}\right)}{30a^6} + \frac{x^3}{60a^3} + \frac{x^4 \coth^{-1}(ax)}{20a^2} + \frac{x^3 \coth^{-1}(ax)^2}{6a^3} + \frac{4x^2 \coth^{-1}(ax)}{15a^4} + \frac{19x}{60a^5} - \frac{19 \tanh^{-1}(ax)}{60a^6} + \frac{x \coth^{-1}(ax)}{60a^6}$$

[Out] (19\*x)/(60\*a^5) + x^3/(60\*a^3) + (4\*x^2\*ArcCoth[a\*x])/(15\*a^4) + (x^4\*ArcCoth[a\*x])/(20\*a^2) + (23\*ArcCoth[a\*x]^2)/(30\*a^6) + (x\*ArcCoth[a\*x]^2)/(2\*a^5) + (x^3\*ArcCoth[a\*x]^2)/(6\*a^3) + (x^5\*ArcCoth[a\*x]^2)/(10\*a) - ArcCoth[a\*x]^3/(6\*a^6) + (x^6\*ArcCoth[a\*x]^3)/6 - (19\*ArcTanh[a\*x])/(60\*a^6) - (23\*ArcCoth[a\*x]\*Log[2/(1 - a\*x)])/(15\*a^6) - (23\*PolyLog[2, 1 - 2/(1 - a\*x)])/(30\*a^6)

**Rubi [A]** time = 0.717193, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 33, number of rules used = 11, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.1$ , Rules used = {5917, 5981, 302, 206, 321, 5985, 5919, 2402, 2315, 5911, 5949}

$$-\frac{23\text{PolyLog}\left(2, 1 - \frac{2}{1-ax}\right)}{30a^6} + \frac{x^3}{60a^3} + \frac{x^4 \coth^{-1}(ax)}{20a^2} + \frac{x^3 \coth^{-1}(ax)^2}{6a^3} + \frac{4x^2 \coth^{-1}(ax)}{15a^4} + \frac{19x}{60a^5} - \frac{19 \tanh^{-1}(ax)}{60a^6} + \frac{x \coth^{-1}(ax)}{60a^6}$$

Antiderivative was successfully verified.

[In] Int[x^5\*ArcCoth[a\*x]^3,x]

[Out] (19\*x)/(60\*a^5) + x^3/(60\*a^3) + (4\*x^2\*ArcCoth[a\*x])/(15\*a^4) + (x^4\*ArcCoth[a\*x])/(20\*a^2) + (23\*ArcCoth[a\*x]^2)/(30\*a^6) + (x\*ArcCoth[a\*x]^2)/(2\*a^5) + (x^3\*ArcCoth[a\*x]^2)/(6\*a^3) + (x^5\*ArcCoth[a\*x]^2)/(10\*a) - ArcCoth[a\*x]^3/(6\*a^6) + (x^6\*ArcCoth[a\*x]^3)/6 - (19\*ArcTanh[a\*x])/(60\*a^6) - (23\*ArcCoth[a\*x]\*Log[2/(1 - a\*x)])/(15\*a^6) - (23\*PolyLog[2, 1 - 2/(1 - a\*x)])/(30\*a^6)

#### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^ (p_.)*((d_.)*(x_.))^ (m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]
```

#### Rule 5981

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)/((d_) + (
e_.)*(x_)^2), x_Symbol] := Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

### Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1)/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1)/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 5985

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_)/((d_) + (e_.)*(x_)^2),
x_Symbol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

### Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] := -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)]]/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

### Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

$c, d, e, f, g, x$  && EqQ[ $c, 2*d$ ] && EqQ[ $e^2*f + d^2*g, 0$ ]

### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

### Rule 5911

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^p, x\_Symbol] := Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

### Rule 5949

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^p/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

### Rubi steps

$$\begin{aligned}
\int x^5 \coth^{-1}(ax)^3 dx &= \frac{1}{6}x^6 \coth^{-1}(ax)^3 - \frac{1}{2}a \int \frac{x^6 \coth^{-1}(ax)^2}{1-a^2x^2} dx \\
&= \frac{1}{6}x^6 \coth^{-1}(ax)^3 + \frac{\int x^4 \coth^{-1}(ax)^2 dx}{2a} - \frac{\int \frac{x^4 \coth^{-1}(ax)^2}{1-a^2x^2} dx}{2a} \\
&= \frac{x^5 \coth^{-1}(ax)^2}{10a} + \frac{1}{6}x^6 \coth^{-1}(ax)^3 - \frac{1}{5} \int \frac{x^5 \coth^{-1}(ax)}{1-a^2x^2} dx + \frac{\int x^2 \coth^{-1}(ax)^2 dx}{2a^3} - \frac{\int \frac{x^2 \coth^{-1}(ax)^2}{1-a^2x^2} dx}{2a^5} \\
&= \frac{x^3 \coth^{-1}(ax)^2}{6a^3} + \frac{x^5 \coth^{-1}(ax)^2}{10a} + \frac{1}{6}x^6 \coth^{-1}(ax)^3 + \frac{\int \coth^{-1}(ax)^2 dx}{2a^5} - \frac{\int \frac{\coth^{-1}(ax)^2}{1-a^2x^2} dx}{2a^5} + \frac{\int \frac{\coth^{-1}(ax)}{1-a^2x^2} dx}{2a^5} \\
&= \frac{x^4 \coth^{-1}(ax)}{20a^2} + \frac{x \coth^{-1}(ax)^2}{2a^5} + \frac{x^3 \coth^{-1}(ax)^2}{6a^3} + \frac{x^5 \coth^{-1}(ax)^2}{10a} - \frac{\coth^{-1}(ax)^3}{6a^6} + \frac{1}{6}x^6 \coth^{-1}(ax)^3 \\
&= \frac{4x^2 \coth^{-1}(ax)}{15a^4} + \frac{x^4 \coth^{-1}(ax)}{20a^2} + \frac{23 \coth^{-1}(ax)^2}{30a^6} + \frac{x \coth^{-1}(ax)^2}{2a^5} + \frac{x^3 \coth^{-1}(ax)^2}{6a^3} + \frac{x^5 \coth^{-1}(ax)^2}{10a} \\
&= \frac{19x}{60a^5} + \frac{x^3}{60a^3} + \frac{4x^2 \coth^{-1}(ax)}{15a^4} + \frac{x^4 \coth^{-1}(ax)}{20a^2} + \frac{23 \coth^{-1}(ax)^2}{30a^6} + \frac{x \coth^{-1}(ax)^2}{2a^5} + \frac{x^3 \coth^{-1}(ax)^2}{6a^3} + \frac{x^5 \coth^{-1}(ax)^2}{10a} \\
&= \frac{19x}{60a^5} + \frac{x^3}{60a^3} + \frac{4x^2 \coth^{-1}(ax)}{15a^4} + \frac{x^4 \coth^{-1}(ax)}{20a^2} + \frac{23 \coth^{-1}(ax)^2}{30a^6} + \frac{x \coth^{-1}(ax)^2}{2a^5} + \frac{x^3 \coth^{-1}(ax)^2}{6a^3} + \frac{x^5 \coth^{-1}(ax)^2}{10a} \\
&= \frac{19x}{60a^5} + \frac{x^3}{60a^3} + \frac{4x^2 \coth^{-1}(ax)}{15a^4} + \frac{x^4 \coth^{-1}(ax)}{20a^2} + \frac{23 \coth^{-1}(ax)^2}{30a^6} + \frac{x \coth^{-1}(ax)^2}{2a^5} + \frac{x^3 \coth^{-1}(ax)^2}{6a^3} + \frac{x^5 \coth^{-1}(ax)^2}{10a}
\end{aligned}$$

**Mathematica [A]** time = 0.525733, size = 117, normalized size = 0.63

$$\frac{46 \text{PolyLog}\left(2, e^{-2 \coth^{-1}(ax)}\right) + ax \left(a^2 x^2 + 19\right) + 10 \left(a^6 x^6 - 1\right) \coth^{-1}(ax)^3 + 2 \left(3 a^5 x^5 + 5 a^3 x^3 + 15 a x - 23\right) \coth^{-1}(ax)}{60 a^6}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^5\*ArcCoth[a\*x]^3,x]

[Out] (a\*x\*(19 + a^2\*x^2) + 2\*(-23 + 15\*a\*x + 5\*a^3\*x^3 + 3\*a^5\*x^5)\*ArcCoth[a\*x]^2 + 10\*(-1 + a^6\*x^6)\*ArcCoth[a\*x]^3 + ArcCoth[a\*x]\*(-19 + 16\*a^2\*x^2 + 3\*a^4\*x^4 - 92\*Log[1 - E^(-2\*ArcCoth[a\*x])])) + 46\*PolyLog[2, E^(-2\*ArcCoth[a\*x])])/(60\*a^6)

**Maple [C]** time = 1.901, size = 1141, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^5 \operatorname{arccoth}(ax)^3, x)$

[Out] 
$$\begin{aligned} & 23/30 \operatorname{arccoth}(ax)^2/a^6 + 1/4/a^6 \operatorname{arccoth}(ax)^2 \ln(ax-1) - 1/4/a^6 \operatorname{arccoth}(ax)^2 \ln(ax+1) - 23/15/a^6 \operatorname{arccoth}(ax) \ln(1 + ((ax-1)/(ax+1))^{1/2}) - 1/4/a^6 \operatorname{arccoth}(ax)^2 \ln((ax-1)/(ax+1)) - 41/120/a^6 / (-1 + ((ax-1)/(ax+1))^{1/2}) * ((ax-1)/(ax+1))^{1/2} - 41/120/a^6 / (((ax-1)/(ax+1))^{1/2} + 1) * ((ax-1)/(ax+1))^{1/2} + 1/80/a^5 / (((ax-1)/(ax+1))^{1/2} * ax + ((ax-1)/(ax+1))^{1/2} + ax) * x - 1/80/a^5 / (((ax-1)/(ax+1))^{1/2} * ax + ((ax-1)/(ax+1))^{1/2} - ax) * x + 1/120/a^6 * ((ax-1)/(ax+1))^{1/2} / (2 * ((ax-1)/(ax+1))^{1/2} * ax + ((ax-1)/(ax+1))^{1/2} - 2 * ax + 1) + 1/120/a^6 * ((ax-1)/(ax+1))^{1/2} / (2 * ((ax-1)/(ax+1))^{1/2} * ax + ((ax-1)/(ax+1))^{1/2} + 2 * ax - 1) - 1/6 \operatorname{arccoth}(ax)^3/a^6 + 1/6 * x^6 \operatorname{arccoth}(ax)^3 + 4/15 * x^2 \operatorname{arccoth}(ax)/a^4 + 1/10 * x^5 \operatorname{arccoth}(ax)^2/a + 1/20 * x^4 \operatorname{arccoth}(ax)/a^2 + 1/2 * x \operatorname{arccoth}(ax)^2/a^5 + 1/6 * x^3 \operatorname{arccoth}(ax)^2/a^3 - 1/80/a^6 / (((ax-1)/(ax+1))^{1/2} * ax + ((ax-1)/(ax+1))^{1/2} + ax) + 1/80/a^6 / (((ax-1)/(ax+1))^{1/2} * ax + ((ax-1)/(ax+1))^{1/2} - ax) - 19/60/a^6 \operatorname{arccoth}(ax) + 23/15/a^6 \operatorname{dilog}(1 / ((ax-1)/(ax+1))^{1/2}) - 23/15/a^6 \operatorname{dilog}(1 + ((ax-1)/(ax+1))^{1/2}) - 1/8 * I/a^6 * \operatorname{Pi} * \operatorname{csgn}(I * (ax+1)/(ax-1) / ((ax+1)/(ax-1) - 1)) * \operatorname{csgn}(I * (ax+1)/(ax-1)) * \operatorname{csgn}(I / ((ax+1)/(ax-1) - 1)) * \operatorname{arccoth}(ax)^2 + 1/4 * I/a^6 * \operatorname{Pi} * \operatorname{csgn}(I / ((ax-1)/(ax+1))^{1/2}) * \operatorname{csgn}(I * (ax+1)/(ax-1))^{2 * \operatorname{arccoth}(ax)^2 + 1/8 * I/a^6 * \operatorname{Pi} * \operatorname{csgn}(I * (ax+1)/(ax-1) / ((ax+1)/(ax-1) - 1))^{2 * \operatorname{csgn}(I * (ax+1)/(ax-1)) * \operatorname{arccoth}(ax)^2 - 1/8 * I/a^6 * \operatorname{Pi} * \operatorname{csgn}(I / ((ax-1)/(ax+1))^{1/2})^{2 * \operatorname{csgn}(I * (ax+1)/(ax-1)) * \operatorname{arccoth}(ax)^2 + 1/8 * I/a^6 * \operatorname{Pi} * \operatorname{csgn}(I * (ax+1)/(ax-1) / ((ax+1)/(ax-1) - 1))^{2 * \operatorname{csgn}(I / ((ax+1)/(ax-1) - 1)) * \operatorname{arccoth}(ax)^2 - 1/8 * I/a^6 * \operatorname{Pi} * \operatorname{csgn}(I * (ax+1)/(ax-1) / ((ax+1)/(ax-1) - 1))^{3 * \operatorname{arccoth}(ax)^2 - 1/8 * I/a^6 * \operatorname{Pi} * \operatorname{csgn}(I * (ax+1)/(ax-1))^{3 * \operatorname{arccoth}(ax)^2 - 1/120/a^5 * ((ax-1)/(ax+1))^{1/2} / (2 * ((ax-1)/(ax+1))^{1/2} * ax + ((ax-1)/(ax+1))^{1/2} - 2 * ax + 1) * x - 1/120/a^5 * ((ax-1)/(ax+1))^{1/2} / (2 * ((ax-1)/(ax+1))^{1/2} * ax + ((ax-1)/(ax+1))^{1/2} + 2 * ax - 1) * x \end{aligned}$$

**Maxima [A]** time = 1.01173, size = 390, normalized size = 2.1

$$\frac{1}{6} x^6 \operatorname{arccoth}(ax)^3 + \frac{1}{60} a \left( \frac{2(3a^4x^5 + 5a^2x^3 + 15x)}{a^6} - \frac{15 \log(ax+1)}{a^7} + \frac{15 \log(ax-1)}{a^7} \right) \operatorname{arccoth}(ax)^2 + \frac{1}{240} a \left( \frac{4a^3x^3 + (15x^2 + 15x + 1)a^2}{a^6} - \frac{15 \log(ax+1)}{a^7} + \frac{15 \log(ax-1)}{a^7} \right) \operatorname{arccoth}(ax) + \frac{1}{240} a \left( \frac{4a^3x^3 + (15x^2 + 15x + 1)a^2}{a^6} - \frac{15 \log(ax+1)}{a^7} + \frac{15 \log(ax-1)}{a^7} \right)$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(x^5\*arccoth(a\*x)^3,x, algorithm="maxima")

[Out]  $\frac{1}{6}x^6\operatorname{arccoth}(ax)^3 + \frac{1}{60}a(2(3a^4x^5 + 5a^2x^3 + 15x)/a^6 - 15\log(ax + 1)/a^7 + 15\log(ax - 1)/a^7)\operatorname{arccoth}(ax)^2 + \frac{1}{240}a(((4a^3x^3 + (15\log(ax - 1) - 46)\log(ax + 1)^2 - 5\log(ax + 1)^3 + 5\log(ax - 1)^3 + 76ax - (15\log(ax - 1)^2 - 92\log(ax - 1))\log(ax + 1) + 46\log(ax - 1)^2 + 38\log(ax - 1))/a - 184(\log(ax - 1)\log(1/2ax + 1/2) + \operatorname{dilog}(-1/2ax + 1/2))/a - 38\log(ax + 1)/a)/a^6 + 2(6a^4x^4 + 32a^2x^2 - 2(15\log(ax - 1) - 46)\log(ax + 1) + 15\log(ax + 1)^2 + 15\log(ax - 1)^2 + 92\log(ax - 1))\operatorname{arccoth}(ax)/a^7)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(x^5 \operatorname{arccoth}(ax)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*arccoth(a\*x)^3,x, algorithm="fricas")

[Out] integral(x^5\*arccoth(a\*x)^3, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^5 \operatorname{arccoth}^3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*acoth(a\*x)\*\*3,x)

[Out] Integral(x\*\*5\*acoth(a\*x)\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^5 \operatorname{arccoth}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccoth(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^5*arccoth(a*x)^3, x)
```

### 3.24 $\int x^4 \coth^{-1}(ax)^3 dx$

**Optimal.** Leaf size=196

$$\frac{3\text{PolyLog}\left(3, 1 - \frac{2}{1-ax}\right)}{10a^5} - \frac{3 \coth^{-1}(ax)\text{PolyLog}\left(2, 1 - \frac{2}{1-ax}\right)}{5a^5} + \frac{x^2}{20a^3} + \frac{\log(1 - a^2x^2)}{2a^5} + \frac{x^3 \coth^{-1}(ax)}{10a^2} + \frac{3x^2 \coth^{-1}(ax)}{10a^3}$$

```
[Out] x^2/(20*a^3) + (9*x*ArcCoth[a*x])/(10*a^4) + (x^3*ArcCoth[a*x])/(10*a^2) -
(9*ArcCoth[a*x]^2)/(20*a^5) + (3*x^2*ArcCoth[a*x]^2)/(10*a^3) + (3*x^4*ArcC
oth[a*x]^2)/(20*a) + ArcCoth[a*x]^3/(5*a^5) + (x^5*ArcCoth[a*x]^3)/5 - (3*A
rcCoth[a*x]^2*Log[2/(1 - a*x)])/(5*a^5) + Log[1 - a^2*x^2]/(2*a^5) - (3*Arc
Coth[a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/(5*a^5) + (3*PolyLog[3, 1 - 2/(1 - a
*x)])/(10*a^5)
```

---

**Rubi [A]** time = 0.580282, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 11, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.1$ , Rules used = {5917, 5981, 266, 43, 5911, 260, 5949, 5985, 5919, 6059, 6610}

$$\frac{3\text{PolyLog}\left(3, 1 - \frac{2}{1-ax}\right)}{10a^5} - \frac{3 \coth^{-1}(ax)\text{PolyLog}\left(2, 1 - \frac{2}{1-ax}\right)}{5a^5} + \frac{x^2}{20a^3} + \frac{\log(1 - a^2x^2)}{2a^5} + \frac{x^3 \coth^{-1}(ax)}{10a^2} + \frac{3x^2 \coth^{-1}(ax)}{10a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcCoth[a*x]^3,x]
```

```
[Out] x^2/(20*a^3) + (9*x*ArcCoth[a*x])/(10*a^4) + (x^3*ArcCoth[a*x])/(10*a^2) -
(9*ArcCoth[a*x]^2)/(20*a^5) + (3*x^2*ArcCoth[a*x]^2)/(10*a^3) + (3*x^4*ArcC
oth[a*x]^2)/(20*a) + ArcCoth[a*x]^3/(5*a^5) + (x^5*ArcCoth[a*x]^3)/5 - (3*A
rcCoth[a*x]^2*Log[2/(1 - a*x)])/(5*a^5) + Log[1 - a^2*x^2]/(2*a^5) - (3*Arc
Coth[a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/(5*a^5) + (3*PolyLog[3, 1 - 2/(1 - a
*x)])/(10*a^5)
```

#### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2
*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 5981

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^p_.)*((f_.)*(x_))^(m_)]/((d_) + (e_.)*(x_)^2), x_Symbol] := Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^p_.), x_Symbol] := Simp[x*(a + b*ArcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

### Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

### Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^p_.)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

### Rule 5985

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^p_.)*(x_)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

### Rule 5919

```

Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)]]/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]

```

### Rule 6059

```

Int[(Log[u_]*)((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcCoth[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]

```

### Rule 6610

```

Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

```

### Rubi steps

$$\begin{aligned}
\int x^4 \coth^{-1}(ax)^3 dx &= \frac{1}{5}x^5 \coth^{-1}(ax)^3 - \frac{1}{5}(3a) \int \frac{x^5 \coth^{-1}(ax)^2}{1-a^2x^2} dx \\
&= \frac{1}{5}x^5 \coth^{-1}(ax)^3 + \frac{3 \int x^3 \coth^{-1}(ax)^2 dx}{5a} - \frac{3 \int \frac{x^3 \coth^{-1}(ax)^2}{1-a^2x^2} dx}{5a} \\
&= \frac{3x^4 \coth^{-1}(ax)^2}{20a} + \frac{1}{5}x^5 \coth^{-1}(ax)^3 - \frac{3}{10} \int \frac{x^4 \coth^{-1}(ax)}{1-a^2x^2} dx + \frac{3 \int x \coth^{-1}(ax)^2 dx}{5a^3} - \frac{3 \int \frac{x \coth^{-1}(ax)}{1-a^2x^2} dx}{5a} \\
&= \frac{3x^2 \coth^{-1}(ax)^2}{10a^3} + \frac{3x^4 \coth^{-1}(ax)^2}{20a} + \frac{\coth^{-1}(ax)^3}{5a^5} + \frac{1}{5}x^5 \coth^{-1}(ax)^3 - \frac{3 \int \frac{\coth^{-1}(ax)^2}{1-ax} dx}{5a^4} + \frac{3 \int \frac{\coth^{-1}(ax)}{1-a^2x^2} dx}{5a} \\
&= \frac{x^3 \coth^{-1}(ax)}{10a^2} + \frac{3x^2 \coth^{-1}(ax)^2}{10a^3} + \frac{3x^4 \coth^{-1}(ax)^2}{20a} + \frac{\coth^{-1}(ax)^3}{5a^5} + \frac{1}{5}x^5 \coth^{-1}(ax)^3 - \frac{3 \coth^{-1}(ax)}{5a} \\
&= \frac{9x \coth^{-1}(ax)}{10a^4} + \frac{x^3 \coth^{-1}(ax)}{10a^2} - \frac{9 \coth^{-1}(ax)^2}{20a^5} + \frac{3x^2 \coth^{-1}(ax)^2}{10a^3} + \frac{3x^4 \coth^{-1}(ax)^2}{20a} + \frac{\coth^{-1}(ax)}{5a} \\
&= \frac{9x \coth^{-1}(ax)}{10a^4} + \frac{x^3 \coth^{-1}(ax)}{10a^2} - \frac{9 \coth^{-1}(ax)^2}{20a^5} + \frac{3x^2 \coth^{-1}(ax)^2}{10a^3} + \frac{3x^4 \coth^{-1}(ax)^2}{20a} + \frac{\coth^{-1}(ax)}{5a} \\
&= \frac{x^2}{20a^3} + \frac{9x \coth^{-1}(ax)}{10a^4} + \frac{x^3 \coth^{-1}(ax)}{10a^2} - \frac{9 \coth^{-1}(ax)^2}{20a^5} + \frac{3x^2 \coth^{-1}(ax)^2}{10a^3} + \frac{3x^4 \coth^{-1}(ax)^2}{20a} + \frac{\coth^{-1}(ax)}{5a}
\end{aligned}$$

**Mathematica [C]** time = 0.566502, size = 175, normalized size = 0.89

$$-24 \coth^{-1}(ax) \text{PolyLog}\left(2, e^{2 \coth^{-1}(ax)}\right) + 12 \text{PolyLog}\left(3, e^{2 \coth^{-1}(ax)}\right) + 2a^2x^2 - 40 \log\left(\frac{1}{ax\sqrt{1-\frac{1}{a^2x^2}}}\right) + 8a^5x^5 \coth^{-1}(ax)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^4\*ArcCoth[a\*x]^3,x]

[Out] (-2 - I\*Pi^3 + 2\*a^2\*x^2 + 36\*a\*x\*ArcCoth[a\*x] + 4\*a^3\*x^3\*ArcCoth[a\*x] - 1  
8\*ArcCoth[a\*x]^2 + 12\*a^2\*x^2\*ArcCoth[a\*x]^2 + 6\*a^4\*x^4\*ArcCoth[a\*x]^2 + 8  
\*ArcCoth[a\*x]^3 + 8\*a^5\*x^5\*ArcCoth[a\*x]^3 - 24\*ArcCoth[a\*x]^2\*Log[1 - E^(2  
\*ArcCoth[a\*x])] - 40\*Log[1/(a\*Sqrt[1 - 1/(a^2\*x^2)]]\*x] - 24\*ArcCoth[a\*x]\*P  
olyLog[2, E^(2\*ArcCoth[a\*x])] + 12\*PolyLog[3, E^(2\*ArcCoth[a\*x])])/(40\*a^5)

**Maple [C]** time = 1.106, size = 806, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*arccoth(a*x)^3,x)`

[Out] 
$$\begin{aligned} & 1/5*\operatorname{arccoth}(a*x)^3/a^5-1/20/a^5-9/20*\operatorname{arccoth}(a*x)^2/a^5+1/a^5*\operatorname{arccoth}(a*x)+ \\ & 6/5/a^5*\operatorname{polylog}(3,1/((a*x-1)/(a*x+1))^{(1/2)})-1/a^5*\ln(1+1/((a*x-1)/(a*x+1)) \\ & ^{(1/2)})-1/a^5*\ln(1/((a*x-1)/(a*x+1))^{(1/2)})-1+6/5/a^5*\operatorname{polylog}(3,-1/((a*x-1) \\ & /((a*x+1))^{(1/2)}))+1/20*x^2/a^3+1/5*x^5*\operatorname{arccoth}(a*x)^3+3/10*x^2*\operatorname{arccoth}(a*x)^ \\ & 2/a^3+9/10*x*\operatorname{arccoth}(a*x)/a^4+1/10*x^3*\operatorname{arccoth}(a*x)/a^2+3/20*x^4*\operatorname{arccoth}(a* \\ & x)^2/a^3+20*I/a^5*\operatorname{Pi}*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))*csgn(I*(a* \\ & x+1)/(a*x-1))*csgn(I/((a*x+1)/(a*x-1)-1))*\operatorname{arccoth}(a*x)^2-3/5/a^5*\operatorname{arccoth}(a* \\ & x)^2*\ln(2)+3/10/a^5*\operatorname{arccoth}(a*x)^2*\ln(a*x-1)+3/10/a^5*\operatorname{arccoth}(a*x)^2*\ln(a*x \\ & +1)+3/10/a^5*\operatorname{arccoth}(a*x)^2*\ln((a*x-1)/(a*x+1))+3/5/a^5*\operatorname{arccoth}(a*x)^2*\ln(( \\ & a*x+1)/(a*x-1)-1)-3/5/a^5*\operatorname{arccoth}(a*x)^2*\ln(1-1/((a*x-1)/(a*x+1))^{(1/2)})-6/ \\ & 5/a^5*\operatorname{arccoth}(a*x)*\operatorname{polylog}(2,1/((a*x-1)/(a*x+1))^{(1/2)})-3/5/a^5*\operatorname{arccoth}(a*x) \\ & )^2*\ln(1+1/((a*x-1)/(a*x+1))^{(1/2)})-6/5/a^5*\operatorname{arccoth}(a*x)*\operatorname{polylog}(2,-1/((a*x \\ & -1)/(a*x+1))^{(1/2)})-3/20*I/a^5*\operatorname{Pi}*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1 \\ & ))^2*csgn(I/((a*x+1)/(a*x-1)-1))*\operatorname{arccoth}(a*x)^2-3/20*I/a^5*\operatorname{Pi}*csgn(I*(a*x+1) \\ & )/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I*(a*x+1)/(a*x-1))*\operatorname{arccoth}(a*x)^2-3/1 \\ & 0*I/a^5*\operatorname{Pi}*csgn(I/((a*x-1)/(a*x+1))^{(1/2)})*csgn(I*(a*x+1)/(a*x-1))^2*\operatorname{arccot} \\ & h(a*x)^2+3/20*I/a^5*\operatorname{Pi}*csgn(I/((a*x-1)/(a*x+1))^{(1/2)})^2*csgn(I*(a*x+1)/(a* \\ & x-1))*\operatorname{arccoth}(a*x)^2+3/20*I/a^5*\operatorname{Pi}*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)- \\ & 1))^3*\operatorname{arccoth}(a*x)^2+3/20*I/a^5*\operatorname{Pi}*csgn(I*(a*x+1)/(a*x-1))^3*\operatorname{arccoth}(a*x)^2 \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2(a^5x^5 + 1)\log(ax + 1)^3 + 3(a^4x^4 + 2a^2x^2 - 2(a^5x^5 - 1)\log(ax - 1))\log(ax + 1)^2}{80a^5} + \frac{1}{8} \int -\frac{5(a^5x^5 + a^4x^4)\log(ax - 1)}{a^5x^5 + a^4x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*arccoth(a*x)^3,x, algorithm="maxima")`

[Out] 
$$\begin{aligned} & 1/80*(2*(a^5*x^5 + 1)*\log(a*x + 1)^3 + 3*(a^4*x^4 + 2*a^2*x^2 - 2*(a^5*x^5 \\ & - 1)*\log(a*x - 1))*\log(a*x + 1)^2)/a^5 + 1/8*\operatorname{integrate}(-1/5*(5*(a^5*x^5 + a \\ & ^4*x^4)*\log(a*x - 1)^3 + 3*(a^4*x^4 + 2*a^2*x^2 - 5*(a^5*x^5 + a^4*x^4)*\log \\ & (a*x - 1)^2 - 2*(a^5*x^5 - 1)*\log(a*x - 1))*\log(a*x + 1))/(a^5*x + a^4), x) \end{aligned}$$

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^4 \operatorname{arccoth}(ax)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*arccoth(a*x)^3,x, algorithm="fricas")`

[Out] `integral(x^4*arccoth(a*x)^3, x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{acoth}^3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*acoth(a*x)**3,x)`

[Out] `Integral(x**4*acoth(a*x)**3, x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{arccoth}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*arccoth(a*x)^3,x, algorithm="giac")`

[Out] `integrate(x^4*arccoth(a*x)^3, x)`



### 3.25 $\int x^3 \coth^{-1}(ax)^3 dx$

**Optimal.** Leaf size=139

$$-\frac{\text{PolyLog}\left(2, 1 - \frac{2}{1-ax}\right)}{a^4} + \frac{x^2 \coth^{-1}(ax)}{4a^2} + \frac{x}{4a^3} - \frac{\tanh^{-1}(ax)}{4a^4} + \frac{3x \coth^{-1}(ax)^2}{4a^3} - \frac{\coth^{-1}(ax)^3}{4a^4} + \frac{\coth^{-1}(ax)^2}{a^4} - \frac{2 \log}{a^4}$$

[Out]  $x/(4*a^3) + (x^2*ArcCoth[a*x])/(4*a^2) + ArcCoth[a*x]^2/a^4 + (3*x*ArcCoth[a*x]^2)/(4*a^3) + (x^3*ArcCoth[a*x]^2)/(4*a) - ArcCoth[a*x]^3/(4*a^4) + (x^4*ArcCoth[a*x]^3)/4 - ArcTanh[a*x]/(4*a^4) - (2*ArcCoth[a*x]*Log[2/(1 - a*x)])/a^4 - PolyLog[2, 1 - 2/(1 - a*x)]/a^4$

**Rubi [A]** time = 0.41668, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 10, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.$ , Rules used = {5917, 5981, 321, 206, 5985, 5919, 2402, 2315, 5911, 5949}

$$-\frac{\text{PolyLog}\left(2, 1 - \frac{2}{1-ax}\right)}{a^4} + \frac{x^2 \coth^{-1}(ax)}{4a^2} + \frac{x}{4a^3} - \frac{\tanh^{-1}(ax)}{4a^4} + \frac{3x \coth^{-1}(ax)^2}{4a^3} - \frac{\coth^{-1}(ax)^3}{4a^4} + \frac{\coth^{-1}(ax)^2}{a^4} - \frac{2 \log}{a^4}$$

Antiderivative was successfully verified.

[In] Int[x^3\*ArcCoth[a\*x]^3,x]

[Out]  $x/(4*a^3) + (x^2*ArcCoth[a*x])/(4*a^2) + ArcCoth[a*x]^2/a^4 + (3*x*ArcCoth[a*x]^2)/(4*a^3) + (x^3*ArcCoth[a*x]^2)/(4*a) - ArcCoth[a*x]^3/(4*a^4) + (x^4*ArcCoth[a*x]^3)/4 - ArcTanh[a*x]/(4*a^4) - (2*ArcCoth[a*x]*Log[2/(1 - a*x)])/a^4 - PolyLog[2, 1 - 2/(1 - a*x)]/a^4$

#### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^ (p_.)*((d_.)*(x_))^(m_.), x_Symbol]
  :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 5981

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^ (p_.)*((f_.)*(x_))^(m_.)/((d_.) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(
```

```
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

### Rule 321

```
Int[((c_.)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

### Rule 5985

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.)*(x_)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

### Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
```

### Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dist[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

### Rule 2315

```
Int[Log[(c_.)*(x_)/((d_) + (e_.)*(x_))], x_Symbol] := -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

### Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*ArcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

### Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.)/((d_.) + (e_.)*(x_.)^2), x_Symbol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
 \int x^3 \coth^{-1}(ax)^3 dx &= \frac{1}{4}x^4 \coth^{-1}(ax)^3 - \frac{1}{4}(3a) \int \frac{x^4 \coth^{-1}(ax)^2}{1 - a^2x^2} dx \\
 &= \frac{1}{4}x^4 \coth^{-1}(ax)^3 + \frac{3 \int x^2 \coth^{-1}(ax)^2 dx}{4a} - \frac{3 \int \frac{x^2 \coth^{-1}(ax)^2}{1 - a^2x^2} dx}{4a} \\
 &= \frac{x^3 \coth^{-1}(ax)^2}{4a} + \frac{1}{4}x^4 \coth^{-1}(ax)^3 - \frac{1}{2} \int \frac{x^3 \coth^{-1}(ax)}{1 - a^2x^2} dx + \frac{3 \int \coth^{-1}(ax)^2 dx}{4a^3} - \frac{3 \int \frac{\coth^{-1}(ax)^2}{1 - a^2x^2} dx}{4a^3} \\
 &= \frac{3x \coth^{-1}(ax)^2}{4a^3} + \frac{x^3 \coth^{-1}(ax)^2}{4a} - \frac{\coth^{-1}(ax)^3}{4a^4} + \frac{1}{4}x^4 \coth^{-1}(ax)^3 + \frac{\int x \coth^{-1}(ax) dx}{2a^2} - \frac{\int \frac{x \coth^{-1}(ax)}{1 - a^2x^2} dx}{2a^2} \\
 &= \frac{x^2 \coth^{-1}(ax)}{4a^2} + \frac{\coth^{-1}(ax)^2}{a^4} + \frac{3x \coth^{-1}(ax)^2}{4a^3} + \frac{x^3 \coth^{-1}(ax)^2}{4a} - \frac{\coth^{-1}(ax)^3}{4a^4} + \frac{1}{4}x^4 \coth^{-1}(ax)^3 \\
 &= \frac{x}{4a^3} + \frac{x^2 \coth^{-1}(ax)}{4a^2} + \frac{\coth^{-1}(ax)^2}{a^4} + \frac{3x \coth^{-1}(ax)^2}{4a^3} + \frac{x^3 \coth^{-1}(ax)^2}{4a} - \frac{\coth^{-1}(ax)^3}{4a^4} + \frac{1}{4}x^4 \coth^{-1}(ax)^3 \\
 &= \frac{x}{4a^3} + \frac{x^2 \coth^{-1}(ax)}{4a^2} + \frac{\coth^{-1}(ax)^2}{a^4} + \frac{3x \coth^{-1}(ax)^2}{4a^3} + \frac{x^3 \coth^{-1}(ax)^2}{4a} - \frac{\coth^{-1}(ax)^3}{4a^4} + \frac{1}{4}x^4 \coth^{-1}(ax)^3
 \end{aligned}$$

**Mathematica [A]** time = 0.299733, size = 88, normalized size = 0.63

$$\frac{4 \operatorname{PolyLog}\left(2, e^{-2 \coth^{-1}(ax)}\right) + \left(a^4 x^4 - 1\right) \coth^{-1}(ax)^3 + \left(a^3 x^3 + 3ax - 4\right) \coth^{-1}(ax)^2 + \coth^{-1}(ax) \left(a^2 x^2 - 8 \log\left(1 - e^{-2 \coth^{-1}(ax)}\right)\right)}{4a^4}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3\*ArcCoth[a\*x]^3,x]

[Out] (a\*x + (-4 + 3\*a\*x + a^3\*x^3)\*ArcCoth[a\*x]^2 + (-1 + a^4\*x^4)\*ArcCoth[a\*x]^3 + ArcCoth[a\*x]\*(-1 + a^2\*x^2 - 8\*Log[1 - E^(-2\*ArcCoth[a\*x])])) + 4\*PolyLog[2, E^(-2\*ArcCoth[a\*x])])/(4\*a^4)

**Maple [C]** time = 0.619, size = 684, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*arccoth(a\*x)^3,x)

[Out] arccoth(a\*x)^2/a^4+3/8/a^4\*arccoth(a\*x)^2\*ln(a\*x-1)-3/8/a^4\*arccoth(a\*x)^2\*ln(a\*x+1)-2/a^4\*arccoth(a\*x)\*ln(1+1/((a\*x-1)/(a\*x+1))^(1/2))-3/8/a^4\*arccoth(a\*x)^2\*ln((a\*x-1)/(a\*x+1))-1/4/a^4/(-1+((a\*x-1)/(a\*x+1))^(1/2))\*((a\*x-1)/(a\*x+1))^(1/2)-1/4/a^4/(((a\*x-1)/(a\*x+1))^(1/2)+1)\*((a\*x-1)/(a\*x+1))^(1/2)-1/4/a^4\*arccoth(a\*x)+1/4\*x^4\*arccoth(a\*x)^3+3/4\*x\*arccoth(a\*x)^2/a^3+1/4\*x^3\*arccoth(a\*x)^2/a+1/4\*x^2\*arccoth(a\*x)/a^2+2/a^4\*dilog(1/((a\*x-1)/(a\*x+1))^(1/2))-2/a^4\*dilog(1+1/((a\*x-1)/(a\*x+1))^(1/2))-3/16\*I/a^4\*Pi\*csgn(I\*(a\*x+1)/(a\*x-1))^3\*arccoth(a\*x)^2-3/16\*I/a^4\*Pi\*csgn(I\*(a\*x+1)/(a\*x-1)/((a\*x+1)/(a\*x-1)-1))^3\*arccoth(a\*x)^2+3/16\*I/a^4\*Pi\*csgn(I\*(a\*x+1)/(a\*x-1)/((a\*x+1)/(a\*x-1)-1))^2\*csgn(I/((a\*x+1)/(a\*x-1)-1))\*arccoth(a\*x)^2-1/4\*arccoth(a\*x)^3/a^4-3/16\*I/a^4\*Pi\*csgn(I/((a\*x-1)/(a\*x+1))^(1/2))^2\*csgn(I\*(a\*x+1)/(a\*x-1))\*arccoth(a\*x)^2+3/8\*I/a^4\*Pi\*csgn(I/((a\*x-1)/(a\*x+1))^(1/2))\*csgn(I\*(a\*x+1)/(a\*x-1))^2\*arccoth(a\*x)^2+3/16\*I/a^4\*Pi\*csgn(I\*(a\*x+1)/(a\*x-1)/((a\*x+1)/(a\*x-1)-1))^2\*csgn(I\*(a\*x+1)/(a\*x-1))\*arccoth(a\*x)^2-3/16\*I/a^4\*Pi\*csgn(I\*(a\*x+1)/(a\*x-1)/((a\*x+1)/(a\*x-1)-1))\*csgn(I\*(a\*x+1)/(a\*x-1))\*csgn(I/((a\*x+1)/(a\*x-1)-1))\*arccoth(a\*x)^2

**Maxima [B]** time = 0.994781, size = 354, normalized size = 2.55

$$\frac{1}{4}x^4 \operatorname{arccoth}(ax)^3 + \frac{1}{8}a \left( \frac{2(a^2x^3 + 3x)}{a^4} - \frac{3 \log(ax+1)}{a^5} + \frac{3 \log(ax-1)}{a^5} \right) \operatorname{arccoth}(ax)^2 + \frac{1}{32}a \left( \frac{(3 \log(ax-1)-8) \log(ax+1)^2 - \log^2(ax-1)}{a^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(a\*x)^3,x, algorithm="maxima")

[Out]  $\frac{1}{4}x^4\operatorname{arccoth}(ax)^3 + \frac{1}{8}a\left(\frac{2(a^2x^3 + 3x)}{a^4} - 3\log(ax + 1)\right)/a^5 + 3\log(ax - 1)/a^5\operatorname{arccoth}(ax)^2 + \frac{1}{32}a\left(\left(\left(3\log(ax - 1) - 8\right)\log(ax + 1)^2 - \log(ax + 1)^3 + \log(ax - 1)^3 + 8ax - (3\log(ax - 1))^2 - 16\log(ax - 1)\right)\log(ax + 1) + 8\log(ax - 1)^2 + 4\log(ax - 1)\right)/a - 32\left(\log(ax - 1)\log\left(\frac{1}{2}ax + \frac{1}{2}\right) + \operatorname{dilog}\left(-\frac{1}{2}ax + \frac{1}{2}\right)\right)/a - 4\log(ax + 1)/a/a^4 + 2\left(4a^2x^2 - 2(3\log(ax - 1) - 8)\log(ax + 1) + 3\log(ax + 1)^2 + 3\log(ax - 1)^2 + 16\log(ax - 1)\right)\operatorname{arccoth}(ax)/a^5$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(x^3 \operatorname{arccoth}(ax)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(a\*x)^3,x, algorithm="fricas")

[Out] integral(x^3\*arccoth(a\*x)^3, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arccoth}^3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*arccoth(a\*x)\*\*3,x)

[Out] Integral(x\*\*3\*arccoth(a\*x)\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arccoth}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*arccoth(a*x)^3, x)
```

### 3.26 $\int x^2 \coth^{-1}(ax)^3 dx$

**Optimal.** Leaf size=149

$$\frac{\text{PolyLog}\left(3, 1 - \frac{2}{1-ax}\right)}{2a^3} - \frac{\coth^{-1}(ax)\text{PolyLog}\left(2, 1 - \frac{2}{1-ax}\right)}{a^3} + \frac{\log(1 - a^2x^2)}{2a^3} + \frac{\coth^{-1}(ax)^3}{3a^3} - \frac{\coth^{-1}(ax)^2}{2a^3} + \frac{x \coth^{-1}(ax)}{a^2}$$

[Out] (x\*ArcCoth[a\*x])/a^2 - ArcCoth[a\*x]^2/(2\*a^3) + (x^2\*ArcCoth[a\*x]^2)/(2\*a) + ArcCoth[a\*x]^3/(3\*a^3) + (x^3\*ArcCoth[a\*x]^3)/3 - (ArcCoth[a\*x]^2\*Log[2/(1 - a\*x)])/a^3 + Log[1 - a^2\*x^2]/(2\*a^3) - (ArcCoth[a\*x]\*PolyLog[2, 1 - 2/(1 - a\*x)])/a^3 + PolyLog[3, 1 - 2/(1 - a\*x)]/(2\*a^3)

**Rubi [A]** time = 0.332522, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.9$ , Rules used = {5917, 5981, 5911, 260, 5949, 5985, 5919, 6059, 6610}

$$\frac{\text{PolyLog}\left(3, 1 - \frac{2}{1-ax}\right)}{2a^3} - \frac{\coth^{-1}(ax)\text{PolyLog}\left(2, 1 - \frac{2}{1-ax}\right)}{a^3} + \frac{\log(1 - a^2x^2)}{2a^3} + \frac{\coth^{-1}(ax)^3}{3a^3} - \frac{\coth^{-1}(ax)^2}{2a^3} + \frac{x \coth^{-1}(ax)}{a^2}$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcCoth[a\*x]^3,x]

[Out] (x\*ArcCoth[a\*x])/a^2 - ArcCoth[a\*x]^2/(2\*a^3) + (x^2\*ArcCoth[a\*x]^2)/(2\*a) + ArcCoth[a\*x]^3/(3\*a^3) + (x^3\*ArcCoth[a\*x]^3)/3 - (ArcCoth[a\*x]^2\*Log[2/(1 - a\*x)])/a^3 + Log[1 - a^2\*x^2]/(2\*a^3) - (ArcCoth[a\*x]\*PolyLog[2, 1 - 2/(1 - a\*x)])/a^3 + PolyLog[3, 1 - 2/(1 - a\*x)]/(2\*a^3)

#### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^ (p_.)*((d_.)*(x_))^(m_.), x_Symbol]
  :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 5981

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^ (p_.)*((f_.)*(x_))^(m_.)/((d_.) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(
```

$d + e*x^2$ ), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]

### Rule 5911

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.), x\_Symbol] := Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rule 5949

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

### Rule 5985

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*(x\_.))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*e\*(p + 1)), x] + Dist[1/(c\*d), Int[(a + b\*ArcCoth[c\*x])^p/(1 - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && IGtQ[p, 0]

### Rule 5919

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[((a + b\*ArcCoth[c\*x])^p\*Log[2/(1 + (e\*x)/d)])/e, x] + Dist[(b\*c\*p)/e, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*Log[2/(1 + (e\*x)/d)]]/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

### Rule 6059

Int[(Log[u\_]\*((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := -Simp[((a + b\*ArcCoth[c\*x])^p\*PolyLog[2, 1 - u])/(2\*c\*d), x] + Dist[(b\*p)/2, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*PolyLog[2, 1 - u])/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c\*x))^2, 0]

### Rule 6610



```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

### Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(ax)^3 dx &= \frac{1}{3}x^3 \coth^{-1}(ax)^3 - a \int \frac{x^3 \coth^{-1}(ax)^2}{1 - a^2x^2} dx \\
&= \frac{1}{3}x^3 \coth^{-1}(ax)^3 + \frac{\int x \coth^{-1}(ax)^2 dx}{a} - \frac{\int \frac{x \coth^{-1}(ax)^2}{1 - a^2x^2} dx}{a} \\
&= \frac{x^2 \coth^{-1}(ax)^2}{2a} + \frac{\coth^{-1}(ax)^3}{3a^3} + \frac{1}{3}x^3 \coth^{-1}(ax)^3 - \frac{\int \frac{\coth^{-1}(ax)^2}{1 - ax} dx}{a^2} - \int \frac{x^2 \coth^{-1}(ax)}{1 - a^2x^2} dx \\
&= \frac{x^2 \coth^{-1}(ax)^2}{2a} + \frac{\coth^{-1}(ax)^3}{3a^3} + \frac{1}{3}x^3 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^2 \log\left(\frac{2}{1 - ax}\right)}{a^3} + \frac{\int \coth^{-1}(ax) dx}{a^2} \\
&= \frac{x \coth^{-1}(ax)}{a^2} - \frac{\coth^{-1}(ax)^2}{2a^3} + \frac{x^2 \coth^{-1}(ax)^2}{2a} + \frac{\coth^{-1}(ax)^3}{3a^3} + \frac{1}{3}x^3 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)}{a} \\
&= \frac{x \coth^{-1}(ax)}{a^2} - \frac{\coth^{-1}(ax)^2}{2a^3} + \frac{x^2 \coth^{-1}(ax)^2}{2a} + \frac{\coth^{-1}(ax)^3}{3a^3} + \frac{1}{3}x^3 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)}{a}
\end{aligned}$$

**Mathematica [C]** time = 0.352743, size = 140, normalized size = 0.94

$$-24 \coth^{-1}(ax) \text{PolyLog}\left(2, e^{2 \coth^{-1}(ax)}\right) + 12 \text{PolyLog}\left(3, e^{2 \coth^{-1}(ax)}\right) - 24 \log\left(\frac{1}{ax \sqrt{1 - \frac{1}{a^2x^2}}}\right) + 8a^3x^3 \coth^{-1}(ax)^3 + 12a$$

---

24a<sup>3</sup>

Warning: Unable to verify antiderivative.

[In] Integrate[x^2\*ArcCoth[a\*x]^3,x]

[Out]  $\frac{((-1)\pi^3 + 24ax \text{ArcCoth}[ax] - 12 \text{ArcCoth}[ax]^2 + 12a^2x^2 \text{ArcCoth}[ax]^2 + 8 \text{ArcCoth}[ax]^3 + 8a^3x^3 \text{ArcCoth}[ax]^3 - 24 \text{ArcCoth}[ax]^2 \text{Log}[1 - E^{(2 \text{ArcCoth}[ax])}] - 24 \text{Log}[1/(a \sqrt{1 - 1/(a^2x^2)})] - 24 \text{ArcCoth}[ax] \text{PolyLog}[2, E^{(2 \text{ArcCoth}[ax])}] + 12 \text{PolyLog}[3, E^{(2 \text{ArcCoth}[ax])}])}{(24a^3)}$

---



---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^2 \operatorname{arccoth}(ax)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccoth(a*x)^3,x, algorithm="fricas")`

[Out] `integral(x^2*arccoth(a*x)^3, x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{acoth}^3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*acoth(a*x)**3,x)`

[Out] `Integral(x**2*acoth(a*x)**3, x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arccoth}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccoth(a*x)^3,x, algorithm="giac")`

[Out] `integrate(x^2*arccoth(a*x)^3, x)`

### 3.27 $\int x \coth^{-1}(ax)^3 dx$

**Optimal.** Leaf size=95

$$-\frac{3\text{PolyLog}\left(2, 1 - \frac{2}{1-ax}\right)}{2a^2} - \frac{\coth^{-1}(ax)^3}{2a^2} + \frac{3\coth^{-1}(ax)^2}{2a^2} - \frac{3\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a^2} + \frac{1}{2}x^2\coth^{-1}(ax)^3 + \frac{3x\coth^{-1}(ax)}{2a}$$

[Out] (3\*ArcCoth[a\*x]^2)/(2\*a^2) + (3\*x\*ArcCoth[a\*x]^2)/(2\*a) - ArcCoth[a\*x]^3/(2\*a^2) + (x^2\*ArcCoth[a\*x]^3)/2 - (3\*ArcCoth[a\*x]\*Log[2/(1 - a\*x)])/a^2 - (3\*PolyLog[2, 1 - 2/(1 - a\*x)])/(2\*a^2)

**Rubi [A]** time = 0.183365, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.$ , Rules used = {5917, 5981, 5911, 5985, 5919, 2402, 2315, 5949}

$$-\frac{3\text{PolyLog}\left(2, 1 - \frac{2}{1-ax}\right)}{2a^2} - \frac{\coth^{-1}(ax)^3}{2a^2} + \frac{3\coth^{-1}(ax)^2}{2a^2} - \frac{3\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a^2} + \frac{1}{2}x^2\coth^{-1}(ax)^3 + \frac{3x\coth^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

[In] Int[x\*ArcCoth[a\*x]^3, x]

[Out] (3\*ArcCoth[a\*x]^2)/(2\*a^2) + (3\*x\*ArcCoth[a\*x]^2)/(2\*a) - ArcCoth[a\*x]^3/(2\*a^2) + (x^2\*ArcCoth[a\*x]^3)/2 - (3\*ArcCoth[a\*x]\*Log[2/(1 - a\*x)])/a^2 - (3\*PolyLog[2, 1 - 2/(1 - a\*x)])/(2\*a^2)

#### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^ (p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 5981

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^ (p_.)*((f_.)*(x_))^(m_.)/((d_.) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
```

]

Rule 5911

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^(p\_.), x\_Symbol] := Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

Rule 5985

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)\*(x\_)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*e\*(p + 1)), x] + Dist[1/(c\*d), Int[(a + b\*ArcCoth[c\*x])^p/(1 - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && IGtQ[p, 0]

Rule 5919

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[((a + b\*ArcCoth[c\*x])^p\*Log[2/(1 + (e\*x)/d)])/e, x] + Dist[(b\*c\*p)/e, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*Log[2/(1 + (e\*x)/d)])/((1 - c^2\*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

Rule 2402

Int[Log[(c\_.)/((d\_) + (e\_.)\*(x\_))]/((f\_) + (g\_.)\*(x\_)^2), x\_Symbol] := -Dist[e/g, Subst[Int[Log[2\*d\*x]/(1 - 2\*d\*x), x], x, 1/(d + e\*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2\*d] && EqQ[e^2\*f + d^2\*g, 0]

Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

Rule 5949

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
\int x \coth^{-1}(ax)^3 dx &= \frac{1}{2}x^2 \coth^{-1}(ax)^3 - \frac{1}{2}(3a) \int \frac{x^2 \coth^{-1}(ax)^2}{1-a^2x^2} dx \\
&= \frac{1}{2}x^2 \coth^{-1}(ax)^3 + \frac{3 \int \coth^{-1}(ax)^2 dx}{2a} - \frac{3 \int \frac{\coth^{-1}(ax)^2}{1-a^2x^2} dx}{2a} \\
&= \frac{3x \coth^{-1}(ax)^2}{2a} - \frac{\coth^{-1}(ax)^3}{2a^2} + \frac{1}{2}x^2 \coth^{-1}(ax)^3 - 3 \int \frac{x \coth^{-1}(ax)}{1-a^2x^2} dx \\
&= \frac{3 \coth^{-1}(ax)^2}{2a^2} + \frac{3x \coth^{-1}(ax)^2}{2a} - \frac{\coth^{-1}(ax)^3}{2a^2} + \frac{1}{2}x^2 \coth^{-1}(ax)^3 - \frac{3 \int \frac{\coth^{-1}(ax)}{1-ax} dx}{a} \\
&= \frac{3 \coth^{-1}(ax)^2}{2a^2} + \frac{3x \coth^{-1}(ax)^2}{2a} - \frac{\coth^{-1}(ax)^3}{2a^2} + \frac{1}{2}x^2 \coth^{-1}(ax)^3 - \frac{3 \coth^{-1}(ax) \log\left(\frac{2}{1-ax}\right)}{a^2} + 3 \\
&= \frac{3 \coth^{-1}(ax)^2}{2a^2} + \frac{3x \coth^{-1}(ax)^2}{2a} - \frac{\coth^{-1}(ax)^3}{2a^2} + \frac{1}{2}x^2 \coth^{-1}(ax)^3 - \frac{3 \coth^{-1}(ax) \log\left(\frac{2}{1-ax}\right)}{a^2} - 3 \\
&= \frac{3 \coth^{-1}(ax)^2}{2a^2} + \frac{3x \coth^{-1}(ax)^2}{2a} - \frac{\coth^{-1}(ax)^3}{2a^2} + \frac{1}{2}x^2 \coth^{-1}(ax)^3 - \frac{3 \coth^{-1}(ax) \log\left(\frac{2}{1-ax}\right)}{a^2} - 3
\end{aligned}$$

**Mathematica [A]** time = 0.140715, size = 68, normalized size = 0.72

$$\frac{3\text{PolyLog}\left(2, e^{-2\coth^{-1}(ax)}\right) + \coth^{-1}(ax)\left(\left(a^2x^2 - 1\right)\coth^{-1}(ax)^2 + 3(ax - 1)\coth^{-1}(ax) - 6\log\left(1 - e^{-2\coth^{-1}(ax)}\right)\right)}{2a^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x\*ArcCoth[a\*x]^3,x]

[Out] (ArcCoth[a\*x]\*(3\*(-1 + a\*x)\*ArcCoth[a\*x] + (-1 + a^2\*x^2)\*ArcCoth[a\*x]^2 - 6\*Log[1 - E^(-2\*ArcCoth[a\*x])]) + 3\*PolyLog[2, E^(-2\*ArcCoth[a\*x])])/(2\*a^2)

**Maple [C]** time = 0.404, size = 3070, normalized size = 32.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(a\*x)^3,x)

```

[Out] 3/2*arccoth(a*x)^2/a^2+3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)
-1))^3*dilog(1/((a*x-1)/(a*x+1))^(1/2))-3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1)
/((a*x+1)/(a*x-1)-1))^3*dilog(1+1/((a*x-1)/(a*x+1))^(1/2))+3/8*I/a^2*Pi*csg
n(I*(a*x+1)/(a*x-1))^3*dilog(1/((a*x-1)/(a*x+1))^(1/2))-3/8*I/a^2*Pi*csgn(I
*(a*x+1)/(a*x-1))^3*dilog(1+1/((a*x-1)/(a*x+1))^(1/2))+3/8*I/a^2*Pi*csgn(I*
(a*x+1)/(a*x-1))^3*polylog(2,1/((a*x-1)/(a*x+1))^(1/2))+3/8*I/a^2*Pi*csgn(I
*(a*x+1)/(a*x-1))^3*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))-3/8*I/a^2*Pi*csgn
(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^3*arccoth(a*x)^2-3/8*I/a^2*Pi*csgn(
I*(a*x+1)/(a*x-1))^3*arccoth(a*x)^2+3/4/a^2*arccoth(a*x)^2*ln(a*x-1)-3/4/a^
2*arccoth(a*x)^2*ln(a*x+1)-3/4/a^2*arccoth(a*x)^2*ln((a*x-1)/(a*x+1))+3/8*I
/a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I*(a*x+1)/(a*x-1))*polylog(2
,1/((a*x-1)/(a*x+1))^(1/2))-3/4*I/a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*cs
gn(I*(a*x+1)/(a*x-1))^2*polylog(2,1/((a*x-1)/(a*x+1))^(1/2))+3/4*I/a^2*Pi*c
sgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/(a*x-1))^2*arccoth(a*x)^2+3/8
*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I*(a*x+1)/(a*x
-1))*arccoth(a*x)^2-3/8*I/a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I*(
a*x+1)/(a*x-1))*arccoth(a*x)^2+3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)
/(a*x-1)-1))^2*csgn(I/((a*x+1)/(a*x-1)-1))*arccoth(a*x)^2-3/4*I/a^2*Pi*csgn
(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/(a*x-1))^2*polylog(2,-1/((a*x-1)
/(a*x+1))^(1/2))-3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2
*csgn(I/((a*x+1)/(a*x-1)-1))*dilog(1/((a*x-1)/(a*x+1))^(1/2))+3/8*I/a^2*Pi*
csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I/((a*x+1)/(a*x-1)-1))*d
ilog(1+1/((a*x-1)/(a*x+1))^(1/2))-3/4*I/a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/
2))*csgn(I*(a*x+1)/(a*x-1))^2*dilog(1/((a*x-1)/(a*x+1))^(1/2))+3/4*I/a^2*Pi
*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/(a*x-1))^2*dilog(1+1/((a*x-
1)/(a*x+1))^(1/2))-3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1)
)^2*csgn(I*(a*x+1)/(a*x-1))*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))-3/8*I/a^2
*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I*(a*x+1)/(a*x-1))*po
lylog(2,1/((a*x-1)/(a*x+1))^(1/2))-3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*
x+1)/(a*x-1)-1))^2*csgn(I/((a*x+1)/(a*x-1)-1))*polylog(2,1/((a*x-1)/(a*x+1)
)^(1/2))+3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^3*arccoth
(a*x)*ln(1-1/((a*x-1)/(a*x+1))^(1/2))+3/8*I/a^2*Pi*csgn(I/((a*x-1)/(a*x+1))
^(1/2))^2*csgn(I*(a*x+1)/(a*x-1))*dilog(1/((a*x-1)/(a*x+1))^(1/2))-3/8*I/a^
2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I*(a*x+1)/(a*x-1))*
dilog(1/((a*x-1)/(a*x+1))^(1/2))+3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+
1)/(a*x-1)-1))^2*csgn(I*(a*x+1)/(a*x-1))*dilog(1+1/((a*x-1)/(a*x+1))^(1/2))
-3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I/((a*x+1)
/(a*x-1)-1))*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))-3/8*I/a^2*Pi*csgn(I/((a*
x-1)/(a*x+1))^(1/2))^2*csgn(I*(a*x+1)/(a*x-1))*dilog(1+1/((a*x-1)/(a*x+1))^(
1/2))+3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1))^3*arccoth(a*x)*ln(1-1/((a*x-1)/
(a*x+1))^(1/2))+3/8*I/a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I*(a*x+
1)/(a*x-1))*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))+3/8*I/a^2*Pi*csgn(I*(a*x+
1)/(a*x-1)/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1))*csgn(I/((a*x+1)/(a*
x-1)-1))*arccoth(a*x)*ln(1-1/((a*x-1)/(a*x+1))^(1/2))-3/a^2*arccoth(a*x)*ln
(1+1/((a*x-1)/(a*x+1))^(1/2))-3/2/a^2*arccoth(a*x)*ln(1-1/((a*x-1)/(a*x+1))

```





**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x \operatorname{arccoth}(ax)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(a\*x)^3,x, algorithm="fricas")

[Out] integral(x\*arccoth(a\*x)^3, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{acoth}^3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(a\*x)\*\*3,x)

[Out] Integral(x\*acoth(a\*x)\*\*3, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arccoth}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(a\*x)^3,x, algorithm="giac")

[Out] integrate(x\*arccoth(a\*x)^3, x)

### 3.28 $\int \coth^{-1}(ax)^3 dx$

**Optimal.** Leaf size=85

$$\frac{3\text{PolyLog}\left(3, 1 - \frac{2}{1-ax}\right)}{2a} - \frac{3 \coth^{-1}(ax)\text{PolyLog}\left(2, 1 - \frac{2}{1-ax}\right)}{a} + x \coth^{-1}(ax)^3 + \frac{\coth^{-1}(ax)^3}{a} - \frac{3 \log\left(\frac{2}{1-ax}\right) \coth^{-1}(ax)}{a}$$

[Out] ArcCoth[a\*x]^3/a + x\*ArcCoth[a\*x]^3 - (3\*ArcCoth[a\*x]^2\*Log[2/(1 - a\*x)])/a - (3\*ArcCoth[a\*x]\*PolyLog[2, 1 - 2/(1 - a\*x)])/a + (3\*PolyLog[3, 1 - 2/(1 - a\*x)])/(2\*a)

**Rubi [A]** time = 0.165484, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.$ , Rules used = {5911, 5985, 5919, 5949, 6059, 6610}

$$\frac{3\text{PolyLog}\left(3, 1 - \frac{2}{1-ax}\right)}{2a} - \frac{3 \coth^{-1}(ax)\text{PolyLog}\left(2, 1 - \frac{2}{1-ax}\right)}{a} + x \coth^{-1}(ax)^3 + \frac{\coth^{-1}(ax)^3}{a} - \frac{3 \log\left(\frac{2}{1-ax}\right) \coth^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]^3,x]

[Out] ArcCoth[a\*x]^3/a + x\*ArcCoth[a\*x]^3 - (3\*ArcCoth[a\*x]^2\*Log[2/(1 - a\*x)])/a - (3\*ArcCoth[a\*x]\*PolyLog[2, 1 - 2/(1 - a\*x)])/a + (3\*PolyLog[3, 1 - 2/(1 - a\*x)])/(2\*a)

#### Rule 5911

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.), x\_Symbol] := Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

#### Rule 5985

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*(x\_.))/((d\_.) + (e\_.)\*(x\_.)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*e\*(p + 1)), x] + Dist[1/(c\*d), Int[(a + b\*ArcCoth[c\*x])^p/(1 - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && IGtQ[p, 0]

#### Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
:> -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)]]/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
```

### Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol]
:> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

### Rule 6059

```
Int[(Log[u_] * ((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> -Simp[((a + b*ArcCoth[c*x])^p * PolyLog[2, 1 - u])/(2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1) * PolyLog[2, 1 - u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

### Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v, x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

### Rubi steps

$$\begin{aligned}
\int \coth^{-1}(ax)^3 dx &= x \coth^{-1}(ax)^3 - (3a) \int \frac{x \coth^{-1}(ax)^2}{1 - a^2x^2} dx \\
&= \frac{\coth^{-1}(ax)^3}{a} + x \coth^{-1}(ax)^3 - 3 \int \frac{\coth^{-1}(ax)^2}{1 - ax} dx \\
&= \frac{\coth^{-1}(ax)^3}{a} + x \coth^{-1}(ax)^3 - \frac{3 \coth^{-1}(ax)^2 \log\left(\frac{2}{1-ax}\right)}{a} + 6 \int \frac{\coth^{-1}(ax) \log\left(\frac{2}{1-ax}\right)}{1 - a^2x^2} dx \\
&= \frac{\coth^{-1}(ax)^3}{a} + x \coth^{-1}(ax)^3 - \frac{3 \coth^{-1}(ax)^2 \log\left(\frac{2}{1-ax}\right)}{a} - \frac{3 \coth^{-1}(ax) \text{Li}_2\left(1 - \frac{2}{1-ax}\right)}{a} + 3 \int \frac{\text{Li}_2}{1 - a^2x^2} dx \\
&= \frac{\coth^{-1}(ax)^3}{a} + x \coth^{-1}(ax)^3 - \frac{3 \coth^{-1}(ax)^2 \log\left(\frac{2}{1-ax}\right)}{a} - \frac{3 \coth^{-1}(ax) \text{Li}_2\left(1 - \frac{2}{1-ax}\right)}{a} + \frac{3 \text{Li}_3\left(1 - \frac{2}{1-ax}\right)}{2a}
\end{aligned}$$

**Mathematica [A]** time = 0.0976196, size = 79, normalized size = 0.93

$$\frac{3 \coth^{-1}(ax) \text{PolyLog}\left(2, e^{2 \coth^{-1}(ax)}\right)}{a} + \frac{3 \text{PolyLog}\left(3, e^{2 \coth^{-1}(ax)}\right)}{2a} + x \coth^{-1}(ax)^3 + \frac{\coth^{-1}(ax)^3}{a} - \frac{3 \coth^{-1}(ax)^2 \log\left(\frac{1 - e^{2 \coth^{-1}(ax)}}{1 + e^{2 \coth^{-1}(ax)}}\right)}{a}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a\*x]^3, x]

[Out] ArcCoth[a\*x]^3/a + x\*ArcCoth[a\*x]^3 - (3\*ArcCoth[a\*x]^2\*Log[1 - E^(2\*ArcCoth[a\*x])])/a - (3\*ArcCoth[a\*x]\*PolyLog[2, E^(2\*ArcCoth[a\*x])])/a + (3\*PolyLog[3, E^(2\*ArcCoth[a\*x])])/(2\*a)

**Maple [B]** time = 0.115, size = 180, normalized size = 2.1

$$x(\operatorname{arccoth}(ax))^3 - 3 \frac{(\operatorname{arccoth}(ax))^2}{a} \ln\left(1 + \frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right) - 3 \frac{(\operatorname{arccoth}(ax))^2}{a} \ln\left(1 - \frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right) + \frac{(\operatorname{arccoth}(ax))^3}{a} - 6 \frac{\operatorname{arccoth}(ax)}{a} \ln\left(\frac{1 - \sqrt{\frac{ax-1}{ax+1}}}{1 + \sqrt{\frac{ax-1}{ax+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)^3, x)

[Out] x\*arccoth(a\*x)^3-3/a\*arccoth(a\*x)^2\*ln(1+1/((a\*x-1)/(a\*x+1))^(1/2))-3/a\*arccoth(a\*x)^2\*ln(1-1/((a\*x-1)/(a\*x+1))^(1/2))+arccoth(a\*x)^3/a-6/a\*arccoth(a\*x)\*polylog(2,-1/((a\*x-1)/(a\*x+1))^(1/2))-6/a\*arccoth(a\*x)\*polylog(2,1/((a\*x-1)/(a\*x+1))^(1/2))+6/a\*polylog(3,-1/((a\*x-1)/(a\*x+1))^(1/2))+6/a\*polylog(3,1/((a\*x-1)/(a\*x+1))^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(ax+1) \log(ax+1)^3 - 3(ax-1) \log(ax+1)^2 \log(ax-1)}{8a} + \frac{1}{8} \int -\frac{(ax+1) \log(ax-1)^3 - 3((ax+1) \log(ax-1))^2 + 2(ax+1) \log(ax-1)}{ax+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^3, x, algorithm="maxima")

```
[Out] 1/8*((a*x + 1)*log(a*x + 1)^3 - 3*(a*x - 1)*log(a*x + 1)^2*log(a*x - 1))/a
+ 1/8*integrate(-((a*x + 1)*log(a*x - 1)^3 - 3*((a*x + 1)*log(a*x - 1)^2 +
2*(a*x - 1)*log(a*x - 1))*log(a*x + 1))/(a*x + 1), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\text{arcoth}(ax)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)^3, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \text{acoth}^3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)**3,x)
```

```
[Out] Integral(acoth(a*x)**3, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \text{arcoth}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)^3, x)
```

$$3.29 \quad \int \frac{\coth^{-1}(ax)^3}{x} dx$$

**Optimal.** Leaf size=150

$$\frac{3}{4}\text{PolyLog}\left(4, 1 - \frac{2}{ax+1}\right) - \frac{3}{4}\text{PolyLog}\left(4, 1 - \frac{2ax}{ax+1}\right) + \frac{3}{2}\coth^{-1}(ax)^2\text{PolyLog}\left(2, 1 - \frac{2}{ax+1}\right) - \frac{3}{2}\coth^{-1}(ax)^2\text{PolyL}$$

```
[Out] 2*ArcCoth[a*x]^3*ArcCoth[1 - 2/(1 - a*x)] + (3*ArcCoth[a*x]^2*PolyLog[2, 1 - 2/(1 + a*x)])/2 - (3*ArcCoth[a*x]^2*PolyLog[2, 1 - (2*a*x)/(1 + a*x)])/2 + (3*ArcCoth[a*x]*PolyLog[3, 1 - 2/(1 + a*x)])/2 - (3*ArcCoth[a*x]*PolyLog[3, 1 - (2*a*x)/(1 + a*x)])/2 + (3*PolyLog[4, 1 - 2/(1 + a*x)])/4 - (3*PolyLog[4, 1 - (2*a*x)/(1 + a*x)])/4
```

---

**Rubi [A]** time = 0.350385, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$ , Rules used = {5915, 6053, 5949, 6057, 6061, 6610}

$$\frac{3}{4}\text{PolyLog}\left(4, 1 - \frac{2}{ax+1}\right) - \frac{3}{4}\text{PolyLog}\left(4, 1 - \frac{2ax}{ax+1}\right) + \frac{3}{2}\coth^{-1}(ax)^2\text{PolyLog}\left(2, 1 - \frac{2}{ax+1}\right) - \frac{3}{2}\coth^{-1}(ax)^2\text{PolyL}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]^3/x, x]
```

```
[Out] 2*ArcCoth[a*x]^3*ArcCoth[1 - 2/(1 - a*x)] + (3*ArcCoth[a*x]^2*PolyLog[2, 1 - 2/(1 + a*x)])/2 - (3*ArcCoth[a*x]^2*PolyLog[2, 1 - (2*a*x)/(1 + a*x)])/2 + (3*ArcCoth[a*x]*PolyLog[3, 1 - 2/(1 + a*x)])/2 - (3*ArcCoth[a*x]*PolyLog[3, 1 - (2*a*x)/(1 + a*x)])/2 + (3*PolyLog[4, 1 - 2/(1 + a*x)])/4 - (3*PolyLog[4, 1 - (2*a*x)/(1 + a*x)])/4
```

### Rule 5915

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_)/(x_), x_Symbol] := Simp[2*(a + b*ArcCoth[c*x])^p*ArcCoth[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b*ArcCoth[c*x])^(p - 1)*ArcCoth[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

### Rule 6053

```
Int[(ArcCoth[u_]*((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.))/((d_.) + (e_.)*(x_)^2), x_Symbol] := Dist[1/2, Int[(Log[SimplifyIntegrand[1 + 1/u, x]]*(a +
```

$b \operatorname{ArcCoth}[c*x]^p / (d + e*x^2), x, x] - \operatorname{Dist}[1/2, \operatorname{Int}[(\operatorname{Log}[\operatorname{SimplifyIntegr} \operatorname{and}[1 - 1/u, x]]*(a + b \operatorname{ArcCoth}[c*x]^p) / (d + e*x^2), x], x] /; \operatorname{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \operatorname{IGtQ}[p, 0] \ \&\& \operatorname{EqQ}[c^2*d + e, 0] \ \&\& \operatorname{EqQ}[u^2 - (1 - 2/(1 - c*x))^2, 0]$

### Rule 5949

$\operatorname{Int}[(a + b \operatorname{ArcCoth}[c*x]^p) / (d + e*x^2), x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(a + b \operatorname{ArcCoth}[c*x]^{p+1}) / (b*c*d*(p+1)), x] /; \operatorname{FreeQ}[\{a, b, c, d, e, p\}, x] \ \&\& \operatorname{EqQ}[c^2*d + e, 0] \ \&\& \operatorname{NeQ}[p, -1]$

### Rule 6057

$\operatorname{Int}[(\operatorname{Log}[u]*(a + b \operatorname{ArcCoth}[c*x]^p) / (d + e*x^2), x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(a + b \operatorname{ArcCoth}[c*x]^p * \operatorname{PolyLog}[2, 1 - u]) / (2*c*d), x] - \operatorname{Dist}[(b*p)/2, \operatorname{Int}[(a + b \operatorname{ArcCoth}[c*x]^{p-1}) * \operatorname{PolyLog}[2, 1 - u]) / (d + e*x^2), x], x] /; \operatorname{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \operatorname{IGtQ}[p, 0] \ \&\& \operatorname{EqQ}[c^2*d + e, 0] \ \&\& \operatorname{EqQ}[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]$

### Rule 6061

$\operatorname{Int}[(a + b \operatorname{ArcCoth}[c*x]^p * \operatorname{PolyLog}[k, u]) / (d + e*x^2), x_{\text{Symbol}}] \rightarrow -\operatorname{Simp}[(a + b \operatorname{ArcCoth}[c*x]^p * \operatorname{PolyLog}[k + 1, u]) / (2*c*d), x] + \operatorname{Dist}[(b*p)/2, \operatorname{Int}[(a + b \operatorname{ArcCoth}[c*x]^{p-1}) * \operatorname{PolyLog}[k + 1, u]) / (d + e*x^2), x], x] /; \operatorname{FreeQ}[\{a, b, c, d, e, k\}, x] \ \&\& \operatorname{IGtQ}[p, 0] \ \&\& \operatorname{EqQ}[c^2*d + e, 0] \ \&\& \operatorname{EqQ}[u^2 - (1 - 2/(1 + c*x))^2, 0]$

### Rule 6610

$\operatorname{Int}[u * \operatorname{PolyLog}[n, v], x_{\text{Symbol}}] \rightarrow \operatorname{With}[\{w = \operatorname{DerivativeDivides}[v, u*v, x]\}, \operatorname{Simp}[w * \operatorname{PolyLog}[n + 1, v], x] /; \operatorname{!FalseQ}[w]] /; \operatorname{FreeQ}[n, x]$

### Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)^3}{x} dx &= 2 \coth^{-1}(ax)^3 \coth^{-1}\left(1 - \frac{2}{1-ax}\right) - (6a) \int \frac{\coth^{-1}(ax)^2 \coth^{-1}\left(1 - \frac{2}{1-ax}\right)}{1-a^2x^2} dx \\
&= 2 \coth^{-1}(ax)^3 \coth^{-1}\left(1 - \frac{2}{1-ax}\right) + (3a) \int \frac{\coth^{-1}(ax)^2 \log\left(\frac{2}{1+ax}\right)}{1-a^2x^2} dx - (3a) \int \frac{\coth^{-1}(ax)^2 \log\left(\frac{2}{1-ax}\right)}{1-a^2x^2} dx \\
&= 2 \coth^{-1}(ax)^3 \coth^{-1}\left(1 - \frac{2}{1-ax}\right) + \frac{3}{2} \coth^{-1}(ax)^2 \text{Li}_2\left(1 - \frac{2}{1+ax}\right) - \frac{3}{2} \coth^{-1}(ax)^2 \text{Li}_2\left(1 - \frac{2ax}{1+ax}\right) \\
&= 2 \coth^{-1}(ax)^3 \coth^{-1}\left(1 - \frac{2}{1-ax}\right) + \frac{3}{2} \coth^{-1}(ax)^2 \text{Li}_2\left(1 - \frac{2}{1+ax}\right) - \frac{3}{2} \coth^{-1}(ax)^2 \text{Li}_2\left(1 - \frac{2ax}{1+ax}\right) \\
&= 2 \coth^{-1}(ax)^3 \coth^{-1}\left(1 - \frac{2}{1-ax}\right) + \frac{3}{2} \coth^{-1}(ax)^2 \text{Li}_2\left(1 - \frac{2}{1+ax}\right) - \frac{3}{2} \coth^{-1}(ax)^2 \text{Li}_2\left(1 - \frac{2ax}{1+ax}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.0758937, size = 156, normalized size = 1.04

$$\frac{1}{64} \left( -96 \coth^{-1}(ax)^2 \text{PolyLog}\left(2, -e^{-2 \coth^{-1}(ax)}\right) - 96 \coth^{-1}(ax)^2 \text{PolyLog}\left(2, e^{2 \coth^{-1}(ax)}\right) - 96 \coth^{-1}(ax) \text{PolyLog}\left(3, -e^{-2 \coth^{-1}(ax)}\right) - 96 \coth^{-1}(ax) \text{PolyLog}\left(3, e^{2 \coth^{-1}(ax)}\right) - 48 \text{PolyLog}\left(4, -e^{-2 \coth^{-1}(ax)}\right) - 48 \text{PolyLog}\left(4, e^{2 \coth^{-1}(ax)}\right) \right) / 64$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a\*x]^3/x,x]

[Out]  $(-\text{Pi}^4 + 32 \text{ArcCoth}[a*x]^4 + 64 \text{ArcCoth}[a*x]^3 \text{Log}[1 + \text{E}^{-2 \text{ArcCoth}[a*x]}] - 64 \text{ArcCoth}[a*x]^3 \text{Log}[1 - \text{E}^{2 \text{ArcCoth}[a*x]}] - 96 \text{ArcCoth}[a*x]^2 \text{PolyLog}[2, -\text{E}^{-2 \text{ArcCoth}[a*x]}] - 96 \text{ArcCoth}[a*x]^2 \text{PolyLog}[2, \text{E}^{2 \text{ArcCoth}[a*x]}] - 96 \text{ArcCoth}[a*x] \text{PolyLog}[3, -\text{E}^{-2 \text{ArcCoth}[a*x]}] + 96 \text{ArcCoth}[a*x] \text{PolyLog}[3, \text{E}^{2 \text{ArcCoth}[a*x]}] - 48 \text{PolyLog}[4, -\text{E}^{-2 \text{ArcCoth}[a*x]}] - 48 \text{PolyLog}[4, \text{E}^{2 \text{ArcCoth}[a*x]}]) / 64$

**Maple [C]** time = 0.253, size = 564, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)^3/x,x)



```
[Out] ln(a*x)*arccoth(a*x)^3+arccoth(a*x)^3*ln((a*x+1)/(a*x-1)-1)+3/2*arccoth(a*x)
)^2*polylog(2,-(a*x+1)/(a*x-1))-3/2*arccoth(a*x)*polylog(3,-(a*x+1)/(a*x-1)
)+3/4*polylog(4,-(a*x+1)/(a*x-1))+1/2*I*Pi*csgn(I/((a*x+1)/(a*x-1)-1)*((a*x
+1)/(a*x-1)+1))^3*arccoth(a*x)^3+1/2*I*Pi*csgn(I*((a*x+1)/(a*x-1)+1))*csgn(
I/((a*x+1)/(a*x-1)-1))*csgn(I/((a*x+1)/(a*x-1)-1)*((a*x+1)/(a*x-1)+1))*arcc
oth(a*x)^3-1/2*I*Pi*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I/((a*x+1)/(a*x-1)-1)*
((a*x+1)/(a*x-1)+1))^2*arccoth(a*x)^3-1/2*I*Pi*csgn(I*((a*x+1)/(a*x-1)+1))*
csgn(I/((a*x+1)/(a*x-1)-1)*((a*x+1)/(a*x-1)+1))^2*arccoth(a*x)^3-arccoth(a*
x)^3*ln(1-1/((a*x-1)/(a*x+1))^(1/2))-3*arccoth(a*x)^2*polylog(2,1/((a*x-1)/
(a*x+1))^(1/2))+6*arccoth(a*x)*polylog(3,1/((a*x-1)/(a*x+1))^(1/2))-6*polylog
(4,1/((a*x-1)/(a*x+1))^(1/2))-arccoth(a*x)^3*ln(1+1/((a*x-1)/(a*x+1))^(1/
2))-3*arccoth(a*x)^2*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))+6*arccoth(a*x)*p
olylog(3,-1/((a*x-1)/(a*x+1))^(1/2))-6*polylog(4,-1/((a*x-1)/(a*x+1))^(1/2)
)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(ax)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x,x, algorithm="maxima")
```

```
[Out] integrate(arccoth(a*x)^3/x, x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arccoth}(ax)^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)^3/x, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^3(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(a\*x)\*\*3/x,x)

[Out] Integral(acoth(a\*x)\*\*3/x, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(ax)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^3/x,x, algorithm="giac")

[Out] integrate(arccoth(a\*x)^3/x, x)

$$3.30 \quad \int \frac{\coth^{-1}(ax)^3}{x^2} dx$$

**Optimal.** Leaf size=79

$$-\frac{3}{2}a \operatorname{PolyLog}\left(3, \frac{2}{ax+1} - 1\right) - 3a \coth^{-1}(ax) \operatorname{PolyLog}\left(2, \frac{2}{ax+1} - 1\right) + a \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{x} + 3a \log\left(2 - \frac{2}{ax+1}\right)$$

[Out] a\*ArcCoth[a\*x]^3 - ArcCoth[a\*x]^3/x + 3\*a\*ArcCoth[a\*x]^2\*Log[2 - 2/(1 + a\*x)] - 3\*a\*ArcCoth[a\*x]\*PolyLog[2, -1 + 2/(1 + a\*x)] - (3\*a\*PolyLog[3, -1 + 2/(1 + a\*x)])/2

**Rubi [A]** time = 0.199122, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$ , Rules used = {5917, 5989, 5933, 5949, 6057, 6610}

$$-\frac{3}{2}a \operatorname{PolyLog}\left(3, \frac{2}{ax+1} - 1\right) - 3a \coth^{-1}(ax) \operatorname{PolyLog}\left(2, \frac{2}{ax+1} - 1\right) + a \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{x} + 3a \log\left(2 - \frac{2}{ax+1}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]^3/x^2,x]

[Out] a\*ArcCoth[a\*x]^3 - ArcCoth[a\*x]^3/x + 3\*a\*ArcCoth[a\*x]^2\*Log[2 - 2/(1 + a\*x)] - 3\*a\*ArcCoth[a\*x]\*PolyLog[2, -1 + 2/(1 + a\*x)] - (3\*a\*PolyLog[3, -1 + 2/(1 + a\*x)])/2

### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
  :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]
```

### Rule 5989

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)), x_Symbol]
  :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/d, Int[(a + b*ArcCoth[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5933

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.)/((x_.)*((d_.) + (e_.)*(x_.))), x
_Symbol] := Simp[((a + b*ArcCoth[c*x])^p*Log[2 - 2/(1 + (e*x)/d)]/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)]
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.)/((d_.) + (e_.)*(x_.)^2), x_Symb
ol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6057

```
Int[(Log[u_] * ((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.))/((d_.) + (e_.)*(x_.)^
2), x_Symbol] := Simp[((a + b*ArcCoth[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)^3}{x^2} dx &= -\frac{\coth^{-1}(ax)^3}{x} + (3a) \int \frac{\coth^{-1}(ax)^2}{x(1-a^2x^2)} dx \\
&= a \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{x} + (3a) \int \frac{\coth^{-1}(ax)^2}{x(1+ax)} dx \\
&= a \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{x} + 3a \coth^{-1}(ax)^2 \log\left(2 - \frac{2}{1+ax}\right) - (6a^2) \int \frac{\coth^{-1}(ax) \log\left(2 - \frac{2}{1+ax}\right)}{1-a^2x^2} dx \\
&= a \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{x} + 3a \coth^{-1}(ax)^2 \log\left(2 - \frac{2}{1+ax}\right) - 3a \coth^{-1}(ax) \text{Li}_2\left(-1 + \frac{2}{1+ax}\right) \\
&= a \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{x} + 3a \coth^{-1}(ax)^2 \log\left(2 - \frac{2}{1+ax}\right) - 3a \coth^{-1}(ax) \text{Li}_2\left(-1 + \frac{2}{1+ax}\right)
\end{aligned}$$



**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^3/x^2,x, algorithm="maxima")

[Out]  $\frac{1}{8}a(\log(ax+1) - \log(x))\log(a)^3 + \frac{3}{8}a\int(x\log(ax-1)/(ax^3+x^2), x)\log(a)^2 - \frac{3}{8}a\int(x\log(x)/(ax^3+x^2), x)\log(a)^2 - \frac{1}{8}(a\log(ax+1) - a\log(x) - 1/x)\log(a)^3 + \frac{3}{4}a^2\int(x^2\log(ax+1)\log(ax-1)/(ax^3+x^2), x) - \frac{3}{2}a^2\int(x^2\log(ax+1)\log(x)/(ax^3+x^2), x) + \frac{3}{4}a\int(x\log(ax-1)\log(x)/(ax^3+x^2), x)\log(a) - \frac{3}{8}a\int(x\log(x)^2/(ax^3+x^2), x)\log(a) + \frac{3}{8}\int(\log(ax-1)/(ax^3+x^2), x)\log(a)^2 - \frac{3}{8}\int(\log(x)/(ax^3+x^2), x)\log(a)^2 + \frac{3}{8}a\int(x\log(ax+1)\log(ax-1)^2/(ax^3+x^2), x) - \frac{3}{8}a\int(x\log(ax-1)^2\log(x)/(ax^3+x^2), x) + \frac{3}{8}a\int(x\log(ax-1)\log(x)^2/(ax^3+x^2), x) - \frac{1}{8}a\int(x\log(x)^3/(ax^3+x^2), x) - \frac{3}{4}a\int(x\log(ax+1)\log(ax-1)/(ax^3+x^2), x) - \frac{3}{8}\int(a^2x\log(ax-1)^2/(ax^3+x^2), x)\log(a) - \frac{3}{8}\int(\log(ax-1)^2/(ax^3+x^2), x)\log(a) + \frac{3}{4}\int(\log(ax-1)\log(x)/(ax^3+x^2), x)\log(a) - \frac{3}{8}\int(\log(x)^2/(ax^3+x^2), x)\log(a) - \frac{3}{8}(a^2\log(ax-1) - a^2\log(x) + a/x)\log(-1/(ax) + 1)^2/a + \frac{1}{8}\log(-1/(ax) + 1)^3/x - \frac{1}{8}((ax+1)\log(ax+1)^3 - 3(2ax\log(x) - (ax-1)\log(ax-1))\log(ax+1)^2)/x + \frac{1}{8}(3(a^3x\log(ax-1)^2 + a^3x\log(x)^2 - 2a^3x\log(x) + 2a^2 - 2(a^3x\log(x) - a^3x)\log(ax-1))\log(-1/(ax) + 1)/(ax) - (a^4x\log(ax-1)^3 - a^4x\log(x)^3 + 3a^4x\log(x)^2 - 6a^4x\log(x) + 6a^3 - 3(a^4x\log(x) - a^4x)\log(ax-1)^2 + 3(a^4x\log(x)^2 - 2a^4x\log(x) + 2a^4x)\log(ax-1))/(a^2x))/a + \frac{3}{8}\int(\log(ax+1)\log(ax-1)^2/(ax^3+x^2), x) - \frac{3}{8}\int(\log(ax-1)^2\log(x)/(ax^3+x^2), x) + \frac{3}{8}\int(\log(ax-1)\log(x)^2/(ax^3+x^2), x) - \frac{1}{8}\int(\log(x)^3/(ax^3+x^2), x)$

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( \frac{\operatorname{arccoth}(ax)^3}{x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^3/x^2,x, algorithm="fricas")

[Out] `integral(arccoth(a*x)^3/x^2, x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^3(ax)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(a*x)**3/x**2,x)`

[Out] `Integral(acoth(a*x)**3/x**2, x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(ax)^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(a*x)^3/x^2,x, algorithm="giac")`

[Out] `integrate(arccoth(a*x)^3/x^2, x)`

### 3.31 $\int \frac{\coth^{-1}(ax)^3}{x^3} dx$

**Optimal.** Leaf size=95

$$-\frac{3}{2}a^2 \text{PolyLog}\left(2, \frac{2}{ax+1} - 1\right) + \frac{1}{2}a^2 \coth^{-1}(ax)^3 + \frac{3}{2}a^2 \coth^{-1}(ax)^2 + 3a^2 \log\left(2 - \frac{2}{ax+1}\right) \coth^{-1}(ax) - \frac{\coth^{-1}(ax)^3}{2x^2}$$

[Out] (3\*a^2\*ArcCoth[a\*x]^2)/2 - (3\*a\*ArcCoth[a\*x]^2)/(2\*x) + (a^2\*ArcCoth[a\*x]^3)/2 - ArcCoth[a\*x]^3/(2\*x^2) + 3\*a^2\*ArcCoth[a\*x]\*Log[2 - 2/(1 + a\*x)] - (3\*a^2\*PolyLog[2, -1 + 2/(1 + a\*x)])/2

**Rubi [A]** time = 0.219236, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$ , Rules used = {5917, 5983, 5989, 5933, 2447, 5949}

$$-\frac{3}{2}a^2 \text{PolyLog}\left(2, \frac{2}{ax+1} - 1\right) + \frac{1}{2}a^2 \coth^{-1}(ax)^3 + \frac{3}{2}a^2 \coth^{-1}(ax)^2 + 3a^2 \log\left(2 - \frac{2}{ax+1}\right) \coth^{-1}(ax) - \frac{\coth^{-1}(ax)^3}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]^3/x^3,x]

[Out] (3\*a^2\*ArcCoth[a\*x]^2)/2 - (3\*a\*ArcCoth[a\*x]^2)/(2\*x) + (a^2\*ArcCoth[a\*x]^3)/2 - ArcCoth[a\*x]^3/(2\*x^2) + 3\*a^2\*ArcCoth[a\*x]\*Log[2 - 2/(1 + a\*x)] - (3\*a^2\*PolyLog[2, -1 + 2/(1 + a\*x)])/2

#### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.)*((d_.)*(x_.))^(m_.), x_Symbol]
  :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 5983

```
Int[(((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.)*((f_.)*(x_.))^(m_.))/((d_.) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcCoth[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```



Rule 5989

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
  x_Symbol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcCoth[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5933

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] := Simp[((a + b*ArcCoth[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)]]
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)^3}{x^3} dx &= -\frac{\coth^{-1}(ax)^3}{2x^2} + \frac{1}{2}(3a) \int \frac{\coth^{-1}(ax)^2}{x^2(1-a^2x^2)} dx \\
&= -\frac{\coth^{-1}(ax)^3}{2x^2} + \frac{1}{2}(3a) \int \frac{\coth^{-1}(ax)^2}{x^2} dx + \frac{1}{2}(3a^3) \int \frac{\coth^{-1}(ax)^2}{1-a^2x^2} dx \\
&= -\frac{3a \coth^{-1}(ax)^2}{2x} + \frac{1}{2}a^2 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{2x^2} + (3a^2) \int \frac{\coth^{-1}(ax)}{x(1-a^2x^2)} dx \\
&= \frac{3}{2}a^2 \coth^{-1}(ax)^2 - \frac{3a \coth^{-1}(ax)^2}{2x} + \frac{1}{2}a^2 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{2x^2} + (3a^2) \int \frac{\coth^{-1}(ax)}{x(1+ax)} dx \\
&= \frac{3}{2}a^2 \coth^{-1}(ax)^2 - \frac{3a \coth^{-1}(ax)^2}{2x} + \frac{1}{2}a^2 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{2x^2} + 3a^2 \coth^{-1}(ax) \log\left(2 - \frac{1}{1+ax}\right) \\
&= \frac{3}{2}a^2 \coth^{-1}(ax)^2 - \frac{3a \coth^{-1}(ax)^2}{2x} + \frac{1}{2}a^2 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{2x^2} + 3a^2 \coth^{-1}(ax) \log\left(2 - \frac{1}{1+ax}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.166635, size = 79, normalized size = 0.83

$$\frac{1}{2} \left( \frac{\coth^{-1}(ax) \left( (a^2x^2 - 1) \coth^{-1}(ax)^2 + 6a^2x^2 \log\left(e^{-2\coth^{-1}(ax)} + 1\right) + 3ax(ax - 1) \coth^{-1}(ax) \right)}{x^2} - 3a^2 \text{PolyLog}\left(2, -e^{-2\coth^{-1}(ax)}\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a\*x]^3/x^3,x]

[Out] ((ArcCoth[a\*x]\*(3\*a\*x\*(-1 + a\*x)\*ArcCoth[a\*x] + (-1 + a^2\*x^2)\*ArcCoth[a\*x]^2 + 6\*a^2\*x^2\*Log[1 + E^(-2\*ArcCoth[a\*x])]))/x^2 - 3\*a^2\*PolyLog[2, -E^(-2\*ArcCoth[a\*x])])/2

**Maple [C]** time = 0.422, size = 3673, normalized size = 38.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)^3/x^3,x)

```

[Out] -3/4*I*a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/(a*x-1))^2*arc
coth(a*x)*ln(1+I/((a*x-1)/(a*x+1))^(1/2))+3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-
1))^3*arccoth(a*x)^2-3/16*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-
1))^3*polylog(2,-(a*x+1)/(a*x-1))+3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x
+1)/(a*x-1)-1))^3*arccoth(a*x)^2+3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1))^3*dil
og(1-I/((a*x-1)/(a*x+1))^(1/2))-3/16*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1))^3*pol
ylog(2,-(a*x+1)/(a*x-1))+3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-
1)-1))^3*dilog(1+I/((a*x-1)/(a*x+1))^(1/2))-3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*
x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I*(a*x+1)/(a*x-1))*arccoth(a*x)*ln(1+I/((a
*x-1)/(a*x+1))^(1/2))-3/4*a^2*arccoth(a*x)^2*ln(a*x-1)+3/4*a^2*arccoth(a*x)
^2*ln(a*x+1)+3/4*a^2*arccoth(a*x)^2*ln((a*x-1)/(a*x+1))+3/2*a^2*arccoth(a*x)
*ln((a*x+1)/(a*x-1)+1)+3/2*a^2*arccoth(a*x)*ln(1+I/((a*x-1)/(a*x+1))^(1/2)
)+3/2*a^2*arccoth(a*x)*ln(1-I/((a*x-1)/(a*x+1))^(1/2))-1/2*arccoth(a*x)^3/x
^2+1/2*a^2*arccoth(a*x)^3+3/2*a^2*dilog(1-I/((a*x-1)/(a*x+1))^(1/2))+3/2*a^
2*dilog(1+I/((a*x-1)/(a*x+1))^(1/2))+3/4*a^2*polylog(2,-(a*x+1)/(a*x-1))+3/
8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-
1))*csgn(I/((a*x+1)/(a*x-1)-1))*arccoth(a*x)*ln(1-I/((a*x-1)/(a*x+1))^(1/2)
)-3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(
a*x-1))*csgn(I/((a*x+1)/(a*x-1)-1))*arccoth(a*x)*ln((a*x+1)/(a*x-1)+1)+3/8*
I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1)
)*csgn(I/((a*x+1)/(a*x-1)-1))*arccoth(a*x)*ln(1+I/((a*x-1)/(a*x+1))^(1/2))-
3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I/((a*x+1)/
(a*x-1)-1))*arccoth(a*x)*ln(1-I/((a*x-1)/(a*x+1))^(1/2))+3/8*I*a^2*Pi*csgn(
I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I*(a*x+1)/(a*x-1))*arccoth(a*
x)*ln((a*x+1)/(a*x-1)+1)+3/8*I*a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csg
n(I*(a*x+1)/(a*x-1))*arccoth(a*x)*ln(1-I/((a*x-1)/(a*x+1))^(1/2))-3/8*I*a^2
*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^3*arccoth(a*x)*ln((a*x+1)/(
a*x-1)+1)+3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^3*arccot
h(a*x)*ln(1+I/((a*x-1)/(a*x+1))^(1/2))+3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/
((a*x+1)/(a*x-1)-1))^3*arccoth(a*x)*ln(1-I/((a*x-1)/(a*x+1))^(1/2))+3/8*I*a
^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/(a*x-1))^2*polylog(2,-
(a*x+1)/(a*x-1))+3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1))^3*arccoth(a*x)*ln(1-I
/((a*x-1)/(a*x+1))^(1/2))+3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1))^3*arccoth(a*
x)*ln(1+I/((a*x-1)/(a*x+1))^(1/2))-3/16*I*a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(
1/2))^2*csgn(I*(a*x+1)/(a*x-1))*polylog(2,-(a*x+1)/(a*x-1))-3/8*I*a^2*Pi*cs
gn(I*(a*x+1)/(a*x-1))^3*arccoth(a*x)*ln((a*x+1)/(a*x-1)+1)-3/4*I*a^2*Pi*csg
n(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/(a*x-1))^2*dilog(1-I/((a*x-1)/(
a*x+1))^(1/2))-3/4*I*a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/
(a*x-1))^2*dilog(1+I/((a*x-1)/(a*x+1))^(1/2))+3/8*I*a^2*Pi*csgn(I/((a*x-1)/
(a*x+1))^(1/2))^2*csgn(I*(a*x+1)/(a*x-1))*dilog(1+I/((a*x-1)/(a*x+1))^(1/2)
)+3/8*I*a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I*(a*x+1)/(a*x-1))*di
log(1-I/((a*x-1)/(a*x+1))^(1/2))-3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+
1)/(a*x-1)-1))^2*csgn(I*(a*x+1)/(a*x-1))*dilog(1-I/((a*x-1)/(a*x+1))^(1/2))
-3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I/((a*x+1)
/(a*x-1)-1))*dilog(1+I/((a*x-1)/(a*x+1))^(1/2))-3/8*I*a^2*Pi*csgn(I*(a*x+1)

```

```

/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I/((a*x+1)/(a*x-1)-1))*dilog(1-I/((a*x-1)/(a*x+1))^(1/2))+3/16*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I*(a*x+1)/(a*x-1))*polylog(2,-(a*x+1)/(a*x-1))+3/16*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I/((a*x+1)/(a*x-1)-1))*polylog(2,-(a*x+1)/(a*x-1))-3/4*I*a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/(a*x-1))^2*arccoth(a*x)^2-3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I*(a*x+1)/(a*x-1))*arccoth(a*x)^2+3/8*I*a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I*(a*x+1)/(a*x-1))*arccoth(a*x)^2+3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^3*dilog(1-I/((a*x-1)/(a*x+1))^(1/2))+3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1))^3*dilog(1+I/((a*x-1)/(a*x+1))^(1/2))-3/2*a^2*arccoth(a*x)^2-3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I/((a*x+1)/(a*x-1)-1))*arccoth(a*x)^2-3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I*(a*x+1)/(a*x-1))*dilog(1+I/((a*x-1)/(a*x+1))^(1/2))-3/4*I*a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/(a*x-1))^2*arccoth(a*x)*ln(1-I/((a*x-1)/(a*x+1))^(1/2))-3/2*a*arccoth(a*x)^2/x+3/4*I*a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/(a*x-1))^2*arccoth(a*x)*ln((a*x+1)/(a*x-1)+1)+3/8*I*a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I*(a*x+1)/(a*x-1))*arccoth(a*x)*ln(1+I/((a*x-1)/(a*x+1))^(1/2))-3/16*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1))*csgn(I/((a*x+1)/(a*x-1)-1))*polylog(2,-(a*x+1)/(a*x-1))+3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1))*csgn(I/((a*x+1)/(a*x-1)-1))*dilog(1+I/((a*x-1)/(a*x+1))^(1/2))+3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1))*csgn(I/((a*x+1)/(a*x-1)-1))*dilog(1-I/((a*x-1)/(a*x+1))^(1/2))+3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1))*csgn(I/((a*x+1)/(a*x-1)-1))*arccoth(a*x)^2-3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I*(a*x+1)/(a*x-1))*arccoth(a*x)*ln(1-I/((a*x-1)/(a*x+1))^(1/2))-3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I/((a*x+1)/(a*x-1)-1))*arccoth(a*x)*ln(1+I/((a*x-1)/(a*x+1))^(1/2))-3/8*I*a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I*(a*x+1)/(a*x-1))*arccoth(a*x)*ln((a*x+1)/(a*x-1)+1)+3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*csgn(I/((a*x+1)/(a*x-1)-1))*arccoth(a*x)*ln((a*x+1)/(a*x-1)+1)

```

---

**Maxima [B]** time = 0.991751, size = 340, normalized size = 3.58

$$\frac{3}{4} \left( a \log(ax+1) - a \log(ax-1) - \frac{2}{x} \right) a \operatorname{arccoth}(ax)^2 - \frac{1}{16} \left( a^2 \frac{3(\log(ax-1) - 2) \log(ax+1)^2 - \log(ax+1)^3 + \log(ax+1)}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^3/x^3,x, algorithm="maxima")

```
[Out] 3/4*(a*log(a*x + 1) - a*log(a*x - 1) - 2/x)*a*arccoth(a*x)^2 - 1/16*(a^2*((
3*(log(a*x - 1) - 2)*log(a*x + 1)^2 - log(a*x + 1)^3 + log(a*x - 1)^3 - 3*(
log(a*x - 1)^2 - 4*log(a*x - 1))*log(a*x + 1) + 6*log(a*x - 1)^2)/a - 24*(1
og(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a + 24*(log(a*x + 1
)*log(x) + dilog(-a*x))/a - 24*(log(-a*x + 1)*log(x) + dilog(a*x))/a) - 6*(
2*(log(a*x - 1) - 2)*log(a*x + 1) - log(a*x + 1)^2 - log(a*x - 1)^2 - 4*log
(a*x - 1) + 8*log(x))*a*arccoth(a*x))*a - 1/2*arccoth(a*x)^3/x^2
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arccoth}(ax)^3}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x^3,x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)^3/x^3, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^3(ax)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)**3/x**3,x)
```

```
[Out] Integral(acoth(a*x)**3/x**3, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(ax)^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x^3,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)^3/x^3, x)
```

$$3.32 \quad \int \frac{\coth^{-1}(ax)^3}{x^4} dx$$

**Optimal.** Leaf size=154

$$-\frac{1}{2}a^3 \text{PolyLog}\left(3, \frac{2}{ax+1} - 1\right) - a^3 \coth^{-1}(ax) \text{PolyLog}\left(2, \frac{2}{ax+1} - 1\right) - \frac{1}{2}a^3 \log(1 - a^2x^2) + a^3 \log(x) + \frac{1}{3}a^3 \coth^{-1}(ax)$$

[Out]  $-\left(\frac{a^2 \text{ArcCoth}[a*x]}{x}\right) + \frac{a^3 \text{ArcCoth}[a*x]^2}{2} - \frac{a \text{ArcCoth}[a*x]^2}{2*x^2} + \frac{a^3 \text{ArcCoth}[a*x]^3}{3} - \frac{\text{ArcCoth}[a*x]^3}{3*x^3} + a^3 \text{Log}[x] - \frac{a^3 \text{Log}[1 - a^2*x^2]}{2} + a^3 \text{ArcCoth}[a*x]^2 \text{Log}\left[2 - \frac{2}{1 + a*x}\right] - a^3 \text{ArcCoth}[a*x] \text{PolyLog}\left[2, -1 + \frac{2}{1 + a*x}\right] - \frac{a^3 \text{PolyLog}\left[3, -1 + \frac{2}{1 + a*x}\right]}{2}$

**Rubi [A]** time = 0.368827, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.1$ , Rules used = {5917, 5983, 266, 36, 29, 31, 5949, 5989, 5933, 6057, 6610}

$$-\frac{1}{2}a^3 \text{PolyLog}\left(3, \frac{2}{ax+1} - 1\right) - a^3 \coth^{-1}(ax) \text{PolyLog}\left(2, \frac{2}{ax+1} - 1\right) - \frac{1}{2}a^3 \log(1 - a^2x^2) + a^3 \log(x) + \frac{1}{3}a^3 \coth^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]^3/x^4, x]

[Out]  $-\left(\frac{a^2 \text{ArcCoth}[a*x]}{x}\right) + \frac{a^3 \text{ArcCoth}[a*x]^2}{2} - \frac{a \text{ArcCoth}[a*x]^2}{2*x^2} + \frac{a^3 \text{ArcCoth}[a*x]^3}{3} - \frac{\text{ArcCoth}[a*x]^3}{3*x^3} + a^3 \text{Log}[x] - \frac{a^3 \text{Log}[1 - a^2*x^2]}{2} + a^3 \text{ArcCoth}[a*x]^2 \text{Log}\left[2 - \frac{2}{1 + a*x}\right] - a^3 \text{ArcCoth}[a*x] \text{PolyLog}\left[2, -1 + \frac{2}{1 + a*x}\right] - \frac{a^3 \text{PolyLog}\left[3, -1 + \frac{2}{1 + a*x}\right]}{2}$

### Rule 5917

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^ (p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcCoth[c\*x])^p)/(d\*(m + 1)), x] - Dist[(b\*c\*p)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 5983

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^ (p\_.)\*((f\_.)\*(x\_))^(m\_.)/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] :> Dist[1/d, Int[(f\*x)^m\*(a + b\*ArcCoth[c\*x])^p, x], x] - Dist[e/(d\*f^2), Int[((f\*x)^(m + 2)\*(a + b\*ArcCoth[c\*x])^p)/(d + e\*x^2), x], x]

2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]

### Rule 266

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[  
Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b,  
m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

### Rule 36

Int[1/(((a\_) + (b\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))), x\_Symbol] := Dist[b/(b\*c  
- a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x],  
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

### Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

### Rule 31

Int[((a\_) + (b\_)\*(x\_))^(p\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x,  
x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 5949

Int[((a\_) + ArcCoth[(c\_)\*(x\_)]\*(b\_))^(p\_)/((d\_) + (e\_)\*(x\_)^2), x\_Symb  
ol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b,  
c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

### Rule 5989

Int[((a\_) + ArcCoth[(c\_)\*(x\_)]\*(b\_))^(p\_)/((x\_)\*((d\_) + (e\_)\*(x\_)^2)),  
x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*d\*(p + 1)), x] + Dist[1/  
d, Int[(a + b\*ArcCoth[c\*x])^p/(x\*(1 + c\*x)), x], x] /; FreeQ[{a, b, c, d, e,  
}, x] && EqQ[c^2\*d + e, 0] && GtQ[p, 0]

### Rule 5933

Int[((a\_) + ArcCoth[(c\_)\*(x\_)]\*(b\_))^(p\_)/((x\_)\*((d\_) + (e\_)\*(x\_))), x  
\_Symbol] := Simp[((a + b\*ArcCoth[c\*x])^p\*Log[2 - 2/(1 + (e\*x)/d)]/d, x] -  
Dist[(b\*c\*p)/d, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*Log[2 - 2/(1 + (e\*x)/d)]  
/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^  
2\*d^2 - e^2, 0]



Rule 6057

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.))/((d_.) + (e_.)*(x_)^
2), x_Symbol] := Simp[((a + b*ArcCoth[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)^3}{x^4} dx &= -\frac{\coth^{-1}(ax)^3}{3x^3} + a \int \frac{\coth^{-1}(ax)^2}{x^3(1-a^2x^2)} dx \\
&= -\frac{\coth^{-1}(ax)^3}{3x^3} + a \int \frac{\coth^{-1}(ax)^2}{x^3} dx + a^3 \int \frac{\coth^{-1}(ax)^2}{x(1-a^2x^2)} dx \\
&= -\frac{a \coth^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^3 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{3x^3} + a^2 \int \frac{\coth^{-1}(ax)}{x^2(1-a^2x^2)} dx + a^3 \int \frac{\coth^{-1}(ax)^2}{x(1+ax)} dx \\
&= -\frac{a \coth^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^3 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{3x^3} + a^3 \coth^{-1}(ax)^2 \log\left(2 - \frac{2}{1+ax}\right) + a^2 \int \frac{\coth^{-1}(ax)}{x^2(1-a^2x^2)} dx \\
&= -\frac{a^2 \coth^{-1}(ax)}{x} + \frac{1}{2}a^3 \coth^{-1}(ax)^2 - \frac{a \coth^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^3 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{3x^3} + a^3 \coth^{-1}(ax) \log\left(2 - \frac{2}{1+ax}\right) \\
&= -\frac{a^2 \coth^{-1}(ax)}{x} + \frac{1}{2}a^3 \coth^{-1}(ax)^2 - \frac{a \coth^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^3 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{3x^3} + a^3 \coth^{-1}(ax) \log\left(2 - \frac{2}{1+ax}\right) \\
&= -\frac{a^2 \coth^{-1}(ax)}{x} + \frac{1}{2}a^3 \coth^{-1}(ax)^2 - \frac{a \coth^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^3 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{3x^3} + a^3 \log\left(2 - \frac{2}{1+ax}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.21288, size = 142, normalized size = 0.92

$$\frac{1}{6} \left( -6a^3 \coth^{-1}(ax) \text{PolyLog}\left(2, -e^{-2 \coth^{-1}(ax)}\right) - 3a^3 \text{PolyLog}\left(3, -e^{-2 \coth^{-1}(ax)}\right) + 6a^3 \log\left(\frac{1}{\sqrt{1 - \frac{1}{a^2x^2}}}\right) + 2a^3 \coth^{-1}(ax) \log\left(2 - \frac{2}{1+ax}\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a\*x]^3/x^4,x]

[Out]  $((-6a^2 \operatorname{ArcCoth}[a*x])/x + 3a^3 \operatorname{ArcCoth}[a*x]^2 - (3a \operatorname{ArcCoth}[a*x]^2)/x^2 + 2a^3 \operatorname{ArcCoth}[a*x]^3 - (2 \operatorname{ArcCoth}[a*x]^3)/x^3 + 6a^3 \operatorname{ArcCoth}[a*x]^2 \operatorname{Log}[1 + E^{(-2 \operatorname{ArcCoth}[a*x])}] + 6a^3 \operatorname{Log}[1/\sqrt{1 - 1/(a^2 x^2)}] - 6a^3 \operatorname{ArcCoth}[a*x] \operatorname{PolyLog}[2, -E^{(-2 \operatorname{ArcCoth}[a*x])}] - 3a^3 \operatorname{PolyLog}[3, -E^{(-2 \operatorname{ArcCoth}[a*x])}])/6$

**Maple [C]** time = 0.657, size = 895, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)^3/x^4,x)

[Out]  $-a^3 \operatorname{arccoth}(a*x) - 1/2 a^3 \operatorname{polylog}(3, -(a*x+1)/(a*x-1)) + a^3 \ln((a*x+1)/(a*x-1)+1) - 1/3 \operatorname{arccoth}(a*x)^3/x^3 - a^2 \operatorname{arccoth}(a*x)/x - 1/2 a^3 \operatorname{arccoth}(a*x)^2 \ln(a*x-1) + a^3 \operatorname{arccoth}(a*x)^2 \ln(a*x) + a^3 \operatorname{arccoth}(a*x) \operatorname{polylog}(2, -(a*x+1)/(a*x-1)) - 1/2 a^3 \operatorname{arccoth}(a*x)^2 \ln(a*x+1) - 1/2 a^3 \operatorname{arccoth}(a*x)^2 \ln((a*x-1)/(a*x+1)) + a^3 \operatorname{arccoth}(a*x)^2 \ln(2) + 1/2 a^3 \operatorname{arccoth}(a*x)^2 - 1/4 I a^3 \operatorname{arccoth}(a*x)^2 \operatorname{Pi} \operatorname{csgn}(I*(a*x+1)/(a*x-1))^3 + 1/2 I a^3 \operatorname{arccoth}(a*x)^2 \operatorname{Pi} \operatorname{csgn}(I/((a*x+1)/(a*x-1)-1))*((a*x+1)/(a*x-1)+1))^3 - 1/2 a^3 \operatorname{arccoth}(a*x)^2/x^2 - 1/3 a^3 \operatorname{arccoth}(a*x)^3 - 1/4 I a^3 \operatorname{arccoth}(a*x)^2 \operatorname{Pi} \operatorname{csgn}(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^3 + 1/2 I a^3 \operatorname{arccoth}(a*x)^2 \operatorname{Pi} \operatorname{csgn}(I/((a*x+1)/(a*x-1)-1))*\operatorname{csgn}(I/((a*x+1)/(a*x-1)-1))*((a*x+1)/(a*x-1)+1))*\operatorname{csgn}(I*((a*x+1)/(a*x-1)+1)) - 1/4 I a^3 \operatorname{arccoth}(a*x)^2 \operatorname{Pi} \operatorname{csgn}(I/((a*x+1)/(a*x-1)-1))*\operatorname{csgn}(I*((a*x+1)/(a*x-1)-1))*\operatorname{csgn}(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))*\operatorname{csgn}(I*(a*x+1)/(a*x-1))+1/4 I a^3 \operatorname{arccoth}(a*x)^2 \operatorname{Pi} \operatorname{csgn}(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2 \operatorname{csgn}(I*(a*x+1)/(a*x-1))+1/4 I a^3 \operatorname{arccoth}(a*x)^2 \operatorname{Pi} \operatorname{csgn}(I/((a*x+1)/(a*x-1)-1))*\operatorname{csgn}(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2 + 1/2 I a^3 \operatorname{arccoth}(a*x)^2 \operatorname{Pi} \operatorname{csgn}(I*(a*x+1)/(a*x-1))^2 \operatorname{csgn}(I/((a*x-1)/(a*x+1))^(1/2)) - 1/4 I a^3 \operatorname{arccoth}(a*x)^2 \operatorname{Pi} \operatorname{csgn}(I*(a*x+1)/(a*x-1))*\operatorname{csgn}(I/((a*x-1)/(a*x+1))^(1/2))^2 - 1/2 I a^3 \operatorname{arccoth}(a*x)^2 \operatorname{Pi} \operatorname{csgn}(I/((a*x+1)/(a*x-1)-1))*\operatorname{csgn}(I/((a*x+1)/(a*x-1)-1))*((a*x+1)/(a*x-1)+1))^2 - 1/2 I a^3 \operatorname{arccoth}(a*x)^2 \operatorname{Pi} \operatorname{csgn}(I/((a*x+1)/(a*x-1)-1))*((a*x+1)/(a*x-1)+1))^2 \operatorname{csgn}(I*(a*x+1)/(a*x-1)+1))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^3/x^4,x, algorithm="maxima")

[Out]  $\frac{1}{4}a^4 \int (x^4 \log(ax+1) \log(ax-1) / (a^5x^5 + x^4), x) - \frac{1}{2}a^4 \int (x^4 \log(ax+1) \log(x) / (a^5x^5 + x^4), x) + \frac{1}{16}(2a^2 \log(ax+1) - 2a^2 \log(x) - (2ax-1)/x^2) a \log(a)^3 + \frac{3}{8}a \int (x \log(ax-1) / (a^5x^5 + x^4), x) \log(a)^2 - \frac{3}{8}a \int (x \log(x) / (a^5x^5 + x^4), x) \log(a)^2 - \frac{1}{48}(6a^3 \log(ax+1) - 6a^3 \log(x) - (6a^2x^2 - 3ax + 2)/x^3) \log(a)^3 + \frac{1}{4}a^2 \int (x^2 \log(ax+1) / (a^5x^5 + x^4), x) + \frac{3}{4}a \int (x \log(ax-1) \log(x) / (a^5x^5 + x^4), x) \log(a) - \frac{3}{8}a \int (x \log(x)^2 / (a^5x^5 + x^4), x) \log(a) + \frac{3}{8} \int (\log(ax-1) / (a^5x^5 + x^4), x) \log(a)^2 - \frac{3}{8} \int (\log(x) / (a^5x^5 + x^4), x) \log(a)^2 + \frac{3}{8}a \int (x \log(ax+1) \log(ax-1)^2 / (a^5x^5 + x^4), x) - \frac{3}{8}a \int (x \log(ax-1)^2 \log(x) / (a^5x^5 + x^4), x) + \frac{3}{8}a \int (x \log(ax-1) \log(x)^2 / (a^5x^5 + x^4), x) - \frac{1}{8}a \int (x \log(x)^3 / (a^5x^5 + x^4), x) - \frac{1}{4}a \int (x \log(ax+1) \log(ax-1) / (a^5x^5 + x^4), x) - \frac{3}{8} \int (ax \log(ax-1)^2 / (a^5x^5 + x^4), x) \log(a) - \frac{3}{8} \int (\log(ax-1)^2 / (a^5x^5 + x^4), x) \log(a) + \frac{3}{4} \int (\log(ax-1) \log(x) / (a^5x^5 + x^4), x) \log(a) - \frac{3}{8} \int (\log(x)^2 / (a^5x^5 + x^4), x) \log(a) - \frac{1}{48}(6a^4 \log(ax-1) - 6a^4 \log(x) + (6a^3x^2 + 3a^2x + 2a)/x^3) \log(-1/(ax) + 1)^2/a + \frac{1}{864}(6(18a^5x^3 \log(ax-1)^2 + 18a^5x^3 \log(x)^2 - 66a^5x^3 \log(x) + 66a^4x^2 + 15a^3x + 4a^2 - 6(6a^5x^3 \log(x) - 11a^5x^3) \log(ax-1)) \log(-1/(ax) + 1) / (a^3x^3) - (36a^6x^3 \log(ax-1)^3 - 36a^6x^3 \log(x)^3 + 198a^6x^3 \log(x)^2 - 510a^6x^3 \log(x) + 510a^5x^2 + 57a^4x + 8a^3 - 18(6a^6x^3 \log(x) - 11a^6x^3) \log(ax-1)^2 + 6(18a^6x^3 \log(x)^2 - 66a^6x^3 \log(x) + 85a^6x^3) \log(ax-1)) / (a^2x^3)) / a + \frac{1}{24} \log(-1/(ax) + 1)^3 / x^3 - \frac{1}{24}((a^3x^3 + 1) \log(ax+1)^3 - 3(2a^3x^3 \log(x) - ax - (a^3x^3 - 1) \log(ax-1)) \log(ax+1)^2) / x^3 + \frac{3}{8} \int (\log(ax+1) \log(ax-1)^2 / (a^5x^5 + x^4), x) - \frac{3}{8} \int (\log(ax-1)^2 \log(x) / (a^5x^5 + x^4), x) + \frac{3}{8} \int (\log(ax-1) \log(x)^2 / (a^5x^5 + x^4), x) - \frac{1}{8} \int (\log(x)^3 / (a^5x^5 + x^4), x)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arccoth}(ax)^3}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^3/x^4,x, algorithm="fricas")

[Out] integral(arccoth(a\*x)^3/x^4, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^3(ax)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(a\*x)\*\*3/x\*\*4,x)

[Out] Integral(acoth(a\*x)\*\*3/x\*\*4, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(ax)^3}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^3/x^4,x, algorithm="giac")

[Out] integrate(arccoth(a\*x)^3/x^4, x)

### 3.33 $\int \frac{\coth^{-1}(ax)^3}{x^5} dx$

**Optimal.** Leaf size=141

$$-a^4 \text{PolyLog}\left(2, \frac{2}{ax+1} - 1\right) - \frac{a^2 \coth^{-1}(ax)}{4x^2} - \frac{a^3}{4x} + \frac{1}{4}a^4 \tanh^{-1}(ax) + \frac{1}{4}a^4 \coth^{-1}(ax)^3 + a^4 \coth^{-1}(ax)^2 - \frac{3a^3 \coth^{-1}(ax)}{4x}$$

[Out]  $-a^3/(4*x) - (a^2*ArcCoth[a*x])/(4*x^2) + a^4*ArcCoth[a*x]^2 - (a*ArcCoth[a*x]^2)/(4*x^3) - (3*a^3*ArcCoth[a*x]^2)/(4*x) + (a^4*ArcCoth[a*x]^3)/4 - ArcCoth[a*x]^3/(4*x^4) + (a^4*ArcTanh[a*x])/4 + 2*a^4*ArcCoth[a*x]*Log[2 - 2/(1 + a*x)] - a^4*PolyLog[2, -1 + 2/(1 + a*x)]$

**Rubi [A]** time = 0.463885, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.8$ , Rules used = {5917, 5983, 325, 206, 5989, 5933, 2447, 5949}

$$-a^4 \text{PolyLog}\left(2, \frac{2}{ax+1} - 1\right) - \frac{a^2 \coth^{-1}(ax)}{4x^2} - \frac{a^3}{4x} + \frac{1}{4}a^4 \tanh^{-1}(ax) + \frac{1}{4}a^4 \coth^{-1}(ax)^3 + a^4 \coth^{-1}(ax)^2 - \frac{3a^3 \coth^{-1}(ax)}{4x}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]^3/x^5, x]

[Out]  $-a^3/(4*x) - (a^2*ArcCoth[a*x])/(4*x^2) + a^4*ArcCoth[a*x]^2 - (a*ArcCoth[a*x]^2)/(4*x^3) - (3*a^3*ArcCoth[a*x]^2)/(4*x) + (a^4*ArcCoth[a*x]^3)/4 - ArcCoth[a*x]^3/(4*x^4) + (a^4*ArcTanh[a*x])/4 + 2*a^4*ArcCoth[a*x]*Log[2 - 2/(1 + a*x)] - a^4*PolyLog[2, -1 + 2/(1 + a*x)]$

#### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
  := Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 5983

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)/((d_.) + (
e_.)*(x_)^2), x_Symbol] := Dist[1/d, Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcCoth[c*x])^p)/(d + e*x^
```

2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]

### Rule 325

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a+b\*x^n)^(p+1))/(a\*c\*(m+1)), x] - Dist[(b\*(m+n\*(p+1)+1))/(a\*c^n\*(m+1)), Int[(c\*x)^(m+n)\*(a+b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 5989

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)/((x\_)\*((d\_) + (e\_.)\*(x\_)^2)), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p+1)/(b\*d\*(p+1)), x] + Dist[1/d, Int[(a + b\*ArcCoth[c\*x])^p/(x\*(1+c\*x)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && GtQ[p, 0]

### Rule 5933

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)/((x\_)\*((d\_) + (e\_.)\*(x\_))), x\_Symbol] := Simp[((a + b\*ArcCoth[c\*x])^p\*Log[2 - 2/(1 + (e\*x)/d)])/d, x] - Dist[(b\*c\*p)/d, Int[((a + b\*ArcCoth[c\*x])^(p-1)\*Log[2 - 2/(1 + (e\*x)/d)]]/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

### Rule 2447

Int[Log[u]\*(Pq\_)^(m\_.), x\_Symbol] := With[{C = FullSimplify[(Pq^m\*(1-u))/D[u, x]]}, Simp[C\*PolyLog[2, 1-u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]

### Rule 5949

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p+1)/(b\*c\*d\*(p+1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)^3}{x^5} dx &= -\frac{\coth^{-1}(ax)^3}{4x^4} + \frac{1}{4}(3a) \int \frac{\coth^{-1}(ax)^2}{x^4(1-a^2x^2)} dx \\
&= -\frac{\coth^{-1}(ax)^3}{4x^4} + \frac{1}{4}(3a) \int \frac{\coth^{-1}(ax)^2}{x^4} dx + \frac{1}{4}(3a^3) \int \frac{\coth^{-1}(ax)^2}{x^2(1-a^2x^2)} dx \\
&= -\frac{a \coth^{-1}(ax)^2}{4x^3} - \frac{\coth^{-1}(ax)^3}{4x^4} + \frac{1}{2}a^2 \int \frac{\coth^{-1}(ax)}{x^3(1-a^2x^2)} dx + \frac{1}{4}(3a^3) \int \frac{\coth^{-1}(ax)^2}{x^2} dx + \frac{1}{4}(3a^5) \int \frac{\coth^{-1}(ax)^2}{x^2(1-a^2x^2)} dx \\
&= -\frac{a \coth^{-1}(ax)^2}{4x^3} - \frac{3a^3 \coth^{-1}(ax)^2}{4x} + \frac{1}{4}a^4 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{4x^4} + \frac{1}{2}a^2 \int \frac{\coth^{-1}(ax)}{x^3} dx + \frac{1}{4}(3a^5) \int \frac{\coth^{-1}(ax)^2}{x^2(1-a^2x^2)} dx \\
&= -\frac{a^2 \coth^{-1}(ax)}{4x^2} + a^4 \coth^{-1}(ax)^2 - \frac{a \coth^{-1}(ax)^2}{4x^3} - \frac{3a^3 \coth^{-1}(ax)^2}{4x} + \frac{1}{4}a^4 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{4x^4} \\
&= -\frac{a^3}{4x} - \frac{a^2 \coth^{-1}(ax)}{4x^2} + a^4 \coth^{-1}(ax)^2 - \frac{a \coth^{-1}(ax)^2}{4x^3} - \frac{3a^3 \coth^{-1}(ax)^2}{4x} + \frac{1}{4}a^4 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{4x^4} \\
&= -\frac{a^3}{4x} - \frac{a^2 \coth^{-1}(ax)}{4x^2} + a^4 \coth^{-1}(ax)^2 - \frac{a \coth^{-1}(ax)^2}{4x^3} - \frac{3a^3 \coth^{-1}(ax)^2}{4x} + \frac{1}{4}a^4 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{4x^4}
\end{aligned}$$

**Mathematica [A]** time = 0.230209, size = 118, normalized size = 0.84

$$\frac{-4a^4x^4 \text{PolyLog}\left(2, -e^{-2\coth^{-1}(ax)}\right) - a^3x^3 + ax(4a^3x^3 - 3a^2x^2 - 1)\coth^{-1}(ax)^2 + (a^4x^4 - 1)\coth^{-1}(ax)^3 + a^2x^2 \coth^{-1}(ax)}{4x^4}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a\*x]^3/x^5, x]

[Out]  $(-(a^3x^3) + a*x*(-1 - 3a^2*x^2 + 4a^3*x^3)*\text{ArcCoth}[a*x]^2 + (-1 + a^4*x^4)*\text{ArcCoth}[a*x]^3 + a^2*x^2*\text{ArcCoth}[a*x]*(-1 + a^2*x^2 + 8a^2*x^2*\text{Log}[1 + E^{(-2*\text{ArcCoth}[a*x])}]) - 4a^4*x^4*\text{PolyLog}[2, -E^{(-2*\text{ArcCoth}[a*x])}])/(4*x^4)$

**Maple [C]** time = 0.536, size = 661, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(a*x)^3/x^5,x)`

[Out]  $\frac{1}{4}a^4 - \frac{3}{8}a^4 \operatorname{arccoth}(ax)^2 \ln(ax-1) + \frac{3}{8}a^4 \operatorname{arccoth}(ax)^2 \ln(ax+1) + 2a^4 \operatorname{arccoth}(ax) \ln(1 + I/\frac{(ax-1)}{(ax+1)})^{1/2} + 3/8 a^4 \operatorname{arccoth}(ax)^2 \ln(\frac{(ax-1)}{(ax+1)}) + 2a^4 \operatorname{arccoth}(ax) \ln(1 - I/\frac{(ax-1)}{(ax+1)})^{1/2} + 1/4 a^4 \operatorname{arccoth}(ax) + 1/4 a^4 \operatorname{arccoth}(ax)^3 + 3/16 I a^4 \operatorname{Pcsgn}(I/\frac{(ax-1)}{(ax+1)})^{1/2})^2 \operatorname{csgn}(I*(ax+1)/(ax-1)) \operatorname{arccoth}(ax)^2 + 3/16 I a^4 \operatorname{Pcsgn}(I*(ax+1)/(ax-1))^3 \operatorname{arccoth}(ax)^2 - 1/4 a^2 \operatorname{arccoth}(ax)/x^2 - 3/8 I a^4 \operatorname{Pcsgn}(I/\frac{(ax-1)}{(ax+1)})^{1/2}) \operatorname{csgn}(I*(ax+1)/(ax-1))^2 \operatorname{arccoth}(ax)^2 + 3/16 I a^4 \operatorname{Pcsgn}(I*(ax+1)/(ax-1)/((ax+1)/(ax-1)-1)) \operatorname{csgn}(I*(ax+1)/(ax-1)) \operatorname{csgn}(I/\frac{(ax+1)}{(ax-1)-1}) \operatorname{arccoth}(ax)^2 + 3/16 I a^4 \operatorname{Pcsgn}(I*(ax+1)/(ax-1)/((ax+1)/(ax-1)-1))^3 \operatorname{arccoth}(ax)^2 - 3/16 I a^4 \operatorname{Pcsgn}(I*(ax+1)/(ax-1)/((ax+1)/(ax-1)-1))^2 \operatorname{csgn}(I/\frac{(ax+1)}{(ax-1)-1}) \operatorname{arccoth}(ax)^2 - 1/4 \operatorname{arccoth}(ax)^3/x^4 - 1/4 a \operatorname{arccoth}(ax)^2/x^3 - 3/4 a^3 \operatorname{arccoth}(ax)^2/x - 1/4 a^3/x - a^4 \operatorname{arccoth}(ax)^2 - 3/16 I a^4 \operatorname{Pcsgn}(I*(ax+1)/(ax-1)/((ax+1)/(ax-1)-1))^2 \operatorname{csgn}(I*(ax+1)/(ax-1)) \operatorname{arccoth}(ax)^2 + 2a^4 \operatorname{dilog}(1 + I/\frac{(ax-1)}{(ax+1)})^{1/2}) + 2a^4 \operatorname{dilog}(1 - I/\frac{(ax-1)}{(ax+1)})^{1/2})$

**Maxima [B]** time = 1.01002, size = 462, normalized size = 3.28

$$\frac{1}{8} \left( 3a^3 \log(ax+1) - 3a^3 \log(ax-1) - \frac{2(3a^2x^2+1)}{x^3} \right) a \operatorname{arccoth}(ax)^2 + \frac{1}{32} \left( \left( 32 \left( \log(ax-1) \log\left(\frac{1}{2}ax + \frac{1}{2}\right) + \operatorname{Li}_2\left(-\frac{1}{2}\right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(a*x)^3/x^5,x, algorithm="maxima")`

[Out]  $\frac{1}{8}(3a^3 \log(ax+1) - 3a^3 \log(ax-1) - \frac{2(3a^2x^2+1)}{x^3}) a \operatorname{arccoth}(ax)^2 + \frac{1}{32}((32(\log(ax-1) \log(\frac{1}{2}ax + \frac{1}{2}) + \operatorname{dilog}(-\frac{1}{2}ax + \frac{1}{2})) a - 32(\log(ax+1) \log(x) + \operatorname{dilog}(-ax)) a + 32(\log(-ax+1) \log(x) + \operatorname{dilog}(ax)) a + 4a \log(ax+1) - 4a \log(ax-1) + (ax \log(ax+1))^3 - ax \log(ax-1)^3 - 8ax \log(ax-1)^2 - (3ax \log(ax-1) - 8ax) \log(ax+1)^2 + (3ax \log(ax-1)^2 - 16ax \log(ax-1)) \log(ax+1) - 8)/x) a^2 + 2(32a^2 \log(x) - (3a^2x^2 \log(ax+1)^2 + 3a^2x^2 \log(ax-1)^2 + 16a^2x^2 \log(ax-1) - 2(3a^2x^2 \log(ax-1) - 8a^2x^2) \log(ax+1) + 4)/x^2) a \operatorname{arccoth}(ax) a - \frac{1}{4} \operatorname{arccoth}(ax)^3/x^4$



**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arccoth}(ax)^3}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^3/x^5,x, algorithm="fricas")

[Out] integral(arccoth(a\*x)^3/x^5, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^3(ax)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(a\*x)\*\*3/x\*\*5,x)

[Out] Integral(acoth(a\*x)\*\*3/x\*\*5, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(ax)^3}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^3/x^5,x, algorithm="giac")

[Out] integrate(arccoth(a\*x)^3/x^5, x)

### 3.34 $\int \frac{\coth^{-1}(cx)^2}{d+ex} dx$

**Optimal.** Leaf size=164

$$\frac{\text{PolyLog}\left(3, 1 - \frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{2e} - \frac{\coth^{-1}(cx)\text{PolyLog}\left(2, 1 - \frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e} + \frac{\text{PolyLog}\left(3, 1 - \frac{2}{cx+1}\right)}{2e} + \frac{\coth^{-1}(cx)\text{PolyLog}\left(2, 1 - \frac{2}{cx+1}\right)}{e}$$

[Out]  $-\left(\frac{\text{ArcCoth}[c*x]^2 \cdot \text{Log}\left[\frac{2}{1+c*x}\right]}{e}\right) + \frac{\text{ArcCoth}[c*x]^2 \cdot \text{Log}\left[\frac{2*c*(d+e*x)}{(c*d+e)*(1+c*x)}\right]}{e} + \frac{\text{ArcCoth}[c*x] \cdot \text{PolyLog}\left[2, 1 - \frac{2}{1+c*x}\right]}{e}$   
 $- \frac{\text{ArcCoth}[c*x] \cdot \text{PolyLog}\left[2, 1 - \frac{2*c*(d+e*x)}{(c*d+e)*(1+c*x)}\right]}{e} + \frac{\text{PolyLog}\left[3, 1 - \frac{2}{1+c*x}\right]}{2*e} - \frac{\text{PolyLog}\left[3, 1 - \frac{2*c*(d+e*x)}{(c*d+e)*(1+c*x)}\right]}{2*e}$

**Rubi [A]** time = 0.0337196, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {5923}

$$\frac{\text{PolyLog}\left(3, 1 - \frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{2e} - \frac{\coth^{-1}(cx)\text{PolyLog}\left(2, 1 - \frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e} + \frac{\text{PolyLog}\left(3, 1 - \frac{2}{cx+1}\right)}{2e} + \frac{\coth^{-1}(cx)\text{PolyLog}\left(2, 1 - \frac{2}{cx+1}\right)}{e}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[c\*x]^2/(d + e\*x), x]

[Out]  $-\left(\frac{\text{ArcCoth}[c*x]^2 \cdot \text{Log}\left[\frac{2}{1+c*x}\right]}{e}\right) + \frac{\text{ArcCoth}[c*x]^2 \cdot \text{Log}\left[\frac{2*c*(d+e*x)}{(c*d+e)*(1+c*x)}\right]}{e} + \frac{\text{ArcCoth}[c*x] \cdot \text{PolyLog}\left[2, 1 - \frac{2}{1+c*x}\right]}{e}$   
 $- \frac{\text{ArcCoth}[c*x] \cdot \text{PolyLog}\left[2, 1 - \frac{2*c*(d+e*x)}{(c*d+e)*(1+c*x)}\right]}{e} + \frac{\text{PolyLog}\left[3, 1 - \frac{2}{1+c*x}\right]}{2*e} - \frac{\text{PolyLog}\left[3, 1 - \frac{2*c*(d+e*x)}{(c*d+e)*(1+c*x)}\right]}{2*e}$

#### Rule 5923

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^2/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :>  
 $-\text{Simp}\left[\frac{(a + b \cdot \text{ArcCoth}[c*x])^2 \cdot \text{Log}\left[\frac{2}{1+c*x}\right]}{e}, x\right] + \left(\text{Simp}\left[\frac{(a + b \cdot \text{ArcCoth}[c*x])^2 \cdot \text{Log}\left[\frac{2*c*(d+e*x)}{(c*d+e)*(1+c*x)}\right]}{e}, x\right] + \text{Simp}\left[\frac{b*(a + b \cdot \text{ArcCoth}[c*x]) \cdot \text{PolyLog}\left[2, 1 - \frac{2}{1+c*x}\right]}{e}, x\right] - \text{Simp}\left[\frac{b*(a + b \cdot \text{ArcCoth}[c*x]) \cdot \text{PolyLog}\left[2, 1 - \frac{2*c*(d+e*x)}{(c*d+e)*(1+c*x)}\right]}{e}, x\right] + \text{Simp}\left[\frac{b^2 \cdot \text{PolyLog}\left[3, 1 - \frac{2}{1+c*x}\right]}{2*e}, x\right] - \text{Simp}\left[\frac{b^2 \cdot \text{PolyLog}\left[3, 1 - \frac{2*c*(d+e*x)}{(c*d+e)*(1+c*x)}\right]}{2*e}, x\right]\right) /;$   
 FreeQ[{a, b, c, d, e}, x] && NeQ[c^2\*d^2 - e^2, 0]

Rubi steps

$$\int \frac{\coth^{-1}(cx)^2}{d+ex} dx = -\frac{\coth^{-1}(cx)^2 \log\left(\frac{2}{1+cx}\right)}{e} + \frac{\coth^{-1}(cx)^2 \log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e} + \frac{\coth^{-1}(cx) \operatorname{Li}_2\left(1 - \frac{2}{1+cx}\right)}{e} - \frac{\coth^{-1}(cx)}{e}$$

**Mathematica [C]** time = 7.68431, size = 741, normalized size = 4.52

$$24(e-cd)(cd+e) \left( 6e \coth^{-1}(cx) \operatorname{PolyLog}\left(2, \frac{(cd+e)e^2 \coth^{-1}(cx)}{cd-e}\right) - 3e \operatorname{PolyLog}\left(3, \frac{(cd+e)e^2 \coth^{-1}(cx)}{cd-e}\right) - 12e \coth^{-1}(cx) \operatorname{PolyLog}\left(2, -e^{\tanh^{-1}\left(\frac{e}{cd}\right) + \coth^{-1}(cx)}\right) - 12e \coth^{-1}(cx) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[c\*x]^2/(d + e\*x), x]

[Out] ((-I)\*e\*Pi^3 + 8\*c\*d\*ArcCoth[c\*x]^3 + 8\*e\*ArcCoth[c\*x]^3 - 24\*e\*ArcCoth[c\*x]^2\*Log[1 - E^(2\*ArcCoth[c\*x])] - 24\*e\*ArcCoth[c\*x]\*PolyLog[2, E^(2\*ArcCoth[c\*x])] + 12\*e\*PolyLog[3, E^(2\*ArcCoth[c\*x])] + (24\*(-(c\*d) + e)\*(c\*d + e)\*(-2\*c\*d\*ArcCoth[c\*x]^3 + 6\*e\*ArcCoth[c\*x]^3 + (4\*c\*d\*Sqrt[1 - e^2/(c^2\*d^2)]\*ArcCoth[c\*x]^3)/E^ArcTanh[e/(c\*d)] + (6\*I)\*e\*Pi\*ArcCoth[c\*x]\*Log[(E^(-ArcCoth[c\*x]) + E^ArcCoth[c\*x])/2] + 6\*e\*ArcCoth[c\*x]^2\*Log[1 + ((c\*d + e)\*E^(2\*ArcCoth[c\*x]))/(-(c\*d) + e)] - 6\*e\*ArcCoth[c\*x]^2\*Log[1 - E^(ArcCoth[c\*x] + ArcTanh[e/(c\*d)])] - 6\*e\*ArcCoth[c\*x]^2\*Log[1 + E^(ArcCoth[c\*x] + ArcTanh[e/(c\*d)])] - 6\*e\*ArcCoth[c\*x]^2\*Log[1 - E^(2\*(ArcCoth[c\*x] + ArcTanh[e/(c\*d)])]) - 12\*e\*ArcCoth[c\*x]\*ArcTanh[e/(c\*d)]\*Log[(I/2)\*E^(-ArcCoth[c\*x] - ArcTanh[e/(c\*d)])\*(-1 + E^(2\*(ArcCoth[c\*x] + ArcTanh[e/(c\*d)]))]) - 6\*e\*ArcCoth[c\*x]^2\*Log[(c\*d\*(-1 + E^(2\*ArcCoth[c\*x])) + e\*(1 + E^(2\*ArcCoth[c\*x]))) / (2\*E^ArcCoth[c\*x])] - (6\*I)\*e\*Pi\*ArcCoth[c\*x]\*Log[1/Sqrt[1 - 1/(c^2\*x^2)]] + 6\*e\*ArcCoth[c\*x]^2\*Log[(d + e\*x)/(Sqrt[1 - 1/(c^2\*x^2)]\*x)] + 12\*e\*ArcCoth[c\*x]\*ArcTanh[e/(c\*d)]\*Log[I\*Sinh[ArcCoth[c\*x] + ArcTanh[e/(c\*d)]]] + 6\*e\*ArcCoth[c\*x]\*PolyLog[2, ((c\*d + e)\*E^(2\*ArcCoth[c\*x]))/(c\*d - e)] - 12\*e\*ArcCoth[c\*x]\*PolyLog[2, -E^(ArcCoth[c\*x] + ArcTanh[e/(c\*d)])] - 12\*e\*ArcCoth[c\*x]\*PolyLog[2, E^(ArcCoth[c\*x] + ArcTanh[e/(c\*d)])] - 6\*e\*ArcCoth[c\*x]\*PolyLog[2, E^(2\*(ArcCoth[c\*x] + ArcTanh[e/(c\*d)])]) - 3\*e\*PolyLog[3, ((c\*d + e)\*E^(2\*ArcCoth[c\*x]))/(c\*d - e)] + 12\*e\*PolyLog[3, -E^(ArcCoth[c\*x] + ArcTanh[e/(c\*d)])] + 12\*e\*PolyLog[3, E^(ArcCoth[c\*x] + ArcTanh[e/(c\*d)])] + 3\*e\*PolyLog[3, E^(2\*(ArcCoth[c\*x] + ArcTanh[e/(c\*d)])])]) / (6\*c^2\*d^2 - 6\*e^2) / (24\*e^2)

**Maple [C]** time = 0.625, size = 926, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(c\*x)^2/(e\*x+d), x)

[Out]  $\ln(c*ex+c*d)/e*\operatorname{arccoth}(c*x)^2-1/e*\operatorname{arccoth}(c*x)^2*\ln(c*d*((c*x+1)/(c*x-1)-1)+((c*x+1)/(c*x-1)+1)*e)+1/e*\operatorname{arccoth}(c*x)^2*\ln((c*x+1)/(c*x-1)-1)-1/e*\operatorname{arccoth}(c*x)^2*\ln(1-1/((c*x-1)/(c*x+1))^{(1/2)})-2/e*\operatorname{arccoth}(c*x)*\operatorname{polylog}(2,1/((c*x-1)/(c*x+1))^{(1/2)})+2/e*\operatorname{polylog}(3,1/((c*x-1)/(c*x+1))^{(1/2)})-1/e*\operatorname{arccoth}(c*x)^2*\ln(1+1/((c*x-1)/(c*x+1))^{(1/2)})-2/e*\operatorname{arccoth}(c*x)*\operatorname{polylog}(2,-1/((c*x-1)/(c*x+1))^{(1/2)})+2/e*\operatorname{polylog}(3,-1/((c*x-1)/(c*x+1))^{(1/2)})-1/2*I/e*\operatorname{Pi}*\operatorname{arccoth}(c*x)^2*\operatorname{csgn}(I*(c*d*((c*x+1)/(c*x-1)-1)+((c*x+1)/(c*x-1)+1)*e)/((c*x+1)/(c*x-1)-1))^2*\operatorname{csgn}(I/((c*x+1)/(c*x-1)-1))+1/2*I/e*\operatorname{Pi}*\operatorname{arccoth}(c*x)^2*\operatorname{csgn}(I*(c*d*((c*x+1)/(c*x-1)-1)+((c*x+1)/(c*x-1)+1)*e)/((c*x+1)/(c*x-1)-1))^3+1/2*I/e*\operatorname{Pi}*\operatorname{arccoth}(c*x)^2*\operatorname{csgn}(I*(c*d*((c*x+1)/(c*x-1)-1)+((c*x+1)/(c*x-1)+1)*e))*\operatorname{csgn}(I*(c*d*((c*x+1)/(c*x-1)-1)+((c*x+1)/(c*x-1)+1)*e)/((c*x+1)/(c*x-1)-1))*\operatorname{csgn}(I/((c*x+1)/(c*x-1)-1))-1/2*I/e*\operatorname{Pi}*\operatorname{arccoth}(c*x)^2*\operatorname{csgn}(I*(c*d*((c*x+1)/(c*x-1)-1)+((c*x+1)/(c*x-1)+1)*e))*\operatorname{csgn}(I*(c*d*((c*x+1)/(c*x-1)-1)+((c*x+1)/(c*x-1)+1)*e)/((c*x+1)/(c*x-1)-1))^2+1/(c*d+e)*\operatorname{arccoth}(c*x)^2*\ln(1-(c*d+e)/(c*d-e)*(c*x+1)/(c*x-1))+1/(c*d+e)*\operatorname{arccoth}(c*x)*\operatorname{polylog}(2,(c*d+e)/(c*d-e)*(c*x+1)/(c*x-1))-1/2/(c*d+e)*\operatorname{polylog}(3,(c*d+e)/(c*d-e)*(c*x+1)/(c*x-1))+c/e*d/(c*d+e)*\operatorname{arccoth}(c*x)^2*\ln(1-(c*d+e)/(c*d-e)*(c*x+1)/(c*x-1))+c/e*d/(c*d+e)*\operatorname{arccoth}(c*x)*\operatorname{polylog}(2,(c*d+e)/(c*d-e)*(c*x+1)/(c*x-1))-1/2*c/e*d/(c*d+e)*\operatorname{polylog}(3,(c*d+e)/(c*d-e)*(c*x+1)/(c*x-1))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(cx)^2}{ex+d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c\*x)^2/(e\*x+d), x, algorithm="maxima")

[Out] integrate(arccoth(c\*x)^2/(e\*x + d), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arccoth}(cx)^2}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c\*x)^2/(e\*x+d),x, algorithm="fricas")

[Out] integral(arccoth(c\*x)^2/(e\*x + d), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^2(cx)}{d+ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(c\*x)\*\*2/(e\*x+d),x)

[Out] Integral(acoth(c\*x)\*\*2/(d + e\*x), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(cx)^2}{ex+d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c\*x)^2/(e\*x+d),x, algorithm="giac")

[Out] integrate(arccoth(c\*x)^2/(e\*x + d), x)

### 3.35 $\int (c + dx^2)^4 \coth^{-1}(ax) dx$

**Optimal.** Leaf size=245

$$\frac{d^2x^4(378a^4c^2 + 180a^2cd + 35d^2)}{1260a^5} + \frac{dx^2(378a^4c^2d + 420a^6c^3 + 180a^2cd^2 + 35d^3)}{630a^7} + \frac{(378a^4c^2d^2 + 420a^6c^3d + 315a^8c^4 + 630a^4cd^2 + 180a^2cd^3 + 35d^4)}{630a^9}$$

[Out] (d\*(420\*a^6\*c^3 + 378\*a^4\*c^2\*d + 180\*a^2\*c\*d^2 + 35\*d^3)\*x^2)/(630\*a^7) + (d^2\*(378\*a^4\*c^2 + 180\*a^2\*c\*d + 35\*d^2)\*x^4)/(1260\*a^5) + (d^3\*(36\*a^2\*c + 7\*d)\*x^6)/(378\*a^3) + (d^4\*x^8)/(72\*a) + c^4\*x\*ArcCoth[a\*x] + (4\*c^3\*d\*x^3\*ArcCoth[a\*x])/3 + (6\*c^2\*d^2\*x^5\*ArcCoth[a\*x])/5 + (4\*c\*d^3\*x^7\*ArcCoth[a\*x])/7 + (d^4\*x^9\*ArcCoth[a\*x])/9 + ((315\*a^8\*c^4 + 420\*a^6\*c^3\*d + 378\*a^4\*c^2\*d^2 + 180\*a^2\*c\*d^3 + 35\*d^4)\*Log[1 - a^2\*x^2])/(630\*a^9)

**Rubi [A]** time = 0.180539, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {194, 5977, 1810, 260}

$$\frac{d^2x^4(378a^4c^2 + 180a^2cd + 35d^2)}{1260a^5} + \frac{dx^2(378a^4c^2d + 420a^6c^3 + 180a^2cd^2 + 35d^3)}{630a^7} + \frac{(378a^4c^2d^2 + 420a^6c^3d + 315a^8c^4 + 630a^4cd^2 + 180a^2cd^3 + 35d^4)}{630a^9}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^2)^4\*ArcCoth[a\*x], x]

[Out] (d\*(420\*a^6\*c^3 + 378\*a^4\*c^2\*d + 180\*a^2\*c\*d^2 + 35\*d^3)\*x^2)/(630\*a^7) + (d^2\*(378\*a^4\*c^2 + 180\*a^2\*c\*d + 35\*d^2)\*x^4)/(1260\*a^5) + (d^3\*(36\*a^2\*c + 7\*d)\*x^6)/(378\*a^3) + (d^4\*x^8)/(72\*a) + c^4\*x\*ArcCoth[a\*x] + (4\*c^3\*d\*x^3\*ArcCoth[a\*x])/3 + (6\*c^2\*d^2\*x^5\*ArcCoth[a\*x])/5 + (4\*c\*d^3\*x^7\*ArcCoth[a\*x])/7 + (d^4\*x^9\*ArcCoth[a\*x])/9 + ((315\*a^8\*c^4 + 420\*a^6\*c^3\*d + 378\*a^4\*c^2\*d^2 + 180\*a^2\*c\*d^3 + 35\*d^4)\*Log[1 - a^2\*x^2])/(630\*a^9)

#### Rule 194

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

#### Rule 5977

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))\*((d\_.) + (e\_.)\*(x\_)^2)^(q\_.), x\_Symbol] := With[{u = IntHide[(d + e\*x^2)^q, x]}, Dist[a + b\*ArcCoth[c\*x], u, x]

] - Dist[b\*c, Int[u/(1 - c^2\*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&  
 (IntegerQ[q] || ILtQ[q + 1/2, 0])

### Rule 1810

Int[(Pq\_)\*((a\_) + (b\_)\*(x\_)^2)^(p\_), x\_Symbol] := Int[ExpandIntegrand[Pq\*  
 (a + b\*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

### Rule 260

Int[(x\_)^(m\_)/((a\_) + (b\_)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveConten  
 t[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rubi steps

$$\begin{aligned} \int (c + dx^2)^4 \coth^{-1}(ax) dx &= c^4 x \coth^{-1}(ax) + \frac{4}{3} c^3 dx^3 \coth^{-1}(ax) + \frac{6}{5} c^2 d^2 x^5 \coth^{-1}(ax) + \frac{4}{7} cd^3 x^7 \coth^{-1}(ax) + \frac{1}{9} d^4 x^9 \coth^{-1}(ax) \\ &= c^4 x \coth^{-1}(ax) + \frac{4}{3} c^3 dx^3 \coth^{-1}(ax) + \frac{6}{5} c^2 d^2 x^5 \coth^{-1}(ax) + \frac{4}{7} cd^3 x^7 \coth^{-1}(ax) + \frac{1}{9} d^4 x^9 \coth^{-1}(ax) \\ &= \frac{d(420a^6c^3 + 378a^4c^2d + 180a^2cd^2 + 35d^3)x^2}{630a^7} + \frac{d^2(378a^4c^2 + 180a^2cd + 35d^2)x^4}{1260a^5} + \frac{d^3(378a^4c^2 + 180a^2cd + 35d^2)x^4}{1260a^5} + \frac{d^3(378a^4c^2 + 180a^2cd + 35d^2)x^4}{1260a^5} \\ &= \frac{d(420a^6c^3 + 378a^4c^2d + 180a^2cd^2 + 35d^3)x^2}{630a^7} + \frac{d^2(378a^4c^2 + 180a^2cd + 35d^2)x^4}{1260a^5} + \frac{d^3(378a^4c^2 + 180a^2cd + 35d^2)x^4}{1260a^5} + \frac{d^3(378a^4c^2 + 180a^2cd + 35d^2)x^4}{1260a^5} \end{aligned}$$

**Mathematica [A]** time = 0.116038, size = 213, normalized size = 0.87

$$\frac{a^2 dx^2 (3a^6 (756c^2 dx^2 + 1680c^3 + 240cd^2 x^4 + 35d^3 x^6)) + 4a^4 d (1134c^2 + 270cdx^2 + 35d^2 x^4) + 30a^2 d^2 (72c + 7dx^2) + 42d^3 x^8}{(7560a^9)}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^2)^4\*ArcCoth[a\*x], x]

[Out] (a^2\*d\*x^2\*(420\*d^3 + 30\*a^2\*d^2\*(72\*c + 7\*d\*x^2) + 4\*a^4\*d\*(1134\*c^2 + 270\*c\*d\*x^2 + 35\*d^2\*x^4) + 3\*a^6\*(1680\*c^3 + 756\*c^2\*d\*x^2 + 240\*c\*d^2\*x^4 + 35\*d^3\*x^6)) + 24\*a^9\*x\*(315\*c^4 + 420\*c^3\*d\*x^2 + 378\*c^2\*d^2\*x^4 + 180\*c\*d^3\*x^6 + 35\*d^4\*x^8)\*ArcCoth[a\*x] + 12\*(315\*a^8\*c^4 + 420\*a^6\*c^3\*d + 378\*a^4\*c^2\*d^2 + 180\*a^2\*c\*d^3 + 35\*d^4)\*Log[1 - a^2\*x^2])/(7560\*a^9)





Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2+c)^4\*arccoth(a\*x),x, algorithm="fricas")

[Out]  $\frac{1}{7560} \cdot (105 \cdot a^8 \cdot d^4 \cdot x^8 + 20 \cdot (36 \cdot a^8 \cdot c \cdot d^3 + 7 \cdot a^6 \cdot d^4) \cdot x^6 + 6 \cdot (378 \cdot a^8 \cdot c^2 \cdot d^2 + 180 \cdot a^6 \cdot c \cdot d^3 + 35 \cdot a^4 \cdot d^4) \cdot x^4 + 12 \cdot (420 \cdot a^8 \cdot c^3 \cdot d + 378 \cdot a^6 \cdot c^2 \cdot d^2 + 180 \cdot a^4 \cdot c \cdot d^3 + 35 \cdot a^2 \cdot d^4) \cdot x^2 + 12 \cdot (315 \cdot a^8 \cdot c^4 + 420 \cdot a^6 \cdot c^3 \cdot d + 378 \cdot a^4 \cdot c^2 \cdot d^2 + 180 \cdot a^2 \cdot c \cdot d^3 + 35 \cdot d^4) \cdot \log(a^2 \cdot x^2 - 1) + 12 \cdot (35 \cdot a^9 \cdot d^4 \cdot x^9 + 180 \cdot a^9 \cdot c \cdot d^3 \cdot x^7 + 378 \cdot a^9 \cdot c^2 \cdot d^2 \cdot x^5 + 420 \cdot a^9 \cdot c^3 \cdot d \cdot x^3 + 315 \cdot a^9 \cdot c^4 \cdot x) \cdot \log\left(\frac{a \cdot x + 1}{a \cdot x - 1}\right) / a^9$

**Sympy [A]** time = 25.0004, size = 427, normalized size = 1.74

$$\left\{ \begin{array}{l} c^4 x \operatorname{acoth}(ax) + \frac{4c^3 dx^3 \operatorname{acoth}(ax)}{3} + \frac{6c^2 d^2 x^5 \operatorname{acoth}(ax)}{5} + \frac{4cd^3 x^7 \operatorname{acoth}(ax)}{7} + \frac{d^4 x^9 \operatorname{acoth}(ax)}{9} + \frac{c^4 \log\left(x - \frac{1}{a}\right)}{a} + \frac{c^4 \operatorname{acoth}(ax)}{a} + \frac{2c^3 dx^2}{3a} + \\ \frac{i\pi \left( c^4 x + \frac{4c^3 dx^3}{3} + \frac{6c^2 d^2 x^5}{5} + \frac{4cd^3 x^7}{7} + \frac{d^4 x^9}{9} \right)}{2} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*2+c)\*\*4\*acoth(a\*x),x)

[Out] Piecewise((c\*\*4\*x\*acoth(a\*x) + 4\*c\*\*3\*d\*x\*\*3\*acoth(a\*x)/3 + 6\*c\*\*2\*d\*\*2\*x\*\*5\*acoth(a\*x)/5 + 4\*c\*d\*\*3\*x\*\*7\*acoth(a\*x)/7 + d\*\*4\*x\*\*9\*acoth(a\*x)/9 + c\*\*4\*log(x - 1/a)/a + c\*\*4\*acoth(a\*x)/a + 2\*c\*\*3\*d\*x\*\*2/(3\*a) + 3\*c\*\*2\*d\*\*2\*x\*\*4/(10\*a) + 2\*c\*d\*\*3\*x\*\*6/(21\*a) + d\*\*4\*x\*\*8/(72\*a) + 4\*c\*\*3\*d\*log(x - 1/a)/(3\*a\*\*3) + 4\*c\*\*3\*d\*acoth(a\*x)/(3\*a\*\*3) + 3\*c\*\*2\*d\*\*2\*x\*\*2/(5\*a\*\*3) + c\*d\*\*3\*x\*\*4/(7\*a\*\*3) + d\*\*4\*x\*\*6/(54\*a\*\*3) + 6\*c\*\*2\*d\*\*2\*log(x - 1/a)/(5\*a\*\*5) + 6\*c\*\*2\*d\*\*2\*acoth(a\*x)/(5\*a\*\*5) + 2\*c\*d\*\*3\*x\*\*2/(7\*a\*\*5) + d\*\*4\*x\*\*4/(36\*a\*\*5) + 4\*c\*d\*\*3\*log(x - 1/a)/(7\*a\*\*7) + 4\*c\*d\*\*3\*acoth(a\*x)/(7\*a\*\*7) + d\*\*4\*x\*\*2/(18\*a\*\*7) + d\*\*4\*log(x - 1/a)/(9\*a\*\*9) + d\*\*4\*acoth(a\*x)/(9\*a\*\*9), Ne(a, 0)), (I\*pi\*(c\*\*4\*x + 4\*c\*\*3\*d\*x\*\*3/3 + 6\*c\*\*2\*d\*\*2\*x\*\*5/5 + 4\*c\*d\*\*3\*x\*\*7/7 + d\*\*4\*x\*\*9/9)/2, True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx^2 + c)^4 \operatorname{arccoth}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arccoth(a*x),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^4*arccoth(a*x), x)
```

### 3.36 $\int (c + dx^2)^3 \coth^{-1}(ax) dx$

**Optimal.** Leaf size=169

$$\frac{dx^2 (35a^4c^2 + 21a^2cd + 5d^2)}{70a^5} + \frac{(35a^4c^2d + 35a^6c^3 + 21a^2cd^2 + 5d^3) \log(1 - a^2x^2)}{70a^7} + \frac{d^2x^4 (21a^2c + 5d)}{140a^3} + c^2dx^3 \coth^{-1}(ax)$$

```
[Out] (d*(35*a^4*c^2 + 21*a^2*c*d + 5*d^2)*x^2)/(70*a^5) + (d^2*(21*a^2*c + 5*d)*
x^4)/(140*a^3) + (d^3*x^6)/(42*a) + c^3*x*ArcCoth[a*x] + c^2*d*x^3*ArcCoth[
a*x] + (3*c*d^2*x^5*ArcCoth[a*x])/5 + (d^3*x^7*ArcCoth[a*x])/7 + ((35*a^6*c
^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*Log[1 - a^2*x^2])/(70*a^7)
```

---

**Rubi [A]** time = 0.1268, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {194, 5977, 1810, 260}

$$\frac{dx^2 (35a^4c^2 + 21a^2cd + 5d^2)}{70a^5} + \frac{(35a^4c^2d + 35a^6c^3 + 21a^2cd^2 + 5d^3) \log(1 - a^2x^2)}{70a^7} + \frac{d^2x^4 (21a^2c + 5d)}{140a^3} + c^2dx^3 \coth^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3*ArcCoth[a*x], x]
```

```
[Out] (d*(35*a^4*c^2 + 21*a^2*c*d + 5*d^2)*x^2)/(70*a^5) + (d^2*(21*a^2*c + 5*d)*
x^4)/(140*a^3) + (d^3*x^6)/(42*a) + c^3*x*ArcCoth[a*x] + c^2*d*x^3*ArcCoth[
a*x] + (3*c*d^2*x^5*ArcCoth[a*x])/5 + (d^3*x^7*ArcCoth[a*x])/7 + ((35*a^6*c
^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*Log[1 - a^2*x^2])/(70*a^7)
```

#### Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

#### Rule 5977

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.)*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] := With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

#### Rule 1810

Int[(Pq\_)\*((a\_) + (b\_)\*(x\_)^2)^(p\_), x\_Symbol] := Int[ExpandIntegrand[Pq\*(a + b\*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

### Rule 260

Int[(x\_)^(m\_)/((a\_) + (b\_)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rubi steps

$$\begin{aligned} \int (c + dx^2)^3 \coth^{-1}(ax) dx &= c^3 x \coth^{-1}(ax) + c^2 dx^3 \coth^{-1}(ax) + \frac{3}{5} cd^2 x^5 \coth^{-1}(ax) + \frac{1}{7} d^3 x^7 \coth^{-1}(ax) - a \int \frac{c^3 x + c^2 dx^3 + \frac{3}{5} cd^2 x^5 + \frac{1}{7} d^3 x^7}{c^3 x^2 + c^2 dx^4 + \frac{3}{5} cd^2 x^6 + \frac{1}{7} d^3 x^8} dx \\ &= c^3 x \coth^{-1}(ax) + c^2 dx^3 \coth^{-1}(ax) + \frac{3}{5} cd^2 x^5 \coth^{-1}(ax) + \frac{1}{7} d^3 x^7 \coth^{-1}(ax) - a \int \left( -\frac{d(35a^4c^2 + 21a^2cd + 5d^2)x^2}{70a^5} + \frac{d^2(21a^2c + 5d)x^4}{140a^3} + \frac{d^3x^6}{42a} \right) dx \\ &= \frac{d(35a^4c^2 + 21a^2cd + 5d^2)x^2}{70a^5} + \frac{d^2(21a^2c + 5d)x^4}{140a^3} + \frac{d^3x^6}{42a} + c^3 x \coth^{-1}(ax) + c^2 dx^3 \coth^{-1}(ax) \\ &= \frac{d(35a^4c^2 + 21a^2cd + 5d^2)x^2}{70a^5} + \frac{d^2(21a^2c + 5d)x^4}{140a^3} + \frac{d^3x^6}{42a} + c^3 x \coth^{-1}(ax) + c^2 dx^3 \coth^{-1}(ax) \end{aligned}$$

**Mathematica [A]** time = 0.0786468, size = 150, normalized size = 0.89

$$\frac{a^2 dx^2 (a^4 (210c^2 + 63cdx^2 + 10d^2x^4) + 3a^2 d (42c + 5dx^2) + 30d^2) + 6 (35a^4c^2d + 35a^6c^3 + 21a^2cd^2 + 5d^3) \log(1 - a^2x^2)}{420a^7}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^2)^3\*ArcCoth[a\*x], x]

[Out] (a^2\*d\*x^2\*(30\*d^2 + 3\*a^2\*d\*(42\*c + 5\*d\*x^2) + a^4\*(210\*c^2 + 63\*c\*d\*x^2 + 10\*d^2\*x^4)) + 12\*a^7\*x\*(35\*c^3 + 35\*c^2\*d\*x^2 + 21\*c\*d^2\*x^4 + 5\*d^3\*x^6) \*ArcCoth[a\*x] + 6\*(35\*a^6\*c^3 + 35\*a^4\*c^2\*d + 21\*a^2\*c\*d^2 + 5\*d^3)\*Log[1 - a^2\*x^2])/(420\*a^7)

**Maple [A]** time = 0.037, size = 233, normalized size = 1.4

$$\frac{d^3 x^7 \operatorname{arccoth}(ax)}{7} + \frac{3 cd^2 x^5 \operatorname{arccoth}(ax)}{5} + c^2 dx^3 \operatorname{arccoth}(ax) + c^3 x \operatorname{arccoth}(ax) + \frac{3 cd^2 x^2}{10 a^3} + \frac{3 cd^2 x^4}{20 a} + \frac{c^2 x^2 d}{2 a} + \frac{d^3 x^6}{42 a} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^2+c)^3*arccoth(a*x),x)`

[Out]  $\frac{1}{7}d^3x^7\operatorname{arccoth}(ax)+\frac{3}{5}c*d^2*x^5\operatorname{arccoth}(ax)+c^2*d*x^3\operatorname{arccoth}(ax)+c^3*x*\operatorname{arccoth}(ax)+\frac{3}{10}/a^3*c*d^2*x^2+\frac{3}{20}/a*c*d^2*x^4+\frac{1}{2}/a*x^2*c^2*d+\frac{1}{42}*d^3*x^6/a+\frac{1}{14}/a^5*d^3*x^2+\frac{1}{2}/a*\ln(a*x-1)*c^3+\frac{1}{2}/a^3*\ln(a*x-1)*c^2*d+\frac{3}{10}/a^5*\ln(a*x-1)*c*d^2+\frac{1}{14}/a^7*\ln(a*x-1)*d^3+\frac{1}{2}/a*\ln(a*x+1)*c^3+\frac{1}{2}/a^3*\ln(a*x+1)*c^2*d+\frac{3}{10}/a^5*\ln(a*x+1)*c*d^2+\frac{1}{14}/a^7*\ln(a*x+1)*d^3+\frac{1}{28}/a^3*x^4*d^3$

**Maxima [A]** time = 0.974128, size = 267, normalized size = 1.58

$$\frac{1}{420} a \left( \frac{10 a^4 d^3 x^6 + 3 (21 a^4 c d^2 + 5 a^2 d^3) x^4 + 6 (35 a^4 c^2 d + 21 a^2 c d^2 + 5 d^3) x^2}{a^6} + \frac{6 (35 a^6 c^3 + 35 a^4 c^2 d + 21 a^2 c d^2 + 5 d^3)}{a^8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^2+c)^3*arccoth(a*x),x, algorithm="maxima")`

[Out]  $\frac{1}{420} a * ((10 * a^4 * d^3 * x^6 + 3 * (21 * a^4 * c * d^2 + 5 * a^2 * d^3) * x^4 + 6 * (35 * a^4 * c^2 * d + 21 * a^2 * c * d^2 + 5 * d^3) * x^2) / a^6 + 6 * (35 * a^6 * c^3 + 35 * a^4 * c^2 * d + 21 * a^2 * c * d^2 + 5 * d^3) * \log(a * x + 1) / a^8 + 6 * (35 * a^6 * c^3 + 35 * a^4 * c^2 * d + 21 * a^2 * c * d^2 + 5 * d^3) * \log(a * x - 1) / a^8 + 1 / 35 * (5 * d^3 * x^7 + 21 * c * d^2 * x^5 + 35 * c^2 * d * x^3 + 35 * c^3 * x) * \operatorname{arccoth}(a * x)$

**Fricas [A]** time = 1.62707, size = 382, normalized size = 2.26

$$\frac{10 a^6 d^3 x^6 + 3 (21 a^6 c d^2 + 5 a^4 d^3) x^4 + 6 (35 a^6 c^2 d + 21 a^4 c d^2 + 5 a^2 d^3) x^2 + 6 (35 a^6 c^3 + 35 a^4 c^2 d + 21 a^2 c d^2 + 5 d^3) \log(a^2 x^2 - 1)}{420 a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^2+c)^3*arccoth(a*x),x, algorithm="fricas")`

[Out]  $\frac{1}{420} * (10 * a^6 * d^3 * x^6 + 3 * (21 * a^6 * c * d^2 + 5 * a^4 * d^3) * x^4 + 6 * (35 * a^6 * c^2 * d + 21 * a^4 * c * d^2 + 5 * a^2 * d^3) * x^2 + 6 * (35 * a^6 * c^3 + 35 * a^4 * c^2 * d + 21 * a^2 * c * d^2 + 5 * d^3) * \log(a^2 * x^2 - 1) + 6 * (5 * a^7 * d^3 * x^7 + 21 * a^7 * c * d^2 * x^5 + 35 * a^7$

$$*c^2*d*x^3 + 35*a^7*c^3*x)*\log((a*x + 1)/(a*x - 1))/a^7$$

**Sympy [A]** time = 7.31205, size = 282, normalized size = 1.67

$$\left\{ \begin{array}{l} c^3 x \operatorname{acoth}(ax) + c^2 dx^3 \operatorname{acoth}(ax) + \frac{3cd^2x^5 \operatorname{acoth}(ax)}{5} + \frac{d^3x^7 \operatorname{acoth}(ax)}{7} + \frac{c^3 \log\left(x - \frac{1}{a}\right)}{a} + \frac{c^3 \operatorname{acoth}(ax)}{a} + \frac{c^2 dx^2}{2a} + \frac{3cd^2x^4}{20a} + \frac{d^3x^6}{42a} + \frac{c^2}{2} \\ \frac{i\pi\left(c^3x + c^2dx^3 + \frac{3cd^2x^5}{5} + \frac{d^3x^7}{7}\right)}{2} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*2+c)\*\*3\*acoth(a\*x),x)

[Out] Piecewise((c\*\*3\*x\*acoth(a\*x) + c\*\*2\*d\*x\*\*3\*acoth(a\*x) + 3\*c\*d\*\*2\*x\*\*5\*acoth(a\*x)/5 + d\*\*3\*x\*\*7\*acoth(a\*x)/7 + c\*\*3\*log(x - 1/a)/a + c\*\*3\*acoth(a\*x)/a + c\*\*2\*d\*x\*\*2/(2\*a) + 3\*c\*d\*\*2\*x\*\*4/(20\*a) + d\*\*3\*x\*\*6/(42\*a) + c\*\*2\*d\*log(x - 1/a)/a\*\*3 + c\*\*2\*d\*acoth(a\*x)/a\*\*3 + 3\*c\*d\*\*2\*x\*\*2/(10\*a\*\*3) + d\*\*3\*x\*\*4/(28\*a\*\*3) + 3\*c\*d\*\*2\*log(x - 1/a)/(5\*a\*\*5) + 3\*c\*d\*\*2\*acoth(a\*x)/(5\*a\*\*5) + d\*\*3\*x\*\*2/(14\*a\*\*5) + d\*\*3\*log(x - 1/a)/(7\*a\*\*7) + d\*\*3\*acoth(a\*x)/(7\*a\*\*7), Ne(a, 0)), (I\*pi\*(c\*\*3\*x + c\*\*2\*d\*x\*\*3 + 3\*c\*d\*\*2\*x\*\*5/5 + d\*\*3\*x\*\*7/7)/2, True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx^2 + c)^3 \operatorname{arccoth}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2+c)^3\*arccoth(a\*x),x, algorithm="giac")

[Out] integrate((d\*x^2 + c)^3\*arccoth(a\*x), x)

### 3.37 $\int (c + dx^2)^2 \coth^{-1}(ax) dx$

**Optimal.** Leaf size=110

$$\frac{(15a^4c^2 + 10a^2cd + 3d^2) \log(1 - a^2x^2)}{30a^5} + \frac{dx^2(10a^2c + 3d)}{30a^3} + c^2x \coth^{-1}(ax) + \frac{2}{3}cdx^3 \coth^{-1}(ax) + \frac{d^2x^4}{20a} + \frac{1}{5}d^2x^5 \coth^{-1}(ax)$$

[Out] (d\*(10\*a^2\*c + 3\*d)\*x^2)/(30\*a^3) + (d^2\*x^4)/(20\*a) + c^2\*x\*ArcCoth[a\*x] + (2\*c\*d\*x^3\*ArcCoth[a\*x])/3 + (d^2\*x^5\*ArcCoth[a\*x])/5 + ((15\*a^4\*c^2 + 10\*a^2\*c\*d + 3\*d^2)\*Log[1 - a^2\*x^2])/(30\*a^5)

**Rubi [A]** time = 0.133164, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {194, 5977, 1594, 1247, 698}

$$\frac{(15a^4c^2 + 10a^2cd + 3d^2) \log(1 - a^2x^2)}{30a^5} + \frac{dx^2(10a^2c + 3d)}{30a^3} + c^2x \coth^{-1}(ax) + \frac{2}{3}cdx^3 \coth^{-1}(ax) + \frac{d^2x^4}{20a} + \frac{1}{5}d^2x^5 \coth^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^2)^2\*ArcCoth[a\*x], x]

[Out] (d\*(10\*a^2\*c + 3\*d)\*x^2)/(30\*a^3) + (d^2\*x^4)/(20\*a) + c^2\*x\*ArcCoth[a\*x] + (2\*c\*d\*x^3\*ArcCoth[a\*x])/3 + (d^2\*x^5\*ArcCoth[a\*x])/5 + ((15\*a^4\*c^2 + 10\*a^2\*c\*d + 3\*d^2)\*Log[1 - a^2\*x^2])/(30\*a^5)

#### Rule 194

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

#### Rule 5977

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(q\_.), x\_Symbol] :> With[{u = IntHide[(d + e\*x^2)^q, x]}, Dist[a + b\*ArcCoth[c\*x], u, x] - Dist[b\*c, Int[u/(1 - c^2\*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])

#### Rule 1594

Int[(u\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.) + (c\_.)\*(x\_)^(r\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(n\*p)\*(a + b\*x^(q - p) + c\*x^(r - p))^n, x] /; FreeQ[{a

, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

### Rule 1247

Int[(x\_)\*((d\_) + (e\_)\*(x\_)^2)^(q\_)\*((a\_) + (b\_)\*(x\_)^2 + (c\_)\*(x\_)^4)^(p\_), x\_Symbol] := Dist[1/2, Subst[Int[(d + e\*x)^q\*(a + b\*x + c\*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

### Rule 698

Int[((d\_) + (e\_)\*(x\_)^m)\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && NeQ[2\*c\*d - b\*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))

### Rubi steps

$$\begin{aligned}
 \int (c + dx^2)^2 \coth^{-1}(ax) dx &= c^2x \coth^{-1}(ax) + \frac{2}{3}cdx^3 \coth^{-1}(ax) + \frac{1}{5}d^2x^5 \coth^{-1}(ax) - a \int \frac{c^2x + \frac{2}{3}cdx^3 + \frac{d^2x^5}{5}}{1 - a^2x^2} dx \\
 &= c^2x \coth^{-1}(ax) + \frac{2}{3}cdx^3 \coth^{-1}(ax) + \frac{1}{5}d^2x^5 \coth^{-1}(ax) - a \int \frac{x \left( c^2 + \frac{2}{3}cdx^2 + \frac{d^2x^4}{5} \right)}{1 - a^2x^2} dx \\
 &= c^2x \coth^{-1}(ax) + \frac{2}{3}cdx^3 \coth^{-1}(ax) + \frac{1}{5}d^2x^5 \coth^{-1}(ax) - \frac{1}{2}a \operatorname{Subst} \left( \int \frac{c^2 + \frac{2cdx}{3} + \frac{d^2x^2}{5}}{1 - a^2x} dx, ax \right) \\
 &= c^2x \coth^{-1}(ax) + \frac{2}{3}cdx^3 \coth^{-1}(ax) + \frac{1}{5}d^2x^5 \coth^{-1}(ax) - \frac{1}{2}a \operatorname{Subst} \left( \int \left( -\frac{d(10a^2c + 3d)}{15a^4} + \frac{2cd}{3(1 - a^2x)} + \frac{d^2x}{5(1 - a^2x)} \right) dx, ax \right) \\
 &= \frac{d(10a^2c + 3d)x^2}{30a^3} + \frac{d^2x^4}{20a} + c^2x \coth^{-1}(ax) + \frac{2}{3}cdx^3 \coth^{-1}(ax) + \frac{1}{5}d^2x^5 \coth^{-1}(ax) + \frac{(15c^2 + 10cdx^2 + 3d^2x^4) \log(1 - a^2x^2)}{60a^5}
 \end{aligned}$$

**Mathematica [A]** time = 0.0538446, size = 98, normalized size = 0.89

$$\frac{(30a^4c^2 + 20a^2cd + 6d^2) \log(1 - a^2x^2) + 4a^5x \coth^{-1}(ax) (15c^2 + 10cdx^2 + 3d^2x^4) + a^2dx^2 (a^2(20c + 3dx^2) + 6d)}{60a^5}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^2)^2\*ArcCoth[a\*x], x]



[Out]  $(a^2 d x^2 (6 d + a^2 (20 c + 3 d x^2)) + 4 a^5 x (15 c^2 + 10 c d x^2 + 3 d^2 x^4) \operatorname{ArcCoth}[a x] + (30 a^4 c^2 + 20 a^2 c d + 6 d^2) \operatorname{Log}[1 - a^2 x^2]) / (60 a^5)$

**Maple [A]** time = 0.036, size = 148, normalized size = 1.4

$$\frac{d^2 x^5 \operatorname{arccoth}(a x)}{5} + \frac{2 c d x^3 \operatorname{arccoth}(a x)}{3} + c^2 x \operatorname{arccoth}(a x) + \frac{d^2 x^4}{20 a} + \frac{c d x^2}{3 a} + \frac{d^2 x^2}{10 a^3} + \frac{\ln(a x - 1) c^2}{2 a} + \frac{\ln(a x - 1) c d}{3 a^3} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^2+c)^2*arccoth(a*x),x)`

[Out]  $1/5 d^2 x^5 \operatorname{arccoth}(a x) + 2/3 c d x^3 \operatorname{arccoth}(a x) + c^2 x \operatorname{arccoth}(a x) + 1/20 d^2 x^4/a + 1/3 a c d x^2 + 1/10 a^3 x^2 d^2 + 1/2 a \ln(a x - 1) c^2 + 1/3 a^3 \ln(a x - 1) c d + 1/10 a^5 \ln(a x - 1) d^2 + 1/2 a \ln(a x + 1) c^2 + 1/3 a^3 \ln(a x + 1) c d + 1/10 a^5 \ln(a x + 1) d^2$

**Maxima [A]** time = 0.973441, size = 177, normalized size = 1.61

$$\frac{1}{60} a \left( \frac{3 a^2 d^2 x^4 + 2 (10 a^2 c d + 3 d^2) x^2}{a^4} + \frac{2 (15 a^4 c^2 + 10 a^2 c d + 3 d^2) \log(a x + 1)}{a^6} + \frac{2 (15 a^4 c^2 + 10 a^2 c d + 3 d^2) \log(a x - 1)}{a^6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^2+c)^2*arccoth(a*x),x, algorithm="maxima")`

[Out]  $1/60 a ((3 a^2 d^2 x^4 + 2 (10 a^2 c d + 3 d^2) x^2) / a^4 + 2 (15 a^4 c^2 + 10 a^2 c d + 3 d^2) \log(a x + 1) / a^6 + 2 (15 a^4 c^2 + 10 a^2 c d + 3 d^2) \log(a x - 1) / a^6) + 1/15 (3 d^2 x^5 + 10 c d x^3 + 15 c^2 x) \operatorname{arccoth}(a x)$

**Fricas [A]** time = 1.5485, size = 258, normalized size = 2.35

$$\frac{3 a^4 d^2 x^4 + 2 (10 a^4 c d + 3 a^2 d^2) x^2 + 2 (15 a^4 c^2 + 10 a^2 c d + 3 d^2) \log(a^2 x^2 - 1) + 2 (3 a^5 d^2 x^5 + 10 a^5 c d x^3 + 15 a^5 c^2 x) \log(a x - 1)}{60 a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2+c)^2\*arccoth(a\*x),x, algorithm="fricas")

[Out] 1/60\*(3\*a^4\*d^2\*x^4 + 2\*(10\*a^4\*c\*d + 3\*a^2\*d^2)\*x^2 + 2\*(15\*a^4\*c^2 + 10\*a^2\*c\*d + 3\*d^2)\*log(a^2\*x^2 - 1) + 2\*(3\*a^5\*d^2\*x^5 + 10\*a^5\*c\*d\*x^3 + 15\*a^5\*c^2\*x)\*log((a\*x + 1)/(a\*x - 1)))/a^5

**Sympy [A]** time = 3.32318, size = 182, normalized size = 1.65

$$\left\{ \begin{array}{l} c^2 x \operatorname{arccoth}(ax) + \frac{2cdx^3 \operatorname{arccoth}(ax)}{3} + \frac{d^2 x^5 \operatorname{arccoth}(ax)}{5} + \frac{c^2 \log\left(x - \frac{1}{a}\right)}{a} + \frac{c^2 \operatorname{arccoth}(ax)}{a} + \frac{cdx^2}{3a} + \frac{d^2 x^4}{20a} + \frac{2cd \log\left(x - \frac{1}{a}\right)}{3a^3} + \frac{2cd \operatorname{arccoth}(ax)}{3a^3} + \frac{d^2 x^5}{10a^5} \\ \frac{i\pi\left(c^2 x + \frac{2cdx^3}{3} + \frac{d^2 x^5}{5}\right)}{2} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*2+c)\*\*2\*acoth(a\*x),x)

[Out] Piecewise((c\*\*2\*x\*acoth(a\*x) + 2\*c\*d\*x\*\*3\*acoth(a\*x)/3 + d\*\*2\*x\*\*5\*acoth(a\*x)/5 + c\*\*2\*log(x - 1/a)/a + c\*\*2\*acoth(a\*x)/a + c\*d\*x\*\*2/(3\*a) + d\*\*2\*x\*\*4/(20\*a) + 2\*c\*d\*log(x - 1/a)/(3\*a\*\*3) + 2\*c\*d\*acoth(a\*x)/(3\*a\*\*3) + d\*\*2\*x\*\*2/(10\*a\*\*3) + d\*\*2\*log(x - 1/a)/(5\*a\*\*5) + d\*\*2\*acoth(a\*x)/(5\*a\*\*5), Ne(a, 0)), (I\*pi\*(c\*\*2\*x + 2\*c\*d\*x\*\*3/3 + d\*\*2\*x\*\*5/5)/2, True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx^2 + c)^2 \operatorname{arccoth}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2+c)^2\*arccoth(a\*x),x, algorithm="giac")

[Out] integrate((d\*x^2 + c)^2\*arccoth(a\*x), x)

### 3.38 $\int (c + dx^2) \coth^{-1}(ax) dx$

**Optimal.** Leaf size=57

$$\frac{(3a^2c + d) \log(1 - a^2x^2)}{6a^3} + cx \coth^{-1}(ax) + \frac{dx^2}{6a} + \frac{1}{3} dx^3 \coth^{-1}(ax)$$

[Out] (d\*x^2)/(6\*a) + c\*x\*ArcCoth[a\*x] + (d\*x^3\*ArcCoth[a\*x])/3 + ((3\*a^2\*c + d)\*Log[1 - a^2\*x^2])/(6\*a^3)

**Rubi [A]** time = 0.0659503, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {5977, 1593, 444, 43}

$$\frac{(3a^2c + d) \log(1 - a^2x^2)}{6a^3} + cx \coth^{-1}(ax) + \frac{dx^2}{6a} + \frac{1}{3} dx^3 \coth^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^2)\*ArcCoth[a\*x], x]

[Out] (d\*x^2)/(6\*a) + c\*x\*ArcCoth[a\*x] + (d\*x^3\*ArcCoth[a\*x])/3 + ((3\*a^2\*c + d)\*Log[1 - a^2\*x^2])/(6\*a^3)

#### Rule 5977

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))\*((d\_.) + (e\_.)\*(x\_)^2)^(q\_.), x\_Symbol] := With[{u = IntHide[(d + e\*x^2)^q, x]}, Dist[a + b\*ArcCoth[c\*x], u, x] - Dist[b\*c, Int[u/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])

#### Rule 1593

Int[(u\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rule 444

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m - n +

1, 0]

**Rule 43**

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

**Rubi steps**

$$\begin{aligned}
\int (c + dx^2) \coth^{-1}(ax) dx &= cx \coth^{-1}(ax) + \frac{1}{3} dx^3 \coth^{-1}(ax) - a \int \frac{cx + \frac{dx^3}{3}}{1 - a^2x^2} dx \\
&= cx \coth^{-1}(ax) + \frac{1}{3} dx^3 \coth^{-1}(ax) - a \int \frac{x \left( c + \frac{dx^2}{3} \right)}{1 - a^2x^2} dx \\
&= cx \coth^{-1}(ax) + \frac{1}{3} dx^3 \coth^{-1}(ax) - \frac{1}{2} a \operatorname{Subst} \left( \int \frac{c + \frac{dx}{3}}{1 - a^2x} dx, x, x^2 \right) \\
&= cx \coth^{-1}(ax) + \frac{1}{3} dx^3 \coth^{-1}(ax) - \frac{1}{2} a \operatorname{Subst} \left( \int \left( -\frac{d}{3a^2} + \frac{-3a^2c - d}{3a^2(-1 + a^2x)} \right) dx, x, x^2 \right) \\
&= \frac{dx^2}{6a} + cx \coth^{-1}(ax) + \frac{1}{3} dx^3 \coth^{-1}(ax) + \frac{(3a^2c + d) \log(1 - a^2x^2)}{6a^3}
\end{aligned}$$

**Mathematica [A]** time = 0.0116903, size = 69, normalized size = 1.21

$$\frac{c \log(1 - a^2x^2)}{2a} + \frac{d \log(1 - a^2x^2)}{6a^3} + cx \coth^{-1}(ax) + \frac{dx^2}{6a} + \frac{1}{3} dx^3 \coth^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^2)\*ArcCoth[a\*x], x]

[Out] (d\*x^2)/(6\*a) + c\*x\*ArcCoth[a\*x] + (d\*x^3\*ArcCoth[a\*x])/3 + (c\*Log[1 - a^2\*x^2])/(2\*a) + (d\*Log[1 - a^2\*x^2])/(6\*a^3)

**Maple [A]** time = 0.038, size = 76, normalized size = 1.3

$$\frac{dx^3 \operatorname{arccoth}(ax)}{3} + cx \operatorname{arccoth}(ax) + \frac{dx^2}{6a} + \frac{\ln(ax - 1)c}{2a} + \frac{\ln(ax - 1)d}{6a^3} + \frac{\ln(ax + 1)c}{2a} + \frac{\ln(ax + 1)d}{6a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^2+c)*arccoth(a*x),x)`

[Out]  $\frac{1}{3}d*x^3*arccoth(a*x)+c*x*arccoth(a*x)+\frac{1}{6}d*x^2/a+\frac{1}{2}/a*\ln(a*x-1)*c+\frac{1}{6}/a^3*\ln(a*x-1)*d+\frac{1}{2}/a*\ln(a*x+1)*c+\frac{1}{6}/a^3*\ln(a*x+1)*d$

**Maxima [A]** time = 0.974406, size = 88, normalized size = 1.54

$$\frac{1}{6}a\left(\frac{dx^2}{a^2} + \frac{(3a^2c + d)\log(ax + 1)}{a^4} + \frac{(3a^2c + d)\log(ax - 1)}{a^4}\right) + \frac{1}{3}(dx^3 + 3cx)\operatorname{arccoth}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^2+c)*arccoth(a*x),x, algorithm="maxima")`

[Out]  $\frac{1}{6}a*(d*x^2/a^2 + (3*a^2*c + d)*\log(a*x + 1)/a^4 + (3*a^2*c + d)*\log(a*x - 1)/a^4) + \frac{1}{3}*(d*x^3 + 3*c*x)*arccoth(a*x)$

**Fricas [A]** time = 1.51526, size = 142, normalized size = 2.49

$$\frac{a^2dx^2 + (3a^2c + d)\log(a^2x^2 - 1) + (a^3dx^3 + 3a^3cx)\log\left(\frac{ax+1}{ax-1}\right)}{6a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^2+c)*arccoth(a*x),x, algorithm="fricas")`

[Out]  $\frac{1}{6}*(a^2*d*x^2 + (3*a^2*c + d)*\log(a^2*x^2 - 1) + (a^3*d*x^3 + 3*a^3*c*x)*\log((a*x + 1)/(a*x - 1)))/a^3$

**Sympy [A]** time = 1.37328, size = 87, normalized size = 1.53

$$\begin{cases} cx \operatorname{acoth}(ax) + \frac{dx^3 \operatorname{acoth}(ax)}{3} + \frac{c \log\left(x - \frac{1}{a}\right)}{a} + \frac{c \operatorname{acoth}(ax)}{a} + \frac{dx^2}{6a} + \frac{d \log\left(x - \frac{1}{a}\right)}{3a^3} + \frac{d \operatorname{acoth}(ax)}{3a^3} & \text{for } a \neq 0 \\ \frac{i\pi\left(cx + \frac{dx^3}{3}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)*acoth(a*x),x)
```

```
[Out] Piecewise((c*x*acoth(a*x) + d*x**3*acoth(a*x)/3 + c*log(x - 1/a)/a + c*acoth(a*x)/a + d*x**2/(6*a) + d*log(x - 1/a)/(3*a**3) + d*acoth(a*x)/(3*a**3), Ne(a, 0)), (I*pi*(c*x + d*x**3/3)/2, True))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx^2 + c) \operatorname{arccoth}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)*arccoth(a*x),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*arccoth(a*x), x)
```

$$3.39 \quad \int \frac{\coth^{-1}(ax)}{c+dx^2} dx$$

**Optimal.** Leaf size=390

$$\frac{i\text{PolyLog}\left(2, 1 + \frac{2\sqrt{c}\sqrt{d}(1-ax)}{(-\sqrt{d+ia}\sqrt{c})(\sqrt{c-i}\sqrt{dx})}\right)}{4\sqrt{c}\sqrt{d}} + \frac{i\text{PolyLog}\left(2, 1 - \frac{2\sqrt{c}\sqrt{d}(ax+1)}{(\sqrt{d+ia}\sqrt{c})(\sqrt{c-i}\sqrt{dx})}\right)}{4\sqrt{c}\sqrt{d}} - \frac{\log\left(1 - \frac{1}{ax}\right) \tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{2\sqrt{c}\sqrt{d}} + \frac{\log\left(\frac{1}{ax} + \frac{1}{\sqrt{c}}\right)}{2\sqrt{c}\sqrt{d}}$$

[Out]  $-(\text{ArcTan}[(\text{Sqrt}[d]*x)/\text{Sqrt}[c]]*\text{Log}[1 - 1/(a*x)])/(2*\text{Sqrt}[c]*\text{Sqrt}[d]) + (\text{ArcTan}[(\text{Sqrt}[d]*x)/\text{Sqrt}[c]]*\text{Log}[1 + 1/(a*x)])/(2*\text{Sqrt}[c]*\text{Sqrt}[d]) + (\text{ArcTan}[(\text{Sqrt}[d]*x)/\text{Sqrt}[c]]*\text{Log}[(-2*\text{Sqrt}[c]*\text{Sqrt}[d]*(1 - a*x))/((I*a*\text{Sqrt}[c] - \text{Sqrt}[d])*(\text{Sqrt}[c] - I*\text{Sqrt}[d]*x))])/(2*\text{Sqrt}[c]*\text{Sqrt}[d]) - (\text{ArcTan}[(\text{Sqrt}[d]*x)/\text{Sqrt}[c]]*\text{Log}[(2*\text{Sqrt}[c]*\text{Sqrt}[d]*(1 + a*x))/((I*a*\text{Sqrt}[c] + \text{Sqrt}[d])*(\text{Sqrt}[c] - I*\text{Sqrt}[d]*x))])/(2*\text{Sqrt}[c]*\text{Sqrt}[d]) - ((I/4)*\text{PolyLog}[2, 1 + (2*\text{Sqrt}[c]*\text{Sqrt}[d]*(1 - a*x))/((I*a*\text{Sqrt}[c] - \text{Sqrt}[d])*(\text{Sqrt}[c] - I*\text{Sqrt}[d]*x))])/(2*\text{Sqrt}[c]*\text{Sqrt}[d]) + ((I/4)*\text{PolyLog}[2, 1 - (2*\text{Sqrt}[c]*\text{Sqrt}[d]*(1 + a*x))/((I*a*\text{Sqrt}[c] + \text{Sqrt}[d])*(\text{Sqrt}[c] - I*\text{Sqrt}[d]*x))])/(2*\text{Sqrt}[c]*\text{Sqrt}[d])$

**Rubi [A]** time = 0.946685, antiderivative size = 390, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 13, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$ , Rules used = {5973, 205, 2470, 12, 260, 6688, 4876, 4848, 2391, 4856, 2402, 2315, 2447}

$$\frac{i\text{PolyLog}\left(2, 1 + \frac{2\sqrt{c}\sqrt{d}(1-ax)}{(-\sqrt{d+ia}\sqrt{c})(\sqrt{c-i}\sqrt{dx})}\right)}{4\sqrt{c}\sqrt{d}} + \frac{i\text{PolyLog}\left(2, 1 - \frac{2\sqrt{c}\sqrt{d}(ax+1)}{(\sqrt{d+ia}\sqrt{c})(\sqrt{c-i}\sqrt{dx})}\right)}{4\sqrt{c}\sqrt{d}} - \frac{\log\left(1 - \frac{1}{ax}\right) \tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{2\sqrt{c}\sqrt{d}} + \frac{\log\left(\frac{1}{ax} + \frac{1}{\sqrt{c}}\right)}{2\sqrt{c}\sqrt{d}}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]/(c + d\*x^2), x]

[Out]  $-(\text{ArcTan}[(\text{Sqrt}[d]*x)/\text{Sqrt}[c]]*\text{Log}[1 - 1/(a*x)])/(2*\text{Sqrt}[c]*\text{Sqrt}[d]) + (\text{ArcTan}[(\text{Sqrt}[d]*x)/\text{Sqrt}[c]]*\text{Log}[1 + 1/(a*x)])/(2*\text{Sqrt}[c]*\text{Sqrt}[d]) + (\text{ArcTan}[(\text{Sqrt}[d]*x)/\text{Sqrt}[c]]*\text{Log}[(-2*\text{Sqrt}[c]*\text{Sqrt}[d]*(1 - a*x))/((I*a*\text{Sqrt}[c] - \text{Sqrt}[d])*(\text{Sqrt}[c] - I*\text{Sqrt}[d]*x))])/(2*\text{Sqrt}[c]*\text{Sqrt}[d]) - (\text{ArcTan}[(\text{Sqrt}[d]*x)/\text{Sqrt}[c]]*\text{Log}[(2*\text{Sqrt}[c]*\text{Sqrt}[d]*(1 + a*x))/((I*a*\text{Sqrt}[c] + \text{Sqrt}[d])*(\text{Sqrt}[c] - I*\text{Sqrt}[d]*x))])/(2*\text{Sqrt}[c]*\text{Sqrt}[d]) - ((I/4)*\text{PolyLog}[2, 1 + (2*\text{Sqrt}[c]*\text{Sqrt}[d]*(1 - a*x))/((I*a*\text{Sqrt}[c] - \text{Sqrt}[d])*(\text{Sqrt}[c] - I*\text{Sqrt}[d]*x))])/(2*\text{Sqrt}[c]*\text{Sqrt}[d]) + ((I/4)*\text{PolyLog}[2, 1 - (2*\text{Sqrt}[c]*\text{Sqrt}[d]*(1 + a*x))/((I*a*\text{Sqrt}[c] + \text{Sqrt}[d])*(\text{Sqrt}[c] - I*\text{Sqrt}[d]*x))])/(2*\text{Sqrt}[c]*\text{Sqrt}[d])$

Rule 5973

$\text{Int}[\text{ArcCoth}[(c\_)(x\_)]/((d\_)+(e\_)(x\_)^2), x\_Symbol] \rightarrow \text{Dist}[1/2, \text{Int}[\text{Log}[1 + 1/(c*x)]/(d + e*x^2), x], x] - \text{Dist}[1/2, \text{Int}[\text{Log}[1 - 1/(c*x)]/(d + e*x^2), x], x] /; \text{FreeQ}\{c, d, e\}, x]$

Rule 205

$\text{Int}[((a_) + (b\_)(x_)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a/b]$

Rule 2470

$\text{Int}(((a_) + \text{Log}[(c\_)((d_) + (e\_)(x_)^n))^p)(b\_)/((f_) + (g_)(x_)^2), x\_Symbol] \rightarrow \text{With}\{u = \text{IntHide}[1/(f + g*x^2), x]\}, \text{Simp}[u*(a + b*\text{Log}[c*(d + e*x^n)^p]), x] - \text{Dist}[b*e*n*p, \text{Int}[(u*x^{n-1})/(d + e*x^n), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, n, p\}, x] \&\& \text{IntegerQ}[n]$

Rule 12

$\text{Int}[(a_)(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& \text{!MatchQ}[u, (b_)(v_)] /; \text{FreeQ}[b, x]$

Rule 260

$\text{Int}[(x_)^m/((a_) + (b_)(x_)^n), x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]]/(b*n), x] /; \text{FreeQ}\{a, b, m, n\}, x] \&\& \text{EqQ}[m, n - 1]$

Rule 6688

$\text{Int}[u_, x\_Symbol] \rightarrow \text{With}\{v = \text{SimplifyIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SimplerIntegrandQ}[v, u, x]$

Rule 4876

$\text{Int}(((a_) + \text{ArcTan}[(c_)(x_)]*(b\_))^p)((f_)(x_)^m)((d_) + (e_)(x_)^q), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*\text{ArcTan}[c*x])^p, (f*x)^m*(d + e*x)^q, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \&\& \text{IGtQ}[p, 0] \&\& \text{IntegerQ}[q] \&\& (\text{GtQ}[q, 0] \parallel \text{NeQ}[a, 0] \parallel \text{IntegerQ}[m])$

Rule 4848

$\text{Int}(((a_) + \text{ArcTan}[(c_)(x_)]*(b_))/(x_), x\_Symbol] \rightarrow \text{Simp}[a*\text{Log}[x], x] + (\text{Dist}[(I*b)/2, \text{Int}[\text{Log}[1 - I*c*x]/x, x], x] - \text{Dist}[(I*b)/2, \text{Int}[\text{Log}[1 +$



$I*c*x]/x, x], x]) /; \text{FreeQ}\{a, b, c\}, x]$

### Rule 2391

$\text{Int}[\text{Log}[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x\_Symbol] :> -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x] \&\& \text{EqQ}[c*d, 1]$

### Rule 4856

$\text{Int}[(a_.) + \text{ArcTan}[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x\_Symbol] :> -\text{Simp}[(a + b*\text{ArcTan}[c*x])*\text{Log}[2/(1 - I*c*x)]/e, x] + (\text{Dist}[(b*c)/e, \text{Int}[\text{Log}[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - \text{Dist}[(b*c)/e, \text{Int}[\text{Log}[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + \text{Simp}[(a + b*\text{ArcTan}[c*x])*\text{Log}[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))]/e, x]) /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[c^2*d^2 + e^2, 0]$

### Rule 2402

$\text{Int}[\text{Log}[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x\_Symbol] :> -\text{Dist}[e/g, \text{Subst}[\text{Int}[\text{Log}[2*d*x]/(1 - 2*d*x)], x], x, 1/(d + e*x)], x] /; \text{FreeQ}\{c, d, e, f, g\}, x] \&\& \text{EqQ}[c, 2*d] \&\& \text{EqQ}[e^2*f + d^2*g, 0]$

### Rule 2315

$\text{Int}[\text{Log}[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x\_Symbol] :> -\text{Simp}[\text{PolyLog}[2, 1 - c*x]/e, x] /; \text{FreeQ}\{c, d, e\}, x] \&\& \text{EqQ}[e + c*d, 0]$

### Rule 2447

$\text{Int}[\text{Log}[u_]*(Pq_)^(m_.), x\_Symbol] :> \text{With}[\{C = \text{FullSimplify}[(Pq^m*(1 - u))/D[u, x]]\}, \text{Simp}[C*\text{PolyLog}[2, 1 - u], x] /; \text{FreeQ}[C, x]] /; \text{IntegerQ}[m] \&\& \text{PolyQ}[Pq, x] \&\& \text{RationalFunctionQ}[u, x] \&\& \text{LeQ}[\text{RationalFunctionExponents}[u, x][[2]], \text{Expon}[Pq, x]]$

### Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)}{c+dx^2} dx &= -\left(\frac{1}{2} \int \frac{\log\left(1-\frac{1}{ax}\right)}{c+dx^2} dx\right) + \frac{1}{2} \int \frac{\log\left(1+\frac{1}{ax}\right)}{c+dx^2} dx \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right) \log\left(1-\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}} + \frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right) \log\left(1+\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}} + \frac{\int \frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}\left(1-\frac{1}{ax}\right)x^2} dx}{2a} + \frac{\int \frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}\left(1+\frac{1}{ax}\right)x^2} dx}{2a} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right) \log\left(1-\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}} + \frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right) \log\left(1+\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}} + \frac{\int \frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{\left(1-\frac{1}{ax}\right)x^2} dx}{2a\sqrt{c}\sqrt{d}} + \frac{\int \frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{\left(1+\frac{1}{ax}\right)x^2} dx}{2a\sqrt{c}\sqrt{d}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right) \log\left(1-\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}} + \frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right) \log\left(1+\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}} + \frac{\int \frac{a \tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{x(-1+ax)} dx}{2a\sqrt{c}\sqrt{d}} + \frac{\int \frac{a \tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{x(1+ax)} dx}{2a\sqrt{c}\sqrt{d}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right) \log\left(1-\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}} + \frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right) \log\left(1+\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}} + \frac{\int \frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{x(-1+ax)} dx}{2\sqrt{c}\sqrt{d}} + \frac{\int \frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{x(1+ax)} dx}{2\sqrt{c}\sqrt{d}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right) \log\left(1-\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}} + \frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right) \log\left(1+\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}} + \frac{\int \left(\frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{x} + \frac{a \tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{-1+ax}\right) dx}{2\sqrt{c}\sqrt{d}} + \frac{\int \left(\frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{x} - \frac{a \tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{1+ax}\right) dx}{2\sqrt{c}\sqrt{d}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right) \log\left(1-\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}} + \frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right) \log\left(1+\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}} + \frac{a \int \frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{-1+ax} dx}{2\sqrt{c}\sqrt{d}} - \frac{a \int \frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{1+ax} dx}{2\sqrt{c}\sqrt{d}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right) \log\left(1-\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}} + \frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right) \log\left(1+\frac{1}{ax}\right)}{2\sqrt{c}\sqrt{d}} + \frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right) \log\left(-\frac{2\sqrt{c}\sqrt{d}(1-ax)}{(ia\sqrt{c}-\sqrt{d})(\sqrt{c}-i\sqrt{dx})}\right)}{2\sqrt{c}\sqrt{d}} - \frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right) \log\left(-\frac{2\sqrt{c}\sqrt{d}(1-ax)}{(ia\sqrt{c}-\sqrt{d})(\sqrt{c}+i\sqrt{dx})}\right)}{2\sqrt{c}\sqrt{d}}
\end{aligned}$$

**Mathematica [A]** time = 1.34434, size = 671, normalized size = 1.72

$$a \left( i \left( \text{PolyLog} \left( 2, \frac{(2i\sqrt{a^2cd+a^2c-d})(\sqrt{a^2cd+iadx})}{(a^2c+d)(\sqrt{a^2cd-iadx})} \right) - \text{PolyLog} \left( 2, \frac{(-2i\sqrt{a^2cd+a^2c-d})(\sqrt{a^2cd+iadx})}{(a^2c+d)(\sqrt{a^2cd-iadx})} \right) \right) - 2i \cos^{-1} \left( \frac{a^2c-d}{a^2c+d} \right) \tan^{-1} \left( \frac{ac}{x\sqrt{a^2cd}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a\*x]/(c + d\*x^2), x]

[Out] (a\*((-2\*I)\*ArcCos[(a^2\*c - d)/(a^2\*c + d)]\*ArcTan[(a\*c)/(Sqrt[a^2\*c\*d]\*x]) + 4\*ArcCoth[a\*x]\*ArcTan[(a\*d\*x)/Sqrt[a^2\*c\*d]] - (ArcCos[(a^2\*c - d)/(a^2\*c + d)] + 2\*ArcTan[(a\*c)/(Sqrt[a^2\*c\*d]\*x]))\*Log[(2\*d\*(a^2\*c - I\*Sqrt[a^2\*c\*d])\*(-1 + a\*x))/((a^2\*c + d)\*(I\*Sqrt[a^2\*c\*d] + a\*d\*x))] - (ArcCos[(a^2\*c - d)/(a^2\*c + d)] - 2\*ArcTan[(a\*c)/(Sqrt[a^2\*c\*d]\*x]))\*Log[(2\*d\*(a^2\*c + I\*Sqrt[a^2\*c\*d])\*(1 + a\*x))/((a^2\*c + d)\*(I\*Sqrt[a^2\*c\*d] + a\*d\*x))] + (ArcCos[(a^2\*c - d)/(a^2\*c + d)] + 2\*(ArcTan[(a\*c)/(Sqrt[a^2\*c\*d]\*x)] + ArcTan[(a\*d\*x)/Sqrt[a^2\*c\*d]])\*Log[(Sqrt[2]\*Sqrt[a^2\*c\*d])/(Sqrt[a^2\*c + d]\*E^ArcCoth[a\*x]\*Sqrt[-(a^2\*c) + d + (a^2\*c + d)\*Cosh[2\*ArcCoth[a\*x]])]) + (ArcCos[(a^2\*c - d)/(a^2\*c + d)] - 2\*(ArcTan[(a\*c)/(Sqrt[a^2\*c\*d]\*x)] + ArcTan[(a\*d\*x)/Sqrt[a^2\*c\*d]])\*Log[(Sqrt[2]\*Sqrt[a^2\*c\*d]\*E^ArcCoth[a\*x])/(Sqrt[a^2\*c + d]\*Sqrt[-(a^2\*c) + d + (a^2\*c + d)\*Cosh[2\*ArcCoth[a\*x]])]) + I\*(-PolyLog[2, ((a^2\*c - d - (2\*I)\*Sqrt[a^2\*c\*d])\*(Sqrt[a^2\*c\*d] + I\*a\*d\*x))/((a^2\*c + d)\*(Sqrt[a^2\*c\*d] - I\*a\*d\*x))] + PolyLog[2, ((a^2\*c - d + (2\*I)\*Sqrt[a^2\*c\*d])\*(Sqrt[a^2\*c\*d] + I\*a\*d\*x))/((a^2\*c + d)\*(Sqrt[a^2\*c\*d] - I\*a\*d\*x))]))/(4\*Sqrt[a^2\*c\*d])

**Maple [B]** time = 0.218, size = 785, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)/(d\*x^2+c), x)

[Out] -1/2\*a^3/d/(a^4\*c^2+2\*a^2\*c\*d+d^2)\*ln(1-(a^2\*c+d)\*(a\*x+1)/(a\*x-1)/(a^2\*c-2\*(-a^2\*c\*d)^(1/2)-d))\*arccoth(a\*x)\*(-a^2\*c\*d)^(1/2)\*c-a/(a^4\*c^2+2\*a^2\*c\*d+d^2)\*ln(1-(a^2\*c+d)\*(a\*x+1)/(a\*x-1)/(a^2\*c-2\*(-a^2\*c\*d)^(1/2)-d))\*arccoth(a\*x)\*(-a^2\*c\*d)^(1/2)+1/2\*a^3/d/(a^4\*c^2+2\*a^2\*c\*d+d^2)\*arccoth(a\*x)^2\*(-a^2\*c\*d)^(1/2)\*c+a/(a^4\*c^2+2\*a^2\*c\*d+d^2)\*arccoth(a\*x)^2\*(-a^2\*c\*d)^(1/2)-1/4\*a^3/d/(a^4\*c^2+2\*a^2\*c\*d+d^2)\*polylog(2, (a^2\*c+d)\*(a\*x+1)/(a\*x-1)/(a^2\*c-2\*

$$(-a^2cd)^{(1/2)-d}) * (-a^2cd)^{(1/2)} * c - 1/2 * a / (a^4c^2 + 2a^2cd + d^2) * \text{polylog}(2, (a^2c+d) * (ax+1) / (ax-1) / (a^2c - 2(-a^2cd)^{(1/2)-d})) * (-a^2cd)^{(1/2)} - 1/2 / a / c / (a^4c^2 + 2a^2cd + d^2) * \ln(1 - (a^2c+d) * (ax+1) / (ax-1) / (a^2c - 2(-a^2cd)^{(1/2)-d})) * \text{arccoth}(ax) * (-a^2cd)^{(1/2)} * d + 1/2 / a / c / (a^4c^2 + 2a^2cd + d^2) * \text{arccoth}(ax)^2 * (-a^2cd)^{(1/2)} * d - 1/4 / a / c / (a^4c^2 + 2a^2cd + d^2) * \text{polylog}(2, (a^2c+d) * (ax+1) / (ax-1) / (a^2c - 2(-a^2cd)^{(1/2)-d})) * (-a^2cd)^{(1/2)} * d + 1/2 / a * (-a^2cd)^{(1/2)} / c / d * \text{arccoth}(ax) * \ln(1 - (a^2c+d) * (ax+1) / (ax-1) / (a^2c + 2(-a^2cd)^{(1/2)-d})) - 1/2 / a * (-a^2cd)^{(1/2)} / c / d * \text{arccoth}(ax)^2 + 1/4 / a * (-a^2cd)^{(1/2)} / c / d * \text{polylog}(2, (a^2c+d) * (ax+1) / (ax-1) / (a^2c + 2(-a^2cd)^{(1/2)-d}))$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(ax)/(d\*x^2+c),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arccoth}(ax)}{dx^2 + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(ax)/(d\*x^2+c),x, algorithm="fricas")

[Out] integral(arccoth(ax)/(d\*x^2 + c), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{acoth}(ax)}{c + dx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoath(a*x)/(d*x**2+c),x)
```

```
[Out] Integral(acoath(a*x)/(c + d*x**2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(ax)}{dx^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoath(a*x)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arccoath(a*x)/(d*x^2 + c), x)
```

$$3.40 \quad \int \frac{\coth^{-1}(ax)}{(c+dx^2)^2} dx$$

**Optimal.** Leaf size=590

$$\frac{i\text{PolyLog}\left(2, \frac{a(\sqrt{c}-i\sqrt{dx})}{a\sqrt{c}-i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}} - \frac{i\text{PolyLog}\left(2, \frac{a(\sqrt{c}-i\sqrt{dx})}{a\sqrt{c}+i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}} + \frac{i\text{PolyLog}\left(2, \frac{a(\sqrt{c}+i\sqrt{dx})}{a\sqrt{c}-i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}} - \frac{i\text{PolyLog}\left(2, \frac{a(\sqrt{c}+i\sqrt{dx})}{a\sqrt{c}+i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}} + \frac{a \log(1 - \dots)}{4c(a^2c \dots)}$$

[Out] (x\*ArcCoth[a\*x])/(2\*c\*(c + d\*x^2)) + (ArcCoth[a\*x]\*ArcTan[(Sqrt[d]\*x)/Sqrt[c]])/(2\*c^(3/2)\*Sqrt[d]) + ((I/8)\*Log[(Sqrt[d]\*(1 - a\*x))/(I\*a\*Sqrt[c] + Sqrt[d])]\*Log[1 - (I\*Sqrt[d]\*x)/Sqrt[c]])/(c^(3/2)\*Sqrt[d]) - ((I/8)\*Log[-((Sqrt[d]\*(1 + a\*x))/(I\*a\*Sqrt[c] - Sqrt[d]))]\*Log[1 - (I\*Sqrt[d]\*x)/Sqrt[c]])/(c^(3/2)\*Sqrt[d]) - ((I/8)\*Log[-((Sqrt[d]\*(1 - a\*x))/(I\*a\*Sqrt[c] - Sqrt[d]))]\*Log[1 + (I\*Sqrt[d]\*x)/Sqrt[c]])/(c^(3/2)\*Sqrt[d]) + ((I/8)\*Log[(Sqrt[d]\*(1 + a\*x))/(I\*a\*Sqrt[c] + Sqrt[d])]\*Log[1 + (I\*Sqrt[d]\*x)/Sqrt[c]])/(c^(3/2)\*Sqrt[d]) + (a\*Log[1 - a^2\*x^2])/(4\*c\*(a^2\*c + d)) - (a\*Log[c + d\*x^2])/(4\*c\*(a^2\*c + d)) + ((I/8)\*PolyLog[2, (a\*(Sqrt[c] - I\*Sqrt[d]\*x))/(a\*Sqrt[c] - I\*Sqrt[d])])/(c^(3/2)\*Sqrt[d]) - ((I/8)\*PolyLog[2, (a\*(Sqrt[c] - I\*Sqrt[d]\*x))/(a\*Sqrt[c] + I\*Sqrt[d])])/(c^(3/2)\*Sqrt[d]) + ((I/8)\*PolyLog[2, (a\*(Sqrt[c] + I\*Sqrt[d]\*x))/(a\*Sqrt[c] - I\*Sqrt[d])])/(c^(3/2)\*Sqrt[d]) - ((I/8)\*PolyLog[2, (a\*(Sqrt[c] + I\*Sqrt[d]\*x))/(a\*Sqrt[c] + I\*Sqrt[d])])/(c^(3/2)\*Sqrt[d])

**Rubi [A]** time = 0.869703, antiderivative size = 590, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 13, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$ , Rules used = {199, 205, 5977, 6725, 517, 444, 36, 31, 4908, 2409, 2394, 2393, 2391}

$$\frac{i\text{PolyLog}\left(2, \frac{a(\sqrt{c}-i\sqrt{dx})}{a\sqrt{c}-i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}} - \frac{i\text{PolyLog}\left(2, \frac{a(\sqrt{c}-i\sqrt{dx})}{a\sqrt{c}+i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}} + \frac{i\text{PolyLog}\left(2, \frac{a(\sqrt{c}+i\sqrt{dx})}{a\sqrt{c}-i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}} - \frac{i\text{PolyLog}\left(2, \frac{a(\sqrt{c}+i\sqrt{dx})}{a\sqrt{c}+i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}} + \frac{a \log(1 - \dots)}{4c(a^2c \dots)}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]/(c + d\*x^2)^2, x]

[Out] (x\*ArcCoth[a\*x])/(2\*c\*(c + d\*x^2)) + (ArcCoth[a\*x]\*ArcTan[(Sqrt[d]\*x)/Sqrt[c]])/(2\*c^(3/2)\*Sqrt[d]) + ((I/8)\*Log[(Sqrt[d]\*(1 - a\*x))/(I\*a\*Sqrt[c] + Sqrt[d])]\*Log[1 - (I\*Sqrt[d]\*x)/Sqrt[c]])/(c^(3/2)\*Sqrt[d]) - ((I/8)\*Log[-((Sqrt[d]\*(1 + a\*x))/(I\*a\*Sqrt[c] - Sqrt[d]))]\*Log[1 - (I\*Sqrt[d]\*x)/Sqrt[c]])/(c^(3/2)\*Sqrt[d]) - ((I/8)\*Log[-((Sqrt[d]\*(1 - a\*x))/(I\*a\*Sqrt[c] - Sqrt[d]))]\*Log[1 + (I\*Sqrt[d]\*x)/Sqrt[c]])/(c^(3/2)\*Sqrt[d]) + ((I/8)\*Log[(Sqrt[d]\*(1 + a\*x))/(I\*a\*Sqrt[c] + Sqrt[d])]\*Log[1 + (I\*Sqrt[d]\*x)/Sqrt[c]])/(c^(3/2)\*Sqrt[d]) + (a\*Log[1 - a^2\*x^2])/(4\*c\*(a^2\*c + d)) - (a\*Log[c + d\*x^2])/(4\*c\*(a^2\*c + d)) + ((I/8)\*PolyLog[2, (a\*(Sqrt[c] - I\*Sqrt[d]\*x))/(a\*Sqrt[c] - I\*Sqrt[d])])/(c^(3/2)\*Sqrt[d]) - ((I/8)\*PolyLog[2, (a\*(Sqrt[c] - I\*Sqrt[d]\*x))/(a\*Sqrt[c] + I\*Sqrt[d])])/(c^(3/2)\*Sqrt[d]) + ((I/8)\*PolyLog[2, (a\*(Sqrt[c] + I\*Sqrt[d]\*x))/(a\*Sqrt[c] - I\*Sqrt[d])])/(c^(3/2)\*Sqrt[d]) - ((I/8)\*PolyLog[2, (a\*(Sqrt[c] + I\*Sqrt[d]\*x))/(a\*Sqrt[c] + I\*Sqrt[d])])/(c^(3/2)\*Sqrt[d])

```

]))*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]]/(c^(3/2)*Sqrt[d]) + ((I/8)*Log[(Sqrt[d]
]*(1 + a*x))/(I*a*Sqrt[c] + Sqrt[d]])*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]]/(c^(3
/2)*Sqrt[d]) + (a*Log[1 - a^2*x^2])/(4*c*(a^2*c + d)) - (a*Log[c + d*x^2])/
(4*c*(a^2*c + d)) + ((I/8)*PolyLog[2, (a*(Sqrt[c] - I*Sqrt[d]*x))/(a*Sqrt[c]
- I*Sqrt[d]))/(c^(3/2)*Sqrt[d]) - ((I/8)*PolyLog[2, (a*(Sqrt[c] - I*Sqrt
[d]*x))/(a*Sqrt[c] + I*Sqrt[d]))/(c^(3/2)*Sqrt[d]) + ((I/8)*PolyLog[2, (a*
(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] - I*Sqrt[d]))/(c^(3/2)*Sqrt[d]) - ((I/
8)*PolyLog[2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] + I*Sqrt[d]))/(c^(3/2
)*Sqrt[d])

```

### Rule 199

```

Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(
p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])

```

### Rule 205

```

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

```

### Rule 5977

```

Int[((a_) + ArcCoth[(c_)*(x_)])*(b_)*((d_) + (e_)*(x_)^2)^(q_), x_Sym
bol] := With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])

```

### Rule 6725

```

Int[(u_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]

```

### Rule 517

```

Int[(u_)*((c_) + (d_)*(x_)^(n_))^(q_)*((a1_) + (b1_)*(x_)^(non2_))^(p
_)*((a2_) + (b2_)*(x_)^(non2_))^(p_), x_Symbol] := Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))

```

Rule 444

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
1, 0]
```

Rule 36

```
Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))), x_Symbol] := Dist[b/(b*c
- a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4908

```
Int[ArcTan[(c_)*(x_)]/((d_) + (e_)*(x_)^2), x_Symbol] := Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 2409

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_)^(n_))]*(b_))^(p_)*((f_) + (g_.
)*(x_)^(r_))^(q_), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x
)^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2394

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_)^(n_))]*(b_))/((f_) + (g_)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_))]*(b_))/((f_) + (g_)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```



Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)}{(c+dx^2)^2} dx &= \frac{x \coth^{-1}(ax)}{2c(c+dx^2)} + \frac{\coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} - a \int \frac{\frac{x}{2c(c+dx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}}{1-a^2x^2} dx \\
&= \frac{x \coth^{-1}(ax)}{2c(c+dx^2)} + \frac{\coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} - a \int \left( \frac{x}{2c(-1+ax)(1+ax)(c+dx^2)} - \frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}(-1+a^2x^2)} \right) dx \\
&= \frac{x \coth^{-1}(ax)}{2c(c+dx^2)} + \frac{\coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} + \frac{a \int \frac{x}{(-1+ax)(1+ax)(c+dx^2)} dx}{2c} + \frac{a \int \frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{-1+a^2x^2} dx}{2c^{3/2}\sqrt{d}} \\
&= \frac{x \coth^{-1}(ax)}{2c(c+dx^2)} + \frac{\coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} + \frac{a \int \frac{x}{(-1+a^2x^2)(c+dx^2)} dx}{2c} + \frac{(ia) \int \frac{\log\left(1-\frac{i\sqrt{dx}}{\sqrt{c}}\right)}{-1+a^2x^2} dx}{4c^{3/2}\sqrt{d}} - \frac{(ia) \int \frac{\log\left(1+\frac{i\sqrt{dx}}{\sqrt{c}}\right)}{-1+a^2x^2} dx}{4c^{3/2}\sqrt{d}} \\
&= \frac{x \coth^{-1}(ax)}{2c(c+dx^2)} + \frac{\coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} + \frac{a \operatorname{Subst}\left(\int \frac{1}{(-1+a^2x)(c+dx)} dx, x, x^2\right)}{4c} + \frac{(ia) \int \left(-\frac{\log\left(1-\frac{i\sqrt{dx}}{\sqrt{c}}\right)}{2(1-ax)}\right) dx}{4c^{3/2}\sqrt{d}} \\
&= \frac{x \coth^{-1}(ax)}{2c(c+dx^2)} + \frac{\coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} - \frac{(ia) \int \frac{\log\left(1-\frac{i\sqrt{dx}}{\sqrt{c}}\right)}{1-ax} dx}{8c^{3/2}\sqrt{d}} - \frac{(ia) \int \frac{\log\left(1-\frac{i\sqrt{dx}}{\sqrt{c}}\right)}{1+ax} dx}{8c^{3/2}\sqrt{d}} + \frac{(ia) \int \frac{\log\left(1+\frac{i\sqrt{dx}}{\sqrt{c}}\right)}{1-ax} dx}{8c^{3/2}\sqrt{d}} \\
&= \frac{x \coth^{-1}(ax)}{2c(c+dx^2)} + \frac{\coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} + \frac{i \log\left(\frac{\sqrt{d}(1-ax)}{ia\sqrt{c}+\sqrt{d}}\right) \log\left(1-\frac{i\sqrt{dx}}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} - \frac{i \log\left(-\frac{\sqrt{d}(1+ax)}{ia\sqrt{c}-\sqrt{d}}\right) \log\left(1-\frac{i\sqrt{dx}}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} \\
&= \frac{x \coth^{-1}(ax)}{2c(c+dx^2)} + \frac{\coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} + \frac{i \log\left(\frac{\sqrt{d}(1-ax)}{ia\sqrt{c}+\sqrt{d}}\right) \log\left(1-\frac{i\sqrt{dx}}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} - \frac{i \log\left(-\frac{\sqrt{d}(1+ax)}{ia\sqrt{c}-\sqrt{d}}\right) \log\left(1-\frac{i\sqrt{dx}}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} \\
&= \frac{x \coth^{-1}(ax)}{2c(c+dx^2)} + \frac{\coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} + \frac{i \log\left(\frac{\sqrt{d}(1-ax)}{ia\sqrt{c}+\sqrt{d}}\right) \log\left(1-\frac{i\sqrt{dx}}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} - \frac{i \log\left(-\frac{\sqrt{d}(1+ax)}{ia\sqrt{c}-\sqrt{d}}\right) \log\left(1-\frac{i\sqrt{dx}}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}}
\end{aligned}$$

**Mathematica [A]** time = 7.81475, size = 753, normalized size = 1.28

$$a \left( i \left( \text{PolyLog} \left( 2, \frac{(-2i\sqrt{a^2cd+a^2c-d})(\sqrt{a^2cd+ia dx})}{(a^2c+d)(\sqrt{a^2cd-ia dx})} \right) - \text{PolyLog} \left( 2, \frac{(2i\sqrt{a^2cd+a^2c-d})(\sqrt{a^2cd+ia dx})}{(a^2c+d)(\sqrt{a^2cd-ia dx})} \right) \right) + 2i \cos^{-1} \left( \frac{a^2c-d}{a^2c+d} \right) \tan^{-1} \left( \frac{ac}{x\sqrt{a^2cd}} \right) - 4 \coth^{-1}(ax) \tan^{-1} \left( \frac{adx}{\sqrt{a^2cd}} \right) + \dots \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a\*x]/(c + d\*x^2)^2, x]

[Out]  $-(a*((2*\text{Log}[1 + ((a^2*c + d)*\text{Cosh}[2*\text{ArcCoth}[a*x]])/(-(a^2*c) + d)])/(a^2*c + d) + ((2*I)*\text{ArcCos}[(a^2*c - d)/(a^2*c + d)]*\text{ArcTan}[(a*c)/(\text{Sqrt}[a^2*c*d]*x)] - 4*\text{ArcCoth}[a*x]*\text{ArcTan}[(a*d*x)/\text{Sqrt}[a^2*c*d]] + (\text{ArcCos}[(a^2*c - d)/(a^2*c + d)] + 2*\text{ArcTan}[(a*c)/(\text{Sqrt}[a^2*c*d]*x)])*\text{Log}[(2*d*(a^2*c - I*\text{Sqrt}[a^2*c*d])*(-1 + a*x))/((a^2*c + d)*(I*\text{Sqrt}[a^2*c*d] + a*d*x))] + (\text{ArcCos}[(a^2*c - d)/(a^2*c + d)] - 2*\text{ArcTan}[(a*c)/(\text{Sqrt}[a^2*c*d]*x)])*\text{Log}[(2*d*(a^2*c + I*\text{Sqrt}[a^2*c*d])*(1 + a*x))/((a^2*c + d)*(I*\text{Sqrt}[a^2*c*d] + a*d*x))] - (\text{ArcCos}[(a^2*c - d)/(a^2*c + d)] + 2*(\text{ArcTan}[(a*c)/(\text{Sqrt}[a^2*c*d]*x)] + \text{ArcTan}[(a*d*x)/\text{Sqrt}[a^2*c*d]])*\text{Log}[(\text{Sqrt}[2]*\text{Sqrt}[a^2*c*d])/(\text{Sqrt}[a^2*c + d]*E^{\text{ArcCoth}[a*x]*\text{Sqrt}[-(a^2*c) + d + (a^2*c + d)*\text{Cosh}[2*\text{ArcCoth}[a*x]])}] - (\text{ArcCos}[(a^2*c - d)/(a^2*c + d)] - 2*(\text{ArcTan}[(a*c)/(\text{Sqrt}[a^2*c*d]*x)] + \text{ArcTan}[(a*d*x)/\text{Sqrt}[a^2*c*d]])*\text{Log}[(\text{Sqrt}[2]*\text{Sqrt}[a^2*c*d]*E^{\text{ArcCoth}[a*x]})/(\text{Sqrt}[a^2*c + d]*\text{Sqrt}[-(a^2*c) + d + (a^2*c + d)*\text{Cosh}[2*\text{ArcCoth}[a*x]])]) + I*(\text{PolyLog}[2, ((a^2*c - d - (2*I)*\text{Sqrt}[a^2*c*d])*(\text{Sqrt}[a^2*c*d] + I*a*d*x))/((a^2*c + d)*(\text{Sqrt}[a^2*c*d] - I*a*d*x))] - \text{PolyLog}[2, ((a^2*c - d + (2*I)*\text{Sqrt}[a^2*c*d])*(\text{Sqrt}[a^2*c*d] + I*a*d*x))/((a^2*c + d)*(\text{Sqrt}[a^2*c*d] - I*a*d*x))])]/\text{Sqrt}[a^2*c*d] - (4*\text{ArcCoth}[a*x]*\text{Sinh}[2*\text{ArcCoth}[a*x]])/(-(a^2*c) + d + (a^2*c + d)*\text{Cosh}[2*\text{ArcCoth}[a*x]])))/(8*c)$

**Maple [B]** time = 0.41, size = 2218, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)/(d\*x^2+c)^2, x)

[Out]  $-1/4*a^3/(a^2*c+d)^2*\ln(a^2*c/(a*x-1)^2*(a*x+1)^2-2*a^2*c*(a*x+1)/(a*x-1)+d/(a*x-1)^2*(a*x+1)^2+a^2*c+2*(a*x+1)/(a*x-1)*d+d)-1/2*a^3/(a^2*c+d)^2*\ln((a$

$$\begin{aligned}
& *x-1)/(a*x+1))-1/4*a^4*(c*d)^{(1/2)}/d/(a^2*c+d)^2*\arctan(a/d*(c*d)^{(1/2)})+1/ \\
& 4*(c*d)^{(1/2)}/c^2*d/(a^2*c+d)^2*\arctan(a/d*(c*d)^{(1/2)})-3/4*a*d*\ln(1-(a^2*c \\
& +d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^{(1/2)}-d))*\operatorname{arccoth}(a*x)/(a^2*c+d)/c/ \\
& (a^4*c^2+2*a^2*c*d+d^2)*(-a^2*c*d)^{(1/2)}-1/4*a^5/d/(a^2*c+d)/(a^4*c^2+2*a^2 \\
& *c*d+d^2)*\ln(1-(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^{(1/2)}-d))*\operatorname{arcc} \\
& \operatorname{oth}(a*x)*(-a^2*c*d)^{(1/2)}*c-1/4/a/c^2*d^2/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2) \\
& *\ln(1-(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^{(1/2)}-d))*\operatorname{arccoth}(a*x)* \\
& (-a^2*c*d)^{(1/2)}+1/4*a^2*(c*d)^{(1/2)}/c/d/(a^2*c+d)*\arctan(a/d*(c*d)^{(1/2)})- \\
& 1/4*a*(-a^2*c*d)^{(1/2)}/c/d/(a^2*c+d)*\operatorname{arccoth}(a*x)^2+1/8*a*(-a^2*c*d)^{(1/2)}/ \\
& c/d/(a^2*c+d)*\operatorname{polylog}(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c+2*(-a^2*c*d)^{(1/2)} \\
& -d))+1/4/a*(-a^2*c*d)^{(1/2)}/c^2/(a^2*c+d)*\operatorname{arccoth}(a*x)*\ln(1-(a^2*c+d)*(a*x+ \\
& 1)/(a*x-1)/(a^2*c+2*(-a^2*c*d)^{(1/2)}-d))-3/4*a^3/(a^2*c+d)/(a^4*c^2+2*a^2*c \\
& *d+d^2)*\ln(1-(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^{(1/2)}-d))*\operatorname{arccot} \\
& \operatorname{h}(a*x)*(-a^2*c*d)^{(1/2)}+1/4*a^2*(c*d)^{(1/2)}/c/d/(a^2*c+d)*\arctan(1/(a^2*c+d \\
& )*d^2/(c*d)^{(1/2)}*x+1/(a^2*c+d)*d/(c*d)^{(1/2)}*a*c+a^2/(a^2*c+d)*(c*d)^{(1/2)} \\
& *x-a/(a^2*c+d)*(c*d)^{(1/2)})-1/2*a/c/(a^2*c+d)^2*d*\ln((a*x-1)/(a*x+1))+1/2*a \\
& ^4*\operatorname{arccoth}(a*x)/(a^2*c+d)/(a^2*d*x^2+a^2*c)*x+1/8/a*(-a^2*c*d)^{(1/2)}/c^2/(a \\
& ^2*c+d)*\operatorname{polylog}(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c+2*(-a^2*c*d)^{(1/2)}-d))-1 \\
& /4/a*(-a^2*c*d)^{(1/2)}/c^2/(a^2*c+d)*\operatorname{arccoth}(a*x)^2-1/4*a/c/(a^2*c+d)^2*d*\ln \\
& (a^2*c/(a*x-1)^2*(a*x+1)^2-2*a^2*c*(a*x+1)/(a*x-1)+d/(a*x-1)^2*(a*x+1)^2+a^ \\
& 2*c+2*(a*x+1)/(a*x-1)*d+d)+3/4*a^3/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*\operatorname{arccot} \\
& \operatorname{h}(a*x)^2*(-a^2*c*d)^{(1/2)}-3/8*a^3/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*\operatorname{polylog} \\
& (2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^{(1/2)}-d))*(-a^2*c*d)^{(1/2)} \\
& +1/4*(c*d)^{(1/2)}/c^2*d/(a^2*c+d)^2*\arctan(1/(a^2*c+d)*d^2/(c*d)^{(1/2)}*x+1/( \\
& a^2*c+d)*d/(c*d)^{(1/2)}*a*c+a^2/(a^2*c+d)*(c*d)^{(1/2)}*x-a/(a^2*c+d)*(c*d)^{(1 \\
& /2)})-1/4*(c*d)^{(1/2)}/c^2/(a^2*c+d)*\arctan(1/(a^2*c+d)*d^2/(c*d)^{(1/2)}*x+1/( \\
& a^2*c+d)*d/(c*d)^{(1/2)}*a*c+a^2/(a^2*c+d)*(c*d)^{(1/2)}*x-a/(a^2*c+d)*(c*d)^{(1 \\
& /2)})-1/2*a^3*\operatorname{arccoth}(a*x)/(a^2*c+d)/(a^2*d*x^2+a^2*c)-1/4*(c*d)^{(1/2)}/c^2/( \\
& a^2*c+d)*\arctan(a/d*(c*d)^{(1/2)})-1/4*a^4*(c*d)^{(1/2)}/d/(a^2*c+d)^2*\arctan(1 \\
& /((a^2*c+d)*d^2/(c*d)^{(1/2)}*x+1/(a^2*c+d)*d/(c*d)^{(1/2)}*a*c+a^2/(a^2*c+d)*(c \\
& *d)^{(1/2)}*x-a/(a^2*c+d)*(c*d)^{(1/2)}))-3/8*a/c*d/(a^2*c+d)/(a^4*c^2+2*a^2*c*d \\
& +d^2)*\operatorname{polylog}(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^{(1/2)}-d))*(-a \\
& ^2*c*d)^{(1/2)}-1/8/a/c^2*d^2/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*\operatorname{polylog}(2,(a^ \\
& 2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^{(1/2)}-d))*(-a^2*c*d)^{(1/2)}+1/4/a \\
& /c^2*d^2/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*\operatorname{arccoth}(a*x)^2*(-a^2*c*d)^{(1/2)}+ \\
& 3/4*a/c*d/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*\operatorname{arccoth}(a*x)^2*(-a^2*c*d)^{(1/2)} \\
& -1/2*a^3*\operatorname{arccoth}(a*x)/c/(a^2*c+d)/(a^2*d*x^2+a^2*c)*d*x^2+1/2*a^2*\operatorname{arccoth}(a \\
& *x)/c/(a^2*c+d)/(a^2*d*x^2+a^2*c)*x*d+1/4*a*(-a^2*c*d)^{(1/2)}/c/d/(a^2*c+d)* \\
& \operatorname{arccoth}(a*x)*\ln(1-(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c+2*(-a^2*c*d)^{(1/2)}-d))-1 \\
& /8*a^5/d/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*\operatorname{polylog}(2,(a^2*c+d)*(a*x+1)/(a*x \\
& -1)/(a^2*c-2*(-a^2*c*d)^{(1/2)}-d))*(-a^2*c*d)^{(1/2)}*c+1/4*a^5/d/(a^2*c+d)/(a \\
& ^4*c^2+2*a^2*c*d+d^2)*\operatorname{arccoth}(a*x)^2*(-a^2*c*d)^{(1/2)}*c
\end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arccoth}(ax)}{d^2x^4 + 2cdx^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c)^2,x, algorithm="fricas")

[Out] integral(arccoth(a\*x)/(d^2\*x^4 + 2\*c\*d\*x^2 + c^2), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(a\*x)/(d\*x\*\*2+c)\*\*2,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arccoth}(ax)}{(dx^2 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)/(d*x^2 + c)^2, x)
```

$$3.41 \quad \int \frac{\coth^{-1}(ax)}{(c+dx^2)^3} dx$$

**Optimal.** Leaf size=657

$$\frac{3i\text{PolyLog}\left(2, \frac{a(\sqrt{c-i\sqrt{d}})}{a\sqrt{c-i\sqrt{d}}}\right)}{32c^{5/2}\sqrt{d}} - \frac{3i\text{PolyLog}\left(2, \frac{a(\sqrt{c-i\sqrt{d}})}{a\sqrt{c+i\sqrt{d}}}\right)}{32c^{5/2}\sqrt{d}} + \frac{3i\text{PolyLog}\left(2, \frac{a(\sqrt{c+i\sqrt{d}})}{a\sqrt{c-i\sqrt{d}}}\right)}{32c^{5/2}\sqrt{d}} - \frac{3i\text{PolyLog}\left(2, \frac{a(\sqrt{c+i\sqrt{d}})}{a\sqrt{c+i\sqrt{d}}}\right)}{32c^{5/2}\sqrt{d}} + \dots$$

[Out]  $a/(8*c*(a^2*c + d)*(c + d*x^2)) + (x*\text{ArcCoth}[a*x])/(4*c*(c + d*x^2)^2) + (3*x*\text{ArcCoth}[a*x])/(8*c^2*(c + d*x^2)) + (3*\text{ArcCoth}[a*x]*\text{ArcTan}[(\text{Sqrt}[d]*x)/\text{Sqrt}[c]])/(8*c^{5/2}*\text{Sqrt}[d]) + (((3*I)/32)*\text{Log}[(\text{Sqrt}[d]*(1 - a*x))/(I*a*\text{Sqrt}[c] + \text{Sqrt}[d])]*\text{Log}[1 - (I*\text{Sqrt}[d]*x)/\text{Sqrt}[c]])/(c^{5/2}*\text{Sqrt}[d]) - (((3*I)/32)*\text{Log}[-((\text{Sqrt}[d]*(1 + a*x))/(I*a*\text{Sqrt}[c] - \text{Sqrt}[d]))]*\text{Log}[1 - (I*\text{Sqrt}[d]*x)/\text{Sqrt}[c]])/(c^{5/2}*\text{Sqrt}[d]) - (((3*I)/32)*\text{Log}[-((\text{Sqrt}[d]*(1 - a*x))/(I*a*\text{Sqrt}[c] - \text{Sqrt}[d]))]*\text{Log}[1 + (I*\text{Sqrt}[d]*x)/\text{Sqrt}[c]])/(c^{5/2}*\text{Sqrt}[d]) + (((3*I)/32)*\text{Log}[(\text{Sqrt}[d]*(1 + a*x))/(I*a*\text{Sqrt}[c] + \text{Sqrt}[d])]*\text{Log}[1 + (I*\text{Sqrt}[d]*x)/\text{Sqrt}[c]])/(c^{5/2}*\text{Sqrt}[d]) + (a*(5*a^2*c + 3*d)*\text{Log}[1 - a^2*x^2])/(16*c^2*(a^2*c + d)^2) - (a*(5*a^2*c + 3*d)*\text{Log}[c + d*x^2])/(16*c^2*(a^2*c + d)^2) + (((3*I)/32)*\text{PolyLog}[2, (a*(\text{Sqrt}[c] - I*\text{Sqrt}[d]*x))/(a*\text{Sqrt}[c] - I*\text{Sqrt}[d])])/(c^{5/2}*\text{Sqrt}[d]) - (((3*I)/32)*\text{PolyLog}[2, (a*(\text{Sqrt}[c] - I*\text{Sqrt}[d]*x))/(a*\text{Sqrt}[c] + I*\text{Sqrt}[d])])/(c^{5/2}*\text{Sqrt}[d]) + (((3*I)/32)*\text{PolyLog}[2, (a*(\text{Sqrt}[c] + I*\text{Sqrt}[d]*x))/(a*\text{Sqrt}[c] - I*\text{Sqrt}[d])])/(c^{5/2}*\text{Sqrt}[d]) - (((3*I)/32)*\text{PolyLog}[2, (a*(\text{Sqrt}[c] + I*\text{Sqrt}[d]*x))/(a*\text{Sqrt}[c] + I*\text{Sqrt}[d])])/(c^{5/2}*\text{Sqrt}[d])$

**Rubi [A]** time = 0.950909, antiderivative size = 657, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 11, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$ , Rules used = {199, 205, 5977, 6725, 571, 77, 4908, 2409, 2394, 2393, 2391}

$$\frac{3i\text{PolyLog}\left(2, \frac{a(\sqrt{c-i\sqrt{d}})}{a\sqrt{c-i\sqrt{d}}}\right)}{32c^{5/2}\sqrt{d}} - \frac{3i\text{PolyLog}\left(2, \frac{a(\sqrt{c-i\sqrt{d}})}{a\sqrt{c+i\sqrt{d}}}\right)}{32c^{5/2}\sqrt{d}} + \frac{3i\text{PolyLog}\left(2, \frac{a(\sqrt{c+i\sqrt{d}})}{a\sqrt{c-i\sqrt{d}}}\right)}{32c^{5/2}\sqrt{d}} - \frac{3i\text{PolyLog}\left(2, \frac{a(\sqrt{c+i\sqrt{d}})}{a\sqrt{c+i\sqrt{d}}}\right)}{32c^{5/2}\sqrt{d}} + \dots$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{ArcCoth}[a*x]/(c + d*x^2)^3, x]$

[Out]  $a/(8*c*(a^2*c + d)*(c + d*x^2)) + (x*\text{ArcCoth}[a*x])/(4*c*(c + d*x^2)^2) + (3*x*\text{ArcCoth}[a*x])/(8*c^2*(c + d*x^2)) + (3*\text{ArcCoth}[a*x]*\text{ArcTan}[(\text{Sqrt}[d]*x)/\text{Sqrt}[c]])/(8*c^{5/2}*\text{Sqrt}[d]) + \dots$

```

qrt[c]]/(8*c^(5/2)*Sqrt[d]) + (((3*I)/32)*Log[(Sqrt[d]*(1 - a*x))/(I*a*Sqr
t[c] + Sqrt[d])]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]]/(c^(5/2)*Sqrt[d]) - (((3*I
)/32)*Log[-((Sqrt[d]*(1 + a*x))/(I*a*Sqrt[c] - Sqrt[d]))]*Log[1 - (I*Sqrt[d
]*x)/Sqrt[c]]/(c^(5/2)*Sqrt[d]) - (((3*I)/32)*Log[-((Sqrt[d]*(1 - a*x))/(I
*a*Sqrt[c] - Sqrt[d]))]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]]/(c^(5/2)*Sqrt[d]) +
(((3*I)/32)*Log[(Sqrt[d]*(1 + a*x))/(I*a*Sqrt[c] + Sqrt[d])]*Log[1 + (I*Sq
rt[d]*x)/Sqrt[c]]/(c^(5/2)*Sqrt[d]) + (a*(5*a^2*c + 3*d)*Log[1 - a^2*x^2]
)/(16*c^2*(a^2*c + d)^2) - (a*(5*a^2*c + 3*d)*Log[c + d*x^2])/(16*c^2*(a^2*c
+ d)^2) + (((3*I)/32)*PolyLog[2, (a*(Sqrt[c] - I*Sqrt[d]*x))/(a*Sqrt[c] -
I*Sqrt[d]))/(c^(5/2)*Sqrt[d]) - (((3*I)/32)*PolyLog[2, (a*(Sqrt[c] - I*Sqr
t[d]*x))/(a*Sqrt[c] + I*Sqrt[d]))/(c^(5/2)*Sqrt[d]) + (((3*I)/32)*PolyLog[
2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] - I*Sqrt[d]))/(c^(5/2)*Sqrt[d])
- (((3*I)/32)*PolyLog[2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] + I*Sqrt[d]
)))/(c^(5/2)*Sqrt[d])

```

### Rule 199

```

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(
p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])

```

### Rule 205

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

```

### Rule 5977

```

Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] := With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])

```

### Rule 6725

```

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]

```

### Rule 571

```

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x

```



$\int (c + dx)^q (e + fx)^r x^n dx$  ; FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && EqQ[m - n + 1, 0]

### Rule 77

$\int ((a + b x) (c + d x)^n (e + f x)^p) dx$  ; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

### Rule 4908

$\int \frac{\text{ArcTan}[c x]}{(d + e x^2)} dx$  ; FreeQ[{c, d, e}, x]

### Rule 2409

$\int ((a + \text{Log}[c(d + e x)^n])^p (f + g x^r)^q) dx$  ; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && IGtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))

### Rule 2394

$\int ((a + \text{Log}[c(d + e x)^n])^p (f + g x)^q) dx$  ; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rule 2393

$\int ((a + \text{Log}[c(d + e x)^n])^p (f + g x)) dx$  ; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

### Rule 2391

$\int \frac{\text{Log}[c(d + e x)^n]}{x} dx$  ; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)}{(c+dx^2)^3} dx &= \frac{x \coth^{-1}(ax)}{4c(c+dx^2)^2} + \frac{3x \coth^{-1}(ax)}{8c^2(c+dx^2)} + \frac{3 \coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}} - a \int \frac{\frac{x}{4c(c+dx^2)^2} + \frac{3x}{8c^2(c+dx^2)} + \frac{3 \tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}}}{1-a^2x^2} dx \\
&= \frac{x \coth^{-1}(ax)}{4c(c+dx^2)^2} + \frac{3x \coth^{-1}(ax)}{8c^2(c+dx^2)} + \frac{3 \coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}} - a \int \left( \frac{x(5c+3dx^2)}{8c^2(-1+a^2x^2)(c+dx^2)^2} - \frac{3 \tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}} \right) dx \\
&= \frac{x \coth^{-1}(ax)}{4c(c+dx^2)^2} + \frac{3x \coth^{-1}(ax)}{8c^2(c+dx^2)} + \frac{3 \coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}} + \frac{a \int \frac{x(5c+3dx^2)}{(-1+a^2x^2)(c+dx^2)^2} dx}{8c^2} + \frac{(3a) \int \frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{-1+a^2x^2} dx}{8c^{5/2}\sqrt{d}} \\
&= \frac{x \coth^{-1}(ax)}{4c(c+dx^2)^2} + \frac{3x \coth^{-1}(ax)}{8c^2(c+dx^2)} + \frac{3 \coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}} + \frac{a \text{ Subst} \left( \int \frac{5c+3dx}{(-1+a^2x)(c+dx)^2} dx, x, x^2 \right)}{16c^2} \\
&= \frac{x \coth^{-1}(ax)}{4c(c+dx^2)^2} + \frac{3x \coth^{-1}(ax)}{8c^2(c+dx^2)} + \frac{3 \coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}} + \frac{a \text{ Subst} \left( \int \left( \frac{a^2(5a^2c+3d)}{(a^2c+d)^2(-1+a^2x)} - \frac{2}{(a^2c+d)} \right) dx, x, x^2 \right)}{16c^2} \\
&= \frac{a}{8c(a^2c+d)(c+dx^2)} + \frac{x \coth^{-1}(ax)}{4c(c+dx^2)^2} + \frac{3x \coth^{-1}(ax)}{8c^2(c+dx^2)} + \frac{3 \coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}} + \frac{a(5a^2c+3d)}{16c^2} \\
&= \frac{a}{8c(a^2c+d)(c+dx^2)} + \frac{x \coth^{-1}(ax)}{4c(c+dx^2)^2} + \frac{3x \coth^{-1}(ax)}{8c^2(c+dx^2)} + \frac{3 \coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}} + \frac{3i \log\left(\frac{\sqrt{d}(1+ia\sqrt{c})}{ia\sqrt{c}}\right)}{3} \\
&= \frac{a}{8c(a^2c+d)(c+dx^2)} + \frac{x \coth^{-1}(ax)}{4c(c+dx^2)^2} + \frac{3x \coth^{-1}(ax)}{8c^2(c+dx^2)} + \frac{3 \coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}} + \frac{3i \log\left(\frac{\sqrt{d}(1+ia\sqrt{c})}{ia\sqrt{c}}\right)}{3} \\
&= \frac{a}{8c(a^2c+d)(c+dx^2)} + \frac{x \coth^{-1}(ax)}{4c(c+dx^2)^2} + \frac{3x \coth^{-1}(ax)}{8c^2(c+dx^2)} + \frac{3 \coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}} + \frac{3i \log\left(\frac{\sqrt{d}(1+ia\sqrt{c})}{ia\sqrt{c}}\right)}{3}
\end{aligned}$$

**Mathematica [B]** time = 12.88, size = 1838, normalized size = 2.8

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a\*x]/(c + d\*x^2)^3,x]

[Out]  $a^5 \left( \frac{-5 \log\left[1 + \frac{(a^2 c + d) \cosh[2 \operatorname{ArcCoth}[a x]]}{-(a^2 c) + d}\right]}{(16 a^2 c (a^2 c + d)^2 - (3 d \log\left[1 + \frac{(a^2 c + d) \cosh[2 \operatorname{ArcCoth}[a x]]}{-(a^2 c) + d}\right]) / (16 a^4 c^2 (a^2 c + d)^2 + (3 ((-2 I) \operatorname{ArcCos}\left[-\frac{-(a^2 c) + d}{(a^2 c + d)}\right]) \operatorname{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d}} x\right] + 4 \operatorname{ArcCoth}[a x] \operatorname{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right] - (\operatorname{ArcCos}\left[-\frac{-(a^2 c) + d}{(a^2 c + d)}\right] - 2 \operatorname{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d}} x\right]) \log\left[1 - \frac{-(a^2 c) + d - (2 I) \sqrt{a^2 c d}}{(2 I) \sqrt{a^2 c d}} (a x)\right]}{(a^2 c + d) (2 d + \frac{(2 I) \sqrt{a^2 c d}}{a x})} + (-\operatorname{ArcCos}\left[-\frac{-(a^2 c) + d}{(a^2 c + d)}\right] - 2 \operatorname{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d}} x\right]) \log\left[1 - \frac{-(a^2 c) + d + (2 I) \sqrt{a^2 c d}}{(2 I) \sqrt{a^2 c d}} (a x)\right]}{(a^2 c + d) (2 d + \frac{(2 I) \sqrt{a^2 c d}}{a x})} + (\operatorname{ArcCos}\left[-\frac{-(a^2 c) + d}{(a^2 c + d)}\right] + (2 I) ((-I) \operatorname{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d}} x\right] - I \operatorname{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right]) \log\left[\frac{\sqrt{2} \sqrt{a^2 c d}}{\sqrt{a^2 c + d}} E^{\operatorname{ArcCoth}[a x]} \sqrt{-(a^2 c) + d + (a^2 c + d) \cosh[2 \operatorname{ArcCoth}[a x]]}\right]} + (\operatorname{ArcCos}\left[-\frac{-(a^2 c) + d}{(a^2 c + d)}\right] - (2 I) ((-I) \operatorname{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d}} x\right] - I \operatorname{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right]) \log\left[\frac{\sqrt{2} \sqrt{a^2 c d}}{\sqrt{a^2 c + d}} E^{\operatorname{ArcCoth}[a x]} \sqrt{-(a^2 c) + d + (a^2 c + d) \cosh[2 \operatorname{ArcCoth}[a x]]}\right]} + I (\operatorname{PolyLog}\left[2, \frac{-(a^2 c) + d - (2 I) \sqrt{a^2 c d}}{(2 d - \frac{(2 I) \sqrt{a^2 c d}}{a x})} / ((a^2 c + d) (2 d + \frac{(2 I) \sqrt{a^2 c d}}{a x}))\right] - \operatorname{PolyLog}\left[2, \frac{-(a^2 c) + d + (2 I) \sqrt{a^2 c d}}{(2 d - \frac{(2 I) \sqrt{a^2 c d}}{a x})} / ((a^2 c + d) (2 d + \frac{(2 I) \sqrt{a^2 c d}}{a x}))\right]) \right) / (32 a^2 c \sqrt{a^2 c d} (a^2 c + d)) + (3 d ((-2 I) \operatorname{ArcCos}\left[-\frac{-(a^2 c) + d}{(a^2 c + d)}\right]) \operatorname{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d}} x\right] + 4 \operatorname{ArcCoth}[a x] \operatorname{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right] - (\operatorname{ArcCos}\left[-\frac{-(a^2 c) + d}{(a^2 c + d)}\right] - 2 \operatorname{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d}} x\right]) \log\left[1 - \frac{-(a^2 c) + d - (2 I) \sqrt{a^2 c d}}{(2 I) \sqrt{a^2 c d}} (a x)\right]}{(a^2 c + d) (2 d + \frac{(2 I) \sqrt{a^2 c d}}{a x})} + (-\operatorname{ArcCos}\left[-\frac{-(a^2 c) + d}{(a^2 c + d)}\right] - 2 \operatorname{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d}} x\right]) \log\left[1 - \frac{-(a^2 c) + d + (2 I) \sqrt{a^2 c d}}{(2 I) \sqrt{a^2 c d}} (a x)\right]}{(a^2 c + d) (2 d + \frac{(2 I) \sqrt{a^2 c d}}{a x})} + (\operatorname{ArcCos}\left[-\frac{-(a^2 c) + d}{(a^2 c + d)}\right] + (2 I) ((-I) \operatorname{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d}} x\right] - I \operatorname{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right]) \log\left[\frac{\sqrt{2} \sqrt{a^2 c d}}{\sqrt{a^2 c + d}} E^{\operatorname{ArcCoth}[a x]} \sqrt{-(a^2 c) + d + (a^2 c + d) \cosh[2 \operatorname{ArcCoth}[a x]]}\right]} + (\operatorname{ArcCos}\left[-\frac{-(a^2 c) + d}{(a^2 c + d)}\right] - (2 I) ((-I) \operatorname{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d}} x\right] - I \operatorname{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right]) \log\left[\frac{\sqrt{2} \sqrt{a^2 c d}}{\sqrt{a^2 c + d}} E^{\operatorname{ArcCoth}[a x]} \sqrt{-(a^2 c) + d + (a^2 c + d) \cosh[2 \operatorname{ArcCoth}[a x]]}\right]} + I (\operatorname{PolyLog}\left[2, \frac{-(a^2 c) + d - (2 I) \sqrt{a^2 c d}}{(2 d - \frac{(2 I) \sqrt{a^2 c d}}{a x})} / ((a^2 c + d) (2 d + \frac{(2 I) \sqrt{a^2 c d}}{a x}))\right] - \operatorname{PolyLog}\left[2, \frac{-(a^2 c) + d + (2 I) \sqrt{a^2 c d}}{(2 d - \frac{(2 I) \sqrt{a^2 c d}}{a x})} / ((a^2 c + d) (2 d + \frac{(2 I) \sqrt{a^2 c d}}{a x}))\right]) \right)$

$$a^2cd)/(ax)) - \text{PolyLog}[2, ((-a^2c) + d + (2I)\sqrt{a^2cd})*(2d - ((2I)\sqrt{a^2cd}))/ax)) / ((a^2c + d)*(2d + ((2I)\sqrt{a^2cd}))/ax)))] / (32a^4c^2\sqrt{a^2cd}*(a^2c + d)) - (d\text{ArcCoth}[ax]*\text{Sinh}[2\text{ArcCoth}[ax]]) / (2a^2c*(a^2c + d)*(-a^2c) + d + a^2c*\text{Cosh}[2\text{ArcCoth}[ax]] + d*\text{Cosh}[2\text{ArcCoth}[ax]])^2) - (2a^2cd - 5a^4c^2\text{ArcCoth}[ax]*\text{Sinh}[2\text{ArcCoth}[ax]] - 8a^2cd*\text{ArcCoth}[ax]*\text{Sinh}[2\text{ArcCoth}[ax]] - 3d^2\text{ArcCoth}[ax]*\text{Sinh}[2\text{ArcCoth}[ax]]) / (8a^4c^2*(a^2c + d)^2*(-a^2c) + d + a^2c*\text{Cosh}[2\text{ArcCoth}[ax]] + d*\text{Cosh}[2\text{ArcCoth}[ax]]))$$

**Maple [B]** time = 0.596, size = 4128, normalized size = 6.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(a*x)/(d*x^2+c)^3,x)`

[Out]  $1/8a^5/(a^4c^2+2a^2cd+d^2)/(a^2dx^2+a^2c)^2d-5/8a^5/(a^4c^2+2a^2cd+d^2)/(a^2c+d)*\ln((a*x-1)/(a*x+1))-5/16a^5/(a^4c^2+2a^2cd+d^2)/(a^2c+d)*\ln(a^2c/(a*x-1)^2*(a*x+1)^2-2a^2c*(a*x+1)/(a*x-1)+d/(a*x-1)^2*(a*x+1)^2+a^2c+2*(a*x+1)/(a*x-1)*d+d)+3/4a^5*\text{arccoth}(a*x)^2/(a^4c^2+2a^2cd+d^2)^2*(-a^2cd)^{1/2}-3/8a^5*\text{polylog}(2,(a^2c+d)*(a*x+1)/(a*x-1)/(a^2c-2*(-a^2cd)^{1/2}-d))/(a^4c^2+2a^2cd+d^2)^2*(-a^2cd)^{1/2}-1/8a^2*(cd)^{1/2}/c^2/(a^4c^2+2a^2cd+d^2)*\arctan(a/d*(cd)^{1/2})-3/16*(cd)^{1/2}/c^3*d/(a^4c^2+2a^2cd+d^2)*\arctan(a/d*(cd)^{1/2})-5/8a^7/(a^4c^2+2a^2cd+d^2)/(a^2dx^2+a^2c)^2c*\text{arccoth}(a*x)-9/8a^3/c/(a^4c^2+2a^2cd+d^2)^2*\ln(1-(a^2c+d)*(a*x+1)/(a*x-1)/(a^2c-2*(-a^2cd)^{1/2}-d))*\text{arccoth}(a*x)*(-a^2cd)^{1/2}*d-1/8a^2*(cd)^{1/2}/c^2/(a^4c^2+2a^2cd+d^2)*\arctan(1/(a^2c+d)*d^2/(cd)^{1/2}*x+1/(a^2c+d)*d/(cd)^{1/2}*a*c+a^2/(a^2c+d)*(cd)^{1/2}*x-a/(a^2c+d)*(cd)^{1/2})-3/8a*(-a^2cd)^{1/2}/c^2/(a^4c^2+2a^2cd+d^2)*\text{arccoth}(a*x)^2-3/8a^5/(a^4c^2+2a^2cd+d^2)/(a^2dx^2+a^2c)^2d*\text{arccoth}(a*x)+3/16a*(-a^2cd)^{1/2}/c^2/(a^4c^2+2a^2cd+d^2)*\text{polylog}(2,(a^2c+d)*(a*x+1)/(a*x-1)/(a^2c+2*(-a^2cd)^{1/2}-d))-3/4a^5*\ln(1-(a^2c+d)*(a*x+1)/(a*x-1)/(a^2c-2*(-a^2cd)^{1/2}-d))*\text{arccoth}(a*x)/(a^4c^2+2a^2cd+d^2)^2*(-a^2cd)^{1/2}-3/16*(cd)^{1/2}/c^3*d/(a^4c^2+2a^2cd+d^2)*\arctan(1/(a^2c+d)*d^2/(cd)^{1/2}*x+1/(a^2c+d)*d/(cd)^{1/2}*a*c+a^2/(a^2c+d)*(cd)^{1/2}*x-a/(a^2c+d)*(cd)^{1/2})-1/8a^7/(a^4c^2+2a^2cd+d^2)/(a^2dx^2+a^2c)^2d*x^2-5/8a^7/(a^4c^2+2a^2cd+d^2)/(a^2dx^2+a^2c)^2/c*\text{arccoth}(a*x)*x^4*d^2-3/4a/c^2*d^2/(a^4c^2+2a^2cd+d^2)^2*\ln(1-(a^2c+d)*(a*x+1)/(a*x-1)/(a^2c-2*(-a^2cd)^{1/2}-d))*\text{arccoth}(a*x)*(-a^2cd)^{1/2}-3/16a^7/d/(a^4c^2+2a^2cd+d^2)^2*\ln(1-(a^2c+d)*(a*x+1)/(a*x-1)/(a^2c-2*(-a^2cd)^{1/2}-d))*\text{arccoth}(a*x)*(-a^2cd)^{1/2}$

$$\begin{aligned}
& 2*c*d)^{(1/2)}*c+3/8*a^8/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2*\operatorname{arccoth}( \\
& a*x)*x^3*d-5/4*a^7/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2*\operatorname{arccoth}(a*x) \\
& *x^2*d+5/4*a^6/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2*\operatorname{arccoth}(a*x)*x*d \\
& +3/16*(c*d)^{(1/2)}/c^3*d^2/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*\operatorname{arctan}(1/(a^2*c \\
& +d)*d^2/(c*d)^{(1/2)}*x+1/(a^2*c+d)*d/(c*d)^{(1/2)}*a*c+a^2/(a^2*c+d)*(c*d)^{(1/ \\
& 2)}*x-a/(a^2*c+d)*(c*d)^{(1/2)})-5/16*a^6*(c*d)^{(1/2)}/d/(a^2*c+d)/(a^4*c^2+2*a \\
& ^2*c*d+d^2)*\operatorname{arctan}(a/d*(c*d)^{(1/2)})+5/16*a^4*(c*d)^{(1/2)}/c/d/(a^4*c^2+2*a^2 \\
& *c*d+d^2)*\operatorname{arctan}(a/d*(c*d)^{(1/2)})-3/16*a^4*(c*d)^{(1/2)}/c/(a^2*c+d)/(a^4*c^2 \\
& +2*a^2*c*d+d^2)*\operatorname{arctan}(a/d*(c*d)^{(1/2)})+3/16*(c*d)^{(1/2)}/c^3*d^2/(a^2*c+d)/ \\
& (a^4*c^2+2*a^2*c*d+d^2)*\operatorname{arctan}(a/d*(c*d)^{(1/2)})-1/8*a^7/(a^4*c^2+2*a^2*c*d+ \\
& d^2)/(a^2*d*x^2+a^2*c)^2/c*d^2*x^4+1/8*a^5/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x \\
& ^2+a^2*c)^2/c*d^2*x^2+9/8*a^3/c*d/(a^4*c^2+2*a^2*c*d+d^2)^2*\operatorname{arccoth}(a*x)^2* \\
& (-a^2*c*d)^{(1/2)}-9/16*a^3/c*d/(a^4*c^2+2*a^2*c*d+d^2)^2*\operatorname{polylog}(2,(a^2*c+d) \\
& *(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^{(1/2)}-d))*(-a^2*c*d)^{(1/2)}-3/8*a/c^2*d \\
& ^2/(a^4*c^2+2*a^2*c*d+d^2)^2*\operatorname{polylog}(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*( \\
& -a^2*c*d)^{(1/2)}-d))*(-a^2*c*d)^{(1/2)}-3/16/a*(-a^2*c*d)^{(1/2)}/c^3*d/(a^4*c^2 \\
& +2*a^2*c*d+d^2)*\operatorname{arccoth}(a*x)^2-a^3/(a^4*c^2+2*a^2*c*d+d^2)/c*d/(a^2*c+d)*\ln \\
& ((a*x-1)/(a*x+1))+3/32*a^3*(-a^2*c*d)^{(1/2)}/c/d/(a^4*c^2+2*a^2*c*d+d^2)*\operatorname{pol} \\
& \operatorname{ylog}(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c+2*(-a^2*c*d)^{(1/2)}-d))-3/32*a^7/d/( \\
& a^4*c^2+2*a^2*c*d+d^2)^2*\operatorname{polylog}(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2 \\
& *c*d)^{(1/2)}-d))*(-a^2*c*d)^{(1/2)}*c+3/16/a/c^3*d^3/(a^4*c^2+2*a^2*c*d+d^2)^2 \\
& *\operatorname{arccoth}(a*x)^2*(-a^2*c*d)^{(1/2)}-5/16*a^6*(c*d)^{(1/2)}/d/(a^2*c+d)/(a^4*c^2+ \\
& 2*a^2*c*d+d^2)*\operatorname{arctan}(1/(a^2*c+d)*d^2/(c*d)^{(1/2)}*x+1/(a^2*c+d)*d/(c*d)^{(1/ \\
& 2)}*a*c+a^2/(a^2*c+d)*(c*d)^{(1/2)}*x-a/(a^2*c+d)*(c*d)^{(1/2)})+3/8*a*(-a^2*c*d \\
& )^2/(c^2/(a^4*c^2+2*a^2*c*d+d^2)*\operatorname{arccoth}(a*x)*\ln(1-(a^2*c+d)*(a*x+1)/(a* \\
& x-1)/(a^2*c+2*(-a^2*c*d)^{(1/2)}-d))-3/32/a/c^3*d^3/(a^4*c^2+2*a^2*c*d+d^2)^2 \\
& *\operatorname{polylog}(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^{(1/2)}-d))*(-a^2*c* \\
& d)^{(1/2)}-3/16*a^3*(-a^2*c*d)^{(1/2)}/c/d/(a^4*c^2+2*a^2*c*d+d^2)*\operatorname{arccoth}(a*x) \\
& ^2-3/8*a/(a^4*c^2+2*a^2*c*d+d^2)/c^2*d^2/(a^2*c+d)*\ln((a*x-1)/(a*x+1))-1/2* \\
& a^3/(a^4*c^2+2*a^2*c*d+d^2)/c*d/(a^2*c+d)*\ln(a^2*c/(a*x-1)^2*(a*x+1)^2-2*a^ \\
& 2*c*(a*x+1)/(a*x-1)+d/(a*x-1)^2*(a*x+1)^2+a^2*c+2*(a*x+1)/(a*x-1)*d+d)-3/16 \\
& *a/(a^4*c^2+2*a^2*c*d+d^2)/c^2*d^2/(a^2*c+d)*\ln(a^2*c/(a*x-1)^2*(a*x+1)^2-2 \\
& *a^2*c*(a*x+1)/(a*x-1)+d/(a*x-1)^2*(a*x+1)^2+a^2*c+2*(a*x+1)/(a*x-1)*d+d)+5 \\
& /8*a^8/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2*c*\operatorname{arccoth}(a*x)*x+3/32/a* \\
& (-a^2*c*d)^{(1/2)}/c^3*d/(a^4*c^2+2*a^2*c*d+d^2)*\operatorname{polylog}(2,(a^2*c+d)*(a*x+1)/ \\
& (a*x-1)/(a^2*c+2*(-a^2*c*d)^{(1/2)}-d))+3/4*a*d^2*\operatorname{arccoth}(a*x)^2/c^2/(a^4*c^2 \\
& +2*a^2*c*d+d^2)^2*(-a^2*c*d)^{(1/2)}+3/16*a^7/d/(a^4*c^2+2*a^2*c*d+d^2)^2*\operatorname{arc} \\
& \operatorname{coth}(a*x)^2*(-a^2*c*d)^{(1/2)}*c-3/16*a^4*(c*d)^{(1/2)}/c/(a^2*c+d)/(a^4*c^2+2* \\
& a^2*c*d+d^2)*\operatorname{arctan}(1/(a^2*c+d)*d^2/(c*d)^{(1/2)}*x+1/(a^2*c+d)*d/(c*d)^{(1/2) \\
& }*a*c+a^2/(a^2*c+d)*(c*d)^{(1/2)}*x-a/(a^2*c+d)*(c*d)^{(1/2)})+5/16*a^4*(c*d)^{(1 \\
& /2)}/c/d/(a^4*c^2+2*a^2*c*d+d^2)*\operatorname{arctan}(1/(a^2*c+d)*d^2/(c*d)^{(1/2)}*x+1/(a^2 \\
& *c+d)*d/(c*d)^{(1/2)}*a*c+a^2/(a^2*c+d)*(c*d)^{(1/2)}*x-a/(a^2*c+d)*(c*d)^{(1/2) \\
& )+3/4*a^6/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2/c*\operatorname{arccoth}(a*x)*x^3*d^ \\
& 2+3/16/a*(-a^2*c*d)^{(1/2)}/c^3*d/(a^4*c^2+2*a^2*c*d+d^2)*\operatorname{arccoth}(a*x)*\ln(1-( \\
& a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c+2*(-a^2*c*d)^{(1/2)}-d))-3/16/a/c^3*d^3/(a^4*
\end{aligned}$$

$$c^2+2a^2cd+d^2)^2 \ln(1-(a^2c+d)(ax+1)/(ax-1)/(a^2c-2(-a^2cd)^{(1/2)}-d)) \operatorname{arccoth}(ax) (-a^2cd)^{(1/2)} + 3/8 a^4 / (a^4c^2+2a^2cd+d^2) / (a^2dx^2+a^2c)^2 / c^2 \operatorname{arccoth}(ax) x^3 d^3 + 3/16 a^3 (-a^2cd)^{(1/2)} / c / d / (a^4c^2+2a^2cd+d^2) \operatorname{arccoth}(ax) \ln(1-(a^2c+d)(ax+1)/(ax-1)/(a^2c+2(-a^2cd)^{(1/2)}-d)) - 3/4 a^5 / (a^4c^2+2a^2cd+d^2) / (a^2dx^2+a^2c)^2 / c \operatorname{arccoth}(ax) x^2 d^2 + 5/8 a^4 / (a^4c^2+2a^2cd+d^2) / (a^2dx^2+a^2c)^2 / c \operatorname{arccoth}(ax) x d^2 + 5/16 a^2 (cd)^{(1/2)} / c^2 d / (a^2c+d) / (a^4c^2+2a^2cd+d^2) \operatorname{arctan}(a/d (cd)^{(1/2)}) - 3/8 a^5 / (a^4c^2+2a^2cd+d^2) / (a^2dx^2+a^2c)^2 / c^2 \operatorname{arccoth}(ax) x^4 d^3 + 5/16 a^2 (cd)^{(1/2)} / c^2 d / (a^2c+d) / (a^4c^2+2a^2cd+d^2) \operatorname{arctan}(1/(a^2c+d) d^2 / (cd)^{(1/2)} x + 1/(a^2c+d) d / (cd)^{(1/2)}) a^2 c + a^2 / (a^2c+d) (cd)^{(1/2)} x - a / (a^2c+d) (cd)^{(1/2)}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(ax)/(d\*x^2+c)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arccoth}(ax)}{d^3x^6 + 3cd^2x^4 + 3c^2dx^2 + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(ax)/(d\*x^2+c)^3,x, algorithm="fricas")

[Out] integral(arccoth(ax)/(d^3\*x^6 + 3\*c\*d^2\*x^4 + 3\*c^2\*d\*x^2 + c^3), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoath(a*x)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(ax)}{(dx^2 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoath(a*x)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(arccoath(a*x)/(d*x^2 + c)^3, x)
```



$$3.42 \quad \int \sqrt{c + dx^2} \coth^{-1}(ax) dx$$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\coth^{-1}(ax)\sqrt{c + dx^2}, x\right)$$

[Out] Unintegrable[Sqrt[c + d\*x^2]\*ArcCoth[a\*x], x]

**Rubi [A]** time = 0.0208016, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \sqrt{c + dx^2} \coth^{-1}(ax) dx$$

Verification is Not applicable to the result.

[In] Int[Sqrt[c + d\*x^2]\*ArcCoth[a\*x], x]

[Out] Defer[Int][Sqrt[c + d\*x^2]\*ArcCoth[a\*x], x]

Rubi steps

$$\int \sqrt{c + dx^2} \coth^{-1}(ax) dx = \int \sqrt{c + dx^2} \coth^{-1}(ax) dx$$

**Mathematica [A]** time = 24.1269, size = 0, normalized size = 0.

$$\int \sqrt{c + dx^2} \coth^{-1}(ax) dx$$

Verification is Not applicable to the result.

[In] Integrate[Sqrt[c + d\*x^2]\*ArcCoth[a\*x], x]

[Out] Integrate[Sqrt[c + d\*x^2]\*ArcCoth[a\*x], x]

**Maple [A]** time = 0.713, size = 0, normalized size = 0.

$$\int \sqrt{dx^2 + c} \operatorname{arccoth}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^2+c)^(1/2)*arccoth(a*x),x)`

[Out] `int((d*x^2+c)^(1/2)*arccoth(a*x),x)`

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^2+c)^(1/2)*arccoth(a*x),x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\sqrt{dx^2 + c} \operatorname{arccoth}(ax), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^2+c)^(1/2)*arccoth(a*x),x, algorithm="fricas")`

[Out] `integral(sqrt(d*x^2 + c)*arccoth(a*x), x)`

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c + dx^2} \operatorname{acoth}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*acoth(a*x),x)
```

```
[Out] Integral(sqrt(c + d*x**2)*acoth(a*x), x)
```

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx^2 + c} \operatorname{arccoth}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*arccoth(a*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)*arccoth(a*x), x)
```

$$3.43 \quad \int \frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}} dx$$

**Optimal.** Leaf size=18

$$\text{Unintegrable}\left(\frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}}, x\right)$$

[Out] Unintegrable[ArcCoth[a\*x]/Sqrt[c + d\*x^2], x]

**Rubi [A]** time = 0.0234482, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}} dx$$

Verification is Not applicable to the result.

[In] Int[ArcCoth[a\*x]/Sqrt[c + d\*x^2], x]

[Out] Defer[Int][ArcCoth[a\*x]/Sqrt[c + d\*x^2], x]

Rubi steps

$$\int \frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}} dx = \int \frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}} dx$$

**Mathematica [A]** time = 4.35927, size = 0, normalized size = 0.

$$\int \frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcCoth[a\*x]/Sqrt[c + d\*x^2], x]

[Out] Integrate[ArcCoth[a\*x]/Sqrt[c + d\*x^2], x]

---

**Maple [A]** time = 0.724, size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(ax) \frac{1}{\sqrt{dx^2 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)/(d\*x^2+c)^(1/2), x)

[Out] int(arccoth(a\*x)/(d\*x^2+c)^(1/2), x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c)^(1/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arccoth}(ax)}{\sqrt{dx^2 + c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c)^(1/2), x, algorithm="fricas")

[Out] integral(arccoth(a\*x)/sqrt(d\*x^2 + c), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}(ax)}{\sqrt{c+dx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(a\*x)/(d\*x\*\*2+c)\*\*(1/2),x)

[Out] Integral(acoth(a\*x)/sqrt(c + d\*x\*\*2), x)

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(ax)}{\sqrt{dx^2+c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c)^(1/2),x, algorithm="giac")

[Out] integrate(arccoth(a\*x)/sqrt(d\*x^2 + c), x)

$$3.44 \quad \int \frac{\coth^{-1}(ax)}{(c+dx^2)^{3/2}} dx$$

**Optimal.** Leaf size=62

$$\frac{x \coth^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{c\sqrt{a^2c+d}}$$

[Out] (x\*ArcCoth[a\*x])/(c\*Sqrt[c + d\*x^2]) - ArcTanh[(a\*Sqrt[c + d\*x^2])/Sqrt[a^2\*c + d]]/(c\*Sqrt[a^2\*c + d])

**Rubi [A]** time = 0.112951, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {191, 5977, 12, 444, 63, 208}

$$\frac{x \coth^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{c\sqrt{a^2c+d}}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]/(c + d\*x^2)^(3/2), x]

[Out] (x\*ArcCoth[a\*x])/(c\*Sqrt[c + d\*x^2]) - ArcTanh[(a\*Sqrt[c + d\*x^2])/Sqrt[a^2\*c + d]]/(c\*Sqrt[a^2\*c + d])

### Rule 191

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

### Rule 5977

Int[((a\_) + ArcCoth[(c\_)\*(x\_)]\*(b\_))\*((d\_) + (e\_)\*(x\_)^2)^(q\_), x\_Symbol] := With[{u = IntHide[(d + e\*x^2)^q, x]}, Dist[a + b\*ArcCoth[c\*x], u, x] - Dist[b\*c, Int[u/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 444

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m - n + 1, 0]

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rubi steps

$$\begin{aligned}
 \int \frac{\coth^{-1}(ax)}{(c + dx^2)^{3/2}} dx &= \frac{x \coth^{-1}(ax)}{c\sqrt{c + dx^2}} - a \int \frac{x}{c(1 - a^2x^2)\sqrt{c + dx^2}} dx \\
 &= \frac{x \coth^{-1}(ax)}{c\sqrt{c + dx^2}} - \frac{a \int \frac{x}{(1 - a^2x^2)\sqrt{c + dx^2}} dx}{c} \\
 &= \frac{x \coth^{-1}(ax)}{c\sqrt{c + dx^2}} - \frac{a \operatorname{Subst}\left(\int \frac{1}{(1 - a^2x)\sqrt{c + dx}} dx, x, x^2\right)}{2c} \\
 &= \frac{x \coth^{-1}(ax)}{c\sqrt{c + dx^2}} - \frac{a \operatorname{Subst}\left(\int \frac{1}{1 + \frac{a^2c}{a} - \frac{a^2x^2}{a}} dx, x, \sqrt{c + dx^2}\right)}{cd} \\
 &= \frac{x \coth^{-1}(ax)}{c\sqrt{c + dx^2}} - \frac{\tanh^{-1}\left(\frac{a\sqrt{c + dx^2}}{\sqrt{a^2c + d}}\right)}{c\sqrt{a^2c + d}}
 \end{aligned}$$



**Mathematica [A]** time = 0.117746, size = 119, normalized size = 1.92

$$\frac{-\log\left(\sqrt{a^2c+d}\sqrt{c+dx^2+ac-dx}\right)-\log\left(\sqrt{a^2c+d}\sqrt{c+dx^2+ac+dx}\right)+\log(1-ax)+\log(ax+1)}{2c} + \frac{2x \operatorname{coth}^{-1}(ax)}{\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a\*x]/(c + d\*x^2)^(3/2), x]

[Out] ((2\*x\*ArcCoth[a\*x])/Sqrt[c + d\*x^2] + (Log[1 - a\*x] + Log[1 + a\*x] - Log[a\*c - d\*x + Sqrt[a^2\*c + d]\*Sqrt[c + d\*x^2]] - Log[a\*c + d\*x + Sqrt[a^2\*c + d]\*Sqrt[c + d\*x^2]])/Sqrt[a^2\*c + d])/(2\*c)

**Maple [F]** time = 0.457, size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(ax) (dx^2 + c)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)/(d\*x^2+c)^(3/2), x)

[Out] int(arccoth(a\*x)/(d\*x^2+c)^(3/2), x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c)^(3/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 1.74724, size = 760, normalized size = 12.26

$$\frac{2(a^2c + d)\sqrt{dx^2 + c}x \log\left(\frac{ax+1}{ax-1}\right) + \sqrt{a^2c + d}(dx^2 + c) \log\left(\frac{a^4d^2x^4 + 8a^4c^2 + 8a^2cd + 2(4a^4cd + 3a^2d^2)x^2 - 4(a^3dx^2 + 2a^3c + ad)\sqrt{a^2c + d}\sqrt{dx^2 + c}}{a^4x^4 - 2a^2x^2 + 1}\right)}{4(a^2c^3 + c^2d + (a^2c^2d + cd^2)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c)^(3/2),x, algorithm="fricas")

[Out] [1/4\*(2\*(a^2\*c + d)\*sqrt(d\*x^2 + c)\*x\*log((a\*x + 1)/(a\*x - 1)) + sqrt(a^2\*c + d)\*(d\*x^2 + c)\*log((a^4\*d^2\*x^4 + 8\*a^4\*c^2 + 8\*a^2\*c\*d + 2\*(4\*a^4\*c\*d + 3\*a^2\*d^2)\*x^2 - 4\*(a^3\*d\*x^2 + 2\*a^3\*c + a\*d)\*sqrt(a^2\*c + d)\*sqrt(d\*x^2 + c) + d^2)/(a^4\*x^4 - 2\*a^2\*x^2 + 1)))/(a^2\*c^3 + c^2\*d + (a^2\*c^2\*d + c\*d^2)\*x^2), 1/2\*((a^2\*c + d)\*sqrt(d\*x^2 + c)\*x\*log((a\*x + 1)/(a\*x - 1)) + sqrt(-a^2\*c - d)\*(d\*x^2 + c)\*arctan(1/2\*(a^2\*d\*x^2 + 2\*a^2\*c + d)\*sqrt(-a^2\*c - d)\*sqrt(d\*x^2 + c)/(a^3\*c^2 + a\*c\*d + (a^3\*c\*d + a\*d^2)\*x^2)))/(a^2\*c^3 + c^2\*d + (a^2\*c^2\*d + c\*d^2)\*x^2)]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}(ax)}{(c + dx^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(a\*x)/(d\*x\*\*2+c)\*\*(3/2),x)

[Out] Integral(acoth(a\*x)/(c + d\*x\*\*2)\*\*(3/2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(ax)}{(dx^2 + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)/(d*x^2 + c)^(3/2), x)
```

$$3.45 \quad \int \frac{\coth^{-1}(ax)}{(c+dx^2)^{5/2}} dx$$

**Optimal.** Leaf size=128

$$-\frac{(3a^2c + 2d) \tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{3c^2(a^2c + d)^{3/2}} + \frac{a}{3c(a^2c + d)\sqrt{c + dx^2}} + \frac{2x \coth^{-1}(ax)}{3c^2\sqrt{c + dx^2}} + \frac{x \coth^{-1}(ax)}{3c(c + dx^2)^{3/2}}$$

[Out] a/(3\*c\*(a^2\*c + d)\*Sqrt[c + d\*x^2]) + (x\*ArcCoth[a\*x])/(3\*c\*(c + d\*x^2)^(3/2)) + (2\*x\*ArcCoth[a\*x])/(3\*c^2\*Sqrt[c + d\*x^2]) - ((3\*a^2\*c + 2\*d)\*ArcTanh[(a\*Sqrt[c + d\*x^2])/Sqrt[a^2\*c + d]])/(3\*c^2\*(a^2\*c + d)^(3/2))

**Rubi [A]** time = 0.339828, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 9, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$ , Rules used = {192, 191, 5977, 6688, 12, 571, 78, 63, 208}

$$-\frac{(3a^2c + 2d) \tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{3c^2(a^2c + d)^{3/2}} + \frac{a}{3c(a^2c + d)\sqrt{c + dx^2}} + \frac{2x \coth^{-1}(ax)}{3c^2\sqrt{c + dx^2}} + \frac{x \coth^{-1}(ax)}{3c(c + dx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]/(c + d\*x^2)^(5/2), x]

[Out] a/(3\*c\*(a^2\*c + d)\*Sqrt[c + d\*x^2]) + (x\*ArcCoth[a\*x])/(3\*c\*(c + d\*x^2)^(3/2)) + (2\*x\*ArcCoth[a\*x])/(3\*c^2\*Sqrt[c + d\*x^2]) - ((3\*a^2\*c + 2\*d)\*ArcTanh[(a\*Sqrt[c + d\*x^2])/Sqrt[a^2\*c + d]])/(3\*c^2\*(a^2\*c + d)^(3/2))

### Rule 192

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

### Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 5977

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symbol]
:> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x]
- Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)
*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)
]^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol]
:> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x]
- Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)),
Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] &&
(!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> With[{p = Denominator[m]},
Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{5/2}} dx &= \frac{x \coth^{-1}(ax)}{3c(c+dx^2)^{3/2}} + \frac{2x \coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - a \int \frac{\frac{x}{3c(c+dx^2)^{3/2}} + \frac{2x}{3c^2\sqrt{c+dx^2}}}{1-a^2x^2} dx \\
&= \frac{x \coth^{-1}(ax)}{3c(c+dx^2)^{3/2}} + \frac{2x \coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - a \int \frac{x(3c+2dx^2)}{3c^2(1-a^2x^2)(c+dx^2)^{3/2}} dx \\
&= \frac{x \coth^{-1}(ax)}{3c(c+dx^2)^{3/2}} + \frac{2x \coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{a \int \frac{x(3c+2dx^2)}{(1-a^2x^2)(c+dx^2)^{3/2}} dx}{3c^2} \\
&= \frac{x \coth^{-1}(ax)}{3c(c+dx^2)^{3/2}} + \frac{2x \coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{a \operatorname{Subst}\left(\int \frac{3c+2dx}{(1-a^2x)(c+dx)^{3/2}} dx, x, x^2\right)}{6c^2} \\
&= \frac{a}{3c(a^2c+d)\sqrt{c+dx^2}} + \frac{x \coth^{-1}(ax)}{3c(c+dx^2)^{3/2}} + \frac{2x \coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{(a(3a^2c+2d)) \operatorname{Subst}\left(\int \frac{1}{(1-a^2x)\sqrt{c+dx}} dx, x, x^2\right)}{6c^2(a^2c+d)} \\
&= \frac{a}{3c(a^2c+d)\sqrt{c+dx^2}} + \frac{x \coth^{-1}(ax)}{3c(c+dx^2)^{3/2}} + \frac{2x \coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{(a(3a^2c+2d)) \operatorname{Subst}\left(\int \frac{1}{1+\frac{a^2c}{d}-\frac{a^2x^2}{d}} dx, x, x^2\right)}{3c^2d(a^2c+d)} \\
&= \frac{a}{3c(a^2c+d)\sqrt{c+dx^2}} + \frac{x \coth^{-1}(ax)}{3c(c+dx^2)^{3/2}} + \frac{2x \coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{(3a^2c+2d) \tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{3c^2(a^2c+d)^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.321201, size = 226, normalized size = 1.77

$$\frac{2ac}{(a^2c+d)\sqrt{c+dx^2}} - \frac{(3a^2c+2d) \log(\sqrt{a^2c+d}\sqrt{c+dx^2}+ac-dx)}{(a^2c+d)^{3/2}} - \frac{(3a^2c+2d) \log(\sqrt{a^2c+d}\sqrt{c+dx^2}+ac+dx)}{(a^2c+d)^{3/2}} + \frac{(3a^2c+2d) \log(1-ax)}{(a^2c+d)^{3/2}} + \frac{(3a^2c+2d) \log(ax+1)}{(a^2c+d)^{3/2}} + \frac{2}{6c^2}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a\*x]/(c + d\*x^2)^(5/2), x]

[Out] ((2\*a\*c)/((a^2\*c + d)\*Sqrt[c + d\*x^2]) + (2\*x\*(3\*c + 2\*d\*x^2)\*ArcCoth[a\*x])/(c + d\*x^2)^(3/2) + ((3\*a^2\*c + 2\*d)\*Log[1 - a\*x])/(a^2\*c + d)^(3/2) + ((3

$$\frac{(a^2c + 2d)\text{Log}[1 + ax]}{(a^2c + d)^{3/2}} - \frac{((3a^2c + 2d)\text{Log}[ac - dx + \sqrt{a^2c + d}]\sqrt{c + dx^2})}{(a^2c + d)^{3/2}} - \frac{((3a^2c + 2d)\text{Log}[ac + dx + \sqrt{a^2c + d}]\sqrt{c + dx^2})}{(a^2c + d)^{3/2}} \frac{1}{(6c^2)}$$

**Maple [F]** time = 0.456, size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(ax) (dx^2 + c)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)/(d\*x^2+c)^(5/2),x)

[Out] int(arccoth(a\*x)/(d\*x^2+c)^(5/2),x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 1.98491, size = 1496, normalized size = 11.69

$$\frac{\left( (3a^2c^3 + (3a^2cd^2 + 2d^3)x^4 + 2c^2d + 2(3a^2c^2d + 2cd^2)x^2)\sqrt{a^2c + d} \log\left( \frac{a^4d^2x^4 + 8a^4c^2 + 8a^2cd + 2(4a^4cd + 3a^2d^2)x^2 - 4(a^3dx^2 + 2a^2c^2)}{a^4x^4 - 2a^2x^2 + 1} \right) \right)}{12(a^4c^6 + 2a^2c^5d + c^4d^2 + (a^4c^3 + 3a^2cd^2 + 2d^3)x^4 + 2c^2d + 2(3a^2c^2d + 2cd^2)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c)^(5/2),x, algorithm="fricas")

```
[Out] [1/12*((3*a^2*c^3 + (3*a^2*c*d^2 + 2*d^3)*x^4 + 2*c^2*d + 2*(3*a^2*c^2*d + 2*c*d^2)*x^2)*sqrt(a^2*c + d)*log((a^4*d^2*x^4 + 8*a^4*c^2 + 8*a^2*c*d + 2*(4*a^4*c*d + 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2 + 2*a^3*c + a*d)*sqrt(a^2*c + d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 - 2*a^2*x^2 + 1)) + 2*(2*a^3*c^3 + 2*a*c^2*d + 2*(a^3*c^2*d + a*c*d^2)*x^2 + (2*(a^4*c^2*d + 2*a^2*c*d^2 + d^3)*x^3 + 3*(a^4*c^3 + 2*a^2*c^2*d + c*d^2)*x)*log((a*x + 1)/(a*x - 1)))*sqrt(d*x^2 + c))/(a^4*c^6 + 2*a^2*c^5*d + c^4*d^2 + (a^4*c^4*d^2 + 2*a^2*c^3*d^3 + c^2*d^4)*x^4 + 2*(a^4*c^5*d + 2*a^2*c^4*d^2 + c^3*d^3)*x^2), 1/6*((3*a^2*c^3 + (3*a^2*c*d^2 + 2*d^3)*x^4 + 2*c^2*d + 2*(3*a^2*c^2*d + 2*c*d^2)*x^2)*sqrt(-a^2*c - d)*arctan(1/2*(a^2*d*x^2 + 2*a^2*c + d)*sqrt(-a^2*c - d)*sqrt(d*x^2 + c)/(a^3*c^2 + a*c*d + (a^3*c*d + a*d^2)*x^2)) + (2*a^3*c^3 + 2*a*c^2*d + 2*(a^3*c^2*d + a*c*d^2)*x^2 + (2*(a^4*c^2*d + 2*a^2*c*d^2 + d^3)*x^3 + 3*(a^4*c^3 + 2*a^2*c^2*d + c*d^2)*x)*log((a*x + 1)/(a*x - 1)))*sqrt(d*x^2 + c))/(a^4*c^6 + 2*a^2*c^5*d + c^4*d^2 + (a^4*c^4*d^2 + 2*a^2*c^3*d^3 + c^2*d^4)*x^4 + 2*(a^4*c^5*d + 2*a^2*c^4*d^2 + c^3*d^3)*x^2)]
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}(ax)}{(c + dx^2)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(acoth(a*x)/(c + d*x**2)**(5/2), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(ax)}{(dx^2 + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)/(d*x^2 + c)^(5/2), x)
```



$$3.46 \quad \int \frac{\coth^{-1}(ax)}{(c+dx^2)^{7/2}} dx$$

**Optimal.** Leaf size=200

$$\frac{(15a^4c^2 + 20a^2cd + 8d^2) \tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{15c^3(a^2c+d)^{5/2}} + \frac{a(7a^2c+4d)}{15c^2(a^2c+d)^2\sqrt{c+dx^2}} + \frac{a}{15c(a^2c+d)(c+dx^2)^{3/2}} + \frac{8x \coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}}$$

[Out] a/(15\*c\*(a^2\*c + d)\*(c + d\*x^2)^(3/2)) + (a\*(7\*a^2\*c + 4\*d))/(15\*c^2\*(a^2\*c + d)^2\*Sqrt[c + d\*x^2]) + (x\*ArcCoth[a\*x])/(5\*c\*(c + d\*x^2)^(5/2)) + (4\*x\*ArcCoth[a\*x])/(15\*c^2\*(c + d\*x^2)^(3/2)) + (8\*x\*ArcCoth[a\*x])/(15\*c^3\*Sqrt[c + d\*x^2]) - ((15\*a^4\*c^2 + 20\*a^2\*c\*d + 8\*d^2)\*ArcTanh[(a\*Sqrt[c + d\*x^2])/Sqrt[a^2\*c + d]])/(15\*c^3\*(a^2\*c + d)^(5/2))

**Rubi [A]** time = 1.04748, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 9, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$ , Rules used = {192, 191, 5977, 6688, 12, 6715, 897, 1261, 208}

$$\frac{(15a^4c^2 + 20a^2cd + 8d^2) \tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{15c^3(a^2c+d)^{5/2}} + \frac{a(7a^2c+4d)}{15c^2(a^2c+d)^2\sqrt{c+dx^2}} + \frac{a}{15c(a^2c+d)(c+dx^2)^{3/2}} + \frac{8x \coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]/(c + d\*x^2)^(7/2), x]

[Out] a/(15\*c\*(a^2\*c + d)\*(c + d\*x^2)^(3/2)) + (a\*(7\*a^2\*c + 4\*d))/(15\*c^2\*(a^2\*c + d)^2\*Sqrt[c + d\*x^2]) + (x\*ArcCoth[a\*x])/(5\*c\*(c + d\*x^2)^(5/2)) + (4\*x\*ArcCoth[a\*x])/(15\*c^2\*(c + d\*x^2)^(3/2)) + (8\*x\*ArcCoth[a\*x])/(15\*c^3\*Sqrt[c + d\*x^2]) - ((15\*a^4\*c^2 + 20\*a^2\*c\*d + 8\*d^2)\*ArcTanh[(a\*Sqrt[c + d\*x^2])/Sqrt[a^2\*c + d]])/(15\*c^3\*(a^2\*c + d)^(5/2))

### Rule 192

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

Rule 191

```
Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5977

```
Int[((a_) + ArcCoth[(c_)*(x_)]*(b_))*((d_) + (e_)*(x_)^2)^(q_), x_Sym
bol] :=> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6688

```
Int[u_, x_Symbol] :=> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :=> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6715

```
Int[(u_)*(x_)^(m_), x_Symbol] :=> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 897

```
Int[((d_) + (e_)*(x_)^(m_))*((f_) + (g_)*(x_)^(n_))*((a_) + (b_)*(x_)
+ (c_)*(x_)^2)^(p_), x_Symbol] :=> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1261

```
Int[((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)^(q_))*((a_) + (b_)*(x_)^2 + (
c_)*(x_)^4)^(p_), x_Symbol] :=> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 208

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{7/2}} dx &= \frac{x \coth^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x \coth^{-1}(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{8x \coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - a \int \frac{\frac{x}{5c(c+dx^2)^{5/2}} + \frac{4x}{15c^2(c+dx^2)^{3/2}} + \frac{8x}{15c^3\sqrt{c+dx^2}}}{1-a^2x^2} dx \\
&= \frac{x \coth^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x \coth^{-1}(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{8x \coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - a \int \frac{x(15c^2+20cdx^2+8d^2x^4)}{15c^3(1-a^2x^2)(c+dx^2)^{5/2}} dx \\
&= \frac{x \coth^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x \coth^{-1}(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{8x \coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a \int \frac{x(15c^2+20cdx^2+8d^2x^4)}{(1-a^2x^2)(c+dx^2)^{5/2}} dx}{15c^3} \\
&= \frac{x \coth^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x \coth^{-1}(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{8x \coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a \operatorname{Subst}\left(\int \frac{15c^2+20cdx+8d^2x^2}{(1-a^2x)(c+dx)^{5/2}} dx, x, x^2\right)}{30c^3} \\
&= \frac{x \coth^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x \coth^{-1}(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{8x \coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a \operatorname{Subst}\left(\int \frac{3c^2+4cx^2+8x^4}{x^4\left(\frac{a^2c+d}{d}-\frac{a^2x^2}{d}\right)} dx, x, \sqrt{c+dx^2}\right)}{15c^3d} \\
&= \frac{x \coth^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x \coth^{-1}(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{8x \coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a \operatorname{Subst}\left(\int \left(\frac{3c^2d}{(a^2c+d)x^4} + \frac{cd(7a^2c+4d)}{(a^2c+d)^2x^2} + \frac{d(15a^4c^2+20a^2cd+8d^2)}{(a^2c+d)^3}\right) dx, x, \sqrt{c+dx^2}\right)}{15c^3d} \\
&= \frac{a}{15c(a^2c+d)(c+dx^2)^{3/2}} + \frac{a(7a^2c+4d)}{15c^2(a^2c+d)^2\sqrt{c+dx^2}} + \frac{x \coth^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x \coth^{-1}(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{8x \coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}} \\
&= \frac{a}{15c(a^2c+d)(c+dx^2)^{3/2}} + \frac{a(7a^2c+4d)}{15c^2(a^2c+d)^2\sqrt{c+dx^2}} + \frac{x \coth^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x \coth^{-1}(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{8x \coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.636018, size = 329, normalized size = 1.64

$$(15a^4c^2 + 20a^2cd + 8d^2) \log(1-ax)(c+dx^2)^{5/2} + (15a^4c^2 + 20a^2cd + 8d^2) \log(ax+1)(c+dx^2)^{5/2} - (15a^4c^2 + 20a^2cd + 8d^2) \log(1-ax^2)(c+dx^2)^{5/2}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a\*x]/(c + d\*x^2)^(7/2),x]

[Out] (2\*a\*c\*Sqrt[a^2\*c + d]\*(c + d\*x^2)\*(d\*(5\*c + 4\*d\*x^2) + a^2\*c\*(8\*c + 7\*d\*x^2)) + 2\*(a^2\*c + d)^(5/2)\*x\*(15\*c^2 + 20\*c\*d\*x^2 + 8\*d^2\*x^4)\*ArcCoth[a\*x] + (15\*a^4\*c^2 + 20\*a^2\*c\*d + 8\*d^2)\*(c + d\*x^2)^(5/2)\*Log[1 - a\*x] + (15\*a^4\*c^2 + 20\*a^2\*c\*d + 8\*d^2)\*(c + d\*x^2)^(5/2)\*Log[1 + a\*x] - (15\*a^4\*c^2 + 20\*a^2\*c\*d + 8\*d^2)\*(c + d\*x^2)^(5/2)\*Log[a\*c - d\*x + Sqrt[a^2\*c + d]\*Sqrt[c + d\*x^2]] - (15\*a^4\*c^2 + 20\*a^2\*c\*d + 8\*d^2)\*(c + d\*x^2)^(5/2)\*Log[a\*c + d\*x + Sqrt[a^2\*c + d]\*Sqrt[c + d\*x^2]])/(30\*c^3\*(a^2\*c + d)^(5/2)\*(c + d\*x^2)^(5/2))

**Maple [F]** time = 0.455, size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(ax) (dx^2 + c)^{-\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)/(d\*x^2+c)^(7/2),x)

[Out] int(arccoth(a\*x)/(d\*x^2+c)^(7/2),x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c)^(7/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.24513, size = 2610, normalized size = 13.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/60*((15*a^4*c^5 + 20*a^2*c^4*d + (15*a^4*c^2*d^3 + 20*a^2*c*d^4 + 8*d^5)
*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 + 20*a^2*c^2*d^3 + 8*c*d^4)*x^4 + 3*(1
5*a^4*c^4*d + 20*a^2*c^3*d^2 + 8*c^2*d^3)*x^2)*sqrt(a^2*c + d)*log((a^4*d^2
*x^4 + 8*a^4*c^2 + 8*a^2*c*d + 2*(4*a^4*c*d + 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2
+ 2*a^3*c + a*d)*sqrt(a^2*c + d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 - 2*a^2*x
^2 + 1)) + 2*(16*a^5*c^5 + 26*a^3*c^4*d + 10*a*c^3*d^2 + 2*(7*a^5*c^3*d^2 +
11*a^3*c^2*d^3 + 4*a*c*d^4)*x^4 + 6*(5*a^5*c^4*d + 8*a^3*c^3*d^2 + 3*a*c^2
*d^3)*x^2 + (8*(a^6*c^3*d^2 + 3*a^4*c^2*d^3 + 3*a^2*c*d^4 + d^5)*x^5 + 20*(
a^6*c^4*d + 3*a^4*c^3*d^2 + 3*a^2*c^2*d^3 + c*d^4)*x^3 + 15*(a^6*c^5 + 3*a^
4*c^4*d + 3*a^2*c^3*d^2 + c^2*d^3)*x)*log((a*x + 1)/(a*x - 1)))*sqrt(d*x^2
+ c))/(a^6*c^9 + 3*a^4*c^8*d + 3*a^2*c^7*d^2 + c^6*d^3 + (a^6*c^6*d^3 + 3*a
^4*c^5*d^4 + 3*a^2*c^4*d^5 + c^3*d^6)*x^6 + 3*(a^6*c^7*d^2 + 3*a^4*c^6*d^3
+ 3*a^2*c^5*d^4 + c^4*d^5)*x^4 + 3*(a^6*c^8*d + 3*a^4*c^7*d^2 + 3*a^2*c^6*d
^3 + c^5*d^4)*x^2), 1/30*((15*a^4*c^5 + 20*a^2*c^4*d + (15*a^4*c^2*d^3 + 20
*a^2*c*d^4 + 8*d^5)*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 + 20*a^2*c^2*d^3 +
8*c*d^4)*x^4 + 3*(15*a^4*c^4*d + 20*a^2*c^3*d^2 + 8*c^2*d^3)*x^2)*sqrt(-a^2
*c - d)*arctan(1/2*(a^2*d*x^2 + 2*a^2*c + d)*sqrt(-a^2*c - d)*sqrt(d*x^2 +
c)/(a^3*c^2 + a*c*d + (a^3*c*d + a*d^2)*x^2)) + (16*a^5*c^5 + 26*a^3*c^4*d
+ 10*a*c^3*d^2 + 2*(7*a^5*c^3*d^2 + 11*a^3*c^2*d^3 + 4*a*c*d^4)*x^4 + 6*(5*
a^5*c^4*d + 8*a^3*c^3*d^2 + 3*a*c^2*d^3)*x^2 + (8*(a^6*c^3*d^2 + 3*a^4*c^2*
d^3 + 3*a^2*c*d^4 + d^5)*x^5 + 20*(a^6*c^4*d + 3*a^4*c^3*d^2 + 3*a^2*c^2*d^
3 + c*d^4)*x^3 + 15*(a^6*c^5 + 3*a^4*c^4*d + 3*a^2*c^3*d^2 + c^2*d^3)*x)*lo
g((a*x + 1)/(a*x - 1)))*sqrt(d*x^2 + c))/(a^6*c^9 + 3*a^4*c^8*d + 3*a^2*c^7
*d^2 + c^6*d^3 + (a^6*c^6*d^3 + 3*a^4*c^5*d^4 + 3*a^2*c^4*d^5 + c^3*d^6)*x^
6 + 3*(a^6*c^7*d^2 + 3*a^4*c^6*d^3 + 3*a^2*c^5*d^4 + c^4*d^5)*x^4 + 3*(a^6*
c^8*d + 3*a^4*c^7*d^2 + 3*a^2*c^6*d^3 + c^5*d^4)*x^2)]
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoath(a*x)/(d*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(ax)}{(dx^2 + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c)^(7/2),x, algorithm="giac")

[Out] integrate(arccoth(a\*x)/(d\*x^2 + c)^(7/2), x)

$$3.47 \quad \int \frac{\coth^{-1}(ax)}{(c+dx^2)^{9/2}} dx$$

**Optimal.** Leaf size=283

$$\frac{a(19a^4c^2 + 22a^2cd + 8d^2)}{35c^3(a^2c + d)^3 \sqrt{c + dx^2}} - \frac{(70a^4c^2d + 35a^6c^3 + 56a^2cd^2 + 16d^3) \tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{35c^4(a^2c + d)^{7/2}} + \frac{a(11a^2c + 6d)}{105c^2(a^2c + d)^2 (c + dx^2)^{3/2}} +$$

[Out] a/(35\*c\*(a^2\*c + d)\*(c + d\*x^2)^(5/2)) + (a\*(11\*a^2\*c + 6\*d))/(105\*c^2\*(a^2\*c + d)^2\*(c + d\*x^2)^(3/2)) + (a\*(19\*a^4\*c^2 + 22\*a^2\*c\*d + 8\*d^2))/(35\*c^3\*(a^2\*c + d)^3\*Sqrt[c + d\*x^2]) + (x\*ArcCoth[a\*x])/(7\*c\*(c + d\*x^2)^(7/2)) + (6\*x\*ArcCoth[a\*x])/(35\*c^2\*(c + d\*x^2)^(5/2)) + (8\*x\*ArcCoth[a\*x])/(35\*c^3\*(c + d\*x^2)^(3/2)) + (16\*x\*ArcCoth[a\*x])/(35\*c^4\*Sqrt[c + d\*x^2]) - ((35\*a^6\*c^3 + 70\*a^4\*c^2\*d + 56\*a^2\*c\*d^2 + 16\*d^3)\*ArcTanh[(a\*Sqrt[c + d\*x^2])/Sqrt[a^2\*c + d]])/(35\*c^4\*(a^2\*c + d)^(7/2))

**Rubi [A]** time = 1.34323, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 9, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$ , Rules used = {192, 191, 5977, 6688, 12, 6715, 1619, 63, 208}

$$\frac{a(19a^4c^2 + 22a^2cd + 8d^2)}{35c^3(a^2c + d)^3 \sqrt{c + dx^2}} - \frac{(70a^4c^2d + 35a^6c^3 + 56a^2cd^2 + 16d^3) \tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{35c^4(a^2c + d)^{7/2}} + \frac{a(11a^2c + 6d)}{105c^2(a^2c + d)^2 (c + dx^2)^{3/2}} +$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]/(c + d\*x^2)^(9/2), x]

[Out] a/(35\*c\*(a^2\*c + d)\*(c + d\*x^2)^(5/2)) + (a\*(11\*a^2\*c + 6\*d))/(105\*c^2\*(a^2\*c + d)^2\*(c + d\*x^2)^(3/2)) + (a\*(19\*a^4\*c^2 + 22\*a^2\*c\*d + 8\*d^2))/(35\*c^3\*(a^2\*c + d)^3\*Sqrt[c + d\*x^2]) + (x\*ArcCoth[a\*x])/(7\*c\*(c + d\*x^2)^(7/2)) + (6\*x\*ArcCoth[a\*x])/(35\*c^2\*(c + d\*x^2)^(5/2)) + (8\*x\*ArcCoth[a\*x])/(35\*c^3\*(c + d\*x^2)^(3/2)) + (16\*x\*ArcCoth[a\*x])/(35\*c^4\*Sqrt[c + d\*x^2]) - ((35\*a^6\*c^3 + 70\*a^4\*c^2\*d + 56\*a^2\*c\*d^2 + 16\*d^3)\*ArcTanh[(a\*Sqrt[c + d\*x^2])/Sqrt[a^2\*c + d]])/(35\*c^4\*(a^2\*c + d)^(7/2))

**Rule 192**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :-> -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p\_)

```
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
&& NeQ[p, -1]
```

### Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

### Rule 5977

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] := With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

### Rule 6688

```
Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

### Rule 6715

```
Int[(u_)*(x_)^(m_), x_Symbol] := Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

### Rule 1619

```
Int[((Px_)*((c_.) + (d_.)*(x_))^(n_.))/((a_.) + (b_.)*(x_)), x_Symbol] := I
nt[ExpandIntegrand[1/Sqrt[c + d*x], (Px*(c + d*x)^(n + 1/2))/(a + b*x), x],
x] /; FreeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[n + 1/2, 0] && GtQ
[Expon[Px, x], 2]
```

### Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```



ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 208

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rubi steps

$$\begin{aligned}
 \int \frac{\coth^{-1}(ax)}{(c+dx^2)^{9/2}} dx &= \frac{x \coth^{-1}(ax)}{7c(c+dx^2)^{7/2}} + \frac{6x \coth^{-1}(ax)}{35c^2(c+dx^2)^{5/2}} + \frac{8x \coth^{-1}(ax)}{35c^3(c+dx^2)^{3/2}} + \frac{16x \coth^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - a \int \frac{x}{7c(c+dx^2)^{7/2} + 35c^2} \\
 &= \frac{x \coth^{-1}(ax)}{7c(c+dx^2)^{7/2}} + \frac{6x \coth^{-1}(ax)}{35c^2(c+dx^2)^{5/2}} + \frac{8x \coth^{-1}(ax)}{35c^3(c+dx^2)^{3/2}} + \frac{16x \coth^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - a \int \frac{x(35c^3+70c^2dx^2)}{35c^4(1-a^2x^2)} \\
 &= \frac{x \coth^{-1}(ax)}{7c(c+dx^2)^{7/2}} + \frac{6x \coth^{-1}(ax)}{35c^2(c+dx^2)^{5/2}} + \frac{8x \coth^{-1}(ax)}{35c^3(c+dx^2)^{3/2}} + \frac{16x \coth^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{a \int \frac{x(35c^3+70c^2dx^2+56cdx^4)}{(1-a^2x^2)(c+dx^2)}}{35c^4} \\
 &= \frac{x \coth^{-1}(ax)}{7c(c+dx^2)^{7/2}} + \frac{6x \coth^{-1}(ax)}{35c^2(c+dx^2)^{5/2}} + \frac{8x \coth^{-1}(ax)}{35c^3(c+dx^2)^{3/2}} + \frac{16x \coth^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{a \operatorname{Subst}\left(\int \frac{35c^3+70c^2dx}{(1-a^2x^2)(c+dx^2)}\right)}{7c} \\
 &= \frac{x \coth^{-1}(ax)}{7c(c+dx^2)^{7/2}} + \frac{6x \coth^{-1}(ax)}{35c^2(c+dx^2)^{5/2}} + \frac{8x \coth^{-1}(ax)}{35c^3(c+dx^2)^{3/2}} + \frac{16x \coth^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{a \operatorname{Subst}\left(\int \left(\frac{5c^3d}{(a^2c+d)(c+dx^2)}\right)\right)}{7c} \\
 &= \frac{a}{35c(a^2c+d)(c+dx^2)^{5/2}} + \frac{a(11a^2c+6d)}{105c^2(a^2c+d)^2(c+dx^2)^{3/2}} + \frac{a(19a^4c^2+22a^2cd+8d^2)}{35c^3(a^2c+d)^3\sqrt{c+dx^2}} + \frac{x \coth^{-1}(ax)}{7c(c+dx^2)} \\
 &= \frac{a}{35c(a^2c+d)(c+dx^2)^{5/2}} + \frac{a(11a^2c+6d)}{105c^2(a^2c+d)^2(c+dx^2)^{3/2}} + \frac{a(19a^4c^2+22a^2cd+8d^2)}{35c^3(a^2c+d)^3\sqrt{c+dx^2}} + \frac{x \coth^{-1}(ax)}{7c(c+dx^2)} \\
 &= \frac{a}{35c(a^2c+d)(c+dx^2)^{5/2}} + \frac{a(11a^2c+6d)}{105c^2(a^2c+d)^2(c+dx^2)^{3/2}} + \frac{a(19a^4c^2+22a^2cd+8d^2)}{35c^3(a^2c+d)^3\sqrt{c+dx^2}} + \frac{x \coth^{-1}(ax)}{7c(c+dx^2)}
 \end{aligned}$$

**Mathematica [A]** time = 0.961486, size = 431, normalized size = 1.52

$$2ac\sqrt{a^2c+d}(c+dx^2)\left(3(19a^4c^2+22a^2cd+8d^2)(c+dx^2)^2+3c^2(a^2c+d)^2+c(11a^2c+6d)(a^2c+d)(c+dx^2)\right)+3(7$$


---

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a\*x]/(c+d\*x^2)^(9/2),x]

[Out] (2\*a\*c\*Sqrt[a^2\*c+d]\*(c+d\*x^2)\*(3\*c^2\*(a^2\*c+d)^2+c\*(a^2\*c+d)\*(11\*a^2\*c+6\*d)\*(c+d\*x^2)+3\*(19\*a^4\*c^2+22\*a^2\*c\*d+8\*d^2)\*(c+d\*x^2)^2)+6\*(a^2\*c+d)^(7/2)\*x\*(35\*c^3+70\*c^2\*d\*x^2+56\*c\*d^2\*x^4+16\*d^3\*x^6)\*ArcCoth[a\*x]+3\*(35\*a^6\*c^3+70\*a^4\*c^2\*d+56\*a^2\*c\*d^2+16\*d^3)\*(c+d\*x^2)^(7/2)\*Log[1-a\*x]+3\*(35\*a^6\*c^3+70\*a^4\*c^2\*d+56\*a^2\*c\*d^2+16\*d^3)\*(c+d\*x^2)^(7/2)\*Log[1+a\*x]-3\*(35\*a^6\*c^3+70\*a^4\*c^2\*d+56\*a^2\*c\*d^2+16\*d^3)\*(c+d\*x^2)^(7/2)\*Log[a\*c-d\*x+Sqrt[a^2\*c+d]\*Sqrt[c+d\*x^2]]-3\*(35\*a^6\*c^3+70\*a^4\*c^2\*d+56\*a^2\*c\*d^2+16\*d^3)\*(c+d\*x^2)^(7/2)\*Log[a\*c+d\*x+Sqrt[a^2\*c+d]\*Sqrt[c+d\*x^2]]/(210\*c^4\*(a^2\*c+d)^(7/2)\*(c+d\*x^2)^(7/2))

---

**Maple [F]** time = 0.461, size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(ax)(dx^2+c)^{-\frac{9}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)/(d\*x^2+c)^(9/2),x)

[Out] int(arccoth(a\*x)/(d\*x^2+c)^(9/2),x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c)^(9/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [B]** time = 3.19344, size = 4140, normalized size = 14.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(a*x)/(d*x^2+c)^(9/2),x, algorithm="fricas")`

[Out] 
$$\begin{aligned} & [1/420*(3*(35*a^6*c^7 + 70*a^4*c^6*d + 56*a^2*c^5*d^2 + (35*a^6*c^3*d^4 + 70*a^4*c^2*d^5 + 56*a^2*c*d^6 + 16*d^7))*x^8 + 16*c^4*d^3 + 4*(35*a^6*c^4*d^3 + 70*a^4*c^3*d^4 + 56*a^2*c^2*d^5 + 16*c*d^6))*x^6 + 6*(35*a^6*c^5*d^2 + 70*a^4*c^4*d^3 + 56*a^2*c^3*d^4 + 16*c^2*d^5))*x^4 + 4*(35*a^6*c^6*d + 70*a^4*c^5*d^2 + 56*a^2*c^4*d^3 + 16*c^3*d^4))*x^2)*\sqrt{a^2*c + d}*\log((a^4*d^2*x^4 + 8*a^4*c^2 + 8*a^2*c*d + 2*(4*a^4*c*d + 3*a^2*d^2))*x^2 - 4*(a^3*d*x^2 + 2*a^3*c + a*d))*\sqrt{a^2*c + d}*\sqrt{d*x^2 + c} + d^2)/(a^4*x^4 - 2*a^2*x^2 + 1)) + 2*(142*a^7*c^7 + 320*a^5*c^6*d + 244*a^3*c^5*d^2 + 66*a*c^4*d^3 + 6*(19*a^7*c^4*d^3 + 41*a^5*c^3*d^4 + 30*a^3*c^2*d^5 + 8*a*c*d^6))*x^6 + 2*(182*a^7*c^5*d^2 + 397*a^5*c^4*d^3 + 293*a^3*c^3*d^4 + 78*a*c^2*d^5))*x^4 + 2*(196*a^7*c^6*d + 434*a^5*c^5*d^2 + 325*a^3*c^4*d^3 + 87*a*c^3*d^4))*x^2 + 3*(16*(a^8*c^4*d^3 + 4*a^6*c^3*d^4 + 6*a^4*c^2*d^5 + 4*a^2*c*d^6 + d^7))*x^7 + 56*(a^8*c^5*d^2 + 4*a^6*c^4*d^3 + 6*a^4*c^3*d^4 + 4*a^2*c^2*d^5 + c*d^6))*x^5 + 70*(a^8*c^6*d + 4*a^6*c^5*d^2 + 6*a^4*c^4*d^3 + 4*a^2*c^3*d^4 + c^2*d^5))*x^3 + 35*(a^8*c^7 + 4*a^6*c^6*d + 6*a^4*c^5*d^2 + 4*a^2*c^4*d^3 + c^3*d^4))*x)*\log((a*x + 1)/(a*x - 1))*\sqrt{d*x^2 + c}]/(a^8*c^12 + 4*a^6*c^11*d + 6*a^4*c^10*d^2 + 4*a^2*c^9*d^3 + c^8*d^4 + (a^8*c^8*d^4 + 4*a^6*c^7*d^5 + 6*a^4*c^6*d^6 + 4*a^2*c^5*d^7 + c^4*d^8))*x^8 + 4*(a^8*c^9*d^3 + 4*a^6*c^8*d^4 + 6*a^4*c^7*d^5 + 4*a^2*c^6*d^6 + c^5*d^7))*x^6 + 6*(a^8*c^10*d^2 + 4*a^6*c^9*d^3 + 6*a^4*c^8*d^4 + 4*a^2*c^7*d^5 + c^6*d^6))*x^4 + 4*(a^8*c^11*d + 4*a^6*c^10*d^2 + 6*a^4*c^9*d^3 + 4*a^2*c^8*d^4 + c^7*d^5))*x^2), 1/210*(3*(35*a^6*c^7 + 70*a^4*c^6*d + 56*a^2*c^5*d^2 + (35*a^6*c^3*d^4 + 70*a^4*c^2*d^5 + 56*a^2*c*d^6 + 16*d^7))*x^8 + 16*c^4*d^3 + 4*(35*a^6*c^4*d^3 + 70*a^4*c^3*d^4 + 56*a^2*c^2*d^5 + 16*c*d^6))*x^6 + 6*(35*a^6*c^5*d^2 + 70*a^4*c^4*d^3 + 56*a^2*c^3*d^4 + 16*c^2*d^5))*x^4 + 4*(35*a^6*c^6*d + 70*a^4*c^5*d^2 + 56*a^2*c^4*d^3 + 16*c^3*d^4))*x^2)*\sqrt{-a^2*c - d}*\arctan(1/2*(a^2*d*x^2 + 2*a^2*c + d))*\sqrt{-a^2*c - d}*\sqrt{d*x^2 + c}]/(a^3*c^2 + a*c*d + (a^3*c*d + a*d^2))*x^2)) + (142*a^7*c^7 + 320*a^5*c^6*d + 244*a^3*c^5*d^2 + 66*a*c^4*d^3 + 6*(19*a^7*c^4*d^3 + 41*a^5*c^3*d^4 + 30*a^3*c^2*d^5 + 8*a*c*d^6))*x^6 + 2*(182*a^7*c^5*d^2 + 397*a^5*c^4*d^3 + 293*a^3*c^3*d^4 + 78*a*c^2*d^5))*x^4 + 2*(196*a^7*c^6*d + 434*a^5*c^5*d^2 + 325*a^3*c^4*d^3 + 87*a*c^3*d^4))*x^2 + 3*(16*(a^8*c^4*d^3 + 4*a^6*c^3*d^4 + 6*a^4*c^2*d^5 + 4*a^2*c*d^6 + d^7))*x^7 \end{aligned}$$

```
+ 56*(a^8*c^5*d^2 + 4*a^6*c^4*d^3 + 6*a^4*c^3*d^4 + 4*a^2*c^2*d^5 + c*d^6)*
x^5 + 70*(a^8*c^6*d + 4*a^6*c^5*d^2 + 6*a^4*c^4*d^3 + 4*a^2*c^3*d^4 + c^2*d
^5)*x^3 + 35*(a^8*c^7 + 4*a^6*c^6*d + 6*a^4*c^5*d^2 + 4*a^2*c^4*d^3 + c^3*d
^4)*x)*log((a*x + 1)/(a*x - 1))*sqrt(d*x^2 + c))/(a^8*c^12 + 4*a^6*c^11*d
+ 6*a^4*c^10*d^2 + 4*a^2*c^9*d^3 + c^8*d^4 + (a^8*c^8*d^4 + 4*a^6*c^7*d^5 +
6*a^4*c^6*d^6 + 4*a^2*c^5*d^7 + c^4*d^8)*x^8 + 4*(a^8*c^9*d^3 + 4*a^6*c^8*
d^4 + 6*a^4*c^7*d^5 + 4*a^2*c^6*d^6 + c^5*d^7)*x^6 + 6*(a^8*c^10*d^2 + 4*a^
6*c^9*d^3 + 6*a^4*c^8*d^4 + 4*a^2*c^7*d^5 + c^6*d^6)*x^4 + 4*(a^8*c^11*d +
4*a^6*c^10*d^2 + 6*a^4*c^9*d^3 + 4*a^2*c^8*d^4 + c^7*d^5)*x^2)]
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)/(d*x**2+c)**(9/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(ax)}{(dx^2 + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)/(d*x^2 + c)^(9/2), x)
```

### 3.48 $\int \sqrt{a - ax^2} \coth^{-1}(x) dx$

**Optimal.** Leaf size=186

$$-\frac{ia\sqrt{1-x^2}\text{PolyLog}\left(2, -\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{2\sqrt{a-ax^2}} + \frac{ia\sqrt{1-x^2}\text{PolyLog}\left(2, \frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{2\sqrt{a-ax^2}} + \frac{1}{2}\sqrt{a-ax^2} + \frac{1}{2}x\sqrt{a-ax^2}\coth^{-1}(x) - \frac{a\sqrt{1-x^2}\text{ta}}{2}$$

[Out] Sqrt[a - a\*x^2]/2 + (x\*Sqrt[a - a\*x^2]\*ArcCoth[x])/2 - (a\*Sqrt[1 - x^2]\*ArcCoth[x]\*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/Sqrt[a - a\*x^2] - ((I/2)\*a\*Sqrt[1 - x^2]\*PolyLog[2, ((-I)\*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a - a\*x^2] + ((I/2)\*a\*Sqrt[1 - x^2]\*PolyLog[2, (I\*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a - a\*x^2]

**Rubi [A]** time = 0.0831836, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {5943, 5955, 5951}

$$-\frac{ia\sqrt{1-x^2}\text{PolyLog}\left(2, -\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{2\sqrt{a-ax^2}} + \frac{ia\sqrt{1-x^2}\text{PolyLog}\left(2, \frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{2\sqrt{a-ax^2}} + \frac{1}{2}\sqrt{a-ax^2} + \frac{1}{2}x\sqrt{a-ax^2}\coth^{-1}(x) - \frac{a\sqrt{1-x^2}\text{ta}}{2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a - a\*x^2]\*ArcCoth[x], x]

[Out] Sqrt[a - a\*x^2]/2 + (x\*Sqrt[a - a\*x^2]\*ArcCoth[x])/2 - (a\*Sqrt[1 - x^2]\*ArcCoth[x]\*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/Sqrt[a - a\*x^2] - ((I/2)\*a\*Sqrt[1 - x^2]\*PolyLog[2, ((-I)\*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a - a\*x^2] + ((I/2)\*a\*Sqrt[1 - x^2]\*PolyLog[2, (I\*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a - a\*x^2]

#### Rule 5943

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^((d\_.) + (e\_.)\*(x\_)^2)^(q\_.), x\_Symbol] :> Simp[(b\*(d + e\*x^2)^q)/(2\*c\*q\*(2\*q + 1)), x] + (Dist[(2\*d\*q)/(2\*q + 1), Int[(d + e\*x^2)^(q - 1)\*(a + b\*ArcCoth[c\*x]), x], x] + Simp[(x\*(d + e\*x^2)^q\*(a + b\*ArcCoth[c\*x])]/(2\*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && GtQ[q, 0]

#### Rule 5955

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)/Sqrt[(d\_.) + (e\_.)\*(x\_)^2], x\_Symbol] :> Dist[Sqrt[1 - c^2\*x^2]/Sqrt[d + e\*x^2], Int[(a + b\*ArcCoth[c\*x])

$\sqrt{p}/\sqrt{1 - c^2x^2}, x, x] /; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{EqQ}[c^2d + e, 0] \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ \text{!GtQ}[d, 0]$

### Rule 5951

$\text{Int}[(a_.) + \text{ArcCoth}[c_.](x_.)](b_.)/\sqrt{(d_.) + (e_.)(x_.)^2}, x\_Symbol] \rightarrow \text{Simp}[(-2*(a + b*\text{ArcCoth}[c*x])*\text{ArcTan}[\sqrt{1 - c*x}/\sqrt{1 + c*x}])/(c*\sqrt{d}), x] + (-\text{Simp}[(I*b*\text{PolyLog}[2, -(I*\sqrt{1 - c*x})/\sqrt{1 + c*x}])]/(c*\sqrt{d}), x] + \text{Simp}[(I*b*\text{PolyLog}[2, (I*\sqrt{1 - c*x})/\sqrt{1 + c*x}])]/(c*\sqrt{d}), x)] /; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{EqQ}[c^2d + e, 0] \ \&\& \ \text{GtQ}[d, 0]$

### Rubi steps

$$\begin{aligned} \int \sqrt{a - ax^2} \coth^{-1}(x) dx &= \frac{1}{2} \sqrt{a - ax^2} + \frac{1}{2} x \sqrt{a - ax^2} \coth^{-1}(x) + \frac{1}{2} a \int \frac{\coth^{-1}(x)}{\sqrt{a - ax^2}} dx \\ &= \frac{1}{2} \sqrt{a - ax^2} + \frac{1}{2} x \sqrt{a - ax^2} \coth^{-1}(x) + \frac{(a\sqrt{1 - x^2}) \int \frac{\coth^{-1}(x)}{\sqrt{1 - x^2}} dx}{2\sqrt{a - ax^2}} \\ &= \frac{1}{2} \sqrt{a - ax^2} + \frac{1}{2} x \sqrt{a - ax^2} \coth^{-1}(x) - \frac{a\sqrt{1 - x^2} \coth^{-1}(x) \tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)}{\sqrt{a - ax^2}} - \frac{ia\sqrt{1 - x^2} \text{Li}_2\left(-\frac{i}{\sqrt{1+x}}\right)}{2\sqrt{a - ax^2}} \end{aligned}$$

**Mathematica [A]** time = 1.03914, size = 125, normalized size = 0.67

---


$$\sqrt{a - ax^2} \left( -4 \text{PolyLog}\left(2, -e^{-\coth^{-1}(x)}\right) + 4 \text{PolyLog}\left(2, e^{-\coth^{-1}(x)}\right) - 2 \coth\left(\frac{1}{2} \coth^{-1}(x)\right) - 4 \coth^{-1}(x) \log\left(1 - e^{-\coth^{-1}(x)}\right) \right)$$


---

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[a - a\*x^2]\*ArcCoth[x], x]

[Out]  $-(\sqrt{a - a*x^2}*(-2*\text{Coth}[\text{ArcCoth}[x]/2] - \text{ArcCoth}[x]*\text{Csch}[\text{ArcCoth}[x]/2])^2 - 4*\text{ArcCoth}[x]*\text{Log}[1 - \text{E}^{\text{ArcCoth}[x]}]) + 4*\text{ArcCoth}[x]*\text{Log}[1 + \text{E}^{\text{ArcCoth}[x]}]) - 4*\text{PolyLog}[2, -\text{E}^{\text{ArcCoth}[x]}] + 4*\text{PolyLog}[2, \text{E}^{\text{ArcCoth}[x]}] - \text{ArcCoth}[x]*\text{Sech}[\text{ArcCoth}[x]/2]^2 + 2*\text{Tanh}[\text{ArcCoth}[x]/2])/(8*\sqrt{1 - x^2})*x$

---

**Maple [A]** time = 0.448, size = 199, normalized size = 1.1

$$\frac{\operatorname{arccoth}(x)x+1}{2}\sqrt{-(-1+x)(1+x)a}-\frac{\operatorname{arccoth}(x)}{-2+2x}\sqrt{-(-1+x)(1+x)a}\sqrt{\frac{-1+x}{1+x}}\ln\left(1+\frac{1}{\sqrt{\frac{-1+x}{1+x}}}\right)-\frac{1}{-2+2x}\sqrt{-(-1+x)(1+x)a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-a\*x^2+a)^(1/2)\*arccoth(x),x)

[Out] 1/2\*(arccoth(x)\*x+1)\*(-(-1+x)\*(1+x)\*a)^(1/2)-1/2\*(-(-1+x)\*(1+x)\*a)^(1/2)\*((  
-1+x)/(1+x))^(1/2)/(-1+x)\*arccoth(x)\*ln(1+1/(((-1+x)/(1+x))^(1/2)))-1/2\*(-(-1  
+x)\*(1+x)\*a)^(1/2)\*(((-1+x)/(1+x))^(1/2)/(-1+x)\*polylog(2,-1/(((-1+x)/(1+x))^(  
1/2))+1/2\*(-(-1+x)\*(1+x)\*a)^(1/2)\*(((-1+x)/(1+x))^(1/2)/(-1+x)\*arccoth(x)\*1  
n(1-1/(((-1+x)/(1+x))^(1/2))+1/2\*(-(-1+x)\*(1+x)\*a)^(1/2)\*(((-1+x)/(1+x))^(1/2)  
)/(-1+x)\*polylog(2,1/(((-1+x)/(1+x))^(1/2))))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a\*x^2+a)^(1/2)\*arccoth(x),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\sqrt{-ax^2+a}\operatorname{arccoth}(x),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a\*x^2+a)^(1/2)\*arccoth(x),x, algorithm="fricas")

[Out] integral(sqrt(-a\*x^2 + a)\*arccoth(x), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a(x-1)(x+1)} \operatorname{acoth}(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a\*x\*\*2+a)\*\*(1/2)\*acoth(x),x)

[Out] Integral(sqrt(-a\*(x - 1)\*(x + 1))\*acoth(x), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-ax^2 + a} \operatorname{arccoth}(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a\*x^2+a)^(1/2)\*arccoth(x),x, algorithm="giac")

[Out] integrate(sqrt(-a\*x^2 + a)\*arccoth(x), x)



$$3.49 \quad \int \frac{\coth^{-1}(x)}{\sqrt{a-ax^2}} dx$$

**Optimal.** Leaf size=144

$$-\frac{i\sqrt{1-x^2}\text{PolyLog}\left(2, -\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{\sqrt{a-ax^2}} + \frac{i\sqrt{1-x^2}\text{PolyLog}\left(2, \frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{\sqrt{a-ax^2}} - \frac{2\sqrt{1-x^2}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)\coth^{-1}(x)}{\sqrt{a-ax^2}}$$

[Out]  $(-2*\text{Sqrt}[1-x^2]*\text{ArcCoth}[x]*\text{ArcTan}[\text{Sqrt}[1-x]/\text{Sqrt}[1+x]])/\text{Sqrt}[a-a*x^2] - (I*\text{Sqrt}[1-x^2]*\text{PolyLog}[2, ((-I)*\text{Sqrt}[1-x])/\text{Sqrt}[1+x]])/\text{Sqrt}[a-a*x^2] + (I*\text{Sqrt}[1-x^2]*\text{PolyLog}[2, (I*\text{Sqrt}[1-x])/\text{Sqrt}[1+x]])/\text{Sqrt}[a-a*x^2]$

**Rubi [A]** time = 0.0495992, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {5955, 5951}

$$-\frac{i\sqrt{1-x^2}\text{PolyLog}\left(2, -\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{\sqrt{a-ax^2}} + \frac{i\sqrt{1-x^2}\text{PolyLog}\left(2, \frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{\sqrt{a-ax^2}} - \frac{2\sqrt{1-x^2}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)\coth^{-1}(x)}{\sqrt{a-ax^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{ArcCoth}[x]/\text{Sqrt}[a-a*x^2], x]$

[Out]  $(-2*\text{Sqrt}[1-x^2]*\text{ArcCoth}[x]*\text{ArcTan}[\text{Sqrt}[1-x]/\text{Sqrt}[1+x]])/\text{Sqrt}[a-a*x^2] - (I*\text{Sqrt}[1-x^2]*\text{PolyLog}[2, ((-I)*\text{Sqrt}[1-x])/\text{Sqrt}[1+x]])/\text{Sqrt}[a-a*x^2] + (I*\text{Sqrt}[1-x^2]*\text{PolyLog}[2, (I*\text{Sqrt}[1-x])/\text{Sqrt}[1+x]])/\text{Sqrt}[a-a*x^2]$

#### Rule 5955

$\text{Int}[(a_.) + \text{ArcCoth}[(c_.)(x_)]*(b_.)]^p/\text{Sqrt}[(d_) + (e_.)(x_)^2], x\_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[1-c^2*x^2]/\text{Sqrt}[d+e*x^2], \text{Int}[(a+b*\text{ArcCoth}[c*x])^p/\text{Sqrt}[1-c^2*x^2], x], x] /;$   $\text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{EqQ}[c^2*d+e, 0] \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ !\text{GtQ}[d, 0]$

#### Rule 5951

$\text{Int}[(a_.) + \text{ArcCoth}[(c_.)(x_)]*(b_.)]/\text{Sqrt}[(d_) + (e_.)(x_)^2], x\_Symbol] \rightarrow \text{Simp}[(-2*(a+b*\text{ArcCoth}[c*x])* \text{ArcTan}[\text{Sqrt}[1-c*x]/\text{Sqrt}[1+c*x]])/(c*$

```
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])]/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

### Rubi steps

$$\int \frac{\coth^{-1}(x)}{\sqrt{a-ax^2}} dx = \frac{\sqrt{1-x^2} \int \frac{\coth^{-1}(x)}{\sqrt{1-x^2}} dx}{\sqrt{a-ax^2}}$$

$$= -\frac{2\sqrt{1-x^2} \coth^{-1}(x) \tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)}{\sqrt{a-ax^2}} - \frac{i\sqrt{1-x^2} \text{Li}_2\left(-\frac{i\sqrt{1-x}}{\sqrt{1+x}}\right)}{\sqrt{a-ax^2}} + \frac{i\sqrt{1-x^2} \text{Li}_2\left(\frac{i\sqrt{1-x}}{\sqrt{1+x}}\right)}{\sqrt{a-ax^2}}$$

**Mathematica [A]** time = 0.112466, size = 77, normalized size = 0.53

$$\frac{\sqrt{a-ax^2} \left( \text{PolyLog}\left(2, -e^{-\coth^{-1}(x)}\right) - \text{PolyLog}\left(2, e^{-\coth^{-1}(x)}\right) + \coth^{-1}(x) \left( \log\left(1 - e^{-\coth^{-1}(x)}\right) - \log\left(e^{-\coth^{-1}(x)} + 1\right) \right) \right)}{a\sqrt{1-\frac{1}{x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[x]/Sqrt[a - a*x^2], x]
```

```
[Out] (Sqrt[a - a*x^2]*(ArcCoth[x]*(Log[1 - E^(-ArcCoth[x])]) - Log[1 + E^(-ArcCot
h[x])]) + PolyLog[2, -E^(-ArcCoth[x])] - PolyLog[2, E^(-ArcCoth[x])])/(a*S
qrt[1 - x^(-2)]*x)
```

**Maple [A]** time = 0.368, size = 190, normalized size = 1.3

$$-\frac{\operatorname{arccoth}(x)}{(-1+x)a} \ln\left(1 + \frac{1}{\sqrt{\frac{-1+x}{1+x}}}\right) \sqrt{\frac{-1+x}{1+x}} \sqrt{-(-1+x)(1+x)a} - \frac{1}{(-1+x)a} \operatorname{polylog}\left(2, -\frac{1}{\sqrt{\frac{-1+x}{1+x}}}\right) \sqrt{\frac{-1+x}{1+x}} \sqrt{-(-1+x)(1+x)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x)/(-a*x^2+a)^(1/2), x)
```

```
[Out] -ln(1+1/((-1+x)/(1+x))^(1/2))*arccoth(x)*((-1+x)/(1+x))^(1/2)*(-(-1+x)*(1+x)
)*a)^(1/2)/(-1+x)/a-polylog(2,-1/((-1+x)/(1+x))^(1/2))*((-1+x)/(1+x))^(1/2)
*(-(-1+x)*(1+x)*a)^(1/2)/(-1+x)/a+ln(1-1/((-1+x)/(1+x))^(1/2))*arccoth(x)*
(-1+x)/(1+x))^(1/2)*(-(-1+x)*(1+x)*a)^(1/2)/(-1+x)/a+polylog(2,1/((-1+x)/(1
+x))^(1/2))*((-1+x)/(1+x))^(1/2)*(-(-1+x)*(1+x)*a)^(1/2)/(-1+x)/a
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-a*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-ax^2+a}\operatorname{arccoth}(x)}{ax^2-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-a*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a*x^2 + a)*arccoth(x)/(a*x^2 - a), x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}(x)}{\sqrt{-a(x-1)(x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x)/(-a*x**2+a)**(1/2),x)
```

[Out] Integral(acoth(x)/sqrt(-a\*(x - 1)\*(x + 1)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(x)}{\sqrt{-ax^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)/(-a\*x^2+a)^(1/2),x, algorithm="giac")

[Out] integrate(arccoth(x)/sqrt(-a\*x^2 + a), x)

$$3.50 \quad \int \frac{\coth^{-1}(x)}{(a-ax^2)^{3/2}} dx$$

Optimal. Leaf size=37

$$\frac{x \coth^{-1}(x)}{a\sqrt{a-ax^2}} - \frac{1}{a\sqrt{a-ax^2}}$$

[Out]  $-(1/(a*\text{Sqrt}[a - a*x^2])) + (x*\text{ArcCoth}[x])/(a*\text{Sqrt}[a - a*x^2])$

**Rubi [A]** time = 0.0254153, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {5959}

$$\frac{x \coth^{-1}(x)}{a\sqrt{a-ax^2}} - \frac{1}{a\sqrt{a-ax^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{ArcCoth}[x]/(a - a*x^2)^{(3/2)}, x]$

[Out]  $-(1/(a*\text{Sqrt}[a - a*x^2])) + (x*\text{ArcCoth}[x])/(a*\text{Sqrt}[a - a*x^2])$

Rule 5959

$\text{Int}[(a_.) + \text{ArcCoth}[(c_.)*(x_)]*(b_.)]/((d_.) + (e_.)*(x_)^2)^{(3/2)}, x_{\text{Symbol}}] :> -\text{Simp}[b/(c*d*\text{Sqrt}[d + e*x^2]), x] + \text{Simp}[(x*(a + b*\text{ArcCoth}[c*x]))/(d*\text{Sqrt}[d + e*x^2]), x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{EqQ}[c^2*d + e, 0]$

Rubi steps

$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{3/2}} dx = -\frac{1}{a\sqrt{a-ax^2}} + \frac{x \coth^{-1}(x)}{a\sqrt{a-ax^2}}$$

**Mathematica [A]** time = 0.0438355, size = 30, normalized size = 0.81

$$\frac{\sqrt{a-ax^2}(1-x \coth^{-1}(x))}{a^2(x^2-1)}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[x]/(a - a\*x^2)^(3/2),x]

[Out] (Sqrt[a - a\*x^2]\*(1 - x\*ArcCoth[x]))/(a^2\*(-1 + x^2))

**Maple [A]** time = 0.226, size = 52, normalized size = 1.4

$$-\frac{\operatorname{arccoth}(x)-1}{(-2+2x)a^2}\sqrt{-(-1+x)(1+x)a}-\frac{\operatorname{arccoth}(x)+1}{(2+2x)a^2}\sqrt{-(-1+x)(1+x)a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(x)/(-a\*x^2+a)^(3/2),x)

[Out] -1/2\*(arccoth(x)-1)\*(-(-1+x)\*(1+x)\*a)^(1/2)/(-1+x)/a^2-1/2\*(arccoth(x)+1)\*(-(-1+x)\*(1+x)\*a)^(1/2)/(1+x)/a^2

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)/(-a\*x^2+a)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.5362, size = 92, normalized size = 2.49

$$-\frac{\sqrt{-ax^2+a}\left(x\log\left(\frac{x+1}{x-1}\right)-2\right)}{2(a^2x^2-a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)/(-a\*x^2+a)^(3/2),x, algorithm="fricas")

[Out]  $-1/2*\sqrt{-a*x^2 + a}*(x*\log((x + 1)/(x - 1)) - 2)/(a^2*x^2 - a^2)$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}(x)}{(-a(x-1)(x+1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(x)/(-a*x**2+a)**(3/2),x)`

[Out] `Integral(acoth(x)/(-a*(x - 1)*(x + 1))**(3/2), x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(x)}{(-ax^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(x)/(-a*x^2+a)^(3/2),x, algorithm="giac")`

[Out] `integrate(arccoth(x)/(-a*x^2 + a)^(3/2), x)`

$$3.51 \quad \int \frac{\coth^{-1}(x)}{(a-ax^2)^{5/2}} dx$$

**Optimal.** Leaf size=83

$$-\frac{2}{3a^2\sqrt{a-ax^2}} + \frac{2x \coth^{-1}(x)}{3a^2\sqrt{a-ax^2}} - \frac{1}{9a(a-ax^2)^{3/2}} + \frac{x \coth^{-1}(x)}{3a(a-ax^2)^{3/2}}$$

[Out]  $-1/(9*a*(a - a*x^2)^{(3/2)}) - 2/(3*a^2*\text{Sqrt}[a - a*x^2]) + (x*\text{ArcCoth}[x])/(3*a*(a - a*x^2)^{(3/2)}) + (2*x*\text{ArcCoth}[x])/(3*a^2*\text{Sqrt}[a - a*x^2])$

**Rubi [A]** time = 0.0534298, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {5961, 5959}

$$-\frac{2}{3a^2\sqrt{a-ax^2}} + \frac{2x \coth^{-1}(x)}{3a^2\sqrt{a-ax^2}} - \frac{1}{9a(a-ax^2)^{3/2}} + \frac{x \coth^{-1}(x)}{3a(a-ax^2)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{ArcCoth}[x]/(a - a*x^2)^{(5/2)}, x]$

[Out]  $-1/(9*a*(a - a*x^2)^{(3/2)}) - 2/(3*a^2*\text{Sqrt}[a - a*x^2]) + (x*\text{ArcCoth}[x])/(3*a*(a - a*x^2)^{(3/2)}) + (2*x*\text{ArcCoth}[x])/(3*a^2*\text{Sqrt}[a - a*x^2])$

#### Rule 5961

$\text{Int}[(a_.) + \text{ArcCoth}[c_.*(x_)]*(b_.)]*((d_.) + (e_.*(x_)^2)^{(q_)}), x\_Symbol] \rightarrow -\text{Simp}[(b*(d + e*x^2)^{(q + 1)})/(4*c*d*(q + 1)^2), x] + (\text{Dist}[(2*q + 3)/(2*d*(q + 1)), \text{Int}[(d + e*x^2)^{(q + 1)}*(a + b*\text{ArcCoth}[c*x]), x], x] - \text{Simp}[(x*(d + e*x^2)^{(q + 1)}*(a + b*\text{ArcCoth}[c*x]))/(2*d*(q + 1)), x]) /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{LtQ}[q, -1] \&\& \text{NeQ}[q, -3/2]$

#### Rule 5959

$\text{Int}[(a_.) + \text{ArcCoth}[c_.*(x_)]*(b_.)]/((d_.) + (e_.*(x_)^2)^{(3/2)}), x\_Symbol] \rightarrow -\text{Simp}[b/(c*d*\text{Sqrt}[d + e*x^2]), x] + \text{Simp}[(x*(a + b*\text{ArcCoth}[c*x]))/(d*\text{Sqrt}[d + e*x^2]), x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{EqQ}[c^2*d + e, 0]$

#### Rubi steps



$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{5/2}} dx = -\frac{1}{9a(a-ax^2)^{3/2}} + \frac{x \coth^{-1}(x)}{3a(a-ax^2)^{3/2}} + \frac{2 \int \frac{\coth^{-1}(x)}{(a-ax^2)^{3/2}} dx}{3a}$$

$$= -\frac{1}{9a(a-ax^2)^{3/2}} - \frac{2}{3a^2\sqrt{a-ax^2}} + \frac{x \coth^{-1}(x)}{3a(a-ax^2)^{3/2}} + \frac{2x \coth^{-1}(x)}{3a^2\sqrt{a-ax^2}}$$

**Mathematica [A]** time = 0.0499682, size = 45, normalized size = 0.54

$$\frac{\sqrt{a-ax^2}(-6x^2 + (6x^3 - 9x)\coth^{-1}(x) + 7)}{9a^3(x^2 - 1)^2}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[x]/(a - a\*x^2)^(5/2), x]

[Out] -(Sqrt[a - a\*x^2]\*(7 - 6\*x^2 + (-9\*x + 6\*x^3)\*ArcCoth[x]))/(9\*a^3\*(-1 + x^2)^2)

**Maple [A]** time = 0.238, size = 112, normalized size = 1.4

$$\frac{(1+x)(-1+3\operatorname{arccoth}(x))\sqrt{-(-1+x)(1+x)a}}{72(-1+x)^2 a^3} - \frac{3\operatorname{arccoth}(x)-3}{(-8+8x)a^3}\sqrt{-(-1+x)(1+x)a} - \frac{3\operatorname{arccoth}(x)+3}{(8+8x)a^3}\sqrt{-(-1+x)(1+x)a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(x)/(-a\*x^2+a)^(5/2), x)

[Out] 1/72\*(1+x)\*(-1+3\*arccoth(x))\*(-(-1+x)\*(1+x)\*a)^(1/2)/(-1+x)^2/a^3-3/8\*(arccoth(x)-1)\*(-(-1+x)\*(1+x)\*a)^(1/2)/(-1+x)/a^3-3/8\*(arccoth(x)+1)\*(-(-1+x)\*(1+x)\*a)^(1/2)/(1+x)/a^3+1/72\*(1+3\*arccoth(x))\*(-1+x)\*(-(-1+x)\*(1+x)\*a)^(1/2)/(-1+x)^2/a^3

**Maxima [A]** time = 1.01709, size = 90, normalized size = 1.08

$$\frac{1}{3} \left( \frac{2x}{\sqrt{-ax^2 + aa^2}} + \frac{x}{(-ax^2 + a)^{\frac{3}{2}} a} \right) \operatorname{arccoth}(x) - \frac{2}{3\sqrt{-ax^2 + aa^2}} - \frac{1}{9(-ax^2 + a)^{\frac{3}{2}} a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)/(-a\*x^2+a)^(5/2),x, algorithm="maxima")

[Out] 1/3\*(2\*x/(sqrt(-a\*x^2 + a)\*a^2) + x/((-a\*x^2 + a)^(3/2)\*a))\*arccoth(x) - 2/3/(sqrt(-a\*x^2 + a)\*a^2) - 1/9/((-a\*x^2 + a)^(3/2)\*a)

**Fricas [A]** time = 1.58402, size = 140, normalized size = 1.69

$$\frac{\sqrt{-ax^2 + a} \left( 12x^2 - 3(2x^3 - 3x) \log\left(\frac{x+1}{x-1}\right) - 14 \right)}{18(a^3x^4 - 2a^3x^2 + a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)/(-a\*x^2+a)^(5/2),x, algorithm="fricas")

[Out] 1/18\*sqrt(-a\*x^2 + a)\*(12\*x^2 - 3\*(2\*x^3 - 3\*x)\*log((x + 1)/(x - 1)) - 14)/(a^3\*x^4 - 2\*a^3\*x^2 + a^3)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}(x)}{(-a(x-1)(x+1))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(x)/(-a\*x\*\*2+a)\*\*(5/2),x)

[Out] Integral(acoth(x)/(-a\*(x - 1)\*(x + 1))\*\*(5/2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(x)}{(-ax^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(x)/(-a*x^2+a)^(5/2),x, algorithm="giac")`

[Out] `integrate(arccoth(x)/(-a*x^2 + a)^(5/2), x)`

$$3.52 \quad \int \frac{\coth^{-1}(x)}{(a-ax^2)^{7/2}} dx$$

**Optimal.** Leaf size=124

$$-\frac{8}{15a^3\sqrt{a-ax^2}} - \frac{4}{45a^2(a-ax^2)^{3/2}} + \frac{8x\coth^{-1}(x)}{15a^3\sqrt{a-ax^2}} + \frac{4x\coth^{-1}(x)}{15a^2(a-ax^2)^{3/2}} - \frac{1}{25a(a-ax^2)^{5/2}} + \frac{x\coth^{-1}(x)}{5a(a-ax^2)^{5/2}}$$

[Out] -1/(25\*a\*(a - a\*x^2)^(5/2)) - 4/(45\*a^2\*(a - a\*x^2)^(3/2)) - 8/(15\*a^3\*Sqrt[a - a\*x^2]) + (x\*ArcCoth[x])/(5\*a\*(a - a\*x^2)^(5/2)) + (4\*x\*ArcCoth[x])/(15\*a^2\*(a - a\*x^2)^(3/2)) + (8\*x\*ArcCoth[x])/(15\*a^3\*Sqrt[a - a\*x^2])

**Rubi [A]** time = 0.0820825, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {5961, 5959}

$$-\frac{8}{15a^3\sqrt{a-ax^2}} - \frac{4}{45a^2(a-ax^2)^{3/2}} + \frac{8x\coth^{-1}(x)}{15a^3\sqrt{a-ax^2}} + \frac{4x\coth^{-1}(x)}{15a^2(a-ax^2)^{3/2}} - \frac{1}{25a(a-ax^2)^{5/2}} + \frac{x\coth^{-1}(x)}{5a(a-ax^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[x]/(a - a\*x^2)^(7/2), x]

[Out] -1/(25\*a\*(a - a\*x^2)^(5/2)) - 4/(45\*a^2\*(a - a\*x^2)^(3/2)) - 8/(15\*a^3\*Sqrt[a - a\*x^2]) + (x\*ArcCoth[x])/(5\*a\*(a - a\*x^2)^(5/2)) + (4\*x\*ArcCoth[x])/(15\*a^2\*(a - a\*x^2)^(3/2)) + (8\*x\*ArcCoth[x])/(15\*a^3\*Sqrt[a - a\*x^2])

### Rule 5961

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))\*((d\_) + (e\_.)\*(x\_)^2)^(q\_), x\_Symbol] :> -Simp[(b\*(d + e\*x^2)^(q + 1))/(4\*c\*d\*(q + 1)^2), x] + (Dist[(2\*q + 3)/(2\*d\*(q + 1)), Int[(d + e\*x^2)^(q + 1)\*(a + b\*ArcCoth[c\*x]), x], x] - Simp[(x\*(d + e\*x^2)^(q + 1)\*(a + b\*ArcCoth[c\*x])]/(2\*d\*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]

### Rule 5959

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))/((d\_) + (e\_.)\*(x\_)^2)^(3/2), x\_Symbol] :> -Simp[b/(c\*d\*Sqrt[d + e\*x^2]), x] + Simp[(x\*(a + b\*ArcCoth[c\*x]))/(d

\*Sqrt[d + e\*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0]

### Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(x)}{(a-ax^2)^{7/2}} dx &= -\frac{1}{25a(a-ax^2)^{5/2}} + \frac{x \coth^{-1}(x)}{5a(a-ax^2)^{5/2}} + \frac{4 \int \frac{\coth^{-1}(x)}{(a-ax^2)^{5/2}} dx}{5a} \\ &= -\frac{1}{25a(a-ax^2)^{5/2}} - \frac{4}{45a^2(a-ax^2)^{3/2}} + \frac{x \coth^{-1}(x)}{5a(a-ax^2)^{5/2}} + \frac{4x \coth^{-1}(x)}{15a^2(a-ax^2)^{3/2}} + \frac{8 \int \frac{\coth^{-1}(x)}{(a-ax^2)^{3/2}} dx}{15a^2} \\ &= -\frac{1}{25a(a-ax^2)^{5/2}} - \frac{4}{45a^2(a-ax^2)^{3/2}} - \frac{8}{15a^3\sqrt{a-ax^2}} + \frac{x \coth^{-1}(x)}{5a(a-ax^2)^{5/2}} + \frac{4x \coth^{-1}(x)}{15a^2(a-ax^2)^{3/2}} + \frac{8x}{15a^2} \end{aligned}$$

**Mathematica [A]** time = 0.0605613, size = 55, normalized size = 0.44

$$\frac{\sqrt{a-ax^2} (120x^4 - 260x^2 - 15(8x^4 - 20x^2 + 15)x \coth^{-1}(x) + 149)}{225a^4(x^2 - 1)^3}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[x]/(a - a\*x^2)^(7/2), x]

[Out] (Sqrt[a - a\*x^2]\*(149 - 260\*x^2 + 120\*x^4 - 15\*x\*(15 - 20\*x^2 + 8\*x^4)\*ArcCoth[x]))/(225\*a^4\*(-1 + x^2)^3)

**Maple [A]** time = 0.261, size = 176, normalized size = 1.4

$$-\frac{(1+x)^2(-1+5 \operatorname{arccoth}(x))\sqrt{-(-1+x)(1+x)a}}{800(-1+x)^3 a^4} + \frac{(5+5x)(-1+3 \operatorname{arccoth}(x))\sqrt{-(-1+x)(1+x)a}}{288(-1+x)^2 a^4} - \frac{5 \operatorname{arccoth}(x)}{(-16+16)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(x)/(-a\*x^2+a)^(7/2), x)

[Out] -1/800\*(1+x)^2\*(-1+5\*arccoth(x))\*(-(-1+x)\*(1+x)\*a)^(1/2)/(-1+x)^3/a^4+5/288\*(1+x)\*(-1+3\*arccoth(x))\*(-(-1+x)\*(1+x)\*a)^(1/2)/(-1+x)^2/a^4-5/16\*(arccoth

$(x-1)*(-(-1+x)*(1+x)*a)^{(1/2)}/(-1+x)/a^4-5/16*(\operatorname{arccoth}(x)+1)*(-(-1+x)*(1+x)*a)^{(1/2)}/(1+x)/a^4+5/288*(1+3*\operatorname{arccoth}(x))*(-1+x)*(-(-1+x)*(1+x)*a)^{(1/2)}/(1+x)^2/a^4-1/800*(1+5*\operatorname{arccoth}(x))*(-1+x)^2*(-(-1+x)*(1+x)*a)^{(1/2)}/(1+x)^3/a^4$

**Maxima [A]** time = 1.0259, size = 134, normalized size = 1.08

$$\frac{1}{15} \left( \frac{8x}{\sqrt{-ax^2+aa^3}} + \frac{4x}{(-ax^2+a)^{\frac{3}{2}}a^2} + \frac{3x}{(-ax^2+a)^{\frac{5}{2}}a} \right) \operatorname{arccoth}(x) - \frac{8}{15\sqrt{-ax^2+aa^3}} - \frac{4}{45(-ax^2+a)^{\frac{3}{2}}a^2} - \frac{1}{25(-ax^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)/(-a\*x^2+a)^(7/2),x, algorithm="maxima")

[Out] 1/15\*(8\*x/(sqrt(-a\*x^2 + a)\*a^3) + 4\*x/((-a\*x^2 + a)^(3/2)\*a^2) + 3\*x/((-a\*x^2 + a)^(5/2)\*a))\*arccoth(x) - 8/15/(sqrt(-a\*x^2 + a)\*a^3) - 4/45/((-a\*x^2 + a)^(3/2)\*a^2) - 1/25/((-a\*x^2 + a)^(5/2)\*a)

**Fricas [A]** time = 1.67926, size = 189, normalized size = 1.52

$$\frac{\left(240x^4 - 520x^2 - 15(8x^5 - 20x^3 + 15x)\log\left(\frac{x+1}{x-1}\right) + 298\right)\sqrt{-ax^2+a}}{450(a^4x^6 - 3a^4x^4 + 3a^4x^2 - a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)/(-a\*x^2+a)^(7/2),x, algorithm="fricas")

[Out] 1/450\*(240\*x^4 - 520\*x^2 - 15\*(8\*x^5 - 20\*x^3 + 15\*x)\*log((x + 1)/(x - 1)) + 298)\*sqrt(-a\*x^2 + a)/(a^4\*x^6 - 3\*a^4\*x^4 + 3\*a^4\*x^2 - a^4)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoath(x)/(-a*x**2+a)**(7/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(x)}{(-ax^2 + a)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoath(x)/(-a*x^2+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(arccoath(x)/(-a*x^2 + a)^(7/2), x)
```

$$3.53 \quad \int \frac{1}{(1-x^2) \coth^{-1}(x)} dx$$

**Optimal.** Leaf size=3

$$\log(\coth^{-1}(x))$$

[Out] Log[ArcCoth[x]]

**Rubi [A]** time = 0.0233692, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {5947}

$$\log(\coth^{-1}(x))$$

Antiderivative was successfully verified.

[In] Int[1/((1 - x^2)\*ArcCoth[x]), x]

[Out] Log[ArcCoth[x]]

#### Rule 5947

```
Int[1/(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol]
  :> Simp[Log[RemoveContent[a + b*ArcCoth[c*x], x]]/(b*c*d), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

#### Rubi steps

$$\int \frac{1}{(1-x^2) \coth^{-1}(x)} dx = \log(\coth^{-1}(x))$$

**Mathematica [A]** time = 0.0250802, size = 3, normalized size = 1.

$$\log(\coth^{-1}(x))$$

Antiderivative was successfully verified.



[In] Integrate[1/((1 - x^2)\*ArcCoth[x]),x]

[Out] Log[ArcCoth[x]]

**Maple [A]** time = 0.043, size = 4, normalized size = 1.3

$$\ln(\operatorname{arccoth}(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^2+1)/arccoth(x),x)

[Out] ln(arccoth(x))

**Maxima [A]** time = 0.975901, size = 4, normalized size = 1.33

$$\log(\operatorname{arccoth}(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+1)/arccoth(x),x, algorithm="maxima")

[Out] log(arccoth(x))

**Fricas [B]** time = 1.50668, size = 36, normalized size = 12.

$$\log\left(\log\left(\frac{x+1}{x-1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+1)/arccoth(x),x, algorithm="fricas")

[Out] log(log((x + 1)/(x - 1)))

**Sympy [A]** time = 0.411158, size = 3, normalized size = 1.

$$\log(\operatorname{acoth}(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x\*\*2+1)/acoth(x), x)

[Out] log(acoth(x))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(x^2 - 1) \operatorname{arccoth}(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+1)/arccoth(x), x, algorithm="giac")

[Out] integrate(-1/((x^2 - 1)\*arccoth(x)), x)

$$3.54 \quad \int \frac{\coth^{-1}(x)^n}{1-x^2} dx$$

Optimal. Leaf size=12

$$\frac{\coth^{-1}(x)^{n+1}}{n+1}$$

[Out] ArcCoth[x]^(1 + n)/(1 + n)

**Rubi [A]** time = 0.0263976, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {5949}

$$\frac{\coth^{-1}(x)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[x]^n/(1 - x^2), x]

[Out] ArcCoth[x]^(1 + n)/(1 + n)

#### Rule 5949

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] :> Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

#### Rubi steps

$$\int \frac{\coth^{-1}(x)^n}{1-x^2} dx = \frac{\coth^{-1}(x)^{1+n}}{1+n}$$

**Mathematica [A]** time = 0.0089221, size = 12, normalized size = 1.

$$\frac{\coth^{-1}(x)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[x]^n/(1 - x^2),x]

[Out] ArcCoth[x]^(1 + n)/(1 + n)

**Maple [A]** time = 0.059, size = 13, normalized size = 1.1

$$\frac{(\operatorname{arccoth}(x))^{1+n}}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(x)^n/(-x^2+1),x)

[Out] arccoth(x)^(1+n)/(1+n)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)^n/(-x^2+1),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 1.61418, size = 182, normalized size = 15.17

$$\frac{\cosh\left(n \log\left(\frac{1}{2} \log\left(\frac{x+1}{x-1}\right)\right)\right) \log\left(\frac{x+1}{x-1}\right) + \log\left(\frac{x+1}{x-1}\right) \sinh\left(n \log\left(\frac{1}{2} \log\left(\frac{x+1}{x-1}\right)\right)\right)}{2(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)^n/(-x^2+1),x, algorithm="fricas")

[Out] 1/2\*(cosh(n\*log(1/2\*log((x + 1)/(x - 1))))\*log((x + 1)/(x - 1)) + log((x + 1)/(x - 1))\*sinh(n\*log(1/2\*log((x + 1)/(x - 1)))))/(n + 1)

---

**Sympy [A]** time = 4.82176, size = 15, normalized size = 1.25

$$\begin{cases} \frac{\operatorname{acoth}^{n+1}(x)}{n+1} & \text{for } n \neq -1 \\ \log(\operatorname{acoth}(x)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(x)\*\*n/(-x\*\*2+1),x)

[Out] Piecewise((acoth(x)\*\*(n + 1)/(n + 1), Ne(n, -1)), (log(acoth(x)), True))

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{\operatorname{arccoth}(x)^n}{x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)^n/(-x^2+1),x, algorithm="giac")

[Out] integrate(-arccoth(x)^n/(x^2 - 1), x)

$$3.55 \quad \int \frac{\coth^{-1}(x)^2}{(1-x^2)^2} dx$$

**Optimal.** Leaf size=62

$$\frac{x}{4(1-x^2)} + \frac{x \coth^{-1}(x)^2}{2(1-x^2)} - \frac{\coth^{-1}(x)}{2(1-x^2)} + \frac{1}{4} \tanh^{-1}(x) + \frac{1}{6} \coth^{-1}(x)^3$$

[Out]  $x/(4*(1 - x^2)) - \text{ArcCoth}[x]/(2*(1 - x^2)) + (x*\text{ArcCoth}[x]^2)/(2*(1 - x^2))$   
 $+ \text{ArcCoth}[x]^3/6 + \text{ArcTanh}[x]/4$

**Rubi [A]** time = 0.0506488, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {5957, 5995, 199, 206}

$$\frac{x}{4(1-x^2)} + \frac{x \coth^{-1}(x)^2}{2(1-x^2)} - \frac{\coth^{-1}(x)}{2(1-x^2)} + \frac{1}{4} \tanh^{-1}(x) + \frac{1}{6} \coth^{-1}(x)^3$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{ArcCoth}[x]^2/(1 - x^2)^2, x]$

[Out]  $x/(4*(1 - x^2)) - \text{ArcCoth}[x]/(2*(1 - x^2)) + (x*\text{ArcCoth}[x]^2)/(2*(1 - x^2))$   
 $+ \text{ArcCoth}[x]^3/6 + \text{ArcTanh}[x]/4$

### Rule 5957

$\text{Int}[(a_.) + \text{ArcCoth}[(c_.)*(x_.)]*(b_.)]^{(p_.)}/((d_.) + (e_.)*(x_.)^2)^2, x\_Symbol] :> \text{Simp}[(x*(a + b*\text{ArcCoth}[c*x])^p)/(2*d*(d + e*x^2)), x] + (-\text{Dist}[(b*c*p)/2, \text{Int}[(x*(a + b*\text{ArcCoth}[c*x])^{(p-1)})/(d + e*x^2)^2, x], x] + \text{Simp}[(a + b*\text{ArcCoth}[c*x])^{(p+1)}/(2*b*c*d^2*(p+1)), x]) /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{GtQ}[p, 0]$

### Rule 5995

$\text{Int}[(a_.) + \text{ArcCoth}[(c_.)*(x_.)]*(b_.)]^{(p_.)*(x_.)*((d_.) + (e_.)*(x_.)^2)^{(q_.)}, x\_Symbol] :> \text{Simp}[(d + e*x^2)^{(q+1)}*(a + b*\text{ArcCoth}[c*x])^p]/(2*e*(q+1)), x] + \text{Dist}[(b*p)/(2*c*(q+1)), \text{Int}[(d + e*x^2)^q*(a + b*\text{ArcCoth}[c*x])^{(p-1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, q\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{GtQ}[p, 0] \&\& \text{NeQ}[q, -1]$

Rule 199

```
Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1)) / (a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1) / (a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x) / Rt[a, 2]]) / (Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(x)^2}{(1-x^2)^2} dx &= \frac{x \coth^{-1}(x)^2}{2(1-x^2)} + \frac{1}{6} \coth^{-1}(x)^3 - \int \frac{x \coth^{-1}(x)}{(1-x^2)^2} dx \\ &= -\frac{\coth^{-1}(x)}{2(1-x^2)} + \frac{x \coth^{-1}(x)^2}{2(1-x^2)} + \frac{1}{6} \coth^{-1}(x)^3 + \frac{1}{2} \int \frac{1}{(1-x^2)^2} dx \\ &= \frac{x}{4(1-x^2)} - \frac{\coth^{-1}(x)}{2(1-x^2)} + \frac{x \coth^{-1}(x)^2}{2(1-x^2)} + \frac{1}{6} \coth^{-1}(x)^3 + \frac{1}{4} \int \frac{1}{1-x^2} dx \\ &= \frac{x}{4(1-x^2)} - \frac{\coth^{-1}(x)}{2(1-x^2)} + \frac{x \coth^{-1}(x)^2}{2(1-x^2)} + \frac{1}{6} \coth^{-1}(x)^3 + \frac{1}{4} \tanh^{-1}(x) \end{aligned}$$

**Mathematica [A]** time = 0.0831476, size = 61, normalized size = 0.98

$$\frac{-3(x^2 - 1) \log(1 - x) + 3(x^2 - 1) \log(x + 1) + 4(x^2 - 1) \coth^{-1}(x)^3 - 6x - 12x \coth^{-1}(x)^2 + 12 \coth^{-1}(x)}{24(x^2 - 1)}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[x]^2/(1 - x^2)^2,x]

[Out] (-6\*x + 12\*ArcCoth[x] - 12\*x\*ArcCoth[x]^2 + 4\*(-1 + x^2)\*ArcCoth[x]^3 - 3\*(-1 + x^2)\*Log[1 - x] + 3\*(-1 + x^2)\*Log[1 + x])/(24\*(-1 + x^2))

**Maple [C]** time = 0.494, size = 707, normalized size = 11.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(x)^2/(-x^2+1)^2,x)`

[Out] 
$$\begin{aligned} & -1/4*\operatorname{arccoth}(x)^2/(-1+x)-1/4*\operatorname{arccoth}(x)^2*\ln(-1+x)-1/4*\operatorname{arccoth}(x)^2/(1+x)+1 \\ & /4*\operatorname{arccoth}(x)^2*\ln(1+x)+1/4*\operatorname{arccoth}(x)^2*\ln((-1+x)/(1+x))+1/24*(6*I*\operatorname{csgn}(I* \\ & (1+x)/(-1+x))^2*\operatorname{csgn}(I/((-1+x)/(1+x))^{(1/2)})*\operatorname{arccoth}(x)^2*\operatorname{Pi}-6*I*\operatorname{arccoth}(x) \\ & ^2*\operatorname{Pi}*\operatorname{csgn}(I/((-1+x)/(1+x))^{(1/2)})*\operatorname{csgn}(I*(1+x)/(-1+x))^2*x^2+3*I*\operatorname{arccoth}(x) \\ & )^2*\operatorname{Pi}*\operatorname{csgn}(I*(1+x)/(-1+x))^3*x^2+3*I*\operatorname{csgn}(I/((1+x)/(-1+x)-1))*\operatorname{csgn}(I*(1+x) \\ & )/(-1+x)/((1+x)/(-1+x)-1))^2*\operatorname{arccoth}(x)^2*\operatorname{Pi}+3*I*\operatorname{arccoth}(x)^2*\operatorname{Pi}*\operatorname{csgn}(I*(1+x) \\ & )/(-1+x)/((1+x)/(-1+x)-1))*\operatorname{csgn}(I*(1+x)/(-1+x))*\operatorname{csgn}(I/((1+x)/(-1+x)-1))*x^2 \\ & -3*I*\operatorname{arccoth}(x)^2*\operatorname{Pi}*\operatorname{csgn}(I*(1+x)/(-1+x)/((1+x)/(-1+x)-1))^2*\operatorname{csgn}(I*(1+x)/ \\ & (-1+x))*x^2+3*I*\operatorname{arccoth}(x)^2*\operatorname{Pi}*\operatorname{csgn}(I/((-1+x)/(1+x))^{(1/2)})^2*\operatorname{csgn}(I*(1+x) \\ & )/(-1+x))*x^2-3*I*\operatorname{csgn}(I*(1+x)/(-1+x))^3*\operatorname{arccoth}(x)^2*\operatorname{Pi}-3*I*\operatorname{arccoth}(x)^2*\operatorname{Pi} \\ & *\operatorname{csgn}(I*(1+x)/(-1+x)/((1+x)/(-1+x)-1))^2*\operatorname{csgn}(I/((1+x)/(-1+x)-1))*x^2-3*I*\operatorname{c} \\ & \operatorname{sgn}(I*(1+x)/(-1+x)/((1+x)/(-1+x)-1))^3*\operatorname{arccoth}(x)^2*\operatorname{Pi}+3*I*\operatorname{csgn}(I*(1+x)/(-1 \\ & +x))*\operatorname{csgn}(I*(1+x)/(-1+x)/((1+x)/(-1+x)-1))^2*\operatorname{arccoth}(x)^2*\operatorname{Pi}+3*I*\operatorname{arccoth}(x) \\ & ^2*\operatorname{Pi}*\operatorname{csgn}(I*(1+x)/(-1+x)/((1+x)/(-1+x)-1))^3*x^2-3*I*\operatorname{csgn}(I*(1+x)/(-1+x))* \\ & \operatorname{csgn}(I/((-1+x)/(1+x))^{(1/2)})^2*\operatorname{arccoth}(x)^2*\operatorname{Pi}-3*I*\operatorname{csgn}(I/((1+x)/(-1+x)-1)) \\ & *\operatorname{csgn}(I*(1+x)/(-1+x))*\operatorname{csgn}(I*(1+x)/(-1+x)/((1+x)/(-1+x)-1))*\operatorname{arccoth}(x)^2*\operatorname{Pi} \\ & +4*x^2*\operatorname{arccoth}(x)^3-4*\operatorname{arccoth}(x)^3+6*\operatorname{arccoth}(x)*x^2+6*\operatorname{arccoth}(x)-6*x)/(-1+x) \\ & )/(1+x) \end{aligned}$$

**Maxima [B]** time = 1.02543, size = 231, normalized size = 3.73

$$-\frac{1}{4}\left(\frac{2x}{x^2-1}-\log(x+1)+\log(x-1)\right)\operatorname{arccoth}(x)^2-\frac{\left((x^2-1)\log(x+1)^2-2(x^2-1)\log(x+1)\log(x-1)+(x^2-1)\log(x-1)^2\right)}{8(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(x)^2/(-x^2+1)^2,x, algorithm="maxima")`

[Out] 
$$\begin{aligned} & -1/4*(2*x/(x^2-1)-\log(x+1)+\log(x-1))*\operatorname{arccoth}(x)^2-1/8*((x^2-1) \\ & )*\log(x+1)^2-2*(x^2-1)*\log(x+1)*\log(x-1)+(x^2-1)*\log(x-1)^2 \\ & -4*\operatorname{arccoth}(x)/(x^2-1)+1/48*((x^2-1)*\log(x+1)^3-3*(x^2-1)*\log \\ & (x+1)^2*\log(x-1)-(x^2-1)*\log(x-1)^3+3*((x^2-1)*\log(x-1)^2+ \\ & 2*x^2-2)*\log(x+1)-6*(x^2-1)*\log(x-1)-12*x)/(x^2-1) \end{aligned}$$



---

**Fricas [A]** time = 1.56907, size = 165, normalized size = 2.66

$$\frac{(x^2 - 1) \log\left(\frac{x+1}{x-1}\right)^3 - 6x \log\left(\frac{x+1}{x-1}\right)^2 + 6(x^2 + 1) \log\left(\frac{x+1}{x-1}\right) - 12x}{48(x^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)^2/(-x^2+1)^2,x, algorithm="fricas")

[Out] 1/48\*((x^2 - 1)\*log((x + 1)/(x - 1))^3 - 6\*x\*log((x + 1)/(x - 1))^2 + 6\*(x^2 + 1)\*log((x + 1)/(x - 1)) - 12\*x)/(x^2 - 1)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^2(x)}{(x-1)^2(x+1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(x)\*\*2/(-x\*\*2+1)\*\*2,x)

[Out] Integral(acoth(x)\*\*2/((x - 1)\*\*2\*(x + 1)\*\*2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(x)^2}{(x^2 - 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)^2/(-x^2+1)^2,x, algorithm="giac")

[Out] integrate(arccoth(x)^2/(x^2 - 1)^2, x)

$$3.56 \quad \int \frac{x \coth^{-1}(x)}{1-x^2} dx$$

**Optimal.** Leaf size=37

$$\frac{1}{2} \text{PolyLog}\left(2, \frac{x+1}{x-1}\right) - \frac{1}{2} \coth^{-1}(x)^2 + \log\left(\frac{2}{1-x}\right) \coth^{-1}(x)$$

[Out]  $-\text{ArcCoth}[x]^2/2 + \text{ArcCoth}[x]*\text{Log}[2/(1-x)] + \text{PolyLog}[2, (1+x)/(-1+x)]/2$

**Rubi [A]** time = 0.0592361, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {5985, 5919, 2402, 2315}

$$\frac{1}{2} \text{PolyLog}\left(2, \frac{x+1}{x-1}\right) - \frac{1}{2} \coth^{-1}(x)^2 + \log\left(\frac{2}{1-x}\right) \coth^{-1}(x)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x*\text{ArcCoth}[x])/(1-x^2), x]$

[Out]  $-\text{ArcCoth}[x]^2/2 + \text{ArcCoth}[x]*\text{Log}[2/(1-x)] + \text{PolyLog}[2, (1+x)/(-1+x)]/2$

#### Rule 5985

$\text{Int}[((a_.) + \text{ArcCoth}[(c_.)*(x_.)]*(b_.))^{(p_.)*(x_.)}/((d_.) + (e_.)*(x_.)^2), x\_Symbol] \rightarrow \text{Simp}[(a + b*\text{ArcCoth}[c*x])^{(p+1)}/(b*e*(p+1)), x] + \text{Dist}[1/(c*d), \text{Int}[(a + b*\text{ArcCoth}[c*x])^p/(1-c*x), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{IGtQ}[p, 0]$

#### Rule 5919

$\text{Int}[((a_.) + \text{ArcCoth}[(c_.)*(x_.)]*(b_.))^{(p_.)}/((d_.) + (e_.)*(x_.)), x\_Symbol] \rightarrow -\text{Simp}[(a + b*\text{ArcCoth}[c*x])^p*\text{Log}[2/(1+(e*x)/d)]/e, x] + \text{Dist}[(b*c*p)/e, \text{Int}[(a + b*\text{ArcCoth}[c*x])^{(p-1)}*\text{Log}[2/(1+(e*x)/d)]/(1-c^2*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{IGtQ}[p, 0] \&\& \text{EqQ}[c^2*d^2 - e^2, 0]$

#### Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

### Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{x \coth^{-1}(x)}{1-x^2} dx &= -\frac{1}{2} \coth^{-1}(x)^2 + \int \frac{\coth^{-1}(x)}{1-x} dx \\ &= -\frac{1}{2} \coth^{-1}(x)^2 + \coth^{-1}(x) \log\left(\frac{2}{1-x}\right) - \int \frac{\log\left(\frac{2}{1-x}\right)}{1-x^2} dx \\ &= -\frac{1}{2} \coth^{-1}(x)^2 + \coth^{-1}(x) \log\left(\frac{2}{1-x}\right) + \text{Subst}\left(\int \frac{\log(2x)}{1-2x} dx, x, \frac{1}{1-x}\right) \\ &= -\frac{1}{2} \coth^{-1}(x)^2 + \coth^{-1}(x) \log\left(\frac{2}{1-x}\right) + \frac{1}{2} \text{Li}_2\left(\frac{1+x}{-1+x}\right) \end{aligned}$$

**Mathematica [A]** time = 0.0491352, size = 34, normalized size = 0.92

$$\frac{1}{2} \left( \coth^{-1}(x) \left( \coth^{-1}(x) + 2 \log\left(1 - e^{-2 \coth^{-1}(x)}\right) \right) - \text{PolyLog}\left(2, e^{-2 \coth^{-1}(x)}\right) \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*ArcCoth[x])/(1 - x^2), x]
```

```
[Out] (ArcCoth[x]*(ArcCoth[x] + 2*Log[1 - E^(-2*ArcCoth[x])]) - PolyLog[2, E^(-2*
ArcCoth[x])])/2
```

**Maple [B]** time = 0.035, size = 75, normalized size = 2.

$$\frac{\operatorname{arccoth}(x) \ln(-1+x)}{2} - \frac{\operatorname{arccoth}(x) \ln(1+x)}{2} - \frac{(\ln(-1+x))^2}{8} + \frac{1}{2} \operatorname{dilog}\left(\frac{1}{2} + \frac{x}{2}\right) + \frac{\ln(-1+x)}{4} \ln\left(\frac{1}{2} + \frac{x}{2}\right) - \frac{1}{4} \left( \ln\left(\frac{1}{2} + \frac{x}{2}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*arccoth(x)/(-x^2+1),x)`

[Out]  $-1/2*\operatorname{arccoth}(x)*\ln(-1+x)-1/2*\operatorname{arccoth}(x)*\ln(1+x)-1/8*\ln(-1+x)^2+1/2*\operatorname{dilog}(1/2+1/2*x)+1/4*\ln(-1+x)*\ln(1/2+1/2*x)-1/4*(\ln(1+x)-\ln(1/2+1/2*x))*\ln(-1/2*x+1/2)+1/8*\ln(1+x)^2$

**Maxima [B]** time = 0.96325, size = 103, normalized size = 2.78

$$\frac{1}{4}(\log(x+1) - \log(x-1))\log(x^2-1) - \frac{1}{2}\operatorname{arccoth}(x)\log(x^2-1) - \frac{1}{8}\log(x+1)^2 - \frac{1}{4}\log(x+1)\log(x-1) + \frac{1}{8}\log(x-1)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccoth(x)/(-x^2+1),x, algorithm="maxima")`

[Out]  $1/4*(\log(x+1) - \log(x-1))*\log(x^2-1) - 1/2*\operatorname{arccoth}(x)*\log(x^2-1) - 1/8*\log(x+1)^2 - 1/4*\log(x+1)*\log(x-1) + 1/8*\log(x-1)^2 + 1/2*\log(x-1)*\log(1/2*x+1/2) + 1/2*\operatorname{dilog}(-1/2*x+1/2)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{x \operatorname{arccoth}(x)}{x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccoth(x)/(-x^2+1),x, algorithm="fricas")`

[Out] `integral(-x*arccoth(x)/(x^2-1), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{x \operatorname{arccoth}(x)}{x^2-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(x)/(-x**2+1),x)
```

```
[Out] -Integral(x*acoth(x)/(x**2 - 1), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{x \operatorname{arccoth}(x)}{x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(x)/(-x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-x*arccoth(x)/(x^2 - 1), x)
```

$$3.57 \quad \int \frac{\coth^{-1}(x)}{1-x^2} dx$$

**Optimal.** Leaf size=8

$$\frac{1}{2} \coth^{-1}(x)^2$$

[Out] ArcCoth[x]^2/2

**Rubi [A]** time = 0.0138965, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {5949}

$$\frac{1}{2} \coth^{-1}(x)^2$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[x]/(1 - x^2), x]

[Out] ArcCoth[x]^2/2

**Rule 5949**

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

**Rubi steps**

$$\int \frac{\coth^{-1}(x)}{1-x^2} dx = \frac{1}{2} \coth^{-1}(x)^2$$

**Mathematica [A]** time = 0.0039697, size = 8, normalized size = 1.

$$\frac{1}{2} \coth^{-1}(x)^2$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[x]/(1 - x^2),x]

[Out] ArcCoth[x]^2/2

**Maple [A]** time = 0.029, size = 13, normalized size = 1.6

$$\operatorname{Artanh}(x) \operatorname{arccoth}(x) - \frac{(\operatorname{Artanh}(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(x)/(-x^2+1),x)

[Out] arctanh(x)\*arccoth(x)-1/2\*arctanh(x)^2

**Maxima [A]** time = 0.953711, size = 8, normalized size = 1.

$$\frac{1}{2} \operatorname{arccoth}(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)/(-x^2+1),x, algorithm="maxima")

[Out] 1/2\*arccoth(x)^2

**Fricas [B]** time = 1.55847, size = 38, normalized size = 4.75

$$\frac{1}{8} \log\left(\frac{x+1}{x-1}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)/(-x^2+1),x, algorithm="fricas")

[Out] 1/8\*log((x + 1)/(x - 1))^2

---

**Sympy [A]** time = 0.817232, size = 5, normalized size = 0.62

$$\frac{\operatorname{acoth}^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(x)/(-x\*\*2+1),x)

[Out] acoth(x)\*\*2/2

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{\operatorname{arccoth}(x)}{x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)/(-x^2+1),x, algorithm="giac")

[Out] integrate(-arccoth(x)/(x^2 - 1), x)



$$3.58 \quad \int \frac{x \coth^{-1}(x)}{(1-x^2)^2} dx$$

**Optimal.** Leaf size=36

$$-\frac{x}{4(1-x^2)} + \frac{\coth^{-1}(x)}{2(1-x^2)} - \frac{1}{4} \tanh^{-1}(x)$$

[Out]  $-x/(4*(1 - x^2)) + \text{ArcCoth}[x]/(2*(1 - x^2)) - \text{ArcTanh}[x]/4$

**Rubi [A]** time = 0.0307194, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {5995, 199, 206}

$$-\frac{x}{4(1-x^2)} + \frac{\coth^{-1}(x)}{2(1-x^2)} - \frac{1}{4} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x*\text{ArcCoth}[x])/(1 - x^2)^2, x]$

[Out]  $-x/(4*(1 - x^2)) + \text{ArcCoth}[x]/(2*(1 - x^2)) - \text{ArcTanh}[x]/4$

### Rule 5995

$\text{Int}[(a_.) + \text{ArcCoth}[(c_.)*(x_)]*(b_.)]^{(p_.)}*(x_)*((d_.) + (e_.)*(x_)^2)^{(q_.)}, x\_Symbol] :> \text{Simp}[(d + e*x^2)^{(q + 1)}*(a + b*\text{ArcCoth}[c*x])^p]/(2*e*(q + 1)), x] + \text{Dist}[(b*p)/(2*c*(q + 1)), \text{Int}[(d + e*x^2)^q*(a + b*\text{ArcCoth}[c*x])^{(p - 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, q\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{GtQ}[p, 0] \&\& \text{NeQ}[q, -1]$

### Rule 199

$\text{Int}[(a_.) + (b_.)*(x_)^{(n_)}]^{(p_.)}, x\_Symbol] :> -\text{Simp}[(x*(a + b*x^n)^{(p + 1)})/(a*n*(p + 1)), x] + \text{Dist}[(n*(p + 1) + 1)/(a*n*(p + 1)), \text{Int}[(a + b*x^n)^{(p + 1)}, x], x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[p, -1] \&\& (\text{IntegerQ}[2*p] || (n == 2 \&\& \text{IntegerQ}[4*p]) || (n == 2 \&\& \text{IntegerQ}[3*p]) || \text{Denominator}[p + 1/n] < \text{Denominator}[p])$

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rubi steps

$$\begin{aligned} \int \frac{x \coth^{-1}(x)}{(1-x^2)^2} dx &= \frac{\coth^{-1}(x)}{2(1-x^2)} - \frac{1}{2} \int \frac{1}{(1-x^2)^2} dx \\ &= -\frac{x}{4(1-x^2)} + \frac{\coth^{-1}(x)}{2(1-x^2)} - \frac{1}{4} \int \frac{1}{1-x^2} dx \\ &= -\frac{x}{4(1-x^2)} + \frac{\coth^{-1}(x)}{2(1-x^2)} - \frac{1}{4} \tanh^{-1}(x) \end{aligned}$$

**Mathematica [A]** time = 0.027431, size = 44, normalized size = 1.22

$$\frac{x}{4(x^2-1)} - \frac{\coth^{-1}(x)}{2(x^2-1)} + \frac{1}{8} \log(1-x) - \frac{1}{8} \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcCoth[x])/(1 - x^2)^2,x]
```

```
[Out] x/(4*(-1 + x^2)) - ArcCoth[x]/(2*(-1 + x^2)) + Log[1 - x]/8 - Log[1 + x]/8
```

**Maple [A]** time = 0.027, size = 39, normalized size = 1.1

$$-\frac{\operatorname{arccoth}(x)}{2x^2-2} + \frac{1}{-8+8x} + \frac{\ln(-1+x)}{8} + \frac{1}{8+8x} - \frac{\ln(1+x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(x)/(-x^2+1)^2,x)
```

```
[Out] -1/2/(x^2-1)*arccoth(x)+1/8/(-1+x)+1/8*ln(-1+x)+1/8/(1+x)-1/8*ln(1+x)
```

**Maxima [A]** time = 0.946376, size = 46, normalized size = 1.28

$$\frac{x}{4(x^2-1)} - \frac{\operatorname{arccoth}(x)}{2(x^2-1)} - \frac{1}{8} \log(x+1) + \frac{1}{8} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(x)/(-x^2+1)^2,x, algorithm="maxima")

[Out] 1/4\*x/(x^2 - 1) - 1/2\*arccoth(x)/(x^2 - 1) - 1/8\*log(x + 1) + 1/8\*log(x - 1)

**Fricas [A]** time = 1.58823, size = 74, normalized size = 2.06

$$\frac{(x^2 + 1) \log\left(\frac{x+1}{x-1}\right) - 2x}{8(x^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(x)/(-x^2+1)^2,x, algorithm="fricas")

[Out] -1/8\*((x^2 + 1)\*log((x + 1)/(x - 1)) - 2\*x)/(x^2 - 1)

**Sympy [A]** time = 0.819492, size = 31, normalized size = 0.86

$$-\frac{x^2 \operatorname{acoth}(x)}{4x^2 - 4} + \frac{x}{4x^2 - 4} - \frac{\operatorname{acoth}(x)}{4x^2 - 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(x)/(-x\*\*2+1)\*\*2,x)

[Out] -x\*\*2\*acoth(x)/(4\*x\*\*2 - 4) + x/(4\*x\*\*2 - 4) - acoth(x)/(4\*x\*\*2 - 4)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x \operatorname{arccoth}(x)}{(x^2 - 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(x)/(-x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(x)/(x^2 - 1)^2, x)
```

$$3.59 \quad \int \frac{\coth^{-1}(x)}{(1-x^2)^2} dx$$

**Optimal.** Leaf size=38

$$-\frac{1}{4(1-x^2)} + \frac{x \coth^{-1}(x)}{2(1-x^2)} + \frac{1}{4} \coth^{-1}(x)^2$$

[Out]  $-1/(4*(1 - x^2)) + (x*ArcCoth[x])/(2*(1 - x^2)) + ArcCoth[x]^2/4$

**Rubi [A]** time = 0.0174931, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {5957, 261}

$$-\frac{1}{4(1-x^2)} + \frac{x \coth^{-1}(x)}{2(1-x^2)} + \frac{1}{4} \coth^{-1}(x)^2$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[x]/(1 - x^2)^2, x]

[Out]  $-1/(4*(1 - x^2)) + (x*ArcCoth[x])/(2*(1 - x^2)) + ArcCoth[x]^2/4$

#### Rule 5957

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)^2)^2, x\_Symbol] :> Simp[(x\*(a + b\*ArcCoth[c\*x])^p)/(2\*d\*(d + e\*x^2)), x] + (-Dist[(b\*c\*p)/2, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(d + e\*x^2)^2, x], x] + Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(2\*b\*c\*d^2\*(p + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && GtQ[p, 0]

#### Rule 261

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] :> Simp[(a + b\*x^n)^(p + 1)/(b\*n\*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

#### Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(x)}{(1-x^2)^2} dx &= \frac{x \coth^{-1}(x)}{2(1-x^2)} + \frac{1}{4} \coth^{-1}(x)^2 - \frac{1}{2} \int \frac{x}{(1-x^2)^2} dx \\ &= -\frac{1}{4(1-x^2)} + \frac{x \coth^{-1}(x)}{2(1-x^2)} + \frac{1}{4} \coth^{-1}(x)^2\end{aligned}$$

**Mathematica [A]** time = 0.0323973, size = 28, normalized size = 0.74

$$\frac{(x^2 - 1) \coth^{-1}(x)^2 - 2x \coth^{-1}(x) + 1}{4(x^2 - 1)}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[x]/(1 - x^2)^2, x]

[Out] (1 - 2\*x\*ArcCoth[x] + (-1 + x^2)\*ArcCoth[x]^2)/(4\*(-1 + x^2))

**Maple [B]** time = 0.043, size = 99, normalized size = 2.6

$$-\frac{\operatorname{arccoth}(x)}{-4+4x} - \frac{\operatorname{arccoth}(x) \ln(-1+x)}{4} - \frac{\operatorname{arccoth}(x)}{4+4x} + \frac{\operatorname{arccoth}(x) \ln(1+x)}{4} + \frac{1}{8} \left( \ln(1+x) - \ln\left(\frac{1}{2} + \frac{x}{2}\right) \right) \ln\left(-\frac{x}{2} + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(x)/(-x^2+1)^2, x)

[Out] -1/4\*arccoth(x)/(-1+x)-1/4\*arccoth(x)\*ln(-1+x)-1/4\*arccoth(x)/(1+x)+1/4\*arccoth(x)\*ln(1+x)+1/8\*(ln(1+x)-ln(1/2+1/2\*x))\*ln(-1/2\*x+1/2)-1/16\*ln(1+x)^2-1/16\*ln(-1+x)^2+1/8\*ln(-1+x)\*ln(1/2+1/2\*x)+1/8/(-1+x)-1/8/(1+x)

**Maxima [B]** time = 0.967018, size = 103, normalized size = 2.71

$$-\frac{1}{4} \left( \frac{2x}{x^2-1} - \log(x+1) + \log(x-1) \right) \operatorname{arccoth}(x) - \frac{(x^2-1) \log(x+1)^2 - 2(x^2-1) \log(x+1) \log(x-1) + (x^2-1) \log(x-1)^2}{16(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)/(-x^2+1)^2,x, algorithm="maxima")

[Out]  $-1/4*(2*x/(x^2 - 1) - \log(x + 1) + \log(x - 1))*\operatorname{arccoth}(x) - 1/16*((x^2 - 1) * \log(x + 1)^2 - 2*(x^2 - 1)*\log(x + 1)*\log(x - 1) + (x^2 - 1)*\log(x - 1)^2 - 4)/(x^2 - 1)$

**Fricas [A]** time = 1.58369, size = 111, normalized size = 2.92

$$\frac{(x^2 - 1) \log\left(\frac{x+1}{x-1}\right)^2 - 4x \log\left(\frac{x+1}{x-1}\right) + 4}{16(x^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)/(-x^2+1)^2,x, algorithm="fricas")

[Out]  $1/16*((x^2 - 1)*\log((x + 1)/(x - 1))^2 - 4*x*\log((x + 1)/(x - 1)) + 4)/(x^2 - 1)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}(x)}{(x-1)^2(x+1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(x)/(-x\*\*2+1)\*\*2,x)

[Out] Integral(acoth(x)/((x - 1)\*\*2\*(x + 1)\*\*2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(x)}{(x^2 - 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(x)/(x^2 - 1)^2, x)
```



$$3.60 \quad \int \frac{x \coth^{-1}(x)}{(1-x^2)^3} dx$$

**Optimal.** Leaf size=50

$$-\frac{3x}{32(1-x^2)} - \frac{x}{16(1-x^2)^2} + \frac{\coth^{-1}(x)}{4(1-x^2)^2} - \frac{3}{32} \tanh^{-1}(x)$$

[Out]  $-x/(16*(1 - x^2)^2) - (3*x)/(32*(1 - x^2)) + \text{ArcCoth}[x]/(4*(1 - x^2)^2) - (3*\text{ArcTanh}[x])/32$

**Rubi [A]** time = 0.0341809, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {5995, 199, 206}

$$-\frac{3x}{32(1-x^2)} - \frac{x}{16(1-x^2)^2} + \frac{\coth^{-1}(x)}{4(1-x^2)^2} - \frac{3}{32} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x*\text{ArcCoth}[x])/(1 - x^2)^3, x]$

[Out]  $-x/(16*(1 - x^2)^2) - (3*x)/(32*(1 - x^2)) + \text{ArcCoth}[x]/(4*(1 - x^2)^2) - (3*\text{ArcTanh}[x])/32$

### Rule 5995

$\text{Int}[(a + \text{ArcCoth}[c*(x)]*(b))^p*(x)*((d) + (e)*(x)^2)^q, x\_Symbol] \rightarrow \text{Simp}[(d + e*x^2)^{q+1}*(a + b*\text{ArcCoth}[c*x])^p]/(2*e*(q+1)), x] + \text{Dist}[(b*p)/(2*c*(q+1)), \text{Int}[(d + e*x^2)^q*(a + b*\text{ArcCoth}[c*x])^{p-1}, x], x] /;$  FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2\*d + e, 0] && GtQ[p, 0] && NeQ[q, -1]

### Rule 199

$\text{Int}[(a + (b)*(x)^n)^p, x\_Symbol] \rightarrow -\text{Simp}[(x*(a + b*x^n)^{p+1})/(a*n*(p+1)), x] + \text{Dist}[(n*(p+1) + 1)/(a*n*(p+1)), \text{Int}[(a + b*x^n)^{p+1}, x], x] /;$  FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2\*p] || (n == 2 && IntegerQ[4\*p]) || (n == 2 && IntegerQ[3\*p]) || Denomin

ator[p + 1/n] < Denominator[p])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned} \int \frac{x \coth^{-1}(x)}{(1-x^2)^3} dx &= \frac{\coth^{-1}(x)}{4(1-x^2)^2} - \frac{1}{4} \int \frac{1}{(1-x^2)^3} dx \\ &= -\frac{x}{16(1-x^2)^2} + \frac{\coth^{-1}(x)}{4(1-x^2)^2} - \frac{3}{16} \int \frac{1}{(1-x^2)^2} dx \\ &= -\frac{x}{16(1-x^2)^2} - \frac{3x}{32(1-x^2)} + \frac{\coth^{-1}(x)}{4(1-x^2)^2} - \frac{3}{32} \int \frac{1}{1-x^2} dx \\ &= -\frac{x}{16(1-x^2)^2} - \frac{3x}{32(1-x^2)} + \frac{\coth^{-1}(x)}{4(1-x^2)^2} - \frac{3}{32} \tanh^{-1}(x) \end{aligned}$$

**Mathematica [A]** time = 0.0505672, size = 50, normalized size = 1.

$$\frac{1}{64} \left( \frac{6x}{x^2-1} - \frac{4x}{(x^2-1)^2} + \frac{16 \coth^{-1}(x)}{(x^2-1)^2} + 3 \log(1-x) - 3 \log(x+1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x\*ArcCoth[x])/(1 - x^2)^3, x]

[Out] ((-4\*x)/(-1 + x^2)^2 + (6\*x)/(-1 + x^2) + (16\*ArcCoth[x])/(-1 + x^2)^2 + 3\*Log[1 - x] - 3\*Log[1 + x])/64

**Maple [A]** time = 0.028, size = 53, normalized size = 1.1

$$\frac{\operatorname{arccoth}(x)}{4(x^2-1)^2} - \frac{1}{64(-1+x)^2} + \frac{3}{-64+64x} + \frac{3 \ln(-1+x)}{64} + \frac{1}{64(1+x)^2} + \frac{3}{64+64x} - \frac{3 \ln(1+x)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*arccoth(x)/(-x^2+1)^3,x)`

[Out]  $1/4/(x^2-1)^2 \operatorname{arccoth}(x) - 1/64/(-1+x)^2 + 3/64/(-1+x) + 3/64 \ln(-1+x) + 1/64/(1+x)^2 + 3/64/(1+x) - 3/64 \ln(1+x)$

**Maxima [A]** time = 0.949001, size = 63, normalized size = 1.26

$$\frac{3x^3 - 5x}{32(x^4 - 2x^2 + 1)} + \frac{\operatorname{arccoth}(x)}{4(x^2 - 1)^2} - \frac{3}{64} \log(x + 1) + \frac{3}{64} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccoth(x)/(-x^2+1)^3,x, algorithm="maxima")`

[Out]  $1/32*(3*x^3 - 5*x)/(x^4 - 2*x^2 + 1) + 1/4*\operatorname{arccoth}(x)/(x^2 - 1)^2 - 3/64*\log(x + 1) + 3/64*\log(x - 1)$

**Fricas [A]** time = 1.59255, size = 111, normalized size = 2.22

$$\frac{6x^3 - (3x^4 - 6x^2 - 5) \log\left(\frac{x+1}{x-1}\right) - 10x}{64(x^4 - 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccoth(x)/(-x^2+1)^3,x, algorithm="fricas")`

[Out]  $1/64*(6*x^3 - (3*x^4 - 6*x^2 - 5)*\log((x + 1)/(x - 1)) - 10*x)/(x^4 - 2*x^2 + 1)$

**Sympy [B]** time = 1.32896, size = 88, normalized size = 1.76

$$-\frac{3x^4 \operatorname{acoth}(x)}{32x^4 - 64x^2 + 32} + \frac{3x^3}{32x^4 - 64x^2 + 32} + \frac{6x^2 \operatorname{acoth}(x)}{32x^4 - 64x^2 + 32} - \frac{5x}{32x^4 - 64x^2 + 32} + \frac{5 \operatorname{acoth}(x)}{32x^4 - 64x^2 + 32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(x)/(-x\*\*2+1)\*\*3,x)

[Out]  $-3x^4 \operatorname{acoth}(x)/(32x^4 - 64x^2 + 32) + 3x^3/(32x^4 - 64x^2 + 32) + 6x^2 \operatorname{acoth}(x)/(32x^4 - 64x^2 + 32) - 5x/(32x^4 - 64x^2 + 32) + 5 \operatorname{acoth}(x)/(32x^4 - 64x^2 + 32)$

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{x \operatorname{arccoth}(x)}{(x^2 - 1)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(x)/(-x^2+1)^3,x, algorithm="giac")

[Out] integrate(-x\*arccoth(x)/(x^2 - 1)^3, x)

$$3.61 \quad \int \frac{\coth^{-1}(x)}{(1-x^2)^3} dx$$

Optimal. Leaf size=67

$$-\frac{3}{16(1-x^2)} - \frac{1}{16(1-x^2)^2} + \frac{3x \coth^{-1}(x)}{8(1-x^2)} + \frac{x \coth^{-1}(x)}{4(1-x^2)^2} + \frac{3}{16} \coth^{-1}(x)^2$$

[Out] -1/(16\*(1 - x^2)^2) - 3/(16\*(1 - x^2)) + (x\*ArcCoth[x])/(4\*(1 - x^2)^2) + (3\*x\*ArcCoth[x])/(8\*(1 - x^2)) + (3\*ArcCoth[x]^2)/16

**Rubi [A]** time = 0.0352498, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {5961, 5957, 261}

$$-\frac{3}{16(1-x^2)} - \frac{1}{16(1-x^2)^2} + \frac{3x \coth^{-1}(x)}{8(1-x^2)} + \frac{x \coth^{-1}(x)}{4(1-x^2)^2} + \frac{3}{16} \coth^{-1}(x)^2$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[x]/(1 - x^2)^3, x]

[Out] -1/(16\*(1 - x^2)^2) - 3/(16\*(1 - x^2)) + (x\*ArcCoth[x])/(4\*(1 - x^2)^2) + (3\*x\*ArcCoth[x])/(8\*(1 - x^2)) + (3\*ArcCoth[x]^2)/16

### Rule 5961

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))\*((d\_) + (e\_.)\*(x\_)^2)^(q\_), x\_Symbol] :> -Simp[(b\*(d + e\*x^2)^(q + 1))/(4\*c\*d\*(q + 1)^2), x] + (Dist[(2\*q + 3)/(2\*d\*(q + 1)), Int[(d + e\*x^2)^(q + 1)\*(a + b\*ArcCoth[c\*x]), x], x] - Simp[(x\*(d + e\*x^2)^(q + 1)\*(a + b\*ArcCoth[c\*x])]/(2\*d\*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]

### Rule 5957

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)^2)^2, x\_Symbol] :> Simp[(x\*(a + b\*ArcCoth[c\*x])^p)/(2\*d\*(d + e\*x^2)), x] + (-Dist[(b\*c\*p)/2, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(d + e\*x^2)^2, x], x] + Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(2\*b\*c\*d^2\*(p + 1)), x]) /; FreeQ[{a, b, c, d,

e}, x] && EqQ[c^2\*d + e, 0] && GtQ[p, 0]

### Rule 261

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a + b\*x^n)^(p + 1)/(b\*n\*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

### Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(x)}{(1-x^2)^3} dx &= -\frac{1}{16(1-x^2)^2} + \frac{x \coth^{-1}(x)}{4(1-x^2)^2} + \frac{3}{4} \int \frac{\coth^{-1}(x)}{(1-x^2)^2} dx \\ &= -\frac{1}{16(1-x^2)^2} + \frac{x \coth^{-1}(x)}{4(1-x^2)^2} + \frac{3x \coth^{-1}(x)}{8(1-x^2)} + \frac{3}{16} \coth^{-1}(x)^2 - \frac{3}{8} \int \frac{x}{(1-x^2)^2} dx \\ &= -\frac{1}{16(1-x^2)^2} - \frac{3}{16(1-x^2)} + \frac{x \coth^{-1}(x)}{4(1-x^2)^2} + \frac{3x \coth^{-1}(x)}{8(1-x^2)} + \frac{3}{16} \coth^{-1}(x)^2 \end{aligned}$$

**Mathematica [A]** time = 0.0523017, size = 43, normalized size = 0.64

$$-\frac{-3x^2 + 2(3x^2 - 5)x \coth^{-1}(x) - 3(x^2 - 1)^2 \coth^{-1}(x)^2 + 4}{16(x^2 - 1)^2}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[x]/(1 - x^2)^3, x]

[Out] -(4 - 3\*x^2 + 2\*x\*(-5 + 3\*x^2)\*ArcCoth[x] - 3\*(-1 + x^2)^2\*ArcCoth[x]^2)/(16\*(-1 + x^2)^2)

**Maple [B]** time = 0.049, size = 131, normalized size = 2.

$$\frac{\operatorname{arccoth}(x)}{16(-1+x)^2} - \frac{3 \operatorname{arccoth}(x)}{-16+16x} - \frac{3 \operatorname{arccoth}(x) \ln(-1+x)}{16} - \frac{\operatorname{arccoth}(x)}{16(1+x)^2} - \frac{3 \operatorname{arccoth}(x)}{16+16x} + \frac{3 \operatorname{arccoth}(x) \ln(1+x)}{16} - \frac{3}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(x)/(-x^2+1)^3,x)`

[Out]  $\frac{1}{16}\operatorname{arccoth}(x)/(-1+x)^2 - \frac{3}{16}\operatorname{arccoth}(x)/(-1+x) - \frac{3}{16}\operatorname{arccoth}(x)\ln(-1+x) - \frac{1}{16}\operatorname{arccoth}(x)/(1+x)^2 - \frac{3}{16}\operatorname{arccoth}(x)/(1+x) + \frac{3}{16}\operatorname{arccoth}(x)\ln(1+x) - \frac{3}{64}\ln(-1+x)^2 + \frac{3}{32}\ln(-1+x)\ln(1/2+1/2*x) + \frac{3}{32}(\ln(1+x) - \ln(1/2+1/2*x))\ln(-1/2*x + 1/2) - \frac{3}{64}\ln(1+x)^2 - \frac{1}{64}/(-1+x)^2 + \frac{7}{64}/(-1+x) - \frac{1}{64}/(1+x)^2 - \frac{7}{64}/(1+x)$

**Maxima [B]** time = 0.961914, size = 159, normalized size = 2.37

$$-\frac{1}{16}\left(\frac{2(3x^3 - 5x)}{x^4 - 2x^2 + 1} - 3\log(x+1) + 3\log(x-1)\right)\operatorname{arccoth}(x) - \frac{3(x^4 - 2x^2 + 1)\log(x+1)^2 - 6(x^4 - 2x^2 + 1)\log(x+1)\log(x-1) + 3(x^4 - 2x^2 + 1)\log(x-1)^2 - 12x^2 + 16}{64(x^4 - 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(x)/(-x^2+1)^3,x, algorithm="maxima")`

[Out]  $-1/16*(2*(3*x^3 - 5*x)/(x^4 - 2*x^2 + 1) - 3*\log(x + 1) + 3*\log(x - 1))*\operatorname{arccoth}(x) - 1/64*(3*(x^4 - 2*x^2 + 1)*\log(x + 1)^2 - 6*(x^4 - 2*x^2 + 1)*\log(x + 1)*\log(x - 1) + 3*(x^4 - 2*x^2 + 1)*\log(x - 1)^2 - 12*x^2 + 16)/(x^4 - 2*x^2 + 1)$

**Fricas [A]** time = 1.58159, size = 165, normalized size = 2.46

$$\frac{3(x^4 - 2x^2 + 1)\log\left(\frac{x+1}{x-1}\right)^2 + 12x^2 - 4(3x^3 - 5x)\log\left(\frac{x+1}{x-1}\right) - 16}{64(x^4 - 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(x)/(-x^2+1)^3,x, algorithm="fricas")`

[Out]  $1/64*(3*(x^4 - 2*x^2 + 1)*\log((x + 1)/(x - 1))^2 + 12*x^2 - 4*(3*x^3 - 5*x)*\log((x + 1)/(x - 1)) - 16)/(x^4 - 2*x^2 + 1)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{\operatorname{acoth}(x)}{x^6 - 3x^4 + 3x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(x)/(-x\*\*2+1)\*\*3,x)

[Out] -Integral(acoth(x)/(x\*\*6 - 3\*x\*\*4 + 3\*x\*\*2 - 1), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{\operatorname{arccoth}(x)}{(x^2 - 1)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)/(-x^2+1)^3,x, algorithm="giac")

[Out] integrate(-arccoth(x)/(x^2 - 1)^3, x)



### 3.62 $\int x^3 \coth^{-1}(a + bx) dx$

**Optimal.** Leaf size=101

$$\frac{(6a^2 + 1)x}{4b^3} + \frac{(a + bx)^3}{12b^4} - \frac{a(a + bx)^2}{2b^4} + \frac{(1 - a)^4 \log(-a - bx + 1)}{8b^4} - \frac{(a + 1)^4 \log(a + bx + 1)}{8b^4} + \frac{1}{4}x^4 \coth^{-1}(a + bx)$$

[Out]  $((1 + 6a^2)x)/(4b^3) - (a(a + bx)^2)/(2b^4) + (a + bx)^3/(12b^4) + (x^4 \text{ArcCoth}[a + bx])/4 + ((1 - a)^4 \text{Log}[1 - a - bx])/(8b^4) - ((1 + a)^4 \text{Log}[1 + a + bx])/(8b^4)$

**Rubi [A]** time = 0.126333, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {6112, 5927, 702, 633, 31}

$$\frac{(6a^2 + 1)x}{4b^3} + \frac{(a + bx)^3}{12b^4} - \frac{a(a + bx)^2}{2b^4} + \frac{(1 - a)^4 \log(-a - bx + 1)}{8b^4} - \frac{(a + 1)^4 \log(a + bx + 1)}{8b^4} + \frac{1}{4}x^4 \coth^{-1}(a + bx)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3 \text{ArcCoth}[a + bx], x]$

[Out]  $((1 + 6a^2)x)/(4b^3) - (a(a + bx)^2)/(2b^4) + (a + bx)^3/(12b^4) + (x^4 \text{ArcCoth}[a + bx])/4 + ((1 - a)^4 \text{Log}[1 - a - bx])/(8b^4) - ((1 + a)^4 \text{Log}[1 + a + bx])/(8b^4)$

#### Rule 6112

$\text{Int}[(a + \text{ArcCoth}[(c + (d \cdot x)] \cdot (b \cdot x))^{(p)} \cdot ((e + (f \cdot x))^{(m)}), x\_Symbol] \rightarrow \text{Dist}[1/d, \text{Subst}[\text{Int}[(d \cdot e - c \cdot f)/d + (f \cdot x)/d]^m \cdot (a + b \cdot \text{ArcCoth}[x])^p, x], x, c + d \cdot x], x] /;$  FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]

#### Rule 5927

$\text{Int}[(a + \text{ArcCoth}[(c \cdot x)] \cdot (b \cdot x)) \cdot ((d + (e \cdot x))^{(q)}), x\_Symbol] \rightarrow \text{Simp}[(d + e \cdot x)^{(q + 1)} \cdot (a + b \cdot \text{ArcCoth}[c \cdot x])]/(e \cdot (q + 1)), x] - \text{Dist}[(b \cdot c)/(e \cdot (q + 1)), \text{Int}[(d + e \cdot x)^{(q + 1)}/(1 - c^2 \cdot x^2), x], x] /;$  FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]

#### Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] := Int[PolynomialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

### Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a*c)]
```

### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rubi steps

$$\begin{aligned}
 \int x^3 \coth^{-1}(a + bx) dx &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b} + \frac{x}{b}\right)^3 \coth^{-1}(x) dx, x, a + bx\right)}{b} \\
 &= \frac{1}{4}x^4 \coth^{-1}(a + bx) - \frac{1}{4} \text{Subst}\left(\int \frac{\left(-\frac{a}{b} + \frac{x}{b}\right)^4}{1 - x^2} dx, x, a + bx\right) \\
 &= \frac{1}{4}x^4 \coth^{-1}(a + bx) - \frac{1}{4} \text{Subst}\left(\int \left(-\frac{1 + 6a^2}{b^4} + \frac{4ax}{b^4} - \frac{x^2}{b^4} + \frac{1 + 6a^2 + a^4 - 4a(1 + a^2)x}{b^4(1 - x^2)}\right) dx, x, a + bx\right) \\
 &= \frac{(1 + 6a^2)x}{4b^3} - \frac{a(a + bx)^2}{2b^4} + \frac{(a + bx)^3}{12b^4} + \frac{1}{4}x^4 \coth^{-1}(a + bx) - \frac{\text{Subst}\left(\int \frac{1 + 6a^2 + a^4 - 4a(1 + a^2)x}{1 - x^2} dx, x, a + bx\right)}{4b^4} \\
 &= \frac{(1 + 6a^2)x}{4b^3} - \frac{a(a + bx)^2}{2b^4} + \frac{(a + bx)^3}{12b^4} + \frac{1}{4}x^4 \coth^{-1}(a + bx) - \frac{(1 - a)^4 \text{Subst}\left(\int \frac{1}{1 - x} dx, x, a + bx\right)}{8b^4} \\
 &= \frac{(1 + 6a^2)x}{4b^3} - \frac{a(a + bx)^2}{2b^4} + \frac{(a + bx)^3}{12b^4} + \frac{1}{4}x^4 \coth^{-1}(a + bx) + \frac{(1 - a)^4 \log(1 - a - bx)}{8b^4} - \frac{(1 + a)^4 \log(1 + a + bx)}{8b^4}
 \end{aligned}$$

**Mathematica [A]** time = 0.038819, size = 81, normalized size = 0.8

$$\frac{6(3a^2 + 1)bx - 6ab^2x^2 + 6b^4x^4 \coth^{-1}(a + bx) + 3(a - 1)^4 \log(-a - bx + 1) - 3(a + 1)^4 \log(a + bx + 1) + 2b^3x^3}{24b^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*ArcCoth[a + b\*x],x]

[Out]  $(6*(1 + 3*a^2)*b*x - 6*a*b^2*x^2 + 2*b^3*x^3 + 6*b^4*x^4*ArcCoth[a + b*x] + 3*(-1 + a)^4*Log[1 - a - b*x] - 3*(1 + a)^4*Log[1 + a + b*x])/(24*b^4)$

**Maple [B]** time = 0.035, size = 199, normalized size = 2.

$$\frac{a}{4b^4} + \frac{x}{4b^3} + \frac{x^4 \operatorname{arccoth}(bx+a)}{4} - \frac{\ln(bx+a+1)}{8b^4} + \frac{\ln(bx+a-1)}{8b^4} + \frac{x^3}{12b} - \frac{ax^2}{4b^2} + \frac{3a^2x}{4b^3} + \frac{13a^3}{12b^4} + \frac{\ln(bx+a-1)a}{8b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*arccoth(b\*x+a),x)

[Out]  $1/4/b^4*a+1/4*x/b^3+1/4*x^4*\operatorname{arccoth}(b*x+a)-1/8/b^4*\ln(b*x+a+1)+1/8/b^4*\ln(b*x+a-1)+1/12*x^3/b-1/4/b^2*x^2*a+3/4/b^3*x*a^2+13/12/b^4*a^3+1/8/b^4*\ln(b*x+a-1)*a^4-1/2/b^4*\ln(b*x+a-1)*a^3+3/4/b^4*\ln(b*x+a-1)*a^2-1/2/b^4*\ln(b*x+a-1)*a-1/8/b^4*\ln(b*x+a+1)*a^4-1/2/b^4*\ln(b*x+a+1)*a^3-3/4/b^4*\ln(b*x+a+1)*a^2-1/2/b^4*\ln(b*x+a+1)*a$

**Maxima [A]** time = 0.959818, size = 143, normalized size = 1.42

$$\frac{1}{4} x^4 \operatorname{arccoth}(bx+a) + \frac{1}{24} b \left( \frac{2(b^2x^3 - 3abx^2 + 3(3a^2 + 1)x)}{b^4} - \frac{3(a^4 + 4a^3 + 6a^2 + 4a + 1) \log(bx+a+1)}{b^5} + \frac{3(a^4 - 4a^3 + 6a^2 - 4a + 1) \log(bx+a-1)}{b^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(b\*x+a),x, algorithm="maxima")

[Out]  $1/4*x^4*\operatorname{arccoth}(b*x + a) + 1/24*b*(2*(b^2*x^3 - 3*a*b*x^2 + 3*(3*a^2 + 1)*x)/b^4 - 3*(a^4 + 4*a^3 + 6*a^2 + 4*a + 1)*\log(b*x + a + 1)/b^5 + 3*(a^4 - 4*a^3 + 6*a^2 - 4*a + 1)*\log(b*x + a - 1)/b^5)$

**Fricas [A]** time = 1.55089, size = 279, normalized size = 2.76

$$\frac{3b^4x^4 \log\left(\frac{bx+a+1}{bx+a-1}\right) + 2b^3x^3 - 6ab^2x^2 + 6(3a^2 + 1)bx - 3(a^4 + 4a^3 + 6a^2 + 4a + 1) \log(bx+a+1) + 3(a^4 - 4a^3 + 6a^2 - 4a + 1) \log(bx+a-1)}{24b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^4*x^4*log((b*x + a + 1)/(b*x + a - 1)) + 2*b^3*x^3 - 6*a*b^2*x^2
+ 6*(3*a^2 + 1)*b*x - 3*(a^4 + 4*a^3 + 6*a^2 + 4*a + 1)*log(b*x + a + 1) +
3*(a^4 - 4*a^3 + 6*a^2 - 4*a + 1)*log(b*x + a - 1))/b^4
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acoth(b*x+a),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arccoth}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*arccoth(b*x + a), x)
```

### 3.63 $\int x^2 \coth^{-1}(a + bx) dx$

**Optimal.** Leaf size=78

$$\frac{(a + bx)^2}{6b^3} - \frac{ax}{b^2} + \frac{(1 - a)^3 \log(-a - bx + 1)}{6b^3} + \frac{(a + 1)^3 \log(a + bx + 1)}{6b^3} + \frac{1}{3}x^3 \coth^{-1}(a + bx)$$

[Out]  $-\frac{(a*x)}{b^2} + \frac{(a + b*x)^2}{(6*b^3)} + \frac{(x^3*ArcCoth[a + b*x])}{3} + \frac{((1 - a)^3*Log[1 - a - b*x])}{(6*b^3)} + \frac{((1 + a)^3*Log[1 + a + b*x])}{(6*b^3)}$

**Rubi [A]** time = 0.101837, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {6112, 5927, 702, 633, 31}

$$\frac{(a + bx)^2}{6b^3} - \frac{ax}{b^2} + \frac{(1 - a)^3 \log(-a - bx + 1)}{6b^3} + \frac{(a + 1)^3 \log(a + bx + 1)}{6b^3} + \frac{1}{3}x^3 \coth^{-1}(a + bx)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*ArcCoth[a + b*x], x]$

[Out]  $-\frac{(a*x)}{b^2} + \frac{(a + b*x)^2}{(6*b^3)} + \frac{(x^3*ArcCoth[a + b*x])}{3} + \frac{((1 - a)^3*Log[1 - a - b*x])}{(6*b^3)} + \frac{((1 + a)^3*Log[1 + a + b*x])}{(6*b^3)}$

#### Rule 6112

$\text{Int}[\frac{(a + ArcCoth[(c + (d*x)]*(b))^{(p)}*((e + (f*x))^{(m)}), x\_Symbol]}{d}, \text{Subst}[\text{Int}[\frac{(d*e - c*f)}{d} + \frac{(f*x)}{d}]^m*(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /;$  FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]

#### Rule 5927

$\text{Int}[\frac{(a + ArcCoth[(c*(x)]*(b))^{(q)}*((d + (e*x))^{(q)}), x\_Symbol]}{d + e*x} + 1*(a + b*ArcCoth[c*x])^{(q)} / (e*(q + 1)), x] - \text{Dist}[\frac{b*c}{e*(q + 1)}, \text{Int}[\frac{(d + e*x)^{(q + 1)}}{(1 - c^2*x^2)}, x], x] /;$  FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]

#### Rule 702

$\text{Int}[\frac{(d + (e*x))^{(m)}}{(a + (c*x)^2)}, x\_Symbol] := \text{Int}[\text{PolynomialDivide}[\frac{(d + e*x)^m}{a + c*x^2}, x], x] /;$  FreeQ[{a, c, d, e}, x] && NeQ[

$c*d^2 + a*e^2, 0] \&\& \text{IGtQ}[m, 1] \&\& (\text{NeQ}[d, 0] \mid\mid \text{GtQ}[m, 2])$

### Rule 633

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (c_.)*(x_.)^2}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-(a*c), 2]\}, \text{Dist}[e/2 + (c*d)/(2*q), \text{Int}[1/(-q + c*x), x], x] + \text{Dist}[e/2 - (c*d)/(2*q), \text{Int}[1/(q + c*x), x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{NiceSqrtQ}[-(a*c)]$

### Rule 31

$\text{Int}[\frac{(a_.) + (b_.)*(x_.)^{-1}}{x\_Symbol}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}[\{a, b\}, x]$

### Rubi steps

$$\begin{aligned} \int x^2 \coth^{-1}(a + bx) dx &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b} + \frac{x}{b}\right)^2 \coth^{-1}(x) dx, x, a + bx\right)}{b} \\ &= \frac{1}{3}x^3 \coth^{-1}(a + bx) - \frac{1}{3} \text{Subst}\left(\int \frac{\left(-\frac{a}{b} + \frac{x}{b}\right)^3}{1 - x^2} dx, x, a + bx\right) \\ &= \frac{1}{3}x^3 \coth^{-1}(a + bx) - \frac{1}{3} \text{Subst}\left(\int \left(\frac{3a}{b^3} - \frac{x}{b^3} - \frac{a(3 + a^2) - (1 + 3a^2)x}{b^3(1 - x^2)}\right) dx, x, a + bx\right) \\ &= -\frac{ax}{b^2} + \frac{(a + bx)^2}{6b^3} + \frac{1}{3}x^3 \coth^{-1}(a + bx) + \frac{\text{Subst}\left(\int \frac{a(3+a^2) - (1+3a^2)x}{1-x^2} dx, x, a + bx\right)}{3b^3} \\ &= -\frac{ax}{b^2} + \frac{(a + bx)^2}{6b^3} + \frac{1}{3}x^3 \coth^{-1}(a + bx) - \frac{(1 - a)^3 \text{Subst}\left(\int \frac{1}{1-x} dx, x, a + bx\right)}{6b^3} - \frac{(1 + a)^3 \text{Subst}\left(\int \frac{1}{1+x} dx, x, a + bx\right)}{6b^3} \\ &= -\frac{ax}{b^2} + \frac{(a + bx)^2}{6b^3} + \frac{1}{3}x^3 \coth^{-1}(a + bx) + \frac{(1 - a)^3 \log(1 - a - bx)}{6b^3} + \frac{(1 + a)^3 \log(1 + a + bx)}{6b^3} \end{aligned}$$

**Mathematica [A]** time = 0.0233845, size = 92, normalized size = 1.18

$$\frac{(-a^3 + 3a^2 - 3a + 1) \log(-a - bx + 1)}{6b^3} + \frac{(a^3 + 3a^2 + 3a + 1) \log(a + bx + 1)}{6b^3} - \frac{2ax}{3b^2} + \frac{1}{3}x^3 \coth^{-1}(a + bx) + \frac{x^2}{6b}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[a + b\*x], x]

[Out]  $(-2ax)/(3b^2) + x^2/(6b) + (x^3 \operatorname{ArcCoth}[a + bx])/3 + ((1 - 3a + 3a^2 - a^3) \operatorname{Log}[1 - a - bx])/(6b^3) + ((1 + 3a + 3a^2 + a^3) \operatorname{Log}[1 + a + bx])/(6b^3)$

**Maple [B]** time = 0.036, size = 146, normalized size = 1.9

$$\frac{x^3 \operatorname{arccoth}(bx + a)}{3} + \frac{x^2}{6b} - \frac{2ax}{3b^2} - \frac{5a^2}{6b^3} - \frac{\ln(bx + a - 1)a^3}{6b^3} + \frac{\ln(bx + a - 1)a^2}{2b^3} - \frac{\ln(bx + a - 1)a}{2b^3} + \frac{\ln(bx + a - 1)}{6b^3} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*arccoth(b*x+a),x)`

[Out]  $1/3x^3 \operatorname{arccoth}(bx+a) + 1/6x^2/b - 2/3ax/b^2 - 5/6/b^3a^2 - 1/6/b^3 \ln(bx+a-1)a^3 + 1/2/b^3 \ln(bx+a-1)a^2 - 1/2/b^3 \ln(bx+a-1)a + 1/6/b^3 \ln(bx+a-1) + 1/6/b^3 \ln(bx+a+1)a^3 + 1/2/b^3 \ln(bx+a+1)a^2 + 1/2/b^3 \ln(bx+a+1)a + 1/6/b^3 \ln(bx+a+1)$

**Maxima [A]** time = 0.958009, size = 107, normalized size = 1.37

$$\frac{1}{3}x^3 \operatorname{arccoth}(bx + a) + \frac{1}{6}b \left( \frac{bx^2 - 4ax}{b^3} + \frac{(a^3 + 3a^2 + 3a + 1) \log(bx + a + 1)}{b^4} - \frac{(a^3 - 3a^2 + 3a - 1) \log(bx + a - 1)}{b^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccoth(b*x+a),x, algorithm="maxima")`

[Out]  $1/3x^3 \operatorname{arccoth}(bx + a) + 1/6b((bx^2 - 4ax)/b^3 + (a^3 + 3a^2 + 3a + 1) \log(bx + a + 1)/b^4 - (a^3 - 3a^2 + 3a - 1) \log(bx + a - 1)/b^4)$

**Fricas [A]** time = 1.61077, size = 213, normalized size = 2.73

$$\frac{b^3x^3 \log\left(\frac{bx+a+1}{bx+a-1}\right) + b^2x^2 - 4abx + (a^3 + 3a^2 + 3a + 1) \log(bx + a + 1) - (a^3 - 3a^2 + 3a - 1) \log(bx + a - 1)}{6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(b^3*x^3*log((b*x + a + 1)/(b*x + a - 1)) + b^2*x^2 - 4*a*b*x + (a^3 +
3*a^2 + 3*a + 1)*log(b*x + a + 1) - (a^3 - 3*a^2 + 3*a - 1)*log(b*x + a - 1
))/b^3
```

**Sympy [A]** time = 3.19662, size = 117, normalized size = 1.5

$$\left\{ \begin{array}{l} \frac{a^3 \operatorname{acoth}(a+bx)}{3b^3} + \frac{a^2 \log(a+bx+1)}{b^3} - \frac{a^2 \operatorname{acoth}(a+bx)}{b^3} - \frac{2ax}{3b^2} + \frac{a \operatorname{acoth}(a+bx)}{b^3} + \frac{x^3 \operatorname{acoth}(a+bx)}{3} + \frac{x^2}{6b} + \frac{\log(a+bx+1)}{3b^3} - \frac{\operatorname{acoth}(a+bx)}{3b^3} \\ \frac{x^3 \operatorname{acoth}(a)}{3} \end{array} \right. \text{for } b \text{ other}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(b*x+a),x)
```

```
[Out] Piecewise((a**3*acoth(a + b*x)/(3*b**3) + a**2*log(a + b*x + 1)/b**3 - a**2
*acoth(a + b*x)/b**3 - 2*a*x/(3*b**2) + a*acoth(a + b*x)/b**3 + x**3*acoth(
a + b*x)/3 + x**2/(6*b) + log(a + b*x + 1)/(3*b**3) - acoth(a + b*x)/(3*b**
3), Ne(b, 0)), (x**3*acoth(a)/3, True))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arccoth}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(b*x + a), x)
```



### 3.64 $\int x \coth^{-1}(a + bx) dx$

**Optimal.** Leaf size=65

$$\frac{(1-a)^2 \log(-a-bx+1)}{4b^2} - \frac{(a+1)^2 \log(a+bx+1)}{4b^2} + \frac{1}{2}x^2 \coth^{-1}(a+bx) + \frac{x}{2b}$$

[Out]  $x/(2*b) + (x^2*ArcCoth[a + b*x])/2 + ((1 - a)^2*Log[1 - a - b*x])/(4*b^2) - ((1 + a)^2*Log[1 + a + b*x])/(4*b^2)$

**Rubi [A]** time = 0.0716315, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$ , Rules used = {6112, 5927, 702, 633, 31}

$$\frac{(1-a)^2 \log(-a-bx+1)}{4b^2} - \frac{(a+1)^2 \log(a+bx+1)}{4b^2} + \frac{1}{2}x^2 \coth^{-1}(a+bx) + \frac{x}{2b}$$

Antiderivative was successfully verified.

[In] Int[x\*ArcCoth[a + b\*x], x]

[Out]  $x/(2*b) + (x^2*ArcCoth[a + b*x])/2 + ((1 - a)^2*Log[1 - a - b*x])/(4*b^2) - ((1 + a)^2*Log[1 + a + b*x])/(4*b^2)$

#### Rule 6112

Int[((a\_.) + ArcCoth[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]

#### Rule 5927

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))\*((d\_.) + (e\_.)\*(x\_.))^(q\_.), x\_Symbol] :> Simp[((d + e\*x)^(q + 1)\*(a + b\*ArcCoth[c\*x]))/(e\*(q + 1)), x] - Dist[(b\*c)/(e\*(q + 1)), Int[(d + e\*x)^(q + 1)/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]

#### Rule 702

Int[((d\_.) + (e\_.)\*(x\_.))^(m\_.)/((a\_.) + (c\_.)\*(x\_.)^2), x\_Symbol] :> Int[PolynomialDivide[(d + e\*x)^m, a + c\*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[

$c*d^2 + a*e^2, 0] \&\& \text{IGtQ}[m, 1] \&\& (\text{NeQ}[d, 0] \mid\mid \text{GtQ}[m, 2])$

### Rule 633

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (c_.)*(x_.)^2}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-(a*c), 2]\}, \text{Dist}[e/2 + (c*d)/(2*q), \text{Int}[1/(-q + c*x), x], x] + \text{Dist}[e/2 - (c*d)/(2*q), \text{Int}[1/(q + c*x), x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{NiceSqrtQ}[-(a*c)]$

### Rule 31

$\text{Int}[\frac{(a_.) + (b_.)*(x_.)^{-1}}{x\_Symbol}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}[\{a, b\}, x]$

### Rubi steps

$$\begin{aligned} \int x \coth^{-1}(a + bx) dx &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b} + \frac{x}{b}\right) \coth^{-1}(x) dx, x, a + bx\right)}{b} \\ &= \frac{1}{2}x^2 \coth^{-1}(a + bx) - \frac{1}{2} \text{Subst}\left(\int \frac{\left(-\frac{a}{b} + \frac{x}{b}\right)^2}{1 - x^2} dx, x, a + bx\right) \\ &= \frac{1}{2}x^2 \coth^{-1}(a + bx) - \frac{1}{2} \text{Subst}\left(\int \left(-\frac{1}{b^2} + \frac{1 + a^2 - 2ax}{b^2(1 - x^2)}\right) dx, x, a + bx\right) \\ &= \frac{x}{2b} + \frac{1}{2}x^2 \coth^{-1}(a + bx) - \frac{\text{Subst}\left(\int \frac{1 + a^2 - 2ax}{1 - x^2} dx, x, a + bx\right)}{2b^2} \\ &= \frac{x}{2b} + \frac{1}{2}x^2 \coth^{-1}(a + bx) - \frac{(1 - a)^2 \text{Subst}\left(\int \frac{1}{1 - x} dx, x, a + bx\right)}{4b^2} + \frac{(1 + a)^2 \text{Subst}\left(\int \frac{1}{-1 - x} dx, x, a + bx\right)}{4b^2} \\ &= \frac{x}{2b} + \frac{1}{2}x^2 \coth^{-1}(a + bx) + \frac{(1 - a)^2 \log(1 - a - bx)}{4b^2} - \frac{(1 + a)^2 \log(1 + a + bx)}{4b^2} \end{aligned}$$

**Mathematica [A]** time = 0.0219382, size = 56, normalized size = 0.86

$$\frac{2b^2x^2 \coth^{-1}(a + bx) + (a - 1)^2 \log(-a - bx + 1) - (a + 1)^2 \log(a + bx + 1) + 2bx}{4b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[a + b\*x], x]

[Out]  $(2bx + 2b^2x^2 \operatorname{ArcCoth}[a + bx] + (-1 + a)^2 \operatorname{Log}[1 - a - bx] - (1 + a)^2 \operatorname{Log}[1 + a + bx]) / (4b^2)$

**Maple [A]** time = 0.033, size = 89, normalized size = 1.4

$$\frac{x^2 \operatorname{arccoth}(bx + a)}{2} - \frac{\operatorname{arccoth}(bx + a) a^2}{2b^2} + \frac{x}{2b} + \frac{a}{2b^2} - \frac{\ln(bx + a - 1) a}{2b^2} + \frac{\ln(bx + a - 1)}{4b^2} - \frac{\ln(bx + a + 1) a}{2b^2} - \frac{\ln(bx + a + 1)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*arccoth(b*x+a),x)`

[Out]  $1/2*x^2*\operatorname{arccoth}(b*x+a)-1/2/b^2*\operatorname{arccoth}(b*x+a)*a^2+1/2*x/b+1/2/b^2*a-1/2/b^2*\ln(b*x+a-1)*a+1/4/b^2*\ln(b*x+a-1)-1/2/b^2*\ln(b*x+a+1)*a-1/4/b^2*\ln(b*x+a+1)$

**Maxima [A]** time = 0.971898, size = 82, normalized size = 1.26

$$\frac{1}{2} x^2 \operatorname{arccoth}(bx + a) + \frac{1}{4} b \left( \frac{2x}{b^2} - \frac{(a^2 + 2a + 1) \log(bx + a + 1)}{b^3} + \frac{(a^2 - 2a + 1) \log(bx + a - 1)}{b^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccoth(b*x+a),x, algorithm="maxima")`

[Out]  $1/2*x^2*\operatorname{arccoth}(b*x + a) + 1/4*b*(2*x/b^2 - (a^2 + 2*a + 1)*\log(b*x + a + 1)/b^3 + (a^2 - 2*a + 1)*\log(b*x + a - 1)/b^3)$

**Fricas [A]** time = 1.59959, size = 176, normalized size = 2.71

$$\frac{b^2 x^2 \log\left(\frac{bx+a+1}{bx+a-1}\right) + 2bx - (a^2 + 2a + 1) \log(bx + a + 1) + (a^2 - 2a + 1) \log(bx + a - 1)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccoth(b*x+a),x, algorithm="fricas")`

[Out]  $\frac{1}{4}*(b^2*x^2*\log((b*x + a + 1)/(b*x + a - 1)) + 2*b*x - (a^2 + 2*a + 1)*\log(b*x + a + 1) + (a^2 - 2*a + 1)*\log(b*x + a - 1))/b^2$

**Sympy [A]** time = 1.60945, size = 76, normalized size = 1.17

$$\begin{cases} -\frac{a^2 \operatorname{acoth}(a+bx)}{2b^2} - \frac{a \log(a+bx+1)}{b^2} + \frac{a \operatorname{acoth}(a+bx)}{b^2} + \frac{x^2 \operatorname{acoth}(a+bx)}{2} + \frac{x}{2b} - \frac{\operatorname{acoth}(a+bx)}{2b^2} & \text{for } b \neq 0 \\ \frac{x^2 \operatorname{acoth}(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*acoth(b*x+a), x)`

[Out] `Piecewise((-a**2*acoth(a + b*x)/(2*b**2) - a*log(a + b*x + 1)/b**2 + a*acoth(a + b*x)/b**2 + x**2*acoth(a + b*x)/2 + x/(2*b) - acoth(a + b*x)/(2*b**2), Ne(b, 0)), (x**2*acoth(a)/2, True))`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arccoth}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccoth(b*x+a), x, algorithm="giac")`

[Out] `integrate(x*arccoth(b*x + a), x)`

### 3.65 $\int \coth^{-1}(a + bx) dx$

**Optimal.** Leaf size=35

$$\frac{\log(1 - (a + bx)^2)}{2b} + \frac{(a + bx) \coth^{-1}(a + bx)}{b}$$

[Out] ((a + b\*x)\*ArcCoth[a + b\*x])/b + Log[1 - (a + b\*x)^2]/(2\*b)

**Rubi [A]** time = 0.0159401, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {6104, 5911, 260}

$$\frac{\log(1 - (a + bx)^2)}{2b} + \frac{(a + bx) \coth^{-1}(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a + b\*x], x]

[Out] ((a + b\*x)\*ArcCoth[a + b\*x])/b + Log[1 - (a + b\*x)^2]/(2\*b)

#### Rule 6104

Int[((a\_.) + ArcCoth[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(p\_.), x\_Symbol] :> Dist[1/d, Subst[Int[(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0]

#### Rule 5911

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.), x\_Symbol] :> Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

#### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

#### Rubi steps

$$\begin{aligned}\int \coth^{-1}(a + bx) dx &= \frac{\text{Subst}\left(\int \coth^{-1}(x) dx, x, a + bx\right)}{b} \\ &= \frac{(a + bx) \coth^{-1}(a + bx)}{b} - \frac{\text{Subst}\left(\int \frac{x}{1-x^2} dx, x, a + bx\right)}{b} \\ &= \frac{(a + bx) \coth^{-1}(a + bx)}{b} + \frac{\log(1 - (a + bx)^2)}{2b}\end{aligned}$$

**Mathematica [A]** time = 0.0145124, size = 43, normalized size = 1.23

$$\frac{(a + 1) \log(a + bx + 1) - (a - 1) \log(-a - bx + 1)}{2b} + x \coth^{-1}(a + bx)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a + b\*x], x]

[Out] x\*ArcCoth[a + b\*x] + (-((-1 + a)\*Log[1 - a - b\*x]) + (1 + a)\*Log[1 + a + b\*x])/(2\*b)

**Maple [A]** time = 0.03, size = 36, normalized size = 1.

$$x \operatorname{arccoth}(bx + a) + \frac{\operatorname{arccoth}(bx + a) a}{b} + \frac{\ln((bx + a)^2 - 1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(b\*x+a), x)

[Out] x\*arccoth(b\*x+a)+1/b\*arccoth(b\*x+a)\*a+1/2/b\*ln((b\*x+a)^2-1)

**Maxima [A]** time = 0.973925, size = 42, normalized size = 1.2

$$\frac{2(bx + a) \operatorname{arccoth}(bx + a) + \log(-(bx + a)^2 + 1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a),x, algorithm="maxima")

[Out]  $1/2*(2*(b*x + a)*\operatorname{arccoth}(b*x + a) + \log(-(b*x + a)^2 + 1))/b$

**Fricas [A]** time = 1.54404, size = 135, normalized size = 3.86

$$\frac{bx \log\left(\frac{bx+a+1}{bx+a-1}\right) + (a+1)\log(bx+a+1) - (a-1)\log(bx+a-1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a),x, algorithm="fricas")

[Out]  $1/2*(b*x*\log((b*x + a + 1)/(b*x + a - 1)) + (a + 1)*\log(b*x + a + 1) - (a - 1)*\log(b*x + a - 1))/b$

**Sympy [A]** time = 0.8191, size = 41, normalized size = 1.17

$$\begin{cases} \frac{a \operatorname{acoth}(a+bx)}{b} + x \operatorname{acoth}(a+bx) + \frac{\log(a+bx+1)}{b} - \frac{\operatorname{acoth}(a+bx)}{b} & \text{for } b \neq 0 \\ x \operatorname{acoth}(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(b\*x+a),x)

[Out] Piecewise((a\*acoth(a + b\*x)/b + x\*acoth(a + b\*x) + log(a + b\*x + 1)/b - acoth(a + b\*x)/b, Ne(b, 0)), (x\*acoth(a), True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a),x, algorithm="giac")

```
[Out] integrate(arccoth(b*x + a), x)
```



$$3.66 \quad \int \frac{\coth^{-1}(a+bx)}{x} dx$$

**Optimal.** Leaf size=92

$$\frac{1}{2}\text{PolyLog}\left(2, 1 - \frac{2}{a+bx+1}\right) - \frac{1}{2}\text{PolyLog}\left(2, 1 - \frac{2bx}{(1-a)(a+bx+1)}\right) + \log\left(\frac{2}{a+bx+1}\right)\left(-\coth^{-1}(a+bx)\right) + \log\left(\frac{2}{a+bx+1}\right)$$

[Out]  $-(\text{ArcCoth}[a + b*x]*\text{Log}[2/(1 + a + b*x)]) + \text{ArcCoth}[a + b*x]*\text{Log}[(2*b*x)/((1 - a)*(1 + a + b*x))] + \text{PolyLog}[2, 1 - 2/(1 + a + b*x)]/2 - \text{PolyLog}[2, 1 - (2*b*x)/((1 - a)*(1 + a + b*x))]/2$

**Rubi [A]** time = 0.0973951, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {6112, 5921, 2402, 2315, 2447}

$$\frac{1}{2}\text{PolyLog}\left(2, 1 - \frac{2}{a+bx+1}\right) - \frac{1}{2}\text{PolyLog}\left(2, 1 - \frac{2bx}{(1-a)(a+bx+1)}\right) + \log\left(\frac{2}{a+bx+1}\right)\left(-\coth^{-1}(a+bx)\right) + \log\left(\frac{2}{a+bx+1}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a + b\*x]/x, x]

[Out]  $-(\text{ArcCoth}[a + b*x]*\text{Log}[2/(1 + a + b*x)]) + \text{ArcCoth}[a + b*x]*\text{Log}[(2*b*x)/((1 - a)*(1 + a + b*x))] + \text{PolyLog}[2, 1 - 2/(1 + a + b*x)]/2 - \text{PolyLog}[2, 1 - (2*b*x)/((1 - a)*(1 + a + b*x))]/2$

### Rule 6112

Int[((a\_.) + ArcCoth[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.), x\_Symbol] := Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]

### Rule 5921

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] := -Simp[((a + b\*ArcCoth[c\*x])\*Log[2/(1 + c\*x)])/e, x] + (Dist[(b\*c)/e, Int[Log[2/(1 + c\*x)]/(1 - c^2\*x^2), x], x] - Dist[(b\*c)/e, Int[Log[(2\*c\*(d + e\*x))/(c\*d + e)\*(1 + c\*x)]/(1 - c^2\*x^2), x], x] + Simp[((a + b\*ArcCoth[c\*x])\*Log[(2\*c\*(d + e\*x))/(c\*d + e)\*(1 + c\*x)])/e, x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2\*d^2 - e^2, 0]

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\coth^{-1}(a+bx)}{x} dx = \frac{\text{Subst}\left(\int \frac{\coth^{-1}(x)}{-\frac{a}{b} + \frac{x}{b}} dx, x, a+bx\right)}{b}$$

$$= -\coth^{-1}(a+bx) \log\left(\frac{2}{1+a+bx}\right) + \coth^{-1}(a+bx) \log\left(\frac{2bx}{(1-a)(1+a+bx)}\right) + \text{Subst}\left(\int \frac{\log\left(\frac{2}{1-u}\right)}{1-u} du, u, \frac{2}{1+a+bx}\right)$$

$$= -\coth^{-1}(a+bx) \log\left(\frac{2}{1+a+bx}\right) + \coth^{-1}(a+bx) \log\left(\frac{2bx}{(1-a)(1+a+bx)}\right) - \frac{1}{2} \text{Li}_2\left(1 - \frac{2}{1+a+bx}\right)$$

$$= -\coth^{-1}(a+bx) \log\left(\frac{2}{1+a+bx}\right) + \coth^{-1}(a+bx) \log\left(\frac{2bx}{(1-a)(1+a+bx)}\right) + \frac{1}{2} \text{Li}_2\left(1 - \frac{2}{1+a+bx}\right)$$

**Mathematica [C]** time = 0.167891, size = 259, normalized size = 2.82

$$\frac{1}{8} \left( -4 \text{PolyLog}\left(2, e^{2 \tanh^{-1}(a) - 2 \tanh^{-1}(a+bx)}\right) - 4 \text{PolyLog}\left(2, -e^{2 \tanh^{-1}(a+bx)}\right) + 4 \left(\tanh^{-1}(a) - \tanh^{-1}(a+bx)\right)^2 - (\pi - 2) \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[a + b*x]/x, x]
```

```
[Out] (ArcCoth[a + b*x] - ArcTanh[a + b*x])*Log[x] + ArcTanh[a + b*x]*(-Log[1/Sqr
t[1 - (a + b*x)^2]] + Log[(-I)*Sinh[ArcTanh[a] - ArcTanh[a + b*x]]]) + (4*(
ArcTanh[a] - ArcTanh[a + b*x])^2 - (Pi - (2*I)*ArcTanh[a + b*x])^2 - 8*(Arc
Tanh[a] - ArcTanh[a + b*x])*Log[1 - E^(2*ArcTanh[a] - 2*ArcTanh[a + b*x])])
- (4*I)*(Pi - (2*I)*ArcTanh[a + b*x])*Log[1 + E^(2*ArcTanh[a + b*x])] + 4*(
I*Pi + 2*ArcTanh[a + b*x])*Log[2/Sqrt[1 - (a + b*x)^2]] + 8*(ArcTanh[a] - A
rcTanh[a + b*x])*Log[(-2*I)*Sinh[ArcTanh[a] - ArcTanh[a + b*x]]] - 4*PolyLo
g[2, E^(2*ArcTanh[a] - 2*ArcTanh[a + b*x])] - 4*PolyLog[2, -E^(2*ArcTanh[a
+ b*x])])]/8
```

**Maple [A]** time = 0.076, size = 81, normalized size = 0.9

$$\ln(bx) \operatorname{arccoth}(bx+a) - \frac{1}{2} \operatorname{dilog}\left(\frac{bx+a+1}{1+a}\right) - \frac{\ln(bx)}{2} \ln\left(\frac{bx+a+1}{1+a}\right) + \frac{1}{2} \operatorname{dilog}\left(\frac{bx+a-1}{a-1}\right) + \frac{\ln(bx)}{2} \ln\left(\frac{bx+a-1}{a-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)/x,x)
```

```
[Out] ln(b*x)*arccoth(b*x+a)-1/2*dilog((b*x+a+1)/(1+a))-1/2*ln(b*x)*ln((b*x+a+1)/
(1+a))+1/2*dilog((b*x+a-1)/(a-1))+1/2*ln(b*x)*ln((b*x+a-1)/(a-1))
```

**Maxima [A]** time = 0.971844, size = 173, normalized size = 1.88

$$-\frac{1}{2}b\left(\frac{\log(bx+a+1)}{b} - \frac{\log(bx+a-1)}{b}\right)\log(x) + \frac{1}{2}b\left(\frac{\log(bx+a+1)\log\left(-\frac{bx+a+1}{a+1}+1\right) + \operatorname{Li}_2\left(\frac{bx+a+1}{a+1}\right)}{b} - \frac{\log(bx+a-1)\log\left(-\frac{bx+a-1}{a-1}+1\right) + \operatorname{Li}_2\left(\frac{bx+a-1}{a-1}\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] -1/2*b*(log(b*x + a + 1)/b - log(b*x + a - 1)/b)*log(x) + 1/2*b*((log(b*x +
a + 1)*log(-(b*x + a + 1)/(a + 1) + 1) + dilog((b*x + a + 1)/(a + 1)))/b -
(log(b*x + a - 1)*log(-(b*x + a - 1)/(a - 1) + 1) + dilog((b*x + a - 1)/(a
- 1)))/b) + arccoth(b*x + a)*log(x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arccoth}(bx+a)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/x,x, algorithm="fricas")

[Out] integral(arccoth(b\*x + a)/x, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}(a+bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(b\*x+a)/x,x)

[Out] Integral(acoth(a + b\*x)/x, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(bx+a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/x,x, algorithm="giac")

[Out] integrate(arccoth(b\*x + a)/x, x)

$$3.67 \quad \int \frac{\coth^{-1}(a+bx)}{x^2} dx$$

**Optimal.** Leaf size=64

$$\frac{b \log(x)}{1-a^2} - \frac{b \log(-a-bx+1)}{2(1-a)} - \frac{b \log(a+bx+1)}{2(a+1)} - \frac{\coth^{-1}(a+bx)}{x}$$

[Out]  $-(\text{ArcCoth}[a + b*x]/x) + (b*\text{Log}[x])/(1 - a^2) - (b*\text{Log}[1 - a - b*x])/(2*(1 - a)) - (b*\text{Log}[1 + a + b*x])/(2*(1 + a))$

**Rubi [A]** time = 0.0510754, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {6110, 371, 706, 31, 633}

$$\frac{b \log(x)}{1-a^2} - \frac{b \log(-a-bx+1)}{2(1-a)} - \frac{b \log(a+bx+1)}{2(a+1)} - \frac{\coth^{-1}(a+bx)}{x}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{ArcCoth}[a + b*x]/x^2, x]$

[Out]  $-(\text{ArcCoth}[a + b*x]/x) + (b*\text{Log}[x])/(1 - a^2) - (b*\text{Log}[1 - a - b*x])/(2*(1 - a)) - (b*\text{Log}[1 + a + b*x])/(2*(1 + a))$

### Rule 6110

$\text{Int}[(a + \text{ArcCoth}[c + (d*x)]*(b))^p * ((e + (f*x))^m), x\_Symbol] \rightarrow \text{Simp}[(e + f*x)^{m+1} * (a + b*\text{ArcCoth}[c + d*x])^p / (f*(m+1)), x] - \text{Dist}[(b*d*p)/(f*(m+1)), \text{Int}[(e + f*x)^{m+1} * (a + b*\text{ArcCoth}[c + d*x])^{p-1} / (1 - (c + d*x)^2), x], x] /;$  FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[m, -1]

### Rule 371

$\text{Int}[(a + (b*v)^n)^p * (x)^m, x\_Symbol] \rightarrow \text{With}\{c = \text{Coefficient}[v, x, 0], d = \text{Coefficient}[v, x, 1]\}, \text{Dist}[1/d^{m+1}, \text{Subst}[\text{Int}[\text{SimplifyIntegrand}[(x - c)^m * (a + b*x^n)^p, x], x], x, v], x] /;$  NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

### Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] := Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
0]
```

### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(a+bx)}{x^2} dx &= -\frac{\coth^{-1}(a+bx)}{x} + b \int \frac{1}{x(1-(a+bx)^2)} dx \\
&= -\frac{\coth^{-1}(a+bx)}{x} + b \operatorname{Subst}\left(\int \frac{1}{(-a+x)(1-x^2)} dx, x, a+bx\right) \\
&= -\frac{\coth^{-1}(a+bx)}{x} + \frac{b \operatorname{Subst}\left(\int \frac{1}{-a+x} dx, x, a+bx\right)}{1-a^2} + \frac{b \operatorname{Subst}\left(\int \frac{a+x}{1-x^2} dx, x, a+bx\right)}{1-a^2} \\
&= -\frac{\coth^{-1}(a+bx)}{x} + \frac{b \log(x)}{1-a^2} + \frac{b \operatorname{Subst}\left(\int \frac{1}{1-x} dx, x, a+bx\right)}{2(1-a)} + \frac{b \operatorname{Subst}\left(\int \frac{1}{-1-x} dx, x, a+bx\right)}{2(1+a)} \\
&= -\frac{\coth^{-1}(a+bx)}{x} + \frac{b \log(x)}{1-a^2} - \frac{b \log(1-a-bx)}{2(1-a)} - \frac{b \log(1+a+bx)}{2(1+a)}
\end{aligned}$$

**Mathematica [A]** time = 0.0505785, size = 55, normalized size = 0.86

$$\frac{b((a+1)\log(-a-bx+1) - (a-1)\log(a+bx+1) - 2\log(x))}{2(a^2-1)} - \frac{\coth^{-1}(a+bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a + b*x]/x^2, x]
```

[Out]  $-(\text{ArcCoth}[a + b*x]/x) + (b*(-2*\text{Log}[x] + (1 + a)*\text{Log}[1 - a - b*x] - (-1 + a)*\text{Log}[1 + a + b*x]))/(2*(-1 + a^2))$

**Maple [A]** time = 0.037, size = 63, normalized size = 1.

$$-\frac{\text{arccoth}(bx + a)}{x} + \frac{b \ln(bx + a - 1)}{2a - 2} - \frac{b \ln(bx)}{(1 + a)(a - 1)} - \frac{b \ln(bx + a + 1)}{2 + 2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(b*x+a)/x^2,x)`

[Out]  $-\text{arccoth}(b*x+a)/x + b/(2*a-2)*\ln(b*x+a-1) - b/(a-1)/(1+a)*\ln(b*x) - b/(2+2*a)*\ln(b*x+a+1)$

**Maxima [A]** time = 0.959147, size = 73, normalized size = 1.14

$$-\frac{1}{2}b\left(\frac{\log(bx + a + 1)}{a + 1} - \frac{\log(bx + a - 1)}{a - 1} + \frac{2 \log(x)}{a^2 - 1}\right) - \frac{\text{arccoth}(bx + a)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(b*x+a)/x^2,x, algorithm="maxima")`

[Out]  $-1/2*b*(\log(b*x + a + 1)/(a + 1) - \log(b*x + a - 1)/(a - 1) + 2*\log(x)/(a^2 - 1)) - \text{arccoth}(b*x + a)/x$

**Fricas [A]** time = 1.67592, size = 192, normalized size = 3.

$$\frac{(a - 1)bx \log(bx + a + 1) - (a + 1)bx \log(bx + a - 1) + 2bx \log(x) + (a^2 - 1) \log\left(\frac{bx+a+1}{bx+a-1}\right)}{2(a^2 - 1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(b*x+a)/x^2,x, algorithm="fricas")`

[Out]  $-1/2*((a - 1)*b*x*\log(b*x + a + 1) - (a + 1)*b*x*\log(b*x + a - 1) + 2*b*x*\log(x) + (a^2 - 1)*\log((b*x + a + 1)/(b*x + a - 1)))/((a^2 - 1)*x)$

**Sympy [A]** time = 2.8849, size = 144, normalized size = 2.25

$$\begin{cases} \frac{b \operatorname{acoth}(bx-1)}{2} - \frac{\operatorname{acoth}(bx-1)}{x} - \frac{1}{2x} & \text{for } a = -1 \\ \frac{b \operatorname{acoth}(bx+1)}{2} - \frac{\operatorname{acoth}(bx+1)}{x} + \frac{1}{2x} & \text{for } a = 1 \\ -\frac{a^2 \operatorname{acoth}(a+bx)}{a^2x-x} - \frac{bx \operatorname{acoth}(a+bx)}{a^2x-x} - \frac{bx \log(x)}{a^2x-x} + \frac{bx \log(a+bx+1)}{a^2x-x} - \frac{bx \operatorname{acoth}(a+bx)}{a^2x-x} + \frac{\operatorname{acoth}(a+bx)}{a^2x-x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(b*x+a)/x**2,x)`

[Out] `Piecewise((b*acoth(b*x - 1)/2 - acoth(b*x - 1)/x - 1/(2*x), Eq(a, -1)), (-b*acoth(b*x + 1)/2 - acoth(b*x + 1)/x + 1/(2*x), Eq(a, 1)), (-a**2*acoth(a + b*x)/(a**2*x - x) - a*b*x*acoth(a + b*x)/(a**2*x - x) - b*x*log(x)/(a**2*x - x) + b*x*log(a + b*x + 1)/(a**2*x - x) - b*x*acoth(a + b*x)/(a**2*x - x) + acoth(a + b*x)/(a**2*x - x), True))`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(bx+a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(b*x+a)/x^2,x, algorithm="giac")`

[Out] `integrate(arccoth(b*x + a)/x^2, x)`



$$3.68 \quad \int \frac{\coth^{-1}(a+bx)}{x^3} dx$$

**Optimal.** Leaf size=90

$$\frac{ab^2 \log(x)}{(1-a^2)^2} - \frac{b}{2(1-a^2)x} - \frac{b^2 \log(-a-bx+1)}{4(1-a)^2} + \frac{b^2 \log(a+bx+1)}{4(a+1)^2} - \frac{\coth^{-1}(a+bx)}{2x^2}$$

[Out]  $-b/(2*(1 - a^2)*x) - \text{ArcCoth}[a + b*x]/(2*x^2) + (a*b^2*\text{Log}[x])/(1 - a^2)^2 - (b^2*\text{Log}[1 - a - b*x])/(4*(1 - a)^2) + (b^2*\text{Log}[1 + a + b*x])/(4*(1 + a)^2)$

**Rubi [A]** time = 0.0997384, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {6110, 371, 710, 801}

$$\frac{ab^2 \log(x)}{(1-a^2)^2} - \frac{b}{2(1-a^2)x} - \frac{b^2 \log(-a-bx+1)}{4(1-a)^2} + \frac{b^2 \log(a+bx+1)}{4(a+1)^2} - \frac{\coth^{-1}(a+bx)}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a + b\*x]/x^3, x]

[Out]  $-b/(2*(1 - a^2)*x) - \text{ArcCoth}[a + b*x]/(2*x^2) + (a*b^2*\text{Log}[x])/(1 - a^2)^2 - (b^2*\text{Log}[1 - a - b*x])/(4*(1 - a)^2) + (b^2*\text{Log}[1 + a + b*x])/(4*(1 + a)^2)$

### Rule 6110

Int[((a\_.) + ArcCoth[(c\_) + (d\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_), x\_Symbol] :> Simp[((e + f\*x)^(m + 1)\*(a + b\*ArcCoth[c + d\*x])^p)/(f\*(m + 1)), x] - Dist[(b\*d\*p)/(f\*(m + 1)), Int[((e + f\*x)^(m + 1)\*(a + b\*ArcCoth[c + d\*x])^(p - 1))/(1 - (c + d\*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[m, -1]

### Rule 371

Int[((a\_) + (b\_.)\*(v\_)^(n\_))^(p\_.)\*(x\_)^(m\_.), x\_Symbol] :> With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m\*(a + b\*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /;

FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

### Rule 710

Int[((d\_) + (e\_)\*(x\_))^(m\_)/((a\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(e\*(d + e\*x)^(m + 1))/((m + 1)\*(c\*d^2 + a\*e^2)), x] + Dist[c/(c\*d^2 + a\*e^2), Int[((d + e\*x)^(m + 1)\*(d - e\*x))/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c\*d^2 + a\*e^2, 0] && LtQ[m, -1]

### Rule 801

Int[(((d\_) + (e\_)\*(x\_))^(m\_)\*((f\_) + (g\_)\*(x\_)))/((a\_) + (c\_)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[((d + e\*x)^m\*(f + g\*x))/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[m]

### Rubi steps

$$\begin{aligned}
 \int \frac{\coth^{-1}(a+bx)}{x^3} dx &= -\frac{\coth^{-1}(a+bx)}{2x^2} + \frac{1}{2}b \int \frac{1}{x^2(1-(a+bx)^2)} dx \\
 &= -\frac{\coth^{-1}(a+bx)}{2x^2} + \frac{1}{2}b^2 \text{Subst} \left( \int \frac{1}{(-a+x)^2(1-x^2)} dx, x, a+bx \right) \\
 &= -\frac{b}{2(1-a^2)x} - \frac{\coth^{-1}(a+bx)}{2x^2} - \frac{b^2 \text{Subst} \left( \int \frac{-a-x}{(-a+x)(1-x^2)} dx, x, a+bx \right)}{2(1-a^2)} \\
 &= -\frac{b}{2(1-a^2)x} - \frac{\coth^{-1}(a+bx)}{2x^2} - \frac{b^2 \text{Subst} \left( \int \left( -\frac{2a}{(-1+a^2)(a-x)} + \frac{-1-a}{2(-1+a)(-1+x)} + \frac{-1+a}{2(1+a)(1+x)} \right) dx, x, a+bx \right)}{2(1-a^2)} \\
 &= -\frac{b}{2(1-a^2)x} - \frac{\coth^{-1}(a+bx)}{2x^2} + \frac{ab^2 \log(x)}{(1-a^2)^2} - \frac{b^2 \log(1-a-bx)}{4(1-a)^2} + \frac{b^2 \log(1+a+bx)}{4(1+a)^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.106819, size = 76, normalized size = 0.84

$$\frac{1}{4} \left( b \left( \frac{4ab \log(x)}{(a^2-1)^2} + \frac{2}{(a^2-1)x} - \frac{b \log(-a-bx+1)}{(a-1)^2} + \frac{b \log(a+bx+1)}{(a+1)^2} \right) - \frac{2 \coth^{-1}(a+bx)}{x^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a + b\*x]/x^3, x]

[Out]  $((-2*\text{ArcCoth}[a + b*x])/x^2 + b*(2/((-1 + a^2)*x) + (4*a*b*\text{Log}[x])/(-1 + a^2)^2 - (b*\text{Log}[1 - a - b*x])/(-1 + a)^2 + (b*\text{Log}[1 + a + b*x])/(1 + a)^2))/4$

**Maple [A]** time = 0.043, size = 82, normalized size = 0.9

$$-\frac{\text{arccoth}(bx+a)}{2x^2} - \frac{b^2 \ln(bx+a-1)}{4(a-1)^2} + \frac{b}{(2a-2)(1+a)x} + \frac{ab^2 \ln(bx)}{(a-1)^2(1+a)^2} + \frac{b^2 \ln(bx+a+1)}{4(1+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(b*x+a)/x^3,x)`

[Out]  $-1/2*\text{arccoth}(b*x+a)/x^2 - 1/4*b^2/(a-1)^2*\ln(b*x+a-1) + 1/2*b/(a-1)/(1+a)/x + b^2*a/(a-1)^2/(1+a)^2*\ln(b*x) + 1/4*b^2*\ln(b*x+a+1)/(1+a)^2$

**Maxima [A]** time = 0.959889, size = 115, normalized size = 1.28

$$\frac{1}{4} \left( \frac{4ab \log(x)}{a^4 - 2a^2 + 1} + \frac{b \log(bx + a + 1)}{a^2 + 2a + 1} - \frac{b \log(bx + a - 1)}{a^2 - 2a + 1} + \frac{2}{(a^2 - 1)x} \right) b - \frac{\text{arccoth}(bx + a)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(b*x+a)/x^3,x, algorithm="maxima")`

[Out]  $1/4*(4*a*b*\log(x)/(a^4 - 2*a^2 + 1) + b*\log(b*x + a + 1)/(a^2 + 2*a + 1) - b*\log(b*x + a - 1)/(a^2 - 2*a + 1) + 2/((a^2 - 1)*x))*b - 1/2*\text{arccoth}(b*x + a)/x^2$

**Fricas [A]** time = 1.75893, size = 279, normalized size = 3.1

$$\frac{(a^2 - 2a + 1)b^2x^2 \log(bx + a + 1) - (a^2 + 2a + 1)b^2x^2 \log(bx + a - 1) + 4ab^2x^2 \log(x) + 2(a^2 - 1)bx - (a^4 - 2a^2 + 1)}{4(a^4 - 2a^2 + 1)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(b*x+a)/x^3,x, algorithm="fricas")`

```
[Out] 1/4*((a^2 - 2*a + 1)*b^2*x^2*log(b*x + a + 1) - (a^2 + 2*a + 1)*b^2*x^2*log
(b*x + a - 1) + 4*a*b^2*x^2*log(x) + 2*(a^2 - 1)*b*x - (a^4 - 2*a^2 + 1)*lo
g((b*x + a + 1)/(b*x + a - 1)))/((a^4 - 2*a^2 + 1)*x^2)
```

**Sympy [A]** time = 3.45566, size = 410, normalized size = 4.56

$$\left\{ \begin{array}{l} \frac{b^2 \operatorname{acoth}(bx-1)}{8} - \frac{b}{8x} - \frac{\operatorname{acoth}(bx-1)}{2x^2} - \frac{1}{8x^2} \\ \frac{b^2 \operatorname{acoth}(bx+1)}{8} - \frac{b}{8x} - \frac{\operatorname{acoth}(bx+1)}{2x^2} + \frac{1}{8x^2} \\ -\frac{a^4 \operatorname{acoth}(a+bx)}{2a^4x^2-4a^2x^2+2x^2} + \frac{a^2b^2x^2 \operatorname{acoth}(a+bx)}{2a^4x^2-4a^2x^2+2x^2} + \frac{a^2bx}{2a^4x^2-4a^2x^2+2x^2} + \frac{2a^2 \operatorname{acoth}(a+bx)}{2a^4x^2-4a^2x^2+2x^2} + \frac{2ab^2x^2 \log(x)}{2a^4x^2-4a^2x^2+2x^2} - \frac{2ab^2x^2 \log(a+bx+1)}{2a^4x^2-4a^2x^2+2x^2} + \frac{2ab^2x^2 \operatorname{acoth}(a+bx)}{2a^4x^2-4a^2x^2+2x^2} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)/x**3,x)
```

```
[Out] Piecewise((b**2*acoth(b*x - 1)/8 - b/(8*x) - acoth(b*x - 1)/(2*x**2) - 1/(8
*x**2), Eq(a, -1)), (b**2*acoth(b*x + 1)/8 - b/(8*x) - acoth(b*x + 1)/(2*x*
*2) + 1/(8*x**2), Eq(a, 1)), (-a**4*acoth(a + b*x)/(2*a**4*x**2 - 4*a**2*x*
*2 + 2*x**2) + a**2*b**2*x**2*acoth(a + b*x)/(2*a**4*x**2 - 4*a**2*x**2 + 2
*x**2) + a**2*b*x/(2*a**4*x**2 - 4*a**2*x**2 + 2*x**2) + 2*a**2*acoth(a + b
*x)/(2*a**4*x**2 - 4*a**2*x**2 + 2*x**2) + 2*a*b**2*x**2*log(x)/(2*a**4*x**
2 - 4*a**2*x**2 + 2*x**2) - 2*a*b**2*x**2*log(a + b*x + 1)/(2*a**4*x**2 - 4
*a**2*x**2 + 2*x**2) + 2*a*b**2*x**2*acoth(a + b*x)/(2*a**4*x**2 - 4*a**2*x
**2 + 2*x**2) + b**2*x**2*acoth(a + b*x)/(2*a**4*x**2 - 4*a**2*x**2 + 2*x**
2) - b*x/(2*a**4*x**2 - 4*a**2*x**2 + 2*x**2) - acoth(a + b*x)/(2*a**4*x**2
- 4*a**2*x**2 + 2*x**2), True))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(bx+a)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a)/x^3, x)
```

### 3.69 $\int x^3 \coth^{-1}(a + bx)^2 dx$

**Optimal.** Leaf size=263

$$\frac{a(a^2 + 1) \operatorname{PolyLog}\left(2, -\frac{a+bx+1}{-a-bx+1}\right)}{b^4} + \frac{(6a^2 + 1) \log(1 - (a + bx)^2)}{4b^4} - \frac{a(a^2 + 1) \coth^{-1}(a + bx)^2}{b^4} - \frac{(a^4 + 6a^2 + 1) \coth^{-1}}{4b^4}$$

```
[Out] -((a*x)/b^3) + (a + b*x)^2/(12*b^4) + ((1 + 6*a^2)*(a + b*x)*ArcCoth[a + b*x])/(2*b^4) - (a*(a + b*x)^2*ArcCoth[a + b*x])/b^4 + ((a + b*x)^3*ArcCoth[a + b*x])/(6*b^4) - (a*(1 + a^2)*ArcCoth[a + b*x]^2)/b^4 - ((1 + 6*a^2 + a^4)*ArcCoth[a + b*x]^2)/(4*b^4) + (x^4*ArcCoth[a + b*x]^2)/4 + (a*ArcTanh[a + b*x])/b^4 + (2*a*(1 + a^2)*ArcCoth[a + b*x]*Log[2/(1 - a - b*x)])/b^4 + Log[1 - (a + b*x)^2]/(12*b^4) + ((1 + 6*a^2)*Log[1 - (a + b*x)^2])/(4*b^4) + (a*(1 + a^2)*PolyLog[2, -((1 + a + b*x)/(1 - a - b*x))])/b^4
```

**Rubi [A]** time = 0.348779, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 15, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.25$ , Rules used = {6112, 5929, 5911, 260, 5917, 321, 206, 266, 43, 6049, 5949, 5985, 5919, 2402, 2315}

$$\frac{a(a^2 + 1) \operatorname{PolyLog}\left(2, -\frac{a+bx+1}{-a-bx+1}\right)}{b^4} + \frac{(6a^2 + 1) \log(1 - (a + bx)^2)}{4b^4} - \frac{a(a^2 + 1) \coth^{-1}(a + bx)^2}{b^4} - \frac{(a^4 + 6a^2 + 1) \coth^{-1}}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCoth[a + b*x]^2, x]
```

```
[Out] -((a*x)/b^3) + (a + b*x)^2/(12*b^4) + ((1 + 6*a^2)*(a + b*x)*ArcCoth[a + b*x])/(2*b^4) - (a*(a + b*x)^2*ArcCoth[a + b*x])/b^4 + ((a + b*x)^3*ArcCoth[a + b*x])/(6*b^4) - (a*(1 + a^2)*ArcCoth[a + b*x]^2)/b^4 - ((1 + 6*a^2 + a^4)*ArcCoth[a + b*x]^2)/(4*b^4) + (x^4*ArcCoth[a + b*x]^2)/4 + (a*ArcTanh[a + b*x])/b^4 + (2*a*(1 + a^2)*ArcCoth[a + b*x]*Log[2/(1 - a - b*x)])/b^4 + Log[1 - (a + b*x)^2]/(12*b^4) + ((1 + 6*a^2)*Log[1 - (a + b*x)^2])/(4*b^4) + (a*(1 + a^2)*PolyLog[2, -((1 + a + b*x)/(1 - a - b*x))])/b^4
```

#### Rule 6112

```
Int[((a_.) + ArcCoth[(c_.) + (d_.)*(x_)])*(b_.))^ (p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
```

Q[p, 0]

### Rule 5929

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*((d\_.) + (e\_.)\*(x\_.))^(q\_.), x\_Symbol] := Simp[((d + e\*x)^(q + 1)\*(a + b\*ArcCoth[c\*x])^p)/(e\*(q + 1)), x] - Dist[(b\*c\*p)/(e\*(q + 1)), Int[ExpandIntegrand[(a + b\*ArcCoth[c\*x])^(p - 1), (d + e\*x)^(q + 1)/(1 - c^2\*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]

### Rule 5911

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.), x\_Symbol] := Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rule 5917

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*((d\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcCoth[c\*x])^p)/(d\*(m + 1)), x] - Dist[(b\*c\*p)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 321

Int[((c\_.)\*(x\_.))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^ (p\_.), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

### Rule 43

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rule 6049

```
Int[(((a_) + ArcCoth[(c_)*(x_)]*(b_))^(p_)*((f_) + (g_)*(x_))^(m_))/
(d_) + (e_)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcCoth[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

### Rule 5949

```
Int[((a_) + ArcCoth[(c_)*(x_)]*(b_))^(p_)/((d_) + (e_)*(x_)^2), x_Symb
ol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

### Rule 5985

```
Int[(((a_) + ArcCoth[(c_)*(x_)]*(b_))^(p_)*(x_))/((d_) + (e_)*(x_)^2),
x_Symbol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

### Rule 5919

```
Int[((a_) + ArcCoth[(c_)*(x_)]*(b_))^(p_)/((d_) + (e_)*(x_)), x_Symbol
] := -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

### Rule 2402

```
Int[Log[(c_)/((d_) + (e_)*(x_))]/((f_) + (g_)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] :> -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

Rubi steps

$$\begin{aligned}
 \int x^3 \coth^{-1}(a + bx)^2 dx &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b} + \frac{x}{b}\right)^3 \coth^{-1}(x)^2 dx, x, a + bx\right)}{b} \\
 &= \frac{1}{4}x^4 \coth^{-1}(a + bx)^2 - \frac{1}{2} \text{Subst}\left(\int \left(-\frac{(1 + 6a^2) \coth^{-1}(x)}{b^4} + \frac{4ax \coth^{-1}(x)}{b^4} - \frac{x^2 \coth^{-1}(x)}{b^4}\right) dx, x, a + bx\right) \\
 &= \frac{1}{4}x^4 \coth^{-1}(a + bx)^2 + \frac{\text{Subst}\left(\int x^2 \coth^{-1}(x) dx, x, a + bx\right)}{2b^4} - \frac{\text{Subst}\left(\int \frac{(1+6a^2+a^4-4a(1+a^2)x) \coth^{-1}(x)}{1-x^2} dx, x, a + bx\right)}{2b^4} \\
 &= \frac{(1 + 6a^2)(a + bx) \coth^{-1}(a + bx)}{2b^4} - \frac{a(a + bx)^2 \coth^{-1}(a + bx)}{b^4} + \frac{(a + bx)^3 \coth^{-1}(a + bx)}{6b^4} + \\
 &= -\frac{ax}{b^3} + \frac{(1 + 6a^2)(a + bx) \coth^{-1}(a + bx)}{2b^4} - \frac{a(a + bx)^2 \coth^{-1}(a + bx)}{b^4} + \frac{(a + bx)^3 \coth^{-1}(a + bx)}{6b^4} \\
 &= -\frac{ax}{b^3} + \frac{(1 + 6a^2)(a + bx) \coth^{-1}(a + bx)}{2b^4} - \frac{a(a + bx)^2 \coth^{-1}(a + bx)}{b^4} + \frac{(a + bx)^3 \coth^{-1}(a + bx)}{6b^4} \\
 &= -\frac{ax}{b^3} + \frac{(a + bx)^2}{12b^4} + \frac{(1 + 6a^2)(a + bx) \coth^{-1}(a + bx)}{2b^4} - \frac{a(a + bx)^2 \coth^{-1}(a + bx)}{b^4} + \frac{(a + bx)^3 \coth^{-1}(a + bx)}{6b^4} \\
 &= -\frac{ax}{b^3} + \frac{(a + bx)^2}{12b^4} + \frac{(1 + 6a^2)(a + bx) \coth^{-1}(a + bx)}{2b^4} - \frac{a(a + bx)^2 \coth^{-1}(a + bx)}{b^4} + \frac{(a + bx)^3 \coth^{-1}(a + bx)}{6b^4} \\
 &= -\frac{ax}{b^3} + \frac{(a + bx)^2}{12b^4} + \frac{(1 + 6a^2)(a + bx) \coth^{-1}(a + bx)}{2b^4} - \frac{a(a + bx)^2 \coth^{-1}(a + bx)}{b^4} + \frac{(a + bx)^3 \coth^{-1}(a + bx)}{6b^4}
 \end{aligned}$$

**Mathematica [A]** time = 1.56441, size = 203, normalized size = 0.77

$$12(a^3 + a) \text{PolyLog}\left(2, e^{-2 \coth^{-1}(a+bx)}\right) + 3(a^4 - 4a^3 + 6a^2 - 4a - b^4 x^4 + 1) \coth^{-1}(a + bx)^2 - 2 \coth^{-1}(a + bx) \left(9a^2 bx^3 + \dots\right)$$

Warning: Unable to verify antiderivative.



```
[In] Integrate[x^3*ArcCoth[a + b*x]^2,x]
```

```
[Out] -(1 + 11*a^2 + 10*a*b*x - b^2*x^2 + 3*(1 - 4*a + 6*a^2 - 4*a^3 + a^4 - b^4*
x^4)*ArcCoth[a + b*x]^2 - 2*ArcCoth[a + b*x]*(9*a + 13*a^3 + 3*b*x + 9*a^2*
b*x - 3*a*b^2*x^2 + b^3*x^3 + 12*(a + a^3)*Log[1 - E^(-2*ArcCoth[a + b*x])])
) + 8*Log[1/((a + b*x)*Sqrt[1 - (a + b*x)^(-2)])] + 36*a^2*Log[1/((a + b*x)
*Sqrt[1 - (a + b*x)^(-2)])] + 12*(a + a^3)*PolyLog[2, E^(-2*ArcCoth[a + b*x
])])/(12*b^4)
```

**Maple [B]** time = 0.056, size = 967, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccoth(b*x+a)^2,x)
```

```
[Out] 1/4/b^4*arccoth(b*x+a)*ln(b*x+a-1)*a^4+1/8/b^4*ln(-1/2*b*x-1/2*a+1/2)*ln(1/
2+1/2*b*x+1/2*a)*a^4+1/2/b^4*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2+1/2*b*x+1/2*a)*a
^3-1/b^4*arccoth(b*x+a)*ln(b*x+a+1)*a-1/b^4*arccoth(b*x+a)*ln(b*x+a+1)*a^3-
1/4/b^4*arccoth(b*x+a)*ln(b*x+a+1)*a^4-11/12/b^4*a^2-1/2/b^2*arccoth(b*x+a)
*x^2*a+1/2/b^4*ln(1/2+1/2*b*x+1/2*a)*ln(b*x+a-1)*a^3+3/2/b^4*arccoth(b*x+a)
*ln(b*x+a-1)*a^2-1/8/b^4*ln(1/2+1/2*b*x+1/2*a)*ln(b*x+a-1)*a^4+3/2/b^3*arcc
oth(b*x+a)*x*a^2-1/b^4*arccoth(b*x+a)*ln(b*x+a-1)*a-1/2/b^4*ln(-1/2*b*x-1/2
*a+1/2)*ln(b*x+a+1)*a-1/b^4*arccoth(b*x+a)*ln(b*x+a-1)*a^3-3/2/b^4*arccoth(
b*x+a)*ln(b*x+a+1)*a^2-3/4/b^4*ln(1/2+1/2*b*x+1/2*a)*ln(b*x+a-1)*a^2-1/2/b^
4*ln(-1/2*b*x-1/2*a+1/2)*ln(b*x+a+1)*a^3+3/2/b^4*ln(b*x+a-1)*a^2-1/2/b^4*ln
(b*x+a-1)*a+3/2/b^4*ln(b*x+a+1)*a^2+1/2/b^4*ln(b*x+a+1)*a-5/6*a*x/b^3+1/2/b
^4*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2+1/2*b*x+1/2*a)*a-1/8/b^4*ln(-1/2*b*x-1/2*a
+1/2)*ln(b*x+a+1)*a^4+3/4/b^4*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2+1/2*b*x+1/2*a)*
a^2+1/2/b^4*ln(1/2+1/2*b*x+1/2*a)*ln(b*x+a-1)*a-3/4/b^4*ln(-1/2*b*x-1/2*a+1
/2)*ln(b*x+a+1)*a^2+1/4*x^4*arccoth(b*x+a)^2+1/b^4*dilog(1/2+1/2*b*x+1/2*a)
*a+1/2/b^4*arccoth(b*x+a)*a+1/4/b^4*arccoth(b*x+a)*ln(b*x+a-1)+1/2/b^3*arcc
oth(b*x+a)*x+3/8/b^4*ln(b*x+a-1)^2*a^2-1/4/b^4*ln(b*x+a-1)^2*a+1/16/b^4*ln(
b*x+a+1)^2*a^4+3/8/b^4*ln(b*x+a+1)^2*a^2+1/4/b^4*ln(b*x+a+1)^2*a+1/16/b^4*ln
(b*x+a-1)^2*a^4-1/4/b^4*ln(b*x+a-1)^2*a^3+1/b^4*dilog(1/2+1/2*b*x+1/2*a)*a
^3-1/8/b^4*ln(-1/2*b*x-1/2*a+1/2)*ln(b*x+a+1)-1/4/b^4*arccoth(b*x+a)*ln(b*x
+a+1)-1/8/b^4*ln(b*x+a-1)*ln(1/2+1/2*b*x+1/2*a)+13/6/b^4*arccoth(b*x+a)*a^3
+1/6/b^4*arccoth(b*x+a)*x^3+1/4/b^4*ln(b*x+a+1)^2*a^3+1/8/b^4*ln(-1/2*b*x-1/2
*a+1/2)*ln(1/2+1/2*b*x+1/2*a)+1/16/b^4*ln(b*x+a+1)^2+1/16/b^4*ln(b*x+a-1)^2
+1/3/b^4*ln(b*x+a+1)+1/3/b^4*ln(b*x+a-1)+1/12*x^2/b^2
```

---

**Maxima [A]** time = 0.988167, size = 432, normalized size = 1.64

$$\frac{1}{4}x^4 \operatorname{arccoth}(bx+a)^2 + \frac{1}{48}b^2 \left( \frac{48(a^3+a) \left( \log(bx+a-1) \log\left(\frac{1}{2}bx + \frac{1}{2}a + \frac{1}{2}\right) + \operatorname{Li}_2\left(-\frac{1}{2}bx - \frac{1}{2}a + \frac{1}{2}\right)\right)}{b^6} + \frac{4(13a^3+18a^2+9a+4) \log(bx+a+1)}{b^6} + \frac{(4b^2x^2 - 40abx + 3(a^4+4a^3+6a^2+4a+1) \log(bx+a+1)^2 - 6(a^4+4a^3+6a^2+4a+1) \log(bx+a+1) \log(bx+a-1) + 3(a^4-4a^3+6a^2-4a+1) \log(bx+a-1)^2 - 4(13a^3-18a^2+9a-4) \log(bx+a-1))}{b^6} + \frac{1}{12}b(2(b^2x^3 - 3abx^2 + 3(3a^2+1)x)/b^4 - 3(a^4+4a^3+6a^2+4a+1) \log(bx+a+1)/b^5 + 3(a^4-4a^3+6a^2-4a+1) \log(bx+a-1)/b^5) \operatorname{arccoth}(bx+a) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(b\*x+a)^2,x, algorithm="maxima")

[Out] 1/4\*x^4\*arccoth(b\*x + a)^2 + 1/48\*b^2\*(48\*(a^3 + a)\*(log(b\*x + a - 1)\*log(1/2\*b\*x + 1/2\*a + 1/2) + dilog(-1/2\*b\*x - 1/2\*a + 1/2))/b^6 + 4\*(13\*a^3 + 18\*a^2 + 9\*a + 4)\*log(b\*x + a + 1)/b^6 + (4\*b^2\*x^2 - 40\*a\*b\*x + 3\*(a^4 + 4\*a^3 + 6\*a^2 + 4\*a + 1)\*log(b\*x + a + 1)^2 - 6\*(a^4 + 4\*a^3 + 6\*a^2 + 4\*a + 1)\*log(b\*x + a + 1)\*log(b\*x + a - 1) + 3\*(a^4 - 4\*a^3 + 6\*a^2 - 4\*a + 1)\*log(b\*x + a - 1)^2 - 4\*(13\*a^3 - 18\*a^2 + 9\*a - 4)\*log(b\*x + a - 1))/b^6 + 1/12\*b\*(2\*(b^2\*x^3 - 3\*a\*b\*x^2 + 3\*(3\*a^2 + 1)\*x)/b^4 - 3\*(a^4 + 4\*a^3 + 6\*a^2 + 4\*a + 1)\*log(b\*x + a + 1)/b^5 + 3\*(a^4 - 4\*a^3 + 6\*a^2 - 4\*a + 1)\*log(b\*x + a - 1)/b^5)\*arccoth(b\*x + a)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(x^3 \operatorname{arccoth}(bx+a)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(b\*x+a)^2,x, algorithm="fricas")

[Out] integral(x^3\*arccoth(b\*x + a)^2, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{acoth}^2(a+bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*acoth(b\*x+a)\*\*2,x)

[Out] Integral(x\*\*3\*acoth(a + b\*x)\*\*2, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arccoth}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(b\*x+a)^2,x, algorithm="giac")

[Out] integrate(x^3\*arccoth(b\*x + a)^2, x)

### 3.70 $\int x^2 \coth^{-1}(a + bx)^2 dx$

**Optimal.** Leaf size=204

$$-\frac{(3a^2 + 1) \operatorname{PolyLog}\left(2, -\frac{a+bx+1}{-a-bx+1}\right)}{3b^3} + \frac{a(a^2 + 3) \coth^{-1}(a + bx)^2}{3b^3} + \frac{(3a^2 + 1) \coth^{-1}(a + bx)^2}{3b^3} - \frac{2(3a^2 + 1) \log\left(\frac{2}{-a-bx+1}\right)}{3b^3}$$

```
[Out] x/(3*b^2) - (2*a*(a + b*x)*ArcCoth[a + b*x])/b^3 + ((a + b*x)^2*ArcCoth[a +
b*x])/(3*b^3) + (a*(3 + a^2)*ArcCoth[a + b*x]^2)/(3*b^3) + ((1 + 3*a^2)*Ar
cCoth[a + b*x]^2)/(3*b^3) + (x^3*ArcCoth[a + b*x]^2)/3 - ArcTanh[a + b*x]/(
3*b^3) - (2*(1 + 3*a^2)*ArcCoth[a + b*x]*Log[2/(1 - a - b*x)])/(3*b^3) - (a
*Log[1 - (a + b*x)^2])/b^3 - ((1 + 3*a^2)*PolyLog[2, -((1 + a + b*x)/(1 - a
- b*x))])/(3*b^3)
```

**Rubi [A]** time = 0.278272, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$ , Rules used = {6112, 5929, 5911, 260, 5917, 321, 206, 6049, 5949, 5985, 5919, 2402, 2315}

$$-\frac{(3a^2 + 1) \operatorname{PolyLog}\left(2, -\frac{a+bx+1}{-a-bx+1}\right)}{3b^3} + \frac{a(a^2 + 3) \coth^{-1}(a + bx)^2}{3b^3} + \frac{(3a^2 + 1) \coth^{-1}(a + bx)^2}{3b^3} - \frac{2(3a^2 + 1) \log\left(\frac{2}{-a-bx+1}\right)}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[a + b*x]^2, x]
```

```
[Out] x/(3*b^2) - (2*a*(a + b*x)*ArcCoth[a + b*x])/b^3 + ((a + b*x)^2*ArcCoth[a +
b*x])/(3*b^3) + (a*(3 + a^2)*ArcCoth[a + b*x]^2)/(3*b^3) + ((1 + 3*a^2)*Ar
cCoth[a + b*x]^2)/(3*b^3) + (x^3*ArcCoth[a + b*x]^2)/3 - ArcTanh[a + b*x]/(
3*b^3) - (2*(1 + 3*a^2)*ArcCoth[a + b*x]*Log[2/(1 - a - b*x)])/(3*b^3) - (a
*Log[1 - (a + b*x)^2])/b^3 - ((1 + 3*a^2)*PolyLog[2, -((1 + a + b*x)/(1 - a
- b*x))])/(3*b^3)
```

#### Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)])*(b_.))^ (p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] := Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

#### Rule 5929

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_), x_Symbol]
:= Simp[((d + e*x)^(q + 1)*(a + b*ArcCoth[c*x])^p)/(e*(q + 1)), x] -
Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcCoth[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

### Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_), x_Symbol] := Simp[x*(a + b*ArcCoth[c*x])^p, x] -
Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

### Rule 260

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)*((d_.)*(x_))^(m_), x_Symbol]
:= Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(d*(m + 1)),
Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x]
&& IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]
```

### Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] -
Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x]
&& IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

### Rule 6049

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)*((f_) + (g_.)*(x_))^(m_))/(d_) + (e_.)*(x_)^2, x_Symbol] := Int[ExpandIntegrand[(a + b*ArcCoth[c*x])^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
```

GtQ[p, 0] && EqQ[c^2\*d + e, 0] && IGtQ[m, 0]

### Rule 5949

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

### Rule 5985

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)\*(x\_)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*e\*(p + 1)), x] + Dist[1/(c\*d), Int[(a + b\*ArcCoth[c\*x])^p/(1 - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && IGtQ[p, 0]

### Rule 5919

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[((a + b\*ArcCoth[c\*x])^p\*Log[2/(1 + (e\*x)/d)])/e, x] + Dist[(b\*c\*p)/e, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*Log[2/(1 + (e\*x)/d)])/((1 - c^2\*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

### Rule 2402

Int[Log[(c\_.)/((d\_) + (e\_.)\*(x\_))]/((f\_) + (g\_.)\*(x\_)^2), x\_Symbol] := -Dist[e/g, Subst[Int[Log[2\*d\*x]/(1 - 2\*d\*x), x], x, 1/(d + e\*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2\*d] && EqQ[e^2\*f + d^2\*g, 0]

### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

### Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(a + bx)^2 dx &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b} + \frac{x}{b}\right)^2 \coth^{-1}(x)^2 dx, x, a + bx\right)}{b} \\
&= \frac{1}{3}x^3 \coth^{-1}(a + bx)^2 - \frac{2}{3} \text{Subst}\left(\int \left(\frac{3a \coth^{-1}(x)}{b^3} - \frac{x \coth^{-1}(x)}{b^3} - \frac{a(3 + a^2) - (1 + 3a^2)}{b^3(1 - x^2)}\right) dx, x, a + bx\right) \\
&= \frac{1}{3}x^3 \coth^{-1}(a + bx)^2 + \frac{2 \text{Subst}\left(\int x \coth^{-1}(x) dx, x, a + bx\right)}{3b^3} + \frac{2 \text{Subst}\left(\int \frac{a(3+a^2) - (1+3a^2)x}{1-x^2} dx, x, a + bx\right)}{3b^3} \\
&= -\frac{2a(a + bx) \coth^{-1}(a + bx)}{b^3} + \frac{(a + bx)^2 \coth^{-1}(a + bx)}{3b^3} + \frac{1}{3}x^3 \coth^{-1}(a + bx)^2 - \frac{\text{Subst}\left(\int \frac{a \log(1-x)}{1-x^2} dx, x, a + bx\right)}{3b^3} \\
&= \frac{x}{3b^2} - \frac{2a(a + bx) \coth^{-1}(a + bx)}{b^3} + \frac{(a + bx)^2 \coth^{-1}(a + bx)}{3b^3} + \frac{1}{3}x^3 \coth^{-1}(a + bx)^2 - \frac{a \log(1 - (a + bx)^2)}{3b^3} \\
&= \frac{x}{3b^2} - \frac{2a(a + bx) \coth^{-1}(a + bx)}{b^3} + \frac{(a + bx)^2 \coth^{-1}(a + bx)}{3b^3} + \frac{a(3 + a^2) \coth^{-1}(a + bx)^2}{3b^3} \\
&= \frac{x}{3b^2} - \frac{2a(a + bx) \coth^{-1}(a + bx)}{b^3} + \frac{(a + bx)^2 \coth^{-1}(a + bx)}{3b^3} + \frac{a(3 + a^2) \coth^{-1}(a + bx)^2}{3b^3} \\
&= \frac{x}{3b^2} - \frac{2a(a + bx) \coth^{-1}(a + bx)}{b^3} + \frac{(a + bx)^2 \coth^{-1}(a + bx)}{3b^3} + \frac{a(3 + a^2) \coth^{-1}(a + bx)^2}{3b^3} \\
&= \frac{x}{3b^2} - \frac{2a(a + bx) \coth^{-1}(a + bx)}{b^3} + \frac{(a + bx)^2 \coth^{-1}(a + bx)}{3b^3} + \frac{a(3 + a^2) \coth^{-1}(a + bx)^2}{3b^3}
\end{aligned}$$

**Mathematica [B]** time = 4.52669, size = 607, normalized size = 2.98

$$(a + bx) \sqrt{1 - \frac{1}{(a + bx)^2}} (1 - (a + bx)^2) \left( \frac{4(3a^2 + 1) \text{PolyLog}\left(2, e^{-2 \coth^{-1}(a + bx)}\right)}{(a + bx)^3 \left(1 - \frac{1}{(a + bx)^2}\right)^{3/2}} + \frac{9a^2 \coth^{-1}(a + bx)^2}{(a + bx) \sqrt{1 - \frac{1}{(a + bx)^2}}} + \frac{-3(a^2 - 1) \coth^{-1}(a + bx)^2 + 6a \coth^{-1}(a + bx)}{\sqrt{1 - \frac{1}{(a + bx)^2}}} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2\*ArcCoth[a + b\*x]^2,x]

[Out] -((a + b\*x)\*Sqrt[1 - (a + b\*x)^(-2)]\*(1 - (a + b\*x)^2)\*((4\*ArcCoth[a + b\*x])^2)/((a + b\*x)\*Sqrt[1 - (a + b\*x)^(-2)]) + (3\*ArcCoth[a + b\*x]^2)/((a + b\*x)\*Sqrt[1 - (a + b\*x)^(-2)])

$$\begin{aligned} & \sqrt{1 - (a + bx)^{-2}} - (12a \operatorname{ArcCoth}[a + bx]^2) / ((a + bx) \sqrt{1 - (a + bx)^{-2}}) + (9a^2 \operatorname{ArcCoth}[a + bx]^2) / ((a + bx) \sqrt{1 - (a + bx)^{-2}}) \\ & + (-1 + 6a \operatorname{ArcCoth}[a + bx] - 3(-1 + a^2) \operatorname{ArcCoth}[a + bx]^2) / \sqrt{1 - (a + bx)^{-2}} + \operatorname{Cosh}[3 \operatorname{ArcCoth}[a + bx]] - 6a \operatorname{ArcCoth}[a + bx] \operatorname{Cosh}[3 \operatorname{ArcCoth}[a + bx]] \\ & + \operatorname{ArcCoth}[a + bx]^2 \operatorname{Cosh}[3 \operatorname{ArcCoth}[a + bx]] + 3a^2 \operatorname{ArcCoth}[a + bx]^2 \operatorname{Cosh}[3 \operatorname{ArcCoth}[a + bx]] + (6 \operatorname{ArcCoth}[a + bx] \operatorname{Log}[1 - E^{-2 \operatorname{ArcCoth}[a + bx]}]) \\ & / ((a + bx) \sqrt{1 - (a + bx)^{-2}}) + (18a^2 \operatorname{ArcCoth}[a + bx] \operatorname{Log}[1 - E^{-2 \operatorname{ArcCoth}[a + bx]}]) / ((a + bx) \sqrt{1 - (a + bx)^{-2}}) \\ & - (18a \operatorname{Log}[1 / ((a + bx) \sqrt{1 - (a + bx)^{-2}})]) / ((a + bx) \sqrt{1 - (a + bx)^{-2}}) + (4(1 + 3a^2) \operatorname{PolyLog}[2, E^{-2 \operatorname{ArcCoth}[a + bx]}]) \\ & / ((a + bx)^3 (1 - (a + bx)^{-2})^{3/2}) - \operatorname{ArcCoth}[a + bx]^2 \operatorname{Sinh}[3 \operatorname{ArcCoth}[a + bx]] - 3a^2 \operatorname{ArcCoth}[a + bx]^2 \operatorname{Sinh}[3 \operatorname{ArcCoth}[a + bx]] \\ & - 2 \operatorname{ArcCoth}[a + bx] \operatorname{Log}[1 - E^{-2 \operatorname{ArcCoth}[a + bx]}] \operatorname{Sinh}[3 \operatorname{ArcCoth}[a + bx]] - 6a^2 \operatorname{ArcCoth}[a + bx] \operatorname{Log}[1 - E^{-2 \operatorname{ArcCoth}[a + bx]}] \operatorname{Sinh}[3 \operatorname{ArcCoth}[a + bx]] \\ & + 6a \operatorname{Log}[1 / ((a + bx) \sqrt{1 - (a + bx)^{-2}})] \operatorname{Sinh}[3 \operatorname{ArcCoth}[a + bx]])) / (12b^3) \end{aligned}$$

**Maple [B]** time = 0.055, size = 729, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\int (x^2 \operatorname{arccoth}(bx+a))^2 dx$

[Out] 
$$\begin{aligned} & -1/b^3 \ln(bx+a-1) a - 1/b^3 \ln(bx+a+1) a + 1/3 x^3 \operatorname{arccoth}(bx+a)^2 + 1/6/b^3 \ln(-1/2bx-1/2a+1/2) \ln(bx+a+1) \\ & + 1/3/b^3 \operatorname{arccoth}(bx+a) \ln(bx+a+1) - 1/6/b^3 \ln(bx+a-1) \ln(1/2+1/2bx+1/2a) - 1/b^3 \operatorname{dilog}(1/2+1/2bx+1/2a) a^2 - 1/12/b^3 \ln(bx+a+1)^2 a^3 \\ & - 1/4/b^3 \ln(bx+a+1)^2 a^2 - 1/4/b^3 \ln(bx+a+1)^2 a - 1/12/b^3 \ln(bx+a-1)^2 a^3 + 1/4/b^3 \ln(bx+a-1)^2 a^2 - 1/4/b^3 \ln(bx+a-1)^2 a - 5/3/b^3 \operatorname{arccoth}(bx+a) a^2 \\ & + 1/3/b \operatorname{arccoth}(bx+a) x^2 + 1/3/b^3 \operatorname{arccoth}(bx+a) \ln(bx+a-1) - 1/6/b^3 \ln(-1/2bx-1/2a+1/2) \ln(1/2+1/2bx+1/2a) \\ & - 1/2/b^3 \ln(1/2+1/2bx+1/2a) \ln(bx+a-1) a^2 + 1/6/b^3 \ln(1/2+1/2bx+1/2a) \ln(bx+a-1) a^3 + 1/3/b^3 \operatorname{arccoth}(bx+a) \ln(bx+a+1) a^3 \\ & - 1/6/b^3 \ln(-1/2bx-1/2a+1/2) \ln(1/2+1/2bx+1/2a) a^3 - 1/2/b^3 \ln(-1/2bx-1/2a+1/2) \ln(1/2+1/2bx+1/2a) a^2 \\ & - 1/b^3 \operatorname{arccoth}(bx+a) \ln(bx+a-1) a - 4/3/b^2 \operatorname{arccoth}(bx+a) x a - 1/2/b^3 \ln(-1/2bx-1/2a+1/2) \ln(1/2+1/2bx+1/2a) a + 1/6/b^3 \ln(-1/2bx-1/2a+1/2) \ln(bx+a+1) a^3 \\ & + 1/2/b^3 \ln(1/2+1/2bx+1/2a) \ln(bx+a-1) a + 1/2/b^3 \ln(-1/2bx-1/2a+1/2) \ln(bx+a+1) a^2 + 1/2/b^3 \ln(-1/2bx-1/2a+1/2) \ln(bx+a+1) a + 1/b^3 \operatorname{arccoth}(bx+a) \ln(bx+a+1) a^2 \\ & + 1/b^3 \operatorname{arccoth}(bx+a) \ln(bx+a+1) a^2 + 1/b^3 \operatorname{arccoth}(bx+a) \ln(bx+a+1) a - 1/3/b^3 \operatorname{arccoth}(bx+a) \ln(bx+a-1) a^3 + 1/b^3 \operatorname{arccoth}(bx+a) \ln(bx+a-1) a^2 \\ & + 1/6/b^3 \ln(bx+a-1) - 1/6/b^3 \ln(bx+a+1) - 1/12/b^3 \ln(bx+a+1)^2 - 1/3/ \end{aligned}$$



$$b^3 \operatorname{dilog}\left(\frac{1}{2} + \frac{1}{2}bx + \frac{1}{2}a\right) + \frac{1}{3}b^3 a + \frac{1}{12}b^3 \ln(bx+a-1)^2 + \frac{1}{3}x/b^2$$

**Maxima [A]** time = 1.00212, size = 350, normalized size = 1.72

$$\frac{1}{3}x^3 \operatorname{arccoth}(bx+a)^2 - \frac{1}{12}b^2 \left( \frac{4(3a^2+1) \left( \log(bx+a-1) \log\left(\frac{1}{2}bx + \frac{1}{2}a + \frac{1}{2}\right) + \operatorname{Li}_2\left(-\frac{1}{2}bx - \frac{1}{2}a + \frac{1}{2}\right) \right)}{b^5} + \frac{2(5a^2+6}{b^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(b\*x+a)^2,x, algorithm="maxima")

[Out]  $\frac{1}{3}x^3 \operatorname{arccoth}(bx+a)^2 - \frac{1}{12}b^2(4(3a^2+1)(\log(bx+a-1)\log(\frac{1}{2}bx + \frac{1}{2}a + \frac{1}{2}) + \operatorname{dilog}(-\frac{1}{2}bx - \frac{1}{2}a + \frac{1}{2}))/b^5 + \frac{2(5a^2+6a+1)\log(bx+a+1)}{b^5} + \frac{(a^3+3a^2+3a+1)\log(bx+a+1)^2 - 2(a^3+3a^2+3a+1)\log(bx+a+1)\log(bx+a-1) + (a^3-3a^2+3a-1)\log(bx+a-1)^2 - 4bx - 2(5a^2-6a+1)\log(bx+a-1)}{b^5} + \frac{1}{3}b((bx^2-4ax)/b^3 + (a^3+3a^2+3a+1)\log(bx+a+1)/b^4 - (a^3-3a^2+3a-1)\log(bx+a-1)/b^4) \operatorname{arccoth}(bx+a)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(x^2 \operatorname{arccoth}(bx+a)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(b\*x+a)^2,x, algorithm="fricas")

[Out] integral(x^2\*arccoth(b\*x + a)^2, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*acoth(b\*x+a)\*\*2,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arccoth}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccoth(b*x+a)^2,x, algorithm="giac")`

[Out] `integrate(x^2*arccoth(b*x + a)^2, x)`

### 3.71 $\int x \coth^{-1}(a + bx)^2 dx$

**Optimal.** Leaf size=136

$$\frac{a \operatorname{PolyLog}\left(2, -\frac{a+bx+1}{-a-bx+1}\right)}{b^2} - \frac{(a^2 + 1) \coth^{-1}(a + bx)^2}{2b^2} + \frac{\log(1 - (a + bx)^2)}{2b^2} - \frac{a \coth^{-1}(a + bx)^2}{b^2} + \frac{(a + bx) \coth^{-1}(a + bx)^2}{b^2}$$

[Out] ((a + b\*x)\*ArcCoth[a + b\*x])/b^2 - (a\*ArcCoth[a + b\*x]^2)/b^2 - ((1 + a^2)\*ArcCoth[a + b\*x]^2)/(2\*b^2) + (x^2\*ArcCoth[a + b\*x]^2)/2 + (2\*a\*ArcCoth[a + b\*x]\*Log[2/(1 - a - b\*x)])/b^2 + Log[1 - (a + b\*x)^2]/(2\*b^2) + (a\*PolyLog[2, -((1 + a + b\*x)/(1 - a - b\*x))])/b^2

**Rubi [A]** time = 0.209095, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.$ , Rules used = {6112, 5929, 5911, 260, 6049, 5949, 5985, 5919, 2402, 2315}

$$\frac{a \operatorname{PolyLog}\left(2, -\frac{a+bx+1}{-a-bx+1}\right)}{b^2} - \frac{(a^2 + 1) \coth^{-1}(a + bx)^2}{2b^2} + \frac{\log(1 - (a + bx)^2)}{2b^2} - \frac{a \coth^{-1}(a + bx)^2}{b^2} + \frac{(a + bx) \coth^{-1}(a + bx)^2}{b^2}$$

Antiderivative was successfully verified.

[In] Int[x\*ArcCoth[a + b\*x]^2,x]

[Out] ((a + b\*x)\*ArcCoth[a + b\*x])/b^2 - (a\*ArcCoth[a + b\*x]^2)/b^2 - ((1 + a^2)\*ArcCoth[a + b\*x]^2)/(2\*b^2) + (x^2\*ArcCoth[a + b\*x]^2)/2 + (2\*a\*ArcCoth[a + b\*x]\*Log[2/(1 - a - b\*x)])/b^2 + Log[1 - (a + b\*x)^2]/(2\*b^2) + (a\*PolyLog[2, -((1 + a + b\*x)/(1 - a - b\*x))])/b^2

#### Rule 6112

Int[((a\_.) + ArcCoth[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]

#### Rule 5929

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*((d\_.) + (e\_.)\*(x\_.))^(q\_.), x\_Symbol] :> Simp[((d + e\*x)^(q + 1)\*(a + b\*ArcCoth[c\*x])^p)/(e\*(q + 1)), x] - Dist[(b\*c\*p)/(e\*(q + 1)), Int[ExpandIntegrand[(a + b\*ArcCoth[c\*x])^(p - 1), (d + e\*x)^(q + 1)/(1 - c^2\*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]

&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]

### Rule 5911

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^ (p\_.), x\_Symbol] := Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rule 6049

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^ (p\_.)\*((f\_) + (g\_.)\*(x\_)^(m\_.)))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[(a + b\*ArcCoth[c\*x])^p/(d + e\*x^2), (f + g\*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[p, 0] && EqQ[c^2\*d + e, 0] && IGtQ[m, 0]

### Rule 5949

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^ (p\_.)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

### Rule 5985

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^ (p\_.)\*(x\_))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*e\*(p + 1)), x] + Dist[1/(c\*d), Int[(a + b\*ArcCoth[c\*x])^p/(1 - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && IGtQ[p, 0]

### Rule 5919

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^ (p\_.)/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[((a + b\*ArcCoth[c\*x])^p\*Log[2/(1 + (e\*x)/d)])/e, x] + Dist[(b\*c\*p)/e, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*Log[2/(1 + (e\*x)/d)]]/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

### Rule 2402

Int[Log[(c\_.)/((d\_) + (e\_.)\*(x\_))]/((f\_) + (g\_.)\*(x\_)^2), x\_Symbol] := -Dist[e/g, Subst[Int[Log[2\*d\*x]/(1 - 2\*d\*x), x], x, 1/(d + e\*x)], x] /; FreeQ[{

$c, d, e, f, g\}, x] \&\& \text{EqQ}[c, 2*d] \&\& \text{EqQ}[e^2*f + d^2*g, 0]$

### Rule 2315

$\text{Int}[\text{Log}[(c_.)*(x_)]/((d_)+(e_)*(x_)), x\_Symbol] \text{ :> } -\text{Simp}[\text{PolyLog}[2, 1 - c*x]/e, x] /; \text{FreeQ}\{c, d, e\}, x] \&\& \text{EqQ}[e + c*d, 0]$

### Rubi steps

$$\begin{aligned}
 \int x \coth^{-1}(a+bx)^2 dx &= \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)\coth^{-1}(x)^2 dx, x, a+bx\right)}{b} \\
 &= \frac{1}{2}x^2 \coth^{-1}(a+bx)^2 - \text{Subst}\left(\int\left(-\frac{\coth^{-1}(x)}{b^2}+\frac{(1+a^2-2ax)\coth^{-1}(x)}{b^2(1-x^2)}\right) dx, x, a+bx\right) \\
 &= \frac{1}{2}x^2 \coth^{-1}(a+bx)^2 + \frac{\text{Subst}\left(\int\coth^{-1}(x) dx, x, a+bx\right)}{b^2} - \frac{\text{Subst}\left(\int\frac{(1+a^2-2ax)\coth^{-1}(x)}{1-x^2} dx, x, a+bx\right)}{b^2} \\
 &= \frac{(a+bx)\coth^{-1}(a+bx)}{b^2} + \frac{1}{2}x^2 \coth^{-1}(a+bx)^2 - \frac{\text{Subst}\left(\int\frac{x}{1-x^2} dx, x, a+bx\right)}{b^2} - \frac{\text{Subst}\left(\int\left(\frac{x\coth^{-1}(x)}{1-x^2}\right) dx, x, a+bx\right)}{b^2} \\
 &= \frac{(a+bx)\coth^{-1}(a+bx)}{b^2} + \frac{1}{2}x^2 \coth^{-1}(a+bx)^2 + \frac{\log(1-(a+bx)^2)}{2b^2} + \frac{(2a)\text{Subst}\left(\int\frac{x\coth^{-1}(x)}{1-x^2} dx, x, a+bx\right)}{b^2} \\
 &= \frac{(a+bx)\coth^{-1}(a+bx)}{b^2} - \frac{a\coth^{-1}(a+bx)^2}{b^2} - \frac{(1+a^2)\coth^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2 \coth^{-1}(a+bx)^2 \\
 &= \frac{(a+bx)\coth^{-1}(a+bx)}{b^2} - \frac{a\coth^{-1}(a+bx)^2}{b^2} - \frac{(1+a^2)\coth^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2 \coth^{-1}(a+bx)^2 \\
 &= \frac{(a+bx)\coth^{-1}(a+bx)}{b^2} - \frac{a\coth^{-1}(a+bx)^2}{b^2} - \frac{(1+a^2)\coth^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2 \coth^{-1}(a+bx)^2 \\
 &= \frac{(a+bx)\coth^{-1}(a+bx)}{b^2} - \frac{a\coth^{-1}(a+bx)^2}{b^2} - \frac{(1+a^2)\coth^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2 \coth^{-1}(a+bx)^2
 \end{aligned}$$

**Mathematica [A]** time = 0.246845, size = 106, normalized size = 0.78

$$\frac{-2a\text{PolyLog}\left(2, e^{-2\coth^{-1}(a+bx)}\right) + (-a^2 + 2a + b^2x^2 - 1)\coth^{-1}(a+bx)^2 - 2\log\left(\frac{1}{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}\right) + 2\coth^{-1}(a+bx)}{2b^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x\*ArcCoth[a + b\*x]^2,x]

[Out]  $((-1 + 2*a - a^2 + b^2*x^2)*ArcCoth[a + b*x]^2 + 2*ArcCoth[a + b*x]*(a + b*x + 2*a*Log[1 - E^{-2*ArcCoth[a + b*x]}])) - 2*Log[1/((a + b*x)*Sqrt[1 - (a + b*x)^{-2}])] - 2*a*PolyLog[2, E^{-2*ArcCoth[a + b*x]}])/(2*b^2)$

**Maple [B]** time = 0.053, size = 365, normalized size = 2.7

$$\frac{x^2 (\operatorname{arccoth}(bx + a))^2}{2} - \frac{(\operatorname{arccoth}(bx + a))^2 a^2}{2b^2} + \frac{x \operatorname{arccoth}(bx + a)}{b} + \frac{\operatorname{arccoth}(bx + a) a}{b^2} - \frac{\operatorname{arccoth}(bx + a) \ln(bx + a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(b\*x+a)^2,x)

[Out]  $1/2*x^2*\operatorname{arccoth}(b*x+a)^2 - 1/2/b^2*\operatorname{arccoth}(b*x+a)^2*a^2 + 1/b*\operatorname{arccoth}(b*x+a)*x + 1/b^2*\operatorname{arccoth}(b*x+a)*a - 1/b^2*\operatorname{arccoth}(b*x+a)*\ln(b*x+a-1)*a + 1/2/b^2*\operatorname{arccoth}(b*x+a)*\ln(b*x+a-1) - 1/b^2*\operatorname{arccoth}(b*x+a)*\ln(b*x+a+1)*a - 1/2/b^2*\operatorname{arccoth}(b*x+a)*\ln(b*x+a+1) + 1/2/b^2*\ln(b*x+a-1) + 1/2/b^2*\ln(b*x+a+1) - 1/4/b^2*\ln(b*x+a-1)^2*a + 1/b^2*\operatorname{dilog}(1/2+1/2*b*x+1/2*a)*a + 1/2/b^2*\ln(1/2+1/2*b*x+1/2*a)*\ln(b*x+a-1)*a + 1/8/b^2*\ln(b*x+a-1)^2 - 1/4/b^2*\ln(b*x+a-1)*\ln(1/2+1/2*b*x+1/2*a) + 1/2/b^2*\ln(-1/2*b*x-1/2*a+1/2)*\ln(1/2+1/2*b*x+1/2*a)*a - 1/2/b^2*\ln(-1/2*b*x-1/2*a+1/2)*\ln(b*x+a+1)*a + 1/4/b^2*\ln(b*x+a+1)^2*a + 1/4/b^2*\ln(-1/2*b*x-1/2*a+1/2)*\ln(1/2+1/2*b*x+1/2*a) - 1/4/b^2*\ln(-1/2*b*x-1/2*a+1/2)*\ln(b*x+a+1) + 1/8/b^2*\ln(b*x+a+1)^2$

**Maxima [A]** time = 0.984672, size = 273, normalized size = 2.01

$$\frac{1}{2} x^2 \operatorname{arccoth}(bx + a)^2 + \frac{1}{8} b^2 \left( \frac{8 \left( \log(bx + a - 1) \log\left(\frac{1}{2}bx + \frac{1}{2}a + \frac{1}{2}\right) + \operatorname{Li}_2\left(-\frac{1}{2}bx - \frac{1}{2}a + \frac{1}{2}\right) \right) a}{b^4} + \frac{4(a + 1) \log(bx + a + 1)}{b^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(b\*x+a)^2,x, algorithm="maxima")

```
[Out] 1/2*x^2*arccoth(b*x + a)^2 + 1/8*b^2*(8*(log(b*x + a - 1)*log(1/2*b*x + 1/2
*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))*a/b^4 + 4*(a + 1)*log(b*x + a +
1)/b^4 + ((a^2 + 2*a + 1)*log(b*x + a + 1)^2 - 2*(a^2 + 2*a + 1)*log(b*x +
a + 1)*log(b*x + a - 1) + (a^2 - 2*a + 1)*log(b*x + a - 1)^2 - 4*(a - 1)*lo
g(b*x + a - 1))/b^4 + 1/2*b*(2*x/b^2 - (a^2 + 2*a + 1)*log(b*x + a + 1)/b^
3 + (a^2 - 2*a + 1)*log(b*x + a - 1)/b^3)*arccoth(b*x + a)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x \operatorname{arccoth}(bx + a)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x*arccoth(b*x + a)^2, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{acoth}^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(b*x+a)**2,x)
```

```
[Out] Integral(x*acoth(a + b*x)**2, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arccoth}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(b*x + a)^2, x)
```

### 3.72 $\int \coth^{-1}(a + bx)^2 dx$

**Optimal.** Leaf size=81

$$-\frac{\text{PolyLog}\left(2, -\frac{a+bx+1}{-a-bx+1}\right)}{b} + \frac{(a+bx)\coth^{-1}(a+bx)^2}{b} + \frac{\coth^{-1}(a+bx)^2}{b} - \frac{2\log\left(\frac{2}{-a-bx+1}\right)\coth^{-1}(a+bx)}{b}$$

[Out] ArcCoth[a + b\*x]^2/b + ((a + b\*x)\*ArcCoth[a + b\*x]^2)/b - (2\*ArcCoth[a + b\*x]\*Log[2/(1 - a - b\*x)])/b - PolyLog[2, -((1 + a + b\*x)/(1 - a - b\*x))]/b

**Rubi [A]** time = 0.0902758, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.75$ , Rules used = {6104, 5911, 5985, 5919, 2402, 2315}

$$-\frac{\text{PolyLog}\left(2, -\frac{a+bx+1}{-a-bx+1}\right)}{b} + \frac{(a+bx)\coth^{-1}(a+bx)^2}{b} + \frac{\coth^{-1}(a+bx)^2}{b} - \frac{2\log\left(\frac{2}{-a-bx+1}\right)\coth^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a + b\*x]^2, x]

[Out] ArcCoth[a + b\*x]^2/b + ((a + b\*x)\*ArcCoth[a + b\*x]^2)/b - (2\*ArcCoth[a + b\*x]\*Log[2/(1 - a - b\*x)])/b - PolyLog[2, -((1 + a + b\*x)/(1 - a - b\*x))]/b

#### Rule 6104

Int[((a\_.) + ArcCoth[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^ (p\_.), x\_Symbol] := Dist[1/d, Subst[Int[(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0]

#### Rule 5911

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.), x\_Symbol] := Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

#### Rule 5985

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*(x\_.))/((d\_.) + (e\_.)\*(x\_.)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*e\*(p + 1)), x] + Dist[1/(c\*d), Int[(a + b\*ArcCoth[c\*x])^p/(1 - c\*x), x], x] /; FreeQ[{a, b, c, d, e}



}, x] && EqQ[c^2\*d + e, 0] && IGtQ[p, 0]

### Rule 5919

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] :> -Simp[((a + b\*ArcCoth[c\*x])^p\*Log[2/(1 + (e\*x)/d)])/e, x] + Dist[(b\*c\*p)/e, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*Log[2/(1 + (e\*x)/d)])/((1 - c^2\*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

### Rule 2402

Int[Log[(c\_.)/((d\_.) + (e\_.)\*(x\_.))]/((f\_.) + (g\_.)\*(x\_.)^2), x\_Symbol] :> -Dist[e/g, Subst[Int[Log[2\*d\*x]/(1 - 2\*d\*x), x], x, 1/(d + e\*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2\*d] && EqQ[e^2\*f + d^2\*g, 0]

### Rule 2315

Int[Log[(c\_.)\*(x\_.)]/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] :> -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

### Rubi steps

$$\begin{aligned}
 \int \coth^{-1}(a + bx)^2 dx &= \frac{\text{Subst}\left(\int \coth^{-1}(x)^2 dx, x, a + bx\right)}{b} \\
 &= \frac{(a + bx) \coth^{-1}(a + bx)^2}{b} - \frac{2 \text{Subst}\left(\int \frac{x \coth^{-1}(x)}{1-x^2} dx, x, a + bx\right)}{b} \\
 &= \frac{\coth^{-1}(a + bx)^2}{b} + \frac{(a + bx) \coth^{-1}(a + bx)^2}{b} - \frac{2 \text{Subst}\left(\int \frac{\coth^{-1}(x)}{1-x} dx, x, a + bx\right)}{b} \\
 &= \frac{\coth^{-1}(a + bx)^2}{b} + \frac{(a + bx) \coth^{-1}(a + bx)^2}{b} - \frac{2 \coth^{-1}(a + bx) \log\left(\frac{2}{1-a-bx}\right)}{b} + \frac{2 \text{Subst}\left(\int \frac{\log}{1-x} dx, x, a + bx\right)}{b} \\
 &= \frac{\coth^{-1}(a + bx)^2}{b} + \frac{(a + bx) \coth^{-1}(a + bx)^2}{b} - \frac{2 \coth^{-1}(a + bx) \log\left(\frac{2}{1-a-bx}\right)}{b} - \frac{2 \text{Subst}\left(\int \frac{\log}{1-x} dx, x, a + bx\right)}{b} \\
 &= \frac{\coth^{-1}(a + bx)^2}{b} + \frac{(a + bx) \coth^{-1}(a + bx)^2}{b} - \frac{2 \coth^{-1}(a + bx) \log\left(\frac{2}{1-a-bx}\right)}{b} - \frac{\text{Li}_2\left(1 - \frac{2}{1-a-bx}\right)}{b}
 \end{aligned}$$

**Mathematica [A]** time = 0.0696864, size = 55, normalized size = 0.68

$$\frac{\text{PolyLog}\left(2, e^{-2 \coth^{-1}(a+bx)}\right) + \coth^{-1}(a+bx) \left( (a+bx-1) \coth^{-1}(a+bx) - 2 \log\left(1 - e^{-2 \coth^{-1}(a+bx)}\right) \right)}{b}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a + b\*x]^2, x]

[Out] (ArcCoth[a + b\*x]\*((-1 + a + b\*x)\*ArcCoth[a + b\*x] - 2\*Log[1 - E^(-2\*ArcCoth[a + b\*x])]) + PolyLog[2, E^(-2\*ArcCoth[a + b\*x])])/b

**Maple [A]** time = 0.102, size = 151, normalized size = 1.9

$$x(\operatorname{arccoth}(bx+a))^2 + \frac{(\operatorname{arccoth}(bx+a))^2 a}{b} - 2 \frac{\operatorname{arccoth}(bx+a)}{b} \ln\left(1 + \frac{1}{\sqrt{\frac{bx+a-1}{bx+a+1}}}\right) - 2 \frac{\operatorname{arccoth}(bx+a)}{b} \ln\left(1 - \frac{1}{\sqrt{\frac{bx+a-1}{bx+a+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(b\*x+a)^2, x)

[Out] x\*arccoth(b\*x+a)^2+1/b\*arccoth(b\*x+a)^2\*a-2/b\*arccoth(b\*x+a)\*ln(1+1/((b\*x+a-1)/(b\*x+a+1))^(1/2))-2/b\*arccoth(b\*x+a)\*ln(1-1/((b\*x+a-1)/(b\*x+a+1))^(1/2))+arccoth(b\*x+a)^2/b-2/b\*polylog(2, -1/((b\*x+a-1)/(b\*x+a+1))^(1/2))-2/b\*polylog(2, 1/((b\*x+a-1)/(b\*x+a+1))^(1/2))

**Maxima [A]** time = 0.97802, size = 188, normalized size = 2.32

$$-\frac{1}{4} b^2 \left( \frac{(a+1) \log(bx+a+1)^2 - 2(a+1) \log(bx+a+1) \log(bx+a-1) + (a-1) \log(bx+a-1)^2}{b^3} + \frac{4 \left( \log(bx+a-1) \log(1/2*b*$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)^2, x, algorithm="maxima")

[Out] -1/4\*b^2\*(((a + 1)\*log(b\*x + a + 1)^2 - 2\*(a + 1)\*log(b\*x + a + 1)\*log(b\*x + a - 1) + (a - 1)\*log(b\*x + a - 1)^2)/b^3 + 4\*(log(b\*x + a - 1)\*log(1/2\*b\*

$$x + 1/2*a + 1/2) + \operatorname{dilog}(-1/2*b*x - 1/2*a + 1/2)/b^3) + b*((a + 1)*\log(b*x + a + 1)/b^2 - (a - 1)*\log(b*x + a - 1)/b^2)*\operatorname{arccoth}(b*x + a) + x*\operatorname{arccoth}(b*x + a)^2$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(\operatorname{arccoth}(bx + a)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(b*x+a)^2,x, algorithm="fricas")`

[Out] `integral(arccoth(b*x + a)^2, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acoth}^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(b*x+a)**2,x)`

[Out] `Integral(acoth(a + b*x)**2, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(b*x+a)^2,x, algorithm="giac")`

[Out] `integrate(arccoth(b*x + a)^2, x)`

$$3.73 \quad \int \frac{\coth^{-1}(a+bx)^2}{x} dx$$

**Optimal.** Leaf size=148

$$\frac{1}{2}\text{PolyLog}\left(3, 1 - \frac{2}{a+bx+1}\right) - \frac{1}{2}\text{PolyLog}\left(3, 1 - \frac{2bx}{(1-a)(a+bx+1)}\right) + \coth^{-1}(a+bx)\text{PolyLog}\left(2, 1 - \frac{2}{a+bx+1}\right) -$$

[Out]  $-(\text{ArcCoth}[a + b*x]^2*\text{Log}[2/(1 + a + b*x)]) + \text{ArcCoth}[a + b*x]^2*\text{Log}[(2*b*x)/((1 - a)*(1 + a + b*x))] + \text{ArcCoth}[a + b*x]*\text{PolyLog}[2, 1 - 2/(1 + a + b*x)] - \text{ArcCoth}[a + b*x]*\text{PolyLog}[2, 1 - (2*b*x)/((1 - a)*(1 + a + b*x))] + \text{PolyLog}[3, 1 - 2/(1 + a + b*x)]/2 - \text{PolyLog}[3, 1 - (2*b*x)/((1 - a)*(1 + a + b*x))]/2$

**Rubi [A]** time = 0.0913941, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {6112, 5923}

$$\frac{1}{2}\text{PolyLog}\left(3, 1 - \frac{2}{a+bx+1}\right) - \frac{1}{2}\text{PolyLog}\left(3, 1 - \frac{2bx}{(1-a)(a+bx+1)}\right) + \coth^{-1}(a+bx)\text{PolyLog}\left(2, 1 - \frac{2}{a+bx+1}\right) -$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a + b\*x]^2/x, x]

[Out]  $-(\text{ArcCoth}[a + b*x]^2*\text{Log}[2/(1 + a + b*x)]) + \text{ArcCoth}[a + b*x]^2*\text{Log}[(2*b*x)/((1 - a)*(1 + a + b*x))] + \text{ArcCoth}[a + b*x]*\text{PolyLog}[2, 1 - 2/(1 + a + b*x)] - \text{ArcCoth}[a + b*x]*\text{PolyLog}[2, 1 - (2*b*x)/((1 - a)*(1 + a + b*x))] + \text{PolyLog}[3, 1 - 2/(1 + a + b*x)]/2 - \text{PolyLog}[3, 1 - (2*b*x)/((1 - a)*(1 + a + b*x))]/2$

#### Rule 6112

Int[((a\_.) + ArcCoth[(c\_.) + (d\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]

#### Rule 5923

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(2/((d\_.) + (e\_.)\*(x\_))), x\_Symbol] := -Simp[((a + b\*ArcCoth[c\*x])^2\*Log[2/(1 + c\*x)])/e, x] + (Simp[(a + b\*ArcC

$\text{oth}[c*x]^2 * \text{Log}[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))]/e, x] + \text{Simp}[(b*(a + b*\text{ArcCoth}[c*x]) * \text{PolyLog}[2, 1 - 2/(1 + c*x)])/e, x] - \text{Simp}[(b*(a + b*\text{ArcCoth}[c*x]) * \text{PolyLog}[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + \text{Simp}[(b^2 * \text{PolyLog}[3, 1 - 2/(1 + c*x)])/(2*e), x] - \text{Simp}[(b^2 * \text{PolyLog}[3, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))]/(2*e), x)] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[c^2*d^2 - e^2, 0]$

### Rubi steps

$$\int \frac{\coth^{-1}(a + bx)^2}{x} dx = \frac{\text{Subst}\left(\int \frac{\coth^{-1}(x)^2}{-\frac{a}{b} + \frac{x}{b}} dx, x, a + bx\right)}{b}$$

$$= -\coth^{-1}(a + bx)^2 \log\left(\frac{2}{1 + a + bx}\right) + \coth^{-1}(a + bx)^2 \log\left(\frac{2bx}{(1 - a)(1 + a + bx)}\right) + \coth^{-1}(a + bx)$$

**Mathematica [C]** time = 3.20549, size = 714, normalized size = 4.82

$$2 \coth^{-1}(a + bx) \text{PolyLog}\left(2, -\sqrt{\frac{a-1}{a+1}} e^{\coth^{-1}(a+bx)}\right) + 2 \coth^{-1}(a + bx) \text{PolyLog}\left(2, \sqrt{\frac{a-1}{a+1}} e^{\coth^{-1}(a+bx)}\right) - \coth^{-1}(a + bx)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a + b\*x]^2/x, x]

[Out]  $(-I/24)*\text{Pi}^3 - (2*\text{ArcCoth}[a + b*x]^3)/3 - (2*a*\text{ArcCoth}[a + b*x]^3)/3 + (2*\text{Sqrt}[1 - a^{(-2)}]*a*\text{E}^{\text{ArcTanh}[a^{(-1)}]}*\text{ArcCoth}[a + b*x]^3)/3 - I*\text{Pi}*\text{ArcCoth}[a + b*x]*\text{Log}[(\text{E}^{(-\text{ArcCoth}[a + b*x])} + \text{E}^{\text{ArcCoth}[a + b*x]})/2] + \text{ArcCoth}[a + b*x]^2*\text{Log}[1 - \text{Sqrt}[(-1 + a)/(1 + a)]]*\text{E}^{\text{ArcCoth}[a + b*x]} + \text{ArcCoth}[a + b*x]^2*\text{Log}[1 + \text{Sqrt}[(-1 + a)/(1 + a)]]*\text{E}^{\text{ArcCoth}[a + b*x]} - \text{ArcCoth}[a + b*x]^2*\text{Log}[1 - \text{E}^{(2*\text{ArcCoth}[a + b*x])}] - \text{ArcCoth}[a + b*x]^2*\text{Log}[1 - ((-1 + a)*\text{E}^{(2*\text{ArcCoth}[a + b*x])})/(1 + a)] + \text{ArcCoth}[a + b*x]^2*\text{Log}[1 - \text{E}^{(2*\text{ArcCoth}[a + b*x] - 2*\text{ArcTanh}[a^{(-1)}])}] - 2*\text{ArcCoth}[a + b*x]*\text{ArcTanh}[a^{(-1)}]*\text{Log}[(I/2)*(\text{E}^{(\text{ArcCoth}[a + b*x] - \text{ArcTanh}[a^{(-1)}])} - \text{E}^{(-\text{ArcCoth}[a + b*x] + \text{ArcTanh}[a^{(-1)}])})] + \text{ArcCoth}[a + b*x]^2*\text{Log}[(-1 - \text{E}^{(2*\text{ArcCoth}[a + b*x])} + a*(-1 + \text{E}^{(2*\text{ArcCoth}[a + b*x])}))]/(2*\text{E}^{\text{ArcCoth}[a + b*x]})] + I*\text{Pi}*\text{ArcCoth}[a + b*x]*\text{Log}[1/\text{Sqrt}[1 - (a + b*x)^{(-2)}]] - \text{ArcCoth}[a + b*x]^2*\text{Log}[-(b*x)/((a + b*x)*\text{Sqrt}[1 - (a + b*x)^{(-2)}])] + 2*\text{ArcCoth}[a + b*x]*\text{ArcTanh}[a^{(-1)}]*\text{Log}[I*\text{Sinh}[\text{ArcCoth}[a + b*x] - \text{ArcTanh}[a^{(-1)}]]] + 2*\text{ArcCoth}[a + b*x]*\text{PolyLog}[2, -(\text{Sqrt}[(-1 + a)/(1 + a)]]*\text{E}^{\text{ArcCoth}[a + b*x]})] + 2*\text{ArcCoth}[a + b*x]*\text{PolyLog}[2, \text{Sqrt}[(-1 + a)/(1 + a)]]*\text{E}^{\text{ArcCoth}[a + b*x]} - \text{ArcCoth}[a + b*x]*\text{PolyLog}[2, \text{E}^{(2*\text{ArcCoth}[a + b*x])}]$

```

oth[a + b*x]]) - ArcCoth[a + b*x]*PolyLog[2, ((-1 + a)*E^(2*ArcCoth[a + b*x
]))/(1 + a)] + ArcCoth[a + b*x]*PolyLog[2, E^(2*ArcCoth[a + b*x] - 2*ArcTan
h[a^(-1)])] - 2*PolyLog[3, -(Sqrt[(-1 + a)/(1 + a)]*E^ArcCoth[a + b*x])] -
2*PolyLog[3, Sqrt[(-1 + a)/(1 + a)]*E^ArcCoth[a + b*x]] + PolyLog[3, E^(2*A
rcCoth[a + b*x])]/2 + PolyLog[3, ((-1 + a)*E^(2*ArcCoth[a + b*x]))/(1 + a)
]/2 - PolyLog[3, E^(2*ArcCoth[a + b*x] - 2*ArcTanh[a^(-1)])]/2

```

**Maple [C]** time = 0.543, size = 985, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(b\*x+a)^2/x,x)

```

[Out] ln(b*x)*arccoth(b*x+a)^2-arccoth(b*x+a)^2*ln(a*((b*x+a+1)/(b*x+a-1)-1)-(b*x
+a+1)/(b*x+a-1)-1)+arccoth(b*x+a)^2*ln((b*x+a+1)/(b*x+a-1)-1)-arccoth(b*x+a
)^2*ln(1-1/((b*x+a-1)/(b*x+a+1))^(1/2))-2*arccoth(b*x+a)*polylog(2,1/((b*x+
a-1)/(b*x+a+1))^(1/2))+2*polylog(3,1/((b*x+a-1)/(b*x+a+1))^(1/2))-arccoth(b
*x+a)^2*ln(1+1/((b*x+a-1)/(b*x+a+1))^(1/2))-2*arccoth(b*x+a)*polylog(2,-1/(
(b*x+a-1)/(b*x+a+1))^(1/2))+2*polylog(3,-1/((b*x+a-1)/(b*x+a+1))^(1/2))-1/2
*I*Pi*arccoth(b*x+a)^2*csgn(I*(a*((b*x+a+1)/(b*x+a-1)-1)-(b*x+a+1)/(b*x+a-1
)-1))*csgn(I*(a*((b*x+a+1)/(b*x+a-1)-1)-(b*x+a+1)/(b*x+a-1)-1)/((b*x+a+1)/(
b*x+a-1)-1))^2+I*Pi*arccoth(b*x+a)^2*csgn(I*(a*((b*x+a+1)/(b*x+a-1)-1)-(b*x
+a+1)/(b*x+a-1)-1)/((b*x+a+1)/(b*x+a-1)-1))^2+1/2*I*Pi*arccoth(b*x+a)^2*csg
n(I*(a*((b*x+a+1)/(b*x+a-1)-1)-(b*x+a+1)/(b*x+a-1)-1))*csgn(I*(a*((b*x+a+1)
/(b*x+a-1)-1)-(b*x+a+1)/(b*x+a-1)-1)/((b*x+a+1)/(b*x+a-1)-1))*csgn(I/(b*x+
a+1)/(b*x+a-1)-1))-I*Pi*arccoth(b*x+a)^2-1/2*I*Pi*arccoth(b*x+a)^2*csgn(I*(
a*((b*x+a+1)/(b*x+a-1)-1)-(b*x+a+1)/(b*x+a-1)-1)/((b*x+a+1)/(b*x+a-1)-1))^2
*csgn(I/(b*x+a+1)/(b*x+a-1)-1))-1/2*I*Pi*arccoth(b*x+a)^2*csgn(I*(a*((b*x+
a+1)/(b*x+a-1)-1)-(b*x+a+1)/(b*x+a-1)-1)/((b*x+a+1)/(b*x+a-1)-1))^3+a/(a-1)
*arccoth(b*x+a)^2*ln(1-(a-1)*(b*x+a+1)/(b*x+a-1)/(1+a))+a/(a-1)*arccoth(b*x
+a)*polylog(2,(a-1)*(b*x+a+1)/(b*x+a-1)/(1+a))-1/2*a/(a-1)*polylog(3,(a-1)*
(b*x+a+1)/(b*x+a-1)/(1+a))-1/(a-1)*arccoth(b*x+a)^2*ln(1-(a-1)*(b*x+a+1)/(b
*x+a-1)/(1+a))-1/(a-1)*arccoth(b*x+a)*polylog(2,(a-1)*(b*x+a+1)/(b*x+a-1)/(
1+a))+1/2/(a-1)*polylog(3,(a-1)*(b*x+a+1)/(b*x+a-1)/(1+a))

```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(bx+a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)^2/x,x, algorithm="maxima")

[Out] integrate(arccoth(b\*x + a)^2/x, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arccoth}(bx+a)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)^2/x,x, algorithm="fricas")

[Out] integral(arccoth(b\*x + a)^2/x, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{acoth}^2(a+bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(b\*x+a)\*\*2/x,x)

[Out] Integral(acoth(a + b\*x)\*\*2/x, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arccoth}(bx+a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)^2/x,x, algorithm="giac")

[Out] integrate(arccoth(b\*x + a)^2/x, x)

$$3.74 \quad \int \frac{\coth^{-1}(a+bx)^2}{x^2} dx$$

**Optimal.** Leaf size=251

$$\frac{b \operatorname{PolyLog}\left(2, 1 - \frac{2}{a+bx+1}\right)}{1-a^2} - \frac{b \operatorname{PolyLog}\left(2, 1 - \frac{2bx}{(1-a)(a+bx+1)}\right)}{1-a^2} + \frac{b \operatorname{PolyLog}\left(2, -\frac{a+bx+1}{-a-bx+1}\right)}{2(1-a)} - \frac{b \operatorname{PolyLog}\left(2, 1 - \frac{2}{a+bx+1}\right)}{2(a+1)} - 2$$

[Out]  $-(\operatorname{ArcCoth}[a + b*x]^2/x) + (b*\operatorname{ArcCoth}[a + b*x]*\operatorname{Log}[2/(1 - a - b*x)])/(1 - a) + (b*\operatorname{ArcCoth}[a + b*x]*\operatorname{Log}[2/(1 + a + b*x)])/(1 + a) - (2*b*\operatorname{ArcCoth}[a + b*x]*\operatorname{Log}[2/(1 + a + b*x)])/(1 - a^2) + (2*b*\operatorname{ArcCoth}[a + b*x]*\operatorname{Log}[(2*b*x)/((1 - a)*(1 + a + b*x))])/(1 - a^2) + (b*\operatorname{PolyLog}[2, -((1 + a + b*x)/(1 - a - b*x))])/(2*(1 - a)) - (b*\operatorname{PolyLog}[2, 1 - 2/(1 + a + b*x)])/(2*(1 + a)) + (b*\operatorname{PolyLog}[2, 1 - 2/(1 + a + b*x)])/(1 - a^2) - (b*\operatorname{PolyLog}[2, 1 - (2*b*x)/((1 - a)*(1 + a + b*x))])/(1 - a^2)$

**Rubi [A]** time = 0.719278, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 15, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.25$ , Rules used = {6110, 371, 706, 31, 633, 6741, 6122, 6688, 12, 6725, 5921, 2402, 2315, 2447, 5919}

$$\frac{b \operatorname{PolyLog}\left(2, 1 - \frac{2}{a+bx+1}\right)}{1-a^2} - \frac{b \operatorname{PolyLog}\left(2, 1 - \frac{2bx}{(1-a)(a+bx+1)}\right)}{1-a^2} + \frac{b \operatorname{PolyLog}\left(2, -\frac{a+bx+1}{-a-bx+1}\right)}{2(1-a)} - \frac{b \operatorname{PolyLog}\left(2, 1 - \frac{2}{a+bx+1}\right)}{2(a+1)} - 2$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{ArcCoth}[a + b*x]^2/x^2, x]$

[Out]  $-(\operatorname{ArcCoth}[a + b*x]^2/x) + (b*\operatorname{ArcCoth}[a + b*x]*\operatorname{Log}[2/(1 - a - b*x)])/(1 - a) + (b*\operatorname{ArcCoth}[a + b*x]*\operatorname{Log}[2/(1 + a + b*x)])/(1 + a) - (2*b*\operatorname{ArcCoth}[a + b*x]*\operatorname{Log}[2/(1 + a + b*x)])/(1 - a^2) + (2*b*\operatorname{ArcCoth}[a + b*x]*\operatorname{Log}[(2*b*x)/((1 - a)*(1 + a + b*x))])/(1 - a^2) + (b*\operatorname{PolyLog}[2, -((1 + a + b*x)/(1 - a - b*x))])/(2*(1 - a)) - (b*\operatorname{PolyLog}[2, 1 - 2/(1 + a + b*x)])/(2*(1 + a)) + (b*\operatorname{PolyLog}[2, 1 - 2/(1 + a + b*x)])/(1 - a^2) - (b*\operatorname{PolyLog}[2, 1 - (2*b*x)/((1 - a)*(1 + a + b*x))])/(1 - a^2)$

**Rule 6110**

$\operatorname{Int}[(a_. + \operatorname{ArcCoth}[c_] + (d_.)*(x_.)]*(b_.)^{(p_.)*((e_. + (f_.)*(x_.))^{(m_.), x\_Symbol]} :> \operatorname{Simp}[(e + f*x)^{(m + 1)}*(a + b*\operatorname{ArcCoth}[c + d*x])^p]/(f*(m + 1)), x] - \operatorname{Dist}[(b*d*p)/(f*(m + 1)), \operatorname{Int}[(e + f*x)^{(m + 1)}*(a + b*\operatorname{ArcCoth}[c + d*x])^p, x]$



$h[c + d*x]^{(p - 1)}/(1 - (c + d*x)^2), x], x] /;$  FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[m, -1]

### Rule 371

Int[((a\_) + (b\_)\*(v\_)^(n\_))^(p\_)\*(x\_)^(m\_), x\_Symbol] :> With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m\*(a + b\*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

### Rule 706

Int[1/(((d\_) + (e\_)\*(x\_))\*((a\_) + (c\_)\*(x\_)^2)), x\_Symbol] :> Dist[e^2/(c\*d^2 + a\*e^2), Int[1/(d + e\*x), x], x] + Dist[1/(c\*d^2 + a\*e^2), Int[(c\*d - c\*e\*x)/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0]

### Rule 31

Int[((a\_) + (b\_)\*(x\_))^(n\_), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 633

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (c\_)\*(x\_)^2), x\_Symbol] :> With[{q = Rt[-(a\*c), 2]}, Dist[e/2 + (c\*d)/(2\*q), Int[1/(-q + c\*x), x], x] + Dist[e/2 - (c\*d)/(2\*q), Int[1/(q + c\*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a\*c)]

### Rule 6741

Int[u\_, x\_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v != u]

### Rule 6122

Int[((a\_) + ArcCoth[(c\_) + (d\_)\*(x\_)]\*(b\_))^(p\_)\*((e\_) + (f\_)\*(x\_))^(m\_)\*((A\_) + (B\_)\*(x\_) + (C\_)\*(x\_)^2)^(q\_), x\_Symbol] :> Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(-(C/d^2) + (C\*x^2)/d^2)^q\*(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] && EqQ[B\*(1 - c^2) + 2\*A\*c\*d, 0] && EqQ[2\*c\*C - B\*d, 0]

### Rule 6688

```
Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]]
```

### Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

### Rule 5921

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.)/((d_) + (e_.)*(x_)), x_Symbol] := -
Simp[((a + b*ArcCoth[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcCoth[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

### Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

### Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

### Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

### Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.)^((p_.)/((d_) + (e_.)*(x_))), x_Symbol
] := -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
```

p)/e, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*Log[2/(1 + (e\*x)/d)])/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{\coth^{-1}(a+bx)^2}{x^2} dx &= -\frac{\coth^{-1}(a+bx)^2}{x} + (2b) \int \frac{\coth^{-1}(a+bx)}{x(1-(a+bx)^2)} dx \\
 &= -\frac{\coth^{-1}(a+bx)^2}{x} + (2b) \int \frac{\coth^{-1}(a+bx)}{x(1-a^2-2abx-b^2x^2)} dx \\
 &= -\frac{\coth^{-1}(a+bx)^2}{x} + 2 \operatorname{Subst} \left( \int \frac{\coth^{-1}(x)}{\left(-\frac{a}{b} + \frac{x}{b}\right)(1-x^2)} dx, x, a+bx \right) \\
 &= -\frac{\coth^{-1}(a+bx)^2}{x} + 2 \operatorname{Subst} \left( \int \frac{b \coth^{-1}(x)}{(-a+x)(1-x^2)} dx, x, a+bx \right) \\
 &= -\frac{\coth^{-1}(a+bx)^2}{x} + (2b) \operatorname{Subst} \left( \int \frac{\coth^{-1}(x)}{(-a+x)(1-x^2)} dx, x, a+bx \right) \\
 &= -\frac{\coth^{-1}(a+bx)^2}{x} + (2b) \operatorname{Subst} \left( \int \left( \frac{\coth^{-1}(x)}{(-1+a^2)(a-x)} + \frac{\coth^{-1}(x)}{2(-1+a)(-1+x)} - \frac{\coth^{-1}(x)}{2(1+a)(1+x)} \right) dx, x, a+bx \right) \\
 &= -\frac{\coth^{-1}(a+bx)^2}{x} - \frac{b \operatorname{Subst} \left( \int \frac{\coth^{-1}(x)}{-1+x} dx, x, a+bx \right)}{1-a} - \frac{b \operatorname{Subst} \left( \int \frac{\coth^{-1}(x)}{1+x} dx, x, a+bx \right)}{1+a} \\
 &= -\frac{\coth^{-1}(a+bx)^2}{x} + \frac{b \coth^{-1}(a+bx) \log\left(\frac{2}{1-a-bx}\right)}{1-a} + \frac{b \coth^{-1}(a+bx) \log\left(\frac{2}{1+a+bx}\right)}{1+a} - \frac{2b \coth^{-1}(a+bx)}{1-a} \\
 &= -\frac{\coth^{-1}(a+bx)^2}{x} + \frac{b \coth^{-1}(a+bx) \log\left(\frac{2}{1-a-bx}\right)}{1-a} + \frac{b \coth^{-1}(a+bx) \log\left(\frac{2}{1+a+bx}\right)}{1+a} - \frac{2b \coth^{-1}(a+bx)}{1-a} \\
 &= -\frac{\coth^{-1}(a+bx)^2}{x} + \frac{b \coth^{-1}(a+bx) \log\left(\frac{2}{1-a-bx}\right)}{1-a} + \frac{b \coth^{-1}(a+bx) \log\left(\frac{2}{1+a+bx}\right)}{1+a} - \frac{2b \coth^{-1}(a+bx)}{1-a}
 \end{aligned}$$

**Mathematica [C]** time = 1.04006, size = 206, normalized size = 0.82

$$bx \operatorname{PolyLog} \left( 2, e^{2 \tanh^{-1}\left(\frac{1}{a}\right) - 2 \coth^{-1}(a+bx)} \right) - \left( \sqrt{1 - \frac{1}{a^2}} abxe^{\tanh^{-1}\left(\frac{1}{a}\right)} + a^2 - 1 \right) \coth^{-1}(a+bx)^2 + bx \coth^{-1}(a+bx) \left( -2 \log \left( \frac{2}{1-a-bx} \right) - 2 \log \left( \frac{2}{1+a+bx} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a + b\*x]^2/x^2,x]

[Out]  $(-((-1 + a^2 + \sqrt{1 - a^2}) * a * b * E^{\text{ArcTanh}[a^{(-1)}] * x} * \text{ArcCoth}[a + b * x]^2) + b * x * \text{ArcCoth}[a + b * x] * ((-1) * \text{Pi} + 2 * \text{ArcTanh}[a^{(-1)}] - 2 * \text{Log}[1 - E^{(-2 * \text{ArcCoth}[a + b * x] + 2 * \text{ArcTanh}[a^{(-1)})}])) + b * x * (I * \text{Pi} * (\text{Log}[1 + E^{(2 * \text{ArcCoth}[a + b * x])}] - \text{Log}[1 / \sqrt{1 - (a + b * x)^{(-2)}}]) + 2 * \text{ArcTanh}[a^{(-1)}] * (\text{Log}[1 - E^{(-2 * \text{ArcCoth}[a + b * x] + 2 * \text{ArcTanh}[a^{(-1)})}])) - \text{Log}[I * \text{Sinh}[\text{ArcCoth}[a + b * x] - \text{ArcTanh}[a^{(-1)}]])) + b * x * \text{PolyLog}[2, E^{(-2 * \text{ArcCoth}[a + b * x] + 2 * \text{ArcTanh}[a^{(-1)})}])) / ((-1 + a^2) * x)$

**Maple [A]** time = 0.096, size = 342, normalized size = 1.4

$$-\frac{(\operatorname{arccoth}(bx+a))^2}{x} + 2 \frac{\operatorname{barccoth}(bx+a) \ln(bx+a-1)}{2a-2} - 2 \frac{\operatorname{barccoth}(bx+a) \ln(bx)}{(1+a)(a-1)} - 2 \frac{\operatorname{barccoth}(bx+a) \ln(bx+a)}{2+2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(b\*x+a)^2/x^2,x)

[Out]  $-\operatorname{arccoth}(b * x + a)^2 / x + 2 * b * \operatorname{arccoth}(b * x + a) / (2 * a - 2) * \ln(b * x + a - 1) - 2 * b * \operatorname{arccoth}(b * x + a) / (a - 1) / (1 + a) * \ln(b * x) - 2 * b * \operatorname{arccoth}(b * x + a) / (2 + 2 * a) * \ln(b * x + a) + b / (a - 1) / (1 + a) * \operatorname{dilog}((b * x + a + 1) / (1 + a)) + b / (a - 1) / (1 + a) * \ln(b * x) * \ln((b * x + a + 1) / (1 + a)) - b / (a - 1) / (1 + a) * \operatorname{dilog}((b * x + a - 1) / (a - 1)) - b / (a - 1) / (1 + a) * \ln(b * x) * \ln((b * x + a - 1) / (a - 1)) + 1 / 2 * b / (1 + a) * \ln(-1 / 2 * b * x - 1 / 2 * a + 1 / 2) * \ln(1 / 2 + 1 / 2 * b * x + 1 / 2 * a) - 1 / 2 * b / (1 + a) * \ln(-1 / 2 * b * x - 1 / 2 * a + 1 / 2) * \ln(b * x + a + 1) + 1 / 2 * b / (1 + a) * \operatorname{dilog}(1 / 2 + 1 / 2 * b * x + 1 / 2 * a) + 1 / 4 * b / (1 + a) * \ln(b * x + a + 1)^2 + 1 / 4 * b / (a - 1) * \ln(b * x + a - 1)^2 - 1 / 2 * b / (a - 1) * \operatorname{dilog}(1 / 2 + 1 / 2 * b * x + 1 / 2 * a) - 1 / 2 * b / (a - 1) * \ln(b * x + a - 1) * \ln(1 / 2 + 1 / 2 * b * x + 1 / 2 * a)$

**Maxima [A]** time = 1.02835, size = 329, normalized size = 1.31

$$\frac{1}{4} b^2 \left( \frac{(a-1) \log(bx+a+1)^2 - 2(a-1) \log(bx+a+1) \log(bx+a-1) + (a+1) \log(bx+a-1)^2}{a^2 b - b} - 4 \left( \log(bx+a-1) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)^2/x^2,x, algorithm="maxima")

```
[Out] 1/4*b^2*((a - 1)*log(b*x + a + 1)^2 - 2*(a - 1)*log(b*x + a + 1)*log(b*x +
a - 1) + (a + 1)*log(b*x + a - 1)^2)/(a^2*b - b) - 4*(log(b*x + a - 1)*log
(1/2*b*x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))/(a^2*b - b) + 4*(l
og(b*x/(a + 1) + 1)*log(x) + dilog(-b*x/(a + 1)))/(a^2*b - b) - 4*(log(b*x/
(a - 1) + 1)*log(x) + dilog(-b*x/(a - 1)))/(a^2*b - b) - b*(log(b*x + a +
1)/(a + 1) - log(b*x + a - 1)/(a - 1) + 2*log(x)/(a^2 - 1))*arccoth(b*x + a
) - arccoth(b*x + a)^2/x
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arccoth}(bx + a)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(arccoth(b*x + a)^2/x^2, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{acoth}^2(a + bx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)**2/x**2,x)
```

```
[Out] Integral(acoth(a + b*x)**2/x**2, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arccoth}(bx + a)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a)^2/x^2, x)
```

$$3.75 \quad \int \frac{\coth^{-1}(a+bx)^2}{x^3} dx$$

**Optimal.** Leaf size=370

$$\frac{ab^2 \text{PolyLog}\left(2, 1 - \frac{2}{a+bx+1}\right)}{(1-a^2)^2} - \frac{ab^2 \text{PolyLog}\left(2, 1 - \frac{2bx}{(1-a)(a+bx+1)}\right)}{(1-a^2)^2} + \frac{b^2 \text{PolyLog}\left(2, -\frac{a+bx+1}{-a-bx+1}\right)}{4(1-a)^2} + \frac{b^2 \text{PolyLog}\left(2, 1 - \frac{2}{a+bx}\right)}{4(a+1)^2}$$

[Out]  $-\left(\frac{b \operatorname{ArcCoth}[a + b x]}{(1 - a^2)x}\right) - \frac{\operatorname{ArcCoth}[a + b x]^2}{2x^2} + (b^2 \operatorname{Log}[x]) / (1 - a^2)^2 + (b^2 \operatorname{ArcCoth}[a + b x] \operatorname{Log}[2 / (1 - a - b x)]) / (2(1 - a)^2) - (b^2 \operatorname{Log}[1 - a - b x]) / (2(1 - a)^2(1 + a)) - (b^2 \operatorname{ArcCoth}[a + b x] \operatorname{Log}[2 / (1 + a + b x)]) / (2(1 + a)^2) - (2ab^2 \operatorname{ArcCoth}[a + b x] \operatorname{Log}[2 / (1 + a + b x)]) / (1 - a^2)^2 + (2ab^2 \operatorname{ArcCoth}[a + b x] \operatorname{Log}[(2bx) / ((1 - a)(1 + a + b x))]) / (1 - a^2)^2 - (b^2 \operatorname{Log}[1 + a + b x]) / (2(1 - a)(1 + a)^2) + (b^2 \operatorname{PolyLog}[2, -((1 + a + b x) / (1 - a - b x))]) / (4(1 - a)^2) + (b^2 \operatorname{PolyLog}[2, 1 - 2 / (1 + a + b x)]) / (4(1 + a)^2) + (ab^2 \operatorname{PolyLog}[2, 1 - 2 / (1 + a + b x)]) / (1 - a^2)^2 - (ab^2 \operatorname{PolyLog}[2, 1 - (2bx) / ((1 - a)(1 + a + b x))]) / (1 - a^2)^2$

**Rubi [A]** time = 0.829986, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 16, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$ , Rules used = {6110, 371, 710, 801, 6741, 6122, 6725, 5927, 706, 31, 633, 5921, 2402, 2315, 2447, 5919}

$$\frac{ab^2 \text{PolyLog}\left(2, 1 - \frac{2}{a+bx+1}\right)}{(1-a^2)^2} - \frac{ab^2 \text{PolyLog}\left(2, 1 - \frac{2bx}{(1-a)(a+bx+1)}\right)}{(1-a^2)^2} + \frac{b^2 \text{PolyLog}\left(2, -\frac{a+bx+1}{-a-bx+1}\right)}{4(1-a)^2} + \frac{b^2 \text{PolyLog}\left(2, 1 - \frac{2}{a+bx}\right)}{4(a+1)^2}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a + b\*x]^2/x^3,x]

[Out]  $-\left(\frac{b \operatorname{ArcCoth}[a + b x]}{(1 - a^2)x}\right) - \frac{\operatorname{ArcCoth}[a + b x]^2}{2x^2} + (b^2 \operatorname{Log}[x]) / (1 - a^2)^2 + (b^2 \operatorname{ArcCoth}[a + b x] \operatorname{Log}[2 / (1 - a - b x)]) / (2(1 - a)^2) - (b^2 \operatorname{Log}[1 - a - b x]) / (2(1 - a)^2(1 + a)) - (b^2 \operatorname{ArcCoth}[a + b x] \operatorname{Log}[2 / (1 + a + b x)]) / (2(1 + a)^2) - (2ab^2 \operatorname{ArcCoth}[a + b x] \operatorname{Log}[2 / (1 + a + b x)]) / (1 - a^2)^2 + (2ab^2 \operatorname{ArcCoth}[a + b x] \operatorname{Log}[(2bx) / ((1 - a)(1 + a + b x))]) / (1 - a^2)^2 - (b^2 \operatorname{Log}[1 + a + b x]) / (2(1 - a)(1 + a)^2) + (b^2 \operatorname{PolyLog}[2, -((1 + a + b x) / (1 - a - b x))]) / (4(1 - a)^2) + (b^2 \operatorname{PolyLog}[2, 1 - 2 / (1 + a + b x)]) / (4(1 + a)^2) + (ab^2 \operatorname{PolyLog}[2, 1 - 2 / (1 + a + b x)]) / (1 - a^2)^2 - (ab^2 \operatorname{PolyLog}[2, 1 - (2bx) / ((1 - a)(1 + a + b x))]) / (1 - a^2)^2$

)]/(1 - a^2)^2

### Rule 6110

Int[((a\_.) + ArcCoth[(c\_) + (d\_.)\*(x\_)]\*(b\_.))^ (p\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[((e + f\*x)^(m + 1)\*(a + b\*ArcCoth[c + d\*x])^p)/(f\*(m + 1)), x] - Dist[(b\*d\*p)/(f\*(m + 1)), Int[((e + f\*x)^(m + 1)\*(a + b\*ArcCoth[c + d\*x])^(p - 1))/(1 - (c + d\*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[m, -1]

### Rule 371

Int[((a\_) + (b\_.)\*(v\_)^(n\_))^(p\_.)\*(x\_)^(m\_.), x\_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m\*(a + b\*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

### Rule 710

Int[((d\_) + (e\_.)\*(x\_))^(m\_)/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(e\*(d + e\*x)^(m + 1))/((m + 1)\*(c\*d^2 + a\*e^2)), x] + Dist[c/(c\*d^2 + a\*e^2), Int[(d + e\*x)^(m + 1)\*(d - e\*x)/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c\*d^2 + a\*e^2, 0] && LtQ[m, -1]

### Rule 801

Int((((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_)))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[((d + e\*x)^m\*(f + g\*x))/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[m]

### Rule 6741

Int[u\_, x\_Symbol] := With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v != u]

### Rule 6122

Int[((a\_.) + ArcCoth[(c\_) + (d\_.)\*(x\_)]\*(b\_.))^ (p\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.)\*((A\_.) + (B\_.)\*(x\_) + (C\_.)\*(x\_)^2)^(q\_.), x\_Symbol] := Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(-(C/d^2) + (C\*x^2)/d^2)^q\*(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] && EqQ[B\*(1 - c^2) + 2\*A\*c\*d, 0] && EqQ[2\*c\*C - B\*d, 0]

### Rule 6725



```
Int[(u_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

### Rule 5927

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))*((d_) + (e_.)*(x_)^(q_.), x_Symbol
] := Simp[((d + e*x)^(q + 1)*(a + b*ArcCoth[c*x]))/(e*(q + 1)), x] - Dist[[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

### Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] := Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
0]
```

### Rule 31

```
Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

### Rule 5921

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] := -
Simp[((a + b*ArcCoth[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcCoth[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

### Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] := -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d))]/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d))]/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(a+bx)^2}{x^3} dx &= -\frac{\coth^{-1}(a+bx)^2}{2x^2} + b \int \frac{\coth^{-1}(a+bx)}{x^2(1-(a+bx)^2)} dx \\
&= -\frac{\coth^{-1}(a+bx)^2}{2x^2} + b \int \frac{\coth^{-1}(a+bx)}{x^2(1-a^2-2abx-b^2x^2)} dx \\
&= -\frac{\coth^{-1}(a+bx)^2}{2x^2} + \text{Subst} \left( \int \frac{\coth^{-1}(x)}{\left(-\frac{a}{b} + \frac{x}{b}\right)^2(1-x^2)} dx, x, a+bx \right) \\
&= -\frac{\coth^{-1}(a+bx)^2}{2x^2} + \text{Subst} \left( \int \left( -\frac{b^2 \coth^{-1}(x)}{(-1+a^2)(a-x)^2} - \frac{2ab^2 \coth^{-1}(x)}{(-1+a^2)^2(a-x)} - \frac{b^2 \coth^{-1}(x)}{2(-1+a)^2(-1+x)} \right) dx, x, a+bx \right) \\
&= -\frac{\coth^{-1}(a+bx)^2}{2x^2} - \frac{b^2 \text{Subst} \left( \int \frac{\coth^{-1}(x)}{-1+x} dx, x, a+bx \right)}{2(1-a)^2} + \frac{b^2 \text{Subst} \left( \int \frac{\coth^{-1}(x)}{1+x} dx, x, a+bx \right)}{2(1+a)^2} \\
&= -\frac{b \coth^{-1}(a+bx)}{(1-a^2)x} - \frac{\coth^{-1}(a+bx)^2}{2x^2} + \frac{b^2 \coth^{-1}(a+bx) \log\left(\frac{2}{1-a-bx}\right)}{2(1-a)^2} - \frac{b^2 \coth^{-1}(a+bx) \log\left(\frac{2}{1+a-bx}\right)}{2(1+a)^2} \\
&= -\frac{b \coth^{-1}(a+bx)}{(1-a^2)x} - \frac{\coth^{-1}(a+bx)^2}{2x^2} + \frac{b^2 \coth^{-1}(a+bx) \log\left(\frac{2}{1-a-bx}\right)}{2(1-a)^2} - \frac{b^2 \coth^{-1}(a+bx) \log\left(\frac{2}{1+a-bx}\right)}{2(1+a)^2} \\
&= -\frac{b \coth^{-1}(a+bx)}{(1-a^2)x} - \frac{\coth^{-1}(a+bx)^2}{2x^2} + \frac{b^2 \log(x)}{(1-a^2)^2} + \frac{b^2 \coth^{-1}(a+bx) \log\left(\frac{2}{1-a-bx}\right)}{2(1-a)^2} - \frac{b^2 \coth^{-1}(a+bx) \log\left(\frac{2}{1+a-bx}\right)}{2(1+a)^2} \\
&= -\frac{b \coth^{-1}(a+bx)}{(1-a^2)x} - \frac{\coth^{-1}(a+bx)^2}{2x^2} + \frac{b^2 \log(x)}{(1-a^2)^2} + \frac{b^2 \coth^{-1}(a+bx) \log\left(\frac{2}{1-a-bx}\right)}{2(1-a)^2} - \frac{b^2 \log(x)}{2(1-a)^2}
\end{aligned}$$

**Mathematica [C]** time = 2.18402, size = 291, normalized size = 0.79

$$-2ab^2x^2 \text{PolyLog} \left( 2, e^{2 \tanh^{-1}\left(\frac{1}{a}\right) - 2 \coth^{-1}(a+bx)} \right) + \left( a^2 \left( b^2 x^2 \left( 2 \sqrt{1 - \frac{1}{a^2}} e^{\tanh^{-1}\left(\frac{1}{a}\right)} - 1 \right) + 2 \right) - a^4 + b^2 x^2 - 1 \right) \coth^{-1}(a+bx)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a + b\*x]^2/x^3,x]

```
[Out] ((-1 - a^4 + b^2*x^2 + a^2*(2 + b^2*(-1 + 2*sqrt[1 - a^(-2)]*E^ArcTanh[a^(-1)])))*x^2))*ArcCoth[a + b*x]^2 + 2*b*x*ArcCoth[a + b*x]*(-1 + a^2 + a*b*x + I*a*b*Pi*x - 2*a*b*x*ArcTanh[a^(-1)] + 2*a*b*x*Log[1 - E^(-2*ArcCoth[a + b*x] + 2*ArcTanh[a^(-1)])]) + 2*b^2*x^2*((-I)*a*Pi*Log[1 + E^(2*ArcCoth[a + b*x])] + I*a*Pi*Log[1/Sqrt[1 - (a + b*x)^(-2)]] + Log[-((b*x)/((a + b*x)*sqrt[1 - (a + b*x)^(-2)])]) - 2*a*ArcTanh[a^(-1)]*(Log[1 - E^(-2*ArcCoth[a + b*x] + 2*ArcTanh[a^(-1)])]) - Log[I*Sinh[ArcCoth[a + b*x] - ArcTanh[a^(-1)]]]) - 2*a*b^2*x^2*PolyLog[2, E^(-2*ArcCoth[a + b*x] + 2*ArcTanh[a^(-1)])])/(2*(-1 + a^2)^2*x^2)
```

**Maple [A]** time = 0.099, size = 467, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)^2/x^3,x)
```

```
[Out] -1/2*arccoth(b*x+a)^2/x^2-1/2*b^2*arccoth(b*x+a)/(a-1)^2*ln(b*x+a-1)+b*arccoth(b*x+a)/(a-1)/(1+a)/x+2*b^2*arccoth(b*x+a)*a/(a-1)^2/(1+a)^2*ln(b*x)+1/2*b^2*arccoth(b*x+a)/(1+a)^2*ln(b*x+a+1)-1/8*b^2/(a-1)^2*ln(b*x+a-1)^2+1/4*b^2/(a-1)^2*dilog(1/2+1/2*b*x+1/2*a)+1/4*b^2/(a-1)^2*ln(b*x+a-1)*ln(1/2+1/2*b*x+1/2*a)-1/4*b^2/(1+a)^2*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2+1/2*b*x+1/2*a)+1/4*b^2/(1+a)^2*ln(-1/2*b*x-1/2*a+1/2)*ln(b*x+a+1)-1/4*b^2/(1+a)^2*dilog(1/2+1/2*b*x+1/2*a)-1/8*b^2/(1+a)^2*ln(b*x+a+1)^2-b^2/(a-1)/(1+a)/(2*a-2)*ln(b*x+a-1)+b^2/(a-1)^2/(1+a)^2*ln(b*x)+b^2/(a-1)/(1+a)/(2+2*a)*ln(b*x+a+1)-b^2*a/(a-1)^2/(1+a)^2*dilog((b*x+a+1)/(1+a))-b^2*a/(a-1)^2/(1+a)^2*ln(b*x)*ln((b*x+a+1)/(1+a))+b^2*a/(a-1)^2/(1+a)^2*dilog((b*x+a-1)/(a-1))+b^2*a/(a-1)^2/(1+a)^2*ln(b*x)*ln((b*x+a-1)/(a-1))
```

**Maxima [A]** time = 1.02374, size = 486, normalized size = 1.31

$$\frac{1}{8} \left( \frac{8 \left( \log(bx + a - 1) \log\left(\frac{1}{2}bx + \frac{1}{2}a + \frac{1}{2}\right) + \text{Li}_2\left(-\frac{1}{2}bx - \frac{1}{2}a + \frac{1}{2}\right) \right) a}{a^4 - 2a^2 + 1} - \frac{8 \left( \log\left(\frac{bx}{a+1} + 1\right) \log(x) + \text{Li}_2\left(-\frac{bx}{a+1}\right) \right) a}{a^4 - 2a^2 + 1} + \frac{8 \left( \log\left(\frac{bx}{a+1} + 1\right) \log(x) + \text{Li}_2\left(-\frac{bx}{a+1}\right) \right) a}{a^4 - 2a^2 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*(8*(log(b*x + a - 1)*log(1/2*b*x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2*
a + 1/2))*a/(a^4 - 2*a^2 + 1) - 8*(log(b*x/(a + 1) + 1)*log(x) + dilog(-b*x
/(a + 1)))*a/(a^4 - 2*a^2 + 1) + 8*(log(b*x/(a - 1) + 1)*log(x) + dilog(-b*
x/(a - 1)))*a/(a^4 - 2*a^2 + 1) - ((a^2 - 2*a + 1)*log(b*x + a + 1)^2 - 2*(
a^2 - 2*a + 1)*log(b*x + a + 1)*log(b*x + a - 1) + (a^2 + 2*a + 1)*log(b*x
+ a - 1)^2)/(a^4 - 2*a^2 + 1) + 4*log(b*x + a + 1)/(a^3 + a^2 - a - 1) - 4*
log(b*x + a - 1)/(a^3 - a^2 - a + 1) + 8*log(x)/(a^4 - 2*a^2 + 1))*b^2 + 1/
2*(4*a*b*log(x)/(a^4 - 2*a^2 + 1) + b*log(b*x + a + 1)/(a^2 + 2*a + 1) - b*
log(b*x + a - 1)/(a^2 - 2*a + 1) + 2/((a^2 - 1)*x))*b*arccoth(b*x + a) - 1/
2*arccoth(b*x + a)^2/x^2
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arccoth}(bx+a)^2}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(arccoth(b*x + a)^2/x^3, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^2(a+bx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)**2/x**3,x)
```

```
[Out] Integral(acoth(a + b*x)**2/x**3, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(bx+a)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a)^2/x^3, x)
```

$$3.76 \quad \int \frac{\coth^{-1}(a+bx)}{c+dx^2} dx$$

**Optimal.** Leaf size=673

$$\frac{\text{PolyLog}\left(2, -\frac{(-a-bx+1)(a^2d+b^2c)}{(a+bx)(-1-a)ad+b^2c-b\sqrt{-c}\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} - \frac{\text{PolyLog}\left(2, -\frac{(-a-bx+1)(a^2d+b^2c)}{(a+bx)(-1-a)ad+b^2c+b\sqrt{-c}\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{\text{PolyLog}\left(2, \frac{(a+bx+1)(a^2d+b^2c)}{(a+bx)(a+1)d+b^2c-b\sqrt{-c}\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}$$

```
[Out] (Log[-((1 - a - b*x)/(a + b*x))]*Log[1 + ((b^2*c + a^2*d)*(1 - a - b*x))/((b^2*c - b*Sqrt[-c]*Sqrt[d] - (1 - a)*a*d)*(a + b*x))])/(4*Sqrt[-c]*Sqrt[d]) - (Log[-((1 - a - b*x)/(a + b*x))]*Log[1 + ((b^2*c + a^2*d)*(1 - a - b*x))/((b^2*c + b*Sqrt[-c]*Sqrt[d] - (1 - a)*a*d)*(a + b*x))])/(4*Sqrt[-c]*Sqrt[d]) + (Log[(1 + a + b*x)/(a + b*x)]*Log[1 - ((b^2*c + a^2*d)*(1 + a + b*x))/((b^2*c - b*Sqrt[-c]*Sqrt[d] + a*(1 + a)*d)*(a + b*x))])/(4*Sqrt[-c]*Sqrt[d]) - (Log[(1 + a + b*x)/(a + b*x)]*Log[1 - ((b^2*c + a^2*d)*(1 + a + b*x))/((b^2*c + b*Sqrt[-c]*Sqrt[d] + a*(1 + a)*d)*(a + b*x))])/(4*Sqrt[-c]*Sqrt[d]) + PolyLog[2, -(((b^2*c + a^2*d)*(1 - a - b*x))/((b^2*c - b*Sqrt[-c]*Sqrt[d] - (1 - a)*a*d)*(a + b*x)))]/(4*Sqrt[-c]*Sqrt[d]) - PolyLog[2, -(((b^2*c + a^2*d)*(1 - a - b*x))/((b^2*c + b*Sqrt[-c]*Sqrt[d] - (1 - a)*a*d)*(a + b*x)))]/(4*Sqrt[-c]*Sqrt[d]) + PolyLog[2, ((b^2*c + a^2*d)*(1 + a + b*x))/((b^2*c - b*Sqrt[-c]*Sqrt[d] + a*(1 + a)*d)*(a + b*x)))]/(4*Sqrt[-c]*Sqrt[d]) - PolyLog[2, ((b^2*c + a^2*d)*(1 + a + b*x))/((b^2*c + b*Sqrt[-c]*Sqrt[d] + a*(1 + a)*d)*(a + b*x)))]/(4*Sqrt[-c]*Sqrt[d])
```

**Rubi [A]** time = 1.09947, antiderivative size = 597, normalized size of antiderivative = 0.89, number of steps used = 37, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {6116, 2513, 2409, 2394, 2393, 2391, 205}

$$-\frac{\text{PolyLog}\left(2, -\frac{\sqrt{d}(-a-bx+1)}{b\sqrt{-c}-(1-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{\text{PolyLog}\left(2, \frac{\sqrt{d}(-a-bx+1)}{(1-a)\sqrt{d}+b\sqrt{-c}}\right)}{4\sqrt{-c}\sqrt{d}} - \frac{\text{PolyLog}\left(2, -\frac{\sqrt{d}(a+bx+1)}{b\sqrt{-c}-(a+1)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{\text{PolyLog}\left(2, \frac{\sqrt{d}(a+bx+1)}{(a+1)\sqrt{d}+b\sqrt{-c}}\right)}{4\sqrt{-c}\sqrt{d}}$$

Warning: Unable to verify antiderivative.

[In] Int[ArcCoth[a + b\*x]/(c + d\*x^2), x]

```
[Out] (ArcTan[(Sqrt[d]*x)/Sqrt[c]]*(Log[-1 + a + b*x] - Log[-((1 - a - b*x)/(a + b*x))]) - Log[a + b*x])/(2*Sqrt[c]*Sqrt[d]) + (ArcTan[(Sqrt[d]*x)/Sqrt[c]]*(Log[a + b*x] - Log[1 + a + b*x] + Log[(1 + a + b*x)/(a + b*x)]))/(2*Sqrt[c]*Sqrt[d]) - (Log[-1 + a + b*x]*Log[(b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] - (1 - a)*Sqrt[d])])/(4*Sqrt[-c]*Sqrt[d]) + (Log[1 + a + b*x]*Log[(b*(Sqrt[
```

$$\begin{aligned}
& -c] - \text{Sqrt}[d]*x))/(b*\text{Sqrt}[-c] + (1 + a)*\text{Sqrt}[d]))/(4*\text{Sqrt}[-c]*\text{Sqrt}[d]) + ( \\
& \text{Log}[-1 + a + b*x]*\text{Log}[(b*(\text{Sqrt}[-c] + \text{Sqrt}[d]*x))/(b*\text{Sqrt}[-c] + (1 - a)*\text{Sqrt} \\
& [d])))/(4*\text{Sqrt}[-c]*\text{Sqrt}[d]) - (\text{Log}[1 + a + b*x]*\text{Log}[(b*(\text{Sqrt}[-c] + \text{Sqrt}[d]* \\
& x))/(b*\text{Sqrt}[-c] - (1 + a)*\text{Sqrt}[d])))/(4*\text{Sqrt}[-c]*\text{Sqrt}[d]) - \text{PolyLog}[2, -((\text{S} \\
& \text{qrt}[d]*(1 - a - b*x))/(b*\text{Sqrt}[-c] - (1 - a)*\text{Sqrt}[d])))/(4*\text{Sqrt}[-c]*\text{Sqrt}[d]) \\
& + \text{PolyLog}[2, (\text{Sqrt}[d]*(1 - a - b*x))/(b*\text{Sqrt}[-c] + (1 - a)*\text{Sqrt}[d])))/(4*\text{S} \\
& \text{qrt}[-c]*\text{Sqrt}[d]) - \text{PolyLog}[2, -((\text{Sqrt}[d]*(1 + a + b*x))/(b*\text{Sqrt}[-c] - (1 + a \\
& )*\text{Sqrt}[d])))/(4*\text{Sqrt}[-c]*\text{Sqrt}[d]) + \text{PolyLog}[2, (\text{Sqrt}[d]*(1 + a + b*x))/(b*S \\
& \text{qrt}[-c] + (1 + a)*\text{Sqrt}[d])))/(4*\text{Sqrt}[-c]*\text{Sqrt}[d])
\end{aligned}$$

### Rule 6116

```

Int[ArcCoth[(c_) + (d_)*(x_)]/((e_) + (f_)*(x_)^(n_)), x_Symbol] := Dist
[1/2, Int[Log[(1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] - Dist[1/2, Int[
Log[(-1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x]
&& RationalQ[n]

```

### Rule 2513

```

Int[Log[(e_)*((f_)*((a_) + (b_)*(x_)^(p_))*((c_) + (d_)*(x_)^(q_))
^(r_)]*(RFx_), x_Symbol] := Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] && !MatchQ[RFx, (u_)*(a + b*x)^(m_)*(c + d*x)^(n_) /; IntegersQ[m, n]
]

```

### Rule 2409

```

Int[((a_) + Log[(c_)*((d_) + (e_)*(x_)^(n_))*((b_))^(p_)*((f_) + (g_
.)*(x_)^(r_))^(q_)], x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))

```

### Rule 2394

```

Int[((a_) + Log[(c_)*((d_) + (e_)*(x_)^(n_))*((b_))]/((f_) + (g_)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)
^n]))/g, x] - Dist[(b*e^n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]

```

### Rule 2393

```

Int[((a_) + Log[(c_)*((d_) + (e_)*(x_))]*(b_)]/((f_) + (g_)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x

```



], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rubi steps

$$\begin{aligned}
 \int \frac{\coth^{-1}(a+bx)}{c+dx^2} dx &= -\left(\frac{1}{2} \int \frac{\log\left(\frac{-1+a+bx}{a+bx}\right)}{c+dx^2} dx\right) + \frac{1}{2} \int \frac{\log\left(\frac{1+a+bx}{a+bx}\right)}{c+dx^2} dx \\
 &= -\left(\frac{1}{2} \int \frac{\log(-1+a+bx)}{c+dx^2} dx\right) + \frac{1}{2} \int \frac{\log(1+a+bx)}{c+dx^2} dx - \frac{1}{2} \left(-\log(-1+a+bx) + \log\left(\frac{-1+a}{a+bx}\right)\right) \\
 &= \frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right) \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2\sqrt{c}\sqrt{d}} + \frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right) \left(\log(a+bx) - \log\left(\frac{-1+a}{a+bx}\right)\right)}{2\sqrt{c}\sqrt{d}} \\
 &= \frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right) \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2\sqrt{c}\sqrt{d}} + \frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right) \left(\log(a+bx) - \log\left(\frac{-1+a}{a+bx}\right)\right)}{2\sqrt{c}\sqrt{d}} \\
 &= \frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right) \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2\sqrt{c}\sqrt{d}} + \frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right) \left(\log(a+bx) - \log\left(\frac{-1+a}{a+bx}\right)\right)}{2\sqrt{c}\sqrt{d}} \\
 &= \frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right) \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2\sqrt{c}\sqrt{d}} + \frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right) \left(\log(a+bx) - \log\left(\frac{-1+a}{a+bx}\right)\right)}{2\sqrt{c}\sqrt{d}} \\
 &= \frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right) \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2\sqrt{c}\sqrt{d}} + \frac{\tan^{-1}\left(\frac{\sqrt{dx}}{\sqrt{c}}\right) \left(\log(a+bx) - \log\left(\frac{-1+a}{a+bx}\right)\right)}{2\sqrt{c}\sqrt{d}}
 \end{aligned}$$

**Mathematica [A]** time = 0.58637, size = 529, normalized size = 0.79

$$\text{PolyLog}\left(2, \frac{b(\sqrt{-c}-\sqrt{dx})}{(a-1)\sqrt{d}+b\sqrt{-c}}\right) - \text{PolyLog}\left(2, \frac{b(\sqrt{-c}-\sqrt{dx})}{(a+1)\sqrt{d}+b\sqrt{-c}}\right) - \text{PolyLog}\left(2, \frac{b(\sqrt{-c}+\sqrt{dx})}{b\sqrt{-c}-(a-1)\sqrt{d}}\right) + \text{PolyLog}\left(2, \frac{b(\sqrt{-c}+\sqrt{dx})}{b\sqrt{-c}-(a+1)\sqrt{d}}\right) + \log$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a + b\*x]/(c + d\*x^2), x]

[Out] (Log[(Sqrt[d]\*(-1 + a + b\*x))/(b\*Sqrt[-c] + (-1 + a)\*Sqrt[d])]\*Log[Sqrt[-c] - Sqrt[d]\*x] - Log[(-1 + a + b\*x)/(a + b\*x)]\*Log[Sqrt[-c] - Sqrt[d]\*x] - Log[(Sqrt[d]\*(1 + a + b\*x))/(b\*Sqrt[-c] + (1 + a)\*Sqrt[d])]\*Log[Sqrt[-c] - Sqrt[d]\*x] + Log[(1 + a + b\*x)/(a + b\*x)]\*Log[Sqrt[-c] - Sqrt[d]\*x] - Log[-(Sqrt[d]\*(-1 + a + b\*x))/(b\*Sqrt[-c] - (-1 + a)\*Sqrt[d])]\*Log[Sqrt[-c] + Sqrt[d]\*x] + Log[(-1 + a + b\*x)/(a + b\*x)]\*Log[Sqrt[-c] + Sqrt[d]\*x] + Log[-((Sqrt[d]\*(1 + a + b\*x))/(b\*Sqrt[-c] - (1 + a)\*Sqrt[d])]\*Log[Sqrt[-c] + Sqrt[d]\*x] - Log[(1 + a + b\*x)/(a + b\*x)]\*Log[Sqrt[-c] + Sqrt[d]\*x] + PolyLog[2, (b\*(Sqrt[-c] - Sqrt[d]\*x))/(b\*Sqrt[-c] + (-1 + a)\*Sqrt[d])] - PolyLog[2, (b\*(Sqrt[-c] - Sqrt[d]\*x))/(b\*Sqrt[-c] + (1 + a)\*Sqrt[d])] - PolyLog[2, (b\*(Sqrt[-c] + Sqrt[d]\*x))/(b\*Sqrt[-c] - (-1 + a)\*Sqrt[d])] + PolyLog[2, (b\*(Sqrt[-c] + Sqrt[d]\*x))/(b\*Sqrt[-c] - (1 + a)\*Sqrt[d])])/(4\*Sqrt[-c]\*Sqrt[d])]

**Maple [B]** time = 0.49, size = 1230, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(b\*x+a)/(d\*x^2+c), x)

[Out] 1/2/b\*(-b^2\*c\*d)^(1/2)/c/d\*arccoth(b\*x+a)\*ln(1-(a^2\*d+b^2\*c-2\*a\*d+d)\*(b\*x+a+1)/(b\*x+a-1)/(a^2\*d+b^2\*c+2\*(-b^2\*c\*d)^(1/2)-d))-1/2/b\*(-b^2\*c\*d)^(1/2)/c/d\*arccoth(b\*x+a)^2+1/4/b\*(-b^2\*c\*d)^(1/2)/c/d\*polylog(2, (a^2\*d+b^2\*c-2\*a\*d+d)\*(b\*x+a+1)/(b\*x+a-1)/(a^2\*d+b^2\*c+2\*(-b^2\*c\*d)^(1/2)-d))-1/2/b\*(-b^2\*c\*d)^(1/2)/c\*ln(1-(a^2\*d+b^2\*c-2\*a\*d+d)\*(b\*x+a+1)/(b\*x+a-1)/(a^2\*d+b^2\*c-2\*(-b^2\*c\*d)^(1/2)-d))\*arccoth(b\*x+a)\*a^2/(a^2\*d+b^2\*c-2\*(-b^2\*c\*d)^(1/2)-d)-1/2\*b\*(-b^2\*c\*d)^(1/2)/d\*ln(1-(a^2\*d+b^2\*c-2\*a\*d+d)\*(b\*x+a+1)/(b\*x+a-1)/(a^2\*d+b^2\*c-2\*(-b^2\*c\*d)^(1/2)-d))\*arccoth(b\*x+a)/(a^2\*d+b^2\*c-2\*(-b^2\*c\*d)^(1/2)-d)+1/2/b\*(-b^2\*c\*d)^(1/2)/c\*arccoth(b\*x+a)^2\*a^2/(a^2\*d+b^2\*c-2\*(-b^2\*c\*d)

$$\begin{aligned} & \frac{1}{2} \sqrt{-d} + \frac{1}{2} b \sqrt{-b^2 c d} / d \operatorname{arccoth}(b x + a) \sqrt{a^2 d + b^2 c - 2(-b^2 c d)} \\ & \sqrt{-d} - \frac{1}{4} b \sqrt{-b^2 c d} / c \operatorname{polylog}(2, (a^2 d + b^2 c - 2 a d + d) (b x + a + 1) / (b x + a - 1) / (a^2 d + b^2 c - 2(-b^2 c d) \sqrt{-d})) \sqrt{a^2 d + b^2 c - 2(-b^2 c d)} \\ & \sqrt{-d} - \frac{1}{4} b \sqrt{-b^2 c d} / d \operatorname{polylog}(2, (a^2 d + b^2 c - 2 a d + d) (b x + a + 1) / (b x + a - 1) / (a^2 d + b^2 c - 2(-b^2 c d) \sqrt{-d})) / (a^2 d + b^2 c - 2(-b^2 c d) \sqrt{-d}) \\ & - b / (a^2 d + b^2 c - 2(-b^2 c d) \sqrt{-d}) \ln(1 - (a^2 d + b^2 c - 2 a d + d) (b x + a + 1) / (b x + a - 1) / (a^2 d + b^2 c - 2(-b^2 c d) \sqrt{-d})) \operatorname{arccoth}(b x + a) + b / (a^2 d + b^2 c - 2(-b^2 c d) \sqrt{-d}) \operatorname{arccoth}(b x + a) \sqrt{a^2 d + b^2 c - 2(-b^2 c d) \sqrt{-d}} \\ & + \frac{1}{2} b \sqrt{-b^2 c d} / c \ln(1 - (a^2 d + b^2 c - 2 a d + d) (b x + a + 1) / (b x + a - 1) / (a^2 d + b^2 c - 2(-b^2 c d) \sqrt{-d})) \operatorname{arccoth}(b x + a) / (a^2 d + b^2 c - 2(-b^2 c d) \sqrt{-d}) \\ & - \frac{1}{2} b \sqrt{-b^2 c d} \sqrt{-d} / c \operatorname{arccoth}(b x + a) \sqrt{a^2 d + b^2 c - 2(-b^2 c d) \sqrt{-d}} - \frac{1}{2} b \sqrt{-b^2 c d} \sqrt{-d} \operatorname{polylog}(2, (a^2 d + b^2 c - 2 a d + d) (b x + a + 1) / (b x + a - 1) / (a^2 d + b^2 c - 2(-b^2 c d) \sqrt{-d})) \\ & + \frac{1}{4} b \sqrt{-b^2 c d} \sqrt{-d} / c \operatorname{polylog}(2, (a^2 d + b^2 c - 2 a d + d) (b x + a + 1) / (b x + a - 1) / (a^2 d + b^2 c - 2(-b^2 c d) \sqrt{-d})) / (a^2 d + b^2 c - 2(-b^2 c d) \sqrt{-d}) \end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(d\*x^2+c),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arccoth}(b x + a)}{d x^2 + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(d\*x^2+c),x, algorithm="fricas")

[Out] integral(arccoth(b\*x + a)/(d\*x^2 + c), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(b\*x+a)/(d\*x\*\*2+c),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(bx+a)}{dx^2+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(d\*x^2+c),x, algorithm="giac")

[Out] integrate(arccoth(b\*x + a)/(d\*x^2 + c), x)

$$3.77 \quad \int \frac{\coth^{-1}(a+bx)}{c+dx} dx$$

**Optimal.** Leaf size=120

$$\frac{\text{PolyLog}\left(2, 1 - \frac{2b(c+dx)}{(a+bx+1)(-ad+bc+d)}\right)}{2d} + \frac{\text{PolyLog}\left(2, 1 - \frac{2}{a+bx+1}\right)}{2d} + \frac{\coth^{-1}(a+bx) \log\left(\frac{2b(c+dx)}{(a+bx+1)(-ad+bc+d)}\right)}{d} - \frac{\log\left(\frac{2}{a+bx+1}\right)}{d}$$

[Out] -((ArcCoth[a + b\*x]\*Log[2/(1 + a + b\*x)])/d) + (ArcCoth[a + b\*x]\*Log[(2\*b\*(c + d\*x))/((b\*c + d - a\*d)\*(1 + a + b\*x))])/d + PolyLog[2, 1 - 2/(1 + a + b\*x)]/(2\*d) - PolyLog[2, 1 - (2\*b\*(c + d\*x))/((b\*c + d - a\*d)\*(1 + a + b\*x))]/(2\*d)

**Rubi [A]** time = 0.126501, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {6112, 5921, 2402, 2315, 2447}

$$\frac{\text{PolyLog}\left(2, 1 - \frac{2b(c+dx)}{(a+bx+1)(-ad+bc+d)}\right)}{2d} + \frac{\text{PolyLog}\left(2, 1 - \frac{2}{a+bx+1}\right)}{2d} + \frac{\coth^{-1}(a+bx) \log\left(\frac{2b(c+dx)}{(a+bx+1)(-ad+bc+d)}\right)}{d} - \frac{\log\left(\frac{2}{a+bx+1}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a + b\*x]/(c + d\*x), x]

[Out] -((ArcCoth[a + b\*x]\*Log[2/(1 + a + b\*x)])/d) + (ArcCoth[a + b\*x]\*Log[(2\*b\*(c + d\*x))/((b\*c + d - a\*d)\*(1 + a + b\*x))])/d + PolyLog[2, 1 - 2/(1 + a + b\*x)]/(2\*d) - PolyLog[2, 1 - (2\*b\*(c + d\*x))/((b\*c + d - a\*d)\*(1 + a + b\*x))]/(2\*d)

### Rule 6112

Int[((a\_.) + ArcCoth[(c\_) + (d\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]

### Rule 5921

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))/((d\_) + (e\_.)\*(x\_)), x\_Symbol] :> -Simp[((a + b\*ArcCoth[c\*x])\*Log[2/(1 + c\*x)])/e, x] + (Dist[(b\*c)/e, Int[Log[2/(1 + c\*x)]/(1 - c^2\*x^2), x], x] - Dist[(b\*c)/e, Int[Log[(2\*c\*(d + e\*x))

$$\frac{1}{((c*d + e)*(1 + c*x))} / (1 - c^2*x^2), x, x] + \text{Simp}[\frac{(a + b*\text{ArcCoth}[c*x]) * \text{Log}[(2*c*(d + e*x)) / ((c*d + e)*(1 + c*x))]}{e}, x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[c^2*d^2 - e^2, 0]$$

### Rule 2402

$$\text{Int}[\text{Log}[(c_.) / ((d_.) + (e_.)*(x_)))] / ((f_.) + (g_.)*(x_)^2), x\_Symbol] \rightarrow -\text{Dist}[e/g, \text{Subst}[\text{Int}[\text{Log}[2*d*x] / (1 - 2*d*x), x], x, 1/(d + e*x)], x] /; \text{FreeQ}\{c, d, e, f, g\}, x] \&\& \text{EqQ}[c, 2*d] \&\& \text{EqQ}[e^2*f + d^2*g, 0]$$

### Rule 2315

$$\text{Int}[\text{Log}[(c_.)*(x_)] / ((d_.) + (e_.)*(x_))], x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, 1 - c*x] / e, x] /; \text{FreeQ}\{c, d, e\}, x] \&\& \text{EqQ}[e + c*d, 0]$$

### Rule 2447

$$\text{Int}[\text{Log}[u_]*(\text{Pq}_.)^{(m_.)}], x\_Symbol] \rightarrow \text{With}\{C = \text{FullSimplify}[(\text{Pq}^m*(1 - u)) / D[u, x]]\}, \text{Simp}[C*\text{PolyLog}[2, 1 - u], x] /; \text{FreeQ}[C, x] /; \text{IntegerQ}[m] \&\& \text{PolyQ}[\text{Pq}, x] \&\& \text{RationalFunctionQ}[u, x] \&\& \text{LeQ}[\text{RationalFunctionExponents}[u, x][[2]], \text{Expon}[\text{Pq}, x]]$$

### Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(a + bx)}{c + dx} dx &= \frac{\text{Subst}\left(\int \frac{\coth^{-1}(x)}{\frac{bc-ad}{b} + \frac{dx}{b}} dx, x, a + bx\right)}{b} \\ &= -\frac{\coth^{-1}(a + bx) \log\left(\frac{2}{1+a+bx}\right)}{d} + \frac{\coth^{-1}(a + bx) \log\left(\frac{2b(c+dx)}{(bc+d-ad)(1+a+bx)}\right)}{d} + \frac{\text{Subst}\left(\int \frac{\log\left(\frac{2}{1+x}\right)}{1-x^2} dx, x\right)}{d} \\ &= -\frac{\coth^{-1}(a + bx) \log\left(\frac{2}{1+a+bx}\right)}{d} + \frac{\coth^{-1}(a + bx) \log\left(\frac{2b(c+dx)}{(bc+d-ad)(1+a+bx)}\right)}{d} - \frac{\text{Li}_2\left(1 - \frac{2b(c+dx)}{(bc+d-ad)(1+a+bx)}\right)}{2d} \\ &= -\frac{\coth^{-1}(a + bx) \log\left(\frac{2}{1+a+bx}\right)}{d} + \frac{\coth^{-1}(a + bx) \log\left(\frac{2b(c+dx)}{(bc+d-ad)(1+a+bx)}\right)}{d} + \frac{\text{Li}_2\left(1 - \frac{2}{1+a+bx}\right)}{2d} - \frac{\text{Li}_2\left(\frac{2b(c+dx)}{(bc+d-ad)(1+a+bx)}\right)}{2d} \end{aligned}$$

**Mathematica [A]** time = 0.0665651, size = 185, normalized size = 1.54

$$-\frac{\text{PolyLog}\left(2, \frac{b(c+dx)}{-ad+bc-d}\right)}{2d} + \frac{\text{PolyLog}\left(2, \frac{b(c+dx)}{-ad+bc+d}\right)}{2d} + \frac{\log(c + dx) \log\left(\frac{d(-a-bx+1)}{-ad+bc+d}\right)}{2d} - \frac{\log\left(\frac{a+bx-1}{a+bx}\right) \log(c + dx)}{2d} - \frac{\log(c + dx)}{2d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a + b\*x]/(c + d\*x),x]

[Out] (Log[(d\*(1 - a - b\*x))/(b\*c + d - a\*d)]\*Log[c + d\*x])/(2\*d) - (Log[(-1 + a + b\*x)/(a + b\*x)]\*Log[c + d\*x])/(2\*d) - (Log[-((d\*(1 + a + b\*x))/(b\*c - d - a\*d))]\*Log[c + d\*x])/(2\*d) + (Log[(1 + a + b\*x)/(a + b\*x)]\*Log[c + d\*x])/(2\*d) - PolyLog[2, (b\*(c + d\*x))/(b\*c - d - a\*d)]/(2\*d) + PolyLog[2, (b\*(c + d\*x))/(b\*c + d - a\*d)]/(2\*d)

**Maple [A]** time = 0.137, size = 176, normalized size = 1.5

$$\frac{\ln(d(bx+a) - ad + bc) \operatorname{arccoth}(bx+a)}{d} - \frac{\ln(d(bx+a) - ad + bc)}{2d} \ln\left(\frac{d(bx+a) + d}{ad - bc + d}\right) - \frac{1}{2d} \operatorname{dilog}\left(\frac{d(bx+a) + d}{ad - bc + d}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(b\*x+a)/(d\*x+c),x)

[Out] ln(d\*(b\*x+a)-a\*d+b\*c)/d\*arccoth(b\*x+a)-1/2/d\*ln(d\*(b\*x+a)-a\*d+b\*c)\*ln((d\*(b\*x+a)+d)/(a\*d-b\*c+d))-1/2/d\*dilog((d\*(b\*x+a)+d)/(a\*d-b\*c+d))+1/2/d\*ln((d\*(b\*x+a)-d)/(a\*d-b\*c-d))\*ln(d\*(b\*x+a)-a\*d+b\*c)+1/2/d\*dilog((d\*(b\*x+a)-d)/(a\*d-b\*c-d))

**Maxima [A]** time = 0.987377, size = 259, normalized size = 2.16

$$-\frac{1}{2}b \left( \frac{\log(bx+a-1) \log\left(\frac{bdx+ad-d}{bc-ad+d} + 1\right) + \operatorname{Li}_2\left(-\frac{bdx+ad-d}{bc-ad+d}\right)}{bd} - \frac{\log(bx+a+1) \log\left(\frac{bdx+ad+d}{bc-ad-d} + 1\right) + \operatorname{Li}_2\left(-\frac{bdx+ad+d}{bc-ad-d}\right)}{bd} \right) -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(d\*x+c),x, algorithm="maxima")

[Out] -1/2\*b\*((log(b\*x + a - 1)\*log((b\*d\*x + a\*d - d)/(b\*c - a\*d + d) + 1) + dilog(-(b\*d\*x + a\*d - d)/(b\*c - a\*d + d)))/(b\*d) - (log(b\*x + a + 1)\*log((b\*d\*x + a\*d + d)/(b\*c - a\*d - d) + 1) + dilog(-(b\*d\*x + a\*d + d)/(b\*c - a\*d - d)))/(b\*d)) - 1/2\*b\*(log(b\*x + a + 1)/b - log(b\*x + a - 1)/b)\*log(d\*x + c)/d

+  $\operatorname{arccoth}(b*x + a)*\log(d*x + c)/d$

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arccoth}(bx + a)}{dx + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(b*x+a)/(d*x+c),x, algorithm="fricas")`

[Out] `integral(arccoth(b*x + a)/(d*x + c), x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(b*x+a)/(d*x+c),x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(bx + a)}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(b*x+a)/(d*x+c),x, algorithm="giac")`

[Out] `integrate(arccoth(b*x + a)/(d*x + c), x)`



$$3.78 \quad \int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x}} dx$$

**Optimal.** Leaf size=292

$$\frac{d\text{PolyLog}\left(2, -\frac{b(cx+d)}{ac-bd+c}\right)}{2c^2} - \frac{d\text{PolyLog}\left(2, \frac{b(cx+d)}{-ac+bd+c}\right)}{2c^2} + \frac{d \log\left(-\frac{-a-bx+1}{a+bx}\right) \log(cx+d)}{2c^2} - \frac{d \log(cx+d) \log\left(\frac{c(-a-bx+1)}{-ac+bd+c}\right)}{2c^2} + \dots$$

[Out] ((1 - a - b\*x)\*Log[-((1 - a - b\*x)/(a + b\*x))])/(2\*b\*c) + Log[a + b\*x]/(2\*b\*c) + Log[1 + a + b\*x]/(2\*b\*c) + ((a + b\*x)\*Log[(1 + a + b\*x)/(a + b\*x)])/(2\*b\*c) - (d\*Log[(c\*(1 - a - b\*x))/(c - a\*c + b\*d)]\*Log[d + c\*x])/(2\*c^2) + (d\*Log[-((1 - a - b\*x)/(a + b\*x))]\*Log[d + c\*x])/(2\*c^2) + (d\*Log[(c\*(1 + a + b\*x))/(c + a\*c - b\*d)]\*Log[d + c\*x])/(2\*c^2) - (d\*Log[(1 + a + b\*x)/(a + b\*x)]\*Log[d + c\*x])/(2\*c^2) + (d\*PolyLog[2, -((b\*(d + c\*x))/(c + a\*c - b\*d))])/(2\*c^2) - (d\*PolyLog[2, (b\*(d + c\*x))/(c - a\*c + b\*d)])/(2\*c^2)

**Rubi [A]** time = 0.499929, antiderivative size = 360, normalized size of antiderivative = 1.23, number of steps used = 37, number of rules used = 10, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$ , Rules used = {6116, 2513, 2409, 2389, 2295, 2394, 2393, 2391, 193, 43}

$$\frac{d\text{PolyLog}\left(2, \frac{c(-a-bx+1)}{-ac+bd+c}\right)}{2c^2} - \frac{d\text{PolyLog}\left(2, \frac{c(a+bx+1)}{ac-bd+c}\right)}{2c^2} + \frac{d \log(a + bx - 1) \log\left(\frac{b(cx+d)}{-ac+bd+c}\right)}{2c^2} - \frac{d \left( \log(a + bx - 1) - \log\left(-\frac{-a-bx+1}{a+bx}\right) \right)}{2c^2}$$

Warning: Unable to verify antiderivative.

[In] Int[ArcCoth[a + b\*x]/(c + d/x), x]

[Out] ((1 - a - b\*x)\*Log[-1 + a + b\*x])/(2\*b\*c) + (x\*(Log[-1 + a + b\*x] - Log[-((1 - a - b\*x)/(a + b\*x))] - Log[a + b\*x]))/(2\*c) + ((1 + a + b\*x)\*Log[1 + a + b\*x])/(2\*b\*c) + (x\*(Log[a + b\*x] - Log[1 + a + b\*x] + Log[(1 + a + b\*x)/(a + b\*x)]))/(2\*c) - (d\*(Log[-1 + a + b\*x] - Log[-((1 - a - b\*x)/(a + b\*x))] - Log[a + b\*x])\*Log[d + c\*x])/(2\*c^2) - (d\*(Log[a + b\*x] - Log[1 + a + b\*x] + Log[(1 + a + b\*x)/(a + b\*x)])\*Log[d + c\*x])/(2\*c^2) - (d\*Log[1 + a + b\*x]\*Log[-((b\*(d + c\*x))/(c + a\*c - b\*d))])/(2\*c^2) + (d\*Log[-1 + a + b\*x]\*Log[(b\*(d + c\*x))/(c - a\*c + b\*d)])/(2\*c^2) + (d\*PolyLog[2, (c\*(1 - a - b\*x))/(c - a\*c + b\*d)])/(2\*c^2) - (d\*PolyLog[2, (c\*(1 + a + b\*x))/(c + a\*c - b\*d)])/(2\*c^2)

**Rule 6116**

```
Int[ArcCoth[(c_) + (d_)*(x_)]/((e_) + (f_)*(x_)^(n_)), x_Symbol] := Dist
[1/2, Int[Log[(1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] - Dist[1/2, Int[
Log[(-1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x]
&& RationalQ[n]
```

### Rule 2513

```
Int[Log[(e_)*((f_)*((a_) + (b_)*(x_))^(p_))*((c_) + (d_)*(x_))^(q_)]
^(r_)]*(RFx_), x_Symbol] := Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r], Int[RFx, x], x)) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] && !MatchQ[RFx, (u_)*(a + b*x)^(m_)*(c + d*x)^(n_)] /; IntegersQ[m, n]
]
```

### Rule 2409

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_))^(n_)]*(b_))^(p_)*((f_) + (g_
)*(x_)^(r_))^(q_), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

### Rule 2389

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_))^(n_)]*(b_))^(p_), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

### Rule 2295

```
Int[Log[(c_)*(x_)^(n_)], x_Symbol] := Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

### Rule 2394

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_))^(n_)]*(b_))/((f_) + (g_)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)
^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_))]*(b_))/((f_) + (g_)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

$(e*f - d*g), 0]$

### Rule 2391

$\text{Int}[\text{Log}[(c\_.)*((d\_.) + (e\_.)*(x\_.)^{(n\_.)})]/(x\_.), x\_Symbol] \text{ :> } -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] \text{ /; FreeQ}\{c, d, e, n\}, x\} \ \&\& \ \text{EqQ}[c*d, 1]$

### Rule 193

$\text{Int}[(a\_.) + (b\_.)*(x\_.)^{(n\_.)}]^{(p\_.)}, x\_Symbol] \text{ :> } \text{Int}[x^{(n*p)}*(b + a/x^n)^p, x] \text{ /; FreeQ}\{a, b\}, x\} \ \&\& \ \text{LtQ}[n, 0] \ \&\& \ \text{IntegerQ}[p]$

### Rule 43

$\text{Int}[(a\_.) + (b\_.)*(x\_.)^{(m\_.)}]^{(c\_.) + (d\_.)*(x\_.)^{(n\_.)}}, x\_Symbol] \text{ :> } \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] \text{ /; FreeQ}\{a, b, c, d, n\}, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\text{!IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0]) \ || \ \text{GtQ}[m + n + 2, 0])$

### Rubi steps

$$\begin{aligned}
 \int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x}} dx &= -\left(\frac{1}{2} \int \frac{\log\left(\frac{-1+a+bx}{a+bx}\right)}{c+\frac{d}{x}} dx\right) + \frac{1}{2} \int \frac{\log\left(\frac{1+a+bx}{a+bx}\right)}{c+\frac{d}{x}} dx \\
 &= -\left(\frac{1}{2} \int \frac{\log(-1+a+bx)}{c+\frac{d}{x}} dx\right) + \frac{1}{2} \int \frac{\log(1+a+bx)}{c+\frac{d}{x}} dx - \frac{1}{2} \left(-\log(-1+a+bx) + \log\left(\frac{-1+a}{a}\right)\right) \\
 &= -\left(\frac{1}{2} \int \left(\frac{\log(-1+a+bx)}{c} - \frac{d \log(-1+a+bx)}{c(d+cx)}\right) dx\right) + \frac{1}{2} \int \left(\frac{\log(1+a+bx)}{c} - \frac{d \log(1+a+bx)}{c(d+cx)}\right) dx \\
 &= -\frac{\int \log(-1+a+bx) dx}{2c} + \frac{\int \log(1+a+bx) dx}{2c} + \frac{d \int \frac{\log(-1+a+bx)}{d+cx} dx}{2c} - \frac{d \int \frac{\log(1+a+bx)}{d+cx} dx}{2c} - \frac{1}{2} \left(\log\left(\frac{-1+a}{a}\right)\right) \\
 &= \frac{x \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2c} + \frac{x \left(\log(a+bx) - \log(1+a+bx) + \log\left(\frac{1+a}{a}\right)\right)}{2c} \\
 &= \frac{(1-a-bx) \log(-1+a+bx)}{2bc} + \frac{x \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2c} + \frac{(1+a) \log(a)}{2c} \\
 &= \frac{(1-a-bx) \log(-1+a+bx)}{2bc} + \frac{x \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2c} + \frac{(1+a) \log(a)}{2c}
 \end{aligned}$$

**Mathematica [C]** time = 4.25848, size = 502, normalized size = 1.72

$$bcd\text{PolyLog}\left(2, \exp\left(2 \tanh^{-1}\left(\frac{c}{ac-bd}\right) - 2 \coth^{-1}(a+bx)\right)\right) - bcd\text{PolyLog}\left(2, e^{-2 \coth^{-1}(a+bx)}\right) + b^2 d^2 \sqrt{1 - \frac{c^2}{(ac-bd)^2}} \coth^{-1}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a + b\*x]/(c + d/x), x]

[Out] (2\*a\*c^2\*ArcCoth[a + b\*x] - I\*b\*c\*d\*Pi\*ArcCoth[a + b\*x] + 2\*b\*c^2\*x\*ArcCoth[a + b\*x] + b\*c\*d\*ArcCoth[a + b\*x]^2 + a\*b\*c\*d\*ArcCoth[a + b\*x]^2 - b^2\*d^2\*ArcCoth[a + b\*x]^2 - a\*b\*c\*d\*Sqrt[1 - c^2/(a\*c - b\*d)^2]\*E^ArcTanh[c/(a\*c - b\*d)]\*ArcCoth[a + b\*x]^2 + b^2\*d^2\*Sqrt[1 - c^2/(a\*c - b\*d)^2]\*E^ArcTanh[c/(a\*c - b\*d)]\*ArcCoth[a + b\*x]^2 + 2\*b\*c\*d\*ArcCoth[a + b\*x]\*ArcTanh[c/(a\*c - b\*d)] + 2\*b\*c\*d\*ArcCoth[a + b\*x]\*Log[1 - E^(-2\*ArcCoth[a + b\*x])] + I\*b\*c\*d\*Pi\*Log[1 + E^(2\*ArcCoth[a + b\*x])] - 2\*b\*c\*d\*ArcCoth[a + b\*x]\*Log[1 - E^(-2\*ArcCoth[a + b\*x] + 2\*ArcTanh[c/(a\*c - b\*d)])] + 2\*b\*c\*d\*ArcTanh[c/(a\*c - b\*d)]\*Log[1 - E^(-2\*ArcCoth[a + b\*x] + 2\*ArcTanh[c/(a\*c - b\*d)])] - I\*b\*c\*d\*Pi\*Log[1/Sqrt[1 - (a + b\*x)^(-2)]] - 2\*c^2\*Log[1/((a + b\*x)\*Sqrt[1 - (a + b\*x)^(-2)])] - 2\*b\*c\*d\*ArcTanh[c/(a\*c - b\*d)]\*Log[I\*Sinh[ArcCoth[a + b\*x]] - ArcTanh[c/(a\*c - b\*d)]] - b\*c\*d\*PolyLog[2, E^(-2\*ArcCoth[a + b\*x])] + b\*c\*d\*PolyLog[2, E^(-2\*ArcCoth[a + b\*x] + 2\*ArcTanh[c/(a\*c - b\*d)])]/(2\*b\*c^3)

**Maple [A]** time = 0.142, size = 297, normalized size = 1.

$$\frac{\operatorname{arccoth}(bx+a)}{c} + \frac{\operatorname{arccoth}(bx+a)a}{bc} - \frac{\operatorname{arccoth}(bx+a)d \ln(c(bx+a) - ac + bd)}{c^2} + \frac{\ln(a^2c^2 - 2abcd + b^2d^2 + 2(c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(b\*x+a)/(c+d/x), x)

[Out] arccoth(b\*x+a)/c\*x+1/b\*arccoth(b\*x+a)/c\*a-arccoth(b\*x+a)\*d/c^2\*ln(c\*(b\*x+a)-a\*c+b\*d)+1/2/b/c\*ln(a^2\*c^2-2\*a\*b\*c\*d+b^2\*d^2+2\*(c\*(b\*x+a)-a\*c+b\*d)\*a\*c-2\*(c\*(b\*x+a)-a\*c+b\*d)\*b\*d+(c\*(b\*x+a)-a\*c+b\*d)^2-c^2)-1/2/c^2\*d\*ln(c\*(b\*x+a)-a\*c+b\*d)\*ln((c\*(b\*x+a)-c)/(a\*c-b\*d-c))-1/2/c^2\*d\*dilog((c\*(b\*x+a)-c)/(a\*c-b\*d-c))+1/2/c^2\*d\*ln(c\*(b\*x+a)-a\*c+b\*d)\*ln((c\*(b\*x+a)+c)/(a\*c-b\*d+c))+1/2/c^2\*d\*dilog((c\*(b\*x+a)+c)/(a\*c-b\*d+c))

---

**Maxima [A]** time = 0.996411, size = 259, normalized size = 0.89

$$\frac{1}{2} b \left( \frac{\left( \log(cx + d) \log\left(\frac{bcx+bd}{ac-bd+c} + 1\right) + \text{Li}_2\left(-\frac{bcx+bd}{ac-bd+c}\right) \right) d}{bc^2} - \frac{\left( \log(cx + d) \log\left(\frac{bcx+bd}{ac-bd-c} + 1\right) + \text{Li}_2\left(-\frac{bcx+bd}{ac-bd-c}\right) \right) d}{bc^2} + \frac{(a+1) \log(bx+a)}{b^2 c} - \frac{(a-1) \log(bx+a-1)}{b^2 c} + \frac{(x/c - d) \log(cx+d)}{c^2} \text{arccoth}(bx+a) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(c+d/x),x, algorithm="maxima")

[Out] 1/2\*b\*((log(c\*x + d)\*log((b\*c\*x + b\*d)/(a\*c - b\*d + c) + 1) + dilog(-(b\*c\*x + b\*d)/(a\*c - b\*d + c)))\*d/(b\*c^2) - (log(c\*x + d)\*log((b\*c\*x + b\*d)/(a\*c - b\*d - c) + 1) + dilog(-(b\*c\*x + b\*d)/(a\*c - b\*d - c)))\*d/(b\*c^2) + (a + 1)\*log(b\*x + a + 1)/(b^2\*c) - (a - 1)\*log(b\*x + a - 1)/(b^2\*c)) + (x/c - d\*log(c\*x + d)/c^2)\*arccoth(b\*x + a)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x \text{arccoth}(bx + a)}{cx + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(c+d/x),x, algorithm="fricas")

[Out] integral(x\*arccoth(b\*x + a)/(c\*x + d), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(b\*x+a)/(c+d/x),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(bx + a)}{c + \frac{d}{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(c+d/x),x, algorithm="giac")

[Out] integrate(arccoth(b\*x + a)/(c + d/x), x)

$$3.79 \quad \int \frac{\coth^{-1}(a+bx)}{c + \frac{d}{x^2}} dx$$

**Optimal.** Leaf size=738

$$\frac{\sqrt{d}\text{PolyLog}\left(2, \frac{\sqrt{-c(-a-bx+1)}}{(1-a)\sqrt{-c-b\sqrt{d}}}\right)}{4(-c)^{3/2}} - \frac{\sqrt{d}\text{PolyLog}\left(2, \frac{\sqrt{-c(-a-bx+1)}}{(1-a)\sqrt{-c+b\sqrt{d}}}\right)}{4(-c)^{3/2}} + \frac{\sqrt{d}\text{PolyLog}\left(2, \frac{\sqrt{-c(a+bx+1)}}{(a+1)\sqrt{-c-b\sqrt{d}}}\right)}{4(-c)^{3/2}} - \frac{\sqrt{d}\text{PolyLog}\left(2, \frac{\sqrt{-c(a+bx+1)}}{(a+1)\sqrt{-c+b\sqrt{d}}}\right)}{4(-c)^{3/2}}$$

```
[Out] ((1 - a - b*x)*Log[-1 + a + b*x])/(2*b*c) + (x*(Log[-1 + a + b*x] - Log[-((1 - a - b*x)/(a + b*x))] - Log[a + b*x]))/(2*c) - (Sqrt[d]*ArcTan[(Sqrt[c]*x)/Sqrt[d]]*(Log[-1 + a + b*x] - Log[-((1 - a - b*x)/(a + b*x))] - Log[a + b*x]))/(2*c^(3/2)) + ((1 + a + b*x)*Log[1 + a + b*x])/(2*b*c) + (x*(Log[a + b*x] - Log[1 + a + b*x] + Log[(1 + a + b*x)/(a + b*x]])))/(2*c) - (Sqrt[d]*ArcTan[(Sqrt[c]*x)/Sqrt[d]]*(Log[a + b*x] - Log[1 + a + b*x] + Log[(1 + a + b*x)/(a + b*x]])))/(2*c^(3/2)) + (Sqrt[d]*Log[-1 + a + b*x]*Log[-((b*(Sqrt[d] - Sqrt[-c]*x))/((1 - a)*Sqrt[-c] - b*Sqrt[d])))]/(4*(-c)^(3/2)) - (Sqrt[d]*Log[1 + a + b*x]*Log[(b*(Sqrt[d] - Sqrt[-c]*x))/((1 + a)*Sqrt[-c] + b*Sqrt[d])))]/(4*(-c)^(3/2)) + (Sqrt[d]*Log[1 + a + b*x]*Log[-((b*(Sqrt[d] + Sqrt[-c]*x))/((1 + a)*Sqrt[-c] - b*Sqrt[d])))]/(4*(-c)^(3/2)) - (Sqrt[d]*Log[-1 + a + b*x]*Log[(b*(Sqrt[d] + Sqrt[-c]*x))/((1 - a)*Sqrt[-c] + b*Sqrt[d])))]/(4*(-c)^(3/2)) + (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(1 - a - b*x))/((1 - a)*Sqrt[-c] - b*Sqrt[d])])/(4*(-c)^(3/2)) - (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(1 - a - b*x))/((1 - a)*Sqrt[-c] + b*Sqrt[d])])/(4*(-c)^(3/2)) + (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(1 + a + b*x))/((1 + a)*Sqrt[-c] - b*Sqrt[d])])/(4*(-c)^(3/2)) - (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(1 + a + b*x))/((1 + a)*Sqrt[-c] + b*Sqrt[d])])/(4*(-c)^(3/2))
```

**Rubi [A]** time = 1.53032, antiderivative size = 738, normalized size of antiderivative = 1., number of steps used = 57, number of rules used = 11, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$ , Rules used = {6116, 2513, 2409, 2389, 2295, 2394, 2393, 2391, 193, 321, 205}

$$\frac{\sqrt{d}\text{PolyLog}\left(2, \frac{\sqrt{-c(-a-bx+1)}}{(1-a)\sqrt{-c-b\sqrt{d}}}\right)}{4(-c)^{3/2}} - \frac{\sqrt{d}\text{PolyLog}\left(2, \frac{\sqrt{-c(-a-bx+1)}}{(1-a)\sqrt{-c+b\sqrt{d}}}\right)}{4(-c)^{3/2}} + \frac{\sqrt{d}\text{PolyLog}\left(2, \frac{\sqrt{-c(a+bx+1)}}{(a+1)\sqrt{-c-b\sqrt{d}}}\right)}{4(-c)^{3/2}} - \frac{\sqrt{d}\text{PolyLog}\left(2, \frac{\sqrt{-c(a+bx+1)}}{(a+1)\sqrt{-c+b\sqrt{d}}}\right)}{4(-c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x]/(c + d/x^2), x]
```

```
[Out] ((1 - a - b*x)*Log[-1 + a + b*x])/(2*b*c) + (x*(Log[-1 + a + b*x] - Log[-((1 - a - b*x)/(a + b*x))] - Log[a + b*x]))/(2*c) - (Sqrt[d]*ArcTan[(Sqrt[c]*x)/Sqrt[d]]*(Log[-1 + a + b*x] - Log[-((1 - a - b*x)/(a + b*x))] - Log[a + b*x]))/(2*c^(3/2)) + ((1 + a + b*x)*Log[1 + a + b*x])/(2*b*c) + (x*(Log[a + b*x] - Log[1 + a + b*x] + Log[(1 + a + b*x)/(a + b*x]])))/(2*c) - (Sqrt[d]*ArcTan[(Sqrt[c]*x)/Sqrt[d]]*(Log[a + b*x] - Log[1 + a + b*x] + Log[(1 + a + b*x)/(a + b*x]])))/(2*c^(3/2)) + (Sqrt[d]*Log[-1 + a + b*x]*Log[-((b*(Sqrt[d] - Sqrt[-c]*x))/((1 - a)*Sqrt[-c] - b*Sqrt[d])))]/(4*(-c)^(3/2)) - (Sqrt[d]*Log[1 + a + b*x]*Log[(b*(Sqrt[d] - Sqrt[-c]*x))/((1 + a)*Sqrt[-c] + b*Sqrt[d])))]/(4*(-c)^(3/2)) + (Sqrt[d]*Log[1 + a + b*x]*Log[-((b*(Sqrt[d] + Sqrt[-c]*x))/((1 + a)*Sqrt[-c] - b*Sqrt[d])))]/(4*(-c)^(3/2)) - (Sqrt[d]*Log[-1 + a + b*x]*Log[(b*(Sqrt[d] + Sqrt[-c]*x))/((1 - a)*Sqrt[-c] + b*Sqrt[d])))]/(4*(-c)^(3/2)) + (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(1 - a - b*x))/((1 - a)*Sqrt[-c] - b*Sqrt[d])])/(4*(-c)^(3/2)) - (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(1 - a - b*x))/((1 - a)*Sqrt[-c] + b*Sqrt[d])])/(4*(-c)^(3/2)) + (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(1 + a + b*x))/((1 + a)*Sqrt[-c] - b*Sqrt[d])])/(4*(-c)^(3/2)) - (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(1 + a + b*x))/((1 + a)*Sqrt[-c] + b*Sqrt[d])])/(4*(-c)^(3/2))
```

$$\begin{aligned} & x)/\text{Sqrt}[d]]*(\text{Log}[-1 + a + b*x] - \text{Log}[-(1 - a - b*x)/(a + b*x)]) - \text{Log}[a + \\ & b*x]))/(2*c^{(3/2)}) + ((1 + a + b*x)*\text{Log}[1 + a + b*x])/(2*b*c) + (x*(\text{Log}[a + \\ & b*x] - \text{Log}[1 + a + b*x] + \text{Log}[(1 + a + b*x)/(a + b*x)])))/(2*c) - (\text{Sqrt}[d]* \\ & \text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d]]*(\text{Log}[a + b*x] - \text{Log}[1 + a + b*x] + \text{Log}[(1 + a + \\ & b*x)/(a + b*x)])))/(2*c^{(3/2)}) + (\text{Sqrt}[d]*\text{Log}[-1 + a + b*x]*\text{Log}[-(b*(\text{Sqrt}[ \\ & d] - \text{Sqrt}[-c]*x))/((1 - a)*\text{Sqrt}[-c] - b*\text{Sqrt}[d])])/(4*(-c)^{(3/2)}) - (\text{Sqrt}[ \\ & d]*\text{Log}[1 + a + b*x]*\text{Log}[(b*(\text{Sqrt}[d] - \text{Sqrt}[-c]*x))/((1 + a)*\text{Sqrt}[-c] + b*\text{Sqr} \\ & \text{rt}[d])])/(4*(-c)^{(3/2)}) + (\text{Sqrt}[d]*\text{Log}[1 + a + b*x]*\text{Log}[-(b*(\text{Sqrt}[d] + \text{Sqr} \\ & \text{t}[-c]*x))/((1 + a)*\text{Sqrt}[-c] - b*\text{Sqrt}[d])])/(4*(-c)^{(3/2)}) - (\text{Sqrt}[d]*\text{Log}[- \\ & 1 + a + b*x]*\text{Log}[(b*(\text{Sqrt}[d] + \text{Sqrt}[-c]*x))/((1 - a)*\text{Sqrt}[-c] + b*\text{Sqrt}[d])]) \\ & )/(4*(-c)^{(3/2)}) + (\text{Sqrt}[d]*\text{PolyLog}[2, (\text{Sqrt}[-c]*(1 - a - b*x))/((1 - a)*\text{Sqr} \\ & \text{t}[-c] - b*\text{Sqrt}[d])])/(4*(-c)^{(3/2)}) - (\text{Sqrt}[d]*\text{PolyLog}[2, (\text{Sqrt}[-c]*(1 - a \\ & - b*x))/((1 - a)*\text{Sqrt}[-c] + b*\text{Sqrt}[d])])/(4*(-c)^{(3/2)}) + (\text{Sqrt}[d]*\text{PolyLog} \\ & [2, (\text{Sqrt}[-c]*(1 + a + b*x))/((1 + a)*\text{Sqrt}[-c] - b*\text{Sqrt}[d])])/(4*(-c)^{(3/2)} \\ & ) - (\text{Sqrt}[d]*\text{PolyLog}[2, (\text{Sqrt}[-c]*(1 + a + b*x))/((1 + a)*\text{Sqrt}[-c] + b*\text{Sqr} \\ & \text{t}[d])])/(4*(-c)^{(3/2)}) \end{aligned}$$

### Rule 6116

```
Int[ArcCoth[(c_) + (d_)*(x_)]/((e_) + (f_)*(x_)^(n_)), x_Symbol] := Dist
[1/2, Int[Log[(1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] - Dist[1/2, Int[
Log[(-1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x]
&& RationalQ[n]
```

### Rule 2513

```
Int[Log[(e_)*((f_)*((a_) + (b_)*(x_))^(p_))*((c_) + (d_)*(x_))^(q_)]
^(r_)]*(RFx_), x_Symbol] := Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] && !MatchQ[RFx, (u_)*(a + b*x)^(m_)*(c + d*x)^(n_) /; IntegersQ[m, n]
]
```

### Rule 2409

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_))^(n_)]*(b_))^(p_)*((f_) + (g_
)*(x_)^(r_))^(q_), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

### Rule 2389

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_))^(n_)]*(b_))^(p_), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```



, b, c, d, e, n, p}, x]

### Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n]))/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))])\*(b\_.)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 193

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[x^(n\*p)\*(b + a/x^n)^p, x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]

### Rule 321

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x^2}} dx &= -\left(\frac{1}{2} \int \frac{\log\left(\frac{-1+a+bx}{a+bx}\right)}{c+\frac{d}{x^2}} dx\right) + \frac{1}{2} \int \frac{\log\left(\frac{1+a+bx}{a+bx}\right)}{c+\frac{d}{x^2}} dx \\
&= -\left(\frac{1}{2} \int \frac{\log(-1+a+bx)}{c+\frac{d}{x^2}} dx\right) + \frac{1}{2} \int \frac{\log(1+a+bx)}{c+\frac{d}{x^2}} dx - \frac{1}{2} \left(-\log(-1+a+bx) + \log\left(\frac{-1+a+bx}{a+bx}\right)\right) \\
&= -\left(\frac{1}{2} \int \left(\frac{\log(-1+a+bx)}{c} - \frac{d \log(-1+a+bx)}{c(d+cx^2)}\right) dx\right) + \frac{1}{2} \int \left(\frac{\log(1+a+bx)}{c} - \frac{d \log(1+a+bx)}{c(d+cx^2)}\right) dx \\
&= \frac{x \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2c} + \frac{x \left(\log(a+bx) - \log(1+a+bx) + \log\left(\frac{1+a+bx}{a+bx}\right)\right)}{2c} \\
&= \frac{x \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2c} - \frac{\sqrt{d} \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right) \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2c^{3/2}} \\
&= \frac{(1-a-bx) \log(-1+a+bx)}{2bc} + \frac{x \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2c} - \frac{\sqrt{d} \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right) \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2c^{3/2}} \\
&= \frac{(1-a-bx) \log(-1+a+bx)}{2bc} + \frac{x \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2c} - \frac{\sqrt{d} \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right) \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2c^{3/2}} \\
&= \frac{(1-a-bx) \log(-1+a+bx)}{2bc} + \frac{x \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2c} - \frac{\sqrt{d} \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right) \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2c^{3/2}} \\
&= \frac{(1-a-bx) \log(-1+a+bx)}{2bc} + \frac{x \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2c} - \frac{\sqrt{d} \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right) \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2c^{3/2}}
\end{aligned}$$

**Mathematica [C]** time = 35.6651, size = 5552, normalized size = 7.52

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a + b\*x]/(c + d/x^2), x]

[Out] Result too large to show

---

**Maple [C]** time = 1.251, size = 19686, normalized size = 26.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(b*x+a)/(c+d/x^2),x)`

[Out] result too large to display

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(b*x+a)/(c+d/x^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2 \operatorname{arccoth}(bx+a)}{cx^2+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(b*x+a)/(c+d/x^2),x, algorithm="fricas")`

[Out] `integral(x^2*arccoth(b*x + a)/(c*x^2 + d), x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoath(b*x+a)/(c+d/x**2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(bx+a)}{c + \frac{d}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoath(b*x+a)/(c+d/x^2),x, algorithm="giac")
```

```
[Out] integrate(arccoath(b*x + a)/(c + d/x^2), x)
```

$$3.80 \quad \int \frac{\coth^{-1}(a+bx)}{c+d\sqrt{x}} dx$$

**Optimal.** Leaf size=619

$$\frac{c \operatorname{PolyLog}\left(2, \frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{bc}-\sqrt{-a-1d}}\right)}{d^2} + \frac{c \operatorname{PolyLog}\left(2, \frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{-a-1d}+\sqrt{bc}}\right)}{d^2} - \frac{c \operatorname{PolyLog}\left(2, \frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{bc}-\sqrt{1-ad}}\right)}{d^2} - \frac{c \operatorname{PolyLog}\left(2, \frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{1-ad}+\sqrt{bc}}\right)}{d^2} + \dots$$

[Out] (2\*sqrt[1 + a]\*ArcTan[(sqrt[b]\*sqrt[x])/sqrt[1 + a]])/(sqrt[b]\*d) - (2\*sqrt[1 - a]\*ArcTanh[(sqrt[b]\*sqrt[x])/sqrt[1 - a]])/(sqrt[b]\*d) + (c\*Log[(d\*(sqrt[-1 - a] - sqrt[b]\*sqrt[x]))/(sqrt[b]\*c + sqrt[-1 - a]\*d)]\*Log[c + d\*sqrt[x]])/d^2 - (c\*Log[(d\*(sqrt[1 - a] - sqrt[b]\*sqrt[x]))/(sqrt[b]\*c + sqrt[1 - a]\*d)]\*Log[c + d\*sqrt[x]])/d^2 + (c\*Log[-((d\*(sqrt[-1 - a] + sqrt[b]\*sqrt[x]))/(sqrt[b]\*c - sqrt[-1 - a]\*d))]\*Log[c + d\*sqrt[x]])/d^2 - (c\*Log[-((d\*(sqrt[1 - a] + sqrt[b]\*sqrt[x]))/(sqrt[b]\*c - sqrt[1 - a]\*d))]\*Log[c + d\*sqrt[x]])/d^2 - (sqrt[x]\*Log[-((1 - a - b\*x)/(a + b\*x))])/d + (c\*Log[c + d\*sqrt[x]]\*Log[-((1 - a - b\*x)/(a + b\*x))])/d^2 + (sqrt[x]\*Log[(1 + a + b\*x)/(a + b\*x)])/d - (c\*Log[c + d\*sqrt[x]]\*Log[(1 + a + b\*x)/(a + b\*x)])/d^2 + (c\*PolyLog[2, (sqrt[b]\*(c + d\*sqrt[x]))/(sqrt[b]\*c - sqrt[-1 - a]\*d)])/d^2 + (c\*PolyLog[2, (sqrt[b]\*(c + d\*sqrt[x]))/(sqrt[b]\*c + sqrt[-1 - a]\*d)])/d^2 - (c\*PolyLog[2, (sqrt[b]\*(c + d\*sqrt[x]))/(sqrt[b]\*c - sqrt[1 - a]\*d)])/d^2 - (c\*PolyLog[2, (sqrt[b]\*(c + d\*sqrt[x]))/(sqrt[b]\*c + sqrt[1 - a]\*d)])/d^2

**Rubi [A]** time = 2.19896, antiderivative size = 619, normalized size of antiderivative = 1., number of steps used = 55, number of rules used = 16, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$ , Rules used = {6116, 190, 43, 2528, 2523, 12, 481, 205, 2524, 2418, 260, 2416, 2394, 2393, 2391, 208}

$$\frac{c \operatorname{PolyLog}\left(2, \frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{bc}-\sqrt{-a-1d}}\right)}{d^2} + \frac{c \operatorname{PolyLog}\left(2, \frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{-a-1d}+\sqrt{bc}}\right)}{d^2} - \frac{c \operatorname{PolyLog}\left(2, \frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{bc}-\sqrt{1-ad}}\right)}{d^2} - \frac{c \operatorname{PolyLog}\left(2, \frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{1-ad}+\sqrt{bc}}\right)}{d^2} + \dots$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a + b\*x]/(c + d\*sqrt[x]), x]

[Out] (2\*sqrt[1 + a]\*ArcTan[(sqrt[b]\*sqrt[x])/sqrt[1 + a]])/(sqrt[b]\*d) - (2\*sqrt[1 - a]\*ArcTanh[(sqrt[b]\*sqrt[x])/sqrt[1 - a]])/(sqrt[b]\*d) + (c\*Log[(d\*(sqrt[-1 - a] - sqrt[b]\*sqrt[x]))/(sqrt[b]\*c + sqrt[-1 - a]\*d)]\*Log[c + d\*sqrt[x]])/d^2 - (c\*Log[(d\*(sqrt[1 - a] - sqrt[b]\*sqrt[x]))/(sqrt[b]\*c + sqrt[1 - a]\*d)]\*Log[c + d\*sqrt[x]])/d^2 + (c\*Log[-((d\*(sqrt[-1 - a] + sqrt[b]\*sqrt[x]))/(sqrt[b]\*c - sqrt[-1 - a]\*d))]\*Log[c + d\*sqrt[x]])/d^2 + (c\*Log[-((d\*(sqrt[1 - a] + sqrt[b]\*sqrt[x]))/(sqrt[b]\*c - sqrt[1 - a]\*d))]\*Log[c + d\*sqrt[x]])/d^2 - (sqrt[x]\*Log[-((1 - a - b\*x)/(a + b\*x))])/d + (c\*Log[c + d\*sqrt[x]]\*Log[-((1 - a - b\*x)/(a + b\*x))])/d^2 + (sqrt[x]\*Log[(1 + a + b\*x)/(a + b\*x)])/d - (c\*Log[c + d\*sqrt[x]]\*Log[(1 + a + b\*x)/(a + b\*x)])/d^2 + (c\*PolyLog[2, (sqrt[b]\*(c + d\*sqrt[x]))/(sqrt[b]\*c - sqrt[-1 - a]\*d)])/d^2 + (c\*PolyLog[2, (sqrt[b]\*(c + d\*sqrt[x]))/(sqrt[b]\*c + sqrt[-1 - a]\*d)])/d^2 - (c\*PolyLog[2, (sqrt[b]\*(c + d\*sqrt[x]))/(sqrt[b]\*c - sqrt[1 - a]\*d)])/d^2 - (c\*PolyLog[2, (sqrt[b]\*(c + d\*sqrt[x]))/(sqrt[b]\*c + sqrt[1 - a]\*d)])/d^2

```
[x]))/(Sqrt[b]*c - Sqrt[-1 - a]*d))*Log[c + d*Sqrt[x]]/d^2 - (c*Log[-((d*
(Sqrt[1 - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c - Sqrt[1 - a]*d))*Log[c + d*Sq
rt[x]]/d^2 - (Sqrt[x]*Log[-((1 - a - b*x)/(a + b*x))])/d + (c*Log[c + d*Sq
rt[x]]*Log[-((1 - a - b*x)/(a + b*x))])/d^2 + (Sqrt[x]*Log[(1 + a + b*x)/(a
+ b*x))]/d - (c*Log[c + d*Sqrt[x]]*Log[(1 + a + b*x)/(a + b*x))]/d^2 + (c*
PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[-1 - a]*d)]/d^2 + (
c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-1 - a]*d)]/d^2 -
(c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[1 - a]*d)]/d^2
- (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[1 - a]*d)]/d^2
```

### Rule 6116

```
Int[ArcCoth[(c_) + (d_)*(x_)]/((e_) + (f_)*(x_)^(n_)), x_Symbol] := Dist
[1/2, Int[Log[(1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] - Dist[1/2, Int[
Log[(-1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x]
&& RationalQ[n]
```

### Rule 190

```
Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
IntegerQ[1/n]
```

### Rule 43

```
Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rule 2528

```
Int[((a_) + Log[(c_)*(RFx_)^(p_)])*(b_)^(n_)*(RGx_), x_Symbol] := With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

### Rule 2523

```
Int[((a_) + Log[(c_)*(RFx_)^(p_)])*(b_)^(n_), x_Symbol] := Simp[x*(a +
b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[(x*(a + b*Log[c*
RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, p}, x] && Ra
tionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 12

$\text{Int}[(a_*)(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)(v_)] /; \text{FreeQ}[b, x]$

Rule 481

$\text{Int}[(e_*)(x_)^{(m_)} / ((a_*) + (b_*)(x_)^{(n_)})(c_*) + (d_*)(x_)^{(n_)}), x\_Symbol] \rightarrow -\text{Dist}[(a_*e^n)/(b*c - a*d), \text{Int}[(e*x)^{(m-n)} / (a + b*x^n), x], x] + \text{Dist}[(c_*e^n)/(b*c - a*d), \text{Int}[(e*x)^{(m-n)} / (c + d*x^n), x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{LeQ}[n, m, 2*n - 1]$

Rule 205

$\text{Int}[(a_*) + (b_*)(x_)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2] * \text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b]$

Rule 2524

$\text{Int}[(a_*) + \text{Log}[(c_*)(\text{RFx}_*)^{(p_)}] * (b_*)^{(n_)} / ((d_*) + (e_*)(x_)), x\_Symbol] \rightarrow \text{Simp}[(\text{Log}[d + e*x] * (a + b * \text{Log}[c * \text{RFx}^p])^n) / e, x] - \text{Dist}[(b*n*p) / e, \text{Int}[(\text{Log}[d + e*x] * (a + b * \text{Log}[c * \text{RFx}^p])^{(n-1)} * D[\text{RFx}, x]) / \text{RFx}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, p\}, x] \ \&\& \ \text{RationalFunctionQ}[\text{RFx}, x] \ \&\& \ \text{IGtQ}[n, 0]$

Rule 2418

$\text{Int}[(a_*) + \text{Log}[(c_*)((d_*) + (e_*)(x_))^{(n_)}] * (b_*)^{(p_)} * (\text{RFx}_*), x\_Symbol] \rightarrow \text{With}[\{u = \text{ExpandIntegrand}[(a + b * \text{Log}[c * (d + e*x)^n])^p, \text{RFx}, x]\}, \text{Int}[u, x] /; \text{SumQ}[u] /; \text{FreeQ}[\{a, b, c, d, e, n\}, x] \ \&\& \ \text{RationalFunctionQ}[\text{RFx}, x] \ \&\& \ \text{IntegerQ}[p]$

Rule 260

$\text{Int}[(x_)^{(m_)} / ((a_*) + (b_*)(x_)^{(n_)}), x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]] / (b*n), x] /; \text{FreeQ}[\{a, b, m, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1]$

Rule 2416

$\text{Int}[(a_*) + \text{Log}[(c_*)((d_*) + (e_*)(x_))^{(n_)}] * (b_*)^{(p_)} * ((h_*)(x_))^{(m_)} * ((f_*) + (g_*)(x_)^{(r_)})^{(q_)}], x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b * \text{Log}[c * (d + e*x)^n])^p, (h*x)^m * (f + g*x^r)^q, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, h, m, n, p, q, r\}, x] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ \text{IntegerQ}[q]$

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rubi steps



$$\begin{aligned}
\int \frac{\coth^{-1}(a+bx)}{c+d\sqrt{x}} dx &= -\left(\frac{1}{2} \int \frac{\log\left(\frac{-1+a+bx}{a+bx}\right)}{c+d\sqrt{x}} dx\right) + \frac{1}{2} \int \frac{\log\left(\frac{1+a+bx}{a+bx}\right)}{c+d\sqrt{x}} dx \\
&= -\text{Subst}\left(\int \frac{x \log\left(\frac{-1+a+bx^2}{a+bx^2}\right)}{c+dx} dx, x, \sqrt{x}\right) + \text{Subst}\left(\int \frac{x \log\left(\frac{1+a+bx^2}{a+bx^2}\right)}{c+dx} dx, x, \sqrt{x}\right) \\
&= -\text{Subst}\left(\int \left(\frac{\log\left(\frac{-1+a+bx^2}{a+bx^2}\right)}{d} - \frac{c \log\left(\frac{-1+a+bx^2}{a+bx^2}\right)}{d(c+dx)}\right) dx, x, \sqrt{x}\right) + \text{Subst}\left(\int \left(\frac{\log\left(\frac{1+a+bx^2}{a+bx^2}\right)}{d} - \frac{c \log\left(\frac{1+a+bx^2}{a+bx^2}\right)}{d(c+dx)}\right) dx, x, \sqrt{x}\right) \\
&= -\frac{\text{Subst}\left(\int \log\left(\frac{-1+a+bx^2}{a+bx^2}\right) dx, x, \sqrt{x}\right)}{d} + \frac{\text{Subst}\left(\int \log\left(\frac{1+a+bx^2}{a+bx^2}\right) dx, x, \sqrt{x}\right)}{d} + \frac{c \text{Subst}\left(\int \frac{\log\left(\frac{-1+a+bx^2}{a+bx^2}\right)}{c+dx} dx, x, \sqrt{x}\right)}{d} \\
&= -\frac{\sqrt{x} \log\left(-\frac{1-a-bx}{a+bx}\right)}{d} + \frac{c \log(c+d\sqrt{x}) \log\left(-\frac{1-a-bx}{a+bx}\right)}{d^2} + \frac{\sqrt{x} \log\left(\frac{1+a+bx}{a+bx}\right)}{d} - \frac{c \log(c+d\sqrt{x}) \log\left(\frac{1+a+bx}{a+bx}\right)}{d^2} \\
&= -\frac{\sqrt{x} \log\left(-\frac{1-a-bx}{a+bx}\right)}{d} + \frac{c \log(c+d\sqrt{x}) \log\left(-\frac{1-a-bx}{a+bx}\right)}{d^2} + \frac{\sqrt{x} \log\left(\frac{1+a+bx}{a+bx}\right)}{d} - \frac{c \log(c+d\sqrt{x}) \log\left(\frac{1+a+bx}{a+bx}\right)}{d^2} \\
&= -\frac{\sqrt{x} \log\left(-\frac{1-a-bx}{a+bx}\right)}{d} + \frac{c \log(c+d\sqrt{x}) \log\left(-\frac{1-a-bx}{a+bx}\right)}{d^2} + \frac{\sqrt{x} \log\left(\frac{1+a+bx}{a+bx}\right)}{d} - \frac{c \log(c+d\sqrt{x}) \log\left(\frac{1+a+bx}{a+bx}\right)}{d^2} \\
&= \frac{2\sqrt{1+a} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{bd}} - \frac{2\sqrt{1-a} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{bd}} - \frac{\sqrt{x} \log\left(-\frac{1-a-bx}{a+bx}\right)}{d} + \frac{c \log(c+d\sqrt{x}) \log\left(-\frac{1-a-bx}{a+bx}\right)}{d^2} \\
&= \frac{2\sqrt{1+a} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{bd}} - \frac{2\sqrt{1-a} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{bd}} - \frac{\sqrt{x} \log\left(-\frac{1-a-bx}{a+bx}\right)}{d} + \frac{c \log(c+d\sqrt{x}) \log\left(-\frac{1-a-bx}{a+bx}\right)}{d^2} \\
&= \frac{2\sqrt{1+a} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{bd}} - \frac{2\sqrt{1-a} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{bd}} + \frac{c \log\left(\frac{d(\sqrt{-1-a}-\sqrt{b}\sqrt{x})}{\sqrt{bc}+\sqrt{-1-ad}}\right) \log(c+d\sqrt{x})}{d^2} - \frac{c \log\left(\frac{d(\sqrt{-1-a}+\sqrt{b}\sqrt{x})}{\sqrt{bc}+\sqrt{-1-ad}}\right) \log(c+d\sqrt{x})}{d^2} \\
&= \frac{2\sqrt{1+a} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{bd}} - \frac{2\sqrt{1-a} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{bd}} + \frac{c \log\left(\frac{d(\sqrt{-1-a}-\sqrt{b}\sqrt{x})}{\sqrt{bc}+\sqrt{-1-ad}}\right) \log(c+d\sqrt{x})}{d^2} - \frac{c \log\left(\frac{d(\sqrt{-1-a}+\sqrt{b}\sqrt{x})}{\sqrt{bc}+\sqrt{-1-ad}}\right) \log(c+d\sqrt{x})}{d^2} \\
&= \frac{2\sqrt{1+a} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{bd}} - \frac{2\sqrt{1-a} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{bd}} + \frac{c \log\left(\frac{d(\sqrt{-1-a}-\sqrt{b}\sqrt{x})}{\sqrt{bc}+\sqrt{-1-ad}}\right) \log(c+d\sqrt{x})}{d^2} - \frac{c \log\left(\frac{d(\sqrt{-1-a}+\sqrt{b}\sqrt{x})}{\sqrt{bc}+\sqrt{-1-ad}}\right) \log(c+d\sqrt{x})}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.666141, size = 575, normalized size = 0.93

$$c \operatorname{PolyLog}\left(2, \frac{\sqrt{b(c+d\sqrt{x})}}{\sqrt{bc-\sqrt{-a-1}d}}\right) + c \operatorname{PolyLog}\left(2, \frac{\sqrt{b(c+d\sqrt{x})}}{\sqrt{-a-1}d+\sqrt{bc}}\right) - c \operatorname{PolyLog}\left(2, \frac{\sqrt{b(c+d\sqrt{x})}}{\sqrt{bc-\sqrt{-a-1}d}}\right) - c \operatorname{PolyLog}\left(2, \frac{\sqrt{b(c+d\sqrt{x})}}{\sqrt{-a-1}d+\sqrt{bc}}\right) + c \log(c)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a + b\*x]/(c + d\*Sqrt[x]),x]

[Out] 
$$\begin{aligned} & ((2*\operatorname{Sqrt}[1 + a]*d*\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[x])/(\operatorname{Sqrt}[1 + a])])/\operatorname{Sqrt}[b] - (2*\operatorname{Sqrt}[1 - a]*d*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[x])/(\operatorname{Sqrt}[1 - a])])/\operatorname{Sqrt}[b] + c*\operatorname{Log}[(d*(\operatorname{Sqrt}[1 - a] - \operatorname{Sqrt}[b]*\operatorname{Sqrt}[x]))/(\operatorname{Sqrt}[b]*c + \operatorname{Sqrt}[-1 - a]*d)]*\operatorname{Log}[c + d*\operatorname{Sqrt}[x]] - c*\operatorname{Log}[(d*(\operatorname{Sqrt}[1 - a] - \operatorname{Sqrt}[b]*\operatorname{Sqrt}[x]))/(\operatorname{Sqrt}[b]*c + \operatorname{Sqrt}[1 - a]*d)]*\operatorname{Log}[c + d*\operatorname{Sqrt}[x]] + c*\operatorname{Log}[(d*(\operatorname{Sqrt}[-1 - a] + \operatorname{Sqrt}[b]*\operatorname{Sqrt}[x]))/(-(\operatorname{Sqrt}[b]*c) + \operatorname{Sqrt}[-1 - a]*d)]*\operatorname{Log}[c + d*\operatorname{Sqrt}[x]] - c*\operatorname{Log}[(d*(\operatorname{Sqrt}[1 - a] + \operatorname{Sqrt}[b]*\operatorname{Sqrt}[x]))/(-(\operatorname{Sqrt}[b]*c) + \operatorname{Sqrt}[1 - a]*d)]*\operatorname{Log}[c + d*\operatorname{Sqrt}[x]] - d*\operatorname{Sqrt}[x]*\operatorname{Log}[(-1 + a + b*x)/(a + b*x)] + c*\operatorname{Log}[c + d*\operatorname{Sqrt}[x]]*\operatorname{Log}[(-1 + a + b*x)/(a + b*x)] + d*\operatorname{Sqrt}[x]*\operatorname{Log}[(1 + a + b*x)/(a + b*x)] - c*\operatorname{Log}[c + d*\operatorname{Sqrt}[x]]*\operatorname{Log}[(1 + a + b*x)/(a + b*x)] + c*\operatorname{PolyLog}[2, (\operatorname{Sqrt}[b]*(c + d*\operatorname{Sqrt}[x]))/(\operatorname{Sqrt}[b]*c - \operatorname{Sqrt}[-1 - a]*d)] + c*\operatorname{PolyLog}[2, (\operatorname{Sqrt}[b]*(c + d*\operatorname{Sqrt}[x]))/(\operatorname{Sqrt}[b]*c + \operatorname{Sqrt}[-1 - a]*d)] - c*\operatorname{PolyLog}[2, (\operatorname{Sqrt}[b]*(c + d*\operatorname{Sqrt}[x]))/(\operatorname{Sqrt}[b]*c - \operatorname{Sqrt}[1 - a]*d)] - c*\operatorname{PolyLog}[2, (\operatorname{Sqrt}[b]*(c + d*\operatorname{Sqrt}[x]))/(\operatorname{Sqrt}[b]*c + \operatorname{Sqrt}[1 - a]*d)]))/d^2 \end{aligned}$$

**Maple [A]** time = 0.191, size = 738, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(b\*x+a)/(c+d\*x^(1/2)),x)

[Out] 
$$\begin{aligned} & 2*\operatorname{arccoth}(b*x+a)*x^{(1/2)}/d-2*\operatorname{arccoth}(b*x+a)*c/d^2*\ln(c+d*x^{(1/2)})+2/(a*b*d^2+b*d^2)^{(1/2)}*\operatorname{arctan}(1/2*(2*b*(c+d*x^{(1/2)})-2*b*c)/(a*b*d^2+b*d^2)^{(1/2)})+2/(a*b*d^2+b*d^2)^{(1/2)}*\operatorname{arctan}(1/2*(2*b*(c+d*x^{(1/2)})-2*b*c)/(a*b*d^2+b*d^2)^{(1/2)})*a+2/(a*b*d^2-b*d^2)^{(1/2)}*\operatorname{arctan}(1/2*(2*b*(c+d*x^{(1/2)})-2*b*c)/(a*b*d^2-b*d^2)^{(1/2)})-2/(a*b*d^2-b*d^2)^{(1/2)}*\operatorname{arctan}(1/2*(2*b*(c+d*x^{(1/2)})-2*b*c)/(a*b*d^2-b*d^2)^{(1/2)})*a+1/d^2*c*\ln(c+d*x^{(1/2)})*\ln((-b*(c+d*x^{(1/2)})+b*c+(-a*b*d^2-b*d^2)^{(1/2)})/(b*c+(-a*b*d^2-b*d^2)^{(1/2)}))+1/d^2*c*\ln(c+d*x^{(1/2)})*\ln((b*(c+d*x^{(1/2)})-b*c+(-a*b*d^2-b*d^2)^{(1/2)})/(-b*c+(-a*b*d^2-b*d^2)^{(1/2)})) \end{aligned}$$

$$\begin{aligned} & ^2)^{(1/2)))+1/d^2*c*dilog((-b*(c+d*x^{(1/2)}))+b*c+(-a*b*d^2-b*d^2)^{(1/2)})/(b* \\ & c+(-a*b*d^2-b*d^2)^{(1/2)}))+1/d^2*c*dilog((b*(c+d*x^{(1/2)})-b*c+(-a*b*d^2-b*d \\ & ^2)^{(1/2)})/(-b*c+(-a*b*d^2-b*d^2)^{(1/2)}))-1/d^2*c*\ln(c+d*x^{(1/2)})*\ln((-b*(c \\ & +d*x^{(1/2)}))+b*c+(-a*b*d^2+b*d^2)^{(1/2)})/(b*c+(-a*b*d^2+b*d^2)^{(1/2)}))-1/d^2 \\ & *c*\ln(c+d*x^{(1/2)})*\ln((b*(c+d*x^{(1/2)})-b*c+(-a*b*d^2+b*d^2)^{(1/2)})/(-b*c+(- \\ & a*b*d^2+b*d^2)^{(1/2)}))-1/d^2*c*dilog((-b*(c+d*x^{(1/2)}))+b*c+(-a*b*d^2+b*d^2) \\ & ^{(1/2)})/(b*c+(-a*b*d^2+b*d^2)^{(1/2)}))-1/d^2*c*dilog((b*(c+d*x^{(1/2)})-b*c+(- \\ & a*b*d^2+b*d^2)^{(1/2)})/(-b*c+(-a*b*d^2+b*d^2)^{(1/2)})) \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(bx+a)}{d\sqrt{x}+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(c+d\*x^(1/2)),x, algorithm="maxima")

[Out] integrate(arccoth(b\*x + a)/(d\*sqrt(x) + c), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{d\sqrt{x}\operatorname{arccoth}(bx+a)-c\operatorname{arccoth}(bx+a)}{d^2x-c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(c+d\*x^(1/2)),x, algorithm="fricas")

[Out] integral((d\*sqrt(x)\*arccoth(b\*x + a) - c\*arccoth(b\*x + a))/(d^2\*x - c^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)/(c+d*x**(1/2)),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(bx+a)}{d\sqrt{x}+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(c+d*x^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a)/(d*sqrt(x) + c), x)
```

$$3.81 \quad \int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}} dx$$

**Optimal.** Leaf size=738

$$-\frac{d^2 \text{PolyLog}\left(2, -\frac{\sqrt{b}(c\sqrt{x}+d)}{\sqrt{-a-1c-\sqrt{bd}}}\right)}{c^3} + \frac{d^2 \text{PolyLog}\left(2, -\frac{\sqrt{b}(c\sqrt{x}+d)}{\sqrt{1-ac-\sqrt{bd}}}\right)}{c^3} - \frac{d^2 \text{PolyLog}\left(2, \frac{\sqrt{b}(c\sqrt{x}+d)}{\sqrt{-a-1c+\sqrt{bd}}}\right)}{c^3} + \frac{d^2 \text{PolyLog}\left(2, \frac{\sqrt{b}(c\sqrt{x}+d)}{\sqrt{1-ac+\sqrt{bd}}}\right)}{c^3}$$

```
[Out] (-2*Sqrt[1 + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[1 + a]]/(Sqrt[b]*c^2) + (2
*Sqrt[1 - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[1 - a]]/(Sqrt[b]*c^2) - (d^2
*Log[(c*(Sqrt[-1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-1 - a]*c + Sqrt[b]*d)]*Log
[d + c*Sqrt[x]])/c^3 + (d^2*Log[(c*(Sqrt[1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[1
- a]*c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 - (d^2*Log[(c*(Sqrt[-1 - a] +
Sqrt[b]*Sqrt[x]))/(Sqrt[-1 - a]*c - Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 +
(d^2*Log[(c*(Sqrt[1 - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[1 - a]*c - Sqrt[b]*d)]*L
og[d + c*Sqrt[x]])/c^3 + ((1 - a)*Log[1 - a - b*x])/(2*b*c) + (d*Sqrt[x]*Lo
g[-((1 - a - b*x)/(a + b*x))])/c^2 - (x*Log[-((1 - a - b*x)/(a + b*x))])/(2
*c) - (d^2*Log[d + c*Sqrt[x]]*Log[-((1 - a - b*x)/(a + b*x))])/c^3 + ((1 +
a)*Log[1 + a + b*x])/(2*b*c) - (d*Sqrt[x]*Log[(1 + a + b*x)/(a + b*x)])/c^2
+ (x*Log[(1 + a + b*x)/(a + b*x)])/(2*c) + (d^2*Log[d + c*Sqrt[x]]*Log[(1
+ a + b*x)/(a + b*x)])/c^3 - (d^2*PolyLog[2, -((Sqrt[b]*(d + c*Sqrt[x]))/(S
qrt[-1 - a]*c - Sqrt[b]*d))])/c^3 + (d^2*PolyLog[2, -((Sqrt[b]*(d + c*Sqrt[
x]))/(Sqrt[1 - a]*c - Sqrt[b]*d))])/c^3 - (d^2*PolyLog[2, (Sqrt[b]*(d + c*S
qrt[x]))/(Sqrt[-1 - a]*c + Sqrt[b]*d))])/c^3 + (d^2*PolyLog[2, (Sqrt[b]*(d +
c*Sqrt[x]))/(Sqrt[1 - a]*c + Sqrt[b]*d))])/c^3
```

**Rubi [A]** time = 2.37149, antiderivative size = 738, normalized size of antiderivative = 1., number of steps used = 65, number of rules used = 19, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.056$ , Rules used = {6116, 190, 44, 2528, 2523, 12, 481, 205, 2525, 446, 72, 2524, 2418, 260, 2416, 2394, 2393, 2391, 208}

$$-\frac{d^2 \text{PolyLog}\left(2, -\frac{\sqrt{b}(c\sqrt{x}+d)}{\sqrt{-a-1c-\sqrt{bd}}}\right)}{c^3} + \frac{d^2 \text{PolyLog}\left(2, -\frac{\sqrt{b}(c\sqrt{x}+d)}{\sqrt{1-ac-\sqrt{bd}}}\right)}{c^3} - \frac{d^2 \text{PolyLog}\left(2, \frac{\sqrt{b}(c\sqrt{x}+d)}{\sqrt{-a-1c+\sqrt{bd}}}\right)}{c^3} + \frac{d^2 \text{PolyLog}\left(2, \frac{\sqrt{b}(c\sqrt{x}+d)}{\sqrt{1-ac+\sqrt{bd}}}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x]/(c + d/Sqrt[x]), x]
```

```
[Out] (-2*Sqrt[1 + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[1 + a]]/(Sqrt[b]*c^2) + (2
*Sqrt[1 - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[1 - a]]/(Sqrt[b]*c^2) - (d^2
```

$$\begin{aligned} & * \text{Log}[(c * (\text{Sqrt}[-1 - a] - \text{Sqrt}[b] * \text{Sqrt}[x])) / (\text{Sqrt}[-1 - a] * c + \text{Sqrt}[b] * d)] * \text{Log} \\ & [d + c * \text{Sqrt}[x]] / c^3 + (d^2 * \text{Log}[(c * (\text{Sqrt}[1 - a] - \text{Sqrt}[b] * \text{Sqrt}[x])) / (\text{Sqrt}[1 \\ & - a] * c + \text{Sqrt}[b] * d)] * \text{Log}[d + c * \text{Sqrt}[x]] / c^3 - (d^2 * \text{Log}[(c * (\text{Sqrt}[-1 - a] + \\ & \text{Sqrt}[b] * \text{Sqrt}[x])) / (\text{Sqrt}[-1 - a] * c - \text{Sqrt}[b] * d)] * \text{Log}[d + c * \text{Sqrt}[x]] / c^3 + \\ & (d^2 * \text{Log}[(c * (\text{Sqrt}[1 - a] + \text{Sqrt}[b] * \text{Sqrt}[x])) / (\text{Sqrt}[1 - a] * c - \text{Sqrt}[b] * d)] * \text{L} \\ & \text{og}[d + c * \text{Sqrt}[x]] / c^3 + ((1 - a) * \text{Log}[1 - a - b * x]) / (2 * b * c) + (d * \text{Sqrt}[x] * \text{Lo} \\ & \text{g}[-((1 - a - b * x) / (a + b * x))]) / c^2 - (x * \text{Log}[-((1 - a - b * x) / (a + b * x))]) / ( \\ & * c) - (d^2 * \text{Log}[d + c * \text{Sqrt}[x]] * \text{Log}[-((1 - a - b * x) / (a + b * x))]) / c^3 + ((1 + \\ & a) * \text{Log}[1 + a + b * x]) / (2 * b * c) - (d * \text{Sqrt}[x] * \text{Log}[(1 + a + b * x) / (a + b * x)]) / c^2 \\ & + (x * \text{Log}[(1 + a + b * x) / (a + b * x)]) / (2 * c) + (d^2 * \text{Log}[d + c * \text{Sqrt}[x]] * \text{Log}[(1 \\ & + a + b * x) / (a + b * x)]) / c^3 - (d^2 * \text{PolyLog}[2, -((\text{Sqrt}[b] * (d + c * \text{Sqrt}[x])) / (\text{S} \\ & \text{qrt}[-1 - a] * c - \text{Sqrt}[b] * d))]) / c^3 + (d^2 * \text{PolyLog}[2, -((\text{Sqrt}[b] * (d + c * \text{Sqrt}[ \\ & x])) / (\text{Sqrt}[1 - a] * c - \text{Sqrt}[b] * d))]) / c^3 - (d^2 * \text{PolyLog}[2, (\text{Sqrt}[b] * (d + c * \text{S} \\ & \text{qrt}[x])) / (\text{Sqrt}[-1 - a] * c + \text{Sqrt}[b] * d))]) / c^3 + (d^2 * \text{PolyLog}[2, (\text{Sqrt}[b] * (d + c * \text{S} \\ & \text{qrt}[x])) / (\text{Sqrt}[1 - a] * c + \text{Sqrt}[b] * d))]) / c^3 \end{aligned}$$

### Rule 6116

```
Int[ArcCoth[(c_) + (d_)*(x_)]/((e_) + (f_)*(x_)^(n_)), x_Symbol] := Dist
[1/2, Int[Log[(1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] - Dist[1/2, Int[
Log[(-1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x]
&& RationalQ[n]
```

### Rule 190

```
Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
IntegerQ[1/n]
```

### Rule 44

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

### Rule 2528

```
Int[((a_) + Log[(c_)*(RFx_)^(p_)])*(b_)^(n_)*(RGx_), x_Symbol] := With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

### Rule 2523

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] := Simp[x*(a +
  b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[(x*(a + b*Log[c*
  RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, p}, x] && Rat
  ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
  Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

### Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.))),
  x_Symbol] := -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
  x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
  eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
  2*n - 1]
```

### Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
  /b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

### Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*
  (a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d
  , e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
  IntegerQ[m]) && NeQ[m, -1]
```

### Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
  *(c + d*x)^q, x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
  b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

### Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
  x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
  /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFX_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[(Log[d + e*x]*(a + b*Log[c*RFX^p])^n)/e, x] - Dist[(b*n*p)/e, Int[(Log[d + e*x]*(a + b*Log[c*RFX^p])^(n - 1)*D[RFX, x])/RFX, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFX_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFX, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFX, x] && IntegerQ[p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_)^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```



Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/  
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(a+bx)}{c + \frac{d}{\sqrt{x}}} dx &= -\left( \frac{1}{2} \int \frac{\log\left(\frac{-1+a+bx}{a+bx}\right)}{c + \frac{d}{\sqrt{x}}} dx \right) + \frac{1}{2} \int \frac{\log\left(\frac{1+a+bx}{a+bx}\right)}{c + \frac{d}{\sqrt{x}}} dx \\
&= -\text{Subst} \left( \int \frac{x^2 \log\left(\frac{-1+a+bx^2}{a+bx^2}\right)}{d+cx} dx, x, \sqrt{x} \right) + \text{Subst} \left( \int \frac{x^2 \log\left(\frac{1+a+bx^2}{a+bx^2}\right)}{d+cx} dx, x, \sqrt{x} \right) \\
&= -\text{Subst} \left( \int \left( -\frac{d \log\left(\frac{-1+a+bx^2}{a+bx^2}\right)}{c^2} + \frac{x \log\left(\frac{-1+a+bx^2}{a+bx^2}\right)}{c} + \frac{d^2 \log\left(\frac{-1+a+bx^2}{a+bx^2}\right)}{c^2(d+cx)} \right) dx, x, \sqrt{x} \right) + \text{Subst} \left( \int \left( \frac{d \log\left(\frac{1+a+bx^2}{a+bx^2}\right)}{c^2} + \frac{x \log\left(\frac{1+a+bx^2}{a+bx^2}\right)}{c} + \frac{d^2 \log\left(\frac{1+a+bx^2}{a+bx^2}\right)}{c^2(d+cx)} \right) dx, x, \sqrt{x} \right) \\
&= -\frac{\text{Subst} \left( \int x \log\left(\frac{-1+a+bx^2}{a+bx^2}\right) dx, x, \sqrt{x} \right)}{c} + \frac{\text{Subst} \left( \int x \log\left(\frac{1+a+bx^2}{a+bx^2}\right) dx, x, \sqrt{x} \right)}{c} + \frac{d \text{Subst} \left( \int \log\left(\frac{-1+a+bx^2}{a+bx^2}\right) dx, x, \sqrt{x} \right)}{c} \\
&= \frac{d\sqrt{x} \log\left(-\frac{1-a-bx}{a+bx}\right)}{c^2} - \frac{x \log\left(-\frac{1-a-bx}{a+bx}\right)}{2c} - \frac{d^2 \log(d+c\sqrt{x}) \log\left(-\frac{1-a-bx}{a+bx}\right)}{c^3} - \frac{d\sqrt{x} \log\left(\frac{1+a+bx}{a+bx}\right)}{c^2} + \frac{x \log\left(\frac{1+a+bx}{a+bx}\right)}{2c} \\
&= \frac{d\sqrt{x} \log\left(-\frac{1-a-bx}{a+bx}\right)}{c^2} - \frac{x \log\left(-\frac{1-a-bx}{a+bx}\right)}{2c} - \frac{d^2 \log(d+c\sqrt{x}) \log\left(-\frac{1-a-bx}{a+bx}\right)}{c^3} - \frac{d\sqrt{x} \log\left(\frac{1+a+bx}{a+bx}\right)}{c^2} + \frac{x \log\left(\frac{1+a+bx}{a+bx}\right)}{2c} \\
&= \frac{d\sqrt{x} \log\left(-\frac{1-a-bx}{a+bx}\right)}{c^2} - \frac{x \log\left(-\frac{1-a-bx}{a+bx}\right)}{2c} - \frac{d^2 \log(d+c\sqrt{x}) \log\left(-\frac{1-a-bx}{a+bx}\right)}{c^3} - \frac{d\sqrt{x} \log\left(\frac{1+a+bx}{a+bx}\right)}{c^2} + \frac{x \log\left(\frac{1+a+bx}{a+bx}\right)}{2c} \\
&= -\frac{2\sqrt{1+ad} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{bc^2}} + \frac{2\sqrt{1-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{bc^2}} + \frac{d\sqrt{x} \log\left(-\frac{1-a-bx}{a+bx}\right)}{c^2} - \frac{x \log\left(-\frac{1-a-bx}{a+bx}\right)}{2c} \\
&= -\frac{2\sqrt{1+ad} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{bc^2}} + \frac{2\sqrt{1-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{bc^2}} + \frac{(1-a) \log(1-a-bx)}{2bc} + \frac{d\sqrt{x} \log\left(-\frac{1-a-bx}{a+bx}\right)}{c^2} \\
&= -\frac{2\sqrt{1+ad} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{bc^2}} + \frac{2\sqrt{1-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{bc^2}} - \frac{d^2 \log\left(\frac{c(\sqrt{-1-a}-\sqrt{b}\sqrt{x})}{\sqrt{-1-ac}+\sqrt{bd}}\right) \log(d+c\sqrt{x})}{c^3} + \frac{x \log\left(-\frac{1-a-bx}{a+bx}\right)}{2c} \\
&= -\frac{2\sqrt{1+ad} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{bc^2}} + \frac{2\sqrt{1-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{bc^2}} - \frac{d^2 \log\left(\frac{c(\sqrt{-1-a}-\sqrt{b}\sqrt{x})}{\sqrt{-1-ac}+\sqrt{bd}}\right) \log(d+c\sqrt{x})}{c^3} + \frac{x \log\left(-\frac{1-a-bx}{a+bx}\right)}{2c} \\
&= -\frac{2\sqrt{1+ad} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{bc^2}} + \frac{2\sqrt{1-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{bc^2}} - \frac{d^2 \log\left(\frac{c(\sqrt{-1-a}-\sqrt{b}\sqrt{x})}{\sqrt{-1-ac}+\sqrt{bd}}\right) \log(d+c\sqrt{x})}{c^3} + \frac{x \log\left(-\frac{1-a-bx}{a+bx}\right)}{2c}
\end{aligned}$$



2)) / (-b\*d + (-a\*b\*c^2 + b\*c^2)^(1/2)) - 1/c^3\*d^2\*ln(d+c\*x^(1/2))\*ln((-b\*(d+c\*x^(1/2))+b\*d+(-a\*b\*c^2-b\*c^2)^(1/2))/(b\*d+(-a\*b\*c^2-b\*c^2)^(1/2)))-1/c^3\*d^2\*ln(d+c\*x^(1/2))\*ln((b\*(d+c\*x^(1/2))-b\*d+(-a\*b\*c^2-b\*c^2)^(1/2))/(-b\*d+(-a\*b\*c^2-b\*c^2)^(1/2)))-1/c^3\*d^2\*dilog((-b\*(d+c\*x^(1/2))+b\*d+(-a\*b\*c^2-b\*c^2)^(1/2))/(b\*d+(-a\*b\*c^2-b\*c^2)^(1/2)))-1/c^3\*d^2\*diolog((b\*(d+c\*x^(1/2))-b\*d+(-a\*b\*c^2-b\*c^2)^(1/2))/(-b\*d+(-a\*b\*c^2-b\*c^2)^(1/2)))-1/2/b/c\*a\*ln(b\*(d+c\*x^(1/2))^2-2\*(d+c\*x^(1/2))\*b\*d+a\*c^2+b\*d^2-c^2)+2/c\*a\*d/(a\*b\*c^2-b\*c^2)^(1/2)\*arctan(1/2\*(2\*b\*(d+c\*x^(1/2))-2\*b\*d)/(a\*b\*c^2-b\*c^2)^(1/2))+1/2/b/c\*ln(b\*(d+c\*x^(1/2))^2-2\*(d+c\*x^(1/2))\*b\*d+a\*c^2+b\*d^2-c^2)-2/c\*d/(a\*b\*c^2-b\*c^2)^(1/2)\*arctan(1/2\*(2\*b\*(d+c\*x^(1/2))-2\*b\*d)/(a\*b\*c^2-b\*c^2)^(1/2))+1/2/b/c\*ln(b\*(d+c\*x^(1/2))^2-2\*(d+c\*x^(1/2))\*b\*d+a\*c^2+b\*d^2+c^2)-2/c\*d/(a\*b\*c^2+b\*c^2)^(1/2)\*arctan(1/2\*(2\*b\*(d+c\*x^(1/2))-2\*b\*d)/(a\*b\*c^2+b\*c^2)^(1/2))+1/2/b/c\*a\*ln(b\*(d+c\*x^(1/2))^2-2\*(d+c\*x^(1/2))\*b\*d+a\*c^2+b\*d^2+c^2)-2/c\*a\*d/(a\*b\*c^2+b\*c^2)^(1/2)\*arctan(1/2\*(2\*b\*(d+c\*x^(1/2))-2\*b\*d)/(a\*b\*c^2+b\*c^2)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(bx + a + 1) \log(bx + a + 1) - (bx + a - 1) \log(bx + a - 1)}{2bc} - \frac{1}{2} \int \frac{d \log(bx + a + 1) - d \log(bx + a - 1)}{c^2 \sqrt{x} + cd} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(c+d/x^(1/2)),x, algorithm="maxima")

[Out] 1/2\*((b\*x + a + 1)\*log(b\*x + a + 1) - (b\*x + a - 1)\*log(b\*x + a - 1))/(b\*c) - 1/2\*integrate((d\*log(b\*x + a + 1) - d\*log(b\*x + a - 1))/(c^2\*sqrt(x) + c\*d), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{cx \operatorname{arccoth}(bx + a) - d\sqrt{x} \operatorname{arccoth}(bx + a)}{c^2x - d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(c+d/x^(1/2)),x, algorithm="fricas")

[Out] integral((c\*x\*arccoth(b\*x + a) - d\*sqrt(x)\*arccoth(b\*x + a))/(c^2\*x - d^2), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoath(b\*x+a)/(c+d/x\*\*(1/2)),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(bx+a)}{c + \frac{d}{\sqrt{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(c+d/x^(1/2)),x, algorithm="giac")

[Out] integrate(arccoth(b\*x + a)/(c + d/sqrt(x)), x)

### 3.82 $\int \frac{\coth^{-1}(d+ex)}{a+bx+cx^2} dx$

**Optimal.** Leaf size=335

$$\frac{\text{PolyLog}\left(2, \frac{2(-e(b-\sqrt{b^2-4ac})-2c(d+ex)+2cd)}{(d+ex+1)(-e\sqrt{b^2-4ac}+be-2cd+2c)} + 1\right)}{2\sqrt{b^2-4ac}} + \frac{\text{PolyLog}\left(2, \frac{2(-e(\sqrt{b^2-4ac}+b)-2c(d+ex)+2cd)}{(d+ex+1)(e(\sqrt{b^2-4ac}+b)+2c(1-d))} + 1\right)}{2\sqrt{b^2-4ac}} + \frac{\coth^{-1}(d+ex) \log\left(\frac{d+ex+1}{d+ex-1}\right)}{\sqrt{b^2-4ac}}$$

[Out] (ArcCoth[d + e\*x]\*Log[(2\*e\*(b - Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))/((2\*c\*(1 - d) + (b - Sqrt[b^2 - 4\*a\*c])\*e)\*(1 + d + e\*x))])/Sqrt[b^2 - 4\*a\*c] - (ArcCoth[d + e\*x]\*Log[(2\*e\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))/((2\*c\*(1 - d) + (b + Sqrt[b^2 - 4\*a\*c])\*e)\*(1 + d + e\*x))])/Sqrt[b^2 - 4\*a\*c] - PolyLog[2, 1 + (2\*(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e - 2\*c\*(d + e\*x)))/((2\*c - 2\*c\*d + b\*e - Sqrt[b^2 - 4\*a\*c])\*e)\*(1 + d + e\*x))]/(2\*Sqrt[b^2 - 4\*a\*c]) + PolyLog[2, 1 + (2\*(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e - 2\*c\*(d + e\*x)))/((2\*c\*(1 - d) + (b + Sqrt[b^2 - 4\*a\*c])\*e)\*(1 + d + e\*x))]/(2\*Sqrt[b^2 - 4\*a\*c])

**Rubi [A]** time = 0.748577, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {618, 206, 6728, 6112, 5921, 2402, 2315, 2447}

$$\frac{\text{PolyLog}\left(2, \frac{2(-e(b-\sqrt{b^2-4ac})-2c(d+ex)+2cd)}{(d+ex+1)(-e\sqrt{b^2-4ac}+be-2cd+2c)} + 1\right)}{2\sqrt{b^2-4ac}} + \frac{\text{PolyLog}\left(2, \frac{2(-e(\sqrt{b^2-4ac}+b)-2c(d+ex)+2cd)}{(d+ex+1)(e(\sqrt{b^2-4ac}+b)+2c(1-d))} + 1\right)}{2\sqrt{b^2-4ac}} + \frac{\coth^{-1}(d+ex) \log\left(\frac{d+ex+1}{d+ex-1}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[d + e\*x]/(a + b\*x + c\*x^2), x]

[Out] (ArcCoth[d + e\*x]\*Log[(2\*e\*(b - Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))/((2\*c\*(1 - d) + (b - Sqrt[b^2 - 4\*a\*c])\*e)\*(1 + d + e\*x))])/Sqrt[b^2 - 4\*a\*c] - (ArcCoth[d + e\*x]\*Log[(2\*e\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))/((2\*c\*(1 - d) + (b + Sqrt[b^2 - 4\*a\*c])\*e)\*(1 + d + e\*x))])/Sqrt[b^2 - 4\*a\*c] - PolyLog[2, 1 + (2\*(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e - 2\*c\*(d + e\*x)))/((2\*c - 2\*c\*d + b\*e - Sqrt[b^2 - 4\*a\*c])\*e)\*(1 + d + e\*x))]/(2\*Sqrt[b^2 - 4\*a\*c]) + PolyLog[2, 1 + (2\*(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e - 2\*c\*(d + e\*x)))/((2\*c\*(1 - d) + (b + Sqrt[b^2 - 4\*a\*c])\*e)\*(1 + d + e\*x))]/(2\*Sqrt[b^2 - 4\*a\*c])

Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 6728

Int[(u\_)/((a\_.) + (b\_.)\*(x\_)^(n\_.) + (c\_.)\*(x\_)^(n2\_.)), x\_Symbol] := With[{v = RationalFunctionExpand[u/(a + b\*x^n + c\*x^(2\*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2\*n] && IGtQ[n, 0]

### Rule 6112

Int[((a\_.) + ArcCoth[(c\_) + (d\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]

### Rule 5921

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[((a + b\*ArcCoth[c\*x])\*Log[2/(1 + c\*x)])/e, x] + (Dist[(b\*c)/e, Int[Log[2/(1 + c\*x)]/(1 - c^2\*x^2), x], x] - Dist[(b\*c)/e, Int[Log[(2\*c\*(d + e\*x))/(c\*d + e)\*(1 + c\*x)]/(1 - c^2\*x^2), x], x] + Simp[((a + b\*ArcCoth[c\*x])\*Log[(2\*c\*(d + e\*x))/(c\*d + e)\*(1 + c\*x)])/e, x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2\*d^2 - e^2, 0]

### Rule 2402

Int[Log[(c\_.)/((d\_) + (e\_.)\*(x\_))]/((f\_) + (g\_.)\*(x\_)^2), x\_Symbol] := -Dist[e/g, Subst[Int[Log[2\*d\*x]/(1 - 2\*d\*x), x], x, 1/(d + e\*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2\*d] && EqQ[e^2\*f + d^2\*g, 0]

### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

### Rule 2447

```
Int[Log[u_]*(Pq_)^(m_), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(d+ex)}{a+bx+cx^2} dx &= \int \left( \frac{2c \coth^{-1}(d+ex)}{\sqrt{b^2-4ac}(b-\sqrt{b^2-4ac}+2cx)} - \frac{2c \coth^{-1}(d+ex)}{\sqrt{b^2-4ac}(b+\sqrt{b^2-4ac}+2cx)} \right) dx \\
&= \frac{(2c) \int \frac{\coth^{-1}(d+ex)}{b-\sqrt{b^2-4ac}+2cx} dx}{\sqrt{b^2-4ac}} - \frac{(2c) \int \frac{\coth^{-1}(d+ex)}{b+\sqrt{b^2-4ac}+2cx} dx}{\sqrt{b^2-4ac}} \\
&= \frac{(2c) \text{Subst} \left( \int \frac{\coth^{-1}(x)}{\frac{-2cd+(b-\sqrt{b^2-4ac})e}{e} + \frac{2cx}{e}} dx, x, d+ex \right)}{\sqrt{b^2-4ac}} - \frac{(2c) \text{Subst} \left( \int \frac{\coth^{-1}(x)}{\frac{-2cd+(b+\sqrt{b^2-4ac})e}{e} + \frac{2cx}{e}} dx, x, d+ex \right)}{\sqrt{b^2-4ac}} \\
&= \frac{\coth^{-1}(d+ex) \log \left( \frac{2e(b-\sqrt{b^2-4ac}+2cx)}{(2c-2cd+be-\sqrt{b^2-4ac})(1+d+ex)} \right)}{\sqrt{b^2-4ac}} - \frac{\coth^{-1}(d+ex) \log \left( \frac{2e(b+\sqrt{b^2-4ac}+2cx)}{(2c(1-d)+(b+\sqrt{b^2-4ac})e)(1+d+ex)} \right)}{\sqrt{b^2-4ac}} \\
&= \frac{\coth^{-1}(d+ex) \log \left( \frac{2e(b-\sqrt{b^2-4ac}+2cx)}{(2c-2cd+be-\sqrt{b^2-4ac})(1+d+ex)} \right)}{\sqrt{b^2-4ac}} - \frac{\coth^{-1}(d+ex) \log \left( \frac{2e(b+\sqrt{b^2-4ac}+2cx)}{(2c(1-d)+(b+\sqrt{b^2-4ac})e)(1+d+ex)} \right)}{\sqrt{b^2-4ac}}
\end{aligned}$$

**Mathematica [A]** time = 0.898204, size = 596, normalized size = 1.78

$$\frac{-\text{PolyLog} \left( 2, \frac{e(\sqrt{b^2-4ac}-b-2cx)}{e(\sqrt{b^2-4ac}-b)+2c(d+1)} \right) + \text{PolyLog} \left( 2, \frac{e(-\sqrt{b^2-4ac}+b+2cx)}{-e\sqrt{b^2-4ac}+be-2cd+2c} \right) - \text{PolyLog} \left( 2, \frac{e(\sqrt{b^2-4ac}+b+2cx)}{e(\sqrt{b^2-4ac}+b)-2c(d-1)} \right) + \text{PolyLog} \left( 2, \frac{e(-\sqrt{b^2-4ac}-b-2cx)}{-e\sqrt{b^2-4ac}-be-2cd-2c} \right)}{\sqrt{b^2-4ac}}$$

Warning: Unable to verify antiderivative.



```
[In] Integrate[ArcCoth[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] (Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[(2*c*(-1 + d + e*x))/(2*c*(-1 + d) + (-b + Sqrt[b^2 - 4*a*c])*e)] - Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[(2*c*(-1 + d + e*x))/(2*c*(-1 + d) - (b + Sqrt[b^2 - 4*a*c])*e)] - Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[(-1 + d + e*x)/(d + e*x)] + Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[(-1 + d + e*x)/(d + e*x)] - Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[(2*c*(1 + d + e*x))/(2*c*(1 + d) + (-b + Sqrt[b^2 - 4*a*c])*e)] + Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[(2*c*(1 + d + e*x))/(2*c*(1 + d) - (b + Sqrt[b^2 - 4*a*c])*e)] + Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[(1 + d + e*x)/(d + e*x)] - Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[(1 + d + e*x)/(d + e*x)] - PolyLog[2, (e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*(1 + d) + (-b + Sqrt[b^2 - 4*a*c])*e)] + PolyLog[2, (e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c - 2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e)] - PolyLog[2, (e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*(-1 + d) + (b + Sqrt[b^2 - 4*a*c])*e)] + PolyLog[2, (e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*(1 + d) + (b + Sqrt[b^2 - 4*a*c])*e)]/(2*Sqrt[b^2 - 4*a*c])
```

**Maple [B]** time = 0.586, size = 2098, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(e*x+d)/(c*x^2+b*x+a),x)
```

```
[Out] 1/e*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*arccoth(e*x+d)*ln(1-(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)*(e*x+d+1)/(e*x+d-1)/(a*e^2-b*e*d+c*d^2+(-e^2*(4*a*c-b^2))^(1/2)-c))-1/e*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*arccoth(e*x+d)^2+1/2/e*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*polylog(2,(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)*(e*x+d+1)/(e*x+d-1)/(a*e^2-b*e*d+c*d^2+(-e^2*(4*a*c-b^2))^(1/2)-c))-e*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*ln(1-(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)*(e*x+d+1)/(e*x+d-1)/(a*e^2-b*e*d+c*d^2+(-e^2*(4*a*c-b^2))^(1/2)-c))*arccoth(e*x+d)*a/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)+(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*ln(1-(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)*(e*x+d+1)/(e*x+d-1)/(a*e^2-b*e*d+c*d^2+(-e^2*(4*a*c-b^2))^(1/2)-c))*arccoth(e*x+d)*b*d/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)-1/e*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*ln(1-(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)*(e*x+d+1)/(e*x+d-1)/(a*e^2-b*e*d+c*d^2+(-e^2*(4*a*c-b^2))^(1/2)-c))*arccoth(e*x+d)*c*d^2/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)+e*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*arccoth(e*x+d)^2*a/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)-(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*arccoth(e*x+d)^2*b*d/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)
```

$$\begin{aligned} &))^{(1/2)-c} + 1/e*(e^{2*(-4*a*c+b^2)})^{(1/2)}/(4*a*c-b^2)*\operatorname{arccoth}(e*x+d)^2*c*d^2 \\ &/ (a*e^{2-b*e*d+c*d^2-(e^{2*(-4*a*c+b^2)})^{(1/2)-c}} - 1/2*e*(e^{2*(-4*a*c+b^2)})^{(1/2)}/(4*a*c-b^2)*\operatorname{polylog}(2, (a*e^{2-b*d*e+c*d^2+b*e-2*c*d+c})*(e*x+d+1)/(e*x+d-1)/(a*e^{2-b*e*d+c*d^2-(e^{2*(4*a*c-b^2)})^{(1/2)-c}})*a/(a*e^{2-b*e*d+c*d^2-(e^{2*(-4*a*c+b^2)})^{(1/2)-c}}) + 1/2*(e^{2*(-4*a*c+b^2)})^{(1/2)}/(4*a*c-b^2)*\operatorname{polylog}(2, (a*e^{2-b*d*e+c*d^2+b*e-2*c*d+c})*(e*x+d+1)/(e*x+d-1)/(a*e^{2-b*e*d+c*d^2-(e^{2*(4*a*c-b^2)})^{(1/2)-c}})*b*d/(a*e^{2-b*e*d+c*d^2-(e^{2*(-4*a*c+b^2)})^{(1/2)-c}}) - 1/2/e*(e^{2*(-4*a*c+b^2)})^{(1/2)}/(4*a*c-b^2)*\operatorname{polylog}(2, (a*e^{2-b*d*e+c*d^2+b*e-2*c*d+c})*(e*x+d+1)/(e*x+d-1)/(a*e^{2-b*e*d+c*d^2-(e^{2*(4*a*c-b^2)})^{(1/2)-c}})*c*d^2/(a*e^{2-b*e*d+c*d^2-(e^{2*(-4*a*c+b^2)})^{(1/2)-c}}) - e/(a*e^{2-b*e*d+c*d^2-(e^{2*(-4*a*c+b^2)})^{(1/2)-c}})*\ln(1-(a*e^{2-b*d*e+c*d^2+b*e-2*c*d+c})*(e*x+d+1)/(e*x+d-1)/(a*e^{2-b*e*d+c*d^2-(e^{2*(4*a*c-b^2)})^{(1/2)-c}})*\operatorname{arccoth}(e*x+d)+1/e*(e^{2*(-4*a*c+b^2)})^{(1/2)}/(4*a*c-b^2)*\ln(1-(a*e^{2-b*d*e+c*d^2+b*e-2*c*d+c})*(e*x+d+1)/(e*x+d-1)/(a*e^{2-b*e*d+c*d^2-(e^{2*(4*a*c-b^2)})^{(1/2)-c}})*\operatorname{arccoth}(e*x+d)*c/(a*e^{2-b*e*d+c*d^2-(e^{2*(-4*a*c+b^2)})^{(1/2)-c}}) + e/(a*e^{2-b*e*d+c*d^2-(e^{2*(-4*a*c+b^2)})^{(1/2)-c}})*\operatorname{arccoth}(e*x+d)^2 - 1/e*(e^{2*(-4*a*c+b^2)})^{(1/2)}/(4*a*c-b^2)*\operatorname{arccoth}(e*x+d)^2*c/(a*e^{2-b*e*d+c*d^2-(e^{2*(-4*a*c+b^2)})^{(1/2)-c}}) - 1/2*e/(a*e^{2-b*e*d+c*d^2-(e^{2*(-4*a*c+b^2)})^{(1/2)-c}})*\operatorname{polylog}(2, (a*e^{2-b*d*e+c*d^2+b*e-2*c*d+c})*(e*x+d+1)/(e*x+d-1)/(a*e^{2-b*e*d+c*d^2-(e^{2*(4*a*c-b^2)})^{(1/2)-c}}) + 1/2/e*(e^{2*(-4*a*c+b^2)})^{(1/2)}/(4*a*c-b^2)*\operatorname{polylog}(2, (a*e^{2-b*d*e+c*d^2+b*e-2*c*d+c})*(e*x+d+1)/(e*x+d-1)/(a*e^{2-b*e*d+c*d^2-(e^{2*(4*a*c-b^2)})^{(1/2)-c}})*c/(a*e^{2-b*e*d+c*d^2-(e^{2*(-4*a*c+b^2)})^{(1/2)-c}}) \end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(e\*x+d)/(c\*x^2+b\*x+a),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arccoth}(ex+d)}{cx^2+bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral(arccoth(e*x + d)/(c*x^2 + b*x + a), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(e*x+d)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(ex + d)}{cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(e*x+d)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate(arccoth(e*x + d)/(c*x^2 + b*x + a), x)
```

### 3.83 $\int x^2 \coth^{-1}(\sqrt{x}) dx$

**Optimal.** Leaf size=51

$$\frac{x^{5/2}}{15} + \frac{x^{3/2}}{9} + \frac{1}{3}x^3 \coth^{-1}(\sqrt{x}) + \frac{\sqrt{x}}{3} - \frac{1}{3} \tanh^{-1}(\sqrt{x})$$

[Out] Sqrt[x]/3 + x^(3/2)/9 + x^(5/2)/15 + (x^3\*ArcCoth[Sqrt[x]])/3 - ArcTanh[Sqrt[x]]/3

**Rubi [A]** time = 0.0147834, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {6098, 50, 63, 206}

$$\frac{x^{5/2}}{15} + \frac{x^{3/2}}{9} + \frac{1}{3}x^3 \coth^{-1}(\sqrt{x}) + \frac{\sqrt{x}}{3} - \frac{1}{3} \tanh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcCoth[Sqrt[x]], x]

[Out] Sqrt[x]/3 + x^(3/2)/9 + x^(5/2)/15 + (x^3\*ArcCoth[Sqrt[x]])/3 - ArcTanh[Sqrt[x]]/3

#### Rule 6098

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)])*(b_.)*((d_.)*(x_)^(m_.)), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 50

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 206

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(\sqrt{x}) dx &= \frac{1}{3}x^3 \coth^{-1}(\sqrt{x}) - \frac{1}{6} \int \frac{x^{5/2}}{1-x} dx \\
&= \frac{x^{5/2}}{15} + \frac{1}{3}x^3 \coth^{-1}(\sqrt{x}) - \frac{1}{6} \int \frac{x^{3/2}}{1-x} dx \\
&= \frac{x^{3/2}}{9} + \frac{x^{5/2}}{15} + \frac{1}{3}x^3 \coth^{-1}(\sqrt{x}) - \frac{1}{6} \int \frac{\sqrt{x}}{1-x} dx \\
&= \frac{\sqrt{x}}{3} + \frac{x^{3/2}}{9} + \frac{x^{5/2}}{15} + \frac{1}{3}x^3 \coth^{-1}(\sqrt{x}) - \frac{1}{6} \int \frac{1}{(1-x)\sqrt{x}} dx \\
&= \frac{\sqrt{x}}{3} + \frac{x^{3/2}}{9} + \frac{x^{5/2}}{15} + \frac{1}{3}x^3 \coth^{-1}(\sqrt{x}) - \frac{1}{3} \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{x}\right) \\
&= \frac{\sqrt{x}}{3} + \frac{x^{3/2}}{9} + \frac{x^{5/2}}{15} + \frac{1}{3}x^3 \coth^{-1}(\sqrt{x}) - \frac{1}{3} \tanh^{-1}(\sqrt{x})
\end{aligned}$$

**Mathematica [A]** time = 0.0175619, size = 59, normalized size = 1.16

$$\frac{1}{90} \left( 6x^{5/2} + 10x^{3/2} + 30x^3 \coth^{-1}(\sqrt{x}) + 30\sqrt{x} + 15 \log(1 - \sqrt{x}) - 15 \log(\sqrt{x} + 1) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[Sqrt[x]], x]
```

```
[Out] (30*Sqrt[x] + 10*x^(3/2) + 6*x^(5/2) + 30*x^3*ArcCoth[Sqrt[x]] + 15*Log[1 -
Sqrt[x]] - 15*Log[1 + Sqrt[x]])/90
```

**Maple [A]** time = 0.036, size = 42, normalized size = 0.8

$$\frac{x^3}{3} \operatorname{arccoth}(\sqrt{x}) + \frac{1}{15} x^{\frac{5}{2}} + \frac{1}{9} x^{\frac{3}{2}} + \frac{1}{3} \sqrt{x} + \frac{1}{6} \ln(-1 + \sqrt{x}) - \frac{1}{6} \ln(1 + \sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*arccoth(x^(1/2)),x)`

[Out] `1/3*x^3*arccoth(x^(1/2))+1/15*x^(5/2)+1/9*x^(3/2)+1/3*x^(1/2)+1/6*ln(-1+x^(1/2))-1/6*ln(1+x^(1/2))`

**Maxima [A]** time = 0.961345, size = 55, normalized size = 1.08

$$\frac{1}{3} x^3 \operatorname{arccoth}(\sqrt{x}) + \frac{1}{15} x^{\frac{5}{2}} + \frac{1}{9} x^{\frac{3}{2}} + \frac{1}{3} \sqrt{x} - \frac{1}{6} \log(\sqrt{x} + 1) + \frac{1}{6} \log(\sqrt{x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccoth(x^(1/2)),x, algorithm="maxima")`

[Out] `1/3*x^3*arccoth(sqrt(x)) + 1/15*x^(5/2) + 1/9*x^(3/2) + 1/3*sqrt(x) - 1/6*log(sqrt(x) + 1) + 1/6*log(sqrt(x) - 1)`

**Fricas [A]** time = 1.60583, size = 111, normalized size = 2.18

$$\frac{1}{6} (x^3 - 1) \log\left(\frac{x + 2\sqrt{x} + 1}{x - 1}\right) + \frac{1}{45} (3x^2 + 5x + 15)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccoth(x^(1/2)),x, algorithm="fricas")`

[Out] `1/6*(x^3 - 1)*log((x + 2*sqrt(x) + 1)/(x - 1)) + 1/45*(3*x^2 + 5*x + 15)*sqrt(x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{acoth}(\sqrt{x}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(x**(1/2)),x)
```

```
[Out] Integral(x**2*acoth(sqrt(x)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arccoth}(\sqrt{x}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(x^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(sqrt(x)), x)
```

### 3.84 $\int x \coth^{-1}(\sqrt{x}) dx$

**Optimal.** Leaf size=42

$$\frac{x^{3/2}}{6} + \frac{1}{2}x^2 \coth^{-1}(\sqrt{x}) + \frac{\sqrt{x}}{2} - \frac{1}{2} \tanh^{-1}(\sqrt{x})$$

[Out] Sqrt[x]/2 + x^(3/2)/6 + (x^2\*ArcCoth[Sqrt[x]])/2 - ArcTanh[Sqrt[x]]/2

**Rubi [A]** time = 0.0107724, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {6098, 50, 63, 206}

$$\frac{x^{3/2}}{6} + \frac{1}{2}x^2 \coth^{-1}(\sqrt{x}) + \frac{\sqrt{x}}{2} - \frac{1}{2} \tanh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[x\*ArcCoth[Sqrt[x]], x]

[Out] Sqrt[x]/2 + x^(3/2)/6 + (x^2\*ArcCoth[Sqrt[x]])/2 - ArcTanh[Sqrt[x]]/2

#### Rule 6098

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)])*(b_.)*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 50

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 63

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```



```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rubi steps

$$\begin{aligned}
 \int x \coth^{-1}(\sqrt{x}) dx &= \frac{1}{2}x^2 \coth^{-1}(\sqrt{x}) - \frac{1}{4} \int \frac{x^{3/2}}{1-x} dx \\
 &= \frac{x^{3/2}}{6} + \frac{1}{2}x^2 \coth^{-1}(\sqrt{x}) - \frac{1}{4} \int \frac{\sqrt{x}}{1-x} dx \\
 &= \frac{\sqrt{x}}{2} + \frac{x^{3/2}}{6} + \frac{1}{2}x^2 \coth^{-1}(\sqrt{x}) - \frac{1}{4} \int \frac{1}{(1-x)\sqrt{x}} dx \\
 &= \frac{\sqrt{x}}{2} + \frac{x^{3/2}}{6} + \frac{1}{2}x^2 \coth^{-1}(\sqrt{x}) - \frac{1}{2} \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{x}\right) \\
 &= \frac{\sqrt{x}}{2} + \frac{x^{3/2}}{6} + \frac{1}{2}x^2 \coth^{-1}(\sqrt{x}) - \frac{1}{2} \tanh^{-1}(\sqrt{x})
 \end{aligned}$$

**Mathematica [A]** time = 0.0145469, size = 52, normalized size = 1.24

$$\frac{1}{12} (2x^{3/2} + 6x^2 \coth^{-1}(\sqrt{x}) + 6\sqrt{x} + 3 \log(1 - \sqrt{x}) - 3 \log(\sqrt{x} + 1))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[Sqrt[x]], x]
```

```
[Out] (6*Sqrt[x] + 2*x^(3/2) + 6*x^2*ArcCoth[Sqrt[x]] + 3*Log[1 - Sqrt[x]] - 3*Lo
g[1 + Sqrt[x]])/12
```

**Maple [A]** time = 0.033, size = 37, normalized size = 0.9

$$\frac{x^2}{2} \operatorname{arccoth}(\sqrt{x}) + \frac{1}{6}x^{3/2} + \frac{1}{2}\sqrt{x} + \frac{1}{4} \ln(-1 + \sqrt{x}) - \frac{1}{4} \ln(1 + \sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*arccoth(x^(1/2)),x)`

[Out]  $\frac{1}{2}x^2 \operatorname{arccoth}(x^{1/2}) + \frac{1}{6}x^{3/2} + \frac{1}{2}x^{1/2} + \frac{1}{4} \ln(-1+x^{1/2}) - \frac{1}{4} \ln(1+x^{1/2})$

**Maxima [A]** time = 0.957906, size = 49, normalized size = 1.17

$$\frac{1}{2}x^2 \operatorname{arccoth}(\sqrt{x}) + \frac{1}{6}x^{3/2} + \frac{1}{2}\sqrt{x} - \frac{1}{4} \log(\sqrt{x} + 1) + \frac{1}{4} \log(\sqrt{x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccoth(x^(1/2)),x, algorithm="maxima")`

[Out]  $\frac{1}{2}x^2 \operatorname{arccoth}(\sqrt{x}) + \frac{1}{6}x^{3/2} + \frac{1}{2}\sqrt{x} - \frac{1}{4} \log(\sqrt{x} + 1) + \frac{1}{4} \log(\sqrt{x} - 1)$

**Fricas [A]** time = 1.53167, size = 95, normalized size = 2.26

$$\frac{1}{4}(x^2 - 1) \log\left(\frac{x + 2\sqrt{x} + 1}{x - 1}\right) + \frac{1}{6}(x + 3)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccoth(x^(1/2)),x, algorithm="fricas")`

[Out]  $\frac{1}{4}(x^2 - 1) \log((x + 2\sqrt{x} + 1)/(x - 1)) + \frac{1}{6}(x + 3)\sqrt{x}$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{acoth}(\sqrt{x}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*acoth(x**(1/2)),x)`

[Out] `Integral(x*acoth(sqrt(x)), x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arccoth}(\sqrt{x}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccoth(x^(1/2)),x, algorithm="giac")`

[Out] `integrate(x*arccoth(sqrt(x)), x)`

### 3.85 $\int \coth^{-1}(\sqrt{x}) dx$

**Optimal.** Leaf size=22

$$\sqrt{x} - \tanh^{-1}(\sqrt{x}) + x \coth^{-1}(\sqrt{x})$$

[Out] Sqrt[x] + x\*ArcCoth[Sqrt[x]] - ArcTanh[Sqrt[x]]

**Rubi [A]** time = 0.006253, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {6092, 50, 63, 206}

$$\sqrt{x} - \tanh^{-1}(\sqrt{x}) + x \coth^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Sqrt[x]], x]

[Out] Sqrt[x] + x\*ArcCoth[Sqrt[x]] - ArcTanh[Sqrt[x]]

#### Rule 6092

Int[ArcCoth[(c\_.)\*(x\_)^(n\_)], x\_Symbol] :> Simp[x\*ArcCoth[c\*x^n], x] - Dist[c\*n, Int[x^n/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{c, n}, x]

#### Rule 50

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[((a + b\*x)^(m + 1)\*(c + d\*x)^n)/(b\*(m + n + 1)), x] + Dist[(n\*(b\*c - a\*d))/(b\*(m + n + 1)), Int[(a + b\*x)^m\*(c + d\*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}\int \coth^{-1}(\sqrt{x}) dx &= x \coth^{-1}(\sqrt{x}) - \frac{1}{2} \int \frac{\sqrt{x}}{1-x} dx \\ &= \sqrt{x} + x \coth^{-1}(\sqrt{x}) - \frac{1}{2} \int \frac{1}{(1-x)\sqrt{x}} dx \\ &= \sqrt{x} + x \coth^{-1}(\sqrt{x}) - \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{x}\right) \\ &= \sqrt{x} + x \coth^{-1}(\sqrt{x}) - \tanh^{-1}(\sqrt{x})\end{aligned}$$

**Mathematica [A]** time = 0.0059219, size = 22, normalized size = 1.

$$\sqrt{x} - \tanh^{-1}(\sqrt{x}) + x \coth^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Sqrt[x]], x]

[Out] Sqrt[x] + x\*ArcCoth[Sqrt[x]] - ArcTanh[Sqrt[x]]

**Maple [A]** time = 0.034, size = 27, normalized size = 1.2

$$x \operatorname{arccoth}(\sqrt{x}) + \sqrt{x} + \frac{1}{2} \ln(-1 + \sqrt{x}) - \frac{1}{2} \ln(1 + \sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(x^(1/2)), x)

[Out] x\*arccoth(x^(1/2))+x^(1/2)+1/2\*ln(-1+x^(1/2))-1/2\*ln(1+x^(1/2))

**Maxima [A]** time = 0.951836, size = 35, normalized size = 1.59

$$x \operatorname{arccoth}(\sqrt{x}) + \sqrt{x} - \frac{1}{2} \log(\sqrt{x} + 1) + \frac{1}{2} \log(\sqrt{x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x^(1/2)),x, algorithm="maxima")

[Out] x\*arccoth(sqrt(x)) + sqrt(x) - 1/2\*log(sqrt(x) + 1) + 1/2\*log(sqrt(x) - 1)

**Fricas [A]** time = 1.63877, size = 76, normalized size = 3.45

$$\frac{1}{2}(x-1) \log\left(\frac{x+2\sqrt{x}+1}{x-1}\right) + \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x^(1/2)),x, algorithm="fricas")

[Out] 1/2\*(x - 1)\*log((x + 2\*sqrt(x) + 1)/(x - 1)) + sqrt(x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acoth}(\sqrt{x}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(x\*\*(1/2)),x)

[Out] Integral(acoth(sqrt(x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(\sqrt{x}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(sqrt(x)), x)
```

$$3.86 \quad \int \frac{\coth^{-1}(\sqrt{x})}{x} dx$$

**Optimal.** Leaf size=19

$$\text{PolyLog}\left(2, -\frac{1}{\sqrt{x}}\right) - \text{PolyLog}\left(2, \frac{1}{\sqrt{x}}\right)$$

[Out] PolyLog[2, -(1/Sqrt[x])] - PolyLog[2, 1/Sqrt[x]]

**Rubi [A]** time = 0.0197034, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {6096, 5913}

$$\text{PolyLog}\left(2, -\frac{1}{\sqrt{x}}\right) - \text{PolyLog}\left(2, \frac{1}{\sqrt{x}}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Sqrt[x]]/x, x]

[Out] PolyLog[2, -(1/Sqrt[x])] - PolyLog[2, 1/Sqrt[x]]

#### Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)])*(b_.)^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcCoth[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

#### Rule 5913

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.)/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b*PolyLog[2, -(c*x)^(-1)])/2, x] - Simp[(b*PolyLog[2, 1/(c*x)])/
2, x]) /; FreeQ[{a, b, c}, x]
```

#### Rubi steps



$$\int \frac{\coth^{-1}(\sqrt{x})}{x} dx = 2 \operatorname{Subst} \left( \int \frac{\coth^{-1}(x)}{x} dx, x, \sqrt{x} \right) \\ = \operatorname{Li}_2 \left( -\frac{1}{\sqrt{x}} \right) - \operatorname{Li}_2 \left( \frac{1}{\sqrt{x}} \right)$$

**Mathematica [A]** time = 0.0101691, size = 19, normalized size = 1.

$$\operatorname{PolyLog} \left( 2, -\frac{1}{\sqrt{x}} \right) - \operatorname{PolyLog} \left( 2, \frac{1}{\sqrt{x}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Sqrt[x]]/x,x]

[Out] PolyLog[2, -(1/Sqrt[x])] - PolyLog[2, 1/Sqrt[x]]

**Maple [B]** time = 0.054, size = 33, normalized size = 1.7

$$\ln(x) \operatorname{arccoth}(\sqrt{x}) - \operatorname{dilog}(\sqrt{x}) - \operatorname{dilog}(1 + \sqrt{x}) - \frac{\ln(x)}{2} \ln(1 + \sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(x^(1/2))/x,x)

[Out] ln(x)\*arccoth(x^(1/2))-dilog(x^(1/2))-dilog(1+x^(1/2))-1/2\*ln(x)\*ln(1+x^(1/2))

**Maxima [B]** time = 0.959881, size = 89, normalized size = 4.68

$$-\frac{1}{2} \left( \log(\sqrt{x} + 1) - \log(\sqrt{x} - 1) \right) \log(x) + \operatorname{arccoth}(\sqrt{x}) \log(x) + \log(-\sqrt{x}) \log(\sqrt{x} + 1) - \frac{1}{2} \log(x) \log(\sqrt{x} - 1) + L$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x^(1/2))/x,x, algorithm="maxima")

```
[Out] -1/2*(log(sqrt(x) + 1) - log(sqrt(x) - 1))*log(x) + arccoth(sqrt(x))*log(x)
+ log(-sqrt(x))*log(sqrt(x) + 1) - 1/2*log(x)*log(sqrt(x) - 1) + dilog(sqrt
t(x) + 1) - dilog(-sqrt(x) + 1)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arccoth}(\sqrt{x})}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(sqrt(x))/x, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{acoth}(\sqrt{x})}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x**(1/2))/x,x)
```

```
[Out] Integral(acoth(sqrt(x))/x, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arccoth}(\sqrt{x})}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(sqrt(x))/x, x)
```

$$3.87 \quad \int \frac{\coth^{-1}(\sqrt{x})}{x^2} dx$$

**Optimal.** Leaf size=25

$$-\frac{1}{\sqrt{x}} + \tanh^{-1}(\sqrt{x}) - \frac{\coth^{-1}(\sqrt{x})}{x}$$

[Out]  $-(1/\text{Sqrt}[x]) - \text{ArcCoth}[\text{Sqrt}[x]]/x + \text{ArcTanh}[\text{Sqrt}[x]]$

**Rubi [A]** time = 0.013518, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {6098, 51, 63, 206}

$$-\frac{1}{\sqrt{x}} + \tanh^{-1}(\sqrt{x}) - \frac{\coth^{-1}(\sqrt{x})}{x}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{ArcCoth}[\text{Sqrt}[x]]/x^2, x]$

[Out]  $-(1/\text{Sqrt}[x]) - \text{ArcCoth}[\text{Sqrt}[x]]/x + \text{ArcTanh}[\text{Sqrt}[x]]$

### Rule 6098

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)])*(b_.)*((d_.)*(x_)^(m_.)), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

### Rule 51

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol]
:> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x]
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

### Rule 63

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/
Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(\sqrt{x})}{x^2} dx &= -\frac{\coth^{-1}(\sqrt{x})}{x} + \frac{1}{2} \int \frac{1}{(1-x)x^{3/2}} dx \\ &= -\frac{1}{\sqrt{x}} - \frac{\coth^{-1}(\sqrt{x})}{x} + \frac{1}{2} \int \frac{1}{(1-x)\sqrt{x}} dx \\ &= -\frac{1}{\sqrt{x}} - \frac{\coth^{-1}(\sqrt{x})}{x} + \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{x}\right) \\ &= -\frac{1}{\sqrt{x}} - \frac{\coth^{-1}(\sqrt{x})}{x} + \tanh^{-1}(\sqrt{x}) \end{aligned}$$

**Mathematica [A]** time = 0.0240027, size = 45, normalized size = 1.8

$$-\frac{1}{\sqrt{x}} - \frac{1}{2} \log(1 - \sqrt{x}) + \frac{1}{2} \log(\sqrt{x} + 1) - \frac{\coth^{-1}(\sqrt{x})}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Sqrt[x]]/x^2, x]
```

```
[Out] -(1/Sqrt[x]) - ArcCoth[Sqrt[x]]/x - Log[1 - Sqrt[x]]/2 + Log[1 + Sqrt[x]]/2
```

**Maple [A]** time = 0.038, size = 32, normalized size = 1.3

$$-\frac{1}{x} \operatorname{arccoth}(\sqrt{x}) - \frac{1}{\sqrt{x}} - \frac{1}{2} \ln(-1 + \sqrt{x}) + \frac{1}{2} \ln(1 + \sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(x^(1/2))/x^2,x)`

[Out] `-arccoth(x^(1/2))/x-1/x^(1/2)-1/2*ln(-1+x^(1/2))+1/2*ln(1+x^(1/2))`

**Maxima [A]** time = 0.969192, size = 42, normalized size = 1.68

$$-\frac{\operatorname{arccoth}(\sqrt{x})}{x} - \frac{1}{\sqrt{x}} + \frac{1}{2} \log(\sqrt{x} + 1) - \frac{1}{2} \log(\sqrt{x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(x^(1/2))/x^2,x, algorithm="maxima")`

[Out] `-arccoth(sqrt(x))/x - 1/sqrt(x) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)`

**Fricas [A]** time = 1.57107, size = 84, normalized size = 3.36

$$\frac{(x-1) \log\left(\frac{x+2\sqrt{x}+1}{x-1}\right) - 2\sqrt{x}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(x^(1/2))/x^2,x, algorithm="fricas")`

[Out] `1/2*((x - 1)*log((x + 2*sqrt(x) + 1)/(x - 1)) - 2*sqrt(x))/x`

**Sympy [B]** time = 3.66259, size = 92, normalized size = 3.68

$$\frac{x^{\frac{5}{2}} \operatorname{acoth}(\sqrt{x})}{x^{\frac{5}{2}} - x^{\frac{3}{2}}} - \frac{2x^{\frac{3}{2}} \operatorname{acoth}(\sqrt{x})}{x^{\frac{5}{2}} - x^{\frac{3}{2}}} + \frac{\sqrt{x} \operatorname{acoth}(\sqrt{x})}{x^{\frac{5}{2}} - x^{\frac{3}{2}}} - \frac{x^2}{x^{\frac{5}{2}} - x^{\frac{3}{2}}} + \frac{x}{x^{\frac{5}{2}} - x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoath(x**(1/2))/x**2,x)
```

```
[Out] x**(5/2)*acoath(sqrt(x))/(x**(5/2) - x**(3/2)) - 2*x**(3/2)*acoath(sqrt(x))/(
x**(5/2) - x**(3/2)) + sqrt(x)*acoath(sqrt(x))/(x**(5/2) - x**(3/2)) - x**2/
(x**(5/2) - x**(3/2)) + x/(x**(5/2) - x**(3/2))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(\sqrt{x})}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoath(x^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccoath(sqrt(x))/x^2, x)
```

$$3.88 \quad \int \frac{\coth^{-1}(\sqrt{x})}{x^3} dx$$

Optimal. Leaf size=42

$$-\frac{1}{6x^{3/2}} - \frac{\coth^{-1}(\sqrt{x})}{2x^2} - \frac{1}{2\sqrt{x}} + \frac{1}{2} \tanh^{-1}(\sqrt{x})$$

[Out] -1/(6\*x^(3/2)) - 1/(2\*Sqrt[x]) - ArcCoth[Sqrt[x]]/(2\*x^2) + ArcTanh[Sqrt[x]]/2

Rubi [A] time = 0.0147932, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {6098, 51, 63, 206}

$$-\frac{1}{6x^{3/2}} - \frac{\coth^{-1}(\sqrt{x})}{2x^2} - \frac{1}{2\sqrt{x}} + \frac{1}{2} \tanh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Sqrt[x]]/x^3, x]

[Out] -1/(6\*x^(3/2)) - 1/(2\*Sqrt[x]) - ArcCoth[Sqrt[x]]/(2\*x^2) + ArcTanh[Sqrt[x]]/2

### Rule 6098

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)])*(b_.))*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

### Rule 51

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(\sqrt{x})}{x^3} dx &= -\frac{\coth^{-1}(\sqrt{x})}{2x^2} + \frac{1}{4} \int \frac{1}{(1-x)x^{5/2}} dx \\
&= -\frac{1}{6x^{3/2}} - \frac{\coth^{-1}(\sqrt{x})}{2x^2} + \frac{1}{4} \int \frac{1}{(1-x)x^{3/2}} dx \\
&= -\frac{1}{6x^{3/2}} - \frac{1}{2\sqrt{x}} - \frac{\coth^{-1}(\sqrt{x})}{2x^2} + \frac{1}{4} \int \frac{1}{(1-x)\sqrt{x}} dx \\
&= -\frac{1}{6x^{3/2}} - \frac{1}{2\sqrt{x}} - \frac{\coth^{-1}(\sqrt{x})}{2x^2} + \frac{1}{2} \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{x}\right) \\
&= -\frac{1}{6x^{3/2}} - \frac{1}{2\sqrt{x}} - \frac{\coth^{-1}(\sqrt{x})}{2x^2} + \frac{1}{2} \tanh^{-1}(\sqrt{x})
\end{aligned}$$

**Mathematica [A]** time = 0.0191363, size = 58, normalized size = 1.38

$$-\frac{1}{6x^{3/2}} - \frac{\coth^{-1}(\sqrt{x})}{2x^2} - \frac{1}{2\sqrt{x}} - \frac{1}{4} \log(1 - \sqrt{x}) + \frac{1}{4} \log(\sqrt{x} + 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Sqrt[x]]/x^3, x]
```

```
[Out] -1/(6*x^(3/2)) - 1/(2*Sqrt[x]) - ArcCoth[Sqrt[x]]/(2*x^2) - Log[1 - Sqrt[x]]/4 + Log[1 + Sqrt[x]]/4
```



---

**Maple [A]** time = 0.035, size = 37, normalized size = 0.9

$$-\frac{1}{2x^2} \operatorname{arccoth}(\sqrt{x}) - \frac{1}{4} \ln(-1 + \sqrt{x}) - \frac{1}{6} x^{-\frac{3}{2}} - \frac{1}{2} \frac{1}{\sqrt{x}} + \frac{1}{4} \ln(1 + \sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(x^(1/2))/x^3,x)

[Out] -1/2\*arccoth(x^(1/2))/x^2-1/4\*ln(-1+x^(1/2))-1/6/x^(3/2)-1/2/x^(1/2)+1/4\*ln(1+x^(1/2))

---

**Maxima [A]** time = 0.985373, size = 49, normalized size = 1.17

$$-\frac{3x+1}{6x^{\frac{3}{2}}} - \frac{\operatorname{arccoth}(\sqrt{x})}{2x^2} + \frac{1}{4} \log(\sqrt{x}+1) - \frac{1}{4} \log(\sqrt{x}-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x^(1/2))/x^3,x, algorithm="maxima")

[Out] -1/6\*(3\*x + 1)/x^(3/2) - 1/2\*arccoth(sqrt(x))/x^2 + 1/4\*log(sqrt(x) + 1) - 1/4\*log(sqrt(x) - 1)

---

**Fricas [A]** time = 1.59093, size = 107, normalized size = 2.55

$$\frac{3(x^2 - 1) \log\left(\frac{x+2\sqrt{x}+1}{x-1}\right) - 2(3x+1)\sqrt{x}}{12x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x^(1/2))/x^3,x, algorithm="fricas")

[Out] 1/12\*(3\*(x^2 - 1)\*log((x + 2\*sqrt(x) + 1)/(x - 1)) - 2\*(3\*x + 1)\*sqrt(x))/x^2

---

**Sympy [B]** time = 9.8067, size = 160, normalized size = 3.81

$$\frac{3x^{\frac{7}{2}} \operatorname{acoth}(\sqrt{x})}{6x^{\frac{7}{2}} - 6x^{\frac{5}{2}}} - \frac{3x^{\frac{5}{2}} \operatorname{acoth}(\sqrt{x})}{6x^{\frac{7}{2}} - 6x^{\frac{5}{2}}} - \frac{3x^{\frac{3}{2}} \operatorname{acoth}(\sqrt{x})}{6x^{\frac{7}{2}} - 6x^{\frac{5}{2}}} + \frac{3\sqrt{x} \operatorname{acoth}(\sqrt{x})}{6x^{\frac{7}{2}} - 6x^{\frac{5}{2}}} - \frac{3x^3}{6x^{\frac{7}{2}} - 6x^{\frac{5}{2}}} + \frac{2x^2}{6x^{\frac{7}{2}} - 6x^{\frac{5}{2}}} + \frac{x}{6x^{\frac{7}{2}} - 6x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(x\*\*(1/2))/x\*\*3,x)

[Out] 3\*x\*\*(7/2)\*acoth(sqrt(x))/(6\*x\*\*(7/2) - 6\*x\*\*(5/2)) - 3\*x\*\*(5/2)\*acoth(sqrt(x))/(6\*x\*\*(7/2) - 6\*x\*\*(5/2)) - 3\*x\*\*(3/2)\*acoth(sqrt(x))/(6\*x\*\*(7/2) - 6\*x\*\*(5/2)) + 3\*sqrt(x)\*acoth(sqrt(x))/(6\*x\*\*(7/2) - 6\*x\*\*(5/2)) - 3\*x\*\*3/(6\*x\*\*(7/2) - 6\*x\*\*(5/2)) + 2\*x\*\*2/(6\*x\*\*(7/2) - 6\*x\*\*(5/2)) + x/(6\*x\*\*(7/2) - 6\*x\*\*(5/2))

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(\sqrt{x})}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x^(1/2))/x^3,x, algorithm="giac")

[Out] integrate(arccoth(sqrt(x))/x^3, x)

### 3.89 $\int x^{3/2} \coth^{-1}(\sqrt{x}) dx$

**Optimal.** Leaf size=38

$$\frac{x^2}{10} + \frac{2}{5}x^{5/2} \coth^{-1}(\sqrt{x}) + \frac{x}{5} + \frac{1}{5} \log(1-x)$$

[Out]  $x/5 + x^2/10 + (2*x^{(5/2)*ArcCoth[Sqrt[x]])/5 + \text{Log}[1 - x]/5$

**Rubi [A]** time = 0.0177347, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {6098, 43}

$$\frac{x^2}{10} + \frac{2}{5}x^{5/2} \coth^{-1}(\sqrt{x}) + \frac{x}{5} + \frac{1}{5} \log(1-x)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^{(3/2)*ArcCoth[Sqrt[x]]}, x]$

[Out]  $x/5 + x^2/10 + (2*x^{(5/2)*ArcCoth[Sqrt[x]])/5 + \text{Log}[1 - x]/5$

#### Rule 6098

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)])*(b_.)*((d_.)*(x_)^(m_.)), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rubi steps

$$\begin{aligned}
\int x^{3/2} \coth^{-1}(\sqrt{x}) dx &= \frac{2}{5} x^{5/2} \coth^{-1}(\sqrt{x}) - \frac{1}{5} \int \frac{x^2}{1-x} dx \\
&= \frac{2}{5} x^{5/2} \coth^{-1}(\sqrt{x}) - \frac{1}{5} \int \left(-1 + \frac{1}{1-x} - x\right) dx \\
&= \frac{x}{5} + \frac{x^2}{10} + \frac{2}{5} x^{5/2} \coth^{-1}(\sqrt{x}) + \frac{1}{5} \log(1-x)
\end{aligned}$$

**Mathematica [A]** time = 0.0154204, size = 31, normalized size = 0.82

$$\frac{1}{10} \left(4x^{5/2} \coth^{-1}(\sqrt{x}) + (x+2)x + 2 \log(1-x)\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)\*ArcCoth[Sqrt[x]],x]

[Out] (x\*(2 + x) + 4\*x^(5/2)\*ArcCoth[Sqrt[x]] + 2\*Log[1 - x])/10

**Maple [A]** time = 0.034, size = 35, normalized size = 0.9

$$\frac{2}{5} x^{\frac{5}{2}} \operatorname{arccoth}(\sqrt{x}) + \frac{x^2}{10} + \frac{x}{5} + \frac{1}{5} \ln(-1 + \sqrt{x}) + \frac{1}{5} \ln(1 + \sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)\*arccoth(x^(1/2)),x)

[Out] 2/5\*x^(5/2)\*arccoth(x^(1/2))+1/10\*x^2+1/5\*x+1/5\*ln(-1+x^(1/2))+1/5\*ln(1+x^(1/2))

**Maxima [A]** time = 0.956905, size = 32, normalized size = 0.84

$$\frac{2}{5} x^{\frac{5}{2}} \operatorname{arccoth}(\sqrt{x}) + \frac{1}{10} x^2 + \frac{1}{5} x + \frac{1}{5} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*arccoth(x^(1/2)),x, algorithm="maxima")

[Out] 2/5\*x^(5/2)\*arccoth(sqrt(x)) + 1/10\*x^2 + 1/5\*x + 1/5\*log(x - 1)

**Fricas [A]** time = 1.57908, size = 111, normalized size = 2.92

$$\frac{1}{5} x^{\frac{5}{2}} \log\left(\frac{x + 2\sqrt{x} + 1}{x - 1}\right) + \frac{1}{10} x^2 + \frac{1}{5} x + \frac{1}{5} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*arccoth(x^(1/2)),x, algorithm="fricas")

[Out] 1/5\*x^(5/2)\*log((x + 2\*sqrt(x) + 1)/(x - 1)) + 1/10\*x^2 + 1/5\*x + 1/5\*log(x - 1)

**Sympy [B]** time = 9.51341, size = 121, normalized size = 3.18

$$\frac{4x^{\frac{7}{2}} \operatorname{acoth}(\sqrt{x})}{10x - 10} - \frac{4x^{\frac{5}{2}} \operatorname{acoth}(\sqrt{x})}{10x - 10} + \frac{x^3}{10x - 10} + \frac{x^2}{10x - 10} + \frac{4x \log(\sqrt{x} + 1)}{10x - 10} - \frac{4x \operatorname{acoth}(\sqrt{x})}{10x - 10} - \frac{4 \log(\sqrt{x} + 1)}{10x - 10} + \frac{4}{10x - 10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(3/2)\*acoth(x\*\*(1/2)),x)

[Out] 4\*x\*\*(7/2)\*acoth(sqrt(x))/(10\*x - 10) - 4\*x\*\*(5/2)\*acoth(sqrt(x))/(10\*x - 10) + x\*\*3/(10\*x - 10) + x\*\*2/(10\*x - 10) + 4\*x\*log(sqrt(x) + 1)/(10\*x - 10) - 4\*x\*acoth(sqrt(x))/(10\*x - 10) - 4\*log(sqrt(x) + 1)/(10\*x - 10) + 4\*acoth(sqrt(x))/(10\*x - 10) - 2/(10\*x - 10)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^{\frac{3}{2}} \operatorname{arccoth}(\sqrt{x}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arccoth(x^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)*arccoth(sqrt(x)), x)
```

### 3.90 $\int \sqrt{x} \coth^{-1}(\sqrt{x}) dx$

**Optimal.** Leaf size=31

$$\frac{2}{3}x^{3/2} \coth^{-1}(\sqrt{x}) + \frac{x}{3} + \frac{1}{3} \log(1-x)$$

[Out]  $x/3 + (2*x^{(3/2)}*ArcCoth[Sqrt[x]])/3 + Log[1 - x]/3$

**Rubi [A]** time = 0.0135038, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {6098, 43}

$$\frac{2}{3}x^{3/2} \coth^{-1}(\sqrt{x}) + \frac{x}{3} + \frac{1}{3} \log(1-x)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[x]*\text{ArcCoth}[\text{Sqrt}[x]], x]$

[Out]  $x/3 + (2*x^{(3/2)}*ArcCoth[Sqrt[x]])/3 + Log[1 - x]/3$

#### Rule 6098

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)])*(b_.))*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rubi steps

$$\begin{aligned}
 \int \sqrt{x} \coth^{-1}(\sqrt{x}) dx &= \frac{2}{3} x^{3/2} \coth^{-1}(\sqrt{x}) - \frac{1}{3} \int \frac{x}{1-x} dx \\
 &= \frac{2}{3} x^{3/2} \coth^{-1}(\sqrt{x}) - \frac{1}{3} \int \left( -1 + \frac{1}{1-x} \right) dx \\
 &= \frac{x}{3} + \frac{2}{3} x^{3/2} \coth^{-1}(\sqrt{x}) + \frac{1}{3} \log(1-x)
 \end{aligned}$$

**Mathematica [A]** time = 0.0101959, size = 25, normalized size = 0.81

$$\frac{1}{3} (2x^{3/2} \coth^{-1}(\sqrt{x}) + x + \log(1-x))$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]\*ArcCoth[Sqrt[x]],x]

[Out] (x + 2\*x^(3/2)\*ArcCoth[Sqrt[x]] + Log[1 - x])/3

**Maple [A]** time = 0.036, size = 30, normalized size = 1.

$$\frac{2}{3} x^{\frac{3}{2}} \operatorname{arccoth}(\sqrt{x}) + \frac{x}{3} + \frac{1}{3} \ln(-1 + \sqrt{x}) + \frac{1}{3} \ln(1 + \sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(x^(1/2))\*x^(1/2),x)

[Out] 2/3\*x^(3/2)\*arccoth(x^(1/2))+1/3\*x+1/3\*ln(-1+x^(1/2))+1/3\*ln(1+x^(1/2))

**Maxima [A]** time = 0.990299, size = 26, normalized size = 0.84

$$\frac{2}{3} x^{\frac{3}{2}} \operatorname{arccoth}(\sqrt{x}) + \frac{1}{3} x + \frac{1}{3} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x^(1/2))\*x^(1/2),x, algorithm="maxima")



[Out]  $\frac{2}{3}x^{3/2}\operatorname{arccoth}(\sqrt{x}) + \frac{1}{3}x + \frac{1}{3}\log(x - 1)$

---

**Fricas [A]** time = 1.65532, size = 96, normalized size = 3.1

$$\frac{1}{3}x^{\frac{3}{2}}\log\left(\frac{x+2\sqrt{x}+1}{x-1}\right) + \frac{1}{3}x + \frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(x^(1/2))*x^(1/2),x, algorithm="fricas")`

[Out]  $\frac{1}{3}x^{3/2}\log((x + 2\sqrt{x} + 1)/(x - 1)) + \frac{1}{3}x + \frac{1}{3}\log(x - 1)$

---

**Sympy [A]** time = 1.60655, size = 39, normalized size = 1.26

$$\frac{2x^{\frac{3}{2}}\operatorname{acoth}(\sqrt{x})}{3} + \frac{x}{3} + \frac{2\log(\sqrt{x}+1)}{3} - \frac{2\operatorname{acoth}(\sqrt{x})}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(x**(1/2))*x**(1/2),x)`

[Out]  $2*x^{3/2}*acoth(sqrt(x))/3 + x/3 + 2*\log(sqrt(x) + 1)/3 - 2*acoth(sqrt(x))/3$

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x} \operatorname{arccoth}(\sqrt{x}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(x^(1/2))*x^(1/2),x, algorithm="giac")`

[Out] `integrate(sqrt(x)*arccoth(sqrt(x)), x)`

$$3.91 \quad \int \frac{\coth^{-1}(\sqrt{x})}{\sqrt{x}} dx$$

**Optimal.** Leaf size=20

$$\log(1-x) + 2\sqrt{x} \coth^{-1}(\sqrt{x})$$

[Out] 2\*Sqrt[x]\*ArcCoth[Sqrt[x]] + Log[1 - x]

**Rubi [A]** time = 0.0092563, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {6098, 31}

$$\log(1-x) + 2\sqrt{x} \coth^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Sqrt[x]]/Sqrt[x], x]

[Out] 2\*Sqrt[x]\*ArcCoth[Sqrt[x]] + Log[1 - x]

#### Rule 6098

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)])*(b_.))*((d_.)*(x_)^(m_.), x_Symbol]
:= Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

#### Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(\sqrt{x})}{\sqrt{x}} dx &= 2\sqrt{x} \coth^{-1}(\sqrt{x}) - \int \frac{1}{1-x} dx \\ &= 2\sqrt{x} \coth^{-1}(\sqrt{x}) + \log(1-x) \end{aligned}$$

**Mathematica [A]** time = 0.007357, size = 20, normalized size = 1.

$$\log(1-x) + 2\sqrt{x} \coth^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Sqrt[x]]/Sqrt[x],x]

[Out] 2\*Sqrt[x]\*ArcCoth[Sqrt[x]] + Log[1 - x]

**Maple [A]** time = 0.036, size = 15, normalized size = 0.8

$$2 \operatorname{arccoth}(\sqrt{x}) \sqrt{x} + \ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(x^(1/2))/x^(1/2),x)

[Out] 2\*arccoth(x^(1/2))\*x^(1/2)+ln(-1+x)

**Maxima [A]** time = 0.967133, size = 22, normalized size = 1.1

$$2\sqrt{x} \operatorname{arccoth}(\sqrt{x}) + \log(-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x^(1/2))/x^(1/2),x, algorithm="maxima")

[Out] 2\*sqrt(x)\*arccoth(sqrt(x)) + log(-x + 1)

**Fricas [A]** time = 1.64507, size = 74, normalized size = 3.7

$$\sqrt{x} \log\left(\frac{x+2\sqrt{x}+1}{x-1}\right) + \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x^(1/2))/x^(1/2),x, algorithm="fricas")

[Out] sqrt(x)\*log((x + 2\*sqrt(x) + 1)/(x - 1)) + log(x - 1)

**Sympy [B]** time = 0.780662, size = 87, normalized size = 4.35

$$\frac{2x^{\frac{3}{2}} \operatorname{acoth}(\sqrt{x})}{x-1} - \frac{2\sqrt{x} \operatorname{acoth}(\sqrt{x})}{x-1} + \frac{2x \log(\sqrt{x}+1)}{x-1} - \frac{2x \operatorname{acoth}(\sqrt{x})}{x-1} - \frac{2 \log(\sqrt{x}+1)}{x-1} + \frac{2 \operatorname{acoth}(\sqrt{x})}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(x\*\*(1/2))/x\*\*(1/2),x)

[Out] 2\*x\*\*(3/2)\*acoth(sqrt(x))/(x - 1) - 2\*sqrt(x)\*acoth(sqrt(x))/(x - 1) + 2\*x\*log(sqrt(x) + 1)/(x - 1) - 2\*x\*acoth(sqrt(x))/(x - 1) - 2\*log(sqrt(x) + 1)/(x - 1) + 2\*acoth(sqrt(x))/(x - 1)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(\sqrt{x})}{\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x^(1/2))/x^(1/2),x, algorithm="giac")

[Out] integrate(arccoth(sqrt(x))/sqrt(x), x)

$$3.92 \quad \int \frac{\coth^{-1}(\sqrt{x})}{x^{3/2}} dx$$

**Optimal.** Leaf size=24

$$-\log(1-x) + \log(x) - \frac{2 \coth^{-1}(\sqrt{x})}{\sqrt{x}}$$

[Out]  $(-2*\text{ArcCoth}[\text{Sqrt}[x]])/\text{Sqrt}[x] - \text{Log}[1 - x] + \text{Log}[x]$

**Rubi [A]** time = 0.0104366, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6098, 36, 31, 29}

$$-\log(1-x) + \log(x) - \frac{2 \coth^{-1}(\sqrt{x})}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{ArcCoth}[\text{Sqrt}[x]]/x^{(3/2)}, x]$

[Out]  $(-2*\text{ArcCoth}[\text{Sqrt}[x]])/\text{Sqrt}[x] - \text{Log}[1 - x] + \text{Log}[x]$

### Rule 6098

$\text{Int}[(a_.) + \text{ArcCoth}[(c_.)*(x_.)^{(n_.)}]*(b_.)]*((d_.)*(x_.))^{(m_.)}, x\_Symbol]$   
 $:\> \text{Simp}[(d*x)^{(m+1)}*(a + b*\text{ArcCoth}[c*x^n])/(d*(m+1)), x] - \text{Dist}[(b*c*n)/(d*(m+1)), \text{Int}[(x^{(n-1)}*(d*x)^{(m+1)})/(1 - c^2*x^{(2*n)}), x], x] /;$   
 $\text{FreeQ}\{a, b, c, d, m, n\}, x \ \&\& \ \text{NeQ}[m, -1]$

### Rule 36

$\text{Int}[1/((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.)), x\_Symbol] :\> \text{Dist}[b/(b*c - a*d), \text{Int}[1/(a + b*x), x], x] - \text{Dist}[d/(b*c - a*d), \text{Int}[1/(c + d*x), x], x] /;$   
 $\text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

### Rule 31

$\text{Int}[(a_.) + (b_.)*(x_.))^{(-1)}, x\_Symbol] :\> \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /;$   
 $\text{FreeQ}\{a, b\}, x]$

Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(\sqrt{x})}{x^{3/2}} dx &= -\frac{2 \coth^{-1}(\sqrt{x})}{\sqrt{x}} + \int \frac{1}{(1-x)x} dx \\ &= -\frac{2 \coth^{-1}(\sqrt{x})}{\sqrt{x}} + \int \frac{1}{1-x} dx + \int \frac{1}{x} dx \\ &= -\frac{2 \coth^{-1}(\sqrt{x})}{\sqrt{x}} - \log(1-x) + \log(x) \end{aligned}$$

**Mathematica [A]** time = 0.0164811, size = 24, normalized size = 1.

$$-\log(1-x) + \log(x) - \frac{2 \coth^{-1}(\sqrt{x})}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Sqrt[x]]/x^(3/2), x]

[Out] (-2\*ArcCoth[Sqrt[x]])/Sqrt[x] - Log[1 - x] + Log[x]

**Maple [A]** time = 0.04, size = 29, normalized size = 1.2

$$-2 \frac{\operatorname{arccoth}(\sqrt{x})}{\sqrt{x}} - \ln(-1 + \sqrt{x}) + \ln(x) - \ln(1 + \sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(x^(1/2))/x^(3/2), x)

[Out] -2\*arccoth(x^(1/2))/x^(1/2) - ln(-1+x^(1/2)) + ln(x) - ln(1+x^(1/2))

**Maxima [A]** time = 0.986294, size = 24, normalized size = 1.

$$-\frac{2 \operatorname{arccoth}(\sqrt{x})}{\sqrt{x}} - \log(x-1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x^(1/2))/x^(3/2),x, algorithm="maxima")

[Out] -2\*arccoth(sqrt(x))/sqrt(x) - log(x - 1) + log(x)

**Fricas [A]** time = 1.63988, size = 99, normalized size = 4.12

$$\frac{x \log(x-1) - x \log(x) + \sqrt{x} \log\left(\frac{x+2\sqrt{x}+1}{x-1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x^(1/2))/x^(3/2),x, algorithm="fricas")

[Out] -(x\*log(x - 1) - x\*log(x) + sqrt(x)\*log((x + 2\*sqrt(x) + 1)/(x - 1)))/x

**Sympy [B]** time = 2.15684, size = 126, normalized size = 5.25

$$-\frac{2x^{\frac{3}{2}} \operatorname{acoth}(\sqrt{x})}{x^2 - x} + \frac{2\sqrt{x} \operatorname{acoth}(\sqrt{x})}{x^2 - x} + \frac{x^2 \log(x)}{x^2 - x} - \frac{2x^2 \log(\sqrt{x} + 1)}{x^2 - x} + \frac{2x^2 \operatorname{acoth}(\sqrt{x})}{x^2 - x} - \frac{x \log(x)}{x^2 - x} + \frac{2x \log(\sqrt{x} + 1)}{x^2 - x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(x\*\*(1/2))/x\*\*(3/2),x)

[Out] -2\*x\*\*(3/2)\*acoth(sqrt(x))/(x\*\*2 - x) + 2\*sqrt(x)\*acoth(sqrt(x))/(x\*\*2 - x) + x\*\*2\*log(x)/(x\*\*2 - x) - 2\*x\*\*2\*log(sqrt(x) + 1)/(x\*\*2 - x) + 2\*x\*\*2\*acoth(sqrt(x))/(x\*\*2 - x) - x\*log(x)/(x\*\*2 - x) + 2\*x\*log(sqrt(x) + 1)/(x\*\*2 - x) - 2\*x\*acoth(sqrt(x))/(x\*\*2 - x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(\sqrt{x})}{x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arccoth(sqrt(x))/x^(3/2), x)
```



$$3.93 \quad \int \frac{\coth^{-1}(ax^5)}{x} dx$$

**Optimal.** Leaf size=28

$$\frac{1}{10} \text{PolyLog}\left(2, -\frac{1}{ax^5}\right) - \frac{1}{10} \text{PolyLog}\left(2, \frac{1}{ax^5}\right)$$

[Out] PolyLog[2, -(1/(a\*x^5))]/10 - PolyLog[2, 1/(a\*x^5)]/10

**Rubi [A]** time = 0.0217677, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {6096, 5913}

$$\frac{1}{10} \text{PolyLog}\left(2, -\frac{1}{ax^5}\right) - \frac{1}{10} \text{PolyLog}\left(2, \frac{1}{ax^5}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x^5]/x,x]

[Out] PolyLog[2, -(1/(a\*x^5))]/10 - PolyLog[2, 1/(a\*x^5)]/10

#### Rule 6096

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)^(n\_)])\*(b\_.))/(x\_), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*ArcCoth[c\*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]

#### Rule 5913

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))/(x\_), x\_Symbol] := Simp[a\*Log[x], x] + (Simp[(b\*PolyLog[2, -(c\*x)^(-1)])/2, x] - Simp[(b\*PolyLog[2, 1/(c\*x)])/2, x]) /; FreeQ[{a, b, c}, x]

#### Rubi steps

$$\int \frac{\coth^{-1}(ax^5)}{x} dx = \frac{1}{5} \text{Subst} \left( \int \frac{\coth^{-1}(ax)}{x} dx, x, x^5 \right) \\ = \frac{1}{10} \text{Li}_2 \left( -\frac{1}{ax^5} \right) - \frac{1}{10} \text{Li}_2 \left( \frac{1}{ax^5} \right)$$

**Mathematica [A]** time = 0.0134413, size = 26, normalized size = 0.93

$$\frac{1}{10} \left( \text{PolyLog} \left( 2, -\frac{1}{ax^5} \right) - \text{PolyLog} \left( 2, \frac{1}{ax^5} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a\*x^5]/x,x]

[Out] (PolyLog[2, -(1/(a\*x^5))] - PolyLog[2, 1/(a\*x^5)])/10

**Maple [C]** time = 0.146, size = 85, normalized size = 3.

$$\ln(x) \operatorname{arccoth}(ax^5) + \frac{1}{2} \sum_{_R1=\text{RootOf}(a\_Z^5-1)} \ln(x) \ln\left(\frac{-R1-x}{-R1}\right) + \operatorname{dilog}\left(\frac{-R1-x}{-R1}\right) - \frac{1}{2} \sum_{_R1=\text{RootOf}(a\_Z^5+1)} \ln(x) \ln\left(\frac{-R1-x}{-R1}\right) + \operatorname{dilog}\left(\frac{-R1-x}{-R1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x^5)/x,x)

[Out] ln(x)\*arccoth(a\*x^5)+1/2\*sum(ln(x)\*ln((-R1-x)/\_R1)+dilog((-R1-x)/\_R1),\_R1=RootOf(\_Z^5\*a-1))-1/2\*sum(ln(x)\*ln((-R1-x)/\_R1)+dilog((-R1-x)/\_R1),\_R1=RootOf(\_Z^5\*a+1))

**Maxima [B]** time = 0.990651, size = 140, normalized size = 5.

$$-\frac{1}{2} a \left( \frac{\log(ax^5+1)}{a} - \frac{\log(ax^5-1)}{a} \right) \log(x) - \frac{1}{10} a \left( \frac{\log(ax^5-1) \log(ax^5) + \text{Li}_2(-ax^5+1)}{a} - \frac{\log(ax^5+1) \log(-ax^5) + \text{Li}_2(-ax^5-1)}{a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x^5)/x,x, algorithm="maxima")

[Out]  $-1/2*a*(\log(a*x^5 + 1)/a - \log(a*x^5 - 1)/a)*\log(x) - 1/10*a*((\log(a*x^5 - 1)*\log(a*x^5) + \operatorname{dilog}(-a*x^5 + 1))/a - (\log(a*x^5 + 1)*\log(-a*x^5) + \operatorname{dilog}(a*x^5 + 1))/a) + \operatorname{arccoth}(a*x^5)*\log(x)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arccoth}(ax^5)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x^5)/x,x, algorithm="fricas")

[Out] integral(arccoth(a\*x^5)/x, x)

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: KeyError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(a\*x\*\*5)/x,x)

[Out] Exception raised: KeyError

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(ax^5)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x^5)/x,x, algorithm="giac")

[Out] integrate(arccoth(a\*x^5)/x, x)

### 3.94 $\int \coth^{-1}\left(\frac{1}{x}\right) dx$

**Optimal.** Leaf size=19

$$\frac{1}{2} \log(1 - x^2) + x \coth^{-1}\left(\frac{1}{x}\right)$$

[Out] x\*ArcCoth[x^(-1)] + Log[1 - x^2]/2

**Rubi [A]** time = 0.0060927, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.75$ , Rules used = {6092, 263, 260}

$$\frac{1}{2} \log(1 - x^2) + x \coth^{-1}\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[x^(-1)],x]

[Out] x\*ArcCoth[x^(-1)] + Log[1 - x^2]/2

#### Rule 6092

Int[ArcCoth[(c\_.)\*(x\_)^(n\_)], x\_Symbol] := Simp[x\*ArcCoth[c\*x^n], x] - Dist[c\*n, Int[x^n/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{c, n}, x]

#### Rule 263

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[x^(m + n\*p)\*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]

#### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

#### Rubi steps

$$\begin{aligned}
\int \coth^{-1}\left(\frac{1}{x}\right) dx &= x \coth^{-1}\left(\frac{1}{x}\right) + \int \frac{1}{\left(1 - \frac{1}{x^2}\right)x} dx \\
&= x \coth^{-1}\left(\frac{1}{x}\right) + \int \frac{x}{-1 + x^2} dx \\
&= x \coth^{-1}\left(\frac{1}{x}\right) + \frac{1}{2} \log(1 - x^2)
\end{aligned}$$

**Mathematica [A]** time = 0.0018144, size = 19, normalized size = 1.

$$\frac{1}{2} \log(1 - x^2) + x \coth^{-1}\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[x^(-1)], x]

[Out] x\*ArcCoth[x^(-1)] + Log[1 - x^2]/2

**Maple [A]** time = 0.056, size = 30, normalized size = 1.6

$$x \operatorname{arccoth}(x^{-1}) + \frac{\ln(x^{-1} - 1)}{2} - \ln(x^{-1}) + \frac{\ln(x^{-1} + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(1/x), x)

[Out] x\*arccoth(1/x)+1/2\*ln(1/x-1)-ln(1/x)+1/2\*ln(1/x+1)

**Maxima [A]** time = 0.976526, size = 20, normalized size = 1.05

$$x \operatorname{arccoth}\left(\frac{1}{x}\right) + \frac{1}{2} \log(x^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1/x),x, algorithm="maxima")

[Out] x\*arccoth(1/x) + 1/2\*log(x^2 - 1)

---

**Fricas [A]** time = 1.85874, size = 65, normalized size = 3.42

$$\frac{1}{2}x \log\left(-\frac{x+1}{x-1}\right) + \frac{1}{2} \log(x^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1/x),x, algorithm="fricas")

[Out] 1/2\*x\*log(-(x + 1)/(x - 1)) + 1/2\*log(x^2 - 1)

---

**Sympy [A]** time = 0.254185, size = 15, normalized size = 0.79

$$x \operatorname{acoth}\left(\frac{1}{x}\right) + \log(x + 1) - \operatorname{acoth}\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(1/x),x)

[Out] x\*acoth(1/x) + log(x + 1) - acoth(1/x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arccoth}\left(\frac{1}{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1/x),x, algorithm="giac")

[Out] integrate(arccoth(1/x), x)

$$3.95 \quad \int \frac{\coth^{-1}(ax^n)}{x} dx$$

**Optimal.** Leaf size=38

$$\frac{\text{PolyLog}\left(2, -\frac{x^{-n}}{a}\right)}{2n} - \frac{\text{PolyLog}\left(2, \frac{x^{-n}}{a}\right)}{2n}$$

[Out] PolyLog[2, -(1/(a\*x^n))]/(2\*n) - PolyLog[2, 1/(a\*x^n)]/(2\*n)

**Rubi [A]** time = 0.023247, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {6096, 5913}

$$\frac{\text{PolyLog}\left(2, -\frac{x^{-n}}{a}\right)}{2n} - \frac{\text{PolyLog}\left(2, \frac{x^{-n}}{a}\right)}{2n}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x^n]/x, x]

[Out] PolyLog[2, -(1/(a\*x^n))]/(2\*n) - PolyLog[2, 1/(a\*x^n)]/(2\*n)

#### Rule 6096

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)^(n\_)])\*(b\_.))/(x\_), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*ArcCoth[c\*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]

#### Rule 5913

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))/(x\_), x\_Symbol] := Simp[a\*Log[x], x] + (Simp[(b\*PolyLog[2, -(c\*x)^(-1)])]/2, x] - Simp[(b\*PolyLog[2, 1/(c\*x)])]/2, x) /; FreeQ[{a, b, c}, x]

#### Rubi steps

$$\int \frac{\operatorname{coth}^{-1}(ax^n)}{x} dx = \frac{\operatorname{Subst}\left(\int \frac{\operatorname{coth}^{-1}(ax)}{x} dx, x, x^n\right)}{n}$$

$$= \frac{\operatorname{Li}_2\left(-\frac{x^{-n}}{a}\right)}{2n} - \frac{\operatorname{Li}_2\left(\frac{x^{-n}}{a}\right)}{2n}$$

**Mathematica [B]** time = 0.0454459, size = 97, normalized size = 2.55

$$\frac{-\operatorname{PolyLog}(2, 1 - ax^n) + \operatorname{PolyLog}(2, ax^n + 1) + n \log(x) \log(ax^n - 1) - n \log(x) \log(ax^n + 1) - \log(ax^n) \log(ax^n - 1)}{2n}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a\*x^n]/x,x]

[Out] (2\*n\*ArcCoth[a\*x^n]\*Log[x] + n\*Log[x]\*Log[-1 + a\*x^n] - Log[a\*x^n]\*Log[-1 + a\*x^n] - n\*Log[x]\*Log[1 + a\*x^n] + Log[-(a\*x^n)]\*Log[1 + a\*x^n] - PolyLog[2, 1 - a\*x^n] + PolyLog[2, 1 + a\*x^n])/(2\*n)

**Maple [A]** time = 0.09, size = 61, normalized size = 1.6

$$\frac{\ln(ax^n) \operatorname{arccoth}(ax^n)}{n} - \frac{\operatorname{dilog}(ax^n)}{2n} - \frac{\operatorname{dilog}(ax^n + 1)}{2n} - \frac{\ln(ax^n) \ln(ax^n + 1)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x^n)/x,x)

[Out] 1/n\*ln(a\*x^n)\*arccoth(a\*x^n)-1/2/n\*dilog(a\*x^n)-1/2/n\*dilog(a\*x^n+1)-1/2/n\*ln(a\*x^n)\*ln(a\*x^n+1)

**Maxima [B]** time = 1.26882, size = 198, normalized size = 5.21

$$-\frac{1}{2} an \left( \frac{\log\left(\frac{ax^n+1}{a}\right)}{an} - \frac{\log\left(\frac{ax^n-1}{a}\right)}{an} \right) \log(x) + \frac{1}{2} an \left( \frac{\log(ax^n+1) \log(x) - \log(ax^n-1) \log(x)}{an} - \frac{n \log(ax^n+1) \log(x) + \dots}{an^2} \right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x^n)/x,x, algorithm="maxima")

[Out]  $-1/2*a*n*(\log((a*x^n + 1)/a)/(a*n) - \log((a*x^n - 1)/a)/(a*n))*\log(x) + 1/2*a*n*((\log(a*x^n + 1)*\log(x) - \log(a*x^n - 1)*\log(x))/(a*n) - (n*\log(a*x^n + 1)*\log(x) + \operatorname{dilog}(-a*x^n))/(a*n^2) + (n*\log(-a*x^n + 1)*\log(x) + \operatorname{dilog}(a*x^n))/(a*n^2)) + \operatorname{arccoth}(a*x^n)*\log(x)$

**Fricas [B]** time = 2.00394, size = 421, normalized size = 11.08

$n \log(a \cosh(n \log(x)) + a \sinh(n \log(x)) + 1) \log(x) - n \log(-a \cosh(n \log(x)) - a \sinh(n \log(x)) + 1) \log(x) - n$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x^n)/x,x, algorithm="fricas")

[Out]  $-1/2*(n*\log(a*\cosh(n*\log(x)) + a*\sinh(n*\log(x)) + 1)*\log(x) - n*\log(-a*\cosh(n*\log(x)) - a*\sinh(n*\log(x)) + 1)*\log(x) - n*\log(x)*\log((a*\cosh(n*\log(x)) + a*\sinh(n*\log(x)) + 1)/(a*\cosh(n*\log(x)) + a*\sinh(n*\log(x)) - 1)) - \operatorname{dilog}(a*\cosh(n*\log(x)) + a*\sinh(n*\log(x))) + \operatorname{dilog}(-a*\cosh(n*\log(x)) - a*\sinh(n*\log(x)))))/n$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}(ax^n)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(a\*x\*\*n)/x,x)

[Out] Integral(acoth(a\*x\*\*n)/x, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(ax^n)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x^n)/x, x)
```

### 3.96 $\int (a + bx) \coth^{-1}(a + bx) dx$

**Optimal.** Leaf size=39

$$-\frac{\tanh^{-1}(a + bx)}{2b} + \frac{(a + bx)^2 \coth^{-1}(a + bx)}{2b} + \frac{x}{2}$$

[Out]  $x/2 + ((a + b*x)^2 * \text{ArcCoth}[a + b*x]) / (2*b) - \text{ArcTanh}[a + b*x] / (2*b)$

**Rubi [A]** time = 0.0224787, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6108, 5917, 321, 206}

$$-\frac{\tanh^{-1}(a + bx)}{2b} + \frac{(a + bx)^2 \coth^{-1}(a + bx)}{2b} + \frac{x}{2}$$

Antiderivative was successfully verified.

[In] `Int[(a + b*x)*ArcCoth[a + b*x], x]`

[Out]  $x/2 + ((a + b*x)^2 * \text{ArcCoth}[a + b*x]) / (2*b) - \text{ArcTanh}[a + b*x] / (2*b)$

#### Rule 6108

```
Int[((a_.) + ArcCoth[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(m_.), x_Symbol]
  := Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] && IGtQ[p, 0]
```

#### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.)*((d_.)*(x_.))^(m_.), x_Symbol]
  := Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]
```

#### Rule 321

```
Int[((c_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.), x_Symbol]
  := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned} \int (a + bx) \coth^{-1}(a + bx) dx &= \frac{\text{Subst}\left(\int x \coth^{-1}(x) dx, x, a + bx\right)}{b} \\ &= \frac{(a + bx)^2 \coth^{-1}(a + bx)}{2b} - \frac{\text{Subst}\left(\int \frac{x^2}{1-x^2} dx, x, a + bx\right)}{2b} \\ &= \frac{x}{2} + \frac{(a + bx)^2 \coth^{-1}(a + bx)}{2b} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, a + bx\right)}{2b} \\ &= \frac{x}{2} + \frac{(a + bx)^2 \coth^{-1}(a + bx)}{2b} - \frac{\tanh^{-1}(a + bx)}{2b} \end{aligned}$$

**Mathematica [A]** time = 0.0320044, size = 66, normalized size = 1.69

$$\frac{a^2 \log(a + bx + 1) - (a^2 - 1) \log(-a - bx + 1) - \log(a + bx + 1) + 2bx(2a + bx) \coth^{-1}(a + bx) + 2bx}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x)\*ArcCoth[a + b\*x], x]

[Out] (2\*b\*x + 2\*b\*x\*(2\*a + b\*x)\*ArcCoth[a + b\*x] - (-1 + a^2)\*Log[1 - a - b\*x] - Log[1 + a + b\*x] + a^2\*Log[1 + a + b\*x])/(4\*b)

**Maple [B]** time = 0.036, size = 70, normalized size = 1.8

$$\frac{\text{barcoth}(bx + a)x^2}{2} + \text{arccoht}(bx + a)xa + \frac{\text{arccoht}(bx + a)a^2}{2b} + \frac{x}{2} + \frac{a}{2b} + \frac{\ln(bx + a - 1)}{4b} - \frac{\ln(bx + a + 1)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x+a)*arccoth(b*x+a),x)`

[Out]  $\frac{1}{2}b \operatorname{arccoth}(bx+a)x^2 + \operatorname{arccoth}(bx+a)x \cdot a + \frac{1}{2} \frac{1}{b} \operatorname{arccoth}(bx+a) \cdot a^2 + \frac{1}{2} \frac{x}{b} + \frac{1}{2} \frac{a}{b} + \frac{1}{4} \frac{1}{b} \ln(bx+a-1) - \frac{1}{4} \frac{1}{b} \ln(bx+a+1)$

**Maxima [A]** time = 0.966219, size = 84, normalized size = 2.15

$$\frac{1}{4}b \left( \frac{2x}{b} + \frac{(a^2-1)\log(bx+a+1)}{b^2} - \frac{(a^2-1)\log(bx+a-1)}{b^2} \right) + \frac{1}{2}(bx^2 + 2ax) \operatorname{arccoth}(bx+a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)*arccoth(b*x+a),x, algorithm="maxima")`

[Out]  $\frac{1}{4}b \left( \frac{2x}{b} + \frac{(a^2-1)\log(bx+a+1)}{b^2} - \frac{(a^2-1)\log(bx+a-1)}{b^2} \right) + \frac{1}{2}(bx^2 + 2ax) \operatorname{arccoth}(bx+a)$

**Fricas [A]** time = 1.92371, size = 108, normalized size = 2.77

$$\frac{2bx + (b^2x^2 + 2abx + a^2 - 1) \log\left(\frac{bx+a+1}{bx+a-1}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)*arccoth(b*x+a),x, algorithm="fricas")`

[Out]  $\frac{1}{4} \left( \frac{2bx + (b^2x^2 + 2abx + a^2 - 1) \log\left(\frac{bx+a+1}{bx+a-1}\right)}{b} \right)$

**Sympy [A]** time = 1.73783, size = 56, normalized size = 1.44

$$\begin{cases} \frac{a^2 \operatorname{acoth}(a+bx)}{2b} + ax \operatorname{acoth}(a+bx) + \frac{bx^2 \operatorname{acoth}(a+bx)}{2} + \frac{x}{2} - \frac{\operatorname{acoth}(a+bx)}{2b} & \text{for } b \neq 0 \\ ax \operatorname{acoth}(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*acoth(b*x+a),x)
```

```
[Out] Piecewise((a**2*acoth(a + b*x)/(2*b) + a*x*acoth(a + b*x) + b*x**2*acoth(a
+ b*x)/2 + x/2 - acoth(a + b*x)/(2*b), Ne(b, 0)), (a*x*acoth(a), True))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx + a) \operatorname{arccoth}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*arccoth(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*arccoth(b*x + a), x)
```

### 3.97 $\int (a + bx)^2 \coth^{-1}(a + bx) dx$

**Optimal.** Leaf size=54

$$\frac{(a + bx)^2}{6b} + \frac{\log(1 - (a + bx)^2)}{6b} + \frac{(a + bx)^3 \coth^{-1}(a + bx)}{3b}$$

[Out]  $(a + b*x)^2/(6*b) + ((a + b*x)^3*ArcCoth[a + b*x])/(3*b) + \text{Log}[1 - (a + b*x)^2]/(6*b)$

**Rubi [A]** time = 0.0479171, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {6108, 5917, 266, 43}

$$\frac{(a + bx)^2}{6b} + \frac{\log(1 - (a + bx)^2)}{6b} + \frac{(a + bx)^3 \coth^{-1}(a + bx)}{3b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x)^2*ArcCoth[a + b*x], x]$

[Out]  $(a + b*x)^2/(6*b) + ((a + b*x)^3*ArcCoth[a + b*x])/(3*b) + \text{Log}[1 - (a + b*x)^2]/(6*b)$

#### Rule 6108

$\text{Int}[(a + ArcCoth[(c + (d*x)]*(b))]^{(p)}*((e + (f*x))^{(m)}), x\_Symbol] \rightarrow \text{Dist}[1/d, \text{Subst}[\text{Int}[(f*x)/d]^{(m)}*(a + b*ArcCoth[x])^{(p)}, x], x, c + d*x], x] /;$  FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d\*e - c\*f, 0] & IGtQ[p, 0]

#### Rule 5917

$\text{Int}[(a + ArcCoth[(c)*(x)]*(b))]^{(p)}*((d)*(x))^{(m)}, x\_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*(a + b*ArcCoth[c*x])^{(p)}/(d*(m+1)), x] - \text{Dist}[(b*c*p)/(d*(m+1)), \text{Int}[(d*x)^{(m+1)}*(a + b*ArcCoth[c*x])^{(p-1)}/(1 - c^2*x^2), x], x] /;$  FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

### Rule 43

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rubi steps

$$\begin{aligned}
\int (a + bx)^2 \coth^{-1}(a + bx) dx &= \frac{\text{Subst}\left(\int x^2 \coth^{-1}(x) dx, x, a + bx\right)}{b} \\
&= \frac{(a + bx)^3 \coth^{-1}(a + bx)}{3b} - \frac{\text{Subst}\left(\int \frac{x^3}{1-x^2} dx, x, a + bx\right)}{3b} \\
&= \frac{(a + bx)^3 \coth^{-1}(a + bx)}{3b} - \frac{\text{Subst}\left(\int \frac{x}{1-x} dx, x, (a + bx)^2\right)}{6b} \\
&= \frac{(a + bx)^3 \coth^{-1}(a + bx)}{3b} - \frac{\text{Subst}\left(\int \left(-1 + \frac{1}{1-x}\right) dx, x, (a + bx)^2\right)}{6b} \\
&= \frac{(a + bx)^2}{6b} + \frac{(a + bx)^3 \coth^{-1}(a + bx)}{3b} + \frac{\log(1 - (a + bx)^2)}{6b}
\end{aligned}$$

**Mathematica [A]** time = 0.0345896, size = 42, normalized size = 0.78

$$\frac{(a + bx)^2 + \log(1 - (a + bx)^2) + 2(a + bx)^3 \coth^{-1}(a + bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*ArcCoth[a + b*x], x]
```

```
[Out] ((a + b*x)^2 + 2*(a + b*x)^3*ArcCoth[a + b*x] + Log[1 - (a + b*x)^2])/(6*b)
```

**Maple [A]** time = 0.034, size = 95, normalized size = 1.8

$$\frac{b^2 \operatorname{arccoth}(bx + a) x^3}{3} + b \operatorname{arccoth}(bx + a) x^2 a + \operatorname{arccoth}(bx + a) x a^2 + \frac{\operatorname{arccoth}(bx + a) a^3}{3b} + \frac{bx^2}{6} + \frac{ax}{3} + \frac{a^2}{6b} + \frac{\ln(bx + a)}{6b}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x+a)^2*arccoth(b*x+a),x)`

[Out]  $\frac{1}{3}b^2\operatorname{arccoth}(b*x+a)*x^3+b*\operatorname{arccoth}(b*x+a)*x^2*a+\operatorname{arccoth}(b*x+a)*x*a^2+\frac{1}{3}b*\operatorname{arccoth}(b*x+a)*a^3+\frac{1}{6}b*x^2+\frac{1}{3}a*x+\frac{1}{6}b*a^2+\frac{1}{6}b*\ln(b*x+a-1)+\frac{1}{6}b*\ln(b*x+a+1)$

**Maxima [A]** time = 0.95623, size = 109, normalized size = 2.02

$$\frac{1}{6}b\left(\frac{bx^2+2ax}{b}+\frac{(a^3+1)\log(bx+a+1)}{b^2}-\frac{(a^3-1)\log(bx+a-1)}{b^2}\right)+\frac{1}{3}(b^2x^3+3abx^2+3a^2x)\operatorname{arccoth}(bx+a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)^2*arccoth(b*x+a),x, algorithm="maxima")`

[Out]  $\frac{1}{6}b*((b*x^2+2*a*x)/b+(a^3+1)*\log(b*x+a+1)/b^2-(a^3-1)*\log(b*x+a-1)/b^2)+\frac{1}{3}*(b^2*x^3+3*a*b*x^2+3*a^2*x)*\operatorname{arccoth}(b*x+a)$

**Fricas [A]** time = 1.87158, size = 211, normalized size = 3.91

$$\frac{b^2x^2+2abx+(a^3+1)\log(bx+a+1)-(a^3-1)\log(bx+a-1)+(b^3x^3+3ab^2x^2+3a^2bx)\log\left(\frac{bx+a+1}{bx+a-1}\right)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)^2*arccoth(b*x+a),x, algorithm="fricas")`

[Out]  $\frac{1}{6}*(b^2*x^2+2*a*b*x+(a^3+1)*\log(b*x+a+1)-(a^3-1)*\log(b*x+a-1)+(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x)*\log((b*x+a+1)/(b*x+a-1)))/b$

**Sympy [A]** time = 3.13341, size = 97, normalized size = 1.8

$$\begin{cases} \frac{a^3 \operatorname{acoth}(a+bx)}{3b} + a^2x \operatorname{acoth}(a+bx) + abx^2 \operatorname{acoth}(a+bx) + \frac{ax}{3} + \frac{b^2x^3 \operatorname{acoth}(a+bx)}{3} + \frac{bx^2}{6} + \frac{\log\left(\frac{a}{b}+x+\frac{1}{b}\right)}{3b} - \frac{\operatorname{acoth}(a+bx)}{3b} \\ a^2x \operatorname{acoth}(a) \end{cases} \text{ for oth}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*acoth(b*x+a),x)
```

```
[Out] Piecewise((a**3*acoth(a + b*x)/(3*b) + a**2*x*acoth(a + b*x) + a*b*x**2*acoth(a + b*x) + a*x/3 + b**2*x**3*acoth(a + b*x)/3 + b*x**2/6 + log(a/b + x + 1/b)/(3*b) - acoth(a + b*x)/(3*b), Ne(b, 0)), (a**2*x*acoth(a), True))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx + a)^2 \operatorname{arccoth}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arccoth(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^2*arccoth(b*x + a), x)
```

$$3.98 \quad \int \frac{\coth^{-1}(a+bx)}{a+bx} dx$$

**Optimal.** Leaf size=35

$$\frac{\text{PolyLog}\left(2, -\frac{1}{a+bx}\right)}{2b} - \frac{\text{PolyLog}\left(2, \frac{1}{a+bx}\right)}{2b}$$

[Out] PolyLog[2, -(a + b\*x)^(-1)]/(2\*b) - PolyLog[2, (a + b\*x)^(-1)]/(2\*b)

**Rubi [A]** time = 0.0254893, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {6108, 5913}

$$\frac{\text{PolyLog}\left(2, -\frac{1}{a+bx}\right)}{2b} - \frac{\text{PolyLog}\left(2, \frac{1}{a+bx}\right)}{2b}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a + b\*x]/(a + b\*x), x]

[Out] PolyLog[2, -(a + b\*x)^(-1)]/(2\*b) - PolyLog[2, (a + b\*x)^(-1)]/(2\*b)

### Rule 6108

Int[((a\_.) + ArcCoth[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Dist[1/d, Subst[Int[((f\*x)/d)^m\*(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d\*e - c\*f, 0] & IGtQ[p, 0]

### Rule 5913

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))/(x\_), x\_Symbol] :> Simp[a\*Log[x], x] + (Simp[(b\*PolyLog[2, -(c\*x)^(-1)])/2, x] - Simp[(b\*PolyLog[2, 1/(c\*x)])/2, x]) /; FreeQ[{a, b, c}, x]

### Rubi steps

$$\int \frac{\coth^{-1}(a+bx)}{a+bx} dx = \frac{\text{Subst}\left(\int \frac{\coth^{-1}(x)}{x} dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Li}_2\left(-\frac{1}{a+bx}\right)}{2b} - \frac{\text{Li}_2\left(\frac{1}{a+bx}\right)}{2b}$$

**Mathematica [B]** time = 0.0254113, size = 286, normalized size = 8.17

$$-\frac{\text{PolyLog}(2, -a - bx)}{2b} + \frac{\text{PolyLog}(2, a + bx)}{2b} - \frac{\log^2\left(\frac{ab - (a-1)b}{b(a+bx)}\right)}{4b} + \frac{\log^2\left(\frac{ab - (a+1)b}{b(a+bx)}\right)}{4b} - \frac{\log\left(\frac{b(a+bx-1)}{(a-1)b-ab}\right)\log\left(\frac{ab - (a-1)b}{b(a+bx)}\right)}{2b} + \dots$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a + b\*x]/(a + b\*x), x]

[Out]  $-(\text{Log}[(b*(-1 + a + b*x))/((-1 + a)*b - a*b)]*\text{Log}[(-((-1 + a)*b) + a*b)/(b*(a + b*x))])/(2*b) - \text{Log}[(-((-1 + a)*b) + a*b)/(b*(a + b*x))]^2/(4*b) + (\text{Log}[(b*(-1 - a - b*x))/((-1 - a)*b + a*b)]*\text{Log}[(a*b - (1 + a)*b)/(b*(a + b*x))])/(2*b) + \text{Log}[(a*b - (1 + a)*b)/(b*(a + b*x))]^2/(4*b) + (\text{Log}[(-((-1 + a)*b) + a*b)/(b*(a + b*x))]*\text{Log}[(-1 + a + b*x)/(a + b*x)])/(2*b) - (\text{Log}[(a*b - (1 + a)*b)/(b*(a + b*x))]*\text{Log}[(1 + a + b*x)/(a + b*x)])/(2*b) - \text{PolyLog}[2, -a - b*x]/(2*b) + \text{PolyLog}[2, a + b*x]/(2*b)$

**Maple [A]** time = 0.042, size = 59, normalized size = 1.7

$$\frac{\ln(bx + a) \operatorname{arccoth}(bx + a)}{b} - \frac{\operatorname{dilog}(bx + a)}{2b} - \frac{\operatorname{dilog}(bx + a + 1)}{2b} - \frac{\ln(bx + a) \ln(bx + a + 1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(b\*x+a)/(b\*x+a), x)

[Out]  $1/b*\ln(b*x+a)*\operatorname{arccoth}(b*x+a) - 1/2/b*\operatorname{dilog}(b*x+a) - 1/2/b*\operatorname{dilog}(b*x+a+1) - 1/2/b*\ln(b*x+a)*\ln(b*x+a+1)$

**Maxima [B]** time = 0.981154, size = 151, normalized size = 4.31

$$-\frac{1}{2}b\left(\frac{\log(bx+a)\log(bx+a-1)+\operatorname{Li}_2(-bx-a+1)}{b^2}-\frac{\log(bx+a+1)\log(-bx-a)+\operatorname{Li}_2(bx+a+1)}{b^2}\right)-\frac{1}{2}\left(\frac{\log(bx+a)}{b}-\log(bx+a-1)/b\right)*\log(bx+a)+\operatorname{arccoth}(bx+a)*\log(bx+a)/b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(b\*x+a),x, algorithm="maxima")

[Out] -1/2\*b\*((log(b\*x + a)\*log(b\*x + a - 1) + dilog(-b\*x - a + 1))/b^2 - (log(b\*x + a + 1)\*log(-b\*x - a) + dilog(b\*x + a + 1))/b^2) - 1/2\*(log(b\*x + a + 1)/b - log(b\*x + a - 1)/b)\*log(b\*x + a) + arccoth(b\*x + a)\*log(b\*x + a)/b

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arccoth}(bx+a)}{bx+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(b\*x+a),x, algorithm="fricas")

[Out] integral(arccoth(b\*x + a)/(b\*x + a), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}(a+bx)}{a+bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(b\*x+a)/(b\*x+a),x)

[Out] Integral(acoth(a + b\*x)/(a + b\*x), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(bx+a)}{bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a)/(b*x + a), x)
```

$$3.99 \quad \int \frac{\coth^{-1}(a+bx)}{(a+bx)^2} dx$$

**Optimal.** Leaf size=48

$$\frac{\log(a+bx)}{b} - \frac{\log(1-(a+bx)^2)}{2b} - \frac{\coth^{-1}(a+bx)}{b(a+bx)}$$

[Out]  $-(\text{ArcCoth}[a + b*x]/(b*(a + b*x))) + \text{Log}[a + b*x]/b - \text{Log}[1 - (a + b*x)^2]/(2*b)$

**Rubi [A]** time = 0.0451322, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {6108, 5917, 266, 36, 31, 29}

$$\frac{\log(a+bx)}{b} - \frac{\log(1-(a+bx)^2)}{2b} - \frac{\coth^{-1}(a+bx)}{b(a+bx)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{ArcCoth}[a + b*x]/(a + b*x)^2, x]$

[Out]  $-(\text{ArcCoth}[a + b*x]/(b*(a + b*x))) + \text{Log}[a + b*x]/b - \text{Log}[1 - (a + b*x)^2]/(2*b)$

### Rule 6108

$\text{Int}[(a_. + \text{ArcCoth}[(c_.) + (d_.)*(x_.)]*(b_.))^{(p_.)*((e_.) + (f_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \text{Dist}[1/d, \text{Subst}[\text{Int}[(f*x)/d]^{m*(a + b*\text{ArcCoth}[x])^p}, x], x, c + d*x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, f, m\}, x$  &&  $\text{EqQ}[d*e - c*f, 0]$  &  $\text{IGtQ}[p, 0]$

### Rule 5917

$\text{Int}[(a_. + \text{ArcCoth}[(c_.)*(x_.)]*(b_.))^{(p_.)*((d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*(a + b*\text{ArcCoth}[c*x])^p/(d*(m+1)), x] - \text{Dist}[(b*c*p)/(d*(m+1)), \text{Int}[(d*x)^{(m+1)}*(a + b*\text{ArcCoth}[c*x])^{(p-1)}]/(1 - c^2*x^2), x], x] /;$   $\text{FreeQ}\{a, b, c, d, m\}, x$  &&  $\text{IGtQ}[p, 0]$  &&  $(\text{EqQ}[p, 1] \mid \mid \text{IntegerQ}[m])$  &&  $\text{NeQ}[m, -1]$

### Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

### Rule 36

```
Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))), x_Symbol] := Dist[b/(b*c
- a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

### Rule 31

```
Int[((a_) + (b_)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 29

```
Int[(x_)^( -1), x_Symbol] := Simp[Log[x], x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(a+bx)}{(a+bx)^2} dx &= \frac{\text{Subst}\left(\int \frac{\coth^{-1}(x)}{x^2} dx, x, a+bx\right)}{b} \\
&= -\frac{\coth^{-1}(a+bx)}{b(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{x(1-x^2)} dx, x, a+bx\right)}{b} \\
&= -\frac{\coth^{-1}(a+bx)}{b(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{(1-x)x} dx, x, (a+bx)^2\right)}{2b} \\
&= -\frac{\coth^{-1}(a+bx)}{b(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{1-x} dx, x, (a+bx)^2\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, (a+bx)^2\right)}{2b} \\
&= -\frac{\coth^{-1}(a+bx)}{b(a+bx)} + \frac{\log(a+bx)}{b} - \frac{\log(1-(a+bx)^2)}{2b}
\end{aligned}$$

**Mathematica [A]** time = 0.0257019, size = 43, normalized size = 0.9

$$-\frac{-2 \log(a+bx) + \log(1-(a+bx)^2) + \frac{2 \coth^{-1}(a+bx)}{a+bx}}{2b}$$

Antiderivative was successfully verified.



[In] Integrate[ArcCoth[a + b\*x]/(a + b\*x)^2,x]

[Out] -((2\*ArcCoth[a + b\*x])/(a + b\*x) - 2\*Log[a + b\*x] + Log[1 - (a + b\*x)^2])/(2\*b)

**Maple [A]** time = 0.04, size = 54, normalized size = 1.1

$$-\frac{\operatorname{arccoth}(bx+a)}{b(bx+a)} - \frac{\ln(bx+a-1)}{2b} + \frac{\ln(bx+a)}{b} - \frac{\ln(bx+a+1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(b\*x+a)/(b\*x+a)^2,x)

[Out] -arccoth(b\*x+a)/b/(b\*x+a)-1/2/b\*ln(b\*x+a-1)+ln(b\*x+a)/b-1/2/b\*ln(b\*x+a+1)

**Maxima [A]** time = 0.97605, size = 72, normalized size = 1.5

$$-\frac{\log(bx+a+1)}{2b} + \frac{\log(bx+a)}{b} - \frac{\log(bx+a-1)}{2b} - \frac{\operatorname{arccoth}(bx+a)}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(b\*x+a)^2,x, algorithm="maxima")

[Out] -1/2\*log(b\*x + a + 1)/b + log(b\*x + a)/b - 1/2\*log(b\*x + a - 1)/b - arccoth(b\*x + a)/((b\*x + a)\*b)

**Fricas [A]** time = 1.91232, size = 171, normalized size = 3.56

$$-\frac{(bx+a)\log(b^2x^2+2abx+a^2-1)-2(bx+a)\log(bx+a)+\log\left(\frac{bx+a+1}{bx+a-1}\right)}{2(b^2x+ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(b\*x+a)^2,x, algorithm="fricas")

[Out]  $-1/2*((b*x + a)*\log(b^2*x^2 + 2*a*b*x + a^2 - 1) - 2*(b*x + a)*\log(b*x + a) + \log((b*x + a + 1)/(b*x + a - 1)))/(b^2*x + a*b)$

**Sympy [A]** time = 4.82632, size = 136, normalized size = 2.83

$$\begin{cases} \frac{a \log\left(\frac{a}{b} + x\right)}{ab + b^2x} - \frac{a \log\left(\frac{a}{b} + x + \frac{1}{b}\right)}{ab + b^2x} + \frac{a \operatorname{acoth}(a + bx)}{ab + b^2x} + \frac{bx \log\left(\frac{a}{b} + x\right)}{ab + b^2x} - \frac{bx \log\left(\frac{a}{b} + x + \frac{1}{b}\right)}{ab + b^2x} + \frac{bx \operatorname{acoth}(a + bx)}{ab + b^2x} - \frac{\operatorname{acoth}(a + bx)}{ab + b^2x} & \text{for } b \neq 0 \\ \frac{x \operatorname{acoth}(a)}{a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(b*x+a)/(b*x+a)**2,x)`

[Out] `Piecewise((a*log(a/b + x)/(a*b + b**2*x) - a*log(a/b + x + 1/b)/(a*b + b**2*x) + a*acoth(a + b*x)/(a*b + b**2*x) + b*x*log(a/b + x)/(a*b + b**2*x) - b*x*log(a/b + x + 1/b)/(a*b + b**2*x) + b*x*acoth(a + b*x)/(a*b + b**2*x) - acoth(a + b*x)/(a*b + b**2*x), Ne(b, 0)), (x*acoth(a)/a**2, True))`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(bx + a)}{(bx + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(b*x+a)/(b*x+a)^2,x, algorithm="giac")`

[Out] `integrate(arccoth(b*x + a)/(b*x + a)^2, x)`

$$3.100 \quad \int \frac{\coth^{-1}(1+x)}{2+2x} dx$$

Optimal. Leaf size=25

$$\frac{1}{4}\text{PolyLog}\left(2, -\frac{1}{x+1}\right) - \frac{1}{4}\text{PolyLog}\left(2, \frac{1}{x+1}\right)$$

[Out] PolyLog[2, -(1 + x)^(-1)]/4 - PolyLog[2, (1 + x)^(-1)]/4

**Rubi [A]** time = 0.0227001, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {6108, 12, 5913}

$$\frac{1}{4}\text{PolyLog}\left(2, -\frac{1}{x+1}\right) - \frac{1}{4}\text{PolyLog}\left(2, \frac{1}{x+1}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[1 + x]/(2 + 2\*x), x]

[Out] PolyLog[2, -(1 + x)^(-1)]/4 - PolyLog[2, (1 + x)^(-1)]/4

### Rule 6108

Int[((a\_.) + ArcCoth[(c\_) + (d\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[1/d, Subst[Int[((f\*x)/d)^m\*(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d\*e - c\*f, 0] & IGtQ[p, 0]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 5913

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))/(x\_), x\_Symbol] :> Simp[a\*Log[x], x] + (Simp[(b\*PolyLog[2, -(c\*x)^(-1)])/2, x] - Simp[(b\*PolyLog[2, 1/(c\*x)])]/2, x) /; FreeQ[{a, b, c}, x]

### Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(1+x)}{2+2x} dx &= \text{Subst} \left( \int \frac{\coth^{-1}(x)}{2x} dx, x, 1+x \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{\coth^{-1}(x)}{x} dx, x, 1+x \right) \\ &= \frac{1}{4} \text{Li}_2 \left( -\frac{1}{1+x} \right) - \frac{1}{4} \text{Li}_2 \left( \frac{1}{1+x} \right) \end{aligned}$$

**Mathematica [B]** time = 0.0131804, size = 117, normalized size = 4.68

$$-\frac{1}{4} \text{PolyLog}(2, -x-1) + \frac{1}{4} \text{PolyLog}(2, x+1) + \frac{1}{8} \log^2 \left( -\frac{1}{x+1} \right) - \frac{1}{8} \log^2 \left( \frac{1}{x+1} \right) + \frac{1}{4} \log(x+2) \log \left( -\frac{1}{x+1} \right) - \frac{1}{4} \log \left( \frac{1}{x+1} \right) \log(x+2)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[1 + x]/(2 + 2\*x), x]

[Out] Log[-(1 + x)^(-1)]^2/8 - (Log[-x]\*Log[(1 + x)^(-1)])/4 - Log[(1 + x)^(-1)]^2/8 + (Log[(1 + x)^(-1)]\*Log[x/(1 + x)])/4 + (Log[-(1 + x)^(-1)]\*Log[2 + x])/4 - (Log[-(1 + x)^(-1)]\*Log[(2 + x)/(1 + x)])/4 - PolyLog[2, -1 - x]/4 + PolyLog[2, 1 + x]/4

**Maple [A]** time = 0.029, size = 34, normalized size = 1.4

$$\frac{\ln(1+x) \operatorname{arccoth}(1+x)}{2} - \frac{\operatorname{dilog}(1+x)}{4} - \frac{\operatorname{dilog}(x+2)}{4} - \frac{\ln(1+x) \ln(x+2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(1+x)/(2+2\*x), x)

[Out] 1/2\*ln(1+x)\*arccoth(1+x)-1/4\*dilog(1+x)-1/4\*dilog(x+2)-1/4\*ln(1+x)\*ln(x+2)

**Maxima [B]** time = 0.978667, size = 78, normalized size = 3.12

$$-\frac{1}{4} (\log(x+2) - \log(x)) \log(x+1) + \frac{1}{2} \operatorname{arccoth}(x+1) \log(x+1) - \frac{1}{4} \log(x+1) \log(x) + \frac{1}{4} \log(x+2) \log(-x-1) -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+x)/(2+2\*x),x, algorithm="maxima")

[Out]  $-1/4*(\log(x + 2) - \log(x))*\log(x + 1) + 1/2*\operatorname{arccoth}(x + 1)*\log(x + 1) - 1/4*\log(x + 1)*\log(x) + 1/4*\log(x + 2)*\log(-x - 1) - 1/4*\operatorname{dilog}(-x) + 1/4*\operatorname{dilog}(x + 2)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arccoth}(x + 1)}{2(x + 1)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+x)/(2+2\*x),x, algorithm="fricas")

[Out] integral(1/2\*arccoth(x + 1)/(x + 1), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{acoth}(x+1)}{x+1} dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(1+x)/(2+2\*x),x)

[Out] Integral(acoth(x + 1)/(x + 1), x)/2

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(x + 1)}{2(x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+x)/(2+2*x),x, algorithm="giac")
```

```
[Out] integrate(1/2*arccoth(x + 1)/(x + 1), x)
```

$$3.101 \quad \int \frac{\coth^{-1}(a+bx)}{\frac{ad}{b}+dx} dx$$

**Optimal.** Leaf size=35

$$\frac{\text{PolyLog}\left(2, -\frac{1}{a+bx}\right)}{2d} - \frac{\text{PolyLog}\left(2, \frac{1}{a+bx}\right)}{2d}$$

[Out] PolyLog[2, -(a + b\*x)^(-1)]/(2\*d) - PolyLog[2, (a + b\*x)^(-1)]/(2\*d)

**Rubi [A]** time = 0.0301393, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {6108, 12, 5913}

$$\frac{\text{PolyLog}\left(2, -\frac{1}{a+bx}\right)}{2d} - \frac{\text{PolyLog}\left(2, \frac{1}{a+bx}\right)}{2d}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a + b\*x]/((a\*d)/b + d\*x), x]

[Out] PolyLog[2, -(a + b\*x)^(-1)]/(2\*d) - PolyLog[2, (a + b\*x)^(-1)]/(2\*d)

### Rule 6108

Int[((a\_.) + ArcCoth[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Dist[1/d, Subst[Int[((f\*x)/d)^m\*(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d\*e - c\*f, 0] && IGtQ[p, 0]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 5913

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))/(x\_), x\_Symbol] :> Simp[a\*Log[x], x] + (Simp[(b\*PolyLog[2, -(c\*x)^(-1)])/2, x] - Simp[(b\*PolyLog[2, 1/(c\*x)])/2, x]) /; FreeQ[{a, b, c}, x]

Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(a+bx)}{\frac{ad}{b} + dx} dx &= \frac{\text{Subst}\left(\int \frac{b \coth^{-1}(x)}{dx} dx, x, a+bx\right)}{b} \\ &= \frac{\text{Subst}\left(\int \frac{\coth^{-1}(x)}{x} dx, x, a+bx\right)}{d} \\ &= \frac{\text{Li}_2\left(-\frac{1}{a+bx}\right)}{2d} - \frac{\text{Li}_2\left(\frac{1}{a+bx}\right)}{2d} \end{aligned}$$

**Mathematica [B]** time = 0.023135, size = 312, normalized size = 8.91

$$b \left( -\frac{\text{PolyLog}(2, -a-bx)}{2bd} + \frac{\text{PolyLog}(2, a+bx)}{2bd} - \frac{\log^2\left(\frac{ab-(a-1)b}{b(a+bx)}\right)}{4bd} + \frac{\log^2\left(\frac{ab-(a+1)b}{b(a+bx)}\right)}{4bd} - \frac{\log\left(\frac{b(a+bx-1)}{(a-1)b-ab}\right) \log\left(\frac{ab-(a-1)b}{b(a+bx)}\right)}{2bd} + \dots \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a + b\*x]/((a\*d)/b + d\*x), x]

[Out]  $b * (-\text{Log}[(b * (-1 + a + b * x)) / ((-1 + a) * b - a * b)] * \text{Log}[(-((-1 + a) * b) + a * b) / (b * (a + b * x))]) / (2 * b * d) - \text{Log}[(-((-1 + a) * b) + a * b) / (b * (a + b * x))]^2 / (4 * b * d) + (\text{Log}[(b * (-1 - a - b * x)) / ((-1 - a) * b + a * b)] * \text{Log}[(a * b - (1 + a) * b) / (b * (a + b * x))]) / (2 * b * d) + \text{Log}[(a * b - (1 + a) * b) / (b * (a + b * x))]^2 / (4 * b * d) + (\text{Log}[(-((-1 + a) * b) + a * b) / (b * (a + b * x))] * \text{Log}[(-1 + a + b * x) / (a + b * x)]) / (2 * b * d) - (\text{Log}[(a * b - (1 + a) * b) / (b * (a + b * x))] * \text{Log}[(1 + a + b * x) / (a + b * x)]) / (2 * b * d) - \text{PolyLog}[2, -a - b * x] / (2 * b * d) + \text{PolyLog}[2, a + b * x] / (2 * b * d)$

**Maple [A]** time = 0.044, size = 59, normalized size = 1.7

$$\frac{\ln(bx+a) \operatorname{arccoth}(bx+a)}{d} - \frac{\operatorname{dilog}(bx+a)}{2d} - \frac{\operatorname{dilog}(bx+a+1)}{2d} - \frac{\ln(bx+a) \ln(bx+a+1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(b\*x+a)/(a\*d/b+d\*x), x)



[Out]  $\frac{1}{d} \ln(bx+a) \operatorname{arccoth}(bx+a) - \frac{1}{2d} \operatorname{dilog}(bx+a) - \frac{1}{2d} \operatorname{dilog}(bx+a+1) - \frac{1}{2d} \ln(bx+a) \ln(bx+a+1)$

**Maxima [B]** time = 0.98962, size = 178, normalized size = 5.09

$$-\frac{1}{2} b \left( \frac{\log(bx+a) \log(bx+a-1) + \operatorname{Li}_2(-bx-a+1)}{bd} - \frac{\log(bx+a+1) \log(-bx-a) + \operatorname{Li}_2(bx+a+1)}{bd} \right) - b \left( \frac{\log(bx+a)}{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")`

[Out]  $-\frac{1}{2} b \left( \frac{\log(bx+a) \log(bx+a-1) + \operatorname{dilog}(-bx-a+1)}{b*d} - \frac{\log(bx+a+1) \log(-bx-a) + \operatorname{dilog}(bx+a+1)}{b*d} \right) - \frac{1}{2} b \left( \frac{\log(bx+a+1)}{b} - \frac{\log(bx+a-1)}{b} \right) \log(d*x + a*d/b) / d + \operatorname{arccoth}(bx+a) \log(d*x + a*d/b) / d$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{b \operatorname{arccoth}(bx+a)}{bdx+ad}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")`

[Out] `integral(b*arccoth(b*x + a)/(b*d*x + a*d), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{b \int \frac{\operatorname{acoth}(a+bx)}{a+bx} dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(b*x+a)/(a*d/b+d*x),x)`

[Out] `b*Integral(acoth(a + b*x)/(a + b*x), x)/d`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(bx + a)}{dx + \frac{ad}{b}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(b*x+a)/(a*d/b+d*x),x, algorithm="giac")`

[Out] `integrate(arccoth(b*x + a)/(d*x + a*d/b), x)`

### 3.102 $\int (e + fx)^3 (a + b \coth^{-1}(c + dx)) dx$

**Optimal.** Leaf size=168

$$\frac{(e + fx)^4 (a + b \coth^{-1}(c + dx))}{4f} + \frac{bfx((6c^2 + 1)f^2 - 12cdef + 6d^2e^2)}{4d^3} + \frac{bf^2(c + dx)^2(de - cf)}{2d^4} - \frac{b(-cf + de - f)^4 \log}{8d^4 f}$$

[Out] (b\*f\*(6\*d^2\*e^2 - 12\*c\*d\*e\*f + (1 + 6\*c^2)\*f^2)\*x)/(4\*d^3) + (b\*f^2\*(d\*e - c\*f)\*(c + d\*x)^2)/(2\*d^4) + (b\*f^3\*(c + d\*x)^3)/(12\*d^4) + ((e + f\*x)^4\*(a + b\*ArcCoth[c + d\*x]))/(4\*f) + (b\*(d\*e + f - c\*f)^4\*Log[1 - c - d\*x])/(8\*d^4\*f) - (b\*(d\*e - f - c\*f)^4\*Log[1 + c + d\*x])/(8\*d^4\*f)

**Rubi [A]** time = 0.341981, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {6112, 5927, 702, 633, 31}

$$\frac{(e + fx)^4 (a + b \coth^{-1}(c + dx))}{4f} + \frac{bfx((6c^2 + 1)f^2 - 12cdef + 6d^2e^2)}{4d^3} + \frac{bf^2(c + dx)^2(de - cf)}{2d^4} - \frac{b(-cf + de - f)^4 \log}{8d^4 f}$$

Antiderivative was successfully verified.

[In] Int[(e + f\*x)^3\*(a + b\*ArcCoth[c + d\*x]),x]

[Out] (b\*f\*(6\*d^2\*e^2 - 12\*c\*d\*e\*f + (1 + 6\*c^2)\*f^2)\*x)/(4\*d^3) + (b\*f^2\*(d\*e - c\*f)\*(c + d\*x)^2)/(2\*d^4) + (b\*f^3\*(c + d\*x)^3)/(12\*d^4) + ((e + f\*x)^4\*(a + b\*ArcCoth[c + d\*x]))/(4\*f) + (b\*(d\*e + f - c\*f)^4\*Log[1 - c - d\*x])/(8\*d^4\*f) - (b\*(d\*e - f - c\*f)^4\*Log[1 + c + d\*x])/(8\*d^4\*f)

#### Rule 6112

Int[((a\_.) + ArcCoth[(c\_.) + (d\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]

#### Rule 5927

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))\*((d\_.) + (e\_.)\*(x\_))^(q\_.), x\_Symbol] := Simp[((d + e\*x)^(q + 1)\*(a + b\*ArcCoth[c\*x]))/(e\*(q + 1)), x] - Dist[(b\*c)/(e\*(q + 1)), Int[(d + e\*x)^(q + 1)/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]

Rule 702

Int[((d\_) + (e\_)\*(x\_))^(m\_)/((a\_) + (c\_)\*(x\_)^2), x\_Symbol] := Int[PolynomialDivide[(d + e\*x)^m, a + c\*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])

Rule 633

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (c\_)\*(x\_)^2), x\_Symbol] := With[{q = Rt[-(a\*c), 2]}, Dist[e/2 + (c\*d)/(2\*q), Int[1/(-q + c\*x), x], x] + Dist[e/2 - (c\*d)/(2\*q), Int[1/(q + c\*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a\*c)]

Rule 31

Int[((a\_) + (b\_)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\begin{aligned}
 \int (e + fx)^3 (a + b \coth^{-1}(c + dx)) dx &= \frac{\text{Subst} \left( \int \left( \frac{de - cf}{d} + \frac{fx}{d} \right)^3 (a + b \coth^{-1}(x)) dx, x, c + dx \right)}{d} \\
 &= \frac{(e + fx)^4 (a + b \coth^{-1}(c + dx))}{4f} - \frac{b \text{Subst} \left( \int \frac{\left( \frac{de - cf}{d} + \frac{fx}{d} \right)^4}{1 - x^2} dx, x, c + dx \right)}{4f} \\
 &= \frac{(e + fx)^4 (a + b \coth^{-1}(c + dx))}{4f} - \frac{b \text{Subst} \left( \int \left( -\frac{f^2(6d^2e^2 - 12cdef + (1 + 6c^2)f^2)}{d^4} - \frac{4f^3(c + dx)}{d^4} \right) dx, x, c + dx \right)}{4f} \\
 &= \frac{bf(6d^2e^2 - 12cdef + (1 + 6c^2)f^2)x}{4d^3} + \frac{bf^2(de - cf)(c + dx)^2}{2d^4} + \frac{bf^3(c + dx)^3}{12d^4} + \\
 &= \frac{bf(6d^2e^2 - 12cdef + (1 + 6c^2)f^2)x}{4d^3} + \frac{bf^2(de - cf)(c + dx)^2}{2d^4} + \frac{bf^3(c + dx)^3}{12d^4} + \\
 &= \frac{bf(6d^2e^2 - 12cdef + (1 + 6c^2)f^2)x}{4d^3} + \frac{bf^2(de - cf)(c + dx)^2}{2d^4} + \frac{bf^3(c + dx)^3}{12d^4} +
 \end{aligned}$$

**Mathematica [A]** time = 0.281329, size = 270, normalized size = 1.61

$$6dx(4ad^3e^3 + bf((3c^2 + 1)f^2 - 8cdef + 6d^2e^2)) + 6d^2fx^2(6ad^2e^2 + bf(2de - cf)) + 2d^3f^2x^3(12ade + bf) + 6ad^4f^3x^4$$

Antiderivative was successfully verified.

[In] Integrate[(e + f\*x)^3\*(a + b\*ArcCoth[c + d\*x]), x]

[Out] (6\*d\*(4\*a\*d^3\*e^3 + b\*f\*(6\*d^2\*e^2 - 8\*c\*d\*e\*f + (1 + 3\*c^2)\*f^2))\*x + 6\*d^2\*f\*(6\*a\*d^2\*e^2 + b\*f\*(2\*d\*e - c\*f))\*x^2 + 2\*d^3\*f^2\*(12\*a\*d\*e + b\*f)\*x^3 + 6\*a\*d^4\*f^3\*x^4 + 6\*b\*d^4\*x\*(4\*e^3 + 6\*e^2\*f\*x + 4\*e\*f^2\*x^2 + f^3\*x^3)\*ArcCoth[c + d\*x] - 3\*b\*(-1 + c)\*(4\*d^3\*e^3 - 6\*(-1 + c)\*d^2\*e^2\*f + 4\*(-1 + c)^2\*d\*e\*f^2 - (-1 + c)^3\*f^3)\*Log[1 - c - d\*x] - 3\*b\*(1 + c)\*(-4\*d^3\*e^3 + 6\*(1 + c)\*d^2\*e^2\*f - 4\*(1 + c)^2\*d\*e\*f^2 + (1 + c)^3\*f^3)\*Log[1 + c + d\*x] / (24\*d^4)

**Maple [B]** time = 0.046, size = 786, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^3\*(a+b\*arccoth(d\*x+c)), x)

[Out] 1/4\*a/f\*e^4+13/12/d^4\*b\*f^3\*c^3+1/4/d^4\*b\*f^3\*c+3/2/d^2\*b\*f\*c\*e^2-5/2/d^3\*b\*f^2\*c^2\*e-1/2/d\*b\*ln(d\*x+c-1)\*c\*e^3+1/2/d\*b\*ln(d\*x+c+1)\*c\*e^3-1/2/d^4\*b\*f^3\*ln(d\*x+c-1)\*c^3-1/8/d^4\*b\*f^3\*ln(d\*x+c+1)\*c^4-3/4/d^2\*b\*f\*ln(d\*x+c+1)\*e^2+1/2/d^3\*b\*f^2\*ln(d\*x+c-1)\*e+1/4\*a\*f^3\*x^4+a\*x\*e^3+1/12/d\*b\*f^3\*x^3+1/2/d\*b\*ln(d\*x+c-1)\*e^3+1/2/d\*b\*ln(d\*x+c+1)\*e^3-1/8/d^4\*b\*f^3\*ln(d\*x+c+1)+1/8/d^4\*b\*f^3\*ln(d\*x+c-1)+1/4\*b\*f^3\*arccoth(d\*x+c)\*x^4+arccoth(d\*x+c)\*x\*b\*e^3-1/8\*b/f\*ln(d\*x+c+1)\*e^4+1/4\*b/f\*arccoth(d\*x+c)\*e^4+1/4\*b/d^3\*f^3\*x+1/8\*b/f\*ln(d\*x+c-1)\*e^4+a\*f^2\*x^3\*e+3/2\*a\*f\*x^2\*e^2+3/4\*b/d^3\*f^3\*c^2\*x+1/2/d^3\*b\*f^2\*ln(d\*x+c+1)\*e+3/4/d^2\*b\*f\*ln(d\*x+c-1)\*e^2+1/8/d^4\*b\*f^3\*ln(d\*x+c-1)\*c^4-1/2/d^4\*b\*f^3\*ln(d\*x+c+1)\*c-3/4/d^4\*b\*f^3\*ln(d\*x+c+1)\*c^2-1/2/d^4\*b\*f^3\*ln(d\*x+c+1)\*c^3-1/2/d^4\*b\*f^3\*ln(d\*x+c-1)\*c+3/4/d^4\*b\*f^3\*ln(d\*x+c-1)\*c^2+b\*f^2\*arccoth(d\*x+c)\*e\*x^3+3/2\*b\*f\*arccoth(d\*x+c)\*e^2\*x^2-1/4/d^2\*b\*f^3\*x^2\*c+1/2/d\*b\*f^2\*e\*x^2+3/2\*b/d\*f\*e^2\*x+3/2/d^3\*b\*f^2\*ln(d\*x+c+1)\*c\*e+3/2/d^3\*b\*f^2\*ln(d\*x+c-1)\*c^2\*e+1/2/d^3\*b\*f^2\*ln(d\*x+c+1)\*c^3\*e-3/2/d^2\*b\*f\*ln(d\*x+c+1)\*c\*e^2-3/4/d^2\*b\*f\*ln(d\*x+c+1)\*c^2\*e^2+3/2/d^3\*b\*f^2\*ln(d\*x+c+1)\*c^2\*e+3/4/d^2\*b\*f\*ln(d\*x+c-1)\*c^2\*e^2-2\*b/d^2\*f^2\*c\*e\*x-3/2/d^2\*b\*f\*ln(d\*x+c-1)\*c\*e^2-1/2/

$$d^3 b f^2 \ln(dx+c-1) c^3 e^{-3/2} / d^3 b f^2 \ln(dx+c-1) c e$$

**Maxima [B]** time = 0.992158, size = 450, normalized size = 2.68

$$\frac{1}{4} a f^3 x^4 + a e f^2 x^3 + \frac{3}{2} a e^2 f x^2 + \frac{3}{4} \left( 2 x^2 \operatorname{arccoth}(dx+c) + d \left( \frac{2x}{d^2} - \frac{(c^2 + 2c + 1) \log(dx+c+1)}{d^3} + \frac{(c^2 - 2c + 1) \log(dx+c-1)}{d^3} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*(a+b\*arccoth(d\*x+c)),x, algorithm="maxima")

[Out] 1/4\*a\*f^3\*x^4 + a\*e\*f^2\*x^3 + 3/2\*a\*e^2\*f\*x^2 + 3/4\*(2\*x^2\*arccoth(d\*x + c) + d\*(2\*x/d^2 - (c^2 + 2\*c + 1)\*log(d\*x + c + 1)/d^3 + (c^2 - 2\*c + 1)\*log(d\*x + c - 1)/d^3))\*b\*e^2\*f + 1/2\*(2\*x^3\*arccoth(d\*x + c) + d\*((d\*x^2 - 4\*c\*x)/d^3 + (c^3 + 3\*c^2 + 3\*c + 1)\*log(d\*x + c + 1)/d^4 - (c^3 - 3\*c^2 + 3\*c - 1)\*log(d\*x + c - 1)/d^4))\*b\*e\*f^2 + 1/24\*(6\*x^4\*arccoth(d\*x + c) + d\*(2\*(d^2\*x^3 - 3\*c\*d\*x^2 + 3\*(3\*c^2 + 1)\*x)/d^4 - 3\*(c^4 + 4\*c^3 + 6\*c^2 + 4\*c + 1)\*log(d\*x + c + 1)/d^5 + 3\*(c^4 - 4\*c^3 + 6\*c^2 - 4\*c + 1)\*log(d\*x + c - 1)/d^5))\*b\*f^3 + a\*e^3\*x + 1/2\*(2\*(d\*x + c)\*arccoth(d\*x + c) + log(-(d\*x + c)^2 + 1))\*b\*e^3/d

**Fricas [B]** time = 2.11405, size = 856, normalized size = 5.1

$$6ad^4f^3x^4 + 2(12ad^4ef^2 + bd^3f^3)x^3 + 6(6ad^4e^2f + 2bd^3ef^2 - bcd^2f^3)x^2 + 6(4ad^4e^3 + 6bd^3e^2f - 8bcd^2ef^2 + (3bc^2 + 3c^3 - 3c^2 - 3c + 1)e^2) + (3bc^2 + 3c^3 - 3c^2 - 3c + 1)e^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*(a+b\*arccoth(d\*x+c)),x, algorithm="fricas")

[Out] 1/24\*(6\*a\*d^4\*f^3\*x^4 + 2\*(12\*a\*d^4\*e\*f^2 + b\*d^3\*f^3)\*x^3 + 6\*(6\*a\*d^4\*e^2\*f + 2\*b\*d^3\*e\*f^2 - b\*c\*d^2\*f^3)\*x^2 + 6\*(4\*a\*d^4\*e^3 + 6\*b\*d^3\*e^2\*f - 8\*b\*c\*d^2\*e\*f^2 + (3\*b\*c^2 + b)\*d\*f^3)\*x + 3\*(4\*(b\*c + b)\*d^3\*e^3 - 6\*(b\*c^2 + 2\*b\*c + b)\*d^2\*e^2\*f + 4\*(b\*c^3 + 3\*b\*c^2 + 3\*b\*c + b)\*d\*e\*f^2 - (b\*c^4 + 4\*b\*c^3 + 6\*b\*c^2 + 4\*b\*c + b)\*f^3)\*log(d\*x + c + 1) - 3\*(4\*(b\*c - b)\*d^3\*e^3 - 6\*(b\*c^2 - 2\*b\*c + b)\*d^2\*e^2\*f + 4\*(b\*c^3 - 3\*b\*c^2 + 3\*b\*c - b)\*d\*e\*f^2 - (b\*c^4 - 4\*b\*c^3 + 6\*b\*c^2 - 4\*b\*c + b)\*f^3)\*log(d\*x + c - 1) + 3\*(b\*d^4\*f^3\*x^4 + 4\*b\*d^4\*e\*f^2\*x^3 + 6\*b\*d^4\*e^2\*f\*x^2 + 4\*b\*d^4\*e^3\*x)\*log((

$d*x + c + 1)/(d*x + c - 1))/d^4$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*(a+b\*acoth(d\*x+c)),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (fx + e)^3 (b \operatorname{arccoth}(dx + c) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*(a+b\*arccoth(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)^3\*(b\*arccoth(d\*x + c) + a), x)

### 3.103 $\int (e + fx)^2 (a + b \coth^{-1}(c + dx)) dx$

**Optimal.** Leaf size=120

$$\frac{(e + fx)^3 (a + b \coth^{-1}(c + dx))}{3f} + \frac{bfx(de - cf)}{d^2} + \frac{b(-cf + de + f)^3 \log(-c - dx + 1)}{6d^3 f} - \frac{b(de - (c + 1)f)^3 \log(c + dx + 1)}{6d^3 f}$$

[Out] (b\*f\*(d\*e - c\*f)\*x)/d^2 + (b\*f^2\*(c + d\*x)^2)/(6\*d^3) + ((e + f\*x)^3\*(a + b\*ArcCoth[c + d\*x]))/(3\*f) + (b\*(d\*e + f - c\*f)^3\*Log[1 - c - d\*x])/(6\*d^3\*f) - (b\*(d\*e - (1 + c)\*f)^3\*Log[1 + c + d\*x])/(6\*d^3\*f)

**Rubi [A]** time = 0.203979, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {6112, 5927, 702, 633, 31}

$$\frac{(e + fx)^3 (a + b \coth^{-1}(c + dx))}{3f} + \frac{bfx(de - cf)}{d^2} + \frac{b(-cf + de + f)^3 \log(-c - dx + 1)}{6d^3 f} - \frac{b(de - (c + 1)f)^3 \log(c + dx + 1)}{6d^3 f}$$

Antiderivative was successfully verified.

[In] Int[(e + f\*x)^2\*(a + b\*ArcCoth[c + d\*x]),x]

[Out] (b\*f\*(d\*e - c\*f)\*x)/d^2 + (b\*f^2\*(c + d\*x)^2)/(6\*d^3) + ((e + f\*x)^3\*(a + b\*ArcCoth[c + d\*x]))/(3\*f) + (b\*(d\*e + f - c\*f)^3\*Log[1 - c - d\*x])/(6\*d^3\*f) - (b\*(d\*e - (1 + c)\*f)^3\*Log[1 + c + d\*x])/(6\*d^3\*f)

#### Rule 6112

Int[((a\_.) + ArcCoth[(c\_) + (d\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]

#### Rule 5927

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))\*((d\_) + (e\_.)\*(x\_))^(q\_.), x\_Symbol] := Simp[((d + e\*x)^(q + 1)\*(a + b\*ArcCoth[c\*x]))/(e\*(q + 1)), x] - Dist[(b\*c)/(e\*(q + 1)), Int[(d + e\*x)^(q + 1)/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]

#### Rule 702



```
Int[((d_) + (e_)*(x_))^(m_)/((a_) + (c_)*(x_)^2), x_Symbol] := Int[PolynomialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

### Rule 633

```
Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a*c)]
```

### Rule 31

```
Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rubi steps

$$\begin{aligned}
 \int (e + fx)^2 (a + b \coth^{-1}(c + dx)) dx &= \frac{\text{Subst} \left( \int \left( \frac{de - cf}{d} + \frac{fx}{d} \right)^2 (a + b \coth^{-1}(x)) dx, x, c + dx \right)}{d} \\
 &= \frac{(e + fx)^3 (a + b \coth^{-1}(c + dx))}{3f} - \frac{b \text{Subst} \left( \int \frac{\left( \frac{de - cf}{d} + \frac{fx}{d} \right)^3}{1 - x^2} dx, x, c + dx \right)}{3f} \\
 &= \frac{(e + fx)^3 (a + b \coth^{-1}(c + dx))}{3f} - \frac{b \text{Subst} \left( \int \left( -\frac{3f^2(de - cf)}{d^3} - \frac{f^3x}{d^3} + \frac{(de - cf)(d^2e^2 - c^2)}{d^3} \right) dx, x, c + dx \right)}{3f} \\
 &= \frac{bf(de - cf)x}{d^2} + \frac{bf^2(c + dx)^2}{6d^3} + \frac{(e + fx)^3 (a + b \coth^{-1}(c + dx))}{3f} - \frac{b \text{Subst} \left( \int \frac{1}{1 - x^2} dx, x, c + dx \right)}{3f} \\
 &= \frac{bf(de - cf)x}{d^2} + \frac{bf^2(c + dx)^2}{6d^3} + \frac{(e + fx)^3 (a + b \coth^{-1}(c + dx))}{3f} - \frac{(b(de + f - c) \coth^{-1}(c + dx) + \frac{1}{2} \ln \left| \frac{c + dx - 1}{c + dx + 1} \right|)}{3f} \\
 &= \frac{bf(de - cf)x}{d^2} + \frac{bf^2(c + dx)^2}{6d^3} + \frac{(e + fx)^3 (a + b \coth^{-1}(c + dx))}{3f} + \frac{b(de + f - c) \coth^{-1}(c + dx) - \frac{1}{2} \ln \left| \frac{c + dx - 1}{c + dx + 1} \right|}{3f}
 \end{aligned}$$

**Mathematica [A]** time = 0.161644, size = 174, normalized size = 1.45

$$\frac{2dx(3ad^2e^2 + bf(3de - 2cf)) + d^2fx^2(6ade + bf) + 2ad^3f^2x^3 + 2bd^3x(3e^2 + 3efx + f^2x^2) \coth^{-1}(c + dx) - b(c - 1) \ln \left| \frac{c + dx - 1}{c + dx + 1} \right|}{6d^3}$$

Antiderivative was successfully verified.

[In] Integrate[(e + f\*x)^2\*(a + b\*ArcCoth[c + d\*x]),x]

[Out] (2\*d\*(3\*a\*d^2\*e^2 + b\*f\*(3\*d\*e - 2\*c\*f))\*x + d^2\*f\*(6\*a\*d\*e + b\*f)\*x^2 + 2\*a\*d^3\*f^2\*x^3 + 2\*b\*d^3\*x\*(3\*e^2 + 3\*e\*f\*x + f^2\*x^2)\*ArcCoth[c + d\*x] - b\*(-1 + c)\*(3\*d^2\*e^2 - 3\*(-1 + c)\*d\*e\*f + (-1 + c)^2\*f^2)\*Log[1 - c - d\*x] + b\*(1 + c)\*(3\*d^2\*e^2 - 3\*(1 + c)\*d\*e\*f + (1 + c)^2\*f^2)\*Log[1 + c + d\*x])/(6\*d^3)

**Maple [B]** time = 0.04, size = 477, normalized size = 4.

$$\frac{bf \ln(dx + c - 1) c^2 e}{2 d^2} + \frac{af^2 x^3}{3} + axe^2 - \frac{2bf^2 cx}{3 d^2} + \frac{bf^2 \ln(dx + c + 1)}{6 d^3} + \frac{b \ln(dx + c - 1) e^2}{2 d} + afx^2 e - \frac{b \ln(dx + c + 1) e^2}{6 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*(a+b\*arccoth(d\*x+c)),x)

[Out] 1/2/d^2\*b\*f\*ln(d\*x+c-1)\*c^2\*e+1/3\*a\*f^2\*x^3+a\*x\*e^2-2/3\*b/d^2\*f^2\*c\*x+1/6/d^3\*b\*f^2\*ln(d\*x+c+1)+1/2/d\*b\*ln(d\*x+c-1)\*e^2+a\*f\*x^2\*e-1/6\*b/f\*ln(d\*x+c+1)\*e^3+1/6\*b/f\*ln(d\*x+c-1)\*e^3+1/3\*b/f\*arccoth(d\*x+c)\*e^3+arccoth(d\*x+c)\*x\*b\*e^2+1/3\*b\*f^2\*arccoth(d\*x+c)\*x^3+1/2/d\*b\*ln(d\*x+c+1)\*e^2+1/6/d^3\*b\*f^2\*ln(d\*x+c-1)+1/6/d\*b\*f^2\*x^2-1/2/d^2\*b\*f\*ln(d\*x+c+1)\*c^2\*e-1/d^2\*b\*f\*ln(d\*x+c+1)\*c\*e-1/d^2\*b\*f\*ln(d\*x+c-1)\*c\*e+1/3\*a/f\*e^3-5/6/d^3\*b\*f^2\*c^2+1/d^2\*b\*f\*c\*e+b/d\*f\*e\*x+1/6/d^3\*b\*f^2\*ln(d\*x+c+1)\*c^3+1/2/d^3\*b\*f^2\*ln(d\*x+c+1)\*c^2-1/2/d^3\*b\*f^2\*ln(d\*x+c-1)\*c-1/6/d^3\*b\*f^2\*ln(d\*x+c-1)\*c^3+1/2/d^3\*b\*f^2\*ln(d\*x+c+1)\*c-1/2/d^2\*b\*f\*ln(d\*x+c+1)\*e+1/2/d^2\*b\*f\*ln(d\*x+c-1)\*e+1/2/d^3\*b\*f^2\*ln(d\*x+c-1)\*c^2+b\*f\*arccoth(d\*x+c)\*e\*x^2-1/2/d\*b\*ln(d\*x+c-1)\*c\*e^2+1/2/d\*b\*ln(d\*x+c+1)\*c\*e^2

**Maxima [A]** time = 0.970765, size = 279, normalized size = 2.32

$$\frac{1}{3} af^2 x^3 + aefx^2 + \frac{1}{2} \left( 2x^2 \operatorname{arccoth}(dx + c) + d \left( \frac{2x}{d^2} - \frac{(c^2 + 2c + 1) \log(dx + c + 1)}{d^3} + \frac{(c^2 - 2c + 1) \log(dx + c - 1)}{d^3} \right) \right) be$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*(a+b\*arccoth(d\*x+c)),x, algorithm="maxima")

```
[Out] 1/3*a*f^2*x^3 + a*e*f*x^2 + 1/2*(2*x^2*arccoth(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*b*e*f + 1/6*(2*x^3*arccoth(d*x + c) + d*((d*x^2 - 4*c*x)/d^3 + (c^3 + 3*c^2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3*c - 1)*log(d*x + c - 1)/d^4))*b*f^2 + a*e^2*x + 1/2*(2*(d*x + c)*arccoth(d*x + c) + log(-(d*x + c)^2 + 1))*b*e^2/d
```

**Fricas [B]** time = 1.96481, size = 548, normalized size = 4.57

$$2ad^3f^2x^3 + (6ad^3ef + bd^2f^2)x^2 + 2(3ad^3e^2 + 3bd^2ef - 2bcd^2f^2)x + (3(bc + b)d^2e^2 - 3(bc^2 + 2bc + b)def + (bc^3 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccoth(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*d^3*f^2*x^3 + (6*a*d^3*e*f + b*d^2*f^2)*x^2 + 2*(3*a*d^3*e^2 + 3*b*d^2*e*f - 2*b*c*d*f^2)*x + (3*(b*c + b)*d^2*e^2 - 3*(b*c^2 + 2*b*c + b)*d*e*f + (b*c^3 + 3*b*c^2 + 3*b*c + b)*f^2)*log(d*x + c + 1) - (3*(b*c - b)*d^2*e^2 - 3*(b*c^2 - 2*b*c + b)*d*e*f + (b*c^3 - 3*b*c^2 + 3*b*c - b)*f^2)*log(d*x + c - 1) + (b*d^3*f^2*x^3 + 3*b*d^3*e*f*x^2 + 3*b*d^3*e^2*x)*log((d*x + c + 1)/(d*x + c - 1))/d^3
```

**Sympy [A]** time = 18.1172, size = 369, normalized size = 3.08

$$\left\{ \begin{array}{l} ae^2x + aefx^2 + \frac{af^2x^3}{3} + \frac{bc^3f^2 \operatorname{acoth}(c+dx)}{3d^3} - \frac{bc^2ef \operatorname{acoth}(c+dx)}{d^2} + \frac{bc^2f^2 \log\left(\frac{c}{d} + x + \frac{1}{d}\right)}{d^3} - \frac{bc^2f^2 \operatorname{acoth}(c+dx)}{d^3} + \frac{bce^2 \operatorname{acoth}(c+dx)}{d} - \frac{2bc^2ef \log\left(\frac{c}{d} + x + \frac{1}{d}\right)}{d^3} \\ (a + b \operatorname{acoth}(c)) \left( e^2x + efx^2 + \frac{f^2x^3}{3} \right) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(a+b*acoth(d*x+c)),x)
```

```
[Out] Piecewise((a*e**2*x + a*e*f*x**2 + a*f**2*x**3/3 + b*c**3*f**2*acoth(c + d*x)/(3*d**3) - b*c**2*e*f*acoth(c + d*x)/d**2 + b*c**2*f**2*log(c/d + x + 1/d)/d**3 - b*c**2*f**2*acoth(c + d*x)/d**3 + b*c*e**2*acoth(c + d*x)/d - 2*b*c*e*f*log(c/d + x + 1/d)/d**2 + 2*b*c*e*f*acoth(c + d*x)/d**2 - 2*b*c*f**2*x/(3*d**2) + b*c*f**2*acoth(c + d*x)/d**3 + b*e**2*x*acoth(c + d*x) + b*e*f*x**2*acoth(c + d*x) + b*f**2*x**3*acoth(c + d*x)/3 + b*e**2*log(c/d + x +
```

```

1/d)/d - b*e**2*acoth(c + d*x)/d + b*e*f*x/d + b*f**2*x**2/(6*d) - b*e*f*a
coth(c + d*x)/d**2 + b*f**2*log(c/d + x + 1/d)/(3*d**3) - b*f**2*acoth(c +
d*x)/(3*d**3), Ne(d, 0)), ((a + b*acoth(c))*(e**2*x + e*f*x**2 + f**2*x**3/
3), True))

```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (fx + e)^2 (b \operatorname{arccoth}(dx + c) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccoth(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*(b*arccoth(d*x + c) + a), x)
```

### 3.104 $\int (e + fx) (a + b \coth^{-1}(c + dx)) dx$

**Optimal.** Leaf size=97

$$\frac{(e + fx)^2 (a + b \coth^{-1}(c + dx))}{2f} + \frac{b(-cf + de + f)^2 \log(-c - dx + 1)}{4d^2 f} - \frac{b(de - (c + 1)f)^2 \log(c + dx + 1)}{4d^2 f} + \frac{bfx}{2d}$$

[Out] (b\*f\*x)/(2\*d) + ((e + f\*x)^2\*(a + b\*ArcCoth[c + d\*x]))/(2\*f) + (b\*(d\*e + f - c\*f)^2\*Log[1 - c - d\*x])/(4\*d^2\*f) - (b\*(d\*e - (1 + c)\*f)^2\*Log[1 + c + d\*x])/(4\*d^2\*f)

**Rubi [A]** time = 0.171195, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {6112, 5927, 702, 633, 31}

$$\frac{(e + fx)^2 (a + b \coth^{-1}(c + dx))}{2f} + \frac{b(-cf + de + f)^2 \log(-c - dx + 1)}{4d^2 f} - \frac{b(de - (c + 1)f)^2 \log(c + dx + 1)}{4d^2 f} + \frac{bfx}{2d}$$

Antiderivative was successfully verified.

[In] Int[(e + f\*x)\*(a + b\*ArcCoth[c + d\*x]),x]

[Out] (b\*f\*x)/(2\*d) + ((e + f\*x)^2\*(a + b\*ArcCoth[c + d\*x]))/(2\*f) + (b\*(d\*e + f - c\*f)^2\*Log[1 - c - d\*x])/(4\*d^2\*f) - (b\*(d\*e - (1 + c)\*f)^2\*Log[1 + c + d\*x])/(4\*d^2\*f)

#### Rule 6112

Int[((a\_.) + ArcCoth[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]

#### Rule 5927

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))\*((d\_.) + (e\_.)\*(x\_.))^(q\_.), x\_Symbol] :> Simp[((d + e\*x)^(q + 1)\*(a + b\*ArcCoth[c\*x]))/(e\*(q + 1)), x] - Dist[(b\*c)/(e\*(q + 1)), Int[(d + e\*x)^(q + 1)/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]

#### Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] := Int[PolynomialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

### Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a*c)]
```

### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rubi steps

$$\begin{aligned}
 \int (e + fx)(a + b \coth^{-1}(c + dx)) dx &= \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d} + \frac{fx}{d}\right)(a + b \coth^{-1}(x)) dx, x, c + dx\right)}{d} \\
 &= \frac{(e + fx)^2 (a + b \coth^{-1}(c + dx))}{2f} - \frac{b \text{Subst}\left(\int \frac{\left(\frac{de-cf}{d} + \frac{fx}{d}\right)^2}{1-x^2} dx, x, c + dx\right)}{2f} \\
 &= \frac{(e + fx)^2 (a + b \coth^{-1}(c + dx))}{2f} - \frac{b \text{Subst}\left(\int \left(-\frac{f^2}{d^2} + \frac{d^2 e^2 - 2cdf + (1+c^2)f^2 + 2f(de-cf)x}{d^2(1-x^2)}\right) dx, x, c + dx\right)}{2f} \\
 &= \frac{bfx}{2d} + \frac{(e + fx)^2 (a + b \coth^{-1}(c + dx))}{2f} - \frac{b \text{Subst}\left(\int \frac{d^2 e^2 - 2cdf + (1+c^2)f^2 + 2f(de-cf)x}{1-x^2} dx, x, c + dx\right)}{2d^2 f} \\
 &= \frac{bfx}{2d} + \frac{(e + fx)^2 (a + b \coth^{-1}(c + dx))}{2f} - \frac{(b(de + f - cf)^2) \text{Subst}\left(\int \frac{1}{1-x} dx, x, c + dx\right)}{4d^2 f} \\
 &= \frac{bfx}{2d} + \frac{(e + fx)^2 (a + b \coth^{-1}(c + dx))}{2f} + \frac{b(de + f - cf)^2 \log(1 - c - dx)}{4d^2 f} - \frac{b(de + f - cf)^2}{4d^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.0427247, size = 138, normalized size = 1.42

$$aex + \frac{1}{2}afx^2 + \frac{b(c^2 - 2c + 1)f \log(-c - dx + 1)}{4d^2} + \frac{b(-c^2 - 2c - 1)f \log(c + dx + 1)}{4d^2} + \frac{be((c + 1) \log(c + dx + 1) - (c - dx) \log(1 - c - dx))}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(e + f\*x)\*(a + b\*ArcCoth[c + d\*x]),x]

[Out] a\*e\*x + (b\*f\*x)/(2\*d) + (a\*f\*x^2)/2 + b\*e\*x\*ArcCoth[c + d\*x] + (b\*f\*x^2\*ArcCoth[c + d\*x])/2 + (b\*(1 - 2\*c + c^2)\*f\*Log[1 - c - d\*x])/(4\*d^2) + (b\*(-1 - 2\*c - c^2)\*f\*Log[1 + c + d\*x])/(4\*d^2) + (b\*e\*(-((-1 + c)\*Log[1 - c - d\*x]) + (1 + c)\*Log[1 + c + d\*x]))/(2\*d)

**Maple [B]** time = 0.04, size = 184, normalized size = 1.9

$$\frac{ax^2f}{2} - \frac{ac^2f}{2d^2} + axe + \frac{ace}{d} + \frac{\operatorname{arccoth}(dx+c)fx^2}{2} - \frac{\operatorname{arccoth}(dx+c)bc^2f}{2d^2} + \operatorname{arccoth}(dx+c)xbe + \frac{\operatorname{arccoth}(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)\*(a+b\*arccoth(d\*x+c)),x)

[Out] 1/2\*a\*x^2\*f-1/2/d^2\*a\*c^2\*f+a\*x\*e+1/d\*a\*c\*e+1/2\*b\*arccoth(d\*x+c)\*f\*x^2-1/2/d^2\*b\*arccoth(d\*x+c)\*f\*c^2+arccoth(d\*x+c)\*x\*b\*e+1/d\*arccoth(d\*x+c)\*b\*c\*e+1/2\*b\*f\*x/d+1/2/d^2\*b\*c\*f-1/2/d^2\*b\*ln(d\*x+c-1)\*c\*f+1/2/d\*b\*ln(d\*x+c-1)\*e+1/4/d^2\*b\*ln(d\*x+c-1)\*f-1/2/d^2\*b\*ln(d\*x+c+1)\*c\*f+1/2/d\*b\*ln(d\*x+c+1)\*e-1/4/d^2\*b\*ln(d\*x+c+1)\*f

**Maxima [A]** time = 0.960865, size = 147, normalized size = 1.52

$$\frac{1}{2}afx^2 + \frac{1}{4}\left(2x^2\operatorname{arccoth}(dx+c) + d\left(\frac{2x}{d^2} - \frac{(c^2+2c+1)\log(dx+c+1)}{d^3} + \frac{(c^2-2c+1)\log(dx+c-1)}{d^3}\right)\right)bf + aex +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*(a+b\*arccoth(d\*x+c)),x, algorithm="maxima")

[Out] 1/2\*a\*f\*x^2 + 1/4\*(2\*x^2\*arccoth(d\*x + c) + d\*(2\*x/d^2 - (c^2 + 2\*c + 1)\*log(d\*x + c + 1)/d^3 + (c^2 - 2\*c + 1)\*log(d\*x + c - 1)/d^3))\*b\*f + a\*e\*x + 1/2\*(2\*(d\*x + c)\*arccoth(d\*x + c) + log(-(d\*x + c)^2 + 1))\*b\*e/d

**Fricas [A]** time = 1.8918, size = 319, normalized size = 3.29

$$\frac{2ad^2fx^2 + 2(2ad^2e + bdf)x + (2(bc + b)de - (bc^2 + 2bc + b)f)\log(dx + c + 1) - (2(bc - b)de - (bc^2 - 2bc + b)f)\log(dx + c - 1)}{4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*(a+b\*arccoth(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{4} * (2 * a * d^2 * f * x^2 + 2 * (2 * a * d^2 * e + b * d * f) * x + (2 * (b * c + b) * d * e - (b * c^2 + 2 * b * c + b) * f) * \log(d * x + c + 1) - (2 * (b * c - b) * d * e - (b * c^2 - 2 * b * c + b) * f) * \log(d * x + c - 1) + (b * d^2 * f * x^2 + 2 * b * d^2 * e * x) * \log((d * x + c + 1) / (d * x + c - 1))) / d^2$

**Sympy [A]** time = 2.6259, size = 173, normalized size = 1.78

$$\left\{ \begin{array}{l} aex + \frac{afx^2}{2} - \frac{bc^2f \operatorname{acoth}(c+dx)}{2d^2} + \frac{bce \operatorname{acoth}(c+dx)}{d} - \frac{bcf \log\left(\frac{c}{d} + x + \frac{1}{d}\right)}{d^2} + \frac{bcf \operatorname{acoth}(c+dx)}{d^2} + bex \operatorname{acoth}(c + dx) + \frac{bf x^2 \operatorname{acoth}(c+dx)}{2} + \frac{bex \operatorname{acoth}(c)}{2} \\ (a + b \operatorname{acoth}(c)) \left( ex + \frac{fx^2}{2} \right) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*(a+b\*acoth(d\*x+c)),x)

[Out] Piecewise((a\*e\*x + a\*f\*x\*\*2/2 - b\*c\*\*2\*f\*acoth(c + d\*x)/(2\*d\*\*2) + b\*c\*e\*acoth(c + d\*x)/d - b\*c\*f\*log(c/d + x + 1/d)/d\*\*2 + b\*c\*f\*acoth(c + d\*x)/d\*\*2 + b\*e\*x\*acoth(c + d\*x) + b\*f\*x\*\*2\*acoth(c + d\*x)/2 + b\*e\*log(c/d + x + 1/d)/d - b\*e\*acoth(c + d\*x)/d + b\*f\*x/(2\*d) - b\*f\*acoth(c + d\*x)/(2\*d\*\*2), Ne(d, 0)), ((a + b\*acoth(c))\*(e\*x + f\*x\*\*2/2), True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (fx + e)(b \operatorname{arccoth}(dx + c) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*(a+b\*arccoth(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)\*(b\*arccoth(d\*x + c) + a), x)



### 3.105 $\int (a + b \coth^{-1}(c + dx)) dx$

**Optimal.** Leaf size=40

$$ax + \frac{b \log(1 - (c + dx)^2)}{2d} + \frac{b(c + dx) \coth^{-1}(c + dx)}{d}$$

[Out] a\*x + (b\*(c + d\*x)\*ArcCoth[c + d\*x])/d + (b\*Log[1 - (c + d\*x)^2])/(2\*d)

**Rubi [A]** time = 0.0245296, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {6104, 5911, 260}

$$ax + \frac{b \log(1 - (c + dx)^2)}{2d} + \frac{b(c + dx) \coth^{-1}(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[a + b\*ArcCoth[c + d\*x],x]

[Out] a\*x + (b\*(c + d\*x)\*ArcCoth[c + d\*x])/d + (b\*Log[1 - (c + d\*x)^2])/(2\*d)

#### Rule 6104

Int[((a\_.) + ArcCoth[(c\_.) + (d\_.)\*(x\_)])\*(b\_.))^(p\_.), x\_Symbol] :> Dist[1/d, Subst[Int[(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0]

#### Rule 5911

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.), x\_Symbol] :> Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

#### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

#### Rubi steps

$$\begin{aligned}
\int (a + b \coth^{-1}(c + dx)) dx &= ax + b \int \coth^{-1}(c + dx) dx \\
&= ax + \frac{b \operatorname{Subst}\left(\int \coth^{-1}(x) dx, x, c + dx\right)}{d} \\
&= ax + \frac{b(c + dx) \coth^{-1}(c + dx)}{d} - \frac{b \operatorname{Subst}\left(\int \frac{x}{1-x^2} dx, x, c + dx\right)}{d} \\
&= ax + \frac{b(c + dx) \coth^{-1}(c + dx)}{d} + \frac{b \log(1 - (c + dx)^2)}{2d}
\end{aligned}$$

**Mathematica [A]** time = 0.0149388, size = 48, normalized size = 1.2

$$ax + \frac{b((c + 1) \log(c + dx + 1) - (c - 1) \log(-c - dx + 1))}{2d} + bx \coth^{-1}(c + dx)$$

Antiderivative was successfully verified.

[In] Integrate[a + b\*ArcCoth[c + d\*x], x]

[Out] a\*x + b\*x\*ArcCoth[c + d\*x] + (b\*(-((-1 + c)\*Log[1 - c - d\*x]) + (1 + c)\*Log[1 + c + d\*x]))/(2\*d)

**Maple [A]** time = 0.033, size = 42, normalized size = 1.1

$$ax + \operatorname{arccoth}(dx + c)x + \frac{\operatorname{arccoth}(dx + c)c}{d} + \frac{b \ln((dx + c)^2 - 1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a+b\*arccoth(d\*x+c), x)

[Out] a\*x+b\*arccoth(d\*x+c)\*x+b/d\*arccoth(d\*x+c)\*c+1/2\*b/d\*ln((d\*x+c)^2-1)

**Maxima [A]** time = 0.970909, size = 49, normalized size = 1.22

$$ax + \frac{(2(dx + c) \operatorname{arccoth}(dx + c) + \log(-(dx + c)^2 + 1))b}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arccoth(d\*x+c),x, algorithm="maxima")

[Out] a\*x + 1/2\*(2\*(d\*x + c)\*arccoth(d\*x + c) + log(-(d\*x + c)^2 + 1))\*b/d

**Fricas [A]** time = 1.91274, size = 157, normalized size = 3.92

$$\frac{bdx \log\left(\frac{dx+c+1}{dx+c-1}\right) + 2adx + (bc+b)\log(dx+c+1) - (bc-b)\log(dx+c-1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arccoth(d\*x+c),x, algorithm="fricas")

[Out] 1/2\*(b\*d\*x\*log((d\*x + c + 1)/(d\*x + c - 1)) + 2\*a\*d\*x + (b\*c + b)\*log(d\*x + c + 1) - (b\*c - b)\*log(d\*x + c - 1))/d

**Sympy [A]** time = 0.761472, size = 46, normalized size = 1.15

$$ax + b \begin{cases} \frac{c \operatorname{acoth}(c+dx)}{d} + x \operatorname{acoth}(c+dx) + \frac{\log(c+dx+1)}{d} - \frac{\operatorname{acoth}(c+dx)}{d} & \text{for } d \neq 0 \\ x \operatorname{acoth}(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*acoth(d\*x+c),x)

[Out] a\*x + b\*Piecewise((c\*acoth(c + d\*x)/d + x\*acoth(c + d\*x) + log(c + d\*x + 1)/d - acoth(c + d\*x)/d, Ne(d, 0)), (x\*acoth(c), True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int b \operatorname{arccoth}(dx+c) + a dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccoth(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(b*arccoth(d*x + c) + a, x)
```

$$3.106 \quad \int \frac{a+b \coth^{-1}(c+dx)}{e+fx} dx$$

**Optimal.** Leaf size=130

$$-\frac{b \operatorname{PolyLog}\left(2, 1 - \frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{2f} + \frac{b \operatorname{PolyLog}\left(2, 1 - \frac{2}{c+dx+1}\right)}{2f} + \frac{(a+b \coth^{-1}(c+dx)) \log\left(\frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{f}$$

[Out] -(((a + b\*ArcCoth[c + d\*x])\*Log[2/(1 + c + d\*x)])/f) + ((a + b\*ArcCoth[c + d\*x])\*Log[(2\*d\*(e + f\*x))/((d\*e + f - c\*f)\*(1 + c + d\*x))])/f + (b\*PolyLog[2, 1 - 2/(1 + c + d\*x)])/(2\*f) - (b\*PolyLog[2, 1 - (2\*d\*(e + f\*x))/((d\*e + f - c\*f)\*(1 + c + d\*x))])/((2\*f))

**Rubi [A]** time = 0.14231, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {6112, 5921, 2402, 2315, 2447}

$$-\frac{b \operatorname{PolyLog}\left(2, 1 - \frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{2f} + \frac{b \operatorname{PolyLog}\left(2, 1 - \frac{2}{c+dx+1}\right)}{2f} + \frac{(a+b \coth^{-1}(c+dx)) \log\left(\frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCoth[c + d\*x])/(e + f\*x), x]

[Out] -(((a + b\*ArcCoth[c + d\*x])\*Log[2/(1 + c + d\*x)])/f) + ((a + b\*ArcCoth[c + d\*x])\*Log[(2\*d\*(e + f\*x))/((d\*e + f - c\*f)\*(1 + c + d\*x))])/f + (b\*PolyLog[2, 1 - 2/(1 + c + d\*x)])/(2\*f) - (b\*PolyLog[2, 1 - (2\*d\*(e + f\*x))/((d\*e + f - c\*f)\*(1 + c + d\*x))])/((2\*f))

### Rule 6112

Int[((a\_.) + ArcCoth[(c\_.) + (d\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]

### Rule 5921

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> -Simp[((a + b\*ArcCoth[c\*x])\*Log[2/(1 + c\*x)])/e, x] + (Dist[(b\*c)/e, Int[Log

```
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcCoth[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

### Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

### Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

### Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

### Rubi steps

$$\int \frac{a + b \coth^{-1}(c + dx)}{e + fx} dx = \frac{\text{Subst}\left(\int \frac{a + b \coth^{-1}(x)}{\frac{de - cf}{d} + \frac{fx}{d}} dx, x, c + dx\right)}{d}$$

$$= -\frac{(a + b \coth^{-1}(c + dx)) \log\left(\frac{2}{1 + c + dx}\right)}{f} + \frac{(a + b \coth^{-1}(c + dx)) \log\left(\frac{2d(e + fx)}{(de + f - cf)(1 + c + dx)}\right)}{f} + \frac{b S}{f}$$

$$= -\frac{(a + b \coth^{-1}(c + dx)) \log\left(\frac{2}{1 + c + dx}\right)}{f} + \frac{(a + b \coth^{-1}(c + dx)) \log\left(\frac{2d(e + fx)}{(de + f - cf)(1 + c + dx)}\right)}{f} - \frac{b L}{f}$$

$$= -\frac{(a + b \coth^{-1}(c + dx)) \log\left(\frac{2}{1 + c + dx}\right)}{f} + \frac{(a + b \coth^{-1}(c + dx)) \log\left(\frac{2d(e + fx)}{(de + f - cf)(1 + c + dx)}\right)}{f} + \frac{b L}{f}$$

**Mathematica [A]** time = 0.11257, size = 206, normalized size = 1.58

$$\frac{b \operatorname{PolyLog}\left(2, \frac{d(e+fx)}{-cf+de-f}\right)}{2f} + \frac{b \operatorname{PolyLog}\left(2, \frac{d(e+fx)}{-cf+de+f}\right)}{2f} + \frac{a \log(e+fx)}{f} + \frac{b \log(e+fx) \log\left(\frac{f(-c-dx+1)}{-cf+de+f}\right)}{2f} - \frac{b \log\left(-\frac{-c-dx+1}{c+dx}\right)}{2f}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCoth[c + d\*x])/(e + f\*x), x]

[Out] (a\*Log[e + f\*x])/f + (b\*Log[(f\*(1 - c - d\*x))/(d\*e + f - c\*f)]\*Log[e + f\*x])/ (2\*f) - (b\*Log[-((1 - c - d\*x)/(c + d\*x))]\*Log[e + f\*x])/ (2\*f) - (b\*Log[-((f\*(1 + c + d\*x))/(d\*e - f - c\*f)]\*Log[e + f\*x])/ (2\*f) + (b\*Log[(1 + c + d\*x)/(c + d\*x)]\*Log[e + f\*x])/ (2\*f) - (b\*PolyLog[2, (d\*(e + f\*x))/(d\*e - f - c\*f)])/ (2\*f) + (b\*PolyLog[2, (d\*(e + f\*x))/(d\*e + f - c\*f)])/ (2\*f)

**Maple [A]** time = 0.153, size = 202, normalized size = 1.6

$$\frac{a \ln((dx + c)f - cf + de)}{f} + \frac{b \ln((dx + c)f - cf + de) \operatorname{arccoth}(dx + c)}{f} - \frac{b \ln((dx + c)f - cf + de)}{2f} \ln\left(\frac{(dx + c)f + cf - de}{cf - de + f}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth(d\*x+c))/(f\*x+e), x)

[Out] a\*ln((d\*x+c)\*f-c\*f+d\*e)/f+b\*ln((d\*x+c)\*f-c\*f+d\*e)/f\*arccoth(d\*x+c)-1/2\*b/f\*ln((d\*x+c)\*f-c\*f+d\*e)\*ln(((d\*x+c)\*f+f)/(c\*f-d\*e+f))-1/2\*b/f\*dilog(((d\*x+c)\*f+f)/(c\*f-d\*e+f))+1/2\*b/f\*ln((d\*x+c)\*f-c\*f+d\*e)\*ln(((d\*x+c)\*f-f)/(c\*f-d\*e-f))+1/2\*b/f\*dilog(((d\*x+c)\*f-f)/(c\*f-d\*e-f))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} b \int \frac{\log\left(\frac{1}{dx+c} + 1\right) - \log\left(-\frac{1}{dx+c} + 1\right)}{fx + e} dx + \frac{a \log(fx + e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))/(f\*x+e), x, algorithm="maxima")

[Out]  $\frac{1}{2}b \int (\log(1/(dx+c)+1) - \log(-1/(dx+c)+1))/(fx+e) dx + a \log(fx+e)/f$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \operatorname{arccoth}(dx+c) + a}{fx+e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccoth(d*x+c))/(f*x+e),x, algorithm="fricas")`

[Out] `integral((b*arccoth(d*x + c) + a)/(f*x + e), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*acoth(d*x+c))/(f*x+e),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccoth}(dx+c) + a}{fx+e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccoth(d*x+c))/(f*x+e),x, algorithm="giac")`

[Out] `integrate((b*arccoth(d*x + c) + a)/(f*x + e), x)`



$$3.107 \quad \int \frac{a+b \coth^{-1}(c+dx)}{(e+fx)^2} dx$$

**Optimal.** Leaf size=115

$$-\frac{a+b \coth^{-1}(c+dx)}{f(e+fx)} - \frac{bd \log(-c-dx+1)}{2f(-cf+de+f)} + \frac{bd \log(c+dx+1)}{2f(-cf+de-f)} - \frac{bd \log(e+fx)}{(-cf+de+f)(de-(c+1)f)}$$

[Out]  $-\left(\frac{a+b \operatorname{ArcCoth}[c+d*x]}{f*(e+f*x)}\right) - \frac{b*d*\operatorname{Log}[1-c-d*x]}{2*f*(d*e+f-c*f)} + \frac{b*d*\operatorname{Log}[1+c+d*x]}{2*f*(d*e-f-c*f)} - \frac{b*d*\operatorname{Log}[e+f*x]}{(d*e+f-c*f)*(d*e-(1+c)*f)}$

**Rubi [A]** time = 0.168724, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {6110, 1982, 705, 31, 632}

$$-\frac{a+b \coth^{-1}(c+dx)}{f(e+fx)} - \frac{bd \log(-c-dx+1)}{2f(-cf+de+f)} + \frac{bd \log(c+dx+1)}{2f(-cf+de-f)} - \frac{bd \log(e+fx)}{(-cf+de+f)(de-(c+1)f)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a+b \operatorname{ArcCoth}[c+d*x])/(e+f*x)^2, x]$

[Out]  $-\left(\frac{a+b \operatorname{ArcCoth}[c+d*x]}{f*(e+f*x)}\right) - \frac{b*d*\operatorname{Log}[1-c-d*x]}{2*f*(d*e+f-c*f)} + \frac{b*d*\operatorname{Log}[1+c+d*x]}{2*f*(d*e-f-c*f)} - \frac{b*d*\operatorname{Log}[e+f*x]}{(d*e+f-c*f)*(d*e-(1+c)*f)}$

### Rule 6110

$\operatorname{Int}[(a_.) + \operatorname{ArcCoth}[(c_.) + (d_.)*(x_.)]*(b_.)]^{(p_.)}*((e_.) + (f_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(e+f*x)^{(m+1)}*(a+b \operatorname{ArcCoth}[c+d*x])^p]/(f*(m+1)), x] - \operatorname{Dist}[(b*d*p)/(f*(m+1)), \operatorname{Int}[(e+f*x)^{(m+1)}*(a+b \operatorname{ArcCoth}[c+d*x])^{(p-1)}]/(1-(c+d*x)^2), x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, e, f, x\} \ \&\& \ \operatorname{IGtQ}[p, 0] \ \&\& \ \operatorname{ILtQ}[m, -1]$

### Rule 1982

$\operatorname{Int}[(u_.)^{(m_.)}*(v_.)^{(p_.)}, x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandToSum}[u, x]^{m*} \operatorname{ExpandToSum}[v, x]^p, x] /;$   $\operatorname{FreeQ}\{m, p, x\} \ \&\& \ \operatorname{LinearQ}[u, x] \ \&\& \ \operatorname{QuadraticQ}[v, x] \ \&\& \ !(\operatorname{LinearMatchQ}[u, x] \ \&\& \ \operatorname{QuadraticMatchQ}[v, x])$

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
  :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \coth^{-1}(c + dx)}{(e + fx)^2} dx &= -\frac{a + b \coth^{-1}(c + dx)}{f(e + fx)} + \frac{(bd) \int \frac{1}{(e+fx)(1-(c+dx)^2)} dx}{f} \\
&= -\frac{a + b \coth^{-1}(c + dx)}{f(e + fx)} + \frac{(bd) \int \frac{1}{(e+fx)(1-c^2-2cdx-d^2x^2)} dx}{f} \\
&= -\frac{a + b \coth^{-1}(c + dx)}{f(e + fx)} + \frac{(bd) \int \frac{-d^2e+2cdf+d^2fx}{1-c^2-2cdx-d^2x^2} dx}{f(-d^2e^2 + 2cdef + (1-c^2)f^2)} + \frac{(bdf) \int \frac{1}{e+fx} dx}{-d^2e^2 + 2cdef + (1-c^2)f^2} \\
&= -\frac{a + b \coth^{-1}(c + dx)}{f(e + fx)} - \frac{bd \log(e + fx)}{(de - f - cf)(de + f - cf)} - \frac{(bd^3) \int \frac{1}{-d-cd-d^2x} dx}{2f(de - f - cf)} + \frac{(bd^3) \int \frac{1}{d-cd-d^2x} dx}{2f(de + f - cf)} \\
&= -\frac{a + b \coth^{-1}(c + dx)}{f(e + fx)} - \frac{bd \log(1 - c - dx)}{2f(de + f - cf)} + \frac{bd \log(1 + c + dx)}{2f(de - f - cf)} - \frac{bd \log(e + fx)}{(de - f - cf)(de + f - cf)}
\end{aligned}$$

**Mathematica [A]** time = 0.200933, size = 125, normalized size = 1.09

$$\frac{1}{2} \left( -\frac{2a}{f(e + fx)} - \frac{2bd \log(e + fx)}{(c^2 - 1)f^2 - 2cdef + d^2e^2} + \frac{bd \log(-c - dx + 1)}{f((c - 1)f - de)} - \frac{bd \log(c + dx + 1)}{f(cf - de + f)} - \frac{2b \coth^{-1}(c + dx)}{f(e + fx)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCoth[c + d\*x])/(e + f\*x)^2,x]

[Out]  $((-2*a)/(f*(e + f*x)) - (2*b*ArcCoth[c + d*x])/(f*(e + f*x)) + (b*d*Log[1 - c - d*x])/(f*(-(d*e) + (-1 + c)*f)) - (b*d*Log[1 + c + d*x])/(f*(-(d*e) + f + c*f)) - (2*b*d*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2))/2$

**Maple [A]** time = 0.043, size = 141, normalized size = 1.2

$$-\frac{ad}{(dfx+de)f} - \frac{bd \operatorname{arccoth}(dx+c)}{(dfx+de)f} + \frac{bd \ln(dx+c-1)}{f(2cf-2de-2f)} - \frac{bd \ln(dx+c+1)}{f(2cf-2de+2f)} - \frac{bd \ln((dx+c)f-cf+de)}{(cf-de-f)(cf-de+f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth(d\*x+c))/(f\*x+e)^2,x)

[Out]  $-d*a/(d*f*x+d*e)/f - d*b/(d*f*x+d*e)/f * \operatorname{arccoth}(d*x+c) + d*b/f/(2*c*f-2*d*e-2*f) * \ln(d*x+c-1) - d*b/f/(2*c*f-2*d*e+2*f) * \ln(d*x+c+1) - d*b/(c*f-d*e-f)/(c*f-d*e+f) * \ln((d*x+c)*f-c*f+d*e)$

**Maxima [A]** time = 0.996587, size = 163, normalized size = 1.42

$$\frac{1}{2} \left( d \left( \frac{\log(dx+c+1)}{def-(c+1)f^2} - \frac{\log(dx+c-1)}{def-(c-1)f^2} - \frac{2 \log(fx+e)}{d^2e^2-2cdef+(c^2-1)f^2} \right) - \frac{2 \operatorname{arccoth}(dx+c)}{f^2x+ef} \right) b - \frac{a}{f^2x+ef}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))/(f\*x+e)^2,x, algorithm="maxima")

[Out]  $1/2*(d*(\log(d*x + c + 1)/(d*e*f - (c + 1)*f^2) - \log(d*x + c - 1)/(d*e*f - (c - 1)*f^2) - 2*\log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 - 1)*f^2)) - 2*arccoth(d*x + c)/(f^2*x + e*f))*b - a/(f^2*x + e*f)$

**Fricas [B]** time = 3.11053, size = 583, normalized size = 5.07

$$\frac{2ad^2e^2 - 4acdef + 2(ac^2 - a)f^2 - (bd^2e^2 - (bc - b)def + (bd^2ef - (bc - b)df^2)x) \log(dx + c + 1) + (bd^2e^2 - (bc + b)def)}{2(d^2e^3f - 2cde^2f^2 + (c^2 - 1)ef^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))/(f\*x+e)^2,x, algorithm="fricas")

[Out] -1/2\*(2\*a\*d^2\*e^2 - 4\*a\*c\*d\*e\*f + 2\*(a\*c^2 - a)\*f^2 - (b\*d^2\*e^2 - (b\*c - b)\*d\*e\*f + (b\*d^2\*e\*f - (b\*c - b)\*d\*f^2)\*x)\*log(d\*x + c + 1) + (b\*d^2\*e^2 - (b\*c + b)\*d\*e\*f + (b\*d^2\*e\*f - (b\*c + b)\*d\*f^2)\*x)\*log(d\*x + c - 1) + 2\*(b\*d\*f^2\*x + b\*d\*e\*f)\*log(f\*x + e) + (b\*d^2\*e^2 - 2\*b\*c\*d\*e\*f + (b\*c^2 - b)\*f^2)\*log((d\*x + c + 1)/(d\*x + c - 1))/(d^2\*e^3\*f - 2\*c\*d\*e^2\*f^2 + (c^2 - 1)\*e\*f^3 + (d^2\*e^2\*f^2 - 2\*c\*d\*e\*f^3 + (c^2 - 1)\*f^4)\*x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth(d\*x+c))/(f\*x+e)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccoth}(dx + c) + a}{(fx + e)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))/(f\*x+e)^2,x, algorithm="giac")

[Out] integrate((b\*arccoth(d\*x + c) + a)/(f\*x + e)^2, x)

$$3.108 \quad \int \frac{a+b \coth^{-1}(c+dx)}{(e+fx)^3} dx$$

**Optimal.** Leaf size=167

$$-\frac{a+b \coth^{-1}(c+dx)}{2f(e+fx)^2} - \frac{bd^2 \log(-c-dx+1)}{4f(-cf+de+f)^2} + \frac{bd^2 \log(c+dx+1)}{4f(-cf+de-f)^2} - \frac{bd^2(de-cf) \log(e+fx)}{(-cf+de+f)^2(de-(c+1)f)^2} + \frac{1}{2(e+fx)(-cf+de+f)}$$

[Out] (b\*d)/(2\*(d\*e + f - c\*f)\*(d\*e - (1 + c)\*f)\*(e + f\*x)) - (a + b\*ArcCoth[c + d\*x])/(2\*f\*(e + f\*x)^2) - (b\*d^2\*Log[1 - c - d\*x])/(4\*f\*(d\*e + f - c\*f)^2) + (b\*d^2\*Log[1 + c + d\*x])/(4\*f\*(d\*e - f - c\*f)^2) - (b\*d^2\*(d\*e - c\*f)\*Log[e + f\*x])/((d\*e + f - c\*f)^2\*(d\*e - (1 + c)\*f)^2)

**Rubi [A]** time = 0.234421, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {6110, 1982, 709, 800}

$$-\frac{a+b \coth^{-1}(c+dx)}{2f(e+fx)^2} - \frac{bd^2 \log(-c-dx+1)}{4f(-cf+de+f)^2} + \frac{bd^2 \log(c+dx+1)}{4f(-cf+de-f)^2} - \frac{bd^2(de-cf) \log(e+fx)}{(-cf+de+f)^2(de-(c+1)f)^2} + \frac{1}{2(e+fx)(-cf+de+f)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCoth[c + d\*x])/(e + f\*x)^3, x]

[Out] (b\*d)/(2\*(d\*e + f - c\*f)\*(d\*e - (1 + c)\*f)\*(e + f\*x)) - (a + b\*ArcCoth[c + d\*x])/(2\*f\*(e + f\*x)^2) - (b\*d^2\*Log[1 - c - d\*x])/(4\*f\*(d\*e + f - c\*f)^2) + (b\*d^2\*Log[1 + c + d\*x])/(4\*f\*(d\*e - f - c\*f)^2) - (b\*d^2\*(d\*e - c\*f)\*Log[e + f\*x])/((d\*e + f - c\*f)^2\*(d\*e - (1 + c)\*f)^2)

### Rule 6110

Int[((a\_.) + ArcCoth[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^(m + 1)\*(a + b\*ArcCoth[c + d\*x])^p)/(f\*(m + 1)), x] - Dist[(b\*d\*p)/(f\*(m + 1)), Int[((e + f\*x)^(m + 1)\*(a + b\*ArcCoth[c + d\*x])^(p - 1))/(1 - (c + d\*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[m, -1]

### Rule 1982

Int[(u\_)^(m\_.)\*(v\_)^(p\_.), x\_Symbol] :> Int[ExpandToSum[u, x]^m\*ExpandToSum[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] && !

(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])

### Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
:> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

### Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

### Rubi steps

$$\begin{aligned}
\int \frac{a + b \coth^{-1}(c + dx)}{(e + fx)^3} dx &= -\frac{a + b \coth^{-1}(c + dx)}{2f(e + fx)^2} + \frac{(bd) \int \frac{1}{(e+fx)^2(1-(c+dx)^2)} dx}{2f} \\
&= -\frac{a + b \coth^{-1}(c + dx)}{2f(e + fx)^2} + \frac{(bd) \int \frac{1}{(e+fx)^2(1-c^2-2cdx-d^2x^2)} dx}{2f} \\
&= \frac{bd}{2(de + f - cf)(de - (1 + c)f)(e + fx)} - \frac{a + b \coth^{-1}(c + dx)}{2f(e + fx)^2} + \frac{(bd) \int \frac{-d(de-2cf)+d^2fx}{(e+fx)(1-c^2-2cdx-d^2x^2)} dx}{2f(-d^2e^2 + 2cdef + (1 - c^2)fx^2)} \\
&= \frac{bd}{2(de + f - cf)(de - (1 + c)f)(e + fx)} - \frac{a + b \coth^{-1}(c + dx)}{2f(e + fx)^2} + \frac{(bd) \int \left( \frac{d^2(-de+(1+c)f)}{2(de+f-cf)(1-c-dx)} + \frac{-d(de-2cf)+d^2fx}{(e+fx)(1-c^2-2cdx-d^2x^2)} \right) dx}{2f(-d^2e^2 + 2cdef + (1 - c^2)fx^2)} \\
&= \frac{bd}{2(de + f - cf)(de - (1 + c)f)(e + fx)} - \frac{a + b \coth^{-1}(c + dx)}{2f(e + fx)^2} - \frac{bd^2 \log(1 - c - dx)}{4f(de + f - cf)^2} + \frac{bd^2 \log(-c - dx + 1)}{4f(-cf + de + f)^2} + \frac{bd^2 \log(-c - dx + 1)}{4f(-cf + de + f)^2}
\end{aligned}$$

**Mathematica [A]** time = 0.346272, size = 174, normalized size = 1.04

$$\frac{1}{4} \left( -\frac{2a}{f(e + fx)^2} + \frac{2bd}{(e + fx)((c^2 - 1)f^2 - 2cdef + d^2e^2)} - \frac{4bd^2(de - cf) \log(e + fx)}{((c^2 - 1)f^2 - 2cdef + d^2e^2)^2} - \frac{bd^2 \log(-c - dx + 1)}{f(-cf + de + f)^2} + \frac{bd^2 \log(-c - dx + 1)}{f(-cf + de + f)^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCoth[c + d\*x])/(e + f\*x)^3,x]

[Out] 
$$\begin{aligned} &((-2*a)/(f*(e + f*x)^2) + (2*b*d)/((d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)*(e + f*x)) - (2*b*ArcCoth[c + d*x])/(f*(e + f*x)^2) - (b*d^2*Log[1 - c - d*x])/((f*(d*e + f - c*f)^2) + (b*d^2*Log[1 + c + d*x])/(f*(-(d*e) + f + c*f)^2) - (4*b*d^2*(d*e - c*f)*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)^2)/4 \end{aligned}$$

**Maple [A]** time = 0.052, size = 236, normalized size = 1.4

$$-\frac{ad^2}{2(dx+de)^2 f} - \frac{bd^2 \operatorname{arccoth}(dx+c)}{2(dx+de)^2 f} - \frac{bd^2 \ln(dx+c-1)}{4f(cf-de-f)^2} + \frac{bd^2 \ln(dx+c+1)}{4f(cf-de+f)^2} + \frac{bd^2}{(2cf-2de-2f)(cf-de+f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth(d\*x+c))/(f\*x+e)^3,x)

[Out] 
$$\begin{aligned} &-1/2*d^2*a/(d*f*x+d*e)^2/f - 1/2*d^2*b/(d*f*x+d*e)^2/f*arccoth(d*x+c) - 1/4*d^2*b/f/(c*f-d*e-f)^2*\ln(d*x+c-1) + 1/4*d^2*b/f/(c*f-d*e+f)^2*\ln(d*x+c+1) + 1/2*d^2*b/(c*f-d*e-f)/(c*f-d*e+f)/(d*f*x+d*e) + d^2*b*f/(c*f-d*e-f)^2/(c*f-d*e+f)^2*\ln((d*x+c)*f-c*f+d*e)*c - d^3*b/(c*f-d*e-f)^2/(c*f-d*e+f)^2*\ln((d*x+c)*f-c*f+d*e)*e \end{aligned}$$

**Maxima [A]** time = 1.04528, size = 393, normalized size = 2.35

$$\frac{1}{4} \left( d \left( \frac{d \log(dx+c+1)}{d^2 e^2 f - 2(c+1) d e f^2 + (c^2 + 2c + 1) f^3} - \frac{d \log(dx+c-1)}{d^2 e^2 f - 2(c-1) d e f^2 + (c^2 - 2c + 1) f^3} - \frac{4(d^2)}{d^4 e^4 - 4 c d^3 e^3 f + 2(3 c^2 - 1)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))/(f\*x+e)^3,x, algorithm="maxima")

[Out] 
$$\begin{aligned} &1/4*(d*(d*log(d*x + c + 1)/(d^2*e^2*f - 2*(c + 1)*d*e*f^2 + (c^2 + 2*c + 1)*f^3) - d*log(d*x + c - 1)/(d^2*e^2*f - 2*(c - 1)*d*e*f^2 + (c^2 - 2*c + 1)*f^3) - 4*(d^2*e - c*d*f)*log(f*x + e)/(d^4*e^4 - 4*c*d^3*e^3*f + 2*(3*c^2 - 1)*d^2*e^2*f^2 - 4*(c^3 - c)*d*e*f^3 + (c^4 - 2*c^2 + 1)*f^4) + 2/(d^2*e^3 - 2*c*d*e^2*f + (c^2 - 1)*e*f^2 + (d^2*e^2*f - 2*c*d*e*f^2 + (c^2 - 1)*f^3)*x)) - 2*arccoth(d*x + c)/(f^3*x^2 + 2*e*f^2*x + e^2*f))*b - 1/2*a/(f^3*x \end{aligned}$$

$$^2 + 2*e*f^2*x + e^2*f)$$

**Fricas [B]** time = 9.18629, size = 1766, normalized size = 10.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))/(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*d^4*e^4 - 2*(4*a*c + b)*d^3*e^3*f + 4*(3*a*c^2 + b*c - a)*d^2*e^2
*f^2 - 2*(4*a*c^3 + b*c^2 - 4*a*c - b)*d*e*f^3 + 2*(a*c^4 - 2*a*c^2 + a)*f^
4 - 2*(b*d^3*e^2*f^2 - 2*b*c*d^2*e*f^3 + (b*c^2 - b)*d*f^4)*x - (b*d^4*e^4
- 2*(b*c - b)*d^3*e^3*f + (b*c^2 - 2*b*c + b)*d^2*e^2*f^2 + (b*d^4*e^2*f^2
- 2*(b*c - b)*d^3*e*f^3 + (b*c^2 - 2*b*c + b)*d^2*f^4)*x^2 + 2*(b*d^4*e^3*f
- 2*(b*c - b)*d^3*e^2*f^2 + (b*c^2 - 2*b*c + b)*d^2*e*f^3)*x*log(d*x + c
+ 1) + (b*d^4*e^4 - 2*(b*c + b)*d^3*e^3*f + (b*c^2 + 2*b*c + b)*d^2*e^2*f^2
+ (b*d^4*e^2*f^2 - 2*(b*c + b)*d^3*e*f^3 + (b*c^2 + 2*b*c + b)*d^2*f^4)*x^
2 + 2*(b*d^4*e^3*f - 2*(b*c + b)*d^3*e^2*f^2 + (b*c^2 + 2*b*c + b)*d^2*e*f^
3)*x*log(d*x + c - 1) + 4*(b*d^3*e^3*f - b*c*d^2*e^2*f^2 + (b*d^3*e*f^3 -
b*c*d^2*f^4)*x^2 + 2*(b*d^3*e^2*f^2 - b*c*d^2*e*f^3)*x)*log(f*x + e) + (b*d
^4*e^4 - 4*b*c*d^3*e^3*f + 2*(3*b*c^2 - b)*d^2*e^2*f^2 - 4*(b*c^3 - b*c)*d*
e*f^3 + (b*c^4 - 2*b*c^2 + b)*f^4)*log((d*x + c + 1)/(d*x + c - 1))/(d^4*e
^6*f - 4*c*d^3*e^5*f^2 + 2*(3*c^2 - 1)*d^2*e^4*f^3 - 4*(c^3 - c)*d*e^3*f^4
+ (c^4 - 2*c^2 + 1)*e^2*f^5 + (d^4*e^4*f^3 - 4*c*d^3*e^3*f^4 + 2*(3*c^2 - 1
)*d^2*e^2*f^5 - 4*(c^3 - c)*d*e*f^6 + (c^4 - 2*c^2 + 1)*f^7)*x^2 + 2*(d^4*e
^5*f^2 - 4*c*d^3*e^4*f^3 + 2*(3*c^2 - 1)*d^2*e^3*f^4 - 4*(c^3 - c)*d*e^2*f^
5 + (c^4 - 2*c^2 + 1)*e*f^6)*x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(d*x+c))/(f*x+e)**3,x)
```

```
[Out] Timed out
```



---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arccoth}(dx + c) + a}{(fx + e)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))/(f\*x+e)^3,x, algorithm="giac")

[Out] integrate((b\*arccoth(d\*x + c) + a)/(f\*x + e)^3, x)

### 3.109 $\int (e + fx)^2 \left( a + b \coth^{-1}(c + dx) \right)^2 dx$

**Optimal.** Leaf size=374

$$\frac{b^2 \left( (3c^2 + 1) f^2 - 6cdef + 3d^2 e^2 \right) \text{PolyLog} \left( 2, -\frac{c+dx+1}{-c-dx+1} \right)}{3d^3} - \frac{(de - cf) \left( (c^2 + 3) f^2 - 2cdef + d^2 e^2 \right) \left( a + b \coth^{-1}(c + dx) \right)^2}{3d^3 f}$$

[Out] (b^2\*f^2\*x)/(3\*d^2) + (2\*a\*b\*f\*(d\*e - c\*f)\*x)/d^2 + (2\*b^2\*f\*(d\*e - c\*f)\*(c + d\*x)\*ArcCoth[c + d\*x])/d^3 + (b\*f^2\*(c + d\*x)^2\*(a + b\*ArcCoth[c + d\*x]))/(3\*d^3) - ((d\*e - c\*f)\*(d^2\*e^2 - 2\*c\*d\*e\*f + (3 + c^2)\*f^2)\*(a + b\*ArcCoth[c + d\*x])^2)/(3\*d^3\*f) + ((3\*d^2\*e^2 - 6\*c\*d\*e\*f + (1 + 3\*c^2)\*f^2)\*(a + b\*ArcCoth[c + d\*x])^2)/(3\*d^3) + ((e + f\*x)^3\*(a + b\*ArcCoth[c + d\*x])^2)/(3\*f) - (b^2\*f^2\*ArcTanh[c + d\*x])/d^3 - (2\*b\*(3\*d^2\*e^2 - 6\*c\*d\*e\*f + (1 + 3\*c^2)\*f^2)\*(a + b\*ArcCoth[c + d\*x])\*Log[2/(1 - c - d\*x)])/(3\*d^3) + (b^2\*f\*(d\*e - c\*f)\*Log[1 - (c + d\*x)^2])/d^3 - (b^2\*(3\*d^2\*e^2 - 6\*c\*d\*e\*f + (1 + 3\*c^2)\*f^2)\*PolyLog[2, -((1 + c + d\*x)/(1 - c - d\*x))])/d^3

**Rubi [A]** time = 0.636536, antiderivative size = 374, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.65$ , Rules used = {6112, 5929, 5911, 260, 5917, 321, 206, 6049, 5949, 5985, 5919, 2402, 2315}

$$\frac{b^2 \left( (3c^2 + 1) f^2 - 6cdef + 3d^2 e^2 \right) \text{PolyLog} \left( 2, -\frac{c+dx+1}{-c-dx+1} \right)}{3d^3} - \frac{(de - cf) \left( (c^2 + 3) f^2 - 2cdef + d^2 e^2 \right) \left( a + b \coth^{-1}(c + dx) \right)^2}{3d^3 f}$$

Antiderivative was successfully verified.

[In] Int[(e + f\*x)^2\*(a + b\*ArcCoth[c + d\*x])^2,x]

[Out] (b^2\*f^2\*x)/(3\*d^2) + (2\*a\*b\*f\*(d\*e - c\*f)\*x)/d^2 + (2\*b^2\*f\*(d\*e - c\*f)\*(c + d\*x)\*ArcCoth[c + d\*x])/d^3 + (b\*f^2\*(c + d\*x)^2\*(a + b\*ArcCoth[c + d\*x]))/(3\*d^3) - ((d\*e - c\*f)\*(d^2\*e^2 - 2\*c\*d\*e\*f + (3 + c^2)\*f^2)\*(a + b\*ArcCoth[c + d\*x])^2)/(3\*d^3\*f) + ((3\*d^2\*e^2 - 6\*c\*d\*e\*f + (1 + 3\*c^2)\*f^2)\*(a + b\*ArcCoth[c + d\*x])^2)/(3\*d^3) + ((e + f\*x)^3\*(a + b\*ArcCoth[c + d\*x])^2)/(3\*f) - (b^2\*f^2\*ArcTanh[c + d\*x])/d^3 - (2\*b\*(3\*d^2\*e^2 - 6\*c\*d\*e\*f + (1 + 3\*c^2)\*f^2)\*(a + b\*ArcCoth[c + d\*x])\*Log[2/(1 - c - d\*x)])/(3\*d^3) + (b^2\*f\*(d\*e - c\*f)\*Log[1 - (c + d\*x)^2])/d^3 - (b^2\*(3\*d^2\*e^2 - 6\*c\*d\*e\*f + (1 + 3\*c^2)\*f^2)\*PolyLog[2, -((1 + c + d\*x)/(1 - c - d\*x))])/d^3

Rule 6112

Int[((a\_.) + ArcCoth[(c\_) + (d\_.)\*(x\_)]\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]

### Rule 5929

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)\*((d\_) + (e\_.)\*(x\_))^(q\_.), x\_Symbol] := Simp[((d + e\*x)^(q + 1)\*(a + b\*ArcCoth[c\*x])^p)/(e\*(q + 1)), x] - Dist[(b\*c\*p)/(e\*(q + 1)), Int[ExpandIntegrand[(a + b\*ArcCoth[c\*x])^(p - 1), (d + e\*x)^(q + 1)/(1 - c^2\*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]

### Rule 5911

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^(p\_.), x\_Symbol] := Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rule 5917

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcCoth[c\*x])^p)/(d\*(m + 1)), x] - Dist[(b\*c\*p)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 321

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

$Q[a, 0] \parallel LtQ[b, 0]$

### Rule 6049

$Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^p)*((f_) + (g_.)*(x_))^{m_})/((d_) + (e_.)*(x_)^2), x\_Symbol] \rightarrow Int[ExpandIntegrand[(a + b*ArcCoth[c*x])^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[\{a, b, c, d, e, f, g\}, x] \&\& IGtQ[p, 0] \&\& EqQ[c^2*d + e, 0] \&\& IGtQ[m, 0]$

### Rule 5949

$Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^p)/((d_) + (e_.)*(x_)^2), x\_Symbol] \rightarrow Simp[(a + b*ArcCoth[c*x])^{p+1}/(b*c*d*(p+1)), x] /; FreeQ[\{a, b, c, d, e, p\}, x] \&\& EqQ[c^2*d + e, 0] \&\& NeQ[p, -1]$

### Rule 5985

$Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^p)*(x_)/((d_) + (e_.)*(x_)^2), x\_Symbol] \rightarrow Simp[(a + b*ArcCoth[c*x])^{p+1}/(b*e*(p+1)), x] + Dist[1/(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[\{a, b, c, d, e\}, x] \&\& EqQ[c^2*d + e, 0] \&\& IGtQ[p, 0]$

### Rule 5919

$Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^p)/((d_) + (e_.)*(x_)), x\_Symbol] \rightarrow -Simp[(a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)]/e, x] + Dist[(b*c*p)/e, Int[(a + b*ArcCoth[c*x])^{p-1}*Log[2/(1 + (e*x)/d)]/(1 - c^2*x^2), x], x] /; FreeQ[\{a, b, c, d, e\}, x] \&\& IGtQ[p, 0] \&\& EqQ[c^2*d^2 - e^2, 0]$

### Rule 2402

$Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x\_Symbol] \rightarrow -Dist[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[\{c, d, e, f, g\}, x] \&\& EqQ[c, 2*d] \&\& EqQ[e^2*f + d^2*g, 0]$

### Rule 2315

$Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x\_Symbol] \rightarrow -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[\{c, d, e\}, x] \&\& EqQ[e + c*d, 0]$

### Rubi steps

$$\begin{aligned}
\int (e + fx)^2 (a + b \coth^{-1}(c + dx))^2 dx &= \frac{\text{Subst}\left(\int \left(\frac{de - cf}{d} + \frac{fx}{d}\right)^2 (a + b \coth^{-1}(x))^2 dx, x, c + dx\right)}{d} \\
&= \frac{(e + fx)^3 (a + b \coth^{-1}(c + dx))^2}{3f} - \frac{(2b) \text{Subst}\left(\int \left(-\frac{3f^2(de - cf)(a + b \coth^{-1}(x))}{d^3} - \frac{2b^2 f^2 (de - cf)(a + b \coth^{-1}(x))^2}{d^3}\right) dx, x, c + dx\right)}{3f} \\
&= \frac{(e + fx)^3 (a + b \coth^{-1}(c + dx))^2}{3f} - \frac{(2b) \text{Subst}\left(\int \frac{(de - cf)(d^2 e^2 - 2cdef + 3f^2 + c^2 f^2)}{d^3} dx, x, c + dx\right)}{3f} \\
&= \frac{2abf(de - cf)x}{d^2} + \frac{bf^2(c + dx)^2 (a + b \coth^{-1}(c + dx))}{3d^3} + \frac{(e + fx)^3 (a + b \coth^{-1}(c + dx))^2}{3f} \\
&= \frac{b^2 f^2 x}{3d^2} + \frac{2abf(de - cf)x}{d^2} + \frac{2b^2 f(de - cf)(c + dx) \coth^{-1}(c + dx)}{d^3} + \frac{bf^2(c + dx)^2}{3f} \\
&= \frac{b^2 f^2 x}{3d^2} + \frac{2abf(de - cf)x}{d^2} + \frac{2b^2 f(de - cf)(c + dx) \coth^{-1}(c + dx)}{d^3} + \frac{bf^2(c + dx)^2}{3f} \\
&= \frac{b^2 f^2 x}{3d^2} + \frac{2abf(de - cf)x}{d^2} + \frac{2b^2 f(de - cf)(c + dx) \coth^{-1}(c + dx)}{d^3} + \frac{bf^2(c + dx)^2}{3f} \\
&= \frac{b^2 f^2 x}{3d^2} + \frac{2abf(de - cf)x}{d^2} + \frac{2b^2 f(de - cf)(c + dx) \coth^{-1}(c + dx)}{d^3} + \frac{bf^2(c + dx)^2}{3f} \\
&= \frac{b^2 f^2 x}{3d^2} + \frac{2abf(de - cf)x}{d^2} + \frac{2b^2 f(de - cf)(c + dx) \coth^{-1}(c + dx)}{d^3} + \frac{bf^2(c + dx)^2}{3f}
\end{aligned}$$

**Mathematica [B]** time = 7.23623, size = 1054, normalized size = 2.82

$$\frac{1}{3}a^2 f^2 x^3 + a^2 e f x^2 + a^2 e^2 x + \frac{1}{3}ab \left( 2x(3e^2 + 3f x e + f^2 x^2) \coth^{-1}(c + dx) + \frac{d f x(6de - 4cf + d f x) - (c - 1)(3d^2 e^2 - 3c d e f + c^2 f^2)}{d^3} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(e + f\*x)^2\*(a + b\*ArcCoth[c + d\*x])^2,x]

[Out]  $a^2e^{2x} + a^2efx^2 + (a^2f^2x^3)/3 + (ab(2x(3e^2 + 3efx + f^2x^2)*\text{ArcCoth}[c + dx] + (dfx(6de - 4cf + d^2fx) - (-1 + c)(3d^2e^2 - 3(-1 + c)d^2ef + (-1 + c)^2f^2)*\text{Log}[1 - c - dx] + (1 + c)(3d^2e^2 - 3(1 + c)d^2ef + (1 + c)^2f^2)*\text{Log}[1 + c + dx])/d^3))/3 + (b^2e^2(1 - (c + dx)^2)*(\text{ArcCoth}[c + dx]*(\text{ArcCoth}[c + dx] - (c + dx)*\text{ArcCoth}[c + dx] + 2*\text{Log}[1 - E^{(-2*\text{ArcCoth}[c + dx])}])) - \text{PolyLog}[2, E^{(-2*\text{ArcCoth}[c + dx])}]))/(d*(c + dx)^2*(1 - (c + dx)^{-2})) - (b^2ef(1 - (c + dx)^2)*(2*c*\text{ArcCoth}[c + dx]^2 + (c + dx)^2*(1 - (c + dx)^{-2})*\text{ArcCoth}[c + dx]^2 - 2*(c + dx)*\text{ArcCoth}[c + dx]*(-1 + c*\text{ArcCoth}[c + dx]) + 4*c*\text{ArcCoth}[c + dx]*\text{Log}[1 - E^{(-2*\text{ArcCoth}[c + dx])}]) - 2*\text{Log}[1/((c + dx)*\text{Sqrt}[1 - (c + dx)^{-2}]]]) - 2*c*\text{PolyLog}[2, E^{(-2*\text{ArcCoth}[c + dx])}]))/(d^2*(c + dx)^2*(1 - (c + dx)^{-2})) - (b^2f^2*(c + dx)*\text{Sqrt}[1 - (c + dx)^{-2}]*(1 - (c + dx)^2)*((4*\text{ArcCoth}[c + dx])/((c + dx)*\text{Sqrt}[1 - (c + dx)^{-2}]) + (3*\text{ArcCoth}[c + dx]^2)/((c + dx)*\text{Sqrt}[1 - (c + dx)^{-2}]) - (12*c*\text{ArcCoth}[c + dx]^2)/((c + dx)*\text{Sqrt}[1 - (c + dx)^{-2}]) + (9*c^2*\text{ArcCoth}[c + dx]^2)/((c + dx)*\text{Sqrt}[1 - (c + dx)^{-2}]) + (-1 + 6*c*\text{ArcCoth}[c + dx] + 3*\text{ArcCoth}[c + dx]^2 - 3*c^2*\text{ArcCoth}[c + dx]^2)/\text{Sqrt}[1 - (c + dx)^{-2}] + \text{Cosh}[3*\text{ArcCoth}[c + dx]] - 6*c*\text{ArcCoth}[c + dx]*\text{Cosh}[3*\text{ArcCoth}[c + dx]] + \text{ArcCoth}[c + dx]^2*\text{Cosh}[3*\text{ArcCoth}[c + dx]] + 3*c^2*\text{ArcCoth}[c + dx]^2*\text{Cosh}[3*\text{ArcCoth}[c + dx]] + (6*\text{ArcCoth}[c + dx]*\text{Log}[1 - E^{(-2*\text{ArcCoth}[c + dx])}]))/((c + dx)*\text{Sqrt}[1 - (c + dx)^{-2}]) + (18*c^2*\text{ArcCoth}[c + dx]*\text{Log}[1 - E^{(-2*\text{ArcCoth}[c + dx])}]))/((c + dx)*\text{Sqrt}[1 - (c + dx)^{-2}]) - (18*c*\text{Log}[1/(c + dx)*\text{Sqrt}[1 - (c + dx)^{-2}]])/((c + dx)*\text{Sqrt}[1 - (c + dx)^{-2}]) + (4*(1 + 3*c^2)*\text{PolyLog}[2, E^{(-2*\text{ArcCoth}[c + dx])}]))/((c + dx)^3*(1 - (c + dx)^{-2}))^{(3/2)} - \text{ArcCoth}[c + dx]^2*\text{Sinh}[3*\text{ArcCoth}[c + dx]] - 3*c^2*\text{ArcCoth}[c + dx]^2*\text{Sinh}[3*\text{ArcCoth}[c + dx]] - 2*\text{ArcCoth}[c + dx]*\text{Log}[1 - E^{(-2*\text{ArcCoth}[c + dx])}])* \text{Sinh}[3*\text{ArcCoth}[c + dx]] - 6*c^2*\text{ArcCoth}[c + dx]*\text{Log}[1 - E^{(-2*\text{ArcCoth}[c + dx])}])* \text{Sinh}[3*\text{ArcCoth}[c + dx]] + 6*c*\text{Log}[1/((c + dx)*\text{Sqrt}[1 - (c + dx)^{-2}])]* \text{Sinh}[3*\text{ArcCoth}[c + dx]]))/((12*d^3)$

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**Maple [B]** time = 0.069, size = 2694, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*(a+b\*arccoth(d\*x+c))^2,x)

[Out]  $-1/d^2*b^2*f*arccoth(d*x+c)*\ln(d*x+c+1)*e+1/6/d^3*b^2*f^2*\ln(d*x+c+1)*\ln(-1/2*d*x-1/2*c+1/2)*c^{-3-1/2}/d^2*b^2*f*\ln(d*x+c+1)*\ln(-1/2*d*x-1/2*c+1/2)*e+1/$

$$\begin{aligned}
& 2/d^2*b^2*f*\ln(1/2+1/2*d*x+1/2*c)*\ln(-1/2*d*x-1/2*c+1/2)*e+1/3*a^2*f^2*x^3+ \\
& a^2*x*e^2+1/3*b^2*f^2*x/d^2+1/3/d*a*b*f^2*x^2+1/3*a^2/f*e^3+1/3/d^3*b^2*f^2 \\
& *c-5/3/d^3*a*b*f^2*c^2+1/2/d^2*b^2*f*\ln(1/2+1/2*d*x+1/2*c)*\ln(-1/2*d*x-1/2* \\
& c+1/2)*c^2*e-1/2/d^2*b^2*f*\ln(d*x+c-1)*\ln(1/2+1/2*d*x+1/2*c)*c^2*e-1/d^2*b^ \\
& 2*f*\operatorname{arccoth}(d*x+c)*\ln(d*x+c+1)*c^2*e-1/2/d^2*b^2*f*\ln(d*x+c+1)*\ln(-1/2*d*x- \\
& 1/2*c+1/2)*c^2*e-2/d^2*a*b*f*\ln(d*x+c-1)*c*e-1/d^2*a*b*f*\ln(d*x+c+1)*c^2*e- \\
& 2/d^2*a*b*f*\ln(d*x+c+1)*c*e+1/d^2*a*b*f*\ln(d*x+c-1)*c^2*e-1/d^2*b^2*f*\ln(d* \\
& x+c+1)*\ln(-1/2*d*x-1/2*c+1/2)*c*e-2/d^2*b^2*f*\operatorname{arccoth}(d*x+c)*\ln(d*x+c-1)*c* \\
& e+1/d^2*b^2*f*\operatorname{arccoth}(d*x+c)*\ln(d*x+c-1)*c^2*e+1/d^2*b^2*f*\ln(1/2+1/2*d*x+1 \\
& /2*c)*\ln(-1/2*d*x-1/2*c+1/2)*c*e-2/d^2*b^2*f*\operatorname{arccoth}(d*x+c)*\ln(d*x+c+1)*c*e \\
& +1/d^2*b^2*f*\ln(d*x+c-1)*\ln(1/2+1/2*d*x+1/2*c)*c*e-1/12/d^3*b^2*f^2*\ln(d*x+ \\
& c+1)^2-1/3/d^3*b^2*f^2*\operatorname{dilog}(1/2+1/2*d*x+1/2*c)+1/6/d^3*b^2*f^2*\ln(d*x+c-1) \\
& -1/4/d*b^2*\ln(d*x+c+1)^2*e^2-1/d*b^2*\operatorname{dilog}(1/2+1/2*d*x+1/2*c)*e^2+1/3*b^2/f \\
& *\operatorname{arccoth}(d*x+c)^2*e^3+1/12*b^2/f*\ln(d*x+c-1)^2*e^3+1/12*b^2/f*\ln(d*x+c+1)^2 \\
& *e^3+1/3*b^2*f^2*\operatorname{arccoth}(d*x+c)^2*x^3+\operatorname{arccoth}(d*x+c)^2*x*b^2*e^2+a^2*f*x^2* \\
& e+1/4/d*b^2*\ln(d*x+c-1)^2*e^2-1/6/d^3*b^2*f^2*\ln(d*x+c+1)+1/12/d^3*b^2*f^2* \\
& \ln(d*x+c-1)^2-5/3/d^3*b^2*f^2*\operatorname{arccoth}(d*x+c)*c^2+1/3/d*b^2*f^2*\operatorname{arccoth}(d*x+ \\
& c)*x^2-1/6/d^3*b^2*f^2*\ln(d*x+c-1)*\ln(1/2+1/2*d*x+1/2*c)+1/3/d^3*a*b*f^2*\ln \\
& (d*x+c+1)+1/6/d^3*b^2*f^2*\ln(d*x+c+1)*\ln(-1/2*d*x-1/2*c+1/2)-1/6/d^3*b^2*f^ \\
& 2*\ln(1/2+1/2*d*x+1/2*c)*\ln(-1/2*d*x-1/2*c+1/2)+1/3/d^3*b^2*f^2*\operatorname{arccoth}(d*x+ \\
& c)*\ln(d*x+c+1)+1/3/d^3*b^2*f^2*\operatorname{arccoth}(d*x+c)*\ln(d*x+c-1)+1/3/d^3*a*b*f^2*\ln \\
& n(d*x+c-1)+1/d*a*b*\ln(d*x+c-1)*e^2+1/d*a*b*\ln(d*x+c+1)*e^2-1/d^3*b^2*f^2*\ln \\
& (d*x+c+1)*c-1/12/d^3*b^2*f^2*\ln(d*x+c-1)^2*c^3-1/12/d^3*b^2*f^2*\ln(d*x+c+1) \\
& ^2*c^3-1/4/d^3*b^2*f^2*\ln(d*x+c+1)^2*c^2-1/4/d^3*b^2*f^2*\ln(d*x+c+1)^2*c+1/ \\
& 4/d^2*b^2*f*\ln(d*x+c-1)^2*e-1/d^3*b^2*f^2*\operatorname{dilog}(1/2+1/2*d*x+1/2*c)*c^2+2/3* \\
& a*b*f^2*\operatorname{arccoth}(d*x+c)*x^3+b^2*f*\operatorname{arccoth}(d*x+c)^2*e*x^2+1/3*b^2/f*\operatorname{arccoth}(d \\
& *x+c)*\ln(d*x+c-1)*e^3-1/6*b^2/f*\ln(d*x+c+1)*\ln(-1/2*d*x-1/2*c+1/2)*e^3+1/6* \\
& b^2/f*\ln(1/2+1/2*d*x+1/2*c)*\ln(-1/2*d*x-1/2*c+1/2)*e^3-1/3*b^2/f*\operatorname{arccoth}(d* \\
& x+c)*\ln(d*x+c+1)*e^3-1/6*b^2/f*\ln(d*x+c-1)*\ln(1/2+1/2*d*x+1/2*c)*e^3+1/3*a* \\
& b/f*\ln(d*x+c-1)*e^3-1/3*a*b/f*\ln(d*x+c+1)*e^3+2/3*a*b/f*\operatorname{arccoth}(d*x+c)*e^3+ \\
& 2*\operatorname{arccoth}(d*x+c)*x*a*b*e^2+1/d^2*b^2*f*\ln(d*x+c-1)*e+1/d^2*b^2*f*\ln(d*x+c+1) \\
& )*e+1/4/d^3*b^2*f^2*\ln(d*x+c-1)^2*c^2-1/4/d^3*b^2*f^2*\ln(d*x+c-1)^2*c+1/4/d \\
& ^2*b^2*f*\ln(d*x+c+1)^2*e-1/d^3*b^2*f^2*\ln(d*x+c-1)*c-1/4/d*b^2*\ln(d*x+c+1)^ \\
& 2*c*e^2-1/4/d*b^2*\ln(d*x+c-1)^2*c*e^2+1/d*b^2*\operatorname{arccoth}(d*x+c)*\ln(d*x+c-1)*e^ \\
& 2+1/2/d*b^2*\ln(d*x+c+1)*\ln(-1/2*d*x-1/2*c+1/2)*e^2-1/2/d*b^2*\ln(1/2+1/2*d*x \\
& +1/2*c)*\ln(-1/2*d*x-1/2*c+1/2)*e^2-1/2/d*b^2*\ln(d*x+c-1)*\ln(1/2+1/2*d*x+1/2 \\
& *c)*e^2+1/d*b^2*\operatorname{arccoth}(d*x+c)*\ln(d*x+c+1)*e^2-4/3*a*b/d^2*f^2*c*x+1/4/d^2* \\
& b^2*f*\ln(d*x+c+1)^2*c^2*e+1/4/d^2*b^2*f*\ln(d*x+c-1)^2*c^2*e-1/2/d^2*b^2*f* \\
& \ln(d*x+c-1)^2*c*e+1/d*a*b*\ln(d*x+c+1)*c*e^2-1/d*a*b*\ln(d*x+c-1)*c*e^2+1/d^2* \\
& a*b*f*\ln(d*x+c-1)*e-4/3/d^2*b^2*f^2*\operatorname{arccoth}(d*x+c)*c*x+2/d^2*a*b*f*e*c-1/d^ \\
& 3*b^2*f^2*\operatorname{arccoth}(d*x+c)*\ln(d*x+c-1)*c+1/3/d^3*b^2*f^2*\operatorname{arccoth}(d*x+c)*\ln(d* \\
& x+c+1)*c^3+1/d^3*b^2*f^2*\operatorname{arccoth}(d*x+c)*\ln(d*x+c+1)*c^2+1/d^3*b^2*f^2*\operatorname{arcco} \\
& th(d*x+c)*\ln(d*x+c+1)*c+2*a*b*f*\operatorname{arccoth}(d*x+c)*e*x^2+2/d*b^2*f*\operatorname{arccoth}(d*x+ \\
& c)*e*x+2/d^2*b^2*f*\operatorname{arccoth}(d*x+c)*e*c+2*a*b/d*f*e*x-1/2/d^3*b^2*f^2*\ln(d*x+ \\
& c-1)*\ln(1/2+1/2*d*x+1/2*c)*c^2-1/3/d^3*b^2*f^2*\operatorname{arccoth}(d*x+c)*\ln(d*x+c-1)*c
\end{aligned}$$

$$\begin{aligned} &^3+1/2/d^3*b^2*f^2*\ln(d*x+c-1)*\ln(1/2+1/2*d*x+1/2*c)*c-1/2/d^2*b^2*f*\ln(d*x \\ &+c-1)*\ln(1/2+1/2*d*x+1/2*c)*e+1/2/d*b^2*\ln(d*x+c-1)*\ln(1/2+1/2*d*x+1/2*c)*c \\ &*e^2+1/d*b^2*\operatorname{arccoth}(d*x+c)*\ln(d*x+c+1)*c*e^2+1/2/d^2*b^2*f*\ln(d*x+c+1)^2*c \\ &*e+2/d^2*b^2*f*\operatorname{dilog}(1/2+1/2*d*x+1/2*c)*c*e+1/d^3*a*b*f^2*\ln(d*x+c-1)*c^2-1 \\ &/d*b^2*\operatorname{arccoth}(d*x+c)*\ln(d*x+c-1)*c*e^2+1/2/d*b^2*\ln(d*x+c+1)*\ln(-1/2*d*x-1 \\ &/2*c+1/2)*c*e^2-1/2/d*b^2*\ln(1/2+1/2*d*x+1/2*c)*\ln(-1/2*d*x-1/2*c+1/2)*c*e^ \\ &2+1/d^2*b^2*f*\operatorname{arccoth}(d*x+c)*\ln(d*x+c-1)*e+1/6/d^3*b^2*f^2*\ln(d*x+c-1)*\ln(1 \\ &/2+1/2*d*x+1/2*c)*c^3+1/d^3*a*b*f^2*\ln(d*x+c+1)*c^2+1/3/d^3*a*b*f^2*\ln(d*x+ \\ &c+1)*c^3-1/d^2*a*b*f*\ln(d*x+c+1)*e-1/3/d^3*a*b*f^2*\ln(d*x+c-1)*c^3+1/d^3*a* \\ &b*f^2*\ln(d*x+c+1)*c-1/d^3*a*b*f^2*\ln(d*x+c-1)*c+1/2/d^3*b^2*f^2*\ln(d*x+c+1) \\ &*\ln(-1/2*d*x-1/2*c+1/2)*c^2+1/2/d^3*b^2*f^2*\ln(d*x+c+1)*\ln(-1/2*d*x-1/2*c+1 \\ &/2)*c-1/6/d^3*b^2*f^2*\ln(1/2+1/2*d*x+1/2*c)*\ln(-1/2*d*x-1/2*c+1/2)*c^3-1/2/ \\ &d^3*b^2*f^2*\ln(1/2+1/2*d*x+1/2*c)*\ln(-1/2*d*x-1/2*c+1/2)*c^2-1/2/d^3*b^2*f^ \\ &2*\ln(1/2+1/2*d*x+1/2*c)*\ln(-1/2*d*x-1/2*c+1/2)*c+1/d^3*b^2*f^2*\operatorname{arccoth}(d*x+ \\ &c)*\ln(d*x+c-1)*c^2 \end{aligned}$$

**Maxima [B]** time = 1.97234, size = 1068, normalized size = 2.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*(a+b\*arccoth(d\*x+c))^2,x, algorithm="maxima")

[Out]  $1/3*a^2*f^2*x^3 + a^2*e*f*x^2 + (2*x^2*\operatorname{arccoth}(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1)*\log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*\log(d*x + c - 1)/d^3))*a*b*e*f + 1/3*(2*x^3*\operatorname{arccoth}(d*x + c) + d*((d*x^2 - 4*c*x)/d^3 + (c^3 + 3*c^2 + 3*c + 1)*\log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3*c - 1)*\log(d*x + c - 1)/d^4))*a*b*f^2 + a^2*e^2*x + (2*(d*x + c)*\operatorname{arccoth}(d*x + c) + \log(-(d*x + c)^2 + 1))*a*b*e^2/d - 1/3*(3*d^2*e^2 - 6*c*d*e*f + 3*c^2*f^2 + f^2)*(\log(d*x + c - 1)*\log(1/2*d*x + 1/2*c + 1/2) + \operatorname{dilog}(-1/2*d*x - 1/2*c + 1/2))*b^2/d^3 - 1/6*(5*c^2*f^2 - 6*d*e*f - 6*(d*e*f - f^2)*c + f^2)*b^2*\log(d*x + c + 1)/d^3 + 1/12*(4*b^2*d*f^2*x + (b^2*d^3*f^2*x^3 + 3*b^2*d^3*e*f*x^2 + 3*b^2*d^3*e^2*x + (c^3*f^2 + 3*d^2*e^2 - 3*(d*e*f - f^2)*c^2 - 3*d*e*f + 3*(d^2*e^2 - 2*d*e*f + f^2)*c + f^2)*b^2)*\log(d*x + c + 1)^2 + (b^2*d^3*f^2*x^3 + 3*b^2*d^3*e*f*x^2 + 3*b^2*d^3*e^2*x + (c^3*f^2 - 3*d^2*e^2 - 3*(d*e*f + f^2)*c^2 - 3*d*e*f + 3*(d^2*e^2 + 2*d*e*f + f^2)*c - f^2)*b^2)*\log(d*x + c - 1)^2 + 2*(b^2*d^2*f^2*x^2 + 2*(3*d^2*e*f - 2*c*d*f^2))*b^2*x - (b^2*d^3*f^2*x^3 + 3*b^2*d^3*e*f*x^2 + 3*b^2*d^3*e^2*x + (c^3*f^2 - 3*d^2*e^2 - 3*(d*e*f + f^2)*c^2 - 3*d*e*f + 3*(d^2*e^2 + 2*d*e*f + f^2)*c - f^2)*b^2)*\log(d*x + c - 1))*\log(d*x + c + 1) - 2*(b^2*d^2*f^2*x^2 + 2*(3*d^2*e*f - 2*c*d*f^2))*b^2*x - (5*c^2*f^2 + 6*d*e*f - 6*(d*e*f + f^2)*c + f^2)*b^2)*\log(d*x + c -$



1))/d^3

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$\int (a^2 f^2 x^2 + 2 a^2 e f x + a^2 e^2 + (b^2 f^2 x^2 + 2 b^2 e f x + b^2 e^2) \operatorname{arccoth}(dx + c)^2 + 2 (ab f^2 x^2 + 2 ab e f x + ab e^2) \operatorname{arccoth}(dx + c)) dx$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*(a+b\*arccoth(d\*x+c))^2,x, algorithm="fricas")

[Out] integral(a^2\*f^2\*x^2 + 2\*a^2\*e\*f\*x + a^2\*e^2 + (b^2\*f^2\*x^2 + 2\*b^2\*e\*f\*x + b^2\*e^2)\*arccoth(d\*x + c)^2 + 2\*(a\*b\*f^2\*x^2 + 2\*a\*b\*e\*f\*x + a\*b\*e^2)\*arccoth(d\*x + c), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*(a+b\*acoth(d\*x+c))\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (fx + e)^2 (b \operatorname{arccoth}(dx + c) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*(a+b\*arccoth(d\*x+c))^2,x, algorithm="giac")

[Out] integrate((f\*x + e)^2\*(b\*arccoth(d\*x + c) + a)^2, x)

### 3.110 $\int (e + fx) (a + b \coth^{-1}(c + dx))^2 dx$

**Optimal.** Leaf size=221

$$\frac{b^2(de - cf)\text{PolyLog}\left(2, -\frac{c+dx+1}{-c-dx+1}\right)}{d^2} - \frac{\left((c^2 + 1)f^2 - 2cdef + d^2e^2\right)(a + b \coth^{-1}(c + dx))^2}{2d^2f} + \frac{(de - cf)(a + b \coth^{-1}(c + dx))^2}{d^2}$$

[Out] (a\*b\*f\*x)/d + (b^2\*f\*(c + d\*x)\*ArcCoth[c + d\*x])/d^2 + ((d\*e - c\*f)\*(a + b\*ArcCoth[c + d\*x])^2)/d^2 - ((d^2\*e^2 - 2\*c\*d\*e\*f + (1 + c^2)\*f^2)\*(a + b\*ArcCoth[c + d\*x])^2)/(2\*d^2\*f) + ((e + f\*x)^2\*(a + b\*ArcCoth[c + d\*x])^2)/(2\*f) - (2\*b\*(d\*e - c\*f)\*(a + b\*ArcCoth[c + d\*x])\*Log[2/(1 - c - d\*x)])/d^2 + (b^2\*f\*Log[1 - (c + d\*x)^2])/(2\*d^2) - (b^2\*(d\*e - c\*f)\*PolyLog[2, -((1 + c + d\*x)/(1 - c - d\*x))])/d^2

**Rubi [A]** time = 0.442664, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$ , Rules used = {6112, 5929, 5911, 260, 6049, 5949, 5985, 5919, 2402, 2315}

$$\frac{b^2(de - cf)\text{PolyLog}\left(2, -\frac{c+dx+1}{-c-dx+1}\right)}{d^2} + \frac{\left(-\frac{(c^2+1)f}{d} + 2ce - \frac{de^2}{f}\right)(a + b \coth^{-1}(c + dx))^2}{2d} + \frac{(de - cf)(a + b \coth^{-1}(c + dx))^2}{d^2}$$

Antiderivative was successfully verified.

[In] Int[(e + f\*x)\*(a + b\*ArcCoth[c + d\*x])^2,x]

[Out] (a\*b\*f\*x)/d + (b^2\*f\*(c + d\*x)\*ArcCoth[c + d\*x])/d^2 + ((d\*e - c\*f)\*(a + b\*ArcCoth[c + d\*x])^2)/d^2 + ((2\*c\*e - (d\*e^2)/f - ((1 + c^2)\*f)/d)\*(a + b\*ArcCoth[c + d\*x])^2)/(2\*d) + ((e + f\*x)^2\*(a + b\*ArcCoth[c + d\*x])^2)/(2\*f) - (2\*b\*(d\*e - c\*f)\*(a + b\*ArcCoth[c + d\*x])\*Log[2/(1 - c - d\*x)])/d^2 + (b^2\*f\*Log[1 - (c + d\*x)^2])/(2\*d^2) - (b^2\*(d\*e - c\*f)\*PolyLog[2, -((1 + c + d\*x)/(1 - c - d\*x))])/d^2

#### Rule 6112

Int[((a\_.) + ArcCoth[(c\_) + (d\_.)\*(x\_)])\*(b\_.))^ (p\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]

Rule 5929

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*((d\_) + (e\_.)\*(x\_.))^(q\_.), x\_Symbol] := Simp[((d + e\*x)^(q + 1)\*(a + b\*ArcCoth[c\*x])^p)/(e\*(q + 1)), x] - Dist[(b\*c\*p)/(e\*(q + 1)), Int[ExpandIntegrand[(a + b\*ArcCoth[c\*x])^(p - 1), (d + e\*x)^(q + 1)/(1 - c^2\*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]

Rule 5911

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.), x\_Symbol] := Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 6049

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*((f\_) + (g\_.)\*(x\_.))^(m\_.))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[(a + b\*ArcCoth[c\*x])^p/(d + e\*x^2), (f + g\*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[p, 0] && EqQ[c^2\*d + e, 0] && IGtQ[m, 0]

Rule 5949

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

Rule 5985

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*(x\_.))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*e\*(p + 1)), x] + Dist[1/(c\*d), Int[(a + b\*ArcCoth[c\*x])^p/(1 - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && IGtQ[p, 0]

Rule 5919

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_.)), x\_Symbol] := -Simp[((a + b\*ArcCoth[c\*x])^p\*Log[2/(1 + (e\*x)/d)])/e, x] + Dist[(b\*c\*p)/e, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*Log[2/(1 + (e\*x)/d)]]/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

]

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}
\int (e + fx)(a + b \coth^{-1}(c + dx))^2 dx &= \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d} + \frac{fx}{d}\right)(a + b \coth^{-1}(x))^2 dx, x, c + dx\right)}{d} \\
&= \frac{(e + fx)^2 (a + b \coth^{-1}(c + dx))^2}{2f} - \frac{b \text{Subst}\left(\int \left(-\frac{f^2(a + b \coth^{-1}(x))}{d^2} + \frac{(d^2 e^2 - 2cdef - (a + b \coth^{-1}(x))^2)}{d^2}\right) dx, x, c + dx\right)}{d^2} \\
&= \frac{(e + fx)^2 (a + b \coth^{-1}(c + dx))^2}{2f} - \frac{b \text{Subst}\left(\int \frac{(d^2 e^2 - 2cdef + (1 + c^2)f^2 + 2f(de - cf)x)(a + b \coth^{-1}(x))^2}{1 - x^2} dx, x, c + dx\right)}{d^2 f} \\
&= \frac{abfx}{d} + \frac{(e + fx)^2 (a + b \coth^{-1}(c + dx))^2}{2f} - \frac{b \text{Subst}\left(\int \frac{\left(d^2 e^2 \left(1 + \frac{f(-2cde + f + c^2 f)}{d^2 e^2}\right)(a + b \coth^{-1}(x))^2\right)}{1 - x^2} dx, x, c + dx\right)}{d^2} \\
&= \frac{abfx}{d} + \frac{b^2 f(c + dx) \coth^{-1}(c + dx)}{d^2} + \frac{(e + fx)^2 (a + b \coth^{-1}(c + dx))^2}{2f} - \frac{(b^2 f(c + dx) \coth^{-1}(c + dx))^2}{d^2} \\
&= \frac{abfx}{d} + \frac{b^2 f(c + dx) \coth^{-1}(c + dx)}{d^2} + \frac{(de - cf)(a + b \coth^{-1}(c + dx))^2}{d^2} - \frac{(d^2 e^2 (1 + \frac{f(-2cde + f + c^2 f)}{d^2 e^2}))^2}{d^2} \\
&= \frac{abfx}{d} + \frac{b^2 f(c + dx) \coth^{-1}(c + dx)}{d^2} + \frac{(de - cf)(a + b \coth^{-1}(c + dx))^2}{d^2} - \frac{(d^2 e^2 (1 + \frac{f(-2cde + f + c^2 f)}{d^2 e^2}))^2}{d^2} \\
&= \frac{abfx}{d} + \frac{b^2 f(c + dx) \coth^{-1}(c + dx)}{d^2} + \frac{(de - cf)(a + b \coth^{-1}(c + dx))^2}{d^2} - \frac{(d^2 e^2 (1 + \frac{f(-2cde + f + c^2 f)}{d^2 e^2}))^2}{d^2} \\
&= \frac{abfx}{d} + \frac{b^2 f(c + dx) \coth^{-1}(c + dx)}{d^2} + \frac{(de - cf)(a + b \coth^{-1}(c + dx))^2}{d^2} - \frac{(d^2 e^2 (1 + \frac{f(-2cde + f + c^2 f)}{d^2 e^2}))^2}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.593743, size = 295, normalized size = 1.33

$$2b^2(de - cf)\text{PolyLog}\left(2, e^{-2\coth^{-1}(c+dx)}\right) - a^2c^2f + 2a^2cde + 2a^2d^2ex + a^2d^2fx^2 + 2b \coth^{-1}(c + dx) \left(- (c + dx)(acf - a^2c^2)\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(e + f\*x)\*(a + b\*ArcCoth[c + d\*x])^2, x]

```
[Out] (2*a^2*c*d*e + 2*a*b*c*f - a^2*c^2*f + 2*a^2*d^2*e*x + 2*a*b*d*f*x + a^2*d^2*f*x^2 + b^2*(-1 + c + d*x)*(2*d*e + f - c*f + d*f*x)*ArcCoth[c + d*x]^2 + 2*b*ArcCoth[c + d*x]*(-(c + d*x)*(-(b*f) + a*c*f - a*d*(2*e + f*x))) - 2*b*(d*e - c*f)*Log[1 - E^(-2*ArcCoth[c + d*x])] + a*b*f*Log[1 - c - d*x] - a*b*f*Log[1 + c + d*x] - 4*a*b*d*e*Log[1/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)])] - 2*b^2*f*Log[1/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)])] + 4*a*b*c*f*Log[1/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)])] + 2*b^2*(d*e - c*f)*PolyLog[2, E^(-2*ArcCoth[c + d*x])])/(2*d^2)
```

**Maple [B]** time = 0.062, size = 857, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(a+b*arccoth(d*x+c))^2,x)
```

```
[Out] -1/4/d^2*b^2*ln(1/2+1/2*d*x+1/2*c)*ln(d*x+c-1)*f+1/d*b^2*arccoth(d*x+c)*f*x+1/d^2*b^2*arccoth(d*x+c)*f*c-1/2/d^2*b^2*arccoth(d*x+c)^2*f*c^2+1/2/d^2*b^2*arccoth(d*x+c)*ln(d*x+c-1)*f+1/d^2*a*b*c*f+1/2/d^2*b^2*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2+1/2*d*x+1/2*c)*c*f-1/d^2*b^2*arccoth(d*x+c)*ln(d*x+c+1)*c*f+1/2*a^2*x^2*f+a^2*x*e-1/d*b^2*dilog(1/2+1/2*d*x+1/2*c)*e-1/4/d*b^2*ln(d*x+c+1)^2*e+1/2/d^2*b^2*ln(d*x+c+1)*f+1/2/d^2*b^2*ln(d*x+c-1)*f+1/8/d^2*b^2*ln(d*x+c+1)^2*f+1/8/d^2*b^2*ln(d*x+c-1)^2*f+1/4/d*b^2*ln(d*x+c-1)^2*e-1/2/d^2*b^2*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)*c*f-1/d^2*a*b*arccoth(d*x+c)*c^2*f+2/d*arccoth(d*x+c)*a*b*c*e-1/d^2*a*b*ln(d*x+c-1)*c*f-1/d^2*a*b*ln(d*x+c+1)*c*f-1/d^2*b^2*arccoth(d*x+c)*ln(d*x+c-1)*c*f+1/2/d^2*b^2*ln(1/2+1/2*d*x+1/2*c)*ln(d*x+c-1)*c*f-1/2/d^2*a^2*c^2*f+1/d*a^2*c*e+1/2*b^2*arccoth(d*x+c)^2*f*x^2+arccoth(d*x+c)^2*x*b^2*e+1/4/d^2*b^2*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2+1/2*d*x+1/2*c)*f-1/4/d^2*b^2*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)*f+a*b*arccoth(d*x+c)*f*x^2+2*arccoth(d*x+c)*x*a*b*e-1/2/d*b^2*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2+1/2*d*x+1/2*c)*e+1/d*b^2*arccoth(d*x+c)*ln(d*x+c-1)*e+1/2/d*b^2*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)*e-1/2/d*b^2*ln(1/2+1/2*d*x+1/2*c)*ln(d*x+c-1)*e+1/d*b^2*arccoth(d*x+c)*ln(d*x+c+1)*e+1/d*arccoth(d*x+c)^2*b^2*c*e+1/d*a*b*ln(d*x+c-1)*e+1/d*a*b*ln(d*x+c+1)*e-1/2/d^2*b^2*arccoth(d*x+c)*ln(d*x+c+1)*f-1/4/d^2*b^2*ln(d*x+c-1)^2*c*f+1/d^2*b^2*dilog(1/2+1/2*d*x+1/2*c)*c*f-1/2/d^2*a*b*ln(d*x+c+1)*f+1/4/d^2*b^2*ln(d*x+c+1)^2*c*f+1/2/d^2*a*b*ln(d*x+c-1)*f+a*b*f*x/d
```

**Maxima [A]** time = 1.96123, size = 540, normalized size = 2.44

$$\frac{1}{2} a^2 f x^2 + \frac{1}{2} \left( 2 x^2 \operatorname{arccoth}(dx + c) + d \left( \frac{2x}{d^2} - \frac{(c^2 + 2c + 1) \log(dx + c + 1)}{d^3} + \frac{(c^2 - 2c + 1) \log(dx + c - 1)}{d^3} \right) \right) abf + a^2 e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*(a+b\*arccoth(d\*x+c))^2,x, algorithm="maxima")

[Out]  $\frac{1}{2} a^2 f x^2 + \frac{1}{2} (2 x^2 \operatorname{arccoth}(dx + c) + d (2 x / d^2 - (c^2 + 2c + 1) \log(dx + c + 1) / d^3 + (c^2 - 2c + 1) \log(dx + c - 1) / d^3)) a b f + a^2 e x + (2 (dx + c) \operatorname{arccoth}(dx + c) + \log(-(dx + c)^2 + 1)) a b e / d - (d e - c f) (\log(dx + c - 1) \log(1/2 dx + 1/2 c + 1/2) + \operatorname{dilog}(-1/2 dx - 1/2 c + 1/2)) b^2 / d^2 + 1/2 (c f + f) b^2 \log(dx + c + 1) / d^2 + 1/8 ((b^2 d^2 f x^2 + 2 b^2 d^2 e x - (c^2 f - 2 (d e - f) c - 2 d e + f) b^2) \log(dx + c + 1)^2 + (b^2 d^2 f x^2 + 2 b^2 d^2 e x - (c^2 f - 2 (d e + f) c + 2 d e + f) b^2) \log(dx + c - 1)^2 + 2 (2 b^2 d f x - (b^2 d^2 f x^2 + 2 b^2 d^2 e x - (c^2 f - 2 (d e + f) c + 2 d e + f) b^2) \log(dx + c - 1)) \log(dx + c + 1) - 4 (b^2 d f x + (c f - f) b^2) \log(dx + c - 1)) / d^2$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(a^2 f x + a^2 e + (b^2 f x + b^2 e) \operatorname{arccoth}(dx + c)^2 + 2(abf x + abe) \operatorname{arccoth}(dx + c), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*(a+b\*arccoth(d\*x+c))^2,x, algorithm="fricas")

[Out]  $\operatorname{integral}(a^2 f x + a^2 e + (b^2 f x + b^2 e) \operatorname{arccoth}(dx + c)^2 + 2(a b f x + a b e) \operatorname{arccoth}(dx + c), x)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{acoth}(c + dx))^2 (e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*(a+b\*acoth(d\*x+c))\*\*2,x)

[Out] Integral((a + b\*acoth(c + d\*x))\*\*2\*(e + f\*x), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (fx + e)(b \operatorname{arccoth}(dx + c) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*(a+b\*arccoth(d\*x+c))^2,x, algorithm="giac")

[Out] integrate((f\*x + e)\*(b\*arccoth(d\*x + c) + a)^2, x)



### 3.111 $\int (a + b \coth^{-1}(c + dx))^2 dx$

**Optimal.** Leaf size=97

$$-\frac{b^2 \text{PolyLog}\left(2, -\frac{c+dx+1}{-c-dx+1}\right)}{d} + \frac{(c+dx)(a+b \coth^{-1}(c+dx))^2}{d} + \frac{(a+b \coth^{-1}(c+dx))^2}{d} - \frac{2b \log\left(\frac{2}{-c-dx+1}\right)(a+b \coth^{-1}(c+dx))}{d}$$

[Out] (a + b\*ArcCoth[c + d\*x])^2/d + ((c + d\*x)\*(a + b\*ArcCoth[c + d\*x])^2)/d - (2\*b\*(a + b\*ArcCoth[c + d\*x])\*Log[2/(1 - c - d\*x)])/d - (b^2\*PolyLog[2, -((1 + c + d\*x)/(1 - c - d\*x))])/d

**Rubi [A]** time = 0.11621, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {6104, 5911, 5985, 5919, 2402, 2315}

$$-\frac{b^2 \text{PolyLog}\left(2, -\frac{c+dx+1}{-c-dx+1}\right)}{d} + \frac{(c+dx)(a+b \coth^{-1}(c+dx))^2}{d} + \frac{(a+b \coth^{-1}(c+dx))^2}{d} - \frac{2b \log\left(\frac{2}{-c-dx+1}\right)(a+b \coth^{-1}(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCoth[c + d\*x])^2, x]

[Out] (a + b\*ArcCoth[c + d\*x])^2/d + ((c + d\*x)\*(a + b\*ArcCoth[c + d\*x])^2)/d - (2\*b\*(a + b\*ArcCoth[c + d\*x])\*Log[2/(1 - c - d\*x)])/d - (b^2\*PolyLog[2, -((1 + c + d\*x)/(1 - c - d\*x))])/d

#### Rule 6104

Int[((a\_.) + ArcCoth[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^p, x\_Symbol] :> Dist[1/d, Subst[Int[(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0]

#### Rule 5911

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^p, x\_Symbol] :> Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcCoth[c\*x])^(p-1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

#### Rule 5985

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
  x_Symbol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

### Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] := -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 - c^2*x^2)
, x), x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

### Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

### Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

### Rubi steps

$$\begin{aligned}
\int (a + b \operatorname{coth}^{-1}(c + dx))^2 dx &= \frac{\operatorname{Subst}\left(\int (a + b \operatorname{coth}^{-1}(x))^2 dx, x, c + dx\right)}{d} \\
&= \frac{(c + dx)(a + b \operatorname{coth}^{-1}(c + dx))^2}{d} - \frac{(2b) \operatorname{Subst}\left(\int \frac{x(a + b \operatorname{coth}^{-1}(x))}{1-x^2} dx, x, c + dx\right)}{d} \\
&= \frac{(a + b \operatorname{coth}^{-1}(c + dx))^2}{d} + \frac{(c + dx)(a + b \operatorname{coth}^{-1}(c + dx))^2}{d} - \frac{(2b) \operatorname{Subst}\left(\int \frac{a + b \operatorname{coth}^{-1}(x)}{1-x} dx\right)}{d} \\
&= \frac{(a + b \operatorname{coth}^{-1}(c + dx))^2}{d} + \frac{(c + dx)(a + b \operatorname{coth}^{-1}(c + dx))^2}{d} - \frac{2b(a + b \operatorname{coth}^{-1}(c + dx))}{d} \\
&= \frac{(a + b \operatorname{coth}^{-1}(c + dx))^2}{d} + \frac{(c + dx)(a + b \operatorname{coth}^{-1}(c + dx))^2}{d} - \frac{2b(a + b \operatorname{coth}^{-1}(c + dx))}{d} \\
&= \frac{(a + b \operatorname{coth}^{-1}(c + dx))^2}{d} + \frac{(c + dx)(a + b \operatorname{coth}^{-1}(c + dx))^2}{d} - \frac{2b(a + b \operatorname{coth}^{-1}(c + dx))}{d}
\end{aligned}$$

**Mathematica [A]** time = 0.148008, size = 111, normalized size = 1.14

$$\frac{b^2 \operatorname{PolyLog}\left(2, e^{-2 \operatorname{coth}^{-1}(c+dx)}\right) + a \left( ac + adx - 2b \log\left(\frac{1}{(c+dx)\sqrt{1-\frac{1}{(c+dx)^2}}}\right) \right) + 2b \operatorname{coth}^{-1}(c + dx) \left( ac + adx - b \log\left(1 - e^{-2 \operatorname{coth}^{-1}(c+dx)}\right) \right)}{d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCoth[c + d\*x])^2, x]

[Out] (b^2\*(-1 + c + d\*x)\*ArcCoth[c + d\*x]^2 + 2\*b\*ArcCoth[c + d\*x]\*(a\*c + a\*d\*x - b\*Log[1 - E^(-2\*ArcCoth[c + d\*x])]) + a\*(a\*c + a\*d\*x - 2\*b\*Log[1/((c + d\*x)\*Sqrt[1 - (c + d\*x)^(-2)])]) + b^2\*PolyLog[2, E^(-2\*ArcCoth[c + d\*x])])/d

**Maple [B]** time = 0.111, size = 226, normalized size = 2.3

$$(\operatorname{arccoth}(dx + c))^2 xb^2 + \frac{(\operatorname{arccoth}(dx + c))^2 b^2 c}{d} + 2 \operatorname{arccoth}(dx + c) xab - 2 \frac{\operatorname{arccoth}(dx + c) b^2}{d} \ln\left(1 - \frac{1}{\sqrt{\frac{dx+c-1}{dx+c+1}}}\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arccoth(d*x+c))^2,x)`

[Out] `arccoth(d*x+c)^2*x*b^2+1/d*arccoth(d*x+c)^2*b^2*c+2*arccoth(d*x+c)*x*a*b-2/d*ln(1-1/((d*x+c-1)/(d*x+c+1))^(1/2))*arccoth(d*x+c)*b^2-2/d*ln(1+1/((d*x+c-1)/(d*x+c+1))^(1/2))*arccoth(d*x+c)*b^2+1/d*b^2*arccoth(d*x+c)^2+2/d*arccoth(d*x+c)*a*b*c+a^2*x-2/d*polylog(2,1/((d*x+c-1)/(d*x+c+1))^(1/2))*b^2-2/d*polylog(2,-1/((d*x+c-1)/(d*x+c+1))^(1/2))*b^2+1/d*a*b*ln((d*x+c)^2-1)+a^2*c/d`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$a^2x + \frac{1}{4}b^2 \left( \frac{dx \log(dx+c-1)^2 + (dx+c+1) \log(dx+c+1)^2 - 2(dx+c-1) \log(dx+c+1) \log(dx+c-1)}{d} + \int \frac{2(c^2}{dx+c-1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccoth(d*x+c))^2,x, algorithm="maxima")`

[Out] `a^2*x + 1/4*b^2*((d*x*log(d*x + c - 1)^2 + (d*x + c + 1)*log(d*x + c + 1)^2 - 2*(d*x + c - 1)*log(d*x + c + 1)*log(d*x + c - 1))/d + integrate(2*(c^2 + (c*d - 3*d)*x - 2*c + 1)*log(d*x + c - 1)/(d^2*x^2 + 2*c*d*x + c^2 - 1), x)) + (2*(d*x + c)*arccoth(d*x + c) + log(-(d*x + c)^2 + 1))*a*b/d`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(b^2 \operatorname{arccoth}(dx+c)^2 + 2ab \operatorname{arccoth}(dx+c) + a^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccoth(d*x+c))^2,x, algorithm="fricas")`

[Out] `integral(b^2*arccoth(d*x + c)^2 + 2*a*b*arccoth(d*x + c) + a^2, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{acoth}(c + dx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth(d\*x+c))\*\*2,x)

[Out] Integral((a + b\*acoth(c + d\*x))\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arccoth}(dx + c) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))^2,x, algorithm="giac")

[Out] integrate((b\*arccoth(d\*x + c) + a)^2, x)

$$3.112 \quad \int \frac{(a + b \coth^{-1}(c + dx))^2}{e + fx} dx$$

**Optimal.** Leaf size=214

$$\frac{b(a + b \coth^{-1}(c + dx)) \operatorname{PolyLog}\left(2, 1 - \frac{2d(e+fx)}{(c+dx+1)(-cf+de+fx)}\right)}{f} + \frac{b \operatorname{PolyLog}\left(2, 1 - \frac{2}{c+dx+1}\right)(a + b \coth^{-1}(c + dx))}{f} - \frac{b^2 \operatorname{PolyLog}\left(3, 1 - \frac{2}{c+dx+1}\right)(a + b \coth^{-1}(c + dx))}{f}$$

```
[Out] -(((a + b*ArcCoth[c + d*x])^2*Log[2/(1 + c + d*x)])/f) + ((a + b*ArcCoth[c + d*x])^2*Log[(2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/f + (b*(a + b*ArcCoth[c + d*x])*PolyLog[2, 1 - 2/(1 + c + d*x)])/f - (b*(a + b*ArcCoth[c + d*x])*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/f + (b^2*PolyLog[3, 1 - 2/(1 + c + d*x)])/(2*f) - (b^2*PolyLog[3, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/(2*f)
```

**Rubi [A]** time = 0.153347, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {6112, 5923}

$$\frac{b(a + b \coth^{-1}(c + dx)) \operatorname{PolyLog}\left(2, 1 - \frac{2d(e+fx)}{(c+dx+1)(-cf+de+fx)}\right)}{f} + \frac{b \operatorname{PolyLog}\left(2, 1 - \frac{2}{c+dx+1}\right)(a + b \coth^{-1}(c + dx))}{f} - \frac{b^2 \operatorname{PolyLog}\left(3, 1 - \frac{2}{c+dx+1}\right)(a + b \coth^{-1}(c + dx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[c + d*x])^2/(e + f*x), x]
```

```
[Out] -(((a + b*ArcCoth[c + d*x])^2*Log[2/(1 + c + d*x)])/f) + ((a + b*ArcCoth[c + d*x])^2*Log[(2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/f + (b*(a + b*ArcCoth[c + d*x])*PolyLog[2, 1 - 2/(1 + c + d*x)])/f - (b*(a + b*ArcCoth[c + d*x])*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/f + (b^2*PolyLog[3, 1 - 2/(1 + c + d*x)])/(2*f) - (b^2*PolyLog[3, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/(2*f)
```

### Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)])*(b_.)^(p_.)*((e_.) + (f_.)*(x_.))^(m_.), x_Symbol] := Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
```

Rule 5923

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^2/((d_.) + (e_.)*(x_.)), x_Symbol] :=
  -Simp[((a + b*ArcCoth[c*x])^2*Log[2/(1 + c*x)])/e, x] + (Simp[((a + b*ArcC
oth[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Simp[(b*(a
+ b*ArcCoth[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e, x] - Simp[(b*(a + b*ArcCo
th[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Sim
p[(b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e), x] - Simp[(b^2*PolyLog[3, 1 - (2
*c*(d + e*x))/((c*d + e)*(1 + c*x)))/(2*e), x]) /; FreeQ[{a, b, c, d, e},
x] && NeQ[c^2*d^2 - e^2, 0]
```

Rubi steps

$$\int \frac{(a + b \coth^{-1}(c + dx))^2}{e + fx} dx = \frac{\text{Subst}\left(\int \frac{(a + b \coth^{-1}(x))^2}{\frac{de - cf}{d} + \frac{fx}{d}} dx, x, c + dx\right)}{d}$$

$$= -\frac{(a + b \coth^{-1}(c + dx))^2 \log\left(\frac{2}{1 + c + dx}\right)}{f} + \frac{(a + b \coth^{-1}(c + dx))^2 \log\left(\frac{2d(e + fx)}{(de + f - cf)(1 + c + dx)}\right)}{f}$$

**Mathematica [C]** time = 16.6836, size = 1721, normalized size = 8.04

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCoth[c + d*x])^2/(e + f*x), x]
```

```
[Out] (a^2*Log[e + f*x])/f + 2*a*b*(((ArcCoth[c + d*x] - ArcTanh[c + d*x])*Log[e
+ f*x])/f - (I*(I*ArcTanh[c + d*x]*(-Log[1/Sqrt[1 - (c + d*x)^2]] + Log[I*S
inh[ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]]]) + ((-I)*(I*ArcTanh[(d*e -
c*f)/f] + I*ArcTanh[c + d*x])^2 - (I/4)*(Pi - (2*I)*ArcTanh[c + d*x])^2 + 2
*(I*ArcTanh[(d*e - c*f)/f] + I*ArcTanh[c + d*x])*Log[1 - E^((2*I)*(I*ArcTan
h[(d*e - c*f)/f] + I*ArcTanh[c + d*x]))] + (Pi - (2*I)*ArcTanh[c + d*x])*Lo
g[1 - E^(I*(Pi - (2*I)*ArcTanh[c + d*x]))] - (Pi - (2*I)*ArcTanh[c + d*x])*
Log[2*Sin[(Pi - (2*I)*ArcTanh[c + d*x])/2]] - 2*(I*ArcTanh[(d*e - c*f)/f] +
I*ArcTanh[c + d*x])*Log[(2*I)*Sinh[ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*
x]]) - I*PolyLog[2, E^((2*I)*(I*ArcTanh[(d*e - c*f)/f] + I*ArcTanh[c + d*x]
))]) - I*PolyLog[2, E^(I*(Pi - (2*I)*ArcTanh[c + d*x]))]/2)/f) - (b^2*(d*e
- c*f + f*(c + d*x))*(1 - (c + d*x)^2)*(-(I*f*Pi^3 - 8*d*e*ArcCoth[c + d*x]
)^3 - 8*f*ArcCoth[c + d*x]^3 + 8*c*f*ArcCoth[c + d*x]^3 + 24*f*ArcCoth[c +
```

$$\begin{aligned}
& d*x]^2*\text{Log}[1 - E^{(2*\text{ArcCoth}[c + d*x])}] + 24*f*\text{ArcCoth}[c + d*x]*\text{PolyLog}[2, E \\
& ^{(2*\text{ArcCoth}[c + d*x])}] - 12*f*\text{PolyLog}[3, E^{(2*\text{ArcCoth}[c + d*x])}])]/(24*f^2) \\
& + ((-(d*e) - f + c*f)*(-(d*e) + f + c*f)*(2*d*e*\text{ArcCoth}[c + d*x]^3 - 6*f*\text{Ar} \\
& \text{cCoth}[c + d*x]^3 - 2*c*f*\text{ArcCoth}[c + d*x]^3 - (4*d*e*\text{Sqrt}[(d^2*e^2 - 2*c*d* \\
& e*f + (-1 + c^2)*f^2)]/(d*e - c*f)^2)*\text{ArcCoth}[c + d*x]^3)/E^{\text{ArcTanh}[f/(d*e - \\
& c*f)]} + (4*c*f*\text{Sqrt}[(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)]/(d*e - c*f)^2)* \\
& \text{ArcCoth}[c + d*x]^3)/E^{\text{ArcTanh}[f/(d*e - c*f)]} + (6*I)*f*\text{Pi}*\text{ArcCoth}[c + d*x]* \\
& \text{Log}[2] - f*\text{ArcCoth}[c + d*x]^2*\text{Log}[64] - (6*I)*f*\text{Pi}*\text{ArcCoth}[c + d*x]*\text{Log}[E^{( \\
& -\text{ArcCoth}[c + d*x])} + E^{\text{ArcCoth}[c + d*x]}] + 6*f*\text{ArcCoth}[c + d*x]^2*\text{Log}[1 - E \\
& ^{(2*(\text{ArcCoth}[c + d*x] + \text{ArcTanh}[f/(d*e - c*f)])} + 12*f*\text{ArcCoth}[c + d*x]*\text{A} \\
& \text{rcTanh}[f/(d*e - c*f)]*\text{Log}[(I/2)*E^{(-\text{ArcCoth}[c + d*x] - \text{ArcTanh}[f/(d*e - c*f \\
& )])}*(-1 + E^{(2*(\text{ArcCoth}[c + d*x] + \text{ArcTanh}[f/(d*e - c*f)])} + 6*f*\text{ArcCoth} \\
& [c + d*x]^2*\text{Log}[-(d*e*(-1 + E^{(2*\text{ArcCoth}[c + d*x])}) + (1 + c + E^{(2*\text{ArcCot} \\
& h[c + d*x]) - c}*E^{(2*\text{ArcCoth}[c + d*x])})*f)/E^{\text{ArcCoth}[c + d*x]}] - 6*f*\text{ArcCo} \\
& \text{th}[c + d*x]^2*\text{Log}[-(d*e*(-1 + E^{(2*\text{ArcCoth}[c + d*x])}) + (-1 - E^{(2*\text{ArcCot} \\
& h[c + d*x]) + c*(-1 + E^{(2*\text{ArcCoth}[c + d*x])})*)*f)/(d*e - (1 + c)*f)] + 6*f* \\
& \text{ArcCoth}[c + d*x]^2*\text{Log}[1 - (E^{\text{ArcCoth}[c + d*x]}*\text{Sqrt}[d*e + f - c*f])/ \text{Sqrt}[d* \\
& e - (1 + c)*f]] + 6*f*\text{ArcCoth}[c + d*x]^2*\text{Log}[1 + (E^{\text{ArcCoth}[c + d*x]}*\text{Sqrt}[d \\
& *e + f - c*f])/ \text{Sqrt}[d*e - (1 + c)*f]] + (6*I)*f*\text{Pi}*\text{ArcCoth}[c + d*x]*\text{Log}[1/\text{S} \\
& \text{qrt}[1 - (c + d*x)^{-2}]] - 6*f*\text{ArcCoth}[c + d*x]^2*\text{Log}[-(f/\text{Sqrt}[1 - (c + d*x) \\
& ]^{-2})) - (d*e)/((c + d*x)*\text{Sqrt}[1 - (c + d*x)^{-2}]) + (c*f)/((c + d*x)*\text{S} \\
& \text{qrt}[1 - (c + d*x)^{-2}])] - 12*f*\text{ArcCoth}[c + d*x]*\text{ArcTanh}[f/(d*e - c*f)]*\text{Log} \\
& [I*\text{Sinh}[\text{ArcCoth}[c + d*x] + \text{ArcTanh}[f/(d*e - c*f)]]] + 6*f*\text{ArcCoth}[c + d*x]* \\
& \text{PolyLog}[2, E^{(2*(\text{ArcCoth}[c + d*x] + \text{ArcTanh}[f/(d*e - c*f)])} - 6*f*\text{ArcCoth} \\
& [c + d*x]*\text{PolyLog}[2, (E^{(2*\text{ArcCoth}[c + d*x])}*(d*e + f - c*f))/(d*e - (1 + c) \\
& *f)] + 12*f*\text{ArcCoth}[c + d*x]*\text{PolyLog}[2, -(E^{\text{ArcCoth}[c + d*x]}*\text{Sqrt}[d*e + f \\
& - c*f])/ \text{Sqrt}[d*e - (1 + c)*f]]] + 12*f*\text{ArcCoth}[c + d*x]*\text{PolyLog}[2, (E^{\text{ArcC} \\
& \text{oth}[c + d*x]}*\text{Sqrt}[d*e + f - c*f])/ \text{Sqrt}[d*e - (1 + c)*f]] - 3*f*\text{PolyLog}[3, E \\
& ^{(2*(\text{ArcCoth}[c + d*x] + \text{ArcTanh}[f/(d*e - c*f)])} + 3*f*\text{PolyLog}[3, (E^{(2*\text{Ar} \\
& \text{cCoth}[c + d*x])}*(d*e + f - c*f))/(d*e - (1 + c)*f)] - 12*f*\text{PolyLog}[3, -(E^ \\
& \text{ArcCoth}[c + d*x]}*\text{Sqrt}[d*e + f - c*f])/ \text{Sqrt}[d*e - (1 + c)*f]]] - 12*f*\text{PolyLo} \\
& \text{g}[3, (E^{\text{ArcCoth}[c + d*x]}*\text{Sqrt}[d*e + f - c*f])/ \text{Sqrt}[d*e - (1 + c)*f]])]/(6*f \\
& ^2*(d*e + f - c*f)*(d*e - (1 + c)*f)))/(d*(c + d*x)^2*(e + f*x)*(1 - (c + \\
& d*x)^{-2}))
\end{aligned}$$


---

**Maple [C]** time = 0.855, size = 1845, normalized size = 8.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth(d\*x+c))^2/(f\*x+e),x)



```
[Out] -1/2*I*b^2/f*Pi*arccoth(d*x+c)^2*csgn(I*((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^2*csgn(I/((d*x+c+1)/(d*x+c-1)-1)-b^2/f*arccoth(d*x+c)^2*ln(1+1/((d*x+c-1)/(d*x+c+1))^(1/2))-2*b^2/f*arccoth(d*x+c)*polylog(2,-1/((d*x+c-1)/(d*x+c+1))^(1/2))-b^2/f*arccoth(d*x+c)^2*ln(1-1/((d*x+c-1)/(d*x+c+1))^(1/2))-2*b^2/f*arccoth(d*x+c)*polylog(2,1/((d*x+c-1)/(d*x+c+1))^(1/2))-b^2/(c*f-d*e-f)*arccoth(d*x+c)^2*ln(1-(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))-b^2/(c*f-d*e-f)*arccoth(d*x+c)*polylog(2,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))+b^2/f*arccoth(d*x+c)^2*ln((d*x+c+1)/(d*x+c-1)-1)-b^2/f*arccoth(d*x+c)^2*ln((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)+b^2*ln((d*x+c)*f-c*f+d*e)/f*arccoth(d*x+c)^2-1/2*b^2*c/(c*f-d*e-f)*polylog(3,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))-a*b/f*dilog((d*x+c)*f+f)/(c*f-d*e+f)+a*b/f*dilog((d*x+c)*f-f)/(c*f-d*e+f)+1/2*I*b^2/f*Pi*arccoth(d*x+c)^2*csgn(I*((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f))*csgn(I*((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^2*csgn(I/((d*x+c+1)/(d*x+c-1)-1))-a*b/f*ln((d*x+c)*f-c*f+d*e)*ln((d*x+c)*f+f)/(c*f-d*e+f)+2*a*b*ln((d*x+c)*f-c*f+d*e)/f*arccoth(d*x+c)+a*b/f*ln((d*x+c)*f-c*f+d*e)*ln((d*x+c)*f-f)/(c*f-d*e-f)+b^2*c/(c*f-d*e-f)*arccoth(d*x+c)^2*ln(1-(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))+b^2*c/(c*f-d*e-f)*arccoth(d*x+c)*polylog(2,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))-I*b^2/f*Pi*arccoth(d*x+c)^2+a^2*ln((d*x+c)*f-c*f+d*e)/f+2*b^2/f*polylog(3,1/((d*x+c-1)/(d*x+c+1))^(1/2))+2*b^2/f*polylog(3,-1/((d*x+c-1)/(d*x+c+1))^(1/2))+1/2*b^2/(c*f-d*e-f)*polylog(3,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))+1/2*d*b^2/f*e/(c*f-d*e-f)*polylog(3,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f)))+I*b^2/f*Pi*arccoth(d*x+c)^2*csgn(I*((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^2-1/2*I*b^2/f*Pi*arccoth(d*x+c)^2*csgn(I*((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^3-d*b^2/f*e/(c*f-d*e-f)*arccoth(d*x+c)^2*ln(1-(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))-d*b^2/f*e/(c*f-d*e-f)*arccoth(d*x+c)*polylog(2,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))-1/2*I*b^2/f*Pi*arccoth(d*x+c)^2*csgn(I*((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f))*csgn(I*((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{a^2 \log(fx + e)}{f} + \int \frac{b^2 \left( \log\left(\frac{1}{dx+c} + 1\right) - \log\left(-\frac{1}{dx+c} + 1\right) \right)^2}{4(fx + e)} + \frac{ab \left( \log\left(\frac{1}{dx+c} + 1\right) - \log\left(-\frac{1}{dx+c} + 1\right) \right)}{fx + e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))^2/(f\*x+e),x, algorithm="maxima")

[Out] a^2\*log(f\*x + e)/f + integrate(1/4\*b^2\*(log(1/(d\*x + c) + 1) - log(-1/(d\*x + c) + 1))^2/(f\*x + e) + a\*b\*(log(1/(d\*x + c) + 1) - log(-1/(d\*x + c) + 1))/(f\*x + e), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \operatorname{arccoth}(dx + c)^2 + 2ab \operatorname{arccoth}(dx + c) + a^2}{fx + e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))^2/(f\*x+e),x, algorithm="fricas")

[Out] integral((b^2\*arccoth(d\*x + c)^2 + 2\*a\*b\*arccoth(d\*x + c) + a^2)/(f\*x + e), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth(d\*x+c))\*\*2/(f\*x+e),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccoth}(dx + c) + a)^2}{fx + e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))^2/(f\*x+e),x, algorithm="giac")

[Out] integrate((b\*arccoth(d\*x + c) + a)^2/(f\*x + e), x)

$$3.113 \quad \int \frac{(a+b \coth^{-1}(c+dx))^2}{(e+fx)^2} dx$$

**Optimal.** Leaf size=480

$$\frac{b^2 d \operatorname{PolyLog}\left(2, -\frac{c+dx+1}{-c-dx+1}\right)}{2f(-cf+de+f)} + \frac{b^2 d \operatorname{PolyLog}\left(2, 1 - \frac{2}{c+dx+1}\right)}{2f(-cf+de-f)} - \frac{b^2 d \operatorname{PolyLog}\left(2, 1 - \frac{2}{c+dx+1}\right)}{(-cf+de+f)(de-(c+1)f)} + \frac{b^2 d \operatorname{PolyLog}\left(2, 1 - \frac{2}{c+dx+1}\right)}{(-cf+de+f)(de-(c+1)f)}$$

```
[Out] -((a + b*ArcCoth[c + d*x])^2/(f*(e + f*x))) + (b^2*d*ArcCoth[c + d*x]*Log[2
/(1 - c - d*x)]/(f*(d*e + f - c*f)) - (a*b*d*Log[1 - c - d*x])/(f*(d*e + f
- c*f)) - (b^2*d*ArcCoth[c + d*x]*Log[2/(1 + c + d*x)]/(f*(d*e - f - c*f)
) + (2*b^2*d*ArcCoth[c + d*x]*Log[2/(1 + c + d*x)]/((d*e + f - c*f)*(d*e -
(1 + c)*f)) + (a*b*d*Log[1 + c + d*x])/(f*(d*e - f - c*f)) + (2*a*b*d*Log[
e + f*x])/(f^2 - (d*e - c*f)^2) - (2*b^2*d*ArcCoth[c + d*x]*Log[(2*d*(e + f
*x)]/((d*e + f - c*f)*(1 + c + d*x)))]/((d*e + f - c*f)*(d*e - (1 + c)*f))
+ (b^2*d*PolyLog[2, -((1 + c + d*x)/(1 - c - d*x))])/(2*f*(d*e + f - c*f))
+ (b^2*d*PolyLog[2, 1 - 2/(1 + c + d*x)]/(2*f*(d*e - f - c*f)) - (b^2*d*Po
lyLog[2, 1 - 2/(1 + c + d*x)]/((d*e + f - c*f)*(d*e - (1 + c)*f)) + (b^2*d
*PolyLog[2, 1 - (2*d*(e + f*x)]/((d*e + f - c*f)*(1 + c + d*x)))]/((d*e + f
- c*f)*(d*e - (1 + c)*f))
```

**Rubi [A]** time = 1.74301, antiderivative size = 485, normalized size of antiderivative = 1.01, number of steps used = 21, number of rules used = 19, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.95$ , Rules used = {6110, 1982, 705, 31, 632, 6741, 6122, 706, 633, 6688, 12, 6725, 72, 6742, 5919, 2402, 2315, 5921, 2447}

$$\frac{b^2 d \operatorname{PolyLog}\left(2, -\frac{c+dx+1}{-c-dx+1}\right)}{2f(-cf+de+f)} + \frac{b^2 d \operatorname{PolyLog}\left(2, 1 - \frac{2}{c+dx+1}\right)}{2f(-cf+de-f)} - \frac{b^2 d \operatorname{PolyLog}\left(2, 1 - \frac{2}{c+dx+1}\right)}{(-cf+de+f)(de-(c+1)f)} + \frac{b^2 d \operatorname{PolyLog}\left(2, 1 - \frac{2}{c+dx+1}\right)}{(-cf+de+f)(de-(c+1)f)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[c + d*x])^2/(e + f*x)^2,x]
```

```
[Out] -((a + b*ArcCoth[c + d*x])^2/(f*(e + f*x))) + (b^2*d*ArcCoth[c + d*x]*Log[2
/(1 - c - d*x)]/(f*(d*e + f - c*f)) - (a*b*d*Log[1 - c - d*x])/(f*(d*e + f
- c*f)) - (b^2*d*ArcCoth[c + d*x]*Log[2/(1 + c + d*x)]/(f*(d*e - f - c*f)
) + (2*b^2*d*ArcCoth[c + d*x]*Log[2/(1 + c + d*x)]/((d*e + f - c*f)*(d*e -
(1 + c)*f)) + (a*b*d*Log[1 + c + d*x])/(f*(d*e - f - c*f)) - (2*a*b*d*Log[
e + f*x])/((d*e + f - c*f)*(d*e - (1 + c)*f)) - (2*b^2*d*ArcCoth[c + d*x]*L
og[(2*d*(e + f*x)]/((d*e + f - c*f)*(1 + c + d*x)))]/((d*e + f - c*f)*(d*e
```

```
- (1 + c)*f)) + (b^2*d*PolyLog[2, -((1 + c + d*x)/(1 - c - d*x))]/(2*f*(d*
e + f - c*f)) + (b^2*d*PolyLog[2, 1 - 2/(1 + c + d*x)])/(2*f*(d*e - f - c*f
)) - (b^2*d*PolyLog[2, 1 - 2/(1 + c + d*x)])/((d*e + f - c*f)*(d*e - (1 + c
)*f)) + (b^2*d*PolyLog[2, 1 - (2*d*(e + f*x))]/((d*e + f - c*f)*(1 + c + d*x
)))]/((d*e + f - c*f)*(d*e - (1 + c)*f))
```

### Rule 6110

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^ (p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] := Simp[((e + f*x)^(m + 1)*(a + b*ArcCoth[c + d*x])^p)/(f*(m
+ 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcCot
h[c + d*x])^(p - 1))/(1 - (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f},
x] && IGtQ[p, 0] && ILtQ[m, -1]
```

### Rule 1982

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] := Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] && !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

### Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
:= Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

### Rule 31

```
Int[((a_) + (b_.)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 6741

```
Int[u_, x_Symbol] := With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v !=
= u]
```

Rule 6122

Int[((a\_.) + ArcCoth[(c\_) + (d\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.)\*((A\_.) + (B\_.)\*(x\_) + (C\_.)\*(x\_)^2)^(q\_.), x\_Symbol] := Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(-(C/d^2) + (C\*x^2)/d^2)^q\*(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] && EqQ[B\*(1 - c^2) + 2\*A\*c\*d, 0] && EqQ[2\*c\*C - B\*d, 0]

Rule 706

Int[1/(((d\_) + (e\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)), x\_Symbol] := Dist[e^2/(c\*d^2 + a\*e^2), Int[1/(d + e\*x), x], x] + Dist[1/(c\*d^2 + a\*e^2), Int[(c\*d - c\*e\*x)/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0]

Rule 633

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := With[{q = Rt[-(a\*c), 2]}, Dist[e/2 + (c\*d)/(2\*q), Int[1/(-q + c\*x), x], x] + Dist[e/2 - (c\*d)/(2\*q), Int[1/(q + c\*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a\*c)]

Rule 6688

Int[u\_, x\_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 6725

Int[(u\_)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := With[{v = RationalFunctionExpand[u/(a + b\*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rule 72

Int[((e\_.) + (f\_.)\*(x\_))^(p\_.)/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Int[ExpandIntegrand[(e + f\*x)^p/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 6742

```
Int[u_, x_Symbol] :=> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^((p_.)/((d_) + (e_.)*(x_))), x_Symbol
] :=> -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)]]/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :=> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :=> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5921

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :=> -
Simp[((a + b*ArcCoth[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcCoth[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :=> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \coth^{-1}(c + dx))^2}{(e + fx)^2} dx &= -\frac{(a + b \coth^{-1}(c + dx))^2}{f(e + fx)} + \frac{(2bd) \int \frac{a + b \coth^{-1}(c + dx)}{(e + fx)(1 - (c + dx)^2)} dx}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^2}{f(e + fx)} + \frac{(2bd) \int \frac{a + b \coth^{-1}(c + dx)}{(e + fx)(1 - c^2 - 2cdx - d^2x^2)} dx}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^2}{f(e + fx)} + \frac{(2b) \text{Subst} \left( \int \frac{a + b \coth^{-1}(x)}{\left(\frac{de - cf}{d} + \frac{fx}{d}\right)(1 - x^2)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^2}{f(e + fx)} + \frac{(2b) \text{Subst} \left( \int \frac{d(a + b \coth^{-1}(x))}{(de - cf + fx)(1 - x^2)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^2}{f(e + fx)} + \frac{(2bd) \text{Subst} \left( \int \frac{a + b \coth^{-1}(x)}{(de - cf + fx)(1 - x^2)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^2}{f(e + fx)} + \frac{(2bd) \text{Subst} \left( \int \left( -\frac{a}{(-1+x)(1+x)(de - cf + fx)} - \frac{b \coth^{-1}(x)}{(-1+x)(1+x)(de - cf + fx)} \right) dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^2}{f(e + fx)} - \frac{(2abd) \text{Subst} \left( \int \frac{1}{(-1+x)(1+x)(de - cf + fx)} dx, x, c + dx \right)}{f} - \frac{(2b^2d) \text{Subst} \left( \int \frac{\coth^{-1}(x)}{(-1+x)(1+x)(de - cf + fx)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^2}{f(e + fx)} - \frac{(2abd) \text{Subst} \left( \int \left( \frac{1}{2(de + f - cf)(-1+x)} + \frac{1}{2(-de + (1+c)f)(1+x)} + \frac{1}{(de + f - cf)(1+x)} \right) dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^2}{f(e + fx)} - \frac{abd \log(1 - c - dx)}{f(de + f - cf)} + \frac{abd \log(1 + c + dx)}{f(de - f - cf)} - \frac{2abd \log(1 + x)}{(de + f - cf)} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^2}{f(e + fx)} + \frac{b^2d \coth^{-1}(c + dx) \log\left(\frac{2}{1 - c - dx}\right)}{f(de + f - cf)} - \frac{abd \log(1 - c - dx)}{f(de + f - cf)} - \frac{b^2d \coth^{-1}(c + dx) \log(1 + x)}{f(de + f - cf)} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^2}{f(e + fx)} + \frac{b^2d \coth^{-1}(c + dx) \log\left(\frac{2}{1 - c - dx}\right)}{f(de + f - cf)} - \frac{abd \log(1 - c - dx)}{f(de + f - cf)} - \frac{b^2d \coth^{-1}(c + dx) \log(1 + x)}{f(de + f - cf)} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^2}{f(e + fx)} + \frac{b^2d \coth^{-1}(c + dx) \log\left(\frac{2}{1 - c - dx}\right)}{f(de + f - cf)} - \frac{abd \log(1 - c - dx)}{f(de + f - cf)} - \frac{b^2d \coth^{-1}(c + dx) \log(1 + x)}{f(de + f - cf)}
\end{aligned}$$





$c)*f-c*f+d*e)-d*b^2/(c*f-d*e-f)/(c*f-d*e+f)*dilog(((d*x+c)*f-f)/(c*f-d*e-f)$   
 $)+1/4*d*b^2/f/(c*f-d*e-f)*ln(d*x+c-1)^2-1/2*d*b^2/f/(c*f-d*e-f)*dilog(1/2+1$   
 $/2*d*x+1/2*c)-1/2*d*b^2/f/(c*f-d*e-f)*ln(d*x+c-1)*ln(1/2+1/2*d*x+1/2*c)-1/2$   
 $*d*b^2/f/(c*f-d*e+f)*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)+1/2*d*b^2/f/(c*f-d*$   
 $e+f)*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2+1/2*d*x+1/2*c)+1/2*d*b^2/f/(c*f-d*e+f)*d$   
 $ilog(1/2+1/2*d*x+1/2*c)+1/4*d*b^2/f/(c*f-d*e+f)*ln(d*x+c+1)^2-2*d*a*b/(d*f*$   
 $x+d*e)/f*arccoth(d*x+c)+2*d*a*b/f/(2*c*f-2*d*e-2*f)*ln(d*x+c-1)-2*d*a*b/f/($   
 $2*c*f-2*d*e+2*f)*ln(d*x+c+1)-2*d*a*b/(c*f-d*e-f)/(c*f-d*e+f)*ln((d*x+c)*f-c$   
 $*f+d*e)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\left( d \left( \frac{\log(dx+c+1)}{def-(c+1)f^2} - \frac{\log(dx+c-1)}{def-(c-1)f^2} - \frac{2 \log(fx+e)}{d^2e^2-2cdef+(c^2-1)f^2} \right) - \frac{2 \operatorname{arccoth}(dx+c)}{f^2x+ef} \right) ab - \frac{1}{4} b^2 \left( \frac{\log(dx+c+1)^2}{f^2x+ef} + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))^2/(f\*x+e)^2,x, algorithm="maxima")

[Out]  $(d*(\log(d*x + c + 1)/(d*e*f - (c + 1)*f^2) - \log(d*x + c - 1)/(d*e*f - (c - 1)*f^2) - 2*\log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 - 1)*f^2)) - 2*\operatorname{arccoth}(d*x + c)/(f^2*x + e*f))*a*b - 1/4*b^2*(\log(d*x + c + 1)^2/(f^2*x + e*f) + \operatorname{integrate}(-((d*f*x + c*f + f)*\log(d*x + c - 1)^2 + 2*(d*f*x + d*e - (d*f*x + c*f + f)*\log(d*x + c - 1))*\log(d*x + c + 1))/(d*f^3*x^3 + c*e^2*f + e^2*f + (2*d*e*f^2 + c*f^3 + f^3)*x^2 + (d*e^2*f + 2*c*e*f^2 + 2*e*f^2)*x), x) - a^2/(f^2*x + e*f)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{b^2 \operatorname{arccoth}(dx+c)^2 + 2ab \operatorname{arccoth}(dx+c) + a^2}{f^2x^2 + 2efx + e^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))^2/(f\*x+e)^2,x, algorithm="fricas")

[Out]  $\operatorname{integral}((b^2*\operatorname{arccoth}(d*x + c)^2 + 2*a*b*\operatorname{arccoth}(d*x + c) + a^2)/(f^2*x^2 + 2*e*f*x + e^2), x)$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth(d\*x+c))\*\*2/(f\*x+e)\*\*2,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccoth}(dx + c) + a)^2}{(fx + e)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))^2/(f\*x+e)^2,x, algorithm="giac")

[Out] integrate((b\*arccoth(d\*x + c) + a)^2/(f\*x + e)^2, x)

### 3.114 $\int (e + fx)^2 (a + b \coth^{-1}(c + dx))^3 dx$

**Optimal.** Leaf size=546

$$\frac{b^2 \left( (3c^2 + 1) f^2 - 6cdef + 3d^2 e^2 \right) \text{PolyLog} \left( 2, 1 - \frac{2}{-c-dx+1} \right) (a + b \coth^{-1}(c + dx))}{d^3} + \frac{b^3 \left( (3c^2 + 1) f^2 - 6cdef + 3d^2 e^2 \right)}{2d^3}$$

```
[Out] (a*b^2*f^2*x)/d^2 + (b^3*f^2*(c + d*x)*ArcCoth[c + d*x])/d^3 - (b*f^2*(a +
b*ArcCoth[c + d*x])^2)/(2*d^3) + (3*b*f*(d*e - c*f)*(a + b*ArcCoth[c + d*x]
)^2)/d^3 + (3*b*f*(d*e - c*f)*(c + d*x)*(a + b*ArcCoth[c + d*x])^2)/d^3 + (
b*f^2*(c + d*x)^2*(a + b*ArcCoth[c + d*x])^2)/(2*d^3) - ((d*e - c*f)*(d^2*e
^2 - 2*c*d*e*f + (3 + c^2)*f^2)*(a + b*ArcCoth[c + d*x])^3)/(3*d^3*f) + ((3
*d^2*e^2 - 6*c*d*e*f + (1 + 3*c^2)*f^2)*(a + b*ArcCoth[c + d*x])^3)/(3*d^3)
+ ((e + f*x)^3*(a + b*ArcCoth[c + d*x])^3)/(3*f) - (6*b^2*f*(d*e - c*f)*(a
+ b*ArcCoth[c + d*x])*Log[2/(1 - c - d*x)])/d^3 - (b*(3*d^2*e^2 - 6*c*d*e*
f + (1 + 3*c^2)*f^2)*(a + b*ArcCoth[c + d*x])^2*Log[2/(1 - c - d*x)])/d^3 +
(b^3*f^2*Log[1 - (c + d*x)^2])/(2*d^3) - (3*b^3*f*(d*e - c*f)*PolyLog[2, -
((1 + c + d*x)/(1 - c - d*x))])/d^3 - (b^2*(3*d^2*e^2 - 6*c*d*e*f + (1 + 3*
c^2)*f^2)*(a + b*ArcCoth[c + d*x])*PolyLog[2, 1 - 2/(1 - c - d*x)])/d^3 + (
b^3*(3*d^2*e^2 - 6*c*d*e*f + (1 + 3*c^2)*f^2)*PolyLog[3, 1 - 2/(1 - c - d*x
)])/d^3
```

**Rubi [A]** time = 1.04283, antiderivative size = 546, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 14, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.7$ , Rules used = {6112, 5929, 5911, 5985, 5919, 2402, 2315, 5917, 5981, 260, 5949, 6049, 6059, 6610}

$$\frac{b^2 \left( (3c^2 + 1) f^2 - 6cdef + 3d^2 e^2 \right) \text{PolyLog} \left( 2, 1 - \frac{2}{-c-dx+1} \right) (a + b \coth^{-1}(c + dx))}{d^3} + \frac{b^3 \left( (3c^2 + 1) f^2 - 6cdef + 3d^2 e^2 \right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*(a + b*ArcCoth[c + d*x])^3,x]
```

```
[Out] (a*b^2*f^2*x)/d^2 + (b^3*f^2*(c + d*x)*ArcCoth[c + d*x])/d^3 - (b*f^2*(a +
b*ArcCoth[c + d*x])^2)/(2*d^3) + (3*b*f*(d*e - c*f)*(a + b*ArcCoth[c + d*x]
)^2)/d^3 + (3*b*f*(d*e - c*f)*(c + d*x)*(a + b*ArcCoth[c + d*x])^2)/d^3 + (
b*f^2*(c + d*x)^2*(a + b*ArcCoth[c + d*x])^2)/(2*d^3) - ((d*e - c*f)*(d^2*e
^2 - 2*c*d*e*f + (3 + c^2)*f^2)*(a + b*ArcCoth[c + d*x])^3)/(3*d^3*f) + ((3
*d^2*e^2 - 6*c*d*e*f + (1 + 3*c^2)*f^2)*(a + b*ArcCoth[c + d*x])^3)/(3*d^3)
```

$$\begin{aligned}
& + ((e + f*x)^3*(a + b*\text{ArcCoth}[c + d*x])^3)/(3*f) - (6*b^2*f*(d*e - c*f)*(a \\
& + b*\text{ArcCoth}[c + d*x])*Log[2/(1 - c - d*x)]/d^3 - (b*(3*d^2*e^2 - 6*c*d*e* \\
& f + (1 + 3*c^2)*f^2)*(a + b*\text{ArcCoth}[c + d*x])^2*Log[2/(1 - c - d*x)]/d^3 + \\
& (b^3*f^2*Log[1 - (c + d*x)^2])/(2*d^3) - (3*b^3*f*(d*e - c*f)*PolyLog[2, - \\
& ((1 + c + d*x)/(1 - c - d*x)))/d^3 - (b^2*(3*d^2*e^2 - 6*c*d*e*f + (1 + 3* \\
& c^2)*f^2)*(a + b*\text{ArcCoth}[c + d*x])*PolyLog[2, 1 - 2/(1 - c - d*x)]/d^3 + ( \\
& b^3*(3*d^2*e^2 - 6*c*d*e*f + (1 + 3*c^2)*f^2)*PolyLog[3, 1 - 2/(1 - c - d*x \\
& )])/(2*d^3)
\end{aligned}$$

### Rule 6112

Int[((a\_.) + ArcCoth[(c\_.) + (d\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]

### Rule 5929

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((d\_.) + (e\_.)\*(x\_))^(q\_.), x\_Symbol] := Simp[((d + e\*x)^(q + 1)\*(a + b\*ArcCoth[c\*x])^p)/(e\*(q + 1)), x] - Dist[(b\*c\*p)/(e\*(q + 1)), Int[ExpandIntegrand[(a + b\*ArcCoth[c\*x])^(p - 1), (d + e\*x)^(q + 1)/(1 - c^2\*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]

### Rule 5911

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.), x\_Symbol] := Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

### Rule 5985

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)\*(x\_))/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*e\*(p + 1)), x] + Dist[1/(c\*d), Int[(a + b\*ArcCoth[c\*x])^p/(1 - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && IGtQ[p, 0]

### Rule 5919

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[((a + b\*ArcCoth[c\*x])^p\*Log[2/(1 + (e\*x)/d)])/e, x] + Dist[(b\*c\*p)/e, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*Log[2/(1 + (e\*x)/d)])/((1 - c^2\*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dist
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^ (p_.)*((d_.)*(x_)^ (m_.)), x_Symbol]
:= Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5981

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^ (p_.)*((f_.)*(x_)^ (m_)))/((d_) + (
e_.)*(x_)^2), x_Symbol] := Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 260

```
Int[(x_)^ (m_.)/((a_) + (b_.)*(x_)^ (n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^ (p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6049

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^ (p_.)*((f_) + (g_.)*(x_)^ (m_.)))/((
d_) + (e_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcCoth[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 6059

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] := -Simp[((a + b*ArcCoth[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int (e + fx)^2 (a + b \coth^{-1}(c + dx))^3 dx &= \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d} + \frac{fx}{d}\right)^2 (a + b \coth^{-1}(x))^3 dx, x, c + dx\right)}{d} \\
&= \frac{(e + fx)^3 (a + b \coth^{-1}(c + dx))^3}{3f} - \frac{b \text{Subst}\left(\int \left(-\frac{3f^2(de-cf)(a+b \coth^{-1}(x))^2}{d^3} - \frac{f^3}{d}\right) dx, x, c + dx\right)}{3f} \\
&= \frac{(e + fx)^3 (a + b \coth^{-1}(c + dx))^3}{3f} - \frac{b \text{Subst}\left(\int \frac{((de-cf)(d^2e^2 - 2cdef + 3f^2 + c^2f^2) + f(3f^2e^2 - 2cde + c^2))}{d^3} dx, x, c + dx\right)}{3f} \\
&= \frac{3bf(de - cf)(c + dx)(a + b \coth^{-1}(c + dx))^2}{d^3} + \frac{bf^2(c + dx)^2 (a + b \coth^{-1}(c + dx))^2}{2d^3} \\
&= \frac{3bf(de - cf)(a + b \coth^{-1}(c + dx))^2}{d^3} + \frac{3bf(de - cf)(c + dx)(a + b \coth^{-1}(c + dx))^2}{d^3} \\
&= \frac{ab^2f^2x}{d^2} - \frac{bf^2(a + b \coth^{-1}(c + dx))^2}{2d^3} + \frac{3bf(de - cf)(a + b \coth^{-1}(c + dx))^2}{d^3} \\
&= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c + dx) \coth^{-1}(c + dx)}{d^3} - \frac{bf^2(a + b \coth^{-1}(c + dx))^2}{2d^3} + \frac{3bf(de - cf)(a + b \coth^{-1}(c + dx))^2}{d^3} \\
&= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c + dx) \coth^{-1}(c + dx)}{d^3} - \frac{bf^2(a + b \coth^{-1}(c + dx))^2}{2d^3} + \frac{3bf(de - cf)(a + b \coth^{-1}(c + dx))^2}{d^3} \\
&= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c + dx) \coth^{-1}(c + dx)}{d^3} - \frac{bf^2(a + b \coth^{-1}(c + dx))^2}{2d^3} + \frac{3bf(de - cf)(a + b \coth^{-1}(c + dx))^2}{d^3}
\end{aligned}$$

**Mathematica [C]** time = 10.423, size = 2594, normalized size = 4.75

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(e + f\*x)^2\*(a + b\*ArcCoth[c + d\*x])^3,x]

```

[Out] (a^2*(a*d^2*e^2 + 3*b*d*e*f - 2*b*c*f^2)*x)/d^2 + (a^2*f*(2*a*d*e + b*f)*x^
2)/(2*d) + (a^3*f^2*x^3)/3 + a^2*b*x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcCoth[c
+ d*x] + ((3*a^2*b*d^2*e^2 - 3*a^2*b*c*d^2*e^2 + 3*a^2*b*d*e*f - 6*a^2*b*c*
d*e*f + 3*a^2*b*c^2*d*e*f + a^2*b*f^2 - 3*a^2*b*c*f^2 + 3*a^2*b*c^2*f^2 - a
^2*b*c^3*f^2)*Log[1 - c - d*x])/(2*d^3) + ((3*a^2*b*d^2*e^2 + 3*a^2*b*c*d^2
*e^2 - 3*a^2*b*d*e*f - 6*a^2*b*c*d*e*f - 3*a^2*b*c^2*d*e*f + a^2*b*f^2 + 3*
a^2*b*c*f^2 + 3*a^2*b*c^2*f^2 + a^2*b*c^3*f^2)*Log[1 + c + d*x])/(2*d^3) +
(3*a*b^2*e^2*(1 - (c + d*x)^2)*(ArcCoth[c + d*x]*(ArcCoth[c + d*x] - (c + d
*x)*ArcCoth[c + d*x] + 2*Log[1 - E^(-2*ArcCoth[c + d*x])])) - PolyLog[2, E^(-
2*ArcCoth[c + d*x])])/(d*(c + d*x)^2*(1 - (c + d*x)^(-2))) - (3*a*b^2*e*f
*(1 - (c + d*x)^2)*(2*c*ArcCoth[c + d*x]^2 + (c + d*x)^2*(1 - (c + d*x)^(-2
))*ArcCoth[c + d*x]^2 - 2*(c + d*x)*ArcCoth[c + d*x]*(-1 + c*ArcCoth[c + d*
x])) + 4*c*ArcCoth[c + d*x]*Log[1 - E^(-2*ArcCoth[c + d*x])] - 2*Log[1/((c +
d*x)*Sqrt[1 - (c + d*x)^(-2)])] - 2*c*PolyLog[2, E^(-2*ArcCoth[c + d*x])])
)/(d^2*(c + d*x)^2*(1 - (c + d*x)^(-2))) + (b^3*e^2*(1 - (c + d*x)^2)*((I/8
)*Pi^3 - ArcCoth[c + d*x]^3 - (c + d*x)*ArcCoth[c + d*x]^3 + 3*ArcCoth[c +
d*x]^2*Log[1 - E^(-2*ArcCoth[c + d*x])] + 3*ArcCoth[c + d*x]*PolyLog[2, E^(2
*ArcCoth[c + d*x])] - (3*PolyLog[3, E^(2*ArcCoth[c + d*x])])]/2))/(d*(c + d*
x)^2*(1 - (c + d*x)^(-2))) - (b^3*e*f*(1 - (c + d*x)^2)*(I*c*Pi^3 - 12*ArcC
oth[c + d*x]^2 + 12*(c + d*x)*ArcCoth[c + d*x]^2 - 8*c*ArcCoth[c + d*x]^3 -
8*c*(c + d*x)*ArcCoth[c + d*x]^3 + 4*(c + d*x)^2*(1 - (c + d*x)^(-2))*ArcC
oth[c + d*x]^3 - 24*ArcCoth[c + d*x]*Log[1 - E^(-2*ArcCoth[c + d*x])] + 24*
c*ArcCoth[c + d*x]^2*Log[1 - E^(-2*ArcCoth[c + d*x])] + 12*PolyLog[2, E^(-2*
ArcCoth[c + d*x])] + 24*c*ArcCoth[c + d*x]*PolyLog[2, E^(2*ArcCoth[c + d*x]
)] - 12*c*PolyLog[3, E^(2*ArcCoth[c + d*x])])/(4*d^2*(c + d*x)^2*(1 - (c +
d*x)^(-2))) - (a*b^2*f^2*(c + d*x)*Sqrt[1 - (c + d*x)^(-2)]*(1 - (c + d*x)
^2)*((4*ArcCoth[c + d*x])/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (3*ArcCoth
[c + d*x]^2)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) - (12*c*ArcCoth[c + d*x]^
2)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (9*c^2*ArcCoth[c + d*x]^2)/((c +
d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (-1 + 6*c*ArcCoth[c + d*x] + 3*ArcCoth[c +
d*x]^2 - 3*c^2*ArcCoth[c + d*x]^2)/Sqrt[1 - (c + d*x)^(-2)] + Cosh[3*ArcCo
th[c + d*x]] - 6*c*ArcCoth[c + d*x]*Cosh[3*ArcCoth[c + d*x]] + ArcCoth[c +
d*x]^2*Cosh[3*ArcCoth[c + d*x]] + 3*c^2*ArcCoth[c + d*x]^2*Cosh[3*ArcCoth[c
+ d*x]] + (6*ArcCoth[c + d*x]*Log[1 - E^(-2*ArcCoth[c + d*x])])/(c + d*x)
*Sqrt[1 - (c + d*x)^(-2)]) + (18*c^2*ArcCoth[c + d*x]*Log[1 - E^(-2*ArcCoth
[c + d*x])])/(c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) - (18*c*Log[1/((c + d*x)*
Sqrt[1 - (c + d*x)^(-2)])])/(c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (4*(1 +
3*c^2)*PolyLog[2, E^(-2*ArcCoth[c + d*x])])/(c + d*x)^3*(1 - (c + d*x)^(-2
))^(3/2)) - ArcCoth[c + d*x]^2*Sinh[3*ArcCoth[c + d*x]] - 3*c^2*ArcCoth[c +
d*x]^2*Sinh[3*ArcCoth[c + d*x]] - 2*ArcCoth[c + d*x]*Log[1 - E^(-2*ArcCoth
[c + d*x])]*Sinh[3*ArcCoth[c + d*x]] - 6*c^2*ArcCoth[c + d*x]*Log[1 - E^(-2
*ArcCoth[c + d*x])]*Sinh[3*ArcCoth[c + d*x]] + 6*c*Log[1/((c + d*x)*Sqrt[1
- (c + d*x)^(-2)])]*Sinh[3*ArcCoth[c + d*x]])/(4*d^3) + (b^3*f^2*(1 - (c +
d*x)^2)*(3*c*PolyLog[2, E^(-2*ArcCoth[c + d*x])] + ((c + d*x)^3*(1 - (c +
d*x)^(-2)))^(3/2)*((-3*I)*Pi^3)/(c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) - ((9*

```



$$\begin{aligned}
& I)c^2\pi^3/((c + d*x)*\text{Sqrt}[1 - (c + d*x)^{-2}]) + (24*\text{ArcCoth}[c + d*x])/S \\
& \text{qrt}[1 - (c + d*x)^{-2}] - (72*c*\text{ArcCoth}[c + d*x]^2)/\text{Sqrt}[1 - (c + d*x)^{-2}] \\
& ] - (48*\text{ArcCoth}[c + d*x]^2)/((c + d*x)*\text{Sqrt}[1 - (c + d*x)^{-2}]) + (216*c*A \\
& \text{rcCoth}[c + d*x]^2)/((c + d*x)*\text{Sqrt}[1 - (c + d*x)^{-2}]) - (24*\text{ArcCoth}[c + d \\
& *x]^3)/\text{Sqrt}[1 - (c + d*x)^{-2}] + (24*c^2*\text{ArcCoth}[c + d*x]^3)/\text{Sqrt}[1 - (c + \\
& d*x)^{-2}] + (24*\text{ArcCoth}[c + d*x]^3)/((c + d*x)*\text{Sqrt}[1 - (c + d*x)^{-2}]) \\
& + (96*c*\text{ArcCoth}[c + d*x]^3)/((c + d*x)*\text{Sqrt}[1 - (c + d*x)^{-2}]) + (72*c^2* \\
& \text{ArcCoth}[c + d*x]^3)/((c + d*x)*\text{Sqrt}[1 - (c + d*x)^{-2}]) - 24*\text{ArcCoth}[c + d \\
& *x]*\text{Cosh}[3*\text{ArcCoth}[c + d*x]] + 72*c*\text{ArcCoth}[c + d*x]^2*\text{Cosh}[3*\text{ArcCoth}[c + d \\
& *x]] - 8*\text{ArcCoth}[c + d*x]^3*\text{Cosh}[3*\text{ArcCoth}[c + d*x]] - 24*c^2*\text{ArcCoth}[c + d \\
& *x]^3*\text{Cosh}[3*\text{ArcCoth}[c + d*x]] + (432*c*\text{ArcCoth}[c + d*x]*\text{Log}[1 - E^{(-2*\text{ArcC} \\
& \text{oth}[c + d*x])}])/((c + d*x)*\text{Sqrt}[1 - (c + d*x)^{-2}]) - (72*\text{ArcCoth}[c + d*x] \\
& ^2*\text{Log}[1 - E^{(2*\text{ArcCoth}[c + d*x])}])/((c + d*x)*\text{Sqrt}[1 - (c + d*x)^{-2}]) - \\
& (216*c^2*\text{ArcCoth}[c + d*x]^2*\text{Log}[1 - E^{(2*\text{ArcCoth}[c + d*x])}])/((c + d*x)*\text{Sqr} \\
& \text{t}[1 - (c + d*x)^{-2}]) - (72*\text{Log}[1/((c + d*x)*\text{Sqrt}[1 - (c + d*x)^{-2}])])/(( \\
& (c + d*x)*\text{Sqrt}[1 - (c + d*x)^{-2}]) + (96*(1 + 3*c^2)*\text{ArcCoth}[c + d*x]*\text{Poly} \\
& \text{Log}[2, E^{(2*\text{ArcCoth}[c + d*x])}])/((c + d*x)^3*(1 - (c + d*x)^{-2})^{(3/2)}) - \\
& (48*(1 + 3*c^2)*\text{PolyLog}[3, E^{(2*\text{ArcCoth}[c + d*x])}])/((c + d*x)^3*(1 - (c + \\
& d*x)^{-2})^{(3/2)}) + I*\pi^3*\text{Sinh}[3*\text{ArcCoth}[c + d*x]] + (3*I)*c^2*\pi^3*\text{Sinh}[3 \\
& *\text{ArcCoth}[c + d*x]] - 72*c*\text{ArcCoth}[c + d*x]^2*\text{Sinh}[3*\text{ArcCoth}[c + d*x]] - 8*A \\
& \text{rcCoth}[c + d*x]^3*\text{Sinh}[3*\text{ArcCoth}[c + d*x]] - 24*c^2*\text{ArcCoth}[c + d*x]^3*\text{Sinh} \\
& [3*\text{ArcCoth}[c + d*x]] - 144*c*\text{ArcCoth}[c + d*x]*\text{Log}[1 - E^{(-2*\text{ArcCoth}[c + d*x} \\
& )}]]*\text{Sinh}[3*\text{ArcCoth}[c + d*x]] + 24*\text{ArcCoth}[c + d*x]^2*\text{Log}[1 - E^{(2*\text{ArcCoth}[c \\
& + d*x])}]]*\text{Sinh}[3*\text{ArcCoth}[c + d*x]] + 72*c^2*\text{ArcCoth}[c + d*x]^2*\text{Log}[1 - E^{(2 \\
& *\text{ArcCoth}[c + d*x])}]]*\text{Sinh}[3*\text{ArcCoth}[c + d*x]] + 24*\text{Log}[1/((c + d*x)*\text{Sqrt}[1 - \\
& (c + d*x)^{-2}])]*\text{Sinh}[3*\text{ArcCoth}[c + d*x]])/96)/(d^3*(c + d*x)^2*(1 - (c \\
& + d*x)^{-2}))
\end{aligned}$$

**Maple [C]** time = 3.685, size = 10477, normalized size = 19.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*(a+b\*arccoth(d\*x+c))^3,x)

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*(a+b\*arccoth(d\*x+c))^3,x, algorithm="maxima")

[Out]  $\frac{1}{3}a^3f^2x^3 + a^3efx^2 + \frac{3}{2}(2x^2\operatorname{arccoth}(dx+c) + d(2x/d^2 - (c^2 + 2c + 1)\log(dx+c+1)/d^3 + (c^2 - 2c + 1)\log(dx+c-1)/d^3))a^2b^2ef + \frac{1}{2}(2x^3\operatorname{arccoth}(dx+c) + d((dx^2 - 4cx)/d^3 + (c^3 + 3c^2 + 3c + 1)\log(dx+c+1)/d^4 - (c^3 - 3c^2 + 3c - 1)\log(dx+c-1)/d^4))a^2b^2f^2 + a^3e^2x + \frac{3}{2}(2(dx+c)\operatorname{arccoth}(dx+c) + \log(-(dx+c)^2 + 1))a^2b^2e^2/d + \frac{1}{24}((b^3d^3f^2x^3 + 3b^3d^3efx^2 + 3b^3d^3e^2x + (c^3f^2 + 3d^2e^2 - 3(d*ef - f^2)*c^2 - 3d*ef + 3(d^2e^2 - 2d*ef + f^2)*c + f^2)*b^3)\log(dx+c+1)^3 + 3(2a^2b^2d^3f^2x^3 + (6a^2b^2d^3ef + b^3d^2f^2)*x^2 + 2(3a^2b^2d^3e^2 + (3d^2ef - 2c*d*f^2)*b^3)*x - (b^3d^3f^2x^3 + 3b^3d^3efx^2 + 3b^3d^3e^2x + (c^3f^2 - 3d^2e^2 - 3(d*ef + f^2)*c^2 - 3d*ef + 3(d^2e^2 + 2d*ef + f^2)*c - f^2)*b^3)\log(dx+c-1))\log(dx+c+1)^2)/d^3 + \operatorname{integrate}(-1/8((b^3d^3f^2x^3 + (2d^3ef + c*d^2f^2 + d^2f^2)*b^3*x^2 + (d^3e^2 + 2c*d^2ef + 2d^2ef)*b^3*x + (c*d^2e^2 + d^2e^2)*b^3)\log(dx+c-1)^3 - 6(a^2b^2d^3f^2x^3 + (2d^3ef + c*d^2f^2 + d^2f^2)*a^2b^2x^2 + (d^3e^2 + 2c*d^2ef + 2d^2ef)*a^2b^2x + (c*d^2e^2 + d^2e^2)*a^2b^2)\log(dx+c-1)^2 + (4a^2b^2d^3f^2x^3 + 2(6a^2b^2d^3ef + b^3d^2f^2)*x^2 - 3(b^3d^3f^2x^3 + (2d^3ef + c*d^2f^2 + d^2f^2)*b^3*x^2 + (d^3e^2 + 2c*d^2ef + 2d^2ef)*b^3*x + (c*d^2e^2 + d^2e^2)*b^3)\log(dx+c-1)^2 + 4(3a^2b^2d^3e^2 + (3d^2ef - 2c*d*f^2)*b^3)*x + 2(6(c*d^2e^2 + d^2e^2)*a^2b^2 - (c^3f^2 - 3d^2e^2 - 3(d*ef + f^2)*c^2 - 3d*ef + 3(d^2e^2 + 2d*ef + f^2)*c - f^2)*b^3 + (6a^2b^2d^3f^2 - b^3d^3f^2)*x^3 - 3(b^3d^3ef - 2(2d^3ef + c*d^2f^2 + d^2f^2)*a^2b^2)*x^2 - 3(b^3d^3e^2 - 2(d^3e^2 + 2c*d^2ef + 2d^2ef)*a^2b^2)*x)\log(dx+c-1))\log(dx+c+1))/(d^3x + c*d^2 + d^2), x)$

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$\operatorname{integral}(a^3f^2x^2 + 2a^3efx + a^3e^2 + (b^3f^2x^2 + 2b^3efx + b^3e^2)\operatorname{arccoth}(dx+c)^3 + 3(ab^2f^2x^2 + 2ab^2efx + ab^2e^2)\operatorname{arccot}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*(a+b\*arccoth(d\*x+c))^3,x, algorithm="fricas")

[Out]  $\operatorname{integral}(a^3f^2x^2 + 2a^3efx + a^3e^2 + (b^3f^2x^2 + 2b^3efx + b^3e^2)\operatorname{arccoth}(dx+c)^3 + 3(a^2b^2f^2x^2 + 2a^2b^2efx + a^2b^2e^2)\operatorname{arccoth}(dx+c)^2 + 3(a^2b^2f^2x^2 + 2a^2b^2efx + a^2b^2e^2)\operatorname{arccot}$

$h(dx + c), x)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*(a+b\*acoth(d\*x+c))\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (fx + e)^2 (b \operatorname{arccoth}(dx + c) + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*(a+b\*arccoth(d\*x+c))^3,x, algorithm="giac")

[Out] integrate((f\*x + e)^2\*(b\*arccoth(d\*x + c) + a)^3, x)

### 3.115 $\int (e + fx) \left( a + b \coth^{-1}(c + dx) \right)^3 dx$

**Optimal.** Leaf size=326

$$\frac{3b^2(de - cf)\text{PolyLog}\left(2, 1 - \frac{2}{-c-dx+1}\right)(a + b \coth^{-1}(c + dx))}{d^2} + \frac{3b^3(de - cf)\text{PolyLog}\left(3, 1 - \frac{2}{-c-dx+1}\right)}{2d^2} - \frac{3b^3f\text{PolyLog}}{2d}$$

```
[Out] (3*b*f*(a + b*ArcCoth[c + d*x])^2)/(2*d^2) + (3*b*f*(c + d*x)*(a + b*ArcCoth[c + d*x])^2)/(2*d^2) + ((d*e - c*f)*(a + b*ArcCoth[c + d*x])^3)/d^2 - ((d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)*(a + b*ArcCoth[c + d*x])^3)/(2*d^2*f) + ((e + f*x)^2*(a + b*ArcCoth[c + d*x])^3)/(2*f) - (3*b^2*f*(a + b*ArcCoth[c + d*x])*Log[2/(1 - c - d*x)])/d^2 - (3*b*(d*e - c*f)*(a + b*ArcCoth[c + d*x])^2*Log[2/(1 - c - d*x)])/d^2 - (3*b^3*f*PolyLog[2, -((1 + c + d*x)/(1 - c - d*x))])/(2*d^2) - (3*b^2*(d*e - c*f)*(a + b*ArcCoth[c + d*x])*PolyLog[2, 1 - 2/(1 - c - d*x)])/d^2 + (3*b^3*(d*e - c*f)*PolyLog[3, 1 - 2/(1 - c - d*x)])/d^2
```

**Rubi [A]** time = 0.720792, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$ , Rules used = {6112, 5929, 5911, 5985, 5919, 2402, 2315, 6049, 5949, 6059, 6610}

$$\frac{3b^2(de - cf)\text{PolyLog}\left(2, 1 - \frac{2}{-c-dx+1}\right)(a + b \coth^{-1}(c + dx))}{d^2} + \frac{3b^3(de - cf)\text{PolyLog}\left(3, 1 - \frac{2}{-c-dx+1}\right)}{2d^2} - \frac{3b^3f\text{PolyLog}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*(a + b*ArcCoth[c + d*x])^3,x]
```

```
[Out] (3*b*f*(a + b*ArcCoth[c + d*x])^2)/(2*d^2) + (3*b*f*(c + d*x)*(a + b*ArcCoth[c + d*x])^2)/(2*d^2) + ((d*e - c*f)*(a + b*ArcCoth[c + d*x])^3)/d^2 + ((2*c*e - (d*e^2)/f - ((1 + c^2)*f)/d)*(a + b*ArcCoth[c + d*x])^3)/(2*d) + ((e + f*x)^2*(a + b*ArcCoth[c + d*x])^3)/(2*f) - (3*b^2*f*(a + b*ArcCoth[c + d*x])*Log[2/(1 - c - d*x)])/d^2 - (3*b*(d*e - c*f)*(a + b*ArcCoth[c + d*x])^2*Log[2/(1 - c - d*x)])/d^2 - (3*b^3*f*PolyLog[2, -((1 + c + d*x)/(1 - c - d*x))])/(2*d^2) - (3*b^2*(d*e - c*f)*(a + b*ArcCoth[c + d*x])*PolyLog[2, 1 - 2/(1 - c - d*x)])/d^2 + (3*b^3*(d*e - c*f)*PolyLog[3, 1 - 2/(1 - c - d*x)])/d^2
```

Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] := Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

### Rule 5929

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] := Simp[((d + e*x)^(q + 1)*(a + b*ArcCoth[c*x])^p)/(e*(q + 1)), x] -
Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcCoth[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

### Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

### Rule 5985

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

### Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] := -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

### Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

### Rule 2315

```
Int[Log[(c_.)*(x_)/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 6049

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((d_) + (e_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcCoth[c*x])^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 5949

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6059

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2), x_Symbol] := -Simp[((a + b*ArcCoth[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v, x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int (e + fx) (a + b \coth^{-1}(c + dx))^3 dx &= \frac{\text{Subst} \left( \int \left( \frac{de - cf}{d} + \frac{fx}{d} \right) (a + b \coth^{-1}(x))^3 dx, x, c + dx \right)}{d} \\
&= \frac{(e + fx)^2 (a + b \coth^{-1}(c + dx))^3}{2f} - \frac{(3b) \text{Subst} \left( \int \left( -\frac{f^2 (a + b \coth^{-1}(x))^2}{d^2} + \frac{(d^2 e^2 - 2)}{d^2} \right) dx, x, c + dx \right)}{2f} \\
&= \frac{(e + fx)^2 (a + b \coth^{-1}(c + dx))^3}{2f} - \frac{(3b) \text{Subst} \left( \int \frac{(d^2 e^2 - 2cdef + (1 + c^2)f^2 + 2f(de - cf)x)}{1 - x^2} dx, x, c + dx \right)}{2d^2 f} \\
&= \frac{3bf(c + dx) (a + b \coth^{-1}(c + dx))^2}{2d^2} + \frac{(e + fx)^2 (a + b \coth^{-1}(c + dx))^3}{2f} - \frac{(3b) \text{Subst} \left( \int \frac{(d^2 e^2 - 2cdef + (1 + c^2)f^2 + 2f(de - cf)x)}{1 - x^2} dx, x, c + dx \right)}{2d^2 f} \\
&= \frac{3bf (a + b \coth^{-1}(c + dx))^2}{2d^2} + \frac{3bf(c + dx) (a + b \coth^{-1}(c + dx))^2}{2d^2} + \frac{(e + fx)^2 (a + b \coth^{-1}(c + dx))^3}{2f} - \frac{(3b) \text{Subst} \left( \int \frac{(d^2 e^2 - 2cdef + (1 + c^2)f^2 + 2f(de - cf)x)}{1 - x^2} dx, x, c + dx \right)}{2d^2 f} \\
&= \frac{3bf (a + b \coth^{-1}(c + dx))^2}{2d^2} + \frac{3bf(c + dx) (a + b \coth^{-1}(c + dx))^2}{2d^2} + \frac{(de - cf) (a + b \coth^{-1}(c + dx))^3}{2d^2} - \frac{(3b) \text{Subst} \left( \int \frac{(d^2 e^2 - 2cdef + (1 + c^2)f^2 + 2f(de - cf)x)}{1 - x^2} dx, x, c + dx \right)}{2d^2 f} \\
&= \frac{3bf (a + b \coth^{-1}(c + dx))^2}{2d^2} + \frac{3bf(c + dx) (a + b \coth^{-1}(c + dx))^2}{2d^2} + \frac{(de - cf) (a + b \coth^{-1}(c + dx))^3}{2d^2} - \frac{(3b) \text{Subst} \left( \int \frac{(d^2 e^2 - 2cdef + (1 + c^2)f^2 + 2f(de - cf)x)}{1 - x^2} dx, x, c + dx \right)}{2d^2 f} \\
&= \frac{3bf (a + b \coth^{-1}(c + dx))^2}{2d^2} + \frac{3bf(c + dx) (a + b \coth^{-1}(c + dx))^2}{2d^2} + \frac{(de - cf) (a + b \coth^{-1}(c + dx))^3}{2d^2} - \frac{(3b) \text{Subst} \left( \int \frac{(d^2 e^2 - 2cdef + (1 + c^2)f^2 + 2f(de - cf)x)}{1 - x^2} dx, x, c + dx \right)}{2d^2 f}
\end{aligned}$$

**Mathematica [C]** time = 1.31426, size = 600, normalized size = 1.84

$$12ab^2de \left( \text{PolyLog} \left( 2, e^{-2 \coth^{-1}(c+dx)} \right) + \coth^{-1}(c + dx) \left( (c + dx - 1) \coth^{-1}(c + dx) - 2 \log \left( 1 - e^{-2 \coth^{-1}(c+dx)} \right) \right) \right) - 12$$

Warning: Unable to verify antiderivative.

[In] Integrate[(e + f\*x)\*(a + b\*ArcCoth[c + d\*x])^3,x]

[Out] (2\*a^2\*(2\*a\*d\*e + 3\*b\*f - 2\*a\*c\*f)\*(c + d\*x) + 2\*a^3\*f\*(c + d\*x)^2 - 6\*a^2\*b\*(c + d\*x)\*(c\*f - d\*(2\*e + f\*x))\*ArcCoth[c + d\*x] + 3\*a^2\*b\*(2\*d\*e + f - 2\*c\*f)\*Log[1 - c - d\*x] + 3\*a^2\*b\*(2\*d\*e - (1 + 2\*c)\*f)\*Log[1 + c + d\*x] + 12\*a\*b^2\*f\*((c + d\*x)\*ArcCoth[c + d\*x] + ((-1 + (c + d\*x)^2)\*ArcCoth[c + d\*x]^2)/2 - Log[1/((c + d\*x)\*Sqrt[1 - (c + d\*x)^(-2)])]) + 12\*a\*b^2\*d\*e\*(ArcCoth[c + d\*x]\*((-1 + c + d\*x)\*ArcCoth[c + d\*x] - 2\*Log[1 - E^(-2\*ArcCoth[c + d\*x])]) + PolyLog[2, E^(-2\*ArcCoth[c + d\*x])]) - 12\*a\*b^2\*c\*f\*(ArcCoth[c + d\*x]\*((-1 + c + d\*x)\*ArcCoth[c + d\*x] - 2\*Log[1 - E^(-2\*ArcCoth[c + d\*x])]) + PolyLog[2, E^(-2\*ArcCoth[c + d\*x])]) + 2\*b^3\*f\*(ArcCoth[c + d\*x]\*(3\*(-1 + c + d\*x)\*ArcCoth[c + d\*x] + (-1 + c^2 + 2\*c\*d\*x + d^2\*x^2)\*ArcCoth[c + d\*x]^2 - 6\*Log[1 - E^(-2\*ArcCoth[c + d\*x])]) + 3\*PolyLog[2, E^(-2\*ArcCoth[c + d\*x])]) + 4\*b^3\*d\*e\*((-I/8)\*Pi^3 + ArcCoth[c + d\*x]^3 + (c + d\*x)\*ArcCoth[c + d\*x]^3 - 3\*ArcCoth[c + d\*x]^2\*Log[1 - E^(2\*ArcCoth[c + d\*x])] - 3\*ArcCoth[c + d\*x]\*PolyLog[2, E^(2\*ArcCoth[c + d\*x])] + (3\*PolyLog[3, E^(2\*ArcCoth[c + d\*x])])/2) - 4\*b^3\*c\*f\*((-I/8)\*Pi^3 + ArcCoth[c + d\*x]^3 + (c + d\*x)\*ArcCoth[c + d\*x]^3 - 3\*ArcCoth[c + d\*x]^2\*Log[1 - E^(2\*ArcCoth[c + d\*x])] - 3\*ArcCoth[c + d\*x]\*PolyLog[2, E^(2\*ArcCoth[c + d\*x])] + (3\*PolyLog[3, E^(2\*ArcCoth[c + d\*x])])/2))/(4\*d^2)

**Maple [C]** time = 1.107, size = 12285, normalized size = 37.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)\*(a+b\*arccoth(d\*x+c))^3,x)

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} a^3 f x^2 + \frac{3}{4} \left( 2 x^2 \operatorname{arccoth}(d x + c) + d \left( \frac{2 x}{d^2} - \frac{(c^2 + 2 c + 1) \log(dx + c + 1)}{d^3} + \frac{(c^2 - 2 c + 1) \log(dx + c - 1)}{d^3} \right) \right) a^2 b f + a^3 e x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*(a+b\*arccoth(d\*x+c))^3,x, algorithm="maxima")



```
[Out] 1/2*a^3*f*x^2 + 3/4*(2*x^2*arccoth(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1)*
log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*a^2*b*f + a^3
*e*x + 3/2*(2*(d*x + c)*arccoth(d*x + c) + log(-(d*x + c)^2 + 1))*a^2*b*e/d
+ 1/16*((b^3*d^2*f*x^2 + 2*b^3*d^2*e*x - (c^2*f - 2*(d*e - f)*c - 2*d*e +
f)*b^3)*log(d*x + c + 1)^3 + 3*(2*a*b^2*d^2*f*x^2 + 2*(2*a*b^2*d^2*e + b^3*
d*f)*x - (b^3*d^2*f*x^2 + 2*b^3*d^2*e*x - (c^2*f - 2*(d*e + f)*c + 2*d*e +
f)*b^3)*log(d*x + c - 1))*log(d*x + c + 1)^2)/d^2 + integrate(-1/8*((b^3*d^
2*f*x^2 + (d^2*e + c*d*f + d*f)*b^3*x + (c*d*e + d*e)*b^3)*log(d*x + c - 1)
^3 - 6*(a*b^2*d^2*f*x^2 + (d^2*e + c*d*f + d*f)*a*b^2*x + (c*d*e + d*e)*a*b
^2)*log(d*x + c - 1)^2 + 3*(2*a*b^2*d^2*f*x^2 - (b^3*d^2*f*x^2 + (d^2*e + c
*d*f + d*f)*b^3*x + (c*d*e + d*e)*b^3)*log(d*x + c - 1)^2 + 2*(2*a*b^2*d^2*
e + b^3*d*f)*x + (4*(c*d*e + d*e)*a*b^2 + (c^2*f - 2*(d*e + f)*c + 2*d*e +
f)*b^3 + (4*a*b^2*d^2*f - b^3*d^2*f)*x^2 - 2*(b^3*d^2*e - 2*(d^2*e + c*d*f
+ d*f)*a*b^2)*x)*log(d*x + c - 1))*log(d*x + c + 1))/(d^2*x + c*d + d), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

integral( $a^3fx + a^3e + (b^3fx + b^3e) \operatorname{arccoth}(dx + c)^3 + 3(ab^2fx + ab^2e) \operatorname{arccoth}(dx + c)^2 + 3(a^2bfx + a^2be) \operatorname{arccoth}(dx + c) + a^3e$ ), x)

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccoth(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*f*x + a^3*e + (b^3*f*x + b^3*e)*arccoth(d*x + c)^3 + 3*(a*b^2*
f*x + a*b^2*e)*arccoth(d*x + c)^2 + 3*(a^2*b*f*x + a^2*b*e)*arccoth(d*x + c
), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*acoth(d*x+c))**3,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (fx + e)(b \operatorname{arccoth}(dx + c) + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccoth(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(b*arccoth(d*x + c) + a)^3, x)
```

### 3.116 $\int (a + b \coth^{-1}(c + dx))^3 dx$

**Optimal.** Leaf size=132

$$\frac{3b^2 \text{PolyLog}\left(2, 1 - \frac{2}{-c-dx+1}\right)(a + b \coth^{-1}(c + dx))}{d} + \frac{3b^3 \text{PolyLog}\left(3, 1 - \frac{2}{-c-dx+1}\right)}{2d} + \frac{(c + dx)(a + b \coth^{-1}(c + dx))}{d}$$

[Out] (a + b\*ArcCoth[c + d\*x])^3/d + ((c + d\*x)\*(a + b\*ArcCoth[c + d\*x])^3)/d - (3\*b\*(a + b\*ArcCoth[c + d\*x])^2\*Log[2/(1 - c - d\*x)]/d - (3\*b^2\*(a + b\*ArcCoth[c + d\*x])\*PolyLog[2, 1 - 2/(1 - c - d\*x)]/d + (3\*b^3\*PolyLog[3, 1 - 2/(1 - c - d\*x)])/(2\*d))

**Rubi [A]** time = 0.227334, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$ , Rules used = {6104, 5911, 5985, 5919, 5949, 6059, 6610}

$$\frac{3b^2 \text{PolyLog}\left(2, 1 - \frac{2}{-c-dx+1}\right)(a + b \coth^{-1}(c + dx))}{d} + \frac{3b^3 \text{PolyLog}\left(3, 1 - \frac{2}{-c-dx+1}\right)}{2d} + \frac{(c + dx)(a + b \coth^{-1}(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCoth[c + d\*x])^3,x]

[Out] (a + b\*ArcCoth[c + d\*x])^3/d + ((c + d\*x)\*(a + b\*ArcCoth[c + d\*x])^3)/d - (3\*b\*(a + b\*ArcCoth[c + d\*x])^2\*Log[2/(1 - c - d\*x)]/d - (3\*b^2\*(a + b\*ArcCoth[c + d\*x])\*PolyLog[2, 1 - 2/(1 - c - d\*x)]/d + (3\*b^3\*PolyLog[3, 1 - 2/(1 - c - d\*x)])/(2\*d))

#### Rule 6104

Int[((a\_.) + ArcCoth[(c\_.) + (d\_.)\*(x\_)])\*(b\_.))^p, x\_Symbol] := Dist[1/d, Subst[Int[(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0]

#### Rule 5911

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^p, x\_Symbol] := Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

Rule 5985

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_)/((d_) + (e_.)*(x_)^2),
  x_Symbol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] := -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[(a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)]]/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6059

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] := -Simp[((a + b*ArcCoth[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[(a + b*ArcCoth[c*x])^(p - 1)*PolyLog[2, 1 - u]]/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int (a + b \coth^{-1}(c + dx))^3 dx &= \frac{\text{Subst} \left( \int (a + b \coth^{-1}(x))^3 dx, x, c + dx \right)}{d} \\
&= \frac{(c + dx)(a + b \coth^{-1}(c + dx))^3}{d} - \frac{(3b) \text{Subst} \left( \int \frac{x(a + b \coth^{-1}(x))^2}{1-x^2} dx, x, c + dx \right)}{d} \\
&= \frac{(a + b \coth^{-1}(c + dx))^3}{d} + \frac{(c + dx)(a + b \coth^{-1}(c + dx))^3}{d} - \frac{(3b) \text{Subst} \left( \int \frac{(a + b \coth^{-1}(x))^2}{1-x} dx, x, c + dx \right)}{d} \\
&= \frac{(a + b \coth^{-1}(c + dx))^3}{d} + \frac{(c + dx)(a + b \coth^{-1}(c + dx))^3}{d} - \frac{3b(a + b \coth^{-1}(c + dx))}{d} \\
&= \frac{(a + b \coth^{-1}(c + dx))^3}{d} + \frac{(c + dx)(a + b \coth^{-1}(c + dx))^3}{d} - \frac{3b(a + b \coth^{-1}(c + dx))}{d} \\
&= \frac{(a + b \coth^{-1}(c + dx))^3}{d} + \frac{(c + dx)(a + b \coth^{-1}(c + dx))^3}{d} - \frac{3b(a + b \coth^{-1}(c + dx))}{d}
\end{aligned}$$

**Mathematica [C]** time = 0.319898, size = 208, normalized size = 1.58

$$\frac{6ab^2 \left( \text{PolyLog} \left( 2, e^{-2 \coth^{-1}(c+dx)} \right) + \coth^{-1}(c + dx) \left( (c + dx - 1) \coth^{-1}(c + dx) - 2 \log \left( 1 - e^{-2 \coth^{-1}(c+dx)} \right) \right) \right) + 2b^3 \left( - \right)}{d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCoth[c + d\*x])^3,x]

[Out] (2\*a^3\*(c + d\*x) + 6\*a^2\*b\*(c + d\*x)\*ArcCoth[c + d\*x] + 3\*a^2\*b\*Log[1 - (c + d\*x)^2] + 6\*a\*b^2\*(ArcCoth[c + d\*x]\*((-1 + c + d\*x)\*ArcCoth[c + d\*x] - 2\*Log[1 - E^(-2\*ArcCoth[c + d\*x])]) + PolyLog[2, E^(-2\*ArcCoth[c + d\*x])]) + 2\*b^3\*((-1/8)\*Pi^3 + ArcCoth[c + d\*x]^3 + (c + d\*x)\*ArcCoth[c + d\*x]^3 - 3\*ArcCoth[c + d\*x]^2\*Log[1 - E^(2\*ArcCoth[c + d\*x])] - 3\*ArcCoth[c + d\*x]\*PolyLog[2, E^(2\*ArcCoth[c + d\*x])] + (3\*PolyLog[3, E^(2\*ArcCoth[c + d\*x])]))/(2\*d)

**Maple [B]** time = 0.14, size = 485, normalized size = 3.7

$$xa^3 + \frac{a^3c}{d} + (\operatorname{arccoth}(dx+c))^3xb^3 + \frac{(\operatorname{arccoth}(dx+c))^3b^3c}{d} - 3\frac{(\operatorname{arccoth}(dx+c))^2b^3}{d} \ln\left(1 + \frac{1}{\sqrt{\frac{dx+c-1}{dx+c+1}}}\right) - 3\frac{(\operatorname{arccoth}(dx+c))b^3}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arccoth(d*x+c))^3,x)`

[Out] `x*a^3+1/d*a^3*c+arccoth(d*x+c)^3*x*b^3+1/d*arccoth(d*x+c)^3*b^3*c-3/d*ln(1+1/((d*x+c-1)/(d*x+c+1))^(1/2))*arccoth(d*x+c)^2*b^3-3/d*ln(1-1/((d*x+c-1)/(d*x+c+1))^(1/2))*arccoth(d*x+c)^2*b^3+1/d*b^3*arccoth(d*x+c)^3-6/d*polylog(2,1/((d*x+c-1)/(d*x+c+1))^(1/2))*arccoth(d*x+c)*b^3-6/d*polylog(2,-1/((d*x+c-1)/(d*x+c+1))^(1/2))*arccoth(d*x+c)*b^3+6/d*polylog(3,-1/((d*x+c-1)/(d*x+c+1))^(1/2))*b^3+6/d*polylog(3,1/((d*x+c-1)/(d*x+c+1))^(1/2))*b^3+3*arccoth(d*x+c)^2*x*a*b^2+3/d*arccoth(d*x+c)^2*a*b^2*c-6/d*ln(1+1/((d*x+c-1)/(d*x+c+1))^(1/2))*arccoth(d*x+c)*a*b^2-6/d*ln(1-1/((d*x+c-1)/(d*x+c+1))^(1/2))*arccoth(d*x+c)*a*b^2+3/d*a*b^2*arccoth(d*x+c)^2-6/d*polylog(2,1/((d*x+c-1)/(d*x+c+1))^(1/2))*a*b^2-6/d*polylog(2,-1/((d*x+c-1)/(d*x+c+1))^(1/2))*a*b^2+3*arccoth(d*x+c)*x*a^2*b+3/d*arccoth(d*x+c)*a^2*b*c+3/2/d*a^2*b*ln((d*x+c)^2-1)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$a^3x + \frac{3(2(dx+c)\operatorname{arccoth}(dx+c) + \log(-(dx+c)^2+1))a^2b}{2d} + \frac{(b^3dx + b^3(c+1))\log(dx+c+1)^3 + 3(2ab^2dx - (b^3d))}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccoth(d*x+c))^3,x, algorithm="maxima")`

[Out] `a^3*x + 3/2*(2*(d*x + c)*arccoth(d*x + c) + log(-(d*x + c)^2 + 1))*a^2*b/d + 1/8*((b^3*d*x + b^3*(c + 1))*log(d*x + c + 1)^3 + 3*(2*a*b^2*d*x - (b^3*d*x + b^3*(c - 1))*log(d*x + c - 1))*log(d*x + c + 1)^2)/d + integrate(-1/8*((b^3*d*x + b^3*(c + 1))*log(d*x + c - 1)^3 - 6*(a*b^2*d*x + a*b^2*(c + 1))*log(d*x + c - 1)^2 + 3*(4*a*b^2*d*x - (b^3*d*x + b^3*(c + 1))*log(d*x + c - 1)^2 + 2*(2*a*b^2*(c + 1) - b^3*(c - 1) + (2*a*b^2*d - b^3*d)*x)*log(d*x + c - 1))*log(d*x + c + 1)/(d*x + c + 1), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(b^3 \operatorname{arccoth}(dx + c)^3 + 3ab^2 \operatorname{arccoth}(dx + c)^2 + 3a^2b \operatorname{arccoth}(dx + c) + a^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))^3,x, algorithm="fricas")

[Out] integral(b^3\*arccoth(d\*x + c)^3 + 3\*a\*b^2\*arccoth(d\*x + c)^2 + 3\*a^2\*b\*arccoth(d\*x + c) + a^3, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{acoth}(c + dx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth(d\*x+c))\*\*3,x)

[Out] Integral((a + b\*acoth(c + d\*x))\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arccoth}(dx + c) + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))^3,x, algorithm="giac")

[Out] integrate((b\*arccoth(d\*x + c) + a)^3, x)

$$3.117 \quad \int \frac{(a + b \coth^{-1}(c + dx))^3}{e + fx} dx$$

**Optimal.** Leaf size=308

$$\frac{3b^2(a + b \coth^{-1}(c + dx)) \operatorname{PolyLog}\left(3, 1 - \frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{2f} + \frac{3b^2 \operatorname{PolyLog}\left(3, 1 - \frac{2}{c+dx+1}\right)(a + b \coth^{-1}(c + dx))}{2f}$$

[Out] -(((a + b\*ArcCoth[c + d\*x])^3\*Log[2/(1 + c + d\*x)])/f) + ((a + b\*ArcCoth[c + d\*x])^3\*Log[(2\*d\*(e + f\*x))/((d\*e + f - c\*f)\*(1 + c + d\*x))])/f + (3\*b\*(a + b\*ArcCoth[c + d\*x])^2\*PolyLog[2, 1 - 2/(1 + c + d\*x)])/(2\*f) - (3\*b\*(a + b\*ArcCoth[c + d\*x])^2\*PolyLog[2, 1 - (2\*d\*(e + f\*x))/((d\*e + f - c\*f)\*(1 + c + d\*x))])/((2\*f) + (3\*b^2\*(a + b\*ArcCoth[c + d\*x])\*PolyLog[3, 1 - 2/(1 + c + d\*x)])/(2\*f) - (3\*b^2\*(a + b\*ArcCoth[c + d\*x])\*PolyLog[3, 1 - (2\*d\*(e + f\*x))/((d\*e + f - c\*f)\*(1 + c + d\*x))])/((2\*f) + (3\*b^3\*PolyLog[4, 1 - 2/(1 + c + d\*x)])/(4\*f) - (3\*b^3\*PolyLog[4, 1 - (2\*d\*(e + f\*x))/((d\*e + f - c\*f)\*(1 + c + d\*x))])/((4\*f)

**Rubi [A]** time = 0.187135, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {6112, 5925}

$$\frac{3b^2(a + b \coth^{-1}(c + dx)) \operatorname{PolyLog}\left(3, 1 - \frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{2f} + \frac{3b^2 \operatorname{PolyLog}\left(3, 1 - \frac{2}{c+dx+1}\right)(a + b \coth^{-1}(c + dx))}{2f}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCoth[c + d\*x])^3/(e + f\*x),x]

[Out] -(((a + b\*ArcCoth[c + d\*x])^3\*Log[2/(1 + c + d\*x)])/f) + ((a + b\*ArcCoth[c + d\*x])^3\*Log[(2\*d\*(e + f\*x))/((d\*e + f - c\*f)\*(1 + c + d\*x))])/f + (3\*b\*(a + b\*ArcCoth[c + d\*x])^2\*PolyLog[2, 1 - 2/(1 + c + d\*x)])/(2\*f) - (3\*b\*(a + b\*ArcCoth[c + d\*x])^2\*PolyLog[2, 1 - (2\*d\*(e + f\*x))/((d\*e + f - c\*f)\*(1 + c + d\*x))])/((2\*f) + (3\*b^2\*(a + b\*ArcCoth[c + d\*x])\*PolyLog[3, 1 - 2/(1 + c + d\*x)])/(2\*f) - (3\*b^2\*(a + b\*ArcCoth[c + d\*x])\*PolyLog[3, 1 - (2\*d\*(e + f\*x))/((d\*e + f - c\*f)\*(1 + c + d\*x))])/((2\*f) + (3\*b^3\*PolyLog[4, 1 - 2/(1 + c + d\*x)])/(4\*f) - (3\*b^3\*PolyLog[4, 1 - (2\*d\*(e + f\*x))/((d\*e + f - c\*f)\*(1 + c + d\*x))])/((4\*f)



Rule 6112

Int[((a\_.) + ArcCoth[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]

Rule 5925

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^3/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] :> -Simp[((a + b\*ArcCoth[c\*x])^3\*Log[2/(1 + c\*x)])/e, x] + (Simp[((a + b\*ArcCoth[c\*x])^3\*Log[(2\*c\*(d + e\*x))/((c\*d + e)\*(1 + c\*x))])/e, x] + Simp[(3\*b\*(a + b\*ArcCoth[c\*x])^2\*PolyLog[2, 1 - 2/(1 + c\*x)]]/(2\*e), x] - Simp[(3\*b\*(a + b\*ArcCoth[c\*x])^2\*PolyLog[2, 1 - (2\*c\*(d + e\*x))/((c\*d + e)\*(1 + c\*x))]]/(2\*e), x] + Simp[(3\*b^2\*(a + b\*ArcCoth[c\*x])\*PolyLog[3, 1 - 2/(1 + c\*x)]]/(2\*e), x] - Simp[(3\*b^2\*(a + b\*ArcCoth[c\*x])\*PolyLog[3, 1 - (2\*c\*(d + e\*x))/((c\*d + e)\*(1 + c\*x))]]/(2\*e), x] + Simp[(3\*b^3\*PolyLog[4, 1 - 2/(1 + c\*x)]]/(4\*e), x] - Simp[(3\*b^3\*PolyLog[4, 1 - (2\*c\*(d + e\*x))/((c\*d + e)\*(1 + c\*x))]]/(4\*e), x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2\*d^2 - e^2, 0]

Rubi steps

$$\int \frac{(a + b \coth^{-1}(c + dx))^3}{e + fx} dx = \frac{\text{Subst}\left(\int \frac{(a + b \coth^{-1}(x))^3}{\frac{de - cf}{d} + \frac{fx}{d}} dx, x, c + dx\right)}{d}$$

$$= -\frac{(a + b \coth^{-1}(c + dx))^3 \log\left(\frac{2}{1 + c + dx}\right)}{f} + \frac{(a + b \coth^{-1}(c + dx))^3 \log\left(\frac{2d(e + fx)}{(de + f - cf)(1 + c + dx)}\right)}{f}$$

**Mathematica [F]** time = 22.4348, size = 0, normalized size = 0.

$$\int \frac{(a + b \coth^{-1}(c + dx))^3}{e + fx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCoth[c + d\*x])^3/(e + f\*x), x]

[Out] Integrate[(a + b\*ArcCoth[c + d\*x])^3/(e + f\*x), x]

**Maple [C]** time = 0.71, size = 3796, normalized size = 12.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((a+b\text{arccoth}(d*x+c))^3/(f*x+e),x)$

[Out] 
$$\begin{aligned} & 3/2*a^2*b/f*dilog(((d*x+c)*f-f)/(c*f-d*e-f))-b^3/(c*f-d*e-f)*\text{arccoth}(d*x+c) \\ & ^3*\ln(1-(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))+3/2*b^3/(c*f-d*e-f)*\text{ar} \\ & \text{ccth}(d*x+c)*\text{polylog}(3,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))-3*b^3/f \\ & *\text{arccoth}(d*x+c)^2*\text{polylog}(2,1/((d*x+c-1)/(d*x+c+1))^{(1/2)})-b^3/f*\text{arccoth}(d* \\ & x+c)^3*\ln(1-1/((d*x+c-1)/(d*x+c+1))^{(1/2)})-3*b^3/f*\text{arccoth}(d*x+c)^2*\text{polylog} \\ & (2,-1/((d*x+c-1)/(d*x+c+1))^{(1/2)})+6*b^3/f*\text{arccoth}(d*x+c)*\text{polylog}(3,-1/((d* \\ & x+c-1)/(d*x+c+1))^{(1/2)})+b^3*\ln((d*x+c)*f-c*f+d*e)/f*\text{arccoth}(d*x+c)^3+b^3/f \\ & *\text{arccoth}(d*x+c)^3*\ln((d*x+c+1)/(d*x+c-1)-1)+6*b^3/f*\text{arccoth}(d*x+c)*\text{polylog} \\ & (3,1/((d*x+c-1)/(d*x+c+1))^{(1/2)})-b^3/f*\text{arccoth}(d*x+c)^3*\ln(1+1/((d*x+c-1)/( \\ & d*x+c+1))^{(1/2)})-b^3/f*\text{arccoth}(d*x+c)^3*\ln(((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-( \\ & d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)-3/2*b^3/(c*f-d*e-f)*\text{arc} \\ & \text{ccth}(d*x+c)^2*\text{polylog}(2,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))+3/4*b^ \\ & 3*c/(c*f-d*e-f)*\text{polylog}(4,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))+6*a* \\ & b^2/f*\text{polylog}(3,1/((d*x+c-1)/(d*x+c+1))^{(1/2)})+6*a*b^2/f*\text{polylog}(3,-1/((d*x \\ & +c-1)/(d*x+c+1))^{(1/2)})+3/2*a*b^2/(c*f-d*e-f)*\text{polylog}(3,(c*f-d*e-f)*(d*x+c+ \\ & 1)/(d*x+c-1)/(c*f-d*e+f))-3/2*a^2*b/f*dilog(((d*x+c)*f+f)/(c*f-d*e+f))+3/2* \\ & b^3*c/(c*f-d*e-f)*\text{arccoth}(d*x+c)^2*\text{polylog}(2,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1 \\ & )/(c*f-d*e+f))-6*a*b^2/f*\text{arccoth}(d*x+c)*\text{polylog}(2,-1/((d*x+c-1)/(d*x+c+1))^{( \\ & 1/2)})-3*a*b^2/f*\text{arccoth}(d*x+c)^2*\ln(1-1/((d*x+c-1)/(d*x+c+1))^{(1/2)})-6*a*b \\ & ^2/f*\text{arccoth}(d*x+c)*\text{polylog}(2,1/((d*x+c-1)/(d*x+c+1))^{(1/2)})-3*a*b^2/(c*f-d \\ & *e-f)*\text{arccoth}(d*x+c)^2*\ln(1-(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))-3* \\ & a*b^2/(c*f-d*e-f)*\text{arccoth}(d*x+c)*\text{polylog}(2,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/ \\ & (c*f-d*e+f))+3*a*b^2/f*\text{arccoth}(d*x+c)^2*\ln((d*x+c+1)/(d*x+c-1)-1)-3*a*b^2/f \\ & *\text{arccoth}(d*x+c)^2*\ln(((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e \\ & d+(-(d*x+c+1)/(d*x+c-1)-1)*f)+b^3*c/(c*f-d*e-f)*\text{arccoth}(d*x+c)^3*\ln(1-(c*f- \\ & d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))-3/2*b^3*c/(c*f-d*e-f)*\text{arccoth}(d*x+c \\ & )*\text{polylog}(3,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))+3*a^2*b*\ln((d*x+c) \\ & *f-c*f+d*e)/f*\text{arccoth}(d*x+c)-I*b^3/f*\text{Pi}*\text{arccoth}(d*x+c)^3-3/2*a^2*b/f*\ln(((d \\ & *x+c)*f+f)/(c*f-d*e+f))*\ln((d*x+c)*f-c*f+d*e)+3/2*a^2*b/f*\ln(((d*x+c)*f-f)/ \\ & (c*f-d*e-f))*\ln((d*x+c)*f-c*f+d*e)+3*a*b^2*\ln((d*x+c)*f-c*f+d*e)/f*\text{arccoth} \\ & (d*x+c)^2-3*a*b^2/f*\text{arccoth}(d*x+c)^2*\ln(1+1/((d*x+c-1)/(d*x+c+1))^{(1/2)})-3/2 \\ & *a*b^2*c/(c*f-d*e-f)*\text{polylog}(3,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f)) \\ & -3*d*a*b^2/f*e/(c*f-d*e-f)*\text{arccoth}(d*x+c)^2*\ln(1-(c*f-d*e-f)*(d*x+c+1)/(d*x \\ & +c-1)/(c*f-d*e+f))-3*d*a*b^2/f*e/(c*f-d*e-f)*\text{arccoth}(d*x+c)*\text{polylog}(2,(c*f- \\ & d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))-3/2*I*a*b^2/f*\text{Pi}*\text{arccoth}(d*x+c)^2*c \\ & \text{sgn}(I*((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/ \end{aligned}$$

$$\begin{aligned}
& ((d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^2*csgn(I/((d*x+c+1)/(d*x+c-1)-1))- \\
& 3/2*I*a*b^2/f*Pi*arccoth(d*x+c)^2*csgn(I*(((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d \\
& *x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f))*csgn(I*(((d*x+c+1)/(d*x \\
& +c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+ \\
& c+1)/(d*x+c-1)-1))^2+1/2*I*b^3/f*Pi*arccoth(d*x+c)^3*csgn(I*(((d*x+c+1)/(d*x \\
& +c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f))*csgn \\
& (I*(((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x \\
& +c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))*csgn(I/((d*x+c+1)/(d*x+c-1)-1))-6*b^3 \\
& /f*polylog(4,1/((d*x+c-1)/(d*x+c+1))^(1/2))+a^3*ln((d*x+c)*f-c*f+d*e)/f-3/4 \\
& *b^3/(c*f-d*e-f)*polylog(4,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))-6*b \\
& ^3/f*polylog(4,-1/((d*x+c-1)/(d*x+c+1))^(1/2))+3*a*b^2*c/(c*f-d*e-f)*arccot \\
& h(d*x+c)^2*ln(1-(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))+3*a*b^2*c/(c*f \\
& -d*e-f)*arccoth(d*x+c)*polylog(2,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f \\
& ))+I*b^3/f*Pi*arccoth(d*x+c)^3*csgn(I*(((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+ \\
& c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^2- \\
& 3/4*d*b^3/f*e/(c*f-d*e-f)*polylog(4,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d* \\
& e+f))-3*I*a*b^2/f*Pi*arccoth(d*x+c)^2-1/2*I*b^3/f*Pi*arccoth(d*x+c)^3*csgn( \\
& I*(((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x \\
& +c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^3-3/2*I*a*b^2/f*Pi*arccoth(d*x+c)^2*cs \\
& gn(I*(((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/( \\
& d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^3-1/2*I*b^3/f*Pi*arccoth(d*x+c)^3*c \\
& sgn(I*(((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/ \\
& (d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^2*csgn(I/((d*x+c+1)/(d*x+c-1)-1))+ \\
& 3/2*d*a*b^2/f*e/(c*f-d*e-f)*polylog(3,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f- \\
& d*e+f))-3/2*d*b^3/f*e/(c*f-d*e-f)*arccoth(d*x+c)^2*polylog(2,(c*f-d*e-f)*(d \\
& *x+c+1)/(d*x+c-1)/(c*f-d*e+f))-d*b^3/f*e/(c*f-d*e-f)*arccoth(d*x+c)^3*ln(1- \\
& (c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))+3/2*d*b^3/f*e/(c*f-d*e-f)*arcc \\
& oth(d*x+c)*polylog(3,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))-1/2*I*b^3 \\
& /f*Pi*arccoth(d*x+c)^3*csgn(I*(((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d* \\
& x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f))*csgn(I*(((d*x+c+1)/(d*x+c-1)-1)*c \\
& f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c \\
& -1)-1))^2+3*I*a*b^2/f*Pi*arccoth(d*x+c)^2*csgn(I*(((d*x+c+1)/(d*x+c-1)-1)*c \\
& *f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+ \\
& c-1)-1))^2+3/2*I*a*b^2/f*Pi*arccoth(d*x+c)^2*csgn(I*(((d*x+c+1)/(d*x+c-1)-1 \\
& )*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f))*csgn(I*(((d* \\
& x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1 \\
& )*f)/((d*x+c+1)/(d*x+c-1)-1))*csgn(I/((d*x+c+1)/(d*x+c-1)-1))
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{a^3 \log(fx + e)}{f} + \int \frac{b^3 \left( \log\left(\frac{1}{dx+c} + 1\right) - \log\left(-\frac{1}{dx+c} + 1\right) \right)^3}{8(fx + e)} + \frac{3ab^2 \left( \log\left(\frac{1}{dx+c} + 1\right) - \log\left(-\frac{1}{dx+c} + 1\right) \right)^2}{4(fx + e)} + \frac{3a^2b \left( \log\left(\frac{1}{dx+c} + 1\right) - \log\left(-\frac{1}{dx+c} + 1\right) \right)}{4(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))^3/(f\*x+e),x, algorithm="maxima")

[Out]  $a^3 \log(fx + e)/f + \int (1/8*b^3*(\log(1/(dx + c) + 1) - \log(-1/(dx + c) + 1))^3/(fx + e) + 3/4*a*b^2*(\log(1/(dx + c) + 1) - \log(-1/(dx + c) + 1))^2/(fx + e) + 3/2*a^2*b*(\log(1/(dx + c) + 1) - \log(-1/(dx + c) + 1)))/(fx + e), x)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \operatorname{arccoth}(dx + c)^3 + 3ab^2 \operatorname{arccoth}(dx + c)^2 + 3a^2b \operatorname{arccoth}(dx + c) + a^3}{fx + e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))^3/(f\*x+e),x, algorithm="fricas")

[Out]  $\text{integral}((b^3*\operatorname{arccoth}(d*x + c)^3 + 3*a*b^2*\operatorname{arccoth}(d*x + c)^2 + 3*a^2*b*\operatorname{arccoth}(d*x + c) + a^3)/(f*x + e), x)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth(d\*x+c))\*3/(f\*x+e),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccoth}(dx + c) + a)^3}{fx + e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^3/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(d*x + c) + a)^3/(f*x + e), x)
```

$$3.118 \quad \int \frac{(a+b \coth^{-1}(c+dx))^3}{(e+fx)^2} dx$$

**Optimal.** Leaf size=1089

result too large to display

```
[Out] -((a + b*ArcCoth[c + d*x])^3/(f*(e + f*x))) + (3*a*b^2*d*ArcCoth[c + d*x]*Log[2/(1 - c - d*x)]/(f*(d*e + f - c*f))) + (3*b^3*d*ArcCoth[c + d*x]^2*Log[2/(1 - c - d*x)]/(2*f*(d*e + f - c*f))) - (3*a^2*b*d*Log[1 - c - d*x])/(2*f*(d*e + f - c*f)) - (3*a*b^2*d*ArcCoth[c + d*x]*Log[2/(1 + c + d*x)]/(f*(d*e - f - c*f))) + (6*a*b^2*d*ArcCoth[c + d*x]*Log[2/(1 + c + d*x)]/((d*e + f - c*f)*(d*e - (1 + c)*f))) - (3*b^3*d*ArcCoth[c + d*x]^2*Log[2/(1 + c + d*x)]/(2*f*(d*e - f - c*f))) + (3*b^3*d*ArcCoth[c + d*x]^2*Log[2/(1 + c + d*x)]/((d*e + f - c*f)*(d*e - (1 + c)*f))) + (3*a^2*b*d*Log[1 + c + d*x])/(2*f*(d*e - f - c*f)) + (3*a^2*b*d*Log[e + f*x])/(f^2 - (d*e - c*f)^2) - (6*a*b^2*d*ArcCoth[c + d*x]*Log[(2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))]) / ((d*e + f - c*f)*(d*e - (1 + c)*f)) - (3*b^3*d*ArcCoth[c + d*x]^2*Log[(2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))]) / ((d*e + f - c*f)*(d*e - (1 + c)*f)) + (3*a*b^2*d*PolyLog[2, -(1 + c + d*x)/(1 - c - d*x)]/(2*f*(d*e + f - c*f))) + (3*b^3*d*ArcCoth[c + d*x]*PolyLog[2, 1 - 2/(1 - c - d*x)]/(2*f*(d*e + f - c*f))) + (3*a*b^2*d*PolyLog[2, 1 - 2/(1 + c + d*x)]/(2*f*(d*e - f - c*f))) - (3*a*b^2*d*PolyLog[2, 1 - 2/(1 + c + d*x)]/((d*e + f - c*f)*(d*e - (1 + c)*f))) + (3*b^3*d*ArcCoth[c + d*x]*PolyLog[2, 1 - 2/(1 + c + d*x)]/(2*f*(d*e - f - c*f))) - (3*b^3*d*ArcCoth[c + d*x]*PolyLog[2, 1 - 2/(1 + c + d*x)]/((d*e + f - c*f)*(d*e - (1 + c)*f))) + (3*a*b^2*d*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))]) / ((d*e + f - c*f)*(d*e - (1 + c)*f))) + (3*b^3*d*ArcCoth[c + d*x]*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))]) / ((d*e + f - c*f)*(d*e - (1 + c)*f))) - (3*b^3*d*PolyLog[3, 1 - 2/(1 - c - d*x)]/(4*f*(d*e + f - c*f))) + (3*b^3*d*PolyLog[3, 1 - 2/(1 + c + d*x)]/(4*f*(d*e - f - c*f))) - (3*b^3*d*PolyLog[3, 1 - 2/(1 + c + d*x)]/(2*(d*e + f - c*f)*(d*e - (1 + c)*f))) + (3*b^3*d*PolyLog[3, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))]) / (2*(d*e + f - c*f)*(d*e - (1 + c)*f))
```

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**Rubi [A]** time = 2.78214, antiderivative size = 1094, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 18, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.9$ , Rules used = {6110, 6741, 6122, 6688, 12, 6725, 72, 6742, 5919, 2402, 2315, 5921, 2447,

5949, 6059, 6610, 6057, 5923}

$$\frac{3d \coth^{-1}(c + dx)^2 \log\left(\frac{2}{-c-dx+1}\right) b^3}{2f(de - cf + f)} - \frac{3d \coth^{-1}(c + dx)^2 \log\left(\frac{2}{c+dx+1}\right) b^3}{2f(de - cf - f)} + \frac{3d \coth^{-1}(c + dx)^2 \log\left(\frac{2}{c+dx+1}\right) b^3}{(de - cf + f)(de - (c + 1)f)} - \frac{3d \coth^{-1}(c + dx)^2 \log\left(\frac{2}{-c-dx+1}\right) b^3}{(de - cf + f)(de - (c + 1)f)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCoth[c + d\*x])^3/(e + f\*x)^2,x]

[Out] -((a + b\*ArcCoth[c + d\*x])^3/(f\*(e + f\*x))) + (3\*a\*b^2\*d\*ArcCoth[c + d\*x]\*Log[2/(1 - c - d\*x)]/(f\*(d\*e + f - c\*f)) + (3\*b^3\*d\*ArcCoth[c + d\*x]^2\*Log[2/(1 - c - d\*x)]/(2\*f\*(d\*e + f - c\*f)) - (3\*a^2\*b\*d\*Log[1 - c - d\*x])/(2\*f\*(d\*e + f - c\*f)) - (3\*a\*b^2\*d\*ArcCoth[c + d\*x]\*Log[2/(1 + c + d\*x)]/(f\*(d\*e - f - c\*f)) + (6\*a\*b^2\*d\*ArcCoth[c + d\*x]\*Log[2/(1 + c + d\*x)]/((d\*e + f - c\*f)\*(d\*e - (1 + c)\*f)) - (3\*b^3\*d\*ArcCoth[c + d\*x]^2\*Log[2/(1 + c + d\*x)]/(2\*f\*(d\*e - f - c\*f)) + (3\*b^3\*d\*ArcCoth[c + d\*x]^2\*Log[2/(1 + c + d\*x)]/((d\*e + f - c\*f)\*(d\*e - (1 + c)\*f)) + (3\*a^2\*b\*d\*Log[1 + c + d\*x])/(2\*f\*(d\*e - f - c\*f)) - (3\*a^2\*b\*d\*Log[e + f\*x])/((d\*e + f - c\*f)\*(d\*e - (1 + c)\*f)) - (6\*a\*b^2\*d\*ArcCoth[c + d\*x]\*Log[(2\*d\*(e + f\*x))/(d\*e + f - c\*f]\*(1 + c + d\*x)))/((d\*e + f - c\*f)\*(d\*e - (1 + c)\*f)) - (3\*b^3\*d\*ArcCoth[c + d\*x]^2\*Log[(2\*d\*(e + f\*x))/(d\*e + f - c\*f]\*(1 + c + d\*x)))/((d\*e + f - c\*f)\*(d\*e - (1 + c)\*f)) + (3\*a\*b^2\*d\*PolyLog[2, -(1 + c + d\*x)/(1 - c - d\*x)]/(2\*f\*(d\*e + f - c\*f)) + (3\*b^3\*d\*ArcCoth[c + d\*x]\*PolyLog[2, 1 - 2/(1 - c - d\*x)]/(2\*f\*(d\*e + f - c\*f)) + (3\*a\*b^2\*d\*PolyLog[2, 1 - 2/(1 + c + d\*x)]/(2\*f\*(d\*e - f - c\*f)) - (3\*a\*b^2\*d\*PolyLog[2, 1 - 2/(1 + c + d\*x)]/((d\*e + f - c\*f)\*(d\*e - (1 + c)\*f)) + (3\*b^3\*d\*ArcCoth[c + d\*x]\*PolyLog[2, 1 - 2/(1 + c + d\*x)]/(2\*f\*(d\*e - f - c\*f)) - (3\*b^3\*d\*ArcCoth[c + d\*x]\*PolyLog[2, 1 - 2/(1 + c + d\*x)]/((d\*e + f - c\*f)\*(d\*e - (1 + c)\*f)) + (3\*a\*b^2\*d\*PolyLog[2, 1 - (2\*d\*(e + f\*x))/(d\*e + f - c\*f]\*(1 + c + d\*x)))/((d\*e + f - c\*f)\*(d\*e - (1 + c)\*f)) + (3\*b^3\*d\*ArcCoth[c + d\*x]\*PolyLog[2, 1 - (2\*d\*(e + f\*x))/(d\*e + f - c\*f]\*(1 + c + d\*x)))/((d\*e + f - c\*f)\*(d\*e - (1 + c)\*f)) - (3\*b^3\*d\*PolyLog[3, 1 - 2/(1 - c - d\*x)]/(4\*f\*(d\*e + f - c\*f)) + (3\*b^3\*d\*PolyLog[3, 1 - 2/(1 + c + d\*x)]/(4\*f\*(d\*e - f - c\*f)) - (3\*b^3\*d\*PolyLog[3, 1 - 2/(1 + c + d\*x)]/(2\*(d\*e + f - c\*f)\*(d\*e - (1 + c)\*f)) + (3\*b^3\*d\*PolyLog[3, 1 - (2\*d\*(e + f\*x))/(d\*e + f - c\*f]\*(1 + c + d\*x)]/(2\*(d\*e + f - c\*f)\*(d\*e - (1 + c)\*f)))

### Rule 6110

Int[((a\_.) + ArcCoth[(c\_) + (d\_.)\*(x\_)])\*(b\_.))^p\_.\*(e\_.) + (f\_.)\*(x\_)^(m\_), x\_Symbol] := Simp[((e + f\*x)^(m + 1)\*(a + b\*ArcCoth[c + d\*x])^p)/(f\*(m + 1)), x] - Dist[(b\*d\*p)/(f\*(m + 1)), Int[((e + f\*x)^(m + 1)\*(a + b\*ArcCoth[c + d\*x])^(p - 1))/(1 - (c + d\*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[m, -1]

Rule 6741

Int[u\_, x\_Symbol] := With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v != u]

Rule 6122

Int[((a\_.) + ArcCoth[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.)\*((A\_.) + (B\_.)\*(x\_.) + (C\_.)\*(x\_.)^2)^(q\_.), x\_Symbol] := Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(-(C/d^2) + (C\*x^2)/d^2)^q\*(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] && EqQ[B\*(1 - c^2) + 2\*A\*c\*d, 0] && EqQ[2\*c\*C - B\*d, 0]

Rule 6688

Int[u\_, x\_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifierIntegrandQ[v, u, x]]

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 6725

Int[(u\_)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := With[{v = RationalFunctionExpand[u/(a + b\*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rule 72

Int[((e\_.) + (f\_.)\*(x\_.))^(p\_.)/(((a\_.) + (b\_.)\*(x\_.))\*((c\_.) + (d\_.)\*(x\_.))), x\_Symbol] := Int[ExpandIntegrand[(e + f\*x)^p/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rule 5919

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] := -Simp[((a + b\*ArcCoth[c\*x])^p\*Log[2/(1 + (e\*x)/d)])/e, x] + Dist[(b\*c^p)/e, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*Log[2/(1 + (e\*x)/d)]]/(1 - c^2\*x^2)



, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

### Rule 2402

Int[Log[(c\_.)/((d\_) + (e\_.)\*(x\_))]/((f\_) + (g\_.)\*(x\_)^2), x\_Symbol] := -Dist[e/g, Subst[Int[Log[2\*d\*x]/(1 - 2\*d\*x), x], x, 1/(d + e\*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2\*d] && EqQ[e^2\*f + d^2\*g, 0]

### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

### Rule 5921

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[((a + b\*ArcCoth[c\*x])\*Log[2/(1 + c\*x)])/e, x] + (Dist[(b\*c)/e, Int[Log[2/(1 + c\*x)]/(1 - c^2\*x^2), x], x] - Dist[(b\*c)/e, Int[Log[(2\*c\*(d + e\*x))/((c\*d + e)\*(1 + c\*x))]/(1 - c^2\*x^2), x], x] + Simp[((a + b\*ArcCoth[c\*x])\*Log[(2\*c\*(d + e\*x))/((c\*d + e)\*(1 + c\*x))])/e, x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2\*d^2 - e^2, 0]

### Rule 2447

Int[Log[u]\*(Pq\_)^(m\_.), x\_Symbol] := With[{C = FullSimplify[(Pq^m\*(1 - u))/D[u, x]]}, Simp[C\*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]

### Rule 5949

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^p\_/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

### Rule 6059

Int[(Log[u]\*((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^p\_/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := -Simp[(a + b\*ArcCoth[c\*x])^p\*PolyLog[2, 1 - u]/(2\*c\*d), x] + Dist[(b\*p)/2, Int[(a + b\*ArcCoth[c\*x])^(p - 1)\*PolyLog[2, 1 - u]/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c\*x))^2, 0]

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6057

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] := Simp[((a + b*ArcCoth[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 5923

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :=
-Simp[((a + b*ArcCoth[c*x])^2*Log[2/(1 + c*x)])/e, x] + (Simp[((a + b*ArcC
oth[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Simp[(b*(a
+ b*ArcCoth[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e, x] - Simp[(b*(a + b*ArcCo
th[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Sim
p[(b^2*PolyLog[3, 1 - 2/(1 + c*x)])/e, x] - Simp[(b^2*PolyLog[3, 1 - (2
*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e},
x] && NeQ[c^2*d^2 - e^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \coth^{-1}(c + dx))^3}{(e + fx)^2} dx &= -\frac{(a + b \coth^{-1}(c + dx))^3}{f(e + fx)} + \frac{(3bd) \int \frac{(a + b \coth^{-1}(c + dx))^2}{(e + fx)(1 - (c + dx)^2)} dx}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^3}{f(e + fx)} + \frac{(3bd) \int \frac{(a + b \coth^{-1}(c + dx))^2}{(e + fx)(1 - c^2 - 2cdx - d^2x^2)} dx}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^3}{f(e + fx)} + \frac{(3b) \text{Subst} \left( \int \frac{(a + b \coth^{-1}(x))^2}{\left(\frac{de - cf}{d} + \frac{fx}{d}\right)(1 - x^2)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^3}{f(e + fx)} + \frac{(3b) \text{Subst} \left( \int \frac{d(a + b \coth^{-1}(x))^2}{(de - cf + fx)(1 - x^2)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^3}{f(e + fx)} + \frac{(3bd) \text{Subst} \left( \int \frac{(a + b \coth^{-1}(x))^2}{(de - cf + fx)(1 - x^2)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^3}{f(e + fx)} + \frac{(3bd) \text{Subst} \left( \int \left( -\frac{a^2}{(-1+x)(1+x)(de - cf + fx)} - \frac{2ab \coth^{-1}(x)}{(-1+x)(1+x)(de - cf + fx)} \right) dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^3}{f(e + fx)} - \frac{(3a^2bd) \text{Subst} \left( \int \frac{1}{(-1+x)(1+x)(de - cf + fx)} dx, x, c + dx \right)}{f} - \frac{(6abd) \text{Subst} \left( \int \frac{\coth^{-1}(x)}{(-1+x)(1+x)(de - cf + fx)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^3}{f(e + fx)} - \frac{(3a^2bd) \text{Subst} \left( \int \left( \frac{1}{2(de + f - cf)(-1+x)} + \frac{1}{2(-de + (1+c)f)(1+x)} + \frac{1}{2(de - f - cf)(1+x)} \right) dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^3}{f(e + fx)} - \frac{3a^2bd \log(1 - c - dx)}{2f(de + f - cf)} + \frac{3a^2bd \log(1 + c + dx)}{2f(de - f - cf)} - \frac{3a^2bd \text{Subst} \left( \int \frac{\coth^{-1}(x)}{(-1+x)(1+x)(de - cf + fx)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^3}{f(e + fx)} + \frac{3ab^2d \coth^{-1}(c + dx) \log\left(\frac{2}{1 - c - dx}\right)}{f(de + f - cf)} + \frac{3b^3d \coth^{-1}(c + dx)^2}{2f(de + f - cf)} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^3}{f(e + fx)} + \frac{3ab^2d \coth^{-1}(c + dx) \log\left(\frac{2}{1 - c - dx}\right)}{f(de + f - cf)} + \frac{3b^3d \coth^{-1}(c + dx)^2}{2f(de + f - cf)} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^3}{f(e + fx)} + \frac{3ab^2d \coth^{-1}(c + dx) \log\left(\frac{2}{1 - c - dx}\right)}{f(de + f - cf)} + \frac{3b^3d \coth^{-1}(c + dx)^2}{2f(de + f - cf)}
\end{aligned}$$

**Mathematica [C]** time = 17.4167, size = 1899, normalized size = 1.74

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCoth[c + d\*x])^3/(e + f\*x)^2,x]

[Out] 
$$-(a^3/(f*(e + f*x))) - (3*a^2*b*ArcCoth[c + d*x])/(f*(e + f*x)) + (3*a^2*b*d*Log[1 - c - d*x])/(2*f*(-(d*e) - f + c*f)) - (3*a^2*b*d*Log[1 + c + d*x])/(2*f*(-(d*e) + f + c*f)) - (3*a^2*b*d*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f - f^2 + c^2*f^2) + (3*a*b^2*(1 - (c + d*x)^2)*(f/Sqrt[1 - (c + d*x)^(-2)] + (d*e - c*f)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]))^2*((E^ArcTanh[f/(-(d*e) + c*f)]*ArcCoth[c + d*x]^2)/((- (d*e) + c*f)*Sqrt[1 - f^2/(d*e - c*f)^2]) + ArcCoth[c + d*x]^2/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]*(f/Sqrt[1 - (c + d*x)^(-2)] + (d*e - c*f)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]))) + (f*(I*Pi*ArcCoth[c + d*x] + 2*ArcCoth[c + d*x]*ArcTanh[f/(d*e - c*f)] - I*Pi*Log[1 + E^(2*ArcCoth[c + d*x])]) + 2*ArcCoth[c + d*x]*Log[1 - E^(-2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)])]) - 2*ArcTanh[f/(-(d*e) + c*f)]*Log[1 - E^(-2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)])]) + I*Pi*Log[1/Sqrt[1 - (c + d*x)^(-2)]] + 2*ArcTanh[f/(-(d*e) + c*f)]*Log[I*Sinh[ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]]] - PolyLog[2, E^(-2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]))])))/(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)))/(d*f*(e + f*x)^2) - (b^3*(1 - (c + d*x)^2)*(f/Sqrt[1 - (c + d*x)^(-2)] + (d*e - c*f)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]))^2*((d*ArcCoth[c + d*x]^3)/(f*(c + d*x)*Sqrt[1 - (c + d*x)^(-2)]*(-(f/Sqrt[1 - (c + d*x)^(-2)]) - (d*e)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)])) + (c*f)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]))) - (d*(2*d*e*ArcCoth[c + d*x]^3 - 6*f*ArcCoth[c + d*x]^3 - 2*c*f*ArcCoth[c + d*x]^3 - (4*d*e*Sqrt[(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)/(d*e - c*f]^2)*ArcCoth[c + d*x]^3)/E^ArcTanh[f/(d*e - c*f)] + (4*c*f*Sqrt[(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)/(d*e - c*f]^2)*ArcCoth[c + d*x]^3)/E^ArcTanh[f/(d*e - c*f)] + (6*I)*f*Pi*ArcCoth[c + d*x]*Log[2] - f*ArcCoth[c + d*x]^2*Log[64] - (6*I)*f*Pi*ArcCoth[c + d*x]*Log[E^(-ArcCoth[c + d*x]) + E^ArcCoth[c + d*x]] + 6*f*ArcCoth[c + d*x]^2*Log[1 - E^(2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)])]) + 12*f*ArcCoth[c + d*x]*ArcTanh[f/(d*e - c*f)]*Log[(I/2)*E^(-ArcCoth[c + d*x] - ArcTanh[f/(d*e - c*f)])*(-1 + E^(2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)])])]) + 6*f*ArcCoth[c + d*x]^2*Log[-((d*e*(-1 + E^(2*ArcCoth[c + d*x])) + (1 + c + E^(2*ArcCoth[c + d*x]) - c*E^(2*ArcCoth[c + d*x]))*f)/E^ArcCoth[c + d*x]]) - 6*f*ArcCoth[c + d*x]^2*Log[-(d*e*(-1 + E^(2*ArcCoth[c + d*x])) + (-1 - E^(2*ArcCoth[c + d*x]) + c*(-1 + E^(2*ArcCoth[c + d*x])))*f)/(d*e - (1 + c)*f)] + 6*f*ArcCoth[c + d*x]^2*Log[1 - (E^ArcCoth[c + d*x]*Sqrt[d*e + f - c*f])/Sqrt[d*e - (1 + c)*f]] + 6*f*ArcCoth[c + d*x]^2*Log[1 + (E^ArcCoth[c + d*x]*Sqrt[d*e + f - c*f])/Sqrt[d*e - (1 + c)*f]] + (6*I)*f*Pi*ArcCoth[c + d*x]*Log[1/Sqrt[1 - (c + d*x)^(-2)]] - 6*f*ArcCoth[c + d*x]^2*Lo$$

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g[-(f/Sqrt[1 - (c + d*x)^(-2)]) - (d*e)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)])
) + (c*f)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)])] - 12*f*ArcCoth[c + d*x]*Arc
Tanh[f/(d*e - c*f)]*Log[I*Sinh[ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]]]
+ 6*f*ArcCoth[c + d*x]*PolyLog[2, E^(2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e -
c*f)))] - 6*f*ArcCoth[c + d*x]*PolyLog[2, (E^(2*ArcCoth[c + d*x])*(d*e +
f - c*f))/(d*e - (1 + c)*f)] + 12*f*ArcCoth[c + d*x]*PolyLog[2, -(E^ArcCot
h[c + d*x]*Sqrt[d*e + f - c*f])/Sqrt[d*e - (1 + c)*f)] + 12*f*ArcCoth[c +
d*x]*PolyLog[2, (E^ArcCoth[c + d*x]*Sqrt[d*e + f - c*f])/Sqrt[d*e - (1 + c)
*f)] - 3*f*PolyLog[3, E^(2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)))] +
3*f*PolyLog[3, (E^(2*ArcCoth[c + d*x])*(d*e + f - c*f))/(d*e - (1 + c)*f)]
- 12*f*PolyLog[3, -(E^ArcCoth[c + d*x]*Sqrt[d*e + f - c*f])/Sqrt[d*e - (1
+ c)*f)] - 12*f*PolyLog[3, (E^ArcCoth[c + d*x]*Sqrt[d*e + f - c*f])/Sqrt[d
*e - (1 + c)*f]])/(2*f*(d*e + f - c*f)*(d*e - (1 + c)*f)))/(d^2*(e + f*x)
^2)

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**Maple [C]** time = 0.844, size = 4619, normalized size = 4.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth(d\*x+c))^3/(f\*x+e)^2,x)

[Out]  $\frac{3}{4}I*d^2*b^3/f/(c*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*e*csgn(I*(d*x+c+1)/(d*x+c-1))^3+3/4*I*d*b^3/(c*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*csgn(I*(d*x+c+1)/(d*x+c-1))/((d*x+c+1)/(d*x+c-1)-1)*csgn(I*(d*x+c+1)/(d*x+c-1))*csgn(I/((d*x+c+1)/(d*x+c-1)-1))+3/4*I*d^2*b^3/f/(c*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*e*csgn(I*(d*x+c+1)/(d*x+c-1))/((d*x+c+1)/(d*x+c-1)-1))^3+3/2*I*d*b^3/(c*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*c*csgn(I/((d*x+c-1)/(d*x+c+1))^(1/2))*csgn(I*(d*x+c+1)/(d*x+c-1))^2-3/2*I*d*b^3/(c*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*csgn(I*((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f))*csgn(I*((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f))/((d*x+c+1)/(d*x+c-1)-1))*csgn(I/((d*x+c+1)/(d*x+c-1)-1))+3/4*I*d*b^3/(c*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*c*csgn(I*(d*x+c+1)/(d*x+c-1))/((d*x+c+1)/(d*x+c-1)-1))^2*csgn(I/((d*x+c+1)/(d*x+c-1)-1))+3/4*I*d*b^3/(c*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*c*csgn(I*(d*x+c+1)/(d*x+c-1))/((d*x+c+1)/(d*x+c-1)-1))^2*csgn(I*(d*x+c+1)/(d*x+c-1))-3/4*I*d*b^3/(c*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*c*csgn(I/((d*x+c-1)/(d*x+c+1))^(1/2))^2*csgn(I*(d*x+c+1)/(d*x+c-1))-d*b^3/(d*f*x+d*e)/f*arccoth(d*x+c)^3-d*a^3/(d*f*x+d*e)/f+3*d^2*b^3/(c*f-d*e-f)^2/(c*f-d*e+f)*e*arccoth(d*x+c)^2*ln(1-(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))+3*d^2*b^3/(c*f-d*e-f)^2/(c*f-d*e+f)*e*arccoth(d*x+c)*poly$

$$\begin{aligned} & \log(2, (c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))+3*I*d*b^3/(c*f-d*e-f)/(c \\ & *f-d*e+f)*\operatorname{arccoth}(d*x+c)^2*\pi+3/4*I*d^2*b^3/f/(c*f-d*e-f)/(c*f-d*e+f)*\operatorname{arcco} \\ & \operatorname{th}(d*x+c)^2*\pi*e*\operatorname{csgn}(I/((d*x+c-1)/(d*x+c+1))^{(1/2)})^2*\operatorname{csgn}(I*(d*x+c+1)/(d* \\ & x+c-1))-3/4*I*d^2*b^3/f/(c*f-d*e-f)/(c*f-d*e+f)*\operatorname{arccoth}(d*x+c)^2*\pi*e*\operatorname{csgn}( \\ & I*(d*x+c+1)/(d*x+c-1)/((d*x+c+1)/(d*x+c-1)-1))^2*\operatorname{csgn}(I/((d*x+c+1)/(d*x+c-1) \\ & )-1))-d*b^3/f*\operatorname{arccoth}(d*x+c)^3/(c*f-d*e+f)-3/4*I*d^2*b^3/f/(c*f-d*e-f)/(c*f \\ & -d*e+f)*\operatorname{arccoth}(d*x+c)^2*\pi*e*\operatorname{csgn}(I*(d*x+c+1)/(d*x+c-1)/((d*x+c+1)/(d*x+c- \\ & 1)-1))^2*\operatorname{csgn}(I*(d*x+c+1)/(d*x+c-1))-3/2*I*d^2*b^3/f/(c*f-d*e-f)/(c*f-d*e+f \\ & )*\operatorname{arccoth}(d*x+c)^2*\pi*e*\operatorname{csgn}(I/((d*x+c-1)/(d*x+c+1))^{(1/2)})*\operatorname{csgn}(I*(d*x+c+1 \\ & )/(d*x+c-1))^2-3*d*b^3*f/(c*f-d*e-f)^2/(c*f-d*e+f)*c*\operatorname{arccoth}(d*x+c)*\operatorname{polylog} \\ & (2, (c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))-3*d*b^3*f/(c*f-d*e-f)^2/(c* \\ & f-d*e+f)*c*\operatorname{arccoth}(d*x+c)^2*\ln(1-(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f \\ & ))-3*I*d*b^3/(c*f-d*e-f)/(c*f-d*e+f)*\operatorname{arccoth}(d*x+c)^2*\pi*\operatorname{csgn}(I*((d*x+c+1) \\ & /((d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/( \\ & (d*x+c+1)/(d*x+c-1)-1))^2+3/2*I*d*b^3/(c*f-d*e-f)/(c*f-d*e+f)*\operatorname{arccoth}(d*x+c \\ & )^2*\pi*\operatorname{csgn}(I*((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d \\ & *x+c+1)/(d*x+c-1)-1)*f)/(d*x+c+1)/(d*x+c-1)-1))^3+3/4*I*d*b^3/(c*f-d*e-f)/ \\ & (c*f-d*e+f)*\operatorname{arccoth}(d*x+c)^2*\pi*\operatorname{csgn}(I*(d*x+c+1)/(d*x+c-1))^3+3/4*I*d*b^3/( \\ & c*f-d*e-f)/(c*f-d*e+f)*\operatorname{arccoth}(d*x+c)^2*\pi*\operatorname{csgn}(I*(d*x+c+1)/(d*x+c-1)/((d*x \\ & +c+1)/(d*x+c-1)-1))^3+3*d*a*b^2/(c*f-d*e-f)/(c*f-d*e+f)*\operatorname{dilog}(((d*x+c)*f+f) \\ & /((c*f-d*e+f))-3*d*a*b^2/(c*f-d*e-f)/(c*f-d*e+f)*\operatorname{dilog}(((d*x+c)*f-f)/(c*f-d* \\ & e-f))+3/4*d*a*b^2/f/(c*f-d*e-f)*\ln(d*x+c-1)^2-3/2*d*a*b^2/f/(c*f-d*e-f)*\operatorname{dil} \\ & \operatorname{og}(1/2+1/2*d*x+1/2*c)+3/2*d*a*b^2/f/(c*f-d*e+f)*\operatorname{dilog}(1/2+1/2*d*x+1/2*c)+3/ \\ & 4*d*a*b^2/f/(c*f-d*e+f)*\ln(d*x+c+1)^2+3*d*a^2*b/f/(2*c*f-2*d*e-2*f)*\ln(d*x+ \\ & c-1)-3/2*d*b^3/f*\operatorname{arccoth}(d*x+c)^2/(c*f-d*e+f)*\ln((d*x+c-1)/(d*x+c+1))-3*d*a \\ & ^2*b/(d*f*x+d*e)/f*\operatorname{arccoth}(d*x+c)-3*d*b^3/f*\operatorname{arccoth}(d*x+c)^2/(2*c*f-2*d*e+2 \\ & *f)*\ln(d*x+c+1)+3*d*b^3/f*\operatorname{arccoth}(d*x+c)^2/(2*c*f-2*d*e-2*f)*\ln(d*x+c-1)-3* \\ & d*b^3*\operatorname{arccoth}(d*x+c)^2/(c*f-d*e-f)/(c*f-d*e+f)*\ln((d*x+c)*f-c*f+d*e)+3*d*b^ \\ & 3*\operatorname{arccoth}(d*x+c)^2/(c*f-d*e-f)/(c*f-d*e+f)*\ln(((d*x+c+1)/(d*x+c-1)-1)*c*f+( \\ & 1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)-3*d*a*b^2/(d*f*x+d*e \\ & )/f*\operatorname{arccoth}(d*x+c)^2-3*d*a^2*b/f/(2*c*f-2*d*e+2*f)*\ln(d*x+c+1)-3*d*a^2*b/(c \\ & *f-d*e-f)/(c*f-d*e+f)*\ln((d*x+c)*f-c*f+d*e)-3/2*d*b^3*f/(c*f-d*e-f)^2/(c*f- \\ & d*e+f)*\operatorname{polylog}(3, (c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))-3*d*b^3/(c*f- \\ & d*e-f)/(c*f-d*e+f)*\operatorname{arccoth}(d*x+c)^2*\ln(2)-3/2*d^2*b^3/(c*f-d*e-f)^2/(c*f-d* \\ & e+f)*e*\operatorname{polylog}(3, (c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))+3/2*I*d*b^3/( \\ & c*f-d*e-f)/(c*f-d*e+f)*\operatorname{arccoth}(d*x+c)^2*\pi*\operatorname{csgn}(I/(((d*x+c+1)/(d*x+c-1)-1)* \\ & c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x \\ & +c-1)-1))^2*\operatorname{csgn}(I/((d*x+c+1)/(d*x+c-1)-1))-3/4*I*d*b^3/(c*f-d*e-f)/(c*f-d* \\ & e+f)*\operatorname{arccoth}(d*x+c)^2*\pi*c*\operatorname{csgn}(I*(d*x+c+1)/(d*x+c-1))^3+3/2*I*d*b^3/(c*f-d \\ & *e-f)/(c*f-d*e+f)*\operatorname{arccoth}(d*x+c)^2*\pi*\operatorname{csgn}(I/(((d*x+c+1)/(d*x+c-1)-1)*c*f+( \\ & 1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f))^2*\operatorname{csgn}(I/((d*x+c+1) \\ & /((d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/(( \\ & d*x+c+1)/(d*x+c-1)-1))^2-3/2*I*d*b^3/(c*f-d*e-f)/(c*f-d*e+f)*\operatorname{arccoth}(d*x+c) \\ & ^2*\pi*\operatorname{csgn}(I/(((d*x+c-1)/(d*x+c+1))^{(1/2)})*\operatorname{csgn}(I*(d*x+c+1)/(d*x+c-1))^2+3/4 \\ & *I*d*b^3/(c*f-d*e-f)/(c*f-d*e+f)*\operatorname{arccoth}(d*x+c)^2*\pi*\operatorname{csgn}(I/((d*x+c-1)/(d*x \\ & \end{aligned}$$

$$\begin{aligned}
& +c+1)^{(1/2)}^2 * \text{csgn}(I*(d*x+c+1)/(d*x+c-1)) - 3/4 * I*d*b^3 / (c*f-d*e-f) / (c*f-d* \\
& e+f) * \text{arccoth}(d*x+c)^2 * \text{Pi} * \text{csgn}(I*(d*x+c+1)/(d*x+c-1) / ((d*x+c+1)/(d*x+c-1) - 1) \\
& )^2 * \text{csgn}(I / ((d*x+c+1)/(d*x+c-1) - 1)) - 3/4 * I*d*b^3 / (c*f-d*e-f) / (c*f-d*e+f) * \text{arc} \\
& \text{coth}(d*x+c)^2 * \text{Pi} * \text{csgn}(I*(d*x+c+1)/(d*x+c-1) / ((d*x+c+1)/(d*x+c-1) - 1))^2 * \text{csgn} \\
& (I*(d*x+c+1)/(d*x+c-1)) - 3/4 * I*d*b^3 / (c*f-d*e-f) / (c*f-d*e+f) * \text{arccoth}(d*x+c)^2 \\
& * \text{Pi} * \text{csgn}(I*(d*x+c+1)/(d*x+c-1) / ((d*x+c+1)/(d*x+c-1) - 1))^3 - 3/4 * I*d*b^3 / (c \\
& *f-d*e-f) / (c*f-d*e+f) * \text{arccoth}(d*x+c)^2 * \text{Pi} * \text{csgn}(I*(d*x+c+1)/(d*x+c-1) / ((d* \\
& x+c+1)/(d*x+c-1) - 1)) * \text{csgn}(I*(d*x+c+1)/(d*x+c-1)) * \text{csgn}(I / ((d*x+c+1)/(d*x+c-1) \\
& ) - 1)) + 3/4 * I*d^2*b^3/f / (c*f-d*e-f) / (c*f-d*e+f) * \text{arccoth}(d*x+c)^2 * \text{Pi} * \text{csgn}(I* \\
& (d*x+c+1)/(d*x+c-1) / ((d*x+c+1)/(d*x+c-1) - 1)) * \text{csgn}(I*(d*x+c+1)/(d*x+c-1)) * \text{cs} \\
& \text{sgn}(I / ((d*x+c+1)/(d*x+c-1) - 1)) + 3/2 * d*b^3 * f / (c*f-d*e-f)^2 / (c*f-d*e+f) * \text{polyl} \\
& \text{og}(3, (c*f-d*e-f) * (d*x+c+1) / (d*x+c-1) / (c*f-d*e+f)) + 6*d*a*b^2 / f * \text{arccoth}(d*x+c \\
& ) / (2*c*f-2*d*e-2*f) * \ln(d*x+c-1) - 6*d*a*b^2 / f * \text{arccoth}(d*x+c) / (2*c*f-2*d*e+2*f \\
& ) * \ln(d*x+c+1) - 6*d*a*b^2 * \text{arccoth}(d*x+c) / (c*f-d*e-f) / (c*f-d*e+f) * \ln((d*x+c)*f \\
& -c*f+d*e) + 3*d*a*b^2 / (c*f-d*e-f) / (c*f-d*e+f) * \ln(((d*x+c)*f+f) / (c*f-d*e+f)) * \ln \\
& ((d*x+c)*f-c*f+d*e) - 3*d*a*b^2 / (c*f-d*e-f) / (c*f-d*e+f) * \ln(((d*x+c)*f-f) / (c* \\
& f-d*e-f)) * \ln(((d*x+c)*f-c*f+d*e) - 3/2*d*a*b^2/f / (c*f-d*e-f) * \ln(d*x+c-1) * \ln(1/ \\
& 2+1/2*d*x+1/2*c) - 3/2*d*a*b^2/f / (c*f-d*e+f) * \ln(-1/2*d*x-1/2*c+1/2) * \ln(d*x+c+ \\
& 1) + 3/2*d*a*b^2/f / (c*f-d*e+f) * \ln(-1/2*d*x-1/2*c+1/2) * \ln(1/2+1/2*d*x+1/2*c) + 3 \\
& *d*b^3*f / (c*f-d*e-f)^2 / (c*f-d*e+f) * \text{arccoth}(d*x+c)^2 * \ln(1 - (c*f-d*e-f) * (d*x+c \\
& +1) / (d*x+c-1) / (c*f-d*e+f)) + 3*d*b^3*f / (c*f-d*e-f)^2 / (c*f-d*e+f) * \text{arccoth}(d*x+ \\
& c) * \text{polylog}(2, (c*f-d*e-f) * (d*x+c+1) / (d*x+c-1) / (c*f-d*e+f))
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))^3/(f\*x+e)^2,x, algorithm="maxima")

[Out] 
$$\begin{aligned}
& 3/2 * (d * (\log(d*x + c + 1) / (d*e*f - (c + 1)*f^2) - \log(d*x + c - 1) / (d*e*f - \\
& (c - 1)*f^2) - 2 * \log(f*x + e) / (d^2*e^2 - 2*c*d*e*f + (c^2 - 1)*f^2)) - 2 * \text{ar} \\
& \text{c} \\
& \text{coth}(d*x + c) / (f^2*x + e*f)) * a^2 * b - a^3 / (f^2*x + e*f) + 1/8 * (((d^2*e*f - \\
& c*d*f^2 + d*f^2) * b^3 * x + (c*d*e*f - c^2*f^2 + d*e*f + f^2) * b^3) * \log(d*x + c \\
& + 1)^3 - 3 * (2 * (d^2*e^2 - 2*c*d*e*f + c^2*f^2 - f^2) * a * b^2 + ((d^2*e*f - c* \\
& d*f^2 - d*f^2) * b^3 * x + (c*d*e*f - c^2*f^2 - d*e*f + f^2) * b^3) * \log(d*x + c - \\
& 1)) * \log(d*x + c + 1)^2) / (d^2*e^3*f - 2*c*d*e^2*f^2 + c^2*e*f^3 - e*f^3 + ( \\
& d^2*e^2*f^2 - 2*c*d*e*f^3 + c^2*f^4 - f^4) * x) + \text{integrate}(-1/8 * (((d^2*e*f - \\
& c*d*f^2 + d*f^2) * b^3 * x + (c*d*e*f - c^2*f^2 + d*e*f + f^2) * b^3) * \log(d*x + \\
& c - 1)^3 - 6 * ((d^2*e*f - c*d*f^2 + d*f^2) * a * b^2 * x + (c*d*e*f - c^2*f^2 + d* \\
& e*f + f^2) * a * b^2) * \log(d*x + c - 1)^2 - 3 * (4 * (d^2*e*f - c*d*f^2 + d*f^2) * a * b
\end{aligned}$$

$$\begin{aligned} &^2*x + 4*(d^2*e^2 - c*d*e*f + d*e*f)*a*b^2 + ((d^2*e*f - c*d*f^2 + d*f^2)*b \\ &^3*x + (c*d*e*f - c^2*f^2 + d*e*f + f^2)*b^3)*\log(d*x + c - 1)^2 + 2*(b^3*d \\ &^2*f^2*x^2 - 2*(c*d*e*f - c^2*f^2 + d*e*f + f^2)*a*b^2 + (c*d*e*f - d*e*f)* \\ &b^3 - (2*(d^2*e*f - c*d*f^2 + d*f^2)*a*b^2 - (d^2*e*f + c*d*f^2 - d*f^2)*b^ \\ &3)*x)*\log(d*x + c - 1))*\log(d*x + c + 1))/(c*d*e^3*f - c^2*e^2*f^2 + d*e^3*f \\ &+ e^2*f^2 + (d^2*e*f^3 - c*d*f^4 + d*f^4)*x^3 + (2*d^2*e^2*f^2 - c*d*e*f^ \\ &3 - c^2*f^4 + 3*d*e*f^3 + f^4)*x^2 + (d^2*e^3*f + c*d*e^2*f^2 - 2*c^2*e*f^3 \\ &+ 3*d*e^2*f^2 + 2*e*f^3)*x), x) \end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \operatorname{arccoth}(dx+c)^3 + 3ab^2 \operatorname{arccoth}(dx+c)^2 + 3a^2b \operatorname{arccoth}(dx+c) + a^3}{f^2x^2 + 2efx + e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))^3/(f\*x+e)^2,x, algorithm="fricas")

[Out] integral((b^3\*arccoth(d\*x + c)^3 + 3\*a\*b^2\*arccoth(d\*x + c)^2 + 3\*a^2\*b\*arccoth(d\*x + c) + a^3)/(f^2\*x^2 + 2\*e\*f\*x + e^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth(d\*x+c))\*\*3/(f\*x+e)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccoth}(dx+c) + a)^3}{(fx+e)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((a+b*arccoth(d*x+c))^3/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(d*x + c) + a)^3/(f*x + e)^2, x)
```

### 3.119 $\int (e + fx)^m (a + b \coth^{-1}(c + dx)) dx$

**Optimal.** Leaf size=162

$$\frac{bd(e + fx)^{m+2} \text{Hypergeometric2F1}\left(1, m + 2, m + 3, \frac{d(e+fx)}{-cf+de-f}\right)}{2f(m+1)(m+2)(de - (c+1)f)} - \frac{bd(e + fx)^{m+2} \text{Hypergeometric2F1}\left(1, m + 2, m + 3, \frac{d(e+fx)}{de-cf-f}\right)}{2f(m+1)(m+2)(-cf + de + f)}$$

```
[Out] ((e + f*x)^(1 + m)*(a + b*ArcCoth[c + d*x]))/(f*(1 + m)) + (b*d*(e + f*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e - f - c*f)]/(2*f*(d*e - (1 + c)*f)*(1 + m)*(2 + m)) - (b*d*(e + f*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e + f - c*f)]/(2*f*(d*e + f - c*f)*(1 + m)*(2 + m))
```

**Rubi [A]** time = 0.246004, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {6112, 5927, 712, 68}

$$\frac{(e + fx)^{m+1} (a + b \coth^{-1}(c + dx))}{f(m+1)} + \frac{bd(e + fx)^{m+2} {}_2F_1\left(1, m + 2; m + 3; \frac{d(e+fx)}{de-cf-f}\right)}{2f(m+1)(m+2)(de - (c+1)f)} - \frac{bd(e + fx)^{m+2} {}_2F_1\left(1, m + 2; m + 3; \frac{d(e+fx)}{de-cf-f}\right)}{2f(m+1)(m+2)(-cf + de + f)}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^m*(a + b*ArcCoth[c + d*x]),x]
```

```
[Out] ((e + f*x)^(1 + m)*(a + b*ArcCoth[c + d*x]))/(f*(1 + m)) + (b*d*(e + f*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e - f - c*f)]/(2*f*(d*e - (1 + c)*f)*(1 + m)*(2 + m)) - (b*d*(e + f*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e + f - c*f)]/(2*f*(d*e + f - c*f)*(1 + m)*(2 + m))
```

#### Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
```

#### Rule 5927

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(q_.), x_Symbol] := Simp[((d + e*x)^(q + 1)*(a + b*ArcCoth[c*x]))/(e*(q + 1)), x] - Dist[
```

$b*c)/(e*(q + 1)), \text{Int}[(d + e*x)^{(q + 1)}/(1 - c^2*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e, q\}, x\} \&\& \text{NeQ}[q, -1]$

### Rule 712

$\text{Int}[(d + e*x)^m/(a + c*x^2), x\_Symbol] :> \text{Int}[\text{Expand} \text{Integrand}[(d + e*x)^m, 1/(a + c*x^2), x], x] /; \text{FreeQ}\{a, c, d, e, m\}, x\} \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& !\text{IntegerQ}[m]$

### Rule 68

$\text{Int}[(a + b*x)^m*(c + d*x)^n, x\_Symbol] :> \text{Simp}[(b*c - a*d)^n*(a + b*x)^{m+1}*\text{Hypergeometric2F1}[-n, m+1, m+2, -(d*(a + b*x)/(b*c - a*d))]/(b^{n+1}*(m+1)), x] /; \text{FreeQ}\{a, b, c, d, m\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& !\text{IntegerQ}[m] \&\& \text{IntegerQ}[n]$

### Rubi steps

$$\begin{aligned} \int (e + fx)^m (a + b \coth^{-1}(c + dx)) dx &= \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d} + \frac{fx}{d}\right)^m (a + b \coth^{-1}(x)) dx, x, c + dx\right)}{d} \\ &= \frac{(e + fx)^{1+m} (a + b \coth^{-1}(c + dx))}{f(1+m)} - \frac{b \text{Subst}\left(\int \frac{\left(\frac{de-cf}{d} + \frac{fx}{d}\right)^{1+m}}{1-x^2} dx, x, c + dx\right)}{f(1+m)} \\ &= \frac{(e + fx)^{1+m} (a + b \coth^{-1}(c + dx))}{f(1+m)} - \frac{b \text{Subst}\left(\int \left(\frac{\left(\frac{de-cf}{d} + \frac{fx}{d}\right)^{1+m}}{2(1-x)} + \frac{\left(\frac{de-cf}{d} + \frac{fx}{d}\right)^{1+m}}{2(1+x)}\right) dx, x, c + dx\right)}{f(1+m)} \\ &= \frac{(e + fx)^{1+m} (a + b \coth^{-1}(c + dx))}{f(1+m)} - \frac{b \text{Subst}\left(\int \frac{\left(\frac{de-cf}{d} + \frac{fx}{d}\right)^{1+m}}{1-x} dx, x, c + dx\right)}{2f(1+m)} \\ &= \frac{(e + fx)^{1+m} (a + b \coth^{-1}(c + dx))}{f(1+m)} + \frac{bd(e + fx)^{2+m} {}_2F_1\left(1, 2+m; 3+m; \frac{d(e+fx)}{de-f}\right)}{2f(de - (1+c)f)(1+m)(2+m)} \end{aligned}$$

**Mathematica [F]** time = 2.35383, size = 0, normalized size = 0.

$$\int (e + fx)^m (a + b \coth^{-1}(c + dx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[(e + f\*x)^m\*(a + b\*ArcCoth[c + d\*x]),x]

[Out] Integrate[(e + f\*x)^m\*(a + b\*ArcCoth[c + d\*x]), x]

**Maple [F]** time = 1.493, size = 0, normalized size = 0.

$$\int (fx + e)^m (a + b \operatorname{arccoth}(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^m\*(a+b\*arccoth(d\*x+c)),x)

[Out] int((f\*x+e)^m\*(a+b\*arccoth(d\*x+c)),x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^m\*(a+b\*arccoth(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left((b \operatorname{arccoth}(dx + c) + a)(fx + e)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^m\*(a+b\*arccoth(d\*x+c)),x, algorithm="fricas")

[Out] integral((b\*arccoth(d\*x + c) + a)\*(f\*x + e)^m, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*m\*(a+b\*acoth(d\*x+c)),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arccoth}(dx + c) + a)(fx + e)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^m\*(a+b\*arccoth(d\*x+c)),x, algorithm="giac")

[Out] integrate((b\*arccoth(d\*x + c) + a)\*(f\*x + e)^m, x)

$$3.120 \quad \int (e + fx)^m (a + b \coth^{-1}(c + dx))^2 dx$$

**Optimal.** Leaf size=22

$$\text{Unintegrable}\left((e + fx)^m (a + b \coth^{-1}(c + dx))^2, x\right)$$

[Out] Unintegrable[(e + f\*x)^m\*(a + b\*ArcCoth[c + d\*x])^2, x]

**Rubi [A]** time = 0.0665056, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (e + fx)^m (a + b \coth^{-1}(c + dx))^2 dx$$

Verification is Not applicable to the result.

[In] Int[(e + f\*x)^m\*(a + b\*ArcCoth[c + d\*x])^2,x]

[Out] Defer[Subst][Defer[Int][((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCoth[x])^2, x], x, c + d\*x]/d

Rubi steps

$$\int (e + fx)^m (a + b \coth^{-1}(c + dx))^2 dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d} + \frac{fx}{d}\right)^m (a + b \coth^{-1}(x))^2 dx, x, c + dx\right)}{d}$$

**Mathematica [A]** time = 2.51712, size = 0, normalized size = 0.

$$\int (e + fx)^m (a + b \coth^{-1}(c + dx))^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[(e + f\*x)^m\*(a + b\*ArcCoth[c + d\*x])^2,x]

[Out] Integrate[(e + f\*x)^m\*(a + b\*ArcCoth[c + d\*x])^2, x]

---

**Maple [A]** time = 1.368, size = 0, normalized size = 0.

$$\int (fx + e)^m (a + b \operatorname{arccoth}(dx + c))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)^m*(a+b*arccoth(d*x+c))^2,x)`

[Out] `int((f*x+e)^m*(a+b*arccoth(d*x+c))^2,x)`

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^m*(a+b*arccoth(d*x+c))^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b^2 \operatorname{arccoth}(dx + c)^2 + 2ab \operatorname{arccoth}(dx + c) + a^2\right)(fx + e)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^m*(a+b*arccoth(d*x+c))^2,x, algorithm="fricas")`

[Out] `integral((b^2*arccoth(d*x + c)^2 + 2*a*b*arccoth(d*x + c) + a^2)*(f*x + e)^m, x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*m\*(a+b\*acoth(d\*x+c))\*\*2,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arccoth}(dx + c) + a)^2 (fx + e)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^m\*(a+b\*arccoth(d\*x+c))^2,x, algorithm="giac")

[Out] integrate((b\*arccoth(d\*x + c) + a)^2\*(f\*x + e)^m, x)



$$3.121 \quad \int (e + fx)^m \left( a + b \coth^{-1}(c + dx) \right)^3 dx$$

**Optimal.** Leaf size=22

$$\text{Unintegrable} \left( (e + fx)^m \left( a + b \coth^{-1}(c + dx) \right)^3, x \right)$$

[Out] Unintegrable[(e + f\*x)^m\*(a + b\*ArcCoth[c + d\*x])^3, x]

**Rubi [A]** time = 0.0654614, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (e + fx)^m \left( a + b \coth^{-1}(c + dx) \right)^3 dx$$

Verification is Not applicable to the result.

[In] Int[(e + f\*x)^m\*(a + b\*ArcCoth[c + d\*x])^3, x]

[Out] Defer[Subst][Defer[Int][((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCoth[x])^3, x], x, c + d\*x]/d

Rubi steps

$$\int (e + fx)^m \left( a + b \coth^{-1}(c + dx) \right)^3 dx = \frac{\text{Subst} \left( \int \left( \frac{de - cf}{d} + \frac{fx}{d} \right)^m \left( a + b \coth^{-1}(x) \right)^3 dx, x, c + dx \right)}{d}$$

**Mathematica [A]** time = 0.342944, size = 0, normalized size = 0.

$$\int (e + fx)^m \left( a + b \coth^{-1}(c + dx) \right)^3 dx$$

Verification is Not applicable to the result.

[In] Integrate[(e + f\*x)^m\*(a + b\*ArcCoth[c + d\*x])^3, x]

[Out] Integrate[(e + f\*x)^m\*(a + b\*ArcCoth[c + d\*x])^3, x]

---

**Maple [A]** time = 1.286, size = 0, normalized size = 0.

$$\int (fx + e)^m (a + b \operatorname{arccoth}(dx + c))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)^m*(a+b*arccoth(d*x+c))^3,x)`

[Out] `int((f*x+e)^m*(a+b*arccoth(d*x+c))^3,x)`

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^m*(a+b*arccoth(d*x+c))^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b^3 \operatorname{arccoth}(dx + c)^3 + 3ab^2 \operatorname{arccoth}(dx + c)^2 + 3a^2b \operatorname{arccoth}(dx + c) + a^3\right)(fx + e)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^m*(a+b*arccoth(d*x+c))^3,x, algorithm="fricas")`

[Out] `integral((b^3*arccoth(d*x + c)^3 + 3*a*b^2*arccoth(d*x + c)^2 + 3*a^2*b*arccoth(d*x + c) + a^3)*(f*x + e)^m, x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)**m*(a+b*acoth(d*x+c))**3,x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arccoth}(dx + c) + a)^3 (fx + e)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^m*(a+b*arccoth(d*x+c))^3,x, algorithm="giac")`

[Out] `integrate((b*arccoth(d*x + c) + a)^3*(f*x + e)^m, x)`

$$3.122 \quad \int \frac{\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2} dx$$

**Optimal.** Leaf size=42

$$\text{Unintegrable}\left(\frac{\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}, x\right)$$

[Out] Unintegrable[(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^n/(1 - c^2\*x^2), x]

**Rubi [A]** time = 0.047806, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^n/(1 - c^2\*x^2), x]

[Out] Defer[Int] [(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^n/(1 - c^2\*x^2), x]

Rubi steps

$$\int \frac{\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2} dx = \int \frac{\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2} dx$$

**Mathematica [A]** time = 0.0983633, size = 0, normalized size = 0.

$$\int \frac{\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^n/(1 - c^2\*x^2), x]

[Out] Integrate[(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^n/(1 - c^2\*x^2), x]

**Maple [A]** time = 1.043, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2 + 1} \left( a + b \operatorname{arccoth} \left( \sqrt{-cx + 1} \frac{1}{\sqrt{cx + 1}} \right) \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^n/(-c^2\*x^2+1), x)

[Out] int((a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^n/(-c^2\*x^2+1), x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$- \int \frac{\left( b \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right) + a \right)^n}{c^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^n/(-c^2\*x^2+1), x, algorithm="maxima")

[Out] -integrate((b\*arccoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) + a)^n/(c^2\*x^2 - 1), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( - \frac{\left( b \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right) + a \right)^n}{c^2x^2 - 1}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="fricas")
```

```
[Out] integral(-(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a + b \operatorname{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^n}{c^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
```

```
[Out] -Integral((a + b*acoth(sqrt(-c*x + 1)/sqrt(c*x + 1)))**n/(c**2*x**2 - 1), x)
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(b \operatorname{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)^n}{c^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] integrate(-(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

$$3.123 \quad \int \frac{\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2} dx$$

**Optimal.** Leaf size=460

$$\frac{3b^2 \text{PolyLog}\left(3, 1 - \frac{2}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}} + 1}\right)\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c} + \frac{3b^2 \text{PolyLog}\left(3, 1 - \frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}} + 1\right)}\right)\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}$$

```
[Out] (-2*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*ArcCoth[1 - 2/(1 - Sqrt[1 - c*x]/Sqrt[1 + c*x])])/c - (3*b*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2, 1 - 2/(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x])])/(2*c) + (3*b*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2, 1 - (2*Sqrt[1 - c*x])/(Sqrt[1 + c*x]*(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x])])]/(2*c) - (3*b^2*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, 1 - 2/(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x])])/(2*c) + (3*b^2*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, 1 - (2*Sqrt[1 - c*x])/(Sqrt[1 + c*x]*(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x])])]/(2*c) - (3*b^3*PolyLog[4, 1 - 2/(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x])])/(4*c) + (3*b^3*PolyLog[4, 1 - (2*Sqrt[1 - c*x])/(Sqrt[1 + c*x]*(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x])])]/(4*c)
```

**Rubi [A]** time = 0.598234, antiderivative size = 460, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$ , Rules used = {6681, 5915, 6053, 5949, 6057, 6061, 6610}

$$\frac{3b^2 \text{PolyLog}\left(3, 1 - \frac{2}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}} + 1}\right)\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c} + \frac{3b^2 \text{PolyLog}\left(3, 1 - \frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}} + 1\right)}\right)\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2), x]
```

```
[Out] (-2*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*ArcCoth[1 - 2/(1 - Sqrt[1 - c*x]/Sqrt[1 + c*x])])/c - (3*b*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2, 1 - 2/(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x])])/(2*c) + (3*b*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2, 1 - (2*Sqrt[1 - c*x])/(Sqrt[1 + c*x]*(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x])])]/(2*c) - (3*b^2*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, 1 - 2/(1 + Sqrt[1 - c*x]
```

$$\frac{1}{\sqrt{1 + cx}} \Big/ (2c) + (3b^2(a + b \operatorname{ArcCoth}[\sqrt{1 - cx}/\sqrt{1 + cx}]) \operatorname{PolyLog}[3, 1 - (2\sqrt{1 - cx})/(\sqrt{1 + cx}(1 + \sqrt{1 - cx})/\sqrt{1 + cx})]) \Big/ (2c) - (3b^3 \operatorname{PolyLog}[4, 1 - 2/(1 + \sqrt{1 - cx})/\sqrt{1 + cx}]) \Big/ (4c) + (3b^3 \operatorname{PolyLog}[4, 1 - (2\sqrt{1 - cx})/(\sqrt{1 + cx}(1 + \sqrt{1 - cx})/\sqrt{1 + cx})]) \Big/ (4c)$$

### Rule 6681

$$\operatorname{Int}[(a + b(F)[((c)\sqrt{(d)} + (e)(x))/\sqrt{(f)} + (g)(x))])^{(n)}/((A) + (C)(x)^2), x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[(2eg)/(C(ef - dg)), \operatorname{Subst}[\operatorname{Int}[(a + bF[cx])^n/x, x], x, \sqrt{d + ex}/\sqrt{f + gx}], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, g, A, C, F\}, x] \&\& \operatorname{EqQ}[Cdf - Aeg, 0] \&\& \operatorname{EqQ}[ef + dg, 0] \&\& \operatorname{IGtQ}[n, 0]$$

### Rule 5915

$$\operatorname{Int}[(a + \operatorname{ArcCoth}[c(x)](b))^{(p)}/(x), x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[2(a + b \operatorname{ArcCoth}[cx])^p \operatorname{ArcCoth}[1 - 2/(1 - cx)], x] - \operatorname{Dist}[2bcp, \operatorname{Int}[(a + b \operatorname{ArcCoth}[cx])^{(p-1)} \operatorname{ArcCoth}[1 - 2/(1 - cx)]/(1 - c^2x^2), x], x] /; \operatorname{FreeQ}\{a, b, c\}, x] \&\& \operatorname{IGtQ}[p, 1]$$

### Rule 6053

$$\operatorname{Int}[(\operatorname{ArcCoth}[u](a + \operatorname{ArcCoth}[c(x)](b))^{(p)}/((d) + (e)(x)^2), x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[1/2, \operatorname{Int}[(\operatorname{Log}[\operatorname{SimplifyIntegrand}[1 + 1/u, x]])(a + b \operatorname{ArcCoth}[cx])^p/(d + ex^2), x], x] - \operatorname{Dist}[1/2, \operatorname{Int}[(\operatorname{Log}[\operatorname{SimplifyIntegrand}[1 - 1/u, x]])(a + b \operatorname{ArcCoth}[cx])^p/(d + ex^2), x], x] /; \operatorname{FreeQ}\{a, b, c, d, e\}, x] \&\& \operatorname{IGtQ}[p, 0] \&\& \operatorname{EqQ}[c^2d + e, 0] \&\& \operatorname{EqQ}[u^2 - (1 - 2/(1 - cx))^2, 0]$$

### Rule 5949

$$\operatorname{Int}[(a + \operatorname{ArcCoth}[c(x)](b))^{(p)}/((d) + (e)(x)^2), x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(a + b \operatorname{ArcCoth}[cx])^{(p+1)}/(bcd(p+1)), x] /; \operatorname{FreeQ}\{a, b, c, d, e, p\}, x] \&\& \operatorname{EqQ}[c^2d + e, 0] \&\& \operatorname{NeQ}[p, -1]$$

### Rule 6057

$$\operatorname{Int}[(\operatorname{Log}[u](a + \operatorname{ArcCoth}[c(x)](b))^{(p)}/((d) + (e)(x)^2), x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(a + b \operatorname{ArcCoth}[cx])^p \operatorname{PolyLog}[2, 1 - u]/(2cd), x] - \operatorname{Dist}[(bp)/2, \operatorname{Int}[(a + b \operatorname{ArcCoth}[cx])^{(p-1)} \operatorname{PolyLog}[2, 1 - u]/(d + ex^2), x], x] /; \operatorname{FreeQ}\{a, b, c, d, e\}, x] \&\& \operatorname{IGtQ}[p, 0] \&\& \operatorname{EqQ}[c^2d + e, 0] \&\& \operatorname{EqQ}[(1 - u)^2 - (1 - 2/(1 + cx))^2, 0]$$



Rule 6061

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^p)\*PolyLog[k\_, u\_]]/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := -Simp[((a + b\*ArcCoth[c\*x])^p\*PolyLog[k + 1, u])/((2\*c\*d), x] + Dist[(b\*p)/2, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*PolyLog[k + 1, u])/((d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && EqQ[c^2\*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c\*x))^2, 0]

Rule 6610

Int[(u\_)\*PolyLog[n\_, v\_], x\_Symbol] := With[{w = DerivativeDivides[v, u\*v, x]}, Simp[w\*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

Rubi steps

$$\begin{aligned}
 \int \frac{\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1 - c^2x^2} dx &= \frac{\text{Subst}\left(\int \frac{(a+b \coth^{-1}(x))^3}{x} dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} \\
 &= -\frac{2\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \coth^{-1}\left(1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(6b) \text{Subst}\left(\int \frac{\coth^{-1}\left(1 - \frac{2}{1-x}\right)(a+b \coth^{-1}(x))^3}{1-x^2} dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} \\
 &= -\frac{2\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \coth^{-1}\left(1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{(3b) \text{Subst}\left(\int \frac{(a+b \coth^{-1}(x))^2 \log\left(\frac{2}{1+x}\right)}{1-x^2} dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} \\
 &= -\frac{2\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \coth^{-1}\left(1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{3b\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \text{Li}_2\left(1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} \\
 &= -\frac{2\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \coth^{-1}\left(1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{3b\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \text{Li}_2\left(1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} \\
 &= -\frac{2\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \coth^{-1}\left(1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{3b\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \text{Li}_2\left(1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c}
 \end{aligned}$$

**Mathematica [F]** time = 0.283333, size = 0, normalized size = 0.

$$\int \frac{\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1 - c^2x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^3/(1 - c^2\*x^2), x]

[Out] Integrate[(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^3/(1 - c^2\*x^2), x]

**Maple [B]** time = 2.439, size = 1492, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^3/(-c^2\*x^2+1), x)

[Out]  $-3*a*b^2/c*\operatorname{arccoth}\left(\frac{-c*x+1}{c*x+1}\right)^2*\ln\left(\frac{1}{\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2}-1}\right)*\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2}+1\right)-3*a*b^2/c*\operatorname{arccoth}\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2})*\operatorname{polylog}\left(2,-1/\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2}-1\right)*\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2}+1\right)+3*a*b^2/c*\operatorname{arccoth}\left(\frac{-c*x+1}{c*x+1}\right)^2*\ln\left(1-1/\left(\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2}-1\right)/\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2}+1\right)\right)^{1/2}+6*a*b^2/c*\operatorname{arccoth}\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2})*\operatorname{polylog}\left(2,1/\left(\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2}-1\right)/\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2}+1\right)\right)^{1/2}+3*a*b^2/c*\operatorname{arccoth}\left(\frac{-c*x+1}{c*x+1}\right)^2*\ln\left(1+1/\left(\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2}-1\right)/\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2}+1\right)\right)^{1/2}+6*a*b^2/c*\operatorname{arccoth}\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2})*\operatorname{polylog}\left(2,-1/\left(\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2}-1\right)/\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2}+1\right)\right)^{1/2}-1/2*a^3/c*\ln(c*x-1)+1/2*a^3/c*\ln(c*x+1)+6*b^3/c*\operatorname{polylog}\left(4,-1/\left(\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2}-1\right)/\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2}+1\right)\right)^{1/2}+6*b^3/c*\operatorname{polylog}\left(4,1/\left(\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2}-1\right)/\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2}+1\right)\right)^{1/2}-3/4*b^3/c*\operatorname{polylog}\left(4,-1/\left(\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2}-1\right)*\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2}+1\right)\right)+b^3/c*\operatorname{arccoth}\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2}\right)^3*\ln\left(1+1/\left(\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2}-1\right)/\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2}+1\right)\right)^{1/2}+3*b^3/c*\operatorname{arccoth}\left(\frac{-c*x+1}{c*x+1}\right)^2*\operatorname{polylog}\left(2,-1/\left(\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2}-1\right)/\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2}+1\right)\right)^{1/2}-6*b^3/c*\operatorname{arccoth}\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2})*\operatorname{polylog}\left(3,-1/\left(\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2}-1\right)/\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2}+1\right)\right)^{1/2}+b^3/c*\operatorname{arccoth}\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2}\right)^3*\ln\left(1-1/\left(\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2}-1\right)/\left(\frac{-c*x+1}{c*x+1}\right)^{1/2}/\left(\frac{c*x+1}{c*x+1}\right)^{1/2}+1\right)\right)^{1/2}+3*b^3/c*\operatorname{arccoth}$

$$\begin{aligned} & ((-c*x+1)^{(1/2)}/(c*x+1)^{(1/2)})^2*\text{polylog}(2,1/(((c*x+1)^{(1/2)}/(c*x+1)^{(1/2)} \\ & -1)/((-c*x+1)^{(1/2)}/(c*x+1)^{(1/2)+1}))^{(1/2)})-6*b^3/c*\text{arccoth}((-c*x+1)^{(1/2)} \\ & /((c*x+1)^{(1/2)}))*\text{polylog}(3,1/(((c*x+1)^{(1/2)}/(c*x+1)^{(1/2)-1)/((-c*x+1)^{(1/2)} \\ & /((c*x+1)^{(1/2)+1}))^{(1/2)})-b^3/c*\text{arccoth}((-c*x+1)^{(1/2)}/(c*x+1)^{(1/2)})^3*1 \\ & n(1/((-c*x+1)^{(1/2)}/(c*x+1)^{(1/2)-1})*((c*x+1)^{(1/2)}/(c*x+1)^{(1/2)+1})+1)-3/ \\ & 2*b^3/c*\text{arccoth}((-c*x+1)^{(1/2)}/(c*x+1)^{(1/2)})^2*\text{polylog}(2,-1/((-c*x+1)^{(1/2)} \\ & )/(c*x+1)^{(1/2)-1})*((c*x+1)^{(1/2)}/(c*x+1)^{(1/2)+1}))+3/2*b^3/c*\text{arccoth}((-c* \\ & x+1)^{(1/2)}/(c*x+1)^{(1/2)})*\text{polylog}(3,-1/((-c*x+1)^{(1/2)}/(c*x+1)^{(1/2)-1})*((c \\ & -c*x+1)^{(1/2)}/(c*x+1)^{(1/2)+1}))-6*a*b^2/c*\text{polylog}(3,-1/(((c*x+1)^{(1/2)}/(c*x \\ & +1)^{(1/2)-1)/((-c*x+1)^{(1/2)}/(c*x+1)^{(1/2)+1}))^{(1/2)})+3/2*a*b^2/c*\text{polylog}(3 \\ & , -1/((-c*x+1)^{(1/2)}/(c*x+1)^{(1/2)-1})*((c*x+1)^{(1/2)}/(c*x+1)^{(1/2)+1}))-6*a* \\ & b^2/c*\text{polylog}(3,1/(((c*x+1)^{(1/2)}/(c*x+1)^{(1/2)-1)/((-c*x+1)^{(1/2)}/(c*x+1) \\ & ^{(1/2)+1}))^{(1/2)})+3*a^2*b/c*\text{dilog}(((c*x+1)^{(1/2)}/(c*x+1)^{(1/2)-1)/((-c*x+1) \\ & )^{(1/2)}/(c*x+1)^{(1/2)+1}))-3/4*a^2*b/c*\text{dilog}(((c*x+1)^{(1/2)}/(c*x+1)^{(1/2)-1} \\ & )^2/((-c*x+1)^{(1/2)}/(c*x+1)^{(1/2)+1})^2) \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} a^3 \left( \frac{\log(cx+1)}{c} - \frac{\log(cx-1)}{c} \right) - \frac{(b^3 \log(cx+1) - b^3 \log(-cx+1)) \log(-\sqrt{cx+1} + \sqrt{-cx+1})^3}{16c} - \int \frac{4(\sqrt{cx+1}b^3}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^3/(-c^2\*x^2+1),x, algorithm="maxima")

[Out] 1/2\*a^3\*(log(c\*x + 1)/c - log(c\*x - 1)/c) - 1/16\*(b^3\*log(c\*x + 1) - b^3\*log(-c\*x + 1))\*log(-sqrt(c\*x + 1) + sqrt(-c\*x + 1))^3/c - integrate(1/32\*(4\*(sqrt(c\*x + 1)\*b^3 - sqrt(-c\*x + 1)\*b^3)\*log(sqrt(c\*x + 1) + sqrt(-c\*x + 1))^3 + 24\*(sqrt(c\*x + 1)\*a\*b^2 - sqrt(-c\*x + 1)\*a\*b^2)\*log(sqrt(c\*x + 1) + sqrt(-c\*x + 1))^2 + 3\*(4\*(sqrt(c\*x + 1)\*b^3 - sqrt(-c\*x + 1)\*b^3)\*log(sqrt(c\*x + 1) + sqrt(-c\*x + 1)) + (8\*a\*b^2 - (b^3\*c\*x - b^3)\*log(c\*x + 1) + (b^3\*c\*x - b^3)\*log(-c\*x + 1))\*sqrt(c\*x + 1) - (8\*a\*b^2 - (b^3\*c\*x + b^3)\*log(c\*x + 1) + (b^3\*c\*x + b^3)\*log(-c\*x + 1))\*sqrt(-c\*x + 1))\*log(-sqrt(c\*x + 1) + sqrt(-c\*x + 1))^2 + 48\*(sqrt(c\*x + 1)\*a^2\*b - sqrt(-c\*x + 1)\*a^2\*b)\*log(sqrt(c\*x + 1) + sqrt(-c\*x + 1)) - 12\*(4\*sqrt(c\*x + 1)\*a^2\*b - 4\*sqrt(-c\*x + 1)\*a^2\*b + (sqrt(c\*x + 1)\*b^3 - sqrt(-c\*x + 1)\*b^3)\*log(sqrt(c\*x + 1) + sqrt(-c\*x + 1))^2 + 4\*(sqrt(c\*x + 1)\*a\*b^2 - sqrt(-c\*x + 1)\*a\*b^2)\*log(sqrt(c\*x + 1) + sqrt(-c\*x + 1))\*log(-sqrt(c\*x + 1) + sqrt(-c\*x + 1)))/((c^2\*x^2 - 1)\*sqrt(c\*x + 1) - (c^2\*x^2 - 1)\*sqrt(-c\*x + 1)), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( -\frac{b^3 \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right)^3 + 3ab^2 \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right)^2 + 3a^2b \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right) + a^3}{c^2x^2 - 1}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^3/(-c^2\*x^2+1),x, algorithm="fricas")

[Out] integral(-(b^3\*arccoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1))^3 + 3\*a\*b^2\*arccoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1))^2 + 3\*a^2\*b\*arccoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) + a^3)/(c^2\*x^2 - 1), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{a^3}{c^2x^2 - 1} dx - \int \frac{b^3 \operatorname{acoth}^3 \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right)}{c^2x^2 - 1} dx - \int \frac{3ab^2 \operatorname{acoth}^2 \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right)}{c^2x^2 - 1} dx - \int \frac{3a^2b \operatorname{acoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right)}{c^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth((-c\*x+1)\*\*(1/2)/(c\*x+1)\*\*(1/2)))\*\*3/(-c\*\*2\*x\*\*2+1),x)

[Out] -Integral(a\*\*3/(c\*\*2\*x\*\*2 - 1), x) - Integral(b\*\*3\*acoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1))\*\*3/(c\*\*2\*x\*\*2 - 1), x) - Integral(3\*a\*b\*\*2\*acoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1))\*\*2/(c\*\*2\*x\*\*2 - 1), x) - Integral(3\*a\*\*2\*b\*acoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1))/(c\*\*2\*x\*\*2 - 1), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(b \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right) + a\right)^3}{c^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^3/(c^2*x^2 - 1), x)
```

$$3.124 \quad \int \frac{\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2} dx$$

**Optimal.** Leaf size=302

$$\frac{b \operatorname{PolyLog}\left(2, 1 - \frac{2}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}} + 1}\right)\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c} + \frac{b \operatorname{PolyLog}\left(2, 1 - \frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}} + 1\right)}\right)\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c} - \frac{b^2 \operatorname{PolyLog}\left(3, 1 - \frac{2}{1 + \sqrt{1-cx}/\sqrt{1+cx}}\right)}{2c} + \frac{b^2 \operatorname{PolyLog}\left(3, 1 - \frac{2\sqrt{1-cx}}{\sqrt{1+cx}\left(1 + \sqrt{1-cx}/\sqrt{1+cx}\right)}\right)}{2c}$$

[Out]  $(-2*(a + b*\operatorname{ArcCoth}[\operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[1 + c*x]])^2*\operatorname{ArcCoth}[1 - 2/(1 - \operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[1 + c*x])])/c - (b*(a + b*\operatorname{ArcCoth}[\operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[1 + c*x]])*\operatorname{PolyLog}[2, 1 - 2/(1 + \operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[1 + c*x])])/c + (b*(a + b*\operatorname{ArcCoth}[\operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[1 + c*x]])*\operatorname{PolyLog}[2, 1 - (2*\operatorname{Sqrt}[1 - c*x])/(\operatorname{Sqrt}[1 + c*x]*(1 + \operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[1 + c*x])])/c - (b^2*\operatorname{PolyLog}[3, 1 - 2/(1 + \operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[1 + c*x])])/(2*c) + (b^2*\operatorname{PolyLog}[3, 1 - (2*\operatorname{Sqrt}[1 - c*x])/(\operatorname{Sqrt}[1 + c*x]*(1 + \operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[1 + c*x])])/(2*c)$

**Rubi [A]** time = 0.346365, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$ , Rules used = {6681, 5915, 6053, 5949, 6057, 6610}

$$\frac{b \operatorname{PolyLog}\left(2, 1 - \frac{2}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}} + 1}\right)\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c} + \frac{b \operatorname{PolyLog}\left(2, 1 - \frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}} + 1\right)}\right)\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c} - \frac{b^2 \operatorname{PolyLog}\left(3, 1 - \frac{2}{1 + \sqrt{1-cx}/\sqrt{1+cx}}\right)}{2c} + \frac{b^2 \operatorname{PolyLog}\left(3, 1 - \frac{2\sqrt{1-cx}}{\sqrt{1+cx}\left(1 + \sqrt{1-cx}/\sqrt{1+cx}\right)}\right)}{2c}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{ArcCoth}[\operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[1 + c*x]])^2/(1 - c^2*x^2), x]$

[Out]  $(-2*(a + b*\operatorname{ArcCoth}[\operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[1 + c*x]])^2*\operatorname{ArcCoth}[1 - 2/(1 - \operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[1 + c*x])])/c - (b*(a + b*\operatorname{ArcCoth}[\operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[1 + c*x]])*\operatorname{PolyLog}[2, 1 - 2/(1 + \operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[1 + c*x])])/c + (b*(a + b*\operatorname{ArcCoth}[\operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[1 + c*x]])*\operatorname{PolyLog}[2, 1 - (2*\operatorname{Sqrt}[1 - c*x])/(\operatorname{Sqrt}[1 + c*x]*(1 + \operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[1 + c*x])])/c - (b^2*\operatorname{PolyLog}[3, 1 - 2/(1 + \operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[1 + c*x])])/(2*c) + (b^2*\operatorname{PolyLog}[3, 1 - (2*\operatorname{Sqrt}[1 - c*x])/(\operatorname{Sqrt}[1 + c*x]*(1 + \operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[1 + c*x])])/(2*c)$

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] := Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

### Rule 5915

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.)^(p_)/(x_), x_Symbol] := Simp[2*(a +
b*ArcCoth[c*x])^p*ArcCoth[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcCoth[c*x])^(p - 1)*ArcCoth[1 - 2/(1 - c*x)]/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

### Rule 6053

```
Int[(ArcCoth[u_] * ((a_.) + ArcCoth[(c_.)*(x_)]) * (b_.)^(p_.)) / ((d_.) + (e_.)*(
x_)^2), x_Symbol] := Dist[1/2, Int[(Log[SimplifyIntegrand[1 + 1/u, x]] * (a +
b*ArcCoth[c*x])^p) / (d + e*x^2), x], x] - Dist[1/2, Int[(Log[SimplifyIntegr
and[1 - 1/u, x]] * (a + b*ArcCoth[c*x])^p) / (d + e*x^2), x], x] /; FreeQ[{a, b
, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 -
c*x))^2, 0]
```

### Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.)^(p_.) / ((d_.) + (e_.)*(x_)^2), x_Symb
ol] := Simp[(a + b*ArcCoth[c*x])^(p + 1) / (b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

### Rule 6057

```
Int[(Log[u_] * ((a_.) + ArcCoth[(c_.)*(x_)]) * (b_.)^(p_.)) / ((d_.) + (e_.)*(x_)^
2), x_Symbol] := Simp[((a + b*ArcCoth[c*x])^p * PolyLog[2, 1 - u]) / (2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1) * PolyLog[2, 1 - u]) / (d +
e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

### Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2} dx &= \frac{\text{Subst}\left(\int \frac{(a+b \coth^{-1}(x))^2}{x} dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} \\
&= -\frac{2\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \coth^{-1}\left(1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(4b) \text{Subst}\left(\int \frac{\coth^{-1}\left(1 - \frac{2}{1-x}\right)(a+b \coth^{-1}(x))^2}{1-x^2} dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} \\
&= -\frac{2\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \coth^{-1}\left(1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{(2b) \text{Subst}\left(\int \frac{(a+b \coth^{-1}(x)) \log\left(\frac{2}{1+x}\right)}{1-x^2} dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} \\
&= -\frac{2\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \coth^{-1}\left(1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{b\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right) \text{Li}_2\left(1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} \\
&= -\frac{2\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \coth^{-1}\left(1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{b\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right) \text{Li}_2\left(1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c}
\end{aligned}$$

**Mathematica [F]** time = 0.510229, size = 0, normalized size = 0.

$$\int \frac{\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^2/(1 - c^2\*x^2), x]

[Out] Integrate[(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^2/(1 - c^2\*x^2), x]

**Maple [B]** time = 1.202, size = 696, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^2/(-c^2\*x^2+1), x)



```
[Out] -1/2*a^2/c*ln(c*x-1)+1/2*a^2/c*ln(c*x+1)+b^2/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+1/(((c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((c*x+1)^(1/2)/(c*x+1)^(1/2)+1)))^(1/2))+2*b^2/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-1/(((c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((c*x+1)^(1/2)/(c*x+1)^(1/2)+1)))^(1/2))-2*b^2/c*polylog(3,-1/(((c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((c*x+1)^(1/2)/(c*x+1)^(1/2)+1)))^(1/2))-b^2/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1/(((c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*((c*x+1)^(1/2)/(c*x+1)^(1/2)+1)+1))-b^2/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-1/(((c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*((c*x+1)^(1/2)/(c*x+1)^(1/2)+1)))+1/2*b^2/c*polylog(3,-1/(((c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*((c*x+1)^(1/2)/(c*x+1)^(1/2)+1)))+b^2/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1-1/(((c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((c*x+1)^(1/2)/(c*x+1)^(1/2)+1)))^(1/2))+2*b^2/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,1/(((c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((c*x+1)^(1/2)/(c*x+1)^(1/2)+1)))^(1/2))-2*b^2/c*polylog(3,1/(((c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((c*x+1)^(1/2)/(c*x+1)^(1/2)+1)))^(1/2))+2*a*b/c*dilog(((c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((c*x+1)^(1/2)/(c*x+1)^(1/2)+1))-1/2*a*b/c*dilog(((c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^2/((c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} a^2 \left( \frac{\log(cx+1)}{c} - \frac{\log(cx-1)}{c} \right) + \frac{(b^2 \log(cx+1) - b^2 \log(-cx+1)) \log(-\sqrt{cx+1} + \sqrt{-cx+1})^2}{8c} + \int -\frac{2(\sqrt{cx+1}b}{$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/8*(b^2*log(c*x + 1) - b^2*log(-c*x + 1))*log(-sqrt(c*x + 1) + sqrt(-c*x + 1))^2/c + integrate(-1/8*(2*(sqrt(c*x + 1)*b^2 - sqrt(-c*x + 1)*b^2)*log(sqrt(c*x + 1) + sqrt(-c*x + 1))^2 + 8*(sqrt(c*x + 1)*a*b - sqrt(-c*x + 1)*a*b)*log(sqrt(c*x + 1) + sqrt(-c*x + 1)) - (4*(sqrt(c*x + 1)*b^2 - sqrt(-c*x + 1)*b^2)*log(sqrt(c*x + 1) + sqrt(-c*x + 1)) + (8*a*b - (b^2*c*x - b^2)*log(c*x + 1) + (b^2*c*x - b^2)*log(-c*x + 1))*sqrt(c*x + 1) - (8*a*b - (b^2*c*x + b^2)*log(c*x + 1) + (b^2*c*x + b^2)*log(-c*x + 1))*sqrt(-c*x + 1))*log(-sqrt(c*x + 1) + sqrt(-c*x + 1)))/((c^2*x^2 - 1)*sqrt(c*x + 1) - (c^2*x^2 - 1)*sqrt(-c*x + 1)), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( -\frac{b^2 \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right)^2 + 2ab \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right) + a^2}{c^2x^2 - 1}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^2/(-c^2\*x^2+1),x, algorithm="fricas")

[Out] integral(-(b^2\*arccoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1))^2 + 2\*a\*b\*arccoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) + a^2)/(c^2\*x^2 - 1), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{a^2}{c^2x^2 - 1} dx - \int \frac{b^2 \operatorname{acoth}^2 \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right)}{c^2x^2 - 1} dx - \int \frac{2ab \operatorname{acoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right)}{c^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth((-c\*x+1)\*\*(1/2)/(c\*x+1)\*\*(1/2)))\*\*2/(-c\*\*2\*x\*\*2+1),x)

[Out] -Integral(a\*\*2/(c\*\*2\*x\*\*2 - 1), x) - Integral(b\*\*2\*acoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1))\*\*2/(c\*\*2\*x\*\*2 - 1), x) - Integral(2\*a\*b\*acoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1))/(c\*\*2\*x\*\*2 - 1), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left( b \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right) + a \right)^2}{c^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^2/(-c^2\*x^2+1),x, algorithm="giac")

```
[Out] integrate(-(b*arccoth(sqrt(-c*x + 1))/sqrt(c*x + 1)) + a)^2/(c^2*x^2 - 1), x  
)
```

$$3.125 \quad \int \frac{a+b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2} dx$$

**Optimal.** Leaf size=89

$$-\frac{b \operatorname{PolyLog}\left(2, -\frac{\sqrt{cx+1}}{\sqrt{1-cx}}\right)}{2c} + \frac{b \operatorname{PolyLog}\left(2, \frac{\sqrt{cx+1}}{\sqrt{1-cx}}\right)}{2c} - \frac{a \log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}$$

[Out] -((a\*Log[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])/c) - (b\*PolyLog[2, -(Sqrt[1 + c\*x]/Sqrt[1 - c\*x])])/(2\*c) + (b\*PolyLog[2, Sqrt[1 + c\*x]/Sqrt[1 - c\*x]])/(2\*c)

**Rubi [A]** time = 0.0586061, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 3, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$ , Rules used = {206, 6681, 5913}

$$-\frac{b \operatorname{PolyLog}\left(2, -\frac{\sqrt{cx+1}}{\sqrt{1-cx}}\right)}{2c} + \frac{b \operatorname{PolyLog}\left(2, \frac{\sqrt{cx+1}}{\sqrt{1-cx}}\right)}{2c} - \frac{a \log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])/(1 - c^2\*x^2), x]

[Out] -((a\*Log[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])/c) - (b\*PolyLog[2, -(Sqrt[1 + c\*x]/Sqrt[1 - c\*x])])/(2\*c) + (b\*PolyLog[2, Sqrt[1 + c\*x]/Sqrt[1 - c\*x]])/(2\*c)

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 6681

Int[((a\_.) + (b\_.)\*(F\_))(((c\_.)\*Sqrt[(d\_.) + (e\_.)\*(x\_)])/Sqrt[(f\_.) + (g\_.)\*(x\_)])^n\_)/((A\_.) + (C\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*e\*g)/(C\*(e\*f - d\*g)), Subst[Int[(a + b\*F[c\*x])^n/x, x], x, Sqrt[d + e\*x]/Sqrt[f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C\*d\*f - A\*e\*g, 0] && EqQ[e\*f + d\*g, 0] && IGtQ[n, 0])

### Rule 5913

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b*PolyLog[2, -(c*x)^(-1)])]/2, x] - Simp[(b*PolyLog[2, 1/(c*x)])]/
2, x]) /; FreeQ[{a, b, c}, x]
```

### Rubi steps

$$\int \frac{a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1 - c^2x^2} dx = -\frac{\text{Subst}\left(\int \frac{a+b \coth^{-1}(x)}{x} dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{a \log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} - \frac{b \text{Li}_2\left(-\frac{\sqrt{1+cx}}{\sqrt{1-cx}}\right)}{2c} + \frac{b \text{Li}_2\left(\frac{\sqrt{1+cx}}{\sqrt{1-cx}}\right)}{2c}$$

**Mathematica [A]** time = 0.414956, size = 98, normalized size = 1.1

$$\frac{b \left( \text{PolyLog}\left(2, -e^{-\tanh^{-1}(cx)}\right) - \text{PolyLog}\left(2, e^{-\tanh^{-1}(cx)}\right) + \tanh^{-1}(cx) \left( 2 \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right) + \log\left(1 - e^{-\tanh^{-1}(cx)}\right) - \log\left(1 - e^{\tanh^{-1}(cx)}\right) \right) \right)}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2), x]
```

```
[Out] (a*ArcTanh[c*x])/c + (b*(ArcTanh[c*x]*(2*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]
]) + Log[1 - E^(-ArcTanh[c*x])] - Log[1 + E^(-ArcTanh[c*x])]) + PolyLog[2,
-E^(-ArcTanh[c*x])] - PolyLog[2, E^(-ArcTanh[c*x])]))/(2*c)
```

**Maple [A]** time = 0.667, size = 119, normalized size = 1.3

$$-\frac{a \ln(cx-1)}{2c} + \frac{a \ln(cx+1)}{2c} + \frac{b}{c} \text{dilog}\left(\left(\sqrt{-cx+1} \frac{1}{\sqrt{cx+1}} - 1\right)\left(\sqrt{-cx+1} \frac{1}{\sqrt{cx+1}} + 1\right)^{-1}\right) - \frac{b}{4c} \text{dilog}\left(\left(\sqrt{-cx+1} \frac{1}{\sqrt{cx+1}} - 1\right)\left(\sqrt{-cx+1} \frac{1}{\sqrt{cx+1}} + 1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1), x)
```

```
[Out] -1/2*a/c*ln(c*x-1)+1/2*a/c*ln(c*x+1)+b/c*dilog(((c*x+1)^(1/2)/(c*x+1)^(1/2)
)-1)/((c*x+1)^(1/2)/(c*x+1)^(1/2)+1))-1/4*b/c*dilog(((c*x+1)^(1/2)/(c*x+1)
)^(1/2)-1)^2/((c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{4} b \left( \frac{(\log(cx+1) - \log(-cx+1)) \log(\sqrt{cx+1} + \sqrt{-cx+1}) - (\log(cx+1) - \log(-cx+1)) \log(-\sqrt{cx+1} + \sqrt{-cx+1})}{c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))/(-c^2\*x^2+1),x, algorith="maxima")

[Out] 1/4\*b\*(((log(c\*x + 1) - log(-c\*x + 1))\*log(sqrt(c\*x + 1) + sqrt(-c\*x + 1)) - (log(c\*x + 1) - log(-c\*x + 1))\*log(-sqrt(c\*x + 1) + sqrt(-c\*x + 1)))/c - 2\*integrate(-1/2\*sqrt(c\*x + 1)\*(log(c\*x + 1) - log(-c\*x + 1))/((c^2\*x^2 - 1)\*sqrt(c\*x + 1) + (c^2\*x^2 - 1)\*sqrt(-c\*x + 1)), x) - 2\*integrate(1/2\*sqrt(c\*x + 1)\*(log(c\*x + 1) - log(-c\*x + 1))/((c^2\*x^2 - 1)\*sqrt(c\*x + 1) - (c^2\*x^2 - 1)\*sqrt(-c\*x + 1)), x)) + 1/2\*a\*(log(c\*x + 1)/c - log(c\*x - 1)/c)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( -\frac{b \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right) + a}{c^2 x^2 - 1}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))/(-c^2\*x^2+1),x, algorith="fricas")

[Out] integral(-(b\*arccoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) + a)/(c^2\*x^2 - 1), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{a}{c^2 x^2 - 1} dx - \int \frac{b \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right)}{c^2 x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth((-c\*x+1)\*\*(1/2)/(c\*x+1)\*\*(1/2)))/(-c\*\*2\*x\*\*2+1),x)

[Out] -Integral(a/(c\*\*2\*x\*\*2 - 1), x) - Integral(b\*acoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1))/(c\*\*2\*x\*\*2 - 1), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{b \operatorname{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a}{c^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))/(-c^2\*x^2+1),x, algorithm="giac")

[Out] integrate(-(b\*arccoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) + a)/(c^2\*x^2 - 1), x)

$$3.126 \quad \int \frac{1}{(1-c^2x^2)\left(a+b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)} dx$$

**Optimal.** Leaf size=42

$$\text{Unintegrable}\left(\frac{1}{(1-c^2x^2)\left(a+b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}, x\right)$$

[Out] Unintegrable[1/((1 - c^2\*x^2)\*(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])) , x]

**Rubi [A]** time = 0.0447068, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((1 - c^2\*x^2)\*(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])), x]

[Out] Defer[Int][1/((1 - c^2\*x^2)\*(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])), x]

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)} dx = \int \frac{1}{(1-c^2x^2)\left(a+b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)} dx$$

**Mathematica [A]** time = 0.0901088, size = 0, normalized size = 0.

$$\int \frac{1}{(1-c^2x^2)\left(a+b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)} dx$$



Verification is Not applicable to the result.

[In] Integrate[1/((1 - c^2\*x^2)\*(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])),x]

[Out] Integrate[1/((1 - c^2\*x^2)\*(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])), x  
]

**Maple [A]** time = 0.983, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2 + 1} \left( a + b \operatorname{arccoth} \left( \sqrt{-cx + 1} \frac{1}{\sqrt{cx + 1}} \right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-c^2\*x^2+1)/(a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))),x)

[Out] int(1/(-c^2\*x^2+1)/(a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$- \int \frac{1}{(c^2x^2 - 1) \left( b \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right) + a \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-c^2\*x^2+1)/(a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))),x, algorithm="maxima")

[Out] -integrate(1/((c^2\*x^2 - 1)\*(b\*arccoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) + a)),  
x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( - \frac{1}{ac^2x^2 + (bc^2x^2 - b) \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right) - a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algorithm="fricas")
```

```
[Out] integral(-1/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) - a), x)
```

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{ac^2x^2 - a + bc^2x^2 \operatorname{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) - b \operatorname{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*acoth((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

```
[Out] -Integral(1/(a*c**2*x**2 - a + b*c**2*x**2*acoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) - b*acoth(sqrt(-c*x + 1)/sqrt(c*x + 1))), x)
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(c^2x^2 - 1)\left(b \operatorname{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algorithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)), x)
```

$$3.127 \quad \int \frac{1}{(1-c^2x^2) \left( a + b \coth^{-1} \left( \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right) \right)^2} dx$$

**Optimal.** Leaf size=42

$$\text{Unintegrable} \left( \frac{1}{(1-c^2x^2) \left( a + b \coth^{-1} \left( \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right) \right)^2}, x \right)$$

[Out] Unintegrable[1/((1 - c^2\*x^2)\*(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^2), x]

**Rubi [A]** time = 0.0426659, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(1-c^2x^2) \left( a + b \coth^{-1} \left( \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right) \right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((1 - c^2\*x^2)\*(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^2), x]

[Out] Defer[Int][1/((1 - c^2\*x^2)\*(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^2), x]

Rubi steps

$$\int \frac{1}{(1-c^2x^2) \left( a + b \coth^{-1} \left( \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right) \right)^2} dx = \int \frac{1}{(1-c^2x^2) \left( a + b \coth^{-1} \left( \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right) \right)^2} dx$$

**Mathematica [A]** time = 0.751716, size = 0, normalized size = 0.

$$\int \frac{1}{(1-c^2x^2) \left( a + b \coth^{-1} \left( \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right) \right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((1 - c^2\*x^2)\*(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]]))^2), x]

[Out] Integrate[1/((1 - c^2\*x^2)\*(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]]))^2), x]

**Maple [A]** time = 0.976, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2 + 1} \left( a + b \operatorname{arccoth} \left( \sqrt{-cx + 1} \frac{1}{\sqrt{cx + 1}} \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-c^2\*x^2+1)/(a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^2,x)

[Out] int(1/(-c^2\*x^2+1)/(a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{4cx}{\sqrt{cx+1}\sqrt{-cx+1}b^2c \log(\sqrt{cx+1} + \sqrt{-cx+1}) - \sqrt{cx+1}\sqrt{-cx+1}b^2c \log(-\sqrt{cx+1} + \sqrt{-cx+1}) + 2\sqrt{cx+1}\sqrt{-cx+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-c^2\*x^2+1)/(a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^2,x, algorithm="maxima")

[Out] 4\*c\*x/(sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*b^2\*c\*log(sqrt(c\*x + 1) + sqrt(-c\*x + 1)) - sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*b^2\*c\*log(-sqrt(c\*x + 1) + sqrt(-c\*x + 1)) + 2\*sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*a\*b\*c) - integrate(-4/((b^2\*c^2\*x^2 - b^2)\*sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*log(sqrt(c\*x + 1) + sqrt(-c\*x + 1)) - (b^2\*c^2\*x^2 - b^2)\*sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*log(-sqrt(c\*x + 1) + sqrt(-c\*x + 1))) + 2\*(a\*b\*c^2\*x^2 - a\*b)\*sqrt(c\*x + 1)\*sqrt(-c\*x + 1)), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{a^2 c^2 x^2 + (b^2 c^2 x^2 - b^2) \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right)^2 - a^2 + 2 (abc^2 x^2 - ab) \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-c^2\*x^2+1)/(a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^2,x, algorithm="fricas")

[Out] integral(-1/(a^2\*c^2\*x^2 + (b^2\*c^2\*x^2 - b^2)\*arccoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1))^2 - a^2 + 2\*(a\*b\*c^2\*x^2 - a\*b)\*arccoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1))), x)

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{a^2 c^2 x^2 - a^2 + 2abc^2 x^2 \operatorname{acoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right) - 2ab \operatorname{acoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right) + b^2 c^2 x^2 \operatorname{acoth}^2 \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right) - b^2 \operatorname{acoth}^2 \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-c\*\*2\*x\*\*2+1)/(a+b\*acoth((-c\*x+1)\*\*(1/2)/(c\*x+1)\*\*(1/2)))\*\*2,x)

[Out] -Integral(1/(a\*\*2\*c\*\*2\*x\*\*2 - a\*\*2 + 2\*a\*b\*c\*\*2\*x\*\*2\*acoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) - 2\*a\*b\*acoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) + b\*\*2\*c\*\*2\*x\*\*2\*acoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1))\*\*2 - b\*\*2\*acoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1))\*\*2), x)

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c^2 x^2 - 1) \left( b \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right) + a \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a  
lgorithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2  
, x)
```

### 3.128 $\int x^m \coth^{-1}(\tanh(a + bx)) dx$

**Optimal.** Leaf size=37

$$\frac{x^{m+1} \coth^{-1}(\tanh(a + bx))}{m + 1} - \frac{bx^{m+2}}{m^2 + 3m + 2}$$

[Out]  $-\left(\frac{b x^{2+m}}{2+3 m+m^2}\right)+\left(x^{1+m} \operatorname{ArcCoth}[\operatorname{Tanh}[a+b x]]\right) / (1+m)$

**Rubi [A]** time = 0.027411, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2168, 30}

$$\frac{x^{m+1} \coth^{-1}(\tanh(a + bx))}{m + 1} - \frac{bx^{m+2}}{m^2 + 3m + 2}$$

Antiderivative was successfully verified.

[In] Int[x^m\*ArcCoth[Tanh[a + b\*x]],x]

[Out]  $-\left(\frac{b x^{2+m}}{2+3 m+m^2}\right)+\left(x^{1+m} \operatorname{ArcCoth}[\operatorname{Tanh}[a+b x]]\right) / (1+m)$

#### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

#### Rule 30

```
Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

#### Rubi steps





$\text{sgn}(I/(\exp(2*b*x+2*a)+1))^{3*m}/(1+m)/(2+m)*x^m$

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 1.63962, size = 72, normalized size = 1.95

$$\frac{((bm + b)x^2 + (am + 2a)x)x^m}{m^2 + 3m + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="fricas")`

[Out]  $((b*m + b)*x^2 + (a*m + 2*a)*x)*x^m/(m^2 + 3*m + 2)$

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m*acoth(tanh(b*x+a)),x)`

[Out] Exception raised: TypeError

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{arccoth}(\tanh(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^m*arccoth(tanh(b*x + a)), x)
```

### 3.129 $\int x^2 \coth^{-1}(\tanh(a + bx)) dx$

**Optimal.** Leaf size=23

$$\frac{1}{3}x^3 \coth^{-1}(\tanh(a + bx)) - \frac{bx^4}{12}$$

[Out]  $-(b*x^4)/12 + (x^3*ArcCoth[Tanh[a + b*x]])/3$

**Rubi [A]** time = 0.0090215, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2168, 30}

$$\frac{1}{3}x^3 \coth^{-1}(\tanh(a + bx)) - \frac{bx^4}{12}$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcCoth[Tanh[a + b\*x]],x]

[Out]  $-(b*x^4)/12 + (x^3*ArcCoth[Tanh[a + b*x]])/3$

#### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

#### Rule 30

```
Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

#### Rubi steps

$$\begin{aligned}\int x^2 \coth^{-1}(\tanh(a + bx)) dx &= \frac{1}{3}x^3 \coth^{-1}(\tanh(a + bx)) - \frac{1}{3}b \int x^3 dx \\ &= -\frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(\tanh(a + bx))\end{aligned}$$

**Mathematica [A]** time = 0.0171361, size = 20, normalized size = 0.87

$$-\frac{1}{12}x^3 (bx - 4 \coth^{-1}(\tanh(a + bx)))$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[Tanh[a + b\*x]],x]

[Out] -(x^3\*(b\*x - 4\*ArcCoth[Tanh[a + b\*x]]))/12

**Maple [B]** time = 0.076, size = 59, normalized size = 2.6

$$\frac{x^3 \operatorname{arccoth}(\tanh(bx + a))}{3} + \frac{1}{3b^3} \left( -\frac{(bx + a)^4}{4} + (bx + a)^3 a - \frac{3a^2 (bx + a)^2}{2} + (bx + a) a^3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(tanh(b\*x+a)),x)

[Out] 1/3\*x^3\*arccoth(tanh(b\*x+a))+1/3/b^3\*(-1/4\*(b\*x+a)^4+(b\*x+a)^3\*a-3/2\*a^2\*(b\*x+a)^2+(b\*x+a)\*a^3)

**Maxima [A]** time = 1.17738, size = 26, normalized size = 1.13

$$-\frac{1}{12}bx^4 + \frac{1}{3}x^3 \operatorname{arccoth}(\tanh(bx + a))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(tanh(b\*x+a)),x, algorithm="maxima")

[Out]  $-1/12*b*x^4 + 1/3*x^3*\operatorname{arccoth}(\tanh(b*x + a))$

---

**Fricas [A]** time = 1.63204, size = 31, normalized size = 1.35

$$\frac{1}{4}bx^4 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccoth(tanh(b*x+a)),x, algorithm="fricas")`

[Out]  $1/4*b*x^4 + 1/3*a*x^3$

---

**Sympy [A]** time = 0.556857, size = 19, normalized size = 0.83

$$-\frac{bx^4}{12} + \frac{x^3 \operatorname{acoth}(\tanh(a + bx))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*acoth(tanh(b*x+a)),x)`

[Out]  $-b*x**4/12 + x**3*acoth(\tanh(a + b*x))/3$

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arccoth}(\tanh(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccoth(tanh(b*x+a)),x, algorithm="giac")`

[Out] `integrate(x^2*arccoth(tanh(b*x + a)), x)`

### 3.130 $\int x \coth^{-1}(\tanh(a + bx)) dx$

**Optimal.** Leaf size=23

$$\frac{1}{2}x^2 \coth^{-1}(\tanh(a + bx)) - \frac{bx^3}{6}$$

[Out]  $-(b*x^3)/6 + (x^2*ArcCoth[Tanh[a + b*x]])/2$

**Rubi [A]** time = 0.0074933, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {6240, 30}

$$\frac{1}{2}x^2 \coth^{-1}(\tanh(a + bx)) - \frac{bx^3}{6}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x*ArcCoth[Tanh[a + b*x]], x]$

[Out]  $-(b*x^3)/6 + (x^2*ArcCoth[Tanh[a + b*x]])/2$

#### Rule 6240

$\text{Int}[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[(e + f*x)^{(m + 1)}*ArcCoth[c + d*Tanh[a + b*x]]/(f*(m + 1)), x] + \text{Dist}[b/(f*(m + 1)), \text{Int}[(e + f*x)^{(m + 1)}/(c - d + c*E^{(2*a + 2*b*x)}), x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, f\}, x$  &&  $\text{IGtQ}[m, 0]$  &&  $\text{EqQ}[(c - d)^2, 1]$

#### Rule 30

$\text{Int}[(x_)^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[x^{(m + 1)}/(m + 1), x] /;$   $\text{FreeQ}[m, x]$  &&  $\text{NeQ}[m, -1]$

#### Rubi steps

$$\begin{aligned} \int x \coth^{-1}(\tanh(a + bx)) dx &= \frac{1}{2}x^2 \coth^{-1}(\tanh(a + bx)) - \frac{1}{2}b \int x^2 dx \\ &= -\frac{bx^3}{6} + \frac{1}{2}x^2 \coth^{-1}(\tanh(a + bx)) \end{aligned}$$

**Mathematica [A]** time = 0.0157631, size = 20, normalized size = 0.87

$$-\frac{1}{6}x^2 (bx - 3 \operatorname{coth}^{-1}(\tanh(a + bx)))$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[Tanh[a + b\*x]],x]

[Out] -(x^2\*(b\*x - 3\*ArcCoth[Tanh[a + b\*x]]))/6

**Maple [B]** time = 0.075, size = 48, normalized size = 2.1

$$\frac{x^2 \operatorname{arccoth}(\tanh(bx + a))}{2} + \frac{1}{2b^2} \left( -\frac{(bx + a)^3}{3} + (bx + a)^2 a - a^2 (bx + a) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(tanh(b\*x+a)),x)

[Out] 1/2\*x^2\*arccoth(tanh(b\*x+a))+1/2/b^2\*(-1/3\*(b\*x+a)^3+(b\*x+a)^2\*a-a^2\*(b\*x+a))

**Maxima [A]** time = 1.17243, size = 26, normalized size = 1.13

$$-\frac{1}{6}bx^3 + \frac{1}{2}x^2 \operatorname{arccoth}(\tanh(bx + a))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(tanh(b\*x+a)),x, algorithm="maxima")

[Out] -1/6\*b\*x^3 + 1/2\*x^2\*arccoth(tanh(b\*x + a))

**Fricas [A]** time = 1.65176, size = 31, normalized size = 1.35

$$\frac{1}{3}bx^3 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(tanh(b\*x+a)),x, algorithm="fricas")

[Out] 1/3\*b\*x^3 + 1/2\*a\*x^2

**Sympy [A]** time = 0.291645, size = 19, normalized size = 0.83

$$-\frac{bx^3}{6} + \frac{x^2 \operatorname{arccoth}(\tanh(a + bx))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(tanh(b\*x+a)),x)

[Out] -b\*x\*\*3/6 + x\*\*2\*acoth(tanh(a + b\*x))/2

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arccoth}(\tanh(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(tanh(b\*x+a)),x, algorithm="giac")

[Out] integrate(x\*arccoth(tanh(b\*x + a)), x)



### 3.131 $\int \coth^{-1}(\tanh(a + bx)) dx$

**Optimal.** Leaf size=16

$$\frac{\coth^{-1}(\tanh(a + bx))^2}{2b}$$

[Out] ArcCoth[Tanh[a + b\*x]]^2/(2\*b)

**Rubi [A]** time = 0.003068, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2157, 30}

$$\frac{\coth^{-1}(\tanh(a + bx))^2}{2b}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]], x]

[Out] ArcCoth[Tanh[a + b\*x]]^2/(2\*b)

#### Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[  
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \coth^{-1}(\tanh(a + bx)) dx &= \frac{\text{Subst}\left(\int x dx, x, \coth^{-1}(\tanh(a + bx))\right)}{b} \\ &= \frac{\coth^{-1}(\tanh(a + bx))^2}{2b} \end{aligned}$$

**Mathematica [A]** time = 0.0083259, size = 18, normalized size = 1.12

$$x \coth^{-1}(\tanh(a + bx)) - \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]], x]

[Out] -(b\*x^2)/2 + x\*ArcCoth[Tanh[a + b\*x]]

**Maple [B]** time = 0.06, size = 32, normalized size = 2.

$$\frac{1}{b} \left( \operatorname{Arctanh}(\tanh(bx + a)) \operatorname{arccoth}(\tanh(bx + a)) - \frac{(\operatorname{Arctanh}(\tanh(bx + a)))^2}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b\*x+a)), x)

[Out] 1/b\*(arctanh(tanh(b\*x+a))\*arccoth(tanh(b\*x+a))-1/2\*arctanh(tanh(b\*x+a))^2)

**Maxima [A]** time = 1.15234, size = 22, normalized size = 1.38

$$-\frac{1}{2}bx^2 + x \operatorname{arccoth}(\tanh(bx + a))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a)), x, algorithm="maxima")

[Out] -1/2\*b\*x^2 + x\*arccoth(tanh(b\*x + a))

**Fricas [A]** time = 1.57382, size = 23, normalized size = 1.44

$$\frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*x
```

**Sympy [A]** time = 0.194614, size = 19, normalized size = 1.19

$$\begin{cases} \frac{\operatorname{acoth}^2(\tanh(a+bx))}{2b} & \text{for } b \neq 0 \\ x \operatorname{acoth}(\tanh(a)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a)),x)
```

```
[Out] Piecewise((acoth(tanh(a + b*x))**2/(2*b), Ne(b, 0)), (x*acoth(tanh(a)), True))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(\tanh(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a)), x)
```

$$3.132 \quad \int \frac{\coth^{-1}(\tanh(a+bx))}{x} dx$$

**Optimal.** Leaf size=21

$$bx - \log(x) (bx - \coth^{-1}(\tanh(a + bx)))$$

[Out] b\*x - (b\*x - ArcCoth[Tanh[a + b\*x]])\*Log[x]

**Rubi [A]** time = 0.0439774, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2158, 29}

$$bx - \log(x) (bx - \coth^{-1}(\tanh(a + bx)))$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]/x,x]

[Out] b\*x - (b\*x - ArcCoth[Tanh[a + b\*x]])\*Log[x]

### Rule 2158

Int[(v\_)/(u\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(b\*x)/a, x] - Dist[(b\*u - a\*v)/a, Int[1/u, x], x] /; NeQ[b\*u - a\*v, 0]] /; PiecewiseLinearQ[u, v, x]

### Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

### Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(\tanh(a + bx))}{x} dx &= bx - (bx - \coth^{-1}(\tanh(a + bx))) \int \frac{1}{x} dx \\ &= bx - (bx - \coth^{-1}(\tanh(a + bx))) \log(x) \end{aligned}$$

**Mathematica [A]** time = 0.0161214, size = 19, normalized size = 0.9

$$\log(x) (\coth^{-1}(\tanh(a + bx)) - bx) + bx$$



**Fricas [A]** time = 1.62091, size = 22, normalized size = 1.05

$$bx + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))/x,x, algorithm="fricas")

[Out] b\*x + a\*log(x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}(\tanh(a + bx))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(tanh(b\*x+a))/x,x)

[Out] Integral(acoth(tanh(a + b\*x))/x, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))/x,x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a))/x, x)

$$3.133 \quad \int \frac{\coth^{-1}(\tanh(a+bx))}{x^2} dx$$

Optimal. Leaf size=17

$$b \log(x) - \frac{\coth^{-1}(\tanh(a+bx))}{x}$$

[Out] `-(ArcCoth[Tanh[a + b*x]]/x) + b*Log[x]`

**Rubi [A]** time = 0.0092769, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2168, 29}

$$b \log(x) - \frac{\coth^{-1}(\tanh(a+bx))}{x}$$

Antiderivative was successfully verified.

[In] `Int[ArcCoth[Tanh[a + b*x]]/x^2,x]`

[Out] `-(ArcCoth[Tanh[a + b*x]]/x) + b*Log[x]`

### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

### Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

### Rubi steps

$$\int \frac{\coth^{-1}(\tanh(a + bx))}{x^2} dx = -\frac{\coth^{-1}(\tanh(a + bx))}{x} + b \int \frac{1}{x} dx$$

$$= -\frac{\coth^{-1}(\tanh(a + bx))}{x} + b \log(x)$$

**Mathematica [A]** time = 0.016392, size = 18, normalized size = 1.06

$$-\frac{\coth^{-1}(\tanh(a + bx))}{x} + b \log(x) + b$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]/x^2,x]

[Out] b - ArcCoth[Tanh[a + b\*x]]/x + b\*Log[x]

**Maple [A]** time = 0.069, size = 20, normalized size = 1.2

$$-\frac{\operatorname{arccoth}(\tanh(bx + a))}{x} + b \ln(bx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b\*x+a))/x^2,x)

[Out] -arccoth(tanh(b\*x+a))/x+b\*ln(b\*x)

**Maxima [A]** time = 1.17533, size = 23, normalized size = 1.35

$$b \log(x) - \frac{\operatorname{arccoth}(\tanh(bx + a))}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))/x^2,x, algorithm="maxima")



[Out]  $b \cdot \log(x) - \operatorname{arccoth}(\tanh(b \cdot x + a)) / x$

---

**Fricas [A]** time = 1.72301, size = 27, normalized size = 1.59

$$\frac{bx \log(x) - a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(tanh(b*x+a))/x^2,x, algorithm="fricas")`

[Out]  $(b \cdot x \cdot \log(x) - a) / x$

---

**Sympy [A]** time = 0.314397, size = 14, normalized size = 0.82

$$b \log(x) - \frac{\operatorname{acoth}(\tanh(a + bx))}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(tanh(b*x+a))/x**2,x)`

[Out]  $b \cdot \log(x) - \operatorname{acoth}(\tanh(a + b \cdot x)) / x$

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(tanh(b*x+a))/x^2,x, algorithm="giac")`

[Out] `integrate(arccoth(tanh(b*x + a))/x^2, x)`

$$3.134 \quad \int \frac{\coth^{-1}(\tanh(a+bx))}{x^3} dx$$

**Optimal.** Leaf size=23

$$-\frac{\coth^{-1}(\tanh(a+bx))}{2x^2} - \frac{b}{2x}$$

[Out] -b/(2\*x) - ArcCoth[Tanh[a + b\*x]]/(2\*x^2)

**Rubi [A]** time = 0.0095948, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2168, 30}

$$-\frac{\coth^{-1}(\tanh(a+bx))}{2x^2} - \frac{b}{2x}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]/x^3,x]

[Out] -b/(2\*x) - ArcCoth[Tanh[a + b\*x]]/(2\*x^2)

### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

### Rule 30

```
Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

### Rubi steps

$$\int \frac{\coth^{-1}(\tanh(a + bx))}{x^3} dx = -\frac{\coth^{-1}(\tanh(a + bx))}{2x^2} + \frac{1}{2}b \int \frac{1}{x^2} dx$$

$$= -\frac{b}{2x} - \frac{\coth^{-1}(\tanh(a + bx))}{2x^2}$$

**Mathematica [A]** time = 0.0135521, size = 18, normalized size = 0.78

$$-\frac{\coth^{-1}(\tanh(a + bx)) + bx}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]/x^3,x]

[Out] -(b\*x + ArcCoth[Tanh[a + b\*x]])/(2\*x^2)

**Maple [A]** time = 0.074, size = 20, normalized size = 0.9

$$-\frac{b}{2x} - \frac{\operatorname{arccoth}(\tanh(bx + a))}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b\*x+a))/x^3,x)

[Out] -1/2\*b/x-1/2\*arccoth(tanh(b\*x+a))/x^2

**Maxima [A]** time = 1.17109, size = 26, normalized size = 1.13

$$-\frac{b}{2x} - \frac{\operatorname{arccoth}(\tanh(bx + a))}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))/x^3,x, algorithm="maxima")

[Out]  $-1/2*b/x - 1/2*\operatorname{arccoth}(\tanh(b*x + a))/x^2$

---

**Fricas [A]** time = 1.59421, size = 30, normalized size = 1.3

$$-\frac{2bx + a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(tanh(b*x+a))/x^3,x, algorithm="fricas")`

[Out]  $-1/2*(2*b*x + a)/x^2$

---

**Sympy [A]** time = 0.773247, size = 19, normalized size = 0.83

$$-\frac{b}{2x} - \frac{\operatorname{acoth}(\tanh(a + bx))}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(tanh(b*x+a))/x**3,x)`

[Out]  $-b/(2*x) - \operatorname{acoth}(\tanh(a + b*x))/(2*x**2)$

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(tanh(b*x+a))/x^3,x, algorithm="giac")`

[Out] `integrate(arccoth(tanh(b*x + a))/x^3, x)`

$$3.135 \quad \int \frac{\coth^{-1}(\tanh(a+bx))}{x^4} dx$$

**Optimal.** Leaf size=23

$$-\frac{\coth^{-1}(\tanh(a+bx))}{3x^3} - \frac{b}{6x^2}$$

[Out] -b/(6\*x^2) - ArcCoth[Tanh[a + b\*x]]/(3\*x^3)

**Rubi [A]** time = 0.0088794, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2168, 30}

$$-\frac{\coth^{-1}(\tanh(a+bx))}{3x^3} - \frac{b}{6x^2}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]/x^4,x]

[Out] -b/(6\*x^2) - ArcCoth[Tanh[a + b\*x]]/(3\*x^3)

### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

### Rule 30

```
Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

### Rubi steps

$$\int \frac{\coth^{-1}(\tanh(a + bx))}{x^4} dx = -\frac{\coth^{-1}(\tanh(a + bx))}{3x^3} + \frac{1}{3}b \int \frac{1}{x^3} dx$$

$$= -\frac{b}{6x^2} - \frac{\coth^{-1}(\tanh(a + bx))}{3x^3}$$

**Mathematica [A]** time = 0.0154072, size = 20, normalized size = 0.87

$$-\frac{2 \coth^{-1}(\tanh(a + bx)) + bx}{6x^3}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]/x^4,x]

[Out] -(b\*x + 2\*ArcCoth[Tanh[a + b\*x]])/(6\*x^3)

**Maple [A]** time = 0.075, size = 20, normalized size = 0.9

$$-\frac{b}{6x^2} - \frac{\operatorname{arccoth}(\tanh(bx + a))}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b\*x+a))/x^4,x)

[Out] -1/6\*b/x^2-1/3\*arccoth(tanh(b\*x+a))/x^3

**Maxima [A]** time = 1.17902, size = 26, normalized size = 1.13

$$-\frac{b}{6x^2} - \frac{\operatorname{arccoth}(\tanh(bx + a))}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))/x^4,x, algorithm="maxima")

[Out]  $-1/6*b/x^2 - 1/3*\operatorname{arccoth}(\tanh(b*x + a))/x^3$

---

**Fricas [A]** time = 1.668, size = 32, normalized size = 1.39

$$-\frac{3bx + 2a}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(tanh(b*x+a))/x^4,x, algorithm="fricas")`

[Out]  $-1/6*(3*b*x + 2*a)/x^3$

---

**Sympy [A]** time = 1.27728, size = 20, normalized size = 0.87

$$-\frac{b}{6x^2} - \frac{\operatorname{acoth}(\tanh(a + bx))}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(tanh(b*x+a))/x**4,x)`

[Out]  $-b/(6*x**2) - \operatorname{acoth}(\tanh(a + b*x))/(3*x**3)$

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(tanh(b*x+a))/x^4,x, algorithm="giac")`

[Out] `integrate(arccoth(tanh(b*x + a))/x^4, x)`

### 3.136 $\int x^m \coth^{-1}(\tanh(a + bx))^2 dx$

**Optimal.** Leaf size=71

$$-\frac{2bx^{m+2} \coth^{-1}(\tanh(a + bx))}{m^2 + 3m + 2} + \frac{x^{m+1} \coth^{-1}(\tanh(a + bx))^2}{m + 1} + \frac{2b^2x^{m+3}}{m^3 + 6m^2 + 11m + 6}$$

[Out]  $(2*b^2*x^(3 + m))/(6 + 11*m + 6*m^2 + m^3) - (2*b*x^(2 + m)*ArcCoth[Tanh[a + b*x]])/(2 + 3*m + m^2) + (x^(1 + m)*ArcCoth[Tanh[a + b*x]]^2)/(1 + m)$

**Rubi [A]** time = 0.0352614, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2168, 30}

$$-\frac{2bx^{m+2} \coth^{-1}(\tanh(a + bx))}{m^2 + 3m + 2} + \frac{x^{m+1} \coth^{-1}(\tanh(a + bx))^2}{m + 1} + \frac{2b^2x^{m+3}}{m^3 + 6m^2 + 11m + 6}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^m * \text{ArcCoth}[\text{Tanh}[a + b*x]]^2, x]$

[Out]  $(2*b^2*x^(3 + m))/(6 + 11*m + 6*m^2 + m^3) - (2*b*x^(2 + m)*ArcCoth[Tanh[a + b*x]])/(2 + 3*m + m^2) + (x^(1 + m)*ArcCoth[Tanh[a + b*x]]^2)/(1 + m)$

#### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

#### Rule 30

```
Int[(x_)^(m_), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

#### Rubi steps



$$\begin{aligned}
\int x^m \coth^{-1}(\tanh(a + bx))^2 dx &= \frac{x^{1+m} \coth^{-1}(\tanh(a + bx))^2}{1+m} - \frac{(2b) \int x^{1+m} \coth^{-1}(\tanh(a + bx)) dx}{1+m} \\
&= -\frac{2bx^{2+m} \coth^{-1}(\tanh(a + bx))}{2+3m+m^2} + \frac{x^{1+m} \coth^{-1}(\tanh(a + bx))^2}{1+m} + \frac{(2b^2) \int x^{2+m} dx}{2+3m+m^2} \\
&= \frac{2b^2 x^{3+m}}{6+11m+6m^2+m^3} - \frac{2bx^{2+m} \coth^{-1}(\tanh(a + bx))}{2+3m+m^2} + \frac{x^{1+m} \coth^{-1}(\tanh(a + bx))^2}{1+m}
\end{aligned}$$

**Mathematica [A]** time = 0.11794, size = 62, normalized size = 0.87

$$\frac{x^{m+1} \left( (m^2 + 5m + 6) \coth^{-1}(\tanh(a + bx))^2 - 2b(m + 3)x \coth^{-1}(\tanh(a + bx)) + 2b^2 x^2 \right)}{(m + 1)(m + 2)(m + 3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*ArcCoth[Tanh[a + b\*x]]^2,x]

[Out] (x^(1 + m)\*(2\*b^2\*x^2 - 2\*b\*(3 + m)\*x\*ArcCoth[Tanh[a + b\*x]] + (6 + 5\*m + m^2)\*ArcCoth[Tanh[a + b\*x]]^2))/((1 + m)\*(2 + m)\*(3 + m))

**Maple [C]** time = 0.773, size = 9175, normalized size = 129.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*arccoth(tanh(b\*x+a))^2,x)

[Out] result too large to display

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*arccoth(tanh(b\*x+a))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.74427, size = 223, normalized size = 3.14

$$\frac{(4(b^2m^2 + 3b^2m + 2b^2)x^3 + 8(abm^2 + 4abm + 3ab)x^2 + (4a^2m^2 - \pi^2(m^2 + 5m + 6) + 20a^2m + 24a^2)x)x^m}{4(m^3 + 6m^2 + 11m + 6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>m</sup>\*arccoth(tanh(b\*x+a))<sup>2</sup>,x, algorithm="fricas")

[Out] 1/4\*(4\*(b<sup>2</sup>\*m<sup>2</sup> + 3\*b<sup>2</sup>\*m + 2\*b<sup>2</sup>)\*x<sup>3</sup> + 8\*(a\*b\*m<sup>2</sup> + 4\*a\*b\*m + 3\*a\*b)\*x<sup>2</sup> + (4\*a<sup>2</sup>\*m<sup>2</sup> - pi<sup>2</sup>\*(m<sup>2</sup> + 5\*m + 6) + 20\*a<sup>2</sup>\*m + 24\*a<sup>2</sup>)\*x)\*x<sup>m</sup>/(m<sup>3</sup> + 6\*m<sup>2</sup> + 11\*m + 6)

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*acoth(tanh(b\*x+a))\*\*2,x)

[Out] Exception raised: TypeError

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{arccoth}(\tanh(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>m</sup>\*arccoth(tanh(b\*x+a))<sup>2</sup>,x, algorithm="giac")

[Out] integrate(x<sup>m</sup>\*arccoth(tanh(b\*x + a))<sup>2</sup>, x)

### 3.137 $\int x^3 \coth^{-1}(\tanh(a + bx))^2 dx$

**Optimal.** Leaf size=42

$$-\frac{1}{10}bx^5 \coth^{-1}(\tanh(a + bx)) + \frac{1}{4}x^4 \coth^{-1}(\tanh(a + bx))^2 + \frac{b^2x^6}{60}$$

[Out] (b^2\*x^6)/60 - (b\*x^5\*ArcCoth[Tanh[a + b\*x]])/10 + (x^4\*ArcCoth[Tanh[a + b\*x]]^2)/4

**Rubi [A]** time = 0.0260149, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2168, 30}

$$-\frac{1}{10}bx^5 \coth^{-1}(\tanh(a + bx)) + \frac{1}{4}x^4 \coth^{-1}(\tanh(a + bx))^2 + \frac{b^2x^6}{60}$$

Antiderivative was successfully verified.

[In] Int[x^3\*ArcCoth[Tanh[a + b\*x]]^2,x]

[Out] (b^2\*x^6)/60 - (b\*x^5\*ArcCoth[Tanh[a + b\*x]])/10 + (x^4\*ArcCoth[Tanh[a + b\*x]]^2)/4

#### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

#### Rule 30

```
Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

#### Rubi steps

$$\begin{aligned}
\int x^3 \coth^{-1}(\tanh(a + bx))^2 dx &= \frac{1}{4}x^4 \coth^{-1}(\tanh(a + bx))^2 - \frac{1}{2}b \int x^4 \coth^{-1}(\tanh(a + bx)) dx \\
&= -\frac{1}{10}bx^5 \coth^{-1}(\tanh(a + bx)) + \frac{1}{4}x^4 \coth^{-1}(\tanh(a + bx))^2 + \frac{1}{10}b^2 \int x^5 dx \\
&= \frac{b^2x^6}{60} - \frac{1}{10}bx^5 \coth^{-1}(\tanh(a + bx)) + \frac{1}{4}x^4 \coth^{-1}(\tanh(a + bx))^2
\end{aligned}$$

**Mathematica [A]** time = 0.0336754, size = 37, normalized size = 0.88

$$\frac{1}{60}x^4 \left( -6bx \coth^{-1}(\tanh(a + bx)) + 15 \coth^{-1}(\tanh(a + bx))^2 + b^2x^2 \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*ArcCoth[Tanh[a + b\*x]]^2,x]

[Out] (x^4\*(b^2\*x^2 - 6\*b\*x\*ArcCoth[Tanh[a + b\*x]] + 15\*ArcCoth[Tanh[a + b\*x]]^2)/60)

**Maple [C]** time = 0.372, size = 3418, normalized size = 81.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*arccoth(tanh(b\*x+a))^2,x)

[Out]  $\frac{1}{20}i\pi b^5 x^5 - \frac{1}{16}\pi^2 x^4 \operatorname{csgn}\left(\frac{i}{\exp(2bx+2a)+1}\right)^6 - \frac{1}{16}\pi^2 x^4 \operatorname{csgn}\left(\frac{i}{\exp(2bx+2a)+1}\right) \operatorname{csgn}(i \exp(2bx+2a)) \operatorname{csgn}\left(\frac{i \exp(2bx+2a)}{\exp(2bx+2a)+1}\right) - \frac{1}{20}i\pi b^5 x^5 \operatorname{csgn}\left(\frac{i}{\exp(2bx+2a)+1}\right)^2 + \frac{1}{8}\pi^2 x^4 \operatorname{csgn}\left(\frac{i}{\exp(2bx+2a)+1}\right)^5 - \frac{1}{16}\pi^2 x^4 + \frac{1}{4}x^4 \ln(\exp(bx+a))^2 + \frac{1}{16}\pi^2 x^4 \operatorname{csgn}\left(\frac{i}{\exp(2bx+2a)+1}\right)^4 \operatorname{csgn}(i \exp(2bx+2a)) \operatorname{csgn}\left(\frac{i \exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^2 - \frac{1}{16}\pi^2 x^4 \operatorname{csgn}\left(\frac{i}{\exp(2bx+2a)+1}\right)^3 \operatorname{csgn}(i \exp(2bx+2a)) \operatorname{csgn}\left(\frac{i \exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^3 + \frac{1}{60}x^6 b^2 - \frac{1}{16}\pi^2 x^4 \operatorname{csgn}\left(\frac{i}{\exp(2bx+2a)+1}\right)^4 - \frac{1}{64}\pi^2 x^4 \operatorname{csgn}(i \exp(2bx+2a)) \operatorname{csgn}\left(\frac{i}{\exp(2bx+2a)+1}\right)^6 - \frac{1}{16}\pi^2 x^4 \operatorname{csgn}\left(\frac{i}{\exp(2bx+2a)+1}\right)^3 \operatorname{csgn}(i \exp(2bx+2a)) \operatorname{csgn}\left(\frac{i \exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^3 - \frac{1}{16}\pi^2 x^4 \operatorname{csgn}\left(\frac{i}{\exp(2bx+2a)+1}\right)^3 \operatorname{csgn}(i \exp(2bx+2a)) \operatorname{csgn}\left(\frac{i \exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^2 - \frac{1}{32}\pi^2 x^4 \operatorname{csgn}(i \exp(2bx+2a)) \operatorname{csgn}(i \exp(2bx+2a)) \operatorname{csgn}\left(\frac{i \exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^3 - \frac{1}{64}\pi^2 x^4 \operatorname{csgn}(i \exp(2bx+2a)) \operatorname{csgn}(i \exp(2bx+2a)) \operatorname{csgn}\left(\frac{i \exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^4 +$



```

*b*x+2*a)+1))^2-1/64*Pi^2*x^4*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x
+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/32*Pi^2*x^4*csgn(I/(
exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*
x+2*a)+1))^3+1/32*Pi^2*x^4*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)
)^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/16*Pi^2*x^4*csgn(I/(exp(2
*b*x+2*a)+1))^3*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a
)+1))+(-1/10*b*x^5-1/8*I*Pi*x^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3
-1/8*I*Pi*x^4*csgn(I*exp(2*b*x+2*a))^3+1/4*I*Pi*x^4*csgn(I*exp(b*x+a))*csgn
(I*exp(2*b*x+2*a))^2+1/8*I*Pi*x^4*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2
*a)/(exp(2*b*x+2*a)+1))^2+1/8*I*Pi*x^4*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/8*I*Pi*x^4*csgn(I/(exp(2*b*x+2*a)+1))*
csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/8*I*Pi*x
^4*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/4*I*Pi*x^4*csgn(I/(exp(2*b
*x+2*a)+1))^3-1/4*I*Pi*x^4+1/4*I*Pi*x^4*csgn(I/(exp(2*b*x+2*a)+1))^2)*ln(ex
p(b*x+a))+1/40*I*Pi*b*x^5*csgn(I*exp(2*b*x+2*a))^3+1/40*I*Pi*b*x^5*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/32*Pi^2*x^4*csgn(I*exp(b*x+a))^2*csgn
(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/16*Pi^2*
x^4*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(
2*b*x+2*a)+1))^2

```

**Maxima [A]** time = 1.3594, size = 49, normalized size = 1.17

$$\frac{1}{60} b^2 x^6 - \frac{1}{10} b x^5 \operatorname{arccoth}(\tanh(bx + a)) + \frac{1}{4} x^4 \operatorname{arccoth}(\tanh(bx + a))^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(tanh(b\*x+a))^2,x, algorithm="maxima")

[Out] 1/60\*b^2\*x^6 - 1/10\*b\*x^5\*arccoth(tanh(b\*x + a)) + 1/4\*x^4\*arccoth(tanh(b\*x + a))^2

**Fricas [A]** time = 1.57062, size = 72, normalized size = 1.71

$$\frac{1}{6} b^2 x^6 + \frac{2}{5} a b x^5 - \frac{1}{16} (\pi^2 - 4 a^2) x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(tanh(b\*x+a))^2,x, algorithm="fricas")

[Out]  $1/6*b^2*x^6 + 2/5*a*b*x^5 - 1/16*(\pi^2 - 4*a^2)*x^4$

**Sympy [A]** time = 3.67829, size = 78, normalized size = 1.86

$$\begin{cases} \frac{x^3 \operatorname{acoth}^3(\tanh(a+bx))}{4} - \frac{x^2 \operatorname{acoth}^4(\tanh(a+bx))}{4b^2} + \frac{x \operatorname{acoth}^5(\tanh(a+bx))}{10b^3} - \frac{\operatorname{acoth}^6(\tanh(a+bx))}{60b^4} & \text{for } b \neq 0 \\ \frac{x^4 \operatorname{acoth}^2(\tanh(a))}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*acoth(tanh(b*x+a))**2,x)`

[Out] `Piecewise((x**3*acoth(tanh(a + b*x))**3/(3*b) - x**2*acoth(tanh(a + b*x))**4/(4*b**2) + x*acoth(tanh(a + b*x))**5/(10*b**3) - acoth(tanh(a + b*x))**6/(60*b**4), Ne(b, 0)), (x**4*acoth(tanh(a))**2/4, True))`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arccoth}(\tanh(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arccoth(tanh(b*x+a))^2,x, algorithm="giac")`

[Out] `integrate(x^3*arccoth(tanh(b*x + a))^2, x)`

### 3.138 $\int x^2 \coth^{-1}(\tanh(a + bx))^2 dx$

**Optimal.** Leaf size=42

$$-\frac{1}{6}bx^4 \coth^{-1}(\tanh(a + bx)) + \frac{1}{3}x^3 \coth^{-1}(\tanh(a + bx))^2 + \frac{b^2x^5}{30}$$

[Out] (b^2\*x^5)/30 - (b\*x^4\*ArcCoth[Tanh[a + b\*x]])/6 + (x^3\*ArcCoth[Tanh[a + b\*x]]^2)/3

**Rubi [A]** time = 0.02533, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2168, 30}

$$-\frac{1}{6}bx^4 \coth^{-1}(\tanh(a + bx)) + \frac{1}{3}x^3 \coth^{-1}(\tanh(a + bx))^2 + \frac{b^2x^5}{30}$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcCoth[Tanh[a + b\*x]]^2,x]

[Out] (b^2\*x^5)/30 - (b\*x^4\*ArcCoth[Tanh[a + b\*x]])/6 + (x^3\*ArcCoth[Tanh[a + b\*x]]^2)/3

#### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

#### Rule 30

```
Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

#### Rubi steps



$$\begin{aligned}
\int x^2 \coth^{-1}(\tanh(a + bx))^2 dx &= \frac{1}{3}x^3 \coth^{-1}(\tanh(a + bx))^2 - \frac{1}{3}(2b) \int x^3 \coth^{-1}(\tanh(a + bx)) dx \\
&= -\frac{1}{6}bx^4 \coth^{-1}(\tanh(a + bx)) + \frac{1}{3}x^3 \coth^{-1}(\tanh(a + bx))^2 + \frac{1}{6}b^2 \int x^4 dx \\
&= \frac{b^2x^5}{30} - \frac{1}{6}bx^4 \coth^{-1}(\tanh(a + bx)) + \frac{1}{3}x^3 \coth^{-1}(\tanh(a + bx))^2
\end{aligned}$$

**Mathematica [A]** time = 0.0561201, size = 37, normalized size = 0.88

$$\frac{1}{30}x^3 \left( -5bx \coth^{-1}(\tanh(a + bx)) + 10 \coth^{-1}(\tanh(a + bx))^2 + b^2x^2 \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[Tanh[a + b\*x]]^2,x]

[Out] (x^3\*(b^2\*x^2 - 5\*b\*x\*ArcCoth[Tanh[a + b\*x]] + 10\*ArcCoth[Tanh[a + b\*x]]^2)/30)

**Maple [C]** time = 0.372, size = 3418, normalized size = 81.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(tanh(b\*x+a))^2,x)

[Out] 
$$\begin{aligned}
& -1/12*\text{Pi}^2*x^3*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^2*\text{csgn}(I*\exp(2*b*x+2*a))*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2 \\
& -1/6*\text{Pi}^2*x^3*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^2*\text{csgn}(I*\exp(b*x+a))*\text{csgn}(I*\exp(2*b*x+2*a))^2 \\
& +1/6*\text{Pi}^2*x^3*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^6 \\
& +1/12*\text{Pi}^2*x^3*\text{csgn}(I*\exp(b*x+a))*\text{csgn}(I*\exp(2*b*x+2*a))^2*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3 \\
& +1/6*\text{Pi}^2*x^3*\text{csgn}(I*\exp(b*x+a))*\text{csgn}(I*\exp(2*b*x+2*a))^2*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^3 \\
& -1/12*\text{Pi}^2*x^3+1/3*x^3*\ln(\exp(b*x+a))^2 \\
& -1/12*\text{Pi}^2*x^3*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^4*\text{csgn}(I*\exp(2*b*x+2*a))*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1)) \\
& -1/12*\text{Pi}^2*x^3*\text{csgn}(I/(\exp(2*b*x+2*a)+1))*\text{csgn}(I*\exp(2*b*x+2*a))*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1)) \\
& -1/48*\text{Pi}^2*x^3*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^2*\text{csgn}(I*\exp(2*b*x+2*a))^2*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2 \\
& +(-1/6*b*x^4-1/3*I*\text{Pi}*x^3*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^3-1/6*I*\text{Pi}*x^
\end{aligned}$$

$$\begin{aligned}
& 3\text{csgn}(I\exp(b*x+a))^2\text{csgn}(I\exp(2*b*x+2*a))-1/6*I*Pi*x^3\text{csgn}(I\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3+1/3*I*Pi*x^3\text{csgn}(I\exp(b*x+a))*\text{csgn}(I\exp(2*b*x+2*a))^2-1/6*I*Pi*x^3\text{csgn}(I/(\exp(2*b*x+2*a)+1))*\text{csgn}(I\exp(2*b*x+2*a))*\text{csgn}(I\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))+1/6*I*Pi*x^3\text{csgn}(I\exp(2*b*x+2*a))*\text{csgn}(I\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2-1/6*I*Pi*x^3\text{csgn}(I\exp(2*b*x+2*a))^3-1/3*I*Pi*x^3+1/6*I*Pi*x^3\text{csgn}(I/(\exp(2*b*x+2*a)+1))*\text{csgn}(I\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2+1/3*I*Pi*x^3\text{csgn}(I/(\exp(2*b*x+2*a)+1))^2*\ln(\exp(b*x+a))-1/12*I*Pi*b*x^4\text{csgn}(I/(\exp(2*b*x+2*a)+1))^2+1/12*I*Pi*b*x^4\text{csgn}(I/(\exp(2*b*x+2*a)+1))^3+1/12*Pi^2*x^3\text{csgn}(I/(\exp(2*b*x+2*a)+1))^4\text{csgn}(I\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2+1/24*Pi^2*x^3\text{csgn}(I\exp(2*b*x+2*a))*\text{csgn}(I\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^5-1/12*Pi^2*x^3\text{csgn}(I\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3\text{csgn}(I/(\exp(2*b*x+2*a)+1))^3+1/12*Pi^2*x^3\text{csgn}(I\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3\text{csgn}(I/(\exp(2*b*x+2*a)+1))^2-1/24*Pi^2*x^3\text{csgn}(I\exp(2*b*x+2*a))^3\text{csgn}(I\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3-1/12*Pi^2*x^3\text{csgn}(I\exp(2*b*x+2*a))^3\text{csgn}(I/(\exp(2*b*x+2*a)+1))^3-1/48*Pi^2*x^3\text{csgn}(I\exp(2*b*x+2*a))^2\text{csgn}(I\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^4+1/12*Pi^2*x^3\text{csgn}(I\exp(b*x+a))*\text{csgn}(I\exp(2*b*x+2*a))^5+1/24*Pi^2*x^3\text{csgn}(I\exp(2*b*x+2*a))^4\text{csgn}(I\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2-1/48*Pi^2*x^3\text{csgn}(I\exp(b*x+a))^4\text{csgn}(I\exp(2*b*x+2*a))^2+1/12*Pi^2*x^3\text{csgn}(I\exp(b*x+a))^3\text{csgn}(I\exp(2*b*x+2*a))^3-1/8*Pi^2*x^3\text{csgn}(I\exp(b*x+a))^2\text{csgn}(I\exp(2*b*x+2*a))^4-1/48*Pi^2*x^3\text{csgn}(I/(\exp(2*b*x+2*a)+1))^2\text{csgn}(I\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^4+1/24*Pi^2*x^3\text{csgn}(I/(\exp(2*b*x+2*a)+1))*\text{csgn}(I\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^5+1/12*Pi^2*x^3\text{csgn}(I\exp(2*b*x+2*a))*\text{csgn}(I\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2-1/12*Pi^2*x^3\text{csgn}(I\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2\text{csgn}(I/(\exp(2*b*x+2*a)+1))^3+1/12*Pi^2*x^3\text{csgn}(I\exp(2*b*x+2*a))^3\text{csgn}(I/(\exp(2*b*x+2*a)+1))^2-1/6*Pi^2*x^3\text{csgn}(I/(\exp(2*b*x+2*a)+1))^3-1/24*I*Pi*b*x^4\text{csgn}(I\exp(2*b*x+2*a))*\text{csgn}(I\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2-1/12*Pi^2*x^3\text{csgn}(I/(\exp(2*b*x+2*a)+1))*\text{csgn}(I\exp(2*b*x+2*a))*\text{csgn}(I\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^4-1/24*Pi^2*x^3\text{csgn}(I\exp(b*x+a))^2\text{csgn}(I\exp(2*b*x+2*a))*\text{csgn}(I\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3-1/12*Pi^2*x^3\text{csgn}(I\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3-1/12*Pi^2*x^3\text{csgn}(I\exp(2*b*x+2*a))^3+1/6*Pi^2*x^3\text{csgn}(I/(\exp(2*b*x+2*a)+1))^2-1/12*Pi^2*x^3\text{csgn}(I\exp(b*x+a))^2\text{csgn}(I\exp(2*b*x+2*a))*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^3-1/12*Pi^2*x^3\text{csgn}(I\exp(b*x+a))*\text{csgn}(I\exp(2*b*x+2*a))^3*\text{csgn}(I\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2+1/12*Pi^2*x^3\text{csgn}(I/(\exp(2*b*x+2*a)+1))^3\text{csgn}(I\exp(2*b*x+2*a))*\text{csgn}(I\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))-1/24*Pi^2*x^3\text{csgn}(I/(\exp(2*b*x+2*a)+1))*\text{csgn}(I\exp(b*x+a))^2\text{csgn}(I\exp(2*b*x+2*a))^2\text{csgn}(I\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))+1/24*Pi^2*x^3\text{csgn}(I/(\exp(2*b*x+2*a)+1))*\text{csgn}(I\exp(b*x+a))^2\text{csgn}(I\exp(2*b*x+2*a))*\text{csgn}(I\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2+1/12*Pi^2*x^3\text{csgn}(I/(\exp(2*b*x+2*a)+1))*\text{csgn}(I\exp(b*x+a))*\text{csgn}(I\exp(2*b*x+2*a))^3\text{csgn}(I\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))-1/12*Pi^2*x^3\text{csgn}(I/(\exp(2*b*x+2*a)+1))*\text{csgn}(I\exp(b*x+a))*\text{csgn}(I\exp(2*b*x+2*a))^2\text{csgn}(I\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2-1/48*Pi^2*x^3\text{csgn}(I\exp(2*b*x+2*a))^6-1/48*Pi^2*x^3\text{csgn}(I\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^6+1/30*x^5*b^2+1/12*Pi^2*x^3\text{csgn}(I/(\exp(2*b*x+2*a)+1))*\text{csgn}(I
\end{aligned}$$

$$\begin{aligned} & \exp(2bx+2a)/(\exp(2bx+2a)+1))^{-2} - 1/12\pi^2 x^3 \operatorname{csgn}(I\exp(bx+a))^{-2} \operatorname{csgn}(I\exp(2bx+2a)) + 1/6\pi^2 x^3 \operatorname{csgn}(I\exp(bx+a)) \operatorname{csgn}(I\exp(2bx+2a))^{-2} \\ & + 1/24\pi^2 I\pi b^2 x^4 \operatorname{csgn}(I/(\exp(2bx+2a)+1)) \operatorname{csgn}(I\exp(2bx+2a)) \operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^{-1} \\ & - 1/12\pi^2 x^3 \operatorname{csgn}(I/(\exp(2bx+2a)+1))^{-4} - 1/24\pi^2 I\pi b^2 x^4 \operatorname{csgn}(I/(\exp(2bx+2a)+1)) \operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^{-2} \\ & + 1/24\pi^2 I\pi b^2 x^4 \operatorname{csgn}(I\exp(bx+a))^{-2} \operatorname{csgn}(I\exp(2bx+2a)) - 1/12\pi^2 I\pi b^2 x^4 \operatorname{csgn}(I\exp(bx+a)) \operatorname{csgn}(I\exp(2bx+2a))^{-2} \\ & + 1/12\pi^2 x^3 \operatorname{csgn}(I/(\exp(2bx+2a)+1))^{-2} \operatorname{csgn}(I\exp(bx+a))^{-2} \operatorname{csgn}(I\exp(2bx+2a)) + 1/24\pi^2 I\pi b^2 x^4 \operatorname{csgn}(I\exp(2bx+2a))^{-3} \\ & + 1/24\pi^2 x^3 \operatorname{csgn}(I/(\exp(2bx+2a)+1))^{-2} \operatorname{csgn}(I\exp(2bx+2a))^{-2} \operatorname{csgn}(I\exp(2bx+2a)) + 1/24\pi^2 I\pi b^2 x^4 \operatorname{csgn}(I\exp(2bx+2a))^{-3} \\ & + 1/24\pi^2 x^3 \operatorname{csgn}(I\exp(bx+a))^{-2} \operatorname{csgn}(I\exp(2bx+2a))^{-2} \operatorname{csgn}(I\exp(2bx+2a)) + 1/24\pi^2 I\pi b^2 x^4 \operatorname{csgn}(I\exp(2bx+2a))^{-3} \\ & + 1/12\pi^2 I\pi b^2 x^4 - 1/24\pi^2 x^3 \operatorname{csgn}(I/(\exp(2bx+2a)+1)) \operatorname{csgn}(I\exp(2bx+2a))^{-4} \operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1)) + 1/24\pi^2 x^3 \operatorname{csgn}(I/(\exp(2bx+2a)+1)) \operatorname{csgn}(I\exp(2bx+2a))^{-3} \\ & \operatorname{csgn}(I\exp(2bx+2a))^{-3} \operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^{-2} + 1/24\pi^2 x^3 \operatorname{csgn}(I/(\exp(2bx+2a)+1)) \operatorname{csgn}(I\exp(2bx+2a))^{-2} \operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^{-3} \\ & + 1/12\pi^2 x^3 \operatorname{csgn}(I/(\exp(2bx+2a)+1))^{-3} \operatorname{csgn}(I\exp(2bx+2a)) \operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^{-2} \end{aligned}$$

**Maxima [A]** time = 1.37034, size = 49, normalized size = 1.17

$$\frac{1}{30} b^2 x^5 - \frac{1}{6} b x^4 \operatorname{arccoth}(\tanh(bx+a)) + \frac{1}{3} x^3 \operatorname{arccoth}(\tanh(bx+a))^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(tanh(b\*x+a))^2,x, algorithm="maxima")

[Out] 1/30\*b^2\*x^5 - 1/6\*b\*x^4\*arccoth(tanh(b\*x + a)) + 1/3\*x^3\*arccoth(tanh(b\*x + a))^2

**Fricas [A]** time = 1.63241, size = 72, normalized size = 1.71

$$\frac{1}{5} b^2 x^5 + \frac{1}{2} a b x^4 - \frac{1}{12} (\pi^2 - 4 a^2) x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(tanh(b\*x+a))^2,x, algorithm="fricas")

[Out]  $\frac{1}{5}b^2x^5 + \frac{1}{2}abx^4 - \frac{1}{12}(\pi^2 - 4a^2)x^3$

**Sympy [A]** time = 1.88637, size = 60, normalized size = 1.43

$$\begin{cases} \frac{x^2 \operatorname{acoth}^3(\tanh(ax+bx))}{3b} - \frac{x \operatorname{acoth}^4(\tanh(ax+bx))}{6b^2} + \frac{\operatorname{acoth}^5(\tanh(ax+bx))}{30b^3} & \text{for } b \neq 0 \\ \frac{x^3 \operatorname{acoth}^2(\tanh(a))}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*acoth(tanh(b*x+a))**2,x)`

[Out] `Piecewise((x**2*acoth(tanh(a + b*x))**3/(3*b) - x*acoth(tanh(a + b*x))**4/(6*b**2) + acoth(tanh(a + b*x))**5/(30*b**3), Ne(b, 0)), (x**3*acoth(tanh(a))**2/3, True))`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arccoth}(\tanh(bx+a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccoth(tanh(b*x+a))^2,x, algorithm="giac")`

[Out] `integrate(x^2*arccoth(tanh(b*x + a))^2, x)`

### 3.139 $\int x \coth^{-1}(\tanh(a + bx))^2 dx$

**Optimal.** Leaf size=34

$$\frac{x \coth^{-1}(\tanh(a + bx))^3}{3b} - \frac{\coth^{-1}(\tanh(a + bx))^4}{12b^2}$$

[Out]  $(x \cdot \text{ArcCoth}[\text{Tanh}[a + b \cdot x]]^3)/(3 \cdot b) - \text{ArcCoth}[\text{Tanh}[a + b \cdot x]]^4/(12 \cdot b^2)$

**Rubi [A]** time = 0.0253549, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {2168, 2157, 30}

$$\frac{x \coth^{-1}(\tanh(a + bx))^3}{3b} - \frac{\coth^{-1}(\tanh(a + bx))^4}{12b^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x \cdot \text{ArcCoth}[\text{Tanh}[a + b \cdot x]]^2, x]$

[Out]  $(x \cdot \text{ArcCoth}[\text{Tanh}[a + b \cdot x]]^3)/(3 \cdot b) - \text{ArcCoth}[\text{Tanh}[a + b \cdot x]]^4/(12 \cdot b^2)$

#### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

#### Rule 2157

```
Int[(u_)^(m_), x_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

#### Rule 30

```
Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x \coth^{-1}(\tanh(a + bx))^2 dx &= \frac{x \coth^{-1}(\tanh(a + bx))^3}{3b} - \frac{\int \coth^{-1}(\tanh(a + bx))^3 dx}{3b} \\
&= \frac{x \coth^{-1}(\tanh(a + bx))^3}{3b} - \frac{\text{Subst}\left(\int x^3 dx, x, \coth^{-1}(\tanh(a + bx))\right)}{3b^2} \\
&= \frac{x \coth^{-1}(\tanh(a + bx))^3}{3b} - \frac{\coth^{-1}(\tanh(a + bx))^4}{12b^2}
\end{aligned}$$

**Mathematica [B]** time = 0.0822737, size = 74, normalized size = 2.18

$$\frac{(a + bx) \left( 4(2a^2 + abx - b^2x^2) \coth^{-1}(\tanh(a + bx)) - (3a - bx)(a + bx)^2 - 6(a - bx) \coth^{-1}(\tanh(a + bx))^2 \right)}{12b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[Tanh[a + b\*x]]^2,x]

[Out] ((a + b\*x)\*(-((3\*a - b\*x)\*(a + b\*x)^2) + 4\*(2\*a^2 + a\*b\*x - b^2\*x^2)\*ArcCot h[Tanh[a + b\*x]] - 6\*(a - b\*x)\*ArcCoth[Tanh[a + b\*x]]^2))/(12\*b^2)

**Maple [C]** time = 0.368, size = 3418, normalized size = 100.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(tanh(b\*x+a))^2,x)

[Out]  $-1/4*\text{Pi}^2*x^2*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^2*\text{csgn}(I*\exp(b*x+a))*\text{csgn}(I*\exp(2*b*x+2*a))^2 - 1/8*\text{Pi}^2*x^2*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^2*\text{csgn}(I*\exp(2*b*x+2*a))*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2 + 1/4*\text{Pi}^2*x^2*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^5 + 1/16*\text{Pi}^2*x^2*\text{csgn}(I*\exp(b*x+a))^2*\text{csgn}(I*\exp(2*b*x+2*a))^2*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2 + 1/16*\text{Pi}^2*x^2*\text{csgn}(I/(\exp(2*b*x+2*a)+1))*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^5 - 1/8*\text{Pi}^2*x^2 + 1/2*x^2*\ln(\exp(b*x+a))^2 + 1/16*\text{Pi}^2*x^2*\text{csgn}(I/(\exp(2*b*x+2*a)+1))*\text{csgn}(I*\exp(2*b*x+2*a))^3*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2 + 1/16*\text{Pi}^2*x^2*\text{csgn}(I/(\exp(2*b*x+2*a)+1))*\text{csgn}(I*\exp(2*b*x+2*a))^2*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2$

$$\begin{aligned}
& *a)+1))^3-1/8\pi^2x^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))^3\operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^3-1/8\pi^2x^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))^3\operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2+1/8\pi^2x^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))^2\operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^3+1/8\pi^2x^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))^2\operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^3-1/8\pi^2x^2\operatorname{csgn}(I\exp(bx+a))^2\operatorname{csgn}(I\exp(2bx+2a))+1/4\pi^2x^2\operatorname{csgn}(I\exp(bx+a))\operatorname{csgn}(I\exp(2bx+2a))^2+1/8\pi^2x^2\operatorname{csgn}(I\exp(2bx+2a))\operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2-1/32\pi^2x^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))^2\operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^4-1/32\pi^2x^2\operatorname{csgn}(I\exp(2bx+2a))^2\operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^4+1/16\pi^2x^2\operatorname{csgn}(I\exp(2bx+2a))\operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^5+1/8\pi^2x^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))\operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2-3/16\pi^2x^2\operatorname{csgn}(I\exp(bx+a))^2\operatorname{csgn}(I\exp(2bx+2a))^4+1/8\pi^2x^2\operatorname{csgn}(I\exp(bx+a))\operatorname{csgn}(I\exp(2bx+2a))^5+1/16\pi^2x^2\operatorname{csgn}(I\exp(2bx+2a))^4\operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2-1/16\pi^2x^2\operatorname{csgn}(I\exp(2bx+2a))^3\operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^3-1/8\pi^2x^2\operatorname{csgn}(I\exp(2bx+2a))^3\operatorname{csgn}(I/(\exp(2bx+2a)+1))^4+1/8\pi^2x^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))^4\operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2-1/32\pi^2x^2\operatorname{csgn}(I\exp(bx+a))^4\operatorname{csgn}(I\exp(2bx+2a))^2+1/8\pi^2x^2\operatorname{csgn}(I\exp(bx+a))^3\operatorname{csgn}(I\exp(2bx+2a))^3-1/32\pi^2x^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))^2\operatorname{csgn}(I\exp(2bx+2a))^2\operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2+1/16\pi^2x^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))^2\operatorname{csgn}(I\exp(2bx+2a))\operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^3-1/16\pi^2x^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))\operatorname{csgn}(I\exp(2bx+2a))^4\operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))-1/4\pi^2x^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))^3-1/8\pi^2x^2\operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^3-1/32\pi^2x^2\operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^6-1/16\pi^2x^2\operatorname{csgn}(I\exp(bx+a))^2\operatorname{csgn}(I\exp(2bx+2a))\operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^3-1/8\pi^2x^2\operatorname{csgn}(I\exp(bx+a))^2\operatorname{csgn}(I\exp(2bx+2a))\operatorname{csgn}(I/(\exp(2bx+2a)+1))^3+1/12I\pi^2x^2\operatorname{csgn}(I\exp(2bx+2a))^3-1/16\pi^2x^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))\operatorname{csgn}(I\exp(bx+a))^2\operatorname{csgn}(I\exp(2bx+2a))^2\operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))+1/16\pi^2x^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))\operatorname{csgn}(I\exp(bx+a))^2\operatorname{csgn}(I\exp(2bx+2a))\operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2-1/8\pi^2x^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))^6-1/8\pi^2x^2\operatorname{csgn}(I\exp(2bx+2a))^3-1/32\pi^2x^2\operatorname{csgn}(I\exp(2bx+2a))^6-1/8\pi^2x^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))^4+1/4\pi^2x^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))^2+(-1/3bx^3+1/4I\pi^2x^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))\operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2-1/4I\pi^2x^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))\operatorname{csgn}(I\exp(2bx+2a))\operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))+1/4I\pi^2x^2\operatorname{csgn}(I\exp(2bx+2a))\operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2-1/2I\pi^2x^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))^3-1/4I\pi^2x^2\operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^3+1/2I\pi^2x^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))^2+1/2I\pi^2x^2\operatorname{csgn}(I\exp(bx+a))\operatorname{csgn}(I\exp(2bx+2a))^2-1/4I\pi^2x^2\operatorname{csgn}(I\exp(2bx+2a))^3-1/2I\pi^2x^2-1/4I\pi^2x^2\operatorname{csgn}(I\exp(bx+a))^2\operatorname{csgn}(I\exp(2bx+2a))\ln(\exp(bx+a))+1/12I\pi^2x^2\operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^3+1/8\pi^2x^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))^2\operatorname{csgn}(I\exp(bx+a))^2\operatorname{csgn}(I\exp(2bx+2a))+1/12x^4b^2+1/8\pi^2x^2\operatorname{csgn}
\end{aligned}$$

$(I/(\exp(2bx+2a)+1))^3 \operatorname{csgn}(I\exp(2bx+2a)) \operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2 + 1/8 \pi^2 x^2 \operatorname{csgn}(I/(\exp(2bx+2a)+1))^3 \operatorname{csgn}(I\exp(2bx+2a)) \operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1)) + 1/6 I \pi b x^3 \operatorname{csgn}(I/(\exp(2bx+2a)+1))^3 + 1/4 \pi^2 x^2 \operatorname{csgn}(I\exp(bx+a)) \operatorname{csgn}(I\exp(2bx+2a))^2 \operatorname{csgn}(I/(\exp(2bx+2a)+1))^3 - 1/8 \pi^2 x^2 \operatorname{csgn}(I\exp(bx+a)) \operatorname{csgn}(I\exp(2bx+2a))^3 \operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2 - 1/8 \pi^2 x^2 \operatorname{csgn}(I/(\exp(2bx+2a)+1)) \operatorname{csgn}(I\exp(2bx+2a)) \operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1)) + 1/6 I \pi b x^3 + 1/12 I \pi b x^3 \operatorname{csgn}(I/(\exp(2bx+2a)+1)) \operatorname{csgn}(I\exp(2bx+2a)) \operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1)) - 1/8 \pi^2 x^2 \operatorname{csgn}(I/(\exp(2bx+2a)+1)) \operatorname{csgn}(I\exp(2bx+2a)) \operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^4 + 1/8 \pi^2 x^2 \operatorname{csgn}(I\exp(bx+a)) \operatorname{csgn}(I\exp(2bx+2a))^2 \operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^3 - 1/6 I \pi b x^3 \operatorname{csgn}(I/(\exp(2bx+2a)+1))^2 - 1/8 \pi^2 x^2 \operatorname{csgn}(I/(\exp(2bx+2a)+1))^4 \operatorname{csgn}(I\exp(2bx+2a)) \operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1)) - 1/6 I \pi b x^3 \operatorname{csgn}(I\exp(bx+a)) \operatorname{csgn}(I\exp(2bx+2a))^2 - 1/12 I \pi b x^3 \operatorname{csgn}(I\exp(2bx+2a)) \operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2 + 1/8 \pi^2 x^2 \operatorname{csgn}(I/(\exp(2bx+2a)+1)) \operatorname{csgn}(I\exp(bx+a)) \operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1)) - 1/8 \pi^2 x^2 \operatorname{csgn}(I/(\exp(2bx+2a)+1)) \operatorname{csgn}(I\exp(bx+a)) \operatorname{csgn}(I\exp(2bx+2a))^2 \operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2 - 1/12 I \pi b x^3 \operatorname{csgn}(I/(\exp(2bx+2a)+1)) \operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2 + 1/12 I \pi b x^3 \operatorname{csgn}(I\exp(bx+a))^2 \operatorname{csgn}(I\exp(2bx+2a))$

**Maxima [A]** time = 1.36817, size = 49, normalized size = 1.44

$$\frac{1}{12} b^2 x^4 - \frac{1}{3} b x^3 \operatorname{arccoth}(\tanh(bx+a)) + \frac{1}{2} x^2 \operatorname{arccoth}(\tanh(bx+a))^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(tanh(b\*x+a))^2,x, algorithm="maxima")

[Out] 1/12\*b^2\*x^4 - 1/3\*b\*x^3\*arccoth(tanh(b\*x + a)) + 1/2\*x^2\*arccoth(tanh(b\*x + a))^2

**Fricas [A]** time = 1.52462, size = 70, normalized size = 2.06

$$\frac{1}{4} b^2 x^4 + \frac{2}{3} a b x^3 - \frac{1}{8} (\pi^2 - 4 a^2) x^2$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(x\*arccoth(tanh(b\*x+a))^2,x, algorithm="fricas")

[Out]  $1/4*b^2*x^4 + 2/3*a*b*x^3 - 1/8*(\pi^2 - 4*a^2)*x^2$

**Sympy [A]** time = 0.586048, size = 37, normalized size = 1.09

$$\frac{b^2x^4}{12} - \frac{bx^3 \operatorname{acoth}(\tanh(a + bx))}{3} + \frac{x^2 \operatorname{acoth}^2(\tanh(a + bx))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(tanh(b\*x+a))\*\*2,x)

[Out]  $b**2*x**4/12 - b*x**3*acoth(\tanh(a + b*x))/3 + x**2*acoth(\tanh(a + b*x))**2/2$

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arccoth}(\tanh(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(tanh(b\*x+a))^2,x, algorithm="giac")

[Out] integrate(x\*arccoth(tanh(b\*x + a))^2, x)

$$3.140 \quad \int \coth^{-1}(\tanh(a + bx))^2 dx$$

**Optimal.** Leaf size=16

$$\frac{\coth^{-1}(\tanh(a + bx))^3}{3b}$$

[Out] ArcCoth[Tanh[a + b\*x]]^3/(3\*b)

**Rubi [A]** time = 0.0053532, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {2157, 30}

$$\frac{\coth^{-1}(\tanh(a + bx))^3}{3b}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^2,x]

[Out] ArcCoth[Tanh[a + b\*x]]^3/(3\*b)

#### Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[  
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \coth^{-1}(\tanh(a + bx))^2 dx &= \frac{\text{Subst}\left(\int x^2 dx, x, \coth^{-1}(\tanh(a + bx))\right)}{b} \\ &= \frac{\coth^{-1}(\tanh(a + bx))^3}{3b} \end{aligned}$$

**Mathematica [A]** time = 0.00554, size = 16, normalized size = 1.

$$\frac{\coth^{-1}(\tanh(a + bx))^3}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^2,x]

[Out] ArcCoth[Tanh[a + b\*x]]^3/(3\*b)

**Maple [A]** time = 0.098, size = 15, normalized size = 0.9

$$\frac{(\operatorname{arccoth}(\tanh(bx + a)))^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b\*x+a))^2,x)

[Out] 1/3\*arccoth(tanh(b\*x+a))^3/b

**Maxima [B]** time = 1.37255, size = 45, normalized size = 2.81

$$\frac{1}{3}b^2x^3 - bx^2 \operatorname{arccoth}(\tanh(bx + a)) + x \operatorname{arccoth}(\tanh(bx + a))^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^2,x, algorithm="maxima")

[Out] 1/3\*b^2\*x^3 - b\*x^2\*arccoth(tanh(b\*x + a)) + x\*arccoth(tanh(b\*x + a))^2

**Fricas [A]** time = 1.62762, size = 62, normalized size = 3.88

$$\frac{1}{3}b^2x^3 + abx^2 - \frac{1}{4}(\pi^2 - 4a^2)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/3*b^2*x^3 + a*b*x^2 - 1/4*(pi^2 - 4*a^2)*x
```

**Sympy [A]** time = 0.414211, size = 20, normalized size = 1.25

$$\begin{cases} \frac{\operatorname{acoth}^3(\tanh(a+bx))}{3^b} & \text{for } b \neq 0 \\ x \operatorname{acoth}^2(\tanh(a)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**2,x)
```

```
[Out] Piecewise((acoth(tanh(a + b*x))**3/(3*b), Ne(b, 0)), (x*acoth(tanh(a))**2, True))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(\tanh(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^2, x)
```

$$3.141 \quad \int \frac{\coth^{-1}(\tanh(a+bx))^2}{x} dx$$

**Optimal.** Leaf size=49

$$-bx(bx - \coth^{-1}(\tanh(a + bx))) + \frac{1}{2} \coth^{-1}(\tanh(a + bx))^2 + \log(x) (bx - \coth^{-1}(\tanh(a + bx)))^2$$

[Out]  $-(b*x*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])) + \text{ArcCoth}[\text{Tanh}[a + b*x]]^2/2 + (b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^2*\text{Log}[x]$

**Rubi [A]** time = 0.0259427, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2159, 2158, 29}

$$-bx(bx - \coth^{-1}(\tanh(a + bx))) + \frac{1}{2} \coth^{-1}(\tanh(a + bx))^2 + \log(x) (bx - \coth^{-1}(\tanh(a + bx)))^2$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{ArcCoth}[\text{Tanh}[a + b*x]]^2/x, x]$

[Out]  $-(b*x*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])) + \text{ArcCoth}[\text{Tanh}[a + b*x]]^2/2 + (b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^2*\text{Log}[x]$

### Rule 2159

$\text{Int}[(v_)^n/(u_), x\_Symbol] \rightarrow \text{With}[\{a = \text{Simplify}[D[u, x]], b = \text{Simplify}[D[v, x]]\}, \text{Simp}[v^n/(a*n), x] - \text{Dist}[(b*u - a*v)/a, \text{Int}[v^{n-1}/u, x], x] /; \text{NeQ}[b*u - a*v, 0] /; \text{PiecewiseLinearQ}[u, v, x] \&\& \text{GtQ}[n, 0] \&\& \text{NeQ}[n, 1]$

### Rule 2158

$\text{Int}[(v_)/(u_), x\_Symbol] \rightarrow \text{With}[\{a = \text{Simplify}[D[u, x]], b = \text{Simplify}[D[v, x]]\}, \text{Simp}[(b*x)/a, x] - \text{Dist}[(b*u - a*v)/a, \text{Int}[1/u, x], x] /; \text{NeQ}[b*u - a*v, 0] /; \text{PiecewiseLinearQ}[u, v, x]$

### Rule 29

$\text{Int}[(x_)^{-1}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[x], x]$

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x} dx &= \frac{1}{2} \coth^{-1}(\tanh(a+bx))^2 - (bx - \coth^{-1}(\tanh(a+bx))) \int \frac{\coth^{-1}(\tanh(a+bx))}{x} dx \\
&= -bx (bx - \coth^{-1}(\tanh(a+bx))) + \frac{1}{2} \coth^{-1}(\tanh(a+bx))^2 - ((bx - \coth^{-1}(\tanh(a+bx))) \int \frac{\coth^{-1}(\tanh(a+bx))}{x} dx) \\
&= -bx (bx - \coth^{-1}(\tanh(a+bx))) + \frac{1}{2} \coth^{-1}(\tanh(a+bx))^2 + (bx - \coth^{-1}(\tanh(a+bx))) \int \frac{\coth^{-1}(\tanh(a+bx))}{x} dx
\end{aligned}$$

**Mathematica [A]** time = 0.0427117, size = 53, normalized size = 1.08

$$\frac{1}{2}(a+bx)^2 - (a+bx)(-2\coth^{-1}(\tanh(a+bx)) + a + 2bx) + \log(bx)(\coth^{-1}(\tanh(a+bx)) - bx)^2$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^2/x,x]

[Out] (a + b\*x)^2/2 - (a + b\*x)\*(a + 2\*b\*x - 2\*ArcCoth[Tanh[a + b\*x]]) + (-b\*x) + ArcCoth[Tanh[a + b\*x]]^2\*Log[b\*x]

**Maple [C]** time = 0.284, size = 3774, normalized size = 77.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b\*x+a))^2/x,x)

[Out]  $-1/8*\ln(x)*\text{Pi}^2*\text{csgn}(I/(\exp(2*b*x+2*a)+1))*\text{csgn}(I*\exp(b*x+a))^2*\text{csgn}(I*\exp(2*b*x+2*a))^2*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))+I*\text{Pi}*b*x*\text{csgn}(I*\exp(b*x+a))*\text{csgn}(I*\exp(2*b*x+2*a))^2-1/2*I*\ln(\exp(b*x+a))*\ln(x)*\text{Pi}*\text{csgn}(I*\exp(2*b*x+2*a))^3-1/2*I*\text{Pi}*x*b*\text{csgn}(I*\exp(2*b*x+2*a))^3-1/2*I*\text{Pi}*x*b*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3-1/8*\ln(x)*\text{Pi}^2*\text{csgn}(I/(\exp(2*b*x+2*a)+1))*\text{csgn}(I*\exp(2*b*x+2*a))^4*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))-1/2*I*\ln(\exp(b*x+a))*\ln(x)*\text{Pi}*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3+1/8*\ln(x)*\text{Pi}^2*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^2*\text{csgn}(I*\exp(2*b*x+2*a))*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3+1/2*I*\ln(x)*\text{Pi}*x*b*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3+1/2*I*\text{Pi}*x*b*\text{csgn}(I*\exp(2*b*x+2*a))*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3+1/2*I*\text{Pi}*x*b*\text{csgn}(I*\exp(2*b*x+2*a))*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3$

$$\begin{aligned}
& \exp(2bx+2a)+1)^{-2-1/2} I \ln(x) \Pi x^* b \operatorname{csgn}(I \exp(2bx+2a)) \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{-2-1/2} I \ln(\exp(bx+a)) \ln(x) \Pi \operatorname{csgn}(I / (\exp(2bx+2a)+1)) \operatorname{csgn}(I \exp(2bx+2a)) \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1)) + b^2 x^2 \ln(x) + 2b \ln(\exp(bx+a)) x - 1/2 \Pi^2 \ln(x) \operatorname{csgn}(I / (\exp(2bx+2a)+1))^{-3+1/2} \Pi^2 \ln(x) \operatorname{csgn}(I / (\exp(2bx+2a)+1))^{-2-1/2} I \Pi x^* b \operatorname{csgn}(I / (\exp(2bx+2a)+1)) \operatorname{csgn}(I \exp(2bx+2a)) \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1)) + 1/2 \ln(x) \Pi^2 \operatorname{csgn}(I / (\exp(2bx+2a)+1))^{-3} \operatorname{csgn}(I \exp(bx+a)) \operatorname{csgn}(I \exp(2bx+2a))^{-2-1/16} \ln(x) \Pi^2 \operatorname{csgn}(I \exp(bx+a))^{-4} \operatorname{csgn}(I \exp(2bx+2a))^{-2+1/4} \ln(x) \Pi^2 \operatorname{csgn}(I \exp(bx+a))^{-3} \operatorname{csgn}(I \exp(2bx+2a))^{-3-1/4} \ln(x) \Pi^2 \operatorname{csgn}(I / (\exp(2bx+2a)+1))^{-3} \operatorname{csgn}(I \exp(bx+a))^{-2} \operatorname{csgn}(I \exp(2bx+2a)) + 1/4 \ln(x) \Pi^2 \operatorname{csgn}(I / (\exp(2bx+2a)+1))^{-3} \operatorname{csgn}(I \exp(2bx+2a)) \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{-2-1/16} \ln(x) \Pi^2 \operatorname{csgn}(I \exp(2bx+2a))^{-2} \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{-4+1/8} \ln(x) \Pi^2 \operatorname{csgn}(I \exp(2bx+2a)) \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{-5-1/4} \ln(x) \Pi^2 \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{-3} \operatorname{csgn}(I / (\exp(2bx+2a)+1))^{-3+1/4} \ln(x) \Pi^2 \operatorname{csgn}(I / (\exp(2bx+2a)+1)) \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{-2-1} \ln(\exp(bx+a)) \ln(x) \Pi \operatorname{csgn}(I / (\exp(2bx+2a)+1))^{-3+1} \ln(\exp(bx+a)) \ln(x) \Pi \operatorname{csgn}(I / (\exp(2bx+2a)+1))^{-2-1} \Pi x^* b \operatorname{csgn}(I / (\exp(2bx+2a)+1))^{-3+1/8} \ln(x) \Pi^2 \operatorname{csgn}(I / (\exp(2bx+2a)+1)) \operatorname{csgn}(I \exp(2bx+2a))^{-3} \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{-2-1/16} \ln(x) \Pi^2 \operatorname{csgn}(I / (\exp(2bx+2a)+1))^{-2} \operatorname{csgn}(I \exp(2bx+2a))^{-2} \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{-2+1} \Pi x^* b \operatorname{csgn}(I / (\exp(2bx+2a)+1))^{-2-1/4} \ln(x) \Pi^2 \operatorname{csgn}(I \exp(bx+a)) \operatorname{csgn}(I \exp(2bx+2a))^{-3} \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{-2-3/2} b^2 x^2 + 1/8 \ln(x) \Pi^2 \operatorname{csgn}(I \exp(bx+a))^{-2} \operatorname{csgn}(I \exp(2bx+2a))^{-2} \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{-2+1/8} \ln(x) \Pi^2 \operatorname{csgn}(I / (\exp(2bx+2a)+1)) \operatorname{csgn}(I \exp(2bx+2a))^{-2} \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{-3-1/4} \ln(x) \Pi^2 \operatorname{csgn}(I / (\exp(2bx+2a)+1)) \operatorname{csgn}(I \exp(2bx+2a)) \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{-1/4} \ln(x) \Pi^2 \operatorname{csgn}(I / (\exp(2bx+2a)+1))^{-4} \operatorname{csgn}(I \exp(2bx+2a)) \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1)) + 1/4 \ln(x) \Pi^2 \operatorname{csgn}(I / (\exp(2bx+2a)+1))^{-3} \operatorname{csgn}(I \exp(2bx+2a)) \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{-1/2} \ln(x) \Pi^2 \operatorname{csgn}(I \exp(bx+a)) \operatorname{csgn}(I \exp(2bx+2a))^{-2} \operatorname{csgn}(I / (\exp(2bx+2a)+1))^{-2-1/4} \ln(x) \Pi^2 \operatorname{csgn}(I \exp(2bx+2a)) \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{-2} \operatorname{csgn}(I / (\exp(2bx+2a)+1))^{-2+1/4} \ln(x) \Pi^2 \operatorname{csgn}(I \exp(bx+a))^{-2} \operatorname{csgn}(I \exp(2bx+2a)) \operatorname{csgn}(I / (\exp(2bx+2a)+1))^{-2+1/2} \ln(x) \Pi^2 \operatorname{csgn}(I / (\exp(2bx+2a)+1))^{-5-1/4} \ln(x) \Pi^2 \operatorname{csgn}(I / (\exp(2bx+2a)+1))^{-4-1/4} \ln(x) \Pi^2 \operatorname{csgn}(I \exp(2bx+2a))^{-3-1/16} \ln(x) \Pi^2 \operatorname{csgn}(I \exp(2bx+2a))^{-6-1/4} \ln(x) \Pi^2 \operatorname{csgn}(I / (\exp(2bx+2a)+1))^{-6-1/4} \ln(x) \Pi^2 \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{-3-1} \ln(\exp(bx+a)) \ln(x) \Pi - I \Pi x^* b + 1/4 \ln(x) \Pi^2 \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{-3} \operatorname{csgn}(I / (\exp(2bx+2a)+1))^{-2-1/16} \ln(x) \Pi^2 \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{-6-3/8} \ln(x) \Pi^2 \operatorname{csgn}(I \exp(bx+a))^{-2} \operatorname{csgn}(I \exp(2bx+2a))^{-4+1/4} \ln(x) \Pi^2 \operatorname{csgn}(I \exp(bx+a)) \operatorname{csgn}(I \exp(2bx+2a))^{-5+1/8} \ln(x) \Pi^2 \operatorname{csgn}(I \exp(2bx+2a))^{-4} \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{-2-1/8} \ln(x) \Pi^2 \operatorname{csgn}(I \exp(2bx+2a))^{-3} \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{-3-1/16} \ln(x) \Pi^2 \operatorname{csgn}(I / (\exp(2bx+2a)+1))^{-2} \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))
\end{aligned}$$

$$\begin{aligned}
& p(2bx+2a+1))^{4+1/4} \ln(x) \pi^2 \operatorname{csgn}(I/(\exp(2bx+2a)+1))^{4} \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{2-1/4} \\
& \ln(x) \pi^2 \operatorname{csgn}(I \exp(2bx+2a))^{3} \operatorname{csgn}(I/(\exp(2bx+2a)+1))^{3-1/4} \\
& \pi^2 \ln(x) \operatorname{csgn}(I \exp(bx+a))^{2} \operatorname{csgn}(I \exp(2bx+2a))^{2+1/4} \\
& \pi^2 \ln(x) \operatorname{csgn}(I \exp(2bx+2a)) \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{2-1/4} \\
& \ln(x) \pi^2 \operatorname{csgn}(I/(\exp(2bx+2a)+1))^{3} \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{2+1/4} \\
& \ln(x) \pi^2 \operatorname{csgn}(I/(\exp(2bx+2a)+1))^{2} \operatorname{csgn}(I \exp(2bx+2a))^{3+1/2} \\
& I \ln(x) \pi \operatorname{csgn}(I/(\exp(2bx+2a)+1)) \operatorname{csgn}(I \exp(2bx+2a)) \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{-2} \\
& b \ln(\exp(bx+a)) \ln(x) \pi \operatorname{csgn}(I/(\exp(2bx+2a)+1)) \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{-5-1/8} \\
& \ln(x) \pi^2 \operatorname{csgn}(I \exp(bx+a))^{2} \operatorname{csgn}(I \exp(2bx+2a)) \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{3+1/4} \\
& \ln(x) \pi^2 \operatorname{csgn}(I \exp(bx+a)) \operatorname{csgn}(I \exp(2bx+2a))^{2} \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{3-1/4} \\
& \ln(x) \pi^2 \operatorname{csgn}(I/(\exp(2bx+2a)+1)) \operatorname{csgn}(I \exp(2bx+2a)) \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{-4} \\
& I \ln(x) \pi \operatorname{csgn}(I/(\exp(2bx+2a)+1))^{2-1/4} \pi^2 \ln(x) + I \ln(x) \pi \operatorname{csgn}(I/(\exp(2bx+2a)+1))^{3+1/4} \\
& \ln(x) \pi^2 \operatorname{csgn}(I/(\exp(2bx+2a)+1)) \operatorname{csgn}(I \exp(bx+a)) \operatorname{csgn}(I \exp(2bx+2a))^{3} \\
& \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{-1/4} \ln(x) \pi^2 \operatorname{csgn}(I/(\exp(2bx+2a)+1)) \operatorname{csgn}(I \exp(bx+a)) \\
& \operatorname{csgn}(I \exp(2bx+2a))^{2} \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{2+1/2} \\
& I \ln(x) \pi \operatorname{csgn}(I \exp(bx+a))^{2} \operatorname{csgn}(I \exp(2bx+2a)) - I \ln(x) \pi \operatorname{csgn}(I \exp(bx+a)) \operatorname{csgn}(I \exp(2bx+2a))^{2+1/2} \\
& I \ln(\exp(bx+a)) \ln(x) \pi \operatorname{csgn}(I/(\exp(2bx+2a)+1)) \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{2-1/2} \\
& I \ln(\exp(bx+a)) \ln(x) \pi \operatorname{csgn}(I \exp(bx+a))^{2} \operatorname{csgn}(I \exp(2bx+2a)) + 1/2 I \ln(x) \pi \operatorname{csgn}(I \exp(2bx+2a))^{3+1/2} \\
& I \ln(\exp(bx+a)) \ln(x) \pi \operatorname{csgn}(I \exp(2bx+2a)) \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{2-1/2} \\
& I \pi \operatorname{csgn}(I \exp(bx+a))^{2} \operatorname{csgn}(I \exp(2bx+2a)) + 1/8 \ln(x) \pi^2 \operatorname{csgn}(I/(\exp(2bx+2a)+1)) \operatorname{csgn}(I \exp(bx+a))^{2} \\
& \operatorname{csgn}(I \exp(2bx+2a)) \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{2+1} \ln(\exp(bx+a)) \ln(x) \pi \operatorname{csgn}(I \exp(bx+a)) \\
& \operatorname{csgn}(I \exp(2bx+2a))^{2+1/2} I \pi \operatorname{csgn}(I/(\exp(2bx+2a)+1)) \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{2-1/2} \\
& I \ln(x) \pi \operatorname{csgn}(I/(\exp(2bx+2a)+1)) \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{2}
\end{aligned}$$

---

**Maxima [C]** time = 2.47337, size = 51, normalized size = 1.04

$$\frac{1}{2} b^2 x^2 + \frac{1}{8} (-8i \pi b + 16 ab)x - \frac{1}{4} (\pi^2 + 4i \pi a - 4a^2) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^2/x,x, algorithm="maxima")

[Out] 1/2\*b^2\*x^2 + 1/8\*(-8\*I\*pi\*b + 16\*a\*b)\*x - 1/4\*(pi^2 + 4\*I\*pi\*a - 4\*a^2)\*lo



$g(x)$

**Fricas [A]** time = 1.65362, size = 69, normalized size = 1.41

$$\frac{1}{2}b^2x^2 + 2abx - \frac{1}{4}(\pi^2 - 4a^2)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^2/x,x, algorithm="fricas")

[Out] 1/2\*b^2\*x^2 + 2\*a\*b\*x - 1/4\*(pi^2 - 4\*a^2)\*log(x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^2(\tanh(a + bx))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(tanh(b\*x+a))\*\*2/x,x)

[Out] Integral(acoth(tanh(a + b\*x))\*\*2/x, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^2/x,x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a))^2/x, x)

$$3.142 \quad \int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2} dx$$

**Optimal.** Leaf size=39

$$-\frac{\coth^{-1}(\tanh(a+bx))^2}{x} - 2b \log(x) (bx - \coth^{-1}(\tanh(a+bx))) + 2b^2x$$

[Out] 2\*b^2\*x - ArcCoth[Tanh[a + b\*x]]^2/x - 2\*b\*(b\*x - ArcCoth[Tanh[a + b\*x]])\*Log[x]

**Rubi [A]** time = 0.0258467, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2168, 2158, 29}

$$-\frac{\coth^{-1}(\tanh(a+bx))^2}{x} - 2b \log(x) (bx - \coth^{-1}(\tanh(a+bx))) + 2b^2x$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^2/x^2,x]

[Out] 2\*b^2\*x - ArcCoth[Tanh[a + b\*x]]^2/x - 2\*b\*(b\*x - ArcCoth[Tanh[a + b\*x]])\*Log[x]

### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

### Rule 2158

```
Int[(v_)/(u_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a*v, 0] /; PiecewiseLinearQ[u, v, x]
```

### Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

### Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2} dx &= -\frac{\coth^{-1}(\tanh(a+bx))^2}{x} + (2b) \int \frac{\coth^{-1}(\tanh(a+bx))}{x} dx \\ &= 2b^2x - \frac{\coth^{-1}(\tanh(a+bx))^2}{x} - (2b(bx - \coth^{-1}(\tanh(a+bx)))) \int \frac{1}{x} dx \\ &= 2b^2x - \frac{\coth^{-1}(\tanh(a+bx))^2}{x} - 2b(bx - \coth^{-1}(\tanh(a+bx))) \log(x) \end{aligned}$$

**Mathematica [A]** time = 0.0489712, size = 37, normalized size = 0.95

$$-\frac{\coth^{-1}(\tanh(a+bx))^2}{x} + 2b(\log(x)+1)\coth^{-1}(\tanh(a+bx)) - 2b^2x\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^2/x^2,x]

[Out] -(ArcCoth[Tanh[a + b\*x]]^2/x) - 2\*b^2\*x\*Log[x] + 2\*b\*ArcCoth[Tanh[a + b\*x]]\*(1 + Log[x])

**Maple [C]** time = 0.21, size = 1095, normalized size = 28.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b\*x+a))^2/x^2,x)

[Out]  $-I\pi \ln(\exp(bx+a))/x \operatorname{csgn}(I \exp(bx+a)) \operatorname{csgn}(I \exp(2bx+2a))^{-2-1/2} I\pi \ln(\exp(bx+a))/x \operatorname{csgn}(I \exp(2bx+2a)) \operatorname{csgn}(I \exp(2bx+2a)/(\exp(2bx+2a)+1))^{2+1/2} I\pi \ln(x) b \operatorname{csgn}(I/(\exp(2bx+2a)+1)) \operatorname{csgn}(I \exp(2bx+2a)/(\exp(2bx+2a)+1))^{2+I\pi \ln(x) b \operatorname{csgn}(I \exp(bx+a)) \operatorname{csgn}(I \exp(2bx+2a))^{2+2b^2x-1/2} I\pi \ln(x) b \operatorname{csgn}(I/(\exp(2bx+2a)+1)) \operatorname{csgn}(I \exp(2bx+2a)) \operatorname{csgn}(I \exp(2bx+2a)/(\exp(2bx+2a)+1))+1/2 I\pi \ln(\exp(bx+a))/x \operatorname{csgn}(I/(\exp(2bx+2a)+1)) \operatorname{csgn}(I \exp(2bx+2a)) \operatorname{csgn}(I \exp(2bx+2a)/(\exp($

$$\begin{aligned}
& 2bx+2a+1)) - 2\ln(x) * x * b^2 + 2\ln(x) * \ln(\exp(bx+a)) * b - I\pi * \ln(x) * b * \operatorname{csgn}(I / (\exp(2bx+2a)+1))^3 - I\pi * \ln(\exp(bx+a)) / x * \operatorname{csgn}(I / (\exp(2bx+2a)+1))^2 + I\pi * \ln(x) * b * \operatorname{csgn}(I / (\exp(2bx+2a)+1))^2 + 1/2 * I\pi * \ln(\exp(bx+a)) / x * \operatorname{csgn}(I * \exp(2bx+2a))^3 + 1/16 * \pi^2 * (-2 * \operatorname{csgn}(I / (\exp(2bx+2a)+1))^2 + \operatorname{csgn}(I / (\exp(2bx+2a)+1)) * \operatorname{csgn}(I * \exp(2bx+2a)) * \operatorname{csgn}(I * \exp(2bx+2a) / (\exp(2bx+2a)+1)) - \operatorname{csgn}(I / (\exp(2bx+2a)+1)) * \operatorname{csgn}(I * \exp(2bx+2a) / (\exp(2bx+2a)+1))^2 + 2 * \operatorname{csgn}(I / (\exp(2bx+2a)+1))^3 + \operatorname{csgn}(I * \exp(bx+a))^2 * \operatorname{csgn}(I * \exp(2bx+2a)) - 2 * \operatorname{csgn}(I * \exp(bx+a)) * \operatorname{csgn}(I * \exp(2bx+2a))^2 + \operatorname{csgn}(I * \exp(2bx+2a))^3 - \operatorname{csgn}(I * \exp(2bx+2a)) * \operatorname{csgn}(I * \exp(2bx+2a) / (\exp(2bx+2a)+1))^2 + \operatorname{csgn}(I * \exp(2bx+2a) / (\exp(2bx+2a)+1))^3 + 2) ^2 / x + 1/2 * I\pi * \ln(\exp(bx+a)) / x * \operatorname{csgn}(I * \exp(bx+a))^2 * \operatorname{csgn}(I * \exp(2bx+2a)) + 1/2 * I\pi * \ln(x) * b * \operatorname{csgn}(I * \exp(2bx+2a)) * \operatorname{csgn}(I * \exp(2bx+2a) / (\exp(2bx+2a)+1))^2 - 1/2 * I\pi * \ln(x) * b * \operatorname{csgn}(I * \exp(bx+a))^2 * \operatorname{csgn}(I * \exp(2bx+2a)) - 1/2 * I\pi * \ln(\exp(bx+a)) / x * \operatorname{csgn}(I / (\exp(2bx+2a)+1)) * \operatorname{csgn}(I * \exp(2bx+2a) / (\exp(2bx+2a)+1))^2 - 1/2 * I\pi * \ln(x) * b * \operatorname{csgn}(I * \exp(2bx+2a))^3 - 1/2 * I\pi * \ln(x) * b * \operatorname{csgn}(I * \exp(2bx+2a) / (\exp(2bx+2a)+1))^3 + 1/2 * I\pi * \ln(\exp(bx+a)) / x * \operatorname{csgn}(I * \exp(2bx+2a) / (\exp(2bx+2a)+1))^3 + I\pi * \ln(\exp(bx+a)) / x - I\pi * \ln(x) * b + I\pi * \ln(\exp(bx+a)) / x * \operatorname{csgn}(I / (\exp(2bx+2a)+1))^3 - 1/x * \ln(\exp(bx+a))^2
\end{aligned}$$

**Maxima [A]** time = 1.19632, size = 73, normalized size = 1.87

$$2b \operatorname{arccoth}(\tanh(bx+a)) \log(x) - 2 \left( b \left( x + \frac{a}{b} \right) \log(x) - b \left( x + \frac{a \log(x)}{b} \right) \right) b - \frac{\operatorname{arccoth}(\tanh(bx+a))^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^2/x^2,x, algorithm="maxima")

[Out] 2\*b\*arccoth(tanh(b\*x + a))\*log(x) - 2\*(b\*(x + a/b)\*log(x) - b\*(x + a\*log(x)/b))\*b - arccoth(tanh(b\*x + a))^2/x

**Fricas [A]** time = 1.67534, size = 69, normalized size = 1.77

$$\frac{4b^2x^2 + 8abx \log(x) + \pi^2 - 4a^2}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^2/x^2,x, algorithm="fricas")

[Out]  $1/4*(4*b^2*x^2 + 8*a*b*x*\log(x) + \pi^2 - 4*a^2)/x$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^2(\tanh(a + bx))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(tanh(b*x+a))**2/x**2,x)`

[Out] `Integral(acoth(tanh(a + b*x))**2/x**2, x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(tanh(b*x+a))^2/x^2,x, algorithm="giac")`

[Out] `integrate(arccoth(tanh(b*x + a))^2/x^2, x)`

$$3.143 \quad \int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^3} dx$$

**Optimal.** Leaf size=36

$$-\frac{\coth^{-1}(\tanh(a+bx))^2}{2x^2} - \frac{b \coth^{-1}(\tanh(a+bx))}{x} + b^2 \log(x)$$

[Out] -((b\*ArcCoth[Tanh[a + b\*x]])/x) - ArcCoth[Tanh[a + b\*x]]^2/(2\*x^2) + b^2\*Log[x]

**Rubi [A]** time = 0.0228216, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2168, 29}

$$-\frac{\coth^{-1}(\tanh(a+bx))^2}{2x^2} - \frac{b \coth^{-1}(\tanh(a+bx))}{x} + b^2 \log(x)$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^2/x^3,x]

[Out] -((b\*ArcCoth[Tanh[a + b\*x]])/x) - ArcCoth[Tanh[a + b\*x]]^2/(2\*x^2) + b^2\*Log[x]

### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

### Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^3} dx &= -\frac{\coth^{-1}(\tanh(a+bx))^2}{2x^2} + b \int \frac{\coth^{-1}(\tanh(a+bx))}{x^2} dx \\
&= -\frac{b \coth^{-1}(\tanh(a+bx))}{x} - \frac{\coth^{-1}(\tanh(a+bx))^2}{2x^2} + b^2 \int \frac{1}{x} dx \\
&= -\frac{b \coth^{-1}(\tanh(a+bx))}{x} - \frac{\coth^{-1}(\tanh(a+bx))^2}{2x^2} + b^2 \log(x)
\end{aligned}$$

**Mathematica [A]** time = 0.0337682, size = 42, normalized size = 1.17

$$-\frac{2bx \coth^{-1}(\tanh(a+bx)) + \coth^{-1}(\tanh(a+bx))^2 - b^2 x^2 (2 \log(x) + 3)}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^2/x^3,x]

[Out]  $-(2*b*x*ArcCoth[Tanh[a + b*x]] + ArcCoth[Tanh[a + b*x]]^2 - b^2*x^2*(3 + 2*Log[x]))/(2*x^2)$

**Maple [C]** time = 0.375, size = 3213, normalized size = 89.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b\*x+a))^2/x^3,x)

[Out]  $-1/2/x^2*\ln(\exp(b*x+a))^2 - 1/4*(4*b*x - 2*I*Pi*csgn(I/(\exp(2*b*x+2*a)+1)))^3 + I*Pi*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2 - I*Pi*csgn(I*\exp(b*x+a))^2*csgn(I*\exp(2*b*x+2*a)) + 2*I*Pi*csgn(I/(\exp(2*b*x+2*a)+1))^2 - 2*I*Pi - I*Pi*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3 - I*Pi*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1)) + 2*I*Pi*csgn(I*\exp(b*x+a))*csgn(I*\exp(2*b*x+2*a))^2 - I*Pi*csgn(I*\exp(2*b*x+2*a))^3 + I*Pi*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2/x^2*\ln(\exp(b*x+a)) + 1/32*(6*Pi^2*csgn(I*\exp(b*x+a))^2*csgn(I*\exp(2*b*x+2*a))^4 + 8*I*Pi*b*x*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1)) + 4*Pi^2 + 4*Pi^2*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3 + Pi^2*csgn(I*\exp(2*b*x+2*a))^6 + 4*Pi^2*csgn(I*e$





$$\begin{aligned} & x+2a+1)^3+2\pi^2\operatorname{csgn}(I\exp(bx+a))^2\operatorname{csgn}(I\exp(2bx+2a))\operatorname{csgn}(I\exp( \\ & 2bx+2a)/(\exp(2bx+2a)+1))^3-8I\pi bxx\operatorname{csgn}(I/(\exp(2bx+2a)+1))\operatorname{csgn} \\ & (I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2-4\pi^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))^2 \\ & * \operatorname{csgn}(I\exp(bx+a))^2\operatorname{csgn}(I\exp(2bx+2a))+4\pi^2\operatorname{csgn}(I/(\exp(2bx+2a)+ \\ & 1))\operatorname{csgn}(I\exp(2bx+2a))\operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^4+\pi^2 \\ & * \operatorname{csgn}(I/(\exp(2bx+2a)+1))^2\operatorname{csgn}(I\exp(2bx+2a))^2\operatorname{csgn}(I\exp(2bx+2a) \\ & )/(\exp(2bx+2a)+1))^2-2\pi^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))\operatorname{csgn}(I\exp(bx+a) \\ & )^2\operatorname{csgn}(I\exp(2bx+2a))\operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2+4\pi \\ & ^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))\operatorname{csgn}(I\exp(bx+a))\operatorname{csgn}(I\exp(2bx+2a))^2* \operatorname{c} \\ & \operatorname{sgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2-4\pi^2\operatorname{csgn}(I/(\exp(2bx+2a)+1) \\ & )\operatorname{csgn}(I\exp(bx+a))\operatorname{csgn}(I\exp(2bx+2a))^3\operatorname{csgn}(I\exp(2bx+2a)/(\exp(2 \\ & bx+2a)+1))+2\pi^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))\operatorname{csgn}(I\exp(bx+a))^2\operatorname{csgn}(I \\ & \exp(2bx+2a))^2\operatorname{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1)))/x^2 \end{aligned}$$

**Maxima [A]** time = 1.40233, size = 46, normalized size = 1.28

$$b^2 \log(x) - \frac{b \operatorname{arccoth}(\tanh(bx + a))}{x} - \frac{\operatorname{arccoth}(\tanh(bx + a))^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^2/x^3,x, algorithm="maxima")

[Out] b^2\*log(x) - b\*arccoth(tanh(b\*x + a))/x - 1/2\*arccoth(tanh(b\*x + a))^2/x^2

**Fricas [A]** time = 1.6086, size = 73, normalized size = 2.03

$$\frac{8b^2x^2 \log(x) - 16abx + \pi^2 - 4a^2}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^2/x^3,x, algorithm="fricas")

[Out] 1/8\*(8\*b^2\*x^2\*log(x) - 16\*a\*b\*x + pi^2 - 4\*a^2)/x^2

**Sympy [A]** time = 0.752501, size = 32, normalized size = 0.89

$$b^2 \log(x) - \frac{b \operatorname{acoth}(\tanh(a + bx))}{x} - \frac{\operatorname{acoth}^2(\tanh(a + bx))}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(tanh(b\*x+a))\*\*2/x\*\*3,x)

[Out] b\*\*2\*log(x) - b\*acoth(tanh(a + b\*x))/x - acoth(tanh(a + b\*x))\*\*2/(2\*x\*\*2)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^2/x^3,x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a))^2/x^3, x)

$$3.144 \quad \int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4} dx$$

**Optimal.** Leaf size=31

$$\frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3(bx - \coth^{-1}(\tanh(a+bx)))}$$

[Out] ArcCoth[Tanh[a + b\*x]]^3/(3\*x^3\*(b\*x - ArcCoth[Tanh[a + b\*x]]))

**Rubi [A]** time = 0.0135441, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2167}

$$\frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3(bx - \coth^{-1}(\tanh(a+bx)))}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^2/x^4,x]

[Out] ArcCoth[Tanh[a + b\*x]]^3/(3\*x^3\*(b\*x - ArcCoth[Tanh[a + b\*x]]))

#### Rule 2167

Int[(u\_)^(m\_)\*(v\_)^(n\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, -Simp[(u^(m + 1)\*v^(n + 1))/((m + 1)\*(b\*u - a\*v)), x] /; NeQ[b\*u - a\*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]

#### Rubi steps

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4} dx = \frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3(bx - \coth^{-1}(\tanh(a+bx)))}$$

**Mathematica [A]** time = 0.0474149, size = 34, normalized size = 1.1

$$\frac{bx \coth^{-1}(\tanh(a+bx)) + \coth^{-1}(\tanh(a+bx))^2 + b^2x^2}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^2/x^4,x]

[Out]  $-(b^2x^2 + b*x*\text{ArcCoth}[\text{Tanh}[a + b*x]] + \text{ArcCoth}[\text{Tanh}[a + b*x]]^2)/(3x^3)$

**Maple [C]** time = 0.357, size = 3217, normalized size = 103.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b\*x+a))^2/x^4,x)

[Out] 
$$-1/3/x^3*\ln(\exp(b*x+a))^2-1/6*(2*b*x+I*\text{Pi}*c\text{sgn}(I*\exp(2*b*x+2*a))*c\text{sgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2-I*\text{Pi}*c\text{sgn}(I*\exp(b*x+a))^2*c\text{sgn}(I*\exp(2*b*x+2*a))-2*I*\text{Pi}+2*I*\text{Pi}*c\text{sgn}(I/(\exp(2*b*x+2*a)+1))^2-I*\text{Pi}*c\text{sgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3-I*\text{Pi}*c\text{sgn}(I/(\exp(2*b*x+2*a)+1))*c\text{sgn}(I*\exp(2*b*x+2*a))*c\text{sgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))-I*\text{Pi}*c\text{sgn}(I*\exp(2*b*x+2*a))^3+2*I*\text{Pi}*c\text{sgn}(I*\exp(b*x+a))*c\text{sgn}(I*\exp(2*b*x+2*a))^2+I*\text{Pi}*c\text{sgn}(I/(\exp(2*b*x+2*a)+1))*c\text{sgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2-2*I*\text{Pi}*c\text{sgn}(I/(\exp(2*b*x+2*a)+1))^3)/x^3*\ln(\exp(b*x+a))-1/48*(-4*I*\text{Pi}*x*b*c\text{sgn}(I*\exp(b*x+a))^2*c\text{sgn}(I*\exp(2*b*x+2*a))-6*\text{Pi}^2*c\text{sgn}(I*\exp(b*x+a))^2*c\text{sgn}(I*\exp(2*b*x+2*a))^4-4*\text{Pi}^2-4*\text{Pi}^2*c\text{sgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3-\text{Pi}^2*c\text{sgn}(I*\exp(2*b*x+2*a))^6-4*\text{Pi}^2*c\text{sgn}(I*\exp(2*b*x+2*a))^3+4*\text{Pi}^2*c\text{sgn}(I*\exp(2*b*x+2*a))*c\text{sgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2+4*\text{Pi}^2*c\text{sgn}(I/(\exp(2*b*x+2*a)+1))^4*c\text{sgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2-4*\text{Pi}^2*c\text{sgn}(I/(\exp(2*b*x+2*a)+1))^3*c\text{sgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3-4*\text{Pi}^2*c\text{sgn}(I*\exp(2*b*x+2*a))^3*c\text{sgn}(I/(\exp(2*b*x+2*a)+1))^3-\text{Pi}^2*c\text{sgn}(I/(\exp(2*b*x+2*a)+1))^2*c\text{sgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^4-4*I*\text{Pi}*b*x*c\text{sgn}(I/(\exp(2*b*x+2*a)+1))*c\text{sgn}(I*\exp(2*b*x+2*a))*c\text{sgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))+4*I*\text{Pi}*b*x*c\text{sgn}(I/(\exp(2*b*x+2*a)+1))*c\text{sgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2+8*I*\text{Pi}*b*x*c\text{sgn}(I*\exp(b*x+a))*c\text{sgn}(I*\exp(2*b*x+2*a))^2+4*I*\text{Pi}*b*x*c\text{sgn}(I*\exp(2*b*x+2*a))*c\text{sgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2-4*I*\text{Pi}*x*b*c\text{sgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3-4*I*\text{Pi}*b*x*c\text{sgn}(I*\exp(2*b*x+2*a))^3-4*\text{Pi}^2*c\text{sgn}(I/(\exp(2*b*x+2*a)+1))^6+2*\text{Pi}^2*c\text{sgn}(I*\exp(2*b*x+2*a))^4*c\text{sgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2-2*\text{Pi}^2*c\text{sgn}(I*\exp(2*b*x+2*a))^3*c\text{sgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3+16*b^2*x^2+4*\text{Pi}^2*c\text{sgn}(I*\exp(2*b*x+2*a))^3*c\text{sgn}(I/(\exp(2*b*x+2*a)+1))^2-4*\text{Pi}^2*c\text{sgn}(I*\exp(2*b*x+2*a))*c\text{sgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2*c\text{sgn}(I/(\exp(2*b*x+2*a)+1))^2+8*I*\text{Pi}*x*b*c\text{sgn}(I/(\exp(2*b*x+2*a)+1))^2-8*I*\text{Pi}*x*b*c\text{sgn}(I/(\exp(2*b*x+2*a)+1))^3+8*\text{Pi}^2*c\text{sgn}(I*\exp(b*x+a))*c\text{sgn}(I*\exp(2*b*x+2*a))^2+8*\text{Pi}^2*c\text{sgn}(I/(\exp(2*b*x+2*a)$$

$$\begin{aligned}
&+1))^{-2-8\pi^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))^{-3+4\pi^2\operatorname{csgn}(I\exp(bx+a))\operatorname{csgn}(I \\
&\exp(2bx+2a))^{-5+2\pi^2\operatorname{csgn}(I\exp(2bx+2a))\operatorname{csgn}(I\exp(2bx+2a))/(\exp \\
&(2bx+2a)+1))^{-5-\pi^2\operatorname{csgn}(I\exp(2bx+2a))^{-2}\operatorname{csgn}(I\exp(2bx+2a))/(\exp( \\
&2bx+2a)+1))^{-4+2\pi^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))\operatorname{csgn}(I\exp(2bx+2a))/(e \\
&\exp(2bx+2a)+1))^{-5-4\pi^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))^{-3}\operatorname{csgn}(I\exp(2bx+2 \\
&a))/(\exp(2bx+2a)+1))^{-2+4\pi^2\operatorname{csgn}(I\exp(2bx+2a))/(\exp(2bx+2a)+1))^{-3} \\
&\operatorname{csgn}(I/(\exp(2bx+2a)+1))^{-2-4\pi^2\operatorname{csgn}(I\exp(bx+a))^{-2}\operatorname{csgn}(I\exp(2bx+2 \\
&a))^{-8}I\pi^x b+4\pi^2\operatorname{csgn}(I\exp(bx+a))^{-3}\operatorname{csgn}(I\exp(2bx+2a))^{-3+4\pi^2\pi^2} \\
&\operatorname{csgn}(I/(\exp(2bx+2a)+1))\operatorname{csgn}(I\exp(2bx+2a))/(\exp(2bx+2a)+1))^{-2+4\pi^2} \\
&\pi^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))^{-3}\operatorname{csgn}(I\exp(2bx+2a))\operatorname{csgn}(I\exp(2bx+2 \\
&a))/(\exp(2bx+2a)+1))^{-\pi^2\operatorname{csgn}(I\exp(bx+a))^{-4}\operatorname{csgn}(I\exp(2bx+2a))^{-2+} \\
&8\pi^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))^{-5-4\pi^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))^{-4+2\pi^2} \\
&\pi^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))^{-2}\operatorname{csgn}(I\exp(2bx+2a))\operatorname{csgn}(I\exp(2bx+2 \\
&a))/(\exp(2bx+2a)+1))^{-3-2\pi^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))\operatorname{csgn}(I\exp(2bx \\
&+2a))^{-4}\operatorname{csgn}(I\exp(2bx+2a))/(\exp(2bx+2a)+1))+2\pi^2\operatorname{csgn}(I/(\exp(2bx \\
&+2a)+1))\operatorname{csgn}(I\exp(2bx+2a))^{-3}\operatorname{csgn}(I\exp(2bx+2a))/(\exp(2bx+2a)+1) \\
&)^{-2+2\pi^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))\operatorname{csgn}(I\exp(2bx+2a))^{-2}\operatorname{csgn}(I\exp(2 \\
&bx+2a))/(\exp(2bx+2a)+1))^{-3-4\pi^2\operatorname{csgn}(I\exp(bx+a))\operatorname{csgn}(I\exp(2bx+2 \\
&a))^{-3}\operatorname{csgn}(I\exp(2bx+2a))/(\exp(2bx+2a)+1))^{-2-8\pi^2\operatorname{csgn}(I/(\exp(2bx \\
&+2a)+1))^{-2}\operatorname{csgn}(I\exp(bx+a))\operatorname{csgn}(I\exp(2bx+2a))^{-2-4\pi^2\operatorname{csgn}(I/(\exp \\
&(2bx+2a)+1))^{-4}\operatorname{csgn}(I\exp(2bx+2a))\operatorname{csgn}(I\exp(2bx+2a))/(\exp(2bx+2 \\
&a)+1))^{-4\pi^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))^{-3}\operatorname{csgn}(I\exp(bx+a))^{-2}\operatorname{csgn}(I\exp \\
&(2bx+2a))+8\pi^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))^{-3}\operatorname{csgn}(I\exp(bx+a))\operatorname{csgn}(I \\
&\exp(2bx+2a))^{-2+4\pi^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))^{-3}\operatorname{csgn}(I\exp(2bx+2a) \\
&)\operatorname{csgn}(I\exp(2bx+2a))/(\exp(2bx+2a)+1))^{-2-\pi^2\operatorname{csgn}(I\exp(2bx+2a))/(e \\
&\exp(2bx+2a)+1))^{-6-4\pi^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))\operatorname{csgn}(I\exp(2bx+2a) \\
&)\operatorname{csgn}(I\exp(2bx+2a))/(\exp(2bx+2a)+1))+2\pi^2\operatorname{csgn}(I\exp(bx+a))^{-2}\operatorname{csg} \\
&n(I\exp(2bx+2a))^{-2}\operatorname{csgn}(I\exp(2bx+2a))/(\exp(2bx+2a)+1))^{-2+4\pi^2\operatorname{cs} \\
&\operatorname{gn}(I\exp(bx+a))\operatorname{csgn}(I\exp(2bx+2a))^{-2}\operatorname{csgn}(I\exp(2bx+2a))/(\exp(2bx+ \\
&2a)+1))^{-3-2\pi^2\operatorname{csgn}(I\exp(bx+a))^{-2}\operatorname{csgn}(I\exp(2bx+2a))\operatorname{csgn}(I\exp(2 \\
&bx+2a))/(\exp(2bx+2a)+1))^{-3+4\pi^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))^{-2}\operatorname{csgn}(I \\
&\exp(bx+a))^{-2}\operatorname{csgn}(I\exp(2bx+2a))-4\pi^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))\operatorname{csgn}( \\
&I\exp(2bx+2a))\operatorname{csgn}(I\exp(2bx+2a))/(\exp(2bx+2a)+1))^{-4-\pi^2\operatorname{csgn}(I/ \\
&\exp(2bx+2a)+1))^{-2}\operatorname{csgn}(I\exp(2bx+2a))^{-2}\operatorname{csgn}(I\exp(2bx+2a))/(\exp(2 \\
&bx+2a)+1))^{-2+2\pi^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))\operatorname{csgn}(I\exp(bx+a))^{-2}\operatorname{csgn}( \\
&I\exp(2bx+2a))\operatorname{csgn}(I\exp(2bx+2a))/(\exp(2bx+2a)+1))^{-2-4\pi^2\operatorname{csgn}(I \\
&/(\exp(2bx+2a)+1))\operatorname{csgn}(I\exp(bx+a))\operatorname{csgn}(I\exp(2bx+2a))^{-2}\operatorname{csgn}(I\exp \\
&(2bx+2a))/(\exp(2bx+2a)+1))^{-2+4\pi^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))\operatorname{csgn}(I \\
&\exp(bx+a))\operatorname{csgn}(I\exp(2bx+2a))^{-3}\operatorname{csgn}(I\exp(2bx+2a))/(\exp(2bx+2a)+ \\
&1))-2\pi^2\operatorname{csgn}(I/(\exp(2bx+2a)+1))\operatorname{csgn}(I\exp(bx+a))^{-2}\operatorname{csgn}(I\exp(2bx \\
&+2a))^{-2}\operatorname{csgn}(I\exp(2bx+2a))/(\exp(2bx+2a)+1)))/x^3
\end{aligned}$$

**Maxima [A]** time = 1.40024, size = 49, normalized size = 1.58

$$-\frac{b^2}{3x} - \frac{b \operatorname{arccoth}(\tanh(bx + a))}{3x^2} - \frac{\operatorname{arccoth}(\tanh(bx + a))^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^2/x^4,x, algorithm="maxima")

[Out] -1/3\*b^2/x - 1/3\*b\*arccoth(tanh(b\*x + a))/x^2 - 1/3\*arccoth(tanh(b\*x + a))^2/x^3

**Fricas [A]** time = 1.58129, size = 68, normalized size = 2.19

$$-\frac{12b^2x^2 + 12abx - \pi^2 + 4a^2}{12x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^2/x^4,x, algorithm="fricas")

[Out] -1/12\*(12\*b^2\*x^2 + 12\*a\*b\*x - pi^2 + 4\*a^2)/x^3

**Sympy [A]** time = 1.18208, size = 37, normalized size = 1.19

$$-\frac{b^2}{3x} - \frac{b \operatorname{acoth}(\tanh(a + bx))}{3x^2} - \frac{\operatorname{acoth}^2(\tanh(a + bx))}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(tanh(b\*x+a))\*\*2/x\*\*4,x)

[Out] -b\*\*2/(3\*x) - b\*acoth(tanh(a + b\*x))/(3\*x\*\*2) - acoth(tanh(a + b\*x))\*\*2/(3\*x\*\*3)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^2/x^4, x)
```

$$3.145 \quad \int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^5} dx$$

**Optimal.** Leaf size=64

$$\frac{\coth^{-1}(\tanh(a+bx))^3}{4x^4 (bx - \coth^{-1}(\tanh(a+bx)))} + \frac{b \coth^{-1}(\tanh(a+bx))^3}{12x^3 (bx - \coth^{-1}(\tanh(a+bx)))^2}$$

[Out] (b\*ArcCoth[Tanh[a + b\*x]]^3)/(12\*x^3\*(b\*x - ArcCoth[Tanh[a + b\*x]])^2) + ArcCoth[Tanh[a + b\*x]]^3/(4\*x^4\*(b\*x - ArcCoth[Tanh[a + b\*x]]))

**Rubi [A]** time = 0.0351377, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2171, 2167}

$$\frac{\coth^{-1}(\tanh(a+bx))^3}{4x^4 (bx - \coth^{-1}(\tanh(a+bx)))} + \frac{b \coth^{-1}(\tanh(a+bx))^3}{12x^3 (bx - \coth^{-1}(\tanh(a+bx)))^2}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^2/x^5, x]

[Out] (b\*ArcCoth[Tanh[a + b\*x]]^3)/(12\*x^3\*(b\*x - ArcCoth[Tanh[a + b\*x]])^2) + ArcCoth[Tanh[a + b\*x]]^3/(4\*x^4\*(b\*x - ArcCoth[Tanh[a + b\*x]]))

### Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dist[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b*u - a*v, 0] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -1]
```

### Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

### Rubi steps



$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^5} dx = \frac{\coth^{-1}(\tanh(a+bx))^3}{4x^4 (bx - \coth^{-1}(\tanh(a+bx)))} + \frac{b \int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4} dx}{4 (bx - \coth^{-1}(\tanh(a+bx)))}$$

$$= \frac{b \coth^{-1}(\tanh(a+bx))^3}{12x^3 (bx - \coth^{-1}(\tanh(a+bx)))^2} + \frac{\coth^{-1}(\tanh(a+bx))^3}{4x^4 (bx - \coth^{-1}(\tanh(a+bx)))}$$

**Mathematica [A]** time = 0.0306651, size = 37, normalized size = 0.58

$$\frac{2bx \coth^{-1}(\tanh(a+bx)) + 3 \coth^{-1}(\tanh(a+bx))^2 + b^2 x^2}{12x^4}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^2/x^5,x]

[Out]  $-(b^2 x^2 + 2 b x \operatorname{ArcCoth}[\operatorname{Tanh}[a + b x]] + 3 \operatorname{ArcCoth}[\operatorname{Tanh}[a + b x]]^2)/(12 x^4)$

**Maple [C]** time = 0.362, size = 3217, normalized size = 50.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b\*x+a))^2/x^5,x)

[Out]  $-1/4/x^4 \ln(\exp(b*x+a))^2 - 1/24*(4*b*x+3*I*Pi*csgn(I*\exp(2*b*x+2*a)))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2 - 3*I*Pi*csgn(I*\exp(b*x+a))^2*csgn(I*\exp(2*b*x+2*a)) - 6*I*Pi - 3*I*Pi*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3 - 3*I*Pi*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1)) + 6*I*Pi*csgn(I/(\exp(2*b*x+2*a)+1))^2 - 3*I*Pi*csgn(I*\exp(2*b*x+2*a))^3 + 6*I*Pi*csgn(I*\exp(b*x+a))*csgn(I*\exp(2*b*x+2*a))^2 + 3*I*Pi*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2 - 6*I*Pi*csgn(I/(\exp(2*b*x+2*a)+1))^3/x^4 \ln(\exp(b*x+a)) - 1/192*(-18*Pi^2*csgn(I*\exp(b*x+a))^2*csgn(I*\exp(2*b*x+2*a))^4 - 12*Pi^2 - 12*Pi^2*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3 - 16*I*Pi*x*b*csgn(I/(\exp(2*b*x+2*a)+1))^3 - 3*Pi^2*csgn(I*\exp(2*b*x+2*a))^6 - 8*I*Pi*b*x*csgn(I*\exp(2*b*x+2*a))^3 - 8*I*Pi*b*x*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3 + 16*I*Pi*x*b*csgn(I/(\exp(2*b*x+2*a)+1))^2 -$



```
*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-6*Pi^
2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*
b*x+2*a)+1))^3+12*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(b*x+a))^2*cs
gn(I*exp(2*b*x+2*a))-12*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*
a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4-3*Pi^2*csgn(I/(exp(2*b*x+2*
a)+1))^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))
^2+6*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+
2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-12*Pi^2*csgn(I/(exp(2*b*x
+2*a)+1))*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)
/(exp(2*b*x+2*a)+1))^2+12*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a)
)*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-6*Pi^2
*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))^2*c
sgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)))/x^4
```

**Maxima [A]** time = 1.40713, size = 49, normalized size = 0.77

$$-\frac{b^2}{12x^2} - \frac{b \operatorname{arccoth}(\tanh(bx+a))}{6x^3} - \frac{\operatorname{arccoth}(\tanh(bx+a))^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^2/x^5,x, algorithm="maxima")

[Out] -1/12\*b^2/x^2 - 1/6\*b\*arccoth(tanh(b\*x + a))/x^3 - 1/4\*arccoth(tanh(b\*x + a))^2/x^4

**Fricas [A]** time = 1.61147, size = 72, normalized size = 1.12

$$-\frac{24b^2x^2 + 32abx - 3\pi^2 + 12a^2}{48x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^2/x^5,x, algorithm="fricas")

[Out] -1/48\*(24\*b^2\*x^2 + 32\*a\*b\*x - 3\*pi^2 + 12\*a^2)/x^4

**Sympy [A]** time = 1.92498, size = 39, normalized size = 0.61

$$-\frac{b^2}{12x^2} - \frac{b \operatorname{acoth}(\tanh(a + bx))}{6x^3} - \frac{\operatorname{acoth}^2(\tanh(a + bx))}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(tanh(b\*x+a))\*\*2/x\*\*5,x)

[Out] -b\*\*2/(12\*x\*\*2) - b\*acoth(tanh(a + b\*x))/(6\*x\*\*3) - acoth(tanh(a + b\*x))\*\*2/(4\*x\*\*4)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^2/x^5,x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a))^2/x^5, x)

### 3.146 $\int x^m \coth^{-1}(\tanh(a + bx))^3 dx$

**Optimal.** Leaf size=110

$$\frac{6b^2x^{m+3} \coth^{-1}(\tanh(a + bx))}{m^3 + 6m^2 + 11m + 6} - \frac{3bx^{m+2} \coth^{-1}(\tanh(a + bx))^2}{m^2 + 3m + 2} + \frac{x^{m+1} \coth^{-1}(\tanh(a + bx))^3}{m + 1} - \frac{6b^3x^{m+4}}{(m + 1)(m^3 + 9m^2 + 14m + 6)}$$

[Out]  $(-6*b^3*x^(4 + m))/((1 + m)*(24 + 26*m + 9*m^2 + m^3)) + (6*b^2*x^(3 + m)*ArcCoth[Tanh[a + b*x]])/(6 + 11*m + 6*m^2 + m^3) - (3*b*x^(2 + m)*ArcCoth[Tanh[a + b*x]]^2)/(2 + 3*m + m^2) + (x^(1 + m)*ArcCoth[Tanh[a + b*x]]^3)/(1 + m)$

**Rubi [A]** time = 0.0593837, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2168, 30}

$$\frac{6b^2x^{m+3} \coth^{-1}(\tanh(a + bx))}{m^3 + 6m^2 + 11m + 6} - \frac{3bx^{m+2} \coth^{-1}(\tanh(a + bx))^2}{m^2 + 3m + 2} + \frac{x^{m+1} \coth^{-1}(\tanh(a + bx))^3}{m + 1} - \frac{6b^3x^{m+4}}{(m + 1)(m^3 + 9m^2 + 14m + 6)}$$

Antiderivative was successfully verified.

[In] Int[x^m\*ArcCoth[Tanh[a + b\*x]]^3,x]

[Out]  $(-6*b^3*x^(4 + m))/((1 + m)*(24 + 26*m + 9*m^2 + m^3)) + (6*b^2*x^(3 + m)*ArcCoth[Tanh[a + b*x]])/(6 + 11*m + 6*m^2 + m^3) - (3*b*x^(2 + m)*ArcCoth[Tanh[a + b*x]]^2)/(2 + 3*m + m^2) + (x^(1 + m)*ArcCoth[Tanh[a + b*x]]^3)/(1 + m)$

#### Rule 2168

Int[(u\_)^(m\_)\*(v\_)^(n\_.), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)\*v^n)/(a\*(m + 1)), x] - Dist[(b\*n)/(a\*(m + 1)), Int[u^(m + 1)\*v^(n - 1), x], x] /; NeQ[b\*u - a\*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2\*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

### Rubi steps

$$\begin{aligned}
 \int x^m \coth^{-1}(\tanh(a + bx))^3 dx &= \frac{x^{1+m} \coth^{-1}(\tanh(a + bx))^3}{1 + m} - \frac{(3b) \int x^{1+m} \coth^{-1}(\tanh(a + bx))^2 dx}{1 + m} \\
 &= -\frac{3bx^{2+m} \coth^{-1}(\tanh(a + bx))^2}{2 + 3m + m^2} + \frac{x^{1+m} \coth^{-1}(\tanh(a + bx))^3}{1 + m} + \frac{(6b^2) \int x^{2+m} \coth^{-1}(\tanh(a + bx)) dx}{2 + 3m + m^2} \\
 &= \frac{6b^2 x^{3+m} \coth^{-1}(\tanh(a + bx))}{6 + 11m + 6m^2 + m^3} - \frac{3bx^{2+m} \coth^{-1}(\tanh(a + bx))^2}{2 + 3m + m^2} + \frac{x^{1+m} \coth^{-1}(\tanh(a + bx))^3}{1 + m} \\
 &= -\frac{6b^3 x^{4+m}}{(4 + m)(6 + 11m + 6m^2 + m^3)} + \frac{6b^2 x^{3+m} \coth^{-1}(\tanh(a + bx))}{6 + 11m + 6m^2 + m^3} - \frac{3bx^{2+m} \coth^{-1}(\tanh(a + bx))^2}{2 + 3m + m^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.113509, size = 97, normalized size = 0.88

$$\frac{x^{m+1} (6b^2(m+4)x^2 \coth^{-1}(\tanh(a + bx)) - 3b(m^2 + 7m + 12)x \coth^{-1}(\tanh(a + bx))^2 + (m^3 + 9m^2 + 26m + 24) \coth^{-1}(\tanh(a + bx)))}{(m+1)(m+2)(m+3)(m+4)}$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*ArcCoth[Tanh[a + b\*x]]^3,x]

[Out] (x^(1 + m)\*(-6\*b^3\*x^3 + 6\*b^2\*(4 + m)\*x^2\*ArcCoth[Tanh[a + b\*x]] - 3\*b\*(12 + 7\*m + m^2)\*x\*ArcCoth[Tanh[a + b\*x]]^2 + (24 + 26\*m + 9\*m^2 + m^3)\*ArcCoth[Tanh[a + b\*x]]^3)/((1 + m)\*(2 + m)\*(3 + m)\*(4 + m))

**Maple [C]** time = 6.384, size = 63382, normalized size = 576.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*arccoth(tanh(b\*x+a))^3,x)

[Out] result too large to display

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>m</sup>\*arccoth(tanh(b\*x+a))<sup>3</sup>,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 1.71674, size = 468, normalized size = 4.25

$$\frac{4(b^3m^3 + 6b^3m^2 + 11b^3m + 6b^3)x^4 + 12(ab^2m^3 + 7ab^2m^2 + 14ab^2m + 8ab^2)x^3 + 3(4a^2bm^3 + 32a^2bm^2 + 76a^2bm + 48a^2b)x^2 + (4a^3m^3 + 36a^3m^2 + 104a^3m - 3\pi^2(a^3m^3 + 9a^3m^2 + 26a^3m + 24a) + 96a^3)x}{4(m^4 + 10m^3 + 35m^2 + 50m + 24)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>m</sup>\*arccoth(tanh(b\*x+a))<sup>3</sup>,x, algorithm="fricas")

[Out] 1/4\*(4\*(b<sup>3</sup>\*m<sup>3</sup> + 6\*b<sup>3</sup>\*m<sup>2</sup> + 11\*b<sup>3</sup>\*m + 6\*b<sup>3</sup>)\*x<sup>4</sup> + 12\*(a\*b<sup>2</sup>\*m<sup>3</sup> + 7\*a\*b<sup>2</sup>\*m<sup>2</sup> + 14\*a\*b<sup>2</sup>\*m + 8\*a\*b<sup>2</sup>)\*x<sup>3</sup> + 3\*(4\*a<sup>2</sup>\*b\*m<sup>3</sup> + 32\*a<sup>2</sup>\*b\*m<sup>2</sup> + 76\*a<sup>2</sup>\*b\*m + 48\*a<sup>2</sup>\*b)\*x<sup>2</sup> + (4\*a<sup>3</sup>\*m<sup>3</sup> + 36\*a<sup>3</sup>\*m<sup>2</sup> + 104\*a<sup>3</sup>\*m - 3\*pi<sup>2</sup>\*(a\*m<sup>3</sup> + 9\*a\*m<sup>2</sup> + 26\*a\*m + 24\*a) + 96\*a<sup>3</sup>)\*x\*x<sup>m</sup>/(m<sup>4</sup> + 10\*m<sup>3</sup> + 35\*m<sup>2</sup> + 50\*m + 24)

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*acoth(tanh(b\*x+a))\*\*3,x)

[Out] Exception raised: TypeError

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{arccoth}(\tanh(bx + a))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*arccoth(tanh(b*x + a))^3, x)
```



### 3.147 $\int x^4 \coth^{-1}(\tanh(a + bx))^3 dx$

**Optimal.** Leaf size=61

$$\frac{1}{35}b^2x^7 \coth^{-1}(\tanh(a + bx)) - \frac{1}{10}bx^6 \coth^{-1}(\tanh(a + bx))^2 + \frac{1}{5}x^5 \coth^{-1}(\tanh(a + bx))^3 - \frac{1}{280}b^3x^8$$

[Out]  $-(b^3x^8)/280 + (b^2x^7\text{ArcCoth}[\text{Tanh}[a + bx]])/35 - (bx^6\text{ArcCoth}[\text{Tanh}[a + bx]]^2)/10 + (x^5\text{ArcCoth}[\text{Tanh}[a + bx]]^3)/5$

**Rubi [A]** time = 0.0411641, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2168, 30}

$$\frac{1}{35}b^2x^7 \coth^{-1}(\tanh(a + bx)) - \frac{1}{10}bx^6 \coth^{-1}(\tanh(a + bx))^2 + \frac{1}{5}x^5 \coth^{-1}(\tanh(a + bx))^3 - \frac{1}{280}b^3x^8$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^4\text{ArcCoth}[\text{Tanh}[a + bx]]^3, x]$

[Out]  $-(b^3x^8)/280 + (b^2x^7\text{ArcCoth}[\text{Tanh}[a + bx]])/35 - (bx^6\text{ArcCoth}[\text{Tanh}[a + bx]]^2)/10 + (x^5\text{ArcCoth}[\text{Tanh}[a + bx]]^3)/5$

#### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

#### Rule 30

```
Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

#### Rubi steps

$$\begin{aligned}
\int x^4 \coth^{-1}(\tanh(a + bx))^3 dx &= \frac{1}{5}x^5 \coth^{-1}(\tanh(a + bx))^3 - \frac{1}{5}(3b) \int x^5 \coth^{-1}(\tanh(a + bx))^2 dx \\
&= -\frac{1}{10}bx^6 \coth^{-1}(\tanh(a + bx))^2 + \frac{1}{5}x^5 \coth^{-1}(\tanh(a + bx))^3 + \frac{1}{5}b^2 \int x^6 \coth^{-1}(\tanh(a + bx)) dx \\
&= \frac{1}{35}b^2x^7 \coth^{-1}(\tanh(a + bx)) - \frac{1}{10}bx^6 \coth^{-1}(\tanh(a + bx))^2 + \frac{1}{5}x^5 \coth^{-1}(\tanh(a + bx))^3 \\
&= -\frac{1}{280}b^3x^8 + \frac{1}{35}b^2x^7 \coth^{-1}(\tanh(a + bx)) - \frac{1}{10}bx^6 \coth^{-1}(\tanh(a + bx))^2 + \frac{1}{5}x^5 \coth^{-1}(\tanh(a + bx))^3
\end{aligned}$$

**Mathematica [A]** time = 0.030511, size = 54, normalized size = 0.89

$$-\frac{1}{280}x^5(-8b^2x^2 \coth^{-1}(\tanh(a + bx)) + 28bx \coth^{-1}(\tanh(a + bx))^2 - 56 \coth^{-1}(\tanh(a + bx))^3 + b^3x^3)$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*ArcCoth[Tanh[a + b\*x]]^3,x]

[Out] -(x^5\*(b^3\*x^3 - 8\*b^2\*x^2\*ArcCoth[Tanh[a + b\*x]] + 28\*b\*x\*ArcCoth[Tanh[a + b\*x]]^2 - 56\*ArcCoth[Tanh[a + b\*x]]^3))/280

**Maple [C]** time = 1.102, size = 18111, normalized size = 296.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*arccoth(tanh(b\*x+a))^3,x)

[Out] result too large to display

**Maxima [A]** time = 1.54533, size = 73, normalized size = 1.2

$$-\frac{1}{10}bx^6 \operatorname{arccoth}(\tanh(bx + a))^2 + \frac{1}{5}x^5 \operatorname{arccoth}(\tanh(bx + a))^3 - \frac{1}{280}(b^2x^8 - 8bx^7 \operatorname{arccoth}(\tanh(bx + a)))b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arccoth(tanh(b\*x+a))^3,x, algorithm="maxima")

[Out]  $-1/10*b*x^6*arccoth(tanh(b*x + a))^2 + 1/5*x^5*arccoth(tanh(b*x + a))^3 - 1/280*(b^2*x^8 - 8*b*x^7*arccoth(tanh(b*x + a)))*b$

**Fricas [A]** time = 1.59236, size = 119, normalized size = 1.95

$$\frac{1}{8} b^3 x^8 + \frac{3}{7} a b^2 x^7 - \frac{1}{8} (\pi^2 b - 4 a^2 b) x^6 - \frac{1}{20} (3 \pi^2 a - 4 a^3) x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arccoth(tanh(b\*x+a))^3,x, algorithm="fricas")

[Out]  $1/8*b^3*x^8 + 3/7*a*b^2*x^7 - 1/8*(pi^2*b - 4*a^2*b)*x^6 - 1/20*(3*pi^2*a - 4*a^3)*x^5$

**Sympy [A]** time = 6.05556, size = 56, normalized size = 0.92

$$-\frac{b^3 x^8}{280} + \frac{b^2 x^7 \operatorname{acoth}(\tanh(a + bx))}{35} - \frac{b x^6 \operatorname{acoth}^2(\tanh(a + bx))}{10} + \frac{x^5 \operatorname{acoth}^3(\tanh(a + bx))}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*acoth(tanh(b\*x+a))\*\*3,x)

[Out]  $-b**3*x**8/280 + b**2*x**7*acoth(tanh(a + b*x))/35 - b*x**6*acoth(tanh(a + b*x))**2/10 + x**5*acoth(tanh(a + b*x))**3/5$

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{arccoth}(\tanh(bx + a))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arccoth(tanh(b\*x+a))^3,x, algorithm="giac")

[Out] integrate(x^4\*arccoth(tanh(b\*x + a))^3, x)

### 3.148 $\int x^3 \coth^{-1}(\tanh(a + bx))^3 dx$

**Optimal.** Leaf size=61

$$\frac{1}{20}b^2x^6 \coth^{-1}(\tanh(a + bx)) - \frac{3}{20}bx^5 \coth^{-1}(\tanh(a + bx))^2 + \frac{1}{4}x^4 \coth^{-1}(\tanh(a + bx))^3 - \frac{1}{140}b^3x^7$$

[Out]  $-(b^3x^7)/140 + (b^2x^6 \operatorname{ArcCoth}[\operatorname{Tanh}[a + bx]])/20 - (3b^2x^5 \operatorname{ArcCoth}[\operatorname{Tanh}[a + bx]]^2)/20 + (x^4 \operatorname{ArcCoth}[\operatorname{Tanh}[a + bx]]^3)/4$

**Rubi [A]** time = 0.0407912, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2168, 30}

$$\frac{1}{20}b^2x^6 \coth^{-1}(\tanh(a + bx)) - \frac{3}{20}bx^5 \coth^{-1}(\tanh(a + bx))^2 + \frac{1}{4}x^4 \coth^{-1}(\tanh(a + bx))^3 - \frac{1}{140}b^3x^7$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^3 \operatorname{ArcCoth}[\operatorname{Tanh}[a + bx]]^3, x]$

[Out]  $-(b^3x^7)/140 + (b^2x^6 \operatorname{ArcCoth}[\operatorname{Tanh}[a + bx]])/20 - (3b^2x^5 \operatorname{ArcCoth}[\operatorname{Tanh}[a + bx]]^2)/20 + (x^4 \operatorname{ArcCoth}[\operatorname{Tanh}[a + bx]]^3)/4$

#### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

#### Rule 30

```
Int[(x_)^(m_), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

#### Rubi steps

$$\begin{aligned}
\int x^3 \coth^{-1}(\tanh(a + bx))^3 dx &= \frac{1}{4}x^4 \coth^{-1}(\tanh(a + bx))^3 - \frac{1}{4}(3b) \int x^4 \coth^{-1}(\tanh(a + bx))^2 dx \\
&= -\frac{3}{20}bx^5 \coth^{-1}(\tanh(a + bx))^2 + \frac{1}{4}x^4 \coth^{-1}(\tanh(a + bx))^3 + \frac{1}{10}(3b^2) \int x^5 \coth^{-1}(\tanh(a + bx)) dx \\
&= \frac{1}{20}b^2x^6 \coth^{-1}(\tanh(a + bx)) - \frac{3}{20}bx^5 \coth^{-1}(\tanh(a + bx))^2 + \frac{1}{4}x^4 \coth^{-1}(\tanh(a + bx))^3 \\
&= -\frac{1}{140}b^3x^7 + \frac{1}{20}b^2x^6 \coth^{-1}(\tanh(a + bx)) - \frac{3}{20}bx^5 \coth^{-1}(\tanh(a + bx))^2 + \frac{1}{4}x^4 \coth^{-1}(\tanh(a + bx))^3
\end{aligned}$$

**Mathematica [A]** time = 0.0271736, size = 54, normalized size = 0.89

$$-\frac{1}{140}x^4(-7b^2x^2 \coth^{-1}(\tanh(a + bx)) + 21bx \coth^{-1}(\tanh(a + bx))^2 - 35 \coth^{-1}(\tanh(a + bx))^3 + b^3x^3)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*ArcCoth[Tanh[a + b\*x]]^3,x]

[Out] -(x^4\*(b^3\*x^3 - 7\*b^2\*x^2\*ArcCoth[Tanh[a + b\*x]] + 21\*b\*x\*ArcCoth[Tanh[a + b\*x]]^2 - 35\*ArcCoth[Tanh[a + b\*x]]^3))/140

**Maple [C]** time = 1.096, size = 18111, normalized size = 296.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*arccoth(tanh(b\*x+a))^3,x)

[Out] result too large to display

**Maxima [A]** time = 1.56117, size = 73, normalized size = 1.2

$$-\frac{3}{20}bx^5 \operatorname{arccoth}(\tanh(bx + a))^2 + \frac{1}{4}x^4 \operatorname{arccoth}(\tanh(bx + a))^3 - \frac{1}{140}(b^2x^7 - 7bx^6 \operatorname{arccoth}(\tanh(bx + a)))b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(tanh(b\*x+a))^3,x, algorithm="maxima")

[Out]  $-3/20*b*x^5*arccoth(tanh(b*x + a))^2 + 1/4*x^4*arccoth(tanh(b*x + a))^3 - 1/140*(b^2*x^7 - 7*b*x^6*arccoth(tanh(b*x + a))) * b$

**Fricas [A]** time = 1.5684, size = 120, normalized size = 1.97

$$\frac{1}{7}b^3x^7 + \frac{1}{2}ab^2x^6 - \frac{3}{20}(\pi^2b - 4a^2b)x^5 - \frac{1}{16}(3\pi^2a - 4a^3)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(tanh(b\*x+a))^3,x, algorithm="fricas")

[Out]  $1/7*b^3*x^7 + 1/2*a*b^2*x^6 - 3/20*(\pi^2*b - 4*a^2*b)*x^5 - 1/16*(3*\pi^2*a - 4*a^3)*x^4$

**Sympy [A]** time = 3.559, size = 58, normalized size = 0.95

$$-\frac{b^3x^7}{140} + \frac{b^2x^6 \operatorname{acoth}(\tanh(a + bx))}{20} - \frac{3bx^5 \operatorname{acoth}^2(\tanh(a + bx))}{20} + \frac{x^4 \operatorname{acoth}^3(\tanh(a + bx))}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*acoth(tanh(b\*x+a))\*\*3,x)

[Out]  $-b**3*x**7/140 + b**2*x**6*acoth(tanh(a + b*x))/20 - 3*b*x**5*acoth(tanh(a + b*x))**2/20 + x**4*acoth(tanh(a + b*x))**3/4$

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arccoth}(\tanh(bx + a))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(tanh(b\*x+a))^3,x, algorithm="giac")

[Out] integrate(x^3\*arccoth(tanh(b\*x + a))^3, x)

### 3.149 $\int x^2 \coth^{-1}(\tanh(a + bx))^3 dx$

**Optimal.** Leaf size=53

$$\frac{\coth^{-1}(\tanh(a + bx))^6}{60b^3} - \frac{x \coth^{-1}(\tanh(a + bx))^5}{10b^2} + \frac{x^2 \coth^{-1}(\tanh(a + bx))^4}{4b}$$

[Out]  $(x^2 \text{ArcCoth}[\text{Tanh}[a + b*x]]^4)/(4*b) - (x \text{ArcCoth}[\text{Tanh}[a + b*x]]^5)/(10*b^2) + \text{ArcCoth}[\text{Tanh}[a + b*x]]^6/(60*b^3)$

**Rubi [A]** time = 0.0312969, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2168, 2157, 30}

$$\frac{\coth^{-1}(\tanh(a + bx))^6}{60b^3} - \frac{x \coth^{-1}(\tanh(a + bx))^5}{10b^2} + \frac{x^2 \coth^{-1}(\tanh(a + bx))^4}{4b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2 \text{ArcCoth}[\text{Tanh}[a + b*x]]^3, x]$

[Out]  $(x^2 \text{ArcCoth}[\text{Tanh}[a + b*x]]^4)/(4*b) - (x \text{ArcCoth}[\text{Tanh}[a + b*x]]^5)/(10*b^2) + \text{ArcCoth}[\text{Tanh}[a + b*x]]^6/(60*b^3)$

#### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

#### Rule 2157

```
Int[(u_)^(m_.), x_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

#### Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

### Rubi steps

$$\begin{aligned} \int x^2 \coth^{-1}(\tanh(a + bx))^3 dx &= \frac{x^2 \coth^{-1}(\tanh(a + bx))^4}{4b} - \frac{\int x \coth^{-1}(\tanh(a + bx))^4 dx}{2b} \\ &= \frac{x^2 \coth^{-1}(\tanh(a + bx))^4}{4b} - \frac{x \coth^{-1}(\tanh(a + bx))^5}{10b^2} + \frac{\int \coth^{-1}(\tanh(a + bx))^5 dx}{10b^2} \\ &= \frac{x^2 \coth^{-1}(\tanh(a + bx))^4}{4b} - \frac{x \coth^{-1}(\tanh(a + bx))^5}{10b^2} + \frac{\text{Subst}\left(\int x^5 dx, x, \coth^{-1}(\tanh(a + bx))\right)}{10b^3} \\ &= \frac{x^2 \coth^{-1}(\tanh(a + bx))^4}{4b} - \frac{x \coth^{-1}(\tanh(a + bx))^5}{10b^2} + \frac{\coth^{-1}(\tanh(a + bx))^6}{60b^3} \end{aligned}$$

**Mathematica [A]** time = 0.0222424, size = 54, normalized size = 1.02

$$-\frac{1}{60}x^3 \left( -6b^2x^2 \coth^{-1}(\tanh(a + bx)) + 15bx \coth^{-1}(\tanh(a + bx))^2 - 20 \coth^{-1}(\tanh(a + bx))^3 + b^3x^3 \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] -(x^3*(b^3*x^3 - 6*b^2*x^2*ArcCoth[Tanh[a + b*x]] + 15*b*x*ArcCoth[Tanh[a + b*x]]^2 - 20*ArcCoth[Tanh[a + b*x]]^3))/60
```

**Maple [C]** time = 1.081, size = 18111, normalized size = 341.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(tanh(b*x+a))^3,x)
```

```
[Out] result too large to display
```



**Maxima [A]** time = 1.54597, size = 73, normalized size = 1.38

$$-\frac{1}{4}bx^4 \operatorname{arccoth}(\tanh(bx+a))^2 + \frac{1}{3}x^3 \operatorname{arccoth}(\tanh(bx+a))^3 - \frac{1}{60}(b^2x^6 - 6bx^5 \operatorname{arccoth}(\tanh(bx+a)))b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(tanh(b\*x+a))^3,x, algorithm="maxima")

[Out] -1/4\*b\*x^4\*arccoth(tanh(b\*x + a))^2 + 1/3\*x^3\*arccoth(tanh(b\*x + a))^3 - 1/60\*(b^2\*x^6 - 6\*b\*x^5\*arccoth(tanh(b\*x + a)))\*b

**Fricas [A]** time = 1.55735, size = 120, normalized size = 2.26

$$\frac{1}{6}b^3x^6 + \frac{3}{5}ab^2x^5 - \frac{3}{16}(\pi^2b - 4a^2b)x^4 - \frac{1}{12}(3\pi^2a - 4a^3)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(tanh(b\*x+a))^3,x, algorithm="fricas")

[Out] 1/6\*b^3\*x^6 + 3/5\*a\*b^2\*x^5 - 3/16\*(pi^2\*b - 4\*a^2\*b)\*x^4 - 1/12\*(3\*pi^2\*a - 4\*a^3)\*x^3

**Sympy [A]** time = 3.3897, size = 60, normalized size = 1.13

$$\begin{cases} \frac{x^2 \operatorname{acoth}^4(\tanh(a+bx))}{4b} - \frac{x \operatorname{acoth}^5(\tanh(a+bx))}{10b^2} + \frac{\operatorname{acoth}^6(\tanh(a+bx))}{60b^3} & \text{for } b \neq 0 \\ \frac{x^3 \operatorname{acoth}^3(\tanh(a))}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*acoth(tanh(b\*x+a))\*\*3,x)

[Out] Piecewise((x\*\*2\*acoth(tanh(a + b\*x))\*\*4/(4\*b) - x\*acoth(tanh(a + b\*x))\*\*5/(10\*b\*\*2) + acoth(tanh(a + b\*x))\*\*6/(60\*b\*\*3), Ne(b, 0)), (x\*\*3\*acoth(tanh(a))\*\*3/3, True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arccoth}(\tanh(bx + a))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(tanh(b*x + a))^3, x)
```

### 3.150 $\int x \coth^{-1}(\tanh(a + bx))^3 dx$

**Optimal.** Leaf size=34

$$\frac{x \coth^{-1}(\tanh(a + bx))^4}{4b} - \frac{\coth^{-1}(\tanh(a + bx))^5}{20b^2}$$

[Out]  $(x \cdot \text{ArcCoth}[\text{Tanh}[a + b \cdot x]]^4)/(4 \cdot b) - \text{ArcCoth}[\text{Tanh}[a + b \cdot x]]^5/(20 \cdot b^2)$

**Rubi [A]** time = 0.0149233, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {2168, 2157, 30}

$$\frac{x \coth^{-1}(\tanh(a + bx))^4}{4b} - \frac{\coth^{-1}(\tanh(a + bx))^5}{20b^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x \cdot \text{ArcCoth}[\text{Tanh}[a + b \cdot x]]^3, x]$

[Out]  $(x \cdot \text{ArcCoth}[\text{Tanh}[a + b \cdot x]]^4)/(4 \cdot b) - \text{ArcCoth}[\text{Tanh}[a + b \cdot x]]^5/(20 \cdot b^2)$

#### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

#### Rule 2157

```
Int[(u_)^(m_), x_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

#### Rule 30

```
Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int x \coth^{-1}(\tanh(a + bx))^3 dx &= \frac{x \coth^{-1}(\tanh(a + bx))^4}{4b} - \frac{\int \coth^{-1}(\tanh(a + bx))^4 dx}{4b} \\ &= \frac{x \coth^{-1}(\tanh(a + bx))^4}{4b} - \frac{\text{Subst}\left(\int x^4 dx, x, \coth^{-1}(\tanh(a + bx))\right)}{4b^2} \\ &= \frac{x \coth^{-1}(\tanh(a + bx))^4}{4b} - \frac{\coth^{-1}(\tanh(a + bx))^5}{20b^2} \end{aligned}$$

**Mathematica [B]** time = 0.0736339, size = 99, normalized size = 2.91

$$\frac{(a + bx) \left( 10(2a^2 + abx - b^2x^2) \coth^{-1}(\tanh(a + bx))^2 + (4a - bx)(a + bx)^3 - 5(3a - bx)(a + bx)^2 \coth^{-1}(\tanh(a + bx)) \right)}{20b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[Tanh[a + b\*x]]^3,x]

[Out] ((a + b\*x)\*((4\*a - b\*x)\*(a + b\*x)^3 - 5\*(3\*a - b\*x)\*(a + b\*x)^2\*ArcCoth[Tanh[a + b\*x]] + 10\*(2\*a^2 + a\*b\*x - b^2\*x^2)\*ArcCoth[Tanh[a + b\*x]]^2 - 10\*(a - b\*x)\*ArcCoth[Tanh[a + b\*x]]^3))/(20\*b^2)

**Maple [C]** time = 1.083, size = 18111, normalized size = 532.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(tanh(b\*x+a))^3,x)

[Out] result too large to display

**Maxima [A]** time = 1.55891, size = 73, normalized size = 2.15

$$-\frac{1}{2}bx^3 \operatorname{arccoth}(\tanh(bx + a))^2 + \frac{1}{2}x^2 \operatorname{arccoth}(\tanh(bx + a))^3 - \frac{1}{20}(b^2x^5 - 5bx^4 \operatorname{arccoth}(\tanh(bx + a)))b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(tanh(b\*x+a))^3,x, algorithm="maxima")

[Out]  $-1/2*b*x^3*arccoth(tanh(b*x + a))^2 + 1/2*x^2*arccoth(tanh(b*x + a))^3 - 1/20*(b^2*x^5 - 5*b*x^4*arccoth(tanh(b*x + a)))*b$

**Fricas [A]** time = 1.50232, size = 117, normalized size = 3.44

$$\frac{1}{5}b^3x^5 + \frac{3}{4}ab^2x^4 - \frac{1}{4}(\pi^2b - 4a^2b)x^3 - \frac{1}{8}(3\pi^2a - 4a^3)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(tanh(b\*x+a))^3,x, algorithm="fricas")

[Out]  $1/5*b^3*x^5 + 3/4*a*b^2*x^4 - 1/4*(\pi^2*b - 4*a^2*b)*x^3 - 1/8*(3*\pi^2*a - 4*a^3)*x^2$

**Sympy [A]** time = 1.73176, size = 41, normalized size = 1.21

$$\begin{cases} \frac{x \operatorname{acoth}^4(\tanh(a+bx))}{4b} - \frac{\operatorname{acoth}^5(\tanh(a+bx))}{20b^2} & \text{for } b \neq 0 \\ \frac{x^2 \operatorname{acoth}^3(\tanh(a))}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(tanh(b\*x+a))\*\*3,x)

[Out] Piecewise((x\*acoth(tanh(a + b\*x))\*\*4/(4\*b) - acoth(tanh(a + b\*x))\*\*5/(20\*b\*2), Ne(b, 0)), (x\*\*2\*acoth(tanh(a))\*\*3/2, True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arccoth}(\tanh(bx + a))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(tanh(b*x + a))^3, x)
```

$$3.151 \quad \int \coth^{-1}(\tanh(a + bx))^3 dx$$

Optimal. Leaf size=16

$$\frac{\coth^{-1}(\tanh(a + bx))^4}{4b}$$

[Out] ArcCoth[Tanh[a + b\*x]]^4/(4\*b)

**Rubi [A]** time = 0.0051923, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {2157, 30}

$$\frac{\coth^{-1}(\tanh(a + bx))^4}{4b}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^3,x]

[Out] ArcCoth[Tanh[a + b\*x]]^4/(4\*b)

#### Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[  
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N  
eQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \coth^{-1}(\tanh(a + bx))^3 dx &= \frac{\text{Subst}\left(\int x^3 dx, x, \coth^{-1}(\tanh(a + bx))\right)}{b} \\ &= \frac{\coth^{-1}(\tanh(a + bx))^4}{4b} \end{aligned}$$

**Mathematica [A]** time = 0.0056073, size = 16, normalized size = 1.

$$\frac{\coth^{-1}(\tanh(a + bx))^4}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^3,x]

[Out] ArcCoth[Tanh[a + b\*x]]^4/(4\*b)

**Maple [A]** time = 0.106, size = 15, normalized size = 0.9

$$\frac{(\operatorname{arccoth}(\tanh(bx + a)))^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b\*x+a))^3,x)

[Out] 1/4\*arccoth(tanh(b\*x+a))^4/b

**Maxima [B]** time = 1.55637, size = 69, normalized size = 4.31

$$-\frac{3}{2}bx^2 \operatorname{arccoth}(\tanh(bx + a))^2 + x \operatorname{arccoth}(\tanh(bx + a))^3 - \frac{1}{4}(b^2x^4 - 4bx^3 \operatorname{arccoth}(\tanh(bx + a)))b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^3,x, algorithm="maxima")

[Out] -3/2\*b\*x^2\*arccoth(tanh(b\*x + a))^2 + x\*arccoth(tanh(b\*x + a))^3 - 1/4\*(b^2\*x^4 - 4\*b\*x^3\*arccoth(tanh(b\*x + a)))\*b

**Fricas [B]** time = 1.59564, size = 109, normalized size = 6.81

$$\frac{1}{4}b^3x^4 + ab^2x^3 - \frac{3}{8}(\pi^2b - 4a^2b)x^2 - \frac{1}{4}(3\pi^2a - 4a^3)x$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/4*b^3*x^4 + a*b^2*x^3 - 3/8*(pi^2*b - 4*a^2*b)*x^2 - 1/4*(3*pi^2*a - 4*a^3)*x
```

**Sympy [A]** time = 0.803763, size = 20, normalized size = 1.25

$$\begin{cases} \frac{\operatorname{acoth}^4(\tanh(a+bx))}{4b} & \text{for } b \neq 0 \\ x \operatorname{acoth}^3(\tanh(a)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**3,x)
```

```
[Out] Piecewise((acoth(tanh(a + b*x))**4/(4*b), Ne(b, 0)), (x*acoth(tanh(a))**3, True))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(\tanh(bx + a))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^3, x)
```

$$3.152 \quad \int \frac{\coth^{-1}(\tanh(a+bx))^3}{x} dx$$

**Optimal.** Leaf size=77

$$bx \left( bx - \coth^{-1}(\tanh(a+bx)) \right)^2 - \frac{1}{2} \coth^{-1}(\tanh(a+bx))^2 \left( bx - \coth^{-1}(\tanh(a+bx)) \right) + \frac{1}{3} \coth^{-1}(\tanh(a+bx))^3 -$$

[Out] b\*x\*(b\*x - ArcCoth[Tanh[a + b\*x]])^2 - ((b\*x - ArcCoth[Tanh[a + b\*x]])\*ArcCoth[Tanh[a + b\*x]]^2)/2 + ArcCoth[Tanh[a + b\*x]]^3/3 - (b\*x - ArcCoth[Tanh[a + b\*x]])^3\*Log[x]

**Rubi [A]** time = 0.0962767, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2159, 2158, 29}

$$bx \left( bx - \coth^{-1}(\tanh(a+bx)) \right)^2 - \frac{1}{2} \coth^{-1}(\tanh(a+bx))^2 \left( bx - \coth^{-1}(\tanh(a+bx)) \right) + \frac{1}{3} \coth^{-1}(\tanh(a+bx))^3 -$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^3/x,x]

[Out] b\*x\*(b\*x - ArcCoth[Tanh[a + b\*x]])^2 - ((b\*x - ArcCoth[Tanh[a + b\*x]])\*ArcCoth[Tanh[a + b\*x]]^2)/2 + ArcCoth[Tanh[a + b\*x]]^3/3 - (b\*x - ArcCoth[Tanh[a + b\*x]])^3\*Log[x]

### Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x] /; NeQ[b*u - a*v, 0] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n, 1]
```

### Rule 2158

```
Int[(v_)/(u_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a*v, 0] /; PiecewiseLinearQ[u, v, x]
```

### Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

### Rubi steps

$$\begin{aligned}
 \int \frac{\coth^{-1}(\tanh(a+bx))^3}{x} dx &= \frac{1}{3} \coth^{-1}(\tanh(a+bx))^3 - (bx - \coth^{-1}(\tanh(a+bx))) \int \frac{\coth^{-1}(\tanh(a+bx))^2}{x} dx \\
 &= -\frac{1}{2} (bx - \coth^{-1}(\tanh(a+bx))) \coth^{-1}(\tanh(a+bx))^2 + \frac{1}{3} \coth^{-1}(\tanh(a+bx))^3 - \\
 &= bx (bx - \coth^{-1}(\tanh(a+bx)))^2 - \frac{1}{2} (bx - \coth^{-1}(\tanh(a+bx))) \coth^{-1}(\tanh(a+bx))^2 \\
 &= bx (bx - \coth^{-1}(\tanh(a+bx)))^2 - \frac{1}{2} (bx - \coth^{-1}(\tanh(a+bx))) \coth^{-1}(\tanh(a+bx))^2
 \end{aligned}$$

**Mathematica [A]** time = 0.0580561, size = 104, normalized size = 1.35

$$(a+bx) \left( a^2 - 3a(-\coth^{-1}(\tanh(a+bx)) + a+bx) + 3(-\coth^{-1}(\tanh(a+bx)) + a+bx)^2 \right) + \frac{1}{3}(a+bx)^3 - \frac{1}{2}(a+bx)^2$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^3/x,x]

[Out] (a + b\*x)^3/3 + (a + b\*x)\*(a^2 - 3\*a\*(a + b\*x - ArcCoth[Tanh[a + b\*x]]) + 3\*(a + b\*x - ArcCoth[Tanh[a + b\*x]])^2) - ((a + b\*x)^2\*(2\*a + 3\*b\*x - 3\*ArcCoth[Tanh[a + b\*x]]))/2 + (-b\*x + ArcCoth[Tanh[a + b\*x]])^3\*Log[b\*x]

**Maple [C]** time = 0.847, size = 21848, normalized size = 283.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b\*x+a))^3/x,x)

[Out] result too large to display

**Maxima [C]** time = 2.51028, size = 101, normalized size = 1.31

$$\frac{1}{3} b^3 x^3 + \frac{1}{24} (-18i \pi b^2 + 36 a b^2) x^2 - \frac{1}{24} (18 \pi^2 b + 72i \pi a b - 72 a^2 b) x + \frac{1}{8} (i \pi^3 - 6 \pi^2 a - 12i \pi a^2 + 8 a^3) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^3/x,x, algorithm="maxima")

[Out] 1/3\*b^3\*x^3 + 1/24\*(-18\*I\*pi\*b^2 + 36\*a\*b^2)\*x^2 - 1/24\*(18\*pi^2\*b + 72\*I\*pi\*a\*b - 72\*a^2\*b)\*x + 1/8\*(I\*pi^3 - 6\*pi^2\*a - 12\*I\*pi\*a^2 + 8\*a^3)\*log(x)

**Fricas [A]** time = 1.69985, size = 119, normalized size = 1.55

$$\frac{1}{3} b^3 x^3 + \frac{3}{2} a b^2 x^2 - \frac{3}{4} (\pi^2 b - 4 a^2 b) x - \frac{1}{4} (3 \pi^2 a - 4 a^3) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^3/x,x, algorithm="fricas")

[Out] 1/3\*b^3\*x^3 + 3/2\*a\*b^2\*x^2 - 3/4\*(pi^2\*b - 4\*a^2\*b)\*x - 1/4\*(3\*pi^2\*a - 4\*a^3)\*log(x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^3(\tanh(a + bx))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(tanh(b\*x+a))\*\*3/x,x)

[Out] Integral(acoth(tanh(a + b\*x))\*\*3/x, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^3/x, x)
```

$$3.153 \quad \int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^2} dx$$

**Optimal.** Leaf size=68

$$-3b^2x \left( bx - \coth^{-1}(\tanh(a+bx)) \right) - \frac{\coth^{-1}(\tanh(a+bx))^3}{x} + \frac{3}{2}b \coth^{-1}(\tanh(a+bx))^2 + 3b \log(x) \left( bx - \coth^{-1}(\tanh(a+bx)) \right)$$

[Out]  $-3*b^2*x*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]]) + (3*b*\text{ArcCoth}[\text{Tanh}[a + b*x]]^2)/2 - \text{ArcCoth}[\text{Tanh}[a + b*x]]^3/x + 3*b*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^2*\text{Log}[x]$

**Rubi [A]** time = 0.0435156, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {2168, 2159, 2158, 29}

$$-3b^2x \left( bx - \coth^{-1}(\tanh(a+bx)) \right) - \frac{\coth^{-1}(\tanh(a+bx))^3}{x} + \frac{3}{2}b \coth^{-1}(\tanh(a+bx))^2 + 3b \log(x) \left( bx - \coth^{-1}(\tanh(a+bx)) \right)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{ArcCoth}[\text{Tanh}[a + b*x]]^3/x^2, x]$

[Out]  $-3*b^2*x*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]]) + (3*b*\text{ArcCoth}[\text{Tanh}[a + b*x]]^2)/2 - \text{ArcCoth}[\text{Tanh}[a + b*x]]^3/x + 3*b*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^2*\text{Log}[x]$

### Rule 2168

$\text{Int}[(u_)^{(m)}*(v_)^{(n)}, x\_Symbol] \rightarrow \text{With}[\{a = \text{Simplify}[D[u, x]], b = \text{Simplify}[D[v, x]]\}, \text{Simp}[(u^{(m+1)}*v^{(n)})/(a*(m+1)), x] - \text{Dist}[(b*n)/(a*(m+1)), \text{Int}[u^{(m+1)}*v^{(n-1)}, x], x] /; \text{NeQ}[b*u - a*v, 0] /; \text{FreeQ}[\{m, n\}, x] \&\& \text{PiecewiseLinearQ}[u, v, x] \&\& \text{NeQ}[m, -1] \&\& ((\text{LtQ}[m, -1] \&\& \text{GtQ}[n, 0] \&\& !(\text{ILtQ}[m+n, -2] \&\& (\text{FractionQ}[m] \parallel \text{GeQ}[2*n+m+1, 0]))) \parallel (\text{IGtQ}[n, 0] \&\& \text{IGtQ}[m, 0] \&\& \text{LeQ}[n, m]) \parallel (\text{IGtQ}[n, 0] \&\& !\text{IntegerQ}[m]) \parallel (\text{ILtQ}[m, 0] \&\& !\text{IntegerQ}[n]))]$

### Rule 2159

$\text{Int}[(v_)^{(n)}/(u_), x\_Symbol] \rightarrow \text{With}[\{a = \text{Simplify}[D[u, x]], b = \text{Simplify}[D[v, x]]\}, \text{Simp}[v^{(n)}/(a*n), x] - \text{Dist}[(b*u - a*v)/a, \text{Int}[v^{(n-1)}/u, x], x] /; \text{NeQ}[b*u - a*v, 0] /; \text{PiecewiseLinearQ}[u, v, x] \&\& \text{GtQ}[n, 0] \&\& \text{NeQ}[n, 1]$

Rule 2158

Int[(v\_)/(u\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(b\*x)/a, x] - Dist[(b\*u - a\*v)/a, Int[1/u, x], x] /; NeQ[b\*u - a\*v, 0]] /; PiecewiseLinearQ[u, v, x]

Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(\tanh(a + bx))^3}{x^2} dx &= -\frac{\coth^{-1}(\tanh(a + bx))^3}{x} + (3b) \int \frac{\coth^{-1}(\tanh(a + bx))^2}{x} dx \\ &= \frac{3}{2}b \coth^{-1}(\tanh(a + bx))^2 - \frac{\coth^{-1}(\tanh(a + bx))^3}{x} - (3b)(bx - \coth^{-1}(\tanh(a + bx))) \\ &= -3b^2x(bx - \coth^{-1}(\tanh(a + bx))) + \frac{3}{2}b \coth^{-1}(\tanh(a + bx))^2 - \frac{\coth^{-1}(\tanh(a + bx))^3}{x} \\ &= -3b^2x(bx - \coth^{-1}(\tanh(a + bx))) + \frac{3}{2}b \coth^{-1}(\tanh(a + bx))^2 - \frac{\coth^{-1}(\tanh(a + bx))^3}{x} \end{aligned}$$

**Mathematica [A]** time = 0.039466, size = 62, normalized size = 0.91

$$-6b^2x \log(x) \coth^{-1}(\tanh(a + bx)) - \frac{\coth^{-1}(\tanh(a + bx))^3}{x} + 3b(\log(x) + 1) \coth^{-1}(\tanh(a + bx))^2 + \frac{3}{2}b^3x^2(2 \log(x))$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^3/x^2,x]

[Out] -(ArcCoth[Tanh[a + b\*x]]^3/x) - 6\*b^2\*x\*ArcCoth[Tanh[a + b\*x]]\*Log[x] + 3\*b\*ArcCoth[Tanh[a + b\*x]]^2\*(1 + Log[x]) + (3\*b^3\*x^2\*(-1 + 2\*Log[x]))/2

**Maple [C]** time = 0.415, size = 7683, normalized size = 113.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(tanh(b*x+a))^3/x^2,x)`

[Out] result too large to display

**Maxima [C]** time = 2.08232, size = 166, normalized size = 2.44

$$3b \operatorname{arccoth}(\tanh(bx+a))^2 \log(x) - \frac{3}{2} \left( 2 \operatorname{arccoth}(\tanh(bx+a))^2 \log(x) - \left( bx^2 + (2i\pi + 4a)x + 2 \left( -\frac{i\pi(bx+a)}{b} - \frac{bx}{b} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(tanh(b*x+a))^3/x^2,x, algorithm="maxima")`

[Out]  $3*b*\operatorname{arccoth}(\tanh(b*x+a))^2*\log(x) - 3/2*(2*\operatorname{arccoth}(\tanh(b*x+a))^2*\log(x) - (b*x^2 + (2*I*\pi + 4*a)*x + 2*(-I*\pi*(b*x+a)/b - (b*x+a)^2/b)*\log(x) + 2*\operatorname{arccoth}(\tanh(b*x+a))^2*\log(x)/b + 2*(I*\pi*a + a^2)*\log(x)/b)*b - \operatorname{arccoth}(\tanh(b*x+a))^3/x$

**Fricas [A]** time = 1.72542, size = 115, normalized size = 1.69

$$\frac{2b^3x^3 + 12ab^2x^2 + 3\pi^2a - 4a^3 - 3(\pi^2b - 4a^2b)x \log(x)}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(tanh(b*x+a))^3/x^2,x, algorithm="fricas")`

[Out]  $1/4*(2*b^3*x^3 + 12*a*b^2*x^2 + 3*\pi^2*a - 4*a^3 - 3*(\pi^2*b - 4*a^2*b)*x*\log(x))/x$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^3(\tanh(a+bx))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(acoth(tanh(b*x+a))**3/x**2,x)
```

```
[Out] Integral(acoth(tanh(a + b*x))**3/x**2, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^3/x^2, x)
```

$$3.154 \quad \int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^3} dx$$

**Optimal.** Leaf size=60

$$-3b^2 \log(x) (bx - \coth^{-1}(\tanh(a+bx))) - \frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2} - \frac{3b \coth^{-1}(\tanh(a+bx))^2}{2x} + 3b^3x$$

[Out] 3\*b^3\*x - (3\*b\*ArcCoth[Tanh[a + b\*x]]^2)/(2\*x) - ArcCoth[Tanh[a + b\*x]]^3/(2\*x^2) - 3\*b^2\*(b\*x - ArcCoth[Tanh[a + b\*x]])\*Log[x]

**Rubi [A]** time = 0.0411344, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2168, 2158, 29}

$$-3b^2 \log(x) (bx - \coth^{-1}(\tanh(a+bx))) - \frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2} - \frac{3b \coth^{-1}(\tanh(a+bx))^2}{2x} + 3b^3x$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^3/x^3,x]

[Out] 3\*b^3\*x - (3\*b\*ArcCoth[Tanh[a + b\*x]]^2)/(2\*x) - ArcCoth[Tanh[a + b\*x]]^3/(2\*x^2) - 3\*b^2\*(b\*x - ArcCoth[Tanh[a + b\*x]])\*Log[x]

### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

### Rule 2158

```
Int[(v_)/(u_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a*v, 0] /; PiecewiseLinearQ[u, v, x]
```

### Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

### Rubi steps

$$\begin{aligned}
 \int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^3} dx &= -\frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2} + \frac{1}{2}(3b) \int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2} dx \\
 &= -\frac{3b \coth^{-1}(\tanh(a+bx))^2}{2x} - \frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2} + (3b^2) \int \frac{\coth^{-1}(\tanh(a+bx))}{x} dx \\
 &= 3b^3x - \frac{3b \coth^{-1}(\tanh(a+bx))^2}{2x} - \frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2} - (3b^2)(bx - \coth^{-1}(\tanh(a+bx))) \\
 &= 3b^3x - \frac{3b \coth^{-1}(\tanh(a+bx))^2}{2x} - \frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2} - 3b^2(bx - \coth^{-1}(\tanh(a+bx)))
 \end{aligned}$$

**Mathematica [A]** time = 0.0363074, size = 66, normalized size = 1.1

$$3b^2 \log(x) (\coth^{-1}(\tanh(a+bx)) - bx) - \frac{(\coth^{-1}(\tanh(a+bx)) - bx)^3}{2x^2} - \frac{3b (\coth^{-1}(\tanh(a+bx)) - bx)^2}{x} + b^3x$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^3/x^3, x]

[Out] b^3\*x - (3\*b\*(-(b\*x) + ArcCoth[Tanh[a + b\*x]]))^2/x - (-(b\*x) + ArcCoth[Tanh[a + b\*x]])^3/(2\*x^2) + 3\*b^2\*(-(b\*x) + ArcCoth[Tanh[a + b\*x]])\*Log[x]

**Maple [C]** time = 0.426, size = 7366, normalized size = 122.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b\*x+a))^3/x^3, x)

[Out] result too large to display

**Maxima [A]** time = 1.40292, size = 97, normalized size = 1.62

$$3 \left( b \operatorname{arccoth}(\tanh(bx + a)) \log(x) - \left( b \left( x + \frac{a}{b} \right) \log(x) - b \left( x + \frac{a \log(x)}{b} \right) \right) b \right) - \frac{3b \operatorname{arccoth}(\tanh(bx + a))^2}{2x} - \frac{\operatorname{arccoth}(t$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^3/x^3,x, algorithm="maxima")

[Out] 3\*(b\*arccoth(tanh(b\*x + a))\*log(x) - (b\*(x + a/b)\*log(x) - b\*(x + a\*log(x)/b))\*b)\*b - 3/2\*b\*arccoth(tanh(b\*x + a))^2/x - 1/2\*arccoth(tanh(b\*x + a))^3/x^2

**Fricas [A]** time = 1.61919, size = 117, normalized size = 1.95

$$\frac{8b^3x^3 + 24ab^2x^2 \log(x) + 3\pi^2a - 4a^3 + 6(\pi^2b - 4a^2b)x}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^3/x^3,x, algorithm="fricas")

[Out] 1/8\*(8\*b^3\*x^3 + 24\*a\*b^2\*x^2\*log(x) + 3\*pi^2\*a - 4\*a^3 + 6\*(pi^2\*b - 4\*a^2\*b\*x)/x^2

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^3(\tanh(a + bx))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(tanh(b\*x+a))\*\*3/x\*\*3,x)

[Out] Integral(acoth(tanh(a + b\*x))\*\*3/x\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^3/x^3,x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a))^3/x^3, x)

$$3.155 \quad \int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^4} dx$$

**Optimal.** Leaf size=55

$$-\frac{b^2 \coth^{-1}(\tanh(a+bx))}{x} - \frac{b \coth^{-1}(\tanh(a+bx))^2}{2x^2} - \frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3} + b^3 \log(x)$$

[Out]  $-(b^2 \text{ArcCoth}[\text{Tanh}[a + b*x]])/x - (b \text{ArcCoth}[\text{Tanh}[a + b*x]]^2)/(2*x^2) - \text{ArcCoth}[\text{Tanh}[a + b*x]]^3/(3*x^3) + b^3 \text{Log}[x]$

**Rubi [A]** time = 0.0381152, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2168, 29}

$$-\frac{b^2 \coth^{-1}(\tanh(a+bx))}{x} - \frac{b \coth^{-1}(\tanh(a+bx))^2}{2x^2} - \frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3} + b^3 \log(x)$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^3/x^4,x]

[Out]  $-(b^2 \text{ArcCoth}[\text{Tanh}[a + b*x]])/x - (b \text{ArcCoth}[\text{Tanh}[a + b*x]]^2)/(2*x^2) - \text{ArcCoth}[\text{Tanh}[a + b*x]]^3/(3*x^3) + b^3 \text{Log}[x]$

### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

### Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(\tanh(a + bx))^3}{x^4} dx &= -\frac{\coth^{-1}(\tanh(a + bx))^3}{3x^3} + b \int \frac{\coth^{-1}(\tanh(a + bx))^2}{x^3} dx \\
&= -\frac{b \coth^{-1}(\tanh(a + bx))^2}{2x^2} - \frac{\coth^{-1}(\tanh(a + bx))^3}{3x^3} + b^2 \int \frac{\coth^{-1}(\tanh(a + bx))}{x^2} dx \\
&= -\frac{b^2 \coth^{-1}(\tanh(a + bx))}{x} - \frac{b \coth^{-1}(\tanh(a + bx))^2}{2x^2} - \frac{\coth^{-1}(\tanh(a + bx))^3}{3x^3} + b^3 \int \frac{1}{x} dx \\
&= -\frac{b^2 \coth^{-1}(\tanh(a + bx))}{x} - \frac{b \coth^{-1}(\tanh(a + bx))^2}{2x^2} - \frac{\coth^{-1}(\tanh(a + bx))^3}{3x^3} + b^3 \log(x)
\end{aligned}$$

**Mathematica [A]** time = 0.0243607, size = 60, normalized size = 1.09

$$\frac{-6b^2x^2 \coth^{-1}(\tanh(a + bx)) - 3bx \coth^{-1}(\tanh(a + bx))^2 - 2 \coth^{-1}(\tanh(a + bx))^3 + b^3x^3(6 \log(x) + 11)}{6x^3}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^3/x^4,x]

[Out]  $(-6*b^2*x^2*ArcCoth[Tanh[a + b*x]] - 3*b*x*ArcCoth[Tanh[a + b*x]]^2 - 2*ArcCoth[Tanh[a + b*x]]^3 + b^3*x^3*(11 + 6*Log[x]))/(6*x^3)$

**Maple [C]** time = 1.385, size = 17237, normalized size = 313.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b\*x+a))^3/x^4,x)

[Out] result too large to display

**Maxima [A]** time = 1.59677, size = 70, normalized size = 1.27

$$\left( b^2 \log(x) - \frac{b \operatorname{arccoth}(\tanh(bx + a))}{x} \right) b - \frac{b \operatorname{arccoth}(\tanh(bx + a))^2}{2x^2} - \frac{\operatorname{arccoth}(\tanh(bx + a))^3}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^3/x^4,x, algorithm="maxima")

[Out] (b^2\*log(x) - b\*arccoth(tanh(b\*x + a))/x)\*b - 1/2\*b\*arccoth(tanh(b\*x + a))^2/x^2 - 1/3\*arccoth(tanh(b\*x + a))^3/x^3

**Fricas [A]** time = 1.65861, size = 120, normalized size = 2.18

$$\frac{24 b^3 x^3 \log(x) - 72 a b^2 x^2 + 6 \pi^2 a - 8 a^3 + 9 (\pi^2 b - 4 a^2 b) x}{24 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^3/x^4,x, algorithm="fricas")

[Out] 1/24\*(24\*b^3\*x^3\*log(x) - 72\*a\*b^2\*x^2 + 6\*pi^2\*a - 8\*a^3 + 9\*(pi^2\*b - 4\*a^2\*b)\*x)/x^3

**Sympy [A]** time = 1.21213, size = 51, normalized size = 0.93

$$b^3 \log(x) - \frac{b^2 \operatorname{acoth}(\tanh(a + bx))}{x} - \frac{b \operatorname{acoth}^2(\tanh(a + bx))}{2x^2} - \frac{\operatorname{acoth}^3(\tanh(a + bx))}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(tanh(b\*x+a))\*\*3/x\*\*4,x)

[Out] b\*\*3\*log(x) - b\*\*2\*acoth(tanh(a + b\*x))/x - b\*acoth(tanh(a + b\*x))\*\*2/(2\*x\*\*2) - acoth(tanh(a + b\*x))\*\*3/(3\*x\*\*3)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^3}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(arccoth(tanh(b*x+a))^3/x^4,x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^3/x^4, x)
```

$$3.156 \quad \int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5} dx$$

**Optimal.** Leaf size=31

$$\frac{\coth^{-1}(\tanh(a+bx))^4}{4x^4 (bx - \coth^{-1}(\tanh(a+bx)))}$$

[Out] ArcCoth[Tanh[a + b\*x]]^4/(4\*x^4\*(b\*x - ArcCoth[Tanh[a + b\*x]]))

**Rubi [A]** time = 0.0137358, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2167}

$$\frac{\coth^{-1}(\tanh(a+bx))^4}{4x^4 (bx - \coth^{-1}(\tanh(a+bx)))}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^3/x^5,x]

[Out] ArcCoth[Tanh[a + b\*x]]^4/(4\*x^4\*(b\*x - ArcCoth[Tanh[a + b\*x]]))

### Rule 2167

Int[(u\_)^(m\_)\*(v\_)^(n\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, -Simp[(u^(m + 1)\*v^(n + 1))/((m + 1)\*(b\*u - a\*v)), x] /; NeQ[b\*u - a\*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]

### Rubi steps

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5} dx = \frac{\coth^{-1}(\tanh(a+bx))^4}{4x^4 (bx - \coth^{-1}(\tanh(a+bx)))}$$

**Mathematica [A]** time = 0.0222898, size = 50, normalized size = 1.61

$$\frac{b^2 x^2 \coth^{-1}(\tanh(a+bx)) + bx \coth^{-1}(\tanh(a+bx))^2 + \coth^{-1}(\tanh(a+bx))^3 + b^3 x^3}{4x^4}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^3/x^5,x]

[Out]  $-(b^3x^3 + b^2x^2\text{ArcCoth}[\text{Tanh}[a + b*x]] + b*x*\text{ArcCoth}[\text{Tanh}[a + b*x]]^2 + \text{ArcCoth}[\text{Tanh}[a + b*x]]^3)/(4*x^4)$

**Maple [C]** time = 1.347, size = 17235, normalized size = 556.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b\*x+a))^3/x^5,x)

[Out] result too large to display

**Maxima [A]** time = 1.59718, size = 72, normalized size = 2.32

$$-\frac{1}{4}b\left(\frac{b^2}{x} + \frac{b \operatorname{arccoth}(\tanh(bx + a))}{x^2}\right) - \frac{b \operatorname{arccoth}(\tanh(bx + a))^2}{4x^3} - \frac{\operatorname{arccoth}(\tanh(bx + a))^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^3/x^5,x, algorithm="maxima")

[Out]  $-1/4*b*(b^2/x + b*\operatorname{arccoth}(\tanh(b*x + a))/x^2) - 1/4*b*\operatorname{arccoth}(\tanh(b*x + a))^2/x^3 - 1/4*\operatorname{arccoth}(\tanh(b*x + a))^3/x^4$

**Fricas [A]** time = 1.60225, size = 112, normalized size = 3.61

$$\frac{16b^3x^3 + 24ab^2x^2 - 3\pi^2a + 4a^3 - 4(\pi^2b - 4a^2b)x}{16x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^3/x^5,x, algorithm="fricas")

[Out]  $-1/16*(16*b^3*x^3 + 24*a*b^2*x^2 - 3*\pi^2*a + 4*a^3 - 4*(\pi^2*b - 4*a^2*b)*x)/x^4$

**Sympy [B]** time = 2.14484, size = 56, normalized size = 1.81

$$-\frac{b^3}{4x} - \frac{b^2 \operatorname{acoth}(\tanh(a + bx))}{4x^2} - \frac{b \operatorname{acoth}^2(\tanh(a + bx))}{4x^3} - \frac{\operatorname{acoth}^3(\tanh(a + bx))}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(tanh(b*x+a))**3/x**5,x)`

[Out]  $-b**3/(4*x) - b**2*acoth(\tanh(a + b*x))/(4*x**2) - b*acoth(\tanh(a + b*x))**2/(4*x**3) - acoth(\tanh(a + b*x))**3/(4*x**4)$

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^3}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(tanh(b*x+a))^3/x^5,x, algorithm="giac")`

[Out] `integrate(arccoth(tanh(b*x + a))^3/x^5, x)`

$$3.157 \quad \int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^6} dx$$

**Optimal.** Leaf size=64

$$\frac{\coth^{-1}(\tanh(a+bx))^4}{5x^5 (bx - \coth^{-1}(\tanh(a+bx)))} + \frac{b \coth^{-1}(\tanh(a+bx))^4}{20x^4 (bx - \coth^{-1}(\tanh(a+bx)))^2}$$

[Out] (b\*ArcCoth[Tanh[a + b\*x]]^4)/(20\*x^4\*(b\*x - ArcCoth[Tanh[a + b\*x]])^2) + ArcCoth[Tanh[a + b\*x]]^4/(5\*x^5\*(b\*x - ArcCoth[Tanh[a + b\*x]]))

**Rubi [A]** time = 0.0335485, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2171, 2167}

$$\frac{\coth^{-1}(\tanh(a+bx))^4}{5x^5 (bx - \coth^{-1}(\tanh(a+bx)))} + \frac{b \coth^{-1}(\tanh(a+bx))^4}{20x^4 (bx - \coth^{-1}(\tanh(a+bx)))^2}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^3/x^6, x]

[Out] (b\*ArcCoth[Tanh[a + b\*x]]^4)/(20\*x^4\*(b\*x - ArcCoth[Tanh[a + b\*x]])^2) + ArcCoth[Tanh[a + b\*x]]^4/(5\*x^5\*(b\*x - ArcCoth[Tanh[a + b\*x]]))

### Rule 2171

Int[(u\_)^(m\_)\*(v\_)^(n\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, -Simp[(u^(m + 1)\*v^(n + 1))/((m + 1)\*(b\*u - a\*v)), x] + Dist[(b\*(m + n + 2))/((m + 1)\*(b\*u - a\*v)), Int[u^(m + 1)\*v^n, x], x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -1]

### Rule 2167

Int[(u\_)^(m\_)\*(v\_)^(n\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, -Simp[(u^(m + 1)\*v^(n + 1))/((m + 1)\*(b\*u - a\*v)), x] /; NeQ[b\*u - a\*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]

### Rubi steps

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^6} dx = \frac{\coth^{-1}(\tanh(a+bx))^4}{5x^5 (bx - \coth^{-1}(\tanh(a+bx)))} + \frac{b \int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5} dx}{5 (bx - \coth^{-1}(\tanh(a+bx)))}$$

$$= \frac{b \coth^{-1}(\tanh(a+bx))^4}{20x^4 (bx - \coth^{-1}(\tanh(a+bx)))^2} + \frac{\coth^{-1}(\tanh(a+bx))^4}{5x^5 (bx - \coth^{-1}(\tanh(a+bx)))}$$

**Mathematica [A]** time = 0.0339501, size = 54, normalized size = 0.84

$$\frac{2b^2x^2 \coth^{-1}(\tanh(a+bx)) + 3bx \coth^{-1}(\tanh(a+bx))^2 + 4 \coth^{-1}(\tanh(a+bx))^3 + b^3x^3}{20x^5}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^3/x^6,x]

[Out]  $-(b^3x^3 + 2b^2x^2 \text{ArcCoth}[\text{Tanh}[a + b*x]] + 3b*x*\text{ArcCoth}[\text{Tanh}[a + b*x]]^2 + 4*\text{ArcCoth}[\text{Tanh}[a + b*x]]^3)/(20*x^5)$

**Maple [C]** time = 1.273, size = 17234, normalized size = 269.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b\*x+a))^3/x^6,x)

[Out] result too large to display

**Maxima [A]** time = 1.59881, size = 73, normalized size = 1.14

$$-\frac{1}{20} b \left( \frac{b^2}{x^2} + \frac{2b \operatorname{arccoth}(\tanh(bx+a))}{x^3} \right) - \frac{3b \operatorname{arccoth}(\tanh(bx+a))^2}{20x^4} - \frac{\operatorname{arccoth}(\tanh(bx+a))^3}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^3/x^6,x, algorithm="maxima")

[Out]  $-1/20*b*(b^2/x^2 + 2*b*\operatorname{arccoth}(\tanh(b*x + a))/x^3) - 3/20*b*\operatorname{arccoth}(\tanh(b*x + a))^2/x^4 - 1/5*\operatorname{arccoth}(\tanh(b*x + a))^3/x^5$

**Fricas [A]** time = 1.69803, size = 116, normalized size = 1.81

$$\frac{40b^3x^3 + 80ab^2x^2 - 12\pi^2a + 16a^3 - 15(\pi^2b - 4a^2b)x}{80x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(tanh(b*x+a))^3/x^6,x, algorithm="fricas")`

[Out]  $-1/80*(40*b^3*x^3 + 80*a*b^2*x^2 - 12*\pi^2*a + 16*a^3 - 15*(\pi^2*b - 4*a^2*b)*x)/x^5$

**Sympy [A]** time = 3.61206, size = 60, normalized size = 0.94

$$\frac{b^3}{20x^2} - \frac{b^2 \operatorname{acoth}(\tanh(a + bx))}{10x^3} - \frac{3b \operatorname{acoth}^2(\tanh(a + bx))}{20x^4} - \frac{\operatorname{acoth}^3(\tanh(a + bx))}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(tanh(b*x+a))**3/x**6,x)`

[Out]  $-b**3/(20*x**2) - b**2*acoth(tanh(a + b*x))/(10*x**3) - 3*b*acoth(tanh(a + b*x))**2/(20*x**4) - acoth(tanh(a + b*x))**3/(5*x**5)$

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^3}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(tanh(b*x+a))^3/x^6,x, algorithm="giac")`

[Out] `integrate(arccoth(tanh(b*x + a))^3/x^6, x)`

$$3.158 \quad \int \frac{x^m}{\coth^{-1}(\tanh(a+bx))} dx$$

Optimal. Leaf size=53

$$-\frac{x^{m+1} \text{Hypergeometric2F1}\left(1, m+1, m+2, \frac{bx}{bx - \coth^{-1}(\tanh(a+bx))}\right)}{(m+1)(bx - \coth^{-1}(\tanh(a+bx)))}$$

[Out]  $-\left(\left(x^{1+m} \text{Hypergeometric2F1}\left[1, 1+m, 2+m, \frac{b*x}{b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]]}\right]\right) / \left((1+m) * (b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])\right)\right)$

**Rubi [A]** time = 0.0284971, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2164}

$$-\frac{x^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{bx}{bx - \coth^{-1}(\tanh(a+bx))}\right)}{(m+1)(bx - \coth^{-1}(\tanh(a+bx)))}$$

Antiderivative was successfully verified.

[In] Int[x^m/ArcCoth[Tanh[a + b\*x]], x]

[Out]  $-\left(\left(x^{1+m} \text{Hypergeometric2F1}\left[1, 1+m, 2+m, \frac{b*x}{b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]]}\right]\right) / \left((1+m) * (b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])\right)\right)$

### Rule 2164

Int[(v\_)^(n\_)/(u\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(v^(n+1)\*Hypergeometric2F1[1, n+1, n+2, -(a\*v)/(b\*u - a\*v)]) / ((n+1)\*(b\*u - a\*v)), x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x] && !IntegerQ[n]

### Rubi steps

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))} dx = -\frac{x^{1+m} {}_2F_1\left(1, 1+m; 2+m; \frac{bx}{bx - \coth^{-1}(\tanh(a+bx))}\right)}{(1+m)(bx - \coth^{-1}(\tanh(a+bx)))}$$



**Mathematica [A]** time = 0.0842978, size = 51, normalized size = 0.96

$$\frac{x^{m+1} \text{Hypergeometric2F1}\left(1, m+1, m+2, -\frac{bx}{\coth^{-1}(\tanh(a+bx))-bx}\right)}{(m+1)(\coth^{-1}(\tanh(a+bx))-bx)}$$

Antiderivative was successfully verified.

[In] Integrate[x^m/ArcCoth[Tanh[a + b\*x]], x]

[Out] (x^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, -((b\*x)/(-b\*x) + ArcCoth[Tanh[a + b\*x]])])/((1 + m)\*(-b\*x) + ArcCoth[Tanh[a + b\*x]])

**Maple [F]** time = 0.346, size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{arccoth}(\tanh(bx + a))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m/arccoth(tanh(b\*x+a)), x)

[Out] int(x^m/arccoth(tanh(b\*x+a)), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{arccoth}(\tanh(bx + a))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/arccoth(tanh(b\*x+a)), x, algorithm="maxima")

[Out] integrate(x^m/arccoth(tanh(b\*x + a)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x^m}{\operatorname{arccoth}(\tanh(bx + a))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>m</sup>/arccoth(tanh(b\*x+a)),x, algorithm="fricas")

[Out] integral(x<sup>m</sup>/arccoth(tanh(b\*x + a)), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{acoth}(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m/acoth(tanh(b\*x+a)),x)

[Out] Integral(x\*\*m/acoth(tanh(a + b\*x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{arccoth}(\tanh(bx + a))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>m</sup>/arccoth(tanh(b\*x+a)),x, algorithm="giac")

[Out] integrate(x<sup>m</sup>/arccoth(tanh(b\*x + a)), x)

$$3.159 \quad \int \frac{x^3}{\coth^{-1}(\tanh(a+bx))} dx$$

**Optimal.** Leaf size=81

$$\frac{x^2 (bx - \coth^{-1}(\tanh(a + bx)))}{2b^2} + \frac{x (bx - \coth^{-1}(\tanh(a + bx)))^2}{b^3} + \frac{(bx - \coth^{-1}(\tanh(a + bx)))^3 \log(\coth^{-1}(\tanh(a + bx)))}{b^4}$$

[Out]  $x^3/(3*b) + (x^2*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]]))/(2*b^2) + (x*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]]^2)/b^3 + ((b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]]))^3*\text{Log}[\text{ArcCoth}[\text{Tanh}[a + b*x]]])/b^4$

**Rubi [A]** time = 0.0588508, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {2159, 2158, 2157, 29}

$$\frac{x^2 (bx - \coth^{-1}(\tanh(a + bx)))}{2b^2} + \frac{x (bx - \coth^{-1}(\tanh(a + bx)))^2}{b^3} + \frac{(bx - \coth^{-1}(\tanh(a + bx)))^3 \log(\coth^{-1}(\tanh(a + bx)))}{b^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3/\text{ArcCoth}[\text{Tanh}[a + b*x]], x]$

[Out]  $x^3/(3*b) + (x^2*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]]))/(2*b^2) + (x*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]]^2)/b^3 + ((b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]]))^3*\text{Log}[\text{ArcCoth}[\text{Tanh}[a + b*x]]])/b^4$

#### Rule 2159

$\text{Int}[(v_)^(n_)/(u_), x\_Symbol] \rightarrow \text{With}[\{a = \text{Simplify}[\text{D}[u, x]], b = \text{Simplify}[\text{D}[v, x]]\}, \text{Simp}[v^n/(a^n), x] - \text{Dist}[(b*u - a*v)/a, \text{Int}[v^(n-1)/u, x], x] /; \text{NeQ}[b*u - a*v, 0] /; \text{PiecewiseLinearQ}[u, v, x] \&\& \text{GtQ}[n, 0] \&\& \text{NeQ}[n, 1]$

#### Rule 2158

$\text{Int}[(v_)/(u_), x\_Symbol] \rightarrow \text{With}[\{a = \text{Simplify}[\text{D}[u, x]], b = \text{Simplify}[\text{D}[v, x]]\}, \text{Simp}[(b*x)/a, x] - \text{Dist}[(b*u - a*v)/a, \text{Int}[1/u, x], x] /; \text{NeQ}[b*u - a*v, 0] /; \text{PiecewiseLinearQ}[u, v, x]$

#### Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[  
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

### Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

### Rubi steps

$$\begin{aligned}
 \int \frac{x^3}{\coth^{-1}(\tanh(a+bx))} dx &= \frac{x^3}{3b} - \frac{(-bx + \coth^{-1}(\tanh(a+bx)))}{b} \int \frac{x^2}{\coth^{-1}(\tanh(a+bx))} dx \\
 &= \frac{x^3}{3b} + \frac{x^2(bx - \coth^{-1}(\tanh(a+bx)))}{2b^2} + \frac{(-bx + \coth^{-1}(\tanh(a+bx)))^2}{b^2} \int \frac{x}{\coth^{-1}(\tanh(a+bx))} dx \\
 &= \frac{x^3}{3b} + \frac{x^2(bx - \coth^{-1}(\tanh(a+bx)))}{2b^2} + \frac{x(bx - \coth^{-1}(\tanh(a+bx)))^2}{b^3} - \frac{(-bx + \coth^{-1}(\tanh(a+bx)))^3}{b^4} \\
 &= \frac{x^3}{3b} + \frac{x^2(bx - \coth^{-1}(\tanh(a+bx)))}{2b^2} + \frac{x(bx - \coth^{-1}(\tanh(a+bx)))^2}{b^3} - \frac{(-bx + \coth^{-1}(\tanh(a+bx)))^3}{b^4} \\
 &= \frac{x^3}{3b} + \frac{x^2(bx - \coth^{-1}(\tanh(a+bx)))}{2b^2} + \frac{x(bx - \coth^{-1}(\tanh(a+bx)))^2}{b^3} + \frac{(bx - \coth^{-1}(\tanh(a+bx)))^3}{b^4}
 \end{aligned}$$

**Mathematica [A]** time = 0.0481871, size = 79, normalized size = 0.98

$$-\frac{x^2(\coth^{-1}(\tanh(a+bx)) - bx)}{2b^2} + \frac{x(\coth^{-1}(\tanh(a+bx)) - bx)^2}{b^3} - \frac{(\coth^{-1}(\tanh(a+bx)) - bx)^3 \log(\coth^{-1}(\tanh(a+bx)))}{b^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/ArcCoth[Tanh[a + b\*x]], x]

[Out] x^3/(3\*b) - (x^2\*(-(b\*x) + ArcCoth[Tanh[a + b\*x]]))/(2\*b^2) + (x\*(-(b\*x) + ArcCoth[Tanh[a + b\*x]])^2)/b^3 - ((-(b\*x) + ArcCoth[Tanh[a + b\*x]])^3\*Log[ArcCoth[Tanh[a + b\*x]]])/b^4

**Maple [C]** time = 4.901, size = 130774, normalized size = 1614.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/arccoth(tanh(b*x+a)),x)`

[Out] result too large to display

**Maxima [C]** time = 1.80687, size = 116, normalized size = 1.43

$$\frac{4b^2x^3 + (3i\pi b - 6ab)x^2 - (3\pi^2 + 12i\pi a - 12a^2)x}{12b^3} - \frac{(i\pi^3 - 6\pi^2a - 12i\pi a^2 + 8a^3)\log(-i\pi + 2bx + 2a)}{8b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/arccoth(tanh(b*x+a)),x, algorithm="maxima")`

[Out]  $\frac{1}{12}*(4*b^2*x^3 + (3*I*pi*b - 6*a*b)*x^2 - (3*pi^2 + 12*I*pi*a - 12*a^2)*x) / b^3 - \frac{1}{8}*(I*pi^3 - 6*pi^2*a - 12*I*pi*a^2 + 8*a^3)*\log(-I*pi + 2*b*x + 2*a) / b^4$

**Fricas [A]** time = 1.5517, size = 293, normalized size = 3.62

$$\frac{8b^3x^3 - 12ab^2x^2 - 6(\pi^2b - 4a^2b)x - 6(\pi^3 - 12\pi a^2)\arctan\left(-\frac{2bx+2a-\sqrt{4b^2x^2+8abx+\pi^2+4a^2}}{\pi}\right) + 3(3\pi^2a - 4a^3)\log(4bx+2a)}{24b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/arccoth(tanh(b*x+a)),x, algorithm="fricas")`

[Out]  $\frac{1}{24}*(8*b^3*x^3 - 12*a*b^2*x^2 - 6*(\pi^2*b - 4*a^2*b)*x - 6*(\pi^3 - 12*\pi*a^2)*\arctan(-(2*b*x + 2*a - \sqrt{4*b^2*x^2 + 8*a*b*x + \pi^2 + 4*a^2}))/\pi) + 3*(3*\pi^2*a - 4*a^3)*\log(4*b^2*x^2 + 8*a*b*x + \pi^2 + 4*a^2))/b^4$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{acoth}(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/acoth(tanh(b*x+a)),x)
```

```
[Out] Integral(x**3/acoth(tanh(a + b*x)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{arccoth}(\tanh(bx + a))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^3/arccoth(tanh(b*x + a)), x)
```

$$3.160 \quad \int \frac{x^2}{\coth^{-1}(\tanh(a+bx))} dx$$

**Optimal.** Leaf size=56

$$\frac{x(bx - \coth^{-1}(\tanh(a+bx)))}{b^2} + \frac{(bx - \coth^{-1}(\tanh(a+bx)))^2 \log(\coth^{-1}(\tanh(a+bx)))}{b^3} + \frac{x^2}{2b}$$

[Out]  $x^2/(2*b) + (x*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]]))/b^2 + ((b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^2*\text{Log}[\text{ArcCoth}[\text{Tanh}[a + b*x]]])/b^3$

**Rubi [A]** time = 0.0346625, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {2159, 2158, 2157, 29}

$$\frac{x(bx - \coth^{-1}(\tanh(a+bx)))}{b^2} + \frac{(bx - \coth^{-1}(\tanh(a+bx)))^2 \log(\coth^{-1}(\tanh(a+bx)))}{b^3} + \frac{x^2}{2b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2/\text{ArcCoth}[\text{Tanh}[a + b*x]], x]$

[Out]  $x^2/(2*b) + (x*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]]))/b^2 + ((b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^2*\text{Log}[\text{ArcCoth}[\text{Tanh}[a + b*x]]])/b^3$

#### Rule 2159

$\text{Int}[(v_)^n/(u_), x\_Symbol] \rightarrow \text{With}[\{a = \text{Simplify}[D[u, x]], b = \text{Simplify}[D[v, x]]\}, \text{Simp}[v^n/(a^n), x] - \text{Dist}[(b*u - a*v)/a, \text{Int}[v^{n-1}/u, x], x] /; \text{NeQ}[b*u - a*v, 0] /; \text{PiecewiseLinearQ}[u, v, x] \&\& \text{GtQ}[n, 0] \&\& \text{NeQ}[n, 1]$

#### Rule 2158

$\text{Int}[(v_)/(u_), x\_Symbol] \rightarrow \text{With}[\{a = \text{Simplify}[D[u, x]], b = \text{Simplify}[D[v, x]]\}, \text{Simp}[(b*x)/a, x] - \text{Dist}[(b*u - a*v)/a, \text{Int}[1/u, x], x] /; \text{NeQ}[b*u - a*v, 0] /; \text{PiecewiseLinearQ}[u, v, x]$

#### Rule 2157

```
Int[(u_)^(m_.), x_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

### Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

### Rubi steps

$$\begin{aligned} \int \frac{x^2}{\coth^{-1}(\tanh(a+bx))} dx &= \frac{x^2}{2b} - \frac{(-bx + \coth^{-1}(\tanh(a+bx))) \int \frac{x}{\coth^{-1}(\tanh(a+bx))} dx}{b} \\ &= \frac{x^2}{2b} + \frac{x(bx - \coth^{-1}(\tanh(a+bx)))}{b^2} + \frac{(-bx + \coth^{-1}(\tanh(a+bx)))^2 \int \frac{1}{\coth^{-1}(\tanh(a+bx))} dx}{b^2} \\ &= \frac{x^2}{2b} + \frac{x(bx - \coth^{-1}(\tanh(a+bx)))}{b^2} + \frac{(-bx + \coth^{-1}(\tanh(a+bx)))^2 \text{Subst}\left(\int \frac{1}{x} dx, x, \coth^{-1}(\tanh(a+bx))\right)}{b^3} \\ &= \frac{x^2}{2b} + \frac{x(bx - \coth^{-1}(\tanh(a+bx)))}{b^2} + \frac{(bx - \coth^{-1}(\tanh(a+bx)))^2 \log(\coth^{-1}(\tanh(a+bx)))}{b^3} \end{aligned}$$

**Mathematica [A]** time = 0.0383789, size = 55, normalized size = 0.98

$$-\frac{x(\coth^{-1}(\tanh(a+bx)) - bx)}{b^2} + \frac{(\coth^{-1}(\tanh(a+bx)) - bx)^2 \log(\coth^{-1}(\tanh(a+bx)))}{b^3} + \frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcCoth[Tanh[a + b*x]], x]
```

```
[Out] x^2/(2*b) - (x*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/b^2 + ((-(b*x) + ArcCoth[
Tanh[a + b*x]])^2*Log[ArcCoth[Tanh[a + b*x]]])/b^3
```

**Maple [C]** time = 1.134, size = 28786, normalized size = 514.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(x^2/arccoth(tanh(b*x+a)),x)`

[Out] result too large to display

**Maxima [C]** time = 1.82435, size = 69, normalized size = 1.23

$$\frac{bx^2 + (i\pi - 2a)x}{2b^2} - \frac{(\pi^2 + 4i\pi a - 4a^2)\log(-i\pi + 2bx + 2a)}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/arccoth(tanh(b*x+a)),x, algorithm="maxima")`

[Out]  $1/2*(b*x^2 + (I*pi - 2*a)*x)/b^2 - 1/4*(pi^2 + 4*I*pi*a - 4*a^2)*\log(-I*pi + 2*b*x + 2*a)/b^3$

**Fricas [A]** time = 1.66004, size = 225, normalized size = 4.02

$$\frac{4b^2x^2 - 8abx - 16\pi a \arctan\left(-\frac{2bx+2a-\sqrt{4b^2x^2+8abx+\pi^2+4a^2}}{\pi}\right) - (\pi^2 - 4a^2)\log(4b^2x^2 + 8abx + \pi^2 + 4a^2)}{8b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/arccoth(tanh(b*x+a)),x, algorithm="fricas")`

[Out]  $1/8*(4*b^2*x^2 - 8*a*b*x - 16*pi*a*arctan(-(2*b*x + 2*a - \sqrt{4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2}))/pi) - (pi^2 - 4*a^2)*\log(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/b^3$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{acoth}(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/acoth(tanh(b*x+a)),x)
```

```
[Out] Integral(x**2/acoth(tanh(a + b*x)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{arccoth}(\tanh(bx + a))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2/arccoth(tanh(b*x + a)), x)
```

$$3.161 \quad \int \frac{x}{\coth^{-1}(\tanh(a+bx))} dx$$

**Optimal.** Leaf size=31

$$\frac{(bx - \coth^{-1}(\tanh(a + bx))) \log(\coth^{-1}(\tanh(a + bx)))}{b^2} + \frac{x}{b}$$

[Out] x/b + ((b\*x - ArcCoth[Tanh[a + b\*x]])\*Log[ArcCoth[Tanh[a + b\*x]]])/b^2

**Rubi [A]** time = 0.0150948, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {2158, 2157, 29}

$$\frac{(bx - \coth^{-1}(\tanh(a + bx))) \log(\coth^{-1}(\tanh(a + bx)))}{b^2} + \frac{x}{b}$$

Antiderivative was successfully verified.

[In] Int[x/ArcCoth[Tanh[a + b\*x]], x]

[Out] x/b + ((b\*x - ArcCoth[Tanh[a + b\*x]])\*Log[ArcCoth[Tanh[a + b\*x]]])/b^2

#### Rule 2158

Int[(v\_)/(u\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(b\*x)/a, x] - Dist[(b\*u - a\*v)/a, Int[1/u, x], x] /; NeQ[b\*u - a\*v, 0]] /; PiecewiseLinearQ[u, v, x]

#### Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

#### Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

#### Rubi steps

$$\begin{aligned}
\int \frac{x}{\coth^{-1}(\tanh(a+bx))} dx &= \frac{x}{b} - \frac{(-bx + \coth^{-1}(\tanh(a+bx)))}{b} \int \frac{1}{\coth^{-1}(\tanh(a+bx))} dx \\
&= \frac{x}{b} - \frac{(-bx + \coth^{-1}(\tanh(a+bx))) \operatorname{Subst}\left(\int \frac{1}{x} dx, x, \coth^{-1}(\tanh(a+bx))\right)}{b^2} \\
&= \frac{x}{b} + \frac{(bx - \coth^{-1}(\tanh(a+bx))) \log(\coth^{-1}(\tanh(a+bx)))}{b^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0221102, size = 31, normalized size = 1.

$$\frac{x}{b} - \frac{(\coth^{-1}(\tanh(a+bx)) - bx) \log(\coth^{-1}(\tanh(a+bx)))}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/ArcCoth[Tanh[a + b\*x]], x]

[Out] x/b - ((-(b\*x) + ArcCoth[Tanh[a + b\*x]])\*Log[ArcCoth[Tanh[a + b\*x]]])/b^2

**Maple [C]** time = 0.314, size = 4303, normalized size = 138.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/arccoth(tanh(b\*x+a)), x)

[Out]  $\frac{1}{b} \ln(-2\pi \operatorname{csgn}(I/(\exp(2bx+2a)+1))^{2+\pi} \operatorname{csgn}(I/(\exp(2bx+2a)+1))) \operatorname{csgn}(I \exp(2bx+2a)) \operatorname{csgn}(I \exp(2bx+2a)/(\exp(2bx+2a)+1)) - \pi \operatorname{csgn}(I/(\exp(2bx+2a)+1)) \operatorname{csgn}(I \exp(2bx+2a)/(\exp(2bx+2a)+1))^{2+2\pi} \operatorname{csgn}(I/(\exp(2bx+2a)+1))^{3+\pi} \operatorname{csgn}(I \exp(bx+a))^{2} \operatorname{csgn}(I \exp(2bx+2a)) - 2\pi \operatorname{csgn}(I \exp(bx+a)) \operatorname{csgn}(I \exp(2bx+2a))^{2+\pi} \operatorname{csgn}(I \exp(2bx+2a))^{3-\pi} \operatorname{csgn}(I \exp(2bx+2a)) \operatorname{csgn}(I \exp(2bx+2a)/(\exp(2bx+2a)+1))^{2+\pi} \operatorname{csgn}(I \exp(2bx+2a)/(\exp(2bx+2a)+1))^{3+4Ibx+4I(\ln(\exp(bx+a))-bx-a)+4I(a+2\pi)x-1/b^2} \ln(-2\pi \operatorname{csgn}(I/(\exp(2bx+2a)+1))^{2+\pi} \operatorname{csgn}(I/(\exp(2bx+2a)+1))) \operatorname{csgn}(I \exp(2bx+2a)) \operatorname{csgn}(I \exp(2bx+2a)/(\exp(2bx+2a)+1)) - \pi \operatorname{csgn}(I/(\exp(2bx+2a)+1)) \operatorname{csgn}(I \exp(2bx+2a)/(\exp(2bx+2a)+1))^{2+2\pi} \operatorname{csgn}(I/(\exp(2bx+2a)+1))^{3+\pi} \operatorname{csgn}(I \exp(bx+a))^{2} \operatorname{csgn}(I \exp(2bx+2a))$



$$\begin{aligned} &)) - \text{Pi} * \text{csgn}(I / (\exp(2*b*x+2*a)+1)) * \text{csgn}(I * \exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1)) ^{2+2*Pi} * \text{csgn}(I / (\exp(2*b*x+2*a)+1)) ^3 + \text{Pi} * \text{csgn}(I * \exp(b*x+a)) ^2 * \text{csgn}(I * \exp(2*b*x+2*a)) - 2*Pi * \text{csgn}(I * \exp(b*x+a)) * \text{csgn}(I * \exp(2*b*x+2*a)) ^2 + \text{Pi} * \text{csgn}(I * \exp(2*b*x+2*a)) ^3 - \text{Pi} * \text{csgn}(I * \exp(2*b*x+2*a)) * \text{csgn}(I * \exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1)) ^2 + \text{Pi} * \text{csgn}(I * \exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1)) ^3 + 4*I*b*x+4*I*(\ln(\exp(b*x+a))-b*x-a)+4*I*a+2*Pi) * \text{Pi} * \text{csgn}(I * \exp(b*x+a)) * \text{csgn}(I * \exp(2*b*x+2*a)) ^2 + 1/4 * I/b^2 * \ln(-2*Pi * \text{csgn}(I / (\exp(2*b*x+2*a)+1)) ^2 + \text{Pi} * \text{csgn}(I / (\exp(2*b*x+2*a)+1)) * \text{csgn}(I * \exp(2*b*x+2*a)) * \text{csgn}(I * \exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1)) - \text{Pi} * \text{csgn}(I / (\exp(2*b*x+2*a)+1)) * \text{csgn}(I * \exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1)) ^2 + 2*Pi * \text{csgn}(I / (\exp(2*b*x+2*a)+1)) ^3 + \text{Pi} * \text{csgn}(I * \exp(b*x+a)) ^2 * \text{csgn}(I * \exp(2*b*x+2*a)) - 2*Pi * \text{csgn}(I * \exp(b*x+a)) * \text{csgn}(I * \exp(2*b*x+2*a)) ^2 + \text{Pi} * \text{csgn}(I * \exp(2*b*x+2*a)) ^3 - \text{Pi} * \text{csgn}(I * \exp(2*b*x+2*a)) * \text{csgn}(I * \exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1)) ^2 + \text{Pi} * \text{csgn}(I * \exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1)) ^3 + 4*I*b*x+4*I*(\ln(\exp(b*x+a))-b*x-a)+4*I*a+2*Pi) * \text{Pi} * \text{csgn}(I * \exp(2*b*x+2*a)) ^3 - 1/4 * I/b^2 * \ln(-2*Pi * \text{csgn}(I / (\exp(2*b*x+2*a)+1)) ^2 + \text{Pi} * \text{csgn}(I / (\exp(2*b*x+2*a)+1)) * \text{csgn}(I * \exp(2*b*x+2*a)) * \text{csgn}(I * \exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1)) - \text{Pi} * \text{csgn}(I / (\exp(2*b*x+2*a)+1)) * \text{csgn}(I * \exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1)) ^2 + 2*Pi * \text{csgn}(I / (\exp(2*b*x+2*a)+1)) ^3 + \text{Pi} * \text{csgn}(I * \exp(b*x+a)) ^2 * \text{csgn}(I * \exp(2*b*x+2*a)) - 2*Pi * \text{csgn}(I * \exp(b*x+a)) * \text{csgn}(I * \exp(2*b*x+2*a)) ^2 + \text{Pi} * \text{csgn}(I * \exp(2*b*x+2*a)) ^3 - \text{Pi} * \text{csgn}(I * \exp(2*b*x+2*a)) * \text{csgn}(I * \exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1)) ^2 + \text{Pi} * \text{csgn}(I * \exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1)) ^3 + 4*I*b*x+4*I*(\ln(\exp(b*x+a))-b*x-a)+4*I*a+2*Pi) * \text{Pi} * \text{csgn}(I * \exp(2*b*x+2*a)) * \text{csgn}(I * \exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1)) ^2 + 1/4 * I/b^2 * \ln(-2*Pi * \text{csgn}(I / (\exp(2*b*x+2*a)+1)) ^2 + \text{Pi} * \text{csgn}(I / (\exp(2*b*x+2*a)+1)) * \text{csgn}(I * \exp(2*b*x+2*a)) * \text{csgn}(I * \exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1)) - \text{Pi} * \text{csgn}(I / (\exp(2*b*x+2*a)+1)) * \text{csgn}(I * \exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1)) ^2 + 2*Pi * \text{csgn}(I / (\exp(2*b*x+2*a)+1)) ^3 + \text{Pi} * \text{csgn}(I * \exp(b*x+a)) ^2 * \text{csgn}(I * \exp(2*b*x+2*a)) - 2*Pi * \text{csgn}(I * \exp(b*x+a)) * \text{csgn}(I * \exp(2*b*x+2*a)) ^2 + \text{Pi} * \text{csgn}(I * \exp(2*b*x+2*a)) ^3 - \text{Pi} * \text{csgn}(I * \exp(2*b*x+2*a)) * \text{csgn}(I * \exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1)) ^2 + \text{Pi} * \text{csgn}(I * \exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1)) ^3 + 4*I*b*x+4*I*(\ln(\exp(b*x+a))-b*x-a)+4*I*a+2*Pi) * \text{Pi} * \text{csgn}(I * \exp(2*b*x+2*a)) * \text{csgn}(I * \exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1)) ^2 + 1/2 * I/b^2 * \ln(-2*Pi * \text{csgn}(I / (\exp(2*b*x+2*a)+1)) ^2 + \text{Pi} * \text{csgn}(I / (\exp(2*b*x+2*a)+1)) * \text{csgn}(I * \exp(2*b*x+2*a)) * \text{csgn}(I * \exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1)) - \text{Pi} * \text{csgn}(I / (\exp(2*b*x+2*a)+1)) * \text{csgn}(I * \exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1)) ^2 + 2*Pi * \text{csgn}(I / (\exp(2*b*x+2*a)+1)) ^3 + \text{Pi} * \text{csgn}(I * \exp(b*x+a)) ^2 * \text{csgn}(I * \exp(2*b*x+2*a)) - 2*Pi * \text{csgn}(I * \exp(b*x+a)) * \text{csgn}(I * \exp(2*b*x+2*a)) ^2 + \text{Pi} * \text{csgn}(I * \exp(2*b*x+2*a)) ^3 - \text{Pi} * \text{csgn}(I * \exp(2*b*x+2*a)) * \text{csgn}(I * \exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1)) ^2 + \text{Pi} * \text{csgn}(I * \exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1)) ^3 + 4*I*b*x+4*I*(\ln(\exp(b*x+a))-b*x-a)+4*I*a+2*Pi) * \text{Pi} * x/b
\end{aligned}$$

**Maxima [C]** time = 1.83026, size = 41, normalized size = 1.32

$$\frac{x}{b} - \frac{(-i\pi + 2a)\log(-i\pi + 2bx + 2a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccoth(tanh(b\*x+a)),x, algorithm="maxima")

[Out]  $x/b - 1/2*(-I*\pi + 2*a)*\log(-I*\pi + 2*b*x + 2*a)/b^2$

**Fricas [B]** time = 1.59347, size = 185, normalized size = 5.97

$$\frac{2bx + 2\pi \arctan\left(-\frac{2bx + 2a - \sqrt{4b^2x^2 + 8abx + \pi^2 + 4a^2}}{\pi}\right) - a \log(4b^2x^2 + 8abx + \pi^2 + 4a^2)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccoth(tanh(b\*x+a)),x, algorithm="fricas")

[Out]  $1/2*(2*b*x + 2*\pi*\arctan(-(2*b*x + 2*a - \sqrt{4*b^2*x^2 + 8*a*b*x + \pi^2 + 4*a^2}))/\pi) - a*\log(4*b^2*x^2 + 8*a*b*x + \pi^2 + 4*a^2))/b^2$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{acoth}(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/acoth(tanh(b\*x+a)),x)

[Out] Integral(x/acoth(tanh(a + b\*x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{arcoth}(\tanh(bx + a))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccoth(tanh(b\*x+a)),x, algorithm="giac")

[Out] integrate(x/arccoth(tanh(b\*x + a)), x)

$$3.162 \quad \int \frac{1}{\coth^{-1}(\tanh(a+bx))} dx$$

**Optimal.** Leaf size=12

$$\frac{\log(\coth^{-1}(\tanh(a+bx)))}{b}$$

[Out] Log[ArcCoth[Tanh[a + b\*x]]]/b

**Rubi [A]** time = 0.0040205, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {2157, 29}

$$\frac{\log(\coth^{-1}(\tanh(a+bx)))}{b}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^(-1), x]

[Out] Log[ArcCoth[Tanh[a + b\*x]]]/b

#### Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[  
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

#### Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

#### Rubi steps

$$\begin{aligned} \int \frac{1}{\coth^{-1}(\tanh(a+bx))} dx &= \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, \coth^{-1}(\tanh(a+bx))\right)}{b} \\ &= \frac{\log(\coth^{-1}(\tanh(a+bx)))}{b} \end{aligned}$$



**Mathematica [A]** time = 0.04961, size = 12, normalized size = 1.

$$\frac{\log(\coth^{-1}(\tanh(a + bx)))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^(-1),x]

[Out] Log[ArcCoth[Tanh[a + b\*x]]]/b

**Maple [A]** time = 0.076, size = 13, normalized size = 1.1

$$\frac{\ln(\operatorname{arccoth}(\tanh(bx + a)))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/arccoth(tanh(b\*x+a)),x)

[Out] ln(arccoth(tanh(b\*x+a)))/b

**Maxima [C]** time = 1.45703, size = 22, normalized size = 1.83

$$\frac{\log\left(-\frac{1}{2}i\pi - bx - a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccoth(tanh(b\*x+a)),x, algorithm="maxima")

[Out] log(-1/2\*I\*pi - b\*x - a)/b

**Fricas [B]** time = 1.63011, size = 63, normalized size = 5.25

$$\frac{\log(4b^2x^2 + 8abx + \pi^2 + 4a^2)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccoth(tanh(b\*x+a)),x, algorithm="fricas")

[Out] 1/2\*log(4\*b^2\*x^2 + 8\*a\*b\*x + pi^2 + 4\*a^2)/b

**Sympy [A]** time = 8.22448, size = 17, normalized size = 1.42

$$\begin{cases} \frac{\log(\operatorname{acoth}(\tanh(a+bx)))}{b} & \text{for } b \neq 0 \\ \frac{x}{\operatorname{acoth}(\tanh(a))} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/acoth(tanh(b\*x+a)),x)

[Out] Piecewise((log(acoth(tanh(a + b\*x)))/b, Ne(b, 0)), (x/acoth(tanh(a)), True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{arccoth}(\tanh(bx + a))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccoth(tanh(b\*x+a)),x, algorithm="giac")

[Out] integrate(1/arccoth(tanh(b\*x + a)), x)

$$3.163 \quad \int \frac{1}{x \coth^{-1}(\tanh(a+bx))} dx$$

Optimal. Leaf size=44

$$\frac{\log(\coth^{-1}(\tanh(a+bx)))}{bx - \coth^{-1}(\tanh(a+bx))} - \frac{\log(x)}{bx - \coth^{-1}(\tanh(a+bx))}$$

[Out]  $-(\text{Log}[x]/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])) + \text{Log}[\text{ArcCoth}[\text{Tanh}[a + b*x]]]/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])$

**Rubi [A]** time = 0.0284736, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2160, 2157, 29}

$$\frac{\log(\coth^{-1}(\tanh(a+bx)))}{bx - \coth^{-1}(\tanh(a+bx))} - \frac{\log(x)}{bx - \coth^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x*\text{ArcCoth}[\text{Tanh}[a + b*x]]), x]$

[Out]  $-(\text{Log}[x]/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])) + \text{Log}[\text{ArcCoth}[\text{Tanh}[a + b*x]]]/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])$

#### Rule 2160

$\text{Int}[1/((u_)*(v_)), x\_Symbol] \text{ :> With}[\{a = \text{Simplify}[D[u, x]], b = \text{Simplify}[D[v, x]]\}, \text{Dist}[b/(b*u - a*v), \text{Int}[1/v, x], x] - \text{Dist}[a/(b*u - a*v), \text{Int}[1/u, x], x] \text{ ; NeQ}[b*u - a*v, 0] \text{ ; PiecewiseLinearQ}[u, v, x]$

#### Rule 2157

$\text{Int}[(u_)^{(m_.)}, x\_Symbol] \text{ :> With}[\{c = \text{Simplify}[D[u, x]]\}, \text{Dist}[1/c, \text{Subst}[\text{Int}[x^m, x], x, u], x] \text{ ; FreeQ}[m, x] \ \&\& \text{PiecewiseLinearQ}[u, x]$

#### Rule 29

$\text{Int}[(x_)^{(-1)}, x\_Symbol] \text{ :> Simp}[\text{Log}[x], x]$

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{x \coth^{-1}(\tanh(a + bx))} dx &= -\frac{\int \frac{1}{x} dx}{bx - \coth^{-1}(\tanh(a + bx))} + \frac{b \int \frac{1}{\coth^{-1}(\tanh(a + bx))} dx}{bx - \coth^{-1}(\tanh(a + bx))} \\
&= -\frac{\log(x)}{bx - \coth^{-1}(\tanh(a + bx))} + \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, \coth^{-1}(\tanh(a + bx))\right)}{bx - \coth^{-1}(\tanh(a + bx))} \\
&= -\frac{\log(x)}{bx - \coth^{-1}(\tanh(a + bx))} + \frac{\log(\coth^{-1}(\tanh(a + bx)))}{bx - \coth^{-1}(\tanh(a + bx))}
\end{aligned}$$

**Mathematica [A]** time = 0.0183468, size = 29, normalized size = 0.66

$$\frac{\log(\coth^{-1}(\tanh(a + bx))) - \log(x)}{bx - \coth^{-1}(\tanh(a + bx))}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*ArcCoth[Tanh[a + b\*x]]),x]

[Out] (-Log[x] + Log[ArcCoth[Tanh[a + b\*x]]])/(b\*x - ArcCoth[Tanh[a + b\*x]])

**Maple [C]** time = 11.445, size = 972, normalized size = 22.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/arccoth(tanh(b\*x+a)),x)

[Out]  $-4*I/(2*Pi*csgn(I/(\exp(2*b*x+2*a)+1)))^3+Pi*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))-Pi*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2+Pi*csgn(I*\exp(2*b*x+2*a))^3-2*Pi*csgn(I*\exp(b*x+a))*csgn(I*\exp(2*b*x+2*a))^2-Pi*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2+Pi*csgn(I*\exp(b*x+a))^2*csgn(I*\exp(2*b*x+2*a))+Pi*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3-4*I*b*x-2*Pi*csgn(I/(\exp(2*b*x+2*a)+1))^2+4*I*ln(\exp(b*x+a))+2*Pi)*ln(-2*Pi*csgn(I/(\exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))-Pi*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(\exp(2*b*x+2*a$

$$\begin{aligned}
& +1))^{3+\pi} \operatorname{csgn}(I \exp(bx+a))^{2\pi} \operatorname{csgn}(I \exp(2bx+2a)) - 2\pi \operatorname{csgn}(I \exp(bx+a)) \\
& \operatorname{csgn}(I \exp(2bx+2a))^{2+\pi} \operatorname{csgn}(I \exp(2bx+2a))^{3-\pi} \operatorname{csgn}(I \exp(2bx+2a)) \\
& \operatorname{csgn}(I \exp(2bx+2a)) \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{2+\pi} \operatorname{csgn}(I \exp(2bx+2a)) \\
& / (\exp(2bx+2a)+1))^{3+4Ibx+4I(\ln(\exp(bx+a))-bx-a)+4Ia+2\pi} + 4I / \\
& (2\pi \operatorname{csgn}(I / (\exp(2bx+2a)+1))^{3+\pi} \operatorname{csgn}(I / (\exp(2bx+2a)+1)) \operatorname{csgn}(I \exp(2bx+2a)) \\
& \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1)) - \pi \operatorname{csgn}(I / (\exp(2bx+2a)+1)) \\
& \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{2+\pi} \operatorname{csgn}(I \exp(2bx+2a))^{3-2\pi} \operatorname{csgn}(I \exp(bx+a)) \\
& \operatorname{csgn}(I \exp(2bx+2a))^{2-\pi} \operatorname{csgn}(I \exp(2bx+2a)) \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{2+\pi} \\
& \operatorname{csgn}(I \exp(bx+a))^{2\pi} \operatorname{csgn}(I \exp(2bx+2a)) + \pi \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{3-4Ibx-2\pi} \\
& \operatorname{csgn}(I / (\exp(2bx+2a)+1))^{2+4I \ln(\exp(bx+a))+2\pi} \ln(x)
\end{aligned}$$

**Maxima [C]** time = 1.81207, size = 50, normalized size = 1.14

$$\frac{2 \log(-i\pi + 2bx + 2a)}{i\pi - 2a} - \frac{2 \log(x)}{i\pi - 2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arccoth(tanh(b\*x+a)),x, algorithm="maxima")

[Out] 2\*log(-I\*pi + 2\*b\*x + 2\*a)/(I\*pi - 2\*a) - 2\*log(x)/(I\*pi - 2\*a)

**Fricas [A]** time = 1.69563, size = 205, normalized size = 4.66

$$\frac{2 \left( 2\pi \arctan \left( -\frac{2bx+2a-\sqrt{4b^2x^2+8abx+\pi^2+4a^2}}{\pi} \right) + a \log(4b^2x^2 + 8abx + \pi^2 + 4a^2) - 2a \log(x) \right)}{\pi^2 + 4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arccoth(tanh(b\*x+a)),x, algorithm="fricas")

[Out] -2\*(2\*pi\*arctan(-(2\*b\*x + 2\*a - sqrt(4\*b^2\*x^2 + 8\*a\*b\*x + pi^2 + 4\*a^2))/pi) + a\*log(4\*b^2\*x^2 + 8\*a\*b\*x + pi^2 + 4\*a^2) - 2\*a\*log(x))/(pi^2 + 4\*a^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{acoth}(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/acoth(tanh(b\*x+a)),x)

[Out] Integral(1/(x\*acoth(tanh(a + b\*x))), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{arccoth}(\tanh(bx + a))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arccoth(tanh(b\*x+a)),x, algorithm="giac")

[Out] integrate(1/(x\*arccoth(tanh(b\*x + a))), x)

$$3.164 \quad \int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))} dx$$

**Optimal.** Leaf size=65

$$\frac{1}{x \left( bx - \coth^{-1}(\tanh(a+bx)) \right)} - \frac{b \log(x)}{\left( bx - \coth^{-1}(\tanh(a+bx)) \right)^2} + \frac{b \log\left(\coth^{-1}(\tanh(a+bx))\right)}{\left( bx - \coth^{-1}(\tanh(a+bx)) \right)^2}$$

[Out] 1/(x\*(b\*x - ArcCoth[Tanh[a + b\*x]])) - (b\*Log[x])/(b\*x - ArcCoth[Tanh[a + b\*x]])^2 + (b\*Log[ArcCoth[Tanh[a + b\*x]]])/(b\*x - ArcCoth[Tanh[a + b\*x]])^2

**Rubi [A]** time = 0.0378546, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {2163, 2160, 2157, 29}

$$\frac{1}{x \left( bx - \coth^{-1}(\tanh(a+bx)) \right)} - \frac{b \log(x)}{\left( bx - \coth^{-1}(\tanh(a+bx)) \right)^2} + \frac{b \log\left(\coth^{-1}(\tanh(a+bx))\right)}{\left( bx - \coth^{-1}(\tanh(a+bx)) \right)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*ArcCoth[Tanh[a + b\*x]]),x]

[Out] 1/(x\*(b\*x - ArcCoth[Tanh[a + b\*x]])) - (b\*Log[x])/(b\*x - ArcCoth[Tanh[a + b\*x]])^2 + (b\*Log[ArcCoth[Tanh[a + b\*x]]])/(b\*x - ArcCoth[Tanh[a + b\*x]])^2

### Rule 2163

Int[(v\_)^(n\_)/(u\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[v^(n + 1)/((n + 1)\*(b\*u - a\*v)), x] - Dist[(a\*(n + 1))/((n + 1)\*(b\*u - a\*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x] && LtQ[n, -1]

### Rule 2160

Int[1/((u\_)\*(v\_)), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Dist[b/(b\*u - a\*v), Int[1/v, x], x] - Dist[a/(b\*u - a\*v), Int[1/u, x], x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x]

### Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[  
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

### Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

### Rubi steps

$$\begin{aligned} \int \frac{1}{x^2 \coth^{-1}(\tanh(a + bx))} dx &= \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))} + \frac{b \int \frac{1}{x \coth^{-1}(\tanh(a + bx))} dx}{bx - \coth^{-1}(\tanh(a + bx))} \\ &= \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))} - \frac{b \int \frac{1}{x} dx}{(bx - \coth^{-1}(\tanh(a + bx)))^2} + \frac{b^2 \int \frac{1}{\coth^{-1}(\tanh(a + bx))} dx}{(bx - \coth^{-1}(\tanh(a + bx)))^2} \\ &= \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))} - \frac{b \log(x)}{(bx - \coth^{-1}(\tanh(a + bx)))^2} + \frac{b \operatorname{Subst}\left(\int \frac{1}{x} dx, x, bx - \coth^{-1}(\tanh(a + bx))\right)}{(bx - \coth^{-1}(\tanh(a + bx)))^2} \\ &= \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))} - \frac{b \log(x)}{(bx - \coth^{-1}(\tanh(a + bx)))^2} + \frac{b \log(\coth^{-1}(\tanh(a + bx)))}{(bx - \coth^{-1}(\tanh(a + bx)))^2} \end{aligned}$$

**Mathematica [A]** time = 0.024122, size = 45, normalized size = 0.69

$$\frac{bx \left( \log(\coth^{-1}(\tanh(a + bx))) - \log(x) + 1 \right) - \coth^{-1}(\tanh(a + bx))}{x \left( \coth^{-1}(\tanh(a + bx)) - bx \right)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*ArcCoth[Tanh[a + b\*x]]),x]

[Out] (-ArcCoth[Tanh[a + b\*x]] + b\*x\*(1 - Log[x] + Log[ArcCoth[Tanh[a + b\*x]]]))/(x\*(-(b\*x) + ArcCoth[Tanh[a + b\*x]])^2)

**Maple [F]** time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \operatorname{arccoth}(\tanh(bx + a))} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/arccoth(tanh(b*x+a)),x)`

[Out] `int(1/x^2/arccoth(tanh(b*x+a)),x)`

**Maxima [C]** time = 1.7929, size = 88, normalized size = 1.35

$$-\frac{4b \log(-i\pi + 2bx + 2a)}{\pi^2 + 4i\pi a - 4a^2} + \frac{4b \log(x)}{\pi^2 + 4i\pi a - 4a^2} + \frac{2}{(i\pi - 2a)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/arccoth(tanh(b*x+a)),x, algorithm="maxima")`

[Out] `-4*b*log(-I*pi + 2*b*x + 2*a)/(pi^2 + 4*I*pi*a - 4*a^2) + 4*b*log(x)/(pi^2 + 4*I*pi*a - 4*a^2) + 2/((I*pi - 2*a)*x)`

**Fricas [B]** time = 1.72745, size = 315, normalized size = 4.85

$$\frac{2 \left( 16 \pi a b x \arctan \left( -\frac{2bx + 2a - \sqrt{4b^2x^2 + 8abx + \pi^2 + 4a^2}}{\pi} \right) - 2\pi^2 a - 8a^3 - (\pi^2 b - 4a^2 b)x \log(4b^2x^2 + 8abx + \pi^2 + 4a^2) + 2 \right)}{(\pi^4 + 8\pi^2 a^2 + 16a^4)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/arccoth(tanh(b*x+a)),x, algorithm="fricas")`

[Out] `2*(16*pi*a*b*x*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/pi) - 2*pi^2*a - 8*a^3 - (pi^2*b - 4*a^2*b)*x*log(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2) + 2*(pi^2*b - 4*a^2*b)*x*log(x))/((pi^4 + 8*pi^2*a^2 + 16*a^4)*x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \operatorname{acoth}(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/acoth(tanh(b\*x+a)),x)

[Out] Integral(1/(x\*\*2\*acoth(tanh(a + b\*x))), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \operatorname{arccoth}(\tanh(bx + a))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arccoth(tanh(b\*x+a)),x, algorithm="giac")

[Out] integrate(1/(x^2\*arccoth(tanh(b\*x + a))), x)

$$3.165 \quad \int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))} dx$$

**Optimal.** Leaf size=92

$$-\frac{b^2 \log(x)}{(bx - \coth^{-1}(\tanh(a+bx)))^3} + \frac{b^2 \log(\coth^{-1}(\tanh(a+bx)))}{(bx - \coth^{-1}(\tanh(a+bx)))^3} + \frac{1}{2x^2 (bx - \coth^{-1}(\tanh(a+bx)))} + \frac{1}{x (bx - \coth^{-1}(\tanh(a+bx)))}$$

[Out] b/(x\*(b\*x - ArcCoth[Tanh[a + b\*x]])^2) + 1/(2\*x^2\*(b\*x - ArcCoth[Tanh[a + b\*x]])) - (b^2\*Log[x])/(b\*x - ArcCoth[Tanh[a + b\*x]])^3 + (b^2\*Log[ArcCoth[Tanh[a + b\*x]]])/(b\*x - ArcCoth[Tanh[a + b\*x]])^3

**Rubi [A]** time = 0.0628528, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {2163, 2160, 2157, 29}

$$-\frac{b^2 \log(x)}{(bx - \coth^{-1}(\tanh(a+bx)))^3} + \frac{b^2 \log(\coth^{-1}(\tanh(a+bx)))}{(bx - \coth^{-1}(\tanh(a+bx)))^3} + \frac{1}{2x^2 (bx - \coth^{-1}(\tanh(a+bx)))} + \frac{1}{x (bx - \coth^{-1}(\tanh(a+bx)))}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*ArcCoth[Tanh[a + b\*x]]),x]

[Out] b/(x\*(b\*x - ArcCoth[Tanh[a + b\*x]])^2) + 1/(2\*x^2\*(b\*x - ArcCoth[Tanh[a + b\*x]])) - (b^2\*Log[x])/(b\*x - ArcCoth[Tanh[a + b\*x]])^3 + (b^2\*Log[ArcCoth[Tanh[a + b\*x]]])/(b\*x - ArcCoth[Tanh[a + b\*x]])^3

### Rule 2163

Int[(v\_)^(n\_)/(u\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[v^(n + 1)/((n + 1)\*(b\*u - a\*v)), x] - Dist[(a\*(n + 1))/((n + 1)\*(b\*u - a\*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x] && LtQ[n, -1]

### Rule 2160

Int[1/((u\_)\*(v\_)), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Dist[b/(b\*u - a\*v), Int[1/v, x], x] - Dist[a/(b\*u - a\*v), Int[1/u, x], x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x]

Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] :=> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[  
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

Rule 29

Int[(x\_)^(-1), x\_Symbol] :=> Simp[Log[x], x]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^3 \coth^{-1}(\tanh(a + bx))} dx &= \frac{1}{2x^2 (bx - \coth^{-1}(\tanh(a + bx)))} - \frac{b \int \frac{1}{x^2 \coth^{-1}(\tanh(a + bx))} dx}{-bx + \coth^{-1}(\tanh(a + bx))} \\
 &= \frac{b}{x (bx - \coth^{-1}(\tanh(a + bx)))^2} + \frac{1}{2x^2 (bx - \coth^{-1}(\tanh(a + bx)))} - \frac{b^2 \log(x)}{(bx - \coth^{-1}(\tanh(a + bx)))} \\
 &= \frac{b}{x (bx - \coth^{-1}(\tanh(a + bx)))^2} + \frac{1}{2x^2 (bx - \coth^{-1}(\tanh(a + bx)))} + \frac{b^2 \log(x)}{(bx - \coth^{-1}(\tanh(a + bx)))} \\
 &= \frac{b}{x (bx - \coth^{-1}(\tanh(a + bx)))^2} + \frac{1}{2x^2 (bx - \coth^{-1}(\tanh(a + bx)))} - \frac{b^2 \log(x)}{(bx - \coth^{-1}(\tanh(a + bx)))} \\
 &= \frac{b}{x (bx - \coth^{-1}(\tanh(a + bx)))^2} + \frac{1}{2x^2 (bx - \coth^{-1}(\tanh(a + bx)))} - \frac{b^2 \log(x)}{(bx - \coth^{-1}(\tanh(a + bx)))}
 \end{aligned}$$

**Mathematica [A]** time = 0.0270553, size = 66, normalized size = 0.72

$$\frac{b^2 x^2 (2 \log(\coth^{-1}(\tanh(a + bx))) - 2 \log(x) + 3) - 4bx \coth^{-1}(\tanh(a + bx)) + \coth^{-1}(\tanh(a + bx))^2}{2x^2 (bx - \coth^{-1}(\tanh(a + bx)))^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*ArcCoth[Tanh[a + b\*x]]),x]

[Out] (-4\*b\*x\*ArcCoth[Tanh[a + b\*x]] + ArcCoth[Tanh[a + b\*x]]^2 + b^2\*x^2\*(3 - 2\*  
Log[x] + 2\*Log[ArcCoth[Tanh[a + b\*x]]]))/(2\*x^2\*(b\*x - ArcCoth[Tanh[a + b\*x]]  
))^3)

---

**Maple [F]** time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \operatorname{arccoth}(\tanh(bx+a))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/arccoth(tanh(b\*x+a)),x)

[Out] int(1/x^3/arccoth(tanh(b\*x+a)),x)

---

**Maxima [C]** time = 1.79593, size = 146, normalized size = 1.59

$$\frac{8b^2 \log(-i\pi + 2bx + 2a)}{-i\pi^3 + 6\pi^2a + 12i\pi a^2 - 8a^3} - \frac{8b^2 \log(x)}{-i\pi^3 + 6\pi^2a + 12i\pi a^2 - 8a^3} - \frac{2(i\pi + 4bx - 2a)}{(2\pi^2 + 8i\pi a - 8a^2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/arccoth(tanh(b\*x+a)),x, algorithm="maxima")

[Out] 8\*b^2\*log(-I\*pi + 2\*b\*x + 2\*a)/(-I\*pi^3 + 6\*pi^2\*a + 12\*I\*pi\*a^2 - 8\*a^3) - 8\*b^2\*log(x)/(-I\*pi^3 + 6\*pi^2\*a + 12\*I\*pi\*a^2 - 8\*a^3) - 2\*(I\*pi + 4\*b\*x - 2\*a)/((2\*pi^2 + 8\*I\*pi\*a - 8\*a^2)\*x^2)

---

**Fricas [B]** time = 1.71336, size = 448, normalized size = 4.87

$$\frac{2 \left( \pi^4 a + 8 \pi^2 a^3 + 16 a^5 - 8 (\pi^3 b^2 - 12 \pi a^2 b^2) x^2 \arctan \left( -\frac{2bx+2a-\sqrt{4b^2x^2+8abx+\pi^2+4a^2}}{\pi} \right) - 4 (3 \pi^2 ab^2 - 4 a^3 b^2) x^2 \log(4 \right)}{(\pi^6 + 12 \pi^4 a^2 + 48 \pi^2 a^4 + 64 a^6) x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/arccoth(tanh(b\*x+a)),x, algorithm="fricas")

[Out] -2\*(pi^4\*a + 8\*pi^2\*a^3 + 16\*a^5 - 8\*(pi^3\*b^2 - 12\*pi\*a^2\*b^2)\*x^2\*arctan(- (2\*b\*x + 2\*a - sqrt(4\*b^2\*x^2 + 8\*a\*b\*x + pi^2 + 4\*a^2))/pi) - 4\*(3\*pi^2\*a\*b^2 - 4\*a^3\*b^2)\*x^2\*log(4\*b^2\*x^2 + 8\*a\*b\*x + pi^2 + 4\*a^2) + 8\*(3\*pi^2\*a

$*b^2 - 4*a^3*b^2)*x^2*\log(x) + 2*(\pi^4*b - 16*a^4*b)*x)/((\pi^6 + 12*\pi^4*a^2 + 48*\pi^2*a^4 + 64*a^6)*x^2)$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \operatorname{acoth}(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/acoth(tanh(b\*x+a)),x)

[Out] Integral(1/(x\*\*3\*acoth(tanh(a + b\*x))), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \operatorname{arccoth}(\tanh(bx + a))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/arccoth(tanh(b\*x+a)),x, algorithm="giac")

[Out] integrate(1/(x^3\*arccoth(tanh(b\*x + a))), x)

$$3.166 \quad \int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^2} dx$$

**Optimal.** Leaf size=65

$$-\frac{x^m \text{Hypergeometric2F1}\left(1, m, m+1, \frac{bx}{bx - \coth^{-1}(\tanh(a+bx))}\right)}{b(bx - \coth^{-1}(\tanh(a+bx)))} - \frac{x^m}{b \coth^{-1}(\tanh(a+bx))}$$

[Out]  $-(x^m/(b*\text{ArcCoth}[\text{Tanh}[a + b*x]])) - (x^m*\text{Hypergeometric2F1}[1, m, 1 + m, (b*x)/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])]/(b*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]]))$

**Rubi [A]** time = 0.0390504, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2168, 2164}

$$-\frac{x^m {}_2F_1\left(1, m; m+1; \frac{bx}{bx - \coth^{-1}(\tanh(a+bx))}\right)}{b(bx - \coth^{-1}(\tanh(a+bx)))} - \frac{x^m}{b \coth^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

[In] Int[x^m/ArcCoth[Tanh[a + b\*x]]^2,x]

[Out]  $-(x^m/(b*\text{ArcCoth}[\text{Tanh}[a + b*x]])) - (x^m*\text{Hypergeometric2F1}[1, m, 1 + m, (b*x)/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])]/(b*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]]))$

### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

### Rule 2164

```
Int[(v_)^(n_)/(u_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(v^(n + 1)*Hypergeometric2F1[1, n + 1, n + 2, -((a*v)/(b*u - a*v))]/((n + 1)*(b*u - a*v)), x] /; NeQ[b*u - a*v, 0] /; PiecewiseLinea
```

rQ[u, v, x] && !IntegerQ[n]

### Rubi steps

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^2} dx = -\frac{x^m}{b \coth^{-1}(\tanh(a+bx))} + \frac{m \int \frac{x^{-1+m}}{\coth^{-1}(\tanh(a+bx))} dx}{b}$$

$$= -\frac{x^m}{b \coth^{-1}(\tanh(a+bx))} - \frac{x^m {}_2F_1\left(1, m; 1+m; \frac{bx}{bx - \coth^{-1}(\tanh(a+bx))}\right)}{b (bx - \coth^{-1}(\tanh(a+bx)))}$$

**Mathematica [A]** time = 0.521421, size = 51, normalized size = 0.78

$$\frac{x^{m+1} \text{Hypergeometric2F1}\left(2, m+1, m+2, -\frac{bx}{\coth^{-1}(\tanh(a+bx))-bx}\right)}{(m+1) \left(\coth^{-1}(\tanh(a+bx)) - bx\right)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^m/ArcCoth[Tanh[a + b\*x]]^2,x]

[Out] (x^(1+m)\*Hypergeometric2F1[2, 1+m, 2+m, -((b\*x)/(-(b\*x) + ArcCoth[Tanh[a + b\*x]]))])/((1+m)\*(-(b\*x) + ArcCoth[Tanh[a + b\*x]])^2)

**Maple [F]** time = 1.83, size = 0, normalized size = 0.

$$\int \frac{x^m}{(\operatorname{arccoth}(\tanh(bx+a)))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m/arccoth(tanh(b\*x+a))^2,x)

[Out] int(x^m/arccoth(tanh(b\*x+a))^2,x)



**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{arccoth}(\tanh(bx + a))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/arccoth(tanh(b\*x+a))^2,x, algorithm="maxima")

[Out] integrate(x^m/arccoth(tanh(b\*x + a))^2, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x^m}{\operatorname{arccoth}(\tanh(bx + a))^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/arccoth(tanh(b\*x+a))^2,x, algorithm="fricas")

[Out] integral(x^m/arccoth(tanh(b\*x + a))^2, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{acoth}^2(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m/acoth(tanh(b\*x+a))\*\*2,x)

[Out] Integral(x\*\*m/acoth(tanh(a + b\*x))\*\*2, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{arccoth}(\tanh(bx + a))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/arccoth(tanh(b*x + a))^2, x)
```

$$3.167 \quad \int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^2} dx$$

**Optimal.** Leaf size=98

$$\frac{2x^2 (bx - \coth^{-1}(\tanh(a + bx)))}{b^3} + \frac{4x (bx - \coth^{-1}(\tanh(a + bx)))^2}{b^4} + \frac{4 (bx - \coth^{-1}(\tanh(a + bx)))^3 \log(\coth^{-1}(\tanh(a + bx)))}{b^5}$$

[Out] (4\*x^3)/(3\*b^2) + (2\*x^2\*(b\*x - ArcCoth[Tanh[a + b\*x]]))/b^3 + (4\*x\*(b\*x - ArcCoth[Tanh[a + b\*x]])^2)/b^4 - x^4/(b\*ArcCoth[Tanh[a + b\*x]]) + (4\*(b\*x - ArcCoth[Tanh[a + b\*x]])^3\*Log[ArcCoth[Tanh[a + b\*x]]])/b^5

**Rubi [A]** time = 0.0808205, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {2168, 2159, 2158, 2157, 29}

$$\frac{2x^2 (bx - \coth^{-1}(\tanh(a + bx)))}{b^3} + \frac{4x (bx - \coth^{-1}(\tanh(a + bx)))^2}{b^4} + \frac{4 (bx - \coth^{-1}(\tanh(a + bx)))^3 \log(\coth^{-1}(\tanh(a + bx)))}{b^5}$$

Antiderivative was successfully verified.

[In] Int[x^4/ArcCoth[Tanh[a + b\*x]]^2,x]

[Out] (4\*x^3)/(3\*b^2) + (2\*x^2\*(b\*x - ArcCoth[Tanh[a + b\*x]]))/b^3 + (4\*x\*(b\*x - ArcCoth[Tanh[a + b\*x]])^2)/b^4 - x^4/(b\*ArcCoth[Tanh[a + b\*x]]) + (4\*(b\*x - ArcCoth[Tanh[a + b\*x]])^3\*Log[ArcCoth[Tanh[a + b\*x]]])/b^5

### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

### Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
```

;/ NeQ[b\*u - a\*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n, 1]

### Rule 2158

Int[(v\_)/(u\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(b\*x)/a, x] - Dist[(b\*u - a\*v)/a, Int[1/u, x], x] /; NeQ[b\*u - a\*v, 0]] /; PiecewiseLinearQ[u, v, x]

### Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

### Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

### Rubi steps

$$\begin{aligned}
 \int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^2} dx &= -\frac{x^4}{b \coth^{-1}(\tanh(a+bx))} + \frac{4 \int \frac{x^3}{\coth^{-1}(\tanh(a+bx))} dx}{b} \\
 &= \frac{4x^3}{3b^2} - \frac{x^4}{b \coth^{-1}(\tanh(a+bx))} - \frac{(4(-bx + \coth^{-1}(\tanh(a+bx)))) \int \frac{x^2}{\coth^{-1}(\tanh(a+bx))} dx}{b^2} \\
 &= \frac{4x^3}{3b^2} + \frac{2x^2(bx - \coth^{-1}(\tanh(a+bx)))}{b^3} - \frac{x^4}{b \coth^{-1}(\tanh(a+bx))} + \frac{(4(-bx + \coth^{-1}(\tanh(a+bx))))^2}{b \coth^{-1}(\tanh(a+bx))} \\
 &= \frac{4x^3}{3b^2} + \frac{2x^2(bx - \coth^{-1}(\tanh(a+bx)))}{b^3} + \frac{4x(bx - \coth^{-1}(\tanh(a+bx)))^2}{b^4} - \frac{1}{b \coth^{-1}(\tanh(a+bx))} \\
 &= \frac{4x^3}{3b^2} + \frac{2x^2(bx - \coth^{-1}(\tanh(a+bx)))}{b^3} + \frac{4x(bx - \coth^{-1}(\tanh(a+bx)))^2}{b^4} - \frac{1}{b \coth^{-1}(\tanh(a+bx))} \\
 &= \frac{4x^3}{3b^2} + \frac{2x^2(bx - \coth^{-1}(\tanh(a+bx)))}{b^3} + \frac{4x(bx - \coth^{-1}(\tanh(a+bx)))^2}{b^4} - \frac{1}{b \coth^{-1}(\tanh(a+bx))}
 \end{aligned}$$

**Mathematica [A]** time = 0.0974712, size = 106, normalized size = 1.08

$$\frac{x^2 (\coth^{-1}(\tanh(a + bx)) - bx)}{b^3} - \frac{(\coth^{-1}(\tanh(a + bx)) - bx)^4}{b^5 \coth^{-1}(\tanh(a + bx))} + \frac{3x (\coth^{-1}(\tanh(a + bx)) - bx)^2}{b^4} - \frac{4 (\coth^{-1}(\tanh(a + bx)) - bx)}{b^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/ArcCoth[Tanh[a + b\*x]]^2,x]

[Out]  $x^3/(3*b^2) - (x^2*(-(b*x) + \text{ArcCoth}[\text{Tanh}[a + b*x]]))/b^3 + (3*x*(-(b*x) + \text{ArcCoth}[\text{Tanh}[a + b*x]]))^2/b^4 - (-(b*x) + \text{ArcCoth}[\text{Tanh}[a + b*x]])^4/(b^5*\text{ArcCoth}[\text{Tanh}[a + b*x]]) - (4*(-(b*x) + \text{ArcCoth}[\text{Tanh}[a + b*x]))^3*\text{Log}[\text{ArcCoth}[\text{Tanh}[a + b*x]])/b^5$

**Maple [C]** time = 6.12, size = 131085, normalized size = 1337.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/arccoth(tanh(b\*x+a))^2,x)

[Out] result too large to display

**Maxima [C]** time = 2.48721, size = 240, normalized size = 2.45

$$\frac{4(16b^4x^4 - 3\pi^4 - 24i\pi^3a + 72\pi^2a^2 + 96i\pi a^3 - 48a^4 + (16i\pi b^3 - 32ab^3)x^3 - (24\pi^2b^2 + 96i\pi ab^2 - 96a^2b^2)x^2 + (16i\pi b^3 - 32ab^3)x - 24\pi^2b^2 - 96i\pi ab^2 + 96a^2b^2)}{192b^6x - 96i\pi b^5 + 192ab^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arccoth(tanh(b\*x+a))^2,x, algorithm="maxima")

[Out]  $4*(16*b^4*x^4 - 3*\pi^4 - 24*I*\pi^3*a + 72*\pi^2*a^2 + 96*I*\pi*a^3 - 48*a^4 + (16*I*\pi*b^3 - 32*a*b^3)*x^3 - (24*\pi^2*b^2 + 96*I*\pi*a*b^2 - 96*a^2*b^2)*x^2 + (18*I*\pi^3*b - 108*\pi^2*a*b - 216*I*\pi*a^2*b + 144*a^3*b)*x)/(192*b^6*x - 96*I*\pi*b^5 + 192*a*b^5) - 1/2*(I*\pi^3 - 6*\pi^2*a - 12*I*\pi*a^2 + 8*a^3)*\log(-I*\pi + 2*b*x + 2*a)/b^5$

---

**Fricas [B]** time = 1.66643, size = 729, normalized size = 7.44

$$16b^5x^5 - 16ab^4x^4 + 9\pi^4a + 24\pi^2a^3 - 48a^5 - 32(\pi^2b^3 - 2a^2b^3)x^3 - 12(7\pi^2ab^2 - 20a^3b^2)x^2 - 12(\pi^4b - 6\pi^2a^2b - 8a^4b)x - 12(\pi^5 - 8\pi^3a^2 - 48\pi a^4 + 4(\pi^3b^2 - 12\pi a^2b^2))x^2 + 8(\pi^3ab - 12\pi a^3b)x \arctan\left(\frac{-(2bx + 2a - \sqrt{4b^2x^2 + 8abx + \pi^2 + 4a^2})}{\pi}\right) + 6(3\pi^4a + 8\pi^2a^3 - 16a^5 + 4(3\pi^2ab^2 - 4a^3b^2))x^2 + 8(3\pi^2a^2b - 4a^4b)x \log(4b^2x^2 + 8abx + \pi^2 + 4a^2) / (4b^7x^2 + 8ab^6x + \pi^2b^5 + 4a^2b^5)$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arccoth(tanh(b\*x+a))^2,x, algorithm="fricas")

[Out] 1/12\*(16\*b^5\*x^5 - 16\*a\*b^4\*x^4 + 9\*pi^4\*a + 24\*pi^2\*a^3 - 48\*a^5 - 32\*(pi^2\*b^3 - 2\*a^2\*b^3)\*x^3 - 12\*(7\*pi^2\*a\*b^2 - 20\*a^3\*b^2)\*x^2 - 12\*(pi^4\*b - 6\*pi^2\*a^2\*b - 8\*a^4\*b)\*x - 12\*(pi^5 - 8\*pi^3\*a^2 - 48\*pi\*a^4 + 4\*(pi^3\*b^2 - 12\*pi\*a^2\*b^2))\*x^2 + 8\*(pi^3\*a\*b - 12\*pi\*a^3\*b)\*x)\*arctan(-(2\*b\*x + 2\*a - sqrt(4\*b^2\*x^2 + 8\*a\*b\*x + pi^2 + 4\*a^2))/pi) + 6\*(3\*pi^4\*a + 8\*pi^2\*a^3 - 16\*a^5 + 4\*(3\*pi^2\*a\*b^2 - 4\*a^3\*b^2))\*x^2 + 8\*(3\*pi^2\*a^2\*b - 4\*a^4\*b)\*x)\*log(4\*b^2\*x^2 + 8\*a\*b\*x + pi^2 + 4\*a^2)/(4\*b^7\*x^2 + 8\*a\*b^6\*x + pi^2\*b^5 + 4\*a^2\*b^5)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{acoth}^2(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4/acoth(tanh(b\*x+a))\*\*2,x)

[Out] Integral(x\*\*4/acoth(tanh(a + b\*x))\*\*2, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{arccoth}(\tanh(bx + a))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(x^4/arccoth(tanh(b*x + a))^2, x)
```

$$3.168 \quad \int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2} dx$$

**Optimal.** Leaf size=75

$$\frac{3x(bx - \coth^{-1}(\tanh(a+bx)))}{b^3} + \frac{3(bx - \coth^{-1}(\tanh(a+bx)))^2 \log(\coth^{-1}(\tanh(a+bx)))}{b^4} - \frac{x^3}{b \coth^{-1}(\tanh(a+bx))}$$

[Out] (3\*x^2)/(2\*b^2) + (3\*x\*(b\*x - ArcCoth[Tanh[a + b\*x]]))/b^3 - x^3/(b\*ArcCoth[Tanh[a + b\*x]]) + (3\*(b\*x - ArcCoth[Tanh[a + b\*x]])^2\*Log[ArcCoth[Tanh[a + b\*x]]])/b^4

**Rubi [A]** time = 0.0537226, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {2168, 2159, 2158, 2157, 29}

$$\frac{3x(bx - \coth^{-1}(\tanh(a+bx)))}{b^3} + \frac{3(bx - \coth^{-1}(\tanh(a+bx)))^2 \log(\coth^{-1}(\tanh(a+bx)))}{b^4} - \frac{x^3}{b \coth^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

[In] Int[x^3/ArcCoth[Tanh[a + b\*x]]^2,x]

[Out] (3\*x^2)/(2\*b^2) + (3\*x\*(b\*x - ArcCoth[Tanh[a + b\*x]]))/b^3 - x^3/(b\*ArcCoth[Tanh[a + b\*x]]) + (3\*(b\*x - ArcCoth[Tanh[a + b\*x]])^2\*Log[ArcCoth[Tanh[a + b\*x]]])/b^4

### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

### Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
```



;/; NeQ[b\*u - a\*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n, 1]

### Rule 2158

Int[(v\_)/(u\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(b\*x)/a, x] - Dist[(b\*u - a\*v)/a, Int[1/u, x], x] /; NeQ[b\*u - a\*v, 0]] /; PiecewiseLinearQ[u, v, x]

### Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

### Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

### Rubi steps

$$\begin{aligned}
 \int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2} dx &= -\frac{x^3}{b \coth^{-1}(\tanh(a+bx))} + \frac{3 \int \frac{x^2}{\coth^{-1}(\tanh(a+bx))} dx}{b} \\
 &= \frac{3x^2}{2b^2} - \frac{x^3}{b \coth^{-1}(\tanh(a+bx))} - \frac{(3(-bx + \coth^{-1}(\tanh(a+bx)))) \int \frac{x}{\coth^{-1}(\tanh(a+bx))}}{b^2} \\
 &= \frac{3x^2}{2b^2} + \frac{3x(bx - \coth^{-1}(\tanh(a+bx)))}{b^3} - \frac{x^3}{b \coth^{-1}(\tanh(a+bx))} + \frac{(3(-bx + \coth^{-1}(\tanh(a+bx)))) \int \frac{x}{\coth^{-1}(\tanh(a+bx))}}{b^2} \\
 &= \frac{3x^2}{2b^2} + \frac{3x(bx - \coth^{-1}(\tanh(a+bx)))}{b^3} - \frac{x^3}{b \coth^{-1}(\tanh(a+bx))} + \frac{(3(-bx + \coth^{-1}(\tanh(a+bx)))) \int \frac{x}{\coth^{-1}(\tanh(a+bx))}}{b^2} \\
 &= \frac{3x^2}{2b^2} + \frac{3x(bx - \coth^{-1}(\tanh(a+bx)))}{b^3} - \frac{x^3}{b \coth^{-1}(\tanh(a+bx))} + \frac{3(bx - \coth^{-1}(\tanh(a+bx))) \log(\coth^{-1}(\tanh(a+bx)))}{b^4}
 \end{aligned}$$

**Mathematica [A]** time = 0.0615903, size = 83, normalized size = 1.11

$$\frac{(\coth^{-1}(\tanh(a+bx)) - bx)^3}{b^4 \coth^{-1}(\tanh(a+bx))} - \frac{2x(\coth^{-1}(\tanh(a+bx)) - bx)}{b^3} + \frac{3(\coth^{-1}(\tanh(a+bx)) - bx)^2 \log(\coth^{-1}(\tanh(a+bx)))}{b^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/ArcCoth[Tanh[a + b\*x]]^2,x]

[Out]  $x^2/(2*b^2) - (2*x*(-(b*x) + \text{ArcCoth}[\text{Tanh}[a + b*x]]))/b^3 + (-(b*x) + \text{ArcCoth}[\text{Tanh}[a + b*x]])^3/(b^4*\text{ArcCoth}[\text{Tanh}[a + b*x]]) + (3*(-(b*x) + \text{ArcCoth}[\text{Tanh}[a + b*x]]))^2*\text{Log}[\text{ArcCoth}[\text{Tanh}[a + b*x]])/b^4$

**Maple [C]** time = 1.408, size = 29109, normalized size = 388.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/arccoth(tanh(b\*x+a))^2,x)

[Out] result too large to display

**Maxima [C]** time = 2.43762, size = 167, normalized size = 2.23

$$\frac{4(4b^3x^3 + i\pi^3 - 6\pi^2a - 12i\pi a^2 + 8a^3 + (6i\pi b^2 - 12ab^2)x^2 + (4\pi^2b + 16i\pi ab - 16a^2b)x) - (3\pi^2 + 12i\pi a - 12a^2)}{32b^5x - 16i\pi b^4 + 32ab^4} - \frac{(3\pi^2 + 12i\pi a - 12a^2)}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arccoth(tanh(b\*x+a))^2,x, algorithm="maxima")

[Out]  $\frac{4*(4*b^3*x^3 + I*\pi^3 - 6*\pi^2*a - 12*I*\pi*a^2 + 8*a^3 + (6*I*\pi*b^2 - 12*a*b^2)*x^2 + (4*\pi^2*b + 16*I*\pi*a*b - 16*a^2*b)*x)/(32*b^5*x - 16*I*\pi*b^4 + 32*a*b^4) - 1/4*(3*\pi^2 + 12*I*\pi*a - 12*a^2)*\log(-I*\pi + 2*b*x + 2*a)/b^4$

**Fricas [B]** time = 1.76149, size = 544, normalized size = 7.25

$$\frac{16b^4x^4 - 32ab^3x^3 - 2\pi^4 + 32a^4 + 4(\pi^2b^2 - 28a^2b^2)x^2 - 8(5\pi^2ab + 4a^3b)x - 48(4\pi ab^2x^2 + 8\pi a^2bx + \pi^3a + 4\pi a^3)}{8(4b^6x^2 + 8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arccoth(tanh(b\*x+a))^2,x, algorithm="fricas")

[Out]  $\frac{1}{8}(16b^4x^4 - 32ab^3x^3 - 2\pi^4 + 32a^4 + 4(\pi^2b^2 - 28a^2b^2)x^2 - 8(5\pi^2ab + 4a^3b)x - 48(4\pi ab^2x^2 + 8\pi a^2bx + \pi^3a + 4\pi a^3))\arctan\left(\frac{-2bx + 2a - \sqrt{4b^2x^2 + 8abx + \pi^2 + 4a^2}}{\pi}\right) - 3(\pi^4 - 16a^4 + 4(\pi^2b^2 - 4a^2b^2)x^2 + 8(\pi^2ab - 4a^3b)x)\log(4b^2x^2 + 8abx + \pi^2 + 4a^2)/(4b^6x^2 + 8ab^5x + \pi^2b^4 + 4a^2b^4)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{acoth}^2(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/acoth(tanh(b\*x+a))\*\*2,x)

[Out] Integral(x\*\*3/acoth(tanh(a + b\*x))\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{arcoth}(\tanh(bx + a))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arccoth(tanh(b\*x+a))^2,x, algorithm="giac")

[Out] integrate(x^3/arccoth(tanh(b\*x + a))^2, x)

$$3.169 \quad \int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2} dx$$

**Optimal.** Leaf size=50

$$\frac{2(bx - \coth^{-1}(\tanh(a+bx))) \log(\coth^{-1}(\tanh(a+bx)))}{b^3} - \frac{x^2}{b \coth^{-1}(\tanh(a+bx))} + \frac{2x}{b^2}$$

[Out] (2\*x)/b^2 - x^2/(b\*ArcCoth[Tanh[a + b\*x]]) + (2\*(b\*x - ArcCoth[Tanh[a + b\*x]])\*Log[ArcCoth[Tanh[a + b\*x]]])/b^3

**Rubi [A]** time = 0.031317, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {2168, 2158, 2157, 29}

$$\frac{2(bx - \coth^{-1}(\tanh(a+bx))) \log(\coth^{-1}(\tanh(a+bx)))}{b^3} - \frac{x^2}{b \coth^{-1}(\tanh(a+bx))} + \frac{2x}{b^2}$$

Antiderivative was successfully verified.

[In] Int[x^2/ArcCoth[Tanh[a + b\*x]]^2,x]

[Out] (2\*x)/b^2 - x^2/(b\*ArcCoth[Tanh[a + b\*x]]) + (2\*(b\*x - ArcCoth[Tanh[a + b\*x]])\*Log[ArcCoth[Tanh[a + b\*x]]])/b^3

#### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

#### Rule 2158

```
Int[(v_)/(u_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a*v, 0] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

Int[(u\_)^(m\_), x\_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[  
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2} dx &= -\frac{x^2}{b \coth^{-1}(\tanh(a+bx))} + \frac{2 \int \frac{x}{\coth^{-1}(\tanh(a+bx))} dx}{b} \\ &= \frac{2x}{b^2} - \frac{x^2}{b \coth^{-1}(\tanh(a+bx))} - \frac{(2(-bx + \coth^{-1}(\tanh(a+bx)))) \int \frac{1}{\coth^{-1}(\tanh(a+bx))} dx}{b^2} \\ &= \frac{2x}{b^2} - \frac{x^2}{b \coth^{-1}(\tanh(a+bx))} - \frac{(2(-bx + \coth^{-1}(\tanh(a+bx)))) \text{Subst}\left(\int \frac{1}{x} dx, x, \coth^{-1}(\tanh(a+bx))\right)}{b^3} \\ &= \frac{2x}{b^2} - \frac{x^2}{b \coth^{-1}(\tanh(a+bx))} + \frac{2(bx - \coth^{-1}(\tanh(a+bx))) \log(\coth^{-1}(\tanh(a+bx)))}{b^3} \end{aligned}$$

**Mathematica [A]** time = 0.0680071, size = 56, normalized size = 1.12

$$\frac{-\frac{(\coth^{-1}(\tanh(a+bx))-bx)^2}{\coth^{-1}(\tanh(a+bx))} + 2(bx - \coth^{-1}(\tanh(a+bx))) \log(\coth^{-1}(\tanh(a+bx))) + bx}{b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/ArcCoth[Tanh[a + b\*x]]^2, x]

[Out] (b\*x - (- (b\*x) + ArcCoth[Tanh[a + b\*x]]))^2/ArcCoth[Tanh[a + b\*x]] + 2\*(b\*x - ArcCoth[Tanh[a + b\*x]])\*Log[ArcCoth[Tanh[a + b\*x]])/b^3

**Maple [C]** time = 0.366, size = 4626, normalized size = 92.5

output too large to display





$$\begin{aligned}
& +\text{Pi}*\text{csgn}(I/(\exp(2*b*x+2*a)+1))*\text{csgn}(I*\exp(2*b*x+2*a))*\text{csgn}(I*\exp(2*b*x+2*a)) \\
& /(\exp(2*b*x+2*a)+1)-\text{Pi}*\text{csgn}(I/(\exp(2*b*x+2*a)+1))*\text{csgn}(I*\exp(2*b*x+2*a))/(e \\
& xp(2*b*x+2*a)+1))^2+2*\text{Pi}*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^3+\text{Pi}*\text{csgn}(I*\exp(b*x+a)) \\
& ^2*\text{csgn}(I*\exp(2*b*x+2*a))-2*\text{Pi}*\text{csgn}(I*\exp(b*x+a))*\text{csgn}(I*\exp(2*b*x+2*a))^2+ \\
& \text{Pi}*\text{csgn}(I*\exp(2*b*x+2*a))^3-\text{Pi}*\text{csgn}(I*\exp(2*b*x+2*a))*\text{csgn}(I*\exp(2*b*x+2*a)) \\
& /(\exp(2*b*x+2*a)+1))^2+\text{Pi}*\text{csgn}(I*\exp(2*b*x+2*a))/(\exp(2*b*x+2*a)+1))^3+4*I*b \\
& *x+4*I*(\ln(\exp(b*x+a))-b*x-a)+4*I*a+2*\text{Pi})*\text{Pi}*\text{csgn}(I*\exp(2*b*x+2*a))/(\exp(2*b \\
& *x+2*a)+1))^3+I/b^3*\ln(-2*\text{Pi}*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^2+\text{Pi}*\text{csgn}(I/(\exp(2* \\
& b*x+2*a)+1))*\text{csgn}(I*\exp(2*b*x+2*a))*\text{csgn}(I*\exp(2*b*x+2*a))/(\exp(2*b*x+2*a)+1 \\
& ))-\text{Pi}*\text{csgn}(I/(\exp(2*b*x+2*a)+1))*\text{csgn}(I*\exp(2*b*x+2*a))/(\exp(2*b*x+2*a)+1))^ \\
& 2+2*\text{Pi}*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^3+\text{Pi}*\text{csgn}(I*\exp(b*x+a))^2*\text{csgn}(I*\exp(2*b* \\
& x+2*a))-2*\text{Pi}*\text{csgn}(I*\exp(b*x+a))*\text{csgn}(I*\exp(2*b*x+2*a))^2+\text{Pi}*\text{csgn}(I*\exp(2*b* \\
& x+2*a))^3-\text{Pi}*\text{csgn}(I*\exp(2*b*x+2*a))*\text{csgn}(I*\exp(2*b*x+2*a))/(\exp(2*b*x+2*a)+1 \\
& ))^2+\text{Pi}*\text{csgn}(I*\exp(2*b*x+2*a))/(\exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(\ln(\exp(b*x \\
& +a))-b*x-a)+4*I*a+2*\text{Pi})*\text{Pi}+2*x/b^2
\end{aligned}$$

**Maxima [C]** time = 2.44275, size = 108, normalized size = 2.16

$$\frac{4(4b^2x^2 + \pi^2 + 4i\pi a - 4a^2 + (-2i\pi b + 4ab)x)}{16b^4x - 8i\pi b^3 + 16ab^3} - \frac{(-i\pi + 2a)\log(-i\pi + 2bx + 2a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arccoth(tanh(b\*x+a))^2,x, algorithm="maxima")

[Out] 4\*(4\*b^2\*x^2 + pi^2 + 4\*I\*pi\*a - 4\*a^2 + (-2\*I\*pi\*b + 4\*a\*b)\*x)/(16\*b^4\*x - 8\*I\*pi\*b^3 + 16\*a\*b^3) - (-I\*pi + 2\*a)\*log(-I\*pi + 2\*b\*x + 2\*a)/b^3

**Fricas [B]** time = 1.63523, size = 416, normalized size = 8.32

$$\frac{4b^3x^3 + 8ab^2x^2 + 2\pi^2bx - \pi^2a - 4a^3 + 2(4\pi b^2x^2 + 8\pi abx + \pi^3 + 4\pi a^2)\arctan\left(-\frac{2bx+2a-\sqrt{4b^2x^2+8abx+\pi^2+4a^2}}{\pi}\right) - (4a}{4b^5x^2 + 8ab^4x + \pi^2b^3 + 4a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arccoth(tanh(b\*x+a))^2,x, algorithm="fricas")

[Out] (4\*b^3\*x^3 + 8\*a\*b^2\*x^2 + 2\*pi^2\*b\*x - pi^2\*a - 4\*a^3 + 2\*(4\*pi\*b^2\*x^2 + 8\*pi\*a\*b\*x + pi^3 + 4\*pi\*a^2)\*arctan(-(2\*b\*x + 2\*a - sqrt(4\*b^2\*x^2 + 8\*a\*b



```
*x + pi^2 + 4*a^2))/pi) - (4*a*b^2*x^2 + 8*a^2*b*x + pi^2*a + 4*a^3)*log(4*
b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/(4*b^5*x^2 + 8*a*b^4*x + pi^2*b^3 + 4*a^
2*b^3)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{acoth}^2(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/acoth(tanh(b*x+a))**2,x)
```

```
[Out] Integral(x**2/acoth(tanh(a + b*x))**2, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{arccoth}(\tanh(bx + a))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/arccoth(tanh(b*x + a))^2, x)
```

$$3.170 \quad \int \frac{x}{\coth^{-1}(\tanh(a+bx))^2} dx$$

**Optimal.** Leaf size=28

$$\frac{\log(\coth^{-1}(\tanh(a+bx)))}{b^2} - \frac{x}{b \coth^{-1}(\tanh(a+bx))}$$

[Out]  $-(x/(b*\text{ArcCoth}[\text{Tanh}[a + b*x]])) + \text{Log}[\text{ArcCoth}[\text{Tanh}[a + b*x]]]/b^2$

**Rubi [A]** time = 0.013374, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {2168, 2157, 29}

$$\frac{\log(\coth^{-1}(\tanh(a+bx)))}{b^2} - \frac{x}{b \coth^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x/\text{ArcCoth}[\text{Tanh}[a + b*x]]^2, x]$

[Out]  $-(x/(b*\text{ArcCoth}[\text{Tanh}[a + b*x]])) + \text{Log}[\text{ArcCoth}[\text{Tanh}[a + b*x]]]/b^2$

### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

### Rule 2157

```
Int[(u_)^(m_), x_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

### Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned} \int \frac{x}{\coth^{-1}(\tanh(a+bx))^2} dx &= -\frac{x}{b \coth^{-1}(\tanh(a+bx))} + \frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))} dx}{b} \\ &= -\frac{x}{b \coth^{-1}(\tanh(a+bx))} + \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, \coth^{-1}(\tanh(a+bx))\right)}{b^2} \\ &= -\frac{x}{b \coth^{-1}(\tanh(a+bx))} + \frac{\log(\coth^{-1}(\tanh(a+bx)))}{b^2} \end{aligned}$$

**Mathematica [A]** time = 0.0654313, size = 27, normalized size = 0.96

$$-\frac{bx}{\coth^{-1}(\tanh(a+bx))} + \frac{\log(\coth^{-1}(\tanh(a+bx))) + 1}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/ArcCoth[Tanh[a + b\*x]]^2,x]

[Out] (1 - (b\*x)/ArcCoth[Tanh[a + b\*x]]) + Log[ArcCoth[Tanh[a + b\*x]])]/b^2

**Maple [C]** time = 0.187, size = 625, normalized size = 22.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/arccoth(tanh(b\*x+a))^2,x)

[Out]  $-4*I*x/b/(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+2*Pi+4*I*ln(exp(b*x+a)))+1/b^2*ln(ln(exp(b*x+a)))-1/4*I*Pi*(-2*csgn(I/(exp(2*b*x+2*a)+1))^2+csgn(I/(exp(2*b*x+2*a)+1)))$

$a+1)) * \operatorname{csgn}(I \exp(2bx+2a)) * \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1)) - \operatorname{csgn}(I / (\exp(2bx+2a)+1)) * \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{2+2} * \operatorname{csgn}(I / (\exp(2bx+2a)+1))^{3+} * \operatorname{csgn}(I \exp(bx+a))^{2} * \operatorname{csgn}(I \exp(2bx+2a)) - 2 * \operatorname{csgn}(I \exp(bx+a)) * \operatorname{csgn}(I \exp(2bx+2a))^{2+} * \operatorname{csgn}(I \exp(2bx+2a))^{3-} * \operatorname{csgn}(I \exp(2bx+2a)) * \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{2+} * \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^{3+2})$

**Maxima [C]** time = 2.4401, size = 63, normalized size = 2.25

$$\frac{4(-i\pi + 2a)}{8b^3x - 4i\pi b^2 + 8ab^2} + \frac{\log(-i\pi + 2bx + 2a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccoth(tanh(b\*x+a))^2,x, algorithm="maxima")

[Out]  $4*(-I\pi + 2a)/(8b^3x - 4I\pi b^2 + 8a*b^2) + \log(-I\pi + 2bx + 2a)/b^2$

**Fricas [B]** time = 1.68068, size = 213, normalized size = 7.61

$$\frac{8abx + 2\pi^2 + 8a^2 + (4b^2x^2 + 8abx + \pi^2 + 4a^2)\log(4b^2x^2 + 8abx + \pi^2 + 4a^2)}{2(4b^4x^2 + 8ab^3x + \pi^2b^2 + 4a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccoth(tanh(b\*x+a))^2,x, algorithm="fricas")

[Out]  $1/2*(8a*b*x + 2\pi^2 + 8a^2 + (4b^2*x^2 + 8a*b*x + \pi^2 + 4a^2)*\log(4b^2*x^2 + 8a*b*x + \pi^2 + 4a^2))/(4b^4*x^2 + 8a*b^3*x + \pi^2*b^2 + 4a^2*b^2)$

**Sympy [A]** time = 16.2979, size = 36, normalized size = 1.29

$$\begin{cases} -\frac{x}{b \operatorname{acoth}(\tanh(a+bx))} + \frac{\log(\operatorname{acoth}(\tanh(a+bx)))}{b^2} & \text{for } b \neq 0 \\ \frac{x^2}{2 \operatorname{acoth}^2(\tanh(a))} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/acoth(tanh(b\*x+a))\*\*2,x)

[Out] Piecewise((-x/(b\*acoth(tanh(a + b\*x))) + log(acoth(tanh(a + b\*x)))/b\*\*2, Ne(b, 0)), (x\*\*2/(2\*acoth(tanh(a))\*\*2), True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{arccoth}(\tanh(bx + a))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccoth(tanh(b\*x+a))^2,x, algorithm="giac")

[Out] integrate(x/arccoth(tanh(b\*x + a))^2, x)

$$3.171 \quad \int \frac{1}{\coth^{-1}(\tanh(a+bx))^2} dx$$

Optimal. Leaf size=14

$$-\frac{1}{b \coth^{-1}(\tanh(a+bx))}$$

[Out] -(1/(b\*ArcCoth[Tanh[a + b\*x]]))

**Rubi [A]** time = 0.0048201, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {2157, 30}

$$-\frac{1}{b \coth^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^(-2), x]

[Out] -(1/(b\*ArcCoth[Tanh[a + b\*x]]))

Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[  
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

Rule 30

Int[(x\_)^(m\_.), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{1}{\coth^{-1}(\tanh(a+bx))^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2} dx, x, \coth^{-1}(\tanh(a+bx))\right)}{b} \\ &= -\frac{1}{b \coth^{-1}(\tanh(a+bx))} \end{aligned}$$

**Mathematica [A]** time = 0.0058844, size = 14, normalized size = 1.

$$-\frac{1}{b \operatorname{coth}^{-1}(\tanh(a + bx))}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^(-2), x]

[Out] -(1/(b\*ArcCoth[Tanh[a + b\*x]]))

**Maple [A]** time = 0.078, size = 15, normalized size = 1.1

$$-\frac{1}{b \operatorname{arccoth}(\tanh(bx + a))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/arccoth(tanh(b\*x+a))^2, x)

[Out] -1/b/arccoth(tanh(b\*x+a))

**Maxima [C]** time = 1.47959, size = 24, normalized size = 1.71

$$\frac{4}{(-2i\pi - 4bx - 4a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccoth(tanh(b\*x+a))^2, x, algorithm="maxima")

[Out] 4/((-2\*I\*pi - 4\*b\*x - 4\*a)\*b)

**Fricas [B]** time = 1.61425, size = 77, normalized size = 5.5

$$-\frac{4(bx + a)}{4b^3x^2 + 8ab^2x + \pi^2b + 4a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccoth(tanh(b\*x+a))^2,x, algorithm="fricas")

[Out] -4\*(b\*x + a)/(4\*b^3\*x^2 + 8\*a\*b^2\*x + pi^2\*b + 4\*a^2\*b)

**Sympy [A]** time = 16.1339, size = 20, normalized size = 1.43

$$\begin{cases} -\frac{1}{b \operatorname{acoth}(\tanh(a+bx))} & \text{for } b \neq 0 \\ \frac{x}{\operatorname{acoth}^2(\tanh(a))} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/acoth(tanh(b\*x+a))\*2,x)

[Out] Piecewise((-1/(b\*acoth(tanh(a + b\*x))), Ne(b, 0)), (x/acoth(tanh(a))\*2, True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{arccoth}(\tanh(bx + a))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccoth(tanh(b\*x+a))^2,x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a))^-2, x)



$$3.172 \quad \int \frac{1}{x \coth^{-1}(\tanh(a+bx))^2} dx$$

**Optimal.** Leaf size=70

$$\frac{1}{(bx - \coth^{-1}(\tanh(a+bx))) \coth^{-1}(\tanh(a+bx))} + \frac{\log(x)}{(bx - \coth^{-1}(\tanh(a+bx)))^2} - \frac{\log(\coth^{-1}(\tanh(a+bx)))}{(bx - \coth^{-1}(\tanh(a+bx)))^2}$$

[Out]  $-(1/((b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])*\text{ArcCoth}[\text{Tanh}[a + b*x]])) + \text{Log}[x]/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^2 - \text{Log}[\text{ArcCoth}[\text{Tanh}[a + b*x]]]/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^2$

**Rubi [A]** time = 0.0479788, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {2163, 2160, 2157, 29}

$$\frac{1}{(bx - \coth^{-1}(\tanh(a+bx))) \coth^{-1}(\tanh(a+bx))} + \frac{\log(x)}{(bx - \coth^{-1}(\tanh(a+bx)))^2} - \frac{\log(\coth^{-1}(\tanh(a+bx)))}{(bx - \coth^{-1}(\tanh(a+bx)))^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x*\text{ArcCoth}[\text{Tanh}[a + b*x]])^2, x]$

[Out]  $-(1/((b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])*\text{ArcCoth}[\text{Tanh}[a + b*x]])) + \text{Log}[x]/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^2 - \text{Log}[\text{ArcCoth}[\text{Tanh}[a + b*x]]]/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^2$

### Rule 2163

$\text{Int}[(v_)^n/(u_), x\_Symbol] \rightarrow \text{With}[\{a = \text{Simplify}[D[u, x]], b = \text{Simplify}[D[v, x]]\}, \text{Simp}[v^{n+1}/((n+1)*(b*u - a*v)), x] - \text{Dist}[(a*(n+1))/((n+1)*(b*u - a*v)), \text{Int}[v^{n+1}/u, x], x] /; \text{NeQ}[b*u - a*v, 0] /; \text{PiecewiseLinearQ}[u, v, x] \&\& \text{LtQ}[n, -1]$

### Rule 2160

$\text{Int}[1/((u_)*(v_)), x\_Symbol] \rightarrow \text{With}[\{a = \text{Simplify}[D[u, x]], b = \text{Simplify}[D[v, x]]\}, \text{Dist}[b/(b*u - a*v), \text{Int}[1/v, x], x] - \text{Dist}[a/(b*u - a*v), \text{Int}[1/u, x], x] /; \text{NeQ}[b*u - a*v, 0] /; \text{PiecewiseLinearQ}[u, v, x]$

Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[  
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

Rubi steps

$$\begin{aligned} \int \frac{1}{x \coth^{-1}(\tanh(a + bx))^2} dx &= -\frac{1}{(bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))} + \frac{\int \frac{1}{x \coth^{-1}(\tanh(a + bx))} dx}{-bx + \coth^{-1}(\tanh(a + bx))} \\ &= -\frac{1}{(bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))} - \frac{\int \frac{1}{x}}{(bx - \coth^{-1}(\tanh(a + bx)))} \\ &= -\frac{1}{(bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))} + \frac{\log(x)}{(bx - \coth^{-1}(\tanh(a + bx)))^2} \\ &= -\frac{1}{(bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))} + \frac{\log(x)}{(bx - \coth^{-1}(\tanh(a + bx)))^2} \end{aligned}$$

**Mathematica [A]** time = 0.0713631, size = 53, normalized size = 0.76

$$\frac{\coth^{-1}(\tanh(a + bx))(-\log(\coth^{-1}(\tanh(a + bx))) + \log(bx) + 1) - bx}{\coth^{-1}(\tanh(a + bx))(\coth^{-1}(\tanh(a + bx)) - bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*ArcCoth[Tanh[a + b\*x]]^2), x]

[Out] (-(b\*x) + ArcCoth[Tanh[a + b\*x]]\*(1 + Log[b\*x] - Log[ArcCoth[Tanh[a + b\*x]]]))/(ArcCoth[Tanh[a + b\*x]]\*(-(b\*x) + ArcCoth[Tanh[a + b\*x]]))^2

**Maple [F]** time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{x (\operatorname{arccoth}(\tanh(bx + a)))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/arccoth(tanh(b*x+a))^2,x)`

[Out] `int(1/x/arccoth(tanh(b*x+a))^2,x)`

**Maxima [C]** time = 2.45303, size = 104, normalized size = 1.49

$$\frac{4 \log(-i \pi + 2 b x + 2 a)}{\pi^2 + 4 i \pi a - 4 a^2} - \frac{4 \log(x)}{\pi^2 + 4 i \pi a - 4 a^2} - \frac{4}{\pi^2 + 4 i \pi a - 4 a^2 + (2 i \pi b - 4 a b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")`

[Out] `4*log(-I*pi + 2*b*x + 2*a)/(pi^2 + 4*I*pi*a - 4*a^2) - 4*log(x)/(pi^2 + 4*I*pi*a - 4*a^2) - 4/(pi^2 + 4*I*pi*a - 4*a^2 + (2*I*pi*b - 4*a*b)*x)`

**Fricas [B]** time = 1.73918, size = 691, normalized size = 9.87

$$\frac{2 \left( 2 \pi^4 - 32 a^4 - 8 (\pi^2 a b + 4 a^3 b) x + 16 (4 \pi a b^2 x^2 + 8 \pi a^2 b x + \pi^3 a + 4 \pi a^3) \arctan \left( -\frac{2 b x + 2 a - \sqrt{4 b^2 x^2 + 8 a b x + \pi^2 + 4 a^2}}{\pi} \right) \right)}{\pi^6 + 12 \pi^4 a^2 + 48 \pi^2 a^4 + 64 a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")`

[Out] `-2*(2*pi^4 - 32*a^4 - 8*(pi^2*a*b + 4*a^3*b)*x + 16*(4*pi*a*b^2*x^2 + 8*pi*a^2*b*x + pi^3*a + 4*pi*a^3)*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/pi) - (pi^4 - 16*a^4 + 4*(pi^2*b^2 - 4*a^2*b^2)*x^2 + 8*(pi^2*a*b - 4*a^3*b)*x)*log(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2) + 2*(pi^4 - 16*a^4 + 4*(pi^2*b^2 - 4*a^2*b^2)*x^2 + 8*(pi^2*a*b - 4*a^3*b)*x)*log(x))/(pi^6 + 12*pi^4*a^2 + 48*pi^2*a^4 + 64*a^6 + 4*(pi^4*b^2 + 8*pi^2*a^2*b^2 + 16*a^4*b^2)*x^2 + 8*(pi^4*a*b + 8*pi^2*a^3*b + 16*a^5*b)*x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{acoth}^2(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/acoth(tanh(b\*x+a))\*\*2,x)

[Out] Integral(1/(x\*acoth(tanh(a + b\*x))\*\*2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{arccoth}(\tanh(bx + a))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arccoth(tanh(b\*x+a))^2,x, algorithm="giac")

[Out] integrate(1/(x\*arccoth(tanh(b\*x + a))^2), x)

$$3.173 \quad \int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^2} dx$$

**Optimal.** Leaf size=102

$$-\frac{2b}{(bx - \coth^{-1}(\tanh(a+bx)))^2 \coth^{-1}(\tanh(a+bx))} + \frac{1}{x(bx - \coth^{-1}(\tanh(a+bx))) \coth^{-1}(\tanh(a+bx))} + \frac{1}{(bx - \coth^{-1}(\tanh(a+bx)))^3}$$

[Out]  $(-2*b)/((b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^2 * \text{ArcCoth}[\text{Tanh}[a + b*x]]) + 1/(x*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]]) * \text{ArcCoth}[\text{Tanh}[a + b*x]]) + (2*b*\text{Log}[x])/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^3 - (2*b*\text{Log}[\text{ArcCoth}[\text{Tanh}[a + b*x]]])/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^3$

**Rubi [A]** time = 0.0626622, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {2171, 2163, 2160, 2157, 29}

$$-\frac{2b}{(bx - \coth^{-1}(\tanh(a+bx)))^2 \coth^{-1}(\tanh(a+bx))} + \frac{1}{x(bx - \coth^{-1}(\tanh(a+bx))) \coth^{-1}(\tanh(a+bx))} + \frac{1}{(bx - \coth^{-1}(\tanh(a+bx)))^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*ArcCoth[Tanh[a + b\*x]]^2), x]

[Out]  $(-2*b)/((b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^2 * \text{ArcCoth}[\text{Tanh}[a + b*x]]) + 1/(x*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]]) * \text{ArcCoth}[\text{Tanh}[a + b*x]]) + (2*b*\text{Log}[x])/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^3 - (2*b*\text{Log}[\text{ArcCoth}[\text{Tanh}[a + b*x]]])/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^3$

### Rule 2171

Int[(u\_)^(m\_)\*(v\_)^(n\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, -Simp[(u^(m+1)\*v^(n+1))/((m+1)\*(b\*u - a\*v)), x] + Dist[(b\*(m+n+2))/((m+1)\*(b\*u - a\*v)), Int[u^(m+1)\*v^n, x], x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x] && NeQ[m+n+2, 0] && LtQ[m, -1]

### Rule 2163

Int[(v\_)^(n\_)/(u\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[v^(n+1)/((n+1)\*(b\*u - a\*v)), x] - Dist[(a\*(n+1))/((n+1)\*(b\*u - a\*v)), Int[v^(n+1), x], x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x] && NeQ[n, 0] && LtQ[n, -1]

+ 1)\*(b\*u - a\*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b\*u - a\*v, 0]] /; PiecewiseLinearQ[u, v, x] && LtQ[n, -1]

### Rule 2160

Int[1/((u\_)\*(v\_)), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Dist[b/(b\*u - a\*v), Int[1/v, x], x] - Dist[a/(b\*u - a\*v), Int[1/u, x], x] /; NeQ[b\*u - a\*v, 0]] /; PiecewiseLinearQ[u, v, x]

### Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

### Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

### Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^2 \coth^{-1}(\tanh(a + bx))^2} dx &= \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))} - \frac{(2b) \int \frac{1}{x \coth^{-1}(\tanh(a + bx))^2} dx}{-bx + \coth^{-1}(\tanh(a + bx))} \\
 &= -\frac{2b}{(bx - \coth^{-1}(\tanh(a + bx)))^2 \coth^{-1}(\tanh(a + bx))} + \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))} \\
 &= -\frac{2b}{(bx - \coth^{-1}(\tanh(a + bx)))^2 \coth^{-1}(\tanh(a + bx))} + \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))} \\
 &= -\frac{2b}{(bx - \coth^{-1}(\tanh(a + bx)))^2 \coth^{-1}(\tanh(a + bx))} + \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))} \\
 &= -\frac{2b}{(bx - \coth^{-1}(\tanh(a + bx)))^2 \coth^{-1}(\tanh(a + bx))} + \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))}
 \end{aligned}$$

**Mathematica [A]** time = 0.0619438, size = 70, normalized size = 0.69

$$\frac{\coth^{-1}(\tanh(a + bx))^2 + 2bx \coth^{-1}(\tanh(a + bx)) (\log(x) - \log(\coth^{-1}(\tanh(a + bx)))) - b^2 x^2}{x (bx - \coth^{-1}(\tanh(a + bx)))^3 \coth^{-1}(\tanh(a + bx))}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*ArcCoth[Tanh[a + b\*x]]^2),x]

[Out]  $(-(b^2*x^2) + \text{ArcCoth}[\text{Tanh}[a + b*x]]^2 + 2*b*x*\text{ArcCoth}[\text{Tanh}[a + b*x]]*(\text{Log}[x] - \text{Log}[\text{ArcCoth}[\text{Tanh}[a + b*x]]]))/(x*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^3*\text{ArcCoth}[\text{Tanh}[a + b*x]])$

**Maple [F]** time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (\operatorname{arccoth}(\tanh(bx + a)))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/arccoth(tanh(b\*x+a))^2,x)

[Out] int(1/x^2/arccoth(tanh(b\*x+a))^2,x)

**Maxima [C]** time = 2.45069, size = 182, normalized size = 1.78

$$-\frac{16b \log(-i\pi + 2bx + 2a)}{-i\pi^3 + 6\pi^2a + 12i\pi a^2 - 8a^3} + \frac{16b \log(x)}{-i\pi^3 + 6\pi^2a + 12i\pi a^2 - 8a^3} - \frac{4(i\pi - 4bx - 2a)}{(2\pi^2b + 8i\pi ab - 8a^2b)x^2 - (i\pi^3 - 6\pi^2a - 12i\pi a^2 + 8a^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arccoth(tanh(b\*x+a))^2,x, algorithm="maxima")

[Out]  $-16*b*\log(-I*\pi + 2*b*x + 2*a)/(-I*\pi^3 + 6*\pi^2*a + 12*I*\pi*a^2 - 8*a^3) + 16*b*\log(x)/(-I*\pi^3 + 6*\pi^2*a + 12*I*\pi*a^2 - 8*a^3) - 4*(I*\pi - 4*b*x - 2*a)/((2*\pi^2*b + 8*I*\pi*a*b - 8*a^2*b)*x^2 - (I*\pi^3 - 6*\pi^2*a - 12*I*\pi*a^2 + 8*a^3)*x)$

**Fricas [B]** time = 1.81465, size = 1065, normalized size = 10.44

$$4 \left( \pi^6 + 4\pi^4 a^2 - 16\pi^2 a^4 - 64a^6 + 8(\pi^4 b^2 - 16a^4 b^2)x^2 + 4(5\pi^4 ab + 8\pi^2 a^3 b - 48a^5 b)x - 8(4(\pi^3 b^3 - 12\pi a^2 b^3)x^3 + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arccoth(tanh(b\*x+a))^2,x, algorithm="fricas")

[Out] 4\*(pi^6 + 4\*pi^4\*a^2 - 16\*pi^2\*a^4 - 64\*a^6 + 8\*(pi^4\*b^2 - 16\*a^4\*b^2)\*x^2 + 4\*(5\*pi^4\*a\*b + 8\*pi^2\*a^3\*b - 48\*a^5\*b)\*x - 8\*(4\*(pi^3\*b^3 - 12\*pi\*a^2\*b^3)\*x^3 + 8\*(pi^3\*a\*b^2 - 12\*pi\*a^3\*b^2)\*x^2 + (pi^5\*b - 8\*pi^3\*a^2\*b - 48\*pi\*a^4\*b)\*x)\*arctan(-(2\*b\*x + 2\*a - sqrt(4\*b^2\*x^2 + 8\*a\*b\*x + pi^2 + 4\*a^2))/pi) - 4\*(4\*(3\*pi^2\*a\*b^3 - 4\*a^3\*b^3)\*x^3 + 8\*(3\*pi^2\*a^2\*b^2 - 4\*a^4\*b^2)\*x^2 + (3\*pi^4\*a\*b + 8\*pi^2\*a^3\*b - 16\*a^5\*b)\*x)\*log(4\*b^2\*x^2 + 8\*a\*b\*x + pi^2 + 4\*a^2) + 8\*(4\*(3\*pi^2\*a\*b^3 - 4\*a^3\*b^3)\*x^3 + 8\*(3\*pi^2\*a^2\*b^2 - 4\*a^4\*b^2)\*x^2 + (3\*pi^4\*a\*b + 8\*pi^2\*a^3\*b - 16\*a^5\*b)\*x)\*log(x))/(4\*(pi^6\*b^2 + 12\*pi^4\*a^2\*b^2 + 48\*pi^2\*a^4\*b^2 + 64\*a^6\*b^2)\*x^3 + 8\*(pi^6\*a\*b + 12\*pi^4\*a^3\*b + 48\*pi^2\*a^5\*b + 64\*a^7\*b)\*x^2 + (pi^8 + 16\*pi^6\*a^2 + 96\*pi^4\*a^4 + 256\*pi^2\*a^6 + 256\*a^8)\*x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \operatorname{acoth}^2(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/acoth(tanh(b\*x+a))\*\*2,x)

[Out] Integral(1/(x\*\*2\*acoth(tanh(a + b\*x))\*\*2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \operatorname{arcoth}(\tanh(bx + a))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arccoth(tanh(b\*x+a))^2,x, algorithm="giac")

[Out] integrate(1/(x^2\*arccoth(tanh(b\*x + a))^2), x)



$$3.174 \quad \int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^2} dx$$

**Optimal.** Leaf size=143

$$\frac{3b^2}{(bx - \coth^{-1}(\tanh(a+bx)))^3 \coth^{-1}(\tanh(a+bx))} + \frac{3b^2 \log(x)}{(bx - \coth^{-1}(\tanh(a+bx)))^4} - \frac{3b^2 \log(\coth^{-1}(\tanh(a+bx)))}{(bx - \coth^{-1}(\tanh(a+bx)))}$$

[Out]  $(-3*b^2)/((b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^3*\text{ArcCoth}[\text{Tanh}[a + b*x]]) + (3*b)/(2*x*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^2*\text{ArcCoth}[\text{Tanh}[a + b*x]]) + 1/(2*x^2*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])*\text{ArcCoth}[\text{Tanh}[a + b*x]]) + (3*b^2*\text{Log}[x])/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^4 - (3*b^2*\text{Log}[\text{ArcCoth}[\text{Tanh}[a + b*x]]])/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^4$

**Rubi [A]** time = 0.0896868, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {2171, 2163, 2160, 2157, 29}

$$\frac{3b^2}{(bx - \coth^{-1}(\tanh(a+bx)))^3 \coth^{-1}(\tanh(a+bx))} + \frac{3b^2 \log(x)}{(bx - \coth^{-1}(\tanh(a+bx)))^4} - \frac{3b^2 \log(\coth^{-1}(\tanh(a+bx)))}{(bx - \coth^{-1}(\tanh(a+bx)))}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*ArcCoth[Tanh[a + b\*x]]^2),x]

[Out]  $(-3*b^2)/((b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^3*\text{ArcCoth}[\text{Tanh}[a + b*x]]) + (3*b)/(2*x*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^2*\text{ArcCoth}[\text{Tanh}[a + b*x]]) + 1/(2*x^2*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])*\text{ArcCoth}[\text{Tanh}[a + b*x]]) + (3*b^2*\text{Log}[x])/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^4 - (3*b^2*\text{Log}[\text{ArcCoth}[\text{Tanh}[a + b*x]]])/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^4$

### Rule 2171

Int[(u\_)^(m\_)\*(v\_)^(n\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, -Simp[(u^(m + 1)\*v^(n + 1))/((m + 1)\*(b\*u - a\*v)), x] + Dist[(b\*(m + n + 2))/((m + 1)\*(b\*u - a\*v)), Int[u^(m + 1)\*v^n, x], x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -1]

### Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n + 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0] /; PiecewiseLinearQ[u, v, x] && LtQ[n, -1]
```

### Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u, x], x] /; NeQ[b*u - a*v, 0] /; PiecewiseLinearQ[u, v, x]
```

### Rule 2157

```
Int[(u_)^(m_), x_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

### Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^3 \coth^{-1}(\tanh(a + bx))^2} dx &= \frac{1}{2x^2 (bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))} + \frac{(3b) \int \frac{1}{x^2 \coth^{-1}(\tanh(a + bx))^2}}{2 (bx - \coth^{-1}(\tanh(a + bx)))} \\
 &= \frac{3b}{2x (bx - \coth^{-1}(\tanh(a + bx)))^2 \coth^{-1}(\tanh(a + bx))} + \frac{3b}{2x^2 (bx - \coth^{-1}(\tanh(a + bx)))} \\
 &= -\frac{3b^2}{(bx - \coth^{-1}(\tanh(a + bx)))^3 \coth^{-1}(\tanh(a + bx))} + \frac{3b}{2x (bx - \coth^{-1}(\tanh(a + bx)))} \\
 &= -\frac{3b^2}{(bx - \coth^{-1}(\tanh(a + bx)))^3 \coth^{-1}(\tanh(a + bx))} + \frac{3b}{2x (bx - \coth^{-1}(\tanh(a + bx)))} \\
 &= -\frac{3b^2}{(bx - \coth^{-1}(\tanh(a + bx)))^3 \coth^{-1}(\tanh(a + bx))} + \frac{3b}{2x (bx - \coth^{-1}(\tanh(a + bx)))} \\
 &= -\frac{3b^2}{(bx - \coth^{-1}(\tanh(a + bx)))^3 \coth^{-1}(\tanh(a + bx))} + \frac{3b}{2x (bx - \coth^{-1}(\tanh(a + bx)))}
 \end{aligned}$$

**Mathematica [A]** time = 0.0456956, size = 92, normalized size = 0.64

$$\frac{-3b^2x^2 \coth^{-1}(\tanh(a+bx))(-2\log(\coth^{-1}(\tanh(a+bx))) + 2\log(x) - 1) - 6bx \coth^{-1}(\tanh(a+bx))^2 + \coth^{-1}(\tanh(a+bx))}{2x^2 \coth^{-1}(\tanh(a+bx))(\coth^{-1}(\tanh(a+bx)) - bx)^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*ArcCoth[Tanh[a + b\*x]]^2), x]

[Out]  $-(2b^3x^3 - 6b^2x^2 \operatorname{ArcCoth}[\operatorname{Tanh}[a + bx]]^2 + \operatorname{ArcCoth}[\operatorname{Tanh}[a + bx]]^3 - 3b^2x^2 \operatorname{ArcCoth}[\operatorname{Tanh}[a + bx]](-1 + 2\operatorname{Log}[x] - 2\operatorname{Log}[\operatorname{ArcCoth}[\operatorname{Tanh}[a + bx]]]))/(2x^2 \operatorname{ArcCoth}[\operatorname{Tanh}[a + bx]](-bx + \operatorname{ArcCoth}[\operatorname{Tanh}[a + bx]])^4)$

**Maple [F]** time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (\operatorname{arccoth}(\tanh(bx+a)))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/arccoth(tanh(b\*x+a))^2, x)

[Out] int(1/x^3/arccoth(tanh(b\*x+a))^2, x)

**Maxima [C]** time = 2.46131, size = 258, normalized size = 1.8

$$\frac{48b^2 \log(-i\pi + 2bx + 2a)}{\pi^4 + 8i\pi^3a - 24\pi^2a^2 - 32i\pi a^3 + 16a^4} + \frac{48b^2 \log(x)}{\pi^4 + 8i\pi^3a - 24\pi^2a^2 - 32i\pi a^3 + 16a^4} - \frac{4(24b^2x^2 \operatorname{arccoth}(\tanh(bx+a)))^2}{(-4i\pi^3b + 24\pi^2ab + 48i\pi a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/arccoth(tanh(b\*x+a))^2, x, algorithm="maxima")

[Out]  $-48b^2 \log(-i\pi + 2bx + 2a)/(\pi^4 + 8i\pi^3a - 24\pi^2a^2 - 32i\pi a^3 + 16a^4) + 48b^2 \log(x)/(\pi^4 + 8i\pi^3a - 24\pi^2a^2 - 32i\pi a^3 + 16a^4) - 4(24b^2x^2 \operatorname{arccoth}(\tanh(bx+a)))^2/((-4i\pi^3b + 24\pi^2ab + 48i\pi a^2b))$

$$+ 16I\pi^3 a - 48\pi^2 a^2 - 64I\pi a^3 + 32a^4)x^2)$$

**Fricas [B]** time = 1.74182, size = 1439, normalized size = 10.06

$$2\left(\pi^8 + 8\pi^6 a^2 - 128\pi^2 a^6 - 256a^8 - 96(3\pi^4 ab^3 + 8\pi^2 a^3 b^3 - 16a^5 b^3)x^3 + 12(\pi^6 b^2 - 44\pi^4 a^2 b^2 - 144\pi^2 a^4 b^2 + 192a^6 b^2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/arccoth(tanh(b\*x+a))^2,x, algorithm="fricas")

[Out]  $2*(\pi^8 + 8\pi^6 a^2 - 128\pi^2 a^6 - 256a^8 - 96*(3\pi^4 a^3 b^3 + 8\pi^2 a^5 b^3 - 16a^7 b^3)*x^3 + 12*(\pi^6 b^2 - 44\pi^4 a^2 b^2 - 144\pi^2 a^4 b^2 + 192a^6 b^2)*x^2 - 8*(5\pi^6 a^3 b + 36\pi^4 a^5 b + 48\pi^2 a^7 b - 64a^9 b)*x + 384*(4*(\pi^3 a^3 b^4 - 4\pi a^5 b^4)*x^4 + 8*(\pi^3 a^2 b^3 - 4\pi a^4 b^3)*x^3 + (\pi^5 a^3 b^2 - 16\pi a^5 b^2)*x^2)*\arctan(-(2bx + 2a - \sqrt{4b^2 x^2 + 8abx + \pi^2 + 4a^2}))/\pi - 12*(4*(\pi^4 b^4 - 24\pi^2 a^2 b^4 + 16a^4 b^4)*x^4 + 8*(\pi^4 a^3 b^3 - 24\pi^2 a^5 b^3 + 16a^7 b^3)*x^3 + (\pi^6 b^2 - 20\pi^4 a^2 b^2 - 80\pi^2 a^4 b^2 + 64a^6 b^2)*x^2)*\log(4b^2 x^2 + 8abx + \pi^2 + 4a^2) + 24*(4*(\pi^4 b^4 - 24\pi^2 a^2 b^4 + 16a^4 b^4)*x^4 + 8*(\pi^4 a^3 b^3 - 24\pi^2 a^5 b^3 + 16a^7 b^3)*x^3 + (\pi^6 b^2 - 20\pi^4 a^2 b^2 - 80\pi^2 a^4 b^2 + 64a^6 b^2)*x^2)*\log(x))/(4*(\pi^8 b^2 + 16\pi^6 a^2 b^2 + 96\pi^4 a^4 b^2 + 256\pi^2 a^6 b^2 + 256a^8 b^2)*x^4 + 8*(\pi^8 a^3 b + 16\pi^6 a^5 b + 96\pi^4 a^7 b + 256\pi^2 a^9 b + 256a^9 b)*x^3 + (\pi^{10} + 20\pi^8 a^2 + 160\pi^6 a^4 + 640\pi^4 a^6 + 1280\pi^2 a^8 + 1024a^{10})*x^2)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \operatorname{acoth}^2(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/acoth(tanh(b\*x+a))\*\*2,x)

[Out] Integral(1/(x\*\*3\*acoth(tanh(a + b\*x))\*\*2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \operatorname{arccoth}(\tanh(bx + a))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/arccoth(tanh(b\*x+a))^2,x, algorithm="giac")

[Out] integrate(1/(x^3\*arccoth(tanh(b\*x + a))^2), x)

$$3.175 \quad \int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^3} dx$$

**Optimal.** Leaf size=94

$$\frac{mx^{m-1} \text{Hypergeometric2F1}\left(1, m-1, m, \frac{bx}{bx - \coth^{-1}(\tanh(a+bx))}\right)}{2b^2 (bx - \coth^{-1}(\tanh(a+bx)))} - \frac{mx^{m-1}}{2b^2 \coth^{-1}(\tanh(a+bx))} - \frac{x^m}{2b \coth^{-1}(\tanh(a+bx))}$$

[Out]  $-x^m/(2*b*\text{ArcCoth}[\text{Tanh}[a + b*x]]^2) - (m*x^{(-1 + m)})/(2*b^2*\text{ArcCoth}[\text{Tanh}[a + b*x]]) - (m*x^{(-1 + m)}*\text{Hypergeometric2F1}[1, -1 + m, m, (b*x)/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])])/(2*b^2*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]]))$

**Rubi [A]** time = 0.0599338, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2168, 2164}

$$\frac{mx^{m-1} {}_2F_1\left(1, m-1; m; \frac{bx}{bx - \coth^{-1}(\tanh(a+bx))}\right)}{2b^2 (bx - \coth^{-1}(\tanh(a+bx)))} - \frac{mx^{m-1}}{2b^2 \coth^{-1}(\tanh(a+bx))} - \frac{x^m}{2b \coth^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

[In] Int[x^m/ArcCoth[Tanh[a + b\*x]]^3,x]

[Out]  $-x^m/(2*b*\text{ArcCoth}[\text{Tanh}[a + b*x]]^2) - (m*x^{(-1 + m)})/(2*b^2*\text{ArcCoth}[\text{Tanh}[a + b*x]]) - (m*x^{(-1 + m)}*\text{Hypergeometric2F1}[1, -1 + m, m, (b*x)/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])])/(2*b^2*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]]))$

### Rule 2168

Int[(u\_)^(m\_)\*(v\_)^(n\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)\*v^n)/(a\*(m + 1)), x] - Dist[(b\*n)/(a\*(m + 1)), Int[u^(m + 1)\*v^(n - 1), x], x] /; NeQ[b\*u - a\*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2\*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))

### Rule 2164

Int[(v\_)^(n\_)/(u\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(v^(n + 1)\*Hypergeometric2F1[1, n + 1, n + 2, -(a\*v)/(b\*u

- a\*v)))]/((n + 1)\*(b\*u - a\*v)), x] /; NeQ[b\*u - a\*v, 0]] /; PiecewiseLinearQ[u, v, x] && !IntegerQ[n]

### Rubi steps

$$\begin{aligned} \int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^3} dx &= -\frac{x^m}{2b \coth^{-1}(\tanh(a+bx))^2} + \frac{m \int \frac{x^{-1+m}}{\coth^{-1}(\tanh(a+bx))^2} dx}{2b} \\ &= -\frac{x^m}{2b \coth^{-1}(\tanh(a+bx))^2} - \frac{mx^{-1+m}}{2b^2 \coth^{-1}(\tanh(a+bx))} - \frac{((1-m)m) \int \frac{x^{-2+m}}{\coth^{-1}(\tanh(a+bx))} dx}{2b^2} \\ &= -\frac{x^m}{2b \coth^{-1}(\tanh(a+bx))^2} - \frac{mx^{-1+m}}{2b^2 \coth^{-1}(\tanh(a+bx))} - \frac{mx^{-1+m} {}_2F_1\left(1, -1+m; m; \frac{bx - \coth^{-1}(\tanh(a+bx))}{\coth^{-1}(\tanh(a+bx))}\right)}{2b^2 (bx - \coth^{-1}(\tanh(a+bx)))} \end{aligned}$$

**Mathematica [A]** time = 0.50711, size = 51, normalized size = 0.54

$$\frac{x^{m+1} \text{Hypergeometric2F1}\left(3, m+1, m+2, -\frac{bx}{\coth^{-1}(\tanh(a+bx))-bx}\right)}{(m+1) \left(\coth^{-1}(\tanh(a+bx)) - bx\right)^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^m/ArcCoth[Tanh[a + b\*x]]^3,x]

[Out] (x^(1+m)\*Hypergeometric2F1[3, 1+m, 2+m, -((b\*x)/(-b\*x) + ArcCoth[Tanh[a + b\*x]])])/((1+m)\*(-b\*x) + ArcCoth[Tanh[a + b\*x]])^3

**Maple [F]** time = 1.891, size = 0, normalized size = 0.

$$\int \frac{x^m}{(\operatorname{arccoth}(\tanh(bx+a)))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m/arccoth(tanh(b\*x+a))^3,x)

[Out] int(x^m/arccoth(tanh(b\*x+a))^3,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{arccoth}(\tanh(bx + a))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/arccoth(tanh(b\*x+a))^3,x, algorithm="maxima")

[Out] integrate(x^m/arccoth(tanh(b\*x + a))^3, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x^m}{\operatorname{arccoth}(\tanh(bx + a))^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/arccoth(tanh(b\*x+a))^3,x, algorithm="fricas")

[Out] integral(x^m/arccoth(tanh(b\*x + a))^3, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{acoth}^3(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m/acoth(tanh(b\*x+a))\*\*3,x)

[Out] Integral(x\*\*m/acoth(tanh(a + b\*x))\*\*3, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{arccoth}(\tanh(bx + a))^3} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(x^m/arccoth(tanh(b*x + a))^3, x)
```

$$3.176 \quad \int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^3} dx$$

**Optimal.** Leaf size=92

$$-\frac{2x^3}{b^2 \coth^{-1}(\tanh(a+bx))} + \frac{6x(bx - \coth^{-1}(\tanh(a+bx)))}{b^4} + \frac{6(bx - \coth^{-1}(\tanh(a+bx)))^2 \log(\coth^{-1}(\tanh(a+bx)))}{b^5}$$

[Out] (3\*x^2)/b^3 + (6\*x\*(b\*x - ArcCoth[Tanh[a + b\*x]]))/b^4 - x^4/(2\*b\*ArcCoth[Tanh[a + b\*x]]^2) - (2\*x^3)/(b^2\*ArcCoth[Tanh[a + b\*x]]) + (6\*(b\*x - ArcCoth[Tanh[a + b\*x]])^2\*Log[ArcCoth[Tanh[a + b\*x]]])/b^5

**Rubi [A]** time = 0.0712986, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {2168, 2159, 2158, 2157, 29}

$$-\frac{2x^3}{b^2 \coth^{-1}(\tanh(a+bx))} + \frac{6x(bx - \coth^{-1}(\tanh(a+bx)))}{b^4} + \frac{6(bx - \coth^{-1}(\tanh(a+bx)))^2 \log(\coth^{-1}(\tanh(a+bx)))}{b^5}$$

Antiderivative was successfully verified.

[In] Int[x^4/ArcCoth[Tanh[a + b\*x]]^3,x]

[Out] (3\*x^2)/b^3 + (6\*x\*(b\*x - ArcCoth[Tanh[a + b\*x]]))/b^4 - x^4/(2\*b\*ArcCoth[Tanh[a + b\*x]]^2) - (2\*x^3)/(b^2\*ArcCoth[Tanh[a + b\*x]]) + (6\*(b\*x - ArcCoth[Tanh[a + b\*x]])^2\*Log[ArcCoth[Tanh[a + b\*x]]])/b^5

### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

### Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
```

]; NeQ[b\*u - a\*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n, 1]

### Rule 2158

Int[(v\_)/(u\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(b\*x)/a, x] - Dist[(b\*u - a\*v)/a, Int[1/u, x], x] /; NeQ[b\*u - a\*v, 0]] /; PiecewiseLinearQ[u, v, x]

### Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

### Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

### Rubi steps

$$\begin{aligned}
 \int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^3} dx &= -\frac{x^4}{2b \coth^{-1}(\tanh(a+bx))^2} + \frac{2 \int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2} dx}{b} \\
 &= -\frac{x^4}{2b \coth^{-1}(\tanh(a+bx))^2} - \frac{2x^3}{b^2 \coth^{-1}(\tanh(a+bx))} + \frac{6 \int \frac{x^2}{\coth^{-1}(\tanh(a+bx))} dx}{b^2} \\
 &= \frac{3x^2}{b^3} - \frac{x^4}{2b \coth^{-1}(\tanh(a+bx))^2} - \frac{2x^3}{b^2 \coth^{-1}(\tanh(a+bx))} - \frac{(6(-bx + \coth^{-1}(\tanh(a+bx))))}{b^2} \\
 &= \frac{3x^2}{b^3} + \frac{6x(bx - \coth^{-1}(\tanh(a+bx)))}{b^4} - \frac{x^4}{2b \coth^{-1}(\tanh(a+bx))^2} - \frac{2x^3}{b^2 \coth^{-1}(\tanh(a+bx))} \\
 &= \frac{3x^2}{b^3} + \frac{6x(bx - \coth^{-1}(\tanh(a+bx)))}{b^4} - \frac{x^4}{2b \coth^{-1}(\tanh(a+bx))^2} - \frac{2x^3}{b^2 \coth^{-1}(\tanh(a+bx))} \\
 &= \frac{3x^2}{b^3} + \frac{6x(bx - \coth^{-1}(\tanh(a+bx)))}{b^4} - \frac{x^4}{2b \coth^{-1}(\tanh(a+bx))^2} - \frac{2x^3}{b^2 \coth^{-1}(\tanh(a+bx))}
 \end{aligned}$$

**Mathematica [A]** time = 0.0418133, size = 114, normalized size = 1.24

$$-\frac{(\coth^{-1}(\tanh(a+bx)) - bx)^4}{2b^5 \coth^{-1}(\tanh(a+bx))^2} + \frac{4(\coth^{-1}(\tanh(a+bx)) - bx)^3}{b^5 \coth^{-1}(\tanh(a+bx))} - \frac{3x(\coth^{-1}(\tanh(a+bx)) - bx)}{b^4} + \frac{6(\coth^{-1}(\tanh(a+bx)) - bx)^2}{b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/ArcCoth[Tanh[a + b\*x]]^3,x]

[Out]  $x^2/(2*b^3) - (3*x*(-(b*x) + \text{ArcCoth}[\text{Tanh}[a + b*x]]))/b^4 + (4*(-(b*x) + \text{ArcCoth}[\text{Tanh}[a + b*x]])^3)/(b^5*\text{ArcCoth}[\text{Tanh}[a + b*x]]) - (-(b*x) + \text{ArcCoth}[\text{Tanh}[a + b*x]])^4/(2*b^5*\text{ArcCoth}[\text{Tanh}[a + b*x]]^2) + (6*(-(b*x) + \text{ArcCoth}[\text{Tanh}[a + b*x]])^2*\text{Log}[\text{ArcCoth}[\text{Tanh}[a + b*x]]])/b^5$

**Maple [C]** time = 1.396, size = 29456, normalized size = 320.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/arccoth(tanh(b\*x+a))^3,x)

[Out] result too large to display

**Maxima [C]** time = 3.54373, size = 266, normalized size = 2.89

$$\frac{8(16b^4x^4 + 7\pi^4 + 56i\pi^3a - 168\pi^2a^2 - 224i\pi a^3 + 112a^4 + (32i\pi b^3 - 64ab^3)x^3 + (44\pi^2b^2 + 176i\pi ab^2 - 176a^2b^2)x^2 + 256b^7x^2 - 64\pi^2b^5 - 256i\pi ab^5 + 256a^2b^5 + (-256i\pi b^6 + 512ab^6)x)}{256b^7x^2 - 64\pi^2b^5 - 256i\pi ab^5 + 256a^2b^5 + (-256i\pi b^6 + 512ab^6)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arccoth(tanh(b\*x+a))^3,x, algorithm="maxima")

[Out]  $8*(16*b^4*x^4 + 7*\pi^4 + 56*I*\pi^3*a - 168*\pi^2*a^2 - 224*I*\pi*a^3 + 112*a^4 + (32*I*\pi*b^3 - 64*a*b^3)*x^3 + (44*\pi^2*b^2 + 176*I*\pi*a*b^2 - 176*a^2*b^2)*x^2 + (4*I*\pi^3*b - 24*\pi^2*a*b - 48*I*\pi*a^2*b + 32*a^3*b)*x)/(256*b^7*x^2 - 64*\pi^2*b^5 - 256*I*\pi*a*b^5 + 256*a^2*b^5 + (-256*I*\pi*b^6 + 512*a*b^6)*x) - 1/2*(3*\pi^2 + 12*I*\pi*a - 12*a^2)*\log(-I*\pi + 2*b*x + 2*a)/b^5$

**Fricas [B]** time = 1.79415, size = 1100, normalized size = 11.96

$$64b^6x^6 - 128ab^5x^5 - 7\pi^6 - 28\pi^4a^2 + 112\pi^2a^4 + 448a^6 + 32(\pi^2b^4 - 36a^2b^4)x^4 - 512(\pi^2ab^3 + 3a^3b^3)x^3 - 32(\pi^4b^2 - 36a^2b^2)x^2 - 32(5\pi^4ab + 12\pi^2a^3b - 32a^5b)x - 96(16\pi ab^4x^4 + 64\pi a^2b^3x^3 + \pi^5a + 8\pi^3a^3 + 16\pi a^5 + 8(\pi^3ab^2 + 12\pi a^3b^2)x^2 + 16(\pi^3a^2b + 4\pi a^4b)x) \arctan\left(\frac{-2bx + 2a - \sqrt{4b^2x^2 + 8abx + \pi^2 + 4a^2}}{\pi}\right) - 6(\pi^6 + 4\pi^4a^2 - 16\pi^2a^4 - 64a^6 + 16(\pi^2b^4 - 4a^2b^4)x^4 + 64(\pi^2ab^3 - 4a^3b^3)x^3 + 8(\pi^4b^2 + 8\pi^2a^2b^2 - 48a^4b^2)x^2 + 16(\pi^4ab - 16a^5b)x) \log(4b^2x^2 + 8abx + \pi^2 + 4a^2) / (16b^9x^4 + 64ab^8x^3 + \pi^4b^5 + 8\pi^2a^2b^5 + 16a^4b^5 + 8(\pi^2b^7 + 12a^2b^7)x^2 + 16(\pi^2ab^6 + 4a^3b^6)x)$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arccoth(tanh(b\*x+a))^3,x, algorithm="fricas")

[Out] 1/8\*(64\*b^6\*x^6 - 128\*a\*b^5\*x^5 - 7\*pi^6 - 28\*pi^4\*a^2 + 112\*pi^2\*a^4 + 448\*a^6 + 32\*(pi^2\*b^4 - 36\*a^2\*b^4)\*x^4 - 512\*(pi^2\*a\*b^3 + 3\*a^3\*b^3)\*x^3 - 32\*(pi^4\*b^2 + 32\*pi^2\*a^2\*b^2)\*x^2 - 32\*(5\*pi^4\*a\*b + 12\*pi^2\*a^3\*b - 32\*a^5\*b)\*x - 96\*(16\*pi\*a\*b^4\*x^4 + 64\*pi\*a^2\*b^3\*x^3 + pi^5\*a + 8\*pi^3\*a^3 + 16\*pi\*a^5 + 8\*(pi^3\*a\*b^2 + 12\*pi\*a^3\*b^2)\*x^2 + 16\*(pi^3\*a^2\*b + 4\*pi\*a^4\*b)\*x)\*arctan(-(2\*b\*x + 2\*a - sqrt(4\*b^2\*x^2 + 8\*a\*b\*x + pi^2 + 4\*a^2))/pi) - 6\*(pi^6 + 4\*pi^4\*a^2 - 16\*pi^2\*a^4 - 64\*a^6 + 16\*(pi^2\*b^4 - 4\*a^2\*b^4)\*x^4 + 64\*(pi^2\*a\*b^3 - 4\*a^3\*b^3)\*x^3 + 8\*(pi^4\*b^2 + 8\*pi^2\*a^2\*b^2 - 48\*a^4\*b^2)\*x^2 + 16\*(pi^4\*a\*b - 16\*a^5\*b)\*x)\*log(4\*b^2\*x^2 + 8\*a\*b\*x + pi^2 + 4\*a^2)/(16\*b^9\*x^4 + 64\*a\*b^8\*x^3 + pi^4\*b^5 + 8\*pi^2\*a^2\*b^5 + 16\*a^4\*b^5 + 8\*(pi^2\*b^7 + 12\*a^2\*b^7)\*x^2 + 16\*(pi^2\*a\*b^6 + 4\*a^3\*b^6)\*x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{arccoth}^3(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4/acoth(tanh(b\*x+a))\*\*3,x)

[Out] Integral(x\*\*4/acoth(tanh(a + b\*x))\*\*3, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{arccoth}(\tanh(bx + a))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(x^4/arccoth(tanh(b*x + a))^3, x)
```

$$3.177 \quad \int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^3} dx$$

**Optimal.** Leaf size=71

$$-\frac{3x^2}{2b^2 \coth^{-1}(\tanh(a+bx))} + \frac{3(bx - \coth^{-1}(\tanh(a+bx))) \log(\coth^{-1}(\tanh(a+bx)))}{b^4} - \frac{x^3}{2b \coth^{-1}(\tanh(a+bx))^2}$$

[Out] (3\*x)/b^3 - x^3/(2\*b\*ArcCoth[Tanh[a + b\*x]]^2) - (3\*x^2)/(2\*b^2\*ArcCoth[Tanh[a + b\*x]]) + (3\*(b\*x - ArcCoth[Tanh[a + b\*x]])\*Log[ArcCoth[Tanh[a + b\*x]]])/b^4

**Rubi [A]** time = 0.0491256, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {2168, 2158, 2157, 29}

$$-\frac{3x^2}{2b^2 \coth^{-1}(\tanh(a+bx))} + \frac{3(bx - \coth^{-1}(\tanh(a+bx))) \log(\coth^{-1}(\tanh(a+bx)))}{b^4} - \frac{x^3}{2b \coth^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

[In] Int[x^3/ArcCoth[Tanh[a + b\*x]]^3,x]

[Out] (3\*x)/b^3 - x^3/(2\*b\*ArcCoth[Tanh[a + b\*x]]^2) - (3\*x^2)/(2\*b^2\*ArcCoth[Tanh[a + b\*x]]) + (3\*(b\*x - ArcCoth[Tanh[a + b\*x]])\*Log[ArcCoth[Tanh[a + b\*x]]])/b^4

### Rule 2168

Int[(u\_)^(m\_)\*(v\_)^(n\_.), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)\*v^n)/(a\*(m + 1)), x] - Dist[(b\*n)/(a\*(m + 1)), Int[u^(m + 1)\*v^(n - 1), x], x] /; NeQ[b\*u - a\*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2\*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))

### Rule 2158

Int[(v\_)/(u\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(b\*x)/a, x] - Dist[(b\*u - a\*v)/a, Int[1/u, x], x] /; NeQ[b\*u - a

\*v, 0]] /; PiecewiseLinearQ[u, v, x]

### Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[  
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

### Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

### Rubi steps

$$\begin{aligned}
 \int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^3} dx &= -\frac{x^3}{2b \coth^{-1}(\tanh(a+bx))^2} + \frac{3 \int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2} dx}{2b} \\
 &= -\frac{x^3}{2b \coth^{-1}(\tanh(a+bx))^2} - \frac{3x^2}{2b^2 \coth^{-1}(\tanh(a+bx))} + \frac{3 \int \frac{x}{\coth^{-1}(\tanh(a+bx))} dx}{b^2} \\
 &= \frac{3x}{b^3} - \frac{x^3}{2b \coth^{-1}(\tanh(a+bx))^2} - \frac{3x^2}{2b^2 \coth^{-1}(\tanh(a+bx))} - \frac{3(-bx + \coth^{-1}(\tanh(a+bx)))^3}{2b^4 \coth^{-1}(\tanh(a+bx))^2} \\
 &= \frac{3x}{b^3} - \frac{x^3}{2b \coth^{-1}(\tanh(a+bx))^2} - \frac{3x^2}{2b^2 \coth^{-1}(\tanh(a+bx))} - \frac{3(-bx + \coth^{-1}(\tanh(a+bx)))^3}{2b^4 \coth^{-1}(\tanh(a+bx))^2} \\
 &= \frac{3x}{b^3} - \frac{x^3}{2b \coth^{-1}(\tanh(a+bx))^2} - \frac{3x^2}{2b^2 \coth^{-1}(\tanh(a+bx))} + \frac{3(bx - \coth^{-1}(\tanh(a+bx)))^3}{2b^4 \coth^{-1}(\tanh(a+bx))^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.0451163, size = 86, normalized size = 1.21

$$\frac{3b^2x^2 \coth^{-1}(\tanh(a+bx)) - bx \coth^{-1}(\tanh(a+bx))^2 (6 \log(\coth^{-1}(\tanh(a+bx))) + 11) + \coth^{-1}(\tanh(a+bx))^3}{2b^4 \coth^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/ArcCoth[Tanh[a + b\*x]]^3,x]

[Out] -(b^3\*x^3 + 3\*b^2\*x^2\*ArcCoth[Tanh[a + b\*x]] + ArcCoth[Tanh[a + b\*x]]^3\*(5 + 6\*Log[ArcCoth[Tanh[a + b\*x]]]) - b\*x\*ArcCoth[Tanh[a + b\*x]]^2\*(11 + 6\*Log[ArcCoth[Tanh[a + b\*x]]]))/(2\*b^4\*ArcCoth[Tanh[a + b\*x]]^2)







$$\begin{aligned}
& (b*x+a)^2 * \text{csgn}(I*\exp(2*b*x+2*a)) - 2*Pi * \text{csgn}(I*\exp(b*x+a)) * \text{csgn}(I*\exp(2*b*x+ \\
& 2*a))^2 + Pi * \text{csgn}(I*\exp(2*b*x+2*a))^3 - Pi * \text{csgn}(I*\exp(2*b*x+2*a)) * \text{csgn}(I*\exp(2* \\
& b*x+2*a) / (\exp(2*b*x+2*a)+1))^2 + Pi * \text{csgn}(I*\exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1)) \\
& ^3 + 4*I*b*x + 4*I*(\ln(\exp(b*x+a)) - b*x - a) + 4*I*a + 2*Pi) * Pi * \text{csgn}(I*\exp(2*b*x+2*a)) \\
& ^3 - 3/4*I/b^4 * \ln(-2*Pi * \text{csgn}(I/(\exp(2*b*x+2*a)+1))^2 + Pi * \text{csgn}(I/(\exp(2*b*x+2*a) \\
& )+1)) * \text{csgn}(I*\exp(2*b*x+2*a)) * \text{csgn}(I*\exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1)) - Pi * \text{c} \\
& \text{sgn}(I/(\exp(2*b*x+2*a)+1)) * \text{csgn}(I*\exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1))^2 + 2*Pi * \\
& \text{csgn}(I/(\exp(2*b*x+2*a)+1))^3 + Pi * \text{csgn}(I*\exp(b*x+a))^2 * \text{csgn}(I*\exp(2*b*x+2*a)) \\
& - 2*Pi * \text{csgn}(I*\exp(b*x+a)) * \text{csgn}(I*\exp(2*b*x+2*a))^2 + Pi * \text{csgn}(I*\exp(2*b*x+2*a)) \\
& ^3 - Pi * \text{csgn}(I*\exp(2*b*x+2*a)) * \text{csgn}(I*\exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1))^2 + Pi \\
& * \text{csgn}(I*\exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1))^3 + 4*I*b*x + 4*I*(\ln(\exp(b*x+a)) - b* \\
& x - a) + 4*I*a + 2*Pi) * Pi * \text{csgn}(I*\exp(2*b*x+2*a)) * \text{csgn}(I*\exp(2*b*x+2*a) / (\exp(2*b*x \\
& +2*a)+1))^2 + 3/4*I/b^4 * \ln(-2*Pi * \text{csgn}(I/(\exp(2*b*x+2*a)+1))^2 + Pi * \text{csgn}(I/(\exp( \\
& 2*b*x+2*a)+1)) * \text{csgn}(I*\exp(2*b*x+2*a)) * \text{csgn}(I*\exp(2*b*x+2*a) / (\exp(2*b*x+2*a) \\
& +1)) - Pi * \text{csgn}(I/(\exp(2*b*x+2*a)+1)) * \text{csgn}(I*\exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1) \\
& )^2 + 2*Pi * \text{csgn}(I/(\exp(2*b*x+2*a)+1))^3 + Pi * \text{csgn}(I*\exp(b*x+a))^2 * \text{csgn}(I*\exp(2* \\
& b*x+2*a)) - 2*Pi * \text{csgn}(I*\exp(b*x+a)) * \text{csgn}(I*\exp(2*b*x+2*a))^2 + Pi * \text{csgn}(I*\exp(2* \\
& b*x+2*a))^3 - Pi * \text{csgn}(I*\exp(2*b*x+2*a)) * \text{csgn}(I*\exp(2*b*x+2*a) / (\exp(2*b*x+2*a) \\
& +1))^2 + Pi * \text{csgn}(I*\exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1))^3 + 4*I*b*x + 4*I*(\ln(\exp(b \\
& *x+a)) - b*x - a) + 4*I*a + 2*Pi) * Pi * \text{csgn}(I*\exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1))^3 + 3/ \\
& 2*I/b^4 * \ln(-2*Pi * \text{csgn}(I/(\exp(2*b*x+2*a)+1))^2 + Pi * \text{csgn}(I/(\exp(2*b*x+2*a)+1)) \\
& ) * \text{csgn}(I*\exp(2*b*x+2*a)) * \text{csgn}(I*\exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1)) - Pi * \text{csgn}(I \\
& /(\exp(2*b*x+2*a)+1)) * \text{csgn}(I*\exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1))^2 + 2*Pi * \text{csgn}( \\
& I/(\exp(2*b*x+2*a)+1))^3 + Pi * \text{csgn}(I*\exp(b*x+a))^2 * \text{csgn}(I*\exp(2*b*x+2*a)) - 2*Pi \\
& * \text{csgn}(I*\exp(b*x+a)) * \text{csgn}(I*\exp(2*b*x+2*a))^2 + Pi * \text{csgn}(I*\exp(2*b*x+2*a))^3 - Pi \\
& * \text{csgn}(I*\exp(2*b*x+2*a)) * \text{csgn}(I*\exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1))^2 + Pi * \text{csgn} \\
& (I*\exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1))^3 + 4*I*b*x + 4*I*(\ln(\exp(b*x+a)) - b*x - a) + \\
& 4*I*a + 2*Pi) * Pi + 3*x/b^3
\end{aligned}$$

**Maxima [C]** time = 3.54468, size = 194, normalized size = 2.73

$$\frac{8(16b^3x^3 - 5i\pi^3 + 30\pi^2a + 60i\pi a^2 - 40a^3 + (-16i\pi b^2 + 32ab^2)x^2 + (8\pi^2b + 32i\pi ab - 32a^2b)x)}{128b^6x^2 - 32\pi^2b^4 - 128i\pi ab^4 + 128a^2b^4 + (-128i\pi b^5 + 256ab^5)x} - \frac{(-3i\pi + 6a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arccoth(tanh(b\*x+a))^3,x, algorithm="maxima")

[Out] 8\*(16\*b^3\*x^3 - 5\*I\*pi^3 + 30\*pi^2\*a + 60\*I\*pi\*a^2 - 40\*a^3 + (-16\*I\*pi\*b^2 + 32\*a\*b^2)\*x^2 + (8\*pi^2\*b + 32\*I\*pi\*a\*b - 32\*a^2\*b)\*x)/(128\*b^6\*x^2 - 32\*pi^2\*b^4 - 128\*I\*pi\*a\*b^4 + 128\*a^2\*b^4 + (-128\*I\*pi\*b^5 + 256\*a\*b^5)\*x) -

$$1/2*(-3*I*pi + 6*a)*\log(-I*pi + 2*b*x + 2*a)/b^4$$

**Fricas [B]** time = 1.81278, size = 936, normalized size = 13.18

$$32b^5x^5 + 128ab^4x^4 - 5\pi^4a - 40\pi^2a^3 - 80a^5 + 8(5\pi^2b^3 + 12a^2b^3)x^3 + 4(11\pi^2ab^2 - 36a^3b^2)x^2 + 2(3\pi^4b - 16\pi^2a^2b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arccoth(tanh(b\*x+a))^3,x, algorithm="fricas")

[Out]  $1/2*(32*b^5*x^5 + 128*a*b^4*x^4 - 5*\pi^4*a - 40*\pi^2*a^3 - 80*a^5 + 8*(5*\pi^2*b^3 + 12*a^2*b^3)*x^3 + 4*(11*\pi^2*a*b^2 - 36*a^3*b^2)*x^2 + 2*(3*\pi^4*b - 16*\pi^2*a^2*b - 112*a^4*b)*x + 6*(16*\pi*b^4*x^4 + 64*\pi*a*b^3*x^3 + \pi^5 + 8*\pi^3*a^2 + 16*\pi*a^4 + 8*(\pi^3*b^2 + 12*\pi*a^2*b^2)*x^2 + 16*(\pi^3*a*b + 4*\pi*a^3*b)*x)*\arctan(-(2*b*x + 2*a - \sqrt{4*b^2*x^2 + 8*a*b*x + \pi^2 + 4*a^2}))/\pi) - 3*(16*a*b^4*x^4 + 64*a^2*b^3*x^3 + \pi^4*a + 8*\pi^2*a^3 + 16*a^5 + 8*(\pi^2*a*b^2 + 12*a^3*b^2)*x^2 + 16*(\pi^2*a^2*b + 4*a^4*b)*x)*\log(4*b^2*x^2 + 8*a*b*x + \pi^2 + 4*a^2))/(16*b^8*x^4 + 64*a*b^7*x^3 + \pi^4*b^4 + 8*\pi^2*a^2*b^4 + 16*a^4*b^4 + 8*(\pi^2*b^6 + 12*a^2*b^6)*x^2 + 16*(\pi^2*a*b^5 + 4*a^3*b^5)*x)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{acoth}^3(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/acoth(tanh(b\*x+a))\*\*3,x)

[Out] Integral(x\*\*3/acoth(tanh(a + b\*x))\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{arccoth}(\tanh(bx + a))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(x^3/arccoth(tanh(b*x + a))^3, x)
```

$$3.178 \quad \int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^3} dx$$

**Optimal.** Leaf size=47

$$-\frac{x}{b^2 \coth^{-1}(\tanh(a+bx))} + \frac{\log(\coth^{-1}(\tanh(a+bx)))}{b^3} - \frac{x^2}{2b \coth^{-1}(\tanh(a+bx))^2}$$

[Out]  $-x^2/(2*b*ArcCoth[Tanh[a + b*x]]^2) - x/(b^2*ArcCoth[Tanh[a + b*x]]) + Log[ArcCoth[Tanh[a + b*x]]]/b^3$

**Rubi [A]** time = 0.0286383, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2168, 2157, 29}

$$-\frac{x}{b^2 \coth^{-1}(\tanh(a+bx))} + \frac{\log(\coth^{-1}(\tanh(a+bx)))}{b^3} - \frac{x^2}{2b \coth^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

[In] Int[x^2/ArcCoth[Tanh[a + b\*x]]^3,x]

[Out]  $-x^2/(2*b*ArcCoth[Tanh[a + b*x]]^2) - x/(b^2*ArcCoth[Tanh[a + b*x]]) + Log[ArcCoth[Tanh[a + b*x]]]/b^3$

### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

### Rule 2157

```
Int[(u_)^(m_), x_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

### Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

### Rubi steps

$$\begin{aligned}
 \int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^3} dx &= -\frac{x^2}{2b \coth^{-1}(\tanh(a+bx))^2} + \frac{\int \frac{x}{\coth^{-1}(\tanh(a+bx))^2} dx}{b} \\
 &= -\frac{x^2}{2b \coth^{-1}(\tanh(a+bx))^2} - \frac{x}{b^2 \coth^{-1}(\tanh(a+bx))} + \frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))} dx}{b^2} \\
 &= -\frac{x^2}{2b \coth^{-1}(\tanh(a+bx))^2} - \frac{x}{b^2 \coth^{-1}(\tanh(a+bx))} + \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, \coth^{-1}(\tanh(a+bx))\right)}{b^3} \\
 &= -\frac{x^2}{2b \coth^{-1}(\tanh(a+bx))^2} - \frac{x}{b^2 \coth^{-1}(\tanh(a+bx))} + \frac{\log(\coth^{-1}(\tanh(a+bx)))}{b^3}
 \end{aligned}$$

**Mathematica [A]** time = 0.0312588, size = 49, normalized size = 1.04

$$\frac{-\frac{b^2 x^2}{\coth^{-1}(\tanh(a+bx))^2} - \frac{2bx}{\coth^{-1}(\tanh(a+bx))} + 2 \log(\coth^{-1}(\tanh(a+bx))) + 3}{2b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/ArcCoth[Tanh[a + b\*x]]^3,x]

[Out] (3 - (b^2\*x^2)/ArcCoth[Tanh[a + b\*x]]^2 - (2\*b\*x)/ArcCoth[Tanh[a + b\*x]] + 2\*Log[ArcCoth[Tanh[a + b\*x]]])/(2\*b^3)

**Maple [C]** time = 0.195, size = 952, normalized size = 20.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/arccoth(tanh(b\*x+a))^3,x)

[Out] -4\*I\*(-2\*Pi\*x\*csgn(I/(exp(2\*b\*x+2\*a)+1))^2+Pi\*x\*csgn(I/(exp(2\*b\*x+2\*a)+1))\*csgn(I\*exp(2\*b\*x+2\*a))\*csgn(I\*exp(2\*b\*x+2\*a)/(exp(2\*b\*x+2\*a)+1))-Pi\*x\*csgn(

$$\begin{aligned} & I/(\exp(2bx+2a)+1) * \operatorname{csgn}(I \exp(2bx+2a)/(\exp(2bx+2a)+1))^{2+2\pi x} * \operatorname{csgn}(I/(\exp(2bx+2a)+1))^{3+\pi x} * \operatorname{csgn}(I \exp(bx+a))^{2+2\pi x} * \operatorname{csgn}(I \exp(2bx+2a))^{2+2\pi x} * \operatorname{csgn}(I \exp(2bx+2a))^{3-\pi x} * \operatorname{csgn}(I \exp(2bx+2a)) * \operatorname{csgn}(I \exp(2bx+2a)/(\exp(2bx+2a)+1))^{2+2\pi x} * \operatorname{csgn}(I \exp(2bx+2a)/(\exp(2bx+2a)+1))^{3+4I x \ln(\exp(bx+a))+2\pi x+2I x^2 b} / b^2 / (-2\pi * \operatorname{csgn}(I/(\exp(2bx+2a)+1))^{2+\pi} * \operatorname{csgn}(I/(\exp(2bx+2a)+1)) * \operatorname{csgn}(I \exp(2bx+2a)) * \operatorname{csgn}(I \exp(2bx+2a)/(\exp(2bx+2a)+1))^{2+\pi} * \operatorname{csgn}(I/(\exp(2bx+2a)+1)) * \operatorname{csgn}(I \exp(2bx+2a)/(\exp(2bx+2a)+1))^{2+2\pi} * \operatorname{csgn}(I/(\exp(2bx+2a)+1))^{3+\pi} * \operatorname{csgn}(I \exp(bx+a))^{2+2\pi} * \operatorname{csgn}(I \exp(2bx+2a))^{2+2\pi} * \operatorname{csgn}(I \exp(2bx+2a))^{3-\pi} * \operatorname{csgn}(I \exp(2bx+2a)) * \operatorname{csgn}(I \exp(2bx+2a)/(\exp(2bx+2a)+1))^{2+2\pi} * \operatorname{csgn}(I \exp(2bx+2a)/(\exp(2bx+2a)+1))^{3+2\pi+4I \ln(\exp(bx+a))}^{2+1/b^3 \ln(\ln(\exp(bx+a)) - 1/4 I \pi (-2 * \operatorname{csgn}(I/(\exp(2bx+2a)+1))^{2+\pi} * \operatorname{csgn}(I/(\exp(2bx+2a)+1)) * \operatorname{csgn}(I \exp(2bx+2a)) * \operatorname{csgn}(I \exp(2bx+2a)/(\exp(2bx+2a)+1)) - \operatorname{csgn}(I/(\exp(2bx+2a)+1)) * \operatorname{csgn}(I \exp(2bx+2a)/(\exp(2bx+2a)+1))^{2+2\pi} * \operatorname{csgn}(I/(\exp(2bx+2a)+1))^{3+\pi} * \operatorname{csgn}(I \exp(bx+a))^{2+2\pi} * \operatorname{csgn}(I \exp(2bx+2a))^{2+2\pi} * \operatorname{csgn}(I \exp(2bx+2a))^{3-\pi} * \operatorname{csgn}(I \exp(2bx+2a)) * \operatorname{csgn}(I \exp(2bx+2a)/(\exp(2bx+2a)+1))^{2+2\pi} * \operatorname{csgn}(I \exp(2bx+2a)/(\exp(2bx+2a)+1))^{3+2\pi}}) \end{aligned}$$

**Maxima [C]** time = 3.57015, size = 127, normalized size = 2.7

$$\frac{8(3\pi^2 + 12i\pi a - 12a^2 + (8i\pi b - 16ab)x)}{64b^5x^2 - 16\pi^2b^3 - 64i\pi ab^3 + 64a^2b^3 + (-64i\pi b^4 + 128ab^4)x} + \frac{\log(-i\pi + 2bx + 2a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arccoth(tanh(b\*x+a))^3,x, algorithm="maxima")

[Out]  $-8(3\pi^2 + 12I\pi a - 12a^2 + (8I\pi b - 16a*b)*x)/(64*b^5*x^2 - 16*\pi^2*b^3 - 64*I*\pi*a*b^3 + 64*a^2*b^3 + (-64*I*\pi*b^4 + 128*a*b^4)*x) + \log(-I*\pi + 2*b*x + 2*a)/b^3$

**Fricas [B]** time = 1.78249, size = 548, normalized size = 11.66

$$\frac{64ab^3x^3 + 3\pi^4 + 24\pi^2a^2 + 48a^4 + 4(5\pi^2b^2 + 44a^2b^2)x^2 + 40(\pi^2ab + 4a^3b)x + (16b^4x^4 + 64ab^3x^3 + \pi^4 + 8\pi^2a^2 + 2(16b^7x^4 + 64ab^6x^3 + \pi^4b^3 + 8\pi^2a^2b^3 + 16a^4b^3 + 8(\pi^2b^5 + 12a^2$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(x^2/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(64*a*b^3*x^3 + 3*pi^4 + 24*pi^2*a^2 + 48*a^4 + 4*(5*pi^2*b^2 + 44*a^2*
b^2)*x^2 + 40*(pi^2*a*b + 4*a^3*b)*x + (16*b^4*x^4 + 64*a*b^3*x^3 + pi^4 +
8*pi^2*a^2 + 16*a^4 + 8*(pi^2*b^2 + 12*a^2*b^2)*x^2 + 16*(pi^2*a*b + 4*a^3*
b)*x)*log(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/(16*b^7*x^4 + 64*a*b^6*x^3 +
pi^4*b^3 + 8*pi^2*a^2*b^3 + 16*a^4*b^3 + 8*(pi^2*b^5 + 12*a^2*b^5)*x^2 + 1
6*(pi^2*a*b^4 + 4*a^3*b^4)*x)
```

**Sympy [A]** time = 24.5137, size = 54, normalized size = 1.15

$$\begin{cases} -\frac{x^2}{2b \operatorname{acoth}^2(\tanh(a+bx))} - \frac{x}{b^2 \operatorname{acoth}(\tanh(a+bx))} + \frac{\log(\operatorname{acoth}(\tanh(a+bx)))}{b^3} & \text{for } b \neq 0 \\ \frac{x^3}{3 \operatorname{acoth}^3(\tanh(a))} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/acoth(tanh(b*x+a))**3,x)
```

```
[Out] Piecewise((-x**2/(2*b*acoth(tanh(a + b*x))**2) - x/(b**2*acoth(tanh(a + b*x)
))) + log(acoth(tanh(a + b*x)))/b**3, Ne(b, 0)), (x**3/(3*acoth(tanh(a))**3
), True))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{arccoth}(\tanh(bx+a))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(x^2/arccoth(tanh(b*x + a))^3, x)
```

$$3.179 \quad \int \frac{x}{\coth^{-1}(\tanh(a+bx))^3} dx$$

**Optimal.** Leaf size=34

$$-\frac{1}{2b^2 \coth^{-1}(\tanh(a+bx))} - \frac{x}{2b \coth^{-1}(\tanh(a+bx))^2}$$

[Out]  $-x/(2*b*ArcCoth[Tanh[a + b*x]]^2) - 1/(2*b^2*ArcCoth[Tanh[a + b*x]])$

**Rubi [A]** time = 0.0141126, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {2168, 2157, 30}

$$-\frac{1}{2b^2 \coth^{-1}(\tanh(a+bx))} - \frac{x}{2b \coth^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x/\text{ArcCoth}[\text{Tanh}[a + b*x]]^3, x]$

[Out]  $-x/(2*b*ArcCoth[Tanh[a + b*x]]^2) - 1/(2*b^2*ArcCoth[Tanh[a + b*x]])$

#### Rule 2168

$\text{Int}[(u_)^{(m_*)}*(v_)^{(n_*)}, x\_Symbol] \rightarrow \text{With}[\{a = \text{Simplify}[D[u, x]], b = \text{Simplify}[D[v, x]]\}, \text{Simp}[(u^{(m+1)}*v^{(n)})/(a*(m+1)), x] - \text{Dist}[(b*n)/(a*(m+1)), \text{Int}[u^{(m+1)}*v^{(n-1)}, x], x] /; \text{NeQ}[b*u - a*v, 0] /; \text{FreeQ}[\{m, n\}, x] \&\& \text{PiecewiseLinearQ}[u, v, x] \&\& \text{NeQ}[m, -1] \&\& ((\text{LtQ}[m, -1] \&\& \text{GtQ}[n, 0] \&\& !(\text{ILtQ}[m+n, -2] \&\& (\text{FractionQ}[m] \mid \text{GeQ}[2*n+m+1, 0]))) \mid (\text{IGtQ}[n, 0] \&\& \text{IGtQ}[m, 0] \&\& \text{LeQ}[n, m]) \mid (\text{IGtQ}[n, 0] \&\& !\text{IntegerQ}[m]) \mid (\text{ILtQ}[m, 0] \&\& !\text{IntegerQ}[n]))]$

#### Rule 2157

$\text{Int}[(u_)^{(m_*)}, x\_Symbol] \rightarrow \text{With}[\{c = \text{Simplify}[D[u, x]]\}, \text{Dist}[1/c, \text{Subst}[\text{Int}[x^m, x], x, u], x] /; \text{FreeQ}[m, x] \&\& \text{PiecewiseLinearQ}[u, x]$

#### Rule 30

$\text{Int}[(x_)^{(m_*)}, x\_Symbol] \rightarrow \text{Simp}[x^{(m+1)}/(m+1), x] /; \text{FreeQ}[m, x] \&\& \text{NeQ}[m, -1]$

Rubi steps

$$\begin{aligned}
\int \frac{x}{\coth^{-1}(\tanh(a+bx))^3} dx &= -\frac{x}{2b \coth^{-1}(\tanh(a+bx))^2} + \frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2} dx}{2b} \\
&= -\frac{x}{2b \coth^{-1}(\tanh(a+bx))^2} + \frac{\text{Subst}\left(\int \frac{1}{x^2} dx, x, \coth^{-1}(\tanh(a+bx))\right)}{2b^2} \\
&= -\frac{x}{2b \coth^{-1}(\tanh(a+bx))^2} - \frac{1}{2b^2 \coth^{-1}(\tanh(a+bx))}
\end{aligned}$$

**Mathematica [A]** time = 0.0493936, size = 27, normalized size = 0.79

$$-\frac{\coth^{-1}(\tanh(a+bx)) + bx}{2b^2 \coth^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/ArcCoth[Tanh[a + b\*x]]^3,x]

[Out] -(b\*x + ArcCoth[Tanh[a + b\*x]])/(2\*b^2\*ArcCoth[Tanh[a + b\*x]]^2)

**Maple [C]** time = 0.171, size = 634, normalized size = 18.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/arccoth(tanh(b\*x+a))^3,x)

[Out]  $-2*I*(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+2*Pi+4*I*b*x+4*I*ln(exp(b*x+a)))/b^2/(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2$

$*a)+1))*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2+2*\text{Pi}*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^3+\text{Pi}*\text{csgn}(I*\exp(b*x+a))^2*\text{csgn}(I*\exp(2*b*x+2*a))-2*\text{Pi}*\text{csgn}(I*\exp(b*x+a))*\text{csgn}(I*\exp(2*b*x+2*a))^2+\text{Pi}*\text{csgn}(I*\exp(2*b*x+2*a))^3-\text{Pi}*\text{csgn}(I*\exp(2*b*x+2*a))*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2+\text{Pi}*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3+2*\text{Pi}+4*I*\ln(\exp(b*x+a)))^2$

**Maxima [C]** time = 3.53148, size = 82, normalized size = 2.41

$$\frac{8i\pi - 32bx - 16a}{32b^4x^2 - 8\pi^2b^2 - 32i\pi ab^2 + 32a^2b^2 + (-32i\pi b^3 + 64ab^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccoth(tanh(b\*x+a))^3,x, algorithm="maxima")

[Out]  $(8*I*\pi - 32*b*x - 16*a)/(32*b^4*x^2 - 8*\pi^2*b^2 - 32*I*\pi*a*b^2 + 32*a^2*b^2 + (-32*I*\pi*b^3 + 64*a*b^3)*x)$

**Fricas [B]** time = 1.64833, size = 262, normalized size = 7.71

$$\frac{2(8b^3x^3 + 20ab^2x^2 + 16a^2bx + \pi^2a + 4a^3)}{16b^6x^4 + 64ab^5x^3 + \pi^4b^2 + 8\pi^2a^2b^2 + 16a^4b^2 + 8(\pi^2b^4 + 12a^2b^4)x^2 + 16(\pi^2ab^3 + 4a^3b^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccoth(tanh(b\*x+a))^3,x, algorithm="fricas")

[Out]  $-2*(8*b^3*x^3 + 20*a*b^2*x^2 + 16*a^2*b*x + \pi^2*a + 4*a^3)/(16*b^6*x^4 + 64*a*b^5*x^3 + \pi^4*b^2 + 8*\pi^2*a^2*b^2 + 16*a^4*b^2 + 8*(\pi^2*b^4 + 12*a^2*b^4)*x^2 + 16*(\pi^2*a*b^3 + 4*a^3*b^3)*x)$

**Sympy [A]** time = 24.1672, size = 42, normalized size = 1.24

$$\begin{cases} \frac{x}{2b \operatorname{acoth}^2(\tanh(a+bx))} - \frac{1}{2b^2 \operatorname{acoth}(\tanh(a+bx))} & \text{for } b \neq 0 \\ \frac{x}{2 \operatorname{acoth}^3(\tanh(a))} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/acoth(tanh(b\*x+a))\*\*3,x)

[Out] Piecewise((-x/(2\*b\*acoth(tanh(a + b\*x))\*\*2) - 1/(2\*b\*\*2\*acoth(tanh(a + b\*x))), Ne(b, 0)), (x\*\*2/(2\*acoth(tanh(a))\*\*3), True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{arccoth}(\tanh(bx + a))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccoth(tanh(b\*x+a))^3,x, algorithm="giac")

[Out] integrate(x/arccoth(tanh(b\*x + a))^3, x)

$$3.180 \quad \int \frac{1}{\coth^{-1}(\tanh(a+bx))^3} dx$$

Optimal. Leaf size=16

$$-\frac{1}{2b \coth^{-1}(\tanh(a+bx))^2}$$

[Out] -1/(2\*b\*ArcCoth[Tanh[a + b\*x]]^2)

**Rubi [A]** time = 0.0053586, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {2157, 30}

$$-\frac{1}{2b \coth^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^(-3), x]

[Out] -1/(2\*b\*ArcCoth[Tanh[a + b\*x]]^2)

#### Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[  
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{1}{\coth^{-1}(\tanh(a+bx))^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3} dx, x, \coth^{-1}(\tanh(a+bx))\right)}{b} \\ &= -\frac{1}{2b \coth^{-1}(\tanh(a+bx))^2} \end{aligned}$$

**Mathematica [A]** time = 0.0055339, size = 16, normalized size = 1.

$$-\frac{1}{2b \coth^{-1}(\tanh(a + bx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^(-3),x]

[Out] -1/(2\*b\*ArcCoth[Tanh[a + b\*x]]^2)

**Maple [A]** time = 0.074, size = 15, normalized size = 0.9

$$-\frac{1}{2b (\operatorname{arccoth}(\tanh(bx + a)))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/arccoth(tanh(b\*x+a))^3,x)

[Out] -1/2/b/arccoth(tanh(b\*x+a))^2

**Maxima [C]** time = 1.47068, size = 41, normalized size = 2.56

$$\frac{8}{(4\pi^2 - 16i\pi(bx + a) - 16(bx + a)^2)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccoth(tanh(b\*x+a))^3,x, algorithm="maxima")

[Out] 8/((4\*pi^2 - 16\*I\*pi\*(b\*x + a) - 16\*(b\*x + a)^2)\*b)

**Fricas [B]** time = 1.60991, size = 227, normalized size = 14.19

$$-\frac{2(4b^2x^2 + 8abx - \pi^2 + 4a^2)}{16b^5x^4 + 64ab^4x^3 + \pi^4b + 8\pi^2a^2b + 16a^4b + 8(\pi^2b^3 + 12a^2b^3)x^2 + 16(\pi^2ab^2 + 4a^3b^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccoth(tanh(b\*x+a))^3,x, algorithm="fricas")

[Out] 
$$\frac{-2(4b^2x^2 + 8abx - \pi^2 + 4a^2)}{(16b^5x^4 + 64ab^4x^3 + \pi^4b + 8\pi^2a^2b + 16a^4b + 8(\pi^2b^3 + 12a^2b^3)x^2 + 16(\pi^2ab^2 + 4a^3b^2)x)}$$

**Sympy [A]** time = 16.4599, size = 24, normalized size = 1.5

$$\begin{cases} -\frac{1}{2b \operatorname{acoth}^2(\tanh(a+bx))} & \text{for } b \neq 0 \\ \frac{x}{\operatorname{acoth}^3(\tanh(a))} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/acoth(tanh(b\*x+a))\*\*3,x)

[Out] Piecewise((-1/(2\*b\*acoth(tanh(a + b\*x))\*\*2), Ne(b, 0)), (x/acoth(tanh(a))\*\*3, True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\operatorname{arcoth}(\tanh(bx+a))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccoth(tanh(b\*x+a))^3,x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a))^-3, x)



$$3.181 \quad \int \frac{1}{x \coth^{-1}(\tanh(a+bx))^3} dx$$

**Optimal.** Leaf size=97

$$\frac{1}{(bx - \coth^{-1}(\tanh(a+bx)))^2 \coth^{-1}(\tanh(a+bx))} - \frac{1}{2(bx - \coth^{-1}(\tanh(a+bx))) \coth^{-1}(\tanh(a+bx))^2} - \frac{1}{(bx - \coth^{-1}(\tanh(a+bx)))^3}$$

[Out]  $-1/(2*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])*\text{ArcCoth}[\text{Tanh}[a + b*x]]^2) + 1/((b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^2*\text{ArcCoth}[\text{Tanh}[a + b*x]]) - \text{Log}[x]/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^3 + \text{Log}[\text{ArcCoth}[\text{Tanh}[a + b*x]]]/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^3$

**Rubi [A]** time = 0.0658827, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {2163, 2160, 2157, 29}

$$\frac{1}{(bx - \coth^{-1}(\tanh(a+bx)))^2 \coth^{-1}(\tanh(a+bx))} - \frac{1}{2(bx - \coth^{-1}(\tanh(a+bx))) \coth^{-1}(\tanh(a+bx))^2} - \frac{1}{(bx - \coth^{-1}(\tanh(a+bx)))^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x*\text{ArcCoth}[\text{Tanh}[a + b*x]])^3, x]$

[Out]  $-1/(2*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])*\text{ArcCoth}[\text{Tanh}[a + b*x]]^2) + 1/((b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^2*\text{ArcCoth}[\text{Tanh}[a + b*x]]) - \text{Log}[x]/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^3 + \text{Log}[\text{ArcCoth}[\text{Tanh}[a + b*x]]]/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^3$

### Rule 2163

$\text{Int}[(v_)^(n_)/(u_), x\_Symbol] := \text{With}[\{a = \text{Simplify}[D[u, x]], b = \text{Simplify}[D[v, x]]\}, \text{Simp}[v^(n+1)/((n+1)*(b*u - a*v)), x] - \text{Dist}[(a*(n+1))/((n+1)*(b*u - a*v)), \text{Int}[v^(n+1)/u, x], x] /; \text{NeQ}[b*u - a*v, 0] /; \text{PiecewiseLinearQ}[u, v, x] \&\& \text{LtQ}[n, -1]$

### Rule 2160

$\text{Int}[1/((u_)*(v_)), x\_Symbol] := \text{With}[\{a = \text{Simplify}[D[u, x]], b = \text{Simplify}[D[v, x]]\}, \text{Dist}[b/(b*u - a*v), \text{Int}[1/v, x], x] - \text{Dist}[a/(b*u - a*v), \text{Int}[1/u, x], x]$

, x], x] /; NeQ[b\*u - a\*v, 0]] /; PiecewiseLinearQ[u, v, x]

### Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[  
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

### Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

### Rubi steps

$$\begin{aligned} \int \frac{1}{x \coth^{-1}(\tanh(a + bx))^3} dx &= -\frac{1}{2(bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))^2} - \frac{\int \frac{1}{x \coth^{-1}(\tanh(a + bx))^2} dx}{bx - \coth^{-1}(\tanh(a + bx))} \\ &= -\frac{1}{2(bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))^2} + \frac{1}{(bx - \coth^{-1}(\tanh(a + bx)))} \\ &= -\frac{1}{2(bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))^2} + \frac{1}{(bx - \coth^{-1}(\tanh(a + bx)))} \\ &= -\frac{1}{2(bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))^2} + \frac{1}{(bx - \coth^{-1}(\tanh(a + bx)))} \\ &= -\frac{1}{2(bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))^2} + \frac{1}{(bx - \coth^{-1}(\tanh(a + bx)))} \end{aligned}$$

**Mathematica [A]** time = 0.106451, size = 74, normalized size = 0.76

$$\frac{-4bx \coth^{-1}(\tanh(a + bx)) + \coth^{-1}(\tanh(a + bx))^2 (-2 \log(\coth^{-1}(\tanh(a + bx))) + 2 \log(bx) + 3) + b^2 x^2}{2 \coth^{-1}(\tanh(a + bx))^2 (\coth^{-1}(\tanh(a + bx)) - bx)^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*ArcCoth[Tanh[a + b\*x]]^3),x]

[Out] (b^2\*x^2 - 4\*b\*x\*ArcCoth[Tanh[a + b\*x]] + ArcCoth[Tanh[a + b\*x]]^2\*(3 + 2\*Log[b\*x] - 2\*Log[ArcCoth[Tanh[a + b\*x]]]))/(2\*ArcCoth[Tanh[a + b\*x]]^2\*(-(b\*

x) + ArcCoth[Tanh[a + b\*x]])^3)

**Maple [F]** time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{x(\operatorname{arccoth}(\tanh(bx+a)))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/arccoth(tanh(b\*x+a))^3,x)

[Out] int(1/x/arccoth(tanh(b\*x+a))^3,x)

**Maxima [C]** time = 3.55119, size = 234, normalized size = 2.41

$$\frac{8(-3i\pi + 4bx + 6a)}{2\pi^4 + 16i\pi^3a - 48\pi^2a^2 - 64i\pi a^3 + 32a^4 - (8\pi^2b^2 + 32i\pi ab^2 - 32a^2b^2)x^2 + (8i\pi^3b - 48\pi^2ab - 96i\pi a^2b + 64a^3b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arccoth(tanh(b\*x+a))^3,x, algorithm="maxima")

[Out]  $8*(-3*I*\pi + 4*b*x + 6*a)/(2*\pi^4 + 16*I*\pi^3*a - 48*\pi^2*a^2 - 64*I*\pi*a^3 + 32*a^4 - (8*\pi^2*b^2 + 32*I*\pi*a*b^2 - 32*a^2*b^2)*x^2 + (8*I*\pi^3*b - 4*8*\pi^2*a*b - 96*I*\pi*a^2*b + 64*a^3*b)*x) + 8*\log(-I*\pi + 2*b*x + 2*a)/(-I*\pi^3 + 6*\pi^2*a + 12*I*\pi*a^2 - 8*a^3) - 8*\log(x)/(-I*\pi^3 + 6*\pi^2*a + 12*I*\pi*a^2 - 8*a^3)$

**Fricas [B]** time = 1.93464, size = 1841, normalized size = 18.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arccoth(tanh(b\*x+a))^3,x, algorithm="fricas")

[Out]  $-8*(9*\pi^6*a + 60*\pi^4*a^3 + 48*\pi^2*a^5 - 192*a^7 + 8*(\pi^4*b^3 - 16*a^4*b^3)*x^3 + 4*(9*\pi^4*a*b^2 + 8*\pi^2*a^3*b^2 - 112*a^5*b^2)*x^2 + 4*(\pi^6*b +$

$$\begin{aligned}
& 16\pi^4 a^2 b + 16\pi^2 a^4 b - 128a^6 b)x - 2(\pi^7 - 4\pi^5 a^2 - 80\pi^3 a^4 - 192\pi a^6 + 16(\pi^3 b^4 - 12\pi a^2 b^4)x^4 + 64(\pi^3 a^3 b^3 - 12\pi a^3 b^3)x^3 + 8(\pi^5 b^2 - 144\pi a^4 b^2)x^2 + 16(\pi^5 a^3 b - 8\pi^3 a^3 b - 48\pi a^5 b)x) \arctan\left(\frac{-2bx + 2a - \sqrt{4b^2 x^2 + 8abx + \pi^2 + 4a^2}}{\pi}\right) - (3\pi^6 a + 20\pi^4 a^3 + 16\pi^2 a^5 - 64a^7 + 16(3\pi^2 a^3 b^4 - 4a^3 b^4)x^4 + 64(3\pi^2 a^2 b^3 - 4a^4 b^3)x^3 + 8(3\pi^4 a^2 b^2 + 32\pi^2 a^3 b^2 - 48a^5 b^2)x^2 + 16(3\pi^4 a^2 b + 8\pi^2 a^4 b - 16a^6 b)x) \log(4b^2 x^2 + 8abx + \pi^2 + 4a^2) + 2(3\pi^6 a + 20\pi^4 a^3 + 16\pi^2 a^5 - 64a^7 + 16(3\pi^2 a^3 b^4 - 4a^3 b^4)x^4 + 64(3\pi^2 a^2 b^3 - 4a^4 b^3)x^3 + 8(3\pi^4 a^2 b^2 + 32\pi^2 a^3 b^2 - 48a^5 b^2)x^2 + 16(3\pi^4 a^2 b + 8\pi^2 a^4 b - 16a^6 b)x) \log(x)) / (\pi^{10} + 20\pi^8 a^2 + 160\pi^6 a^4 + 640\pi^4 a^6 + 1280\pi^2 a^8 + 1024a^{10} + 16(\pi^6 b^4 + 12\pi^4 a^2 b^4 + 48\pi^2 a^4 b^4 + 64a^6 b^4)x^4 + 64(\pi^6 a^3 b^3 + 12\pi^4 a^3 b^3 + 48\pi^2 a^5 b^3 + 64a^7 b^3)x^3 + 8(\pi^8 b^2 + 24\pi^6 a^2 b^2 + 192\pi^4 a^4 b^2 + 640\pi^2 a^6 b^2 + 768a^8 b^2)x^2 + 16(\pi^8 a^3 b + 16\pi^6 a^3 b + 96\pi^4 a^5 b + 256\pi^2 a^7 b + 256a^9 b)x)
\end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{acoth}^3(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/acoth(tanh(b\*x+a))\*\*3,x)

[Out] Integral(1/(x\*acoth(tanh(a + b\*x))\*\*3), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{arccoth}(\tanh(bx + a))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arccoth(tanh(b\*x+a))^3,x, algorithm="giac")

[Out] integrate(1/(x\*arccoth(tanh(b\*x + a))^3), x)

$$3.182 \quad \int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^3} dx$$

**Optimal.** Leaf size=131

$$\frac{3b}{(bx - \coth^{-1}(\tanh(a+bx)))^3 \coth^{-1}(\tanh(a+bx))} - \frac{3b}{2(bx - \coth^{-1}(\tanh(a+bx)))^2 \coth^{-1}(\tanh(a+bx))^2} + \frac{1}{x(bx - \coth^{-1}(\tanh(a+bx)))}$$

[Out]  $(-3*b)/(2*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^2*\text{ArcCoth}[\text{Tanh}[a + b*x]]^2) + 1/(x*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])*\text{ArcCoth}[\text{Tanh}[a + b*x]]^2) + (3*b)/((b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^3*\text{ArcCoth}[\text{Tanh}[a + b*x]]) - (3*b*\text{Log}[x])/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^4 + (3*b*\text{Log}[\text{ArcCoth}[\text{Tanh}[a + b*x]]])/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^4$

**Rubi [A]** time = 0.0874151, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {2171, 2163, 2160, 2157, 29}

$$\frac{3b}{(bx - \coth^{-1}(\tanh(a+bx)))^3 \coth^{-1}(\tanh(a+bx))} - \frac{3b}{2(bx - \coth^{-1}(\tanh(a+bx)))^2 \coth^{-1}(\tanh(a+bx))^2} + \frac{1}{x(bx - \coth^{-1}(\tanh(a+bx)))}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*ArcCoth[Tanh[a + b\*x]]^3),x]

[Out]  $(-3*b)/(2*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^2*\text{ArcCoth}[\text{Tanh}[a + b*x]]^2) + 1/(x*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])*\text{ArcCoth}[\text{Tanh}[a + b*x]]^2) + (3*b)/((b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^3*\text{ArcCoth}[\text{Tanh}[a + b*x]]) - (3*b*\text{Log}[x])/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^4 + (3*b*\text{Log}[\text{ArcCoth}[\text{Tanh}[a + b*x]]])/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^4$

### Rule 2171

Int[(u\_)^(m\_)\*(v\_)^(n\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, -Simp[(u^(m + 1)\*v^(n + 1))/((m + 1)\*(b\*u - a\*v)), x] + Dist[(b\*(m + n + 2))/((m + 1)\*(b\*u - a\*v)), Int[u^(m + 1)\*v^n, x], x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -1]

### Rule 2163

Int[(v\_)^(n\_)/(u\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[v^(n + 1)/((n + 1)\*(b\*u - a\*v)), x] - Dist[(a\*(n + 1))/((n + 1)\*(b\*u - a\*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x] && LtQ[n, -1]

### Rule 2160

Int[1/((u\_)\*(v\_)), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Dist[b/(b\*u - a\*v), Int[1/v, x], x] - Dist[a/(b\*u - a\*v), Int[1/u, x], x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x]

### Rule 2157

Int[(u\_)^(m\_), x\_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

### Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

### Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^2 \coth^{-1}(\tanh(a + bx))^3} dx &= \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))^2} - \frac{(3b) \int \frac{1}{x \coth^{-1}(\tanh(a + bx))^3} dx}{-bx + \coth^{-1}(\tanh(a + bx))} \\
 &= -\frac{3b}{2 (bx - \coth^{-1}(\tanh(a + bx)))^2 \coth^{-1}(\tanh(a + bx))^2} + \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))} \\
 &= -\frac{3b}{2 (bx - \coth^{-1}(\tanh(a + bx)))^2 \coth^{-1}(\tanh(a + bx))^2} + \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))} \\
 &= -\frac{3b}{2 (bx - \coth^{-1}(\tanh(a + bx)))^2 \coth^{-1}(\tanh(a + bx))^2} + \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))} \\
 &= -\frac{3b}{2 (bx - \coth^{-1}(\tanh(a + bx)))^2 \coth^{-1}(\tanh(a + bx))^2} + \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))} \\
 &= -\frac{3b}{2 (bx - \coth^{-1}(\tanh(a + bx)))^2 \coth^{-1}(\tanh(a + bx))^2} + \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))}
 \end{aligned}$$

**Mathematica [A]** time = 0.050491, size = 93, normalized size = 0.71

$$\frac{-6b^2x^2 \coth^{-1}(\tanh(a + bx)) + 2 \coth^{-1}(\tanh(a + bx))^3 + 3bx \coth^{-1}(\tanh(a + bx))^2 \left(-2 \log\left(\coth^{-1}(\tanh(a + bx))\right)\right)}{2x \coth^{-1}(\tanh(a + bx))^2 \left(\coth^{-1}(\tanh(a + bx)) - bx\right)^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*ArcCoth[Tanh[a + b\*x]]^3),x]

[Out] -(b^3\*x^3 - 6\*b^2\*x^2\*ArcCoth[Tanh[a + b\*x]] + 2\*ArcCoth[Tanh[a + b\*x]]^3 + 3\*b\*x\*ArcCoth[Tanh[a + b\*x]]^2\*(1 + 2\*Log[x] - 2\*Log[ArcCoth[Tanh[a + b\*x]]]))/(2\*x\*ArcCoth[Tanh[a + b\*x]]^2\*(-(b\*x) + ArcCoth[Tanh[a + b\*x]])^4)

**Maple [F]** time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (\operatorname{arccoth}(\tanh(bx + a)))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/arccoth(tanh(b\*x+a))^3,x)

[Out] int(1/x^2/arccoth(tanh(b\*x+a))^3,x)

**Maxima [C]** time = 3.5656, size = 328, normalized size = 2.5

$$\frac{48b \log(-i\pi + 2bx + 2a)}{\pi^4 + 8i\pi^3a - 24\pi^2a^2 - 32i\pi a^3 + 16a^4} - \frac{48b \log(x)}{\pi^4 + 8i\pi^3a - 24\pi^2a^2 - 32i\pi a^3 + 16a^4} + \frac{(-4i\pi^3b^2 + 24\pi^2ab^2 + 48i\pi a^2b^2)}{\pi^4 + 8i\pi^3a - 24\pi^2a^2 - 32i\pi a^3 + 16a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arccoth(tanh(b\*x+a))^3,x, algorithm="maxima")

[Out] 48\*b\*log(-I\*pi + 2\*b\*x + 2\*a)/(pi^4 + 8\*I\*pi^3\*a - 24\*pi^2\*a^2 - 32\*I\*pi\*a^3 + 16\*a^4) - 48\*b\*log(x)/(pi^4 + 8\*I\*pi^3\*a - 24\*pi^2\*a^2 - 32\*I\*pi\*a^3 + 16\*a^4) + 8\*(12\*b^2\*x^2 - pi^2 - 4\*I\*pi\*a + 4\*a^2 + (-9\*I\*pi\*b + 18\*a\*b)\*x)/((-4\*I\*pi^3\*b^2 + 24\*pi^2\*a\*b^2 + 48\*I\*pi\*a^2\*b^2 - 32\*a^3\*b^2)\*x^3 - (4\*pi^4\*b + 32\*I\*pi^3\*a\*b - 96\*pi^2\*a^2\*b - 128\*I\*pi\*a^3\*b + 64\*a^4\*b)\*x^2 + (I

$\pi^5 - 10\pi^4 a - 40I\pi^3 a^2 + 80\pi^2 a^3 + 80I\pi a^4 - 32a^5)x)$

**Fricas [B]** time = 2.11983, size = 2475, normalized size = 18.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arccoth(tanh(b\*x+a))^3,x, algorithm="fricas")

[Out]  $8(6\pi^8 a + 64\pi^6 a^3 + 192\pi^4 a^5 - 512a^9 + 96(3\pi^4 a^2 b^4 + 8\pi^2 a^3 b^4 - 16a^5 b^4)x^4 - 12(\pi^6 b^3 - 92\pi^4 a^2 b^3 - 272\pi^2 a^4 b^3 + 448a^6 b^3)x^3 + 8(11\pi^6 a b^2 + 228\pi^4 a^3 b^2 + 528\pi^2 a^5 b^2 - 832a^7 b^2)x^2 - (5\pi^8 b - 176\pi^6 a^2 b - 1440\pi^4 a^4 b - 1792\pi^2 a^6 b + 3328a^8 b)x - 96(16(\pi^3 a^2 b^5 - 4\pi a^3 b^5)x^5 + 64(\pi^3 a^2 b^4 - 4\pi a^4 b^4)x^4 + 8(\pi^5 a^2 b^3 + 8\pi^3 a^3 b^3 - 48\pi a^5 b^3)x^3 + 16(\pi^5 a^2 b^2 - 16\pi a^6 b^2)x^2 + (\pi^7 a^2 b + 4\pi^5 a^3 b - 16\pi^3 a^5 b - 64\pi a^7 b)x) \arctan\left(\frac{-2bx + 2a - \sqrt{4b^2 x^2 + 8abx + \pi^2 + 4a^2}}{\pi}\right) + 3(16(\pi^4 b^5 - 24\pi^2 a^2 b^5 + 16a^4 b^5)x^5 + 64(\pi^4 a^2 b^4 - 24\pi^2 a^3 b^4 + 16a^5 b^4)x^4 + 8(\pi^6 b^3 - 12\pi^4 a^2 b^3 - 272\pi^2 a^4 b^3 + 192a^6 b^3)x^3 + 16(\pi^6 a^2 b^2 - 20\pi^4 a^3 b^2 - 80\pi^2 a^5 b^2 + 64a^7 b^2)x^2 + (\pi^8 b - 16\pi^6 a^2 b - 160\pi^4 a^4 b - 256\pi^2 a^6 b + 256a^8 b)x) \log(4b^2 x^2 + 8abx + \pi^2 + 4a^2) - 6(16(\pi^4 b^5 - 24\pi^2 a^2 b^5 + 16a^4 b^5)x^5 + 64(\pi^4 a^2 b^4 - 24\pi^2 a^3 b^4 + 16a^5 b^4)x^4 + 8(\pi^6 b^3 - 12\pi^4 a^2 b^3 - 272\pi^2 a^4 b^3 + 192a^6 b^3)x^3 + 16(\pi^6 a^2 b^2 - 20\pi^4 a^3 b^2 - 80\pi^2 a^5 b^2 + 64a^7 b^2)x^2 + (\pi^8 b - 16\pi^6 a^2 b - 160\pi^4 a^4 b - 256\pi^2 a^6 b + 256a^8 b)x) \log(x)) / (16(\pi^8 b^4 + 16\pi^6 a^2 b^4 + 96\pi^4 a^4 b^4 + 256\pi^2 a^6 b^4 + 256a^8 b^4)x^5 + 64(\pi^8 a^2 b^3 + 16\pi^6 a^3 b^3 + 96\pi^4 a^5 b^3 + 256\pi^2 a^7 b^3 + 256a^9 b^3)x^4 + 8(\pi^{10} b^2 + 28\pi^8 a^2 b^2 + 288\pi^6 a^4 b^2 + 1408\pi^4 a^6 b^2 + 3328\pi^2 a^8 b^2 + 3072a^{10} b^2)x^3 + 16(\pi^{10} a^2 b + 20\pi^8 a^3 b + 160\pi^6 a^5 b + 640\pi^4 a^7 b + 1280\pi^2 a^9 b + 1024a^{11} b)x^2 + (\pi^{12} + 24\pi^{10} a^2 + 240\pi^8 a^4 + 1280\pi^6 a^6 + 3840\pi^4 a^8 + 6144\pi^2 a^{10} + 4096a^{12})x)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \operatorname{acoth}^3(\tanh(a + bx))} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/acoth(tanh(b\*x+a))\*\*3,x)

[Out] Integral(1/(x\*\*2\*acoth(tanh(a + b\*x))\*\*3), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \operatorname{arccoth}(\tanh(bx + a))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arccoth(tanh(b\*x+a))^3,x, algorithm="giac")

[Out] integrate(1/(x^2\*arccoth(tanh(b\*x + a))^3), x)

$$3.183 \quad \int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^3} dx$$

**Optimal.** Leaf size=170

$$\frac{6b^2}{(bx - \coth^{-1}(\tanh(a+bx)))^4 \coth^{-1}(\tanh(a+bx))} - \frac{3b^2}{(bx - \coth^{-1}(\tanh(a+bx)))^3 \coth^{-1}(\tanh(a+bx))^2} - \frac{1}{(bx - \coth^{-1}(\tanh(a+bx)))^2 \coth^{-1}(\tanh(a+bx))}$$

[Out]  $(-3*b^2)/((b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^3 * \text{ArcCoth}[\text{Tanh}[a + b*x]]^2) + (2*b)/(x*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^2 * \text{ArcCoth}[\text{Tanh}[a + b*x]]^2) + 1/(2*x^2*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]]) * \text{ArcCoth}[\text{Tanh}[a + b*x]]^2) + (6*b^2)/((b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^4 * \text{ArcCoth}[\text{Tanh}[a + b*x]]) - (6*b^2 * \text{Log}[x])/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^5 + (6*b^2 * \text{Log}[\text{ArcCoth}[\text{Tanh}[a + b*x]]])/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^5$

**Rubi [A]** time = 0.126766, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {2171, 2163, 2160, 2157, 29}

$$\frac{6b^2}{(bx - \coth^{-1}(\tanh(a+bx)))^4 \coth^{-1}(\tanh(a+bx))} - \frac{3b^2}{(bx - \coth^{-1}(\tanh(a+bx)))^3 \coth^{-1}(\tanh(a+bx))^2} - \frac{1}{(bx - \coth^{-1}(\tanh(a+bx)))^2 \coth^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*ArcCoth[Tanh[a + b\*x]]^3),x]

[Out]  $(-3*b^2)/((b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^3 * \text{ArcCoth}[\text{Tanh}[a + b*x]]^2) + (2*b)/(x*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^2 * \text{ArcCoth}[\text{Tanh}[a + b*x]]^2) + 1/(2*x^2*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]]) * \text{ArcCoth}[\text{Tanh}[a + b*x]]^2) + (6*b^2)/((b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^4 * \text{ArcCoth}[\text{Tanh}[a + b*x]]) - (6*b^2 * \text{Log}[x])/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^5 + (6*b^2 * \text{Log}[\text{ArcCoth}[\text{Tanh}[a + b*x]]])/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])^5$

### Rule 2171

Int[(u\_)^(m\_)\*(v\_)^(n\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, -Simp[(u^(m+1)\*v^(n+1))/((m+1)\*(b\*u - a\*v)), x] + Dist[(b\*(m+n+2))/((m+1)\*(b\*u - a\*v)), Int[u^(m+1)\*v^n, x], x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x] && NeQ[m+n+2, 0] && LtQ[m, -1]

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_), x_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 \coth^{-1}(\tanh(a + bx))^3} dx &= \frac{1}{2x^2 (bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))^2} + \frac{(2b) \int \frac{1}{x^2 \coth^{-1}(\tanh(a + bx))^3}}{bx - \coth^{-1}(\tanh(a + bx))} \\
&= \frac{2b}{x (bx - \coth^{-1}(\tanh(a + bx)))^2 \coth^{-1}(\tanh(a + bx))^2} + \frac{2b}{2x^2 (bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))^2} \\
&= -\frac{3b^2}{(bx - \coth^{-1}(\tanh(a + bx)))^3 \coth^{-1}(\tanh(a + bx))^2} + \frac{2b}{x (bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))^2} \\
&= -\frac{3b^2}{(bx - \coth^{-1}(\tanh(a + bx)))^3 \coth^{-1}(\tanh(a + bx))^2} + \frac{2b}{x (bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))^2} \\
&= -\frac{3b^2}{(bx - \coth^{-1}(\tanh(a + bx)))^3 \coth^{-1}(\tanh(a + bx))^2} + \frac{2b}{x (bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))^2} \\
&= -\frac{3b^2}{(bx - \coth^{-1}(\tanh(a + bx)))^3 \coth^{-1}(\tanh(a + bx))^2} + \frac{2b}{x (bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))^2} \\
&= -\frac{3b^2}{(bx - \coth^{-1}(\tanh(a + bx)))^3 \coth^{-1}(\tanh(a + bx))^2} + \frac{2b}{x (bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))^2} \\
&= -\frac{3b^2}{(bx - \coth^{-1}(\tanh(a + bx)))^3 \coth^{-1}(\tanh(a + bx))^2} + \frac{2b}{x (bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0433847, size = 107, normalized size = 0.63

$$\frac{8b^3 x^3 \coth^{-1}(\tanh(a + bx)) - 12b^2 x^2 \coth^{-1}(\tanh(a + bx))^2 (\log(x) - \log(\coth^{-1}(\tanh(a + bx)))) - 8bx \coth^{-1}(\tanh(a + bx))}{2x^2 (bx - \coth^{-1}(\tanh(a + bx)))^5 \coth^{-1}(\tanh(a + bx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*ArcCoth[Tanh[a + b\*x]]^3),x]

[Out]  $(-b^4 x^4 + 8b^3 x^3 \text{ArcCoth}[\text{Tanh}[a + b x]] - 8b^2 x^2 \text{ArcCoth}[\text{Tanh}[a + b x]]^2 + \text{ArcCoth}[\text{Tanh}[a + b x]]^3 - 12b^2 x^2 \text{ArcCoth}[\text{Tanh}[a + b x]]^2 (\log[x] - \log[\text{ArcCoth}[\text{Tanh}[a + b x]])]) / (2x^2 (bx - \text{ArcCoth}[\text{Tanh}[a + b x]])^5 \text{ArcCoth}[\text{Tanh}[a + b x]]^2)$

**Maple [F]** time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (\operatorname{arccoth}(\tanh(bx + a)))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/arccoth(tanh(b\*x+a))^3,x)

[Out] int(1/x^3/arccoth(tanh(b\*x+a))^3,x)

**Maxima [C]** time = 3.5402, size = 448, normalized size = 2.64

$$\frac{192 b^2 \log(-i \pi + 2 b x + 2 a)}{i \pi^5 - 10 \pi^4 a - 40 i \pi^3 a^2 + 80 \pi^2 a^3 + 80 i \pi a^4 - 32 a^5} - \frac{192 b^2 \log(x)}{i \pi^5 - 10 \pi^4 a - 40 i \pi^3 a^2 + 80 \pi^2 a^3 + 80 i \pi a^4 - 32 a^5} + \frac{1}{8 \pi^4 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/arccoth(tanh(b\*x+a))^3,x, algorithm="maxima")

[Out] 192\*b^2\*log(-I\*pi + 2\*b\*x + 2\*a)/(I\*pi^5 - 10\*pi^4\*a - 40\*I\*pi^3\*a^2 + 80\*pi^2\*a^3 + 80\*I\*pi\*a^4 - 32\*a^5) - 192\*b^2\*log(x)/(I\*pi^5 - 10\*pi^4\*a - 40\*I\*pi^3\*a^2 + 80\*pi^2\*a^3 + 80\*I\*pi\*a^4 - 32\*a^5) + 8\*(96\*b^3\*x^3 - I\*pi^3 + 6\*pi^2\*a + 12\*I\*pi\*a^2 - 8\*a^3 + (-72\*I\*pi\*b^2 + 144\*a\*b^2)\*x^2 - (8\*pi^2\*b + 32\*I\*pi\*a\*b - 32\*a^2\*b)\*x)/((8\*pi^4\*b^2 + 64\*I\*pi^3\*a\*b^2 - 192\*pi^2\*a^2\*b^2 - 256\*I\*pi\*a^3\*b^2 + 128\*a^4\*b^2)\*x^4 + (-8\*I\*pi^5\*b + 80\*pi^4\*a\*b + 320\*I\*pi^3\*a^2\*b - 640\*pi^2\*a^3\*b - 640\*I\*pi\*a^4\*b + 256\*a^5\*b)\*x^3 - (2\*pi^6 + 24\*I\*pi^5\*a - 120\*pi^4\*a^2 - 320\*I\*pi^3\*a^3 + 480\*pi^2\*a^4 + 384\*I\*pi\*a^5 - 128\*a^6)\*x^2)

**Fricas [B]** time = 2.23993, size = 3069, normalized size = 18.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/arccoth(tanh(b\*x+a))^3,x, algorithm="fricas")

```
[Out] 8*(3*pi^10*a + 44*pi^8*a^3 + 224*pi^6*a^5 + 384*pi^4*a^7 - 256*pi^2*a^9 - 1
024*a^11 + 192*(pi^6*b^5 - 20*pi^4*a^2*b^5 - 80*pi^2*a^4*b^5 + 64*a^6*b^5)*
x^5 + 96*(11*pi^6*a*b^4 - 140*pi^4*a^3*b^4 - 624*pi^2*a^5*b^4 + 448*a^7*b^4
)*x^4 + 16*(5*pi^8*b^3 + 16*pi^6*a^2*b^3 - 1440*pi^4*a^4*b^3 - 4864*pi^2*a^
6*b^3 + 3328*a^8*b^3)*x^3 + 4*(65*pi^8*a*b^2 - 272*pi^6*a^3*b^2 - 4896*pi^4
*a^5*b^2 - 9472*pi^2*a^7*b^2 + 6400*a^9*b^2)*x^2 + 2*(3*pi^10*b - 12*pi^8*a
^2*b - 416*pi^6*a^4*b - 1920*pi^4*a^6*b - 2304*pi^2*a^8*b + 1024*a^10*b)*x
- 48*(16*(pi^5*b^6 - 40*pi^3*a^2*b^6 + 80*pi*a^4*b^6)*x^6 + 64*(pi^5*a*b^5
- 40*pi^3*a^3*b^5 + 80*pi*a^5*b^5)*x^5 + 8*(pi^7*b^4 - 28*pi^5*a^2*b^4 - 40
0*pi^3*a^4*b^4 + 960*pi*a^6*b^4)*x^4 + 16*(pi^7*a*b^3 - 36*pi^5*a^3*b^3 - 8
0*pi^3*a^5*b^3 + 320*pi*a^7*b^3)*x^3 + (pi^9*b^2 - 32*pi^7*a^2*b^2 - 224*pi
^5*a^4*b^2 + 1280*pi*a^8*b^2)*x^2)*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 +
8*a*b*x + pi^2 + 4*a^2))/pi) - 24*(16*(5*pi^4*a*b^6 - 40*pi^2*a^3*b^6 + 16*
a^5*b^6)*x^6 + 64*(5*pi^4*a^2*b^5 - 40*pi^2*a^4*b^5 + 16*a^6*b^5)*x^5 + 8*(
5*pi^6*a*b^4 + 20*pi^4*a^3*b^4 - 464*pi^2*a^5*b^4 + 192*a^7*b^4)*x^4 + 16*(
5*pi^6*a^2*b^3 - 20*pi^4*a^4*b^3 - 144*pi^2*a^6*b^3 + 64*a^8*b^3)*x^3 + (5*
pi^8*a*b^2 - 224*pi^4*a^5*b^2 - 512*pi^2*a^7*b^2 + 256*a^9*b^2)*x^2)*log(4*
b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2) + 48*(16*(5*pi^4*a*b^6 - 40*pi^2*a^3*b^6
+ 16*a^5*b^6)*x^6 + 64*(5*pi^4*a^2*b^5 - 40*pi^2*a^4*b^5 + 16*a^6*b^5)*x^5
+ 8*(5*pi^6*a*b^4 + 20*pi^4*a^3*b^4 - 464*pi^2*a^5*b^4 + 192*a^7*b^4)*x^4 +
16*(5*pi^6*a^2*b^3 - 20*pi^4*a^4*b^3 - 144*pi^2*a^6*b^3 + 64*a^8*b^3)*x^3
+ (5*pi^8*a*b^2 - 224*pi^4*a^5*b^2 - 512*pi^2*a^7*b^2 + 256*a^9*b^2)*x^2)*l
og(x))/(16*(pi^10*b^4 + 20*pi^8*a^2*b^4 + 160*pi^6*a^4*b^4 + 640*pi^4*a^6*b
^4 + 1280*pi^2*a^8*b^4 + 1024*a^10*b^4)*x^6 + 64*(pi^10*a*b^3 + 20*pi^8*a^3
*b^3 + 160*pi^6*a^5*b^3 + 640*pi^4*a^7*b^3 + 1280*pi^2*a^9*b^3 + 1024*a^11*
b^3)*x^5 + 8*(pi^12*b^2 + 32*pi^10*a^2*b^2 + 400*pi^8*a^4*b^2 + 2560*pi^6*a
^6*b^2 + 8960*pi^4*a^8*b^2 + 16384*pi^2*a^10*b^2 + 12288*a^12*b^2)*x^4 + 16
*(pi^12*a*b + 24*pi^10*a^3*b + 240*pi^8*a^5*b + 1280*pi^6*a^7*b + 3840*pi^4
*a^9*b + 6144*pi^2*a^11*b + 4096*a^13*b)*x^3 + (pi^14 + 28*pi^12*a^2 + 336*
pi^10*a^4 + 2240*pi^8*a^6 + 8960*pi^6*a^8 + 21504*pi^4*a^10 + 28672*pi^2*a^
12 + 16384*a^14)*x^2)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \operatorname{acoth}^3(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/acoth(tanh(b*x+a))**3,x)
```

```
[Out] Integral(1/(x**3*acoth(tanh(a + b*x))**3), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \operatorname{arccoth}(\tanh(bx + a))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/arccoth(tanh(b*x+a))^3,x, algorithm="giac")`

[Out] `integrate(1/(x^3*arccoth(tanh(b*x + a))^3), x)`

### 3.184 $\int x^m \coth^{-1}(\tanh(a + bx))^n dx$

**Optimal.** Leaf size=79

$$\frac{x^m \left( \frac{bx}{bx - \coth^{-1}(\tanh(a+bx))} \right)^{-m} \coth^{-1}(\tanh(a+bx))^{n+1} \text{Hypergeometric2F1} \left( -m, n+1, n+2, -\frac{\coth^{-1}(\tanh(a+bx))}{bx - \coth^{-1}(\tanh(a+bx))} \right)}{b(n+1)}$$

[Out] (x^m\*ArcCoth[Tanh[a + b\*x]]^(1 + n)\*Hypergeometric2F1[-m, 1 + n, 2 + n, -(ArcCoth[Tanh[a + b\*x]]/(b\*x - ArcCoth[Tanh[a + b\*x]])))]/(b\*(1 + n)\*((b\*x)/(b\*x - ArcCoth[Tanh[a + b\*x]]))^m)

**Rubi [A]** time = 0.0494153, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2173}

$$\frac{x^m \left( \frac{bx}{bx - \coth^{-1}(\tanh(a+bx))} \right)^{-m} \coth^{-1}(\tanh(a+bx))^{n+1} {}_2F_1 \left( -m, n+1; n+2; -\frac{\coth^{-1}(\tanh(a+bx))}{bx - \coth^{-1}(\tanh(a+bx))} \right)}{b(n+1)}$$

Antiderivative was successfully verified.

[In] Int[x^m\*ArcCoth[Tanh[a + b\*x]]^n,x]

[Out] (x^m\*ArcCoth[Tanh[a + b\*x]]^(1 + n)\*Hypergeometric2F1[-m, 1 + n, 2 + n, -(ArcCoth[Tanh[a + b\*x]]/(b\*x - ArcCoth[Tanh[a + b\*x]])))]/(b\*(1 + n)\*((b\*x)/(b\*x - ArcCoth[Tanh[a + b\*x]]))^m)

#### Rule 2173

Int[(u\_)^(m\_)\*(v\_)^(n\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^m\*v^(n + 1)\*Hypergeometric2F1[-m, n + 1, n + 2, -(a\*v)/(b\*u - a\*v)]]/(b\*(n + 1)\*((b\*u)/(b\*u - a\*v))^m), x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x] && !IntegerQ[m] && !IntegerQ[n]

#### Rubi steps

$$\int x^m \coth^{-1}(\tanh(a + bx))^n dx = \frac{x^m \left( \frac{bx}{bx - \coth^{-1}(\tanh(a+bx))} \right)^{-m} \coth^{-1}(\tanh(a+bx))^{1+n} {}_2F_1 \left( -m, 1 + n; 2 + n; -\frac{\coth^{-1}(\tanh(a+bx))}{bx - \coth^{-1}(\tanh(a+bx))} \right)}{b(1 + n)}$$



**Mathematica [A]** time = 0.132399, size = 71, normalized size = 0.9

$$x^{m+1} \coth^{-1}(\tanh(a + bx))^n \left( \frac{bx}{\coth^{-1}(\tanh(a+bx))-bx} + 1 \right)^{-n} \text{Hypergeometric2F1} \left( m + 1, -n, m + 2, -\frac{bx}{\coth^{-1}(\tanh(a+bx))-bx} \right)$$


---


$$m + 1$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*ArcCoth[Tanh[a + b\*x]]^n,x]

[Out] (x^(1 + m)\*ArcCoth[Tanh[a + b\*x]]^n\*Hypergeometric2F1[1 + m, -n, 2 + m, -((b\*x)/(-(b\*x) + ArcCoth[Tanh[a + b\*x]]))])/((1 + m)\*(1 + (b\*x)/(-(b\*x) + ArcCoth[Tanh[a + b\*x]]))^n)

**Maple [F]** time = 2.566, size = 0, normalized size = 0.

$$\int x^m (\operatorname{arccoth}(\tanh(bx + a)))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*arccoth(tanh(b\*x+a))^n,x)

[Out] int(x^m\*arccoth(tanh(b\*x+a))^n,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{arccoth}(\tanh(bx + a))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*arccoth(tanh(b\*x+a))^n,x, algorithm="maxima")

[Out] integrate(x^m\*arccoth(tanh(b\*x + a))^n, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(x^m \operatorname{arccoth}(\tanh(bx + a))^n, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>m</sup>\*arccoth(tanh(b\*x+a))<sup>n</sup>,x, algorithm="fricas")

[Out] integral(x<sup>m</sup>\*arccoth(tanh(b\*x + a))<sup>n</sup>, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{arccoth}^n(\tanh(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*acoth(tanh(b\*x+a))\*\*n,x)

[Out] Integral(x\*\*m\*acoth(tanh(a + b\*x))\*\*n, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{arccoth}(\tanh(bx + a))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>m</sup>\*arccoth(tanh(b\*x+a))<sup>n</sup>,x, algorithm="giac")

[Out] integrate(x<sup>m</sup>\*arccoth(tanh(b\*x + a))<sup>n</sup>, x)

### 3.185 $\int x^4 \coth^{-1}(\tanh(a + bx))^n dx$

**Optimal.** Leaf size=165

$$-\frac{4x^3 \coth^{-1}(\tanh(a + bx))^{n+2}}{b^2(n+1)(n+2)} + \frac{12x^2 \coth^{-1}(\tanh(a + bx))^{n+3}}{b^3(n+1)(n+2)(n+3)} - \frac{24x \coth^{-1}(\tanh(a + bx))^{n+4}}{b^4(n+1)(n+2)(n+3)(n+4)} + \frac{24 \coth^{-1}(\tanh(a + bx))^{n+5}}{b^5(n+1)(n+2)(n+3)(n+4)(n+5)}$$

```
[Out] (x^4*ArcCoth[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (4*x^3*ArcCoth[Tanh[a +
b*x]]^(2 + n))/(b^2*(1 + n)*(2 + n)) + (12*x^2*ArcCoth[Tanh[a + b*x]]^(3 +
n))/(b^3*(1 + n)*(2 + n)*(3 + n)) - (24*x*ArcCoth[Tanh[a + b*x]]^(4 + n))/(
b^4*(1 + n)*(2 + n)*(3 + n)*(4 + n)) + (24*ArcCoth[Tanh[a + b*x]]^(5 + n))/(
b^5*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n))
```

---

**Rubi [A]** time = 0.133038, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2168, 2157, 30}

$$-\frac{4x^3 \coth^{-1}(\tanh(a + bx))^{n+2}}{b^2(n+1)(n+2)} + \frac{12x^2 \coth^{-1}(\tanh(a + bx))^{n+3}}{b^3(n+1)(n+2)(n+3)} - \frac{24x \coth^{-1}(\tanh(a + bx))^{n+4}}{b^4(n+1)(n+2)(n+3)(n+4)} + \frac{24 \coth^{-1}(\tanh(a + bx))^{n+5}}{b^5(n+1)(n+2)(n+3)(n+4)(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcCoth[Tanh[a + b*x]]^n,x]
```

```
[Out] (x^4*ArcCoth[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (4*x^3*ArcCoth[Tanh[a +
b*x]]^(2 + n))/(b^2*(1 + n)*(2 + n)) + (12*x^2*ArcCoth[Tanh[a + b*x]]^(3 +
n))/(b^3*(1 + n)*(2 + n)*(3 + n)) - (24*x*ArcCoth[Tanh[a + b*x]]^(4 + n))/(
b^4*(1 + n)*(2 + n)*(3 + n)*(4 + n)) + (24*ArcCoth[Tanh[a + b*x]]^(5 + n))/(
b^5*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n))
```

#### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] := With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n},
x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
&& !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

#### Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] :=> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[  
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] :=> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

### Rubi steps

$$\begin{aligned}
 \int x^4 \coth^{-1}(\tanh(a + bx))^n dx &= \frac{x^4 \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{4 \int x^3 \coth^{-1}(\tanh(a + bx))^{1+n} dx}{b(1+n)} \\
 &= \frac{x^4 \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{4x^3 \coth^{-1}(\tanh(a + bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12 \int x^2 \coth^{-1}(\tanh(a + bx))^{2+n} dx}{b^2(1+n)(2+n)} \\
 &= \frac{x^4 \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{4x^3 \coth^{-1}(\tanh(a + bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12x^2 \coth^{-1}(\tanh(a + bx))^{2+n}}{b^3(1+n)(2+n)(3+n)} \\
 &= \frac{x^4 \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{4x^3 \coth^{-1}(\tanh(a + bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12x^2 \coth^{-1}(\tanh(a + bx))^{2+n}}{b^3(1+n)(2+n)(3+n)} \\
 &= \frac{x^4 \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{4x^3 \coth^{-1}(\tanh(a + bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12x^2 \coth^{-1}(\tanh(a + bx))^{2+n}}{b^3(1+n)(2+n)(3+n)} \\
 &= \frac{x^4 \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{4x^3 \coth^{-1}(\tanh(a + bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12x^2 \coth^{-1}(\tanh(a + bx))^{2+n}}{b^3(1+n)(2+n)(3+n)}
 \end{aligned}$$

**Mathematica [A]** time = 0.103393, size = 146, normalized size = 0.88

$$\frac{\coth^{-1}(\tanh(a + bx))^{n+1} (-4b^3 (n^3 + 12n^2 + 47n + 60) x^3 \coth^{-1}(\tanh(a + bx)) + 12b^2 (n^2 + 9n + 20) x^2 \coth^{-1}(\tanh(a + bx)) - 12b (n + 1) x \coth^{-1}(\tanh(a + bx)) + 12 \coth^{-1}(\tanh(a + bx)))}{b^5 (n + 1)(n + 2)(n + 3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*ArcCoth[Tanh[a + b\*x]]^n, x]

[Out] (ArcCoth[Tanh[a + b\*x]]^(1 + n)\*(b^4\*(120 + 154\*n + 71\*n^2 + 14\*n^3 + n^4)\*  
x^4 - 4\*b^3\*(60 + 47\*n + 12\*n^2 + n^3)\*x^3\*ArcCoth[Tanh[a + b\*x]] + 12\*b^2\*  
(20 + 9\*n + n^2)\*x^2\*ArcCoth[Tanh[a + b\*x]]^2 - 24\*b\*(5 + n)\*x\*ArcCoth[Tanh  
[a + b\*x]]^3 + 24\*ArcCoth[Tanh[a + b\*x]]^4)/(b^5\*(1 + n)\*(2 + n)\*(3 + n)\*(  
4 + n)\*(5 + n))

**Maple [B]** time = 87.445, size = 2984280, normalized size = 18086.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^4 \cdot \text{arccoth}(\tanh(b \cdot x + a))^n, x)$

[Out] result too large to display

**Maxima [C]** time = 1.81928, size = 513, normalized size = 3.11

$(4(n^4 + 10n^3 + 35n^2 + 50n + 24)b^5x^5 - 3i\pi^5 + 30\pi^4a + 120i\pi^3a^2 - 240\pi^2a^3 - 240i\pi a^4 + 96a^5 - 2(i\pi(n^4 + 6n^3$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(x^4 \cdot \text{arccoth}(\tanh(b \cdot x + a))^n, x, \text{algorithm}="maxima")$

[Out]  $(4(n^4 + 10n^3 + 35n^2 + 50n + 24)b^5x^5 - 3I\pi^5 + 30\pi^4a + 120I\pi^3a^2 - 240\pi^2a^3 - 240I\pi a^4 + 96a^5 - 2(I\pi(n^4 + 6n^3 + 11n^2 + 6n)b^4 - 2(n^4 + 6n^3 + 11n^2 + 6n)ab^4)x^4 + (4\pi^2(n^3 + 3n^2 + 2n)b^3 + 16I\pi(n^3 + 3n^2 + 2n)ab^3 - 16(n^3 + 3n^2 + 2n)a^2b^3)x^3 + (6I\pi^3(n^2 + n)b^2 - 36\pi^2(n^2 + n)ab^2 - 72I\pi(n^2 + n)a^2b^2 + 48(n^2 + n)a^3b^2)x^2 - (6\pi^4bn + 48I\pi^3abn - 144\pi^2a^2bn - 192I\pi a^3bn + 96a^4bn)x)(\cosh(-n \log(-I\pi + 2bx + 2a)) - \sinh(-n \log(-I\pi + 2bx + 2a)))/((2^{n+2})n^5 + 15 \cdot 2^{n+2}n^4 + 85 \cdot 2^{n+2}n^3 + 225 \cdot 2^{n+2}n^2 + 137 \cdot 2^{n+3}n + 15 \cdot 2^{n+5})b^5)$

**Fricas [B]** time = 1.72095, size = 1288, normalized size = 7.81

$2(2(b^5n^4 + 10b^5n^3 + 35b^5n^2 + 50b^5n + 24b^5)x^5 + 15\pi^4a - 120\pi^2a^3 + 48a^5 + 2(ab^4n^4 + 6ab^4n^3 + 11ab^4n^2 + 6ab^4n$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(tanh(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(2*(b^5*n^4 + 10*b^5*n^3 + 35*b^5*n^2 + 50*b^5*n + 24*b^5)*x^5 + 15*
pi^4*a - 120*pi^2*a^3 + 48*a^5 + 2*(a*b^4*n^4 + 6*a*b^4*n^3 + 11*a*b^4*n^2
+ 6*a*b^4*n)*x^4 - 2*(4*a^2*b^3*n^3 + 12*a^2*b^3*n^2 + 8*a^2*b^3*n - pi^2*(
b^3*n^3 + 3*b^3*n^2 + 2*b^3*n))*x^3 + 6*(4*a^3*b^2*n^2 + 4*a^3*b^2*n - 3*pi
^2*(a*b^2*n^2 + a*b^2*n))*x^2 - 3*(pi^4*b*n - 24*pi^2*a^2*b*n + 16*a^4*b*n)
*x)*(b^2*x^2 + 2*a*b*x + 1/4*pi^2 + a^2)^(1/2*n)*cos(2*n*arctan(-2*b*x/pi -
2*a/pi + sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2)/pi)) - (2*pi*(b^4*n^4 +
6*b^4*n^3 + 11*b^4*n^2 + 6*b^4*n)*x^4 + 3*pi^5 - 120*pi^3*a^2 + 240*pi*a^4
- 16*pi*(a*b^3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 6*(pi^3*(b^2*n^2 + b^2*
n) - 12*pi*(a^2*b^2*n^2 + a^2*b^2*n))*x^2 + 48*(pi^3*a*b*n - 4*pi*a^3*b*n)*
x)*(b^2*x^2 + 2*a*b*x + 1/4*pi^2 + a^2)^(1/2*n)*sin(2*n*arctan(-2*b*x/pi -
2*a/pi + sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2)/pi)))/(b^5*n^5 + 15*b^5*n
^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acoth(tanh(b*x+a))**n,x)
```

```
[Out] Exception raised: TypeError
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{arccoth}(\tanh(bx + a))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(tanh(b*x+a))^n,x, algorithm="giac")
```

```
[Out] integrate(x^4*arccoth(tanh(b*x + a))^n, x)
```

### 3.186 $\int x^3 \coth^{-1}(\tanh(a + bx))^n dx$

**Optimal.** Leaf size=121

$$-\frac{3x^2 \coth^{-1}(\tanh(a + bx))^{n+2}}{b^2(n+1)(n+2)} + \frac{6x \coth^{-1}(\tanh(a + bx))^{n+3}}{b^3(n+1)(n+2)(n+3)} - \frac{6 \coth^{-1}(\tanh(a + bx))^{n+4}}{b^4(n+1)(n+2)(n+3)(n+4)} + \frac{x^3 \coth^{-1}(\tanh(a + bx))^{n+5}}{b(n+1)}$$

```
[Out] (x^3*ArcCoth[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (3*x^2*ArcCoth[Tanh[a +
b*x]]^(2 + n))/(b^2*(1 + n)*(2 + n)) + (6*x*ArcCoth[Tanh[a + b*x]]^(3 + n))
/(b^3*(1 + n)*(2 + n)*(3 + n)) - (6*ArcCoth[Tanh[a + b*x]]^(4 + n))/(b^4*(1
+ n)*(2 + n)*(3 + n)*(4 + n))
```

**Rubi [A]** time = 0.0796348, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2168, 2157, 30}

$$-\frac{3x^2 \coth^{-1}(\tanh(a + bx))^{n+2}}{b^2(n+1)(n+2)} + \frac{6x \coth^{-1}(\tanh(a + bx))^{n+3}}{b^3(n+1)(n+2)(n+3)} - \frac{6 \coth^{-1}(\tanh(a + bx))^{n+4}}{b^4(n+1)(n+2)(n+3)(n+4)} + \frac{x^3 \coth^{-1}(\tanh(a + bx))^{n+5}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCoth[Tanh[a + b*x]]^n, x]
```

```
[Out] (x^3*ArcCoth[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (3*x^2*ArcCoth[Tanh[a +
b*x]]^(2 + n))/(b^2*(1 + n)*(2 + n)) + (6*x*ArcCoth[Tanh[a + b*x]]^(3 + n))
/(b^3*(1 + n)*(2 + n)*(3 + n)) - (6*ArcCoth[Tanh[a + b*x]]^(4 + n))/(b^4*(1
+ n)*(2 + n)*(3 + n)*(4 + n))
```

#### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n},
x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
&& !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

#### Rule 2157

```
Int[(u_)^(m_), x_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

`Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]`

Rubi steps

$$\begin{aligned}
 \int x^3 \coth^{-1}(\tanh(a + bx))^n dx &= \frac{x^3 \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{3 \int x^2 \coth^{-1}(\tanh(a + bx))^{1+n} dx}{b(1+n)} \\
 &= \frac{x^3 \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{3x^2 \coth^{-1}(\tanh(a + bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{6 \int x \coth^{-1}(\tanh(a + bx))^n dx}{b^2(1+n)(2+n)} \\
 &= \frac{x^3 \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{3x^2 \coth^{-1}(\tanh(a + bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{6x \coth^{-1}(\tanh(a + bx))^n}{b^3(1+n)(2+n)(3+n)} \\
 &= \frac{x^3 \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{3x^2 \coth^{-1}(\tanh(a + bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{6x \coth^{-1}(\tanh(a + bx))^n}{b^3(1+n)(2+n)(3+n)} \\
 &= \frac{x^3 \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{3x^2 \coth^{-1}(\tanh(a + bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{6x \coth^{-1}(\tanh(a + bx))^n}{b^3(1+n)(2+n)(3+n)}
 \end{aligned}$$

**Mathematica [A]** time = 0.0773392, size = 106, normalized size = 0.88

$$\frac{\coth^{-1}(\tanh(a + bx))^{n+1} (-3b^2(n^2 + 7n + 12)x^2 \coth^{-1}(\tanh(a + bx)) + 6b(n + 4)x \coth^{-1}(\tanh(a + bx))^2 - 6 \coth^{-1}(\tanh(a + bx)))}{b^4(n + 1)(n + 2)(n + 3)(n + 4)}$$

Antiderivative was successfully verified.

[In] `Integrate[x^3*ArcCoth[Tanh[a + b*x]]^n,x]`

[Out] `(ArcCoth[Tanh[a + b*x]]^(1 + n)*(b^3*(24 + 26*n + 9*n^2 + n^3)*x^3 - 3*b^2*(12 + 7*n + n^2)*x^2*ArcCoth[Tanh[a + b*x]] + 6*b*(4 + n)*x*ArcCoth[Tanh[a + b*x]]^2 - 6*ArcCoth[Tanh[a + b*x]]^3)/(b^4*(1 + n)*(2 + n)*(3 + n)*(4 + n))`

**Maple [B]** time = 36.951, size = 953037, normalized size = 7876.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(x^3*arccoth(tanh(b*x+a))^n,x)`

[Out] result too large to display

**Maxima [C]** time = 1.79852, size = 344, normalized size = 2.84

$$\frac{(8(n^3 + 6n^2 + 11n + 6)b^4x^4 - 3\pi^4 - 24i\pi^3a + 72\pi^2a^2 + 96i\pi a^3 - 48a^4 + (-4i\pi(n^3 + 3n^2 + 2n)b^3 + 8(n^3 + 3n^2 + 2n)a^2b^3)x^3 + (6\pi^2(n^2 + n)b^2 + 24i\pi(n^2 + n)a^2b^2 - 24(n^2 + n)a^2b^2)x^2 + (6i\pi^3bn - 36\pi^2abn - 72i\pi a^2bn + 48a^3bn)x)(\cosh(-n\log(-i\pi + 2bx + 2a)) - \sinh(-n\log(-i\pi + 2bx + 2a)))}{(2^{n+3}n^4 + 5 \cdot 2^{n+4}n^3 + 35 \cdot 2^{n+3}n^2 + 25 \cdot 2^{n+4}n + 3 \cdot 2^{n+6})b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arccoth(tanh(b*x+a))^n,x, algorithm="maxima")`

[Out]  $(8(n^3 + 6n^2 + 11n + 6)b^4x^4 - 3\pi^4 - 24i\pi^3a + 72\pi^2a^2 + 96i\pi a^3 - 48a^4 + (-4i\pi(n^3 + 3n^2 + 2n)b^3 + 8(n^3 + 3n^2 + 2n)a^2b^3)x^3 + (6\pi^2(n^2 + n)b^2 + 24i\pi(n^2 + n)a^2b^2 - 24(n^2 + n)a^2b^2)x^2 + (6i\pi^3bn - 36\pi^2abn - 72i\pi a^2bn + 48a^3bn)x)(\cosh(-n\log(-i\pi + 2bx + 2a)) - \sinh(-n\log(-i\pi + 2bx + 2a))) / ((2^{n+3}n^4 + 5 \cdot 2^{n+4}n^3 + 35 \cdot 2^{n+3}n^2 + 25 \cdot 2^{n+4}n + 3 \cdot 2^{n+6})b^4)$

**Fricas [B]** time = 1.76948, size = 917, normalized size = 7.58

$$(8(b^4n^3 + 6b^4n^2 + 11b^4n + 6b^4)x^4 - 3\pi^4 + 72\pi^2a^2 - 48a^4 + 8(ab^3n^3 + 3ab^3n^2 + 2ab^3n)x^3 - 6(4a^2b^2n^2 + 4a^2b^2n + 4a^2b^2)x^2 - 12(3\pi^2abn - 4a^3bn)x)(b^2x^2 + 2abx + 1/4\pi^2 + a^2)^{(1/2)n} \cos(2n \arctan(-2bx/\pi - 2a/\pi + \sqrt{4b^2x^2 + 8abx + \pi^2 + 4a^2})/\pi) - 2(2\pi(b^3n^3 + 3b^3n^2 + 2b^3n)x^3 + 12\pi^3a - 48\pi a^3 - 12\pi(ab^2n^2 + ab^2n)x^2 - 3(\pi^3bn - 12\pi a^2bn)x)(b^2x^2 + 2abx + 1/4\pi^2 + a^2)^{(1/2)n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arccoth(tanh(b*x+a))^n,x, algorithm="fricas")`

[Out]  $1/8((8(b^4n^3 + 6b^4n^2 + 11b^4n + 6b^4)x^4 - 3\pi^4 + 72\pi^2a^2 - 48a^4 + 8(ab^3n^3 + 3ab^3n^2 + 2ab^3n)x^3 - 6(4a^2b^2n^2 + 4a^2b^2n + 4a^2b^2)x^2 - 12(3\pi^2abn - 4a^3bn)x)(b^2x^2 + 2abx + 1/4\pi^2 + a^2)^{(1/2)n} \cos(2n \arctan(-2bx/\pi - 2a/\pi + \sqrt{4b^2x^2 + 8abx + \pi^2 + 4a^2})/\pi) - 2(2\pi(b^3n^3 + 3b^3n^2 + 2b^3n)x^3 + 12\pi^3a - 48\pi a^3 - 12\pi(ab^2n^2 + ab^2n)x^2 - 3(\pi^3bn - 12\pi a^2bn)x)(b^2x^2 + 2abx + 1/4\pi^2 + a^2)^{(1/2)n}$

$$a^2)^{(1/2*n)*\sin(2*n*\arctan(-2*b*x/\pi - 2*a/\pi + \sqrt{4*b^2*x^2 + 8*a*b*x + \pi^2 + 4*a^2})/\pi)))/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)$$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*acoth(tanh(b\*x+a))\*\*n,x)

[Out] Exception raised: TypeError

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arccoth}(\tanh(bx + a))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(tanh(b\*x+a))^n,x, algorithm="giac")

[Out] integrate(x^3\*arccoth(tanh(b\*x + a))^n, x)

### 3.187 $\int x^2 \coth^{-1}(\tanh(a + bx))^n dx$

**Optimal.** Leaf size=82

$$-\frac{2x \coth^{-1}(\tanh(a + bx))^{n+2}}{b^2(n+1)(n+2)} + \frac{2 \coth^{-1}(\tanh(a + bx))^{n+3}}{b^3(n+1)(n+2)(n+3)} + \frac{x^2 \coth^{-1}(\tanh(a + bx))^{n+1}}{b(n+1)}$$

[Out]  $(x^2 \operatorname{ArcCoth}[\operatorname{Tanh}[a + b x]]^{(1 + n)}) / (b(1 + n)) - (2 x \operatorname{ArcCoth}[\operatorname{Tanh}[a + b x]]^{(2 + n)}) / (b^2(1 + n)(2 + n)) + (2 \operatorname{ArcCoth}[\operatorname{Tanh}[a + b x]]^{(3 + n)}) / (b^3(1 + n)(2 + n)(3 + n))$

**Rubi [A]** time = 0.0461923, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2168, 2157, 30}

$$-\frac{2x \coth^{-1}(\tanh(a + bx))^{n+2}}{b^2(n+1)(n+2)} + \frac{2 \coth^{-1}(\tanh(a + bx))^{n+3}}{b^3(n+1)(n+2)(n+3)} + \frac{x^2 \coth^{-1}(\tanh(a + bx))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2 \operatorname{ArcCoth}[\operatorname{Tanh}[a + b x]]^n, x]$

[Out]  $(x^2 \operatorname{ArcCoth}[\operatorname{Tanh}[a + b x]]^{(1 + n)}) / (b(1 + n)) - (2 x \operatorname{ArcCoth}[\operatorname{Tanh}[a + b x]]^{(2 + n)}) / (b^2(1 + n)(2 + n)) + (2 \operatorname{ArcCoth}[\operatorname{Tanh}[a + b x]]^{(3 + n)}) / (b^3(1 + n)(2 + n)(3 + n))$

#### Rule 2168

$\operatorname{Int}[(u_)^{(m)}(v_)^{(n)}, x\_Symbol] := \operatorname{With}[\{a = \operatorname{Simplify}[D[u, x]], b = \operatorname{Simplify}[D[v, x]]\}, \operatorname{Simp}[(u^{(m+1)}v^{(n)}) / (a(m+1)), x] - \operatorname{Dist}[(b^n) / (a(m+1))], \operatorname{Int}[u^{(m+1)}v^{(n-1)}, x], x] /; \operatorname{NeQ}[b u - a v, 0] /; \operatorname{FreeQ}[\{m, n\}, x] \&\& \operatorname{PiecewiseLinearQ}[u, v, x] \&\& \operatorname{NeQ}[m, -1] \&\& ((\operatorname{LtQ}[m, -1] \&\& \operatorname{GtQ}[n, 0] \&\& !(\operatorname{ILtQ}[m+n, -2] \&\& (\operatorname{FractionQ}[m] \mid \operatorname{GeQ}[2m+m+1, 0]))) \mid (\operatorname{IGtQ}[n, 0] \&\& \operatorname{IGtQ}[m, 0] \&\& \operatorname{LeQ}[n, m]) \mid (\operatorname{IGtQ}[n, 0] \&\& !\operatorname{IntegerQ}[m]) \mid (\operatorname{ILtQ}[m, 0] \&\& !\operatorname{IntegerQ}[n]))$

#### Rule 2157

$\operatorname{Int}[(u_)^{(m)}, x\_Symbol] := \operatorname{With}[\{c = \operatorname{Simplify}[D[u, x]]\}, \operatorname{Dist}[1/c, \operatorname{Subst}[\operatorname{Int}[x^m, x], x, u], x] /; \operatorname{FreeQ}[m, x] \&\& \operatorname{PiecewiseLinearQ}[u, x]$

Rule 30

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned}
 \int x^2 \coth^{-1}(\tanh(a + bx))^n dx &= \frac{x^2 \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{2 \int x \coth^{-1}(\tanh(a + bx))^{1+n} dx}{b(1+n)} \\
 &= \frac{x^2 \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{2x \coth^{-1}(\tanh(a + bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{2 \int \coth^{-1}(\tanh(a + bx))^{2+n} dx}{b^2(1+n)(2+n)} \\
 &= \frac{x^2 \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{2x \coth^{-1}(\tanh(a + bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{2 \text{Subst}\left(\int x^{2+n} dx, x, \coth^{-1}(\tanh(a + bx))\right)}{b^3(1+n)} \\
 &= \frac{x^2 \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{2x \coth^{-1}(\tanh(a + bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{2 \coth^{-1}(\tanh(a + bx))^{3+n}}{b^3(1+n)(2+n)(3+n)}
 \end{aligned}$$

**Mathematica [A]** time = 0.0645757, size = 71, normalized size = 0.87

$$\frac{\coth^{-1}(\tanh(a + bx))^{n+1} \left( -2b(n+3)x \coth^{-1}(\tanh(a + bx)) + 2 \coth^{-1}(\tanh(a + bx))^2 + b^2(n^2 + 5n + 6)x^2 \right)}{b^3(n+1)(n+2)(n+3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[Tanh[a + b\*x]]^n,x]

[Out] (ArcCoth[Tanh[a + b\*x]]^(1 + n)\*(b^2\*(6 + 5\*n + n^2)\*x^2 - 2\*b\*(3 + n)\*x\*ArcCoth[Tanh[a + b\*x]] + 2\*ArcCoth[Tanh[a + b\*x]]^2))/(b^3\*(1 + n)\*(2 + n)\*(3 + n))

**Maple [C]** time = 18.674, size = 252344, normalized size = 3077.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(tanh(b\*x+a))^n,x)

[Out] result too large to display

**Maxima [C]** time = 1.79112, size = 223, normalized size = 2.72

$$\frac{(4(n^2 + 3n + 2)b^3x^3 + i\pi^3 - 6\pi^2a - 12i\pi a^2 + 8a^3 + (-2i\pi(n^2 + n)b^2 + 4(n^2 + n)ab^2)x^2 + (2\pi^2bn + 8i\pi abn - 8a^3))}{(2^{n+2}n^3 + 3 \cdot 2^{n+3}n^2 + 11 \cdot 2^{n+2}n + 3 \cdot 2^{n+3})b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(tanh(b\*x+a))^n,x, algorithm="maxima")

$$\frac{(4(n^2 + 3n + 2)b^3x^3 + I\pi^3 - 6\pi^2a - 12I\pi a^2 + 8a^3 + (-2I\pi(n^2 + n)b^2 + 4(n^2 + n)a*b^2)*x^2 + (2\pi^2*b*n + 8I\pi*a*b*n - 8*a^2*b*n)*x)*(cosh(-n*\log(-I*\pi + 2*b*x + 2*a)) - sinh(-n*\log(-I*\pi + 2*b*x + 2*a)))/((2^(n + 2)*n^3 + 3*2^(n + 3)*n^2 + 11*2^(n + 2)*n + 3*2^(n + 3))*b^3)$$

**Fricas [B]** time = 1.72839, size = 649, normalized size = 7.91

$$2\left(2(b^3n^2 + 3b^3n + 2b^3)x^3 - 3\pi^2a + 4a^3 + 2(ab^2n^2 + ab^2n)x^2 + (\pi^2bn - 4a^2bn)x\right)\left(b^2x^2 + 2abx + \frac{1}{4}\pi^2 + a^2\right)^{\frac{1}{2}n} \cos$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(tanh(b\*x+a))^n,x, algorithm="fricas")

$$\frac{1/4*(2*(2*(b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 3*\pi^2*a + 4*a^3 + 2*(a*b^2*n^2 + a*b^2*n)*x^2 + (\pi^2*b*n - 4*a^2*b*n)*x)*(b^2*x^2 + 2*a*b*x + 1/4*\pi^2 + a^2)^{(1/2)*n}*\cos(2*n*\arctan(-2*b*x/\pi - 2*a/\pi + \sqrt{4*b^2*x^2 + 8*a*b*x + \pi^2 + 4*a^2})/\pi) + (8*\pi*a*b*n*x - 2*\pi*(b^2*n^2 + b^2*n)*x^2 + \pi^3 - 12*\pi*a^2)*(b^2*x^2 + 2*a*b*x + 1/4*\pi^2 + a^2)^{(1/2)*n}*\sin(2*n*\arctan(-2*b*x/\pi - 2*a/\pi + \sqrt{4*b^2*x^2 + 8*a*b*x + \pi^2 + 4*a^2})/\pi)))/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3)$$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*acoth(tanh(b*x+a))**n,x)`

[Out] Exception raised: TypeError

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arccoth}(\tanh(bx + a))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccoth(tanh(b*x+a))^n,x, algorithm="giac")`

[Out] `integrate(x^2*arccoth(tanh(b*x + a))^n, x)`

### 3.188 $\int x \coth^{-1}(\tanh(a + bx))^n dx$

**Optimal.** Leaf size=48

$$\frac{x \coth^{-1}(\tanh(a + bx))^{n+1}}{b(n+1)} - \frac{\coth^{-1}(\tanh(a + bx))^{n+2}}{b^2(n+1)(n+2)}$$

[Out] (x\*ArcCoth[Tanh[a + b\*x]]^(1 + n))/(b\*(1 + n)) - ArcCoth[Tanh[a + b\*x]]^(2 + n)/(b^2\*(1 + n)\*(2 + n))

**Rubi [A]** time = 0.0200901, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {2168, 2157, 30}

$$\frac{x \coth^{-1}(\tanh(a + bx))^{n+1}}{b(n+1)} - \frac{\coth^{-1}(\tanh(a + bx))^{n+2}}{b^2(n+1)(n+2)}$$

Antiderivative was successfully verified.

[In] Int[x\*ArcCoth[Tanh[a + b\*x]]^n,x]

[Out] (x\*ArcCoth[Tanh[a + b\*x]]^(1 + n))/(b\*(1 + n)) - ArcCoth[Tanh[a + b\*x]]^(2 + n)/(b^2\*(1 + n)\*(2 + n))

#### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

#### Rule 2157

```
Int[(u_)^(m_), x_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

#### Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

### Rubi steps

$$\begin{aligned} \int x \coth^{-1}(\tanh(a + bx))^n dx &= \frac{x \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{\int \coth^{-1}(\tanh(a + bx))^{1+n} dx}{b(1+n)} \\ &= \frac{x \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{\text{Subst}\left(\int x^{1+n} dx, x, \coth^{-1}(\tanh(a + bx))\right)}{b^2(1+n)} \\ &= \frac{x \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{\coth^{-1}(\tanh(a + bx))^{2+n}}{b^2(1+n)(2+n)} \end{aligned}$$

**Mathematica [A]** time = 0.0428894, size = 41, normalized size = 0.85

$$\frac{(b(n+2)x - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))^{n+1}}{b^2(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[Tanh[a + b*x]]^n,x]
```

```
[Out] ((b*(2 + n)*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]^(1 + n))/(b^2*(1 + n)*(2 + n))
```

**Maple [C]** time = 15.611, size = 71611, normalized size = 1491.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(tanh(b*x+a))^n,x)
```

```
[Out] result too large to display
```



**Maxima [C]** time = 1.79195, size = 138, normalized size = 2.88

$$\frac{(4b^2(n+1)x^2 + \pi^2 + 4i\pi a - 4a^2 - 2(i\pi bn - 2abn)x)(\cosh(-n \log(-i\pi + 2bx + 2a)) - \sinh(-n \log(-i\pi + 2bx + 2a)))}{(2^{n+2}n^2 + 3 \cdot 2^{n+2}n + 2^{n+3})b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(tanh(b\*x+a))^n,x, algorithm="maxima")

[Out] (4\*b^2\*(n + 1)\*x^2 + pi^2 + 4\*I\*pi\*a - 4\*a^2 - 2\*(I\*pi\*b\*n - 2\*a\*b\*n)\*x)\*(cosh(-n\*log(-I\*pi + 2\*b\*x + 2\*a)) - sinh(-n\*log(-I\*pi + 2\*b\*x + 2\*a)))/((2^(n + 2)\*n^2 + 3\*2^(n + 2)\*n + 2^(n + 3))\*b^2)

**Fricas [B]** time = 1.60396, size = 489, normalized size = 10.19

$$\frac{(4abnx + 4(b^2n + b^2)x^2 + \pi^2 - 4a^2)\left(b^2x^2 + 2abx + \frac{1}{4}\pi^2 + a^2\right)^{\frac{1}{2}n} \cos\left(2n \arctan\left(-\frac{2bx}{\pi} - \frac{2a}{\pi} + \frac{\sqrt{4b^2x^2 + 8abx + \pi^2 + 4a^2}}{\pi}\right)\right)}{4(b^2n^2 + 3b^2n + 2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(tanh(b\*x+a))^n,x, algorithm="fricas")

[Out] 1/4\*((4\*a\*b\*n\*x + 4\*(b^2\*n + b^2)\*x^2 + pi^2 - 4\*a^2)\*(b^2\*x^2 + 2\*a\*b\*x + 1/4\*pi^2 + a^2)^(1/2\*n)\*cos(2\*n\*arctan(-2\*b\*x/pi - 2\*a/pi + sqrt(4\*b^2\*x^2 + 8\*a\*b\*x + pi^2 + 4\*a^2)/pi)) - 2\*(pi\*b\*n\*x - 2\*pi\*a)\*(b^2\*x^2 + 2\*a\*b\*x + 1/4\*pi^2 + a^2)^(1/2\*n)\*sin(2\*n\*arctan(-2\*b\*x/pi - 2\*a/pi + sqrt(4\*b^2\*x^2 + 8\*a\*b\*x + pi^2 + 4\*a^2)/pi)))/(b^2\*n^2 + 3\*b^2\*n + 2\*b^2)

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(tanh(b\*x+a))\*\*n,x)

[Out] Exception raised: TypeError

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arccoth}(\tanh(bx + a))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(tanh(b*x+a))^n,x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(tanh(b*x + a))^n, x)
```

$$3.189 \quad \int \coth^{-1}(\tanh(a + bx))^n dx$$

Optimal. Leaf size=20

$$\frac{\coth^{-1}(\tanh(a + bx))^{n+1}}{b(n+1)}$$

[Out] ArcCoth[Tanh[a + b\*x]]^(1 + n)/(b\*(1 + n))

**Rubi [A]** time = 0.0065514, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {2157, 30}

$$\frac{\coth^{-1}(\tanh(a + bx))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^n, x]

[Out] ArcCoth[Tanh[a + b\*x]]^(1 + n)/(b\*(1 + n))

#### Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[  
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \coth^{-1}(\tanh(a + bx))^n dx &= \frac{\text{Subst}\left(\int x^n dx, x, \coth^{-1}(\tanh(a + bx))\right)}{b} \\ &= \frac{\coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} \end{aligned}$$

**Mathematica [A]** time = 0.0142894, size = 20, normalized size = 1.

$$\frac{\coth^{-1}(\tanh(a + bx))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^n,x]

[Out] ArcCoth[Tanh[a + b\*x]]^(1 + n)/(b\*(1 + n))

**Maple [A]** time = 0.087, size = 21, normalized size = 1.1

$$\frac{(\operatorname{arccoth}(\tanh(bx + a)))^{1+n}}{b(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b\*x+a))^n,x)

[Out] arccoth(tanh(b\*x+a))^(1+n)/b/(1+n)

**Maxima [C]** time = 1.73917, size = 88, normalized size = 4.4

$$\frac{(-i\pi + 2bx + 2a)(\cosh(-n \log(-i\pi + 2bx + 2a)) - \sinh(-n \log(-i\pi + 2bx + 2a)))}{(2^{n+1}n + 2^{n+1})b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^n,x, algorithm="maxima")

[Out] (-I\*pi + 2\*b\*x + 2\*a)\*(cosh(-n\*log(-I\*pi + 2\*b\*x + 2\*a)) - sinh(-n\*log(-I\*pi + 2\*b\*x + 2\*a)))/((2^(n + 1)\*n + 2^(n + 1))\*b)

**Fricas [B]** time = 1.7029, size = 389, normalized size = 19.45

$$\frac{2(bx + a)\left(b^2x^2 + 2abx + \frac{1}{4}\pi^2 + a^2\right)^{\frac{1}{2}n} \cos\left(2n \arctan\left(-\frac{2bx}{\pi} - \frac{2a}{\pi} + \frac{\sqrt{4b^2x^2 + 8abx + \pi^2 + 4a^2}}{\pi}\right)\right) - \pi\left(b^2x^2 + 2abx + \frac{1}{4}\pi^2 + a^2\right)}{2(bn + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^n,x, algorithm="fricas")

[Out]  $\frac{1}{2} * (2 * (b * x + a) * (b^2 * x^2 + 2 * a * b * x + 1/4 * \pi^2 + a^2)^{(1/2 * n)} * \cos(2 * n * \arctan(-2 * b * x / \pi - 2 * a / \pi + \sqrt{4 * b^2 * x^2 + 8 * a * b * x + \pi^2 + 4 * a^2} / \pi)) - \pi * (b^2 * x^2 + 2 * a * b * x + 1/4 * \pi^2 + a^2)^{(1/2 * n)} * \sin(2 * n * \arctan(-2 * b * x / \pi - 2 * a / \pi + \sqrt{4 * b^2 * x^2 + 8 * a * b * x + \pi^2 + 4 * a^2} / \pi))) / (b * n + b)$

**Sympy [A]** time = 1.04688, size = 51, normalized size = 2.55

$$\begin{cases} \frac{x}{\operatorname{acoth}(\tanh(a))} & \text{for } b = 0 \wedge n = -1 \\ x \operatorname{acoth}^n(\tanh(a)) & \text{for } b = 0 \\ \frac{\log(\operatorname{acoth}(\tanh(a+bx)))}{b} & \text{for } n = -1 \\ \frac{\operatorname{acoth}(\tanh(a+bx)) \operatorname{acoth}^n(\tanh(a+bx))}{bn+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(tanh(b\*x+a))^n,x)

[Out] Piecewise((x/acoth(tanh(a)), Eq(b, 0) & Eq(n, -1)), (x\*acoth(tanh(a))^n, Eq(b, 0)), (log(acoth(tanh(a + b\*x)))/b, Eq(n, -1)), (acoth(tanh(a + b\*x))\*acoth(tanh(a + b\*x))^n/(b\*n + b), True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(\tanh(bx + a))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^n,x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a))^n, x)

$$3.190 \quad \int \frac{\coth^{-1}(\tanh(a+bx))^n}{x} dx$$

**Optimal.** Leaf size=64

$$\frac{\coth^{-1}(\tanh(a+bx))^{n+1} \text{Hypergeometric2F1}\left(1, n+1, n+2, -\frac{\coth^{-1}(\tanh(a+bx))}{bx - \coth^{-1}(\tanh(a+bx))}\right)}{(n+1)(bx - \coth^{-1}(\tanh(a+bx)))}$$

[Out] (ArcCoth[Tanh[a + b\*x]]^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, -(ArcCot h[Tanh[a + b\*x]]/(b\*x - ArcCoth[Tanh[a + b\*x]]))])/((1 + n)\*(b\*x - ArcCoth[ Tanh[a + b\*x]]))

**Rubi [A]** time = 0.0208313, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2164}

$$\frac{\coth^{-1}(\tanh(a+bx))^{n+1} {}_2F_1\left(1, n+1; n+2; -\frac{\coth^{-1}(\tanh(a+bx))}{bx - \coth^{-1}(\tanh(a+bx))}\right)}{(n+1)(bx - \coth^{-1}(\tanh(a+bx)))}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^n/x, x]

[Out] (ArcCoth[Tanh[a + b\*x]]^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, -(ArcCot h[Tanh[a + b\*x]]/(b\*x - ArcCoth[Tanh[a + b\*x]]))])/((1 + n)\*(b\*x - ArcCoth[ Tanh[a + b\*x]]))

### Rule 2164

Int[(v\_)^(n\_)/(u\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(v^(n + 1)\*Hypergeometric2F1[1, n + 1, n + 2, -((a\*v)/(b\*u - a\*v))])/((n + 1)\*(b\*u - a\*v)), x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinea rQ[u, v, x] && !IntegerQ[n]

### Rubi steps

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x} dx = \frac{\coth^{-1}(\tanh(a+bx))^{1+n} {}_2F_1\left(1, 1+n; 2+n; -\frac{\coth^{-1}(\tanh(a+bx))}{bx - \coth^{-1}(\tanh(a+bx))}\right)}{(1+n)(bx - \coth^{-1}(\tanh(a+bx)))}$$

**Mathematica [A]** time = 0.0811981, size = 60, normalized size = 0.94

$$\frac{\coth^{-1}(\tanh(a + bx))^n \left( \frac{\coth^{-1}(\tanh(a + bx))}{bx} \right)^{-n} \text{Hypergeometric2F1} \left( -n, -n, 1 - n, 1 - \frac{\coth^{-1}(\tanh(a + bx))}{bx} \right)}{n}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^n/x,x]

[Out] (ArcCoth[Tanh[a + b\*x]]^n\*Hypergeometric2F1[-n, -n, 1 - n, 1 - ArcCoth[Tanh[a + b\*x]]/(b\*x)]/(b\*x))/(n\*(ArcCoth[Tanh[a + b\*x]]/(b\*x))^n)

**Maple [F]** time = 0.855, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{arccoth}(\tanh(bx + a)))^n}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b\*x+a))^n/x,x)

[Out] int(arccoth(tanh(b\*x+a))^n/x,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^n}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^n/x,x, algorithm="maxima")

[Out] integrate(arccoth(tanh(b\*x + a))^n/x, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{\operatorname{arccoth}(\tanh(bx + a))^n}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^n/x,x, algorithm="fricas")

[Out] integral(arccoth(tanh(b\*x + a))^n/x, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^n(\tanh(a + bx))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(tanh(b\*x+a))\*\*n/x,x)

[Out] Integral(acoth(tanh(a + b\*x))\*\*n/x, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(\tanh(bx + a))^n}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^n/x,x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a))^n/x, x)



$$3.191 \quad \int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^2} dx$$

**Optimal.** Leaf size=71

$$\frac{b \coth^{-1}(\tanh(a+bx))^n \text{Hypergeometric2F1}\left(1, n, n+1, -\frac{\coth^{-1}(\tanh(a+bx))}{bx - \coth^{-1}(\tanh(a+bx))}\right)}{bx - \coth^{-1}(\tanh(a+bx))} - \frac{\coth^{-1}(\tanh(a+bx))^n}{x}$$

[Out]  $-(\text{ArcCoth}[\text{Tanh}[a + b*x]]^n/x) + (b*\text{ArcCoth}[\text{Tanh}[a + b*x]]^n*\text{Hypergeometric2F1}[1, n, 1 + n, -(\text{ArcCoth}[\text{Tanh}[a + b*x]]/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]]))]/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]]))$

**Rubi [A]** time = 0.0412581, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2168, 2164}

$$\frac{b \coth^{-1}(\tanh(a+bx))^n {}_2F_1\left(1, n; n+1; -\frac{\coth^{-1}(\tanh(a+bx))}{bx - \coth^{-1}(\tanh(a+bx))}\right)}{bx - \coth^{-1}(\tanh(a+bx))} - \frac{\coth^{-1}(\tanh(a+bx))^n}{x}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^n/x^2,x]

[Out]  $-(\text{ArcCoth}[\text{Tanh}[a + b*x]]^n/x) + (b*\text{ArcCoth}[\text{Tanh}[a + b*x]]^n*\text{Hypergeometric2F1}[1, n, 1 + n, -(\text{ArcCoth}[\text{Tanh}[a + b*x]]/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]]))]/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]]))$

### Rule 2168

Int[(u\_)^(m\_)\*(v\_)^(n\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m+1)\*v^n)/(a\*(m+1)), x] - Dist[(b\*n)/(a\*(m+1)), Int[u^(m+1)\*v^(n-1), x], x] /; NeQ[b\*u - a\*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m+n, -2] && (FractionQ[m] || GeQ[2\*n+m+1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))

### Rule 2164

Int[(v\_)^(n\_)/(u\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(v^(n+1)\*Hypergeometric2F1[1, n+1, n+2, -((a\*v)/(b\*u

- a\*v)))]/((n + 1)\*(b\*u - a\*v)), x] /; NeQ[b\*u - a\*v, 0]] /; PiecewiseLinearQ[u, v, x] && !IntegerQ[n]

### Rubi steps

$$\int \frac{\coth^{-1}(\tanh(a + bx))^n}{x^2} dx = -\frac{\coth^{-1}(\tanh(a + bx))^n}{x} + (bn) \int \frac{\coth^{-1}(\tanh(a + bx))^{-1+n}}{x} dx$$

$$= -\frac{\coth^{-1}(\tanh(a + bx))^n}{x} + \frac{b \coth^{-1}(\tanh(a + bx))^n {}_2F_1\left(1, n; 1 + n; -\frac{\coth^{-1}(\tanh(a + bx))}{bx - \coth^{-1}(\tanh(a + bx))}\right)}{bx - \coth^{-1}(\tanh(a + bx))}$$

**Mathematica [A]** time = 0.0404399, size = 67, normalized size = 0.94

$$\frac{\coth^{-1}(\tanh(a + bx))^n \left(\frac{\coth^{-1}(\tanh(a + bx))}{bx}\right)^{-n} \text{Hypergeometric2F1}\left(1 - n, -n, 2 - n, 1 - \frac{\coth^{-1}(\tanh(a + bx))}{bx}\right)}{(n - 1)x}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^n/x^2,x]

[Out] (ArcCoth[Tanh[a + b\*x]]^n\*Hypergeometric2F1[1 - n, -n, 2 - n, 1 - ArcCoth[Tanh[a + b\*x]]/(b\*x)]/((b\*x)))/((-1 + n)\*x\*(ArcCoth[Tanh[a + b\*x]]/(b\*x))^n)

**Maple [F]** time = 1.964, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{arccoth}(\tanh(bx + a)))^n}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b\*x+a))^n/x^2,x)

[Out] int(arccoth(tanh(b\*x+a))^n/x^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^n}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^n/x^2,x, algorithm="maxima")

[Out] integrate(arccoth(tanh(b\*x + a))^n/x^2, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arccoth}(\tanh(bx + a))^n}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^n/x^2,x, algorithm="fricas")

[Out] integral(arccoth(tanh(b\*x + a))^n/x^2, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{acoth}^n(\tanh(a + bx))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(tanh(b\*x+a))^n/x\*\*2,x)

[Out] Integral(acoth(tanh(a + b\*x))^n/x\*\*2, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arccoth}(\tanh(bx + a))^n}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^n/x^2,x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a))^n/x^2, x)

$$3.192 \quad \int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^3} dx$$

**Optimal.** Leaf size=101

$$\frac{b^2 n \coth^{-1}(\tanh(a+bx))^{n-1} \text{Hypergeometric2F1}\left(1, n-1, n, -\frac{\coth^{-1}(\tanh(a+bx))}{bx - \coth^{-1}(\tanh(a+bx))}\right)}{2(bx - \coth^{-1}(\tanh(a+bx)))} - \frac{\coth^{-1}(\tanh(a+bx))^n}{2x^2} - \frac{bn \coth^{-1}(\tanh(a+bx))}{2x}$$

[Out]  $-(b*n*\text{ArcCoth}[\text{Tanh}[a + b*x]]^{(-1 + n)})/(2*x) - \text{ArcCoth}[\text{Tanh}[a + b*x]]^n/(2*x^2) + (b^2*n*\text{ArcCoth}[\text{Tanh}[a + b*x]]^{(-1 + n)}*\text{Hypergeometric2F1}[1, -1 + n, n, -(\text{ArcCoth}[\text{Tanh}[a + b*x]]/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])))]/(2*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]]))$

**Rubi [A]** time = 0.0692208, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2168, 2164}

$$\frac{b^2 n \coth^{-1}(\tanh(a+bx))^{n-1} {}_2F_1\left(1, n-1; n; -\frac{\coth^{-1}(\tanh(a+bx))}{bx - \coth^{-1}(\tanh(a+bx))}\right)}{2(bx - \coth^{-1}(\tanh(a+bx)))} - \frac{\coth^{-1}(\tanh(a+bx))^n}{2x^2} - \frac{bn \coth^{-1}(\tanh(a+bx))}{2x}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^n/x^3,x]

[Out]  $-(b*n*\text{ArcCoth}[\text{Tanh}[a + b*x]]^{(-1 + n)})/(2*x) - \text{ArcCoth}[\text{Tanh}[a + b*x]]^n/(2*x^2) + (b^2*n*\text{ArcCoth}[\text{Tanh}[a + b*x]]^{(-1 + n)}*\text{Hypergeometric2F1}[1, -1 + n, n, -(\text{ArcCoth}[\text{Tanh}[a + b*x]]/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])))]/(2*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]]))$

### Rule 2168

Int[(u\_)^(m\_)\*(v\_)^(n\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)\*v^n)/(a\*(m + 1)), x] - Dist[(b\*n)/(a\*(m + 1)), Int[u^(m + 1)\*v^(n - 1), x], x] /; NeQ[b\*u - a\*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2\*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))

### Rule 2164

```
Int[(v_)^(n_)/(u_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)*Hypergeometric2F1[1, n + 1, n + 2, -((a*v)/(b*u
- a*v))])/((n + 1)*(b*u - a*v)), x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] && !IntegerQ[n]
```

### Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(\tanh(a + bx))^n}{x^3} dx &= -\frac{\coth^{-1}(\tanh(a + bx))^n}{2x^2} + \frac{1}{2}(bn) \int \frac{\coth^{-1}(\tanh(a + bx))^{-1+n}}{x^2} dx \\ &= -\frac{bn \coth^{-1}(\tanh(a + bx))^{-1+n}}{2x} - \frac{\coth^{-1}(\tanh(a + bx))^n}{2x^2} - \frac{1}{2}(b^2(1 - n)n) \int \frac{\coth^{-1}(\tanh(a + bx))^{-1+n}}{x} dx \\ &= -\frac{bn \coth^{-1}(\tanh(a + bx))^{-1+n}}{2x} - \frac{\coth^{-1}(\tanh(a + bx))^n}{2x^2} + \frac{b^2n \coth^{-1}(\tanh(a + bx))^{-1+n}}{2(bx - a)} \end{aligned}$$

**Mathematica [A]** time = 0.0411354, size = 67, normalized size = 0.66

$$\frac{\coth^{-1}(\tanh(a + bx))^n \left( \frac{\coth^{-1}(\tanh(a + bx))}{bx} \right)^{-n} \text{Hypergeometric2F1} \left( 2 - n, -n, 3 - n, 1 - \frac{\coth^{-1}(\tanh(a + bx))}{bx} \right)}{(n - 2)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^n/x^3,x]
```

```
[Out] (ArcCoth[Tanh[a + b*x]]^n*Hypergeometric2F1[2 - n, -n, 3 - n, 1 - ArcCoth[Tanh[a + b*x]]/(b*x)])/((-2 + n)*x^2*(ArcCoth[Tanh[a + b*x]]/(b*x))^n)
```

**Maple [F]** time = 2.029, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{arccoth}(\tanh(bx + a)))^n}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^n/x^3,x)
```

```
[Out] int(arccoth(tanh(b*x+a))^n/x^3,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^n}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^n/x^3,x, algorithm="maxima")

[Out] integrate(arccoth(tanh(b\*x + a))^n/x^3, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arccoth}(\tanh(bx + a))^n}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^n/x^3,x, algorithm="fricas")

[Out] integral(arccoth(tanh(b\*x + a))^n/x^3, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}^n(\tanh(a + bx))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(tanh(b\*x+a))\*\*n/x\*\*3,x)

[Out] Integral(acoth(tanh(a + b\*x))\*\*n/x\*\*3, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^n}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^n/x^3,x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^n/x^3, x)
```

### 3.193 $\int x^m \coth^{-1}(\tanh(a + bx)) dx$

**Optimal.** Leaf size=37

$$\frac{x^{m+1} \coth^{-1}(\tanh(a + bx))}{m + 1} - \frac{bx^{m+2}}{m^2 + 3m + 2}$$

[Out]  $-\left(\frac{b x^{2+m}}{2+3 m+m^2}\right)+\left(x^{1+m} \operatorname{ArcCoth}[\operatorname{Tanh}[a+b x]]\right) / (1+m)$

**Rubi [A]** time = 0.0100579, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2168, 30}

$$\frac{x^{m+1} \coth^{-1}(\tanh(a + bx))}{m + 1} - \frac{bx^{m+2}}{m^2 + 3m + 2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[x^m \operatorname{ArcCoth}[\operatorname{Tanh}[a+b x]], x\right]$

[Out]  $-\left(\frac{b x^{2+m}}{2+3 m+m^2}\right)+\left(x^{1+m} \operatorname{ArcCoth}[\operatorname{Tanh}[a+b x]]\right) / (1+m)$

#### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

#### Rule 30

```
Int[(x_)^(m_), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

#### Rubi steps



$$\int x^m \coth^{-1}(\tanh(a + bx)) dx = \frac{x^{1+m} \coth^{-1}(\tanh(a + bx))}{1+m} - \frac{b \int x^{1+m} dx}{1+m}$$

$$= -\frac{bx^{2+m}}{2+3m+m^2} + \frac{x^{1+m} \coth^{-1}(\tanh(a + bx))}{1+m}$$

**Mathematica [A]** time = 0.033024, size = 34, normalized size = 0.92

$$x^m \left( \frac{x \left( \coth^{-1}(\tanh(a + bx)) - bx \right)}{m+1} + \frac{bx^2}{m+2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*ArcCoth[Tanh[a + b\*x]],x]

[Out] x^m\*((b\*x^2)/(2 + m) + (x\*(-(b\*x) + ArcCoth[Tanh[a + b\*x]])))/(1 + m))

**Maple [C]** time = 0.159, size = 676, normalized size = 18.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*arccoth(tanh(b\*x+a)),x)

[Out] 1/(1+m)\*x\*x^m\*ln(exp(b\*x+a))-1/4\*x\*(4\*b\*x-I\*Pi\*csgn(I/(exp(2\*b\*x+2\*a)+1))\*csgn(I\*exp(2\*b\*x+2\*a)/(exp(2\*b\*x+2\*a)+1))^2\*m-4\*I\*Pi\*csgn(I\*exp(b\*x+a))\*csgn(I\*exp(2\*b\*x+2\*a))^2-2\*I\*Pi\*csgn(I\*exp(b\*x+a))\*csgn(I\*exp(2\*b\*x+2\*a))^2\*m+I\*Pi\*csgn(I\*exp(b\*x+a))^2\*csgn(I\*exp(2\*b\*x+2\*a))\*m+I\*Pi\*csgn(I\*exp(2\*b\*x+2\*a))^3\*m+4\*I\*Pi+2\*I\*Pi\*m+2\*I\*Pi\*csgn(I\*exp(2\*b\*x+2\*a)/(exp(2\*b\*x+2\*a)+1))^3-2\*I\*Pi\*csgn(I/(exp(2\*b\*x+2\*a)+1))^2\*m+2\*I\*Pi\*csgn(I/(exp(2\*b\*x+2\*a)+1))^3\*m-I\*Pi\*csgn(I\*exp(2\*b\*x+2\*a))\*csgn(I\*exp(2\*b\*x+2\*a)/(exp(2\*b\*x+2\*a)+1))^2\*m+2\*I\*Pi\*csgn(I\*exp(2\*b\*x+2\*a))^3+I\*Pi\*csgn(I\*exp(2\*b\*x+2\*a)/(exp(2\*b\*x+2\*a)+1))^3\*m+2\*I\*Pi\*csgn(I\*exp(b\*x+a))^2\*csgn(I\*exp(2\*b\*x+2\*a))-4\*I\*Pi\*csgn(I/(exp(2\*b\*x+2\*a)+1))^2-2\*I\*Pi\*csgn(I/(exp(2\*b\*x+2\*a)+1))\*csgn(I\*exp(2\*b\*x+2\*a)/(exp(2\*b\*x+2\*a)+1))^2+2\*I\*Pi\*csgn(I/(exp(2\*b\*x+2\*a)+1))\*csgn(I\*exp(2\*b\*x+2\*a))\*csgn(I\*exp(2\*b\*x+2\*a)/(exp(2\*b\*x+2\*a)+1))+4\*I\*Pi\*csgn(I/(exp(2\*b\*x+2\*a)+1))^3-2\*I\*Pi\*csgn(I\*exp(2\*b\*x+2\*a))\*csgn(I\*exp(2\*b\*x+2\*a)/(exp(2\*b\*x+2\*a)+1))^2+I\*Pi\*csgn(I/(exp(2\*b\*x+2\*a)+1))\*csgn(I\*exp(2\*b\*x+2\*a))\*csgn(I\*exp(2\*b

$$*x+2*a)/(exp(2*b*x+2*a)+1))*m)/(1+m)/(2+m)*x^m$$


---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*arccoth(tanh(b\*x+a)),x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 1.63888, size = 72, normalized size = 1.95

$$\frac{((bm + b)x^2 + (am + 2a)x)x^m}{m^2 + 3m + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*arccoth(tanh(b\*x+a)),x, algorithm="fricas")

[Out] ((b\*m + b)\*x^2 + (a\*m + 2\*a)\*x)\*x^m/(m^2 + 3\*m + 2)

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*acoth(tanh(b\*x+a)),x)

[Out] Exception raised: TypeError

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{arccoth}(\tanh(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^m*arccoth(tanh(b*x + a)), x)
```

### 3.194 $\int x^2 \coth^{-1}(\coth(a + bx)) dx$

**Optimal.** Leaf size=23

$$\frac{1}{3}x^3 \coth^{-1}(\coth(a + bx)) - \frac{bx^4}{12}$$

[Out]  $-(b*x^4)/12 + (x^3*ArcCoth[Coth[a + b*x]])/3$

**Rubi [A]** time = 0.014254, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2168, 30}

$$\frac{1}{3}x^3 \coth^{-1}(\coth(a + bx)) - \frac{bx^4}{12}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*ArcCoth[Coth[a + b*x]],x]$

[Out]  $-(b*x^4)/12 + (x^3*ArcCoth[Coth[a + b*x]])/3$

#### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

#### Rule 30

```
Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

#### Rubi steps

$$\begin{aligned}\int x^2 \coth^{-1}(\coth(a + bx)) dx &= \frac{1}{3}x^3 \coth^{-1}(\coth(a + bx)) - \frac{1}{3}b \int x^3 dx \\ &= -\frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(\coth(a + bx))\end{aligned}$$

**Mathematica [A]** time = 0.0213134, size = 20, normalized size = 0.87

$$-\frac{1}{12}x^3 (bx - 4 \coth^{-1}(\coth(a + bx)))$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[Coth[a + b\*x]], x]

[Out] -(x^3\*(b\*x - 4\*ArcCoth[Coth[a + b\*x]]))/12

**Maple [B]** time = 0.092, size = 59, normalized size = 2.6

$$\frac{x^3 \operatorname{arccoth}(\coth(bx + a))}{3} + \frac{1}{3b^3} \left( -\frac{(bx + a)^4}{4} + (bx + a)^3 a - \frac{3a^2 (bx + a)^2}{2} + (bx + a) a^3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(coth(b\*x+a)), x)

[Out] 1/3\*x^3\*arccoth(coth(b\*x+a))+1/3/b^3\*(-1/4\*(b\*x+a)^4+(b\*x+a)^3\*a-3/2\*a^2\*(b\*x+a)^2+(b\*x+a)\*a^3)

**Maxima [A]** time = 0.945767, size = 18, normalized size = 0.78

$$\frac{1}{4}bx^4 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(coth(b\*x+a)), x, algorithm="maxima")

[Out]  $\frac{1}{4}bx^4 + \frac{1}{3}ax^3$

**Fricas [A]** time = 1.34393, size = 31, normalized size = 1.35

$$\frac{1}{4}x^4b + \frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccoth(coth(b*x+a)),x, algorithm="fricas")`

[Out]  $\frac{1}{4}x^4b + \frac{1}{3}x^3a$

**Sympy [A]** time = 91.8494, size = 39, normalized size = 1.7

$$\begin{cases} 0 & \text{for } a = \log(-e^{-bx}) \vee a = \log(e^{-bx}) \\ -\frac{bx^4}{12} + \frac{x^3 \operatorname{acoth}\left(\frac{1}{\tanh(a+bx)}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*acoth(coth(b*x+a)),x)`

[Out] `Piecewise((0, Eq(a, log(exp(-b*x))) | Eq(a, log(-exp(-b*x))))), (-b*x**4/12 + x**3*acoth(1/tanh(a + b*x))/3, True))`

**Giac [A]** time = 1.17349, size = 18, normalized size = 0.78

$$\frac{1}{4}bx^4 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccoth(coth(b*x+a)),x, algorithm="giac")`

[Out]  $\frac{1}{4}bx^4 + \frac{1}{3}ax^3$

### 3.195 $\int x \coth^{-1}(\coth(a + bx)) dx$

**Optimal.** Leaf size=23

$$\frac{1}{2}x^2 \coth^{-1}(\coth(a + bx)) - \frac{bx^3}{6}$$

[Out]  $-(b*x^3)/6 + (x^2*ArcCoth[Coth[a + b*x]])/2$

**Rubi [A]** time = 0.0077072, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {6242, 30}

$$\frac{1}{2}x^2 \coth^{-1}(\coth(a + bx)) - \frac{bx^3}{6}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x*ArcCoth[Coth[a + b*x]], x]$

[Out]  $-(b*x^3)/6 + (x^2*ArcCoth[Coth[a + b*x]])/2$

#### Rule 6242

$\text{Int}[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]]*(d_.)]*((e_.) + (f_.)*(x_))^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[(e + f*x)^{(m + 1)}*ArcCoth[c + d*Coth[a + b*x]]]/(f*(m + 1)), x] + \text{Dist}[b/(f*(m + 1)), \text{Int}[(e + f*x)^{(m + 1)}/(c - d - c*E^{(2*a + 2*b*x)})], x], x] /;$  FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d)^2, 1]

#### Rule 30

$\text{Int}[(x_)^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[x^{(m + 1)}/(m + 1), x] /;$  FreeQ[m, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int x \coth^{-1}(\coth(a + bx)) dx &= \frac{1}{2}x^2 \coth^{-1}(\coth(a + bx)) - \frac{1}{2}b \int x^2 dx \\ &= -\frac{bx^3}{6} + \frac{1}{2}x^2 \coth^{-1}(\coth(a + bx)) \end{aligned}$$

**Mathematica [A]** time = 0.015436, size = 20, normalized size = 0.87

$$-\frac{1}{6}x^2 (bx - 3 \coth^{-1}(\coth(a + bx)))$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[Coth[a + b\*x]],x]

[Out] -(x^2\*(b\*x - 3\*ArcCoth[Coth[a + b\*x]]))/6

**Maple [B]** time = 0.075, size = 48, normalized size = 2.1

$$\frac{x^2 \operatorname{arccoth}(\coth(bx + a))}{2} + \frac{1}{2b^2} \left( -\frac{(bx + a)^3}{3} + (bx + a)^2 a - a^2 (bx + a) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(coth(b\*x+a)),x)

[Out] 1/2\*x^2\*arccoth(coth(b\*x+a))+1/2/b^2\*(-1/3\*(b\*x+a)^3+(b\*x+a)^2\*a-a^2\*(b\*x+a))

**Maxima [A]** time = 0.933736, size = 18, normalized size = 0.78

$$\frac{1}{3}bx^3 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(coth(b\*x+a)),x, algorithm="maxima")

[Out] 1/3\*b\*x^3 + 1/2\*a\*x^2

**Fricas [A]** time = 1.26847, size = 31, normalized size = 1.35

$$\frac{1}{3}x^3b + \frac{1}{2}x^2a$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(coth(b\*x+a)),x, algorithm="fricas")

[Out]  $1/3*x^3*b + 1/2*x^2*a$

**Sympy [A]** time = 37.9831, size = 60, normalized size = 2.61

$$\begin{cases} 0 & \text{for } a = \log(-e^{-bx}) \vee a = \log(e^{-bx}) \\ \frac{x^2 \operatorname{acoth}(\operatorname{coth}(a))}{2b} & \text{for } b = 0 \\ \frac{x \operatorname{acoth}^2\left(\frac{1}{\tanh(a+bx)}\right)}{2b} - \frac{\operatorname{acoth}^3\left(\frac{1}{\tanh(a+bx)}\right)}{6b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(coth(b\*x+a)),x)

[Out] Piecewise((0, Eq(a, log(exp(-b\*x))) | Eq(a, log(-exp(-b\*x))))), (x\*\*2\*acoth(coth(a))/2, Eq(b, 0)), (x\*acoth(1/tanh(a + b\*x))\*\*2/(2\*b) - acoth(1/tanh(a + b\*x))\*\*3/(6\*b\*\*2), True))

**Giac [A]** time = 1.13859, size = 18, normalized size = 0.78

$$\frac{1}{3}bx^3 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(coth(b\*x+a)),x, algorithm="giac")

[Out]  $1/3*b*x^3 + 1/2*a*x^2$

### 3.196 $\int \coth^{-1}(\coth(a + bx)) dx$

**Optimal.** Leaf size=16

$$\frac{\coth^{-1}(\coth(a + bx))^2}{2b}$$

[Out] ArcCoth[Coth[a + b\*x]]^2/(2\*b)

**Rubi [A]** time = 0.0031558, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2157, 30}

$$\frac{\coth^{-1}(\coth(a + bx))^2}{2b}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Coth[a + b\*x]], x]

[Out] ArcCoth[Coth[a + b\*x]]^2/(2\*b)

#### Rule 2157

Int[(u\_)^(m\_), x\_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[  
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

#### Rule 30

Int[(x\_)^(m\_), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \coth^{-1}(\coth(a + bx)) dx &= \frac{\text{Subst}\left(\int x dx, x, \coth^{-1}(\coth(a + bx))\right)}{b} \\ &= \frac{\coth^{-1}(\coth(a + bx))^2}{2b} \end{aligned}$$

**Mathematica [A]** time = 0.0059037, size = 18, normalized size = 1.12

$$x \coth^{-1}(\coth(a + bx)) - \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Coth[a + b\*x]],x]

[Out] -(b\*x^2)/2 + x\*ArcCoth[Coth[a + b\*x]]

**Maple [B]** time = 0.064, size = 32, normalized size = 2.

$$\frac{1}{b} \left( \operatorname{Artanh}(\coth(bx + a)) \operatorname{arccoth}(\coth(bx + a)) - \frac{(\operatorname{Artanh}(\coth(bx + a)))^2}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(coth(b\*x+a)),x)

[Out] 1/b\*(arctanh(coth(b\*x+a))\*arccoth(coth(b\*x+a))-1/2\*arctanh(coth(b\*x+a))^2)

**Maxima [A]** time = 0.934248, size = 14, normalized size = 0.88

$$\frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(coth(b\*x+a)),x, algorithm="maxima")

[Out] 1/2\*b\*x^2 + a\*x

**Fricas [A]** time = 1.28358, size = 23, normalized size = 1.44

$$\frac{1}{2}x^2b + xa$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(coth(b\*x+a)),x, algorithm="fricas")

[Out] 1/2\*x^2\*b + x\*a

**Sympy [A]** time = 3.98976, size = 36, normalized size = 2.25

$$\begin{cases} 0 & \text{for } a = \log(-e^{-bx}) \vee a = \log(e^{-bx}) \\ -\frac{bx^2}{2} + x \operatorname{acoth}\left(\frac{1}{\tanh(a+bx)}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(coth(b\*x+a)),x)

[Out] Piecewise((0, Eq(a, log(exp(-b\*x))) | Eq(a, log(-exp(-b\*x))))), (-b\*x\*\*2/2 + x\*acoth(1/tanh(a + b\*x)), True))

**Giac [A]** time = 1.12394, size = 14, normalized size = 0.88

$$\frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(coth(b\*x+a)),x, algorithm="giac")

[Out] 1/2\*b\*x^2 + a\*x

$$3.197 \quad \int \frac{\coth^{-1}(\coth(a+bx))}{x} dx$$

Optimal. Leaf size=21

$$bx - \log(x) \left( bx - \coth^{-1}(\coth(a + bx)) \right)$$

[Out] b\*x - (b\*x - ArcCoth[Coth[a + b\*x]])\*Log[x]

**Rubi [A]** time = 0.0309233, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2158, 29}

$$bx - \log(x) \left( bx - \coth^{-1}(\coth(a + bx)) \right)$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Coth[a + b\*x]]/x,x]

[Out] b\*x - (b\*x - ArcCoth[Coth[a + b\*x]])\*Log[x]

#### Rule 2158

Int[(v\_)/(u\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(b\*x)/a, x] - Dist[(b\*u - a\*v)/a, Int[1/u, x], x] /; NeQ[b\*u - a\*v, 0]] /; PiecewiseLinearQ[u, v, x]

#### Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

#### Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(\coth(a + bx))}{x} dx &= bx - \left( bx - \coth^{-1}(\coth(a + bx)) \right) \int \frac{1}{x} dx \\ &= bx - \left( bx - \coth^{-1}(\coth(a + bx)) \right) \log(x) \end{aligned}$$

**Mathematica [A]** time = 0.0166107, size = 19, normalized size = 0.9

$$\log(x) \left( \coth^{-1}(\coth(a + bx)) - bx \right) + bx$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Coth[a + b\*x]]/x,x]

[Out] b\*x + (-(b\*x) + ArcCoth[Coth[a + b\*x]])\*Log[x]

**Maple [A]** time = 0.073, size = 27, normalized size = 1.3

$$bx + a \ln(x) + \ln(x) (\operatorname{arccoth}(\operatorname{coth}(bx + a)) - bx - a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(coth(b\*x+a))/x,x)

[Out] b\*x+a\*ln(x)+ln(x)\*(arccoth(coth(b\*x+a))-b\*x-a)

**Maxima [A]** time = 0.944969, size = 11, normalized size = 0.52

$$bx + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(coth(b\*x+a))/x,x, algorithm="maxima")

[Out] b\*x + a\*log(x)

**Fricas [A]** time = 1.48964, size = 22, normalized size = 1.05

$$bx + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(coth(b\*x+a))/x,x, algorithm="fricas")

[Out] b\*x + a\*log(x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}(\operatorname{coth}(a + bx))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(coth(b\*x+a))/x,x)

[Out] Integral(acoth(coth(a + b\*x))/x, x)

---

**Giac [A]** time = 1.13719, size = 12, normalized size = 0.57

$$bx + a \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(coth(b\*x+a))/x,x, algorithm="giac")

[Out] b\*x + a\*log(abs(x))

$$3.198 \quad \int \frac{\coth^{-1}(\coth(ax+bx))}{x^2} dx$$

**Optimal.** Leaf size=17

$$b \log(x) - \frac{\coth^{-1}(\coth(a+bx))}{x}$$

[Out] -(ArcCoth[Coth[a + b\*x]]/x) + b\*Log[x]

**Rubi [A]** time = 0.008717, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2168, 29}

$$b \log(x) - \frac{\coth^{-1}(\coth(a+bx))}{x}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Coth[a + b\*x]]/x^2,x]

[Out] -(ArcCoth[Coth[a + b\*x]]/x) + b\*Log[x]

### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

### Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

### Rubi steps



$$\int \frac{\coth^{-1}(\coth(a + bx))}{x^2} dx = -\frac{\coth^{-1}(\coth(a + bx))}{x} + b \int \frac{1}{x} dx$$

$$= -\frac{\coth^{-1}(\coth(a + bx))}{x} + b \log(x)$$

**Mathematica [A]** time = 0.0160234, size = 18, normalized size = 1.06

$$-\frac{\coth^{-1}(\coth(a + bx))}{x} + b \log(x) + b$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Coth[a + b\*x]]/x^2,x]

[Out] b - ArcCoth[Coth[a + b\*x]]/x + b\*Log[x]

**Maple [A]** time = 0.074, size = 20, normalized size = 1.2

$$-\frac{\operatorname{arccoth}(\coth(bx + a))}{x} + b \ln(bx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(coth(b\*x+a))/x^2,x)

[Out] -arccoth(coth(b\*x+a))/x+b\*ln(b\*x)

**Maxima [A]** time = 0.941743, size = 15, normalized size = 0.88

$$b \log(x) - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(coth(b\*x+a))/x^2,x, algorithm="maxima")

[Out]  $b \cdot \log(x) - a/x$

**Fricas [A]** time = 1.61768, size = 27, normalized size = 1.59

$$\frac{bx \log(x) - a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(coth(b*x+a))/x^2,x, algorithm="fricas")`

[Out]  $(b \cdot x \cdot \log(x) - a)/x$

**Sympy [A]** time = 12.1868, size = 34, normalized size = 2.

$$\begin{cases} 0 & \text{for } a = \log(-e^{-bx}) \vee a = \log(e^{-bx}) \\ b \log(x) - \frac{\operatorname{acoth}\left(\frac{1}{\tanh(a+bx)}\right)}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(coth(b*x+a))/x**2,x)`

[Out] `Piecewise((0, Eq(a, log(exp(-b*x))) | Eq(a, log(-exp(-b*x))))), (b*log(x) - acoth(1/tanh(a + b*x))/x, True))`

**Giac [A]** time = 1.16178, size = 16, normalized size = 0.94

$$b \log(|x|) - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(coth(b*x+a))/x^2,x, algorithm="giac")`

[Out]  $b \cdot \log(\operatorname{abs}(x)) - a/x$

$$3.199 \quad \int \frac{\coth^{-1}(\coth(a+bx))}{x^3} dx$$

**Optimal.** Leaf size=23

$$-\frac{\coth^{-1}(\coth(a+bx))}{2x^2} - \frac{b}{2x}$$

[Out]  $-b/(2*x) - \text{ArcCoth}[\text{Coth}[a + b*x]]/(2*x^2)$

**Rubi [A]** time = 0.009241, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2168, 30}

$$-\frac{\coth^{-1}(\coth(a+bx))}{2x^2} - \frac{b}{2x}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{ArcCoth}[\text{Coth}[a + b*x]]/x^3, x]$

[Out]  $-b/(2*x) - \text{ArcCoth}[\text{Coth}[a + b*x]]/(2*x^2)$

### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

### Rule 30

```
Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

### Rubi steps

$$\int \frac{\coth^{-1}(\coth(a + bx))}{x^3} dx = -\frac{\coth^{-1}(\coth(a + bx))}{2x^2} + \frac{1}{2}b \int \frac{1}{x^2} dx$$

$$= -\frac{b}{2x} - \frac{\coth^{-1}(\coth(a + bx))}{2x^2}$$

**Mathematica [A]** time = 0.0141821, size = 18, normalized size = 0.78

$$-\frac{\coth^{-1}(\coth(a + bx)) + bx}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Coth[a + b\*x]]/x^3,x]

[Out] -(b\*x + ArcCoth[Coth[a + b\*x]])/(2\*x^2)

**Maple [A]** time = 0.076, size = 20, normalized size = 0.9

$$-\frac{b}{2x} - \frac{\operatorname{arccoth}(\coth(bx + a))}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(coth(b\*x+a))/x^3,x)

[Out] -1/2\*b/x-1/2\*arccoth(coth(b\*x+a))/x^2

**Maxima [A]** time = 0.944455, size = 15, normalized size = 0.65

$$-\frac{2bx + a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(coth(b\*x+a))/x^3,x, algorithm="maxima")

[Out]  $-1/2*(2*b*x + a)/x^2$

**Fricas [A]** time = 1.47088, size = 30, normalized size = 1.3

$$-\frac{2bx + a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(coth(b*x+a))/x^3,x, algorithm="fricas")`

[Out]  $-1/2*(2*b*x + a)/x^2$

**Sympy [A]** time = 77.2892, size = 39, normalized size = 1.7

$$\begin{cases} 0 & \text{for } a = \log(-e^{-bx}) \vee a = \log(e^{-bx}) \\ -\frac{b}{2x} - \frac{\operatorname{acoth}\left(\frac{1}{\tanh(a+bx)}\right)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(coth(b*x+a))/x**3,x)`

[Out] `Piecewise((0, Eq(a, log(exp(-b*x))) | Eq(a, log(-exp(-b*x))))), (-b/(2*x) - acoth(1/tanh(a + b*x))/(2*x**2), True))`

**Giac [A]** time = 1.1404, size = 15, normalized size = 0.65

$$-\frac{2bx + a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(coth(b*x+a))/x^3,x, algorithm="giac")`

[Out]  $-1/2*(2*b*x + a)/x^2$

### 3.200 $\int \coth^{-1}(\cosh(x)) dx$

**Optimal.** Leaf size=27

$$-\text{PolyLog}(2, -e^x) + \text{PolyLog}(2, e^x) - 2x \tanh^{-1}(e^x) + x \coth^{-1}(\cosh(x))$$

[Out] x\*ArcCoth[Cosh[x]] - 2\*x\*ArcTanh[E^x] - PolyLog[2, -E^x] + PolyLog[2, E^x]

**Rubi [A]** time = 0.0342236, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$ , Rules used = {6272, 4182, 2279, 2391}

$$-\text{PolyLog}(2, -e^x) + \text{PolyLog}(2, e^x) - 2x \tanh^{-1}(e^x) + x \coth^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Cosh[x]],x]

[Out] x\*ArcCoth[Cosh[x]] - 2\*x\*ArcTanh[E^x] - PolyLog[2, -E^x] + PolyLog[2, E^x]

#### Rule 6272

Int[ArcCoth[u\_], x\_Symbol] := Simp[x\*ArcCoth[u], x] - Int[SimplifyIntegrand[(x\*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]

#### Rule 4182

Int[csc[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)])/(f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

#### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2391

```
Int[Log[(c_.)*(d_) + (e_.)*(x_)^(n_.)]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned} \int \coth^{-1}(\cosh(x)) dx &= x \coth^{-1}(\cosh(x)) + \int x \operatorname{csch}(x) dx \\ &= x \coth^{-1}(\cosh(x)) - 2x \tanh^{-1}(e^x) - \int \log(1 - e^x) dx + \int \log(1 + e^x) dx \\ &= x \coth^{-1}(\cosh(x)) - 2x \tanh^{-1}(e^x) - \operatorname{Subst}\left(\int \frac{\log(1-x)}{x} dx, x, e^x\right) + \operatorname{Subst}\left(\int \frac{\log(1+x)}{x} dx, x, e^x\right) \\ &= x \coth^{-1}(\cosh(x)) - 2x \tanh^{-1}(e^x) - \operatorname{Li}_2(-e^x) + \operatorname{Li}_2(e^x) \end{aligned}$$

**Mathematica [A]** time = 0.0196667, size = 47, normalized size = 1.74

$$\operatorname{PolyLog}(2, -e^{-x}) - \operatorname{PolyLog}(2, e^{-x}) + x(\log(1 - e^{-x}) - \log(e^{-x} + 1)) + x \coth^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Cosh[x]], x]
```

```
[Out] x*ArcCoth[Cosh[x]] + x*(Log[1 - E^(-x)] - Log[1 + E^(-x)]) + PolyLog[2, -E^(-x)] - PolyLog[2, E^(-x)]
```

**Maple [A]** time = 0.052, size = 21, normalized size = 0.8

$$x \operatorname{arccoth}(\cosh(x)) + 2 \operatorname{dilog}(e^{-x}) - \frac{\operatorname{dilog}(e^{-2x})}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(cosh(x)), x)
```

```
[Out] x*arccoth(cosh(x))+2*dilog(exp(-x))-1/2*dilog(exp(-2*x))
```

**Maxima [A]** time = 1.15314, size = 45, normalized size = 1.67

$$x \operatorname{arccoth}(\cosh(x)) - x \log(e^x + 1) + x \log(-e^x + 1) - \operatorname{Li}_2(-e^x) + \operatorname{Li}_2(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(cosh(x)),x, algorithm="maxima")

[Out]  $x \operatorname{arccoth}(\cosh(x)) - x \log(e^x + 1) + x \log(-e^x + 1) - \operatorname{dilog}(-e^x) + \operatorname{dilog}(e^x)$

**Fricas [B]** time = 1.66712, size = 213, normalized size = 7.89

$\frac{1}{2} x \log\left(\frac{\cosh(x) + 1}{\cosh(x) - 1}\right) - x \log(\cosh(x) + \sinh(x) + 1) + x \log(-\cosh(x) - \sinh(x) + 1) + \operatorname{Li}_2(\cosh(x) + \sinh(x)) - \operatorname{Li}_2(-\cosh(x) - \sinh(x))$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(cosh(x)),x, algorithm="fricas")

[Out]  $\frac{1}{2} x \log\left(\frac{\cosh(x) + 1}{\cosh(x) - 1}\right) - x \log(\cosh(x) + \sinh(x) + 1) + x \log(-\cosh(x) - \sinh(x) + 1) + \operatorname{dilog}(\cosh(x) + \sinh(x)) - \operatorname{dilog}(-\cosh(x) - \sinh(x))$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acoth}(\cosh(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(cosh(x)),x)

[Out] Integral(acoth(cosh(x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(\cosh(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(arccoth(cosh(x)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(cosh(x)), x)
```

### 3.201 $\int x \coth^{-1}(\cosh(x)) dx$

**Optimal.** Leaf size=51

$$-x\text{PolyLog}(2, -e^x) + x\text{PolyLog}(2, e^x) + \text{PolyLog}(3, -e^x) - \text{PolyLog}(3, e^x) - x^2 \tanh^{-1}(e^x) + \frac{1}{2}x^2 \coth^{-1}(\cosh(x))$$

[Out] (x^2\*ArcCoth[Cosh[x]])/2 - x^2\*ArcTanh[E^x] - x\*PolyLog[2, -E^x] + x\*PolyLog[2, E^x] + PolyLog[3, -E^x] - PolyLog[3, E^x]

**Rubi [A]** time = 0.0664171, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 5,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.$ , Rules used = {6274, 4182, 2531, 2282, 6589}

$$-x\text{PolyLog}(2, -e^x) + x\text{PolyLog}(2, e^x) + \text{PolyLog}(3, -e^x) - \text{PolyLog}(3, e^x) - x^2 \tanh^{-1}(e^x) + \frac{1}{2}x^2 \coth^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

[In] Int[x\*ArcCoth[Cosh[x]],x]

[Out] (x^2\*ArcCoth[Cosh[x]])/2 - x^2\*ArcTanh[E^x] - x\*PolyLog[2, -E^x] + x\*PolyLog[2, E^x] + PolyLog[3, -E^x] - PolyLog[3, E^x]

#### Rule 6274

```
Int[((a_.) + ArcCoth[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[
((c + d*x)^(m + 1)*(a + b*ArcCoth[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)),
Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x]
]; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&& !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m + 1, x]]
```

#### Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[
(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)]/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I),
Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)], x], x] + Dist[(d*m)/(f*fz*I),
Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)], x], x)) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

#### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int x \coth^{-1}(\cosh(x)) dx &= \frac{1}{2}x^2 \coth^{-1}(\cosh(x)) + \frac{1}{2} \int x^2 \operatorname{csch}(x) dx \\
&= \frac{1}{2}x^2 \coth^{-1}(\cosh(x)) - x^2 \tanh^{-1}(e^x) - \int x \log(1 - e^x) dx + \int x \log(1 + e^x) dx \\
&= \frac{1}{2}x^2 \coth^{-1}(\cosh(x)) - x^2 \tanh^{-1}(e^x) - x \operatorname{Li}_2(-e^x) + x \operatorname{Li}_2(e^x) + \int \operatorname{Li}_2(-e^x) dx - \int \operatorname{Li}_2(e^x) dx \\
&= \frac{1}{2}x^2 \coth^{-1}(\cosh(x)) - x^2 \tanh^{-1}(e^x) - x \operatorname{Li}_2(-e^x) + x \operatorname{Li}_2(e^x) + \operatorname{Subst}\left(\int \frac{\operatorname{Li}_2(-x)}{x} dx, x, e^x\right) \\
&= \frac{1}{2}x^2 \coth^{-1}(\cosh(x)) - x^2 \tanh^{-1}(e^x) - x \operatorname{Li}_2(-e^x) + x \operatorname{Li}_2(e^x) + \operatorname{Li}_3(-e^x) - \operatorname{Li}_3(e^x)
\end{aligned}$$

**Mathematica [A]** time = 0.0135662, size = 81, normalized size = 1.59

$$\frac{1}{2} (2x \operatorname{PolyLog}(2, -e^{-x}) - 2x \operatorname{PolyLog}(2, e^{-x}) + 2 \operatorname{PolyLog}(3, -e^{-x}) - 2 \operatorname{PolyLog}(3, e^{-x}) + x^2 \log(1 - e^{-x}) - x^2 \log(e^{-x}))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[Cosh[x]], x]
```

[Out]  $(x^2 \operatorname{ArcCoth}[\operatorname{Cosh}[x]] + x^2 \operatorname{Log}[1 - E^{-x}] - x^2 \operatorname{Log}[1 + E^{-x}] + 2x \operatorname{PolyLog}[2, -E^{-x}] - 2x \operatorname{PolyLog}[2, E^{-x}] + 2 \operatorname{PolyLog}[3, -E^{-x}] - 2 \operatorname{PolyLog}[3, E^{-x}])/2$

**Maple [C]** time = 0.154, size = 449, normalized size = 8.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{int}(x \operatorname{arccoth}(\operatorname{cosh}(x)), x)$

[Out]  $x \operatorname{polylog}(2, \exp(x)) - x \operatorname{polylog}(2, -\exp(x)) + 1/8 I \pi \operatorname{csgn}(I(\exp(x)-1)^2)^3 x^2 - 1/8 I \pi \operatorname{csgn}(I \exp(-x)) \operatorname{csgn}(I \exp(-x)(\exp(x)-1)^2)^2 x^2 + 1/8 I \pi \operatorname{csgn}(I(\exp(x)-1)^2) \operatorname{csgn}(I \exp(-x)) \operatorname{csgn}(I \exp(-x)(\exp(x)-1)^2) x^2 - 1/8 I \pi \operatorname{csgn}(I(\exp(x)-1)^2) \operatorname{csgn}(I \exp(-x)(\exp(x)-1)^2)^2 x^2 - 1/8 I \pi \operatorname{csgn}(I(\exp(x)+1)^2) \operatorname{csgn}(I \exp(-x)) \operatorname{csgn}(I \exp(-x)(\exp(x)+1)^2) x^2 + 1/8 I \pi \operatorname{csgn}(I \exp(-x)(\exp(x)-1)^2)^3 x^2 + 1/8 I \pi \operatorname{csgn}(I(\exp(x)+1)^2) \operatorname{csgn}(I \exp(-x)(\exp(x)+1)^2)^2 x^2 + 1/4 I \pi \operatorname{csgn}(I(\exp(x)+1)) \operatorname{csgn}(I(\exp(x)+1)^2)^2 x^2 - 1/8 I \pi \operatorname{csgn}(I \exp(-x)(\exp(x)+1)^2)^3 x^2 + 1/8 I \pi \operatorname{csgn}(I \exp(-x)) \operatorname{csgn}(I \exp(-x)(\exp(x)+1)^2)^2 x^2 + \operatorname{polylog}(3, -\exp(x)) - \operatorname{polylog}(3, \exp(x)) - 1/8 I \pi \operatorname{csgn}(I(\exp(x)+1)^2)^3 x^2 + 1/2 x^2 \ln(1 - \exp(x)) - 1/2 x^2 \ln(\exp(x) - 1) - 1/4 I \pi \operatorname{csgn}(I(\exp(x)-1)) \operatorname{csgn}(I(\exp(x)-1)^2)^2 x^2 + 1/8 I \pi \operatorname{csgn}(I(\exp(x)-1))^2 \operatorname{csgn}(I(\exp(x)-1)^2) x^2$

**Maxima [A]** time = 1.18363, size = 76, normalized size = 1.49

$\frac{1}{2} x^2 \operatorname{arccoth}(\operatorname{cosh}(x)) - \frac{1}{2} x^2 \log(e^x + 1) + \frac{1}{2} x^2 \log(-e^x + 1) - x \operatorname{Li}_2(-e^x) + x \operatorname{Li}_2(e^x) + \operatorname{Li}_3(-e^x) - \operatorname{Li}_3(e^x)$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}(x \operatorname{arccoth}(\operatorname{cosh}(x)), x, \operatorname{algorithm}="maxima")$

[Out]  $1/2 x^2 \operatorname{arccoth}(\operatorname{cosh}(x)) - 1/2 x^2 \log(e^x + 1) + 1/2 x^2 \log(-e^x + 1) - x \operatorname{dilog}(-e^x) + x \operatorname{dilog}(e^x) + \operatorname{polylog}(3, -e^x) - \operatorname{polylog}(3, e^x)$

**Fricas [C]** time = 1.61809, size = 325, normalized size = 6.37

$$\frac{1}{4}x^2 \log\left(\frac{\cosh(x)+1}{\cosh(x)-1}\right) - \frac{1}{2}x^2 \log(\cosh(x)+\sinh(x)+1) + \frac{1}{2}x^2 \log(-\cosh(x)-\sinh(x)+1) + x\text{Li}_2(\cosh(x)+\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(cosh(x)),x, algorithm="fricas")

[Out] 1/4\*x^2\*log((cosh(x) + 1)/(cosh(x) - 1)) - 1/2\*x^2\*log(cosh(x) + sinh(x) + 1) + 1/2\*x^2\*log(-cosh(x) - sinh(x) + 1) + x\*dilog(cosh(x) + sinh(x)) - x\*dilog(-cosh(x) - sinh(x)) - polylog(3, cosh(x) + sinh(x)) + polylog(3, -cosh(x) - sinh(x))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{acoth}(\cosh(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(cosh(x)),x)

[Out] Integral(x\*acoth(cosh(x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arccoth}(\cosh(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(cosh(x)),x, algorithm="giac")

[Out] integrate(x\*arccoth(cosh(x)), x)

### 3.202 $\int x^2 \coth^{-1}(\cosh(x)) dx$

**Optimal.** Leaf size=77

$-x^2 \text{PolyLog}(2, -e^x) + x^2 \text{PolyLog}(2, e^x) + 2x \text{PolyLog}(3, -e^x) - 2x \text{PolyLog}(3, e^x) - 2 \text{PolyLog}(4, -e^x) + 2 \text{PolyLog}(4, e^x)$

[Out]  $(x^3 \text{ArcCoth}[\text{Cosh}[x]])/3 - (2x^3 \text{ArcTanh}[E^x])/3 - x^2 \text{PolyLog}[2, -E^x] + x^2 \text{PolyLog}[2, E^x] + 2x \text{PolyLog}[3, -E^x] - 2x \text{PolyLog}[3, E^x] - 2 \text{PolyLog}[4, -E^x] + 2 \text{PolyLog}[4, E^x]$

**Rubi [A]** time = 0.0900436, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$ , Rules used = {6274, 4182, 2531, 6609, 2282, 6589}

$-x^2 \text{PolyLog}(2, -e^x) + x^2 \text{PolyLog}(2, e^x) + 2x \text{PolyLog}(3, -e^x) - 2x \text{PolyLog}(3, e^x) - 2 \text{PolyLog}(4, -e^x) + 2 \text{PolyLog}(4, e^x)$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2 \text{ArcCoth}[\text{Cosh}[x]], x]$

[Out]  $(x^3 \text{ArcCoth}[\text{Cosh}[x]])/3 - (2x^3 \text{ArcTanh}[E^x])/3 - x^2 \text{PolyLog}[2, -E^x] + x^2 \text{PolyLog}[2, E^x] + 2x \text{PolyLog}[3, -E^x] - 2x \text{PolyLog}[3, E^x] - 2 \text{PolyLog}[4, -E^x] + 2 \text{PolyLog}[4, E^x]$

#### Rule 6274

$\text{Int}[(a + \text{ArcCoth}[u] \cdot b) \cdot (c + d \cdot x)^m, x\_Symbol] \rightarrow \text{Simp}[(c + d \cdot x)^{m+1} \cdot (a + b \cdot \text{ArcCoth}[u]) / (d \cdot (m+1)), x] - \text{Dist}[b / (d \cdot (m+1)), \text{Int}[\text{SimplifyIntegrand}[(c + d \cdot x)^{m+1} \cdot D[u, x]] / (1 - u^2), x], x] /;$  FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] && !FunctionOfQ[(c + d \cdot x)^{m+1}, u, x] && FalseQ[PowerVariableExpn[u, m + 1, x]]

#### Rule 4182

$\text{Int}[\text{csc}[e + (\text{Complex}[0, fz]) \cdot (f \cdot x)] \cdot (c + d \cdot x)^m, x\_Symbol] \rightarrow \text{Simp}[(-2 \cdot (c + d \cdot x)^m \cdot \text{ArcTanh}[E^{-(I \cdot e) + f \cdot fz \cdot x}]) / (f \cdot fz \cdot I), x] + (-\text{Dist}[(d \cdot m) / (f \cdot fz \cdot I), \text{Int}[(c + d \cdot x)^{m-1} \cdot \text{Log}[1 - E^{-(I \cdot e) + f \cdot fz \cdot x}], x], x] + \text{Dist}[(d \cdot m) / (f \cdot fz \cdot I), \text{Int}[(c + d \cdot x)^{m-1} \cdot \text{Log}[1 + E^{-(I \cdot e) + f \cdot fz \cdot x}], x], x]) /;$  FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*(f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(\cosh(x)) dx &= \frac{1}{3}x^3 \coth^{-1}(\cosh(x)) + \frac{1}{3} \int x^3 \operatorname{csch}(x) dx \\
&= \frac{1}{3}x^3 \coth^{-1}(\cosh(x)) - \frac{2}{3}x^3 \tanh^{-1}(e^x) - \int x^2 \log(1 - e^x) dx + \int x^2 \log(1 + e^x) dx \\
&= \frac{1}{3}x^3 \coth^{-1}(\cosh(x)) - \frac{2}{3}x^3 \tanh^{-1}(e^x) - x^2 \operatorname{Li}_2(-e^x) + x^2 \operatorname{Li}_2(e^x) + 2 \int x \operatorname{Li}_2(-e^x) dx - 2 \int x \operatorname{Li}_2(e^x) dx \\
&= \frac{1}{3}x^3 \coth^{-1}(\cosh(x)) - \frac{2}{3}x^3 \tanh^{-1}(e^x) - x^2 \operatorname{Li}_2(-e^x) + x^2 \operatorname{Li}_2(e^x) + 2x \operatorname{Li}_3(-e^x) - 2x \operatorname{Li}_3(e^x) \\
&= \frac{1}{3}x^3 \coth^{-1}(\cosh(x)) - \frac{2}{3}x^3 \tanh^{-1}(e^x) - x^2 \operatorname{Li}_2(-e^x) + x^2 \operatorname{Li}_2(e^x) + 2x \operatorname{Li}_3(-e^x) - 2x \operatorname{Li}_3(e^x) \\
&= \frac{1}{3}x^3 \coth^{-1}(\cosh(x)) - \frac{2}{3}x^3 \tanh^{-1}(e^x) - x^2 \operatorname{Li}_2(-e^x) + x^2 \operatorname{Li}_2(e^x) + 2x \operatorname{Li}_3(-e^x) - 2x \operatorname{Li}_3(e^x)
\end{aligned}$$

**Mathematica [A]** time = 0.0286605, size = 109, normalized size = 1.42

$$\frac{1}{24} (24x^2 \operatorname{PolyLog}(2, -e^{-x}) + 24x^2 \operatorname{PolyLog}(2, e^x) + 48x \operatorname{PolyLog}(3, -e^{-x}) - 48x \operatorname{PolyLog}(3, e^x) + 48 \operatorname{PolyLog}(4, -e^{-x}) - 48 \operatorname{PolyLog}(4, e^x))$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[Cosh[x]],x]

[Out] (Pi^4 - 2\*x^4 + 8\*x^3\*ArcCoth[Cosh[x]] - 8\*x^3\*Log[1 + E^(-x)] + 8\*x^3\*Log[1 - E^x] + 24\*x^2\*PolyLog[2, -E^(-x)] + 24\*x^2\*PolyLog[2, E^x] + 48\*x\*PolyLog[3, -E^(-x)] - 48\*x\*PolyLog[3, E^x] + 48\*PolyLog[4, -E^(-x)] + 48\*PolyLog[4, E^x])/24

**Maple [C]** time = 0.141, size = 471, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(cosh(x)),x)

[Out] x^2\*polylog(2,exp(x))-x^2\*polylog(2,-exp(x))-2\*polylog(4,-exp(x))+2\*polylog(4,exp(x))-1/12\*I\*Pi\*csgn(I\*(exp(x)-1)^2)\*csgn(I\*exp(-x)\*(exp(x)-1)^2)\*x^3+1/12\*I\*Pi\*csgn(I\*(exp(x)-1)^2)\*csgn(I\*(exp(x)-1)^2)\*x^3-1/12\*I\*Pi\*csgn(I\*(exp(x)+1)^2)\*csgn(I\*(exp(x)+1)^2)\*x^3+1/6\*I\*Pi\*csgn(I\*(exp(x)+1))\*csgn(I\*(



```

exp(x)+1)^2)^2*x^3+1/12*I*Pi*csgn(I*(exp(x)-1)^2)*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-1)^2)*x^3+1/3*x^3*ln(1-exp(x))-1/3*x^3*ln(exp(x)-1)+2*x*polylog(3,-exp(x))-2*x*polylog(3,exp(x))+1/12*I*Pi*csgn(I*(exp(x)-1)^2)^3*x^3-1/12*I*Pi*csgn(I*exp(-x)*(exp(x)+1)^2)^3*x^3-1/6*I*Pi*csgn(I*(exp(x)-1))*csgn(I*(exp(x)-1)^2)^2*x^3+1/12*I*Pi*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)+1)^2)^2*x^3-1/12*I*Pi*csgn(I*(exp(x)+1)^2)^3*x^3-1/12*I*Pi*csgn(I*(exp(x)+1)^2)*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)+1)^2)*x^3+1/12*I*Pi*csgn(I*exp(-x)*(exp(x)-1)^2)^3*x^3+1/12*I*Pi*csgn(I*(exp(x)+1)^2)*csgn(I*exp(-x)*(exp(x)+1)^2)^2*x^3-1/12*I*Pi*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-1)^2)^2*x^3

```

**Maxima [A]** time = 1.17831, size = 105, normalized size = 1.36

$$\frac{1}{3} x^3 \operatorname{arccoth}(\cosh(x)) - \frac{1}{3} x^3 \log(e^x + 1) + \frac{1}{3} x^3 \log(-e^x + 1) - x^2 \operatorname{Li}_2(-e^x) + x^2 \operatorname{Li}_2(e^x) + 2x \operatorname{Li}_3(-e^x) - 2x \operatorname{Li}_3(e^x) - 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(cosh(x)) - 1/3*x^3*log(e^x + 1) + 1/3*x^3*log(-e^x + 1) - x^2*dilog(-e^x) + x^2*dilog(e^x) + 2*x*polylog(3, -e^x) - 2*x*polylog(3, e^x) - 2*polylog(4, -e^x) + 2*polylog(4, e^x)
```

**Fricas [C]** time = 1.77078, size = 435, normalized size = 5.65

$$\frac{1}{6} x^3 \log\left(\frac{\cosh(x) + 1}{\cosh(x) - 1}\right) - \frac{1}{3} x^3 \log(\cosh(x) + \sinh(x) + 1) + \frac{1}{3} x^3 \log(-\cosh(x) - \sinh(x) + 1) + x^2 \operatorname{Li}_2(\cosh(x) + \sinh(x)) - x^2 \operatorname{Li}_2(-\cosh(x) - \sinh(x)) + 2x \operatorname{Li}_3(\cosh(x) + \sinh(x)) - 2x \operatorname{Li}_3(-\cosh(x) - \sinh(x)) - 2 \operatorname{Li}_4(\cosh(x) + \sinh(x)) + 2 \operatorname{Li}_4(-\cosh(x) - \sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/6*x^3*log((cosh(x) + 1)/(cosh(x) - 1)) - 1/3*x^3*log(cosh(x) + sinh(x) + 1) + 1/3*x^3*log(-cosh(x) - sinh(x) + 1) + x^2*dilog(cosh(x) + sinh(x)) - x^2*dilog(-cosh(x) - sinh(x)) - 2*x*polylog(3, cosh(x) + sinh(x)) + 2*x*polylog(3, -cosh(x) - sinh(x)) + 2*polylog(4, cosh(x) + sinh(x)) - 2*polylog(4, -cosh(x) - sinh(x))
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{acoth}(\cosh(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*acoth(cosh(x)),x)

[Out] Integral(x\*\*2\*acoth(cosh(x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arccoth}(\cosh(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(cosh(x)),x, algorithm="giac")

[Out] integrate(x^2\*arccoth(cosh(x)), x)

### 3.203 $\int x^2 \coth^{-1}(c + d \tanh(a + bx)) dx$

**Optimal.** Leaf size=307

$$-\frac{x \operatorname{PolyLog}\left(3, -\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2} + \frac{x \operatorname{PolyLog}\left(3, -\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2} + \frac{\operatorname{PolyLog}\left(4, -\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^3} - \frac{\operatorname{PolyLog}\left(4, -\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^3}$$

[Out]  $(x^3 \operatorname{ArcCoth}[c + d \operatorname{Tanh}[a + b x]])/3 + (x^3 \operatorname{Log}[1 + ((1 - c - d)E^{(2a + 2bx)})/(1 - c + d)])/6 - (x^3 \operatorname{Log}[1 + ((1 + c + d)E^{(2a + 2bx)})/(1 + c - d)])/6 + (x^2 \operatorname{PolyLog}[2, -(((1 - c - d)E^{(2a + 2bx)})/(1 - c + d))])/(4b) - (x^2 \operatorname{PolyLog}[2, -(((1 + c + d)E^{(2a + 2bx)})/(1 + c - d))])/(4b) - (x \operatorname{PolyLog}[3, -(((1 - c - d)E^{(2a + 2bx)})/(1 - c + d))])/(4b^2) + (x \operatorname{PolyLog}[3, -(((1 + c + d)E^{(2a + 2bx)})/(1 + c - d))])/(4b^2) + \operatorname{PolyLog}[4, -(((1 - c - d)E^{(2a + 2bx)})/(1 - c + d))]/(8b^3) - \operatorname{PolyLog}[4, -(((1 + c + d)E^{(2a + 2bx)})/(1 + c - d))]/(8b^3)$

**Rubi [A]** time = 0.464891, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {6244, 2190, 2531, 6609, 2282, 6589}

$$-\frac{x \operatorname{PolyLog}\left(3, -\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2} + \frac{x \operatorname{PolyLog}\left(3, -\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2} + \frac{\operatorname{PolyLog}\left(4, -\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^3} - \frac{\operatorname{PolyLog}\left(4, -\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2 \operatorname{ArcCoth}[c + d \operatorname{Tanh}[a + b x]], x]$

[Out]  $(x^3 \operatorname{ArcCoth}[c + d \operatorname{Tanh}[a + b x]])/3 + (x^3 \operatorname{Log}[1 + ((1 - c - d)E^{(2a + 2bx)})/(1 - c + d)])/6 - (x^3 \operatorname{Log}[1 + ((1 + c + d)E^{(2a + 2bx)})/(1 + c - d)])/6 + (x^2 \operatorname{PolyLog}[2, -(((1 - c - d)E^{(2a + 2bx)})/(1 - c + d))])/(4b) - (x^2 \operatorname{PolyLog}[2, -(((1 + c + d)E^{(2a + 2bx)})/(1 + c - d))])/(4b) - (x \operatorname{PolyLog}[3, -(((1 - c - d)E^{(2a + 2bx)})/(1 - c + d))])/(4b^2) + (x \operatorname{PolyLog}[3, -(((1 + c + d)E^{(2a + 2bx)})/(1 + c - d))])/(4b^2) + \operatorname{PolyLog}[4, -(((1 - c - d)E^{(2a + 2bx)})/(1 - c + d))]/(8b^3) - \operatorname{PolyLog}[4, -(((1 + c + d)E^{(2a + 2bx)})/(1 + c - d))]/(8b^3)$

#### Rule 6244

$\operatorname{Int}[\operatorname{ArcCoth}[(c_.) + (d_.) \operatorname{Tanh}[(a_.) + (b_.)(x_.)]] * ((e_.) + (f_.)(x_.))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(e + f x)^{(m + 1)} \operatorname{ArcCoth}[c + d \operatorname{Tanh}[a + b x]] / (f$

```
(m + 1)), x] + (Dist[(b*(1 - c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*a + 2*b*x))/(1 - c + d + (1 - c - d)*E^(2*a + 2*b*x)), x], x] - Dist[(b*(1 + c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*a + 2*b*x))/(1 + c - d + (1 + c + d)*E^(2*a + 2*b*x)), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

### Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)*((c_) + (d_)*(x_)^(m_)))/((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*((f_) + (g_)*(x_)^(m_)), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

### Rule 6609

```
Int[((e_) + (f_)*(x_)^(m_))*PolyLog[n_, (d_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(p_))], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))* (F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_)^(p_))]/((d_) + (e_)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(c + d \tanh(a + bx)) dx &= \frac{1}{3}x^3 \coth^{-1}(c + d \tanh(a + bx)) + \frac{1}{3}(b(1 - c - d)) \int \frac{e^{2a+2bx}x^3}{1 - c + d + (1 - c - d)e^{2a+2bx}} \\
&= \frac{1}{3}x^3 \coth^{-1}(c + d \tanh(a + bx)) + \frac{1}{6}x^3 \log\left(1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{6}x^3 \log\left(1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) \\
&= \frac{1}{3}x^3 \coth^{-1}(c + d \tanh(a + bx)) + \frac{1}{6}x^3 \log\left(1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{6}x^3 \log\left(1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) \\
&= \frac{1}{3}x^3 \coth^{-1}(c + d \tanh(a + bx)) + \frac{1}{6}x^3 \log\left(1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{6}x^3 \log\left(1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) \\
&= \frac{1}{3}x^3 \coth^{-1}(c + d \tanh(a + bx)) + \frac{1}{6}x^3 \log\left(1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{6}x^3 \log\left(1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) \\
&= \frac{1}{3}x^3 \coth^{-1}(c + d \tanh(a + bx)) + \frac{1}{6}x^3 \log\left(1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{6}x^3 \log\left(1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.452205, size = 345, normalized size = 1.12

$$-6b^2x^2\text{PolyLog}\left(2, \frac{(c-d-1)(\sinh(2(a+bx))-\cosh(2(a+bx)))}{c+d-1}\right) + 6b^2x^2\text{PolyLog}\left(2, \frac{(c-d+1)(\sinh(2(a+bx))-\cosh(2(a+bx)))}{c+d+1}\right) - 6bx\text{PolyLog}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[c + d\*Tanh[a + b\*x]], x]

[Out] (x^3\*ArcCoth[c + d\*Tanh[a + b\*x]])/3 + (4\*b^3\*x^3\*Log[1 + ((-1 + c - d)\*(Cosh[2\*(a + b\*x)] - Sinh[2\*(a + b\*x)])]/(-1 + c + d)] - 4\*b^3\*x^3\*Log[1 + ((1 + c - d)\*(Cosh[2\*(a + b\*x)] - Sinh[2\*(a + b\*x)])]/(1 + c + d)] - 6\*b^2\*x^2\*PolyLog[2, ((-1 + c - d)\*(-Cosh[2\*(a + b\*x)] + Sinh[2\*(a + b\*x)])]/(-1 + c + d)] + 6\*b^2\*x^2\*PolyLog[2, ((1 + c - d)\*(-Cosh[2\*(a + b\*x)] + Sinh[2\*(a + b\*x)])]/(1 + c + d)] - 6\*b\*x\*PolyLog[3, ((-1 + c - d)\*(-Cosh[2\*(a + b\*x)] + Sinh[2\*(a + b\*x)])]/(-1 + c + d)] + 6\*b\*x\*PolyLog[3, ((1 + c - d)\*(-Cosh[2\*(a + b\*x)] + Sinh[2\*(a + b\*x)])]/(1 + c + d)] - 3\*PolyLog[4, ((-1 + c - d)\*(-Cosh[2\*(a + b\*x)] + Sinh[2\*(a + b\*x)])]/(-1 + c + d)] + 3\*PolyLog[4, ((1 + c - d)\*(-Cosh[2\*(a + b\*x)] + Sinh[2\*(a + b\*x)])]/(1 + c + d)]/(24\*b^3)

---

**Maple [C]** time = 5.085, size = 5294, normalized size = 17.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*arccoth(c+d*tanh(b*x+a)),x)`

[Out] result too large to display

---

**Maxima [A]** time = 2.28858, size = 379, normalized size = 1.23

$$\frac{1}{3}x^3 \operatorname{arccoth}(d \tanh(bx + a) + c) - \frac{1}{18}bd \left( \frac{4b^3x^3 \log\left(\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1} + 1\right) + 6b^2x^2 \operatorname{Li}_2\left(-\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right) - 6bx \operatorname{Li}_3\left(-\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right)}{b^4d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccoth(c+d*tanh(b*x+a)),x, algorithm="maxima")`

[Out] `1/3*x^3*arccoth(d*tanh(b*x + a) + c) - 1/18*b*d*((4*b^3*x^3*log((c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 6*b^2*x^2*dilog(-(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)) - 6*b*x*polylog(3, -(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)) + 3*polylog(4, -(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^4*d) - (4*b^3*x^3*log((c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1) + 1) + 6*b^2*x^2*dilog(-(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)) - 6*b*x*polylog(3, -(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)) + 3*polylog(4, -(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)))/(b^4*d)`

---

**Fricas [C]** time = 2.06443, size = 2587, normalized size = 8.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccoth(c+d*tanh(b*x+a)),x, algorithm="fricas")`

```
[Out] 1/6*(b^3*x^3*log(((c + 1)*cosh(b*x + a) + d*sinh(b*x + a))/((c - 1)*cosh(b*x + a) + d*sinh(b*x + a))) - 3*b^2*x^2*dilog(sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 3*b^2*x^2*dilog(-sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(-sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + a^3*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d + 1))) + a^3*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) - 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d + 1))) - a^3*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt(-(c + d - 1)/(c - d - 1))) - a^3*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) - 2*(c - d - 1)*sqrt(-(c + d - 1)/(c - d - 1))) + 6*b*x*polylog(3, sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 6*b*x*polylog(3, -sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 6*b*x*polylog(3, sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) - 6*b*x*polylog(3, -sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) - (b^3*x^3 + a^3)*log(sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b^3*x^3 + a^3)*log(-sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b^3*x^3 + a^3)*log(sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b^3*x^3 + a^3)*log(-sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 6*polylog(4, sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 6*polylog(4, -sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 6*polylog(4, sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + 6*polylog(4, -sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(c+d*tanh(b*x+a)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arccoth}(d \tanh(bx + a) + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(c+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(d*tanh(b*x + a) + c), x)
```



### 3.204 $\int x \coth^{-1}(c + d \tanh(a + bx)) dx$

**Optimal.** Leaf size=231

$$-\frac{\text{PolyLog}\left(3, -\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^2} + \frac{\text{PolyLog}\left(3, -\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^2} + \frac{x\text{PolyLog}\left(2, -\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b} - \frac{x\text{PolyLog}\left(2, -\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}$$

[Out]  $(x^2 \text{ArcCoth}[c + d \text{Tanh}[a + b x]])/2 + (x^2 \text{Log}[1 + ((1 - c - d)E^{(2a + 2bx)})/(1 - c + d)])/4 - (x^2 \text{Log}[1 + ((1 + c + d)E^{(2a + 2bx)})/(1 + c - d)])/4 + (x \text{PolyLog}[2, -(((1 - c - d)E^{(2a + 2bx)})/(1 - c + d))])/(4b) - (x \text{PolyLog}[2, -(((1 + c + d)E^{(2a + 2bx)})/(1 + c - d))])/(4b) - \text{PolyLog}[3, -(((1 - c - d)E^{(2a + 2bx)})/(1 - c + d))]/(8b^2) + \text{PolyLog}[3, -(((1 + c + d)E^{(2a + 2bx)})/(1 + c - d))]/(8b^2)$

**Rubi [A]** time = 0.379466, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {6244, 2190, 2531, 2282, 6589}

$$-\frac{\text{PolyLog}\left(3, -\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^2} + \frac{\text{PolyLog}\left(3, -\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^2} + \frac{x\text{PolyLog}\left(2, -\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b} - \frac{x\text{PolyLog}\left(2, -\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x \text{ArcCoth}[c + d \text{Tanh}[a + b x]], x]$

[Out]  $(x^2 \text{ArcCoth}[c + d \text{Tanh}[a + b x]])/2 + (x^2 \text{Log}[1 + ((1 - c - d)E^{(2a + 2bx)})/(1 - c + d)])/4 - (x^2 \text{Log}[1 + ((1 + c + d)E^{(2a + 2bx)})/(1 + c - d)])/4 + (x \text{PolyLog}[2, -(((1 - c - d)E^{(2a + 2bx)})/(1 - c + d))])/(4b) - (x \text{PolyLog}[2, -(((1 + c + d)E^{(2a + 2bx)})/(1 + c - d))])/(4b) - \text{PolyLog}[3, -(((1 - c - d)E^{(2a + 2bx)})/(1 - c + d))]/(8b^2) + \text{PolyLog}[3, -(((1 + c + d)E^{(2a + 2bx)})/(1 + c - d))]/(8b^2)$

#### Rule 6244

$\text{Int}[\text{ArcCoth}[(c_.) + (d_.) \text{Tanh}[(a_.) + (b_.)(x_.)]] * ((e_.) + (f_.)(x_.))^{(m_.)}, x\_Symbol] :> \text{Simp}[(e + f x)^{(m + 1)} \text{ArcCoth}[c + d \text{Tanh}[a + b x]] / (f^{(m + 1)}, x) + (\text{Dist}[(b(1 - c - d)) / (f^{(m + 1)})], \text{Int}[(e + f x)^{(m + 1)} E^{(2a + 2bx)} / (1 - c + d + (1 - c - d) E^{(2a + 2bx)}), x], x) - \text{Dist}[(b(1 + c + d)) / (f^{(m + 1)})], \text{Int}[(e + f x)^{(m + 1)} E^{(2a + 2bx)} / (1 + c - d + (1 + c + d) E^{(2a + 2bx)}), x], x) /; \text{FreeQ}[a, b, c, d, e, f], x] \&$

& IGtQ[m, 0] && NeQ[(c - d)^2, 1]

### Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*(a_) + (b_)*(x_)))^(n_)]*((f_) + (g_)
*(x_))^(m_), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int x \coth^{-1}(c + d \tanh(a + bx)) dx &= \frac{1}{2}x^2 \coth^{-1}(c + d \tanh(a + bx)) + \frac{1}{2}(b(1 - c - d)) \int \frac{e^{2a+2bx}x^2}{1 - c + d + (1 - c - d)e^{2a+2bx}} \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \tanh(a + bx)) + \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \tanh(a + bx)) + \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \tanh(a + bx)) + \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \tanh(a + bx)) + \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.163279, size = 259, normalized size = 1.12

---


$$-2bx \operatorname{PolyLog}\left(2, \frac{(c-d-1)(\sinh(2(a+bx))-\cosh(2(a+bx)))}{c+d-1}\right) + 2bx \operatorname{PolyLog}\left(2, \frac{(c-d+1)(\sinh(2(a+bx))-\cosh(2(a+bx)))}{c+d+1}\right) - \operatorname{PolyLog}\left(3, \frac{(c-d-1)(\sinh(2(a+bx))-\cosh(2(a+bx)))}{c+d-1}\right) + \operatorname{PolyLog}\left(3, \frac{(c-d+1)(\sinh(2(a+bx))-\cosh(2(a+bx)))}{c+d+1}\right)$$


---

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[c + d\*Tanh[a + b\*x]], x]

[Out] (x^2\*ArcCoth[c + d\*Tanh[a + b\*x]])/2 + (2\*b^2\*x^2\*Log[1 + ((-1 + c - d)\*(Cosh[2\*(a + b\*x)] - Sinh[2\*(a + b\*x)])))/(-1 + c + d)] - 2\*b^2\*x^2\*Log[1 + ((1 + c - d)\*(Cosh[2\*(a + b\*x)] - Sinh[2\*(a + b\*x)]))]/(1 + c + d)] - 2\*b\*x\*PolyLog[2, ((-1 + c - d)\*(-Cosh[2\*(a + b\*x)] + Sinh[2\*(a + b\*x)])))/(-1 + c + d)] + 2\*b\*x\*PolyLog[2, ((1 + c - d)\*(-Cosh[2\*(a + b\*x)] + Sinh[2\*(a + b\*x)]))]/(1 + c + d)] - PolyLog[3, ((-1 + c - d)\*(-Cosh[2\*(a + b\*x)] + Sinh[2\*(a + b\*x)])))/(-1 + c + d)] + PolyLog[3, ((1 + c - d)\*(-Cosh[2\*(a + b\*x)] + Sinh[2\*(a + b\*x)]))]/(1 + c + d)]/(8\*b^2)

**Maple [C]** time = 4.741, size = 4990, normalized size = 21.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*arccoth(c+d*tanh(b*x+a)),x)`

[Out] 
$$-1/8*I*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*((exp(2*b*x+2*a)+1)*c+(exp(2*b*x+2*a)-1)*d-exp(2*b*x+2*a)-1)/(exp(2*b*x+2*a)+1))^2+1/8*I*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*((exp(2*b*x+2*a)+1)*c+(exp(2*b*x+2*a)-1)*d+exp(2*b*x+2*a)+1)/(exp(2*b*x+2*a)+1))^2-1/4*x^2*ln((exp(2*b*x+2*a)+1)*c+(exp(2*b*x+2*a)-1)*d-exp(2*b*x+2*a)-1)+1/8*I*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*((exp(2*b*x+2*a)+1)*c+(exp(2*b*x+2*a)-1)*d-exp(2*b*x+2*a)-1))*csgn(I*((exp(2*b*x+2*a)+1)*c+(exp(2*b*x+2*a)-1)*d-exp(2*b*x+2*a)-1)/(exp(2*b*x+2*a)+1))+1/8*I*Pi*x^2*csgn(I*((exp(2*b*x+2*a)+1)*c+(exp(2*b*x+2*a)-1)*d-exp(2*b*x+2*a)-1)/(exp(2*b*x+2*a)+1))^3-1/4/b^2*a^2/(1+c+d)*ln(exp(2*b*x+2*a)*c+exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+c-d+1)-1/8*I*Pi*x^2*csgn(I*((exp(2*b*x+2*a)+1)*c+(exp(2*b*x+2*a)-1)*d+exp(2*b*x+2*a)+1)/(exp(2*b*x+2*a)+1))^3+1/2/b*a*c/(1+c+d)*ln((c*exp(b*x+a)+exp(b*x+a)*d+(-(1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/(-(1+c-d)*(1+c+d))^(1/2))*x-1/2/b*d/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(-1-c+d))*x*a-1/2/b*c/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(-1-c+d))*x*a+1/4*x^2*ln((exp(2*b*x+2*a)+1)*c+(exp(2*b*x+2*a)-1)*d+exp(2*b*x+2*a)+1)+1/2/b*a*d/(1+c+d)*ln((-c*exp(b*x+a)-exp(b*x+a)*d+(-(1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/(-(1+c-d)*(1+c+d))^(1/2))*x+1/2/b*a*d/(1+c+d)*ln((c*exp(b*x+a)+exp(b*x+a)*d+(-(1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/(-(1+c-d)*(1+c+d))^(1/2))*x+1/2/b*a*c/(1+c+d)*ln((-c*exp(b*x+a)-exp(b*x+a)*d+(-(1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/(-(1+c-d)*(1+c+d))^(1/2))*x-1/4/b/(1+c+d)*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(-1-c+d))*x+1/2/b^2*a^2/(1+c+d)*ln((-c*exp(b*x+a)-exp(b*x+a)*d+(-(1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/(-(1+c-d)*(1+c+d))^(1/2))+1/2/b^2*a^2/(1+c+d)*ln((c*exp(b*x+a)+exp(b*x+a)*d+(-(1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/(-(1+c-d)*(1+c+d))^(1/2))-1/4/b^2/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(-1-c+d))*a^2-1/4/b^2/(1+c+d)*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(-1-c+d))*a+1/8/b^2*c/(1+c+d)*polylog(3,(1+c+d)*exp(2*b*x+2*a)/(-1-c+d))+1/2/b^2*a/(1+c+d)*dilog((-c*exp(b*x+a)-exp(b*x+a)*d+(-(1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/(-(1+c-d)*(1+c+d))^(1/2))-1/8/b^2*c/(c+d-1)*polylog(3,(c+d-1)*exp(2*b*x+2*a)/(1-c+d))-1/8/b^2*d/(c+d-1)*polylog(3,(c+d-1)*exp(2*b*x+2*a)/(1-c+d))+1/2/b^2*a/(c+d-1)*dilog((-c*exp(b*x+a)-exp(b*x+a)*d+(-(c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/(-(c-d-1)*(c+d-1))^(1/2))+1/2/b^2*a/(c+d-1)*dilog((c*exp(b*x+a)+exp(b*x+a)*d+(-(c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/(-(c-d-1)*(c+d-1))^(1/2))-1/4/b^2/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(1-c+d))*a^2-1/4/b/(c+d-1)*polylog(2,(c+d-1)*exp(2*b*x+2*a)/(1-c+d))*x-1/4/b^2/(c+d-1)*polylog(2,(c+d-1)*exp(2*b*x+2*a)/(1-c+d))*a+1/2/b^2*a^2/(c+d-1)*ln((-c*exp(b*x+a)-exp(b*x+a)*d+(-(c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/(-(c-d-1)*(c+d-1))^(1/2))+1/2/b^2*a^2/(c+d-1)*ln((c*exp(b*x+a)+exp(b*x+a)*d+(-(c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/(-(c-d-1)*(c+d-1))^(1/2))+1/4*c/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(1-c+d))*x^2+1/4*d/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(1-c+d))*x^2+1/8/b^2/(c+d-1)*polylog(3,(c+d-1)*exp(2*b*x+2*a)/(1-c+d))-1/4/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(1-c+d))*x^2+1/2/b^2*a/(1+c+d)*dilog((c*exp(b*x+a)+exp(b*x+a)*d+(-(1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/(-(1+c-d)*(1+c+d))^(1/2))+1/8/b^2*d/(1+c+d)*polylog(3,$$

$$\begin{aligned}
& (1+c+d) \exp(2bx+2a) / (-1-c+d) - 1/4 d / (1+c+d) \ln(1 - (1+c+d) \exp(2bx+2a) / (-1-c+d)) \\
& x^2 - 1/4 c / (1+c+d) \ln(1 - (1+c+d) \exp(2bx+2a) / (-1-c+d)) x^2 - 1/2 b \\
& * d a / (c+d-1) \ln((-c \exp(bx+a) - \exp(bx+a) * d + (-c-d-1) * (c+d-1))^{1/2} + \exp(bx+a)) \\
& / (-c-d-1) * (c+d-1)^{1/2} * x - 1/2 / b * d a / (c+d-1) \ln((c \exp(bx+a) + \exp(bx+a) * d + (-c-d-1) * (c+d-1))^{1/2} - \exp(bx+a)) \\
& / (-c-d-1) * (c+d-1)^{1/2} * x + 1/2 / b * c / (c+d-1) \ln(1 - (c+d-1) \exp(2bx+2a) / (1-c+d)) * x a + 1/2 / b * d / (c+d-1) \ln(1 - (c+d-1) \exp(2bx+2a) / (1-c+d)) * x a - 1/2 / b * c a / (c+d-1) \ln((-c \exp(bx+a) - \exp(bx+a) * d + (-c-d-1) * (c+d-1))^{1/2} + \exp(bx+a)) / (-c-d-1) * (c+d-1)^{1/2} * x - 1/2 / b * c a / (c+d-1) \ln((c \exp(bx+a) + \exp(bx+a) * d + (-c-d-1) * (c+d-1))^{1/2} - \exp(bx+a)) / (-c-d-1) * (c+d-1)^{1/2} * x - 1/4 / b^2 a^2 / (c+d-1) \ln(\exp(2bx+2a) * c + \exp(2bx+2a) * d - \exp(2bx+2a) + c - d - 1) + 1/2 / b^2 a^2 d / (1+c+d) \ln((-c \exp(bx+a) - \exp(bx+a) * d + (-1+c-d) * (1+c+d))^{1/2} - \exp(bx+a)) / (-1+c-d) * (1+c+d)^{1/2}) + 1/2 / b^2 a^2 d / (1+c+d) \ln((c \exp(bx+a) + \exp(bx+a) * d + (-1+c-d) * (1+c+d))^{1/2} + \exp(bx+a)) / (-1+c-d) * (1+c+d)^{1/2}) + 1/2 / b^2 a^2 c / (1+c+d) \ln((-c \exp(bx+a) - \exp(bx+a) * d + (-1+c-d) * (1+c+d))^{1/2} - \exp(bx+a)) / (-1+c-d) * (1+c+d)^{1/2}) + 1/2 / b^2 a^2 c / (1+c+d) \ln((c \exp(bx+a) + \exp(bx+a) * d + (-1+c-d) * (1+c+d))^{1/2} + \exp(bx+a)) / (-1+c-d) * (1+c+d)^{1/2}) - 1/4 / b^2 d / (1+c+d) \ln(1 - (1+c+d) \exp(2bx+2a) / (-1-c+d)) a^2 - 1/4 / b^2 d / (1+c+d) \operatorname{polylog}(2, (1+c+d) \exp(2bx+2a) / (-1-c+d)) a - 1/4 / b^2 c / (1+c+d) \ln(1 - (1+c+d) \exp(2bx+2a) / (-1-c+d)) a^2 - 1/4 / b^2 c / (1+c+d) \operatorname{polylog}(2, (1+c+d) \exp(2bx+2a) / (-1-c+d)) a + 1/2 / b^2 a c / (1+c+d) \operatorname{dilog}((-c \exp(bx+a) - \exp(bx+a) * d + (-1+c-d) * (1+c+d))^{1/2} - \exp(bx+a)) / (-1+c-d) * (1+c+d)^{1/2}) + 1/2 / b^2 a c / (1+c+d) \operatorname{dilog}((c \exp(bx+a) + \exp(bx+a) * d + (-1+c-d) * (1+c+d))^{1/2} + \exp(bx+a)) / (-1+c-d) * (1+c+d)^{1/2}) + 1/2 / b^2 a d / (1+c+d) \operatorname{dilog}((-c \exp(bx+a) - \exp(bx+a) * d + (-1+c-d) * (1+c+d))^{1/2} - \exp(bx+a)) / (-1+c-d) * (1+c+d)^{1/2}) + 1/2 / b^2 a d / (1+c+d) \operatorname{dilog}((c \exp(bx+a) + \exp(bx+a) * d + (-1+c-d) * (1+c+d))^{1/2} + \exp(bx+a)) / (-1+c-d) * (1+c+d)^{1/2}) + 1/2 / b a / (1+c+d) \ln((-c \exp(bx+a) - \exp(bx+a) * d + (-1+c-d) * (1+c+d))^{1/2} - \exp(bx+a)) / (-1+c-d) * (1+c+d)^{1/2} * x + 1/2 / b a / (1+c+d) \ln((c \exp(bx+a) + \exp(bx+a) * d + (-1+c-d) * (1+c+d))^{1/2} + \exp(bx+a)) / (-1+c-d) * (1+c+d)^{1/2} * x - 1/4 / b d / (1+c+d) \operatorname{polylog}(2, (1+c+d) \exp(2bx+2a) / (-1-c+d)) * x - 1/4 / b c / (1+c+d) \operatorname{polylog}(2, (1+c+d) \exp(2bx+2a) / (-1-c+d)) * x - 1/2 / b / (1+c+d) \ln(1 - (1+c+d) \exp(2bx+2a) / (-1-c+d)) * x a - 1/4 / b^2 a^2 c / (1+c+d) \ln(\exp(2bx+2a) * c + \exp(2bx+2a) * d + \exp(2bx+2a) + c - d + 1) - 1/4 / b^2 a^2 d / (1+c+d) \ln(\exp(2bx+2a) * c + \exp(2bx+2a) * d + \exp(2bx+2a) + c - d + 1) - 1/4 / (1+c+d) \ln(1 - (1+c+d) \exp(2bx+2a) / (-1-c+d)) * x^2 + 1/8 / b^2 / (1+c+d) \operatorname{polylog}(3, (1+c+d) \exp(2bx+2a) / (-1-c+d)) - 1/8 * I * \pi * x^2 * \operatorname{csgn}(I * ((\exp(2bx+2a) + 1) * c + (\exp(2bx+2a) - 1) * d - \exp(2bx+2a) - 1)) * \operatorname{csgn}(I * ((\exp(2bx+2a) + 1) * c + (\exp(2bx+2a) - 1) * d - \exp(2bx+2a) - 1)) / (\exp(2bx+2a) + 1))^{2+1/8} * I * \pi * x^2 * \operatorname{csgn}(I * ((\exp(2bx+2a) + 1) * c + (\exp(2bx+2a) - 1) * d + \exp(2bx+2a) + 1)) * \operatorname{csgn}(I * ((\exp(2bx+2a) + 1) * c + (\exp(2bx+2a) - 1) * d + \exp(2bx+2a) + 1)) / (\exp(2bx+2a) + 1))^{2-1/2} / b^2 d a / (c+d-1) \operatorname{dilog}((-c \exp(bx+a) - \exp(bx+a) * d + (-c-d-1) * (c+d-1))^{1/2} + \exp(bx+a)) / (-c-d-1) * (c+d-1)^{1/2} - 1/2 / b^2 d a / (c+d-1) \operatorname{dilog}((c \exp(bx+a) + \exp(bx+a) * d + (-c-d-1) * (c+d-1))^{1/2} - \exp(bx+a)) / (-c-d-1) * (c+d-1)^{1/2}) + 1/4 / b^2 c / (c+d-1) \ln(1 - (c+d-1) \exp(2bx+2a) / (1-c+d)) a^2 - 1/2 / b^2 c a / (c+d-1) \operatorname{dilog}((c \exp(bx+a) + \exp(bx+a) * d + (-c-d-1) * (c+d-1))^{1/2} - \exp(bx+a)) / (-c-d-1) * (c+d-1)^{1/2} - \exp(bx+a)) / (-c-d-1) * (c+d-1)^{1/2}
\end{aligned}$$



**Fricas [C]** time = 1.89422, size = 2114, normalized size = 9.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(c+d\*tanh(b\*x+a)),x, algorithm="fricas")

[Out]  $\frac{1}{4}*(b^2*x^2*\log(((c + 1)*\cosh(b*x + a) + d*\sinh(b*x + a))/((c - 1)*\cosh(b*x + a) + d*\sinh(b*x + a))) - 2*b*x*dilog(\sqrt{-(c + d + 1)/(c - d + 1)}*(\cosh(b*x + a) + \sinh(b*x + a))) - 2*b*x*dilog(-\sqrt{-(c + d + 1)/(c - d + 1)}*(\cosh(b*x + a) + \sinh(b*x + a))) + 2*b*x*dilog(\sqrt{-(c + d - 1)/(c - d - 1)}*(\cosh(b*x + a) + \sinh(b*x + a))) + 2*b*x*dilog(-\sqrt{-(c + d - 1)/(c - d - 1)}*(\cosh(b*x + a) + \sinh(b*x + a))) - a^2*\log(2*(c + d + 1)*\cosh(b*x + a) + 2*(c + d + 1)*\sinh(b*x + a) + 2*(c - d + 1)*\sqrt{-(c + d + 1)/(c - d + 1)}) - a^2*\log(2*(c + d + 1)*\cosh(b*x + a) + 2*(c + d + 1)*\sinh(b*x + a) - 2*(c - d + 1)*\sqrt{-(c + d + 1)/(c - d + 1)}) + a^2*\log(2*(c + d - 1)*\cosh(b*x + a) + 2*(c + d - 1)*\sinh(b*x + a) + 2*(c - d - 1)*\sqrt{-(c + d - 1)/(c - d - 1)}) + a^2*\log(2*(c + d - 1)*\cosh(b*x + a) + 2*(c + d - 1)*\sinh(b*x + a) - 2*(c - d - 1)*\sqrt{-(c + d - 1)/(c - d - 1)}) - (b^2*x^2 - a^2)*\log(\sqrt{-(c + d + 1)/(c - d + 1)}*(\cosh(b*x + a) + \sinh(b*x + a)) + 1) - (b^2*x^2 - a^2)*\log(-\sqrt{-(c + d + 1)/(c - d + 1)}*(\cosh(b*x + a) + \sinh(b*x + a)) + 1) + (b^2*x^2 - a^2)*\log(\sqrt{-(c + d - 1)/(c - d - 1)}*(\cosh(b*x + a) + \sinh(b*x + a)) + 1) + (b^2*x^2 - a^2)*\log(-\sqrt{-(c + d - 1)/(c - d - 1)}*(\cosh(b*x + a) + \sinh(b*x + a)) + 1) + 2*\text{polylog}(3, \sqrt{-(c + d + 1)/(c - d + 1)}*(\cosh(b*x + a) + \sinh(b*x + a))) + 2*\text{polylog}(3, -\sqrt{-(c + d + 1)/(c - d + 1)}*(\cosh(b*x + a) + \sinh(b*x + a))) - 2*\text{polylog}(3, \sqrt{-(c + d - 1)/(c - d - 1)}*(\cosh(b*x + a) + \sinh(b*x + a))) - 2*\text{polylog}(3, -\sqrt{-(c + d - 1)/(c - d - 1)}*(\cosh(b*x + a) + \sinh(b*x + a))))/b^2$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(c+d\*tanh(b\*x+a)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arccoth}(d \tanh(bx + a) + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(c+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(d*tanh(b*x + a) + c), x)
```



### 3.205 $\int \coth^{-1}(c + d \tanh(a + bx)) dx$

**Optimal.** Leaf size=150

$$\frac{\text{PolyLog}\left(2, -\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b} - \frac{\text{PolyLog}\left(2, -\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b} + \frac{1}{2}x \log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1} + 1\right) - \frac{1}{2}x \log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1} + 1\right)$$

```
[Out] x*ArcCoth[c + d*Tanh[a + b*x]] + (x*Log[1 + ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/2 - (x*Log[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/2 + PolyLog[2, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))]/(4*b) - PolyLog[2, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))]/(4*b)
```

**Rubi [A]** time = 0.231966, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {6236, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2, -\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b} - \frac{\text{PolyLog}\left(2, -\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b} + \frac{1}{2}x \log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1} + 1\right) - \frac{1}{2}x \log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1} + 1\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[c + d*Tanh[a + b*x]], x]
```

```
[Out] x*ArcCoth[c + d*Tanh[a + b*x]] + (x*Log[1 + ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/2 - (x*Log[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/2 + PolyLog[2, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))]/(4*b) - PolyLog[2, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))]/(4*b)
```

#### Rule 6236

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] := Simp[x*ArcCoth[c + d*Tanh[a + b*x]], x] + (Dist[b*(1 - c - d), Int[(x*E^(2*a + 2*b*x))/(1 - c + d + (1 - c - d)*E^(2*a + 2*b*x)), x], x] - Dist[b*(1 + c + d), Int[(x*E^(2*a + 2*b*x))/(1 + c - d + (1 + c + d)*E^(2*a + 2*b*x)), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, 1]
```

#### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.), x_Symbol] := Simp
```

```

[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Dist
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

```

### Rule 2279

```

Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

```

### Rule 2391

```

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

```

### Rubi steps

$$\begin{aligned}
\int \coth^{-1}(c + d \tanh(a + bx)) dx &= x \coth^{-1}(c + d \tanh(a + bx)) + (b(1 - c - d)) \int \frac{e^{2a+2bx} x}{1 - c + d + (1 - c - d)e^{2a+2bx}} dx - (b \\
&= x \coth^{-1}(c + d \tanh(a + bx)) + \frac{1}{2} x \log \left( 1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d} \right) - \frac{1}{2} x \log \left( 1 + \frac{(1 + c - d)e^{2a+2bx}}{1 - c + d} \right) \\
&= x \coth^{-1}(c + d \tanh(a + bx)) + \frac{1}{2} x \log \left( 1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d} \right) - \frac{1}{2} x \log \left( 1 + \frac{(1 + c - d)e^{2a+2bx}}{1 - c + d} \right) \\
&= x \coth^{-1}(c + d \tanh(a + bx)) + \frac{1}{2} x \log \left( 1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d} \right) - \frac{1}{2} x \log \left( 1 + \frac{(1 + c - d)e^{2a+2bx}}{1 - c + d} \right)
\end{aligned}$$

**Mathematica [A]** time = 1.24638, size = 131, normalized size = 0.87

$$\frac{\text{PolyLog}\left(2, -\frac{(c+d-1)e^{2(a+bx)}}{c-d-1}\right) - \text{PolyLog}\left(2, -\frac{(c+d+1)e^{2(a+bx)}}{c-d+1}\right) + 2bx \left( \log\left(\frac{(c+d-1)e^{2(a+bx)}}{c-d-1} + 1\right) - \log\left(\frac{(c+d+1)e^{2(a+bx)}}{c-d+1} + 1\right) \right)}{4b} + x \coth^{-1}(c + d \tanh(a + bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[c + d*Tanh[a + b*x]], x]
```

```
[Out] x*ArcCoth[c + d*Tanh[a + b*x]] + (2*b*x*(Log[1 + ((-1 + c + d)*E^(2*(a + b*x)))/(-1 + c - d)] - Log[1 + ((1 + c + d)*E^(2*(a + b*x)))/(1 + c - d)]) + PolyLog[2, -(((1 + c + d)*E^(2*(a + b*x)))/(-1 + c - d))] - PolyLog[2, -((
```

$$(1 + c + d) * E^{(2 * (a + b * x))} / (1 + c - d) ] / (4 * b)$$

**Maple [B]** time = 0.164, size = 306, normalized size = 2.

$$\frac{\operatorname{arccoth}(c + d \tanh(bx + a)) \ln(d \tanh(bx + a) - d)}{2b} + \frac{\operatorname{arccoth}(c + d \tanh(bx + a)) \ln(d \tanh(bx + a) + d)}{2b} + \frac{1}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(c+d\*tanh(b\*x+a)),x)

[Out]  $-1/2/b * \operatorname{arccoth}(c + d * \tanh(b * x + a)) * \ln(d * \tanh(b * x + a) - d) + 1/2/b * \operatorname{arccoth}(c + d * \tanh(b * x + a)) * \ln(d * \tanh(b * x + a) + d) + 1/4/b * \operatorname{dilog}((d * \tanh(b * x + a) + c + 1) / (1 + c + d)) + 1/4/b * \ln(d * \tanh(b * x + a) - d) * \ln((d * \tanh(b * x + a) + c + 1) / (1 + c + d)) - 1/4/b * \operatorname{dilog}((d * \tanh(b * x + a) + c - 1) / (c + d - 1)) - 1/4/b * \ln(d * \tanh(b * x + a) - d) * \ln((d * \tanh(b * x + a) + c - 1) / (c + d - 1)) - 1/4/b * \operatorname{dilog}((d * \tanh(b * x + a) + c + 1) / (1 + c - d)) - 1/4/b * \ln(d * \tanh(b * x + a) + d) * \ln((d * \tanh(b * x + a) + c + 1) / (1 + c - d)) + 1/4/b * \operatorname{dilog}((d * \tanh(b * x + a) + c - 1) / (c - d - 1)) + 1/4/b * \ln(d * \tanh(b * x + a) + d) * \ln((d * \tanh(b * x + a) + c - 1) / (c - d - 1))$

**Maxima [A]** time = 2.1111, size = 192, normalized size = 1.28

$$-\frac{1}{4}bd \left( \frac{2bx \log\left(\frac{(c+d+1)e^{2bx+2a}}{c-d+1} + 1\right) + \operatorname{Li}_2\left(-\frac{(c+d+1)e^{2bx+2a}}{c-d+1}\right)}{b^2d} - \frac{2bx \log\left(\frac{(c+d-1)e^{2bx+2a}}{c-d-1} + 1\right) + \operatorname{Li}_2\left(-\frac{(c+d-1)e^{2bx+2a}}{c-d-1}\right)}{b^2d} \right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c+d\*tanh(b\*x+a)),x, algorithm="maxima")

[Out]  $-1/4 * b * d * ((2 * b * x * \log((c + d + 1) * e^{(2 * b * x + 2 * a)} / (c - d + 1) + 1) + \operatorname{dilog}(-(c + d + 1) * e^{(2 * b * x + 2 * a)} / (c - d + 1))) / (b^2 * d) - (2 * b * x * \log((c + d - 1) * e^{(2 * b * x + 2 * a)} / (c - d - 1) + 1) + \operatorname{dilog}(-(c + d - 1) * e^{(2 * b * x + 2 * a)} / (c - d - 1))) / (b^2 * d) + x * \operatorname{arccoth}(d * \tanh(b * x + a) + c)$

**Fricas [B]** time = 1.85118, size = 1601, normalized size = 10.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x*log(((c + 1)*cosh(b*x + a) + d*sinh(b*x + a))/((c - 1)*cosh(b*x + a) + d*sinh(b*x + a))) + a*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d + 1))) + a*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) - 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d + 1))) - a*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt(-(c + d - 1)/(c - d - 1))) - a*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) - 2*(c - d - 1)*sqrt(-(c + d - 1)/(c - d - 1))) - (b*x + a)*log(sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x + a)*log(sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x + a)*log(-sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - dilog(sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) - dilog(-sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + dilog(sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + dilog(-sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))))/b
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoath(c+d*tanh(b*x+a)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(d \tanh(bx + a) + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*tanh(b*x + a) + c), x)
```

$$3.206 \quad \int \frac{\coth^{-1}(c+d \tanh(a+bx))}{x} dx$$

**Optimal.** Leaf size=17

$$\text{CannotIntegrate}\left(\frac{\coth^{-1}(d \tanh(a + bx) + c)}{x}, x\right)$$

[Out] CannotIntegrate[ArcCoth[c + d\*Tanh[a + b\*x]]/x, x]

**Rubi [A]** time = 0.0662613, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\coth^{-1}(c + d \tanh(a + bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[ArcCoth[c + d\*Tanh[a + b\*x]]/x,x]

[Out] Defer[Int][ArcCoth[c + d\*Tanh[a + b\*x]]/x, x]

Rubi steps

$$\int \frac{\coth^{-1}(c + d \tanh(a + bx))}{x} dx = \int \frac{\coth^{-1}(c + d \tanh(a + bx))}{x} dx$$

**Mathematica [A]** time = 5.26462, size = 0, normalized size = 0.

$$\int \frac{\coth^{-1}(c + d \tanh(a + bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcCoth[c + d\*Tanh[a + b\*x]]/x,x]

[Out] Integrate[ArcCoth[c + d\*Tanh[a + b\*x]]/x, x]

---

**Maple [A]** time = 0.331, size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(c + d \tanh(bx + a))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(c+d*tanh(b*x+a))/x,x)`

[Out] `int(arccoth(c+d*tanh(b*x+a))/x,x)`

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(d \tanh(bx + a) + c)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(c+d*tanh(b*x+a))/x,x, algorithm="maxima")`

[Out] `integrate(arccoth(d*tanh(b*x + a) + c)/x, x)`

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arccoth}(d \tanh(bx + a) + c)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(c+d*tanh(b*x+a))/x,x, algorithm="fricas")`

[Out] `integral(arccoth(d*tanh(b*x + a) + c)/x, x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(c+d*tanh(b*x+a))/x,x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(d \tanh(bx + a) + c)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*tanh(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*tanh(b*x + a) + c)/x, x)
```



### 3.207 $\int x^3 \coth^{-1}(1 + d + d \tanh(a + bx)) dx$

**Optimal.** Leaf size=155

$$\frac{3x^2 \text{PolyLog}(3, -(d+1)e^{2a+2bx})}{8b^2} - \frac{3x \text{PolyLog}(4, -(d+1)e^{2a+2bx})}{8b^3} + \frac{3 \text{PolyLog}(5, -(d+1)e^{2a+2bx})}{16b^4} - \frac{x^3 \text{PolyLog}(2, -(d+1)e^{2a+2bx})}{4b^4}$$

```
[Out] (b*x^5)/20 + (x^4*ArcCoth[1 + d + d*Tanh[a + b*x]])/4 - (x^4*Log[1 + (1 + d)*E^(2*a + 2*b*x)])/8 - (x^3*PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))])/(4*b) + (3*x^2*PolyLog[3, -((1 + d)*E^(2*a + 2*b*x))])/(8*b^2) - (3*x*PolyLog[4, -((1 + d)*E^(2*a + 2*b*x))])/(8*b^3) + (3*PolyLog[5, -((1 + d)*E^(2*a + 2*b*x))])/(16*b^4)
```

**Rubi [A]** time = 0.30719, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {6240, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2 \text{PolyLog}(3, -(d+1)e^{2a+2bx})}{8b^2} - \frac{3x \text{PolyLog}(4, -(d+1)e^{2a+2bx})}{8b^3} + \frac{3 \text{PolyLog}(5, -(d+1)e^{2a+2bx})}{16b^4} - \frac{x^3 \text{PolyLog}(2, -(d+1)e^{2a+2bx})}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCoth[1 + d + d*Tanh[a + b*x]], x]
```

```
[Out] (b*x^5)/20 + (x^4*ArcCoth[1 + d + d*Tanh[a + b*x]])/4 - (x^4*Log[1 + (1 + d)*E^(2*a + 2*b*x)])/8 - (x^3*PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))])/(4*b) + (3*x^2*PolyLog[3, -((1 + d)*E^(2*a + 2*b*x))])/(8*b^2) - (3*x*PolyLog[4, -((1 + d)*E^(2*a + 2*b*x))])/(8*b^3) + (3*PolyLog[5, -((1 + d)*E^(2*a + 2*b*x))])/(16*b^4)
```

#### Rule 6240

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Tanh[a + b*x]])/(f*(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d)^2, 1]
```

#### Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_))))^(n_.), x_Symbol] := Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
```

$b/a, \text{Int}[(c + d*x)^m*(F^{(g*(e + f*x)))^n}/(a + b*(F^{(g*(e + f*x)))^n}), x], x] /;$  FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2190

$\text{Int}[(F^{(g_*)*(e_*) + (f_*)*(x_*)})^{(n_*)}*((c_*) + (d_*)*(x_*)^{(m_*)})/((a_*) + (b_*)*(F^{(g_*)*(e_*) + (f_*)*(x_*)})^{(n_*)}), x\_Symbol] :> \text{Simp}[(c + d*x)^m*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a]/(b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a], x], x] /;$  FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

$\text{Int}[\text{Log}[1 + (e_*)*(F^{(c_*)*(a_*) + (b_*)*(x_*)})^{(n_*)}]*((f_*) + (g_*)*(x_*)^{(m_*)}), x\_Symbol] :> -\text{Simp}[(f + g*x)^m*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x)))^n})]/(b*c*n*\text{Log}[F]), x] + \text{Dist}[(g*m)/(b*c*n*\text{Log}[F]), \text{Int}[(f + g*x)^{(m-1)}*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x)))^n})], x], x] /;$  FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 6609

$\text{Int}[(e_*) + (f_*)*(x_*)^{(m_*)}*\text{PolyLog}[n_, (d_*)*(F^{(c_*)*(a_*) + (b_*)*(x_*)})^{(p_*)}], x\_Symbol] :> \text{Simp}[(e + f*x)^m*\text{PolyLog}[n + 1, d*(F^{(c*(a + b*x)))^p}]/(b*c*p*\text{Log}[F]), x] - \text{Dist}[(f*m)/(b*c*p*\text{Log}[F]), \text{Int}[(e + f*x)^{(m-1)}*\text{PolyLog}[n + 1, d*(F^{(c*(a + b*x)))^p}], x], x] /;$  FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

### Rule 2282

$\text{Int}[u_, x\_Symbol] :> \text{With}[\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x]] /;$  FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_\*)\*(v\_)^{(n\_)})^{(m\_)} /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^{(c\_\*)\*(a\_\*) + (b\_\*)\*x})\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

$\text{Int}[\text{PolyLog}[n_, (c_*)*(a_*) + (b_*)*(x_*)^{(p_*)}]/((d_*) + (e_*)*(x_*)), x\_Symbol] :> \text{Simp}[\text{PolyLog}[n + 1, c*(a + b*x)^p]/(e*p), x] /;$  FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
\int x^3 \coth^{-1}(1 + d + d \tanh(a + bx)) dx &= \frac{1}{4} x^4 \coth^{-1}(1 + d + d \tanh(a + bx)) + \frac{1}{4} b \int \frac{x^4}{1 + (1 + d)e^{2a+2bx}} dx \\
&= \frac{bx^5}{20} + \frac{1}{4} x^4 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{4} (b(1 + d)) \int \frac{e^{2a+2bx} x^4}{1 + (1 + d)e^{2a+2bx}} dx \\
&= \frac{bx^5}{20} + \frac{1}{4} x^4 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{8} x^4 \log(1 + (1 + d)e^{2a+2bx}) + \dots \\
&= \frac{bx^5}{20} + \frac{1}{4} x^4 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{8} x^4 \log(1 + (1 + d)e^{2a+2bx}) - \dots \\
&= \frac{bx^5}{20} + \frac{1}{4} x^4 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{8} x^4 \log(1 + (1 + d)e^{2a+2bx}) - \dots \\
&= \frac{bx^5}{20} + \frac{1}{4} x^4 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{8} x^4 \log(1 + (1 + d)e^{2a+2bx}) - \dots \\
&= \frac{bx^5}{20} + \frac{1}{4} x^4 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{8} x^4 \log(1 + (1 + d)e^{2a+2bx}) - \dots \\
&= \frac{bx^5}{20} + \frac{1}{4} x^4 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{8} x^4 \log(1 + (1 + d)e^{2a+2bx}) - \dots \\
&= \frac{bx^5}{20} + \frac{1}{4} x^4 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{8} x^4 \log(1 + (1 + d)e^{2a+2bx}) - \dots
\end{aligned}$$

**Mathematica [A]** time = 0.191028, size = 144, normalized size = 0.93

$$\frac{1}{16} \left( \frac{6x^2 \text{PolyLog}\left(3, -\frac{e^{-2(a+bx)}}{d+1}\right)}{b^2} + \frac{6x \text{PolyLog}\left(4, -\frac{e^{-2(a+bx)}}{d+1}\right)}{b^3} + \frac{3 \text{PolyLog}\left(5, -\frac{e^{-2(a+bx)}}{d+1}\right)}{b^4} + \frac{4x^3 \text{PolyLog}\left(2, -\frac{e^{-2(a+bx)}}{d+1}\right)}{b} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*ArcCoth[1 + d + d\*Tanh[a + b\*x]], x]

[Out] (4\*x^4\*ArcCoth[1 + d + d\*Tanh[a + b\*x]] - 2\*x^4\*Log[1 + 1/((1 + d)\*E^(2\*(a + b\*x)))] + (4\*x^3\*PolyLog[2, -(1/((1 + d)\*E^(2\*(a + b\*x))))])/b + (6\*x^2\*PolyLog[3, -(1/((1 + d)\*E^(2\*(a + b\*x))))])/b^2 + (6\*x\*PolyLog[4, -(1/((1 + d)\*E^(2\*(a + b\*x))))])/b^3 + (3\*PolyLog[5, -(1/((1 + d)\*E^(2\*(a + b\*x))))])/b^4)/16

**Maple [C]** time = 34.221, size = 1726, normalized size = 11.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^3 \operatorname{arccoth}(1+d \tanh(bx+a)), x)$

[Out]  $\frac{1}{16} I x^4 \operatorname{Pisgn}\left(\frac{I}{\exp(2bx+2a)+1}\right) \operatorname{csgn}(I \exp(2bx+2a)) \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1)) - \frac{1}{8} b^{-4} a^4 / (1+d) \ln(\exp(2bx+2a) d + \exp(2bx+2a)+1) + \frac{1}{2} b^{-4} d a^3 / (1+d) \operatorname{dilog}(1 + \exp(bx+a) (-d-1)^{1/2}) + \frac{1}{2} b^{-4} d a^3 / (1+d) \operatorname{dilog}(1 - \exp(bx+a) (-d-1)^{1/2}) - \frac{3}{8} b^{-4} d a^4 / (1+d) \ln(1 + (1+d) \exp(2bx+2a)) - \frac{1}{4} b^{-4} d a^3 / (1+d) \operatorname{polylog}(2, -(1+d) \exp(2bx+2a)) + \frac{1}{2} b^{-4} d a^4 / (1+d) \ln(1 + \exp(bx+a) (-d-1)^{1/2}) + \frac{1}{2} b^{-4} d a^4 / (1+d) \ln(1 - \exp(bx+a) (-d-1)^{1/2}) + \frac{1}{2} b^{-3} a^3 / (1+d) \ln(1 + \exp(bx+a) (-d-1)^{1/2}) x - \frac{1}{4} b d / (1+d) \operatorname{polylog}(2, -(1+d) \exp(2bx+2a)) x^3 + \frac{3}{8} b^2 d / (1+d) \operatorname{polylog}(3, -(1+d) \exp(2bx+2a)) x^2 + \frac{1}{2} b^3 a^3 / (1+d) \ln(1 - \exp(bx+a) (-d-1)^{1/2}) x - \frac{1}{2} b^3 a^3 / (1+d) \ln(1 + (1+d) \exp(2bx+2a)) x - \frac{3}{8} b^3 d / (1+d) \operatorname{polylog}(4, -(1+d) \exp(2bx+2a)) x - \frac{1}{16} I x^4 \operatorname{Pisgn}\left(\frac{I}{\exp(2bx+2a)+1}\right) (\exp(2bx+2a) d + \exp(2bx+2a)+1)^3 + \frac{1}{16} I x^4 \operatorname{Pisgn}(I \exp(bx+a))^2 \operatorname{csgn}(I \exp(2bx+2a)) - \frac{1}{8} I x^4 \operatorname{Pisgn}(I \exp(bx+a)) \operatorname{csgn}(I \exp(2bx+2a))^2 + \frac{1}{16} I x^4 \operatorname{Pisgn}(I d / (\exp(2bx+2a)+1) \exp(2bx+2a))^3 - \frac{1}{2} b^{-3} d a^3 / (1+d) \ln(1 + (1+d) \exp(2bx+2a)) x + \frac{1}{2} b^{-3} d a^3 / (1+d) \ln(1 + \exp(bx+a) (-d-1)^{1/2}) x + \frac{1}{2} b^{-3} d a^3 / (1+d) \ln(1 - \exp(bx+a) (-d-1)^{1/2}) x + \frac{1}{16} I x^4 \operatorname{Pisgn}(I \exp(2bx+2a))^3 + \frac{1}{16} I x^4 \operatorname{Pisgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^3 + \frac{1}{20} b x^5 - \frac{1}{16} I x^4 \operatorname{Pisgn}(I d) \operatorname{csgn}(I d / (\exp(2bx+2a)+1) \exp(2bx+2a))^2 - \frac{1}{16} I x^4 \operatorname{Pisgn}(I / (\exp(2bx+2a)+1)) \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^2 - \frac{1}{16} I x^4 \operatorname{Pisgn}(I \exp(2bx+2a)) \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1))^2 + \frac{1}{16} I x^4 \operatorname{Pisgn}(I d) \operatorname{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1)) \operatorname{csgn}(I d / (\exp(2bx+2a)+1) \exp(2bx+2a)) + \frac{1}{8} x^4 \ln(\exp(2bx+2a) d + \exp(2bx+2a)+1) - \frac{1}{8} d / (1+d) \ln(1 + (1+d) \exp(2bx+2a)) x^4 + \frac{1}{2} b^{-4} a^4 / (1+d) \ln(1 - \exp(bx+a) (-d-1)^{1/2}) - \frac{3}{8} b^{-4} a^4 / (1+d) \ln(1 + (1+d) \exp(2bx+2a)) - \frac{1}{4} b / (1+d) \operatorname{polylog}(2, -(1+d) \exp(2bx+2a)) x^3 - \frac{1}{4} b^4 / (1+d) \operatorname{polylog}(2, -(1+d) \exp(2bx+2a)) a^3 + \frac{3}{8} b^2 / (1+d) \operatorname{polylog}(3, -(1+d) \exp(2bx+2a)) x^2 - \frac{3}{8} b^3 / (1+d) \operatorname{polylog}(4, -(1+d) \exp(2bx+2a)) x + \frac{1}{2} b^4 a^4 / (1+d) \ln(1 + \exp(bx+a) (-d-1)^{1/2}) + \frac{3}{16} b^4 d / (1+d) \operatorname{polylog}(5, -(1+d) \exp(2bx+2a)) + \frac{1}{2} b^4 a^3 / (1+d) \operatorname{dilog}(1 + \exp(bx+a) (-d-1)^{1/2}) + \frac{1}{2} b^4 a^3 / (1+d) \operatorname{dilog}(1 - \exp(bx+a) (-d-1)^{1/2}) - \frac{1}{16} I x^4 \operatorname{Pisgn}(I \exp(2bx+2a) / (\exp(2bx+2a)+1)) \operatorname{csgn}(I d / (\exp(2bx+2a)+1) \exp(2bx+2a))^2 + \frac{1}{16} I x^4 \operatorname{Pisgn}(I (\exp(2bx+2a) d + \exp(2bx+2a)+1)) \operatorname{csgn}(I / (\exp(2bx+2a)+1) (\exp(2bx+2a) d + \exp(2bx+2a)+1))^2 - \frac{1}{8} / (1+d) \ln(1 + (1+d) \exp(2bx+2a)) x^4 + \frac{3}{16} b^4 / (1+d) \operatorname{polylog}(5, -(1+d) \exp(2bx+2a)) - \frac{1}{4} x^4 \ln(\exp(bx+a)) - \frac{1}{8} x^4 \ln(d) + \frac{1}{16} I x^4 \operatorname{Pisgn}(I / (\exp(2bx+2a)+1)) \operatorname{csgn}(I / (\exp(2bx+2a)+1) (\exp(2bx+2a) d + \exp(2bx+2a)+1))^2 - \frac{1}{16} I x^4 \operatorname{Pisgn}(I / (\exp(2bx+2a)+1)) \operatorname{csgn}(I (\exp(2bx+2a) d + \exp(2bx+2a)+1)) \operatorname{csgn}(I / (\exp(2bx+2a)+1) (\exp(2bx+2a) d + \exp(2bx+2a)+1)) - \frac{1}{8} b^{-4} d a^4 / (1+d) \ln(\exp(2bx+2a) d + \exp(2bx+2a)+1)$

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**Maxima [A]** time = 3.35491, size = 201, normalized size = 1.3

$$\frac{1}{4} x^4 \operatorname{arccoth}(d \tanh(bx + a) + d + 1) + \frac{1}{40} \left( \frac{2x^5}{d} - \frac{5(2b^4 x^4 \log((d+1)e^{2bx+2a}) + 1) + 4b^3 x^3 \operatorname{Li}_2(-(d+1)e^{2bx+2a})}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(1+d+d\*tanh(b\*x+a)),x, algorithm="maxima")

[Out] 1/4\*x^4\*arccoth(d\*tanh(b\*x + a) + d + 1) + 1/40\*(2\*x^5/d - 5\*(2\*b^4\*x^4\*log((d + 1)\*e^(2\*b\*x + 2\*a) + 1) + 4\*b^3\*x^3\*dilog(-(d + 1)\*e^(2\*b\*x + 2\*a)) - 6\*b^2\*x^2\*polylog(3, -(d + 1)\*e^(2\*b\*x + 2\*a)) + 6\*b\*x\*polylog(4, -(d + 1)\*e^(2\*b\*x + 2\*a)) - 3\*polylog(5, -(d + 1)\*e^(2\*b\*x + 2\*a)))/(b^5\*d)\*b\*d

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**Fricas [C]** time = 1.88589, size = 1377, normalized size = 8.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(1+d+d\*tanh(b\*x+a)),x, algorithm="fricas")

[Out] 1/40\*(2\*b^5\*x^5 + 5\*b^4\*x^4\*log(((d + 2)\*cosh(b\*x + a) + d\*sinh(b\*x + a))/(d\*cosh(b\*x + a) + d\*sinh(b\*x + a))) - 20\*b^3\*x^3\*dilog(1/2\*sqrt(-4\*d - 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 20\*b^3\*x^3\*dilog(-1/2\*sqrt(-4\*d - 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 5\*a^4\*log(2\*(d + 1)\*cosh(b\*x + a) + 2\*(d + 1)\*sinh(b\*x + a) + sqrt(-4\*d - 4)) - 5\*a^4\*log(2\*(d + 1)\*cosh(b\*x + a) + 2\*(d + 1)\*sinh(b\*x + a) - sqrt(-4\*d - 4)) + 60\*b^2\*x^2\*polylog(3, 1/2\*sqrt(-4\*d - 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) + 60\*b^2\*x^2\*polylog(3, -1/2\*sqrt(-4\*d - 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 120\*b\*x\*polylog(4, 1/2\*sqrt(-4\*d - 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 120\*b\*x\*polylog(4, -1/2\*sqrt(-4\*d - 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 5\*(b^4\*x^4 - a^4)\*log(1/2\*sqrt(-4\*d - 4)\*(cosh(b\*x + a) + sinh(b\*x + a)) + 1) - 5\*(b^4\*x^4 - a^4)\*log(-1/2\*sqrt(-4\*d - 4)\*(cosh(b\*x + a) + sinh(b\*x + a)) + 1) + 120\*polylog(5, 1/2\*sqrt(-4\*d - 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) + 120\*polylog(5, -1/2\*sqrt(-4\*d - 4)\*(cosh(b\*x + a) + sinh(b\*x + a))))/b^4

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**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*acoth(1+d+d\*tanh(b\*x+a)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arccoth}(d \tanh(bx + a) + d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(1+d+d\*tanh(b\*x+a)),x, algorithm="giac")

[Out] integrate(x^3\*arccoth(d\*tanh(b\*x + a) + d + 1), x)

### 3.208 $\int x^2 \coth^{-1}(1 + d + d \tanh(a + bx)) dx$

**Optimal.** Leaf size=128

$$\frac{x \operatorname{PolyLog}\left(3, -(d+1)e^{2a+2bx}\right)}{4b^2} - \frac{\operatorname{PolyLog}\left(4, -(d+1)e^{2a+2bx}\right)}{8b^3} - \frac{x^2 \operatorname{PolyLog}\left(2, -(d+1)e^{2a+2bx}\right)}{4b} - \frac{1}{6} x^3 \log\left((d+1)e^{2a}\right)$$

[Out] (b\*x^4)/12 + (x^3\*ArcCoth[1 + d + d\*Tanh[a + b\*x]])/3 - (x^3\*Log[1 + (1 + d)\*E^(2\*a + 2\*b\*x)])/6 - (x^2\*PolyLog[2, -((1 + d)\*E^(2\*a + 2\*b\*x))])/(4\*b) + (x\*PolyLog[3, -((1 + d)\*E^(2\*a + 2\*b\*x))])/(4\*b^2) - PolyLog[4, -((1 + d)\*E^(2\*a + 2\*b\*x))]/(8\*b^3)

**Rubi [A]** time = 0.269003, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {6240, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{x \operatorname{PolyLog}\left(3, -(d+1)e^{2a+2bx}\right)}{4b^2} - \frac{\operatorname{PolyLog}\left(4, -(d+1)e^{2a+2bx}\right)}{8b^3} - \frac{x^2 \operatorname{PolyLog}\left(2, -(d+1)e^{2a+2bx}\right)}{4b} - \frac{1}{6} x^3 \log\left((d+1)e^{2a}\right)$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcCoth[1 + d + d\*Tanh[a + b\*x]], x]

[Out] (b\*x^4)/12 + (x^3\*ArcCoth[1 + d + d\*Tanh[a + b\*x]])/3 - (x^3\*Log[1 + (1 + d)\*E^(2\*a + 2\*b\*x)])/6 - (x^2\*PolyLog[2, -((1 + d)\*E^(2\*a + 2\*b\*x))])/(4\*b) + (x\*PolyLog[3, -((1 + d)\*E^(2\*a + 2\*b\*x))])/(4\*b^2) - PolyLog[4, -((1 + d)\*E^(2\*a + 2\*b\*x))]/(8\*b^3)

#### Rule 6240

Int[ArcCoth[(c\_.) + (d\_.)\*Tanh[(a\_.) + (b\_.)\*(x\_)]]\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((e + f\*x)^(m + 1)\*ArcCoth[c + d\*Tanh[a + b\*x]])/(f\*(m + 1)), x] + Dist[b/(f\*(m + 1)), Int[(e + f\*x)^(m + 1)/(c - d + c\*E^(2\*a + 2\*b\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d)^2, 1]

#### Rule 2184

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.), x\_Symbol] := Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x],

x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_)]/((a\_) + (b\_)\*((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_)\*((F\_)^((c\_)\*(a\_) + (b\_)\*(x\_)))^(n\_)]\*((f\_) + (g\_)\*(x\_))^(m\_), x\_Symbol] :> -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n])]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 6609

Int[((e\_) + (f\_)\*(x\_))^(m\_)\*PolyLog[n\_, (d\_)\*((F\_)^((c\_)\*(a\_) + (b\_)\*(x\_)))^(p\_)], x\_Symbol] :> Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p])/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\* (F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]

### Rule 6589

Int[PolyLog[n\_, (c\_)\*((a\_) + (b\_)\*(x\_))^(p\_)]/((d\_) + (e\_)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps



$$\begin{aligned}
\int x^2 \coth^{-1}(1 + d + d \tanh(a + bx)) dx &= \frac{1}{3} x^3 \coth^{-1}(1 + d + d \tanh(a + bx)) + \frac{1}{3} b \int \frac{x^3}{1 + (1 + d)e^{2a+2bx}} dx \\
&= \frac{bx^4}{12} + \frac{1}{3} x^3 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{3} (b(1 + d)) \int \frac{e^{2a+2bx} x^3}{1 + (1 + d)e^{2a+2bx}} dx \\
&= \frac{bx^4}{12} + \frac{1}{3} x^3 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 + d)e^{2a+2bx}) + \dots \\
&= \frac{bx^4}{12} + \frac{1}{3} x^3 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 + d)e^{2a+2bx}) - \dots \\
&= \frac{bx^4}{12} + \frac{1}{3} x^3 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 + d)e^{2a+2bx}) - \dots \\
&= \frac{bx^4}{12} + \frac{1}{3} x^3 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 + d)e^{2a+2bx}) - \dots \\
&= \frac{bx^4}{12} + \frac{1}{3} x^3 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 + d)e^{2a+2bx}) - \dots \\
&= \frac{bx^4}{12} + \frac{1}{3} x^3 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 + d)e^{2a+2bx}) - \dots
\end{aligned}$$

**Mathematica [A]** time = 0.113944, size = 118, normalized size = 0.92

$$\frac{1}{24} \left( \frac{6x \operatorname{PolyLog}\left(3, -\frac{e^{-2(a+bx)}}{d+1}\right)}{b^2} + \frac{3 \operatorname{PolyLog}\left(4, -\frac{e^{-2(a+bx)}}{d+1}\right)}{b^3} + \frac{6x^2 \operatorname{PolyLog}\left(2, -\frac{e^{-2(a+bx)}}{d+1}\right)}{b} - 4x^3 \log\left(\frac{e^{-2(a+bx)}}{d+1} + 1\right) + 8x^3 \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[1 + d + d\*Tanh[a + b\*x]], x]

[Out] (8\*x^3\*ArcCoth[1 + d + d\*Tanh[a + b\*x]] - 4\*x^3\*Log[1 + 1/((1 + d)\*E^(2\*(a + b\*x)))] + (6\*x^2\*PolyLog[2, -(1/((1 + d)\*E^(2\*(a + b\*x))))])/b + (6\*x\*PolyLog[3, -(1/((1 + d)\*E^(2\*(a + b\*x))))])/b^2 + (3\*PolyLog[4, -(1/((1 + d)\*E^(2\*(a + b\*x))))])/b^3)/24

**Maple [C]** time = 37.152, size = 1667, normalized size = 13.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(1+d+d\*tanh(b\*x+a)), x)

```
[Out] -1/3*x^3*ln(exp(b*x+a))-1/6*x^3*ln(d)-1/4/b*d/(1+d)*polylog(2,-(1+d)*exp(2*
b*x+2*a))*x^2-1/6*I*x^3*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/12
*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+
1))^2-1/2/b^3*a^3/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))-1/8/b^3*d/(1+d)*polyl
og(4,-(1+d)*exp(2*b*x+2*a))-1/6*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x^3-1/2/
b^3*a^2/(1+d)*dilog(1+exp(b*x+a)*(-d-1)^(1/2))-1/2/b^3*a^2/(1+d)*dilog(1-ex
p(b*x+a)*(-d-1)^(1/2))+1/3/b^3/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*a^3-1/4/b/(
1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*x^2+1/4/b^3/(1+d)*polylog(2,-(1+d)*ex
p(2*b*x+2*a))*a^2+1/4/b^2/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2*a))*x-1/2/b^3*
a^3/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))+1/4/b^3*d/(1+d)*polylog(2,-(1+d)*ex
p(2*b*x+2*a))*a^2+1/4/b^2*d/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2*a))*x-1/2/b^
2*a^2/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))*x-1/2/b^2*a^2/(1+d)*ln(1-exp(b*x+
a)*(-d-1)^(1/2))*x-1/2/b^3*d*a^3/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))-1/2/b^
3*d*a^3/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))-1/2/b^3*d*a^2/(1+d)*dilog(1+exp
(b*x+a)*(-d-1)^(1/2))-1/2/b^3*d*a^2/(1+d)*dilog(1-exp(b*x+a)*(-d-1)^(1/2))+
1/2/b^2/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x*a^2+1/3/b^3*d/(1+d)*ln(1+(1+d)*e
xp(2*b*x+2*a))*a^3+1/12*b*x^4+1/6/b^3*a^3/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b
*x+2*a)+1)+1/6/b^3*d*a^3/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1)-1/12*I
*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d/(exp(2*b*x+2*a)+
1)*exp(2*b*x+2*a))^2-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+
2*a)/(exp(2*b*x+2*a)+1))^2+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I/
(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))^2-1/2/b^2*d*a^2/(1+
d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))*x+1/2/b^2*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a
))*x*a^2-1/2/b^2*d*a^2/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))*x-1/8/b^3/(1+d)*
polylog(4,-(1+d)*exp(2*b*x+2*a))-1/6/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x^3+1
/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))^3+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(
exp(2*b*x+2*a)+1))^3-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a
)*d+exp(2*b*x+2*a)+1))^3+1/12*I*x^3*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*
x+2*a))+1/12*I*x^3*Pi*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))*csgn(I/(e
xp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))^2-1/12*I*x^3*Pi*csgn(
I*d)*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2+1/6*x^3*ln(exp(2*b*x+2*a
)*d+exp(2*b*x+2*a)+1)+1/12*I*x^3*Pi*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2
*a))^3-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)*d+ex
p(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a
)+1))+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I
*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/12*I*x^3*Pi*csgn(I*d)*csgn(I*exp(2*b*
x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))
```

**Maxima [A]** time = 3.46662, size = 169, normalized size = 1.32

$$\frac{1}{3} x^3 \operatorname{arccoth}(d \tanh(bx + a) + d + 1) + \frac{1}{36} \left( \frac{3x^4}{d} - \frac{2(4b^3x^3 \log((d+1)e^{2bx+2a}) + 1) + 6b^2x^2 \operatorname{Li}_2(-(d+1)e^{2bx+2a})}{b^4d} \right) - e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(1+d+d\*tanh(b\*x+a)),x, algorithm="maxima")

[Out]  $\frac{1}{3}x^3 \operatorname{arccoth}(d \tanh(bx + a) + d + 1) + \frac{1}{36} \left( 3x^4/d - 2(4b^3x^3 \log((d + 1)e^{2bx + 2a} + 1) + 6b^2x^2 \operatorname{dilog}(-(d + 1)e^{2bx + 2a}) - 6bx \operatorname{polylog}(3, -(d + 1)e^{2bx + 2a})) + 3 \operatorname{polylog}(4, -(d + 1)e^{2bx + 2a})) \right) / (b^4d) * b * d$

**Fricas [C]** time = 1.76501, size = 1160, normalized size = 9.06

$$\frac{b^4x^4 + 2b^3x^3 \log\left(\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 6b^2x^2\operatorname{Li}_2\left(\frac{1}{2}\sqrt{-4d-4}(\cosh(bx+a)+\sinh(bx+a))\right) - 6b^2x^2\operatorname{Li}_2\left(-\frac{1}{2}\right)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(1+d+d\*tanh(b\*x+a)),x, algorithm="fricas")

[Out]  $\frac{1}{12} \left( b^4x^4 + 2b^3x^3 \log((d + 2)\cosh(bx + a) + d\sinh(bx + a)) / (d\cosh(bx + a) + d\sinh(bx + a)) - 6b^2x^2 \operatorname{dilog}(1/2\sqrt{-4d - 4}(\cosh(bx + a) + \sinh(bx + a))) - 6b^2x^2 \operatorname{dilog}(-1/2\sqrt{-4d - 4}(\cosh(bx + a) + \sinh(bx + a))) + 2a^3 \log(2(d + 1)\cosh(bx + a) + 2(d + 1)\sinh(bx + a) + \sqrt{-4d - 4}) + 2a^3 \log(2(d + 1)\cosh(bx + a) + 2(d + 1)\sinh(bx + a) - \sqrt{-4d - 4}) + 12bx \operatorname{polylog}(3, 1/2\sqrt{-4d - 4}(\cosh(bx + a) + \sinh(bx + a))) + 12bx \operatorname{polylog}(3, -1/2\sqrt{-4d - 4}(\cosh(bx + a) + \sinh(bx + a))) - 2(b^3x^3 + a^3) \log(1/2\sqrt{-4d - 4}(\cosh(bx + a) + \sinh(bx + a)) + 1) - 2(b^3x^3 + a^3) \log(-1/2\sqrt{-4d - 4}(\cosh(bx + a) + \sinh(bx + a)) + 1) - 12 \operatorname{polylog}(4, 1/2\sqrt{-4d - 4}(\cosh(bx + a) + \sinh(bx + a))) - 12 \operatorname{polylog}(4, -1/2\sqrt{-4d - 4}(\cosh(bx + a) + \sinh(bx + a))) \right) / b^3$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*acoth(1+d+d\*tanh(b\*x+a)),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arccoth}(d \tanh(bx + a) + d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(1+d\*d\*tanh(b\*x+a)),x, algorithm="giac")

[Out] integrate(x^2\*arccoth(d\*tanh(b\*x + a) + d + 1), x)

### 3.209 $\int x \coth^{-1}(1 + d + d \tanh(a + bx)) dx$

**Optimal.** Leaf size=101

$$\frac{\text{PolyLog}\left(3, -(d+1)e^{2a+2bx}\right)}{8b^2} - \frac{x\text{PolyLog}\left(2, -(d+1)e^{2a+2bx}\right)}{4b} - \frac{1}{4}x^2 \log\left((d+1)e^{2a+2bx} + 1\right) + \frac{1}{2}x^2 \coth^{-1}(d \tanh(a + bx))$$

[Out] (b\*x^3)/6 + (x^2\*ArcCoth[1 + d + d\*Tanh[a + b\*x]])/2 - (x^2\*Log[1 + (1 + d)\*E^(2\*a + 2\*b\*x)])/4 - (x\*PolyLog[2, -((1 + d)\*E^(2\*a + 2\*b\*x))])/(4\*b) + PolyLog[3, -((1 + d)\*E^(2\*a + 2\*b\*x))]/(8\*b^2)

**Rubi [A]** time = 0.230903, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {6240, 2184, 2190, 2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3, -(d+1)e^{2a+2bx}\right)}{8b^2} - \frac{x\text{PolyLog}\left(2, -(d+1)e^{2a+2bx}\right)}{4b} - \frac{1}{4}x^2 \log\left((d+1)e^{2a+2bx} + 1\right) + \frac{1}{2}x^2 \coth^{-1}(d \tanh(a + bx))$$

Antiderivative was successfully verified.

[In] Int[x\*ArcCoth[1 + d + d\*Tanh[a + b\*x]],x]

[Out] (b\*x^3)/6 + (x^2\*ArcCoth[1 + d + d\*Tanh[a + b\*x]])/2 - (x^2\*Log[1 + (1 + d)\*E^(2\*a + 2\*b\*x)])/4 - (x\*PolyLog[2, -((1 + d)\*E^(2\*a + 2\*b\*x))])/(4\*b) + PolyLog[3, -((1 + d)\*E^(2\*a + 2\*b\*x))]/(8\*b^2)

#### Rule 6240

Int[ArcCoth[(c\_.) + (d\_.)\*Tanh[(a\_.) + (b\_.)\*(x\_)]]\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((e + f\*x)^(m + 1)\*ArcCoth[c + d\*Tanh[a + b\*x]])/(f\*(m + 1)), x] + Dist[b/(f\*(m + 1)), Int[(e + f\*x)^(m + 1)/(c - d + c\*E^(2\*a + 2\*b\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d)^2, 1]

#### Rule 2184

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.), x\_Symbol] := Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[(c + d\*x)^m\*(F^(g\*(e + f\*x)))^n/(a + b\*(F^(g\*(e + f\*x))))^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*(a_) + (b_)*(x_)))^(n_)]*((f_) + (g_)
*(x_))^(m_), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x \coth^{-1}(1 + d + d \tanh(a + bx)) dx &= \frac{1}{2}x^2 \coth^{-1}(1 + d + d \tanh(a + bx)) + \frac{1}{2}b \int \frac{x^2}{1 + (1 + d)e^{2a+2bx}} dx \\
&= \frac{bx^3}{6} + \frac{1}{2}x^2 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{2}(b(1 + d)) \int \frac{e^{2a+2bx}x^2}{1 + (1 + d)e^{2a+2bx}} dx \\
&= \frac{bx^3}{6} + \frac{1}{2}x^2 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{4}x^2 \log(1 + (1 + d)e^{2a+2bx}) + \frac{1}{2} \\
&= \frac{bx^3}{6} + \frac{1}{2}x^2 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{4}x^2 \log(1 + (1 + d)e^{2a+2bx}) - \frac{x}{2} \\
&= \frac{bx^3}{6} + \frac{1}{2}x^2 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{4}x^2 \log(1 + (1 + d)e^{2a+2bx}) - \frac{x}{2} \\
&= \frac{bx^3}{6} + \frac{1}{2}x^2 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{4}x^2 \log(1 + (1 + d)e^{2a+2bx}) - \frac{x}{2}
\end{aligned}$$

**Mathematica [A]** time = 0.0958579, size = 91, normalized size = 0.9

$$\frac{2bx \operatorname{PolyLog}\left(2, -\frac{e^{-2(a+bx)}}{d+1}\right) + \operatorname{PolyLog}\left(3, -\frac{e^{-2(a+bx)}}{d+1}\right) + 2b^2x^2\left(2 \coth^{-1}(d \tanh(a + bx) + d + 1) - \log\left(\frac{e^{-2(a+bx)}}{d+1} + 1\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[1 + d + d\*Tanh[a + b\*x]], x]

[Out] (2\*b^2\*x^2\*(2\*ArcCoth[1 + d + d\*Tanh[a + b\*x]] - Log[1 + 1/((1 + d)\*E^(2\*(a + b\*x)))])) + 2\*b\*x\*PolyLog[2, -(1/((1 + d)\*E^(2\*(a + b\*x))))] + PolyLog[3, -(1/((1 + d)\*E^(2\*(a + b\*x))))]/(8\*b^2)

**Maple [C]** time = 10.194, size = 1584, normalized size = 15.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(1+d+d\*tanh(b\*x+a)), x)

[Out] -1/8\*I\*x^2\*Pi\*csgn(I\*exp(2\*b\*x+2\*a))\*csgn(I\*exp(2\*b\*x+2\*a)/(exp(2\*b\*x+2\*a)+1))^2+1/6\*b\*x^3-1/4/b^2\*a^2/(1+d)\*ln(exp(2\*b\*x+2\*a)\*d+exp(2\*b\*x+2\*a)+1)+1/2

$$\begin{aligned}
& /b^2*a/(1+d)*\operatorname{dilog}(1-\exp(b*x+a)*(-d-1)^{(1/2)})-1/4/b^2/(1+d)*\ln(1+(1+d)*\exp(2*b*x+2*a))*a^2-1/4/b/(1+d)*\operatorname{polylog}(2,-(1+d)*\exp(2*b*x+2*a))*x-1/4/b^2/(1+d) \\
& )*\operatorname{polylog}(2,-(1+d)*\exp(2*b*x+2*a))*a+1/2/b^2*a^2/(1+d)*\ln(1+\exp(b*x+a)*(-d-1)^{(1/2)})+1/2/b^2*a^2/(1+d)*\ln(1-\exp(b*x+a)*(-d-1)^{(1/2)})+1/8/b^2*d/(1+d)*\operatorname{polylog}(3,-(1+d)*\exp(2*b*x+2*a))+1/2/b^2*a/(1+d)*\operatorname{dilog}(1+\exp(b*x+a)*(-d-1)^{(1/2)})-1/4*d/(1+d)*\ln(1+(1+d)*\exp(2*b*x+2*a))*x^2+1/4*x^2*\ln(\exp(2*b*x+2*a)*d+\exp(2*b*x+2*a)+1)-1/4/b^2*d*a^2/(1+d)*\ln(\exp(2*b*x+2*a)*d+\exp(2*b*x+2*a)+1)+1/8*I*x^2*Pi*csgn(I*(\exp(2*b*x+2*a)*d+\exp(2*b*x+2*a)+1))*csgn(I/(\exp(2*b*x+2*a)+1))*(\exp(2*b*x+2*a)*d+\exp(2*b*x+2*a)+1)^2-1/8*I*x^2*Pi*csgn(I*d)*csgn(I*d/(\exp(2*b*x+2*a)+1)*\exp(2*b*x+2*a))^2-1/2*x^2*\ln(\exp(b*x+a))-1/4*x^2*\ln(d)+1/8*I*x^2*Pi*csgn(I*d/(\exp(2*b*x+2*a)+1)*\exp(2*b*x+2*a))^3-1/8*I*x^2*Pi*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))*csgn(I*d/(\exp(2*b*x+2*a)+1)*\exp(2*b*x+2*a))^2-1/4/(1+d)*\ln(1+(1+d)*\exp(2*b*x+2*a))*x^2+1/8/b^2/(1+d)*\operatorname{polylog}(3,-(1+d)*\exp(2*b*x+2*a))+1/8*I*x^2*Pi*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I/(\exp(2*b*x+2*a)+1)*(\exp(2*b*x+2*a)*d+\exp(2*b*x+2*a)+1))^2+1/8*I*x^2*Pi*csgn(I*\exp(2*b*x+2*a))^3+1/8*I*x^2*Pi*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3-1/8*I*x^2*Pi*csgn(I/(\exp(2*b*x+2*a)+1)*(\exp(2*b*x+2*a)*d+\exp(2*b*x+2*a)+1))^3+1/2/b^2*d*a/(1+d)*\operatorname{dilog}(1+\exp(b*x+a)*(-d-1)^{(1/2)})+1/2/b^2*d*a/(1+d)*\operatorname{dilog}(1-\exp(b*x+a)*(-d-1)^{(1/2)})-1/2/b/(1+d)*\ln(1+(1+d)*\exp(2*b*x+2*a))*x*a-1/4/b^2*d/(1+d)*\ln(1+(1+d)*\exp(2*b*x+2*a))*a^2-1/4/b*d/(1+d)*\operatorname{polylog}(2,-(1+d)*\exp(2*b*x+2*a))*x-1/4/b^2*d/(1+d)*\operatorname{polylog}(2,-(1+d)*\exp(2*b*x+2*a))*a+1/2/b*a/(1+d)*\ln(1+\exp(b*x+a)*(-d-1)^{(1/2)})*x+1/2/b*a/(1+d)*\ln(1-\exp(b*x+a)*(-d-1)^{(1/2)})*x+1/2/b^2*d*a^2/(1+d)*\ln(1+\exp(b*x+a)*(-d-1)^{(1/2)})+1/2/b^2*d*a^2/(1+d)*\ln(1-\exp(b*x+a)*(-d-1)^{(1/2)})-1/8*I*x^2*Pi*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2+1/8*I*x^2*Pi*csgn(I*\exp(b*x+a))^2*csgn(I*\exp(2*b*x+2*a))-1/4*I*x^2*Pi*csgn(I*\exp(b*x+a))*csgn(I*\exp(2*b*x+2*a))^2+1/8*I*x^2*Pi*csgn(I*d)*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))*csgn(I*d/(\exp(2*b*x+2*a)+1)*\exp(2*b*x+2*a))+1/8*I*x^2*Pi*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))-1/8*I*x^2*Pi*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*(\exp(2*b*x+2*a)*d+\exp(2*b*x+2*a)+1))*csgn(I/(\exp(2*b*x+2*a)+1)*(\exp(2*b*x+2*a)*d+\exp(2*b*x+2*a)+1))-1/2/b*d/(1+d)*\ln(1+(1+d)*\exp(2*b*x+2*a))*x*a+1/2/b*d*a/(1+d)*\ln(1+\exp(b*x+a)*(-d-1)^{(1/2)})*x+1/2/b*d*a/(1+d)*\ln(1-\exp(b*x+a)*(-d-1)^{(1/2)})*x
\end{aligned}$$

**Maxima [A]** time = 3.48612, size = 136, normalized size = 1.35

$$\frac{1}{24} \left( \frac{4x^3}{d} - \frac{3(2b^2x^2 \log((d+1)e^{2bx+2a}) + 1) + 2bx \operatorname{Li}_2(-(d+1)e^{2bx+2a}) - \operatorname{Li}_3(-(d+1)e^{2bx+2a})}{b^3d} \right) bd + \frac{1}{2} x^2 \operatorname{arccoth}($$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1+d\*d\*tanh(b\*x+a)),x, algorithm="maxima")



[Out]  $\frac{1}{24} \left( \frac{4x^3}{d} - 3(2b^2x^2 \log((d+1)e^{(2bx+2a)} + 1) + 2bx \operatorname{dilog}(-(d+1)e^{(2bx+2a)}) - \operatorname{polylog}(3, -(d+1)e^{(2bx+2a)})) / (b^3d) \right) * b^2d + \frac{1}{2} x^2 \operatorname{arccoth}(d \tanh(bx+a) + d + 1)$

**Fricas [C]** time = 1.74289, size = 953, normalized size = 9.44

$2b^3x^3 + 3b^2x^2 \log\left(\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 6bx\operatorname{Li}_2\left(\frac{1}{2}\sqrt{-4d-4}(\cosh(bx+a)+\sinh(bx+a))\right) - 6bx\operatorname{Li}_2\left(-\frac{1}{2}\sqrt{-4d-4}(\cosh(bx+a)+\sinh(bx+a))\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="fricas")`

[Out]  $\frac{1}{12} \left( 2b^3x^3 + 3b^2x^2 \log\left(\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 6bx \operatorname{dilog}\left(\frac{1}{2}\sqrt{-4d-4}(\cosh(bx+a)+\sinh(bx+a))\right) - 6bx \operatorname{dilog}\left(-\frac{1}{2}\sqrt{-4d-4}(\cosh(bx+a)+\sinh(bx+a))\right) - 3a^2 \log(2(d+1)\cosh(bx+a)+2(d+1)\sinh(bx+a)+\sqrt{-4d-4}) - 3a^2 \log(2(d+1)\cosh(bx+a)+2(d+1)\sinh(bx+a)-\sqrt{-4d-4}) - 3(b^2x^2-a^2) \log\left(\frac{1}{2}\sqrt{-4d-4}(\cosh(bx+a)+\sinh(bx+a))+1\right) - 3(b^2x^2-a^2) \log\left(-\frac{1}{2}\sqrt{-4d-4}(\cosh(bx+a)+\sinh(bx+a))+1\right) + 6 \operatorname{polylog}(3, \frac{1}{2}\sqrt{-4d-4}(\cosh(bx+a)+\sinh(bx+a))) + 6 \operatorname{polylog}(3, -\frac{1}{2}\sqrt{-4d-4}(\cosh(bx+a)+\sinh(bx+a))) \right) / b^2$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*acoth(1+d+d*tanh(b*x+a)),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arccoth}(d \tanh(bx+a) + d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1+d*d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(d*tanh(b*x + a) + d + 1), x)
```

### 3.210 $\int \coth^{-1}(1 + d + d \tanh(a + bx)) dx$

**Optimal.** Leaf size=69

$$-\frac{\text{PolyLog}\left(2, -(d+1)e^{2a+2bx}\right)}{4b} - \frac{1}{2}x \log\left((d+1)e^{2a+2bx} + 1\right) + x \coth^{-1}(d \tanh(a + bx) + d + 1) + \frac{bx^2}{2}$$

[Out] (b\*x^2)/2 + x\*ArcCoth[1 + d + d\*Tanh[a + b\*x]] - (x\*Log[1 + (1 + d)\*E^(2\*a + 2\*b\*x)])/2 - PolyLog[2, -((1 + d)\*E^(2\*a + 2\*b\*x))]/(4\*b)

**Rubi [A]** time = 0.140006, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {6232, 2184, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2, -(d+1)e^{2a+2bx}\right)}{4b} - \frac{1}{2}x \log\left((d+1)e^{2a+2bx} + 1\right) + x \coth^{-1}(d \tanh(a + bx) + d + 1) + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[1 + d + d\*Tanh[a + b\*x]], x]

[Out] (b\*x^2)/2 + x\*ArcCoth[1 + d + d\*Tanh[a + b\*x]] - (x\*Log[1 + (1 + d)\*E^(2\*a + 2\*b\*x)])/2 - PolyLog[2, -((1 + d)\*E^(2\*a + 2\*b\*x))]/(4\*b)

#### Rule 6232

Int[ArcCoth[(c\_.) + (d\_.)\*Tanh[(a\_.) + (b\_.)\*(x\_)]], x\_Symbol] := Simp[x\*ArcCoth[c + d\*Tanh[a + b\*x]], x] + Dist[b, Int[x/(c - d + c\*E^(2\*a + 2\*b\*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]

#### Rule 2184

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2190

Int[((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[(c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a]/(b\*f\*g\*n\*Log[F]), x] - Di

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^(n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
)^(n)], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
 \int \coth^{-1}(1 + d + d \tanh(a + bx)) dx &= x \coth^{-1}(1 + d + d \tanh(a + bx)) + b \int \frac{x}{1 + (1 + d)e^{2a+2bx}} dx \\
 &= \frac{bx^2}{2} + x \coth^{-1}(1 + d + d \tanh(a + bx)) - (b(1 + d)) \int \frac{e^{2a+2bx} x}{1 + (1 + d)e^{2a+2bx}} dx \\
 &= \frac{bx^2}{2} + x \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{2} x \log(1 + (1 + d)e^{2a+2bx}) + \frac{1}{2} \int \log \left( \frac{\text{Subst}\left(\frac{x}{1 + (1 + d)e^{2a+2bx}}, e^{2a+2bx}\right)}{1 + (1 + d)e^{2a+2bx}} \right) dx \\
 &= \frac{bx^2}{2} + x \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{2} x \log(1 + (1 + d)e^{2a+2bx}) + \frac{\text{Li}_2(-1 + (1 + d)e^{2a+2bx})}{2}
 \end{aligned}$$

**Mathematica [B]** time = 0.858181, size = 201, normalized size = 2.91

---


$$-2\text{PolyLog}\left(2, -\sqrt{-d-1}e^{a+bx}\right) - 2\text{PolyLog}\left(2, \sqrt{-d-1}e^{a+bx}\right) - 2\log\left(e^{a+bx}\right)\log\left(1 - \sqrt{-d-1}e^{a+bx}\right) - 2\log\left(e^{a+bx}\right)\log\left(1 + \sqrt{-d-1}e^{a+bx}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[1 + d + d*Tanh[a + b*x]], x]
```

```
[Out] x*ArcCoth[1 + d + d*Tanh[a + b*x]] + (b^2*x^2 + Log[E^(a + b*x)]^2 - 2*Log[E^(a + b*x)]*Log[1 - Sqrt[-1 - d]*E^(a + b*x)] - 2*Log[E^(a + b*x)]*Log[1 + Sqrt[-1 - d]*E^(a + b*x)] + 2*Log[E^(a + b*x)]*Log[E^(-a - b*x) + (1 + d)*E^(a + b*x)] - 2*b*x*Log[(2 + d)*Cosh[a + b*x] + d*Sinh[a + b*x]] - 2*PolyL
```

$\log[2, -(\text{Sqrt}[-1 - d]*E^{(a + b*x)})] - 2*\text{PolyLog}[2, \text{Sqrt}[-1 - d]*E^{(a + b*x)}] / (4*b)$

**Maple [B]** time = 0.156, size = 247, normalized size = 3.6

$$\frac{\text{arccoth}(1 + d + d \tanh(bx + a)) \ln(d \tanh(bx + a) - d)}{2b} + \frac{\text{arccoth}(1 + d + d \tanh(bx + a)) \ln(d \tanh(bx + a) + d)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(1+d+d*tanh(b*x+a)),x)`

[Out]  $-1/2/b*\text{arccoth}(1+d+d*\tanh(b*x+a))*\ln(d*\tanh(b*x+a)-d)+1/2/b*\text{arccoth}(1+d+d*\tanh(b*x+a))*\ln(d*\tanh(b*x+a)+d)+1/4/b*\text{dilog}((d*\tanh(b*x+a)+d+2)/(2*d+2))+1/4/b*\ln(d*\tanh(b*x+a)-d)*\ln((d*\tanh(b*x+a)+d+2)/(2*d+2))-1/4/b*\text{dilog}(1/2*(d*\tanh(b*x+a)+d)/d)-1/4/b*\ln(d*\tanh(b*x+a)-d)*\ln(1/2*(d*\tanh(b*x+a)+d)/d)+1/8/b*\ln(d*\tanh(b*x+a)+d)^2-1/4/b*\text{dilog}(1+1/2*d*\tanh(b*x+a)+1/2*d)-1/4/b*\ln(d*\tanh(b*x+a)+d)*\ln(1+1/2*d*\tanh(b*x+a)+1/2*d)$

**Maxima [A]** time = 3.44926, size = 97, normalized size = 1.41

$$\frac{1}{4} b d \left( \frac{2x^2}{d} - \frac{2bx \log((d+1)e^{2bx+2a} + 1) + \text{Li}_2(-(d+1)e^{2bx+2a})}{b^2 d} \right) + x \text{arccoth}(d \tanh(bx + a) + d + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(1+d+d*tanh(b*x+a)),x, algorithm="maxima")`

[Out]  $1/4*b*d*(2*x^2/d - (2*b*x*\log((d + 1)*e^{(2*b*x + 2*a) + 1) + \text{dilog}(-(d + 1)*e^{(2*b*x + 2*a)})))/(b^2*d)) + x*\text{arccoth}(d*\tanh(b*x + a) + d + 1)$

**Fricas [B]** time = 2.13949, size = 707, normalized size = 10.25

$$\frac{b^2 x^2 + bx \log\left(\frac{(d+2) \cosh(bx+a)+d \sinh(bx+a)}{d \cosh(bx+a)+d \sinh(bx+a)}\right) + a \log\left(2(d+1) \cosh(bx+a) + 2(d+1) \sinh(bx+a) + \sqrt{-4d-4}\right) + a \log\left(2(d+1) \cosh(bx+a) - 2(d+1) \sinh(bx+a) + \sqrt{-4d-4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+d*d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + b*x*log(((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/(d*cosh(b*x + a) + d*sinh(b*x + a))) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a) + sqrt(-4*d - 4)) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a) - sqrt(-4*d - 4)) - (b*x + a)*log(1/2*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-1/2*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - dilog(1/2*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) - dilog(-1/2*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acoth}(d \tanh(a + bx) + d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1+d*d*tanh(b*x+a)),x)
```

```
[Out] Integral(acoth(d*tanh(a + b*x) + d + 1), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}(d \tanh(bx + a) + d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+d*d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*tanh(b*x + a) + d + 1), x)
```

$$3.211 \quad \int \frac{\coth^{-1}(1+d+d \tanh(ax))}{x} dx$$

**Optimal.** Leaf size=18

$$\text{CannotIntegrate}\left(\frac{\coth^{-1}(d \tanh(ax) + d + 1)}{x}, x\right)$$

[Out] CannotIntegrate[ArcCoth[1 + d + d\*Tanh[a + b\*x]]/x, x]

**Rubi [A]** time = 0.0772992, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\coth^{-1}(1 + d + d \tanh(ax))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[ArcCoth[1 + d + d\*Tanh[a + b\*x]]/x,x]

[Out] Defer[Int][ArcCoth[1 + d + d\*Tanh[a + b\*x]]/x, x]

Rubi steps

$$\int \frac{\coth^{-1}(1 + d + d \tanh(ax))}{x} dx = \int \frac{\coth^{-1}(1 + d + d \tanh(ax))}{x} dx$$

**Mathematica [A]** time = 2.97231, size = 0, normalized size = 0.

$$\int \frac{\coth^{-1}(1 + d + d \tanh(ax))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcCoth[1 + d + d\*Tanh[a + b\*x]]/x,x]

[Out] Integrate[ArcCoth[1 + d + d\*Tanh[a + b\*x]]/x, x]

---

**Maple [A]** time = 0.364, size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(1 + d + d \tanh(bx + a))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(1+d+d\*tanh(b\*x+a))/x,x)

[Out] int(arccoth(1+d+d\*tanh(b\*x+a))/x,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(d \tanh(bx + a) + d + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+d+d\*tanh(b\*x+a))/x,x, algorithm="maxima")

[Out] integrate(arccoth(d\*tanh(b\*x + a) + d + 1)/x, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arccoth}(d \tanh(bx + a) + d + 1)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+d+d\*tanh(b\*x+a))/x,x, algorithm="fricas")

[Out] integral(arccoth(d\*tanh(b\*x + a) + d + 1)/x, x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}(d \tanh(a + bx) + d + 1)}{x} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(1+d*d*tanh(b*x+a))/x,x)`

[Out] `Integral(acoth(d*tanh(a + b*x) + d + 1)/x, x)`

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(d \tanh(bx + a) + d + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(1+d*d*tanh(b*x+a))/x,x, algorithm="giac")`

[Out] `integrate(arccoth(d*tanh(b*x + a) + d + 1)/x, x)`

### 3.212 $\int x^3 \coth^{-1}(1 - d - d \tanh(a + bx)) dx$

**Optimal.** Leaf size=168

$$\frac{3x^2 \text{PolyLog}\left(3, -(1-d)e^{2a+2bx}\right)}{8b^2} - \frac{3x \text{PolyLog}\left(4, -(1-d)e^{2a+2bx}\right)}{8b^3} + \frac{3 \text{PolyLog}\left(5, -(1-d)e^{2a+2bx}\right)}{16b^4} - \frac{x^3 \text{PolyLog}\left(2, -(1-d)e^{2a+2bx}\right)}{4b}$$

[Out] (b\*x^5)/20 + (x^4\*ArcCoth[1 - d - d\*Tanh[a + b\*x]])/4 - (x^4\*Log[1 + (1 - d)\*E^(2\*a + 2\*b\*x)])/8 - (x^3\*PolyLog[2, -((1 - d)\*E^(2\*a + 2\*b\*x))])/(4\*b) + (3\*x^2\*PolyLog[3, -((1 - d)\*E^(2\*a + 2\*b\*x))])/(8\*b^2) - (3\*x\*PolyLog[4, -((1 - d)\*E^(2\*a + 2\*b\*x))])/(8\*b^3) + (3\*PolyLog[5, -((1 - d)\*E^(2\*a + 2\*b\*x))])/(16\*b^4)

**Rubi [A]** time = 0.30328, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {6240, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2 \text{PolyLog}\left(3, -(1-d)e^{2a+2bx}\right)}{8b^2} - \frac{3x \text{PolyLog}\left(4, -(1-d)e^{2a+2bx}\right)}{8b^3} + \frac{3 \text{PolyLog}\left(5, -(1-d)e^{2a+2bx}\right)}{16b^4} - \frac{x^3 \text{PolyLog}\left(2, -(1-d)e^{2a+2bx}\right)}{4b}$$

Antiderivative was successfully verified.

[In] Int[x^3\*ArcCoth[1 - d - d\*Tanh[a + b\*x]], x]

[Out] (b\*x^5)/20 + (x^4\*ArcCoth[1 - d - d\*Tanh[a + b\*x]])/4 - (x^4\*Log[1 + (1 - d)\*E^(2\*a + 2\*b\*x)])/8 - (x^3\*PolyLog[2, -((1 - d)\*E^(2\*a + 2\*b\*x))])/(4\*b) + (3\*x^2\*PolyLog[3, -((1 - d)\*E^(2\*a + 2\*b\*x))])/(8\*b^2) - (3\*x\*PolyLog[4, -((1 - d)\*E^(2\*a + 2\*b\*x))])/(8\*b^3) + (3\*PolyLog[5, -((1 - d)\*E^(2\*a + 2\*b\*x))])/(16\*b^4)

#### Rule 6240

Int[ArcCoth[(c\_.) + (d\_.)\*Tanh[(a\_.) + (b\_.)\*(x\_)]]\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^(m + 1)\*ArcCoth[c + d\*Tanh[a + b\*x]])/(f\*(m + 1)), x] + Dist[b/(f\*(m + 1)), Int[(e + f\*x)^(m + 1)/(c - d + c\*E^(2\*a + 2\*b\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d)^2, 1]

#### Rule 2184

Int[(((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[

$b/a$ , Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_)]/((a\_) + (b\_)\*((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_)\*((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_)))^(n\_))]\*(f\_) + (g\_)\*(x\_)]^(m\_), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 6609

Int[((e\_) + (f\_)\*(x\_))^(m\_)\*PolyLog[n\_, (d\_)\*((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_)))^(p\_)]], x\_Symbol] := Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p])/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\* (F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_)\*((a\_) + (b\_)\*(x\_))^(p\_)]/((d\_) + (e\_)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
\int x^3 \coth^{-1}(1-d-d \tanh(a+bx)) dx &= \frac{1}{4}x^4 \coth^{-1}(1-d-d \tanh(a+bx)) + \frac{1}{4}b \int \frac{x^4}{1+(1-d)e^{2a+2bx}} dx \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{4}(b(1-d)) \int \frac{e^{2a+2bx}x^4}{1+(1-d)e^{2a+2bx}} dx \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{8}x^4 \log(1+(1-d)e^{2a+2bx}) + \frac{1}{2} \int \frac{x^3}{1+(1-d)e^{2a+2bx}} dx \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{8}x^4 \log(1+(1-d)e^{2a+2bx}) - \frac{x^3}{2} \int \frac{1}{1+(1-d)e^{2a+2bx}} dx \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{8}x^4 \log(1+(1-d)e^{2a+2bx}) - \frac{x^3}{2} \int \frac{1}{1+(1-d)e^{2a+2bx}} dx \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{8}x^4 \log(1+(1-d)e^{2a+2bx}) - \frac{x^3}{2} \int \frac{1}{1+(1-d)e^{2a+2bx}} dx \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{8}x^4 \log(1+(1-d)e^{2a+2bx}) - \frac{x^3}{2} \int \frac{1}{1+(1-d)e^{2a+2bx}} dx \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{8}x^4 \log(1+(1-d)e^{2a+2bx}) - \frac{x^3}{2} \int \frac{1}{1+(1-d)e^{2a+2bx}} dx \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{8}x^4 \log(1+(1-d)e^{2a+2bx}) - \frac{x^3}{2} \int \frac{1}{1+(1-d)e^{2a+2bx}} dx
\end{aligned}$$

**Mathematica [A]** time = 0.185126, size = 144, normalized size = 0.86

$$\frac{1}{16} \left( \frac{6x^2 \text{PolyLog}\left(3, \frac{e^{-2(a+bx)}}{d-1}\right)}{b^2} + \frac{6x \text{PolyLog}\left(4, \frac{e^{-2(a+bx)}}{d-1}\right)}{b^3} + \frac{3 \text{PolyLog}\left(5, \frac{e^{-2(a+bx)}}{d-1}\right)}{b^4} + \frac{4x^3 \text{PolyLog}\left(2, \frac{e^{-2(a+bx)}}{d-1}\right)}{b} - 2x^4 \log \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*ArcCoth[1-d-d\*Tanh[a+b\*x]],x]

[Out] (4\*x^4\*ArcCoth[1-d-d\*Tanh[a+b\*x]] - 2\*x^4\*Log[1-1/((-1+d)\*E^(2\*(a+b\*x))]) + (4\*x^3\*PolyLog[2, 1/((-1+d)\*E^(2\*(a+b\*x))])]/b + (6\*x^2\*PolyLog[3, 1/((-1+d)\*E^(2\*(a+b\*x))])]/b^2 + (6\*x\*PolyLog[4, 1/((-1+d)\*E^(2\*(a+b\*x))])]/b^3 + (3\*PolyLog[5, 1/((-1+d)\*E^(2\*(a+b\*x))])]/b^4)/16

**Maple [C]** time = 39.194, size = 1802, normalized size = 10.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^3 \cdot \text{arccoth}(1-d-d \cdot \tanh(b \cdot x+a)), x)$

[Out] 
$$\begin{aligned} & -1/16 \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1)) \cdot \text{csgn}(I \cdot (\exp(2 \cdot b \cdot x + 2 \cdot a) \cdot d - \exp(2 \cdot b \cdot x + 2 \cdot a) - 1)) \cdot \text{csgn}(I / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1) \cdot (\exp(2 \cdot b \cdot x + 2 \cdot a) \cdot d - \exp(2 \cdot b \cdot x + 2 \cdot a) - 1)) + 1/16 \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1)) \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a) / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1)) + 3/16 \cdot b^4 \cdot d / (d-1) \cdot \text{polylog}(5, (d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) - 1/2 \cdot b^4 \cdot a^3 / (d-1) \cdot \text{dilog}(1 - \exp(b \cdot x + a) \cdot (d-1)^{(1/2)}) + 1/2 \cdot b^4 \cdot d \cdot a^3 / (d-1) \cdot \text{dilog}(1 - \exp(b \cdot x + a) \cdot (d-1)^{(1/2)}) - 3/16 \cdot b^4 / (d-1) \cdot \text{polylog}(5, (d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) + 1/8 / (d-1) \cdot \ln(1 - (d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot x^4 + 3/8 \cdot b^4 / (d-1) \cdot \ln(1 - (d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot a^4 + 1/4 \cdot b / (d-1) \cdot \text{polylog}(2, (d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot x^3 + 1/4 \cdot b^4 / (d-1) \cdot \text{polylog}(2, (d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot a^3 - 1/2 \cdot b^4 \cdot a^4 / (d-1) \cdot \ln(1 - \exp(b \cdot x + a) \cdot (d-1)^{(1/2)}) + 1/2 \cdot b^4 \cdot d \cdot a^3 / (d-1) \cdot \text{dilog}(1 + \exp(b \cdot x + a) \cdot (d-1)^{(1/2)}) - 1/2 \cdot b^3 \cdot a^3 / (d-1) \cdot \ln(1 - \exp(b \cdot x + a) \cdot (d-1)^{(1/2)}) \cdot x - 3/8 \cdot b^4 \cdot d / (d-1) \cdot \ln(1 - (d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot a^4 - 1/4 \cdot b \cdot d / (d-1) \cdot \text{polylog}(2, (d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot x^3 - 1/4 \cdot b^4 \cdot d / (d-1) \cdot \text{polylog}(2, (d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot a^3 + 3/8 \cdot b^2 \cdot d / (d-1) \cdot \text{polylog}(3, (d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot x^2 - 3/8 \cdot b^3 \cdot d / (d-1) \cdot \text{polylog}(4, (d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot x + 1/2 \cdot b^4 \cdot d \cdot a^4 / (d-1) \cdot \ln(1 - \exp(b \cdot x + a) \cdot (d-1)^{(1/2)}) + 1/2 \cdot b^4 \cdot d \cdot a^4 / (d-1) \cdot \ln(1 + \exp(b \cdot x + a) \cdot (d-1)^{(1/2)}) + 1/2 \cdot b^3 / (d-1) \cdot \ln(1 - (d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot x \cdot a^3 + 1/16 \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I \cdot \exp(b \cdot x + a))^2 \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) - 1/8 \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I \cdot \exp(b \cdot x + a)) \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a))^2 + 1/16 \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a))^3 + 1/16 \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a) / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1))^3 + 1/20 \cdot b \cdot x^5 - 1/16 \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I \cdot d / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a))^3 - 1/16 \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I \cdot d) \cdot \text{csgn}(I \cdot d / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a))^2 + 1/16 \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1)) \cdot \text{csgn}(I / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1) \cdot (\exp(2 \cdot b \cdot x + 2 \cdot a) \cdot d - \exp(2 \cdot b \cdot x + 2 \cdot a) - 1))^2 + 1/16 \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I \cdot (\exp(2 \cdot b \cdot x + 2 \cdot a) \cdot d - \exp(2 \cdot b \cdot x + 2 \cdot a) - 1)) \cdot \text{csgn}(I / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1) \cdot (\exp(2 \cdot b \cdot x + 2 \cdot a) \cdot d - \exp(2 \cdot b \cdot x + 2 \cdot a) - 1))^2 - 1/16 \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1)) \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a) / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1))^2 - 1/16 \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a) / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1))^2 + 1/16 \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1) \cdot (\exp(2 \cdot b \cdot x + 2 \cdot a) \cdot d - \exp(2 \cdot b \cdot x + 2 \cdot a) - 1))^3 - 1/2 \cdot b^4 \cdot a^3 / (d-1) \cdot \text{dilog}(1 + \exp(b \cdot x + a) \cdot (d-1)^{(1/2)}) + 1/8 \cdot x^4 \cdot \ln(\exp(2 \cdot b \cdot x + 2 \cdot a) \cdot d - \exp(2 \cdot b \cdot x + 2 \cdot a) - 1) - 1/2 \cdot b^4 \cdot a^4 / (d-1) \cdot \ln(1 + \exp(b \cdot x + a) \cdot (d-1)^{(1/2)}) - 1/8 \cdot d / (d-1) \cdot \ln(1 - (d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot x^4 + 1/16 \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I \cdot d) \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a) / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1)) \cdot \text{csgn}(I \cdot d / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) - 1/8 \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1) \cdot (\exp(2 \cdot b \cdot x + 2 \cdot a) \cdot d - \exp(2 \cdot b \cdot x + 2 \cdot a) - 1))^2 + 1/8 \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I \cdot d / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a))^2 - 1/8 \cdot b^4 \cdot d \cdot a^4 / (d-1) \cdot \ln(\exp(2 \cdot b \cdot x + 2 \cdot a) \cdot d - \exp(2 \cdot b \cdot x + 2 \cdot a) - 1) - 1/16 \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a) / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1)) \cdot \text{csgn}(I \cdot d / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a))^2 + 1/8 \cdot b^4 \cdot a^4 / (d-1) \cdot \ln(\exp(2 \cdot b \cdot x + 2 \cdot a) \cdot d - \exp(2 \cdot b \cdot x + 2 \cdot a) - 1) - 1/4 \cdot x^4 \cdot \ln(\exp(b \cdot x + a)) - 1/8 \cdot x^4 \cdot \ln(d) - 3/8 \cdot b^2 / (d-1) \cdot \text{polylog}(3, (d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot x^2 + 3/8 \cdot b^3 / (d-1) \cdot \text{polylog}(4, (d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot x - 1/2 \cdot b^3 \cdot d / (d-1) \cdot \ln(1 - (d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot x \cdot a^3 + 1/2 \cdot b^3 \cdot d \cdot a^3 / (d-1) \cdot \ln(1 - \exp(b \cdot x + a) \cdot (d-1)^{(1/2)}) \cdot x + 1/2 \cdot b^3 \cdot d \cdot a^3 / (d-1) \cdot \ln(1 + \exp(b \cdot x + a) \cdot (d-1)^{(1/2)}) \cdot x \end{aligned}$$

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**Maxima [A]** time = 3.4718, size = 197, normalized size = 1.17

$$-\frac{1}{4}x^4 \operatorname{arccoth}(d \tanh(bx + a) + d - 1) + \frac{1}{40} \left( \frac{2x^5}{d} - \frac{5(2b^4x^4 \log(-(d-1)e^{2bx+2a}) + 1) + 4b^3x^3 \operatorname{Li}_2((d-1)e^{2bx+2a})}{d} \right) -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(1-d-d\*tanh(b\*x+a)),x, algorithm="maxima")

[Out] -1/4\*x^4\*arccoth(d\*tanh(b\*x + a) + d - 1) + 1/40\*(2\*x^5/d - 5\*(2\*b^4\*x^4\*log(-(d - 1)\*e^(2\*b\*x + 2\*a) + 1) + 4\*b^3\*x^3\*dilog((d - 1)\*e^(2\*b\*x + 2\*a)) - 6\*b^2\*x^2\*polylog(3, (d - 1)\*e^(2\*b\*x + 2\*a)) + 6\*b\*x\*polylog(4, (d - 1)\*e^(2\*b\*x + 2\*a)) - 3\*polylog(5, (d - 1)\*e^(2\*b\*x + 2\*a)))/(b^5\*d))\*b\*d

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**Fricas [C]** time = 2.09938, size = 1280, normalized size = 7.62

$$2b^5x^5 - 5b^4x^4 \log\left(\frac{d \cosh(bx+a) + d \sinh(bx+a)}{(d-2) \cosh(bx+a) + d \sinh(bx+a)}\right) - 20b^3x^3 \operatorname{Li}_2(\sqrt{d-1}(\cosh(bx+a) + \sinh(bx+a))) - 20b^3x^3 \operatorname{Li}_2(-\sqrt{d-1}(\cosh(bx+a) + \sinh(bx+a)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(1-d-d\*tanh(b\*x+a)),x, algorithm="fricas")

[Out] 1/40\*(2\*b^5\*x^5 - 5\*b^4\*x^4\*log((d\*cosh(b\*x + a) + d\*sinh(b\*x + a))/((d - 2)\*cosh(b\*x + a) + d\*sinh(b\*x + a))) - 20\*b^3\*x^3\*dilog(sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 20\*b^3\*x^3\*dilog(-sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 5\*a^4\*log(2\*(d - 1)\*cosh(b\*x + a) + 2\*(d - 1)\*sinh(b\*x + a) + 2\*sqrt(d - 1)) - 5\*a^4\*log(2\*(d - 1)\*cosh(b\*x + a) + 2\*(d - 1)\*sinh(b\*x + a) - 2\*sqrt(d - 1)) + 60\*b^2\*x^2\*polylog(3, sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) + 60\*b^2\*x^2\*polylog(3, -sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 120\*b\*x\*polylog(4, sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 120\*b\*x\*polylog(4, -sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 5\*(b^4\*x^4 - a^4)\*log(sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a)) + 1) - 5\*(b^4\*x^4 - a^4)\*log(-sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a)) + 1) + 120\*polylog(5, sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) + 120\*polylog(5, -sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a))))/b^4

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**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*acoth(1-d-d\*tanh(b\*x+a)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arccoth}(-d \tanh(bx + a) - d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(1-d-d\*tanh(b\*x+a)),x, algorithm="giac")

[Out] integrate(x^3\*arccoth(-d\*tanh(b\*x + a) - d + 1), x)

### 3.213 $\int x^2 \coth^{-1}(1 - d - d \tanh(a + bx)) dx$

**Optimal.** Leaf size=139

$$\frac{x \operatorname{PolyLog}\left(3, -(1-d)e^{2a+2bx}\right)}{4b^2} - \frac{\operatorname{PolyLog}\left(4, -(1-d)e^{2a+2bx}\right)}{8b^3} - \frac{x^2 \operatorname{PolyLog}\left(2, -(1-d)e^{2a+2bx}\right)}{4b} - \frac{1}{6} x^3 \log\left((1-d)e^{2a+2bx}\right)$$

[Out] (b\*x^4)/12 + (x^3\*ArcCoth[1 - d - d\*Tanh[a + b\*x]])/3 - (x^3\*Log[1 + (1 - d)\*E^(2\*a + 2\*b\*x)])/6 - (x^2\*PolyLog[2, -((1 - d)\*E^(2\*a + 2\*b\*x))])/(4\*b) + (x\*PolyLog[3, -((1 - d)\*E^(2\*a + 2\*b\*x))])/(4\*b^2) - PolyLog[4, -((1 - d)\*E^(2\*a + 2\*b\*x))]/(8\*b^3)

**Rubi [A]** time = 0.268458, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {6240, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{x \operatorname{PolyLog}\left(3, -(1-d)e^{2a+2bx}\right)}{4b^2} - \frac{\operatorname{PolyLog}\left(4, -(1-d)e^{2a+2bx}\right)}{8b^3} - \frac{x^2 \operatorname{PolyLog}\left(2, -(1-d)e^{2a+2bx}\right)}{4b} - \frac{1}{6} x^3 \log\left((1-d)e^{2a+2bx}\right)$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcCoth[1 - d - d\*Tanh[a + b\*x]], x]

[Out] (b\*x^4)/12 + (x^3\*ArcCoth[1 - d - d\*Tanh[a + b\*x]])/3 - (x^3\*Log[1 + (1 - d)\*E^(2\*a + 2\*b\*x)])/6 - (x^2\*PolyLog[2, -((1 - d)\*E^(2\*a + 2\*b\*x))])/(4\*b) + (x\*PolyLog[3, -((1 - d)\*E^(2\*a + 2\*b\*x))])/(4\*b^2) - PolyLog[4, -((1 - d)\*E^(2\*a + 2\*b\*x))]/(8\*b^3)

#### Rule 6240

Int[ArcCoth[(c\_.) + (d\_.)\*Tanh[(a\_.) + (b\_.)\*(x\_)]]\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^(m + 1)\*ArcCoth[c + d\*Tanh[a + b\*x]])/(f\*(m + 1)), x] + Dist[b/(f\*(m + 1)), Int[(e + f\*x)^(m + 1)/(c - d + c\*E^(2\*a + 2\*b\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d)^2, 1]

#### Rule 2184

Int[(((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[(((c + d\*x)^m\*(F^(g\*(e + f\*x))))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x],



x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_)]/((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_)\*((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_))))^(n\_)]\*(f\_) + (g\_)\*(x\_)]^(m\_), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 6609

Int[((e\_) + (f\_)\*(x\_))^(m\_)\*PolyLog[n\_, (d\_)\*((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_))))^(p\_)], x\_Symbol] := Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p])/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\*(F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_)\*((a\_) + (b\_)\*(x\_))^(p\_)]/((d\_) + (e\_)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(1-d-d \tanh(a+bx)) dx &= \frac{1}{3}x^3 \coth^{-1}(1-d-d \tanh(a+bx)) + \frac{1}{3}b \int \frac{x^3}{1+(1-d)e^{2a+2bx}} dx \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{3}(b(1-d)) \int \frac{e^{2a+2bx}x^3}{1+(1-d)e^{2a+2bx}} dx \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{6}x^3 \log(1+(1-d)e^{2a+2bx}) + \frac{1}{2} \dots \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{6}x^3 \log(1+(1-d)e^{2a+2bx}) - \frac{x^2}{2} \dots \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{6}x^3 \log(1+(1-d)e^{2a+2bx}) - \frac{x^2}{2} \dots \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{6}x^3 \log(1+(1-d)e^{2a+2bx}) - \frac{x^2}{2} \dots \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{6}x^3 \log(1+(1-d)e^{2a+2bx}) - \frac{x^2}{2} \dots
\end{aligned}$$

**Mathematica [A]** time = 0.102574, size = 119, normalized size = 0.86

$$\frac{1}{24} \left( \frac{6x \text{PolyLog}\left(3, \frac{e^{-2(a+bx)}}{d-1}\right)}{b^2} + \frac{3 \text{PolyLog}\left(4, \frac{e^{-2(a+bx)}}{d-1}\right)}{b^3} + \frac{6x^2 \text{PolyLog}\left(2, \frac{e^{-2(a+bx)}}{d-1}\right)}{b} - 4x^3 \log\left(1 - \frac{e^{-2(a+bx)}}{d-1}\right) + 8x^3 \coth^{-1}\left(\frac{1-d-d \tanh(a+bx)}{1-d}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[1 - d - d\*Tanh[a + b\*x]],x]

[Out] (8\*x^3\*ArcCoth[1 - d - d\*Tanh[a + b\*x]] - 4\*x^3\*Log[1 - 1/((-1 + d)\*E^(2\*(a + b\*x))]) + (6\*x^2\*PolyLog[2, 1/((-1 + d)\*E^(2\*(a + b\*x))])]/b + (6\*x\*PolyLog[3, 1/((-1 + d)\*E^(2\*(a + b\*x))])]/b^2 + (3\*PolyLog[4, 1/((-1 + d)\*E^(2\*(a + b\*x))])]/b^3)/24

**Maple [C]** time = 33.599, size = 1745, normalized size = 12.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(1-d-d\*tanh(b\*x+a)),x)

```

[Out] -1/6*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1)
)^2-1/2/b^3*d*a^2/(d-1)*dilog(1-exp(b*x+a)*(d-1)^(1/2))-1/2/b^3*d*a^2/(d-1)
*dilog(1+exp(b*x+a)*(d-1)^(1/2))+1/3/b^3*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))
*a^3-1/4/b*d/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*x^2+1/4/b^3*d/(d-1)*poly
log(2,(d-1)*exp(2*b*x+2*a))*a^2+1/4/b^2*d/(d-1)*polylog(3,(d-1)*exp(2*b*x+2
*a))*x-1/2/b^2/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x*a^2+1/2/b^2*a^2/(d-1)*ln(
1-exp(b*x+a)*(d-1)^(1/2))*x-1/2/b^3*d*a^3/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2)
)-1/2/b^3*d*a^3/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))-1/3*x^3*ln(exp(b*x+a))-1
/6*x^3*ln(d)-1/6*I*x^3*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/12*
I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1
))^2+1/12*b*x^4+1/8/b^3/(d-1)*polylog(4,(d-1)*exp(2*b*x+2*a))+1/6/(d-1)*ln(
1-(d-1)*exp(2*b*x+2*a))*x^3+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I
/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))^2+1/12*I*x^3*Pi*cs
gn(I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*
b*x+2*a)*d-exp(2*b*x+2*a)-1))^2-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*
b*x+2*a)+1))*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2+1/2/b^2*a^2/(d-1
)*ln(1+exp(b*x+a)*(d-1)^(1/2))*x+1/6*x^3*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)
-1)+1/6/b^3*d*a^3/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1)-1/12*I*x^3*Pi
*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/12*I*
x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1
))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))-1/6/b^3*a
^3/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1)+1/12*I*x^3*Pi*csgn(I/(exp(2*
b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))^3+1/12*I*x^3*Pi*csgn(I*exp
(2*b*x+2*a))^3+1/6*I*x^3*Pi*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2-1
/12*I*x^3*Pi*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^3+1/12*I*x^3*Pi*cs
gn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/6*d/(d-1)*ln(1-(d-1)*exp(2*b*x+
2*a))*x^3-1/3/b^3/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*a^3+1/4/b/(d-1)*polylog(
2,(d-1)*exp(2*b*x+2*a))*x^2-1/4/b^3/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*a
^2-1/4/b^2/(d-1)*polylog(3,(d-1)*exp(2*b*x+2*a))*x+1/2/b^3*a^3/(d-1)*ln(1-e
xp(b*x+a)*(d-1)^(1/2))+1/2/b^3*a^3/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))-1/8/b
^3*d/(d-1)*polylog(4,(d-1)*exp(2*b*x+2*a))+1/2/b^3*a^2/(d-1)*dilog(1-exp(b*
x+a)*(d-1)^(1/2))+1/2/b^3*a^2/(d-1)*dilog(1+exp(b*x+a)*(d-1)^(1/2))+1/12*I*
x^3*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/12*I*x^3*Pi*csgn(I*d)*
csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2+1/12*I*x^3*Pi*csgn(I/(exp(2*b
*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1
))+1/12*I*x^3*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*
d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))+1/2/b^2*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2
*a))*x*a^2-1/2/b^2*d*a^2/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))*x-1/2/b^2*d*a^2
/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))*x

```

**Maxima [A]** time = 3.50631, size = 166, normalized size = 1.19

$$-\frac{1}{3}x^3 \operatorname{arccoth}(d \tanh(bx + a) + d - 1) + \frac{1}{36} \left( \frac{3x^4}{d} - \frac{2(4b^3x^3 \log(-(d-1)e^{(2bx+2a)} + 1) + 6b^2x^2 \operatorname{Li}_2((d-1)e^{(2bx+2a)}) - b^4d)}{b^4d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(1-d-d\*tanh(b\*x+a)),x, algorithm="maxima")

[Out] -1/3\*x^3\*arccoth(d\*tanh(b\*x + a) + d - 1) + 1/36\*(3\*x^4/d - 2\*(4\*b^3\*x^3\*log(-(d - 1)\*e^(2\*b\*x + 2\*a) + 1) + 6\*b^2\*x^2\*dilog((d - 1)\*e^(2\*b\*x + 2\*a)) - 6\*b\*x\*polylog(3, (d - 1)\*e^(2\*b\*x + 2\*a)) + 3\*polylog(4, (d - 1)\*e^(2\*b\*x + 2\*a)))/(b^4\*d))\*b\*d

**Fricas [C]** time = 2.13774, size = 1081, normalized size = 7.78

$$b^4x^4 - 2b^3x^3 \log\left(\frac{d \cosh(bx+a) + d \sinh(bx+a)}{(d-2) \cosh(bx+a) + d \sinh(bx+a)}\right) - 6b^2x^2 \operatorname{Li}_2\left(\sqrt{d-1}(\cosh(bx+a) + \sinh(bx+a))\right) - 6b^2x^2 \operatorname{Li}_2\left(-\sqrt{d-1}(\cosh(bx+a) + \sinh(bx+a))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(1-d-d\*tanh(b\*x+a)),x, algorithm="fricas")

[Out] 1/12\*(b^4\*x^4 - 2\*b^3\*x^3\*log((d\*cosh(b\*x + a) + d\*sinh(b\*x + a))/((d - 2)\*cosh(b\*x + a) + d\*sinh(b\*x + a))) - 6\*b^2\*x^2\*dilog(sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 6\*b^2\*x^2\*dilog(-sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a)))) + 2\*a^3\*log(2\*(d - 1)\*cosh(b\*x + a) + 2\*(d - 1)\*sinh(b\*x + a) + 2\*sqrt(d - 1)) + 2\*a^3\*log(2\*(d - 1)\*cosh(b\*x + a) + 2\*(d - 1)\*sinh(b\*x + a) - 2\*sqrt(d - 1)) + 12\*b\*x\*polylog(3, sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) + 12\*b\*x\*polylog(3, -sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 2\*(b^3\*x^3 + a^3)\*log(sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a)) + 1) - 2\*(b^3\*x^3 + a^3)\*log(-sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a)) + 1) - 12\*polylog(4, sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 12\*polylog(4, -sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a))))/b^3

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(1-d-d*tanh(b*x+a)),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arccoth}(-d \tanh(bx + a) - d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(-d*tanh(b*x + a) - d + 1), x)
```

### 3.214 $\int x \coth^{-1}(1 - d - d \tanh(a + bx)) dx$

**Optimal.** Leaf size=110

$$\frac{\text{PolyLog}\left(3, -(1-d)e^{2a+2bx}\right)}{8b^2} - \frac{x\text{PolyLog}\left(2, -(1-d)e^{2a+2bx}\right)}{4b} - \frac{1}{4}x^2 \log\left((1-d)e^{2a+2bx} + 1\right) + \frac{1}{2}x^2 \coth^{-1}(d(-\tanh(a + bx)))$$

[Out] (b\*x^3)/6 + (x^2\*ArcCoth[1 - d - d\*Tanh[a + b\*x]])/2 - (x^2\*Log[1 + (1 - d)\*E^(2\*a + 2\*b\*x)])/4 - (x\*PolyLog[2, -((1 - d)\*E^(2\*a + 2\*b\*x))])/(4\*b) + PolyLog[3, -((1 - d)\*E^(2\*a + 2\*b\*x))]/(8\*b^2)

**Rubi [A]** time = 0.238174, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$ , Rules used = {6240, 2184, 2190, 2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3, -(1-d)e^{2a+2bx}\right)}{8b^2} - \frac{x\text{PolyLog}\left(2, -(1-d)e^{2a+2bx}\right)}{4b} - \frac{1}{4}x^2 \log\left((1-d)e^{2a+2bx} + 1\right) + \frac{1}{2}x^2 \coth^{-1}(d(-\tanh(a + bx)))$$

Antiderivative was successfully verified.

[In] Int[x\*ArcCoth[1 - d - d\*Tanh[a + b\*x]], x]

[Out] (b\*x^3)/6 + (x^2\*ArcCoth[1 - d - d\*Tanh[a + b\*x]])/2 - (x^2\*Log[1 + (1 - d)\*E^(2\*a + 2\*b\*x)])/4 - (x\*PolyLog[2, -((1 - d)\*E^(2\*a + 2\*b\*x))])/(4\*b) + PolyLog[3, -((1 - d)\*E^(2\*a + 2\*b\*x))]/(8\*b^2)

#### Rule 6240

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Tanh[a + b*x]])/(f*(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d)^2, 1]
```

#### Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.), x_Symbol] := Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n])]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x \coth^{-1}(1-d-d \tanh(a+bx)) dx &= \frac{1}{2}x^2 \coth^{-1}(1-d-d \tanh(a+bx)) + \frac{1}{2}b \int \frac{x^2}{1+(1-d)e^{2a+2bx}} dx \\
&= \frac{bx^3}{6} + \frac{1}{2}x^2 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{2}(b(1-d)) \int \frac{e^{2a+2bx}x^2}{1+(1-d)e^{2a+2bx}} dx \\
&= \frac{bx^3}{6} + \frac{1}{2}x^2 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{4}x^2 \log(1+(1-d)e^{2a+2bx}) + \frac{1}{2} \int \frac{x^2 e^{2a+2bx}}{1+(1-d)e^{2a+2bx}} dx \\
&= \frac{bx^3}{6} + \frac{1}{2}x^2 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{4}x^2 \log(1+(1-d)e^{2a+2bx}) - \frac{x \operatorname{Li}_2(-e^{2a+2bx})}{4} \\
&= \frac{bx^3}{6} + \frac{1}{2}x^2 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{4}x^2 \log(1+(1-d)e^{2a+2bx}) - \frac{x \operatorname{Li}_2(-e^{2a+2bx})}{4} \\
&= \frac{bx^3}{6} + \frac{1}{2}x^2 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{4}x^2 \log(1+(1-d)e^{2a+2bx}) - \frac{x \operatorname{Li}_2(-e^{2a+2bx})}{4}
\end{aligned}$$

**Mathematica [A]** time = 0.0907819, size = 93, normalized size = 0.85

$$\frac{2bx \operatorname{PolyLog}\left(2, \frac{e^{-2(a+bx)}}{d-1}\right) + \operatorname{PolyLog}\left(3, \frac{e^{-2(a+bx)}}{d-1}\right) + 2b^2x^2 \left(2 \coth^{-1}(d(-\tanh(a+bx)) - d + 1) - \log\left(1 - \frac{e^{-2(a+bx)}}{d-1}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[1 - d - d\*Tanh[a + b\*x]], x]

[Out] (2\*b^2\*x^2\*(2\*ArcCoth[1 - d - d\*Tanh[a + b\*x]] - Log[1 - 1/((-1 + d)\*E^(2\*(a + b\*x))])) + 2\*b\*x\*PolyLog[2, 1/((-1 + d)\*E^(2\*(a + b\*x))]] + PolyLog[3, 1/((-1 + d)\*E^(2\*(a + b\*x))]])/(8\*b^2)

**Maple [C]** time = 16.123, size = 1664, normalized size = 15.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(1-d-d\*tanh(b\*x+a)), x)

[Out] 1/2/b^2\*d\*a^2/(d-1)\*ln(1+exp(b\*x+a)\*(d-1)^(1/2))+1/2/b^2\*d\*a/(d-1)\*dilog(1-exp(b\*x+a)\*(d-1)^(1/2))+1/2/b^2\*d\*a/(d-1)\*dilog(1+exp(b\*x+a)\*(d-1)^(1/2))-1



$$\begin{aligned}
& /4/b^2*d/(d-1)*\ln(1-(d-1)*\exp(2*b*x+2*a))*a^{-2-1/4}/b*d/(d-1)*\text{polylog}(2, (d-1) \\
& * \exp(2*b*x+2*a)) * x - 1/8*I*x^2*Pi*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a) \\
& )/( \exp(2*b*x+2*a)+1) )^2 + 1/6*b*x^3 + 1/8*I*x^2*Pi*csgn(I*( \exp(2*b*x+2*a)*d - \exp \\
& (2*b*x+2*a)-1) ) * csgn(I/( \exp(2*b*x+2*a)+1) * ( \exp(2*b*x+2*a)*d - \exp(2*b*x+2*a)- \\
& 1) )^2 + 1/4/b^2/(d-1)*\ln(1-(d-1)*\exp(2*b*x+2*a))*a^{-2+1/4}/b/(d-1)*\text{polylog}(2, (d \\
& -1)*\exp(2*b*x+2*a)) * x + 1/4/b^2/(d-1)*\text{polylog}(2, (d-1)*\exp(2*b*x+2*a)) * a^{-1/2}/b \\
& ^2*a^2/(d-1)*\ln(1-\exp(b*x+a)*(d-1)^{(1/2)}) - 1/2/b^2*a^2/(d-1)*\ln(1+\exp(b*x+a) \\
& *(d-1)^{(1/2)}) + 1/8/b^2*d/(d-1)*\text{polylog}(3, (d-1)*\exp(2*b*x+2*a)) - 1/2/b^2*a/(d- \\
& 1)*\text{dilog}(1-\exp(b*x+a)*(d-1)^{(1/2)}) - 1/2/b^2*a/(d-1)*\text{dilog}(1+\exp(b*x+a)*(d-1) \\
& ^{(1/2)}) - 1/4*d/(d-1)*\ln(1-(d-1)*\exp(2*b*x+2*a)) * x^2 + 1/4/b^2*a^2/(d-1)*\ln(\exp \\
& (2*b*x+2*a)*d - \exp(2*b*x+2*a)-1) + 1/4*I*x^2*Pi*csgn(I*d/( \exp(2*b*x+2*a)+1) * \exp \\
& (2*b*x+2*a)) ^2 + 1/4/(d-1)*\ln(1-(d-1)*\exp(2*b*x+2*a)) * x^{-2} - 1/8/b^2/(d-1)*\text{poly} \\
& \text{log}(3, (d-1)*\exp(2*b*x+2*a)) - 1/8*I*x^2*Pi*csgn(I*d)*csgn(I*d/( \exp(2*b*x+2*a) \\
& +1) * \exp(2*b*x+2*a)) ^2 - 1/2*x^2*\ln(\exp(b*x+a)) - 1/4*x^2*\ln(d) + 1/8*I*x^2*Pi*csgn \\
& (I/( \exp(2*b*x+2*a)+1) ) * csgn(I/( \exp(2*b*x+2*a)+1) * ( \exp(2*b*x+2*a)*d - \exp(2*b \\
& *x+2*a)-1) )^2 - 1/4/b^2*d/(d-1)*\text{polylog}(2, (d-1)*\exp(2*b*x+2*a)) * a + 1/2/b/(d-1) \\
& *\ln(1-(d-1)*\exp(2*b*x+2*a)) * x * a^{-1/2}/b*a/(d-1)*\ln(1-\exp(b*x+a)*(d-1)^{(1/2)}) * \\
& x - 1/2/b*a/(d-1)*\ln(1+\exp(b*x+a)*(d-1)^{(1/2)}) * x + 1/2/b^2*d*a^2/(d-1)*\ln(1-\exp \\
& (b*x+a)*(d-1)^{(1/2)}) - 1/8*I*x^2*Pi*csgn(I*\exp(2*b*x+2*a)/( \exp(2*b*x+2*a)+1) ) \\
& *csgn(I*d/( \exp(2*b*x+2*a)+1) * \exp(2*b*x+2*a)) ^2 + 1/4*x^2*\ln(\exp(2*b*x+2*a)*d - \\
& \exp(2*b*x+2*a)-1) - 1/4/b^2*d*a^2/(d-1)*\ln(\exp(2*b*x+2*a)*d - \exp(2*b*x+2*a)-1) \\
& + 1/8*I*x^2*Pi*csgn(I*\exp(2*b*x+2*a)) ^3 + 1/8*I*x^2*Pi*csgn(I*\exp(2*b*x+2*a)/( \\
& \exp(2*b*x+2*a)+1) )^3 + 1/8*I*x^2*Pi*csgn(I/( \exp(2*b*x+2*a)+1) * ( \exp(2*b*x+2*a) \\
& *d - \exp(2*b*x+2*a)-1) )^3 - 1/4*I*x^2*Pi*csgn(I/( \exp(2*b*x+2*a)+1) * ( \exp(2*b*x+2 \\
& *a)*d - \exp(2*b*x+2*a)-1) )^2 - 1/8*I*x^2*Pi*csgn(I*d/( \exp(2*b*x+2*a)+1) * \exp(2*b \\
& *x+2*a)) ^3 - 1/8*I*x^2*Pi*csgn(I/( \exp(2*b*x+2*a)+1) ) * csgn(I*\exp(2*b*x+2*a)/( \exp \\
& (2*b*x+2*a)+1) )^2 + 1/8*I*x^2*Pi*csgn(I*\exp(b*x+a)) ^2 * csgn(I*\exp(2*b*x+2*a) \\
& ) - 1/4*I*x^2*Pi*csgn(I*\exp(b*x+a)) * csgn(I*\exp(2*b*x+2*a)) ^2 - 1/8*I*x^2*Pi*csgn \\
& (I/( \exp(2*b*x+2*a)+1) ) * csgn(I*( \exp(2*b*x+2*a)*d - \exp(2*b*x+2*a)-1) ) * csgn(I/ \\
& ( \exp(2*b*x+2*a)+1) * ( \exp(2*b*x+2*a)*d - \exp(2*b*x+2*a)-1) ) + 1/8*I*x^2*Pi*csgn(I \\
& *d)*csgn(I*\exp(2*b*x+2*a)/( \exp(2*b*x+2*a)+1) ) * csgn(I*d/( \exp(2*b*x+2*a)+1) * \exp \\
& (2*b*x+2*a)) + 1/8*I*x^2*Pi*csgn(I/( \exp(2*b*x+2*a)+1) ) * csgn(I*\exp(2*b*x+2*a) \\
& )) * csgn(I*\exp(2*b*x+2*a)/( \exp(2*b*x+2*a)+1) ) - 1/2/b*d/(d-1)*\ln(1-(d-1)*\exp(2 \\
& *b*x+2*a)) * x * a + 1/2/b*d*a/(d-1)*\ln(1-\exp(b*x+a)*(d-1)^{(1/2)}) * x + 1/2/b*d*a/(d- \\
& 1)*\ln(1+\exp(b*x+a)*(d-1)^{(1/2)}) * x
\end{aligned}$$

**Maxima [A]** time = 3.50123, size = 135, normalized size = 1.23

$$\frac{1}{24} \left( \frac{4x^3}{d} - \frac{3(2b^2x^2 \log(-(d-1)e^{2bx+2a}) + 1) + 2bx \text{Li}_2((d-1)e^{2bx+2a}) - \text{Li}_3((d-1)e^{2bx+2a})}{b^3d} \right) bd - \frac{1}{2} x^2 \text{arcoth}(d)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1-d-d\*tanh(b\*x+a)),x, algorithm="maxima")

[Out]  $\frac{1}{24} \left( \frac{4x^3}{d} - 3(2b^2x^2 \log(-(d-1)e^{(2bx+a)} + 1) + 2bx \operatorname{dilog}((d-1)e^{(2bx+a)} - \operatorname{polylog}(3, (d-1)e^{(2bx+a)})))/(b^3d)) \right) b^3d - \frac{1}{2}x^2 \operatorname{arccoth}(d \tanh(bx+a) + d - 1)$

**Fricas [C]** time = 2.06429, size = 894, normalized size = 8.13

$$2b^3x^3 - 3b^2x^2 \log\left(\frac{d \cosh(bx+a) + d \sinh(bx+a)}{(d-2) \cosh(bx+a) + d \sinh(bx+a)}\right) - 6bx \operatorname{Li}_2\left(\sqrt{d-1}(\cosh(bx+a) + \sinh(bx+a))\right) - 6bx \operatorname{Li}_2\left(-\sqrt{d-1}(\cosh(bx+a) + \sinh(bx+a))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1-d-d\*tanh(b\*x+a)),x, algorithm="fricas")

[Out]  $\frac{1}{12} \left( 2b^3x^3 - 3b^2x^2 \log\left(\frac{d \cosh(bx+a) + d \sinh(bx+a)}{(d-2) \cosh(bx+a) + d \sinh(bx+a)}\right) - 6bx \operatorname{dilog}(\sqrt{d-1}(\cosh(bx+a) + \sinh(bx+a))) - 6bx \operatorname{dilog}(-\sqrt{d-1}(\cosh(bx+a) + \sinh(bx+a))) - 3a^2 \log(2(d-1)\cosh(bx+a) + 2(d-1)\sinh(bx+a) + 2\sqrt{d-1}) - 3a^2 \log(2(d-1)\cosh(bx+a) + 2(d-1)\sinh(bx+a) - 2\sqrt{d-1}) - 3(b^2x^2 - a^2) \log(\sqrt{d-1}(\cosh(bx+a) + \sinh(bx+a)) + 1) - 3(b^2x^2 - a^2) \log(-\sqrt{d-1}(\cosh(bx+a) + \sinh(bx+a)) + 1) + 6 \operatorname{polylog}(3, \sqrt{d-1}(\cosh(bx+a) + \sinh(bx+a))) + 6 \operatorname{polylog}(3, -\sqrt{d-1}(\cosh(bx+a) + \sinh(bx+a))) \right) / b^2$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(1-d-d\*tanh(b\*x+a)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arccoth}(-d \tanh(bx + a) - d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(-d*tanh(b*x + a) - d + 1), x)
```

### 3.215 $\int \coth^{-1}(1 - d - d \tanh(a + bx)) dx$

**Optimal.** Leaf size=76

$$-\frac{\text{PolyLog}\left(2, -(1-d)e^{2a+2bx}\right)}{4b} - \frac{1}{2}x \log\left((1-d)e^{2a+2bx} + 1\right) + x \coth^{-1}(d - \tanh(a + bx)) - d + 1 + \frac{bx^2}{2}$$

[Out] (b\*x^2)/2 + x\*ArcCoth[1 - d - d\*Tanh[a + b\*x]] - (x\*Log[1 + (1 - d)\*E^(2\*a + 2\*b\*x)])/2 - PolyLog[2, -((1 - d)\*E^(2\*a + 2\*b\*x))]/(4\*b)

**Rubi [A]** time = 0.149527, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6232, 2184, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2, -(1-d)e^{2a+2bx}\right)}{4b} - \frac{1}{2}x \log\left((1-d)e^{2a+2bx} + 1\right) + x \coth^{-1}(d - \tanh(a + bx)) - d + 1 + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[1 - d - d\*Tanh[a + b\*x]], x]

[Out] (b\*x^2)/2 + x\*ArcCoth[1 - d - d\*Tanh[a + b\*x]] - (x\*Log[1 + (1 - d)\*E^(2\*a + 2\*b\*x)])/2 - PolyLog[2, -((1 - d)\*E^(2\*a + 2\*b\*x))]/(4\*b)

#### Rule 6232

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] := Simp[x*ArcCoth[c + d*Tanh[a + b*x]], x] + Dist[b, Int[x/(c - d + c*E^(2*a + 2*b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

#### Rule 2184

```
Int[(((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

#### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned} \int \coth^{-1}(1 - d - d \tanh(a + bx)) dx &= x \coth^{-1}(1 - d - d \tanh(a + bx)) + b \int \frac{x}{1 + (1 - d)e^{2a+2bx}} dx \\ &= \frac{bx^2}{2} + x \coth^{-1}(1 - d - d \tanh(a + bx)) - (b(1 - d)) \int \frac{e^{2a+2bx} x}{1 + (1 - d)e^{2a+2bx}} dx \\ &= \frac{bx^2}{2} + x \coth^{-1}(1 - d - d \tanh(a + bx)) - \frac{1}{2} x \log(1 + (1 - d)e^{2a+2bx}) + \frac{1}{2} \int \log \left( \frac{1 + (1 - d)e^{2a+2bx}}{1 + (1 - d)} \right) dx \\ &= \frac{bx^2}{2} + x \coth^{-1}(1 - d - d \tanh(a + bx)) - \frac{1}{2} x \log(1 + (1 - d)e^{2a+2bx}) + \frac{\text{Subst}\left(\int \log(u) du, u, 1 + (1 - d)e^{2a+2bx}\right)}{2} \\ &= \frac{bx^2}{2} + x \coth^{-1}(1 - d - d \tanh(a + bx)) - \frac{1}{2} x \log(1 + (1 - d)e^{2a+2bx}) - \frac{\text{Li}_2\left(-\frac{1 + (1 - d)e^{2a+2bx}}{1 + (1 - d)}\right)}{2} \end{aligned}$$

**Mathematica [B]** time = 0.890278, size = 200, normalized size = 2.63

$$\frac{-2\text{PolyLog}\left(2, -\sqrt{d-1}e^{a+bx}\right) - 2\text{PolyLog}\left(2, \sqrt{d-1}e^{a+bx}\right) - 2\log\left(e^{a+bx}\right)\log\left(1 - \sqrt{d-1}e^{a+bx}\right) - 2\log\left(e^{a+bx}\right)\log\left(\sqrt{d-1}e^{a+bx}\right)}{2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[1 - d - d*Tanh[a + b*x]], x]
```

```
[Out] x*ArcCoth[1 - d - d*Tanh[a + b*x]] + (b^2*x^2 + Log[E^(a + b*x)]^2 - 2*Log[E^(a + b*x)]*Log[1 - Sqrt[-1 + d]*E^(a + b*x)] - 2*Log[E^(a + b*x)]*Log[1 + Sqrt[-1 + d]*E^(a + b*x)] + 2*Log[E^(a + b*x)]*Log[E^(-a - b*x)*(-1 + (-1 + d)*E^(2*(a + b*x)))] - 2*b*x*Log[(-2 + d)*Cosh[a + b*x] + d*Sinh[a + b*x]])/2
```

$] - 2*\text{PolyLog}[2, -(\text{Sqrt}[-1 + d]*E^{(a + b*x)})] - 2*\text{PolyLog}[2, \text{Sqrt}[-1 + d]*E^{(a + b*x)}]/(4*b)$

**Maple [B]** time = 0.158, size = 271, normalized size = 3.6

$$\frac{\text{arccoth}(1 - d - d \tanh(bx + a)) \ln(-d \tanh(bx + a) - d)}{2b} - \frac{\text{arccoth}(1 - d - d \tanh(bx + a)) \ln(-d \tanh(bx + a) + d)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(1-d-d*tanh(b*x+a)),x)`

[Out]  $\frac{1}{2} \frac{1}{b} \text{arccoth}(1-d-d*\tanh(b*x+a)) * \ln(-d*\tanh(b*x+a)-d) - \frac{1}{2} \frac{1}{b} \text{arccoth}(1-d-d*\tanh(b*x+a)) * \ln(-d*\tanh(b*x+a)+d) + \frac{1}{8} \frac{1}{b} \ln(-d*\tanh(b*x+a)-d)^2 - \frac{1}{4} \frac{1}{b} \text{dilog}(1 - \frac{1}{2}d*\tanh(b*x+a) - \frac{1}{2}d) - \frac{1}{4} \frac{1}{b} \ln(-d*\tanh(b*x+a)-d) * \ln(1 - \frac{1}{2}d*\tanh(b*x+a) - \frac{1}{2}d) - \frac{1}{4} \frac{1}{b} \text{dilog}(-\frac{1}{2}*(-d*\tanh(b*x+a)-d)/d) - \frac{1}{4} \frac{1}{b} \ln(-d*\tanh(b*x+a)+d) * \ln(-\frac{1}{2}*(-d*\tanh(b*x+a)-d)/d) + \frac{1}{4} \frac{1}{b} \text{dilog}((-d*\tanh(b*x+a)-d+2)/(-2*d+2)) + \frac{1}{4} \frac{1}{b} \ln(-d*\tanh(b*x+a)+d) * \ln((-d*\tanh(b*x+a)-d+2)/(-2*d+2))$

**Maxima [A]** time = 3.90124, size = 99, normalized size = 1.3

$$\frac{1}{4} b d \left( \frac{2x^2}{d} - \frac{2bx \log(-(d-1)e^{2bx+2a} + 1) + \text{Li}_2((d-1)e^{2bx+2a})}{b^2 d} \right) - x \text{arccoth}(d \tanh(bx + a) + d - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(1-d-d*tanh(b*x+a)),x, algorithm="maxima")`

[Out]  $\frac{1}{4} * b * d * (2*x^2/d - (2*b*x*\log(-(d-1)*e^{(2*b*x+2*a)} + 1) + \text{dilog}((d-1)*e^{(2*b*x+2*a)})) / (b^2*d)) - x*\text{arccoth}(d*\tanh(b*x+a) + d - 1)$

**Fricas [B]** time = 1.96049, size = 667, normalized size = 8.78

$$\frac{b^2 x^2 - bx \log\left(\frac{d \cosh(bx+a) + d \sinh(bx+a)}{(d-2) \cosh(bx+a) + d \sinh(bx+a)}\right) + a \log\left(2(d-1) \cosh(bx+a) + 2(d-1) \sinh(bx+a) + 2\sqrt{d-1}\right) + a \log\left(2(d-1) \cosh(bx+a) - 2(d-1) \sinh(bx+a) + 2\sqrt{d-1}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-d-d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - b*x*log((d*cosh(b*x + a) + d*sinh(b*x + a))/((d - 2)*cosh(b*x + a) + d*sinh(b*x + a))) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a) + 2*sqrt(d - 1)) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a) - 2*sqrt(d - 1)) - (b*x + a)*log(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - dilog(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) - dilog(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acoth}(-d \tanh(a + bx) - d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1-d-d*tanh(b*x+a)),x)
```

```
[Out] Integral(acoth(-d*tanh(a + b*x) - d + 1), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}(-d \tanh(bx + a) - d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-d-d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(-d*tanh(b*x + a) - d + 1), x)
```

$$3.216 \quad \int \frac{\coth^{-1}(1-d-d \tanh(a+bx))}{x} dx$$

**Optimal.** Leaf size=21

$$\text{CannotIntegrate}\left(\frac{\coth^{-1}(d(-\tanh(a+bx))-d+1)}{x}, x\right)$$

[Out] CannotIntegrate[ArcCoth[1 - d - d\*Tanh[a + b\*x]]/x, x]

**Rubi [A]** time = 0.0645733, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\coth^{-1}(1-d-d \tanh(a+bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[ArcCoth[1 - d - d\*Tanh[a + b\*x]]/x,x]

[Out] Defer[Int][ArcCoth[1 - d - d\*Tanh[a + b\*x]]/x, x]

Rubi steps

$$\int \frac{\coth^{-1}(1-d-d \tanh(a+bx))}{x} dx = \int \frac{\coth^{-1}(1-d-d \tanh(a+bx))}{x} dx$$

**Mathematica [A]** time = 3.11442, size = 0, normalized size = 0.

$$\int \frac{\coth^{-1}(1-d-d \tanh(a+bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcCoth[1 - d - d\*Tanh[a + b\*x]]/x,x]

[Out] Integrate[ArcCoth[1 - d - d\*Tanh[a + b\*x]]/x, x]



---

**Maple [A]** time = 0.372, size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(1 - d - d \tanh(bx + a))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(1-d-d\*tanh(b\*x+a))/x,x)

[Out] int(arccoth(1-d-d\*tanh(b\*x+a))/x,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$- \int \frac{\operatorname{arccoth}(d \tanh(bx + a) + d - 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-d-d\*tanh(b\*x+a))/x,x, algorithm="maxima")

[Out] -integrate(arccoth(d\*tanh(b\*x + a) + d - 1)/x, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\operatorname{arccoth}(d \tanh(bx + a) + d - 1)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-d-d\*tanh(b\*x+a))/x,x, algorithm="fricas")

[Out] integral(-arccoth(d\*tanh(b\*x + a) + d - 1)/x, x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}(-d \tanh(a + bx) - d + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(1-d-d\*tanh(b\*x+a))/x,x)

[Out] Integral(acoth(-d\*tanh(a + b\*x) - d + 1)/x, x)

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(-d \tanh(bx + a) - d + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-d-d\*tanh(b\*x+a))/x,x, algorithm="giac")

[Out] integrate(arccoth(-d\*tanh(b\*x + a) - d + 1)/x, x)

### 3.217 $\int x^2 \coth^{-1}(c + d \coth(a + bx)) dx$

**Optimal.** Leaf size=303

$$-\frac{x \operatorname{PolyLog}\left(3, \frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2} + \frac{x \operatorname{PolyLog}\left(3, \frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2} + \frac{\operatorname{PolyLog}\left(4, \frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^3} - \frac{\operatorname{PolyLog}\left(4, \frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^3}$$

[Out]  $(x^3 \operatorname{ArcCoth}[c + d \operatorname{Coth}[a + b x]])/3 + (x^3 \operatorname{Log}[1 - ((1 - c - d)E^{(2a + 2bx)})/(1 - c + d)])/6 - (x^3 \operatorname{Log}[1 - ((1 + c + d)E^{(2a + 2bx)})/(1 + c - d)])/6 + (x^2 \operatorname{PolyLog}[2, ((1 - c - d)E^{(2a + 2bx)})/(1 - c + d)])/4b - (x^2 \operatorname{PolyLog}[2, ((1 + c + d)E^{(2a + 2bx)})/(1 + c - d)])/4b - (x \operatorname{PolyLog}[3, ((1 - c - d)E^{(2a + 2bx)})/(1 - c + d)])/4b^2 + (x \operatorname{PolyLog}[3, ((1 + c + d)E^{(2a + 2bx)})/(1 + c - d)])/4b^2 + \operatorname{PolyLog}[4, ((1 - c - d)E^{(2a + 2bx)})/(1 - c + d)])/8b^3 - \operatorname{PolyLog}[4, ((1 + c + d)E^{(2a + 2bx)})/(1 + c - d)])/8b^3$

**Rubi [A]** time = 0.465445, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {6246, 2190, 2531, 6609, 2282, 6589}

$$-\frac{x \operatorname{PolyLog}\left(3, \frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2} + \frac{x \operatorname{PolyLog}\left(3, \frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2} + \frac{\operatorname{PolyLog}\left(4, \frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^3} - \frac{\operatorname{PolyLog}\left(4, \frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2 \operatorname{ArcCoth}[c + d \operatorname{Coth}[a + b x]], x]$

[Out]  $(x^3 \operatorname{ArcCoth}[c + d \operatorname{Coth}[a + b x]])/3 + (x^3 \operatorname{Log}[1 - ((1 - c - d)E^{(2a + 2bx)})/(1 - c + d)])/6 - (x^3 \operatorname{Log}[1 - ((1 + c + d)E^{(2a + 2bx)})/(1 + c - d)])/6 + (x^2 \operatorname{PolyLog}[2, ((1 - c - d)E^{(2a + 2bx)})/(1 - c + d)])/4b - (x^2 \operatorname{PolyLog}[2, ((1 + c + d)E^{(2a + 2bx)})/(1 + c - d)])/4b - (x \operatorname{PolyLog}[3, ((1 - c - d)E^{(2a + 2bx)})/(1 - c + d)])/4b^2 + (x \operatorname{PolyLog}[3, ((1 + c + d)E^{(2a + 2bx)})/(1 + c - d)])/4b^2 + \operatorname{PolyLog}[4, ((1 - c - d)E^{(2a + 2bx)})/(1 - c + d)])/8b^3 - \operatorname{PolyLog}[4, ((1 + c + d)E^{(2a + 2bx)})/(1 + c - d)])/8b^3$

#### Rule 6246

$\operatorname{Int}[\operatorname{ArcCoth}[(c_.) + \operatorname{Coth}[(a_.) + (b_.)(x_.)](d_.)]*((e_.) + (f_.)(x_.))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(e + f x)^{(m + 1)} \operatorname{ArcCoth}[c + d \operatorname{Coth}[a + b x]]]/(f$

```
(m + 1)), x] + (-Dist[(b*(1 - c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E
^(2*a + 2*b*x))/(1 - c + d - (1 - c - d)*E^(2*a + 2*b*x)), x], x] + Dist[(b
*(1 + c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*a + 2*b*x))/(1 + c -
d - (1 + c + d)*E^(2*a + 2*b*x)), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

### Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)
*(x_))^(m_), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6609

```
Int[((e_) + (f_)*(x_))^(m_)*PolyLog[n_, (d_)*((F_)^((c_)*((a_) + (b_
)*(x_))))^(p_)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_) ]/((d_) + (e_)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(c + d \coth(a + bx)) dx &= \frac{1}{3}x^3 \coth^{-1}(c + d \coth(a + bx)) - \frac{1}{3}(b(1 - c - d)) \int \frac{e^{2a+2bx}x^3}{1 - c + d + (-1 + c + d)e^{2a+2bx}} dx \\
&= \frac{1}{3}x^3 \coth^{-1}(c + d \coth(a + bx)) + \frac{1}{6}x^3 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{6}x^3 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) \\
&= \frac{1}{3}x^3 \coth^{-1}(c + d \coth(a + bx)) + \frac{1}{6}x^3 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{6}x^3 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) \\
&= \frac{1}{3}x^3 \coth^{-1}(c + d \coth(a + bx)) + \frac{1}{6}x^3 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{6}x^3 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) \\
&= \frac{1}{3}x^3 \coth^{-1}(c + d \coth(a + bx)) + \frac{1}{6}x^3 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{6}x^3 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) \\
&= \frac{1}{3}x^3 \coth^{-1}(c + d \coth(a + bx)) + \frac{1}{6}x^3 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{6}x^3 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.39741, size = 353, normalized size = 1.17

$$-6b^2x^2\text{PolyLog}\left(2, \frac{(c-d-1)(\cosh(2(a+bx))-\sinh(2(a+bx)))}{c+d-1}\right) + 6b^2x^2\text{PolyLog}\left(2, \frac{(c-d+1)(\cosh(2(a+bx))-\sinh(2(a+bx)))}{c+d+1}\right) - 6bx\text{PolyLog}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2\*ArcCoth[c + d\*Coth[a + b\*x]], x]

[Out] (x^3\*ArcCoth[c + d\*Coth[a + b\*x]])/3 + (4\*b^3\*x^3\*Log[(2\*(Cosh[a + b\*x] - Sinh[a + b\*x])\*(d\*Cosh[a + b\*x] + (-1 + c)\*Sinh[a + b\*x]))/(-1 + c + d)] - 4\*b^3\*x^3\*Log[1 + ((1 + c - d)\*(-Cosh[2\*(a + b\*x)] + Sinh[2\*(a + b\*x)])]/(1 + c + d)] - 6\*b^2\*x^2\*PolyLog[2, ((-1 + c - d)\*(Cosh[2\*(a + b\*x)] - Sinh[2\*(a + b\*x)])]/(-1 + c + d)] + 6\*b^2\*x^2\*PolyLog[2, ((1 + c - d)\*(Cosh[2\*(a + b\*x)] - Sinh[2\*(a + b\*x)])]/(1 + c + d)] - 6\*b\*x\*PolyLog[3, ((-1 + c - d)\*(Cosh[2\*(a + b\*x)] - Sinh[2\*(a + b\*x)])]/(-1 + c + d)] + 6\*b\*x\*PolyLog[3, ((1 + c - d)\*(Cosh[2\*(a + b\*x)] - Sinh[2\*(a + b\*x)])]/(1 + c + d)] - 3\*PolyLog[4, ((-1 + c - d)\*(Cosh[2\*(a + b\*x)] - Sinh[2\*(a + b\*x)])]/(-1 + c + d)] + 3\*PolyLog[4, ((1 + c - d)\*(Cosh[2\*(a + b\*x)] - Sinh[2\*(a + b\*x)])]/(1 + c + d))]/(24\*b^3)

---

**Maple [C]** time = 5.121, size = 5222, normalized size = 17.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*arccoth(c+d*coth(b*x+a)),x)`

[Out] result too large to display

---

**Maxima [A]** time = 2.28051, size = 374, normalized size = 1.23

$$\frac{1}{3}x^3 \operatorname{arccoth}(d \coth(bx + a) + c) - \frac{1}{18}bd \left( \frac{4b^3x^3 \log\left(-\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1} + 1\right) + 6b^2x^2 \operatorname{Li}_2\left(\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right) - 6bx \operatorname{Li}_3\left(\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right)}{b^4d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccoth(c+d*coth(b*x+a)),x, algorithm="maxima")`

[Out] `1/3*x^3*arccoth(d*coth(b*x + a) + c) - 1/18*b*d*((4*b^3*x^3*log(-(c + d + 1)*e^(2*b*x + 2*a))/(c - d + 1) + 1) + 6*b^2*x^2*dilog((c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)) - 6*b*x*polylog(3, (c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)) + 3*polylog(4, (c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^4*d) - (4*b^3*x^3*log(-(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1) + 1) + 6*b^2*x^2*dilog((c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)) - 6*b*x*polylog(3, (c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)) + 3*polylog(4, (c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)))/(b^4*d)`

---

**Fricas [C]** time = 2.32668, size = 2560, normalized size = 8.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccoth(c+d*coth(b*x+a)),x, algorithm="fricas")`

```
[Out] 1/6*(b^3*x^3*log((d*cosh(b*x + a) + (c + 1)*sinh(b*x + a))/(d*cosh(b*x + a)
+ (c - 1)*sinh(b*x + a))) - 3*b^2*x^2*dilog(sqrt((c + d + 1)/(c - d + 1))*
(cosh(b*x + a) + sinh(b*x + a))) - 3*b^2*x^2*dilog(-sqrt((c + d + 1)/(c - d
+ 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(sqrt((c + d - 1)/
(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(-sqrt((c +
d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + a^3*log(2*(c + d + 1)
)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt((c + d +
1)/(c - d + 1))) + a^3*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sin
h(b*x + a) - 2*(c - d + 1)*sqrt((c + d + 1)/(c - d + 1))) - a^3*log(2*(c +
d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt((c
+ d - 1)/(c - d - 1))) - a^3*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)
)*sinh(b*x + a) - 2*(c - d - 1)*sqrt((c + d - 1)/(c - d - 1))) + 6*b*x*poly
log(3, sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 6*b
*x*polylog(3, -sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)
)) - 6*b*x*polylog(3, sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b
*x + a))) - 6*b*x*polylog(3, -sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a)
+ sinh(b*x + a))) - (b^3*x^3 + a^3)*log(sqrt((c + d + 1)/(c - d + 1))*(cosh
(b*x + a) + sinh(b*x + a)) + 1) - (b^3*x^3 + a^3)*log(-sqrt((c + d + 1)/(c
- d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b^3*x^3 + a^3)*log(sqrt((
c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b^3*x^3 + a
^3)*log(-sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1)
- 6*polylog(4, sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a
))) - 6*polylog(4, -sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x
+ a))) + 6*polylog(4, sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(
b*x + a))) + 6*polylog(4, -sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + s
inh(b*x + a))))/b^3
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(c+d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arccoth}(d \coth(bx + a) + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(c+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(d*coth(b*x + a) + c), x)
```



### 3.218 $\int x \coth^{-1}(c + d \coth(a + bx)) dx$

**Optimal.** Leaf size=229

$$-\frac{\text{PolyLog}\left(3, \frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^2} + \frac{\text{PolyLog}\left(3, \frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^2} + \frac{x \text{PolyLog}\left(2, \frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b} - \frac{x \text{PolyLog}\left(2, \frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}$$

[Out]  $(x^2 \text{ArcCoth}[c + d \text{Coth}[a + b*x]])/2 + (x^2 \text{Log}[1 - ((1 - c - d)*E^{(2*a + 2*b*x)})/(1 - c + d)])/4 - (x^2 \text{Log}[1 - ((1 + c + d)*E^{(2*a + 2*b*x)})/(1 + c - d)])/4 + (x \text{PolyLog}[2, ((1 - c - d)*E^{(2*a + 2*b*x)})/(1 - c + d)])/ (4*b) - (x \text{PolyLog}[2, ((1 + c + d)*E^{(2*a + 2*b*x)})/(1 + c - d)])/ (4*b) - \text{PolyLog}[3, ((1 - c - d)*E^{(2*a + 2*b*x)})/(1 - c + d)]/(8*b^2) + \text{PolyLog}[3, ((1 + c + d)*E^{(2*a + 2*b*x)})/(1 + c - d)]/(8*b^2)$

**Rubi [A]** time = 0.382299, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {6246, 2190, 2531, 2282, 6589}

$$-\frac{\text{PolyLog}\left(3, \frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^2} + \frac{\text{PolyLog}\left(3, \frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^2} + \frac{x \text{PolyLog}\left(2, \frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b} - \frac{x \text{PolyLog}\left(2, \frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x \text{ArcCoth}[c + d \text{Coth}[a + b*x]], x]$

[Out]  $(x^2 \text{ArcCoth}[c + d \text{Coth}[a + b*x]])/2 + (x^2 \text{Log}[1 - ((1 - c - d)*E^{(2*a + 2*b*x)})/(1 - c + d)])/4 - (x^2 \text{Log}[1 - ((1 + c + d)*E^{(2*a + 2*b*x)})/(1 + c - d)])/4 + (x \text{PolyLog}[2, ((1 - c - d)*E^{(2*a + 2*b*x)})/(1 - c + d)])/ (4*b) - (x \text{PolyLog}[2, ((1 + c + d)*E^{(2*a + 2*b*x)})/(1 + c - d)])/ (4*b) - \text{PolyLog}[3, ((1 - c - d)*E^{(2*a + 2*b*x)})/(1 - c + d)]/(8*b^2) + \text{PolyLog}[3, ((1 + c + d)*E^{(2*a + 2*b*x)})/(1 + c - d)]/(8*b^2)$

#### Rule 6246

$\text{Int}[\text{ArcCoth}[(c_.) + \text{Coth}[(a_.) + (b_.)(x_.)]*(d_.)]*((e_.) + (f_.)(x_.))^{(m_.)}, x\_Symbol] :> \text{Simp}[(e + f*x)^{(m + 1)} \text{ArcCoth}[c + d \text{Coth}[a + b*x]]/(f*(m + 1)), x] + (-\text{Dist}[(b*(1 - c - d))/(f*(m + 1)], \text{Int}[(e + f*x)^{(m + 1)} * E^{(2*a + 2*b*x)}/(1 - c + d - (1 - c - d)*E^{(2*a + 2*b*x)})], x], x] + \text{Dist}[(b*(1 + c + d))/(f*(m + 1)], \text{Int}[(e + f*x)^{(m + 1)} * E^{(2*a + 2*b*x)}/(1 + c - d - (1 + c + d)*E^{(2*a + 2*b*x)})], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x]$

&& IGtQ[m, 0] && NeQ[(c - d)^2, 1]

### Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*(a_) + (b_)*(x_)))^(n_)]*((f_) + (g_)
*(x_))^(m_), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int x \coth^{-1}(c + d \coth(a + bx)) dx &= \frac{1}{2}x^2 \coth^{-1}(c + d \coth(a + bx)) - \frac{1}{2}(b(1 - c - d)) \int \frac{e^{2a+2bx}x^2}{1 - c + d + (-1 + c + d)e^{2a+2bx}} \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \coth(a + bx)) + \frac{1}{4}x^2 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{4}x^2 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \coth(a + bx)) + \frac{1}{4}x^2 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{4}x^2 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \coth(a + bx)) + \frac{1}{4}x^2 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{4}x^2 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \coth(a + bx)) + \frac{1}{4}x^2 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{4}x^2 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.158329, size = 267, normalized size = 1.17

$$-2bx \operatorname{PolyLog}\left(2, \frac{(c-d-1)(\cosh(2(a+bx))-\sinh(2(a+bx)))}{c+d-1}\right) + 2bx \operatorname{PolyLog}\left(2, \frac{(c-d+1)(\cosh(2(a+bx))-\sinh(2(a+bx)))}{c+d+1}\right) - \operatorname{PolyLog}\left(3, \frac{(c-d-1)(\cosh(2(a+bx))-\sinh(2(a+bx)))}{c+d-1}\right) + \operatorname{PolyLog}\left(3, \frac{(c-d+1)(\cosh(2(a+bx))-\sinh(2(a+bx)))}{c+d+1}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x\*ArcCoth[c + d\*Coth[a + b\*x]], x]

[Out] (x^2\*ArcCoth[c + d\*Coth[a + b\*x]])/2 + (2\*b^2\*x^2\*Log[(2\*(Cosh[a + b\*x] - Sinh[a + b\*x])\*(d\*Cosh[a + b\*x] + (-1 + c)\*Sinh[a + b\*x]))/(-1 + c + d)] - 2\*b^2\*x^2\*Log[1 + ((1 + c - d)\*(-Cosh[2\*(a + b\*x)] + Sinh[2\*(a + b\*x)])/(1 + c + d)] - 2\*b\*x\*PolyLog[2, ((-1 + c - d)\*(Cosh[2\*(a + b\*x)] - Sinh[2\*(a + b\*x)])/(-1 + c + d)] + 2\*b\*x\*PolyLog[2, ((1 + c - d)\*(Cosh[2\*(a + b\*x)] - Sinh[2\*(a + b\*x)])/(1 + c + d)] - PolyLog[3, ((-1 + c - d)\*(Cosh[2\*(a + b\*x)] - Sinh[2\*(a + b\*x)])/(-1 + c + d)] + PolyLog[3, ((1 + c - d)\*(Cosh[2\*(a + b\*x)] - Sinh[2\*(a + b\*x)])/(1 + c + d))]/(8\*b^2)

**Maple [C]** time = 4.785, size = 4918, normalized size = 21.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.



$$\begin{aligned}
& /b^2a^2/(1+c+d)*\ln((-c*\exp(b*x+a)-\exp(b*x+a)*d+((1+c-d)*(1+c+d))^{(1/2)}-\exp(b*x+a))/((1+c-d)*(1+c+d))^{(1/2)})+1/2/b^2a^2/(1+c+d)*\ln((c*\exp(b*x+a)+\exp(b*x+a)*d+((1+c-d)*(1+c+d))^{(1/2)}+\exp(b*x+a))/((1+c-d)*(1+c+d))^{(1/2)})-1/4/b^2/(1+c+d)*\ln(1-(1+c+d)*\exp(2*b*x+2*a)/(1+c-d))*a^2-1/4/b^2/(1+c+d)*\text{polylog}(2,(1+c+d)*\exp(2*b*x+2*a)/(1+c-d))*a+1/2/b^2a/(1+c+d)*\text{dilog}((c*\exp(b*x+a)+\exp(b*x+a)*d+((1+c-d)*(1+c+d))^{(1/2)}+\exp(b*x+a))/((1+c-d)*(1+c+d))^{(1/2)})+1/8/b^2c/(1+c+d)*\text{polylog}(3,(1+c+d)*\exp(2*b*x+2*a)/(1+c-d))+1/8/b^2d/(1+c+d)*\text{polylog}(3,(1+c+d)*\exp(2*b*x+2*a)/(1+c-d))+1/2/b^2a/(1+c+d)*\text{dilog}((-c*\exp(b*x+a)-\exp(b*x+a)*d+((1+c-d)*(1+c+d))^{(1/2)}-\exp(b*x+a))/((1+c-d)*(1+c+d))^{(1/2)})-1/4*x^2*\ln((\exp(2*b*x+2*a)-1)*c+(\exp(2*b*x+2*a)+1)*d-\exp(2*b*x+2*a)+1)-1/2/b*c/(1+c+d)*\ln(1-(1+c+d)*\exp(2*b*x+2*a)/(1+c-d))*x*a+1/2/b*a*c/(1+c+d)*\ln((-c*\exp(b*x+a)-\exp(b*x+a)*d+((1+c-d)*(1+c+d))^{(1/2)}-\exp(b*x+a))/((1+c-d)*(1+c+d))^{(1/2)})*x+1/2/b*a*c/(1+c+d)*\ln((c*\exp(b*x+a)+\exp(b*x+a)*d+((1+c-d)*(1+c+d))^{(1/2)}+\exp(b*x+a))/((1+c-d)*(1+c+d))^{(1/2)})*x+1/2/b*a*d/(1+c+d)*\ln((-c*\exp(b*x+a)-\exp(b*x+a)*d+((1+c-d)*(1+c+d))^{(1/2)}-\exp(b*x+a))/((1+c-d)*(1+c+d))^{(1/2)})*x+1/2/b*a*d/(1+c+d)*\ln((c*\exp(b*x+a)+\exp(b*x+a)*d+((1+c-d)*(1+c+d))^{(1/2)}+\exp(b*x+a))/((1+c-d)*(1+c+d))^{(1/2)})*x+1/4/b^2a^2*c/(c+d-1)*\ln(\exp(2*b*x+2*a)*c+\exp(2*b*x+2*a)*d-\exp(2*b*x+2*a)-c+d+1)+1/4/b^2*d*a^2/(c+d-1)*\ln(\exp(2*b*x+2*a)*c+\exp(2*b*x+2*a)*d-\exp(2*b*x+2*a)-c+d+1)+1/8*I*P i*x^2*csgn(I*((\exp(2*b*x+2*a)-1)*c+(\exp(2*b*x+2*a)+1)*d+\exp(2*b*x+2*a)-1))*csgn(I*((\exp(2*b*x+2*a)-1)*c+(\exp(2*b*x+2*a)+1)*d+\exp(2*b*x+2*a)-1)/(\exp(2*b*x+2*a)-1))^2+1/2/b*c/(c+d-1)*\ln(1-(c+d-1)*\exp(2*b*x+2*a)/(c+d-1))*x*a+1/2/b*d/(c+d-1)*\ln(1-(c+d-1)*\exp(2*b*x+2*a)/(c+d-1))*x*a-1/2/b*c*a/(c+d-1)*\ln((-c*\exp(b*x+a)-\exp(b*x+a)*d+((c+d-1)*(c+d-1))^{(1/2)}+\exp(b*x+a))/((c+d-1)*(c+d-1))^{(1/2)})*x-1/2/b*c*a/(c+d-1)*\ln((c*\exp(b*x+a)+\exp(b*x+a)*d+((c+d-1)*(c+d-1))^{(1/2)}-\exp(b*x+a))/((c+d-1)*(c+d-1))^{(1/2)})*x-1/2/b*d*a/(c+d-1)*\ln((-c*\exp(b*x+a)-\exp(b*x+a)*d+((c+d-1)*(c+d-1))^{(1/2)}+\exp(b*x+a))/((c+d-1)*(c+d-1))^{(1/2)})*x-1/2/b*d*a/(c+d-1)*\ln((c*\exp(b*x+a)+\exp(b*x+a)*d+((c+d-1)*(c+d-1))^{(1/2)}-\exp(b*x+a))/((c+d-1)*(c+d-1))^{(1/2)})*x-1/4/b^2a^2/(c+d-1)*\ln(\exp(2*b*x+2*a)*c+\exp(2*b*x+2*a)*d-\exp(2*b*x+2*a)-c+d+1)-1/4/b^2a^2*c/(1+c+d)*\ln(\exp(2*b*x+2*a)*c+\exp(2*b*x+2*a)*d+\exp(2*b*x+2*a)-c+d-1)-1/4/b^2a^2*d/(1+c+d)*\ln(\exp(2*b*x+2*a)*c+\exp(2*b*x+2*a)*d+\exp(2*b*x+2*a)-c+d-1)-1/8*I*P i*x^2*csgn(I*((\exp(2*b*x+2*a)-1)*c+(\exp(2*b*x+2*a)+1)*d+\exp(2*b*x+2*a)-1)/(\exp(2*b*x+2*a)-1))^3+1/8*I*P i*x^2*csgn(I*((\exp(2*b*x+2*a)-1)*c+(\exp(2*b*x+2*a)+1)*d-\exp(2*b*x+2*a)+1)/(\exp(2*b*x+2*a)-1))^3-1/4/(1+c+d)*\ln(1-(1+c+d)*\exp(2*b*x+2*a)/(1+c-d))*x^2+1/8/b^2/(1+c+d)*\text{polylog}(3,(1+c+d)*\exp(2*b*x+2*a)/(1+c-d))-1/8*I*P i*x^2*csgn(I*((\exp(2*b*x+2*a)-1)*c+(\exp(2*b*x+2*a)+1)*d-\exp(2*b*x+2*a)+1))*csgn(I*((\exp(2*b*x+2*a)-1)*c+(\exp(2*b*x+2*a)+1)*d-\exp(2*b*x+2*a)+1)/(\exp(2*b*x+2*a)-1))^2-1/2/b^2*c*a^2/(c+d-1)*\ln((-c*\exp(b*x+a)-\exp(b*x+a)*d+((c+d-1)*(c+d-1))^{(1/2)}+\exp(b*x+a))/((c+d-1)*(c+d-1))^{(1/2)})-1/2/b^2*c*a^2/(c+d-1)*\ln((c*\exp(b*x+a)+\exp(b*x+a)*d+((c+d-1)*(c+d-1))^{(1/2)}-\exp(b*x+a))/((c+d-1)*(c+d-1))^{(1/2)})-1/2/b^2*d*a^2/(c+d-1)*\ln((-c*\exp(b*x+a)-\exp(b*x+a)*d+((c+d-1)*(c+d-1))^{(1/2)}+\exp(b*x+a))/((c+d-1)*(c+d-1))^{(1/2)})-1/2/b^2*d*a^2/(c+d-1)*\ln((c*\exp(b*x+a)+\exp(b*x+a)*d+((c+d-1)*(c+d-1))^{(1/2)}-\exp(b*x+a))/((c+d-1)*(c+d-1))^{(1/2)})-1/2/b^2*c*a/(c+d-1)*\text{dilog}((-c*\exp(b*x+a)-
\end{aligned}$$

$$\begin{aligned} & \exp(b*x+a)*d+((c-d-1)*(c+d-1))^{(1/2)+\exp(b*x+a)} / ((c-d-1)*(c+d-1))^{(1/2)} - 1 \\ & / 2/b^2*c*a/(c+d-1)*\operatorname{dilog}((c*\exp(b*x+a)+\exp(b*x+a)*d+((c-d-1)*(c+d-1))^{(1/2)} \\ & -\exp(b*x+a)) / ((c-d-1)*(c+d-1))^{(1/2)} - 1/2/b^2*d*a/(c+d-1)*\operatorname{dilog}((-c*\exp(b*x \\ & +a)-\exp(b*x+a)*d+((c-d-1)*(c+d-1))^{(1/2)+\exp(b*x+a)} / ((c-d-1)*(c+d-1))^{(1/2)} \\ & )) - 1/2/b^2*d*a/(c+d-1)*\operatorname{dilog}((c*\exp(b*x+a)+\exp(b*x+a)*d+((c-d-1)*(c+d-1))^{( \\ & 1/2)-\exp(b*x+a)} / ((c-d-1)*(c+d-1))^{(1/2)})) + 1/4/b^2*c/(c+d-1)*\ln(1-(c+d-1)*\exp \\ & (2*b*x+2*a)/(c-d-1))*a^2+1/4/b*c/(c+d-1)*\operatorname{polylog}(2,(c+d-1)*\exp(2*b*x+2*a)/ \\ & (c-d-1))*x+1/4/b^2*c/(c+d-1)*\operatorname{polylog}(2,(c+d-1)*\exp(2*b*x+2*a)/(c-d-1))*a+1/ \\ & 4/b^2*d/(c+d-1)*\ln(1-(c+d-1)*\exp(2*b*x+2*a)/(c-d-1))*a^2+1/4/b*d/(c+d-1)*\operatorname{po \\ & lylog}(2,(c+d-1)*\exp(2*b*x+2*a)/(c-d-1))*x+1/4/b^2*d/(c+d-1)*\operatorname{polylog}(2,(c+d- \\ & 1)*\exp(2*b*x+2*a)/(c-d-1))*a-1/2/b/(c+d-1)*\ln(1-(c+d-1)*\exp(2*b*x+2*a)/(c-d \\ & -1))*x*a+1/2/b*a/(c+d-1)*\ln((-c*\exp(b*x+a)-\exp(b*x+a)*d+((c-d-1)*(c+d-1))^{( \\ & 1/2)+\exp(b*x+a)} / ((c-d-1)*(c+d-1))^{(1/2)}))*x+1/2/b*a/(c+d-1)*\ln((c*\exp(b*x+a \\ & )+\exp(b*x+a)*d+((c-d-1)*(c+d-1))^{(1/2)-\exp(b*x+a)} / ((c-d-1)*(c+d-1))^{(1/2)})) \\ & *x+1/8*I*Pi*x^2*\operatorname{csgn}(I/(\exp(2*b*x+2*a)-1))*\operatorname{csgn}(I*((\exp(2*b*x+2*a)-1)*c+(\exp \\ & (2*b*x+2*a)+1)*d-\exp(2*b*x+2*a)+1))*\operatorname{csgn}(I*((\exp(2*b*x+2*a)-1)*c+(\exp(2*b* \\ & x+2*a)+1)*d-\exp(2*b*x+2*a)+1)/(\exp(2*b*x+2*a)-1))-1/8*I*Pi*x^2*\operatorname{csgn}(I/(\exp( \\ & 2*b*x+2*a)-1))*\operatorname{csgn}(I*((\exp(2*b*x+2*a)-1)*c+(\exp(2*b*x+2*a)+1)*d+\exp(2*b*x+ \\ & 2*a)-1))*\operatorname{csgn}(I*((\exp(2*b*x+2*a)-1)*c+(\exp(2*b*x+2*a)+1)*d+\exp(2*b*x+2*a)-1 \\ & ))/(\exp(2*b*x+2*a)-1)) \end{aligned}$$

**Maxima [A]** time = 2.22562, size = 288, normalized size = 1.26

$$-\frac{1}{8}bd \left( \frac{2b^2x^2 \log\left(-\frac{(c+d+1)e^{2bx+2a}}{c-d+1} + 1\right) + 2bx \operatorname{Li}_2\left(\frac{(c+d+1)e^{2bx+2a}}{c-d+1}\right) - \operatorname{Li}_3\left(\frac{(c+d+1)e^{2bx+2a}}{c-d+1}\right)}{b^3d} - \frac{2b^2x^2 \log\left(-\frac{(c+d-1)e^{2bx+2a}}{c-d-1} + 1\right)}{b^3d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(c+d\*coth(b\*x+a)),x, algorithm="maxima")

[Out]  $-1/8*b*d*((2*b^2*x^2*\log(-(c+d+1)*e^{(2*b*x+2*a)/(c-d+1)}+1)+2*b*x*\operatorname{dilog}((c+d+1)*e^{(2*b*x+2*a)/(c-d+1)}-\operatorname{polylog}(3,(c+d+1)*e^{(2*b*x+2*a)/(c-d+1)})))/(b^3*d)-(2*b^2*x^2*\log(-(c+d-1)*e^{(2*b*x+2*a)/(c-d-1)}+1)+2*b*x*\operatorname{dilog}((c+d-1)*e^{(2*b*x+2*a)/(c-d-1)}-\operatorname{polylog}(3,(c+d-1)*e^{(2*b*x+2*a)/(c-d-1)})))/(b^3*d))+1/2*x^2*\operatorname{arccoth}(d*\operatorname{coth}(b*x+a)+c)$

**Fricas [C]** time = 2.23779, size = 2093, normalized size = 9.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccoth(c+d*coth(b*x+a)),x, algorithm="fricas")`

[Out] 
$$\frac{1}{4} \cdot (b^2 x^2 \log((d \cosh(bx + a) + (c + 1) \sinh(bx + a)) / (d \cosh(bx + a) + (c - 1) \sinh(bx + a))) - 2bx \operatorname{dilog}(\sqrt{(c + d + 1)/(c - d + 1)}) (\cosh(bx + a) + \sinh(bx + a)) - 2bx \operatorname{dilog}(-\sqrt{(c + d + 1)/(c - d + 1)}) (\cosh(bx + a) + \sinh(bx + a)) + 2bx \operatorname{dilog}(\sqrt{(c + d - 1)/(c - d - 1)}) (\cosh(bx + a) + \sinh(bx + a)) + 2bx \operatorname{dilog}(-\sqrt{(c + d - 1)/(c - d - 1)}) (\cosh(bx + a) + \sinh(bx + a)) - a^2 \log(2(c + d + 1) \cosh(bx + a) + 2(c + d + 1) \sinh(bx + a) + 2(c - d + 1) \sqrt{(c + d + 1)/(c - d + 1)}) - a^2 \log(2(c + d + 1) \cosh(bx + a) + 2(c + d + 1) \sinh(bx + a) - 2(c - d + 1) \sqrt{(c + d + 1)/(c - d + 1)}) + a^2 \log(2(c + d - 1) \cosh(bx + a) + 2(c + d - 1) \sinh(bx + a) - 2(c - d - 1) \sqrt{(c + d - 1)/(c - d - 1)}) + a^2 \log(2(c + d - 1) \cosh(bx + a) + 2(c + d - 1) \sinh(bx + a) - 2(c - d - 1) \sqrt{(c + d - 1)/(c - d - 1)}) - (b^2 x^2 - a^2) \log(\sqrt{(c + d + 1)/(c - d + 1)} (\cosh(bx + a) + \sinh(bx + a)) + 1) - (b^2 x^2 - a^2) \log(-\sqrt{(c + d + 1)/(c - d + 1)} (\cosh(bx + a) + \sinh(bx + a)) + 1) + (b^2 x^2 - a^2) \log(\sqrt{(c + d - 1)/(c - d - 1)} (\cosh(bx + a) + \sinh(bx + a)) + 1) + (b^2 x^2 - a^2) \log(-\sqrt{(c + d - 1)/(c - d - 1)} (\cosh(bx + a) + \sinh(bx + a)) + 1) + 2 \operatorname{polylog}(3, \sqrt{(c + d + 1)/(c - d + 1)}) (\cosh(bx + a) + \sinh(bx + a)) + 2 \operatorname{polylog}(3, -\sqrt{(c + d + 1)/(c - d + 1)}) (\cosh(bx + a) + \sinh(bx + a)) - 2 \operatorname{polylog}(3, \sqrt{(c + d - 1)/(c - d - 1)}) (\cosh(bx + a) + \sinh(bx + a)) - 2 \operatorname{polylog}(3, -\sqrt{(c + d - 1)/(c - d - 1)}) (\cosh(bx + a) + \sinh(bx + a))) / b^2$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*acoth(c+d*coth(b*x+a)),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arccoth}(d \coth(bx + a) + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(c+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(d*coth(b*x + a) + c), x)
```



### 3.219 $\int \coth^{-1}(c + d \coth(a + bx)) dx$

**Optimal.** Leaf size=150

$$\frac{\text{PolyLog}\left(2, \frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b} - \frac{\text{PolyLog}\left(2, \frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b} + \frac{1}{2}x \log\left(1 - \frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right) - \frac{1}{2}x \log\left(1 - \frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)$$

[Out] x\*ArcCoth[c + d\*Coth[a + b\*x]] + (x\*Log[1 - ((1 - c - d)\*E^(2\*a + 2\*b\*x))/(1 - c + d)])/2 - (x\*Log[1 - ((1 + c + d)\*E^(2\*a + 2\*b\*x))/(1 + c - d)])/2 + PolyLog[2, ((1 - c - d)\*E^(2\*a + 2\*b\*x))/(1 - c + d)]/(4\*b) - PolyLog[2, ((1 + c + d)\*E^(2\*a + 2\*b\*x))/(1 + c - d)]/(4\*b)

**Rubi [A]** time = 0.234753, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {6238, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2, \frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b} - \frac{\text{PolyLog}\left(2, \frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b} + \frac{1}{2}x \log\left(1 - \frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right) - \frac{1}{2}x \log\left(1 - \frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[c + d\*Coth[a + b\*x]],x]

[Out] x\*ArcCoth[c + d\*Coth[a + b\*x]] + (x\*Log[1 - ((1 - c - d)\*E^(2\*a + 2\*b\*x))/(1 - c + d)])/2 - (x\*Log[1 - ((1 + c + d)\*E^(2\*a + 2\*b\*x))/(1 + c - d)])/2 + PolyLog[2, ((1 - c - d)\*E^(2\*a + 2\*b\*x))/(1 - c + d)]/(4\*b) - PolyLog[2, ((1 + c + d)\*E^(2\*a + 2\*b\*x))/(1 + c - d)]/(4\*b)

#### Rule 6238

Int[ArcCoth[(c\_.) + Coth[(a\_.) + (b\_.)\*(x\_)]]\*(d\_.)], x\_Symbol] := Simp[x\*ArcCoth[c + d\*Coth[a + b\*x]], x] + (-Dist[b\*(1 - c - d), Int[(x\*E^(2\*a + 2\*b\*x))/(1 - c + d - (1 - c - d)\*E^(2\*a + 2\*b\*x)), x], x] + Dist[b\*(1 + c + d), Int[(x\*E^(2\*a + 2\*b\*x))/(1 + c - d - (1 + c + d)\*E^(2\*a + 2\*b\*x)), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, 1]

#### Rule 2190

Int[(((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*(F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)], x\_Symbol] := Simp

```

[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

```

### Rule 2279

```

Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

```

### Rule 2391

```

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

```

### Rubi steps

$$\begin{aligned}
\int \coth^{-1}(c + d \coth(a + bx)) dx &= x \coth^{-1}(c + d \coth(a + bx)) - (b(1 - c - d)) \int \frac{e^{2a+2bx} x}{1 - c + d + (-1 + c + d)e^{2a+2bx}} dx + \dots \\
&= x \coth^{-1}(c + d \coth(a + bx)) + \frac{1}{2} x \log \left( 1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d} \right) - \frac{1}{2} x \log \left( 1 - \frac{(1 + c + d)e^{2a+2bx}}{1 - c + d} \right) \\
&= x \coth^{-1}(c + d \coth(a + bx)) + \frac{1}{2} x \log \left( 1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d} \right) - \frac{1}{2} x \log \left( 1 - \frac{(1 + c + d)e^{2a+2bx}}{1 - c + d} \right) \\
&= x \coth^{-1}(c + d \coth(a + bx)) + \frac{1}{2} x \log \left( 1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d} \right) - \frac{1}{2} x \log \left( 1 - \frac{(1 + c + d)e^{2a+2bx}}{1 - c + d} \right)
\end{aligned}$$

**Mathematica [A]** time = 1.12623, size = 131, normalized size = 0.87

$$x \coth^{-1}(d \coth(a + bx) + c) - \frac{-\text{PolyLog}\left(2, \frac{(c+d-1)e^{2(a+bx)}}{c-d-1}\right) + \text{PolyLog}\left(2, \frac{(c+d+1)e^{2(a+bx)}}{c-d+1}\right) - 2bx \left(\log\left(1 - \frac{(c+d-1)e^{2(a+bx)}}{c-d-1}\right) - \log\left(1 - \frac{(c+d+1)e^{2(a+bx)}}{c-d+1}\right)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[c + d*Coth[a + b*x]], x]
```

```
[Out] x*ArcCoth[c + d*Coth[a + b*x]] - (-2*b*x*(Log[1 - ((-1 + c + d)*E^(2*(a + b
*x)))/(-1 + c - d)] - Log[1 - ((1 + c + d)*E^(2*(a + b*x)))/(1 + c - d)]) -
PolyLog[2, ((-1 + c + d)*E^(2*(a + b*x)))/(-1 + c - d)] + PolyLog[2, ((1 +

```

$$(c + d) * E^{(2 * (a + b * x))} / (1 + c - d)] / (4 * b)$$

**Maple [B]** time = 0.171, size = 306, normalized size = 2.

$$\frac{\operatorname{arccoth}(c + d \operatorname{coth}(bx + a)) \ln(d \operatorname{coth}(bx + a) - d)}{2b} + \frac{\operatorname{arccoth}(c + d \operatorname{coth}(bx + a)) \ln(d \operatorname{coth}(bx + a) + d)}{2b} + \frac{1}{4b} \operatorname{dilog}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(c+d*coth(b*x+a)),x)`

[Out]  $-1/2/b * \operatorname{arccoth}(c + d * \operatorname{coth}(b * x + a)) * \ln(d * \operatorname{coth}(b * x + a) - d) + 1/2/b * \operatorname{arccoth}(c + d * \operatorname{coth}(b * x + a)) * \ln(d * \operatorname{coth}(b * x + a) + d) + 1/4/b * \operatorname{dilog}((d * \operatorname{coth}(b * x + a) + c + 1) / (1 + c + d)) + 1/4/b * \ln(d * \operatorname{coth}(b * x + a) - d) * \ln((d * \operatorname{coth}(b * x + a) + c + 1) / (1 + c + d)) - 1/4/b * \operatorname{dilog}((d * \operatorname{coth}(b * x + a) + c - 1) / (c + d - 1)) - 1/4/b * \ln(d * \operatorname{coth}(b * x + a) - d) * \ln((d * \operatorname{coth}(b * x + a) + c - 1) / (c + d - 1)) + 1/4/b * \operatorname{dilog}((d * \operatorname{coth}(b * x + a) + c - 1) / (c - d - 1)) + 1/4/b * \ln(d * \operatorname{coth}(b * x + a) + d) * \ln((d * \operatorname{coth}(b * x + a) + c - 1) / (c - d - 1)) - 1/4/b * \operatorname{dilog}((d * \operatorname{coth}(b * x + a) + c + 1) / (1 + c - d)) - 1/4/b * \ln(d * \operatorname{coth}(b * x + a) + d) * \ln((d * \operatorname{coth}(b * x + a) + c + 1) / (1 + c - d))$

**Maxima [A]** time = 2.27015, size = 192, normalized size = 1.28

$$-\frac{1}{4} b d \left( \frac{2 b x \log\left(-\frac{(c+d+1)e^{2bx+2a}}{c-d+1} + 1\right) + \operatorname{Li}_2\left(\frac{(c+d+1)e^{2bx+2a}}{c-d+1}\right)}{b^2 d} - \frac{2 b x \log\left(-\frac{(c+d-1)e^{2bx+2a}}{c-d-1} + 1\right) + \operatorname{Li}_2\left(\frac{(c+d-1)e^{2bx+2a}}{c-d-1}\right)}{b^2 d} \right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(c+d*coth(b*x+a)),x, algorithm="maxima")`

[Out]  $-1/4 * b * d * ((2 * b * x * \log(-(c + d + 1) * e^{(2 * b * x + 2 * a)} / (c - d + 1) + 1) + \operatorname{dilog}((c + d + 1) * e^{(2 * b * x + 2 * a)} / (c - d + 1))) / (b^2 * d) - (2 * b * x * \log(-(c + d - 1) * e^{(2 * b * x + 2 * a)} / (c - d - 1) + 1) + \operatorname{dilog}((c + d - 1) * e^{(2 * b * x + 2 * a)} / (c - d - 1))) / (b^2 * d) + x * \operatorname{arccoth}(d * \operatorname{coth}(b * x + a) + c)$

**Fricas [B]** time = 2.16709, size = 1585, normalized size = 10.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c+d\*coth(b\*x+a)),x, algorithm="fricas")

[Out]  $\frac{1}{2}(b*x*\log((d*\cosh(b*x + a) + (c + 1)*\sinh(b*x + a))/(d*\cosh(b*x + a) + (c - 1)*\sinh(b*x + a))) + a*\log(2*(c + d + 1)*\cosh(b*x + a) + 2*(c + d + 1)*\sinh(b*x + a) + 2*(c - d + 1)*\sqrt{(c + d + 1)/(c - d + 1)}) + a*\log(2*(c + d + 1)*\cosh(b*x + a) + 2*(c + d + 1)*\sinh(b*x + a) - 2*(c - d + 1)*\sqrt{(c + d + 1)/(c - d + 1)}) - a*\log(2*(c + d - 1)*\cosh(b*x + a) + 2*(c + d - 1)*\sinh(b*x + a) + 2*(c - d - 1)*\sqrt{(c + d - 1)/(c - d - 1)}) - a*\log(2*(c + d - 1)*\cosh(b*x + a) + 2*(c + d - 1)*\sinh(b*x + a) - 2*(c - d - 1)*\sqrt{(c + d - 1)/(c - d - 1)}) - (b*x + a)*\log(\sqrt{(c + d + 1)/(c - d + 1)}*(\cosh(b*x + a) + \sinh(b*x + a)) + 1) - (b*x + a)*\log(-\sqrt{(c + d + 1)/(c - d + 1)}*(\cosh(b*x + a) + \sinh(b*x + a)) + 1) + (b*x + a)*\log(\sqrt{(c + d - 1)/(c - d - 1)}*(\cosh(b*x + a) + \sinh(b*x + a)) + 1) + (b*x + a)*\log(-\sqrt{(c + d - 1)/(c - d - 1)}*(\cosh(b*x + a) + \sinh(b*x + a)) + 1) - \operatorname{dilog}(\sqrt{(c + d + 1)/(c - d + 1)}*(\cosh(b*x + a) + \sinh(b*x + a))) - \operatorname{dilog}(-\sqrt{(c + d + 1)/(c - d + 1)}*(\cosh(b*x + a) + \sinh(b*x + a))) + \operatorname{dilog}(\sqrt{(c + d - 1)/(c - d - 1)}*(\cosh(b*x + a) + \sinh(b*x + a))) + \operatorname{dilog}(-\sqrt{(c + d - 1)/(c - d - 1)}*(\cosh(b*x + a) + \sinh(b*x + a))))/b$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoath(c+d\*coth(b\*x+a)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(d \coth(bx + a) + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c+d\*coth(b\*x+a)),x, algorithm="giac")

```
[Out] integrate(arccoth(d*coth(b*x + a) + c), x)
```

$$3.220 \quad \int \frac{\coth^{-1}(c+d \coth(a+bx))}{x} dx$$

**Optimal.** Leaf size=17

$$\text{CannotIntegrate}\left(\frac{\coth^{-1}(d \coth(a+bx)+c)}{x}, x\right)$$

[Out] CannotIntegrate[ArcCoth[c + d\*Coth[a + b\*x]]/x, x]

**Rubi [A]** time = 0.147025, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\coth^{-1}(c+d \coth(a+bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[ArcCoth[c + d\*Coth[a + b\*x]]/x,x]

[Out] Defer[Int][ArcCoth[c + d\*Coth[a + b\*x]]/x, x]

Rubi steps

$$\int \frac{\coth^{-1}(c+d \coth(a+bx))}{x} dx = \int \frac{\coth^{-1}(c+d \coth(a+bx))}{x} dx$$

**Mathematica [A]** time = 5.47975, size = 0, normalized size = 0.

$$\int \frac{\coth^{-1}(c+d \coth(a+bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcCoth[c + d\*Coth[a + b\*x]]/x,x]

[Out] Integrate[ArcCoth[c + d\*Coth[a + b\*x]]/x, x]

---

**Maple [A]** time = 0.337, size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(c + d \coth(bx + a))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(c+d\*coth(b\*x+a))/x,x)

[Out] int(arccoth(c+d\*coth(b\*x+a))/x,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(d \coth(bx + a) + c)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c+d\*coth(b\*x+a))/x,x, algorithm="maxima")

[Out] integrate(arccoth(d\*coth(b\*x + a) + c)/x, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arccoth}(d \coth(bx + a) + c)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c+d\*coth(b\*x+a))/x,x, algorithm="fricas")

[Out] integral(arccoth(d\*coth(b\*x + a) + c)/x, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(c+d\*coth(b\*x+a))/x,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(d \coth(bx + a) + c)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c+d\*coth(b\*x+a))/x,x, algorithm="giac")

[Out] integrate(arccoth(d\*coth(b\*x + a) + c)/x, x)



### 3.221 $\int x^3 \coth^{-1}(1 + d + d \coth(a + bx)) dx$

**Optimal.** Leaf size=152

$$\frac{3x^2 \text{PolyLog}\left(3, (d+1)e^{2a+2bx}\right)}{8b^2} - \frac{3x \text{PolyLog}\left(4, (d+1)e^{2a+2bx}\right)}{8b^3} + \frac{3 \text{PolyLog}\left(5, (d+1)e^{2a+2bx}\right)}{16b^4} - \frac{x^3 \text{PolyLog}\left(2, (d+1)e^{2a+2bx}\right)}{4b}$$

[Out] (b\*x^5)/20 + (x^4\*ArcCoth[1 + d + d\*Coth[a + b\*x]])/4 - (x^4\*Log[1 - (1 + d)\*E^(2\*a + 2\*b\*x)])/8 - (x^3\*PolyLog[2, (1 + d)\*E^(2\*a + 2\*b\*x)])/(4\*b) + (3\*x^2\*PolyLog[3, (1 + d)\*E^(2\*a + 2\*b\*x)])/(8\*b^2) - (3\*x\*PolyLog[4, (1 + d)\*E^(2\*a + 2\*b\*x)])/(8\*b^3) + (3\*PolyLog[5, (1 + d)\*E^(2\*a + 2\*b\*x)])/(16\*b^4)

**Rubi [A]** time = 0.30751, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {6242, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2 \text{PolyLog}\left(3, (d+1)e^{2a+2bx}\right)}{8b^2} - \frac{3x \text{PolyLog}\left(4, (d+1)e^{2a+2bx}\right)}{8b^3} + \frac{3 \text{PolyLog}\left(5, (d+1)e^{2a+2bx}\right)}{16b^4} - \frac{x^3 \text{PolyLog}\left(2, (d+1)e^{2a+2bx}\right)}{4b}$$

Antiderivative was successfully verified.

[In] Int[x^3\*ArcCoth[1 + d + d\*Coth[a + b\*x]], x]

[Out] (b\*x^5)/20 + (x^4\*ArcCoth[1 + d + d\*Coth[a + b\*x]])/4 - (x^4\*Log[1 - (1 + d)\*E^(2\*a + 2\*b\*x)])/8 - (x^3\*PolyLog[2, (1 + d)\*E^(2\*a + 2\*b\*x)])/(4\*b) + (3\*x^2\*PolyLog[3, (1 + d)\*E^(2\*a + 2\*b\*x)])/(8\*b^2) - (3\*x\*PolyLog[4, (1 + d)\*E^(2\*a + 2\*b\*x)])/(8\*b^3) + (3\*PolyLog[5, (1 + d)\*E^(2\*a + 2\*b\*x)])/(16\*b^4)

#### Rule 6242

Int[ArcCoth[(c\_.) + Coth[(a\_.) + (b\_.)\*(x\_)]\*(d\_.)]\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((e + f\*x)^(m + 1)\*ArcCoth[c + d\*Coth[a + b\*x]])/(f\*(m + 1)), x] + Dist[b/(f\*(m + 1)), Int[(e + f\*x)^(m + 1)/(c - d - c\*E^(2\*a + 2\*b\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d)^2, 1]

#### Rule 2184

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.), x\_Symbol] := Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[

$b/a, \text{Int}[(c + d*x)^m*(F^{(g*(e + f*x)))^n}/(a + b*(F^{(g*(e + f*x)))^n}), x], x] /;$  FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2190

$\text{Int}[(F^{(g_*)*(e_*) + (f_*)*(x_*)})^{(n_*)}*((c_*) + (d_*)*(x_*)^{(m_*)})/((a_*) + (b_*)*(F^{(g_*)*(e_*) + (f_*)*(x_*)})^{(n_*)}), x\_Symbol] :> \text{Simp}[(c + d*x)^m*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a]/(b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a], x], x] /;$  FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

$\text{Int}[\text{Log}[1 + (e_*)*(F^{(c_*)*(a_*) + (b_*)*(x_*)})^{(n_*)}]*((f_*) + (g_*)*(x_*)^{(m_*)}), x\_Symbol] :> -\text{Simp}[(f + g*x)^m*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x)))^n})]/(b*c*n*\text{Log}[F]), x] + \text{Dist}[(g*m)/(b*c*n*\text{Log}[F]), \text{Int}[(f + g*x)^{(m-1)}*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x)))^n})], x], x] /;$  FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 6609

$\text{Int}[(e_*) + (f_*)*(x_*)^{(m_*)}*\text{PolyLog}[n_*, (d_*)*(F^{(c_*)*(a_*) + (b_*)*(x_*)})^{(p_*)}], x\_Symbol] :> \text{Simp}[(e + f*x)^m*\text{PolyLog}[n + 1, d*(F^{(c*(a + b*x)))^p}]/(b*c*p*\text{Log}[F]), x] - \text{Dist}[(f*m)/(b*c*p*\text{Log}[F]), \text{Int}[(e + f*x)^{(m-1)}*\text{PolyLog}[n + 1, d*(F^{(c*(a + b*x)))^p}], x], x] /;$  FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

### Rule 2282

$\text{Int}[u_*, x\_Symbol] :> \text{With}[\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x]] /;$  FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_\*)\*(v\_\*)^{(n\_\*)})^{(m\_\*)} /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n]] && !MatchQ[u, E^{(c\_\*)\*(a\_\*) + (b\_\*)\*x})\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

$\text{Int}[\text{PolyLog}[n_*, (c_*)*(a_*) + (b_*)*(x_*)^{(p_*)}]/((d_*) + (e_*)*(x_*)), x\_Symbol] :> \text{Simp}[\text{PolyLog}[n + 1, c*(a + b*x)^p]/(e*p), x] /;$  FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
\int x^3 \coth^{-1}(1 + d + d \coth(a + bx)) dx &= \frac{1}{4} x^4 \coth^{-1}(1 + d + d \coth(a + bx)) + \frac{1}{4} b \int \frac{x^4}{1 + (-1 - d)e^{2a+2bx}} dx \\
&= \frac{bx^5}{20} + \frac{1}{4} x^4 \coth^{-1}(1 + d + d \coth(a + bx)) + \frac{1}{4} (b(1 + d)) \int \frac{e^{2a+2bx} x^4}{1 + (-1 - d)e^{2a+2bx}} dx \\
&= \frac{bx^5}{20} + \frac{1}{4} x^4 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{8} x^4 \log(1 - (1 + d)e^{2a+2bx}) + \frac{1}{2} x^4 \log(1 - (1 + d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4} x^4 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{8} x^4 \log(1 - (1 + d)e^{2a+2bx}) - \frac{1}{8} x^4 \log(1 - (1 + d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4} x^4 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{8} x^4 \log(1 - (1 + d)e^{2a+2bx}) - \frac{1}{8} x^4 \log(1 - (1 + d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4} x^4 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{8} x^4 \log(1 - (1 + d)e^{2a+2bx}) - \frac{1}{8} x^4 \log(1 - (1 + d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4} x^4 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{8} x^4 \log(1 - (1 + d)e^{2a+2bx}) - \frac{1}{8} x^4 \log(1 - (1 + d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4} x^4 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{8} x^4 \log(1 - (1 + d)e^{2a+2bx}) - \frac{1}{8} x^4 \log(1 - (1 + d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4} x^4 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{8} x^4 \log(1 - (1 + d)e^{2a+2bx}) - \frac{1}{8} x^4 \log(1 - (1 + d)e^{2a+2bx})
\end{aligned}$$

**Mathematica [A]** time = 0.174493, size = 141, normalized size = 0.93

$$\frac{1}{16} \left( \frac{6x^2 \text{PolyLog}\left(3, \frac{e^{-2(a+bx)}}{d+1}\right)}{b^2} + \frac{6x \text{PolyLog}\left(4, \frac{e^{-2(a+bx)}}{d+1}\right)}{b^3} + \frac{3 \text{PolyLog}\left(5, \frac{e^{-2(a+bx)}}{d+1}\right)}{b^4} + \frac{4x^3 \text{PolyLog}\left(2, \frac{e^{-2(a+bx)}}{d+1}\right)}{b} - 2x^4 \log\left(1 - (1 + d)e^{2a+2bx}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*ArcCoth[1 + d + d\*Coth[a + b\*x]], x]

[Out] (4\*x^4\*ArcCoth[1 + d + d\*Coth[a + b\*x]] - 2\*x^4\*Log[1 - 1/((1 + d)\*E^(2\*(a + b\*x)))] + (4\*x^3\*PolyLog[2, 1/((1 + d)\*E^(2\*(a + b\*x)))]/b + (6\*x^2\*PolyLog[3, 1/((1 + d)\*E^(2\*(a + b\*x)))]/b^2 + (6\*x\*PolyLog[4, 1/((1 + d)\*E^(2\*(a + b\*x)))]/b^3 + (3\*PolyLog[5, 1/((1 + d)\*E^(2\*(a + b\*x)))]/b^4)/16 - 2\*x^4\*Log[1 - (1 + d)\*E^(2\*(a + b\*x))])

**Maple [C]** time = 32.161, size = 1698, normalized size = 11.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^3 \text{arccoth}(1+d+d \coth(b*x+a)), x)$

[Out] 
$$\begin{aligned} & -1/8/b^4*d*a^4/(1+d)*\ln(\exp(2*b*x+2*a)*d+\exp(2*b*x+2*a)-1)+1/2/b^3*a^3/(1+d) \\ & *\ln(1-\exp(b*x+a)*(1+d)^{(1/2)})*x+1/2/b^3*a^3/(1+d)*\ln(1+\exp(b*x+a)*(1+d)^{(1/2)}) \\ & *x-1/2/b^3*a^3/(1+d)*\ln(1-(1+d)*\exp(2*b*x+2*a))*x-1/4/b*d/(1+d)*\text{polylog} \\ & (2, (1+d)*\exp(2*b*x+2*a))*x^3+3/8/b^2*d/(1+d)*\text{polylog}(3, (1+d)*\exp(2*b*x+2*a)) \\ & *x^2-3/8/b^3*d/(1+d)*\text{polylog}(4, (1+d)*\exp(2*b*x+2*a))*x+1/2/b^4*d*a^3/(1+d) \\ & *\text{dilog}(1-\exp(b*x+a)*(1+d)^{(1/2)})+1/2/b^4*d*a^3/(1+d)*\text{dilog}(1+\exp(b*x+a)*(1+d)^{(1/2)}) \\ & +1/2/b^4*d*a^4/(1+d)*\ln(1+\exp(b*x+a)*(1+d)^{(1/2)})-3/8/b^4*d*a^4/(1+d) \\ & *\ln(1-(1+d)*\exp(2*b*x+2*a))-1/4/b^4*d*a^3/(1+d)*\text{polylog}(2, (1+d)*\exp(2*b*x+2*a)) \\ & -1/16*I*x^4*Pi*csgn(I/(\exp(2*b*x+2*a)-1))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)-1))^2 \\ & +1/16*I*x^4*Pi*csgn(I/(\exp(2*b*x+2*a)-1))*csgn(I/(\exp(2*b*x+2*a)-1))*(\exp(2*b*x+2*a)*d+\exp(2*b*x+2*a)-1))^2 \\ & +1/16*I*x^4*Pi*csgn(I*(\exp(2*b*x+2*a)*d+\exp(2*b*x+2*a)-1))*csgn(I/(\exp(2*b*x+2*a)-1)*(\exp(2*b*x+2*a)*d+\exp(2*b*x+2*a)-1))^2 \\ & -1/16*I*x^4*Pi*csgn(I*d)*csgn(I*d/(\exp(2*b*x+2*a)-1)*\exp(2*b*x+2*a))^2 \\ & -1/16*I*x^4*Pi*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)-1))*csgn(I*d/(\exp(2*b*x+2*a)-1)*\exp(2*b*x+2*a))^2 \\ & +1/2/b^3*d*a^3/(1+d)*\ln(1+\exp(b*x+a)*(1+d)^{(1/2)})*x-1/2/b^3*d*a^3/(1+d)*\ln(1-(1+d)*\exp(2*b*x+2*a))*x \\ & +1/2/b^3*d*a^3/(1+d)*\ln(1-\exp(b*x+a)*(1+d)^{(1/2)})*x+1/2/b^4*d*a^4/(1+d)*\ln(1-\exp(b*x+a)*(1+d)^{(1/2)}) \\ & -1/16*I*x^4*Pi*csgn(I/(\exp(2*b*x+2*a)-1))*(\exp(2*b*x+2*a)*d+\exp(2*b*x+2*a)-1))^3 \\ & +1/16*I*x^4*Pi*csgn(I*\exp(b*x+a))^2*csgn(I*\exp(2*b*x+2*a))-1/8*I*x^4*Pi*csgn(I*\exp(b*x+a))*csgn(I*\exp(2*b*x+2*a))^2 \\ & +1/16*I*x^4*Pi*csgn(I*\exp(2*b*x+2*a))^3+1/20*b*x^5+1/16*I*x^4*Pi*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)-1))^3 \\ & +1/8*x^4*\ln(\exp(2*b*x+2*a)*d+\exp(2*b*x+2*a)-1)-1/8/b^4*a^4/(1+d)*\ln(\exp(2*b*x+2*a)*d+\exp(2*b*x+2*a)-1) \\ & +3/16/b^4/(1+d)*\text{polylog}(5, (1+d)*\exp(2*b*x+2*a))-1/8/(1+d)*\ln(1-(1+d)*\exp(2*b*x+2*a))*x^4-1/8*d/(1+d)*\ln(1-(1+d)*\exp(2*b*x+2*a))*x^4 \\ & -1/4/b^4*a^3/(1+d)*\text{polylog}(2, (1+d)*\exp(2*b*x+2*a))*x^4-1/4/b^4*a^3/(1+d)*\text{polylog}(2, (1+d)*\exp(2*b*x+2*a)) \\ & -1/4/b/(1+d)*\text{polylog}(2, (1+d)*\exp(2*b*x+2*a))*x^3+3/8/b^2/(1+d)*\text{polylog}(3, (1+d)*\exp(2*b*x+2*a))*x^2-3/8/b^3/(1+d)*\text{polylog}(4, (1+d)*\exp(2*b*x+2*a))*x \\ & +1/2/b^4*a^3/(1+d)*\text{dilog}(1-\exp(b*x+a)*(1+d)^{(1/2)})+1/2/b^4*a^3/(1+d)*\text{dilog}(1+\exp(b*x+a)*(1+d)^{(1/2)}) \\ & +3/16/b^4*d/(1+d)*\text{polylog}(5, (1+d)*\exp(2*b*x+2*a))+1/2/b^4*a^4/(1+d)*\ln(1-\exp(b*x+a)*(1+d)^{(1/2)}) \\ & +1/2/b^4*a^4/(1+d)*\ln(1+\exp(b*x+a)*(1+d)^{(1/2)})-3/8/b^4*a^4/(1+d)*\ln(1-(1+d)*\exp(2*b*x+2*a))+1/16*I*x^4*Pi*csgn(I*d/(\exp(2*b*x+2*a)-1)*\exp(2*b*x+2*a))^3 \\ & -1/4*x^4*\ln(\exp(b*x+a))-1/8*x^4*\ln(d)-1/16*I*x^4*Pi*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)-1))^2 \\ & +1/16*I*x^4*Pi*csgn(I*\exp(2*b*x+2*a))*csgn(I/(\exp(2*b*x+2*a)-1))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)-1)) \\ & +1/16*I*x^4*Pi*csgn(I*d)*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)-1))*csgn(I*d/(\exp(2*b*x+2*a)-1)*\exp(2*b*x+2*a)) \\ & -1/16*I*x^4*Pi*csgn(I/(\exp(2*b*x+2*a)-1))*csgn(I*(\exp(2*b*x+2*a)*d+\exp(2*b*x+2*a)-1))*csgn(I/(\exp(2*b*x+2*a)-1)*(\exp(2*b*x+2*a)*d+\exp(2*b*x+2*a)-1)) \end{aligned}$$

**Maxima [A]** time = 3.42852, size = 197, normalized size = 1.3

$$\frac{1}{4}x^4 \operatorname{arccoth}(d \coth(bx + a) + d + 1) + \frac{1}{40} \left( \frac{2x^5}{d} - \frac{5(2b^4x^4 \log(-(d+1)e^{(2bx+2a)} + 1) + 4b^3x^3 \operatorname{Li}_2((d+1)e^{(2bx+2a)}) - \dots}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(1+d+d\*coth(b\*x+a)),x, algorithm="maxima")

[Out] 1/4\*x^4\*arccoth(d\*coth(b\*x + a) + d + 1) + 1/40\*(2\*x^5/d - 5\*(2\*b^4\*x^4\*log(-(d + 1)\*e^(2\*b\*x + 2\*a) + 1) + 4\*b^3\*x^3\*dilog((d + 1)\*e^(2\*b\*x + 2\*a)) - 6\*b^2\*x^2\*polylog(3, (d + 1)\*e^(2\*b\*x + 2\*a)) + 6\*b\*x\*polylog(4, (d + 1)\*e^(2\*b\*x + 2\*a)) - 3\*polylog(5, (d + 1)\*e^(2\*b\*x + 2\*a)))/(b^5\*d)\*b\*d

**Fricas [C]** time = 2.26028, size = 1280, normalized size = 8.42

$$2b^5x^5 + 5b^4x^4 \log\left(\frac{d \cosh(bx+a) + (d+2) \sinh(bx+a)}{d \cosh(bx+a) + d \sinh(bx+a)}\right) - 20b^3x^3 \operatorname{Li}_2\left(\sqrt{d+1}(\cosh(bx+a) + \sinh(bx+a))\right) - 20b^3x^3 \operatorname{Li}_2\left(-\sqrt{d+1}(\cosh(bx+a) + \sinh(bx+a))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(1+d+d\*coth(b\*x+a)),x, algorithm="fricas")

[Out] 1/40\*(2\*b^5\*x^5 + 5\*b^4\*x^4\*log((d\*cosh(b\*x + a) + (d + 2)\*sinh(b\*x + a))/(d\*cosh(b\*x + a) + d\*sinh(b\*x + a))) - 20\*b^3\*x^3\*dilog(sqrt(d + 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 20\*b^3\*x^3\*dilog(-sqrt(d + 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 5\*a^4\*log(2\*(d + 1)\*cosh(b\*x + a) + 2\*(d + 1)\*sinh(b\*x + a) + 2\*sqrt(d + 1)) - 5\*a^4\*log(2\*(d + 1)\*cosh(b\*x + a) + 2\*(d + 1)\*sinh(b\*x + a) - 2\*sqrt(d + 1)) + 60\*b^2\*x^2\*polylog(3, sqrt(d + 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) + 60\*b^2\*x^2\*polylog(3, -sqrt(d + 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 120\*b\*x\*polylog(4, sqrt(d + 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 120\*b\*x\*polylog(4, -sqrt(d + 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 5\*(b^4\*x^4 - a^4)\*log(sqrt(d + 1)\*(cosh(b\*x + a) + sinh(b\*x + a)) + 1) - 5\*(b^4\*x^4 - a^4)\*log(-sqrt(d + 1)\*(cosh(b\*x + a) + sinh(b\*x + a)) + 1) + 120\*polylog(5, sqrt(d + 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) + 120\*polylog(5, -sqrt(d + 1)\*(cosh(b\*x + a) + sinh(b\*x + a))))/b^4

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acoth(1+d*d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arccoth}(d \coth(bx + a) + d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(1+d*d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^3*arccoth(d*coth(b*x + a) + d + 1), x)
```

### 3.222 $\int x^2 \coth^{-1}(1 + d + d \coth(a + bx)) dx$

**Optimal.** Leaf size=126

$$\frac{x \operatorname{PolyLog}\left(3, (d+1)e^{2a+2bx}\right)}{4b^2} - \frac{\operatorname{PolyLog}\left(4, (d+1)e^{2a+2bx}\right)}{8b^3} - \frac{x^2 \operatorname{PolyLog}\left(2, (d+1)e^{2a+2bx}\right)}{4b} - \frac{1}{6} x^3 \log\left(1 - (d+1)e^{2a+2bx}\right)$$

[Out] (b\*x^4)/12 + (x^3\*ArcCoth[1 + d + d\*Coth[a + b\*x]])/3 - (x^3\*Log[1 - (1 + d)\*E^(2\*a + 2\*b\*x)])/6 - (x^2\*PolyLog[2, (1 + d)\*E^(2\*a + 2\*b\*x)])/(4\*b) + (x\*PolyLog[3, (1 + d)\*E^(2\*a + 2\*b\*x)])/(4\*b^2) - PolyLog[4, (1 + d)\*E^(2\*a + 2\*b\*x)]/(8\*b^3)

**Rubi [A]** time = 0.26909, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {6242, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{x \operatorname{PolyLog}\left(3, (d+1)e^{2a+2bx}\right)}{4b^2} - \frac{\operatorname{PolyLog}\left(4, (d+1)e^{2a+2bx}\right)}{8b^3} - \frac{x^2 \operatorname{PolyLog}\left(2, (d+1)e^{2a+2bx}\right)}{4b} - \frac{1}{6} x^3 \log\left(1 - (d+1)e^{2a+2bx}\right)$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcCoth[1 + d + d\*Coth[a + b\*x]], x]

[Out] (b\*x^4)/12 + (x^3\*ArcCoth[1 + d + d\*Coth[a + b\*x]])/3 - (x^3\*Log[1 - (1 + d)\*E^(2\*a + 2\*b\*x)])/6 - (x^2\*PolyLog[2, (1 + d)\*E^(2\*a + 2\*b\*x)])/(4\*b) + (x\*PolyLog[3, (1 + d)\*E^(2\*a + 2\*b\*x)])/(4\*b^2) - PolyLog[4, (1 + d)\*E^(2\*a + 2\*b\*x)]/(8\*b^3)

#### Rule 6242

Int[ArcCoth[(c\_.) + Coth[(a\_.) + (b\_.)\*(x\_)]\*(d\_.)]\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^(m + 1)\*ArcCoth[c + d\*Coth[a + b\*x]])/(f\*(m + 1)), x] + Dist[b/(f\*(m + 1)), Int[(e + f\*x)^(m + 1)/(c - d - c\*E^(2\*a + 2\*b\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d)^2, 1]

#### Rule 2184

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x],

x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_))/  
((a\_) + (b\_)\*((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] :> Simp  
[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Di  
st[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)  
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_)\*((F\_)^((c\_)\*(a\_) + (b\_)\*(x\_)))^(n\_)]\*((f\_) + (g\_  
)\*(x\_))^(m\_), x\_Symbol] :> -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)  
))^n])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m -  
1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f  
, g, n}, x] && GtQ[m, 0]

### Rule 6609

Int[((e\_) + (f\_)\*(x\_))^(m\_)\*PolyLog[n\_, (d\_)\*((F\_)^((c\_)\*(a\_) + (b\_  
)\*(x\_)))^(p\_)], x\_Symbol] :> Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a  
+ b\*x)))^p])/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(  
m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c,  
d, e, f, n, p}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]  
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi  
onOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_) /; FreeQ[  
{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\*  
(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_)\*((a\_) + (b\_)\*(x\_))^(p\_)]/((d\_) + (e\_)\*(x\_)), x\_S  
ymbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d  
, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps



$$\begin{aligned}
\int x^2 \coth^{-1}(1 + d + d \coth(a + bx)) dx &= \frac{1}{3} x^3 \coth^{-1}(1 + d + d \coth(a + bx)) + \frac{1}{3} b \int \frac{x^3}{1 + (-1 - d)e^{2a+2bx}} dx \\
&= \frac{bx^4}{12} + \frac{1}{3} x^3 \coth^{-1}(1 + d + d \coth(a + bx)) + \frac{1}{3} (b(1 + d)) \int \frac{e^{2a+2bx} x^3}{1 + (-1 - d)e^{2a+2bx}} dx \\
&= \frac{bx^4}{12} + \frac{1}{3} x^3 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{6} x^3 \log(1 - (1 + d)e^{2a+2bx}) + \frac{1}{2} x^3 \log(1 - (1 + d)e^{2a+2bx}) \\
&= \frac{bx^4}{12} + \frac{1}{3} x^3 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{6} x^3 \log(1 - (1 + d)e^{2a+2bx}) - \frac{1}{6} x^3 \log(1 - (1 + d)e^{2a+2bx}) \\
&= \frac{bx^4}{12} + \frac{1}{3} x^3 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{6} x^3 \log(1 - (1 + d)e^{2a+2bx}) - \frac{1}{6} x^3 \log(1 - (1 + d)e^{2a+2bx}) \\
&= \frac{bx^4}{12} + \frac{1}{3} x^3 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{6} x^3 \log(1 - (1 + d)e^{2a+2bx}) - \frac{1}{6} x^3 \log(1 - (1 + d)e^{2a+2bx}) \\
&= \frac{bx^4}{12} + \frac{1}{3} x^3 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{6} x^3 \log(1 - (1 + d)e^{2a+2bx}) - \frac{1}{6} x^3 \log(1 - (1 + d)e^{2a+2bx})
\end{aligned}$$

**Mathematica [A]** time = 0.102156, size = 116, normalized size = 0.92

$$\frac{1}{24} \left( \frac{6x \operatorname{PolyLog}\left(3, \frac{e^{-2(a+bx)}}{d+1}\right)}{b^2} + \frac{3 \operatorname{PolyLog}\left(4, \frac{e^{-2(a+bx)}}{d+1}\right)}{b^3} + \frac{6x^2 \operatorname{PolyLog}\left(2, \frac{e^{-2(a+bx)}}{d+1}\right)}{b} - 4x^3 \log\left(1 - \frac{e^{-2(a+bx)}}{d+1}\right) + 8x^3 \coth^{-1}\left(1 + d + d \coth(a + bx)\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[1 + d + d\*Coth[a + b\*x]], x]

[Out] (8\*x^3\*ArcCoth[1 + d + d\*Coth[a + b\*x]] - 4\*x^3\*Log[1 - 1/((1 + d)\*E^(2\*(a + b\*x)))] + (6\*x^2\*PolyLog[2, 1/((1 + d)\*E^(2\*(a + b\*x)))]/b + (6\*x\*PolyLog[3, 1/((1 + d)\*E^(2\*(a + b\*x)))]/b^2 + (3\*PolyLog[4, 1/((1 + d)\*E^(2\*(a + b\*x)))]/b^3)/24

**Maple [C]** time = 35.69, size = 1641, normalized size = 13.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(1+d+d\*coth(b\*x+a)), x)

```
[Out] -1/4/b*d/(1+d)*polylog(2, (1+d)*exp(2*b*x+2*a))*x^2-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^3+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^3-1/3*x^3*ln(exp(b*x+a))-1/6*x^3*ln(d)-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2-1/12*I*x^3*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^2-1/6*I*x^3*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/12*b*x^4+1/6/b^3*a^3/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1)+1/6*x^3*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1)+1/12*I*x^3*Pi*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^2-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/6/b^3*d*a^3/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1)+1/12*I*x^3*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^3-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))-1/2/b^3*d*a^2/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))+1/2/b^2/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x*a^2+1/3/b^3*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a^3-1/2/b^3*d*a^3/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))-1/2/b^3*d*a^3/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))+1/12*I*x^3*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))+1/4/b^3*d/(1+d)*polylog(2, (1+d)*exp(2*b*x+2*a))*a^2+1/4/b^2*d/(1+d)*polylog(3, (1+d)*exp(2*b*x+2*a))*x-1/2/b^2*a^2/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))*x-1/2/b^2*a^2/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))*x-1/2/b^3*d*a^2/(1+d)*dilog(1-exp(b*x+a)*(1+d)^(1/2))+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))^3+1/4/b^2/(1+d)*polylog(3, (1+d)*exp(2*b*x+2*a))*x-1/2/b^3*a^3/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))-1/2/b^3*a^3/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))-1/6*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x^3-1/8/b^3*d/(1+d)*polylog(4, (1+d)*exp(2*b*x+2*a))-1/2/b^3*a^2/(1+d)*dilog(1-exp(b*x+a)*(1+d)^(1/2))-1/2/b^3*a^2/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))+1/3/b^3/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a^3-1/4/b/(1+d)*polylog(2, (1+d)*exp(2*b*x+2*a))*x^2+1/4/b^3/(1+d)*polylog(2, (1+d)*exp(2*b*x+2*a))*a^2+1/12*I*x^3*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/6/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x^3-1/8/b^3/(1+d)*polylog(4, (1+d)*exp(2*b*x+2*a))-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2+1/2/b^2*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x*a^2-1/2/b^2*d*a^2/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))*x-1/2/b^2*d*a^2/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))*x
```

**Maxima [A]** time = 3.5198, size = 166, normalized size = 1.32

$$\frac{1}{3} x^3 \operatorname{arcoth}(d \coth(bx + a) + d + 1) + \frac{1}{36} \left( \frac{3x^4}{d} - \frac{2(4b^3x^3 \log(-(d+1)e^{2bx+2a}) + 1) + 6b^2x^2 \operatorname{Li}_2((d+1)e^{2bx+2a})}{b^4d} \right) - \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+d*d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(d*coth(b*x + a) + d + 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*log
(-(d + 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog((d + 1)*e^(2*b*x + 2*a)) -
6*b*x*polylog(3, (d + 1)*e^(2*b*x + 2*a)) + 3*polylog(4, (d + 1)*e^(2*b*x
+ 2*a)))/(b^4*d))*b*d
```

**Fricas [C]** time = 2.04236, size = 1081, normalized size = 8.58

$$b^4x^4 + 2b^3x^3 \log\left(\frac{d \cosh(bx+a) + (d+2) \sinh(bx+a)}{d \cosh(bx+a) + d \sinh(bx+a)}\right) - 6b^2x^2 \operatorname{Li}_2\left(\sqrt{d+1}(\cosh(bx+a) + \sinh(bx+a))\right) - 6b^2x^2 \operatorname{Li}_2\left(-\sqrt{d+1}(\cosh(bx+a) + \sinh(bx+a))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+d*d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(b^4*x^4 + 2*b^3*x^3*log((d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/(d*
cosh(b*x + a) + d*sinh(b*x + a))) - 6*b^2*x^2*dilog(sqrt(d + 1)*(cosh(b*x +
a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-sqrt(d + 1)*(cosh(b*x + a) + sinh(
b*x + a))) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a) +
2*sqrt(d + 1)) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a
) - 2*sqrt(d + 1)) + 12*b*x*polylog(3, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*
x + a))) + 12*b*x*polylog(3, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)))
- 2*(b^3*x^3 + a^3)*log(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) -
2*(b^3*x^3 + a^3)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 1
2*polylog(4, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) - 12*polylog(4, -
sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(1+d*d*coth(b*x+a)),x)
```

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arccoth}(d \coth(bx + a) + d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccoth(1+d*d*coth(b*x+a)),x, algorithm="giac")`

[Out] `integrate(x^2*arccoth(d*coth(b*x + a) + d + 1), x)`

### 3.223 $\int x \coth^{-1}(1 + d + d \coth(a + bx)) dx$

**Optimal.** Leaf size=100

$$\frac{\text{PolyLog}\left(3, (d+1)e^{2a+2bx}\right)}{8b^2} - \frac{x \text{PolyLog}\left(2, (d+1)e^{2a+2bx}\right)}{4b} - \frac{1}{4}x^2 \log\left(1 - (d+1)e^{2a+2bx}\right) + \frac{1}{2}x^2 \coth^{-1}(d \coth(a + bx))$$

[Out] (b\*x^3)/6 + (x^2\*ArcCoth[1 + d + d\*Coth[a + b\*x]])/2 - (x^2\*Log[1 - (1 + d)\*E^(2\*a + 2\*b\*x)])/4 - (x\*PolyLog[2, (1 + d)\*E^(2\*a + 2\*b\*x)])/(4\*b) + PolyLog[3, (1 + d)\*E^(2\*a + 2\*b\*x)]/(8\*b^2)

**Rubi [A]** time = 0.24039, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {6242, 2184, 2190, 2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3, (d+1)e^{2a+2bx}\right)}{8b^2} - \frac{x \text{PolyLog}\left(2, (d+1)e^{2a+2bx}\right)}{4b} - \frac{1}{4}x^2 \log\left(1 - (d+1)e^{2a+2bx}\right) + \frac{1}{2}x^2 \coth^{-1}(d \coth(a + bx))$$

Antiderivative was successfully verified.

[In] Int[x\*ArcCoth[1 + d + d\*Coth[a + b\*x]],x]

[Out] (b\*x^3)/6 + (x^2\*ArcCoth[1 + d + d\*Coth[a + b\*x]])/2 - (x^2\*Log[1 - (1 + d)\*E^(2\*a + 2\*b\*x)])/4 - (x\*PolyLog[2, (1 + d)\*E^(2\*a + 2\*b\*x)])/(4\*b) + PolyLog[3, (1 + d)\*E^(2\*a + 2\*b\*x)]/(8\*b^2)

#### Rule 6242

Int[ArcCoth[(c\_.) + Coth[(a\_.) + (b\_.)\*(x\_)]]\*(d\_.)]\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((e + f\*x)^(m + 1)\*ArcCoth[c + d\*Coth[a + b\*x]])/(f\*(m + 1)), x] + Dist[b/(f\*(m + 1)), Int[(e + f\*x)^(m + 1)/(c - d - c\*E^(2\*a + 2\*b\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d)^2, 1]

#### Rule 2184

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.), x\_Symbol] := Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[(c + d\*x)^m\*(F^(g\*(e + f\*x)))^n]/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*(a_) + (b_)*(x_)))^(n_)]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x \coth^{-1}(1 + d + d \coth(a + bx)) dx &= \frac{1}{2} x^2 \coth^{-1}(1 + d + d \coth(a + bx)) + \frac{1}{2} b \int \frac{x^2}{1 + (-1 - d)e^{2a+2bx}} dx \\
&= \frac{bx^3}{6} + \frac{1}{2} x^2 \coth^{-1}(1 + d + d \coth(a + bx)) + \frac{1}{2} (b(1 + d)) \int \frac{e^{2a+2bx} x^2}{1 + (-1 - d)e^{2a+2bx}} dx \\
&= \frac{bx^3}{6} + \frac{1}{2} x^2 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{4} x^2 \log(1 - (1 + d)e^{2a+2bx}) + \frac{1}{2} \int \frac{e^{2a+2bx} x^2}{1 + (-1 - d)e^{2a+2bx}} dx \\
&= \frac{bx^3}{6} + \frac{1}{2} x^2 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{4} x^2 \log(1 - (1 + d)e^{2a+2bx}) - \frac{1}{2} \int \frac{e^{2a+2bx} x^2}{1 + (-1 - d)e^{2a+2bx}} dx \\
&= \frac{bx^3}{6} + \frac{1}{2} x^2 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{4} x^2 \log(1 - (1 + d)e^{2a+2bx}) - \frac{1}{2} \int \frac{e^{2a+2bx} x^2}{1 + (-1 - d)e^{2a+2bx}} dx \\
&= \frac{bx^3}{6} + \frac{1}{2} x^2 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{4} x^2 \log(1 - (1 + d)e^{2a+2bx}) - \frac{1}{2} \int \frac{e^{2a+2bx} x^2}{1 + (-1 - d)e^{2a+2bx}} dx
\end{aligned}$$

**Mathematica [A]** time = 0.092782, size = 90, normalized size = 0.9

$$\frac{2bx \text{PolyLog}\left(2, \frac{e^{-2(a+bx)}}{d+1}\right) + \text{PolyLog}\left(3, \frac{e^{-2(a+bx)}}{d+1}\right) + 2b^2 x^2 \left(2 \coth^{-1}(d \coth(a + bx) + d + 1) - \log\left(1 - \frac{e^{-2(a+bx)}}{d+1}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[1 + d + d\*Coth[a + b\*x]], x]

[Out] (2\*b^2\*x^2\*(2\*ArcCoth[1 + d + d\*Coth[a + b\*x]] - Log[1 - 1/((1 + d)\*E^(2\*(a + b\*x)))])) + 2\*b\*x\*PolyLog[2, 1/((1 + d)\*E^(2\*(a + b\*x)))] + PolyLog[3, 1/((1 + d)\*E^(2\*(a + b\*x)))]/(8\*b^2)

**Maple [C]** time = 15.664, size = 1560, normalized size = 15.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(1+d+d\*coth(b\*x+a)), x)

[Out] 1/6\*b\*x^3+1/8/b^2/(1+d)\*polylog(3, (1+d)\*exp(2\*b\*x+2\*a))-1/4/(1+d)\*ln(1-(1+d)\*exp(2\*b\*x+2\*a))\*x^2-1/8\*I\*x^2\*Pi\*csgn(I\*exp(2\*b\*x+2\*a)/(exp(2\*b\*x+2\*a)-1))

```

)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/8*I*x^2*Pi*csgn(I/(exp(2*
b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))
^2+1/8*I*x^2*Pi*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b
*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^2-1/8*I*x^2*Pi*csgn(I*exp(2
*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2-1/2*x^2*ln(exp(b*x+a
))-1/4*x^2*ln(d)-1/2/b/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x*a+1/4*x^2*ln(exp(
2*b*x+2*a)*d+exp(2*b*x+2*a)-1)-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn
(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2-1/8*I*x^2*Pi*csgn(I*d)*csgn(I*d/(ex
p(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/4*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x^
2+1/8/b^2*d/(1+d)*polylog(3,(1+d)*exp(2*b*x+2*a))+1/2/b^2*a/(1+d)*dilog(1-e
xp(b*x+a)*(1+d)^(1/2))+1/2/b^2*a/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))-1/4/
b^2/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a^2-1/4/b/(1+d)*polylog(2,(1+d)*exp(2*
b*x+2*a))*x-1/4/b^2/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*a+1/2/b^2*a^2/(1+
d)*ln(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b^2*a^2/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/
2))-1/4/b^2*a^2/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1)-1/4/b^2*d/(1+d)
*ln(1-(1+d)*exp(2*b*x+2*a))*a^2-1/4/b*d/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a
))*x-1/4/b^2*d/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*a+1/2/b*a/(1+d)*ln(1-e
xp(b*x+a)*(1+d)^(1/2))*x+1/2/b*a/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))*x+1/2/b
^2*d*a^2/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b^2*d*a^2/(1+d)*ln(1+exp(b*
x+a)*(1+d)^(1/2))+1/2/b^2*d*a/(1+d)*dilog(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b^2
*d*a/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))-1/4/b^2*d*a^2/(1+d)*ln(exp(2*b*x
+2*a)*d+exp(2*b*x+2*a)-1)+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a))^3-1/8*I*x^2*P
i*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^3+1/8*I*x^
2*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^3+1/8*I*x^2*Pi*csgn(I*exp(
2*b*x+2*a)/(exp(2*b*x+2*a)-1))^3+1/8*I*x^2*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp
(2*b*x+2*a))-1/4*I*x^2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/8
*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a
)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))+1/8*I*
x^2*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2
*a)/(exp(2*b*x+2*a)-1))+1/8*I*x^2*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2
*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))-1/2/b*d/(1+d)*ln(
1-(1+d)*exp(2*b*x+2*a))*x*a+1/2/b*d*a/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))*x+
1/2/b*d*a/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))*x

```

**Maxima [A]** time = 3.7136, size = 135, normalized size = 1.35

$$\frac{1}{24} \left( \frac{4x^3}{d} - \frac{3(2b^2x^2 \log(-(d+1)e^{2bx+2a}) + 1) + 2bx \operatorname{Li}_2((d+1)e^{2bx+2a}) - \operatorname{Li}_3((d+1)e^{2bx+2a})}{b^3d} \right) bd + \frac{1}{2} x^2 \operatorname{arccoth}(d)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1+d\*d\*coth(b\*x+a)),x, algorithm="maxima")



```
[Out] 1/24*(4*x^3/d - 3*(2*b^2*x^2*log(-(d + 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dilog((d + 1)*e^(2*b*x + 2*a)) - polylog(3, (d + 1)*e^(2*b*x + 2*a)))/(b^3*d))*
b*d + 1/2*x^2*arcoth(d*coth(b*x + a) + d + 1)
```

**Fricas [C]** time = 1.97112, size = 894, normalized size = 8.94

$$2b^3x^3 + 3b^2x^2 \log\left(\frac{d \cosh(bx+a) + (d+2) \sinh(bx+a)}{d \cosh(bx+a) + d \sinh(bx+a)}\right) - 6bx \operatorname{Li}_2\left(\sqrt{d+1}(\cosh(bx+a) + \sinh(bx+a))\right) - 6bx \operatorname{Li}_2\left(-\sqrt{d+1}(\cosh(bx+a) + \sinh(bx+a))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcoth(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 + 3*b^2*x^2*log((d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/(d*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b*x*dilog(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) - 6*b*x*dilog(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a) + 2*sqrt(d + 1)) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a) - 2*sqrt(d + 1)) - 3*(b^2*x^2 - a^2)*log(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 3*(b^2*x^2 - a^2)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 6*polylog(3, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) + 6*polylog(3, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(1+d+d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arcoth}(d \coth(bx + a) + d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1+d*d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(d*coth(b*x + a) + d + 1), x)
```

### 3.224 $\int \coth^{-1}(1 + d + d \coth(a + bx)) dx$

**Optimal.** Leaf size=69

$$-\frac{\text{PolyLog}\left(2, (d+1)e^{2a+2bx}\right)}{4b} - \frac{1}{2}x \log\left(1 - (d+1)e^{2a+2bx}\right) + x \coth^{-1}(d \coth(a + bx) + d + 1) + \frac{bx^2}{2}$$

[Out] (b\*x^2)/2 + x\*ArcCoth[1 + d + d\*Coth[a + b\*x]] - (x\*Log[1 - (1 + d)\*E^(2\*a + 2\*b\*x)])/2 - PolyLog[2, (1 + d)\*E^(2\*a + 2\*b\*x)]/(4\*b)

**Rubi [A]** time = 0.144776, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {6234, 2184, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2, (d+1)e^{2a+2bx}\right)}{4b} - \frac{1}{2}x \log\left(1 - (d+1)e^{2a+2bx}\right) + x \coth^{-1}(d \coth(a + bx) + d + 1) + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[1 + d + d\*Coth[a + b\*x]], x]

[Out] (b\*x^2)/2 + x\*ArcCoth[1 + d + d\*Coth[a + b\*x]] - (x\*Log[1 - (1 + d)\*E^(2\*a + 2\*b\*x)])/2 - PolyLog[2, (1 + d)\*E^(2\*a + 2\*b\*x)]/(4\*b)

#### Rule 6234

Int[ArcCoth[(c\_.) + Coth[(a\_.) + (b\_.)\*(x\_.)]\*(d\_.)], x\_Symbol] := Simp[x\*ArcCoth[c + d\*Coth[a + b\*x]], x] + Dist[b, Int[x/(c - d - c\*E^(2\*a + 2\*b\*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]

#### Rule 2184

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_.))))^(n\_.), x\_Symbol] := Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2190

Int[((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_.)))^(n\_.)\*((c\_.) + (d\_.)\*(x\_.))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_.))))^(n\_.), x\_Symbol] := Simp[(c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a]/(b\*f\*g\*n\*Log[F]), x] - Di

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^(n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
)^(n)], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
 \int \coth^{-1}(1 + d + d \coth(a + bx)) dx &= x \coth^{-1}(1 + d + d \coth(a + bx)) + b \int \frac{x}{1 + (-1 - d)e^{2a+2bx}} dx \\
 &= \frac{bx^2}{2} + x \coth^{-1}(1 + d + d \coth(a + bx)) + (b(1 + d)) \int \frac{e^{2a+2bx} x}{1 + (-1 - d)e^{2a+2bx}} dx \\
 &= \frac{bx^2}{2} + x \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{2} x \log(1 - (1 + d)e^{2a+2bx}) + \frac{1}{2} \int \log \left( \frac{1 + (-1 - d)e^{2a+2bx}}{1 - (1 + d)e^{2a+2bx}} \right) dx \\
 &= \frac{bx^2}{2} + x \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{2} x \log(1 - (1 + d)e^{2a+2bx}) + \frac{\text{Subst}\left(\int \frac{1}{1 - (1 + d)e^{2a+2bx}} dx\right)}{2} \\
 &= \frac{bx^2}{2} + x \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{2} x \log(1 - (1 + d)e^{2a+2bx}) - \frac{\text{Li}_2\left(\frac{1 - (1 + d)e^{2a+2bx}}{2}\right)}{4}
 \end{aligned}$$

**Mathematica [B]** time = 0.786928, size = 197, normalized size = 2.86

---


$$-2\text{PolyLog}\left(2, -\sqrt{d+1}e^{a+bx}\right) - 2\text{PolyLog}\left(2, \sqrt{d+1}e^{a+bx}\right) - 2\log\left(e^{a+bx}\right)\log\left(1 - \sqrt{d+1}e^{a+bx}\right) - 2\log\left(e^{a+bx}\right)\log\left(\sqrt{d+1}e^{a+bx}\right)$$


---

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[1 + d + d*Coth[a + b*x]], x]
```

```
[Out] x*ArcCoth[1 + d + d*Coth[a + b*x]] + (b^2*x^2 + Log[E^(a + b*x)]^2 - 2*Log[E^(a + b*x)]*Log[1 - Sqrt[1 + d]*E^(a + b*x)] - 2*Log[E^(a + b*x)]*Log[1 + Sqrt[1 + d]*E^(a + b*x)] + 2*Log[E^(a + b*x)]*Log[E^(-a - b*x)*(-1 + (1 + d)*E^(2*(a + b*x)))] - 2*b*x*Log[d*Cosh[a + b*x] + (2 + d)*Sinh[a + b*x]] -
```

$2*\text{PolyLog}[2, -(\text{Sqrt}[1 + d]*E^{(a + b*x)})] - 2*\text{PolyLog}[2, \text{Sqrt}[1 + d]*E^{(a + b*x)}]/(4*b)$

**Maple [B]** time = 0.161, size = 247, normalized size = 3.6

$$\frac{\text{arccoth}(1 + d + d\coth(bx + a)) \ln(d\coth(bx + a) + d)}{2b} - \frac{\text{arccoth}(1 + d + d\coth(bx + a)) \ln(d\coth(bx + a) - d)}{2b} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(1+d+d*coth(b*x+a)),x)`

[Out]  $\frac{1}{2} \frac{1}{b} \text{arccoth}(1+d+d*\coth(b*x+a)) * \ln(d*\coth(b*x+a)+d) - \frac{1}{2} \frac{1}{b} \text{arccoth}(1+d+d*\coth(b*x+a)) * \ln(d*\coth(b*x+a)-d) + \frac{1}{4} \frac{1}{b} \text{dilog}((d*\coth(b*x+a)+d+2)/(2*d+2)) + \frac{1}{4} \frac{1}{b} \ln(d*\coth(b*x+a)-d) * \ln((d*\coth(b*x+a)+d+2)/(2*d+2)) - \frac{1}{4} \frac{1}{b} \text{dilog}(1/2*(d*\coth(b*x+a)+d)/d) - \frac{1}{4} \frac{1}{b} \ln(d*\coth(b*x+a)-d) * \ln(1/2*(d*\coth(b*x+a)+d)/d) + \frac{1}{8} \frac{1}{b} \ln(d*\coth(b*x+a)+d)^2 - \frac{1}{4} \frac{1}{b} \text{dilog}(1+1/2*d*\coth(b*x+a)+1/2*d) - \frac{1}{4} \frac{1}{b} \ln(d*\coth(b*x+a)+d) * \ln(1+1/2*d*\coth(b*x+a)+1/2*d)$

**Maxima [A]** time = 3.59745, size = 97, normalized size = 1.41

$$\frac{1}{4} b d \left( \frac{2x^2}{d} - \frac{2bx \log(-(d+1)e^{(2bx+2a)} + 1) + \text{Li}_2((d+1)e^{(2bx+2a)})}{b^2 d} \right) + x \text{arccoth}(d \coth(bx + a) + d + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(1+d+d*coth(b*x+a)),x, algorithm="maxima")`

[Out]  $\frac{1}{4} * b * d * (2 * x^2 / d - (2 * b * x * \log(-(d + 1) * e^{(2 * b * x + 2 * a)} + 1) + \text{dilog}((d + 1) * e^{(2 * b * x + 2 * a)})) / (b^2 * d)) + x * \text{arccoth}(d * \coth(b * x + a) + d + 1)$

**Fricas [B]** time = 2.05268, size = 667, normalized size = 9.67

$$\frac{b^2 x^2 + b x \log\left(\frac{d \cosh(bx+a) + (d+2) \sinh(bx+a)}{d \cosh(bx+a) + d \sinh(bx+a)}\right) + a \log\left(2(d+1) \cosh(bx+a) + 2(d+1) \sinh(bx+a) + 2\sqrt{d+1}\right) + a \log\left(\dots\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+d*d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + b*x*log((d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/(d*cosh(b*x + a) + d*sinh(b*x + a))) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a) + 2*sqrt(d + 1)) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a) - 2*sqrt(d + 1)) - (b*x + a)*log(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - dilog(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) - dilog(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acoth}(d \operatorname{coth}(a + bx) + d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1+d*d*coth(b*x+a)),x)
```

```
[Out] Integral(acoth(d*coth(a + b*x) + d + 1), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(d \operatorname{coth}(bx + a) + d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+d*d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*coth(b*x + a) + d + 1), x)
```

$$3.225 \quad \int \frac{\coth^{-1}(1+d+d \coth(ax))}{x} dx$$

**Optimal.** Leaf size=18

$$\text{CannotIntegrate}\left(\frac{\coth^{-1}(d \coth(ax) + d + 1)}{x}, x\right)$$

[Out] CannotIntegrate[ArcCoth[1 + d + d\*Coth[a + b\*x]]/x, x]

**Rubi [A]** time = 0.0609226, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\coth^{-1}(1 + d + d \coth(ax))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[ArcCoth[1 + d + d\*Coth[a + b\*x]]/x,x]

[Out] Defer[Int][ArcCoth[1 + d + d\*Coth[a + b\*x]]/x, x]

Rubi steps

$$\int \frac{\coth^{-1}(1 + d + d \coth(ax))}{x} dx = \int \frac{\coth^{-1}(1 + d + d \coth(ax))}{x} dx$$

**Mathematica [A]** time = 3.14883, size = 0, normalized size = 0.

$$\int \frac{\coth^{-1}(1 + d + d \coth(ax))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcCoth[1 + d + d\*Coth[a + b\*x]]/x,x]

[Out] Integrate[ArcCoth[1 + d + d\*Coth[a + b\*x]]/x, x]

---

**Maple [A]** time = 0.372, size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(1 + d + d \coth(bx + a))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(1+d+d\*coth(b\*x+a))/x,x)

[Out] int(arccoth(1+d+d\*coth(b\*x+a))/x,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(d \coth(bx + a) + d + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+d+d\*coth(b\*x+a))/x,x, algorithm="maxima")

[Out] integrate(arccoth(d\*coth(b\*x + a) + d + 1)/x, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arccoth}(d \coth(bx + a) + d + 1)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+d+d\*coth(b\*x+a))/x,x, algorithm="fricas")

[Out] integral(arccoth(d\*coth(b\*x + a) + d + 1)/x, x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}(d \coth(a + bx) + d + 1)}{x} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(1+d+d*coth(b*x+a))/x,x)`

[Out] `Integral(acoth(d*coth(a + b*x) + d + 1)/x, x)`

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(d \coth(bx + a) + d + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(1+d+d*coth(b*x+a))/x,x, algorithm="giac")`

[Out] `integrate(arccoth(d*coth(b*x + a) + d + 1)/x, x)`

### 3.226 $\int x^3 \coth^{-1}(1 - d - d \coth(a + bx)) dx$

**Optimal.** Leaf size=165

$$\frac{3x^2 \text{PolyLog}\left(3, (1-d)e^{2a+2bx}\right)}{8b^2} - \frac{3x \text{PolyLog}\left(4, (1-d)e^{2a+2bx}\right)}{8b^3} + \frac{3 \text{PolyLog}\left(5, (1-d)e^{2a+2bx}\right)}{16b^4} - \frac{x^3 \text{PolyLog}\left(2, (1-d)\right)}{4b}$$

[Out] (b\*x^5)/20 + (x^4\*ArcCoth[1 - d - d\*Coth[a + b\*x]])/4 - (x^4\*Log[1 - (1 - d)\*E^(2\*a + 2\*b\*x)])/8 - (x^3\*PolyLog[2, (1 - d)\*E^(2\*a + 2\*b\*x)])/(4\*b) + (3\*x^2\*PolyLog[3, (1 - d)\*E^(2\*a + 2\*b\*x)])/(8\*b^2) - (3\*x\*PolyLog[4, (1 - d)\*E^(2\*a + 2\*b\*x)])/(8\*b^3) + (3\*PolyLog[5, (1 - d)\*E^(2\*a + 2\*b\*x)])/(16\*b^4)

**Rubi [A]** time = 0.309702, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {6242, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2 \text{PolyLog}\left(3, (1-d)e^{2a+2bx}\right)}{8b^2} - \frac{3x \text{PolyLog}\left(4, (1-d)e^{2a+2bx}\right)}{8b^3} + \frac{3 \text{PolyLog}\left(5, (1-d)e^{2a+2bx}\right)}{16b^4} - \frac{x^3 \text{PolyLog}\left(2, (1-d)\right)}{4b}$$

Antiderivative was successfully verified.

[In] Int[x^3\*ArcCoth[1 - d - d\*Coth[a + b\*x]],x]

[Out] (b\*x^5)/20 + (x^4\*ArcCoth[1 - d - d\*Coth[a + b\*x]])/4 - (x^4\*Log[1 - (1 - d)\*E^(2\*a + 2\*b\*x)])/8 - (x^3\*PolyLog[2, (1 - d)\*E^(2\*a + 2\*b\*x)])/(4\*b) + (3\*x^2\*PolyLog[3, (1 - d)\*E^(2\*a + 2\*b\*x)])/(8\*b^2) - (3\*x\*PolyLog[4, (1 - d)\*E^(2\*a + 2\*b\*x)])/(8\*b^3) + (3\*PolyLog[5, (1 - d)\*E^(2\*a + 2\*b\*x)])/(16\*b^4)

#### Rule 6242

Int[ArcCoth[(c\_.) + Coth[(a\_.) + (b\_.)\*(x\_.)]\*(d\_.)]\*((e\_.) + (f\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^(m + 1)\*ArcCoth[c + d\*Coth[a + b\*x]])/(f\*(m + 1)), x] + Dist[b/(f\*(m + 1)), Int[(e + f\*x)^(m + 1)/(c - d - c\*E^(2\*a + 2\*b\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d)^2, 1]

#### Rule 2184

Int[(((c\_.) + (d\_.)\*(x\_.))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_.))))^(n\_.), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[

$b/a, \text{Int}[(c + dx)^m (F^{g(e+fx)})^n / (a + b(F^{g(e+fx)})^n), x], x] /;$  FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2190

$\text{Int}[(F^{g(e+fx)})^n (c + dx)^m / (a + b(F^{g(e+fx)})^n), x\_Symbol] \rightarrow \text{Simp}[(c + dx)^m \text{Log}[1 + (b(F^{g(e+fx)})^n)/a] / (b f g^n \text{Log}[F]), x] - \text{Dist}[(d m) / (b f g^n \text{Log}[F]), \text{Int}[(c + dx)^{m-1} \text{Log}[1 + (b(F^{g(e+fx)})^n)/a], x], x] /;$  FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

$\text{Int}[\text{Log}[1 + (e + f x) (F^{c(a+bx)})^n] (f + g x)^m, x\_Symbol] \rightarrow -\text{Simp}[(f + g x)^m \text{PolyLog}[2, -(e(F^{c(a+bx)})^n)] / (b c^n \text{Log}[F]), x] + \text{Dist}[(g m) / (b c^n \text{Log}[F]), \text{Int}[(f + g x)^{m-1} \text{PolyLog}[2, -(e(F^{c(a+bx)})^n)], x], x] /;$  FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 6609

$\text{Int}[(e + f x)^m \text{PolyLog}[n, d(F^{c(a+bx)})^p], x\_Symbol] \rightarrow \text{Simp}[(e + f x)^m \text{PolyLog}[n + 1, d(F^{c(a+bx)})^p] / (b c^p \text{Log}[F]), x] - \text{Dist}[(f m) / (b c^p \text{Log}[F]), \text{Int}[(e + f x)^{m-1} \text{PolyLog}[n + 1, d(F^{c(a+bx)})^p], x], x] /;$  FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

### Rule 2282

$\text{Int}[u, x\_Symbol] \rightarrow \text{With}[v = \text{FunctionOfExponential}[u, x], \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /;$  FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*(a\_)\*(v\_)^n]^m /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^{(c\_)\*(a\_)+(b\_)\*x}\*(F\_)[v\_]] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]

### Rule 6589

$\text{Int}[\text{PolyLog}[n, c(a+bx)^p] / (d + e x), x\_Symbol] \rightarrow \text{Simp}[\text{PolyLog}[n + 1, c(a+bx)^p] / (e p), x] /;$  FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
\int x^3 \coth^{-1}(1-d-d \coth(a+bx)) dx &= \frac{1}{4}x^4 \coth^{-1}(1-d-d \coth(a+bx)) + \frac{1}{4}b \int \frac{x^4}{1+(-1+d)e^{2a+2bx}} dx \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \coth(a+bx)) + \frac{1}{4}(b(1-d)) \int \frac{e^{2a+2bx} x^4}{1+(-1+d)e^{2a+2bx}} dx \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \coth(a+bx)) - \frac{1}{8}x^4 \log(1-(1-d)e^{2a+2bx}) + \frac{1}{2} \int \frac{x^3 L}{1+(-1+d)e^{2a+2bx}} dx \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \coth(a+bx)) - \frac{1}{8}x^4 \log(1-(1-d)e^{2a+2bx}) - \frac{x^3 L}{8} \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \coth(a+bx)) - \frac{1}{8}x^4 \log(1-(1-d)e^{2a+2bx}) - \frac{x^3 L}{8} \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \coth(a+bx)) - \frac{1}{8}x^4 \log(1-(1-d)e^{2a+2bx}) - \frac{x^3 L}{8} \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \coth(a+bx)) - \frac{1}{8}x^4 \log(1-(1-d)e^{2a+2bx}) - \frac{x^3 L}{8} \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \coth(a+bx)) - \frac{1}{8}x^4 \log(1-(1-d)e^{2a+2bx}) - \frac{x^3 L}{8}
\end{aligned}$$

**Mathematica [A]** time = 0.187995, size = 147, normalized size = 0.89

$$\frac{1}{16} \left( \frac{6x^2 \text{PolyLog}\left(3, -\frac{e^{-2(a+bx)}}{d-1}\right)}{b^2} + \frac{6x \text{PolyLog}\left(4, -\frac{e^{-2(a+bx)}}{d-1}\right)}{b^3} + \frac{3 \text{PolyLog}\left(5, -\frac{e^{-2(a+bx)}}{d-1}\right)}{b^4} + \frac{4x^3 \text{PolyLog}\left(2, -\frac{e^{-2(a+bx)}}{d-1}\right)}{b} - 2 \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*ArcCoth[1-d-d\*Coth[a+b\*x]],x]

[Out] (4\*x^4\*ArcCoth[1-d-d\*Coth[a+b\*x]] - 2\*x^4\*Log[1+1/((-1+d)\*E^(2\*(a+b\*x)))] + (4\*x^3\*PolyLog[2,-(1/((-1+d)\*E^(2\*(a+b\*x))))])/b + (6\*x^2\*PolyLog[3,-(1/((-1+d)\*E^(2\*(a+b\*x))))])/b^2 + (6\*x\*PolyLog[4,-(1/((-1+d)\*E^(2\*(a+b\*x))))])/b^3 + (3\*PolyLog[5,-(1/((-1+d)\*E^(2\*(a+b\*x))))])/b^4)/16

**Maple [C]** time = 37.705, size = 1830, normalized size = 11.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^3 \cdot \text{arccoth}(1-d-d \cdot \text{coth}(b \cdot x+a)), x)$

[Out] 
$$\begin{aligned} & -1/16 \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I \cdot d) \cdot \text{csgn}(I \cdot d / (\exp(2 \cdot b \cdot x + 2 \cdot a) - 1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a))^{-2-1/16} \\ & \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a) / (\exp(2 \cdot b \cdot x + 2 \cdot a) - 1)) \cdot \text{csgn}(I \cdot d / (\exp(2 \cdot b \cdot x + 2 \cdot a) \\ & - 1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a))^{-2+1/2} / b^4 \cdot d \cdot a^3 / (d-1) \cdot \text{dilog}(1 + \exp(b \cdot x + a) \cdot (1-d)^{(1/2)}) - \\ & 3/8 / b^4 \cdot d / (d-1) \cdot \ln(1 + (d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot a^4 - 1/4 / b \cdot d / (d-1) \cdot \text{polylog}(2, -(d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot x^3 - \\ & 1/4 / b^4 \cdot d / (d-1) \cdot \text{polylog}(2, -(d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot a^3 + 3/8 / b^2 \cdot d / (d-1) \cdot \text{polylog}(3, -(d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot x^2 - \\ & 3/8 / b^3 \cdot d / (d-1) \cdot \text{polylog}(4, -(d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot x - 1/16 \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I / (\exp(2 \cdot b \cdot x + 2 \cdot a) - 1)) \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a) / (\exp(2 \cdot b \cdot x + 2 \cdot a) - 1))^{-2-1/16} \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I \cdot d / (\exp(2 \cdot b \cdot x + 2 \cdot a) - 1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a))^{-3+1/16} \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I \cdot \exp(b \cdot x + a))^2 \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a))^{-1} / 8 \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I \cdot \exp(b \cdot x + a)) \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a))^{-2+1/16} \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a))^{-3+1/20} \cdot b \cdot x^5 + 1/8 / b^4 \cdot a^4 / (d-1) \cdot \ln(\exp(2 \cdot b \cdot x + 2 \cdot a) \cdot d - \exp(2 \cdot b \cdot x + 2 \cdot a) + 1) - 1/2 / b^4 \cdot a^3 / (d-1) \cdot \text{dilog}(1 + \exp(b \cdot x + a) \cdot (1-d)^{(1/2)}) - 1/2 / b^4 \cdot a^3 / (d-1) \cdot \text{dilog}(1 - \exp(b \cdot x + a) \cdot (1-d)^{(1/2)}) + 3/16 / b^4 \cdot d / (d-1) \cdot \text{polylog}(5, -(d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) - 1/2 / b^4 \cdot a^4 / (d-1) \cdot \ln(1 - \exp(b \cdot x + a) \cdot (1-d)^{(1/2)}) + 3/8 / b^4 \cdot a^4 / (d-1) \cdot \ln(1 + (d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) + 1/4 / b^4 \cdot a^3 / (d-1) \cdot \text{polylog}(2, -(d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) - 1/2 / b^4 \cdot a^4 / (d-1) \cdot \ln(1 + \exp(b \cdot x + a) \cdot (1-d)^{(1/2)}) + 1/4 / b / (d-1) \cdot \text{polylog}(2, -(d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot x^3 - 3/8 / b^2 / (d-1) \cdot \text{polylog}(3, -(d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot x^2 + 3/8 / b^3 / (d-1) \cdot \text{polylog}(4, -(d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot x - 1/8 \cdot d / (d-1) \cdot \ln(1 + (d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot x^4 + 1/8 / (d-1) \cdot \ln(1 + (d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot x^4 - 3/16 / b^4 / (d-1) \cdot \text{polylog}(5, -(d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) + 1/2 / b^4 \cdot d \cdot a^3 / (d-1) \cdot \text{dilog}(1 - \exp(b \cdot x + a) \cdot (1-d)^{(1/2)}) - 1/2 / b^3 \cdot a^3 / (d-1) \cdot \ln(1 - \exp(b \cdot x + a) \cdot (1-d)^{(1/2)}) \cdot x + 1/2 / b^4 \cdot d \cdot a^4 / (d-1) \cdot \ln(1 + \exp(b \cdot x + a) \cdot (1-d)^{(1/2)}) + 1/2 / b^4 \cdot d \cdot a^4 / (d-1) \cdot \ln(1 - \exp(b \cdot x + a) \cdot (1-d)^{(1/2)}) + 1/2 / b^3 \cdot a^3 / (d-1) \cdot \ln(1 + (d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot x - 1/2 / b^3 \cdot a^3 / (d-1) \cdot \ln(1 + \exp(b \cdot x + a) \cdot (1-d)^{(1/2)}) \cdot x - 1/8 \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I / (\exp(2 \cdot b \cdot x + 2 \cdot a) - 1)) \cdot (\exp(2 \cdot b \cdot x + 2 \cdot a) \cdot d - \exp(2 \cdot b \cdot x + 2 \cdot a) + 1))^{-2+1/16} \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a) / (\exp(2 \cdot b \cdot x + 2 \cdot a) - 1))^{-3+1/16} \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I / (\exp(2 \cdot b \cdot x + 2 \cdot a) - 1) \cdot (\exp(2 \cdot b \cdot x + 2 \cdot a) \cdot d - \exp(2 \cdot b \cdot x + 2 \cdot a) + 1))^{-3+1/8} \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I \cdot d / (\exp(2 \cdot b \cdot x + 2 \cdot a) - 1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a))^{-2-1/4} \cdot x^4 \cdot \ln(\exp(b \cdot x + a)) - 1/8 \cdot x^4 \cdot \ln(d) - 1/8 / b^4 \cdot d \cdot a^4 / (d-1) \cdot \ln(\exp(2 \cdot b \cdot x + 2 \cdot a) \cdot d - \exp(2 \cdot b \cdot x + 2 \cdot a) + 1) - 1/16 \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a) / (\exp(2 \cdot b \cdot x + 2 \cdot a) - 1))^{-2+1/16} \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I \cdot (\exp(2 \cdot b \cdot x + 2 \cdot a) \cdot d - \exp(2 \cdot b \cdot x + 2 \cdot a) + 1)) \cdot \text{csgn}(I / (\exp(2 \cdot b \cdot x + 2 \cdot a) - 1) \cdot (\exp(2 \cdot b \cdot x + 2 \cdot a) \cdot d - \exp(2 \cdot b \cdot x + 2 \cdot a) + 1))^{-2+1/16} \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I / (\exp(2 \cdot b \cdot x + 2 \cdot a) - 1)) \cdot \text{csgn}(I / (\exp(2 \cdot b \cdot x + 2 \cdot a) - 1) \cdot (\exp(2 \cdot b \cdot x + 2 \cdot a) \cdot d - \exp(2 \cdot b \cdot x + 2 \cdot a) + 1))^{-2+1/8} \cdot x^4 \cdot \ln(\exp(2 \cdot b \cdot x + 2 \cdot a) \cdot d - \exp(2 \cdot b \cdot x + 2 \cdot a) + 1) + 1/2 / b^3 \cdot d \cdot a^3 / (d-1) \cdot \ln(1 + \exp(b \cdot x + a) \cdot (1-d)^{(1/2)}) \cdot x + 1/2 / b^3 \cdot d \cdot a^3 / (d-1) \cdot \ln(1 - \exp(b \cdot x + a) \cdot (1-d)^{(1/2)}) \cdot x - 1/2 / b^3 \cdot d / (d-1) \cdot \ln(1 + (d-1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot x \cdot a^3 - 1/16 \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I / (\exp(2 \cdot b \cdot x + 2 \cdot a) - 1)) \cdot \text{csgn}(I \cdot (\exp(2 \cdot b \cdot x + 2 \cdot a) \cdot d - \exp(2 \cdot b \cdot x + 2 \cdot a) + 1)) \cdot \text{csgn}(I / (\exp(2 \cdot b \cdot x + 2 \cdot a) - 1) \cdot (\exp(2 \cdot b \cdot x + 2 \cdot a) \cdot d - \exp(2 \cdot b \cdot x + 2 \cdot a) + 1))) + 1/16 \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot \text{csgn}(I / (\exp(2 \cdot b \cdot x + 2 \cdot a) - 1)) \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a) / (\exp(2 \cdot b \cdot x + 2 \cdot a) - 1)) + 1/16 \cdot I \cdot x^4 \cdot \text{Pi} \cdot \text{csgn}(I \cdot d) \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a) / (\exp(2 \cdot b \cdot x + 2 \cdot a) - 1)) \cdot \text{csgn}(I \cdot d / (\exp(2 \cdot b \cdot x + 2 \cdot a) - 1) \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \end{aligned}$$

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**Maxima [A]** time = 3.71832, size = 201, normalized size = 1.22

$$-\frac{1}{4}x^4 \operatorname{arccoth}(d \coth(bx + a) + d - 1) + \frac{1}{40} \left( \frac{2x^5}{d} - \frac{5(2b^4x^4 \log((d-1)e^{2bx+2a}) + 1) + 4b^3x^3 \operatorname{Li}_2(-(d-1)e^{2bx+2a})}{d} \right) -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(1-d-d\*coth(b\*x+a)),x, algorithm="maxima")

[Out] -1/4\*x^4\*arccoth(d\*coth(b\*x + a) + d - 1) + 1/40\*(2\*x^5/d - 5\*(2\*b^4\*x^4\*log((d - 1)\*e^(2\*b\*x + 2\*a) + 1) + 4\*b^3\*x^3\*dilog(-(d - 1)\*e^(2\*b\*x + 2\*a)) - 6\*b^2\*x^2\*polylog(3, -(d - 1)\*e^(2\*b\*x + 2\*a)) + 6\*b\*x\*polylog(4, -(d - 1)\*e^(2\*b\*x + 2\*a)) - 3\*polylog(5, -(d - 1)\*e^(2\*b\*x + 2\*a)))/(b^5\*d))\*b\*d

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**Fricas [C]** time = 2.14981, size = 1377, normalized size = 8.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(1-d-d\*coth(b\*x+a)),x, algorithm="fricas")

[Out] 1/40\*(2\*b^5\*x^5 - 5\*b^4\*x^4\*log((d\*cosh(b\*x + a) + d\*sinh(b\*x + a))/(d\*cosh(b\*x + a) + (d - 2)\*sinh(b\*x + a))) - 20\*b^3\*x^3\*dilog(1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 20\*b^3\*x^3\*dilog(-1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 5\*a^4\*log(2\*(d - 1)\*cosh(b\*x + a) + 2\*(d - 1)\*sinh(b\*x + a) + sqrt(-4\*d + 4)) - 5\*a^4\*log(2\*(d - 1)\*cosh(b\*x + a) + 2\*(d - 1)\*sinh(b\*x + a) - sqrt(-4\*d + 4)) + 60\*b^2\*x^2\*polylog(3, 1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) + 60\*b^2\*x^2\*polylog(3, -1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 120\*b\*x\*polylog(4, 1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 120\*b\*x\*polylog(4, -1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 5\*(b^4\*x^4 - a^4)\*log(1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a)) + 1) - 5\*(b^4\*x^4 - a^4)\*log(-1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a)) + 1) + 120\*polylog(5, 1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) + 120\*polylog(5, -1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a))))/b^4

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**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*acoth(1-d-d\*coth(b\*x+a)),x)

[Out] Timed out

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**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arccoth}(-d \coth(bx + a) - d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(1-d-d\*coth(b\*x+a)),x, algorithm="giac")

[Out] integrate(x^3\*arccoth(-d\*coth(b\*x + a) - d + 1), x)

### 3.227 $\int x^2 \coth^{-1}(1 - d - d \coth(a + bx)) dx$

**Optimal.** Leaf size=137

$$\frac{x \operatorname{PolyLog}\left(3, (1-d)e^{2a+2bx}\right)}{4b^2} - \frac{\operatorname{PolyLog}\left(4, (1-d)e^{2a+2bx}\right)}{8b^3} - \frac{x^2 \operatorname{PolyLog}\left(2, (1-d)e^{2a+2bx}\right)}{4b} - \frac{1}{6} x^3 \log\left(1 - (1-d)e^{2a+2bx}\right)$$

[Out] (b\*x^4)/12 + (x^3\*ArcCoth[1 - d - d\*Coth[a + b\*x]])/3 - (x^3\*Log[1 - (1 - d)\*E^(2\*a + 2\*b\*x)])/6 - (x^2\*PolyLog[2, (1 - d)\*E^(2\*a + 2\*b\*x)])/(4\*b) + (x\*PolyLog[3, (1 - d)\*E^(2\*a + 2\*b\*x)])/(4\*b^2) - PolyLog[4, (1 - d)\*E^(2\*a + 2\*b\*x)]/(8\*b^3)

**Rubi [A]** time = 0.270101, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {6242, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{x \operatorname{PolyLog}\left(3, (1-d)e^{2a+2bx}\right)}{4b^2} - \frac{\operatorname{PolyLog}\left(4, (1-d)e^{2a+2bx}\right)}{8b^3} - \frac{x^2 \operatorname{PolyLog}\left(2, (1-d)e^{2a+2bx}\right)}{4b} - \frac{1}{6} x^3 \log\left(1 - (1-d)e^{2a+2bx}\right)$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcCoth[1 - d - d\*Coth[a + b\*x]],x]

[Out] (b\*x^4)/12 + (x^3\*ArcCoth[1 - d - d\*Coth[a + b\*x]])/3 - (x^3\*Log[1 - (1 - d)\*E^(2\*a + 2\*b\*x)])/6 - (x^2\*PolyLog[2, (1 - d)\*E^(2\*a + 2\*b\*x)])/(4\*b) + (x\*PolyLog[3, (1 - d)\*E^(2\*a + 2\*b\*x)])/(4\*b^2) - PolyLog[4, (1 - d)\*E^(2\*a + 2\*b\*x)]/(8\*b^3)

#### Rule 6242

Int[ArcCoth[(c\_.) + Coth[(a\_.) + (b\_.)\*(x\_)]\*(d\_.)]\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^(m + 1)\*ArcCoth[c + d\*Coth[a + b\*x]])/(f\*(m + 1)), x] + Dist[b/(f\*(m + 1)), Int[(e + f\*x)^(m + 1)/(c - d - c\*E^(2\*a + 2\*b\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d)^2, 1]

#### Rule 2184

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x],



x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_)]/((a\_) + (b\_)\*((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_)\*((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_)))^(n\_))]\*(f\_) + (g\_)\*(x\_)]^(m\_), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 6609

Int[((e\_) + (f\_)\*(x\_)]^(m\_)\*PolyLog[n\_, (d\_)\*((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_)))^(p\_)]], x\_Symbol] := Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p])/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))]^(m\_) /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\* (F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_)\*((a\_) + (b\_)\*(x\_)]^(p\_)]/((d\_) + (e\_)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(1-d-d \coth(a+bx)) dx &= \frac{1}{3}x^3 \coth^{-1}(1-d-d \coth(a+bx)) + \frac{1}{3}b \int \frac{x^3}{1+(-1+d)e^{2a+2bx}} dx \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(1-d-d \coth(a+bx)) + \frac{1}{3}(b(1-d)) \int \frac{e^{2a+2bx}x^3}{1+(-1+d)e^{2a+2bx}} dx \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(1-d-d \coth(a+bx)) - \frac{1}{6}x^3 \log(1-(1-d)e^{2a+2bx}) + \frac{1}{2} \int \frac{x^2 dx}{1+(-1+d)e^{2a+2bx}} \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(1-d-d \coth(a+bx)) - \frac{1}{6}x^3 \log(1-(1-d)e^{2a+2bx}) - \frac{x^2 \operatorname{Li}_2(-1+d)e^{2a+2bx}}{2(1+d)} \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(1-d-d \coth(a+bx)) - \frac{1}{6}x^3 \log(1-(1-d)e^{2a+2bx}) - \frac{x^2 \operatorname{Li}_2(-1+d)e^{2a+2bx}}{2(1+d)} \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(1-d-d \coth(a+bx)) - \frac{1}{6}x^3 \log(1-(1-d)e^{2a+2bx}) - \frac{x^2 \operatorname{Li}_2(-1+d)e^{2a+2bx}}{2(1+d)}
\end{aligned}$$

**Mathematica [A]** time = 0.11101, size = 121, normalized size = 0.88

$$\frac{1}{24} \left( \frac{6x \operatorname{PolyLog}\left(3, -\frac{e^{-2(a+bx)}}{d-1}\right)}{b^2} + \frac{3 \operatorname{PolyLog}\left(4, -\frac{e^{-2(a+bx)}}{d-1}\right)}{b^3} + \frac{6x^2 \operatorname{PolyLog}\left(2, -\frac{e^{-2(a+bx)}}{d-1}\right)}{b} - 4x^3 \log\left(\frac{e^{-2(a+bx)}}{d-1} + 1\right) + 8x^3 \coth^{-1}\left(\frac{e^{-2(a+bx)}}{d-1} + 1\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[1 - d - d\*Coth[a + b\*x]], x]

[Out] (8\*x^3\*ArcCoth[1 - d - d\*Coth[a + b\*x]] - 4\*x^3\*Log[1 + 1/((-1 + d)\*E^(2\*(a + b\*x))]) + (6\*x^2\*PolyLog[2, -(1/((-1 + d)\*E^(2\*(a + b\*x)))])/b + (6\*x\*PolyLog[3, -(1/((-1 + d)\*E^(2\*(a + b\*x)))])/b^2 + (3\*PolyLog[4, -(1/((-1 + d)\*E^(2\*(a + b\*x)))])/b^3)/24

**Maple [C]** time = 37.721, size = 1771, normalized size = 12.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(1-d-d\*coth(b\*x+a)), x)

```

[Out] 1/12*I*x^3*Pi*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x
+2*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))^2+1/6/b^3*d*a^3/(d-1)*ln(exp(
2*b*x+2*a)*d-exp(2*b*x+2*a)+1)+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csg
n(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))^2-1/12*I*x^3*Pi
*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^3+1/12*I*x^3*Pi*csgn(I*exp(2*b
*x+2*a)/(exp(2*b*x+2*a)-1))^3-1/3*x^3*ln(exp(b*x+a))-1/6*x^3*ln(d)+1/6*I*x^
3*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/12*I*x^3*Pi*csgn(I/(ex
p(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2-1/12*I*x^3*Pi*
csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/6*I*x^3*Pi*csgn(I
*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/12*b*x^4+1/6*x^3*ln(exp(2*b*x+2*a)*
d-exp(2*b*x+2*a)+1)-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))
*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/2/b^3*d*a^2/(d-1)*dilog(1+
exp(b*x+a)*(1-d)^(1/2))-1/2/b^3*d*a^2/(d-1)*dilog(1-exp(b*x+a)*(1-d)^(1/2))
-1/2/b^3*d*a^3/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))-1/2/b^3*d*a^3/(d-1)*ln(1-
exp(b*x+a)*(1-d)^(1/2))+1/3/b^3*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a^3-1/4/
b*d/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*x^2+1/4/b^3*d/(d-1)*polylog(2,-(
d-1)*exp(2*b*x+2*a))*a^2+1/4/b^2*d/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))*x
-1/2/b^2/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x*a^2+1/2/b^2*a^2/(d-1)*ln(1+exp(
b*x+a)*(1-d)^(1/2))*x+1/2/b^2*a^2/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))*x-1/6/
b^3*a^3/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1)+1/12*I*x^3*Pi*csgn(I/(e
xp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))^3-1/6*d/(d-1)*ln(1+(d
-1)*exp(2*b*x+2*a))*x^3-1/8/b^3*d/(d-1)*polylog(4,-(d-1)*exp(2*b*x+2*a))+1/
2/b^3*a^2/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^3*a^2/(d-1)*dilog(1-e
xp(b*x+a)*(1-d)^(1/2))-1/3/b^3/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a^3+1/4/b/(
d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*x^2-1/4/b^3/(d-1)*polylog(2,-(d-1)*ex
p(2*b*x+2*a))*a^2-1/4/b^2/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))*x+1/2/b^3*
a^3/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^3*a^3/(d-1)*ln(1-exp(b*x+a)*(1
-d)^(1/2))+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)-1))*
csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))+1/12*I*x^3*Pi*csgn(I*d)*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a)
)-1/6*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1
))^2+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))^3+1/12*I*x^3*Pi*csgn(I*exp(b*x+a)
)^2*csgn(I*exp(2*b*x+2*a))+1/6/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^3+1/8/b^3
/(d-1)*polylog(4,-(d-1)*exp(2*b*x+2*a))-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)
)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2-1/12*I*x^3*Pi*csgn(I/(exp(2*b
*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2
*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))+1/2/b^2*d/(d-1)*ln(1+(d-1)*exp(
2*b*x+2*a))*x*a^2-1/2/b^2*d*a^2/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))*x-1/2/b^
2*d*a^2/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))*x

```

**Maxima [A]** time = 3.56393, size = 169, normalized size = 1.23

$$-\frac{1}{3}x^3 \operatorname{arccoth}(d \coth(bx + a) + d - 1) + \frac{1}{36} \left( \frac{3x^4}{d} - \frac{2(4b^3x^3 \log((d-1)e^{2bx+2a}) + 1) + 6b^2x^2 \operatorname{Li}_2(-(d-1)e^{2bx+2a})}{b^4d} \right) -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(1-d-d\*coth(b\*x+a)),x, algorithm="maxima")

[Out] -1/3\*x^3\*arccoth(d\*coth(b\*x + a) + d - 1) + 1/36\*(3\*x^4/d - 2\*(4\*b^3\*x^3\*log((d - 1)\*e^(2\*b\*x + 2\*a) + 1) + 6\*b^2\*x^2\*dilog(-(d - 1)\*e^(2\*b\*x + 2\*a)) - 6\*b\*x\*polylog(3, -(d - 1)\*e^(2\*b\*x + 2\*a)) + 3\*polylog(4, -(d - 1)\*e^(2\*b\*x + 2\*a)))/(b^4\*d))\*b\*d

**Fricas [C]** time = 2.05805, size = 1160, normalized size = 8.47

$$b^4x^4 - 2b^3x^3 \log\left(\frac{d \cosh(bx+a) + d \sinh(bx+a)}{d \cosh(bx+a) + (d-2) \sinh(bx+a)}\right) - 6b^2x^2 \operatorname{Li}_2\left(\frac{1}{2} \sqrt{-4d+4}(\cosh(bx+a) + \sinh(bx+a))\right) - 6b^2x^2 \operatorname{Li}_2\left(-\frac{1}{2} \sqrt{-4d+4}(\cosh(bx+a) + \sinh(bx+a))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(1-d-d\*coth(b\*x+a)),x, algorithm="fricas")

[Out] 1/12\*(b^4\*x^4 - 2\*b^3\*x^3\*log((d\*cosh(b\*x + a) + d\*sinh(b\*x + a))/(d\*cosh(b\*x + a) + (d - 2)\*sinh(b\*x + a))) - 6\*b^2\*x^2\*dilog(1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 6\*b^2\*x^2\*dilog(-1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) + 2\*a^3\*log(2\*(d - 1)\*cosh(b\*x + a) + 2\*(d - 1)\*sinh(b\*x + a) + sqrt(-4\*d + 4)) + 2\*a^3\*log(2\*(d - 1)\*cosh(b\*x + a) + 2\*(d - 1)\*sinh(b\*x + a) - sqrt(-4\*d + 4)) + 12\*b\*x\*polylog(3, 1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) + 12\*b\*x\*polylog(3, -1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 2\*(b^3\*x^3 + a^3)\*log(1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a)) + 1) - 2\*(b^3\*x^3 + a^3)\*log(-1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a)) + 1) - 12\*polylog(4, 1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 12\*polylog(4, -1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a))))/b^3

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(1-d-d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arccoth}(-d \coth(bx + a) - d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1-d-d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(-d*coth(b*x + a) - d + 1), x)
```

### 3.228 $\int x \coth^{-1}(1 - d - d \coth(a + bx)) dx$

**Optimal.** Leaf size=109

$$\frac{\text{PolyLog}\left(3, (1-d)e^{2a+2bx}\right)}{8b^2} - \frac{x \text{PolyLog}\left(2, (1-d)e^{2a+2bx}\right)}{4b} - \frac{1}{4}x^2 \log\left(1 - (1-d)e^{2a+2bx}\right) + \frac{1}{2}x^2 \coth^{-1}(d - \coth(a + bx))$$

[Out] (b\*x^3)/6 + (x^2\*ArcCoth[1 - d - d\*Coth[a + b\*x]])/2 - (x^2\*Log[1 - (1 - d)\*E^(2\*a + 2\*b\*x)])/4 - (x\*PolyLog[2, (1 - d)\*E^(2\*a + 2\*b\*x)])/(4\*b) + PolyLog[3, (1 - d)\*E^(2\*a + 2\*b\*x)]/(8\*b^2)

**Rubi [A]** time = 0.237728, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$ , Rules used = {6242, 2184, 2190, 2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3, (1-d)e^{2a+2bx}\right)}{8b^2} - \frac{x \text{PolyLog}\left(2, (1-d)e^{2a+2bx}\right)}{4b} - \frac{1}{4}x^2 \log\left(1 - (1-d)e^{2a+2bx}\right) + \frac{1}{2}x^2 \coth^{-1}(d - \coth(a + bx))$$

Antiderivative was successfully verified.

[In] Int[x\*ArcCoth[1 - d - d\*Coth[a + b\*x]], x]

[Out] (b\*x^3)/6 + (x^2\*ArcCoth[1 - d - d\*Coth[a + b\*x]])/2 - (x^2\*Log[1 - (1 - d)\*E^(2\*a + 2\*b\*x)])/4 - (x\*PolyLog[2, (1 - d)\*E^(2\*a + 2\*b\*x)])/(4\*b) + PolyLog[3, (1 - d)\*E^(2\*a + 2\*b\*x)]/(8\*b^2)

#### Rule 6242

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_.)]*(d_.)]*((e_.) + (f_.)*(x_.))^(m_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Coth[a + b*x]])/(f*(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d)^2, 1]
```

#### Rule 2184

```
Int[((c_.) + (d_.)*(x_.))^(m_.)/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_.))))^(n_.), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x \coth^{-1}(1-d-d \coth(a+bx)) dx &= \frac{1}{2}x^2 \coth^{-1}(1-d-d \coth(a+bx)) + \frac{1}{2}b \int \frac{x^2}{1+(-1+d)e^{2a+2bx}} dx \\
&= \frac{bx^3}{6} + \frac{1}{2}x^2 \coth^{-1}(1-d-d \coth(a+bx)) + \frac{1}{2}(b(1-d)) \int \frac{e^{2a+2bx}x^2}{1+(-1+d)e^{2a+2bx}} dx \\
&= \frac{bx^3}{6} + \frac{1}{2}x^2 \coth^{-1}(1-d-d \coth(a+bx)) - \frac{1}{4}x^2 \log(1-(1-d)e^{2a+2bx}) + \frac{1}{2} \int \frac{x^2 e^{2a+2bx}}{1+(-1+d)e^{2a+2bx}} dx \\
&= \frac{bx^3}{6} + \frac{1}{2}x^2 \coth^{-1}(1-d-d \coth(a+bx)) - \frac{1}{4}x^2 \log(1-(1-d)e^{2a+2bx}) - \frac{x \operatorname{Li}_2}{2} \\
&= \frac{bx^3}{6} + \frac{1}{2}x^2 \coth^{-1}(1-d-d \coth(a+bx)) - \frac{1}{4}x^2 \log(1-(1-d)e^{2a+2bx}) - \frac{x \operatorname{Li}_2}{2} \\
&= \frac{bx^3}{6} + \frac{1}{2}x^2 \coth^{-1}(1-d-d \coth(a+bx)) - \frac{1}{4}x^2 \log(1-(1-d)e^{2a+2bx}) - \frac{x \operatorname{Li}_2}{2}
\end{aligned}$$

**Mathematica [A]** time = 0.112547, size = 94, normalized size = 0.86

$$\frac{2bx \operatorname{PolyLog}\left(2, -\frac{e^{-2(a+bx)}}{d-1}\right) + \operatorname{PolyLog}\left(3, -\frac{e^{-2(a+bx)}}{d-1}\right) + 2b^2x^2\left(2 \coth^{-1}(d(-\coth(a+bx)) - d + 1) - \log\left(\frac{e^{-2(a+bx)}}{d-1} + 1\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[1 - d - d\*Coth[a + b\*x]], x]

[Out] (2\*b^2\*x^2\*(2\*ArcCoth[1 - d - d\*Coth[a + b\*x]] - Log[1 + 1/((-1 + d)\*E^(2\*(a + b\*x))])) + 2\*b\*x\*PolyLog[2, -(1/((-1 + d)\*E^(2\*(a + b\*x)))] + PolyLog[3, -(1/((-1 + d)\*E^(2\*(a + b\*x)))])/(8\*b^2)

**Maple [C]** time = 19.147, size = 1688, normalized size = 15.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(1-d-d\*coth(b\*x+a)), x)

[Out] -1/4/b^2\*d\*a^2/(d-1)\*ln(exp(2\*b\*x+2\*a)\*d-exp(2\*b\*x+2\*a)+1)-1/8\*I\*x^2\*Pi\*csgn(I\*d/(exp(2\*b\*x+2\*a)-1)\*exp(2\*b\*x+2\*a))^3+1/6\*b\*x^3+1/8\*I\*x^2\*Pi\*csgn(I\*(e



```

xp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)
)*d-exp(2*b*x+2*a)+1))^2-1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)
-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/8*I*x^2*Pi*csgn(I*exp(
2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2-1/2*x^2*ln(exp(b*x+
a))-1/4*x^2*ln(d)+1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x
+2*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))^2+1/4*x^2*ln(exp(2*b*x+2*a)*d
-exp(2*b*x+2*a)+1)-1/2/b^2*a/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))-1/4*d/(d
-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^2-1/2/b^2*a/(d-1)*dilog(1-exp(b*x+a)*(1-d)
^(1/2))+1/4/b^2/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a^2+1/4/b/(d-1)*polylog(2,
-(d-1)*exp(2*b*x+2*a))*x+1/4/b^2/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*a-1
/2/b^2*a^2/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))-1/2/b^2*a^2/(d-1)*ln(1-exp(b*
x+a)*(1-d)^(1/2))+1/8/b^2*d/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))-1/8*I*x^
2*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2
-1/8*I*x^2*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/4*I
*x^2*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/4/(d-1)*ln(1+(d-1)*
exp(2*b*x+2*a))*x^2-1/8/b^2/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))+1/8*I*x^
2*Pi*csgn(I*exp(2*b*x+2*a))^3-1/4*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2
*b*x+2*a)*d-exp(2*b*x+2*a)+1))^2+1/4/b^2*a^2/(d-1)*ln(exp(2*b*x+2*a)*d-exp(
2*b*x+2*a)+1)+1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d-exp(
2*b*x+2*a)+1))^3+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^3+1
/2/b^2*d*a/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^2*d*a/(d-1)*dilog(1-
exp(b*x+a)*(1-d)^(1/2))-1/4/b^2*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a^2-1/4/
b*d/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*x-1/4/b^2*d/(d-1)*polylog(2,-(d-
1)*exp(2*b*x+2*a))*a+1/2/b/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x*a-1/2/b*a/(d-
1)*ln(1+exp(b*x+a)*(1-d)^(1/2))*x-1/2/b*a/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2)
)*x+1/2/b^2*d*a^2/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^2*d*a^2/(d-1)*ln
(1-exp(b*x+a)*(1-d)^(1/2))+1/8*I*x^2*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b
*x+2*a))-1/4*I*x^2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/8*I*x^2
*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)
/(exp(2*b*x+2*a)-1))+1/8*I*x^2*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*
x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))-1/2/b*d/(d-1)*ln(1+(
d-1)*exp(2*b*x+2*a))*x*a+1/2/b*d*a/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))*x+1/2
/b*d*a/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))*x-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+
2*a)-1))*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)
-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))

```

---

**Maxima [A]** time = 3.53762, size = 136, normalized size = 1.25

$$\frac{1}{24} \left( \frac{4x^3}{d} - \frac{3(2b^2x^2 \log((d-1)e^{(2bx+2a)} + 1) + 2bx \operatorname{Li}_2(-(d-1)e^{(2bx+2a)}) - \operatorname{Li}_3(-(d-1)e^{(2bx+2a)}))}{b^3d} \right) bd - \frac{1}{2} x^2 \operatorname{arccoth}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1-d-d\*coth(b\*x+a)),x, algorithm="maxima")

[Out]  $\frac{1}{24} \left( \frac{4x^3}{d} - 3(2b^2x^2 \log((d-1)e^{2bx+2a} + 1) + 2bx \operatorname{dilog}(-(d-1)e^{2bx+2a}) - \operatorname{polylog}(3, -(d-1)e^{2bx+2a})) \right) / (b^3d) + bx^2 - \frac{1}{2}x^2 \operatorname{arccoth}(d \operatorname{coth}(bx+a) + d - 1)$

**Fricas [C]** time = 1.84068, size = 953, normalized size = 8.74

$$2b^3x^3 - 3b^2x^2 \log\left(\frac{d \cosh(bx+a) + d \sinh(bx+a)}{d \cosh(bx+a) + (d-2) \sinh(bx+a)}\right) - 6bx \operatorname{Li}_2\left(\frac{1}{2} \sqrt{-4d+4} (\cosh(bx+a) + \sinh(bx+a))\right) - 6bx \operatorname{Li}_2\left(-\frac{1}{2} \sqrt{-4d+4} (\cosh(bx+a) + \sinh(bx+a))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1-d-d\*coth(b\*x+a)),x, algorithm="fricas")

[Out]  $\frac{1}{12} \left( 2b^3x^3 - 3b^2x^2 \log\left(\frac{d \cosh(bx+a) + d \sinh(bx+a)}{d \cosh(bx+a) + (d-2) \sinh(bx+a)}\right) - 6bx \operatorname{dilog}\left(\frac{1}{2} \sqrt{-4d+4} (\cosh(bx+a) + \sinh(bx+a))\right) - 6bx \operatorname{dilog}\left(-\frac{1}{2} \sqrt{-4d+4} (\cosh(bx+a) + \sinh(bx+a))\right) - 3a^2 \log(2(d-1) \cosh(bx+a) + 2(d-1) \sinh(bx+a) + \sqrt{-4d+4}) - 3a^2 \log(2(d-1) \cosh(bx+a) + 2(d-1) \sinh(bx+a) - \sqrt{-4d+4}) - 3(b^2x^2 - a^2) \log\left(\frac{1}{2} \sqrt{-4d+4} (\cosh(bx+a) + \sinh(bx+a)) + 1\right) - 3(b^2x^2 - a^2) \log\left(-\frac{1}{2} \sqrt{-4d+4} (\cosh(bx+a) + \sinh(bx+a)) + 1\right) + 6 \operatorname{polylog}(3, \frac{1}{2} \sqrt{-4d+4} (\cosh(bx+a) + \sinh(bx+a))) + 6 \operatorname{polylog}(3, -\frac{1}{2} \sqrt{-4d+4} (\cosh(bx+a) + \sinh(bx+a))) \right) / b^2$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(1-d-d\*coth(b\*x+a)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arccoth}(-d \coth(bx + a) - d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1-d-d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(-d*coth(b*x + a) - d + 1), x)
```

### 3.229 $\int \coth^{-1}(1 - d - d \coth(a + bx)) dx$

**Optimal.** Leaf size=76

$$-\frac{\text{PolyLog}\left(2, (1-d)e^{2a+2bx}\right)}{4b} - \frac{1}{2}x \log\left(1 - (1-d)e^{2a+2bx}\right) + x \coth^{-1}(d(-\coth(a+bx)) - d + 1) + \frac{bx^2}{2}$$

[Out] (b\*x^2)/2 + x\*ArcCoth[1 - d - d\*Coth[a + b\*x]] - (x\*Log[1 - (1 - d)\*E^(2\*a + 2\*b\*x)])/2 - PolyLog[2, (1 - d)\*E^(2\*a + 2\*b\*x)]/(4\*b)

**Rubi [A]** time = 0.144469, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6234, 2184, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2, (1-d)e^{2a+2bx}\right)}{4b} - \frac{1}{2}x \log\left(1 - (1-d)e^{2a+2bx}\right) + x \coth^{-1}(d(-\coth(a+bx)) - d + 1) + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[1 - d - d\*Coth[a + b\*x]], x]

[Out] (b\*x^2)/2 + x\*ArcCoth[1 - d - d\*Coth[a + b\*x]] - (x\*Log[1 - (1 - d)\*E^(2\*a + 2\*b\*x)])/2 - PolyLog[2, (1 - d)\*E^(2\*a + 2\*b\*x)]/(4\*b)

#### Rule 6234

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*ArcCoth[c + d*Coth[a + b*x]], x] + Dist[b, Int[x/(c - d - c*E^(2*a + 2*b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

#### Rule 2184

```
Int[(((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

#### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned} \int \coth^{-1}(1 - d - d \coth(a + bx)) dx &= x \coth^{-1}(1 - d - d \coth(a + bx)) + b \int \frac{x}{1 + (-1 + d)e^{2a+2bx}} dx \\ &= \frac{bx^2}{2} + x \coth^{-1}(1 - d - d \coth(a + bx)) + (b(1 - d)) \int \frac{e^{2a+2bx} x}{1 + (-1 + d)e^{2a+2bx}} dx \\ &= \frac{bx^2}{2} + x \coth^{-1}(1 - d - d \coth(a + bx)) - \frac{1}{2} x \log(1 - (1 - d)e^{2a+2bx}) + \frac{1}{2} \int \log \left( \frac{\text{Subst}\left(\frac{x}{1 + (-1 + d)e^{2a+2bx}}, x, e^{2a+2bx}\right)}{1 + (-1 + d)e^{2a+2bx}} \right) dx \\ &= \frac{bx^2}{2} + x \coth^{-1}(1 - d - d \coth(a + bx)) - \frac{1}{2} x \log(1 - (1 - d)e^{2a+2bx}) + \frac{\text{Li}_2(1 - (1 - d)e^{2a+2bx})}{2} \end{aligned}$$

**Mathematica [B]** time = 0.749857, size = 208, normalized size = 2.74

---


$$-2\text{PolyLog}\left(2, -\sqrt{1 - de^{a+bx}}\right) - 2\text{PolyLog}\left(2, \sqrt{1 - de^{a+bx}}\right) - 2\log\left(e^{a+bx}\right)\log\left(1 - \sqrt{1 - de^{a+bx}}\right) - 2\log\left(e^{a+bx}\right)\log\left(\sqrt{1 - de^{a+bx}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[1 - d - d*Coth[a + b*x]], x]
```

```
[Out] x*ArcCoth[1 - d - d*Coth[a + b*x]] + (b^2*x^2 + Log[E^(a + b*x)]^2 - 2*Log[E^(a + b*x)]*Log[1 - Sqrt[1 - d]*E^(a + b*x)] - 2*Log[E^(a + b*x)]*Log[1 + Sqrt[1 - d]*E^(a + b*x)] + 2*Log[E^(a + b*x)]*Log[E^(-a - b*x)*(1 + (-1 + d)*E^(2*(a + b*x)))] - 2*b*x*Log[d*Cosh[a + b*x] + (-2 + d)*Sinh[a + b*x]] -
```

$$2*\text{PolyLog}[2, -(\text{Sqrt}[1 - d]*E^{(a + b*x)})] - 2*\text{PolyLog}[2, \text{Sqrt}[1 - d]*E^{(a + b*x)}]/(4*b)$$

**Maple [B]** time = 0.175, size = 271, normalized size = 3.6

$$\frac{\text{arccoth}(1 - d - d\coth(bx + a)) \ln(-d\coth(bx + a) - d)}{2b} - \frac{\text{arccoth}(1 - d - d\coth(bx + a)) \ln(-d\coth(bx + a) + d)}{2b} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(1-d-d\*coth(b\*x+a)),x)

[Out] 1/2/b\*arccoth(1-d-d\*coth(b\*x+a))\*ln(-d\*coth(b\*x+a)-d)-1/2/b\*arccoth(1-d-d\*coth(b\*x+a))\*ln(-d\*coth(b\*x+a)+d)+1/8/b\*ln(-d\*coth(b\*x+a)-d)^2-1/4/b\*dilog(1-1/2\*d\*coth(b\*x+a)-1/2\*d)-1/4/b\*ln(-d\*coth(b\*x+a)-d)\*ln(1-1/2\*d\*coth(b\*x+a)-1/2\*d)-1/4/b\*dilog(-1/2\*(-d\*coth(b\*x+a)-d)/d)-1/4/b\*ln(-d\*coth(b\*x+a)+d)\*ln(-1/2\*(-d\*coth(b\*x+a)-d)/d)+1/4/b\*dilog((-d\*coth(b\*x+a)-d+2)/(-2\*d+2))+1/4/b\*ln(-d\*coth(b\*x+a)+d)\*ln((-d\*coth(b\*x+a)-d+2)/(-2\*d+2))

**Maxima [A]** time = 3.52798, size = 99, normalized size = 1.3

$$\frac{1}{4}bd\left(\frac{2x^2}{d} - \frac{2bx \log((d-1)e^{(2bx+2a)} + 1) + \text{Li}_2(-(d-1)e^{(2bx+2a)})}{b^2d}\right) - x \text{arccoth}(d \coth(bx + a) + d - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-d-d\*coth(b\*x+a)),x, algorithm="maxima")

[Out] 1/4\*b\*d\*(2\*x^2/d - (2\*b\*x\*log((d - 1)\*e^(2\*b\*x + 2\*a) + 1) + dilog(-(d - 1)\*e^(2\*b\*x + 2\*a)))/(b^2\*d)) - x\*arccoth(d\*coth(b\*x + a) + d - 1)

**Fricas [B]** time = 1.74687, size = 707, normalized size = 9.3

$$\frac{b^2x^2 - bx \log\left(\frac{d \cosh(bx+a)+d \sinh(bx+a)}{d \cosh(bx+a)+(d-2) \sinh(bx+a)}\right) + a \log\left(2(d-1) \cosh(bx+a) + 2(d-1) \sinh(bx+a) + \sqrt{-4d+4}\right) + a \log\left(2(d-1) \cosh(bx+a) - 2(d-1) \sinh(bx+a) + \sqrt{-4d+4}\right)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-d-d\*coth(b\*x+a)),x, algorithm="fricas")

[Out]  $\frac{1}{2}(b^2x^2 - b*x*\log((d*\cosh(b*x + a) + d*\sinh(b*x + a))/(d*\cosh(b*x + a) + (d - 2)*\sinh(b*x + a))) + a*\log(2*(d - 1)*\cosh(b*x + a) + 2*(d - 1)*\sinh(b*x + a) + \sqrt{-4*d + 4}) + a*\log(2*(d - 1)*\cosh(b*x + a) + 2*(d - 1)*\sinh(b*x + a) - \sqrt{-4*d + 4}) - (b*x + a)*\log(1/2*\sqrt{-4*d + 4}*(\cosh(b*x + a) + \sinh(b*x + a)) + 1) - (b*x + a)*\log(-1/2*\sqrt{-4*d + 4}*(\cosh(b*x + a) + \sinh(b*x + a)) + 1) - \operatorname{dilog}(1/2*\sqrt{-4*d + 4}*(\cosh(b*x + a) + \sinh(b*x + a))) - \operatorname{dilog}(-1/2*\sqrt{-4*d + 4}*(\cosh(b*x + a) + \sinh(b*x + a))))/b$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acoth}(-d \operatorname{coth}(a + bx) - d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(1-d-d\*coth(b\*x+a)),x)

[Out] Integral(acoth(-d\*coth(a + b\*x) - d + 1), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(-d \operatorname{coth}(bx + a) - d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-d-d\*coth(b\*x+a)),x, algorithm="giac")

[Out] integrate(arccoth(-d\*coth(b\*x + a) - d + 1), x)

$$3.230 \quad \int \frac{\coth^{-1}(1-d-d \coth(ax))}{x} dx$$

**Optimal.** Leaf size=21

$$\text{CannotIntegrate}\left(\frac{\coth^{-1}(d(-\coth(ax)) - d + 1)}{x}, x\right)$$

[Out] CannotIntegrate[ArcCoth[1 - d - d\*Coth[a + b\*x]]/x, x]

**Rubi [A]** time = 0.0834828, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\coth^{-1}(1-d-d \coth(ax))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[ArcCoth[1 - d - d\*Coth[a + b\*x]]/x,x]

[Out] Defer[Int][ArcCoth[1 - d - d\*Coth[a + b\*x]]/x, x]

Rubi steps

$$\int \frac{\coth^{-1}(1-d-d \coth(ax))}{x} dx = \int \frac{\coth^{-1}(1-d-d \coth(ax))}{x} dx$$

**Mathematica [A]** time = 3.29888, size = 0, normalized size = 0.

$$\int \frac{\coth^{-1}(1-d-d \coth(ax))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcCoth[1 - d - d\*Coth[a + b\*x]]/x,x]

[Out] Integrate[ArcCoth[1 - d - d\*Coth[a + b\*x]]/x, x]



---

**Maple [A]** time = 0.392, size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(1-d-d\coth(bx+a))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(1-d-d\*coth(b\*x+a))/x,x)

[Out] int(arccoth(1-d-d\*coth(b\*x+a))/x,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{\operatorname{arccoth}(d\coth(bx+a)+d-1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-d-d\*coth(b\*x+a))/x,x, algorithm="maxima")

[Out] -integrate(arccoth(d\*coth(b\*x + a) + d - 1)/x, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\operatorname{arccoth}(d\coth(bx+a)+d-1)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-d-d\*coth(b\*x+a))/x,x, algorithm="fricas")

[Out] integral(-arccoth(d\*coth(b\*x + a) + d - 1)/x, x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}(-d\coth(a+bx)-d+1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(1-d-d\*coth(b\*x+a))/x,x)

[Out] Integral(acoth(-d\*coth(a + b\*x) - d + 1)/x, x)

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcoth}(-d \coth(bx + a) - d + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-d-d\*coth(b\*x+a))/x,x, algorithm="giac")

[Out] integrate(arccoth(-d\*coth(b\*x + a) - d + 1)/x, x)

### 3.231 $\int (e + fx)^3 \coth^{-1}(\tan(a + bx)) dx$

**Optimal.** Leaf size=302

$$\frac{3if^2(e + fx)\text{PolyLog}\left(4, -ie^{2i(a+bx)}\right)}{8b^3} - \frac{3if^2(e + fx)\text{PolyLog}\left(4, ie^{2i(a+bx)}\right)}{8b^3} + \frac{3f(e + fx)^2\text{PolyLog}\left(3, -ie^{2i(a+bx)}\right)}{8b^2} - \frac{3f(e + fx)^2\text{PolyLog}\left(3, ie^{2i(a+bx)}\right)}{8b^2}$$

```
[Out] ((e + f*x)^4*ArcCoth[Tan[a + b*x]])/(4*f) + ((I/4)*(e + f*x)^4*ArcTan[E^((2
*I)*(a + b*x))])/f - ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (3*f*(e +
f*x)^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (3*f*(e + f*x)^2*Pol
yLog[3, I*E^((2*I)*(a + b*x))])/(8*b^2) + (((3*I)/8)*f^2*(e + f*x)*PolyLog[
4, (-I)*E^((2*I)*(a + b*x))])/b^3 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E
^((2*I)*(a + b*x))])/b^3 - (3*f^3*PolyLog[5, (-I)*E^((2*I)*(a + b*x))])/(16
*b^4) + (3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```

**Rubi [A]** time = 0.24248, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {6252, 4181, 2531, 6609, 2282, 6589}

$$\frac{3if^2(e + fx)\text{PolyLog}\left(4, -ie^{2i(a+bx)}\right)}{8b^3} - \frac{3if^2(e + fx)\text{PolyLog}\left(4, ie^{2i(a+bx)}\right)}{8b^3} + \frac{3f(e + fx)^2\text{PolyLog}\left(3, -ie^{2i(a+bx)}\right)}{8b^2} - \frac{3f(e + fx)^2\text{PolyLog}\left(3, ie^{2i(a+bx)}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*ArcCoth[Tan[a + b*x]], x]
```

```
[Out] ((e + f*x)^4*ArcCoth[Tan[a + b*x]])/(4*f) + ((I/4)*(e + f*x)^4*ArcTan[E^((2
*I)*(a + b*x))])/f - ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (3*f*(e +
f*x)^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (3*f*(e + f*x)^2*Pol
yLog[3, I*E^((2*I)*(a + b*x))])/(8*b^2) + (((3*I)/8)*f^2*(e + f*x)*PolyLog[
4, (-I)*E^((2*I)*(a + b*x))])/b^3 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E
^((2*I)*(a + b*x))])/b^3 - (3*f^3*PolyLog[5, (-I)*E^((2*I)*(a + b*x))])/(16
*b^4) + (3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```

#### Rule 6252

```
Int[ArcCoth[Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcCoth[Tan[a + b*x]]/(f*(m + 1)), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
```

e, f}, x] && IGtQ[m, 0]

### Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol]
:> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f, x] + (-Dist
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_.)))^(n_.)]*((f_.) + (g_.)
*(x_.))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_.))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_.)))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int (e + fx)^3 \coth^{-1}(\tan(a + bx)) dx &= \frac{(e + fx)^4 \coth^{-1}(\tan(a + bx))}{4f} - \frac{b \int (e + fx)^4 \sec(2a + 2bx) dx}{4f} \\
&= \frac{(e + fx)^4 \coth^{-1}(\tan(a + bx))}{4f} + \frac{i(e + fx)^4 \tan^{-1}(e^{2i(a+bx)})}{4f} + \frac{1}{2} \int (e + fx)^3 \log \\
&= \frac{(e + fx)^4 \coth^{-1}(\tan(a + bx))}{4f} + \frac{i(e + fx)^4 \tan^{-1}(e^{2i(a+bx)})}{4f} - \frac{i(e + fx)^3 \text{Li}_2(-ie^{2i(a+bx)})}{4b} \\
&= \frac{(e + fx)^4 \coth^{-1}(\tan(a + bx))}{4f} + \frac{i(e + fx)^4 \tan^{-1}(e^{2i(a+bx)})}{4f} - \frac{i(e + fx)^3 \text{Li}_2(-ie^{2i(a+bx)})}{4b} \\
&= \frac{(e + fx)^4 \coth^{-1}(\tan(a + bx))}{4f} + \frac{i(e + fx)^4 \tan^{-1}(e^{2i(a+bx)})}{4f} - \frac{i(e + fx)^3 \text{Li}_2(-ie^{2i(a+bx)})}{4b} \\
&= \frac{(e + fx)^4 \coth^{-1}(\tan(a + bx))}{4f} + \frac{i(e + fx)^4 \tan^{-1}(e^{2i(a+bx)})}{4f} - \frac{i(e + fx)^3 \text{Li}_2(-ie^{2i(a+bx)})}{4b} \\
&= \frac{(e + fx)^4 \coth^{-1}(\tan(a + bx))}{4f} + \frac{i(e + fx)^4 \tan^{-1}(e^{2i(a+bx)})}{4f} - \frac{i(e + fx)^3 \text{Li}_2(-ie^{2i(a+bx)})}{4b} \\
&= \frac{(e + fx)^4 \coth^{-1}(\tan(a + bx))}{4f} + \frac{i(e + fx)^4 \tan^{-1}(e^{2i(a+bx)})}{4f} - \frac{i(e + fx)^3 \text{Li}_2(-ie^{2i(a+bx)})}{4b}
\end{aligned}$$

**Mathematica [B]** time = 0.314003, size = 654, normalized size = 2.17

$$\frac{1}{4}x(6e^2fx + 4e^3 + 4ef^2x^2 + f^3x^3) \coth^{-1}(\tan(a + bx)) + \frac{6b^2e^2f \text{PolyLog}(3, -ie^{2i(a+bx)}) - 6b^2e^2f \text{PolyLog}(3, ie^{2i(a+bx)})}{4}$$

Antiderivative was successfully verified.

[In] Integrate[(e + f\*x)^3\*ArcCoth[Tan[a + b\*x]], x]

[Out] (x\*(4\*e^3 + 6\*e^2\*f\*x + 4\*e\*f^2\*x^2 + f^3\*x^3)\*ArcCoth[Tan[a + b\*x]])/4 + (-8\*b^4\*e^3\*x\*Log[1 - I\*E^((2\*I)\*(a + b\*x))] - 12\*b^4\*e^2\*f\*x^2\*Log[1 - I\*E^((2\*I)\*(a + b\*x))] - 8\*b^4\*e\*f^2\*x^3\*Log[1 - I\*E^((2\*I)\*(a + b\*x))] - 2\*b^4\*f^3\*x^4\*Log[1 - I\*E^((2\*I)\*(a + b\*x))] + 8\*b^4\*e^3\*x\*Log[1 + I\*E^((2\*I)\*(a + b\*x))] + 12\*b^4\*e^2\*f\*x^2\*Log[1 + I\*E^((2\*I)\*(a + b\*x))] + 8\*b^4\*e\*f^2\*x^3\*Log[1 + I\*E^((2\*I)\*(a + b\*x))] + 2\*b^4\*f^3\*x^4\*Log[1 + I\*E^((2\*I)\*(a + b\*x))] - (4\*I)\*b^3\*(e + f\*x)^3\*PolyLog[2, (-I)\*E^((2\*I)\*(a + b\*x))] + (4\*I)\*b^3\*(e + f\*x)^3\*PolyLog[2, I\*E^((2\*I)\*(a + b\*x))] + 6\*b^2\*e^2\*f\*PolyLog[3, (-I)\*E^((2\*I)\*(a + b\*x))] + 12\*b^2\*e\*f^2\*x\*PolyLog[3, (-I)\*E^((2\*I)\*(a + b\*x))] + 6\*b^2\*f^3\*x^2\*PolyLog[3, (-I)\*E^((2\*I)\*(a + b\*x))] - 6\*b^2\*e^2\*f\*PolyLog[3, I\*E^((2\*I)\*(a + b\*x))] - 12\*b^2\*e\*f^2\*x\*PolyLog[3, I\*E^((2\*I)\*(a + b\*x))] - 6\*b^2\*f^3\*x^2\*PolyLog[3, I\*E^((2\*I)\*(a + b\*x))] + (6\*I)\*b\*e\*f^2\*PolyLog[4, (-I)\*E^((2\*I)\*(a + b\*x))] + (6\*I)\*b\*f^3\*x\*PolyLog[4, (-I)\*E^((2\*I)\*(a + b\*x))] - (6\*I)\*b\*e\*f^2\*PolyLog[4, I\*E^((2\*I)\*(a + b\*x))] - (6\*I)\*b\*f^3

```
3*x*PolyLog[4, I*E^((2*I)*(a + b*x))] - 3*f^3*PolyLog[5, (-I)*E^((2*I)*(a +
b*x))] + 3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))]/(16*b^4)
```

**Maple [C]** time = 8.453, size = 7429, normalized size = 24.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*arccoth(tan(b*x+a)),x)
```

```
[Out] result too large to display
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{16} (f^3x^4 + 4ef^2x^3 + 6e^2fx^2 + 4e^3x) \log(2 \cos(2bx + 2a)^2 + 2 \sin(2bx + 2a)^2 + 4 \sin(2bx + 2a) + 2) - \frac{1}{16} (f^3x^4 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arccoth(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/16*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*log(2*cos(2*b*x + 2*a)
^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/16*(f^3*x^4 + 4*e*f
^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*
a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(1/2*((b*f^3*x^4 + 4*b*e*f^2*x^3
+ 6*b*e^2*f*x^2 + 4*b*e^3*x)*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f^3*x^4
+ 4*b*e*f^2*x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*sin(4*b*x + 4*a)*sin(2*b*x +
2*a) + (b*f^3*x^4 + 4*b*e*f^2*x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*cos(2*b*x +
2*a))/(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x
)
```

**Fricas [C]** time = 2.5959, size = 4514, normalized size = 14.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*arccoth(tan(b\*x+a)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/32*(3*f^3*\text{polylog}(5, (I*\tan(b*x + a)^2 + 2*\tan(b*x + a) - I)/(\tan(b*x + a)^2 + 1)) - 3*f^3*\text{polylog}(5, (I*\tan(b*x + a)^2 - 2*\tan(b*x + a) - I)/(\tan(b*x + a)^2 + 1)) + 3*f^3*\text{polylog}(5, (-I*\tan(b*x + a)^2 + 2*\tan(b*x + a) + I)/(\tan(b*x + a)^2 + 1)) - 3*f^3*\text{polylog}(5, (-I*\tan(b*x + a)^2 - 2*\tan(b*x + a) + I)/(\tan(b*x + a)^2 + 1)) - (4*I*b^3*f^3*x^3 + 12*I*b^3*e*f^2*x^2 + 12*I*b^3*e^2*f*x + 4*I*b^3*e^3)*\text{dilog}(-((I + 1)*\tan(b*x + a)^2 + 2*\tan(b*x + a) - I + 1)/(\tan(b*x + a)^2 + 1) + 1) - (4*I*b^3*f^3*x^3 + 12*I*b^3*e*f^2*x^2 + 12*I*b^3*e^2*f*x + 4*I*b^3*e^3)*\text{dilog}(-((I + 1)*\tan(b*x + a)^2 - 2*\tan(b*x + a) - I + 1)/(\tan(b*x + a)^2 + 1) + 1) - (-4*I*b^3*f^3*x^3 - 12*I*b^3*e*f^2*x^2 - 12*I*b^3*e^2*f*x - 4*I*b^3*e^3)*\text{dilog}(-(-(I - 1)*\tan(b*x + a)^2 + 2*\tan(b*x + a) + I + 1)/(\tan(b*x + a)^2 + 1) + 1) - (-4*I*b^3*f^3*x^3 - 12*I*b^3*e*f^2*x^2 - 12*I*b^3*e^2*f*x - 4*I*b^3*e^3)*\text{dilog}(-(-(I - 1)*\tan(b*x + a)^2 - 2*\tan(b*x + a) + I + 1)/(\tan(b*x + a)^2 + 1) + 1) + 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*\log(((I + 1)*\tan(b*x + a)^2 + 2*\tan(b*x + a) - I + 1)/(\tan(b*x + a)^2 + 1)) - 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*\log(((I + 1)*\tan(b*x + a)^2 + 2*I*\tan(b*x + a) + I - 1)/(\tan(b*x + a)^2 + 1)) + 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*\log(((I + 1)*\tan(b*x + a)^2 - 2*I*\tan(b*x + a) + I - 1)/(\tan(b*x + a)^2 + 1)) - 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*\log(((I + 1)*\tan(b*x + a)^2 - 2*\tan(b*x + a) - I + 1)/(\tan(b*x + a)^2 + 1)) + 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*\log((- (I - 1)*\tan(b*x + a)^2 + 2*\tan(b*x + a) + I + 1)/(\tan(b*x + a)^2 + 1)) - 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*\log((- (I - 1)*\tan(b*x + a)^2 - 2*\tan(b*x + a) + I + 1)/(\tan(b*x + a)^2 + 1)) - 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*\log(((I - 1)*\tan(b*x + a)^2 + 2*I*\tan(b*x + a) + I + 1)/(\tan(b*x + a)^2 + 1)) + 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*\log(((I - 1)*\tan(b*x + a)^2 - 2*I*\tan(b*x + a) + I + 1)/(\tan(b*x + a)^2 + 1)) - 4*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x) * \log((\tan(b*x + a) + 1)/(\tan(b*x + a) - 1)) - (6*I*b*f^3*x + 6*I*b*e*f^2)*\text{polylog}(4, (I*\tan(b*x + a)^2 + 2*\tan(b*x + a) - I)/(\tan(b*x + a)^2 + 1)) - (6*I*b*f^3*x + 6*I*b*e*f^2)*\text{polylog}(4, (I*\tan(b*x + a)^2 - 2*\tan(b*x + a) - I)/(\tan(b*x + a)^2 + 1)) - (-6*I*b*f^3*x - 6*I*b*e*f^2)*\text{polylog}(4, (-I*\tan(b*x + a)^2 + 2*\tan(b*x + a) + I)/(\tan(b*x + a)^2 + 1)) - (-6*I*b*f^3*x - 6*I*b*e*f^2)*\text{polylog}(4, (-I*\tan(b*x + a)^2 - 2*\tan(b*x + a) + I)/(\tan(b*x + a)^2 + 1)) - 6*(b^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*\text{polylog}(3, (I*\tan(b*x + a)^2 + 2*\tan(b*x + a) - I)/(\tan(b*x + a)^2 + 1)) + 6*(b^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*\text{polylog}(3, (I*\tan(b*x + a)^2 - 2*\tan(b*x + a) - I)/(\tan(b*x + a)^2 + 1)) - 6*(b^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*\text{polylo} \end{aligned}$$

$$g(3, (-I*\tan(b*x + a)^2 + 2*\tan(b*x + a) + I)/(\tan(b*x + a)^2 + 1)) + 6*(b^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*\text{polylog}(3, (-I*\tan(b*x + a)^2 - 2*\tan(b*x + a) + I)/(\tan(b*x + a)^2 + 1)))/b^4$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (e + fx)^3 \operatorname{acoth}(\tan(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*acoth(tan(b\*x+a)),x)

[Out] Integral((e + f\*x)\*\*3\*acoth(tan(a + b\*x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (fx + e)^3 \operatorname{arccoth}(\tan(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*arccoth(tan(b\*x+a)),x, algorithm="giac")

[Out] integrate((f\*x + e)^3\*arccoth(tan(b\*x + a)), x)



### 3.232 $\int (e + fx)^2 \coth^{-1}(\tan(a + bx)) dx$

**Optimal.** Leaf size=234

$$\frac{f(e + fx)\text{PolyLog}(3, -ie^{2i(a+bx)})}{4b^2} - \frac{f(e + fx)\text{PolyLog}(3, ie^{2i(a+bx)})}{4b^2} + \frac{if^2\text{PolyLog}(4, -ie^{2i(a+bx)})}{8b^3} - \frac{if^2\text{PolyLog}(4, ie^{2i(a+bx)})}{8b^3}$$

```
[Out] ((e + f*x)^3*ArcCoth[Tan[a + b*x]])/(3*f) + ((I/3)*(e + f*x)^3*ArcTan[E^((2
*I)*(a + b*x))])/f - ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*(e + f*
x)*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(4*b^2) - (f*(e + f*x)*PolyLog[3,
I*E^((2*I)*(a + b*x))])/(4*b^2) + ((I/8)*f^2*PolyLog[4, (-I)*E^((2*I)*(a +
b*x))])/b^3 - ((I/8)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])/b^3
```

**Rubi [A]** time = 0.17167, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {6252, 4181, 2531, 6609, 2282, 6589}

$$\frac{f(e + fx)\text{PolyLog}(3, -ie^{2i(a+bx)})}{4b^2} - \frac{f(e + fx)\text{PolyLog}(3, ie^{2i(a+bx)})}{4b^2} + \frac{if^2\text{PolyLog}(4, -ie^{2i(a+bx)})}{8b^3} - \frac{if^2\text{PolyLog}(4, ie^{2i(a+bx)})}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*ArcCoth[Tan[a + b*x]], x]
```

```
[Out] ((e + f*x)^3*ArcCoth[Tan[a + b*x]])/(3*f) + ((I/3)*(e + f*x)^3*ArcTan[E^((2
*I)*(a + b*x))])/f - ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*(e + f*
x)*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(4*b^2) - (f*(e + f*x)*PolyLog[3,
I*E^((2*I)*(a + b*x))])/(4*b^2) + ((I/8)*f^2*PolyLog[4, (-I)*E^((2*I)*(a +
b*x))])/b^3 - ((I/8)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])/b^3
```

#### Rule 6252

```
Int[ArcCoth[Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcCoth[Tan[a + b*x]]/(f*(m + 1)), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
e, f}, x] && IGtQ[m, 0]
```

#### Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol]
:> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f, x] + (-Dist
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_.)))^(n_.)]*((f_.) + (g_.)
*(x_.))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_.))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_.)))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int (e + fx)^2 \coth^{-1}(\tan(a + bx)) dx &= \frac{(e + fx)^3 \coth^{-1}(\tan(a + bx))}{3f} - \frac{b \int (e + fx)^3 \sec(2a + 2bx) dx}{3f} \\
&= \frac{(e + fx)^3 \coth^{-1}(\tan(a + bx))}{3f} + \frac{i(e + fx)^3 \tan^{-1}(e^{2i(a+bx)})}{3f} + \frac{1}{2} \int (e + fx)^2 \log \\
&= \frac{(e + fx)^3 \coth^{-1}(\tan(a + bx))}{3f} + \frac{i(e + fx)^3 \tan^{-1}(e^{2i(a+bx)})}{3f} - \frac{i(e + fx)^2 \text{Li}_2(-ie^{2i(a+bx)})}{4b} \\
&= \frac{(e + fx)^3 \coth^{-1}(\tan(a + bx))}{3f} + \frac{i(e + fx)^3 \tan^{-1}(e^{2i(a+bx)})}{3f} - \frac{i(e + fx)^2 \text{Li}_2(-ie^{2i(a+bx)})}{4b} \\
&= \frac{(e + fx)^3 \coth^{-1}(\tan(a + bx))}{3f} + \frac{i(e + fx)^3 \tan^{-1}(e^{2i(a+bx)})}{3f} - \frac{i(e + fx)^2 \text{Li}_2(-ie^{2i(a+bx)})}{4b} \\
&= \frac{(e + fx)^3 \coth^{-1}(\tan(a + bx))}{3f} + \frac{i(e + fx)^3 \tan^{-1}(e^{2i(a+bx)})}{3f} - \frac{i(e + fx)^2 \text{Li}_2(-ie^{2i(a+bx)})}{4b}
\end{aligned}$$

**Mathematica [A]** time = 0.188441, size = 409, normalized size = 1.75

$$\frac{1}{3}x(3e^2 + 3efx + f^2x^2) \coth^{-1}(\tan(a + bx)) + \frac{-6ib^2(e + fx)^2 \text{PolyLog}(2, -ie^{2i(a+bx)}) + 6ib^2(e + fx)^2 \text{PolyLog}(2, ie^{2i(a+bx)})}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[(e + f\*x)^2\*ArcCoth[Tan[a + b\*x]],x]

[Out] (x\*(3\*e^2 + 3\*e\*f\*x + f^2\*x^2)\*ArcCoth[Tan[a + b\*x]])/3 + (-12\*b^3\*e^2\*x\*Log[1 - I\*E^((2\*I)\*(a + b\*x))] - 12\*b^3\*e\*f\*x^2\*Log[1 - I\*E^((2\*I)\*(a + b\*x))] - 4\*b^3\*f^2\*x^3\*Log[1 - I\*E^((2\*I)\*(a + b\*x))] + 12\*b^3\*e^2\*x\*Log[1 + I\*E^((2\*I)\*(a + b\*x))] + 12\*b^3\*e\*f\*x^2\*Log[1 + I\*E^((2\*I)\*(a + b\*x))] + 4\*b^3\*f^2\*x^3\*Log[1 + I\*E^((2\*I)\*(a + b\*x))] - (6\*I)\*b^2\*(e + f\*x)^2\*PolyLog[2, (-I)\*E^((2\*I)\*(a + b\*x))] + (6\*I)\*b^2\*(e + f\*x)^2\*PolyLog[2, I\*E^((2\*I)\*(a + b\*x))] + 6\*b\*e\*f\*PolyLog[3, (-I)\*E^((2\*I)\*(a + b\*x))] + 6\*b\*f^2\*x\*PolyLog[3, (-I)\*E^((2\*I)\*(a + b\*x))] - 6\*b\*e\*f\*PolyLog[3, I\*E^((2\*I)\*(a + b\*x))] - 6\*b\*f^2\*x\*PolyLog[3, I\*E^((2\*I)\*(a + b\*x))] + (3\*I)\*f^2\*PolyLog[4, (-I)\*E^((2\*I)\*(a + b\*x))] - (3\*I)\*f^2\*PolyLog[4, I\*E^((2\*I)\*(a + b\*x))])/(24\*b^3)

**Maple [C]** time = 13.508, size = 5543, normalized size = 23.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)^2*arccoth(tan(b*x+a)),x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{12} (f^2x^3 + 3efx^2 + 3e^2x) \log(2 \cos(2bx + 2a)^2 + 2 \sin(2bx + 2a)^2 + 4 \sin(2bx + 2a) + 2) - \frac{1}{12} (f^2x^3 + 3efx^2 + 3e^2x) \log(2 \cos(2bx + 2a)^2 + 2 \sin(2bx + 2a)^2 - 4 \sin(2bx + 2a) + 2) - \int \frac{(bf^2x^3 + 3b*efx^2 + 3b*e^2x) \cos(4bx + 4a) \cos(2bx + 2a) + (bf^2x^3 + 3b*efx^2 + 3b*e^2x) \sin(4bx + 4a) \sin(2bx + 2a) + (bf^2x^3 + 3b*efx^2 + 3b*e^2x) \cos(2bx + 2a)}{(\cos(4bx + 4a)^2 + \sin(4bx + 4a)^2 + 2 \cos(4bx + 4a) + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^2*arccoth(tan(b*x+a)),x, algorithm="maxima")`

[Out] `1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(2/3*((b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(2*b*x + 2*a))/(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)`

**Fricas [C]** time = 2.20852, size = 3357, normalized size = 14.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^2*arccoth(tan(b*x+a)),x, algorithm="fricas")`

[Out] `1/48*(3*I*f^2*polylog(4, (I*tan(b*x + a)^2 + 2*tan(b*x + a) - I)/(tan(b*x + a)^2 + 1)) + 3*I*f^2*polylog(4, (I*tan(b*x + a)^2 - 2*tan(b*x + a) - I)/(tan(b*x + a)^2 + 1)) - 3*I*f^2*polylog(4, (-I*tan(b*x + a)^2 + 2*tan(b*x + a) + I)/(tan(b*x + a)^2 + 1)) - 3*I*f^2*polylog(4, (-I*tan(b*x + a)^2 - 2*tan(b*x + a) + I)/(tan(b*x + a)^2 + 1)) + (6*I*b^2*f^2*x^2 + 12*I*b^2*e*f*x + 6*I*b^2*e^2)*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1) + 1) + (6*I*b^2*f^2*x^2 + 12*I*b^2*e*f*x + 6*I*b^2*e^2)*dilog(-((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1))`

+ 1) + (-6\*I\*b^2\*f^2\*x^2 - 12\*I\*b^2\*e\*f\*x - 6\*I\*b^2\*e^2)\*dilog(-(-(I - 1)\*tan(b\*x + a)^2 + 2\*tan(b\*x + a) + I + 1)/(tan(b\*x + a)^2 + 1) + 1) + (-6\*I\*b^2\*f^2\*x^2 - 12\*I\*b^2\*e\*f\*x - 6\*I\*b^2\*e^2)\*dilog(-(-(I - 1)\*tan(b\*x + a)^2 - 2\*tan(b\*x + a) + I + 1)/(tan(b\*x + a)^2 + 1) + 1) - 4\*(b^3\*f^2\*x^3 + 3\*b^3\*e\*f\*x^2 + 3\*b^3\*e^2\*x + 3\*a\*b^2\*e^2 - 3\*a^2\*b\*e\*f + a^3\*f^2)\*log(((I + 1)\*tan(b\*x + a)^2 + 2\*tan(b\*x + a) - I + 1)/(tan(b\*x + a)^2 + 1)) + 4\*(3\*a\*b^2\*e^2 - 3\*a^2\*b\*e\*f + a^3\*f^2)\*log(((I + 1)\*tan(b\*x + a)^2 + 2\*I\*tan(b\*x + a) + I - 1)/(tan(b\*x + a)^2 + 1)) - 4\*(3\*a\*b^2\*e^2 - 3\*a^2\*b\*e\*f + a^3\*f^2)\*log(((I + 1)\*tan(b\*x + a)^2 - 2\*I\*tan(b\*x + a) + I - 1)/(tan(b\*x + a)^2 + 1)) + 4\*(b^3\*f^2\*x^3 + 3\*b^3\*e\*f\*x^2 + 3\*b^3\*e^2\*x + 3\*a\*b^2\*e^2 - 3\*a^2\*b\*e\*f + a^3\*f^2)\*log(((I + 1)\*tan(b\*x + a)^2 - 2\*tan(b\*x + a) - I + 1)/(tan(b\*x + a)^2 + 1)) - 4\*(b^3\*f^2\*x^3 + 3\*b^3\*e\*f\*x^2 + 3\*b^3\*e^2\*x + 3\*a\*b^2\*e^2 - 3\*a^2\*b\*e\*f + a^3\*f^2)\*log((- (I - 1)\*tan(b\*x + a)^2 + 2\*tan(b\*x + a) + I + 1)/(tan(b\*x + a)^2 + 1)) + 4\*(b^3\*f^2\*x^3 + 3\*b^3\*e\*f\*x^2 + 3\*b^3\*e^2\*x + 3\*a\*b^2\*e^2 - 3\*a^2\*b\*e\*f + a^3\*f^2)\*log((- (I - 1)\*tan(b\*x + a)^2 - 2\*tan(b\*x + a) + I + 1)/(tan(b\*x + a)^2 + 1)) + 4\*(3\*a\*b^2\*e^2 - 3\*a^2\*b\*e\*f + a^3\*f^2)\*log(((I - 1)\*tan(b\*x + a)^2 + 2\*I\*tan(b\*x + a) + I + 1)/(tan(b\*x + a)^2 + 1)) - 4\*(3\*a\*b^2\*e^2 - 3\*a^2\*b\*e\*f + a^3\*f^2)\*log(((I - 1)\*tan(b\*x + a)^2 - 2\*I\*tan(b\*x + a) + I + 1)/(tan(b\*x + a)^2 + 1)) + 8\*(b^3\*f^2\*x^3 + 3\*b^3\*e\*f\*x^2 + 3\*b^3\*e^2\*x)\*log((tan(b\*x + a) + 1)/(tan(b\*x + a) - 1)) + 6\*(b\*f^2\*x + b\*e\*f)\*polylog(3, (I\*tan(b\*x + a)^2 + 2\*tan(b\*x + a) - I)/(tan(b\*x + a)^2 + 1)) - 6\*(b\*f^2\*x + b\*e\*f)\*polylog(3, (I\*tan(b\*x + a)^2 - 2\*tan(b\*x + a) - I)/(tan(b\*x + a)^2 + 1)) + 6\*(b\*f^2\*x + b\*e\*f)\*polylog(3, (-I\*tan(b\*x + a)^2 + 2\*tan(b\*x + a) + I)/(tan(b\*x + a)^2 + 1)) - 6\*(b\*f^2\*x + b\*e\*f)\*polylog(3, (-I\*tan(b\*x + a)^2 - 2\*tan(b\*x + a) + I)/(tan(b\*x + a)^2 + 1)))/b^3

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (e + fx)^2 \operatorname{acoth}(\tan(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*acoth(tan(b\*x+a)),x)

[Out] Integral((e + f\*x)\*\*2\*acoth(tan(a + b\*x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (fx + e)^2 \operatorname{arcoth}(\tan(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arccoth(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*arccoth(tan(b*x + a)), x)
```

### 3.233 $\int (e + fx) \coth^{-1}(\tan(a + bx)) dx$

**Optimal.** Leaf size=162

$$\frac{f \operatorname{PolyLog}\left(3, -ie^{2i(a+bx)}\right)}{8b^2} - \frac{f \operatorname{PolyLog}\left(3, ie^{2i(a+bx)}\right)}{8b^2} - \frac{i(e + fx) \operatorname{PolyLog}\left(2, -ie^{2i(a+bx)}\right)}{4b} + \frac{i(e + fx) \operatorname{PolyLog}\left(2, ie^{2i(a+bx)}\right)}{4b}$$

```
[Out] ((e + f*x)^2*ArcCoth[Tan[a + b*x]])/(2*f) + ((I/2)*(e + f*x)^2*ArcTan[E^((2
*I)*(a + b*x))])/f - ((I/4)*(e + f*x)*PolyLog[2, (-I)*E^((2*I)*(a + b*x))])
/b + ((I/4)*(e + f*x)*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*PolyLog[3,
(-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (f*PolyLog[3, I*E^((2*I)*(a + b*x))])/
(8*b^2)
```

**Rubi [A]** time = 0.111402, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {6252, 4181, 2531, 2282, 6589}

$$\frac{f \operatorname{PolyLog}\left(3, -ie^{2i(a+bx)}\right)}{8b^2} - \frac{f \operatorname{PolyLog}\left(3, ie^{2i(a+bx)}\right)}{8b^2} - \frac{i(e + fx) \operatorname{PolyLog}\left(2, -ie^{2i(a+bx)}\right)}{4b} + \frac{i(e + fx) \operatorname{PolyLog}\left(2, ie^{2i(a+bx)}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*ArcCoth[Tan[a + b*x]], x]
```

```
[Out] ((e + f*x)^2*ArcCoth[Tan[a + b*x]])/(2*f) + ((I/2)*(e + f*x)^2*ArcTan[E^((2
*I)*(a + b*x))])/f - ((I/4)*(e + f*x)*PolyLog[2, (-I)*E^((2*I)*(a + b*x))])
/b + ((I/4)*(e + f*x)*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*PolyLog[3,
(-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (f*PolyLog[3, I*E^((2*I)*(a + b*x))])/
(8*b^2)
```

#### Rule 6252

```
Int[ArcCoth[Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_)^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcCoth[Tan[a + b*x]])/(f*(m + 1)), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
e, f}, x] && IGtQ[m, 0]
```

#### Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]]*((c_.) + (d_.)*(x_)^(m_.), x_Symbol]
:> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
```

```
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int (e + fx) \coth^{-1}(\tan(a + bx)) dx &= \frac{(e + fx)^2 \coth^{-1}(\tan(a + bx))}{2f} - \frac{b \int (e + fx)^2 \sec(2a + 2bx) dx}{2f} \\
&= \frac{(e + fx)^2 \coth^{-1}(\tan(a + bx))}{2f} + \frac{i(e + fx)^2 \tan^{-1}(e^{2i(a+bx)})}{2f} + \frac{1}{2} \int (e + fx) \log(1 - \\
&= \frac{(e + fx)^2 \coth^{-1}(\tan(a + bx))}{2f} + \frac{i(e + fx)^2 \tan^{-1}(e^{2i(a+bx)})}{2f} - \frac{i(e + fx) \text{Li}_2(-ie^{2i(a+bx)})}{4b} \\
&= \frac{(e + fx)^2 \coth^{-1}(\tan(a + bx))}{2f} + \frac{i(e + fx)^2 \tan^{-1}(e^{2i(a+bx)})}{2f} - \frac{i(e + fx) \text{Li}_2(-ie^{2i(a+bx)})}{4b} \\
&= \frac{(e + fx)^2 \coth^{-1}(\tan(a + bx))}{2f} + \frac{i(e + fx)^2 \tan^{-1}(e^{2i(a+bx)})}{2f} - \frac{i(e + fx) \text{Li}_2(-ie^{2i(a+bx)})}{4b}
\end{aligned}$$



**Mathematica [A]** time = 0.132029, size = 263, normalized size = 1.62

$$-be \left( \frac{i \operatorname{PolyLog}(2, -ie^{i(2a+2bx)})}{4b^2} - \frac{i \operatorname{PolyLog}(2, ie^{i(2a+2bx)})}{4b^2} - \frac{ix \tan^{-1}(e^{2ia+2ibx})}{b} \right) + \frac{f(2ibx \operatorname{PolyLog}(2, -\sin(2(a+bx)))}{b^2} - ((I/4) \operatorname{PolyLog}(2, (-I)E^{I(2a+2bx)})))/b^2 - ((I/4) \operatorname{PolyLog}(2, IE^{I(2a+2bx)})))/b^2 + (f((4I)b^2 x^2 \operatorname{ArcTan}[\cos[2(a+bx)] + I \sin[2(a+bx)]] + (2I)b^2 x \operatorname{PolyLog}(2, I \cos[2(a+bx)] - \sin[2(a+bx)]) - (2I)b^2 x \operatorname{PolyLog}(2, (-I) \cos[2(a+bx)] + \sin[2(a+bx)]) - \operatorname{PolyLog}(3, I \cos[2(a+bx)] - \sin[2(a+bx)]) + \operatorname{PolyLog}(3, (-I) \cos[2(a+bx)] + \sin[2(a+bx)])))/(8b^2)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(e + f\*x)\*ArcCoth[Tan[a + b\*x]], x]

[Out] e\*x\*ArcCoth[Tan[a + b\*x]] + (f\*x^2\*ArcCoth[Tan[a + b\*x]])/2 - b\*e\*(((-I)\*x\*ArcTan[E^((2\*I)\*a + (2\*I)\*b\*x)])/b + ((I/4)\*PolyLog[2, (-I)\*E^(I\*(2\*a + 2\*b\*x))])/b^2 - ((I/4)\*PolyLog[2, I\*E^(I\*(2\*a + 2\*b\*x))])/b^2 + (f\*((4\*I)\*b^2\*x^2\*ArcTan[Cos[2\*(a + b\*x)] + I\*Sin[2\*(a + b\*x)]] + (2\*I)\*b\*x\*PolyLog[2, I\*Cos[2\*(a + b\*x)] - Sin[2\*(a + b\*x)]] - (2\*I)\*b\*x\*PolyLog[2, (-I)\*Cos[2\*(a + b\*x)] + Sin[2\*(a + b\*x)]] - PolyLog[3, I\*Cos[2\*(a + b\*x)] - Sin[2\*(a + b\*x)]] + PolyLog[3, (-I)\*Cos[2\*(a + b\*x)] + Sin[2\*(a + b\*x)])))/(8\*b^2)

**Maple [C]** time = 10.639, size = 2543, normalized size = 15.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)\*arccoth(tan(b\*x+a)), x)

[Out] 
$$-1/8*I*Pi*f*csgn(I*(\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))+1))^3*x^2-1/4*I*P$$

$$i*x*e*csgn(I*(\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))+1))^3-1/4*\ln(\exp(2*I*(b*x+a))-I)*x^2*f-1/2*\ln(\exp(2*I*(b*x+a))-I)*x*e-1/2*I/b^2*f*a*dilog(((I)^(1/2)-\exp(I*(b*x+a)))/(-I)^(1/2))-1/2*I/b^2*f*a*dilog(((I)^(1/2)+\exp(I*(b*x+a)))/(-I)^(1/2))+1/2*I/b*e*(I*b*x+I*a)*\ln(((I)^(1/2)-\exp(I*(b*x+a)))/(-I)^(1/2))-1/2*I/b*e*dilog(1+\exp(I*(b*x+a)))*(-1)^(3/4))-1/2*I/b*e*dilog(1-\exp(I*(b*x+a)))*(-1)^(3/4))+1/2*I/b*e*dilog(((I)^(1/2)-\exp(I*(b*x+a)))/(-I)^(1/2))+1/4*I*Pi*x*e*csgn(I*(\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x+a))+1))^3-1/4*I*P$$

$$i*x*e*csgn((1+I)*(\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))+1))^3+1/8*f*polylog(3,-I*\exp(2*I*(b*x+a)))/b^2-1/8*f*polylog(3,I*\exp(2*I*(b*x+a)))/b^2+1/4/b^2*f*(I*b*x+I*a)^2*\ln(1-I*\exp(2*I*(b*x+a)))+1/4/b^2*f*(I*b*x+I*a)*polylog(2,I*\exp(2*I*(b*x+a)))-1/4/b^2*f*(I*b*x+I*a)^2*\ln(1+I*\exp(2*I*(b*x+a)))-1/4/b^2*f*(I*b*x+I*a)*polylog(2,-I*\exp(2*I*(b*x+a)))+1/4*I*Pi*x*e+1/8*I*Pi*f*x^2+1/4*I*Pi*x*e*csgn((1-I)*(\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x+a))+1))^3+1/2*I/b$$

$$*e*dilog(((I)^(1/2)+\exp(I*(b*x+a)))/(-I)^(1/2))+1/4/b^2*f*a^2*\ln(-\exp(2*I*(b*x+a))+I)-1/2/b*e*a*\ln(-\exp(2*I*(b*x+a))+I)+1/2/b*a*e*\ln(\exp(2*I*(b*x+a)))$$

$$\begin{aligned}
& +I)-1/4/b^2*f*a^2*\ln(\exp(2*I*(b*x+a))+I)+1/8*I*Pi*f*csgn((1+I)*(\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))+1))^2*x^2-1/4*I*Pi*x*e*csgn((1-I)*(\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x+a))+1))^2-1/8*I*Pi*f*csgn((1-I)*(\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x+a))+1))^2*x^2+1/2*I/b*e*(I*b*x+I*a)*\ln(((1-I)^{(1/2)}+\exp(I*(b*x+a))))/(-I)^{(1/2)}-1/2*I/b*e*(I*b*x+I*a)*\ln(1+\exp(I*(b*x+a)))*(-1)^{(3/4)}-1/2*I/b*e*(I*b*x+I*a)*\ln(1-\exp(I*(b*x+a)))*(-1)^{(3/4)}+1/2*I/b^2*f*a*dilog(1+\exp(I*(b*x+a)))*(-1)^{(3/4)}+(1/4*f*x^2+1/2*e*x)*\ln(\exp(2*I*(b*x+a))+I)+1/2*I/b^2*f*a*dilog(1-\exp(I*(b*x+a)))*(-1)^{(3/4)}+1/8*I*Pi*f*csgn((1-I)*(\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x+a))+1))^3*x^2-1/8*I*Pi*f*csgn(I/(\exp(2*I*(b*x+a))+1))*csgn(I*(\exp(2*I*(b*x+a))+I))*csgn(I*(\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))+1))*x^2-1/4*I*Pi*x*e*csgn(I/(\exp(2*I*(b*x+a))+1))*csgn(I*(\exp(2*I*(b*x+a))+I))*csgn(I*(\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))+1))+1/4*I*Pi*x*e*csgn((1+I)*(\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))+1))^2-1/4*I*Pi*x*e*csgn(I*(\exp(2*I*(b*x+a))-I))*csgn(I*(\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x+a))+1))^2-1/4*I*Pi*x*e*csgn(I*(\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x+a))+1))*csgn((1-I)*(\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x+a))+1))^2-1/8*I*Pi*f*csgn(I*(\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x+a))+1))*csgn((1-I)*(\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x+a))+1))^2*x^2+1/4*I*Pi*x*e*csgn(I*(\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x+a))+1))*csgn((1-I)*(\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x+a))+1))-1/8*I*Pi*f*csgn(I*(\exp(2*I*(b*x+a))-I))*csgn(I*(\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x+a))+1))^2*x^2-1/8*I*Pi*f*csgn(I*(\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))+1))*csgn((1+I)*(\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))+1))*x^2+1/8*I*Pi*f*csgn(I*(\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x+a))+1))*csgn((1-I)*(\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x+a))+1))*x^2-1/4*I*Pi*x*e*csgn(I*(\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))+1))*csgn((1+I)*(\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))+1))+1/8*I*Pi*f*csgn(I/(\exp(2*I*(b*x+a))+1))*csgn(I*(\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))+1))^2*x^2+1/8*I*Pi*f*csgn(I*(\exp(2*I*(b*x+a))+I))*csgn(I*(\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))+1))^2*x^2+1/4*I*Pi*x*e*csgn(I/(\exp(2*I*(b*x+a))+1))*csgn(I*(\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))+1))^2+1/4*I*Pi*x*e*csgn(I*(\exp(2*I*(b*x+a))+I))*csgn(I*(\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))+1))^2+1/4*I*Pi*x*e*csgn(I*(\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))+1))*csgn((1+I)*(\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))+1))^2+1/8*I*Pi*f*csgn(I*(\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))+1))*csgn((1+I)*(\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))+1))^2*x^2-1/2*I/b^2*f*a*(I*b*x+I*a)*\ln(((1-I)^{(1/2)}-\exp(I*(b*x+a))))/(-I)^{(1/2)}-1/2*I/b^2*f*a*(I*b*x+I*a)*\ln(((1-I)^{(1/2)}+\exp(I*(b*x+a))))/(-I)^{(1/2)}-1/8*I*Pi*f*csgn(I/(\exp(2*I*(b*x+a))+1))*csgn(I*(\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x+a))+1))^2*x^2-1/4*I*Pi*x*e*csgn(I/(\exp(2*I*(b*x+a))+1))*csgn(I*(\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x+a))+1))^2+1/2*I/b^2*f*a*(I*b*x+I*a)*\ln(1+\exp(I*(b*x+a)))*(-1)^{(3/4)}+1/2*I/b^2*f*a*(I*b*x+I*a)*\ln(1-\exp(I*(b*x+a)))*(-1)^{(3/4)}+1/8*I*Pi*f*csgn(I*(\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x+a))+1))^3*x^2-1/8*I*Pi*f*csgn((1+I)*(\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))+1))^3*x^2+1/8*I*Pi*f*csgn(I/(\exp(2*I*(b*x+a))+1))*csgn(I*(\exp(2*I*(b*x+a))-I))*csgn(I*(\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x+a))+1))*x^2+1/4*I*Pi*x*e*csgn(I/(\exp(2*I*(b*x+a))+1))*csgn(I*(\exp(2*I*(b*x+a))-I))*csgn(I*(\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x+a))+1))
\end{aligned}$$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{8} (fx^2 + 2ex) \log(2 \cos(2bx + 2a)^2 + 2 \sin(2bx + 2a)^2 + 4 \sin(2bx + 2a) + 2) - \frac{1}{8} (fx^2 + 2ex) \log(2 \cos(2bx + 2a)^2 + 2 \sin(2bx + 2a)^2 - 4 \sin(2bx + 2a) + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*arccoth(tan(b\*x+a)),x, algorithm="maxima")

[Out] 1/8\*(f\*x^2 + 2\*e\*x)\*log(2\*cos(2\*b\*x + 2\*a)^2 + 2\*sin(2\*b\*x + 2\*a)^2 + 4\*sin(2\*b\*x + 2\*a) + 2) - 1/8\*(f\*x^2 + 2\*e\*x)\*log(2\*cos(2\*b\*x + 2\*a)^2 + 2\*sin(2\*b\*x + 2\*a)^2 - 4\*sin(2\*b\*x + 2\*a) + 2) - integrate(((b\*f\*x^2 + 2\*b\*e\*x)\*cos(4\*b\*x + 4\*a)\*cos(2\*b\*x + 2\*a) + (b\*f\*x^2 + 2\*b\*e\*x)\*sin(4\*b\*x + 4\*a)\*sin(2\*b\*x + 2\*a) + (b\*f\*x^2 + 2\*b\*e\*x)\*cos(2\*b\*x + 2\*a))/(cos(4\*b\*x + 4\*a)^2 + sin(4\*b\*x + 4\*a)^2 + 2\*cos(4\*b\*x + 4\*a) + 1), x)

---

**Fricas [C]** time = 2.27982, size = 2313, normalized size = 14.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*arccoth(tan(b\*x+a)),x, algorithm="fricas")

[Out] 1/16\*((2\*I\*b\*f\*x + 2\*I\*b\*e)\*dilog(-((I + 1)\*tan(b\*x + a)^2 + 2\*tan(b\*x + a) - I + 1)/(tan(b\*x + a)^2 + 1) + 1) + (2\*I\*b\*f\*x + 2\*I\*b\*e)\*dilog(-((I + 1)\*tan(b\*x + a)^2 - 2\*tan(b\*x + a) - I + 1)/(tan(b\*x + a)^2 + 1) + 1) + (-2\*I\*b\*f\*x - 2\*I\*b\*e)\*dilog(-(-(I - 1)\*tan(b\*x + a)^2 + 2\*tan(b\*x + a) + I + 1)/(tan(b\*x + a)^2 + 1) + 1) + (-2\*I\*b\*f\*x - 2\*I\*b\*e)\*dilog(-(-(I - 1)\*tan(b\*x + a)^2 - 2\*tan(b\*x + a) + I + 1)/(tan(b\*x + a)^2 + 1) + 1) - 2\*(b^2\*f\*x^2 + 2\*b^2\*e\*x + 2\*a\*b\*e - a^2\*f)\*log(((I + 1)\*tan(b\*x + a)^2 + 2\*tan(b\*x + a) - I + 1)/(tan(b\*x + a)^2 + 1)) + 2\*(2\*a\*b\*e - a^2\*f)\*log(((I + 1)\*tan(b\*x + a)^2 + 2\*I\*tan(b\*x + a) + I - 1)/(tan(b\*x + a)^2 + 1)) - 2\*(2\*a\*b\*e - a^2\*f)\*log(((I + 1)\*tan(b\*x + a)^2 - 2\*I\*tan(b\*x + a) + I - 1)/(tan(b\*x + a)^2 + 1)) + 2\*(b^2\*f\*x^2 + 2\*b^2\*e\*x + 2\*a\*b\*e - a^2\*f)\*log(((I + 1)\*tan(b\*x + a)^2 - 2\*tan(b\*x + a) - I + 1)/(tan(b\*x + a)^2 + 1)) - 2\*(b^2\*f\*x^2 + 2\*b^2\*e\*x + 2\*a\*b\*e - a^2\*f)\*log((- (I - 1)\*tan(b\*x + a)^2 + 2\*tan(b\*x + a) + I + 1)/(tan(b\*x + a)^2 + 1)) + 2\*(b^2\*f\*x^2 + 2\*b^2\*e\*x + 2\*a\*b\*e - a^2\*f)\*log(((I + 1)\*tan(b\*x + a)^2 + 2\*I\*tan(b\*x + a) + I + 1)/(tan(b\*x + a)^2 + 1)) + 2\*(2\*a\*b\*e - a^2\*f)\*log(((I - 1)\*tan(b\*x + a)^2 + 2\*I\*tan(b\*x + a) + I + 1)/(tan(b\*x + a)^2 + 1))

$$\begin{aligned} & 1)/(\tan(b*x + a)^2 + 1)) - 2*(2*a*b*e - a^2*f)*\log(((I - 1)*\tan(b*x + a)^2 \\ & - 2*I*\tan(b*x + a) + I + 1)/(\tan(b*x + a)^2 + 1)) + 4*(b^2*f*x^2 + 2*b^2*e \\ & *x)*\log((\tan(b*x + a) + 1)/(\tan(b*x + a) - 1)) + f*\text{polylog}(3, (I*\tan(b*x + \\ & a)^2 + 2*\tan(b*x + a) - I)/(\tan(b*x + a)^2 + 1)) - f*\text{polylog}(3, (I*\tan(b*x \\ & + a)^2 - 2*\tan(b*x + a) - I)/(\tan(b*x + a)^2 + 1)) + f*\text{polylog}(3, (-I*\tan(b \\ & *x + a)^2 + 2*\tan(b*x + a) + I)/(\tan(b*x + a)^2 + 1)) - f*\text{polylog}(3, (-I*\tan \\ & n(b*x + a)^2 - 2*\tan(b*x + a) + I)/(\tan(b*x + a)^2 + 1))) / b^2 \end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (e + fx) \operatorname{acoth}(\tan(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*acoth(tan(b\*x+a)),x)

[Out] Integral((e + f\*x)\*acoth(tan(a + b\*x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (fx + e) \operatorname{arccoth}(\tan(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*arccoth(tan(b\*x+a)),x, algorithm="giac")

[Out] integrate((f\*x + e)\*arccoth(tan(b\*x + a)), x)

### 3.234 $\int \coth^{-1}(\tan(a + bx)) dx$

**Optimal.** Leaf size=79

$$-\frac{i\text{PolyLog}\left(2, -ie^{2i(a+bx)}\right)}{4b} + \frac{i\text{PolyLog}\left(2, ie^{2i(a+bx)}\right)}{4b} + ix \tan^{-1}\left(e^{2i(a+bx)}\right) + x \coth^{-1}(\tan(a + bx))$$

[Out] x\*ArcCoth[Tan[a + b\*x]] + I\*x\*ArcTan[E^((2\*I)\*(a + b\*x))] - ((I/4)\*PolyLog[2, (-I)\*E^((2\*I)\*(a + b\*x))])/b + ((I/4)\*PolyLog[2, I\*E^((2\*I)\*(a + b\*x))])/b

**Rubi [A]** time = 0.047899, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {6248, 4181, 2279, 2391}

$$-\frac{i\text{PolyLog}\left(2, -ie^{2i(a+bx)}\right)}{4b} + \frac{i\text{PolyLog}\left(2, ie^{2i(a+bx)}\right)}{4b} + ix \tan^{-1}\left(e^{2i(a+bx)}\right) + x \coth^{-1}(\tan(a + bx))$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tan[a + b\*x]], x]

[Out] x\*ArcCoth[Tan[a + b\*x]] + I\*x\*ArcTan[E^((2\*I)\*(a + b\*x))] - ((I/4)\*PolyLog[2, (-I)\*E^((2\*I)\*(a + b\*x))])/b + ((I/4)\*PolyLog[2, I\*E^((2\*I)\*(a + b\*x))])/b

#### Rule 6248

Int[ArcCoth[Tan[(a\_.) + (b\_.)\*(x\_)]], x\_Symbol] := Simp[x\*ArcCoth[Tan[a + b\*x]], x] - Dist[b, Int[x\*Sec[2\*a + 2\*b\*x], x], x] /; FreeQ[{a, b}, x]

#### Rule 4181

Int[csc[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(I\*k\*Pi)\*E^(I\*(e + f\*x))])/f, x] + (-Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Log[1 - E^(I\*k\*Pi)\*E^(I\*(e + f\*x))], x], x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Log[1 + E^(I\*k\*Pi)\*E^(I\*(e + f\*x))], x], x) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

#### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned} \int \coth^{-1}(\tan(a + bx)) dx &= x \coth^{-1}(\tan(a + bx)) - b \int x \sec(2a + 2bx) dx \\ &= x \coth^{-1}(\tan(a + bx)) + ix \tan^{-1}(e^{2i(a+bx)}) + \frac{1}{2} \int \log(1 - ie^{i(2a+2bx)}) dx - \frac{1}{2} \int \log(1 + ie^{i(2a+2bx)}) dx \\ &= x \coth^{-1}(\tan(a + bx)) + ix \tan^{-1}(e^{2i(a+bx)}) - \frac{i \operatorname{Subst}\left(\int \frac{\log(1-ix)}{x} dx, x, e^{i(2a+2bx)}\right)}{4b} + \frac{i \operatorname{Subst}\left(\int \frac{\log(1+ix)}{x} dx, x, e^{i(2a+2bx)}\right)}{4b} \\ &= x \coth^{-1}(\tan(a + bx)) + ix \tan^{-1}(e^{2i(a+bx)}) - \frac{i \operatorname{Li}_2(-ie^{2i(a+bx)})}{4b} + \frac{i \operatorname{Li}_2(ie^{2i(a+bx)})}{4b} \end{aligned}$$

**Mathematica [A]** time = 0.0230426, size = 78, normalized size = 0.99

$$\frac{-i \operatorname{PolyLog}\left(2, -ie^{2i(a+bx)}\right) + i \operatorname{PolyLog}\left(2, ie^{2i(a+bx)}\right) + 4bx \left(\coth^{-1}(\tan(a + bx)) + i \tan^{-1}(e^{2i(a+bx)})\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tan[a + b*x]], x]
```

```
[Out] (4*b*x*(ArcCoth[Tan[a + b*x]] + I*ArcTan[E^((2*I)*(a + b*x))]) - I*PolyLog[
2, (-I)*E^((2*I)*(a + b*x))] + I*PolyLog[2, I*E^((2*I)*(a + b*x))])/(4*b)
```

**Maple [B]** time = 0.158, size = 178, normalized size = 2.3

$$\frac{\arctan(\tan(bx + a)) \operatorname{arccoth}(\tan(bx + a))}{b} + \frac{\arctan(\tan(bx + a))}{2b} \ln\left(1 + \frac{i(1 + i \tan(bx + a))^2}{1 + (\tan(bx + a))^2}\right) - \frac{i}{b} \operatorname{polylog}\left(2, \frac{-i}{1 + (\tan(bx + a))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.



$$\begin{aligned}
& b*x + a)^2 - 2*\tan(b*x + a) + I + 1)/(\tan(b*x + a)^2 + 1)) + 2*a*\log(((I - \\
& 1)*\tan(b*x + a)^2 + 2*I*\tan(b*x + a) + I + 1)/(\tan(b*x + a)^2 + 1)) - 2*a*1 \\
& \log(((I - 1)*\tan(b*x + a)^2 - 2*I*\tan(b*x + a) + I + 1)/(\tan(b*x + a)^2 + 1) \\
& ) + I*\operatorname{dilog}(-((I + 1)*\tan(b*x + a)^2 + 2*\tan(b*x + a) - I + 1)/(\tan(b*x + a \\
& )^2 + 1) + 1) + I*\operatorname{dilog}(-((I + 1)*\tan(b*x + a)^2 - 2*\tan(b*x + a) - I + 1)/ \\
& (\tan(b*x + a)^2 + 1) + 1) - I*\operatorname{dilog}(-(-(I - 1)*\tan(b*x + a)^2 + 2*\tan(b*x + \\
& a) + I + 1)/(\tan(b*x + a)^2 + 1) + 1) - I*\operatorname{dilog}(-(-(I - 1)*\tan(b*x + a)^2 \\
& - 2*\tan(b*x + a) + I + 1)/(\tan(b*x + a)^2 + 1) + 1))/b
\end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acoth}(\tan(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(tan(b\*x+a)),x)

[Out] Integral(acoth(tan(a + b\*x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(\tan(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tan(b\*x+a)),x, algorithm="giac")

[Out] integrate(arccoth(tan(b\*x + a)), x)



$$3.235 \quad \int \frac{\coth^{-1}(\tan(a+bx))}{e+fx} dx$$

Optimal. Leaf size=17

$$\text{CannotIntegrate}\left(\frac{\coth^{-1}(\tan(a+bx))}{e+fx}, x\right)$$

[Out] CannotIntegrate[ArcCoth[Tan[a + b\*x]]/(e + f\*x), x]

Rubi [A] time = 0.0410895, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx} dx$$

Verification is Not applicable to the result.

[In] Int[ArcCoth[Tan[a + b\*x]]/(e + f\*x), x]

[Out] Defer[Int][ArcCoth[Tan[a + b\*x]]/(e + f\*x), x]

Rubi steps

$$\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx} dx = \int \frac{\coth^{-1}(\tan(a+bx))}{e+fx} dx$$

Mathematica [A] time = 5.06394, size = 0, normalized size = 0.

$$\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcCoth[Tan[a + b\*x]]/(e + f\*x), x]

[Out] Integrate[ArcCoth[Tan[a + b\*x]]/(e + f\*x), x]

**Maple [A]** time = 1.157, size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(\tan(bx + a))}{fx + e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tan(b\*x+a))/(f\*x+e),x)

[Out] int(arccoth(tan(b\*x+a))/(f\*x+e),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(\tan(bx + a))}{fx + e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tan(b\*x+a))/(f\*x+e),x, algorithm="maxima")

[Out] integrate(arccoth(tan(b\*x + a))/(f\*x + e), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arccoth}(\tan(bx + a))}{fx + e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tan(b\*x+a))/(f\*x+e),x, algorithm="fricas")

[Out] integral(arccoth(tan(b\*x + a))/(f\*x + e), x)

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}(\tan(a + bx))}{e + fx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(tan(b\*x+a))/(f\*x+e), x)

[Out] Integral(acoth(tan(a + b\*x))/(e + f\*x), x)

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(\tan(bx + a))}{fx + e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tan(b\*x+a))/(f\*x+e), x, algorithm="giac")

[Out] integrate(arccoth(tan(b\*x + a))/(f\*x + e), x)

### 3.236 $\int x^2 \coth^{-1}(c + d \tan(a + bx)) dx$

**Optimal.** Leaf size=395

$$\frac{x \operatorname{PolyLog}\left(3, -\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b^2} - \frac{x \operatorname{PolyLog}\left(3, -\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b^2} + \frac{i \operatorname{PolyLog}\left(4, -\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{8b^3} - \frac{i \operatorname{PolyLog}\left(4, -\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{8b^3}$$

```
[Out] (x^3*ArcCoth[c + d*Tan[a + b*x]])/3 + (x^3*Log[1 + ((1 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d)])/6 - (x^3*Log[1 + ((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d)])/6 - ((I/4)*x^2*PolyLog[2, -(((1 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/b + ((I/4)*x^2*PolyLog[2, -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/b + (x*PolyLog[3, -(((1 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/(4*b^2) - (x*PolyLog[3, -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/(4*b^2) + ((I/8)*PolyLog[4, -(((1 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/b^3 - ((I/8)*PolyLog[4, -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/b^3
```

---

**Rubi [A]** time = 0.500079, antiderivative size = 395, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {6268, 2190, 2531, 6609, 2282, 6589}

$$\frac{x \operatorname{PolyLog}\left(3, -\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b^2} - \frac{x \operatorname{PolyLog}\left(3, -\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b^2} + \frac{i \operatorname{PolyLog}\left(4, -\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{8b^3} - \frac{i \operatorname{PolyLog}\left(4, -\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[c + d*Tan[a + b*x]], x]
```

```
[Out] (x^3*ArcCoth[c + d*Tan[a + b*x]])/3 + (x^3*Log[1 + ((1 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d)])/6 - (x^3*Log[1 + ((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d)])/6 - ((I/4)*x^2*PolyLog[2, -(((1 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/b + ((I/4)*x^2*PolyLog[2, -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/b + (x*PolyLog[3, -(((1 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/(4*b^2) - (x*PolyLog[3, -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/(4*b^2) + ((I/8)*PolyLog[4, -(((1 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/b^3 - ((I/8)*PolyLog[4, -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/b^3
```

Rule 6268

```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Tan[a + b*x]])/(f*(m + 1)), x] + (-Dist[(I*b*(1 + c - I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*I*a + 2*I*b*x))/(1 + c + I*d + (1 + c - I*d)*E^(2*I*a + 2*I*b*x)), x] + Dist[(I*b*(1 - c + I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*I*a + 2*I*b*x))/(1 - c - I*d + (1 - c + I*d)*E^(2*I*a + 2*I*b*x)), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, 1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.), x_Symbol] := Simp[(((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a)]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x))))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_.)*((a_.)*(v_)^(n_.))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))* (F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
```

```

ybol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]

```

### Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(c + d \tan(a + bx)) dx &= \frac{1}{3} x^3 \coth^{-1}(c + d \tan(a + bx)) + \frac{1}{3} (b(i(1-c) - d)) \int \frac{e^{2ia+2ibx} x^3}{1-c-id + (1-c+id)e^{2ia+2ibx}} dx \\
&= \frac{1}{3} x^3 \coth^{-1}(c + d \tan(a + bx)) + \frac{1}{6} x^3 \log \left( 1 + \frac{(1-c+id)e^{2ia+2ibx}}{1-c-id} \right) - \frac{1}{6} x^3 \log \left( 1 + \frac{(1-c-id)e^{2ia+2ibx}}{1-c+id} \right) \\
&= \frac{1}{3} x^3 \coth^{-1}(c + d \tan(a + bx)) + \frac{1}{6} x^3 \log \left( 1 + \frac{(1-c+id)e^{2ia+2ibx}}{1-c-id} \right) - \frac{1}{6} x^3 \log \left( 1 + \frac{(1-c-id)e^{2ia+2ibx}}{1-c+id} \right) \\
&= \frac{1}{3} x^3 \coth^{-1}(c + d \tan(a + bx)) + \frac{1}{6} x^3 \log \left( 1 + \frac{(1-c+id)e^{2ia+2ibx}}{1-c-id} \right) - \frac{1}{6} x^3 \log \left( 1 + \frac{(1-c-id)e^{2ia+2ibx}}{1-c+id} \right) \\
&= \frac{1}{3} x^3 \coth^{-1}(c + d \tan(a + bx)) + \frac{1}{6} x^3 \log \left( 1 + \frac{(1-c+id)e^{2ia+2ibx}}{1-c-id} \right) - \frac{1}{6} x^3 \log \left( 1 + \frac{(1-c-id)e^{2ia+2ibx}}{1-c+id} \right) \\
&= \frac{1}{3} x^3 \coth^{-1}(c + d \tan(a + bx)) + \frac{1}{6} x^3 \log \left( 1 + \frac{(1-c+id)e^{2ia+2ibx}}{1-c-id} \right) - \frac{1}{6} x^3 \log \left( 1 + \frac{(1-c-id)e^{2ia+2ibx}}{1-c+id} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.315059, size = 346, normalized size = 0.88

$$\frac{1}{3} x^3 \coth^{-1}(d \tan(a + bx) + c) + \frac{-6ib^2 x^2 \text{PolyLog}\left(2, \frac{(-c+id+1)e^{2i(a+bx)}}{c+id-1}\right) + 6ib^2 x^2 \text{PolyLog}\left(2, -\frac{(c-id+1)e^{2i(a+bx)}}{c+id+1}\right) + 6bx \text{PolyLog}\left(1, \frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) - 6bx \text{PolyLog}\left(1, \frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)}{1}$$

Antiderivative was successfully verified.

```

[In] Integrate[x^2*ArcCoth[c + d*Tan[a + b*x]], x]

```

```

[Out] (x^3*ArcCoth[c + d*Tan[a + b*x]])/3 + (4*b^3*x^3*Log[1 + ((-1 + c - I*d)*E^((2*I)*(a + b*x)))/(-1 + c + I*d)] - 4*b^3*x^3*Log[1 + ((1 + c - I*d)*E^((2*I)*(a + b*x)))/(1 + c + I*d)] - (6*I)*b^2*x^2*PolyLog[2, ((1 - c + I*d)*E^((2*I)*(a + b*x)))/(-1 + c + I*d)] + (6*I)*b^2*x^2*PolyLog[2, -(((1 + c - I*d)*E^((2*I)*(a + b*x)))/(1 + c + I*d))] + 6*b*x*PolyLog[3, ((1 - c + I*d)*E^((2*I)*(a + b*x)))/(-1 + c + I*d)] - 6*b*x*PolyLog[3, -(((1 + c - I*d)*E^((2*I)*(a + b*x)))/(1 + c + I*d))] + (3*I)*PolyLog[4, ((1 - c + I*d)*E^((2*I)*(a + b*x)))/(-1 + c + I*d)] - (3*I)*PolyLog[4, -(((1 + c - I*d)*E^((2*I)*(a + b*x)))/(-1 + c + I*d))]

```

$*(a + b*x)))/(1 + c + I*d)))]/(24*b^3)$

**Maple [C]** time = 6.798, size = 6820, normalized size = 17.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*arccoth(c+d*tan(b*x+a)),x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccoth(c+d*tan(b*x+a)),x, algorithm="maxima")`

[Out] 
$$\frac{1}{12}x^3 \log((c^2 + d^2 + 2c + 1)\cos(2bx + 2a)^2 + 4(c + 1)d\sin(2bx + 2a) + (c^2 + d^2 + 2c + 1)\sin(2bx + 2a)^2 + c^2 + d^2 + 2(c^2 - d^2 + 2c + 1)\cos(2bx + 2a) + 2c + 1) - \frac{1}{12}x^3 \log((c^2 + d^2 - 2c + 1)\cos(2bx + 2a)^2 + 4(c - 1)d\sin(2bx + 2a) + (c^2 + d^2 - 2c + 1)\sin(2bx + 2a)^2 + c^2 + d^2 + 2(c^2 - d^2 - 2c + 1)\cos(2bx + 2a) - 2c + 1) - 4bd \int (-\frac{1}{3}(2(c^2 + d^2 - 1)x^3 \cos(2bx + 2a)^2 + 2cdx^3 \sin(2bx + 2a) + 2(c^2 + d^2 - 1)x^3 \sin(2bx + 2a)^2 + (c^2 - d^2 - 1)x^3 \cos(2bx + 2a) - (2cdx^3 \sin(2bx + 2a) - (c^2 - d^2 - 1)x^3 \cos(2bx + 2a))\cos(4bx + 4a) + (2cdx^3 \cos(2bx + 2a) + (c^2 - d^2 - 1)x^3 \sin(2bx + 2a))\sin(4bx + 4a)) / (c^4 + d^4 + 2(c^2 + 1)d^2 + (c^4 + d^4 + 2(c^2 + 1)d^2 - 2c^2 + 1)\cos(4bx + 4a)^2 + 4(c^4 + d^4 + 2(c^2 - 1)d^2 - 2c^2 + 1)\cos(2bx + 2a)^2 + (c^4 + d^4 + 2(c^2 + 1)d^2 - 2c^2 + 1)\sin(4bx + 4a)^2 + 4(c^4 + d^4 + 2(c^2 - 1)d^2 - 2c^2 + 1)\sin(2bx + 2a)^2 - 2c^2 + 2(c^4 + d^4 - 2(3c^2 - 1)d^2 - 2c^2 + 2(c^4 - d^4 - 2c^2 + 1)\cos(2bx + 2a) - 4(c*d^3 + (c^3 - c)*d)\sin(2bx + 2a) + 1)\cos(4bx + 4a) + 4(c^4 - d^4 - 2c^2 + 1)\cos(2bx + 2a) - 4(2cd^3 - 2(c^3 - c)*d - 2(c*d^3 + (c^3 - c)*d)\cos(2bx + 2a) - (c^4 - d^4 - 2c^2 + 1)\sin(2bx + 2a))\sin(4bx + 4a) + 8(c*d^3 + (c^3 - c)*d)\sin(2bx + 2a) + 1), x)$$





$$\begin{aligned}
& 4*I*c + 2*I)*\tan(b*x + a) + 4*c + 2)/((c^2 + d^2 + 2*c + 1)*\tan(b*x + a)^2 \\
& + c^2 + d^2 + 2*c + 1)) - 4*(b^3*x^3 + a^3)*\log((( -2*I*(c + 1)*d + 2*d^2)* \\
& \tan(b*x + a)^2 + 2*c^2 + 2*I*(c + 1)*d + (-2*I*c^2 + 4*(c + 1)*d + 2*I*d^2 \\
& - 4*I*c - 2*I)*\tan(b*x + a) + 4*c + 2)/((c^2 + d^2 + 2*c + 1)*\tan(b*x + a)^2 \\
& + c^2 + d^2 + 2*c + 1)) + 4*(b^3*x^3 + a^3)*\log(((2*I*(c - 1)*d + 2*d^2)* \\
& \tan(b*x + a)^2 + 2*c^2 - 2*I*(c - 1)*d + (2*I*c^2 + 4*(c - 1)*d - 2*I*d^2 - \\
& 4*I*c + 2*I)*\tan(b*x + a) - 4*c + 2)/((c^2 + d^2 - 2*c + 1)*\tan(b*x + a)^2 \\
& + c^2 + d^2 - 2*c + 1)) + 4*(b^3*x^3 + a^3)*\log((( -2*I*(c - 1)*d + 2*d^2)* \\
& \tan(b*x + a)^2 + 2*c^2 + 2*I*(c - 1)*d + (-2*I*c^2 + 4*(c - 1)*d + 2*I*d^2 \\
& + 4*I*c - 2*I)*\tan(b*x + a) - 4*c + 2)/((c^2 + d^2 - 2*c + 1)*\tan(b*x + a)^2 \\
& + c^2 + d^2 - 2*c + 1)) + 3*I*\text{polylog}(4, ((c^2 + 2*I*(c + 1)*d - d^2 + 2* \\
& c + 1)*\tan(b*x + a)^2 - c^2 - 2*I*(c + 1)*d + d^2 + (2*I*c^2 - 4*(c + 1)*d \\
& - 2*I*d^2 + 4*I*c + 2*I)*\tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*\tan \\
& (b*x + a)^2 + c^2 + d^2 + 2*c + 1)) - 3*I*\text{polylog}(4, ((c^2 - 2*I*(c + 1)*d \\
& - d^2 + 2*c + 1)*\tan(b*x + a)^2 - c^2 + 2*I*(c + 1)*d + d^2 + (-2*I*c^2 - 4 \\
& *(c + 1)*d + 2*I*d^2 - 4*I*c - 2*I)*\tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2 \\
& *c + 1)*\tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) - 3*I*\text{polylog}(4, ((c^2 + 2*I \\
& *(c - 1)*d - d^2 - 2*c + 1)*\tan(b*x + a)^2 - c^2 - 2*I*(c - 1)*d + d^2 + (2 \\
& *I*c^2 - 4*(c - 1)*d - 2*I*d^2 - 4*I*c + 2*I)*\tan(b*x + a) + 2*c - 1)/((c^2 \\
& + d^2 - 2*c + 1)*\tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1)) + 3*I*\text{polylog}(4, ( \\
& (c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*\tan(b*x + a)^2 - c^2 + 2*I*(c - 1)*d \\
& + d^2 + (-2*I*c^2 - 4*(c - 1)*d + 2*I*d^2 + 4*I*c - 2*I)*\tan(b*x + a) + 2*c \\
& - 1)/((c^2 + d^2 - 2*c + 1)*\tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1)))/b^3
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*acoth(c+d\*tan(b\*x+a)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arccoth}(d \tan(bx + a) + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(c+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(d*tan(b*x + a) + c), x)
```

### 3.237 $\int x \coth^{-1}(c + d \tan(a + bx)) dx$

**Optimal.** Leaf size=295

$$\frac{\text{PolyLog}\left(3, -\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{8b^2} - \frac{\text{PolyLog}\left(3, -\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{8b^2} - \frac{ix \text{PolyLog}\left(2, -\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b} + \frac{ix \text{PolyLog}\left(2, -\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b}$$

[Out]  $(x^2 \text{ArcCoth}[c + d \text{Tan}[a + b*x]])/2 + (x^2 \text{Log}[1 + ((1 - c + I*d)*E^{((2*I)*a + (2*I)*b*x)})/(1 - c - I*d)])/4 - (x^2 \text{Log}[1 + ((1 + c - I*d)*E^{((2*I)*a + (2*I)*b*x)})/(1 + c + I*d)])/4 - ((I/4)*x*\text{PolyLog}[2, -(((1 - c + I*d)*E^{((2*I)*a + (2*I)*b*x)})/(1 - c - I*d))])/b + ((I/4)*x*\text{PolyLog}[2, -(((1 + c - I*d)*E^{((2*I)*a + (2*I)*b*x)})/(1 + c + I*d))])/b + \text{PolyLog}[3, -(((1 - c + I*d)*E^{((2*I)*a + (2*I)*b*x)})/(1 - c - I*d))]/(8*b^2) - \text{PolyLog}[3, -(((1 + c - I*d)*E^{((2*I)*a + (2*I)*b*x)})/(1 + c + I*d))]/(8*b^2)$

**Rubi [A]** time = 0.396999, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {6268, 2190, 2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3, -\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{8b^2} - \frac{\text{PolyLog}\left(3, -\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{8b^2} - \frac{ix \text{PolyLog}\left(2, -\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b} + \frac{ix \text{PolyLog}\left(2, -\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x*\text{ArcCoth}[c + d*\text{Tan}[a + b*x]], x]$

[Out]  $(x^2 \text{ArcCoth}[c + d \text{Tan}[a + b*x]])/2 + (x^2 \text{Log}[1 + ((1 - c + I*d)*E^{((2*I)*a + (2*I)*b*x)})/(1 - c - I*d)])/4 - (x^2 \text{Log}[1 + ((1 + c - I*d)*E^{((2*I)*a + (2*I)*b*x)})/(1 + c + I*d)])/4 - ((I/4)*x*\text{PolyLog}[2, -(((1 - c + I*d)*E^{((2*I)*a + (2*I)*b*x)})/(1 - c - I*d))])/b + ((I/4)*x*\text{PolyLog}[2, -(((1 + c - I*d)*E^{((2*I)*a + (2*I)*b*x)})/(1 + c + I*d))])/b + \text{PolyLog}[3, -(((1 - c + I*d)*E^{((2*I)*a + (2*I)*b*x)})/(1 - c - I*d))]/(8*b^2) - \text{PolyLog}[3, -(((1 + c - I*d)*E^{((2*I)*a + (2*I)*b*x)})/(1 + c + I*d))]/(8*b^2)$

#### Rule 6268

$\text{Int}[\text{ArcCoth}[(c_.) + (d_.)*\text{Tan}[(a_.) + (b_.)*(x_.)]]*((e_.) + (f_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[(e + f*x)^{(m+1)}*\text{ArcCoth}[c + d*\text{Tan}[a + b*x]]/(f*(m+1)), x] + (-\text{Dist}[(I*b*(1 + c - I*d))/(f*(m+1)), \text{Int}[(e + f*x)^{(m+1)}*E^{(2*I*a + 2*I*b*x)})/(1 + c + I*d + (1 + c - I*d)*E^{(2*I*a + 2*I*b*x)}], x]$

```
, x] + Dist[(I*b*(1 - c + I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*I*
a + 2*I*b*x))/(1 - c - I*d + (1 - c + I*d)*E^(2*I*a + 2*I*b*x)), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, 1]
```

### Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int x \coth^{-1}(c + d \tan(a + bx)) dx &= \frac{1}{2}x^2 \coth^{-1}(c + d \tan(a + bx)) + \frac{1}{2}(b(i(1 - c) - d)) \int \frac{e^{2ia+2ibx}x^2}{1 - c - id + (1 - c + id)e^{2ia+2ibx}} \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \tan(a + bx)) + \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id}\right) - \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id}\right) \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \tan(a + bx)) + \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id}\right) - \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id}\right) \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \tan(a + bx)) + \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id}\right) - \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id}\right) \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \tan(a + bx)) + \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id}\right) - \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.140406, size = 257, normalized size = 0.87

$$\frac{1}{2}x^2 \coth^{-1}(d \tan(a + bx) + c) + \frac{-2ibx \operatorname{PolyLog}\left(2, \frac{(-c+id+1)e^{2i(a+bx)}}{c+id-1}\right) + 2ibx \operatorname{PolyLog}\left(2, -\frac{(c-id+1)e^{2i(a+bx)}}{c+id+1}\right) + \operatorname{PolyLog}\left(3, \frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) - \operatorname{PolyLog}\left(3, -\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)}{(8b^2)}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[c + d\*Tan[a + b\*x]],x]

[Out] (x^2\*ArcCoth[c + d\*Tan[a + b\*x]])/2 + (2\*b^2\*x^2\*Log[1 + ((-1 + c - I\*d)\*E^((2\*I)\*(a + b\*x)))/(-1 + c + I\*d)] - 2\*b^2\*x^2\*Log[1 + ((1 + c - I\*d)\*E^((2\*I)\*(a + b\*x)))/(1 + c + I\*d)] - (2\*I)\*b\*x\*PolyLog[2, ((1 - c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(-1 + c + I\*d)] + (2\*I)\*b\*x\*PolyLog[2, -(((1 + c - I\*d)\*E^((2\*I)\*(a + b\*x)))/(1 + c + I\*d))] + PolyLog[3, ((1 - c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(-1 + c + I\*d)] - PolyLog[3, -(((1 + c - I\*d)\*E^((2\*I)\*(a + b\*x)))/(1 + c + I\*d))]/(8\*b^2)

**Maple [C]** time = 7.796, size = 6482, normalized size = 22.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(c+d\*tan(b\*x+a)),x)

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(c+d\*tan(b\*x+a)),x, algorithm="maxima")

[Out] 
$$-2*b*d*\integrate(-(2*(c^2 + d^2 - 1)*x^2*\cos(2*b*x + 2*a)^2 + 2*c*d*x^2*\sin(2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^2*\sin(2*b*x + 2*a)^2 + (c^2 - d^2 - 1)*x^2*\cos(2*b*x + 2*a) - (2*c*d*x^2*\sin(2*b*x + 2*a) - (c^2 - d^2 - 1)*x^2*\cos(2*b*x + 2*a))*\cos(4*b*x + 4*a) + (2*c*d*x^2*\cos(2*b*x + 2*a) + (c^2 - d^2 - 1)*x^2*\sin(2*b*x + 2*a))*\sin(4*b*x + 4*a))/(c^4 + d^4 + 2*(c^2 + 1)*d^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*\cos(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*\cos(2*b*x + 2*a)^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*\sin(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*\sin(2*b*x + 2*a)^2 - 2*c^2 + 2*(c^4 + d^4 - 2*(3*c^2 - 1)*d^2 - 2*c^2 + 2*(c^4 - d^4 - 2*c^2 + 1)*\cos(2*b*x + 2*a) - 4*(c*d^3 + (c^3 - c)*d)*\sin(2*b*x + 2*a) + 1)*\cos(4*b*x + 4*a) + 4*(c^4 - d^4 - 2*c^2 + 1)*\cos(2*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3 - c)*d - 2*(c*d^3 + (c^3 - c)*d)*\cos(2*b*x + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*\sin(2*b*x + 2*a))*\sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 - c)*d)*\sin(2*b*x + 2*a) + 1), x) + 1/8*x^2*\log((c^2 + d^2 + 2*c + 1)*\cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*\sin(2*b*x + 2*a) + (c^2 + d^2 + 2*c + 1)*\sin(2*b*x + 2*a)^2 + c^2 + d^2 + 2*(c^2 - d^2 + 2*c + 1)*\cos(2*b*x + 2*a) + 2*c + 1) - 1/8*x^2*\log((c^2 + d^2 - 2*c + 1)*\cos(2*b*x + 2*a)^2 + 4*(c - 1)*d*\sin(2*b*x + 2*a) + (c^2 + d^2 - 2*c + 1)*\sin(2*b*x + 2*a)^2 + c^2 + d^2 + 2*(c^2 - d^2 - 2*c + 1)*\cos(2*b*x + 2*a) - 2*c + 1)$$

**Fricas [C]** time = 2.49072, size = 4545, normalized size = 15.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(c+d\*tan(b\*x+a)),x, algorithm="fricas")

[Out] 
$$1/16*(4*b^2*x^2*\log((d*\tan(b*x + a) + c + 1)/(d*\tan(b*x + a) + c - 1)) + 2*I*b*x*dilog(-((2*I*(c + 1)*d + 2*d^2)*\tan(b*x + a)^2 + 2*c^2 - 2*I*(c + 1)*$$

$$\begin{aligned}
& d + (2Ic^2 + 4(c+1)d - 2Id^2 + 4Ic + 2I)\tan(bx+a) + 4c + 2 \\
& /((c^2 + d^2 + 2c + 1)\tan(bx+a)^2 + c^2 + d^2 + 2c + 1) + 1) - 2Ibx \\
& x \operatorname{dilog}(-((-2I(c+1)d + 2d^2)\tan(bx+a)^2 + 2c^2 + 2I(c+1)d + \\
& (-2Ic^2 + 4(c+1)d + 2Id^2 - 4Ic - 2I)\tan(bx+a) + 4c + 2)/((c^2 + d^2 + 2c + 1)\tan(bx+a)^2 + c^2 + d^2 + 2c + 1) + 1) - 2Ibx \\
& x \operatorname{dilog}(-((2I(c-1)d + 2d^2)\tan(bx+a)^2 + 2c^2 - 2I(c-1)d + (2 \\
& Ic^2 + 4(c-1)d - 2Id^2 - 4Ic + 2I)\tan(bx+a) - 4c + 2)/((c^2 + d^2 - 2c + 1)\tan(bx+a)^2 + c^2 + d^2 - 2c + 1) + 1) + 2Ibx \\
& x \operatorname{dilog}(-((-2I(c-1)d + 2d^2)\tan(bx+a)^2 + 2c^2 + 2I(c-1)d + (-2I \\
& c^2 + 4(c-1)d + 2Id^2 + 4Ic - 2I)\tan(bx+a) - 4c + 2)/((c^2 + d^2 - 2c + 1)\tan(bx+a)^2 + c^2 + d^2 - 2c + 1) + 1) - 2a^2 \log(((I \\
& (c+1)d + d^2)\tan(bx+a)^2 - c^2 + I(c+1)d + (Ic^2 + Id^2 + 2Ic \\
& + I)\tan(bx+a) - 2c - 1)/(\tan(bx+a)^2 + 1)) - 2a^2 \log(((I(c+1) \\
& )d - d^2)\tan(bx+a)^2 + c^2 + I(c+1)d + (Ic^2 + Id^2 + 2Ic + I) \\
& \tan(bx+a) + 2c + 1)/(\tan(bx+a)^2 + 1)) + 2a^2 \log(((I(c-1)d + d^2)\tan(bx+a)^2 - c^2 + I(c-1)d + (Ic^2 + Id^2 - 2Ic + I)\tan(b \\
& x+a) + 2c - 1)/(\tan(bx+a)^2 + 1)) + 2a^2 \log(((I(c-1)d - d^2)\tan(bx+a)^2 + c^2 + I(c-1)d + (Ic^2 + Id^2 - 2Ic + I)\tan(bx+a) \\
& ) - 2c + 1)/(\tan(bx+a)^2 + 1)) - 2(b^2x^2 - a^2) \log(((2I(c+1)d \\
& + 2d^2)\tan(bx+a)^2 + 2c^2 - 2I(c+1)d + (2Ic^2 + 4(c+1)d - \\
& 2Id^2 + 4Ic + 2I)\tan(bx+a) + 4c + 2)/((c^2 + d^2 + 2c + 1)\tan(b \\
& x+a)^2 + c^2 + d^2 + 2c + 1)) - 2(b^2x^2 - a^2) \log((-2I(c+1)d \\
& + 2d^2)\tan(bx+a)^2 + 2c^2 + 2I(c+1)d + (-2Ic^2 + 4(c+1)d + \\
& 2Id^2 - 4Ic - 2I)\tan(bx+a) + 4c + 2)/((c^2 + d^2 + 2c + 1)\tan( \\
& bx+a)^2 + c^2 + d^2 + 2c + 1)) + 2(b^2x^2 - a^2) \log(((2I(c-1)d \\
& + 2d^2)\tan(bx+a)^2 + 2c^2 - 2I(c-1)d + (2Ic^2 + 4(c-1)d - \\
& 2Id^2 - 4Ic + 2I)\tan(bx+a) - 4c + 2)/((c^2 + d^2 - 2c + 1)\tan(b \\
& x+a)^2 + c^2 + d^2 - 2c + 1)) + 2(b^2x^2 - a^2) \log((-2I(c-1)d \\
& + 2d^2)\tan(bx+a)^2 + 2c^2 + 2I(c-1)d + (-2Ic^2 + 4(c-1)d + \\
& 2Id^2 + 4Ic - 2I)\tan(bx+a) - 4c + 2)/((c^2 + d^2 - 2c + 1)\tan( \\
& bx+a)^2 + c^2 + d^2 - 2c + 1)) - \operatorname{polylog}(3, ((c^2 + 2I(c+1)d - d^2 \\
& + 2c + 1)\tan(bx+a)^2 - c^2 - 2I(c+1)d + d^2 + (2Ic^2 - 4(c+ \\
& 1)d - 2Id^2 + 4Ic + 2I)\tan(bx+a) - 2c - 1)/((c^2 + d^2 + 2c + 1) \\
& \tan(bx+a)^2 + c^2 + d^2 + 2c + 1)) - \operatorname{polylog}(3, ((c^2 - 2I(c+1)d \\
& - d^2 + 2c + 1)\tan(bx+a)^2 - c^2 + 2I(c+1)d + d^2 + (-2Ic^2 - \\
& 4(c+1)d + 2Id^2 - 4Ic - 2I)\tan(bx+a) - 2c - 1)/((c^2 + d^2 + 2c + 1) \\
& \tan(bx+a)^2 + c^2 + d^2 + 2c + 1)) + \operatorname{polylog}(3, ((c^2 + 2I(c \\
& - 1)d - d^2 - 2c + 1)\tan(bx+a)^2 - c^2 - 2I(c-1)d + d^2 + (2Ic \\
& c^2 - 4(c-1)d - 2Id^2 - 4Ic + 2I)\tan(bx+a) + 2c - 1)/((c^2 + \\
& d^2 - 2c + 1)\tan(bx+a)^2 + c^2 + d^2 - 2c + 1)) + \operatorname{polylog}(3, ((c^2 - \\
& 2I(c-1)d - d^2 - 2c + 1)\tan(bx+a)^2 - c^2 + 2I(c-1)d + d^2 + \\
& (-2Ic^2 - 4(c-1)d + 2Id^2 + 4Ic - 2I)\tan(bx+a) + 2c - 1)/((c^2 + d^2 - 2c + 1)\tan(bx+a)^2 + c^2 + d^2 - 2c + 1))) / b^2
\end{aligned}$$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(c+d\*tan(b\*x+a)),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arccoth}(d \tan(bx + a) + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(c+d\*tan(b\*x+a)),x, algorithm="giac")

[Out] integrate(x\*arccoth(d\*tan(b\*x + a) + c), x)



### 3.238 $\int \coth^{-1}(c + d \tan(a + bx)) dx$

**Optimal.** Leaf size=194

$$-\frac{i \operatorname{PolyLog}\left(2, -\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b} + \frac{i \operatorname{PolyLog}\left(2, -\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b} + \frac{1}{2}x \log\left(1 + \frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right) - \frac{1}{2}x \log\left(1 + \frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)$$

```
[Out] x*ArcCoth[c + d*Tan[a + b*x]] + (x*Log[1 + ((1 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d)])/2 - (x*Log[1 + ((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d)])/2 - ((I/4)*PolyLog[2, -(((1 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/b + ((I/4)*PolyLog[2, -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/b
```

**Rubi [A]** time = 0.250733, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {6260, 2190, 2279, 2391}

$$-\frac{i \operatorname{PolyLog}\left(2, -\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b} + \frac{i \operatorname{PolyLog}\left(2, -\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b} + \frac{1}{2}x \log\left(1 + \frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right) - \frac{1}{2}x \log\left(1 + \frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[c + d*Tan[a + b*x]], x]
```

```
[Out] x*ArcCoth[c + d*Tan[a + b*x]] + (x*Log[1 + ((1 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d)])/2 - (x*Log[1 + ((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d)])/2 - ((I/4)*PolyLog[2, -(((1 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/b + ((I/4)*PolyLog[2, -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/b
```

#### Rule 6260

```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] := Simp[x*ArcCoth[c + d*Tan[a + b*x]], x] + (-Dist[I*b*(1 + c - I*d), Int[(x*E^(2*I*a + 2*I*b*x))/(1 + c + I*d + (1 + c - I*d)*E^(2*I*a + 2*I*b*x)), x], x] + Dist[I*b*(1 - c + I*d), Int[(x*E^(2*I*a + 2*I*b*x))/(1 - c - I*d + (1 - c + I*d)*E^(2*I*a + 2*I*b*x)), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c + I*d)^2, 1]
```

#### Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned} \int \coth^{-1}(c + d \tan(a + bx)) dx &= x \coth^{-1}(c + d \tan(a + bx)) + (b(i(1 - c) - d)) \int \frac{e^{2ia+2ibx} x}{1 - c - id + (1 - c + id)e^{2ia+2ibx}} dx - \\ &= x \coth^{-1}(c + d \tan(a + bx)) + \frac{1}{2} x \log \left( 1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id} \right) - \frac{1}{2} x \log \left( 1 + \frac{(1 + c - id)e^{2ia+2ibx}}{1 - c - id} \right) \\ &= x \coth^{-1}(c + d \tan(a + bx)) + \frac{1}{2} x \log \left( 1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id} \right) - \frac{1}{2} x \log \left( 1 + \frac{(1 + c - id)e^{2ia+2ibx}}{1 - c - id} \right) \\ &= x \coth^{-1}(c + d \tan(a + bx)) + \frac{1}{2} x \log \left( 1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id} \right) - \frac{1}{2} x \log \left( 1 + \frac{(1 + c - id)e^{2ia+2ibx}}{1 - c - id} \right) \end{aligned}$$

**Mathematica [B]** time = 13.2235, size = 4654, normalized size = 23.99

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[c + d*Tan[a + b*x]], x]
```

```
[Out] x*ArcCoth[c + d*Tan[a + b*x]] + (d*(-(a*Log[-(Sec[(a + b*x)/2]^2*((-1 + c)*
Cos[a + b*x] + d*Sin[a + b*x])))] + a*Log[Sec[(a + b*x)/2]^2*(Cos[a + b*x]
```

$$\begin{aligned}
& + c \cos[a + b*x] + d \sin[a + b*x] + (a + b*x) \log[-d + \sqrt{1 - 2*c + c^2 + d^2}] / (-1 + c) + \tan[(a + b*x)/2] + I \log[(-1 + c) * (1 + I \tan[(a + b*x)/2])] / (-1 + c + I*d - I \sqrt{1 - 2*c + c^2 + d^2}) * \log[-d + \sqrt{1 - 2*c + c^2 + d^2}] / (-1 + c) + \tan[(a + b*x)/2] - I \log[-((1 + c) * (I + \tan[(a + b*x)/2]))] / (I - I*c - d + \sqrt{1 - 2*c + c^2 + d^2}) * \log[-d + \sqrt{1 - 2*c + c^2 + d^2}] / (-1 + c) + \tan[(a + b*x)/2] + (a + b*x) \log[(d + \sqrt{1 - 2*c + c^2 + d^2}) / (1 - c) + \tan[(a + b*x)/2]] + I \log[(-1 + c) * (-I + \tan[(a + b*x)/2])] / (I - I*c + d + \sqrt{1 - 2*c + c^2 + d^2}) * \log[(d + \sqrt{1 - 2*c + c^2 + d^2}) / (1 - c) + \tan[(a + b*x)/2]] - I \log[(-1 + c) * (I + \tan[(a + b*x)/2])] / (-I + I*c + d + \sqrt{1 - 2*c + c^2 + d^2}) * \log[(d + \sqrt{1 - 2*c + c^2 + d^2}) / (1 - c) + \tan[(a + b*x)/2]] - (a + b*x) \log[-(d + \sqrt{1 + 2*c + c^2 + d^2}) / (1 + c)] + \tan[(a + b*x)/2] - I \log[(1 + c) * (-I + \tan[(a + b*x)/2])] / (-I - I*c + d + \sqrt{1 + 2*c + c^2 + d^2}) * \log[-(d + \sqrt{1 + 2*c + c^2 + d^2}) / (1 + c)] + \tan[(a + b*x)/2] + I \log[(1 + c) * (I + \tan[(a + b*x)/2])] / (I + I*c + d + \sqrt{1 + 2*c + c^2 + d^2}) * \log[-(d + \sqrt{1 + 2*c + c^2 + d^2}) / (1 + c)] + \tan[(a + b*x)/2] - (a + b*x) \log[-(d + \sqrt{1 + 2*c + c^2 + d^2}) / (1 + c)] + \tan[(a + b*x)/2] + I \log[(1 + c) * (1 - I \tan[(a + b*x)/2])] / (1 + c - I*d + I \sqrt{1 + 2*c + c^2 + d^2}) * \log[-d + \sqrt{1 + 2*c + c^2 + d^2} + (1 + c) * \tan[(a + b*x)/2]] / (1 + c) - I \log[(1 + c) * (1 + I \tan[(a + b*x)/2])] / (1 + c + I*d - I \sqrt{1 + 2*c + c^2 + d^2}) * \log[-d + \sqrt{1 + 2*c + c^2 + d^2} + (1 + c) * \tan[(a + b*x)/2]] / (1 + c) + I \operatorname{PolyLog}[2, (d + \sqrt{1 - 2*c + c^2 + d^2}) - (-1 + c) * \tan[(a + b*x)/2]] / (I - I*c + d + \sqrt{1 - 2*c + c^2 + d^2}) - I \operatorname{PolyLog}[2, (d + \sqrt{1 - 2*c + c^2 + d^2}) - (-1 + c) * \tan[(a + b*x)/2]] / (-I + I*c + d + \sqrt{1 - 2*c + c^2 + d^2}) - I \operatorname{PolyLog}[2, (-d + \sqrt{1 - 2*c + c^2 + d^2}) + (-1 + c) * \tan[(a + b*x)/2]] / (I - I*c - d + \sqrt{1 - 2*c + c^2 + d^2}) + I \operatorname{PolyLog}[2, (-d + \sqrt{1 - 2*c + c^2 + d^2}) + (-1 + c) * \tan[(a + b*x)/2]] / (-I + I*c - d + \sqrt{1 - 2*c + c^2 + d^2}) - I \operatorname{PolyLog}[2, (d + \sqrt{1 + 2*c + c^2 + d^2}) - (1 + c) * \tan[(a + b*x)/2]] / (-I - I*c + d + \sqrt{1 + 2*c + c^2 + d^2}) + I \operatorname{PolyLog}[2, (d + \sqrt{1 + 2*c + c^2 + d^2}) - (1 + c) * \tan[(a + b*x)/2]] / (I + I*c + d + \sqrt{1 + 2*c + c^2 + d^2}) + I \operatorname{PolyLog}[2, (-d + \sqrt{1 + 2*c + c^2 + d^2}) + (1 + c) * \tan[(a + b*x)/2]] / (-I - I*c - d + \sqrt{1 + 2*c + c^2 + d^2}) - I \operatorname{PolyLog}[2, (-d + \sqrt{1 + 2*c + c^2 + d^2}) + (1 + c) * \tan[(a + b*x)/2]] / (I + I*c - d + \sqrt{1 + 2*c + c^2 + d^2})] * ((-2*a) / (b * (-1 + c^2 + d^2 - \cos[2*(a + b*x)] + c^2 * \cos[2*(a + b*x)] - d^2 * \cos[2*(a + b*x)] + 2*c*d * \sin[2*(a + b*x)])) + (2*(a + b*x)) / (b * (-1 + c^2 + d^2 - \cos[2*(a + b*x)] + c^2 * \cos[2*(a + b*x)] - d^2 * \cos[2*(a + b*x)] + 2*c*d * \sin[2*(a + b*x)])))] / (\log[-d + \sqrt{1 - 2*c + c^2 + d^2}] / (-1 + c) + \tan[(a + b*x)/2] + \log[(d + \sqrt{1 - 2*c + c^2 + d^2}) / (1 - c) + \tan[(a + b*x)/2]] - \log[-(d + \sqrt{1 + 2*c + c^2 + d^2}) / (1 + c)] + \tan[(a + b*x)/2] - \log[-(d + \sqrt{1 + 2*c + c^2 + d^2}) + (1 + c) * \tan[(a + b*x)/2]] / (1 + c)] + (\log[-d + \sqrt{1 + 2*c + c^2 + d^2} + (1 + c) * \tan[(a + b*x)/2]] / (1 + c)) * \operatorname{Sec}[(a + b*x)/2]^2 / (2*(1 - I \tan[(a + b*x)/2])) - (\log[-d + \sqrt{1 - 2*c + c^2 + d^2}] / (-1 + c) + \tan[(a + b*x)/2]) * \operatorname{Sec}[(a + b*x)/2]^2 / (2*(1 + I \tan[(a + b*x)/2])) + (\log[-d + \sqrt{1 + 2*c + c^2 + d^2} + (1 + c) * \tan[(a + b*x)/2]] / (
\end{aligned}$$

$$\begin{aligned}
& 1 + c)] * \text{Sec}[(a + b*x)/2]^2 / (2*(1 + I*\text{Tan}[(a + b*x)/2])) + ((I/2)*\text{Log}[(d + \\
& \text{Sqrt}[1 - 2*c + c^2 + d^2]) / (1 - c) + \text{Tan}[(a + b*x)/2]] * \text{Sec}[(a + b*x)/2]^2 / \\
& (-I + \text{Tan}[(a + b*x)/2]) - ((I/2)*\text{Log}[-((d + \text{Sqrt}[1 + 2*c + c^2 + d^2]) / (1 + \\
& c)) + \text{Tan}[(a + b*x)/2]] * \text{Sec}[(a + b*x)/2]^2 / (-I + \text{Tan}[(a + b*x)/2]) - ((I/ \\
& 2)*\text{Log}[(-d + \text{Sqrt}[1 - 2*c + c^2 + d^2]) / (-1 + c) + \text{Tan}[(a + b*x)/2]] * \text{Sec}[(a \\
& + b*x)/2]^2 / (I + \text{Tan}[(a + b*x)/2]) - ((I/2)*\text{Log}[(d + \text{Sqrt}[1 - 2*c + c^2 + \\
& d^2]) / (1 - c) + \text{Tan}[(a + b*x)/2]] * \text{Sec}[(a + b*x)/2]^2 / (I + \text{Tan}[(a + b*x)/2 \\
& ]) + ((I/2)*\text{Log}[-((d + \text{Sqrt}[1 + 2*c + c^2 + d^2]) / (1 + c)) + \text{Tan}[(a + b*x)/ \\
& 2]] * \text{Sec}[(a + b*x)/2]^2 / (I + \text{Tan}[(a + b*x)/2]) + ((a + b*x)*\text{Sec}[(a + b*x)/2 \\
& ]^2) / (2*((-d + \text{Sqrt}[1 - 2*c + c^2 + d^2]) / (-1 + c) + \text{Tan}[(a + b*x)/2])) + ( \\
& (I/2)*\text{Log}[((-1 + c)*(1 + I*\text{Tan}[(a + b*x)/2])) / (-1 + c + I*d - I*\text{Sqrt}[1 - 2* \\
& c + c^2 + d^2])] * \text{Sec}[(a + b*x)/2]^2 / ((-d + \text{Sqrt}[1 - 2*c + c^2 + d^2]) / (-1 \\
& + c) + \text{Tan}[(a + b*x)/2]) - ((I/2)*\text{Log}[-(((-1 + c)*(I + \text{Tan}[(a + b*x)/2])) / ( \\
& I - I*c - d + \text{Sqrt}[1 - 2*c + c^2 + d^2]))] * \text{Sec}[(a + b*x)/2]^2 / ((-d + \text{Sqrt}[ \\
& 1 - 2*c + c^2 + d^2]) / (-1 + c) + \text{Tan}[(a + b*x)/2]) + ((a + b*x)*\text{Sec}[(a + b* \\
& x)/2]^2) / (2*((d + \text{Sqrt}[1 - 2*c + c^2 + d^2]) / (1 - c) + \text{Tan}[(a + b*x)/2])) + \\
& ((I/2)*\text{Log}[((-1 + c)*(-I + \text{Tan}[(a + b*x)/2])) / (I - I*c + d + \text{Sqrt}[1 - 2*c \\
& + c^2 + d^2])] * \text{Sec}[(a + b*x)/2]^2 / ((d + \text{Sqrt}[1 - 2*c + c^2 + d^2]) / (1 - c) \\
& + \text{Tan}[(a + b*x)/2]) - ((I/2)*\text{Log}[((-1 + c)*(I + \text{Tan}[(a + b*x)/2])) / (-I + I \\
& *c + d + \text{Sqrt}[1 - 2*c + c^2 + d^2])] * \text{Sec}[(a + b*x)/2]^2 / ((d + \text{Sqrt}[1 - 2*c \\
& + c^2 + d^2]) / (1 - c) + \text{Tan}[(a + b*x)/2]) - ((a + b*x)*\text{Sec}[(a + b*x)/2]^2) \\
& / (2*(-((d + \text{Sqrt}[1 + 2*c + c^2 + d^2]) / (1 + c)) + \text{Tan}[(a + b*x)/2])) - ((I/ \\
& 2)*\text{Log}[((1 + c)*(-I + \text{Tan}[(a + b*x)/2])) / (-I - I*c + d + \text{Sqrt}[1 + 2*c + c^2 \\
& + d^2])] * \text{Sec}[(a + b*x)/2]^2 / (-((d + \text{Sqrt}[1 + 2*c + c^2 + d^2]) / (1 + c)) + \\
& \text{Tan}[(a + b*x)/2]) + ((I/2)*\text{Log}[((1 + c)*(I + \text{Tan}[(a + b*x)/2])) / (I + I*c + \\
& d + \text{Sqrt}[1 + 2*c + c^2 + d^2])] * \text{Sec}[(a + b*x)/2]^2 / (-((d + \text{Sqrt}[1 + 2*c + \\
& c^2 + d^2]) / (1 + c)) + \text{Tan}[(a + b*x)/2]) + ((I/2)*(-1 + c)*\text{Log}[1 - (d + \text{Sqr \\
& rt}[1 - 2*c + c^2 + d^2] - (-1 + c)*\text{Tan}[(a + b*x)/2]) / (I - I*c + d + \text{Sqrt}[1 \\
& - 2*c + c^2 + d^2])] * \text{Sec}[(a + b*x)/2]^2 / (d + \text{Sqrt}[1 - 2*c + c^2 + d^2] - ( \\
& -1 + c)*\text{Tan}[(a + b*x)/2]) - ((I/2)*(-1 + c)*\text{Log}[1 - (d + \text{Sqrt}[1 - 2*c + c^2 \\
& + d^2] - (-1 + c)*\text{Tan}[(a + b*x)/2]) / (-I + I*c + d + \text{Sqrt}[1 - 2*c + c^2 + d \\
& ^2])] * \text{Sec}[(a + b*x)/2]^2 / (d + \text{Sqrt}[1 - 2*c + c^2 + d^2] - (-1 + c)*\text{Tan}[(a \\
& + b*x)/2]) + ((I/2)*(-1 + c)*\text{Log}[1 - (-d + \text{Sqrt}[1 - 2*c + c^2 + d^2] + (-1 \\
& + c)*\text{Tan}[(a + b*x)/2]) / (I - I*c - d + \text{Sqrt}[1 - 2*c + c^2 + d^2])] * \text{Sec}[(a + \\
& b*x)/2]^2 / (-d + \text{Sqrt}[1 - 2*c + c^2 + d^2] + (-1 + c)*\text{Tan}[(a + b*x)/2]) - ( \\
& (I/2)*(-1 + c)*\text{Log}[1 - (-d + \text{Sqrt}[1 - 2*c + c^2 + d^2] + (-1 + c)*\text{Tan}[(a + \\
& b*x)/2]) / (-I + I*c - d + \text{Sqrt}[1 - 2*c + c^2 + d^2])] * \text{Sec}[(a + b*x)/2]^2 / (- \\
& d + \text{Sqrt}[1 - 2*c + c^2 + d^2] + (-1 + c)*\text{Tan}[(a + b*x)/2]) - ((I/2)*(1 + c) \\
& * \text{Log}[1 - (d + \text{Sqrt}[1 + 2*c + c^2 + d^2] - (1 + c)*\text{Tan}[(a + b*x)/2]) / (-I - I \\
& *c + d + \text{Sqrt}[1 + 2*c + c^2 + d^2])] * \text{Sec}[(a + b*x)/2]^2 / (d + \text{Sqrt}[1 + 2*c \\
& + c^2 + d^2] - (1 + c)*\text{Tan}[(a + b*x)/2]) + ((I/2)*(1 + c)*\text{Log}[1 - (d + \text{Sqrt} \\
& [1 + 2*c + c^2 + d^2] - (1 + c)*\text{Tan}[(a + b*x)/2]) / (I + I*c + d + \text{Sqrt}[1 + 2 \\
& *c + c^2 + d^2])] * \text{Sec}[(a + b*x)/2]^2 / (d + \text{Sqrt}[1 + 2*c + c^2 + d^2] - (1 + \\
& c)*\text{Tan}[(a + b*x)/2]) - ((1 + c)*(a + b*x)*\text{Sec}[(a + b*x)/2]^2) / (2*(-d + \text{Sqr \\
& t}[1 + 2*c + c^2 + d^2] + (1 + c)*\text{Tan}[(a + b*x)/2])) + ((I/2)*(1 + c)*\text{Log}[(
\end{aligned}$$

$$\begin{aligned}
& (1 + c) * (1 - I * \tan[(a + b * x) / 2]) / (1 + c - I * d + I * \sqrt{1 + 2 * c + c^2 + d^2}) \\
& ) * \sec[(a + b * x) / 2]^2 / (-d + \sqrt{1 + 2 * c + c^2 + d^2} + (1 + c) * \tan[(a + b * x) / 2]) - ((I / 2) * (1 + c) * \log[((1 + c) * (1 + I * \tan[(a + b * x) / 2])) / (1 + c + I * \\
& d - I * \sqrt{1 + 2 * c + c^2 + d^2})] * \sec[(a + b * x) / 2]^2 / (-d + \sqrt{1 + 2 * c + c^2 + d^2} + (1 + c) * \tan[(a + b * x) / 2]) - ((I / 2) * (1 + c) * \log[1 - (-d + \sqrt{1 + 2 * c + c^2 + d^2} + (1 + c) * \tan[(a + b * x) / 2]) / (-I - I * c - d + \sqrt{1 + 2 * c + c^2 + d^2})] * \sec[(a + b * x) / 2]^2 / (-d + \sqrt{1 + 2 * c + c^2 + d^2} + (1 + c) * \tan[(a + b * x) / 2]) + ((I / 2) * (1 + c) * \log[1 - (-d + \sqrt{1 + 2 * c + c^2 + d^2} + (1 + c) * \tan[(a + b * x) / 2]) / (I + I * c - d + \sqrt{1 + 2 * c + c^2 + d^2})] * \sec[(a + b * x) / 2]^2 / (-d + \sqrt{1 + 2 * c + c^2 + d^2} + (1 + c) * \tan[(a + b * x) / 2]) + (a * \cos[(a + b * x) / 2]^2 * (-\sec[(a + b * x) / 2]^2 * (d * \cos[a + b * x] - (-1 + c) * \sin[a + b * x])) - \sec[(a + b * x) / 2]^2 * ((-1 + c) * \cos[a + b * x] + d * \sin[a + b * x]) * \tan[(a + b * x) / 2]) / ((-1 + c) * \cos[a + b * x] + d * \sin[a + b * x]) + (a * \cos[(a + b * x) / 2]^2 * (\sec[(a + b * x) / 2]^2 * (d * \cos[a + b * x] - \sin[a + b * x] - c * \sin[a + b * x]) + \sec[(a + b * x) / 2]^2 * (\cos[a + b * x] + c * \cos[a + b * x] + d * \sin[a + b * x])) * \tan[(a + b * x) / 2]) / (\cos[a + b * x] + c * \cos[a + b * x] + d * \sin[a + b * x]))
\end{aligned}$$

**Maple [B]** time = 0.131, size = 612, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(c+d\*tan(b\*x+a)),x)

[Out]  $1/b * \arctan(\tan(b * x + a)) * \operatorname{arccoth}(c + d * \tan(b * x + a)) - 1/2 * \arctan((c + d * \tan(b * x + a)) / d - c / d) * \ln(d * ((c + d * \tan(b * x + a)) / d - c / d) + c + 1) + 1/2 * \arctan((c + d * \tan(b * x + a)) / d - c / d) * \ln(d * ((c + d * \tan(b * x + a)) / d - c / d) + c - 1) + 1/4 * I / b * \ln(d * ((c + d * \tan(b * x + a)) / d - c / d) + c - 1) * \ln((I * d - d * ((c + d * \tan(b * x + a)) / d - c / d)) / (I * d + c - 1)) - 1/4 * I / b * \ln(d * ((c + d * \tan(b * x + a)) / d - c / d) + c - 1) * \ln((I * d + d * ((c + d * \tan(b * x + a)) / d - c / d)) / (1 - c + I * d)) + 1/4 * I / b * \operatorname{dilog}((I * d - d * ((c + d * \tan(b * x + a)) / d - c / d)) / (I * d + c - 1)) - 1/4 * I / b * \operatorname{dilog}((I * d + d * ((c + d * \tan(b * x + a)) / d - c / d)) / (1 - c + I * d)) - 1/4 * I / b * \ln(d * ((c + d * \tan(b * x + a)) / d - c / d) + c + 1) * \ln((I * d - d * ((c + d * \tan(b * x + a)) / d - c / d)) / (1 + c + I * d)) + 1/4 * I / b * \ln(d * ((c + d * \tan(b * x + a)) / d - c / d) + c + 1) * \ln((I * d + d * ((c + d * \tan(b * x + a)) / d - c / d)) / (I * d - c - 1)) - 1/4 * I / b * \operatorname{dilog}((I * d - d * ((c + d * \tan(b * x + a)) / d - c / d)) / (1 + c + I * d)) + 1/4 * I / b * \operatorname{dilog}((I * d + d * ((c + d * \tan(b * x + a)) / d - c / d)) / (I * d - c - 1))$

**Maxima [B]** time = 2.12998, size = 502, normalized size = 2.59

$$4(bx + a) \operatorname{arccoth}(d \tan(bx + a) + c) + \left( \arctan\left(\frac{d^2 \tan(bx + a) + (c + 1)d}{c^2 + d^2 + 2c + 1}\right), \frac{(c + 1)d \tan(bx + a) + c^2 + 2c + 1}{c^2 + d^2 + 2c + 1} \right) - \arctan\left(\frac{d^2 \tan(bx + a) + (c - 1)d}{c^2 + d^2 - 2c + 1}\right),$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(b*x + a)*arccoth(d*tan(b*x + a) + c) + (arctan2((d^2*tan(b*x + a) +
(c + 1)*d)/(c^2 + d^2 + 2*c + 1), ((c + 1)*d*tan(b*x + a) + c^2 + 2*c + 1)
/(c^2 + d^2 + 2*c + 1)) - arctan2((d^2*tan(b*x + a) + (c - 1)*d)/(c^2 + d^2
- 2*c + 1), ((c - 1)*d*tan(b*x + a) + c^2 - 2*c + 1)/(c^2 + d^2 - 2*c + 1)
))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log((d^2*tan(b*x + a)^2 + 2*(c + 1)*
d*tan(b*x + a) + c^2 + 2*c + 1)/(c^2 + d^2 + 2*c + 1)) + (b*x + a)*log((d^2
*tan(b*x + a)^2 + 2*(c - 1)*d*tan(b*x + a) + c^2 - 2*c + 1)/(c^2 + d^2 - 2*
c + 1)) - I*dilog(-(I*d*tan(b*x + a) - d)/(I*c + d + I)) + I*dilog(-(I*d*ta
n(b*x + a) - d)/(I*c + d - I)) - I*dilog((I*d*tan(b*x + a) + d)/(-I*c + d +
I)) + I*dilog((I*d*tan(b*x + a) + d)/(-I*c + d - I)))/b
```

**Fricas [B]** time = 2.43372, size = 3210, normalized size = 16.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*log((d*tan(b*x + a) + c + 1)/(d*tan(b*x + a) + c - 1)) - 2*(b*x
+ a)*log(((2*I*(c + 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 - 2*I*(c + 1)*d +
(2*I*c^2 + 4*(c + 1)*d - 2*I*d^2 + 4*I*c + 2*I)*tan(b*x + a) + 4*c + 2)/((c
^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) - 2*(b*x + a)*lo
g(((2*I*(c + 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 + 2*I*(c + 1)*d + (-2*I*
c^2 + 4*(c + 1)*d + 2*I*d^2 - 4*I*c - 2*I)*tan(b*x + a) + 4*c + 2)/((c^2 +
d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) + 2*(b*x + a)*log(((2
*I*(c - 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 - 2*I*(c - 1)*d + (2*I*c^2 + 4
*(c - 1)*d - 2*I*d^2 - 4*I*c + 2*I)*tan(b*x + a) - 4*c + 2)/((c^2 + d^2 - 2
*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1)) + 2*(b*x + a)*log(((2*I*(c
- 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 + 2*I*(c - 1)*d + (-2*I*c^2 + 4*(c -
1)*d + 2*I*d^2 + 4*I*c - 2*I)*tan(b*x + a) - 4*c + 2)/((c^2 + d^2 - 2*c +
1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1)) + 2*a*log(((I*(c + 1)*d + d^2)*ta
n(b*x + a)^2 - c^2 + I*(c + 1)*d + (I*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a)
- 2*c - 1)/(tan(b*x + a)^2 + 1)) + 2*a*log(((I*(c + 1)*d - d^2)*tan(b*x +
a)^2 + c^2 + I*(c + 1)*d + (I*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a) + 2*c +
1)/(tan(b*x + a)^2 + 1)) - 2*a*log(((I*(c - 1)*d + d^2)*tan(b*x + a)^2 - c
^2 + I*(c - 1)*d + (I*c^2 + I*d^2 - 2*I*c + I)*tan(b*x + a) + 2*c - 1)/(tan
(b*x + a)^2 + 1)) - 2*a*log(((I*(c - 1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*(
```

$$\begin{aligned}
& (c - 1)d + (Ic^2 + Id^2 - 2Ic + I)\tan(bx + a) - 2c + 1)/(\tan(bx + a) \\
& ^2 + 1) + I\operatorname{dilog}(-((2I(c + 1)d + 2d^2)\tan(bx + a)^2 + 2c^2 - 2I \\
& (c + 1)d + (2Ic^2 + 4(c + 1)d - 2Id^2 + 4Ic + 2I)\tan(bx + a) + \\
& 4c + 2)/((c^2 + d^2 + 2c + 1)\tan(bx + a)^2 + c^2 + d^2 + 2c + 1) + 1) \\
& - I\operatorname{dilog}(-((-2I(c + 1)d + 2d^2)\tan(bx + a)^2 + 2c^2 + 2I(c + 1)d \\
& + (-2Ic^2 + 4(c + 1)d + 2Id^2 - 4Ic - 2I)\tan(bx + a) + 4c + 2) \\
& /((c^2 + d^2 + 2c + 1)\tan(bx + a)^2 + c^2 + d^2 + 2c + 1) + 1) - I\operatorname{dilo} \\
& g(-((2I(c - 1)d + 2d^2)\tan(bx + a)^2 + 2c^2 - 2I(c - 1)d + (2Ic \\
& ^2 + 4(c - 1)d - 2Id^2 - 4Ic + 2I)\tan(bx + a) - 4c + 2)/((c^2 + d \\
& ^2 - 2c + 1)\tan(bx + a)^2 + c^2 + d^2 - 2c + 1) + 1) + I\operatorname{dilog}(-((-2I \\
& (c - 1)d + 2d^2)\tan(bx + a)^2 + 2c^2 + 2I(c - 1)d + (-2Ic^2 + 4(c \\
& - 1)d + 2Id^2 + 4Ic - 2I)\tan(bx + a) - 4c + 2)/((c^2 + d^2 - 2c \\
& + 1)\tan(bx + a)^2 + c^2 + d^2 - 2c + 1) + 1))/b
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(c+d\*tan(b\*x+a)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}(d \tan(bx + a) + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c+d\*tan(b\*x+a)),x, algorithm="giac")

[Out] integrate(arccoth(d\*tan(b\*x + a) + c), x)

$$3.239 \quad \int \frac{\coth^{-1}(c+d \tan(a+bx))}{x} dx$$

**Optimal.** Leaf size=17

$$\text{CannotIntegrate}\left(\frac{\coth^{-1}(d \tan(a + bx) + c)}{x}, x\right)$$

[Out] CannotIntegrate[ArcCoth[c + d\*Tan[a + b\*x]]/x, x]

**Rubi [A]** time = 0.145621, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\coth^{-1}(c + d \tan(a + bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[ArcCoth[c + d\*Tan[a + b\*x]]/x,x]

[Out] Defer[Int][ArcCoth[c + d\*Tan[a + b\*x]]/x, x]

Rubi steps

$$\int \frac{\coth^{-1}(c + d \tan(a + bx))}{x} dx = \int \frac{\coth^{-1}(c + d \tan(a + bx))}{x} dx$$

**Mathematica [A]** time = 0.355154, size = 0, normalized size = 0.

$$\int \frac{\coth^{-1}(c + d \tan(a + bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcCoth[c + d\*Tan[a + b\*x]]/x,x]

[Out] Integrate[ArcCoth[c + d\*Tan[a + b\*x]]/x, x]



---

**Maple [A]** time = 0.382, size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(c + d \tan(bx + a))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(c+d\*tan(b\*x+a))/x,x)

[Out] int(arccoth(c+d\*tan(b\*x+a))/x,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(d \tan(bx + a) + c)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c+d\*tan(b\*x+a))/x,x, algorithm="maxima")

[Out] integrate(arccoth(d\*tan(b\*x + a) + c)/x, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arccoth}(d \tan(bx + a) + c)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c+d\*tan(b\*x+a))/x,x, algorithm="fricas")

[Out] integral(arccoth(d\*tan(b\*x + a) + c)/x, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(c+d*tan(b*x+a))/x,x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(d \tan(bx + a) + c)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*tan(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*tan(b*x + a) + c)/x, x)
```

### 3.240 $\int x^2 \coth^{-1}(1 - id + d \tan(a + bx)) dx$

**Optimal.** Leaf size=170

$$\frac{x \operatorname{PolyLog}\left(3, -(1 - id)e^{2ia+2ibx}\right)}{4b^2} - \frac{i \operatorname{PolyLog}\left(4, -(1 - id)e^{2ia+2ibx}\right)}{8b^3} + \frac{ix^2 \operatorname{PolyLog}\left(2, -(1 - id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{6}x^3 \log(1 -$$

[Out] (I/12)\*b\*x^4 + (x^3\*ArcCoth[1 - I\*d + d\*Tan[a + b\*x]])/3 - (x^3\*Log[1 + (1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/6 + ((I/4)\*x^2\*PolyLog[2, -((1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))])/b - (x\*PolyLog[3, -((1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))])/(4\*b^2) - ((I/8)\*PolyLog[4, -((1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))])/b^3

**Rubi [A]** time = 0.304345, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$ , Rules used = {6264, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{x \operatorname{PolyLog}\left(3, -(1 - id)e^{2ia+2ibx}\right)}{4b^2} - \frac{i \operatorname{PolyLog}\left(4, -(1 - id)e^{2ia+2ibx}\right)}{8b^3} + \frac{ix^2 \operatorname{PolyLog}\left(2, -(1 - id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{6}x^3 \log(1 -$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcCoth[1 - I\*d + d\*Tan[a + b\*x]],x]

[Out] (I/12)\*b\*x^4 + (x^3\*ArcCoth[1 - I\*d + d\*Tan[a + b\*x]])/3 - (x^3\*Log[1 + (1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/6 + ((I/4)\*x^2\*PolyLog[2, -((1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))])/b - (x\*PolyLog[3, -((1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))])/(4\*b^2) - ((I/8)\*PolyLog[4, -((1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))])/b^3

#### Rule 6264

Int[ArcCoth[(c\_.) + (d\_.)\*Tan[(a\_.) + (b\_.)\*(x\_)]]\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((e + f\*x)^(m + 1)\*ArcCoth[c + d\*Tan[a + b\*x]])/(f\*(m + 1)), x] + Dist[(I\*b)/(f\*(m + 1)), Int[(e + f\*x)^(m + 1)/(c + I\*d + c\*E^(2\*I\*a + 2\*I\*b\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c + I\*d)^2, 1]

#### Rule 2184

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.), x\_Symbol] := Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x],

x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_)]/((a\_) + (b\_)\*((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_)\*((F\_)^((c\_)\*(a\_) + (b\_)\*(x\_)))^(n\_)]\*((f\_) + (g\_)\*(x\_))^(m\_), x\_Symbol] :> -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 6609

Int[((e\_) + (f\_)\*(x\_))^(m\_)\*PolyLog[n\_, (d\_)\*((F\_)^((c\_)\*(a\_) + (b\_)\*(x\_)))^(p\_)], x\_Symbol] :> Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p])/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\* (F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]

### Rule 6589

Int[PolyLog[n\_, (c\_)\*((a\_) + (b\_)\*(x\_))^(p\_)]/((d\_) + (e\_)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(1 - id + d \tan(a + bx)) dx &= \frac{1}{3} x^3 \coth^{-1}(1 - id + d \tan(a + bx)) + \frac{1}{3} (ib) \int \frac{x^3}{1 + (1 - id)e^{2ia+2ibx}} dx \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 - id + d \tan(a + bx)) - \frac{1}{3} (b(i + d)) \int \frac{e^{2ia+2ibx} x^3}{1 + (1 - id)e^{2ia+2ibx}} dx \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 - id + d \tan(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 - id)e^{2ia+2ibx}) \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 - id + d \tan(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 - id)e^{2ia+2ibx}) \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 - id + d \tan(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 - id)e^{2ia+2ibx}) \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 - id + d \tan(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 - id)e^{2ia+2ibx}) \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 - id + d \tan(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 - id)e^{2ia+2ibx}) \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 - id + d \tan(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 - id)e^{2ia+2ibx})
\end{aligned}$$

**Mathematica [A]** time = 0.187783, size = 155, normalized size = 0.91

$$\frac{1}{3} x^3 \coth^{-1}(d \tan(a + bx) - id + 1) - \frac{6ib^2 x^2 \text{PolyLog}\left(2, -\frac{ie^{-2i(a+bx)}}{d+i}\right) + 6bx \text{PolyLog}\left(3, -\frac{ie^{-2i(a+bx)}}{d+i}\right) - 3i \text{PolyLog}\left(4, -\frac{ie^{-2i(a+bx)}}{d+i}\right)}{24b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[1 - I\*d + d\*Tan[a + b\*x]], x]

[Out] (x^3\*ArcCoth[1 - I\*d + d\*Tan[a + b\*x]])/3 - (4\*b^3\*x^3\*Log[1 + I/((I + d)\*E^((2\*I)\*(a + b\*x)))] + (6\*I)\*b^2\*x^2\*PolyLog[2, (-I)/((I + d)\*E^((2\*I)\*(a + b\*x)))] + 6\*b\*x\*PolyLog[3, (-I)/((I + d)\*E^((2\*I)\*(a + b\*x)))] - (3\*I)\*PolyLog[4, (-I)/((I + d)\*E^((2\*I)\*(a + b\*x)))])/(24\*b^3)

**Maple [C]** time = 30.058, size = 2339, normalized size = 13.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(1-I\*d+d\*tan(b\*x+a)), x)

[Out] 
$$\begin{aligned}
& -1/2/b^2*a^2*d/(I+d)*\ln(1+I*\exp(I*(b*x+a))*(-I*(I+d))^{(1/2)})*x+1/2/b^2*d/(I+d)*\ln(1-I*(I+d)*\exp(2*I*(b*x+a)))*x*a^2-1/2/b^2*a^2*d/(I+d)*\ln(1-I*\exp(I*(b*x+a))*(-I*(I+d))^{(1/2)})*x-1/6*x^3*\ln(d)-1/12*I*x^3*\text{P}\i\text{c}\text{s}\text{g}\text{n}(I*d/(\exp(2*I*(b*x+a))+1)*\exp(2*I*(b*x+a)))^2+1/12*I*b*x^4-1/8*I/b^3*d/(I+d)*\text{p}\text{o}\text{l}\text{y}\text{l}\text{o}\text{g}(4,I*(I+d)*\exp(2*I*(b*x+a)))+1/6*I/b^3*a^3/(I+d)*\ln(I*\exp(2*I*(b*x+a))+\exp(2*I*(b*x+a))*d+I)+1/3*I/b^3/(I+d)*\ln(1-I*(I+d)*\exp(2*I*(b*x+a)))*a^3-1/4*I/b^2/(I+d)*\text{p}\text{o}\text{l}\text{y}\text{l}\text{o}\text{g}(3,I*(I+d)*\exp(2*I*(b*x+a)))*x-1/2*I/b^3*a^3/(I+d)*\ln(1-I*\exp(I*(b*x+a))*(-I*(I+d))^{(1/2)})-1/6*I/(I+d)*\ln(1-I*(I+d)*\exp(2*I*(b*x+a)))*x^3-1/2/b^3*a^2/(I+d)*\text{d}\text{i}\text{l}\text{o}\text{g}(1-I*\exp(I*(b*x+a))*(-I*(I+d))^{(1/2)})-1/2/b^3*a^2/(I+d)*\text{d}\text{i}\text{l}\text{o}\text{g}(1+I*\exp(I*(b*x+a))*(-I*(I+d))^{(1/2)})-1/12*I*x^3*\text{P}\i\text{c}\text{s}\text{g}\text{n}(I*\exp(2*I*(b*x+a)))*\text{c}\text{s}\text{g}\text{n}(I*\exp(2*I*(b*x+a)))/(\exp(2*I*(b*x+a))+1)^2+1/12*I*x^3*\text{P}\i\text{c}\text{s}\text{g}\text{n}(I/(\exp(2*I*(b*x+a))+1))*\text{c}\text{s}\text{g}\text{n}(I*(I*\exp(2*I*(b*x+a))+\exp(2*I*(b*x+a))*d+I)/(\exp(2*I*(b*x+a))+1))^2+1/8/b^3/(I+d)*\text{p}\text{o}\text{l}\text{y}\text{l}\text{o}\text{g}(4,I*(I+d)*\exp(2*I*(b*x+a)))-1/3*x^3*\ln(\exp(I*(b*x+a)))+1/6*x^3*\ln(I*\exp(2*I*(b*x+a))+\exp(2*I*(b*x+a))*d+I)+1/4*I/b*d/(I+d)*\text{p}\text{o}\text{l}\text{y}\text{l}\text{o}\text{g}(2,I*(I+d)*\exp(2*I*(b*x+a)))*x^2+1/12*I*x^3*\text{P}\i\text{c}\text{s}\text{g}\text{n}(I*d)*\text{c}\text{s}\text{g}\text{n}(I*\exp(2*I*(b*x+a)))/(\exp(2*I*(b*x+a))+1))*\text{c}\text{s}\text{g}\text{n}(I*d/(\exp(2*I*(b*x+a))+1)*\exp(2*I*(b*x+a)))+1/12*I*x^3*\text{P}\i\text{c}\text{s}\text{g}\text{n}(I*\exp(I*(b*x+a)))^2*\text{c}\text{s}\text{g}\text{n}(I*\exp(2*I*(b*x+a)))-1/6*I*x^3*\text{P}\i\text{c}\text{s}\text{g}\text{n}(I*\exp(I*(b*x+a)))*\text{c}\text{s}\text{g}\text{n}(I*\exp(2*I*(b*x+a)))^2-1/12*I*x^3*\text{P}\i\text{c}\text{s}\text{g}\text{n}(I*d)*\text{c}\text{s}\text{g}\text{n}(I*d/(\exp(2*I*(b*x+a))+1)*\exp(2*I*(b*x+a)))^2-1/12*I*x^3*\text{P}\i\text{c}\text{s}\text{g}\text{n}(I*\exp(2*I*(b*x+a)))/(\exp(2*I*(b*x+a))+1))*\text{c}\text{s}\text{g}\text{n}(I*d/(\exp(2*I*(b*x+a))+1)*\exp(2*I*(b*x+a)))^2-1/12*I*x^3*\text{P}\i\text{c}\text{s}\text{g}\text{n}(I/(\exp(2*I*(b*x+a))+1))*\text{c}\text{s}\text{g}\text{n}(I*\exp(2*I*(b*x+a)))/(\exp(2*I*(b*x+a))+1))^2+1/2*I/b^3*a^2*d/(I+d)*\text{d}\text{i}\text{l}\text{o}\text{g}(1-I*\exp(I*(b*x+a))*(-I*(I+d))^{(1/2)})+1/2*I/b^3*a^2*d/(I+d)*\text{d}\text{i}\text{l}\text{o}\text{g}(1+I*\exp(I*(b*x+a))*(-I*(I+d))^{(1/2)})+1/2*I/b^2/(I+d)*\ln(1-I*(I+d)*\exp(2*I*(b*x+a)))*x*a^2-1/4/b^2*d/(I+d)*\text{p}\text{o}\text{l}\text{y}\text{l}\text{o}\text{g}(3,I*(I+d)*\exp(2*I*(b*x+a)))*x-1/2/b^3*a^3*d/(I+d)*\ln(1-I*\exp(I*(b*x+a))*(-I*(I+d))^{(1/2)})-1/2/b^3*a^3*d/(I+d)*\ln(1+I*\exp(I*(b*x+a))*(-I*(I+d))^{(1/2)})+1/6/b^3*a^3*d/(I+d)*\ln(I*\exp(2*I*(b*x+a))+\exp(2*I*(b*x+a))*d+I)+1/3/b^3*d/(I+d)*\ln(1-I*(I+d)*\exp(2*I*(b*x+a)))*a^3+1/12*I*x^3*\text{P}\i\text{c}\text{s}\text{g}\text{n}(I*d/(\exp(2*I*(b*x+a))+1)*\exp(2*I*(b*x+a)))^3-1/12*I*x^3*\text{P}\i\text{c}\text{s}\text{g}\text{n}(I*(I*\exp(2*I*(b*x+a))+\exp(2*I*(b*x+a))*d+I)/(\exp(2*I*(b*x+a))+1))^3-1/12*I*x^3*\text{P}\i\text{c}\text{s}\text{g}\text{n}(I*(I*\exp(2*I*(b*x+a))+\exp(2*I*(b*x+a))*d+I)/(\exp(2*I*(b*x+a))+1))*\text{c}\text{s}\text{g}\text{n}((I*\exp(2*I*(b*x+a))+\exp(2*I*(b*x+a))*d+I)/(\exp(2*I*(b*x+a))+1))+1/12*I*x^3*\text{P}\i\text{c}\text{s}\text{g}\text{n}(I*d/(\exp(2*I*(b*x+a))+1)*\exp(2*I*(b*x+a)))*\text{c}\text{s}\text{g}\text{n}(d/(\exp(2*I*(b*x+a))+1)*\exp(2*I*(b*x+a)))-1/4*I/b^3*d/(I+d)*\text{p}\text{o}\text{l}\text{y}\text{l}\text{o}\text{g}(2,I*(I+d)*\exp(2*I*(b*x+a)))*a^2-1/2*I/b^2*a^2/(I+d)*\ln(1-I*\exp(I*(b*x+a))*(-I*(I+d))^{(1/2)})*x-1/2*I/b^3*a^3/(I+d)*\ln(1+I*\exp(I*(b*x+a))*(-I*(I+d))^{(1/2)})+1/12*I*x^3*\text{P}\i\text{c}\text{s}\text{g}\text{n}(I*(I*\exp(2*I*(b*x+a))+\exp(2*I*(b*x+a))*d+I))*\text{c}\text{s}\text{g}\text{n}(I*(I*\exp(2*I*(b*x+a))+\exp(2*I*(b*x+a))*d+I)/(\exp(2*I*(b*x+a))+1))^2+1/12*I*x^3*\text{P}\i\text{c}\text{s}\text{g}\text{n}(I*(I*\exp(2*I*(b*x+a))+\exp(2*I*(b*x+a))*d+I)/(\exp(2*I*(b*x+a))+1))*\text{c}\text{s}\text{g}\text{n}((I*\exp(2*I*(b*x+a))+\exp(2*I*(b*x+a))*d+I)/(\exp(2*I*(b*x+a))+1))^2-1/2*I/b^2*a^2/(I+d)*\ln(1+I*\exp(I*(b*x+a))*(-I*(I+d))^{(1/2)})*x-1/4/b/(I+d)*\text{p}\text{o}\text{l}\text{y}\text{l}\text{o}\text{g}(2,I*(I+d)*\exp(2*I*(b*x+a)))*x^2+1/4/b^3/(I+d)*\text{p}\text{o}\text{l}\text{y}\text{l}\text{o}\text{g}(2,I*(I+d)*\exp(2*I*(b*x+a)))*a^2-1/6*d/(I+d)*\ln(1-I*(I+d)*\exp(2*I*(b*x+a)))*x^3+1/12*I*x^3*\text{P}\i\text{c}\text{s}\text{g}\text{n}((I*\exp(2*I*(b*x+a))+\exp(2*I*(b*x+a))*d+I)/(\exp(2*I*(b*x+a))+1))^2-1/12*I*x^3*\text{P}\i\text{c}\text{s}\text{g}\text{n}(d/(\exp(2*I*(b*x+a))+1))*
\end{aligned}$$

$$\frac{\exp(2I(b*x+a))^{2+1/12} I x^3 \text{P}i \text{csgn}(d/(\exp(2I(b*x+a))+1) \exp(2I(b*x+a)))^{3-1/12} I x^3 \text{P}i \text{csgn}((I \exp(2I(b*x+a))+\exp(2I(b*x+a))*d+I)/(\exp(2I(b*x+a))+1))^{3+1/12} I x^3 \text{P}i \text{csgn}(I \exp(2I(b*x+a)))^{3+1/12} I x^3 \text{P}i \text{csgn}(I \exp(2I(b*x+a))/(\exp(2I(b*x+a))+1))^{3-1/12} I x^3 \text{P}i \text{csgn}(I/(\exp(2I(b*x+a))+1)) \text{csgn}(I*(I \exp(2I(b*x+a))+\exp(2I(b*x+a))*d+I)) \text{csgn}(I*(I \exp(2I(b*x+a))+\exp(2I(b*x+a))*d+I)/(\exp(2I(b*x+a))+1)) + 1/12 I x^3 \text{P}i \text{csgn}(I/(\exp(2I(b*x+a))+1)) \text{csgn}(I \exp(2I(b*x+a))) \text{csgn}(I \exp(2I(b*x+a)))/(\exp(2I(b*x+a))+1))$$

**Maxima [B]** time = 1.2286, size = 460, normalized size = 2.71

$$\frac{12((bx+a)^3 - 3(bx+a)^2 a + 3(bx+a)a^2) \operatorname{arccoth}(d \tan(bx+a) - i d + 1)}{b^2} - \frac{-3i(bx+a)^4 + 12i(bx+a)^3 a - 18i(bx+a)^2 a^2 + (8i(bx+a)^3 - 18i(bx+a)^2 a + 18i(bx+a)a^2) \operatorname{arctan}(\dots)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(1-I\*d+d\*tan(b\*x+a)),x, algorithm="maxima")

[Out] 1/36\*(12\*((b\*x + a)^3 - 3\*(b\*x + a)^2\*a + 3\*(b\*x + a)\*a^2)\*arccoth(d\*tan(b\*x + a) - I\*d + 1)/b^2 - (-3\*I\*(b\*x + a)^4 + 12\*I\*(b\*x + a)^3\*a - 18\*I\*(b\*x + a)^2\*a^2 + (8\*I\*(b\*x + a)^3 - 18\*I\*(b\*x + a)^2\*a + 18\*I\*(b\*x + a)\*a^2)\*arctan2(-d\*cos(2\*b\*x + 2\*a) + sin(2\*b\*x + 2\*a), d\*sin(2\*b\*x + 2\*a) + cos(2\*b\*x + 2\*a) + 1) + (-12\*I\*(b\*x + a)^2 + 18\*I\*(b\*x + a)\*a - 9\*I\*a^2)\*dilog((I\*d - 1)\*e^(2\*I\*b\*x + 2\*I\*a)) + (4\*(b\*x + a)^3 - 9\*(b\*x + a)^2\*a + 9\*(b\*x + a)\*a^2)\*log((d^2 + 1)\*cos(2\*b\*x + 2\*a)^2 + (d^2 + 1)\*sin(2\*b\*x + 2\*a)^2 + 2\*d\*sin(2\*b\*x + 2\*a) + 2\*cos(2\*b\*x + 2\*a) + 1) + 3\*(4\*b\*x + a)\*polylog(3, (I\*d - 1)\*e^(2\*I\*b\*x + 2\*I\*a)) + 6\*I\*polylog(4, (I\*d - 1)\*e^(2\*I\*b\*x + 2\*I\*a)))/b^2)/b

**Fricas [C]** time = 1.83746, size = 980, normalized size = 5.76

$$i b^4 x^4 + 2 b^3 x^3 \log\left(\frac{((d+i)e^{2i bx+2i a}+i)e^{-2i bx-2i a}}{d}\right) + 6i b^2 x^2 \operatorname{Li}_2\left(\frac{1}{2} \sqrt{4i d - 4e^{i bx+i a}}\right) + 6i b^2 x^2 \operatorname{Li}_2\left(-\frac{1}{2} \sqrt{4i d - 4e^{i bx+i a}}\right) - i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(1-I\*d+d\*tan(b\*x+a)),x, algorithm="fricas")

```
[Out] 1/12*(I*b^4*x^4 + 2*b^3*x^3*log(((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)/d) + 6*I*b^2*x^2*dilog(1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) + 6*I*b^2*x^2*dilog(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - I*a^4 + 2*a^3*log(((2*d + 2*I)*e^(I*b*x + I*a) + I*sqrt(4*I*d - 4))/(2*d + 2*I)) + 2*a^3*log(((2*d + 2*I)*e^(I*b*x + I*a) - I*sqrt(4*I*d - 4))/(2*d + 2*I)) - 12*b*x*polylog(3, 1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - 12*b*x*polylog(3, -1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) - 12*I*polylog(4, 1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - 12*I*polylog(4, -1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)))/b^3
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(1-I*d+d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arccoth}(d \tan(bx + a) - id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(d*tan(b*x + a) - I*d + 1), x)
```



### 3.241 $\int x \coth^{-1}(1 - id + d \tan(a + bx)) dx$

**Optimal.** Leaf size=133

$$-\frac{\text{PolyLog}\left(3, -(1-id)e^{2ia+2ibx}\right)}{8b^2} + \frac{ix\text{PolyLog}\left(2, -(1-id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{4}x^2 \log\left(1 + (1-id)e^{2ia+2ibx}\right) + \frac{1}{2}x^2 \coth^{-1}(d \tan(a + bx))$$

[Out] (I/6)\*b\*x^3 + (x^2\*ArcCoth[1 - I\*d + d\*Tan[a + b\*x]])/2 - (x^2\*Log[1 + (1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/4 + ((I/4)\*x\*PolyLog[2, -((1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))])/b - PolyLog[3, -((1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))]/(8\*b^2)

**Rubi [A]** time = 0.253474, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6264, 2184, 2190, 2531, 2282, 6589}

$$-\frac{\text{PolyLog}\left(3, -(1-id)e^{2ia+2ibx}\right)}{8b^2} + \frac{ix\text{PolyLog}\left(2, -(1-id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{4}x^2 \log\left(1 + (1-id)e^{2ia+2ibx}\right) + \frac{1}{2}x^2 \coth^{-1}(d \tan(a + bx))$$

Antiderivative was successfully verified.

[In] Int[x\*ArcCoth[1 - I\*d + d\*Tan[a + b\*x]], x]

[Out] (I/6)\*b\*x^3 + (x^2\*ArcCoth[1 - I\*d + d\*Tan[a + b\*x]])/2 - (x^2\*Log[1 + (1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/4 + ((I/4)\*x\*PolyLog[2, -((1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))])/b - PolyLog[3, -((1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))]/(8\*b^2)

#### Rule 6264

Int[ArcCoth[(c\_.) + (d\_.)\*Tan[(a\_.) + (b\_.)\*(x\_)]]\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^(m + 1)\*ArcCoth[c + d\*Tan[a + b\*x]])/(f\*(m + 1)), x] + Dist[(I\*b)/(f\*(m + 1)), Int[(e + f\*x)^(m + 1)/(c + I\*d + c\*E^(2\*I\*a + 2\*I\*b\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c + I\*d)^2, 1]

#### Rule 2184

Int[(((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x],

x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_)]/((a\_) + (b\_)\*((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_)\*((F\_)^((c\_)\*(a\_) + (b\_)\*(x\_)))^(n\_)]\*((f\_) + (g\_)\*(x\_))^(m\_), x\_Symbol] :> -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n])]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_)\*((a\_) + (b\_)\*(x\_))^(p\_)]/((d\_) + (e\_)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
\int x \coth^{-1}(1 - id + d \tan(a + bx)) dx &= \frac{1}{2}x^2 \coth^{-1}(1 - id + d \tan(a + bx)) + \frac{1}{2}(ib) \int \frac{x^2}{1 + (1 - id)e^{2ia+2ibx}} dx \\
&= \frac{1}{6}ibx^3 + \frac{1}{2}x^2 \coth^{-1}(1 - id + d \tan(a + bx)) - \frac{1}{2}(b(i + d)) \int \frac{e^{2ia+2ibx}x^2}{1 + (1 - id)e^{2ia+2ibx}} dx \\
&= \frac{1}{6}ibx^3 + \frac{1}{2}x^2 \coth^{-1}(1 - id + d \tan(a + bx)) - \frac{1}{4}x^2 \log(1 + (1 - id)e^{2ia+2ibx}) + \\
&= \frac{1}{6}ibx^3 + \frac{1}{2}x^2 \coth^{-1}(1 - id + d \tan(a + bx)) - \frac{1}{4}x^2 \log(1 + (1 - id)e^{2ia+2ibx}) + \\
&= \frac{1}{6}ibx^3 + \frac{1}{2}x^2 \coth^{-1}(1 - id + d \tan(a + bx)) - \frac{1}{4}x^2 \log(1 + (1 - id)e^{2ia+2ibx}) + \\
&= \frac{1}{6}ibx^3 + \frac{1}{2}x^2 \coth^{-1}(1 - id + d \tan(a + bx)) - \frac{1}{4}x^2 \log(1 + (1 - id)e^{2ia+2ibx}) +
\end{aligned}$$

**Mathematica [A]** time = 0.10564, size = 119, normalized size = 0.89

$$\frac{1}{2}x^2 \coth^{-1}(d \tan(a + bx) - id + 1) - \frac{2ibx \operatorname{PolyLog}\left(2, -\frac{ie^{-2i(a+bx)}}{d+i}\right) + \operatorname{PolyLog}\left(3, -\frac{ie^{-2i(a+bx)}}{d+i}\right) + 2b^2x^2 \log\left(1 + \frac{ie^{-2i(a+bx)}}{d+i}\right)}{8b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[1 - I\*d + d\*Tan[a + b\*x]], x]

[Out] (x^2\*ArcCoth[1 - I\*d + d\*Tan[a + b\*x]])/2 - (2\*b^2\*x^2\*Log[1 + I/((I + d)\*E^((2\*I)\*(a + b\*x)))] + (2\*I)\*b\*x\*PolyLog[2, (-I)/((I + d)\*E^((2\*I)\*(a + b\*x)))] + PolyLog[3, (-I)/((I + d)\*E^((2\*I)\*(a + b\*x)))])/(8\*b^2)

**Maple [C]** time = 17.08, size = 2249, normalized size = 16.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(1-I\*d+d\*tan(b\*x+a)), x)

[Out] 1/8\*I\*x^2\*Pi\*csgn(I\*exp(2\*I\*(b\*x+a))/(exp(2\*I\*(b\*x+a))+1))^3-1/8\*I\*x^2\*Pi\*csgn((I\*exp(2\*I\*(b\*x+a))+exp(2\*I\*(b\*x+a))\*d+I)/(exp(2\*I\*(b\*x+a))+1))^3+1/8\*I

$$\begin{aligned}
& x^2 \pi \operatorname{csgn}(I d / (\exp(2 I (b x+a))+1) \exp(2 I (b x+a)))^3 + 1/4 x^2 \ln(I \exp(2 I (b x+a)) + \exp(2 I (b x+a)) d + I) - 1/8 I x^2 \pi \operatorname{csgn}(d / (\exp(2 I (b x+a))+1) \exp(2 I (b x+a)))^2 + 1/8 I x^2 \pi \operatorname{csgn}(d / (\exp(2 I (b x+a))+1) \exp(2 I (b x+a)))^3 - 1/8 I / b^2 / (I+d) \operatorname{polylog}(3, I (I+d) \exp(2 I (b x+a))) - 1/4 d / (I+d) \ln(1 - I (I+d) \exp(2 I (b x+a))) x^2 + 1/2 / b^2 a / (I+d) \operatorname{dilog}(1 - I \exp(I (b x+a)) * (-I (I+d))^{1/2}) + 1/2 / b^2 a / (I+d) \operatorname{dilog}(1 + I \exp(I (b x+a)) * (-I (I+d))^{1/2}) + 1/6 I b x^3 - 1/4 I x^2 \pi \operatorname{csgn}(I \exp(I (b x+a))) * \operatorname{csgn}(I \exp(2 I (b x+a)))^2 - 1/8 I x^2 \pi \operatorname{csgn}(I (I \exp(2 I (b x+a)) + \exp(2 I (b x+a)) d + I) / (\exp(2 I (b x+a)) + 1)) * \operatorname{csgn}((I \exp(2 I (b x+a)) + \exp(2 I (b x+a)) d + I) / (\exp(2 I (b x+a)) + 1)) - 1/4 b / (I+d) \operatorname{polylog}(2, I (I+d) \exp(2 I (b x+a))) x - 1/4 b^2 / (I+d) \operatorname{polylog}(2, I (I+d) \exp(2 I (b x+a))) a - 1/8 b^2 d / (I+d) \operatorname{polylog}(3, I (I+d) \exp(2 I (b x+a))) - 1/8 I x^2 \pi \operatorname{csgn}(I (I \exp(2 I (b x+a)) + \exp(2 I (b x+a)) d + I) / (\exp(2 I (b x+a)) + 1))^3 + 1/8 I x^2 \pi \operatorname{csgn}((I \exp(2 I (b x+a)) + \exp(2 I (b x+a)) d + I) / (\exp(2 I (b x+a)) + 1))^2 - 1/4 x^2 \ln(d) + 1/8 I x^2 \pi \operatorname{csgn}(I \exp(I (b x+a)))^2 * \operatorname{csgn}(I \exp(2 I (b x+a))) + 1/8 I x^2 \pi \operatorname{csgn}(I \exp(2 I (b x+a)))^3 - 1/4 I / (I+d) \ln(1 - I (I+d) \exp(2 I (b x+a))) x^2 - 1/2 x^2 \ln(\exp(I (b x+a))) + 1/2 I / b a / (I+d) \ln(1 - I \exp(I (b x+a)) * (-I (I+d))^{1/2}) x + 1/2 I / b a / (I+d) \ln(1 + I \exp(I (b x+a)) * (-I (I+d))^{1/2}) x - 1/2 b d / (I+d) \ln(1 - I (I+d) \exp(2 I (b x+a))) x a + 1/2 b a d / (I+d) \ln(1 - I \exp(I (b x+a)) * (-I (I+d))^{1/2}) x + 1/2 b a d / (I+d) \ln(1 + I \exp(I (b x+a)) * (-I (I+d))^{1/2}) x - 1/8 I x^2 \pi \operatorname{csgn}(I d / (\exp(2 I (b x+a)) + 1) \exp(2 I (b x+a))) * \operatorname{csgn}(d / (\exp(2 I (b x+a)) + 1) \exp(2 I (b x+a)))^2 + 1/8 I x^2 \pi \operatorname{csgn}(I (I \exp(2 I (b x+a)) + \exp(2 I (b x+a)) d + I) / (\exp(2 I (b x+a)) + 1)) * \operatorname{csgn}((I \exp(2 I (b x+a)) + \exp(2 I (b x+a)) d + I) / (\exp(2 I (b x+a)) + 1))^2 + 1/8 I x^2 \pi \operatorname{csgn}(I (I \exp(2 I (b x+a)) + \exp(2 I (b x+a)) d + I)) * \operatorname{csgn}(I (I \exp(2 I (b x+a)) + \exp(2 I (b x+a)) d + I) / (\exp(2 I (b x+a)) + 1))^2 - 1/2 I / b^2 a d / (I+d) \operatorname{dilog}(1 - I \exp(I (b x+a)) * (-I (I+d))^{1/2}) - 1/2 I / b^2 a d / (I+d) \operatorname{dilog}(1 + I \exp(I (b x+a)) * (-I (I+d))^{1/2}) - 1/8 I x^2 \pi \operatorname{csgn}(I \exp(2 I (b x+a))) * \operatorname{csgn}(I \exp(2 I (b x+a)) / (\exp(2 I (b x+a)) + 1))^2 - 1/8 I x^2 \pi \operatorname{csgn}(I d) * \operatorname{csgn}(I d / (\exp(2 I (b x+a)) + 1) \exp(2 I (b x+a)))^2 + 1/8 I x^2 \pi \operatorname{csgn}(I d / (\exp(2 I (b x+a)) + 1) \exp(2 I (b x+a))) * \operatorname{csgn}(d / (\exp(2 I (b x+a)) + 1) \exp(2 I (b x+a))) - 1/8 I x^2 \pi \operatorname{csgn}(I / (\exp(2 I (b x+a)) + 1)) * \operatorname{csgn}(I \exp(2 I (b x+a)) / (\exp(2 I (b x+a)) + 1))^2 - 1/4 b^2 a^2 d / (I+d) \ln(I \exp(2 I (b x+a)) + \exp(2 I (b x+a)) d + I) - 1/4 b^2 d / (I+d) \ln(1 - I (I+d) \exp(2 I (b x+a))) a^2 + 1/2 b^2 a^2 d / (I+d) \ln(1 - I \exp(I (b x+a)) * (-I (I+d))^{1/2}) + 1/2 b^2 a^2 d / (I+d) \ln(1 + I \exp(I (b x+a)) * (-I (I+d))^{1/2}) - 1/4 I / b^2 a^2 / (I+d) \ln(I \exp(2 I (b x+a)) + \exp(2 I (b x+a)) d + I) - 1/2 I / b / (I+d) \ln(1 - I (I+d) \exp(2 I (b x+a))) x a + 1/4 I / b d / (I+d) \operatorname{polylog}(2, I (I+d) \exp(2 I (b x+a))) x + 1/4 I / b^2 d / (I+d) \operatorname{polylog}(2, I (I+d) \exp(2 I (b x+a))) a + 1/8 I x^2 \pi \operatorname{csgn}(I / (\exp(2 I (b x+a)) + 1)) * \operatorname{csgn}(I \exp(2 I (b x+a))) * \operatorname{csgn}(I \exp(2 I (b x+a)) / (\exp(2 I (b x+a)) + 1)) + 1/8 I x^2 \pi \operatorname{csgn}(I d) * \operatorname{csgn}(I \exp(2 I (b x+a)) / (\exp(2 I (b x+a)) + 1)) * \operatorname{csgn}(I d / (\exp(2 I (b x+a)) + 1) \exp(2 I (b x+a))) - 1/4 I / b^2 / (I+d) \ln(1 - I (I+d) \exp(2 I (b x+a))) a^2 + 1/2 I / b^2 a^2 / (I+d) \ln(1 - I \exp(I (b x+a)) * (-I (I+d))^{1/2}) - 1/8 I x^2 \pi \operatorname{csgn}(I \exp(2 I (b x+a)) / (\exp(2 I (b x+a)) + 1)) * \operatorname{csgn}(I d / (\exp(2 I (b x+a)) + 1) \exp(2 I (b x+a)))^2 + 1/8 I x^2 \pi \operatorname{csgn}(I / (\exp(2 I (b x+a)) + 1)) * \operatorname{csgn}(I (I \exp(2 I (b x+a)) + \exp(2 I (b x+a)) d + I) / (\exp(2 I (b x+a)) + 1))
\end{aligned}$$

$$\begin{aligned} & \frac{1}{2} \sqrt{\frac{I}{b^2 a^2 (I+d)}} \ln(1 + I \exp(I(bx+a))) (-I(I+d))^{1/2} - \frac{1}{8} I x^2 \pi \\ & * \operatorname{csgn}\left(\frac{I}{\exp(2I(bx+a))+1}\right) * \operatorname{csgn}\left(\frac{I(I \exp(2I(bx+a))+\exp(2I(bx+a))*d}{+I})\right) * \operatorname{csgn}\left(\frac{I(I \exp(2I(bx+a))+\exp(2I(bx+a))*d+I)}{(\exp(2I(bx+a))+1)}\right) \end{aligned}$$

**Maxima [B]** time = 1.1443, size = 333, normalized size = 2.5

$$\frac{12((bx+a)^2 - 2(bx+a)a) \operatorname{arccoth}(d \tan(bx+a) - id + 1)}{b} - \frac{-4i(bx+a)^3 + 12i(bx+a)^2 a - 6i b x \operatorname{Li}_2((i d - 1) e^{2i b x + 2i a}) + (6i(bx+a)^2 - 12i(bx+a)a) \arctan(-d \cos(2bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1-I\*d+d\*tan(b\*x+a)),x, algorithm="maxima")

[Out]  $\frac{1}{24} * (12 * ((b * x + a)^2 - 2 * (b * x + a) * a) * \operatorname{arccoth}(d * \tan(b * x + a) - I * d + 1) / b - (-4 * I * (b * x + a)^3 + 12 * I * (b * x + a)^2 * a - 6 * I * b * x * \operatorname{dilog}((I * d - 1) * e^{(2 * I * b * x + 2 * I * a)}) + (6 * I * (b * x + a)^2 - 12 * I * (b * x + a) * a) * \operatorname{arctan}2(-d * \cos(2 * b * x + 2 * a) + \sin(2 * b * x + 2 * a), d * \sin(2 * b * x + 2 * a) + \cos(2 * b * x + 2 * a) + 1) + 3 * ((b * x + a)^2 - 2 * (b * x + a) * a) * \log((d^2 + 1) * \cos(2 * b * x + 2 * a)^2 + (d^2 + 1) * \sin(2 * b * x + 2 * a)^2 + 2 * d * \sin(2 * b * x + 2 * a) + 2 * \cos(2 * b * x + 2 * a) + 1) + 3 * \operatorname{polylog}(3, (I * d - 1) * e^{(2 * I * b * x + 2 * I * a)})) / b) / b$

**Fricas [C]** time = 1.73063, size = 811, normalized size = 6.1

$$2i b^3 x^3 + 3b^2 x^2 \log\left(\frac{((d+i)e^{2ibx+2ia}+i)e^{-2ibx-2ia}}{d}\right) + 2ia^3 + 6i b x \operatorname{Li}_2\left(\frac{1}{2} \sqrt{4id-4} e^{(ibx+ia)}\right) + 6i b x \operatorname{Li}_2\left(-\frac{1}{2} \sqrt{4id-4} e^{(ibx+ia)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1-I\*d+d\*tan(b\*x+a)),x, algorithm="fricas")

[Out]  $\frac{1}{12} * (2 * I * b^3 * x^3 + 3 * b^2 * x^2 * \log(((d + I) * e^{(2 * I * b * x + 2 * I * a)} + I) * e^{(-2 * I * b * x - 2 * I * a) / d}) + 2 * I * a^3 + 6 * I * b * x * \operatorname{dilog}(1/2 * \sqrt{4 * I * d - 4} * e^{(I * b * x + I * a)}) + 6 * I * b * x * \operatorname{dilog}(-1/2 * \sqrt{4 * I * d - 4} * e^{(I * b * x + I * a)}) - 3 * a^2 * \log(((2 * d + 2 * I) * e^{(I * b * x + I * a)} + I * \sqrt{4 * I * d - 4}) / (2 * d + 2 * I)) - 3 * a^2 * \log(((2 * d + 2 * I) * e^{(I * b * x + I * a)} - I * \sqrt{4 * I * d - 4}) / (2 * d + 2 * I)) - 3 * (b^2 * x^2 - a^2) * \log(1/2 * \sqrt{4 * I * d - 4} * e^{(I * b * x + I * a)} + 1) - 3 * (b^2 * x^2 - a^2) * \log(-1/2 * \sqrt{4 * I * d - 4} * e^{(I * b * x + I * a)} + 1) - 6 * \operatorname{polylog}(3, 1/2 * \sqrt{4 * I * d - 4} * e^{(I * b * x + I * a)}) - 6 * \operatorname{polylog}(3, -1/2 * \sqrt{4 * I * d - 4} * e^{(I * b * x + I * a)})) / b^2$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(1-I\*d+d\*tan(b\*x+a)),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arccoth}(d \tan(bx + a) - id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1-I\*d+d\*tan(b\*x+a)),x, algorithm="giac")

[Out] integrate(x\*arccoth(d\*tan(b\*x + a) - I\*d + 1), x)

### 3.242 $\int \coth^{-1}(1 - id + d \tan(a + bx)) dx$

**Optimal.** Leaf size=93

$$\frac{i\text{PolyLog}\left(2, -(1 - id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{2}x \log\left(1 + (1 - id)e^{2ia+2ibx}\right) + x \coth^{-1}(d \tan(a + bx) - id + 1) + \frac{1}{2}ibx^2$$

[Out] (I/2)\*b\*x^2 + x\*ArcCoth[1 - I\*d + d\*Tan[a + b\*x]] - (x\*Log[1 + (1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/2 + ((I/4)\*PolyLog[2, -((1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))])/b

**Rubi [A]** time = 0.154592, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {6256, 2184, 2190, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2, -(1 - id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{2}x \log\left(1 + (1 - id)e^{2ia+2ibx}\right) + x \coth^{-1}(d \tan(a + bx) - id + 1) + \frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[1 - I\*d + d\*Tan[a + b\*x]], x]

[Out] (I/2)\*b\*x^2 + x\*ArcCoth[1 - I\*d + d\*Tan[a + b\*x]] - (x\*Log[1 + (1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/2 + ((I/4)\*PolyLog[2, -((1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))])/b

#### Rule 6256

Int[ArcCoth[(c\_.) + (d\_.)\*Tan[(a\_.) + (b\_.)\*(x\_)]], x\_Symbol] := Simp[x\*ArcCoth[c + d\*Tan[a + b\*x]], x] + Dist[I\*b, Int[x/(c + I\*d + c\*E^(2\*I\*a + 2\*I\*b\*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I\*d)^2, 1]

#### Rule 2184

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned} \int \coth^{-1}(1 - id + d \tan(a + bx)) dx &= x \coth^{-1}(1 - id + d \tan(a + bx)) + (ib) \int \frac{x}{1 + (1 - id)e^{2ia+2ibx}} dx \\ &= \frac{1}{2} ibx^2 + x \coth^{-1}(1 - id + d \tan(a + bx)) - (b(i + d)) \int \frac{e^{2ia+2ibx} x}{1 + (1 - id)e^{2ia+2ibx}} dx \\ &= \frac{1}{2} ibx^2 + x \coth^{-1}(1 - id + d \tan(a + bx)) - \frac{1}{2} x \log(1 + (1 - id)e^{2ia+2ibx}) + \frac{1}{2} \int \log \left( \frac{1 + (1 - id)e^{2ia+2ibx}}{1 + (1 - id)} \right) dx \\ &= \frac{1}{2} ibx^2 + x \coth^{-1}(1 - id + d \tan(a + bx)) - \frac{1}{2} x \log(1 + (1 - id)e^{2ia+2ibx}) - \frac{i \operatorname{Subst}[\operatorname{Li}_2(-\frac{1 + (1 - id)e^{2ia+2ibx}}{1 + (1 - id)})]}{2b} \\ &= \frac{1}{2} ibx^2 + x \coth^{-1}(1 - id + d \tan(a + bx)) - \frac{1}{2} x \log(1 + (1 - id)e^{2ia+2ibx}) + \frac{i \operatorname{Li}_2\left(-\frac{1 + (1 - id)e^{2ia+2ibx}}{1 + (1 - id)}\right)}{2b} \end{aligned}$$

**Mathematica [B]** time = 2.85825, size = 766, normalized size = 8.24

$$\frac{x \sec^2(a + bx)(\cos(bx) + i \sin(bx))(\sin(bx) + i \cos(bx))(d \sin(a + bx) + (2 - id) \cos(a + bx)) \left( \operatorname{PolyLog}\left(2, -\frac{1}{2}(\cos(a) + i \sin(a))\right) \right)}{(\tan(a + bx) - i)(d \tan(a + bx) - id + 2)(id \sin(a + bx) + (d + 2i) \cos(a + bx)) \left( \frac{\sec^2(bx) \log\left(\frac{\sec(bx)(d \sin(a + bx) + (2 - id) \cos(a + bx))}{2c}\right)}{\tan(bx)} \right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[1 - I*d + d*Tan[a + b*x]], x]
```



```
[Out] x*ArcCoth[1 - I*d + d*Tan[a + b*x]] + (x*((2*I)*b*x*Log[2*Cos[b*x]*(Cos[b*x]
- I*Sin[b*x])] - Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((2*I + d)*Cos[a + b*x]
+ I*d*Sin[a + b*x]))/(2*(I + d))]*Log[1 - I*Tan[b*x]] + Log[(Sec[b*x]*(2
- I*d)*Cos[a + b*x] + d*Sin[a + b*x])/(2*Cos[a] - (2*I)*Sin[a])]*Log[1 +
I*Tan[b*x]] - PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] - PolyLog[2, (Sec[b*x]
*(d*Cos[a] + I*(2*I + d)*Sin[a])*((Cos[a + b*x] - I*Sin[a + b*x]))/(2*(I + d
))] + PolyLog[2, -((Cos[a] + I*Sin[a])*((d*Cos[a] + I*(2*I + d)*Sin[a]))*(-I
+ Tan[b*x]))/2]*Sec[a + b*x]^2*(Cos[b*x] + I*Sin[b*x])*(I*Cos[b*x] + Sin[b
*x])*((2 - I*d)*Cos[a + b*x] + d*Sin[a + b*x]))/(((2*I + d)*Cos[a + b*x] +
I*d*Sin[a + b*x])*((I*Log[1 + I*Tan[b*x]])*Sec[b*x]*(d*Cos[a] + I*(2*I + d)*
Sin[a]))/((2*I + d)*Cos[a + b*x] + I*d*Sin[a + b*x]) + (Log[1 - I*Tan[b*x]]
*Sec[b*x]*((-I)*d*Cos[a] + (2*I + d)*Sin[a]))/((2*I + d)*Cos[a + b*x] + I*d
*Sin[a + b*x]) + 2*b*x*(1 - I*Tan[b*x])) + (Log[(Sec[b*x]*((2 - I*d)*Cos[a +
b*x] + d*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*Sec[b*x]^2)/(-I + Tan[b
*x]) - (Log[1 + ((Cos[a] + I*Sin[a])*((d*Cos[a] + I*(2*I + d)*Sin[a]))*(-I +
Tan[b*x]))/2]*Sec[b*x]^2)/(-I + Tan[b*x]) + Log[(Sec[b*x]*(Cos[a] - I*Sin[a
])*((2*I + d)*Cos[a + b*x] + I*d*Sin[a + b*x]))/(2*(I + d))]*(-I + Tan[b*x]
) - (Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((2*I + d)*Cos[a + b*x] + I*d*Sin[a
+ b*x]))/(2*(I + d))]*Sec[b*x]^2)/(I + Tan[b*x]))*(-I + Tan[a + b*x])*(2 -
I*d + d*Tan[a + b*x]))
```

**Maple [B]** time = 0.175, size = 292, normalized size = 3.1

$$\frac{\frac{i}{2} \operatorname{arccoth}(1 - id + d \tan(bx + a)) \ln(id + d \tan(bx + a))}{b} - \frac{\frac{i}{2} \operatorname{arccoth}(1 - id + d \tan(bx + a)) \ln(-id + d \tan(bx + a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1-I*d+d*tan(b*x+a)),x)
```

```
[Out] 1/2*I/b*arccoth(1-I*d+d*tan(b*x+a))*ln(I*d+d*tan(b*x+a))-1/2*I/b*arccoth(1-
I*d+d*tan(b*x+a))*ln(-I*d+d*tan(b*x+a))-1/8*I/b*ln(-I*d+d*tan(b*x+a))^2+1/4
*I/b*dilog(1-1/2*I*d+1/2*d*tan(b*x+a))+1/4*I/b*ln(-I*d+d*tan(b*x+a))*ln(1-1
/2*I*d+1/2*d*tan(b*x+a))+1/4*I/b*dilog(1/2*I*(-I*d+d*tan(b*x+a))/d)+1/4*I/b
*ln(I*d+d*tan(b*x+a))*ln(1/2*I*(-I*d+d*tan(b*x+a))/d)-1/4*I/b*dilog((2-I*d+
d*tan(b*x+a))/(-2*I*d+2))-1/4*I/b*ln(I*d+d*tan(b*x+a))*ln((2-I*d+d*tan(b*x+
a))/(-2*I*d+2))
```

**Maxima [B]** time = 1.55359, size = 358, normalized size = 3.85

$$4(bx + a)d \left( \frac{\log(d \tan(bx+a) - id + 2)}{d} - \frac{\log(\tan(bx+a) - i)}{d} \right) - d \left( \frac{2i \left( \log(d \tan(bx+a) - id + 2) \log\left(-\frac{id \tan(bx+a) + d + 2i}{2d + 2i} + 1\right) + \operatorname{Li}_2\left(\frac{id \tan(bx+a) + d + 2i}{2d + 2i}\right) \right)}{d} - \frac{2i \log(\tan(bx+a) - i)}{d} \right)$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-I\*d+d\*tan(b\*x+a)),x, algorithm="maxima")

[Out] -1/8\*(4\*(b\*x + a)\*d\*(log(d\*tan(b\*x + a) - I\*d + 2)/d - log(tan(b\*x + a) - I)/d) - d\*(2\*I\*(log(d\*tan(b\*x + a) - I\*d + 2)\*log(-(I\*d\*tan(b\*x + a) + d + 2\*I)/(2\*d + 2\*I) + 1) + dilog((I\*d\*tan(b\*x + a) + d + 2\*I)/(2\*d + 2\*I)))/d - (2\*I\*log(d\*tan(b\*x + a) - I\*d + 2)\*log(tan(b\*x + a) - I) - I\*log(tan(b\*x + a) - I)^2)/d + 2\*I\*(log(1/2\*d\*tan(b\*x + a) - 1/2\*I\*d + 1)\*log(tan(b\*x + a) - I) + dilog(-1/2\*d\*tan(b\*x + a) + 1/2\*I\*d))/d - 2\*I\*(log(tan(b\*x + a) - I)\*log(-1/2\*I\*tan(b\*x + a) + 1/2) + dilog(1/2\*I\*tan(b\*x + a) + 1/2))/d - 8\*(b\*x + a)\*arccoth(d\*tan(b\*x + a) - I\*d + 1))/b

---

**Fricas [B]** time = 1.71042, size = 603, normalized size = 6.48

$$ib^2x^2 + bx \log\left(\frac{((d+i)e^{2ibx+2ia}+i)e^{(-2ibx-2ia)}}{d}\right) - ia^2 - (bx+a) \log\left(\frac{1}{2}\sqrt{4id-4}e^{(ibx+ia)}+1\right) - (bx+a) \log\left(-\frac{1}{2}\sqrt{4id-4}e^{ibx}+1\right)$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-I\*d+d\*tan(b\*x+a)),x, algorithm="fricas")

[Out] 1/2\*(I\*b^2\*x^2 + b\*x\*log(((d + I)\*e^(2\*I\*b\*x + 2\*I\*a) + I)\*e^(-2\*I\*b\*x - 2\*I\*a)/d) - I\*a^2 - (b\*x + a)\*log(1/2\*sqrt(4\*I\*d - 4)\*e^(I\*b\*x + I\*a) + 1) - (b\*x + a)\*log(-1/2\*sqrt(4\*I\*d - 4)\*e^(I\*b\*x + I\*a) + 1) + a\*log(((2\*d + 2\*I)\*e^(I\*b\*x + I\*a) + I\*sqrt(4\*I\*d - 4))/(2\*d + 2\*I)) + a\*log(((2\*d + 2\*I)\*e^(I\*b\*x + I\*a) - I\*sqrt(4\*I\*d - 4))/(2\*d + 2\*I)) + I\*dilog(1/2\*sqrt(4\*I\*d - 4)\*e^(I\*b\*x + I\*a)) + I\*dilog(-1/2\*sqrt(4\*I\*d - 4)\*e^(I\*b\*x + I\*a)))/b

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1-I*d+d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(d \tan(bx + a) - id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*tan(b*x + a) - I*d + 1), x)
```

$$3.243 \quad \int \frac{\coth^{-1}(1-id+d \tan(ax))}{x} dx$$

**Optimal.** Leaf size=22

$$\text{CannotIntegrate}\left(\frac{\coth^{-1}(d \tan(ax) - id + 1)}{x}, x\right)$$

[Out] CannotIntegrate[ArcCoth[1 - I\*d + d\*Tan[a + b\*x]]/x, x]

**Rubi [A]** time = 0.0885803, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\coth^{-1}(1 - id + d \tan(ax))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[ArcCoth[1 - I\*d + d\*Tan[a + b\*x]]/x,x]

[Out] Defer[Int][ArcCoth[1 - I\*d + d\*Tan[a + b\*x]]/x, x]

Rubi steps

$$\int \frac{\coth^{-1}(1 - id + d \tan(ax))}{x} dx = \int \frac{\coth^{-1}(1 - id + d \tan(ax))}{x} dx$$

**Mathematica [A]** time = 0.727147, size = 0, normalized size = 0.

$$\int \frac{\coth^{-1}(1 - id + d \tan(ax))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcCoth[1 - I\*d + d\*Tan[a + b\*x]]/x,x]

[Out] Integrate[ArcCoth[1 - I\*d + d\*Tan[a + b\*x]]/x, x]

---

**Maple [A]** time = 0.444, size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(1 - id + d \tan(bx + a))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(1-I\*d+d\*tan(b\*x+a))/x,x)

[Out] int(arccoth(1-I\*d+d\*tan(b\*x+a))/x,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$-ibx + \frac{1}{4}(-i\pi - 4ia - 2 \log(d)) \log(x) - \frac{1}{2}i \int \frac{\arctan(-d \cos(2bx + 2a) + \sin(2bx + 2a), -d \sin(2bx + 2a) - \cos(2bx + 2a) - 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-I\*d+d\*tan(b\*x+a))/x,x, algorithm="maxima")

[Out] -I\*b\*x + 1/4\*(-I\*pi - 4\*I\*a - 2\*log(d))\*log(x) - 1/2\*I\*integrate(arctan2(-d\*cos(2\*b\*x + 2\*a) + sin(2\*b\*x + 2\*a), -d\*sin(2\*b\*x + 2\*a) - cos(2\*b\*x + 2\*a) - 1)/x, x) + 1/4\*integrate(log((d^2 + 1)\*cos(2\*b\*x + 2\*a)^2 + (d^2 + 1)\*sin(2\*b\*x + 2\*a)^2 + 2\*d\*sin(2\*b\*x + 2\*a) + 2\*cos(2\*b\*x + 2\*a) + 1)/x, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\log\left(\frac{(d+i)e^{(2i bx+2i a)+i}e^{(-2i bx-2i a)}}{d}\right)}{2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-I\*d+d\*tan(b\*x+a))/x,x, algorithm="fricas")

[Out] integral(1/2\*log(((d + I)\*e^(2\*I\*b\*x + 2\*I\*a) + I)\*e^(-2\*I\*b\*x - 2\*I\*a)/d)/x, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(1-I\*d+d\*tan(b\*x+a))/x,x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(d \tan(bx + a) - id + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-I\*d+d\*tan(b\*x+a))/x,x, algorithm="giac")

[Out] integrate(arccoth(d\*tan(b\*x + a) - I\*d + 1)/x, x)

### 3.244 $\int x^2 \coth^{-1}(1 + id - d \tan(a + bx)) dx$

**Optimal.** Leaf size=171

$$\frac{x \operatorname{PolyLog}\left(3, -(1 + id)e^{2ia+2ibx}\right)}{4b^2} - \frac{i \operatorname{PolyLog}\left(4, -(1 + id)e^{2ia+2ibx}\right)}{8b^3} + \frac{ix^2 \operatorname{PolyLog}\left(2, -(1 + id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{6}x^3 \log(1$$

[Out] (I/12)\*b\*x^4 + (x^3\*ArcCoth[1 + I\*d - d\*Tan[a + b\*x]])/3 - (x^3\*Log[1 + (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/6 + ((I/4)\*x^2\*PolyLog[2, -((1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))])/b - (x\*PolyLog[3, -((1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))])/(4\*b^2) - ((I/8)\*PolyLog[4, -((1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))])/b^3

**Rubi [A]** time = 0.297572, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6264, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{x \operatorname{PolyLog}\left(3, -(1 + id)e^{2ia+2ibx}\right)}{4b^2} - \frac{i \operatorname{PolyLog}\left(4, -(1 + id)e^{2ia+2ibx}\right)}{8b^3} + \frac{ix^2 \operatorname{PolyLog}\left(2, -(1 + id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{6}x^3 \log(1$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcCoth[1 + I\*d - d\*Tan[a + b\*x]],x]

[Out] (I/12)\*b\*x^4 + (x^3\*ArcCoth[1 + I\*d - d\*Tan[a + b\*x]])/3 - (x^3\*Log[1 + (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/6 + ((I/4)\*x^2\*PolyLog[2, -((1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))])/b - (x\*PolyLog[3, -((1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))])/(4\*b^2) - ((I/8)\*PolyLog[4, -((1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))])/b^3

#### Rule 6264

Int[ArcCoth[(c\_.) + (d\_.)\*Tan[(a\_.) + (b\_.)\*(x\_)]]\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((e + f\*x)^(m + 1)\*ArcCoth[c + d\*Tan[a + b\*x]])/(f\*(m + 1)), x] + Dist[(I\*b)/(f\*(m + 1)), Int[(e + f\*x)^(m + 1)/(c + I\*d + c\*E^(2\*I\*a + 2\*I\*b\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c + I\*d)^2, 1]

#### Rule 2184

Int[(((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[(((c + d\*x)^m\*(F^(g\*(e + f\*x))))^n)/(a + b\*(F^(g\*(e + f\*x))))^n, x],

x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_))/  
((a\_) + (b\_)\*((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] :> Simp  
[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Di  
st[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)  
))^(n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_)\*((F\_)^((c\_)\*(a\_) + (b\_)\*(x\_)))^(n\_)]\*((f\_) + (g\_  
)\*(x\_))^(m\_), x\_Symbol] :> -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)  
))^(n)])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m -  
1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^(n)]], x], x] /; FreeQ[{F, a, b, c, e, f  
, g, n}, x] && GtQ[m, 0]

### Rule 6609

Int[((e\_) + (f\_)\*(x\_))^(m\_)\*PolyLog[n\_, (d\_)\*((F\_)^((c\_)\*(a\_) + (b\_  
)\*(x\_)))^(p\_)], x\_Symbol] :> Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a  
+ b\*x)))^p])/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(  
m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c,  
d, e, f, n, p}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]  
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi  
onOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[  
{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\*  
(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_)\*((a\_) + (b\_)\*(x\_))^(p\_)]/((d\_) + (e\_)\*(x\_)), x\_S  
ymbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d  
, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps



$$\begin{aligned}
\int x^2 \coth^{-1}(1 + id - d \tan(a + bx)) dx &= \frac{1}{3} x^3 \coth^{-1}(1 + id - d \tan(a + bx)) + \frac{1}{3} (ib) \int \frac{x^3}{1 + (1 + id)e^{2ia+2ibx}} dx \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 + id - d \tan(a + bx)) - \frac{1}{3} (b(i - d)) \int \frac{e^{2ia+2ibx} x^3}{1 + (1 + id)e^{2ia+2ibx}} dx \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 + id - d \tan(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 + id)e^{2ia+2ibx}) \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 + id - d \tan(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 + id)e^{2ia+2ibx}) \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 + id - d \tan(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 + id)e^{2ia+2ibx}) \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 + id - d \tan(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 + id)e^{2ia+2ibx}) \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 + id - d \tan(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 + id)e^{2ia+2ibx})
\end{aligned}$$

**Mathematica [A]** time = 0.176928, size = 156, normalized size = 0.91

$$\frac{1}{3} x^3 \coth^{-1}(d(-\tan(a + bx)) + id + 1) - \frac{6ib^2 x^2 \text{PolyLog}\left(2, \frac{ie^{-2i(a+bx)}}{d-i}\right) + 6bx \text{PolyLog}\left(3, \frac{ie^{-2i(a+bx)}}{d-i}\right) - 3i \text{PolyLog}\left(4, \frac{ie^{-2i(a+bx)}}{d-i}\right)}{24b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[1 + I\*d - d\*Tan[a + b\*x]], x]

[Out] (x^3\*ArcCoth[1 + I\*d - d\*Tan[a + b\*x]])/3 - (4\*b^3\*x^3\*Log[1 - I/((-I + d)\*E^((2\*I)\*(a + b\*x)))] + (6\*I)\*b^2\*x^2\*PolyLog[2, I/((-I + d)\*E^((2\*I)\*(a + b\*x)))] + 6\*b\*x\*PolyLog[3, I/((-I + d)\*E^((2\*I)\*(a + b\*x)))] - (3\*I)\*PolyLog[4, I/((-I + d)\*E^((2\*I)\*(a + b\*x)))])/(24\*b^3)

**Maple [C]** time = 29.632, size = 2449, normalized size = 14.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(1+I\*d-d\*tan(b\*x+a)), x)

```

[Out] -1/2*I/b^3*a^2*d/(-d+I)*dilog(1-I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))+1/2*I/b
^2/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b*x+a)))*x*a^2-1/6*x^3*ln(d)-1/12*I*x^3*Pi
*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))+1
)*exp(2*I*(b*x+a)))^2+1/12*I*b*x^4-1/2/b^2*d/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(
b*x+a)))*x*a^2+1/2/b^2*a^2*d/(-d+I)*ln(1+I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2)
)*x+1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I
*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+
a))+1))^2-1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(e
xp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I
*(b*x+a))+1))+1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b
*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2+1/12*I*x^3*Pi*csgn(I
*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))*d-I*ex
p(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2-1/2*I/b^2*a^2/(-d+I)*ln(1+I*exp(I
*(b*x+a))*(-I*(-d+I))^(1/2))*x+1/4*I/b^3*d/(-d+I)*polylog(2,I*(-d+I)*exp(2*
I*(b*x+a)))*a^2-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+
a))/(exp(2*I*(b*x+a))+1))^2-1/3*x^3*ln(exp(I*(b*x+a)))+1/6*x^3*ln(exp(2*I*(
b*x+a))*d-I*exp(2*I*(b*x+a))-I)-1/2*I/b^3*a^3/(-d+I)*ln(1-I*exp(I*(b*x+a))*
(-I*(-d+I))^(1/2))+1/6*I/b^3*a^3/(-d+I)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+
a))*d+I)+1/8*I/b^3*d/(-d+I)*polylog(4,I*(-d+I)*exp(2*I*(b*x+a)))+1/12*I*x^3
*Pi*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3+
1/6*d/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b*x+a)))*x^3-1/2/b^3*a^2/(-d+I)*dilog(1
+I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))-1/2/b^3*a^2/(-d+I)*dilog(1-I*exp(I*(b*
x+a))*(-I*(-d+I))^(1/2))-1/4/b/(-d+I)*polylog(2,I*(-d+I)*exp(2*I*(b*x+a)))*
x^2+1/4/b^3/(-d+I)*polylog(2,I*(-d+I)*exp(2*I*(b*x+a)))*a^2-1/12*I*x^3*Pi*c
sgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3+1/3
*I/b^3/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b*x+a)))*a^3+1/12*I*x^3*Pi*csgn(I*d)*c
sgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I*(b*x+a))+1)*
exp(2*I*(b*x+a)))+1/12*I*x^3*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*
x+a)))-1/6*I*x^3*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2-1/12*
I*x^3*Pi*csgn(I*d)*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2-1/12*I
*x^3*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I*(b*
x+a))+1)*exp(2*I*(b*x+a)))^2-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csg
n(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^2+1/8/b^3/(-d+I)*polylog(4,I*(-d
+I)*exp(2*I*(b*x+a)))+1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(
b*x+a)))^3-1/6*I/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b*x+a)))*x^3+1/2/b^2*a^2*d/(
-d+I)*ln(1-I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))*x-1/4*I/b*d/(-d+I)*polylog(2
,I*(-d+I)*exp(2*I*(b*x+a)))*x^2-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*
csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))*
d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))-1/2*I/b^2*a^2/(-d+I)*ln(1-I*
exp(I*(b*x+a))*(-I*(-d+I))^(1/2))*x+1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a)
+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))+1/2/b^3
*a^3*d/(-d+I)*ln(1+I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))+1/2/b^3*a^3*d/(-d+I)
*ln(1-I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))+1/4/b^2*d/(-d+I)*polylog(3,I*(-d+
I)*exp(2*I*(b*x+a)))*x-1/6/b^3*a^3*d/(-d+I)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(
b*x+a))*d+I)-1/3/b^3*d/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b*x+a)))*a^3-1/2*I/b^3

```

```
*a^2*d/(-d+I)*dilog(1+I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))-1/4*I/b^2/(-d+I)*
polylog(3,I*(-d+I)*exp(2*I*(b*x+a)))*x-1/2*I/b^3*a^3/(-d+I)*ln(1+I*exp(I*(b
*x+a))*(-I*(-d+I))^(1/2))-1/12*I*x^3*Pi*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*
(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2+1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+a))+
1)*exp(2*I*(b*x+a)))^2-1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b
*x+a)))^3+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))^3+1/12*I*x^3*Pi*csgn(I*exp
(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^3+1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a)
)+1))*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1)
)
```

**Maxima [B]** time = 1.21992, size = 459, normalized size = 2.68

$$\frac{12((bx+a)^3-3(bx+a)^2a+3(bx+a)a^2)\operatorname{arccoth}(d\tan(bx+a)-id-1)}{b^2} + \frac{-3i(bx+a)^4+12i(bx+a)^3a-18i(bx+a)^2a^2+(8i(bx+a)^3-18i(bx+a)^2a+18i(bx+a)a^2)\operatorname{arccot}(d\tan(bx+a)-id-1)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arccoth(d*tan(b
*x + a) - I*d - 1)/b^2 + (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
+ a)^2*a^2 + (8*I*(b*x + a)^3 - 18*I*(b*x + a)^2*a + 18*I*(b*x + a)*a^2)*a
rctan2(d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + cos(2*b
*x + 2*a) + 1) + (-12*I*(b*x + a)^2 + 18*I*(b*x + a)*a - 9*I*a^2)*dilog((-I
*d - 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(b*x +
a)*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^2 - 2
*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, (-
I*d - 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (-I*d - 1)*e^(2*I*b*x + 2*I*
a)))/b^2)/b
```

**Fricas [C]** time = 1.89045, size = 992, normalized size = 5.8

$$ib^4x^4 - 2b^3x^3 \log\left(\frac{de^{2ibx+2ia}}{(d-i)e^{2ibx+2ia}-i}\right) + 6ib^2x^2\operatorname{Li}_2\left(\frac{1}{2}\sqrt{-4id-4e^{ibx+ia}}\right) + 6ib^2x^2\operatorname{Li}_2\left(-\frac{1}{2}\sqrt{-4id-4e^{ibx+ia}}\right) - ia^4 + 2a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(I*b^4*x^4 - 2*b^3*x^3*log(d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*x + 2*I*a) - I)) + 6*I*b^2*x^2*dilog(1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) + 6*I*b^2*x^2*dilog(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - I*a^4 + 2*a^3*log(((2*d - 2*I)*e^(I*b*x + I*a) + I*sqrt(-4*I*d - 4))/(2*d - 2*I)) + 2*a^3*log(((2*d - 2*I)*e^(I*b*x + I*a) - I*sqrt(-4*I*d - 4))/(2*d - 2*I)) - 12*b*x*polylog(3, 1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - 12*b*x*polylog(3, -1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1) - 12*I*polylog(4, 1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - 12*I*polylog(4, -1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)))/b^3
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(1+I*d-d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arccoth}(-d \tan(bx + a) + id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(-d*tan(b*x + a) + I*d + 1), x)
```

### 3.245 $\int x \coth^{-1}(1 + id - d \tan(a + bx)) dx$

**Optimal.** Leaf size=134

$$-\frac{\text{PolyLog}\left(3, -(1 + id)e^{2ia+2ibx}\right)}{8b^2} + \frac{ix\text{PolyLog}\left(2, -(1 + id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{4}x^2 \log\left(1 + (1 + id)e^{2ia+2ibx}\right) + \frac{1}{2}x^2 \coth^{-1}(d(-$$

[Out] (I/6)\*b\*x^3 + (x^2\*ArcCoth[1 + I\*d - d\*Tan[a + b\*x]])/2 - (x^2\*Log[1 + (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/4 + ((I/4)\*x\*PolyLog[2, -((1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))])/b - PolyLog[3, -((1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))]/(8\*b^2)

**Rubi [A]** time = 0.24538, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {6264, 2184, 2190, 2531, 2282, 6589}

$$-\frac{\text{PolyLog}\left(3, -(1 + id)e^{2ia+2ibx}\right)}{8b^2} + \frac{ix\text{PolyLog}\left(2, -(1 + id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{4}x^2 \log\left(1 + (1 + id)e^{2ia+2ibx}\right) + \frac{1}{2}x^2 \coth^{-1}(d(-$$

Antiderivative was successfully verified.

[In] Int[x\*ArcCoth[1 + I\*d - d\*Tan[a + b\*x]], x]

[Out] (I/6)\*b\*x^3 + (x^2\*ArcCoth[1 + I\*d - d\*Tan[a + b\*x]])/2 - (x^2\*Log[1 + (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/4 + ((I/4)\*x\*PolyLog[2, -((1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))])/b - PolyLog[3, -((1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))]/(8\*b^2)

#### Rule 6264

Int[ArcCoth[(c\_.) + (d\_.)\*Tan[(a\_.) + (b\_.)\*(x\_)]]\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^(m + 1)\*ArcCoth[c + d\*Tan[a + b\*x]])/(f\*(m + 1)), x] + Dist[(I\*b)/(f\*(m + 1)), Int[(e + f\*x)^(m + 1)/(c + I\*d + c\*E^(2\*I\*a + 2\*I\*b\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c + I\*d)^2, 1]

#### Rule 2184

Int[(((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[(((c + d\*x)^m\*(F^(g\*(e + f\*x))))^n)/(a + b\*(F^(g\*(e + f\*x))))^n, x],

x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_)]/((a\_) + (b\_)\*((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_)\*((F\_)^((c\_)\*(a\_) + (b\_)\*(x\_)))^(n\_)]\*((f\_) + (g\_)\*(x\_))^(m\_), x\_Symbol] :> -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n])]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_)\*((a\_) + (b\_)\*(x\_))^(p\_)]/((d\_) + (e\_)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
\int x \coth^{-1}(1 + id - d \tan(a + bx)) dx &= \frac{1}{2}x^2 \coth^{-1}(1 + id - d \tan(a + bx)) + \frac{1}{2}(ib) \int \frac{x^2}{1 + (1 + id)e^{2ia+2ibx}} dx \\
&= \frac{1}{6}ibx^3 + \frac{1}{2}x^2 \coth^{-1}(1 + id - d \tan(a + bx)) - \frac{1}{2}(b(i - d)) \int \frac{e^{2ia+2ibx}x^2}{1 + (1 + id)e^{2ia+2ibx}} dx \\
&= \frac{1}{6}ibx^3 + \frac{1}{2}x^2 \coth^{-1}(1 + id - d \tan(a + bx)) - \frac{1}{4}x^2 \log(1 + (1 + id)e^{2ia+2ibx}) + \frac{1}{4}x^2 \log(1 + (1 + id)e^{2ia+2ibx}) \\
&= \frac{1}{6}ibx^3 + \frac{1}{2}x^2 \coth^{-1}(1 + id - d \tan(a + bx)) - \frac{1}{4}x^2 \log(1 + (1 + id)e^{2ia+2ibx}) + \frac{1}{4}x^2 \log(1 + (1 + id)e^{2ia+2ibx}) \\
&= \frac{1}{6}ibx^3 + \frac{1}{2}x^2 \coth^{-1}(1 + id - d \tan(a + bx)) - \frac{1}{4}x^2 \log(1 + (1 + id)e^{2ia+2ibx}) + \frac{1}{4}x^2 \log(1 + (1 + id)e^{2ia+2ibx}) \\
&= \frac{1}{6}ibx^3 + \frac{1}{2}x^2 \coth^{-1}(1 + id - d \tan(a + bx)) - \frac{1}{4}x^2 \log(1 + (1 + id)e^{2ia+2ibx}) + \frac{1}{4}x^2 \log(1 + (1 + id)e^{2ia+2ibx})
\end{aligned}$$

**Mathematica [A]** time = 0.108779, size = 120, normalized size = 0.9

$$\frac{1}{2}x^2 \coth^{-1}(d(-\tan(a + bx)) + id + 1) - \frac{2ibx \operatorname{PolyLog}\left(2, \frac{ie^{-2i(a+bx)}}{d-i}\right) + \operatorname{PolyLog}\left(3, \frac{ie^{-2i(a+bx)}}{d-i}\right) + 2b^2x^2 \log\left(1 - \frac{ie^{-2i(a+bx)}}{d-i}\right)}{8b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[1 + I\*d - d\*Tan[a + b\*x]], x]

[Out] (x^2\*ArcCoth[1 + I\*d - d\*Tan[a + b\*x]])/2 - (2\*b^2\*x^2\*Log[1 - I/((-I + d)\*E^((2\*I)\*(a + b\*x)))] + (2\*I)\*b\*x\*PolyLog[2, I/((-I + d)\*E^((2\*I)\*(a + b\*x)))] + PolyLog[3, I/((-I + d)\*E^((2\*I)\*(a + b\*x)))])/(8\*b^2)

**Maple [C]** time = 13.712, size = 2351, normalized size = 17.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(1+I\*d-d\*tan(b\*x+a)), x)

[Out] -1/8\*I\*x^2\*Pi\*csgn(I/(exp(2\*I\*(b\*x+a))+1))\*csgn(I\*(exp(2\*I\*(b\*x+a))\*d-I\*exp(2\*I\*(b\*x+a))-I))\*csgn(I\*(exp(2\*I\*(b\*x+a))\*d-I\*exp(2\*I\*(b\*x+a))-I)/(exp(2\*I

$$\begin{aligned}
&*(b*x+a)+1)+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^3- \\
&1/8*I*x^2*Pi*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a) \\
&)+1))^2+1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^3+1/8 \\
&*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2-1/8*I*x^2*Pi*csgn \\
&(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3+1/6*I* \\
&b*x^3-1/4*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2+1/8*I* \\
&x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b \\
&*x+a))-I)/(exp(2*I*(b*x+a))+1))^2-1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))+1)* \\
&exp(2*I*(b*x+a)))^3-1/8*I/b^2/(-d+I)*polylog(3,I*(-d+I)*exp(2*I*(b*x+a)))-1 \\
&/4*I/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b*x+a)))*x^2-1/4*x^2*ln(d)+1/8*I*x^2*Pi* \\
&csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))+1/4*x^2*ln(exp(2*I*(b*x+a) \\
&))*d-I*exp(2*I*(b*x+a))-I)+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))^3+1/8*I*x^ \\
&2*Pi*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3 \\
&+1/8/b^2*d/(-d+I)*polylog(3,I*(-d+I)*exp(2*I*(b*x+a)))+1/2/b^2*a/(-d+I)*dil \\
&og(1+I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))+1/2/b^2*a/(-d+I)*dilog(1-I*exp(I*( \\
&b*x+a))*(-I*(-d+I))^(1/2))-1/4/b/(-d+I)*polylog(2,I*(-d+I)*exp(2*I*(b*x+a) \\
&))*x-1/4/b^2/(-d+I)*polylog(2,I*(-d+I)*exp(2*I*(b*x+a)))*a+1/4*d/(-d+I)*ln(1 \\
&-I*(-d+I)*exp(2*I*(b*x+a)))*x^2-1/2*x^2*ln(exp(I*(b*x+a)))+1/2*I/b*a/(-d+I) \\
&*ln(1-I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))*x-1/2/b*a*d/(-d+I)*ln(1+I*exp(I*( \\
&b*x+a))*(-I*(-d+I))^(1/2))*x-1/2/b*a*d/(-d+I)*ln(1-I*exp(I*(b*x+a))*(-I*(-d \\
&+I))^(1/2))*x-1/4*I/b^2*d/(-d+I)*polylog(2,I*(-d+I)*exp(2*I*(b*x+a)))*a-1/2 \\
&*I/b/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b*x+a)))*x*a+1/2*I/b*a/(-d+I)*ln(1+I*exp \\
&(I*(b*x+a))*(-I*(-d+I))^(1/2))*x+1/2*I/b^2*a*d/(-d+I)*dilog(1-I*exp(I*(b*x+ \\
&a))*(-I*(-d+I))^(1/2))+1/8*I*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b \\
&*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a) \\
&)+1))^2-1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn( \\
&d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2+1/2*I/b^2*a*d/(-d+I)*dilog(1+I*exp \\
&(I*(b*x+a))*(-I*(-d+I))^(1/2))-1/4*I/b*d/(-d+I)*polylog(2,I*(-d+I)*exp(2* \\
&I*(b*x+a)))*x+1/2/b*d/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b*x+a)))*x*a-1/8*I*x^2* \\
&Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*c \\
&sgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))-1/8*I*x \\
&^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))/(exp(2*I*(b*x+a))+1) \\
&)^2-1/8*I*x^2*Pi*csgn(I*d)*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^ \\
&2+1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp( \\
&2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))-1/4*I/b^2/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b \\
&*x+a)))*a^2-1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a) \\
&))/(exp(2*I*(b*x+a))+1))^2+1/2*I/b^2*a^2/(-d+I)*ln(1+I*exp(I*(b*x+a))*(-I*( \\
&-d+I))^(1/2))+1/2*I/b^2*a^2/(-d+I)*ln(1-I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2)) \\
&-1/4*I/b^2*a^2/(-d+I)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d+I)+1/4/b^2*d \\
&/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b*x+a)))*a^2-1/2/b^2*a^2*d/(-d+I)*ln(1+I*exp \\
&(I*(b*x+a))*(-I*(-d+I))^(1/2))-1/2/b^2*a^2*d/(-d+I)*ln(1-I*exp(I*(b*x+a))* \\
&-I*(-d+I))^(1/2))+1/4/b^2*a^2*d/(-d+I)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a) \\
&))*d+I)+1/8*I*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp( \\
&2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b \\
&*x+a))+1))^2+1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+
\end{aligned}$$



a)))\*csgn(I\*exp(2\*I\*(b\*x+a))/(exp(2\*I\*(b\*x+a))+1))+1/8\*I\*x^2\*Pi\*csgn(I\*d)\*csgn(I\*exp(2\*I\*(b\*x+a))/(exp(2\*I\*(b\*x+a))+1))\*csgn(I\*d/(exp(2\*I\*(b\*x+a))+1)\*exp(2\*I\*(b\*x+a)))-1/8\*I\*x^2\*Pi\*csgn(I\*exp(2\*I\*(b\*x+a))/(exp(2\*I\*(b\*x+a))+1))\*csgn(I\*d/(exp(2\*I\*(b\*x+a))+1)\*exp(2\*I\*(b\*x+a)))^2

**Maxima [B]** time = 1.15077, size = 332, normalized size = 2.48

$$\frac{12((bx+a)^2-2(bx+a)a)\operatorname{arccoth}(d\tan(bx+a)-id-1)}{b} + \frac{-4i(bx+a)^3+12i(bx+a)^2a-6ibx\operatorname{Li}_2((-id-1)e^{2ibx+2ia})+(6i(bx+a)^2-12i(bx+a)a)\arctan(d\cos(2bx+2a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1+I\*d-d\*tan(b\*x+a)),x, algorithm="maxima")

[Out]  $-1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*\operatorname{arccoth}(d*\tan(b*x + a) - I*d - 1)/b + (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*\operatorname{dilog}((-I*d - 1)*e^{(2*I*b*x + 2*I*a)}) + (6*I*(b*x + a)^2 - 12*I*(b*x + a)*a)*\arctan2(d*\cos(2*b*x + 2*a) + \sin(2*b*x + 2*a), -d*\sin(2*b*x + 2*a) + \cos(2*b*x + 2*a) + 1) + 3*((b*x + a)^2 - 2*(b*x + a)*a)*\log((d^2 + 1)*\cos(2*b*x + 2*a)^2 + (d^2 + 1)*\sin(2*b*x + 2*a)^2 - 2*d*\sin(2*b*x + 2*a) + 2*\cos(2*b*x + 2*a) + 1) + 3*\operatorname{polylog}(3, (-I*d - 1)*e^{(2*I*b*x + 2*I*a)}))/b/b$

**Fricas [C]** time = 1.778, size = 821, normalized size = 6.13

$$2i b^3 x^3 - 3 b^2 x^2 \log\left(\frac{d e^{(2i b x + 2i a)}}{(d - i) e^{(2i b x + 2i a) - i}}\right) + 2i a^3 + 6i b x \operatorname{Li}_2\left(\frac{1}{2} \sqrt{-4i d - 4} e^{(i b x + i a)}\right) + 6i b x \operatorname{Li}_2\left(-\frac{1}{2} \sqrt{-4i d - 4} e^{(i b x + i a)}\right) - 3 a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1+I\*d-d\*tan(b\*x+a)),x, algorithm="fricas")

[Out]  $1/12*(2*I*b^3*x^3 - 3*b^2*x^2*\log(d*e^{(2*I*b*x + 2*I*a)})/((d - I)*e^{(2*I*b*x + 2*I*a)} - I)) + 2*I*a^3 + 6*I*b*x*\operatorname{dilog}(1/2*\sqrt{-4*I*d - 4}*e^{(I*b*x + I*a)}) + 6*I*b*x*\operatorname{dilog}(-1/2*\sqrt{-4*I*d - 4}*e^{(I*b*x + I*a)}) - 3*a^2*\log(((2*d - 2*I)*e^{(I*b*x + I*a)} + I*\sqrt{-4*I*d - 4})/(2*d - 2*I)) - 3*a^2*\log(((2*d - 2*I)*e^{(I*b*x + I*a)} - I*\sqrt{-4*I*d - 4})/(2*d - 2*I)) - 3*(b^2*x^2 - a^2)*\log(1/2*\sqrt{-4*I*d - 4}*e^{(I*b*x + I*a)} + 1) - 3*(b^2*x^2 - a^2)*\log(-1/2*\sqrt{-4*I*d - 4}*e^{(I*b*x + I*a)} + 1) - 6*\operatorname{polylog}(3, 1/2*\sqrt{-4*I*d - 4}*e^{(I*b*x + I*a)}) - 6*\operatorname{polylog}(3, -1/2*\sqrt{-4*I*d - 4}*e^{(I*b*x + I*a)})$

))/b^2

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(1+I\*d-d\*tan(b\*x+a)),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arccoth}(-d \tan(bx + a) + id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1+I\*d-d\*tan(b\*x+a)),x, algorithm="giac")

[Out] integrate(x\*arccoth(-d\*tan(b\*x + a) + I\*d + 1), x)

### 3.246 $\int \coth^{-1}(1 + id - d \tan(a + bx)) dx$

**Optimal.** Leaf size=94

$$\frac{i \operatorname{PolyLog}\left(2, -(1 + id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{2}x \log\left(1 + (1 + id)e^{2ia+2ibx}\right) + x \coth^{-1}(d(-\tan(a + bx)) + id + 1) + \frac{1}{2}ibx^2$$

[Out] (I/2)\*b\*x^2 + x\*ArcCoth[1 + I\*d - d\*Tan[a + b\*x]] - (x\*Log[1 + (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/2 + ((I/4)\*PolyLog[2, -((1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))])/b

**Rubi [A]** time = 0.156441, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {6256, 2184, 2190, 2279, 2391}

$$\frac{i \operatorname{PolyLog}\left(2, -(1 + id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{2}x \log\left(1 + (1 + id)e^{2ia+2ibx}\right) + x \coth^{-1}(d(-\tan(a + bx)) + id + 1) + \frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[1 + I\*d - d\*Tan[a + b\*x]], x]

[Out] (I/2)\*b\*x^2 + x\*ArcCoth[1 + I\*d - d\*Tan[a + b\*x]] - (x\*Log[1 + (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/2 + ((I/4)\*PolyLog[2, -((1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))])/b

#### Rule 6256

Int[ArcCoth[(c\_.) + (d\_.)\*Tan[(a\_.) + (b\_.)\*(x\_)]], x\_Symbol] := Simp[x\*ArcCoth[c + d\*Tan[a + b\*x]], x] + Dist[I\*b, Int[x/(c + I\*d + c\*E^(2\*I\*a + 2\*I\*b\*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I\*d)^2, 1]

#### Rule 2184

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.), x\_Symbol] := Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned} \int \coth^{-1}(1 + id - d \tan(a + bx)) dx &= x \coth^{-1}(1 + id - d \tan(a + bx)) + (ib) \int \frac{x}{1 + (1 + id)e^{2ia+2ibx}} dx \\ &= \frac{1}{2} ibx^2 + x \coth^{-1}(1 + id - d \tan(a + bx)) - (b(i - d)) \int \frac{e^{2ia+2ibx}}{1 + (1 + id)e^{2ia+2ibx}} dx \\ &= \frac{1}{2} ibx^2 + x \coth^{-1}(1 + id - d \tan(a + bx)) - \frac{1}{2} x \log(1 + (1 + id)e^{2ia+2ibx}) + \frac{1}{2} \int \log \left( \frac{1 + (1 + id)e^{2ia+2ibx}}{1 + (1 + id)} \right) dx \\ &= \frac{1}{2} ibx^2 + x \coth^{-1}(1 + id - d \tan(a + bx)) - \frac{1}{2} x \log(1 + (1 + id)e^{2ia+2ibx}) - \frac{i \operatorname{Subst} \left( \int \frac{1}{1 + (1 + id)e^{2ia+2ibx}} dx \right)}{2} \\ &= \frac{1}{2} ibx^2 + x \coth^{-1}(1 + id - d \tan(a + bx)) - \frac{1}{2} x \log(1 + (1 + id)e^{2ia+2ibx}) + \frac{i \operatorname{Li}_2 \left( -\frac{1 + (1 + id)e^{2ia+2ibx}}{1 + (1 + id)} \right)}{2} \end{aligned}$$

**Mathematica [B]** time = 2.82527, size = 723, normalized size = 7.69

$$x \coth^{-1}(d(-\tan(a + bx)) + id + 1) - \frac{x \sec(a + bx)(\cos(bx) + i \sin(bx))(\sin(bx) + i \cos(bx)) \left( -\operatorname{PolyLog} \left( 2, \frac{1}{2}(\cos(a) + i \sin(a)) \frac{1 + (1 + id)e^{2ia+2ibx}}{1 + (1 + id)} \right) \right)}{(\tan(a + bx) - i)(id \sin(a + bx) + (d - 2i) \cos(a + bx))} \left( -\frac{\sec^2(bx) \log \left( \frac{1 + (1 + id)e^{2ia+2ibx}}{1 + (1 + id)} \right)}{2} \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[1 + I*d - d*Tan[a + b*x]], x]
```

```
[Out] x*ArcCoth[1 + I*d - d*Tan[a + b*x]] - (x*((-2*I)*b*x*Log[2*Cos[b*x]*(Cos[b*x] - I*Sin[b*x])] + Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-2*I + d)*Cos[a + b*x] + I*d*Sin[a + b*x]))/(2*(-I + d))]*Log[1 - I*Tan[b*x]] - Log[(Sec[b*x]*((2 + I*d)*Cos[a + b*x] - d*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*Log[1 + I*Tan[b*x]] + PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] + PolyLog[2, (Sec[b*x]*(d*Cos[a] + (2 + I*d)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*(-I + d))] - PolyLog[2, ((Cos[a] + I*Sin[a])*(d*Cos[a] + (2 + I*d)*Sin[a])*(-I + Tan[b*x]))/2])*Sec[a + b*x]*(Cos[b*x] + I*Sin[b*x])*(I*Cos[b*x] + Sin[b*x]))/((( -2*I + d)*Cos[a + b*x] + I*d*Sin[a + b*x])*((I*Log[1 - I*Tan[b*x]])*Sec[b*x]*(d*Cos[a] + (2 + I*d)*Sin[a]))/((-2*I + d)*Cos[a + b*x] + I*d*Sin[a + b*x]) + (Log[1 + I*Tan[b*x]]*Sec[b*x]*((-I)*d*Cos[a] + (-2*I + d)*Sin[a]))/((-2*I + d)*Cos[a + b*x] + I*d*Sin[a + b*x]) - (Log[(Sec[b*x]*((2 + I*d)*Cos[a + b*x] - d*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*Sec[b*x]^2)/(-I + Tan[b*x]) + (Log[1 - ((Cos[a] + I*Sin[a])*(d*Cos[a] + (2 + I*d)*Sin[a])*(-I + Tan[b*x]))/2]*Sec[b*x]^2)/(-I + Tan[b*x]) - Log[1 - (Sec[b*x]*(d*Cos[a] + (2 + I*d)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*(-I + d))]*(-I + Tan[b*x]) + (Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-2*I + d)*Cos[a + b*x] + I*d*Sin[a + b*x]))/(2*(-I + d))]*Sec[b*x]^2)/(I + Tan[b*x]) + (2*I)*b*x*(I + Tan[b*x]))*(-I + Tan[a + b*x]))
```

**Maple [B]** time = 0.178, size = 297, normalized size = 3.2

$$\frac{\frac{i}{2} \operatorname{arccoth}(1 + id - d \tan(bx + a)) \ln(id + d \tan(bx + a))}{b} - \frac{\frac{i}{2} \operatorname{arccoth}(1 + id - d \tan(bx + a)) \ln(id - d \tan(bx + a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1+I*d-d*tan(b*x+a)),x)
```

```
[Out] 1/2*I/b*arccoth(1+I*d-d*tan(b*x+a))*ln(I*d+d*tan(b*x+a))-1/2*I/b*arccoth(1+I*d-d*tan(b*x+a))*ln(I*d-d*tan(b*x+a))-1/8*I/b*ln(I*d-d*tan(b*x+a))^2+1/4*I/b*dilog(1+1/2*I*d-1/2*d*tan(b*x+a))+1/4*I/b*ln(I*d-d*tan(b*x+a))*ln(1+1/2*I*d-1/2*d*tan(b*x+a))+1/4*I/b*dilog(1/2*I*(-I*d+d*tan(b*x+a))/d)+1/4*I/b*ln(I*d+d*tan(b*x+a))*ln(1/2*I*(-I*d+d*tan(b*x+a))/d)-1/4*I/b*dilog((-2-I*d+d*tan(b*x+a))/(-2*I*d-2))-1/4*I/b*ln(I*d+d*tan(b*x+a))*ln((-2-I*d+d*tan(b*x+a))/(-2*I*d-2))
```

**Maxima [B]** time = 1.59644, size = 355, normalized size = 3.78

$$4(bx + a)d \left( \frac{\log(d \tan(bx+a) - id - 2)}{d} - \frac{\log(\tan(bx+a) - i)}{d} \right) + d \left( -\frac{2i \left( \log(d \tan(bx+a) - id - 2) \log\left(-\frac{id \tan(bx+a) + d - 2i}{2d - 2i} + 1\right) + \text{Li}_2\left(\frac{id \tan(bx+a) + d - 2i}{2d - 2i}\right) \right)}{d} + 2i \right)$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+I\*d-d\*tan(b\*x+a)),x, algorithm="maxima")

[Out]  $-1/8*(4*(b*x + a)*d*(\log(d*\tan(b*x + a) - I*d - 2)/d - \log(\tan(b*x + a) - I)/d) + d*(-2*I*(\log(d*\tan(b*x + a) - I*d - 2)*\log(-(I*d*\tan(b*x + a) + d - 2*I)/(2*d - 2*I) + 1) + \text{dilog}((I*d*\tan(b*x + a) + d - 2*I)/(2*d - 2*I)))/d + (2*I*\log(d*\tan(b*x + a) - I*d - 2)*\log(\tan(b*x + a) - I) - I*\log(\tan(b*x + a) - I)^2)/d - 2*I*(\log(-1/2*d*\tan(b*x + a) + 1/2*I*d + 1)*\log(\tan(b*x + a) - I) + \text{dilog}(1/2*d*\tan(b*x + a) - 1/2*I*d))/d + 2*I*(\log(\tan(b*x + a) - I)*\log(-1/2*I*\tan(b*x + a) + 1/2) + \text{dilog}(1/2*I*\tan(b*x + a) + 1/2))/d) + 8*(b*x + a)*\text{arccoth}(d*\tan(b*x + a) - I*d - 1))/b$

---

**Fricas [B]** time = 1.66666, size = 610, normalized size = 6.49

$$ib^2x^2 - bx \log\left(\frac{de^{2ibx+2ia}}{(d-i)e^{2ibx+2ia}-i}\right) - ia^2 - (bx+a) \log\left(\frac{1}{2}\sqrt{-4id-4e^{ibx+ia}}+1\right) - (bx+a) \log\left(-\frac{1}{2}\sqrt{-4id-4e^{ibx+ia}}+1\right)$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+I\*d-d\*tan(b\*x+a)),x, algorithm="fricas")

[Out]  $1/2*(I*b^2*x^2 - b*x*\log(d*e^{(2*I*b*x + 2*I*a)/((d - I)*e^{(2*I*b*x + 2*I*a) - I})} - I*a^2 - (b*x + a)*\log(1/2*\text{sqrt}(-4*I*d - 4)*e^{(I*b*x + I*a) + 1}) - (b*x + a)*\log(-1/2*\text{sqrt}(-4*I*d - 4)*e^{(I*b*x + I*a) + 1}) + a*\log(((2*d - 2*I)*e^{(I*b*x + I*a) + I*\text{sqrt}(-4*I*d - 4)})/(2*d - 2*I)) + a*\log(((2*d - 2*I)*e^{(I*b*x + I*a) - I*\text{sqrt}(-4*I*d - 4)})/(2*d - 2*I)) + I*\text{dilog}(1/2*\text{sqrt}(-4*I*d - 4)*e^{(I*b*x + I*a)}) + I*\text{dilog}(-1/2*\text{sqrt}(-4*I*d - 4)*e^{(I*b*x + I*a)}))/b$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1+I*d-d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}(-d \tan (b x+a)+i d+1) d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(-d*tan(b*x + a) + I*d + 1), x)
```

$$3.247 \quad \int \frac{\coth^{-1}(1+id-d \tan(a+bx))}{x} dx$$

**Optimal.** Leaf size=23

$$\text{CannotIntegrate}\left(\frac{\coth^{-1}(d(-\tan(a+bx))+id+1)}{x}, x\right)$$

[Out] CannotIntegrate[ArcCoth[1 + I\*d - d\*Tan[a + b\*x]]/x, x]

**Rubi [A]** time = 0.0846946, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\coth^{-1}(1+id-d \tan(a+bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[ArcCoth[1 + I\*d - d\*Tan[a + b\*x]]/x,x]

[Out] Defer[Int][ArcCoth[1 + I\*d - d\*Tan[a + b\*x]]/x, x]

Rubi steps

$$\int \frac{\coth^{-1}(1+id-d \tan(a+bx))}{x} dx = \int \frac{\coth^{-1}(1+id-d \tan(a+bx))}{x} dx$$

**Mathematica [A]** time = 0.720247, size = 0, normalized size = 0.

$$\int \frac{\coth^{-1}(1+id-d \tan(a+bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcCoth[1 + I\*d - d\*Tan[a + b\*x]]/x,x]

[Out] Integrate[ArcCoth[1 + I\*d - d\*Tan[a + b\*x]]/x, x]



---

**Maple [A]** time = 0.44, size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(1 + id - d \tan(bx + a))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(1+I\*d-d\*tan(b\*x+a))/x,x)

[Out] int(arccoth(1+I\*d-d\*tan(b\*x+a))/x,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$-ibx + \frac{1}{4}(-i\pi - 4ia - 2 \log(d)) \log(x) + \frac{1}{2}i \int \frac{\arctan(d \cos(2bx + 2a) + \sin(2bx + 2a), -d \sin(2bx + 2a) + \cos(2bx + 2a))}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+I\*d-d\*tan(b\*x+a))/x,x, algorithm="maxima")

[Out] -I\*b\*x + 1/4\*(-I\*pi - 4\*I\*a - 2\*log(d))\*log(x) + 1/2\*I\*integrate(arctan2(d\*cos(2\*b\*x + 2\*a) + sin(2\*b\*x + 2\*a), -d\*sin(2\*b\*x + 2\*a) + cos(2\*b\*x + 2\*a) + 1)/x, x) + 1/4\*integrate(log((d^2 + 1)\*cos(2\*b\*x + 2\*a)^2 + (d^2 + 1)\*sin(2\*b\*x + 2\*a)^2 - 2\*d\*sin(2\*b\*x + 2\*a) + 2\*cos(2\*b\*x + 2\*a) + 1)/x, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\log\left(\frac{de^{(2ibx+2ia)}}{(d-i)e^{(2ibx+2ia)}-i}\right)}{2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+I\*d-d\*tan(b\*x+a))/x,x, algorithm="fricas")

[Out] integral(-1/2\*log(d\*e^(2\*I\*b\*x + 2\*I\*a)/((d - I)\*e^(2\*I\*b\*x + 2\*I\*a) - I))/x, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(1+I\*d-d\*tan(b\*x+a))/x,x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}\left(\frac{-d \tan(bx+a) + id + 1}{x}\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+I\*d-d\*tan(b\*x+a))/x,x, algorithm="giac")

[Out] integrate(arccoth(-d\*tan(b\*x + a) + I\*d + 1)/x, x)

### 3.248 $\int (e + fx)^3 \coth^{-1}(\cot(a + bx)) dx$

**Optimal.** Leaf size=302

$$\frac{3if^2(e + fx)\text{PolyLog}\left(4, -ie^{2i(a+bx)}\right)}{8b^3} - \frac{3if^2(e + fx)\text{PolyLog}\left(4, ie^{2i(a+bx)}\right)}{8b^3} + \frac{3f(e + fx)^2\text{PolyLog}\left(3, -ie^{2i(a+bx)}\right)}{8b^2} - \frac{3f(e + fx)^2\text{PolyLog}\left(3, ie^{2i(a+bx)}\right)}{8b^2}$$

```
[Out] ((e + f*x)^4*ArcCoth[Cot[a + b*x]])/(4*f) + ((I/4)*(e + f*x)^4*ArcTan[E^((2*I)*(a + b*x))])/f - ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))])/b + ((I/4)*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (3*f*(e + f*x)^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (3*f*(e + f*x)^2*PolyLog[3, I*E^((2*I)*(a + b*x))])/(8*b^2) + (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, (-I)*E^((2*I)*(a + b*x))])/b^3 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E^((2*I)*(a + b*x))])/b^3 - (3*f^3*PolyLog[5, (-I)*E^((2*I)*(a + b*x))])/(16*b^4) + (3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```

**Rubi [A]** time = 0.235077, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {6254, 4181, 2531, 6609, 2282, 6589}

$$\frac{3if^2(e + fx)\text{PolyLog}\left(4, -ie^{2i(a+bx)}\right)}{8b^3} - \frac{3if^2(e + fx)\text{PolyLog}\left(4, ie^{2i(a+bx)}\right)}{8b^3} + \frac{3f(e + fx)^2\text{PolyLog}\left(3, -ie^{2i(a+bx)}\right)}{8b^2} - \frac{3f(e + fx)^2\text{PolyLog}\left(3, ie^{2i(a+bx)}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*ArcCoth[Cot[a + b*x]], x]
```

```
[Out] ((e + f*x)^4*ArcCoth[Cot[a + b*x]])/(4*f) + ((I/4)*(e + f*x)^4*ArcTan[E^((2*I)*(a + b*x))])/f - ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))])/b + ((I/4)*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (3*f*(e + f*x)^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (3*f*(e + f*x)^2*PolyLog[3, I*E^((2*I)*(a + b*x))])/(8*b^2) + (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, (-I)*E^((2*I)*(a + b*x))])/b^3 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E^((2*I)*(a + b*x))])/b^3 - (3*f^3*PolyLog[5, (-I)*E^((2*I)*(a + b*x))])/(16*b^4) + (3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```

#### Rule 6254

```
Int[ArcCoth[Cot[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcCoth[Cot[a + b*x]]/(f*(m + 1)), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
```

e, f}, x] && IGtQ[m, 0]

### Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol]
:> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Dist
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_.)))^(n_.)]*((f_.) + (g_.)
*(x_.))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])/ (b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_.))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_.)))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/ (b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int (e+fx)^3 \coth^{-1}(\cot(a+bx)) dx &= \frac{(e+fx)^4 \coth^{-1}(\cot(a+bx))}{4f} - \frac{b \int (e+fx)^4 \sec(2a+2bx) dx}{4f} \\
&= \frac{(e+fx)^4 \coth^{-1}(\cot(a+bx))}{4f} + \frac{i(e+fx)^4 \tan^{-1}(e^{2i(a+bx)})}{4f} + \frac{1}{2} \int (e+fx)^3 \log\left(\frac{e^{2i(a+bx)} - 1}{e^{2i(a+bx)} + 1}\right) dx \\
&= \frac{(e+fx)^4 \coth^{-1}(\cot(a+bx))}{4f} + \frac{i(e+fx)^4 \tan^{-1}(e^{2i(a+bx)})}{4f} - \frac{i(e+fx)^3 \operatorname{Li}_2(-ie^{2i(a+bx)})}{4b} \\
&= \frac{(e+fx)^4 \coth^{-1}(\cot(a+bx))}{4f} + \frac{i(e+fx)^4 \tan^{-1}(e^{2i(a+bx)})}{4f} - \frac{i(e+fx)^3 \operatorname{Li}_2(-ie^{2i(a+bx)})}{4b} \\
&= \frac{(e+fx)^4 \coth^{-1}(\cot(a+bx))}{4f} + \frac{i(e+fx)^4 \tan^{-1}(e^{2i(a+bx)})}{4f} - \frac{i(e+fx)^3 \operatorname{Li}_2(-ie^{2i(a+bx)})}{4b} \\
&= \frac{(e+fx)^4 \coth^{-1}(\cot(a+bx))}{4f} + \frac{i(e+fx)^4 \tan^{-1}(e^{2i(a+bx)})}{4f} - \frac{i(e+fx)^3 \operatorname{Li}_2(-ie^{2i(a+bx)})}{4b} \\
&= \frac{(e+fx)^4 \coth^{-1}(\cot(a+bx))}{4f} + \frac{i(e+fx)^4 \tan^{-1}(e^{2i(a+bx)})}{4f} - \frac{i(e+fx)^3 \operatorname{Li}_2(-ie^{2i(a+bx)})}{4b}
\end{aligned}$$

**Mathematica [B]** time = 0.286991, size = 654, normalized size = 2.17

$$\frac{1}{4}x(6e^2fx + 4e^3 + 4ef^2x^2 + f^3x^3) \coth^{-1}(\cot(a+bx)) + \frac{6b^2e^2f \operatorname{PolyLog}(3, -ie^{2i(a+bx)}) - 6b^2e^2f \operatorname{PolyLog}(3, ie^{2i(a+bx)})}{4}$$

Antiderivative was successfully verified.

[In] Integrate[(e + f\*x)^3\*ArcCoth[Cot[a + b\*x]], x]

[Out] (x\*(4\*e^3 + 6\*e^2\*f\*x + 4\*e\*f^2\*x^2 + f^3\*x^3)\*ArcCoth[Cot[a + b\*x]])/4 + (-8\*b^4\*e^3\*x\*Log[1 - I\*E^((2\*I)\*(a + b\*x))] - 12\*b^4\*e^2\*f\*x^2\*Log[1 - I\*E^((2\*I)\*(a + b\*x))] - 8\*b^4\*e\*f^2\*x^3\*Log[1 - I\*E^((2\*I)\*(a + b\*x))] - 2\*b^4\*f^3\*x^4\*Log[1 - I\*E^((2\*I)\*(a + b\*x))] + 8\*b^4\*e^3\*x\*Log[1 + I\*E^((2\*I)\*(a + b\*x))] + 12\*b^4\*e^2\*f\*x^2\*Log[1 + I\*E^((2\*I)\*(a + b\*x))] + 8\*b^4\*e\*f^2\*x^3\*Log[1 + I\*E^((2\*I)\*(a + b\*x))] + 2\*b^4\*f^3\*x^4\*Log[1 + I\*E^((2\*I)\*(a + b\*x))] - (4\*I)\*b^3\*(e + f\*x)^3\*PolyLog[2, (-I)\*E^((2\*I)\*(a + b\*x))] + (4\*I)\*b^3\*(e + f\*x)^3\*PolyLog[2, I\*E^((2\*I)\*(a + b\*x))] + 6\*b^2\*e^2\*f\*PolyLog[3, (-I)\*E^((2\*I)\*(a + b\*x))] + 12\*b^2\*e\*f^2\*x\*PolyLog[3, (-I)\*E^((2\*I)\*(a + b\*x))] + 6\*b^2\*f^3\*x^2\*PolyLog[3, (-I)\*E^((2\*I)\*(a + b\*x))] - 6\*b^2\*e^2\*f\*PolyLog[3, I\*E^((2\*I)\*(a + b\*x))] - 12\*b^2\*e\*f^2\*x\*PolyLog[3, I\*E^((2\*I)\*(a + b\*x))] - 6\*b^2\*f^3\*x^2\*PolyLog[3, I\*E^((2\*I)\*(a + b\*x))] + (6\*I)\*b\*e\*f^2\*PolyLog[4, (-I)\*E^((2\*I)\*(a + b\*x))] + (6\*I)\*b\*f^3\*x\*PolyLog[4, (-I)\*E^((2\*I)\*(a + b\*x))] - (6\*I)\*b\*e\*f^2\*PolyLog[4, I\*E^((2\*I)\*(a + b\*x))] - (6\*I)\*b\*f^3

```
3*x*PolyLog[4, I*E^((2*I)*(a + b*x))] - 3*f^3*PolyLog[5, (-I)*E^((2*I)*(a +
b*x))] + 3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))]/(16*b^4)
```

**Maple [C]** time = 6.052, size = 7429, normalized size = 24.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*arccoth(cot(b*x+a)),x)
```

```
[Out] result too large to display
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{16} (f^3x^4 + 4ef^2x^3 + 6e^2fx^2 + 4e^3x) \log(2 \cos(2bx + 2a)^2 + 2 \sin(2bx + 2a)^2 + 4 \sin(2bx + 2a) + 2) - \frac{1}{16} (f^3x^4 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arccoth(cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/16*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*log(2*cos(2*b*x + 2*a)
^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/16*(f^3*x^4 + 4*e*f
^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*
a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(1/2*((b*f^3*x^4 + 4*b*e*f^2*x^3
+ 6*b*e^2*f*x^2 + 4*b*e^3*x)*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f^3*x^4
+ 4*b*e*f^2*x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*sin(4*b*x + 4*a)*sin(2*b*x +
2*a) + (b*f^3*x^4 + 4*b*e*f^2*x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*cos(2*b*x +
2*a))/(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x
)
```

**Fricas [C]** time = 2.68441, size = 3677, normalized size = 12.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*arccoth(cot(b\*x+a)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/32*(3*f^3*\text{polylog}(5, I*\cos(2*b*x + 2*a) + \sin(2*b*x + 2*a)) - 3*f^3*\text{poly} \\ & \log(5, I*\cos(2*b*x + 2*a) - \sin(2*b*x + 2*a)) + 3*f^3*\text{polylog}(5, -I*\cos(2*b \\ & *x + 2*a) + \sin(2*b*x + 2*a)) - 3*f^3*\text{polylog}(5, -I*\cos(2*b*x + 2*a) - \sin( \\ & 2*b*x + 2*a)) - (4*I*b^3*f^3*x^3 + 12*I*b^3*e*f^2*x^2 + 12*I*b^3*e^2*f*x + \\ & 4*I*b^3*e^3)*\text{dilog}(I*\cos(2*b*x + 2*a) + \sin(2*b*x + 2*a)) - (4*I*b^3*f^3*x^ \\ & 3 + 12*I*b^3*e*f^2*x^2 + 12*I*b^3*e^2*f*x + 4*I*b^3*e^3)*\text{dilog}(I*\cos(2*b*x \\ & + 2*a) - \sin(2*b*x + 2*a)) - (-4*I*b^3*f^3*x^3 - 12*I*b^3*e*f^2*x^2 - 12*I* \\ & b^3*e^2*f*x - 4*I*b^3*e^3)*\text{dilog}(-I*\cos(2*b*x + 2*a) + \sin(2*b*x + 2*a)) - \\ & (-4*I*b^3*f^3*x^3 - 12*I*b^3*e*f^2*x^2 - 12*I*b^3*e^2*f*x - 4*I*b^3*e^3)*\text{di} \\ & \log(-I*\cos(2*b*x + 2*a) - \sin(2*b*x + 2*a)) - 4*(b^4*f^3*x^4 + 4*b^4*e*f^2*x \\ & x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x)*\log((\cos(2*b*x + 2*a) + \sin(2*b*x + 2* \\ & a) + 1)/(\cos(2*b*x + 2*a) - \sin(2*b*x + 2*a) + 1)) - 2*(4*a*b^3*e^3 - 6*a^2 \\ & *b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*\log(\cos(2*b*x + 2*a) + I*\sin(2*b*x + \\ & 2*a) + I) + 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*\log \\ & (\cos(2*b*x + 2*a) - I*\sin(2*b*x + 2*a) + I) + 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x \\ & x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3 \\ & *b*e*f^2 - a^4*f^3)*\log(I*\cos(2*b*x + 2*a) + \sin(2*b*x + 2*a) + 1) - 2*(b^4 \\ & *f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - \\ & 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*\log(I*\cos(2*b*x + 2*a) - \sin(2*b \\ & *x + 2*a) + 1) + 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4 \\ & *e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*\log(-I*co \\ & s(2*b*x + 2*a) + \sin(2*b*x + 2*a) + 1) - 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + \\ & 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e* \\ & f^2 - a^4*f^3)*\log(-I*\cos(2*b*x + 2*a) - \sin(2*b*x + 2*a) + 1) - 2*(4*a*b^3 \\ & *e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*\log(-\cos(2*b*x + 2*a) + I \\ & *\sin(2*b*x + 2*a) + I) + 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - \\ & a^4*f^3)*\log(-\cos(2*b*x + 2*a) - I*\sin(2*b*x + 2*a) + I) - (-6*I*b*f^3*x - \\ & 6*I*b*e*f^2)*\text{polylog}(4, I*\cos(2*b*x + 2*a) + \sin(2*b*x + 2*a)) - (-6*I*b*f \\ & ^3*x - 6*I*b*e*f^2)*\text{polylog}(4, I*\cos(2*b*x + 2*a) - \sin(2*b*x + 2*a)) - (6* \\ & I*b*f^3*x + 6*I*b*e*f^2)*\text{polylog}(4, -I*\cos(2*b*x + 2*a) + \sin(2*b*x + 2*a)) \\ & - (6*I*b*f^3*x + 6*I*b*e*f^2)*\text{polylog}(4, -I*\cos(2*b*x + 2*a) - \sin(2*b*x + \\ & 2*a)) - 6*(b^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*\text{polylog}(3, I*\cos(2*b*x \\ & + 2*a) + \sin(2*b*x + 2*a)) + 6*(b^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*\text{p} \\ & \text{olylog}(3, I*\cos(2*b*x + 2*a) - \sin(2*b*x + 2*a)) - 6*(b^2*f^3*x^2 + 2*b^2*e \\ & *f^2*x + b^2*e^2*f)*\text{polylog}(3, -I*\cos(2*b*x + 2*a) + \sin(2*b*x + 2*a)) + 6* \\ & (b^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*\text{polylog}(3, -I*\cos(2*b*x + 2*a) - \\ & \sin(2*b*x + 2*a))/b^4 \end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (e + fx)^3 \operatorname{acoth}(\cot(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*acoth(cot(b\*x+a)),x)

[Out] Integral((e + f\*x)\*\*3\*acoth(cot(a + b\*x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (fx + e)^3 \operatorname{arccoth}(\cot(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*arccoth(cot(b\*x+a)),x, algorithm="giac")

[Out] integrate((f\*x + e)^3\*arccoth(cot(b\*x + a)), x)



### 3.249 $\int (e + fx)^2 \coth^{-1}(\cot(a + bx)) dx$

**Optimal.** Leaf size=234

$$\frac{f(e + fx)\text{PolyLog}(3, -ie^{2i(a+bx)})}{4b^2} - \frac{f(e + fx)\text{PolyLog}(3, ie^{2i(a+bx)})}{4b^2} + \frac{if^2\text{PolyLog}(4, -ie^{2i(a+bx)})}{8b^3} - \frac{if^2\text{PolyLog}(4, ie^{2i(a+bx)})}{8b^3}$$

```
[Out] ((e + f*x)^3*ArcCoth[Cot[a + b*x]])/(3*f) + ((I/3)*(e + f*x)^3*ArcTan[E^((2
*I)*(a + b*x))])/f - ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^((2*I)*(a + b*x)
)])/b + ((I/4)*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*(e + f*
x)*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(4*b^2) - (f*(e + f*x)*PolyLog[3,
I*E^((2*I)*(a + b*x))])/(4*b^2) + ((I/8)*f^2*PolyLog[4, (-I)*E^((2*I)*(a +
b*x))])/b^3 - ((I/8)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])/b^3
```

**Rubi [A]** time = 0.170911, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {6254, 4181, 2531, 6609, 2282, 6589}

$$\frac{f(e + fx)\text{PolyLog}(3, -ie^{2i(a+bx)})}{4b^2} - \frac{f(e + fx)\text{PolyLog}(3, ie^{2i(a+bx)})}{4b^2} + \frac{if^2\text{PolyLog}(4, -ie^{2i(a+bx)})}{8b^3} - \frac{if^2\text{PolyLog}(4, ie^{2i(a+bx)})}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*ArcCoth[Cot[a + b*x]], x]
```

```
[Out] ((e + f*x)^3*ArcCoth[Cot[a + b*x]])/(3*f) + ((I/3)*(e + f*x)^3*ArcTan[E^((2
*I)*(a + b*x))])/f - ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^((2*I)*(a + b*x)
)])/b + ((I/4)*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*(e + f*
x)*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(4*b^2) - (f*(e + f*x)*PolyLog[3,
I*E^((2*I)*(a + b*x))])/(4*b^2) + ((I/8)*f^2*PolyLog[4, (-I)*E^((2*I)*(a +
b*x))])/b^3 - ((I/8)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])/b^3
```

#### Rule 6254

```
Int[ArcCoth[Cot[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcCoth[Cot[a + b*x]]/(f*(m + 1)), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
e, f}, x] && IGtQ[m, 0]
```

#### Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol]
:> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f, x] + (-Dist
[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_.)))^(n_.)]*((f_.) + (g_.)
*(x_.))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_.))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_.)))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int (e + fx)^2 \coth^{-1}(\cot(a + bx)) dx &= \frac{(e + fx)^3 \coth^{-1}(\cot(a + bx))}{3f} - \frac{b \int (e + fx)^3 \sec(2a + 2bx) dx}{3f} \\
&= \frac{(e + fx)^3 \coth^{-1}(\cot(a + bx))}{3f} + \frac{i(e + fx)^3 \tan^{-1}(e^{2i(a+bx)})}{3f} + \frac{1}{2} \int (e + fx)^2 \log(1 - I e^{((2I)(a + bx))}) \\
&= \frac{(e + fx)^3 \coth^{-1}(\cot(a + bx))}{3f} + \frac{i(e + fx)^3 \tan^{-1}(e^{2i(a+bx)})}{3f} - \frac{i(e + fx)^2 \text{Li}_2(-ie^{2i(a+bx)})}{4b} \\
&= \frac{(e + fx)^3 \coth^{-1}(\cot(a + bx))}{3f} + \frac{i(e + fx)^3 \tan^{-1}(e^{2i(a+bx)})}{3f} - \frac{i(e + fx)^2 \text{Li}_2(-ie^{2i(a+bx)})}{4b} \\
&= \frac{(e + fx)^3 \coth^{-1}(\cot(a + bx))}{3f} + \frac{i(e + fx)^3 \tan^{-1}(e^{2i(a+bx)})}{3f} - \frac{i(e + fx)^2 \text{Li}_2(-ie^{2i(a+bx)})}{4b} \\
&= \frac{(e + fx)^3 \coth^{-1}(\cot(a + bx))}{3f} + \frac{i(e + fx)^3 \tan^{-1}(e^{2i(a+bx)})}{3f} - \frac{i(e + fx)^2 \text{Li}_2(-ie^{2i(a+bx)})}{4b}
\end{aligned}$$

**Mathematica [A]** time = 0.174746, size = 409, normalized size = 1.75

$$\frac{1}{3}x(3e^2 + 3efx + f^2x^2) \coth^{-1}(\cot(a + bx)) + \frac{-6ib^2(e + fx)^2 \text{PolyLog}(2, -ie^{2i(a+bx)}) + 6ib^2(e + fx)^2 \text{PolyLog}(2, ie^{2i(a+bx)})}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[(e + f\*x)^2\*ArcCoth[Cot[a + b\*x]],x]

[Out] (x\*(3\*e^2 + 3\*e\*f\*x + f^2\*x^2)\*ArcCoth[Cot[a + b\*x]])/3 + (-12\*b^3\*e^2\*x\*Log[1 - I\*E^((2\*I)\*(a + b\*x))] - 12\*b^3\*e\*f\*x^2\*Log[1 - I\*E^((2\*I)\*(a + b\*x))] - 4\*b^3\*f^2\*x^3\*Log[1 - I\*E^((2\*I)\*(a + b\*x))] + 12\*b^3\*e^2\*x\*Log[1 + I\*E^((2\*I)\*(a + b\*x))] + 12\*b^3\*e\*f\*x^2\*Log[1 + I\*E^((2\*I)\*(a + b\*x))] + 4\*b^3\*f^2\*x^3\*Log[1 + I\*E^((2\*I)\*(a + b\*x))] - (6\*I)\*b^2\*(e + f\*x)^2\*PolyLog[2, (-I)\*E^((2\*I)\*(a + b\*x))] + (6\*I)\*b^2\*(e + f\*x)^2\*PolyLog[2, I\*E^((2\*I)\*(a + b\*x))] + 6\*b\*e\*f\*PolyLog[3, (-I)\*E^((2\*I)\*(a + b\*x))] + 6\*b\*f^2\*x\*PolyLog[3, (-I)\*E^((2\*I)\*(a + b\*x))] - 6\*b\*e\*f\*PolyLog[3, I\*E^((2\*I)\*(a + b\*x))] - 6\*b\*f^2\*x\*PolyLog[3, I\*E^((2\*I)\*(a + b\*x))] + (3\*I)\*f^2\*PolyLog[4, (-I)\*E^((2\*I)\*(a + b\*x))] - (3\*I)\*f^2\*PolyLog[4, I\*E^((2\*I)\*(a + b\*x))])/(24\*b^3)

**Maple [C]** time = 13.674, size = 5543, normalized size = 23.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)^2*arccoth(cot(b*x+a)),x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{12} (f^2x^3 + 3efx^2 + 3e^2x) \log(2 \cos(2bx + 2a)^2 + 2 \sin(2bx + 2a)^2 + 4 \sin(2bx + 2a) + 2) - \frac{1}{12} (f^2x^3 + 3efx^2 + 3e^2x) \log(2 \cos(2bx + 2a)^2 + 2 \sin(2bx + 2a)^2 - 4 \sin(2bx + 2a) + 2) - \int \frac{(bf^2x^3 + 3befe^2x^2 + 3be^2x) \cos(4bx + 4a) \cos(2bx + 2a) + (bf^2x^3 + 3befe^2x^2 + 3be^2x) \sin(4bx + 4a) \sin(2bx + 2a) + (bf^2x^3 + 3befe^2x^2 + 3be^2x) \cos(2bx + 2a)}{(\cos(4bx + 4a)^2 + \sin(4bx + 4a)^2 + 2 \cos(4bx + 4a) + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^2*arccoth(cot(b*x+a)),x, algorithm="maxima")`

[Out] `1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(2/3*((b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(2*b*x + 2*a))/(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)`

**Fricas [C]** time = 2.43799, size = 2646, normalized size = 11.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^2*arccoth(cot(b*x+a)),x, algorithm="fricas")`

[Out] `1/48*(-3*I*f^2*polylog(4, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - 3*I*f^2*polylog(4, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + 3*I*f^2*polylog(4, -I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + 3*I*f^2*polylog(4, -I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + (6*I*b^2*f^2*x^2 + 12*I*b^2*e*f*x + 6*I*b^2*e^2)*dilog(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + (6*I*b^2*f^2*x^2 + 12*I*b^2*e*f*x + 6*I*b^2*e^2)*dilog(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + (-6*I*b^2*f^2*x^2 - 12*I*b^2*e*f*x - 6*I*b^2*e^2)*dilog(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + (-6*I*b^2*f^2*x^2 - 12*I*b^2*e*f*x - 6*I*b^2*e^2)*dilog(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))`

$s(2bx + 2a) - \sin(2bx + 2a)) + 8*(b^3f^2x^3 + 3b^3e*fx^2 + 3b^3e^2x)*\log((\cos(2bx + 2a) + \sin(2bx + 2a) + 1)/(\cos(2bx + 2a) - \sin(2bx + 2a) + 1)) + 4*(3a*b^2e^2 - 3a^2*b*ef + a^3*f^2)*\log(\cos(2bx + 2a) + I*\sin(2bx + 2a) + I) - 4*(3a*b^2e^2 - 3a^2*b*ef + a^3*f^2)*\log(\cos(2bx + 2a) - I*\sin(2bx + 2a) + I) - 4*(b^3f^2x^3 + 3b^3e*fx^2 + 3b^3e^2x + 3a*b^2e^2 - 3a^2*b*ef + a^3*f^2)*\log(I*\cos(2bx + 2a) + \sin(2bx + 2a) + 1) + 4*(b^3f^2x^3 + 3b^3e*fx^2 + 3b^3e^2x + 3a*b^2e^2 - 3a^2*b*ef + a^3*f^2)*\log(I*\cos(2bx + 2a) - \sin(2bx + 2a) + 1) - 4*(b^3f^2x^3 + 3b^3e*fx^2 + 3b^3e^2x + 3a*b^2e^2 - 3a^2*b*ef + a^3*f^2)*\log(-I*\cos(2bx + 2a) + \sin(2bx + 2a) + 1) + 4*(b^3f^2x^3 + 3b^3e*fx^2 + 3b^3e^2x + 3a*b^2e^2 - 3a^2*b*ef + a^3*f^2)*\log(-I*\cos(2bx + 2a) - \sin(2bx + 2a) + 1) + 4*(3a*b^2e^2 - 3a^2*b*ef + a^3*f^2)*\log(-\cos(2bx + 2a) + I*\sin(2bx + 2a) + I) - 4*(3a*b^2e^2 - 3a^2*b*ef + a^3*f^2)*\log(-\cos(2bx + 2a) - I*\sin(2bx + 2a) + I) + 6*(b*f^2x + b*ef)*polylog(3, I*\cos(2bx + 2a) + \sin(2bx + 2a)) - 6*(b*f^2x + b*ef)*polylog(3, I*\cos(2bx + 2a) - \sin(2bx + 2a)) + 6*(b*f^2x + b*ef)*polylog(3, -I*\cos(2bx + 2a) + \sin(2bx + 2a)) - 6*(b*f^2x + b*ef)*polylog(3, -I*\cos(2bx + 2a) - \sin(2bx + 2a)))/b^3$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (e + fx)^2 \operatorname{acoth}(\cot(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*acoth(cot(b\*x+a)),x)

[Out] Integral((e + f\*x)\*\*2\*acoth(cot(a + b\*x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (fx + e)^2 \operatorname{arccoth}(\cot(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*arccoth(cot(b\*x+a)),x, algorithm="giac")

[Out] integrate((f\*x + e)^2\*arccoth(cot(b\*x + a)), x)

### 3.250 $\int (e + fx) \coth^{-1}(\cot(a + bx)) dx$

**Optimal.** Leaf size=162

$$\frac{f \operatorname{PolyLog}\left(3, -ie^{2i(a+bx)}\right)}{8b^2} - \frac{f \operatorname{PolyLog}\left(3, ie^{2i(a+bx)}\right)}{8b^2} - \frac{i(e+fx) \operatorname{PolyLog}\left(2, -ie^{2i(a+bx)}\right)}{4b} + \frac{i(e+fx) \operatorname{PolyLog}\left(2, ie^{2i(a+bx)}\right)}{4b}$$

[Out]  $((e + f*x)^2 \operatorname{ArcCoth}[\operatorname{Cot}[a + b*x]])/(2*f) + ((I/2)*(e + f*x)^2 \operatorname{ArcTan}[E^{((2*I)*(a + b*x))}]/f - ((I/4)*(e + f*x)*\operatorname{PolyLog}[2, (-I)*E^{((2*I)*(a + b*x))}]]/b + ((I/4)*(e + f*x)*\operatorname{PolyLog}[2, I*E^{((2*I)*(a + b*x))}]]/b + (f*\operatorname{PolyLog}[3, (-I)*E^{((2*I)*(a + b*x))}]]/(8*b^2) - (f*\operatorname{PolyLog}[3, I*E^{((2*I)*(a + b*x))}]]/(8*b^2))$

**Rubi [A]** time = 0.110149, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {6254, 4181, 2531, 2282, 6589}

$$\frac{f \operatorname{PolyLog}\left(3, -ie^{2i(a+bx)}\right)}{8b^2} - \frac{f \operatorname{PolyLog}\left(3, ie^{2i(a+bx)}\right)}{8b^2} - \frac{i(e+fx) \operatorname{PolyLog}\left(2, -ie^{2i(a+bx)}\right)}{4b} + \frac{i(e+fx) \operatorname{PolyLog}\left(2, ie^{2i(a+bx)}\right)}{4b}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e + f*x)*\operatorname{ArcCoth}[\operatorname{Cot}[a + b*x]], x]$

[Out]  $((e + f*x)^2 \operatorname{ArcCoth}[\operatorname{Cot}[a + b*x]])/(2*f) + ((I/2)*(e + f*x)^2 \operatorname{ArcTan}[E^{((2*I)*(a + b*x))}]/f - ((I/4)*(e + f*x)*\operatorname{PolyLog}[2, (-I)*E^{((2*I)*(a + b*x))}]]/b + ((I/4)*(e + f*x)*\operatorname{PolyLog}[2, I*E^{((2*I)*(a + b*x))}]]/b + (f*\operatorname{PolyLog}[3, (-I)*E^{((2*I)*(a + b*x))}]]/(8*b^2) - (f*\operatorname{PolyLog}[3, I*E^{((2*I)*(a + b*x))}]]/(8*b^2))$

#### Rule 6254

$\operatorname{Int}[\operatorname{ArcCoth}[\operatorname{Cot}[(a_.) + (b_.)*(x_.)]]*((e_.) + (f_.)*(x_.))^{(m_.)}, x\_Symbol]$   
 $\rightarrow \operatorname{Simp}[(e + f*x)^{(m + 1)}*\operatorname{ArcCoth}[\operatorname{Cot}[a + b*x]]/(f*(m + 1)), x] - \operatorname{Dist}[b/(f*(m + 1)), \operatorname{Int}[(e + f*x)^{(m + 1)}*\operatorname{Sec}[2*a + 2*b*x], x], x] /;$   $\operatorname{FreeQ}\{a, b, e, f\}, x \ \&\& \ \operatorname{IGtQ}[m, 0]$

#### Rule 4181

$\operatorname{Int}[\operatorname{csc}[(e_.) + \operatorname{Pi}*(k_.) + (f_.)*(x_.)]]*((c_.) + (d_.)*(x_.))^{(m_.)}, x\_Symbol]$   
 $\rightarrow \operatorname{Simp}[(-2*(c + d*x)^m*\operatorname{ArcTanh}[E^{(I*k*Pi)}*E^{(I*(e + f*x))}]]/f, x] + (-\operatorname{Di}$

```
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)^v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int (e + fx) \coth^{-1}(\cot(a + bx)) dx &= \frac{(e + fx)^2 \coth^{-1}(\cot(a + bx))}{2f} - \frac{b \int (e + fx)^2 \sec(2a + 2bx) dx}{2f} \\
&= \frac{(e + fx)^2 \coth^{-1}(\cot(a + bx))}{2f} + \frac{i(e + fx)^2 \tan^{-1}(e^{2i(a+bx)})}{2f} + \frac{1}{2} \int (e + fx) \log(1 \\
&= \frac{(e + fx)^2 \coth^{-1}(\cot(a + bx))}{2f} + \frac{i(e + fx)^2 \tan^{-1}(e^{2i(a+bx)})}{2f} - \frac{i(e + fx) \operatorname{Li}_2(-ie^{2i(a+bx)})}{4b} \\
&= \frac{(e + fx)^2 \coth^{-1}(\cot(a + bx))}{2f} + \frac{i(e + fx)^2 \tan^{-1}(e^{2i(a+bx)})}{2f} - \frac{i(e + fx) \operatorname{Li}_2(-ie^{2i(a+bx)})}{4b} \\
&= \frac{(e + fx)^2 \coth^{-1}(\cot(a + bx))}{2f} + \frac{i(e + fx)^2 \tan^{-1}(e^{2i(a+bx)})}{2f} - \frac{i(e + fx) \operatorname{Li}_2(-ie^{2i(a+bx)})}{4b}
\end{aligned}$$

**Mathematica [A]** time = 0.120911, size = 263, normalized size = 1.62

$$-be \left( \frac{i \operatorname{PolyLog}(2, -ie^{i(2a+2bx)})}{4b^2} - \frac{i \operatorname{PolyLog}(2, ie^{i(2a+2bx)})}{4b^2} - \frac{ix \tan^{-1}(e^{2ia+2ibx})}{b} \right) + \frac{f(2ibx \operatorname{PolyLog}(2, -\sin(2(a+bx))) + \dots}{\dots}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(e + f\*x)\*ArcCoth[Cot[a + b\*x]], x]

[Out] e\*x\*ArcCoth[Cot[a + b\*x]] + (f\*x^2\*ArcCoth[Cot[a + b\*x]])/2 - b\*e\*((( -I)\*x\*ArcTan[E^((2\*I)\*a + (2\*I)\*b\*x)]/b + ((I/4)\*PolyLog[2, (-I)\*E^(I\*(2\*a + 2\*b\*x))])/b^2 - ((I/4)\*PolyLog[2, I\*E^(I\*(2\*a + 2\*b\*x))])/b^2) + (f\*((4\*I)\*b^2\*x^2\*ArcTan[Cos[2\*(a + b\*x)] + I\*Sin[2\*(a + b\*x)]] + (2\*I)\*b\*x\*PolyLog[2, I\*Cos[2\*(a + b\*x)] - Sin[2\*(a + b\*x)]] - (2\*I)\*b\*x\*PolyLog[2, (-I)\*Cos[2\*(a + b\*x)] + Sin[2\*(a + b\*x)]] - PolyLog[3, I\*Cos[2\*(a + b\*x)] - Sin[2\*(a + b\*x)]] + PolyLog[3, (-I)\*Cos[2\*(a + b\*x)] + Sin[2\*(a + b\*x)]]))/ (8\*b^2)

**Maple [C]** time = 10.737, size = 2543, normalized size = 15.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)\*arccoth(cot(b\*x+a)), x)

[Out] -1/4\*ln(exp(2\*I\*(b\*x+a))-I)\*x^2\*f-1/2\*ln(exp(2\*I\*(b\*x+a))-I)\*x\*e-1/2\*I/b^2\*f\*a\*dilog((( -I)^(1/2)-exp(I\*(b\*x+a)))/(-I)^(1/2))-1/2\*I/b^2\*f\*a\*dilog((( -I)^(1/2)+exp(I\*(b\*x+a)))/(-I)^(1/2))+1/2\*I/b\*e\*(I\*b\*x+I\*a)\*ln((( -I)^(1/2)-exp(I\*(b\*x+a)))/(-I)^(1/2))-1/2\*I/b\*e\*dilog(1+exp(I\*(b\*x+a))\*(-1)^(3/4))-1/2\*I/b\*e\*dilog(1-exp(I\*(b\*x+a))\*(-1)^(3/4))+1/2\*I/b\*e\*dilog((( -I)^(1/2)-exp(I\*(b\*x+a)))/(-I)^(1/2))+1/8\*f\*polylog(3, -I\*exp(2\*I\*(b\*x+a)))/b^2-1/8\*f\*polylog(3, I\*exp(2\*I\*(b\*x+a)))/b^2+1/4/b^2\*f\*(I\*b\*x+I\*a)^2\*ln(1-I\*exp(2\*I\*(b\*x+a)))+1/4/b^2\*f\*(I\*b\*x+I\*a)\*polylog(2, I\*exp(2\*I\*(b\*x+a)))-1/4/b^2\*f\*(I\*b\*x+I\*a)^2\*ln(1+I\*exp(2\*I\*(b\*x+a)))-1/4/b^2\*f\*(I\*b\*x+I\*a)\*polylog(2, -I\*exp(2\*I\*(b\*x+a)))+1/8\*I\*Pi\*f\*csgn(I\*(exp(2\*I\*(b\*x+a))-I)/(exp(2\*I\*(b\*x+a))-1))^3\*x^2+1/4\*I\*Pi\*x\*e\*csgn((1-I)\*(exp(2\*I\*(b\*x+a))+I)/(exp(2\*I\*(b\*x+a))-1))^3+1/8\*I\*Pi\*f\*csgn((1-I)\*(exp(2\*I\*(b\*x+a))+I)/(exp(2\*I\*(b\*x+a))-1))^3\*x^2-1/4\*I\*Pi\*x\*e\*csgn(I/(exp(2\*I\*(b\*x+a))-1))\*csgn(I\*(exp(2\*I\*(b\*x+a))-I)/(exp(2\*I\*(b\*x+a))-1))^2-1/4\*I\*Pi\*x\*e-1/8\*I\*Pi\*f\*x^2-1/8\*I\*Pi\*f\*csgn(I/(exp(2\*I\*(b\*x+a))-1))\*csgn(I\*(exp(2\*I\*(b\*x+a))-I)/(exp(2\*I\*(b\*x+a))-1))^2\*x^2+1/2\*I/b\*e\*dilog((( -I)^(1/2)+exp(I\*(b\*x+a)))/(-I)^(1/2))+1/4/b^2\*f\*a^2\*ln(-exp(2\*I\*(b\*x+a))+I)-1



$$\begin{aligned}
& /2/b^2 * a * \ln(-\exp(2*I*(b*x+a))+I)+1/2/b^2 * a * \ln(\exp(2*I*(b*x+a))+I)-1/4/b^2 * f \\
& * a^2 * \ln(\exp(2*I*(b*x+a))+I)+1/2*I/b^2 * e * (I*b*x+I*a) * \ln((( -I)^{(1/2)} + \exp(I*(b*x \\
& +a)))) / (-I)^{(1/2)} - 1/2*I/b^2 * e * (I*b*x+I*a) * \ln(1+\exp(I*(b*x+a)) * (-1)^{(3/4)}) - 1/2 \\
& * I/b^2 * e * (I*b*x+I*a) * \ln(1-\exp(I*(b*x+a)) * (-1)^{(3/4)}) + 1/2*I/b^2 * f * a * \operatorname{dilog}(1+\exp \\
& (I*(b*x+a)) * (-1)^{(3/4)}) + (1/4*f*x^2+1/2*e*x) * \ln(\exp(2*I*(b*x+a))+I)+1/8*I*P \\
& i*f * \operatorname{csgn}(I*(\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))-1)) * \operatorname{csgn}((1-I)*(\exp(2*I*( \\
& b*x+a))+I)/(\exp(2*I*(b*x+a))-1))^2 * x^2+1/4*I*Pi*x*e * \operatorname{csgn}(I/(\exp(2*I*(b*x+a) \\
& )-1)) * \operatorname{csgn}(I*(\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))-1))^2+1/4*I*Pi*x*e * \operatorname{csgn} \\
& (I*(\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x+a))-1))^3+1/8*I*Pi*f * \operatorname{csgn}(I*(\exp(2*I*( \\
& b*x+a))+I)) * \operatorname{csgn}(I*(\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))-1))^2 * x^2+1/2*I/ \\
& b^2 * f * a * \operatorname{dilog}(1-\exp(I*(b*x+a)) * (-1)^{(3/4)}) + 1/4*I*Pi*x*e * \operatorname{csgn}(I*(\exp(2*I*(b* \\
& x+a))-I)) * \operatorname{csgn}(I/(\exp(2*I*(b*x+a))-1)) * \operatorname{csgn}(I*(\exp(2*I*(b*x+a))-I)/(\exp(2*I \\
& *(b*x+a))-1))+1/8*I*Pi*f * \operatorname{csgn}(I*(\exp(2*I*(b*x+a))-I)) * \operatorname{csgn}(I/(\exp(2*I*(b*x+ \\
& a))-1)) * \operatorname{csgn}(I*(\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x+a))-1)) * x^2-1/2*I/b^2 * f * a \\
& * (I*b*x+I*a) * \ln((( -I)^{(1/2)} - \exp(I*(b*x+a)))) / (-I)^{(1/2)} - 1/2*I/b^2 * f * a * (I*b \\
& x+I*a) * \ln((( -I)^{(1/2)} + \exp(I*(b*x+a)))) / (-I)^{(1/2)} - 1/4*I*Pi*x*e * \operatorname{csgn}(I*(\exp( \\
& 2*I*(b*x+a))-I)) * \operatorname{csgn}(I*(\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x+a))-1))^2-1/8*I* \\
& Pi*f * \operatorname{csgn}(I*(\exp(2*I*(b*x+a))-I)) * \operatorname{csgn}(I*(\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x \\
& +a))-1))^2 * x^2+1/8*I*Pi*f * \operatorname{csgn}(I*(\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x+a))-1)) \\
& * \operatorname{csgn}((1+I)*(\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x+a))-1)) * x^2+1/4*I*Pi*x*e * \operatorname{csgn} \\
& (I*(\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))-1)) * \operatorname{csgn}((1-I)*(\exp(2*I*(b*x+a)) \\
& +I)/(\exp(2*I*(b*x+a))-1))^2+1/4*I*Pi*x*e * \operatorname{csgn}(I*(\exp(2*I*(b*x+a))+I)) * \operatorname{csgn}( \\
& I*(\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))-1))^2+1/8*I*Pi*f * \operatorname{csgn}(I/(\exp(2*I*( \\
& b*x+a))-1)) * \operatorname{csgn}(I*(\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))-1))^2 * x^2+1/2*I/b \\
& ^2 * f * a * (I*b*x+I*a) * \ln(1+\exp(I*(b*x+a)) * (-1)^{(3/4)}) + 1/2*I/b^2 * f * a * (I*b*x+I*a \\
& ) * \ln(1-\exp(I*(b*x+a)) * (-1)^{(3/4)}) - 1/4*I*Pi*x*e * \operatorname{csgn}((1-I)*(\exp(2*I*(b*x+a)) \\
& +I)/(\exp(2*I*(b*x+a))-1))^2-1/8*I*Pi*f * \operatorname{csgn}(I*(\exp(2*I*(b*x+a))+I)/(\exp(2*I \\
& *(b*x+a))-1))^3 * x^2-1/4*I*Pi*x*e * \operatorname{csgn}(I*(\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+ \\
& a))-1))^3+1/4*I*Pi*x*e * \operatorname{csgn}(I*(\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x+a))-1)) * \operatorname{cs \\
& gn}((1+I)*(\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x+a))-1))+1/4*I*Pi*x*e * \operatorname{csgn}((1+I) \\
& *(\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x+a))-1))^2+1/8*I*Pi*f * \operatorname{csgn}((1+I)*(\exp(2* \\
& I*(b*x+a))-I)/(\exp(2*I*(b*x+a))-1))^2 * x^2-1/8*I*Pi*f * \operatorname{csgn}((1-I)*(\exp(2*I*(b \\
& *x+a))+I)/(\exp(2*I*(b*x+a))-1))^2 * x^2-1/8*I*Pi*f * \operatorname{csgn}((1+I)*(\exp(2*I*(b*x+a) \\
& )-I)/(\exp(2*I*(b*x+a))-1))^3 * x^2-1/4*I*Pi*x*e * \operatorname{csgn}((1+I)*(\exp(2*I*(b*x+a)) \\
& -I)/(\exp(2*I*(b*x+a))-1))^3-1/8*I*Pi*f * \operatorname{csgn}(I*(\exp(2*I*(b*x+a))+I)) * \operatorname{csgn}(I/ \\
& (\exp(2*I*(b*x+a))-1)) * \operatorname{csgn}(I*(\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))-1)) * x^2 \\
& -1/4*I*Pi*x*e * \operatorname{csgn}(I*(\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x+a))-1)) * \operatorname{csgn}((1+I)* \\
& (\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x+a))-1))^2-1/4*I*Pi*x*e * \operatorname{csgn}(I*(\exp(2*I*( \\
& b*x+a))+I)/(\exp(2*I*(b*x+a))-1)) * \operatorname{csgn}((1-I)*(\exp(2*I*(b*x+a))+I)/(\exp(2*I*( \\
& b*x+a))-1))-1/4*I*Pi*x*e * \operatorname{csgn}(I*(\exp(2*I*(b*x+a))+I)) * \operatorname{csgn}(I/(\exp(2*I*(b*x+ \\
& a))-1)) * \operatorname{csgn}(I*(\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))-1))-1/8*I*Pi*f * \operatorname{csgn}(I \\
& *(\exp(2*I*(b*x+a))-I)/(\exp(2*I*(b*x+a))-1)) * \operatorname{csgn}((1+I)*(\exp(2*I*(b*x+a))-I) \\
& /(\exp(2*I*(b*x+a))-1))^2 * x^2-1/8*I*Pi*f * \operatorname{csgn}(I*(\exp(2*I*(b*x+a))+I)/(\exp(2* \\
& I*(b*x+a))-1)) * \operatorname{csgn}((1-I)*(\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))-1)) * x^2
\end{aligned}$$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{8} (fx^2 + 2ex) \log(2 \cos(2bx + 2a)^2 + 2 \sin(2bx + 2a)^2 + 4 \sin(2bx + 2a) + 2) - \frac{1}{8} (fx^2 + 2ex) \log(2 \cos(2bx + 2a)^2 + 2 \sin(2bx + 2a)^2 - 4 \sin(2bx + 2a) + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*arccoth(cot(b\*x+a)),x, algorithm="maxima")

[Out] 1/8\*(f\*x^2 + 2\*e\*x)\*log(2\*cos(2\*b\*x + 2\*a)^2 + 2\*sin(2\*b\*x + 2\*a)^2 + 4\*sin(2\*b\*x + 2\*a) + 2) - 1/8\*(f\*x^2 + 2\*e\*x)\*log(2\*cos(2\*b\*x + 2\*a)^2 + 2\*sin(2\*b\*x + 2\*a)^2 - 4\*sin(2\*b\*x + 2\*a) + 2) - integrate(((b\*f\*x^2 + 2\*b\*e\*x)\*cos(4\*b\*x + 4\*a)\*cos(2\*b\*x + 2\*a) + (b\*f\*x^2 + 2\*b\*e\*x)\*sin(4\*b\*x + 4\*a)\*sin(2\*b\*x + 2\*a) + (b\*f\*x^2 + 2\*b\*e\*x)\*cos(2\*b\*x + 2\*a))/(cos(4\*b\*x + 4\*a)^2 + sin(4\*b\*x + 4\*a)^2 + 2\*cos(4\*b\*x + 4\*a) + 1), x)

---

**Fricas [C]** time = 2.13758, size = 1724, normalized size = 10.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*arccoth(cot(b\*x+a)),x, algorithm="fricas")

[Out] 1/16\*((2\*I\*b\*f\*x + 2\*I\*b\*e)\*dilog(I\*cos(2\*b\*x + 2\*a) + sin(2\*b\*x + 2\*a)) + (2\*I\*b\*f\*x + 2\*I\*b\*e)\*dilog(I\*cos(2\*b\*x + 2\*a) - sin(2\*b\*x + 2\*a)) + (-2\*I\*b\*f\*x - 2\*I\*b\*e)\*dilog(-I\*cos(2\*b\*x + 2\*a) + sin(2\*b\*x + 2\*a)) + (-2\*I\*b\*f\*x - 2\*I\*b\*e)\*dilog(-I\*cos(2\*b\*x + 2\*a) - sin(2\*b\*x + 2\*a)) + 4\*(b^2\*f\*x^2 + 2\*b^2\*e\*x)\*log((cos(2\*b\*x + 2\*a) + sin(2\*b\*x + 2\*a) + 1)/(cos(2\*b\*x + 2\*a) - sin(2\*b\*x + 2\*a) + 1)) + 2\*(2\*a\*b\*e - a^2\*f)\*log(cos(2\*b\*x + 2\*a) + I\*sin(2\*b\*x + 2\*a) + I) - 2\*(2\*a\*b\*e - a^2\*f)\*log(cos(2\*b\*x + 2\*a) - I\*sin(2\*b\*x + 2\*a) + I) - 2\*(b^2\*f\*x^2 + 2\*b^2\*e\*x + 2\*a\*b\*e - a^2\*f)\*log(I\*cos(2\*b\*x + 2\*a) + sin(2\*b\*x + 2\*a) + 1) + 2\*(b^2\*f\*x^2 + 2\*b^2\*e\*x + 2\*a\*b\*e - a^2\*f)\*log(I\*cos(2\*b\*x + 2\*a) - sin(2\*b\*x + 2\*a) + 1) - 2\*(b^2\*f\*x^2 + 2\*b^2\*e\*x + 2\*a\*b\*e - a^2\*f)\*log(-I\*cos(2\*b\*x + 2\*a) + sin(2\*b\*x + 2\*a) + 1) + 2\*(b^2\*f\*x^2 + 2\*b^2\*e\*x + 2\*a\*b\*e - a^2\*f)\*log(-I\*cos(2\*b\*x + 2\*a) - sin(2\*b\*x + 2\*a) + 1) + 2\*(2\*a\*b\*e - a^2\*f)\*log(-cos(2\*b\*x + 2\*a) + I\*sin(2\*b\*x + 2\*a) + I) - 2\*(2\*a\*b\*e - a^2\*f)\*log(-cos(2\*b\*x + 2\*a) - I\*sin(2\*b\*x + 2\*a) + I) + f\*polylog(3, I\*cos(2\*b\*x + 2\*a) + sin(2\*b\*x + 2\*a)) - f\*polylog(3, I\*cos(2\*b\*x + 2\*a) - sin(2\*b\*x + 2\*a)) + f\*polylog(3, -I\*cos(2\*b\*x + 2\*a) + si

$n(2bx + 2a) - f \cdot \text{polylog}(3, -I \cdot \cos(2bx + 2a) - \sin(2bx + 2a)) / b^2$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (e + fx) \operatorname{acoth}(\cot(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*acoth(cot(b\*x+a)),x)

[Out] Integral((e + f\*x)\*acoth(cot(a + b\*x)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (fx + e) \operatorname{arccoth}(\cot(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*arccoth(cot(b\*x+a)),x, algorithm="giac")

[Out] integrate((f\*x + e)\*arccoth(cot(b\*x + a)), x)

### 3.251 $\int \coth^{-1}(\cot(a + bx)) dx$

**Optimal.** Leaf size=79

$$-\frac{i\text{PolyLog}\left(2, -ie^{2i(a+bx)}\right)}{4b} + \frac{i\text{PolyLog}\left(2, ie^{2i(a+bx)}\right)}{4b} + ix \tan^{-1}\left(e^{2i(a+bx)}\right) + x \coth^{-1}(\cot(a + bx))$$

[Out] x\*ArcCoth[Cot[a + b\*x]] + I\*x\*ArcTan[E^((2\*I)\*(a + b\*x))] - ((I/4)\*PolyLog[2, (-I)\*E^((2\*I)\*(a + b\*x))])/b + ((I/4)\*PolyLog[2, I\*E^((2\*I)\*(a + b\*x))])/b

**Rubi [A]** time = 0.048343, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {6250, 4181, 2279, 2391}

$$-\frac{i\text{PolyLog}\left(2, -ie^{2i(a+bx)}\right)}{4b} + \frac{i\text{PolyLog}\left(2, ie^{2i(a+bx)}\right)}{4b} + ix \tan^{-1}\left(e^{2i(a+bx)}\right) + x \coth^{-1}(\cot(a + bx))$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Cot[a + b\*x]], x]

[Out] x\*ArcCoth[Cot[a + b\*x]] + I\*x\*ArcTan[E^((2\*I)\*(a + b\*x))] - ((I/4)\*PolyLog[2, (-I)\*E^((2\*I)\*(a + b\*x))])/b + ((I/4)\*PolyLog[2, I\*E^((2\*I)\*(a + b\*x))])/b

#### Rule 6250

Int[ArcCoth[Cot[(a\_.) + (b\_.)\*(x\_)]], x\_Symbol] := Simp[x\*ArcCoth[Cot[a + b\*x]], x] - Dist[b, Int[x\*Sec[2\*a + 2\*b\*x], x], x] /; FreeQ[{a, b}, x]

#### Rule 4181

Int[csc[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(I\*k\*Pi)\*E^(I\*(e + f\*x))])/f, x] + (-Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Log[1 - E^(I\*k\*Pi)\*E^(I\*(e + f\*x))], x], x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Log[1 + E^(I\*k\*Pi)\*E^(I\*(e + f\*x))], x], x) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

#### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned} \int \coth^{-1}(\cot(a + bx)) dx &= x \coth^{-1}(\cot(a + bx)) - b \int x \sec(2a + 2bx) dx \\ &= x \coth^{-1}(\cot(a + bx)) + ix \tan^{-1}(e^{2i(a+bx)}) + \frac{1}{2} \int \log(1 - ie^{i(2a+2bx)}) dx - \frac{1}{2} \int \log(1 + ie^{i(2a+2bx)}) dx \\ &= x \coth^{-1}(\cot(a + bx)) + ix \tan^{-1}(e^{2i(a+bx)}) - \frac{i \operatorname{Subst}\left(\int \frac{\log(1-ix)}{x} dx, x, e^{i(2a+2bx)}\right)}{4b} + \frac{i \operatorname{Subst}\left(\int \frac{\log(1+ix)}{x} dx, x, e^{i(2a+2bx)}\right)}{4b} \\ &= x \coth^{-1}(\cot(a + bx)) + ix \tan^{-1}(e^{2i(a+bx)}) - \frac{i \operatorname{Li}_2(-ie^{2i(a+bx)})}{4b} + \frac{i \operatorname{Li}_2(ie^{2i(a+bx)})}{4b} \end{aligned}$$

**Mathematica [A]** time = 0.0317604, size = 78, normalized size = 0.99

$$\frac{-i \operatorname{PolyLog}\left(2, -ie^{2i(a+bx)}\right) + i \operatorname{PolyLog}\left(2, ie^{2i(a+bx)}\right) + 4bx \left(\coth^{-1}(\cot(a + bx)) + i \tan^{-1}(e^{2i(a+bx)})\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Cot[a + b*x]], x]
```

```
[Out] (4*b*x*(ArcCoth[Cot[a + b*x]] + I*ArcTan[E^((2*I)*(a + b*x))]) - I*PolyLog[
2, (-I)*E^((2*I)*(a + b*x))] + I*PolyLog[2, I*E^((2*I)*(a + b*x))])/(4*b)
```

**Maple [B]** time = 0.161, size = 265, normalized size = 3.4

$$\frac{\operatorname{arccoth}(\cot(bx + a)) \pi}{2b} + \frac{\operatorname{arccoth}(\cot(bx + a)) \operatorname{arccot}(\cot(bx + a))}{b} - \frac{\pi}{4b} \ln\left(1 + \frac{i(1 + i \cot(bx + a))^2}{(\cot(bx + a))^2 + 1}\right) + \frac{\operatorname{arccot}(\cot(bx + a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(cot(b\*x+a)),x)

[Out]  $-1/2/b*\operatorname{arccoth}(\cot(b*x+a))*\pi+1/b*\operatorname{arccoth}(\cot(b*x+a))*\operatorname{arccot}(\cot(b*x+a))-1/4/b*\ln(1+I*(1+I*\cot(b*x+a))^2/(\cot(b*x+a)^2+1))*\pi+1/2/b*\ln(1+I*(1+I*\cot(b*x+a))^2/(\cot(b*x+a)^2+1))*\operatorname{arccot}(\cot(b*x+a))+1/4*I/b*\operatorname{polylog}(2,-I*(1+I*\cot(b*x+a))^2/(\cot(b*x+a)^2+1))+1/4/b*\ln(1-I*(1+I*\cot(b*x+a))^2/(\cot(b*x+a)^2+1))*\pi-1/2/b*\ln(1-I*(1+I*\cot(b*x+a))^2/(\cot(b*x+a)^2+1))*\operatorname{arccot}(\cot(b*x+a))-1/4*I/b*\operatorname{polylog}(2,I*(1+I*\cot(b*x+a))^2/(\cot(b*x+a)^2+1))$

**Maxima [B]** time = 1.68979, size = 248, normalized size = 3.14

$4(bx+a)\operatorname{arccoth}\left(\frac{1}{\tan(bx+a)}\right)+\left(\arctan\left(\frac{1}{2}\tan(bx+a)+\frac{1}{2},\frac{1}{2}\tan(bx+a)+\frac{1}{2}\right)-\arctan\left(\frac{1}{2}\tan(bx+a)-\frac{1}{2},-\frac{1}{2}\tan(bx+a)+\frac{1}{2}\right)\right)\log(\tan(bx+a)^2+1)-(bx+a)\log\left(\frac{1}{2}\tan(bx+a)^2+\tan(bx+a)+\frac{1}{2}\right)+(bx+a)\log\left(\frac{1}{2}\tan(bx+a)^2-\tan(bx+a)+\frac{1}{2}\right)-I*\operatorname{dilog}\left(\frac{1}{2}I+\frac{1}{2}\right)*\tan(bx+a)-\frac{1}{2}I+\frac{1}{2}+I*\operatorname{dilog}\left(-\frac{1}{2}I-\frac{1}{2}\right)*\tan(bx+a)+\frac{1}{2}I+\frac{1}{2}+I*\operatorname{dilog}\left(\frac{1}{2}I-\frac{1}{2}\right)*\tan(bx+a)+\frac{1}{2}I+\frac{1}{2}-I*\operatorname{dilog}\left(-\frac{1}{2}I+\frac{1}{2}\right)*\tan(bx+a)-\frac{1}{2}I+\frac{1}{2})/b$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(cot(b\*x+a)),x, algorithm="maxima")

[Out]  $1/4*(4*(bx+a)*\operatorname{arccoth}(1/\tan(bx+a))+(\arctan2(1/2*\tan(bx+a)+1/2,1/2*\tan(bx+a)+1/2)-\arctan2(1/2*\tan(bx+a)-1/2,-1/2*\tan(bx+a)+1/2))*\log(\tan(bx+a)^2+1)-(bx+a)*\log(1/2*\tan(bx+a)^2+\tan(bx+a)+1/2)+(bx+a)*\log(1/2*\tan(bx+a)^2-\tan(bx+a)+1/2)-I*\operatorname{dilog}((1/2*I+1/2)*\tan(bx+a)-1/2*I+1/2)+I*\operatorname{dilog}(-(1/2*I-1/2)*\tan(bx+a)+1/2*I+1/2)+I*\operatorname{dilog}((1/2*I-1/2)*\tan(bx+a)+1/2*I+1/2)-I*\operatorname{dilog}(-(1/2*I+1/2)*\tan(bx+a)-1/2*I+1/2))/b$

**Fricas [B]** time = 1.83244, size = 1029, normalized size = 13.03

$4bx\log\left(\frac{\cos(2bx+2a)+\sin(2bx+2a)+1}{\cos(2bx+2a)-\sin(2bx+2a)+1}\right)+2a\log(\cos(2bx+2a)+i\sin(2bx+2a)+i)-2a\log(\cos(2bx+2a)-i\sin(2bx+2a)+i)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(cot(b\*x+a)),x, algorithm="fricas")

[Out]  $1/8*(4*b*x*\log((\cos(2*b*x+2*a)+\sin(2*b*x+2*a)+1)/(\cos(2*b*x+2*a)-\sin(2*b*x+2*a)+1))+2*a*\log(\cos(2*b*x+2*a)+I*\sin(2*b*x+2*a)+I)-2*a*\log(\cos(2*b*x+2*a)-I*\sin(2*b*x+2*a)+I)-2*(b*x+a)*\log(I$

```
*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1) + 2*(b*x + a)*log(I*cos(2*b*x + 2
*a) - sin(2*b*x + 2*a) + 1) - 2*(b*x + a)*log(-I*cos(2*b*x + 2*a) + sin(2*b
*x + 2*a) + 1) + 2*(b*x + a)*log(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1
) + 2*a*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + I) - 2*a*log(-cos(2*b*
x + 2*a) - I*sin(2*b*x + 2*a) + I) + I*dilog(I*cos(2*b*x + 2*a) + sin(2*b*x
+ 2*a)) + I*dilog(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - I*dilog(-I*cos(
2*b*x + 2*a) + sin(2*b*x + 2*a)) - I*dilog(-I*cos(2*b*x + 2*a) - sin(2*b*x
+ 2*a)))/b
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acoth}(\cot(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(cot(b\*x+a)),x)

[Out] Integral(acoth(cot(a + b\*x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(\cot(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(cot(b\*x+a)),x, algorithm="giac")

[Out] integrate(arccoth(cot(b\*x + a)), x)

$$3.252 \quad \int \frac{\coth^{-1}(\cot(a+bx))}{e+fx} dx$$

**Optimal.** Leaf size=17

$$\text{CannotIntegrate}\left(\frac{\coth^{-1}(\cot(a+bx))}{e+fx}, x\right)$$

[Out] CannotIntegrate[ArcCoth[Cot[a + b\*x]]/(e + f\*x), x]

**Rubi [A]** time = 0.0417984, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx} dx$$

Verification is Not applicable to the result.

[In] Int[ArcCoth[Cot[a + b\*x]]/(e + f\*x), x]

[Out] Defer[Int][ArcCoth[Cot[a + b\*x]]/(e + f\*x), x]

Rubi steps

$$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx} dx = \int \frac{\coth^{-1}(\cot(a+bx))}{e+fx} dx$$

**Mathematica [A]** time = 0.107466, size = 0, normalized size = 0.

$$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcCoth[Cot[a + b\*x]]/(e + f\*x), x]



[Out] Integrate[ArcCoth[Cot[a + b\*x]]/(e + f\*x), x]

---

**Maple [A]** time = 1.173, size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(\cot(bx + a))}{fx + e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(cot(b\*x+a))/(f\*x+e),x)

[Out] int(arccoth(cot(b\*x+a))/(f\*x+e),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(\cot(bx + a))}{fx + e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(cot(b\*x+a))/(f\*x+e),x, algorithm="maxima")

[Out] integrate(arccoth(cot(b\*x + a))/(f\*x + e), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arccoth}(\cot(bx + a))}{fx + e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(cot(b\*x+a))/(f\*x+e),x, algorithm="fricas")

[Out] integral(arccoth(cot(b\*x + a))/(f\*x + e), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acoth}(\cot(a + bx))}{e + fx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(cot(b\*x+a))/(f\*x+e),x)

[Out] Integral(acoth(cot(a + b\*x))/(e + f\*x), x)

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(\cot(bx + a))}{fx + e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(cot(b\*x+a))/(f\*x+e),x, algorithm="giac")

[Out] integrate(arccoth(cot(b\*x + a))/(f\*x + e), x)

### 3.253 $\int x^2 \coth^{-1}(c + d \cot(a + bx)) dx$

**Optimal.** Leaf size=391

$$\frac{x \operatorname{PolyLog}\left(3, \frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b^2} - \frac{x \operatorname{PolyLog}\left(3, \frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b^2} + \frac{i \operatorname{PolyLog}\left(4, \frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{8b^3} - \frac{i \operatorname{PolyLog}\left(4, \frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{8b^3}$$

[Out]  $(x^3 \operatorname{ArcCoth}[c + d \operatorname{Cot}[a + b x]])/3 + (x^3 \operatorname{Log}[1 - ((1 - c - I d) E^{((2 I) a + (2 I) b x)})/(1 - c + I d)])/6 - (x^3 \operatorname{Log}[1 - ((1 + c + I d) E^{((2 I) a + (2 I) b x)})/(1 + c - I d)])/6 - ((I/4) x^2 \operatorname{PolyLog}[2, ((1 - c - I d) E^{((2 I) a + (2 I) b x)})/(1 - c + I d)])/b + ((I/4) x^2 \operatorname{PolyLog}[2, ((1 + c + I d) E^{((2 I) a + (2 I) b x)})/(1 + c - I d)])/b + (x \operatorname{PolyLog}[3, ((1 - c - I d) E^{((2 I) a + (2 I) b x)})/(1 - c + I d)])/ (4 b^2) - (x \operatorname{PolyLog}[3, ((1 + c + I d) E^{((2 I) a + (2 I) b x)})/(1 + c - I d)])/ (4 b^2) + ((I/8) \operatorname{PolyLog}[4, ((1 - c - I d) E^{((2 I) a + (2 I) b x)})/(1 - c + I d)])/b^3 - ((I/8) \operatorname{PolyLog}[4, ((1 + c + I d) E^{((2 I) a + (2 I) b x)})/(1 + c - I d)])/b^3$

**Rubi [A]** time = 0.503235, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {6270, 2190, 2531, 6609, 2282, 6589}

$$\frac{x \operatorname{PolyLog}\left(3, \frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b^2} - \frac{x \operatorname{PolyLog}\left(3, \frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b^2} + \frac{i \operatorname{PolyLog}\left(4, \frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{8b^3} - \frac{i \operatorname{PolyLog}\left(4, \frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{8b^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2 \operatorname{ArcCoth}[c + d \operatorname{Cot}[a + b x]], x]$

[Out]  $(x^3 \operatorname{ArcCoth}[c + d \operatorname{Cot}[a + b x]])/3 + (x^3 \operatorname{Log}[1 - ((1 - c - I d) E^{((2 I) a + (2 I) b x)})/(1 - c + I d)])/6 - (x^3 \operatorname{Log}[1 - ((1 + c + I d) E^{((2 I) a + (2 I) b x)})/(1 + c - I d)])/6 - ((I/4) x^2 \operatorname{PolyLog}[2, ((1 - c - I d) E^{((2 I) a + (2 I) b x)})/(1 - c + I d)])/b + ((I/4) x^2 \operatorname{PolyLog}[2, ((1 + c + I d) E^{((2 I) a + (2 I) b x)})/(1 + c - I d)])/b + (x \operatorname{PolyLog}[3, ((1 - c - I d) E^{((2 I) a + (2 I) b x)})/(1 - c + I d)])/ (4 b^2) - (x \operatorname{PolyLog}[3, ((1 + c + I d) E^{((2 I) a + (2 I) b x)})/(1 + c - I d)])/ (4 b^2) + ((I/8) \operatorname{PolyLog}[4, ((1 - c - I d) E^{((2 I) a + (2 I) b x)})/(1 - c + I d)])/b^3 - ((I/8) \operatorname{PolyLog}[4, ((1 + c + I d) E^{((2 I) a + (2 I) b x)})/(1 + c - I d)])/b^3$

Rule 6270

```

Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Cot[a + b*x]])/(f*(m + 1)), x] + (-Dist[(I*b*(1 - c - I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*I*a + 2*I*b*x))/(1 - c + I*d - (1 - c - I*d)*E^(2*I*a + 2*I*b*x)), x], x] + Dist[(I*b*(1 + c + I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*I*a + 2*I*b*x))/(1 + c - I*d - (1 + c + I*d)*E^(2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, 1]

```

### Rule 2190

```

Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.), x_Symbol]
:> Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

```

### Rule 2531

```

Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol]
:> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

```

### Rule 6609

```

Int[(((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol]
:> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

```

### Rule 2282

```

Int[u_, x_Symbol]
:> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

```

### Rule 6589

```

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d}

```

, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
 \int x^2 \coth^{-1}(c + d \cot(a + bx)) dx &= \frac{1}{3}x^3 \coth^{-1}(c + d \cot(a + bx)) + \frac{1}{3}(b(i(1 + c) - d)) \int \frac{e^{2ia+2ibx}x^3}{1 + c - id + (-1 - c - id)e^{2ia}} \\
 &= \frac{1}{3}x^3 \coth^{-1}(c + d \cot(a + bx)) + \frac{1}{6}x^3 \log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right) - \frac{1}{6}x^3 \log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right) \\
 &= \frac{1}{3}x^3 \coth^{-1}(c + d \cot(a + bx)) + \frac{1}{6}x^3 \log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right) - \frac{1}{6}x^3 \log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right) \\
 &= \frac{1}{3}x^3 \coth^{-1}(c + d \cot(a + bx)) + \frac{1}{6}x^3 \log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right) - \frac{1}{6}x^3 \log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right) \\
 &= \frac{1}{3}x^3 \coth^{-1}(c + d \cot(a + bx)) + \frac{1}{6}x^3 \log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right) - \frac{1}{6}x^3 \log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right) \\
 &= \frac{1}{3}x^3 \coth^{-1}(c + d \cot(a + bx)) + \frac{1}{6}x^3 \log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right) - \frac{1}{6}x^3 \log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right)
 \end{aligned}$$

**Mathematica [A]** time = 0.306333, size = 339, normalized size = 0.87

$$\frac{1}{3}x^3 \coth^{-1}(d \cot(a + bx) + c) + \frac{-6ib^2x^2 \text{PolyLog}\left(2, \frac{(c+id-1)e^{2i(a+bx)}}{c-id-1}\right) + 6ib^2x^2 \text{PolyLog}\left(2, \frac{(c+id+1)e^{2i(a+bx)}}{c-id+1}\right) + 6bx \text{PolyLog}\left(3, \frac{(c+id-1)e^{2i(a+bx)}}{c-id-1}\right) + 6bx \text{PolyLog}\left(3, \frac{(c+id+1)e^{2i(a+bx)}}{c-id+1}\right)}{24b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[c + d\*Cot[a + b\*x]], x]

[Out] (x^3\*ArcCoth[c + d\*Cot[a + b\*x]])/3 + (4\*b^3\*x^3\*Log[1 - ((-1 + c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(-1 + c - I\*d)] - 4\*b^3\*x^3\*Log[1 - ((1 + c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(1 + c - I\*d)] - (6\*I)\*b^2\*x^2\*PolyLog[2, ((-1 + c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(-1 + c - I\*d)] + (6\*I)\*b^2\*x^2\*PolyLog[2, ((1 + c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(1 + c - I\*d)] + 6\*b\*x\*PolyLog[3, ((-1 + c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(-1 + c - I\*d)] - 6\*b\*x\*PolyLog[3, ((1 + c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(1 + c - I\*d)] + (3\*I)\*PolyLog[4, ((-1 + c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(-1 + c - I\*d)] - (3\*I)\*PolyLog[4, ((1 + c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(1 + c - I\*d)])/(24\*b^3)

---

**Maple [C]** time = 6.855, size = 6662, normalized size = 17.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*arccoth(c+d*cot(b*x+a)),x)`

[Out] result too large to display

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccoth(c+d*cot(b*x+a)),x, algorithm="maxima")`

[Out] 
$$\frac{1}{12}x^3 \log((c^2 + d^2 + 2c + 1)\cos(2bx + 2a)^2 + 4(c + 1)d\sin(2bx + 2a) + (c^2 + d^2 + 2c + 1)\sin(2bx + 2a)^2 + c^2 + d^2 - 2(c^2 - d^2 + 2c + 1)\cos(2bx + 2a) + 2c + 1) - \frac{1}{12}x^3 \log((c^2 + d^2 - 2c + 1)\cos(2bx + 2a)^2 + 4(c - 1)d\sin(2bx + 2a) + (c^2 + d^2 - 2c + 1)\sin(2bx + 2a)^2 + c^2 + d^2 - 2(c^2 - d^2 - 2c + 1)\cos(2bx + 2a) - 2c + 1) - 4bd \int \frac{1}{3}((c^2 + d^2 - 1)x^3 \cos(2bx + 2a)^2 + 2cdx^3 \sin(2bx + 2a) + 2(c^2 + d^2 - 1)x^3 \sin(2bx + 2a)^2 - (c^2 - d^2 - 1)x^3 \cos(2bx + 2a) - (2cdx^3 \sin(2bx + 2a) + (c^2 - d^2 - 1)x^3 \cos(2bx + 2a))\cos(4bx + 4a) + (2cdx^3 \cos(2bx + 2a) - (c^2 - d^2 - 1)x^3 \sin(2bx + 2a))\sin(4bx + 4a)) / (c^4 + d^4 + 2(c^2 + 1)d^2 + (c^4 + d^4 + 2(c^2 + 1)d^2 - 2c^2 + 1)\cos(4bx + 4a)^2 + 4(c^4 + d^4 + 2(c^2 - 1)d^2 - 2c^2 + 1)\cos(2bx + 2a)^2 + (c^4 + d^4 + 2(c^2 + 1)d^2 - 2c^2 + 1)\sin(4bx + 4a)^2 + 4(c^4 + d^4 + 2(c^2 - 1)d^2 - 2c^2 + 1)\sin(2bx + 2a)^2 - 2c^2 + 2(c^4 + d^4 - 2(3c^2 - 1)d^2 - 2c^2 - 2(c^4 - d^4 - 2c^2 + 1)\cos(2bx + 2a) - 4(cd^3 + (c^3 - c)d)\sin(2bx + 2a) + 1)\cos(4bx + 4a) - 4(c^4 - d^4 - 2c^2 + 1)\cos(2bx + 2a) + 4(2cd^3 - 2(c^3 - c)d + 2(cd^3 + (c^3 - c)d)\cos(2bx + 2a) - (c^4 - d^4 - 2c^2 + 1)\sin(2bx + 2a))\sin(4bx + 4a) + 8(cd^3 + (c^3 - c)d)\sin(2bx + 2a) + 1), x)$$

---



$$\begin{aligned} &^3 + a^3) \log((c^2 + d^2 - (c^2 - 2I(c-1)d - d^2 - 2c + 1)\cos(2bx + 2a) + (Ic^2 + 2(c-1)d - Id^2 - 2Ic + I)\sin(2bx + 2a) - 2c + 1)/(c^2 + d^2 - 2c + 1)) - 3I \operatorname{polylog}(4, ((c^2 + 2I(c+1)d - d^2 + 2c + 1)\cos(2bx + 2a) + (Ic^2 - 2(c+1)d - Id^2 + 2Ic + I)\sin(2bx + 2a))/(c^2 + d^2 + 2c + 1)) + 3I \operatorname{polylog}(4, ((c^2 - 2I(c+1)d - d^2 + 2c + 1)\cos(2bx + 2a) + (-Ic^2 - 2(c+1)d + Id^2 - 2Ic - I)\sin(2bx + 2a))/(c^2 + d^2 + 2c + 1)) + 3I \operatorname{polylog}(4, ((c^2 + 2I(c-1)d - d^2 - 2c + 1)\cos(2bx + 2a) + (Ic^2 - 2(c-1)d - Id^2 - 2Ic + I)\sin(2bx + 2a))/(c^2 + d^2 - 2c + 1)) - 3I \operatorname{polylog}(4, ((c^2 - 2I(c-1)d - d^2 - 2c + 1)\cos(2bx + 2a) + (-Ic^2 - 2(c-1)d + Id^2 + 2Ic - I)\sin(2bx + 2a))/(c^2 + d^2 - 2c + 1)))/b^3 \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*acoth(c+d\*cot(b\*x+a)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arccoth}(d \cot(bx + a) + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(c+d\*cot(b\*x+a)),x, algorithm="giac")

[Out] integrate(x^2\*arccoth(d\*cot(b\*x + a) + c), x)



### 3.254 $\int x \coth^{-1}(c + d \cot(a + bx)) dx$

**Optimal.** Leaf size=293

$$\frac{\text{PolyLog}\left(3, \frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{8b^2} - \frac{\text{PolyLog}\left(3, \frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{8b^2} - \frac{ix \text{PolyLog}\left(2, \frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b} + \frac{ix \text{PolyLog}\left(2, \frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b}$$

```
[Out] (x^2*ArcCoth[c + d*Cot[a + b*x]])/2 + (x^2*Log[1 - ((1 - c - I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 - c + I*d)])/4 - (x^2*Log[1 - ((1 + c + I*d)*E^((2*I)*a
+ (2*I)*b*x))/(1 + c - I*d)])/4 - ((I/4)*x*PolyLog[2, ((1 - c - I*d)*E^((2*
I)*a + (2*I)*b*x))/(1 - c + I*d)])/b + ((I/4)*x*PolyLog[2, ((1 + c + I*d)*E
^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/b + PolyLog[3, ((1 - c - I*d)*E^((2
*I)*a + (2*I)*b*x))/(1 - c + I*d)]/(8*b^2) - PolyLog[3, ((1 + c + I*d)*E^((
2*I)*a + (2*I)*b*x))/(1 + c - I*d)]/(8*b^2)
```

**Rubi [A]** time = 0.40347, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {6270, 2190, 2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3, \frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{8b^2} - \frac{\text{PolyLog}\left(3, \frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{8b^2} - \frac{ix \text{PolyLog}\left(2, \frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b} + \frac{ix \text{PolyLog}\left(2, \frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[c + d*Cot[a + b*x]], x]
```

```
[Out] (x^2*ArcCoth[c + d*Cot[a + b*x]])/2 + (x^2*Log[1 - ((1 - c - I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 - c + I*d)])/4 - (x^2*Log[1 - ((1 + c + I*d)*E^((2*I)*a
+ (2*I)*b*x))/(1 + c - I*d)])/4 - ((I/4)*x*PolyLog[2, ((1 - c - I*d)*E^((2*
I)*a + (2*I)*b*x))/(1 - c + I*d)])/b + ((I/4)*x*PolyLog[2, ((1 + c + I*d)*E
^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/b + PolyLog[3, ((1 - c - I*d)*E^((2
*I)*a + (2*I)*b*x))/(1 - c + I*d)]/(8*b^2) - PolyLog[3, ((1 + c + I*d)*E^((
2*I)*a + (2*I)*b*x))/(1 + c - I*d)]/(8*b^2)
```

#### Rule 6270

```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Cot[a + b*x]])/(f*(m
+ 1)), x] + (-Dist[(I*b*(1 - c - I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)
*E^(2*I*a + 2*I*b*x))/(1 - c + I*d - (1 - c - I*d)*E^(2*I*a + 2*I*b*x)), x]
```

```
, x] + Dist[(I*b*(1 + c + I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*I*
a + 2*I*b*x))/(1 + c - I*d - (1 + c + I*d)*E^(2*I*a + 2*I*b*x)), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, 1]
```

### Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n])]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int x \coth^{-1}(c + d \cot(a + bx)) dx &= \frac{1}{2}x^2 \coth^{-1}(c + d \cot(a + bx)) + \frac{1}{2}(b(i(1 + c) - d)) \int \frac{e^{2ia+2ibx}x^2}{1 + c - id + (-1 - c - id)e^{2ia+}} \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \cot(a + bx)) + \frac{1}{4}x^2 \log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right) - \frac{1}{4}x^2 \log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right) \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \cot(a + bx)) + \frac{1}{4}x^2 \log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right) - \frac{1}{4}x^2 \log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right) \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \cot(a + bx)) + \frac{1}{4}x^2 \log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right) - \frac{1}{4}x^2 \log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right) \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \cot(a + bx)) + \frac{1}{4}x^2 \log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right) - \frac{1}{4}x^2 \log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.125174, size = 253, normalized size = 0.86

$$\frac{1}{2}x^2 \coth^{-1}(d \cot(a + bx) + c) + \frac{-2ibx \operatorname{PolyLog}\left(2, \frac{(c+id-1)e^{2i(a+bx)}}{c-id-1}\right) + 2ibx \operatorname{PolyLog}\left(2, \frac{(c+id+1)e^{2i(a+bx)}}{c-id+1}\right) + \operatorname{PolyLog}\left(3, \frac{(c+id-1)e^{2i(a+bx)}}{c-id-1}\right) + \operatorname{PolyLog}\left(3, \frac{(c+id+1)e^{2i(a+bx)}}{c-id+1}\right)}{8b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[c + d\*Cot[a + b\*x]],x]

[Out] (x^2\*ArcCoth[c + d\*Cot[a + b\*x]])/2 + (2\*b^2\*x^2\*Log[1 - ((-1 + c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(-1 + c - I\*d)] - 2\*b^2\*x^2\*Log[1 - ((1 + c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(1 + c - I\*d)] - (2\*I)\*b\*x\*PolyLog[2, ((-1 + c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(-1 + c - I\*d)] + (2\*I)\*b\*x\*PolyLog[2, ((1 + c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(1 + c - I\*d)] + PolyLog[3, ((-1 + c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(-1 + c - I\*d)] - PolyLog[3, ((1 + c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(1 + c - I\*d)])/(8\*b^2)

**Maple [C]** time = 8.394, size = 6312, normalized size = 21.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(c+d\*cot(b\*x+a)),x)

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(c+d\*cot(b\*x+a)),x, algorithm="maxima")

[Out] 
$$-2*b*d*\integrate((2*(c^2 + d^2 - 1)*x^2*\cos(2*b*x + 2*a)^2 + 2*c*d*x^2*\sin(2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^2*\sin(2*b*x + 2*a)^2 - (c^2 - d^2 - 1)*x^2*\cos(2*b*x + 2*a) - (2*c*d*x^2*\sin(2*b*x + 2*a) + (c^2 - d^2 - 1)*x^2*\cos(2*b*x + 2*a))*\cos(4*b*x + 4*a) + (2*c*d*x^2*\cos(2*b*x + 2*a) - (c^2 - d^2 - 1)*x^2*\sin(2*b*x + 2*a))*\sin(4*b*x + 4*a))/(c^4 + d^4 + 2*(c^2 + 1)*d^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*\cos(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*\cos(2*b*x + 2*a)^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*\sin(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*\sin(2*b*x + 2*a)^2 - 2*c^2 + 2*(c^4 + d^4 - 2*(3*c^2 - 1)*d^2 - 2*c^2 - 2*(c^4 - d^4 - 2*c^2 + 1)*\cos(2*b*x + 2*a) - 4*(c*d^3 + (c^3 - c)*d)*\sin(2*b*x + 2*a) + 1)*\cos(4*b*x + 4*a) - 4*(c^4 - d^4 - 2*c^2 + 1)*\cos(2*b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 - c)*d + 2*(c*d^3 + (c^3 - c)*d)*\cos(2*b*x + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*\sin(2*b*x + 2*a))*\sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 - c)*d)*\sin(2*b*x + 2*a) + 1), x) + 1/8*x^2*\log((c^2 + d^2 + 2*c + 1)*\cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*\sin(2*b*x + 2*a) + (c^2 + d^2 + 2*c + 1)*\sin(2*b*x + 2*a)^2 + c^2 + d^2 - 2*(c^2 - d^2 + 2*c + 1)*\cos(2*b*x + 2*a) + 2*c + 1) - 1/8*x^2*\log((c^2 + d^2 - 2*c + 1)*\cos(2*b*x + 2*a)^2 + 4*(c - 1)*d*\sin(2*b*x + 2*a) + (c^2 + d^2 - 2*c + 1)*\sin(2*b*x + 2*a)^2 + c^2 + d^2 - 2*(c^2 - d^2 - 2*c + 1)*\cos(2*b*x + 2*a) - 2*c + 1)$$

**Fricas [C]** time = 2.83357, size = 3838, normalized size = 13.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(c+d\*cot(b\*x+a)),x, algorithm="fricas")

[Out] 
$$1/16*(4*b^2*x^2*\log((d*\cos(2*b*x + 2*a) + (c + 1)*\sin(2*b*x + 2*a) + d)/(d*\cos(2*b*x + 2*a) + (c - 1)*\sin(2*b*x + 2*a) + d)) + 2*I*b*x*dilog(-(c^2 + d$$

$$\begin{aligned}
&^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*\cos(2*b*x + 2*a) + (-I*c^2 + 2*(c + 1)*d + I*d^2 - 2*I*c - I)*\sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1) + 1) - 2*I*b*x*\operatorname{dilog}(-(c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*\cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*\sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1) + 1) - 2*I*b*x*\operatorname{dilog}(-(c^2 + d^2 - (c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*\cos(2*b*x + 2*a) + (-I*c^2 + 2*(c - 1)*d + I*d^2 + 2*I*c - I)*\sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1) + 1) + 2*I*b*x*\operatorname{dilog}(-(c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*\cos(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*\sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1) + 1) - 2*a^2*\log(1/2*c^2 + I*(c + 1)*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*c + 1)*\cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*c + I)*\sin(2*b*x + 2*a) + c + 1/2) + 2*a^2*\log(1/2*c^2 + I*(c - 1)*d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*c + 1)*\cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*c + I)*\sin(2*b*x + 2*a) - c + 1/2) - 2*a^2*\log(-1/2*c^2 + I*(c + 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*c + 1)*\cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*c + I)*\sin(2*b*x + 2*a) - c - 1/2) + 2*a^2*\log(-1/2*c^2 + I*(c - 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*c + 1)*\cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*c + I)*\sin(2*b*x + 2*a) + c - 1/2) - 2*(b^2*x^2 - a^2)*\log((c^2 + d^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*\cos(2*b*x + 2*a) + (-I*c^2 + 2*(c + 1)*d + I*d^2 - 2*I*c - I)*\sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1)) - 2*(b^2*x^2 - a^2)*\log((c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*\cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*\sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1)) + 2*(b^2*x^2 - a^2)*\log((c^2 + d^2 - (c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*\cos(2*b*x + 2*a) + (-I*c^2 + 2*(c - 1)*d + I*d^2 + 2*I*c - I)*\sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1)) + 2*(b^2*x^2 - a^2)*\log((c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*\cos(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*\sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1)) - \operatorname{polylog}(3, ((c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*\cos(2*b*x + 2*a) + (I*c^2 - 2*(c + 1)*d - I*d^2 + 2*I*c + I)*\sin(2*b*x + 2*a))/(c^2 + d^2 + 2*c + 1)) - \operatorname{polylog}(3, ((c^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*\cos(2*b*x + 2*a) + (-I*c^2 - 2*(c + 1)*d + I*d^2 - 2*I*c - I)*\sin(2*b*x + 2*a))/(c^2 + d^2 + 2*c + 1)) + \operatorname{polylog}(3, ((c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*\cos(2*b*x + 2*a) + (I*c^2 - 2*(c - 1)*d - I*d^2 - 2*I*c + I)*\sin(2*b*x + 2*a))/(c^2 + d^2 - 2*c + 1)) + \operatorname{polylog}(3, ((c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*\cos(2*b*x + 2*a) + (-I*c^2 - 2*(c - 1)*d + I*d^2 + 2*I*c - I)*\sin(2*b*x + 2*a))/(c^2 + d^2 - 2*c + 1)))/b^2
\end{aligned}$$


---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(c+d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arccoth}(d \cot(bx + a) + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(c+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(d*cot(b*x + a) + c), x)
```

### 3.255 $\int \coth^{-1}(c + d \cot(a + bx)) dx$

**Optimal.** Leaf size=194

$$-\frac{i \operatorname{PolyLog}\left(2, \frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b} + \frac{i \operatorname{PolyLog}\left(2, \frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b} + \frac{1}{2}x \log\left(1 - \frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right) - \frac{1}{2}x \log\left(1 - \frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)$$

```
[Out] x*ArcCoth[c + d*Cot[a + b*x]] + (x*Log[1 - ((1 - c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/2 - (x*Log[1 - ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/2 - ((I/4)*PolyLog[2, ((1 - c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/b + ((I/4)*PolyLog[2, ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/b
```

**Rubi [A]** time = 0.249007, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {6262, 2190, 2279, 2391}

$$-\frac{i \operatorname{PolyLog}\left(2, \frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b} + \frac{i \operatorname{PolyLog}\left(2, \frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b} + \frac{1}{2}x \log\left(1 - \frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right) - \frac{1}{2}x \log\left(1 - \frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[c + d*Cot[a + b*x]],x]
```

```
[Out] x*ArcCoth[c + d*Cot[a + b*x]] + (x*Log[1 - ((1 - c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/2 - (x*Log[1 - ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/2 - ((I/4)*PolyLog[2, ((1 - c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/b + ((I/4)*PolyLog[2, ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/b
```

#### Rule 6262

```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_.)]*(d_.)], x_Symbol] :> Simp[x*ArcCoth[c + d*Cot[a + b*x]], x] + (-Dist[I*b*(1 - c - I*d), Int[(x*E^(2*I*a + 2*I*b*x))/(1 - c + I*d - (1 - c - I*d)*E^(2*I*a + 2*I*b*x)), x], x] + Dist[I*b*(1 + c + I*d), Int[(x*E^(2*I*a + 2*I*b*x))/(1 + c - I*d - (1 + c + I*d)*E^(2*I*a + 2*I*b*x)), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - I*d)^2, 1]
```

#### Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned} \int \coth^{-1}(c + d \cot(a + bx)) dx &= x \coth^{-1}(c + d \cot(a + bx)) + (b(i(1 + c) - d)) \int \frac{e^{2ia+2ibx} x}{1 + c - id + (-1 - c - id)e^{2ia+2ibx}} dx \\ &= x \coth^{-1}(c + d \cot(a + bx)) + \frac{1}{2} x \log \left( 1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id} \right) - \frac{1}{2} x \log \left( 1 - \frac{(1 + c + id)e^{2ia+2ibx}}{1 + c + id} \right) \\ &= x \coth^{-1}(c + d \cot(a + bx)) + \frac{1}{2} x \log \left( 1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id} \right) - \frac{1}{2} x \log \left( 1 - \frac{(1 + c + id)e^{2ia+2ibx}}{1 + c + id} \right) \\ &= x \coth^{-1}(c + d \cot(a + bx)) + \frac{1}{2} x \log \left( 1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id} \right) - \frac{1}{2} x \log \left( 1 - \frac{(1 + c + id)e^{2ia+2ibx}}{1 + c + id} \right) \end{aligned}$$

**Mathematica [B]** time = 12.9929, size = 4463, normalized size = 23.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[c + d*Cot[a + b*x]], x]
```

```
[Out] x*ArcCoth[c + d*Cot[a + b*x]] - (d*(a*Log[-(Sec[(a + b*x)/2]^2*(d*Cos[a + b
*x] + (-1 + c)*Sin[a + b*x]))] - a*Log[-(Sec[(a + b*x)/2]^2*(d*Cos[a + b*x]
```



$$\begin{aligned}
& + \sin[a + bx] + c \sin[a + bx])) - (a + bx) \log\left[-\frac{(-1 + c + \sqrt{1 - 2c + c^2 + d^2})}{d} + \tan\left[\frac{a + bx}{2}\right] - I \log\left[\frac{d(-I + \tan\left[\frac{a + bx}{2}\right])}{(-1 + c - Id + \sqrt{1 - 2c + c^2 + d^2})}\right] \log\left[-\frac{(-1 + c + \sqrt{1 - 2c + c^2 + d^2})}{d} + \tan\left[\frac{a + bx}{2}\right] + I \log\left[\frac{d(I + \tan\left[\frac{a + bx}{2}\right])}{(-1 + c + Id + \sqrt{1 - 2c + c^2 + d^2})}\right] \log\left[-\frac{(-1 + c + \sqrt{1 - 2c + c^2 + d^2})}{d} + \tan\left[\frac{a + bx}{2}\right] + (a + bx) \log\left[-\frac{(1 + c + \sqrt{1 + 2c + c^2 + d^2})}{d} + \tan\left[\frac{a + bx}{2}\right] + I \log\left[\frac{d(-I + \tan\left[\frac{a + bx}{2}\right])}{(1 + c - Id + \sqrt{1 + 2c + c^2 + d^2})}\right] \log\left[-\frac{(1 + c + \sqrt{1 + 2c + c^2 + d^2})}{d} + \tan\left[\frac{a + bx}{2}\right] - I \log\left[\frac{d(I + \tan\left[\frac{a + bx}{2}\right])}{(1 + c + Id + \sqrt{1 + 2c + c^2 + d^2})}\right] \log\left[-\frac{(1 + c + \sqrt{1 + 2c + c^2 + d^2})}{d} + \tan\left[\frac{a + bx}{2}\right] - (a + bx) \log\left[\frac{(1 - c + \sqrt{1 - 2c + c^2 + d^2}) + d \tan\left[\frac{a + bx}{2}\right]}{d} - I \log\left[\frac{d(-I + \tan\left[\frac{a + bx}{2}\right])}{(1 - c + Id + \sqrt{1 - 2c + c^2 + d^2})}\right] \log\left[\frac{(1 - c + \sqrt{1 - 2c + c^2 + d^2}) + d \tan\left[\frac{a + bx}{2}\right]}{d} + I \log\left[\frac{d(I + \tan\left[\frac{a + bx}{2}\right])}{(1 - c - Id + \sqrt{1 - 2c + c^2 + d^2})}\right] \log\left[\frac{(1 - c + \sqrt{1 - 2c + c^2 + d^2}) + d \tan\left[\frac{a + bx}{2}\right]}{d} + (a + bx) \log\left[\frac{(-1 - c + \sqrt{1 + 2c + c^2 + d^2}) + d \tan\left[\frac{a + bx}{2}\right]}{d} + I \log\left[\frac{d(-I + \tan\left[\frac{a + bx}{2}\right])}{(-1 - c + Id + \sqrt{1 + 2c + c^2 + d^2})}\right] \log\left[\frac{(-1 - c + \sqrt{1 + 2c + c^2 + d^2}) + d \tan\left[\frac{a + bx}{2}\right]}{d} - I \log\left[\frac{d(I + \tan\left[\frac{a + bx}{2}\right])}{(-1 - c - Id + \sqrt{1 + 2c + c^2 + d^2})}\right] \log\left[\frac{(-1 - c + \sqrt{1 + 2c + c^2 + d^2}) + d \tan\left[\frac{a + bx}{2}\right]}{d} - I \text{PolyLog}\left[2, \frac{(-1 + c + \sqrt{1 - 2c + c^2 + d^2}) - d \tan\left[\frac{a + bx}{2}\right]}{(-1 + c - Id + \sqrt{1 - 2c + c^2 + d^2})}\right] + I \text{PolyLog}\left[2, \frac{(-1 + c + \sqrt{1 - 2c + c^2 + d^2}) - d \tan\left[\frac{a + bx}{2}\right]}{(-1 + c + Id + \sqrt{1 - 2c + c^2 + d^2})}\right] - I \text{PolyLog}\left[2, \frac{(1 + c - \sqrt{1 + 2c + c^2 + d^2}) - d \tan\left[\frac{a + bx}{2}\right]}{(1 + c + Id - \sqrt{1 + 2c + c^2 + d^2})}\right] + I \text{PolyLog}\left[2, \frac{(1 + c + \sqrt{1 + 2c + c^2 + d^2}) - d \tan\left[\frac{a + bx}{2}\right]}{(1 + c - Id + \sqrt{1 + 2c + c^2 + d^2})}\right] - I \text{PolyLog}\left[2, \frac{(1 + c + \sqrt{1 + 2c + c^2 + d^2}) - d \tan\left[\frac{a + bx}{2}\right]}{(1 + c + Id + \sqrt{1 + 2c + c^2 + d^2})}\right] + I \text{PolyLog}\left[2, \frac{(1 - c + \sqrt{1 - 2c + c^2 + d^2}) + d \tan\left[\frac{a + bx}{2}\right]}{(1 - c - Id + \sqrt{1 - 2c + c^2 + d^2})}\right] - I \text{PolyLog}\left[2, \frac{(1 - c + \sqrt{1 - 2c + c^2 + d^2}) + d \tan\left[\frac{a + bx}{2}\right]}{(1 - c + Id + \sqrt{1 - 2c + c^2 + d^2})}\right] + I \text{PolyLog}\left[2, \frac{(-1 - c + \sqrt{1 + 2c + c^2 + d^2}) + d \tan\left[\frac{a + bx}{2}\right]}{(-1 - c + Id + \sqrt{1 + 2c + c^2 + d^2})}\right] \right] \left( \frac{2a}{b(1 - c^2 - d^2 - \cos[2(a + bx)])} + c^2 \cos[2(a + bx)] - d^2 \cos[2(a + bx)] - 2cd \sin[2(a + bx)] \right) - \frac{2(a + bx)}{b(1 - c^2 - d^2 - \cos[2(a + bx)] + c^2 \cos[2(a + bx)] - d^2 \cos[2(a + bx)] - 2cd \sin[2(a + bx)])} \left( -\log\left[-\frac{(-1 + c + \sqrt{1 - 2c + c^2 + d^2})}{d} + \tan\left[\frac{a + bx}{2}\right] + \log\left[-\frac{(1 + c + \sqrt{1 + 2c + c^2 + d^2})}{d} + \tan\left[\frac{a + bx}{2}\right] - \log\left[\frac{(1 - c + \sqrt{1 - 2c + c^2 + d^2}) + d \tan\left[\frac{a + bx}{2}\right]}{d} + \log\left[\frac{(-1 - c + \sqrt{1 + 2c + c^2 + d^2}) + d \tan\left[\frac{a + bx}{2}\right]}{d} - \left(\frac{I}{2}\right) \log\left[-\frac{(-1 + c + \sqrt{1 - 2c + c^2 + d^2})}{d} + \tan\left[\frac{a + bx}{2}\right] \right] \sec\left[\frac{a + bx}{2}\right]^2 / (-I + \tan\left[\frac{a + bx}{2}\right]) + \left(\frac{I}{2}\right) \log\left[-\frac{(1 + c + \sqrt{1 + 2c + c^2 + d^2})}{d} + \tan\left[\frac{a + bx}{2}\right] \right] \sec\left[\frac{a + bx}{2}\right]^2 / (-I + \tan\left[\frac{a + bx}{2}\right]) - \left(\frac{I}{2}\right) \log\left[\frac{(1 - c + \sqrt{1 - 2c + c^2 + d^2}) + d \tan\left[\frac{a + bx}{2}\right]}{d} \right] \sec\left[\frac{a + bx}{2}\right]^2 / (-I + \tan\left[\frac{a + bx}{2}\right]) + \left(\frac{I}{2}\right) \log\left[\frac{(-1 - c + \sqrt{1 + 2c + c^2 + d^2}) + d \tan\left[\frac{a + bx}{2}\right]}{d} \right] \right)
\end{aligned}$$



$$\begin{aligned} & (a + b*x)/2)^2)/(-1 - c + \text{Sqrt}[1 + 2*c + c^2 + d^2] + d*\text{Tan}[(a + b*x)/2]) - \\ & ((I/2)*d*\text{Log}[-((d*(I + \text{Tan}[(a + b*x)/2])))/(-1 - c - I*d + \text{Sqrt}[1 + 2*c + c^2 + d^2]))] * \text{Sec}[(a + b*x)/2]^2)/(-1 - c + \text{Sqrt}[1 + 2*c + c^2 + d^2] + d*\text{Tan} \\ & [(a + b*x)/2]) - ((I/2)*d*\text{Log}[1 - (-1 - c + \text{Sqrt}[1 + 2*c + c^2 + d^2] + d*\text{Tan} \\ & [(a + b*x)/2])]/(-1 - c + I*d + \text{Sqrt}[1 + 2*c + c^2 + d^2])) * \text{Sec}[(a + b*x)/ \\ & 2]^2)/(-1 - c + \text{Sqrt}[1 + 2*c + c^2 + d^2] + d*\text{Tan}[(a + b*x)/2]) - (a*\text{Cos}[(a \\ & + b*x)/2]^2*(-(\text{Sec}[(a + b*x)/2]^2*((-1 + c)*\text{Cos}[a + b*x] - d*\text{Sin}[a + b*x]) \\ & ) - \text{Sec}[(a + b*x)/2]^2*(d*\text{Cos}[a + b*x] + (-1 + c)*\text{Sin}[a + b*x])* \text{Tan}[(a + b* \\ & x)/2])))/(d*\text{Cos}[a + b*x] + (-1 + c)*\text{Sin}[a + b*x]) + (a*\text{Cos}[(a + b*x)/2]^2*(- \\ & (\text{Sec}[(a + b*x)/2]^2*(\text{Cos}[a + b*x] + c*\text{Cos}[a + b*x] - d*\text{Sin}[a + b*x])) - \text{Sec} \\ & [(a + b*x)/2]^2*(d*\text{Cos}[a + b*x] + \text{Sin}[a + b*x] + c*\text{Sin}[a + b*x])* \text{Tan}[(a + b \\ & *x)/2])))/(d*\text{Cos}[a + b*x] + \text{Sin}[a + b*x] + c*\text{Sin}[a + b*x])) \end{aligned}$$

**Maple [B]** time = 0.139, size = 629, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(c+d\*cot(b\*x+a)),x)

[Out] 
$$\begin{aligned} & -1/2/b*\text{arccoth}(c+d*\text{cot}(b*x+a))*\text{Pi}+1/b*\text{arccoth}(c+d*\text{cot}(b*x+a))*\text{arccot}(\text{cot}(b* \\ & x+a))+1/2/b*\text{arctan}((c+d*\text{cot}(b*x+a))/d-c/d)*\text{ln}(d*((c+d*\text{cot}(b*x+a))/d-c/d)+c+ \\ & 1)-1/2/b*\text{arctan}((c+d*\text{cot}(b*x+a))/d-c/d)*\text{ln}(d*((c+d*\text{cot}(b*x+a))/d-c/d)+c-1)- \\ & 1/4*I/b*\text{ln}(d*((c+d*\text{cot}(b*x+a))/d-c/d)+c-1)*\text{ln}((I*d-d*((c+d*\text{cot}(b*x+a))/d-c/ \\ & d))/(I*d+c-1))+1/4*I/b*\text{ln}(d*((c+d*\text{cot}(b*x+a))/d-c/d)+c-1)*\text{ln}((I*d+d*((c+d*c \\ & ot}(b*x+a))/d-c/d))/(1-c+I*d))-1/4*I/b*\text{dilog}((I*d-d*((c+d*\text{cot}(b*x+a))/d-c/d) \\ & )/(I*d+c-1))+1/4*I/b*\text{dilog}((I*d+d*((c+d*\text{cot}(b*x+a))/d-c/d))/(1-c+I*d))+1/4* \\ & I/b*\text{ln}(d*((c+d*\text{cot}(b*x+a))/d-c/d)+c+1)*\text{ln}((I*d-d*((c+d*\text{cot}(b*x+a))/d-c/d))/ \\ & (1+c+I*d))-1/4*I/b*\text{ln}(d*((c+d*\text{cot}(b*x+a))/d-c/d)+c+1)*\text{ln}((I*d+d*((c+d*\text{cot}(b \\ & *x+a))/d-c/d))/(I*d-c-1))+1/4*I/b*\text{dilog}((I*d-d*((c+d*\text{cot}(b*x+a))/d-c/d))/(1 \\ & +c+I*d))-1/4*I/b*\text{dilog}((I*d+d*((c+d*\text{cot}(b*x+a))/d-c/d))/(I*d-c-1)) \end{aligned}$$

**Maxima [B]** time = 1.89117, size = 529, normalized size = 2.73

$$4(bx + a) \operatorname{arccoth}\left(c + \frac{d}{\tan(bx+a)}\right) + \left(\arctan\left(\frac{(c+1)d+(c^2+2c+1)\tan(bx+a)}{c^2+d^2+2c+1}, \frac{(c+1)d \tan(bx+a)+d^2}{c^2+d^2+2c+1}\right) - \arctan\left(\frac{(c-1)d+(c^2-2c+1)\tan(bx+a)}{c^2+d^2-2c+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c+d\*cot(b\*x+a)),x, algorithm="maxima")

[Out]  $\frac{1}{4} \cdot (4 \cdot (b \cdot x + a) \cdot \operatorname{arccoth}(c + d/\tan(b \cdot x + a)) + (\arctan2(((c + 1) \cdot d + (c^2 + 2 \cdot c + 1) \cdot \tan(b \cdot x + a))/(c^2 + d^2 + 2 \cdot c + 1), ((c + 1) \cdot d \cdot \tan(b \cdot x + a) + d^2)/(c^2 + d^2 + 2 \cdot c + 1)) - \arctan2(((c - 1) \cdot d + (c^2 - 2 \cdot c + 1) \cdot \tan(b \cdot x + a))/(c^2 + d^2 - 2 \cdot c + 1), ((c - 1) \cdot d \cdot \tan(b \cdot x + a) + d^2)/(c^2 + d^2 - 2 \cdot c + 1)))) \cdot \log(\tan(b \cdot x + a)^2 + 1) - (b \cdot x + a) \cdot \log((2 \cdot (c + 1) \cdot d \cdot \tan(b \cdot x + a) + (c^2 + 2 \cdot c + 1) \cdot \tan(b \cdot x + a)^2 + d^2)/(c^2 + d^2 + 2 \cdot c + 1)) + (b \cdot x + a) \cdot \log((2 \cdot (c - 1) \cdot d \cdot \tan(b \cdot x + a) + (c^2 - 2 \cdot c + 1) \cdot \tan(b \cdot x + a)^2 + d^2)/(c^2 + d^2 - 2 \cdot c + 1)) + I \cdot \operatorname{dilog}(-((c + 1) \cdot \tan(b \cdot x + a) - I \cdot c - I)/(I \cdot c + d + I)) - I \cdot \operatorname{dilog}(-((c - 1) \cdot \tan(b \cdot x + a) - I \cdot c + I)/(I \cdot c + d - I)) + I \cdot \operatorname{dilog}(-((c - 1) \cdot \tan(b \cdot x + a) + I \cdot c - I)/(-I \cdot c + d + I)) - I \cdot \operatorname{dilog}(-((c + 1) \cdot \tan(b \cdot x + a) + I \cdot c + I)/(-I \cdot c + d - I)))/b$

**Fricas [B]** time = 2.78503, size = 2919, normalized size = 15.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c+d\*cot(b\*x+a)),x, algorithm="fricas")

[Out]  $\frac{1}{8} \cdot (4 \cdot b \cdot x \cdot \log((d \cdot \cos(2 \cdot b \cdot x + 2 \cdot a) + (c + 1) \cdot \sin(2 \cdot b \cdot x + 2 \cdot a) + d)/(d \cdot \cos(2 \cdot b \cdot x + 2 \cdot a) + (c - 1) \cdot \sin(2 \cdot b \cdot x + 2 \cdot a) + d)) + 2 \cdot a \cdot \log(1/2 \cdot c^2 + I \cdot (c + 1) \cdot d - 1/2 \cdot d^2 - 1/2 \cdot (c^2 + d^2 + 2 \cdot c + 1) \cdot \cos(2 \cdot b \cdot x + 2 \cdot a) + 1/2 \cdot (I \cdot c^2 + I \cdot d^2 + 2 \cdot I \cdot c + I) \cdot \sin(2 \cdot b \cdot x + 2 \cdot a) + c + 1/2) - 2 \cdot a \cdot \log(1/2 \cdot c^2 + I \cdot (c - 1) \cdot d - 1/2 \cdot d^2 - 1/2 \cdot (c^2 + d^2 - 2 \cdot c + 1) \cdot \cos(2 \cdot b \cdot x + 2 \cdot a) + 1/2 \cdot (I \cdot c^2 + I \cdot d^2 - 2 \cdot I \cdot c + I) \cdot \sin(2 \cdot b \cdot x + 2 \cdot a) - c + 1/2) + 2 \cdot a \cdot \log(-1/2 \cdot c^2 + I \cdot (c + 1) \cdot d + 1/2 \cdot d^2 + 1/2 \cdot (c^2 + d^2 + 2 \cdot c + 1) \cdot \cos(2 \cdot b \cdot x + 2 \cdot a) + 1/2 \cdot (I \cdot c^2 + I \cdot d^2 + 2 \cdot I \cdot c + I) \cdot \sin(2 \cdot b \cdot x + 2 \cdot a) - c - 1/2) - 2 \cdot a \cdot \log(-1/2 \cdot c^2 + I \cdot (c - 1) \cdot d + 1/2 \cdot d^2 + 1/2 \cdot (c^2 + d^2 - 2 \cdot c + 1) \cdot \cos(2 \cdot b \cdot x + 2 \cdot a) + 1/2 \cdot (I \cdot c^2 + I \cdot d^2 - 2 \cdot I \cdot c + I) \cdot \sin(2 \cdot b \cdot x + 2 \cdot a) + c - 1/2) - 2 \cdot (b \cdot x + a) \cdot \log((c^2 + d^2 - (c^2 + 2 \cdot I \cdot (c + 1) \cdot d - d^2 + 2 \cdot c + 1) \cdot \cos(2 \cdot b \cdot x + 2 \cdot a) + (-I \cdot c^2 + 2 \cdot (c + 1) \cdot d + I \cdot d^2 - 2 \cdot I \cdot c - I) \cdot \sin(2 \cdot b \cdot x + 2 \cdot a) + 2 \cdot c + 1)/(c^2 + d^2 + 2 \cdot c + 1)) - 2 \cdot (b \cdot x + a) \cdot \log((c^2 + d^2 - (c^2 - 2 \cdot I \cdot (c + 1) \cdot d - d^2 + 2 \cdot c + 1) \cdot \cos(2 \cdot b \cdot x + 2 \cdot a) + (I \cdot c^2 + 2 \cdot (c + 1) \cdot d - I \cdot d^2 + 2 \cdot I \cdot c + I) \cdot \sin(2 \cdot b \cdot x + 2 \cdot a) + 2 \cdot c + 1)/(c^2 + d^2 + 2 \cdot c + 1)) + 2 \cdot (b \cdot x + a) \cdot \log((c^2 + d^2 - (c^2 + 2 \cdot I \cdot (c - 1) \cdot d - d^2 - 2 \cdot c + 1) \cdot \cos(2 \cdot b \cdot x + 2 \cdot a) + (-I \cdot c^2 + 2 \cdot (c - 1) \cdot d + I \cdot d^2 + 2 \cdot I \cdot c - I) \cdot \sin(2 \cdot b \cdot x + 2 \cdot a) - 2 \cdot c + 1)/(c^2 + d^2 - 2 \cdot c + 1)) + 2 \cdot (b \cdot x + a) \cdot \log((c^2 + d^2 - (c^2 - 2 \cdot I \cdot (c - 1) \cdot d - d^2 - 2 \cdot c + 1) \cdot \cos(2 \cdot b \cdot x + 2 \cdot a) + (I \cdot c^2 + 2 \cdot (c - 1) \cdot d - I \cdot d^2 - 2 \cdot I \cdot c + I) \cdot \sin(2 \cdot b \cdot x + 2 \cdot a) - 2 \cdot c + 1)/(c^2 + d^2 - 2 \cdot c + 1)) + I \cdot \operatorname{dilog}(-c^2 + d^2 - (c^2 + 2 \cdot I \cdot (c + 1) \cdot d - d^2 + 2 \cdot c$

$$\begin{aligned}
& + 1) \cos(2bx + 2a) + (-Ic^2 + 2(c+1)d + Id^2 - 2Ic - I) \sin(2bx + 2a) + 2c + 1) / (c^2 + d^2 + 2c + 1) + 1) - I \operatorname{dilog}(-(c^2 + d^2 - (c^2 - 2I(c+1)d - d^2 + 2c + 1) \cos(2bx + 2a) + (Ic^2 + 2(c+1)d - Id^2 + 2Ic + I) \sin(2bx + 2a) + 2c + 1) / (c^2 + d^2 + 2c + 1) + 1) \\
& - I \operatorname{dilog}(-(c^2 + d^2 - (c^2 + 2I(c-1)d - d^2 - 2c + 1) \cos(2bx + 2a) + (-Ic^2 + 2(c-1)d + Id^2 + 2Ic - I) \sin(2bx + 2a) - 2c + 1) / (c^2 + d^2 - 2c + 1) + 1) + I \operatorname{dilog}(-(c^2 + d^2 - (c^2 - 2I(c-1)d - d^2 - 2c + 1) \cos(2bx + 2a) + (Ic^2 + 2(c-1)d - Id^2 - 2Ic + I) \sin(2bx + 2a) - 2c + 1) / (c^2 + d^2 - 2c + 1) + 1)) / b
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(c+d\*cot(b\*x+a)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(d \cot(bx + a) + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c+d\*cot(b\*x+a)),x, algorithm="giac")

[Out] integrate(arccoth(d\*cot(b\*x + a) + c), x)

$$3.256 \quad \int \frac{\coth^{-1}(c+d \cot(a+bx))}{x} dx$$

**Optimal.** Leaf size=17

$$\text{CannotIntegrate}\left(\frac{\coth^{-1}(d \cot(a+bx)+c)}{x}, x\right)$$

[Out] CannotIntegrate[ArcCoth[c + d\*Cot[a + b\*x]]/x, x]

**Rubi [A]** time = 0.0933881, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\coth^{-1}(c+d \cot(a+bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[ArcCoth[c + d\*Cot[a + b\*x]]/x,x]

[Out] Defer[Int][ArcCoth[c + d\*Cot[a + b\*x]]/x, x]

Rubi steps

$$\int \frac{\coth^{-1}(c+d \cot(a+bx))}{x} dx = \int \frac{\coth^{-1}(c+d \cot(a+bx))}{x} dx$$

**Mathematica [A]** time = 0.347825, size = 0, normalized size = 0.

$$\int \frac{\coth^{-1}(c+d \cot(a+bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcCoth[c + d\*Cot[a + b\*x]]/x,x]

[Out] Integrate[ArcCoth[c + d\*Cot[a + b\*x]]/x, x]

---

**Maple [A]** time = 0.391, size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(c + d \cot(bx + a))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(c+d*cot(b*x+a))/x,x)`

[Out] `int(arccoth(c+d*cot(b*x+a))/x,x)`

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(d \cot(bx + a) + c)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(c+d*cot(b*x+a))/x,x, algorithm="maxima")`

[Out] `integrate(arccoth(d*cot(b*x + a) + c)/x, x)`

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arccoth}(d \cot(bx + a) + c)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(c+d*cot(b*x+a))/x,x, algorithm="fricas")`

[Out] `integral(arccoth(d*cot(b*x + a) + c)/x, x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(c+d*cot(b*x+a))/x,x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(d \cot (bx + a) + c)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*cot(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*cot(b*x + a) + c)/x, x)
```



### 3.257 $\int x^2 \coth^{-1}(1 + id + d \cot(a + bx)) dx$

**Optimal.** Leaf size=168

$$\frac{x \operatorname{PolyLog}\left(3, (1 + id)e^{2ia+2ibx}\right)}{4b^2} - \frac{i \operatorname{PolyLog}\left(4, (1 + id)e^{2ia+2ibx}\right)}{8b^3} + \frac{ix^2 \operatorname{PolyLog}\left(2, (1 + id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{6}x^3 \log\left(1 - (1 + I*d)*E^{\left((2*I)*a + (2*I)*b*x\right)}\right)$$

[Out] (I/12)\*b\*x^4 + (x^3\*ArcCoth[1 + I\*d + d\*Cot[a + b\*x]])/3 - (x^3\*Log[1 - (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)])/6 + ((I/4)\*x^2\*PolyLog[2, (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)])/b - (x\*PolyLog[3, (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)])/(4\*b^2) - ((I/8)\*PolyLog[4, (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)])/b^3

**Rubi [A]** time = 0.305705, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$ , Rules used = {6266, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{x \operatorname{PolyLog}\left(3, (1 + id)e^{2ia+2ibx}\right)}{4b^2} - \frac{i \operatorname{PolyLog}\left(4, (1 + id)e^{2ia+2ibx}\right)}{8b^3} + \frac{ix^2 \operatorname{PolyLog}\left(2, (1 + id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{6}x^3 \log\left(1 - (1 + I*d)*E^{\left((2*I)*a + (2*I)*b*x\right)}\right)$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcCoth[1 + I\*d + d\*Cot[a + b\*x]],x]

[Out] (I/12)\*b\*x^4 + (x^3\*ArcCoth[1 + I\*d + d\*Cot[a + b\*x]])/3 - (x^3\*Log[1 - (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)])/6 + ((I/4)\*x^2\*PolyLog[2, (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)])/b - (x\*PolyLog[3, (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)])/(4\*b^2) - ((I/8)\*PolyLog[4, (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)])/b^3

#### Rule 6266

Int[ArcCoth[(c\_.) + Cot[(a\_.) + (b\_.)\*(x\_)]]\*(d\_.)]\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^(m + 1)\*ArcCoth[c + d\*Cot[a + b\*x]])/(f\*(m + 1)), x] + Dist[(I\*b)/(f\*(m + 1)), Int[(e + f\*x)^(m + 1)/(c - I\*d - c\*E^(2\*I\*a + 2\*I\*b\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - I\*d)^2, 1]

#### Rule 2184

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x],

x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_))/  
((a\_) + (b\_)\*((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] :> Simp  
[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Di  
st[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)  
))^(n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_)\*((F\_)^((c\_)\*(a\_) + (b\_)\*(x\_)))^(n\_)]\*((f\_) + (g\_  
)\*(x\_))^(m\_), x\_Symbol] :> -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)  
))^(n)])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m -  
1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^(n)]], x], x] /; FreeQ[{F, a, b, c, e, f  
, g, n}, x] && GtQ[m, 0]

### Rule 6609

Int[((e\_) + (f\_)\*(x\_))^(m\_)\*PolyLog[n\_, (d\_)\*((F\_)^((c\_)\*(a\_) + (b\_  
)\*(x\_)))^(p\_)], x\_Symbol] :> Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a  
+ b\*x)))^p])/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(  
m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c,  
d, e, f, n, p}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]  
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi  
onOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[  
{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\*  
(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_)\*((a\_) + (b\_)\*(x\_))^(p\_)]/((d\_) + (e\_)\*(x\_)), x\_S  
ymbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d  
, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(1 + id + d \cot(a + bx)) dx &= \frac{1}{3} x^3 \coth^{-1}(1 + id + d \cot(a + bx)) + \frac{1}{3} (ib) \int \frac{x^3}{1 + (-1 - id)e^{2ia+2ibx}} dx \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 + id + d \cot(a + bx)) + \frac{1}{3} (b(i - d)) \int \frac{e^{2ia+2ibx} x^3}{1 + (-1 - id)e^{2ia+2ibx}} dx \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 + id + d \cot(a + bx)) - \frac{1}{6} x^3 \log(1 - (1 + id)e^{2ia+2ibx}) \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 + id + d \cot(a + bx)) - \frac{1}{6} x^3 \log(1 - (1 + id)e^{2ia+2ibx}) \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 + id + d \cot(a + bx)) - \frac{1}{6} x^3 \log(1 - (1 + id)e^{2ia+2ibx}) \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 + id + d \cot(a + bx)) - \frac{1}{6} x^3 \log(1 - (1 + id)e^{2ia+2ibx}) \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 + id + d \cot(a + bx)) - \frac{1}{6} x^3 \log(1 - (1 + id)e^{2ia+2ibx})
\end{aligned}$$

**Mathematica [A]** time = 0.177359, size = 155, normalized size = 0.92

$$\frac{1}{3} x^3 \coth^{-1}(d \cot(a + bx) + id + 1) - \frac{6ib^2 x^2 \text{PolyLog}\left(2, -\frac{ie^{-2i(a+bx)}}{d-i}\right) + 6bx \text{PolyLog}\left(3, -\frac{ie^{-2i(a+bx)}}{d-i}\right) - 3i \text{PolyLog}\left(4, -\frac{ie^{-2i(a+bx)}}{d-i}\right)}{24b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[1 + I\*d + d\*Cot[a + b\*x]], x]

[Out] (x^3\*ArcCoth[1 + I\*d + d\*Cot[a + b\*x]])/3 - (4\*b^3\*x^3\*Log[1 + I/((-I + d)\*E^((2\*I)\*(a + b\*x)))] + (6\*I)\*b^2\*x^2\*PolyLog[2, (-I)/((-I + d)\*E^((2\*I)\*(a + b\*x)))] + 6\*b\*x\*PolyLog[3, (-I)/((-I + d)\*E^((2\*I)\*(a + b\*x)))] - (3\*I)\*PolyLog[4, (-I)/((-I + d)\*E^((2\*I)\*(a + b\*x)))])/(24\*b^3)

**Maple [C]** time = 29.797, size = 2449, normalized size = 14.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(1+I\*d+d\*cot(b\*x+a)), x)

```
[Out] -1/6*x^3*ln(d)+1/12*I*b*x^4+1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(
2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b
*x+a))+I)/(exp(2*I*(b*x+a))-1))^2-1/2*I/b^2*a^2/(-d+I)*ln(1+I*exp(I*(b*x+a)
)*(I*(-d+I))^(1/2))*x-1/2*I/b^3*a^2*d/(-d+I)*dilog(1-I*exp(I*(b*x+a))*(I*(-
d+I))^(1/2))-1/2*I/b^3*a^2*d/(-d+I)*dilog(1+I*exp(I*(b*x+a))*(I*(-d+I))^(1/
2))-1/4*I/b*d/(-d+I)*polylog(2,-I*(-d+I)*exp(2*I*(b*x+a)))*x^2+1/2/b^3*a^3*
d/(-d+I)*ln(1-I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))+1/2/b^3*a^3*d/(-d+I)*ln(1+
I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))-1/6/b^3*a^3*d/(-d+I)*ln(I*exp(2*I*(b*x+a
))-exp(2*I*(b*x+a))*d-I)+1/4/b^2*d/(-d+I)*polylog(3,-I*(-d+I)*exp(2*I*(b*x+
a)))*x+1/6*I/b^3*a^3/(-d+I)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d-I)+1/8
/b^3/(-d+I)*polylog(4,-I*(-d+I)*exp(2*I*(b*x+a)))-1/3*x^3*ln(exp(I*(b*x+a)
))-1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(exp
(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a)
)-1))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1)
)^2+1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+1/12*I*x^3
*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))-1/6*I*x^3*Pi*csgn(I*exp
(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2-1/6*I/(-d+I)*ln(1+I*(-d+I)*exp(2*
I*(b*x+a)))*x^3-1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))
^3-1/12*I*x^3*Pi*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b
*x+a))-1))^2-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*
x+a))*d-I*exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a)
)+I)/(exp(2*I*(b*x+a))-1))+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp
(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a)))/(exp(2*I*(b*x+a))-1))+1/12*I*x^
3*Pi*csgn(I*d)*csgn(I*exp(2*I*(b*x+a)))/(exp(2*I*(b*x+a))-1))*csgn(I*d/(exp(
2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))-1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-
I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*d-I*exp(
2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))+1/2/b^2*a^2*d/(-d+I)*ln(1+I*exp(I*(b*
x+a))*(I*(-d+I))^(1/2))*x+1/4*I/b^3*d/(-d+I)*polylog(2,-I*(-d+I)*exp(2*I*(b
*x+a)))*a^2+1/2*I/b^2/(-d+I)*ln(1+I*(-d+I)*exp(2*I*(b*x+a)))*x*a^2-1/2*I/b^
2*a^2/(-d+I)*ln(1-I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))*x+1/8*I/b^3*d/(-d+I)*p
olylog(4,-I*(-d+I)*exp(2*I*(b*x+a)))+1/12*I*x^3*Pi*csgn((exp(2*I*(b*x+a))*d
-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^3-1/12*I*x^3*Pi*csgn(I*(exp(2*
I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^3+1/12*I*x^3*Pi*cs
gn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))-1)*exp
(2*I*(b*x+a)))+1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a)
)+I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1)
)^2+1/6*x^3*ln(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)-1/12*I*x^3*Pi*csgn(
I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(
2*I*(b*x+a)))^2-1/3/b^3*d/(-d+I)*ln(1+I*(-d+I)*exp(2*I*(b*x+a)))*a^3-1/4*I/
b^2/(-d+I)*polylog(3,-I*(-d+I)*exp(2*I*(b*x+a)))*x-1/2*I/b^3*a^3/(-d+I)*ln(
1-I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))-1/2*I/b^3*a^3/(-d+I)*ln(1+I*exp(I*(b*x
+a))*(I*(-d+I))^(1/2))-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*
I*(b*x+a))/(exp(2*I*(b*x+a))-1))^2-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1
))*csgn(I*exp(2*I*(b*x+a)))/(exp(2*I*(b*x+a))-1))^2-1/12*I*x^3*Pi*csgn(I*d)*
csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+1/3*I/b^3/(-d+I)*ln(1+I*(
```

$$\begin{aligned}
& -d+I) \exp(2*I*(b*x+a)) * a^{-3-1/2} / b^3 * a^2 / (-d+I) * \operatorname{dilog}(1-I \exp(I*(b*x+a)) * (I * \\
& (-d+I))^{1/2}) - 1/2 / b^3 * a^2 / (-d+I) * \operatorname{dilog}(1+I \exp(I*(b*x+a)) * (I * (-d+I))^{1/2}) \\
& ) - 1/4 / b / (-d+I) * \operatorname{polylog}(2, -I * (-d+I) \exp(2*I*(b*x+a))) * x^2 + 1/4 / b^3 / (-d+I) * \operatorname{polylog}(2, -I * (-d+I) \exp(2*I*(b*x+a))) * a^2 + 1/12 * I * x^3 * \operatorname{Pisgn}(I \exp(2*I*(b*x+a)))^3 - 1/2 / b^2 * d / (-d+I) * \ln(1+I * (-d+I) \exp(2*I*(b*x+a))) * x * a^2 + 1/2 / b^2 * a^2 * d / (-d+I) * \ln(1-I \exp(I*(b*x+a)) * (I * (-d+I))^{1/2}) * x + 1/6 * d / (-d+I) * \ln(1+I * (-d+I) \exp(2*I*(b*x+a))) * x^3 + 1/12 * I * x^3 * \operatorname{Pisgn}(I \exp(2*I*(b*x+a))) / (\exp(2*I*(b*x+a)) - 1))^3 + 1/12 * I * x^3 * \operatorname{Pisgn}(I * d / (\exp(2*I*(b*x+a)) - 1) * \exp(2*I*(b*x+a)))^3
\end{aligned}$$

**Maxima [B]** time = 1.17363, size = 462, normalized size = 2.75

$$\frac{12((bx+a)^3 - 3(bx+a)^2 a + 3(bx+a)a^2) \operatorname{arccoth}(d \cot(bx+a) + i d + 1)}{b^2} - \frac{-3i(bx+a)^4 + 12i(bx+a)^3 a - 18i(bx+a)^2 a^2 + (-8i(bx+a)^3 + 18i(bx+a)^2 a - 18i(bx+a)a^2) \operatorname{arctan}(\dots)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(1+I\*d+d\*cot(b\*x+a)),x, algorithm="maxima")

[Out] 1/36\*(12\*((b\*x + a)^3 - 3\*(b\*x + a)^2\*a + 3\*(b\*x + a)\*a^2)\*arccoth(d\*cot(b\*x + a) + I\*d + 1)/b^2 - (-3\*I\*(b\*x + a)^4 + 12\*I\*(b\*x + a)^3\*a - 18\*I\*(b\*x + a)^2\*a^2 + (-8\*I\*(b\*x + a)^3 + 18\*I\*(b\*x + a)^2\*a - 18\*I\*(b\*x + a)\*a^2)\*arctan2(d\*cos(2\*b\*x + 2\*a) + sin(2\*b\*x + 2\*a), d\*sin(2\*b\*x + 2\*a) - cos(2\*b\*x + 2\*a) + 1) + (-12\*I\*(b\*x + a)^2 + 18\*I\*(b\*x + a)\*a - 9\*I\*a^2)\*dilog((I\*d + 1)\*e^(2\*I\*b\*x + 2\*I\*a)) + (4\*(b\*x + a)^3 - 9\*(b\*x + a)^2\*a + 9\*(b\*x + a)\*a^2)\*log((d^2 + 1)\*cos(2\*b\*x + 2\*a)^2 + (d^2 + 1)\*sin(2\*b\*x + 2\*a)^2 + 2\*d\*sin(2\*b\*x + 2\*a) - 2\*cos(2\*b\*x + 2\*a) + 1) + 3\*(4\*b\*x + a)\*polylog(3, (I\*d + 1)\*e^(2\*I\*b\*x + 2\*I\*a)) + 6\*I\*polylog(4, (I\*d + 1)\*e^(2\*I\*b\*x + 2\*I\*a)))/b^2)/b

**Fricas [C]** time = 1.77333, size = 505, normalized size = 3.01

$$\frac{2i b^4 x^4 + 4 b^3 x^3 \log\left(\frac{(d-i)e^{(2i b x + 2i a) + i} e^{(-2i b x - 2i a)}}{d}\right) + 6i b^2 x^2 \operatorname{Li}_2(-(-i d - 1)e^{(2i b x + 2i a)}) - 2i a^4 + 4 a^3 \log\left(\frac{(d-i)e^{(2i b x + 2i a) + i}}{d-i}\right) - 6}{24 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(1+I\*d+d\*cot(b\*x+a)),x, algorithm="fricas")

```
[Out] 1/24*(2*I*b^4*x^4 + 4*b^3*x^3*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I
*b*x - 2*I*a)/d) + 6*I*b^2*x^2*dilog((-I*d - 1)*e^(2*I*b*x + 2*I*a)) - 2*I
*a^4 + 4*a^3*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)/(d - I)) - 6*b*x*polylog
(3, (I*d + 1)*e^(2*I*b*x + 2*I*a)) - 4*(b^3*x^3 + a^3)*log((-I*d - 1)*e^(2*
I*b*x + 2*I*a) + 1) - 3*I*polylog(4, (I*d + 1)*e^(2*I*b*x + 2*I*a)))/b^3
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(1+I*d+d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arccoth}(d \cot(bx + a) + id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(d*cot(b*x + a) + I*d + 1), x)
```

### 3.258 $\int x \coth^{-1}(1 + id + d \cot(a + bx)) dx$

**Optimal.** Leaf size=132

$$-\frac{\text{PolyLog}\left(3, (1 + id)e^{2ia+2ibx}\right)}{8b^2} + \frac{ix\text{PolyLog}\left(2, (1 + id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{4}x^2 \log\left(1 - (1 + id)e^{2ia+2ibx}\right) + \frac{1}{2}x^2 \coth^{-1}(d \cot(a + bx))$$

[Out] (I/6)\*b\*x^3 + (x^2\*ArcCoth[1 + I\*d + d\*Cot[a + b\*x]])/2 - (x^2\*Log[1 - (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/4 + ((I/4)\*x\*PolyLog[2, (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/b - PolyLog[3, (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/(8\*b^2))

**Rubi [A]** time = 0.256505, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6266, 2184, 2190, 2531, 2282, 6589}

$$-\frac{\text{PolyLog}\left(3, (1 + id)e^{2ia+2ibx}\right)}{8b^2} + \frac{ix\text{PolyLog}\left(2, (1 + id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{4}x^2 \log\left(1 - (1 + id)e^{2ia+2ibx}\right) + \frac{1}{2}x^2 \coth^{-1}(d \cot(a + bx))$$

Antiderivative was successfully verified.

[In] Int[x\*ArcCoth[1 + I\*d + d\*Cot[a + b\*x]], x]

[Out] (I/6)\*b\*x^3 + (x^2\*ArcCoth[1 + I\*d + d\*Cot[a + b\*x]])/2 - (x^2\*Log[1 - (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/4 + ((I/4)\*x\*PolyLog[2, (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/b - PolyLog[3, (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/(8\*b^2))

#### Rule 6266

Int[ArcCoth[(c\_.) + Cot[(a\_.) + (b\_.)\*(x\_.)]\*(d\_.)]\*((e\_.) + (f\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^(m + 1)\*ArcCoth[c + d\*Cot[a + b\*x]])/(f\*(m + 1)), x] + Dist[(I\*b)/(f\*(m + 1)), Int[(e + f\*x)^(m + 1)/(c - I\*d - c\*E^(2\*I\*a + 2\*I\*b\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - I\*d)^2, 1]

#### Rule 2184

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_.))))^(n\_.), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[(c + d\*x)^m\*(F^(g\*(e + f\*x)))^n/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*(a_) + (b_)*(x_)))^(n_)]*((f_) + (g_)
*(x_))^(m_), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps



$$\begin{aligned}
\int x \coth^{-1}(1 + id + d \cot(a + bx)) dx &= \frac{1}{2}x^2 \coth^{-1}(1 + id + d \cot(a + bx)) + \frac{1}{2}(ib) \int \frac{x^2}{1 + (-1 - id)e^{2ia+2ibx}} dx \\
&= \frac{1}{6}ibx^3 + \frac{1}{2}x^2 \coth^{-1}(1 + id + d \cot(a + bx)) + \frac{1}{2}(b(i - d)) \int \frac{e^{2ia+2ibx}x^2}{1 + (-1 - id)e^{2ia+2ibx}} dx \\
&= \frac{1}{6}ibx^3 + \frac{1}{2}x^2 \coth^{-1}(1 + id + d \cot(a + bx)) - \frac{1}{4}x^2 \log(1 - (1 + id)e^{2ia+2ibx}) + \frac{1}{4}x^2 \log(1 - (-1 - id)e^{2ia+2ibx}) \\
&= \frac{1}{6}ibx^3 + \frac{1}{2}x^2 \coth^{-1}(1 + id + d \cot(a + bx)) - \frac{1}{4}x^2 \log(1 - (1 + id)e^{2ia+2ibx}) + \frac{1}{4}x^2 \log(1 - (-1 - id)e^{2ia+2ibx}) \\
&= \frac{1}{6}ibx^3 + \frac{1}{2}x^2 \coth^{-1}(1 + id + d \cot(a + bx)) - \frac{1}{4}x^2 \log(1 - (1 + id)e^{2ia+2ibx}) + \frac{1}{4}x^2 \log(1 - (-1 - id)e^{2ia+2ibx}) \\
&= \frac{1}{6}ibx^3 + \frac{1}{2}x^2 \coth^{-1}(1 + id + d \cot(a + bx)) - \frac{1}{4}x^2 \log(1 - (1 + id)e^{2ia+2ibx}) + \frac{1}{4}x^2 \log(1 - (-1 - id)e^{2ia+2ibx})
\end{aligned}$$

**Mathematica [A]** time = 0.108773, size = 119, normalized size = 0.9

$$\frac{1}{2}x^2 \coth^{-1}(d \cot(a + bx) + id + 1) - \frac{2ibx \operatorname{PolyLog}\left(2, -\frac{ie^{-2i(a+bx)}}{d-i}\right) + \operatorname{PolyLog}\left(3, -\frac{ie^{-2i(a+bx)}}{d-i}\right) + 2b^2x^2 \log\left(1 + \frac{ie^{-2i(a+bx)}}{d-i}\right)}{8b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[1 + I\*d + d\*Cot[a + b\*x]], x]

[Out] (x^2\*ArcCoth[1 + I\*d + d\*Cot[a + b\*x]])/2 - (2\*b^2\*x^2\*Log[1 + I/((-I + d)\*E^((2\*I)\*(a + b\*x)))] + (2\*I)\*b\*x\*PolyLog[2, (-I)/((-I + d)\*E^((2\*I)\*(a + b\*x)))] + PolyLog[3, (-I)/((-I + d)\*E^((2\*I)\*(a + b\*x)))])/(8\*b^2)

**Maple [C]** time = 17.385, size = 2351, normalized size = 17.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(1+I\*d+d\*cot(b\*x+a)), x)

[Out] 1/2\*I/b^2\*a^2/(-d+I)\*ln(1+I\*exp(I\*(b\*x+a))\*(I\*(-d+I))^(1/2))+1/8\*I\*x^2\*Pi\*csgn(I/(exp(2\*I\*(b\*x+a))-1))\*csgn(I\*(exp(2\*I\*(b\*x+a))\*d-I\*exp(2\*I\*(b\*x+a)))+I

$$\begin{aligned}
& ) / (\exp(2*I*(b*x+a)) - 1)^{-2-1/4*I/b^2*a^2/(-d+I)*\ln(I*\exp(2*I*(b*x+a)) - \exp(2*I*(b*x+a))*d-I) - 1/4*I/b^2/(-d+I)*\ln(1+I*(-d+I)*\exp(2*I*(b*x+a)))*a^{2+1/2*I/b^2*a^2/(-d+I)*\ln(1-I*\exp(I*(b*x+a)))*(I*(-d+I))^{(1/2)}} + 1/4/b^2*a^2*d/(-d+I)*\ln(I*\exp(2*I*(b*x+a)) - \exp(2*I*(b*x+a))*d-I) - 1/2/b^2*a^2*d/(-d+I)*\ln(1-I*\exp(I*(b*x+a)))*(I*(-d+I))^{(1/2)} - 1/2/b^2*a^2*d/(-d+I)*\ln(1+I*\exp(I*(b*x+a)))*(I*(-d+I))^{(1/2)} + 1/4/b^2*d/(-d+I)*\ln(1+I*(-d+I)*\exp(2*I*(b*x+a)))*a^{2-1/8*I*x^2*Pi*csgn(I*(\exp(2*I*(b*x+a))*d-I*\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))-1))*csgn((\exp(2*I*(b*x+a))*d-I*\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))-1)) + 1/8*I*x^2*Pi*csgn(I*\exp(2*I*(b*x+a)))/(\exp(2*I*(b*x+a))-1)^3 + 1/8*I*x^2*Pi*csgn(I*d/(\exp(2*I*(b*x+a))-1)*\exp(2*I*(b*x+a)))^3 - 1/8*I*x^2*Pi*csgn((\exp(2*I*(b*x+a))*d-I*\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))-1))^{2+1/4*x^2*\ln(\exp(2*I*(b*x+a))*d-I*\exp(2*I*(b*x+a))+I) - 1/4/b/(-d+I)*\text{polylog}(2, -I*(-d+I)*\exp(2*I*(b*x+a)))*x - 1/4/b^2/(-d+I)*\text{polylog}(2, -I*(-d+I)*\exp(2*I*(b*x+a)))*a + 1/8/b^2*d/(-d+I)*\text{polylog}(3, -I*(-d+I)*\exp(2*I*(b*x+a))) - 1/8*I*x^2*Pi*csgn(I*(\exp(2*I*(b*x+a))*d-I*\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))-1))^{3-1/8*I/b^2/(-d+I)*\text{polylog}(3, -I*(-d+I)*\exp(2*I*(b*x+a))) - 1/4*I/(-d+I)*\ln(1+I*(-d+I)*\exp(2*I*(b*x+a)))*x^{2+1/6*I*b*x^3-1/4*I*x^2*Pi*csgn(I*\exp(I*(b*x+a)))*csgn(I*\exp(2*I*(b*x+a)))^{2-1/8*I*x^2*Pi*csgn(I*d)*csgn(I*d/(\exp(2*I*(b*x+a))-1)*\exp(2*I*(b*x+a)))^{2-1/8*I*x^2*Pi*csgn(I*\exp(2*I*(b*x+a)))/(\exp(2*I*(b*x+a))-1))*csgn(I*d/(\exp(2*I*(b*x+a))-1)*\exp(2*I*(b*x+a)))^{2-1/8*I*x^2*Pi*csgn(d/(\exp(2*I*(b*x+a))-1)*\exp(2*I*(b*x+a)))^{3-1/4*x^2*\ln(d) + 1/8*I*x^2*Pi*csgn(I*\exp(I*(b*x+a)))^{2*csgn(I*\exp(2*I*(b*x+a))) - 1/8*I*x^2*Pi*csgn(I*\exp(2*I*(b*x+a)))*csgn(I*\exp(2*I*(b*x+a)))/(\exp(2*I*(b*x+a))-1))^{2-1/8*I*x^2*Pi*csgn(I/(\exp(2*I*(b*x+a))-1))*csgn(I*\exp(2*I*(b*x+a)))/(\exp(2*I*(b*x+a))-1))^{2-1/8*I*x^2*Pi*csgn(I*d/(\exp(2*I*(b*x+a))-1)*\exp(2*I*(b*x+a)))*csgn(d/(\exp(2*I*(b*x+a))-1)*\exp(2*I*(b*x+a)))^{2+1/8*I*x^2*Pi*csgn(I*d/(\exp(2*I*(b*x+a))-1)*\exp(2*I*(b*x+a)))*csgn(d/(\exp(2*I*(b*x+a))-1)*\exp(2*I*(b*x+a)))+1/8*I*x^2*Pi*csgn(I*\exp(2*I*(b*x+a)))^{3+1/8*I*x^2*Pi*csgn(I*(\exp(2*I*(b*x+a))*d-I*\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))-1))*csgn((\exp(2*I*(b*x+a))*d-I*\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))-1))^{2-1/2*x^2*\ln(\exp(I*(b*x+a)))+1/8*I*x^2*Pi*csgn((\exp(2*I*(b*x+a))*d-I*\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))-1))^{3+1/4*d/(-d+I)*\ln(1+I*(-d+I)*\exp(2*I*(b*x+a)))*x^{2+1/2/b^2*a/(-d+I)*\text{dilog}(1-I*\exp(I*(b*x+a)))*(I*(-d+I))^{(1/2)}} + 1/2/b^2*a/(-d+I)*\text{dilog}(1+I*\exp(I*(b*x+a)))*(I*(-d+I))^{(1/2)}} + 1/8*I*x^2*Pi*csgn(d/(\exp(2*I*(b*x+a))-1)*\exp(2*I*(b*x+a)))^{2+1/8*I*x^2*Pi*csgn(I*(\exp(2*I*(b*x+a))*d-I*\exp(2*I*(b*x+a))+I))*csgn(I*(\exp(2*I*(b*x+a))*d-I*\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))-1))^{2+1/2/b*d/(-d+I)*\ln(1+I*(-d+I)*\exp(2*I*(b*x+a)))*x*a - 1/2/b*a*d/(-d+I)*\ln(1-I*\exp(I*(b*x+a)))*(I*(-d+I))^{(1/2)}}*x - 1/2/b*a*d/(-d+I)*\ln(1+I*\exp(I*(b*x+a)))*(I*(-d+I))^{(1/2)}}*x + 1/2*I/b*a/(-d+I)*\ln(1-I*\exp(I*(b*x+a)))*(I*(-d+I))^{(1/2)}}*x + 1/2*I/b*a/(-d+I)*\ln(1+I*\exp(I*(b*x+a)))*(I*(-d+I))^{(1/2)}}*x - 1/4*I/b*d/(-d+I)*\text{polylog}(2, -I*(-d+I)*\exp(2*I*(b*x+a)))*x - 1/4*I/b^2*d/(-d+I)*\text{polylog}(2, -I*(-d+I)*\exp(2*I*(b*x+a)))*a + 1/8*I*x^2*Pi*csgn(I*d)*csgn(I*\exp(2*I*(b*x+a)))/(\exp(2*I*(b*x+a))-1))*csgn(I*d/(\exp(2*I*(b*x+a))-1)*\exp(2*I*(b*x+a)))+1/8*I*x^2*Pi*csgn(I*\exp(2*I*(b*x+a)))*csgn(I/(\exp(2*I*(b*x+a))-1))*csgn(I*\exp(2*I*(b*x+a)))/(\exp(2*I*(b*x+a))-1)) - 1/2*I/b/(-d+I)*\ln(1+I*(-d+I)*\exp(2*I*(b*x+a)))*x*a + 1/2*I/b^2*a*d/(-d+I)*
\end{aligned}$$

$\operatorname{dilog}(1-I\exp(I*(b*x+a))*(I*(-d+I))^{(1/2)})+1/2*I/b^2*a*d/(-d+I)*\operatorname{dilog}(1+I\exp(I*(b*x+a))*(I*(-d+I))^{(1/2)})-1/8*I*x^2*\pi*c\operatorname{sgn}(I/(\exp(2*I*(b*x+a))-1))*c\operatorname{sgn}(I*(\exp(2*I*(b*x+a))*d-I*\exp(2*I*(b*x+a))+I))*c\operatorname{sgn}(I*(\exp(2*I*(b*x+a))*d-I*\exp(2*I*(b*x+a))+I)/(\exp(2*I*(b*x+a))-1))$

**Maxima [B]** time = 1.1394, size = 335, normalized size = 2.54

$$\frac{12((bx+a)^2-2(bx+a)a)\operatorname{arccoth}(d\cot(bx+a)+id+1)}{b} - \frac{-4i(bx+a)^3+12i(bx+a)^2a-6ibx\operatorname{Li}_2((id+1)e^{2ibx+2ia})+(-6i(bx+a)^2+12i(bx+a)a)\arctan(d\cos(2bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1+I\*d+d\*cot(b\*x+a)),x, algorithm="maxima")

[Out]  $1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*\operatorname{arccoth}(d*\cot(b*x + a) + I*d + 1)/b - (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*\operatorname{dilog}((I*d + 1)*e^{(2*I*b*x + 2*I*a)}) + (-6*I*(b*x + a)^2 + 12*I*(b*x + a)*a)*\operatorname{arctan}2(d*\cos(2*b*x + 2*a) + \sin(2*b*x + 2*a), d*\sin(2*b*x + 2*a) - \cos(2*b*x + 2*a) + 1) + 3*((b*x + a)^2 - 2*(b*x + a)*a)*\log((d^2 + 1)*\cos(2*b*x + 2*a)^2 + (d^2 + 1)*\sin(2*b*x + 2*a)^2 + 2*d*\sin(2*b*x + 2*a) - 2*\cos(2*b*x + 2*a) + 1) + 3*\operatorname{polylog}(3, (I*d + 1)*e^{(2*I*b*x + 2*I*a)}))/b/b$

**Fricas [C]** time = 1.69142, size = 429, normalized size = 3.25

$$\frac{4i b^3 x^3 + 6 b^2 x^2 \log\left(\frac{((d-i)e^{2ibx+2ia}+i)e^{(-2ibx-2ia)}}{d}\right) + 4i a^3 + 6i b x \operatorname{Li}_2(-(-id-1)e^{2ibx+2ia}) - 6 a^2 \log\left(\frac{(d-i)e^{2ibx+2ia}+i}{d-i}\right) - 6}{24 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1+I\*d+d\*cot(b\*x+a)),x, algorithm="fricas")

[Out]  $1/24*(4*I*b^3*x^3 + 6*b^2*x^2*\log(((d - I)*e^{(2*I*b*x + 2*I*a)} + I)*e^{(-2*I*b*x - 2*I*a)/d}) + 4*I*a^3 + 6*I*b*x*\operatorname{dilog}(-(-I*d - 1)*e^{(2*I*b*x + 2*I*a)}) - 6*a^2*\log(((d - I)*e^{(2*I*b*x + 2*I*a)} + I)/(d - I)) - 6*(b^2*x^2 - a^2)*\log((-I*d - 1)*e^{(2*I*b*x + 2*I*a)} + 1) - 3*\operatorname{polylog}(3, (I*d + 1)*e^{(2*I*b*x + 2*I*a)}))/b^2$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(1+I\*d+d\*cot(b\*x+a)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arccoth}(d \cot(bx + a) + id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1+I\*d+d\*cot(b\*x+a)),x, algorithm="giac")

[Out] integrate(x\*arccoth(d\*cot(b\*x + a) + I\*d + 1), x)

### 3.259 $\int \coth^{-1}(1 + id + d \cot(a + bx)) dx$

**Optimal.** Leaf size=93

$$\frac{i\text{PolyLog}\left(2, (1 + id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{2}x \log\left(1 - (1 + id)e^{2ia+2ibx}\right) + x \coth^{-1}(d \cot(a + bx) + id + 1) + \frac{1}{2}ibx^2$$

[Out] (I/2)\*b\*x^2 + x\*ArcCoth[1 + I\*d + d\*Cot[a + b\*x]] - (x\*Log[1 - (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/2 + ((I/4)\*PolyLog[2, (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)])/b

**Rubi [A]** time = 0.158715, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {6258, 2184, 2190, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2, (1 + id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{2}x \log\left(1 - (1 + id)e^{2ia+2ibx}\right) + x \coth^{-1}(d \cot(a + bx) + id + 1) + \frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[1 + I\*d + d\*Cot[a + b\*x]], x]

[Out] (I/2)\*b\*x^2 + x\*ArcCoth[1 + I\*d + d\*Cot[a + b\*x]] - (x\*Log[1 - (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/2 + ((I/4)\*PolyLog[2, (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)])/b

#### Rule 6258

Int[ArcCoth[(c\_.) + Cot[(a\_.) + (b\_.)\*(x\_)]]\*(d\_.)], x\_Symbol] :> Simp[x\*ArcCoth[c + d\*Cot[a + b\*x]], x] + Dist[I\*b, Int[x/(c - I\*d - c\*E^(2\*I\*a + 2\*I\*b\*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I\*d)^2, 1]

#### Rule 2184

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^(n_))], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
\int \coth^{-1}(1 + id + d \cot(a + bx)) dx &= x \coth^{-1}(1 + id + d \cot(a + bx)) + (ib) \int \frac{x}{1 + (-1 - id)e^{2ia+2ibx}} dx \\
&= \frac{1}{2}ibx^2 + x \coth^{-1}(1 + id + d \cot(a + bx)) + (b(i - d)) \int \frac{e^{2ia+2ibx}x}{1 + (-1 - id)e^{2ia+2ibx}} dx \\
&= \frac{1}{2}ibx^2 + x \coth^{-1}(1 + id + d \cot(a + bx)) - \frac{1}{2}x \log(1 - (1 + id)e^{2ia+2ibx}) + \frac{1}{2} \int \log \dots \\
&= \frac{1}{2}ibx^2 + x \coth^{-1}(1 + id + d \cot(a + bx)) - \frac{1}{2}x \log(1 - (1 + id)e^{2ia+2ibx}) - \frac{i \operatorname{Subst}}{\dots} \\
&= \frac{1}{2}ibx^2 + x \coth^{-1}(1 + id + d \cot(a + bx)) - \frac{1}{2}x \log(1 - (1 + id)e^{2ia+2ibx}) + \frac{i \operatorname{Li}_2}{\dots}
\end{aligned}$$

**Mathematica [B]** time = 3.36414, size = 709, normalized size = 7.62

$$\frac{x \csc^2(a + bx)(\cos(bx) - i \sin(bx))(\cos(bx) + i \sin(bx)) \left( i \operatorname{PolyLog} \left( 2, \frac{\cos(a) - i \sin(a)}{\cos(a + bx) - i \sin(a + bx)} \right) \right)}{(\cot(a + bx) + i)(d \cot(a + bx) + id + 2) \left( \frac{(d-2i) \cos(a+bx)(\log(1-i \tan(bx)) - \log(1+i \tan(bx)))}{d \cos(a+bx) + (2+id) \sin(a+bx)} + \frac{d \sin(a+bx)(\log(1-i \tan(bx)) - \log(1+i \tan(bx)))}{(d-2i) \sin(a+bx) - id \cos(a+bx)} \right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[1 + I*d + d*Cot[a + b*x]], x]
```

```
[Out] x*ArcCoth[1 + I*d + d*Cot[a + b*x]] + (x*Csc[a + b*x]^2*(2*b*x*Log[2*Cos[b*x]
*(Cos[b*x] - I*Sin[b*x])] + I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*(d*Cos[a
+ b*x] + (2 + I*d)*Sin[a + b*x]))/(2*(-I + d))]*Log[1 - I*Tan[b*x]] - I*Log
[(Sec[b*x]*((-I)*Cos[a] + Sin[a])*(d*Cos[a + b*x] + (2 + I*d)*Sin[a + b*x])
]/2)*Log[1 + I*Tan[b*x]] + I*PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] - I*Pol
yLog[2, (Sec[b*x]*((2 + I*d)*Cos[a] - d*Sin[a])*(Cos[a + b*x] + I*Sin[a + b
*x]))/2] + I*PolyLog[2, ((Cos[a] - I*Sin[a])*((-2 - I*d)*Cos[a] + d*Sin[a])
*(I + Tan[b*x]))/(2*(-I + d))]*(Cos[b*x] - I*Sin[b*x])*(Cos[b*x] + I*Sin[b
*x]))/((I + Cot[a + b*x])*(2 + I*d + d*Cot[a + b*x])*((2*I)*b*x + Log[1 + (
Sec[b*x]*((-2 - I*d)*Cos[a] + d*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2]
- Log[(Sec[b*x]*((-I)*Cos[a] + Sin[a])*(d*Cos[a + b*x] + (2 + I*d)*Sin[a +
b*x]))/2] + ((-2*I + d)*Cos[a + b*x]*(Log[1 - I*Tan[b*x]] - Log[1 + I*Tan[
b*x]])))/(d*Cos[a + b*x] + (2 + I*d)*Sin[a + b*x]) + (d*(Log[1 - I*Tan[b*x]]
- Log[1 + I*Tan[b*x]])*Sin[a + b*x])/((-I)*d*Cos[a + b*x] + (-2*I + d)*Sin
[a + b*x]) + 2*b*x*Tan[b*x] - I*Log[1 + (Sec[b*x]*((-2 - I*d)*Cos[a] + d*Si
n[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2]*Tan[b*x] + I*Log[(Sec[b*x]*((-I)*
Cos[a] + Sin[a])*(d*Cos[a + b*x] + (2 + I*d)*Sin[a + b*x]))/2]*Tan[b*x] - I
*Log[1 - I*Tan[b*x]]*Tan[b*x] + I*Log[1 + I*Tan[b*x]]*Tan[b*x]))
```

**Maple [B]** time = 0.181, size = 299, normalized size = 3.2

$$\frac{\frac{i}{2} \operatorname{arccoth}(1 + id + d \cot(bx + a)) \ln(id - d \cot(bx + a))}{b} - \frac{\frac{i}{2} \operatorname{arccoth}(1 + id + d \cot(bx + a)) \ln(id + d \cot(bx + a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1+I*d+d*cot(b*x+a)),x)
```

```
[Out] 1/2*I/b*arccoth(1+I*d+d*cot(b*x+a))*ln(I*d-d*cot(b*x+a))-1/2*I/b*arccoth(1+
I*d+d*cot(b*x+a))*ln(I*d+d*cot(b*x+a))-1/8*I/b*ln(I*d+d*cot(b*x+a))^2+1/4*I
/b*dilog(1+1/2*I*d+1/2*d*cot(b*x+a))+1/4*I/b*ln(I*d+d*cot(b*x+a))*ln(1+1/2*
I*d+1/2*d*cot(b*x+a))+1/4*I/b*dilog(1/2*I*(-I*d-d*cot(b*x+a))/d)+1/4*I/b*ln
(I*d-d*cot(b*x+a))*ln(1/2*I*(-I*d-d*cot(b*x+a))/d)-1/4*I/b*dilog((-2-I*d-d*
cot(b*x+a))/(-2*I*d-2))-1/4*I/b*ln(I*d-d*cot(b*x+a))*ln((-2-I*d-d*cot(b*x+a
)))/(-2*I*d-2))
```

**Maxima [B]** time = 1.54505, size = 386, normalized size = 4.15

$$4(bx + a)d \left( \frac{\log((id+2)\tan(bx+a)+d)}{d} - \frac{\log(i\tan(bx+a)+1)}{d} \right) + d \left( -\frac{2i \left( \log((id+2)\tan(bx+a)+d) \log\left(\frac{(d-2i)\tan(bx+a)-id}{2id+2} + 1\right) + \text{Li}_2\left(-\frac{(d-2i)\tan(bx+a)-id}{2id+2}\right) \right)}{d} \right)$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+I\*d+d\*cot(b\*x+a)),x, algorithm="maxima")

[Out] 
$$-1/8*(4*(b*x + a)*d*(\log((I*d + 2)*\tan(b*x + a) + d)/d - \log(I*\tan(b*x + a) + 1)/d) + d*(-2*I*(\log((I*d + 2)*\tan(b*x + a) + d)*\log(((d - 2*I)*\tan(b*x + a) - I*d)/(2*I*d + 2) + 1) + \text{dilog}(-((d - 2*I)*\tan(b*x + a) - I*d)/(2*I*d + 2)))/d - 2*I*(\log(1/2*(d - 2*I)*\tan(b*x + a) - 1/2*I*d)*\log(I*\tan(b*x + a) + 1) + \text{dilog}(-1/2*(d - 2*I)*\tan(b*x + a) + 1/2*I*d + 1))/d + (2*I*\log((I*d + 2)*\tan(b*x + a) + d)*\log(I*\tan(b*x + a) + 1) - I*\log(I*\tan(b*x + a) + 1)^2)/d + 2*I*(\log(I*\tan(b*x + a) + 1)*\log(-1/2*I*\tan(b*x + a) + 1/2) + \text{dilog}(1/2*I*\tan(b*x + a) + 1/2))/d - 8*(b*x + a)*\text{arccoth}(I*d + d/\tan(b*x + a) + 1))/b$$

---

**Fricas [A]** time = 1.77423, size = 339, normalized size = 3.65

$$2ib^2x^2 + 2bx \log\left(\frac{((d-i)e^{2ibx+2ia}+i)e^{-2ibx-2ia}}{d}\right) - 2ia^2 - 2(bx+a) \log((-id-1)e^{2ibx+2ia}+1) + 2a \log\left(\frac{(d-i)e^{2ibx+2ia}+i}{d-i}\right) + 4b$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+I\*d+d\*cot(b\*x+a)),x, algorithm="fricas")

[Out] 
$$1/4*(2*I*b^2*x^2 + 2*b*x*\log(((d - I)*e^{(2*I*b*x + 2*I*a)} + I)*e^{-(2*I*b*x - 2*I*a)}/d) - 2*I*a^2 - 2*(b*x + a)*\log((-I*d - 1)*e^{(2*I*b*x + 2*I*a)} + 1) + 2*a*\log(((d - I)*e^{(2*I*b*x + 2*I*a)} + I)/(d - I)) + I*\text{dilog}(-(-I*d - 1)*e^{(2*I*b*x + 2*I*a)}))/b$$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(acoth(1+I*d+d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(d \cot (bx+a)+i d+1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*cot(b*x + a) + I*d + 1), x)
```

$$3.260 \quad \int \frac{\coth^{-1}(1+id+d \cot(a+bx))}{x} dx$$

**Optimal.** Leaf size=22

$$\text{CannotIntegrate}\left(\frac{\coth^{-1}(d \cot(a+bx) + id + 1)}{x}, x\right)$$

[Out] CannotIntegrate[ArcCoth[1 + I\*d + d\*Cot[a + b\*x]]/x, x]

**Rubi [A]** time = 0.0877378, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\coth^{-1}(1 + id + d \cot(a + bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[ArcCoth[1 + I\*d + d\*Cot[a + b\*x]]/x,x]

[Out] Defer[Int][ArcCoth[1 + I\*d + d\*Cot[a + b\*x]]/x, x]

Rubi steps

$$\int \frac{\coth^{-1}(1 + id + d \cot(a + bx))}{x} dx = \int \frac{\coth^{-1}(1 + id + d \cot(a + bx))}{x} dx$$

**Mathematica [A]** time = 0.673287, size = 0, normalized size = 0.

$$\int \frac{\coth^{-1}(1 + id + d \cot(a + bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcCoth[1 + I\*d + d\*Cot[a + b\*x]]/x,x]

[Out] Integrate[ArcCoth[1 + I\*d + d\*Cot[a + b\*x]]/x, x]

---

**Maple [A]** time = 0.444, size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(1 + id + d \cot(bx + a))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(1+I*d+d*cot(b*x+a))/x,x)`

[Out] `int(arccoth(1+I*d+d*cot(b*x+a))/x,x)`

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$-ibx + \frac{1}{4}(-i\pi - 4ia - 2 \log(d)) \log(x) + \frac{1}{2}i \int \frac{\arctan(d \cos(2bx + 2a) + \sin(2bx + 2a))}{x} dx - d \sin(2bx + 2a) + \cos(2bx + 2a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(1+I*d+d*cot(b*x+a))/x,x, algorithm="maxima")`

[Out] `-I*b*x + 1/4*(-I*pi - 4*I*a - 2*log(d))*log(x) + 1/2*I*integrate(arctan2(d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + cos(2*b*x + 2*a) - 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1)/x, x)`

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{\log \left( \frac{(d-i)e^{2ibx+2ia} + i}{d} e^{(-2ibx-2ia)} \right)}{2x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(1+I*d+d*cot(b*x+a))/x,x, algorithm="fricas")`

[Out] `integral(1/2*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)/d)/x, x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(1+I\*d+d\*cot(b\*x+a))/x,x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(d \cot(bx + a) + id + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+I\*d+d\*cot(b\*x+a))/x,x, algorithm="giac")

[Out] integrate(arccoth(d\*cot(b\*x + a) + I\*d + 1)/x, x)

### 3.261 $\int x^2 \coth^{-1}(1 - id - d \cot(a + bx)) dx$

**Optimal.** Leaf size=169

$$\frac{x \operatorname{PolyLog}\left(3, (1 - id)e^{2ia+2ibx}\right)}{4b^2} - \frac{i \operatorname{PolyLog}\left(4, (1 - id)e^{2ia+2ibx}\right)}{8b^3} + \frac{ix^2 \operatorname{PolyLog}\left(2, (1 - id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{6} x^3 \log(1 - (1 - I*d)*E^{((2*I)*a + (2*I)*b*x)})$$

[Out] (I/12)\*b\*x^4 + (x^3\*ArcCoth[1 - I\*d - d\*Cot[a + b\*x]])/3 - (x^3\*Log[1 - (1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)])/6 + ((I/4)\*x^2\*PolyLog[2, (1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)])/b - (x\*PolyLog[3, (1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)])/(4\*b^2) - ((I/8)\*PolyLog[4, (1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)])/b^3

**Rubi [A]** time = 0.294771, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6266, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{x \operatorname{PolyLog}\left(3, (1 - id)e^{2ia+2ibx}\right)}{4b^2} - \frac{i \operatorname{PolyLog}\left(4, (1 - id)e^{2ia+2ibx}\right)}{8b^3} + \frac{ix^2 \operatorname{PolyLog}\left(2, (1 - id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{6} x^3 \log(1 - (1 - I*d)*E^{((2*I)*a + (2*I)*b*x)})$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcCoth[1 - I\*d - d\*Cot[a + b\*x]],x]

[Out] (I/12)\*b\*x^4 + (x^3\*ArcCoth[1 - I\*d - d\*Cot[a + b\*x]])/3 - (x^3\*Log[1 - (1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)])/6 + ((I/4)\*x^2\*PolyLog[2, (1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)])/b - (x\*PolyLog[3, (1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)])/(4\*b^2) - ((I/8)\*PolyLog[4, (1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)])/b^3

#### Rule 6266

Int[ArcCoth[(c\_.) + Cot[(a\_.) + (b\_.)\*(x\_)]]\*(d\_.)]\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^(m + 1)\*ArcCoth[c + d\*Cot[a + b\*x]])/(f\*(m + 1)), x] + Dist[(I\*b)/(f\*(m + 1)), Int[(e + f\*x)^(m + 1)/(c - I\*d - c\*E^(2\*I\*a + 2\*I\*b\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - I\*d)^2, 1]

#### Rule 2184

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x],

x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_))/  
((a\_) + (b\_)\*((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] :> Simp  
[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Di  
st[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)  
))^(n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_)\*((F\_)^((c\_)\*(a\_) + (b\_)\*(x\_)))^(n\_)]\*((f\_) + (g\_  
)\*(x\_))^(m\_), x\_Symbol] :> -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)  
))^(n))])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m -  
1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^(n))], x], x] /; FreeQ[{F, a, b, c, e, f  
, g, n}, x] && GtQ[m, 0]

### Rule 6609

Int[((e\_) + (f\_)\*(x\_))^(m\_)\*PolyLog[n\_, (d\_)\*((F\_)^((c\_)\*(a\_) + (b\_  
)\*(x\_)))^(p\_)], x\_Symbol] :> Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a  
+ b\*x)))^p])/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(  
m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c,  
d, e, f, n, p}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]  
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi  
onOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[  
{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\*  
(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_)\*((a\_) + (b\_)\*(x\_))^(p\_)]/((d\_) + (e\_)\*(x\_)), x\_S  
ymbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d  
, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(1 - id - d \cot(a + bx)) dx &= \frac{1}{3} x^3 \coth^{-1}(1 - id - d \cot(a + bx)) + \frac{1}{3} (ib) \int \frac{x^3}{1 + (-1 + id)e^{2ia+2ibx}} dx \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 - id - d \cot(a + bx)) + \frac{1}{3} (b(i + d)) \int \frac{e^{2ia+2ibx} x^3}{1 + (-1 + id)e^{2ia+2ibx}} dx \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 - id - d \cot(a + bx)) - \frac{1}{6} x^3 \log(1 - (1 - id)e^{2ia+2ibx}) + \frac{1}{6} x^3 \log(1 - (-1 + id)e^{2ia+2ibx}) \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 - id - d \cot(a + bx)) - \frac{1}{6} x^3 \log(1 - (1 - id)e^{2ia+2ibx}) + \frac{1}{6} x^3 \log(1 - (-1 + id)e^{2ia+2ibx}) \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 - id - d \cot(a + bx)) - \frac{1}{6} x^3 \log(1 - (1 - id)e^{2ia+2ibx}) + \frac{1}{6} x^3 \log(1 - (-1 + id)e^{2ia+2ibx}) \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 - id - d \cot(a + bx)) - \frac{1}{6} x^3 \log(1 - (1 - id)e^{2ia+2ibx}) + \frac{1}{6} x^3 \log(1 - (-1 + id)e^{2ia+2ibx}) \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 - id - d \cot(a + bx)) - \frac{1}{6} x^3 \log(1 - (1 - id)e^{2ia+2ibx}) + \frac{1}{6} x^3 \log(1 - (-1 + id)e^{2ia+2ibx})
\end{aligned}$$

**Mathematica [A]** time = 0.196546, size = 155, normalized size = 0.92

$$\frac{1}{3} x^3 \coth^{-1}(d(-\cot(a + bx)) - id + 1) - \frac{6ib^2 x^2 \text{PolyLog}\left(2, \frac{ie^{-2i(a+bx)}}{d+i}\right) + 6bx \text{PolyLog}\left(3, \frac{ie^{-2i(a+bx)}}{d+i}\right) - 3i \text{PolyLog}\left(4, \frac{ie^{-2i(a+bx)}}{d+i}\right)}{24b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[1 - I\*d - d\*Cot[a + b\*x]], x]

[Out] (x^3\*ArcCoth[1 - I\*d - d\*Cot[a + b\*x]])/3 - (4\*b^3\*x^3\*Log[1 + 1/((-1 + I\*d)\*E^((2\*I)\*(a + b\*x)))] + (6\*I)\*b^2\*x^2\*PolyLog[2, I/((I + d)\*E^((2\*I)\*(a + b\*x)))] + 6\*b\*x\*PolyLog[3, I/((I + d)\*E^((2\*I)\*(a + b\*x)))] - (3\*I)\*PolyLog[4, I/((I + d)\*E^((2\*I)\*(a + b\*x)))])/(24\*b^3)

**Maple [C]** time = 29.139, size = 2339, normalized size = 13.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(1-I\*d-d\*cot(b\*x+a)), x)

```
[Out] -1/6*x^3*ln(d)+1/12*I*b*x^4+1/4*I/b*d/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*x^2+1/12*I*x^3*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^2-1/3*x^3*ln(exp(I*(b*x+a)))-1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^3-1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-1/12*I*x^3*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^3+1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^3-1/2/b^3*a^2/(I+d)*dilog(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/2/b^3*a^2/(I+d)*dilog(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/4/b/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*x^2+1/4/b^3/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*a^2-1/6*d/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*x^3+1/6*x^3*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)-1/4*I/b^3*d/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*a^2+1/12*I*x^3*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))-1/6*I*x^3*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2-1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+1/2*I/b^3*a^2*d/(I+d)*dilog(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/2*I/b^3*a^3/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/8*I/b^3*d/(I+d)*polylog(4,-I*(I+d)*exp(2*I*(b*x+a)))+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))+1/12*I*x^3*Pi*csgn(I*d)*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a))+1/6*I/b^3*a^3/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)+1/3*I/b^3/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*a^3-1/4*I/b^2/(I+d)*polylog(3,-I*(I+d)*exp(2*I*(b*x+a)))*x-1/2*I/b^3*a^3/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))+1/2*I/b^3*a^2*d/(I+d)*dilog(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))+1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))+1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^2-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))/(exp(2*I*(b*x+a))-1))*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+1/8/b^3/(I+d)*polylog(4,-I*(I+d)*exp(2*I*(b*x+a)))-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^2-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^2-1/12*I*x^3*Pi*csgn(I*d)*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^2-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))-1/6*I/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*x^3+1/2*I/b^2/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*x*a^2+1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^2-1/2/b^3*a^3*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/2/b^3*a^3*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))+1/6/b^3*a^3*d/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)+1/3/b^3*d/(I+d)*ln(1+I*(I
```



d)\*exp(2\*I\*(b\*x+a))\*a^3-1/4/b^2\*d/(I+d)\*polylog(3,-I\*(I+d)\*exp(2\*I\*(b\*x+a)))\*x+1/12\*I\*x^3\*Pi\*csgn(I\*exp(2\*I\*(b\*x+a)))^3-1/2\*I/b^2\*a^2/(I+d)\*ln(1-I\*exp(I\*(b\*x+a)))\*(I\*(I+d))^(1/2))\*x-1/2/b^2\*a^2\*d/(I+d)\*ln(1-I\*exp(I\*(b\*x+a)))\*(I\*(I+d))^(1/2))\*x-1/2/b^2\*a^2\*d/(I+d)\*ln(1+I\*exp(I\*(b\*x+a)))\*(I\*(I+d))^(1/2))\*x+1/2/b^2\*d/(I+d)\*ln(1+I\*(I+d)\*exp(2\*I\*(b\*x+a)))\*x\*a^2+1/12\*I\*x^3\*Pi\*csgn(I\*exp(2\*I\*(b\*x+a)))/(exp(2\*I\*(b\*x+a))-1)^3+1/12\*I\*x^3\*Pi\*csgn(I\*d/(exp(2\*I\*(b\*x+a))-1)\*exp(2\*I\*(b\*x+a)))^3-1/2\*I/b^2\*a^2/(I+d)\*ln(1+I\*exp(I\*(b\*x+a)))\*(I\*(I+d))^(1/2))\*x

**Maxima [B]** time = 1.37798, size = 463, normalized size = 2.74

$$\frac{12((bx+a)^3-3(bx+a)^2a+3(bx+a)a^2)\operatorname{arccoth}(d\cot(bx+a)+id-1)}{b^2} + \frac{-3i(bx+a)^4+12i(bx+a)^3a-18i(bx+a)^2a^2+(-8i(bx+a)^3+18i(bx+a)^2a-18i(bx+a)a^2)\operatorname{arctan2}(-d\cos(2bx+2a)+\sin(2bx+2a),-d\sin(2bx+2a)-\cos(2bx+2a)+1)+(-12I(bx+a)^2+18I(bx+a)a-9Ia^2)*\operatorname{dilog}((-Id+1)*e^{(2Ibx+2Ia)})+(4(bx+a)^3-9(bx+a)^2a+9(bx+a)a^2)*\log((d^2+1)\cos(2bx+2a)^2+(d^2+1)\sin(2bx+2a)^2-2d\sin(2bx+2a)-2\cos(2bx+2a)+1)+3(4bx+a)*\operatorname{polylog}(3,(-Id+1)*e^{(2Ibx+2Ia)})+6I*\operatorname{polylog}(4,(-Id+1)*e^{(2Ibx+2Ia)}))}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(1-I\*d-d\*cot(b\*x+a)),x, algorithm="maxima")

[Out] -1/36\*(12\*((b\*x + a)^3 - 3\*(b\*x + a)^2\*a + 3\*(b\*x + a)\*a^2)\*arccoth(d\*cot(b\*x + a) + I\*d - 1)/b^2 + (-3\*I\*(b\*x + a)^4 + 12\*I\*(b\*x + a)^3\*a - 18\*I\*(b\*x + a)^2\*a^2 + (-8\*I\*(b\*x + a)^3 + 18\*I\*(b\*x + a)^2\*a - 18\*I\*(b\*x + a)\*a^2)\*arctan2(-d\*cos(2\*b\*x + 2\*a) + sin(2\*b\*x + 2\*a), -d\*sin(2\*b\*x + 2\*a) - cos(2\*b\*x + 2\*a) + 1) + (-12\*I\*(b\*x + a)^2 + 18\*I\*(b\*x + a)\*a - 9\*I\*a^2)\*dilog((-I\*d + 1)\*e^(2\*I\*b\*x + 2\*I\*a)) + (4\*(b\*x + a)^3 - 9\*(b\*x + a)^2\*a + 9\*(b\*x + a)\*a^2)\*log((d^2 + 1)\*cos(2\*b\*x + 2\*a)^2 + (d^2 + 1)\*sin(2\*b\*x + 2\*a)^2 - 2\*d\*sin(2\*b\*x + 2\*a) - 2\*cos(2\*b\*x + 2\*a) + 1) + 3\*(4\*b\*x + a)\*polylog(3, (-I\*d + 1)\*e^(2\*I\*b\*x + 2\*I\*a)) + 6\*I\*polylog(4, (-I\*d + 1)\*e^(2\*I\*b\*x + 2\*I\*a)))/b^2)/b

**Fricas [C]** time = 1.68903, size = 504, normalized size = 2.98

$$\frac{2i b^4 x^4 - 4 b^3 x^3 \log\left(\frac{d e^{(2i b x + 2i a)}}{(d+i)e^{(2i b x + 2i a)} - i}\right) + 6i b^2 x^2 \operatorname{Li}_2\left(-i(d-1)e^{(2i b x + 2i a)}\right) - 2i a^4 + 4 a^3 \log\left(\frac{(d+i)e^{(2i b x + 2i a)} - i}{d+i}\right) - 6 b x \operatorname{polylog}\left(3, \frac{(d+i)e^{(2i b x + 2i a)} - i}{d+i}\right)}{24 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(1-I\*d-d\*cot(b\*x+a)),x, algorithm="fricas")

[Out] 1/24\*(2\*I\*b^4\*x^4 - 4\*b^3\*x^3\*log(d\*e^(2\*I\*b\*x + 2\*I\*a))/((d + I)\*e^(2\*I\*b\*x + 2\*I\*a) - I)) + 6\*I\*b^2\*x^2\*dilog(-(I\*d - 1)\*e^(2\*I\*b\*x + 2\*I\*a)) - 2\*I\*a

$$\begin{aligned} &^4 + 4*a^3*\log(((d + I)*e^{(2*I*b*x + 2*I*a)} - I)/(d + I)) - 6*b*x*polylog(3 \\ &, (-I*d + 1)*e^{(2*I*b*x + 2*I*a)}) - 4*(b^3*x^3 + a^3)*\log((I*d - 1)*e^{(2*I* \\ &b*x + 2*I*a)} + 1) - 3*I*polylog(4, (-I*d + 1)*e^{(2*I*b*x + 2*I*a)}))/b^3 \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*acoth(1-I\*d-d\*cot(b\*x+a)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arccoth}(-d \cot(bx + a) - id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(1-I\*d-d\*cot(b\*x+a)),x, algorithm="giac")

[Out] integrate(x^2\*arccoth(-d\*cot(b\*x + a) - I\*d + 1), x)

### 3.262 $\int x \coth^{-1}(1 - id - d \cot(a + bx)) dx$

**Optimal.** Leaf size=133

$$-\frac{\text{PolyLog}\left(3, (1 - id)e^{2ia+2ibx}\right)}{8b^2} + \frac{ix\text{PolyLog}\left(2, (1 - id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{4}x^2 \log\left(1 - (1 - id)e^{2ia+2ibx}\right) + \frac{1}{2}x^2 \coth^{-1}(d(-\cot(a + bx)))$$

[Out] (I/6)\*b\*x^3 + (x^2\*ArcCoth[1 - I\*d - d\*Cot[a + b\*x]])/2 - (x^2\*Log[1 - (1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)])/4 + ((I/4)\*x\*PolyLog[2, (1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)])/b - PolyLog[3, (1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/(8\*b^2)

**Rubi [A]** time = 0.247589, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {6266, 2184, 2190, 2531, 2282, 6589}

$$-\frac{\text{PolyLog}\left(3, (1 - id)e^{2ia+2ibx}\right)}{8b^2} + \frac{ix\text{PolyLog}\left(2, (1 - id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{4}x^2 \log\left(1 - (1 - id)e^{2ia+2ibx}\right) + \frac{1}{2}x^2 \coth^{-1}(d(-\cot(a + bx)))$$

Antiderivative was successfully verified.

[In] Int[x\*ArcCoth[1 - I\*d - d\*Cot[a + b\*x]], x]

[Out] (I/6)\*b\*x^3 + (x^2\*ArcCoth[1 - I\*d - d\*Cot[a + b\*x]])/2 - (x^2\*Log[1 - (1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)])/4 + ((I/4)\*x\*PolyLog[2, (1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)])/b - PolyLog[3, (1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/(8\*b^2)

#### Rule 6266

Int[ArcCoth[(c\_.) + Cot[(a\_.) + (b\_.)\*(x\_.)]\*(d\_.)]\*((e\_.) + (f\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^(m + 1)\*ArcCoth[c + d\*Cot[a + b\*x]])/(f\*(m + 1)), x] + Dist[(I\*b)/(f\*(m + 1)), Int[(e + f\*x)^(m + 1)/(c - I\*d - c\*E^(2\*I\*a + 2\*I\*b\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - I\*d)^2, 1]

#### Rule 2184

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_.))))^(n\_.), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[(c + d\*x)^m\*(F^(g\*(e + f\*x)))^n/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*(a_) + (b_)*(x_)))^(n_)]*((f_) + (g_)
*(x_))^(m_), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x \coth^{-1}(1 - id - d \cot(a + bx)) dx &= \frac{1}{2}x^2 \coth^{-1}(1 - id - d \cot(a + bx)) + \frac{1}{2}(ib) \int \frac{x^2}{1 + (-1 + id)e^{2ia+2ibx}} dx \\
&= \frac{1}{6}ibx^3 + \frac{1}{2}x^2 \coth^{-1}(1 - id - d \cot(a + bx)) + \frac{1}{2}(b(i + d)) \int \frac{e^{2ia+2ibx}x^2}{1 + (-1 + id)e^{2ia+2ibx}} dx \\
&= \frac{1}{6}ibx^3 + \frac{1}{2}x^2 \coth^{-1}(1 - id - d \cot(a + bx)) - \frac{1}{4}x^2 \log\left(1 - (1 - id)e^{2ia+2ibx}\right) + \frac{1}{2}ibx^2 \log\left(1 - (1 - id)e^{2ia+2ibx}\right) \\
&= \frac{1}{6}ibx^3 + \frac{1}{2}x^2 \coth^{-1}(1 - id - d \cot(a + bx)) - \frac{1}{4}x^2 \log\left(1 - (1 - id)e^{2ia+2ibx}\right) + \frac{1}{2}ibx^2 \log\left(1 - (1 - id)e^{2ia+2ibx}\right) \\
&= \frac{1}{6}ibx^3 + \frac{1}{2}x^2 \coth^{-1}(1 - id - d \cot(a + bx)) - \frac{1}{4}x^2 \log\left(1 - (1 - id)e^{2ia+2ibx}\right) + \frac{1}{2}ibx^2 \log\left(1 - (1 - id)e^{2ia+2ibx}\right) \\
&= \frac{1}{6}ibx^3 + \frac{1}{2}x^2 \coth^{-1}(1 - id - d \cot(a + bx)) - \frac{1}{4}x^2 \log\left(1 - (1 - id)e^{2ia+2ibx}\right) + \frac{1}{2}ibx^2 \log\left(1 - (1 - id)e^{2ia+2ibx}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.103554, size = 119, normalized size = 0.89

$$\frac{1}{2}x^2 \coth^{-1}(d(-\cot(a + bx)) - id + 1) - \frac{2ibx \operatorname{PolyLog}\left(2, \frac{ie^{-2i(a+bx)}}{d+i}\right) + \operatorname{PolyLog}\left(3, \frac{ie^{-2i(a+bx)}}{d+i}\right) + 2b^2x^2 \log\left(1 + \frac{e^{-2i(a+bx)}}{-1+id}\right)}{8b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[1 - I\*d - d\*Cot[a + b\*x]], x]

[Out] (x^2\*ArcCoth[1 - I\*d - d\*Cot[a + b\*x]])/2 - (2\*b^2\*x^2\*Log[1 + 1/((-1 + I\*d)\*E^((2\*I)\*(a + b\*x)))] + (2\*I)\*b\*x\*PolyLog[2, I/((I + d)\*E^((2\*I)\*(a + b\*x)))] + PolyLog[3, I/((I + d)\*E^((2\*I)\*(a + b\*x)))])/(8\*b^2)

**Maple [C]** time = 20.552, size = 2249, normalized size = 16.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(1-I\*d-d\*cot(b\*x+a)), x)

[Out] 1/8\*I\*x^2\*Pi\*csgn(I\*exp(2\*I\*(b\*x+a))/(exp(2\*I\*(b\*x+a))-1))^3+1/8\*I\*x^2\*Pi\*csgn(I\*d/(exp(2\*I\*(b\*x+a))-1)\*exp(2\*I\*(b\*x+a)))^3+1/2\*I/b^2\*a^2/(I+d)\*ln(1+I

$$\begin{aligned}
& * \exp(I*(b*x+a)) * (I*(I+d))^{(1/2)} + 1/2*I/b^2*a^2/(I+d) * \ln(1-I*\exp(I*(b*x+a)) * \\
& (I*(I+d))^{(1/2)}) - 1/4*I/b^2*a^2/(I+d) * \ln(I*\exp(2*I*(b*x+a)) + \exp(2*I*(b*x+a)) \\
& * d - I) - 1/8/b^2*d/(I+d) * \text{polylog}(3, -I*(I+d)*\exp(2*I*(b*x+a))) - 1/4/b^2/(I+d) * \text{poly} \\
& \text{log}(2, -I*(I+d)*\exp(2*I*(b*x+a))) * a + 1/2/b^2*a/(I+d) * \text{dilog}(1+I*\exp(I*(b*x+a) \\
& )) * (I*(I+d))^{(1/2)} - 1/4*d/(I+d) * \ln(1+I*(I+d)*\exp(2*I*(b*x+a))) * x^{2+1/2}/b^2* \\
& a/(I+d) * \text{dilog}(1-I*\exp(I*(b*x+a)) * (I*(I+d))^{(1/2)}) + 1/4*x^2 * \ln(I*\exp(2*I*(b*x \\
& +a)) + \exp(2*I*(b*x+a)) * d - I) + 1/8*I*x^2 * \text{Pisgn}(I*(I*\exp(2*I*(b*x+a)) + \exp(2*I* \\
& (b*x+a)) * d - I)) * \text{csgn}(I*(I*\exp(2*I*(b*x+a)) + \exp(2*I*(b*x+a)) * d - I) / (\exp(2*I*(b \\
& *x+a)) - 1))^{2+1/6} * I*b*x^{3+1/2}/b^2*a^2*d/(I+d) * \ln(1-I*\exp(I*(b*x+a)) * (I*(I+d) \\
& ))^{(1/2)} - 1/4*I*x^2 * \text{Pisgn}(I*\exp(I*(b*x+a))) * \text{csgn}(I*\exp(2*I*(b*x+a)))^{2-1/8} \\
& * I*x^2 * \text{Pisgn}(I*d) * \text{csgn}(I*d / (\exp(2*I*(b*x+a)) - 1) * \exp(2*I*(b*x+a)))^{2-1/8} * I \\
& * x^2 * \text{Pisgn}(I*\exp(2*I*(b*x+a)) / (\exp(2*I*(b*x+a)) - 1)) * \text{csgn}(I*d / (\exp(2*I*(b* \\
& x+a)) - 1) * \exp(2*I*(b*x+a)))^{2-1/4} / b^2*d/(I+d) * \ln(1+I*(I+d)*\exp(2*I*(b*x+a))) \\
& * a^{2+1/2}/b^2*a^2*d/(I+d) * \ln(1+I*\exp(I*(b*x+a)) * (I*(I+d))^{(1/2)}) - 1/4*x^2 * \ln(d \\
& + 1/8*I*x^2 * \text{Pisgn}(I*\exp(I*(b*x+a)))^{2} * \text{csgn}(I*\exp(2*I*(b*x+a))) + 1/8*I*x^2 \\
& * \text{Pisgn}(I*(I*\exp(2*I*(b*x+a)) + \exp(2*I*(b*x+a)) * d - I) / (\exp(2*I*(b*x+a)) - 1)) * \\
& \text{csgn}((I*\exp(2*I*(b*x+a)) + \exp(2*I*(b*x+a)) * d - I) / (\exp(2*I*(b*x+a)) - 1))^{2+1/8} * \\
& I*x^2 * \text{Pisgn}(I / (\exp(2*I*(b*x+a)) - 1)) * \text{csgn}(I*(I*\exp(2*I*(b*x+a)) + \exp(2*I*(b \\
& *x+a)) * d - I) / (\exp(2*I*(b*x+a)) - 1))^{2-1/8} * I/b^2/(I+d) * \text{polylog}(3, -I*(I+d)*\exp( \\
& 2*I*(b*x+a))) - 1/4*I/(I+d) * \ln(1+I*(I+d)*\exp(2*I*(b*x+a))) * x^{2-1/8} * I*x^2 * \text{Pisgn} \\
& \text{sgn}(I*\exp(2*I*(b*x+a))) * \text{csgn}(I*\exp(2*I*(b*x+a)) / (\exp(2*I*(b*x+a)) - 1))^{2-1/8} \\
& * I*x^2 * \text{Pisgn}(I / (\exp(2*I*(b*x+a)) - 1)) * \text{csgn}(I*\exp(2*I*(b*x+a)) / (\exp(2*I*(b* \\
& x+a)) - 1))^{2-1/8} * I*x^2 * \text{Pisgn}(I*d / (\exp(2*I*(b*x+a)) - 1) * \exp(2*I*(b*x+a))) * \text{cs} \\
& \text{gn}(d / (\exp(2*I*(b*x+a)) - 1) * \exp(2*I*(b*x+a)))^{2+1/8} * I*x^2 * \text{Pisgn}(I*d / (\exp(2* \\
& I*(b*x+a)) - 1) * \exp(2*I*(b*x+a))) * \text{csgn}(d / (\exp(2*I*(b*x+a)) - 1) * \exp(2*I*(b*x+a) \\
& )) - 1/4/b^2*a^2*d/(I+d) * \ln(I*\exp(2*I*(b*x+a)) + \exp(2*I*(b*x+a)) * d - I) + 1/8*I*x^ \\
& 2 * \text{Pisgn}(I*\exp(2*I*(b*x+a)))^3 - 1/4/b/(I+d) * \text{polylog}(2, -I*(I+d)*\exp(2*I*(b*x \\
& +a))) * x - 1/8*I*x^2 * \text{Pisgn}((I*\exp(2*I*(b*x+a)) + \exp(2*I*(b*x+a)) * d - I) / (\exp(2* \\
& I*(b*x+a)) - 1))^{3-1/8} * I*x^2 * \text{Pisgn}(I*(I*\exp(2*I*(b*x+a)) + \exp(2*I*(b*x+a)) * d \\
& - I) / (\exp(2*I*(b*x+a)) - 1))^{3-1/2} * x^2 * \ln(\exp(I*(b*x+a))) - 1/2*I/b^2*a*d/(I+d) * \\
& \text{dilog}(1+I*\exp(I*(b*x+a)) * (I*(I+d))^{(1/2)}) - 1/4*I/b^2/(I+d) * \ln(1+I*(I+d)*\exp( \\
& 2*I*(b*x+a))) * a^{2-1/8} * I*x^2 * \text{Pisgn}(I*(I*\exp(2*I*(b*x+a)) + \exp(2*I*(b*x+a)) * \\
& d - I) / (\exp(2*I*(b*x+a)) - 1)) * \text{csgn}((I*\exp(2*I*(b*x+a)) + \exp(2*I*(b*x+a)) * d - I) / ( \\
& \exp(2*I*(b*x+a)) - 1)) + 1/8*I*x^2 * \text{Pisgn}(d / (\exp(2*I*(b*x+a)) - 1) * \exp(2*I*(b*x+ \\
& a)))^{3-1/8} * I*x^2 * \text{Pisgn}(d / (\exp(2*I*(b*x+a)) - 1) * \exp(2*I*(b*x+a)))^{2+1/8} * I*x \\
& ^2 * \text{Pisgn}((I*\exp(2*I*(b*x+a)) + \exp(2*I*(b*x+a)) * d - I) / (\exp(2*I*(b*x+a)) - 1))^{2+1/8} * I*x \\
& ^2 * \text{Pisgn}(I*d) * \text{csgn}(I*\exp(2*I*(b*x+a)) / (\exp(2*I*(b*x+a)) - 1)) * \text{csgn} \\
& (I*d / (\exp(2*I*(b*x+a)) - 1) * \exp(2*I*(b*x+a))) - 1/2*I/b/(I+d) * \ln(1+I*(I+d)*\exp( \\
& 2*I*(b*x+a))) * x * a + 1/4*I/b*d/(I+d) * \text{polylog}(2, -I*(I+d)*\exp(2*I*(b*x+a))) * x + 1/ \\
& 4*I/b^2*d/(I+d) * \text{polylog}(2, -I*(I+d)*\exp(2*I*(b*x+a))) * a - 1/2/b*d/(I+d) * \ln(1+I \\
& *(I+d)*\exp(2*I*(b*x+a))) * x * a + 1/2/b*a*d/(I+d) * \ln(1+I*\exp(I*(b*x+a)) * (I*(I+d) \\
& ))^{(1/2)} * x + 1/2/b*a*d/(I+d) * \ln(1-I*\exp(I*(b*x+a)) * (I*(I+d))^{(1/2)}) * x - 1/2*I/b \\
& ^2*a*d/(I+d) * \text{dilog}(1-I*\exp(I*(b*x+a)) * (I*(I+d))^{(1/2)}) + 1/2*I/b*a/(I+d) * \ln(1 \\
& +I*\exp(I*(b*x+a)) * (I*(I+d))^{(1/2)}) * x + 1/2*I/b*a/(I+d) * \ln(1-I*\exp(I*(b*x+a)) * \\
& (I*(I+d))^{(1/2)}) * x - 1/8*I*x^2 * \text{Pisgn}(I / (\exp(2*I*(b*x+a)) - 1)) * \text{csgn}(I*(I*\exp(
\end{aligned}$$

$$2*I*(b*x+a))+\exp(2*I*(b*x+a))*d-I))*\operatorname{csgn}(I*(I*\exp(2*I*(b*x+a))+\exp(2*I*(b*x+a))*d-I)/(\exp(2*I*(b*x+a))-1))+1/8*I*x^2*\operatorname{Pi}*\operatorname{csgn}(I*\exp(2*I*(b*x+a)))*\operatorname{csgn}(I/(\exp(2*I*(b*x+a))-1))*\operatorname{csgn}(I*\exp(2*I*(b*x+a)))/(\exp(2*I*(b*x+a))-1))$$

**Maxima [B]** time = 1.28095, size = 336, normalized size = 2.53

$$\frac{12((bx+a)^2-2(bx+a)a)\operatorname{arccoth}(d\cot(bx+a)+id-1)}{b} + \frac{-4i(bx+a)^3+12i(bx+a)^2a-6ibx\operatorname{Li}_2((-id+1)e^{2ibx+2ia})+(-6i(bx+a)^2+12i(bx+a)a)\operatorname{arctan}(-d\cos(2bx+2a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1-I\*d-d\*cot(b\*x+a)),x, algorithm="maxima")

[Out]  $-1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*\operatorname{arccoth}(d*\cot(b*x + a) + I*d - 1)/b + (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*\operatorname{dilog}((-I*d + 1)*e^{(2*I*b*x + 2*I*a)}) + (-6*I*(b*x + a)^2 + 12*I*(b*x + a)*a)*\operatorname{arctan}2(-d*\cos(2*b*x + 2*a) + \sin(2*b*x + 2*a), -d*\sin(2*b*x + 2*a) - \cos(2*b*x + 2*a) + 1) + 3*((b*x + a)^2 - 2*(b*x + a)*a)*\log((d^2 + 1)*\cos(2*b*x + 2*a)^2 + (d^2 + 1)*\sin(2*b*x + 2*a)^2 - 2*d*\sin(2*b*x + 2*a) - 2*\cos(2*b*x + 2*a) + 1) + 3*\operatorname{polylog}(3, (-I*d + 1)*e^{(2*I*b*x + 2*I*a)}))/b/b$

**Fricas [C]** time = 1.77153, size = 427, normalized size = 3.21

$$\frac{4i b^3 x^3 - 6 b^2 x^2 \log\left(\frac{d e^{2i b x + 2i a}}{(d+i)e^{2i b x + 2i a} - i}\right) + 4i a^3 + 6i b x \operatorname{Li}_2(-i d - 1) e^{2i b x + 2i a} - 6 a^2 \log\left(\frac{(d+i)e^{2i b x + 2i a} - i}{d+i}\right) - 6(b^2 x^2 - a^2) \log(d)}{24 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1-I\*d-d\*cot(b\*x+a)),x, algorithm="fricas")

[Out]  $1/24*(4*I*b^3*x^3 - 6*b^2*x^2*\log(d*e^{(2*I*b*x + 2*I*a)})/((d + I)*e^{(2*I*b*x + 2*I*a)} - I)) + 4*I*a^3 + 6*I*b*x*\operatorname{dilog}(-(I*d - 1)*e^{(2*I*b*x + 2*I*a)}) - 6*a^2*\log(((d + I)*e^{(2*I*b*x + 2*I*a)} - I)/(d + I)) - 6*(b^2*x^2 - a^2)*\log((I*d - 1)*e^{(2*I*b*x + 2*I*a)} + 1) - 3*\operatorname{polylog}(3, (-I*d + 1)*e^{(2*I*b*x + 2*I*a)}))/b^2$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(1-I\*d-d\*cot(b\*x+a)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arccoth}(-d \cot(bx + a) - id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1-I\*d-d\*cot(b\*x+a)),x, algorithm="giac")

[Out] integrate(x\*arccoth(-d\*cot(b\*x + a) - I\*d + 1), x)



### 3.263 $\int \coth^{-1}(1 - id - d \cot(a + bx)) dx$

**Optimal.** Leaf size=94

$$\frac{i\text{PolyLog}\left(2, (1 - id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{2}x \log\left(1 - (1 - id)e^{2ia+2ibx}\right) + x \coth^{-1}(d(-\cot(a + bx)) - id + 1) + \frac{1}{2}ibx^2$$

[Out] (I/2)\*b\*x^2 + x\*ArcCoth[1 - I\*d - d\*Cot[a + b\*x]] - (x\*Log[1 - (1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/2 + ((I/4)\*PolyLog[2, (1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/b

**Rubi [A]** time = 0.150208, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {6258, 2184, 2190, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2, (1 - id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{2}x \log\left(1 - (1 - id)e^{2ia+2ibx}\right) + x \coth^{-1}(d(-\cot(a + bx)) - id + 1) + \frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[1 - I\*d - d\*Cot[a + b\*x]], x]

[Out] (I/2)\*b\*x^2 + x\*ArcCoth[1 - I\*d - d\*Cot[a + b\*x]] - (x\*Log[1 - (1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/2 + ((I/4)\*PolyLog[2, (1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/b

#### Rule 6258

Int[ArcCoth[(c\_.) + Cot[(a\_.) + (b\_.)\*(x\_)]]\*(d\_.)], x\_Symbol] :> Simp[x\*ArcCoth[c + d\*Cot[a + b\*x]], x] + Dist[I\*b, Int[x/(c - I\*d - c\*E^(2\*I\*a + 2\*I\*b\*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I\*d)^2, 1]

#### Rule 2184

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned} \int \coth^{-1}(1 - id - d \cot(a + bx)) dx &= x \coth^{-1}(1 - id - d \cot(a + bx)) + (ib) \int \frac{x}{1 + (-1 + id)e^{2ia+2ibx}} dx \\ &= \frac{1}{2}ibx^2 + x \coth^{-1}(1 - id - d \cot(a + bx)) + (b(i + d)) \int \frac{e^{2ia+2ibx} x}{1 + (-1 + id)e^{2ia+2ibx}} dx \\ &= \frac{1}{2}ibx^2 + x \coth^{-1}(1 - id - d \cot(a + bx)) - \frac{1}{2}x \log(1 - (1 - id)e^{2ia+2ibx}) + \frac{1}{2} \int \log \\ &= \frac{1}{2}ibx^2 + x \coth^{-1}(1 - id - d \cot(a + bx)) - \frac{1}{2}x \log(1 - (1 - id)e^{2ia+2ibx}) - \frac{i \text{Subst}}{2} \\ &= \frac{1}{2}ibx^2 + x \coth^{-1}(1 - id - d \cot(a + bx)) - \frac{1}{2}x \log(1 - (1 - id)e^{2ia+2ibx}) + \frac{i \text{Li}_2(1 - (1 - id)e^{2ia+2ibx})}{2} \end{aligned}$$

**Mathematica [B]** time = 2.63817, size = 605, normalized size = 6.44

$$\frac{x \csc^2(a + bx)(\cos(bx) - i \sin(bx))(\cos(bx) + i \sin(bx)) \left( i \text{PolyLog} \left( 2, \frac{(\cos(a) - i \sin(a))(\tan(bx) + i)(d \sin(a) + (2 - id) \cos(a))}{2(d + i)} \right) - i \text{PolyLog} \left( 2, \frac{(\cos(a) + i \sin(a))(\tan(bx) - i)(d \sin(a) + (2 + id) \cos(a))}{2(d - i)} \right) \right)}{(\cot(a + bx) + i)(d \cot(a + bx) + id - 2) \left( \frac{\sec^2(bx) \log \left( \frac{1 - (1 - id)e^{2ia+2ibx}}{1 - (1 + id)e^{2ia+2ibx}} \right)}{2} \right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[1 - I*d - d*Cot[a + b*x]], x]
```

```
[Out] x*ArcCoth[1 - I*d - d*Cot[a + b*x]] + (x*Csc[a + b*x]^2*(2*b*x*Log[2*Cos[b*x]
*(Cos[b*x] - I*Sin[b*x])] + I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*(d*Cos[a
+ b*x] + I*(2*I + d)*Sin[a + b*x])]/(2*(I + d))]*Log[1 - I*Tan[b*x]] - I*Lo
g[(I*Sec[b*x]*(d*Cos[a + b*x] + I*(2*I + d)*Sin[a + b*x])]/(2*Cos[a] - (2*I
)*Sin[a])]*Log[1 + I*Tan[b*x]] + I*PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] -
I*PolyLog[2, (Sec[b*x]*((2 - I*d)*Cos[a] + d*Sin[a])*(Cos[a + b*x] + I*Sin
[a + b*x]))/2] + I*PolyLog[2, ((Cos[a] - I*Sin[a])*((2 - I*d)*Cos[a] + d*Si
n[a])*(I + Tan[b*x]))/(2*(I + d))])*(Cos[b*x] - I*Sin[b*x])*(Cos[b*x] + I*S
in[b*x]))/((I + Cot[a + b*x])*(-2 + I*d + d*Cot[a + b*x])*(-((Log[1 - I*Tan
[b*x]]*Sec[b*x]*((2*I + d)*Cos[a] + I*d*Sin[a]))/(d*Cos[a + b*x] + I*(2*I +
d)*Sin[a + b*x])) + (Log[1 + I*Tan[b*x]]*Sec[b*x]*((2*I + d)*Cos[a] + I*d*
Sin[a]))/(d*Cos[a + b*x] + I*(2*I + d)*Sin[a + b*x]) + (Log[(I*Sec[b*x]*(d*
Cos[a + b*x] + I*(2*I + d)*Sin[a + b*x])]/(2*Cos[a] - (2*I)*Sin[a])]*Sec[b*
x]^2)/(1 + I*Tan[b*x]) - 2*b*x*(I + Tan[b*x]) + I*Log[1 - (Sec[b*x]*((2 - I
*d)*Cos[a] + d*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2]*(I + Tan[b*x])))
```

**Maple [B]** time = 0.193, size = 304, normalized size = 3.2

$$\frac{-\frac{i}{2}\operatorname{arccoth}(1-id-d\cot(bx+a))\ln(-id-d\cot(bx+a))}{b} + \frac{\frac{i}{2}\operatorname{arccoth}(1-id-d\cot(bx+a))\ln(id-d\cot(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1-I*d-d*cot(b*x+a)),x)
```

```
[Out] -1/2*I/b*arccoth(1-I*d-d*cot(b*x+a))*ln(-I*d-d*cot(b*x+a))+1/2*I/b*arccoth(
1-I*d-d*cot(b*x+a))*ln(I*d-d*cot(b*x+a))-1/8*I/b*ln(-I*d-d*cot(b*x+a))^2+1/
4*I/b*dilog(1-1/2*I*d-1/2*d*cot(b*x+a))+1/4*I/b*ln(-I*d-d*cot(b*x+a))*ln(1-
1/2*I*d-1/2*d*cot(b*x+a))+1/4*I/b*dilog(1/2*I*(-I*d-d*cot(b*x+a))/d)+1/4*I/
b*ln(I*d-d*cot(b*x+a))*ln(1/2*I*(-I*d-d*cot(b*x+a))/d)-1/4*I/b*dilog((2-I*d
-d*cot(b*x+a))/(-2*I*d+2))-1/4*I/b*ln(I*d-d*cot(b*x+a))*ln((2-I*d-d*cot(b*x
+a))/(-2*I*d+2))
```

**Maxima [B]** time = 1.75784, size = 389, normalized size = 4.14

$$4(bx+a)d\left(\frac{\log((id-2)\tan(bx+a)+d)}{d} - \frac{\log(i\tan(bx+a)+1)}{d}\right) - d\left(\frac{2i\left(\log((id-2)\tan(bx+a)+d)\log\left(\frac{(d+2i)\tan(bx+a)-id}{2id-2}+1\right)+\operatorname{Li}_2\left(-\frac{(d+2i)\tan(bx+a)-id}{2id-2}\right)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-I\*d-d\*cot(b\*x+a)),x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -1/8*(4*(b*x + a)*d*(\log((I*d - 2)*\tan(b*x + a) + d)/d - \log(I*\tan(b*x + a) \\ & + 1)/d) - d*(2*I*(\log((I*d - 2)*\tan(b*x + a) + d)*\log(((d + 2*I)*\tan(b*x + \\ & a) - I*d)/(2*I*d - 2) + 1) + \operatorname{dilog}(-((d + 2*I)*\tan(b*x + a) - I*d)/(2*I*d \\ & - 2)))/d + 2*I*(\log(-1/2*(d + 2*I)*\tan(b*x + a) + 1/2*I*d)*\log(I*\tan(b*x + \\ & a) + 1) + \operatorname{dilog}(1/2*(d + 2*I)*\tan(b*x + a) - 1/2*I*d + 1))/d - (2*I*\log((I* \\ & d - 2)*\tan(b*x + a) + d)*\log(I*\tan(b*x + a) + 1) - I*\log(I*\tan(b*x + a) + 1 \\ & )^2)/d - 2*I*(\log(I*\tan(b*x + a) + 1)*\log(-1/2*I*\tan(b*x + a) + 1/2) + \operatorname{dilo} \\ & g(1/2*I*\tan(b*x + a) + 1/2))/d) + 8*(b*x + a)*\operatorname{arccoth}(I*d + d/\tan(b*x + a) \\ & - 1))/b \end{aligned}$$

**Fricas [A]** time = 1.74486, size = 335, normalized size = 3.56

$$\frac{2i b^2 x^2 - 2 b x \log\left(\frac{d e^{2i b x + 2i a}}{(d+i)e^{2i b x + 2i a} - i}\right) - 2i a^2 - 2(bx + a) \log\left((id - 1)e^{2i b x + 2i a} + 1\right) + 2a \log\left(\frac{(d+i)e^{2i b x + 2i a} - i}{d+i}\right) + i \operatorname{Li}_2\left(-\left(id - \frac{d e^{2i b x + 2i a}}{(d+i)e^{2i b x + 2i a} - i}\right)\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-I\*d-d\*cot(b\*x+a)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & 1/4*(2*I*b^2*x^2 - 2*b*x*\log(d*e^{(2*I*b*x + 2*I*a)}/((d + I)*e^{(2*I*b*x + 2* \\ & I*a) - I})) - 2*I*a^2 - 2*(b*x + a)*\log((I*d - 1)*e^{(2*I*b*x + 2*I*a) + 1) + \\ & 2*a*\log(((d + I)*e^{(2*I*b*x + 2*I*a) - I})/(d + I)) + I*\operatorname{dilog}(-(I*d - 1)*e^{(2*I*b*x + 2*I*a)}))/b \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoath(1-I\*d-d\*cot(b\*x+a)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(-d \cot (bx+a)-i d+1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-I\*d-d\*cot(b\*x+a)),x, algorithm="giac")

[Out] integrate(arccoth(-d\*cot(b\*x + a) - I\*d + 1), x)

$$3.264 \quad \int \frac{\coth^{-1}(1-id-d \cot(a+bx))}{x} dx$$

**Optimal.** Leaf size=23

$$\text{CannotIntegrate}\left(\frac{\coth^{-1}(d(-\cot(a+bx))-id+1)}{x}, x\right)$$

[Out] CannotIntegrate[ArcCoth[1 - I\*d - d\*Cot[a + b\*x]]/x, x]

**Rubi [A]** time = 0.0800919, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\coth^{-1}(1-id-d \cot(a+bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[ArcCoth[1 - I\*d - d\*Cot[a + b\*x]]/x,x]

[Out] Defer[Int][ArcCoth[1 - I\*d - d\*Cot[a + b\*x]]/x, x]

Rubi steps

$$\int \frac{\coth^{-1}(1-id-d \cot(a+bx))}{x} dx = \int \frac{\coth^{-1}(1-id-d \cot(a+bx))}{x} dx$$

**Mathematica [A]** time = 0.749697, size = 0, normalized size = 0.

$$\int \frac{\coth^{-1}(1-id-d \cot(a+bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcCoth[1 - I\*d - d\*Cot[a + b\*x]]/x,x]

[Out] Integrate[ArcCoth[1 - I\*d - d\*Cot[a + b\*x]]/x, x]

---

**Maple [A]** time = 0.457, size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}(1 - id - d \cot(bx + a))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(1-I\*d-d\*cot(b\*x+a))/x,x)

[Out] int(arccoth(1-I\*d-d\*cot(b\*x+a))/x,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$-ibx + \frac{1}{4}(-i\pi - 4ia - 2 \log(d)) \log(x) - \frac{1}{2}i \int \frac{\arctan(-d \cos(2bx + 2a) + \sin(2bx + 2a), -d \sin(2bx + 2a) - \cos(2bx + 2a) + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-I\*d-d\*cot(b\*x+a))/x,x, algorithm="maxima")

[Out] -I\*b\*x + 1/4\*(-I\*pi - 4\*I\*a - 2\*log(d))\*log(x) - 1/2\*I\*integrate(arctan2(-d\*cos(2\*b\*x + 2\*a) + sin(2\*b\*x + 2\*a), -d\*sin(2\*b\*x + 2\*a) - cos(2\*b\*x + 2\*a) + 1)/x, x) + 1/4\*integrate(log((d^2 + 1)\*cos(2\*b\*x + 2\*a)^2 + (d^2 + 1)\*sin(2\*b\*x + 2\*a)^2 - 2\*d\*sin(2\*b\*x + 2\*a) - 2\*cos(2\*b\*x + 2\*a) + 1)/x, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\log\left(\frac{de^{(2ibx+2ia)}}{(d+i)e^{(2ibx+2ia)}-i}\right)}{2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-I\*d-d\*cot(b\*x+a))/x,x, algorithm="fricas")

[Out] integral(-1/2\*log(d\*e^(2\*I\*b\*x + 2\*I\*a)/((d + I)\*e^(2\*I\*b\*x + 2\*I\*a) - I))/x, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(1-I\*d-d\*cot(b\*x+a))/x,x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccoth}\left(\frac{-d \cot(bx + a) - id + 1}{x}\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-I\*d-d\*cot(b\*x+a))/x,x, algorithm="giac")

[Out] integrate(arccoth(-d\*cot(b\*x + a) - I\*d + 1)/x, x)



$$3.265 \quad \int \frac{(a+b \coth^{-1}(cx^n))(d+e \log(fx^m))}{x} dx$$

**Optimal.** Leaf size=160

$$\frac{bd \operatorname{PolyLog}\left(2, -\frac{x^{-n}}{c}\right)}{2n} - \frac{bd \operatorname{PolyLog}\left(2, \frac{x^{-n}}{c}\right)}{2n} + \frac{be \log(fx^m) \operatorname{PolyLog}\left(2, -\frac{x^{-n}}{c}\right)}{2n} - \frac{be \log(fx^m) \operatorname{PolyLog}\left(2, \frac{x^{-n}}{c}\right)}{2n} + \frac{be \log(fx^m) \operatorname{PolyLog}\left(3, -\frac{x^{-n}}{c}\right)}{2n^2} - \frac{be \log(fx^m) \operatorname{PolyLog}\left(3, \frac{x^{-n}}{c}\right)}{2n^2}$$

[Out] a\*d\*Log[x] + (a\*e\*Log[f\*x^m]^2)/(2\*m) + (b\*d\*PolyLog[2, -(1/(c\*x^n))])/(2\*n) + (b\*e\*Log[f\*x^m]\*PolyLog[2, -(1/(c\*x^n))])/(2\*n) - (b\*d\*PolyLog[2, 1/(c\*x^n)])/(2\*n) - (b\*e\*Log[f\*x^m]\*PolyLog[2, 1/(c\*x^n)])/(2\*n) + (b\*e\*m\*PolyLog[3, -(1/(c\*x^n))])/(2\*n^2) - (b\*e\*m\*PolyLog[3, 1/(c\*x^n)])/(2\*n^2)

**Rubi [A]** time = 0.573596, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2301, 6742, 6096, 5913, 6072, 6070, 2374, 6589}

$$\frac{bd \operatorname{PolyLog}\left(2, -\frac{x^{-n}}{c}\right)}{2n} - \frac{bd \operatorname{PolyLog}\left(2, \frac{x^{-n}}{c}\right)}{2n} + \frac{be \log(fx^m) \operatorname{PolyLog}\left(2, -\frac{x^{-n}}{c}\right)}{2n} - \frac{be \log(fx^m) \operatorname{PolyLog}\left(2, \frac{x^{-n}}{c}\right)}{2n} + \frac{be \log(fx^m) \operatorname{PolyLog}\left(3, -\frac{x^{-n}}{c}\right)}{2n^2} - \frac{be \log(fx^m) \operatorname{PolyLog}\left(3, \frac{x^{-n}}{c}\right)}{2n^2}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*ArcCoth[c\*x^n])\*(d + e\*Log[f\*x^m]))/x, x]

[Out] a\*d\*Log[x] + (a\*e\*Log[f\*x^m]^2)/(2\*m) + (b\*d\*PolyLog[2, -(1/(c\*x^n))])/(2\*n) + (b\*e\*Log[f\*x^m]\*PolyLog[2, -(1/(c\*x^n))])/(2\*n) - (b\*d\*PolyLog[2, 1/(c\*x^n)])/(2\*n) - (b\*e\*Log[f\*x^m]\*PolyLog[2, 1/(c\*x^n)])/(2\*n) + (b\*e\*m\*PolyLog[3, -(1/(c\*x^n))])/(2\*n^2) - (b\*e\*m\*PolyLog[3, 1/(c\*x^n)])/(2\*n^2)

### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n]^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

### Rule 6096

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)/(x\_), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*ArcCoth[c\*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]

### Rule 5913

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.)/(x\_), x\_Symbol] := Simp[a\*Log[x], x] + (Simp[(b\*PolyLog[2, -(c\*x)^(-1)])/2, x] - Simp[(b\*PolyLog[2, 1/(c\*x)])/2, x]) /; FreeQ[{a, b, c}, x]

### Rule 6072

Int[(Log[(d\_.)\*(x\_)^(m\_.)]\*(ArcCoth[(c\_.)\*(x\_)^(n\_.)]\*(b\_.) + (a\_.)))/(x\_), x\_Symbol] := Dist[a, Int[Log[d\*x^m]/x, x], x] + Dist[b, Int[(Log[d\*x^m]\*ArcCoth[c\*x^n])/x, x], x] /; FreeQ[{a, b, c, d, m, n}, x]

### Rule 6070

Int[(ArcCoth[(c\_.)\*(x\_)^(n\_.)]\*Log[(d\_.)\*(x\_)^(m\_.)])/(x\_), x\_Symbol] := Dist[1/2, Int[(Log[d\*x^m]\*Log[1 + 1/(c\*x^n)])/x, x], x] - Dist[1/2, Int[(Log[d\*x^m]\*Log[1 - 1/(c\*x^n)])/x, x], x] /; FreeQ[{c, d, m, n}, x]

### Rule 2374

Int[(Log[(d\_.)\*((e\_.) + (f\_.)\*(x\_)^(m\_.))]\*((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)^(p\_.)))/(x\_), x\_Symbol] := -Simp[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^p)/m, x] + Dist[(b\*n\*p)/m, Int[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d\*e, 1]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \coth^{-1}(cx^n))(d + e \log(fx^m))}{x} dx &= \int \left( \frac{d(a + b \coth^{-1}(cx^n))}{x} + \frac{e(a + b \coth^{-1}(cx^n)) \log(fx^m)}{x} \right) dx \\
&= d \int \frac{a + b \coth^{-1}(cx^n)}{x} dx + e \int \frac{(a + b \coth^{-1}(cx^n)) \log(fx^m)}{x} dx \\
&= (ae) \int \frac{\log(fx^m)}{x} dx + (be) \int \frac{\coth^{-1}(cx^n) \log(fx^m)}{x} dx + \frac{d \operatorname{Subst}\left(\int \frac{1}{x} dx, cx^n, x\right)}{n} \\
&= ad \log(x) + \frac{ae \log^2(fx^m)}{2m} + \frac{bd \operatorname{Li}_2\left(-\frac{x^{-n}}{c}\right)}{2n} - \frac{bd \operatorname{Li}_2\left(\frac{x^{-n}}{c}\right)}{2n} - \frac{1}{2}(be) \int \frac{1}{x} dx \\
&= ad \log(x) + \frac{ae \log^2(fx^m)}{2m} + \frac{bd \operatorname{Li}_2\left(-\frac{x^{-n}}{c}\right)}{2n} + \frac{be \log(fx^m) \operatorname{Li}_2\left(-\frac{x^{-n}}{c}\right)}{2n} \\
&= ad \log(x) + \frac{ae \log^2(fx^m)}{2m} + \frac{bd \operatorname{Li}_2\left(-\frac{x^{-n}}{c}\right)}{2n} + \frac{be \log(fx^m) \operatorname{Li}_2\left(-\frac{x^{-n}}{c}\right)}{2n}
\end{aligned}$$

**Mathematica [C]** time = 0.315276, size = 131, normalized size = 0.82

$$\frac{bcx^n (d + e \log(fx^m)) \operatorname{HypergeometricPFQ}\left(\left\{\frac{1}{2}, \frac{1}{2}, 1\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, c^2 x^{2n}\right)}{n} - \frac{bcemx^n \operatorname{HypergeometricPFQ}\left(\left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 1\right\}\right)}{n^2}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*ArcCoth[c\*x^n])\*(d + e\*Log[f\*x^m]))/x,x]

[Out] -((b\*c\*e\*m\*x^n\*HypergeometricPFQ[{1/2, 1/2, 1/2, 1}, {3/2, 3/2, 3/2}, c^2\*x^(2\*n)])/n^2) + (b\*c\*x^n\*HypergeometricPFQ[{1/2, 1/2, 1}, {3/2, 3/2}, c^2\*x^(2\*n)]\*(d + e\*Log[f\*x^m]))/n - ((a + b\*ArcCoth[c\*x^n] - b\*ArcTanh[c\*x^n])\*Log[x]\*(e\*m\*Log[x] - 2\*(d + e\*Log[f\*x^m])))/2

**Maple [C]** time = 0.47, size = 920, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth(c\*x^n))\*(d+e\*ln(f\*x^m))/x,x)

```
[Out] 1/4*I/n*dilog(c*x^n)*Pi*b*e*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/2/n*dilog
(c*x^n+1)*b*d+1/n*ln(x^n)*a*d+1/2*e*a/m*ln(x^m)^2+1/4*I/n*Pi*dilog(c*x^n+1)
*b*e*csgn(I*f*x^m)^3+1/2*I/n*Pi*ln(x^n)*a*e*csgn(I*x^m)*csgn(I*f*x^m)^2-1/4
*I/n*Pi*dilog(c*x^n+1)*b*e*csgn(I*x^m)*csgn(I*f*x^m)^2+1/4*I/n*dilog(c*x^n)
*Pi*b*e*csgn(I*f*x^m)^3-1/4*I/n*ln(c*x^n)*Pi*ln(c*x^n-1)*b*e*csgn(I*f)*csgn
(I*f*x^m)^2-1/4*I/n*ln(c*x^n)*Pi*ln(c*x^n-1)*b*e*csgn(I*x^m)*csgn(I*f*x^m)^
2+1/4*I/n*Pi*dilog(c*x^n+1)*b*e*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/2*I/n
*Pi*ln(x^n)*a*e*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/2*I/n*Pi*ln(x^n)*a*e*
csgn(I*f)*csgn(I*f*x^m)^2-1/4*I/n*dilog(c*x^n)*Pi*b*e*csgn(I*x^m)*csgn(I*f*
x^m)^2-1/2*e*b/n*m*ln(x)*polylog(2,-c*x^n)+1/2*e*b/n*dilog(c*x^n+1)*m*ln(x)
+1/2*e*b/n*m*ln(x)*polylog(2,c*x^n)-1/2*e*b/n*ln(1-c*x^n)*ln(c*x^n)*ln(x^m)
+1/2*e*b/n*dilog(c*x^n)*m*ln(x)-1/4*I/n*Pi*dilog(c*x^n+1)*b*e*csgn(I*f)*csg
n(I*f*x^m)^2-1/2*e*b/n*dilog(c*x^n)*ln(x^m)-1/2/n*ln(f)*dilog(c*x^n+1)*b*e+
1/n*ln(f)*ln(x^n)*a*e+1/2*e*b*ln(x^m)*ln(1-c*x^n)*ln(x)-1/2/n*dilog(c*x^n)*
ln(f)*b*e-1/2*e*b*ln(c*x^n-1)*ln(x^m)*ln(x)-1/4*e*b*m*ln(x)^2*ln(1-c*x^n)-1
/2*e*b/n*dilog(c*x^n+1)*ln(x^m)-1/2*e*b/n^2*m*polylog(3,c*x^n)+1/4*e*b*ln(c
*x^n-1)*ln(x)^2*m-1/2/n*ln(c*x^n)*ln(c*x^n-1)*b*d+1/2*e*b/n^2*m*polylog(3,-
c*x^n)-1/2/n*ln(c*x^n)*ln(f)*ln(c*x^n-1)*b*e+1/4*I/n*ln(c*x^n)*Pi*ln(c*x^n-
1)*b*e*csgn(I*f*x^m)^3-1/4*I/n*dilog(c*x^n)*Pi*b*e*csgn(I*f)*csgn(I*f*x^m)^
2+1/2*e*b/n*ln(1-c*x^n)*ln(c*x^n)*m*ln(x)-1/2*I/n*Pi*ln(x^n)*a*e*csgn(I*f*x
^m)^3-1/2/n*dilog(c*x^n)*b*d+1/4*I/n*ln(c*x^n)*Pi*ln(c*x^n-1)*b*e*csgn(I*f)
*csgn(I*x^m)*csgn(I*f*x^m)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{ae \log(fx^m)^2}{2m} + ad \log(x) - \frac{1}{4} (bem \log(x)^2 - 2be \log(x) \log(x^m) - 2(e \log(f) + d)b \log(x)) \log(cx^n + 1) + \frac{1}{4} (bem \log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*e*log(f*x^m)^2/m + a*d*log(x) - 1/4*(b*e*m*log(x)^2 - 2*b*e*log(x)*lo
g(x^m) - 2*(e*log(f) + d)*b*log(x))*log(c*x^n + 1) + 1/4*(b*e*m*log(x)^2 -
2*b*e*log(x)*log(x^m) - 2*(e*log(f) + d)*b*log(x))*log(c*x^n - 1) + integra
te(1/2*(2*b*c*e*n*x^n*log(x)*log(x^m) - (b*c*e*m*n*log(x)^2 - 2*(e*n*log(f)
+ d*n)*b*c*log(x))*x^n)/(c^2*x*x^(2*n) - x), x)
```

**Fricas [C]** time = 1.92843, size = 979, normalized size = 6.12

$2 a e m n^2 \log(x)^2 - 2 b e m \operatorname{polylog}(3, c \cosh(n \log(x)) + c \sinh(n \log(x))) + 2 b e m \operatorname{polylog}(3, -c \cosh(n \log(x)) - c \sinh(n \log(x)))$

---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x^n))\*(d+e\*log(f\*x^m))/x,x, algorithm="fricas")

[Out]  $\frac{1}{4} * (2 * a * e * m * n^2 * \log(x)^2 - 2 * b * e * m * \operatorname{polylog}(3, c * \cosh(n * \log(x)) + c * \sinh(n * \log(x))) + 2 * b * e * m * \operatorname{polylog}(3, -c * \cosh(n * \log(x)) - c * \sinh(n * \log(x))) + 2 * (b * e * m * n * \log(x) + b * e * n * \log(f) + b * d * n) * \operatorname{dilog}(c * \cosh(n * \log(x)) + c * \sinh(n * \log(x))) - 2 * (b * e * m * n * \log(x) + b * e * n * \log(f) + b * d * n) * \operatorname{dilog}(-c * \cosh(n * \log(x)) - c * \sinh(n * \log(x))) - (b * e * m * n^2 * \log(x)^2 + 2 * (b * e * n^2 * \log(f) + b * d * n^2) * \log(x)) * \log(c * \cosh(n * \log(x)) + c * \sinh(n * \log(x)) + 1) + (b * e * m * n^2 * \log(x)^2 + 2 * (b * e * n^2 * \log(f) + b * d * n^2) * \log(x)) * \log(-c * \cosh(n * \log(x)) - c * \sinh(n * \log(x)) + 1) + 4 * (a * e * n^2 * \log(f) + a * d * n^2) * \log(x) + (b * e * m * n^2 * \log(x)^2 + 2 * (b * e * n^2 * \log(f) + b * d * n^2) * \log(x)) * \log((c * \cosh(n * \log(x)) + c * \sinh(n * \log(x)) + 1) / (c * \cosh(n * \log(x)) + c * \sinh(n * \log(x)) - 1))) / n^2$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth(c\*x\*\*n))\*(d+e\*ln(f\*x\*\*m))/x,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccoth}(c x^n) + a)(e \log(f x^m) + d)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x^n))\*(d+e\*log(f\*x^m))/x,x, algorithm="giac")

[Out] integrate((b\*arccoth(c\*x^n) + a)\*(e\*log(f\*x^m) + d)/x, x)

### 3.266 $\int x^5 (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2x^2)) dx$

**Optimal.** Leaf size=297

$$\frac{1}{6}x^6(a + b \coth^{-1}(cx))(e \log(1 - c^2x^2) + d) - \frac{ex^4(a + b \coth^{-1}(cx))}{12c^2} - \frac{ex^2(a + b \coth^{-1}(cx))}{6c^4} - \frac{e \log(1 - c^2x^2)(a + b \coth^{-1}(cx))}{6c^6}$$

[Out] (b\*(3\*d - e)\*x)/(18\*c^5) - (137\*b\*e\*x)/(180\*c^5) + (b\*(3\*d - e)\*x^3)/(54\*c^3) - (47\*b\*e\*x^3)/(540\*c^3) + (b\*(3\*d - e)\*x^5)/(90\*c) - (b\*e\*x^5)/(75\*c) - (e\*x^2\*(a + b\*ArcCoth[c\*x]))/(6\*c^4) - (e\*x^4\*(a + b\*ArcCoth[c\*x]))/(12\*c^2) - (e\*x^6\*(a + b\*ArcCoth[c\*x]))/18 - (b\*(3\*d - e)\*ArcTanh[c\*x])/(18\*c^6) + (137\*b\*e\*ArcTanh[c\*x])/(180\*c^6) + (b\*e\*x\*Log[1 - c^2\*x^2])/(6\*c^5) + (b\*e\*x^3\*Log[1 - c^2\*x^2])/(18\*c^3) + (b\*e\*x^5\*Log[1 - c^2\*x^2])/(30\*c) - (e\*(a + b\*ArcCoth[c\*x])\*Log[1 - c^2\*x^2])/(6\*c^6) + (x^6\*(a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/6

**Rubi [A]** time = 0.385512, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 11, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$ , Rules used = {2454, 2395, 43, 6084, 321, 207, 302, 2528, 2448, 206, 2455}

$$\frac{1}{6}x^6(a + b \coth^{-1}(cx))(e \log(1 - c^2x^2) + d) - \frac{ex^4(a + b \coth^{-1}(cx))}{12c^2} - \frac{ex^2(a + b \coth^{-1}(cx))}{6c^4} - \frac{e \log(1 - c^2x^2)(a + b \coth^{-1}(cx))}{6c^6}$$

Antiderivative was successfully verified.

[In] Int[x^5\*(a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]),x]

[Out] (b\*(3\*d - e)\*x)/(18\*c^5) - (137\*b\*e\*x)/(180\*c^5) + (b\*(3\*d - e)\*x^3)/(54\*c^3) - (47\*b\*e\*x^3)/(540\*c^3) + (b\*(3\*d - e)\*x^5)/(90\*c) - (b\*e\*x^5)/(75\*c) - (e\*x^2\*(a + b\*ArcCoth[c\*x]))/(6\*c^4) - (e\*x^4\*(a + b\*ArcCoth[c\*x]))/(12\*c^2) - (e\*x^6\*(a + b\*ArcCoth[c\*x]))/18 - (b\*(3\*d - e)\*ArcTanh[c\*x])/(18\*c^6) + (137\*b\*e\*ArcTanh[c\*x])/(180\*c^6) + (b\*e\*x\*Log[1 - c^2\*x^2])/(6\*c^5) + (b\*e\*x^3\*Log[1 - c^2\*x^2])/(18\*c^3) + (b\*e\*x^5\*Log[1 - c^2\*x^2])/(30\*c) - (e\*(a + b\*ArcCoth[c\*x])\*Log[1 - c^2\*x^2])/(6\*c^6) + (x^6\*(a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/6

#### Rule 2454

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&

!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e^n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

### Rule 6084

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))\*((d\_.) + Log[(f\_.) + (g\_.)\*(x\_)^2]\*(e\_.)\*(x\_)^(m\_.), x\_Symbol] := With[{u = IntHide[x^m\*(d + e\*Log[f + g\*x^2]), x]}, Dist[a + b\*ArcCoth[c\*x], u, x] - Dist[b\*c, Int[ExpandIntegrand[u/(1 - c^2\*x^2), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)/2, 0]

### Rule 321

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 207

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTanh[(Rt[b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

### Rule 302

Int[(x\_)^(m\_)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := Int[PolynomialDivide[x^m, a + b\*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2\*n - 1]

Rule 2528

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With
[{u = ExpandIntegrand[(a + b*Log[c*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunc
tionQ[RGx, x] && IGtQ[n, 0]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] := Simp[x*Log[c*(d
+ e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_)^
(m_.), x_Symbol] := Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
+ 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps



$$\begin{aligned}
\int x^5 (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2 x^2)) dx &= -\frac{ex^2 (a + b \coth^{-1}(cx))}{6c^4} - \frac{ex^4 (a + b \coth^{-1}(cx))}{12c^2} - \frac{1}{18} ex^6 (a + b \coth^{-1}(cx)) \\
&= -\frac{ex^2 (a + b \coth^{-1}(cx))}{6c^4} - \frac{ex^4 (a + b \coth^{-1}(cx))}{12c^2} - \frac{1}{18} ex^6 (a + b \coth^{-1}(cx)) \\
&= -\frac{bex}{6c^5} - \frac{ex^2 (a + b \coth^{-1}(cx))}{6c^4} - \frac{ex^4 (a + b \coth^{-1}(cx))}{12c^2} - \frac{1}{18} ex^6 (a + b \coth^{-1}(cx)) \\
&= \frac{b(3d - e)x}{18c^5} - \frac{bex}{4c^5} + \frac{b(3d - e)x^3}{54c^3} - \frac{bex^3}{36c^3} + \frac{b(3d - e)x^5}{90c} - \frac{ex^2 (a + b \coth^{-1}(cx))}{6c^4} \\
&= \frac{b(3d - e)x}{18c^5} - \frac{bex}{4c^5} + \frac{b(3d - e)x^3}{54c^3} - \frac{bex^3}{36c^3} + \frac{b(3d - e)x^5}{90c} - \frac{ex^2 (a + b \coth^{-1}(cx))}{6c^4} \\
&= \frac{b(3d - e)x}{18c^5} - \frac{7bex}{12c^5} + \frac{b(3d - e)x^3}{54c^3} - \frac{bex^3}{36c^3} + \frac{b(3d - e)x^5}{90c} - \frac{ex^2 (a + b \coth^{-1}(cx))}{6c^4} \\
&= \frac{b(3d - e)x}{18c^5} - \frac{137bex}{180c^5} + \frac{b(3d - e)x^3}{54c^3} - \frac{47bex^3}{540c^3} + \frac{b(3d - e)x^5}{90c} - \frac{bex^5}{75c} \\
&= \frac{b(3d - e)x}{18c^5} - \frac{137bex}{180c^5} + \frac{b(3d - e)x^3}{54c^3} - \frac{47bex^3}{540c^3} + \frac{b(3d - e)x^5}{90c} - \frac{bex^5}{75c}
\end{aligned}$$

**Mathematica [A]** time = 0.171192, size = 236, normalized size = 0.79

$$20e \log(1 - c^2 x^2) (15ac^6 x^6 + bcx(3c^4 x^4 + 5c^2 x^2 + 15) + 15b(c^6 x^6 - 1) \coth^{-1}(cx)) + 15 \log(1 - cx)(-20ae + 10bd - 49e)$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]),x]

[Out] (30\*b\*c\*(10\*d - 49\*e)\*x - 300\*a\*c^2\*e\*x^2 + 10\*b\*c^3\*(10\*d - 19\*e)\*x^3 - 150\*a\*c^4\*e\*x^4 + 4\*b\*c^5\*(15\*d - 11\*e)\*x^5 + 100\*a\*c^6\*(3\*d - e)\*x^6 - 50\*b\*c^2\*x^2\*(-6\*c^4\*d\*x^4 + e\*(6 + 3\*c^2\*x^2 + 2\*c^4\*x^4))\*ArcCoth[c\*x] + 15\*(10\*b\*d - 20\*a\*e - 49\*b\*e)\*Log[1 - c\*x] - 15\*(10\*b\*d + 20\*a\*e - 49\*b\*e)\*Log[1 + c\*x] + 20\*e\*(15\*a\*c^6\*x^6 + b\*c\*x\*(15 + 5\*c^2\*x^2 + 3\*c^4\*x^4) + 15\*b\*(-1 + c^6\*x^6))\*ArcCoth[c\*x])\*Log[1 - c^2\*x^2])/(1800\*c^6)



$$\begin{aligned}
& /((c*x+1)/(c*x-1)-1)^2 * csgn(I*(c*x+1)/(c*x-1)) - 1/12 * I/c^6 * b * arccoth(c*x) * e \\
& * Pi * csgn(I/((c*x+1)/(c*x-1)-1)^2 * csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1 \\
& )^2)^2 - 1/6 * I/c^5 * b * csgn(I*((c*x+1)/(c*x-1)-1)^2 * csgn(I*((c*x+1)/(c*x-1)- \\
& 1)) * Pi * x * e + 1/6 * I/c^6 * b * arccoth(c*x) * e * Pi * csgn(I*((c*x+1)/(c*x-1)-1)^2 * cs \\
& gn(I*((c*x+1)/(c*x-1)-1)) - 1/6 * I/c^6 * b * arccoth(c*x) * e * Pi * csgn(I/((c*x-1)/(c* \\
& x+1))^(1/2)) * csgn(I*(c*x+1)/(c*x-1))^2 + 1/12 * I/c^6 * b * arccoth(c*x) * Pi * e * csgn( \\
& I/((c*x-1)/(c*x+1))^(1/2))^2 * csgn(I*(c*x+1)/(c*x-1)) + 1/12 * I/c^5 * b * csgn(I*(( \\
& c*x+1)/(c*x-1)-1)^2 * csgn(I*((c*x+1)/(c*x-1)-1))^2 * Pi * x * e - 1/36 * I/c^3 * b * csgn \\
& (I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2 * csgn(I*(c*x+1)/(c*x-1)) * csgn(I/(( \\
& c*x+1)/(c*x-1)-1)^2 * Pi * x^3 * e - 1/12 * I/c^5 * b * csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/ \\
& (c*x-1)-1)^2 * csgn(I*(c*x+1)/(c*x-1)) * csgn(I/((c*x+1)/(c*x-1)-1)^2 * Pi * x * e + \\
& 1/12 * I/c^6 * b * arccoth(c*x) * e * Pi * csgn(I/((c*x+1)/(c*x-1)-1)^2 * csgn(I*(c*x+1) \\
& /((c*x-1)/((c*x+1)/(c*x-1)-1)^2 * csgn(I*(c*x+1)/(c*x-1)) - 1/60 * I/c * b * csgn(I*( \\
& c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2 * csgn(I*(c*x+1)/(c*x-1)) * csgn(I/((c*x+ \\
& 1)/(c*x-1)-1)^2 * Pi * x^5 * e - 1/12 * I * b * csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)- \\
& 1)^2 * csgn(I*(c*x+1)/(c*x-1)) * csgn(I/((c*x+1)/(c*x-1)-1)^2 * Pi * arccoth(c*x) \\
& * x^6 * e - 1/12 * I/c^6 * b * arccoth(c*x) * e * Pi * csgn(I*((c*x+1)/(c*x-1)-1)^2)^3 - 1/12 * \\
& I/c^6 * b * arccoth(c*x) * Pi * e * csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3 + 1 \\
& /6 * I/c^6 * b * arccoth(c*x) * e * Pi * csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2 - \\
& 23/180 * I/c^6 * b * e * Pi * csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2 * csgn( \\
& I*(c*x+1)/(c*x-1)) + 23/180 * I/c^6 * b * e * Pi * csgn(I/((c*x-1)/(c*x+1))^(1/2))^2 * cs \\
& gn(I*(c*x+1)/(c*x-1)) - 23/90 * I/c^6 * b * e * Pi + 1/36 * I/c^3 * b * csgn(I*((c*x+1)/(c*x- \\
& 1)-1)^2)^3 * Pi * x^3 * e + 1/12 * I/c^6 * b * arccoth(c*x) * e * Pi * csgn(I*(c*x+1)/(c*x-1))^ \\
& 3 + 1/12 * I * b * csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3 * Pi * arccoth(c*x) * \\
& x^6 * e - 1/6 * I * b * csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2 * Pi * arccoth(c* \\
& x) * x^6 * e - 1/12 * I * b * csgn(I*(c*x+1)/(c*x-1))^3 * Pi * arccoth(c*x) * x^6 * e + 1/12 * I * b * \\
& csgn(I*((c*x+1)/(c*x-1)-1)^2)^3 * Pi * arccoth(c*x) * x^6 * e + 1/12 * I/c^5 * b * csgn(I*( \\
& (c*x+1)/(c*x-1)-1)^2)^3 * Pi * x * e - 23/90 * I/c^6 * b * e * Pi * csgn(I/((c*x-1)/(c*x+1))^ \\
& (1/2)) * csgn(I*(c*x+1)/(c*x-1))^2 - 23/180 * I/c^6 * b * e * Pi * csgn(I/((c*x+1)/(c*x-1) \\
& )-1)^2 * csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2 - 23/180 * I/c^6 * b * e * Pi \\
& * csgn(I*((c*x+1)/(c*x-1)-1)^2 * csgn(I*((c*x+1)/(c*x-1)-1))^2 + 23/90 * I/c^6 * b * \\
& e * Pi * csgn(I*((c*x+1)/(c*x-1)-1)^2)^2 * csgn(I*((c*x+1)/(c*x-1)-1)) - 1/60 * I/c * b \\
& * csgn(I*(c*x+1)/(c*x-1))^3 * Pi * x^5 * e - 1/36 * I/c^3 * b * csgn(I*(c*x+1)/(c*x-1))^3 * \\
& Pi * x^3 * e - 1/12 * I/c^5 * b * csgn(I*(c*x+1)/(c*x-1))^3 * Pi * x * e + 1/60 * I/c * b * csgn(I*(c \\
& *x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3 * Pi * x^5 * e + 1/36 * I/c^3 * b * csgn(I*(c*x+1) \\
& /((c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3 * Pi * x^3 * e + 1/12 * I/c^5 * b * csgn(I*(c*x+1)/(c*x \\
& -1)/((c*x+1)/(c*x-1)-1)^2)^3 * Pi * x * e - 1/30 * I/c * b * csgn(I*(c*x+1)/(c*x-1)/((c*x \\
& +1)/(c*x-1)-1)^2)^2 * Pi * x^5 * e - 1/18 * I/c^3 * b * csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/( \\
& c*x-1)-1)^2)^2 * Pi * x^3 * e - 1/6 * I/c^5 * b * csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1) \\
& -1)^2)^2 * Pi * x * e + 1/60 * I/c * b * csgn(I*((c*x+1)/(c*x-1)-1)^2)^3 * Pi * x^5 * e - 23/180 * \\
& I/c^6 * b * e * Pi * csgn(I*((c*x+1)/(c*x-1)-1)^2)^3 + 23/180 * I/c^6 * b * e * Pi * csgn(I*(c* \\
& x+1)/(c*x-1))^3 - 23/180 * I/c^6 * b * e * Pi * csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1) \\
& -1)^2)^3 + 23/90 * I/c^6 * b * e * Pi * csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2 - \\
& 1/6 * I/c^6 * b * Pi * e * arccoth(c*x) + 1/30 * I/c * b * Pi * x^5 * e + 1/18 * I/c^3 * b * Pi * x^3 * e + 1/ \\
& 6 * I/c^5 * b * Pi * x * e + 1/6 * I * b * Pi * arccoth(c*x) * x^6 * e - 1/90 * I/c^6 * b * e * (15 * arccoth(c*x)
\end{aligned}$$

) $x^5c^5+15\operatorname{arccoth}(cx)x^4c^4+3c^4x^4+15\operatorname{arccoth}(cx)x^3c^3+3c^3x^3+15\operatorname{arccoth}(cx)x^2c^2+8c^2x^2+15\operatorname{arccoth}(cx)xc+8cx+15\operatorname{arccoth}(cx)+23)(cx-1)\ln((cx-1)/(cx+1))-1/12ae/c^2x^4-1/6ae/c^4x^2+1/6ae^e x^6\ln(-c^2x^2+1)-1/6ae/c^6\ln(c^2x^2-1)-1/6/c^6b\operatorname{arccoth}(cx)*d+1/18/c^3b x^3d+1/6/c^5b x*d+1/30/c b x^5d-1/18b\operatorname{arccoth}(cx)x^6e+1/6b\operatorname{arccoth}(cx)x^6d+239/180/c^6b\operatorname{arccoth}(cx)e-23/45/c^6b e\ln(2)$

**Maxima [C]** time = 1.11322, size = 447, normalized size = 1.51

$$\frac{1}{6}adx^6 + \frac{1}{36}\left(6x^6\log(-c^2x^2+1) - c^2\left(\frac{2c^4x^6+3c^2x^4+6x^2}{c^6} + \frac{6\log(c^2x^2-1)}{c^8}\right)\right)be\operatorname{arccoth}(cx) + \frac{1}{180}\left(30x^6\operatorname{arccoth}(cx)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate( $x^5(a+b\operatorname{arccoth}(cx))(d+e\log(-c^2x^2+1))$ ),x, algorithm="maxima")

[Out]  $1/6ad x^6 + 1/36(6x^6\log(-c^2x^2+1) - c^2((2c^4x^6+3c^2x^4+6x^2)/c^6 + 6\log(c^2x^2-1)/c^8))b e\operatorname{arccoth}(cx) + 1/180(30x^6\operatorname{arccoth}(cx) + c(2(3c^4x^5+5c^2x^3+15x)/c^6 - 15\log(cx+1)/c^7 + 15\log(cx-1)/c^7))b d + 1/36(6x^6\log(-c^2x^2+1) - c^2((2c^4x^6+3c^2x^4+6x^2)/c^6 + 6\log(c^2x^2-1)/c^8))a e - 1/1800(4(-15I\pi c^5+11c^5)x^5 + 10(-10I\pi c^3+19c^3)x^3 + 30(-10I\pi c+49c)x - (-150I\pi+60c^5x^5+100c^3x^3+300cx+735)\log(cx+1) - (150I\pi+60c^5x^5+100c^3x^3+300cx-735)\log(cx-1))b e/c^6$

**Fricas [A]** time = 1.69584, size = 579, normalized size = 1.95

$$150ac^4ex^4 - 100(3ac^6d - ac^6e)x^6 + 300ac^2ex^2 - 4(15bc^5d - 11bc^5e)x^5 - 10(10bc^3d - 19bc^3e)x^3 - 30(10bcd - 49bce)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate( $x^5(a+b\operatorname{arccoth}(cx))(d+e\log(-c^2x^2+1))$ ),x, algorithm="fricas")

[Out]  $-1/1800(150ac^4ex^4 - 100(3ac^6d - ac^6e)x^6 + 300ac^2ex^2 - 4(15bc^5d - 11bc^5e)x^5 - 10(10bc^3d - 19bc^3e)x^3 - 30(10bcd - 49bce))$

$$10*b*c*d - 49*b*c*e)*x - 20*(15*a*c^6*e*x^6 + 3*b*c^5*e*x^5 + 5*b*c^3*e*x^3 + 15*b*c*e*x - 15*a*e)*\log(-c^2*x^2 + 1) + 5*(15*b*c^4*e*x^4 - 10*(3*b*c^6*d - b*c^6*e)*x^6 + 30*b*c^2*e*x^2 + 30*b*d - 147*b*e - 30*(b*c^6*e*x^6 - b*e)*\log(-c^2*x^2 + 1))*\log((c*x + 1)/(c*x - 1))/c^6$$

**Sympy [A]** time = 113.193, size = 362, normalized size = 1.22

$$\left\{ \frac{adx^6}{6} + \frac{aex^6 \log(-c^2x^2+1)}{6} - \frac{aex^6}{18} - \frac{aex^4}{12c^2} - \frac{aex^2}{6c^4} - \frac{ae \log(-c^2x^2+1)}{6c^6} + \frac{bdx^6 \operatorname{arccoth}(cx)}{6} + \frac{bex^6 \log(-c^2x^2+1) \operatorname{arccoth}(cx)}{6} - \frac{bex^6 \operatorname{arccoth}(cx)}{18} + \frac{bdx^6 \left(a + \frac{i\pi b}{2}\right)}{6} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(a+b\*acoth(c\*x))\*(d+e\*ln(-c\*\*2\*x\*\*2+1)),x)

[Out] Piecewise((a\*d\*x\*\*6/6 + a\*e\*x\*\*6\*log(-c\*\*2\*x\*\*2 + 1)/6 - a\*e\*x\*\*6/18 - a\*e\*x\*\*4/(12\*c\*\*2) - a\*e\*x\*\*2/(6\*c\*\*4) - a\*e\*log(-c\*\*2\*x\*\*2 + 1)/(6\*c\*\*6) + b\*d\*x\*\*6\*acoth(c\*x)/6 + b\*e\*x\*\*6\*log(-c\*\*2\*x\*\*2 + 1)\*acoth(c\*x)/6 - b\*e\*x\*\*6\*a\*coth(c\*x)/18 + b\*d\*x\*\*5/(30\*c) + b\*e\*x\*\*5\*log(-c\*\*2\*x\*\*2 + 1)/(30\*c) - 11\*b\*e\*x\*\*5/(450\*c) - b\*e\*x\*\*4\*acoth(c\*x)/(12\*c\*\*2) + b\*d\*x\*\*3/(18\*c\*\*3) + b\*e\*x\*\*3\*log(-c\*\*2\*x\*\*2 + 1)/(18\*c\*\*3) - 19\*b\*e\*x\*\*3/(180\*c\*\*3) - b\*e\*x\*\*2\*acoth(c\*x)/(6\*c\*\*4) + b\*d\*x/(6\*c\*\*5) + b\*e\*x\*log(-c\*\*2\*x\*\*2 + 1)/(6\*c\*\*5) - 49\*b\*e\*x/(60\*c\*\*5) - b\*d\*acoth(c\*x)/(6\*c\*\*6) - b\*e\*log(-c\*\*2\*x\*\*2 + 1)\*acoth(c\*x)/(6\*c\*\*6) + 49\*b\*e\*acoth(c\*x)/(60\*c\*\*6), Ne(c, 0)), (d\*x\*\*6\*(a + I\*pi\*b/2)/6, True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arccoth}(cx) + a)(e \log(-c^2x^2 + 1) + d)x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1)),x, algorithm="giac")

[Out] integrate((b\*arccoth(c\*x) + a)\*(e\*log(-c^2\*x^2 + 1) + d)\*x^5, x)

### 3.267 $\int x^3 (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2x^2)) dx$

**Optimal.** Leaf size=225

$$\frac{1}{4}x^4(a + b \coth^{-1}(cx))(e \log(1 - c^2x^2) + d) - \frac{ex^2(a + b \coth^{-1}(cx))}{4c^2} - \frac{e \log(1 - c^2x^2)(a + b \coth^{-1}(cx))}{4c^4} - \frac{1}{8}ex^4(a + b \coth^{-1}(cx))$$

[Out] (b\*(2\*d - 3\*e)\*x)/(8\*c^3) - (2\*b\*e\*x)/(3\*c^3) + (b\*(2\*d - e)\*x^3)/(24\*c) - (b\*e\*x^3)/(18\*c) - (e\*x^2\*(a + b\*ArcCoth[c\*x]))/(4\*c^2) - (e\*x^4\*(a + b\*ArcCoth[c\*x]))/8 - (b\*(2\*d - 3\*e)\*ArcTanh[c\*x])/(8\*c^4) + (2\*b\*e\*ArcTanh[c\*x])/(3\*c^4) + (b\*e\*x\*Log[1 - c^2\*x^2])/(4\*c^3) + (b\*e\*x^3\*Log[1 - c^2\*x^2])/(12\*c) - (e\*(a + b\*ArcCoth[c\*x])\*Log[1 - c^2\*x^2])/(4\*c^4) + (x^4\*(a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/4

**Rubi [A]** time = 0.256242, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$ , Rules used = {2454, 2395, 43, 6084, 459, 321, 206, 2471, 2448, 2455, 302}

$$\frac{1}{4}x^4(a + b \coth^{-1}(cx))(e \log(1 - c^2x^2) + d) - \frac{ex^2(a + b \coth^{-1}(cx))}{4c^2} - \frac{e \log(1 - c^2x^2)(a + b \coth^{-1}(cx))}{4c^4} - \frac{1}{8}ex^4(a + b \coth^{-1}(cx))$$

Antiderivative was successfully verified.

[In] Int[x^3\*(a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]),x]

[Out] (b\*(2\*d - 3\*e)\*x)/(8\*c^3) - (2\*b\*e\*x)/(3\*c^3) + (b\*(2\*d - e)\*x^3)/(24\*c) - (b\*e\*x^3)/(18\*c) - (e\*x^2\*(a + b\*ArcCoth[c\*x]))/(4\*c^2) - (e\*x^4\*(a + b\*ArcCoth[c\*x]))/8 - (b\*(2\*d - 3\*e)\*ArcTanh[c\*x])/(8\*c^4) + (2\*b\*e\*ArcTanh[c\*x])/(3\*c^4) + (b\*e\*x\*Log[1 - c^2\*x^2])/(4\*c^3) + (b\*e\*x^3\*Log[1 - c^2\*x^2])/(12\*c) - (e\*(a + b\*ArcCoth[c\*x])\*Log[1 - c^2\*x^2])/(4\*c^4) + (x^4\*(a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/4

#### Rule 2454

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

#### Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_))^(q_.), x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n])/(g*(q + 1)), x] - Dist[(b*e^n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && NeQ[q, -1]
```

### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rule 6084

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(e_.)*(x_)^(m_.), x_Symbol] := With[{u = IntHide[x^m*(d + e*Log[f + g*x^2]), x]}, Dist[a + b*ArcCoth[c*x], u, x] - Dist[b*c, Int[ExpandIntegrand[u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)/2, 0]
```

### Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

### Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

### Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
(g_.)*(x_)^(s_))^(r_.), x_Symbol] := With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p]]^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && IntegerQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s, 0] && LtQ[r, 0]))
```

### Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] := Simp[x*Log[c*(d
+ e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

### Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_)^(
m_.), x_Symbol] := Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
+ 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

### Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

### Rubi steps



$$\begin{aligned}
\int x^3 (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2x^2)) dx &= -\frac{ex^2 (a + b \coth^{-1}(cx))}{4c^2} - \frac{1}{8}ex^4 (a + b \coth^{-1}(cx)) - \frac{e (a + b \coth^{-1}(cx))}{8c} \\
&= -\frac{ex^2 (a + b \coth^{-1}(cx))}{4c^2} - \frac{1}{8}ex^4 (a + b \coth^{-1}(cx)) - \frac{e (a + b \coth^{-1}(cx))}{8c} \\
&= \frac{b(2d - e)x^3}{24c} - \frac{ex^2 (a + b \coth^{-1}(cx))}{4c^2} - \frac{1}{8}ex^4 (a + b \coth^{-1}(cx)) - \frac{e (a + b \coth^{-1}(cx))}{8c} \\
&= \frac{b(2d - 3e)x}{8c^3} + \frac{b(2d - e)x^3}{24c} - \frac{ex^2 (a + b \coth^{-1}(cx))}{4c^2} - \frac{1}{8}ex^4 (a + b \coth^{-1}(cx)) - \frac{e (a + b \coth^{-1}(cx))}{8c} \\
&= \frac{b(2d - 3e)x}{8c^3} + \frac{b(2d - e)x^3}{24c} - \frac{ex^2 (a + b \coth^{-1}(cx))}{4c^2} - \frac{1}{8}ex^4 (a + b \coth^{-1}(cx)) - \frac{e (a + b \coth^{-1}(cx))}{8c} \\
&= \frac{b(2d - 3e)x}{8c^3} - \frac{bex}{2c^3} + \frac{b(2d - e)x^3}{24c} - \frac{ex^2 (a + b \coth^{-1}(cx))}{4c^2} - \frac{1}{8}ex^4 (a + b \coth^{-1}(cx)) - \frac{e (a + b \coth^{-1}(cx))}{8c} \\
&= \frac{b(2d - 3e)x}{8c^3} - \frac{2bex}{3c^3} + \frac{b(2d - e)x^3}{24c} - \frac{bex^3}{18c} - \frac{ex^2 (a + b \coth^{-1}(cx))}{4c^2} - \frac{e (a + b \coth^{-1}(cx))}{8c} \\
&= \frac{b(2d - 3e)x}{8c^3} - \frac{2bex}{3c^3} + \frac{b(2d - e)x^3}{24c} - \frac{bex^3}{18c} - \frac{ex^2 (a + b \coth^{-1}(cx))}{4c^2} - \frac{e (a + b \coth^{-1}(cx))}{8c}
\end{aligned}$$

**Mathematica [A]** time = 0.144613, size = 192, normalized size = 0.85

$$\frac{12e \log(1 - c^2x^2) (3ac^4x^4 + bcx(c^2x^2 + 3) + 3b(c^4x^4 - 1) \coth^{-1}(cx)) + 3 \log(1 - cx)(-12ae + 6bd - 25be) - 3 \log(cx) (3ac^4x^4 + bcx(c^2x^2 + 3) + 3b(c^4x^4 - 1) \coth^{-1}(cx))}{144c^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]),x]

[Out] (6\*b\*c\*(6\*d - 25\*e)\*x - 36\*a\*c^2\*e\*x^2 + 2\*b\*c^3\*(6\*d - 7\*e)\*x^3 + 18\*a\*c^4\*(2\*d - e)\*x^4 - 18\*b\*c^2\*x^2\*(-2\*c^2\*d\*x^2 + e\*(2 + c^2\*x^2))\*ArcCoth[c\*x] + 3\*(6\*b\*d - 12\*a\*e - 25\*b\*e)\*Log[1 - c\*x] - 3\*(6\*b\*d + 12\*a\*e - 25\*b\*e)\*Log[1 + c\*x] + 12\*e\*(3\*a\*c^4\*x^4 + b\*c\*x\*(3 + c^2\*x^2) + 3\*b\*(-1 + c^4\*x^4))\*ArcCoth[c\*x])\*Log[1 - c^2\*x^2])/(144\*c^4)

**Maple [C]** time = 6.22, size = 3320, normalized size = 14.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^3*(a+b*\text{arccoth}(c*x))*(d+e*\ln(-c^2*x^2+1)),x)$

[Out] 
$$\begin{aligned} & -1/8*a*x^4*e-25/24*b*e*x/c^3-7/72*b*e*x^3/c+1/4*x^4*a*d+1/12/c*b*x^3*d+1/4/ \\ & c^3*b*x*d-1/4/c^4*b*\text{arccoth}(c*x)*d+41/24/c^4*b*\text{arccoth}(c*x)*e+1/4*b*\text{arccoth} \\ & (c*x)*x^4*d-1/8*b*\text{arccoth}(c*x)*x^4*e+41/36/c^4*b*e-2/3/c^4*b*e*\ln(2)-1/3*I/ \\ & c^4*b*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn(I*(c*x+1)/(c*x-1))^2+1/24*I \\ & /c*b*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^3*x^3*e+1/24*I/c*b*Pi*csgn(I*(c*x+1)/ \\ & (c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3*x^3*e-1/3/c^4*b*d-1/4*a*e/c^2*x^2+1/4*a*e* \\ & x^4*\ln(-c^2*x^2+1)-1/4*a*e/c^4*\ln(c^2*x^2-1)+1/4*I/c^3*b*Pi*x*e-1/4*I/c^4*b \\ & *Pi*e*\text{arccoth}(c*x)+1/6/c*b*\ln(2)*x^3*e-1/6/c*b*\ln((c*x+1)/(c*x-1)-1)*x^3*e- \\ & 1/4/c^2*b*\text{arccoth}(c*x)*x^2*e+1/2/c^3*b*\ln(2)*x*e-1/2/c^3*b*\ln((c*x+1)/(c*x- \\ & 1)-1)*x*e-1/2/c^4*b*\text{arccoth}(c*x)*\ln(2)*e+1/2/c^4*b*\text{arccoth}(c*x)*e*\ln((c*x+1 \\ & )/(c*x-1)-1)+1/2*b*\ln(2)*\text{arccoth}(c*x)*x^4*e-1/2*b*\ln((c*x+1)/(c*x-1)-1)*\text{arcc} \\ & \text{oth}(c*x)*x^4*e-1/3*I/c^4*b*e*Pi-1/24*I/c*b*Pi*csgn(I/((c*x+1)/(c*x-1)-1)^2 \\ & )*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1))*x^3 \\ & *e-1/8*I/c^3*b*Pi*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c* \\ & x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1))*x*e+1/8*I/c^4*b*\text{arccoth}(c*x)*e*P \\ & i*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^ \\ & 2)*csgn(I*(c*x+1)/(c*x-1))-1/8*I*b*Pi*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I* \\ & (c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1))*\text{arccoth}(c*x) \\ & *x^4*e-1/12/c^4*b*e*(3*\text{arccoth}(c*x)*x^3*c^3+3*\text{arccoth}(c*x)*x^2*c^2+c^2*x^2+ \\ & 3*\text{arccoth}(c*x)*x*c+c*x+3*\text{arccoth}(c*x)+4)*(c*x-1)*\ln((c*x-1)/(c*x+1))+1/4*I* \\ & b*Pi*\text{arccoth}(c*x)*x^4*e+1/12*I/c*b*Pi*x^3*e-1/6*I/c^4*b*e*Pi*csgn(I*((c*x+1 \\ & )/(c*x-1)-1)^2)^3-1/6*I/c^4*b*e*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)- \\ & 1)^2)^3+1/6*I/c^4*b*e*Pi*csgn(I*(c*x+1)/(c*x-1))^3+1/3*I/c^4*b*e*Pi*csgn(I* \\ & (c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2+1/8*I*b*Pi*csgn(I*((c*x+1)/(c*x-1) \\ & -1)^2)^3*\text{arccoth}(c*x)*x^4*e+1/8*I*b*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x \\ & -1)-1)^2)^3*\text{arccoth}(c*x)*x^4*e-1/8*I*b*Pi*csgn(I*(c*x+1)/(c*x-1))^3*\text{arccoth} \\ & (c*x)*x^4*e-1/4*I*b*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*\text{arcc} \\ & \text{oth}(c*x)*x^4*e-1/4*I/c^3*b*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2) \\ & ^2*x*e+1/8*I/c^4*b*\text{arccoth}(c*x)*Pi*e*csgn(I*(c*x+1)/(c*x-1))^3+1/3*I/c^4*b* \\ & e*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*((c*x+1)/(c*x-1)-1))-1/6*I/c^4* \\ & b*e*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)*csgn(I*((c*x+1)/(c*x-1)-1))^2-1/24*I/c \\ & *b*Pi*csgn(I*(c*x+1)/(c*x-1))^3*x^3*e-1/12*I/c*b*Pi*csgn(I*(c*x+1)/(c*x-1)/ \\ & ((c*x+1)/(c*x-1)-1)^2)^2*x^3*e+1/8*I/c^3*b*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2) \\ & ^3*x*e+1/8*I/c^3*b*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3*x*e-1 \\ & /8*I/c^3*b*Pi*csgn(I*(c*x+1)/(c*x-1))^3*x*e-1/8*I/c^4*b*\text{arccoth}(c*x)*Pi*e*c \\ & sgn(I*((c*x+1)/(c*x-1)-1)^2)^3-1/8*I/c^4*b*\text{arccoth}(c*x)*e*Pi*csgn(I*(c*x+1) \\ & /((c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3-1/6*I/c^4*b*e*Pi*csgn(I/((c*x+1)/(c*x-1)- \\ & 1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2-1/6*I/c^4*b*e*Pi*csgn \\ & (I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*(c*x+1)/(c*x-1))+1/4*I/c \\ & ^4*b*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*Pi*e*\text{arccoth}(c*x)+1/6* \end{aligned}$$

$$\begin{aligned} & I/c^4*b*e*Pi*csgn(I/((c*x-1)/(c*x+1))^{(1/2)})^2*csgn(I*(c*x+1)/(c*x-1))-1/8* \\ & I/c^4*b*arccoth(c*x)*e*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)*csgn(I*((c*x+1)/(c* \\ & x-1)-1))^{2-1/8*I/c^4*b*arccoth(c*x)*e*Pi*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn \\ & (I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^{2-1/4*I/c^4*b*arccoth(c*x)*e*Pi*c \\ & sgn(I/((c*x-1)/(c*x+1))^{(1/2)})*csgn(I*(c*x+1)/(c*x-1))^{2+1/6*I/c^4*b*e*Pi*c \\ & sgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)* \\ & csgn(I*(c*x+1)/(c*x-1))-1/8*I/c^4*b*arccoth(c*x)*e*Pi*csgn(I*(c*x+1)/(c*x-1) \\ & )/((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*(c*x+1)/(c*x-1))+1/8*I/c^4*b*arccoth(c*x) \\ & *e*Pi*csgn(I/((c*x-1)/(c*x+1))^{(1/2)})^2*csgn(I*(c*x+1)/(c*x-1))+1/8*I*b*Pi* \\ & csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2) \\ & ^2*arccoth(c*x)*x^4*e+1/8*I*b*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1) \\ & ^2)^2*csgn(I*(c*x+1)/(c*x-1))*arccoth(c*x)*x^4*e-1/8*I*b*Pi*csgn(I/((c*x-1) \\ & /((c*x+1))^{(1/2)})^2*csgn(I*(c*x+1)/(c*x-1))*arccoth(c*x)*x^4*e+1/4*I*b*Pi*cs \\ & gn(I/((c*x-1)/(c*x+1))^{(1/2)})*csgn(I*(c*x+1)/(c*x-1))^2*arccoth(c*x)*x^4*e- \\ & 1/4*I*b*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*((c*x+1)/(c*x-1)-1))*arcc \\ & oth(c*x)*x^4*e+1/8*I*b*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)*csgn(I*((c*x+1)/(c* \\ & x-1)-1))^2*arccoth(c*x)*x^4*e-1/12*I/c*b*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2 \\ & *csgn(I*((c*x+1)/(c*x-1)-1))*x^3*e+1/24*I/c*b*Pi*csgn(I*((c*x+1)/(c*x-1)-1) \\ & ^2)*csgn(I*((c*x+1)/(c*x-1)-1))^2*x^3*e+1/24*I/c*b*Pi*csgn(I/((c*x+1)/(c*x- \\ & 1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*x^3*e+1/24*I/c*b*P \\ & i*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*(c*x+1)/(c*x-1))*x \\ & ^3*e-1/24*I/c*b*Pi*csgn(I/((c*x-1)/(c*x+1))^{(1/2)})^2*csgn(I*(c*x+1)/(c*x-1) \\ & )*x^3*e+1/12*I/c*b*Pi*csgn(I/((c*x-1)/(c*x+1))^{(1/2)})*csgn(I*(c*x+1)/(c*x-1) \\ & ))^2*x^3*e-1/4*I/c^3*b*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*((c*x+1)/( \\ & c*x-1)-1))*x*e+1/8*I/c^3*b*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)*csgn(I*((c*x+1) \\ & /((c*x-1)-1))^2*x*e+1/8*I/c^3*b*Pi*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x \\ & +1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*x*e+1/8*I/c^3*b*Pi*csgn(I*(c*x+1)/(c*x \\ & -1)/((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*(c*x+1)/(c*x-1))*x*e-1/8*I/c^3*b*Pi*cs \\ & gn(I/((c*x-1)/(c*x+1))^{(1/2)})^2*csgn(I*(c*x+1)/(c*x-1))*x*e+1/4*I/c^3*b*Pi*c \\ & sgn(I/((c*x-1)/(c*x+1))^{(1/2)})*csgn(I*(c*x+1)/(c*x-1))^2*x*e+1/4*I/c^4*b*ar \\ & ccoth(c*x)*e*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*((c*x+1)/(c*x-1)-1)) \end{aligned}$$

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**Maxima [C]** time = 1.07594, size = 366, normalized size = 1.63

$$\frac{1}{4} adx^4 + \frac{1}{8} \left( 2x^4 \log(-c^2x^2 + 1) - c^2 \left( \frac{c^2x^4 + 2x^2}{c^4} + \frac{2 \log(c^2x^2 - 1)}{c^6} \right) \right) be \operatorname{arccoth}(cx) + \frac{1}{24} \left( 6x^4 \operatorname{arccoth}(cx) + c \left( \frac{2(c^2x^2 - 1)}{c^4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1)),x, algorithm="maxima")

[Out]  $\frac{1}{4}ad^2x^4 + \frac{1}{8}(2x^4 \log(-c^2x^2 + 1) - c^2((c^2x^4 + 2x^2)/c^4 + 2 \log(c^2x^2 - 1)/c^6))b^2e \operatorname{arccoth}(cx) + \frac{1}{24}(6x^4 \operatorname{arccoth}(cx) + c(2(c^2x^3 + 3x)/c^4 - 3 \log(cx + 1)/c^5 + 3 \log(cx - 1)/c^5))b^2d + \frac{1}{8}(2x^4 \log(-c^2x^2 + 1) - c^2((c^2x^4 + 2x^2)/c^4 + 2 \log(c^2x^2 - 1)/c^6))a^2e - \frac{1}{144}(2(-6I\pi c^3 + 7c^3)x^3 + 6(-6I\pi c + 25c)x - (18I\pi + 12c^3x^3 + 36cx + 75) \log(cx + 1) - (18I\pi + 12c^3x^3 + 36cx - 75) \log(cx - 1))b^2e/c^4$

**Fricas [A]** time = 1.68059, size = 440, normalized size = 1.96

$$\frac{36ac^2ex^2 - 18(2ac^4d - ac^4e)x^4 - 2(6bc^3d - 7bc^3e)x^3 - 6(6bcd - 25bce)x - 12(3ac^4ex^4 + bc^3ex^3 + 3bcex - 3ae) \log(cx)}{144c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas")`

[Out]  $-\frac{1}{144}(36a^2c^2ex^2 - 18(2a^2c^4d - a^2c^4e)x^4 - 2(6b^2c^3d - 7b^2c^3e)x^3 - 6(6b^2c^3d - 25b^2c^3e)x - 12(3a^2c^4ex^4 + b^2c^3ex^3 + 3b^2c^3ex - 3a^2e) \log(-c^2x^2 + 1) + 3(6b^2c^2ex^2 - 3(2b^2c^4d - b^2c^4e)x^4 + 6b^2d - 25b^2e - 6(b^2c^4ex^4 - b^2e) \log(-c^2x^2 + 1)) \log((cx + 1)/(cx - 1))) / c^4$

**Sympy [A]** time = 33.8454, size = 286, normalized size = 1.27

$$\left\{ \begin{array}{l} \frac{adx^4}{4} + \frac{aex^4 \log(-c^2x^2+1)}{4} - \frac{aex^4}{8} - \frac{aex^2}{4c^2} - \frac{ae \log(-c^2x^2+1)}{4c^4} + \frac{bdx^4 \operatorname{acoth}(cx)}{4} + \frac{bex^4 \log(-c^2x^2+1) \operatorname{acoth}(cx)}{4} - \frac{bex^4 \operatorname{acoth}(cx)}{8} + \frac{bdx^3}{12c} + \frac{bex^3}{4} \\ \frac{dx^4 \left(a + \frac{ib}{2}\right)}{4} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1)),x)`

[Out] `Piecewise((a*d*x**4/4 + a*e*x**4*log(-c**2*x**2 + 1)/4 - a*e*x**4/8 - a*e*x**2/(4*c**2) - a*e*log(-c**2*x**2 + 1)/(4*c**4) + b*d*x**4*acoth(c*x)/4 + b*e*x**4*log(-c**2*x**2 + 1)*acoth(c*x)/4 - b*e*x**4*acoth(c*x)/8 + b*d*x**3/(12*c) + b*e*x**3*log(-c**2*x**2 + 1)/(12*c) - 7*b*e*x**3/(72*c) - b*e*x**2*acoth(c*x)/(4*c**2) + b*d*x/(4*c**3) + b*e*x*log(-c**2*x**2 + 1)/(4*c**3))`

- 25\*b\*e\*x/(24\*c\*\*3) - b\*d\*acoth(c\*x)/(4\*c\*\*4) - b\*e\*log(-c\*\*2\*x\*\*2 + 1)\*a  
 coth(c\*x)/(4\*c\*\*4) + 25\*b\*e\*acoth(c\*x)/(24\*c\*\*4), Ne(c, 0)), (d\*x\*\*4\*(a + I  
 \*pi\*b/2)/4, True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arccoth}(cx) + a)(e \log(-c^2x^2 + 1) + d)x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1)),x, algorithm="giac")

[Out] integrate((b\*arccoth(c\*x) + a)\*(e\*log(-c^2\*x^2 + 1) + d)\*x^3, x)

### 3.268 $\int x \left( a + b \coth^{-1}(cx) \right) \left( d + e \log \left( 1 - c^2 x^2 \right) \right) dx$

**Optimal.** Leaf size=140

$$\frac{e(1-c^2x^2)\log(1-c^2x^2)(a+b\coth^{-1}(cx))}{2c^2} + \frac{1}{2}dx^2(a+b\coth^{-1}(cx)) - \frac{1}{2}ex^2(a+b\coth^{-1}(cx)) - \frac{b(d-e)\tanh^{-1}(cx)}{2c^2}$$

[Out] (b\*(d - e)\*x)/(2\*c) - (b\*e\*x)/c + (d\*x^2\*(a + b\*ArcCoth[c\*x]))/2 - (e\*x^2\*(a + b\*ArcCoth[c\*x]))/2 - (b\*(d - e)\*ArcTanh[c\*x])/(2\*c^2) + (b\*e\*ArcTanh[c\*x])/c^2 + (b\*e\*x\*Log[1 - c^2\*x^2])/(2\*c) - (e\*(1 - c^2\*x^2)\*(a + b\*ArcCoth[c\*x])\*Log[1 - c^2\*x^2])/(2\*c^2)

**Rubi [A]** time = 0.118628, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 8, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.32$ , Rules used = {2454, 2389, 2295, 6084, 321, 207, 2448, 206}

$$\frac{e(1-c^2x^2)\log(1-c^2x^2)(a+b\coth^{-1}(cx))}{2c^2} + \frac{1}{2}dx^2(a+b\coth^{-1}(cx)) - \frac{1}{2}ex^2(a+b\coth^{-1}(cx)) - \frac{b(d-e)\tanh^{-1}(cx)}{2c^2}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]),x]

[Out] (b\*(d - e)\*x)/(2\*c) - (b\*e\*x)/c + (d\*x^2\*(a + b\*ArcCoth[c\*x]))/2 - (e\*x^2\*(a + b\*ArcCoth[c\*x]))/2 - (b\*(d - e)\*ArcTanh[c\*x])/(2\*c^2) + (b\*e\*ArcTanh[c\*x])/c^2 + (b\*e\*x\*Log[1 - c^2\*x^2])/(2\*c) - (e\*(1 - c^2\*x^2)\*(a + b\*ArcCoth[c\*x])\*Log[1 - c^2\*x^2])/(2\*c^2)

#### Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.)]*(b_.))^(q_.)*(x_)^(m_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Log[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

#### Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.)]*(b_.))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]
```

Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

Rule 6084

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.)\*((d\_.) + Log[(f\_.) + (g\_.)\*(x\_)^2])\*(e\_.)\*(x\_)^(m\_.), x\_Symbol] := With[{u = IntHide[x^m\*(d + e\*Log[f + g\*x^2]), x]}, Dist[a + b\*ArcCoth[c\*x], u, x] - Dist[b\*c, Int[ExpandIntegrand[u/(1 - c^2\*x^2), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)/2, 0]

Rule 321

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 207

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTanh[(Rt[b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2448

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)], x\_Symbol] := Simp[x\*Log[c\*(d + e\*x^n)^p], x] - Dist[e\*n\*p, Int[x^n/(d + e\*x^n), x], x] /; FreeQ[{c, d, e, n, p}, x]

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps





$$\begin{aligned}
& c*x)*\text{Pi}*\text{csgn}(I*(c*x+1)/(c*x-1))^3*x^2*e^{-1/2}*I*b*\text{arccoth}(c*x)*\text{Pi}*\text{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*x^2*e^{-1/2}*I/c*b*\text{Pi}*\text{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*x^2*e^{-1/2}*I/c^2*b*\text{arccoth}(c*x)*e*\text{Pi}*\text{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2-1/4*I/c^2*b*e*\text{Pi}*\text{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*\text{csgn}(I*(c*x+1)/(c*x-1))-1/4*I/c^2*b*e*\text{Pi}*\text{csgn}(I/(c*x+1)/(c*x-1)-1)^2)*\text{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2+1/2*I/c^2*b*e*\text{Pi}*\text{csgn}(I*((c*x+1)/(c*x-1)-1)^2)^2*\text{csgn}(I*((c*x+1)/(c*x-1)-1))-1/4*I/c^2*b*e*\text{Pi}*\text{csgn}(I*((c*x+1)/(c*x-1)-1)^2)*\text{csgn}(I*((c*x+1)/(c*x-1)-1))^2+1/2*a*e/c^2+1/2*d*a*x^2+1/2*a*e*\ln(-c^2*x^2+1)*x^2-1/2*a*e/c^2*\ln(-c^2*x^2+1)+1/2/c*b*x*d+3/2/c^2*b*e^{-1/c^2}*b*e*\ln(2)-3/2*b*e*x/c-1/2/c^2*b*d-1/2/c^2*b*\text{arccoth}(c*x)*d+5/2/c^2*b*\text{arccoth}(c*x)*e+1/2*b*\text{arccoth}(c*x)*x^2*d-1/2*b*\text{arccoth}(c*x)*x^2*e^{-1/2}*I*b*\text{arccoth}(c*x)*\text{Pi}*\text{csgn}(I*((c*x+1)/(c*x-1)-1)^2)^2*\text{csgn}(I*((c*x+1)/(c*x-1)-1))*x^2*e+1/4*I*b*\text{arccoth}(c*x)*\text{Pi}*\text{csgn}(I*((c*x+1)/(c*x-1)-1)^2)^2*\text{csgn}(I*((c*x+1)/(c*x-1)-1))^2*x^2*e^{-1/4}*I*b*\text{arccoth}(c*x)*\text{Pi}*\text{csgn}(I/((c*x-1)/(c*x+1))^{(1/2)})^2*\text{csgn}(I*(c*x+1)/(c*x-1))*x^2*e+1/2*I*b*\text{arccoth}(c*x)*\text{Pi}*\text{csgn}(I/((c*x-1)/(c*x+1))^{(1/2)})*\text{csgn}(I*(c*x+1)/(c*x-1))^2*x^2*e+1/4*I/c*b*\text{Pi}*\text{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*\text{csgn}(I*(c*x+1)/(c*x-1))*x^2*e+1/4*I/c*b*\text{Pi}*\text{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*\text{csgn}(I/((c*x+1)/(c*x-1)-1)^2)*x^2*e^{-1/2}*I/c*b*\text{Pi}*\text{csgn}(I*((c*x+1)/(c*x-1)-1)^2)^2*\text{csgn}(I*((c*x+1)/(c*x-1)-1))*x^2*e+1/4*I/c*b*\text{Pi}*\text{csgn}(I*((c*x+1)/(c*x-1)-1)^2)*\text{csgn}(I*((c*x+1)/(c*x-1)-1))^2*x^2*e^{-1/4}*I/c*b*\text{Pi}*\text{csgn}(I/((c*x-1)/(c*x+1))^{(1/2)})^2*\text{csgn}(I*(c*x+1)/(c*x-1))*x^2*e+1/2*I/c*b*\text{Pi}*\text{csgn}(I/((c*x-1)/(c*x+1))^{(1/2)})*\text{csgn}(I*(c*x+1)/(c*x-1))^2*x^2*e^{-1/4}*I/c^2*b*\text{arccoth}(c*x)*e*\text{Pi}*\text{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2+1/2*I/c^2*b*\text{arccoth}(c*x)*e*\text{Pi}*\text{csgn}(I*((c*x+1)/(c*x-1)-1)^2)^2*\text{csgn}(I*((c*x+1)/(c*x-1)-1))-1/4*I/c^2*b*\text{arccoth}(c*x)*e*\text{Pi}*\text{csgn}(I*((c*x+1)/(c*x-1)-1)^2)*\text{csgn}(I*((c*x+1)/(c*x-1)-1))^2+1/4*I/c^2*b*\text{arccoth}(c*x)*e*\text{Pi}*\text{csgn}(I/((c*x-1)/(c*x+1))^{(1/2)})^2*\text{csgn}(I*(c*x+1)/(c*x-1))-1/2*I/c^2*b*\text{arccoth}(c*x)*e*\text{Pi}*\text{csgn}(I/((c*x-1)/(c*x+1))^{(1/2)})*\text{csgn}(I*(c*x+1)/(c*x-1))^2+1/4*I/c^2*b*e*\text{Pi}*\text{csgn}(I/((c*x+1)/(c*x-1)-1)^2)*\text{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*\text{csgn}(I*(c*x+1)/(c*x-1))+1/4*I*b*\text{arccoth}(c*x)*\text{Pi}*\text{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*\text{csgn}(I*(c*x+1)/(c*x-1))*x^2*e+1/4*I*b*\text{arccoth}(c*x)*\text{Pi}*\text{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*\text{csgn}(I/((c*x+1)/(c*x-1)-1)^2)*x^2*e^{-1/2}*I/c^2*b*\text{Pi}*e^{-1/2/c^2}*b*e*(\text{arccoth}(c*x)*x*c+\text{arccoth}(c*x)+1)*(c*x-1)*\ln((c*x-1)/(c*x+1))-1/4*I/c^2*b*e*\text{Pi}*\text{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3-1/4*I/c^2*b*e*\text{Pi}*\text{csgn}(I*((c*x+1)/(c*x-1)-1)^2)^3+1/4*I/c^2*b*\text{Pi}*e*\text{csgn}(I*(c*x+1)/(c*x-1))^3+1/2*I*b*\text{arccoth}(c*x)*\text{Pi}*x^2*e+1/2*I/c^2*b*e*\text{Pi}*\text{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2+1/2*I/c*b*\text{Pi}*x^2*e^{-1/2}*I/c^2*b*\text{Pi}*e*\text{arccoth}(c*x)+b*\ln(2)*\text{arccoth}(c*x)*x^2*e-b*\text{arccoth}(c*x)*\ln((c*x+1)/(c*x-1)-1)*x^2*e+1/c*b*\ln(2)*x^2*e^{-1/c*b*\ln((c*x+1)/(c*x-1)-1)}*x^2*e^{-1/c^2*b*\text{arccoth}(c*x)*e*\ln(2)+1/c^2*b*\text{arccoth}(c*x)*e*\ln((c*x+1)/(c*x-1)-1)}+1/4*I/c^2*b*\text{arccoth}(c*x)*e*\text{Pi}*\text{csgn}(I/((c*x+1)/(c*x-1)-1)^2)*\text{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*\text{csgn}(I*(c*x+1)/(c*x-1))-1/4*I*b*\text{arccoth}(c*x)*\text{Pi}*\text{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*\text{csgn}(I*(c*x+1)/
\end{aligned}$$

$$(c*x-1))*\operatorname{csgn}(I/((c*x+1)/(c*x-1)-1)^2)*x^2*e^{-1/4*I/c*b*\Pi*\operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*\operatorname{csgn}(I*(c*x+1)/(c*x-1))*\operatorname{csgn}(I/((c*x+1)/(c*x-1)-1)^2)*x*e^{1/4*I/c^2*b*\Pi*e*\operatorname{csgn}(I/((c*x-1)/(c*x+1))^{(1/2)})^2*\operatorname{csgn}(I*(c*x+1)/(c*x-1))-1/2*I/c^2*b*\Pi*e*\operatorname{csgn}(I/((c*x-1)/(c*x+1))^{(1/2)})*\operatorname{csgn}(I*(c*x+1)/(c*x-1))^{2+1/4*I/c*b*\Pi*\operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3*x*e^{1/4*I/c*b*\Pi*\operatorname{csgn}(I*(c*x+1)/(c*x-1)-1)^2)^3*x*e^{-1/4*I/c^2*b*\operatorname{arccoth}(c*x)*e*\Pi*\operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3-1/4*I/c^2*b*\operatorname{arccoth}(c*x)*e*\Pi*\operatorname{csgn}(I*((c*x+1)/(c*x-1)-1)^2)^3}$$

**Maxima [A]** time = 1.02803, size = 231, normalized size = 1.65

$$\frac{1}{2} adx^2 + \frac{1}{4} \left( 2x^2 \operatorname{arccoth}(cx) + c \left( \frac{2x}{c^2} - \frac{\log(cx+1)}{c^3} + \frac{\log(cx-1)}{c^3} \right) \right) bd - \frac{(c^2x^2 - (c^2x^2 - 1) \log(-c^2x^2 + 1) - 1) be \operatorname{arccoth}(cx)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1)),x, algorithm="maxima")

[Out] 1/2\*a\*d\*x^2 + 1/4\*(2\*x^2\*arccoth(c\*x) + c\*(2\*x/c^2 - log(c\*x + 1)/c^3 + log(c\*x - 1)/c^3))\*b\*d - 1/2\*(c^2\*x^2 - (c^2\*x^2 - 1)\*log(-c^2\*x^2 + 1) - 1)\*b\*e\*arccoth(c\*x)/c^2 - 1/2\*(c^2\*x^2 - (c^2\*x^2 - 1)\*log(-c^2\*x^2 + 1) - 1)\*a\*e/c^2 - 1/2\*(3\*c\*x - (c\*x + 1)\*log(c\*x + 1) - (c\*x - 1)\*log(-c\*x + 1))\*b\*e/c^2

**Fricas [A]** time = 1.62476, size = 297, normalized size = 2.12

$$\frac{2(ac^2d - ac^2e)x^2 + 2(bcd - 3bce)x + 2(ac^2ex^2 + bcex - ae) \log(-c^2x^2 + 1) + ((bc^2d - bc^2e)x^2 - bd + 3be + (bc^2ex^2 - bcd - 3bce)) \log((c*x + 1)/(c*x - 1))}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1)),x, algorithm="fricas")

[Out] 1/4\*(2\*(a\*c^2\*d - a\*c^2\*e)\*x^2 + 2\*(b\*c\*d - 3\*b\*c\*e)\*x + 2\*(a\*c^2\*e\*x^2 + b\*c\*e\*x - a\*e)\*log(-c^2\*x^2 + 1) + ((b\*c^2\*d - b\*c^2\*e)\*x^2 - b\*d + 3\*b\*e + (b\*c^2\*e\*x^2 - b\*c\*d - 3\*b\*c\*e)\*log(-c^2\*x^2 + 1))\*log((c\*x + 1)/(c\*x - 1))/c^2

**Sympy [A]** time = 12.6635, size = 209, normalized size = 1.49

$$\left\{ \begin{array}{l} \frac{adx^2}{2} + \frac{aex^2 \log(-c^2x^2+1)}{2} - \frac{aex^2}{2} - \frac{ae \log(-c^2x^2+1)}{2c^2} + \frac{bdx^2 \operatorname{acoth}(cx)}{2} + \frac{bex^2 \log(-c^2x^2+1) \operatorname{acoth}(cx)}{2} - \frac{bex^2 \operatorname{acoth}(cx)}{2} + \frac{bdx}{2c} + \frac{bex \log(-c^2x^2+1)}{2c} \\ \frac{dx^2 \left(a + \frac{i\pi b}{2}\right)}{2} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*acoth(c\*x))\*(d+e\*ln(-c\*\*2\*x\*\*2+1)),x)

[Out] Piecewise((a\*d\*x\*\*2/2 + a\*e\*x\*\*2\*log(-c\*\*2\*x\*\*2 + 1)/2 - a\*e\*x\*\*2/2 - a\*e\*log(-c\*\*2\*x\*\*2 + 1)/(2\*c\*\*2) + b\*d\*x\*\*2\*acoth(c\*x)/2 + b\*e\*x\*\*2\*log(-c\*\*2\*x\*\*2 + 1)\*acoth(c\*x)/2 - b\*e\*x\*\*2\*acoth(c\*x)/2 + b\*d\*x/(2\*c) + b\*e\*x\*log(-c\*\*2\*x\*\*2 + 1)/(2\*c) - 3\*b\*e\*x/(2\*c) - b\*d\*acoth(c\*x)/(2\*c\*\*2) - b\*e\*log(-c\*\*2\*x\*\*2 + 1)\*acoth(c\*x)/(2\*c\*\*2) + 3\*b\*e\*acoth(c\*x)/(2\*c\*\*2), Ne(c, 0)), (d\*x\*\*2\*(a + I\*pi\*b/2)/2, True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arccoth}(cx) + a)(e \log(-c^2x^2 + 1) + d)x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1)),x, algorithm="giac")

[Out] integrate((b\*arccoth(c\*x) + a)\*(e\*log(-c^2\*x^2 + 1) + d)\*x, x)

$$3.269 \quad \int \frac{(a+b \coth^{-1}(cx))(d+e \log(1-c^2x^2))}{x} dx$$

**Optimal.** Leaf size=381

$$-\frac{1}{2}ae \operatorname{PolyLog}(2, c^2x^2) + \frac{1}{2}be \log(-c^2x^2) \operatorname{PolyLog}\left(2, -\frac{1}{cx}\right) - \frac{1}{2}be \left(\log(-c^2x^2) - \log(1-c^2x^2) + \log\left(1 - \frac{1}{cx}\right) + \log\left(-\frac{1}{cx}\right)\right)$$

```
[Out] -(b*e*Log[1 + 1/(c*x)]^2*Log[-(1/(c*x))])/2 + (b*e*Log[1 - 1/(c*x)]^2*Log[1/(c*x)])/2 + a*d*Log[x] - b*e*Log[(c + x^(-1))/c]*PolyLog[2, (c + x^(-1))/c] + b*e*Log[1 - 1/(c*x)]*PolyLog[2, 1 - 1/(c*x)] + (b*d*PolyLog[2, -(1/(c*x))])/2 + (b*e*Log[-(c^2*x^2)]*PolyLog[2, -(1/(c*x))])/2 - (b*e*(Log[1 - 1/(c*x)] + Log[1 + 1/(c*x)] + Log[-(c^2*x^2)] - Log[1 - c^2*x^2])*PolyLog[2, -(1/(c*x))])/2 - (b*d*PolyLog[2, 1/(c*x)])/2 - (b*e*Log[-(c^2*x^2)]*PolyLog[2, 1/(c*x)])/2 + (b*e*(Log[1 - 1/(c*x)] + Log[1 + 1/(c*x)] + Log[-(c^2*x^2)] - Log[1 - c^2*x^2])*PolyLog[2, 1/(c*x)])/2 - (a*e*PolyLog[2, c^2*x^2])/2 + b*e*PolyLog[3, (c + x^(-1))/c] - b*e*PolyLog[3, 1 - 1/(c*x)] + b*e*PolyLog[3, -(1/(c*x))] - b*e*PolyLog[3, 1/(c*x)]
```

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**Rubi [A]** time = 0.439495, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 11, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$ , Rules used = {6080, 5913, 6078, 2391, 6076, 2454, 2396, 2433, 2374, 6589, 6070}

$$-\frac{1}{2}ae \operatorname{PolyLog}(2, c^2x^2) + \frac{1}{2}be \log(-c^2x^2) \operatorname{PolyLog}\left(2, -\frac{1}{cx}\right) - \frac{1}{2}be \left(\log(-c^2x^2) - \log(1-c^2x^2) + \log\left(1 - \frac{1}{cx}\right) + \log\left(-\frac{1}{cx}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x, x]
```

```
[Out] -(b*e*Log[1 + 1/(c*x)]^2*Log[-(1/(c*x))])/2 + (b*e*Log[1 - 1/(c*x)]^2*Log[1/(c*x)])/2 + a*d*Log[x] - b*e*Log[(c + x^(-1))/c]*PolyLog[2, (c + x^(-1))/c] + b*e*Log[1 - 1/(c*x)]*PolyLog[2, 1 - 1/(c*x)] + (b*d*PolyLog[2, -(1/(c*x))])/2 + (b*e*Log[-(c^2*x^2)]*PolyLog[2, -(1/(c*x))])/2 - (b*e*(Log[1 - 1/(c*x)] + Log[1 + 1/(c*x)] + Log[-(c^2*x^2)] - Log[1 - c^2*x^2])*PolyLog[2, -(1/(c*x))])/2 - (b*d*PolyLog[2, 1/(c*x)])/2 - (b*e*Log[-(c^2*x^2)]*PolyLog[2, 1/(c*x)])/2 + (b*e*(Log[1 - 1/(c*x)] + Log[1 + 1/(c*x)] + Log[-(c^2*x^2)] - Log[1 - c^2*x^2])*PolyLog[2, 1/(c*x)])/2 - (a*e*PolyLog[2, c^2*x^2])/2 + b*e*PolyLog[3, (c + x^(-1))/c] - b*e*PolyLog[3, 1 - 1/(c*x)] + b*e*PolyLog[3, -(1/(c*x))] - b*e*PolyLog[3, 1/(c*x)]
```

Rule 6080

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))\*(Log[(f\_.) + (g\_.)\*(x\_)^2]\*(e\_.) + (d\_)))/(x\_), x\_Symbol] := Dist[d, Int[(a + b\*ArcCoth[c\*x])/x, x], x] + Dist[e, Int[(Log[f + g\*x^2]\*(a + b\*ArcCoth[c\*x]))/x, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]

Rule 5913

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[a\*Log[x], x] + (Simp[(b\*PolyLog[2, -(c\*x)^(-1)])/2, x] - Simp[(b\*PolyLog[2, 1/(c\*x)])/2, x]) /; FreeQ[{a, b, c}, x]

Rule 6078

Int[(Log[(f\_.) + (g\_.)\*(x\_)^2]\*(ArcCoth[(c\_.)\*(x\_.)]\*(b\_.) + (a\_)))/(x\_), x\_Symbol] := Dist[a, Int[Log[f + g\*x^2]/x, x], x] + Dist[b, Int[(Log[f + g\*x^2]\*ArcCoth[c\*x])/x, x], x] /; FreeQ[{a, b, c, f, g}, x]

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 6076

Int[(ArcCoth[(c\_.)\*(x\_.)]\*Log[(f\_.) + (g\_.)\*(x\_)^2])/x, x\_Symbol] := Dist[Log[f + g\*x^2] - Log[-(c^2\*x^2)] - Log[1 - 1/(c\*x)] - Log[1 + 1/(c\*x)], Int[ArcCoth[c\*x]/x, x], x] + (-Dist[1/2, Int[Log[1 - 1/(c\*x)]^2/x, x], x] + Dist[1/2, Int[Log[1 + 1/(c\*x)]^2/x, x], x] + Int[(Log[-(c^2\*x^2)]\*ArcCoth[c\*x])/x, x]) /; FreeQ[{c, f, g}, x] && EqQ[c^2\*f + g, 0]

Rule 2454

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))^(p\_.)]\*(b\_.))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

Rule 2396

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))^(p\_.)]\*(b\_.))/(f\_. + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^p/g, x] - Dist[(b\*e\*n\*p)/g, Int[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]

$(a + b \log[c(d + ex)^n])^{p-1} / (d + ex), x, x] /;$  FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[p, 1]

### Rule 2433

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_))^(m\_.)]\*(g\_.))\*((k\_.) + (l\_.)\*(x\_))^(r\_.), x\_Symbol] :> Dist[1/e, Subst[Int[((k\*x)/d)^r\*(a + b\*Log[c\*x^n])^p\*(f + g\*Log[h\*(e\*i - d\*j)/e + (j\*x)/e]^m), x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e\*k - d\*l, 0]

### Rule 2374

Int[(Log[(d\_.)\*((e\_) + (f\_.)\*(x\_))^(m\_.))]\*((a\_.) + Log[(c\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)/(x\_), x\_Symbol] :> -Simp[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^p)/m, x] + Dist[(b\*n\*p)/m, Int[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^(p-1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d\*e, 1]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 6070

Int[(ArcCoth[(c\_.)\*(x\_))^(n\_.)]\*Log[(d\_.)\*(x\_))^(m\_.)]/(x\_), x\_Symbol] :> Dist[1/2, Int[(Log[d\*x^m]\*Log[1 + 1/(c\*x^n)])/x, x], x] - Dist[1/2, Int[(Log[d\*x^m]\*Log[1 - 1/(c\*x^n)])/x, x], x] /; FreeQ[{c, d, m, n}, x]

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{x} dx &= d \int \frac{a + b \coth^{-1}(cx)}{x} dx + e \int \frac{(a + b \coth^{-1}(cx)) \log(1 - c^2x^2)}{x} dx \\
&= ad \log(x) + \frac{1}{2}bd\text{Li}_2\left(-\frac{1}{cx}\right) - \frac{1}{2}bd\text{Li}_2\left(\frac{1}{cx}\right) + (ae) \int \frac{\log(1 - c^2x^2)}{x} dx \\
&= ad \log(x) + \frac{1}{2}bd\text{Li}_2\left(-\frac{1}{cx}\right) - \frac{1}{2}bd\text{Li}_2\left(\frac{1}{cx}\right) - \frac{1}{2}ae\text{Li}_2(c^2x^2) - \frac{1}{2}(be) \int \log\left(1 - \frac{1}{cx}\right) \log\left(1 + \frac{1}{cx}\right) dx \\
&= ad \log(x) + \frac{1}{2}bd\text{Li}_2\left(-\frac{1}{cx}\right) - \frac{1}{2}be \left( \log\left(1 - \frac{1}{cx}\right) + \log\left(1 + \frac{1}{cx}\right) \right) + \log\left(1 - \frac{1}{cx}\right) \log\left(1 + \frac{1}{cx}\right) \\
&= -\frac{1}{2}be \log^2\left(1 + \frac{1}{cx}\right) \log\left(-\frac{1}{cx}\right) + \frac{1}{2}be \log^2\left(1 - \frac{1}{cx}\right) \log\left(\frac{1}{cx}\right) + ad \log(x) \\
&= -\frac{1}{2}be \log^2\left(1 + \frac{1}{cx}\right) \log\left(-\frac{1}{cx}\right) + \frac{1}{2}be \log^2\left(1 - \frac{1}{cx}\right) \log\left(\frac{1}{cx}\right) + ad \log(x) \\
&= -\frac{1}{2}be \log^2\left(1 + \frac{1}{cx}\right) \log\left(-\frac{1}{cx}\right) + \frac{1}{2}be \log^2\left(1 - \frac{1}{cx}\right) \log\left(\frac{1}{cx}\right) + ad \log(x) \\
&= -\frac{1}{2}be \log^2\left(1 + \frac{1}{cx}\right) \log\left(-\frac{1}{cx}\right) + \frac{1}{2}be \log^2\left(1 - \frac{1}{cx}\right) \log\left(\frac{1}{cx}\right) + ad \log(x)
\end{aligned}$$

**Mathematica [F]** time = 0.212522, size = 0, normalized size = 0.

$$\int \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/x,x]

[Out] Integrate[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/x, x]

**Maple [C]** time = 1.104, size = 864, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth(c\*x))\*(d+e\*ln(-c^2\*x^2+1))/x,x)

[Out] dilog(c\*x)\*a\*e-1/2\*dilog(c\*x)\*b\*d+ln(c\*x)\*a\*d+1/2\*b\*e\*ln(-c\*x)\*ln(c\*x+1)^2+  
 b\*e\*ln(c\*x+1)\*polylog(2,c\*x+1)-1/4\*I\*Pi\*ln(c\*x)\*ln(c\*x-1)\*b\*e\*csgn(I\*(c\*x-1))  
 \*csgn(I\*(c\*x-1)\*(c\*x+1))^2-1/4\*I\*Pi\*ln(c\*x)\*ln(c\*x-1)\*b\*e\*csgn(I\*(c\*x+1))  
 \*csgn(I\*(c\*x-1)\*(c\*x+1))^2+1/4\*I\*Pi\*dilog(c\*x)\*b\*e\*csgn(I\*(c\*x-1))\*csgn(I\*(c\*x+1))  
 \*csgn(I\*(c\*x-1)\*(c\*x+1))+b\*e\*polylog(3,-c\*x+1)-b\*e\*polylog(3,c\*x+1)-  
 (-1/2\*I\*Pi\*b\*e\*csgn(I\*(c\*x-1)\*(c\*x+1))^2-1/4\*I\*Pi\*b\*e\*csgn(I\*(c\*x-1))\*csgn(I\*(c\*x+1))  
 \*csgn(I\*(c\*x-1)\*(c\*x+1))+1/4\*I\*Pi\*b\*e\*csgn(I\*(c\*x-1))\*csgn(I\*(c\*x-1)\*(c\*x+1))^2+1/4\*I  
 \*Pi\*b\*e\*csgn(I\*(c\*x-1)\*(c\*x+1))^3+1/2\*I\*e\*Pi\*b+a\*e+1/2\*b\*d)\*dilog(c\*x+1)+ln  
 (c\*x)\*ln(c\*x-1)\*a\*e-1/2\*ln(c\*x)\*ln(c\*x-1)\*b\*d-1/2\*ln(c\*x)\*ln(c\*x-1)^2\*b\*e-l  
 n(c\*x-1)\*polylog(2,-c\*x+1)\*b\*e-1/2\*I\*Pi\*ln(c\*x)\*a\*e\*csgn(I\*(c\*x-1))\*csgn(I\*(c\*x+1))  
 \*csgn(I\*(c\*x-1)\*(c\*x+1))-1/4\*I\*Pi\*ln(c\*x)\*ln(c\*x-1)\*b\*e\*csgn(I\*(c\*x-1)\*(c\*x+1))^3+1/2\*I\*Pi\*ln(c\*x)  
 \*a\*e\*csgn(I\*(c\*x-1)\*(c\*x+1))^3-1/4\*I\*Pi\*dilo  
 g(c\*x)\*b\*e\*csgn(I\*(c\*x-1)\*(c\*x+1))^3+1/4\*I\*Pi\*ln(c\*x)\*ln(c\*x-1)\*b\*e\*csgn(I\*(c\*x-1))  
 \*csgn(I\*(c\*x+1))\*csgn(I\*(c\*x-1)\*(c\*x+1))+1/2\*I\*Pi\*ln(c\*x)\*ln(c\*x-1)  
 \*b\*e\*csgn(I\*(c\*x-1)\*(c\*x+1))^2-1/4\*I\*Pi\*dilog(c\*x)\*b\*e\*csgn(I\*(c\*x-1))\*csgn(I\*(c\*x-1)  
 \*(c\*x+1))^2-1/4\*I\*Pi\*dilog(c\*x)\*b\*e\*csgn(I\*(c\*x+1))\*csgn(I\*(c\*x-1)  
 \*(c\*x+1))^2+1/2\*I\*Pi\*ln(c\*x)\*a\*e\*csgn(I\*(c\*x-1))\*csgn(I\*(c\*x-1)\*(c\*x+1))^2  
 +1/2\*I\*Pi\*ln(c\*x)\*a\*e\*csgn(I\*(c\*x+1))\*csgn(I\*(c\*x-1)\*(c\*x+1))^2+I\*Pi\*ln(c\*x)  
 \*a\*e-I\*Pi\*ln(c\*x)\*a\*e\*csgn(I\*(c\*x-1)\*(c\*x+1))^2+1/2\*I\*Pi\*dilog(c\*x)\*b\*e\*cs  
 gn(I\*(c\*x-1)\*(c\*x+1))^2-1/2\*I\*dilog(c\*x)\*Pi\*b\*e-1/2\*I\*ln(c\*x)\*Pi\*ln(c\*x-1)\*  
 b\*e

**Maxima [C]** time = 1.77925, size = 225, normalized size = 0.59

$$i \pi a e \log(x) - \frac{1}{2} \left( \log(cx-1)^2 \log(cx) + 2 \operatorname{Li}_2(-cx+1) \log(cx-1) - 2 \operatorname{Li}_3(-cx+1) \right) b e + \frac{1}{2} \left( \log(cx+1)^2 \log(-cx) + 2 \operatorname{Li}_2(-cx+1) \log(cx+1) - 2 \operatorname{Li}_3(-cx+1) \right) b e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1))/x,x, algorithm="maxima")

[Out] I\*pi\*a\*e\*log(x) - 1/2\*(log(c\*x - 1)^2\*log(c\*x) + 2\*dilog(-c\*x + 1)\*log(c\*x  
 - 1) - 2\*polylog(3, -c\*x + 1))\*b\*e + 1/2\*(log(c\*x + 1)^2\*log(-c\*x) + 2\*dilo  
 g(c\*x + 1)\*log(c\*x + 1) - 2\*polylog(3, c\*x + 1))\*b\*e + a\*d\*log(x) - 1/2\*(I\*  
 pi\*b\*e + b\*d - 2\*a\*e)\*(log(c\*x - 1)\*log(c\*x) + dilog(-c\*x + 1)) - 1/2\*(-I\*p  
 i\*b\*e - b\*d - 2\*a\*e)\*(log(c\*x + 1)\*log(-c\*x) + dilog(c\*x + 1))



**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bd \operatorname{arccoth}(cx) + ad + (be \operatorname{arccoth}(cx) + ae) \log(-c^2x^2 + 1)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x,x, algorithm="fricas")`

[Out] `integral((b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(-c^2*x^2 + 1))/x, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{acoth}(cx)) (d + e \log(-c^2x^2 + 1))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1))/x,x)`

[Out] `Integral((a + b*acoth(c*x))*(d + e*log(-c**2*x**2 + 1))/x, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccoth}(cx) + a)(e \log(-c^2x^2 + 1) + d)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x,x, algorithm="giac")`

[Out] `integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x, x)`

$$3.270 \quad \int \frac{(a+b \coth^{-1}(cx))(d+e \log(1-c^2x^2))}{x^3} dx$$

**Optimal.** Leaf size=247

$$\frac{1}{2}bc^2e \operatorname{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right) + \frac{1}{2}bc^2e \operatorname{PolyLog}\left(2, \frac{2}{cx+1} - 1\right) - \frac{(a+b \coth^{-1}(cx))(e \log(1-c^2x^2) + d)}{2x^2} + \frac{1}{2}c^2e(a+b)$$

[Out]  $-(b*c^2*e*ArcCoth[c*x]^2)/2 - (b*c^2*e*ArcTanh[c*x]^2)/2 - a*c^2*e*Log[x] + b*c^2*e*ArcTanh[c*x]*Log[2/(1-c*x)] + ((a+b)*c^2*e*Log[1-c*x])/2 + (a-b)*c^2*e*Log[1+c*x])/2 - (b*c*(d+e*Log[1-c^2*x^2]))/(2*x) - ((a+b*ArcCoth[c*x])*(d+e*Log[1-c^2*x^2]))/(2*x^2) + (b*c^2*ArcTanh[c*x]*(d+e*Log[1-c^2*x^2]))/2 - b*c^2*e*ArcCoth[c*x]*Log[2-2/(1+c*x)] + (b*c^2*e*PolyLog[2, 1-2/(1-c*x)])/2 + (b*c^2*e*PolyLog[2, -1+2/(1+c*x)])/2$

**Rubi [A]** time = 0.487551, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.482$ , Rules used = {5917, 325, 206, 6086, 6725, 801, 5989, 5933, 2447, 5984, 5918, 2402, 2315}

$$\frac{1}{2}bc^2e \operatorname{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right) + \frac{1}{2}bc^2e \operatorname{PolyLog}\left(2, \frac{2}{cx+1} - 1\right) - \frac{(a+b \coth^{-1}(cx))(e \log(1-c^2x^2) + d)}{2x^2} + \frac{1}{2}c^2e(a+b)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a+b*ArcCoth[c*x])*(d+e*Log[1-c^2*x^2])/x^3, x]$

[Out]  $-(b*c^2*e*ArcCoth[c*x]^2)/2 - (b*c^2*e*ArcTanh[c*x]^2)/2 - a*c^2*e*Log[x] + b*c^2*e*ArcTanh[c*x]*Log[2/(1-c*x)] + ((a+b)*c^2*e*Log[1-c*x])/2 + (a-b)*c^2*e*Log[1+c*x])/2 - (b*c*(d+e*Log[1-c^2*x^2]))/(2*x) - ((a+b*ArcCoth[c*x])*(d+e*Log[1-c^2*x^2]))/(2*x^2) + (b*c^2*ArcTanh[c*x]*(d+e*Log[1-c^2*x^2]))/2 - b*c^2*e*ArcCoth[c*x]*Log[2-2/(1+c*x)] + (b*c^2*e*PolyLog[2, 1-2/(1-c*x)])/2 + (b*c^2*e*PolyLog[2, -1+2/(1+c*x)])/2$

**Rule 5917**

$\operatorname{Int}[(a_.) + ArcCoth[(c_.)*(x_.)]*(b_.)]^{(p_.)}*((d_.)*(x_.))^{(m_.)}, x\_Symbol]$   
 $\rightarrow \operatorname{Simp}[(d*x)^{(m+1)}*(a+b*ArcCoth[c*x])^p/(d*(m+1)), x] - \operatorname{Dist}[(b*c*p)/(d*(m+1)), \operatorname{Int}[(d*x)^{(m+1)}*(a+b*ArcCoth[c*x])^{(p-1)}/(1-c^2*$

$x^2$ ), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 325

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a+b\*x^n)^(p+1))/(a\*c\*(m+1)), x] - Dist[(b\*(m+n\*(p+1)+1))/(a\*c^n\*(m+1)), Int[(c\*x)^(m+n)\*(a+b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 6086

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))\*((d\_.) + Log[(f\_.) + (g\_.)\*(x\_)^2]\*(e\_.)\*(x\_)^(m\_.)), x\_Symbol] := With[{u = IntHide[x^m\*(a + b\*ArcCoth[c\*x]), x]}, Dist[d + e\*Log[f + g\*x^2], u, x] - Dist[2\*e\*g, Int[ExpandIntegrand[(x\*u)/(f + g\*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[m] && NeQ[m, -1]

### Rule 6725

Int[(u\_)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := With[{v = RationalFunctionExpand[u/(a + b\*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

### Rule 801

Int[(((d\_.) + (e\_.)\*(x\_)^(m\_))\*((f\_.) + (g\_.)\*(x\_)))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[((d + e\*x)^m\*(f + g\*x))/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[m]

### Rule 5989

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)/((x\_)\*((d\_) + (e\_.)\*(x\_)^2)), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p+1)/(b\*d\*(p+1)), x] + Dist[1/d, Int[(a + b\*ArcCoth[c\*x])^p/(x\*(1 + c\*x)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && GtQ[p, 0]

Rule 5933

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.)/((x_.)*((d_) + (e_.)*(x_.))), x
_Symbol] := Simp[((a + b*ArcCoth[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/
(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*(x_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_.)), x_Symbol
] := -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_.))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_.)/((d_) + (e_.)*(x_.)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{x^3} dx &= -\frac{bc(d + e \log(1 - c^2x^2))}{2x} - \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{2x^2} \\
&= -\frac{bc(d + e \log(1 - c^2x^2))}{2x} - \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{2x^2} \\
&= -\frac{1}{2}bc^2e \tanh^{-1}(cx)^2 - \frac{bc(d + e \log(1 - c^2x^2))}{2x} - \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{2x^2} \\
&= -\frac{1}{2}bc^2e \tanh^{-1}(cx)^2 + bc^2e \tanh^{-1}(cx) \log\left(\frac{2}{1 - cx}\right) - \frac{bc(d + e \log(1 - c^2x^2))}{2x} \\
&= -\frac{1}{2}bc^2e \coth^{-1}(cx)^2 - \frac{1}{2}bc^2e \tanh^{-1}(cx)^2 + bc^2e \tanh^{-1}(cx) \log\left(\frac{2}{1 - cx}\right) \\
&= -\frac{1}{2}bc^2e \coth^{-1}(cx)^2 - \frac{1}{2}bc^2e \tanh^{-1}(cx)^2 - ac^2e \log(x) + bc^2e \tanh^{-1}(cx) \log\left(\frac{2}{1 - cx}\right) \\
&= -\frac{1}{2}bc^2e \coth^{-1}(cx)^2 - \frac{1}{2}bc^2e \tanh^{-1}(cx)^2 - ac^2e \log(x) + bc^2e \tanh^{-1}(cx) \log\left(\frac{2}{1 - cx}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.149537, size = 161, normalized size = 0.65

$$\frac{1}{2} \left( -bc^2e \left( \text{PolyLog}\left(2, -\frac{1}{cx}\right) - \text{PolyLog}\left(2, \frac{1}{cx}\right) \right) - \frac{e \log(1 - c^2x^2) \left( a + (b - bc^2x^2) \coth^{-1}(cx) + bcx \right)}{x^2} + c^2e(a + b) \log(x) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/x^3,x]

[Out] (-(a\*d)/x^2) - 2\*a\*c^2\*e\*Log[x] + (a + b)\*c^2\*e\*Log[1 - c\*x] + (a - b)\*c^2\*e\*Log[1 + c\*x] - (b\*d\*(2\*ArcCoth[c\*x] + c\*x\*(2 + c\*x\*Log[1 - c\*x] - c\*x\*Log[1 + c\*x])))/(2\*x^2) - (e\*(a + b\*c\*x + (b - b\*c^2\*x^2)\*ArcCoth[c\*x])\*Log[1 - c^2\*x^2])/x^2 - b\*c^2\*e\*(PolyLog[2, -(1/(c\*x))] - PolyLog[2, 1/(c\*x)]))/2

**Maple [F]** time = 2.582, size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{arccoth}(cx))(d + e \ln(-c^2x^2 + 1))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^3,x)`

[Out] `int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^3,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{4} \left( \left( c \log(cx+1) - c \log(cx-1) - \frac{2}{x} \right) c - \frac{2 \operatorname{arccoth}(cx)}{x^2} \right) bd + \frac{1}{2} \left( c^2 (\log(c^2x^2-1) - \log(x^2)) - \frac{\log(-c^2x^2+1)}{x^2} \right) ae - \frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^3,x, algorithm="maxima")`

[Out] `1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arccoth(c*x)/x^2)*b*d + 1/2*(c^2*(log(c^2*x^2 - 1) - log(x^2)) - log(-c^2*x^2 + 1)/x^2)*a*e - 1/4*b*e*(log(c*x + 1)^2/x^2 - 2*integrate(-((c*x + 1)*log(c*x - 1)^2 - (I*pi + (I*pi*c + c)*x)*log(c*x + 1) - (-I*pi - I*pi*c*x)*log(c*x - 1))/(c*x^4 + x^3), x)) - 1/2*a*d/x^2`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{bd \operatorname{arccoth}(cx) + ad + (be \operatorname{arccoth}(cx) + ae) \log(-c^2x^2 + 1)}{x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^3,x, algorithm="fricas")`

[Out] `integral((b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(-c^2*x^2 + 1))/x^3, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{acoth}(cx)) (d + e \log(-c^2 x^2 + 1))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth(c\*x))\*(d+e\*ln(-c\*\*2\*x\*\*2+1))/x\*\*3,x)

[Out] Integral((a + b\*acoth(c\*x))\*(d + e\*log(-c\*\*2\*x\*\*2 + 1))/x\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccoth}(cx) + a)(e \log(-c^2 x^2 + 1) + d)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1))/x^3,x, algorithm="giac")

[Out] integrate((b\*arccoth(c\*x) + a)\*(e\*log(-c^2\*x^2 + 1) + d)/x^3, x)

$$3.271 \quad \int \frac{(a+b \coth^{-1}(cx))(d+e \log(1-c^2x^2))}{x^5} dx$$

**Optimal.** Leaf size=339

$$\frac{1}{4}bc^4e \operatorname{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right) + \frac{1}{4}bc^4e \operatorname{PolyLog}\left(2, \frac{2}{cx+1} - 1\right) - \frac{(a+b \coth^{-1}(cx))(e \log(1-c^2x^2)+d)}{4x^4} + \frac{1}{12}c^4e(3a +$$

[Out] (a\*c^2\*e)/(4\*x^2) + (5\*b\*c^3\*e)/(12\*x) + (b\*c^2\*e\*ArcCoth[c\*x])/(4\*x^2) - (b\*c^4\*e\*ArcCoth[c\*x]^2)/4 - (b\*c^4\*e\*ArcTanh[c\*x])/4 - (b\*c^4\*e\*ArcTanh[c\*x]^2)/4 - (a\*c^4\*e\*Log[x])/2 + (b\*c^4\*e\*ArcTanh[c\*x]\*Log[2/(1-c\*x)])/2 + ((3\*a+4\*b)\*c^4\*e\*Log[1-c\*x])/12 + ((3\*a-4\*b)\*c^4\*e\*Log[1+c\*x])/12 - (b\*c\*(d+e\*Log[1-c^2\*x^2]))/(12\*x^3) - (b\*c^3\*(d+e\*Log[1-c^2\*x^2]))/(4\*x) - ((a+b\*ArcCoth[c\*x])\*(d+e\*Log[1-c^2\*x^2]))/(4\*x^4) + (b\*c^4\*ArcTanh[c\*x]\*(d+e\*Log[1-c^2\*x^2]))/4 - (b\*c^4\*e\*ArcCoth[c\*x]\*Log[2-2/(1+c\*x)])/2 + (b\*c^4\*e\*PolyLog[2, 1-2/(1-c\*x)])/4 + (b\*c^4\*e\*PolyLog[2, -1+2/(1+c\*x)])/4

**Rubi [A]** time = 0.717232, antiderivative size = 339, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.518$ , Rules used = {5917, 325, 206, 6086, 6725, 1802, 5983, 5989, 5933, 2447, 5984, 5918, 2402, 2315}

$$\frac{1}{4}bc^4e \operatorname{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right) + \frac{1}{4}bc^4e \operatorname{PolyLog}\left(2, \frac{2}{cx+1} - 1\right) - \frac{(a+b \coth^{-1}(cx))(e \log(1-c^2x^2)+d)}{4x^4} + \frac{1}{12}c^4e(3a +$$

Antiderivative was successfully verified.

[In] Int[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/x^5, x]

[Out] (a\*c^2\*e)/(4\*x^2) + (5\*b\*c^3\*e)/(12\*x) + (b\*c^2\*e\*ArcCoth[c\*x])/(4\*x^2) - (b\*c^4\*e\*ArcCoth[c\*x]^2)/4 - (b\*c^4\*e\*ArcTanh[c\*x])/4 - (b\*c^4\*e\*ArcTanh[c\*x]^2)/4 - (a\*c^4\*e\*Log[x])/2 + (b\*c^4\*e\*ArcTanh[c\*x]\*Log[2/(1-c\*x)])/2 + ((3\*a+4\*b)\*c^4\*e\*Log[1-c\*x])/12 + ((3\*a-4\*b)\*c^4\*e\*Log[1+c\*x])/12 - (b\*c\*(d+e\*Log[1-c^2\*x^2]))/(12\*x^3) - (b\*c^3\*(d+e\*Log[1-c^2\*x^2]))/(4\*x) - ((a+b\*ArcCoth[c\*x])\*(d+e\*Log[1-c^2\*x^2]))/(4\*x^4) + (b\*c^4\*ArcTanh[c\*x]\*(d+e\*Log[1-c^2\*x^2]))/4 - (b\*c^4\*e\*ArcCoth[c\*x]\*Log[2-2/(1+c\*x)])/2 + (b\*c^4\*e\*PolyLog[2, 1-2/(1-c\*x)])/4 + (b\*c^4\*e\*PolyLog[2, -1+2/(1+c\*x)])/4

**Rule 5917**



```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
  := Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

### Rule 325

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 6086

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.)*(x_)^(m_.), x_Symbol] := With[{u = IntHide[x^m*(a + b*ArcCoth[c*x]),
x]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[(x
*u)/(f + g*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ
[m] && NeQ[m, -1]
```

### Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

### Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

### Rule 5983

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)/((d_) + (
e_.)*(x_)^2), x_Symbol] := Dist[1/d, Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcCoth[c*x])^p)/(d + e*x^
```

2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]

### Rule 5989

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)/((x\_)\*((d\_) + (e\_.)\*(x\_)^2)), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*d\*(p + 1)), x] + Dist[1/d, Int[(a + b\*ArcCoth[c\*x])^p/(x\*(1 + c\*x)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && GtQ[p, 0]

### Rule 5933

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)/((x\_)\*((d\_) + (e\_.)\*(x\_))), x\_Symbol] := Simp[((a + b\*ArcCoth[c\*x])^p\*Log[2 - 2/(1 + (e\*x)/d)])/d, x] - Dist[(b\*c\*p)/d, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*Log[2 - 2/(1 + (e\*x)/d)])/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

### Rule 2447

Int[Log[u\_]\*(Pq\_)^(m\_.), x\_Symbol] := With[{C = FullSimplify[(Pq^m\*(1 - u))/D[u, x]]}, Simp[C\*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]

### Rule 5984

Int[(((a\_.) + ArcTanh[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)\*(x\_))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcTanh[c\*x])^(p + 1)/(b\*e\*(p + 1)), x] + Dist[1/(c\*d), Int[(a + b\*ArcTanh[c\*x])^p/(1 - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && IGtQ[p, 0]

### Rule 5918

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[((a + b\*ArcTanh[c\*x])^p\*Log[2/(1 + (e\*x)/d)])/e, x] + Dist[(b\*c\*p)/e, Int[((a + b\*ArcTanh[c\*x])^(p - 1)\*Log[2/(1 + (e\*x)/d)])/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

### Rule 2402

Int[Log[(c\_.)/((d\_) + (e\_.)\*(x\_))]/((f\_.) + (g\_.)\*(x\_)^2), x\_Symbol] := -Dist[e/g, Subst[Int[Log[2\*d\*x]/(1 - 2\*d\*x), x], x, 1/(d + e\*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2\*d] && EqQ[e^2\*f + d^2\*g, 0]

Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] :> -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{x^5} dx &= -\frac{bc(d + e \log(1 - c^2x^2))}{12x^3} - \frac{bc^3(d + e \log(1 - c^2x^2))}{4x} - \frac{(a + b \coth^{-1}(cx))}{x^4} \\
 &= -\frac{bc(d + e \log(1 - c^2x^2))}{12x^3} - \frac{bc^3(d + e \log(1 - c^2x^2))}{4x} - \frac{(a + b \coth^{-1}(cx))}{x^4} \\
 &= -\frac{1}{4}bc^4e \tanh^{-1}(cx)^2 - \frac{bc(d + e \log(1 - c^2x^2))}{12x^3} - \frac{bc^3(d + e \log(1 - c^2x^2))}{4x} \\
 &= -\frac{1}{4}bc^4e \tanh^{-1}(cx)^2 + \frac{1}{2}bc^4e \tanh^{-1}(cx) \log\left(\frac{2}{1 - cx}\right) - \frac{bc(d + e \log(1 - c^2x^2))}{12x^3} \\
 &= -\frac{1}{4}bc^4e \tanh^{-1}(cx)^2 + \frac{1}{2}bc^4e \tanh^{-1}(cx) \log\left(\frac{2}{1 - cx}\right) - \frac{bc(d + e \log(1 - c^2x^2))}{12x^3} \\
 &= \frac{ac^2e}{4x^2} + \frac{bc^3e}{6x} + \frac{bc^2e \coth^{-1}(cx)}{4x^2} - \frac{1}{4}bc^4e \coth^{-1}(cx)^2 - \frac{1}{4}bc^4e \tanh^{-1}(cx) \\
 &= \frac{ac^2e}{4x^2} + \frac{5bc^3e}{12x} + \frac{bc^2e \coth^{-1}(cx)}{4x^2} - \frac{1}{4}bc^4e \coth^{-1}(cx)^2 - \frac{1}{4}bc^4e \tanh^{-1}(cx) \\
 &= \frac{ac^2e}{4x^2} + \frac{5bc^3e}{12x} + \frac{bc^2e \coth^{-1}(cx)}{4x^2} - \frac{1}{4}bc^4e \coth^{-1}(cx)^2 - \frac{1}{4}bc^4e \tanh^{-1}(cx)
 \end{aligned}$$

**Mathematica [A]** time = 0.144746, size = 307, normalized size = 0.91

$$-\frac{1}{4}bc^4e \left( \text{PolyLog}\left(2, -\frac{1}{cx}\right) - \text{PolyLog}\left(2, \frac{1}{cx}\right) \right) + \frac{e \log(1 - c^2x^2)(-3a - 3bc^3x^3 + 3bc^4x^4 \coth^{-1}(cx) - bcx - 3b \coth^{-1}(cx))}{12x^4}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/x^5, x]

[Out] -(a\*d)/(4\*x^4) + (a\*c^2\*e)/(4\*x^2) + (b\*c^3\*e)/(6\*x) - (a\*c^4\*e\*Log[x])/2 + ((3\*a\*c^4\*e + 4\*b\*c^4\*e)\*Log[1 - c\*x])/12 - (b\*c^4\*e\*(-ArcCoth[c\*x])/(2\*c^2

$*x^2) + (-1/(c*x)) - \text{Log}[1 - c*x]/2 + \text{Log}[1 + c*x]/2)/2) + b*c^4*d*(-\text{ArcCoth}[c*x]/(4*c^4*x^4) + (-1/(3*c^3*x^3) - 1/(c*x) - \text{Log}[1 - c*x]/2 + \text{Log}[1 + c*x]/2)/4) + ((3*a*c^4*e - 4*b*c^4*e)*\text{Log}[1 + c*x])/12 + (e*(-3*a - b*c*x - 3*b*c^3*x^3 - 3*b*\text{ArcCoth}[c*x] + 3*b*c^4*x^4*\text{ArcCoth}[c*x])*\text{Log}[1 - c^2*x^2])/(12*x^4) - (b*c^4*e*(\text{PolyLog}[2, -1/(c*x)]) - \text{PolyLog}[2, 1/(c*x)]))/4$

**Maple [F]** time = 6.231, size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{arccoth}(cx)) (d + e \ln(-c^2 x^2 + 1))}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth(c\*x))\*(d+e\*ln(-c^2\*x^2+1))/x^5,x)

[Out] int((a+b\*arccoth(c\*x))\*(d+e\*ln(-c^2\*x^2+1))/x^5,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{24} \left( \left( 3c^3 \log(cx+1) - 3c^3 \log(cx-1) - \frac{2(3c^2x^2+1)}{x^3} \right) c - \frac{6 \operatorname{arccoth}(cx)}{x^4} \right) bd + \frac{1}{4} \left( c^2 \log(c^2x^2-1) - c^2 \log(x^2) + \frac{1}{x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1))/x^5,x, algorithm="maxima")

[Out] 1/24\*((3\*c^3\*log(c\*x + 1) - 3\*c^3\*log(c\*x - 1) - 2\*(3\*c^2\*x^2 + 1)/x^3)\*c - 6\*arccoth(c\*x)/x^4)\*b\*d + 1/4\*((c^2\*log(c^2\*x^2 - 1) - c^2\*log(x^2) + 1/x^2)\*c^2 - log(-c^2\*x^2 + 1)/x^4)\*a\*e - 1/8\*b\*e\*(log(c\*x + 1)^2/x^4 - 4\*integrate(-1/2\*(2\*(c\*x + 1)\*log(c\*x - 1)^2 - (2\*I\*pi + (2\*I\*pi\*c + c)\*x)\*log(c\*x + 1) - (-2\*I\*pi - 2\*I\*pi\*c\*x)\*log(c\*x - 1))/(c\*x^6 + x^5), x)) - 1/4\*a\*d/x^4

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{bd \operatorname{arccoth}(cx) + ad + (be \operatorname{arccoth}(cx) + ae) \log(-c^2 x^2 + 1)}{x^5}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1))/x^5,x, algorithm="fricas")

[Out] integral((b\*d\*arccoth(c\*x) + a\*d + (b\*e\*arccoth(c\*x) + a\*e)\*log(-c^2\*x^2 + 1))/x^5, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{arccoth}(cx)) (d + e \log(-c^2 x^2 + 1))}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth(c\*x))\*(d+e\*ln(-c\*\*2\*x\*\*2+1))/x\*\*5,x)

[Out] Integral((a + b\*acoth(c\*x))\*(d + e\*log(-c\*\*2\*x\*\*2 + 1))/x\*\*5, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccoth}(cx) + a)(e \log(-c^2 x^2 + 1) + d)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1))/x^5,x, algorithm="giac")

[Out] integrate((b\*arccoth(c\*x) + a)\*(e\*log(-c^2\*x^2 + 1) + d)/x^5, x)

### 3.272 $\int x^4 (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2x^2)) dx$

**Optimal.** Leaf size=315

$$\frac{1}{5}x^5 (a + b \coth^{-1}(cx)) (e \log(1 - c^2x^2) + d) - \frac{e(4a + 3b) \log(1 - cx)}{20c^5} + \frac{e(4a - 3b) \log(cx + 1)}{20c^5} - \frac{2aex^3}{15c^2} - \frac{2aex}{5c^4} - \frac{2}{25}aex^5$$

[Out]  $(-2*a*e*x)/(5*c^4) - (77*b*e*x^2)/(300*c^3) - (2*a*e*x^3)/(15*c^2) - (9*b*e*x^4)/(200*c) - (2*a*e*x^5)/25 - (2*b*e*x*ArcCoth[c*x])/(5*c^4) - (2*b*e*x^3*ArcCoth[c*x])/(15*c^2) - (2*b*e*x^5*ArcCoth[c*x])/25 + (b*e*ArcCoth[c*x]^2)/(5*c^5) - ((4*a + 3*b)*e*Log[1 - c*x])/(20*c^5) + ((4*a - 3*b)*e*Log[1 + c*x])/(20*c^5) - (23*b*e*Log[1 - c^2*x^2])/(75*c^5) - (b*e*Log[1 - c^2*x^2]^2)/(20*c^5) + (b*x^2*(d + e*Log[1 - c^2*x^2]))/(10*c^3) + (b*x^4*(d + e*Log[1 - c^2*x^2]))/(20*c) + (x^5*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/5 + (b*Log[1 - c^2*x^2]*(d + e*Log[1 - c^2*x^2]))/(10*c^5)$

**Rubi [A]** time = 0.749021, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 15, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$ , Rules used = {5917, 266, 43, 6086, 6725, 1802, 633, 31, 5981, 5911, 260, 5949, 2475, 2390, 2301}

$$\frac{1}{5}x^5 (a + b \coth^{-1}(cx)) (e \log(1 - c^2x^2) + d) - \frac{e(4a + 3b) \log(1 - cx)}{20c^5} + \frac{e(4a - 3b) \log(cx + 1)}{20c^5} - \frac{2aex^3}{15c^2} - \frac{2aex}{5c^4} - \frac{2}{25}aex^5$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^4*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]$

[Out]  $(-2*a*e*x)/(5*c^4) - (77*b*e*x^2)/(300*c^3) - (2*a*e*x^3)/(15*c^2) - (9*b*e*x^4)/(200*c) - (2*a*e*x^5)/25 - (2*b*e*x*ArcCoth[c*x])/(5*c^4) - (2*b*e*x^3*ArcCoth[c*x])/(15*c^2) - (2*b*e*x^5*ArcCoth[c*x])/25 + (b*e*ArcCoth[c*x]^2)/(5*c^5) - ((4*a + 3*b)*e*Log[1 - c*x])/(20*c^5) + ((4*a - 3*b)*e*Log[1 + c*x])/(20*c^5) - (23*b*e*Log[1 - c^2*x^2])/(75*c^5) - (b*e*Log[1 - c^2*x^2]^2)/(20*c^5) + (b*x^2*(d + e*Log[1 - c^2*x^2]))/(10*c^3) + (b*x^4*(d + e*Log[1 - c^2*x^2]))/(20*c) + (x^5*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/5 + (b*Log[1 - c^2*x^2]*(d + e*Log[1 - c^2*x^2]))/(10*c^5)$

**Rule 5917**

$\text{Int}[(a_. + \text{ArcCoth}[(c_.)*(x_.)]*(b_.))^{(p_.)*((d_.)*(x_.))^{(m_.)}, x\_Symbol]$   
 $\rightarrow \text{Simp}[(d*x)^{(m+1)}*(a + b*ArcCoth[c*x])^p/(d*(m+1)), x] - \text{Dist}[(b*c*p)/(d*(m+1)), \text{Int}[(d*x)^{(m+1)}*(a + b*ArcCoth[c*x])^{(p-1)}/(1 - c^2*$

$x^2$ ), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 266

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

### Rule 43

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

### Rule 6086

Int[((a\_) + ArcCoth[(c\_)\*(x\_)])\*(b\_))\*((d\_) + Log[(f\_) + (g\_)\*(x\_)^2]\*(e\_))\*(x\_)^(m\_), x\_Symbol] := With[{u = IntHide[x^m\*(a + b\*ArcCoth[c\*x]), x]}, Dist[d + e\*Log[f + g\*x^2], u, x] - Dist[2\*e\*g, Int[ExpandIntegrand[(x\*u)/(f + g\*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[m] && NeQ[m, -1]

### Rule 6725

Int[(u\_)/((a\_) + (b\_)\*(x\_)^(n\_)), x\_Symbol] := With[{v = RationalFunctionExpand[u/(a + b\*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

### Rule 1802

Int[(Pq\_)\*((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^2)^(p\_), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*Pq\*(a + b\*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

### Rule 633

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (c\_)\*(x\_)^2), x\_Symbol] := With[{q = Rt[-(a\*c), 2]}, Dist[e/2 + (c\*d)/(2\*q), Int[1/(-q + c\*x), x], x] + Dist[e/2 - (c\*d)/(2\*q), Int[1/(q + c\*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a\*c)]

### Rule 31

```
Int[((a_) + (b_)*(x_))^-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 5981

```
Int[(((a_) + ArcCoth[(c_)*(x_)]*(b_))^(p_)*((f_)*(x_))^(m_))/((d_) + (
e_)*(x_)^2), x_Symbol] := Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

### Rule 5911

```
Int[((a_) + ArcCoth[(c_)*(x_)]*(b_))^(p_), x_Symbol] := Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

### Rule 260

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

### Rule 5949

```
Int[((a_) + ArcCoth[(c_)*(x_)]*(b_))^(p_)/((d_) + (e_)*(x_)^2), x_Symb
ol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

### Rule 2475

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_)^(n_))^(p_)]*(b_))^(q_)*(x_)^(m
_)*((f_) + (g_)*(x_)^(s_))^(r_), x_Symbol] := Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
|| IGtQ[q, 0])
```

### Rule 2390

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_)^(n_))^(p_)]*(b_))^(q_)*((f_) + (g_
)*(x_)^(q_)), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```



Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] :> Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

Rubi steps

$$\begin{aligned}
 \int x^4 (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2 x^2)) dx &= \frac{bx^2 (d + e \log(1 - c^2 x^2))}{10c^3} + \frac{bx^4 (d + e \log(1 - c^2 x^2))}{20c} + \frac{1}{5} x^5 (a + b) \\
 &= \frac{bx^2 (d + e \log(1 - c^2 x^2))}{10c^3} + \frac{bx^4 (d + e \log(1 - c^2 x^2))}{20c} + \frac{1}{5} x^5 (a + b) \\
 &= \frac{bx^2 (d + e \log(1 - c^2 x^2))}{10c^3} + \frac{bx^4 (d + e \log(1 - c^2 x^2))}{20c} + \frac{1}{5} x^5 (a + b) \\
 &= \frac{bx^2 (d + e \log(1 - c^2 x^2))}{10c^3} + \frac{bx^4 (d + e \log(1 - c^2 x^2))}{20c} + \frac{1}{5} x^5 (a + b) \\
 &= -\frac{be \log^2(1 - c^2 x^2)}{20c^5} + \frac{bx^2 (d + e \log(1 - c^2 x^2))}{10c^3} + \frac{bx^4 (d + e \log(1 - c^2 x^2))}{20c} \\
 &= -\frac{2aex}{5c^4} - \frac{3bex^2}{20c^3} - \frac{2aex^3}{15c^2} - \frac{bex^4}{40c} - \frac{2}{25} aex^5 - \frac{2}{25} bex^5 \coth^{-1}(cx) - \frac{2}{25} bex^5 \log(1 - c^2 x^2) \\
 &= -\frac{2aex}{5c^4} - \frac{3bex^2}{20c^3} - \frac{2aex^3}{15c^2} - \frac{bex^4}{40c} - \frac{2}{25} aex^5 - \frac{2bex^3 \coth^{-1}(cx)}{15c^2} - \frac{2}{25} bex^3 \log(1 - c^2 x^2) \\
 &= -\frac{2aex}{5c^4} - \frac{3bex^2}{20c^3} - \frac{2aex^3}{15c^2} - \frac{bex^4}{40c} - \frac{2}{25} aex^5 - \frac{2bex \coth^{-1}(cx)}{5c^4} - \frac{2}{25} bex \log(1 - c^2 x^2) \\
 &= -\frac{2aex}{5c^4} - \frac{19bex^2}{100c^3} - \frac{2aex^3}{15c^2} - \frac{9bex^4}{200c} - \frac{2}{25} aex^5 - \frac{2bex \coth^{-1}(cx)}{5c^4} - \frac{2}{25} bex \log(1 - c^2 x^2) \\
 &= -\frac{2aex}{5c^4} - \frac{77bex^2}{300c^3} - \frac{2aex^3}{15c^2} - \frac{9bex^4}{200c} - \frac{2}{25} aex^5 - \frac{2bex \coth^{-1}(cx)}{5c^4} - \frac{2}{25} bex \log(1 - c^2 x^2)
 \end{aligned}$$

**Mathematica [A]** time = 0.148721, size = 236, normalized size = 0.75

$$\frac{30c^2 e x^2 \log(1 - c^2 x^2) (4ac^3 x^3 + b(c^2 x^2 + 2)) + 4bc^3 x^3 \coth^{-1}(cx) + 2 \log(1 - cx) (-60ae + 30bd - 137be) + 2 \log(cx + 1) (-60ae + 30bd - 137be)}{10c^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]),x]

[Out] (-240\*a\*c\*e\*x + 2\*b\*c^2\*(30\*d - 77\*e)\*x^2 - 80\*a\*c^3\*e\*x^3 + 3\*b\*c^4\*(10\*d - 9\*e)\*x^4 + 24\*a\*c^5\*(5\*d - 2\*e)\*x^5 - 8\*b\*c\*x\*(-15\*c^4\*d\*x^4 + 2\*e\*(15 + 5\*c^2\*x^2 + 3\*c^4\*x^4))\*ArcCoth[c\*x] + 120\*b\*e\*ArcCoth[c\*x]^2 + 2\*(30\*b\*d - 60\*a\*e - 137\*b\*e)\*Log[1 - c\*x] + 2\*(30\*b\*d + 60\*a\*e - 137\*b\*e)\*Log[1 + c\*x] + 30\*c^2\*e\*x^2\*(4\*a\*c^3\*x^3 + b\*(2 + c^2\*x^2) + 4\*b\*c^3\*x^3\*ArcCoth[c\*x])\*Log[1 - c^2\*x^2] + 30\*b\*e\*Log[1 - c^2\*x^2]^2)/(600\*c^5)

**Maple [C]** time = 1.72, size = 4194, normalized size = 13.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(a+b\*arccoth(c\*x))\*(d+e\*ln(-c^2\*x^2+1)),x)

[Out] 1/10/c^3\*b\*x^2\*d+1/20/c\*b\*x^4\*d+1/5\*b\*arccoth(c\*x)\*x^5\*d-46/75/c^5\*b\*arccoth(c\*x)\*e-1/5/c^5\*b\*ln((c\*x+1)/(c\*x-1)-1)\*d+1/5/c^5\*b\*e\*ln((c\*x+1)/(c\*x-1)-1)^2+137/150/c^5\*b\*ln((c\*x+1)/(c\*x-1)-1)\*e+1/5/c^5\*b\*arccoth(c\*x)\*d-2/5\*b\*e\*x\*arccoth(c\*x)/c^4-2/15\*b\*e\*x^3\*arccoth(c\*x)/c^2+181/600/c^5\*b\*e-2/5\*a\*e\*x/c^4-77/300\*b\*e\*x^2/c^3-2/15\*a\*e\*x^3/c^2-9/200\*b\*e\*x^4/c-2/25\*b\*e\*x^5\*arccoth(c\*x)+1/20\*I/c^3\*b\*csgn(I\*(c\*x+1)/(c\*x-1))\*csgn(I\*(c\*x+1)/(c\*x-1)/((c\*x+1)/(c\*x-1)-1)^2)^2\*Pi\*x^2\*e+1/40\*I/c\*b\*csgn(I\*((c\*x+1)/(c\*x-1)-1)^2)\*csgn(I\*((c\*x+1)/(c\*x-1)-1)^2\*Pi\*x^4\*e+1/20\*I/c^3\*b\*csgn(I\*((c\*x+1)/(c\*x-1)-1)^2)\*csgn(I\*((c\*x+1)/(c\*x-1)-1)^2\*Pi\*x^2\*e+1/20\*I/c\*b\*csgn(I/((c\*x-1)/(c\*x+1))^(1/2))\*csgn(I\*(c\*x+1)/(c\*x-1))^2\*Pi\*x^4\*e+1/10\*I/c^3\*b\*csgn(I/((c\*x-1)/(c\*x+1))^(1/2))\*csgn(I\*(c\*x+1)/(c\*x-1))^2\*Pi\*x^2\*e-1/40\*I/c\*b\*csgn(I/((c\*x-1)/(c\*x+1))^(1/2))^2\*csgn(I\*(c\*x+1)/(c\*x-1))\*Pi\*x^4\*e-2/25\*a\*e\*x^5+1/10\*I/c^5\*b\*Pi\*ln((c\*x+1)/(c\*x-1)-1)\*e\*csgn(I/((c\*x+1)/(c\*x-1)-1)^2)\*csgn(I\*(c\*x+1)/(c\*x-1)/((c\*x+1)/(c\*x-1)-1)^2)\*csgn(I\*(c\*x+1)/(c\*x-1))-1/10\*I\*b\*arccoth(c\*x)\*csgn(I\*(c\*x+1)/(c\*x-1))\*csgn(I\*(c\*x+1)/(c\*x-1)/((c\*x+1)/(c\*x-1)-1)^2)\*csgn(I/((c\*x+1)/(c\*x-1)-1)^2)\*Pi\*x^5\*e-3/20/c^5\*b\*d-1/10\*I/c^5\*b\*arccoth(c\*x)\*e\*Pi\*csgn(I/((c\*x+1)/(c\*x-1)-1)^2)\*csgn(I\*(c\*x+1)/(c\*x-1)/((c\*x+1)/(c\*x-1)-1)^2)\*csgn(I\*(c\*x+1)/(c\*x-1))-1/40\*I/c\*b\*csgn(I\*(c\*x+1)/(c\*x-1))\*csgn(I\*(c\*x+1)/(c\*x-1)/((c\*x+1)/(c\*x-1)-1)^2)\*csgn(I/((c\*x+1)/(c\*x-1)-1)^2)\*Pi\*x^4\*e-1/20\*I/c^3\*b\*csgn(I\*(c\*x+1)/(c\*x-1))\*csgn(I\*(c\*x+1)/(c\*x-1)/((c\*x+1)/(c\*x-1)-1)^2)\*csgn(I/((c\*x+1)/(c\*x-1)-1)^2)\*Pi\*x^2\*e+1/5\*x^5\*a\*d-3/40\*I/c^5\*b\*Pi\*e\*csgn(I/((c\*x+1)/(c\*x-1)-1)^2)\*csgn(I\*(c\*x+1)/(c\*x-1)/((c\*x+1)/(c\*x-1)-1)^2)^2-3/40\*I/c^5\*b\*Pi\*e\*csgn(I\*(c\*x+1)/(c\*x-1)/((c\*x+1)/(c\*x-1)-1)^2)^2\*csgn(I\*(c\*x+1)/(c\*x-1))-3/40\*I/c^5\*b\*Pi\*e\*csgn(I\*((c\*x+1)/(c\*x-1)-1)^2)\*csgn(I\*((c\*x+1)/(c\*x-1)-1)^2-1/10\*I/c^5\*b\*arccoth(c\*x)\*e\*Pi\*csgn(I\*(c\*x+1)/(c\*x-1))^2

$$\begin{aligned}
& 3-3/20*I/c^5*b*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn(I*(c*x+1)/(c*x-1)) \\
& ^2-1/5*I/c^5*b*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*Pi*e*arccoth \\
& (c*x)+1/5*I/c^5*b*Pi*ln((c*x+1)/(c*x-1)-1)*e*csgn(I*(c*x+1)/(c*x-1)/((c*x+1) \\
& )/(c*x-1)-1)^2)^2-1/40*I/c*b*csgn(I*(c*x+1)/(c*x-1))^3*Pi*x^4*e-1/20*I/c^3* \\
& b*csgn(I*(c*x+1)/(c*x-1))^3*Pi*x^2*e+1/10*I/c^5*b*Pi*ln((c*x+1)/(c*x-1)-1)* \\
& e*csgn(I*(c*x+1)/(c*x-1))^3+1/10*I/c^5*b*arccoth(c*x)*Pi*e*csgn(I*(c*x+1)/( \\
& c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3-1/20*I/c*b*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/ \\
& (c*x-1)-1)^2)^2*Pi*x^4*e-1/10*I/c^3*b*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x- \\
& 1)-1)^2)^2*Pi*x^2*e+1/40*I/c*b*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2 \\
& )^3*Pi*x^4*e+1/20*I/c^3*b*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3*P \\
& i*x^2*e-1/5*I*b*arccoth(c*x)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2 \\
& ^2*Pi*x^5*e+1/10*I*b*arccoth(c*x)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1) \\
& ^2)^3*Pi*x^5*e+1/10*I*b*arccoth(c*x)*csgn(I*((c*x+1)/(c*x-1)-1)^2)^3*Pi*x^5 \\
& *e+1/10*I/c^5*b*arccoth(c*x)*e*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^3+3/20*I/c^ \\
& 5*b*Pi*e*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*((c*x+1)/(c*x-1)-1))+1/5*a* \\
& e*x^5*ln(-c^2*x^2+1)-1/5*a*e/c^5*ln(c*x-1)+1/5*a*e/c^5*ln(c*x+1)-1/10*I*b*a \\
& rccoth(c*x)*csgn(I*(c*x+1)/(c*x-1))^3*Pi*x^5*e-1/10/c*b*ln((c*x+1)/(c*x-1)- \\
& 1)*x^4*e-1/5/c^3*b*ln((c*x+1)/(c*x-1)-1)*x^2*e+1/10/c*b*ln(2)*x^4*e+1/5/c^3 \\
& *b*ln(2)*x^2*e+2/5/c^5*b*arccoth(c*x)*ln(2)*e-2/5/c^5*b*ln(2)*ln((c*x+1)/(c \\
& *x-1)-1)*e-2/5*b*arccoth(c*x)*ln((c*x+1)/(c*x-1)-1)*x^5*e+2/5*b*arccoth(c*x \\
& )*ln(2)*x^5*e-3/20*I/c^5*b*Pi*e-1/20*I/c^3*b*csgn(I/((c*x-1)/(c*x+1))^(1/2) \\
& )^2*csgn(I*(c*x+1)/(c*x-1))*Pi*x^2*e+1/10*I/c^5*b*arccoth(c*x)*e*Pi*csgn(I* \\
& ((c*x+1)/(c*x-1)-1)^2)*csgn(I*((c*x+1)/(c*x-1)-1))^2-1/10*I/c^5*b*Pi*ln((c* \\
& x+1)/(c*x-1)-1)*e*csgn(I*((c*x+1)/(c*x-1)-1)^2)*csgn(I*((c*x+1)/(c*x-1)-1)) \\
& ^2+1/10*I/c^5*b*arccoth(c*x)*e*Pi*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x \\
& +1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2+3/40*I/c^5*b*e*Pi*csgn(I/((c*x+1)/(c*x \\
& -1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x \\
& -1))+1/5*I/c^5*b*arccoth(c*x)*Pi*e*csgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn(I*( \\
& c*x+1)/(c*x-1))^2-1/10*I/c^5*b*arccoth(c*x)*e*Pi*csgn(I/((c*x-1)/(c*x+1))^( \\
& 1/2))^2*csgn(I*(c*x+1)/(c*x-1))-1/10*I/c^5*b*Pi*ln((c*x+1)/(c*x-1)-1)*e*csg \\
& n(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2- \\
& 1/5*I/c^5*b*Pi*ln((c*x+1)/(c*x-1)-1)*e*csgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn \\
& (I*(c*x+1)/(c*x-1))^2+1/5*I/c^5*b*Pi*ln((c*x+1)/(c*x-1)-1)*e*csgn(I*((c*x+1) \\
& )/(c*x-1)-1)^2)^2*csgn(I*((c*x+1)/(c*x-1)-1))-1/20*I/c*b*csgn(I*((c*x+1)/(c \\
& *x-1)-1)^2)^2*csgn(I*((c*x+1)/(c*x-1)-1))*Pi*x^4*e-1/10*I/c^3*b*csgn(I*((c* \\
& x+1)/(c*x-1)-1)^2)^2*csgn(I*((c*x+1)/(c*x-1)-1))*Pi*x^2*e-1/10*I/c^5*b*Pi*l \\
& n((c*x+1)/(c*x-1)-1)*e*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*csgn \\
& (I*(c*x+1)/(c*x-1))+1/10*I/c^5*b*Pi*ln((c*x+1)/(c*x-1)-1)*e*csgn(I/((c*x-1) \\
& )/(c*x+1))^(1/2))^2*csgn(I*(c*x+1)/(c*x-1))+1/10*I*b*arccoth(c*x)*csgn(I*(c* \\
& x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*csgn(I/((c*x+1)/(c*x-1)-1)^2)*Pi*x^5 \\
& *e+1/10*I*b*arccoth(c*x)*csgn(I*((c*x+1)/(c*x-1)-1)^2)*csgn(I*((c*x+1)/(c*x- \\
& 1)-1))^2*Pi*x^5*e+1/5*I*b*arccoth(c*x)*csgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn \\
& (I*(c*x+1)/(c*x-1))^2*Pi*x^5*e-1/10*I*b*arccoth(c*x)*csgn(I/((c*x-1)/(c*x+1) \\
& ))^(1/2))^2*csgn(I*(c*x+1)/(c*x-1))*Pi*x^5*e+1/10*I*b*arccoth(c*x)*csgn(I*( \\
& c*x+1)/(c*x-1))*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*Pi*x^5*e-1/
\end{aligned}$$

$$\begin{aligned}
& 5*I*b*arccoth(c*x)*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*((c*x+1)/(c*x-1)-1)) \\
& *Pi*x^5*e^{-1/5}*I/c^5*b*arccoth(c*x)*Pi*e*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2 \\
& csgn(I*((c*x+1)/(c*x-1)-1))+1/10*I/c^5*b*arccoth(c*x)*Pi*e*csgn(I*(c*x+1)/(c*x-1)) \\
& /((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*(c*x+1)/(c*x-1))+1/40*I/c*b*csgn(I*(c*x+1)/(c*x-1)) \\
& /((c*x+1)/(c*x-1)-1)^2)^2*csgn(I/((c*x+1)/(c*x-1)-1)^2)*Pi*x^4*e^{1/20}*I/c^3*b*csgn(I*(c*x+1)/(c*x-1)) \\
& /((c*x+1)/(c*x-1)-1)^2)^2*csgn(I/((c*x+1)/(c*x-1)-1)^2)*Pi*x^2*e^{1/40}*I/c*b*csgn(I*(c*x+1)/(c*x-1)) \\
& *csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*Pi*x^4*e^{1/40}*I/c*b*csgn(I*((c*x+1)/(c*x-1)-1)^2)^3 \\
& *Pi*x^4*e^{1/20}*I/c^3*b*csgn(I*((c*x+1)/(c*x-1)-1)^2)^3*Pi*x^2*e^{-1/10}*I/c^5*b*Pi*ln((c*x+1)/(c*x-1)-1) \\
& *e*csgn(I*((c*x+1)/(c*x-1)-1)^2)^3-1/10*I/c^5*b*Pi*ln((c*x+1)/(c*x-1)-1)*e*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3 \\
& +3/40*I/c^5*b*e*Pi*csgn(I/((c*x-1)/(c*x+1))^{1/2})^2*csgn(I*(c*x+1)/(c*x-1))-3/10/c^5*b*e*ln(2)-1/20/c^5*b*e*(4*arccoth(c*x)*x^5*c^5+c^4*x^4+2*c^2*x^2+4*arccoth(c*x)-4*ln((c*x+1)/(c*x-1)-1)-3)*ln((c*x-1)/(c*x+1))+1/5*I*b*arccoth(c*x)*Pi*x^5*e^{1/5}*I/c^5*b*arccoth(c*x)*Pi*e^{-1/5}*I/c^5*b*Pi*ln((c*x+1)/(c*x-1)-1)*e^{-3/40}*I/c^5*b*Pi*e*csgn(I*((c*x+1)/(c*x-1)-1)^2)^3+3/40*I/c^5*b*e*Pi*csgn(I*(c*x+1)/(c*x-1))^3+3/20*I/c^5*b*Pi*e*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2-3/40*I/c^5*b*Pi*e*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3+1/20*I/c*b*Pi*x^4*e^{1/10}*I/c^3*b*Pi*x^2*e
\end{aligned}$$

**Maxima [C]** time = 1.10989, size = 428, normalized size = 1.36

$$\frac{1}{5} adx^5 + \frac{1}{75} \left( 15x^5 \log(-c^2x^2 + 1) - c^2 \left( \frac{2(3c^4x^5 + 5c^2x^3 + 15x)}{c^6} - \frac{15 \log(cx + 1)}{c^7} + \frac{15 \log(cx - 1)}{c^7} \right) \right) be \operatorname{arccoth}(cx) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1)),x, algorithm="maxima")

[Out] 1/5\*a\*d\*x^5 + 1/75\*(15\*x^5\*log(-c^2\*x^2 + 1) - c^2\*(2\*(3\*c^4\*x^5 + 5\*c^2\*x^3 + 15\*x)/c^6 - 15\*log(c\*x + 1)/c^7 + 15\*log(c\*x - 1)/c^7))\*b\*e\*arccoth(c\*x) + 1/20\*(4\*x^5\*arccoth(c\*x) + c\*((c^2\*x^4 + 2\*x^2)/c^4 + 2\*log(c^2\*x^2 - 1)/c^6))\*b\*d + 1/75\*(15\*x^5\*log(-c^2\*x^2 + 1) - c^2\*(2\*(3\*c^4\*x^5 + 5\*c^2\*x^3 + 15\*x)/c^6 - 15\*log(c\*x + 1)/c^7 + 15\*log(c\*x - 1)/c^7))\*a\*e - 1/600\*(3\*(-10\*I\*pi\*c^4 + 9\*c^4)\*x^4 + 2\*(-30\*I\*pi\*c^2 + 77\*c^2)\*x^2 - (60\*I\*pi + 30\*c^4\*x^4 + 60\*c^2\*x^2 + 120\*log(c\*x - 1) - 274)\*log(c\*x + 1) - (60\*I\*pi + 30\*c^4\*x^4 + 60\*c^2\*x^2 - 274)\*log(c\*x - 1))\*b\*e/c^5

**Fricas [A]** time = 1.67589, size = 587, normalized size = 1.86

$$80ac^3ex^3 - 24(5ac^5d - 2ac^5e)x^5 - 3(10bc^4d - 9bc^4e)x^4 + 240acex - 30be \log(-c^2x^2 + 1)^2 - 30be \log\left(\frac{cx+1}{cx-1}\right)^2 - 2$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1)),x, algorithm="fricas")

[Out] 
$$-1/600*(80*a*c^3*e*x^3 - 24*(5*a*c^5*d - 2*a*c^5*e)*x^5 - 3*(10*b*c^4*d - 9*b*c^4*e)*x^4 + 240*a*c*e*x - 30*b*e*\log(-c^2*x^2 + 1)^2 - 30*b*e*\log((c*x + 1)/(c*x - 1))^2 - 2*(30*b*c^2*d - 77*b*c^2*e)*x^2 - 2*(60*a*c^5*e*x^5 + 15*b*c^4*e*x^4 + 30*b*c^2*e*x^2 + 30*b*d - 137*b*e)*\log(-c^2*x^2 + 1) - 4*(15*b*c^5*e*x^5*\log(-c^2*x^2 + 1) - 10*b*c^3*e*x^3 + 3*(5*b*c^5*d - 2*b*c^5*e)*x^5 - 30*b*c*e*x + 30*a*e)*\log((c*x + 1)/(c*x - 1)))/c^5$$

---

**Sympy [A]** time = 43.4042, size = 345, normalized size = 1.1

$$\left\{ \begin{array}{l} \frac{adx^5}{5} + \frac{aex^5 \log(-c^2x^2+1)}{5} - \frac{2aex^5}{25} - \frac{2aex^3}{15c^2} - \frac{2aex}{5c^4} + \frac{2ae \operatorname{acoth}(cx)}{5c^5} + \frac{bdx^5 \operatorname{acoth}(cx)}{5} + \frac{bex^5 \log(-c^2x^2+1) \operatorname{acoth}(cx)}{5} - \frac{2bex^5 \operatorname{acoth}(cx)}{25} + \frac{bdx^5 \left(a + \frac{i\pi b}{2}\right)}{5} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(a+b\*acoth(c\*x))\*(d+e\*ln(-c\*\*2\*x\*\*2+1)),x)

[Out] 
$$\text{Piecewise}((a*d*x**5/5 + a*e*x**5*\log(-c**2*x**2 + 1)/5 - 2*a*e*x**5/25 - 2*a*e*x**3/(15*c**2) - 2*a*e*x/(5*c**4) + 2*a*e*\operatorname{acoth}(c*x)/(5*c**5) + b*d*x**5*\operatorname{acoth}(c*x)/5 + b*e*x**5*\log(-c**2*x**2 + 1)*\operatorname{acoth}(c*x)/5 - 2*b*e*x**5*\operatorname{acoth}(c*x)/25 + b*d*x**4/(20*c) + b*e*x**4*\log(-c**2*x**2 + 1)/(20*c) - 9*b*e*x**4/(200*c) - 2*b*e*x**3*\operatorname{acoth}(c*x)/(15*c**2) + b*d*x**2/(10*c**3) + b*e*x**2*\log(-c**2*x**2 + 1)/(10*c**3) - 77*b*e*x**2/(300*c**3) - 2*b*e*x*\operatorname{acoth}(c*x)/(5*c**4) + b*d*\log(-c**2*x**2 + 1)/(10*c**5) + b*e*\log(-c**2*x**2 + 1)**2/(20*c**5) - 137*b*e*\log(-c**2*x**2 + 1)/(300*c**5) + b*e*\operatorname{acoth}(c*x)**2/(5*c**5), \text{Ne}(c, 0)), (d*x**5*(a + I*pi*b/2)/5, \text{True}))$$

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arccoth}(cx) + a)(e \log(-c^2x^2 + 1) + d)x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)*x^4, x)
```

$$3.273 \quad \int x^2 \left( a + b \coth^{-1}(cx) \right) \left( d + e \log \left( 1 - c^2 x^2 \right) \right) dx$$

**Optimal.** Leaf size=247

$$\frac{1}{3}x^3 \left( a + b \coth^{-1}(cx) \right) \left( e \log \left( 1 - c^2 x^2 \right) + d \right) - \frac{e(2a+b) \log(1-cx)}{6c^3} + \frac{e(2a-b) \log(cx+1)}{6c^3} - \frac{2aex}{3c^2} - \frac{2}{9}aex^3 + \frac{bx^2(e \log(1-c^2x^2))}{3}$$

```
[Out] (-2*a*e*x)/(3*c^2) - (5*b*e*x^2)/(18*c) - (2*a*e*x^3)/9 - (2*b*e*x*ArcCoth[
c*x])/(3*c^2) - (2*b*e*x^3*ArcCoth[c*x])/9 + (b*e*ArcCoth[c*x]^2)/(3*c^3) -
((2*a + b)*e*Log[1 - c*x])/(6*c^3) + ((2*a - b)*e*Log[1 + c*x])/(6*c^3) -
(4*b*e*Log[1 - c^2*x^2])/(9*c^3) - (b*e*Log[1 - c^2*x^2]^2)/(12*c^3) + (b*x
^2*(d + e*Log[1 - c^2*x^2]))/(6*c) + (x^3*(a + b*ArcCoth[c*x])*(d + e*Log[1
- c^2*x^2]))/3 + (b*Log[1 - c^2*x^2]*(d + e*Log[1 - c^2*x^2]))/(6*c^3)
```

**Rubi [A]** time = 0.620957, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 15, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$ , Rules used = {5917, 266, 43, 6086, 6725, 801, 633, 31, 5981, 5911, 260, 5949, 2475, 2390, 2301}

$$\frac{1}{3}x^3 \left( a + b \coth^{-1}(cx) \right) \left( e \log \left( 1 - c^2 x^2 \right) + d \right) - \frac{e(2a+b) \log(1-cx)}{6c^3} + \frac{e(2a-b) \log(cx+1)}{6c^3} - \frac{2aex}{3c^2} - \frac{2}{9}aex^3 + \frac{bx^2(e \log(1-c^2x^2))}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (-2*a*e*x)/(3*c^2) - (5*b*e*x^2)/(18*c) - (2*a*e*x^3)/9 - (2*b*e*x*ArcCoth[
c*x])/(3*c^2) - (2*b*e*x^3*ArcCoth[c*x])/9 + (b*e*ArcCoth[c*x]^2)/(3*c^3) -
((2*a + b)*e*Log[1 - c*x])/(6*c^3) + ((2*a - b)*e*Log[1 + c*x])/(6*c^3) -
(4*b*e*Log[1 - c^2*x^2])/(9*c^3) - (b*e*Log[1 - c^2*x^2]^2)/(12*c^3) + (b*x
^2*(d + e*Log[1 - c^2*x^2]))/(6*c) + (x^3*(a + b*ArcCoth[c*x])*(d + e*Log[1
- c^2*x^2]))/3 + (b*Log[1 - c^2*x^2]*(d + e*Log[1 - c^2*x^2]))/(6*c^3)
```

### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^ (p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b,
m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6086

```
Int[((a_) + ArcCoth[(c_)*(x_)])*(b_))*((d_) + Log[(f_) + (g_)*(x_)^2]*
(e_)*(x_)^(m_), x_Symbol] := With[{u = IntHide[x^m*(a + b*ArcCoth[c*x]),
x]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[(x
*u)/(f + g*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ
[m] && NeQ[m, -1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 801

```
Int[(((d_) + (e_)*(x_)^(m_))*((f_) + (g_)*(x_)))/((a_) + (c_)*(x_)^2),
x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 633

```
Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```



Rule 5981

```
Int[(((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.)*((f_.)*(x_.))^(m_.))/((d_.) + (
e_.)*(x_)^2), x_Symbol] := Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.)/((d_.) + (e_.)*(x_)^2), x_Symb
ol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_)^(s_.))^(r_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
|| IGtQ[q, 0])
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((f_.) + (g_.
)*(x_)^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int x^2 (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2 x^2)) dx &= \frac{bx^2 (d + e \log(1 - c^2 x^2))}{6c} + \frac{1}{3} x^3 (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2 x^2)) \\
&= \frac{bx^2 (d + e \log(1 - c^2 x^2))}{6c} + \frac{1}{3} x^3 (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2 x^2)) \\
&= \frac{bx^2 (d + e \log(1 - c^2 x^2))}{6c} + \frac{1}{3} x^3 (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2 x^2)) \\
&= \frac{bx^2 (d + e \log(1 - c^2 x^2))}{6c} + \frac{1}{3} x^3 (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2 x^2)) \\
&= -\frac{be \log^2(1 - c^2 x^2)}{12c^3} + \frac{bx^2 (d + e \log(1 - c^2 x^2))}{6c} + \frac{1}{3} x^3 (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2 x^2)) \\
&= -\frac{2aex}{3c^2} - \frac{bex^2}{6c} - \frac{2}{9} aex^3 - \frac{2}{9} bex^3 \coth^{-1}(cx) - \frac{be \log^2(1 - c^2 x^2)}{12c^3} + \frac{bx^2 (d + e \log(1 - c^2 x^2))}{6c} \\
&= -\frac{2aex}{3c^2} - \frac{bex^2}{6c} - \frac{2}{9} aex^3 - \frac{2bex \coth^{-1}(cx)}{3c^2} - \frac{2}{9} bex^3 \coth^{-1}(cx) + \frac{bx^2 (d + e \log(1 - c^2 x^2))}{6c} \\
&= -\frac{2aex}{3c^2} - \frac{bex^2}{6c} - \frac{2}{9} aex^3 - \frac{2bex \coth^{-1}(cx)}{3c^2} - \frac{2}{9} bex^3 \coth^{-1}(cx) + \frac{bx^2 (d + e \log(1 - c^2 x^2))}{6c} \\
&= -\frac{2aex}{3c^2} - \frac{5bex^2}{18c} - \frac{2}{9} aex^3 - \frac{2bex \coth^{-1}(cx)}{3c^2} - \frac{2}{9} bex^3 \coth^{-1}(cx) + \frac{bx^2 (d + e \log(1 - c^2 x^2))}{6c}
\end{aligned}$$

**Mathematica [A]** time = 0.123003, size = 183, normalized size = 0.74

$$\frac{6c^2 e x^2 \log(1 - c^2 x^2) (2acx + 2bcx \coth^{-1}(cx) + b) + 2 \log(1 - cx) (-6ae + 3bd - 11be) + 2 \log(cx + 1) (6ae + 3bd - 11be)}{1}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]),x]

[Out] (-24\*a\*c\*e\*x + 2\*b\*c^2\*(3\*d - 5\*e)\*x^2 + 4\*a\*c^3\*(3\*d - 2\*e)\*x^3 + 4\*b\*c\*x\*(3\*c^2\*d\*x^2 - 2\*e\*(3 + c^2\*x^2))\*ArcCoth[c\*x] + 12\*b\*e\*ArcCoth[c\*x]^2 + 2\*(3\*b\*d - 6\*a\*e - 11\*b\*e)\*Log[1 - c\*x] + 2\*(3\*b\*d + 6\*a\*e - 11\*b\*e)\*Log[1 + c\*x] + 6\*c^2\*e\*x^2\*(b + 2\*a\*c\*x + 2\*b\*c\*x\*ArcCoth[c\*x])\*Log[1 - c^2\*x^2] +

$$3*b*e*\text{Log}[1 - c^2*x^2]^2)/(36*c^3)$$

**Maple [C]** time = 1.083, size = 3515, normalized size = 14.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^2*(a+b*\text{arccoth}(c*x))*(d+e*\ln(-c^2*x^2+1)), x)$

[Out] 
$$\begin{aligned} & -2/3*b*e*x*\text{arccoth}(c*x)/c^2-2/3*a*e*x/c^2-5/18*b*e*x^2/c^2-9*b*e*x^3*\text{arccot} \\ & h(c*x)+1/3*a*e*x^3*\ln(-c^2*x^2+1)-1/3*a*e/c^3*\ln(c*x-1)+1/3*a*e/c^3*\ln(c*x+ \\ & 1)+1/6/c*b*x^2*d+1/3/c^3*b*e*\ln((c*x+1)/(c*x-1)-1)^2-1/3/c^3*b*\ln((c*x+1)/( \\ & c*x-1)-1)*d+11/9/c^3*b*\ln((c*x+1)/(c*x-1)-1)*e+1/3/c^3*b*\text{arccoth}(c*x)*d-8/9 \\ & /c^3*b*e*\text{arccoth}(c*x)+1/3*b*\text{arccoth}(c*x)*x^3*d-1/6/c^3*b*d+5/18/c^3*b*e-2/9 \\ & *a*e*x^3+1/3*x^3*a*d-1/3/c^3*b*e*\ln(2)+1/3*I/c^3*b*e*\ln((c*x+1)/(c*x-1)-1)* \\ & \text{Pi}*c\text{sgn}(I*((c*x+1)/(c*x-1)-1)^2)^2*c\text{sgn}(I*((c*x+1)/(c*x-1)-1))-1/6*I/c^3*b* \\ & \text{Pi}*c\text{sgn}(I*((c*x+1)/(c*x-1)-1)^2)^2*c\text{sgn}(I*((c*x+1)/(c*x-1)-1))^2-1/6*I/c^3*b*\text{arccoth}(c*x)*e*\text{Pi}*c\text{sgn}(I/((c*x-1)/(c*x+1))^(1/2))^2 \\ & *c\text{sgn}(I*(c*x+1)/(c*x-1))+1/3*I/c^3*b*\text{arccoth}(c*x)*e*\text{Pi}*c\text{sgn}(I/((c*x-1)/(c*x \\ & +1))^(1/2))^2*c\text{sgn}(I*(c*x+1)/(c*x-1))^2+1/6*I/c^3*b*\text{arccoth}(c*x)*e*\text{Pi}*c\text{sgn}(I* \\ & (c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*c\text{sgn}(I*(c*x+1)/(c*x-1))+1/6*I/c^3*b \\ & *\text{arccoth}(c*x)*e*\text{Pi}*c\text{sgn}(I/((c*x+1)/(c*x-1)-1)^2)*c\text{sgn}(I*(c*x+1)/(c*x-1)/(( \\ & c*x+1)/(c*x-1)-1)^2)^2-1/3*I/c^3*b*\text{arccoth}(c*x)*e*\text{Pi}*c\text{sgn}(I*((c*x+1)/(c*x-1 \\ & )-1)^2)^2*c\text{sgn}(I*((c*x+1)/(c*x-1)-1))+1/6*I/c^3*b*\text{arccoth}(c*x)*\text{Pi}*e*c\text{sgn}(I* \\ & ((c*x+1)/(c*x-1)-1)^2)*c\text{sgn}(I*((c*x+1)/(c*x-1)-1))^2+1/12*I/c^3*b*e*\text{Pi}*c\text{sgn} \\ & (I/((c*x+1)/(c*x-1)-1)^2)*c\text{sgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*c\text{sg} \\ & n(I*(c*x+1)/(c*x-1))+1/6*I*b*\text{arccoth}(c*x)*\text{Pi}*c\text{sgn}(I*((c*x+1)/(c*x-1)-1)^2)* \\ & c\text{sgn}(I*((c*x+1)/(c*x-1)-1))^2*x^3*e-1/12*I/c*b*\text{Pi}*c\text{sgn}(I/((c*x-1)/(c*x+1))^( \\ & 1/2))^2*c\text{sgn}(I*(c*x+1)/(c*x-1))*x^2*e-1/6*I/c^3*b*\text{Pi}*e-1/6*I*b*\text{arccoth}(c*x \\ & )*\text{Pi}*c\text{sgn}(I*(c*x+1)/(c*x-1))*c\text{sgn}(I/((c*x+1)/(c*x-1)-1)^2)*c\text{sgn}(I*(c*x+1)/( \\ & c*x-1)/((c*x+1)/(c*x-1)-1)^2)*x^3*e-1/12*I/c*b*\text{Pi}*c\text{sgn}(I*(c*x+1)/(c*x-1))*c \\ & \text{sgn}(I/((c*x+1)/(c*x-1)-1)^2)*c\text{sgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)* \\ & x^2*e+1/6*I/c^3*b*c\text{sgn}(I*(c*x+1)/(c*x-1))*c\text{sgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/( \\ & c*x-1)-1)^2)*c\text{sgn}(I/((c*x+1)/(c*x-1)-1)^2)*e*\ln((c*x+1)/(c*x-1)-1)*\text{Pi}-1/6*I \\ & /c^3*b*\text{arccoth}(c*x)*e*\text{Pi}*c\text{sgn}(I/((c*x+1)/(c*x-1)-1)^2)*c\text{sgn}(I*(c*x+1)/(c*x- \\ & 1)/((c*x+1)/(c*x-1)-1)^2)*c\text{sgn}(I*(c*x+1)/(c*x-1))-1/3/c*b*\ln((c*x+1)/(c*x-1 \\ & )-1)*x^2*e+1/3/c*b*\ln(2)*x^2*e-2/3/c^3*b*\ln(2)*\ln((c*x+1)/(c*x-1)-1)*e+2/3/ \\ & c^3*b*\text{arccoth}(c*x)*\ln(2)*e-2/3*b*\ln((c*x+1)/(c*x-1)-1)*\text{arccoth}(c*x)*x^3*e+2 \\ & /3*b*\text{arccoth}(c*x)*\ln(2)*x^3*e+1/6/c^3*b*e*(-2*\text{arccoth}(c*x)*x^3*c^3-c^2*x^2+ \\ & 2*\ln((c*x+1)/(c*x-1)-1)-2*\text{arccoth}(c*x)+1)*\ln((c*x-1)/(c*x+1))+1/6*I/c*b*\text{Pi}* \\ & c\text{sgn}(I/((c*x-1)/(c*x+1))^(1/2))^2*c\text{sgn}(I*(c*x+1)/(c*x-1))^2*x^2*e+1/12*I/c*b* \end{aligned}$$

$$\begin{aligned}
& \text{Pi} * \text{csgn}(I * (c * x + 1) / (c * x - 1)) * \text{csgn}(I * (c * x + 1) / (c * x - 1) / ((c * x + 1) / (c * x - 1) - 1)^2)^2 * \\
& x^{2 * e + 1} / 12 * I / c * b * \text{Pi} * \text{csgn}(I / ((c * x + 1) / (c * x - 1) - 1)^2) * \text{csgn}(I * (c * x + 1) / (c * x - 1) / (( \\
& c * x + 1) / (c * x - 1) - 1)^2)^2 * x^{2 * e - 1} / 6 * I / c * b * \text{Pi} * \text{csgn}(I * ((c * x + 1) / (c * x - 1) - 1)^2)^2 * c \\
& \text{sgn}(I * ((c * x + 1) / (c * x - 1) - 1)) * x^{2 * e + 1} / 12 * I / c * b * \text{Pi} * \text{csgn}(I * ((c * x + 1) / (c * x - 1) - 1)^2) \\
& ) * \text{csgn}(I * ((c * x + 1) / (c * x - 1) - 1))^2 * x^{2 * e - 1} / 6 * I * b * \text{arccoth}(c * x) * \text{Pi} * \text{csgn}(I / ((c * x - \\
& 1) / (c * x + 1))^{(1/2)})^2 * \text{csgn}(I * (c * x + 1) / (c * x - 1)) * x^{3 * e + 1} / 3 * I * b * \text{arccoth}(c * x) * \text{Pi} * \\
& \text{csgn}(I / ((c * x - 1) / (c * x + 1))^{(1/2)}) * \text{csgn}(I * (c * x + 1) / (c * x - 1))^2 * x^{3 * e + 1} / 6 * I * b * \text{arc} \\
& \text{coth}(c * x) * \text{Pi} * \text{csgn}(I * (c * x + 1) / (c * x - 1)) * \text{csgn}(I * (c * x + 1) / (c * x - 1) / ((c * x + 1) / (c * x - 1) \\
& ) - 1)^2)^2 * x^{3 * e + 1} / 6 * I * b * \text{arccoth}(c * x) * \text{Pi} * \text{csgn}(I / ((c * x + 1) / (c * x - 1) - 1)^2) * \text{csgn}( \\
& I * (c * x + 1) / (c * x - 1) / ((c * x + 1) / (c * x - 1) - 1)^2)^2 * x^{3 * e - 1} / 3 * I * b * \text{arccoth}(c * x) * \text{Pi} * \text{csgn} \\
& (I * ((c * x + 1) / (c * x - 1) - 1)^2)^2 * \text{csgn}(I * ((c * x + 1) / (c * x - 1) - 1)) * x^{3 * e + 1} / 6 * I / c^3 * b \\
& * e * \ln((c * x + 1) / (c * x - 1) - 1) * \text{Pi} * \text{csgn}(I / ((c * x - 1) / (c * x + 1))^{(1/2)})^2 * \text{csgn}(I * (c * x + 1 \\
& ) / (c * x - 1)) - 1 / 3 * I / c^3 * b * e * \ln((c * x + 1) / (c * x - 1) - 1) * \text{Pi} * \text{csgn}(I / ((c * x - 1) / (c * x + 1))^{( \\
& 1/2)}) * \text{csgn}(I * (c * x + 1) / (c * x - 1))^2 - 1 / 6 * I / c^3 * b * e * \ln((c * x + 1) / (c * x - 1) - 1) * \text{Pi} * \text{csgn} \\
& (I * (c * x + 1) / (c * x - 1) / ((c * x + 1) / (c * x - 1) - 1)^2)^2 * \text{csgn}(I * (c * x + 1) / (c * x - 1)) - 1 / 6 * I / \\
& c^3 * b * \text{csgn}(I / ((c * x + 1) / (c * x - 1) - 1)^2) * e * \ln((c * x + 1) / (c * x - 1) - 1) * \text{Pi} * \text{csgn}(I * (c * x + \\
& 1) / (c * x - 1) / ((c * x + 1) / (c * x - 1) - 1)^2)^2 + 1 / 6 * I / c^3 * b * \text{arccoth}(c * x) * e * \text{Pi} * \text{csgn}(I * (c \\
& * x + 1) / (c * x - 1) / ((c * x + 1) / (c * x - 1) - 1)^2)^3 + 1 / 6 * I / c^3 * b * \text{arccoth}(c * x) * \text{Pi} * e * \text{csgn}(I \\
& * ((c * x + 1) / (c * x - 1) - 1)^2)^3 + 1 / 3 * I / c^3 * b * e * \ln((c * x + 1) / (c * x - 1) - 1) * \text{Pi} * \text{csgn}(I * (c * \\
& x + 1) / (c * x - 1) / ((c * x + 1) / (c * x - 1) - 1)^2)^2 - 1 / 3 * I / c^3 * b * \text{arccoth}(c * x) * e * \text{Pi} * \text{csgn}(I * \\
& (c * x + 1) / (c * x - 1) / ((c * x + 1) / (c * x - 1) - 1)^2)^2 + 1 / 12 * I / c^3 * b * \text{Pi} * e * \text{csgn}(I / ((c * x - 1) / \\
& (c * x + 1))^{(1/2)})^2 * \text{csgn}(I * (c * x + 1) / (c * x - 1)) - 1 / 6 * I / c^3 * b * \text{Pi} * e * \text{csgn}(I / ((c * x - 1) / \\
& (c * x + 1))^{(1/2)}) * \text{csgn}(I * (c * x + 1) / (c * x - 1))^2 - 1 / 12 * I / c^3 * b * \text{Pi} * e * \text{csgn}(I * (c * x + 1) / \\
& (c * x - 1) / ((c * x + 1) / (c * x - 1) - 1)^2)^2 * \text{csgn}(I * (c * x + 1) / (c * x - 1)) - 1 / 6 * I * b * \text{arccoth}(c * \\
& x) * \text{Pi} * \text{csgn}(I * (c * x + 1) / (c * x - 1))^3 * x^{3 * e + 1} / 6 * I * b * \text{arccoth}(c * x) * \text{Pi} * \text{csgn}(I * (c * x + 1 \\
& ) / (c * x - 1) / ((c * x + 1) / (c * x - 1) - 1)^2)^3 * x^{3 * e + 1} / 6 * I * b * \text{arccoth}(c * x) * \text{Pi} * \text{csgn}(I * ((c \\
& * x + 1) / (c * x - 1) - 1)^2)^3 * x^{3 * e - 1} / 3 * I * b * \text{arccoth}(c * x) * \text{Pi} * \text{csgn}(I * (c * x + 1) / (c * x - 1) / \\
& ((c * x + 1) / (c * x - 1) - 1)^2)^2 * x^{3 * e - 1} / 12 * I / c^3 * b * e * \text{Pi} * \text{csgn}(I / ((c * x + 1) / (c * x - 1) - 1) \\
& ^2) * \text{csgn}(I * (c * x + 1) / (c * x - 1) / ((c * x + 1) / (c * x - 1) - 1)^2)^2 + 1 / 6 * I / c^3 * b * \text{Pi} * e * \text{csgn}(I \\
& * ((c * x + 1) / (c * x - 1) - 1)^2)^2 * \text{csgn}(I * ((c * x + 1) / (c * x - 1) - 1)) - 1 / 12 * I / c^3 * b * \text{Pi} * e * \text{csgn} \\
& (I * ((c * x + 1) / (c * x - 1) - 1)^2) * \text{csgn}(I * ((c * x + 1) / (c * x - 1) - 1))^2 - 1 / 12 * I / c * b * \text{Pi} * \text{csgn} \\
& (I * (c * x + 1) / (c * x - 1))^3 * x^{2 * e + 1} / 12 * I / c * b * \text{Pi} * \text{csgn}(I * (c * x + 1) / (c * x - 1) / ((c * x + 1) / \\
& c * x - 1) - 1)^2)^3 * x^{2 * e + 1} / 12 * I / c * b * \text{Pi} * \text{csgn}(I * ((c * x + 1) / (c * x - 1) - 1)^2)^3 * x^{2 * e - 1} / \\
& 6 * I / c * b * \text{Pi} * \text{csgn}(I * (c * x + 1) / (c * x - 1) / ((c * x + 1) / (c * x - 1) - 1)^2)^2 * x^{2 * e + 1} / 6 * I / c^3 * \\
& b * e * \ln((c * x + 1) / (c * x - 1) - 1) * \text{Pi} * \text{csgn}(I * (c * x + 1) / (c * x - 1))^3 - 1 / 6 * I / c^3 * b * e * \ln((c * \\
& x + 1) / (c * x - 1) - 1) * \text{Pi} * \text{csgn}(I * (c * x + 1) / (c * x - 1) / ((c * x + 1) / (c * x - 1) - 1)^2)^3 - 1 / 6 * I / c^ \\
& 3 * b * e * \ln((c * x + 1) / (c * x - 1) - 1) * \text{Pi} * \text{csgn}(I * ((c * x + 1) / (c * x - 1) - 1)^2)^3 - 1 / 6 * I / c^3 * b * \\
& \text{arccoth}(c * x) * e * \text{Pi} * \text{csgn}(I * (c * x + 1) / (c * x - 1))^3 + 1 / 3 * I * b * \text{arccoth}(c * x) * \text{Pi} * x^{3 * e + 1} \\
& / 6 * I / c * b * \text{Pi} * x^{2 * e + 1} / 12 * I / c^3 * b * \text{Pi} * e * \text{csgn}(I * (c * x + 1) / (c * x - 1))^3 - 1 / 12 * I / c^3 * b * \\
& \text{Pi} * e * \text{csgn}(I * (c * x + 1) / (c * x - 1) / ((c * x + 1) / (c * x - 1) - 1)^2)^3 - 1 / 12 * I / c^3 * b * \text{Pi} * e * \text{csgn} \\
& (I * ((c * x + 1) / (c * x - 1) - 1)^2)^3 + 1 / 6 * I / c^3 * b * \text{Pi} * e * \text{csgn}(I * (c * x + 1) / (c * x - 1) / ((c * x + 1 \\
& ) / (c * x - 1) - 1)^2)^2 - 1 / 3 * I / c^3 * b * e * \ln((c * x + 1) / (c * x - 1) - 1) * \text{Pi} + 1 / 3 * I / c^3 * b * \text{arccot} \\
& h(c * x) * \text{Pi} * e
\end{aligned}$$

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**Maxima [C]** time = 1.19273, size = 340, normalized size = 1.38

$$\frac{1}{3} adx^3 + \frac{1}{9} \left( 3x^3 \log(-c^2x^2 + 1) - c^2 \left( \frac{2(c^2x^3 + 3x)}{c^4} - \frac{3 \log(cx + 1)}{c^5} + \frac{3 \log(cx - 1)}{c^5} \right) \right) be \operatorname{arccoth}(cx) + \frac{1}{6} \left( 2x^3 \operatorname{arccoth}(cx) + \frac{1}{3} \left( 3x^3 \log(-c^2x^2 + 1) - c^2 \left( \frac{2(c^2x^3 + 3x)}{c^4} - \frac{3 \log(cx + 1)}{c^5} + \frac{3 \log(cx - 1)}{c^5} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1)),x, algorithm="maxima")

[Out] 1/3\*a\*d\*x^3 + 1/9\*(3\*x^3\*log(-c^2\*x^2 + 1) - c^2\*(2\*(c^2\*x^3 + 3\*x)/c^4 - 3\*log(c\*x + 1)/c^5 + 3\*log(c\*x - 1)/c^5))\*b\*e\*arccoth(c\*x) + 1/6\*(2\*x^3\*arccoth(c\*x) + c\*(x^2/c^2 + log(c^2\*x^2 - 1)/c^4))\*b\*d + 1/9\*(3\*x^3\*log(-c^2\*x^2 + 1) - c^2\*(2\*(c^2\*x^3 + 3\*x)/c^4 - 3\*log(c\*x + 1)/c^5 + 3\*log(c\*x - 1)/c^5))\*a\*e + 1/18\*((3\*I\*pi\*c^2 - 5\*c^2)\*x^2 + (3\*I\*pi + 3\*c^2\*x^2 + 6\*log(c\*x - 1) - 11)\*log(c\*x + 1) + (3\*I\*pi + 3\*c^2\*x^2 - 11)\*log(c\*x - 1))\*b\*e/c^3

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**Fricas [A]** time = 1.72752, size = 452, normalized size = 1.83

$$\frac{24acex - 4(3ac^3d - 2ac^3e)x^3 - 3be \log(-c^2x^2 + 1)^2 - 3be \log\left(\frac{cx+1}{cx-1}\right)^2 - 2(3bc^2d - 5bc^2e)x^2 - 2(6ac^3ex^3 + 3bc^2ex^2 - 3ac^3d - 2ac^3e)x - 3be \log(-c^2x^2 + 1) - 3be \log\left(\frac{cx+1}{cx-1}\right)}{36c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1)),x, algorithm="fricas")

[Out] -1/36\*(24\*a\*c\*e\*x - 4\*(3\*a\*c^3\*d - 2\*a\*c^3\*e)\*x^3 - 3\*b\*e\*log(-c^2\*x^2 + 1)^2 - 3\*b\*e\*log((c\*x + 1)/(c\*x - 1))^2 - 2\*(3\*b\*c^2\*d - 5\*b\*c^2\*e)\*x^2 - 2\*(6\*a\*c^3\*e\*x^3 + 3\*b\*c^2\*e\*x^2 + 3\*b\*d - 11\*b\*e)\*log(-c^2\*x^2 + 1) - 2\*(3\*b\*c^3\*e\*x^3\*log(-c^2\*x^2 + 1) - 6\*b\*c\*e\*x + (3\*b\*c^3\*d - 2\*b\*c^3\*e)\*x^3 + 6\*a\*e)\*log((c\*x + 1)/(c\*x - 1)))/c^3

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**Sympy [A]** time = 15.7038, size = 265, normalized size = 1.07

$$\frac{\left\{ \begin{array}{l} \frac{adx^3}{3} + \frac{aex^3 \log(-c^2x^2+1)}{3} - \frac{2aex^3}{9} - \frac{2aex}{3c^2} + \frac{2ae \operatorname{acoth}(cx)}{3c^3} + \frac{bdx^3 \operatorname{acoth}(cx)}{3} + \frac{bex^3 \log(-c^2x^2+1) \operatorname{acoth}(cx)}{3} - \frac{2bex^3 \operatorname{acoth}(cx)}{9} + \frac{bdx^2}{6c} + \frac{be}{3} \\ \frac{dx^3 \left(a + \frac{ib}{2}\right)}{3} \end{array} \right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**3/3 + a*e*x**3*log(-c**2*x**2 + 1)/3 - 2*a*e*x**3/9 - 2*a
*e*x/(3*c**2) + 2*a*e*acoth(c*x)/(3*c**3) + b*d*x**3*acoth(c*x)/3 + b*e*x**
3*log(-c**2*x**2 + 1)*acoth(c*x)/3 - 2*b*e*x**3*acoth(c*x)/9 + b*d*x**2/(6*
c) + b*e*x**2*log(-c**2*x**2 + 1)/(6*c) - 5*b*e*x**2/(18*c) - 2*b*e*x*acoth
(c*x)/(3*c**2) + b*d*log(-c**2*x**2 + 1)/(6*c**3) + b*e*log(-c**2*x**2 + 1)
**2/(12*c**3) - 11*b*e*log(-c**2*x**2 + 1)/(18*c**3) + b*e*acoth(c*x)**2/(3
*c**3), Ne(c, 0)), (d*x**3*(a + I*pi*b/2)/3, True))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arccoth}(cx) + a)(e \log(-c^2x^2 + 1) + d)x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)*x^2, x)
```

### 3.274 $\int (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2x^2)) dx$

**Optimal.** Leaf size=104

$$x(a + b \coth^{-1}(cx))(e \log(1 - c^2x^2) + d) + \frac{e(a + b \coth^{-1}(cx))^2}{bc} - 2aex + \frac{b(e \log(1 - c^2x^2) + d)^2}{4ce} - \frac{be \log(1 - c^2x^2)}{c}$$

[Out]  $-2*a*e*x - 2*b*e*x*ArcCoth[c*x] + (e*(a + b*ArcCoth[c*x])^2)/(b*c) - (b*e*Log[1 - c^2*x^2])/c + x*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]) + (b*(d + e*Log[1 - c^2*x^2])^2)/(4*c*e)$

**Rubi [A]** time = 0.198226, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6074, 2475, 2390, 2301, 5981, 5911, 260, 5949}

$$x(a + b \coth^{-1}(cx))(e \log(1 - c^2x^2) + d) + \frac{e(a + b \coth^{-1}(cx))^2}{bc} - 2aex + \frac{b(e \log(1 - c^2x^2) + d)^2}{4ce} - \frac{be \log(1 - c^2x^2)}{c}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]), x]$

[Out]  $-2*a*e*x - 2*b*e*x*ArcCoth[c*x] + (e*(a + b*ArcCoth[c*x])^2)/(b*c) - (b*e*Log[1 - c^2*x^2])/c + x*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]) + (b*(d + e*Log[1 - c^2*x^2])^2)/(4*c*e)$

#### Rule 6074

$\text{Int}[(a + ArcCoth[(c_*)(x_)]*(b_))*((d_.) + Log[(f_.) + (g_*)(x_)^2])*(e_.), x\_Symbol] \rightarrow \text{Simp}[x*(d + e*Log[f + g*x^2])*(a + b*ArcCoth[c*x]), x] + (-\text{Dist}[b*c, \text{Int}[(x*(d + e*Log[f + g*x^2]))/(1 - c^2*x^2), x], x] - \text{Dist}[2*e*g, \text{Int}[(x^2*(a + b*ArcCoth[c*x]))/(f + g*x^2), x], x]) /; \text{FreeQ}\{a, b, c, d, e, f, g\}, x]$

#### Rule 2475

$\text{Int}[(a + Log[(c_)*((d_.) + (e_*)(x_)^(n_))^(p_.)]*(b_))^(q_.)*(x_)^(m_)*((f_.) + (g_*)(x_)^(s_))^(r_.), x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^*(\text{Simplify}[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, n, p, q, r, s\}, x \&\& \text{IntegerQ}[r] \&\& \text{IntegerQ}[s/n] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]] \&\& (\text{GtQ}[(m + 1)/n, 0])$

|| IGtQ[q, 0])

### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

### Rule 5981

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((f\_.)\*(x\_)^(m\_))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Dist[f^2/e, Int[(f\*x)^(m - 2)\*(a + b\*ArcCoth[c\*x])^p, x], x] - Dist[(d\*f^2)/e, Int[((f\*x)^(m - 2)\*(a + b\*ArcCoth[c\*x])^p)/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]

### Rule 5911

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.), x\_Symbol] := Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

### Rule 260

Int[(x\_)^(m\_)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rule 5949

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

### Rubi steps



$$\begin{aligned}
\int (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2x^2)) dx &= x(a + b \coth^{-1}(cx)) (d + e \log(1 - c^2x^2)) - (bc) \int \frac{x(d + e \log(1 - c^2x^2))}{1 - c^2x^2} dx \\
&= x(a + b \coth^{-1}(cx)) (d + e \log(1 - c^2x^2)) - \frac{1}{2}(bc) \text{Subst} \left( \int \frac{d + e \log u}{1 - u} du \right) \\
&= -2aex + \frac{e(a + b \coth^{-1}(cx))^2}{bc} + x(a + b \coth^{-1}(cx)) (d + e \log(1 - c^2x^2)) \\
&= -2aex - 2bex \coth^{-1}(cx) + \frac{e(a + b \coth^{-1}(cx))^2}{bc} + x(a + b \coth^{-1}(cx)) (d + e \log(1 - c^2x^2)) \\
&= -2aex - 2bex \coth^{-1}(cx) + \frac{e(a + b \coth^{-1}(cx))^2}{bc} - \frac{be \log(1 - c^2x^2)}{c} + bex \log(1 - c^2x^2)
\end{aligned}$$

**Mathematica [A]** time = 0.0166919, size = 144, normalized size = 1.38

$$aex \log(1 - c^2x^2) + \frac{2ae \tanh^{-1}(cx)}{c} + adx - 2aex + \frac{bd \log(1 - c^2x^2)}{2c} + \frac{be \log^2(1 - c^2x^2)}{4c} - \frac{be \log(1 - c^2x^2)}{c} + bex \log(1 - c^2x^2)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]), x]

[Out] a\*d\*x - 2\*a\*e\*x + b\*d\*x\*ArcCoth[c\*x] - 2\*b\*e\*x\*ArcCoth[c\*x] + (b\*e\*ArcCoth[c\*x]^2)/c + (2\*a\*e\*ArcTanh[c\*x])/c + (b\*d\*Log[1 - c^2\*x^2])/(2\*c) - (b\*e\*Log[1 - c^2\*x^2])/c + a\*e\*x\*Log[1 - c^2\*x^2] + b\*e\*x\*ArcCoth[c\*x]\*Log[1 - c^2\*x^2] + (b\*e\*Log[1 - c^2\*x^2]^2)/(4\*c)

**Maple [C]** time = 0.566, size = 2210, normalized size = 21.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth(c\*x))\*(d+e\*ln(-c^2\*x^2+1)), x)

[Out] 1/c\*b\*e\*ln((c\*x+1)/(c\*x-1)-1)^2-1/c\*b\*ln((c\*x+1)/(c\*x-1)-1)\*d+2/c\*b\*ln((c\*x+1)/(c\*x-1)-1)\*e+1/c\*b\*arccoth(c\*x)\*d-2/c\*b\*e\*arccoth(c\*x)+b\*arccoth(c\*x)\*x\*d+ln((c\*x-1)/(c\*x+1))\*(-arccoth(c\*x)\*x\*c+ln((c\*x+1)/(c\*x-1)-1)-arccoth(c\*x

$$\begin{aligned}
&)) * b * e / c - 2 / c * b * \ln(2) * \ln((c * x + 1) / (c * x - 1) - 1) * e + 2 / c * b * \operatorname{arccoth}(c * x) * e * \ln(2) - 2 * b \\
&* \ln((c * x + 1) / (c * x - 1) - 1) * \operatorname{arccoth}(c * x) * x * e + 2 * b * \operatorname{arccoth}(c * x) * \ln(2) * x * e - 2 * a * x * e - \\
&2 * b * e * x * \operatorname{arccoth}(c * x) + a * d * x + a * e * x * \ln(-c^2 * x^2 + 1) - a * e / c * \ln(c * x - 1) + a * e / c * \ln(c * \\
&x + 1) + 1 / 2 * I / c * b * \pi * \ln((c * x + 1) / (c * x - 1) - 1) * e * \operatorname{csgn}(I * (c * x + 1) / (c * x - 1))^{3 - 1 / 2} * I / c \\
&* b * \pi * \ln((c * x + 1) / (c * x - 1) - 1) * e * \operatorname{csgn}(I * ((c * x + 1) / (c * x - 1) - 1)^2)^{3 - 1 / 2} * I / c * b * \pi * \\
&\ln((c * x + 1) / (c * x - 1) - 1) * e * \operatorname{csgn}(I * (c * x + 1) / (c * x - 1) / ((c * x + 1) / (c * x - 1) - 1)^2)^{3 - 1 / 2} \\
&* I / c * b * \operatorname{arccoth}(c * x) * e * \pi * \operatorname{csgn}(I * (c * x + 1) / (c * x - 1))^{3 + 1 / 2} * I / c * b * \operatorname{arccoth}(c * x) * e \\
&* \pi * \operatorname{csgn}(I * ((c * x + 1) / (c * x - 1) - 1)^2)^{3 + 1 / 2} * I / c * b * \operatorname{arccoth}(c * x) * e * \pi * \operatorname{csgn}(I * (c * x \\
&+ 1) / (c * x - 1) / ((c * x + 1) / (c * x - 1) - 1)^2)^{3 - 1 / 2} * I / c * b * \operatorname{csgn}(I * (c * x + 1) / (c * x - 1) / ((c * x \\
&+ 1) / (c * x - 1) - 1)^2)^2 * \operatorname{csgn}(I / ((c * x + 1) / (c * x - 1) - 1)^2) * e * \ln((c * x + 1) / (c * x - 1) - 1) * \pi \\
&- 1 / 2 * I / c * b * \operatorname{arccoth}(c * x) * e * \pi * \operatorname{csgn}(I / ((c * x - 1) / (c * x + 1))^{(1 / 2)})^2 * \operatorname{csgn}(I * (c * x \\
&+ 1) / (c * x - 1)) + I / c * b * \operatorname{csgn}(I * (c * x + 1) / (c * x - 1) / ((c * x + 1) / (c * x - 1) - 1)^2)^2 * e * \ln((c * \\
&x + 1) / (c * x - 1) - 1) * \pi - I / c * b * \operatorname{csgn}(I * (c * x + 1) / (c * x - 1) / ((c * x + 1) / (c * x - 1) - 1)^2)^2 * \pi \\
&* e * \operatorname{arccoth}(c * x) - 1 / 2 * I * b * \pi * \operatorname{arccoth}(c * x) * \operatorname{csgn}(I * (c * x + 1) / (c * x - 1))^{3 * x * e + 1 / 2} * I \\
&* b * \pi * \operatorname{arccoth}(c * x) * \operatorname{csgn}(I * ((c * x + 1) / (c * x - 1) - 1)^2)^{3 * x * e + 1 / 2} * I * b * \pi * \operatorname{arccoth}(c * \\
&x) * \operatorname{csgn}(I * (c * x + 1) / (c * x - 1) / ((c * x + 1) / (c * x - 1) - 1)^2)^{3 * x * e - I * b * \pi * \operatorname{arccoth}(c * x) \\
&* \operatorname{csgn}(I * (c * x + 1) / (c * x - 1) / ((c * x + 1) / (c * x - 1) - 1)^2)^2 * x * e + 1 / 2 * I / c * b * \operatorname{csgn}(I * (c * x + \\
&1) / (c * x - 1)) * \operatorname{csgn}(I * (c * x + 1) / (c * x - 1) / ((c * x + 1) / (c * x - 1) - 1)^2) * \operatorname{csgn}(I / ((c * x + 1) / ( \\
&c * x - 1) - 1)^2) * e * \ln((c * x + 1) / (c * x - 1) - 1) * \pi - 1 / 2 * I / c * b * \operatorname{arccoth}(c * x) * e * \pi * \operatorname{csgn}(I / \\
&((c * x + 1) / (c * x - 1) - 1)^2) * \operatorname{csgn}(I * (c * x + 1) / (c * x - 1) / ((c * x + 1) / (c * x - 1) - 1)^2) * \operatorname{csgn}(I \\
&* (c * x + 1) / (c * x - 1)) - 1 / 2 * I * b * \pi * \operatorname{arccoth}(c * x) * \operatorname{csgn}(I * (c * x + 1) / (c * x - 1)) * \operatorname{csgn}(I / (( \\
&c * x + 1) / (c * x - 1) - 1)^2) * \operatorname{csgn}(I * (c * x + 1) / (c * x - 1) / ((c * x + 1) / (c * x - 1) - 1)^2) * x * e + I / c * \\
&b * \operatorname{arccoth}(c * x) * e * \pi * \operatorname{csgn}(I / ((c * x - 1) / (c * x + 1))^{(1 / 2)}) * \operatorname{csgn}(I * (c * x + 1) / (c * x - 1)) \\
&^2 + I * b * \pi * \operatorname{arccoth}(c * x) * \operatorname{csgn}(I / ((c * x - 1) / (c * x + 1))^{(1 / 2)}) * \operatorname{csgn}(I * (c * x + 1) / (c * x - \\
&1))^{2 * x * e + I / c * b * \operatorname{csgn}(I * ((c * x + 1) / (c * x - 1) - 1)) * \operatorname{csgn}(I * ((c * x + 1) / (c * x - 1) - 1)^2)^2 \\
&* e * \ln((c * x + 1) / (c * x - 1) - 1) * \pi + 1 / 2 * I / c * b * \operatorname{arccoth}(c * x) * e * \pi * \operatorname{csgn}(I * (c * x + 1) / (c * x \\
&- 1) / ((c * x + 1) / (c * x - 1) - 1)^2)^2 * \operatorname{csgn}(I * (c * x + 1) / (c * x - 1)) - I / c * b * \operatorname{arccoth}(c * x) * e * \pi \\
&* \operatorname{csgn}(I * ((c * x + 1) / (c * x - 1) - 1)^2)^2 * \operatorname{csgn}(I * ((c * x + 1) / (c * x - 1) - 1)) + 1 / 2 * I / c * b * \operatorname{arc} \\
&\operatorname{coth}(c * x) * e * \pi * \operatorname{csgn}(I * ((c * x + 1) / (c * x - 1) - 1)^2) * \operatorname{csgn}(I * ((c * x + 1) / (c * x - 1) - 1))^{2 + \\
&1 / 2 * I / c * b * \operatorname{arccoth}(c * x) * e * \pi * \operatorname{csgn}(I / ((c * x + 1) / (c * x - 1) - 1)^2) * \operatorname{csgn}(I * (c * x + 1) / (c \\
&* x - 1) / ((c * x + 1) / (c * x - 1) - 1)^2)^2 - 1 / 2 * I * b * \pi * \operatorname{arccoth}(c * x) * \operatorname{csgn}(I / ((c * x - 1) / (c * x \\
&+ 1))^{(1 / 2)})^2 * \operatorname{csgn}(I * (c * x + 1) / (c * x - 1)) * x * e + 1 / 2 * I * b * \pi * \operatorname{arccoth}(c * x) * \operatorname{csgn}(I * (c \\
&* x + 1) / (c * x - 1)) * \operatorname{csgn}(I * (c * x + 1) / (c * x - 1) / ((c * x + 1) / (c * x - 1) - 1)^2)^2 * x * e - I * b * \pi * a \\
&\operatorname{rccoth}(c * x) * \operatorname{csgn}(I * ((c * x + 1) / (c * x - 1) - 1)^2)^2 * \operatorname{csgn}(I * ((c * x + 1) / (c * x - 1) - 1)) * x * e \\
&+ 1 / 2 * I * b * \pi * \operatorname{arccoth}(c * x) * \operatorname{csgn}(I * ((c * x + 1) / (c * x - 1) - 1)^2) * \operatorname{csgn}(I * ((c * x + 1) / (c * x \\
&- 1) - 1))^{2 * x * e + 1 / 2 * I * b * \pi * \operatorname{arccoth}(c * x) * \operatorname{csgn}(I / ((c * x + 1) / (c * x - 1) - 1)^2) * \operatorname{csgn}(I * \\
&(c * x + 1) / (c * x - 1) / ((c * x + 1) / (c * x - 1) - 1)^2)^2 * x * e + 1 / 2 * I / c * b * \operatorname{csgn}(I * (c * x + 1) / (c * x - \\
&1)) * \operatorname{csgn}(I / ((c * x - 1) / (c * x + 1))^{(1 / 2)})^2 * e * \ln((c * x + 1) / (c * x - 1) - 1) * \pi - I / c * b * \operatorname{csgn} \\
&(I * (c * x + 1) / (c * x - 1))^{2 * \operatorname{csgn}(I / ((c * x - 1) / (c * x + 1))^{(1 / 2)})} * e * \ln((c * x + 1) / (c * x - 1) - \\
&1) * \pi - 1 / 2 * I / c * b * \operatorname{csgn}(I * (c * x + 1) / (c * x - 1)) * \operatorname{csgn}(I * (c * x + 1) / (c * x - 1) / ((c * x + 1) / (c * \\
&x - 1) - 1)^2)^2 * e * \ln((c * x + 1) / (c * x - 1) - 1) * \pi - 1 / 2 * I / c * b * \operatorname{csgn}(I * ((c * x + 1) / (c * x - 1) - 1 \\
&))^{2 * \operatorname{csgn}(I * ((c * x + 1) / (c * x - 1) - 1)^2) * e * \ln((c * x + 1) / (c * x - 1) - 1) * \pi + I / c * b * \pi * e * \operatorname{ar} \\
&\operatorname{ccoth}(c * x) + I * b * \pi * \operatorname{arccoth}(c * x) * x * e - I / c * b * e * \ln((c * x + 1) / (c * x - 1) - 1) * \pi
\end{aligned}$$

---

**Maxima [C]** time = 1.14086, size = 240, normalized size = 2.31

$$-\left(c^2\left(\frac{2x}{c^2} - \frac{\log(cx+1)}{c^3} + \frac{\log(cx-1)}{c^3}\right) - x \log(-c^2x^2+1)\right) be \operatorname{arccoth}(cx) - \left(c^2\left(\frac{2x}{c^2} - \frac{\log(cx+1)}{c^3} + \frac{\log(cx-1)}{c^3}\right) - x \log(-c^2x^2+1)\right) be \operatorname{arccoth}(cx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1)),x, algorithm="maxima")

[Out]  $-(c^2*(2*x/c^2 - \log(c*x + 1)/c^3 + \log(c*x - 1)/c^3) - x*\log(-c^2*x^2 + 1)) * b*e*\operatorname{arccoth}(c*x) - (c^2*(2*x/c^2 - \log(c*x + 1)/c^3 + \log(c*x - 1)/c^3) - x*\log(-c^2*x^2 + 1)) * a*e + a*d*x + 1/2*(2*c*x*\operatorname{arccoth}(c*x) + \log(-c^2*x^2 + 1)) * b*d/c + 1/2*((I*\pi + 2*\log(c*x - 1) - 2)*\log(c*x + 1) + (I*\pi - 2)*\log(c*x - 1)) * b*e/c$

---

**Fricas [A]** time = 1.69862, size = 304, normalized size = 2.92

$$\frac{be \log(-c^2x^2+1)^2 + be \log\left(\frac{cx+1}{cx-1}\right)^2 + 4(acd - 2ace)x + 2(2acex + bd - 2be) \log(-c^2x^2+1) + 2(bcex \log(-c^2x^2+1) + 2a*c*d - 2*a*c*e)*x + 2*(2*a*c*e*x + b*d - 2*b*e)*\log(-c^2*x^2 + 1) + 2*(b*c*e*x*\log(-c^2*x^2 + 1) + 2*a*e + (b*c*d - 2*b*c*e)*x)*\log((c*x + 1)/(c*x - 1))}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1)),x, algorithm="fricas")

[Out]  $1/4*(b*e*\log(-c^2*x^2 + 1)^2 + b*e*\log((c*x + 1)/(c*x - 1))^2 + 4*(a*c*d - 2*a*c*e)*x + 2*(2*a*c*e*x + b*d - 2*b*e)*\log(-c^2*x^2 + 1) + 2*(b*c*e*x*\log(-c^2*x^2 + 1) + 2*a*e + (b*c*d - 2*b*c*e)*x)*\log((c*x + 1)/(c*x - 1)))/c$

---

**Sympy [A]** time = 9.84375, size = 155, normalized size = 1.49

$$\left\{ \begin{array}{l} dx + aex \log(-c^2x^2+1) - 2aex + \frac{2ae \operatorname{arccoth}(cx)}{c} + bdx \operatorname{arccoth}(cx) + bex \log(-c^2x^2+1) \operatorname{arccoth}(cx) - 2bex \operatorname{arccoth}(cx) + \\ dx \left( a + \frac{i\pi b}{2} \right) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth(c\*x))\*(d+e\*ln(-c\*\*2\*x\*\*2+1)),x)

```
[Out] Piecewise((a*d*x + a*e*x*log(-c**2*x**2 + 1) - 2*a*e*x + 2*a*e*acoth(c*x)/c
+ b*d*x*acoth(c*x) + b*e*x*log(-c**2*x**2 + 1)*acoth(c*x) - 2*b*e*x*acoth(
c*x) + b*d*log(-c**2*x**2 + 1)/(2*c) + b*e*log(-c**2*x**2 + 1)**2/(4*c) - b
*e*log(-c**2*x**2 + 1)/c + b*e*acoth(c*x)**2/c, Ne(c, 0)), (d*x*(a + I*pi*b
/2), True))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arccoth}(cx) + a)(e \log(-c^2x^2 + 1) + d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d), x)
```

$$3.275 \quad \int \frac{(a+b \coth^{-1}(cx))(d+e \log(1-c^2x^2))}{x^2} dx$$

**Optimal.** Leaf size=105

$$-\frac{1}{2}bce \operatorname{PolyLog}\left(2, \frac{1}{1-c^2x^2}\right) - \frac{(a+b \coth^{-1}(cx))(e \log(1-c^2x^2)+d)}{x} - \frac{ce(a+b \coth^{-1}(cx))^2}{b} + \frac{1}{2}bc \log\left(1 - \frac{1}{1-c^2}\right)$$

[Out] -((c\*e\*(a + b\*ArcCoth[c\*x])^2)/b) - ((a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/x + (b\*c\*(d + e\*Log[1 - c^2\*x^2])\*Log[1 - (1 - c^2\*x^2)^(-1)])/2 - (b\*c\*e\*PolyLog[2, (1 - c^2\*x^2)^(-1)])/2

**Rubi [A]** time = 0.269076, antiderivative size = 94, normalized size of antiderivative = 0.9, number of steps used = 8, number of rules used = 8, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$ , Rules used = {6082, 2475, 2411, 2344, 2301, 2316, 2315, 5949}

$$-\frac{1}{2}bce \operatorname{PolyLog}\left(2, c^2x^2\right) - \frac{(a+b \coth^{-1}(cx))(e \log(1-c^2x^2)+d)}{x} - \frac{ce(a+b \coth^{-1}(cx))^2}{b} - \frac{bc(e \log(1-c^2x^2)+d)}{4e}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/x^2, x]

[Out] -((c\*e\*(a + b\*ArcCoth[c\*x])^2)/b) + b\*c\*d\*Log[x] - ((a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/x - (b\*c\*(d + e\*Log[1 - c^2\*x^2])^2)/(4\*e) - (b\*c\*e\*PolyLog[2, c^2\*x^2])/2

### Rule 6082

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))\*((d\_.) + Log[(f\_.) + (g\_.)\*(x\_.)^2]\* (e\_.)\*(x\_.)^(m\_.), x\_Symbol] :> Simp[(x^(m + 1)\*(d + e\*Log[f + g\*x^2])\*(a + b\*ArcCoth[c\*x]))/(m + 1), x] + (-Dist[(b\*c)/(m + 1), Int[(x^(m + 1)\*(d + e\*Log[f + g\*x^2]))/(1 - c^2\*x^2), x], x] - Dist[(2\*e\*g)/(m + 1), Int[(x^(m + 2)\*(a + b\*ArcCoth[c\*x]))/(f + g\*x^2), x], x]) /; FreeQ[{a, b, c, d, e, f, g}, x] && ILtQ[m/2, 0]

### Rule 2475

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.)^(n\_.))^(p\_.)]\*(b\_.))^(q\_.)\*(x\_.)^(m\_.)\*((f\_.) + (g\_.)\*(x\_.)^(s\_.))^(r\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(f + g\*x^(s/n))^r\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x

, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])

### Rule 2411

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.)\*((h\_.) + (i\_.)\*(x\_))^(r\_.), x\_Symbol] := Dist[1/e, Subst[Int[((g\*x)/e)^q\*((e\*h - d\*i)/e + (i\*x)/e)^r\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e\*f - d\*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2\*r]

### Rule 2344

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/((x\_)\*((d\_) + (e\_.)\*(x\_))), x\_Symbol] := Dist[1/d, Int[(a + b\*Log[c\*x^n])^p/x, x], x] - Dist[e/d, Int[(a + b\*Log[c\*x^n])^p/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0]

### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

### Rule 2316

Int[((a\_.) + Log[(c\_.)\*(x\_)]\*(b\_.))/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(a + b\*Log[-((c\*d)/e)])\*Log[d + e\*x])/e, x] + Dist[b, Int[Log[-((e\*x)/d)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c\*d)/e), 0]

### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

### Rule 5949

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{x^2} dx &= -\frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{x} + (bc) \int \frac{d + e \log(1 - c^2x^2)}{x(1 - c^2x^2)} \\
&= -\frac{ce(a + b \coth^{-1}(cx))^2}{b} - \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{x} + \\
&= -\frac{ce(a + b \coth^{-1}(cx))^2}{b} - \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{x} - \\
&= -\frac{ce(a + b \coth^{-1}(cx))^2}{b} - \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{x} - \\
&= -\frac{ce(a + b \coth^{-1}(cx))^2}{b} + bcd \log(x) - \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{x} \\
&= -\frac{ce(a + b \coth^{-1}(cx))^2}{b} + bcd \log(x) - \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{x}
\end{aligned}$$

**Mathematica [B]** time = 0.203594, size = 332, normalized size = 3.16

$$\frac{4bcex \operatorname{PolyLog}(2, -cx) + 4bcex \operatorname{PolyLog}(2, cx) - 2bcex \operatorname{PolyLog}\left(2, \frac{1}{2} - \frac{cx}{2}\right) - 2bcex \operatorname{PolyLog}\left(2, \frac{1}{2}(cx + 1)\right) + 4ae \log(x)}{4x}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/x^2,x]

[Out]  $-(4*a*d + 4*b*d*ArcCoth[c*x] + 4*b*c*e*x*ArcCoth[c*x]^2 + 8*a*c*e*x*ArcTanh[c*x] - 4*b*c*d*x*Log[x] - b*c*e*x*Log[-c^{(-1)} + x]^2 - b*c*e*x*Log[c^{(-1)} + x]^2 - 2*b*c*e*x*Log[c^{(-1)} + x]*Log[(1 - c*x)/2] + 4*b*c*e*x*Log[x]*Log[1 - c*x] - 2*b*c*e*x*Log[-c^{(-1)} + x]*Log[(1 + c*x)/2] + 4*b*c*e*x*Log[x]*Log[1 + c*x] + 4*a*e*Log[1 - c^2*x^2] + 2*b*c*d*x*Log[1 - c^2*x^2] + 4*b*e*ArcCoth[c*x]*Log[1 - c^2*x^2] - 4*b*c*e*x*Log[x]*Log[1 - c^2*x^2] + 2*b*c*e*x*Log[-c^{(-1)} + x]*Log[1 - c^2*x^2] + 2*b*c*e*x*Log[c^{(-1)} + x]*Log[1 - c^2*x^2] + 4*b*c*e*x*PolyLog[2, -(c*x)] + 4*b*c*e*x*PolyLog[2, c*x] - 2*b*c*e*x*PolyLog[2, 1/2 - (c*x)/2] - 2*b*c*e*x*PolyLog[2, (1 + c*x)/2])/(4*x)$

**Maple [F]** time = 1.642, size = 0, normalized size = 0.

$$\int \frac{(a + \operatorname{arccoth}(cx))(d + e \ln(-c^2x^2 + 1))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth(c\*x))\*(d+e\*ln(-c^2\*x^2+1))/x^2,x)

[Out] int((a+b\*arccoth(c\*x))\*(d+e\*ln(-c^2\*x^2+1))/x^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2} \left( c(\log(c^2x^2 - 1) - \log(x^2)) + \frac{2 \operatorname{arccoth}(cx)}{x} \right) bd - \left( c^2 \left( \frac{\log(cx + 1)}{c} - \frac{\log(cx - 1)}{c} \right) + \frac{\log(-c^2x^2 + 1)}{x} \right) ae - \frac{1}{2} be \left( \frac{\log}{x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1))/x^2,x, algorithm="maxima")

[Out] -1/2\*(c\*(log(c^2\*x^2 - 1) - log(x^2)) + 2\*arccoth(c\*x)/x)\*b\*d - (c^2\*(log(c\*x + 1)/c - log(c\*x - 1)/c) + log(-c^2\*x^2 + 1)/x)\*a\*e - 1/2\*b\*e\*(log(c\*x + 1)^2/x - integrate(-((c\*x + 1)\*log(c\*x - 1)^2 - (I\*pi + (I\*pi\*c + 2\*c)\*x)\*log(c\*x + 1) - (-I\*pi - I\*pi\*c\*x)\*log(c\*x - 1))/(c\*x^3 + x^2), x)) - a\*d/x

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{bd \operatorname{arccoth}(cx) + ad + (be \operatorname{arccoth}(cx) + ae) \log(-c^2x^2 + 1)}{x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1))/x^2,x, algorithm="fricas")

[Out] integral((b\*d\*arccoth(c\*x) + a\*d + (b\*e\*arccoth(c\*x) + a\*e)\*log(-c^2\*x^2 + 1))/x^2, x)



---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{acoth}(cx))(d + e \log(-c^2x^2 + 1))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth(c\*x))\*(d+e\*ln(-c\*\*2\*x\*\*2+1))/x\*\*2,x)

[Out] Integral((a + b\*acoth(c\*x))\*(d + e\*log(-c\*\*2\*x\*\*2 + 1))/x\*\*2, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccoth}(cx) + a)(e \log(-c^2x^2 + 1) + d)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1))/x^2,x, algorithm="giac")

[Out] integrate((b\*arccoth(c\*x) + a)\*(e\*log(-c^2\*x^2 + 1) + d)/x^2, x)

$$3.276 \quad \int \frac{(a+b \coth^{-1}(cx))(d+e \log(1-c^2x^2))}{x^4} dx$$

**Optimal.** Leaf size=197

$$-\frac{1}{6}bc^3e \operatorname{PolyLog}\left(2, \frac{1}{1-c^2x^2}\right) - \frac{(a+b \coth^{-1}(cx))(e \log(1-c^2x^2)+d)}{3x^3} - \frac{c^3e(a+b \coth^{-1}(cx))^2}{3b} + \frac{2c^2e(a+b \coth^{-1}(cx))}{3x}$$

[Out] (2\*c^2\*e\*(a + b\*ArcCoth[c\*x]))/(3\*x) - (c^3\*e\*(a + b\*ArcCoth[c\*x])^2)/(3\*b) - b\*c^3\*e\*Log[x] + (b\*c^3\*e\*Log[1 - c^2\*x^2])/3 - (b\*c\*(1 - c^2\*x^2)\*(d + e\*Log[1 - c^2\*x^2]))/(6\*x^2) - ((a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/(3\*x^3) + (b\*c^3\*(d + e\*Log[1 - c^2\*x^2])\*Log[1 - (1 - c^2\*x^2)^(-1)])/6 - (b\*c^3\*e\*PolyLog[2, (1 - c^2\*x^2)^(-1)])/6

**Rubi [A]** time = 0.463391, antiderivative size = 191, normalized size of antiderivative = 0.97, number of steps used = 17, number of rules used = 16, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.593$ , Rules used = {6082, 2475, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 5983, 5917, 266, 36, 29, 5949}

$$-\frac{1}{6}bc^3e \operatorname{PolyLog}\left(2, c^2x^2\right) - \frac{(a+b \coth^{-1}(cx))(e \log(1-c^2x^2)+d)}{3x^3} - \frac{c^3e(a+b \coth^{-1}(cx))^2}{3b} + \frac{2c^2e(a+b \coth^{-1}(cx))}{3x}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/x^4, x]

[Out] (2\*c^2\*e\*(a + b\*ArcCoth[c\*x]))/(3\*x) - (c^3\*e\*(a + b\*ArcCoth[c\*x])^2)/(3\*b) + (b\*c^3\*d\*Log[x])/3 - b\*c^3\*e\*Log[x] + (b\*c^3\*e\*Log[1 - c^2\*x^2])/3 - (b\*c\*(1 - c^2\*x^2)\*(d + e\*Log[1 - c^2\*x^2]))/(6\*x^2) - ((a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/(3\*x^3) - (b\*c^3\*(d + e\*Log[1 - c^2\*x^2])^2)/(12\*e) - (b\*c^3\*e\*PolyLog[2, c^2\*x^2])/6

### Rule 6082

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))\*((d\_.) + Log[(f\_.) + (g\_.)\*(x\_.)^2]\*(e\_.))\*(x\_.)^(m\_.), x\_Symbol] := Simp[(x^(m + 1)\*(d + e\*Log[f + g\*x^2])\*(a + b\*ArcCoth[c\*x]))/(m + 1), x] + (-Dist[(b\*c)/(m + 1), Int[(x^(m + 1)\*(d + e\*Log[f + g\*x^2]))/(1 - c^2\*x^2), x], x] - Dist[(2\*e\*g)/(m + 1), Int[(x^(m + 2)\*(a + b\*ArcCoth[c\*x]))/(f + g\*x^2), x], x]) /; FreeQ[{a, b, c, d, e, f, g}, x] && ILtQ[m/2, 0]

Rule 2475

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))^(q\_.)\*(x\_)^(m\_.)\*((f\_) + (g\_.)\*(x\_)^(s\_))^(r\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(f + g\*x^(s/n))^r\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])

Rule 2411

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_)^(q\_.))\*((h\_.) + (i\_.)\*(x\_)^(r\_.), x\_Symbol] := Dist[1/e, Subst[Int[((g\*x)/e)^q\*((e\*h - d\*i)/e + (i\*x)/e)^r\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e\*f - d\*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2\*r]

Rule 2347

Int[(((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_) + (e\_.)\*(x\_)^(q\_)))/(x\_), x\_Symbol] := Dist[1/d, Int[((d + e\*x)^(q + 1)\*(a + b\*Log[c\*x^n])^p)/x, x], x] - Dist[e/d, Int[(d + e\*x)^q\*(a + b\*Log[c\*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2\*q]

Rule 2344

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/((x\_)\*((d\_) + (e\_.)\*(x\_))), x\_Symbol] := Dist[1/d, Int[(a + b\*Log[c\*x^n])^p/x, x], x] - Dist[e/d, Int[(a + b\*Log[c\*x^n])^p/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0]

Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

Rule 2316

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := Simp[((a + b\*Log[-((c\*d)/e)])\*Log[d + e\*x])/e, x] + Dist[b, Int[Log[-((e\*x)/d)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c\*d)/e), 0]

Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 -

$c*x]/e, x] /; \text{FreeQ}\{c, d, e\}, x] \&\& \text{EqQ}[e + c*d, 0]$

### Rule 2314

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_)^(n_.)]*(b_.)]*((d_.) + (e_.)*(x_)^(r_.))^(q_.), x\_Symbol] \rightarrow \text{Simp}[(x*(d + e*x^r)^(q + 1)*(a + b*\text{Log}[c*x^n]))/d, x] - \text{Dist}[(b*n)/d, \text{Int}[(d + e*x^r)^(q + 1), x], x] /; \text{FreeQ}\{a, b, c, d, e, n, q, r\}, x] \&\& \text{EqQ}[r*(q + 1) + 1, 0]$

### Rule 31

$\text{Int}[(a_.) + (b_.)*(x_)^(-1), x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}\{a, b\}, x]$

### Rule 5983

$\text{Int}[(a_.) + \text{ArcCoth}[(c_.)*(x_)]*(b_.)]^(p_.)*((f_.)*(x_)^(m_.))/((d_.) + (e_.)*(x_)^2), x\_Symbol] \rightarrow \text{Dist}[1/d, \text{Int}[(f*x)^m*(a + b*\text{ArcCoth}[c*x])^p, x], x] - \text{Dist}[e/(d*f^2), \text{Int}[(f*x)^(m + 2)*(a + b*\text{ArcCoth}[c*x])^p]/(d + e*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{GtQ}[p, 0] \&\& \text{LtQ}[m, -1]$

### Rule 5917

$\text{Int}[(a_.) + \text{ArcCoth}[(c_.)*(x_)]*(b_.)]^(p_.)*((d_.)*(x_)^(m_.)), x\_Symbol] \rightarrow \text{Simp}[(d*x)^(m + 1)*(a + b*\text{ArcCoth}[c*x])^p/(d*(m + 1)), x] - \text{Dist}[(b*c*p)/(d*(m + 1)), \text{Int}[(d*x)^(m + 1)*(a + b*\text{ArcCoth}[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x] \&\& \text{IGtQ}[p, 0] \&\& (\text{EqQ}[p, 1] \mid \mid \text{IntegerQ}[m]) \&\& \text{NeQ}[m, -1]$

### Rule 266

$\text{Int}[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p}, x], x, x^n], x] /; \text{FreeQ}\{a, b, m, n, p\}, x] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

### Rule 36

$\text{Int}[1/((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_)), x\_Symbol] \rightarrow \text{Dist}[b/(b*c - a*d), \text{Int}[1/(a + b*x), x], x] - \text{Dist}[d/(b*c - a*d), \text{Int}[1/(c + d*x), x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0]$

### Rule 29

$\text{Int}[(x_)^(-1), x\_Symbol] \rightarrow \text{Simp}[\text{Log}[x], x]$

Rule 5949

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] :> Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{x^4} dx &= -\frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{3x^3} + \frac{1}{3}(bc) \int \frac{d + e \log(1 - c^2x^2)}{x^3(1 - c^2x^2)} \\
 &= -\frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{3x^3} + \frac{1}{6}(bc) \operatorname{Subst}\left(\int \frac{d + e \log(1 - c^2x^2)}{x^2(1 - cx)} dx, cx, x\right) \\
 &= \frac{2c^2e(a + b \coth^{-1}(cx))}{3x} - \frac{c^3e(a + b \coth^{-1}(cx))^2}{3b} - \frac{(a + b \coth^{-1}(cx))}{3x} \\
 &= \frac{2c^2e(a + b \coth^{-1}(cx))}{3x} - \frac{c^3e(a + b \coth^{-1}(cx))^2}{3b} - \frac{(a + b \coth^{-1}(cx))}{3x} \\
 &= \frac{2c^2e(a + b \coth^{-1}(cx))}{3x} - \frac{c^3e(a + b \coth^{-1}(cx))^2}{3b} - \frac{bc(1 - c^2x^2)(d + e \log(1 - c^2x^2))}{6x} \\
 &= \frac{2c^2e(a + b \coth^{-1}(cx))}{3x} - \frac{c^3e(a + b \coth^{-1}(cx))^2}{3b} + \frac{1}{3}bc^3d \log(x) - bc^3e \log(x) \\
 &= \frac{2c^2e(a + b \coth^{-1}(cx))}{3x} - \frac{c^3e(a + b \coth^{-1}(cx))^2}{3b} + \frac{1}{3}bc^3d \log(x) - bc^3e \log(x)
 \end{aligned}$$

**Mathematica [B]** time = 0.382553, size = 457, normalized size = 2.32

$$\frac{1}{6} \left( -2bc^3e \operatorname{PolyLog}(2, -cx) - 2bc^3e \operatorname{PolyLog}(2, cx) + bc^3e \operatorname{PolyLog}\left(2, \frac{1}{2} - \frac{cx}{2}\right) + bc^3e \operatorname{PolyLog}\left(2, \frac{1}{2}(cx + 1)\right) - \frac{2ae \log(1 - c^2x^2)}{3x} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/x^4, x]

```
[Out] ((-2*a*d)/x^3 - (b*c*d)/x^2 + (4*a*c^2*e)/x - (2*b*d*ArcCoth[c*x])/x^3 + (4
*b*c^2*e*ArcCoth[c*x])/x - 2*b*c^3*e*ArcCoth[c*x]^2 - 4*a*c^3*e*ArcTanh[c*x
] - 4*b*c^3*e*Log[1/Sqrt[1 - 1/(c^2*x^2)]] + 2*b*c^3*d*Log[x] - 2*b*c^3*e*L
og[x] + (b*c^3*e*Log[-c^(-1) + x]^2)/2 + (b*c^3*e*Log[c^(-1) + x]^2)/2 + b*
c^3*e*Log[c^(-1) + x]*Log[(1 - c*x)/2] - 2*b*c^3*e*Log[x]*Log[1 - c*x] + b*
c^3*e*Log[-c^(-1) + x]*Log[(1 + c*x)/2] - 2*b*c^3*e*Log[x]*Log[1 + c*x] - b
*c^3*d*Log[1 - c^2*x^2] + b*c^3*e*Log[1 - c^2*x^2] - (2*a*e*Log[1 - c^2*x^2
])/x^3 - (b*c*e*Log[1 - c^2*x^2])/x^2 - (2*b*e*ArcCoth[c*x]*Log[1 - c^2*x^2
])/x^3 + 2*b*c^3*e*Log[x]*Log[1 - c^2*x^2] - b*c^3*e*Log[-c^(-1) + x]*Log[1
- c^2*x^2] - b*c^3*e*Log[c^(-1) + x]*Log[1 - c^2*x^2] - 2*b*c^3*e*PolyLog[
2, -(c*x)] - 2*b*c^3*e*PolyLog[2, c*x] + b*c^3*e*PolyLog[2, 1/2 - (c*x)/2]
+ b*c^3*e*PolyLog[2, (1 + c*x)/2])/6
```

**Maple [F]** time = 4.322, size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{arccoth}(cx)) (d + e \ln(-c^2 x^2 + 1))}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^4,x)
```

```
[Out] int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^4,x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{6} \left( \left( c^2 \log(c^2 x^2 - 1) - c^2 \log(x^2) + \frac{1}{x^2} \right) c + \frac{2 \operatorname{arccoth}(cx)}{x^3} \right) b d - \frac{1}{3} \left( \left( c \log(cx + 1) - c \log(cx - 1) - \frac{2}{x} \right) c^2 + \frac{\log(-c^2 x^2 + 1)}{x^3} \right) a e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^4,x, algorithm="maxima
")
```

```
[Out] -1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arccoth(c*x)/x^3)
*b*d - 1/3*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c^2 + log(-c^2*x^2 + 1)
/x^3)*a*e - 1/6*b*e*(log(c*x + 1)^2/x^3 - 3*integrate(-1/3*(3*(c*x + 1)*log
(c*x - 1)^2 - (3*I*pi + (3*I*pi*c + 2*c)*x)*log(c*x + 1) - (-3*I*pi - 3*I*p
```

$i*c*x)*\log(c*x - 1))/(c*x^5 + x^4), x)) - 1/3*a*d/x^3$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bd \operatorname{arccoth}(cx) + ad + (be \operatorname{arccoth}(cx) + ae) \log(-c^2x^2 + 1)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1))/x^4,x, algorithm="fricas")

[Out] integral((b\*d\*arccoth(c\*x) + a\*d + (b\*e\*arccoth(c\*x) + a\*e)\*log(-c^2\*x^2 + 1))/x^4, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{acoth}(cx))(d + e \log(-c^2x^2 + 1))}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth(c\*x))\*(d+e\*ln(-c\*\*2\*x\*\*2+1))/x\*\*4,x)

[Out] Integral((a + b\*acoth(c\*x))\*(d + e\*log(-c\*\*2\*x\*\*2 + 1))/x\*\*4, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccoth}(cx) + a)(e \log(-c^2x^2 + 1) + d)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1))/x^4,x, algorithm="giac")

[Out] integrate((b\*arccoth(c\*x) + a)\*(e\*log(-c^2\*x^2 + 1) + d)/x^4, x)

$$3.277 \quad \int \frac{(a+b \coth^{-1}(cx))(d+e \log(1-c^2x^2))}{x^6} dx$$

**Optimal.** Leaf size=256

$$-\frac{1}{10}bc^5e \operatorname{PolyLog}\left(2, \frac{1}{1-c^2x^2}\right) - \frac{(a+b \coth^{-1}(cx))(e \log(1-c^2x^2)+d)}{5x^5} + \frac{2c^2e(a+b \coth^{-1}(cx))}{15x^3} - \frac{c^5e(a+b \coth^{-1}(cx))}{5b}$$

[Out] (7\*b\*c^3\*e)/(60\*x^2) + (2\*c^2\*e\*(a + b\*ArcCoth[c\*x]))/(15\*x^3) + (2\*c^4\*e\*(a + b\*ArcCoth[c\*x]))/(5\*x) - (c^5\*e\*(a + b\*ArcCoth[c\*x])^2)/(5\*b) - (5\*b\*c^5\*e\*Log[x])/6 + (19\*b\*c^5\*e\*Log[1 - c^2\*x^2])/60 - (b\*c\*(d + e\*Log[1 - c^2\*x^2]))/(20\*x^4) - (b\*c^3\*(1 - c^2\*x^2)\*(d + e\*Log[1 - c^2\*x^2]))/(10\*x^2) - ((a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/(5\*x^5) + (b\*c^5\*(d + e\*Log[1 - c^2\*x^2])\*Log[1 - (1 - c^2\*x^2)^(-1)])/10 - (b\*c^5\*e\*PolyLog[2, (1 - c^2\*x^2)^(-1)])/10

**Rubi [A]** time = 0.670072, antiderivative size = 250, normalized size of antiderivative = 0.98, number of steps used = 26, number of rules used = 18, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {6082, 2475, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2319, 44, 5983, 5917, 266, 36, 29, 5949}

$$-\frac{1}{10}bc^5e \operatorname{PolyLog}\left(2, c^2x^2\right) - \frac{(a+b \coth^{-1}(cx))(e \log(1-c^2x^2)+d)}{5x^5} + \frac{2c^2e(a+b \coth^{-1}(cx))}{15x^3} - \frac{c^5e(a+b \coth^{-1}(cx))}{5b}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/x^6, x]

[Out] (7\*b\*c^3\*e)/(60\*x^2) + (2\*c^2\*e\*(a + b\*ArcCoth[c\*x]))/(15\*x^3) + (2\*c^4\*e\*(a + b\*ArcCoth[c\*x]))/(5\*x) - (c^5\*e\*(a + b\*ArcCoth[c\*x])^2)/(5\*b) + (b\*c^5\*d\*Log[x])/5 - (5\*b\*c^5\*e\*Log[x])/6 + (19\*b\*c^5\*e\*Log[1 - c^2\*x^2])/60 - (b\*c\*(d + e\*Log[1 - c^2\*x^2]))/(20\*x^4) - (b\*c^3\*(1 - c^2\*x^2)\*(d + e\*Log[1 - c^2\*x^2]))/(10\*x^2) - ((a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/(5\*x^5) - (b\*c^5\*(d + e\*Log[1 - c^2\*x^2])^2)/(20\*e) - (b\*c^5\*e\*PolyLog[2, c^2\*x^2])/10

**Rule 6082**

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))\*((d\_.) + Log[(f\_.) + (g\_.)\*(x\_.)^2]\*(e\_.))\*(x\_.)^(m\_.), x\_Symbol] := Simp[(x^(m + 1)\*(d + e\*Log[f + g\*x^2])\*(a + b\*ArcCoth[c\*x]))/(m + 1), x] + (-Dist[(b\*c)/(m + 1), Int[(x^(m + 1)\*(d + e



$\text{Log}[f + g*x^2]/(1 - c^2*x^2), x], x] - \text{Dist}[(2*e*g)/(m + 1), \text{Int}[(x^{(m + 2)}*(a + b*\text{ArcCoth}[c*x]))/(f + g*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g\}, x] \ \&\& \ \text{ILtQ}[m/2, 0]$

### Rule 2475

$\text{Int}[(a + \text{Log}[c*((d) + (e)*(x)^{(n)})^{(p)}])*(b)^{(q)}*(x)^{(m)}*((f) + (g)*(x)^{(s)})^{(r)}], x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(f + g*x^{(s/n)})^r*(a + b*\text{Log}[c*(d + e*x)^p])^q}, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, n, p, q, r, s\}, x] \ \&\& \ \text{IntegerQ}[r] \ \&\& \ \text{IntegerQ}[s/n] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]] \ \&\& \ (\text{GtQ}[(m + 1)/n, 0] \ || \ \text{IGtQ}[q, 0])$

### Rule 2411

$\text{Int}[(a + \text{Log}[c*((d) + (e)*(x))^{(n)}])*(b)^{(p)}*((f) + (g)*(x))^{(q)}*((h) + (i)*(x))^{(r)}], x\_Symbol] \rightarrow \text{Dist}[1/e, \text{Subst}[\text{Int}[(g*x/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*\text{Log}[c*x^n])^p], x], x, d + e*x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h, i, n, p, q, r\}, x] \ \&\& \ \text{EqQ}[e*f - d*g, 0] \ \&\& \ (\text{IGtQ}[p, 0] \ || \ \text{IGtQ}[r, 0]) \ \&\& \ \text{IntegerQ}[2*r]$

### Rule 2347

$\text{Int}[(a + \text{Log}[c*(x)^{(n)}])*(b)^{(p)}*((d) + (e)*(x))^{(q)}]/(x), x\_Symbol] \rightarrow \text{Dist}[1/d, \text{Int}[(d + e*x)^{(q + 1)}*(a + b*\text{Log}[c*x^n])^p/x], x], x] - \text{Dist}[e/d, \text{Int}[(d + e*x)^q*(a + b*\text{Log}[c*x^n])^p], x], x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x] \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ \text{LtQ}[q, -1] \ \&\& \ \text{IntegerQ}[2*q]$

### Rule 2344

$\text{Int}[(a + \text{Log}[c*(x)^{(n)}])*(b)^{(p)}/((x)*((d) + (e)*(x))), x\_Symbol] \rightarrow \text{Dist}[1/d, \text{Int}[(a + b*\text{Log}[c*x^n])^p/x], x], x] - \text{Dist}[e/d, \text{Int}[(a + b*\text{Log}[c*x^n])^p/(d + e*x)], x], x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x] \ \&\& \ \text{IGtQ}[p, 0]$

### Rule 2301

$\text{Int}[(a + \text{Log}[c*(x)^{(n)}])*(b)]/(x), x\_Symbol] \rightarrow \text{Simp}[(a + b*\text{Log}[c*x^n])^2/(2*b*n), x] /; \text{FreeQ}\{a, b, c, n\}, x]$

### Rule 2316

$\text{Int}[(a + \text{Log}[c*(x)]*(b)]/((d) + (e)*(x)), x\_Symbol] \rightarrow \text{Simp}[(a + b*\text{Log}[-(c*d)/e])*Log[d + e*x])/e, x] + \text{Dist}[b, \text{Int}[Log[-(e*x)/d]]/$

$(d + e*x), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{GtQ}[-((c*d)/e), 0]$

### Rule 2315

$\text{Int}[\text{Log}[(c_.)*(x_.)]/((d_.) + (e_.)*(x_.)), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, 1 - c*x]/e, x] /; \text{FreeQ}[\{c, d, e\}, x] \ \&\& \ \text{EqQ}[e + c*d, 0]$

### Rule 2314

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]*((d_.) + (e_.)*(x_.)^{(r_.)})^{(q_.)}, x\_Symbol] \rightarrow \text{Simp}[(x*(d + e*x^r)^{(q + 1)}*(a + b*\text{Log}[c*x^n]))/d, x] - \text{Dist}[(b*n)/d, \text{Int}[(d + e*x^r)^{(q + 1)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, n, q, r\}, x] \ \&\& \ \text{EqQ}[r*(q + 1) + 1, 0]$

### Rule 31

$\text{Int}[(a_.) + (b_.)*(x_.)^{-1}], x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}[\{a, b\}, x]$

### Rule 2319

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]^{(p_.)}*((d_.) + (e_.)*(x_.))^{(q_.)}, x\_Symbol] \rightarrow \text{Simp}[(d + e*x)^{(q + 1)}*(a + b*\text{Log}[c*x^n])^p/(e*(q + 1)), x] - \text{Dist}[(b*n*p)/(e*(q + 1)), \text{Int}[(d + e*x)^{(q + 1)}*(a + b*\text{Log}[c*x^n])^{(p - 1)}], x], x] /; \text{FreeQ}[\{a, b, c, d, e, n, p, q\}, x] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{NeQ}[q, -1] \ \&\& \ (\text{EqQ}[p, 1] \ || \ (\text{IntegersQ}[2*p, 2*q] \ \&\& \ !\text{IGtQ}[q, 0]) \ || \ (\text{EqQ}[p, 2] \ \&\& \ \text{NeQ}[q, 1]))$

### Rule 44

$\text{Int}[(a_.) + (b_.)*(x_.)^{(m_.)}*((c_.) + (d_.)*(x_.)^{(n_.)}), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{ILtQ}[m, 0] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ !(\text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[m + n + 2, 0])$

### Rule 5983

$\text{Int}[(a_.) + \text{ArcCoth}[(c_.)*(x_.)]*(b_.)]^{(p_.)}*((f_.)*(x_.))^{(m_.)}/((d_.) + (e_.)*(x_.)^2), x\_Symbol] \rightarrow \text{Dist}[1/d, \text{Int}[(f*x)^m*(a + b*\text{ArcCoth}[c*x])^p, x], x] - \text{Dist}[e/(d*f^2), \text{Int}[(f*x)^{(m + 2)}*(a + b*\text{ArcCoth}[c*x])^p]/(d + e*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{LtQ}[m, -1]$

### Rule 5917

```

Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
  := Simp[(((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[(((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]

```

### Rule 266

```

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

```

### Rule 36

```

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c
- a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

```

### Rule 29

```

Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]

```

### Rule 5949

```

Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{x^6} dx &= -\frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{5x^5} + \frac{1}{5}(bc) \int \frac{d + e \log(1 - c^2x^2)}{x^5(1 - c^2x^2)} \\
&= -\frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{5x^5} + \frac{1}{10}(bc) \text{Subst} \left( \int \frac{d + e \log}{x^3(1 - c^2x^2)} \right. \\
&= \frac{2c^2e(a + b \coth^{-1}(cx))}{15x^3} - \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{5x^5} - \frac{b}{5} \\
&= \frac{2c^2e(a + b \coth^{-1}(cx))}{15x^3} + \frac{2c^4e(a + b \coth^{-1}(cx))}{5x} - \frac{c^5e(a + b \coth^{-1}(cx))}{5b} \\
&= \frac{2c^2e(a + b \coth^{-1}(cx))}{15x^3} + \frac{2c^4e(a + b \coth^{-1}(cx))}{5x} - \frac{c^5e(a + b \coth^{-1}(cx))}{5b} \\
&= \frac{bc^3e}{15x^2} + \frac{2c^2e(a + b \coth^{-1}(cx))}{15x^3} + \frac{2c^4e(a + b \coth^{-1}(cx))}{5x} - \frac{c^5e(a + b \coth^{-1}(cx))}{5b} \\
&= \frac{7bc^3e}{60x^2} + \frac{2c^2e(a + b \coth^{-1}(cx))}{15x^3} + \frac{2c^4e(a + b \coth^{-1}(cx))}{5x} - \frac{c^5e(a + b \coth^{-1}(cx))}{5b} \\
&= \frac{7bc^3e}{60x^2} + \frac{2c^2e(a + b \coth^{-1}(cx))}{15x^3} + \frac{2c^4e(a + b \coth^{-1}(cx))}{5x} - \frac{c^5e(a + b \coth^{-1}(cx))}{5b}
\end{aligned}$$

**Mathematica [F]** time = 0.279638, size = 0, normalized size = 0.

$$\int \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{x^6} dx$$

Verification is Not applicable to the result.

[In] Integrate[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/x^6, x]

[Out] Integrate[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/x^6, x]

**Maple [F]** time = 4.318, size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{arccoth}(cx)) (d + e \ln(-c^2 x^2 + 1))}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth(c\*x))\*(d+e\*ln(-c^2\*x^2+1))/x^6,x)

[Out] int((a+b\*arccoth(c\*x))\*(d+e\*ln(-c^2\*x^2+1))/x^6,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{20} \left( \left( 2c^4 \log(c^2 x^2 - 1) - 2c^4 \log(x^2) + \frac{2c^2 x^2 + 1}{x^4} \right) c + \frac{4 \operatorname{arccoth}(cx)}{x^5} \right) bd - \frac{1}{15} \left( \left( 3c^3 \log(cx + 1) - 3c^3 \log(cx - 1) - \right. \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1))/x^6,x, algorithm="maxima")

[Out] -1/20\*((2\*c^4\*log(c^2\*x^2 - 1) - 2\*c^4\*log(x^2) + (2\*c^2\*x^2 + 1)/x^4)\*c + 4\*arccoth(c\*x)/x^5)\*b\*d - 1/15\*((3\*c^3\*log(c\*x + 1) - 3\*c^3\*log(c\*x - 1) - 2\*(3\*c^2\*x^2 + 1)/x^3)\*c^2 + 3\*log(-c^2\*x^2 + 1)/x^5)\*a\*e - 1/10\*b\*e\*(log(c\*x + 1)^2/x^5 - 5\*integrate(-1/5\*(5\*(c\*x + 1)\*log(c\*x - 1)^2 - (5\*I\*pi + (5\*I\*pi\*c + 2\*c)\*x)\*log(c\*x + 1) - (-5\*I\*pi - 5\*I\*pi\*c\*x)\*log(c\*x - 1))/(c\*x^7 + x^6), x)) - 1/5\*a\*d/x^5

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{bd \operatorname{arccoth}(cx) + ad + (be \operatorname{arccoth}(cx) + ae) \log(-c^2 x^2 + 1)}{x^6}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1))/x^6,x, algorithm="fricas")

[Out] integral((b\*d\*arccoth(c\*x) + a\*d + (b\*e\*arccoth(c\*x) + a\*e)\*log(-c^2\*x^2 + 1))/x^6, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth(c\*x))\*(d+e\*ln(-c\*\*2\*x\*\*2+1))/x\*\*6,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccoth}(cx) + a)(e \log(-c^2x^2 + 1) + d)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1))/x^6,x, algorithm="giac")

[Out] integrate((b\*arccoth(c\*x) + a)\*(e\*log(-c^2\*x^2 + 1) + d)/x^6, x)

### 3.278 $\int x \left( a + b \coth^{-1}(cx) \right) \left( d + e \log \left( f + gx^2 \right) \right) dx$

**Optimal.** Leaf size=512

$$\frac{be(c^2f + g) \operatorname{PolyLog}\left(2, 1 - \frac{2}{cx+1}\right)}{2c^2g} - \frac{be(c^2f + g) \operatorname{PolyLog}\left(2, 1 - \frac{2c(\sqrt{-f}-\sqrt{gx})}{(cx+1)(c\sqrt{-f}-\sqrt{g})}\right)}{4c^2g} - \frac{be(c^2f + g) \operatorname{PolyLog}\left(2, 1 - \frac{2c}{cx+1}\right)}{4c^2g}$$

[Out] (b\*(d - e)\*x)/(2\*c) - (b\*e\*x)/c + (d\*x^2\*(a + b\*ArcCoth[c\*x]))/2 - (e\*x^2\*(a + b\*ArcCoth[c\*x]))/2 + (b\*e\*Sqrt[f]\*ArcTan[(Sqrt[g]\*x)/Sqrt[f]])/(c\*Sqrt[g]) - (b\*(d - e)\*ArcTanh[c\*x])/(2\*c^2) - (b\*e\*(c^2\*f + g)\*ArcTanh[c\*x]\*Log[2/(1 + c\*x)])/(c^2\*g) + (b\*e\*(c^2\*f + g)\*ArcTanh[c\*x]\*Log[(2\*c\*(Sqrt[-f] - Sqrt[g]\*x))/((c\*Sqrt[-f] - Sqrt[g])\*(1 + c\*x))])/(2\*c^2\*g) + (b\*e\*(c^2\*f + g)\*ArcTanh[c\*x]\*Log[(2\*c\*(Sqrt[-f] + Sqrt[g]\*x))/((c\*Sqrt[-f] + Sqrt[g])\*(1 + c\*x))])/(2\*c^2\*g) + (b\*e\*x\*Log[f + g\*x^2])/(2\*c) + (e\*(f + g\*x^2)\*(a + b\*ArcCoth[c\*x])\*Log[f + g\*x^2])/(2\*g) - (b\*e\*(c^2\*f + g)\*ArcTanh[c\*x]\*Log[f + g\*x^2])/(2\*c^2\*g) + (b\*e\*(c^2\*f + g)\*PolyLog[2, 1 - 2/(1 + c\*x)])/(2\*c^2\*g) - (b\*e\*(c^2\*f + g)\*PolyLog[2, 1 - (2\*c\*(Sqrt[-f] - Sqrt[g]\*x))/((c\*Sqrt[-f] - Sqrt[g])\*(1 + c\*x))])/(4\*c^2\*g) - (b\*e\*(c^2\*f + g)\*PolyLog[2, 1 - (2\*c\*(Sqrt[-f] + Sqrt[g]\*x))/((c\*Sqrt[-f] + Sqrt[g])\*(1 + c\*x))])/(4\*c^2\*g)

**Rubi [A]** time = 0.76723, antiderivative size = 512, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 17, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.773$ , Rules used = {2454, 2389, 2295, 6084, 321, 207, 517, 2528, 2448, 205, 2470, 12, 5992, 5920, 2402, 2315, 2447}

$$\frac{be(c^2f + g) \operatorname{PolyLog}\left(2, 1 - \frac{2}{cx+1}\right)}{2c^2g} - \frac{be(c^2f + g) \operatorname{PolyLog}\left(2, 1 - \frac{2c(\sqrt{-f}-\sqrt{gx})}{(cx+1)(c\sqrt{-f}-\sqrt{g})}\right)}{4c^2g} - \frac{be(c^2f + g) \operatorname{PolyLog}\left(2, 1 - \frac{2c}{cx+1}\right)}{4c^2g}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*ArcCoth[c\*x])\*(d + e\*Log[f + g\*x^2]),x]

[Out] (b\*(d - e)\*x)/(2\*c) - (b\*e\*x)/c + (d\*x^2\*(a + b\*ArcCoth[c\*x]))/2 - (e\*x^2\*(a + b\*ArcCoth[c\*x]))/2 + (b\*e\*Sqrt[f]\*ArcTan[(Sqrt[g]\*x)/Sqrt[f]])/(c\*Sqrt[g]) - (b\*(d - e)\*ArcTanh[c\*x])/(2\*c^2) - (b\*e\*(c^2\*f + g)\*ArcTanh[c\*x]\*Log[2/(1 + c\*x)])/(c^2\*g) + (b\*e\*(c^2\*f + g)\*ArcTanh[c\*x]\*Log[(2\*c\*(Sqrt[-f] - Sqrt[g]\*x))/((c\*Sqrt[-f] - Sqrt[g])\*(1 + c\*x))])/(2\*c^2\*g) + (b\*e\*(c^2\*f + g)\*ArcTanh[c\*x]\*Log[(2\*c\*(Sqrt[-f] + Sqrt[g]\*x))/((c\*Sqrt[-f] + Sqrt[g])\*(1 + c\*x))])/(2\*c^2\*g) + (b\*e\*x\*Log[f + g\*x^2])/(2\*c) + (e\*(f + g\*x^2)\*(a + b

\*ArcCoth[c\*x])\*Log[f + g\*x^2])/(2\*g) - (b\*e\*(c^2\*f + g)\*ArcTanh[c\*x]\*Log[f + g\*x^2])/(2\*c^2\*g) + (b\*e\*(c^2\*f + g)\*PolyLog[2, 1 - 2/(1 + c\*x)])/(2\*c^2\*g) - (b\*e\*(c^2\*f + g)\*PolyLog[2, 1 - (2\*c\*(Sqrt[-f] - Sqrt[g]\*x))/((c\*Sqrt[-f] - Sqrt[g])\*(1 + c\*x)))]/(4\*c^2\*g) - (b\*e\*(c^2\*f + g)\*PolyLog[2, 1 - (2\*c\*(Sqrt[-f] + Sqrt[g]\*x))/((c\*Sqrt[-f] + Sqrt[g])\*(1 + c\*x)))]/(4\*c^2\*g)

#### Rule 2454

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

#### Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))^(p\_.), x\_Symbol] := Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

#### Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

#### Rule 6084

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.)\*((d\_.) + Log[(f\_.) + (g\_.)\*(x\_)^2]\*(e\_.)\*(x\_)^(m\_.), x\_Symbol] := With[{u = IntHide[x^m\*(d + e\*Log[f + g\*x^2]), x]}, Dist[a + b\*ArcCoth[c\*x], u, x] - Dist[b\*c, Int[ExpandIntegrand[u/(1 - c^2\*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)/2, 0]

#### Rule 321

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 207

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTanh[(Rt[b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a



, 0] || GtQ[b, 0])

### Rule 517

Int[(u\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.))^(q\_.)\*((a1\_.) + (b1\_.)\*(x\_)^(non2\_.))^(p\_.)\*((a2\_.) + (b2\_.)\*(x\_)^(non2\_.))^(p\_.), x\_Symbol] := Int[u\*(a1\*a2 + b1\*b2\*x^n)^p\*(c + d\*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]))

### Rule 2528

Int[((a\_.) + Log[(c\_.)\*(RFX\_)^(p\_.)]\*(b\_.))^(n\_.)\*(RGx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*RFX^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFX, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

### Rule 2448

Int[Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))^(p\_.)], x\_Symbol] := Simp[x\*Log[c\*(d + e\*x^n)^p], x] - Dist[e\*n\*p, Int[x^n/(d + e\*x^n), x], x] /; FreeQ[{c, d, e, n, p}, x]

### Rule 205

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 2470

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))^(p\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)^2), x\_Symbol] := With[{u = IntHide[1/(f + g\*x^2), x]}, Simp[u\*(a + b\*Log[c\*(d + e\*x^n)^p]), x] - Dist[b\*e\*n\*p, Int[(u\*x^(n - 1))/(d + e\*x^n), x], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 5992

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.)\*(x\_)^(m\_.))/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[a + b\*ArcTanh[c\*x], x^m/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0])

)

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))/((d_) + (e_.)*(x_.)), x_Symbol] := -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_.))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_.)]/((d_) + (e_.)*(x_.)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

Rubi steps



**Mathematica [A]** time = 4.25167, size = 677, normalized size = 1.32

$$bc^2ef \operatorname{PolyLog}\left(2, \frac{(c^2f+g)e^{2\operatorname{coth}^{-1}(cx)}}{c^2f+2c\sqrt{-f}\sqrt{g-g}}\right) + 2be(c^2f+g) \operatorname{PolyLog}\left(2, e^{-2\operatorname{coth}^{-1}(cx)}\right) + be(c^2f+g) \operatorname{PolyLog}\left(2, \frac{(c^2f+g)e^{2\operatorname{coth}^{-1}(cx)}}{c^2f-2c\sqrt{-f}\sqrt{g-g}}\right)$$


---

Warning: Unable to verify antiderivative.

[In] Integrate[x\*(a + b\*ArcCoth[c\*x])\*(d + e\*Log[f + g\*x^2]), x]

[Out] (2\*b\*c\*d\*g\*x - 6\*b\*c\*e\*g\*x + 2\*a\*c^2\*d\*g\*x^2 - 2\*a\*c^2\*e\*g\*x^2 - 2\*b\*d\*g\*ArcCoth[c\*x] + 2\*b\*e\*g\*ArcCoth[c\*x] + 2\*b\*c^2\*d\*g\*x^2\*ArcCoth[c\*x] - 2\*b\*c^2\*e\*g\*x^2\*ArcCoth[c\*x] - 4\*b\*c^2\*e\*f\*ArcCoth[c\*x]^2 - 4\*b\*e\*g\*ArcCoth[c\*x]^2 + 4\*b\*c\*e\*Sqrt[f]\*Sqrt[g]\*ArcTan[(Sqrt[g]\*x)/Sqrt[f]] - 4\*b\*c^2\*e\*f\*ArcCoth[c\*x]\*Log[1 - E^(-2\*ArcCoth[c\*x])] - 4\*b\*e\*g\*ArcCoth[c\*x]\*Log[1 - E^(-2\*ArcCoth[c\*x])] + 2\*b\*c^2\*e\*f\*ArcCoth[c\*x]\*Log[1 + (E^(2\*ArcCoth[c\*x])\*(c^2\*f + g))/(-c^2\*f - 2\*c\*Sqrt[-f]\*Sqrt[g] + g)] + 2\*b\*e\*g\*ArcCoth[c\*x]\*Log[1 + (E^(2\*ArcCoth[c\*x])\*(c^2\*f + g))/(-c^2\*f - 2\*c\*Sqrt[-f]\*Sqrt[g] + g)] + 2\*b\*c^2\*e\*f\*ArcCoth[c\*x]\*Log[1 + (E^(2\*ArcCoth[c\*x])\*(c^2\*f + g))/(-c^2\*f + 2\*c\*Sqrt[-f]\*Sqrt[g] + g)] + 2\*a\*c^2\*e\*f\*Log[f + g\*x^2] + 2\*b\*c\*e\*g\*x\*Log[f + g\*x^2] + 2\*a\*c^2\*e\*g\*x^2\*Log[f + g\*x^2] - 2\*b\*e\*g\*ArcCoth[c\*x]\*Log[f + g\*x^2] + 2\*b\*c^2\*e\*g\*x^2\*ArcCoth[c\*x]\*Log[f + g\*x^2] + 2\*b\*e\*(c^2\*f + g)\*PolyLog[2, E^(-2\*ArcCoth[c\*x])] + b\*e\*(c^2\*f + g)\*PolyLog[2, (E^(2\*ArcCoth[c\*x])\*(c^2\*f + g))/(c^2\*f - 2\*c\*Sqrt[-f]\*Sqrt[g] - g)] + b\*c^2\*e\*f\*PolyLog[2, (E^(2\*ArcCoth[c\*x])\*(c^2\*f + g))/(c^2\*f + 2\*c\*Sqrt[-f]\*Sqrt[g] - g)] + b\*e\*g\*PolyLog[2, (E^(2\*ArcCoth[c\*x])\*(c^2\*f + g))/(c^2\*f + 2\*c\*Sqrt[-f]\*Sqrt[g] - g)]/(4\*c^2\*g)

---

**Maple [C]** time = 1.996, size = 8491, normalized size = 16.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arccoth(c\*x))\*(d+e\*ln(g\*x^2+f)), x)

[Out] result too large to display

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccoth(c\*x))\*(d+e\*log(g\*x^2+f)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(b dx \operatorname{arccoth}(cx) + a dx + (b e x \operatorname{arccoth}(cx) + a e x) \log(g x^2 + f), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccoth(c\*x))\*(d+e\*log(g\*x^2+f)),x, algorithm="fricas")

[Out] integral(b\*d\*x\*arccoth(c\*x) + a\*d\*x + (b\*e\*x\*arccoth(c\*x) + a\*e\*x)\*log(g\*x^2 + f), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*acoth(c\*x))\*(d+e\*ln(g\*x\*\*2+f)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arccoth}(cx) + a)(e \log(gx^2 + f) + d)x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccoth(c*x))*(d+e*log(g*x^2+f)),x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(g*x^2 + f) + d)*x, x)
```

$$3.279 \quad \int \left( a + b \coth^{-1}(cx) \right) \left( d + e \log \left( f + gx^2 \right) \right) dx$$

**Optimal.** Leaf size=546

$$\frac{be \operatorname{PolyLog} \left( 2, \frac{c^2(f+gx^2)}{c^2f+g} \right)}{2c} - \frac{ibe \sqrt{f} \operatorname{PolyLog} \left( 2, 1 + \frac{2\sqrt{f}\sqrt{g}(1-cx)}{(-\sqrt{g}+ic\sqrt{f})(\sqrt{f}-i\sqrt{gx})} \right)}{2\sqrt{g}} + \frac{ibe \sqrt{f} \operatorname{PolyLog} \left( 2, 1 - \frac{2\sqrt{f}\sqrt{g}(cx+1)}{(\sqrt{g}+ic\sqrt{f})(\sqrt{f}-i\sqrt{gx})} \right)}{2\sqrt{g}} +$$

```
[Out] -2*a*e*x - 2*b*e*x*ArcCoth[c*x] + (2*a*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]
)/Sqrt[g] - (b*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[1 - 1/(c*x)])/Sqr
t[g] + (b*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[1 + 1/(c*x)])/Sqrt[g] +
(b*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(1 - c*x)
)/((I*c*Sqrt[f] - Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[g] - (b*e*Sqrt[f
]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(1 + c*x))/((I*c*Sqrt[
f] + Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[g] - (b*e*Log[1 - c^2*x^2])/c
+ x*(a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]) + (b*Log[(g*(1 - c^2*x^2)
)/(c^2*f + g)]*(d + e*Log[f + g*x^2]))/(2*c) + (b*e*PolyLog[2, (c^2*(f + g*x
^2))/(c^2*f + g)])/(2*c) - ((I/2)*b*e*Sqrt[f]*PolyLog[2, 1 + (2*Sqrt[f]*Sqr
t[g]*(1 - c*x))/((I*c*Sqrt[f] - Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[g]
+ ((I/2)*b*e*Sqrt[f]*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(1 + c*x))/((I*c*Sq
rt[f] + Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[g]
```

**Rubi [A]** time = 1.38741, antiderivative size = 546, normalized size of antiderivative = 1., number of steps used = 38, number of rules used = 20, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.952$ , Rules used = {6074, 2475, 2394, 2393, 2391, 5981, 5911, 260, 5975, 205, 5973, 2470, 12, 6688, 4876, 4848, 4856, 2402, 2315, 2447}

$$\frac{be \operatorname{PolyLog} \left( 2, \frac{c^2(f+gx^2)}{c^2f+g} \right)}{2c} - \frac{ibe \sqrt{f} \operatorname{PolyLog} \left( 2, 1 + \frac{2\sqrt{f}\sqrt{g}(1-cx)}{(-\sqrt{g}+ic\sqrt{f})(\sqrt{f}-i\sqrt{gx})} \right)}{2\sqrt{g}} + \frac{ibe \sqrt{f} \operatorname{PolyLog} \left( 2, 1 - \frac{2\sqrt{f}\sqrt{g}(cx+1)}{(\sqrt{g}+ic\sqrt{f})(\sqrt{f}-i\sqrt{gx})} \right)}{2\sqrt{g}} +$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]),x]
```

```
[Out] -2*a*e*x - 2*b*e*x*ArcCoth[c*x] + (2*a*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]
)/Sqrt[g] - (b*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[1 - 1/(c*x)])/Sqr
t[g] + (b*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[1 + 1/(c*x)])/Sqrt[g] +
(b*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(1 - c*x)
)/((I*c*Sqrt[f] - Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[g] - (b*e*Sqrt[f
]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(1 + c*x))/((I*c*Sqrt[
f] + Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[g] - (b*e*Log[1 - c^2*x^2])/c
+ x*(a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]) + (b*Log[(g*(1 - c^2*x^2)
)/(c^2*f + g)]*(d + e*Log[f + g*x^2]))/(2*c) + (b*e*PolyLog[2, (c^2*(f + g*x
^2))/(c^2*f + g)])/(2*c) - ((I/2)*b*e*Sqrt[f]*PolyLog[2, 1 + (2*Sqrt[f]*Sqr
t[g]*(1 - c*x))/((I*c*Sqrt[f] - Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[g]
+ ((I/2)*b*e*Sqrt[f]*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(1 + c*x))/((I*c*Sq
rt[f] + Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[g]
```

$$\begin{aligned} & f] + \text{Sqrt}[g])*(\text{Sqrt}[f] - I*\text{Sqrt}[g]*x)))/\text{Sqrt}[g] - (b*e*\text{Log}[1 - c^2*x^2])/c \\ & + x*(a + b*\text{ArcCoth}[c*x])*(d + e*\text{Log}[f + g*x^2]) + (b*\text{Log}[(g*(1 - c^2*x^2)) \\ & / (c^2*f + g)]*(d + e*\text{Log}[f + g*x^2]))/(2*c) + (b*e*\text{PolyLog}[2, (c^2*(f + g*x \\ & ^2))/(c^2*f + g)])/(2*c) - ((I/2)*b*e*\text{Sqrt}[f]*\text{PolyLog}[2, 1 + (2*\text{Sqrt}[f]*\text{Sqr} \\ & t[g]*(1 - c*x))/((I*c*\text{Sqrt}[f] - \text{Sqrt}[g])*(\text{Sqrt}[f] - I*\text{Sqrt}[g]*x)))]/\text{Sqrt}[g] \\ & + ((I/2)*b*e*\text{Sqrt}[f]*\text{PolyLog}[2, 1 - (2*\text{Sqrt}[f]*\text{Sqrt}[g]*(1 + c*x))/((I*c*\text{Sqr} \\ & t[f] + \text{Sqrt}[g])*(\text{Sqrt}[f] - I*\text{Sqrt}[g]*x)))])/\text{Sqrt}[g] \end{aligned}$$
Rule 6074

$$\begin{aligned} & \text{Int}[(a_.) + \text{ArcCoth}[c_.*(x_.)]*(b_.)]*((d_.) + \text{Log}[(f_.) + (g_.)*(x_.)^2]* \\ & (e_.)), x\_Symbol] \text{:>} \text{Simp}[x*(d + e*\text{Log}[f + g*x^2])*(a + b*\text{ArcCoth}[c*x]), x] \\ & + (-\text{Dist}[b*c, \text{Int}[(x*(d + e*\text{Log}[f + g*x^2]))/(1 - c^2*x^2), x], x] - \text{Dist}[ \\ & 2*e*g, \text{Int}[(x^2*(a + b*\text{ArcCoth}[c*x]))/(f + g*x^2), x], x]) /; \text{FreeQ}\{a, b, \\ & c, d, e, f, g\}, x] \end{aligned}$$
Rule 2475

$$\begin{aligned} & \text{Int}[(a_.) + \text{Log}[c_.*((d_.) + (e_.)*(x_.)^{n_.})^{p_.}]*b_.)]^{q_.)*(x_.)^{m \\ & _.*((f_.) + (g_.)*(x_.)^{s_.})^{r_.}, x\_Symbol] \text{:>} \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simp} \\ & lify[(m + 1)/n] - 1)*(f + g*x^{(s/n)})^r*(a + b*\text{Log}[c*(d + e*x)^p])^q}, x], x \\ & , x^n], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, n, p, q, r, s\}, x] \&\& \text{IntegerQ} \\ & [r] \&\& \text{IntegerQ}[s/n] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]] \&\& (\text{GtQ}[(m + 1)/n, 0] \\ & || \text{IGtQ}[q, 0]) \end{aligned}$$
Rule 2394

$$\begin{aligned} & \text{Int}[(a_.) + \text{Log}[c_.*((d_.) + (e_.)*(x_.)^{n_.})]*b_.)]/((f_.) + (g_.)*(x_. \\ & )), x\_Symbol] \text{:>} \text{Simp}[(\text{Log}[(e*(f + g*x))/(e*f - d*g)]*(a + b*\text{Log}[c*(d + e*x) \\ & ^n]))/g, x] - \text{Dist}[(b*e^n)/g, \text{Int}[\text{Log}[(e*(f + g*x))/(e*f - d*g)]/(d + e*x) \\ & , x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, n\}, x] \&\& \text{NeQ}[e*f - d*g, 0] \end{aligned}$$
Rule 2393

$$\begin{aligned} & \text{Int}[(a_.) + \text{Log}[c_.*((d_.) + (e_.)*(x_.))]*(b_.)]/((f_.) + (g_.)*(x_.)), x\_ \\ & Symbol] \text{:>} \text{Dist}[1/g, \text{Subst}[\text{Int}[(a + b*\text{Log}[1 + (c*e*x)/g])/x, x], x, f + g*x \\ & ], x] /; \text{FreeQ}\{a, b, c, d, e, f, g\}, x] \&\& \text{NeQ}[e*f - d*g, 0] \&\& \text{EqQ}[g + c* \\ & (e*f - d*g), 0] \end{aligned}$$
Rule 2391

$$\begin{aligned} & \text{Int}[\text{Log}[(c_.)*((d_.) + (e_.)*(x_.)^{n_.})]/(x_.), x\_Symbol] \text{:>} -\text{Simp}[\text{PolyLog}[2 \\ & , -(c*e*x^n)]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x] \&\& \text{EqQ}[c*d, 1] \end{aligned}$$



Rule 5981

```
Int[(((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.)*((f_.)*(x_.))^(m_)))/((d_.) + (
e_.)*(x_)^2), x_Symbol] := Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5975

```
Int[(ArcCoth[(c_.)*(x_.)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :=>
Dist[a, Int[1/(d + e*x^2), x], x] + Dist[b, Int[ArcCoth[c*x]/(d + e*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :=> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5973

```
Int[ArcCoth[(c_.)*(x_.)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :=> Dist[1/2, Int[
Log[1 + 1/(c*x)]/(d + e*x^2), x], x] - Dist[1/2, Int[Log[1 - 1/(c*x)]/(d +
e*x^2), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :=> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p], x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :=> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 6688

```
Int[u_, x_Symbol] :=> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

### Rule 4876

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :=> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

### Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :=> Simp[a*Log[x], x]
+ (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x)) /; FreeQ[{a, b, c}, x]
```

### Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :=> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))]/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

### Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :=> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

### Rule 2315

```
Int[Log[(c_.)*(x_)/((d_) + (e_.)*(x_)), x_Symbol] :=> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

### Rule 2447

```
Int[Log[u]*(Pq_)^(m_.), x_Symbol] :=> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] &&
```

```
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,  
x][[2]], Expon[Pq, x]]
```

Rubi steps



**Mathematica [B]** time = 3.21188, size = 1287, normalized size = 2.36

result too large to display

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCoth[c\*x])\*(d + e\*Log[f + g\*x^2]),x]

[Out] a\*d\*x - 2\*a\*e\*x + b\*d\*x\*ArcCoth[c\*x] + (2\*a\*e\*sqrt[f]\*ArcTan[(sqrt[g]\*x)/sqrt[f]])/sqrt[g] + (b\*d\*Log[1 - c^2\*x^2])/(2\*c) + a\*e\*x\*Log[f + g\*x^2] + b\*e\*(x\*ArcCoth[c\*x] + Log[1 - c^2\*x^2]/(2\*c))\*Log[f + g\*x^2] + (b\*e\*(-4\*c\*x\*ArcCoth[c\*x] + 4\*Log[1/(c\*sqrt[1 - 1/(c^2\*x^2)])\*x]) + (sqrt[c^2\*f\*g]\*((-2\*I)\*ArcCos[(c^2\*f - g)/(c^2\*f + g)]\*ArcTan[sqrt[c^2\*f\*g]/(c\*g\*x)] + 4\*ArcCoth[c\*x]\*ArcTan[(c\*g\*x)/sqrt[c^2\*f\*g]] - (ArcCos[(c^2\*f - g)/(c^2\*f + g)] + 2\*ArcTan[sqrt[c^2\*f\*g]/(c\*g\*x)])\*Log[((2\*I)\*g\*(I\*c^2\*f + sqrt[c^2\*f\*g])\*(-1 + 1/(c\*x))]/((c^2\*f + g)\*(g + (I\*sqrt[c^2\*f\*g])/(c\*x)))) - (ArcCos[(c^2\*f - g)/(c^2\*f + g)] - 2\*ArcTan[sqrt[c^2\*f\*g]/(c\*g\*x)])\*Log[(2\*g\*(c^2\*f + I\*sqrt[c^2\*f\*g])\*(1 + 1/(c\*x))]/((c^2\*f + g)\*(g + (I\*sqrt[c^2\*f\*g])/(c\*x)))) + (ArcCos[(c^2\*f - g)/(c^2\*f + g)] + 2\*(ArcTan[sqrt[c^2\*f\*g]/(c\*g\*x)] + ArcTan[(c\*g\*x)/sqrt[c^2\*f\*g]]))\*Log[(sqrt[2]\*sqrt[c^2\*f\*g])/(E^ArcCoth[c\*x]\*sqrt[c^2\*f + g]\*sqrt[-(c^2\*f) + g + (c^2\*f + g)\*Cosh[2\*ArcCoth[c\*x]])]) + (ArcCos[(c^2\*f - g)/(c^2\*f + g)] - 2\*(ArcTan[sqrt[c^2\*f\*g]/(c\*g\*x)] + ArcTan[(c\*g\*x)/sqrt[c^2\*f\*g]]))\*Log[(sqrt[2]\*E^ArcCoth[c\*x]\*sqrt[c^2\*f\*g])/(sqrt[c^2\*f + g]\*sqrt[-(c^2\*f) + g + (c^2\*f + g)\*Cosh[2\*ArcCoth[c\*x]])]) + I\*(-PolyLog[2, ((-c^2\*f) + g + (2\*I)\*sqrt[c^2\*f\*g])\*(g - (I\*sqrt[c^2\*f\*g])/(c\*x))]/((c^2\*f + g)\*(g + (I\*sqrt[c^2\*f\*g])/(c\*x)))) + PolyLog[2, ((c^2\*f - g + (2\*I)\*sqrt[c^2\*f\*g])\*(I\*g + sqrt[c^2\*f\*g]/(c\*x)))/((c^2\*f + g)\*((-I)\*g + sqrt[c^2\*f\*g]/(c\*x)))]/g)/(2\*c) - (b\*e\*g\*(((-Log[-c^(-1) + x] - Log[c^(-1) + x] + Log[1 - c^2\*x^2])\*Log[f + g\*x^2])/(2\*g) + (Log[-c^(-1) + x]\*Log[1 - (sqrt[g]\*(-c^(-1) + x))/((-I)\*sqrt[f] - sqrt[g]/c)] + PolyLog[2, (sqrt[g]\*(-c^(-1) + x))/((-I)\*sqrt[f] - sqrt[g]/c)]/(2\*g) + (Log[-c^(-1) + x]\*Log[1 - (sqrt[g]\*(-c^(-1) + x))/(I\*sqrt[f] - sqrt[g]/c)] + PolyLog[2, (sqrt[g]\*(-c^(-1) + x))/(I\*sqrt[f] - sqrt[g]/c)]/(2\*g) + (Log[c^(-1) + x]\*Log[1 - (sqrt[g]\*(c^(-1) + x))/((-I)\*sqrt[f] + sqrt[g]/c)] + PolyLog[2, (sqrt[g]\*(c^(-1) + x))/((-I)\*sqrt[f] + sqrt[g]/c)]/(2\*g) + (Log[c^(-1) + x]\*Log[1 - (sqrt[g]\*(c^(-1) + x))/(I\*sqrt[f] + sqrt[g]/c)] + PolyLog[2, (sqrt[g]\*(c^(-1) + x))/(I\*sqrt[f] + sqrt[g]/c)]/(2\*g)))/c

**Maple [F]** time = 1.994, size = 0, normalized size = 0.

$$\int (a + b \operatorname{arccoth}(cx)) (d + e \ln(gx^2 + f)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(c*x))*(d+e*ln(g*x^2+f)),x)
```

```
[Out] int((a+b*arccoth(c*x))*(d+e*ln(g*x^2+f)),x)
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(bd \operatorname{arccoth}(cx) + ad + (be \operatorname{arccoth}(cx) + ae) \log(gx^2 + f), x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f)),x, algorithm="fricas")
```

```
[Out] integral(b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(g*x^2 + f),
x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(c*x))*(d+e*ln(g*x**2+f)),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arccoth}(cx) + a)(e \log(gx^2 + f) + d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(g\*x^2+f)),x, algorithm="giac")

[Out] integrate((b\*arccoth(c\*x) + a)\*(e\*log(g\*x^2 + f) + d), x)

$$3.280 \quad \int \frac{(a+b \coth^{-1}(cx))(d+e \log(f+gx^2))}{x} dx$$

**Optimal.** Leaf size=100

$$\frac{1}{2}ae \operatorname{PolyLog}\left(2, \frac{gx^2}{f} + 1\right) + be \operatorname{CannotIntegrate}\left(\frac{\coth^{-1}(cx) \log(f+gx^2)}{x}, x\right) + \frac{1}{2}bd \operatorname{PolyLog}\left(2, -\frac{1}{cx}\right) - \frac{1}{2}bd \operatorname{PolyLog}\left(2, \frac{1}{cx}\right)$$

[Out] b\*e\*CannotIntegrate[(ArcCoth[c\*x]\*Log[f + g\*x^2])/x, x] + a\*d\*Log[x] + (a\*e\*Log[-((g\*x^2)/f)]\*Log[f + g\*x^2])/2 + (b\*d\*PolyLog[2, -(1/(c\*x))])/2 - (b\*d\*PolyLog[2, 1/(c\*x))]/2 + (a\*e\*PolyLog[2, 1 + (g\*x^2)/f])/2

**Rubi [A]** time = 0.251519, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(a + b \coth^{-1}(cx))(d + e \log(f + gx^2))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[f + g\*x^2]))/x,x]

[Out] a\*d\*Log[x] + (a\*e\*Log[-((g\*x^2)/f)]\*Log[f + g\*x^2])/2 + (b\*d\*PolyLog[2, -(1/(c\*x))])/2 - (b\*d\*PolyLog[2, 1/(c\*x))]/2 + (a\*e\*PolyLog[2, 1 + (g\*x^2)/f])/2 + b\*e\*Defer[Int] [(ArcCoth[c\*x]\*Log[f + g\*x^2])/x, x]

Rubi steps



$$\begin{aligned}
\int \frac{(a + b \coth^{-1}(cx))(d + e \log(f + gx^2))}{x} dx &= d \int \frac{a + b \coth^{-1}(cx)}{x} dx + e \int \frac{(a + b \coth^{-1}(cx)) \log(f + gx^2)}{x} dx \\
&= ad \log(x) + \frac{1}{2}bd\text{Li}_2\left(-\frac{1}{cx}\right) - \frac{1}{2}bd\text{Li}_2\left(\frac{1}{cx}\right) + (ae) \int \frac{\log(f + gx^2)}{x} dx \\
&= ad \log(x) + \frac{1}{2}bd\text{Li}_2\left(-\frac{1}{cx}\right) - \frac{1}{2}bd\text{Li}_2\left(\frac{1}{cx}\right) + \frac{1}{2}(ae) \text{Subst}\left(\int \frac{\log(f + gx^2)}{x} dx, x, \sqrt{f + gx^2}\right) \\
&= ad \log(x) + \frac{1}{2}ae \log\left(-\frac{gx^2}{f}\right) \log(f + gx^2) + \frac{1}{2}bd\text{Li}_2\left(-\frac{1}{cx}\right) - \frac{1}{2}bd\text{Li}_2\left(\frac{1}{cx}\right) \\
&= ad \log(x) + \frac{1}{2}ae \log\left(-\frac{gx^2}{f}\right) \log(f + gx^2) + \frac{1}{2}bd\text{Li}_2\left(-\frac{1}{cx}\right) - \frac{1}{2}bd\text{Li}_2\left(\frac{1}{cx}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.27258, size = 0, normalized size = 0.

$$\int \frac{(a + b \coth^{-1}(cx))(d + e \log(f + gx^2))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[f + g\*x^2]))/x,x]

[Out] Integrate[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[f + g\*x^2]))/x, x]

**Maple [A]** time = 0.74, size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{arccoth}(cx))(d + e \ln(gx^2 + f))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth(c\*x))\*(d+e\*ln(g\*x^2+f))/x,x)

[Out] int((a+b\*arccoth(c\*x))\*(d+e\*ln(g\*x^2+f))/x,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$ad \log(x) + \int \frac{be \left( \log\left(\frac{1}{cx} + 1\right) - \log\left(-\frac{1}{cx} + 1\right) \right) \log(gx^2 + f)}{2x} + \frac{bd \left( \log\left(\frac{1}{cx} + 1\right) - \log\left(-\frac{1}{cx} + 1\right) \right)}{2x} + \frac{ae \log(gx^2 + f)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(g\*x^2+f))/x,x, algorithm="maxima")

[Out] a\*d\*log(x) + integrate(1/2\*b\*e\*(log(1/(c\*x) + 1) - log(-1/(c\*x) + 1))\*log(g\*x^2 + f)/x + 1/2\*b\*d\*(log(1/(c\*x) + 1) - log(-1/(c\*x) + 1))/x + a\*e\*log(g\*x^2 + f)/x, x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{bd \operatorname{arccoth}(cx) + ad + (be \operatorname{arccoth}(cx) + ae) \log(gx^2 + f)}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(g\*x^2+f))/x,x, algorithm="fricas")

[Out] integral((b\*d\*arccoth(c\*x) + a\*d + (b\*e\*arccoth(c\*x) + a\*e)\*log(g\*x^2 + f))/x, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth(c\*x))\*(d+e\*ln(g\*x\*\*2+f))/x,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccoth}(cx) + a)(e \log(gx^2 + f) + d)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(g*x^2 + f) + d)/x, x)
```

$$3.281 \quad \int \frac{(a+b \coth^{-1}(cx))(d+e \log(f+gx^2))}{x^2} dx$$

**Optimal.** Leaf size=560

$$-\frac{1}{2}bce \operatorname{PolyLog}\left(2, \frac{c^2(f+gx^2)}{c^2f+g}\right) + \frac{1}{2}bce \operatorname{PolyLog}\left(2, \frac{gx^2}{f} + 1\right) - \frac{ibe\sqrt{g} \operatorname{PolyLog}\left(2, 1 + \frac{2\sqrt{f}\sqrt{g}(1-cx)}{(-\sqrt{g}+ic\sqrt{f})(\sqrt{f}-i\sqrt{gx})}\right)}{2\sqrt{f}} + \frac{ibe\sqrt{g} \operatorname{PolyLog}\left(2, 1 + \frac{2\sqrt{f}\sqrt{g}(1-cx)}{(\sqrt{g}+ic\sqrt{f})(\sqrt{f}+i\sqrt{gx})}\right)}{2\sqrt{f}}$$

```
[Out] (2*a*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] - (b*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[1 - 1/(c*x)])/Sqrt[f] + (b*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[1 + 1/(c*x)])/Sqrt[f] + (b*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(1 - c*x))/((I*c*Sqrt[f] - Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[f] - (b*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(1 + c*x))/((I*c*Sqrt[f] + Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[f] - ((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x + (b*c*Log[-((g*x^2)/f)]*(d + e*Log[f + g*x^2]))/2 - (b*c*Log[(g*(1 - c^2*x^2))/(c^2*f + g)]*(d + e*Log[f + g*x^2]))/2 - (b*c*e*PolyLog[2, (c^2*(f + g*x^2))/(c^2*f + g)])/2 + (b*c*e*PolyLog[2, 1 + (g*x^2)/f])/2 - ((I/2)*b*e*Sqrt[g]*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(1 - c*x))/((I*c*Sqrt[f] - Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[f] + ((I/2)*b*e*Sqrt[g]*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(1 + c*x))/((I*c*Sqrt[f] + Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[f]
```

**Rubi [A]** time = 1.26144, antiderivative size = 560, normalized size of antiderivative = 1., number of steps used = 38, number of rules used = 22, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$ , Rules used = {6082, 2475, 36, 29, 31, 2416, 2394, 2315, 2393, 2391, 5975, 205, 5973, 2470, 12, 260, 6688, 4876, 4848, 4856, 2402, 2447}

$$-\frac{1}{2}bce \operatorname{PolyLog}\left(2, \frac{c^2(f+gx^2)}{c^2f+g}\right) + \frac{1}{2}bce \operatorname{PolyLog}\left(2, \frac{gx^2}{f} + 1\right) - \frac{ibe\sqrt{g} \operatorname{PolyLog}\left(2, 1 + \frac{2\sqrt{f}\sqrt{g}(1-cx)}{(-\sqrt{g}+ic\sqrt{f})(\sqrt{f}-i\sqrt{gx})}\right)}{2\sqrt{f}} + \frac{ibe\sqrt{g} \operatorname{PolyLog}\left(2, 1 + \frac{2\sqrt{f}\sqrt{g}(1-cx)}{(\sqrt{g}+ic\sqrt{f})(\sqrt{f}+i\sqrt{gx})}\right)}{2\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x^2,x]
```

```
[Out] (2*a*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] - (b*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[1 - 1/(c*x)])/Sqrt[f] + (b*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[1 + 1/(c*x)])/Sqrt[f] + (b*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(1 - c*x))/((I*c*Sqrt[f] - Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[f] - (b*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(1 + c*x))/((I*c*Sqrt[f] + Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[f] - ((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x + (b*c*Log[-((g*x^2)/f)]*(d + e*Log[f + g*x^2]))/2 - (b*c*Log[(g*(1 - c^2*x^2))/(c^2*f + g)]*(d + e*Log[f + g*x^2]))/2 - (b*c*e*PolyLog[2, (c^2*(f + g*x^2))/(c^2*f + g)])/2 + (b*c*e*PolyLog[2, 1 + (g*x^2)/f])/2 - ((I/2)*b*e*Sqrt[g]*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(1 - c*x))/((I*c*Sqrt[f] - Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[f] + ((I/2)*b*e*Sqrt[g]*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(1 + c*x))/((I*c*Sqrt[f] + Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[f]
```

```

] - I*Sqrt[g]*x))/Sqrt[f] - (b*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[
(2*Sqrt[f]*Sqrt[g]*(1 + c*x))/((I*c*Sqrt[f] + Sqrt[g])*(Sqrt[f] - I*Sqrt[g]
*x)))]/Sqrt[f] - ((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x + (b*c*Log
[-((g*x^2)/f)]*(d + e*Log[f + g*x^2]))/2 - (b*c*Log[(g*(1 - c^2*x^2))/(c^2*
f + g)]*(d + e*Log[f + g*x^2]))/2 - (b*c*e*PolyLog[2, (c^2*(f + g*x^2))/(c^
2*f + g)]/2 + (b*c*e*PolyLog[2, 1 + (g*x^2)/f])/2 - ((I/2)*b*e*Sqrt[g]*Pol
yLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(1 - c*x))/((I*c*Sqrt[f] - Sqrt[g])*(Sqrt[f]
- I*Sqrt[g]*x)))]/Sqrt[f] + ((I/2)*b*e*Sqrt[g]*PolyLog[2, 1 - (2*Sqrt[f]*S
qrt[g]*(1 + c*x))/((I*c*Sqrt[f] + Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x)))]/Sqrt[
f]

```

### Rule 6082

```

Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.)*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.)*(x_)^(m_.), x_Symbol] := Simp[(x^(m + 1)*(d + e*Log[f + g*x^2])*(a +
b*ArcCoth[c*x]))/(m + 1), x] + (-Dist[(b*c)/(m + 1), Int[(x^(m + 1)*(d + e
*Log[f + g*x^2]))/(1 - c^2*x^2), x], x] - Dist[(2*e*g)/(m + 1), Int[(x^(m +
2)*(a + b*ArcCoth[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b, c, d, e, f,
g}, x] && ILtQ[m/2, 0]

```

### Rule 2475

```

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
|| IGtQ[q, 0])

```

### Rule 36

```

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c
- a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

```

### Rule 29

```

Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]

```

### Rule 31

```

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]

```

Rule 2416

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((h\_.)\*(x\_))^(m\_.)\*((f\_) + (g\_.)\*(x\_)^(r\_.))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n]]^p, (h\*x)^m\*(f + g\*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n]))/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 5975

Int[(ArcCoth[(c\_.)\*(x\_)]\*(b\_.) + (a\_.))/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] := Dist[a, Int[1/(d + e\*x^2), x], x] + Dist[b, Int[ArcCoth[c\*x]/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 5973

Int[ArcCoth[(c\_.)\*(x\_)]/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] := Dist[1/2, Int[Log[1 + 1/(c\*x)]/(d + e\*x^2), x], x] - Dist[1/2, Int[Log[1 - 1/(c\*x)]/(d + e\*x^2), x], x] /; FreeQ[{c, d, e}, x]

### Rule 2470

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)^2), x\_Symbol] := With[{u = IntHide[1/(f + g\*x^2), x]}, Simp[u\*(a + b\*Log[c\*(d + e\*x^n)^p]), x] - Dist[b\*e\*n\*p, Int[(u\*x^(n - 1))/(d + e\*x^n), x], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rule 6688

Int[u\_, x\_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]

### Rule 4876

Int[((a\_.) + ArcTan[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)\*((f\_.)\*(x\_)^(m\_.))\*((d\_.) + (e\_.)\*(x\_)^(q\_.)), x\_Symbol] := Int[ExpandIntegrand[(a + b\*ArcTan[c\*x])^p, (f\*x)^m\*(d + e\*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])

### Rule 4848

Int[((a\_.) + ArcTan[(c\_.)\*(x\_)]\*(b\_.))/(x\_), x\_Symbol] := Simp[a\*Log[x], x] + (Dist[(I\*b)/2, Int[Log[1 - I\*c\*x]/x, x], x] - Dist[(I\*b)/2, Int[Log[1 + I\*c\*x]/x, x], x]) /; FreeQ[{a, b, c}, x]

### Rule 4856

Int[((a\_.) + ArcTan[(c\_.)\*(x\_)]\*(b\_.))/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[((a + b\*ArcTan[c\*x])\*Log[2/(1 - I\*c\*x)])/e, x] + (Dist[(b\*c)/e, Int[Log[2/(1 - I\*c\*x)]/(1 + c^2\*x^2), x], x] - Dist[(b\*c)/e, Int[Log[(2\*c\*(d + e\*x))/(c\*d + I\*e)\*(1 - I\*c\*x)]/(1 + c^2\*x^2), x], x] + Simp[(a + b\*ArcTan[c

```
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))]/e, x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

### Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

### Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]]
```

### Rubi steps



$$\begin{aligned}
\int \frac{(a + b \coth^{-1}(cx))(d + e \log(f + gx^2))}{x^2} dx &= -\frac{(a + b \coth^{-1}(cx))(d + e \log(f + gx^2))}{x} + (bc) \int \frac{d + e \log(f + gx^2)}{x(1 - c^2x^2)} \\
&= -\frac{(a + b \coth^{-1}(cx))(d + e \log(f + gx^2))}{x} + \frac{1}{2}(bc) \text{Subst} \left( \int \frac{d + e \log(f + gx^2)}{x(1 - c^2x^2)} \right) \\
&= \frac{2ae\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{(a + b \coth^{-1}(cx))(d + e \log(f + gx^2))}{x} + \frac{1}{2}(bc) \\
&= \frac{2ae\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right) \log\left(1 - \frac{1}{cx}\right)}{\sqrt{f}} + \frac{be\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right)}{\sqrt{f}} \\
&= \frac{2ae\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right) \log\left(1 - \frac{1}{cx}\right)}{\sqrt{f}} + \frac{be\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right)}{\sqrt{f}} \\
&= \frac{2ae\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right) \log\left(1 - \frac{1}{cx}\right)}{\sqrt{f}} + \frac{be\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right)}{\sqrt{f}} \\
&= \frac{2ae\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right) \log\left(1 - \frac{1}{cx}\right)}{\sqrt{f}} + \frac{be\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right)}{\sqrt{f}} \\
&= \frac{2ae\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right) \log\left(1 - \frac{1}{cx}\right)}{\sqrt{f}} + \frac{be\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right)}{\sqrt{f}} \\
&= \frac{2ae\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right) \log\left(1 - \frac{1}{cx}\right)}{\sqrt{f}} + \frac{be\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right)}{\sqrt{f}} \\
&= \frac{2ae\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right) \log\left(1 - \frac{1}{cx}\right)}{\sqrt{f}} + \frac{be\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right)}{\sqrt{f}} \\
&= \frac{2ae\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right) \log\left(1 - \frac{1}{cx}\right)}{\sqrt{f}} + \frac{be\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right)}{\sqrt{f}} \\
&= \frac{2ae\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right) \log\left(1 - \frac{1}{cx}\right)}{\sqrt{f}} + \frac{be\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right)}{\sqrt{f}}
\end{aligned}$$

**Mathematica [B]** time = 3.56189, size = 1236, normalized size = 2.21

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[f + g\*x^2]))/x^2,x]

[Out] 
$$\begin{aligned} & -((a*d)/x) - (b*d*ArcCoth[c*x])/x + b*c*d*Log[x] - (b*c*d*Log[1 - c^2*x^2]) \\ & /2 + a*e*((2*sqrt[g]*ArcTan[(sqrt[g]*x)/sqrt[f]])/sqrt[f] - Log[f + g*x^2]/ \\ & x) + (b*e*(-(((2*ArcCoth[c*x] + c*x*(-2*Log[x] + Log[1 - c^2*x^2]))*Log[f + \\ & g*x^2])/x) - 2*c*(Log[x]*(Log[1 - (I*sqrt[g]*x)/sqrt[f]] + Log[1 + (I*sqrt \\ & [g]*x)/sqrt[f]]) + PolyLog[2, ((-I)*sqrt[g]*x)/sqrt[f]] + PolyLog[2, (I*sqrt \\ & [g]*x)/sqrt[f]]) + c*(Log[-c^(-1) + x]*Log[(c*(sqrt[f] - I*sqrt[g]*x))/(c* \\ & sqrt[f] - I*sqrt[g]]) + Log[-c^(-1) + x]*Log[(c*(sqrt[f] - I*sqrt[g]*x))/(c* \\ & sqrt[f] + I*sqrt[g]]) + Log[-c^(-1) + x]*Log[(c*(sqrt[f] + I*sqrt[g]*x))/(c \\ & *sqrt[f] + I*sqrt[g])) - (Log[-c^(-1) + x] + Log[c^(-1) + x] - Log[1 - c^2* \\ & x^2])*Log[f + g*x^2] + Log[c^(-1) + x]*Log[1 - (sqrt[g]*(1 + c*x))/(I*c*sqrt \\ & [f] + sqrt[g])] + PolyLog[2, (c*sqrt[g]*(c^(-1) + x))/(I*c*sqrt[f] + sqrt[ \\ & g])] + PolyLog[2, (I*sqrt[g]*(-1 + c*x))/(c*sqrt[f] - I*sqrt[g])] + PolyLog \\ & [2, ((-I)*sqrt[g]*(-1 + c*x))/(c*sqrt[f] + I*sqrt[g])] + PolyLog[2, (I*sqrt \\ & [g]*(1 + c*x))/(c*sqrt[f] + I*sqrt[g])) - (c*g*((2*I)*ArcCos[(c^2*f - g)/( \\ & c^2*f + g)]*ArcTan[(c*f)/(sqrt[c^2*f*g]*x)] - 4*ArcCoth[c*x]*ArcTan[(c*g*x) \\ & /sqrt[c^2*f*g]] + (ArcCos[(c^2*f - g)/(c^2*f + g)] + 2*ArcTan[(c*f)/(sqrt[ \\ & c^2*f*g]*x)])*Log[(2*g*(c^2*f - I*sqrt[c^2*f*g])*(-1 + c*x))/((c^2*f + g)*(I \\ & *sqrt[c^2*f*g] + c*g*x))] + (ArcCos[(c^2*f - g)/(c^2*f + g)] - 2*ArcTan[(c* \\ & f)/(sqrt[c^2*f*g]*x)]*Log[(2*g*(c^2*f + I*sqrt[c^2*f*g])*(1 + c*x))/((c^2* \\ & f + g)*(I*sqrt[c^2*f*g] + c*g*x))] - (ArcCos[(c^2*f - g)/(c^2*f + g)] + 2*( \\ & ArcTan[(c*f)/(sqrt[c^2*f*g]*x)] + ArcTan[(c*g*x)/sqrt[c^2*f*g]]))*Log[(sqrt[ \\ & 2]*sqrt[c^2*f*g])/(E^ArcCoth[c*x]*sqrt[c^2*f + g]*sqrt[-(c^2*f) + g + (c^2* \\ & *f + g)*Cosh[2*ArcCoth[c*x]])] - (ArcCos[(c^2*f - g)/(c^2*f + g)] - 2*(Arc \\ & Tan[(c*f)/(sqrt[c^2*f*g]*x)] + ArcTan[(c*g*x)/sqrt[c^2*f*g]]))*Log[(sqrt[ \\ & 2]*E^ArcCoth[c*x]*sqrt[c^2*f*g])/(sqrt[c^2*f + g]*sqrt[-(c^2*f) + g + (c^2*f \\ & + g)*Cosh[2*ArcCoth[c*x]])] + I*(PolyLog[2, ((c^2*f - g - (2*I)*sqrt[c^2*f \\ & *g])*(sqrt[c^2*f*g] + I*c*g*x))/((c^2*f + g)*(sqrt[c^2*f*g] - I*c*g*x))] - \\ & PolyLog[2, ((c^2*f - g + (2*I)*sqrt[c^2*f*g])*(sqrt[c^2*f*g] + I*c*g*x))/(( \\ & c^2*f + g)*(sqrt[c^2*f*g] - I*c*g*x)))]))/sqrt[c^2*f*g])/2 \end{aligned}$$

**Maple [F]** time = 0.832, size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{arccoth}(cx)) (d + e \ln(gx^2 + f))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arccoth(c*x))*(d+e*ln(g*x^2+f))/x^2,x)`

[Out] `int((a+b*arccoth(c*x))*(d+e*ln(g*x^2+f))/x^2,x)`

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bd \operatorname{arccoth}(cx) + ad + (be \operatorname{arccoth}(cx) + ae) \log(gx^2 + f)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="fricas")`

[Out] `integral((b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(g*x^2 + f))/x^2, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*acoth(c*x))*(d+e*ln(g*x**2+f))/x**2,x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccoth}(cx) + a)(e \log(gx^2 + f) + d)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(g\*x^2+f))/x^2,x, algorithm="giac")

[Out] integrate((b\*arccoth(c\*x) + a)\*(e\*log(g\*x^2 + f) + d)/x^2, x)

$$3.282 \quad \int \frac{(a+b \coth^{-1}(cx))(d+e \log(f+gx^2))}{x^3} dx$$

**Optimal.** Leaf size=712

$$\frac{1}{4}bc^2e \operatorname{PolyLog}\left(2, 1 - \frac{2c(\sqrt{-f} - \sqrt{gx})}{(cx+1)(c\sqrt{-f} - \sqrt{g})}\right) + \frac{1}{4}bc^2e \operatorname{PolyLog}\left(2, 1 - \frac{2c(\sqrt{-f} + \sqrt{gx})}{(cx+1)(c\sqrt{-f} + \sqrt{g})}\right) - \frac{1}{2}bc^2e \operatorname{PolyLog}\left(2, 1 - \frac{2c(\sqrt{-f} - \sqrt{gx})}{(cx+1)(c\sqrt{-f} - \sqrt{g})}\right)$$

```
[Out] (b*c*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] + (a*e*g*Log[x])/f + (b
*e*g*ArcCoth[c*x]*Log[2/(1+c*x)])/f + b*c^2*e*ArcTanh[c*x]*Log[2/(1+c*x
)] - (b*e*g*ArcCoth[c*x]*Log[(2*c*(Sqrt[-f]-Sqrt[g]*x))/((c*Sqrt[-f]-Sqrt
[g])*x)*(1+c*x))]/(2*f) - (b*c^2*e*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f]-Sqrt
[g])*x))/((c*Sqrt[-f]-Sqrt[g])*x)*(1+c*x)))/2 - (b*e*g*ArcCoth[c*x]*Log[(2
*c*(Sqrt[-f]+Sqrt[g]*x))/((c*Sqrt[-f]+Sqrt[g])*x)*(1+c*x))]/(2*f) - (b*
c^2*e*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f]+Sqrt[g])*x))/((c*Sqrt[-f]+Sqrt[g])*
x)*(1+c*x)))/2 - (a*e*g*Log[f+g*x^2])/f - (b*c*(d+e*Log[f+g*x^2]))/x - ((a
+b*ArcCoth[c*x])*(d+e*Log[f+g*x^2]))/x^2 + (b*c^2*ArcTanh[c*x]*(d+e*Log[f
+g*x^2]))/2 + (b*e*g*PolyLog[2,-(1/(c*x))])/f - (b*e*g*PolyLog[2,1/(c*x)])/f
- (b*c^2*e*PolyLog[2,1-2/(1+c*x)])/2 - (b*e*g*PolyLog[2,1-2/(1+c*x)])/f + (b*c^2*e*PolyLog[2,1-
(2*c*(Sqrt[-f]-Sqrt[g]*x))/((c*Sqrt[-f]-Sqrt[g])*x)*(1+c*x)))/4 + (b*e*
g*PolyLog[2,1-(2*c*(Sqrt[-f]-Sqrt[g]*x))/((c*Sqrt[-f]-Sqrt[g])*x)*(1+c*x)])/4
+ (b*c^2*e*PolyLog[2,1-(2*c*(Sqrt[-f]+Sqrt[g]*x))/((c*Sqrt[-f]+Sqrt[g])*x)*(1+c*x)])/4
+ (b*e*g*PolyLog[2,1-(2*c*(Sqrt[-f]+Sqrt[g]*x))/((c*Sqrt[-f]+Sqrt[g])*x)*(1+c*x)])/4
```

**Rubi [A]** time = 1.11314, antiderivative size = 712, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 17, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.708$ , Rules used = {5917, 325, 206, 6086, 6725, 801, 635, 205, 260, 5993, 5913, 5921, 2402, 2315, 2447, 5992, 5920}

$$\frac{1}{4}bc^2e \operatorname{PolyLog}\left(2, 1 - \frac{2c(\sqrt{-f} - \sqrt{gx})}{(cx+1)(c\sqrt{-f} - \sqrt{g})}\right) + \frac{1}{4}bc^2e \operatorname{PolyLog}\left(2, 1 - \frac{2c(\sqrt{-f} + \sqrt{gx})}{(cx+1)(c\sqrt{-f} + \sqrt{g})}\right) - \frac{1}{2}bc^2e \operatorname{PolyLog}\left(2, 1 - \frac{2c(\sqrt{-f} - \sqrt{gx})}{(cx+1)(c\sqrt{-f} - \sqrt{g})}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x^3,x]
```

```
[Out] (b*c*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] + (a*e*g*Log[x])/f + (b
*e*g*ArcCoth[c*x]*Log[2/(1+c*x)])/f + b*c^2*e*ArcTanh[c*x]*Log[2/(1+c*x
```

```

)] - (b*e*g*ArcCoth[c*x]*Log[(2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))]/(2*f) - (b*c^2*e*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))]/2 - (b*e*g*ArcCoth[c*x]*Log[(2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1 + c*x))]/(2*f) - (b*c^2*e*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1 + c*x))]/2 - (a*e*g*Log[f + g*x^2])/((2*f) - (b*c*(d + e*Log[f + g*x^2]))/(2*x) - ((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/(2*x^2) + (b*c^2*ArcTanh[c*x]*(d + e*Log[f + g*x^2]))/2 + (b*e*g*PolyLog[2, -(1/(c*x))])/((2*f) - (b*e*g*PolyLog[2, 1/(c*x))]/(2*f) - (b*c^2*e*PolyLog[2, 1 - 2/(1 + c*x)])/2 - (b*e*g*PolyLog[2, 1 - 2/(1 + c*x)])/(2*f) + (b*c^2*e*PolyLog[2, 1 - (2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))])/4 + (b*e*g*PolyLog[2, 1 - (2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))])/((4*f) + (b*c^2*e*PolyLog[2, 1 - (2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1 + c*x))])/4 + (b*e*g*PolyLog[2, 1 - (2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1 + c*x))])/((4*f)

```

### Rule 5917

```

Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^((p_.)*((d_.)*(x_.))^((m_.)), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

```

### Rule 325

```

Int[((c_.)*(x_.))^((m_.)*((a_.) + (b_.)*(x_.)^((n_.))^((p_.)), x_Symbol]
:> Simp[((c*x)^(m + 1)*(a + b*x^n)^p)/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

```

### Rule 206

```

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol]
:> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

```

### Rule 6086

```

Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(e_.))*(x_)^((m_.)), x_Symbol]
:> With[{u = IntHide[x^m*(a + b*ArcCoth[c*x]), x]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[(x*u)/(f + g*x^2), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ

```

[m] && NeQ[m, -1]

### Rule 6725

Int[(u\_)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := With[{v = RationalFunctionExpand[u/(a + b\*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

### Rule 801

Int[(((d\_.) + (e\_.)\*(x\_)^(m\_))\*((f\_.) + (g\_.)\*(x\_)))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[((d + e\*x)^m\*(f + g\*x))/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[m]

### Rule 635

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a\*c)]

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rule 5993

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))\*(x\_)^(m\_.)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[a + b\*ArcCoth[c\*x], x^m/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0])

### Rule 5913

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.)/(x\_), x\_Symbol] := Simp[a\*Log[x], x] + (Simp[(b\*PolyLog[2, -(c\*x)^(-1)])/2, x] - Simp[(b\*PolyLog[2, 1/(c\*x)])/2, x]) /; FreeQ[{a, b, c}, x]

### Rule 5921

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))/((d_) + (e_.)*(x_.)), x_Symbol] := -
Simp[((a + b*ArcCoth[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcCoth[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

### Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_.))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

### Rule 2315

```
Int[Log[(c_.)*(x_.)/((d_) + (e_.)*(x_.)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

### Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

### Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] := Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0]
)
```

### Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))/((d_) + (e_.)*(x_.)), x_Symbol] := -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

### Rubi steps



$$\begin{aligned}
\int \frac{(a + b \coth^{-1}(cx))(d + e \log(f + gx^2))}{x^3} dx &= -\frac{bc(d + e \log(f + gx^2))}{2x} - \frac{(a + b \coth^{-1}(cx))(d + e \log(f + gx^2))}{2x^2} + \\
&= -\frac{bc(d + e \log(f + gx^2))}{2x} - \frac{(a + b \coth^{-1}(cx))(d + e \log(f + gx^2))}{2x^2} + \\
&= -\frac{bc(d + e \log(f + gx^2))}{2x} - \frac{(a + b \coth^{-1}(cx))(d + e \log(f + gx^2))}{2x^2} + \\
&= -\frac{bc(d + e \log(f + gx^2))}{2x} - \frac{(a + b \coth^{-1}(cx))(d + e \log(f + gx^2))}{2x^2} + \\
&= bc^2 e \tanh^{-1}(cx) \log\left(\frac{2}{1+cx}\right) - \frac{1}{2} bc^2 e \tanh^{-1}(cx) \log\left(\frac{2c(\sqrt{-f}-\sqrt{g})}{(c\sqrt{-f}-\sqrt{g})}\right) + \\
&= \frac{aeg \log(x)}{f} + bc^2 e \tanh^{-1}(cx) \log\left(\frac{2}{1+cx}\right) - \frac{1}{2} bc^2 e \tanh^{-1}(cx) \log\left(\frac{2c(\sqrt{-f}-\sqrt{g})}{(c\sqrt{-f}-\sqrt{g})}\right) + \\
&= \frac{aeg \log(x)}{f} + bc^2 e \tanh^{-1}(cx) \log\left(\frac{2}{1+cx}\right) - \frac{1}{2} bc^2 e \tanh^{-1}(cx) \log\left(\frac{2c(\sqrt{-f}-\sqrt{g})}{(c\sqrt{-f}-\sqrt{g})}\right) + \\
&= \frac{bce\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right)}{\sqrt{f}} + \frac{aeg \log(x)}{f} + bc^2 e \tanh^{-1}(cx) \log\left(\frac{2}{1+cx}\right) - \frac{1}{2} bc^2 e \tanh^{-1}(cx) \log\left(\frac{2c(\sqrt{-f}-\sqrt{g})}{(c\sqrt{-f}-\sqrt{g})}\right) + \\
&= \frac{bce\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right)}{\sqrt{f}} + \frac{aeg \log(x)}{f} + \frac{beg \coth^{-1}(cx) \log\left(\frac{2}{1+cx}\right)}{f} + bc^2 e \tanh^{-1}(cx) \log\left(\frac{2}{1+cx}\right) + \\
&= \frac{bce\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right)}{\sqrt{f}} + \frac{aeg \log(x)}{f} + \frac{beg \coth^{-1}(cx) \log\left(\frac{2}{1+cx}\right)}{f} + bc^2 e \tanh^{-1}(cx) \log\left(\frac{2}{1+cx}\right) + \\
&= \frac{bce\sqrt{g} \tan^{-1}\left(\frac{\sqrt{gx}}{\sqrt{f}}\right)}{\sqrt{f}} + \frac{aeg \log(x)}{f} + \frac{beg \coth^{-1}(cx) \log\left(\frac{2}{1+cx}\right)}{f} + bc^2 e \tanh^{-1}(cx) \log\left(\frac{2}{1+cx}\right) +
\end{aligned}$$

**Mathematica [C]** time = 5.57142, size = 1318, normalized size = 1.85

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[f + g\*x^2]))/x^3,x]

[Out] 
$$-(2*a*d*f - 4*b*c*e*\sqrt{f}*\sqrt{g}*x^2*\text{ArcTan}[\frac{\sqrt{g}*x}{\sqrt{f}}] - 4*a*e*g*x^2*\text{Log}[x] + 2*a*e*g*x^2*\text{Log}[f + g*x^2] + 2*e*f*(a + b*c*x + (b - b*c^2*x^2)*\text{ArcCoth}[c*x])*\text{Log}[f + g*x^2] + b*c^2*e*f*x^2*(-4*\text{ArcCoth}[c*x]^2 - 4*\text{ArcCoth}[c*x]*\text{Log}[1 - E^{(-2*\text{ArcCoth}[c*x])}] + 2*\text{ArcCoth}[c*x]*\text{Log}[1 + (E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g))]/(-c^2*f) - 2*c*\sqrt{-f}*\sqrt{g} + g)] + 2*\text{ArcCoth}[c*x]*\text{Log}[1 + (E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g))]/(-c^2*f) + 2*c*\sqrt{-f}*\sqrt{g} + g)] + 2*\text{PolyLog}[2, E^{(-2*\text{ArcCoth}[c*x])}] + \text{PolyLog}[2, (E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g))/(c^2*f - 2*c*\sqrt{-f}*\sqrt{g} - g)] + \text{PolyLog}[2, (E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g))/(c^2*f + 2*c*\sqrt{-f}*\sqrt{g} - g)]) - b*d*(-2*(c*f*x + g*x^2*\text{ArcCoth}[c*x]^2 + \text{ArcCoth}[c*x]*(f - c^2*f*x^2 + 2*g*x^2*\text{Log}[1 + E^{(-2*\text{ArcCoth}[c*x])}])) - g*x^2*\text{PolyLog}[2, -E^{(-2*\text{ArcCoth}[c*x])}]) + g*x^2*(2*\text{ArcCoth}[c*x]*(-\text{ArcCoth}[c*x] + \text{Log}[1 + (E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g))]/(-c^2*f) - 2*c*\sqrt{-f}*\sqrt{g} + g)] + \text{Log}[1 + (E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g))]/(-c^2*f) + 2*c*\sqrt{-f}*\sqrt{g} + g)] + \text{PolyLog}[2, (E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g))/(c^2*f - 2*c*\sqrt{-f}*\sqrt{g} - g)] + \text{PolyLog}[2, (E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g))/(c^2*f + 2*c*\sqrt{-f}*\sqrt{g} - g)]) + b*d*g*x^2*(2*\text{ArcCoth}[c*x]^2 - (4*I)*\text{ArcSin}[\sqrt{g/(c^2*f + g)}]*\text{ArcTanh}[(c*f)/(\sqrt{-(c^2*f*g)}*x)] - 2*\text{ArcCoth}[c*x]*(\text{ArcCoth}[c*x] + 2*\text{Log}[1 + E^{(-2*\text{ArcCoth}[c*x])}])) + 2*(\text{ArcCoth}[c*x] - I*\text{ArcSin}[\sqrt{g/(c^2*f + g)}])* \text{Log}[(c^2*(-1 + E^{(2*\text{ArcCoth}[c*x])})*f + g + E^{(2*\text{ArcCoth}[c*x])}*g - 2*\sqrt{-(c^2*f*g)}]/(E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g))] + 2*(\text{ArcCoth}[c*x] + I*\text{ArcSin}[\sqrt{g/(c^2*f + g)}])* \text{Log}[(c^2*(-1 + E^{(2*\text{ArcCoth}[c*x])})*f + g + E^{(2*\text{ArcCoth}[c*x])}*g + 2*\sqrt{-(c^2*f*g)}]/(E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g))] + 2*\text{PolyLog}[2, -E^{(-2*\text{ArcCoth}[c*x])}] - \text{PolyLog}[2, (c^2*f - g + 2*\sqrt{-(c^2*f*g)}]/(E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g))] - \text{PolyLog}[2, -((-c^2*f) + g + 2*\sqrt{-(c^2*f*g)}]/(E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g)))) + b*e*g*x^2*(2*\text{ArcCoth}[c*x]^2 - (4*I)*\text{ArcSin}[\sqrt{g/(c^2*f + g)}]*\text{ArcTanh}[(c*f)/(\sqrt{-(c^2*f*g)}*x)] - 2*\text{ArcCoth}[c*x]*(\text{ArcCoth}[c*x] + 2*\text{Log}[1 + E^{(-2*\text{ArcCoth}[c*x])}])) + 2*(\text{ArcCoth}[c*x] - I*\text{ArcSin}[\sqrt{g/(c^2*f + g)}])* \text{Log}[(c^2*(-1 + E^{(2*\text{ArcCoth}[c*x])})*f + g + E^{(2*\text{ArcCoth}[c*x])}*g - 2*\sqrt{-(c^2*f*g)}]/(E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g))] + 2*(\text{ArcCoth}[c*x] + I*\text{ArcSin}[\sqrt{g/(c^2*f + g)}])* \text{Log}[(c^2*(-1 + E^{(2*\text{ArcCoth}[c*x])})*f + g + E^{(2*\text{ArcCoth}[c*x])}*g + 2*\sqrt{-(c^2*f*g)}]/(E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g))] + 2*\text{PolyLog}[2, -E^{(-2*\text{ArcCoth}[c*x])}] - \text{PolyLog}[2, (c^2*f - g + 2*\sqrt{-(c^2*f*g)}]/(E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g))] - \text{PolyLog}[2, -((-c^2*f) + g + 2*\sqrt{-(c^2*f*g)}]/(E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g)))))/(4*f*x^2)$$

---

**Maple [A]** time = 3.198, size = 937, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((a+b*\text{arccoth}(c*x))*(d+e*\ln(g*x^2+f))/x^3,x)$

[Out] 
$$\begin{aligned} & -1/2*b*c*d/x+a*e*g*\ln(x)/f-1/2*a*e*g*\ln(g*x^2+f)/f-1/2/x^2*a*d-1/2*g*b*e/f* \\ & \text{dilog}(c*x+1)-1/4*b*e*\text{dilog}((c*(-f*g)^{(1/2)}-(c*x+1)*g+g)/(c*(-f*g)^{(1/2)}+g)) \\ & *c^2-1/4*b*e*\text{dilog}((c*(-f*g)^{(1/2)}+(c*x+1)*g-g)/(c*(-f*g)^{(1/2)}-g))*c^2+1/4 \\ & *d*c^2*b*\ln(c*x+1)-1/4*d*b*\ln(c*x+1)/x^2+1/4*b*e*\text{dilog}((c*(-f*g)^{(1/2)}-(c*x \\ & -1)*g-g)/(c*(-f*g)^{(1/2)}-g))*c^2+1/4*b*e*\text{dilog}((c*(-f*g)^{(1/2)}+(c*x-1)*g+g) \\ & /((c*(-f*g)^{(1/2)}+g))*c^2-1/4*g*b*e/f*\ln(c*x+1)*\ln((c*(-f*g)^{(1/2)}-(c*x+1)*g \\ & +g)/(c*(-f*g)^{(1/2)}+g))-1/4*g*b*e/f*\ln(c*x+1)*\ln((c*(-f*g)^{(1/2)}+(c*x+1)*g- \\ & g)/(c*(-f*g)^{(1/2)}-g))+g*e*b*c/(f*g)^{(1/2)}*\arctan(x*g/(f*g)^{(1/2)})-1/4*d*c^ \\ & 2*b*\ln(c*x-1)+1/4*d*b*\ln(c*x-1)/x^2-1/4*b*e*\ln(c*x+1)*\ln((c*(-f*g)^{(1/2)}-(c \\ & *x+1)*g+g)/(c*(-f*g)^{(1/2)}+g))*c^2-1/4*b*e*\ln(c*x+1)*\ln((c*(-f*g)^{(1/2)}+(c \\ & x+1)*g-g)/(c*(-f*g)^{(1/2)}-g))*c^2-1/4*g*b*e/f*\text{dilog}((c*(-f*g)^{(1/2)}-(c*x+1) \\ & *g+g)/(c*(-f*g)^{(1/2)}+g))-1/4*g*b*e/f*\text{dilog}((c*(-f*g)^{(1/2)}+(c*x+1)*g-g)/(c \\ & *(-f*g)^{(1/2)}-g))+1/4*b*e*\ln(c*x-1)*\ln((c*(-f*g)^{(1/2)}-(c*x-1)*g-g)/(c*(-f* \\ & g)^{(1/2)}-g))*c^2+1/4*b*e*\ln(c*x-1)*\ln((c*(-f*g)^{(1/2)}+(c*x-1)*g+g)/(c*(-f*g) \\ & )^{(1/2)}+g))*c^2+1/4*g*b*e/f*\text{dilog}((c*(-f*g)^{(1/2)}-(c*x-1)*g-g)/(c*(-f*g)^{(1 \\ & /2)}-g))+1/4*g*b*e/f*\text{dilog}((c*(-f*g)^{(1/2)}+(c*x-1)*g+g)/(c*(-f*g)^{(1/2)}+g))- \\ & 1/2*g*b*e/f*\text{dilog}(c*x)+(-1/4*b*e/x^2*\ln(c*x+1)-1/4*e*(c^2*b*\ln(c*x-1)*x^2-c \\ & ^2*b*\ln(c*x+1)*x^2+2*x*b*c-b*\ln(c*x-1)+2*a)/x^2)*\ln(g*x^2+f)+1/4*g*b*e/f*\ln \\ & (c*x-1)*\ln((c*(-f*g)^{(1/2)}-(c*x-1)*g-g)/(c*(-f*g)^{(1/2)}-g))+1/4*g*b*e/f*\ln \\ & (c*x-1)*\ln((c*(-f*g)^{(1/2)}+(c*x-1)*g+g)/(c*(-f*g)^{(1/2)}+g))-1/2*g*b*e/f*\ln(c \\ & *x-1)*\ln(c*x) \end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((a+b*\text{arccoth}(c*x))*(d+e*\log(g*x^2+f))/x^3,x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bd \operatorname{arccoth}(cx) + ad + (be \operatorname{arccoth}(cx) + ae) \log(gx^2 + f)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*d*arccoth(c*x) + a*d + (b*e*arccoth(c*x) + a*e)*log(g*x^2 + f))
/x^3, x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(c*x))*(d+e*ln(g*x**2+f))/x**3,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arccoth}(cx) + a)(e \log(gx^2 + f) + d)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(g*x^2 + f) + d)/x^3, x)
```

### 3.283 $\int \coth^{-1}(e^x) dx$

Optimal. Leaf size=25

$$\frac{1}{2}\text{PolyLog}(2, -e^{-x}) - \frac{1}{2}\text{PolyLog}(2, e^{-x})$$

[Out] PolyLog[2, -E^(-x)]/2 - PolyLog[2, E^(-x)]/2

Rubi [A] time = 0.0121473, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {2282, 5913}

$$\frac{1}{2}\text{PolyLog}(2, -e^{-x}) - \frac{1}{2}\text{PolyLog}(2, e^{-x})$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[E^x], x]

[Out] PolyLog[2, -E^(-x)]/2 - PolyLog[2, E^(-x)]/2

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 5913

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))/(x_), x_Symbol] := Simp[a*Log[x], x] + (Simp[(b*PolyLog[2, -(c*x)^(-1)])/2, x] - Simp[(b*PolyLog[2, 1/(c*x)])/2, x]) /; FreeQ[{a, b, c}, x]
```

#### Rubi steps

$$\int \coth^{-1}(e^x) dx = \text{Subst} \left( \int \frac{\coth^{-1}(x)}{x} dx, x, e^x \right) \\ = \frac{\text{Li}_2(-e^{-x})}{2} - \frac{\text{Li}_2(e^{-x})}{2}$$

**Mathematica [B]** time = 0.0343158, size = 51, normalized size = 2.04

$$-\frac{1}{2}\text{PolyLog}(2, -e^x) + \frac{1}{2}\text{PolyLog}(2, e^x) + \frac{1}{2}x \log(1 - e^x) - \frac{1}{2}x \log(e^x + 1) + x \coth^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[E^x], x]

[Out] x\*ArcCoth[E^x] + (x\*Log[1 - E^x])/2 - (x\*Log[1 + E^x])/2 - PolyLog[2, -E^x]/2 + PolyLog[2, E^x]/2

**Maple [A]** time = 0.052, size = 31, normalized size = 1.2

$$\ln(e^x) \operatorname{arccoth}(e^x) - \frac{\operatorname{dilog}(e^x)}{2} - \frac{\operatorname{dilog}(e^x + 1)}{2} - \frac{\ln(e^x) \ln(e^x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(exp(x)), x)

[Out] ln(exp(x))\*arccoth(exp(x))-1/2\*dilog(exp(x))-1/2\*dilog(exp(x)+1)-1/2\*ln(exp(x))\*ln(exp(x)+1)

**Maxima [B]** time = 1.11888, size = 78, normalized size = 3.12

$$-\frac{1}{2}x(\log(e^x + 1) - \log(e^x - 1)) + x \operatorname{arccoth}(e^x) + \frac{1}{2} \log(-e^x) \log(e^x + 1) - \frac{1}{2}x \log(e^x - 1) + \frac{1}{2} \text{Li}_2(e^x + 1) - \frac{1}{2} \text{Li}_2(-e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(exp(x)), x, algorithm="maxima")

[Out]  $-1/2*x*(\log(e^x + 1) - \log(e^x - 1)) + x*\operatorname{arccoth}(e^x) + 1/2*\log(-e^x)*\log(e^x + 1) - 1/2*x*\log(e^x - 1) + 1/2*\operatorname{dilog}(e^x + 1) - 1/2*\operatorname{dilog}(-e^x + 1)$

**Fricas [B]** time = 1.77649, size = 262, normalized size = 10.48

$\frac{1}{2}x \log\left(\frac{\cosh(x) + \sinh(x) + 1}{\cosh(x) + \sinh(x) - 1}\right) - \frac{1}{2}x \log(\cosh(x) + \sinh(x) + 1) + \frac{1}{2}x \log(-\cosh(x) - \sinh(x) + 1) + \frac{1}{2}\operatorname{Li}_2(\cosh(x) + \sinh(x) + 1) - \frac{1}{2}\operatorname{Li}_2(-\cosh(x) - \sinh(x) + 1)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(exp(x)),x, algorithm="fricas")`

[Out]  $1/2*x*\log((\cosh(x) + \sinh(x) + 1)/(\cosh(x) + \sinh(x) - 1)) - 1/2*x*\log(\cosh(x) + \sinh(x) + 1) + 1/2*x*\log(-\cosh(x) - \sinh(x) + 1) + 1/2*\operatorname{dilog}(\cosh(x) + \sinh(x)) - 1/2*\operatorname{dilog}(-\cosh(x) - \sinh(x))$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acoth}(e^x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(exp(x)),x)`

[Out] `Integral(acoth(exp(x)), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(e^x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(exp(x)),x, algorithm="giac")`

[Out] `integrate(arccoth(e^x), x)`

### 3.284 $\int x \coth^{-1}(e^x) dx$

**Optimal.** Leaf size=51

$$\frac{1}{2}x\text{PolyLog}(2, -e^{-x}) - \frac{1}{2}x\text{PolyLog}(2, e^{-x}) + \frac{1}{2}\text{PolyLog}(3, -e^{-x}) - \frac{1}{2}\text{PolyLog}(3, e^{-x})$$

[Out] (x\*PolyLog[2, -E^(-x)])/2 - (x\*PolyLog[2, E^(-x)])/2 + PolyLog[3, -E^(-x)]/2 - PolyLog[3, E^(-x)]/2

**Rubi [A]** time = 0.0474447, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {6214, 2531, 2282, 6589}

$$\frac{1}{2}x\text{PolyLog}(2, -e^{-x}) - \frac{1}{2}x\text{PolyLog}(2, e^{-x}) + \frac{1}{2}\text{PolyLog}(3, -e^{-x}) - \frac{1}{2}\text{PolyLog}(3, e^{-x})$$

Antiderivative was successfully verified.

[In] Int[x\*ArcCoth[E^x], x]

[Out] (x\*PolyLog[2, -E^(-x)])/2 - (x\*PolyLog[2, E^(-x)])/2 + PolyLog[3, -E^(-x)]/2 - PolyLog[3, E^(-x)]/2

#### Rule 6214

```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Dist[1/2, Int
[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IGtQ[m, 0]
```

#### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

#### Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
```



```

onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

```

### Rule 6589

```

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]

```

### Rubi steps

$$\begin{aligned}
\int x \coth^{-1}(e^x) dx &= -\left(\frac{1}{2} \int x \log(1 - e^{-x}) dx\right) + \frac{1}{2} \int x \log(1 + e^{-x}) dx \\
&= \frac{1}{2} x \text{Li}_2(-e^{-x}) - \frac{1}{2} x \text{Li}_2(e^{-x}) - \frac{1}{2} \int \text{Li}_2(-e^{-x}) dx + \frac{1}{2} \int \text{Li}_2(e^{-x}) dx \\
&= \frac{1}{2} x \text{Li}_2(-e^{-x}) - \frac{1}{2} x \text{Li}_2(e^{-x}) + \frac{1}{2} \text{Subst}\left(\int \frac{\text{Li}_2(-x)}{x} dx, x, e^{-x}\right) - \frac{1}{2} \text{Subst}\left(\int \frac{\text{Li}_2(x)}{x} dx, x, e^{-x}\right) \\
&= \frac{1}{2} x \text{Li}_2(-e^{-x}) - \frac{1}{2} x \text{Li}_2(e^{-x}) + \frac{\text{Li}_3(-e^{-x})}{2} - \frac{\text{Li}_3(e^{-x})}{2}
\end{aligned}$$

**Mathematica [A]** time = 0.0223064, size = 71, normalized size = 1.39

$$\frac{1}{4} \left( -2x \text{PolyLog}(2, -e^x) + 2x \text{PolyLog}(2, e^x) + 2 \text{PolyLog}(3, -e^x) - 2 \text{PolyLog}(3, e^x) + x^2 \log(1 - e^x) - x^2 \log(e^x + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[E^x], x]

[Out] (2\*x^2\*ArcCoth[E^x] + x^2\*Log[1 - E^x] - x^2\*Log[1 + E^x] - 2\*x\*PolyLog[2, -E^x] + 2\*x\*PolyLog[2, E^x] + 2\*PolyLog[3, -E^x] - 2\*PolyLog[3, E^x])/4

**Maple [A]** time = 0.049, size = 62, normalized size = 1.2

$$\frac{x^2 \operatorname{arccoth}(e^x)}{2} - \frac{x^2 \ln(e^x + 1)}{4} - \frac{x \operatorname{polylog}(2, -e^x)}{2} + \frac{\operatorname{polylog}(3, -e^x)}{2} + \frac{x^2 \ln(1 - e^x)}{4} + \frac{x \operatorname{polylog}(2, e^x)}{2} - \frac{\operatorname{polylog}(3, e^x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*arccoth(exp(x)),x)`

[Out]  $\frac{1}{2}x^2 \operatorname{arccoth}(\exp(x)) - \frac{1}{4}x^2 \ln(\exp(x)+1) - \frac{1}{2}x \operatorname{polylog}(2, -\exp(x)) + \frac{1}{2} \operatorname{polylog}(3, -\exp(x)) + \frac{1}{4}x^2 \ln(1-\exp(x)) + \frac{1}{2}x \operatorname{polylog}(2, \exp(x)) - \frac{1}{2} \operatorname{polylog}(3, \exp(x))$

**Maxima [A]** time = 1.1231, size = 80, normalized size = 1.57

$$\frac{1}{2} x^2 \operatorname{arccoth}(e^x) - \frac{1}{4} x^2 \log(e^x + 1) + \frac{1}{4} x^2 \log(-e^x + 1) - \frac{1}{2} x \operatorname{Li}_2(-e^x) + \frac{1}{2} x \operatorname{Li}_2(e^x) + \frac{1}{2} \operatorname{Li}_3(-e^x) - \frac{1}{2} \operatorname{Li}_3(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccoth(exp(x)),x, algorithm="maxima")`

[Out]  $\frac{1}{2}x^2 \operatorname{arccoth}(e^x) - \frac{1}{4}x^2 \log(e^x + 1) + \frac{1}{4}x^2 \log(-e^x + 1) - \frac{1}{2}x \operatorname{dilog}(-e^x) + \frac{1}{2}x \operatorname{dilog}(e^x) + \frac{1}{2} \operatorname{polylog}(3, -e^x) - \frac{1}{2} \operatorname{polylog}(3, e^x)$

**Fricas [C]** time = 1.6609, size = 374, normalized size = 7.33

$$\frac{1}{4} x^2 \log\left(\frac{\cosh(x) + \sinh(x) + 1}{\cosh(x) + \sinh(x) - 1}\right) - \frac{1}{4} x^2 \log(\cosh(x) + \sinh(x) + 1) + \frac{1}{4} x^2 \log(-\cosh(x) - \sinh(x) + 1) + \frac{1}{2} x \operatorname{Li}_2(\cosh(x) + \sinh(x)) - \frac{1}{2} x \operatorname{Li}_2(-\cosh(x) - \sinh(x)) + \frac{1}{2} \operatorname{polylog}(3, \cosh(x) + \sinh(x)) - \frac{1}{2} \operatorname{polylog}(3, -\cosh(x) - \sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccoth(exp(x)),x, algorithm="fricas")`

[Out]  $\frac{1}{4}x^2 \log\left(\frac{\cosh(x) + \sinh(x) + 1}{\cosh(x) + \sinh(x) - 1}\right) - \frac{1}{4}x^2 \log(\cosh(x) + \sinh(x) + 1) + \frac{1}{4}x^2 \log(-\cosh(x) - \sinh(x) + 1) + \frac{1}{2}x \operatorname{dilog}(\cosh(x) + \sinh(x)) - \frac{1}{2}x \operatorname{dilog}(-\cosh(x) - \sinh(x)) - \frac{1}{2} \operatorname{polylog}(3, \cosh(x) + \sinh(x)) + \frac{1}{2} \operatorname{polylog}(3, -\cosh(x) - \sinh(x))$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{acoth}(e^x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(exp(x)),x)
```

```
[Out] Integral(x*acoth(exp(x)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arccoth}(e^x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(exp(x)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(e^x), x)
```

### 3.285 $\int x^2 \coth^{-1}(e^x) dx$

**Optimal.** Leaf size=70

$$\frac{1}{2}x^2\text{PolyLog}(2, -e^{-x}) - \frac{1}{2}x^2\text{PolyLog}(2, e^{-x}) + x\text{PolyLog}(3, -e^{-x}) - x\text{PolyLog}(3, e^{-x}) + \text{PolyLog}(4, -e^{-x}) - \text{PolyLog}(4, e^{-x})$$

[Out] (x^2\*PolyLog[2, -E^(-x)])/2 - (x^2\*PolyLog[2, E^(-x)])/2 + x\*PolyLog[3, -E^(-x)] - x\*PolyLog[3, E^(-x)] + PolyLog[4, -E^(-x)] - PolyLog[4, E^(-x)]

**Rubi [A]** time = 0.0730378, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$ , Rules used = {6214, 2531, 6609, 2282, 6589}

$$\frac{1}{2}x^2\text{PolyLog}(2, -e^{-x}) - \frac{1}{2}x^2\text{PolyLog}(2, e^{-x}) + x\text{PolyLog}(3, -e^{-x}) - x\text{PolyLog}(3, e^{-x}) + \text{PolyLog}(4, -e^{-x}) - \text{PolyLog}(4, e^{-x})$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcCoth[E^x], x]

[Out] (x^2\*PolyLog[2, -E^(-x)])/2 - (x^2\*PolyLog[2, E^(-x)])/2 + x\*PolyLog[3, -E^(-x)] - x\*PolyLog[3, E^(-x)] + PolyLog[4, -E^(-x)] - PolyLog[4, E^(-x)]

#### Rule 6214

```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Dist[1/2, Int
[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IGtQ[m, 0]
```

#### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

#### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
```

```
+ b*x)))^p]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned} \int x^2 \coth^{-1}(e^x) dx &= -\left(\frac{1}{2} \int x^2 \log(1 - e^{-x}) dx\right) + \frac{1}{2} \int x^2 \log(1 + e^{-x}) dx \\ &= \frac{1}{2} x^2 \text{Li}_2(-e^{-x}) - \frac{1}{2} x^2 \text{Li}_2(e^{-x}) - \int x \text{Li}_2(-e^{-x}) dx + \int x \text{Li}_2(e^{-x}) dx \\ &= \frac{1}{2} x^2 \text{Li}_2(-e^{-x}) - \frac{1}{2} x^2 \text{Li}_2(e^{-x}) + x \text{Li}_3(-e^{-x}) - x \text{Li}_3(e^{-x}) - \int \text{Li}_3(-e^{-x}) dx + \int \text{Li}_3(e^{-x}) dx \\ &= \frac{1}{2} x^2 \text{Li}_2(-e^{-x}) - \frac{1}{2} x^2 \text{Li}_2(e^{-x}) + x \text{Li}_3(-e^{-x}) - x \text{Li}_3(e^{-x}) + \text{Subst}\left(\int \frac{\text{Li}_3(-x)}{x} dx, x, e^{-x}\right) - \text{Subst}\left(\int \frac{\text{Li}_3(x)}{x} dx, x, e^{-x}\right) \\ &= \frac{1}{2} x^2 \text{Li}_2(-e^{-x}) - \frac{1}{2} x^2 \text{Li}_2(e^{-x}) + x \text{Li}_3(-e^{-x}) - x \text{Li}_3(e^{-x}) + \text{Li}_4(-e^{-x}) - \text{Li}_4(e^{-x}) \end{aligned}$$

**Mathematica [A]** time = 0.0193149, size = 93, normalized size = 1.33

$$\frac{1}{6} \left( -3x^2 \text{PolyLog}(2, -e^x) + 3x^2 \text{PolyLog}(2, e^x) + 6x \text{PolyLog}(3, -e^x) - 6x \text{PolyLog}(3, e^x) - 6 \text{PolyLog}(4, -e^x) + 6 \text{PolyLog}(4, e^x) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[E^x], x]
```

```
[Out] (2*x^3*ArcCoth[E^x] + x^3*Log[1 - E^x] - x^3*Log[1 + E^x] - 3*x^2*PolyLog[2, -E^x] + 3*x^2*PolyLog[2, E^x] + 6*x*PolyLog[3, -E^x] - 6*x*PolyLog[3, E^x]
```

] - 6\*PolyLog[4, -E^x] + 6\*PolyLog[4, E^x])/6

**Maple [A]** time = 0.046, size = 79, normalized size = 1.1

$$\frac{x^3 \operatorname{arccoth}(e^x)}{3} - \frac{x^3 \ln(e^x + 1)}{6} - \frac{x^2 \operatorname{polylog}(2, -e^x)}{2} + x \operatorname{polylog}(3, -e^x) - \operatorname{polylog}(4, -e^x) + \frac{x^3 \ln(1 - e^x)}{6} + \frac{x^2 \operatorname{polylog}(2, e^x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(exp(x)),x)

[Out] 1/3\*x^3\*arccoth(exp(x))-1/6\*x^3\*ln(exp(x)+1)-1/2\*x^2\*polylog(2,-exp(x))+x\*polylog(3,-exp(x))-polylog(4,-exp(x))+1/6\*x^3\*ln(1-exp(x))+1/2\*x^2\*polylog(2,exp(x))-x\*polylog(3,exp(x))+polylog(4,exp(x))

**Maxima [A]** time = 1.10916, size = 103, normalized size = 1.47

$$\frac{1}{3} x^3 \operatorname{arccoth}(e^x) - \frac{1}{6} x^3 \log(e^x + 1) + \frac{1}{6} x^3 \log(-e^x + 1) - \frac{1}{2} x^2 \operatorname{Li}_2(-e^x) + \frac{1}{2} x^2 \operatorname{Li}_2(e^x) + x \operatorname{Li}_3(-e^x) - x \operatorname{Li}_3(e^x) - \operatorname{Li}_4(-e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(exp(x)),x, algorithm="maxima")

[Out] 1/3\*x^3\*arccoth(e^x) - 1/6\*x^3\*log(e^x + 1) + 1/6\*x^3\*log(-e^x + 1) - 1/2\*x^2\*dilog(-e^x) + 1/2\*x^2\*dilog(e^x) + x\*polylog(3, -e^x) - x\*polylog(3, e^x) - polylog(4, -e^x) + polylog(4, e^x)

**Fricas [C]** time = 1.77765, size = 462, normalized size = 6.6

$$\frac{1}{6} x^3 \log\left(\frac{\cosh(x) + \sinh(x) + 1}{\cosh(x) + \sinh(x) - 1}\right) - \frac{1}{6} x^3 \log(\cosh(x) + \sinh(x) + 1) + \frac{1}{6} x^3 \log(-\cosh(x) - \sinh(x) + 1) + \frac{1}{2} x^2 \operatorname{Li}_2(\cosh(x) + \sinh(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(exp(x)),x, algorithm="fricas")

```
[Out] 1/6*x^3*log((cosh(x) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) - 1/6*x^3*log(cosh(x) + sinh(x) + 1) + 1/6*x^3*log(-cosh(x) - sinh(x) + 1) + 1/2*x^2*dilog(cosh(x) + sinh(x)) - 1/2*x^2*dilog(-cosh(x) - sinh(x)) - x*polylog(3, cosh(x) + sinh(x)) + x*polylog(3, -cosh(x) - sinh(x)) + polylog(4, cosh(x) + sinh(x)) - polylog(4, -cosh(x) - sinh(x))
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{acoth}(e^x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(exp(x)),x)
```

```
[Out] Integral(x**2*acoth(exp(x)), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arccoth}(e^x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(exp(x)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(e^x), x)
```

### 3.286 $\int \coth^{-1}(e^{a+bx}) dx$

**Optimal.** Leaf size=41

$$\frac{\text{PolyLog}(2, -e^{-a-bx})}{2b} - \frac{\text{PolyLog}(2, e^{-a-bx})}{2b}$$

[Out] PolyLog[2, -E^(-a - b\*x)]/(2\*b) - PolyLog[2, E^(-a - b\*x)]/(2\*b)

**Rubi [A]** time = 0.0164433, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {2282, 5913}

$$\frac{\text{PolyLog}(2, -e^{-a-bx})}{2b} - \frac{\text{PolyLog}(2, e^{-a-bx})}{2b}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[E^(a + b\*x)], x]

[Out] PolyLog[2, -E^(-a - b\*x)]/(2\*b) - PolyLog[2, E^(-a - b\*x)]/(2\*b)

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 5913

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] := Simp[a*Log[x], x] + (Simp[(b*PolyLog[2, -(c*x)^(-1)])/2, x] - Simp[(b*PolyLog[2, 1/(c*x)])/2, x]) /; FreeQ[{a, b, c}, x]
```

#### Rubi steps



$$\int \coth^{-1}(e^{a+bx}) dx = \frac{\text{Subst}\left(\int \frac{\coth^{-1}(x)}{x} dx, x, e^{a+bx}\right)}{b}$$

$$= \frac{\text{Li}_2(-e^{-a-bx})}{2b} - \frac{\text{Li}_2(e^{-a-bx})}{2b}$$

**Mathematica [A]** time = 0.0725544, size = 68, normalized size = 1.66

$$\frac{-\text{PolyLog}\left(2, -e^{a+bx}\right) + \text{PolyLog}\left(2, e^{a+bx}\right) + bx\left(\log\left(1 - e^{a+bx}\right) - \log\left(e^{a+bx} + 1\right) + 2 \coth^{-1}\left(e^{a+bx}\right)\right)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[E^(a + b\*x)], x]

[Out] (b\*x\*(2\*ArcCoth[E^(a + b\*x)] + Log[1 - E^(a + b\*x)] - Log[1 + E^(a + b\*x)]) - PolyLog[2, -E^(a + b\*x)] + PolyLog[2, E^(a + b\*x)])/(2\*b)

**Maple [A]** time = 0.072, size = 67, normalized size = 1.6

$$\frac{\ln(e^{bx+a}) \operatorname{arccoth}(e^{bx+a})}{b} - \frac{\operatorname{dilog}(e^{bx+a})}{2b} - \frac{\operatorname{dilog}(e^{bx+a} + 1)}{2b} - \frac{\ln(e^{bx+a}) \ln(e^{bx+a} + 1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(exp(b\*x+a)), x)

[Out] 1/b\*ln(exp(b\*x+a))\*arccoth(exp(b\*x+a))-1/2/b\*dilog(exp(b\*x+a))-1/2/b\*dilog(exp(b\*x+a)+1)-1/2/b\*ln(exp(b\*x+a))\*ln(exp(b\*x+a)+1)

**Maxima [B]** time = 1.07206, size = 144, normalized size = 3.51

$$\frac{(bx + a) \operatorname{arccoth}(e^{(bx+a)})}{b} - \frac{(bx + a)(\log(e^{(bx+a)} + 1) - \log(e^{(bx+a)} - 1)) - \log(-e^{(bx+a)}) \log(e^{(bx+a)} + 1) + (bx + a) \log(e^{(bx+a)})}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(exp(b\*x+a)),x, algorithm="maxima")

[Out] (b\*x + a)\*arccoth(e^(b\*x + a))/b - 1/2\*((b\*x + a)\*(log(e^(b\*x + a) + 1) - log(e^(b\*x + a) - 1)) - log(-e^(b\*x + a))\*log(e^(b\*x + a) + 1) + (b\*x + a)\*log(e^(b\*x + a) - 1) - dilog(e^(b\*x + a) + 1) + dilog(-e^(b\*x + a) + 1))/b

**Fricas [B]** time = 1.67866, size = 417, normalized size = 10.17

$bx \log\left(\frac{\cosh(bx+a)+\sinh(bx+a)+1}{\cosh(bx+a)+\sinh(bx+a)-1}\right) - bx \log(\cosh(bx+a) + \sinh(bx+a) + 1) - a \log(\cosh(bx+a) + \sinh(bx+a) - 1) +$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(exp(b\*x+a)),x, algorithm="fricas")

[Out] 1/2\*(b\*x\*log((cosh(b\*x + a) + sinh(b\*x + a) + 1)/(cosh(b\*x + a) + sinh(b\*x + a) - 1)) - b\*x\*log(cosh(b\*x + a) + sinh(b\*x + a) + 1) - a\*log(cosh(b\*x + a) + sinh(b\*x + a) - 1) + (b\*x + a)\*log(-cosh(b\*x + a) - sinh(b\*x + a) + 1) + dilog(cosh(b\*x + a) + sinh(b\*x + a)) - dilog(-cosh(b\*x + a) - sinh(b\*x + a)))/b

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acoth}(e^{a+bx}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(exp(b\*x+a)),x)

[Out] Integral(acoth(exp(a + b\*x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}(e^{(bx+a)}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(e^(b*x + a)), x)
```

### 3.287 $\int x \coth^{-1} \left( e^{a+bx} \right) dx$

**Optimal.** Leaf size=83

$$\frac{\text{PolyLog}\left(3, -e^{-a-bx}\right)}{2b^2} - \frac{\text{PolyLog}\left(3, e^{-a-bx}\right)}{2b^2} + \frac{x\text{PolyLog}\left(2, -e^{-a-bx}\right)}{2b} - \frac{x\text{PolyLog}\left(2, e^{-a-bx}\right)}{2b}$$

[Out] (x\*PolyLog[2, -E^(-a - b\*x)])/(2\*b) - (x\*PolyLog[2, E^(-a - b\*x)])/(2\*b) + PolyLog[3, -E^(-a - b\*x)]/(2\*b^2) - PolyLog[3, E^(-a - b\*x)]/(2\*b^2)

**Rubi [A]** time = 0.0642786, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {6214, 2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3, -e^{-a-bx}\right)}{2b^2} - \frac{\text{PolyLog}\left(3, e^{-a-bx}\right)}{2b^2} + \frac{x\text{PolyLog}\left(2, -e^{-a-bx}\right)}{2b} - \frac{x\text{PolyLog}\left(2, e^{-a-bx}\right)}{2b}$$

Antiderivative was successfully verified.

[In] Int[x\*ArcCoth[E^(a + b\*x)], x]

[Out] (x\*PolyLog[2, -E^(-a - b\*x)])/(2\*b) - (x\*PolyLog[2, E^(-a - b\*x)])/(2\*b) + PolyLog[3, -E^(-a - b\*x)]/(2\*b^2) - PolyLog[3, E^(-a - b\*x)]/(2\*b^2)

#### Rule 6214

```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Dist[1/2, Int
[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IGtQ[m, 0]
```

#### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned} \int x \coth^{-1}(e^{a+bx}) dx &= -\left(\frac{1}{2} \int x \log(1 - e^{-a-bx}) dx\right) + \frac{1}{2} \int x \log(1 + e^{-a-bx}) dx \\ &= \frac{x \operatorname{Li}_2(-e^{-a-bx})}{2b} - \frac{x \operatorname{Li}_2(e^{-a-bx})}{2b} - \frac{\int \operatorname{Li}_2(-e^{-a-bx}) dx}{2b} + \frac{\int \operatorname{Li}_2(e^{-a-bx}) dx}{2b} \\ &= \frac{x \operatorname{Li}_2(-e^{-a-bx})}{2b} - \frac{x \operatorname{Li}_2(e^{-a-bx})}{2b} + \frac{\operatorname{Subst}\left(\int \frac{\operatorname{Li}_2(-x)}{x} dx, x, e^{-a-bx}\right)}{2b^2} - \frac{\operatorname{Subst}\left(\int \frac{\operatorname{Li}_2(x)}{x} dx, x, e^{-a-bx}\right)}{2b^2} \\ &= \frac{x \operatorname{Li}_2(-e^{-a-bx})}{2b} - \frac{x \operatorname{Li}_2(e^{-a-bx})}{2b} + \frac{\operatorname{Li}_3(-e^{-a-bx})}{2b^2} - \frac{\operatorname{Li}_3(e^{-a-bx})}{2b^2} \end{aligned}$$

**Mathematica [A]** time = 0.0309224, size = 113, normalized size = 1.36

$$\frac{-2bx \operatorname{PolyLog}(2, -e^{a+bx}) + 2bx \operatorname{PolyLog}(2, e^{a+bx}) + 2 \operatorname{PolyLog}(3, -e^{a+bx}) - 2 \operatorname{PolyLog}(3, e^{a+bx}) + b^2 x^2 \log(1 - e^{a+bx})}{4b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[E^(a + b\*x)], x]

[Out] (2\*b^2\*x^2\*ArcCoth[E^(a + b\*x)] + b^2\*x^2\*Log[1 - E^(a + b\*x)] - b^2\*x^2\*Lo
g[1 + E^(a + b\*x)] - 2\*b\*x\*PolyLog[2, -E^(a + b\*x)] + 2\*b\*x\*PolyLog[2, E^(a
+ b\*x)] + 2\*PolyLog[3, -E^(a + b\*x)] - 2\*PolyLog[3, E^(a + b\*x)])/(4\*b^2)

**Maple [B]** time = 0.07, size = 153, normalized size = 1.8

$$\frac{x^2 \operatorname{arccoth}(e^{bx+a})}{2} - \frac{\ln(e^{bx+a} + 1) x^2}{4} + \frac{\ln(e^{bx+a} + 1) a^2}{4b^2} - \frac{\operatorname{polylog}(2, -e^{bx+a}) x}{2b} + \frac{\operatorname{polylog}(3, -e^{bx+a})}{2b^2} + \frac{\ln(1 - e^{bx+a})}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*arccoth(exp(b*x+a)),x)`

[Out]  $\frac{1}{2}x^2 \operatorname{arccoth}(\exp(bx+a)) - \frac{1}{4} \ln(\exp(bx+a)+1)x^2 + \frac{1}{4b^2} \ln(\exp(bx+a)+1)a^2 - \frac{1}{2b} \operatorname{polylog}(2, -\exp(bx+a))x + \frac{1}{2b^2} \operatorname{polylog}(3, -\exp(bx+a)) + \frac{1}{4} \ln(1-\exp(bx+a))x^2 - \frac{1}{4b^2} \ln(1-\exp(bx+a))a^2 + \frac{1}{2b} \operatorname{polylog}(2, \exp(bx+a))x - \frac{1}{2b^2} \operatorname{polylog}(3, \exp(bx+a)) - \frac{1}{2b^2} a^2 \operatorname{arctanh}(\exp(bx+a))$

**Maxima [A]** time = 1.1348, size = 146, normalized size = 1.76

$$\frac{1}{2} x^2 \operatorname{arccoth}(e^{(bx+a)}) - \frac{1}{4} b \left( \frac{b^2 x^2 \log(e^{(bx+a)} + 1) + 2 bx \operatorname{Li}_2(-e^{(bx+a)}) - 2 \operatorname{Li}_3(-e^{(bx+a)})}{b^3} - \frac{b^2 x^2 \log(-e^{(bx+a)} + 1) + 2 bx \operatorname{Li}_2(-e^{(bx+a)})}{b^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccoth(exp(b*x+a)),x, algorithm="maxima")`

[Out]  $\frac{1}{2}x^2 \operatorname{arccoth}(e^{(bx+a)}) - \frac{1}{4}b * ((b^2x^2 \log(e^{(bx+a)} + 1) + 2bx * \operatorname{dilog}(-e^{(bx+a)}) - 2 \operatorname{polylog}(3, -e^{(bx+a)})) / b^3 - (b^2x^2 \log(-e^{(bx+a)} + 1) + 2bx * \operatorname{dilog}(e^{(bx+a)}) - 2 \operatorname{polylog}(3, e^{(bx+a)})) / b^3)$

**Fricas [C]** time = 1.61181, size = 583, normalized size = 7.02

$$b^2 x^2 \log\left(\frac{\cosh(bx+a) + \sinh(bx+a) + 1}{\cosh(bx+a) + \sinh(bx+a) - 1}\right) - b^2 x^2 \log(\cosh(bx+a) + \sinh(bx+a) + 1) + 2 bx \operatorname{Li}_2(\cosh(bx+a) + \sinh(bx+a))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccoth(exp(b*x+a)),x, algorithm="fricas")`

[Out]  $\frac{1}{4} * (b^2 x^2 \log((\cosh(bx+a) + \sinh(bx+a) + 1) / (\cosh(bx+a) + \sinh(bx+a) - 1))) - b^2 x^2 \log(\cosh(bx+a) + \sinh(bx+a) + 1) + 2bx * \operatorname{dilog}(\cosh(bx+a) + \sinh(bx+a)) - 2bx * \operatorname{dilog}(-\cosh(bx+a) - \sinh(bx+a)) + a^2 \log(\cosh(bx+a) + \sinh(bx+a) - 1) + (b^2 x^2 - a^2) \log(-\cosh(bx+a) - \sinh(bx+a) + 1) - 2 \operatorname{polylog}(3, \cosh(bx+a) + \sinh(bx+a)) + 2 \operatorname{polylog}(3, -\cosh(bx+a) - \sinh(bx+a))) / b^2$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{acoth}(e^a e^{bx}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*acoth(exp(b*x+a)),x)`

[Out] `Integral(x*acoth(exp(a)*exp(b*x)), x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arccoth}(e^{(bx+a)}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccoth(exp(b*x+a)),x, algorithm="giac")`

[Out] `integrate(x*arccoth(e^(b*x + a)), x)`

### 3.288 $\int x^2 \coth^{-1}(e^{a+bx}) dx$

**Optimal.** Leaf size=119

$$\frac{x \operatorname{PolyLog}(3, -e^{-a-bx})}{b^2} - \frac{x \operatorname{PolyLog}(3, e^{-a-bx})}{b^2} + \frac{\operatorname{PolyLog}(4, -e^{-a-bx})}{b^3} - \frac{\operatorname{PolyLog}(4, e^{-a-bx})}{b^3} + \frac{x^2 \operatorname{PolyLog}(2, -e^{-a-bx})}{2b}$$

[Out] (x^2\*PolyLog[2, -E^(-a - b\*x)])/(2\*b) - (x^2\*PolyLog[2, E^(-a - b\*x)])/(2\*b) + (x\*PolyLog[3, -E^(-a - b\*x)])/b^2 - (x\*PolyLog[3, E^(-a - b\*x)])/b^2 + PolyLog[4, -E^(-a - b\*x)]/b^3 - PolyLog[4, E^(-a - b\*x)]/b^3

**Rubi [A]** time = 0.0994868, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {6214, 2531, 6609, 2282, 6589}

$$\frac{x \operatorname{PolyLog}(3, -e^{-a-bx})}{b^2} - \frac{x \operatorname{PolyLog}(3, e^{-a-bx})}{b^2} + \frac{\operatorname{PolyLog}(4, -e^{-a-bx})}{b^3} - \frac{\operatorname{PolyLog}(4, e^{-a-bx})}{b^3} + \frac{x^2 \operatorname{PolyLog}(2, -e^{-a-bx})}{2b}$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcCoth[E^(a + b\*x)], x]

[Out] (x^2\*PolyLog[2, -E^(-a - b\*x)])/(2\*b) - (x^2\*PolyLog[2, E^(-a - b\*x)])/(2\*b) + (x\*PolyLog[3, -E^(-a - b\*x)])/b^2 - (x\*PolyLog[3, E^(-a - b\*x)])/b^2 + PolyLog[4, -E^(-a - b\*x)]/b^3 - PolyLog[4, E^(-a - b\*x)]/b^3

#### Rule 6214

```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Dist[1/2, Int
[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IGtQ[m, 0]
```

#### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```



Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(e^{a+bx}) dx &= -\left(\frac{1}{2} \int x^2 \log(1 - e^{-a-bx}) dx\right) + \frac{1}{2} \int x^2 \log(1 + e^{-a-bx}) dx \\
&= \frac{x^2 \text{Li}_2(-e^{-a-bx})}{2b} - \frac{x^2 \text{Li}_2(e^{-a-bx})}{2b} - \frac{\int x \text{Li}_2(-e^{-a-bx}) dx}{b} + \frac{\int x \text{Li}_2(e^{-a-bx}) dx}{b} \\
&= \frac{x^2 \text{Li}_2(-e^{-a-bx})}{2b} - \frac{x^2 \text{Li}_2(e^{-a-bx})}{2b} + \frac{x \text{Li}_3(-e^{-a-bx})}{b^2} - \frac{x \text{Li}_3(e^{-a-bx})}{b^2} - \frac{\int \text{Li}_3(-e^{-a-bx}) dx}{b^2} + \frac{\int \text{Li}_3(e^{-a-bx}) dx}{b^2} \\
&= \frac{x^2 \text{Li}_2(-e^{-a-bx})}{2b} - \frac{x^2 \text{Li}_2(e^{-a-bx})}{2b} + \frac{x \text{Li}_3(-e^{-a-bx})}{b^2} - \frac{x \text{Li}_3(e^{-a-bx})}{b^2} + \frac{\text{Subst}\left(\int \frac{\text{Li}_3(-x)}{x} dx, x, e^{-a-bx}\right)}{b^3} \\
&= \frac{x^2 \text{Li}_2(-e^{-a-bx})}{2b} - \frac{x^2 \text{Li}_2(e^{-a-bx})}{2b} + \frac{x \text{Li}_3(-e^{-a-bx})}{b^2} - \frac{x \text{Li}_3(e^{-a-bx})}{b^2} + \frac{\text{Li}_4(-e^{-a-bx})}{b^3} - \frac{\text{Li}_4(e^{-a-bx})}{b^3}
\end{aligned}$$

**Mathematica [A]** time = 0.0298635, size = 149, normalized size = 1.25

$$\frac{-3b^2 x^2 \text{PolyLog}(2, -e^{a+bx}) + 3b^2 x^2 \text{PolyLog}(2, e^{a+bx}) + 6bx \text{PolyLog}(3, -e^{a+bx}) - 6bx \text{PolyLog}(3, e^{a+bx}) - 6 \text{PolyLog}(4, -e^{a+bx}) + 6 \text{PolyLog}(4, e^{a+bx})}{6b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[E^(a + b\*x)],x]

[Out] (2\*b^3\*x^3\*ArcCoth[E^(a + b\*x)] + b^3\*x^3\*Log[1 - E^(a + b\*x)] - b^3\*x^3\*Log[1 + E^(a + b\*x)] - 3\*b^2\*x^2\*PolyLog[2, -E^(a + b\*x)] + 3\*b^2\*x^2\*PolyLog[2, E^(a + b\*x)] + 6\*b\*x\*PolyLog[3, -E^(a + b\*x)] - 6\*b\*x\*PolyLog[3, E^(a + b\*x)] - 6\*PolyLog[4, -E^(a + b\*x)] + 6\*PolyLog[4, E^(a + b\*x)])/(6\*b^3)

**Maple [A]** time = 0.076, size = 185, normalized size = 1.6

$$\frac{x^3 \operatorname{arccoth}(e^{bx+a})}{3} + \frac{\operatorname{polylog}(2, e^{bx+a}) x^2}{2b} - \frac{\operatorname{polylog}(3, e^{bx+a}) x}{b^2} - \frac{\ln(e^{bx+a} + 1) x^3}{6} - \frac{\operatorname{polylog}(2, -e^{bx+a}) x^2}{2b} + \frac{\operatorname{polylog}(4, -e^{bx+a}) x}{b^3} - \frac{\operatorname{polylog}(4, e^{bx+a}) x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(exp(b\*x+a)),x)

[Out] 1/3\*x^3\*arccoth(exp(b\*x+a))+1/2/b\*polylog(2,exp(b\*x+a))\*x^2-1/b^2\*polylog(3,exp(b\*x+a))\*x-1/6\*ln(exp(b\*x+a)+1)\*x^3-1/2/b\*polylog(2,-exp(b\*x+a))\*x^2+1/b^2\*polylog(3,-exp(b\*x+a))\*x+1/6\*ln(1-exp(b\*x+a))\*x^3-1/6/b^3\*ln(exp(b\*x+a)+1)\*a^3+1/6/b^3\*ln(1-exp(b\*x+a))\*a^3+1/3/b^3\*a^3\*arctanh(exp(b\*x+a))-1/b^3\*polylog(4,-exp(b\*x+a))+1/b^3\*polylog(4,exp(b\*x+a))

**Maxima [A]** time = 1.12452, size = 192, normalized size = 1.61

$$\frac{1}{3} x^3 \operatorname{arccoth}(e^{(bx+a)}) - \frac{1}{6} b \left( \frac{b^3 x^3 \log(e^{(bx+a)} + 1) + 3 b^2 x^2 \operatorname{Li}_2(-e^{(bx+a)}) - 6 b x \operatorname{Li}_3(-e^{(bx+a)}) + 6 \operatorname{Li}_4(-e^{(bx+a)})}{b^4} - \frac{b^3 x^3 \log(-e^{(bx+a)} + 1) + 3 b^2 x^2 \operatorname{Li}_2(e^{(bx+a)}) - 6 b x \operatorname{Li}_3(e^{(bx+a)}) + 6 \operatorname{Li}_4(e^{(bx+a)})}{b^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(exp(b\*x+a)),x, algorithm="maxima")

[Out] 1/3\*x^3\*arccoth(e^(b\*x + a)) - 1/6\*b\*((b^3\*x^3\*log(e^(b\*x + a) + 1) + 3\*b^2\*x^2\*dilog(-e^(b\*x + a)) - 6\*b\*x\*polylog(3, -e^(b\*x + a)) + 6\*polylog(4, -e^(b\*x + a)))/b^4 - (b^3\*x^3\*log(-e^(b\*x + a) + 1) + 3\*b^2\*x^2\*dilog(e^(b\*x + a)) - 6\*b\*x\*polylog(3, e^(b\*x + a)) + 6\*polylog(4, e^(b\*x + a)))/b^4)

**Fricas [C]** time = 1.75526, size = 730, normalized size = 6.13

$$b^3 x^3 \log\left(\frac{\cosh(bx+a)+\sinh(bx+a)+1}{\cosh(bx+a)+\sinh(bx+a)-1}\right) - b^3 x^3 \log(\cosh(bx+a) + \sinh(bx+a) + 1) + 3 b^2 x^2 \text{Li}_2(\cosh(bx+a) + \sinh(bx+a))$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(exp(b\*x+a)),x, algorithm="fricas")

[Out] 1/6\*(b^3\*x^3\*log((cosh(b\*x + a) + sinh(b\*x + a) + 1)/(cosh(b\*x + a) + sinh(b\*x + a) - 1)) - b^3\*x^3\*log(cosh(b\*x + a) + sinh(b\*x + a) + 1) + 3\*b^2\*x^2\*dilog(cosh(b\*x + a) + sinh(b\*x + a)) - 3\*b^2\*x^2\*dilog(-cosh(b\*x + a) - sinh(b\*x + a)) - a^3\*log(cosh(b\*x + a) + sinh(b\*x + a) - 1) - 6\*b\*x\*polylog(3, cosh(b\*x + a) + sinh(b\*x + a)) + 6\*b\*x\*polylog(3, -cosh(b\*x + a) - sinh(b\*x + a)) + (b^3\*x^3 + a^3)\*log(-cosh(b\*x + a) - sinh(b\*x + a) + 1) + 6\*polylog(4, cosh(b\*x + a) + sinh(b\*x + a)) - 6\*polylog(4, -cosh(b\*x + a) - sinh(b\*x + a)))/b^3

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*acoth(exp(b\*x+a)),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arccoth}(e^{(bx+a)}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(exp(b\*x+a)),x, algorithm="giac")

[Out] integrate(x^2\*arccoth(e^(b\*x + a)), x)

### 3.289 $\int \coth^{-1} (a + bf^{c+dx}) dx$

**Optimal.** Leaf size=168

$$\frac{\text{PolyLog}\left(2, 1 - \frac{2}{a+bf^{c+dx}+1}\right)}{2d \log(f)} - \frac{\text{PolyLog}\left(2, 1 - \frac{2bf^{c+dx}}{(1-a)(a+bf^{c+dx}+1)}\right)}{2d \log(f)} - \frac{\log\left(\frac{2}{a+bf^{c+dx}+1}\right) \coth^{-1}(a + bf^{c+dx})}{d \log(f)} + \frac{\log\left(\frac{2bf^{c+dx}}{(1-a)(a+bf^{c+dx}+1)}\right)}{d \log(f)}$$

```
[Out] -((ArcCoth[a + b*f^(c + d*x)]*Log[2/(1 + a + b*f^(c + d*x))])/(d*Log[f])) +
(ArcCoth[a + b*f^(c + d*x)]*Log[(2*b*f^(c + d*x))/((1 - a)*(1 + a + b*f^(c
+ d*x)))]/(d*Log[f]) + PolyLog[2, 1 - 2/(1 + a + b*f^(c + d*x))]/(2*d*Log
[f]) - PolyLog[2, 1 - (2*b*f^(c + d*x))/((1 - a)*(1 + a + b*f^(c + d*x)))]/
(2*d*Log[f]))
```

**Rubi [A]** time = 0.132084, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {2282, 6112, 5921, 2402, 2315, 2447}

$$\frac{\text{PolyLog}\left(2, 1 - \frac{2}{a+bf^{c+dx}+1}\right)}{2d \log(f)} - \frac{\text{PolyLog}\left(2, 1 - \frac{2bf^{c+dx}}{(1-a)(a+bf^{c+dx}+1)}\right)}{2d \log(f)} - \frac{\log\left(\frac{2}{a+bf^{c+dx}+1}\right) \coth^{-1}(a + bf^{c+dx})}{d \log(f)} + \frac{\log\left(\frac{2bf^{c+dx}}{(1-a)(a+bf^{c+dx}+1)}\right)}{d \log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*f^(c + d*x)], x]
```

```
[Out] -((ArcCoth[a + b*f^(c + d*x)]*Log[2/(1 + a + b*f^(c + d*x))])/(d*Log[f])) +
(ArcCoth[a + b*f^(c + d*x)]*Log[(2*b*f^(c + d*x))/((1 - a)*(1 + a + b*f^(c
+ d*x)))]/(d*Log[f]) + PolyLog[2, 1 - 2/(1 + a + b*f^(c + d*x))]/(2*d*Log
[f]) - PolyLog[2, 1 - (2*b*f^(c + d*x))/((1 - a)*(1 + a + b*f^(c + d*x)))]/
(2*d*Log[f]))
```

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] := Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

### Rule 5921

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] := -
Simp[((a + b*ArcCoth[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcCoth[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

### Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

### Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

### Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

### Rubi steps

$$\begin{aligned}
\int \coth^{-1}(a + bf^{c+dx}) dx &= \frac{\text{Subst}\left(\int \frac{\coth^{-1}(a+bx)}{x} dx, x, f^{c+dx}\right)}{d \log(f)} \\
&= \frac{\text{Subst}\left(\int \frac{\coth^{-1}(x)}{-\frac{a}{b} + \frac{x}{b}} dx, x, a + bf^{c+dx}\right)}{bd \log(f)} \\
&= -\frac{\coth^{-1}(a + bf^{c+dx}) \log\left(\frac{2}{1+bf^{c+dx}}\right)}{d \log(f)} + \frac{\coth^{-1}(a + bf^{c+dx}) \log\left(\frac{2bf^{c+dx}}{(1-a)(1+bf^{c+dx})}\right)}{d \log(f)} + \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, \frac{2bf^{c+dx}}{(1-a)(1+bf^{c+dx})}\right)}{d \log(f)} \\
&= -\frac{\coth^{-1}(a + bf^{c+dx}) \log\left(\frac{2}{1+bf^{c+dx}}\right)}{d \log(f)} + \frac{\coth^{-1}(a + bf^{c+dx}) \log\left(\frac{2bf^{c+dx}}{(1-a)(1+bf^{c+dx})}\right)}{d \log(f)} - \frac{\text{Li}_2\left(\frac{2bf^{c+dx}}{(1-a)(1+bf^{c+dx})}\right)}{d \log(f)} \\
&= -\frac{\coth^{-1}(a + bf^{c+dx}) \log\left(\frac{2}{1+bf^{c+dx}}\right)}{d \log(f)} + \frac{\coth^{-1}(a + bf^{c+dx}) \log\left(\frac{2bf^{c+dx}}{(1-a)(1+bf^{c+dx})}\right)}{d \log(f)} + \frac{\text{Li}_2\left(\frac{2bf^{c+dx}}{(1-a)(1+bf^{c+dx})}\right)}{2d \log(f)}
\end{aligned}$$

**Mathematica [A]** time = 0.0750964, size = 108, normalized size = 0.64

$$\frac{\text{PolyLog}\left(2, -\frac{bf^{c+dx}}{a-1}\right) - \text{PolyLog}\left(2, -\frac{bf^{c+dx}}{a+1}\right) + dx \log(f) \left(\log\left(\frac{a+bf^{c+dx}-1}{a-1}\right) - \log\left(\frac{a+bf^{c+dx}+1}{a+1}\right) + 2 \coth^{-1}(a + bf^{c+dx})\right)}{2d \log(f)}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a + b\*f^(c + d\*x)], x]

[Out] (d\*x\*Log[f]\*(2\*ArcCoth[a + b\*f^(c + d\*x)] + Log[(-1 + a + b\*f^(c + d\*x))]/(-1 + a)] - Log[(1 + a + b\*f^(c + d\*x))/(1 + a)] + PolyLog[2, -((b\*f^(c + d\*x)))/(-1 + a))] - PolyLog[2, -((b\*f^(c + d\*x))/(1 + a))]/(2\*d\*Log[f])

**Maple [A]** time = 0.199, size = 164, normalized size = 1.

$$\frac{\ln(bf^{dx+c}) \operatorname{arccoth}(a + bf^{dx+c})}{d \ln(f)} - \frac{1}{2d \ln(f)} \operatorname{dilog}\left(\frac{1 + a + bf^{dx+c}}{1 + a}\right) - \frac{\ln(bf^{dx+c})}{2d \ln(f)} \ln\left(\frac{1 + a + bf^{dx+c}}{1 + a}\right) + \frac{1}{2d \ln(f)} \operatorname{dilog}\left(\frac{1 + a + bf^{dx+c}}{1 + a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a+b\*f^(d\*x+c)),x)

[Out]  $\frac{1}{d \ln(f)} \ln(b f^{d x+c}) \operatorname{arccoth}(a+b f^{d x+c}) - \frac{1}{2} \frac{1}{d \ln(f)} \operatorname{dilog}\left(\frac{1+a+b f^{d x+c}}{1+a}\right) - \frac{1}{2} \frac{1}{d \ln(f)} \ln(b f^{d x+c}) \ln\left(\frac{1+a+b f^{d x+c}}{1+a}\right) + \frac{1}{2} \frac{1}{d \ln(f)} \operatorname{dilog}\left(\frac{b f^{d x+c}+a-1}{a-1}\right) + \frac{1}{2} \frac{1}{d \ln(f)} \ln(b f^{d x+c}) \ln\left(\frac{b f^{d x+c}+a-1}{a-1}\right)$

**Maxima [A]** time = 1.08897, size = 284, normalized size = 1.69

$$\frac{\operatorname{arccoth}(b f^{d x+c}+a) \log(f^{d x+c})}{d \log(f)} - \frac{b \left( \frac{\log(b f^{d x+c}+a+1)}{b} - \frac{\log(b f^{d x+c}+a-1)}{b} \right) \log(f^{d x+c}) - b \left( \frac{\log(b f^{d x+c}+a+1) \log\left(-\frac{b f^{d x+c}+a+1}{a+1}+1\right) + \operatorname{Li}_2\left(-\frac{b f^{d x+c}+a+1}{a+1}\right)}{b} \right)}{2 d \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a+b\*f^(d\*x+c)),x, algorithm="maxima")

[Out]  $\operatorname{arccoth}(b f^{d x+c}+a) \log(f^{d x+c}) / (d \log(f)) - \frac{1}{2} \left( \frac{b \log(b f^{d x+c}+a+1)}{b} - \frac{\log(b f^{d x+c}+a-1)}{b} \right) \log(f^{d x+c}) - b \left( \frac{\log(b f^{d x+c}+a+1) \log\left(-\frac{b f^{d x+c}+a+1}{a+1}+1\right) + \operatorname{dilog}\left(\frac{b f^{d x+c}+a+1}{a+1}\right)}{b} - \frac{(\log(b f^{d x+c}+a-1) \log\left(-\frac{b f^{d x+c}+a-1}{a-1}+1\right) + \operatorname{dilog}\left(\frac{b f^{d x+c}+a-1}{a-1}\right))}{b} \right) / (d \log(f))$

**Fricas [A]** time = 1.60135, size = 899, normalized size = 5.35

$$\frac{d x \log(f) \log\left(\frac{b \cosh((d x+c) \log(f))+b \sinh((d x+c) \log(f))+a+1}{b \cosh((d x+c) \log(f))+b \sinh((d x+c) \log(f))+a-1}\right) + c \log\left(b \cosh((d x+c) \log(f))+b \sinh((d x+c) \log(f))+a+1\right) + c \log\left(b \cosh((d x+c) \log(f))+b \sinh((d x+c) \log(f))+a-1\right)}{2 d \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a+b\*f^(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{2} \frac{d x \log(f) \log\left(\frac{b \cosh((d x+c) \log(f))+b \sinh((d x+c) \log(f))+a+1}{b \cosh((d x+c) \log(f))+b \sinh((d x+c) \log(f))+a-1}\right) + c \log\left(b \cosh((d x+c) \log(f))+b \sinh((d x+c) \log(f))+a+1\right) + c \log\left(b \cosh((d x+c) \log(f))+b \sinh((d x+c) \log(f))+a-1\right)}{2 d \log(f)}$

```
log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)*log(f) - (
d*x + c)*log(f)*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) +
a + 1)/(a + 1)) + (d*x + c)*log(f)*log((b*cosh((d*x + c)*log(f)) + b*sinh((
d*x + c)*log(f)) + a - 1)/(a - 1)) - dilog(-(b*cosh((d*x + c)*log(f)) + b*s
inh((d*x + c)*log(f)) + a + 1)/(a + 1) + 1) + dilog(-(b*cosh((d*x + c)*log(
f)) + b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1) + 1))/(d*log(f))
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a+b*f**(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(bf^{dx+c} + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a+b*f^(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*f^(d*x + c) + a), x)
```



### 3.290 $\int x \coth^{-1} \left( a + b f^{c+dx} \right) dx$

**Optimal.** Leaf size=216

$$-\frac{\text{PolyLog}\left(3, \frac{b f^{c+dx}}{1-a}\right)}{2d^2 \log^2(f)} + \frac{\text{PolyLog}\left(3, -\frac{b f^{c+dx}}{a+1}\right)}{2d^2 \log^2(f)} + \frac{x \text{PolyLog}\left(2, \frac{b f^{c+dx}}{1-a}\right)}{2d \log(f)} - \frac{x \text{PolyLog}\left(2, -\frac{b f^{c+dx}}{a+1}\right)}{2d \log(f)} + \frac{1}{4} x^2 \log\left(1 - \frac{b f^{c+dx}}{1-a}\right)$$

[Out] (x^2\*Log[1 - (b\*f^(c + d\*x))/(1 - a)]/4 - (x^2\*Log[1 + (b\*f^(c + d\*x))/(1 + a)]/4 - (x^2\*Log[1 - (a + b\*f^(c + d\*x))^(-1)]/4 + (x^2\*Log[1 + (a + b\*f^(c + d\*x))^(-1)]/4 + (x\*PolyLog[2, (b\*f^(c + d\*x))/(1 - a)]/(2\*d\*Log[f]) - (x\*PolyLog[2, -((b\*f^(c + d\*x))/(1 + a))]/(2\*d\*Log[f]) - PolyLog[3, (b\*f^(c + d\*x))/(1 - a)]/(2\*d^2\*Log[f]^2) + PolyLog[3, -((b\*f^(c + d\*x))/(1 + a))]/(2\*d^2\*Log[f]^2)

**Rubi [A]** time = 2.71421, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 8, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {6214, 2551, 12, 6742, 2190, 2531, 2282, 6589}

$$-\frac{\text{PolyLog}\left(3, \frac{b f^{c+dx}}{1-a}\right)}{2d^2 \log^2(f)} + \frac{\text{PolyLog}\left(3, -\frac{b f^{c+dx}}{a+1}\right)}{2d^2 \log^2(f)} + \frac{x \text{PolyLog}\left(2, \frac{b f^{c+dx}}{1-a}\right)}{2d \log(f)} - \frac{x \text{PolyLog}\left(2, -\frac{b f^{c+dx}}{a+1}\right)}{2d \log(f)} + \frac{1}{4} x^2 \log\left(1 - \frac{b f^{c+dx}}{1-a}\right)$$

Antiderivative was successfully verified.

[In] Int[x\*ArcCoth[a + b\*f^(c + d\*x)],x]

[Out] (x^2\*Log[1 - (b\*f^(c + d\*x))/(1 - a)]/4 - (x^2\*Log[1 + (b\*f^(c + d\*x))/(1 + a)]/4 - (x^2\*Log[1 - (a + b\*f^(c + d\*x))^(-1)]/4 + (x^2\*Log[1 + (a + b\*f^(c + d\*x))^(-1)]/4 + (x\*PolyLog[2, (b\*f^(c + d\*x))/(1 - a)]/(2\*d\*Log[f]) - (x\*PolyLog[2, -((b\*f^(c + d\*x))/(1 + a))]/(2\*d\*Log[f]) - PolyLog[3, (b\*f^(c + d\*x))/(1 - a)]/(2\*d^2\*Log[f]^2) + PolyLog[3, -((b\*f^(c + d\*x))/(1 + a))]/(2\*d^2\*Log[f]^2)

#### Rule 6214

Int[ArcCoth[(a\_.) + (b\_.)\*(f\_)^(c\_. + (d\_.)\*(x\_))]\*(x\_)^(m\_.), x\_Symbol]  
 :> Dist[1/2, Int[x^m\*Log[1 + 1/(a + b\*f^(c + d\*x))], x], x] - Dist[1/2, Int[x^m\*Log[1 - 1/(a + b\*f^(c + d\*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] && IGtQ[m, 0]

#### Rule 2551

```
Int[Log[u]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[((a + b*x)^(m + 1)
)*Log[u]]/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x \coth^{-1}(a + b f^{c+dx}) dx &= -\left(\frac{1}{2} \int x \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) dx\right) + \frac{1}{2} \int x \log\left(1 + \frac{1}{a + b f^{c+dx}}\right) dx \\
&= -\frac{1}{4} x^2 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{4} x^2 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{4} \int \frac{b d f^{c+dx} x^2 \log(f)}{(-1 + a + b f^{c+dx})(a + b f^{c+dx})} dx \\
&= -\frac{1}{4} x^2 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{4} x^2 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{4} (b d \log(f)) \int \frac{f^{c+dx}}{(-1 + a + b f^{c+dx})} dx \\
&= -\frac{1}{4} x^2 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{4} x^2 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{4} (b d \log(f)) \int \left(\frac{f^{c+dx} x^2}{-a - b f^{c+dx}} + \frac{f^{c+dx} x^2}{-1 + a + b f^{c+dx}}\right) dx \\
&= -\frac{1}{4} x^2 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{4} x^2 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{4} (b d \log(f)) \int \frac{f^{c+dx} x^2}{-1 + a + b f^{c+dx}} dx \\
&= \frac{1}{4} x^2 \log\left(1 - \frac{b f^{c+dx}}{1 - a}\right) - \frac{1}{4} x^2 \log\left(1 + \frac{b f^{c+dx}}{1 + a}\right) - \frac{1}{4} x^2 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{4} x^2 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right) \\
&= \frac{1}{4} x^2 \log\left(1 - \frac{b f^{c+dx}}{1 - a}\right) - \frac{1}{4} x^2 \log\left(1 + \frac{b f^{c+dx}}{1 + a}\right) - \frac{1}{4} x^2 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{4} x^2 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right) \\
&= \frac{1}{4} x^2 \log\left(1 - \frac{b f^{c+dx}}{1 - a}\right) - \frac{1}{4} x^2 \log\left(1 + \frac{b f^{c+dx}}{1 + a}\right) - \frac{1}{4} x^2 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{4} x^2 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right) \\
&= \frac{1}{4} x^2 \log\left(1 - \frac{b f^{c+dx}}{1 - a}\right) - \frac{1}{4} x^2 \log\left(1 + \frac{b f^{c+dx}}{1 + a}\right) - \frac{1}{4} x^2 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{4} x^2 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.104145, size = 177, normalized size = 0.82

$$\frac{-2 \operatorname{PolyLog}\left(3, -\frac{b f^{c+dx}}{a-1}\right) + 2 \operatorname{PolyLog}\left(3, -\frac{b f^{c+dx}}{a+1}\right) + 2 d x \log(f) \operatorname{PolyLog}\left(2, -\frac{b f^{c+dx}}{a-1}\right) - 2 d x \log(f) \operatorname{PolyLog}\left(2, -\frac{b f^{c+dx}}{a+1}\right)}{4 d^2 \log^2(f)}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[a + b\*f^(c + d\*x)], x]

[Out] (2\*d^2\*x^2\*ArcCoth[a + b\*f^(c + d\*x)]\*Log[f]^2 + d^2\*x^2\*Log[f]^2\*Log[1 + (b\*f^(c + d\*x))/(-1 + a)] - d^2\*x^2\*Log[f]^2\*Log[1 + (b\*f^(c + d\*x))/(1 + a)] + 2\*d\*x\*Log[f]\*PolyLog[2, -((b\*f^(c + d\*x))/(-1 + a))] - 2\*d\*x\*Log[f]\*PolyLog[2, -((b\*f^(c + d\*x))/(1 + a))] - 2\*PolyLog[3, -((b\*f^(c + d\*x))/(-1 + a))] + 2\*PolyLog[3, -((b\*f^(c + d\*x))/(1 + a))])/(4\*d^2\*log^2(f))

a))] + 2\*PolyLog[3, -((b\*f^(c + d\*x))/(1 + a))]/(4\*d^2\*Log[f]^2)

**Maple [B]** time = 0.174, size = 590, normalized size = 2.7

$$-\frac{x^2 \ln(bf^{dx+c} + a - 1)}{4} + \frac{x^2 \ln(1 + a + bf^{dx+c})}{4} - \frac{x^2 \ln\left(1 - \frac{bf^{dx}fc}{-1-a}\right)}{4} - \frac{cx \ln\left(1 - \frac{bf^{dx}fc}{-1-a}\right)}{2d} - \frac{c^2 \ln\left(1 - \frac{bf^{dx}fc}{-1-a}\right)}{4d^2} - \frac{c^3 \ln\left(1 - \frac{bf^{dx}fc}{-1-a}\right)}{4d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(a+b\*f^(d\*x+c)),x)

[Out] -1/4\*x^2\*ln(b\*f^(d\*x+c)+a-1)+1/4\*x^2\*ln(1+a+b\*f^(d\*x+c))-1/4\*ln(1-b\*f^(d\*x)\*f^c/(-1-a))\*x^2-1/2/d\*ln(1-b\*f^(d\*x)\*f^c/(-1-a))\*x\*c-1/4/d^2\*ln(1-b\*f^(d\*x)\*f^c/(-1-a))\*c^2-1/2/d\*ln(f)/d\*polylog(2,b\*f^(d\*x)\*f^c/(-1-a))\*x-1/2/d\*ln(f)/d^2\*polylog(2,b\*f^(d\*x)\*f^c/(-1-a))\*c+1/2/d\*ln(f)^2/d^2\*polylog(3,b\*f^(d\*x)\*f^c/(-1-a))-1/4/d^2\*c^2\*ln(1+a+b\*f^(d\*x)\*f^c)+1/2/d\*ln(f)/d^2\*c\*dilog((1+a+b\*f^(d\*x)\*f^c)/(1+a))+1/2/d\*c\*ln((1+a+b\*f^(d\*x)\*f^c)/(1+a))\*x+1/2/d^2\*c^2\*ln((1+a+b\*f^(d\*x)\*f^c)/(1+a))+1/4\*ln(1-b\*f^(d\*x)\*f^c/(1-a))\*x^2+1/2/d\*ln(1-b\*f^(d\*x)\*f^c/(1-a))\*x\*c+1/4/d^2\*ln(1-b\*f^(d\*x)\*f^c/(1-a))\*c^2+1/2/d\*ln(f)/d\*polylog(2,b\*f^(d\*x)\*f^c/(1-a))\*x+1/2/d\*ln(f)/d^2\*polylog(2,b\*f^(d\*x)\*f^c/(1-a))\*c-1/2/d\*ln(f)^2/d^2\*polylog(3,b\*f^(d\*x)\*f^c/(1-a))+1/4/d^2\*c^2\*ln(b\*f^(d\*x)\*f^c+a-1)-1/2/d\*ln(f)/d^2\*c\*dilog((b\*f^(d\*x)\*f^c+a-1)/(a-1))-1/2/d\*c\*ln((b\*f^(d\*x)\*f^c+a-1)/(a-1))\*x-1/2/d^2\*c^2\*ln((b\*f^(d\*x)\*f^c+a-1)/(a-1))

**Maxima [A]** time = 1.28441, size = 262, normalized size = 1.21

$$-\frac{1}{4}bd \left( \frac{\log\left(\frac{bf^{dx}fc}{a+1} + 1\right) \log(f^{dx})^2 + 2 \operatorname{Li}_2\left(-\frac{bf^{dx}fc}{a+1}\right) \log(f^{dx}) - 2 \operatorname{Li}_3\left(-\frac{bf^{dx}fc}{a+1}\right)}{bd^3 \log(f)^3} - \frac{\log\left(\frac{bf^{dx}fc}{a-1} + 1\right) \log(f^{dx})^2 + 2 \operatorname{Li}_2\left(-\frac{bf^{dx}fc}{a-1}\right)}{bd^3 \log(f)^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(a+b\*f^(d\*x+c)),x, algorithm="maxima")

[Out] -1/4\*b\*d\*((log(b\*f^(d\*x)\*f^c/(a + 1) + 1)\*log(f^(d\*x))^2 + 2\*dilog(-b\*f^(d\*x)\*f^c/(a + 1))\*log(f^(d\*x)) - 2\*polylog(3, -b\*f^(d\*x)\*f^c/(a + 1)))/(b\*d^3\*log(f)^3) - (log(b\*f^(d\*x)\*f^c/(a - 1) + 1)\*log(f^(d\*x))^2 + 2\*dilog(-b\*f^(d\*x)\*f^c/(a - 1))\*log(f^(d\*x)) - 2\*polylog(3, -b\*f^(d\*x)\*f^c/(a - 1)))/(b\*d^3\*log(f)^3)

$$d^3 \log(f)^3) \log(f) + 1/2 x^2 \operatorname{arccoth}(b f^{(dx+c)} + a)$$

**Fricas [C]** time = 1.6202, size = 1193, normalized size = 5.52

$$d^2 x^2 \log(f)^2 \log\left(\frac{b \cosh((dx+c)\log(f)) + b \sinh((dx+c)\log(f)) + a + 1}{b \cosh((dx+c)\log(f)) + b \sinh((dx+c)\log(f)) + a - 1}\right) - c^2 \log\left(b \cosh((dx+c)\log(f)) + b \sinh((dx+c)\log(f))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(a+b\*f^(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{4} (d^2 x^2 \log(f)^2 \log((b \cosh((d*x+c)\log(f)) + b \sinh((d*x+c)\log(f)) + a + 1) / (b \cosh((d*x+c)\log(f)) + b \sinh((d*x+c)\log(f)) + a - 1)) - c^2 \log(b \cosh((d*x+c)\log(f)) + b \sinh((d*x+c)\log(f)) + a + 1) \log(f)^2 + c^2 \log(b \cosh((d*x+c)\log(f)) + b \sinh((d*x+c)\log(f)) + a - 1) \log(f)^2 - 2 d x \operatorname{dilog}(-(b \cosh((d*x+c)\log(f)) + b \sinh((d*x+c)\log(f)) + a + 1) / (a + 1) + 1) \log(f) + 2 d x \operatorname{dilog}(-(b \cosh((d*x+c)\log(f)) + b \sinh((d*x+c)\log(f)) + a - 1) / (a - 1) + 1) \log(f) - (d^2 x^2 - c^2) \log(f)^2 \log((b \cosh((d*x+c)\log(f)) + b \sinh((d*x+c)\log(f)) + a + 1) / (a + 1)) + (d^2 x^2 - c^2) \log(f)^2 \log((b \cosh((d*x+c)\log(f)) + b \sinh((d*x+c)\log(f)) + a - 1) / (a - 1)) + 2 \operatorname{polylog}(3, -(b \cosh((d*x+c)\log(f)) + b \sinh((d*x+c)\log(f)) / (a + 1)) - 2 \operatorname{polylog}(3, -(b \cosh((d*x+c)\log(f)) + b \sinh((d*x+c)\log(f)) / (a - 1))) / (d^2 \log(f)^2)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(a+b\*f\*\*(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arccoth}(b f^{dx+c} + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(a+b*f^(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(b*f^(d*x + c) + a), x)
```

### 3.291 $\int x^2 \coth^{-1} \left( a + b f^{c+dx} \right) dx$

**Optimal.** Leaf size=269

$$\frac{x \operatorname{PolyLog}\left(3, \frac{b f^{c+dx}}{1-a}\right)}{d^2 \log^2(f)} + \frac{x \operatorname{PolyLog}\left(3, -\frac{b f^{c+dx}}{a+1}\right)}{d^2 \log^2(f)} + \frac{\operatorname{PolyLog}\left(4, \frac{b f^{c+dx}}{1-a}\right)}{d^3 \log^3(f)} - \frac{\operatorname{PolyLog}\left(4, -\frac{b f^{c+dx}}{a+1}\right)}{d^3 \log^3(f)} + \frac{x^2 \operatorname{PolyLog}\left(2, \frac{b f^{c+dx}}{1-a}\right)}{2d \log(f)}$$

```
[Out] (x^3*Log[1 - (b*f^(c + d*x))/(1 - a)])/6 - (x^3*Log[1 + (b*f^(c + d*x))/(1 + a)])/6 - (x^3*Log[1 - (a + b*f^(c + d*x))^(-1)])/6 + (x^3*Log[1 + (a + b*f^(c + d*x))^(-1)])/6 + (x^2*PolyLog[2, (b*f^(c + d*x))/(1 - a)])/(2*d*Log[f]) - (x^2*PolyLog[2, -((b*f^(c + d*x))/(1 + a))])/(2*d*Log[f]) - (x*PolyLog[3, (b*f^(c + d*x))/(1 - a)])/(d^2*Log[f]^2) + (x*PolyLog[3, -((b*f^(c + d*x))/(1 + a))])/(d^2*Log[f]^2) + PolyLog[4, (b*f^(c + d*x))/(1 - a)]/(d^3*Log[f]^3) - PolyLog[4, -((b*f^(c + d*x))/(1 + a))]/(d^3*Log[f]^3)
```

**Rubi [A]** time = 2.58565, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 9, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$ , Rules used = {6214, 2551, 12, 6742, 2190, 2531, 6609, 2282, 6589}

$$\frac{x \operatorname{PolyLog}\left(3, \frac{b f^{c+dx}}{1-a}\right)}{d^2 \log^2(f)} + \frac{x \operatorname{PolyLog}\left(3, -\frac{b f^{c+dx}}{a+1}\right)}{d^2 \log^2(f)} + \frac{\operatorname{PolyLog}\left(4, \frac{b f^{c+dx}}{1-a}\right)}{d^3 \log^3(f)} - \frac{\operatorname{PolyLog}\left(4, -\frac{b f^{c+dx}}{a+1}\right)}{d^3 \log^3(f)} + \frac{x^2 \operatorname{PolyLog}\left(2, \frac{b f^{c+dx}}{1-a}\right)}{2d \log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[a + b*f^(c + d*x)], x]
```

```
[Out] (x^3*Log[1 - (b*f^(c + d*x))/(1 - a)])/6 - (x^3*Log[1 + (b*f^(c + d*x))/(1 + a)])/6 - (x^3*Log[1 - (a + b*f^(c + d*x))^(-1)])/6 + (x^3*Log[1 + (a + b*f^(c + d*x))^(-1)])/6 + (x^2*PolyLog[2, (b*f^(c + d*x))/(1 - a)])/(2*d*Log[f]) - (x^2*PolyLog[2, -((b*f^(c + d*x))/(1 + a))])/(2*d*Log[f]) - (x*PolyLog[3, (b*f^(c + d*x))/(1 - a)])/(d^2*Log[f]^2) + (x*PolyLog[3, -((b*f^(c + d*x))/(1 + a))])/(d^2*Log[f]^2) + PolyLog[4, (b*f^(c + d*x))/(1 - a)]/(d^3*Log[f]^3) - PolyLog[4, -((b*f^(c + d*x))/(1 + a))]/(d^3*Log[f]^3)
```

#### Rule 6214

```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Dist[1/2, Int[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
```

IGtQ[m, 0]

### Rule 2551

Int[Log[u\_\*((a\_.) + (b\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((a + b\*x)^(m + 1)\*Log[u])/(b\*(m + 1)), x] - Dist[1/(b\*(m + 1)), Int[SimplifyIntegrand[((a + b\*x)^(m + 1)\*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunctionFreeQ[u, x] && NeQ[m, -1]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 6742

Int[u\_, x\_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

### Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 6609

Int[((e\_.) + (f\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (d\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(p\_.), x\_Symbol] :> Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p])/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

### Rule 2282



```

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

```

### Rule 6589

```

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]

```

### Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(a + b f^{c+dx}) dx &= -\left(\frac{1}{2} \int x^2 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) dx\right) + \frac{1}{2} \int x^2 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right) dx \\
&= -\frac{1}{6} x^3 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{6} x^3 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{6} \int \frac{b d f^{c+dx} x^3 \log(f)}{(-1 + a + b f^{c+dx})(a + b f^{c+dx})} dx \\
&= -\frac{1}{6} x^3 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{6} x^3 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{6} (b d \log(f)) \int \frac{f^c}{(-1 + a + b f^{c+dx})} dx \\
&= -\frac{1}{6} x^3 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{6} x^3 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{6} (b d \log(f)) \int \left(\frac{f^{c+dx} x^3}{-a - b f^{c+dx}}\right) dx \\
&= -\frac{1}{6} x^3 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{6} x^3 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{6} (b d \log(f)) \int \frac{f^{c+dx} x^3}{-1 + a + b f^{c+dx}} dx \\
&= \frac{1}{6} x^3 \log\left(1 - \frac{b f^{c+dx}}{1 - a}\right) - \frac{1}{6} x^3 \log\left(1 + \frac{b f^{c+dx}}{1 + a}\right) - \frac{1}{6} x^3 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{6} x^3 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right) \\
&= \frac{1}{6} x^3 \log\left(1 - \frac{b f^{c+dx}}{1 - a}\right) - \frac{1}{6} x^3 \log\left(1 + \frac{b f^{c+dx}}{1 + a}\right) - \frac{1}{6} x^3 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{6} x^3 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right) \\
&= \frac{1}{6} x^3 \log\left(1 - \frac{b f^{c+dx}}{1 - a}\right) - \frac{1}{6} x^3 \log\left(1 + \frac{b f^{c+dx}}{1 + a}\right) - \frac{1}{6} x^3 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{6} x^3 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right) \\
&= \frac{1}{6} x^3 \log\left(1 - \frac{b f^{c+dx}}{1 - a}\right) - \frac{1}{6} x^3 \log\left(1 + \frac{b f^{c+dx}}{1 + a}\right) - \frac{1}{6} x^3 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{6} x^3 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right) \\
&= \frac{1}{6} x^3 \log\left(1 - \frac{b f^{c+dx}}{1 - a}\right) - \frac{1}{6} x^3 \log\left(1 + \frac{b f^{c+dx}}{1 + a}\right) - \frac{1}{6} x^3 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{6} x^3 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.0697542, size = 235, normalized size = 0.87

$$3d^2x^2 \log^2(f) \operatorname{PolyLog}\left(2, -\frac{bf^{c+dx}}{a-1}\right) - 3d^2x^2 \log^2(f) \operatorname{PolyLog}\left(2, -\frac{bf^{c+dx}}{a+1}\right) + 6 \operatorname{PolyLog}\left(4, -\frac{bf^{c+dx}}{a-1}\right) - 6 \operatorname{PolyLog}\left(4, -\frac{bf^{c+dx}}{a+1}\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[a + b\*f^(c + d\*x)], x]

[Out] (2\*d^3\*x^3\*ArcCoth[a + b\*f^(c + d\*x)]\*Log[f]^3 + d^3\*x^3\*Log[f]^3\*Log[1 + (b\*f^(c + d\*x))/(-1 + a)] - d^3\*x^3\*Log[f]^3\*Log[1 + (b\*f^(c + d\*x))/(1 + a)] + 3\*d^2\*x^2\*Log[f]^2\*PolyLog[2, -((b\*f^(c + d\*x))/(-1 + a))] - 3\*d^2\*x^2\*Log[f]^2\*PolyLog[2, -((b\*f^(c + d\*x))/(1 + a))] - 6\*d\*x\*Log[f]\*PolyLog[3, -((b\*f^(c + d\*x))/(-1 + a))] + 6\*d\*x\*Log[f]\*PolyLog[3, -((b\*f^(c + d\*x))/(1 + a))] + 6\*PolyLog[4, -((b\*f^(c + d\*x))/(-1 + a))] - 6\*PolyLog[4, -((b\*f^(c + d\*x))/(1 + a))])/(6\*d^3\*Log[f]^3)

**Maple [B]** time = 0.175, size = 666, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(a+b\*f^(d\*x+c)), x)

[Out] -1/6\*x^3\*ln(b\*f^(d\*x+c)+a-1)+1/6\*x^3\*ln(1+a+b\*f^(d\*x+c))-1/6\*ln(1-b\*f^(d\*x)\*f^c/(-1-a))\*x^3+1/2/d^2\*ln(1-b\*f^(d\*x)\*f^c/(-1-a))\*x\*c^2+1/3/d^3\*ln(1-b\*f^(d\*x)\*f^c/(-1-a))\*c^3-1/2/ln(f)/d\*polylog(2,b\*f^(d\*x)\*f^c/(-1-a))\*x^2+1/2/ln(f)/d^3\*polylog(2,b\*f^(d\*x)\*f^c/(-1-a))\*c^2+1/ln(f)^2/d^2\*polylog(3,b\*f^(d\*x)\*f^c/(-1-a))\*x-1/ln(f)^3/d^3\*polylog(4,b\*f^(d\*x)\*f^c/(-1-a))+1/6/d^3\*c^3\*ln(1+a+b\*f^(d\*x)\*f^c)-1/2/ln(f)/d^3\*c^2\*dilog((1+a+b\*f^(d\*x)\*f^c)/(1+a))-1/2/d^2\*c^2\*ln((1+a+b\*f^(d\*x)\*f^c)/(1+a))\*x-1/2/d^3\*c^3\*ln((1+a+b\*f^(d\*x)\*f^c)/(1+a))+1/6\*ln(1-b\*f^(d\*x)\*f^c/(1-a))\*x^3-1/2/d^2\*ln(1-b\*f^(d\*x)\*f^c/(1-a))\*x\*c^2-1/3/d^3\*ln(1-b\*f^(d\*x)\*f^c/(1-a))\*c^3+1/2/ln(f)/d\*polylog(2,b\*f^(d\*x)\*f^c/(1-a))\*x^2-1/2/ln(f)/d^3\*polylog(2,b\*f^(d\*x)\*f^c/(1-a))\*c^2-1/ln(f)^2/d^2\*polylog(3,b\*f^(d\*x)\*f^c/(1-a))\*x+1/ln(f)^3/d^3\*polylog(4,b\*f^(d\*x)\*f^c/(1-a))-1/6/d^3\*c^3\*ln(b\*f^(d\*x)\*f^c+a-1)+1/2/ln(f)/d^3\*c^2\*dilog((b\*f^(d\*x)\*f^c+a-1)/(a-1))+1/2/d^2\*c^2\*ln((b\*f^(d\*x)\*f^c+a-1)/(a-1))\*x+1/2/d^3\*c^3\*ln((b\*f^(d\*x)\*f^c+a-1)/(a-1))

---

**Maxima [A]** time = 1.27328, size = 338, normalized size = 1.26

$$\frac{1}{3}x^3 \operatorname{arccoth}(bf^{dx+c} + a) - \frac{1}{6}bd \left( \frac{\log\left(\frac{bf^{dx}f^c}{a+1} + 1\right) \log(f^{dx})^3 + 3 \operatorname{Li}_2\left(-\frac{bf^{dx}f^c}{a+1}\right) \log(f^{dx})^2 - 6 \log(f^{dx}) \operatorname{Li}_3\left(-\frac{bf^{dx}f^c}{a+1}\right) + 6 \operatorname{Li}_4\left(-\frac{bf^{dx}f^c}{a+1}\right)}{bd^4 \log(f)^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(a+b\*f^(d\*x+c)),x, algorithm="maxima")

[Out] 1/3\*x^3\*arccoth(b\*f^(d\*x + c) + a) - 1/6\*b\*d\*((log(b\*f^(d\*x)\*f^c/(a + 1) + 1)\*log(f^(d\*x))^3 + 3\*dilog(-b\*f^(d\*x)\*f^c/(a + 1))\*log(f^(d\*x))^2 - 6\*log(f^(d\*x))\*polylog(3, -b\*f^(d\*x)\*f^c/(a + 1)) + 6\*polylog(4, -b\*f^(d\*x)\*f^c/(a + 1)))/(b\*d^4\*log(f)^4) - (log(b\*f^(d\*x)\*f^c/(a - 1) + 1)\*log(f^(d\*x))^3 + 3\*dilog(-b\*f^(d\*x)\*f^c/(a - 1))\*log(f^(d\*x))^2 - 6\*log(f^(d\*x))\*polylog(3, -b\*f^(d\*x)\*f^c/(a - 1)) + 6\*polylog(4, -b\*f^(d\*x)\*f^c/(a - 1)))/(b\*d^4\*log(f)^4)\*log(f)

---

**Fricas [C]** time = 1.91391, size = 1453, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(a+b\*f^(d\*x+c)),x, algorithm="fricas")

[Out] 1/6\*(d^3\*x^3\*log(f)^3\*log((b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)) + a + 1)/(b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)) + a - 1)) - 3\*d^2\*x^2\*dilog(-(b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)) + a + 1)/(a + 1) + 1)\*log(f)^2 + 3\*d^2\*x^2\*dilog(-(b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)) + a - 1)/(a - 1) + 1)\*log(f)^2 + c^3\*log(b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)) + a + 1)\*log(f)^3 - c^3\*log(b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)) + a - 1)\*log(f)^3 - (d^3\*x^3 + c^3)\*log(f)^3\*log((b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)) + a + 1)/(a + 1)) + (d^3\*x^3 + c^3)\*log(f)^3\*log((b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)) + a - 1)/(a - 1)) + 6\*d\*x\*log(f)\*polylog(3, -(b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)))/(a + 1)) - 6\*d\*x\*log(f)\*polylog(3, -(b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)))/(a - 1)) - 6\*polylog(4, -(b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)))/(a + 1)) + 6\*polylog(4, -(b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)))/(a - 1))

$(a - 1)))/(d^3 \log(f)^3)$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*acoth(a+b*f**(d*x+c)),x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arccoth}(bf^{dx+c} + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccoth(a+b*f^(d*x+c)),x, algorithm="giac")`

[Out] `integrate(x^2*arccoth(b*f^(d*x + c) + a), x)`

$$3.292 \quad \int \frac{1}{(a-ax^2)(b-2b \coth^{-1}(x))} dx$$

**Optimal.** Leaf size=17

$$-\frac{\log(1-2 \coth^{-1}(x))}{2ab}$$

[Out] -Log[1 - 2\*ArcCoth[x]]/(2\*a\*b)

**Rubi [A]** time = 0.0443519, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$ , Rules used = {5947}

$$-\frac{\log(1-2 \coth^{-1}(x))}{2ab}$$

Antiderivative was successfully verified.

[In] Int[1/((a - a\*x^2)\*(b - 2\*b\*ArcCoth[x])), x]

[Out] -Log[1 - 2\*ArcCoth[x]]/(2\*a\*b)

#### Rule 5947

Int[1/(((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))\*((d\_.) + (e\_.)\*(x\_)^2)), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*ArcCoth[c\*x], x]]/(b\*c\*d), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0]

#### Rubi steps

$$\int \frac{1}{(a-ax^2)(b-2b \coth^{-1}(x))} dx = -\frac{\log(1-2 \coth^{-1}(x))}{2ab}$$

**Mathematica [A]** time = 0.0524332, size = 17, normalized size = 1.

$$-\frac{\log(2 \coth^{-1}(x) - 1)}{2ab}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a - a\*x^2)\*(b - 2\*b\*ArcCoth[x])),x]

[Out] -Log[-1 + 2\*ArcCoth[x]]/(2\*a\*b)

**Maple [A]** time = 0.072, size = 19, normalized size = 1.1

$$\frac{\ln(2 \operatorname{arccoth}(x) - b)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-a\*x^2+a)/(b-2\*b\*arccoth(x)),x)

[Out] -1/2/a\*ln(2\*b\*arccoth(x)-b)/b

**Maxima [A]** time = 1.12692, size = 28, normalized size = 1.65

$$\frac{\log(\log(x+1) - \log(x-1) - 1)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a\*x^2+a)/(b-2\*b\*arccoth(x)),x, algorithm="maxima")

[Out] -1/2\*log(log(x + 1) - log(x - 1) - 1)/(a\*b)

**Fricas [A]** time = 1.64775, size = 57, normalized size = 3.35

$$\frac{\log\left(\log\left(\frac{x+1}{x-1}\right) - 1\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a\*x^2+a)/(b-2\*b\*arccoth(x)),x, algorithm="fricas")

[Out]  $-1/2*\log(\log((x + 1)/(x - 1)) - 1)/(a*b)$

---

**Sympy [A]** time = 1.31523, size = 14, normalized size = 0.82

$$-\frac{\log\left(\operatorname{acoth}(x) - \frac{1}{2}\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-a*x**2+a)/(b-2*b*acoth(x)),x)`

[Out]  $-\log(\operatorname{acoth}(x) - 1/2)/(2*a*b)$

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax^2 - a)(2b \operatorname{arccoth}(x) - b)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-a*x^2+a)/(b-2*b*arccoth(x)),x, algorithm="giac")`

[Out] `integrate(1/((a*x^2 - a)*(2*b*arccoth(x) - b)), x)`

### 3.293 $\int x^3 \coth^{-1}(a + bx^4) dx$

**Optimal.** Leaf size=44

$$\frac{\log\left(1 - (a + bx^4)^2\right)}{8b} + \frac{(a + bx^4) \coth^{-1}(a + bx^4)}{4b}$$

[Out]  $((a + b*x^4)*\text{ArcCoth}[a + b*x^4])/(4*b) + \text{Log}[1 - (a + b*x^4)^2]/(8*b)$

**Rubi [A]** time = 0.0523872, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6715, 6104, 5911, 260}

$$\frac{\log\left(1 - (a + bx^4)^2\right)}{8b} + \frac{(a + bx^4) \coth^{-1}(a + bx^4)}{4b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3*\text{ArcCoth}[a + b*x^4], x]$

[Out]  $((a + b*x^4)*\text{ArcCoth}[a + b*x^4])/(4*b) + \text{Log}[1 - (a + b*x^4)^2]/(8*b)$

#### Rule 6715

$\text{Int}[(u_)*(x_)^(m_.), x\_Symbol] \rightarrow \text{Dist}[1/(m + 1), \text{Subst}[\text{Int}[\text{SubstFor}[x^(m + 1), u, x], x, x^(m + 1)], x] /; \text{FreeQ}[m, x] \ \&\& \ \text{NeQ}[m, -1] \ \&\& \ \text{FunctionOfQ}[x^(m + 1), u, x]$

#### Rule 6104

$\text{Int}[(a_.) + \text{ArcCoth}[(c_) + (d_.)*(x_)]*(b_.)]^(p_.), x\_Symbol] \rightarrow \text{Dist}[1/d, \text{Subst}[\text{Int}[(a + b*\text{ArcCoth}[x])^p, x], x, c + d*x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{IGtQ}[p, 0]$

#### Rule 5911

$\text{Int}[(a_.) + \text{ArcCoth}[(c_.)*(x_)]*(b_.)]^(p_.), x\_Symbol] \rightarrow \text{Simp}[x*(a + b*\text{ArcCoth}[c*x])^p, x] - \text{Dist}[b*c*p, \text{Int}[(x*(a + b*\text{ArcCoth}[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; \text{FreeQ}\{a, b, c\}, x] \ \&\& \ \text{IGtQ}[p, 0]$



Rule 260

`Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]`

Rubi steps

$$\begin{aligned}
 \int x^3 \coth^{-1}(a + bx^4) dx &= \frac{1}{4} \text{Subst} \left( \int \coth^{-1}(a + bx) dx, x, x^4 \right) \\
 &= \frac{\text{Subst} \left( \int \coth^{-1}(x) dx, x, a + bx^4 \right)}{4b} \\
 &= \frac{(a + bx^4) \coth^{-1}(a + bx^4)}{4b} - \frac{\text{Subst} \left( \int \frac{x}{1-x^2} dx, x, a + bx^4 \right)}{4b} \\
 &= \frac{(a + bx^4) \coth^{-1}(a + bx^4)}{4b} + \frac{\log \left( 1 - (a + bx^4)^2 \right)}{8b}
 \end{aligned}$$

**Mathematica [A]** time = 0.016213, size = 39, normalized size = 0.89

$$\frac{\log \left( 1 - (a + bx^4)^2 \right) + 2(a + bx^4) \coth^{-1}(a + bx^4)}{8b}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*ArcCoth[a + b\*x^4], x]

[Out] (2\*(a + b\*x^4)\*ArcCoth[a + b\*x^4] + Log[1 - (a + b\*x^4)^2])/(8\*b)

**Maple [A]** time = 0.078, size = 46, normalized size = 1.1

$$\frac{\operatorname{arccoth}(bx^4 + a)x^4}{4} + \frac{\operatorname{arccoth}(bx^4 + a)a}{4b} + \frac{\ln \left( (bx^4 + a)^2 - 1 \right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*arccoth(b\*x^4+a), x)

[Out] 1/4\*arccoth(b\*x^4+a)\*x^4+1/4/b\*arccoth(b\*x^4+a)\*a+1/8/b\*ln((b\*x^4+a)^2-1)

---

**Maxima [A]** time = 1.04667, size = 50, normalized size = 1.14

$$\frac{2(bx^4 + a) \operatorname{arccoth}(bx^4 + a) + \log\left(-\left(bx^4 + a\right)^2 + 1\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(b\*x^4+a),x, algorithm="maxima")

[Out] 1/8\*(2\*(b\*x^4 + a)\*arccoth(b\*x^4 + a) + log(-(b\*x^4 + a)^2 + 1))/b

---

**Fricas [A]** time = 1.82153, size = 149, normalized size = 3.39

$$\frac{bx^4 \log\left(\frac{bx^4+a+1}{bx^4+a-1}\right) + (a+1) \log(bx^4 + a + 1) - (a-1) \log(bx^4 + a - 1)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(b\*x^4+a),x, algorithm="fricas")

[Out] 1/8\*(b\*x^4\*log((b\*x^4 + a + 1)/(b\*x^4 + a - 1)) + (a + 1)\*log(b\*x^4 + a + 1) - (a - 1)\*log(b\*x^4 + a - 1))/b

---

**Sympy [A]** time = 9.03658, size = 60, normalized size = 1.36

$$\begin{cases} \frac{a \operatorname{acoth}(a+bx^4)}{4b} + \frac{x^4 \operatorname{acoth}(a+bx^4)}{4} + \frac{\log(a+bx^4+1)}{4b} - \frac{\operatorname{acoth}(a+bx^4)}{4b} & \text{for } b \neq 0 \\ \frac{x^4 \operatorname{acoth}(a)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*acoth(b\*x\*\*4+a),x)

[Out] Piecewise((a\*acoth(a + b\*x\*\*4)/(4\*b) + x\*\*4\*acoth(a + b\*x\*\*4)/4 + log(a + b\*x\*\*4 + 1)/(4\*b) - acoth(a + b\*x\*\*4)/(4\*b), Ne(b, 0)), (x\*\*4\*acoth(a)/4, True))

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arccoth}(bx^4 + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arccoth(b*x^4+a),x, algorithm="giac")`

[Out] `integrate(x^3*arccoth(b*x^4 + a), x)`

### 3.294 $\int x^{-1+n} \coth^{-1}(a + bx^n) dx$

**Optimal.** Leaf size=47

$$\frac{\log(1 - (a + bx^n)^2)}{2bn} + \frac{(a + bx^n) \coth^{-1}(a + bx^n)}{bn}$$

[Out]  $((a + b*x^n)*\text{ArcCoth}[a + b*x^n])/(b*n) + \text{Log}[1 - (a + b*x^n)^2]/(2*b*n)$

**Rubi [A]** time = 0.0552597, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {6715, 6104, 5911, 260}

$$\frac{\log(1 - (a + bx^n)^2)}{2bn} + \frac{(a + bx^n) \coth^{-1}(a + bx^n)}{bn}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^{(-1 + n)*\text{ArcCoth}[a + b*x^n]}, x]$

[Out]  $((a + b*x^n)*\text{ArcCoth}[a + b*x^n])/(b*n) + \text{Log}[1 - (a + b*x^n)^2]/(2*b*n)$

#### Rule 6715

$\text{Int}[(u_)*(x_)^{(m_.)}, x\_Symbol] := \text{Dist}[1/(m + 1), \text{Subst}[\text{Int}[\text{SubstFor}[x^{(m + 1)}, u, x], x], x, x^{(m + 1)}], x] /;$   $\text{FreeQ}[m, x] \ \&\& \ \text{NeQ}[m, -1] \ \&\& \ \text{FunctionOfQ}[x^{(m + 1)}, u, x]$

#### Rule 6104

$\text{Int}[(a_.) + \text{ArcCoth}[(c_.) + (d_.)*(x_)]*(b_.))^{(p_.)}, x\_Symbol] := \text{Dist}[1/d, \text{Subst}[\text{Int}[(a + b*\text{ArcCoth}[x])^p, x], x, c + d*x], x] /;$   $\text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{IGtQ}[p, 0]$

#### Rule 5911

$\text{Int}[(a_.) + \text{ArcCoth}[(c_.)*(x_)]*(b_.))^{(p_.)}, x\_Symbol] := \text{Simp}[x*(a + b*\text{ArcCoth}[c*x])^p, x] - \text{Dist}[b*c*p, \text{Int}[(x*(a + b*\text{ArcCoth}[c*x])^{(p - 1)})/(1 - c^2*x^2), x], x] /;$   $\text{FreeQ}\{a, b, c\}, x] \ \&\& \ \text{IGtQ}[p, 0]$

#### Rule 260

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

### Rubi steps

$$\begin{aligned} \int x^{-1+n} \coth^{-1}(a + bx^n) dx &= \frac{\text{Subst}\left(\int \coth^{-1}(a + bx) dx, x, x^n\right)}{n} \\ &= \frac{\text{Subst}\left(\int \coth^{-1}(x) dx, x, a + bx^n\right)}{bn} \\ &= \frac{(a + bx^n) \coth^{-1}(a + bx^n)}{bn} - \frac{\text{Subst}\left(\int \frac{x}{1-x^2} dx, x, a + bx^n\right)}{bn} \\ &= \frac{(a + bx^n) \coth^{-1}(a + bx^n)}{bn} + \frac{\log\left(1 - (a + bx^n)^2\right)}{2bn} \end{aligned}$$

**Mathematica [A]** time = 0.0357095, size = 42, normalized size = 0.89

$$\frac{\log\left(1 - (a + bx^n)^2\right) + 2(a + bx^n) \coth^{-1}(a + bx^n)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*ArcCoth[a + b*x^n], x]
```

```
[Out] (2*(a + b*x^n)*ArcCoth[a + b*x^n] + Log[1 - (a + b*x^n)^2])/(2*b*n)
```

**Maple [B]** time = 0.119, size = 118, normalized size = 2.5

$$\frac{x^n \ln(1 + a + bx^n)}{2n} - \frac{x^n \ln(-1 + a + bx^n)}{2n} - \frac{a}{2bn} \ln\left(x^n + \frac{a-1}{b}\right) + \frac{a}{2bn} \ln\left(x^n + \frac{1+a}{b}\right) + \frac{1}{2bn} \ln\left(x^n + \frac{a-1}{b}\right) + \frac{1}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*arccoth(a+b*x^n), x)
```

```
[Out] 1/2/n*x^n*ln(1+a+b*x^n)-1/2/n*x^n*ln(-1+a+b*x^n)-1/2/b/n*ln(x^n+(a-1)/b)*a+1/2/b/n*ln(x^n+(1+a)/b)*a+1/2/b/n*ln(x^n+(a-1)/b)+1/2/b/n*ln(x^n+(1+a)/b)
```

---

**Maxima [A]** time = 1.06526, size = 54, normalized size = 1.15

$$\frac{2 (bx^n + a) \operatorname{arccoth}(bx^n + a) + \log(-(bx^n + a)^2 + 1)}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-1+n)</sup>\*arccoth(a+b\*x<sup>n</sup>),x, algorithm="maxima")

[Out] 1/2\*(2\*(b\*x<sup>n</sup> + a)\*arccoth(b\*x<sup>n</sup> + a) + log(-(b\*x<sup>n</sup> + a)<sup>2</sup> + 1))/(b\*n)

---

**Fricas [B]** time = 1.85904, size = 359, normalized size = 7.64

$$\frac{(a + 1) \log(b \cosh(n \log(x)) + b \sinh(n \log(x)) + a + 1) - (a - 1) \log(b \cosh(n \log(x)) + b \sinh(n \log(x)) + a - 1) + (b \cosh(n \log(x)) + b \sinh(n \log(x)) + a + 1) \operatorname{arccoth}(b \cosh(n \log(x)) + b \sinh(n \log(x)) + a + 1) - (b \cosh(n \log(x)) + b \sinh(n \log(x)) + a - 1) \operatorname{arccoth}(b \cosh(n \log(x)) + b \sinh(n \log(x)) + a - 1))}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-1+n)</sup>\*arccoth(a+b\*x<sup>n</sup>),x, algorithm="fricas")

[Out] 1/2\*((a + 1)\*log(b\*cosh(n\*log(x)) + b\*sinh(n\*log(x)) + a + 1) - (a - 1)\*log(b\*cosh(n\*log(x)) + b\*sinh(n\*log(x)) + a - 1) + (b\*cosh(n\*log(x)) + b\*sinh(n\*log(x)) + a + 1)\*arccoth(b\*cosh(n\*log(x)) + b\*sinh(n\*log(x)) + a + 1) - (b\*cosh(n\*log(x)) + b\*sinh(n\*log(x)) + a - 1)\*arccoth(b\*cosh(n\*log(x)) + b\*sinh(n\*log(x)) + a - 1)))/(b\*n)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>\*\*(-1+n)</sup>\*acoth(a+b\*x<sup>\*\*n</sup>),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^{n-1} \operatorname{arccoth}(bx^n + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arccoth(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(n - 1)*arccoth(b*x^n + a), x)
```

### 3.295 $\int e^{c(a+bx)} \coth^{-1}(\sinh(ac + bcx)) dx$

**Optimal.** Leaf size=107

$$\frac{(1 - \sqrt{2}) \log(-e^{2c(a+bx)} + 3 - 2\sqrt{2})}{2bc} + \frac{(1 + \sqrt{2}) \log(-e^{2c(a+bx)} + 3 + 2\sqrt{2})}{2bc} + \frac{e^{ac+bcx} \coth^{-1}(\sinh(c(a + bx)))}{bc}$$

[Out] (E^(a\*c + b\*c\*x)\*ArcCoth[Sinh[c\*(a + b\*x)]])/(b\*c) + ((1 - Sqrt[2])\*Log[3 - 2\*Sqrt[2] - E^(2\*c\*(a + b\*x))])/(2\*b\*c) + ((1 + Sqrt[2])\*Log[3 + 2\*Sqrt[2] - E^(2\*c\*(a + b\*x))])/(2\*b\*c)

**Rubi [A]** time = 0.1546, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$ , Rules used = {2194, 6276, 2282, 12, 1247, 632, 31}

$$\frac{(1 - \sqrt{2}) \log(-e^{2c(a+bx)} + 3 - 2\sqrt{2})}{2bc} + \frac{(1 + \sqrt{2}) \log(-e^{2c(a+bx)} + 3 + 2\sqrt{2})}{2bc} + \frac{e^{ac+bcx} \coth^{-1}(\sinh(c(a + bx)))}{bc}$$

Antiderivative was successfully verified.

[In] Int[E^(c\*(a + b\*x))\*ArcCoth[Sinh[a\*c + b\*c\*x]],x]

[Out] (E^(a\*c + b\*c\*x)\*ArcCoth[Sinh[c\*(a + b\*x)]])/(b\*c) + ((1 - Sqrt[2])\*Log[3 - 2\*Sqrt[2] - E^(2\*c\*(a + b\*x))])/(2\*b\*c) + ((1 + Sqrt[2])\*Log[3 + 2\*Sqrt[2] - E^(2\*c\*(a + b\*x))])/(2\*b\*c)

#### Rule 2194

Int[((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.), x\_Symbol] := Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

#### Rule 6276

Int[((a\_.) + ArcCoth[u\_]\*(b\_.))\*(v\_), x\_Symbol] := With[{w = IntHide[v, x]}, Dist[a + b\*ArcCoth[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w\*D[u, x])/(1 - u^2), x], x], x] /; InverseFunctionFreeQ[w, x] /; FreeQ[{a, b}, x] && InverseFunctionFreeQ[u, x] && !MatchQ[v, ((c\_.) + (d\_.)\*x)^(m\_.) /; FreeQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v\*(a + b\*ArcCoth[u]), x]]

#### Rule 2282



```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

### Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 31

```
Int[((a_) + (b_.)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)} \coth^{-1}(\sinh(ac+bcx)) dx &= \frac{\text{Subst}\left(\int e^x \coth^{-1}(\sinh(x)) dx, x, ac+bcx\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\sinh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{e^x \cosh(x)}{1-\sinh^2(x)} dx, x, ac+bcx\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\sinh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{2x(-1-x^2)}{1-6x^2+x^4} dx, x, e^{ac+bcx}\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\sinh(c(a+bx)))}{bc} - \frac{2 \text{Subst}\left(\int \frac{x(-1-x^2)}{1-6x^2+x^4} dx, x, e^{ac+bcx}\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\sinh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{-1-x}{1-6x+x^2} dx, x, e^{2ac+2bcx}\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\sinh(c(a+bx)))}{bc} + \frac{(1-\sqrt{2}) \text{Subst}\left(\int \frac{1}{-3+2\sqrt{2}+x} dx, x, e^{2ac+2bcx}\right)}{2bc} + \\
&= \frac{e^{ac+bcx} \coth^{-1}(\sinh(c(a+bx)))}{bc} + \frac{(1-\sqrt{2}) \log(3-2\sqrt{2}-e^{2ac+2bcx})}{2bc} + \frac{(1+\sqrt{2})}{2bc}
\end{aligned}$$

**Mathematica [A]** time = 0.164186, size = 153, normalized size = 1.43

$$\frac{\log(-2e^{c(a+bx)} - e^{2c(a+bx)} + 1) + \log(2e^{c(a+bx)} - e^{2c(a+bx)} + 1) - 2\sqrt{2} \tanh^{-1}\left(\frac{e^{c(a+bx)}-1}{\sqrt{2}}\right) + 2\sqrt{2} \tanh^{-1}\left(\frac{e^{c(a+bx)}+1}{\sqrt{2}}\right) - 2e^{c(a+bx)}}{2bc}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^(c\*(a + b\*x))\*ArcCoth[Sinh[a\*c + b\*c\*x]], x]

[Out] (-2\*E^(c\*(a + b\*x))\*ArcCoth[1/(2\*E^(c\*(a + b\*x)))] - E^(c\*(a + b\*x))/2] - 2\*  
Sqrt[2]\*ArcTanh[(-1 + E^(c\*(a + b\*x)))/Sqrt[2]] + 2\*Sqrt[2]\*ArcTanh[(1 + E^(  
c\*(a + b\*x)))/Sqrt[2]] + Log[1 - 2\*E^(c\*(a + b\*x)) - E^(2\*c\*(a + b\*x))] +  
Log[1 + 2\*E^(c\*(a + b\*x)) - E^(2\*c\*(a + b\*x))]/(2\*b\*c)

**Maple [C]** time = 0.329, size = 794, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c\*(b\*x+a))\*arccoth(sinh(b\*c\*x+a\*c)),x)

[Out]  $\frac{1}{2} \frac{1}{b} \frac{1}{c} \exp(c(bx+a)) \ln(\exp(2c(bx+a)) + 2\exp(c(bx+a)) - 1) - \frac{1}{4} \frac{1}{b} \frac{1}{c} \text{Pi} * \text{csgn}(I \exp(-c(bx+a)) * (-\exp(2c(bx+a)) + 2\exp(c(bx+a)) + 1)) ^3 \exp(c(bx+a)) - \frac{1}{4} \frac{1}{b} \frac{1}{c} \text{Pi} * \text{csgn}(I \exp(-c(bx+a))) * \text{csgn}(I \exp(-c(bx+a)) * (-\exp(2c(bx+a)) + 2\exp(c(bx+a)) + 1)) ^2 \exp(c(bx+a)) + \frac{1}{4} \frac{1}{b} \frac{1}{c} \text{Pi} * \text{csgn}(I(-\exp(2c(bx+a)) + 2\exp(c(bx+a)) + 1)) * \text{csgn}(I \exp(-c(bx+a)) * (-\exp(2c(bx+a)) + 2\exp(c(bx+a)) + 1)) ^2 \exp(c(bx+a)) + \frac{1}{4} \frac{1}{b} \frac{1}{c} \text{Pi} * \text{csgn}(I(-\exp(2c(bx+a)) + 2\exp(c(bx+a)) + 1)) * \text{csgn}(I \exp(-c(bx+a))) * \text{csgn}(I \exp(-c(bx+a)) * (-\exp(2c(bx+a)) + 2\exp(c(bx+a)) + 1)) * \exp(c(bx+a)) + \frac{1}{4} \frac{1}{b} \frac{1}{c} \text{Pi} * \text{csgn}(I(\exp(2c(bx+a)) + 2\exp(c(bx+a)) - 1)) * \text{csgn}(I \exp(-c(bx+a)) * (\exp(2c(bx+a)) + 2\exp(c(bx+a)) - 1)) ^2 \exp(c(bx+a)) - \frac{1}{4} \frac{1}{b} \frac{1}{c} \text{Pi} * \text{csgn}(I(\exp(2c(bx+a)) + 2\exp(c(bx+a)) - 1)) * \text{csgn}(I \exp(-c(bx+a))) * \text{csgn}(I \exp(-c(bx+a)) * (\exp(2c(bx+a)) + 2\exp(c(bx+a)) - 1)) * \exp(c(bx+a)) - \frac{1}{4} \frac{1}{b} \frac{1}{c} \text{Pi} * \text{csgn}(I \exp(-c(bx+a)) * (\exp(2c(bx+a)) + 2\exp(c(bx+a)) - 1)) ^3 \exp(c(bx+a)) + \frac{1}{4} \frac{1}{b} \frac{1}{c} \text{Pi} * \text{csgn}(I \exp(-c(bx+a))) * \text{csgn}(I \exp(-c(bx+a)) * (\exp(2c(bx+a)) + 2\exp(c(bx+a)) - 1)) ^2 \exp(c(bx+a)) - \frac{1}{2} \frac{1}{b} \frac{1}{c} \exp(c(bx+a)) * \ln(\exp(2c(bx+a)) - 2\exp(c(bx+a)) - 1) + \frac{1}{2} \frac{1}{b} \frac{1}{c} \ln(\exp(2c(bx+a)) - (1+2^{(1/2)})^2) * 2^{(1/2)} - \frac{1}{2} \frac{1}{b} \frac{1}{c} \ln(\exp(2c(bx+a)) - (2^{(1/2)} - 1)^2) * 2^{(1/2)} - 2a/b + \frac{1}{2} \frac{1}{b} \frac{1}{c} \ln(\exp(2c(bx+a)) - (1+2^{(1/2)})^2) + \frac{1}{2} \frac{1}{b} \frac{1}{c} \ln(\exp(2c(bx+a)) - (2^{(1/2)} - 1)^2)$

**Maxima [B]** time = 1.61425, size = 248, normalized size = 2.32

$$\frac{\operatorname{arccoth}(\sinh(bc x + ac)) e^{(bx+ac)c}}{bc} + \frac{\sqrt{2} \log\left(-\frac{\sqrt{2}-e^{(bcx+ac)+1}}{\sqrt{2}+e^{(bcx+ac)-1}}\right)}{2bc} - \frac{\sqrt{2} \log\left(-\frac{\sqrt{2}-e^{(bcx+ac)-1}}{\sqrt{2}+e^{(bcx+ac)+1}}\right)}{2bc} + \frac{\log\left(e^{2bcx+2ac} + 2e^{(bcx+ac)} - 1\right)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*(b\*x+a))\*arccoth(sinh(b\*c\*x+a\*c)),x, algorithm="maxima")

[Out]  $\operatorname{arccoth}(\sinh(bcx + ac)) * e^{(bx+a)c} / (bc) + \frac{1}{2} \sqrt{2} * \log(-(\sqrt{2} - e^{(bcx+a)c}) / (\sqrt{2} + e^{(bcx+a)c} - 1)) / (bc) - \frac{1}{2} \sqrt{2} * \log(-(\sqrt{2} - e^{(bcx+a)c} - 1) / (\sqrt{2} + e^{(bcx+a)c} + 1)) / (bc) + \frac{1}{2} * \log(e^{(2bcx+2ac)} + 2e^{(bcx+a)c} - 1) / (bc) + \frac{1}{2} * \log(e^{(2bcx+2ac)} - 2e^{(bcx+a)c} - 1) / (bc)$

**Fricas [B]** time = 1.74446, size = 624, normalized size = 5.83

$$\frac{(\cosh(bc x + ac) + \sinh(bc x + ac)) \log\left(\frac{\sinh(bc x + ac) + 1}{\sinh(bc x + ac) - 1}\right) + \sqrt{2} \log\left(\frac{3(2\sqrt{2}+3) \cosh(bc x + ac)^2 - 4(3\sqrt{2}+4) \cosh(bc x + ac) \sinh(bc x + ac) + 3}{\cosh(bc x + ac)^2 + \sinh(bc x + ac)^2 - 3}\right)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arcoth(sinh(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log((sinh(b*c*x + a*c) + 1)/(sinh(b*c*x + a*c) - 1)) + sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(b*c*x + a*c)^2 - 4*(3*sqrt(2) + 4)*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2*sqrt(2) + 3)*sinh(b*c*x + a*c)^2 - 2*sqrt(2) - 3)/(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 - 3)) + log(2*(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 - 3)/(cosh(b*c*x + a*c)^2 - 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b*c*x + a*c)^2)))/(b*c)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*acoth(sinh(b*c*x+a*c)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}(\sinh(bc x + ac)) e^{(bx+a)c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arcoth(sinh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] integrate(arcoth(sinh(b*c*x + a*c))*e^((b*x + a)*c), x)
```

### 3.296 $\int e^{c(a+bx)} \coth^{-1}(\cosh(ac + bcx)) dx$

**Optimal.** Leaf size=49

$$\frac{\log(1 - e^{2c(a+bx)})}{bc} + \frac{e^{ac+bcx} \coth^{-1}(\cosh(c(a + bx)))}{bc}$$

[Out]  $(E^{(a*c + b*c*x)*ArcCoth[Cosh[c*(a + b*x)]])/(b*c) + \text{Log}[1 - E^{(2*c*(a + b*x))}]/(b*c)$

**Rubi [A]** time = 0.0793503, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {2194, 6276, 2282, 12, 260}

$$\frac{\log(1 - e^{2c(a+bx)})}{bc} + \frac{e^{ac+bcx} \coth^{-1}(\cosh(c(a + bx)))}{bc}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[E^{(c*(a + b*x))*ArcCoth[Cosh[a*c + b*c*x]]}, x]$

[Out]  $(E^{(a*c + b*c*x)*ArcCoth[Cosh[c*(a + b*x)]])/(b*c) + \text{Log}[1 - E^{(2*c*(a + b*x))}]/(b*c)$

#### Rule 2194

$\text{Int}[(F^{((c_.)*((a_.) + (b_.)*(x_)))})^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(F^{(c*(a + b*x)})^{n}/(b*c*n*\text{Log}[F]), x] /; \text{FreeQ}\{F, a, b, c, n\}, x]$

#### Rule 6276

$\text{Int}[(a_.) + \text{ArcCoth}[u_]*(b_.)*(v_), x\_Symbol] \rightarrow \text{With}\{w = \text{IntHide}[v, x]\}, \text{Dist}[a + b*\text{ArcCoth}[u], w, x] - \text{Dist}[b, \text{Int}[\text{SimplifyIntegrand}[(w*D[u, x])/(1 - u^2), x], x], x] /; \text{InverseFunctionFreeQ}[w, x] /; \text{FreeQ}\{a, b\}, x \&\& \text{InverseFunctionFreeQ}[u, x] \&\& \text{!MatchQ}[v, ((c_.) + (d_.)*x)^{(m_.)} /; \text{FreeQ}\{c, d, m\}, x] \&\& \text{FalseQ}[\text{FunctionOfLinear}[v*(a + b*\text{ArcCoth}[u]), x]]$

#### Rule 2282

$\text{Int}[u_, x\_Symbol] \rightarrow \text{With}\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \text{FunctionOfExponentialQ}[u, x] \&\& \text{!MatchQ}[u, (w_)*((a_.)*(v_))^{(n_)}^{(m_)} /; \text{FreeQ}$

```
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

### Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)} \coth^{-1}(\cosh(ac+bcx)) dx &= \frac{\text{Subst}\left(\int e^x \coth^{-1}(\cosh(x)) dx, x, ac+bcx\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\cosh(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int e^x \text{csch}(x) dx, x, ac+bcx\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\cosh(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{2x}{-1+x^2} dx, x, e^{ac+bcx}\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\cosh(c(a+bx)))}{bc} + \frac{2 \text{Subst}\left(\int \frac{x}{-1+x^2} dx, x, e^{ac+bcx}\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\cosh(c(a+bx)))}{bc} + \frac{\log\left(1 - e^{2c(a+bx)}\right)}{bc}
\end{aligned}$$

**Mathematica [A]** time = 0.0855697, size = 60, normalized size = 1.22

$$\frac{\log\left(1 - e^{2c(a+bx)}\right) + e^{c(a+bx)} \coth^{-1}\left(\frac{1}{2}e^{-c(a+bx)}\left(e^{2c(a+bx)} + 1\right)\right)}{bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcCoth[Cosh[a*c + b*c*x]], x]
```

```
[Out] (E^(c*(a + b*x))*ArcCoth[(1 + E^(2*c*(a + b*x)))/(2*E^(c*(a + b*x)))] + Log
[1 - E^(2*c*(a + b*x))]/(b*c)
```

**Maple [C]** time = 0.281, size = 824, normalized size = 16.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(c*(b*x+a))*arccoth(cosh(b*c*x+a*c)),x)`

[Out] 
$$\begin{aligned} & 1/b/c*\exp(c*(b*x+a))*\ln(\exp(c*(b*x+a))+1)-1/4*I/b/c*Pi*csgn(I*\exp(-c*(b*x+a))) \\ & *(\exp(c*(b*x+a))+1)^2)^3*\exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(\exp(c*(b*x+a))) \\ & +1)^2)*csgn(I*\exp(-c*(b*x+a))*(\exp(c*(b*x+a))+1)^2)^2*\exp(c*(b*x+a))-1/4* \\ & I/b/c*Pi*csgn(I*(\exp(c*(b*x+a))+1)^2)^3*\exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I* \\ & \exp(-c*(b*x+a)))*csgn(I*\exp(-c*(b*x+a))*(\exp(c*(b*x+a))+1)^2)^2*\exp(c*(b*x+ \\ & a))-1/4*I/b/c*Pi*csgn(I*\exp(-c*(b*x+a)))*csgn(I*(\exp(c*(b*x+a))+1)^2)*csgn( \\ & I*\exp(-c*(b*x+a))*(\exp(c*(b*x+a))+1)^2)*\exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I* \\ & (\exp(c*(b*x+a))-1)^2)*csgn(I*(\exp(c*(b*x+a))-1)^2)*\exp(c*(b*x+a))+1/2*I/b/c \\ & *Pi*csgn(I*(\exp(c*(b*x+a))+1))*csgn(I*(\exp(c*(b*x+a))+1)^2)^2*\exp(c*(b*x+a) \\ & )-1/2*I/b/c*Pi*csgn(I*(\exp(c*(b*x+a))-1))*csgn(I*(\exp(c*(b*x+a))-1)^2)^2*ex \\ & p(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*\exp(-c*(b*x+a)))*csgn(I*(\exp(c*(b*x+a))-1) \\ & ^2)*csgn(I*\exp(-c*(b*x+a))*(\exp(c*(b*x+a))-1)^2)*\exp(c*(b*x+a))-1/4*I/b/c*P \\ & i*csgn(I*\exp(-c*(b*x+a)))*csgn(I*\exp(-c*(b*x+a))*(\exp(c*(b*x+a))-1)^2)^2*ex \\ & p(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(\exp(c*(b*x+a))-1)^2)*csgn(I*\exp(-c*(b*x+a) \\ & ))*(\exp(c*(b*x+a))-1)^2)^2*\exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(\exp(c*(b*x+a) \\ & ))-1)^2)^3*\exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(\exp(c*(b*x+a))+1))^2*csgn(I* \\ & (\exp(c*(b*x+a))+1)^2)*\exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*\exp(-c*(b*x+a))* \\ & (\exp(c*(b*x+a))-1)^2)^3*\exp(c*(b*x+a))-1/b/c*\exp(c*(b*x+a))*\ln(\exp(c*(b*x+a) \\ & )-1)-2*a/b+1/b/c*\ln(\exp(2*c*(b*x+a))-1) \end{aligned}$$

**Maxima [A]** time = 1.1183, size = 86, normalized size = 1.76

$$\frac{\operatorname{arccoth}(\cosh(bc x + ac)) e^{(bx+a)c}}{bc} + \frac{\log(e^{(bcx+ac)} + 1)}{bc} + \frac{\log(e^{(bcx+ac)} - 1)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(c*(b*x+a))*arccoth(cosh(b*c*x+a*c)),x, algorithm="maxima")`

[Out] 
$$\operatorname{arccoth}(\cosh(b*c*x + a*c))*e^{((b*x + a)*c)/(b*c)} + \log(e^{(b*c*x + a*c)} + 1)/(b*c) + \log(e^{(b*c*x + a*c)} - 1)/(b*c)$$

**Fricas [A]** time = 1.65442, size = 236, normalized size = 4.82

$$\frac{(\cosh(bc x + ac) + \sinh(bc x + ac)) \log\left(\frac{\cosh(bc x + ac) + 1}{\cosh(bc x + ac) - 1}\right) + 2 \log\left(\frac{2 \sinh(bc x + ac)}{\cosh(bc x + ac) - \sinh(bc x + ac)}\right)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*(b\*x+a))\*arcoth(cosh(b\*c\*x+a\*c)),x, algorithm="fricas")

[Out] 1/2\*((cosh(b\*c\*x + a\*c) + sinh(b\*c\*x + a\*c))\*log((cosh(b\*c\*x + a\*c) + 1)/(cosh(b\*c\*x + a\*c) - 1)) + 2\*log(2\*sinh(b\*c\*x + a\*c)/(cosh(b\*c\*x + a\*c) - sinh(b\*c\*x + a\*c))))/(b\*c)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*(b\*x+a))\*acoth(cosh(b\*c\*x+a\*c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}(\cosh(bc x + ac)) e^{(bx+a)c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*(b\*x+a))\*arcoth(cosh(b\*c\*x+a\*c)),x, algorithm="giac")

[Out] integrate(arcoth(cosh(b\*c\*x + a\*c))\*e^((b\*x + a)\*c), x)



$$3.297 \quad \int e^{c(a+bx)} \coth^{-1}(\tanh(ac + bcx)) dx$$

Optimal. Leaf size=45

$$\frac{e^{ac+bcx} \coth^{-1}(\tanh(c(a + bx)))}{bc} - \frac{e^{ac+bcx}}{bc}$$

[Out]  $-(E^{(a*c + b*c*x)/(b*c)}) + (E^{(a*c + b*c*x)*ArcCoth[Tanh[c*(a + b*x)]])/(b*c)$

**Rubi [A]** time = 0.0596168, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2194, 6276}

$$\frac{e^{ac+bcx} \coth^{-1}(\tanh(c(a + bx)))}{bc} - \frac{e^{ac+bcx}}{bc}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[E^{(c*(a + b*x))*ArcCoth[Tanh[a*c + b*c*x]]}, x]$

[Out]  $-(E^{(a*c + b*c*x)/(b*c)}) + (E^{(a*c + b*c*x)*ArcCoth[Tanh[c*(a + b*x)]])/(b*c)$

#### Rule 2194

$\text{Int}[(F^{((c_.) * ((a_.) + (b_.) * (x_))))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[F^{(c*(a + b*x))}]^n / (b*c*n*\text{Log}[F]), x] /; \text{FreeQ}\{F, a, b, c, n\}, x]$

#### Rule 6276

$\text{Int}[(a_.) + \text{ArcCoth}[u_]* (b_.) * (v_), x\_Symbol] \rightarrow \text{With}\{w = \text{IntHide}[v, x]\}, \text{Dist}[a + b*\text{ArcCoth}[u], w, x] - \text{Dist}[b, \text{Int}[\text{SimplifyIntegrand}[(w*D[u, x]) / (1 - u^2), x], x], x] /; \text{InverseFunctionFreeQ}[w, x] /; \text{FreeQ}\{a, b\}, x \&\& \text{InverseFunctionFreeQ}[u, x] \&\& !\text{MatchQ}[v, ((c_.) + (d_.) * x)^{(m_.)} /; \text{FreeQ}\{c, d, m\}, x] \&\& \text{FalseQ}[\text{FunctionOfLinear}[v*(a + b*\text{ArcCoth}[u]), x]]$

#### Rubi steps

$$\begin{aligned} \int e^{c(a+bx)} \coth^{-1}(\tanh(ac + bcx)) dx &= \frac{\text{Subst}\left(\int e^x \coth^{-1}(\tanh(x)) dx, x, ac + bcx\right)}{bc} \\ &= \frac{e^{ac+bcx} \coth^{-1}(\tanh(c(a + bx)))}{bc} - \frac{\text{Subst}\left(\int e^x dx, x, ac + bcx\right)}{bc} \\ &= -\frac{e^{ac+bcx}}{bc} + \frac{e^{ac+bcx} \coth^{-1}(\tanh(c(a + bx)))}{bc} \end{aligned}$$

**Mathematica [A]** time = 0.0752806, size = 46, normalized size = 1.02

$$\frac{e^{c(a+bx)} \left( \coth^{-1} \left( \frac{e^{2c(a+bx)} - 1}{e^{2c(a+bx)} + 1} \right) - 1 \right)}{bc}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^(c\*(a + b\*x))\*ArcCoth[Tanh[a\*c + b\*c\*x]], x]

[Out] (E^(c\*(a + b\*x))\*(-1 + ArcCoth[(-1 + E^(2\*c\*(a + b\*x))]/(1 + E^(2\*c\*(a + b\*x)))))/(b\*c)

**Maple [C]** time = 0.25, size = 351, normalized size = 7.8

$$\frac{e^{c(bx+a)} \ln(e^{c(bx+a)})}{bc} + \frac{i}{4} \frac{e^{c(bx+a)}}{bc} \left( -2\pi \left( \text{csgn} \left( \frac{i}{e^{2c(bx+a)} + 1} \right) \right)^3 + 2\pi \left( \text{csgn} \left( \frac{i}{e^{2c(bx+a)} + 1} \right) \right)^2 - \pi \left( \text{csgn} \left( \frac{ie^{2c(bx+a)}}{e^{2c(bx+a)} + 1} \right) \right)^3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c\*(b\*x+a))\*arccoth(tanh(b\*c\*x+a\*c)), x)

[Out] 1/b/c\*exp(c\*(b\*x+a))\*ln(exp(c\*(b\*x+a)))+1/4\*I\*(-2\*Pi\*csgn(I/(exp(2\*c\*(b\*x+a))+1))^3+2\*Pi\*csgn(I/(exp(2\*c\*(b\*x+a))+1))^2-Pi\*csgn(I\*exp(2\*c\*(b\*x+a))/(exp(2\*c\*(b\*x+a))+1))^3+Pi\*csgn(I\*exp(2\*c\*(b\*x+a))/(exp(2\*c\*(b\*x+a))+1))^2\*csgn(I\*exp(2\*c\*(b\*x+a)))+Pi\*csgn(I\*exp(2\*c\*(b\*x+a))/(exp(2\*c\*(b\*x+a))+1))^2\*csgn(I/(exp(2\*c\*(b\*x+a))+1))-Pi\*csgn(I\*exp(2\*c\*(b\*x+a))/(exp(2\*c\*(b\*x+a))+1))\*csgn(I\*exp(2\*c\*(b\*x+a)))\*csgn(I/(exp(2\*c\*(b\*x+a))+1))-Pi\*csgn(I\*exp(c\*(b\*x+a)))^2\*csgn(I\*exp(2\*c\*(b\*x+a)))+2\*Pi\*csgn(I\*exp(c\*(b\*x+a)))\*csgn(I\*exp(2\*c\*(b\*x+a)))^2-Pi\*csgn(I\*exp(2\*c\*(b\*x+a)))^3+4\*I-2\*Pi)/b/c\*exp(c\*(b\*x+a))

---

**Maxima [A]** time = 1.05829, size = 58, normalized size = 1.29

$$\frac{\operatorname{arccoth}(\tanh(bc x + ac)) e^{(bc x + ac)}}{bc} - \frac{e^{(bc x + ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(c*(b*x+a))*arccoth(tanh(b*c*x+a*c)),x, algorithm="maxima")`

[Out] `arccoth(tanh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - e^(b*c*x + a*c)/(b*c)`

---

**Fricas [A]** time = 1.7068, size = 55, normalized size = 1.22

$$\frac{(bc x + ac - 1)e^{(bc x + ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(c*(b*x+a))*arccoth(tanh(b*c*x+a*c)),x, algorithm="fricas")`

[Out] `(b*c*x + a*c - 1)*e^(b*c*x + a*c)/(b*c)`

---

**Sympy [A]** time = 25.7929, size = 63, normalized size = 1.4

$$\begin{cases} \frac{i\pi x}{2} & \text{for } c = 0 \wedge (b = 0 \vee c = 0) \\ x e^{ac} \operatorname{acoth}(\tanh(ac)) & \text{for } b = 0 \\ \frac{e^{ac} e^{bcx} \operatorname{acoth}(\tanh(ac+bcx))}{bc} - \frac{e^{ac} e^{bcx}}{bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(c*(b*x+a))*acoth(tanh(b*c*x+a*c)),x)`

[Out] `Piecewise((I*pi*x/2, Eq(c, 0) & (Eq(b, 0) | Eq(c, 0))), (x*exp(a*c)*acoth(tanh(a*c)), Eq(b, 0)), (exp(a*c)*exp(b*c*x)*acoth(tanh(a*c + b*c*x))/(b*c) - exp(a*c)*exp(b*c*x)/(b*c), True))`

---

**Giac** [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(\tanh(bc x + ac)) e^{(bx+a)c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(tanh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*c*x + a*c))*e^((b*x + a)*c), x)
```

$$3.298 \quad \int e^{c(a+bx)} \coth^{-1}(\coth(ac + bcx)) dx$$

Optimal. Leaf size=45

$$\frac{e^{ac+bcx} \coth^{-1}(\coth(c(a + bx)))}{bc} - \frac{e^{ac+bcx}}{bc}$$

[Out]  $-(E^{(a*c + b*c*x)/(b*c)}) + (E^{(a*c + b*c*x)*ArcCoth[Coth[c*(a + b*x)]])/(b*c)$

**Rubi [A]** time = 0.0586381, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2194, 6276}

$$\frac{e^{ac+bcx} \coth^{-1}(\coth(c(a + bx)))}{bc} - \frac{e^{ac+bcx}}{bc}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[E^{(c*(a + b*x))*ArcCoth[Coth[a*c + b*c*x]]}, x]$

[Out]  $-(E^{(a*c + b*c*x)/(b*c)}) + (E^{(a*c + b*c*x)*ArcCoth[Coth[c*(a + b*x)]])/(b*c)$

#### Rule 2194

$\text{Int}[(F^{((c_.)*(a_.) + (b_.)*(x_.)))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[F^{(c*(a + b*x))}]^n/(b*c*n*\text{Log}[F]), x] /; \text{FreeQ}\{F, a, b, c, n\}, x]$

#### Rule 6276

$\text{Int}[(a_.) + \text{ArcCoth}[u_]*(b_.)]*(v_), x\_Symbol] \rightarrow \text{With}\{w = \text{IntHide}[v, x]\}, \text{Dist}[a + b*\text{ArcCoth}[u], w, x] - \text{Dist}[b, \text{Int}[\text{SimplifyIntegrand}[(w*D[u, x])/(1 - u^2), x], x], x] /; \text{InverseFunctionFreeQ}[w, x] /; \text{FreeQ}\{a, b\}, x \&\& \text{InverseFunctionFreeQ}[u, x] \&\& !\text{MatchQ}[v, ((c_.) + (d_.)*x)^{(m_.)} /; \text{FreeQ}\{c, d, m\}, x] \&\& \text{FalseQ}[\text{FunctionOfLinear}[v*(a + b*\text{ArcCoth}[u]), x]]]$

#### Rubi steps

$$\begin{aligned} \int e^{c(a+bx)} \coth^{-1}(\coth(ac + bcx)) dx &= \frac{\text{Subst}\left(\int e^x \coth^{-1}(\coth(x)) dx, x, ac + bcx\right)}{bc} \\ &= \frac{e^{ac+bcx} \coth^{-1}(\coth(c(a + bx)))}{bc} - \frac{\text{Subst}\left(\int e^x dx, x, ac + bcx\right)}{bc} \\ &= -\frac{e^{ac+bcx}}{bc} + \frac{e^{ac+bcx} \coth^{-1}(\coth(c(a + bx)))}{bc} \end{aligned}$$

**Mathematica [A]** time = 0.0797835, size = 46, normalized size = 1.02

$$\frac{e^{c(a+bx)} \left( \coth^{-1} \left( \frac{e^{2c(a+bx)} + 1}{e^{2c(a+bx)} - 1} \right) - 1 \right)}{bc}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^(c\*(a + b\*x))\*ArcCoth[Coth[a\*c + b\*c\*x]], x]

[Out] (E^(c\*(a + b\*x))\*(-1 + ArcCoth[(1 + E^(2\*c\*(a + b\*x)))/(-1 + E^(2\*c\*(a + b\*x))]))/(b\*c)

**Maple [A]** time = 0.069, size = 68, normalized size = 1.5

$$\frac{(xbc + ac) e^{xbc+ac} - e^{xbc+ac} + e^{xbc+ac} (\operatorname{arccoth}(\coth(xbc + ac)) - xbc - ac)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c\*(b\*x+a))\*arccoth(coth(b\*c\*x+a\*c)), x)

[Out] 1/b/c\*((b\*c\*x+a\*c)\*exp(b\*c\*x+a\*c)-exp(b\*c\*x+a\*c)+exp(b\*c\*x+a\*c)\*(arccoth(coth(b\*c\*x+a\*c))-x\*b\*c-a\*c))

**Maxima [A]** time = 1.1811, size = 57, normalized size = 1.27

$$\frac{ae^{(bcx+ac)}}{b} + \frac{(bcxe^{(ac)} - e^{(ac)})e^{(bcx)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(c*(b*x+a))*arccoth(coth(b*c*x+a*c)),x, algorithm="maxima")`

[Out]  $a*e^{(b*c*x + a*c)}/b + (b*c*x*e^{(a*c)} - e^{(a*c)})*e^{(b*c*x)}/(b*c)$

**Fricas [A]** time = 1.51155, size = 55, normalized size = 1.22

$$\frac{(bcx + ac - 1)e^{(bcx+ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(c*(b*x+a))*arccoth(coth(b*c*x+a*c)),x, algorithm="fricas")`

[Out]  $(b*c*x + a*c - 1)*e^{(b*c*x + a*c)}/(b*c)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(c*(b*x+a))*acoth(coth(b*c*x+a*c)),x)`

[Out] Timed out

**Giac [A]** time = 1.18341, size = 47, normalized size = 1.04

$$\frac{(b^2c^2x + abc^2 - bc)e^{(bcx+ac)}}{b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(c*(b*x+a))*arccoth(coth(b*c*x+a*c)),x, algorithm="giac")`

[Out]  $(b^2*c^2*x + a*b*c^2 - b*c)*e^{(b*c*x + a*c)}/(b^2*c^2)$

### 3.299 $\int e^{c(a+bx)} \coth^{-1}(\operatorname{sech}(ac + bcx)) dx$

**Optimal.** Leaf size=49

$$\frac{\log(1 - e^{2c(a+bx)})}{bc} + \frac{e^{ac+bcx} \coth^{-1}(\operatorname{sech}(c(a + bx)))}{bc}$$

[Out] (E^(a\*c + b\*c\*x)\*ArcCoth[Sech[c\*(a + b\*x)]])/(b\*c) + Log[1 - E^(2\*c\*(a + b\*x))]/(b\*c)

**Rubi [A]** time = 0.0706529, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {2194, 6276, 2282, 12, 260}

$$\frac{\log(1 - e^{2c(a+bx)})}{bc} + \frac{e^{ac+bcx} \coth^{-1}(\operatorname{sech}(c(a + bx)))}{bc}$$

Antiderivative was successfully verified.

[In] Int[E^(c\*(a + b\*x))\*ArcCoth[Sech[a\*c + b\*c\*x]],x]

[Out] (E^(a\*c + b\*c\*x)\*ArcCoth[Sech[c\*(a + b\*x)]])/(b\*c) + Log[1 - E^(2\*c\*(a + b\*x))]/(b\*c)

#### Rule 2194

Int[((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.), x\_Symbol] := Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

#### Rule 6276

Int[((a\_.) + ArcCoth[u\_]\*(b\_.))\*(v\_), x\_Symbol] := With[{w = IntHide[v, x]}, Dist[a + b\*ArcCoth[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w\*D[u, x])/(1 - u^2), x], x], x] /; InverseFunctionFreeQ[w, x] /; FreeQ[{a, b}, x] && InverseFunctionFreeQ[u, x] && !MatchQ[v, ((c\_.) + (d\_.)\*x)^(m\_.) /; FreeQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v\*(a + b\*ArcCoth[u]), x]]

#### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_) /; FreeQ[



```
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] :=> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :=> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

### Rubi steps

$$\begin{aligned}
 \int e^{c(a+bx)} \coth^{-1}(\operatorname{sech}(ac+bcx)) dx &= \frac{\operatorname{Subst}\left(\int e^x \coth^{-1}(\operatorname{sech}(x)) dx, x, ac+bcx\right)}{bc} \\
 &= \frac{e^{ac+bcx} \coth^{-1}(\operatorname{sech}(c(a+bx)))}{bc} + \frac{\operatorname{Subst}\left(\int e^x \operatorname{csch}(x) dx, x, ac+bcx\right)}{bc} \\
 &= \frac{e^{ac+bcx} \coth^{-1}(\operatorname{sech}(c(a+bx)))}{bc} + \frac{\operatorname{Subst}\left(\int \frac{2x}{-1+x^2} dx, x, e^{ac+bcx}\right)}{bc} \\
 &= \frac{e^{ac+bcx} \coth^{-1}(\operatorname{sech}(c(a+bx)))}{bc} + \frac{2 \operatorname{Subst}\left(\int \frac{x}{-1+x^2} dx, x, e^{ac+bcx}\right)}{bc} \\
 &= \frac{e^{ac+bcx} \coth^{-1}(\operatorname{sech}(c(a+bx)))}{bc} + \frac{\log\left(1 - e^{2c(a+bx)}\right)}{bc}
 \end{aligned}$$

**Mathematica [A]** time = 0.0829819, size = 59, normalized size = 1.2

$$\frac{\log\left(1 - e^{2c(a+bx)}\right) + e^{c(a+bx)} \coth^{-1}\left(\frac{2e^{c(a+bx)}}{e^{2c(a+bx)}+1}\right)}{bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcCoth[Sech[a*c + b*c*x]], x]
```

```
[Out] (E^(c*(a + b*x))*ArcCoth[(2*E^(c*(a + b*x)))/(1 + E^(2*c*(a + b*x)))] + Log
[1 - E^(2*c*(a + b*x))]/(b*c)
```

**Maple [C]** time = 0.292, size = 939, normalized size = 19.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\exp(c*(b*x+a))*\text{arccoth}(\text{sech}(b*c*x+a*c)),x)$

[Out] 
$$\begin{aligned} & -1/4*I/b/c*Pi*csgn(I*(\exp(c*(b*x+a))+1)^2)^3*\exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(\exp(c*(b*x+a))+1)^2)*csgn(I*(\exp(c*(b*x+a))+1)^2/(\exp(2*c*(b*x+a))+1)^2*\exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I/(\exp(2*c*(b*x+a))+1))*csgn(I*(\exp(c*(b*x+a))+1)^2/(\exp(2*c*(b*x+a))+1)^2*\exp(c*(b*x+a))+1/2*I/b/c*Pi*csgn(I*(\exp(c*(b*x+a))+1))*csgn(I*(\exp(c*(b*x+a))+1)^2)^2*\exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(\exp(c*(b*x+a))-1)^2)*csgn(I*(\exp(c*(b*x+a))-1)^2/(\exp(2*c*(b*x+a))+1))^2*\exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(\exp(c*(b*x+a))+1)^2)*csgn(I/(\exp(2*c*(b*x+a))+1))*csgn(I*(\exp(c*(b*x+a))+1)^2/(\exp(2*c*(b*x+a))+1))*\exp(c*(b*x+a))-1/2*I/b/c*Pi*csgn(I*(\exp(c*(b*x+a))-1))*csgn(I*(\exp(c*(b*x+a))-1)^2)^2*\exp(c*(b*x+a))+1/2*I/b/c*Pi*csgn(I*(\exp(c*(b*x+a))-1)^2/(\exp(2*c*(b*x+a))+1))^2*\exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(\exp(c*(b*x+a))+1)^2)*csgn(I*(\exp(c*(b*x+a))+1)^2)*\exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I/(\exp(2*c*(b*x+a))+1))*csgn(I*(\exp(c*(b*x+a))-1)^2/(\exp(2*c*(b*x+a))+1))^2*\exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(\exp(c*(b*x+a))-1)^2)*csgn(I/(\exp(2*c*(b*x+a))+1))*csgn(I*(\exp(c*(b*x+a))-1)^2/(\exp(2*c*(b*x+a))+1))*\exp(c*(b*x+a))-1/2*I/b/c*\exp(c*(b*x+a))*Pi+1/4*I/b/c*Pi*csgn(I*(\exp(c*(b*x+a))-1)^2)^3*\exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(\exp(c*(b*x+a))+1)^2/(\exp(2*c*(b*x+a))+1))^3*\exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(\exp(c*(b*x+a))-1)^2/(\exp(2*c*(b*x+a))+1))^3*\exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(\exp(c*(b*x+a))-1))^2*csgn(I*(\exp(c*(b*x+a))-1)^2)*\exp(c*(b*x+a))-1/b/c*\exp(c*(b*x+a))*\ln(\exp(c*(b*x+a))-1)+1/b/c*\exp(c*(b*x+a))*\ln(\exp(c*(b*x+a))+1)-2*a/b+1/b/c*\ln(\exp(2*c*(b*x+a))-1) \end{aligned}$$

**Maxima [A]** time = 1.03876, size = 86, normalized size = 1.76

$$\frac{\text{arccoth}(\text{sech}(bcx+ac))e^{(bx+a)c}}{bc} + \frac{\log(e^{(bcx+ac)}+1)}{bc} + \frac{\log(e^{(bcx+ac)}-1)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\exp(c*(b*x+a))*\text{arccoth}(\text{sech}(b*c*x+a*c)),x, \text{algorithm}=\text{"maxima"})$

[Out]  $\text{arccoth}(\text{sech}(b*c*x+a*c))*e^{(b*x+a)*c}/(b*c) + \log(e^{(b*c*x+a*c)}+1)/(b*c) + \log(e^{(b*c*x+a*c)}-1)/(b*c)$

---

**Fricas [A]** time = 1.75632, size = 238, normalized size = 4.86

$$\frac{(\cosh(bc x + ac) + \sinh(bc x + ac)) \log\left(-\frac{\cosh(bc x + ac) + 1}{\cosh(bc x + ac) - 1}\right) + 2 \log\left(\frac{2 \sinh(bc x + ac)}{\cosh(bc x + ac) - \sinh(bc x + ac)}\right)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*(b\*x+a))\*arcoth(sech(b\*c\*x+a\*c)),x, algorithm="fricas")

[Out] 1/2\*((cosh(b\*c\*x + a\*c) + sinh(b\*c\*x + a\*c))\*log(-(cosh(b\*c\*x + a\*c) + 1)/(cosh(b\*c\*x + a\*c) - 1)) + 2\*log(2\*sinh(b\*c\*x + a\*c)/(cosh(b\*c\*x + a\*c) - sinh(b\*c\*x + a\*c))))/(b\*c)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*(b\*x+a))\*acoth(sech(b\*c\*x+a\*c)),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arcoth}(\operatorname{sech}(bcx + ac)) e^{((bx+a)c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*(b\*x+a))\*arcoth(sech(b\*c\*x+a\*c)),x, algorithm="giac")

[Out] integrate(arcoth(sech(b\*c\*x + a\*c))\*e^((b\*x + a)\*c), x)

### 3.300 $\int e^{c(a+bx)} \coth^{-1}(\operatorname{csch}(ac + bcx)) dx$

**Optimal.** Leaf size=107

$$\frac{(1 - \sqrt{2}) \log(-e^{2c(a+bx)} + 3 - 2\sqrt{2})}{2bc} + \frac{(1 + \sqrt{2}) \log(-e^{2c(a+bx)} + 3 + 2\sqrt{2})}{2bc} + \frac{e^{ac+bcx} \coth^{-1}(\operatorname{csch}(c(a + bx)))}{bc}$$

[Out] (E^(a\*c + b\*c\*x)\*ArcCoth[Csch[c\*(a + b\*x)]])/(b\*c) + ((1 - Sqrt[2])\*Log[3 - 2\*Sqrt[2] - E^(2\*c\*(a + b\*x))])/(2\*b\*c) + ((1 + Sqrt[2])\*Log[3 + 2\*Sqrt[2] - E^(2\*c\*(a + b\*x))])/(2\*b\*c)

**Rubi [A]** time = 0.147599, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$ , Rules used = {2194, 6276, 2282, 12, 1247, 632, 31}

$$\frac{(1 - \sqrt{2}) \log(-e^{2c(a+bx)} + 3 - 2\sqrt{2})}{2bc} + \frac{(1 + \sqrt{2}) \log(-e^{2c(a+bx)} + 3 + 2\sqrt{2})}{2bc} + \frac{e^{ac+bcx} \coth^{-1}(\operatorname{csch}(c(a + bx)))}{bc}$$

Antiderivative was successfully verified.

[In] Int[E^(c\*(a + b\*x))\*ArcCoth[Csch[a\*c + b\*c\*x]], x]

[Out] (E^(a\*c + b\*c\*x)\*ArcCoth[Csch[c\*(a + b\*x)]])/(b\*c) + ((1 - Sqrt[2])\*Log[3 - 2\*Sqrt[2] - E^(2\*c\*(a + b\*x))])/(2\*b\*c) + ((1 + Sqrt[2])\*Log[3 + 2\*Sqrt[2] - E^(2\*c\*(a + b\*x))])/(2\*b\*c)

#### Rule 2194

Int[((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.), x\_Symbol] := Simp[(F^(c\*(a + b\*x)))^n/(b\*c\*n\*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

#### Rule 6276

Int[((a\_.) + ArcCoth[u\_]\*(b\_.))\*(v\_), x\_Symbol] := With[{w = IntHide[v, x]}, Dist[a + b\*ArcCoth[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w\*D[u, x])/(1 - u^2), x], x], x] /; InverseFunctionFreeQ[w, x] /; FreeQ[{a, b}, x] && InverseFunctionFreeQ[u, x] && !MatchQ[v, ((c\_.) + (d\_.)\*x)^(m\_.) /; FreeQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v\*(a + b\*ArcCoth[u]), x]]

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

### Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 31

```
Int[((a_) + (b_.)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)} \coth^{-1}(\operatorname{csch}(ac+bcx)) dx &= \frac{\operatorname{Subst}\left(\int e^x \coth^{-1}(\operatorname{csch}(x)) dx, x, ac+bcx\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\operatorname{csch}(c(a+bx)))}{bc} + \frac{\operatorname{Subst}\left(\int \frac{e^x \coth(x) \operatorname{csch}(x)}{1-\operatorname{csch}^2(x)} dx, x, ac+bcx\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\operatorname{csch}(c(a+bx)))}{bc} + \frac{\operatorname{Subst}\left(\int \frac{2x(1+x^2)}{1-6x^2+x^4} dx, x, e^{ac+bcx}\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\operatorname{csch}(c(a+bx)))}{bc} + \frac{2 \operatorname{Subst}\left(\int \frac{x(1+x^2)}{1-6x^2+x^4} dx, x, e^{ac+bcx}\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\operatorname{csch}(c(a+bx)))}{bc} + \frac{\operatorname{Subst}\left(\int \frac{1+x}{1-6x+x^2} dx, x, e^{2ac+2bcx}\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\operatorname{csch}(c(a+bx)))}{bc} + \frac{(1-\sqrt{2}) \operatorname{Subst}\left(\int \frac{1}{-3+2\sqrt{2}+x} dx, x, e^{2ac+2bcx}\right)}{2bc} + \\
&= \frac{e^{ac+bcx} \coth^{-1}(\operatorname{csch}(c(a+bx)))}{bc} + \frac{(1-\sqrt{2}) \log(3-2\sqrt{2}-e^{2ac+2bcx})}{2bc} + \frac{(1+\sqrt{2})}{2bc}
\end{aligned}$$

**Mathematica [A]** time = 0.156532, size = 150, normalized size = 1.4

$$\frac{\log(-2e^{c(a+bx)} - e^{2c(a+bx)} + 1) + \log(2e^{c(a+bx)} - e^{2c(a+bx)} + 1) - 2\sqrt{2} \tanh^{-1}\left(\frac{e^{c(a+bx)}-1}{\sqrt{2}}\right) + 2\sqrt{2} \tanh^{-1}\left(\frac{e^{c(a+bx)}+1}{\sqrt{2}}\right) + 2e^{c(a+bx)}}{2bc}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^(c\*(a + b\*x))\*ArcCoth[Csch[a\*c + b\*c\*x]], x]

[Out] (2\*E^(c\*(a + b\*x))\*ArcCoth[(2\*E^(c\*(a + b\*x)))/(-1 + E^(2\*c\*(a + b\*x))]) - 2\*Sqrt[2]\*ArcTanh[(-1 + E^(c\*(a + b\*x)))/Sqrt[2]] + 2\*Sqrt[2]\*ArcTanh[(1 + E^(c\*(a + b\*x)))/Sqrt[2]] + Log[1 - 2\*E^(c\*(a + b\*x)) - E^(2\*c\*(a + b\*x))] + Log[1 + 2\*E^(c\*(a + b\*x)) - E^(2\*c\*(a + b\*x))]/(2\*b\*c)

**Maple [C]** time = 0.333, size = 920, normalized size = 8.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.



**Fricas [B]** time = 1.79002, size = 625, normalized size = 5.84

$$\frac{(\cosh(bc x + ac) + \sinh(bc x + ac)) \log\left(-\frac{\sinh(bc x + ac) + 1}{\sinh(bc x + ac) - 1}\right) + \sqrt{2} \log\left(\frac{3(2\sqrt{2} + 3) \cosh(bc x + ac)^2 - 4(3\sqrt{2} + 4) \cosh(bc x + ac) \sinh(bc x + ac) + 3}{\cosh(bc x + ac)^2 + \sinh(bc x + ac)^2 - 3}\right)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*(b\*x+a))\*arccoth(csch(b\*c\*x+a\*c)),x, algorithm="fricas")

[Out] 1/2\*((cosh(b\*c\*x + a\*c) + sinh(b\*c\*x + a\*c))\*log(-(sinh(b\*c\*x + a\*c) + 1)/(sinh(b\*c\*x + a\*c) - 1)) + sqrt(2)\*log((3\*(2\*sqrt(2) + 3)\*cosh(b\*c\*x + a\*c)^2 - 4\*(3\*sqrt(2) + 4)\*cosh(b\*c\*x + a\*c)\*sinh(b\*c\*x + a\*c) + 3\*(2\*sqrt(2) + 3)\*sinh(b\*c\*x + a\*c)^2 - 2\*sqrt(2) - 3)/(cosh(b\*c\*x + a\*c)^2 + sinh(b\*c\*x + a\*c)^2 - 3)) + log(2\*(cosh(b\*c\*x + a\*c)^2 + sinh(b\*c\*x + a\*c)^2 - 3)/(cosh(b\*c\*x + a\*c)^2 - 2\*cosh(b\*c\*x + a\*c)\*sinh(b\*c\*x + a\*c) + sinh(b\*c\*x + a\*c)^2)))/(b\*c)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$e^{ac} \int e^{bcx} \operatorname{acoth}(\operatorname{csch}(ac + bcx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*(b\*x+a))\*acoth(csch(b\*c\*x+a\*c)),x)

[Out] exp(a\*c)\*Integral(exp(b\*c\*x)\*acoth(csch(a\*c + b\*c\*x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arccoth}(\operatorname{csch}(bcx + ac)) e^{(bx+a)c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*(b\*x+a))\*arccoth(csch(b\*c\*x+a\*c)),x, algorithm="giac")

[Out] integrate(arccoth(csch(b\*c\*x + a\*c))\*e^((b\*x + a)\*c), x)



# Chapter 4

## Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*      is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*      antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```

22     If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25     "C"],
26 If[FreeQ[result,Integrate] && FreeQ[result,Int],
27     "C",
28     "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
38 (*5 = hyperpergeometric function*)
39 (*6 = appell function*)
40 (*7 = rootsum function*)
41 (*8 = integrate function*)
42 (*9 = unknown function*)
43
44
45 ExpnType[expn_] :=
46     If[AtomQ[expn],
47         1,
48     If[ListQ[expn],
49         Max[Map[ExpnType,expn]],
50     If[Head[expn]===Power,
51         If[IntegerQ[expn[[2]]],
52             ExpnType[expn[[1]]],
53         If[Head[expn[[2]]]===Rational,
54             If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
55                 1,
56                 Max[ExpnType[expn[[1]],2]],
57             Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
58     If[Head[expn]===Plus || Head[expn]===Times,
59         Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
60     If[ElementaryFunctionQ[Head[expn]],
61         Max[3,ExpnType[expn[[1]]],
62     If[SpecialFunctionQ[Head[expn]],
63         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
64     If[HypergeometricFunctionQ[Head[expn]],
65         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
66     If[AppellFunctionQ[Head[expn]],
67         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
68     If[Head[expn]===RootSum,

```

```

69   Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
70   If[Head[expn]===Integrate || Head[expn]===Int,
71     Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
72   9]]]]]]]]]]
73
74
75 ElementaryFunctionQ[func_] :=
76   MemberQ[{
77     Exp, Log,
78     Sin, Cos, Tan, Cot, Sec, Csc,
79     ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
80     Sinh, Cosh, Tanh, Coth, Sech, CsCh,
81     ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsCh
82   }, func]
83
84
85 SpecialFunctionQ[func_] :=
86   MemberQ[{
87     Erf, Erfc, Erfi,
88     FresnelS, FresnelC,
89     ExpIntegralE, ExpIntegralEi, LogIntegral,
90     SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
91     Gamma, LogGamma, PolyGamma,
92     Zeta, PolyLog, ProductLog,
93     EllipticF, EllipticE, EllipticPi
94   }, func]
95
96
97 HypergeometricFunctionQ[func_] :=
98   MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]
99
100
101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1}, func]

```

## 4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 #
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 #
11 see problem 156, file Apostol_Problems

```

```

11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;
14
15     leaf_count_result:=leafcount(result);
16     #do NOT call ExpnType() if leaf size is too large. Recursion problem
17     if leaf_count_result > 500000 then
18         return "B";
19     fi;
20
21     leaf_count_optimal:=leafcount(optimal);
22
23     ExpnType_result:=ExpnType(result);
24     ExpnType_optimal:=ExpnType(optimal);
25
26     if debug then
27         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
28     fi;
29
30 # If result and optimal are mathematical expressions,
31 # GradeAntiderivative[result,optimal] returns
32 #   "F" if the result fails to integrate an expression that
33 #     is integrable
34 #   "C" if result involves higher level functions than necessary
35 #   "B" if result is more than twice the size of the optimal
36 #     antiderivative
37 #   "A" if result can be considered optimal
38
39 #This check below actually is not needed, since I only
40 #call this grading only for passed integrals. i.e. I check
41 #for "F" before calling this. But no harm of keeping it here.
42 #just in case.
43
44
45 if not type(result,freeof('int')) then
46     return "F";
47 end if;
48
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then

```

```
56     if debug then
57         print("both result and optimal complex");
58     fi;
59     #both result and optimal complex
60     if leaf_count_result<=2*leaf_count_optimal then
61         return "A";
62     else
63         return "B";
64     end if
65 else #result contains complex but optimal is not
66     if debug then
67         print("result contains complex but optimal is not");
68     fi;
69     return "C";
70 end if
71 else # result do not contain complex
72     # this assumes optimal do not as well
73     if debug then
74         print("result do not contain complex, this assumes optimal do not
as well");
75     fi;
76     if leaf_count_result<=2*leaf_count_optimal then
77         if debug then
78             print("leaf_count_result<=2*leaf_count_optimal");
79         fi;
80         return "A";
81     else
82         if debug then
83             print("leaf_count_result>2*leaf_count_optimal");
84         fi;
85         return "B";
86     end if
87 end if
88 else #ExpnType(result) > ExpnType(optimal)
89     if debug then
90         print("ExpnType(result) > ExpnType(optimal)");
91     fi;
92     return "C";
93 end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417
```

```

102 is_contains_complex:= proc(expression)
103   return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)
119   if type(expn,'atomic') then
120     1
121   elif type(expn,'list') then
122     apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124     if type(op(1,expn),'rational') then
125       1
126     else
127       max(2,ExpnType(op(1,expn)))
128     end if
129   elif type(expn,'^^') then
130     if type(op(2,expn),'integer') then
131       ExpnType(op(1,expn))
132     elif type(op(2,expn),'rational') then
133       if type(op(1,expn),'rational') then
134         1
135       else
136         max(2,ExpnType(op(1,expn)))
137       end if
138     else
139       max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140     end if
141   elif type(expn,'+'') or type(expn,'*') then
142     max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144     max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146     max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148     max(5,apply(max,map(ExpnType,[op(expn)])))

```

```

149   elif AppellFunctionQ(op(0,expn)) then
150       max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152       max(8,apply(max,map(ExpnType,[op(expn)]))) else
153       9
154   end if
155 end proc:
156
157
158 ElementaryFunctionQ := proc(func)
159     member(func,[
160         exp,log,ln,
161         sin,cos,tan,cot,sec,csc,
162         arcsin,arccos,arctan,arccot,arcsec,arccsc,
163         sinh,cosh,tanh,coth,sech,csch,
164         arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
165 end proc:
166
167 SpecialFunctionQ := proc(func)
168     member(func,[
169         erf,erfc,erfi,
170         FresnelS,FresnelC,
171         Ei,Ei,Li,Si,Ci,Shi,Chi,
172         GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
173         EllipticF,EllipticE,EllipticPi])
174 end proc:
175
176 HypergeometricFunctionQ := proc(func)
177     member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
178 end proc:
179
180 AppellFunctionQ := proc(func)
181     member(func,[AppellF1])
182 end proc:
183
184 # u is a sum or product. rest(u) returns all but the
185 # first term or factor of u.
186 rest := proc(u) local v;
187     if nops(u)=2 then
188         op(2,u)
189     else
190         apply(op(0,u),op(2..nops(u),u))
191     end if
192 end proc:
193
194 #leafcount(u) returns the number of nodes in u.
195 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple

```

```

196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

### 4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #           Port of original Maple grading function by
3 #           Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #           added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                    asinh,acosh,atanh,acoth,asech,acsch
25                    ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                    gamma,loggamma,digamma,zeta,polylog,LambertW,
31                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                    ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:

```



```

42     if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43         return True
44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,'^^')
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
72 ))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+' or type
77 (expn,'*')
78         m1 = expnType(expn.args[0])
79         m2 = expnType(list(expn.args[1:]))
80         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82         return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84         m1 = max(map(expnType, list(expn.args)))
85         return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88         m1 = max(map(expnType, list(expn.args)))

```

```

85     return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
86 elif is_appell_function(expn.func):
87     m1 = max(map(expnType, list(expn.args)))
88     return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
89 elif isinstance(expn,RootSum):
90     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
91     return max(7,m1)
92 elif str(expn).find("Integral") != -1:
93     m1 = max(map(expnType, list(expn.args)))
94     return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
95 else:
96     return 9
97
98 #main function
99 def grade_antiderivative(result,optimal):
100
101     leaf_count_result  = leaf_count(result)
102     leaf_count_optimal = leaf_count(optimal)
103
104     expnType_result  = expnType(result)
105     expnType_optimal = expnType(optimal)
106
107     if str(result).find("Integral") != -1:
108         return "F"
109
110     if expnType_result <= expnType_optimal:
111         if result.has(I):
112             if optimal.has(I): #both result and optimal complex
113                 if leaf_count_result <= 2*leaf_count_optimal:
114                     return "A"
115                 else:
116                     return "B"
117             else: #result contains complex but optimal is not
118                 return "C"
119         else: # result do not contain complex, this assumes optimal do not as
well
120             if leaf_count_result <= 2*leaf_count_optimal:
121                 return "A"
122             else:
123                 return "B"
124     else:
125         return "C"

```

## 4.0.4 SageMath grading function

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by

```

2 #           Albert Rich to use with Sagemath. This is used to
3 #           grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #           'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
33         flatten(tree(anti))))
34         return round(1.35*len(flatten(tree(anti)))) #fudge factor
35         #since this estimate of leaf count is bit lower than
36         #what it should be compared to Mathematica's
37
38 def is_sqrt(expr):
39     debug=False;
40     if expr.operator() == operator.pow: #isinstance(expr,Pow):
41         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
42             if debug: print ("expr is sqrt")
43             return True
44         else:
45             return False
46     else:
47         return False

```

```

48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func ," is special_function")
83         else:
84             print ("func ", func ," is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
91     ']
92
93 def is_appell_function(func):

```

```

93     return func.name() in ['hypergeometric']    #[appellf1] can't find this in
          sagemath
94
95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
104             return expn in expn.parent().base_ring() or expn in expn.parent().
          gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list:    #isinstance(expn,list):
121         return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
          Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0]))    #max(2,expnType(expn.
          args[0]))
127     elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer:    #isinstance(expn.args[1],Integer)
129             return expnType(expn.operands()[0])    #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational:    #isinstance(expn.args[1],
          Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
          Rational)
132                 return 1

```

```

133         else:
134             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137         elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138             m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139             m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140             return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
141         elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
142             return max(3,expnType(expn.operands()[0]))
143         elif is_special_function(expn.operator()): #is_special_function(expn.func)
144             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
145             return max(4,m1) #max(4,m1)
146         elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
148             return max(5,m1) #max(5,m1)
149         elif is_appell_function(expn.operator()):
150             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
151             return max(6,m1) #max(6,m1)
152         elif str(expn).find("Integral") != -1: #this will never happen, since it
153             #is checked before calling the grading function that is passed.
154             #but kept it here.
155             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
156             return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
157         else:
158             return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

```

```
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
176                     expnType_optimal)
177
178     if expnType_result <= expnType_optimal:
179         if result.has(I):
180             if optimal.has(I): #both result and optimal complex
181                 if leaf_count_result <= 2*leaf_count_optimal:
182                     return "A"
183             else:
184                 return "B"
185         else: #result contains complex but optimal is not
186             return "C"
187     else: # result do not contain complex, this assumes optimal do not as
188         well
189         if leaf_count_result <= 2*leaf_count_optimal:
190             return "A"
191         else:
192             return "B"
193     else:
194         return "C"
```