Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.3-Inverse-hyperbolic-tangent/7.3.7-Inverse-hyperbolic-tangent-functions

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [361]. This is test number [197].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (361)	% 0. (0)
Mathematica	% 100. (361)	% 0. (0)
Maple	% 94.74 (342)	% 5.26 (19)
Maxima	% 65.93 (238)	% 34.07 (123)
Fricas	% 93.63 (338)	% 6.37 (23)
Sympy	% 22.16 (80)	% 77.84 (281)
Giac	% 71.19 (257)	% 28.81 (104)

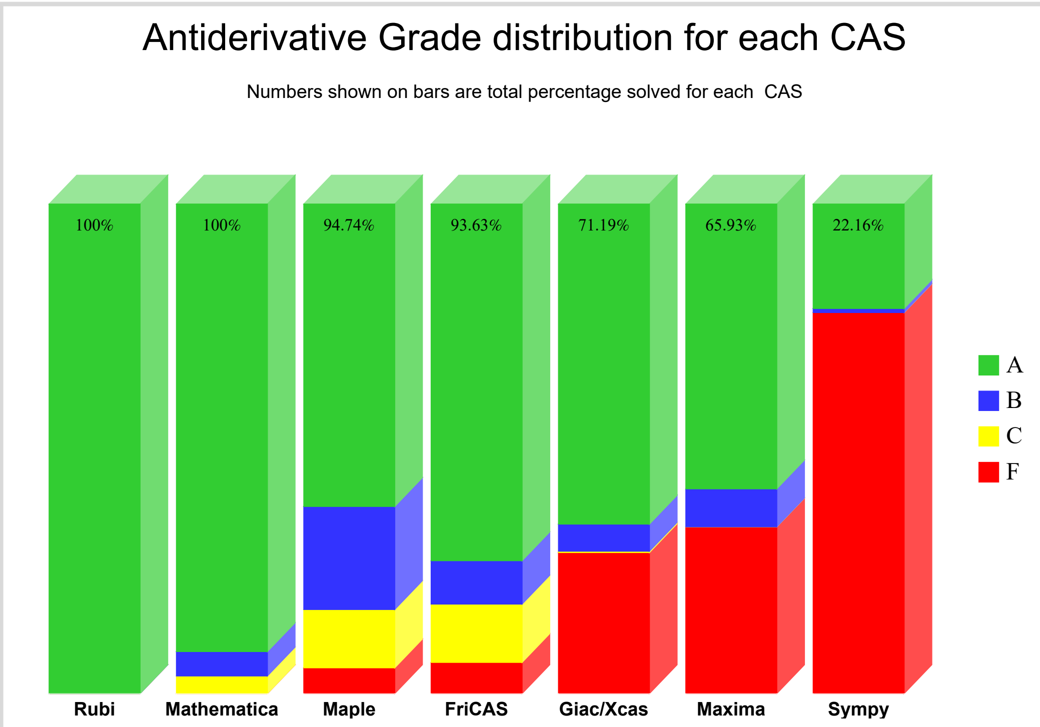
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

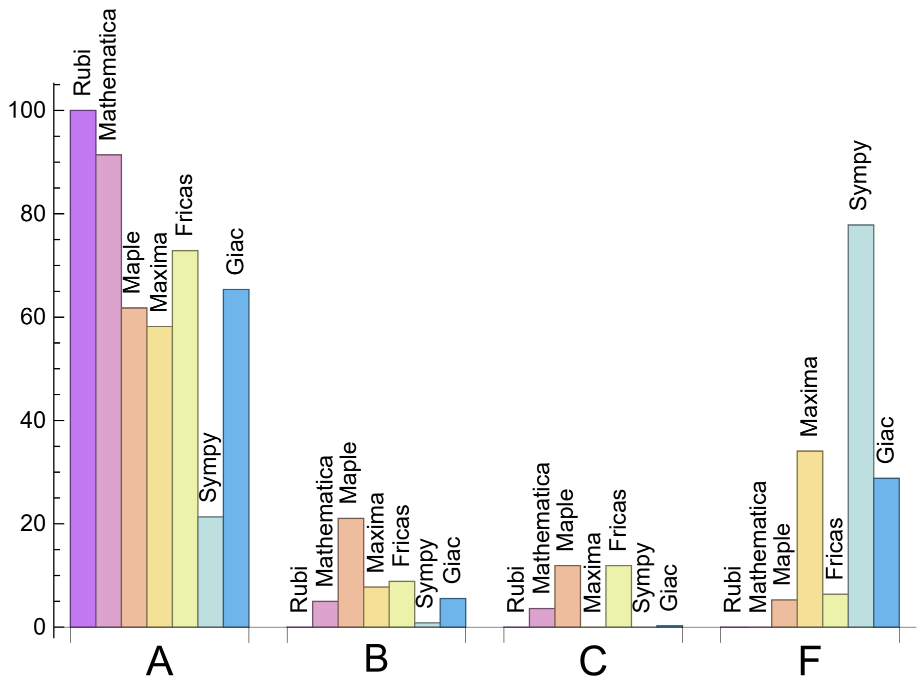
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	91.41	4.99	3.6	0.
Maple	61.77	21.05	11.91	5.26
Maxima	58.17	7.76	0.	34.07
Fricas	72.85	8.86	11.91	6.37
Sympy	21.33	0.83	0.	77.84
Giac	65.37	5.54	0.28	28.81

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.08	95.79	0.96	80.	1.
Mathematica	1.04	119.43	1.13	74.	0.88
Maple	1.05	477.97	3.2	105.	1.11
Maxima	1.57	98.32	1.27	72.	1.14
Fricas	1.88	418.88	3.46	162.	2.3
Sympy	17.1	44.56	1.12	42.	1.03
Giac	1.09	73.4	1.07	61.	0.78

1.4 list of integrals that has no closed form antiderivative

{30, 34, 35, 287, 292, 297, 301, 306, 311, 316, 320, 324, 328, 333, 337, 341, 345}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {4, 33, 291, 296, 298, 299, 305, 310, 314, 319, 323, 327, 331, 336, 340, 344, 355, 356, 357, 358, 359, 360}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

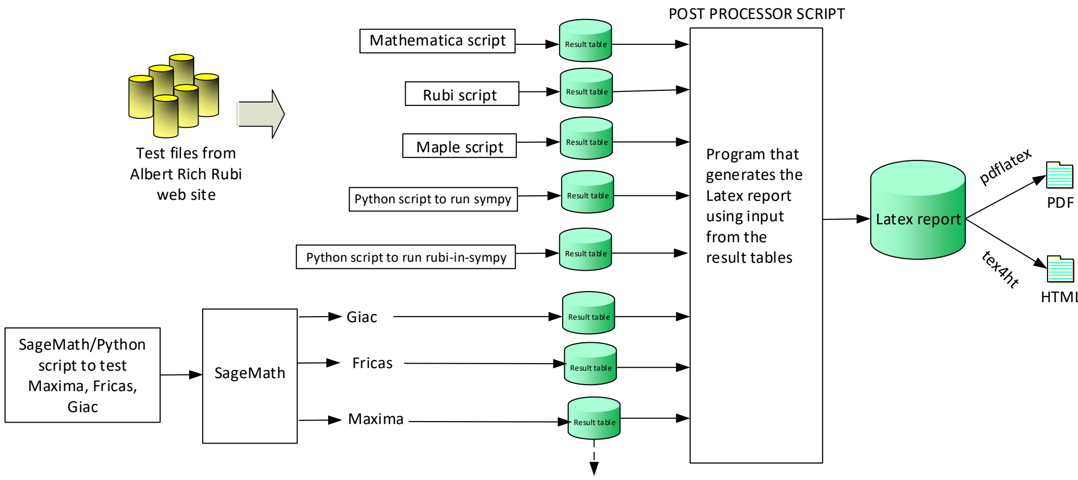
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 79, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293, 294, 295, 297, 298, 299, 300, 301, 302, 303, 304, 306, 307, 308, 309, 311, 313, 314, 315, 316, 317, 318, 320, 321, 322, 324, 325, 326, 328, 330, 331, 332, 333, 334, 335, 337, 338, 339, 341, 342, 343, 345, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360 }

B grade: { 47, 57, 71, 78, 84, 291, 296, 305, 310, 312, 319, 323, 327, 329, 336, 340, 344, 346 }

C grade: { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 361 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 13, 14, 15, 28, 30, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 79, 80, 81, 82, 83, 88, 89, 90, 91, 92, 97, 98, 99, 100, 101, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 152, 153, 154, 155, 156, 157, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 195, 196, 197, 198, 202, 203, 204, 205, 206, 209, 210, 211, 212, 213, 214, 217, 219, 220, 221, 222, 228, 229, 230, 231, 237, 238, 239, 240, 243, 244, 245, 246, 247, 248, 251, 252, 253, 254, 255, 256, 259, 261, 262, 263, 264, 270, 275, 276, 277, 278, 279, 280, 281, 287, 292, 297, 301, 306, 311, 316, 320, 324, 328, 333, 337, 341, 345, 346, 347, 348, 352, 357 }

B grade: { 9, 10, 11, 12, 29, 31, 32, 71, 78, 84, 86, 87, 94, 95, 96, 103, 104, 105, 134, 149, 150, 151, 158, 159, 191, 192, 193, 194, 199, 200, 201, 207, 208, 215, 216, 218, 223, 224, 225, 226, 227, 232, 233, 234, 235, 236, 241, 242, 249, 250, 257, 258, 260, 266, 267, 268, 269, 286, 291, 296, 300, 305, 310, 315, 319, 323, 327, 332, 336, 340, 344, 349, 350, 351, 353, 354 }

C grade: { 274, 282, 283, 284, 285, 288, 289, 290, 293, 294, 295, 298, 299, 302, 303, 304, 307, 308, 309, 312, 313, 314, 317, 318, 321, 322, 325, 326, 329, 330, 331, 334, 335, 338, 339, 342, 343, 355, 356, 358, 359, 360, 361 }

F grade: { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 85, 93, 102, 265, 271, 272, 273 }

2.1.4 Maxima

A grade: { 5, 6, 7, 8, 9, 10, 11, 28, 29, 30, 34, 35, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 49, 50, 51, 52, 53, 55, 56, 57, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 73, 74, 75, 76, 77, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 120, 121, 122, 123, 124, 129, 130, 131, 132, 133, 140, 141, 142, 143, 144, 149, 150, 151, 152, 153, 158, 159, 160, 161, 162, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 219, 220, 221, 222, 228, 229, 230, 231, 237, 238, 239, 240, 245, 246, 247, 248, 254, 255, 256, 262, 263, 264, 266, 267, 268, 269, 270, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 316, 320, 324, 328, 333, 337, 341, 345, 348, 350, 351, 352, 353, 354, 356, 357, 358, 359

B grade: { 12, 48, 58, 71, 72, 78, 84, 315, 319, 321, 322, 323, 325, 326, 327, 332, 336, 338, 339, 340, 342, 343, 344, 346, 347, 349, 355, 360 }

C grade: { }

F grade: { 1, 2, 3, 4, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 31, 32, 33, 36, 44, 54, 65, 85, 93, 102, 116, 117, 118, 119, 125, 126, 127, 128, 134, 135, 136, 137, 138, 139, 145, 146, 147, 148, 154, 155, 156, 157, 163, 164, 165, 166, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 223, 224, 225, 226, 227, 232, 233, 234, 235, 236, 241, 242, 243, 244, 249, 250, 251, 252, 253, 257, 258, 259, 260, 261, 265, 271, 272, 273, 274, 312, 313, 314, 317, 318, 329, 330, 331, 334, 335, 361 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 28, 30, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 269, 270, 274, 275, 276, 277, 278, 279, 280, 287, 292, 297, 301, 306, 311, 316, 320, 324, 328, 333, 337, 340, 341, 344, 345, 352, 356, 357, 358, 359 }

B grade: { 13, 14, 15, 29, 44, 54, 58, 65, 72, 84, 133, 162, 266, 267, 268, 281, 286, 291, 296, 300, 305, 310, 315, 319, 323, 327, 332, 336, 346, 349, 355, 360 }

C grade: { 282, 283, 284, 285, 288, 289, 290, 293, 294, 295, 298, 299, 302, 303, 304, 307, 308, 309, 312, 313, 314, 317, 318, 321, 322, 325, 326, 329, 330, 331, 334, 335, 338, 339, 342, 343, 347, 348, 350, 351, 353, 354, 361 }

F grade: { 4, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 31, 32, 33, 85, 93, 102, 265, 271, 272, 273 }

2.1.6 Sympy

A grade: { 1, 2, 3, 9, 10, 11, 12, 28, 30, 34, 35, 37, 38, 39, 41, 42, 43, 45, 46, 47, 48, 51, 52, 53, 55, 56, 57, 58, 62, 64, 66, 67, 68, 69, 70, 72, 77, 79, 80, 81, 82, 83, 84, 89, 97, 98, 105, 106, 107, 115, 123, 124, 144, 152, 153, 159, 160, 161, 162, 169, 172, 173, 181, 189, 270, 276, 277, 279, 280, 287, 292, 297, 306, 311, 316, 333, 357 }

B grade: { 63, 71, 78 }

C grade: { }

F grade: { 4, 5, 6, 7, 8, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 31, 32, 33, 36, 40, 44, 49, 50, 54, 59, 60, 61, 65, 73, 74, 75, 76, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 168, 170, 171, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 275, 278, 281, 282, 283, 284, 285, 286, 288, 289, 290, 291, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 305, 307, 308, 309, 310, 312, 313, 314, 315, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 358, 359, 360, 361 }

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A grade: { 5, 6, 7, 8, 24, 28, 29, 30, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 125, 126, 127, 128, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 224, 228, 229, 230, 231, 232, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 267, 268, 269, 270, 287, 292, 297, 301, 306, 311, 316, 320, 324, 328, 333, 337, 341, 345, 355, 357, 358, 360 }

B grade: { 58, 72, 84, 122, 123, 124, 129, 130, 131, 132, 133, 219, 266, 275, 276, 277, 279, 280, 356, 359 }

C grade: { 278 }

F grade: { 1, 2, 3, 4, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 31, 32, 33, 44, 54, 65, 85, 93, 102, 225, 226, 227, 233, 234, 235, 236, 265, 271, 272, 273, 274, 281, 282, 283, 284, 285, 286, 288, 289, 290, 291, 293, 294, 295, 296, 298, 299, 300, 302, 303, 304, 305, 307, 308, 309, 310, 312, 313, 314, 315, 317, 318, 319, 321, 322, 323, 325, 326, 327, 329, 330, 331, 332, 334, 335, 336, 338, 339, 340, 342, 343, 344, 346, 347, 348, 349, 350, 351, 352, 353, 354, 361 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	99	172	0	205	121	0
normalized size	1	1.	0.78	1.35	0.	1.61	0.95	0.
time (sec)	N/A	0.052	0.072	0.036	0.	2.208	11.81	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	88	134	0	176	95	0
normalized size	1	1.	0.87	1.33	0.	1.74	0.94	0.
time (sec)	N/A	0.039	0.048	0.031	0.	2.102	4.849	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	76	97	0	142	66	0
normalized size	1	1.	1.01	1.29	0.	1.89	0.88	0.
time (sec)	N/A	0.023	0.031	0.03	0.	2.172	0.879	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	238	238	167	209	0	0	0	0
normalized size	1	1.	0.7	0.88	0.	0.	0.	0.
time (sec)	N/A	0.158	2.164	0.325	0.	0.	0.	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	50	60	69	134	0	96
normalized size	1	1.	0.94	1.13	1.3	2.53	0.	1.81
time (sec)	N/A	0.019	0.036	0.031	1.105	2.227	0.	1.25

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	63	62	82	162	0	144
normalized size	1	1.	0.8	0.78	1.04	2.05	0.	1.82
time (sec)	N/A	0.026	0.04	0.03	0.974	2.248	0.	1.402

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	74	110	138	189	0	181
normalized size	1	1.	0.7	1.05	1.31	1.8	0.	1.72
time (sec)	N/A	0.037	0.048	0.032	0.976	2.339	0.	1.427

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	85	158	169	212	0	217
normalized size	1	1.	0.65	1.21	1.29	1.62	0.	1.66
time (sec)	N/A	0.049	0.054	0.034	0.979	2.435	0.	1.548

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	79	224	209	205	116	0
normalized size	1	1.	0.69	1.96	1.83	1.8	1.02	0.
time (sec)	N/A	0.064	0.055	0.037	0.987	2.202	19.253	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	68	176	171	182	90	0
normalized size	1	1.	0.75	1.93	1.88	2.	0.99	0.
time (sec)	N/A	0.049	0.057	0.033	0.983	2.162	5.203	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	56	128	134	155	65	0
normalized size	1	1.	0.82	1.88	1.97	2.28	0.96	0.
time (sec)	N/A	0.035	0.047	0.03	0.976	2.114	8.623	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	76	88	124	36	0
normalized size	1	1.	1.	1.9	2.2	3.1	0.9	0.
time (sec)	N/A	0.009	0.01	0.029	0.977	2.136	1.493	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	61	53	0	637	0	0
normalized size	1	1.	1.11	0.96	0.	11.58	0.	0.
time (sec)	N/A	0.031	0.042	0.031	0.	2.413	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	92	90	0	783	0	0
normalized size	1	1.	1.08	1.06	0.	9.21	0.	0.
time (sec)	N/A	0.045	0.088	0.032	0.	2.545	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	107	130	0	872	0	0
normalized size	1	1.	0.96	1.17	0.	7.86	0.	0.
time (sec)	N/A	0.057	0.11	0.033	0.	2.637	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	161	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	0.542	0.898	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	147	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	0.372	0.865	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	135	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	0.246	1.036	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	111	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.119	0.848	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	142	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	0.195	0.842	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	154	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	0.287	0.851	0.	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	163	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	0.38	0.835	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	124	0	0	0	0	0
normalized size	1	1.	0.42	0.	0.	0.	0.	0.
time (sec)	N/A	0.187	0.106	0.936	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	109	0	0	0	0	1
normalized size	1	1.	0.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.157	0.09	0.848	0.	0.	0.	1.29

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	85	0	0	0	0	0
normalized size	1	1.	0.37	0.	0.	0.	0.	0.
time (sec)	N/A	0.136	0.104	0.944	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	118	0	0	0	0	0
normalized size	1	1.	0.43	0.	0.	0.	0.	0.
time (sec)	N/A	0.162	0.105	0.819	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	302	131	0	0	0	0	0
normalized size	1	1.	0.43	0.	0.	0.	0.	0.
time (sec)	N/A	0.181	0.1	0.843	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	39	48	50	150	60	89
normalized size	1	1.	0.89	1.09	1.14	3.41	1.36	2.02
time (sec)	N/A	0.049	0.017	0.027	0.95	1.948	7.843	1.121

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	42	121	54	360	0	93
normalized size	1	1.	0.89	2.57	1.15	7.66	0.	1.98
time (sec)	N/A	0.049	0.037	0.06	0.972	1.906	0.	1.161

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.091	0.803	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	409	409	482	1449	0	0	0	0
normalized size	1	1.	1.18	3.54	0.	0.	0.	0.
time (sec)	N/A	0.506	0.174	1.372	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	324	676	0	0	0	0
normalized size	1	1.	1.21	2.52	0.	0.	0.	0.
time (sec)	N/A	0.311	0.101	0.904	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	89	43	118	0	0	0	0
normalized size	1	1.	0.48	1.33	0.	0.	0.	0.
time (sec)	N/A	0.051	0.263	0.514	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.09	0.78	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.744	0.756	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	34	41	0	155	0	58
normalized size	1	1.	0.92	1.11	0.	4.19	0.	1.57
time (sec)	N/A	0.025	0.062	0.039	0.	1.994	0.	1.133

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	26	31	19	18
normalized size	1	1.	0.87	0.87	1.13	1.35	0.83	0.78
time (sec)	N/A	0.008	0.016	0.032	1.152	1.598	0.813	1.134

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	26	31	39	18
normalized size	1	1.	0.87	0.87	1.13	1.35	1.7	0.78
time (sec)	N/A	0.007	0.015	0.03	1.143	1.619	3.212	1.127

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	15	22	23	19	14
normalized size	1	1.	1.12	0.94	1.38	1.44	1.19	0.88
time (sec)	N/A	0.003	0.01	0.027	1.144	1.694	1.279	1.12

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	21	46	22	0	12
normalized size	1	1.	0.9	1.	2.19	1.05	0.	0.57
time (sec)	N/A	0.036	0.014	0.036	0.973	1.822	0.	1.118

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	18	18	23	27	14	16
normalized size	1	1.	1.06	1.06	1.35	1.59	0.82	0.94
time (sec)	N/A	0.008	0.015	0.032	1.156	1.744	0.606	1.124

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	18	20	26	30	19	15
normalized size	1	1.	0.78	0.87	1.13	1.3	0.83	0.65
time (sec)	N/A	0.009	0.014	0.033	1.164	1.66	1.1	1.139

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	26	32	20	18
normalized size	1	1.	0.87	0.87	1.13	1.39	0.87	0.78
time (sec)	N/A	0.008	0.015	0.033	1.155	1.871	7.862	1.128

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	62	98	0	356	0	0
normalized size	1	1.	0.87	1.38	0.	5.01	0.	0.
time (sec)	N/A	0.032	0.108	0.04	0.	2.223	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	37	38	49	55	37	32
normalized size	1	1.	0.88	0.9	1.17	1.31	0.88	0.76
time (sec)	N/A	0.023	0.03	0.036	1.349	1.959	2.629	1.151

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	37	38	49	55	37	32
normalized size	1	1.	0.88	0.9	1.17	1.31	0.88	0.76
time (sec)	N/A	0.022	0.048	0.035	1.345	1.535	1.741	1.141

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	74	38	49	55	37	32
normalized size	1	1.	2.18	1.12	1.44	1.62	1.09	0.94
time (sec)	N/A	0.021	0.075	0.035	1.355	1.439	0.826	1.142

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	45	42	20	27
normalized size	1	1.	1.	0.94	2.81	2.62	1.25	1.69
time (sec)	N/A	0.004	0.006	0.027	1.39	1.513	2.319	1.147

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	53	55	27	49	0	28
normalized size	1	1.	1.08	1.12	0.55	1.	0.	0.57
time (sec)	N/A	0.034	0.041	0.036	2.359	1.478	0.	1.126

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	37	42	73	49	0	28
normalized size	1	1.	0.95	1.08	1.87	1.26	0.	0.72
time (sec)	N/A	0.024	0.045	0.04	1.194	1.488	0.	1.132

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	42	35	46	59	32	30
normalized size	1	1.	1.17	0.97	1.28	1.64	0.89	0.83
time (sec)	N/A	0.021	0.033	0.036	1.377	1.504	0.823	1.188

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	34	38	49	51	37	30
normalized size	1	1.	1.1	1.23	1.58	1.65	1.19	0.97
time (sec)	N/A	0.013	0.039	0.035	1.371	1.52	2.068	1.113

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	64	37	38	49	55	39	32
normalized size	1	1.52	0.88	0.9	1.17	1.31	0.93	0.76
time (sec)	N/A	0.03	0.029	0.035	1.369	1.464	8.015	1.179

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	97	177	0	657	0	0
normalized size	1	1.	0.88	1.61	0.	5.97	0.	0.
time (sec)	N/A	0.058	0.104	0.041	0.	1.679	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	54	56	73	80	80	47
normalized size	1	1.	0.89	0.92	1.2	1.31	1.31	0.77
time (sec)	N/A	0.044	0.023	0.038	1.563	1.422	6.785	1.142

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	54	56	73	80	56	47
normalized size	1	1.	1.02	1.06	1.38	1.51	1.06	0.89
time (sec)	N/A	0.029	0.022	0.036	1.561	1.49	2.119	1.154

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	99	56	73	74	56	46
normalized size	1	1.	2.91	1.65	2.15	2.18	1.65	1.35
time (sec)	N/A	0.014	0.073	0.037	1.547	1.426	6.831	1.128

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	69	66	20	42
normalized size	1	1.	1.	0.94	4.31	4.12	1.25	2.62
time (sec)	N/A	0.004	0.006	0.028	1.531	1.413	3.007	1.11

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	104	92	42	73	0	43
normalized size	1	1.	1.35	1.19	0.55	0.95	0.	0.56
time (sec)	N/A	0.081	0.058	0.037	2.45	1.577	0.	1.155

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	62	76	88	78	0	45
normalized size	1	1.	0.91	1.12	1.29	1.15	0.	0.66
time (sec)	N/A	0.041	0.038	0.04	2.587	1.506	0.	1.11

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	66	59	97	81	0	42
normalized size	1	1.	1.1	0.98	1.62	1.35	0.	0.7
time (sec)	N/A	0.04	0.044	0.042	1.427	1.524	0.	1.146

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	60	52	70	85	51	47
normalized size	1	1.	1.09	0.95	1.27	1.55	0.93	0.85
time (sec)	N/A	0.037	0.024	0.037	1.595	1.442	6.218	1.145

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	50	56	72	73	56	45
normalized size	1	1.	1.61	1.81	2.32	2.35	1.81	1.45
time (sec)	N/A	0.013	0.022	0.039	1.594	1.424	4.615	1.134

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	54	56	73	81	60	47
normalized size	1	1.	0.84	0.88	1.14	1.27	0.94	0.73
time (sec)	N/A	0.032	0.04	0.038	1.589	1.442	3.858	1.142

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	137	278	0	1076	0	0
normalized size	1	1.	0.89	1.81	0.	6.99	0.	0.
time (sec)	N/A	0.099	0.142	0.042	0.	1.593	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	71	74	97	108	75	62
normalized size	1	1.	0.89	0.92	1.21	1.35	0.94	0.78
time (sec)	N/A	0.063	0.062	0.04	1.738	1.559	32.549	1.126

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	71	74	97	107	76	62
normalized size	1	1.	0.89	0.92	1.21	1.34	0.95	0.78
time (sec)	N/A	0.056	0.031	0.04	1.738	1.512	18.93	1.122

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	71	74	97	104	78	62
normalized size	1	1.	0.89	0.92	1.21	1.3	0.98	0.78
time (sec)	N/A	0.055	0.033	0.04	1.778	1.465	11.288	1.141

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	71	74	97	99	75	61
normalized size	1	1.	0.99	1.03	1.35	1.38	1.04	0.85
time (sec)	N/A	0.047	0.024	0.042	1.735	1.389	6.741	1.138

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	71	74	97	99	60	61
normalized size	1	1.	1.34	1.4	1.83	1.87	1.13	1.15
time (sec)	N/A	0.03	0.051	0.043	1.747	1.494	7.451	1.176

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	125	74	97	104	76	62
normalized size	1	1.	3.68	2.18	2.85	3.06	2.24	1.82
time (sec)	N/A	0.014	0.084	0.04	1.742	1.496	2.47	1.101

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	93	85	20	57
normalized size	1	1.	1.	0.94	5.81	5.31	1.25	3.56
time (sec)	N/A	0.005	0.005	0.027	1.733	1.535	2.017	1.13

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	175	127	57	95	0	58
normalized size	1	1.	1.67	1.21	0.54	0.9	0.	0.55
time (sec)	N/A	0.065	0.091	0.038	2.404	1.616	0.	1.111

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	85	112	104	103	0	59
normalized size	1	1.	0.89	1.18	1.09	1.08	0.	0.62
time (sec)	N/A	0.064	0.076	0.04	2.61	1.485	0.	1.19

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	81	93	112	101	0	58
normalized size	1	1.	0.93	1.07	1.29	1.16	0.	0.67
time (sec)	N/A	0.062	0.035	0.043	2.771	1.575	0.	1.165

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	82	76	123	105	0	57
normalized size	1	1.	1.06	0.99	1.6	1.36	0.	0.74
time (sec)	N/A	0.058	0.048	0.046	1.612	1.547	0.	1.135

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	78	69	97	112	70	62
normalized size	1	1.	1.05	0.93	1.31	1.51	0.95	0.84
time (sec)	N/A	0.054	0.03	0.041	1.796	1.564	2.293	1.126

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	66	74	95	97	75	59
normalized size	1	1.	2.13	2.39	3.06	3.13	2.42	1.9
time (sec)	N/A	0.013	0.052	0.044	1.798	1.485	4.027	1.125

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	71	74	97	104	78	62
normalized size	1	1.	1.11	1.16	1.52	1.62	1.22	0.97
time (sec)	N/A	0.031	0.031	0.042	1.791	1.457	6.675	1.168

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	71	74	97	109	80	62
normalized size	1	1.	0.72	0.76	0.99	1.11	0.82	0.63
time (sec)	N/A	0.053	0.033	0.04	1.798	1.523	11.641	1.158

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	132	71	74	97	111	76	62
normalized size	1	1.65	0.89	0.92	1.21	1.39	0.95	0.78
time (sec)	N/A	0.075	0.034	0.04	1.793	1.408	16.137	1.103

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	71	74	97	112	76	62
normalized size	1	1.	0.89	0.92	1.21	1.4	0.95	0.78
time (sec)	N/A	0.055	0.06	0.04	1.783	1.462	25.871	1.121

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	71	74	97	116	78	62
normalized size	1	1.	0.89	0.92	1.21	1.45	0.98	0.78
time (sec)	N/A	0.056	0.032	0.04	1.801	1.467	78.113	1.166

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	177	110	149	149	41	92
normalized size	1	1.	5.21	3.24	4.38	4.38	1.21	2.71
time (sec)	N/A	0.014	0.118	0.048	2.149	1.459	11.737	1.131

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	51	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.085	0.4	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	79	202	57	92	0	58
normalized size	1	1.	0.98	2.49	0.7	1.14	0.	0.72
time (sec)	N/A	0.058	0.044	0.034	1.781	1.485	0.	1.127

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	55	111	39	68	0	41
normalized size	1	1.	0.98	1.98	0.7	1.21	0.	0.73
time (sec)	N/A	0.035	0.032	0.033	1.768	1.508	0.	1.133

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	49	24	38	0	26
normalized size	1	1.	1.	1.58	0.77	1.23	0.	0.84
time (sec)	N/A	0.015	0.023	0.033	1.766	1.532	0.	1.137

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	18	22	17	15
normalized size	1	1.	1.	1.08	1.5	1.83	1.42	1.25
time (sec)	N/A	0.004	0.047	0.027	1.461	1.451	9.014	1.156

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	29	43	24	38	0	27
normalized size	1	1.	0.66	0.98	0.55	0.86	0.	0.61
time (sec)	N/A	0.027	0.019	0.038	1.778	1.558	0.	1.157

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	45	64	38	61	0	41
normalized size	1	1.	0.69	0.98	0.58	0.94	0.	0.63
time (sec)	N/A	0.044	0.026	0.079	1.791	1.515	0.	1.143

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	66	87	54	103	0	61
normalized size	1	1.	0.72	0.95	0.59	1.12	0.	0.66
time (sec)	N/A	0.068	0.034	0.081	1.796	1.515	0.	1.135

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	51	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.489	1.622	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	106	350	95	155	0	84
normalized size	1	1.	1.08	3.57	0.97	1.58	0.	0.86
time (sec)	N/A	0.081	0.097	0.042	2.436	1.519	0.	1.161

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	83	223	80	132	0	65
normalized size	1	1.	1.11	2.97	1.07	1.76	0.	0.87
time (sec)	N/A	0.053	0.052	0.041	2.44	1.513	0.	1.117

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	56	127	59	97	0	46
normalized size	1	1.	1.12	2.54	1.18	1.94	0.	0.92
time (sec)	N/A	0.034	0.063	0.04	2.405	1.523	0.	1.159

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	56	35	62	36	32
normalized size	1	1.	0.96	2.	1.25	2.21	1.29	1.14
time (sec)	N/A	0.014	0.051	0.039	2.447	1.5	15.179	1.122

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	16	24	20	16
normalized size	1	1.	1.	1.07	1.14	1.71	1.43	1.14
time (sec)	N/A	0.005	0.006	0.027	1.472	1.441	14.972	1.143

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	53	67	38	89	0	42
normalized size	1	1.	0.76	0.96	0.54	1.27	0.	0.6
time (sec)	N/A	0.046	0.062	0.066	2.449	1.597	0.	1.159

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	70	91	61	138	0	61
normalized size	1	1.	0.69	0.89	0.6	1.35	0.	0.6
time (sec)	N/A	0.072	0.06	0.09	2.426	1.509	0.	1.148

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	92	116	86	177	0	86
normalized size	1	1.	0.64	0.81	0.6	1.24	0.	0.6
time (sec)	N/A	0.096	0.048	0.089	2.423	1.504	0.	1.154

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	51	0	0	0	0	0
normalized size	1	1.	0.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	0.48	1.675	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	114	371	109	200	0	82
normalized size	1	1.	1.24	4.03	1.18	2.17	0.	0.89
time (sec)	N/A	0.073	0.042	0.041	3.541	1.5	0.	1.16

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	86	239	93	176	0	59
normalized size	1	1.	1.21	3.37	1.31	2.48	0.	0.83
time (sec)	N/A	0.048	0.065	0.04	3.483	1.478	0.	1.185

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	49	136	65	132	54	50
normalized size	1	1.	1.04	2.89	1.38	2.81	1.15	1.06
time (sec)	N/A	0.029	0.032	0.04	3.478	1.472	22.172	1.137

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	27	43	43	68	42	24
normalized size	1	1.	0.79	1.26	1.26	2.	1.24	0.71
time (sec)	N/A	0.014	0.049	0.037	3.531	1.559	22.14	1.138

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	16	49	24	16
normalized size	1	1.	1.	0.94	1.	3.06	1.5	1.
time (sec)	N/A	0.004	0.007	0.027	1.475	1.492	15.312	1.121

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	74	92	69	182	0	58
normalized size	1	1.	0.76	0.95	0.71	1.88	0.	0.6
time (sec)	N/A	0.066	0.097	0.068	3.542	1.528	0.	1.153

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	93	117	93	232	0	81
normalized size	1	1.	0.71	0.89	0.71	1.77	0.	0.62
time (sec)	N/A	0.093	0.043	0.085	3.525	1.522	0.	1.144

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	107	145	116	269	0	99
normalized size	1	1.	0.63	0.85	0.68	1.58	0.	0.58
time (sec)	N/A	0.121	0.048	0.087	3.519	1.558	0.	1.131

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	83	154	86	151	0	82
normalized size	1	1.	0.82	1.52	0.85	1.5	0.	0.81
time (sec)	N/A	0.065	0.032	0.066	1.793	1.496	0.	1.178

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	66	124	72	120	0	66
normalized size	1	1.	0.82	1.55	0.9	1.5	0.	0.82
time (sec)	N/A	0.049	0.032	0.047	1.801	1.525	0.	1.186

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	49	69	57	97	0	50
normalized size	1	1.	0.83	1.17	0.97	1.64	0.	0.85
time (sec)	N/A	0.03	0.027	0.046	1.794	1.542	0.	1.175

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	32	42	41	70	0	34
normalized size	1	1.	0.84	1.11	1.08	1.84	0.	0.89
time (sec)	N/A	0.014	0.049	0.046	1.764	1.528	0.	1.123

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	16	31	26	24
normalized size	1	1.	1.	0.83	0.89	1.72	1.44	1.33
time (sec)	N/A	0.005	0.006	0.029	1.685	1.506	0.755	1.151

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	61	54	0	188	0	43
normalized size	1	1.	0.97	0.86	0.	2.98	0.	0.68
time (sec)	N/A	0.06	0.073	0.128	0.	1.557	0.	1.157

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	65	63	0	225	0	55
normalized size	1	1.	0.98	0.95	0.	3.41	0.	0.83
time (sec)	N/A	0.032	0.037	0.126	0.	1.777	0.	1.18

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	89	92	0	292	0	89
normalized size	1	1.	0.71	0.74	0.	2.34	0.	0.71
time (sec)	N/A	0.072	0.097	0.127	0.	1.72	0.	1.17

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	115	185	0	347	0	113
normalized size	1	1.	0.64	1.03	0.	1.94	0.	0.63
time (sec)	N/A	0.121	0.098	0.129	0.	1.856	0.	1.168

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	83	154	86	180	0	203
normalized size	1	1.	0.82	1.52	0.85	1.78	0.	2.01
time (sec)	N/A	0.067	0.037	0.033	1.78	1.682	0.	1.171

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	66	124	72	147	0	169
normalized size	1	1.	0.82	1.55	0.9	1.84	0.	2.11
time (sec)	N/A	0.046	0.033	0.033	1.765	1.799	0.	1.159

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	49	69	57	120	0	136
normalized size	1	1.	0.83	1.17	0.97	2.03	0.	2.31
time (sec)	N/A	0.029	0.03	0.033	1.768	1.789	0.	1.143

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	32	42	42	92	49	104
normalized size	1	1.	0.84	1.11	1.11	2.42	1.29	2.74
time (sec)	N/A	0.014	0.057	0.036	1.762	1.921	49.994	1.156

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	16	63	26	62
normalized size	1	1.	1.	0.83	0.89	3.5	1.44	3.44
time (sec)	N/A	0.005	0.006	0.027	1.693	2.023	19.827	1.138

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	80	131	0	228	0	77
normalized size	1	1.	0.88	1.44	0.	2.51	0.	0.85
time (sec)	N/A	0.05	0.069	0.107	0.	2.12	0.	1.159

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	79	85	0	247	0	93
normalized size	1	1.	0.98	1.05	0.	3.05	0.	1.15
time (sec)	N/A	0.047	0.035	0.118	0.	2.212	0.	1.157

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	88	91	0	296	0	99
normalized size	1	1.	0.96	0.99	0.	3.22	0.	1.08
time (sec)	N/A	0.049	0.059	0.115	0.	2.141	0.	1.173

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	117	116	0	351	0	126
normalized size	1	1.	0.8	0.79	0.	2.4	0.	0.86
time (sec)	N/A	0.094	0.092	0.115	0.	2.188	0.	1.203

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	83	154	86	205	0	324
normalized size	1	1.	0.82	1.52	0.85	2.03	0.	3.21
time (sec)	N/A	0.065	0.038	0.035	1.831	2.046	0.	1.17

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	66	124	72	171	0	277
normalized size	1	1.	0.82	1.55	0.9	2.14	0.	3.46
time (sec)	N/A	0.047	0.037	0.036	1.781	2.032	0.	1.185

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	49	69	57	146	0	227
normalized size	1	1.	0.83	1.17	0.97	2.47	0.	3.85
time (sec)	N/A	0.029	0.031	0.033	1.801	1.982	0.	1.181

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	32	42	42	116	0	178
normalized size	1	1.	0.84	1.11	1.11	3.05	0.	4.68
time (sec)	N/A	0.014	0.06	0.033	1.784	2.049	0.	1.117

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	16	85	0	116
normalized size	1	1.	1.	0.83	0.89	4.72	0.	6.44
time (sec)	N/A	0.005	0.007	0.029	1.716	1.973	0.	1.12

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	99	222	0	288	0	99
normalized size	1	1.	0.82	1.83	0.	2.38	0.	0.82
time (sec)	N/A	0.07	0.07	0.109	0.	2.092	0.	1.171

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	106	193	0	309	0	120
normalized size	1	1.	0.96	1.75	0.	2.81	0.	1.09
time (sec)	N/A	0.069	0.053	0.116	0.	2.113	0.	1.142

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	108	142	0	320	0	124
normalized size	1	1.	0.98	1.29	0.	2.91	0.	1.13
time (sec)	N/A	0.067	0.044	0.115	0.	2.111	0.	1.152

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	107	144	0	350	0	119
normalized size	1	1.	0.95	1.27	0.	3.1	0.	1.05
time (sec)	N/A	0.068	0.07	0.118	0.	2.129	0.	1.171

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	134	169	0	413	0	146
normalized size	1	1.	0.8	1.01	0.	2.47	0.	0.87
time (sec)	N/A	0.117	0.111	0.118	0.	2.161	0.	1.166

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	150	262	0	462	0	166
normalized size	1	1.	0.68	1.19	0.	2.09	0.	0.75
time (sec)	N/A	0.174	0.123	0.122	0.	2.281	0.	1.165

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	83	153	86	126	0	82
normalized size	1	1.	0.84	1.55	0.87	1.27	0.	0.83
time (sec)	N/A	0.065	0.043	0.061	1.797	2.082	0.	1.163

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	66	123	72	96	0	66
normalized size	1	1.	0.87	1.62	0.95	1.26	0.	0.87
time (sec)	N/A	0.048	0.037	0.063	1.807	2.045	0.	1.17

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	49	68	57	73	0	50
normalized size	1	1.	0.86	1.19	1.	1.28	0.	0.88
time (sec)	N/A	0.031	0.034	0.06	1.807	2.375	0.	1.185

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	32	56	41	47	0	31
normalized size	1	1.	0.89	1.56	1.14	1.31	0.	0.86
time (sec)	N/A	0.015	0.051	0.062	1.774	2.338	0.	1.147

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	16	26	24	16
normalized size	1	1.	1.	0.94	1.	1.62	1.5	1.
time (sec)	N/A	0.004	0.006	0.032	1.7	2.054	18.52	1.168

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	47	42	0	142	0	28
normalized size	1	1.	0.96	0.86	0.	2.9	0.	0.57
time (sec)	N/A	0.016	0.047	0.136	0.	2.107	0.	1.123

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	78	95	0	232	0	63
normalized size	1	1.	0.83	1.01	0.	2.47	0.	0.67
time (sec)	N/A	0.053	0.056	0.138	0.	2.156	0.	1.14

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	98	148	0	301	0	93
normalized size	1	1.	0.62	0.94	0.	1.91	0.	0.59
time (sec)	N/A	0.096	0.084	0.147	0.	2.202	0.	1.164

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	117	200	0	356	0	113
normalized size	1	1.	0.55	0.94	0.	1.68	0.	0.53
time (sec)	N/A	0.151	0.107	0.148	0.	2.155	0.	1.146

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	83	319	86	138	0	104
normalized size	1	1.	0.87	3.36	0.91	1.45	0.	1.09
time (sec)	N/A	0.067	0.044	0.042	1.786	2.343	0.	1.161

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	66	201	70	108	0	82
normalized size	1	1.	0.89	2.72	0.95	1.46	0.	1.11
time (sec)	N/A	0.049	0.04	0.038	1.797	2.384	0.	1.174

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	49	106	55	85	0	62
normalized size	1	1.	0.89	1.93	1.	1.55	0.	1.13
time (sec)	N/A	0.03	0.032	0.04	1.785	2.268	0.	1.155

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	29	40	41	61	46	39
normalized size	1	1.	0.85	1.18	1.21	1.79	1.35	1.15
time (sec)	N/A	0.015	0.057	0.037	1.784	2.118	35.22	1.152

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	16	43	26	16
normalized size	1	1.	1.	0.94	1.	2.69	1.62	1.
time (sec)	N/A	0.005	0.007	0.027	1.701	2.005	35.141	1.138

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	75	68	0	266	0	50
normalized size	1	1.	0.96	0.87	0.	3.41	0.	0.64
time (sec)	N/A	0.036	0.091	0.115	0.	2.07	0.	1.159

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	91	105	0	346	0	86
normalized size	1	1.	0.73	0.85	0.	2.79	0.	0.69
time (sec)	N/A	0.074	0.083	0.104	0.	2.113	0.	1.166

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	115	131	0	420	0	108
normalized size	1	1.	0.6	0.69	0.	2.2	0.	0.57
time (sec)	N/A	0.128	0.116	0.127	0.	2.136	0.	1.177

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	133	186	0	478	0	128
normalized size	1	1.	0.54	0.76	0.	1.95	0.	0.52
time (sec)	N/A	0.183	0.113	0.128	0.	2.122	0.	1.146

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	83	295	86	161	0	101
normalized size	1	1.	0.84	2.98	0.87	1.63	0.	1.02
time (sec)	N/A	0.066	0.048	0.045	1.788	2.077	0.	1.15

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	65	186	70	131	90	80
normalized size	1	1.	0.86	2.45	0.92	1.72	1.18	1.05
time (sec)	N/A	0.049	0.039	0.042	1.791	1.938	92.422	1.162

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	48	91	57	111	71	53
normalized size	1	1.	0.81	1.54	0.97	1.88	1.2	0.9
time (sec)	N/A	0.03	0.036	0.043	1.797	1.952	92.143	1.165

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	31	42	42	89	51	27
normalized size	1	1.	0.82	1.11	1.11	2.34	1.34	0.71
time (sec)	N/A	0.014	0.055	0.039	1.767	2.055	85.1	1.142

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	16	68	27	16
normalized size	1	1.	1.	0.83	0.89	3.78	1.5	0.89
time (sec)	N/A	0.005	0.007	0.029	1.723	2.102	70.418	1.146

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	91	93	0	409	0	61
normalized size	1	1.	0.84	0.86	0.	3.79	0.	0.56
time (sec)	N/A	0.054	0.143	0.115	0.	2.134	0.	1.151

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	113	130	0	494	0	88
normalized size	1	1.	0.73	0.84	0.	3.19	0.	0.57
time (sec)	N/A	0.1	0.127	0.123	0.	2.158	0.	1.154

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	133	157	0	566	0	126
normalized size	1	1.	0.59	0.7	0.	2.53	0.	0.56
time (sec)	N/A	0.152	0.116	0.128	0.	2.155	0.	1.154

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	150	211	0	613	0	155
normalized size	1	1.	0.54	0.76	0.	2.21	0.	0.56
time (sec)	N/A	0.222	0.195	0.125	0.	2.173	0.	1.151

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	23	20	26	47	0	18
normalized size	1	1.	0.85	0.74	0.96	1.74	0.	0.67
time (sec)	N/A	0.01	0.036	0.039	0.979	1.988	0.	1.146

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	23	20	26	46	0	18
normalized size	1	1.	0.85	0.74	0.96	1.7	0.	0.67
time (sec)	N/A	0.009	0.026	0.038	0.983	2.041	0.	1.122

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	23	20	26	46	26	18
normalized size	1	1.	0.85	0.74	0.96	1.7	0.96	0.67
time (sec)	N/A	0.01	0.026	0.037	0.978	1.995	26.399	1.134

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	23	20	26	43	0	18
normalized size	1	1.	0.85	0.74	0.96	1.59	0.	0.67
time (sec)	N/A	0.008	0.026	0.036	0.989	1.988	0.	1.117

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	20	26	34	0	18
normalized size	1	1.	0.92	0.8	1.04	1.36	0.	0.72
time (sec)	N/A	0.008	0.019	0.04	0.989	1.991	0.	1.134

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	26	28	22	18
normalized size	1	1.	0.87	0.87	1.13	1.22	0.96	0.78
time (sec)	N/A	0.008	0.02	0.034	0.982	2.086	1.642	1.159

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	21	20	26	35	27	15
normalized size	1	1.	0.78	0.74	0.96	1.3	1.	0.56
time (sec)	N/A	0.009	0.018	0.037	0.986	2.028	21.236	1.154

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	23	20	26	39	0	18
normalized size	1	1.	0.85	0.74	0.96	1.44	0.	0.67
time (sec)	N/A	0.009	0.021	0.039	0.98	1.978	0.	1.164

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	40	38	49	77	0	32
normalized size	1	1.	0.83	0.79	1.02	1.6	0.	0.67
time (sec)	N/A	0.023	0.06	0.042	1.019	1.97	0.	1.139

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	40	38	49	74	0	32
normalized size	1	1.	0.83	0.79	1.02	1.54	0.	0.67
time (sec)	N/A	0.022	0.044	0.042	1.022	1.961	0.	1.117

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	40	38	49	73	0	32
normalized size	1	1.	0.83	0.79	1.02	1.52	0.	0.67
time (sec)	N/A	0.023	0.04	0.039	1.015	2.006	0.	1.13

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	40	38	49	70	0	32
normalized size	1	1.	0.83	0.79	1.02	1.46	0.	0.67
time (sec)	N/A	0.022	0.048	0.04	1.033	2.005	0.	1.125

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	40	47	49	62	0	32
normalized size	1	1.	0.87	1.02	1.07	1.35	0.	0.7
time (sec)	N/A	0.021	0.031	0.04	1.028	1.999	0.	1.166

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	40	37	49	55	0	32
normalized size	1	1.	0.91	0.84	1.11	1.25	0.	0.73
time (sec)	N/A	0.022	0.042	0.04	1.018	2.006	0.	1.152

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	40	38	49	55	48	31
normalized size	1	1.	0.83	0.79	1.02	1.15	1.	0.65
time (sec)	N/A	0.023	0.044	0.041	1.044	1.935	22.546	1.132

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	40	38	49	63	0	32
normalized size	1	1.	0.83	0.79	1.02	1.31	0.	0.67
time (sec)	N/A	0.023	0.043	0.043	1.058	2.085	0.	1.145

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	57	56	74	105	0	47
normalized size	1	1.	0.83	0.81	1.07	1.52	0.	0.68
time (sec)	N/A	0.039	0.029	0.046	1.062	1.926	0.	1.142

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	57	56	74	104	0	47
normalized size	1	1.	0.83	0.81	1.07	1.51	0.	0.68
time (sec)	N/A	0.038	0.039	0.044	1.065	2.041	0.	1.137

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	57	56	74	103	0	47
normalized size	1	1.	0.83	0.81	1.07	1.49	0.	0.68
time (sec)	N/A	0.037	0.032	0.046	1.066	2.027	0.	1.175

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	57	56	74	97	0	47
normalized size	1	1.	0.83	0.81	1.07	1.41	0.	0.68
time (sec)	N/A	0.037	0.028	0.046	1.053	2.035	0.	1.162

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	57	69	74	85	0	47
normalized size	1	1.	0.88	1.06	1.14	1.31	0.	0.72
time (sec)	N/A	0.036	0.028	0.039	1.055	1.93	0.	1.128

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	57	64	74	78	0	47
normalized size	1	1.	0.9	1.02	1.17	1.24	0.	0.75
time (sec)	N/A	0.036	0.031	0.043	1.049	1.926	0.	1.118

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	55	55	74	74	66	46
normalized size	1	1.	0.85	0.85	1.14	1.14	1.02	0.71
time (sec)	N/A	0.038	0.029	0.043	1.069	1.972	26.945	1.164

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	57	56	74	78	0	46
normalized size	1	1.	0.83	0.81	1.07	1.13	0.	0.67
time (sec)	N/A	0.038	0.031	0.044	1.088	1.964	0.	1.129

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	129	481	0	366	0	95
normalized size	1	1.	0.9	3.36	0.	2.56	0.	0.66
time (sec)	N/A	0.127	0.058	0.132	0.	2.093	0.	1.128

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	108	330	0	308	0	80
normalized size	1	1.	0.93	2.84	0.	2.66	0.	0.69
time (sec)	N/A	0.079	0.106	0.126	0.	2.201	0.	1.132

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	86	207	0	244	0	61
normalized size	1	1.	0.97	2.33	0.	2.74	0.	0.69
time (sec)	N/A	0.056	0.089	0.126	0.	2.109	0.	1.119

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	62	112	0	189	0	42
normalized size	1	1.	0.97	1.75	0.	2.95	0.	0.66
time (sec)	N/A	0.033	0.043	0.126	0.	2.089	0.	1.133

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	51	41	0	163	0	24
normalized size	1	1.	0.96	0.77	0.	3.08	0.	0.45
time (sec)	N/A	0.018	0.027	0.126	0.	2.324	0.	1.112

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	73	76	0	207	0	42
normalized size	1	1.	0.96	1.	0.	2.72	0.	0.55
time (sec)	N/A	0.036	0.06	0.13	0.	2.109	0.	1.147

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	89	98	0	275	0	55
normalized size	1	1.	0.88	0.97	0.	2.72	0.	0.54
time (sec)	N/A	0.057	0.172	0.137	0.	2.017	0.	1.14

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	107	120	0	336	0	70
normalized size	1	1.	0.84	0.94	0.	2.62	0.	0.55
time (sec)	N/A	0.084	0.146	0.134	0.	2.206	0.	1.158

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	144	452	0	427	0	103
normalized size	1	1.	1.07	3.35	0.	3.16	0.	0.76
time (sec)	N/A	0.103	0.187	0.136	0.	2.107	0.	1.121

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	119	294	0	366	0	88
normalized size	1	1.	1.1	2.72	0.	3.39	0.	0.81
time (sec)	N/A	0.076	0.134	0.139	0.	2.068	0.	1.141

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	81	160	0	300	0	62
normalized size	1	1.	0.98	1.93	0.	3.61	0.	0.75
time (sec)	N/A	0.054	0.077	0.135	0.	2.033	0.	1.136

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	70	61	0	277	0	49
normalized size	1	1.	0.96	0.84	0.	3.79	0.	0.67
time (sec)	N/A	0.034	0.06	0.135	0.	2.126	0.	1.132

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	80	82	0	274	0	47
normalized size	1	1.	0.82	0.85	0.	2.82	0.	0.48
time (sec)	N/A	0.054	0.06	0.132	0.	2.057	0.	1.182

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	104	105	0	323	0	66
normalized size	1	1.	0.87	0.88	0.	2.69	0.	0.55
time (sec)	N/A	0.08	0.104	0.124	0.	2.1	0.	1.153

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	120	128	0	402	0	78
normalized size	1	1.	0.83	0.88	0.	2.77	0.	0.54
time (sec)	N/A	0.106	0.199	0.18	0.	2.081	0.	1.122

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	139	151	0	460	0	95
normalized size	1	1.	0.81	0.88	0.	2.67	0.	0.55
time (sec)	N/A	0.137	0.267	0.178	0.	2.192	0.	1.124

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	147	418	0	509	0	104
normalized size	1	1.	1.09	3.1	0.	3.77	0.	0.77
time (sec)	N/A	0.097	0.112	0.14	0.	2.092	0.	1.144

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	104	249	0	443	0	80
normalized size	1	1.	0.95	2.26	0.	4.03	0.	0.73
time (sec)	N/A	0.071	0.088	0.138	0.	2.034	0.	1.135

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	96	85	0	423	0	63
normalized size	1	1.	0.98	0.87	0.	4.32	0.	0.64
time (sec)	N/A	0.053	0.073	0.145	0.	2.067	0.	1.112

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	107	98	0	412	0	70
normalized size	1	1.	0.86	0.78	0.	3.3	0.	0.56
time (sec)	N/A	0.071	0.132	0.135	0.	2.037	0.	1.131

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	118	112	0	423	0	63
normalized size	1	1.	0.78	0.74	0.	2.78	0.	0.41
time (sec)	N/A	0.097	0.079	0.132	0.	2.102	0.	1.163

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	141	181	0	466	0	80
normalized size	1	1.	0.8	1.03	0.	2.65	0.	0.45
time (sec)	N/A	0.128	0.167	0.142	0.	2.139	0.	1.11

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	156	207	0	545	0	96
normalized size	1	1.	0.78	1.03	0.	2.71	0.	0.48
time (sec)	N/A	0.159	0.256	0.142	0.	2.129	0.	1.15

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	174	229	0	598	0	108
normalized size	1	1.	0.76	1.	0.	2.62	0.	0.47
time (sec)	N/A	0.202	0.327	0.145	0.	2.163	0.	1.145

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	104	304	0	362	0	81
normalized size	1	1.	0.73	2.14	0.	2.55	0.	0.57
time (sec)	N/A	0.078	0.086	0.167	0.	2.13	0.	1.168

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	84	174	0	304	0	65
normalized size	1	1.	0.81	1.67	0.	2.92	0.	0.62
time (sec)	N/A	0.051	0.069	0.141	0.	2.072	0.	1.155

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	62	75	0	251	0	49
normalized size	1	1.	1.02	1.23	0.	4.11	0.	0.8
time (sec)	N/A	0.029	0.039	0.069	0.	2.159	0.	1.149

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	52	149	0	243	0	77
normalized size	1	1.	1.06	3.04	0.	4.96	0.	1.57
time (sec)	N/A	0.029	0.038	0.144	0.	2.063	0.	1.168

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	34	29	20	46	0	80
normalized size	1	1.	0.97	0.83	0.57	1.31	0.	2.29
time (sec)	N/A	0.014	0.031	0.138	1.483	1.997	0.	1.141

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	48	59	46	84	0	151
normalized size	1	1.	0.67	0.82	0.64	1.17	0.	2.1
time (sec)	N/A	0.033	0.034	0.154	1.482	2.159	0.	1.156

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	66	105	61	112	0	186
normalized size	1	1.	0.6	0.95	0.55	1.02	0.	1.69
time (sec)	N/A	0.054	0.056	0.143	1.481	2.255	0.	1.185

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	82	151	76	134	0	224
normalized size	1	1.	0.55	1.02	0.51	0.91	0.	1.51
time (sec)	N/A	0.078	0.061	0.158	1.476	2.189	0.	1.198

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	122	471	0	412	0	198
normalized size	1	1.	0.69	2.66	0.	2.33	0.	1.12
time (sec)	N/A	0.104	0.089	0.115	0.	2.309	0.	1.204

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	105	304	0	363	0	165
normalized size	1	1.	0.76	2.19	0.	2.61	0.	1.19
time (sec)	N/A	0.071	0.071	0.119	0.	2.327	0.	1.2

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	83	165	0	311	0	0
normalized size	1	1.	0.82	1.63	0.	3.08	0.	0.
time (sec)	N/A	0.048	0.058	0.042	0.	2.273	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	77	280	0	289	0	0
normalized size	1	1.	0.95	3.46	0.	3.57	0.	0.
time (sec)	N/A	0.056	0.052	0.117	0.	2.445	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	74	315	0	302	0	0
normalized size	1	1.	1.06	4.5	0.	4.31	0.	0.
time (sec)	N/A	0.042	0.042	0.135	0.	2.312	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	34	29	20	78	0	45
normalized size	1	1.	0.97	0.83	0.57	2.23	0.	1.29
time (sec)	N/A	0.014	0.036	0.146	1.483	2.308	0.	1.338

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	48	59	46	105	0	80
normalized size	1	1.	0.67	0.82	0.64	1.46	0.	1.11
time (sec)	N/A	0.033	0.041	0.129	1.469	2.183	0.	1.257

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	66	105	61	135	0	105
normalized size	1	1.	0.6	0.95	0.55	1.23	0.	0.95
time (sec)	N/A	0.052	0.045	0.136	1.485	2.029	0.	1.336

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	82	151	76	162	0	131
normalized size	1	1.	0.55	1.02	0.51	1.09	0.	0.89
time (sec)	N/A	0.076	0.066	0.153	1.5	1.958	0.	1.336

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	121	471	0	425	0	275
normalized size	1	1.	0.7	2.71	0.	2.44	0.	1.58
time (sec)	N/A	0.098	0.082	0.117	0.	2.218	0.	1.312

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	101	286	0	365	0	0
normalized size	1	1.	0.74	2.1	0.	2.68	0.	0.
time (sec)	N/A	0.069	0.058	0.046	0.	2.17	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	101	460	0	351	0	0
normalized size	1	1.	0.83	3.8	0.	2.9	0.	0.
time (sec)	N/A	0.066	0.071	0.116	0.	2.114	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	97	501	0	362	0	0
normalized size	1	1.	0.92	4.73	0.	3.42	0.	0.
time (sec)	N/A	0.063	0.064	0.119	0.	2.153	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	95	532	0	365	0	0
normalized size	1	1.	1.02	5.72	0.	3.92	0.	0.
time (sec)	N/A	0.059	0.051	0.124	0.	2.084	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	34	29	20	100	0	45
normalized size	1	1.	0.97	0.83	0.57	2.86	0.	1.29
time (sec)	N/A	0.013	0.042	0.121	1.475	2.108	0.	1.243

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	48	59	46	130	0	80
normalized size	1	1.	0.67	0.82	0.64	1.81	0.	1.11
time (sec)	N/A	0.033	0.045	0.134	1.482	2.066	0.	1.271

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	66	105	61	162	0	105
normalized size	1	1.	0.6	0.95	0.55	1.47	0.	0.95
time (sec)	N/A	0.057	0.045	0.163	1.473	2.048	0.	1.194

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	82	151	76	186	0	131
normalized size	1	1.	0.55	1.02	0.51	1.26	0.	0.89
time (sec)	N/A	0.079	0.071	0.241	1.481	2.298	0.	1.229

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	105	304	0	369	0	86
normalized size	1	1.	0.72	2.1	0.	2.54	0.	0.59
time (sec)	N/A	0.081	0.085	0.171	0.	2.457	0.	1.159

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	88	174	0	316	0	70
normalized size	1	1.	0.82	1.63	0.	2.95	0.	0.65
time (sec)	N/A	0.052	0.076	0.167	0.	2.296	0.	1.165

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	66	80	0	255	0	51
normalized size	1	1.	1.05	1.27	0.	4.05	0.	0.81
time (sec)	N/A	0.028	0.054	0.173	0.	2.15	0.	1.173

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	33	24	0	162	0	31
normalized size	1	1.	1.1	0.8	0.	5.4	0.	1.03
time (sec)	N/A	0.012	0.03	0.095	0.	2.075	0.	1.149

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	29	20	41	0	41
normalized size	1	1.	0.97	0.88	0.61	1.24	0.	1.24
time (sec)	N/A	0.013	0.039	0.17	1.479	2.062	0.	1.146

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	46	59	45	61	0	74
normalized size	1	1.	0.64	0.82	0.62	0.85	0.	1.03
time (sec)	N/A	0.032	0.041	0.168	1.483	2.123	0.	1.156

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	66	105	61	88	0	104
normalized size	1	1.	0.6	0.95	0.55	0.8	0.	0.95
time (sec)	N/A	0.056	0.048	0.171	1.488	2.002	0.	1.174

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	82	151	74	109	0	139
normalized size	1	1.	0.55	1.02	0.5	0.74	0.	0.94
time (sec)	N/A	0.083	0.052	0.169	1.501	2.099	0.	1.17

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	122	428	0	485	0	101
normalized size	1	1.	0.73	2.58	0.	2.92	0.	0.61
time (sec)	N/A	0.107	0.102	0.124	0.	2.159	0.	1.195

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	104	261	0	429	0	85
normalized size	1	1.	0.81	2.04	0.	3.35	0.	0.66
time (sec)	N/A	0.076	0.094	0.119	0.	2.224	0.	1.204

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	81	130	0	363	0	65
normalized size	1	1.	0.94	1.51	0.	4.22	0.	0.76
time (sec)	N/A	0.049	0.078	0.121	0.	2.219	0.	1.208

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	55	42	0	308	0	53
normalized size	1	1.	1.06	0.81	0.	5.92	0.	1.02
time (sec)	N/A	0.026	0.048	0.125	0.	2.106	0.	1.221

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	29	0	53	0	20
normalized size	1	1.	0.97	0.88	0.	1.61	0.	0.61
time (sec)	N/A	0.013	0.031	0.057	0.	1.988	0.	1.17

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	43	59	43	78	0	68
normalized size	1	1.	0.63	0.87	0.63	1.15	0.	1.
time (sec)	N/A	0.033	0.047	0.146	1.499	2.03	0.	1.178

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	64	105	61	104	0	144
normalized size	1	1.	0.58	0.95	0.55	0.95	0.	1.31
time (sec)	N/A	0.054	0.049	0.148	1.505	2.057	0.	1.276

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	80	151	73	128	0	217
normalized size	1	1.	0.54	1.02	0.49	0.86	0.	1.47
time (sec)	N/A	0.08	0.062	0.119	1.507	1.998	0.	1.401

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	121	348	0	575	0	101
normalized size	1	1.	0.79	2.27	0.	3.76	0.	0.66
time (sec)	N/A	0.096	0.105	0.125	0.	2.15	0.	1.209

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	101	180	0	512	0	82
normalized size	1	1.	0.91	1.62	0.	4.61	0.	0.74
time (sec)	N/A	0.066	0.09	0.127	0.	2.099	0.	1.202

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	78	59	0	451	0	66
normalized size	1	1.	1.04	0.79	0.	6.01	0.	0.88
time (sec)	N/A	0.042	0.064	0.118	0.	2.126	0.	1.281

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	34	92	0	77	0	20
normalized size	1	1.	0.97	2.63	0.	2.2	0.	0.57
time (sec)	N/A	0.012	0.042	0.118	0.	2.093	0.	1.162

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	47	58	0	99	0	34
normalized size	1	1.	0.66	0.82	0.	1.39	0.	0.48
time (sec)	N/A	0.032	0.035	0.043	0.	2.079	0.	1.218

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	66	104	61	128	0	84
normalized size	1	1.	0.62	0.98	0.58	1.21	0.	0.79
time (sec)	N/A	0.055	0.052	0.119	1.514	2.165	0.	1.21

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	79	150	76	150	0	161
normalized size	1	1.	0.54	1.03	0.52	1.03	0.	1.1
time (sec)	N/A	0.079	0.059	0.118	1.514	2.081	0.	1.322

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	100	196	90	181	0	234
normalized size	1	1.	0.54	1.05	0.48	0.97	0.	1.26
time (sec)	N/A	0.104	0.065	0.12	1.513	2.073	0.	1.442

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	71	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.128	1.392	0.	0.	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	146	654	188	788	0	448
normalized size	1	1.	0.88	3.96	1.14	4.78	0.	2.72
time (sec)	N/A	0.134	0.104	0.049	1.793	2.183	0.	1.178

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	106	492	136	536	0	305
normalized size	1	1.	0.88	4.07	1.12	4.43	0.	2.52
time (sec)	N/A	0.075	0.077	0.046	1.813	2.162	0.	1.214

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	71	315	92	347	0	189
normalized size	1	1.	0.87	3.84	1.12	4.23	0.	2.3
time (sec)	N/A	0.046	0.058	0.044	1.799	2.13	0.	1.17

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	41	175	57	198	0	103
normalized size	1	1.	0.85	3.65	1.19	4.12	0.	2.15
time (sec)	N/A	0.019	0.042	0.042	1.782	2.083	0.	1.162

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	28	104	51	38
normalized size	1	1.	1.	1.05	1.4	5.2	2.55	1.9
time (sec)	N/A	0.007	0.015	0.027	1.696	2.339	1.469	1.16

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	60	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.082	0.654	0.	0.	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	67	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.04	0.836	0.	0.	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	67	0	0	0	0	0
normalized size	1	1.	0.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	0.052	0.913	0.	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	34	676	0	72	0	0
normalized size	1	1.	0.92	18.27	0.	1.95	0.	0.
time (sec)	N/A	0.018	0.053	0.24	0.	2.142	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	26	31	0	96
normalized size	1	1.	0.87	0.87	1.13	1.35	0.	4.17
time (sec)	N/A	0.02	0.024	0.034	1.133	2.085	0.	1.176

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	26	31	70	96
normalized size	1	1.	0.87	0.87	1.13	1.35	3.04	4.17
time (sec)	N/A	0.007	0.014	0.032	1.143	2.123	124.543	1.202

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	15	22	23	46	89
normalized size	1	1.	1.12	0.94	1.38	1.44	2.88	5.56
time (sec)	N/A	0.003	0.007	0.026	1.125	2.106	9.436	1.2

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	21	46	22	0	20
normalized size	1	1.	0.9	1.	2.19	1.05	0.	0.95
time (sec)	N/A	0.03	0.016	0.039	0.953	2.135	0.	1.169

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	18	18	23	27	42	95
normalized size	1	1.	1.06	1.06	1.35	1.59	2.47	5.59
time (sec)	N/A	0.008	0.015	0.033	1.139	2.084	17.095	1.187

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	18	20	26	30	49	96
normalized size	1	1.	0.78	0.87	1.13	1.3	2.13	4.17
time (sec)	N/A	0.009	0.014	0.032	1.147	2.12	142.164	1.19

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	47	21	45	215	0	0
normalized size	1	1.	1.74	0.78	1.67	7.96	0.	0.
time (sec)	N/A	0.032	0.021	0.04	1.122	2.237	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	81	479	76	327	0	0
normalized size	1	1.	1.59	9.39	1.49	6.41	0.	0.
time (sec)	N/A	0.061	0.015	0.23	1.164	2.147	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	109	501	105	436	0	0
normalized size	1	1.	1.42	6.51	1.36	5.66	0.	0.
time (sec)	N/A	0.086	0.031	0.151	1.158	2.232	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	345	5366	379	2588	0	0
normalized size	1	1.	1.12	17.48	1.23	8.43	0.	0.
time (sec)	N/A	0.469	11.565	3.866	2.092	2.575	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	259	5062	290	2115	0	0
normalized size	1	1.	1.12	21.91	1.26	9.16	0.	0.
time (sec)	N/A	0.371	9.429	5.313	2.11	2.261	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	131	306	192	1602	0	0
normalized size	1	1.	0.87	2.04	1.28	10.68	0.	0.
time (sec)	N/A	0.222	6.085	0.158	2.097	2.108	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.141	13.658	0.31	0.	0.	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	144	1769	201	1378	0	0
normalized size	1	1.	0.93	11.41	1.3	8.89	0.	0.
time (sec)	N/A	0.296	4.363	11.484	3.28	2.003	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	118	1710	169	1161	0	0
normalized size	1	1.	0.92	13.36	1.32	9.07	0.	0.
time (sec)	N/A	0.254	4.879	14.46	3.272	1.987	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	91	1627	136	954	0	0
normalized size	1	1.	0.9	16.11	1.35	9.45	0.	0.
time (sec)	N/A	0.224	4.922	3.849	3.252	2.015	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	69	69	201	247	97	709	0	0
normalized size	1	1.	2.91	3.58	1.41	10.28	0.	0.
time (sec)	N/A	0.137	4.316	0.116	3.263	2.056	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	4.235	0.31	0.	0.	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	144	1773	197	1281	0	0
normalized size	1	1.	0.86	10.55	1.17	7.62	0.	0.
time (sec)	N/A	0.297	4.375	15.613	3.278	2.009	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	119	1716	166	1083	0	0
normalized size	1	1.	0.86	12.35	1.19	7.79	0.	0.
time (sec)	N/A	0.258	5.035	13.838	3.27	2.024	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	93	1635	135	895	0	0
normalized size	1	1.	0.85	14.86	1.23	8.14	0.	0.
time (sec)	N/A	0.224	5.022	6.896	3.251	1.926	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	76	76	200	280	99	668	0	0
normalized size	1	1.	2.63	3.68	1.3	8.79	0.	0.
time (sec)	N/A	0.138	4.275	0.147	3.287	1.969	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	5.204	0.329	0.	0.	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	303	303	353	5294	374	2561	0	0
normalized size	1	1.	1.17	17.47	1.23	8.45	0.	0.
time (sec)	N/A	0.457	10.053	3.53	2.129	2.572	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	229	229	267	4990	288	2094	0	0
normalized size	1	1.	1.17	21.79	1.26	9.14	0.	0.
time (sec)	N/A	0.372	8.601	5.571	2.133	2.482	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	131	306	192	1586	0	0
normalized size	1	1.	0.87	2.04	1.28	10.57	0.	0.
time (sec)	N/A	0.227	4.782	0.137	2.096	2.307	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.141	13.623	0.329	0.	0.	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	141	1741	197	1281	0	0
normalized size	1	1.	0.93	11.45	1.3	8.43	0.	0.
time (sec)	N/A	0.304	4.291	13.915	3.269	2.091	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	116	1684	166	1083	0	0
normalized size	1	1.	0.92	13.37	1.32	8.6	0.	0.
time (sec)	N/A	0.259	4.844	16.177	3.315	1.967	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	90	1603	135	895	0	0
normalized size	1	1.	0.9	16.03	1.35	8.95	0.	0.
time (sec)	N/A	0.23	4.876	7.336	3.294	2.057	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	69	69	197	247	97	668	0	0
normalized size	1	1.	2.86	3.58	1.41	9.68	0.	0.
time (sec)	N/A	0.142	3.455	0.108	3.251	1.931	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	4.187	0.349	0.	0.	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	147	1801	201	1378	0	0
normalized size	1	1.	0.89	10.92	1.22	8.35	0.	0.
time (sec)	N/A	0.299	4.28	14.384	3.312	1.982	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	121	1742	169	1161	0	0
normalized size	1	1.	0.88	12.72	1.23	8.47	0.	0.
time (sec)	N/A	0.264	4.947	15.92	3.292	2.08	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	94	1659	136	954	0	0
normalized size	1	1.	0.86	15.22	1.25	8.75	0.	0.
time (sec)	N/A	0.231	4.798	9.189	3.272	1.917	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	76	76	208	280	99	709	0	0
normalized size	1	1.	2.74	3.68	1.3	9.33	0.	0.
time (sec)	N/A	0.137	4.055	0.107	3.275	2.007	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	4.308	0.322	0.	0.	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	302	654	7429	0	4516	0	0
normalized size	1	1.	2.17	24.6	0.	14.95	0.	0.
time (sec)	N/A	0.232	1.193	4.766	0.	2.886	0.	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	409	5543	0	3359	0	0
normalized size	1	1.	1.75	23.69	0.	14.35	0.	0.
time (sec)	N/A	0.165	0.648	8.575	0.	2.487	0.	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	162	162	263	2543	0	2314	0	0
normalized size	1	1.	1.62	15.7	0.	14.28	0.	0.
time (sec)	N/A	0.108	0.292	4.416	0.	2.161	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	74	178	246	1494	0	0
normalized size	1	1.	0.94	2.25	3.11	18.91	0.	0.
time (sec)	N/A	0.048	0.027	0.138	1.595	2.295	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	5.063	0.602	0.	0.	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	395	346	6967	0	5873	0	0
normalized size	1	1.	0.88	17.64	0.	14.87	0.	0.
time (sec)	N/A	0.499	0.853	4.986	0.	2.978	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	257	6593	0	4547	0	0
normalized size	1	1.	0.87	22.35	0.	15.41	0.	0.
time (sec)	N/A	0.403	0.583	9.453	0.	2.738	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	4654	612	502	3212	0	0
normalized size	1	1.	23.99	3.15	2.59	16.56	0.	0.
time (sec)	N/A	0.239	32.49	0.09	1.804	2.771	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.15	4.54	0.375	0.	0.	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	155	2346	460	981	0	0
normalized size	1	1.	0.91	13.8	2.71	5.77	0.	0.
time (sec)	N/A	0.292	0.411	14.966	1.17	2.086	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	119	2256	333	813	0	0
normalized size	1	1.	0.89	16.96	2.5	6.11	0.	0.
time (sec)	N/A	0.246	0.29	7.477	1.086	1.95	0.	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	766	292	355	605	0	0
normalized size	1	1.	8.24	3.14	3.82	6.51	0.	0.
time (sec)	N/A	0.151	12.926	0.134	1.486	1.991	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	1.043	0.388	0.	0.	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	156	2456	459	994	0	0
normalized size	1	1.	0.91	14.36	2.68	5.81	0.	0.
time (sec)	N/A	0.289	0.392	14.246	1.182	2.082	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	120	2358	332	822	0	0
normalized size	1	1.	0.9	17.6	2.48	6.13	0.	0.
time (sec)	N/A	0.244	0.299	3.953	1.096	2.007	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	94	94	723	328	358	612	0	0
normalized size	1	1.	7.69	3.49	3.81	6.51	0.	0.
time (sec)	N/A	0.148	15.109	0.136	1.5	1.942	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.998	0.381	0.	0.	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	302	654	7429	0	3679	0	0
normalized size	1	1.	2.17	24.6	0.	12.18	0.	0.
time (sec)	N/A	0.233	0.301	4.522	0.	2.895	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	409	5543	0	2647	0	0
normalized size	1	1.	1.75	23.69	0.	11.31	0.	0.
time (sec)	N/A	0.17	0.188	8.488	0.	2.666	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	162	162	263	2544	0	1725	0	0
normalized size	1	1.	1.62	15.7	0.	10.65	0.	0.
time (sec)	N/A	0.109	0.123	5.203	0.	2.369	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	74	422	248	1030	0	0
normalized size	1	1.	0.94	5.34	3.14	13.04	0.	0.
time (sec)	N/A	0.046	0.037	0.109	1.627	2.268	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.889	1.075	0.	0.	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	339	6775	0	4749	0	0
normalized size	1	1.	0.87	17.33	0.	12.15	0.	0.
time (sec)	N/A	0.486	0.903	5.454	0.	3.3	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	253	6425	0	3839	0	0
normalized size	1	1.	0.86	21.93	0.	13.1	0.	0.
time (sec)	N/A	0.4	0.541	9.795	0.	3.221	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	4463	629	529	2920	0	0
normalized size	1	1.	23.01	3.24	2.73	15.05	0.	0.
time (sec)	N/A	0.241	31.437	0.088	1.885	2.976	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.142	4.909	0.371	0.	0.	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	155	2456	462	506	0	0
normalized size	1	1.	0.92	14.62	2.75	3.01	0.	0.
time (sec)	N/A	0.296	0.417	16.132	1.186	2.009	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	119	2358	335	431	0	0
normalized size	1	1.	0.9	17.86	2.54	3.27	0.	0.
time (sec)	N/A	0.246	0.298	9.899	1.108	1.815	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	709	299	389	340	0	0
normalized size	1	1.	7.62	3.22	4.18	3.66	0.	0.
time (sec)	N/A	0.149	36.436	0.124	1.537	1.822	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	0.954	0.395	0.	0.	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	155	2346	463	505	0	0
normalized size	1	1.	0.92	13.88	2.74	2.99	0.	0.
time (sec)	N/A	0.295	0.405	18.589	1.18	1.938	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	119	2256	336	428	0	0
normalized size	1	1.	0.89	16.96	2.53	3.22	0.	0.
time (sec)	N/A	0.25	0.297	10.754	1.104	2.149	0.	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	94	94	605	335	386	336	0	0
normalized size	1	1.	6.44	3.56	4.11	3.57	0.	0.
time (sec)	N/A	0.149	27.178	0.122	1.515	2.3	0.	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	1.04	0.387	0.	0.	0.	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	51	31	78	263	0	0
normalized size	1	1.	2.43	1.48	3.71	12.52	0.	0.
time (sec)	N/A	0.012	0.035	0.044	0.964	1.581	0.	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	71	62	80	375	0	0
normalized size	1	1.	1.65	1.44	1.86	8.72	0.	0.
time (sec)	N/A	0.043	0.026	0.036	0.963	1.603	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	93	79	103	463	0	0
normalized size	1	1.	1.6	1.36	1.78	7.98	0.	0.
time (sec)	N/A	0.067	0.028	0.036	0.962	1.686	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	68	67	144	419	0	0
normalized size	1	1.	1.94	1.91	4.11	11.97	0.	0.
time (sec)	N/A	0.014	0.077	0.042	0.962	1.548	0.	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	113	153	146	585	0	0
normalized size	1	1.	1.59	2.15	2.06	8.24	0.	0.
time (sec)	N/A	0.058	0.052	0.042	1.008	1.617	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	149	185	192	732	0	0
normalized size	1	1.	1.48	1.83	1.9	7.25	0.	0.
time (sec)	N/A	0.092	0.049	0.048	1.037	1.63	0.	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	108	164	284	900	0	0
normalized size	1	1.	0.64	0.98	1.69	5.36	0.	0.
time (sec)	N/A	0.13	0.077	0.076	0.996	1.801	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	177	596	262	1195	0	0
normalized size	1	1.	0.84	2.82	1.24	5.66	0.	0.
time (sec)	N/A	0.15	0.118	0.131	1.075	1.748	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	235	672	338	1454	0	0
normalized size	1	1.	0.89	2.55	1.28	5.51	0.	0.
time (sec)	N/A	0.197	0.088	0.126	1.064	1.607	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	153	868	248	625	0	220
normalized size	1	1.	1.43	8.11	2.32	5.84	0.	2.06
time (sec)	N/A	0.171	0.175	0.473	1.483	1.763	0.	1.273

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	49	49	60	887	86	238	0	198
normalized size	1	1.	1.22	18.1	1.76	4.86	0.	4.04
time (sec)	N/A	0.075	0.088	0.375	1.003	1.76	0.	1.235

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	45	45	46	68	58	112	58	47
normalized size	1	1.	1.02	1.51	1.29	2.49	1.29	1.04
time (sec)	N/A	0.054	0.082	0.038	0.994	1.733	11.378	1.105

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	45	45	46	351	58	55	0	54
normalized size	1	1.	1.02	7.8	1.29	1.22	0.	1.2
time (sec)	N/A	0.054	0.083	0.257	0.986	1.932	0.	1.193

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	49	49	59	872	86	236	0	132
normalized size	1	1.	1.2	17.8	1.76	4.82	0.	2.69
time (sec)	N/A	0.068	0.083	0.335	0.999	1.944	0.	1.181

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	150	842	248	624	0	234
normalized size	1	1.	1.4	7.87	2.32	5.83	0.	2.19
time (sec)	N/A	0.17	0.172	0.385	1.48	1.761	0.	1.256

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	114	668	0	980	0	0
normalized size	1	1.	0.84	4.91	0.	7.21	0.	0.
time (sec)	N/A	0.538	0.283	0.285	0.	1.75	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [281] had the largest ratio of [1.333]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	4	1.	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2	A	5	4	1.	23	0.174
3	A	4	4	1.	21	0.19
4	A	8	8	1.	23	0.348
5	A	2	2	1.	23	0.087
6	A	3	3	1.	23	0.13
7	A	4	3	1.	23	0.13
8	A	5	3	1.	23	0.13
9	A	4	3	1.	23	0.13
10	A	4	3	1.	23	0.13
11	A	4	3	1.	23	0.13
12	A	2	2	1.	19	0.105
13	A	4	4	1.	23	0.174
14	A	5	5	1.	23	0.217
15	A	6	5	1.	23	0.217
16	A	6	4	1.	25	0.16
17	A	5	4	1.	25	0.16
18	A	4	4	1.	25	0.16
19	A	3	3	1.	25	0.12
20	A	4	4	1.	25	0.16
21	A	5	4	1.	25	0.16
22	A	6	4	1.	25	0.16
23	A	7	6	1.	25	0.24
24	A	6	6	1.	25	0.24
25	A	5	5	1.	25	0.2
26	A	6	6	1.	25	0.24
27	A	7	6	1.	25	0.24
28	A	4	4	1.	12	0.333
29	A	4	4	1.	14	0.286
30	A	0	0	0.	0	0.
31	A	9	7	1.	40	0.175
32	A	7	6	1.	40	0.15
33	A	2	3	1.	38	0.079

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
34	A	0	0	0.	0	0.
35	A	0	0	0.	0	0.
36	A	2	2	1.	11	0.182
37	A	2	2	1.	11	0.182
38	A	2	2	1.	9	0.222
39	A	2	2	1.	7	0.286
40	A	2	2	1.	11	0.182
41	A	2	2	1.	11	0.182
42	A	2	2	1.	11	0.182
43	A	2	2	1.	11	0.182
44	A	3	2	1.	13	0.154
45	A	3	2	1.	13	0.154
46	A	3	2	1.	13	0.154
47	A	3	3	1.	11	0.273
48	A	2	2	1.	9	0.222
49	A	3	3	1.	13	0.231
50	A	3	3	1.	13	0.231
51	A	3	2	1.	13	0.154
52	A	1	1	1.	13	0.077
53	A	2	2	1.52	13	0.154
54	A	4	2	1.	13	0.154
55	A	4	2	1.	13	0.154
56	A	4	3	1.	13	0.231
57	A	3	3	1.	11	0.273
58	A	2	2	1.	9	0.222
59	A	4	3	1.	13	0.231
60	A	4	4	1.	13	0.308
61	A	4	3	1.	13	0.231
62	A	4	2	1.	13	0.154
63	A	1	1	1.	13	0.077
64	A	2	2	1.	13	0.154
65	A	5	2	1.	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
66	A	5	2	1.	13	0.154
67	A	5	2	1.	13	0.154
68	A	5	2	1.	13	0.154
69	A	5	3	1.	13	0.231
70	A	4	3	1.	13	0.231
71	A	3	3	1.	11	0.273
72	A	2	2	1.	9	0.222
73	A	5	3	1.	13	0.231
74	A	5	4	1.	13	0.308
75	A	5	4	1.	13	0.308
76	A	5	3	1.	13	0.231
77	A	5	2	1.	13	0.154
78	A	1	1	1.	13	0.077
79	A	2	2	1.	13	0.154
80	A	3	2	1.	13	0.154
81	A	4	2	1.65	13	0.154
82	A	5	2	1.	13	0.154
83	A	5	2	1.	13	0.154
84	A	3	3	1.	11	0.273
85	A	1	1	1.	13	0.077
86	A	5	4	1.	13	0.308
87	A	4	4	1.	13	0.308
88	A	3	3	1.	11	0.273
89	A	2	2	1.	9	0.222
90	A	4	3	1.	13	0.231
91	A	5	4	1.	13	0.308
92	A	6	4	1.	13	0.308
93	A	2	2	1.	13	0.154
94	A	6	5	1.	13	0.385
95	A	5	5	1.	13	0.385
96	A	4	4	1.	13	0.308
97	A	3	3	1.	11	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
98	A	2	2	1.	9	0.222
99	A	5	4	1.	13	0.308
100	A	6	5	1.	13	0.385
101	A	7	5	1.	13	0.385
102	A	3	2	1.	13	0.154
103	A	6	5	1.	13	0.385
104	A	5	4	1.	13	0.308
105	A	4	3	1.	13	0.231
106	A	3	3	1.	11	0.273
107	A	2	2	1.	9	0.222
108	A	6	4	1.	13	0.308
109	A	7	5	1.	13	0.385
110	A	8	5	1.	13	0.385
111	A	6	3	1.	15	0.2
112	A	5	3	1.	15	0.2
113	A	4	3	1.	15	0.2
114	A	3	3	1.	13	0.231
115	A	2	2	1.	11	0.182
116	A	2	2	1.	15	0.133
117	A	2	2	1.	15	0.133
118	A	4	3	1.	15	0.2
119	A	6	3	1.	15	0.2
120	A	6	3	1.	15	0.2
121	A	5	3	1.	15	0.2
122	A	4	3	1.	15	0.2
123	A	3	3	1.	13	0.231
124	A	2	2	1.	11	0.182
125	A	3	2	1.	15	0.133
126	A	3	3	1.	15	0.2
127	A	3	2	1.	15	0.133
128	A	5	3	1.	15	0.2
129	A	6	3	1.	15	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	5	3	1.	15	0.2
131	A	4	3	1.	15	0.2
132	A	3	3	1.	13	0.231
133	A	2	2	1.	11	0.182
134	A	4	2	1.	15	0.133
135	A	4	3	1.	15	0.2
136	A	4	3	1.	15	0.2
137	A	4	2	1.	15	0.133
138	A	6	3	1.	15	0.2
139	A	8	3	1.	15	0.2
140	A	6	3	1.	15	0.2
141	A	5	3	1.	15	0.2
142	A	4	3	1.	15	0.2
143	A	3	3	1.	13	0.231
144	A	2	2	1.	11	0.182
145	A	1	1	1.	15	0.067
146	A	3	3	1.	15	0.2
147	A	5	3	1.	15	0.2
148	A	7	3	1.	15	0.2
149	A	6	3	1.	15	0.2
150	A	5	3	1.	15	0.2
151	A	4	3	1.	15	0.2
152	A	3	3	1.	13	0.231
153	A	2	2	1.	11	0.182
154	A	2	2	1.	15	0.133
155	A	4	3	1.	15	0.2
156	A	6	3	1.	15	0.2
157	A	8	3	1.	15	0.2
158	A	6	3	1.	15	0.2
159	A	5	3	1.	15	0.2
160	A	4	3	1.	15	0.2
161	A	3	3	1.	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
162	A	2	2	1.	11	0.182
163	A	3	2	1.	15	0.133
164	A	5	3	1.	15	0.2
165	A	7	3	1.	15	0.2
166	A	9	3	1.	15	0.2
167	A	2	2	1.	13	0.154
168	A	2	2	1.	13	0.154
169	A	2	2	1.	13	0.154
170	A	2	2	1.	13	0.154
171	A	2	2	1.	13	0.154
172	A	2	2	1.	13	0.154
173	A	2	2	1.	13	0.154
174	A	2	2	1.	13	0.154
175	A	3	2	1.	15	0.133
176	A	3	2	1.	15	0.133
177	A	3	2	1.	15	0.133
178	A	3	2	1.	15	0.133
179	A	3	2	1.	15	0.133
180	A	3	2	1.	15	0.133
181	A	3	2	1.	15	0.133
182	A	3	2	1.	15	0.133
183	A	4	2	1.	15	0.133
184	A	4	2	1.	15	0.133
185	A	4	2	1.	15	0.133
186	A	4	2	1.	15	0.133
187	A	4	2	1.	15	0.133
188	A	4	2	1.	15	0.133
189	A	4	2	1.	15	0.133
190	A	4	2	1.	15	0.133
191	A	5	2	1.	15	0.133
192	A	4	2	1.	15	0.133
193	A	3	2	1.	15	0.133

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
194	A	2	2	1.	15	0.133
195	A	1	1	1.	15	0.067
196	A	2	2	1.	15	0.133
197	A	3	2	1.	15	0.133
198	A	4	2	1.	15	0.133
199	A	5	3	1.	15	0.2
200	A	4	3	1.	15	0.2
201	A	3	3	1.	15	0.2
202	A	2	2	1.	15	0.133
203	A	3	3	1.	15	0.2
204	A	4	3	1.	15	0.2
205	A	5	3	1.	15	0.2
206	A	6	3	1.	15	0.2
207	A	5	3	1.	15	0.2
208	A	4	3	1.	15	0.2
209	A	3	2	1.	15	0.133
210	A	4	3	1.	15	0.2
211	A	5	3	1.	15	0.2
212	A	6	3	1.	15	0.2
213	A	7	3	1.	15	0.2
214	A	8	3	1.	15	0.2
215	A	4	2	1.	17	0.118
216	A	3	2	1.	17	0.118
217	A	2	2	1.	17	0.118
218	A	2	2	1.	17	0.118
219	A	1	1	1.	17	0.059
220	A	2	2	1.	17	0.118
221	A	3	2	1.	17	0.118
222	A	4	2	1.	17	0.118
223	A	5	2	1.	17	0.118
224	A	4	2	1.	17	0.118
225	A	3	2	1.	17	0.118

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
226	A	3	3	1.	17	0.176
227	A	3	2	1.	17	0.118
228	A	1	1	1.	17	0.059
229	A	2	2	1.	17	0.118
230	A	3	2	1.	17	0.118
231	A	4	2	1.	17	0.118
232	A	5	2	1.	17	0.118
233	A	4	2	1.	17	0.118
234	A	4	3	1.	17	0.176
235	A	4	3	1.	17	0.176
236	A	4	2	1.	17	0.118
237	A	1	1	1.	17	0.059
238	A	2	2	1.	17	0.118
239	A	3	2	1.	17	0.118
240	A	4	2	1.	17	0.118
241	A	4	2	1.	17	0.118
242	A	3	2	1.	17	0.118
243	A	2	2	1.	17	0.118
244	A	1	1	1.	17	0.059
245	A	1	1	1.	17	0.059
246	A	2	2	1.	17	0.118
247	A	3	2	1.	17	0.118
248	A	4	2	1.	17	0.118
249	A	5	3	1.	17	0.176
250	A	4	3	1.	17	0.176
251	A	3	3	1.	17	0.176
252	A	2	2	1.	17	0.118
253	A	1	1	1.	17	0.059
254	A	2	2	1.	17	0.118
255	A	3	2	1.	17	0.118
256	A	4	2	1.	17	0.118
257	A	5	3	1.	17	0.176

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
258	A	4	3	1.	17	0.176
259	A	3	2	1.	17	0.118
260	A	1	1	1.	17	0.059
261	A	2	2	1.	17	0.118
262	A	3	2	1.	17	0.118
263	A	4	2	1.	17	0.118
264	A	5	2	1.	17	0.118
265	A	1	1	1.	13	0.077
266	A	6	3	1.	13	0.231
267	A	5	3	1.	13	0.231
268	A	4	3	1.	13	0.231
269	A	3	3	1.	11	0.273
270	A	2	2	1.	9	0.222
271	A	1	1	1.	13	0.077
272	A	2	2	1.	13	0.154
273	A	3	2	1.	13	0.154
274	A	2	2	1.	11	0.182
275	A	2	2	1.	11	0.182
276	A	2	2	1.	9	0.222
277	A	2	2	1.	7	0.286
278	A	2	2	1.	11	0.182
279	A	2	2	1.	11	0.182
280	A	2	2	1.	11	0.182
281	A	6	4	1.	3	1.333
282	A	8	5	1.	5	1.
283	A	10	6	1.	7	0.857
284	A	11	6	1.	15	0.4
285	A	9	5	1.	13	0.385
286	A	7	4	1.	11	0.364
287	A	0	0	0.	0	0.
288	A	8	7	1.	16	0.438
289	A	7	7	1.	16	0.438

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
290	A	6	6	1.	14	0.429
291	A	5	5	1.	12	0.417
292	A	0	0	0.	0	0.
293	A	8	7	1.	19	0.368
294	A	7	7	1.	19	0.368
295	A	6	6	1.	17	0.353
296	A	5	5	1.	15	0.333
297	A	0	0	0.	0	0.
298	A	11	6	1.	15	0.4
299	A	9	5	1.	13	0.385
300	A	7	4	1.	11	0.364
301	A	0	0	0.	0	0.
302	A	8	7	1.	16	0.438
303	A	7	7	1.	16	0.438
304	A	6	6	1.	14	0.429
305	A	5	5	1.	12	0.417
306	A	0	0	0.	0	0.
307	A	8	7	1.	19	0.368
308	A	7	7	1.	19	0.368
309	A	6	6	1.	17	0.353
310	A	5	5	1.	15	0.333
311	A	0	0	0.	0	0.
312	A	12	6	1.	15	0.4
313	A	10	6	1.	15	0.4
314	A	8	5	1.	13	0.385
315	A	6	4	1.	7	0.571
316	A	0	0	0.	0	0.
317	A	11	6	1.	15	0.4
318	A	9	5	1.	13	0.385
319	A	7	4	1.	11	0.364
320	A	0	0	0.	0	0.
321	A	7	7	1.	20	0.35

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
322	A	6	6	1.	18	0.333
323	A	5	5	1.	16	0.312
324	A	0	0	0.	0	0.
325	A	7	7	1.	21	0.333
326	A	6	6	1.	19	0.316
327	A	5	5	1.	17	0.294
328	A	0	0	0.	0	0.
329	A	12	6	1.	15	0.4
330	A	10	6	1.	15	0.4
331	A	8	5	1.	13	0.385
332	A	6	4	1.	7	0.571
333	A	0	0	0.	0	0.
334	A	11	6	1.	15	0.4
335	A	9	5	1.	13	0.385
336	A	7	4	1.	11	0.364
337	A	0	0	0.	0	0.
338	A	7	7	1.	20	0.35
339	A	6	6	1.	18	0.333
340	A	5	5	1.	16	0.312
341	A	0	0	0.	0	0.
342	A	7	7	1.	21	0.333
343	A	6	6	1.	19	0.316
344	A	5	5	1.	17	0.294
345	A	0	0	0.	0	0.
346	A	2	2	1.	4	0.5
347	A	7	4	1.	6	0.667
348	A	9	5	1.	8	0.625
349	A	2	2	1.	8	0.25
350	A	7	4	1.	10	0.4
351	A	9	5	1.	12	0.417
352	A	6	6	1.	12	0.5
353	A	9	5	1.	14	0.357

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
354	A	11	6	1.	16	0.375
355	A	8	7	1.	20	0.35
356	A	5	5	1.	20	0.25
357	A	3	2	1.	20	0.1
358	A	3	2	1.	20	0.1
359	A	5	5	1.	20	0.25
360	A	8	7	1.	20	0.35
361	A	11	8	1.	24	0.333

Chapter 3

Listing of integrals

3.1 $\int x^5 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=127

$$-\frac{5d^2x\sqrt{d+ex^2}}{96e^{5/2}}+\frac{5d^3\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{96e^3}+\frac{5dx^3\sqrt{d+ex^2}}{144e^{3/2}}-\frac{x^5\sqrt{d+ex^2}}{36\sqrt{e}}+\frac{1}{6}x^6\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

```
[Out] (-5*d^2*x*Sqrt[d + e*x^2])/(96*e^(5/2)) + (5*d*x^3*Sqrt[d + e*x^2])/(144*e^
(3/2)) - (x^5*Sqrt[d + e*x^2])/(36*Sqrt[e]) + (5*d^3*ArcTanh[(Sqrt[e]*x)/Sq
rt[d + e*x^2]])/(96*e^3) + (x^6*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/6
```

Rubi [A] time = 0.0524512, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6221, 321, 217, 206}

$$-\frac{5d^2x\sqrt{d+ex^2}}{96e^{5/2}}+\frac{5d^3\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{96e^3}+\frac{5dx^3\sqrt{d+ex^2}}{144e^{3/2}}-\frac{x^5\sqrt{d+ex^2}}{36\sqrt{e}}+\frac{1}{6}x^6\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (-5*d^2*x*Sqrt[d + e*x^2])/(96*e^(5/2)) + (5*d*x^3*Sqrt[d + e*x^2])/(144*e^
(3/2)) - (x^5*Sqrt[d + e*x^2])/(36*Sqrt[e]) + (5*d^3*ArcTanh[(Sqrt[e]*x)/Sq
```

```
rt[d + e*x^2]])/(96*e^3) + (x^6*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/6
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^5 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{6}x^6 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{6}\sqrt{e}\int \frac{x^6}{\sqrt{d+ex^2}}\,dx$$

$$= -\frac{x^5\sqrt{d+ex^2}}{36\sqrt{e}} + \frac{1}{6}x^6\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) + \frac{(5d)\int \frac{x^4}{\sqrt{d+ex^2}}\,dx}{36\sqrt{e}}$$

$$= \frac{5dx^3\sqrt{d+ex^2}}{144e^{3/2}} - \frac{x^5\sqrt{d+ex^2}}{36\sqrt{e}} + \frac{1}{6}x^6\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{\left(5d^2\right)\int \frac{x^2}{\sqrt{d+ex^2}}\,dx}{48e^{3/2}}$$

$$= -\frac{5d^2x\sqrt{d+ex^2}}{96e^{5/2}} + \frac{5dx^3\sqrt{d+ex^2}}{144e^{3/2}} - \frac{x^5\sqrt{d+ex^2}}{36\sqrt{e}} + \frac{1}{6}x^6\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) + \frac{\left(5d^3\right)\int \ldots}{96e^{5\ldots}}$$

$$= -\frac{5d^2x\sqrt{d+ex^2}}{96e^{5/2}} + \frac{5dx^3\sqrt{d+ex^2}}{144e^{3/2}} - \frac{x^5\sqrt{d+ex^2}}{36\sqrt{e}} + \frac{1}{6}x^6\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) + \frac{\left(5d^3\right)\text{Sub}\ldots}{\ldots}$$

$$= -\frac{5d^2x\sqrt{d+ex^2}}{96e^{5/2}} + \frac{5dx^3\sqrt{d+ex^2}}{144e^{3/2}} - \frac{x^5\sqrt{d+ex^2}}{36\sqrt{e}} + \frac{5d^3\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{96e^3} + \frac{1}{6}x^6\tanh^{-1}\Big(\ldots$$

Mathematica [A] time = 0.0719379, size = 99, normalized size = 0.78

$$\frac{\sqrt{e}x\sqrt{d+ex^2}\left(-15d^2+10dex^2-8e^2x^4\right)+15d^3\log\left(\sqrt{d+ex^2}+\sqrt{e}x\right)+48e^3x^6\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{288e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (Sqrt[e]*x*Sqrt[d + e*x^2]*(-15*d^2 + 10*d*e*x^2 - 8*e^2*x^4) + 48*e^3*x^6*
ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] + 15*d^3*Log[Sqrt[e]*x + Sqrt[d + e*x^
2]])/(288*e^3)
```

Maple [A] time = 0.036, size = 172, normalized size = 1.4

$$\frac{x^6}{6}\text{Artanh}\left(x\sqrt{e}\frac{1}{\sqrt{ex^2+d}}\right) + \frac{x^7}{48\,d}\sqrt{e}\sqrt{ex^2+d} - \frac{7\,x^5}{288}\sqrt{ex^2+d}\frac{1}{\sqrt{e}} + \frac{35\,dx^3}{1152}\sqrt{ex^2+d}e^{-\frac{3}{2}} - \frac{5\,d^2x}{128}\sqrt{ex^2+d}e^{-\frac{5}{2}} + \frac{5\,d^3}{96\,e^3}\,\text{l}\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] 1/6*x^6*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+1/48*e^(1/2)/d*x^7*(e*x^2+d)^(1/
2)-7/288*x^5*(e*x^2+d)^(1/2)/e^(1/2)+35/1152*d*x^3*(e*x^2+d)^(1/2)/e^(3/2)-
5/128*d^2*x*(e*x^2+d)^(1/2)/e^(5/2)+5/96/e^3*d^3*ln(x*e^(1/2)+(e*x^2+d)^(1/
2))-1/48/e^(1/2)/d*x^5*(e*x^2+d)^(3/2)+5/288/e^(3/2)*x^3*(e*x^2+d)^(3/2)-5/
384/e^(5/2)*d*x*(e*x^2+d)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{12}\,x^6\log\left(\sqrt{e}x+\sqrt{ex^2+d}\right)-\frac{1}{12}\,x^6\log\left(-\sqrt{e}x+\sqrt{ex^2+d}\right)-\frac{1}{2}\,d\sqrt{e}\int-\frac{\sqrt{ex^2+d}x^6}{3\left(e^2x^4+dex^2-\left(ex^2+d\right)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/12*x^6*log(sqrt(e)*x + sqrt(e*x^2 + d)) - 1/12*x^6*log(-sqrt(e)*x + sqrt(
e*x^2 + d)) - 1/2*d*sqrt(e)*integrate(-1/3*sqrt(e*x^2 + d)*x^6/(e^2*x^4 + d
*e*x^2 - (e*x^2 + d)^2), x)
```

Fricas [A] time = 2.20834, size = 205, normalized size = 1.61

$$-\frac{2\left(8\,e^2x^5-10\,dex^3+15\,d^2x\right)\sqrt{ex^2+d}\sqrt{e}-3\left(16\,e^3x^6+5\,d^3\right)\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)}{576\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/576*(2*(8*e^2*x^5 - 10*d*e*x^3 + 15*d^2*x)*sqrt(e*x^2 + d)*sqrt(e) - 3*(
16*e^3*x^6 + 5*d^3)*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d))/e^3
```

Sympy [A] time = 11.8099, size = 121, normalized size = 0.95

$$\begin{cases} \frac{5d^3 \operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{96e^3} - \frac{5d^2x\sqrt{d+ex^2}}{96e^{\frac{5}{2}}} + \frac{5dx^3\sqrt{d+ex^2}}{144e^{\frac{3}{2}}} + \frac{x^6 \operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{6} - \frac{x^5\sqrt{d+ex^2}}{36\sqrt{e}} & \text{for } e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((5*d**3*atanh(sqrt(e)*x/sqrt(d + e*x**2))/(96*e**3) - 5*d**2*x*sq
rt(d + e*x**2)/(96*e**(5/2)) + 5*d*x**3*sqrt(d + e*x**2)/(144*e**(3/2)) + x
**6*atanh(sqrt(e)*x/sqrt(d + e*x**2))/6 - x**5*sqrt(d + e*x**2)/(36*sqrt(e)
), Ne(e, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\left[\text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, -\frac{1}{2}\,de^{\frac{1}{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, -1/2*d*e^(1/2)]
```

3.2 $\int x^3 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=101

$$-\frac{3d^2 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{32e^2}+\frac{3dx\sqrt{d+ex^2}}{32e^{3/2}}-\frac{x^3\sqrt{d+ex^2}}{16\sqrt{e}}+\frac{1}{4}x^4 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

```
[Out] (3*d*x*Sqrt[d + e*x^2])/(32*e^(3/2)) - (x^3*Sqrt[d + e*x^2])/(16*Sqrt[e]) -
 (3*d^2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(32*e^2) + (x^4*ArcTanh[(Sqrt
[e]*x)/Sqrt[d + e*x^2]])/4
```

Rubi [A] time = 0.0388202, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6221, 321, 217, 206}

$$-\frac{3d^2 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{32e^2}+\frac{3dx\sqrt{d+ex^2}}{32e^{3/2}}-\frac{x^3\sqrt{d+ex^2}}{16\sqrt{e}}+\frac{1}{4}x^4 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (3*d*x*Sqrt[d + e*x^2])/(32*e^(3/2)) - (x^3*Sqrt[d + e*x^2])/(16*Sqrt[e]) -
 (3*d^2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(32*e^2) + (x^4*ArcTanh[(Sqrt
[e]*x)/Sqrt[d + e*x^2]])/4
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)dx &= \frac{1}{4}x^4\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{4}\sqrt{e}\int\frac{x^4}{\sqrt{d+ex^2}}dx\\
&= -\frac{x^3\sqrt{d+ex^2}}{16\sqrt{e}}+\frac{1}{4}x^4\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)+\frac{(3d)\int\frac{x^2}{\sqrt{d+ex^2}}dx}{16\sqrt{e}}\\
&= \frac{3dx\sqrt{d+ex^2}}{32e^{3/2}}-\frac{x^3\sqrt{d+ex^2}}{16\sqrt{e}}+\frac{1}{4}x^4\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\left(3d^2\right)\int\frac{1}{\sqrt{d+ex^2}}dx}{32e^{3/2}}\\
&= \frac{3dx\sqrt{d+ex^2}}{32e^{3/2}}-\frac{x^3\sqrt{d+ex^2}}{16\sqrt{e}}+\frac{1}{4}x^4\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\left(3d^2\right)\text{Subst}\left(\int\frac{1}{1-ex^2}dx,x,\frac{\quad}{\sqrt{d}}\right.}{32e^{3/2}}\\
&= \frac{3dx\sqrt{d+ex^2}}{32e^{3/2}}-\frac{x^3\sqrt{d+ex^2}}{16\sqrt{e}}-\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{32e^2}+\frac{1}{4}x^4\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0481845, size = 88, normalized size = 0.87

$$\frac{-3d^2\log\left(\sqrt{d+ex^2}+\sqrt{e}x\right)+8e^2x^4\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)+\sqrt{e}x\left(3d-2ex^2\right)\sqrt{d+ex^2}}{32e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (Sqrt[e]*x*(3*d - 2*e*x^2)*Sqrt[d + e*x^2] + 8*e^2*x^4*ArcTanh[(Sqrt[e]*x)/
Sqrt[d + e*x^2]] - 3*d^2*Log[Sqrt[e]*x + Sqrt[d + e*x^2]])/(32*e^2)
```

Maple [A] time = 0.031, size = 134, normalized size = 1.3

$$\frac{x^4}{4}\mathrm{Artanh}\left(x\sqrt{e}\frac{1}{\sqrt{ex^2+d}}\right)+\frac{x^5}{24\,d}\sqrt{e}\sqrt{ex^2+d}-\frac{5\,x^3}{96}\sqrt{ex^2+d}\frac{1}{\sqrt{e}}+\frac{dx}{16}\sqrt{ex^2+d}e^{-\frac{3}{2}}-\frac{3\,d^2}{32\,e^2}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)-\frac{x^3}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] 1/4*x^4*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+1/24*e^(1/2)/d*x^5*(e*x^2+d)^(1/
2)-5/96*x^3*(e*x^2+d)^(1/2)/e^(1/2)+1/16*d*x*(e*x^2+d)^(1/2)/e^(3/2)-3/32/e
^2*d^2*ln(x*e^(1/2)+(e*x^2+d)^(1/2))-1/24/e^(1/2)/d*x^3*(e*x^2+d)^(3/2)+1/3
2/e^(3/2)*x*(e*x^2+d)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{8}\,x^4\log\left(\sqrt{e}x+\sqrt{ex^2+d}\right)-\frac{1}{8}\,x^4\log\left(-\sqrt{e}x+\sqrt{ex^2+d}\right)-\frac{1}{2}\,d\sqrt{e}\int-\frac{\sqrt{ex^2+d}x^4}{2\left(e^2x^4+dex^2-\left(ex^2+d\right)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/8*x^4*log(sqrt(e)*x + sqrt(e*x^2 + d)) - 1/8*x^4*log(-sqrt(e)*x + sqrt(e*
x^2 + d)) - 1/2*d*sqrt(e)*integrate(-1/2*sqrt(e*x^2 + d)*x^4/(e^2*x^4 + d*e
*x^2 - (e*x^2 + d)^2), x)
```

Fricas [A] time = 2.10243, size = 176, normalized size = 1.74

$$-\frac{2\left(2\,ex^3-3\,dx\right)\sqrt{ex^2+d}\sqrt{e}-\left(8\,e^2x^4-3\,d^2\right)\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)}{64\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/64*(2*(2*e*x^3 - 3*d*x)*sqrt(e*x^2 + d)*sqrt(e) - (8*e^2*x^4 - 3*d^2)*lo
g((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d))/e^2
```

Sympy [A] time = 4.84909, size = 95, normalized size = 0.94

$$\begin{cases} -\frac{3d^2 \operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{32e^2} + \frac{3dx\sqrt{d+ex^2}}{32e^{\frac{3}{2}}} + \frac{x^4 \operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4} - \frac{x^3\sqrt{d+ex^2}}{16\sqrt{e}} & \text{for } e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((-3*d**2*atanh(sqrt(e)*x/sqrt(d + e*x**2))/(32*e**2) + 3*d*x*sqrt
(d + e*x**2)/(32*e**(3/2)) + x**4*atanh(sqrt(e)*x/sqrt(d + e*x**2))/4 - x**
3*sqrt(d + e*x**2)/(16*sqrt(e)), Ne(e, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\left[\text{undef}, \text{undef}, \text{undef}, -\frac{1}{2}\, de^{\frac{1}{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, -1/2*d*e^(1/2)]
```

3.3 $\int x \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=75

$$-\frac{x\sqrt{d+ex^2}}{4\sqrt{e}}+\frac{1}{2}x^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)+\frac{d\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4e}$$

```
[Out] -(x*Sqrt[d + e*x^2])/(4*Sqrt[e]) + (d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])
/(4*e) + (x^2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/2
```

Rubi [A] time = 0.0225233, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {6221, 321, 217, 206}

$$-\frac{x\sqrt{d+ex^2}}{4\sqrt{e}}+\frac{1}{2}x^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)+\frac{d\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] -(x*Sqrt[d + e*x^2])/(4*Sqrt[e]) + (d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])
/(4*e) + (x^2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/2
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)dx &= \frac{1}{2}x^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{2}\sqrt{e}\int\frac{x^2}{\sqrt{d+ex^2}}\,dx\\
&= -\frac{x\sqrt{d+ex^2}}{4\sqrt{e}}+\frac{1}{2}x^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)+\frac{d\int\frac{1}{\sqrt{d+ex^2}}\,dx}{4\sqrt{e}}\\
&= -\frac{x\sqrt{d+ex^2}}{4\sqrt{e}}+\frac{1}{2}x^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)+\frac{d\,\mathrm{Subst}\left(\int\frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{4\sqrt{e}}\\
&= -\frac{x\sqrt{d+ex^2}}{4\sqrt{e}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4e}+\frac{1}{2}x^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0312259, size = 76, normalized size = 1.01

$$-\frac{x\sqrt{d+ex^2}}{4\sqrt{e}}+\frac{d\log\left(\sqrt{d+ex^2}+\sqrt{e}x\right)}{4e}+\frac{1}{2}x^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] -(x*Sqrt[d + e*x^2])/(4*Sqrt[e]) + (x^2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]
])/2 + (d*Log[Sqrt[e]*x + Sqrt[d + e*x^2]])/(4*e)
```

Maple [A] time = 0.03, size = 97, normalized size = 1.3

$$\frac{x^2}{2}\mathrm{Artanh}\left(x\sqrt{e}\frac{1}{\sqrt{ex^2+d}}\right)+\frac{x^3}{8d}\sqrt{e}\sqrt{ex^2+d}-\frac{x}{8}\sqrt{ex^2+d}\frac{1}{\sqrt{e}}+\frac{d}{4e}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)-\frac{x}{8d}\left(ex^2+d\right)^{\frac{3}{2}}\frac{1}{\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] 1/2*x^2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+1/8*e^(1/2)/d*x^3*(e*x^2+d)^(1/2
)-1/8*x*(e*x^2+d)^(1/2)/e^(1/2)+1/4/e*d*ln(x*e^(1/2)+(e*x^2+d)^(1/2))-1/8/e
^(1/2)/d*x*(e*x^2+d)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}\,x^{2}\log\left(\sqrt{e}x+\sqrt{ex^{2}+d}\right)-\frac{1}{4}\,x^{2}\log\left(-\sqrt{e}x+\sqrt{ex^{2}+d}\right)-\frac{1}{2}\,d\sqrt{e}\int-\frac{\sqrt{ex^{2}+d}x^{2}}{e^{2}x^{4}+dex^{2}-\left(ex^{2}+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/4*x^2*log(sqrt(e)*x + sqrt(e*x^2 + d)) - 1/4*x^2*log(-sqrt(e)*x + sqrt(e*
x^2 + d)) - 1/2*d*sqrt(e)*integrate(-sqrt(e*x^2 + d)*x^2/(e^2*x^4 + d*e*x^2
 - (e*x^2 + d)^2), x)
```

Fricas [A] time = 2.17179, size = 142, normalized size = 1.89

$$-\frac{2\,\sqrt{ex^{2}+d}\sqrt{e}x-\left(2\,ex^{2}+d\right)\log\left(\frac{2\,ex^{2}+2\,\sqrt{ex^{2}+d}\sqrt{e}x+d}{d}\right)}{8\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(e*x^2 + d)*sqrt(e)*x - (2*e*x^2 + d)*log((2*e*x^2 + 2*sqrt(e*x
^2 + d)*sqrt(e)*x + d)/d))/e
```

Sympy [A] time = 0.878603, size = 66, normalized size = 0.88

$$\begin{cases} \frac{d \operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4e} + \frac{x^2 \operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2} - \frac{x\sqrt{d+ex^2}}{4\sqrt{e}} & \text{for } e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((d*atanh(sqrt(e)*x/sqrt(d + e*x**2))/(4*e) + x**2*atanh(sqrt(e)*x
/sqrt(d + e*x**2))/2 - x*sqrt(d + e*x**2)/(4*sqrt(e)), Ne(e, 0)), (0, True)
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\left[\text{undef}, \text{undef}, -\frac{1}{2}\, de^{\frac{1}{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] [undef, undef, -1/2*d*e^(1/2)]
```

3.4 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x} dx$

Optimal. Leaf size=238

$$\frac{\sqrt{d}\sqrt{\frac{ex^2}{d}+1}\text{PolyLog}\left(2,e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{d+ex^2}}-\frac{\sqrt{d}\sqrt{\frac{ex^2}{d}+1}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{d+ex^2}}+\frac{\sqrt{d}\sqrt{\frac{ex^2}{d}+1}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{d+ex^2}}-$$

```
[Out] -(Sqrt[d]*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]^2)/(2*Sqrt[d + e
*x^2]) + (Sqrt[d]*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]*Log[1 -
E^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])])/Sqrt[d + e*x^2] - (Sqrt[d]*Sqrt[1 + (e
*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]*Log[x])/Sqrt[d + e*x^2] + ArcTanh[(Sq
rt[e]*x)/Sqrt[d + e*x^2]]*Log[x] + (Sqrt[d]*Sqrt[1 + (e*x^2)/d]*PolyLog[2,
E^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])])/(2*Sqrt[d + e*x^2])
```

Rubi [A] time = 0.157596, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6219, 2327, 2325, 5659, 3716, 2190, 2279, 2391}

$$\frac{\sqrt{d}\sqrt{\frac{ex^2}{d}+1}\text{PolyLog}\left(2,e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{d+ex^2}}-\frac{\sqrt{d}\sqrt{\frac{ex^2}{d}+1}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{d+ex^2}}+\frac{\sqrt{d}\sqrt{\frac{ex^2}{d}+1}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{d+ex^2}}-$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x,x]
```


```
[Out] -(Sqrt[d]*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]^2)/(2*Sqrt[d + e
*x^2]) + (Sqrt[d]*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]*Log[1 -
E^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])])/Sqrt[d + e*x^2] - (Sqrt[d]*Sqrt[1 + (e
*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]*Log[x])/Sqrt[d + e*x^2] + ArcTanh[(Sq
rt[e]*x)/Sqrt[d + e*x^2]]*Log[x] + (Sqrt[d]*Sqrt[1 + (e*x^2)/d]*PolyLog[2,
E^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])])/(2*Sqrt[d + e*x^2])
```


Rule 6219

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]/(x_), x_Symbol] :> Sim
p[ArcTanh[(c*x)/Sqrt[a + b*x^2]]*Log[x], x] - Dist[c, Int[Log[x]/Sqrt[a + b
```

```
*x^2], x], x] /; FreeQ[{a, b, c}, x] && EqQ[b, c^2]
```

Rule 2327

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Dist[Sqrt[1 + (e*x^2)/d]/Sqrt[d + e*x^2], Int[(a + b*Log[c*x^n])/Sqr
t[1 + (e*x^2)/d], x], x] /; FreeQ[{a, b, c, d, e, n}, x] &&  !GtQ[d, 0]
```

Rule 2325

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[(ArcSinh[(Rt[e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/Rt[e, 2], x]
 - Dist[(b*n)/Rt[e, 2], Int[ArcSinh[(Rt[e, 2]*x)/Sqrt[d]]/x, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && GtQ[d, 0] && PosQ[e]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx = \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\log(x) - \sqrt{e}\int \frac{\log(x)}{\sqrt{d+ex^2}}\,dx$$

$$= \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\log(x) - \frac{\left(\sqrt{e}\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{\log(x)}{\sqrt{1+\frac{ex^2}{d}}}\,dx}{\sqrt{d+ex^2}}$$

$$= -\frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log(x)}{\sqrt{d+ex^2}} + \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\log(x) + \frac{\left(\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}\,d}{\sqrt{d+ex^2}}$$

$$= -\frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log(x)}{\sqrt{d+ex^2}} + \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\log(x) + \frac{\left(\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\right)\text{Subst}\left(\int x\,\text{co}\right.}{\sqrt{d+}}$$

$$= -\frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{d+ex^2}} - \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log(x)}{\sqrt{d+ex^2}} + \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\log(x) -$$

$$= -\frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{d+ex^2}} - \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}{}$$

$$= -\frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{d+ex^2}} - \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}{}$$

$$= -\frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{d+ex^2}} - \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}{}$$

Mathematica [A] time = 2.16352, size = 167, normalized size = 0.7

$$\frac{\sqrt{e}\sqrt{\frac{ex^2}{d}+1}\left(-\text{PolyLog}\left(2,e^{-2\sinh^{-1}\left(x\sqrt{\frac{e}{d}}\right)}\right)-2\log(x)\log\left(\sqrt{\frac{ex^2}{d}+1}+x\sqrt{\frac{e}{d}}\right)+\sinh^{-1}\left(x\sqrt{\frac{e}{d}}\right)^2+2\sinh^{-1}\left(x\sqrt{\frac{e}{d}}\right)\log\left(1\right.\right.}{2\sqrt{\frac{e}{d}}\sqrt{d+ex^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x,x]
```

```
[Out] ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]*Log[x] + (Sqrt[e]*Sqrt[1 + (e*x^2)/d]*
(ArcSinh[Sqrt[e/d]*x]^2 + 2*ArcSinh[Sqrt[e/d]*x]*Log[1 - E^(-2*ArcSinh[Sqrt
[e/d]*x])] - 2*Log[x]*Log[Sqrt[e/d]*x + Sqrt[1 + (e*x^2)/d]] - PolyLog[2, E
^(-2*ArcSinh[Sqrt[e/d]*x])]))/(2*Sqrt[e/d]*Sqrt[d + e*x^2])
```

Maple [A] time = 0.325, size = 209, normalized size = 0.9

$$-\frac{1}{2}\left(\operatorname{Artanh}\left(x\sqrt{e}\frac{1}{\sqrt{ex^2+d}}\right)\right)^2+\operatorname{Artanh}\left(x\sqrt{e}\frac{1}{\sqrt{ex^2+d}}\right)\ln\left(1+\left(x\sqrt{e}\frac{1}{\sqrt{ex^2+d}}+1\right)\frac{1}{\sqrt{-\frac{ex^2}{ex^2+d}+1}}\right)+\operatorname{polylog}\left(2,-\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x,x)
```

```
[Out] -1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))^2+arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)
)*ln(1+(x*e^(1/2)/(e*x^2+d)^(1/2)+1)/(-x^2*e/(e*x^2+d)+1)^(1/2))+polylog(2,
-(x*e^(1/2)/(e*x^2+d)^(1/2)+1)/(-x^2*e/(e*x^2+d)+1)^(1/2))+arctanh(x*e^(1/2
)/(e*x^2+d)^(1/2))*ln(1-(x*e^(1/2)/(e*x^2+d)^(1/2)+1)/(-x^2*e/(e*x^2+d)+1)^
(1/2))+polylog(2,(x*e^(1/2)/(e*x^2+d)^(1/2)+1)/(-x^2*e/(e*x^2+d)+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x,x, algorithm="fricas")
```

```
[Out] integral(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x,x)
```

```
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x, x)
```

3.5 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx$

Optimal. Leaf size=53

$$-\frac{\sqrt{e}\sqrt{d+ex^2}}{2dx}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2x^2}$$

```
[Out] -(Sqrt[e]*Sqrt[d + e*x^2])/(2*d*x) - ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/(
2*x^2)
```

Rubi [A] time = 0.0186087, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6221, 264}

$$-\frac{\sqrt{e}\sqrt{d+ex^2}}{2dx}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^3,x]
```

```
[Out] -(Sqrt[e]*Sqrt[d + e*x^2])/(2*d*x) - ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/(
2*x^2)
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx = -\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2x^2} + \frac{1}{2}\sqrt{e}\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx$$
$$= -\frac{\sqrt{e}\sqrt{d+ex^2}}{2dx} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2x^2}$$

Mathematica [A] time = 0.036218, size = 50, normalized size = 0.94

$$-\frac{\sqrt{e}x\sqrt{d+ex^2}+d\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2dx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^3,x]
```

```
[Out] -(Sqrt[e]*x*Sqrt[d + e*x^2] + d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*d*
x^2)
```

Maple [A] time = 0.031, size = 60, normalized size = 1.1

$$-\frac{1}{2\,x^2}\mathrm{Artanh}\left(x\sqrt{e}\frac{1}{\sqrt{ex^2+d}}\right)-\frac{1}{2\,d^2x}\sqrt{e}\left(ex^2+d\right)^{\frac{3}{2}}+\frac{x}{2\,d^2}e^{\frac{3}{2}}\sqrt{ex^2+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^3,x)
```

```
[Out] -1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^2-1/2*e^(1/2)/d^2/x*(e*x^2+d)^(3/
2)+1/2*e^(3/2)/d^2*x*(e*x^2+d)^(1/2)
```

Maxima [A] time = 1.10477, size = 69, normalized size = 1.3

$$-\frac{\mathrm{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{2\,x^2}-\frac{e^{\frac{3}{2}}x^2+d\sqrt{e}}{2\,\sqrt{ex^2+d}dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^2 - 1/2*(e^(3/2)*x^2 + d*sqrt(e))
/(sqrt(e*x^2 + d)*d*x)
```

Fricas [A] time = 2.22698, size = 134, normalized size = 2.53

$$-\frac{2\sqrt{ex^2+d}\sqrt{e}x + d\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)}{4\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(e*x^2 + d)*sqrt(e)*x + d*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt
(e)*x + d)/d))/(d*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**3,x)
```

```
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**3, x)
```

Giac [A] time = 1.24982, size = 96, normalized size = 1.81

$$\frac{e}{\left(xe^{\frac{1}{2}} - \sqrt{x^2e+d}\right)^2 - d} - \frac{\log\left(-\frac{\frac{xe^{\frac{1}{2}}}{\sqrt{x^2e+d}}+1}{\frac{xe^{\frac{1}{2}}}{\sqrt{x^2e+d}}-1}\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] e/((x*e^(1/2) - sqrt(x^2*e + d))^2 - d) - 1/4*log(-(x*e^(1/2)/sqrt(x^2*e +
d) + 1)/(x*e^(1/2)/sqrt(x^2*e + d) - 1))/x^2
```

3.6 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^5} dx$

Optimal. Leaf size=79

$$\frac{e^{3/2}\sqrt{d+ex^2}}{6d^2x} - \frac{\sqrt{e}\sqrt{d+ex^2}}{12dx^3} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4x^4}$$

```
[Out] -(Sqrt[e]*Sqrt[d + e*x^2])/(12*d*x^3) + (e^(3/2)*Sqrt[d + e*x^2])/(6*d^2*x)
 - ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/(4*x^4)
```

Rubi [A] time = 0.0262587, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6221, 271, 264}

$$\frac{e^{3/2}\sqrt{d+ex^2}}{6d^2x} - \frac{\sqrt{e}\sqrt{d+ex^2}}{12dx^3} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^5,x]
```

```
[Out] -(Sqrt[e]*Sqrt[d + e*x^2])/(12*d*x^3) + (e^(3/2)*Sqrt[d + e*x^2])/(6*d^2*x)
 - ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/(4*x^4)
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx &= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4x^4}+\frac{1}{4}\sqrt{e}\int \frac{1}{x^4\sqrt{d+ex^2}}\,dx \\
&= -\frac{\sqrt{e}\sqrt{d+ex^2}}{12dx^3}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4x^4}-\frac{e^{3/2}\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{6d} \\
&= -\frac{\sqrt{e}\sqrt{d+ex^2}}{12dx^3}+\frac{e^{3/2}\sqrt{d+ex^2}}{6d^2x}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4x^4}
\end{aligned}$$

Mathematica [A] time = 0.0404816, size = 63, normalized size = 0.8

$$\frac{\sqrt{e}x\sqrt{d+ex^2}\left(2ex^2-d\right)-3d^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{12d^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^5,x]
```

```
[Out] (Sqrt[e]*x*Sqrt[d + e*x^2]*(-d + 2*e*x^2) - 3*d^2*ArcTanh[(Sqrt[e]*x)/Sqrt[
d + e*x^2]])/(12*d^2*x^4)
```

Maple [A] time = 0.03, size = 62, normalized size = 0.8

$$-\frac{1}{4x^4}\text{Artanh}\left(x\sqrt{e}\frac{1}{\sqrt{ex^2+d}}\right)+\frac{1}{4d^2x}e^{\frac{3}{2}}\sqrt{ex^2+d}-\frac{1}{12d^2x^3}\sqrt{e}\left(ex^2+d\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^5,x)
```

```
[Out] -1/4*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^4+1/4*e^(3/2)*(e*x^2+d)^(1/2)/d^2
/x-1/12*e^(1/2)/d^2/x^3*(e*x^2+d)^(3/2)
```

Maxima [A] time = 0.973572, size = 82, normalized size = 1.04

$$\frac{\sqrt{ex^2+d}e^{\frac{3}{2}}}{4\,d^2x}-\frac{\left(ex^2+d\right)^{\frac{3}{2}}\sqrt{e}}{12\,d^2x^3}-\frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(e*x^2 + d)*e^(3/2)/(d^2*x) - 1/12*(e*x^2 + d)^(3/2)*sqrt(e)/(d^2*x
^3) - 1/4*arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^4
```

Fricas [A] time = 2.24824, size = 162, normalized size = 2.05

$$-\frac{3\,d^2\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)-2\left(2\,ex^3-dx\right)\sqrt{ex^2+d}\sqrt{e}}{24\,d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^5,x, algorithm="fricas")
```

```
[Out] -1/24*(3*d^2*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) - 2*(2*e*x^
3 - d*x)*sqrt(e*x^2 + d)*sqrt(e))/(d^2*x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**5,x)
```

```
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**5, x)
```

Giac [A] time = 1.40209, size = 144, normalized size = 1.82

$$\frac{\left(3\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2de-d^2e\right)e}{3\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2-d\right)^3d}-\frac{\log\left(-\frac{\frac{xe^{\frac{1}{2}}}{\sqrt{x^2e+d}}+1}{\frac{xe^{\frac{1}{2}}}{\sqrt{x^2e+d}}-1}\right)}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^5,x, algorithm="giac")
```

```
[Out] 1/3*(3*(x*e^(1/2) - sqrt(x^2*e + d))^2*d*e - d^2*e)*e/(((x*e^(1/2) - sqrt(x
^2*e + d))^2 - d)^3*d) - 1/8*log(-(x*e^(1/2)/sqrt(x^2*e + d) + 1)/(x*e^(1/2
)/sqrt(x^2*e + d) - 1))/x^4
```

3.7 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx$

Optimal. Leaf size=105

$$-\frac{4e^{5/2}\sqrt{d+ex^2}}{45d^3x}+\frac{2e^{3/2}\sqrt{d+ex^2}}{45d^2x^3}-\frac{\sqrt{e}\sqrt{d+ex^2}}{30dx^5}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{6x^6}$$

```
[Out] -(Sqrt[e]*Sqrt[d + e*x^2])/(30*d*x^5) + (2*e^(3/2)*Sqrt[d + e*x^2])/(45*d^2
*x^3) - (4*e^(5/2)*Sqrt[d + e*x^2])/(45*d^3*x) - ArcTanh[(Sqrt[e]*x)/Sqrt[d
 + e*x^2]]/(6*x^6)
```

Rubi [A] time = 0.0369368, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6221, 271, 264}

$$-\frac{4e^{5/2}\sqrt{d+ex^2}}{45d^3x}+\frac{2e^{3/2}\sqrt{d+ex^2}}{45d^2x^3}-\frac{\sqrt{e}\sqrt{d+ex^2}}{30dx^5}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^7,x]
```

```
[Out] -(Sqrt[e]*Sqrt[d + e*x^2])/(30*d*x^5) + (2*e^(3/2)*Sqrt[d + e*x^2])/(45*d^2
*x^3) - (4*e^(5/2)*Sqrt[d + e*x^2])/(45*d^3*x) - ArcTanh[(Sqrt[e]*x)/Sqrt[d
 + e*x^2]]/(6*x^6)
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
```

```
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx &= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{6x^6}+\frac{1}{6}\sqrt{e}\int \frac{1}{x^6\sqrt{d+ex^2}}\,dx\\
&= -\frac{\sqrt{e}\sqrt{d+ex^2}}{30dx^5}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{6x^6}-\frac{\left(2e^{3/2}\right)\int \frac{1}{x^4\sqrt{d+ex^2}}\,dx}{15d}\\
&= -\frac{\sqrt{e}\sqrt{d+ex^2}}{30dx^5}+\frac{2e^{3/2}\sqrt{d+ex^2}}{45d^2x^3}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{6x^6}+\frac{\left(4e^{5/2}\right)\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{45d^2}\\
&= -\frac{\sqrt{e}\sqrt{d+ex^2}}{30dx^5}+\frac{2e^{3/2}\sqrt{d+ex^2}}{45d^2x^3}-\frac{4e^{5/2}\sqrt{d+ex^2}}{45d^3x}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{6x^6}
\end{aligned}
$$

Mathematica [A] time = 0.0482387, size = 74, normalized size = 0.7

$$\frac{\sqrt{e}x\sqrt{d+ex^2}\left(-3d^2+4dex^2-8e^2x^4\right)-15d^3\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{90d^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^7,x]
```

```
[Out] (Sqrt[e]*x*Sqrt[d + e*x^2]*(-3*d^2 + 4*d*e*x^2 - 8*e^2*x^4) - 15*d^3*ArcTan
h[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(90*d^3*x^6)
```

Maple [A] time = 0.032, size = 110, normalized size = 1.1

$$-\frac{1}{6x^6}\mathrm{Artanh}\left(x\sqrt{e}\frac{1}{\sqrt{ex^2+d}}\right)-\frac{1}{6d}e^{\frac{3}{2}}\left(-\frac{1}{3dx^3}\sqrt{ex^2+d}+\frac{2e}{3d^2x}\sqrt{ex^2+d}\right)+\frac{1}{6d}\sqrt{e}\left(-\frac{1}{5dx^5}\left(ex^2+d\right)^{\frac{3}{2}}+\frac{2e}{15d^2x^3}\left(ex^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^7,x)
```

```
[Out] -1/6*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^6-1/6*e^(3/2)/d*(-1/3/d/x^3*(e*x^
2+d)^(1/2)+2/3*e/d^2/x*(e*x^2+d)^(1/2))+1/6*e^(1/2)/d*(-1/5/d/x^5*(e*x^2+d)
^(3/2)+2/15*e/d^2/x^3*(e*x^2+d)^(3/2))
```

Maxima [A] time = 0.976213, size = 138, normalized size = 1.31

$$-\frac{\left(2\,e^2x^4+dex^2-d^2\right)e^{\frac{3}{2}}}{18\,\sqrt{ex^2+d}d^3x^3}-\frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{6\,x^6}+\frac{\left(2\,e^2x^4-dex^2-3\,d^2\right)\sqrt{ex^2+d}\sqrt{e}}{90\,d^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^7,x, algorithm="maxima")
```

```
[Out] -1/18*(2*e^2*x^4 + d*e*x^2 - d^2)*e^(3/2)/(sqrt(e*x^2 + d)*d^3*x^3) - 1/6*a
rctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^6 + 1/90*(2*e^2*x^4 - d*e*x^2 - 3*d^2)*
sqrt(e*x^2 + d)*sqrt(e)/(d^3*x^5)
```

Fricas [A] time = 2.33867, size = 189, normalized size = 1.8

$$-\frac{15\,d^3\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)+2\left(8\,e^2x^5-4\,dex^3+3\,d^2x\right)\sqrt{ex^2+d}\sqrt{e}}{180\,d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^7,x, algorithm="fricas")
```

```
[Out] -1/180*(15*d^3*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) + 2*(8*e^
2*x^5 - 4*d*e*x^3 + 3*d^2*x)*sqrt(e*x^2 + d)*sqrt(e))/(d^3*x^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**7,x)
```

```
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**7, x)
```

Giac [A] time = 1.42731, size = 181, normalized size = 1.72

$$\frac{8\left(10\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^4d^2e^2-5\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2d^3e^2+d^4e^2\right)e}{45\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2-d\right)^5d^2}-\frac{\log\left(-\frac{\frac{xe^{\frac{1}{2}}}{\sqrt{x^2e+d}}+1}{\frac{xe^{\frac{1}{2}}}{\sqrt{x^2e+d}}-1}\right)}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^7,x, algorithm="giac")
```

```
[Out] 8/45*(10*(x*e^(1/2) - sqrt(x^2*e + d))^4*d^2*e^2 - 5*(x*e^(1/2) - sqrt(x^2*
e + d))^2*d^3*e^2 + d^4*e^2)*e/(((x*e^(1/2) - sqrt(x^2*e + d))^2 - d)^5*d^2
) - 1/12*log(-(x*e^(1/2)/sqrt(x^2*e + d) + 1)/(x*e^(1/2)/sqrt(x^2*e + d) -
1))/x^6
```

3.8 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx$

Optimal. Leaf size=131

$$\frac{2e^{7/2}\sqrt{d+ex^2}}{35d^4x} - \frac{e^{5/2}\sqrt{d+ex^2}}{35d^3x^3} + \frac{3e^{3/2}\sqrt{d+ex^2}}{140d^2x^5} - \frac{\sqrt{e}\sqrt{d+ex^2}}{56dx^7} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8x^8}$$

```
[Out] -(Sqrt[e]*Sqrt[d + e*x^2])/(56*d*x^7) + (3*e^(3/2)*Sqrt[d + e*x^2])/(140*d^
2*x^5) - (e^(5/2)*Sqrt[d + e*x^2])/(35*d^3*x^3) + (2*e^(7/2)*Sqrt[d + e*x^2
])/(35*d^4*x) - ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/(8*x^8)
```

Rubi [A] time = 0.048865, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6221, 271, 264}

$$\frac{2e^{7/2}\sqrt{d+ex^2}}{35d^4x} - \frac{e^{5/2}\sqrt{d+ex^2}}{35d^3x^3} + \frac{3e^{3/2}\sqrt{d+ex^2}}{140d^2x^5} - \frac{\sqrt{e}\sqrt{d+ex^2}}{56dx^7} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^9,x]
```

```
[Out] -(Sqrt[e]*Sqrt[d + e*x^2])/(56*d*x^7) + (3*e^(3/2)*Sqrt[d + e*x^2])/(140*d^
2*x^5) - (e^(5/2)*Sqrt[d + e*x^2])/(35*d^3*x^3) + (2*e^(7/2)*Sqrt[d + e*x^2
])/(35*d^4*x) - ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/(8*x^8)
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
```

```
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx &= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8x^8}+\frac{1}{8}\sqrt{e}\int\frac{1}{x^8\sqrt{d+ex^2}}\,dx\\
&= -\frac{\sqrt{e}\sqrt{d+ex^2}}{56dx^7}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8x^8}-\frac{\left(3e^{3/2}\right)\int\frac{1}{x^6\sqrt{d+ex^2}}\,dx}{28d}\\
&= -\frac{\sqrt{e}\sqrt{d+ex^2}}{56dx^7}+\frac{3e^{3/2}\sqrt{d+ex^2}}{140d^2x^5}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8x^8}+\frac{\left(3e^{5/2}\right)\int\frac{1}{x^4\sqrt{d+ex^2}}\,dx}{35d^2}\\
&= -\frac{\sqrt{e}\sqrt{d+ex^2}}{56dx^7}+\frac{3e^{3/2}\sqrt{d+ex^2}}{140d^2x^5}-\frac{e^{5/2}\sqrt{d+ex^2}}{35d^3x^3}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8x^8}-\frac{\left(2e^{7/2}\right)\int\frac{1}{x^2\sqrt{d+ex^2}}\,dx}{35d^3}\\
&= -\frac{\sqrt{e}\sqrt{d+ex^2}}{56dx^7}+\frac{3e^{3/2}\sqrt{d+ex^2}}{140d^2x^5}-\frac{e^{5/2}\sqrt{d+ex^2}}{35d^3x^3}+\frac{2e^{7/2}\sqrt{d+ex^2}}{35d^4x}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8x^8}
\end{aligned}$$

Mathematica [A] time = 0.0535337, size = 85, normalized size = 0.65

$$\frac{\sqrt{e}x\sqrt{d+ex^2}\left(6d^2ex^2-5d^3-8de^2x^4+16e^3x^6\right)-35d^4\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{280d^4x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^9,x]
```

```
[Out] (Sqrt[e]*x*Sqrt[d + e*x^2]*(-5*d^3 + 6*d^2*e*x^2 - 8*d*e^2*x^4 + 16*e^3*x^6
) - 35*d^4*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(280*d^4*x^8)
```

Maple [A] time = 0.034, size = 158, normalized size = 1.2

$$-\frac{1}{8x^8}\mathrm{Artanh}\left(x\sqrt{e}\frac{1}{\sqrt{ex^2+d}}\right)-\frac{1}{8d}e^{\frac{3}{2}}\left(-\frac{1}{5dx^5}\sqrt{ex^2+d}-\frac{4e}{5d}\left(-\frac{1}{3dx^3}\sqrt{ex^2+d}+\frac{2e}{3d^2x}\sqrt{ex^2+d}\right)\right)+\frac{1}{8d}\sqrt{e}\left(-\frac{1}{7dx^7}\left(e\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^9,x)
```

```
[Out] -1/8*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^8-1/8*e^(3/2)/d*(-1/5/d/x^5*(e*x^
2+d)^(1/2)-4/5*e/d*(-1/3/d/x^3*(e*x^2+d)^(1/2)+2/3*e/d^2/x*(e*x^2+d)^(1/2))
)+1/8*e^(1/2)/d*(-1/7/d/x^7*(e*x^2+d)^(3/2)-4/7*e/d*(-1/5/d/x^5*(e*x^2+d)^(
3/2)+2/15*e/d^2/x^3*(e*x^2+d)^(3/2)))
```

Maxima [A] time = 0.979119, size = 169, normalized size = 1.29

$$\frac{\left(8\,e^{3}x^{6}+4\,de^{2}x^{4}-d^{2}ex^{2}+3\,d^{3}\right)e^{\frac{3}{2}}}{120\,\sqrt{ex^{2}+d}d^{4}x^{5}}-\frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^{2}+d}}\right)}{8\,x^{8}}-\frac{\left(8\,e^{3}x^{6}-4\,de^{2}x^{4}+3\,d^{2}ex^{2}+15\,d^{3}\right)\sqrt{ex^{2}+d}\sqrt{e}}{840\,d^{4}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^9,x, algorithm="maxima")
```

```
[Out] 1/120*(8*e^3*x^6 + 4*d*e^2*x^4 - d^2*e*x^2 + 3*d^3)*e^(3/2)/(sqrt(e*x^2 + d
)*d^4*x^5) - 1/8*arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^8 - 1/840*(8*e^3*x^6
- 4*d*e^2*x^4 + 3*d^2*e*x^2 + 15*d^3)*sqrt(e*x^2 + d)*sqrt(e)/(d^4*x^7)
```

Fricas [A] time = 2.43524, size = 212, normalized size = 1.62

$$-\frac{35\,d^{4}\log\left(\frac{2\,ex^{2}+2\,\sqrt{ex^{2}+d}\sqrt{e}x+d}{d}\right)-2\left(16\,e^{3}x^{7}-8\,de^{2}x^{5}+6\,d^{2}ex^{3}-5\,d^{3}x\right)\sqrt{ex^{2}+d}\sqrt{e}}{560\,d^{4}x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^9,x, algorithm="fricas")
```

```
[Out] -1/560*(35*d^4*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) - 2*(16*e
^3*x^7 - 8*d*e^2*x^5 + 6*d^2*e*x^3 - 5*d^3*x)*sqrt(e*x^2 + d)*sqrt(e))/(d^4
*x^8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**9,x)
```

```
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**9, x)
```

Giac [A] time = 1.54826, size = 217, normalized size = 1.66

$$-\frac{\log\left(-\frac{\frac{xe^{\frac{1}{2}}}{\sqrt{x^2e+d}}+1}{\frac{xe^{\frac{1}{2}}}{\sqrt{x^2e+d}}-1}\right)}{16\,x^8}+\frac{4\left(35\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^6d^3e^3-21\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^4d^4e^3+7\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2d^5e^3-d^6e^3\right)e}{35\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2-d\right)^7d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^9,x, algorithm="giac")
```

```
[Out] -1/16*log(-(x*e^(1/2)/sqrt(x^2*e + d) + 1)/(x*e^(1/2)/sqrt(x^2*e + d) - 1))
/x^8 + 4/35*(35*(x*e^(1/2) - sqrt(x^2*e + d))^6*d^3*e^3 - 21*(x*e^(1/2) - s
qrt(x^2*e + d))^4*d^4*e^3 + 7*(x*e^(1/2) - sqrt(x^2*e + d))^2*d^5*e^3 - d^6
*e^3)*e/(((x*e^(1/2) - sqrt(x^2*e + d))^2 - d)^7*d^3)
```

3.9 $\int x^6 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=114

$$-\frac{d^2\left(d+ex^2\right)^{3/2}}{7e^{7/2}}+\frac{d^3\sqrt{d+ex^2}}{7e^{7/2}}-\frac{\left(d+ex^2\right)^{7/2}}{49e^{7/2}}+\frac{3d\left(d+ex^2\right)^{5/2}}{35e^{7/2}}+\frac{1}{7}x^7\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

```
[Out] (d^3*Sqrt[d + e*x^2])/(7*e^(7/2)) - (d^2*(d + e*x^2)^(3/2))/(7*e^(7/2)) + (
3*d*(d + e*x^2)^(5/2))/(35*e^(7/2)) - (d + e*x^2)^(7/2)/(49*e^(7/2)) + (x^7
*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/7
```

Rubi [A] time = 0.0639015, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6221, 266, 43}

$$-\frac{d^2\left(d+ex^2\right)^{3/2}}{7e^{7/2}}+\frac{d^3\sqrt{d+ex^2}}{7e^{7/2}}-\frac{\left(d+ex^2\right)^{7/2}}{49e^{7/2}}+\frac{3d\left(d+ex^2\right)^{5/2}}{35e^{7/2}}+\frac{1}{7}x^7\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^6*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (d^3*Sqrt[d + e*x^2])/(7*e^(7/2)) - (d^2*(d + e*x^2)^(3/2))/(7*e^(7/2)) + (
3*d*(d + e*x^2)^(5/2))/(35*e^(7/2)) - (d + e*x^2)^(7/2)/(49*e^(7/2)) + (x^7
*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/7
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^6\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)dx &= \frac{1}{7}x^7\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{7}\sqrt{e}\int\frac{x^7}{\sqrt{d+ex^2}}\,dx\\ &= \frac{1}{7}x^7\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{14}\sqrt{e}\,\text{Subst}\left(\int\frac{x^3}{\sqrt{d+ex}}\,dx,x,x^2\right)\\ &= \frac{1}{7}x^7\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{14}\sqrt{e}\,\text{Subst}\left(\int\left(-\frac{d^3}{e^3\sqrt{d+ex}}+\frac{3d^2\sqrt{d+ex}}{e^3}-\frac{3d(d+ex)^{3/2}}{e^3}+\right.\right.\\ &= \frac{d^3\sqrt{d+ex^2}}{7e^{7/2}}-\frac{d^2\left(d+ex^2\right)^{3/2}}{7e^{7/2}}+\frac{3d\left(d+ex^2\right)^{5/2}}{35e^{7/2}}-\frac{\left(d+ex^2\right)^{7/2}}{49e^{7/2}}+\frac{1}{7}x^7\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0553371, size = 79, normalized size = 0.69

$$\frac{\sqrt{d+ex^2}\left(-8d^2ex^2+16d^3+6de^2x^4-5e^3x^6\right)}{245e^{7/2}}+\frac{1}{7}x^7\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (Sqrt[d + e*x^2]*(16*d^3 - 8*d^2*e*x^2 + 6*d*e^2*x^4 - 5*e^3*x^6))/(245*e^(
7/2)) + (x^7*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/7
```

Maple [B] time = 0.037, size = 224, normalized size = 2.

$$\frac{x^7}{7}\text{Artanh}\left(x\sqrt{e}\frac{1}{\sqrt{ex^2+d}}\right)+\frac{1}{7d}e^{\frac{3}{2}}\left(\frac{x^8}{9e}\sqrt{ex^2+d}-\frac{8d}{9e}\left(\frac{x^6}{7e}\sqrt{ex^2+d}-\frac{6d}{7e}\left(\frac{x^4}{5e}\sqrt{ex^2+d}-\frac{4d}{5e}\left(\frac{x^2}{3e}\sqrt{ex^2+d}-\frac{2d}{3e^2}\sqrt{ex^2}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] 1/7*x^7*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+1/7*e^(3/2)/d*(1/9*x^8/e*(e*x^2+
d)^(1/2)-8/9*d/e*(1/7*x^6/e*(e*x^2+d)^(1/2)-6/7*d/e*(1/5*x^4/e*(e*x^2+d)^(1
/2)-4/5*d/e*(1/3*x^2/e*(e*x^2+d)^(1/2)-2/3*d/e^2*(e*x^2+d)^(1/2)))))-1/7*e^
(1/2)/d*(1/9*x^6*(e*x^2+d)^(3/2)/e-2/3*d/e*(1/7*x^4*(e*x^2+d)^(3/2)/e-4/7*d
/e*(1/5*x^2*(e*x^2+d)^(3/2)/e-2/15*d/e^2*(e*x^2+d)^(3/2))))
```

Maxima [A] time = 0.986734, size = 209, normalized size = 1.83

$$\frac{1}{7}\,x^7\,\mathrm{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)-\frac{35\left(ex^2+d\right)^{\frac{9}{2}}-135\left(ex^2+d\right)^{\frac{7}{2}}d+189\left(ex^2+d\right)^{\frac{5}{2}}d^2-105\left(ex^2+d\right)^{\frac{3}{2}}d^3}{2205\,de^{\frac{7}{2}}}+\frac{35\left(ex^2+d\right)^{\frac{9}{2}}-1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/7*x^7*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)) - 1/2205*(35*(e*x^2 + d)^(9/2) -
 135*(e*x^2 + d)^(7/2)*d + 189*(e*x^2 + d)^(5/2)*d^2 - 105*(e*x^2 + d)^(3/2
)*d^3)/(d*e^(7/2)) + 1/2205*(35*(e*x^2 + d)^(9/2) - 180*(e*x^2 + d)^(7/2)*d
 + 378*(e*x^2 + d)^(5/2)*d^2 - 420*(e*x^2 + d)^(3/2)*d^3 + 315*sqrt(e*x^2 +
 d)*d^4)/(d*e^(7/2))
```

Fricas [A] time = 2.20161, size = 205, normalized size = 1.8

$$\frac{35\,e^4x^7\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)-2\left(5\,e^3x^6-6\,de^2x^4+8\,d^2ex^2-16\,d^3\right)\sqrt{ex^2+d}\sqrt{e}}{490\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/490*(35*e^4*x^7*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) - 2*(5
*e^3*x^6 - 6*d*e^2*x^4 + 8*d^2*e*x^2 - 16*d^3)*sqrt(e*x^2 + d)*sqrt(e))/e^4
```

Sympy [A] time = 19.2527, size = 116, normalized size = 1.02

$$\begin{cases} \frac{16d^3\sqrt{d+ex^2}}{245e^{\frac{7}{2}}} - \frac{8d^2x^2\sqrt{d+ex^2}}{245e^{\frac{5}{2}}} + \frac{6dx^4\sqrt{d+ex^2}}{245e^{\frac{3}{2}}} + \frac{x^7\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{7} - \frac{x^6\sqrt{d+ex^2}}{49\sqrt{e}} & \text{for } e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((16*d**3*sqrt(d + e*x**2)/(245*e**(7/2)) - 8*d**2*x**2*sqrt(d + e
*x**2)/(245*e**(5/2)) + 6*d*x**4*sqrt(d + e*x**2)/(245*e**(3/2)) + x**7*ata
nh(sqrt(e)*x/sqrt(d + e*x**2))/7 - x**6*sqrt(d + e*x**2)/(49*sqrt(e)), Ne(e
, 0)), (0, True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.10 $\int x^4 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=91

$$-\frac{d^2\sqrt{d+ex^2}}{5e^{5/2}}-\frac{\left(d+ex^2\right)^{5/2}}{25e^{5/2}}+\frac{2d\left(d+ex^2\right)^{3/2}}{15e^{5/2}}+\frac{1}{5}x^5\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

```
[Out] -(d^2*Sqrt[d + e*x^2])/(5*e^(5/2)) + (2*d*(d + e*x^2)^(3/2))/(15*e^(5/2)) -
 (d + e*x^2)^(5/2)/(25*e^(5/2)) + (x^5*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]
)/5
```

Rubi [A] time = 0.0491054, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6221, 266, 43}

$$-\frac{d^2\sqrt{d+ex^2}}{5e^{5/2}}-\frac{\left(d+ex^2\right)^{5/2}}{25e^{5/2}}+\frac{2d\left(d+ex^2\right)^{3/2}}{15e^{5/2}}+\frac{1}{5}x^5\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] -(d^2*Sqrt[d + e*x^2])/(5*e^(5/2)) + (2*d*(d + e*x^2)^(3/2))/(15*e^(5/2)) -
 (d + e*x^2)^(5/2)/(25*e^(5/2)) + (x^5*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]
)/5
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx &= \frac{1}{5}x^5 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{5}\sqrt{e}\int \frac{x^5}{\sqrt{d+ex^2}}\,dx \\ &= \frac{1}{5}x^5 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{10}\sqrt{e}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{d+ex}}\,dx, x, x^2\right) \\ &= \frac{1}{5}x^5 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{10}\sqrt{e}\,\text{Subst}\left(\int \left(\frac{d^2}{e^2\sqrt{d+ex}} - \frac{2d\sqrt{d+ex}}{e^2} + \frac{(d+ex)^{3/2}}{e^2}\right) dx, x \\ &= -\frac{d^2\sqrt{d+ex^2}}{5e^{5/2}} + \frac{2d\left(d+ex^2\right)^{3/2}}{15e^{5/2}} - \frac{\left(d+ex^2\right)^{5/2}}{25e^{5/2}} + \frac{1}{5}x^5 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0572996, size = 68, normalized size = 0.75

$$\frac{1}{5}x^5 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{\sqrt{d+ex^2}\left(8d^2-4dex^2+3e^2x^4\right)}{75e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] -(Sqrt[d + e*x^2]*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4))/(75*e^(5/2)) + (x^5*ArcT
anh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/5
```

Maple [B] time = 0.033, size = 176, normalized size = 1.9

$$\frac{x^5}{5}\text{Artanh}\left(x\sqrt{e}\frac{1}{\sqrt{ex^2+d}}\right) + \frac{1}{5d}e^{\frac{3}{2}}\left(\frac{x^6}{7e}\sqrt{ex^2+d} - \frac{6d}{7e}\left(\frac{x^4}{5e}\sqrt{ex^2+d} - \frac{4d}{5e}\left(\frac{x^2}{3e}\sqrt{ex^2+d} - \frac{2d}{3e^2}\sqrt{ex^2+d}\right)\right)\right) - \frac{1}{5d}\sqrt{e}\left(\frac{x}{7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] 1/5*x^5*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+1/5*e^(3/2)/d*(1/7*x^6/e*(e*x^2+
d)^(1/2)-6/7*d/e*(1/5*x^4/e*(e*x^2+d)^(1/2)-4/5*d/e*(1/3*x^2/e*(e*x^2+d)^(1
/2)-2/3*d/e^2*(e*x^2+d)^(1/2))))-1/5*e^(1/2)/d*(1/7*x^4*(e*x^2+d)^(3/2)/e-4
/7*d/e*(1/5*x^2*(e*x^2+d)^(3/2)/e-2/15*d/e^2*(e*x^2+d)^(3/2)))
```

Maxima [A] time = 0.983253, size = 171, normalized size = 1.88

$$\frac{1}{5}\,x^5\,\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)-\frac{15\left(ex^2+d\right)^{\frac{7}{2}}-42\left(ex^2+d\right)^{\frac{5}{2}}d+35\left(ex^2+d\right)^{\frac{3}{2}}d^2}{525\,de^{\frac{5}{2}}}+\frac{5\left(ex^2+d\right)^{\frac{7}{2}}-21\left(ex^2+d\right)^{\frac{5}{2}}d+35\left(ex^2\right.}{175\,de^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)) - 1/525*(15*(e*x^2 + d)^(7/2) -
42*(e*x^2 + d)^(5/2)*d + 35*(e*x^2 + d)^(3/2)*d^2)/(d*e^(5/2)) + 1/175*(5*(
e*x^2 + d)^(7/2) - 21*(e*x^2 + d)^(5/2)*d + 35*(e*x^2 + d)^(3/2)*d^2 - 35*s
qrt(e*x^2 + d)*d^3)/(d*e^(5/2))
```

Fricas [A] time = 2.16161, size = 182, normalized size = 2.

$$\frac{15\,e^3x^5\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)-2\left(3\,e^2x^4-4\,dex^2+8\,d^2\right)\sqrt{ex^2+d}\sqrt{e}}{150\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/150*(15*e^3*x^5*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) - 2*(3
*e^2*x^4 - 4*d*e*x^2 + 8*d^2)*sqrt(e*x^2 + d)*sqrt(e))/e^3
```

Sympy [A] time = 5.2025, size = 90, normalized size = 0.99

$$\begin{cases} -\frac{8d^2\sqrt{d+ex^2}}{75e^{\frac{5}{2}}} + \frac{4dx^2\sqrt{d+ex^2}}{75e^{\frac{3}{2}}} + \frac{x^5 \operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5} - \frac{x^4\sqrt{d+ex^2}}{25\sqrt{e}} & \text{for } e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((-8*d**2*sqrt(d + e*x**2)/(75*e**(5/2)) + 4*d*x**2*sqrt(d + e*x**
2)/(75*e**(3/2)) + x**5*atanh(sqrt(e)*x/sqrt(d + e*x**2))/5 - x**4*sqrt(d +
 e*x**2)/(25*sqrt(e)), Ne(e, 0)), (0, True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.11 $\int x^2 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=68

$$-\frac{\left(d+ex^2\right)^{3/2}}{9e^{3/2}}+\frac{d\sqrt{d+ex^2}}{3e^{3/2}}+\frac{1}{3}x^3\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

```
[Out] (d*Sqrt[d + e*x^2])/(3*e^(3/2)) - (d + e*x^2)^(3/2)/(9*e^(3/2)) + (x^3*ArcT
anh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/3
```

Rubi [A] time = 0.0347721, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6221, 266, 43}

$$-\frac{\left(d+ex^2\right)^{3/2}}{9e^{3/2}}+\frac{d\sqrt{d+ex^2}}{3e^{3/2}}+\frac{1}{3}x^3\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (d*Sqrt[d + e*x^2])/(3*e^(3/2)) - (d + e*x^2)^(3/2)/(9*e^(3/2)) + (x^3*ArcT
anh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/3
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)dx &= \frac{1}{3}x^3\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{3}\sqrt{e}\int\frac{x^3}{\sqrt{d+ex^2}}\,dx\\ &= \frac{1}{3}x^3\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{6}\sqrt{e}\,\text{Subst}\left(\int\frac{x}{\sqrt{d+ex}}\,dx,x,x^2\right)\\ &= \frac{1}{3}x^3\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{6}\sqrt{e}\,\text{Subst}\left(\int\left(-\frac{d}{e\sqrt{d+ex}}+\frac{\sqrt{d+ex}}{e}\right)dx,x,x^2\right)\\ &= \frac{d\sqrt{d+ex^2}}{3e^{3/2}}-\frac{\left(d+ex^2\right)^{3/2}}{9e^{3/2}}+\frac{1}{3}x^3\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0465507, size = 56, normalized size = 0.82

$$\frac{1}{9}\left(\frac{\left(2d-ex^2\right)\sqrt{d+ex^2}}{e^{3/2}}+3x^3\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (((2*d - e*x^2)*Sqrt[d + e*x^2])/e^(3/2) + 3*x^3*ArcTanh[(Sqrt[e]*x)/Sqrt[d
 + e*x^2]])/9
```

Maple [B] time = 0.03, size = 128, normalized size = 1.9

$$\frac{x^3}{3}\text{Artanh}\left(x\sqrt{e}\frac{1}{\sqrt{ex^2+d}}\right)+\frac{1}{3d}e^{\frac{3}{2}}\left(\frac{x^4}{5e}\sqrt{ex^2+d}-\frac{4d}{5e}\left(\frac{x^2}{3e}\sqrt{ex^2+d}-\frac{2d}{3e^2}\sqrt{ex^2+d}\right)\right)-\frac{1}{3d}\sqrt{e}\left(\frac{x^2}{5e}\left(ex^2+d\right)^{\frac{3}{2}}-\frac{2d}{15e^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] 1/3*x^3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+1/3*e^(3/2)/d*(1/5*x^4/e*(e*x^2+
d)^(1/2)-4/5*d/e*(1/3*x^2/e*(e*x^2+d)^(1/2)-2/3*d/e^2*(e*x^2+d)^(1/2)))-1/3
*e^(1/2)/d*(1/5*x^2*(e*x^2+d)^(3/2)/e-2/15*d/e^2*(e*x^2+d)^(3/2))
```

Maxima [A] time = 0.976157, size = 134, normalized size = 1.97

$$\frac{1}{3}\,x^3\,\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)-\frac{3\left(ex^2+d\right)^{\frac{5}{2}}-5\left(ex^2+d\right)^{\frac{3}{2}}d}{45\,de^{\frac{3}{2}}}+\frac{3\left(ex^2+d\right)^{\frac{5}{2}}-10\left(ex^2+d\right)^{\frac{3}{2}}d+15\sqrt{ex^2+d}d^2}{45\,de^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)) - 1/45*(3*(e*x^2 + d)^(5/2) - 5*
(e*x^2 + d)^(3/2)*d)/(d*e^(3/2)) + 1/45*(3*(e*x^2 + d)^(5/2) - 10*(e*x^2 +
d)^(3/2)*d + 15*sqrt(e*x^2 + d)*d^2)/(d*e^(3/2))
```

Fricas [A] time = 2.11433, size = 155, normalized size = 2.28

$$\frac{3\,e^2x^3\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)-2\,\sqrt{ex^2+d}\left(ex^2-2\,d\right)\sqrt{e}}{18\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/18*(3*e^2*x^3*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) - 2*sqrt
(e*x^2 + d)*(e*x^2 - 2*d)*sqrt(e))/e^2
```

Sympy [A] time = 8.62313, size = 65, normalized size = 0.96

$$\begin{cases} \frac{2d\sqrt{d+ex^2}}{9e^{\frac{3}{2}}}+\frac{x^3\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3}-\frac{x^2\sqrt{d+ex^2}}{9\sqrt{e}} & \text{for}\ e\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((2*d*sqrt(d + e*x**2)/(9*e**(3/2)) + x**3*atanh(sqrt(e)*x/sqrt(d
+ e*x**2))/3 - x**2*sqrt(d + e*x**2)/(9*sqrt(e)), Ne(e, 0)), (0, True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.12 $\int \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=40

$$x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{d+ex^2}}{\sqrt{e}}$$

```
[Out] -(Sqrt[d + e*x^2]/Sqrt[e]) + x*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]
```

Rubi [A] time = 0.0086467, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6217, 261}

$$x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{d+ex^2}}{\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] -(Sqrt[d + e*x^2]/Sqrt[e]) + x*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]
```

Rule 6217

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]], x_Symbol] :> Simp[x*A
rcTanh[(c*x)/Sqrt[a + b*x^2]], x] - Dist[c, Int[x/Sqrt[a + b*x^2], x], x] /
; FreeQ[{a, b, c}, x] && EqQ[b, c^2]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \sqrt{e}\int \frac{x}{\sqrt{d+ex^2}}\,dx$$
$$= -\frac{\sqrt{d+ex^2}}{\sqrt{e}} + x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

Mathematica [A] time = 0.010424, size = 40, normalized size = 1.

$$x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{\sqrt{d+ex^2}}{\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] -(Sqrt[d + e*x^2]/Sqrt[e]) + x*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]
```

Maple [B] time = 0.029, size = 76, normalized size = 1.9

$$x\mathrm{Artanh}\left(x\sqrt{e}\frac{1}{\sqrt{ex^2+d}}\right) + \frac{1}{d}e^{\frac{3}{2}}\left(\frac{x^2}{3e}\sqrt{ex^2+d} - \frac{2d}{3e^2}\sqrt{ex^2+d}\right) - \frac{1}{3d}\left(ex^2+d\right)^{\frac{3}{2}}\frac{1}{\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] x*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+e^(3/2)/d*(1/3*x^2/e*(e*x^2+d)^(1/2)-2
/3*d/e^2*(e*x^2+d)^(1/2))-1/3/e^(1/2)/d*(e*x^2+d)^(3/2)
```

Maxima [B] time = 0.976917, size = 88, normalized size = 2.2

$$x\,\mathrm{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right) - \frac{\left(ex^2+d\right)^{\frac{3}{2}}}{3d\sqrt{e}} + \frac{\left(ex^2+d\right)^{\frac{3}{2}} - 3\sqrt{ex^2+d}d}{3d\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] x*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)) - 1/3*(e*x^2 + d)^(3/2)/(d*sqrt(e)) +
1/3*((e*x^2 + d)^(3/2) - 3*sqrt(e*x^2 + d)*d)/(d*sqrt(e))
```

Fricas [A] time = 2.13618, size = 124, normalized size = 3.1

$$\frac{ex\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)-2\,\sqrt{ex^2+d}\sqrt{e}}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*(e*x*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) - 2*sqrt(e*x^2
+ d)*sqrt(e))/e
```

Sympy [A] time = 1.49326, size = 36, normalized size = 0.9

$$\begin{cases} x\,\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{d+ex^2}}{\sqrt{e}} & \text{for}\ e\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((x*atanh(sqrt(e)*x/sqrt(d + e*x**2)) - sqrt(d + e*x**2)/sqrt(e),
Ne(e, 0)), (0, True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.13 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^2} dx$

Optimal. Leaf size=55

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}$$

```
[Out] -(ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x) - (Sqrt[e]*ArcTanh[Sqrt[d + e*x^2
]/Sqrt[d]])/Sqrt[d]
```

Rubi [A] time = 0.0310416, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6221, 266, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^2,x]
```

```
[Out] -(ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x) - (Sqrt[e]*ArcTanh[Sqrt[d + e*x^2
]/Sqrt[d]])/Sqrt[d]
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx &= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x}+\sqrt{e}\int \frac{1}{x\sqrt{d+ex^2}}\,dx\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x}+\frac{1}{2}\sqrt{e}\,\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx,x,x^2\right)\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex^2}\right)}{\sqrt{e}}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}
\end{aligned}$$

Mathematica [A] time = 0.0422285, size = 61, normalized size = 1.11

$$\frac{\sqrt{e}\left(\log(x)-\log\left(\sqrt{d}\sqrt{d+ex^2}+d\right)\right)}{\sqrt{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^2,x]
```

```
[Out] -(ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x) + (Sqrt[e]*(Log[x] - Log[d + Sqrt
[d]*Sqrt[d + e*x^2]]))/Sqrt[d]
```

Maple [A] time = 0.031, size = 53, normalized size = 1.

$$-\frac{1}{x}\mathrm{Artanh}\left(x\sqrt{e}\frac{1}{\sqrt{ex^2+d}}\right)-\sqrt{e}\ln\left(\frac{1}{x}\left(2\,d+2\,\sqrt{d}\sqrt{ex^2+d}\right)\right)\frac{1}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^2,x)
```

```
[Out] -arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x-e^(1/2)/d^(1/2)*ln((2*d+2*d^(1/2)*(e*
x^2+d)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$d\sqrt{e}\int-\frac{\sqrt{ex^2+d}}{e^2x^5+dex^3-\left(ex^3+dx\right)\left(ex^2+d\right)}\,dx-\frac{\log\left(\sqrt{e}x+\sqrt{ex^2+d}\right)-\log\left(-\sqrt{e}x+\sqrt{ex^2+d}\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] d*sqrt(e)*integrate(-sqrt(e*x^2 + d)/(e^2*x^5 + d*e*x^3 - (e*x^3 + d*x)*(e*
x^2 + d)), x) - 1/2*(log(sqrt(e)*x + sqrt(e*x^2 + d)) - log(-sqrt(e)*x + sq
rt(e*x^2 + d)))/x
```

Fricas [B] time = 2.41289, size = 637, normalized size = 11.58

$$\left[\frac{x\sqrt{\frac{e}{d}}\log\left(-\frac{e^2x^2-2\,\sqrt{ex^2+d}d\sqrt{e}\sqrt{\frac{e}{d}}+2\,de}{x^2}\right)+(x-1)\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{ex+d}}{d}\right)-x\log\left(\frac{ex+\sqrt{ex^2+d}\sqrt{e}}{x}\right)+x\log\left(\frac{ex-\sqrt{ex^2+d}\sqrt{e}}{x}\right)}{2\,x},\ \frac{2\,x\sqrt{\cdots}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(x*sqrt(e/d)*log(-(e^2*x^2 - 2*sqrt(e*x^2 + d)*d*sqrt(e)*sqrt(e/d) + 2
*d*e)/x^2) + (x - 1)*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) - x
*log((e*x + sqrt(e*x^2 + d)*sqrt(e))/x) + x*log((e*x - sqrt(e*x^2 + d)*sqrt
(e))/x))/x, 1/2*(2*x*sqrt(-e/d)*arctan(sqrt(e*x^2 + d)*d*sqrt(e)*sqrt(-e/d)
/(e^2*x^2 + d*e)) + (x - 1)*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)
/d) - x*log((e*x + sqrt(e*x^2 + d)*sqrt(e))/x) + x*log((e*x - sqrt(e*x^2 +
d)*sqrt(e))/x))/x]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**2,x)
```

```
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.14 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^4} dx$

Optimal. Leaf size=85

$$\frac{e^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{3/2}}-\frac{\sqrt{e}\sqrt{d+ex^2}}{6dx^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^3}$$

```
[Out] -(Sqrt[e]*Sqrt[d + e*x^2])/(6*d*x^2) - ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]
/(3*x^3) + (e^(3/2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(6*d^(3/2))
```

Rubi [A] time = 0.0447361, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6221, 266, 51, 63, 208}

$$\frac{e^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{3/2}}-\frac{\sqrt{e}\sqrt{d+ex^2}}{6dx^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^4,x]
```

```
[Out] -(Sqrt[e]*Sqrt[d + e*x^2])/(6*d*x^2) - ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]
/(3*x^3) + (e^(3/2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(6*d^(3/2))
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx &= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^3}+\frac{1}{3}\sqrt{e}\int\frac{1}{x^3\sqrt{d+ex^2}}\,dx\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^3}+\frac{1}{6}\sqrt{e}\,\mathrm{Subst}\left(\int\frac{1}{x^2\sqrt{d+ex}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{e}\sqrt{d+ex^2}}{6dx^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^3}-\frac{e^{3/2}\,\mathrm{Subst}\left(\int\frac{1}{x\sqrt{d+ex}}\,dx,x,x^2\right)}{12d}\\
&= -\frac{\sqrt{e}\sqrt{d+ex^2}}{6dx^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^3}-\frac{\sqrt{e}\,\mathrm{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex^2}\right)}{6d}\\
&= -\frac{\sqrt{e}\sqrt{d+ex^2}}{6dx^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^3}+\frac{e^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0881199, size = 92, normalized size = 1.08

$$
-\frac{\frac{\sqrt{e}x\left(\sqrt{d}\sqrt{d+ex^2}-ex^2\log\left(\sqrt{d}\sqrt{d+ex^2}+d\right)+ex^2\log(x)\right)}{d^{3/2}}+2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{6x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^4,x]
```

```
[Out] -(2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] + (Sqrt[e]*x*(Sqrt[d]*Sqrt[d + e*x
^2] + e*x^2*Log[x] - e*x^2*Log[d + Sqrt[d]*Sqrt[d + e*x^2]]))/d^(3/2))/(6*x
^3)
```

Maple [A] time = 0.032, size = 90, normalized size = 1.1

$$-\frac{1}{3x^3}\mathrm{Artanh}\left(x\sqrt{e}\frac{1}{\sqrt{ex^2+d}}\right)+\frac{1}{6}e^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2d+2\sqrt{d}\sqrt{ex^2+d}\right)\right)d^{-\frac{3}{2}}-\frac{1}{6d^2x^2}\sqrt{e}\left(ex^2+d\right)^{\frac{3}{2}}+\frac{1}{6d^2}e^{\frac{3}{2}}\sqrt{ex^2+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^4,x)
```

```
[Out] -1/3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^3+1/6*e^(3/2)/d^(3/2)*ln((2*d+2*d
^(1/2)*(e*x^2+d)^(1/2))/x)-1/6*e^(1/2)/d^2/x^2*(e*x^2+d)^(3/2)+1/6*e^(3/2)/
d^2*(e*x^2+d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$d\sqrt{e}\int-\frac{\sqrt{ex^2+d}}{3\left(e^2x^7+dex^5-\left(ex^5+dx^3\right)\left(ex^2+d\right)\right)}\,dx-\frac{\log\left(\sqrt{e}x+\sqrt{ex^2+d}\right)-\log\left(-\sqrt{e}x+\sqrt{ex^2+d}\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^4,x, algorithm="maxima")
```

```
[Out] d*sqrt(e)*integrate(-1/3*sqrt(e*x^2 + d)/(e^2*x^7 + d*e*x^5 - (e*x^5 + d*x^
3)*(e*x^2 + d)), x) - 1/6*(log(sqrt(e)*x + sqrt(e*x^2 + d)) - log(-sqrt(e)*
x + sqrt(e*x^2 + d)))/x^3
```

Fricas [B] time = 2.54526, size = 783, normalized size = 9.21

$$\left[\frac{ex^3\sqrt{\frac{e}{d}}\log\left(-\frac{e^2x^2+2\sqrt{ex^2+d}d\sqrt{e}\sqrt{\frac{e}{d}}+2de}{x^2}\right)-2dx^3\log\left(\frac{ex+\sqrt{ex^2+d}\sqrt{e}}{x}\right)+2dx^3\log\left(\frac{ex-\sqrt{ex^2+d}\sqrt{e}}{x}\right)-2\sqrt{ex^2+d}\sqrt{e}x+2\left(dx^3-d\right)}{12dx^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^4,x, algorithm="fricas")
```

```
[Out] [1/12*(e*x^3*sqrt(e/d)*log(-(e^2*x^2 + 2*sqrt(e*x^2 + d)*d*sqrt(e)*sqrt(e/d
) + 2*d*e)/x^2) - 2*d*x^3*log((e*x + sqrt(e*x^2 + d)*sqrt(e))/x) + 2*d*x^3*
log((e*x - sqrt(e*x^2 + d)*sqrt(e))/x) - 2*sqrt(e*x^2 + d)*sqrt(e)*x + 2*(d
*x^3 - d)*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d))/(d*x^3), -1/6
*(e*x^3*sqrt(-e/d)*arctan(sqrt(e*x^2 + d)*d*sqrt(e)*sqrt(-e/d)/(e^2*x^2 + d
*e)) + d*x^3*log((e*x + sqrt(e*x^2 + d)*sqrt(e))/x) - d*x^3*log((e*x - sqrt
(e*x^2 + d)*sqrt(e))/x) + sqrt(e*x^2 + d)*sqrt(e)*x - (d*x^3 - d)*log((2*e*
x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d))/(d*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**4,x)
```

```
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.15 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^6} dx$

Optimal. Leaf size=111

$$\frac{3e^{3/2}\sqrt{d+ex^2}}{40d^2x^2} - \frac{3e^{5/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{40d^{5/2}} - \frac{\sqrt{e}\sqrt{d+ex^2}}{20dx^4} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^5}$$

```
[Out] -(Sqrt[e]*Sqrt[d + e*x^2])/(20*d*x^4) + (3*e^(3/2)*Sqrt[d + e*x^2])/(40*d^2
*x^2) - ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/(5*x^5) - (3*e^(5/2)*ArcTanh[S
qrt[d + e*x^2]/Sqrt[d]])/(40*d^(5/2))
```

Rubi [A] time = 0.0570099, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6221, 266, 51, 63, 208}

$$\frac{3e^{3/2}\sqrt{d+ex^2}}{40d^2x^2} - \frac{3e^{5/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{40d^{5/2}} - \frac{\sqrt{e}\sqrt{d+ex^2}}{20dx^4} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^6,x]
```

```
[Out] -(Sqrt[e]*Sqrt[d + e*x^2])/(20*d*x^4) + (3*e^(3/2)*Sqrt[d + e*x^2])/(40*d^2
*x^2) - ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/(5*x^5) - (3*e^(5/2)*ArcTanh[S
qrt[d + e*x^2]/Sqrt[d]])/(40*d^(5/2))
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx &= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^5}+\frac{1}{5}\sqrt{e}\int \frac{1}{x^5\sqrt{d+ex^2}}\,dx\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^5}+\frac{1}{10}\sqrt{e}\,\mathrm{Subst}\left(\int \frac{1}{x^3\sqrt{d+ex}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{e}\sqrt{d+ex^2}}{20dx^4}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^5}-\frac{\left(3e^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{x^2\sqrt{d+ex}}\,dx,x,x^2\right)}{40d}\\
&= -\frac{\sqrt{e}\sqrt{d+ex^2}}{20dx^4}+\frac{3e^{3/2}\sqrt{d+ex^2}}{40d^2x^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^5}+\frac{\left(3e^{5/2}\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx,x,x^2\right)}{80d^2}\\
&= -\frac{\sqrt{e}\sqrt{d+ex^2}}{20dx^4}+\frac{3e^{3/2}\sqrt{d+ex^2}}{40d^2x^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^5}+\frac{\left(3e^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex^2}\right)}{40d^2}\\
&= -\frac{\sqrt{e}\sqrt{d+ex^2}}{20dx^4}+\frac{3e^{3/2}\sqrt{d+ex^2}}{40d^2x^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^5}-\frac{3e^{5/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{40d^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.109866, size = 107, normalized size = 0.96

$$\frac{\frac{\sqrt{e}x\left(-3e^2x^4\log\left(\sqrt{d}\sqrt{d+ex^2}+d\right)+\sqrt{d}\sqrt{d+ex^2}\left(3ex^2-2d\right)+3e^2x^4\log(x)\right)}{d^{5/2}}-8\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{40x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^6,x]
```

```
[Out] (-8*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] + (Sqrt[e]*x*(Sqrt[d]*Sqrt[d + e*x
^2]*(-2*d + 3*e*x^2) + 3*e^2*x^4*Log[x] - 3*e^2*x^4*Log[d + Sqrt[d]*Sqrt[d
+ e*x^2]]))/d^(5/2))/(40*x^5)
```

Maple [A] time = 0.033, size = 130, normalized size = 1.2

$$-\frac{1}{5x^5}\mathrm{Artanh}\left(x\sqrt{e}\frac{1}{\sqrt{ex^2+d}}\right)+\frac{1}{10d^2x^2}e^{\frac{3}{2}}\sqrt{ex^2+d}-\frac{3}{40}e^{\frac{5}{2}}\ln\left(\frac{1}{x}\left(2d+2\sqrt{d}\sqrt{ex^2+d}\right)\right)d^{-\frac{5}{2}}-\frac{1}{20d^2x^4}\sqrt{e}\left(ex^2+d\right)^{\frac{3}{2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^6,x)
```

```
[Out] -1/5*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^5+1/10*e^(3/2)*(e*x^2+d)^(1/2)/d^
2/x^2-3/40*e^(5/2)/d^(5/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x)-1/20*e^(1/
2)/d^2/x^4*(e*x^2+d)^(3/2)+1/40*e^(3/2)/d^3/x^2*(e*x^2+d)^(3/2)-1/40*e^(5/2
)/d^3*(e*x^2+d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$d\sqrt{e}\int-\frac{\sqrt{ex^2+d}}{5\left(e^2x^9+dex^7-\left(ex^7+dx^5\right)\left(ex^2+d\right)\right)}\,dx-\frac{\log\left(\sqrt{e}x+\sqrt{ex^2+d}\right)-\log\left(-\sqrt{e}x+\sqrt{ex^2+d}\right)}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^6,x, algorithm="maxima")
```

```
[Out] d*sqrt(e)*integrate(-1/5*sqrt(e*x^2 + d)/(e^2*x^9 + d*e*x^7 - (e*x^7 + d*x^
5)*(e*x^2 + d)), x) - 1/10*(log(sqrt(e)*x + sqrt(e*x^2 + d)) - log(-sqrt(e)
```

```
*x + sqrt(e*x^2 + d)))/x^5
```

Fricas [B] time = 2.63659, size = 872, normalized size = 7.86

$$\left[\frac{3\,e^2x^5\sqrt{\frac{e}{d}}\log\left(-\frac{e^2x^2-2\,\sqrt{ex^2+d}d\sqrt{e}\sqrt{\frac{e}{d}}+2\,de}{x^2}\right)-8\,d^2x^5\log\left(\frac{ex+\sqrt{ex^2+d}\sqrt{e}}{x}\right)+8\,d^2x^5\log\left(\frac{ex-\sqrt{ex^2+d}\sqrt{e}}{x}\right)+2\left(3\,ex^3-2\,dx\right)\sqrt{ex^2+}}{80\,d^2x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^6,x, algorithm="fricas")
```

```
[Out] [1/80*(3*e^2*x^5*sqrt(e/d)*log(-(e^2*x^2 - 2*sqrt(e*x^2 + d)*d*sqrt(e)*sqrt
(e/d) + 2*d*e)/x^2) - 8*d^2*x^5*log((e*x + sqrt(e*x^2 + d)*sqrt(e))/x) + 8*
d^2*x^5*log((e*x - sqrt(e*x^2 + d)*sqrt(e))/x) + 2*(3*e*x^3 - 2*d*x)*sqrt(e
*x^2 + d)*sqrt(e) + 8*(d^2*x^5 - d^2)*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt
(e)*x + d)/d))/(d^2*x^5), 1/40*(3*e^2*x^5*sqrt(-e/d)*arctan(sqrt(e*x^2 + d)
*d*sqrt(e)*sqrt(-e/d)/(e^2*x^2 + d*e)) - 4*d^2*x^5*log((e*x + sqrt(e*x^2 +
d)*sqrt(e))/x) + 4*d^2*x^5*log((e*x - sqrt(e*x^2 + d)*sqrt(e))/x) + (3*e*x^
3 - 2*d*x)*sqrt(e*x^2 + d)*sqrt(e) + 4*(d^2*x^5 - d^2)*log((2*e*x^2 + 2*sqr
t(e*x^2 + d)*sqrt(e)*x + d)/d))/(d^2*x^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**6,x)
```

```
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**6, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^6,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.16 $\int x^{9/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)dx$

Optimal. Leaf size=196

$$\frac{30d^{11/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{847e^{11/4}\sqrt{d+ex^2}}-\frac{60d^2\sqrt{x}\sqrt{d+ex^2}}{847e^{5/2}}+\frac{36dx^{5/2}\sqrt{d+ex^2}}{847e^{3/2}}-\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{e}}$$

```
[Out] (-60*d^2*Sqrt[x]*Sqrt[d + e*x^2])/(847*e^(5/2)) + (36*d*x^(5/2)*Sqrt[d + e*
x^2])/(847*e^(3/2)) - (4*x^(9/2)*Sqrt[d + e*x^2])/(121*Sqrt[e]) + (2*x^(11/
2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/11 + (30*d^(11/4)*(Sqrt[d] + Sqrt[
e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)
*Sqrt[x])/d^(1/4)], 1/2])/(847*e^(11/4)*Sqrt[d + e*x^2])
```

Rubi [A] time = 0.115093, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6221, 321, 329, 220}

$$-\frac{60d^2\sqrt{x}\sqrt{d+ex^2}}{847e^{5/2}}+\frac{30d^{11/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{847e^{11/4}\sqrt{d+ex^2}}+\frac{36dx^{5/2}\sqrt{d+ex^2}}{847e^{3/2}}-\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{e}}+\frac{2}{11}x$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (-60*d^2*Sqrt[x]*Sqrt[d + e*x^2])/(847*e^(5/2)) + (36*d*x^(5/2)*Sqrt[d + e*
x^2])/(847*e^(3/2)) - (4*x^(9/2)*Sqrt[d + e*x^2])/(121*Sqrt[e]) + (2*x^(11/
2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/11 + (30*d^(11/4)*(Sqrt[d] + Sqrt[
e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)
*Sqrt[x])/d^(1/4)], 1/2])/(847*e^(11/4)*Sqrt[d + e*x^2])
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int x^{9/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)dx=\frac{2}{11}x^{11/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{11}\left(2\sqrt{e}\right)\int\frac{x^{11/2}}{\sqrt{d+ex^2}}dx$$

$$=-\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{e}}+\frac{2}{11}x^{11/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)+\frac{(18d)\int\frac{x^{7/2}}{\sqrt{d+ex^2}}dx}{121\sqrt{e}}$$

$$=\frac{36dx^{5/2}\sqrt{d+ex^2}}{847e^{3/2}}-\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{e}}+\frac{2}{11}x^{11/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\left(90d^2\right)\int\frac{x^{3/2}}{\sqrt{d+ex^2}}dx}{847e^{3/2}}$$

$$=-\frac{60d^2\sqrt{x}\sqrt{d+ex^2}}{847e^{5/2}}+\frac{36dx^{5/2}\sqrt{d+ex^2}}{847e^{3/2}}-\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{e}}+\frac{2}{11}x^{11/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)+$$

$$=-\frac{60d^2\sqrt{x}\sqrt{d+ex^2}}{847e^{5/2}}+\frac{36dx^{5/2}\sqrt{d+ex^2}}{847e^{3/2}}-\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{e}}+\frac{2}{11}x^{11/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)+$$

$$=-\frac{60d^2\sqrt{x}\sqrt{d+ex^2}}{847e^{5/2}}+\frac{36dx^{5/2}\sqrt{d+ex^2}}{847e^{3/2}}-\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{e}}+\frac{2}{11}x^{11/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)+$$

Mathematica [C] time = 0.541893, size = 161, normalized size = 0.82

$$\frac{2}{847}\sqrt{x}\left(77x^5\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2\sqrt{d+ex^2}\left(15d^2-9dex^2+7e^2x^4\right)}{e^{5/2}}\right)+\frac{60d^{5/2}x\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{\frac{d}{ex^2}+1}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\right.\right.}{847e^2\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (2*Sqrt[x]*((-2*Sqrt[d + e*x^2]*(15*d^2 - 9*d*e*x^2 + 7*e^2*x^4))/e^(5/2) +
 77*x^5*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]))/847 + (60*d^(5/2)*Sqrt[(I*Sq
rt[d])/Sqrt[e]]*Sqrt[1 + d/(e*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/
Sqrt[e]]/Sqrt[x]], -1])/(847*e^2*Sqrt[d + e*x^2])
```

Maple [F] time = 0.898, size = 0, normalized size = 0.

$$\int x^{\frac{9}{2}}\operatorname{Artanh}\left(x\sqrt{e}\frac{1}{\sqrt{ex^2+d}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] int(x^(9/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{11}x^{\frac{11}{2}}\log\left(\sqrt{e}x+\sqrt{ex^2+d}\right)-\frac{1}{11}x^{\frac{11}{2}}\log\left(-\sqrt{e}x+\sqrt{ex^2+d}\right)-2d\sqrt{e}\int-\frac{xe^{\left(\frac{1}{2}\log\left(ex^2+d\right)+\frac{9}{2}\log(x)\right)}}{11\left(e^2x^4+dex^2-\left(ex^2+d\right)^2\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/11*x^(11/2)*log(sqrt(e)*x + sqrt(e*x^2 + d)) - 1/11*x^(11/2)*log(-sqrt(e)
*x + sqrt(e*x^2 + d)) - 2*d*sqrt(e)*integrate(-1/11*x*e^(1/2*log(e*x^2 + d)
```

```
+ 9/2*log(x))/(e^2*x^4 + d*e*x^2 - (e*x^2 + d)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^{\frac{9}{2}}\,\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] integral(x^(9/2)*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\left[\text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, 4\,de^{\frac{1}{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, undef, undef, undef, undef, 4*d*e^(1/2)
]
```

3.17 $\int x^{5/2} \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=168

$$-\frac{10d^{7/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{147e^{7/4}\sqrt{d+ex^2}}+\frac{20d\sqrt{x}\sqrt{d+ex^2}}{147e^{3/2}}-\frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{e}}+\frac{2}{7}x^{7/2}\tanh^{-1}\left(\frac{}{\sqrt{}}\right.$$

```
[Out] (20*d*Sqrt[x]*Sqrt[d + e*x^2])/(147*e^(3/2)) - (4*x^(5/2)*Sqrt[d + e*x^2])/
(49*Sqrt[e]) + (2*x^(7/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/7 - (10*d^(
7/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*Ellipt
icF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(147*e^(7/4)*Sqrt[d + e*x^2]
)
```

Rubi [A] time = 0.0863851, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6221, 321, 329, 220}

$$-\frac{10d^{7/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{147e^{7/4}\sqrt{d+ex^2}}+\frac{20d\sqrt{x}\sqrt{d+ex^2}}{147e^{3/2}}-\frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{e}}+\frac{2}{7}x^{7/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (20*d*Sqrt[x]*Sqrt[d + e*x^2])/(147*e^(3/2)) - (4*x^(5/2)*Sqrt[d + e*x^2])/
(49*Sqrt[e]) + (2*x^(7/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/7 - (10*d^(
7/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*Ellipt
icF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(147*e^(7/4)*Sqrt[d + e*x^2]
)
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int x^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)dx = \frac{2}{7}x^{7/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{7}\left(2\sqrt{e}\right)\int\frac{x^{7/2}}{\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{e}} + \frac{2}{7}x^{7/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) + \frac{(10d)\int\frac{x^{3/2}}{\sqrt{d+ex^2}}\,dx}{49\sqrt{e}}$$

$$= \frac{20d\sqrt{x}\sqrt{d+ex^2}}{147e^{3/2}} - \frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{e}} + \frac{2}{7}x^{7/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{\left(10d^2\right)\int\frac{1}{\sqrt{x}\sqrt{d+ex^2}}\,dx}{147e^{3/2}}$$

$$= \frac{20d\sqrt{x}\sqrt{d+ex^2}}{147e^{3/2}} - \frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{e}} + \frac{2}{7}x^{7/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{\left(20d^2\right)\text{Subst}\left(\int\frac{1}{\sqrt{d+ex^{\cdots}}}\cdots\right.}{147e^{3/2}}$$

$$= \frac{20d\sqrt{x}\sqrt{d+ex^2}}{147e^{3/2}} - \frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{e}} + \frac{2}{7}x^{7/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{10d^{7/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\cdots}}{147e^{\cdots}}$$

Mathematica [C] time = 0.372306, size = 147, normalized size = 0.88

$$\frac{2}{147}\sqrt{x}\left(\frac{2\left(5d-3ex^2\right)\sqrt{d+ex^2}}{e^{3/2}}+21x^3\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\right)+\frac{20\sqrt{d}x\left(\frac{i\sqrt{d}}{\sqrt{e}}\right)^{5/2}\sqrt{\frac{d}{ex^2}+1}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right),-1\right)}{147\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (2*Sqrt[x]*((2*(5*d - 3*e*x^2)*Sqrt[d + e*x^2])/e^(3/2) + 21*x^3*ArcTanh[(S
qrt[e]*x)/Sqrt[d + e*x^2]]))/147 + (20*Sqrt[d]*((I*Sqrt[d])/Sqrt[e])^(5/2)*
Sqrt[1 + d/(e*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]
], -1])/(147*Sqrt[d + e*x^2])
```

Maple [F] time = 0.865, size = 0, normalized size = 0.

$$\int x^{\frac{5}{2}}\operatorname{Artanh}\left(x\sqrt{e}\frac{1}{\sqrt{ex^2+d}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] int(x^(5/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{7}x^{\frac{7}{2}}\log\left(\sqrt{e}x+\sqrt{ex^2+d}\right)-\frac{1}{7}x^{\frac{7}{2}}\log\left(-\sqrt{e}x+\sqrt{ex^2+d}\right)-2\,d\sqrt{e}\int-\frac{xe^{\left(\frac{1}{2}\log\left(ex^2+d\right)+\frac{5}{2}\log(x)\right)}}{7\left(e^2x^4+dex^2-\left(ex^2+d\right)^2\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/7*x^(7/2)*log(sqrt(e)*x + sqrt(e*x^2 + d)) - 1/7*x^(7/2)*log(-sqrt(e)*x +
 sqrt(e*x^2 + d)) - 2*d*sqrt(e)*integrate(-1/7*x*e^(1/2*log(e*x^2 + d) + 5/
```

```
2*log(x))/(e^2*x^4 + d*e*x^2 - (e*x^2 + d)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^{\frac{5}{2}}\,\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")

[Out] integral(x^(5/2)*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\left[\text{undef},\text{undef},\text{undef},\text{undef},\text{undef},4\,de^{\frac{1}{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")

[Out] [undef, undef, undef, undef, undef, 4*d*e^(1/2)]
```

3.18 $\int \sqrt{x}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)dx$

Optimal. Leaf size=142

$$\frac{2d^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{9e^{3/4}\sqrt{d+ex^2}}-\frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{e}}+\frac{2}{3}x^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

```
[Out] (-4*Sqrt[x]*Sqrt[d + e*x^2])/(9*Sqrt[e]) + (2*x^(3/2)*ArcTanh[(Sqrt[e]*x)/S
qrt[d + e*x^2]])/3 + (2*d^(3/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqr
t[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(
9*e^(3/4)*Sqrt[d + e*x^2])
```

Rubi [A] time = 0.0705534, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6221, 321, 329, 220}

$$\frac{2d^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{9e^{3/4}\sqrt{d+ex^2}}-\frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{e}}+\frac{2}{3}x^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (-4*Sqrt[x]*Sqrt[d + e*x^2])/(9*Sqrt[e]) + (2*x^(3/2)*ArcTanh[(Sqrt[e]*x)/S
qrt[d + e*x^2]])/3 + (2*d^(3/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqr
t[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(
9*e^(3/4)*Sqrt[d + e*x^2])
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \sqrt{x}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)dx = \frac{2}{3}x^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{3}\left(2\sqrt{e}\right)\int\frac{x^{3/2}}{\sqrt{d+ex^2}}dx$$

$$= -\frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{e}} + \frac{2}{3}x^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) + \frac{(2d)\int\frac{1}{\sqrt{x}\sqrt{d+ex^2}}dx}{9\sqrt{e}}$$

$$= -\frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{e}} + \frac{2}{3}x^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) + \frac{(4d)\,\mathrm{Subst}\left(\int\frac{1}{\sqrt{d+ex^4}}dx, x, \sqrt{x}\right)}{9\sqrt{e}}$$

$$= -\frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{e}} + \frac{2}{3}x^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) + \frac{2d^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\right.\right.}{9e^{3/4}\sqrt{d+ex^2}}$$

Mathematica [C] time = 0.245893, size = 135, normalized size = 0.95

$$\frac{2}{9}\sqrt{x}\left(3x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{2\sqrt{d+ex^2}}{\sqrt{e}}\right) + \frac{4\sqrt{d}x\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{\frac{d}{ex^2}+1}\,\mathrm{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right), -1\right)}{9\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (2*Sqrt[x]*((-2*Sqrt[d + e*x^2])/Sqrt[e] + 3*x*ArcTanh[(Sqrt[e]*x)/Sqrt[d +
 e*x^2]]))/9 + (4*Sqrt[d]*Sqrt[(I*Sqrt[d])/Sqrt[e]]*Sqrt[1 + d/(e*x^2)]*x*E
llipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])/(9*Sqrt[d + e*x
^2])
```

Maple [F] time = 1.036, size = 0, normalized size = 0.

$$\int \sqrt{x}\operatorname{Artanh}\left(x\sqrt{e}\frac{1}{\sqrt{ex^2+d}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] int(x^(1/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-2\,d\sqrt{e}\int -\frac{xe^{\left(\frac{1}{2}\,\log\left(ex^2+d\right)+\frac{1}{2}\,\log(x)\right)}}{3\left(e^2x^4+dex^2-\left(ex^2+d\right)^2\right)}\,dx+\frac{1}{3}\,x^{\frac{3}{2}}\log\left(\sqrt{e}x+\sqrt{ex^2+d}\right)-\frac{1}{3}\,x^{\frac{3}{2}}\log\left(-\sqrt{e}x+\sqrt{ex^2+d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] -2*d*sqrt(e)*integrate(-1/3*x*e^(1/2*log(e*x^2 + d) + 1/2*log(x))/(e^2*x^4
+ d*e*x^2 - (e*x^2 + d)^2), x) + 1/3*x^(3/2)*log(sqrt(e)*x + sqrt(e*x^2 + d
)) - 1/3*x^(3/2)*log(-sqrt(e)*x + sqrt(e*x^2 + d))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{x}\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x)*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x}\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Integral(sqrt(x)*atanh(sqrt(e)*x/sqrt(d + e*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\left[\text{undef}, \text{undef}, \text{undef}, \text{undef}, 4\,de^{\frac{1}{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, 4*d*e^(1/2)]
```

3.19 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx$

Optimal. Leaf size=113

$$\frac{2\sqrt[4]{e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{d+ex^2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}$$

```
[Out] (-2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[x] + (2*e^(1/4)*(Sqrt[d] + S
qrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(
1/4)*Sqrt[x])/d^(1/4)], 1/2])/(d^(1/4)*Sqrt[d + e*x^2])
```

Rubi [A] time = 0.0577945, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {6221, 329, 220}

$$\frac{2\sqrt[4]{e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{d+ex^2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(3/2),x]
```


```
[Out] (-2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[x] + (2*e^(1/4)*(Sqrt[d] + S
qrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(
1/4)*Sqrt[x])/d^(1/4)], 1/2])/(d^(1/4)*Sqrt[d + e*x^2])
```


Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \left(2\sqrt{e}\right)\int \frac{1}{\sqrt{x}\sqrt{d+ex^2}}\,dx\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \left(4\sqrt{e}\right)\text{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \frac{2\sqrt[4]{e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{d+ex^2}}\end{aligned}$$

Mathematica [C] time = 0.118777, size = 111, normalized size = 0.98

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \frac{4i\sqrt{e}x\sqrt{\frac{d}{ex^2}+1}\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right), -1\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(3/2),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[x] + ((4*I)*Sqrt[e]*Sqrt[1 +
 d/(e*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])/
(Sqrt[(I*Sqrt[d])/Sqrt[e]]*Sqrt[d + e*x^2])
```

Maple [F] time = 0.848, size = 0, normalized size = 0.

$$\int \operatorname{Artanh}\left(x\sqrt{e}\frac{1}{\sqrt{ex^2+d}}\right)x^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x)
```

```
[Out] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2\,d\sqrt{e}\int -\frac{\sqrt{ex^2+d}x}{\left(e^2x^4+dex^2\right)x^{\frac{3}{2}}-\left(ex^2+d\right)e^{\left(\log\left(ex^2+d\right)+\frac{3}{2}\,\log(x)\right)}}\,dx-\frac{\log\left(\sqrt{e}x+\sqrt{ex^2+d}\right)}{\sqrt{x}}+\frac{\log\left(-\sqrt{e}x+\sqrt{ex^2+d}\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2*d*sqrt(e)*integrate(-sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e*x^2)*x^(3/2) - (e*
x^2 + d)*e^(log(e*x^2 + d) + 3/2*log(x))), x) - log(sqrt(e)*x + sqrt(e*x^2
+ d))/sqrt(x) + log(-sqrt(e)*x + sqrt(e*x^2 + d))/sqrt(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x, algorithm="fricas")
```

```
[Out] integral(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**(3/2),x)
```

```
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(3/2), x)
```

3.20 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}} \, dx$

Optimal. Leaf size=145

$$-\frac{2e^{5/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{15d^{5/4}\sqrt{d+ex^2}}-\frac{4\sqrt{e}\sqrt{d+ex^2}}{15dx^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}$$

```
[Out] (-4*Sqrt[e]*Sqrt[d + e*x^2])/(15*d*x^(3/2)) - (2*ArcTanh[(Sqrt[e]*x)/Sqrt[d
 + e*x^2]])/(5*x^(5/2)) - (2*e^(5/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)
/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/
2])/(15*d^(5/4)*Sqrt[d + e*x^2])
```

Rubi [A] time = 0.0717091, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6221, 325, 329, 220}

$$-\frac{2e^{5/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{15d^{5/4}\sqrt{d+ex^2}}-\frac{4\sqrt{e}\sqrt{d+ex^2}}{15dx^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(7/2),x]
```


```
[Out] (-4*Sqrt[e]*Sqrt[d + e*x^2])/(15*d*x^(3/2)) - (2*ArcTanh[(Sqrt[e]*x)/Sqrt[d
 + e*x^2]])/(5*x^(5/2)) - (2*e^(5/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)
/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/
2])/(15*d^(5/4)*Sqrt[d + e*x^2])
```


Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}+\frac{1}{5}\left(2\sqrt{e}\right)\int\frac{1}{x^{5/2}\sqrt{d+ex^2}}\,dx\\
&= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{15dx^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}-\frac{\left(2e^{3/2}\right)\int\frac{1}{\sqrt{x}\sqrt{d+ex^2}}\,dx}{15d}\\
&= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{15dx^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}-\frac{\left(4e^{3/2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{d+ex^4}}\,dx,x,\sqrt{x}\right)}{15d}\\
&= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{15dx^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}-\frac{2e^{5/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{15d^{5/4}\sqrt{d+ex^2}}
\end{aligned}
$$

Mathematica [C] time = 0.194816, size = 142, normalized size = 0.98

$$
-\frac{2\left(2\sqrt{e}x\sqrt{d+ex^2}+3d\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\right)}{15dx^{5/2}}-\frac{4e^2x\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{\frac{d}{ex^2}+1}\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right),-1\right)}{15d^{3/2}\sqrt{d+ex^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(7/2),x]
```

```
[Out] (-2*(2*Sqrt[e]*x*Sqrt[d + e*x^2] + 3*d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]
))/(15*d*x^(5/2)) - (4*Sqrt[(I*Sqrt[d])/Sqrt[e]]*e^2*Sqrt[1 + d/(e*x^2)]*x*
EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])/(15*d^(3/2)*Sq
rt[d + e*x^2])
```

Maple [F] time = 0.842, size = 0, normalized size = 0.

$$\int \mathrm{Artanh}\left(x\sqrt{e}\frac{1}{\sqrt{ex^2+d}}\right)x^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x)
```

```
[Out] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2\,d\sqrt{e}\int -\frac{\sqrt{ex^2+d}x}{5\left(\left(e^2x^4+dex^2\right)x^{\frac{7}{2}}-\left(ex^2+d\right)e^{\left(\log\left(ex^2+d\right)+\frac{7}{2}\log(x)\right)}\right)}\,dx-\frac{\log\left(\sqrt{e}x+\sqrt{ex^2+d}\right)}{5\,x^{\frac{5}{2}}}+\frac{\log\left(-\sqrt{e}x+\sqrt{ex^2+d}\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2*d*sqrt(e)*integrate(-1/5*sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e*x^2)*x^(7/2) -
 (e*x^2 + d)*e^(log(e*x^2 + d) + 7/2*log(x))), x) - 1/5*log(sqrt(e)*x + sqr
t(e*x^2 + d))/x^(5/2) + 1/5*log(-sqrt(e)*x + sqrt(e*x^2 + d))/x^(5/2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x, algorithm="fricas")
```

```
[Out] integral(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(7/2), x)
```

3.21 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx$

Optimal. Leaf size=173

$$\frac{10e^{9/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{189d^{9/4}\sqrt{d+ex^2}}+\frac{20e^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}}-\frac{4\sqrt{e}\sqrt{d+ex^2}}{63dx^{7/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}}$$

```
[Out] (-4*Sqrt[e]*Sqrt[d + e*x^2])/(63*d*x^(7/2)) + (20*e^(3/2)*Sqrt[d + e*x^2])/
(189*d^2*x^(3/2)) - (2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(9*x^(9/2)) +
(10*e^(9/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]
*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(189*d^(9/4)*Sqrt[d +
 e*x^2])
```

Rubi [A] time = 0.087056, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6221, 325, 329, 220}

$$\frac{20e^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}}+\frac{10e^{9/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{189d^{9/4}\sqrt{d+ex^2}}-\frac{4\sqrt{e}\sqrt{d+ex^2}}{63dx^{7/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(11/2),x]
```

```
[Out] (-4*Sqrt[e]*Sqrt[d + e*x^2])/(63*d*x^(7/2)) + (20*e^(3/2)*Sqrt[d + e*x^2])/
(189*d^2*x^(3/2)) - (2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(9*x^(9/2)) +
(10*e^(9/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]
*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(189*d^(9/4)*Sqrt[d +
 e*x^2])
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx = -\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}}+\frac{1}{9}\left(2\sqrt{e}\right)\int \frac{1}{x^{9/2}\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{63dx^{7/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}}-\frac{\left(10e^{3/2}\right)\int \frac{1}{x^{5/2}\sqrt{d+ex^2}}\,dx}{63d}$$

$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{63dx^{7/2}}+\frac{20e^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}}+\frac{\left(10e^{5/2}\right)\int \frac{1}{\sqrt{x}\sqrt{d+ex^2}}\,dx}{189d^2}$$

$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{63dx^{7/2}}+\frac{20e^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}}+\frac{\left(20e^{5/2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}\,dx,x,\sqrt{x}\right)}{189d^2}$$

$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{63dx^{7/2}}+\frac{20e^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}}+\frac{10e^{9/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\,\mathrm{ta}\right.}{189d^{9/4}\sqrt{d+ex^2}}$$

Mathematica [C] time = 0.287303, size = 154, normalized size = 0.89

$$\frac{4\sqrt{e}x\sqrt{d+ex^2}\left(5ex^2-3d\right)-42d^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{189d^2x^{9/2}}+\frac{20e^3x\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{\frac{d}{ex^2}+1}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right),-1\right)}{189d^{5/2}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(11/2),x]
```

```
[Out] (4*Sqrt[e]*x*Sqrt[d + e*x^2]*(-3*d + 5*e*x^2) - 42*d^2*ArcTanh[(Sqrt[e]*x)/
Sqrt[d + e*x^2]])/(189*d^2*x^(9/2)) + (20*Sqrt[(I*Sqrt[d])/Sqrt[e]]*e^3*Sqr
t[1 + d/(e*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]],
-1])/(189*d^(5/2)*Sqrt[d + e*x^2])
```

Maple [F] time = 0.851, size = 0, normalized size = 0.

$$\int \operatorname{Artanh}\left(x\sqrt{e}\frac{1}{\sqrt{ex^2+d}}\right)x^{-\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x)
```

```
[Out] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2\,d\sqrt{e}\int -\frac{\sqrt{ex^2+d}x}{9\left(\left(e^2x^4+dex^2\right)x^{\frac{11}{2}}-\left(ex^2+d\right)e^{\left(\log\left(ex^2+d\right)+\frac{11}{2}\,\log(x)\right)}\right)}\,dx-\frac{\log\left(\sqrt{e}x+\sqrt{ex^2+d}\right)}{9\,x^{\frac{9}{2}}}+\frac{\log\left(-\sqrt{e}x+\sqrt{ex^2+d}\right)}{9\,x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x, algorithm="maxima"
)
```

```
[Out] 2*d*sqrt(e)*integrate(-1/9*sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e*x^2)*x^(11/2)
- (e*x^2 + d)*e^(log(e*x^2 + d) + 11/2*log(x))), x) - 1/9*log(sqrt(e)*x + s
qrt(e*x^2 + d))/x^(9/2) + 1/9*log(-sqrt(e)*x + sqrt(e*x^2 + d))/x^(9/2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{11}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x, algorithm="fricas"
)
```

```
[Out] integral(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{11}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(11/2), x)
```

3.22 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx$

Optimal. Leaf size=201

$$-\frac{30e^{13/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{1001d^{13/4}\sqrt{d+ex^2}}-\frac{60e^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}}+\frac{36e^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}}-\frac{4\sqrt{e}\sqrt{d+ex^2}}{143dx^{11/2}}-$$

```
[Out] (-4*Sqrt[e]*Sqrt[d + e*x^2])/(143*d*x^(11/2)) + (36*e^(3/2)*Sqrt[d + e*x^2]
)/(1001*d^2*x^(7/2)) - (60*e^(5/2)*Sqrt[d + e*x^2])/(1001*d^3*x^(3/2)) - (2
*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(13*x^(13/2)) - (30*e^(13/4)*(Sqrt[d
] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan
[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(1001*d^(13/4)*Sqrt[d + e*x^2])
```

Rubi [A] time = 0.102399, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6221, 325, 329, 220}

$$-\frac{60e^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}}+\frac{36e^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}}-\frac{30e^{13/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{1001d^{13/4}\sqrt{d+ex^2}}-\frac{4\sqrt{e}\sqrt{d+ex^2}}{143dx^{11/2}}-\frac{2\tanh^{-1}}{13x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(15/2),x]
```


```
[Out] (-4*Sqrt[e]*Sqrt[d + e*x^2])/(143*d*x^(11/2)) + (36*e^(3/2)*Sqrt[d + e*x^2]
)/(1001*d^2*x^(7/2)) - (60*e^(5/2)*Sqrt[d + e*x^2])/(1001*d^3*x^(3/2)) - (2
*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(13*x^(13/2)) - (30*e^(13/4)*(Sqrt[d
] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan
[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(1001*d^(13/4)*Sqrt[d + e*x^2])
```


Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx = -\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} + \frac{1}{13}\left(2\sqrt{e}\right)\int \frac{1}{x^{13/2}\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{143dx^{11/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} - \frac{\left(18e^{3/2}\right)\int \frac{1}{x^{9/2}\sqrt{d+ex^2}}\,dx}{143d}$$

$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{143dx^{11/2}} + \frac{36e^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} + \frac{\left(90e^{5/2}\right)\int \frac{1}{x^{5/2}\sqrt{d+ex^2}}\,dx}{1001d^2}$$

$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{143dx^{11/2}} + \frac{36e^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}} - \frac{60e^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} - \frac{\left(30e^{7/2}\right)\int \frac{1}{\sqrt{x}\sqrt{d+ex^2}}}{1001d^3}$$

$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{143dx^{11/2}} + \frac{36e^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}} - \frac{60e^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} - \frac{\left(60e^{7/2}\right)\mathrm{Subst}\left(\int \right.}{1001}$$

$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{143dx^{11/2}} + \frac{36e^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}} - \frac{60e^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} - \frac{30e^{13/4}\left(\sqrt{d}+\sqrt{e}x\right)}{10}$$

Mathematica [C] time = 0.379867, size = 163, normalized size = 0.81

$$\frac{2\left(-\frac{30e^4x^{15/2}\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{\frac{d}{ex^2}+1}\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right),-1\right)}{d^{7/2}\sqrt{d+ex^2}}-\frac{2\sqrt{e}x\sqrt{d+ex^2}\left(7d^2-9dex^2+15e^2x^4\right)}{d^3}-77\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\right)}{1001x^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(15/2),x]
```

```
[Out] (2*((-2*Sqrt[e]*x*Sqrt[d + e*x^2]*(7*d^2 - 9*d*e*x^2 + 15*e^2*x^4))/d^3 - 7
7*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] - (30*Sqrt[(I*Sqrt[d])/Sqrt[e]]*e^4*
Sqrt[1 + d/(e*x^2)]*x^(15/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/
Sqrt[x]], -1])/(d^(7/2)*Sqrt[d + e*x^2])))/(1001*x^(13/2))
```

Maple [F] time = 0.835, size = 0, normalized size = 0.

$$\int \text{Artanh}\left(x\sqrt{e}\frac{1}{\sqrt{ex^2+d}}\right)x^{-\frac{15}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x)
```

```
[Out] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2\,d\sqrt{e}\int -\frac{\sqrt{ex^2+d}x}{13\left(\left(e^2x^4+dex^2\right)x^{\frac{15}{2}}-\left(ex^2+d\right)e^{\left(\log\left(ex^2+d\right)+\frac{15}{2}\log(x)\right)}\right)}\,dx-\frac{\log\left(\sqrt{e}x+\sqrt{ex^2+d}\right)}{13\,x^{\frac{13}{2}}}+\frac{\log\left(-\sqrt{e}x+\sqrt{ex^2+d}\right)}{13\,x^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x, algorithm="maxima"
)
```

```
[Out] 2*d*sqrt(e)*integrate(-1/13*sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e*x^2)*x^(15/2)
 - (e*x^2 + d)*e^(log(e*x^2 + d) + 15/2*log(x))), x) - 1/13*log(sqrt(e)*x +
 sqrt(e*x^2 + d))/x^(13/2) + 1/13*log(-sqrt(e)*x + sqrt(e*x^2 + d))/x^(13/2
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{15}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x, algorithm="fricas"
)
```

```
[Out] integral(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(15/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**(15/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{15}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(15/2), x)
```

3.23 $\int x^{7/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)dx$

Optimal. Leaf size=297

$$-\frac{14d^{9/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{135e^{9/4}\sqrt{d+ex^2}}-\frac{28d^2\sqrt{x}\sqrt{d+ex^2}}{135e^2\left(\sqrt{d}+\sqrt{e}x\right)}+\frac{28d^{9/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(\right.}{135e^{9/4}\sqrt{d+ex^2}}$$

```
[Out] (28*d*x^(3/2)*Sqrt[d + e*x^2])/(405*e^(3/2)) - (4*x^(7/2)*Sqrt[d + e*x^2])/
(81*Sqrt[e]) - (28*d^2*Sqrt[x]*Sqrt[d + e*x^2])/(135*e^2*(Sqrt[d] + Sqrt[e]
*x)) + (2*x^(9/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/9 + (28*d^(9/4)*(Sq
rt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE[2*Ar
cTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(135*e^(9/4)*Sqrt[d + e*x^2]) - (14*
d^(9/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*Ell
ipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(135*e^(9/4)*Sqrt[d + e*x
^2])
```

Rubi [A] time = 0.187218, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {6221, 321, 329, 305, 220, 1196}

$$-\frac{28d^2\sqrt{x}\sqrt{d+ex^2}}{135e^2\left(\sqrt{d}+\sqrt{e}x\right)}-\frac{14d^{9/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{135e^{9/4}\sqrt{d+ex^2}}+\frac{28d^{9/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\tan^{-1}\left(\right.\right.}{135e^{9/4}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (28*d*x^(3/2)*Sqrt[d + e*x^2])/(405*e^(3/2)) - (4*x^(7/2)*Sqrt[d + e*x^2])/
(81*Sqrt[e]) - (28*d^2*Sqrt[x]*Sqrt[d + e*x^2])/(135*e^2*(Sqrt[d] + Sqrt[e]
*x)) + (2*x^(9/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/9 + (28*d^(9/4)*(Sq
rt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE[2*Ar
cTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(135*e^(9/4)*Sqrt[d + e*x^2]) - (14*
d^(9/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*Ell
ipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(135*e^(9/4)*Sqrt[d + e*x
^2])
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int x^{7/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)dx = \frac{2}{9}x^{9/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{9}\left(2\sqrt{e}\right)\int \frac{x^{9/2}}{\sqrt{d+ex^2}}dx$$

$$= -\frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{e}} + \frac{2}{9}x^{9/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) + \frac{(14d)\int \frac{x^{5/2}}{\sqrt{d+ex^2}}dx}{81\sqrt{e}}$$

$$= \frac{28dx^{3/2}\sqrt{d+ex^2}}{405e^{3/2}} - \frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{e}} + \frac{2}{9}x^{9/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{\left(14d^2\right)\int \frac{\sqrt{x}}{\sqrt{d+ex^2}}dx}{135e^{3/2}}$$

$$= \frac{28dx^{3/2}\sqrt{d+ex^2}}{405e^{3/2}} - \frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{e}} + \frac{2}{9}x^{9/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{\left(28d^2\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{d+ex}}\right.}{135e^{3/2}}$$

$$= \frac{28dx^{3/2}\sqrt{d+ex^2}}{405e^{3/2}} - \frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{e}} + \frac{2}{9}x^{9/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{\left(28d^{5/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{d+}}\right.}{135e^2}$$

$$= \frac{28dx^{3/2}\sqrt{d+ex^2}}{405e^{3/2}} - \frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{e}} - \frac{28d^2\sqrt{x}\sqrt{d+ex^2}}{135e^2\left(\sqrt{d}+\sqrt{e}x\right)} + \frac{2}{9}x^{9/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) + \cdots$$

Mathematica [C] time = 0.105563, size = 124, normalized size = 0.42

$$\frac{2x^{3/2}\left(-14d^2\sqrt{\frac{ex^2}{d}+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{ex^2}{d}\right)+14d^2+45e^{3/2}x^3\sqrt{d+ex^2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)+4dex^2-10e^2x^4\right.}{405e^{3/2}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (2*x^(3/2)*(14*d^2 + 4*d*e*x^2 - 10*e^2*x^4 + 45*e^(3/2)*x^3*Sqrt[d + e*x^2
]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] - 14*d^2*Sqrt[1 + (e*x^2)/d]*Hyperge
ometric2F1[1/2, 3/4, 7/4, -((e*x^2)/d)]))/(405*e^(3/2)*Sqrt[d + e*x^2])
```

Maple [F] time = 0.936, size = 0, normalized size = 0.

$$\int x^{\frac{7}{2}}\text{Artanh}\left(x\sqrt{e}\frac{1}{\sqrt{ex^2+d}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] int(x^(7/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{9}x^{\frac{9}{2}}\log\left(\sqrt{e}x+\sqrt{ex^2+d}\right)-\frac{1}{9}x^{\frac{9}{2}}\log\left(-\sqrt{e}x+\sqrt{ex^2+d}\right)-2d\sqrt{e}\int-\frac{xe^{\left(\frac{1}{2}\log\left(ex^2+d\right)+\frac{7}{2}\log(x)\right)}}{9\left(e^2x^4+dex^2-\left(ex^2+d\right)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/9*x^(9/2)*log(sqrt(e)*x + sqrt(e*x^2 + d)) - 1/9*x^(9/2)*log(-sqrt(e)*x +
 sqrt(e*x^2 + d)) - 2*d*sqrt(e)*integrate(-1/9*x*e^(1/2*log(e*x^2 + d) + 7/
2*log(x))/(e^2*x^4 + d*e*x^2 - (e*x^2 + d)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^{\frac{7}{2}}\,\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] integral(x^(7/2)*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\left[\text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, \text{undef}, 4\, de^{\frac{1}{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, undef, undef, 4*d*e^(1/2)]
```

3.24 $\int x^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)dx$

Optimal. Leaf size=269

$$\frac{6d^{5/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{25e^{5/4}\sqrt{d+ex^2}}-\frac{12d^{5/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{25e^{5/4}\sqrt{d+ex^2}}-$$

```
[Out] (-4*x^(3/2)*Sqrt[d + e*x^2])/(25*Sqrt[e]) + (12*d*Sqrt[x]*Sqrt[d + e*x^2])/
(25*e*(Sqrt[d] + Sqrt[e]*x)) + (2*x^(5/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^
2]])/5 - (12*d^(5/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt
[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(25*e^(5/4)*
Sqrt[d + e*x^2]) + (6*d^(5/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[
d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(25
*e^(5/4)*Sqrt[d + e*x^2])
```

Rubi [A] time = 0.157495, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {6221, 321, 329, 305, 220, 1196}

$$\frac{6d^{5/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{25e^{5/4}\sqrt{d+ex^2}}-\frac{12d^{5/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{25e^{5/4}\sqrt{d+ex^2}}-\frac{4x^{3/2}\sqrt{d-}}{25\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (-4*x^(3/2)*Sqrt[d + e*x^2])/(25*Sqrt[e]) + (12*d*Sqrt[x]*Sqrt[d + e*x^2])/
(25*e*(Sqrt[d] + Sqrt[e]*x)) + (2*x^(5/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^
2]])/5 - (12*d^(5/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt
[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(25*e^(5/4)*
Sqrt[d + e*x^2]) + (6*d^(5/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[
d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(25
*e^(5/4)*Sqrt[d + e*x^2])
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
```

```
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int x^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)dx = \frac{2}{5}x^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{5}\left(2\sqrt{e}\right)\int\frac{x^{5/2}}{\sqrt{d+ex^2}}dx$$

$$= -\frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{e}} + \frac{2}{5}x^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) + \frac{(6d)\int\frac{\sqrt{x}}{\sqrt{d+ex^2}}dx}{25\sqrt{e}}$$

$$= -\frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{e}} + \frac{2}{5}x^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) + \frac{(12d)\,\text{Subst}\left(\int\frac{x^2}{\sqrt{d+ex^4}}dx, x, \sqrt{x}\right)}{25\sqrt{e}}$$

$$= -\frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{e}} + \frac{2}{5}x^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) + \frac{\left(12d^{3/2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{d+ex^4}}dx, x, \sqrt{x}\right)}{25e} - \cdots$$

$$= -\frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{e}} + \frac{12d\sqrt{x}\sqrt{d+ex^2}}{25e\left(\sqrt{d}+\sqrt{e}x\right)} + \frac{2}{5}x^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{12d^{5/4}\left(\sqrt{d}+\sqrt{e}x\right)\cdots}{2\cdots}$$

Mathematica [C] time = 0.0904489, size = 109, normalized size = 0.41

$$\frac{2x^{3/2}\left(2d\sqrt{\frac{ex^2}{d}+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{ex^2}{d}\right) - 2\left(d+ex^2\right) + 5\sqrt{e}x\sqrt{d+ex^2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\right)}{25\sqrt{e}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (2*x^(3/2)*(-2*(d + e*x^2) + 5*Sqrt[e]*x*Sqrt[d + e*x^2]*ArcTanh[(Sqrt[e]*x
)/Sqrt[d + e*x^2]] + 2*d*Sqrt[1 + (e*x^2)/d]*Hypergeometric2F1[1/2, 3/4, 7/
4, -((e*x^2)/d)]))/(25*Sqrt[e]*Sqrt[d + e*x^2])
```

Maple [F] time = 0.848, size = 0, normalized size = 0.

$$\int x^{\frac{3}{2}}\text{Artanh}\left(x\sqrt{e}\frac{1}{\sqrt{ex^2+d}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] int(x^(3/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{5}\,x^{\frac{5}{2}}\log\left(\sqrt{e}x+\sqrt{ex^2+d}\right)-\frac{1}{5}\,x^{\frac{5}{2}}\log\left(-\sqrt{e}x+\sqrt{ex^2+d}\right)-2\,d\sqrt{e}\int-\frac{xe^{\left(\frac{1}{2}\,\log\left(ex^2+d\right)+\frac{3}{2}\,\log(x)\right)}}{5\left(e^2x^4+dex^2-\left(ex^2+d\right)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/5*x^(5/2)*log(sqrt(e)*x + sqrt(e*x^2 + d)) - 1/5*x^(5/2)*log(-sqrt(e)*x +
 sqrt(e*x^2 + d)) - 2*d*sqrt(e)*integrate(-1/5*x*e^(1/2*log(e*x^2 + d) + 3/
2*log(x))/(e^2*x^4 + d*e*x^2 - (e*x^2 + d)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^{\frac{3}{2}}\,\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] integral(x^(3/2)*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{\frac{3}{2}}\,\text{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Integral(x**(3/2)*atanh(sqrt(e)*x/sqrt(d + e*x**2)), x)
```

Giac [A] time = 1.29049, size = 1, normalized size = 0.

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] +Infinity
```

3.25 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=232

$$-\frac{2\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{\sqrt[4]{e}\sqrt{d+ex^2}}-\frac{4\sqrt{x}\sqrt{d+ex^2}}{\sqrt{d}+\sqrt{e}x}+2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)+\frac{4\sqrt[4]{d}\left(\sqrt{d}+\right.}{}$$

```
[Out] (-4*Sqrt[x]*Sqrt[d + e*x^2])/(Sqrt[d] + Sqrt[e]*x) + 2*Sqrt[x]*ArcTanh[(Sqr
t[e]*x)/Sqrt[d + e*x^2]] + (4*d^(1/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2
)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1
/2])/(e^(1/4)*Sqrt[d + e*x^2]) - (2*d^(1/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d +
 e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/
4)], 1/2])/(e^(1/4)*Sqrt[d + e*x^2])
```

Rubi [A] time = 0.135636, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6221, 329, 305, 220, 1196}

$$-\frac{4\sqrt{x}\sqrt{d+ex^2}}{\sqrt{d}+\sqrt{e}x}+2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{\sqrt[4]{e}\sqrt{d+ex^2}}+\frac{4\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{}{(}}}{\sqrt[4]{}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/Sqrt[x],x]
```

```
[Out] (-4*Sqrt[x]*Sqrt[d + e*x^2])/(Sqrt[d] + Sqrt[e]*x) + 2*Sqrt[x]*ArcTanh[(Sqr
t[e]*x)/Sqrt[d + e*x^2]] + (4*d^(1/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2
)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1
/2])/(e^(1/4)*Sqrt[d + e*x^2]) - (2*d^(1/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d +
 e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/
4)], 1/2])/(e^(1/4)*Sqrt[d + e*x^2])
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
```

```
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right) - \left(2\sqrt{e}\right)\int \frac{\sqrt{x}}{\sqrt{d+ex^2}}\,dx$$
$$= 2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right) - \left(4\sqrt{e}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)$$
$$= 2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right) - \left(4\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right) + \left(4\sqrt{d}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{e}x^2}{\sqrt{d}}}{\sqrt{d+ex^4}}\right.$$
$$= -\frac{4\sqrt{x}\sqrt{d+ex^2}}{\sqrt{d}+\sqrt{ex}} + 2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right) + \frac{4\sqrt[4]{d}\left(\sqrt{d}+\sqrt{ex}\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{ex}\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right.}{\sqrt[4]{e}\sqrt{d+ex^2}}$$

Mathematica [C] time = 0.10397, size = 85, normalized size = 0.37

$$2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right) - \frac{4\sqrt{e}x^{3/2}\sqrt{\frac{ex^2}{d}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{ex^2}{d}\right)}{3\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] - (4*Sqrt[e]*x^(3/2)*Sqrt[1
+ (e*x^2)/d]*Hypergeometric2F1[1/2, 3/4, 7/4, -((e*x^2)/d)])/(3*Sqrt[d + e*
x^2])
```

Maple [F] time = 0.944, size = 0, normalized size = 0.

$$\int \text{Artanh}\left(x\sqrt{e}\frac{1}{\sqrt{ex^2+d}}\right)\frac{1}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x)
```

```
[Out] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-2\,d\sqrt{e}\int \frac{\sqrt{ex^2+d}\,x}{\left(ex^2+d\right)e^{\left(\log\left(ex^2+d\right)+\frac{1}{2}\log\left(x\right)\right)}-\left(e^2x^4+dex^2\right)\sqrt{x}}\,dx+\sqrt{x}\log\left(\sqrt{e}x+\sqrt{ex^2+d}\right)-\sqrt{x}\log\left(-\sqrt{e}x+\sqrt{ex^2+d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] -2*d*sqrt(e)*integrate(sqrt(e*x^2 + d)*x/((e*x^2 + d)*e^(log(e*x^2 + d) + 1
/2*log(x)) - (e^2*x^4 + d*e*x^2)*sqrt(x)), x) + sqrt(x)*log(sqrt(e)*x + sqr
t(e*x^2 + d)) - sqrt(x)*log(-sqrt(e)*x + sqrt(e*x^2 + d))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{\sqrt{x}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] integral(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/sqrt(x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**(1/2),x)
```

```
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/sqrt(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/sqrt(x), x)
```

3.26 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx$

Optimal. Leaf size=272

$$\frac{2e^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{3d^{3/4}\sqrt{d+ex^2}}-\frac{4e^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{3d^{3/4}\sqrt{d+ex^2}}+\frac{4e}{3d}$$

```
[Out] (-4*Sqrt[e]*Sqrt[d + e*x^2])/(3*d*Sqrt[x]) + (4*e*Sqrt[x]*Sqrt[d + e*x^2])/
(3*d*(Sqrt[d] + Sqrt[e]*x)) - (2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(3*x
^(3/2)) - (4*e^(3/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt
[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(3*d^(3/4)*S
qrt[d + e*x^2]) + (2*e^(3/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d
] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(3*d
^(3/4)*Sqrt[d + e*x^2])
```

Rubi [A] time = 0.161741, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {6221, 325, 329, 305, 220, 1196}

$$\frac{2e^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{3d^{3/4}\sqrt{d+ex^2}}-\frac{4e^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{3d^{3/4}\sqrt{d+ex^2}}+\frac{4e\sqrt{x}\sqrt{d+}}{3d\left(\sqrt{d}+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(5/2),x]
```

```
[Out] (-4*Sqrt[e]*Sqrt[d + e*x^2])/(3*d*Sqrt[x]) + (4*e*Sqrt[x]*Sqrt[d + e*x^2])/
(3*d*(Sqrt[d] + Sqrt[e]*x)) - (2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(3*x
^(3/2)) - (4*e^(3/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt
[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(3*d^(3/4)*S
qrt[d + e*x^2]) + (2*e^(3/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d
] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(3*d
^(3/4)*Sqrt[d + e*x^2])
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx = -\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} + \frac{1}{3}\left(2\sqrt{e}\right)\int \frac{1}{x^{3/2}\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{3d\sqrt{x}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} + \frac{\left(2e^{3/2}\right)\int \frac{\sqrt{x}}{\sqrt{d+ex^2}}\,dx}{3d}$$

$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{3d\sqrt{x}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} + \frac{\left(4e^{3/2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)}{3d}$$

$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{3d\sqrt{x}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} + \frac{(4e)\,\text{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)}{3\sqrt{d}} - \frac{(4e)\,\text{Subst}\left(\int \frac{1-\frac{\sqrt{e}x}{\sqrt{d}}}{\sqrt{d+ex}}\right.}{3\sqrt{d}}$$

$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{3d\sqrt{x}} + \frac{4e\sqrt{x}\sqrt{d+ex^2}}{3d\left(\sqrt{d}+\sqrt{e}x\right)} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} - \frac{4e^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\,\text{ta}\right.}{3d^{3/4}\sqrt{d+ex^2}}$$

Mathematica [C] time = 0.104923, size = 118, normalized size = 0.43

$$\frac{4e^{3/2}x^{3/2}\sqrt{\frac{ex^2}{d}+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{ex^2}{d}\right)}{9d\sqrt{d+ex^2}} - \frac{4\sqrt{e}\sqrt{d+ex^2}}{3d\sqrt{x}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(5/2),x]
```

```
[Out] (-4*Sqrt[e]*Sqrt[d + e*x^2])/(3*d*Sqrt[x]) - (2*ArcTanh[(Sqrt[e]*x)/Sqrt[d
+ e*x^2]])/(3*x^(3/2)) + (4*e^(3/2)*x^(3/2)*Sqrt[1 + (e*x^2)/d]*Hypergeomet
ric2F1[1/2, 3/4, 7/4, -((e*x^2)/d)])/(9*d*Sqrt[d + e*x^2])
```

Maple [F] time = 0.819, size = 0, normalized size = 0.

$$\int \text{Artanh}\left(x\sqrt{e}\frac{1}{\sqrt{ex^2+d}}\right)x^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x)
```

```
[Out] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2\,d\sqrt{e}\int -\frac{\sqrt{ex^2+d}x}{3\left(\left(e^2x^4+dex^2\right)x^{\frac{5}{2}}-\left(ex^2+d\right)e^{\left(\log\left(ex^2+d\right)+\frac{5}{2}\,\log(x)\right)}\right)}\,dx-\frac{\log\left(\sqrt{e}x+\sqrt{ex^2+d}\right)}{3\,x^{\frac{3}{2}}}+\frac{\log\left(-\sqrt{e}x+\sqrt{ex^2+d}\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2*d*sqrt(e)*integrate(-1/3*sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e*x^2)*x^(5/2) -
 (e*x^2 + d)*e^(log(e*x^2 + d) + 5/2*log(x))), x) - 1/3*log(sqrt(e)*x + sqr
t(e*x^2 + d))/x^(3/2) + 1/3*log(-sqrt(e)*x + sqrt(e*x^2 + d))/x^(3/2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x, algorithm="fricas")
```

```
[Out] integral(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(5/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**(5/2),x)
```

```
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(5/2), x)
```

3.27 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx$

Optimal. Leaf size=302

$$-\frac{6e^{7/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{35d^{7/4}\sqrt{d+ex^2}}-\frac{12e^2\sqrt{x}\sqrt{d+ex^2}}{35d^2\left(\sqrt{d}+\sqrt{e}x\right)}+\frac{12e^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}}+\frac{12e^{7/4}\left(\sqrt{d}+\sqrt{e}x\right)\cdots}{\cdots}$$

```
[Out] (-4*Sqrt[e]*Sqrt[d + e*x^2])/(35*d*x^(5/2)) + (12*e^(3/2)*Sqrt[d + e*x^2])/
(35*d^2*Sqrt[x]) - (12*e^2*Sqrt[x]*Sqrt[d + e*x^2])/(35*d^2*(Sqrt[d] + Sqrt
[e]*x)) - (2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(7*x^(7/2)) + (12*e^(7/4
)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE
[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(35*d^(7/4)*Sqrt[d + e*x^2]) -
(6*e^(7/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*
EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(35*d^(7/4)*Sqrt[d + e
*x^2])
```

Rubi [A] time = 0.181199, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {6221, 325, 329, 305, 220, 1196}

$$-\frac{12e^2\sqrt{x}\sqrt{d+ex^2}}{35d^2\left(\sqrt{d}+\sqrt{e}x\right)}+\frac{12e^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}}-\frac{6e^{7/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{35d^{7/4}\sqrt{d+ex^2}}+\frac{12e^{7/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+e\cdots}{\left(\sqrt{d}+\cdots\right.}}\cdots}{35d^{7/4}\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(9/2),x]
```


```
[Out] (-4*Sqrt[e]*Sqrt[d + e*x^2])/(35*d*x^(5/2)) + (12*e^(3/2)*Sqrt[d + e*x^2])/
(35*d^2*Sqrt[x]) - (12*e^2*Sqrt[x]*Sqrt[d + e*x^2])/(35*d^2*(Sqrt[d] + Sqrt
[e]*x)) - (2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(7*x^(7/2)) + (12*e^(7/4
)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE
[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(35*d^(7/4)*Sqrt[d + e*x^2]) -
(6*e^(7/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*
EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(35*d^(7/4)*Sqrt[d + e
*x^2])
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx = -\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} + \frac{1}{7}\left(2\sqrt{e}\right)\int \frac{1}{x^{7/2}\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{35dx^{5/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} - \frac{\left(6e^{3/2}\right)\int \frac{1}{x^{3/2}\sqrt{d+ex^2}}\,dx}{35d}$$

$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{35dx^{5/2}} + \frac{12e^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} - \frac{\left(6e^{5/2}\right)\int \frac{\sqrt{x}}{\sqrt{d+ex^2}}\,dx}{35d^2}$$

$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{35dx^{5/2}} + \frac{12e^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} - \frac{\left(12e^{5/2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)}{35d^2}$$

$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{35dx^{5/2}} + \frac{12e^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} - \frac{\left(12e^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)}{35d^{3/2}} + \cdots$$

$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{35dx^{5/2}} + \frac{12e^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}} - \frac{12e^2\sqrt{x}\sqrt{d+ex^2}}{35d^2\left(\sqrt{d}+\sqrt{e}x\right)} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} + \frac{12e^{7/4}\left(\sqrt{d}+\sqrt{e}x \cdots\right.}{\cdots}$$

Mathematica [C] time = 0.0995408, size = 131, normalized size = 0.43

$$\frac{-4e^{5/2}x^5\sqrt{\frac{ex^2}{d}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{ex^2}{d}\right) + 4\sqrt{e}x\left(-d^2+2dex^2+3e^2x^4\right) - 10d^2\sqrt{d+ex^2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{35d^2x^{7/2}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(9/2),x]
```

```
[Out] (4*Sqrt[e]*x*(-d^2 + 2*d*e*x^2 + 3*e^2*x^4) - 10*d^2*Sqrt[d + e*x^2]*ArcTan
h[(Sqrt[e]*x)/Sqrt[d + e*x^2]] - 4*e^(5/2)*x^5*Sqrt[1 + (e*x^2)/d]*Hypergeo
metric2F1[1/2, 3/4, 7/4, -((e*x^2)/d)])/(35*d^2*x^(7/2)*Sqrt[d + e*x^2])
```

Maple [F] time = 0.843, size = 0, normalized size = 0.

$$\int \text{Artanh}\left(x\sqrt{e}\frac{1}{\sqrt{ex^2+d}}\right)x^{-\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x)
```

```
[Out] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2\,d\sqrt{e}\int -\frac{\sqrt{ex^2+d}\,x}{7\left(\left(e^2x^4+dex^2\right)x^{\frac{9}{2}}-\left(ex^2+d\right)e^{\left(\log\left(ex^2+d\right)+\frac{9}{2}\,\log(x)\right)}\right)}\,dx-\frac{\log\left(\sqrt{e}x+\sqrt{ex^2+d}\right)}{7\,x^{\frac{7}{2}}}+\frac{\log\left(-\sqrt{e}x+\sqrt{ex^2+d}\right)}{7\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x, algorithm="maxima")
```

```
[Out] 2*d*sqrt(e)*integrate(-1/7*sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e*x^2)*x^(9/2) -
 (e*x^2 + d)*e^(log(e*x^2 + d) + 9/2*log(x))), x) - 1/7*log(sqrt(e)*x + sqr
t(e*x^2 + d))/x^(7/2) + 1/7*log(-sqrt(e)*x + sqrt(e*x^2 + d))/x^(7/2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x, algorithm="fricas")
```

```
[Out] integral(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(9/2), x)
```

3.28 $\int x^3 \tanh^{-1}\left(a+bx^4\right)\,dx$

Optimal. Leaf size=44

$$\frac{\log\left(1-\left(a+bx^4\right)^2\right)}{8b}+\frac{\left(a+bx^4\right)\tanh^{-1}\left(a+bx^4\right)}{4b}$$

```
[Out] ((a + b*x^4)*ArcTanh[a + b*x^4])/(4*b) + Log[1 - (a + b*x^4)^2]/(8*b)
```

Rubi [A] time = 0.0493132, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6715, 6103, 5910, 260}

$$\frac{\log\left(1-\left(a+bx^4\right)^2\right)}{8b}+\frac{\left(a+bx^4\right)\tanh^{-1}\left(a+bx^4\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[a + b*x^4],x]
```

```
[Out] ((a + b*x^4)*ArcTanh[a + b*x^4])/(4*b) + Log[1 - (a + b*x^4)^2]/(8*b)
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}
, x] && IGtQ[p, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int x^3 \tanh^{-1}\left(a+bx^4\right) dx &= \frac{1}{4}\operatorname{Subst}\left(\int \tanh^{-1}(a+bx)\,dx, x, x^4\right)\\ &= \frac{\operatorname{Subst}\left(\int \tanh^{-1}(x)\,dx, x, a+bx^4\right)}{4b}\\ &= \frac{\left(a+bx^4\right)\tanh^{-1}\left(a+bx^4\right)}{4b} - \frac{\operatorname{Subst}\left(\int \frac{x}{1-x^2}\,dx, x, a+bx^4\right)}{4b}\\ &= \frac{\left(a+bx^4\right)\tanh^{-1}\left(a+bx^4\right)}{4b} + \frac{\log\left(1-\left(a+bx^4\right)^2\right)}{8b}\end{aligned}$$

Mathematica [A] time = 0.0173853, size = 39, normalized size = 0.89

$$\frac{\log\left(1-\left(a+bx^4\right)^2\right)+2\left(a+bx^4\right)\tanh^{-1}\left(a+bx^4\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[a + b*x^4],x]
```

```
[Out] (2*(a + b*x^4)*ArcTanh[a + b*x^4] + Log[1 - (a + b*x^4)^2])/(8*b)
```

Maple [A] time = 0.027, size = 48, normalized size = 1.1

$$\frac{\operatorname{Artanh}\left(bx^4+a\right)x^4}{4}+\frac{\operatorname{Artanh}\left(bx^4+a\right)a}{4\,b}+\frac{\ln\left(1-\left(bx^4+a\right)^2\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(b*x^4+a),x)
```

```
[Out] 1/4*arctanh(b*x^4+a)*x^4+1/4/b*arctanh(b*x^4+a)*a+1/8*ln(1-(b*x^4+a)^2)/b
```

Maxima [A] time = 0.950051, size = 50, normalized size = 1.14

$$\frac{2\left(bx^4+a\right)\operatorname{artanh}\left(bx^4+a\right)+\log\left(-\left(bx^4+a\right)^2+1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*(2*(b*x^4 + a)*arctanh(b*x^4 + a) + log(-(b*x^4 + a)^2 + 1))/b
```

Fricas [A] time = 1.94777, size = 150, normalized size = 3.41

$$\frac{bx^4\log\left(-\frac{bx^4+a+1}{bx^4+a-1}\right)+(a+1)\log\left(bx^4+a+1\right)-(a-1)\log\left(bx^4+a-1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/8*(b*x^4*log(-(b*x^4 + a + 1)/(b*x^4 + a - 1)) + (a + 1)*log(b*x^4 + a +
1) - (a - 1)*log(b*x^4 + a - 1))/b
```

Sympy [A] time = 7.84333, size = 60, normalized size = 1.36

$$\begin{cases}\frac{a\operatorname{atanh}\left(a+bx^4\right)}{4b}+\frac{x^4\operatorname{atanh}\left(a+bx^4\right)}{4}+\frac{\log\left(a+bx^4+1\right)}{4b}-\frac{\operatorname{atanh}\left(a+bx^4\right)}{4b} & \text{for } b\neq 0\\ \frac{x^4\operatorname{atanh}(a)}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(b*x**4+a),x)
```

```
[Out] Piecewise((a*atanh(a + b*x**4)/(4*b) + x**4*atanh(a + b*x**4)/4 + log(a + b
*x**4 + 1)/(4*b) - atanh(a + b*x**4)/(4*b), Ne(b, 0)), (x**4*atanh(a)/4, Tr
ue))
```

Giac [A] time = 1.12077, size = 89, normalized size = 2.02

$$\frac{1}{8}\,x^4\log\left(-\frac{bx^4+a+1}{bx^4+a-1}\right)+\frac{1}{8}\,b\left(\frac{(a+1)\log\left(\left|bx^4+a+1\right|\right)}{b^2}-\frac{(a-1)\log\left(\left|bx^4+a-1\right|\right)}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*x^4*log(-(b*x^4 + a + 1)/(b*x^4 + a - 1)) + 1/8*b*((a + 1)*log(abs(b*x^
4 + a + 1))/b^2 - (a - 1)*log(abs(b*x^4 + a - 1))/b^2)
```

3.29 $\int x^{-1+n}\tanh^{-1}(a+bx^n)\,dx$

Optimal. Leaf size=47

$$\frac{\log\left(1-(a+bx^n)^2\right)}{2bn}+\frac{(a+bx^n)\tanh^{-1}(a+bx^n)}{bn}$$

```
[Out] ((a + b*x^n)*ArcTanh[a + b*x^n])/(b*n) + Log[1 - (a + b*x^n)^2]/(2*b*n)
```

Rubi [A] time = 0.0493811, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6715, 6103, 5910, 260}

$$\frac{\log\left(1-(a+bx^n)^2\right)}{2bn}+\frac{(a+bx^n)\tanh^{-1}(a+bx^n)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*ArcTanh[a + b*x^n],x]
```

```
[Out] ((a + b*x^n)*ArcTanh[a + b*x^n])/(b*n) + Log[1 - (a + b*x^n)^2]/(2*b*n)
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}
, x] && IGtQ[p, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int x^{-1+n}\tanh^{-1}(a+bx^n)\,dx &= \frac{\text{Subst}\left(\int \tanh^{-1}(a+bx)\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \tanh^{-1}(x)\,dx,x,a+bx^n\right)}{bn}\\ &= \frac{(a+bx^n)\tanh^{-1}(a+bx^n)}{bn}-\frac{\text{Subst}\left(\int \frac{x}{1-x^2}\,dx,x,a+bx^n\right)}{bn}\\ &= \frac{(a+bx^n)\tanh^{-1}(a+bx^n)}{bn}+\frac{\log\left(1-(a+bx^n)^2\right)}{2bn}\end{aligned}$$

Mathematica [A] time = 0.0368803, size = 42, normalized size = 0.89

$$\frac{\log\left(1-(a+bx^n)^2\right)+2(a+bx^n)\tanh^{-1}(a+bx^n)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*ArcTanh[a + b*x^n],x]
```

```
[Out] (2*(a + b*x^n)*ArcTanh[a + b*x^n] + Log[1 - (a + b*x^n)^2])/(2*b*n)
```

Maple [B] time = 0.06, size = 121, normalized size = 2.6

$$\frac{x^n\ln(1+a+bx^n)}{2n}-\frac{x^n\ln(1-a-bx^n)}{2n}+\frac{a}{2bn}\ln\left(x^n+\frac{1+a}{b}\right)-\frac{a}{2bn}\ln\left(x^n+\frac{a-1}{b}\right)+\frac{1}{2bn}\ln\left(x^n+\frac{1+a}{b}\right)+\frac{1}{2bn}\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*arctanh(a+b*x^n),x)
```

```
[Out] 1/2/n*x^n*ln(1+a+b*x^n)-1/2/n*x^n*ln(1-a-b*x^n)+1/2/b/n*ln(x^n+(1+a)/b)*a-1
/2/b/n*ln(x^n+(a-1)/b)*a+1/2/b/n*ln(x^n+(1+a)/b)+1/2/b/n*ln(x^n+(a-1)/b)
```

Maxima [A] time = 0.972472, size = 54, normalized size = 1.15

$$\frac{2\,(bx^n + a)\,\operatorname{artanh}\,(bx^n + a) + \log\left(-(bx^n + a)^2 + 1\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arctanh(a+b*x^n),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*x^n + a)*arctanh(b*x^n + a) + log(-(b*x^n + a)^2 + 1))/(b*n)
```

Fricas [B] time = 1.9057, size = 360, normalized size = 7.66

$$\frac{(a+1)\log\left(b\cosh\left(n\log\left(x\right)\right) + b\sinh\left(n\log\left(x\right)\right) + a + 1\right) - (a-1)\log\left(b\cosh\left(n\log\left(x\right)\right) + b\sinh\left(n\log\left(x\right)\right) + a - 1\right) +}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arctanh(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*((a + 1)*log(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a + 1) - (a - 1)*log
(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a - 1) + (b*cosh(n*log(x)) + b*sinh(
n*log(x)))*log(-(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a + 1)/(b*cosh(n*log
(x)) + b*sinh(n*log(x)) + a - 1)))/(b*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*atanh(a+b*x**n),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16071, size = 93, normalized size = 1.98

$$\frac{b\left(\frac{(a+1)\log(|bx^n+a+1|)}{b^2} - \frac{(a-1)\log(|bx^n+a-1|)}{b^2}\right) + x^n\log\left(-\frac{bx^n+a+1}{bx^n+a-1}\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arctanh(a+b*x^n),x, algorithm="giac")
```

```
[Out] 1/2*(b*((a + 1)*log(abs(b*x^n + a + 1))/b^2 - (a - 1)*log(abs(b*x^n + a - 1
))/b^2) + x^n*log(-(b*x^n + a + 1)/(b*x^n + a - 1)))/n
```

3.30 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

Optimal. Leaf size=42

$$\text{Unintegrable}\left(\frac{\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^n}{1-c^2x^2},x\right)$$

```
[Out] Unintegrable[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),
x]
```

Rubi [A] time = 0.0443373, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

```
[Out] Defer[Int][(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int \frac{\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Mathematica [A] time = 0.0907897, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

```
[Out] Integrate[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Maple [A] time = 0.803, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2+1}\left(a+b\mathrm{Artanh}\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

```
[Out] int((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(b\,\mathrm{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] -integrate((b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(b\,\mathrm{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="fricas")
```

```
[Out] integral(-(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a + b \operatorname{atanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^n}{c^2x^2 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
```

```
[Out] -Integral((a + b*atanh(sqrt(-c*x + 1)/sqrt(c*x + 1)))**n/(c**2*x**2 - 1), x
)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(b \operatorname{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)^n}{c^2x^2 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] integrate(-(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

3.31 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

Optimal. Leaf size=409

$$-\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}+\frac{3b^2\text{PolyLog}\left(3,\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}-1\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}+\frac{3b\text{PolyL}}{}$$

```
[Out] (-2*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*ArcTanh[1 - 2/(1 - Sqrt[
1 - c*x]/Sqrt[1 + c*x])])/c + (3*b*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])^2*PolyLog[2, 1 - 2/(1 - Sqrt[1 - c*x]/Sqrt[1 + c*x])])/(2*c) - (3*b*(a
 + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2, -1 + 2/(1 - Sqrt[1
- c*x]/Sqrt[1 + c*x])])/(2*c) - (3*b^2*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1
+ c*x]])*PolyLog[3, 1 - 2/(1 - Sqrt[1 - c*x]/Sqrt[1 + c*x])])/(2*c) + (3*b^
2*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, -1 + 2/(1 - Sqrt[
1 - c*x]/Sqrt[1 + c*x])])/(2*c) + (3*b^3*PolyLog[4, 1 - 2/(1 - Sqrt[1 - c*x
]/Sqrt[1 + c*x])])/(4*c) - (3*b^3*PolyLog[4, -1 + 2/(1 - Sqrt[1 - c*x]/Sqrt
[1 + c*x])])/(4*c)
```

Rubi [A] time = 0.506209, antiderivative size = 409, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {6681, 5914, 6052, 5948, 6058, 6062, 6610}

$$-\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}+\frac{3b^2\text{PolyLog}\left(3,\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}-1\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}+\frac{3b\text{PolyL}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] (-2*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*ArcTanh[1 - 2/(1 - Sqrt[
1 - c*x]/Sqrt[1 + c*x])])/c + (3*b*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])^2*PolyLog[2, 1 - 2/(1 - Sqrt[1 - c*x]/Sqrt[1 + c*x])])/(2*c) - (3*b*(a
 + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2, -1 + 2/(1 - Sqrt[1
- c*x]/Sqrt[1 + c*x])])/(2*c) - (3*b^2*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1
+ c*x]])*PolyLog[3, 1 - 2/(1 - Sqrt[1 - c*x]/Sqrt[1 + c*x])])/(2*c) + (3*b^
2*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, -1 + 2/(1 - Sqrt[
1 - c*x]/Sqrt[1 + c*x])])/(2*c) + (3*b^3*PolyLog[4, 1 - 2/(1 - Sqrt[1 - c*x
```

]/Sqrt[1 + c*x])])/(4*c) - (3*b^3*PolyLog[4, -1 + 2/(1 - Sqrt[1 - c*x]/Sqrt[1 + c*x])])/(4*c)

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f - d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a + b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d + e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
```

```
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a+b\tanh^{-1}(x))^3}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(6b)\,\text{Subst}\left(\int \frac{\tanh^{-1}\left(1-\frac{2}{1-x}\right)(a+b\tanh}{1-x^2}\right.}{c}$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(3b)\,\text{Subst}\left(\int \frac{(a+b\tanh^{-1}(x))^2\log\left(2-\right.}{1-x^2}\right.}{c}$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{3b\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\text{Li}_2\left(1-\right.}{2c}$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{3b\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\text{Li}_2\left(1-\right.}{2c}$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{3b\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\text{Li}_2\left(1-\right.}{2c}$$

Mathematica [A] time = 0.174075, size = 482, normalized size = 1.18

$$-\frac{-6b^2\text{PolyLog}\left(3,-\frac{\sqrt{1-cx}+\sqrt{cx+1}}{\sqrt{1-cx}-\sqrt{cx+1}}\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)+6b^2\text{PolyLog}\left(3,\frac{\sqrt{1-cx}+\sqrt{cx+1}}{\sqrt{1-cx}-\sqrt{cx+1}}\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)+6b\text{P}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] -(8*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*ArcTanh[1 - 2/(1 - Sqrt[
1 - c*x]/Sqrt[1 + c*x])] + 6*b*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])
^2*PolyLog[2, -((Sqrt[1 - c*x] + Sqrt[1 + c*x])/(Sqrt[1 - c*x] - Sqrt[1 + c
*x]))] - 6*b*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2, (Sqr
t[1 - c*x] + Sqrt[1 + c*x])/(Sqrt[1 - c*x] - Sqrt[1 + c*x])] - 6*b^2*(a + b
*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, -((Sqrt[1 - c*x] + Sqrt[1
 + c*x])/(Sqrt[1 - c*x] - Sqrt[1 + c*x]))] + 6*b^2*(a + b*ArcTanh[Sqrt[1 -
c*x]/Sqrt[1 + c*x]])*PolyLog[3, (Sqrt[1 - c*x] + Sqrt[1 + c*x])/(Sqrt[1 - c
*x] - Sqrt[1 + c*x])] + 3*b^3*PolyLog[4, -((Sqrt[1 - c*x] + Sqrt[1 + c*x])/
(Sqrt[1 - c*x] - Sqrt[1 + c*x]))] - 3*b^3*PolyLog[4, (Sqrt[1 - c*x] + Sqrt[
1 + c*x])/(Sqrt[1 - c*x] - Sqrt[1 + c*x])])/(4*c)
```

Maple [B] time = 1.372, size = 1449, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x)
```

```
[Out] -6*b^3/c*polylog(4,((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(
1/2))-6*b^3/c*polylog(4,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1
)+1)^(1/2))+3/4*b^3/c*polylog(4,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2/(-(-c*x
+1)/(c*x+1)+1))-1/2*a^3/c*ln(c*x-1)+1/2*a^3/c*ln(c*x+1)-3*a^2*b/c*dilog(1/(
(-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2*(-(-c*x+1)/(c*x+1)+1))-3*b^3/c*arctanh((-
c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(
-(-c*x+1)/(c*x+1)+1)^(1/2))+6*a*b^2/c*polylog(3,-((-c*x+1)^(1/2)/(c*x+1)^(1
/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))-3/2*a*b^2/c*polylog(3,-((-c*x+1)^(1/2)/
(c*x+1)^(1/2)+1)^2/(-(-c*x+1)/(c*x+1)+1))+6*a*b^2/c*polylog(3,((-c*x+1)^(1/
2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))+6*b^3/c*arctanh((-c*x+1)^(
1/2)/(c*x+1)^(1/2))*polylog(3,((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(
c*x+1)+1)^(1/2))-b^3/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1+((-c*x+
1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))+3/4*a^2*b/c*dilog(1/
((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^4*(-(-c*x+1)/(c*x+1)+1)^2)-b^3/c*arctanh((
-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*
x+1)/(c*x+1)+1)^(1/2))-3*b^3/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*poly
log(2,((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))-3/2*b^3
/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,-((-c*x+1)^(1/2)/(c*x+1)
^(1/2)+1)^2/(-(-c*x+1)/(c*x+1)+1))+3/2*b^3/c*arctanh((-c*x+1)^(1/2)/(c*x+1)
^(1/2))^2*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2/(-(-c*x+1)/(c*x+1)+
```

```
1))+6*b^3/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,-((-c*x+1)^(1/2
)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))+b^3/c*arctanh((-c*x+1)^(1/2
)/(c*x+1)^(1/2))^3*ln(((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2/(-(-c*x+1)/(c*x+1)
+1)+1)+3*a*b^2/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(((-c*x+1)^(1/2)
/(c*x+1)^(1/2)+1)^2/(-(-c*x+1)/(c*x+1)+1)+1)+3*a*b^2/c*arctanh((-c*x+1)^(1/
2)/(c*x+1)^(1/2))*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2/(-(-c*x+1)/
(c*x+1)+1))-3*a*b^2/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1-((-c*x+1
)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))-6*a*b^2/c*arctanh((-c
*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c
*x+1)/(c*x+1)+1)^(1/2))-3*a*b^2/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*l
n(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))-6*a*b^2/c
*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(
1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a^3\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)-\frac{\left(b^3\log(cx+1)-b^3\log(-cx+1)\right)\log\left(\sqrt{cx+1}-\sqrt{-cx+1}\right)^3}{16c}-\int\frac{4\left(\sqrt{cx+1}b^3-\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] 1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) - 1/16*(b^3*log(c*x + 1) - b^3*lo
g(-c*x + 1))*log(sqrt(c*x + 1) - sqrt(-c*x + 1))^3/c - integrate(1/32*(4*(s
qrt(c*x + 1)*b^3 - sqrt(-c*x + 1)*b^3)*log(sqrt(c*x + 1) + sqrt(-c*x + 1))^
3 + 24*(sqrt(c*x + 1)*a*b^2 - sqrt(-c*x + 1)*a*b^2)*log(sqrt(c*x + 1) + sqr
t(-c*x + 1))^2 + 3*(4*(sqrt(c*x + 1)*b^3 - sqrt(-c*x + 1)*b^3)*log(sqrt(c*x
 + 1) + sqrt(-c*x + 1)) + (8*a*b^2 - (b^3*c*x - b^3)*log(c*x + 1) + (b^3*c*
x - b^3)*log(-c*x + 1))*sqrt(c*x + 1) - (8*a*b^2 - (b^3*c*x + b^3)*log(c*x
+ 1) + (b^3*c*x + b^3)*log(-c*x + 1))*sqrt(-c*x + 1))*log(sqrt(c*x + 1) - s
qrt(-c*x + 1))^2 + 48*(sqrt(c*x + 1)*a^2*b - sqrt(-c*x + 1)*a^2*b)*log(sqrt
(c*x + 1) + sqrt(-c*x + 1)) - 12*(4*sqrt(c*x + 1)*a^2*b - 4*sqrt(-c*x + 1)*
a^2*b + (sqrt(c*x + 1)*b^3 - sqrt(-c*x + 1)*b^3)*log(sqrt(c*x + 1) + sqrt(-
c*x + 1))^2 + 4*(sqrt(c*x + 1)*a*b^2 - sqrt(-c*x + 1)*a*b^2)*log(sqrt(c*x +
 1) + sqrt(-c*x + 1)))*log(sqrt(c*x + 1) - sqrt(-c*x + 1)))/((c^2*x^2 - 1)*
sqrt(c*x + 1) - (c^2*x^2 - 1)*sqrt(-c*x + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{b^{3}\,\mathrm{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^{3}+3\,ab^{2}\,\mathrm{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^{2}+3\,a^{2}b\,\mathrm{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a^{3}}{c^{2}x^{2}-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="fricas")
```

```
[Out] integral(-(b^3*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b^2*arctanh(sq
rt(-c*x + 1)/sqrt(c*x + 1))^2 + 3*a^2*b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1
)) + a^3)/(c^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{a^{3}}{c^{2}x^{2}-1}\,dx-\int\frac{b^{3}\,\mathrm{atanh}^{3}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^{2}x^{2}-1}\,dx-\int\frac{3ab^{2}\,\mathrm{atanh}^{2}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^{2}x^{2}-1}\,dx-\int\frac{3a^{2}b\,\mathrm{atanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a**3/(c**2*x**2 - 1), x) - Integral(b**3*atanh(sqrt(-c*x + 1)/sqr
t(c*x + 1))**3/(c**2*x**2 - 1), x) - Integral(3*a*b**2*atanh(sqrt(-c*x + 1)
/sqrt(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(3*a**2*b*atanh(sqrt(-c*x
+ 1)/sqrt(c*x + 1))/(c**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{\left(b\,\mathrm{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^{3}}{c^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] integrate(-(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^3/(c^2*x^2 - 1), x
)
```

3.32 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

Optimal. Leaf size=268

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}-\frac{b\text{PolyLog}\left(2,\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}-1\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}-\frac{b^2\text{PolyLog}\left(\right.}{}$$

```
[Out] (-2*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*ArcTanh[1 - 2/(1 - Sqrt[
1 - c*x]/Sqrt[1 + c*x])])/c + (b*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]
])*PolyLog[2, 1 - 2/(1 - Sqrt[1 - c*x]/Sqrt[1 + c*x])])/c - (b*(a + b*ArcTa
nh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, -1 + 2/(1 - Sqrt[1 - c*x]/Sqrt[
1 + c*x])])/c - (b^2*PolyLog[3, 1 - 2/(1 - Sqrt[1 - c*x]/Sqrt[1 + c*x])])/(
2*c) + (b^2*PolyLog[3, -1 + 2/(1 - Sqrt[1 - c*x]/Sqrt[1 + c*x])])/(2*c)
```

Rubi [A] time = 0.31119, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6681, 5914, 6052, 5948, 6058, 6610}

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}-\frac{b\text{PolyLog}\left(2,\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}-1\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}-\frac{b^2\text{PolyLog}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] (-2*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*ArcTanh[1 - 2/(1 - Sqrt[
1 - c*x]/Sqrt[1 + c*x])])/c + (b*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]
])*PolyLog[2, 1 - 2/(1 - Sqrt[1 - c*x]/Sqrt[1 + c*x])])/c - (b*(a + b*ArcTa
nh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, -1 + 2/(1 - Sqrt[1 - c*x]/Sqrt[
1 + c*x])])/c - (b^2*PolyLog[3, 1 - 2/(1 - Sqrt[1 - c*x]/Sqrt[1 + c*x])])/(
2*c) + (b^2*PolyLog[3, -1 + 2/(1 - Sqrt[1 - c*x]/Sqrt[1 + c*x])])/(2*c)
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
```

```
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a+b\tanh^{-1}(x))^2}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(4b)\,\text{Subst}\left(\int \frac{\tanh^{-1}\left(1-\frac{2}{1-x}\right)(a+b\tan}{1-x^2}\right.}{c}$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(2b)\,\text{Subst}\left(\int \frac{(a+b\tanh^{-1}(x))\log\left(2-\right.}{1-x^2}\right.}{c}$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{b\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{Li}_2\left(1-\right.}{c}$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{b\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{Li}_2\left(1-\right.}{c}$$

Mathematica [A] time = 0.100641, size = 324, normalized size = 1.21

$$-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{1-cx}+\sqrt{cx+1}}{\sqrt{1-cx}-\sqrt{cx+1}}\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)-b\text{PolyLog}\left(2,\frac{\sqrt{1-cx}+\sqrt{cx+1}}{\sqrt{1-cx}-\sqrt{cx+1}}\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)-\frac{1}{2}b^2\text{Poly}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] -((2*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*ArcTanh[1 - 2/(1 - Sqrt
[1 - c*x]/Sqrt[1 + c*x])] + b*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*
PolyLog[2, -((Sqrt[1 - c*x] + Sqrt[1 + c*x])/(Sqrt[1 - c*x] - Sqrt[1 + c*x]
))] - b*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, (Sqrt[1 - c
*x] + Sqrt[1 + c*x])/(Sqrt[1 - c*x] - Sqrt[1 + c*x])] - (b^2*PolyLog[3, -((
Sqrt[1 - c*x] + Sqrt[1 + c*x])/(Sqrt[1 - c*x] - Sqrt[1 + c*x]))])/2 + (b^2*
PolyLog[3, (Sqrt[1 - c*x] + Sqrt[1 + c*x])/(Sqrt[1 - c*x] - Sqrt[1 + c*x])]
)/2)/c)
```

Maple [B] time = 0.904, size = 676, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x)
```

```
[Out] -1/2*a^2/c*ln(c*x-1)+1/2*a^2/c*ln(c*x+1)-b^2/c*arctanh((-c*x+1)^(1/2)/(c*x+
1)^(1/2))^2*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/
2))-2*b^2/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-((-c*x+1)^(1/2
)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))+2*b^2/c*polylog(3,-((-c*x+1
)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))+b^2/c*arctanh((-c*x+1
)^(1/2)/(c*x+1)^(1/2))^2*ln(((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2/(-(-c*x+1)/(
c*x+1)+1)+1)+b^2/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-((-c*x+
1)^(1/2)/(c*x+1)^(1/2)+1)^2/(-(-c*x+1)/(c*x+1)+1))-1/2*b^2/c*polylog(3,-((-
c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2/(-(-c*x+1)/(c*x+1)+1))-b^2/c*arctanh((-c*x+
1)^(1/2)/(c*x+1)^(1/2))^2*ln(1-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/
(c*x+1)+1)^(1/2))-2*b^2/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,(
(-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))+2*b^2/c*polylo
g(3,((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))-2*a*b/c*d
ilog(1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2*(-(-c*x+1)/(c*x+1)+1))+1/2*a*b/c*
dilog(1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^4*(-(-c*x+1)/(c*x+1)+1)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a^2\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)+\frac{\left(b^2\log(cx+1)-b^2\log(-cx+1)\right)\log\left(\sqrt{cx+1}-\sqrt{-cx+1}\right)^2}{8c}+\int-\frac{2\left(\sqrt{cx+1}b^2-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] 1/2*a^2*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/8*(b^2*log(c*x + 1) - b^2*log
(-c*x + 1))*log(sqrt(c*x + 1) - sqrt(-c*x + 1))^2/c + integrate(-1/8*(2*(sq
rt(c*x + 1)*b^2 - sqrt(-c*x + 1)*b^2)*log(sqrt(c*x + 1) + sqrt(-c*x + 1))^2
 + 8*(sqrt(c*x + 1)*a*b - sqrt(-c*x + 1)*a*b)*log(sqrt(c*x + 1) + sqrt(-c*x
 + 1)) - (4*(sqrt(c*x + 1)*b^2 - sqrt(-c*x + 1)*b^2)*log(sqrt(c*x + 1) + sq
rt(-c*x + 1)) + (8*a*b - (b^2*c*x - b^2)*log(c*x + 1) + (b^2*c*x - b^2)*log
(-c*x + 1))*sqrt(c*x + 1) - (8*a*b - (b^2*c*x + b^2)*log(c*x + 1) + (b^2*c*
```

```
x + b^2)*log(-c*x + 1))*sqrt(-c*x + 1))*log(sqrt(c*x + 1) - sqrt(-c*x + 1))
)/((c^2*x^2 - 1)*sqrt(c*x + 1) - (c^2*x^2 - 1)*sqrt(-c*x + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b^2 \operatorname{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 + 2\,ab \operatorname{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a^2}{c^2x^2 - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="fricas")
```

```
[Out] integral(-(b^2*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b*arctanh(sqrt
(-c*x + 1)/sqrt(c*x + 1)) + a^2)/(c^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a^2}{c^2x^2 - 1}\,dx - \int \frac{b^2 \operatorname{atanh}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2 - 1}\,dx - \int \frac{2ab \operatorname{atanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a**2/(c**2*x**2 - 1), x) - Integral(b**2*atanh(sqrt(-c*x + 1)/sqr
t(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(2*a*b*atanh(sqrt(-c*x + 1)/sq
rt(c*x + 1))/(c**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(b \operatorname{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)^2}{c^2x^2 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] integrate(-(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2/(c^2*x^2 - 1), x
)
```

3.33 $\int \frac{a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

Optimal. Leaf size=89

$$\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{2c}-\frac{b\text{PolyLog}\left(2,\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{2c}-\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}$$

```
[Out] -((a*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/c) + (b*PolyLog[2, -(Sqrt[1 - c*x]/S
qrt[1 + c*x])])/(2*c) - (b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(2*c)
```

Rubi [A] time = 0.0506488, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {206, 6681, 5912}

$$\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{2c}-\frac{b\text{PolyLog}\left(2,\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{2c}-\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] -((a*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/c) + (b*PolyLog[2, -(Sqrt[1 - c*x]/S
qrt[1 + c*x])])/(2*c) - (b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(2*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{a+b\tanh^{-1}(x)}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$
$$= -\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} + \frac{b\text{Li}_2\left(-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c} - \frac{b\text{Li}_2\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c}$$

Mathematica [A] time = 0.263249, size = 43, normalized size = 0.48

$$\frac{b\left(\text{PolyLog}\left(2,-e^{-\tanh^{-1}(cx)}\right)-\text{PolyLog}\left(2,e^{-\tanh^{-1}(cx)}\right)\right)}{2c}+\frac{a\tanh^{-1}(cx)}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] (a*ArcTanh[c*x])/c + (b*(PolyLog[2, -E^(-ArcTanh[c*x])] - PolyLog[2, E^(-Ar
cTanh[c*x])]))/(2*c)
```

Maple [A] time = 0.514, size = 118, normalized size = 1.3

$$-\frac{a\ln(cx-1)}{2c}+\frac{a\ln(cx+1)}{2c}-\frac{b}{c}\text{dilog}\left(\left(-\frac{-cx+1}{cx+1}+1\right)\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}+1\right)^{-2}\right)+\frac{b}{4c}\text{dilog}\left(\left(-\frac{-cx+1}{cx+1}+1\right)^2\left(\sqrt{-cx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x)
```

```
[Out] -1/2*a/c*ln(c*x-1)+1/2*a/c*ln(c*x+1)-b/c*dilog(1/((-c*x+1)^(1/2)/(c*x+1)^(1
/2)+1)^2*(-(-c*x+1)/(c*x+1)+1))+1/4*b/c*dilog(1/((-c*x+1)^(1/2)/(c*x+1)^(1/
2)+1)^4*(-(-c*x+1)/(c*x+1)+1)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}b\left(\frac{(\log(cx+1)-\log(-cx+1))\log\left(\sqrt{cx+1}+\sqrt{-cx+1}\right)-(\log(cx+1)-\log(-cx+1))\log\left(\sqrt{cx+1}-\sqrt{-cx+1}\right)}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="maxima")
```

```
[Out] 1/4*b*(((log(c*x + 1) - log(-c*x + 1))*log(sqrt(c*x + 1) + sqrt(-c*x + 1))
- (log(c*x + 1) - log(-c*x + 1))*log(sqrt(c*x + 1) - sqrt(-c*x + 1)))/c - 2
*integrate(-1/2*sqrt(c*x + 1)*(log(c*x + 1) - log(-c*x + 1))/((c^2*x^2 - 1)
*sqrt(c*x + 1) + (c^2*x^2 - 1)*sqrt(-c*x + 1)), x) - 2*integrate(1/2*sqrt(c
*x + 1)*(log(c*x + 1) - log(-c*x + 1))/((c^2*x^2 - 1)*sqrt(c*x + 1) - (c^2*
x^2 - 1)*sqrt(-c*x + 1)), x)) + 1/2*a*(log(c*x + 1)/c - log(c*x - 1)/c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{b\,\mathrm{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="fricas")
```

```
[Out] integral(-(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{a}{c^2x^2-1}\,dx-\int\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a/(c**2*x**2 - 1), x) - Integral(b*atanh(sqrt(-c*x + 1)/sqrt(c*x
+ 1))/(c**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{b \operatorname{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a}{c^2 x^2 - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="giac")
```

```
[Out] integrate(-(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

3.34 $\int \frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

Optimal. Leaf size=42

$$\text{Unintegrable}\left(\frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)},x\right)$$

```
[Out] Unintegrable[1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]]))
, x]
```

Rubi [A] time = 0.04332, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),
x]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx=\int \frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Mathematica [A] time = 0.0904258, size = 0, normalized size = 0.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x
]
```

Maple [A] time = 0.78, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2+1} \left(a + b\text{Artanh}\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(c^2x^2-1\right)\left(b\,\text{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="maxima")
```

```
[Out] -integrate(1/((c^2*x^2 - 1)*(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
 x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{ac^2x^2+\left(bc^2x^2-b\right)\text{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="fricas")
```

```
[Out] integral(-1/(a*c^2*x^2 + (b*c^2*x^2 - b)*arctanh(sqrt(-c*x + 1)/sqrt(c*x +
1)) - a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{ac^2x^2 - a + bc^2x^2 \operatorname{atanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) - b \operatorname{atanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*atanh((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

```
[Out] -Integral(1/(a*c**2*x**2 - a + b*c**2*x**2*atanh(sqrt(-c*x + 1)/sqrt(c*x +
1)) - b*atanh(sqrt(-c*x + 1)/sqrt(c*x + 1))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(c^2x^2 - 1)\left(b \operatorname{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
 x)
```

3.35 $\int \frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$

Optimal. Leaf size=42

$$\text{Unintegrable}\left(\frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2},x\right)$$

[Out] Unintegrable[1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2), x]

Rubi [A] time = 0.0417539, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]

[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2), x]

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Mathematica [A] time = 0.744021, size = 0, normalized size = 0.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
 x]
```

Maple [A] time = 0.756, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2+1}\left(a+b\mathrm{Artanh}\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)^{-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{4\,cx}{\sqrt{cx+1}\sqrt{-cx+1}b^2c\log\left(\sqrt{cx+1}+\sqrt{-cx+1}\right)-\sqrt{cx+1}\sqrt{-cx+1}b^2c\log\left(\sqrt{cx+1}-\sqrt{-cx+1}\right)+2\sqrt{cx+1}\sqrt{-cx+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="maxima")
```

```
[Out] 4*c*x/(sqrt(c*x + 1)*sqrt(-c*x + 1)*b^2*c*log(sqrt(c*x + 1) + sqrt(-c*x + 1
)) - sqrt(c*x + 1)*sqrt(-c*x + 1)*b^2*c*log(sqrt(c*x + 1) - sqrt(-c*x + 1))
 + 2*sqrt(c*x + 1)*sqrt(-c*x + 1)*a*b*c) - integrate(-4/((b^2*c^2*x^2 - b^2
)*sqrt(c*x + 1)*sqrt(-c*x + 1)*log(sqrt(c*x + 1) + sqrt(-c*x + 1)) - (b^2*c
^2*x^2 - b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*log(sqrt(c*x + 1) - sqrt(-c*x +
1)) + 2*(a*b*c^2*x^2 - a*b)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{a^{2}c^{2}x^{2}+\left(b^{2}c^{2}x^{2}-b^{2}\right)\operatorname{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^{2}-a^{2}+2\left(abc^{2}x^{2}-ab\right)\operatorname{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="fricas")
```

```
[Out] integral(-1/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arctanh(sqrt(-c*x + 1)/sqrt(
c*x + 1))^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arctanh(sqrt(-c*x + 1)/sqrt(c*x +
 1))), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int\frac{1}{a^{2}c^{2}x^{2}-a^{2}+2abc^{2}x^{2}\operatorname{atanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-2ab\operatorname{atanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+b^{2}c^{2}x^{2}\operatorname{atanh}^{2}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-b^{2}\operatorname{atanh}^{2}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*atanh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,x
)
```

```
[Out] -Integral(1/(a**2*c**2*x**2 - a**2 + 2*a*b*c**2*x**2*atanh(sqrt(-c*x + 1)/s
qrt(c*x + 1)) - 2*a*b*atanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + b**2*c**2*x**2*
atanh(sqrt(-c*x + 1)/sqrt(c*x + 1))**2 - b**2*atanh(sqrt(-c*x + 1)/sqrt(c*x
 + 1))**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int-\frac{1}{\left(c^{2}x^{2}-1\right)\left(b\operatorname{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2
), x)
```

3.36 $\int x^m \tanh^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=37

$$\frac{x^{m+1}\tanh^{-1}(\tanh(a+bx))}{m+1}-\frac{bx^{m+2}}{m^2+3m+2}$$

```
[Out] -((b*x^(2 + m))/(2 + 3*m + m^2)) + (x^(1 + m)*ArcTanh[Tanh[a + b*x]])/(1 +
m)
```

Rubi [A] time = 0.0254027, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 30}

$$\frac{x^{m+1}\tanh^{-1}(\tanh(a+bx))}{m+1}-\frac{bx^{m+2}}{m^2+3m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] -((b*x^(2 + m))/(2 + 3*m + m^2)) + (x^(1 + m)*ArcTanh[Tanh[a + b*x]])/(1 +
m)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^m \tanh^{-1}(\tanh(a+bx))\,dx = \frac{x^{1+m}\tanh^{-1}(\tanh(a+bx))}{1+m} - \frac{b\int x^{1+m}\,dx}{1+m}$$
$$= -\frac{bx^{2+m}}{2+3m+m^2} + \frac{x^{1+m}\tanh^{-1}(\tanh(a+bx))}{1+m}$$

Mathematica [A] time = 0.0623129, size = 34, normalized size = 0.92

$$x^m\left(\frac{x\left(\tanh^{-1}(\tanh(a+bx))-bx\right)}{m+1}+\frac{bx^2}{m+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] x^m*((b*x^2)/(2 + m) + (x*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/(1 + m))
```

Maple [A] time = 0.039, size = 41, normalized size = 1.1

$$\frac{bx^2 e^{m\ln(x)}}{2+m} + \frac{(\mathrm{Artanh}\,(\tanh\,(bx+a)) - bx)\,xe^{m\ln(x)}}{1+m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctanh(tanh(b*x+a)),x)
```

```
[Out] b/(2+m)*x^2*exp(m*ln(x))+(arctanh(tanh(b*x+a))-b*x)/(1+m)*x*exp(m*ln(x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.99437, size = 155, normalized size = 4.19

$$\frac{\left((bm+b)x^2+(am+2a)x\right)\cosh\left(m\log\left(x\right)\right)+\left((bm+b)x^2+(am+2a)x\right)\sinh\left(m\log\left(x\right)\right)}{m^2+3m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] (((b*m + b)*x^2 + (a*m + 2*a)*x)*cosh(m*log(x)) + ((b*m + b)*x^2 + (a*m + 2
*a)*x)*sinh(m*log(x)))/(m^2 + 3*m + 2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atanh(tanh(b*x+a)),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.13281, size = 58, normalized size = 1.57

$$\frac{bmx^2x^m+amxx^m+bx^2x^m+2\,axx^m}{m^2+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] (b*m*x^2*x^m + a*m*x*x^m + b*x^2*x^m + 2*a*x*x^m)/(m^2 + 3*m + 2)
```

3.37 $\int x^2 \tanh^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=23

$$\frac{1}{3}x^3\tanh^{-1}(\tanh(a+bx)) - \frac{bx^4}{12}$$

```
[Out] -(b*x^4)/12 + (x^3*ArcTanh[Tanh[a + b*x]])/3
```

Rubi [A] time = 0.0081557, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 30}

$$\frac{1}{3}x^3\tanh^{-1}(\tanh(a+bx)) - \frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] -(b*x^4)/12 + (x^3*ArcTanh[Tanh[a + b*x]])/3
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^2 \tanh^{-1}(\tanh(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(\tanh(a+bx)) - \frac{1}{3}b\int x^3\,dx$$
$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(\tanh(a+bx))$$

Mathematica [A] time = 0.0163977, size = 20, normalized size = 0.87

$$-\frac{1}{12}x^3\left(bx - 4\tanh^{-1}(\tanh(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] -(x^3*(b*x - 4*ArcTanh[Tanh[a + b*x]]))/12
```

Maple [A] time = 0.032, size = 20, normalized size = 0.9

$$-\frac{bx^4}{12} + \frac{x^3 \mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(tanh(b*x+a)),x)
```

```
[Out] -1/12*b*x^4+1/3*x^3*arctanh(tanh(b*x+a))
```

Maxima [A] time = 1.15154, size = 26, normalized size = 1.13

$$-\frac{1}{12}\,bx^4 + \frac{1}{3}\,x^3\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/12*b*x^4 + 1/3*x^3*arctanh(tanh(b*x + a))
```

Fricas [A] time = 1.598, size = 31, normalized size = 1.35

$$\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

Sympy [A] time = 0.812558, size = 19, normalized size = 0.83

$$-\frac{bx^4}{12}+\frac{x^3\,\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(tanh(b*x+a)),x)
```

```
[Out] -b*x**4/12 + x**3*atanh(tanh(a + b*x))/3
```

Giac [A] time = 1.1336, size = 18, normalized size = 0.78

$$\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

3.38 $\int x \tanh^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=23

$$\frac{1}{2}x^2\tanh^{-1}(\tanh(a+bx))-\frac{bx^3}{6}$$

```
[Out] -(b*x^3)/6 + (x^2*ArcTanh[Tanh[a + b*x]])/2
```

Rubi [A] time = 0.0068111, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6239, 30}

$$\frac{1}{2}x^2\tanh^{-1}(\tanh(a+bx))-\frac{bx^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] -(b*x^3)/6 + (x^2*ArcTanh[Tanh[a + b*x]])/2
```

Rule 6239

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Tanh[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\tanh^{-1}(\tanh(a+bx))\,dx &= \frac{1}{2}x^2\tanh^{-1}(\tanh(a+bx))-\frac{1}{2}b\int x^2\,dx\\ &= -\frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(\tanh(a+bx))\end{aligned}$$

Mathematica [A] time = 0.0150803, size = 20, normalized size = 0.87

$$-\frac{1}{6}x^2\left(bx-3\tanh^{-1}(\tanh(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] -(x^2*(b*x - 3*ArcTanh[Tanh[a + b*x]]))/6
```

Maple [A] time = 0.03, size = 20, normalized size = 0.9

$$-\frac{bx^3}{6}+\frac{x^2\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(tanh(b*x+a)),x)
```

```
[Out] -1/6*b*x^3+1/2*x^2*arctanh(tanh(b*x+a))
```

Maxima [A] time = 1.14261, size = 26, normalized size = 1.13

$$-\frac{1}{6}\,bx^3+\frac{1}{2}\,x^2\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/6*b*x^3 + 1/2*x^2*arctanh(tanh(b*x + a))
```

Fricas [A] time = 1.61921, size = 31, normalized size = 1.35

$$\frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

Sympy [A] time = 3.21176, size = 39, normalized size = 1.7

$$\begin{cases} \frac{x\,\mathrm{atanh}^2\left(\tanh\left(a+bx\right)\right)}{2b} - \frac{\mathrm{atanh}^3\left(\tanh\left(a+bx\right)\right)}{6b^2} & \text{for } b \neq 0 \\ \frac{x^2\,\mathrm{atanh}\left(\tanh\left(a\right)\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(tanh(b*x+a)),x)
```

```
[Out] Piecewise((x*atanh(tanh(a + b*x))**2/(2*b) - atanh(tanh(a + b*x))**3/(6*b**
2), Ne(b, 0)), (x**2*atanh(tanh(a))/2, True))
```

Giac [A] time = 1.12741, size = 18, normalized size = 0.78

$$\frac{1}{3}\,bx^3 + \frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

3.39 $\int \tanh^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=16

$$\frac{\tanh^{-1}(\tanh(a+bx))^2}{2b}$$

```
[Out] ArcTanh[Tanh[a + b*x]]^2/(2*b)
```

Rubi [A] time = 0.0032771, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2157, 30}

$$\frac{\tanh^{-1}(\tanh(a+bx))^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^2/(2*b)
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(\tanh(a+bx))\,dx &= \frac{\text{Subst}\left(\int x\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{\tanh^{-1}(\tanh(a+bx))^2}{2b}\end{aligned}$$

Mathematica [A] time = 0.0100272, size = 18, normalized size = 1.12

$$x\tanh^{-1}(\tanh(a+bx))-\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] -(b*x^2)/2 + x*ArcTanh[Tanh[a + b*x]]
```

Maple [A] time = 0.027, size = 15, normalized size = 0.9

$$\frac{(\mathrm{Artanh}\,(\tanh\,(bx+a)))^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a)),x)
```

```
[Out] 1/2*arctanh(tanh(b*x+a))^2/b
```

Maxima [A] time = 1.14429, size = 22, normalized size = 1.38

$$-\frac{1}{2}\,bx^2+x\,\mathrm{artanh}\,(\tanh\,(bx+a))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/2*b*x^2 + x*arctanh(tanh(b*x + a))
```

Fricas [A] time = 1.69433, size = 23, normalized size = 1.44

$$\frac{1}{2}\,bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*x
```

Sympy [A] time = 1.27905, size = 19, normalized size = 1.19

$$\begin{cases} \frac{\operatorname{atanh}^{2}\left(\tanh\left(a+bx\right)\right)}{2b} & \text{for}\ b \neq 0 \\ x \operatorname{atanh}\left(\tanh\left(a\right)\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a)),x)
```

```
[Out] Piecewise((atanh(tanh(a + b*x))**2/(2*b), Ne(b, 0)), (x*atanh(tanh(a)), Tru
e))
```

Giac [A] time = 1.12007, size = 14, normalized size = 0.88

$$\frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x
```

3.40 $\int \frac{\tanh^{-1}(\tanh(a+bx))}{x}\,dx$

Optimal. Leaf size=21

$$bx - \log(x)\left(bx - \tanh^{-1}(\tanh(a+bx))\right)$$

```
[Out] b*x - (b*x - ArcTanh[Tanh[a + b*x]])*Log[x]
```

Rubi [A] time = 0.0358928, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2158, 29}

$$bx - \log(x)\left(bx - \tanh^{-1}(\tanh(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]/x,x]
```

```
[Out] b*x - (b*x - ArcTanh[Tanh[a + b*x]])*Log[x]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned} \int \frac{\tanh^{-1}(\tanh(a+bx))}{x}\,dx &= bx - \left(bx - \tanh^{-1}(\tanh(a+bx))\right)\int \frac{1}{x}\,dx \\ &= bx - \left(bx - \tanh^{-1}(\tanh(a+bx))\right)\log(x) \end{aligned}$$

Mathematica [A] time = 0.0139229, size = 19, normalized size = 0.9

$$\log(x)\left(\tanh^{-1}(\tanh(a+bx)) - bx\right) + bx$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]/x,x]
```

```
[Out] b*x + (-(b*x) + ArcTanh[Tanh[a + b*x]])*Log[x]
```

Maple [A] time = 0.036, size = 21, normalized size = 1.

$$\ln(x)\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-\ln(x)\,xb+bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))/x,x)
```

```
[Out] ln(x)*arctanh(tanh(b*x+a))-ln(x)*x*b+b*x
```

Maxima [A] time = 0.973203, size = 46, normalized size = 2.19

$$-b\left(x+\frac{a}{b}\right)\log(x)+b\left(x+\frac{a\log(x)}{b}\right)+\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -b*(x + a/b)*log(x) + b*(x + a*log(x)/b) + arctanh(tanh(b*x + a))*log(x)
```

Fricas [A] time = 1.82228, size = 22, normalized size = 1.05

$$bx+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] b*x + a*log(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}\left(\tanh\left(a + bx\right)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))/x,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))/x, x)
```

Giac [A] time = 1.11829, size = 12, normalized size = 0.57

$$bx + a\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x,x, algorithm="giac")
```

```
[Out] b*x + a*log(abs(x))
```

3.41 $\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^2} dx$

Optimal. Leaf size=17

$$b\log(x) - \frac{\tanh^{-1}(\tanh(a+bx))}{x}$$

```
[Out] -(ArcTanh[Tanh[a + b*x]]/x) + b*Log[x]
```

Rubi [A] time = 0.0080839, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 29}

$$b\log(x) - \frac{\tanh^{-1}(\tanh(a+bx))}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]/x^2,x]
```

```
[Out] -(ArcTanh[Tanh[a + b*x]]/x) + b*Log[x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^2}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))}{x} + b\int \frac{1}{x}\,dx$$
$$= -\frac{\tanh^{-1}(\tanh(a+bx))}{x} + b\log(x)$$

Mathematica [A] time = 0.0154614, size = 18, normalized size = 1.06

$$-\frac{\tanh^{-1}(\tanh(a+bx))}{x} + b\log(x) + b$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]/x^2,x]
```

```
[Out] b - ArcTanh[Tanh[a + b*x]]/x + b*Log[x]
```

Maple [A] time = 0.032, size = 18, normalized size = 1.1

$$-\frac{\operatorname{Artanh}(\tanh(bx+a))}{x} + b\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))/x^2,x)
```

```
[Out] -arctanh(tanh(b*x+a))/x+b*ln(x)
```

Maxima [A] time = 1.15594, size = 23, normalized size = 1.35

$$b\log(x) - \frac{\operatorname{artanh}(\tanh(bx+a))}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^2,x, algorithm="maxima")
```

```
[Out] b*log(x) - arctanh(tanh(b*x + a))/x
```

Fricas [A] time = 1.74436, size = 27, normalized size = 1.59

$$\frac{bx\log(x) - a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^2,x, algorithm="fricas")
```

```
[Out] (b*x*log(x) - a)/x
```

Sympy [A] time = 0.60585, size = 14, normalized size = 0.82

$$b\log(x) - \frac{\operatorname{atanh}(\tanh(a + bx))}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))/x**2,x)
```

```
[Out] b*log(x) - atanh(tanh(a + b*x))/x
```

Giac [A] time = 1.12392, size = 16, normalized size = 0.94

$$b\log(|x|) - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^2,x, algorithm="giac")
```

```
[Out] b*log(abs(x)) - a/x
```

3.42 $\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^3} \, dx$

Optimal. Leaf size=23

$$-\frac{\tanh^{-1}(\tanh(a+bx))}{2x^2}-\frac{b}{2x}$$

```
[Out] -b/(2*x) - ArcTanh[Tanh[a + b*x]]/(2*x^2)
```

Rubi [A] time = 0.0089049, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 30}

$$-\frac{\tanh^{-1}(\tanh(a+bx))}{2x^2}-\frac{b}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]/x^3,x]
```

```
[Out] -b/(2*x) - ArcTanh[Tanh[a + b*x]]/(2*x^2)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^3}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))}{2x^2} + \frac{1}{2}b\int \frac{1}{x^2}\,dx$$
$$= -\frac{b}{2x} - \frac{\tanh^{-1}(\tanh(a+bx))}{2x^2}$$

Mathematica [A] time = 0.0139429, size = 18, normalized size = 0.78

$$-\frac{\tanh^{-1}(\tanh(a+bx)) + bx}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]/x^3,x]
```

```
[Out] -(b*x + ArcTanh[Tanh[a + b*x]])/(2*x^2)
```

Maple [A] time = 0.033, size = 20, normalized size = 0.9

$$-\frac{b}{2\,x} - \frac{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))/x^3,x)
```

```
[Out] -1/2*b/x-1/2*arctanh(tanh(b*x+a))/x^2
```

Maxima [A] time = 1.16409, size = 26, normalized size = 1.13

$$-\frac{b}{2\,x} - \frac{\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b/x - 1/2*arctanh(tanh(b*x + a))/x^2
```

Fricas [A] time = 1.66006, size = 30, normalized size = 1.3

$$-\frac{2\,bx+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x + a)/x^2
```

Sympy [A] time = 1.1003, size = 19, normalized size = 0.83

$$-\frac{b}{2x}-\frac{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))/x**3,x)
```

```
[Out] -b/(2*x) - atanh(tanh(a + b*x))/(2*x**2)
```

Giac [A] time = 1.1388, size = 15, normalized size = 0.65

$$-\frac{2\,bx+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x + a)/x^2
```

3.43 $\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^4} \, dx$

Optimal. Leaf size=23

$$-\frac{\tanh^{-1}(\tanh(a+bx))}{3x^3}-\frac{b}{6x^2}$$

```
[Out] -b/(6*x^2) - ArcTanh[Tanh[a + b*x]]/(3*x^3)
```

Rubi [A] time = 0.0081961, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 30}

$$-\frac{\tanh^{-1}(\tanh(a+bx))}{3x^3}-\frac{b}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]/x^4,x]
```

```
[Out] -b/(6*x^2) - ArcTanh[Tanh[a + b*x]]/(3*x^3)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^4}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))}{3x^3} + \frac{1}{3}b\int \frac{1}{x^3}\,dx$$
$$= -\frac{b}{6x^2} - \frac{\tanh^{-1}(\tanh(a+bx))}{3x^3}$$

Mathematica [A] time = 0.0146518, size = 20, normalized size = 0.87

$$-\frac{2\tanh^{-1}(\tanh(a+bx)) + bx}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]/x^4,x]
```

```
[Out] -(b*x + 2*ArcTanh[Tanh[a + b*x]])/(6*x^3)
```

Maple [A] time = 0.033, size = 20, normalized size = 0.9

$$-\frac{b}{6\,x^2} - \frac{\mathrm{Artanh}\,(\tanh\,(bx+a))}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))/x^4,x)
```

```
[Out] -1/6*b/x^2-1/3*arctanh(tanh(b*x+a))/x^3
```

Maxima [A] time = 1.15543, size = 26, normalized size = 1.13

$$-\frac{b}{6\,x^2} - \frac{\mathrm{artanh}\,(\tanh\,(bx+a))}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*b/x^2 - 1/3*arctanh(tanh(b*x + a))/x^3
```

Fricas [A] time = 1.87144, size = 32, normalized size = 1.39

$$-\frac{3\,bx+2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b*x + 2*a)/x^3
```

Sympy [A] time = 7.86199, size = 20, normalized size = 0.87

$$-\frac{b}{6x^2}-\frac{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))/x**4,x)
```

```
[Out] -b/(6*x**2) - atanh(tanh(a + b*x))/(3*x**3)
```

Giac [A] time = 1.12839, size = 18, normalized size = 0.78

$$-\frac{3\,bx+2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*b*x + 2*a)/x^3
```

3.44 $\int x^m \tanh^{-1}(\tanh(a+bx))^2\,dx$

Optimal. Leaf size=71

$$-\frac{2bx^{m+2}\tanh^{-1}(\tanh(a+bx))}{m^2+3m+2}+\frac{x^{m+1}\tanh^{-1}(\tanh(a+bx))^2}{m+1}+\frac{2b^2x^{m+3}}{m^3+6m^2+11m+6}$$

```
[Out] (2*b^2*x^(3 + m))/(6 + 11*m + 6*m^2 + m^3) - (2*b*x^(2 + m)*ArcTanh[Tanh[a
+ b*x]])/(2 + 3*m + m^2) + (x^(1 + m)*ArcTanh[Tanh[a + b*x]]^2)/(1 + m)
```

Rubi [A] time = 0.0321608, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$-\frac{2bx^{m+2}\tanh^{-1}(\tanh(a+bx))}{m^2+3m+2}+\frac{x^{m+1}\tanh^{-1}(\tanh(a+bx))^2}{m+1}+\frac{2b^2x^{m+3}}{m^3+6m^2+11m+6}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (2*b^2*x^(3 + m))/(6 + 11*m + 6*m^2 + m^3) - (2*b*x^(2 + m)*ArcTanh[Tanh[a
+ b*x]])/(2 + 3*m + m^2) + (x^(1 + m)*ArcTanh[Tanh[a + b*x]]^2)/(1 + m)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^m \tanh^{-1}(\tanh(a+bx))^2\,dx = \frac{x^{1+m}\tanh^{-1}(\tanh(a+bx))^2}{1+m} - \frac{(2b)\int x^{1+m}\tanh^{-1}(\tanh(a+bx))\,dx}{1+m}$$
$$= -\frac{2bx^{2+m}\tanh^{-1}(\tanh(a+bx))}{2+3m+m^2} + \frac{x^{1+m}\tanh^{-1}(\tanh(a+bx))^2}{1+m} + \frac{\left(2b^2\right)\int x^{2+m}\,dx}{2+3m+m^2}$$
$$= \frac{2b^2x^{3+m}}{6+11m+6m^2+m^3} - \frac{2bx^{2+m}\tanh^{-1}(\tanh(a+bx))}{2+3m+m^2} + \frac{x^{1+m}\tanh^{-1}(\tanh(a+bx)}{1+m}$$

Mathematica [A] time = 0.107658, size = 62, normalized size = 0.87

$$\frac{x^{m+1}\left(\left(m^2+5m+6\right)\tanh^{-1}(\tanh(a+bx))^2 - 2b(m+3)x\tanh^{-1}(\tanh(a+bx)) + 2b^2x^2\right)}{(m+1)(m+2)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (x^(1 + m)*(2*b^2*x^2 - 2*b*(3 + m)*x*ArcTanh[Tanh[a + b*x]] + (6 + 5*m + m
^2)*ArcTanh[Tanh[a + b*x]]^2))/((1 + m)*(2 + m)*(3 + m))
```

Maple [A] time = 0.04, size = 98, normalized size = 1.4

$$\frac{x^3b^2e^{m\ln(x)}}{3+m} + \frac{\left(a^2+2\,a\,(\mathrm{Artanh}\,(\tanh\,(bx+a)) - bx - a) + (\mathrm{Artanh}\,(\tanh\,(bx+a)) - bx - a)^2\right)xe^{m\ln(x)}}{1+m} + 2\,\frac{b\,(\mathrm{Artar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctanh(tanh(b*x+a))^2,x)
```

```
[Out] b^2/(3+m)*x^3*exp(m*ln(x))+(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(t
anh(b*x+a))-b*x-a)^2)/(1+m)*x*exp(m*ln(x))+2*b*(arctanh(tanh(b*x+a))-b*x)/(
2+m)*x^2*exp(m*ln(x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.22331, size = 356, normalized size = 5.01

$$\frac{\left(\left(b^2m^2 + 3\,b^2m + 2\,b^2\right)x^3 + 2\left(abm^2 + 4\,abm + 3\,ab\right)x^2 + \left(a^2m^2 + 5\,a^2m + 6\,a^2\right)x\right)\cosh\left(m\log\left(x\right)\right) + \left(\left(b^2m^2 + 3\,b^2m + 2\right.\right.}{m^3 + 6\,m^2 + 11\,m + 6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] (((b^2*m^2 + 3*b^2*m + 2*b^2)*x^3 + 2*(a*b*m^2 + 4*a*b*m + 3*a*b)*x^2 + (a^
2*m^2 + 5*a^2*m + 6*a^2)*x)*cosh(m*log(x)) + ((b^2*m^2 + 3*b^2*m + 2*b^2)*x
^3 + 2*(a*b*m^2 + 4*a*b*m + 3*a*b)*x^2 + (a^2*m^2 + 5*a^2*m + 6*a^2)*x)*sin
h(m*log(x)))/(m^3 + 6*m^2 + 11*m + 6)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atanh(tanh(b*x+a))**2,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{artanh}\left(\tanh\left(bx + a\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*arctanh(tanh(b*x + a))^2, x)
```

3.45 $\int x^3 \tanh^{-1}(\tanh(a+bx))^2 \, dx$

Optimal. Leaf size=42

$$-\frac{1}{10}bx^5\tanh^{-1}(\tanh(a+bx))+\frac{1}{4}x^4\tanh^{-1}(\tanh(a+bx))^2+\frac{b^2x^6}{60}$$

```
[Out] (b^2*x^6)/60 - (b*x^5*ArcTanh[Tanh[a + b*x]])/10 + (x^4*ArcTanh[Tanh[a + b*
x]]^2)/4
```

Rubi [A] time = 0.0234797, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$-\frac{1}{10}bx^5\tanh^{-1}(\tanh(a+bx))+\frac{1}{4}x^4\tanh^{-1}(\tanh(a+bx))^2+\frac{b^2x^6}{60}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (b^2*x^6)/60 - (b*x^5*ArcTanh[Tanh[a + b*x]])/10 + (x^4*ArcTanh[Tanh[a + b*
x]]^2)/4
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^3 \tanh^{-1}(\tanh(a+bx))^2\,dx &= \frac{1}{4}x^4\tanh^{-1}(\tanh(a+bx))^2 - \frac{1}{2}b\int x^4\tanh^{-1}(\tanh(a+bx))\,dx \\
&= -\frac{1}{10}bx^5\tanh^{-1}(\tanh(a+bx)) + \frac{1}{4}x^4\tanh^{-1}(\tanh(a+bx))^2 + \frac{1}{10}b^2\int x^5\,dx \\
&= \frac{b^2x^6}{60} - \frac{1}{10}bx^5\tanh^{-1}(\tanh(a+bx)) + \frac{1}{4}x^4\tanh^{-1}(\tanh(a+bx))^2
\end{aligned}
$$

Mathematica [A] time = 0.0295993, size = 37, normalized size = 0.88

$$\frac{1}{60}x^4\left(-6bx\tanh^{-1}(\tanh(a+bx)) + 15\tanh^{-1}(\tanh(a+bx))^2 + b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (x^4*(b^2*x^2 - 6*b*x*ArcTanh[Tanh[a + b*x]] + 15*ArcTanh[Tanh[a + b*x]]^2)
)/60
```

Maple [A] time = 0.036, size = 38, normalized size = 0.9

$$\frac{x^4\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^2}{4} - \frac{b}{2}\left(\frac{x^5\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}{5} - \frac{x^6b}{30}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(tanh(b*x+a))^2,x)
```

```
[Out] 1/4*x^4*arctanh(tanh(b*x+a))^2-1/2*b*(1/5*x^5*arctanh(tanh(b*x+a))-1/30*x^6
*b)
```

Maxima [A] time = 1.34876, size = 49, normalized size = 1.17

$$\frac{1}{60}b^2x^6 - \frac{1}{10}bx^5\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right) + \frac{1}{4}x^4\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/60*b^2*x^6 - 1/10*b*x^5*arctanh(tanh(b*x + a)) + 1/4*x^4*arctanh(tanh(b*x
 + a))^2
```

Fricas [A] time = 1.95857, size = 55, normalized size = 1.31

$$\frac{1}{6}\,b^2x^6 + \frac{2}{5}\,abx^5 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

Sympy [A] time = 2.62917, size = 37, normalized size = 0.88

$$\frac{b^2x^6}{60} - \frac{bx^5\,\mathrm{atanh}\,(\tanh{(a+bx)})}{10} + \frac{x^4\,\mathrm{atanh}^2\,(\tanh{(a+bx)})}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(tanh(b*x+a))**2,x)
```

```
[Out] b**2*x**6/60 - b*x**5*atanh(tanh(a + b*x))/10 + x**4*atanh(tanh(a + b*x))**
2/4
```

Giac [A] time = 1.15066, size = 32, normalized size = 0.76

$$\frac{1}{6}\,b^2x^6 + \frac{2}{5}\,abx^5 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

3.46 $\int x^2 \tanh^{-1}(\tanh(a+bx))^2 \, dx$

Optimal. Leaf size=42

$$-\frac{1}{6}bx^4 \tanh^{-1}(\tanh(a+bx)) + \frac{1}{3}x^3 \tanh^{-1}(\tanh(a+bx))^2 + \frac{b^2x^5}{30}$$

```
[Out] (b^2*x^5)/30 - (b*x^4*ArcTanh[Tanh[a + b*x]])/6 + (x^3*ArcTanh[Tanh[a + b*x
]]^2)/3
```

Rubi [A] time = 0.0221278, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$-\frac{1}{6}bx^4 \tanh^{-1}(\tanh(a+bx)) + \frac{1}{3}x^3 \tanh^{-1}(\tanh(a+bx))^2 + \frac{b^2x^5}{30}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (b^2*x^5)/30 - (b*x^4*ArcTanh[Tanh[a + b*x]])/6 + (x^3*ArcTanh[Tanh[a + b*x
]]^2)/3
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^2 \tanh^{-1}(\tanh(a+bx))^2\,dx = \frac{1}{3}x^3\tanh^{-1}(\tanh(a+bx))^2 - \frac{1}{3}(2b)\int x^3\tanh^{-1}(\tanh(a+bx))\,dx$$
$$= -\frac{1}{6}bx^4\tanh^{-1}(\tanh(a+bx)) + \frac{1}{3}x^3\tanh^{-1}(\tanh(a+bx))^2 + \frac{1}{6}b^2\int x^4\,dx$$
$$= \frac{b^2x^5}{30} - \frac{1}{6}bx^4\tanh^{-1}(\tanh(a+bx)) + \frac{1}{3}x^3\tanh^{-1}(\tanh(a+bx))^2$$

Mathematica [A] time = 0.0478986, size = 37, normalized size = 0.88

$$\frac{1}{30}x^3\left(-5bx\tanh^{-1}(\tanh(a+bx)) + 10\tanh^{-1}(\tanh(a+bx))^2 + b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (x^3*(b^2*x^2 - 5*b*x*ArcTanh[Tanh[a + b*x]] + 10*ArcTanh[Tanh[a + b*x]]^2)
)/30
```

Maple [A] time = 0.035, size = 38, normalized size = 0.9

$$\frac{x^3\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^2}{3} - \frac{2\,b}{3}\left(\frac{x^4\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}{4} - \frac{bx^5}{20}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(tanh(b*x+a))^2,x)
```

```
[Out] 1/3*x^3*arctanh(tanh(b*x+a))^2-2/3*b*(1/4*x^4*arctanh(tanh(b*x+a))-1/20*b*x
^5)
```

Maxima [A] time = 1.34477, size = 49, normalized size = 1.17

$$\frac{1}{30}\,b^2x^5 - \frac{1}{6}\,bx^4\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right) + \frac{1}{3}\,x^3\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/30*b^2*x^5 - 1/6*b*x^4*arctanh(tanh(b*x + a)) + 1/3*x^3*arctanh(tanh(b*x
+ a))^2
```

Fricas [A] time = 1.53462, size = 55, normalized size = 1.31

$$\frac{1}{5}b^2x^5 + \frac{1}{2}abx^4 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

Sympy [A] time = 1.74069, size = 37, normalized size = 0.88

$$\frac{b^2x^5}{30} - \frac{bx^4\operatorname{atanh}(\tanh(a+bx))}{6} + \frac{x^3\operatorname{atanh}^2(\tanh(a+bx))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(tanh(b*x+a))**2,x)
```

```
[Out] b**2*x**5/30 - b*x**4*atanh(tanh(a + b*x))/6 + x**3*atanh(tanh(a + b*x))**2
/3
```

Giac [A] time = 1.14106, size = 32, normalized size = 0.76

$$\frac{1}{5}b^2x^5 + \frac{1}{2}abx^4 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

3.47 $\int x \tanh^{-1}(\tanh(a+bx))^2 \, dx$

Optimal. Leaf size=34

$$\frac{x\tanh^{-1}(\tanh(a+bx))^3}{3b}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{12b^2}$$

```
[Out] (x*ArcTanh[Tanh[a + b*x]]^3)/(3*b) - ArcTanh[Tanh[a + b*x]]^4/(12*b^2)
```

Rubi [A] time = 0.020971, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2168, 2157, 30}

$$\frac{x\tanh^{-1}(\tanh(a+bx))^3}{3b}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (x*ArcTanh[Tanh[a + b*x]]^3)/(3*b) - ArcTanh[Tanh[a + b*x]]^4/(12*b^2)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\tanh^{-1}(\tanh(a+bx))^2\,dx &= \frac{x\tanh^{-1}(\tanh(a+bx))^3}{3b}-\frac{\int \tanh^{-1}(\tanh(a+bx))^3\,dx}{3b}\\ &= \frac{x\tanh^{-1}(\tanh(a+bx))^3}{3b}-\frac{\text{Subst}\left(\int x^3\,dx,x,\tanh^{-1}(\tanh(a+bx))\right)}{3b^2}\\ &= \frac{x\tanh^{-1}(\tanh(a+bx))^3}{3b}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{12b^2}\end{aligned}$$

Mathematica [B] time = 0.0751046, size = 74, normalized size = 2.18

$$\frac{(a+bx)\left(4\left(2a^2+abx-b^2x^2\right)\tanh^{-1}(\tanh(a+bx))-(3a-bx)(a+bx)^2-6(a-bx)\tanh^{-1}(\tanh(a+bx))^2\right)}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] ((a + b*x)*(-((3*a - b*x)*(a + b*x)^2) + 4*(2*a^2 + a*b*x - b^2*x^2)*ArcTan
h[Tanh[a + b*x]] - 6*(a - b*x)*ArcTanh[Tanh[a + b*x]]^2))/(12*b^2)
```

Maple [A] time = 0.035, size = 38, normalized size = 1.1

$$\frac{x^2\left(\text{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^2}{2}-b\left(-\frac{bx^4}{12}+\frac{x^3\text{Artanh}\left(\tanh\left(bx+a\right)\right)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(tanh(b*x+a))^2,x)
```

```
[Out] 1/2*x^2*arctanh(tanh(b*x+a))^2-b*(-1/12*b*x^4+1/3*x^3*arctanh(tanh(b*x+a)))
```

Maxima [A] time = 1.35548, size = 49, normalized size = 1.44

$$\frac{1}{12}\,b^2x^4-\frac{1}{3}\,bx^3\,\text{artanh}\left(\tanh\left(bx+a\right)\right)+\frac{1}{2}\,x^2\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/12*b^2*x^4 - 1/3*b*x^3*arctanh(tanh(b*x + a)) + 1/2*x^2*arctanh(tanh(b*x
+ a))^2
```

Fricas [A] time = 1.43912, size = 55, normalized size = 1.62

$$\frac{1}{4}b^2x^4 + \frac{2}{3}abx^3 + \frac{1}{2}a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/4*b^2*x^4 + 2/3*a*b*x^3 + 1/2*a^2*x^2
```

Sympy [A] time = 0.825643, size = 37, normalized size = 1.09

$$\frac{b^2x^4}{12} - \frac{bx^3\operatorname{atanh}(\tanh(a+bx))}{3} + \frac{x^2\operatorname{atanh}^2(\tanh(a+bx))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(tanh(b*x+a))**2,x)
```

```
[Out] b**2*x**4/12 - b*x**3*atanh(tanh(a + b*x))/3 + x**2*atanh(tanh(a + b*x))**2
/2
```

Giac [A] time = 1.14194, size = 32, normalized size = 0.94

$$\frac{1}{4}b^2x^4 + \frac{2}{3}abx^3 + \frac{1}{2}a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 1/4*b^2*x^4 + 2/3*a*b*x^3 + 1/2*a^2*x^2
```

3.48 $\int \tanh^{-1}(\tanh(a+bx))^2\,dx$

Optimal. Leaf size=16

$$\frac{\tanh^{-1}(\tanh(a+bx))^3}{3b}$$

[Out] ArcTanh[Tanh[a + b*x]]^3/(3*b)

Rubi [A] time = 0.0043591, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2157, 30}

$$\frac{\tanh^{-1}(\tanh(a+bx))^3}{3b}$$

Antiderivative was successfully verified.

[In] Int[ArcTanh[Tanh[a + b*x]]^2,x]

[Out] ArcTanh[Tanh[a + b*x]]^3/(3*b)

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(\tanh(a+bx))^2\,dx &= \frac{\text{Subst}\left(\int x^2\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{\tanh^{-1}(\tanh(a+bx))^3}{3b}\end{aligned}$$

Mathematica [A] time = 0.0057246, size = 16, normalized size = 1.

$$\frac{\tanh^{-1}(\tanh(a+bx))^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^3/(3*b)
```

Maple [A] time = 0.027, size = 15, normalized size = 0.9

$$\frac{(\operatorname{Artanh}(\tanh(bx+a)))^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2,x)
```

```
[Out] 1/3*arctanh(tanh(b*x+a))^3/b
```

Maxima [B] time = 1.39033, size = 45, normalized size = 2.81

$$\frac{1}{3}b^2x^3 - bx^2\operatorname{artanh}(\tanh(bx+a)) + x\operatorname{artanh}(\tanh(bx+a))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3 - b*x^2*arctanh(tanh(b*x + a)) + x*arctanh(tanh(b*x + a))^2
```

Fricas [A] time = 1.51263, size = 42, normalized size = 2.62

$$\frac{1}{3}b^2x^3 + abx^2 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

Sympy [A] time = 2.31926, size = 20, normalized size = 1.25

$$\begin{cases} \frac{\operatorname{atanh}^{3}{\left(\tanh{\left(a+bx \right)} \right)}}{3b} & \text{for}\: b \neq 0 \\ x \operatorname{atanh}^{2}{\left(\tanh{\left(a \right)} \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2,x)
```

```
[Out] Piecewise((atanh(tanh(a + b*x))**3/(3*b), Ne(b, 0)), (x*atanh(tanh(a))**2,
True))
```

Giac [A] time = 1.14693, size = 27, normalized size = 1.69

$$\frac{1}{3}\,b^2x^3 + abx^2 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

3.49 $\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x} dx$

Optimal. Leaf size=49

$$-bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+\frac{1}{2}\tanh^{-1}(\tanh(a+bx))^2+\log(x)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2$$

```
[Out] -(b*x*(b*x - ArcTanh[Tanh[a + b*x]])) + ArcTanh[Tanh[a + b*x]]^2/2 + (b*x -
 ArcTanh[Tanh[a + b*x]])^2*Log[x]
```

Rubi [A] time = 0.033532, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2159, 2158, 29}

$$-bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+\frac{1}{2}\tanh^{-1}(\tanh(a+bx))^2+\log(x)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^2/x,x]
```

```
[Out] -(b*x*(b*x - ArcTanh[Tanh[a + b*x]])) + ArcTanh[Tanh[a + b*x]]^2/2 + (b*x -
 ArcTanh[Tanh[a + b*x]])^2*Log[x]
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x}\,dx = \frac{1}{2}\tanh^{-1}(\tanh(a+bx))^2 - \left(bx-\tanh^{-1}(\tanh(a+bx))\right)\int \frac{\tanh^{-1}(\tanh(a+bx))}{x}\,dx$$
$$= -bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right) + \frac{1}{2}\tanh^{-1}(\tanh(a+bx))^2 - \left(\left(bx-\tanh^{-1}(\tanh(a\right.\right.$$
$$= -bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right) + \frac{1}{2}\tanh^{-1}(\tanh(a+bx))^2 + \left(bx-\tanh^{-1}(\tanh(a+\right.$$

Mathematica [A] time = 0.0408171, size = 53, normalized size = 1.08

$$\frac{1}{2}(a+bx)^2 - (a+bx)\left(-2\tanh^{-1}(\tanh(a+bx)) + a + 2bx\right) + \log(bx)\left(\tanh^{-1}(\tanh(a+bx)) - bx\right)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^2/x,x]
```

```
[Out] (a + b*x)^2/2 - (a + b*x)*(a + 2*b*x - 2*ArcTanh[Tanh[a + b*x]]) + (-(b*x)
+ ArcTanh[Tanh[a + b*x]])^2*Log[b*x]
```

Maple [A] time = 0.036, size = 55, normalized size = 1.1

$$\ln(x)\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^2 + b^2x^2\ln(x) - \frac{3\,b^2x^2}{2} - 2\,b\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\ln(x)\,x + 2\,b\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2/x,x)
```

```
[Out] ln(x)*arctanh(tanh(b*x+a))^2+b^2*x^2*ln(x)-3/2*b^2*x^2-2*b*arctanh(tanh(b*x
+a))*ln(x)*x+2*b*arctanh(tanh(b*x+a))*x
```

Maxima [A] time = 2.35913, size = 27, normalized size = 0.55

$$\frac{1}{2}\,b^2x^2 + 2\,abx + a^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2 + 2*a*b*x + a^2*log(x)
```

Fricas [A] time = 1.47833, size = 49, normalized size = 1.

$$\frac{1}{2}\,b^2x^2 + 2\,abx + a^2\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x,x, algorithm="fricas")
```

```
[Out] 1/2*b^2*x^2 + 2*a*b*x + a^2*log(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^2\left(\tanh\left(a + bx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2/x,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**2/x, x)
```

Giac [A] time = 1.12558, size = 28, normalized size = 0.57

$$\frac{1}{2}\,b^2x^2 + 2\,abx + a^2\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x,x, algorithm="giac")
```

```
[Out] 1/2*b^2*x^2 + 2*a*b*x + a^2*log(abs(x))
```

3.50 $\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^2} dx$

Optimal. Leaf size=39

$$-\frac{\tanh^{-1}(\tanh(a+bx))^2}{x} - 2b\log(x)\left(bx - \tanh^{-1}(\tanh(a+bx))\right) + 2b^2x$$

```
[Out] 2*b^2*x - ArcTanh[Tanh[a + b*x]]^2/x - 2*b*(b*x - ArcTanh[Tanh[a + b*x]])*L
og[x]
```

Rubi [A] time = 0.0243156, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2158, 29}

$$-\frac{\tanh^{-1}(\tanh(a+bx))^2}{x} - 2b\log(x)\left(bx - \tanh^{-1}(\tanh(a+bx))\right) + 2b^2x$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^2/x^2,x]
```

```
[Out] 2*b^2*x - ArcTanh[Tanh[a + b*x]]^2/x - 2*b*(b*x - ArcTanh[Tanh[a + b*x]])*L
og[x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^2}\,dx &= -\frac{\tanh^{-1}(\tanh(a+bx))^2}{x} + (2b)\int \frac{\tanh^{-1}(\tanh(a+bx))}{x}\,dx\\ &= 2b^2x - \frac{\tanh^{-1}(\tanh(a+bx))^2}{x} - \left(2b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\right)\int \frac{1}{x}\,dx\\ &= 2b^2x - \frac{\tanh^{-1}(\tanh(a+bx))^2}{x} - 2b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log(x)\end{aligned}$$

Mathematica [A] time = 0.0449575, size = 37, normalized size = 0.95

$$-\frac{\tanh^{-1}(\tanh(a+bx))^2}{x} + 2b(\log(x)+1)\tanh^{-1}(\tanh(a+bx)) - 2b^2x\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^2/x^2,x]
```

```
[Out] -(ArcTanh[Tanh[a + b*x]]^2/x) - 2*b^2*x*Log[x] + 2*b*ArcTanh[Tanh[a + b*x]]
*(1 + Log[x])
```

Maple [A] time = 0.04, size = 42, normalized size = 1.1

$$-\frac{(\mathrm{Artanh}(\tanh(bx+a)))^2}{x} - 2\ln(x)xb^2 + 2\ln(x)\mathrm{Artanh}(\tanh(bx+a))b + 2b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2/x^2,x)
```

```
[Out] -arctanh(tanh(b*x+a))^2/x-2*ln(x)*x*b^2+2*ln(x)*arctanh(tanh(b*x+a))*b+2*b^
2*x
```

Maxima [A] time = 1.19392, size = 73, normalized size = 1.87

$$2\,b\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)\log\left(x\right)-2\left(b\left(x+\frac{a}{b}\right)\log\left(x\right)-b\left(x+\frac{a\log\left(x\right)}{b}\right)\right)b-\frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^2,x, algorithm="maxima")
```

```
[Out] 2*b*arctanh(tanh(b*x + a))*log(x) - 2*(b*(x + a/b)*log(x) - b*(x + a*log(x)
/b))*b - arctanh(tanh(b*x + a))^2/x
```

Fricas [A] time = 1.48802, size = 49, normalized size = 1.26

$$\frac{b^{2}x^{2}+2\,abx\log\left(x\right)-a^{2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^2,x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + 2*a*b*x*log(x) - a^2)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atanh}^{2}\left(\tanh\left(a+bx\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2/x**2,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**2/x**2, x)
```

Giac [A] time = 1.13201, size = 28, normalized size = 0.72

$$b^{2}x+2\,ab\log\left(|x|\right)-\frac{a^{2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^2,x, algorithm="giac")
```

```
[Out] b^2*x + 2*a*b*log(abs(x)) - a^2/x
```

3.51 $\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^3}\,dx$

Optimal. Leaf size=36

$$-\frac{\tanh^{-1}(\tanh(a+bx))^2}{2x^2}-\frac{b\tanh^{-1}(\tanh(a+bx))}{x}+b^2\log(x)$$

```
[Out] -((b*ArcTanh[Tanh[a + b*x]])/x) - ArcTanh[Tanh[a + b*x]]^2/(2*x^2) + b^2*Lo
g[x]
```

Rubi [A] time = 0.0211679, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 29}

$$-\frac{\tanh^{-1}(\tanh(a+bx))^2}{2x^2}-\frac{b\tanh^{-1}(\tanh(a+bx))}{x}+b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^2/x^3,x]
```

```
[Out] -((b*ArcTanh[Tanh[a + b*x]])/x) - ArcTanh[Tanh[a + b*x]]^2/(2*x^2) + b^2*Lo
g[x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^3}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^2}{2x^2} + b\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^2}\,dx$$
$$= -\frac{b\tanh^{-1}(\tanh(a+bx))}{x} - \frac{\tanh^{-1}(\tanh(a+bx))^2}{2x^2} + b^2\int \frac{1}{x}\,dx$$
$$= -\frac{b\tanh^{-1}(\tanh(a+bx))}{x} - \frac{\tanh^{-1}(\tanh(a+bx))^2}{2x^2} + b^2\log(x)$$

Mathematica [A] time = 0.0326993, size = 42, normalized size = 1.17

$$-\frac{2bx\tanh^{-1}(\tanh(a+bx)) + \tanh^{-1}(\tanh(a+bx))^2 - b^2x^2(2\log(x)+3)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^2/x^3,x]
```

```
[Out] -(2*b*x*ArcTanh[Tanh[a + b*x]] + ArcTanh[Tanh[a + b*x]]^2 - b^2*x^2*(3 + 2*
Log[x]))/(2*x^2)
```

Maple [A] time = 0.036, size = 35, normalized size = 1.

$$-\frac{b\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}{x} - \frac{\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^2}{2\,x^2} + b^2\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2/x^3,x)
```

```
[Out] -b*arctanh(tanh(b*x+a))/x-1/2*arctanh(tanh(b*x+a))^2/x^2+b^2*ln(x)
```

Maxima [A] time = 1.37652, size = 46, normalized size = 1.28

$$b^2\log\left(x\right) - \frac{b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)}{x} - \frac{\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^3,x, algorithm="maxima")
```

```
[Out] b^2*log(x) - b*arctanh(tanh(b*x + a))/x - 1/2*arctanh(tanh(b*x + a))^2/x^2
```

Fricas [A] time = 1.5044, size = 59, normalized size = 1.64

$$\frac{2 b^2 x^2 \log(x) - 4 abx - a^2}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)/x^2
```

Sympy [A] time = 0.822608, size = 32, normalized size = 0.89

$$b^2 \log(x) - \frac{b \operatorname{atanh}(\tanh(a + bx))}{x} - \frac{\operatorname{atanh}^2(\tanh(a + bx))}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2/x**3,x)
```

```
[Out] b**2*log(x) - b*atanh(tanh(a + b*x))/x - atanh(tanh(a + b*x))**2/(2*x**2)
```

Giac [A] time = 1.18836, size = 30, normalized size = 0.83

$$b^2 \log(|x|) - \frac{4 abx + a^2}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^3,x, algorithm="giac")
```

```
[Out] b^2*log(abs(x)) - 1/2*(4*a*b*x + a^2)/x^2
```

3.52 $\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^4} dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}(\tanh(a+bx))^3}{3x^3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] ArcTanh[Tanh[a + b*x]]^3/(3*x^3*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rubi [A] time = 0.0126636, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2167}

$$\frac{\tanh^{-1}(\tanh(a+bx))^3}{3x^3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^2/x^4,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^3/(3*x^3*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^4} dx = \frac{\tanh^{-1}(\tanh(a+bx))^3}{3x^3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A] time = 0.0387074, size = 34, normalized size = 1.1

$$-\frac{bx\tanh^{-1}(\tanh(a+bx))+\tanh^{-1}(\tanh(a+bx))^2+b^2x^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^2/x^4,x]
```

```
[Out] -(b^2*x^2 + b*x*ArcTanh[Tanh[a + b*x]] + ArcTanh[Tanh[a + b*x]]^2)/(3*x^3)
```

Maple [A] time = 0.035, size = 38, normalized size = 1.2

$$-\frac{(\mathrm{Artanh}\,(\tanh\,(bx+a)))^2}{3\,x^3}+\frac{2\,b}{3}\left(-\frac{b}{2\,x}-\frac{\mathrm{Artanh}\,(\tanh\,(bx+a))}{2\,x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2/x^4,x)
```

```
[Out] -1/3*arctanh(tanh(b*x+a))^2/x^3+2/3*b*(-1/2*b/x-1/2*arctanh(tanh(b*x+a))/x^
2)
```

Maxima [A] time = 1.37101, size = 49, normalized size = 1.58

$$-\frac{b^2}{3\,x}-\frac{b\,\mathrm{artanh}\,(\tanh\,(bx+a))}{3\,x^2}-\frac{\mathrm{artanh}\,(\tanh\,(bx+a))^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^2/x - 1/3*b*arctanh(tanh(b*x + a))/x^2 - 1/3*arctanh(tanh(b*x + a))^
2/x^3
```

Fricas [A] time = 1.52019, size = 51, normalized size = 1.65

$$-\frac{3\,b^2x^2+3\,abx+a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/x^3
```

Sympy [A] time = 2.06786, size = 37, normalized size = 1.19

$$-\frac{b^2}{3x} - \frac{b\,\mathrm{atanh}\,(\tanh\,(a+bx))}{3x^2} - \frac{\mathrm{atanh}^2\,(\tanh\,(a+bx))}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2/x**4,x)
```

```
[Out] -b**2/(3*x) - b*atanh(tanh(a + b*x))/(3*x**2) - atanh(tanh(a + b*x))**2/(3*
x**3)
```

Giac [A] time = 1.11319, size = 30, normalized size = 0.97

$$-\frac{3\,b^2x^2 + 3\,abx + a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/x^3
```

3.53 $\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^5}\,dx$

Optimal. Leaf size=42

$$-\frac{b\tanh^{-1}(\tanh(a+bx))}{6x^3}-\frac{\tanh^{-1}(\tanh(a+bx))^2}{4x^4}-\frac{b^2}{12x^2}$$

```
[Out] -b^2/(12*x^2) - (b*ArcTanh[Tanh[a + b*x]])/(6*x^3) - ArcTanh[Tanh[a + b*x]]
^2/(4*x^4)
```

Rubi [A] time = 0.0303805, antiderivative size = 64, normalized size of antiderivative = 1.52, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2171, 2167}

$$\frac{\tanh^{-1}(\tanh(a+bx))^3}{4x^4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{b\tanh^{-1}(\tanh(a+bx))^3}{12x^3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^2/x^5,x]
```

```
[Out] (b*ArcTanh[Tanh[a + b*x]]^3)/(12*x^3*(b*x - ArcTanh[Tanh[a + b*x]])^2) + Ar
cTanh[Tanh[a + b*x]]^3/(4*x^4*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^5}\,dx = \frac{\tanh^{-1}(\tanh(a+bx))^3}{4x^4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{b\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^4}\,dx}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$
$$= \frac{b\tanh^{-1}(\tanh(a+bx))^3}{12x^3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{\tanh^{-1}(\tanh(a+bx))^3}{4x^4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A] time = 0.0289113, size = 37, normalized size = 0.88

$$-\frac{2bx\tanh^{-1}(\tanh(a+bx)) + 3\tanh^{-1}(\tanh(a+bx))^2 + b^2x^2}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^2/x^5,x]
```

```
[Out] -(b^2*x^2 + 2*b*x*ArcTanh[Tanh[a + b*x]] + 3*ArcTanh[Tanh[a + b*x]]^2)/(12*
x^4)
```

Maple [A] time = 0.035, size = 38, normalized size = 0.9

$$-\frac{(\mathrm{Artanh}\,(\tanh\,(bx+a)))^2}{4\,x^4} + \frac{b}{2}\left(-\frac{b}{6\,x^2} - \frac{\mathrm{Artanh}\,(\tanh\,(bx+a))}{3\,x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2/x^5,x)
```

```
[Out] -1/4*arctanh(tanh(b*x+a))^2/x^4+1/2*b*(-1/6*b/x^2-1/3*arctanh(tanh(b*x+a))/
x^3)
```

Maxima [A] time = 1.36886, size = 49, normalized size = 1.17

$$-\frac{b^2}{12\,x^2} - \frac{b\,\mathrm{artanh}\,(\tanh\,(bx+a))}{6\,x^3} - \frac{\mathrm{artanh}\,(\tanh\,(bx+a))^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/12*b^2/x^2 - 1/6*b*arctanh(tanh(b*x + a))/x^3 - 1/4*arctanh(tanh(b*x + a
))^2/x^4
```

Fricas [A] time = 1.46397, size = 55, normalized size = 1.31

$$-\frac{6\,b^2x^2+8\,abx+3\,a^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/x^4
```

Sympy [A] time = 8.01543, size = 39, normalized size = 0.93

$$-\frac{b^2}{12x^2}-\frac{b\,\text{atanh}\,(\tanh\,(a+bx))}{6x^3}-\frac{\text{atanh}^2\,(\tanh\,(a+bx))}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2/x**5,x)
```

```
[Out] -b**2/(12*x**2) - b*atanh(tanh(a + b*x))/(6*x**3) - atanh(tanh(a + b*x))**2
/(4*x**4)
```

Giac [A] time = 1.17859, size = 32, normalized size = 0.76

$$-\frac{6\,b^2x^2+8\,abx+3\,a^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^5,x, algorithm="giac")
```

```
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/x^4
```

3.54 $\int x^m \tanh^{-1}(\tanh(a+bx))^3\,dx$

Optimal. Leaf size=110

$$\frac{6b^2x^{m+3}\tanh^{-1}(\tanh(a+bx))}{m^3+6m^2+11m+6}-\frac{3bx^{m+2}\tanh^{-1}(\tanh(a+bx))^2}{m^2+3m+2}+\frac{x^{m+1}\tanh^{-1}(\tanh(a+bx))^3}{m+1}-\frac{6b^3x^{m+}}{(m+1)\left(m^3+9m^2\right.}$$

```
[Out] (-6*b^3*x^(4 + m))/((1 + m)*(24 + 26*m + 9*m^2 + m^3)) + (6*b^2*x^(3 + m)*A
rcTanh[Tanh[a + b*x]])/(6 + 11*m + 6*m^2 + m^3) - (3*b*x^(2 + m)*ArcTanh[Ta
nh[a + b*x]]^2)/(2 + 3*m + m^2) + (x^(1 + m)*ArcTanh[Tanh[a + b*x]]^3)/(1 +
 m)
```

Rubi [A] time = 0.0576198, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$\frac{6b^2x^{m+3}\tanh^{-1}(\tanh(a+bx))}{m^3+6m^2+11m+6}-\frac{3bx^{m+2}\tanh^{-1}(\tanh(a+bx))^2}{m^2+3m+2}+\frac{x^{m+1}\tanh^{-1}(\tanh(a+bx))^3}{m+1}-\frac{6b^3x^{m+}}{(m+1)\left(m^3+9m^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (-6*b^3*x^(4 + m))/((1 + m)*(24 + 26*m + 9*m^2 + m^3)) + (6*b^2*x^(3 + m)*A
rcTanh[Tanh[a + b*x]])/(6 + 11*m + 6*m^2 + m^3) - (3*b*x^(2 + m)*ArcTanh[Ta
nh[a + b*x]]^2)/(2 + 3*m + m^2) + (x^(1 + m)*ArcTanh[Tanh[a + b*x]]^3)/(1 +
 m)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^m \tanh^{-1}(\tanh(a+bx))^3\,dx = \frac{x^{1+m}\tanh^{-1}(\tanh(a+bx))^3}{1+m} - \frac{(3b)\int x^{1+m}\tanh^{-1}(\tanh(a+bx))^2\,dx}{1+m}$$
$$= -\frac{3bx^{2+m}\tanh^{-1}(\tanh(a+bx))^2}{2+3m+m^2} + \frac{x^{1+m}\tanh^{-1}(\tanh(a+bx))^3}{1+m} + \frac{\left(6b^2\right)\int x^{2+m}\tanh$$
$$= \frac{6b^2x^{3+m}\tanh^{-1}(\tanh(a+bx))}{6+11m+6m^2+m^3} - \frac{3bx^{2+m}\tanh^{-1}(\tanh(a+bx))^2}{2+3m+m^2} + \frac{x^{1+m}\tanh^{-1}(\tan}{1+m}$$
$$= -\frac{6b^3x^{4+m}}{(4+m)\left(6+11m+6m^2+m^3\right)} + \frac{6b^2x^{3+m}\tanh^{-1}(\tanh(a+bx))}{6+11m+6m^2+m^3} - \frac{3bx^{2+m}\tanh^{-1}(}{2+3m}$$

Mathematica [A] time = 0.103638, size = 97, normalized size = 0.88

$$\frac{x^{m+1}\left(6b^2(m+4)x^2\tanh^{-1}(\tanh(a+bx)) - 3b\left(m^2+7m+12\right)x\tanh^{-1}(\tanh(a+bx))^2 + \left(m^3+9m^2+26m+24\right)\tan}{(m+1)(m+2)(m+3)(m+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (x^(1 + m)*(-6*b^3*x^3 + 6*b^2*(4 + m)*x^2*ArcTanh[Tanh[a + b*x]] - 3*b*(12
 + 7*m + m^2)*x*ArcTanh[Tanh[a + b*x]]^2 + (24 + 26*m + 9*m^2 + m^3)*ArcTan
h[Tanh[a + b*x]]^3))/((1 + m)*(2 + m)*(3 + m)*(4 + m))
```

Maple [A] time = 0.041, size = 177, normalized size = 1.6

$$\frac{x^4b^3e^{m\ln(x)}}{4+m} + \frac{\left(a^3+3a^2(\text{Artanh}(\tanh(bx+a))-bx-a)+3a(\text{Artanh}(\tanh(bx+a))-bx-a)^2+(\text{Artanh}(\tanh(}{1+m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctanh(tanh(b*x+a))^3,x)
```

```
[Out] b^3/(4+m)*x^4*exp(m*ln(x))+(a^3+3*a^2*(arctanh(tanh(b*x+a))-b*x-a)+3*a*(arc
tanh(tanh(b*x+a))-b*x-a)^2+(arctanh(tanh(b*x+a))-b*x-a)^3)/(1+m)*x*exp(m*ln
```

```
(x))+3*b*(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)
^2)/(2+m)*x^2*exp(m*ln(x))+3*b^2*(arctanh(tanh(b*x+a))-b*x)/(3+m)*x^3*exp(m
*ln(x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.67885, size = 657, normalized size = 5.97

$$\frac{\left(\left(b^3m^3 + 6\,b^3m^2 + 11\,b^3m + 6\,b^3\right)x^4 + 3\left(ab^2m^3 + 7\,ab^2m^2 + 14\,ab^2m + 8\,ab^2\right)x^3 + 3\left(a^2bm^3 + 8\,a^2bm^2 + 19\,a^2bm + 12\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] (((b^3*m^3 + 6*b^3*m^2 + 11*b^3*m + 6*b^3)*x^4 + 3*(a*b^2*m^3 + 7*a*b^2*m^2
 + 14*a*b^2*m + 8*a*b^2)*x^3 + 3*(a^2*b*m^3 + 8*a^2*b*m^2 + 19*a^2*b*m + 12
*a^2*b)*x^2 + (a^3*m^3 + 9*a^3*m^2 + 26*a^3*m + 24*a^3)*x)*cosh(m*log(x)) +
 ((b^3*m^3 + 6*b^3*m^2 + 11*b^3*m + 6*b^3)*x^4 + 3*(a*b^2*m^3 + 7*a*b^2*m^2
 + 14*a*b^2*m + 8*a*b^2)*x^3 + 3*(a^2*b*m^3 + 8*a^2*b*m^2 + 19*a^2*b*m + 12
*a^2*b)*x^2 + (a^3*m^3 + 9*a^3*m^2 + 26*a^3*m + 24*a^3)*x)*sinh(m*log(x)))/
(m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atanh(tanh(b*x+a))**3,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{artanh}(\tanh(bx+a))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*arctanh(tanh(b*x + a))^3, x)
```

3.55 $\int x^3 \tanh^{-1}(\tanh(a+bx))^3\,dx$

Optimal. Leaf size=61

$$\frac{1}{20}b^2x^6\tanh^{-1}(\tanh(a+bx))-\frac{3}{20}bx^5\tanh^{-1}(\tanh(a+bx))^2+\frac{1}{4}x^4\tanh^{-1}(\tanh(a+bx))^3-\frac{1}{140}b^3x^7$$

```
[Out] -(b^3*x^7)/140 + (b^2*x^6*ArcTanh[Tanh[a + b*x]])/20 - (3*b*x^5*ArcTanh[Tan
h[a + b*x]]^2)/20 + (x^4*ArcTanh[Tanh[a + b*x]]^3)/4
```

Rubi [A] time = 0.04446, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$\frac{1}{20}b^2x^6\tanh^{-1}(\tanh(a+bx))-\frac{3}{20}bx^5\tanh^{-1}(\tanh(a+bx))^2+\frac{1}{4}x^4\tanh^{-1}(\tanh(a+bx))^3-\frac{1}{140}b^3x^7$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] -(b^3*x^7)/140 + (b^2*x^6*ArcTanh[Tanh[a + b*x]])/20 - (3*b*x^5*ArcTanh[Tan
h[a + b*x]]^2)/20 + (x^4*ArcTanh[Tanh[a + b*x]]^3)/4
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^3 \tanh^{-1}(\tanh(a+bx))^3\,dx = \frac{1}{4}x^4\tanh^{-1}(\tanh(a+bx))^3 - \frac{1}{4}(3b)\int x^4\tanh^{-1}(\tanh(a+bx))^2\,dx$$
$$= -\frac{3}{20}bx^5\tanh^{-1}(\tanh(a+bx))^2 + \frac{1}{4}x^4\tanh^{-1}(\tanh(a+bx))^3 + \frac{1}{10}\left(3b^2\right)\int x^5\tanh^{-}$$
$$= \frac{1}{20}b^2x^6\tanh^{-1}(\tanh(a+bx)) - \frac{3}{20}bx^5\tanh^{-1}(\tanh(a+bx))^2 + \frac{1}{4}x^4\tanh^{-1}(\tanh(a$$
$$= -\frac{1}{140}b^3x^7 + \frac{1}{20}b^2x^6\tanh^{-1}(\tanh(a+bx)) - \frac{3}{20}bx^5\tanh^{-1}(\tanh(a+bx))^2 + \frac{1}{4}x^4\tan$$

Mathematica [A] time = 0.0232976, size = 54, normalized size = 0.89

$$-\frac{1}{140}x^4\left(-7b^2x^2\tanh^{-1}(\tanh(a+bx)) + 21bx\tanh^{-1}(\tanh(a+bx))^2 - 35\tanh^{-1}(\tanh(a+bx))^3 + b^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] -(x^4*(b^3*x^3 - 7*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 21*b*x*ArcTanh[Tanh[a +
 b*x]]^2 - 35*ArcTanh[Tanh[a + b*x]]^3))/140
```

Maple [A] time = 0.038, size = 56, normalized size = 0.9

$$\frac{x^4\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^3}{4} - \frac{3b}{4}\left(\frac{x^5\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^2}{5} - \frac{2b}{5}\left(\frac{x^6\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}{6} - \frac{x^7b}{42}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(tanh(b*x+a))^3,x)
```

```
[Out] 1/4*x^4*arctanh(tanh(b*x+a))^3-3/4*b*(1/5*x^5*arctanh(tanh(b*x+a))^2-2/5*b*
(1/6*x^6*arctanh(tanh(b*x+a))-1/42*x^7*b))
```

Maxima [A] time = 1.56287, size = 73, normalized size = 1.2

$$-\frac{3}{20}bx^5\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2 + \frac{1}{4}x^4\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^3 - \frac{1}{140}\left(b^2x^7 - 7bx^6\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -3/20*b*x^5*arctanh(tanh(b*x + a))^2 + 1/4*x^4*arctanh(tanh(b*x + a))^3 - 1
/140*(b^2*x^7 - 7*b*x^6*arctanh(tanh(b*x + a)))*b
```

Fricas [A] time = 1.42188, size = 80, normalized size = 1.31

$$\frac{1}{7}b^3x^7 + \frac{1}{2}ab^2x^6 + \frac{3}{5}a^2bx^5 + \frac{1}{4}a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/7*b^3*x^7 + 1/2*a*b^2*x^6 + 3/5*a^2*b*x^5 + 1/4*a^3*x^4
```

Sympy [A] time = 6.78503, size = 80, normalized size = 1.31

$$\begin{cases} \frac{x^3 \operatorname{atanh}^4(\tanh(a+bx))}{4b} - \frac{3x^2 \operatorname{atanh}^5(\tanh(a+bx))}{20b^2} + \frac{x \operatorname{atanh}^6(\tanh(a+bx))}{20b^3} - \frac{\operatorname{atanh}^7(\tanh(a+bx))}{140b^4} & \text{for } b \neq 0 \\ \frac{x^4 \operatorname{atanh}^3(\tanh(a))}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(tanh(b*x+a))**3,x)
```

```
[Out] Piecewise((x**3*atanh(tanh(a + b*x))**4/(4*b) - 3*x**2*atanh(tanh(a + b*x))
**5/(20*b**2) + x*atanh(tanh(a + b*x))**6/(20*b**3) - atanh(tanh(a + b*x))*
*7/(140*b**4), Ne(b, 0)), (x**4*atanh(tanh(a))**3/4, True))
```

Giac [A] time = 1.14231, size = 47, normalized size = 0.77

$$\frac{1}{7}b^3x^7 + \frac{1}{2}ab^2x^6 + \frac{3}{5}a^2bx^5 + \frac{1}{4}a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 1/7*b^3*x^7 + 1/2*a*b^2*x^6 + 3/5*a^2*b*x^5 + 1/4*a^3*x^4
```

3.56 $\int x^2 \tanh^{-1}(\tanh(a+bx))^3 \, dx$

Optimal. Leaf size=53

$$\frac{\tanh^{-1}(\tanh(a+bx))^6}{60b^3} - \frac{x\tanh^{-1}(\tanh(a+bx))^5}{10b^2} + \frac{x^2\tanh^{-1}(\tanh(a+bx))^4}{4b}$$

```
[Out] (x^2*ArcTanh[Tanh[a + b*x]]^4)/(4*b) - (x*ArcTanh[Tanh[a + b*x]]^5)/(10*b^2
) + ArcTanh[Tanh[a + b*x]]^6/(60*b^3)
```

Rubi [A] time = 0.0286999, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2157, 30}

$$\frac{\tanh^{-1}(\tanh(a+bx))^6}{60b^3} - \frac{x\tanh^{-1}(\tanh(a+bx))^5}{10b^2} + \frac{x^2\tanh^{-1}(\tanh(a+bx))^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (x^2*ArcTanh[Tanh[a + b*x]]^4)/(4*b) - (x*ArcTanh[Tanh[a + b*x]]^5)/(10*b^2
) + ArcTanh[Tanh[a + b*x]]^6/(60*b^3)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^2\tanh^{-1}(\tanh(a+bx))^3\,dx = \frac{x^2\tanh^{-1}(\tanh(a+bx))^4}{4b} - \frac{\int x\tanh^{-1}(\tanh(a+bx))^4\,dx}{2b}$$
$$= \frac{x^2\tanh^{-1}(\tanh(a+bx))^4}{4b} - \frac{x\tanh^{-1}(\tanh(a+bx))^5}{10b^2} + \frac{\int \tanh^{-1}(\tanh(a+bx))^5\,dx}{10b^2}$$
$$= \frac{x^2\tanh^{-1}(\tanh(a+bx))^4}{4b} - \frac{x\tanh^{-1}(\tanh(a+bx))^5}{10b^2} + \frac{\text{Subst}\left(\int x^5\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{10b^3}$$
$$= \frac{x^2\tanh^{-1}(\tanh(a+bx))^4}{4b} - \frac{x\tanh^{-1}(\tanh(a+bx))^5}{10b^2} + \frac{\tanh^{-1}(\tanh(a+bx))^6}{60b^3}$$

Mathematica [A] time = 0.0220387, size = 54, normalized size = 1.02

$$-\frac{1}{60}x^3\left(-6b^2x^2\tanh^{-1}(\tanh(a+bx))+15bx\tanh^{-1}(\tanh(a+bx))^2-20\tanh^{-1}(\tanh(a+bx))^3+b^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] -(x^3*(b^3*x^3 - 6*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 15*b*x*ArcTanh[Tanh[a +
 b*x]]^2 - 20*ArcTanh[Tanh[a + b*x]]^3))/60
```

Maple [A] time = 0.036, size = 56, normalized size = 1.1

$$\frac{x^3(\text{Artanh}(\tanh(bx+a)))^3}{3} - b\left(\frac{x^4(\text{Artanh}(\tanh(bx+a)))^2}{4} - \frac{b}{2}\left(\frac{x^5\text{Artanh}(\tanh(bx+a))}{5} - \frac{x^6b}{30}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(tanh(b*x+a))^3,x)
```

```
[Out] 1/3*x^3*arctanh(tanh(b*x+a))^3-b*(1/4*x^4*arctanh(tanh(b*x+a))^2-1/2*b*(1/5
*x^5*arctanh(tanh(b*x+a))-1/30*x^6*b))
```

Maxima [A] time = 1.56134, size = 73, normalized size = 1.38

$$-\frac{1}{4}\,bx^4\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2+\frac{1}{3}\,x^3\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^3-\frac{1}{60}\,\left(b^2x^6-6\,bx^5\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -1/4*b*x^4*arctanh(tanh(b*x + a))^2 + 1/3*x^3*arctanh(tanh(b*x + a))^3 - 1/
60*(b^2*x^6 - 6*b*x^5*arctanh(tanh(b*x + a)))*b
```

Fricas [A] time = 1.4896, size = 80, normalized size = 1.51

$$\frac{1}{6}\,b^3x^6+\frac{3}{5}\,ab^2x^5+\frac{3}{4}\,a^2bx^4+\frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/6*b^3*x^6 + 3/5*a*b^2*x^5 + 3/4*a^2*b*x^4 + 1/3*a^3*x^3
```

Sympy [A] time = 2.11896, size = 56, normalized size = 1.06

$$-\frac{b^3x^6}{60}+\frac{b^2x^5\,\mathrm{atanh}\left(\tanh\left(a+bx\right)\right)}{10}-\frac{bx^4\,\mathrm{atanh}^2\left(\tanh\left(a+bx\right)\right)}{4}+\frac{x^3\,\mathrm{atanh}^3\left(\tanh\left(a+bx\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(tanh(b*x+a))**3,x)
```

```
[Out] -b**3*x**6/60 + b**2*x**5*atanh(tanh(a + b*x))/10 - b*x**4*atanh(tanh(a + b
*x))**2/4 + x**3*atanh(tanh(a + b*x))**3/3
```

Giac [A] time = 1.15411, size = 47, normalized size = 0.89

$$\frac{1}{6}\,b^3x^6+\frac{3}{5}\,ab^2x^5+\frac{3}{4}\,a^2bx^4+\frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 1/6*b^3*x^6 + 3/5*a*b^2*x^5 + 3/4*a^2*b*x^4 + 1/3*a^3*x^3
```

3.57 $\int x \tanh^{-1}(\tanh(a+bx))^3 \, dx$

Optimal. Leaf size=34

$$\frac{x \tanh^{-1}(\tanh(a+bx))^4}{4b} - \frac{\tanh^{-1}(\tanh(a+bx))^5}{20b^2}$$

```
[Out] (x*ArcTanh[Tanh[a + b*x]]^4)/(4*b) - ArcTanh[Tanh[a + b*x]]^5/(20*b^2)
```

Rubi [A] time = 0.0139659, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2168, 2157, 30}

$$\frac{x \tanh^{-1}(\tanh(a+bx))^4}{4b} - \frac{\tanh^{-1}(\tanh(a+bx))^5}{20b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (x*ArcTanh[Tanh[a + b*x]]^4)/(4*b) - ArcTanh[Tanh[a + b*x]]^5/(20*b^2)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x\tanh^{-1}(\tanh(a+bx))^3\,dx = \frac{x\tanh^{-1}(\tanh(a+bx))^4}{4b} - \frac{\int \tanh^{-1}(\tanh(a+bx))^4\,dx}{4b}$$
$$= \frac{x\tanh^{-1}(\tanh(a+bx))^4}{4b} - \frac{\text{Subst}\left(\int x^4\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{4b^2}$$
$$= \frac{x\tanh^{-1}(\tanh(a+bx))^4}{4b} - \frac{\tanh^{-1}(\tanh(a+bx))^5}{20b^2}$$

Mathematica [B] time = 0.0727119, size = 99, normalized size = 2.91

$$\frac{(a+bx)\left(10\left(2a^2+abx-b^2x^2\right)\tanh^{-1}(\tanh(a+bx))^2+(4a-bx)(a+bx)^3-5(3a-bx)(a+bx)^2\tanh^{-1}(\tanh(a+bx))\right.}{20b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] ((a + b*x)*((4*a - b*x)*(a + b*x)^3 - 5*(3*a - b*x)*(a + b*x)^2*ArcTanh[Tan
h[a + b*x]] + 10*(2*a^2 + a*b*x - b^2*x^2)*ArcTanh[Tanh[a + b*x]]^2 - 10*(a
 - b*x)*ArcTanh[Tanh[a + b*x]]^3))/(20*b^2)
```

Maple [A] time = 0.037, size = 56, normalized size = 1.7

$$\frac{x^2\left(\text{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^3}{2} - \frac{3\,b}{2}\left(\frac{x^3\left(\text{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^2}{3} - \frac{2\,b}{3}\left(\frac{x^4\text{Artanh}\left(\tanh\left(bx+a\right)\right)}{4} - \frac{bx^5}{20}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(tanh(b*x+a))^3,x)
```

```
[Out] 1/2*x^2*arctanh(tanh(b*x+a))^3-3/2*b*(1/3*x^3*arctanh(tanh(b*x+a))^2-2/3*b*
(1/4*x^4*arctanh(tanh(b*x+a))-1/20*b*x^5))
```

Maxima [A] time = 1.54725, size = 73, normalized size = 2.15

$$-\frac{1}{2}\,bx^3\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^2 + \frac{1}{2}\,x^2\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^3 - \frac{1}{20}\left(b^2x^5 - 5\,bx^4\,\text{artanh}\left(\tanh\left(bx+a\right)\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -1/2*b*x^3*arctanh(tanh(b*x + a))^2 + 1/2*x^2*arctanh(tanh(b*x + a))^3 - 1/
20*(b^2*x^5 - 5*b*x^4*arctanh(tanh(b*x + a)))*b
```

Fricas [A] time = 1.42563, size = 74, normalized size = 2.18

$$\frac{1}{5}\,b^3x^5+\frac{3}{4}\,ab^2x^4+a^2bx^3+\frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/5*b^3*x^5 + 3/4*a*b^2*x^4 + a^2*b*x^3 + 1/2*a^3*x^2
```

Sympy [A] time = 6.83063, size = 56, normalized size = 1.65

$$-\frac{b^3x^5}{20}+\frac{b^2x^4\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{4}-\frac{bx^3\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{2}+\frac{x^2\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(tanh(b*x+a))**3,x)
```

```
[Out] -b**3*x**5/20 + b**2*x**4*atanh(tanh(a + b*x))/4 - b*x**3*atanh(tanh(a + b*
x))**2/2 + x**2*atanh(tanh(a + b*x))**3/2
```

Giac [A] time = 1.12829, size = 46, normalized size = 1.35

$$\frac{1}{5}\,b^3x^5+\frac{3}{4}\,ab^2x^4+a^2bx^3+\frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 1/5*b^3*x^5 + 3/4*a*b^2*x^4 + a^2*b*x^3 + 1/2*a^3*x^2
```

3.58 $\int \tanh^{-1}(\tanh(a+bx))^3\,dx$

Optimal. Leaf size=16

$$\frac{\tanh^{-1}(\tanh(a+bx))^4}{4b}$$

```
[Out] ArcTanh[Tanh[a + b*x]]^4/(4*b)
```

Rubi [A] time = 0.0044888, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2157, 30}

$$\frac{\tanh^{-1}(\tanh(a+bx))^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^4/(4*b)
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(\tanh(a+bx))^3\,dx &= \frac{\text{Subst}\left(\int x^3\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{\tanh^{-1}(\tanh(a+bx))^4}{4b}\end{aligned}$$

Mathematica [A] time = 0.0057057, size = 16, normalized size = 1.

$$\frac{\tanh^{-1}(\tanh(a+bx))^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^4/(4*b)
```

Maple [A] time = 0.028, size = 15, normalized size = 0.9

$$\frac{\left(\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^4}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3,x)
```

```
[Out] 1/4*arctanh(tanh(b*x+a))^4/b
```

Maxima [B] time = 1.53072, size = 69, normalized size = 4.31

$$-\frac{3}{2}\,bx^2\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2+x\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3-\frac{1}{4}\left(b^2x^4-4\,bx^3\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -3/2*b*x^2*arctanh(tanh(b*x + a))^2 + x*arctanh(tanh(b*x + a))^3 - 1/4*(b^2
*x^4 - 4*b*x^3*arctanh(tanh(b*x + a)))*b
```

Fricas [B] time = 1.41287, size = 66, normalized size = 4.12

$$\frac{1}{4}\,b^3x^4+ab^2x^3+\frac{3}{2}\,a^2bx^2+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

Sympy [A] time = 3.0068, size = 20, normalized size = 1.25

$$\begin{cases} \frac{\operatorname{atanh}^{4}{\left(\tanh{\left(a+bx \right)} \right)}}{4b} & \text{for}\: b \neq 0 \\ x \operatorname{atanh}^{3}{\left(\tanh{\left(a \right)} \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3,x)
```

```
[Out] Piecewise((atanh(tanh(a + b*x))**4/(4*b), Ne(b, 0)), (x*atanh(tanh(a))**3,
True))
```

Giac [B] time = 1.11, size = 42, normalized size = 2.62

$$\frac{1}{4} b^{3} x^{4} + a b^{2} x^{3} + \frac{3}{2} a^{2} b x^{2} + a^{3} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

3.59 $\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x} \, dx$

Optimal. Leaf size=77

$$bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2-\frac{1}{2}\tanh^{-1}(\tanh(a+bx))^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+\frac{1}{3}\tanh^{-1}(\tanh(a+bx))^3 -$$

```
[Out] b*x*(b*x - ArcTanh[Tanh[a + b*x]])^2 - ((b*x - ArcTanh[Tanh[a + b*x]])*ArcT
anh[Tanh[a + b*x]]^2)/2 + ArcTanh[Tanh[a + b*x]]^3/3 - (b*x - ArcTanh[Tanh[
a + b*x]])^3*Log[x]
```

Rubi [A] time = 0.0809734, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2159, 2158, 29}

$$bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2-\frac{1}{2}\tanh^{-1}(\tanh(a+bx))^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+\frac{1}{3}\tanh^{-1}(\tanh(a+bx))^3 -$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3/x,x]
```

```
[Out] b*x*(b*x - ArcTanh[Tanh[a + b*x]])^2 - ((b*x - ArcTanh[Tanh[a + b*x]])*ArcT
anh[Tanh[a + b*x]]^2)/2 + ArcTanh[Tanh[a + b*x]]^3/3 - (b*x - ArcTanh[Tanh[
a + b*x]])^3*Log[x]
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x}\,dx = \frac{1}{3}\tanh^{-1}(\tanh(a+bx))^3 - \left(bx-\tanh^{-1}(\tanh(a+bx))\right)\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x}\,dx$$
$$= -\frac{1}{2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2 + \frac{1}{3}\tanh^{-1}(\tanh(a+bx))^3 -$$
$$= bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 - \frac{1}{2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+b$$
$$= bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 - \frac{1}{2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+b$$

Mathematica [A] time = 0.0576073, size = 104, normalized size = 1.35

$$(a+bx)\left(a^2-3a\left(-\tanh^{-1}(\tanh(a+bx))+a+bx\right)+3\left(-\tanh^{-1}(\tanh(a+bx))+a+bx\right)^2\right)+\frac{1}{3}(a+bx)^3-\frac{1}{2}(a+bx$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3/x,x]
```

```
[Out] (a + b*x)^3/3 + (a + b*x)*(a^2 - 3*a*(a + b*x - ArcTanh[Tanh[a + b*x]]) + 3
*(a + b*x - ArcTanh[Tanh[a + b*x]])^2) - ((a + b*x)^2*(2*a + 3*b*x - 3*ArcT
anh[Tanh[a + b*x]]))/2 + (-(b*x) + ArcTanh[Tanh[a + b*x]])^3*Log[b*x]
```

Maple [A] time = 0.037, size = 92, normalized size = 1.2

$$\ln(x)\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^3+3\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\ln(x)\,x^2b^2-\frac{9\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)x^2b^2}{2}-3\,b\,(\mathrm{Artanl}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3/x,x)
```

```
[Out] ln(x)*arctanh(tanh(b*x+a))^3+3*arctanh(tanh(b*x+a))*ln(x)*x^2*b^2-9/2*arcta
nh(tanh(b*x+a))*x^2*b^2-3*b*arctanh(tanh(b*x+a))^2*ln(x)*x-b^3*x^3*ln(x)+11
/6*x^3*b^3+3*b*arctanh(tanh(b*x+a))^2*x
```

Maxima [A] time = 2.44981, size = 42, normalized size = 0.55

$$\frac{1}{3}\,b^3x^3 + \frac{3}{2}\,ab^2x^2 + 3\,a^2bx + a^3\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(x)
```

Fricas [A] time = 1.57665, size = 73, normalized size = 0.95

$$\frac{1}{3}\,b^3x^3 + \frac{3}{2}\,ab^2x^2 + 3\,a^2bx + a^3\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x,x, algorithm="fricas")
```

```
[Out] 1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^{3}\left(\tanh\left(a + bx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3/x,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**3/x, x)
```

Giac [A] time = 1.15529, size = 43, normalized size = 0.56

$$\frac{1}{3}\,b^3x^3 + \frac{3}{2}\,ab^2x^2 + 3\,a^2bx + a^3\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x,x, algorithm="giac")
```

```
[Out] 1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(abs(x))
```

3.60 $\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^2} dx$

Optimal. Leaf size=68

$$-3b^2x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)-\frac{\tanh^{-1}(\tanh(a+bx))^3}{x}+\frac{3}{2}b\tanh^{-1}(\tanh(a+bx))^2+3b\log(x)\left(bx-\tanh^{-1}(\tan$$

```
[Out] -3*b^2*x*(b*x - ArcTanh[Tanh[a + b*x]]) + (3*b*ArcTanh[Tanh[a + b*x]]^2)/2
- ArcTanh[Tanh[a + b*x]]^3/x + 3*b*(b*x - ArcTanh[Tanh[a + b*x]])^2*Log[x]
```

Rubi [A] time = 0.0412704, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2168, 2159, 2158, 29}

$$-3b^2x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)-\frac{\tanh^{-1}(\tanh(a+bx))^3}{x}+\frac{3}{2}b\tanh^{-1}(\tanh(a+bx))^2+3b\log(x)\left(bx-\tanh^{-1}(\tan$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3/x^2,x]
```


```
[Out] -3*b^2*x*(b*x - ArcTanh[Tanh[a + b*x]]) + (3*b*ArcTanh[Tanh[a + b*x]]^2)/2
- ArcTanh[Tanh[a + b*x]]^3/x + 3*b*(b*x - ArcTanh[Tanh[a + b*x]])^2*Log[x]
```


Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^2}\,dx &= -\frac{\tanh^{-1}(\tanh(a+bx))^3}{x} + (3b)\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x}\,dx\\ &= \frac{3}{2}b\tanh^{-1}(\tanh(a+bx))^2 - \frac{\tanh^{-1}(\tanh(a+bx))^3}{x} - \left(3b\left(bx - \tanh^{-1}(\tanh(a+bx\right.\right.\\ &= -3b^2x\left(bx - \tanh^{-1}(\tanh(a+bx))\right) + \frac{3}{2}b\tanh^{-1}(\tanh(a+bx))^2 - \frac{\tanh^{-1}(\tanh(a+}{x}\\ &= -3b^2x\left(bx - \tanh^{-1}(\tanh(a+bx))\right) + \frac{3}{2}b\tanh^{-1}(\tanh(a+bx))^2 - \frac{\tanh^{-1}(\tanh(a+}{x}\end{aligned}$$

Mathematica [A] time = 0.0381505, size = 62, normalized size = 0.91

$$-6b^2x\log(x)\tanh^{-1}(\tanh(a+bx)) - \frac{\tanh^{-1}(\tanh(a+bx))^3}{x} + 3b(\log(x)+1)\tanh^{-1}(\tanh(a+bx))^2 + \frac{3}{2}b^3x^2(2\log(x$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3/x^2,x]
```

```
[Out] -(ArcTanh[Tanh[a + b*x]]^3/x) - 6*b^2*x*ArcTanh[Tanh[a + b*x]]*Log[x] + 3*b
*ArcTanh[Tanh[a + b*x]]^2*(1 + Log[x]) + (3*b^3*x^2*(-1 + 2*Log[x]))/2
```

Maple [A] time = 0.04, size = 76, normalized size = 1.1

$$-\frac{(\operatorname{Artanh}(\tanh(bx+a)))^3}{x} + 3\ln(x)(\operatorname{Artanh}(\tanh(bx+a)))^2 b + 3b^3x^2\ln(x) - \frac{9b^3x^2}{2} - 6b^2\operatorname{Artanh}(\tanh(bx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3/x^2,x)
```

```
[Out] -arctanh(tanh(b*x+a))^3/x+3*ln(x)*arctanh(tanh(b*x+a))^2*b+3*b^3*x^2*ln(x)-
9/2*b^3*x^2-6*b^2*arctanh(tanh(b*x+a))*ln(x)*x+6*b^2*arctanh(tanh(b*x+a))*x
```

Maxima [A] time = 2.58652, size = 88, normalized size = 1.29

$$3\,b\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^{2}\log\left(x\right)+\frac{3}{2}\left(b^{2}x^{2}+4\,abx+2\,a^{2}\log\left(x\right)-2\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^{2}\log\left(x\right)\right)b-\frac{\text{artanh}\left(\tanh\left(bx+a\right)\right)^{3}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^2,x, algorithm="maxima")
```

```
[Out] 3*b*arctanh(tanh(b*x + a))^2*log(x) + 3/2*(b^2*x^2 + 4*a*b*x + 2*a^2*log(x)
 - 2*arctanh(tanh(b*x + a))^2*log(x))*b - arctanh(tanh(b*x + a))^3/x
```

Fricas [A] time = 1.50572, size = 78, normalized size = 1.15

$$\frac{b^{3}x^{3}+6\,ab^{2}x^{2}+6\,a^{2}bx\log\left(x\right)-2\,a^{3}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 + 6*a*b^2*x^2 + 6*a^2*b*x*log(x) - 2*a^3)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^{3}\left(\tanh\left(a+bx\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3/x**2,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**3/x**2, x)
```

Giac [A] time = 1.10957, size = 45, normalized size = 0.66

$$\frac{1}{2}b^3x^2 + 3ab^2x + 3a^2b\log(|x|) - \frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^2,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^2 + 3*a*b^2*x + 3*a^2*b*log(abs(x)) - a^3/x
```

3.61 $\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^3} dx$

Optimal. Leaf size=60

$$-3b^2\log(x)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)-\frac{\tanh^{-1}(\tanh(a+bx))^3}{2x^2}-\frac{3b\tanh^{-1}(\tanh(a+bx))^2}{2x}+3b^3x$$

```
[Out] 3*b^3*x - (3*b*ArcTanh[Tanh[a + b*x]]^2)/(2*x) - ArcTanh[Tanh[a + b*x]]^3/(
2*x^2) - 3*b^2*(b*x - ArcTanh[Tanh[a + b*x]])*Log[x]
```

Rubi [A] time = 0.0396589, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2158, 29}

$$-3b^2\log(x)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)-\frac{\tanh^{-1}(\tanh(a+bx))^3}{2x^2}-\frac{3b\tanh^{-1}(\tanh(a+bx))^2}{2x}+3b^3x$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3/x^3,x]
```

```
[Out] 3*b^3*x - (3*b*ArcTanh[Tanh[a + b*x]]^2)/(2*x) - ArcTanh[Tanh[a + b*x]]^3/(
2*x^2) - 3*b^2*(b*x - ArcTanh[Tanh[a + b*x]])*Log[x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^3}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^3}{2x^2} + \frac{1}{2}(3b)\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^2}\,dx$$
$$= -\frac{3b\tanh^{-1}(\tanh(a+bx))^2}{2x} - \frac{\tanh^{-1}(\tanh(a+bx))^3}{2x^2} + \left(3b^2\right)\int \frac{\tanh^{-1}(\tanh(a+bx$$
$$= 3b^3x - \frac{3b\tanh^{-1}(\tanh(a+bx))^2}{2x} - \frac{\tanh^{-1}(\tanh(a+bx))^3}{2x^2} - \left(3b^2\left(bx - \tanh^{-1}(\tanh\right.\right.$$
$$= 3b^3x - \frac{3b\tanh^{-1}(\tanh(a+bx))^2}{2x} - \frac{\tanh^{-1}(\tanh(a+bx))^3}{2x^2} - 3b^2\left(bx - \tanh^{-1}(\tanh\right.$$

Mathematica [A] time = 0.0444846, size = 66, normalized size = 1.1

$$3b^2\log(x)\left(\tanh^{-1}(\tanh(a+bx)) - bx\right) - \frac{\left(\tanh^{-1}(\tanh(a+bx)) - bx\right)^3}{2x^2} - \frac{3b\left(\tanh^{-1}(\tanh(a+bx)) - bx\right)^2}{x} + b^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3/x^3,x]
```

```
[Out] b^3*x - (3*b*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)/x - (-(b*x) + ArcTanh[Tan
h[a + b*x]])^3/(2*x^2) + 3*b^2*(-(b*x) + ArcTanh[Tanh[a + b*x]])*Log[x]
```

Maple [A] time = 0.042, size = 59, normalized size = 1.

$$-\frac{(\mathrm{Artanh}(\tanh(bx+a)))^3}{2x^2} - \frac{3b(\mathrm{Artanh}(\tanh(bx+a)))^2}{2x} - 3\ln(x)xb^3 + 3\,\mathrm{Artanh}(\tanh(bx+a))\ln(x)b^2 + 3b^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3/x^3,x)
```

```
[Out] -1/2*arctanh(tanh(b*x+a))^3/x^2-3/2*b*arctanh(tanh(b*x+a))^2/x-3*ln(x)*x*b^
3+3*arctanh(tanh(b*x+a))*ln(x)*b^2+3*b^3*x
```

Maxima [A] time = 1.42748, size = 97, normalized size = 1.62

$$3\left(b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)\log\left(x\right)-\left(b\left(x+\frac{a}{b}\right)\log\left(x\right)-b\left(x+\frac{a\log\left(x\right)}{b}\right)\right)b\right)b-\frac{3\,b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^{2}}{2\,x}-\frac{\mathrm{artanh}\left(t$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^3,x, algorithm="maxima")
```

```
[Out] 3*(b*arctanh(tanh(b*x + a))*log(x) - (b*(x + a/b)*log(x) - b*(x + a*log(x)/
b))*b)*b - 3/2*b*arctanh(tanh(b*x + a))^2/x - 1/2*arctanh(tanh(b*x + a))^3/
x^2
```

Fricas [A] time = 1.52351, size = 81, normalized size = 1.35

$$\frac{2\,b^{3}x^{3}+6\,ab^{2}x^{2}\log\left(x\right)-6\,a^{2}bx-a^{3}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*x^3 + 6*a*b^2*x^2*log(x) - 6*a^2*b*x - a^3)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\mathrm{atanh}^{3}\left(\tanh\left(a+bx\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3/x**3,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**3/x**3, x)
```

Giac [A] time = 1.14616, size = 42, normalized size = 0.7

$$b^{3}x+3\,ab^{2}\log\left(|x|\right)-\frac{6\,a^{2}bx+a^{3}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^3,x, algorithm="giac")
```

```
[Out] b^3*x + 3*a*b^2*log(abs(x)) - 1/2*(6*a^2*b*x + a^3)/x^2
```

3.62 $\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^4} dx$

Optimal. Leaf size=55

$$-\frac{b^2\tanh^{-1}(\tanh(a+bx))}{x}-\frac{b\tanh^{-1}(\tanh(a+bx))^2}{2x^2}-\frac{\tanh^{-1}(\tanh(a+bx))^3}{3x^3}+b^3\log(x)$$

```
[Out] -((b^2*ArcTanh[Tanh[a + b*x]])/x) - (b*ArcTanh[Tanh[a + b*x]]^2)/(2*x^2) -
ArcTanh[Tanh[a + b*x]]^3/(3*x^3) + b^3*Log[x]
```

Rubi [A] time = 0.0368214, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 29}

$$-\frac{b^2\tanh^{-1}(\tanh(a+bx))}{x}-\frac{b\tanh^{-1}(\tanh(a+bx))^2}{2x^2}-\frac{\tanh^{-1}(\tanh(a+bx))^3}{3x^3}+b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3/x^4,x]
```

```
[Out] -((b^2*ArcTanh[Tanh[a + b*x]])/x) - (b*ArcTanh[Tanh[a + b*x]]^2)/(2*x^2) -
ArcTanh[Tanh[a + b*x]]^3/(3*x^3) + b^3*Log[x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^4}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^3}{3x^3} + b\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^3}\,dx$$
$$= -\frac{b\tanh^{-1}(\tanh(a+bx))^2}{2x^2} - \frac{\tanh^{-1}(\tanh(a+bx))^3}{3x^3} + b^2\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^2}\,dx$$
$$= -\frac{b^2\tanh^{-1}(\tanh(a+bx))}{x} - \frac{b\tanh^{-1}(\tanh(a+bx))^2}{2x^2} - \frac{\tanh^{-1}(\tanh(a+bx))^3}{3x^3} + b^3$$
$$= -\frac{b^2\tanh^{-1}(\tanh(a+bx))}{x} - \frac{b\tanh^{-1}(\tanh(a+bx))^2}{2x^2} - \frac{\tanh^{-1}(\tanh(a+bx))^3}{3x^3} + b^3$$

Mathematica [A] time = 0.0241028, size = 60, normalized size = 1.09

$$\frac{-6b^2x^2\tanh^{-1}(\tanh(a+bx)) - 3bx\tanh^{-1}(\tanh(a+bx))^2 - 2\tanh^{-1}(\tanh(a+bx))^3 + b^3x^3(6\log(x)+11)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3/x^4,x]
```

```
[Out] (-6*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 3*b*x*ArcTanh[Tanh[a + b*x]]^2 - 2*Arc
Tanh[Tanh[a + b*x]]^3 + b^3*x^3*(11 + 6*Log[x]))/(6*x^3)
```

Maple [A] time = 0.037, size = 52, normalized size = 1.

$$-\frac{b^2\mathrm{Artanh}(\tanh(bx+a))}{x} - \frac{b(\mathrm{Artanh}(\tanh(bx+a)))^2}{2x^2} - \frac{(\mathrm{Artanh}(\tanh(bx+a)))^3}{3x^3} + b^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3/x^4,x)
```

```
[Out] -b^2*arctanh(tanh(b*x+a))/x-1/2*b*arctanh(tanh(b*x+a))^2/x^2-1/3*arctanh(ta
nh(b*x+a))^3/x^3+b^3*ln(x)
```

Maxima [A] time = 1.59454, size = 70, normalized size = 1.27

$$\left(b^2\log(x) - \frac{b\,\mathrm{artanh}(\tanh(bx+a))}{x}\right)b - \frac{b\,\mathrm{artanh}(\tanh(bx+a))^2}{2x^2} - \frac{\mathrm{artanh}(\tanh(bx+a))^3}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^4,x, algorithm="maxima")
```

```
[Out] (b^2*log(x) - b*arctanh(tanh(b*x + a))/x)*b - 1/2*b*arctanh(tanh(b*x + a))^
2/x^2 - 1/3*arctanh(tanh(b*x + a))^3/x^3
```

Fricas [A] time = 1.44155, size = 85, normalized size = 1.55

$$\frac{6\,b^3x^3\log\left(x\right)-18\,ab^2x^2-9\,a^2bx-2\,a^3}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*x^3*log(x) - 18*a*b^2*x^2 - 9*a^2*b*x - 2*a^3)/x^3
```

Sympy [A] time = 6.21766, size = 51, normalized size = 0.93

$$b^3\log\left(x\right)-\frac{b^2\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{x}-\frac{b\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{2x^2}-\frac{\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3/x**4,x)
```

```
[Out] b**3*log(x) - b**2*atanh(tanh(a + b*x))/x - b*atanh(tanh(a + b*x))**2/(2*x*
*2) - atanh(tanh(a + b*x))**3/(3*x**3)
```

Giac [A] time = 1.14549, size = 47, normalized size = 0.85

$$b^3\log\left(|x|\right)-\frac{18\,ab^2x^2+9\,a^2bx+2\,a^3}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^4,x, algorithm="giac")
```

```
[Out] b^3*log(abs(x)) - 1/6*(18*a*b^2*x^2 + 9*a^2*b*x + 2*a^3)/x^3
```

3.63 $\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^5}\,dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}(\tanh(a+bx))^4}{4x^4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] ArcTanh[Tanh[a + b*x]]^4/(4*x^4*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rubi [A] time = 0.0132947, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2167}

$$\frac{\tanh^{-1}(\tanh(a+bx))^4}{4x^4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3/x^5,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^4/(4*x^4*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^5}\,dx=\frac{\tanh^{-1}(\tanh(a+bx))^4}{4x^4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A] time = 0.0224993, size = 50, normalized size = 1.61

$$-\frac{b^2x^2\tanh^{-1}(\tanh(a+bx))+bx\tanh^{-1}(\tanh(a+bx))^2+\tanh^{-1}(\tanh(a+bx))^3+b^3x^3}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3/x^5,x]
```

```
[Out] -(b^3*x^3 + b^2*x^2*ArcTanh[Tanh[a + b*x]] + b*x*ArcTanh[Tanh[a + b*x]]^2 +
 ArcTanh[Tanh[a + b*x]]^3)/(4*x^4)
```

Maple [A] time = 0.039, size = 56, normalized size = 1.8

$$-\frac{(\mathrm{Artanh}(\tanh(bx+a)))^3}{4x^4}+\frac{3b}{4}\left(-\frac{(\mathrm{Artanh}(\tanh(bx+a)))^2}{3x^3}+\frac{2b}{3}\left(-\frac{b}{2x}-\frac{\mathrm{Artanh}(\tanh(bx+a))}{2x^2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3/x^5,x)
```

```
[Out] -1/4*arctanh(tanh(b*x+a))^3/x^4+3/4*b*(-1/3*arctanh(tanh(b*x+a))^2/x^3+2/3*
b*(-1/2*b/x-1/2*arctanh(tanh(b*x+a))/x^2))
```

Maxima [A] time = 1.59351, size = 72, normalized size = 2.32

$$-\frac{1}{4}b\left(\frac{b^2}{x}+\frac{b\,\mathrm{artanh}(\tanh(bx+a))}{x^2}\right)-\frac{b\,\mathrm{artanh}(\tanh(bx+a))^2}{4x^3}-\frac{\mathrm{artanh}(\tanh(bx+a))^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*b*(b^2/x + b*arctanh(tanh(b*x + a))/x^2) - 1/4*b*arctanh(tanh(b*x + a)
)^2/x^3 - 1/4*arctanh(tanh(b*x + a))^3/x^4
```

Fricas [A] time = 1.42449, size = 73, normalized size = 2.35

$$-\frac{4b^3x^3+6ab^2x^2+4a^2bx+a^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^5,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/x^4
```

Sympy [B] time = 4.61478, size = 56, normalized size = 1.81

$$-\frac{b^3}{4x} - \frac{b^2 \operatorname{atanh}(\tanh(a+bx))}{4x^2} - \frac{b \operatorname{atanh}^2(\tanh(a+bx))}{4x^3} - \frac{\operatorname{atanh}^3(\tanh(a+bx))}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3/x**5,x)
```

```
[Out] -b**3/(4*x) - b**2*atanh(tanh(a + b*x))/(4*x**2) - b*atanh(tanh(a + b*x))**
2/(4*x**3) - atanh(tanh(a + b*x))**3/(4*x**4)
```

Giac [A] time = 1.1343, size = 45, normalized size = 1.45

$$-\frac{4 b^3 x^3 + 6 a b^2 x^2 + 4 a^2 b x + a^3}{4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^5,x, algorithm="giac")
```

```
[Out] -1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/x^4
```

3.64 $\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^6} dx$

Optimal. Leaf size=64

$$\frac{\tanh^{-1}(\tanh(a+bx))^4}{5x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{b\tanh^{-1}(\tanh(a+bx))^4}{20x^4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

```
[Out] (b*ArcTanh[Tanh[a + b*x]]^4)/(20*x^4*(b*x - ArcTanh[Tanh[a + b*x]])^2) + Ar
cTanh[Tanh[a + b*x]]^4/(5*x^5*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rubi [A] time = 0.0319722, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2171, 2167}

$$\frac{\tanh^{-1}(\tanh(a+bx))^4}{5x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{b\tanh^{-1}(\tanh(a+bx))^4}{20x^4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3/x^6,x]
```

```
[Out] (b*ArcTanh[Tanh[a + b*x]]^4)/(20*x^4*(b*x - ArcTanh[Tanh[a + b*x]])^2) + Ar
cTanh[Tanh[a + b*x]]^4/(5*x^5*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^6}\,dx = \frac{\tanh^{-1}(\tanh(a+bx))^4}{5x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{b\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^5}\,dx}{5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$
$$= \frac{b\tanh^{-1}(\tanh(a+bx))^4}{20x^4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{\tanh^{-1}(\tanh(a+bx))^4}{5x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A] time = 0.0398583, size = 54, normalized size = 0.84

$$-\frac{2b^2x^2\tanh^{-1}(\tanh(a+bx)) + 3bx\tanh^{-1}(\tanh(a+bx))^2 + 4\tanh^{-1}(\tanh(a+bx))^3 + b^3x^3}{20x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3/x^6,x]
```

```
[Out] -(b^3*x^3 + 2*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 3*b*x*ArcTanh[Tanh[a + b*x]]
^2 + 4*ArcTanh[Tanh[a + b*x]]^3)/(20*x^5)
```

Maple [A] time = 0.038, size = 56, normalized size = 0.9

$$-\frac{(\mathrm{Artanh}(\tanh(bx+a)))^3}{5x^5} + \frac{3b}{5}\left(-\frac{(\mathrm{Artanh}(\tanh(bx+a)))^2}{4x^4} + \frac{b}{2}\left(-\frac{b}{6x^2} - \frac{\mathrm{Artanh}(\tanh(bx+a))}{3x^3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3/x^6,x)
```

```
[Out] -1/5*arctanh(tanh(b*x+a))^3/x^5+3/5*b*(-1/4*arctanh(tanh(b*x+a))^2/x^4+1/2*
b*(-1/6*b/x^2-1/3*arctanh(tanh(b*x+a))/x^3))
```

Maxima [A] time = 1.58905, size = 73, normalized size = 1.14

$$-\frac{1}{20}b\left(\frac{b^2}{x^2} + \frac{2b\,\mathrm{artanh}(\tanh(bx+a))}{x^3}\right) - \frac{3b\,\mathrm{artanh}(\tanh(bx+a))^2}{20x^4} - \frac{\mathrm{artanh}(\tanh(bx+a))^3}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^6,x, algorithm="maxima")
```

```
[Out] -1/20*b*(b^2/x^2 + 2*b*arctanh(tanh(b*x + a))/x^3) - 3/20*b*arctanh(tanh(b*
x + a))^2/x^4 - 1/5*arctanh(tanh(b*x + a))^3/x^5
```

Fricas [A] time = 1.44183, size = 81, normalized size = 1.27

$$-\frac{10\,b^3x^3 + 20\,ab^2x^2 + 15\,a^2bx + 4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^6,x, algorithm="fricas")
```

```
[Out] -1/20*(10*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 4*a^3)/x^5
```

Sympy [A] time = 3.85814, size = 60, normalized size = 0.94

$$-\frac{b^3}{20x^2} - \frac{b^2\operatorname{atanh}(\tanh(a+bx))}{10x^3} - \frac{3b\operatorname{atanh}^2(\tanh(a+bx))}{20x^4} - \frac{\operatorname{atanh}^3(\tanh(a+bx))}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3/x**6,x)
```

```
[Out] -b**3/(20*x**2) - b**2*atanh(tanh(a + b*x))/(10*x**3) - 3*b*atanh(tanh(a +
b*x))**2/(20*x**4) - atanh(tanh(a + b*x))**3/(5*x**5)
```

Giac [A] time = 1.14204, size = 47, normalized size = 0.73

$$-\frac{10\,b^3x^3 + 20\,ab^2x^2 + 15\,a^2bx + 4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^6,x, algorithm="giac")
```

```
[Out] -1/20*(10*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 4*a^3)/x^5
```

3.65 $\int x^m \tanh^{-1}(\tanh(a+bx))^4\,dx$

Optimal. Leaf size=154

$$\frac{12b^2x^{m+3}\tanh^{-1}(\tanh(a+bx))^2}{m^3+6m^2+11m+6}-\frac{24b^3x^{m+4}\tanh^{-1}(\tanh(a+bx))}{(m+1)\left(m^3+9m^2+26m+24\right)}-\frac{4bx^{m+2}\tanh^{-1}(\tanh(a+bx))^3}{m^2+3m+2}+\frac{x^{m+1}\tanh^{-1}(t}{m}$$

```
[Out] (24*b^4*x^(5 + m))/((1 + m)*(2 + m)*(3 + m)*(20 + 9*m + m^2)) - (24*b^3*x^(
4 + m)*ArcTanh[Tanh[a + b*x]])/((1 + m)*(24 + 26*m + 9*m^2 + m^3)) + (12*b^
2*x^(3 + m)*ArcTanh[Tanh[a + b*x]]^2)/(6 + 11*m + 6*m^2 + m^3) - (4*b*x^(2
+ m)*ArcTanh[Tanh[a + b*x]]^3)/(2 + 3*m + m^2) + (x^(1 + m)*ArcTanh[Tanh[a
+ b*x]]^4)/(1 + m)
```

Rubi [A] time = 0.0989838, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$\frac{12b^2x^{m+3}\tanh^{-1}(\tanh(a+bx))^2}{m^3+6m^2+11m+6}-\frac{24b^3x^{m+4}\tanh^{-1}(\tanh(a+bx))}{(m+1)\left(m^3+9m^2+26m+24\right)}-\frac{4bx^{m+2}\tanh^{-1}(\tanh(a+bx))^3}{m^2+3m+2}+\frac{x^{m+1}\tanh^{-1}(t}{m}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (24*b^4*x^(5 + m))/((1 + m)*(2 + m)*(3 + m)*(20 + 9*m + m^2)) - (24*b^3*x^(
4 + m)*ArcTanh[Tanh[a + b*x]])/((1 + m)*(24 + 26*m + 9*m^2 + m^3)) + (12*b^
2*x^(3 + m)*ArcTanh[Tanh[a + b*x]]^2)/(6 + 11*m + 6*m^2 + m^3) - (4*b*x^(2
+ m)*ArcTanh[Tanh[a + b*x]]^3)/(2 + 3*m + m^2) + (x^(1 + m)*ArcTanh[Tanh[a
+ b*x]]^4)/(1 + m)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^m \tanh^{-1}(\tanh(a+bx))^4\,dx = \frac{x^{1+m}\tanh^{-1}(\tanh(a+bx))^4}{1+m} - \frac{(4b)\int x^{1+m}\tanh^{-1}(\tanh(a+bx))^3\,dx}{1+m}$$
$$= -\frac{4bx^{2+m}\tanh^{-1}(\tanh(a+bx))^3}{2+3m+m^2} + \frac{x^{1+m}\tanh^{-1}(\tanh(a+bx))^4}{1+m} + \frac{\left(12b^2\right)\int x^{2+m}\,\mathrm{ta}}{2\,\cdot}$$
$$= \frac{12b^2x^{3+m}\tanh^{-1}(\tanh(a+bx))^2}{6+11m+6m^2+m^3} - \frac{4bx^{2+m}\tanh^{-1}(\tanh(a+bx))^3}{2+3m+m^2} + \frac{x^{1+m}\tanh^{-1}(}{1}$$
$$= -\frac{24b^3x^{4+m}\tanh^{-1}(\tanh(a+bx))}{(4+m)\left(6+11m+6m^2+m^3\right)} + \frac{12b^2x^{3+m}\tanh^{-1}(\tanh(a+bx))^2}{6+11m+6m^2+m^3} - \frac{4bx^{2+m}\,\mathrm{ta}}{2}$$
$$= \frac{24b^4x^{5+m}}{(4+m)(5+m)\left(6+11m+6m^2+m^3\right)} - \frac{24b^3x^{4+m}\tanh^{-1}(\tanh(a+bx))}{(4+m)\left(6+11m+6m^2+m^3\right)} + \frac{12b^2x^{3}}{6}$$

Mathematica [A] time = 0.142082, size = 137, normalized size = 0.89

$$\frac{x^{m+1}\left(12b^2\left(m^2+9m+20\right)x^2\tanh^{-1}(\tanh(a+bx))^2 - 24b^3(m+5)x^3\tanh^{-1}(\tanh(a+bx)) - 4b\left(m^3+12m^2+47m\right.}{(m+1)(m+2)(m+3)(m}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (x^(1 + m)*(24*b^4*x^4 - 24*b^3*(5 + m)*x^3*ArcTanh[Tanh[a + b*x]] + 12*b^2
*(20 + 9*m + m^2)*x^2*ArcTanh[Tanh[a + b*x]]^2 - 4*b*(60 + 47*m + 12*m^2 +
m^3)*x*ArcTanh[Tanh[a + b*x]]^3 + (120 + 154*m + 71*m^2 + 14*m^3 + m^4)*Arc
Tanh[Tanh[a + b*x]]^4))/((1 + m)*(2 + m)*(3 + m)*(4 + m)*(5 + m))
```

Maple [A] time = 0.042, size = 278, normalized size = 1.8

$$\frac{b^4x^5\mathrm{e}^{m\ln(x)}}{5+m} + \frac{\left(a^4+4a^3(\mathrm{Artanh}(\tanh(bx+a))-bx-a)+6a^2(\mathrm{Artanh}(\tanh(bx+a))-bx-a)^2+4a(\mathrm{Artanh}(\mathrm{t}\right.}{1+m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctanh(tanh(b*x+a))^4,x)
```

```
[Out] b^4/(5+m)*x^5*exp(m*ln(x))+(a^4+4*a^3*(arctanh(tanh(b*x+a))-b*x-a)+6*a^2*(a
rctanh(tanh(b*x+a))-b*x-a)^2+4*a*(arctanh(tanh(b*x+a))-b*x-a)^3+(arctanh(ta
nh(b*x+a))-b*x-a)^4)/(1+m)*x*exp(m*ln(x))+4*b*(a^3+3*a^2*(arctanh(tanh(b*x+
a))-b*x-a)+3*a*(arctanh(tanh(b*x+a))-b*x-a)^2+(arctanh(tanh(b*x+a))-b*x-a)^
3)/(2+m)*x^2*exp(m*ln(x))+6*b^2*(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arct
anh(tanh(b*x+a))-b*x-a)^2)/(3+m)*x^3*exp(m*ln(x))+4*b^3*(arctanh(tanh(b*x+a
))-b*x)/(4+m)*x^4*exp(m*ln(x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.593, size = 1076, normalized size = 6.99

$$\frac{\left(\left(b^4m^4 + 10\,b^4m^3 + 35\,b^4m^2 + 50\,b^4m + 24\,b^4\right)x^5 + 4\left(ab^3m^4 + 11\,ab^3m^3 + 41\,ab^3m^2 + 61\,ab^3m + 30\,ab^3\right)x^4 + 6\left(a^2b^2m^4\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

```
[Out] (((b^4*m^4 + 10*b^4*m^3 + 35*b^4*m^2 + 50*b^4*m + 24*b^4)*x^5 + 4*(a*b^3*m^
4 + 11*a*b^3*m^3 + 41*a*b^3*m^2 + 61*a*b^3*m + 30*a*b^3)*x^4 + 6*(a^2*b^2*m
^4 + 12*a^2*b^2*m^3 + 49*a^2*b^2*m^2 + 78*a^2*b^2*m + 40*a^2*b^2)*x^3 + 4*(
a^3*b*m^4 + 13*a^3*b*m^3 + 59*a^3*b*m^2 + 107*a^3*b*m + 60*a^3*b)*x^2 + (a^
4*m^4 + 14*a^4*m^3 + 71*a^4*m^2 + 154*a^4*m + 120*a^4)*x)*cosh(m*log(x)) +
((b^4*m^4 + 10*b^4*m^3 + 35*b^4*m^2 + 50*b^4*m + 24*b^4)*x^5 + 4*(a*b^3*m^4
 + 11*a*b^3*m^3 + 41*a*b^3*m^2 + 61*a*b^3*m + 30*a*b^3)*x^4 + 6*(a^2*b^2*m^
4 + 12*a^2*b^2*m^3 + 49*a^2*b^2*m^2 + 78*a^2*b^2*m + 40*a^2*b^2)*x^3 + 4*(a
^3*b*m^4 + 13*a^3*b*m^3 + 59*a^3*b*m^2 + 107*a^3*b*m + 60*a^3*b)*x^2 + (a^4
*m^4 + 14*a^4*m^3 + 71*a^4*m^2 + 154*a^4*m + 120*a^4)*x)*sinh(m*log(x)))/(m
```

```
^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atanh(tanh(b*x+a))**4,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{artanh}(\tanh(bx+a))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^4,x, algorithm="giac")
```

```
[Out] integrate(x^m*arctanh(tanh(b*x + a))^4, x)
```

3.66 $\int x^6 \tanh^{-1}(\tanh(a+bx))^4\,dx$

Optimal. Leaf size=80

$$-\frac{1}{210}b^3x^{10}\tanh^{-1}(\tanh(a+bx))+\frac{1}{42}b^2x^9\tanh^{-1}(\tanh(a+bx))^2-\frac{1}{14}bx^8\tanh^{-1}(\tanh(a+bx))^3+\frac{1}{7}x^7\tanh^{-1}(\tanh$$

```
[Out] (b^4*x^11)/2310 - (b^3*x^10*ArcTanh[Tanh[a + b*x]])/210 + (b^2*x^9*ArcTanh[
Tanh[a + b*x]]^2)/42 - (b*x^8*ArcTanh[Tanh[a + b*x]]^3)/14 + (x^7*ArcTanh[T
anh[a + b*x]]^4)/7
```

Rubi [A] time = 0.0632376, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$-\frac{1}{210}b^3x^{10}\tanh^{-1}(\tanh(a+bx))+\frac{1}{42}b^2x^9\tanh^{-1}(\tanh(a+bx))^2-\frac{1}{14}bx^8\tanh^{-1}(\tanh(a+bx))^3+\frac{1}{7}x^7\tanh^{-1}(\tanh$$

Antiderivative was successfully verified.

```
[In] Int[x^6*ArcTanh[Tanh[a + b*x]]^4,x]
```


```
[Out] (b^4*x^11)/2310 - (b^3*x^10*ArcTanh[Tanh[a + b*x]])/210 + (b^2*x^9*ArcTanh[
Tanh[a + b*x]]^2)/42 - (b*x^8*ArcTanh[Tanh[a + b*x]]^3)/14 + (x^7*ArcTanh[T
anh[a + b*x]]^4)/7
```


Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^6 \tanh^{-1}(\tanh(a+bx))^4\,dx = \frac{1}{7}x^7 \tanh^{-1}(\tanh(a+bx))^4 - \frac{1}{7}(4b)\int x^7 \tanh^{-1}(\tanh(a+bx))^3\,dx$$
$$= -\frac{1}{14}bx^8 \tanh^{-1}(\tanh(a+bx))^3 + \frac{1}{7}x^7 \tanh^{-1}(\tanh(a+bx))^4 + \frac{1}{14}\left(3b^2\right)\int x^8 \tanh$$
$$= \frac{1}{42}b^2x^9 \tanh^{-1}(\tanh(a+bx))^2 - \frac{1}{14}bx^8 \tanh^{-1}(\tanh(a+bx))^3 + \frac{1}{7}x^7 \tanh^{-1}(\tanh$$
$$= -\frac{1}{210}b^3x^{10} \tanh^{-1}(\tanh(a+bx)) + \frac{1}{42}b^2x^9 \tanh^{-1}(\tanh(a+bx))^2 - \frac{1}{14}bx^8 \tanh^{-1}$$
$$= \frac{b^4x^{11}}{2310} - \frac{1}{210}b^3x^{10} \tanh^{-1}(\tanh(a+bx)) + \frac{1}{42}b^2x^9 \tanh^{-1}(\tanh(a+bx))^2 - \frac{1}{14}bx^8$$

Mathematica [A] time = 0.0622043, size = 71, normalized size = 0.89

$$\frac{x^7\left(-11b^3x^3 \tanh^{-1}(\tanh(a+bx)) + 55b^2x^2 \tanh^{-1}(\tanh(a+bx))^2 - 165bx \tanh^{-1}(\tanh(a+bx))^3 + 330 \tanh^{-1}(\tan\right.}{2310}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (x^7*(b^4*x^4 - 11*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 55*b^2*x^2*ArcTanh[Tanh
[a + b*x]]^2 - 165*b*x*ArcTanh[Tanh[a + b*x]]^3 + 330*ArcTanh[Tanh[a + b*x]
]^4))/2310
```

Maple [A] time = 0.04, size = 74, normalized size = 0.9

$$\frac{x^7 (\mathrm{Artanh}(\tanh(bx+a)))^4}{7} - \frac{4b}{7}\left(\frac{x^8 (\mathrm{Artanh}(\tanh(bx+a)))^3}{8} - \frac{3b}{8}\left(\frac{x^9 (\mathrm{Artanh}(\tanh(bx+a)))^2}{9} - \frac{2b}{9}\left(\frac{x^{10}\mathrm{Art}}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*arctanh(tanh(b*x+a))^4,x)
```

```
[Out] 1/7*x^7*arctanh(tanh(b*x+a))^4-4/7*b*(1/8*x^8*arctanh(tanh(b*x+a))^3-3/8*b*
(1/9*x^9*arctanh(tanh(b*x+a))^2-2/9*b*(1/10*x^10*arctanh(tanh(b*x+a))-1/110
*x^11*b)))
```

Maxima [A] time = 1.73779, size = 97, normalized size = 1.21

$$-\frac{1}{14}\,bx^8\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3+\frac{1}{7}\,x^7\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^4+\frac{1}{2310}\left(55\,bx^9\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2+\left(b^2x^{11}-11\,b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

```
[Out] -1/14*b*x^8*arctanh(tanh(b*x + a))^3 + 1/7*x^7*arctanh(tanh(b*x + a))^4 + 1
/2310*(55*b*x^9*arctanh(tanh(b*x + a))^2 + (b^2*x^11 - 11*b*x^10*arctanh(ta
nh(b*x + a)))*b)*b
```

Fricas [A] time = 1.55855, size = 108, normalized size = 1.35

$$\frac{1}{11}\,b^4x^{11}+\frac{2}{5}\,ab^3x^{10}+\frac{2}{3}\,a^2b^2x^9+\frac{1}{2}\,a^3bx^8+\frac{1}{7}\,a^4x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

```
[Out] 1/11*b^4*x^11 + 2/5*a*b^3*x^10 + 2/3*a^2*b^2*x^9 + 1/2*a^3*b*x^8 + 1/7*a^4*
x^7
```

Sympy [A] time = 32.5487, size = 75, normalized size = 0.94

$$\frac{b^4x^{11}}{2310}-\frac{b^3x^{10}\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{210}+\frac{b^2x^9\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{42}-\frac{bx^8\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{14}+\frac{x^7\operatorname{atanh}^4\left(\tanh\left(a\right.\right.}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*atanh(tanh(b*x+a))**4,x)
```

```
[Out] b**4*x**11/2310 - b**3*x**10*atanh(tanh(a + b*x))/210 + b**2*x**9*atanh(tan
h(a + b*x))**2/42 - b*x**8*atanh(tanh(a + b*x))**3/14 + x**7*atanh(tanh(a +
 b*x))**4/7
```

Giac [A] time = 1.12612, size = 62, normalized size = 0.78

$$\frac{1}{11}\,b^4x^{11} + \frac{2}{5}\,ab^3x^{10} + \frac{2}{3}\,a^2b^2x^9 + \frac{1}{2}\,a^3bx^8 + \frac{1}{7}\,a^4x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arctanh(tanh(b*x+a))^4,x, algorithm="giac")
```

```
[Out] 1/11*b^4*x^11 + 2/5*a*b^3*x^10 + 2/3*a^2*b^2*x^9 + 1/2*a^3*b*x^8 + 1/7*a^4*
x^7
```

3.67 $\int x^5 \tanh^{-1}(\tanh(a+bx))^4 \, dx$

Optimal. Leaf size=80

$$-\frac{1}{126}b^3x^9\tanh^{-1}(\tanh(a+bx))+\frac{1}{28}b^2x^8\tanh^{-1}(\tanh(a+bx))^2-\frac{2}{21}bx^7\tanh^{-1}(\tanh(a+bx))^3+\frac{1}{6}x^6\tanh^{-1}(\tanh($$

```
[Out] (b^4*x^10)/1260 - (b^3*x^9*ArcTanh[Tanh[a + b*x]])/126 + (b^2*x^8*ArcTanh[T
anh[a + b*x]]^2)/28 - (2*b*x^7*ArcTanh[Tanh[a + b*x]]^3)/21 + (x^6*ArcTanh[
Tanh[a + b*x]]^4)/6
```

Rubi [A] time = 0.0564928, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$-\frac{1}{126}b^3x^9\tanh^{-1}(\tanh(a+bx))+\frac{1}{28}b^2x^8\tanh^{-1}(\tanh(a+bx))^2-\frac{2}{21}bx^7\tanh^{-1}(\tanh(a+bx))^3+\frac{1}{6}x^6\tanh^{-1}(\tanh($$

Antiderivative was successfully verified.

```
[In] Int[x^5*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (b^4*x^10)/1260 - (b^3*x^9*ArcTanh[Tanh[a + b*x]])/126 + (b^2*x^8*ArcTanh[T
anh[a + b*x]]^2)/28 - (2*b*x^7*ArcTanh[Tanh[a + b*x]]^3)/21 + (x^6*ArcTanh[
Tanh[a + b*x]]^4)/6
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^5 \tanh^{-1}(\tanh(a+bx))^4\,dx = \frac{1}{6}x^6 \tanh^{-1}(\tanh(a+bx))^4 - \frac{1}{3}(2b)\int x^6 \tanh^{-1}(\tanh(a+bx))^3\,dx$$
$$= -\frac{2}{21}bx^7 \tanh^{-1}(\tanh(a+bx))^3 + \frac{1}{6}x^6 \tanh^{-1}(\tanh(a+bx))^4 + \frac{1}{7}\left(2b^2\right)\int x^7 \tanh$$
$$= \frac{1}{28}b^2x^8 \tanh^{-1}(\tanh(a+bx))^2 - \frac{2}{21}bx^7 \tanh^{-1}(\tanh(a+bx))^3 + \frac{1}{6}x^6 \tanh^{-1}(\tanh$$
$$= -\frac{1}{126}b^3x^9 \tanh^{-1}(\tanh(a+bx)) + \frac{1}{28}b^2x^8 \tanh^{-1}(\tanh(a+bx))^2 - \frac{2}{21}bx^7 \tanh^{-1}($$
$$= \frac{b^4x^{10}}{1260} - \frac{1}{126}b^3x^9 \tanh^{-1}(\tanh(a+bx)) + \frac{1}{28}b^2x^8 \tanh^{-1}(\tanh(a+bx))^2 - \frac{2}{21}bx^7 \mathrm{ta}$$

Mathematica [A] time = 0.0308633, size = 71, normalized size = 0.89

$$\frac{x^6\left(-10b^3x^3 \tanh^{-1}(\tanh(a+bx)) + 45b^2x^2 \tanh^{-1}(\tanh(a+bx))^2 - 120bx \tanh^{-1}(\tanh(a+bx))^3 + 210 \tanh^{-1}(\mathrm{tan}\right.}{1260}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (x^6*(b^4*x^4 - 10*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 45*b^2*x^2*ArcTanh[Tanh
[a + b*x]]^2 - 120*b*x*ArcTanh[Tanh[a + b*x]]^3 + 210*ArcTanh[Tanh[a + b*x]
]^4))/1260
```

Maple [A] time = 0.04, size = 74, normalized size = 0.9

$$\frac{x^6 (\mathrm{Artanh}(\tanh(bx+a)))^4}{6} - \frac{2b}{3}\left(\frac{x^7(\mathrm{Artanh}(\tanh(bx+a)))^3}{7} - \frac{3b}{7}\left(\frac{x^8(\mathrm{Artanh}(\tanh(bx+a)))^2}{8} - \frac{b}{4}\left(\frac{x^9 \mathrm{Artan}}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arctanh(tanh(b*x+a))^4,x)
```

```
[Out] 1/6*x^6*arctanh(tanh(b*x+a))^4-2/3*b*(1/7*x^7*arctanh(tanh(b*x+a))^3-3/7*b*
(1/8*x^8*arctanh(tanh(b*x+a))^2-1/4*b*(1/9*x^9*arctanh(tanh(b*x+a))-1/90*x^
10*b)))
```

Maxima [A] time = 1.73772, size = 97, normalized size = 1.21

$$-\frac{2}{21}\,bx^7\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^3+\frac{1}{6}\,x^6\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^4+\frac{1}{1260}\left(45\,bx^8\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2+\left(b^2x^{10}-10\,b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

```
[Out] -2/21*b*x^7*arctanh(tanh(b*x + a))^3 + 1/6*x^6*arctanh(tanh(b*x + a))^4 + 1
/1260*(45*b*x^8*arctanh(tanh(b*x + a))^2 + (b^2*x^10 - 10*b*x^9*arctanh(tan
h(b*x + a)))*b)*b
```

Fricas [A] time = 1.51244, size = 107, normalized size = 1.34

$$\frac{1}{10}\,b^4x^{10}+\frac{4}{9}\,ab^3x^9+\frac{3}{4}\,a^2b^2x^8+\frac{4}{7}\,a^3bx^7+\frac{1}{6}\,a^4x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

```
[Out] 1/10*b^4*x^10 + 4/9*a*b^3*x^9 + 3/4*a^2*b^2*x^8 + 4/7*a^3*b*x^7 + 1/6*a^4*x
^6
```

Sympy [A] time = 18.9303, size = 76, normalized size = 0.95

$$\frac{b^4x^{10}}{1260}-\frac{b^3x^9\,\mathrm{atanh}\left(\tanh\left(a+bx\right)\right)}{126}+\frac{b^2x^8\,\mathrm{atanh}^2\left(\tanh\left(a+bx\right)\right)}{28}-\frac{2bx^7\,\mathrm{atanh}^3\left(\tanh\left(a+bx\right)\right)}{21}+\frac{x^6\,\mathrm{atanh}^4\left(\tanh\left(a\right.\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*atanh(tanh(b*x+a))**4,x)
```

```
[Out] b**4*x**10/1260 - b**3*x**9*atanh(tanh(a + b*x))/126 + b**2*x**8*atanh(tanh
(a + b*x))**2/28 - 2*b*x**7*atanh(tanh(a + b*x))**3/21 + x**6*atanh(tanh(a
+ b*x))**4/6
```

<u>Giac</u> [A] time = 1.12196, size = 62, normalized size = 0.78

$$\frac{1}{10}\,b^4x^{10} + \frac{4}{9}\,ab^3x^9 + \frac{3}{4}\,a^2b^2x^8 + \frac{4}{7}\,a^3bx^7 + \frac{1}{6}\,a^4x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctanh(tanh(b*x+a))^4,x, algorithm="giac")
```

```
[Out] 1/10*b^4*x^10 + 4/9*a*b^3*x^9 + 3/4*a^2*b^2*x^8 + 4/7*a^3*b*x^7 + 1/6*a^4*x
^6
```

3.68 $\int x^4 \tanh^{-1}(\tanh(a+bx))^4\,dx$

Optimal. Leaf size=80

$$-\frac{1}{70}b^3x^8\tanh^{-1}(\tanh(a+bx))+\frac{2}{35}b^2x^7\tanh^{-1}(\tanh(a+bx))^2-\frac{2}{15}bx^6\tanh^{-1}(\tanh(a+bx))^3+\frac{1}{5}x^5\tanh^{-1}(\tanh(a$$

```
[Out] (b^4*x^9)/630 - (b^3*x^8*ArcTanh[Tanh[a + b*x]])/70 + (2*b^2*x^7*ArcTanh[Ta
nh[a + b*x]]^2)/35 - (2*b*x^6*ArcTanh[Tanh[a + b*x]]^3)/15 + (x^5*ArcTanh[T
anh[a + b*x]]^4)/5
```

Rubi [A] time = 0.0547148, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$-\frac{1}{70}b^3x^8\tanh^{-1}(\tanh(a+bx))+\frac{2}{35}b^2x^7\tanh^{-1}(\tanh(a+bx))^2-\frac{2}{15}bx^6\tanh^{-1}(\tanh(a+bx))^3+\frac{1}{5}x^5\tanh^{-1}(\tanh(a$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (b^4*x^9)/630 - (b^3*x^8*ArcTanh[Tanh[a + b*x]])/70 + (2*b^2*x^7*ArcTanh[Ta
nh[a + b*x]]^2)/35 - (2*b*x^6*ArcTanh[Tanh[a + b*x]]^3)/15 + (x^5*ArcTanh[T
anh[a + b*x]]^4)/5
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^4 \tanh^{-1}(\tanh(a+bx))^4\,dx = \frac{1}{5}x^5 \tanh^{-1}(\tanh(a+bx))^4 - \frac{1}{5}(4b)\int x^5 \tanh^{-1}(\tanh(a+bx))^3\,dx$$
$$= -\frac{2}{15}bx^6 \tanh^{-1}(\tanh(a+bx))^3 + \frac{1}{5}x^5 \tanh^{-1}(\tanh(a+bx))^4 + \frac{1}{5}\left(2b^2\right)\int x^6 \tanh^{-1}$$
$$= \frac{2}{35}b^2x^7 \tanh^{-1}(\tanh(a+bx))^2 - \frac{2}{15}bx^6 \tanh^{-1}(\tanh(a+bx))^3 + \frac{1}{5}x^5 \tanh^{-1}(\tanh$$
$$= -\frac{1}{70}b^3x^8 \tanh^{-1}(\tanh(a+bx)) + \frac{2}{35}b^2x^7 \tanh^{-1}(\tanh(a+bx))^2 - \frac{2}{15}bx^6 \tanh^{-1}(t$$
$$= \frac{b^4x^9}{630} - \frac{1}{70}b^3x^8 \tanh^{-1}(\tanh(a+bx)) + \frac{2}{35}b^2x^7 \tanh^{-1}(\tanh(a+bx))^2 - \frac{2}{15}bx^6 \tan$$

Mathematica [A] time = 0.0329736, size = 71, normalized size = 0.89

$$\frac{1}{630}x^5\left(-9b^3x^3\tanh^{-1}(\tanh(a+bx)) + 36b^2x^2\tanh^{-1}(\tanh(a+bx))^2 - 84bx\tanh^{-1}(\tanh(a+bx))^3 + 126\tanh^{-1}(t\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (x^5*(b^4*x^4 - 9*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 36*b^2*x^2*ArcTanh[Tanh[
a + b*x]]^2 - 84*b*x*ArcTanh[Tanh[a + b*x]]^3 + 126*ArcTanh[Tanh[a + b*x]]^
4))/630
```

Maple [A] time = 0.04, size = 74, normalized size = 0.9

$$\frac{x^5(\mathrm{Artanh}(\tanh(bx+a)))^4}{5} - \frac{4b}{5}\left(\frac{x^6(\mathrm{Artanh}(\tanh(bx+a)))^3}{6} - \frac{b}{2}\left(\frac{x^7(\mathrm{Artanh}(\tanh(bx+a)))^2}{7} - \frac{2b}{7}\left(\frac{x^8\mathrm{Artan}}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctanh(tanh(b*x+a))^4,x)
```

```
[Out] 1/5*x^5*arctanh(tanh(b*x+a))^4-4/5*b*(1/6*x^6*arctanh(tanh(b*x+a))^3-1/2*b*
(1/7*x^7*arctanh(tanh(b*x+a))^2-2/7*b*(1/8*x^8*arctanh(tanh(b*x+a))-1/72*x^
9*b)))
```

Maxima [A] time = 1.77846, size = 97, normalized size = 1.21

$$-\frac{2}{15}\,bx^6\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3+\frac{1}{5}\,x^5\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^4+\frac{1}{630}\left(36\,bx^7\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2+\left(b^2x^9-9\,bx^8\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

```
[Out] -2/15*b*x^6*arctanh(tanh(b*x + a))^3 + 1/5*x^5*arctanh(tanh(b*x + a))^4 + 1
/630*(36*b*x^7*arctanh(tanh(b*x + a))^2 + (b^2*x^9 - 9*b*x^8*arctanh(tanh(b
*x + a)))*b)*b
```

Fricas [A] time = 1.46481, size = 104, normalized size = 1.3

$$\frac{1}{9}\,b^4x^9+\frac{1}{2}\,ab^3x^8+\frac{6}{7}\,a^2b^2x^7+\frac{2}{3}\,a^3bx^6+\frac{1}{5}\,a^4x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

```
[Out] 1/9*b^4*x^9 + 1/2*a*b^3*x^8 + 6/7*a^2*b^2*x^7 + 2/3*a^3*b*x^6 + 1/5*a^4*x^5
```

Sympy [A] time = 11.2878, size = 78, normalized size = 0.98

$$\frac{b^4x^9}{630}-\frac{b^3x^8\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{70}+\frac{2b^2x^7\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{35}-\frac{2bx^6\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{15}+\frac{x^5\operatorname{atanh}^4\left(\tanh\left(a\right.\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atanh(tanh(b*x+a))**4,x)
```

```
[Out] b**4*x**9/630 - b**3*x**8*atanh(tanh(a + b*x))/70 + 2*b**2*x**7*atanh(tanh(
a + b*x))**2/35 - 2*b*x**6*atanh(tanh(a + b*x))**3/15 + x**5*atanh(tanh(a +
 b*x))**4/5
```

Giac [A] time = 1.14128, size = 62, normalized size = 0.78

$$\frac{1}{9}\,b^{4}x^{9} + \frac{1}{2}\,ab^{3}x^{8} + \frac{6}{7}\,a^{2}b^{2}x^{7} + \frac{2}{3}\,a^{3}bx^{6} + \frac{1}{5}\,a^{4}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^4,x, algorithm="giac")
```

```
[Out] 1/9*b^4*x^9 + 1/2*a*b^3*x^8 + 6/7*a^2*b^2*x^7 + 2/3*a^3*b*x^6 + 1/5*a^4*x^5
```

3.69 $\int x^3 \tanh^{-1}(\tanh(a+bx))^4 \, dx$

Optimal. Leaf size=72

$$-\frac{x^2 \tanh^{-1}(\tanh(a+bx))^6}{10b^2}-\frac{\tanh^{-1}(\tanh(a+bx))^8}{280b^4}+\frac{x \tanh^{-1}(\tanh(a+bx))^7}{35b^3}+\frac{x^3 \tanh^{-1}(\tanh(a+bx))^5}{5b}$$

```
[Out] (x^3*ArcTanh[Tanh[a + b*x]]^5)/(5*b) - (x^2*ArcTanh[Tanh[a + b*x]]^6)/(10*b
^2) + (x*ArcTanh[Tanh[a + b*x]]^7)/(35*b^3) - ArcTanh[Tanh[a + b*x]]^8/(280
*b^4)
```

Rubi [A] time = 0.0466606, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2157, 30}

$$-\frac{x^2 \tanh^{-1}(\tanh(a+bx))^6}{10b^2}-\frac{\tanh^{-1}(\tanh(a+bx))^8}{280b^4}+\frac{x \tanh^{-1}(\tanh(a+bx))^7}{35b^3}+\frac{x^3 \tanh^{-1}(\tanh(a+bx))^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (x^3*ArcTanh[Tanh[a + b*x]]^5)/(5*b) - (x^2*ArcTanh[Tanh[a + b*x]]^6)/(10*b
^2) + (x*ArcTanh[Tanh[a + b*x]]^7)/(35*b^3) - ArcTanh[Tanh[a + b*x]]^8/(280
*b^4)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^3\tanh^{-1}(\tanh(a+bx))^4\,dx = \frac{x^3\tanh^{-1}(\tanh(a+bx))^5}{5b} - \frac{3\int x^2\tanh^{-1}(\tanh(a+bx))^5\,dx}{5b}$$
$$= \frac{x^3\tanh^{-1}(\tanh(a+bx))^5}{5b} - \frac{x^2\tanh^{-1}(\tanh(a+bx))^6}{10b^2} + \frac{\int x\tanh^{-1}(\tanh(a+bx))}{5b^2}$$
$$= \frac{x^3\tanh^{-1}(\tanh(a+bx))^5}{5b} - \frac{x^2\tanh^{-1}(\tanh(a+bx))^6}{10b^2} + \frac{x\tanh^{-1}(\tanh(a+bx))^7}{35b^3}$$
$$= \frac{x^3\tanh^{-1}(\tanh(a+bx))^5}{5b} - \frac{x^2\tanh^{-1}(\tanh(a+bx))^6}{10b^2} + \frac{x\tanh^{-1}(\tanh(a+bx))^7}{35b^3}$$
$$= \frac{x^3\tanh^{-1}(\tanh(a+bx))^5}{5b} - \frac{x^2\tanh^{-1}(\tanh(a+bx))^6}{10b^2} + \frac{x\tanh^{-1}(\tanh(a+bx))^7}{35b^3}$$

Mathematica [A] time = 0.0238973, size = 71, normalized size = 0.99

$$\frac{1}{280}x^4\left(-8b^3x^3\tanh^{-1}(\tanh(a+bx)) + 28b^2x^2\tanh^{-1}(\tanh(a+bx))^2 - 56bx\tanh^{-1}(\tanh(a+bx))^3 + 70\tanh^{-1}(\tan\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (x^4*(b^4*x^4 - 8*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 28*b^2*x^2*ArcTanh[Tanh[
a + b*x]]^2 - 56*b*x*ArcTanh[Tanh[a + b*x]]^3 + 70*ArcTanh[Tanh[a + b*x]]^4
))/280
```

Maple [A] time = 0.042, size = 74, normalized size = 1.

$$\frac{x^4(\mathrm{Artanh}(\tanh(bx+a)))^4}{4} - b\left(\frac{x^5(\mathrm{Artanh}(\tanh(bx+a)))^3}{5} - \frac{3b}{5}\left(\frac{x^6(\mathrm{Artanh}(\tanh(bx+a)))^2}{6} - \frac{b}{3}\left(\frac{x^7\mathrm{Artanh}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(tanh(b*x+a))^4,x)
```

```
[Out] 1/4*x^4*arctanh(tanh(b*x+a))^4-b*(1/5*x^5*arctanh(tanh(b*x+a))^3-3/5*b*(1/6
*x^6*arctanh(tanh(b*x+a))^2-1/3*b*(1/7*x^7*arctanh(tanh(b*x+a))-1/56*x^8*b)
))
```

Maxima [A] time = 1.73531, size = 97, normalized size = 1.35

$$-\frac{1}{5}\,bx^5\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^3+\frac{1}{4}\,x^4\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^4+\frac{1}{280}\left(28\,bx^6\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^2+\left(b^2x^8-8\,bx^7\,\text{a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

```
[Out] -1/5*b*x^5*arctanh(tanh(b*x + a))^3 + 1/4*x^4*arctanh(tanh(b*x + a))^4 + 1/
280*(28*b*x^6*arctanh(tanh(b*x + a))^2 + (b^2*x^8 - 8*b*x^7*arctanh(tanh(b*
x + a)))*b)*b
```

Fricas [A] time = 1.38852, size = 99, normalized size = 1.38

$$\frac{1}{8}\,b^4x^8+\frac{4}{7}\,ab^3x^7+a^2b^2x^6+\frac{4}{5}\,a^3bx^5+\frac{1}{4}\,a^4x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

```
[Out] 1/8*b^4*x^8 + 4/7*a*b^3*x^7 + a^2*b^2*x^6 + 4/5*a^3*b*x^5 + 1/4*a^4*x^4
```

Sympy [A] time = 6.74145, size = 75, normalized size = 1.04

$$\frac{b^4x^8}{280}-\frac{b^3x^7\,\text{atanh}\left(\tanh\left(a+bx\right)\right)}{35}+\frac{b^2x^6\,\text{atanh}^2\left(\tanh\left(a+bx\right)\right)}{10}-\frac{bx^5\,\text{atanh}^3\left(\tanh\left(a+bx\right)\right)}{5}+\frac{x^4\,\text{atanh}^4\left(\tanh\left(a+\right.\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(tanh(b*x+a))**4,x)
```

```
[Out] b**4*x**8/280 - b**3*x**7*atanh(tanh(a + b*x))/35 + b**2*x**6*atanh(tanh(a
+ b*x))**2/10 - b*x**5*atanh(tanh(a + b*x))**3/5 + x**4*atanh(tanh(a + b*x)
)**4/4
```

Giac [A] time = 1.13812, size = 61, normalized size = 0.85

$$\frac{1}{8}b^4x^8 + \frac{4}{7}ab^3x^7 + a^2b^2x^6 + \frac{4}{5}a^3bx^5 + \frac{1}{4}a^4x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^4,x, algorithm="giac")
```

```
[Out] 1/8*b^4*x^8 + 4/7*a*b^3*x^7 + a^2*b^2*x^6 + 4/5*a^3*b*x^5 + 1/4*a^4*x^4
```

3.70 $\int x^2 \tanh^{-1}(\tanh(a+bx))^4 \, dx$

Optimal. Leaf size=53

$$\frac{\tanh^{-1}(\tanh(a+bx))^7}{105b^3} - \frac{x\tanh^{-1}(\tanh(a+bx))^6}{15b^2} + \frac{x^2\tanh^{-1}(\tanh(a+bx))^5}{5b}$$

```
[Out] (x^2*ArcTanh[Tanh[a + b*x]]^5)/(5*b) - (x*ArcTanh[Tanh[a + b*x]]^6)/(15*b^2
) + ArcTanh[Tanh[a + b*x]]^7/(105*b^3)
```

Rubi [A] time = 0.0297045, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2157, 30}

$$\frac{\tanh^{-1}(\tanh(a+bx))^7}{105b^3} - \frac{x\tanh^{-1}(\tanh(a+bx))^6}{15b^2} + \frac{x^2\tanh^{-1}(\tanh(a+bx))^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (x^2*ArcTanh[Tanh[a + b*x]]^5)/(5*b) - (x*ArcTanh[Tanh[a + b*x]]^6)/(15*b^2
) + ArcTanh[Tanh[a + b*x]]^7/(105*b^3)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^2 \tanh^{-1}(\tanh(a+bx))^4\,dx = \frac{x^2 \tanh^{-1}(\tanh(a+bx))^5}{5b} - \frac{2\int x \tanh^{-1}(\tanh(a+bx))^5\,dx}{5b}$$
$$= \frac{x^2 \tanh^{-1}(\tanh(a+bx))^5}{5b} - \frac{x \tanh^{-1}(\tanh(a+bx))^6}{15b^2} + \frac{\int \tanh^{-1}(\tanh(a+bx))^6\,d}{15b^2}$$
$$= \frac{x^2 \tanh^{-1}(\tanh(a+bx))^5}{5b} - \frac{x \tanh^{-1}(\tanh(a+bx))^6}{15b^2} + \frac{\mathrm{Subst}\left(\int x^6\,dx, x, \tanh^{-1}(\right.}{15b^3}$$
$$= \frac{x^2 \tanh^{-1}(\tanh(a+bx))^5}{5b} - \frac{x \tanh^{-1}(\tanh(a+bx))^6}{15b^2} + \frac{\tanh^{-1}(\tanh(a+bx))^7}{105b^3}$$

Mathematica [A] time = 0.0509028, size = 71, normalized size = 1.34

$$\frac{1}{105}x^3\left(-7b^3x^3 \tanh^{-1}(\tanh(a+bx)) + 21b^2x^2 \tanh^{-1}(\tanh(a+bx))^2 - 35bx \tanh^{-1}(\tanh(a+bx))^3 + 35 \tanh^{-1}(\tan\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (x^3*(b^4*x^4 - 7*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 21*b^2*x^2*ArcTanh[Tanh[
a + b*x]]^2 - 35*b*x*ArcTanh[Tanh[a + b*x]]^3 + 35*ArcTanh[Tanh[a + b*x]]^4
))/105
```

Maple [A] time = 0.043, size = 74, normalized size = 1.4

$$\frac{x^3(\mathrm{Artanh}(\tanh(bx+a)))^4}{3} - \frac{4b}{3}\left(\frac{x^4(\mathrm{Artanh}(\tanh(bx+a)))^3}{4} - \frac{3b}{4}\left(\frac{x^5(\mathrm{Artanh}(\tanh(bx+a)))^2}{5} - \frac{2b}{5}\left(\frac{x^6\mathrm{Arta}}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(tanh(b*x+a))^4,x)
```

```
[Out] 1/3*x^3*arctanh(tanh(b*x+a))^4-4/3*b*(1/4*x^4*arctanh(tanh(b*x+a))^3-3/4*b*
(1/5*x^5*arctanh(tanh(b*x+a))^2-2/5*b*(1/6*x^6*arctanh(tanh(b*x+a))-1/42*x^
```

```
7*b)))
```

Maxima [A] time = 1.74675, size = 97, normalized size = 1.83

$$-\frac{1}{3}\,bx^4\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^3+\frac{1}{3}\,x^3\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^4+\frac{1}{105}\left(21\,bx^5\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2+\left(b^2x^7-7\,bx^6\,\mathrm{a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

```
[Out] -1/3*b*x^4*arctanh(tanh(b*x + a))^3 + 1/3*x^3*arctanh(tanh(b*x + a))^4 + 1/
105*(21*b*x^5*arctanh(tanh(b*x + a))^2 + (b^2*x^7 - 7*b*x^6*arctanh(tanh(b*
x + a)))*b)*b
```

Fricas [A] time = 1.49441, size = 99, normalized size = 1.87

$$\frac{1}{7}\,b^4x^7+\frac{2}{3}\,ab^3x^6+\frac{6}{5}\,a^2b^2x^5+a^3bx^4+\frac{1}{3}\,a^4x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

```
[Out] 1/7*b^4*x^7 + 2/3*a*b^3*x^6 + 6/5*a^2*b^2*x^5 + a^3*b*x^4 + 1/3*a^4*x^3
```

Sympy [A] time = 7.45079, size = 60, normalized size = 1.13

$$\begin{cases} \frac{x^2\,\mathrm{atanh}^5\left(\tanh\left(a+bx\right)\right)}{5b}-\frac{x\,\mathrm{atanh}^6\left(\tanh\left(a+bx\right)\right)}{15b^2}+\frac{\mathrm{atanh}^7\left(\tanh\left(a+bx\right)\right)}{105b^3} & \text{for } b\neq 0\\ \frac{x^3\,\mathrm{atanh}^4\left(\tanh\left(a\right)\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(tanh(b*x+a))**4,x)
```

```
[Out] Piecewise((x**2*atanh(tanh(a + b*x))**5/(5*b) - x*atanh(tanh(a + b*x))**6/(
15*b**2) + atanh(tanh(a + b*x))**7/(105*b**3), Ne(b, 0)), (x**3*atanh(tanh(
```

```
a))**4/3, True))
```

Giac [A] time = 1.17556, size = 61, normalized size = 1.15

$$\frac{1}{7}\,b^4x^7 + \frac{2}{3}\,ab^3x^6 + \frac{6}{5}\,a^2b^2x^5 + a^3bx^4 + \frac{1}{3}\,a^4x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^4,x, algorithm="giac")
```

```
[Out] 1/7*b^4*x^7 + 2/3*a*b^3*x^6 + 6/5*a^2*b^2*x^5 + a^3*b*x^4 + 1/3*a^4*x^3
```

3.71 $\int x \tanh^{-1}(\tanh(a+bx))^4 \, dx$

Optimal. Leaf size=34

$$\frac{x\tanh^{-1}(\tanh(a+bx))^5}{5b}-\frac{\tanh^{-1}(\tanh(a+bx))^6}{30b^2}$$

```
[Out] (x*ArcTanh[Tanh[a + b*x]]^5)/(5*b) - ArcTanh[Tanh[a + b*x]]^6/(30*b^2)
```

Rubi [A] time = 0.0139257, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2168, 2157, 30}

$$\frac{x\tanh^{-1}(\tanh(a+bx))^5}{5b}-\frac{\tanh^{-1}(\tanh(a+bx))^6}{30b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (x*ArcTanh[Tanh[a + b*x]]^5)/(5*b) - ArcTanh[Tanh[a + b*x]]^6/(30*b^2)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x\tanh^{-1}(\tanh(a+bx))^4\,dx = \frac{x\tanh^{-1}(\tanh(a+bx))^5}{5b} - \frac{\int \tanh^{-1}(\tanh(a+bx))^5\,dx}{5b}$$
$$= \frac{x\tanh^{-1}(\tanh(a+bx))^5}{5b} - \frac{\text{Subst}\left(\int x^5\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{5b^2}$$
$$= \frac{x\tanh^{-1}(\tanh(a+bx))^5}{5b} - \frac{\tanh^{-1}(\tanh(a+bx))^6}{30b^2}$$

Mathematica [B] time = 0.0841169, size = 125, normalized size = 3.68

$$-\frac{(a+bx)\left(-20\left(2a^2+abx-b^2x^2\right)\tanh^{-1}(\tanh(a+bx))^3+(5a-bx)(a+bx)^4-6(4a-bx)(a+bx)^3\tanh^{-1}(\tanh(a+\right.}{30b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] -((a + b*x)*((5*a - b*x)*(a + b*x)^4 - 6*(4*a - b*x)*(a + b*x)^3*ArcTanh[Ta
nh[a + b*x]] + 15*(3*a - b*x)*(a + b*x)^2*ArcTanh[Tanh[a + b*x]]^2 - 20*(2*
a^2 + a*b*x - b^2*x^2)*ArcTanh[Tanh[a + b*x]]^3 + 15*(a - b*x)*ArcTanh[Tanh
[a + b*x]]^4))/(30*b^2)
```

Maple [B] time = 0.04, size = 74, normalized size = 2.2

$$\frac{x^2\left(\text{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^4}{2} - 2b\left(1/3\,x^3\left(\text{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^3 - b\left(1/4\,x^4\left(\text{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^2 - 1/2\,b\left(1/5\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(tanh(b*x+a))^4,x)
```

```
[Out] 1/2*x^2*arctanh(tanh(b*x+a))^4-2*b*(1/3*x^3*arctanh(tanh(b*x+a))^3-b*(1/4*x
^4*arctanh(tanh(b*x+a))^2-1/2*b*(1/5*x^5*arctanh(tanh(b*x+a))-1/30*x^6*b)))
```

Maxima [B] time = 1.74207, size = 97, normalized size = 2.85

$$-\frac{2}{3}bx^3\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^3 + \frac{1}{2}x^2\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^4 + \frac{1}{30}\left(15\,bx^4\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^2 + \left(b^2x^6 - 6\,bx^5\,\text{a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

```
[Out] -2/3*b*x^3*arctanh(tanh(b*x + a))^3 + 1/2*x^2*arctanh(tanh(b*x + a))^4 + 1/
30*(15*b*x^4*arctanh(tanh(b*x + a))^2 + (b^2*x^6 - 6*b*x^5*arctanh(tanh(b*x
 + a)))*b)*b
```

Fricas [A] time = 1.49553, size = 104, normalized size = 3.06

$$\frac{1}{6}\,b^{4}x^{6}+\frac{4}{5}\,ab^{3}x^{5}+\frac{3}{2}\,a^{2}b^{2}x^{4}+\frac{4}{3}\,a^{3}bx^{3}+\frac{1}{2}\,a^{4}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

```
[Out] 1/6*b^4*x^6 + 4/5*a*b^3*x^5 + 3/2*a^2*b^2*x^4 + 4/3*a^3*b*x^3 + 1/2*a^4*x^2
```

Sympy [B] time = 2.47037, size = 76, normalized size = 2.24

$$\frac{b^{4}x^{6}}{30}-\frac{b^{3}x^{5}\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{5}+\frac{b^{2}x^{4}\operatorname{atanh}^{2}\left(\tanh\left(a+bx\right)\right)}{2}-\frac{2bx^{3}\operatorname{atanh}^{3}\left(\tanh\left(a+bx\right)\right)}{3}+\frac{x^{2}\operatorname{atanh}^{4}\left(\tanh\left(a\right.\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(tanh(b*x+a))**4,x)
```

```
[Out] b**4*x**6/30 - b**3*x**5*atanh(tanh(a + b*x))/5 + b**2*x**4*atanh(tanh(a +
b*x))**2/2 - 2*b*x**3*atanh(tanh(a + b*x))**3/3 + x**2*atanh(tanh(a + b*x))
**4/2
```

Giac [A] time = 1.10129, size = 62, normalized size = 1.82

$$\frac{1}{6}\,b^{4}x^{6}+\frac{4}{5}\,ab^{3}x^{5}+\frac{3}{2}\,a^{2}b^{2}x^{4}+\frac{4}{3}\,a^{3}bx^{3}+\frac{1}{2}\,a^{4}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^4,x, algorithm="giac")

[Out] 1/6*b^4*x^6 + 4/5*a*b^3*x^5 + 3/2*a^2*b^2*x^4 + 4/3*a^3*b*x^3 + 1/2*a^4*x^2
```

3.72 $\int \tanh^{-1}(\tanh(a+bx))^4\,dx$

Optimal. Leaf size=16

$$\frac{\tanh^{-1}(\tanh(a+bx))^5}{5b}$$

```
[Out] ArcTanh[Tanh[a + b*x]]^5/(5*b)
```

Rubi [A] time = 0.0046208, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2157, 30}

$$\frac{\tanh^{-1}(\tanh(a+bx))^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^5/(5*b)
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(\tanh(a+bx))^4\,dx &= \frac{\text{Subst}\left(\int x^4\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{\tanh^{-1}(\tanh(a+bx))^5}{5b}\end{aligned}$$

Mathematica [A] time = 0.0054625, size = 16, normalized size = 1.

$$\frac{\tanh^{-1}(\tanh(a+bx))^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^5/(5*b)
```

Maple [A] time = 0.027, size = 15, normalized size = 0.9

$$\frac{(\operatorname{Artanh}(\tanh(bx+a)))^5}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4,x)
```

```
[Out] 1/5*arctanh(tanh(b*x+a))^5/b
```

Maxima [B] time = 1.7329, size = 93, normalized size = 5.81

$$-2\,bx^2\operatorname{artanh}(\tanh(bx+a))^3 + x\operatorname{artanh}(\tanh(bx+a))^4 + \frac{1}{5}\left(10\,bx^3\operatorname{artanh}(\tanh(bx+a))^2 + \left(b^2x^5 - 5\,bx^4\operatorname{artan}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

```
[Out] -2*b*x^2*arctanh(tanh(b*x + a))^3 + x*arctanh(tanh(b*x + a))^4 + 1/5*(10*b*
x^3*arctanh(tanh(b*x + a))^2 + (b^2*x^5 - 5*b*x^4*arctanh(tanh(b*x + a)))*b
)*b
```

Fricas [B] time = 1.53508, size = 85, normalized size = 5.31

$$\frac{1}{5}\,b^4x^5 + ab^3x^4 + 2\,a^2b^2x^3 + 2\,a^3bx^2 + a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

```
[Out] 1/5*b^4*x^5 + a*b^3*x^4 + 2*a^2*b^2*x^3 + 2*a^3*b*x^2 + a^4*x
```

Sympy [A] time = 2.01654, size = 20, normalized size = 1.25

$$\begin{cases} \frac{\operatorname{atanh}^{5}{\left(\tanh{\left(a + b x \right)} \right)}}{5 b} & \text{for}\: b \neq 0 \\ x \operatorname{atanh}^{4}{\left(\tanh{\left(a \right)} \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4,x)
```

```
[Out] Piecewise((atanh(tanh(a + b*x))**5/(5*b), Ne(b, 0)), (x*atanh(tanh(a))**4,
True))
```

Giac [B] time = 1.13025, size = 57, normalized size = 3.56

$$\frac{1}{5}\,b^{4}x^{5} + ab^{3}x^{4} + 2\,a^{2}b^{2}x^{3} + 2\,a^{3}bx^{2} + a^{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4,x, algorithm="giac")
```

```
[Out] 1/5*b^4*x^5 + a*b^3*x^4 + 2*a^2*b^2*x^3 + 2*a^3*b*x^2 + a^4*x
```

3.73 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x} \, dx$

Optimal. Leaf size=105

$$-bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3+\frac{1}{2}\tanh^{-1}(\tanh(a+bx))^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2-\frac{1}{3}\tanh^{-1}(\tanh(a+bx$$

```
[Out] -(b*x*(b*x - ArcTanh[Tanh[a + b*x]])^3) + ((b*x - ArcTanh[Tanh[a + b*x]])^2
*ArcTanh[Tanh[a + b*x]]^2)/2 - ((b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh
[a + b*x]]^3)/3 + ArcTanh[Tanh[a + b*x]]^4/4 + (b*x - ArcTanh[Tanh[a + b*x]
])^4*Log[x]
```

Rubi [A] time = 0.064792, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2159, 2158, 29}

$$-bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3+\frac{1}{2}\tanh^{-1}(\tanh(a+bx))^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2-\frac{1}{3}\tanh^{-1}(\tanh(a+bx$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x,x]
```

```
[Out] -(b*x*(b*x - ArcTanh[Tanh[a + b*x]])^3) + ((b*x - ArcTanh[Tanh[a + b*x]])^2
*ArcTanh[Tanh[a + b*x]]^2)/2 - ((b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh
[a + b*x]]^3)/3 + ArcTanh[Tanh[a + b*x]]^4/4 + (b*x - ArcTanh[Tanh[a + b*x]
])^4*Log[x]
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x}\,dx = \frac{1}{4}\tanh^{-1}(\tanh(a+bx))^4 - \left(bx - \tanh^{-1}(\tanh(a+bx))\right)\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x}\,dx$$
$$= -\frac{1}{3}\left(bx - \tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^3 + \frac{1}{4}\tanh^{-1}(\tanh(a+bx))^4 - ($$
$$= \frac{1}{2}\left(bx - \tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^2 - \frac{1}{3}\left(bx - \tanh^{-1}(\tanh(a+bx)\right.$$
$$= -bx\left(bx - \tanh^{-1}(\tanh(a+bx))\right)^3 + \frac{1}{2}\left(bx - \tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+$$
$$= -bx\left(bx - \tanh^{-1}(\tanh(a+bx))\right)^3 + \frac{1}{2}\left(bx - \tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+$$

Mathematica [A] time = 0.0914317, size = 175, normalized size = 1.67

$$\frac{1}{2}(a+bx)^2\left(a^2 - 4a\left(-\tanh^{-1}(\tanh(a+bx)) + a + bx\right) + 6\left(-\tanh^{-1}(\tanh(a+bx)) + a + bx\right)^2\right) + (a+bx)\left(-4a^2\left(-\tan\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x,x]
```

```
[Out] (a + b*x)^4/4 + ((a + b*x)^2*(a^2 - 4*a*(a + b*x - ArcTanh[Tanh[a + b*x]])
+ 6*(a + b*x - ArcTanh[Tanh[a + b*x]])^2))/2 + (a + b*x)*(a^3 - 4*a^2*(a +
b*x - ArcTanh[Tanh[a + b*x]]) + 6*a*(a + b*x - ArcTanh[Tanh[a + b*x]])^2 -
4*(a + b*x - ArcTanh[Tanh[a + b*x]])^3) - ((a + b*x)^3*(3*a + 4*b*x - 4*Arc
Tanh[Tanh[a + b*x]]))/3 + (-(b*x) + ArcTanh[Tanh[a + b*x]])^4*Log[b*x]
```

Maple [A] time = 0.038, size = 127, normalized size = 1.2

$$\ln(x)\left(\operatorname{Artanh}(\tanh(bx+a))\right)^4 - 4\operatorname{Artanh}(\tanh(bx+a))\ln(x)x^3b^3 + 6\left(\operatorname{Artanh}(\tanh(bx+a))\right)^2\ln(x)x^2b^2 + \frac{22\,A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x,x)
```

```
[Out] ln(x)*arctanh(tanh(b*x+a))^4-4*arctanh(tanh(b*x+a))*ln(x)*x^3*b^3+6*arctanh
(tanh(b*x+a))^2*ln(x)*x^2*b^2+22/3*arctanh(tanh(b*x+a))*x^3*b^3-9*arctanh(t
anh(b*x+a))^2*x^2*b^2-4*b*arctanh(tanh(b*x+a))^3*ln(x)*x+b^4*x^4*ln(x)-25/1
2*x^4*b^4+4*b*arctanh(tanh(b*x+a))^3*x
```

Maxima [A] time = 2.40378, size = 57, normalized size = 0.54

$$\frac{1}{4}b^4x^4 + \frac{4}{3}ab^3x^3 + 3a^2b^2x^2 + 4a^3bx + a^4\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x,x, algorithm="maxima")
```

```
[Out] 1/4*b^4*x^4 + 4/3*a*b^3*x^3 + 3*a^2*b^2*x^2 + 4*a^3*b*x + a^4*log(x)
```

Fricas [A] time = 1.61591, size = 95, normalized size = 0.9

$$\frac{1}{4}b^4x^4 + \frac{4}{3}ab^3x^3 + 3a^2b^2x^2 + 4a^3bx + a^4\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x,x, algorithm="fricas")
```

```
[Out] 1/4*b^4*x^4 + 4/3*a*b^3*x^3 + 3*a^2*b^2*x^2 + 4*a^3*b*x + a^4*log(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^4(\tanh(a + bx))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**4/x, x)
```

Giac [A] time = 1.1105, size = 58, normalized size = 0.55

$$\frac{1}{4}\,b^4x^4 + \frac{4}{3}\,ab^3x^3 + 3\,a^2b^2x^2 + 4\,a^3bx + a^4\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x,x, algorithm="giac")
```

```
[Out] 1/4*b^4*x^4 + 4/3*a*b^3*x^3 + 3*a^2*b^2*x^2 + 4*a^3*b*x + a^4*log(abs(x))
```

3.74 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^2} dx$

Optimal. Leaf size=95

$$4b^2x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2-\frac{\tanh^{-1}(\tanh(a+bx))^4}{x}+\frac{4}{3}b\tanh^{-1}(\tanh(a+bx))^3-2b\left(bx-\tanh^{-1}(\tanh(a+bx))\right.$$

```
[Out] 4*b^2*x*(b*x - ArcTanh[Tanh[a + b*x]])^2 - 2*b*(b*x - ArcTanh[Tanh[a + b*x]
])*ArcTanh[Tanh[a + b*x]]^2 + (4*b*ArcTanh[Tanh[a + b*x]]^3)/3 - ArcTanh[Ta
nh[a + b*x]]^4/x - 4*b*(b*x - ArcTanh[Tanh[a + b*x]])^3*Log[x]
```

Rubi [A] time = 0.0638568, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2168, 2159, 2158, 29}

$$4b^2x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2-\frac{\tanh^{-1}(\tanh(a+bx))^4}{x}+\frac{4}{3}b\tanh^{-1}(\tanh(a+bx))^3-2b\left(bx-\tanh^{-1}(\tanh(a+bx))\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x^2,x]
```


```
[Out] 4*b^2*x*(b*x - ArcTanh[Tanh[a + b*x]])^2 - 2*b*(b*x - ArcTanh[Tanh[a + b*x]
])*ArcTanh[Tanh[a + b*x]]^2 + (4*b*ArcTanh[Tanh[a + b*x]]^3)/3 - ArcTanh[Ta
nh[a + b*x]]^4/x - 4*b*(b*x - ArcTanh[Tanh[a + b*x]])^3*Log[x]
```


Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
```

```
1]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^2}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^4}{x} + (4b)\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x}\,dx$$
$$= \frac{4}{3}b\tanh^{-1}(\tanh(a+bx))^3 - \frac{\tanh^{-1}(\tanh(a+bx))^4}{x} - \left(4b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\right.$$
$$= -2b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2 + \frac{4}{3}b\tanh^{-1}(\tanh(a+bx))^3 -$$
$$= 4b^2x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 - 2b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+$$
$$= 4b^2x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 - 2b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+$$

Mathematica [A] time = 0.0759046, size = 85, normalized size = 0.89

$$6b^3x^2(2\log(x)-1)\tanh^{-1}(\tanh(a+bx)) - 12b^2x\log(x)\tanh^{-1}(\tanh(a+bx))^2 - \frac{\tanh^{-1}(\tanh(a+bx))^4}{x} + 4b(\log(x)+$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x^2,x]
```

```
[Out] -(ArcTanh[Tanh[a + b*x]]^4/x) + (2*b^4*x^3*(5 - 6*Log[x]))/3 - 12*b^2*x*Arc
Tanh[Tanh[a + b*x]]^2*Log[x] + 4*b*ArcTanh[Tanh[a + b*x]]^3*(1 + Log[x]) +
6*b^3*x^2*ArcTanh[Tanh[a + b*x]]*(-1 + 2*Log[x])
```

Maple [A] time = 0.04, size = 112, normalized size = 1.2

$$-\frac{(\text{Artanh}(\tanh(bx+a)))^4}{x}+4\ln(x)(\text{Artanh}(\tanh(bx+a)))^3b+12\,\text{Artanh}(\tanh(bx+a))\ln(x)x^2b^3-18\,\text{Artar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x^2,x)
```

```
[Out] -arctanh(tanh(b*x+a))^4/x+4*ln(x)*arctanh(tanh(b*x+a))^3*b+12*arctanh(tanh(
b*x+a))*ln(x)*x^2*b^3-18*arctanh(tanh(b*x+a))*x^2*b^3-12*b^2*arctanh(tanh(b
*x+a))^2*ln(x)*x-4*b^4*x^3*ln(x)+22/3*x^3*b^4+12*b^2*arctanh(tanh(b*x+a))^2
*x
```

Maxima [A] time = 2.61009, size = 104, normalized size = 1.09

$$4\,b\,\text{artanh}(\tanh(bx+a))^3\log(x)-\frac{\text{artanh}(\tanh(bx+a))^4}{x}+\frac{2}{3}\left(2\,b^3x^3+9\,ab^2x^2+18\,a^2bx+6\,a^3\log(x)-6\,\text{artanl}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^2,x, algorithm="maxima")
```

```
[Out] 4*b*arctanh(tanh(b*x + a))^3*log(x) - arctanh(tanh(b*x + a))^4/x + 2/3*(2*b
^3*x^3 + 9*a*b^2*x^2 + 18*a^2*b*x + 6*a^3*log(x) - 6*arctanh(tanh(b*x + a))
^3*log(x))*b
```

Fricas [A] time = 1.48507, size = 103, normalized size = 1.08

$$\frac{b^4x^4+6\,ab^3x^3+18\,a^2b^2x^2+12\,a^3bx\log(x)-3\,a^4}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*x^4 + 6*a*b^3*x^3 + 18*a^2*b^2*x^2 + 12*a^3*b*x*log(x) - 3*a^4)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^4\left(\tanh\left(a+bx\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x**2,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**4/x**2, x)
```

Giac [A] time = 1.18977, size = 59, normalized size = 0.62

$$\frac{1}{3}\,b^4x^3 + 2\,ab^3x^2 + 6\,a^2b^2x + 4\,a^3b\log\left(|x|\right) - \frac{a^4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^2,x, algorithm="giac")
```

```
[Out] 1/3*b^4*x^3 + 2*a*b^3*x^2 + 6*a^2*b^2*x + 4*a^3*b*log(abs(x)) - a^4/x
```

3.75 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^3}\,dx$

Optimal. Leaf size=87

$$3b^2\tanh^{-1}(\tanh(a+bx))^2-6b^3x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+6b^2\log(x)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2-\frac{\tanh^{-1}(\mathrm{t}}{}$$

```
[Out] -6*b^3*x*(b*x - ArcTanh[Tanh[a + b*x]]) + 3*b^2*ArcTanh[Tanh[a + b*x]]^2 -
(2*b*ArcTanh[Tanh[a + b*x]]^3)/x - ArcTanh[Tanh[a + b*x]]^4/(2*x^2) + 6*b^2
*(b*x - ArcTanh[Tanh[a + b*x]])^2*Log[x]
```

Rubi [A] time = 0.062113, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2168, 2159, 2158, 29}

$$3b^2\tanh^{-1}(\tanh(a+bx))^2-6b^3x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+6b^2\log(x)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2-\frac{\tanh^{-1}(\mathrm{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x^3,x]
```

```
[Out] -6*b^3*x*(b*x - ArcTanh[Tanh[a + b*x]]) + 3*b^2*ArcTanh[Tanh[a + b*x]]^2 -
(2*b*ArcTanh[Tanh[a + b*x]]^3)/x - ArcTanh[Tanh[a + b*x]]^4/(2*x^2) + 6*b^2
*(b*x - ArcTanh[Tanh[a + b*x]])^2*Log[x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
```

```
1]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^3}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^4}{2x^2} + (2b)\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^2}\,dx$$
$$= -\frac{2b\tanh^{-1}(\tanh(a+bx))^3}{x} - \frac{\tanh^{-1}(\tanh(a+bx))^4}{2x^2} + \left(6b^2\right)\int \frac{\tanh^{-1}(\tanh(a+bx))}{x}$$
$$= 3b^2\tanh^{-1}(\tanh(a+bx))^2 - \frac{2b\tanh^{-1}(\tanh(a+bx))^3}{x} - \frac{\tanh^{-1}(\tanh(a+bx))^4}{2x^2} - \left(6b\right.$$
$$= -6b^3x\left(bx-\tanh^{-1}(\tanh(a+bx))\right) + 3b^2\tanh^{-1}(\tanh(a+bx))^2 - \frac{2b\tanh^{-1}(\tanh(a}{x}$$
$$= -6b^3x\left(bx-\tanh^{-1}(\tanh(a+bx))\right) + 3b^2\tanh^{-1}(\tanh(a+bx))^2 - \frac{2b\tanh^{-1}(\tanh(a}{x}$$

Mathematica [A] time = 0.0345662, size = 81, normalized size = 0.93

$$-6b^3x(2\log(x)+1)\tanh^{-1}(\tanh(a+bx)) + 3b^2(2\log(x)+3)\tanh^{-1}(\tanh(a+bx))^2 - \frac{\tanh^{-1}(\tanh(a+bx))^4}{2x^2} - \frac{2b\tan}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x^3,x]
```

```
[Out] (-2*b*ArcTanh[Tanh[a + b*x]]^3)/x - ArcTanh[Tanh[a + b*x]]^4/(2*x^2) + 6*b^
4*x^2*Log[x] - 6*b^3*x*ArcTanh[Tanh[a + b*x]]*(1 + 2*Log[x]) + 3*b^2*ArcTan
h[Tanh[a + b*x]]^2*(3 + 2*Log[x])
```

Maple [A] time = 0.043, size = 93, normalized size = 1.1

$$-\frac{(\mathrm{Artanh}(\tanh(bx+a)))^4}{2x^2}-2\,\frac{b(\mathrm{Artanh}(\tanh(bx+a)))^3}{x}+6\,b^2\ln(x)(\mathrm{Artanh}(\tanh(bx+a)))^2+6\,b^4x^2\ln(x)-9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x^3,x)
```

```
[Out] -1/2*arctanh(tanh(b*x+a))^4/x^2-2*b*arctanh(tanh(b*x+a))^3/x+6*b^2*ln(x)*ar
ctanh(tanh(b*x+a))^2+6*b^4*x^2*ln(x)-9*x^2*b^4-12*b^3*arctanh(tanh(b*x+a))*
ln(x)*x+12*b^3*arctanh(tanh(b*x+a))*x
```

Maxima [A] time = 2.77082, size = 112, normalized size = 1.29

$$-\frac{2\,b\,\mathrm{artanh}(\tanh(bx+a))^3}{x}+3\left(2\,b\,\mathrm{artanh}(\tanh(bx+a))^2\log(x)+\left(b^2x^2+4\,abx+2\,a^2\log(x)-2\,\mathrm{artanh}(\tanh(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^3,x, algorithm="maxima")
```

```
[Out] -2*b*arctanh(tanh(b*x + a))^3/x + 3*(2*b*arctanh(tanh(b*x + a))^2*log(x) +
(b^2*x^2 + 4*a*b*x + 2*a^2*log(x) - 2*arctanh(tanh(b*x + a))^2*log(x))*b)*b
 - 1/2*arctanh(tanh(b*x + a))^4/x^2
```

Fricas [A] time = 1.57488, size = 101, normalized size = 1.16

$$\frac{b^4x^4+8\,ab^3x^3+12\,a^2b^2x^2\log(x)-8\,a^3bx-a^4}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*x^4 + 8*a*b^3*x^3 + 12*a^2*b^2*x^2*log(x) - 8*a^3*b*x - a^4)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^4(\tanh(a+bx))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x**3,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**4/x**3, x)
```

Giac [A] time = 1.16512, size = 58, normalized size = 0.67

$$\frac{1}{2}\,b^4x^2+4\,ab^3x+6\,a^2b^2\log\left(|x|\right)-\frac{8\,a^3bx+a^4}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^3,x, algorithm="giac")
```

```
[Out] 1/2*b^4*x^2 + 4*a*b^3*x + 6*a^2*b^2*log(abs(x)) - 1/2*(8*a^3*b*x + a^4)/x^2
```

3.76 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^4} dx$

Optimal. Leaf size=77

$$-\frac{2b^2\tanh^{-1}(\tanh(a+bx))^2}{x}-4b^3\log(x)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)-\frac{2b\tanh^{-1}(\tanh(a+bx))^3}{3x^2}-\frac{\tanh^{-1}(\tanh(a}{3x^3}$$

```
[Out] 4*b^4*x - (2*b^2*ArcTanh[Tanh[a + b*x]]^2)/x - (2*b*ArcTanh[Tanh[a + b*x]]^
3)/(3*x^2) - ArcTanh[Tanh[a + b*x]]^4/(3*x^3) - 4*b^3*(b*x - ArcTanh[Tanh[a
 + b*x]])*Log[x]
```

Rubi [A] time = 0.0582334, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2158, 29}

$$-\frac{2b^2\tanh^{-1}(\tanh(a+bx))^2}{x}-4b^3\log(x)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)-\frac{2b\tanh^{-1}(\tanh(a+bx))^3}{3x^2}-\frac{\tanh^{-1}(\tanh(a}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x^4,x]
```

```
[Out] 4*b^4*x - (2*b^2*ArcTanh[Tanh[a + b*x]]^2)/x - (2*b*ArcTanh[Tanh[a + b*x]]^
3)/(3*x^2) - ArcTanh[Tanh[a + b*x]]^4/(3*x^3) - 4*b^3*(b*x - ArcTanh[Tanh[a
 + b*x]])*Log[x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^4}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^4}{3x^3} + \frac{1}{3}(4b)\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^3}\,dx$$
$$= -\frac{2b\tanh^{-1}(\tanh(a+bx))^3}{3x^2} - \frac{\tanh^{-1}(\tanh(a+bx))^4}{3x^3} + \left(2b^2\right)\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^2}$$
$$= -\frac{2b^2\tanh^{-1}(\tanh(a+bx))^2}{x} - \frac{2b\tanh^{-1}(\tanh(a+bx))^3}{3x^2} - \frac{\tanh^{-1}(\tanh(a+bx))^4}{3x^3} + ($$
$$= 4b^4x - \frac{2b^2\tanh^{-1}(\tanh(a+bx))^2}{x} - \frac{2b\tanh^{-1}(\tanh(a+bx))^3}{3x^2} - \frac{\tanh^{-1}(\tanh(a+bx}{3x^3}$$
$$= 4b^4x - \frac{2b^2\tanh^{-1}(\tanh(a+bx))^2}{x} - \frac{2b\tanh^{-1}(\tanh(a+bx))^3}{3x^2} - \frac{\tanh^{-1}(\tanh(a+bx}{3x^3}$$

Mathematica [A] time = 0.0481872, size = 82, normalized size = 1.06

$$-\frac{6b^2x^2\tanh^{-1}(\tanh(a+bx))^2 - 2b^3x^3(6\log(x)+11)\tanh^{-1}(\tanh(a+bx)) + 2bx\tanh^{-1}(\tanh(a+bx))^3 + \tanh^{-1}(\tan}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x^4,x]
```

```
[Out] -(6*b^2*x^2*ArcTanh[Tanh[a + b*x]]^2 + 2*b*x*ArcTanh[Tanh[a + b*x]]^3 + Arc
Tanh[Tanh[a + b*x]]^4 + 2*b^4*x^4*(5 + 6*Log[x]) - 2*b^3*x^3*ArcTanh[Tanh[a
 + b*x]]*(11 + 6*Log[x]))/(3*x^3)
```

Maple [A] time = 0.046, size = 76, normalized size = 1.

$$-\frac{(\mathrm{Artanh}(\tanh(bx+a)))^4}{3x^3} - \frac{2b(\mathrm{Artanh}(\tanh(bx+a)))^3}{3x^2} - 2\frac{b^2(\mathrm{Artanh}(\tanh(bx+a)))^2}{x} - 4\ln(x)xb^4 + 4\,\mathrm{Artan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x^4,x)
```

```
[Out] -1/3*arctanh(tanh(b*x+a))^4/x^3-2/3*b*arctanh(tanh(b*x+a))^3/x^2-2*b^2*arct
anh(tanh(b*x+a))^2/x-4*ln(x)*x*b^4+4*arctanh(tanh(b*x+a))*ln(x)*b^3+4*b^4*x
```

Maxima [A] time = 1.61241, size = 123, normalized size = 1.6

$$2\left(2\left(b\operatorname{artanh}(\tanh(bx+a))\log(x)-\left(b\left(x+\frac{a}{b}\right)\log(x)-b\left(x+\frac{a\log(x)}{b}\right)\right)b\right)b-\frac{b\operatorname{artanh}(\tanh(bx+a))^2}{x}\right)b-\frac{2\,b\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^4,x, algorithm="maxima")
```

```
[Out] 2*(2*(b*arctanh(tanh(b*x + a))*log(x) - (b*(x + a/b)*log(x) - b*(x + a*log(
x)/b))*b)*b - b*arctanh(tanh(b*x + a))^2/x)*b - 2/3*b*arctanh(tanh(b*x + a)
)^3/x^2 - 1/3*arctanh(tanh(b*x + a))^4/x^3
```

Fricas [A] time = 1.54661, size = 105, normalized size = 1.36

$$\frac{3\,b^4x^4+12\,ab^3x^3\log(x)-18\,a^2b^2x^2-6\,a^3bx-a^4}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b^4*x^4 + 12*a*b^3*x^3*log(x) - 18*a^2*b^2*x^2 - 6*a^3*b*x - a^4)/x^
3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atanh}^4(\tanh(a+bx))}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x**4,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**4/x**4, x)
```

Giac [A] time = 1.13538, size = 57, normalized size = 0.74

$$b^4x + 4\,ab^3\log\left(|x|\right) - \frac{18\,a^2b^2x^2 + 6\,a^3bx + a^4}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^4,x, algorithm="giac")
```

```
[Out] b^4*x + 4*a*b^3*log(abs(x)) - 1/3*(18*a^2*b^2*x^2 + 6*a^3*b*x + a^4)/x^3
```

3.77 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^5} dx$

Optimal. Leaf size=74

$$-\frac{b^2 \tanh^{-1}(\tanh(a+bx))^2}{2x^2} - \frac{b^3 \tanh^{-1}(\tanh(a+bx))}{x} - \frac{b \tanh^{-1}(\tanh(a+bx))^3}{3x^3} - \frac{\tanh^{-1}(\tanh(a+bx))^4}{4x^4} + b^4 \log$$

```
[Out] -((b^3*ArcTanh[Tanh[a + b*x]])/x) - (b^2*ArcTanh[Tanh[a + b*x]]^2)/(2*x^2)
- (b*ArcTanh[Tanh[a + b*x]]^3)/(3*x^3) - ArcTanh[Tanh[a + b*x]]^4/(4*x^4) +
 b^4*Log[x]
```

Rubi [A] time = 0.05442, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 29}

$$-\frac{b^2 \tanh^{-1}(\tanh(a+bx))^2}{2x^2} - \frac{b^3 \tanh^{-1}(\tanh(a+bx))}{x} - \frac{b \tanh^{-1}(\tanh(a+bx))^3}{3x^3} - \frac{\tanh^{-1}(\tanh(a+bx))^4}{4x^4} + b^4 \log$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x^5,x]
```

```
[Out] -((b^3*ArcTanh[Tanh[a + b*x]])/x) - (b^2*ArcTanh[Tanh[a + b*x]]^2)/(2*x^2)
- (b*ArcTanh[Tanh[a + b*x]]^3)/(3*x^3) - ArcTanh[Tanh[a + b*x]]^4/(4*x^4) +
 b^4*Log[x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^5}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^4}{4x^4} + b\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^4}\,dx$$
$$= -\frac{b\tanh^{-1}(\tanh(a+bx))^3}{3x^3} - \frac{\tanh^{-1}(\tanh(a+bx))^4}{4x^4} + b^2\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^3}\,dx$$
$$= -\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{2x^2} - \frac{b\tanh^{-1}(\tanh(a+bx))^3}{3x^3} - \frac{\tanh^{-1}(\tanh(a+bx))^4}{4x^4} + b^3$$
$$= -\frac{b^3\tanh^{-1}(\tanh(a+bx))}{x} - \frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{2x^2} - \frac{b\tanh^{-1}(\tanh(a+bx))^3}{3x^3} - $$
$$= -\frac{b^3\tanh^{-1}(\tanh(a+bx))}{x} - \frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{2x^2} - \frac{b\tanh^{-1}(\tanh(a+bx))^3}{3x^3} - $$

Mathematica [A] time = 0.0301265, size = 78, normalized size = 1.05

$$-\frac{12b^3x^3\tanh^{-1}(\tanh(a+bx)) + 6b^2x^2\tanh^{-1}(\tanh(a+bx))^2 + 4bx\tanh^{-1}(\tanh(a+bx))^3 + 3\tanh^{-1}(\tanh(a+bx))^4}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x^5,x]
```

```
[Out] -(12*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 6*b^2*x^2*ArcTanh[Tanh[a + b*x]]^2 +
4*b*x*ArcTanh[Tanh[a + b*x]]^3 + 3*ArcTanh[Tanh[a + b*x]]^4 - b^4*x^4*(25 +
 12*Log[x]))/(12*x^4)
```

Maple [A] time = 0.041, size = 69, normalized size = 0.9

$$-\frac{b^3\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}{x} - \frac{b^2\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^2}{2\,x^2} - \frac{b\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^3}{3\,x^3} - \frac{\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^4}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x^5,x)
```

```
[Out] -b^3*arctanh(tanh(b*x+a))/x-1/2*b^2*arctanh(tanh(b*x+a))^2/x^2-1/3*b*arctan
h(tanh(b*x+a))^3/x^3-1/4*arctanh(tanh(b*x+a))^4/x^4+b^4*ln(x)
```

Maxima [A] time = 1.79604, size = 97, normalized size = 1.31

$$\frac{1}{2}\left(2\left(b^2\log(x) - \frac{b\,\mathrm{artanh}(\tanh(bx+a))}{x}\right)b - \frac{b\,\mathrm{artanh}(\tanh(bx+a))^2}{x^2}\right)b - \frac{b\,\mathrm{artanh}(\tanh(bx+a))^3}{3\,x^3} - \frac{\mathrm{artanh}(\mathrm{ta}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b^2*log(x) - b*arctanh(tanh(b*x + a))/x)*b - b*arctanh(tanh(b*x + a
))^2/x^2)*b - 1/3*b*arctanh(tanh(b*x + a))^3/x^3 - 1/4*arctanh(tanh(b*x + a
))^4/x^4
```

Fricas [A] time = 1.56394, size = 112, normalized size = 1.51

$$\frac{12\,b^4x^4\log(x) - 48\,ab^3x^3 - 36\,a^2b^2x^2 - 16\,a^3bx - 3\,a^4}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(12*b^4*x^4*log(x) - 48*a*b^3*x^3 - 36*a^2*b^2*x^2 - 16*a^3*b*x - 3*a^
4)/x^4
```

Sympy [A] time = 2.29266, size = 70, normalized size = 0.95

$$b^4\log(x) - \frac{b^3\,\mathrm{atanh}(\tanh(a+bx))}{x} - \frac{b^2\,\mathrm{atanh}^2(\tanh(a+bx))}{2x^2} - \frac{b\,\mathrm{atanh}^3(\tanh(a+bx))}{3x^3} - \frac{\mathrm{atanh}^4(\tanh(a+bx}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x**5,x)
```

```
[Out] b**4*log(x) - b**3*atanh(tanh(a + b*x))/x - b**2*atanh(tanh(a + b*x))**2/(2
*x**2) - b*atanh(tanh(a + b*x))**3/(3*x**3) - atanh(tanh(a + b*x))**4/(4*x*
*4)
```

Giac [A] time = 1.12572, size = 62, normalized size = 0.84

$$b^4 \log(|x|) - \frac{48\, ab^3x^3 + 36\, a^2b^2x^2 + 16\, a^3bx + 3\, a^4}{12\, x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^5,x, algorithm="giac")
```

```
[Out] b^4*log(abs(x)) - 1/12*(48*a*b^3*x^3 + 36*a^2*b^2*x^2 + 16*a^3*b*x + 3*a^4)
/x^4
```

3.78 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^6}\,dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}(\tanh(a+bx))^5}{5x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] ArcTanh[Tanh[a + b*x]]^5/(5*x^5*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rubi [A] time = 0.0131038, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2167}

$$\frac{\tanh^{-1}(\tanh(a+bx))^5}{5x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x^6,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^5/(5*x^5*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^6}\,dx=\frac{\tanh^{-1}(\tanh(a+bx))^5}{5x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [B] time = 0.052177, size = 66, normalized size = 2.13

$$-\frac{b^3x^3\tanh^{-1}(\tanh(a+bx))+b^2x^2\tanh^{-1}(\tanh(a+bx))^2+bx\tanh^{-1}(\tanh(a+bx))^3+\tanh^{-1}(\tanh(a+bx))^4+b^4}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x^6,x]
```

```
[Out] -(b^4*x^4 + b^3*x^3*ArcTanh[Tanh[a + b*x]] + b^2*x^2*ArcTanh[Tanh[a + b*x]]
^2 + b*x*ArcTanh[Tanh[a + b*x]]^3 + ArcTanh[Tanh[a + b*x]]^4)/(5*x^5)
```

Maple [B] time = 0.044, size = 74, normalized size = 2.4

$$-\frac{(\text{Artanh}(\tanh(bx+a)))^4}{5x^5}+\frac{4b}{5}\left(-\frac{(\text{Artanh}(\tanh(bx+a)))^3}{4x^4}+\frac{3b}{4}\left(-\frac{(\text{Artanh}(\tanh(bx+a)))^2}{3x^3}+\frac{2b}{3}\left(-\frac{b}{2x}-\text{Art}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x^6,x)
```

```
[Out] -1/5*arctanh(tanh(b*x+a))^4/x^5+4/5*b*(-1/4*arctanh(tanh(b*x+a))^3/x^4+3/4*
b*(-1/3*arctanh(tanh(b*x+a))^2/x^3+2/3*b*(-1/2*b/x-1/2*arctanh(tanh(b*x+a))
/x^2)))
```

Maxima [B] time = 1.79837, size = 95, normalized size = 3.06

$$-\frac{1}{5}\left(b\left(\frac{b^2}{x}+\frac{b\,\text{artanh}(\tanh(bx+a))}{x^2}\right)+\frac{b\,\text{artanh}(\tanh(bx+a))^2}{x^3}\right)b-\frac{b\,\text{artanh}(\tanh(bx+a))^3}{5x^4}-\frac{\text{artanh}(\tanh(bx}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^6,x, algorithm="maxima")
```

```
[Out] -1/5*(b*(b^2/x + b*arctanh(tanh(b*x + a))/x^2) + b*arctanh(tanh(b*x + a))^2
/x^3)*b - 1/5*b*arctanh(tanh(b*x + a))^3/x^4 - 1/5*arctanh(tanh(b*x + a))^4
/x^5
```

Fricas [A] time = 1.48467, size = 97, normalized size = 3.13

$$-\frac{5b^4x^4+10ab^3x^3+10a^2b^2x^2+5a^3bx+a^4}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^6,x, algorithm="fricas")
```

```
[Out] -1/5*(5*b^4*x^4 + 10*a*b^3*x^3 + 10*a^2*b^2*x^2 + 5*a^3*b*x + a^4)/x^5
```

Sympy [B] time = 4.02722, size = 75, normalized size = 2.42

$$-\frac{b^4}{5x} - \frac{b^3 \operatorname{atanh}(\tanh(a+bx))}{5x^2} - \frac{b^2 \operatorname{atanh}^2(\tanh(a+bx))}{5x^3} - \frac{b \operatorname{atanh}^3(\tanh(a+bx))}{5x^4} - \frac{\operatorname{atanh}^4(\tanh(a+bx))}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x**6,x)
```

```
[Out] -b**4/(5*x) - b**3*atanh(tanh(a + b*x))/(5*x**2) - b**2*atanh(tanh(a + b*x)
)**2/(5*x**3) - b*atanh(tanh(a + b*x))**3/(5*x**4) - atanh(tanh(a + b*x))**
4/(5*x**5)
```

Giac [A] time = 1.12471, size = 59, normalized size = 1.9

$$-\frac{5\,b^4x^4 + 10\,ab^3x^3 + 10\,a^2b^2x^2 + 5\,a^3bx + a^4}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^6,x, algorithm="giac")
```

```
[Out] -1/5*(5*b^4*x^4 + 10*a*b^3*x^3 + 10*a^2*b^2*x^2 + 5*a^3*b*x + a^4)/x^5
```

3.79 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^7} dx$

Optimal. Leaf size=64

$$\frac{\tanh^{-1}(\tanh(a+bx))^5}{6x^6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{b\tanh^{-1}(\tanh(a+bx))^5}{30x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

```
[Out] (b*ArcTanh[Tanh[a + b*x]]^5)/(30*x^5*(b*x - ArcTanh[Tanh[a + b*x]])^2) + Ar
cTanh[Tanh[a + b*x]]^5/(6*x^6*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rubi [A] time = 0.0313652, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2171, 2167}

$$\frac{\tanh^{-1}(\tanh(a+bx))^5}{6x^6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{b\tanh^{-1}(\tanh(a+bx))^5}{30x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x^7,x]
```

```
[Out] (b*ArcTanh[Tanh[a + b*x]]^5)/(30*x^5*(b*x - ArcTanh[Tanh[a + b*x]])^2) + Ar
cTanh[Tanh[a + b*x]]^5/(6*x^6*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^7}\,dx = \frac{\tanh^{-1}(\tanh(a+bx))^5}{6x^6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{b\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^6}\,dx}{6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$
$$= \frac{b\tanh^{-1}(\tanh(a+bx))^5}{30x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{\tanh^{-1}(\tanh(a+bx))^5}{6x^6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A] time = 0.0313331, size = 71, normalized size = 1.11

$$-\frac{2b^3x^3\tanh^{-1}(\tanh(a+bx)) + 3b^2x^2\tanh^{-1}(\tanh(a+bx))^2 + 4bx\tanh^{-1}(\tanh(a+bx))^3 + 5\tanh^{-1}(\tanh(a+bx))}{30x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x^7,x]
```

```
[Out] -(b^4*x^4 + 2*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 3*b^2*x^2*ArcTanh[Tanh[a + b
*x]]^2 + 4*b*x*ArcTanh[Tanh[a + b*x]]^3 + 5*ArcTanh[Tanh[a + b*x]]^4)/(30*x
^6)
```

Maple [A] time = 0.042, size = 74, normalized size = 1.2

$$-\frac{(\mathrm{Artanh}(\tanh(bx+a)))^4}{6x^6} + \frac{2b}{3}\left(-\frac{(\mathrm{Artanh}(\tanh(bx+a)))^3}{5x^5} + \frac{3b}{5}\left(-\frac{(\mathrm{Artanh}(\tanh(bx+a)))^2}{4x^4} + \frac{b}{2}\left(-\frac{b}{6x^2} - \mathrm{Ar}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x^7,x)
```

```
[Out] -1/6*arctanh(tanh(b*x+a))^4/x^6+2/3*b*(-1/5*arctanh(tanh(b*x+a))^3/x^5+3/5*
b*(-1/4*arctanh(tanh(b*x+a))^2/x^4+1/2*b*(-1/6*b/x^2-1/3*arctanh(tanh(b*x+a
))/x^3)))
```

Maxima [A] time = 1.79142, size = 97, normalized size = 1.52

$$-\frac{1}{30}\left(b\left(\frac{b^2}{x^2} + \frac{2b\,\mathrm{artanh}(\tanh(bx+a))}{x^3}\right) + \frac{3b\,\mathrm{artanh}(\tanh(bx+a))^2}{x^4}\right)b - \frac{2b\,\mathrm{artanh}(\tanh(bx+a))^3}{15x^5} - \frac{\mathrm{artanh}(\mathrm{ta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^7,x, algorithm="maxima")
```

```
[Out] -1/30*(b*(b^2/x^2 + 2*b*arctanh(tanh(b*x + a))/x^3) + 3*b*arctanh(tanh(b*x
+ a))^2/x^4)*b - 2/15*b*arctanh(tanh(b*x + a))^3/x^5 - 1/6*arctanh(tanh(b*x
 + a))^4/x^6
```

Fricas [A] time = 1.45738, size = 104, normalized size = 1.62

$$-\frac{15\,b^4x^4 + 40\,ab^3x^3 + 45\,a^2b^2x^2 + 24\,a^3bx + 5\,a^4}{30\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^7,x, algorithm="fricas")
```

```
[Out] -1/30*(15*b^4*x^4 + 40*a*b^3*x^3 + 45*a^2*b^2*x^2 + 24*a^3*b*x + 5*a^4)/x^6
```

Sympy [A] time = 6.67547, size = 78, normalized size = 1.22

$$-\frac{b^4}{30x^2} - \frac{b^3\operatorname{atanh}(\tanh(a+bx))}{15x^3} - \frac{b^2\operatorname{atanh}^2(\tanh(a+bx))}{10x^4} - \frac{2b\operatorname{atanh}^3(\tanh(a+bx))}{15x^5} - \frac{\operatorname{atanh}^4(\tanh(a+bx))}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x**7,x)
```

```
[Out] -b**4/(30*x**2) - b**3*atanh(tanh(a + b*x))/(15*x**3) - b**2*atanh(tanh(a +
 b*x))**2/(10*x**4) - 2*b*atanh(tanh(a + b*x))**3/(15*x**5) - atanh(tanh(a
+ b*x))**4/(6*x**6)
```

Giac [A] time = 1.16793, size = 62, normalized size = 0.97

$$-\frac{15\,b^4x^4 + 40\,ab^3x^3 + 45\,a^2b^2x^2 + 24\,a^3bx + 5\,a^4}{30\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^7,x, algorithm="giac")

[Out] -1/30*(15*b^4*x^4 + 40*a*b^3*x^3 + 45*a^2*b^2*x^2 + 24*a^3*b*x + 5*a^4)/x^6
```

3.80 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^8} dx$

Optimal. Leaf size=98

$$\frac{b^2 \tanh^{-1}(\tanh(a+bx))^5}{105x^5 \left(bx - \tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{\tanh^{-1}(\tanh(a+bx))^5}{7x^7 \left(bx - \tanh^{-1}(\tanh(a+bx))\right)} + \frac{b \tanh^{-1}(\tanh(a+bx))^5}{21x^6 \left(bx - \tanh^{-1}(\tanh(a+bx))\right)^2}$$

```
[Out] (b^2*ArcTanh[Tanh[a + b*x]]^5)/(105*x^5*(b*x - ArcTanh[Tanh[a + b*x]])^3) +
 (b*ArcTanh[Tanh[a + b*x]]^5)/(21*x^6*(b*x - ArcTanh[Tanh[a + b*x]])^2) + A
rcTanh[Tanh[a + b*x]]^5/(7*x^7*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rubi [A] time = 0.0531644, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2171, 2167}

$$\frac{b^2 \tanh^{-1}(\tanh(a+bx))^5}{105x^5 \left(bx - \tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{\tanh^{-1}(\tanh(a+bx))^5}{7x^7 \left(bx - \tanh^{-1}(\tanh(a+bx))\right)} + \frac{b \tanh^{-1}(\tanh(a+bx))^5}{21x^6 \left(bx - \tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x^8,x]
```

```
[Out] (b^2*ArcTanh[Tanh[a + b*x]]^5)/(105*x^5*(b*x - ArcTanh[Tanh[a + b*x]])^3) +
 (b*ArcTanh[Tanh[a + b*x]]^5)/(21*x^6*(b*x - ArcTanh[Tanh[a + b*x]])^2) + A
rcTanh[Tanh[a + b*x]]^5/(7*x^7*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
```

```
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^8}\,dx = \frac{\tanh^{-1}(\tanh(a+bx))^5}{7x^7\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(2b)\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^7}\,dx}{7\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

$$= \frac{b\tanh^{-1}(\tanh(a+bx))^5}{21x^6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{\tanh^{-1}(\tanh(a+bx))^5}{7x^7\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{b^2\int \frac{\tan}{}}{21\left(bx-\text{ta}\right.}$$

$$= \frac{b^2\tanh^{-1}(\tanh(a+bx))^5}{105x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{b\tanh^{-1}(\tanh(a+bx))^5}{21x^6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{\tan}{7x^7\left(bx\right.}$$

Mathematica [A] time = 0.033165, size = 71, normalized size = 0.72

$$-\frac{3b^3x^3\tanh^{-1}(\tanh(a+bx)) + 6b^2x^2\tanh^{-1}(\tanh(a+bx))^2 + 10bx\tanh^{-1}(\tanh(a+bx))^3 + 15\tanh^{-1}(\tanh(a+bx}{105x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x^8,x]
```

```
[Out] -(b^4*x^4 + 3*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 6*b^2*x^2*ArcTanh[Tanh[a + b
*x]]^2 + 10*b*x*ArcTanh[Tanh[a + b*x]]^3 + 15*ArcTanh[Tanh[a + b*x]]^4)/(10
5*x^7)
```

Maple [A] time = 0.04, size = 74, normalized size = 0.8

$$-\frac{(\text{Artanh}(\tanh(bx+a)))^4}{7x^7} + \frac{4b}{7}\left(-\frac{(\text{Artanh}(\tanh(bx+a)))^3}{6x^6} + \frac{b}{2}\left(-\frac{(\text{Artanh}(\tanh(bx+a)))^2}{5x^5} + \frac{2b}{5}\left(-\frac{\text{Artanh}(\text{t}}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x^8,x)
```

```
[Out] -1/7*arctanh(tanh(b*x+a))^4/x^7+4/7*b*(-1/6*arctanh(tanh(b*x+a))^3/x^6+1/2*
b*(-1/5*arctanh(tanh(b*x+a))^2/x^5+2/5*b*(-1/4*arctanh(tanh(b*x+a))/x^4-1/1
```

```
2*b/x^3)))
```

Maxima [A] time = 1.79776, size = 97, normalized size = 0.99

$$-\frac{1}{105}\left(b\left(\frac{b^2}{x^3}+\frac{3\,b\,\text{artanh}\left(\tanh\left(bx+a\right)\right)}{x^4}\right)+\frac{6\,b\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^2}{x^5}\right)b-\frac{2\,b\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^3}{21\,x^6}-\frac{\text{artanh}\left(\text{ta}\right.}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^8,x, algorithm="maxima")
```

```
[Out] -1/105*(b*(b^2/x^3 + 3*b*arctanh(tanh(b*x + a))/x^4) + 6*b*arctanh(tanh(b*x
 + a))^2/x^5)*b - 2/21*b*arctanh(tanh(b*x + a))^3/x^6 - 1/7*arctanh(tanh(b*
x + a))^4/x^7
```

Fricas [A] time = 1.52279, size = 109, normalized size = 1.11

$$-\frac{35\,b^4x^4+105\,ab^3x^3+126\,a^2b^2x^2+70\,a^3bx+15\,a^4}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^8,x, algorithm="fricas")
```

```
[Out] -1/105*(35*b^4*x^4 + 105*a*b^3*x^3 + 126*a^2*b^2*x^2 + 70*a^3*b*x + 15*a^4)
/x^7
```

Sympy [A] time = 11.6407, size = 80, normalized size = 0.82

$$-\frac{b^4}{105x^3}-\frac{b^3\,\text{atanh}\left(\tanh\left(a+bx\right)\right)}{35x^4}-\frac{2b^2\,\text{atanh}^2\left(\tanh\left(a+bx\right)\right)}{35x^5}-\frac{2b\,\text{atanh}^3\left(\tanh\left(a+bx\right)\right)}{21x^6}-\frac{\text{atanh}^4\left(\tanh\left(a+bx\right)\right.}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x**8,x)
```

```
[Out] -b**4/(105*x**3) - b**3*atanh(tanh(a + b*x))/(35*x**4) - 2*b**2*atanh(tanh(
a + b*x))**2/(35*x**5) - 2*b*atanh(tanh(a + b*x))**3/(21*x**6) - atanh(tanh
```

```
(a + b*x))**4/(7*x**7)
```

Giac [A] time = 1.15779, size = 62, normalized size = 0.63

$$-\frac{35 b^4 x^4 + 105 a b^3 x^3 + 126 a^2 b^2 x^2 + 70 a^3 b x + 15 a^4}{105 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^8,x, algorithm="giac")
```

```
[Out] -1/105*(35*b^4*x^4 + 105*a*b^3*x^3 + 126*a^2*b^2*x^2 + 70*a^3*b*x + 15*a^4)
/x^7
```

3.81 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^9}\,dx$

Optimal. Leaf size=80

$$-\frac{b^3\tanh^{-1}(\tanh(a+bx))}{70x^5}-\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{28x^6}-\frac{b\tanh^{-1}(\tanh(a+bx))^3}{14x^7}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{8x^8}-\frac{b^4}{280x^4}$$

```
[Out] -b^4/(280*x^4) - (b^3*ArcTanh[Tanh[a + b*x]])/(70*x^5) - (b^2*ArcTanh[Tanh[
a + b*x]]^2)/(28*x^6) - (b*ArcTanh[Tanh[a + b*x]]^3)/(14*x^7) - ArcTanh[Tan
h[a + b*x]]^4/(8*x^8)
```

Rubi [A] time = 0.0754094, antiderivative size = 132, normalized size of antiderivative = 1.65, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2171, 2167}

$$\frac{b^2\tanh^{-1}(\tanh(a+bx))^5}{56x^6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{b^3\tanh^{-1}(\tanh(a+bx))^5}{280x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{\tanh^{-1}(\tanh(a+bx))^5}{8x^8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{3}{56x^7}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x^9,x]
```

```
[Out] (b^3*ArcTanh[Tanh[a + b*x]]^5)/(280*x^5*(b*x - ArcTanh[Tanh[a + b*x]])^4) +
 (b^2*ArcTanh[Tanh[a + b*x]]^5)/(56*x^6*(b*x - ArcTanh[Tanh[a + b*x]])^3) +
 (3*b*ArcTanh[Tanh[a + b*x]]^5)/(56*x^7*(b*x - ArcTanh[Tanh[a + b*x]])^2) +
 ArcTanh[Tanh[a + b*x]]^5/(8*x^8*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
```

```
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^9}\,dx = \frac{\tanh^{-1}(\tanh(a+bx))^5}{8x^8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(3b)\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^8}\,dx}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

$$= \frac{3b\tanh^{-1}(\tanh(a+bx))^5}{56x^7\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{\tanh^{-1}(\tanh(a+bx))^5}{8x^8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{\left(3b^2\right)\int}{28\left(bx-\mathrm{ta}\right.}$$

$$= \frac{b^2\tanh^{-1}(\tanh(a+bx))^5}{56x^6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{3b\tanh^{-1}(\tanh(a+bx))^5}{56x^7\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{\mathrm{tan}}{8x^8\left(bx-\right.}$$

$$= \frac{b^3\tanh^{-1}(\tanh(a+bx))^5}{280x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4} + \frac{b^2\tanh^{-1}(\tanh(a+bx))^5}{56x^6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{3b}{56x^7\left(b\right.}$$

Mathematica [A] time = 0.0335632, size = 71, normalized size = 0.89

$$-\frac{4b^3x^3\tanh^{-1}(\tanh(a+bx)) + 10b^2x^2\tanh^{-1}(\tanh(a+bx))^2 + 20bx\tanh^{-1}(\tanh(a+bx))^3 + 35\tanh^{-1}(\tanh(a+b}{280x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x^9,x]
```

```
[Out] -(b^4*x^4 + 4*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 10*b^2*x^2*ArcTanh[Tanh[a +
b*x]]^2 + 20*b*x*ArcTanh[Tanh[a + b*x]]^3 + 35*ArcTanh[Tanh[a + b*x]]^4)/(2
80*x^8)
```

Maple [A] time = 0.04, size = 74, normalized size = 0.9

$$-\frac{(\mathrm{Artanh}(\tanh(bx+a)))^4}{8x^8} + \frac{b}{2}\left(-\frac{(\mathrm{Artanh}(\tanh(bx+a)))^3}{7x^7} + \frac{3b}{7}\left(-\frac{(\mathrm{Artanh}(\tanh(bx+a)))^2}{6x^6} + \frac{b}{3}\left(-\frac{\mathrm{Artanh}(\mathrm{ta}}{5}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x^9,x)
```

```
[Out] -1/8*arctanh(tanh(b*x+a))^4/x^8+1/2*b*(-1/7/x^7*arctanh(tanh(b*x+a))^3+3/7*
b*(-1/6/x^6*arctanh(tanh(b*x+a))^2+1/3*b*(-1/5/x^5*arctanh(tanh(b*x+a))-1/2
0/x^4*b)))
```

Maxima [A] time = 1.79259, size = 97, normalized size = 1.21

$$-\frac{1}{280}\left(b\left(\frac{b^2}{x^4}+\frac{4\,b\,\text{artanh}\,(\tanh\,(bx+a))}{x^5}\right)+\frac{10\,b\,\text{artanh}\,(\tanh\,(bx+a))^2}{x^6}\right)b-\frac{b\,\text{artanh}\,(\tanh\,(bx+a))^3}{14\,x^7}-\frac{\text{artanh}\,(\text{ta}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^9,x, algorithm="maxima")
```

```
[Out] -1/280*(b*(b^2/x^4 + 4*b*arctanh(tanh(b*x + a))/x^5) + 10*b*arctanh(tanh(b*
x + a))^2/x^6)*b - 1/14*b*arctanh(tanh(b*x + a))^3/x^7 - 1/8*arctanh(tanh(b
*x + a))^4/x^8
```

Fricas [A] time = 1.40823, size = 111, normalized size = 1.39

$$-\frac{70\,b^4x^4+224\,ab^3x^3+280\,a^2b^2x^2+160\,a^3bx+35\,a^4}{280\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^9,x, algorithm="fricas")
```

```
[Out] -1/280*(70*b^4*x^4 + 224*a*b^3*x^3 + 280*a^2*b^2*x^2 + 160*a^3*b*x + 35*a^4
)/x^8
```

Sympy [A] time = 16.1373, size = 76, normalized size = 0.95

$$-\frac{b^4}{280x^4}-\frac{b^3\,\text{atanh}\,(\tanh\,(a+bx))}{70x^5}-\frac{b^2\,\text{atanh}^2\,(\tanh\,(a+bx))}{28x^6}-\frac{b\,\text{atanh}^3\,(\tanh\,(a+bx))}{14x^7}-\frac{\text{atanh}^4\,(\tanh\,(a+bx))}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x**9,x)
```

```
[Out] -b**4/(280*x**4) - b**3*atanh(tanh(a + b*x))/(70*x**5) - b**2*atanh(tanh(a
+ b*x))**2/(28*x**6) - b*atanh(tanh(a + b*x))**3/(14*x**7) - atanh(tanh(a +
 b*x))**4/(8*x**8)
```

Giac [A] time = 1.10287, size = 62, normalized size = 0.78

$$-\frac{70\, b^4 x^4 + 224\, a b^3 x^3 + 280\, a^2 b^2 x^2 + 160\, a^3 b x + 35\, a^4}{280\, x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^9,x, algorithm="giac")
```

```
[Out] -1/280*(70*b^4*x^4 + 224*a*b^3*x^3 + 280*a^2*b^2*x^2 + 160*a^3*b*x + 35*a^4
)/x^8
```

3.82 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^{10}}\,dx$

Optimal. Leaf size=80

$$-\frac{b^3\tanh^{-1}(\tanh(a+bx))}{126x^6}-\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{42x^7}-\frac{b\tanh^{-1}(\tanh(a+bx))^3}{18x^8}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{9x^9}-\frac{b^4}{630x^5}$$

```
[Out] -b^4/(630*x^5) - (b^3*ArcTanh[Tanh[a + b*x]])/(126*x^6) - (b^2*ArcTanh[Tanh
[a + b*x]]^2)/(42*x^7) - (b*ArcTanh[Tanh[a + b*x]]^3)/(18*x^8) - ArcTanh[Ta
nh[a + b*x]]^4/(9*x^9)
```

Rubi [A] time = 0.0554779, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$-\frac{b^3\tanh^{-1}(\tanh(a+bx))}{126x^6}-\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{42x^7}-\frac{b\tanh^{-1}(\tanh(a+bx))^3}{18x^8}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{9x^9}-\frac{b^4}{630x^5}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x^10,x]
```

```
[Out] -b^4/(630*x^5) - (b^3*ArcTanh[Tanh[a + b*x]])/(126*x^6) - (b^2*ArcTanh[Tanh
[a + b*x]]^2)/(42*x^7) - (b*ArcTanh[Tanh[a + b*x]]^3)/(18*x^8) - ArcTanh[Ta
nh[a + b*x]]^4/(9*x^9)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^{10}}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^4}{9x^9} + \frac{1}{9}(4b)\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^9}\,dx$$
$$= -\frac{b\tanh^{-1}(\tanh(a+bx))^3}{18x^8} - \frac{\tanh^{-1}(\tanh(a+bx))^4}{9x^9} + \frac{1}{6}b^2\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^8}$$
$$= -\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{42x^7} - \frac{b\tanh^{-1}(\tanh(a+bx))^3}{18x^8} - \frac{\tanh^{-1}(\tanh(a+bx))^4}{9x^9} + \frac{1}{2}$$
$$= -\frac{b^3\tanh^{-1}(\tanh(a+bx))}{126x^6} - \frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{42x^7} - \frac{b\tanh^{-1}(\tanh(a+bx))^3}{18x^8} -$$
$$= -\frac{b^4}{630x^5} - \frac{b^3\tanh^{-1}(\tanh(a+bx))}{126x^6} - \frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{42x^7} - \frac{b\tanh^{-1}(\tanh(a+}{18x^8}$$

Mathematica [A] time = 0.0599034, size = 71, normalized size = 0.89

$$-\frac{5b^3x^3\tanh^{-1}(\tanh(a+bx)) + 15b^2x^2\tanh^{-1}(\tanh(a+bx))^2 + 35bx\tanh^{-1}(\tanh(a+bx))^3 + 70\tanh^{-1}(\tanh(a+b}{630x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x^10,x]
```

```
[Out] -(b^4*x^4 + 5*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 15*b^2*x^2*ArcTanh[Tanh[a +
b*x]]^2 + 35*b*x*ArcTanh[Tanh[a + b*x]]^3 + 70*ArcTanh[Tanh[a + b*x]]^4)/(6
30*x^9)
```

Maple [A] time = 0.04, size = 74, normalized size = 0.9

$$-\frac{(\mathrm{Artanh}(\tanh(bx+a)))^4}{9x^9} + \frac{4b}{9}\left(-\frac{(\mathrm{Artanh}(\tanh(bx+a)))^3}{8x^8} + \frac{3b}{8}\left(-\frac{(\mathrm{Artanh}(\tanh(bx+a)))^2}{7x^7} + \frac{2b}{7}\left(-\frac{\mathrm{Artanh}}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x^10,x)
```

```
[Out] -1/9*arctanh(tanh(b*x+a))^4/x^9+4/9*b*(-1/8/x^8*arctanh(tanh(b*x+a))^3+3/8*
b*(-1/7/x^7*arctanh(tanh(b*x+a))^2+2/7*b*(-1/6/x^6*arctanh(tanh(b*x+a))-1/3
0/x^5*b)))
```

Maxima [A] time = 1.78264, size = 97, normalized size = 1.21

$$-\frac{1}{630}\left(b\left(\frac{b^2}{x^5}+\frac{5\,b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)}{x^6}\right)+\frac{15\,b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2}{x^7}\right)b-\frac{b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^3}{18\,x^8}-\frac{\mathrm{artanh}\left(\tan\right.}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^10,x, algorithm="maxima")
```

```
[Out] -1/630*(b*(b^2/x^5 + 5*b*arctanh(tanh(b*x + a))/x^6) + 15*b*arctanh(tanh(b*
x + a))^2/x^7)*b - 1/18*b*arctanh(tanh(b*x + a))^3/x^8 - 1/9*arctanh(tanh(b
*x + a))^4/x^9
```

Fricas [A] time = 1.46187, size = 112, normalized size = 1.4

$$-\frac{126\,b^4x^4+420\,ab^3x^3+540\,a^2b^2x^2+315\,a^3bx+70\,a^4}{630\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^10,x, algorithm="fricas")
```

```
[Out] -1/630*(126*b^4*x^4 + 420*a*b^3*x^3 + 540*a^2*b^2*x^2 + 315*a^3*b*x + 70*a^
4)/x^9
```

Sympy [A] time = 25.8709, size = 76, normalized size = 0.95

$$-\frac{b^4}{630x^5}-\frac{b^3\,\mathrm{atanh}\left(\tanh\left(a+bx\right)\right)}{126x^6}-\frac{b^2\,\mathrm{atanh}^2\left(\tanh\left(a+bx\right)\right)}{42x^7}-\frac{b\,\mathrm{atanh}^3\left(\tanh\left(a+bx\right)\right)}{18x^8}-\frac{\mathrm{atanh}^4\left(\tanh\left(a+bx\right)\right)}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x**10,x)
```

```
[Out] -b**4/(630*x**5) - b**3*atanh(tanh(a + b*x))/(126*x**6) - b**2*atanh(tanh(a
 + b*x))**2/(42*x**7) - b*atanh(tanh(a + b*x))**3/(18*x**8) - atanh(tanh(a
+ b*x))**4/(9*x**9)
```

Giac [A] time = 1.12085, size = 62, normalized size = 0.78

$$-\frac{126\,b^4x^4+420\,ab^3x^3+540\,a^2b^2x^2+315\,a^3bx+70\,a^4}{630\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^10,x, algorithm="giac")
```

```
[Out] -1/630*(126*b^4*x^4 + 420*a*b^3*x^3 + 540*a^2*b^2*x^2 + 315*a^3*b*x + 70*a^
4)/x^9
```

3.83 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^{11}}\,dx$

Optimal. Leaf size=80

$$-\frac{b^3\tanh^{-1}(\tanh(a+bx))}{210x^7}-\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{60x^8}-\frac{2b\tanh^{-1}(\tanh(a+bx))^3}{45x^9}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{10x^{10}}-\frac{b^4}{1260x^6}$$

```
[Out] -b^4/(1260*x^6) - (b^3*ArcTanh[Tanh[a + b*x]])/(210*x^7) - (b^2*ArcTanh[Tan
h[a + b*x]]^2)/(60*x^8) - (2*b*ArcTanh[Tanh[a + b*x]]^3)/(45*x^9) - ArcTanh
[Tanh[a + b*x]]^4/(10*x^10)
```

Rubi [A] time = 0.0561019, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$-\frac{b^3\tanh^{-1}(\tanh(a+bx))}{210x^7}-\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{60x^8}-\frac{2b\tanh^{-1}(\tanh(a+bx))^3}{45x^9}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{10x^{10}}-\frac{b^4}{1260x^6}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x^11,x]
```

```
[Out] -b^4/(1260*x^6) - (b^3*ArcTanh[Tanh[a + b*x]])/(210*x^7) - (b^2*ArcTanh[Tan
h[a + b*x]]^2)/(60*x^8) - (2*b*ArcTanh[Tanh[a + b*x]]^3)/(45*x^9) - ArcTanh
[Tanh[a + b*x]]^4/(10*x^10)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^{11}}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^4}{10x^{10}} + \frac{1}{5}(2b)\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^{10}}\,dx$$
$$= -\frac{2b\tanh^{-1}(\tanh(a+bx))^3}{45x^9} - \frac{\tanh^{-1}(\tanh(a+bx))^4}{10x^{10}} + \frac{1}{15}\left(2b^2\right)\int \frac{\tanh^{-1}(\tanh(a+}{x^9}$$
$$= -\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{60x^8} - \frac{2b\tanh^{-1}(\tanh(a+bx))^3}{45x^9} - \frac{\tanh^{-1}(\tanh(a+bx))^4}{10x^{10}} +$$
$$= -\frac{b^3\tanh^{-1}(\tanh(a+bx))}{210x^7} - \frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{60x^8} - \frac{2b\tanh^{-1}(\tanh(a+bx))^3}{45x^9} -$$
$$= -\frac{b^4}{1260x^6} - \frac{b^3\tanh^{-1}(\tanh(a+bx))}{210x^7} - \frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{60x^8} - \frac{2b\tanh^{-1}(\tanh(}{45x^9}$$

Mathematica [A] time = 0.0324927, size = 71, normalized size = 0.89

$$-\frac{6b^3x^3\tanh^{-1}(\tanh(a+bx)) + 21b^2x^2\tanh^{-1}(\tanh(a+bx))^2 + 56bx\tanh^{-1}(\tanh(a+bx))^3 + 126\tanh^{-1}(\tanh(a+}{1260x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x^11,x]
```

```
[Out] -(b^4*x^4 + 6*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 21*b^2*x^2*ArcTanh[Tanh[a +
b*x]]^2 + 56*b*x*ArcTanh[Tanh[a + b*x]]^3 + 126*ArcTanh[Tanh[a + b*x]]^4)/(
1260*x^10)
```

Maple [A] time = 0.04, size = 74, normalized size = 0.9

$$-\frac{(\operatorname{Artanh}(\tanh(bx+a)))^4}{10x^{10}} + \frac{2b}{5}\left(-\frac{(\operatorname{Artanh}(\tanh(bx+a)))^3}{9x^9} + \frac{b}{3}\left(-\frac{(\operatorname{Artanh}(\tanh(bx+a)))^2}{8x^8} + \frac{b}{4}\left(-\frac{\operatorname{Artanh}(\mathrm{ta}}{7}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x^11,x)
```

```
[Out] -1/10*arctanh(tanh(b*x+a))^4/x^10+2/5*b*(-1/9/x^9*arctanh(tanh(b*x+a))^3+1/
3*b*(-1/8/x^8*arctanh(tanh(b*x+a))^2+1/4*b*(-1/7/x^7*arctanh(tanh(b*x+a))-1
/42/x^6*b)))
```

Maxima [A] time = 1.80116, size = 97, normalized size = 1.21

$$-\frac{1}{1260}\left(b\left(\frac{b^2}{x^6}+\frac{6\,b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)}{x^7}\right)+\frac{21\,b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2}{x^8}\right)b-\frac{2\,b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^3}{45\,x^9}-\frac{\mathrm{artanh}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^11,x, algorithm="maxima")
```

```
[Out] -1/1260*(b*(b^2/x^6 + 6*b*arctanh(tanh(b*x + a))/x^7) + 21*b*arctanh(tanh(b
*x + a))^2/x^8)*b - 2/45*b*arctanh(tanh(b*x + a))^3/x^9 - 1/10*arctanh(tanh
(b*x + a))^4/x^10
```

Fricas [A] time = 1.46743, size = 116, normalized size = 1.45

$$-\frac{210\,b^4x^4+720\,ab^3x^3+945\,a^2b^2x^2+560\,a^3bx+126\,a^4}{1260\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^11,x, algorithm="fricas")
```

```
[Out] -1/1260*(210*b^4*x^4 + 720*a*b^3*x^3 + 945*a^2*b^2*x^2 + 560*a^3*b*x + 126*
a^4)/x^10
```

Sympy [A] time = 78.1129, size = 78, normalized size = 0.98

$$-\frac{b^4}{1260x^6}-\frac{b^3\,\mathrm{atanh}\left(\tanh\left(a+bx\right)\right)}{210x^7}-\frac{b^2\,\mathrm{atanh}^2\left(\tanh\left(a+bx\right)\right)}{60x^8}-\frac{2b\,\mathrm{atanh}^3\left(\tanh\left(a+bx\right)\right)}{45x^9}-\frac{\mathrm{atanh}^4\left(\tanh\left(a+bx\right)\right.}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x**11,x)
```

```
[Out] -b**4/(1260*x**6) - b**3*atanh(tanh(a + b*x))/(210*x**7) - b**2*atanh(tanh(
a + b*x))**2/(60*x**8) - 2*b*atanh(tanh(a + b*x))**3/(45*x**9) - atanh(tanh
(a + b*x))**4/(10*x**10)
```

Giac [A] time = 1.16586, size = 62, normalized size = 0.78

$$-\frac{210 \, b^{4} x^{4} + 720 \, a b^{3} x^{3} + 945 \, a^{2} b^{2} x^{2} + 560 \, a^{3} b x + 126 \, a^{4}}{1260 \, x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^11,x, algorithm="giac")
```

```
[Out] -1/1260*(210*b^4*x^4 + 720*a*b^3*x^3 + 945*a^2*b^2*x^2 + 560*a^3*b*x + 126*
a^4)/x^10
```

3.84 $\int x \tanh^{-1}(\tanh(a+bx))^6 \, dx$

Optimal. Leaf size=34

$$\frac{x\tanh^{-1}(\tanh(a+bx))^7}{7b}-\frac{\tanh^{-1}(\tanh(a+bx))^8}{56b^2}$$

```
[Out] (x*ArcTanh[Tanh[a + b*x]]^7)/(7*b) - ArcTanh[Tanh[a + b*x]]^8/(56*b^2)
```

Rubi [A] time = 0.0137937, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2168, 2157, 30}

$$\frac{x\tanh^{-1}(\tanh(a+bx))^7}{7b}-\frac{\tanh^{-1}(\tanh(a+bx))^8}{56b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[Tanh[a + b*x]]^6,x]
```

```
[Out] (x*ArcTanh[Tanh[a + b*x]]^7)/(7*b) - ArcTanh[Tanh[a + b*x]]^8/(56*b^2)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x \tanh^{-1}(\tanh(a+bx))^6\,dx = \frac{x\tanh^{-1}(\tanh(a+bx))^7}{7b} - \frac{\int \tanh^{-1}(\tanh(a+bx))^7\,dx}{7b}$$
$$= \frac{x\tanh^{-1}(\tanh(a+bx))^7}{7b} - \frac{\text{Subst}\left(\int x^7\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{7b^2}$$
$$= \frac{x\tanh^{-1}(\tanh(a+bx))^7}{7b} - \frac{\tanh^{-1}(\tanh(a+bx))^8}{56b^2}$$

Mathematica [B] time = 0.1182, size = 177, normalized size = 5.21

$$-\frac{(a+bx)\left(-56\left(2a^2+abx-b^2x^2\right)\tanh^{-1}(\tanh(a+bx))^5+(7a-bx)(a+bx)^6-8(6a-bx)(a+bx)^5\tanh^{-1}(\tanh(a+$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[Tanh[a + b*x]]^6,x]
```

```
[Out] -((a + b*x)*((7*a - b*x)*(a + b*x)^6 - 8*(6*a - b*x)*(a + b*x)^5*ArcTanh[Ta
nh[a + b*x]] + 28*(5*a - b*x)*(a + b*x)^4*ArcTanh[Tanh[a + b*x]]^2 - 56*(4*
a - b*x)*(a + b*x)^3*ArcTanh[Tanh[a + b*x]]^3 + 70*(3*a - b*x)*(a + b*x)^2*
ArcTanh[Tanh[a + b*x]]^4 - 56*(2*a^2 + a*b*x - b^2*x^2)*ArcTanh[Tanh[a + b*
x]]^5 + 28*(a - b*x)*ArcTanh[Tanh[a + b*x]]^6))/(56*b^2)
```

Maple [B] time = 0.048, size = 110, normalized size = 3.2

$$\frac{x^2\left(\text{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^6}{2} - 3\,b\left(1/3\,x^3\left(\text{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^5 - 5/3\,b\left(1/4\,x^4\left(\text{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^4 - b\left(1/5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(tanh(b*x+a))^6,x)
```

```
[Out] 1/2*x^2*arctanh(tanh(b*x+a))^6-3*b*(1/3*x^3*arctanh(tanh(b*x+a))^5-5/3*b*(1
/4*x^4*arctanh(tanh(b*x+a))^4-b*(1/5*x^5*arctanh(tanh(b*x+a))^3-3/5*b*(1/6*
x^6*arctanh(tanh(b*x+a))^2-1/3*b*(1/7*x^7*arctanh(tanh(b*x+a))-1/56*x^8*b))
)))
```

Maxima [B] time = 2.14864, size = 149, normalized size = 4.38

$$-bx^3 \operatorname{artanh}(\tanh(bx+a))^5 + \frac{1}{2}x^2 \operatorname{artanh}(\tanh(bx+a))^6 + \frac{1}{56}\Big(70\,bx^4 \operatorname{artanh}(\tanh(bx+a))^4 - \Big(56\,bx^5 \operatorname{artanh}(\text{ta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^6,x, algorithm="maxima")
```

```
[Out] -b*x^3*arctanh(tanh(b*x + a))^5 + 1/2*x^2*arctanh(tanh(b*x + a))^6 + 1/56*(
70*b*x^4*arctanh(tanh(b*x + a))^4 - (56*b*x^5*arctanh(tanh(b*x + a))^3 - (2
8*b*x^6*arctanh(tanh(b*x + a))^2 + (b^2*x^8 - 8*b*x^7*arctanh(tanh(b*x + a)
))*b)*b)*b)*b
```

Fricas [B] time = 1.45888, size = 149, normalized size = 4.38

$$\frac{1}{8}\,b^6x^8 + \frac{6}{7}\,ab^5x^7 + \frac{5}{2}\,a^2b^4x^6 + 4\,a^3b^3x^5 + \frac{15}{4}\,a^4b^2x^4 + 2\,a^5bx^3 + \frac{1}{2}\,a^6x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^6,x, algorithm="fricas")
```

```
[Out] 1/8*b^6*x^8 + 6/7*a*b^5*x^7 + 5/2*a^2*b^4*x^6 + 4*a^3*b^3*x^5 + 15/4*a^4*b^
2*x^4 + 2*a^5*b*x^3 + 1/2*a^6*x^2
```

Sympy [A] time = 11.7372, size = 41, normalized size = 1.21

$$\begin{cases} \frac{x \operatorname{atanh}^{7}(\tanh(a+bx))}{7b} - \frac{\operatorname{atanh}^{8}(\tanh(a+bx))}{56b^2} & \text{for } b \neq 0 \\ \frac{x^2 \operatorname{atanh}^{6}(\tanh(a))}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(tanh(b*x+a))**6,x)
```

```
[Out] Piecewise((x*atanh(tanh(a + b*x))**7/(7*b) - atanh(tanh(a + b*x))**8/(56*b*
*2), Ne(b, 0)), (x**2*atanh(tanh(a))**6/2, True))
```

Giac [B] time = 1.13139, size = 92, normalized size = 2.71

$$\frac{1}{8}\,b^6x^8 + \frac{6}{7}\,ab^5x^7 + \frac{5}{2}\,a^2b^4x^6 + 4\,a^3b^3x^5 + \frac{15}{4}\,a^4b^2x^4 + 2\,a^5bx^3 + \frac{1}{2}\,a^6x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^6,x, algorithm="giac")
```

```
[Out] 1/8*b^6*x^8 + 6/7*a*b^5*x^7 + 5/2*a^2*b^4*x^6 + 4*a^3*b^3*x^5 + 15/4*a^4*b^
2*x^4 + 2*a^5*b*x^3 + 1/2*a^6*x^2
```

3.85 $\int \frac{x^m}{\tanh^{-1}(\tanh(a+bx))} \, dx$

Optimal. Leaf size=53

$$-\frac{x^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{(m+1)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] -((x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (b*x)/(b*x - ArcTanh[Tanh[a
 + b*x]])])/((1 + m)*(b*x - ArcTanh[Tanh[a + b*x]])))
```

Rubi [A] time = 0.0256857, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2164}

$$-\frac{x^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{(m+1)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] -((x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (b*x)/(b*x - ArcTanh[Tanh[a
 + b*x]])])/((1 + m)*(b*x - ArcTanh[Tanh[a + b*x]])))
```

Rule 2164

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)*Hypergeometric2F1[1, n + 1, n + 2, -((a*v)/(b*u
- a*v))])/((n + 1)*(b*u - a*v)), x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{x^m}{\tanh^{-1}(\tanh(a+bx))} \, dx = -\frac{x^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{(1+m)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A] time = 0.0845128, size = 51, normalized size = 0.96

$$\frac{x^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{bx}{\tanh^{-1}(\tanh(a+bx))-bx}\right)}{(m+1)\left(\tanh^{-1}(\tanh(a+bx))-bx\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((b*x)/(-(b*x) + ArcTanh[Tan
h[a + b*x]]))])/((1 + m)*(-(b*x) + ArcTanh[Tanh[a + b*x]]))
```

Maple [F] time = 0.4, size = 0, normalized size = 0.

$$\int \frac{x^m}{\text{Artanh}\left(\tanh\left(bx+a\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arctanh(tanh(b*x+a)),x)
```

```
[Out] int(x^m/arctanh(tanh(b*x+a)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\text{artanh}\left(\tanh\left(bx+a\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] integrate(x^m/arctanh(tanh(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{\text{artanh}\left(\tanh\left(bx+a\right)\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] integral(x^m/arctanh(tanh(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(x**m/atanh(tanh(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^m/arctanh(tanh(b*x + a)), x)
```

3.86 $\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))} dx$

Optimal. Leaf size=81

$$\frac{x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{2b^2}+\frac{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^3}+\frac{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\log\left(\tanh^{-1}(\tanh\right.}{b^4}$$

```
[Out] x^3/(3*b) + (x^2*(b*x - ArcTanh[Tanh[a + b*x]]))/(2*b^2) + (x*(b*x - ArcTan
h[Tanh[a + b*x]])^2)/b^3 + ((b*x - ArcTanh[Tanh[a + b*x]])^3*Log[ArcTanh[Ta
nh[a + b*x]]])/b^4
```

Rubi [A] time = 0.058285, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2159, 2158, 2157, 29}

$$\frac{x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{2b^2}+\frac{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^3}+\frac{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\log\left(\tanh^{-1}(\tanh\right.}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] x^3/(3*b) + (x^2*(b*x - ArcTanh[Tanh[a + b*x]]))/(2*b^2) + (x*(b*x - ArcTan
h[Tanh[a + b*x]])^2)/b^3 + ((b*x - ArcTanh[Tanh[a + b*x]])^3*Log[ArcTanh[Ta
nh[a + b*x]]])/b^4
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{x^3}{3b} - \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$

$$= \frac{x^3}{3b} + \frac{x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{2b^2} + \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2\int \frac{x}{\tanh^{-1}(\tanh(a+$$

$$= \frac{x^3}{3b} + \frac{x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{2b^2} + \frac{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^3} - \frac{\left(-bx+\tanh\right.}{}$$

$$= \frac{x^3}{3b} + \frac{x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{2b^2} + \frac{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^3} - \frac{\left(-bx+\tanh\right.}{}$$

$$= \frac{x^3}{3b} + \frac{x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{2b^2} + \frac{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^3} + \frac{\left(bx-\tanh^{-}\right.}{}$$

Mathematica [A] time = 0.04354, size = 79, normalized size = 0.98

$$-\frac{x^2\left(\tanh^{-1}(\tanh(a+bx))-bx\right)}{2b^2} + \frac{x\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}{b^3} - \frac{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^3\log\left(\tanh^{-1}(\tanh\right.}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] x^3/(3*b) - (x^2*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/(2*b^2) + (x*(-(b*x) +
ArcTanh[Tanh[a + b*x]])^2)/b^3 - ((-(b*x) + ArcTanh[Tanh[a + b*x]])^3*Log[A
rcTanh[Tanh[a + b*x]]])/b^4
```

Maple [B] time = 0.034, size = 202, normalized size = 2.5

$$\frac{x^3}{3\,b} - \frac{ax^2}{2\,b^2} - \frac{x^2\,(\mathrm{Artanh}\,(\tanh\,(bx+a)) - bx - a)}{2\,b^2} + \frac{a^2x}{b^3} + 2\,\frac{a\,(\mathrm{Artanh}\,(\tanh\,(bx+a)) - bx - a)\,x}{b^3} + \frac{(\mathrm{Artanh}\,(\tanh\,(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arctanh(tanh(b*x+a)),x)
```

```
[Out] 1/3*x^3/b-1/2/b^2*a*x^2-1/2/b^2*x^2*(arctanh(tanh(b*x+a))-b*x-a)+1/b^3*x*a^
2+2/b^3*a*(arctanh(tanh(b*x+a))-b*x-a)*x+1/b^3*(arctanh(tanh(b*x+a))-b*x-a)
^2*x-1/b^4*ln(arctanh(tanh(b*x+a)))*a^3-3/b^4*ln(arctanh(tanh(b*x+a)))*a^2*
(arctanh(tanh(b*x+a))-b*x-a)-3/b^4*ln(arctanh(tanh(b*x+a)))*a*(arctanh(tanh
(b*x+a))-b*x-a)^2-1/b^4*ln(arctanh(tanh(b*x+a)))*(arctanh(tanh(b*x+a))-b*x-
a)^3
```

Maxima [A] time = 1.78117, size = 57, normalized size = 0.7

$$-\frac{a^3 \log(bx+a)}{b^4} + \frac{2\,b^2x^3 - 3\,abx^2 + 6\,a^2x}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -a^3*log(b*x + a)/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

Fricas [A] time = 1.48546, size = 92, normalized size = 1.14

$$\frac{2\,b^3x^3 - 3\,ab^2x^2 + 6\,a^2bx - 6\,a^3\log(bx+a)}{6\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))/b^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{atanh}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(x**3/atanh(tanh(a + b*x)), x)
```

Giac [A] time = 1.1273, size = 58, normalized size = 0.72

$$-\frac{a^3 \log(|bx+a|)}{b^4}+\frac{2\,b^2x^3-3\,abx^2+6\,a^2x}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] -a^3*log(abs(b*x + a))/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

3.87 $\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))} dx$

Optimal. Leaf size=56

$$\frac{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^2}+\frac{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^3}+\frac{x^2}{2b}$$

```
[Out] x^2/(2*b) + (x*(b*x - ArcTanh[Tanh[a + b*x]]))/b^2 + ((b*x - ArcTanh[Tanh[a
 + b*x]])^2*Log[ArcTanh[Tanh[a + b*x]]])/b^3
```

Rubi [A] time = 0.0346303, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2159, 2158, 2157, 29}

$$\frac{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^2}+\frac{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^3}+\frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] x^2/(2*b) + (x*(b*x - ArcTanh[Tanh[a + b*x]]))/b^2 + ((b*x - ArcTanh[Tanh[a
 + b*x]])^2*Log[ArcTanh[Tanh[a + b*x]]])/b^3
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{x^2}{2b} - \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\int \frac{x}{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$

$$= \frac{x^2}{2b} + \frac{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^2} + \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2 \int \frac{1}{\tanh^{-1}(\tanh(a+b}}{b^2}$$

$$= \frac{x^2}{2b} + \frac{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^2} + \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2 \operatorname{Subst}\left(\int \frac{1}{x}\,dx, x}{b^3}$$

$$= \frac{x^2}{2b} + \frac{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^2} + \frac{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 \log\left(\tanh^{-1}(\tanh}{b^3}$$

Mathematica [A] time = 0.0322665, size = 55, normalized size = 0.98

$$-\frac{x\left(\tanh^{-1}(\tanh(a+bx))-bx\right)}{b^2} + \frac{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2 \log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^3} + \frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] x^2/(2*b) - (x*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/b^2 + ((-(b*x) + ArcTanh[
Tanh[a + b*x]])^2*Log[ArcTanh[Tanh[a + b*x]]])/b^3
```

Maple [B] time = 0.033, size = 111, normalized size = 2.

$$\frac{x^2}{2\,b} - \frac{ax}{b^2} - \frac{(\mathrm{Artanh}\,(\tanh\,(bx+a))-bx-a)\,x}{b^2} + \frac{\ln\,(\mathrm{Artanh}\,(\tanh\,(bx+a)))\,a^2}{b^3} + 2\,\frac{\ln\,(\mathrm{Artanh}\,(\tanh\,(bx+a)))\,a\,(\mathrm{Ar}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arctanh(tanh(b*x+a)),x)
```

```
[Out] 1/2*x^2/b-1/b^2*a*x-1/b^2*(arctanh(tanh(b*x+a))-b*x-a)*x+1/b^3*ln(arctanh(t
anh(b*x+a)))*a^2+2/b^3*ln(arctanh(tanh(b*x+a)))*a*(arctanh(tanh(b*x+a))-b*x
-a)+1/b^3*ln(arctanh(tanh(b*x+a)))*(arctanh(tanh(b*x+a))-b*x-a)^2
```

Maxima [A] time = 1.76817, size = 39, normalized size = 0.7

$$\frac{a^2 \log(bx+a)}{b^3} + \frac{bx^2 - 2\,ax}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] a^2*log(b*x + a)/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

Fricas [A] time = 1.50836, size = 68, normalized size = 1.21

$$\frac{b^2x^2 - 2\,abx + 2\,a^2 \log(bx+a)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{atanh}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(x**2/atanh(tanh(a + b*x)), x)
```

Giac [A] time = 1.13289, size = 41, normalized size = 0.73

$$\frac{a^2 \log(|bx + a|)}{b^3} + \frac{bx^2 - 2\,ax}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] a^2*log(abs(b*x + a))/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

3.88 $\int \frac{x}{\tanh^{-1}(\tanh(a+bx))}\, dx$

Optimal. Leaf size=31

$$\frac{(bx-\tanh^{-1}(\tanh(a+bx)))\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^2}+\frac{x}{b}$$

```
[Out] x/b + ((b*x - ArcTanh[Tanh[a + b*x]])*Log[ArcTanh[Tanh[a + b*x]]])/b^2
```

Rubi [A] time = 0.0150204, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2158, 2157, 29}

$$\frac{(bx-\tanh^{-1}(\tanh(a+bx)))\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^2}+\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] x/b + ((b*x - ArcTanh[Tanh[a + b*x]])*Log[ArcTanh[Tanh[a + b*x]]])/b^2
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{x}{\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{x}{b} - \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\int \frac{1}{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$
$$= \frac{x}{b} - \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b^2}$$
$$= \frac{x}{b} + \frac{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^2}$$

Mathematica [A] time = 0.0232616, size = 31, normalized size = 1.

$$\frac{x}{b} - \frac{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] x/b - ((-(b*x) + ArcTanh[Tanh[a + b*x]])*Log[ArcTanh[Tanh[a + b*x]]])/b^2
```

Maple [A] time = 0.033, size = 49, normalized size = 1.6

$$\frac{x}{b} - \frac{\ln\left(\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)\right)a}{b^2} - \frac{\ln\left(\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)\right)\left(\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)-bx-a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctanh(tanh(b*x+a)),x)
```

```
[Out] x/b-1/b^2*ln(arctanh(tanh(b*x+a)))*a-1/b^2*ln(arctanh(tanh(b*x+a)))*(arctan
h(tanh(b*x+a))-b*x-a)
```

Maxima [A] time = 1.76601, size = 24, normalized size = 0.77

$$\frac{x}{b} - \frac{a\log\left(bx+a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] x/b - a*log(b*x + a)/b^2
```

Fricas [A] time = 1.53216, size = 38, normalized size = 1.23

$$\frac{bx - a\log(bx + a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] (b*x - a*log(b*x + a))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\operatorname{atanh}(\tanh(a + bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(x/atanh(tanh(a + b*x)), x)
```

Giac [A] time = 1.13733, size = 26, normalized size = 0.84

$$\frac{x}{b} - \frac{a\log(|bx + a|)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] x/b - a*log(abs(b*x + a))/b^2
```

3.89 $\int \frac{1}{\tanh^{-1}(\tanh(a+bx))} \, dx$

Optimal. Leaf size=12

$$\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b}$$

```
[Out] Log[ArcTanh[Tanh[a + b*x]]]/b
```

Rubi [A] time = 0.0043055, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2157, 29}

$$\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(-1),x]
```

```
[Out] Log[ArcTanh[Tanh[a + b*x]]]/b
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\tanh^{-1}(\tanh(a+bx))} \, dx &= \frac{\text{Subst}\left(\int \frac{1}{x} \, dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b} \\
&= \frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0466461, size = 12, normalized size = 1.

$$\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(-1),x]
```

```
[Out] Log[ArcTanh[Tanh[a + b*x]]]/b
```

Maple [A] time = 0.027, size = 13, normalized size = 1.1

$$\frac{\ln\left(\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctanh(tanh(b*x+a)),x)
```

```
[Out] ln(arctanh(tanh(b*x+a)))/b
```

Maxima [A] time = 1.46089, size = 18, normalized size = 1.5

$$\frac{\log\left(-bx-a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] log(-b*x - a)/b
```

Fricas [A] time = 1.45088, size = 22, normalized size = 1.83

$$\frac{\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] log(b*x + a)/b
```

Sympy [A] time = 9.01365, size = 17, normalized size = 1.42

$$\begin{cases} \frac{\log\left(\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)\right)}{b} & \text{for}\ b \neq 0 \\ \frac{x}{\operatorname{atanh}\left(\tanh\left(a\right)\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atanh(tanh(b*x+a)),x)
```

```
[Out] Piecewise((log(atanh(tanh(a + b*x)))/b, Ne(b, 0)), (x/atanh(tanh(a)), True)
)
```

Giac [A] time = 1.15638, size = 15, normalized size = 1.25

$$\frac{\log\left(|bx + a|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b
```

3.90 $\int \frac{1}{x \tanh^{-1}(\tanh(a+bx))} dx$

Optimal. Leaf size=44

$$\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{bx-\tanh^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\tanh^{-1}(\tanh(a+bx))}$$

```
[Out] -(Log[x]/(b*x - ArcTanh[Tanh[a + b*x]])) + Log[ArcTanh[Tanh[a + b*x]]]/(b*x
 - ArcTanh[Tanh[a + b*x]])
```

Rubi [A] time = 0.0265642, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2160, 2157, 29}

$$\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{bx-\tanh^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] -(Log[x]/(b*x - ArcTanh[Tanh[a + b*x]])) + Log[ArcTanh[Tanh[a + b*x]]]/(b*x
 - ArcTanh[Tanh[a + b*x]])
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))}\,dx = -\frac{\int \frac{1}{x}\,dx}{bx-\tanh^{-1}(\tanh(a+bx))} + \frac{b\int \frac{1}{\tanh^{-1}(\tanh(a+bx))}\,dx}{bx-\tanh^{-1}(\tanh(a+bx))}$$
$$= -\frac{\log(x)}{bx-\tanh^{-1}(\tanh(a+bx))} + \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{bx-\tanh^{-1}(\tanh(a+bx))}$$
$$= -\frac{\log(x)}{bx-\tanh^{-1}(\tanh(a+bx))} + \frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{bx-\tanh^{-1}(\tanh(a+bx))}$$

Mathematica [A] time = 0.0188116, size = 29, normalized size = 0.66

$$\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)-\log(x)}{bx-\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] (-Log[x] + Log[ArcTanh[Tanh[a + b*x]]])/(b*x - ArcTanh[Tanh[a + b*x]])
```

Maple [A] time = 0.038, size = 43, normalized size = 1.

$$\frac{\ln(x)}{\text{Artanh}(\tanh(bx+a))-bx} - \frac{\ln(\text{Artanh}(\tanh(bx+a)))}{\text{Artanh}(\tanh(bx+a))-bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctanh(tanh(b*x+a)),x)
```

```
[Out] 1/(arctanh(tanh(b*x+a))-b*x)*ln(x)-1/(arctanh(tanh(b*x+a))-b*x)*ln(arctanh(
tanh(b*x+a)))
```

Maxima [A] time = 1.77811, size = 24, normalized size = 0.55

$$-\frac{\log(bx+a)}{a} + \frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -log(b*x + a)/a + log(x)/a
```

Fricas [A] time = 1.55827, size = 38, normalized size = 0.86

$$-\frac{\log(bx+a)-\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] -(log(b*x + a) - log(x))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{atanh}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(1/(x*atanh(tanh(a + b*x))), x)
```

Giac [A] time = 1.15711, size = 27, normalized size = 0.61

$$-\frac{\log(|bx+a|)}{a}+\frac{\log(|x|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a + log(abs(x))/a
```

3.91 $\int \frac{1}{x^2 \tanh^{-1}(\tanh(a+bx))} \, dx$

Optimal. Leaf size=65

$$\frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{b\log(x)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{b\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

```
[Out] 1/(x*(b*x - ArcTanh[Tanh[a + b*x]])) - (b*Log[x])/(b*x - ArcTanh[Tanh[a + b
*x]])^2 + (b*Log[ArcTanh[Tanh[a + b*x]]])/(b*x - ArcTanh[Tanh[a + b*x]])^2
```

Rubi [A] time = 0.0443764, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2163, 2160, 2157, 29}

$$\frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{b\log(x)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{b\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] 1/(x*(b*x - ArcTanh[Tanh[a + b*x]])) - (b*Log[x])/(b*x - ArcTanh[Tanh[a + b
*x]])^2 + (b*Log[ArcTanh[Tanh[a + b*x]]])/(b*x - ArcTanh[Tanh[a + b*x]])^2
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x^2 \tanh^{-1}(\tanh(a+bx))}\,dx = \frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{b\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))}\,dx}{bx-\tanh^{-1}(\tanh(a+bx))}$$

$$= \frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{b\int \frac{1}{x}\,dx}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{b^2\int \frac{1}{\tanh^{-1}(\tan}}{\left(bx-\tanh^{-1}(\tan\right.}$$

$$= \frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{b\log(x)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{b\,\mathrm{Subst}\left(\int \frac{1}{x}\,dx,\right.}{\left(bx-\tanh\right.}$$

$$= \frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{b\log(x)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{b\log\left(\tanh^{-1}(\mathrm{ta}\right.}{\left(bx-\tanh^{-1}(\tan\right.}$$

Mathematica [A] time = 0.0257177, size = 45, normalized size = 0.69

$$\frac{bx\left(\log\left(\tanh^{-1}(\tanh(a+bx))\right)-\log(x)+1\right)-\tanh^{-1}(\tanh(a+bx))}{x\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] (-ArcTanh[Tanh[a + b*x]] + b*x*(1 - Log[x] + Log[ArcTanh[Tanh[a + b*x]]]))/
(x*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

Maple [A] time = 0.079, size = 64, normalized size = 1.

$$\frac{b\ln(\mathrm{Artanh}(\tanh(bx+a)))}{(\mathrm{Artanh}(\tanh(bx+a))-bx)^2} - \frac{1}{(\mathrm{Artanh}(\tanh(bx+a))-bx)\,x} - \frac{b\ln(x)}{(\mathrm{Artanh}(\tanh(bx+a))-bx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arctanh(tanh(b*x+a)),x)
```

```
[Out] 1/(arctanh(tanh(b*x+a))-b*x)^2*b*ln(arctanh(tanh(b*x+a)))-1/(arctanh(tanh(b
*x+a))-b*x)/x-1/(arctanh(tanh(b*x+a))-b*x)^2*b*ln(x)
```

Maxima [A] time = 1.79131, size = 38, normalized size = 0.58

$$\frac{b\log(bx+a)}{a^2} - \frac{b\log(x)}{a^2} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)
```

Fricas [A] time = 1.5147, size = 61, normalized size = 0.94

$$\frac{bx\log(bx+a) - bx\log(x) - a}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] (b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\operatorname{atanh}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(1/(x**2*atanh(tanh(a + b*x))), x)
```

Giac [A] time = 1.14328, size = 41, normalized size = 0.63

$$\frac{b\log(|bx+a|)}{a^2}-\frac{b\log(|x|)}{a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x)
```

3.92 $\int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))} \, dx$

Optimal. Leaf size=92

$$-\frac{b^2 \log(x)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{b^2 \log\left(\tanh^{-1}(\tanh(a+bx))\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{1}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{\quad}{x\left(bx-\tanh^{-1}\right.}$$

```
[Out] b/(x*(b*x - ArcTanh[Tanh[a + b*x]])^2) + 1/(2*x^2*(b*x - ArcTanh[Tanh[a + b
*x]])) - (b^2*Log[x])/(b*x - ArcTanh[Tanh[a + b*x]])^3 + (b^2*Log[ArcTanh[T
anh[a + b*x]]])/(b*x - ArcTanh[Tanh[a + b*x]])^3
```

Rubi [A] time = 0.0681549, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2163, 2160, 2157, 29}

$$-\frac{b^2 \log(x)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{b^2 \log\left(\tanh^{-1}(\tanh(a+bx))\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{1}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{\quad}{x\left(bx-\tanh^{-1}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] b/(x*(b*x - ArcTanh[Tanh[a + b*x]])^2) + 1/(2*x^2*(b*x - ArcTanh[Tanh[a + b
*x]])) - (b^2*Log[x])/(b*x - ArcTanh[Tanh[a + b*x]])^3 + (b^2*Log[ArcTanh[T
anh[a + b*x]]])/(b*x - ArcTanh[Tanh[a + b*x]])^3
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))}\,dx = \frac{1}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{b\int \frac{1}{x^2\tanh^{-1}(\tanh(a+bx))}\,dx}{-bx+\tanh^{-1}(\tanh(a+bx))}$$

$$= \frac{b}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{1}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{}{\left(bx-\tanh^{-1}\right.}$$

$$= \frac{b}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{1}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{}{\left(bx-\tanh^{-1}\right.}$$

$$= \frac{b}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{1}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{b^2}{\left(bx-\tanh^{-1}\right.}$$

$$= \frac{b}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{1}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{b^2}{\left(bx-\tanh^{-1}\right.}$$

Mathematica [A] time = 0.0341613, size = 66, normalized size = 0.72

$$\frac{b^2x^2\left(2\log\left(\tanh^{-1}(\tanh(a+bx))\right)-2\log(x)+3\right)-4bx\tanh^{-1}(\tanh(a+bx))+\tanh^{-1}(\tanh(a+bx))^2}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] (-4*b*x*ArcTanh[Tanh[a + b*x]] + ArcTanh[Tanh[a + b*x]]^2 + b^2*x^2*(3 - 2*
Log[x] + 2*Log[ArcTanh[Tanh[a + b*x]]]))/(2*x^2*(b*x - ArcTanh[Tanh[a + b*x
]])^3)
```

Maple [A] time = 0.081, size = 87, normalized size = 1.

$$-\frac{b^2 \ln(\operatorname{Artanh}(\tanh(bx+a)))}{(\operatorname{Artanh}(\tanh(bx+a)) - bx)^3} - \frac{1}{(2\operatorname{Artanh}(\tanh(bx+a)) - 2bx)\,x^2} + \frac{b^2 \ln(x)}{(\operatorname{Artanh}(\tanh(bx+a)) - bx)^3} + \frac{}{(\operatorname{Artanh}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/arctanh(tanh(b*x+a)),x)
```

```
[Out] -1/(arctanh(tanh(b*x+a))-b*x)^3*b^2*ln(arctanh(tanh(b*x+a)))-1/2/(arctanh(t
anh(b*x+a))-b*x)/x^2+1/(arctanh(tanh(b*x+a))-b*x)^3*b^2*ln(x)+1/(arctanh(ta
nh(b*x+a))-b*x)^2*b/x
```

Maxima [A] time = 1.79578, size = 54, normalized size = 0.59

$$-\frac{b^2 \log(bx+a)}{a^3} + \frac{b^2 \log(x)}{a^3} + \frac{2\,bx - a}{2\,a^2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -b^2*log(b*x + a)/a^3 + b^2*log(x)/a^3 + 1/2*(2*b*x - a)/(a^2*x^2)
```

Fricas [A] time = 1.51525, size = 103, normalized size = 1.12

$$-\frac{2\,b^2x^2 \log(bx+a) - 2\,b^2x^2 \log(x) - 2\,abx + a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*x^2*log(b*x + a) - 2*b^2*x^2*log(x) - 2*a*b*x + a^2)/(a^3*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \operatorname{atanh}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(1/(x**3*atanh(tanh(a + b*x))), x)
```

Giac [A] time = 1.13528, size = 61, normalized size = 0.66

$$-\frac{b^2\log(|bx+a|)}{a^3}+\frac{b^2\log(|x|)}{a^3}+\frac{2\,abx-a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] -b^2*log(abs(b*x + a))/a^3 + b^2*log(abs(x))/a^3 + 1/2*(2*a*b*x - a^2)/(a^3
*x^2)
```

3.93 $\int \frac{x^m}{\tanh^{-1}(\tanh(a+bx))^2}\,dx$

Optimal. Leaf size=65

$$-\frac{x^m \text{Hypergeometric2F1}\left(1,m,m+1,\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{x^m}{b\tanh^{-1}(\tanh(a+bx))}$$

```
[Out] -(x^m/(b*ArcTanh[Tanh[a + b*x]])) - (x^m*Hypergeometric2F1[1, m, 1 + m, (b*
x)/(b*x - ArcTanh[Tanh[a + b*x]])])/(b*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rubi [A] time = 0.0360839, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 2164}

$$-\frac{x^m\,{}_2F_1\left(1,m;m+1;\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{x^m}{b\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] -(x^m/(b*ArcTanh[Tanh[a + b*x]])) - (x^m*Hypergeometric2F1[1, m, 1 + m, (b*
x)/(b*x - ArcTanh[Tanh[a + b*x]])])/(b*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2164

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)*Hypergeometric2F1[1, n + 1, n + 2, -((a*v)/(b*u
- a*v))])/((n + 1)*(b*u - a*v)), x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
```

```
rQ[u, v, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{x^m}{\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{x^m}{b\tanh^{-1}(\tanh(a+bx))}+\frac{m\int \frac{x^{-1+m}}{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$
$$= -\frac{x^m}{b\tanh^{-1}(\tanh(a+bx))}-\frac{x^m\,{}_2F_1\left(1,m;1+m;\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A] time = 0.48889, size = 51, normalized size = 0.78

$$\frac{x^{m+1}\text{Hypergeometric2F1}\left(2,m+1,m+2,-\frac{bx}{\tanh^{-1}(\tanh(a+bx))-bx}\right)}{(m+1)\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((b*x)/(-(b*x) + ArcTanh[Tan
h[a + b*x]]))])/((1 + m)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

Maple [F] time = 1.622, size = 0, normalized size = 0.

$$\int \frac{x^m}{(\text{Artanh}\,(\tanh\,(bx+a)))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] int(x^m/arctanh(tanh(b*x+a))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{artanh}(\tanh(bx+a))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m/arctanh(tanh(b*x + a))^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x^m}{\operatorname{artanh}(\tanh(bx+a))^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] integral(x^m/arctanh(tanh(b*x + a))^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{atanh}^2(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(x**m/atanh(tanh(a + b*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{artanh}(\tanh(bx+a))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/arctanh(tanh(b*x + a))^2, x)
```

3.94 $\int \frac{x^4}{\tanh^{-1}(\tanh(a+bx))^2} dx$

Optimal. Leaf size=98

$$\frac{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3}+\frac{4x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^4}+\frac{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\log\left(\tanh^{-1}(\mathrm{ta}\right.}{b^5}$$

```
[Out] (4*x^3)/(3*b^2) + (2*x^2*(b*x - ArcTanh[Tanh[a + b*x]]))/b^3 + (4*x*(b*x -
ArcTanh[Tanh[a + b*x]])^2)/b^4 - x^4/(b*ArcTanh[Tanh[a + b*x]]) + (4*(b*x -
 ArcTanh[Tanh[a + b*x]])^3*Log[ArcTanh[Tanh[a + b*x]]])/b^5
```

Rubi [A] time = 0.0806588, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2168, 2159, 2158, 2157, 29}

$$\frac{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3}+\frac{4x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^4}+\frac{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\log\left(\tanh^{-1}(\mathrm{ta}\right.}{b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcTanh[Tanh[a + b*x]]^2,x]
```


```
[Out] (4*x^3)/(3*b^2) + (2*x^2*(b*x - ArcTanh[Tanh[a + b*x]]))/b^3 + (4*x*(b*x -
ArcTanh[Tanh[a + b*x]])^2)/b^4 - x^4/(b*ArcTanh[Tanh[a + b*x]]) + (4*(b*x -
 ArcTanh[Tanh[a + b*x]])^3*Log[ArcTanh[Tanh[a + b*x]]])/b^5
```


Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
```

```
/; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{x^4}{\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{x^4}{b\tanh^{-1}(\tanh(a+bx))} + \frac{4\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$

$$= \frac{4x^3}{3b^2} - \frac{x^4}{b\tanh^{-1}(\tanh(a+bx))} - \frac{\left(4\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\right)\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx)}}{b^2}$$

$$= \frac{4x^3}{3b^2} + \frac{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3} - \frac{x^4}{b\tanh^{-1}(\tanh(a+bx))} + \frac{\left(4\left(-bx+\tanh\right.\right.}{}$$

$$= \frac{4x^3}{3b^2} + \frac{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3} + \frac{4x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^4} - \frac{}{b\tanh}$$

$$= \frac{4x^3}{3b^2} + \frac{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3} + \frac{4x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^4} - \frac{}{b\tanh}$$

$$= \frac{4x^3}{3b^2} + \frac{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3} + \frac{4x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^4} - \frac{}{b\tanh}$$

Mathematica [A] time = 0.0972113, size = 106, normalized size = 1.08

$$-\frac{x^2\left(\tanh^{-1}(\tanh(a+bx))-bx\right)}{b^3}-\frac{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^4}{b^5\tanh^{-1}(\tanh(a+bx))}+\frac{3x\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}{b^4}-\frac{4\left(\tanh^{-1}(\tan\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] x^3/(3*b^2) - (x^2*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/b^3 + (3*x*(-(b*x) +
ArcTanh[Tanh[a + b*x]])^2)/b^4 - (-(b*x) + ArcTanh[Tanh[a + b*x]])^4/(b^5*A
rcTanh[Tanh[a + b*x]]) - (4*(-(b*x) + ArcTanh[Tanh[a + b*x]])^3*Log[ArcTanh
[Tanh[a + b*x]]])/b^5
```

Maple [B] time = 0.042, size = 350, normalized size = 3.6

$$\frac{x^3}{3b^2}-\frac{ax^2}{b^3}-\frac{x^2(\mathrm{Artanh}(\tanh(bx+a))-bx-a)}{b^3}+3\frac{a^2x}{b^4}+6\frac{a(\mathrm{Artanh}(\tanh(bx+a))-bx-a)x}{b^4}+3\frac{(\mathrm{Artanh}(\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] 1/3*x^3/b^2-1/b^3*a*x^2-1/b^3*x^2*(arctanh(tanh(b*x+a))-b*x-a)+3/b^4*x*a^2+
6/b^4*a*(arctanh(tanh(b*x+a))-b*x-a)*x+3/b^4*(arctanh(tanh(b*x+a))-b*x-a)^2
*x-4/b^5*ln(arctanh(tanh(b*x+a)))*a^3-12/b^5*ln(arctanh(tanh(b*x+a)))*a^2*(
arctanh(tanh(b*x+a))-b*x-a)-12/b^5*ln(arctanh(tanh(b*x+a)))*a*(arctanh(tanh
(b*x+a))-b*x-a)^2-4/b^5*ln(arctanh(tanh(b*x+a)))*(arctanh(tanh(b*x+a))-b*x-
a)^3-1/b^5/arctanh(tanh(b*x+a))*a^4-4/b^5/arctanh(tanh(b*x+a))*a^3*(arctanh
(tanh(b*x+a))-b*x-a)-6/b^5/arctanh(tanh(b*x+a))*a^2*(arctanh(tanh(b*x+a))-b
*x-a)^2-4/b^5/arctanh(tanh(b*x+a))*a*(arctanh(tanh(b*x+a))-b*x-a)^3-1/b^5/a
rctanh(tanh(b*x+a))*(arctanh(tanh(b*x+a))-b*x-a)^4
```

Maxima [A] time = 2.43619, size = 95, normalized size = 0.97

$$\frac{b^4x^4-2\,ab^3x^3+6\,a^2b^2x^2+9\,a^3bx-3\,a^4}{3\left(b^6x+ab^5\right)}-\frac{4\,a^3\log\left(bx+a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4)/(b^6*x + a*
b^5) - 4*a^3*log(b*x + a)/b^5
```

Fricas [A] time = 1.51892, size = 155, normalized size = 1.58

$$\frac{b^4x^4 - 2\,ab^3x^3 + 6\,a^2b^2x^2 + 9\,a^3bx - 3\,a^4 - 12\left(a^3bx + a^4\right)\log\left(bx + a\right)}{3\left(b^6x + ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4 - 12*(a^3*b*
x + a^4)*log(b*x + a))/(b^6*x + a*b^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{atanh}^2\left(\tanh\left(a + bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(x**4/atanh(tanh(a + b*x))**2, x)
```

Giac [A] time = 1.16089, size = 84, normalized size = 0.86

$$-\frac{4\,a^3\log\left(|bx + a|\right)}{b^5} - \frac{a^4}{(bx + a)b^5} + \frac{b^4x^3 - 3\,ab^3x^2 + 9\,a^2b^2x}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] -4*a^3*log(abs(b*x + a))/b^5 - a^4/((b*x + a)*b^5) + 1/3*(b^4*x^3 - 3*a*b^3
*x^2 + 9*a^2*b^2*x)/b^6
```

3.95 $\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))^2} \, dx$

Optimal. Leaf size=75

$$\frac{3x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3}+\frac{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^4}-\frac{x^3}{b\tanh^{-1}(\tanh(a+\ldots}$$

```
[Out] (3*x^2)/(2*b^2) + (3*x*(b*x - ArcTanh[Tanh[a + b*x]]))/b^3 - x^3/(b*ArcTanh
[Tanh[a + b*x]]) + (3*(b*x - ArcTanh[Tanh[a + b*x]])^2*Log[ArcTanh[Tanh[a +
 b*x]]])/b^4
```

Rubi [A] time = 0.0529525, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2168, 2159, 2158, 2157, 29}

$$\frac{3x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3}+\frac{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^4}-\frac{x^3}{b\tanh^{-1}(\tanh(a+\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (3*x^2)/(2*b^2) + (3*x*(b*x - ArcTanh[Tanh[a + b*x]]))/b^3 - x^3/(b*ArcTanh
[Tanh[a + b*x]]) + (3*(b*x - ArcTanh[Tanh[a + b*x]])^2*Log[ArcTanh[Tanh[a +
 b*x]]])/b^4
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
```

```
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))^2}\,dx &= -\frac{x^3}{b\tanh^{-1}(\tanh(a+bx))}+\frac{3\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}\\ &= \frac{3x^2}{2b^2}-\frac{x^3}{b\tanh^{-1}(\tanh(a+bx))}-\frac{\left(3\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\right)\int \frac{x}{\tanh^{-1}(\tanh(a+bx))}\end{aligned}$$

$$= \frac{3x^2}{2b^2}+\frac{3x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{x^3}{b\tanh^{-1}(\tanh(a+bx))}+\frac{\left(3\left(-bx+\tanh^{-1}(\right.\right.}{}$$

$$= \frac{3x^2}{2b^2}+\frac{3x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{x^3}{b\tanh^{-1}(\tanh(a+bx))}+\frac{\left(3\left(-bx+\tanh^{-1}(\right.\right.}{}$$

$$= \frac{3x^2}{2b^2}+\frac{3x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{x^3}{b\tanh^{-1}(\tanh(a+bx))}+\frac{3\left(bx-\tanh^{-1}(\tan\right.}{}$$

Mathematica [A] time = 0.052245, size = 83, normalized size = 1.11

$$\frac{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^3}{b^4\tanh^{-1}(\tanh(a+bx))}-\frac{2x\left(\tanh^{-1}(\tanh(a+bx))-bx\right)}{b^3}+\frac{3\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2\log\left(\tanh^{-1}(\tanh(\right.}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] x^2/(2*b^2) - (2*x*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/b^3 + (-(b*x) + ArcTa
nh[Tanh[a + b*x]])^3/(b^4*ArcTanh[Tanh[a + b*x]]) + (3*(-(b*x) + ArcTanh[Ta
nh[a + b*x]])^2*Log[ArcTanh[Tanh[a + b*x]]])/b^4
```

Maple [B] time = 0.041, size = 223, normalized size = 3.

$$\frac{x^2}{2\,b^2}-2\,\frac{ax}{b^3}-2\,\frac{\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx-a\right)x}{b^3}+\frac{a^3}{b^4\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}+3\,\frac{a^2\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx}{b^4\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] 1/2*x^2/b^2-2/b^3*a*x-2/b^3*(arctanh(tanh(b*x+a))-b*x-a)*x+1/b^4/arctanh(ta
nh(b*x+a))*a^3+3/b^4/arctanh(tanh(b*x+a))*a^2*(arctanh(tanh(b*x+a))-b*x-a)+
3/b^4/arctanh(tanh(b*x+a))*a*(arctanh(tanh(b*x+a))-b*x-a)^2+1/b^4/arctanh(t
anh(b*x+a))*(arctanh(tanh(b*x+a))-b*x-a)^3+3/b^4*ln(arctanh(tanh(b*x+a)))*a
^2+6/b^4*ln(arctanh(tanh(b*x+a)))*a*(arctanh(tanh(b*x+a))-b*x-a)+3/b^4*ln(a
rctanh(tanh(b*x+a)))*(arctanh(tanh(b*x+a))-b*x-a)^2
```

Maxima [A] time = 2.43982, size = 80, normalized size = 1.07

$$\frac{b^3x^3-3\,ab^2x^2-4\,a^2bx+2\,a^3}{2\left(b^5x+ab^4\right)}+\frac{3\,a^2\log\left(bx+a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3)/(b^5*x + a*b^4) + 3*a^2*log
(b*x + a)/b^4
```

Fricas [A] time = 1.51337, size = 132, normalized size = 1.76

$$\frac{b^3x^3-3\,ab^2x^2-4\,a^2bx+2\,a^3+6\left(a^2bx+a^3\right)\log\left(bx+a\right)}{2\left(b^5x+ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3 + 6*(a^2*b*x + a^3)*log(b*x
+ a))/(b^5*x + a*b^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(x**3/atanh(tanh(a + b*x))**2, x)
```

Giac [A] time = 1.11688, size = 65, normalized size = 0.87

$$\frac{3\,a^2\log\left(|bx+a|\right)}{b^4}+\frac{a^3}{(bx+a)b^4}+\frac{b^2x^2-4\,abx}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 3*a^2*log(abs(b*x + a))/b^4 + a^3/((b*x + a)*b^4) + 1/2*(b^2*x^2 - 4*a*b*x)
/b^4
```

3.96 $\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))^2}\,dx$

Optimal. Leaf size=50

$$\frac{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{x^2}{b\tanh^{-1}(\tanh(a+bx))}+\frac{2x}{b^2}$$

```
[Out] (2*x)/b^2 - x^2/(b*ArcTanh[Tanh[a + b*x]]) + (2*(b*x - ArcTanh[Tanh[a + b*x
]])*Log[ArcTanh[Tanh[a + b*x]]])/b^3
```

Rubi [A] time = 0.0340511, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2168, 2158, 2157, 29}

$$\frac{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{x^2}{b\tanh^{-1}(\tanh(a+bx))}+\frac{2x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (2*x)/b^2 - x^2/(b*ArcTanh[Tanh[a + b*x]]) + (2*(b*x - ArcTanh[Tanh[a + b*x
]])*Log[ArcTanh[Tanh[a + b*x]]])/b^3
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{x^2}{b\tanh^{-1}(\tanh(a+bx))} + \frac{2\int \frac{x}{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$

$$= \frac{2x}{b^2} - \frac{x^2}{b\tanh^{-1}(\tanh(a+bx))} - \frac{\left(2\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\right)\int \frac{1}{\tanh^{-1}(\tanh(a+bx))}\,d}{b^2}$$

$$= \frac{2x}{b^2} - \frac{x^2}{b\tanh^{-1}(\tanh(a+bx))} - \frac{\left(2\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\right)\mathrm{Subst}\left(\int \frac{1}{x}\,dx, x, \mathrm{ta}\right.}{b^3}$$

$$= \frac{2x}{b^2} - \frac{x^2}{b\tanh^{-1}(\tanh(a+bx))} + \frac{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log\left(\tanh^{-1}(\tanh(a+b\right.}{b^3}$$

Mathematica [A] time = 0.0634883, size = 56, normalized size = 1.12

$$\frac{-\frac{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}{\tanh^{-1}(\tanh(a+bx))} + 2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log\left(\tanh^{-1}(\tanh(a+bx))\right) + bx}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (b*x - (-(b*x) + ArcTanh[Tanh[a + b*x]])^2/ArcTanh[Tanh[a + b*x]] + 2*(b*x
- ArcTanh[Tanh[a + b*x]])*Log[ArcTanh[Tanh[a + b*x]]])/b^3
```

Maple [B] time = 0.04, size = 127, normalized size = 2.5

$$\frac{x}{b^2} - 2\,\frac{\ln(\mathrm{Artanh}(\tanh(bx+a)))\,a}{b^3} - 2\,\frac{\ln(\mathrm{Artanh}(\tanh(bx+a)))(\mathrm{Artanh}(\tanh(bx+a))-bx-a)}{b^3} - \frac{a}{b^3\mathrm{Artanh}(\mathrm{t}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] x/b^2-2/b^3*ln(arctanh(tanh(b*x+a)))*a-2/b^3*ln(arctanh(tanh(b*x+a)))*(arct
anh(tanh(b*x+a))-b*x-a)-1/b^3/arctanh(tanh(b*x+a))*a^2-2/b^3/arctanh(tanh(b
*x+a))*a*(arctanh(tanh(b*x+a))-b*x-a)-1/b^3/arctanh(tanh(b*x+a))*(arctanh(t
anh(b*x+a))-b*x-a)^2
```

Maxima [A] time = 2.40512, size = 59, normalized size = 1.18

$$\frac{b^2x^2 + abx - a^2}{b^4x + ab^3} - \frac{2\,a\log\left(bx + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] (b^2*x^2 + a*b*x - a^2)/(b^4*x + a*b^3) - 2*a*log(b*x + a)/b^3
```

Fricas [A] time = 1.52317, size = 97, normalized size = 1.94

$$\frac{b^2x^2 + abx - a^2 - 2\left(abx + a^2\right)\log\left(bx + a\right)}{b^4x + ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))/(b^4*x + a*b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{atanh}^2\left(\tanh\left(a + bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(x**2/atanh(tanh(a + b*x))**2, x)
```

Giac [A] time = 1.15878, size = 46, normalized size = 0.92

$$\frac{x}{b^2} - \frac{2\,a\log\left(|bx + a|\right)}{b^3} - \frac{a^2}{(bx + a)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] x/b^2 - 2*a*log(abs(b*x + a))/b^3 - a^2/((b*x + a)*b^3)
```

3.97 $\int \frac{x}{\tanh^{-1}(\tanh(a+bx))^2}\,dx$

Optimal. Leaf size=28

$$\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^2}-\frac{x}{b\tanh^{-1}(\tanh(a+bx))}$$

```
[Out] -(x/(b*ArcTanh[Tanh[a + b*x]])) + Log[ArcTanh[Tanh[a + b*x]]]/b^2
```

Rubi [A] time = 0.0140836, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2168, 2157, 29}

$$\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^2}-\frac{x}{b\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] -(x/(b*ArcTanh[Tanh[a + b*x]])) + Log[ArcTanh[Tanh[a + b*x]]]/b^2
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{x}{\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{x}{b\tanh^{-1}(\tanh(a+bx))} + \frac{\int \frac{1}{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$
$$= -\frac{x}{b\tanh^{-1}(\tanh(a+bx))} + \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b^2}$$
$$= -\frac{x}{b\tanh^{-1}(\tanh(a+bx))} + \frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^2}$$

Mathematica [A] time = 0.0508815, size = 27, normalized size = 0.96

$$\frac{-\frac{bx}{\tanh^{-1}(\tanh(a+bx))} + \log\left(\tanh^{-1}(\tanh(a+bx))\right) + 1}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (1 - (b*x)/ArcTanh[Tanh[a + b*x]] + Log[ArcTanh[Tanh[a + b*x]]])/b^2
```

Maple [A] time = 0.039, size = 56, normalized size = 2.

$$\frac{\ln(\text{Artanh}(\tanh(bx+a)))}{b^2} + \frac{a}{b^2\text{Artanh}(\tanh(bx+a))} + \frac{\text{Artanh}(\tanh(bx+a)) - bx - a}{b^2\text{Artanh}(\tanh(bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] ln(arctanh(tanh(b*x+a)))/b^2+1/b^2/arctanh(tanh(b*x+a))*a+1/b^2/arctanh(tan
h(b*x+a))*(arctanh(tanh(b*x+a))-b*x-a)
```

Maxima [A] time = 2.44653, size = 35, normalized size = 1.25

$$\frac{a}{b^3x + ab^2} + \frac{\log(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] a/(b^3*x + a*b^2) + log(b*x + a)/b^2
```

Fricas [A] time = 1.50046, size = 62, normalized size = 2.21

$$\frac{(bx+a)\log(bx+a)+a}{b^3x+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] ((b*x + a)*log(b*x + a) + a)/(b^3*x + a*b^2)
```

Sympy [A] time = 15.1792, size = 36, normalized size = 1.29

$$\begin{cases} -\frac{x}{b\operatorname{atanh}(\tanh(a+bx))}+\frac{\log(\operatorname{atanh}(\tanh(a+bx)))}{b^2} & \text{for } b\neq 0 \\ \frac{x^2}{2\operatorname{atanh}^2(\tanh(a))} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Piecewise((-x/(b*atanh(tanh(a + b*x))) + log(atanh(tanh(a + b*x)))/b**2, Ne
(b, 0)), (x**2/(2*atanh(tanh(a))**2), True))
```

Giac [A] time = 1.12223, size = 32, normalized size = 1.14

$$\frac{\log(|bx+a|)}{b^2}+\frac{a}{(bx+a)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b^2 + a/((b*x + a)*b^2)
```

3.98 $\int \frac{1}{\tanh^{-1}(\tanh(a+bx))^2} dx$

Optimal. Leaf size=14

$$-\frac{1}{b\tanh^{-1}(\tanh(a+bx))}$$

```
[Out] -(1/(b*ArcTanh[Tanh[a + b*x]]))
```

Rubi [A] time = 0.0048739, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2157, 30}

$$-\frac{1}{b\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(-2),x]

[Out] -(1/(b*ArcTanh[Tanh[a + b*x]]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\tanh^{-1}(\tanh(a+bx))^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2} dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= -\frac{1}{b\tanh^{-1}(\tanh(a+bx))}\end{aligned}$$

Mathematica [A] time = 0.0057617, size = 14, normalized size = 1.

$$-\frac{1}{b\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(-2),x]
```

```
[Out] -(1/(b*ArcTanh[Tanh[a + b*x]]))
```

Maple [A] time = 0.027, size = 15, normalized size = 1.1

$$-\frac{1}{b\text{Artanh}\left(\tanh\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] -1/b/arctanh(tanh(b*x+a))
```

Maxima [A] time = 1.47179, size = 16, normalized size = 1.14

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] -1/((b*x + a)*b)
```

Fricas [A] time = 1.44121, size = 24, normalized size = 1.71

$$-\frac{1}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] -1/(b^2*x + a*b)
```

Sympy [A] time = 14.972, size = 20, normalized size = 1.43

$$\begin{cases} -\frac{1}{b \operatorname{atanh}\left(\tanh\left(a+bx\right)\right)} & \text{for}\ b \neq 0 \\ \frac{x}{\operatorname{atanh}^{2}\left(\tanh\left(a\right)\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Piecewise((-1/(b*atanh(tanh(a + b*x))), Ne(b, 0)), (x/atanh(tanh(a))**2, Tr
ue))
```

Giac [A] time = 1.14276, size = 16, normalized size = 1.14

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] -1/((b*x + a)*b)
```

3.99 $\int \frac{1}{x \tanh^{-1}(\tanh(a+bx))^2} \, dx$

Optimal. Leaf size=70

$$-\frac{1}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))}+\frac{\log(x)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}-\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

```
[Out] -(1/((b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]])) + Log[x]/(b*x
- ArcTanh[Tanh[a + b*x]])^2 - Log[ArcTanh[Tanh[a + b*x]]]/(b*x - ArcTanh[Ta
nh[a + b*x]])^2
```

Rubi [A] time = 0.0459563, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2163, 2160, 2157, 29}

$$-\frac{1}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))}+\frac{\log(x)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}-\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] -(1/((b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]])) + Log[x]/(b*x
- ArcTanh[Tanh[a + b*x]])^2 - Log[ArcTanh[Tanh[a + b*x]]]/(b*x - ArcTanh[Ta
nh[a + b*x]])^2
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{1}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))} + \frac{\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))}\,dx}{-bx+\tanh^{-1}(\tanh(a+bx)}$$

$$= -\frac{1}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))} - \frac{\int}{\left(bx-\tanh^{-1}(\tanh(a+bx)\right)}$$

$$= -\frac{1}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))} + \frac{\log(x)}{\left(bx-\tanh^{-1}(\tanh(a+bx)\right)}$$

$$= -\frac{1}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))} + \frac{\log(x)}{\left(bx-\tanh^{-1}(\tanh(a+bx)\right)}$$

Mathematica [A] time = 0.062484, size = 53, normalized size = 0.76

$$\frac{\tanh^{-1}(\tanh(a+bx))\left(-\log\left(\tanh^{-1}(\tanh(a+bx))\right)+\log(bx)+1\right)-bx}{\tanh^{-1}(\tanh(a+bx))\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] (-(b*x) + ArcTanh[Tanh[a + b*x]]*(1 + Log[b*x] - Log[ArcTanh[Tanh[a + b*x]]
]))/(ArcTanh[Tanh[a + b*x]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

Maple [A] time = 0.066, size = 67, normalized size = 1.

$$-\frac{\ln(\mathrm{Artanh}(\tanh(bx+a)))}{(\mathrm{Artanh}(\tanh(bx+a))-bx)^2} + \frac{1}{(\mathrm{Artanh}(\tanh(bx+a))-bx)\,\mathrm{Artanh}(\tanh(bx+a))} + \frac{\ln(x)}{(\mathrm{Artanh}(\tanh(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] -1/(arctanh(tanh(b*x+a))-b*x)^2*ln(arctanh(tanh(b*x+a)))+1/(arctanh(tanh(b*
x+a))-b*x)/arctanh(tanh(b*x+a))+1/(arctanh(tanh(b*x+a))-b*x)^2*ln(x)
```

Maxima [A] time = 2.44898, size = 38, normalized size = 0.54

$$\frac{1}{abx + a^2} - \frac{\log(bx + a)}{a^2} + \frac{\log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/(a*b*x + a^2) - log(b*x + a)/a^2 + log(x)/a^2
```

Fricas [A] time = 1.59652, size = 89, normalized size = 1.27

$$-\frac{(bx + a)\log(bx + a) - (bx + a)\log(x) - a}{a^2bx + a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] -((b*x + a)*log(b*x + a) - (b*x + a)*log(x) - a)/(a^2*b*x + a^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{atanh}^2(\tanh(a + bx))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(1/(x*atanh(tanh(a + b*x))**2), x)
```

Giac [A] time = 1.15898, size = 42, normalized size = 0.6

$$-\frac{\log\left(|bx+a|\right)}{a^2}+\frac{\log\left(|x|\right)}{a^2}+\frac{1}{(bx+a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a^2 + log(abs(x))/a^2 + 1/((b*x + a)*a)
```

3.100 $\int \frac{1}{x^2 \tanh^{-1}(\tanh(a+bx))^2} \, dx$

Optimal. Leaf size=102

$$-\frac{2b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 \tanh^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))}+\frac{}{(bx-}$$

```
[Out] (-2*b)/((b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]) + 1/(x*(b*
x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]) + (2*b*Log[x])/(b*x - A
rcTanh[Tanh[a + b*x]])^3 - (2*b*Log[ArcTanh[Tanh[a + b*x]]])/(b*x - ArcTanh
[Tanh[a + b*x]])^3
```

Rubi [A] time = 0.0716322, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2171, 2163, 2160, 2157, 29}

$$-\frac{2b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 \tanh^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))}+\frac{}{(bx-}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] (-2*b)/((b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]) + 1/(x*(b*
x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]) + (2*b*Log[x])/(b*x - A
rcTanh[Tanh[a + b*x]])^3 - (2*b*Log[ArcTanh[Tanh[a + b*x]]])/(b*x - ArcTanh
[Tanh[a + b*x]])^3
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
```

```
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x^2\tanh^{-1}(\tanh(a+bx))^2}\,dx = \frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))} - \frac{(2b)\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))^2}\,d}{-bx+\tanh^{-1}(\tanh(a+bx}$$

$$= -\frac{2b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))} + \frac{}{x\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

$$= -\frac{2b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))} + \frac{}{x\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

$$= -\frac{2b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))} + \frac{}{x\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

$$= -\frac{2b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))} + \frac{}{x\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

Mathematica [A] time = 0.0604331, size = 70, normalized size = 0.69

$$\frac{\tanh^{-1}(\tanh(a+bx))^2+2bx\tanh^{-1}(\tanh(a+bx))\left(\log(x)-\log\left(\tanh^{-1}(\tanh(a+bx))\right)\right)-b^2x^2}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] (-(b^2*x^2) + ArcTanh[Tanh[a + b*x]]^2 + 2*b*x*ArcTanh[Tanh[a + b*x]]*(Log[
x] - Log[ArcTanh[Tanh[a + b*x]]]))/(x*(b*x - ArcTanh[Tanh[a + b*x]])^3*ArcT
anh[Tanh[a + b*x]])
```

Maple [A] time = 0.09, size = 91, normalized size = 0.9

$$-\frac{b}{(\mathrm{Artanh}(\tanh(bx+a))-bx)^2\,\mathrm{Artanh}(\tanh(bx+a))}+2\,\frac{b\ln(\mathrm{Artanh}(\tanh(bx+a)))}{(\mathrm{Artanh}(\tanh(bx+a))-bx)^3}-\frac{1}{(\mathrm{Artanh}(\tanh(bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] -1/(arctanh(tanh(b*x+a))-b*x)^2*b/arctanh(tanh(b*x+a))+2/(arctanh(tanh(b*x+
a))-b*x)^3*b*ln(arctanh(tanh(b*x+a)))-1/(arctanh(tanh(b*x+a))-b*x)^2/x-2/(a
rctanh(tanh(b*x+a))-b*x)^3*b*ln(x)
```

Maxima [A] time = 2.42605, size = 61, normalized size = 0.6

$$-\frac{2\,bx+a}{a^2bx^2+a^3x}+\frac{2\,b\log(bx+a)}{a^3}-\frac{2\,b\log(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] -(2*b*x + a)/(a^2*b*x^2 + a^3*x) + 2*b*log(b*x + a)/a^3 - 2*b*log(x)/a^3
```

Fricas [A] time = 1.50922, size = 138, normalized size = 1.35

$$-\frac{2\,abx+a^2-2\left(b^2x^2+abx\right)\log(bx+a)+2\left(b^2x^2+abx\right)\log(x)}{a^3bx^2+a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log(b*x + a) + 2*(b^2*x^2 + a*b*x)*lo
g(x))/(a^3*b*x^2 + a^4*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \operatorname{atanh}^2(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(1/(x**2*atanh(tanh(a + b*x))**2), x)
```

Giac [A] time = 1.14788, size = 61, normalized size = 0.6

$$\frac{2\,b\log(|bx+a|)}{a^3} - \frac{2\,b\log(|x|)}{a^3} - \frac{2\,bx+a}{\left(bx^2+ax\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 2*b*log(abs(b*x + a))/a^3 - 2*b*log(abs(x))/a^3 - (2*b*x + a)/((b*x^2 + a*x
)*a^2)
```

3.101 $\int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))^2} \, dx$

Optimal. Leaf size=143

$$-\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3 \tanh^{-1}(\tanh(a+bx))}+\frac{3b^2 \log(x)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}-\frac{3b^2 \log\left(\tanh^{-1}(\tanh(a+bx\right.}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right.}$$

```
[Out] (-3*b^2)/((b*x - ArcTanh[Tanh[a + b*x]])^3*ArcTanh[Tanh[a + b*x]]) + (3*b)/
(2*x*(b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]) + 1/(2*x^2*(b
*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]) + (3*b^2*Log[x])/(b*x
- ArcTanh[Tanh[a + b*x]])^4 - (3*b^2*Log[ArcTanh[Tanh[a + b*x]]])/(b*x - Ar
cTanh[Tanh[a + b*x]])^4
```

Rubi [A] time = 0.0961146, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2171, 2163, 2160, 2157, 29}

$$-\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3 \tanh^{-1}(\tanh(a+bx))}+\frac{3b^2 \log(x)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}-\frac{3b^2 \log\left(\tanh^{-1}(\tanh(a+bx\right.}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] (-3*b^2)/((b*x - ArcTanh[Tanh[a + b*x]])^3*ArcTanh[Tanh[a + b*x]]) + (3*b)/
(2*x*(b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]) + 1/(2*x^2*(b
*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]) + (3*b^2*Log[x])/(b*x
- ArcTanh[Tanh[a + b*x]])^4 - (3*b^2*Log[ArcTanh[Tanh[a + b*x]]])/(b*x - Ar
cTanh[Tanh[a + b*x]])^4
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x^3\tanh^{-1}(\tanh(a+bx))^2}\,dx = \frac{1}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))} + \frac{(3b)\int \frac{1}{x^2\tanh^{-1}(\tanh(a+bx}}{2\left(bx-\tanh^{-1}(\tanh(a\right.}$$

$$= \frac{3b}{2x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))} + \frac{}{2x^2\left(bx-\tanh^{-1}(\tanh(\right.}$$

$$= -\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))} + \frac{}{2x\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

$$= -\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))} + \frac{}{2x\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

$$= -\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))} + \frac{}{2x\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

$$= -\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))} + \frac{}{2x\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

Mathematica [A] time = 0.0483696, size = 92, normalized size = 0.64

$$-\frac{-3b^2x^2\tanh^{-1}(\tanh(a+bx))\left(-2\log\left(\tanh^{-1}(\tanh(a+bx))\right)+2\log(x)-1\right)-6bx\tanh^{-1}(\tanh(a+bx))^2+\tanh^{-1}(\cdots}{2x^2\tanh^{-1}(\tanh(a+bx))\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] -(2*b^3*x^3 - 6*b*x*ArcTanh[Tanh[a + b*x]]^2 + ArcTanh[Tanh[a + b*x]]^3 - 3
*b^2*x^2*ArcTanh[Tanh[a + b*x]]*(-1 + 2*Log[x] - 2*Log[ArcTanh[Tanh[a + b*x
]]]))/(2*x^2*ArcTanh[Tanh[a + b*x]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4)
```

Maple [A] time = 0.089, size = 116, normalized size = 0.8

$$-3\,\frac{b^2\ln\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)}{\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^4}+\frac{b^2}{\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^3\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}-\frac{1}{2\,\left(\mathrm{Artanh}\left(\tanh\left(bx+\cdots\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] -3/(arctanh(tanh(b*x+a))-b*x)^4*b^2*ln(arctanh(tanh(b*x+a)))+1/(arctanh(tan
h(b*x+a))-b*x)^3*b^2/arctanh(tanh(b*x+a))-1/2/(arctanh(tanh(b*x+a))-b*x)^2/
x^2+3/(arctanh(tanh(b*x+a))-b*x)^4*b^2*ln(x)+2/(arctanh(tanh(b*x+a))-b*x)^3
*b/x
```

Maxima [A] time = 2.42335, size = 86, normalized size = 0.6

$$\frac{6\,b^2x^2+3\,abx-a^2}{2\left(a^3bx^3+a^4x^2\right)}-\frac{3\,b^2\log\left(bx+a\right)}{a^4}+\frac{3\,b^2\log\left(x\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(6*b^2*x^2 + 3*a*b*x - a^2)/(a^3*b*x^3 + a^4*x^2) - 3*b^2*log(b*x + a)/
a^4 + 3*b^2*log(x)/a^4
```

Fricas [A] time = 1.5043, size = 177, normalized size = 1.24

$$\frac{6\,ab^2x^2 + 3\,a^2bx - a^3 - 6\,\left(b^3x^3 + ab^2x^2\right)\log\left(bx + a\right) + 6\,\left(b^3x^3 + ab^2x^2\right)\log\left(x\right)}{2\,\left(a^4bx^3 + a^5x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 - 6*(b^3*x^3 + a*b^2*x^2)*log(b*x + a) +
 6*(b^3*x^3 + a*b^2*x^2)*log(x))/(a^4*b*x^3 + a^5*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\,\operatorname{atanh}^2\left(\tanh\left(a + bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(1/(x**3*atanh(tanh(a + b*x))**2), x)
```

Giac [A] time = 1.15376, size = 86, normalized size = 0.6

$$-\frac{3\,b^2\log\left(|bx + a|\right)}{a^4} + \frac{3\,b^2\log\left(|x|\right)}{a^4} + \frac{6\,ab^2x^2 + 3\,a^2bx - a^3}{2\,(bx + a)a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] -3*b^2*log(abs(b*x + a))/a^4 + 3*b^2*log(abs(x))/a^4 + 1/2*(6*a*b^2*x^2 + 3
*a^2*b*x - a^3)/((b*x + a)*a^4*x^2)
```

3.102 $\int \frac{x^m}{\tanh^{-1}(\tanh(a+bx))^3} \, dx$

Optimal. Leaf size=94

$$-\frac{mx^{m-1}\text{Hypergeometric2F1}\left(1,m-1,m,\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{2b^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{mx^{m-1}}{2b^2\tanh^{-1}(\tanh(a+bx))}-\frac{x^m}{2b\tanh^{-1}(\tanh(a+bx$$

```
[Out] -x^m/(2*b*ArcTanh[Tanh[a + b*x]]^2) - (m*x^(-1 + m))/(2*b^2*ArcTanh[Tanh[a
+ b*x]]) - (m*x^(-1 + m)*Hypergeometric2F1[1, -1 + m, m, (b*x)/(b*x - ArcTa
nh[Tanh[a + b*x]])])/(2*b^2*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rubi [A] time = 0.0606245, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 2164}

$$-\frac{mx^{m-1}\,{}_2F_1\left(1,m-1;m;\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{2b^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{mx^{m-1}}{2b^2\tanh^{-1}(\tanh(a+bx))}-\frac{x^m}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] -x^m/(2*b*ArcTanh[Tanh[a + b*x]]^2) - (m*x^(-1 + m))/(2*b^2*ArcTanh[Tanh[a
+ b*x]]) - (m*x^(-1 + m)*Hypergeometric2F1[1, -1 + m, m, (b*x)/(b*x - ArcTa
nh[Tanh[a + b*x]])])/(2*b^2*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2164

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)*Hypergeometric2F1[1, n + 1, n + 2, -((a*v)/(b*u
```

```
- a*v))])/((n + 1)*(b*u - a*v)), x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{x^m}{\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{x^m}{2b\tanh^{-1}(\tanh(a+bx))^2} + \frac{m\int \frac{x^{-1+m}}{\tanh^{-1}(\tanh(a+bx))^2}\,dx}{2b}$$

$$= -\frac{x^m}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{mx^{-1+m}}{2b^2\tanh^{-1}(\tanh(a+bx))} - \frac{((1-m)m)\int \frac{x^{-2+m}}{\tanh^{-1}(\tanh(a+$$

$$= -\frac{x^m}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{mx^{-1+m}}{2b^2\tanh^{-1}(\tanh(a+bx))} - \frac{mx^{-1+m}\,{}_2F_1\left(1,-1+m;m;\right.}{2b^2\left(bx-\tanh^{-1}(\mathrm{t}\right.}$$

Mathematica [A] time = 0.47992, size = 51, normalized size = 0.54

$$\frac{x^{m+1}\text{Hypergeometric2F1}\left(3,m+1,m+2,-\frac{bx}{\tanh^{-1}(\tanh(a+bx))-bx}\right)}{(m+1)\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[3, 1 + m, 2 + m, -((b*x)/(-(b*x) + ArcTanh[Tan
h[a + b*x]]))])/((1 + m)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^3)
```

Maple [F] time = 1.675, size = 0, normalized size = 0.

$$\int \frac{x^m}{(\mathrm{Artanh}\,(\tanh\,(bx+a)))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arctanh(tanh(b*x+a))^3,x)
```

```
[Out] int(x^m/arctanh(tanh(b*x+a))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{artanh}(\tanh(bx+a))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] integrate(x^m/arctanh(tanh(b*x + a))^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{\operatorname{artanh}(\tanh(bx+a))^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] integral(x^m/arctanh(tanh(b*x + a))^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{atanh}^3(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(x**m/atanh(tanh(a + b*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\operatorname{artanh}(\tanh(bx+a))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(x^m/arctanh(tanh(b*x + a))^3, x)
```

3.103 $\int \frac{x^4}{\tanh^{-1}(\tanh(a+bx))^3} dx$

Optimal. Leaf size=92

$$-\frac{2x^3}{b^2\tanh^{-1}(\tanh(a+bx))}+\frac{6x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^4}+\frac{6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\log\left(\tanh^{-1}(\tanh(a+\right.}{b^5}$$

```
[Out] (3*x^2)/b^3 + (6*x*(b*x - ArcTanh[Tanh[a + b*x]]))/b^4 - x^4/(2*b*ArcTanh[T
anh[a + b*x]]^2) - (2*x^3)/(b^2*ArcTanh[Tanh[a + b*x]]) + (6*(b*x - ArcTanh
[Tanh[a + b*x]])^2*Log[ArcTanh[Tanh[a + b*x]]])/b^5
```

Rubi [A] time = 0.0727309, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2168, 2159, 2158, 2157, 29}

$$-\frac{2x^3}{b^2\tanh^{-1}(\tanh(a+bx))}+\frac{6x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^4}+\frac{6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\log\left(\tanh^{-1}(\tanh(a+\right.}{b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (3*x^2)/b^3 + (6*x*(b*x - ArcTanh[Tanh[a + b*x]]))/b^4 - x^4/(2*b*ArcTanh[T
anh[a + b*x]]^2) - (2*x^3)/(b^2*ArcTanh[Tanh[a + b*x]]) + (6*(b*x - ArcTanh
[Tanh[a + b*x]])^2*Log[ArcTanh[Tanh[a + b*x]]])/b^5
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
```

```
/; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{x^4}{\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{x^4}{2b\tanh^{-1}(\tanh(a+bx))^2} + \frac{2\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))^2}\,dx}{b}$$

$$= -\frac{x^4}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{2x^3}{b^2\tanh^{-1}(\tanh(a+bx))} + \frac{6\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))}\,dx}{b^2}$$

$$= \frac{3x^2}{b^3} - \frac{x^4}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{2x^3}{b^2\tanh^{-1}(\tanh(a+bx))} - \frac{\left(6\left(-bx+\tanh^{-1}(\tan\right.\right.}{}$$

$$= \frac{3x^2}{b^3} + \frac{6x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^4} - \frac{x^4}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{2x^3}{b^2\tanh^{-1}(\tan}$$

$$= \frac{3x^2}{b^3} + \frac{6x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^4} - \frac{x^4}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{2x^3}{b^2\tanh^{-1}(\tan}$$

$$= \frac{3x^2}{b^3} + \frac{6x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^4} - \frac{x^4}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{2x^3}{b^2\tanh^{-1}(\tan}$$

Mathematica [A] time = 0.0418385, size = 114, normalized size = 1.24

$$-\frac{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^4}{2b^5\tanh^{-1}(\tanh(a+bx))^2} + \frac{4\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^3}{b^5\tanh^{-1}(\tanh(a+bx))} - \frac{3x\left(\tanh^{-1}(\tanh(a+bx))-bx\right)}{b^4} + \frac{6\left(\tanh^{-1}(\ta\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] x^2/(2*b^3) - (3*x*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/b^4 + (4*(-(b*x) + Ar
cTanh[Tanh[a + b*x]])^3)/(b^5*ArcTanh[Tanh[a + b*x]]) - (-(b*x) + ArcTanh[T
anh[a + b*x]])^4/(2*b^5*ArcTanh[Tanh[a + b*x]]^2) + (6*(-(b*x) + ArcTanh[Ta
nh[a + b*x]])^2*Log[ArcTanh[Tanh[a + b*x]]])/b^5
```

Maple [B] time = 0.041, size = 371, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arctanh(tanh(b*x+a))^3,x)
```

```
[Out] 1/2*x^2/b^3-3/b^4*a*x-3/b^4*(arctanh(tanh(b*x+a))-b*x-a)*x+6/b^5*ln(arctanh
(tanh(b*x+a)))*a^2+12/b^5*ln(arctanh(tanh(b*x+a)))*a*(arctanh(tanh(b*x+a))-
b*x-a)+6/b^5*ln(arctanh(tanh(b*x+a)))*(arctanh(tanh(b*x+a))-b*x-a)^2+4/b^5/
arctanh(tanh(b*x+a))*a^3+12/b^5/arctanh(tanh(b*x+a))*a^2*(arctanh(tanh(b*x+
a))-b*x-a)+12/b^5/arctanh(tanh(b*x+a))*a*(arctanh(tanh(b*x+a))-b*x-a)^2+4/b
^5/arctanh(tanh(b*x+a))*(arctanh(tanh(b*x+a))-b*x-a)^3-1/2/b^5/arctanh(tanh
(b*x+a))^2*a^4-2/b^5/arctanh(tanh(b*x+a))^2*a^3*(arctanh(tanh(b*x+a))-b*x-a
)-3/b^5/arctanh(tanh(b*x+a))^2*a^2*(arctanh(tanh(b*x+a))-b*x-a)^2-2/b^5/arc
tanh(tanh(b*x+a))^2*a*(arctanh(tanh(b*x+a))-b*x-a)^3-1/2/b^5/arctanh(tanh(b
*x+a))^2*(arctanh(tanh(b*x+a))-b*x-a)^4
```

Maxima [A] time = 3.54089, size = 109, normalized size = 1.18

$$\frac{b^4x^4 - 4\,ab^3x^3 - 11\,a^2b^2x^2 + 2\,a^3bx + 7\,a^4}{2\left(b^7x^2 + 2\,ab^6x + a^2b^5\right)} + \frac{6\,a^2\log\left(bx + a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^4*x^4 - 4*a*b^3*x^3 - 11*a^2*b^2*x^2 + 2*a^3*b*x + 7*a^4)/(b^7*x^2 +
 2*a*b^6*x + a^2*b^5) + 6*a^2*log(b*x + a)/b^5
```

Fricas [A] time = 1.50021, size = 200, normalized size = 2.17

$$\frac{b^4x^4 - 4\,ab^3x^3 - 11\,a^2b^2x^2 + 2\,a^3bx + 7\,a^4 + 12\left(a^2b^2x^2 + 2\,a^3bx + a^4\right)\log\left(bx + a\right)}{2\left(b^7x^2 + 2\,ab^6x + a^2b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*x^4 - 4*a*b^3*x^3 - 11*a^2*b^2*x^2 + 2*a^3*b*x + 7*a^4 + 12*(a^2*b
^2*x^2 + 2*a^3*b*x + a^4)*log(b*x + a))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{atanh}^3\left(\tanh\left(a + bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(x**4/atanh(tanh(a + b*x))**3, x)
```

Giac [A] time = 1.1599, size = 82, normalized size = 0.89

$$\frac{6\,a^2\log\left(|bx + a|\right)}{b^5} + \frac{b^3x^2 - 6\,ab^2x}{2\,b^6} + \frac{8\,a^3bx + 7\,a^4}{2\left(bx + a\right)^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 6*a^2*log(abs(b*x + a))/b^5 + 1/2*(b^3*x^2 - 6*a*b^2*x)/b^6 + 1/2*(8*a^3*b*
x + 7*a^4)/((b*x + a)^2*b^5)
```

3.104 $\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=71

$$-\frac{3x^2}{2b^2\tanh^{-1}(\tanh(a+bx))}+\frac{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^4}-\frac{x^3}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

```
[Out] (3*x)/b^3 - x^3/(2*b*ArcTanh[Tanh[a + b*x]]^2) - (3*x^2)/(2*b^2*ArcTanh[Tan
h[a + b*x]]) + (3*(b*x - ArcTanh[Tanh[a + b*x]])*Log[ArcTanh[Tanh[a + b*x]]
])/b^4
```

Rubi [A] time = 0.0483379, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2168, 2158, 2157, 29}

$$-\frac{3x^2}{2b^2\tanh^{-1}(\tanh(a+bx))}+\frac{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^4}-\frac{x^3}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (3*x)/b^3 - x^3/(2*b*ArcTanh[Tanh[a + b*x]]^2) - (3*x^2)/(2*b^2*ArcTanh[Tan
h[a + b*x]]) + (3*(b*x - ArcTanh[Tanh[a + b*x]])*Log[ArcTanh[Tanh[a + b*x]]
])/b^4
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
```

```
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{x^3}{2b\tanh^{-1}(\tanh(a+bx))^2} + \frac{3\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))^2}\,dx}{2b}$$

$$= -\frac{x^3}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{3x^2}{2b^2\tanh^{-1}(\tanh(a+bx))} + \frac{3\int \frac{x}{\tanh^{-1}(\tanh(a+bx))}\,dx}{b^2}$$

$$= \frac{3x}{b^3} - \frac{x^3}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{3x^2}{2b^2\tanh^{-1}(\tanh(a+bx))} - \frac{\left(3\left(-bx+\tanh^{-1}(\tan\right.\right.}{}$$

$$= \frac{3x}{b^3} - \frac{x^3}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{3x^2}{2b^2\tanh^{-1}(\tanh(a+bx))} - \frac{\left(3\left(-bx+\tanh^{-1}(\tan\right.\right.}{}$$

$$= \frac{3x}{b^3} - \frac{x^3}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{3x^2}{2b^2\tanh^{-1}(\tanh(a+bx))} + \frac{3\left(bx-\tanh^{-1}(\tanh(\right.}{}$$

Mathematica [A] time = 0.0654912, size = 86, normalized size = 1.21

$$-\frac{3b^2x^2\tanh^{-1}(\tanh(a+bx)) - bx\tanh^{-1}(\tanh(a+bx))^2\left(6\log\left(\tanh^{-1}(\tanh(a+bx))\right)+11\right)+\tanh^{-1}(\tanh(a+bx}{2b^4\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] -(b^3*x^3 + 3*b^2*x^2*ArcTanh[Tanh[a + b*x]] + ArcTanh[Tanh[a + b*x]]^3*(5
+ 6*Log[ArcTanh[Tanh[a + b*x]]]) - b*x*ArcTanh[Tanh[a + b*x]]^2*(11 + 6*Log
[ArcTanh[Tanh[a + b*x]]]))/(2*b^4*ArcTanh[Tanh[a + b*x]]^2)
```

Maple [B] time = 0.04, size = 239, normalized size = 3.4

$$\frac{x}{b^3}-3\frac{a^2}{b^4\text{Artanh}\left(\tanh\left(bx+a\right)\right)}-6\frac{a\left(\text{Artanh}\left(\tanh\left(bx+a\right)\right)-bx-a\right)}{b^4\text{Artanh}\left(\tanh\left(bx+a\right)\right)}-3\frac{\left(\text{Artanh}\left(\tanh\left(bx+a\right)\right)-bx-a\right)^2}{b^4\text{Artanh}\left(\tanh\left(bx+a\right)\right)}-3\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arctanh(tanh(b*x+a))^3,x)
```

```
[Out] x/b^3-3/b^4/arctanh(tanh(b*x+a))*a^2-6/b^4/arctanh(tanh(b*x+a))*a*(arctanh(
tanh(b*x+a))-b*x-a)-3/b^4/arctanh(tanh(b*x+a))*(arctanh(tanh(b*x+a))-b*x-a)
^2-3/b^4*ln(arctanh(tanh(b*x+a)))*a-3/b^4*ln(arctanh(tanh(b*x+a)))*(arctanh
(tanh(b*x+a))-b*x-a)+1/2/b^4/arctanh(tanh(b*x+a))^2*a^3+3/2/b^4/arctanh(tan
h(b*x+a))^2*a^2*(arctanh(tanh(b*x+a))-b*x-a)+3/2/b^4/arctanh(tanh(b*x+a))^2
*a*(arctanh(tanh(b*x+a))-b*x-a)^2+1/2/b^4/arctanh(tanh(b*x+a))^2*(arctanh(t
anh(b*x+a))-b*x-a)^3
```

Maxima [A] time = 3.48334, size = 93, normalized size = 1.31

$$\frac{2\,b^3x^3+4\,ab^2x^2-4\,a^2bx-5\,a^3}{2\left(b^6x^2+2\,ab^5x+a^2b^4\right)}-\frac{3\,a\log\left(bx+a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^3*x^3 + 4*a*b^2*x^2 - 4*a^2*b*x - 5*a^3)/(b^6*x^2 + 2*a*b^5*x + a^
2*b^4) - 3*a*log(b*x + a)/b^4
```

Fricas [A] time = 1.47845, size = 176, normalized size = 2.48

$$\frac{2\,b^3x^3+4\,ab^2x^2-4\,a^2bx-5\,a^3-6\left(ab^2x^2+2\,a^2bx+a^3\right)\log\left(bx+a\right)}{2\left(b^6x^2+2\,ab^5x+a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*x^3 + 4*a*b^2*x^2 - 4*a^2*b*x - 5*a^3 - 6*(a*b^2*x^2 + 2*a^2*b*x
 + a^3)*log(b*x + a))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{atanh}^3(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(x**3/atanh(tanh(a + b*x))**3, x)
```

Giac [A] time = 1.18488, size = 59, normalized size = 0.83

$$\frac{x}{b^3} - \frac{3\,a\log(|bx+a|)}{b^4} - \frac{6\,a^2bx + 5\,a^3}{2\,(bx+a)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] x/b^3 - 3*a*log(abs(b*x + a))/b^4 - 1/2*(6*a^2*b*x + 5*a^3)/((b*x + a)^2*b^
4)
```

3.105 $\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))^3} \, dx$

Optimal. Leaf size=47

$$-\frac{x}{b^2 \tanh^{-1}(\tanh(a+bx))}+\frac{\log \left(\tanh^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{x^2}{2 b \tanh^{-1}(\tanh(a+bx))^2}$$

```
[Out] -x^2/(2*b*ArcTanh[Tanh[a + b*x]]^2) - x/(b^2*ArcTanh[Tanh[a + b*x]]) + Log[
ArcTanh[Tanh[a + b*x]]]/b^3
```

Rubi [A] time = 0.0287172, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2157, 29}

$$-\frac{x}{b^2 \tanh^{-1}(\tanh(a+bx))}+\frac{\log \left(\tanh^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{x^2}{2 b \tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] -x^2/(2*b*ArcTanh[Tanh[a + b*x]]^2) - x/(b^2*ArcTanh[Tanh[a + b*x]]) + Log[
ArcTanh[Tanh[a + b*x]]]/b^3
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{x^2}{2b\tanh^{-1}(\tanh(a+bx))^2} + \frac{\int \frac{x}{\tanh^{-1}(\tanh(a+bx))^2}\,dx}{b}$$

$$= -\frac{x^2}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{x}{b^2\tanh^{-1}(\tanh(a+bx))} + \frac{\int \frac{1}{\tanh^{-1}(\tanh(a+bx))}\,dx}{b^2}$$

$$= -\frac{x^2}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{x}{b^2\tanh^{-1}(\tanh(a+bx))} + \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \tanh^{-1}(\text{ta}\right.}{b^3}$$

$$= -\frac{x^2}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{x}{b^2\tanh^{-1}(\tanh(a+bx))} + \frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^3}$$

Mathematica [A] time = 0.0316702, size = 49, normalized size = 1.04

$$\frac{-\frac{b^2x^2}{\tanh^{-1}(\tanh(a+bx))^2} - \frac{2bx}{\tanh^{-1}(\tanh(a+bx))} + 2\log\left(\tanh^{-1}(\tanh(a+bx))\right) + 3}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (3 - (b^2*x^2)/ArcTanh[Tanh[a + b*x]]^2 - (2*b*x)/ArcTanh[Tanh[a + b*x]] +
2*Log[ArcTanh[Tanh[a + b*x]]])/(2*b^3)
```

Maple [B] time = 0.04, size = 136, normalized size = 2.9

$$2\,\frac{a}{b^3\text{Artanh}\,(\tanh\,(bx+a))} + 2\,\frac{\text{Artanh}\,(\tanh\,(bx+a)) - bx - a}{b^3\text{Artanh}\,(\tanh\,(bx+a))} + \frac{\ln\,(\text{Artanh}\,(\tanh\,(bx+a)))}{b^3} - \frac{a^2}{2\,b^3\,(\text{Artanh}\,(\text{tanl}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arctanh(tanh(b*x+a))^3,x)
```

```
[Out] 2/b^3/arctanh(tanh(b*x+a))*a+2/b^3/arctanh(tanh(b*x+a))*(arctanh(tanh(b*x+a
))-b*x-a)+ln(arctanh(tanh(b*x+a)))/b^3-1/2/b^3/arctanh(tanh(b*x+a))^2*a^2-1
/b^3/arctanh(tanh(b*x+a))^2*a*(arctanh(tanh(b*x+a))-b*x-a)-1/2/b^3/arctanh(
tanh(b*x+a))^2*(arctanh(tanh(b*x+a))-b*x-a)^2
```

Maxima [A] time = 3.47843, size = 65, normalized size = 1.38

$$\frac{4\,abx+3\,a^2}{2\left(b^5x^2+2\,ab^4x+a^2b^3\right)}+\frac{\log\left(bx+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*b*x + 3*a^2)/(b^5*x^2 + 2*a*b^4*x + a^2*b^3) + log(b*x + a)/b^3
```

Fricas [A] time = 1.47192, size = 132, normalized size = 2.81

$$\frac{4\,abx+3\,a^2+2\left(b^2x^2+2\,abx+a^2\right)\log\left(bx+a\right)}{2\left(b^5x^2+2\,ab^4x+a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*b*x + 3*a^2 + 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a))/(b^5*x^2 +
 2*a*b^4*x + a^2*b^3)
```

Sympy [A] time = 22.1723, size = 54, normalized size = 1.15

$$\begin{cases} -\frac{x^2}{2b\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}-\frac{x}{b^2\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}+\frac{\log\left(\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)\right)}{b^3} & \text{for}\ b\neq 0\\ \frac{x^3}{3\operatorname{atanh}^3\left(\tanh\left(a\right)\right)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Piecewise((-x**2/(2*b*atanh(tanh(a + b*x))**2) - x/(b**2*atanh(tanh(a + b*x
))) + log(atanh(tanh(a + b*x)))/b**3, Ne(b, 0)), (x**3/(3*atanh(tanh(a))**3
), True))
```

Giac [A] time = 1.13653, size = 50, normalized size = 1.06

$$\frac{\log\left(|bx+a|\right)}{b^3}+\frac{4\,ax+\frac{3\,a^2}{b}}{2\,(bx+a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b^3 + 1/2*(4*a*x + 3*a^2/b)/((b*x + a)^2*b^2)
```

3.106 $\int \frac{x}{\tanh^{-1}(\tanh(a+bx))^3} dx$

Optimal. Leaf size=34

$$-\frac{1}{2b^2\tanh^{-1}(\tanh(a+bx))}-\frac{x}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

```
[Out] -x/(2*b*ArcTanh[Tanh[a + b*x]]^2) - 1/(2*b^2*ArcTanh[Tanh[a + b*x]])
```

Rubi [A] time = 0.0144834, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2168, 2157, 30}

$$-\frac{1}{2b^2\tanh^{-1}(\tanh(a+bx))}-\frac{x}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] -x/(2*b*ArcTanh[Tanh[a + b*x]]^2) - 1/(2*b^2*ArcTanh[Tanh[a + b*x]])
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x}{\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{x}{2b\tanh^{-1}(\tanh(a+bx))^2} + \frac{\int \frac{1}{\tanh^{-1}(\tanh(a+bx))^2}\,dx}{2b}$$
$$= -\frac{x}{2b\tanh^{-1}(\tanh(a+bx))^2} + \frac{\text{Subst}\left(\int \frac{1}{x^2}\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{2b^2}$$
$$= -\frac{x}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{1}{2b^2\tanh^{-1}(\tanh(a+bx))}$$

Mathematica [A] time = 0.0486065, size = 27, normalized size = 0.79

$$-\frac{\tanh^{-1}(\tanh(a+bx)) + bx}{2b^2\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] -(b*x + ArcTanh[Tanh[a + b*x]])/(2*b^2*ArcTanh[Tanh[a + b*x]]^2)
```

Maple [A] time = 0.037, size = 43, normalized size = 1.3

$$-\frac{bx - \text{Artanh}(\tanh(bx+a))}{2b^2(\text{Artanh}(\tanh(bx+a)))^2} - \frac{1}{b^2\text{Artanh}(\tanh(bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctanh(tanh(b*x+a))^3,x)
```

```
[Out] -1/2*(b*x-arctanh(tanh(b*x+a)))/b^2/arctanh(tanh(b*x+a))^2-1/b^2/arctanh(ta
nh(b*x+a))
```

Maxima [A] time = 3.53057, size = 43, normalized size = 1.26

$$-\frac{2\,bx + a}{2\left(b^4x^2 + 2\,ab^3x + a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Fricas [A] time = 1.55933, size = 68, normalized size = 2.

$$-\frac{2\,bx+a}{2\left(b^4x^2+2\,ab^3x+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Sympy [A] time = 22.1402, size = 42, normalized size = 1.24

$$\begin{cases} -\frac{x}{2b\,\mathrm{atanh}^2\left(\tanh\left(a+bx\right)\right)}-\frac{1}{2b^2\,\mathrm{atanh}\left(\tanh\left(a+bx\right)\right)} & \text{for } b\neq 0\\ \frac{x^2}{2\,\mathrm{atanh}^3\left(\tanh\left(a\right)\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Piecewise((-x/(2*b*atanh(tanh(a + b*x))**2) - 1/(2*b**2*atanh(tanh(a + b*x)
)), Ne(b, 0)), (x**2/(2*atanh(tanh(a))**3), True))
```

Giac [A] time = 1.13751, size = 24, normalized size = 0.71

$$-\frac{2\,bx+a}{2\,(bx+a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^3,x, algorithm="giac")

[Out] -1/2*(2*b*x + a)/((b*x + a)^2*b^2)
```

3.107 $\int \frac{1}{\tanh^{-1}(\tanh(a+bx))^3} \, dx$

Optimal. Leaf size=16

$$-\frac{1}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

```
[Out] -1/(2*b*ArcTanh[Tanh[a + b*x]]^2)
```

Rubi [A] time = 0.004478, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2157, 30}

$$-\frac{1}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(-3),x]
```

```
[Out] -1/(2*b*ArcTanh[Tanh[a + b*x]]^2)
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\tanh^{-1}(\tanh(a+bx))^3} \, dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3} \, dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= -\frac{1}{2b\tanh^{-1}(\tanh(a+bx))^2}\end{aligned}$$

Mathematica [A] time = 0.006647, size = 16, normalized size = 1.

$$-\frac{1}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(-3),x]
```

```
[Out] -1/(2*b*ArcTanh[Tanh[a + b*x]]^2)
```

Maple [A] time = 0.027, size = 15, normalized size = 0.9

$$-\frac{1}{2\,b\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctanh(tanh(b*x+a))^3,x)
```

```
[Out] -1/2/b/arctanh(tanh(b*x+a))^2
```

Maxima [A] time = 1.47506, size = 16, normalized size = 1.

$$-\frac{1}{2\,(bx+a)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -1/2/((b*x + a)^2*b)
```

Fricas [A] time = 1.49182, size = 49, normalized size = 3.06

$$-\frac{1}{2\left(b^3x^2+2\,ab^2x+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] -1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Sympy [A] time = 15.3124, size = 24, normalized size = 1.5

$$\begin{cases} -\frac{1}{2b\operatorname{atanh}^{2}\left(\tanh\left(a+bx\right)\right)} & \text{for } b \neq 0 \\ \frac{x}{\operatorname{atanh}^{3}\left(\tanh\left(a\right)\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Piecewise((-1/(2*b*atanh(tanh(a + b*x))**2), Ne(b, 0)), (x/atanh(tanh(a))**
3, True))
```

Giac [A] time = 1.12099, size = 16, normalized size = 1.

$$-\frac{1}{2\,(bx+a)^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] -1/2/((b*x + a)^2*b)
```

3.108 $\int \frac{1}{x \tanh^{-1}(\tanh(a+bx))^3} dx$

Optimal. Leaf size=97

$$\frac{1}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 \tanh^{-1}(\tanh(a+bx))} - \frac{1}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2} - \frac{}{\left(bx-\right.}$$

```
[Out] -1/(2*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^2) + 1/((b*x -
ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]) - Log[x]/(b*x - ArcTanh[T
anh[a + b*x]])^3 + Log[ArcTanh[Tanh[a + b*x]]]/(b*x - ArcTanh[Tanh[a + b*x]
])^3
```

Rubi [A] time = 0.0663209, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2163, 2160, 2157, 29}

$$\frac{1}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 \tanh^{-1}(\tanh(a+bx))} - \frac{1}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2} - \frac{}{\left(bx-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] -1/(2*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^2) + 1/((b*x -
ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]) - Log[x]/(b*x - ArcTanh[T
anh[a + b*x]])^3 + Log[ArcTanh[Tanh[a + b*x]]]/(b*x - ArcTanh[Tanh[a + b*x]
])^3
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
```

```
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{1}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2} - \frac{\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))^2}\,dx}{bx-\tanh^{-1}(\tanh(a+bx))}$$

$$= -\frac{1}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2} + \frac{1}{\left(bx-\tanh^{-1}(\tanh(a+bx\right.}$$

$$= -\frac{1}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2} + \frac{1}{\left(bx-\tanh^{-1}(\tanh(a+bx\right.}$$

$$= -\frac{1}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2} + \frac{1}{\left(bx-\tanh^{-1}(\tanh(a+bx\right.}$$

$$= -\frac{1}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2} + \frac{1}{\left(bx-\tanh^{-1}(\tanh(a+bx\right.}$$

Mathematica [A] time = 0.0966306, size = 74, normalized size = 0.76

$$\frac{-4bx\tanh^{-1}(\tanh(a+bx))+\tanh^{-1}(\tanh(a+bx))^2\left(-2\log\left(\tanh^{-1}(\tanh(a+bx))\right)+2\log(bx)+3\right)+b^2x^2}{2\tanh^{-1}(\tanh(a+bx))^2\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] (b^2*x^2 - 4*b*x*ArcTanh[Tanh[a + b*x]] + ArcTanh[Tanh[a + b*x]]^2*(3 + 2*L
og[b*x] - 2*Log[ArcTanh[Tanh[a + b*x]]]))/(2*ArcTanh[Tanh[a + b*x]]^2*(-(b*
```

```
x) + ArcTanh[Tanh[a + b*x]])^3)
```

Maple [A] time = 0.068, size = 92, normalized size = 1.

$$-\frac{\ln(\operatorname{Artanh}(\tanh(bx+a)))}{(\operatorname{Artanh}(\tanh(bx+a))-bx)^3}+\frac{1}{(\operatorname{Artanh}(\tanh(bx+a))-bx)^2\operatorname{Artanh}(\tanh(bx+a))}+\frac{}{(2\operatorname{Artanh}(\tanh(bx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctanh(tanh(b*x+a))^3,x)
```

```
[Out] -1/(arctanh(tanh(b*x+a))-b*x)^3*ln(arctanh(tanh(b*x+a)))+1/(arctanh(tanh(b*
x+a))-b*x)^2/arctanh(tanh(b*x+a))+1/2/(arctanh(tanh(b*x+a))-b*x)/arctanh(ta
nh(b*x+a))^2+1/(arctanh(tanh(b*x+a))-b*x)^3*ln(x)
```

Maxima [A] time = 3.54231, size = 69, normalized size = 0.71

$$\frac{2\,bx+3\,a}{2\left(a^2b^2x^2+2\,a^3bx+a^4\right)}-\frac{\log(bx+a)}{a^3}+\frac{\log(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x + 3*a)/(a^2*b^2*x^2 + 2*a^3*b*x + a^4) - log(b*x + a)/a^3 + log(
x)/a^3
```

Fricas [A] time = 1.52844, size = 182, normalized size = 1.88

$$\frac{2\,abx+3\,a^2-2\left(b^2x^2+2\,abx+a^2\right)\log(bx+a)+2\left(b^2x^2+2\,abx+a^2\right)\log(x)}{2\left(a^3b^2x^2+2\,a^4bx+a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*x + 3*a^2 - 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a) + 2*(b^2*x^
2 + 2*a*b*x + a^2)*log(x))/(a^3*b^2*x^2 + 2*a^4*b*x + a^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{atanh}^3(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(1/(x*atanh(tanh(a + b*x))**3), x)
```

Giac [A] time = 1.15298, size = 58, normalized size = 0.6

$$-\frac{\log(|bx + a|)}{a^3} + \frac{\log(|x|)}{a^3} + \frac{2\,abx + 3\,a^2}{2\,(bx + a)^2 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a^3 + log(abs(x))/a^3 + 1/2*(2*a*b*x + 3*a^2)/((b*x + a)
^2*a^3)
```

3.109 $\int \frac{1}{x^2 \tanh^{-1}(\tanh(a+bx))^3} dx$

Optimal. Leaf size=131

$$\frac{3b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3 \tanh^{-1}(\tanh(a+bx))}-\frac{3b}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 \tanh^{-1}(\tanh(a+bx))^2}+\frac{}{x\left(b\right.}$$

```
[Out] (-3*b)/(2*(b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^2) + 1/(x
*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^2) + (3*b)/((b*x - A
rcTanh[Tanh[a + b*x]])^3*ArcTanh[Tanh[a + b*x]]) - (3*b*Log[x])/(b*x - ArcT
anh[Tanh[a + b*x]])^4 + (3*b*Log[ArcTanh[Tanh[a + b*x]]])/(b*x - ArcTanh[Ta
nh[a + b*x]])^4
```

Rubi [A] time = 0.0932824, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2171, 2163, 2160, 2157, 29}

$$\frac{3b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3 \tanh^{-1}(\tanh(a+bx))}-\frac{3b}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 \tanh^{-1}(\tanh(a+bx))^2}+\frac{}{x\left(b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*ArcTanh[Tanh[a + b*x]]^3),x]
```


```
[Out] (-3*b)/(2*(b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^2) + 1/(x
*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^2) + (3*b)/((b*x - A
rcTanh[Tanh[a + b*x]])^3*ArcTanh[Tanh[a + b*x]]) - (3*b*Log[x])/(b*x - ArcT
anh[Tanh[a + b*x]])^4 + (3*b*Log[ArcTanh[Tanh[a + b*x]]])/(b*x - ArcTanh[Ta
nh[a + b*x]])^4
```


Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x^2 \tanh^{-1}(\tanh(a+bx))^3}\,dx = \frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2} - \frac{(3b)\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))^3}\,d}{-bx+\tanh^{-1}(\tanh(a+bx}$$

$$= -\frac{3b}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

$$= -\frac{3b}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

$$= -\frac{3b}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

$$= -\frac{3b}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

$$= -\frac{3b}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

Mathematica [A] time = 0.0428006, size = 93, normalized size = 0.71

$$-\frac{-6b^2x^2\tanh^{-1}(\tanh(a+bx))+2\tanh^{-1}(\tanh(a+bx))^3+3bx\tanh^{-1}(\tanh(a+bx))^2\left(-2\log\left(\tanh^{-1}(\tanh(a+bx\right.\right.}{2x\tanh^{-1}(\tanh(a+bx))^2\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] -(b^3*x^3 - 6*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 2*ArcTanh[Tanh[a + b*x]]^3 +
 3*b*x*ArcTanh[Tanh[a + b*x]]^2*(1 + 2*Log[x] - 2*Log[ArcTanh[Tanh[a + b*x]
]]))/(2*x*ArcTanh[Tanh[a + b*x]]^2*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4)
```

Maple [A] time = 0.085, size = 117, normalized size = 0.9

$$-\frac{1}{\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^3x}-3\frac{b\ln\left(x\right)}{\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^4}-\frac{b}{2\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^2\left(\mathrm{Artanh}\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arctanh(tanh(b*x+a))^3,x)
```

```
[Out] -1/(arctanh(tanh(b*x+a))-b*x)^3/x-3/(arctanh(tanh(b*x+a))-b*x)^4*b*ln(x)-1/
2/(arctanh(tanh(b*x+a))-b*x)^2*b/arctanh(tanh(b*x+a))^2+3/(arctanh(tanh(b*x
+a))-b*x)^4*b*ln(arctanh(tanh(b*x+a)))-2/(arctanh(tanh(b*x+a))-b*x)^3*b/arc
tanh(tanh(b*x+a))
```

Maxima [A] time = 3.52496, size = 93, normalized size = 0.71

$$-\frac{6\,b^2x^2+9\,abx+2\,a^2}{2\left(a^3b^2x^3+2\,a^4bx^2+a^5x\right)}+\frac{3\,b\log\left(bx+a\right)}{a^4}-\frac{3\,b\log\left(x\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(6*b^2*x^2 + 9*a*b*x + 2*a^2)/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x) + 3*
b*log(b*x + a)/a^4 - 3*b*log(x)/a^4
```

Fricas [A] time = 1.52248, size = 232, normalized size = 1.77

$$-\frac{6\,ab^2x^2 + 9\,a^2bx + 2\,a^3 - 6\left(b^3x^3 + 2\,ab^2x^2 + a^2bx\right)\log\left(bx + a\right) + 6\left(b^3x^3 + 2\,ab^2x^2 + a^2bx\right)\log\left(x\right)}{2\left(a^4b^2x^3 + 2\,a^5bx^2 + a^6x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(6*a*b^2*x^2 + 9*a^2*b*x + 2*a^3 - 6*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)
*log(b*x + a) + 6*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)*log(x))/(a^4*b^2*x^3 +
2*a^5*b*x^2 + a^6*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \operatorname{atanh}^3\left(\tanh\left(a + bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(1/(x**2*atanh(tanh(a + b*x))**3), x)
```

Giac [A] time = 1.144, size = 81, normalized size = 0.62

$$\frac{3\,b\log\left(|bx + a|\right)}{a^4} - \frac{3\,b\log\left(|x|\right)}{a^4} - \frac{6\,ab^2x^2 + 9\,a^2bx + 2\,a^3}{2\left(bx + a\right)^2a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 3*b*log(abs(b*x + a))/a^4 - 3*b*log(abs(x))/a^4 - 1/2*(6*a*b^2*x^2 + 9*a^2*
b*x + 2*a^3)/((b*x + a)^2*a^4*x)
```

3.110 $\int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))^3} \, dx$

Optimal. Leaf size=170

$$\frac{6b^2}{\left(bx - \tanh^{-1}(\tanh(a+bx))\right)^4 \tanh^{-1}(\tanh(a+bx))} - \frac{3b^2}{\left(bx - \tanh^{-1}(\tanh(a+bx))\right)^3 \tanh^{-1}(\tanh(a+bx))^2} - \frac{}{\left(bx - \right.}$$

```
[Out] (-3*b^2)/((b*x - ArcTanh[Tanh[a + b*x]])^3*ArcTanh[Tanh[a + b*x]]^2) + (2*b
)/(x*(b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^2) + 1/(2*x^2*
(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^2) + (6*b^2)/((b*x -
ArcTanh[Tanh[a + b*x]])^4*ArcTanh[Tanh[a + b*x]]) - (6*b^2*Log[x])/(b*x - A
rcTanh[Tanh[a + b*x]])^5 + (6*b^2*Log[ArcTanh[Tanh[a + b*x]]])/(b*x - ArcTa
nh[Tanh[a + b*x]])^5
```

Rubi [A] time = 0.120783, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2171, 2163, 2160, 2157, 29}

$$\frac{6b^2}{\left(bx - \tanh^{-1}(\tanh(a+bx))\right)^4 \tanh^{-1}(\tanh(a+bx))} - \frac{3b^2}{\left(bx - \tanh^{-1}(\tanh(a+bx))\right)^3 \tanh^{-1}(\tanh(a+bx))^2} - \frac{}{\left(bx - \right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*ArcTanh[Tanh[a + b*x]]^3),x]
```



```
[Out] (-3*b^2)/((b*x - ArcTanh[Tanh[a + b*x]])^3*ArcTanh[Tanh[a + b*x]]^2) + (2*b
)/(x*(b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^2) + 1/(2*x^2*
(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^2) + (6*b^2)/((b*x -
ArcTanh[Tanh[a + b*x]])^4*ArcTanh[Tanh[a + b*x]]) - (6*b^2*Log[x])/(b*x - A
rcTanh[Tanh[a + b*x]])^5 + (6*b^2*Log[ArcTanh[Tanh[a + b*x]]])/(b*x - ArcTa
nh[Tanh[a + b*x]])^5
```


Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))^3}\,dx = \frac{1}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2} + \frac{(2b)\int \frac{1}{x^2\tanh^{-1}(\tanh(a+bx}{bx-\tanh^{-1}(\tanh(a+}$$

$$= \frac{2b}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{2x^2\left(bx-\tanh^{-1}(\tanh(\right.}$$

$$= -\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

$$= -\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

$$= -\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

$$= -\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

$$= -\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

Mathematica [A] time = 0.0484153, size = 107, normalized size = 0.63

$$\frac{8b^3x^3\tanh^{-1}(\tanh(a+bx)) - 12b^2x^2\tanh^{-1}(\tanh(a+bx))^2\left(\log(x)-\log\left(\tanh^{-1}(\tanh(a+bx))\right)\right) - 8bx\tanh^{-1}(\tan}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^5\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] (-(b^4*x^4) + 8*b^3*x^3*ArcTanh[Tanh[a + b*x]] - 8*b*x*ArcTanh[Tanh[a + b*x
]]^3 + ArcTanh[Tanh[a + b*x]]^4 - 12*b^2*x^2*ArcTanh[Tanh[a + b*x]]^2*(Log[
x] - Log[ArcTanh[Tanh[a + b*x]]]))/(2*x^2*(b*x - ArcTanh[Tanh[a + b*x]])^5*
ArcTanh[Tanh[a + b*x]]^2)
```

Maple [A] time = 0.087, size = 145, normalized size = 0.9

$$-\frac{1}{2\left(\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{3}x^{2}}+6\frac{b^{2}\ln\left(x\right)}{\left(\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{5}}+3\frac{b}{\left(\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{4}x}-6\frac{b^{2}}{(\mathrm{A}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/arctanh(tanh(b*x+a))^3,x)
```

```
[Out] -1/2/(arctanh(tanh(b*x+a))-b*x)^3/x^2+6/(arctanh(tanh(b*x+a))-b*x)^5*b^2*ln
(x)+3/(arctanh(tanh(b*x+a))-b*x)^4*b/x-6/(arctanh(tanh(b*x+a))-b*x)^5*b^2*l
n(arctanh(tanh(b*x+a)))+3/(arctanh(tanh(b*x+a))-b*x)^4*b^2/arctanh(tanh(b*x
+a))+1/2/(arctanh(tanh(b*x+a))-b*x)^3*b^2/arctanh(tanh(b*x+a))^2
```

Maxima [A] time = 3.51906, size = 116, normalized size = 0.68

$$\frac{12\,b^{3}x^{3}+18\,ab^{2}x^{2}+4\,a^{2}bx-a^{3}}{2\left(a^{4}b^{2}x^{4}+2\,a^{5}bx^{3}+a^{6}x^{2}\right)}-\frac{6\,b^{2}\log\left(bx+a\right)}{a^{5}}+\frac{6\,b^{2}\log\left(x\right)}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(12*b^3*x^3 + 18*a*b^2*x^2 + 4*a^2*b*x - a^3)/(a^4*b^2*x^4 + 2*a^5*b*x^
3 + a^6*x^2) - 6*b^2*log(b*x + a)/a^5 + 6*b^2*log(x)/a^5
```

Fricas [A] time = 1.55764, size = 269, normalized size = 1.58

$$\frac{12\,ab^{3}x^{3}+18\,a^{2}b^{2}x^{2}+4\,a^{3}bx-a^{4}-12\left(b^{4}x^{4}+2\,ab^{3}x^{3}+a^{2}b^{2}x^{2}\right)\log\left(bx+a\right)+12\left(b^{4}x^{4}+2\,ab^{3}x^{3}+a^{2}b^{2}x^{2}\right)\log\left(x\right)}{2\left(a^{5}b^{2}x^{4}+2\,a^{6}bx^{3}+a^{7}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(12*a*b^3*x^3 + 18*a^2*b^2*x^2 + 4*a^3*b*x - a^4 - 12*(b^4*x^4 + 2*a*b^
3*x^3 + a^2*b^2*x^2)*log(b*x + a) + 12*(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x^2
)*log(x))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \operatorname{atanh}^3(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(1/(x**3*atanh(tanh(a + b*x))**3), x)
```

Giac [A] time = 1.13097, size = 99, normalized size = 0.58

$$-\frac{6\,b^2\log(|bx+a|)}{a^5} + \frac{6\,b^2\log(|x|)}{a^5} + \frac{12\,b^3x^3 + 18\,ab^2x^2 + 4\,a^2bx - a^3}{2\left(bx^2+ax\right)^2 a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] -6*b^2*log(abs(b*x + a))/a^5 + 6*b^2*log(abs(x))/a^5 + 1/2*(12*b^3*x^3 + 18
*a*b^2*x^2 + 4*a^2*b*x - a^3)/((b*x^2 + a*x)^2*a^4)
```

3.111 $\int x^4 \sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=101

$$\frac{32x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^3}-\frac{16x^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2}+\frac{256\tanh^{-1}(\tanh(a+bx))^{11/2}}{3465b^5}-\frac{128x\tanh^{-1}(\tanh(a+}{315b^4}$$

```
[Out] (2*x^4*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b) - (16*x^3*ArcTanh[Tanh[a + b*x]]
^(5/2))/(15*b^2) + (32*x^2*ArcTanh[Tanh[a + b*x]]^(7/2))/(35*b^3) - (128*x*
ArcTanh[Tanh[a + b*x]]^(9/2))/(315*b^4) + (256*ArcTanh[Tanh[a + b*x]]^(11/2
))/(3465*b^5)
```

Rubi [A] time = 0.0645386, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2157, 30}

$$\frac{32x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^3}-\frac{16x^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2}+\frac{256\tanh^{-1}(\tanh(a+bx))^{11/2}}{3465b^5}-\frac{128x\tanh^{-1}(\tanh(a+}{315b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*x^4*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b) - (16*x^3*ArcTanh[Tanh[a + b*x]]
^(5/2))/(15*b^2) + (32*x^2*ArcTanh[Tanh[a + b*x]]^(7/2))/(35*b^3) - (128*x*
ArcTanh[Tanh[a + b*x]]^(9/2))/(315*b^4) + (256*ArcTanh[Tanh[a + b*x]]^(11/2
))/(3465*b^5)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^4\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx &= \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{8\int x^3\tanh^{-1}(\tanh(a+bx))^{3/2}\,dx}{3b}\\
&= \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2} + \frac{16\int x^2\tanh^{-1}(\tan}{5b^{}}\\
&= \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2} + \frac{32x^2\tanh^{-1}(\tanh}{35b^3}\\
&= \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2} + \frac{32x^2\tanh^{-1}(\tanh}{35b^3}\\
&= \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2} + \frac{32x^2\tanh^{-1}(\tanh}{35b^3}\\
&= \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2} + \frac{32x^2\tanh^{-1}(\tanh}{35b^3}\end{aligned}$$

Mathematica [A] time = 0.0319772, size = 83, normalized size = 0.82

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}\left(-1848b^3x^3\tanh^{-1}(\tanh(a+bx))+1584b^2x^2\tanh^{-1}(\tanh(a+bx))^2-704bx\tanh^{-1}(\tanh}{3465b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(3/2)*(1155*b^4*x^4 - 1848*b^3*x^3*ArcTanh[Tanh[a
 + b*x]] + 1584*b^2*x^2*ArcTanh[Tanh[a + b*x]]^2 - 704*b*x*ArcTanh[Tanh[a +
 b*x]]^3 + 128*ArcTanh[Tanh[a + b*x]]^4))/(3465*b^5)
```

Maple [A] time = 0.066, size = 154, normalized size = 1.5

$$2\,\frac{1/11\,(\mathrm{Artanh}(\tanh(bx+a)))^{11/2}+1/9\,(-4\,\mathrm{Artanh}(\tanh(bx+a))+4\,bx)(\mathrm{Artanh}(\tanh(bx+a)))^{9/2}+1/7\,\left(2\,(bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 2/b^5*(1/11*arctanh(tanh(b*x+a))^(11/2)+1/9*(-4*arctanh(tanh(b*x+a))+4*b*x)
*arctanh(tanh(b*x+a))^(9/2)+1/7*(2*(b*x-arctanh(tanh(b*x+a)))^2+(-2*arctanh
(tanh(b*x+a))+2*b*x)^2)*arctanh(tanh(b*x+a))^(7/2)+2/5*(b*x-arctanh(tanh(b*
x+a)))^2*(-2*arctanh(tanh(b*x+a))+2*b*x)*arctanh(tanh(b*x+a))^(5/2)+1/3*(b*
x-arctanh(tanh(b*x+a)))^4*arctanh(tanh(b*x+a))^(3/2))
```

Maxima [A] time = 1.79295, size = 86, normalized size = 0.85

$$\frac{2\left(315\,b^5x^5+35\,ab^4x^4-40\,a^2b^3x^3+48\,a^3b^2x^2-64\,a^4bx+128\,a^5\right)\sqrt{bx+a}}{3465\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3465*(315*b^5*x^5 + 35*a*b^4*x^4 - 40*a^2*b^3*x^3 + 48*a^3*b^2*x^2 - 64*a
^4*b*x + 128*a^5)*sqrt(b*x + a)/b^5
```

Fricas [A] time = 1.49619, size = 151, normalized size = 1.5

$$\frac{2\left(315\,b^5x^5+35\,ab^4x^4-40\,a^2b^3x^3+48\,a^3b^2x^2-64\,a^4bx+128\,a^5\right)\sqrt{bx+a}}{3465\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*b^5*x^5 + 35*a*b^4*x^4 - 40*a^2*b^3*x^3 + 48*a^3*b^2*x^2 - 64*a
^4*b*x + 128*a^5)*sqrt(b*x + a)/b^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(atanh(tanh(a + b*x))), x)
```

Giac [A] time = 1.17788, size = 82, normalized size = 0.81

$$\frac{2\left(315\,(bx+a)^{\frac{11}{2}}-1540\,(bx+a)^{\frac{9}{2}}a+2970\,(bx+a)^{\frac{7}{2}}a^{2}-2772\,(bx+a)^{\frac{5}{2}}a^{3}+1155\,(bx+a)^{\frac{3}{2}}a^{4}\right)}{3465\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3465*(315*(b*x + a)^(11/2) - 1540*(b*x + a)^(9/2)*a + 2970*(b*x + a)^(7/2
)*a^2 - 2772*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4)/b^5
```

3.112 $\int x^3 \sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=80

$$-\frac{4x^2 \tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^2} - \frac{32 \tanh^{-1}(\tanh(a+bx))^{9/2}}{315b^4} + \frac{16x \tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^3} + \frac{2x^3 \tanh^{-1}(\tanh(a+bx}{3b}$$

```
[Out] (2*x^3*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b) - (4*x^2*ArcTanh[Tanh[a + b*x]]^
(5/2))/(5*b^2) + (16*x*ArcTanh[Tanh[a + b*x]]^(7/2))/(35*b^3) - (32*ArcTanh
[Tanh[a + b*x]]^(9/2))/(315*b^4)
```

Rubi [A] time = 0.0487165, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2157, 30}

$$-\frac{4x^2 \tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^2} - \frac{32 \tanh^{-1}(\tanh(a+bx))^{9/2}}{315b^4} + \frac{16x \tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^3} + \frac{2x^3 \tanh^{-1}(\tanh(a+bx}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*x^3*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b) - (4*x^2*ArcTanh[Tanh[a + b*x]]^
(5/2))/(5*b^2) + (16*x*ArcTanh[Tanh[a + b*x]]^(7/2))/(35*b^3) - (32*ArcTanh
[Tanh[a + b*x]]^(9/2))/(315*b^4)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{2\int x^2\tanh^{-1}(\tanh(a+bx))^{3/2}\,dx}{b}$$
$$= \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{4x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^2} + \frac{8\int x\tanh^{-1}(\tanh(}{5b^2}$$
$$= \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{4x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^2} + \frac{16x\tanh^{-1}(\tanh(a}{35b^3}$$
$$= \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{4x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^2} + \frac{16x\tanh^{-1}(\tanh(a}{35b^3}$$
$$= \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{4x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^2} + \frac{16x\tanh^{-1}(\tanh(a}{35b^3}$$

Mathematica [A] time = 0.0322845, size = 66, normalized size = 0.82

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}\left(-126b^2x^2\tanh^{-1}(\tanh(a+bx))+72bx\tanh^{-1}(\tanh(a+bx))^2-16\tanh^{-1}(\tanh(a+bx))\right.}{315b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(3/2)*(105*b^3*x^3 - 126*b^2*x^2*ArcTanh[Tanh[a +
 b*x]] + 72*b*x*ArcTanh[Tanh[a + b*x]]^2 - 16*ArcTanh[Tanh[a + b*x]]^3))/(3
15*b^4)
```

Maple [A] time = 0.047, size = 124, normalized size = 1.6

$$2\,\frac{1/9\,(\mathrm{Artanh}\,(\tanh\,(bx+a)))^{9/2}+1/7\,(-3\,\mathrm{Artanh}\,(\tanh\,(bx+a))+3\,bx)\,(\mathrm{Artanh}\,(\tanh\,(bx+a)))^{7/2}+1/5\,\left((bx-\mathrm{A}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 2/b^4*(1/9*arctanh(tanh(b*x+a))^(9/2)+1/7*(-3*arctanh(tanh(b*x+a))+3*b*x)*a
rctanh(tanh(b*x+a))^(7/2)+1/5*((b*x-arctanh(tanh(b*x+a)))*(-2*arctanh(tanh(
b*x+a))+2*b*x)+(b*x-arctanh(tanh(b*x+a)))^2)*arctanh(tanh(b*x+a))^(5/2)+1/3
*(b*x-arctanh(tanh(b*x+a)))^3*arctanh(tanh(b*x+a))^(3/2))
```

Maxima [A] time = 1.80073, size = 72, normalized size = 0.9

$$\frac{2\left(35\,b^4x^4+5\,ab^3x^3-6\,a^2b^2x^2+8\,a^3bx-16\,a^4\right)\sqrt{bx+a}}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqrt(
b*x + a)/b^4
```

Fricas [A] time = 1.52478, size = 120, normalized size = 1.5

$$\frac{2\left(35\,b^4x^4+5\,ab^3x^3-6\,a^2b^2x^2+8\,a^3bx-16\,a^4\right)\sqrt{bx+a}}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqrt(
b*x + a)/b^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(atanh(tanh(a + b*x))), x)
```

Giac [A] time = 1.18603, size = 66, normalized size = 0.82

$$\frac{2\left(35\,(bx+a)^{\frac{9}{2}}-135\,(bx+a)^{\frac{7}{2}}a+189\,(bx+a)^{\frac{5}{2}}a^{2}-105\,(bx+a)^{\frac{3}{2}}a^{3}\right)}{315\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(35*(b*x + a)^(9/2) - 135*(b*x + a)^(7/2)*a + 189*(b*x + a)^(5/2)*a^2
 - 105*(b*x + a)^(3/2)*a^3)/b^4
```

3.113 $\int x^2 \sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=59

$$\frac{16\tanh^{-1}(\tanh(a+bx))^{7/2}}{105b^3}-\frac{8x\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2}+\frac{2x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}$$

```
[Out] (2*x^2*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b) - (8*x*ArcTanh[Tanh[a + b*x]]^(5
/2))/(15*b^2) + (16*ArcTanh[Tanh[a + b*x]]^(7/2))/(105*b^3)
```

Rubi [A] time = 0.0297581, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2157, 30}

$$\frac{16\tanh^{-1}(\tanh(a+bx))^{7/2}}{105b^3}-\frac{8x\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2}+\frac{2x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*x^2*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b) - (8*x*ArcTanh[Tanh[a + b*x]]^(5
/2))/(15*b^2) + (16*ArcTanh[Tanh[a + b*x]]^(7/2))/(105*b^3)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{4\int x\tanh^{-1}(\tanh(a+bx))^{3/2}\,dx}{3b}$$
$$= \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{8x\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2} + \frac{8\int \tanh^{-1}(\tanh(a+}{15b^2}$$
$$= \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{8x\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2} + \frac{8\,\text{Subst}\left(\int x^{5/2}\,dx, x\right.}{1}$$
$$= \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{8x\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2} + \frac{16\tanh^{-1}(\tanh(a+}{105b^3}$$

Mathematica [A] time = 0.0274241, size = 49, normalized size = 0.83

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}\left(-28bx\tanh^{-1}(\tanh(a+bx))+8\tanh^{-1}(\tanh(a+bx))^2+35b^2x^2\right)}{105b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(3/2)*(35*b^2*x^2 - 28*b*x*ArcTanh[Tanh[a + b*x]]
 + 8*ArcTanh[Tanh[a + b*x]]^2))/(105*b^3)
```

Maple [A] time = 0.046, size = 69, normalized size = 1.2

$$2\,\frac{1/7\,(\text{Artanh}(\tanh(bx+a)))^{7/2}+1/5\,(-2\,\text{Artanh}(\tanh(bx+a))+2\,bx)(\text{Artanh}(\tanh(bx+a)))^{5/2}+1/3\,(bx-\text{A}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 2/b^3*(1/7*arctanh(tanh(b*x+a))^(7/2)+1/5*(-2*arctanh(tanh(b*x+a))+2*b*x)*a
rctanh(tanh(b*x+a))^(5/2)+1/3*(b*x-arctanh(tanh(b*x+a)))^2*arctanh(tanh(b*x
```

```
+a))^(3/2))
```

Maxima [A] time = 1.79401, size = 57, normalized size = 0.97

$$\frac{2\left(15\,b^{3}x^{3}+3\,ab^{2}x^{2}-4\,a^{2}bx+8\,a^{3}\right)\sqrt{bx+a}}{105\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a)/b^3
```

Fricas [A] time = 1.542, size = 97, normalized size = 1.64

$$\frac{2\left(15\,b^{3}x^{3}+3\,ab^{2}x^{2}-4\,a^{2}bx+8\,a^{3}\right)\sqrt{bx+a}}{105\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a)/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{2}\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(atanh(tanh(a + b*x))), x)
```

Giac [A] time = 1.17487, size = 50, normalized size = 0.85

$$\frac{2\left(15\,(bx+a)^{\frac{7}{2}}-42\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^{2}\right)}{105\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(15*(b*x + a)^(7/2) - 42*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2)/
b^3
```

3.114 $\int x\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=38

$$\frac{2x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}-\frac{4\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2}$$

```
[Out] (2*x*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b) - (4*ArcTanh[Tanh[a + b*x]]^(5/2))
/(15*b^2)
```

Rubi [A] time = 0.0140504, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2157, 30}

$$\frac{2x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}-\frac{4\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*x*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b) - (4*ArcTanh[Tanh[a + b*x]]^(5/2))
/(15*b^2)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx &= \frac{2x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{2\int \tanh^{-1}(\tanh(a+bx))^{3/2}\,dx}{3b}\\ &= \frac{2x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{2\,\text{Subst}\left(\int x^{3/2}\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{3b^2}\\ &= \frac{2x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{4\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2}\end{aligned}$$

Mathematica [A] time = 0.0491836, size = 32, normalized size = 0.84

$$\frac{2\left(5bx-2\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*(5*b*x - 2*ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2))/(15*b^2
)
```

Maple [A] time = 0.046, size = 42, normalized size = 1.1

$$2\,\frac{1/5\,\left(\text{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{5/2}+1/3\,\left(bx-\text{Artanh}\left(\tanh\left(bx+a\right)\right)\right)\left(\text{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{3/2}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 2/b^2*(1/5*arctanh(tanh(b*x+a))^(5/2)+1/3*(b*x-arctanh(tanh(b*x+a)))*arctan
h(tanh(b*x+a))^(3/2))
```

Maxima [A] time = 1.76433, size = 41, normalized size = 1.08

$$\frac{2\left(3\,b^2x^2+abx-2\,a^2\right)\sqrt{bx+a}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)/b^2
```

Fricas [A] time = 1.52821, size = 70, normalized size = 1.84

$$\frac{2\left(3\,b^2x^2+abx-2\,a^2\right)\sqrt{bx+a}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(x*sqrt(atanh(tanh(a + b*x))), x)
```

Giac [A] time = 1.12343, size = 34, normalized size = 0.89

$$\frac{2\left(3\,(bx+a)^{\frac{5}{2}}-5\,(bx+a)^{\frac{3}{2}}a\right)}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*(b*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a)/b^2
```

3.115 $\int \sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=18

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}$$

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b)
```

Rubi [A] time = 0.0047037, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2157, 30}

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b)
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx &= \frac{\text{Subst}\left(\int \sqrt{x}\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}\end{aligned}$$

Mathematica [A] time = 0.0064963, size = 18, normalized size = 1.

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b)
```

Maple [A] time = 0.029, size = 15, normalized size = 0.8

$$\frac{2}{3\,b}\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 2/3*arctanh(tanh(b*x+a))^(3/2)/b
```

Maxima [A] time = 1.6853, size = 16, normalized size = 0.89

$$\frac{2\left(bx+a\right)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b
```

Fricas [A] time = 1.50621, size = 31, normalized size = 1.72

$$\frac{2\left(bx+a\right)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b
```

Sympy [A] time = 0.754845, size = 26, normalized size = 1.44

$$\begin{cases} \frac{2 \operatorname{atanh}^{\frac{3}{2}}{\left(\tanh{\left(a+bx \right)} \right)}}{3b} & \text{for}\: b \neq 0 \\ x \sqrt{\operatorname{atanh}{\left(\tanh{\left(a \right)} \right)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Piecewise((2*atanh(tanh(a + b*x))**(3/2)/(3*b), Ne(b, 0)), (x*sqrt(atanh(ta
nh(a))), True))
```

Giac [A] time = 1.15123, size = 24, normalized size = 1.33

$$\frac{\sqrt{2}(2\,bx + 2\,a)^{\frac{3}{2}}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(2*b*x + 2*a)^(3/2)/b
```

3.116 $\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x} dx$

Optimal. Leaf size=63

$$2\sqrt{\tanh^{-1}(\tanh(a+bx))}-2\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)$$

```
[Out] -2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*
Sqrt[b*x - ArcTanh[Tanh[a + b*x]]] + 2*Sqrt[ArcTanh[Tanh[a + b*x]]]
```

Rubi [A] time = 0.0595282, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2159, 2161}

$$2\sqrt{\tanh^{-1}(\tanh(a+bx))}-2\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x,x]
```

```
[Out] -2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*
Sqrt[b*x - ArcTanh[Tanh[a + b*x]]] + 2*Sqrt[ArcTanh[Tanh[a + b*x]]]
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x}\,dx = 2\sqrt{\tanh^{-1}(\tanh(a+bx))} - \left(bx-\tanh^{-1}(\tanh(a+bx))\right)\int \frac{1}{x\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

$$= -2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}+2\sqrt{\tanh^{-1}(\text{t}}$$

Mathematica [A] time = 0.0733578, size = 61, normalized size = 0.97

$$2\sqrt{\tanh^{-1}(\tanh(a+bx))}-2\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x,x]
```

```
[Out] 2*Sqrt[ArcTanh[Tanh[a + b*x]]] - 2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqr
t[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]
```

Maple [A] time = 0.128, size = 54, normalized size = 0.9

$$2\sqrt{\text{Artanh}(\tanh(bx+a))}-2\sqrt{\text{Artanh}(\tanh(bx+a))-bx}\,\text{Artanh}\left(\frac{\sqrt{\text{Artanh}(\tanh(bx+a))}}{\sqrt{\text{Artanh}(\tanh(bx+a))-bx}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2)/x,x)
```

```
[Out] 2*arctanh(tanh(b*x+a))^(1/2)-2*(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arc
tanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\text{artanh}(\tanh(bx+a))}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arctanh(tanh(b*x + a)))/x, x)
```

Fricas [A] time = 1.55731, size = 188, normalized size = 2.98

$$\left[\sqrt{a}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a}, 2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+2\sqrt{bx+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x + a), 2*
sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + 2*sqrt(b*x + a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\operatorname{atanh}(\tanh(a+bx))}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(atanh(tanh(a + b*x)))/x, x)
```

Giac [A] time = 1.15686, size = 43, normalized size = 0.68

$$\frac{2a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*x + a)
```

3.117 $\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^2}\,dx$

Optimal. Leaf size=66

$$\frac{b\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x}$$

```
[Out] (b*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])
/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]] - Sqrt[ArcTanh[Tanh[a + b*x]]]/x
```

Rubi [A] time = 0.0315191, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 2161}

$$\frac{b\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^2,x]
```

```
[Out] (b*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])
/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]] - Sqrt[ArcTanh[Tanh[a + b*x]]]/x
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^2}\,dx = -\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x} + \frac{1}{2}b\int \frac{1}{x\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$$
$$= \frac{b\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}} - \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x}$$

Mathematica [A] time = 0.0371004, size = 65, normalized size = 0.98

$$-\frac{b\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}} - \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^2,x]
```

```
[Out] -(Sqrt[ArcTanh[Tanh[a + b*x]]]/x) - (b*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]
/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x
]]]
```

Maple [A] time = 0.126, size = 63, normalized size = 1.

$$2\,b\left(-1/2\,\frac{\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}}{bx}-1/2\,\frac{1}{\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx}}\mathrm{Artanh}\left(\frac{\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}}{\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2)/x^2,x)
```

```
[Out] 2*b*(-1/2*arctanh(tanh(b*x+a))^(1/2)/b/x-1/2/(arctanh(tanh(b*x+a))-b*x)^(1/
2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arctanh(tanh(b*x + a)))/x^2, x)
```

Fricas [A] time = 1.77652, size = 225, normalized size = 3.41

$$\left[\frac{\sqrt{a}bx\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}a}{2ax}, \frac{\sqrt{-a}bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-\sqrt{bx+a}a}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x
 + a)*a)/(a*x), (sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(b*x +
 a)*a)/(a*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(atanh(tanh(a + b*x)))/x**2, x)
```

Giac [A] time = 1.18025, size = 55, normalized size = 0.83

$$\frac{\frac{b^2 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\sqrt{bx+a}b}{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] (b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - sqrt(b*x + a)*b/x)/b
```

3.118 $\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^3} dx$

Optimal. Leaf size=125

$$\frac{b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}-\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{2x^2}$$

```
[Out] (b^2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]
])/(4*(b*x - ArcTanh[Tanh[a + b*x]])^(3/2)) - b/(4*x*Sqrt[ArcTanh[Tanh[a +
b*x]]]) + b^2/(4*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]
]) - Sqrt[ArcTanh[Tanh[a + b*x]]]/(2*x^2)
```

Rubi [A] time = 0.0716427, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2163, 2161}

$$\frac{b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}-\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^3,x]
```

```
[Out] (b^2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]
])/(4*(b*x - ArcTanh[Tanh[a + b*x]])^(3/2)) - b/(4*x*Sqrt[ArcTanh[Tanh[a +
b*x]]]) + b^2/(4*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]
]) - Sqrt[ArcTanh[Tanh[a + b*x]]]/(2*x^2)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
```

```
[m, 0] &&  !IntegerQ[n]))
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^3}\,dx = -\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{2x^2}+\frac{1}{4}b\int \frac{1}{x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$$

$$= -\frac{b}{4x\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{2x^2}-\frac{1}{8}b^2\int \frac{1}{x\tanh^{-1}(\tanh(a+ \cdots}$$

$$= -\frac{b}{4x\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx)} \cdots}$$

$$= \frac{b^2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}-\frac{b}{4x\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{\cdots}{4\left(bx-\tanh^{-1}(\tan \cdots\right.}$$

Mathematica [A] time = 0.0970263, size = 89, normalized size = 0.71

$$\frac{1}{4}\left(\frac{b^2\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{3/2}}+\frac{\left(\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}-2\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^3,x]
```

```
[Out] (((-2 + (b*x)/(b*x - ArcTanh[Tanh[a + b*x]]))*Sqrt[ArcTanh[Tanh[a + b*x]]])
/x^2 + (b^2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh
[a + b*x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2))/4
```

Maple [A] time = 0.127, size = 92, normalized size = 0.7

$$2\,b^{2}\left(\frac{1}{b^{2}x^{2}}\left(-1/8\,\frac{\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{3/2}}{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx}-1/8\,\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}\right)+1/8\,\frac{1}{\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{3/2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2)/x^3,x)
```

```
[Out] 2*b^2*((-1/8/(arctanh(tanh(b*x+a))-b*x)*arctanh(tanh(b*x+a))^(3/2)-1/8*arct
anh(tanh(b*x+a))^(1/2))/b^2/x^2+1/8/(arctanh(tanh(b*x+a))-b*x)^(3/2)*arctan
h(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))
```

Maramaxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arctanh(tanh(b*x + a)))/x^3, x)
```

Fricas [A] time = 1.72048, size = 292, normalized size = 2.34

$$\left[\frac{\sqrt{a}b^{2}x^{2}\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)-2\left(abx+2\,a^{2}\right)\sqrt{bx+a}}{8\,a^{2}x^{2}},-\frac{\sqrt{-a}b^{2}x^{2}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(abx+2\,a^{2}\right)\sqrt{bx+a}}{4\,a^{2}x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b^2*x^2*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(a*b
*x + 2*a^2)*sqrt(b*x + a))/(a^2*x^2), -1/4*(sqrt(-a)*b^2*x^2*arctan(sqrt(b*
x + a)*sqrt(-a)/a) + (a*b*x + 2*a^2)*sqrt(b*x + a))/(a^2*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(atanh(tanh(a + b*x)))/x**3, x)
```

Giac [A] time = 1.16984, size = 89, normalized size = 0.71

$$-\frac{\frac{b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{(bx+a)^{\frac{3}{2}}b^3+\sqrt{bx+a}ab^3}{ab^2x^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + ((b*x + a)^(3/2)*b^
3 + sqrt(b*x + a)*a*b^3)/(a*b^2*x^2))/b
```

3.119 $\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^4} dx$

Optimal. Leaf size=179

$$\frac{b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}+$$

```
[Out] (b^3*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]
])/(8*(b*x - ArcTanh[Tanh[a + b*x]])^(5/2)) + b^2/(24*x*ArcTanh[Tanh[a + b*
x]]^(3/2)) - b^3/(24*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^
(3/2)) - b/(12*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]]) + b^3/(8*(b*x - ArcTanh[Ta
nh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b*x]]]) - Sqrt[ArcTanh[Tanh[a + b*x]]
]/(3*x^3)
```

Rubi [A] time = 0.120568, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2163, 2161}

$$\frac{b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^4,x]
```

```
[Out] (b^3*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]
])/(8*(b*x - ArcTanh[Tanh[a + b*x]])^(5/2)) + b^2/(24*x*ArcTanh[Tanh[a + b*
x]]^(3/2)) - b^3/(24*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^
(3/2)) - b/(12*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]]) + b^3/(8*(b*x - ArcTanh[Ta
nh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b*x]]]) - Sqrt[ArcTanh[Tanh[a + b*x]]
]/(3*x^3)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
```

```
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^4}\,dx = -\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3x^3} + \frac{1}{6}b\int \frac{1}{x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$$

$$= -\frac{b}{12x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3x^3} - \frac{1}{24}b^2\int \frac{1}{x^2\tanh^{-1}(\tanh}$$

$$= \frac{b^2}{24x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{b}{12x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{\sqrt{\tanh^{-1}(\tanh(a+b}}{3x^3}$$

$$= \frac{b^2}{24x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx}$$

$$= \frac{b^2}{24x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx}$$

$$= \frac{b^3\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}} + \frac{b^2}{24x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{}{24\left(bx-\tanh^{-1}\right.}$$

<u>Mathematica</u> [A] time = 0.0981389, size = 115, normalized size = 0.64

$$\frac{1}{24}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(14bx\tanh^{-1}(\tanh(a+bx)) - 8\tanh^{-1}(\tanh(a+bx))^2 - 3b^2x^2\right)}{x^3\left(\tanh^{-1}(\tanh(a+bx)) - bx\right)^2} - \frac{3b^3\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}}}{\sqrt{\tanh^{-1}(}}\right.}{\left(\tanh^{-1}(\tanh(a+b\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^4,x]
```

```
[Out] ((-3*b^3*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a
+ b*x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2) + (Sqrt[ArcTanh[Tanh[a +
 b*x]]]*(-3*b^2*x^2 + 14*b*x*ArcTanh[Tanh[a + b*x]] - 8*ArcTanh[Tanh[a + b*
x]]^2))/(x^3*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2))/24
```

Maple [A] time = 0.129, size = 185, normalized size = 1.

$$2\,b^3\left(\frac{1}{x^3b^3}\left(1/16\,\frac{(\mathrm{Artanh}\,(\tanh\,(bx+a)))^{5/2}}{a^2+2\,a\,(\mathrm{Artanh}\,(\tanh\,(bx+a))-bx-a)+(\mathrm{Artanh}\,(\tanh\,(bx+a))-bx-a)^2}-1/6\,\frac{(\mathrm{Artanh}\,(\tanh}{\mathrm{Artanh}\,(\tanh\,(b}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2)/x^4,x)
```

```
[Out] 2*b^3*((1/16/(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*
x-a)^2)*arctanh(tanh(b*x+a))^(5/2)-1/6/(arctanh(tanh(b*x+a))-b*x)*arctanh(t
anh(b*x+a))^(3/2)-1/16*arctanh(tanh(b*x+a))^(1/2))/b^3/x^3-1/16/(a^2+2*a*(a
rctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)/(arctanh(tanh(b*
x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b
*x)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\mathrm{artanh}\,(\tanh\,(bx+a))}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arctanh(tanh(b*x + a)))/x^4, x)
```

Fricas [A] time = 1.85557, size = 347, normalized size = 1.94

$$\left[\frac{3\,\sqrt{a}b^3x^3\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\left(3\,ab^2x^2-2\,a^2bx-8\,a^3\right)\sqrt{bx+a}}{48\,a^3x^3},\frac{3\,\sqrt{-a}b^3x^3\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(3\,ab^2x^2-2\,a^2bx\right.}{24\,a^3x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(a)*b^3*x^3*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(
3*a*b^2*x^2 - 2*a^2*b*x - 8*a^3)*sqrt(b*x + a))/(a^3*x^3), 1/24*(3*sqrt(-a)
```

```
*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (3*a*b^2*x^2 - 2*a^2*b*x - 8*a^
3)*sqrt(b*x + a))/(a^3*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(atanh(tanh(a + b*x)))/x**4, x)
```

Giac [A] time = 1.16832, size = 113, normalized size = 0.63

$$\frac{\frac{3\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{3\,(bx+a)^{\frac{5}{2}}b^4-8\,(bx+a)^{\frac{3}{2}}ab^4-3\,\sqrt{bx+a}a^2b^4}{a^2b^3x^3}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/24*(3*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x + a)^(5
/2)*b^4 - 8*(b*x + a)^(3/2)*a*b^4 - 3*sqrt(b*x + a)*a^2*b^4)/(a^2*b^3*x^3))
/b
```

3.120 $\int x^4 \tanh^{-1}(\tanh(a+bx))^{3/2}\,dx$

Optimal. Leaf size=101

$$\frac{32x^2\tanh^{-1}(\tanh(a+bx))^{9/2}}{105b^3}-\frac{16x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2}+\frac{256\tanh^{-1}(\tanh(a+bx))^{13/2}}{15015b^5}-\frac{128x\tanh^{-1}(\tanh(a}{1155b^4}$$

```
[Out] (2*x^4*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b) - (16*x^3*ArcTanh[Tanh[a + b*x]]
^(7/2))/(35*b^2) + (32*x^2*ArcTanh[Tanh[a + b*x]]^(9/2))/(105*b^3) - (128*x
*ArcTanh[Tanh[a + b*x]]^(11/2))/(1155*b^4) + (256*ArcTanh[Tanh[a + b*x]]^(1
3/2))/(15015*b^5)
```

Rubi [A] time = 0.0672489, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2157, 30}

$$\frac{32x^2\tanh^{-1}(\tanh(a+bx))^{9/2}}{105b^3}-\frac{16x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2}+\frac{256\tanh^{-1}(\tanh(a+bx))^{13/2}}{15015b^5}-\frac{128x\tanh^{-1}(\tanh(a}{1155b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*x^4*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b) - (16*x^3*ArcTanh[Tanh[a + b*x]]
^(7/2))/(35*b^2) + (32*x^2*ArcTanh[Tanh[a + b*x]]^(9/2))/(105*b^3) - (128*x
*ArcTanh[Tanh[a + b*x]]^(11/2))/(1155*b^4) + (256*ArcTanh[Tanh[a + b*x]]^(1
3/2))/(15015*b^5)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^4 \tanh^{-1}(\tanh(a+bx))^{3/2}\,dx = \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{8\int x^3\tanh^{-1}(\tanh(a+bx))^{5/2}\,dx}{5b}$$
$$= \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{48\int x^2\tanh^{-1}(\mathrm{ta}}{35}$$
$$= \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{32x^2\tanh^{-1}(\mathrm{tanh}}{105b^3}$$
$$= \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{32x^2\tanh^{-1}(\mathrm{tanh}}{105b^3}$$
$$= \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{32x^2\tanh^{-1}(\mathrm{tanh}}{105b^3}$$
$$= \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{32x^2\tanh^{-1}(\mathrm{tanh}}{105b^3}$$

Mathematica [A] time = 0.0371079, size = 83, normalized size = 0.82

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}\left(-3432b^3x^3\tanh^{-1}(\tanh(a+bx)) + 2288b^2x^2\tanh^{-1}(\tanh(a+bx))^2 - 832bx\tanh^{-1}(\mathrm{tanh}\right.}{15015b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(5/2)*(3003*b^4*x^4 - 3432*b^3*x^3*ArcTanh[Tanh[a
 + b*x]] + 2288*b^2*x^2*ArcTanh[Tanh[a + b*x]]^2 - 832*b*x*ArcTanh[Tanh[a +
 b*x]]^3 + 128*ArcTanh[Tanh[a + b*x]]^4))/(15015*b^5)
```

Maple [A] time = 0.033, size = 154, normalized size = 1.5

$$2\,\frac{1/13\,(\mathrm{Artanh}\,(\tanh\,(bx+a)))^{13/2} + 1/11\,(-4\,\mathrm{Artanh}\,(\tanh\,(bx+a)) + 4\,bx)\,(\mathrm{Artanh}\,(\tanh\,(bx+a)))^{11/2} + 1/9\,\left(2\,(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 2/b^5*(1/13*arctanh(tanh(b*x+a))^(13/2)+1/11*(-4*arctanh(tanh(b*x+a))+4*b*x
)*arctanh(tanh(b*x+a))^(11/2)+1/9*(2*(b*x-arctanh(tanh(b*x+a)))^2+(-2*arcta
nh(tanh(b*x+a))+2*b*x)^2)*arctanh(tanh(b*x+a))^(9/2)+2/7*(b*x-arctanh(tanh(
b*x+a)))^2*(-2*arctanh(tanh(b*x+a))+2*b*x)*arctanh(tanh(b*x+a))^(7/2)+1/5*(
b*x-arctanh(tanh(b*x+a)))^4*arctanh(tanh(b*x+a))^(5/2))
```

Maxima [A] time = 1.78044, size = 86, normalized size = 0.85

$$\frac{2\left(1155\,b^{5}x^{5}+315\,ab^{4}x^{4}-280\,a^{2}b^{3}x^{3}+240\,a^{3}b^{2}x^{2}-192\,a^{4}bx+128\,a^{5}\right)(bx+a)^{\frac{3}{2}}}{15015\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15015*(1155*b^5*x^5 + 315*a*b^4*x^4 - 280*a^2*b^3*x^3 + 240*a^3*b^2*x^2 -
 192*a^4*b*x + 128*a^5)*(b*x + a)^(3/2)/b^5
```

Fricas [A] time = 1.68228, size = 180, normalized size = 1.78

$$\frac{2\left(1155\,b^{6}x^{6}+1470\,ab^{5}x^{5}+35\,a^{2}b^{4}x^{4}-40\,a^{3}b^{3}x^{3}+48\,a^{4}b^{2}x^{2}-64\,a^{5}bx+128\,a^{6}\right)\sqrt{bx+a}}{15015\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15015*(1155*b^6*x^6 + 1470*a*b^5*x^5 + 35*a^2*b^4*x^4 - 40*a^3*b^3*x^3 +
48*a^4*b^2*x^2 - 64*a^5*b*x + 128*a^6)*sqrt(b*x + a)/b^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17114, size = 203, normalized size = 2.01

$$\frac{\sqrt{2}\left(\frac{13\sqrt{2}\left(315\,(bx+a)^{\frac{11}{2}}-1540\,(bx+a)^{\frac{9}{2}}a+2970\,(bx+a)^{\frac{7}{2}}a^2-2772\,(bx+a)^{\frac{5}{2}}a^3+1155\,(bx+a)^{\frac{3}{2}}a^4\right)a}{b^4}+\frac{5\sqrt{2}\left(693\,(bx+a)^{\frac{13}{2}}-4095\,(bx+a)^{\frac{11}{2}}a+10010\,(bx+a)^{\frac{9}{2}}a^2-\right.}{b^4}\right.}{45045\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/45045*sqrt(2)*(13*sqrt(2)*(315*(b*x + a)^(11/2) - 1540*(b*x + a)^(9/2)*a
+ 2970*(b*x + a)^(7/2)*a^2 - 2772*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2
)*a^4)*a/b^4 + 5*sqrt(2)*(693*(b*x + a)^(13/2) - 4095*(b*x + a)^(11/2)*a +
10010*(b*x + a)^(9/2)*a^2 - 12870*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2
)*a^4 - 3003*(b*x + a)^(3/2)*a^5)/b^4)/b
```

3.121 $\int x^3 \tanh^{-1}(\tanh(a+bx))^{3/2}\,dx$

Optimal. Leaf size=80

$$-\frac{12x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2}-\frac{32\tanh^{-1}(\tanh(a+bx))^{11/2}}{1155b^4}+\frac{16x\tanh^{-1}(\tanh(a+bx))^{9/2}}{105b^3}+\frac{2x^3\tanh^{-1}(\tanh(a+\ }{5b}$$

```
[Out] (2*x^3*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b) - (12*x^2*ArcTanh[Tanh[a + b*x]]
^(7/2))/(35*b^2) + (16*x*ArcTanh[Tanh[a + b*x]]^(9/2))/(105*b^3) - (32*ArcT
anh[Tanh[a + b*x]]^(11/2))/(1155*b^4)
```

Rubi [A] time = 0.0460745, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2157, 30}

$$-\frac{12x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2}-\frac{32\tanh^{-1}(\tanh(a+bx))^{11/2}}{1155b^4}+\frac{16x\tanh^{-1}(\tanh(a+bx))^{9/2}}{105b^3}+\frac{2x^3\tanh^{-1}(\tanh(a+\ }{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*x^3*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b) - (12*x^2*ArcTanh[Tanh[a + b*x]]
^(7/2))/(35*b^2) + (16*x*ArcTanh[Tanh[a + b*x]]^(9/2))/(105*b^3) - (32*ArcT
anh[Tanh[a + b*x]]^(11/2))/(1155*b^4)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^3 \tanh^{-1}(\tanh(a+bx))^{3/2}\,dx = \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{6\int x^2\tanh^{-1}(\tanh(a+bx))^{5/2}\,dx}{5b}$$
$$= \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{12x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{24\int x\tanh^{-1}(\tan}{35l}$$
$$= \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{12x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{16x\tanh^{-1}(\tanh(}{105b^3}$$
$$= \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{12x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{16x\tanh^{-1}(\tanh(}{105b^3}$$
$$= \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{12x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{16x\tanh^{-1}(\tanh(}{105b^3}$$

Mathematica [A] time = 0.0334686, size = 66, normalized size = 0.82

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}\left(-198b^2x^2\tanh^{-1}(\tanh(a+bx)) + 88bx\tanh^{-1}(\tanh(a+bx))^2 - 16\tanh^{-1}(\tanh(a+bx))\right.}{1155b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(5/2)*(231*b^3*x^3 - 198*b^2*x^2*ArcTanh[Tanh[a +
 b*x]] + 88*b*x*ArcTanh[Tanh[a + b*x]]^2 - 16*ArcTanh[Tanh[a + b*x]]^3))/(1
155*b^4)
```

Maple [A] time = 0.033, size = 124, normalized size = 1.6

$$2\,\frac{1/11\,(\mathrm{Artanh}\,(\tanh\,(bx+a)))^{11/2} + 1/9\,(-3\,\mathrm{Artanh}\,(\tanh\,(bx+a)) + 3\,bx)\,(\mathrm{Artanh}\,(\tanh\,(bx+a)))^{9/2} + 1/7\,\left((bx-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 2/b^4*(1/11*arctanh(tanh(b*x+a))^(11/2)+1/9*(-3*arctanh(tanh(b*x+a))+3*b*x)
*arctanh(tanh(b*x+a))^(9/2)+1/7*((b*x-arctanh(tanh(b*x+a)))*(-2*arctanh(tan
h(b*x+a))+2*b*x)+(b*x-arctanh(tanh(b*x+a)))^2)*arctanh(tanh(b*x+a))^(7/2)+1
/5*(b*x-arctanh(tanh(b*x+a)))^3*arctanh(tanh(b*x+a))^(5/2))
```

Maxima [A] time = 1.76476, size = 72, normalized size = 0.9

$$\frac{2\left(105\,b^4x^4 + 35\,ab^3x^3 - 30\,a^2b^2x^2 + 24\,a^3bx - 16\,a^4\right)(bx+a)^{\frac{3}{2}}}{1155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(105*b^4*x^4 + 35*a*b^3*x^3 - 30*a^2*b^2*x^2 + 24*a^3*b*x - 16*a^4)*
(b*x + a)^(3/2)/b^4
```

Fricas [A] time = 1.79939, size = 147, normalized size = 1.84

$$\frac{2\left(105\,b^5x^5 + 140\,ab^4x^4 + 5\,a^2b^3x^3 - 6\,a^3b^2x^2 + 8\,a^4bx - 16\,a^5\right)\sqrt{bx+a}}{1155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*a^3*b^2*x^2 + 8*a^4
*b*x - 16*a^5)*sqrt(b*x + a)/b^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(x**3*atanh(tanh(a + b*x))**(3/2), x)
```

Giac [A] time = 1.15894, size = 169, normalized size = 2.11

$$\frac{\sqrt{2}\left(\frac{11\sqrt{2}\left(35\,(bx+a)^{\frac{9}{2}}-135\,(bx+a)^{\frac{7}{2}}a+189\,(bx+a)^{\frac{5}{2}}a^{2}-105\,(bx+a)^{\frac{3}{2}}a^{3}\right)a}{b^{3}}+\frac{\sqrt{2}\left(315\,(bx+a)^{\frac{11}{2}}-1540\,(bx+a)^{\frac{9}{2}}a+2970\,(bx+a)^{\frac{7}{2}}a^{2}-2772\,(bx+a)^{\frac{5}{2}}a^{3}+1155\,(bx\cdots\right.}{b^{3}}\right.}{3465\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/3465*sqrt(2)*(11*sqrt(2)*(35*(b*x + a)^(9/2) - 135*(b*x + a)^(7/2)*a + 18
9*(b*x + a)^(5/2)*a^2 - 105*(b*x + a)^(3/2)*a^3)*a/b^3 + sqrt(2)*(315*(b*x
+ a)^(11/2) - 1540*(b*x + a)^(9/2)*a + 2970*(b*x + a)^(7/2)*a^2 - 2772*(b*x
 + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4)/b^3)/b
```

3.122 $\int x^2 \tanh^{-1}(\tanh(a+bx))^{3/2}\,dx$

Optimal. Leaf size=59

$$\frac{16\tanh^{-1}(\tanh(a+bx))^{9/2}}{315b^3}-\frac{8x\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2}+\frac{2x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}$$

```
[Out] (2*x^2*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b) - (8*x*ArcTanh[Tanh[a + b*x]]^(7
/2))/(35*b^2) + (16*ArcTanh[Tanh[a + b*x]]^(9/2))/(315*b^3)
```

Rubi [A] time = 0.0292661, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2157, 30}

$$\frac{16\tanh^{-1}(\tanh(a+bx))^{9/2}}{315b^3}-\frac{8x\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2}+\frac{2x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*x^2*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b) - (8*x*ArcTanh[Tanh[a + b*x]]^(7
/2))/(35*b^2) + (16*ArcTanh[Tanh[a + b*x]]^(9/2))/(315*b^3)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^2 \tanh^{-1}(\tanh(a+bx))^{3/2}\,dx = \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{4\int x\tanh^{-1}(\tanh(a+bx))^{5/2}\,dx}{5b}$$
$$= \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{8x\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{8\int \tanh^{-1}(\tanh(a}{35b^2}$$
$$= \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{8x\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{8\,\mathrm{Subst}\left(\int x^{7/2}\,dx, x\right.}{}$$
$$= \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{8x\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{16\tanh^{-1}(\tanh(a+}{315b^3}$$

Mathematica [A] time = 0.0303672, size = 49, normalized size = 0.83

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}\left(-36bx\tanh^{-1}(\tanh(a+bx))+8\tanh^{-1}(\tanh(a+bx))^2+63b^2x^2\right)}{315b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(5/2)*(63*b^2*x^2 - 36*b*x*ArcTanh[Tanh[a + b*x]]
 + 8*ArcTanh[Tanh[a + b*x]]^2))/(315*b^3)
```

Maple [A] time = 0.033, size = 69, normalized size = 1.2

$$2\,\frac{1/9\,(\mathrm{Artanh}(\tanh(bx+a)))^{9/2}+1/7\,(-2\,\mathrm{Artanh}(\tanh(bx+a))+2\,bx)(\mathrm{Artanh}(\tanh(bx+a)))^{7/2}+1/5\,(bx-\mathrm{A}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 2/b^3*(1/9*arctanh(tanh(b*x+a))^(9/2)+1/7*(-2*arctanh(tanh(b*x+a))+2*b*x)*a
rctanh(tanh(b*x+a))^(7/2)+1/5*(b*x-arctanh(tanh(b*x+a)))^2*arctanh(tanh(b*x
```

```
+a))^(5/2))
```

Maxima [A] time = 1.76832, size = 57, normalized size = 0.97

$$\frac{2\left(35\,b^3x^3+15\,ab^2x^2-12\,a^2bx+8\,a^3\right)(bx+a)^{\frac{3}{2}}}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*b^3*x^3 + 15*a*b^2*x^2 - 12*a^2*b*x + 8*a^3)*(b*x + a)^(3/2)/b^3
```

Fricas [A] time = 1.78853, size = 120, normalized size = 2.03

$$\frac{2\left(35\,b^4x^4+50\,ab^3x^3+3\,a^2b^2x^2-4\,a^3bx+8\,a^4\right)\sqrt{bx+a}}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 + 50*a*b^3*x^3 + 3*a^2*b^2*x^2 - 4*a^3*b*x + 8*a^4)*sqrt(
b*x + a)/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\,\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(x**2*atanh(tanh(a + b*x))**(3/2), x)
```

Giac [B] time = 1.14326, size = 136, normalized size = 2.31

$$\frac{\sqrt{2}\left(\frac{3\sqrt{2}\left(15\,(bx+a)^{\frac{7}{2}}-42\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^{2}\right)a}{b^{2}}+\frac{\sqrt{2}\left(35\,(bx+a)^{\frac{9}{2}}-135\,(bx+a)^{\frac{7}{2}}a+189\,(bx+a)^{\frac{5}{2}}a^{2}-105\,(bx+a)^{\frac{3}{2}}a^{3}\right)}{b^{2}}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/315*sqrt(2)*(3*sqrt(2)*(15*(b*x + a)^(7/2) - 42*(b*x + a)^(5/2)*a + 35*(b
*x + a)^(3/2)*a^2)*a/b^2 + sqrt(2)*(35*(b*x + a)^(9/2) - 135*(b*x + a)^(7/2
)*a + 189*(b*x + a)^(5/2)*a^2 - 105*(b*x + a)^(3/2)*a^3)/b^2)/b
```

3.123 $\int x \tanh^{-1}(\tanh(a+bx))^{3/2}\,dx$

Optimal. Leaf size=38

$$\frac{2x\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}-\frac{4\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2}$$

```
[Out] (2*x*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b) - (4*ArcTanh[Tanh[a + b*x]]^(7/2))
/(35*b^2)
```

Rubi [A] time = 0.0141962, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2157, 30}

$$\frac{2x\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}-\frac{4\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*x*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b) - (4*ArcTanh[Tanh[a + b*x]]^(7/2))
/(35*b^2)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\tanh^{-1}(\tanh(a+bx))^{3/2}\,dx &= \frac{2x\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}-\frac{2\int\tanh^{-1}(\tanh(a+bx))^{5/2}\,dx}{5b}\\ &= \frac{2x\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}-\frac{2\,\text{Subst}\left(\int x^{5/2}\,dx,x,\tanh^{-1}(\tanh(a+bx))\right)}{5b^2}\\ &= \frac{2x\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}-\frac{4\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2}\end{aligned}$$

Mathematica [A] time = 0.0568584, size = 32, normalized size = 0.84

$$\frac{2\left(7bx-2\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{5/2}}{35b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*(7*b*x - 2*ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(5/2))/(35*b^2
)
```

Maple [A] time = 0.036, size = 42, normalized size = 1.1

$$2\,\frac{1/7\,(\text{Artanh}\,(\tanh\,(bx+a)))^{7/2}+1/5\,(bx-\text{Artanh}\,(\tanh\,(bx+a)))\,(\text{Artanh}\,(\tanh\,(bx+a)))^{5/2}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 2/b^2*(1/7*arctanh(tanh(b*x+a))^(7/2)+1/5*(b*x-arctanh(tanh(b*x+a)))*arctan
h(tanh(b*x+a))^(5/2))
```

Maxima [A] time = 1.76248, size = 42, normalized size = 1.11

$$\frac{2\left(5\,b^2x^2 + 3\,abx - 2\,a^2\right)(bx + a)^{\frac{3}{2}}}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*b^2*x^2 + 3*a*b*x - 2*a^2)*(b*x + a)^(3/2)/b^2
```

Fricas [A] time = 1.92149, size = 92, normalized size = 2.42

$$\frac{2\left(5\,b^3x^3 + 8\,ab^2x^2 + a^2bx - 2\,a^3\right)\sqrt{bx + a}}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^3 + 8*a*b^2*x^2 + a^2*b*x - 2*a^3)*sqrt(b*x + a)/b^2
```

Sympy [A] time = 49.9938, size = 49, normalized size = 1.29

$$\begin{cases} \frac{2x\,\text{atanh}^{\frac{5}{2}}\left(\tanh\left(a+bx\right)\right)}{5b} - \frac{4\,\text{atanh}^{\frac{7}{2}}\left(\tanh\left(a+bx\right)\right)}{35b^2} & \text{for}\ b \neq 0 \\ \frac{x^2\,\text{atanh}^{\frac{3}{2}}\left(\tanh\left(a\right)\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Piecewise((2*x*atanh(tanh(a + b*x))**(5/2)/(5*b) - 4*atanh(tanh(a + b*x))**
(7/2)/(35*b**2), Ne(b, 0)), (x**2*atanh(tanh(a))**(3/2)/2, True))
```

Giac [B] time = 1.15601, size = 104, normalized size = 2.74

$$\frac{\sqrt{2}\left(\frac{7\sqrt{2}\left(3(bx+a)^{\frac{5}{2}}-5(bx+a)^{\frac{3}{2}}a\right)a}{b}+\frac{\sqrt{2}\left(15(bx+a)^{\frac{7}{2}}-42(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^2\right)}{b}\right)}{105b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/105*sqrt(2)*(7*sqrt(2)*(3*(b*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a)*a/b + sq
rt(2)*(15*(b*x + a)^(7/2) - 42*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2)/
b)/b
```

3.124 $\int \tanh^{-1}(\tanh(a+bx))^{3/2}\,dx$

Optimal. Leaf size=18

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}$$

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b)
```

Rubi [A] time = 0.0047478, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2157, 30}

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b)
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(\tanh(a+bx))^{3/2}\,dx &= \frac{\text{Subst}\left(\int x^{3/2}\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}\end{aligned}$$

Mathematica [A] time = 0.0061363, size = 18, normalized size = 1.

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b)
```

Maple [A] time = 0.027, size = 15, normalized size = 0.8

$$\frac{2}{5b}\left(\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 2/5*arctanh(tanh(b*x+a))^(5/2)/b
```

Maxima [A] time = 1.69263, size = 16, normalized size = 0.89

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x + a)^(5/2)/b
```

Fricas [A] time = 2.02259, size = 63, normalized size = 3.5

$$\frac{2\left(b^2x^2+2\,abx+a^2\right)\sqrt{bx+a}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/b
```

Sympy [A] time = 19.8266, size = 26, normalized size = 1.44

$$\begin{cases} \frac{2\operatorname{atanh}^{\frac{5}{2}}\left(\tanh\left(a+bx\right)\right)}{5b} & \text{for } b \neq 0 \\ x\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a\right)\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Piecewise((2*atanh(tanh(a + b*x))**(5/2)/(5*b), Ne(b, 0)), (x*atanh(tanh(a)
)**(3/2), True))
```

Giac [B] time = 1.13809, size = 62, normalized size = 3.44

$$\frac{\sqrt{2}\left(5\sqrt{2}(bx+a)^{\frac{3}{2}}a+\sqrt{2}\left(3(bx+a)^{\frac{5}{2}}-5(bx+a)^{\frac{3}{2}}a\right)\right)}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/15*sqrt(2)*(5*sqrt(2)*(b*x + a)^(3/2)*a + sqrt(2)*(3*(b*x + a)^(5/2) - 5*
(b*x + a)^(3/2)*a))/b
```

3.125 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x} dx$

Optimal. Leaf size=91

$$-2\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+\frac{2}{3}\tanh^{-1}(\tanh(a+bx))^{3/2}+2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)$$

```
[Out] 2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*(
b*x - ArcTanh[Tanh[a + b*x]])^(3/2) - 2*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt
[ArcTanh[Tanh[a + b*x]]] + (2*ArcTanh[Tanh[a + b*x]]^(3/2))/3
```

Rubi [A] time = 0.0497527, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2159, 2161}

$$-2\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+\frac{2}{3}\tanh^{-1}(\tanh(a+bx))^{3/2}+2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x,x]
```

```
[Out] 2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*(
b*x - ArcTanh[Tanh[a + b*x]])^(3/2) - 2*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt
[ArcTanh[Tanh[a + b*x]]] + (2*ArcTanh[Tanh[a + b*x]]^(3/2))/3
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
```

```
arQ[u, v, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x}\,dx = \frac{2}{3}\tanh^{-1}(\tanh(a+bx))^{3/2} - \left(bx-\tanh^{-1}(\tanh(a+bx))\right)\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x}$$
$$= -2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))} + \frac{2}{3}\tanh^{-1}(\tanh(a+bx))^{3/2}$$
$$= 2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2} - 2\left(bx-\tanh\right.$$

Mathematica [A] time = 0.0693455, size = 80, normalized size = 0.88

$$-\frac{2}{3}\left(-4\tanh^{-1}(\tanh(a+bx))^{3/2}+3bx\sqrt{\tanh^{-1}(\tanh(a+bx))}+3\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)\left(\tanh^{-1}(\tan\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x,x]
```

```
[Out] (-2*(3*b*x*Sqrt[ArcTanh[Tanh[a + b*x]]] - 4*ArcTanh[Tanh[a + b*x]]^(3/2) +
3*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]
]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2)))/3
```

Maple [A] time = 0.107, size = 131, normalized size = 1.4

$$\frac{2}{3}\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{\frac{3}{2}}+2a\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}+2\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx-a\right)\sqrt{\mathrm{Artanh}\left(\tanh\left(bx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x,x)
```

```
[Out] 2/3*arctanh(tanh(b*x+a))^(3/2)+2*a*arctanh(tanh(b*x+a))^(1/2)+2*(arctanh(ta
nh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/2)-2*(a^2+2*a*(arctanh(tanh(b*x+a
```

```
))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)*
arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(3/2)/x, x)
```

Fricas [A] time = 2.11955, size = 228, normalized size = 2.51

$$\left[a^{\frac{3}{2}}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\frac{2}{3}\left(bx+4a\right)\sqrt{bx+a},2\sqrt{-a}a\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\frac{2}{3}\left(bx+4a\right)\sqrt{bx+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [a^(3/2)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2/3*(b*x + 4*a)*sqr
t(b*x + a), 2*sqrt(-a)*a*arctan(sqrt(b*x + a)*sqrt(-a)/a) + 2/3*(b*x + 4*a)
*sqrt(b*x + a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**(3/2)/x, x)
```

Giac [A] time = 1.15854, size = 77, normalized size = 0.85

$$\frac{1}{3}\sqrt{2}\left(\frac{3\sqrt{2}a^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\sqrt{2}(bx+a)^{\frac{3}{2}}+3\sqrt{2}\sqrt{bx+a}a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(3*sqrt(2)*a^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + sqrt(2
)*(b*x + a)^(3/2) + 3*sqrt(2)*sqrt(b*x + a)*a)
```

3.126 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^2} dx$

Optimal. Leaf size=81

$$-\frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x}+3b\sqrt{\tanh^{-1}(\tanh(a+bx))}-3b\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a}}{\sqrt{bx-\tanh^{-1}(\tanh}}\right.$$

```
[Out] -3*b*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]
]*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]] + 3*b*Sqrt[ArcTanh[Tanh[a + b*x]]] - A
rcTanh[Tanh[a + b*x]]^(3/2)/x
```

Rubi [A] time = 0.0474029, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2159, 2161}

$$-\frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x}+3b\sqrt{\tanh^{-1}(\tanh(a+bx))}-3b\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a}}{\sqrt{bx-\tanh^{-1}(\tanh}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^2,x]
```

```
[Out] -3*b*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]
]*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]] + 3*b*Sqrt[ArcTanh[Tanh[a + b*x]]] - A
rcTanh[Tanh[a + b*x]]^(3/2)/x
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^2}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x}+\frac{1}{2}(3b)\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x}\,dx$$
$$= 3b\sqrt{\tanh^{-1}(\tanh(a+bx))}-\frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x}-\frac{1}{2}\Big(3b\Big(bx-\tanh^{-1}(\tanh(a+$$
$$= -3b\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}+3b\sqrt{\tanh^{-1}}$$

Mathematica [A] time = 0.0350054, size = 79, normalized size = 0.98

$$-\frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x}+3b\sqrt{\tanh^{-1}(\tanh(a+bx))}-3b\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)\sqrt{\tanh^{-1}(\tanh(a-}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^2,x]
```

```
[Out] 3*b*Sqrt[ArcTanh[Tanh[a + b*x]]] - ArcTanh[Tanh[a + b*x]]^(3/2)/x - 3*b*Arc
Tanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*Sq
rt[-(b*x) + ArcTanh[Tanh[a + b*x]]]
```

Maple [A] time = 0.118, size = 85, normalized size = 1.1

$$2b\left(\sqrt{\operatorname{Artanh}(\tanh(bx+a))}+\frac{(-1/2\operatorname{Artanh}(\tanh(bx+a))+1/2\,bx)\sqrt{\operatorname{Artanh}(\tanh(bx+a))}}{bx}-3/2\sqrt{\operatorname{Artanh}(\tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x^2,x)
```

```
[Out] 2*b*(arctanh(tanh(b*x+a))^(1/2)+(-1/2*arctanh(tanh(b*x+a))+1/2*b*x)*arctanh
(tanh(b*x+a))^(1/2)/b/x-3/2*(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctan
h(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(\tanh(bx+a))^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(3/2)/x^2, x)
```

Fricas [A] time = 2.21208, size = 247, normalized size = 3.05

$$\left[\frac{3\sqrt{a}bx\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2(2bx-a)\sqrt{bx+a}}{2x}, \frac{3\sqrt{-a}bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+(2bx-a)\sqrt{bx+a}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*sqrt(a)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(2*b*x
 - a)*sqrt(b*x + a))/x, (3*sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) +
(2*b*x - a)*sqrt(b*x + a))/x]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x**2,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**(3/2)/x**2, x)
```

Giac [A] time = 1.15655, size = 93, normalized size = 1.15

$$\frac{\sqrt{2}\left(\frac{3\sqrt{2}ab^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{2}\sqrt{bx+a}b^2-\frac{\sqrt{2}\sqrt{bx+a}ab}{x}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(3*sqrt(2)*a*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*sq
rt(2)*sqrt(b*x + a)*b^2 - sqrt(2)*sqrt(b*x + a)*a*b/x)/b
```

3.127 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^3} dx$

Optimal. Leaf size=92

$$\frac{3b^2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{2x^2}-\frac{3b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{4x}$$

```
[Out] (3*b^2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]
]]])/(4*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]) - (3*b*Sqrt[ArcTanh[Tanh[a + b*
x]]])/(4*x) - ArcTanh[Tanh[a + b*x]]^(3/2)/(2*x^2)
```

Rubi [A] time = 0.0490936, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 2161}

$$\frac{3b^2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{2x^2}-\frac{3b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^3,x]
```

```
[Out] (3*b^2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]
]]])/(4*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]) - (3*b*Sqrt[ArcTanh[Tanh[a + b*
x]]])/(4*x) - ArcTanh[Tanh[a + b*x]]^(3/2)/(2*x^2)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^3}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{2x^2} + \frac{1}{4}(3b)\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^2}\,dx$$
$$= -\frac{3b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{4x} - \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{2x^2} + \frac{1}{8}\left(3b^2\right)\int \frac{1}{x\sqrt{\tanh^{-1}(\tanh}}$$
$$= \frac{3b^2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}} - \frac{3b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{4x} - \frac{\tanh^{-1}(\tanh(a+bx}{2x^2}$$

Mathematica [A] time = 0.0594817, size = 88, normalized size = 0.96

$$\frac{1}{4}\left(-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}} - \frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^2} - \frac{3b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^3,x]
```

```
[Out] ((-3*b*Sqrt[ArcTanh[Tanh[a + b*x]]])/x - (2*ArcTanh[Tanh[a + b*x]]^(3/2))/x
^2 - (3*b^2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh
[a + b*x]]]])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]])/4
```

Maple [A] time = 0.115, size = 91, normalized size = 1.

$$2\,b^2\left(\frac{-5/8\,\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{3/2}+\left(3/8\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-3/8\,bx\right)\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}}{b^2x^2} - 3/8\,\frac{}{\sqrt{\mathrm{Artar}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x^3,x)
```

```
[Out] 2*b^2*((-5/8*arctanh(tanh(b*x+a))^(3/2)+(3/8*arctanh(tanh(b*x+a))-3/8*b*x)*
arctanh(tanh(b*x+a))^(1/2))/b^2/x^2-3/8/(arctanh(tanh(b*x+a))-b*x)^(1/2)*ar
ctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(3/2)/x^3, x)
```

Fricas [A] time = 2.14147, size = 296, normalized size = 3.22

$$\left[\frac{3\sqrt{a}b^{2}x^{2}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(5abx+2a^{2}\right)\sqrt{bx+a}}{8ax^{2}},\frac{3\sqrt{-a}b^{2}x^{2}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-\left(5abx+2a^{2}\right)\sqrt{bx+a}}{4ax^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(5
*a*b*x + 2*a^2)*sqrt(b*x + a))/(a*x^2), 1/4*(3*sqrt(-a)*b^2*x^2*arctan(sqrt
(b*x + a)*sqrt(-a)/a) - (5*a*b*x + 2*a^2)*sqrt(b*x + a))/(a*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x**3,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**(3/2)/x**3, x)
```

Giac [A] time = 1.17312, size = 99, normalized size = 1.08

$$\frac{\sqrt{2}\left(\frac{3\sqrt{2}b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\sqrt{2}\left(5(bx+a)^{\frac{3}{2}}b^3 - 3\sqrt{bx+a}ab^3\right)}{b^2x^2}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(3*sqrt(2)*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - sqrt(2
)*(5*(b*x + a)^(3/2)*b^3 - 3*sqrt(b*x + a)*a*b^3)/(b^2*x^2))/b
```

3.128 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^4}\,dx$

Optimal. Leaf size=146

$$\frac{b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{b^2}{8x\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{b^3\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}$$

```
[Out] (b^3*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]
])/(8*(b*x - ArcTanh[Tanh[a + b*x]])^(3/2)) - b^2/(8*x*Sqrt[ArcTanh[Tanh[a
+ b*x]]]) + b^3/(8*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x
]]]) - (b*Sqrt[ArcTanh[Tanh[a + b*x]]])/(4*x^2) - ArcTanh[Tanh[a + b*x]]^(3
/2)/(3*x^3)
```

Rubi [A] time = 0.0935789, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2163, 2161}

$$\frac{b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{b^2}{8x\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{b^3\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^4,x]
```

```
[Out] (b^3*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]
])/(8*(b*x - ArcTanh[Tanh[a + b*x]])^(3/2)) - b^2/(8*x*Sqrt[ArcTanh[Tanh[a
+ b*x]]]) + b^3/(8*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x
]]]) - (b*Sqrt[ArcTanh[Tanh[a + b*x]]])/(4*x^2) - ArcTanh[Tanh[a + b*x]]^(3
/2)/(3*x^3)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
```

```
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^4}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^3} + \frac{1}{2}b\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^3}\,dx$$

$$= -\frac{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{4x^2} - \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^3} + \frac{1}{8}b^2\int \frac{1}{x^2\sqrt{\tanh^{-1}(\tanh(a+}$$

$$= -\frac{b^2}{8x\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{4x^2} - \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^3}$$

$$= -\frac{b^2}{8x\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx}}$$

$$= \frac{b^3\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}} - \frac{b^2}{8x\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{}{8\left(bx-\tanh^{-1}(\tan\right.}$$

Mathematica [A] time = 0.0918898, size = 117, normalized size = 0.8

$$\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-\frac{b^2}{8x\left(\tanh^{-1}(\tanh(a+bx))-bx\right)}-\frac{\tanh^{-1}(\tanh(a+bx))-bx}{3x^3}-\frac{7b}{12x^2}\right)+\frac{b^3\tanh^{-1}\left(\frac{\sqrt{\tan}}{\sqrt{\tanh}}\right.}{8\left(\tanh^{-1}(\tanh(\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^4,x]
```

```
[Out] (b^3*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*
x]]]])/(8*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2)) + Sqrt[ArcTanh[Tanh[a +
b*x]]]*((-7*b)/(12*x^2) - b^2/(8*x*(-(b*x) + ArcTanh[Tanh[a + b*x]])) - (-(
b*x) + ArcTanh[Tanh[a + b*x]])/(3*x^3))
```

Maple [A] time = 0.115, size = 116, normalized size = 0.8

$$2\,b^3\left(\frac{1}{x^3b^3}\left(-1/16\,\frac{(\mathrm{Artanh}\,(\tanh\,(bx+a)))^{5/2}}{\mathrm{Artanh}\,(\tanh\,(bx+a))-bx}-1/6\,(\mathrm{Artanh}\,(\tanh\,(bx+a)))^{3/2}+(1/16\,\mathrm{Artanh}\,(\tanh\,(bx+a))-1/1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x^4,x)
```

```
[Out] 2*b^3*((-1/16/(arctanh(tanh(b*x+a))-b*x)*arctanh(tanh(b*x+a))^(5/2)-1/6*arc
tanh(tanh(b*x+a))^(3/2)+(1/16*arctanh(tanh(b*x+a))-1/16*b*x)*arctanh(tanh(b
*x+a))^(1/2))/b^3/x^3+1/16/(arctanh(tanh(b*x+a))-b*x)^(3/2)*arctanh(arctanh
(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{artanh}\,(\tanh\,(bx+a))^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(3/2)/x^4, x)
```

Fricas [A] time = 2.18842, size = 351, normalized size = 2.4

$$\left[\frac{3\sqrt{a}b^3x^3\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(3ab^2x^2+14a^2bx+8a^3\right)\sqrt{bx+a}}{48a^2x^3}, -\frac{3\sqrt{-a}b^3x^3\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(3ab^2x^2+14a\right.}{24a^2x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(a)*b^3*x^3*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(
3*a*b^2*x^2 + 14*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a^2*x^3), -1/24*(3*sqrt(-
a)*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (3*a*b^2*x^2 + 14*a^2*b*x + 8
*a^3)*sqrt(b*x + a))/(a^2*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x**4,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**(3/2)/x**4, x)
```

Giac [A] time = 1.20262, size = 126, normalized size = 0.86

$$-\frac{\sqrt{2}\left(\frac{3\sqrt{2}b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{\sqrt{2}\left(3(bx+a)^{\frac{5}{2}}b^4+8(bx+a)^{\frac{3}{2}}ab^4-3\sqrt{bx+a}a^2b^4\right)}{ab^3x^3}\right)}{48b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/48*sqrt(2)*(3*sqrt(2)*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) +
sqrt(2)*(3*(b*x + a)^(5/2)*b^4 + 8*(b*x + a)^(3/2)*a*b^4 - 3*sqrt(b*x + a)*
a^2*b^4)/(a*b^3*x^3))/b
```

3.129 $\int x^4 \tanh^{-1}(\tanh(a+bx))^{5/2}\,dx$

Optimal. Leaf size=101

$$\frac{32x^2\tanh^{-1}(\tanh(a+bx))^{11/2}}{231b^3}-\frac{16x^3\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2}+\frac{256\tanh^{-1}(\tanh(a+bx))^{15/2}}{45045b^5}-\frac{128x\tanh^{-1}(\tanh(a}{3003b^4}$$

```
[Out] (2*x^4*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b) - (16*x^3*ArcTanh[Tanh[a + b*x]]
^(9/2))/(63*b^2) + (32*x^2*ArcTanh[Tanh[a + b*x]]^(11/2))/(231*b^3) - (128*
x*ArcTanh[Tanh[a + b*x]]^(13/2))/(3003*b^4) + (256*ArcTanh[Tanh[a + b*x]]^(
15/2))/(45045*b^5)
```

Rubi [A] time = 0.0647503, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2157, 30}

$$\frac{32x^2\tanh^{-1}(\tanh(a+bx))^{11/2}}{231b^3}-\frac{16x^3\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2}+\frac{256\tanh^{-1}(\tanh(a+bx))^{15/2}}{45045b^5}-\frac{128x\tanh^{-1}(\tanh(a}{3003b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (2*x^4*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b) - (16*x^3*ArcTanh[Tanh[a + b*x]]
^(9/2))/(63*b^2) + (32*x^2*ArcTanh[Tanh[a + b*x]]^(11/2))/(231*b^3) - (128*
x*ArcTanh[Tanh[a + b*x]]^(13/2))/(3003*b^4) + (256*ArcTanh[Tanh[a + b*x]]^(
15/2))/(45045*b^5)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^4 \tanh^{-1}(\tanh(a+bx))^{5/2}\,dx = \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{8\int x^3\tanh^{-1}(\tanh(a+bx))^{7/2}\,dx}{7b}$$
$$= \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2} + \frac{16\int x^2\tanh^{-1}(\text{ta}}{21}$$
$$= \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2} + \frac{32x^2\tanh^{-1}(\text{tanh}}{231b^3}$$
$$= \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2} + \frac{32x^2\tanh^{-1}(\text{tanh}}{231b^3}$$
$$= \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2} + \frac{32x^2\tanh^{-1}(\text{tanh}}{231b^3}$$
$$= \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2} + \frac{32x^2\tanh^{-1}(\text{tanh}}{231b^3}$$

Mathematica [A] time = 0.0381337, size = 83, normalized size = 0.82

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}\left(-5720b^3x^3\tanh^{-1}(\tanh(a+bx))+3120b^2x^2\tanh^{-1}(\tanh(a+bx))^2-960bx\tanh^{-1}(\text{tanh}\right.}{45045b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(7/2)*(6435*b^4*x^4 - 5720*b^3*x^3*ArcTanh[Tanh[a
 + b*x]] + 3120*b^2*x^2*ArcTanh[Tanh[a + b*x]]^2 - 960*b*x*ArcTanh[Tanh[a +
 b*x]]^3 + 128*ArcTanh[Tanh[a + b*x]]^4))/(45045*b^5)
```

Maple [A] time = 0.035, size = 154, normalized size = 1.5

$$2\,\frac{1/15\,(\mathrm{Artanh}(\tanh(bx+a)))^{15/2}+1/13\,(-4\,\mathrm{Artanh}(\tanh(bx+a))+4\,bx)(\mathrm{Artanh}(\tanh(bx+a)))^{13/2}+1/11\,\big(2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] 2/b^5*(1/15*arctanh(tanh(b*x+a))^(15/2)+1/13*(-4*arctanh(tanh(b*x+a))+4*b*x
)*arctanh(tanh(b*x+a))^(13/2)+1/11*(2*(b*x-arctanh(tanh(b*x+a)))^2+(-2*arct
anh(tanh(b*x+a))+2*b*x)^2)*arctanh(tanh(b*x+a))^(11/2)+2/9*(b*x-arctanh(tan
h(b*x+a)))^2*(-2*arctanh(tanh(b*x+a))+2*b*x)*arctanh(tanh(b*x+a))^(9/2)+1/7
*(b*x-arctanh(tanh(b*x+a)))^4*arctanh(tanh(b*x+a))^(7/2))
```

Maxima [A] time = 1.83062, size = 86, normalized size = 0.85

$$\frac{2\left(3003\,b^5x^5+1155\,ab^4x^4-840\,a^2b^3x^3+560\,a^3b^2x^2-320\,a^4bx+128\,a^5\right)(bx+a)^{\frac{5}{2}}}{45045\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(3003*b^5*x^5 + 1155*a*b^4*x^4 - 840*a^2*b^3*x^3 + 560*a^3*b^2*x^2
- 320*a^4*b*x + 128*a^5)*(b*x + a)^(5/2)/b^5
```

Fricas [A] time = 2.04585, size = 205, normalized size = 2.03

$$\frac{2\left(3003\,b^7x^7+7161\,ab^6x^6+4473\,a^2b^5x^5+35\,a^3b^4x^4-40\,a^4b^3x^3+48\,a^5b^2x^2-64\,a^6bx+128\,a^7\right)\sqrt{bx+a}}{45045\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3003*b^7*x^7 + 7161*a*b^6*x^6 + 4473*a^2*b^5*x^5 + 35*a^3*b^4*x^4
- 40*a^4*b^3*x^3 + 48*a^5*b^2*x^2 - 64*a^6*b*x + 128*a^7)*sqrt(b*x + a)/b^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17011, size = 324, normalized size = 3.21

$$\frac{\sqrt{2}\left(\frac{13\sqrt{2}\left(315\,(bx+a)^{\frac{11}{2}}-1540\,(bx+a)^{\frac{9}{2}}a+2970\,(bx+a)^{\frac{7}{2}}a^2-2772\,(bx+a)^{\frac{5}{2}}a^3+1155\,(bx+a)^{\frac{3}{2}}a^4\right)a^2}{b^4}+\frac{10\sqrt{2}\left(693\,(bx+a)^{\frac{13}{2}}-4095\,(bx+a)^{\frac{11}{2}}a+10010\,(bx+a)^{\frac{9}{2}}\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/45045*sqrt(2)*(13*sqrt(2)*(315*(b*x + a)^(11/2) - 1540*(b*x + a)^(9/2)*a
+ 2970*(b*x + a)^(7/2)*a^2 - 2772*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2
)*a^4)*a^2/b^4 + 10*sqrt(2)*(693*(b*x + a)^(13/2) - 4095*(b*x + a)^(11/2)*a
 + 10010*(b*x + a)^(9/2)*a^2 - 12870*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(
5/2)*a^4 - 3003*(b*x + a)^(3/2)*a^5)*a/b^4 + sqrt(2)*(3003*(b*x + a)^(15/2)
 - 20790*(b*x + a)^(13/2)*a + 61425*(b*x + a)^(11/2)*a^2 - 100100*(b*x + a)
^(9/2)*a^3 + 96525*(b*x + a)^(7/2)*a^4 - 54054*(b*x + a)^(5/2)*a^5 + 15015*
(b*x + a)^(3/2)*a^6)/b^4)/b
```

3.130 $\int x^3 \tanh^{-1}(\tanh(a+bx))^{5/2}\,dx$

Optimal. Leaf size=80

$$-\frac{4x^2\tanh^{-1}(\tanh(a+bx))^{9/2}}{21b^2}-\frac{32\tanh^{-1}(\tanh(a+bx))^{13/2}}{3003b^4}+\frac{16x\tanh^{-1}(\tanh(a+bx))^{11/2}}{231b^3}+\frac{2x^3\tanh^{-1}(\tanh(a+$$

```
[Out] (2*x^3*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b) - (4*x^2*ArcTanh[Tanh[a + b*x]]^
(9/2))/(21*b^2) + (16*x*ArcTanh[Tanh[a + b*x]]^(11/2))/(231*b^3) - (32*ArcT
anh[Tanh[a + b*x]]^(13/2))/(3003*b^4)
```

Rubi [A] time = 0.0468241, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2157, 30}

$$-\frac{4x^2\tanh^{-1}(\tanh(a+bx))^{9/2}}{21b^2}-\frac{32\tanh^{-1}(\tanh(a+bx))^{13/2}}{3003b^4}+\frac{16x\tanh^{-1}(\tanh(a+bx))^{11/2}}{231b^3}+\frac{2x^3\tanh^{-1}(\tanh(a+$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (2*x^3*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b) - (4*x^2*ArcTanh[Tanh[a + b*x]]^
(9/2))/(21*b^2) + (16*x*ArcTanh[Tanh[a + b*x]]^(11/2))/(231*b^3) - (32*ArcT
anh[Tanh[a + b*x]]^(13/2))/(3003*b^4)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^3 \tanh^{-1}(\tanh(a+bx))^{5/2}\,dx = \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{6\int x^2\tanh^{-1}(\tanh(a+bx))^{7/2}\,dx}{7b}$$
$$= \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{4x^2\tanh^{-1}(\tanh(a+bx))^{9/2}}{21b^2} + \frac{8\int x\tanh^{-1}(\tanh}{21b^2}$$
$$= \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{4x^2\tanh^{-1}(\tanh(a+bx))^{9/2}}{21b^2} + \frac{16x\tanh^{-1}(\tanh(a}{231b^3}$$
$$= \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{4x^2\tanh^{-1}(\tanh(a+bx))^{9/2}}{21b^2} + \frac{16x\tanh^{-1}(\tanh(a}{231b^3}$$
$$= \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{4x^2\tanh^{-1}(\tanh(a+bx))^{9/2}}{21b^2} + \frac{16x\tanh^{-1}(\tanh(a}{231b^3}$$

Mathematica [A] time = 0.0369652, size = 66, normalized size = 0.82

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}\left(-286b^2x^2\tanh^{-1}(\tanh(a+bx))+104bx\tanh^{-1}(\tanh(a+bx))^2-16\tanh^{-1}(\tanh(a+bx)\right.}{3003b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(7/2)*(429*b^3*x^3 - 286*b^2*x^2*ArcTanh[Tanh[a +
 b*x]] + 104*b*x*ArcTanh[Tanh[a + b*x]]^2 - 16*ArcTanh[Tanh[a + b*x]]^3))/(
3003*b^4)
```

Maple [A] time = 0.036, size = 124, normalized size = 1.6

$$2\,\frac{1/13\,(\mathrm{Artanh}\,(\tanh\,(bx+a)))^{13/2}+1/11\,(-3\,\mathrm{Artanh}\,(\tanh\,(bx+a))+3\,bx)\,(\mathrm{Artanh}\,(\tanh\,(bx+a)))^{11/2}+1/9\,\left((bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] 2/b^4*(1/13*arctanh(tanh(b*x+a))^(13/2)+1/11*(-3*arctanh(tanh(b*x+a))+3*b*x
)*arctanh(tanh(b*x+a))^(11/2)+1/9*((b*x-arctanh(tanh(b*x+a)))*(-2*arctanh(t
anh(b*x+a))+2*b*x)+(b*x-arctanh(tanh(b*x+a)))^2)*arctanh(tanh(b*x+a))^(9/2)
+1/7*(b*x-arctanh(tanh(b*x+a)))^3*arctanh(tanh(b*x+a))^(7/2))
```

Maxima [A] time = 1.78086, size = 72, normalized size = 0.9

$$\frac{2\left(231\,b^{4}x^{4}+105\,ab^{3}x^{3}-70\,a^{2}b^{2}x^{2}+40\,a^{3}bx-16\,a^{4}\right)(bx+a)^{\frac{5}{2}}}{3003\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3003*(231*b^4*x^4 + 105*a*b^3*x^3 - 70*a^2*b^2*x^2 + 40*a^3*b*x - 16*a^4)
*(b*x + a)^(5/2)/b^4
```

Fricas [A] time = 2.03198, size = 171, normalized size = 2.14

$$\frac{2\left(231\,b^{6}x^{6}+567\,ab^{5}x^{5}+371\,a^{2}b^{4}x^{4}+5\,a^{3}b^{3}x^{3}-6\,a^{4}b^{2}x^{2}+8\,a^{5}bx-16\,a^{6}\right)\sqrt{bx+a}}{3003\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(231*b^6*x^6 + 567*a*b^5*x^5 + 371*a^2*b^4*x^4 + 5*a^3*b^3*x^3 - 6*a
^4*b^2*x^2 + 8*a^5*b*x - 16*a^6)*sqrt(b*x + a)/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18499, size = 277, normalized size = 3.46

$$\frac{\sqrt{2}\left(\frac{143\sqrt{2}\left(35\,(bx+a)^{\frac{9}{2}}-135\,(bx+a)^{\frac{7}{2}}a+189\,(bx+a)^{\frac{5}{2}}a^{2}-105\,(bx+a)^{\frac{3}{2}}a^{3}\right)a^{2}}{b^{3}}+\frac{26\sqrt{2}\left(315\,(bx+a)^{\frac{11}{2}}-1540\,(bx+a)^{\frac{9}{2}}a+2970\,(bx+a)^{\frac{7}{2}}a^{2}-2772\,(bx+a)^{\frac{5}{2}}a^{3}+115\right.}{b^{3}}}{45045\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/45045*sqrt(2)*(143*sqrt(2)*(35*(b*x + a)^(9/2) - 135*(b*x + a)^(7/2)*a +
189*(b*x + a)^(5/2)*a^2 - 105*(b*x + a)^(3/2)*a^3)*a^2/b^3 + 26*sqrt(2)*(31
5*(b*x + a)^(11/2) - 1540*(b*x + a)^(9/2)*a + 2970*(b*x + a)^(7/2)*a^2 - 27
72*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4)*a/b^3 + 5*sqrt(2)*(693*(
b*x + a)^(13/2) - 4095*(b*x + a)^(11/2)*a + 10010*(b*x + a)^(9/2)*a^2 - 128
70*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 3003*(b*x + a)^(3/2)*a^
5)/b^3)/b
```

3.131 $\int x^2 \tanh^{-1}(\tanh(a+bx))^{5/2}\,dx$

Optimal. Leaf size=59

$$\frac{16\tanh^{-1}(\tanh(a+bx))^{11/2}}{693b^3}-\frac{8x\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2}+\frac{2x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}$$

```
[Out] (2*x^2*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b) - (8*x*ArcTanh[Tanh[a + b*x]]^(9
/2))/(63*b^2) + (16*ArcTanh[Tanh[a + b*x]]^(11/2))/(693*b^3)
```

Rubi [A] time = 0.02927, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2157, 30}

$$\frac{16\tanh^{-1}(\tanh(a+bx))^{11/2}}{693b^3}-\frac{8x\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2}+\frac{2x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (2*x^2*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b) - (8*x*ArcTanh[Tanh[a + b*x]]^(9
/2))/(63*b^2) + (16*ArcTanh[Tanh[a + b*x]]^(11/2))/(693*b^3)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2 \tanh^{-1}(\tanh(a+bx))^{5/2}\,dx &= \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{4\int x\tanh^{-1}(\tanh(a+bx))^{7/2}\,dx}{7b}\\ &= \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{8x\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2} + \frac{8\int \tanh^{-1}(\tanh(a}{63b^2}\\ &= \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{8x\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2} + \frac{8\,\mathrm{Subst}\left(\int x^{9/2}\,dx, x\right.}{}\\ &= \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{8x\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2} + \frac{16\tanh^{-1}(\tanh(a+}{693b^3}\end{aligned}$$

Mathematica [A] time = 0.0312256, size = 49, normalized size = 0.83

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}\left(-44bx\tanh^{-1}(\tanh(a+bx))+8\tanh^{-1}(\tanh(a+bx))^2+99b^2x^2\right)}{693b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(7/2)*(99*b^2*x^2 - 44*b*x*ArcTanh[Tanh[a + b*x]]
 + 8*ArcTanh[Tanh[a + b*x]]^2))/(693*b^3)
```

Maple [A] time = 0.033, size = 69, normalized size = 1.2

$$2\,\frac{1/11\,\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{11/2}+1/9\,\left(-2\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)+2\,bx\right)\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{9/2}+1/7\,\left(bx-\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] 2/b^3*(1/11*arctanh(tanh(b*x+a))^(11/2)+1/9*(-2*arctanh(tanh(b*x+a))+2*b*x)
*arctanh(tanh(b*x+a))^(9/2)+1/7*(b*x-arctanh(tanh(b*x+a)))^2*arctanh(tanh(b
```

```
*x+a))^(7/2))
```

Maxima [A] time = 1.80105, size = 57, normalized size = 0.97

$$\frac{2\left(63\,b^3x^3+35\,ab^2x^2-20\,a^2bx+8\,a^3\right)(bx+a)^{\frac{5}{2}}}{693\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/693*(63*b^3*x^3 + 35*a*b^2*x^2 - 20*a^2*b*x + 8*a^3)*(b*x + a)^(5/2)/b^3
```

Fricas [A] time = 1.98195, size = 146, normalized size = 2.47

$$\frac{2\left(63\,b^5x^5+161\,ab^4x^4+113\,a^2b^3x^3+3\,a^3b^2x^2-4\,a^4bx+8\,a^5\right)\sqrt{bx+a}}{693\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*b^5*x^5 + 161*a*b^4*x^4 + 113*a^2*b^3*x^3 + 3*a^3*b^2*x^2 - 4*a^4
*b*x + 8*a^5)*sqrt(b*x + a)/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18087, size = 227, normalized size = 3.85

$$\frac{\sqrt{2}\left(\frac{33\sqrt{2}\left(15(bx+a)^{\frac{7}{2}}-42(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^{2}\right)a^{2}}{b^{2}}+\frac{22\sqrt{2}\left(35(bx+a)^{\frac{9}{2}}-135(bx+a)^{\frac{7}{2}}a+189(bx+a)^{\frac{5}{2}}a^{2}-105(bx+a)^{\frac{3}{2}}a^{3}\right)a}{b^{2}}+\frac{\sqrt{2}\left(315(bx+a)^{\frac{11}{2}}-1540\right.}{}\right.}{3465\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/3465*sqrt(2)*(33*sqrt(2)*(15*(b*x + a)^(7/2) - 42*(b*x + a)^(5/2)*a + 35*
(b*x + a)^(3/2)*a^2)*a^2/b^2 + 22*sqrt(2)*(35*(b*x + a)^(9/2) - 135*(b*x +
a)^(7/2)*a + 189*(b*x + a)^(5/2)*a^2 - 105*(b*x + a)^(3/2)*a^3)*a/b^2 + sqr
t(2)*(315*(b*x + a)^(11/2) - 1540*(b*x + a)^(9/2)*a + 2970*(b*x + a)^(7/2)*
a^2 - 2772*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4)/b^2)/b
```

3.132 $\int x \tanh^{-1}(\tanh(a+bx))^{5/2}\,dx$

Optimal. Leaf size=38

$$\frac{2x\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}-\frac{4\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2}$$

```
[Out] (2*x*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b) - (4*ArcTanh[Tanh[a + b*x]]^(9/2))
/(63*b^2)
```

Rubi [A] time = 0.0142888, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2157, 30}

$$\frac{2x\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}-\frac{4\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (2*x*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b) - (4*ArcTanh[Tanh[a + b*x]]^(9/2))
/(63*b^2)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\tanh^{-1}(\tanh(a+bx))^{5/2}\,dx &= \frac{2x\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}-\frac{2\int \tanh^{-1}(\tanh(a+bx))^{7/2}\,dx}{7b}\\ &= \frac{2x\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}-\frac{2\,\mathrm{Subst}\left(\int x^{7/2}\,dx,x,\tanh^{-1}(\tanh(a+bx))\right)}{7b^2}\\ &= \frac{2x\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}-\frac{4\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2}\end{aligned}$$

Mathematica [A] time = 0.0598802, size = 32, normalized size = 0.84

$$\frac{2\left(9bx-2\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{7/2}}{63b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (2*(9*b*x - 2*ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(7/2))/(63*b^2
)
```

Maple [A] time = 0.033, size = 42, normalized size = 1.1

$$2\,\frac{1/9\,\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{9/2}+1/7\,\left(bx-\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{7/2}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] 2/b^2*(1/9*arctanh(tanh(b*x+a))^(9/2)+1/7*(b*x-arctanh(tanh(b*x+a)))*arctan
h(tanh(b*x+a))^(7/2))
```

Maxima [A] time = 1.78357, size = 42, normalized size = 1.11

$$\frac{2\left(7\,b^2x^2+5\,abx-2\,a^2\right)(bx+a)^{\frac{5}{2}}}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/63*(7*b^2*x^2 + 5*a*b*x - 2*a^2)*(b*x + a)^(5/2)/b^2
```

Fricas [A] time = 2.04863, size = 116, normalized size = 3.05

$$\frac{2\left(7\,b^4x^4+19\,ab^3x^3+15\,a^2b^2x^2+a^3bx-2\,a^4\right)\sqrt{bx+a}}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/63*(7*b^4*x^4 + 19*a*b^3*x^3 + 15*a^2*b^2*x^2 + a^3*b*x - 2*a^4)*sqrt(b*x
 + a)/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.11733, size = 178, normalized size = 4.68

$$\frac{\sqrt{2}\left(\frac{21\sqrt{2}\left(3\,(bx+a)^{\frac{5}{2}}-5\,(bx+a)^{\frac{3}{2}}a\right)a^2}{b}+\frac{6\sqrt{2}\left(15\,(bx+a)^{\frac{7}{2}}-42\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2\right)a}{b}+\frac{\sqrt{2}\left(35\,(bx+a)^{\frac{9}{2}}-135\,(bx+a)^{\frac{7}{2}}a+189\,(bx+a)^{\frac{5}{2}}a^2-105\,(bx+a)^{\frac{3}{2}}a^3\right)}{b}\right.}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/315*sqrt(2)*(21*sqrt(2)*(3*(b*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a)*a^2/b +
 6*sqrt(2)*(15*(b*x + a)^(7/2) - 42*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*
a^2)*a/b + sqrt(2)*(35*(b*x + a)^(9/2) - 135*(b*x + a)^(7/2)*a + 189*(b*x +
 a)^(5/2)*a^2 - 105*(b*x + a)^(3/2)*a^3)/b)/b
```

3.133 $\int \tanh^{-1}(\tanh(a+bx))^{5/2}\,dx$

Optimal. Leaf size=18

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}$$

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b)
```

Rubi [A] time = 0.0046375, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2157, 30}

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2),x]

[Out] (2*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b)
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(\tanh(a+bx))^{5/2}\,dx &= \frac{\text{Subst}\left(\int x^{5/2}\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}\end{aligned}$$

Mathematica [A] time = 0.0065517, size = 18, normalized size = 1.

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b)
```

Maple [A] time = 0.029, size = 15, normalized size = 0.8

$$\frac{2}{7\,b}\left(\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] 2/7*arctanh(tanh(b*x+a))^(7/2)/b
```

Maxima [A] time = 1.71599, size = 16, normalized size = 0.89

$$\frac{2\left(bx+a\right)^{\frac{7}{2}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*x + a)^(7/2)/b
```

Fricas [B] time = 1.97339, size = 85, normalized size = 4.72

$$\frac{2\left(b^{3}x^{3}+3\,ab^{2}x^{2}+3\,a^{2}bx+a^{3}\right)\sqrt{bx+a}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b*x + a)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.12041, size = 116, normalized size = 6.44

$$\frac{\sqrt{2}\left(35\sqrt{2}(bx+a)^{\frac{3}{2}}a^2+14\sqrt{2}\left(3\,(bx+a)^{\frac{5}{2}}-5\,(bx+a)^{\frac{3}{2}}a\right)a+\sqrt{2}\left(15\,(bx+a)^{\frac{7}{2}}-42\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2\right)\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/105*sqrt(2)*(35*sqrt(2)*(b*x + a)^(3/2)*a^2 + 14*sqrt(2)*(3*(b*x + a)^(5/
2) - 5*(b*x + a)^(3/2)*a)*a + sqrt(2)*(15*(b*x + a)^(7/2) - 42*(b*x + a)^(5
/2)*a + 35*(b*x + a)^(3/2)*a^2))/b
```

3.134 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x}\,dx$

Optimal. Leaf size=121

$$2\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2-\frac{2}{3}\tanh^{-1}(\tanh(a+bx))^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+\frac{2}{5}$$

```
[Out] -2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*
(b*x - ArcTanh[Tanh[a + b*x]])^(5/2) + 2*(b*x - ArcTanh[Tanh[a + b*x]])^2*S
qrt[ArcTanh[Tanh[a + b*x]]] - (2*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tan
h[a + b*x]]^(3/2))/3 + (2*ArcTanh[Tanh[a + b*x]]^(5/2))/5
```

Rubi [A] time = 0.0702375, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2159, 2161}

$$2\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2-\frac{2}{3}\tanh^{-1}(\tanh(a+bx))^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+\frac{2}{5}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x,x]
```


```
[Out] -2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*
(b*x - ArcTanh[Tanh[a + b*x]])^(5/2) + 2*(b*x - ArcTanh[Tanh[a + b*x]])^2*S
qrt[ArcTanh[Tanh[a + b*x]]] - (2*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tan
h[a + b*x]]^(3/2))/3 + (2*ArcTanh[Tanh[a + b*x]]^(5/2))/5
```


Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
```

```
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x}\,dx = \frac{2}{5}\tanh^{-1}(\tanh(a+bx))^{5/2} - \left(bx-\tanh^{-1}(\tanh(a+bx))\right)\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x}$$

$$= -\frac{2}{3}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2} + \frac{2}{5}\tanh^{-1}(\tanh(a+bx))^{5/}$$

$$= 2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{2}{3}\left(bx-\tanh^{-1}(\tanh(a+b\right.$$

$$= -2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2} + 2\left(bx-\tan\right.$$

Mathematica [A] time = 0.0699247, size = 99, normalized size = 0.82

$$\frac{2}{15}\left(15b^2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))} + 23\tanh^{-1}(\tanh(a+bx))^{5/2} - 35bx\tanh^{-1}(\tanh(a+bx))^{3/2} - 15\tanh^{-1}\left(\frac{\sqrt{\tan}}{\sqrt{\tanh}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x,x]
```

```
[Out] (2*(15*b^2*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]] - 35*b*x*ArcTanh[Tanh[a + b*x]]
^(3/2) + 23*ArcTanh[Tanh[a + b*x]]^(5/2) - 15*ArcTanh[Sqrt[ArcTanh[Tanh[a +
 b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*(-(b*x) + ArcTanh[Tanh[a + b
*x]])^(5/2)))/15
```

Maple [B] time = 0.109, size = 222, normalized size = 1.8

$$\frac{2}{5}\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{\frac{5}{2}} + \frac{2a}{3}\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{\frac{3}{2}} + \frac{2\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right) - 2bx - 2a}{3}\left(\mathrm{Artanh}\left(\tanh\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x,x)
```

```
[Out] 2/5*arctanh(tanh(b*x+a))^(5/2)+2/3*arctanh(tanh(b*x+a))^(3/2)*a+2/3*arctanh
(tanh(b*x+a))^(3/2)*(arctanh(tanh(b*x+a))-b*x-a)+2*arctanh(tanh(b*x+a))^(1/
2)*a^2+4*a*(arctanh(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/2)+2*(arcta
nh(tanh(b*x+a))-b*x-a)^2*arctanh(tanh(b*x+a))^(1/2)-2*(a^3+3*a^2*(arctanh(t
anh(b*x+a))-b*x-a)+3*a*(arctanh(tanh(b*x+a))-b*x-a)^2+(arctanh(tanh(b*x+a))
-b*x-a)^3)/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1
/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(\tanh(bx+a))^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(5/2)/x, x)
```

Fricas [A] time = 2.0917, size = 288, normalized size = 2.38

$$\left[a^{\frac{5}{2}}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\frac{2}{15}\left(3b^2x^2+11abx+23a^2\right)\sqrt{bx+a}, 2\sqrt{-a}a^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\frac{2}{15}\left(3b^2x^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [a^(5/2)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2/15*(3*b^2*x^2 + 1
1*a*b*x + 23*a^2)*sqrt(b*x + a), 2*sqrt(-a)*a^2*arctan(sqrt(b*x + a)*sqrt(-
a)/a) + 2/15*(3*b^2*x^2 + 11*a*b*x + 23*a^2)*sqrt(b*x + a)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17054, size = 99, normalized size = 0.82

$$\frac{1}{15}\sqrt{2}\left(\frac{15\sqrt{2}a^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+3\sqrt{2}(bx+a)^{\frac{5}{2}}+5\sqrt{2}(bx+a)^{\frac{3}{2}}a+15\sqrt{2}\sqrt{bx+a}a^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x,x, algorithm="giac")
```

```
[Out] 1/15*sqrt(2)*(15*sqrt(2)*a^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 3*sq
rt(2)*(b*x + a)^(5/2) + 5*sqrt(2)*(b*x + a)^(3/2)*a + 15*sqrt(2)*sqrt(b*x +
 a)*a^2)
```

3.135 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^2} dx$

Optimal. Leaf size=110

$$-\frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x}+\frac{5}{3}b\tanh^{-1}(\tanh(a+bx))^{3/2}-5b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}+$$

```
[Out] 5*b*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]
*(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) - 5*b*(b*x - ArcTanh[Tanh[a + b*x]])*
Sqrt[ArcTanh[Tanh[a + b*x]]] + (5*b*ArcTanh[Tanh[a + b*x]]^(3/2))/3 - ArcTa
nh[Tanh[a + b*x]]^(5/2)/x
```

Rubi [A] time = 0.0689825, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2159, 2161}

$$-\frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x}+\frac{5}{3}b\tanh^{-1}(\tanh(a+bx))^{3/2}-5b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}+$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^2,x]
```


```
[Out] 5*b*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]
*(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) - 5*b*(b*x - ArcTanh[Tanh[a + b*x]])*
Sqrt[ArcTanh[Tanh[a + b*x]]] + (5*b*ArcTanh[Tanh[a + b*x]]^(3/2))/3 - ArcTa
nh[Tanh[a + b*x]]^(5/2)/x
```


Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^2}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x} + \frac{1}{2}(5b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x}\,dx$$

$$= \frac{5}{3}b\tanh^{-1}(\tanh(a+bx))^{3/2} - \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x} - \frac{1}{2}\left(5b\left(bx-\tanh^{-1}(\tanh(a\right.\right.$$

$$= -5b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))} + \frac{5}{3}b\tanh^{-1}(\tanh(a+bx))$$

$$= 5b\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2} - 5b\left(bx-\text{ta}\right.$$

Mathematica [A] time = 0.0530697, size = 106, normalized size = 0.96

$$\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(\frac{14}{3}b\left(\tanh^{-1}(\tanh(a+bx))-bx\right) - \frac{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}{x} + \frac{2b^2x}{3}\right) - 5b\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{\text{ta}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^2,x]
```

```
[Out] -5*b*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*
x]]]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2) + Sqrt[ArcTanh[Tanh[a + b*x]]
]*((2*b^2*x)/3 + (14*b*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/3 - (-(b*x) + Arc
```

```
Tanh[Tanh[a + b*x]])^2/x)
```

Maple [A] time = 0.116, size = 193, normalized size = 1.8

$$2b\left(1/3\,(\mathrm{Artanh}\,(\tanh\,(bx+a)))^{3/2}+2\,a\sqrt{\mathrm{Artanh}\,(\tanh\,(bx+a))}+2\,(\mathrm{Artanh}\,(\tanh\,(bx+a))-bx-a)\,\sqrt{\mathrm{Artanh}\,(\tanh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^2,x)
```

```
[Out] 2*b*(1/3*arctanh(tanh(b*x+a))^(3/2)+2*a*arctanh(tanh(b*x+a))^(1/2)+2*(arcta
nh(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/2)+(-1/2*a^2-a*(arctanh(tanh
(b*x+a))-b*x-a)-1/2*(arctanh(tanh(b*x+a))-b*x-a)^2)*arctanh(tanh(b*x+a))^(1
/2)/b/x-5/2*(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x
-a)^2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/
(arctanh(tanh(b*x+a))-b*x)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{artanh}\,(\tanh\,(bx+a))^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(5/2)/x^2, x)
```

Fricas [A] time = 2.1134, size = 309, normalized size = 2.81

$$\left[\frac{15\,a^{\frac{3}{2}}bx\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\left(2\,b^2x^2+14\,abx-3\,a^2\right)\sqrt{bx+a}}{6\,x},\frac{15\,\sqrt{-a}abx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(2\,b^2x^2+14\,abx-\right.}{3\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*a^(3/2)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(2*b^
2*x^2 + 14*a*b*x - 3*a^2)*sqrt(b*x + a))/x, 1/3*(15*sqrt(-a)*a*b*x*arctan(s
qrt(b*x + a)*sqrt(-a)/a) + (2*b^2*x^2 + 14*a*b*x - 3*a^2)*sqrt(b*x + a))/x]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14152, size = 120, normalized size = 1.09

$$\frac{\sqrt{2}\left(\frac{15\sqrt{2}a^2b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2\sqrt{2}(bx+a)^{\frac{3}{2}}b^2 + 12\sqrt{2}\sqrt{bx+a}ab^2 - \frac{3\sqrt{2}\sqrt{bx+a}a^2b}{x}\right)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(15*sqrt(2)*a^2*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2
*sqrt(2)*(b*x + a)^(3/2)*b^2 + 12*sqrt(2)*sqrt(b*x + a)*a*b^2 - 3*sqrt(2)*s
qrt(b*x + a)*a^2*b/x)/b
```

3.136 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^3}\,dx$

Optimal. Leaf size=110

$$\frac{15}{4}b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}-\frac{15}{4}b^2\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)-\frac{\tanh^{-1}(\tan}{2}$$

```
[Out] (-15*b^2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*
x]]]]*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]])/4 + (15*b^2*Sqrt[ArcTanh[Tanh[a +
 b*x]]])/4 - (5*b*ArcTanh[Tanh[a + b*x]]^(3/2))/(4*x) - ArcTanh[Tanh[a + b*
x]]^(5/2)/(2*x^2)
```

Rubi [A] time = 0.0667365, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2159, 2161}

$$\frac{15}{4}b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}-\frac{15}{4}b^2\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)-\frac{\tanh^{-1}(\tan}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^3,x]
```



```
[Out] (-15*b^2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*
x]]]]*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]])/4 + (15*b^2*Sqrt[ArcTanh[Tanh[a +
 b*x]]])/4 - (5*b*ArcTanh[Tanh[a + b*x]]^(3/2))/(4*x) - ArcTanh[Tanh[a + b*
x]]^(5/2)/(2*x^2)
```


Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^3}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{2x^2} + \frac{1}{4}(5b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^2}\,dx$$

$$= -\frac{5b\tanh^{-1}(\tanh(a+bx))^{3/2}}{4x} - \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{2x^2} + \frac{1}{8}\left(15b^2\right)\int \frac{\sqrt{\tanh^{-1}(\tan}}{x}$$

$$= \frac{15}{4}b^2\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{5b\tanh^{-1}(\tanh(a+bx))^{3/2}}{4x} - \frac{\tanh^{-1}(\tanh(a+bx))^{5/}}{2x^2}$$

$$= -\frac{15}{4}b^2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))} + \frac{15}{4}b^2\sqrt{\text{ta}}$$

Mathematica [A] time = 0.043991, size = 108, normalized size = 0.98

$$-\frac{-15b^2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))} + 15b^2x^2\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)\sqrt{\tanh^{-1}(\tanh(a+bx))-bx} + 2\tanh^{-1}(\text{tan}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^3,x]
```

```
[Out] -(-15*b^2*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]] + 5*b*x*ArcTanh[Tanh[a + b*x]]^(
3/2) + 2*ArcTanh[Tanh[a + b*x]]^(5/2) + 15*b^2*x^2*ArcTanh[Sqrt[ArcTanh[Tan
h[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*Sqrt[-(b*x) + ArcTanh[T
```

```
anh[a + b*x]]])/(4*x^2)
```

Maple [A] time = 0.115, size = 142, normalized size = 1.3

$$2b^2\left(\sqrt{\operatorname{Artanh}(\tanh(bx+a))}+\frac{1}{b^2x^2}\left(\left(-\frac{9\operatorname{Artanh}(\tanh(bx+a))}{8}+\frac{9bx}{8}\right)(\operatorname{Artanh}(\tanh(bx+a)))^{3/2}+\left(\frac{7a^2}{8}+7/\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^3,x)
```

```
[Out] 2*b^2*(arctanh(tanh(b*x+a))^(1/2)+((-9/8*arctanh(tanh(b*x+a))+9/8*b*x)*arct
anh(tanh(b*x+a))^(3/2)+(7/8*a^2+7/4*a*(arctanh(tanh(b*x+a))-b*x-a)+7/8*(arc
tanh(tanh(b*x+a))-b*x-a)^2)*arctanh(tanh(b*x+a))^(1/2))/b^2/x^2-15/8*(arcta
nh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh
(b*x+a))-b*x)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(\tanh(bx+a))^{\frac{5}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(5/2)/x^3, x)
```

Fricas [A] time = 2.11054, size = 320, normalized size = 2.91

$$\left[\frac{15\sqrt{a}b^2x^2\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(8b^2x^2-9abx-2a^2\right)\sqrt{bx+a}}{8x^2},\frac{15\sqrt{-a}b^2x^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(8b^2x^2-9abx\right.}{4x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(
8*b^2*x^2 - 9*a*b*x - 2*a^2)*sqrt(b*x + a))/x^2, 1/4*(15*sqrt(-a)*b^2*x^2*a
rctan(sqrt(b*x + a)*sqrt(-a)/a) + (8*b^2*x^2 - 9*a*b*x - 2*a^2)*sqrt(b*x +
a))/x^2]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15224, size = 124, normalized size = 1.13

$$\frac{\sqrt{2}\left(\frac{15\sqrt{2}ab^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+8\sqrt{2}\sqrt{bx+a}b^3-\frac{\sqrt{2}\left(9(bx+a)^{\frac{3}{2}}ab^3-7\sqrt{bx+a}a^2b^3\right)}{b^2x^2}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(15*sqrt(2)*a*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 8*s
qrt(2)*sqrt(b*x + a)*b^3 - sqrt(2)*(9*(b*x + a)^(3/2)*a*b^3 - 7*sqrt(b*x +
a)*a^2*b^3)/(b^2*x^2))/b
```

3.137 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^4}\,dx$

Optimal. Leaf size=113

$$-\frac{5b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{8x}+\frac{5b^3\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{5b\tanh^{-1}(\tanh(a+bx))^{3/2}}{12x^2}-\frac{\tanh^{-1}(\tanh(a+b$$

```
[Out] (5*b^3*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]
]]])/(8*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]) - (5*b^2*Sqrt[ArcTanh[Tanh[a +
b*x]]])/(8*x) - (5*b*ArcTanh[Tanh[a + b*x]]^(3/2))/(12*x^2) - ArcTanh[Tanh[
a + b*x]]^(5/2)/(3*x^3)
```

Rubi [A] time = 0.0675483, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 2161}

$$-\frac{5b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{8x}+\frac{5b^3\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{5b\tanh^{-1}(\tanh(a+bx))^{3/2}}{12x^2}-\frac{\tanh^{-1}(\tanh(a+b$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^4,x]
```


```
[Out] (5*b^3*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]
]]])/(8*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]) - (5*b^2*Sqrt[ArcTanh[Tanh[a +
b*x]]])/(8*x) - (5*b*ArcTanh[Tanh[a + b*x]]^(3/2))/(12*x^2) - ArcTanh[Tanh[
a + b*x]]^(5/2)/(3*x^3)
```


Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
```

```
[m, 0] &&  !IntegerQ[n]))
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^4}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{3x^3} + \frac{1}{6}(5b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^3}\,dx$$

$$= -\frac{5b\tanh^{-1}(\tanh(a+bx))^{3/2}}{12x^2} - \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{3x^3} + \frac{1}{8}\left(5b^2\right)\int \frac{\sqrt{\tanh^{-1}(\tanh}}{x^2}$$

$$= -\frac{5b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{8x} - \frac{5b\tanh^{-1}(\tanh(a+bx))^{3/2}}{12x^2} - \frac{\tanh^{-1}(\tanh(a+bx))^{5}}{3x^3}$$

$$= \frac{5b^3\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}} - \frac{5b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{8x} - \frac{5b\tanh^{-1}(\tanh(a}{12x^2}$$

Mathematica [A] time = 0.0703738, size = 107, normalized size = 0.95

$$\frac{1}{24}\left(-\frac{15b^3\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}} - \frac{15b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x} - \frac{10b\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^2} - \frac{8\tanh^{-1}(\mathrm{ta}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^4,x]
```

```
[Out] ((-15*b^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/x - (10*b*ArcTanh[Tanh[a + b*x]]^(3
/2))/x^2 - (8*ArcTanh[Tanh[a + b*x]]^(5/2))/x^3 - (15*b^3*ArcTanh[Sqrt[ArcT
anh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]])/Sqrt[-(b*x) + A
rcTanh[Tanh[a + b*x]]])/24
```

Maple [A] time = 0.118, size = 144, normalized size = 1.3

$$2\,b^{3}\left(\frac{1}{x^{3}b^{3}}\left(-\frac{11\,(\mathrm{Artanh}\,(\tanh\,(bx+a)))^{5/2}}{16}+(5/6\,\mathrm{Artanh}\,(\tanh\,(bx+a))-5/6\,bx)\,(\mathrm{Artanh}\,(\tanh\,(bx+a)))^{3/2}+\left(-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^4,x)
```

```
[Out] 2*b^3*((-11/16*arctanh(tanh(b*x+a))^(5/2)+(5/6*arctanh(tanh(b*x+a))-5/6*b*x
)*arctanh(tanh(b*x+a))^(3/2)+(-5/16*a^2-5/8*a*(arctanh(tanh(b*x+a))-b*x-a)-
5/16*(arctanh(tanh(b*x+a))-b*x-a)^2)*arctanh(tanh(b*x+a))^(1/2))/b^3/x^3-5/
16/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arc
tanh(tanh(b*x+a))-b*x)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{artanh}\,(\tanh\,(bx+a))^{\frac{5}{2}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(5/2)/x^4, x)
```

Fricas [A] time = 2.12887, size = 350, normalized size = 3.1

$$\left[\frac{15\,\sqrt{a}b^{3}x^{3}\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)-2\left(33\,ab^{2}x^{2}+26\,a^{2}bx+8\,a^{3}\right)\sqrt{bx+a}}{48\,ax^{3}},\frac{15\,\sqrt{-a}b^{3}x^{3}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-\left(33\,ab^{2}x^{2}\right.}{24\,ax^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(a)*b^3*x^3*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*
(33*a*b^2*x^2 + 26*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a*x^3), 1/24*(15*sqrt(-
a)*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) - (33*a*b^2*x^2 + 26*a^2*b*x +
8*a^3)*sqrt(b*x + a))/(a*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17132, size = 119, normalized size = 1.05

$$\frac{\sqrt{2}\left(\frac{15\,\sqrt{2}b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\sqrt{2}\left(33\,(bx+a)^{\frac{5}{2}}b^4-40\,(bx+a)^{\frac{3}{2}}ab^4+15\,\sqrt{bx+a}a^2b^4\right)}{b^3x^3}\right)}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/48*sqrt(2)*(15*sqrt(2)*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - sqrt
(2)*(33*(b*x + a)^(5/2)*b^4 - 40*(b*x + a)^(3/2)*a*b^4 + 15*sqrt(b*x + a)*a
^2*b^4)/(b^3*x^3))/b
```

3.138 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^5} dx$

Optimal. Leaf size=167

$$-\frac{5b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{32x^2}+\frac{5b^4}{64\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{5b^3}{64x\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] (5*b^4*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]
]]])/(64*(b*x - ArcTanh[Tanh[a + b*x]])^(3/2)) - (5*b^3)/(64*x*Sqrt[ArcTanh
[Tanh[a + b*x]]]) + (5*b^4)/(64*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh
[Tanh[a + b*x]]]) - (5*b^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(32*x^2) - (5*b*Ar
cTanh[Tanh[a + b*x]]^(3/2))/(24*x^3) - ArcTanh[Tanh[a + b*x]]^(5/2)/(4*x^4)
```

Rubi [A] time = 0.116921, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2163, 2161}

$$-\frac{5b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{32x^2}+\frac{5b^4}{64\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{5b^3}{64x\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^5,x]
```


```
[Out] (5*b^4*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]
]]])/(64*(b*x - ArcTanh[Tanh[a + b*x]])^(3/2)) - (5*b^3)/(64*x*Sqrt[ArcTanh
[Tanh[a + b*x]]]) + (5*b^4)/(64*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh
[Tanh[a + b*x]]]) - (5*b^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(32*x^2) - (5*b*Ar
cTanh[Tanh[a + b*x]]^(3/2))/(24*x^3) - ArcTanh[Tanh[a + b*x]]^(5/2)/(4*x^4)
```


Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
```

```
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^5}\,dx &= -\frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{4x^4}+\frac{1}{8}(5b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^4}\,dx \\
&= -\frac{5b\tanh^{-1}(\tanh(a+bx))^{3/2}}{24x^3}-\frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{4x^4}+\frac{1}{16}\left(5b^2\right)\int \frac{\sqrt{\tanh^{-1}(\tan}}{x^3} \\
&= -\frac{5b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{32x^2}-\frac{5b\tanh^{-1}(\tanh(a+bx))^{3/2}}{24x^3}-\frac{\tanh^{-1}(\tanh(a+bx))^{5}}{4x^4} \\
&= -\frac{5b^3}{64x\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{5b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{32x^2}-\frac{5b\tanh^{-1}(\tanh(a+b}{24x^3} \\
&= -\frac{5b^3}{64x\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{5b^4}{64\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+b}} \\
&= \frac{5b^4\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{64\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}-\frac{5b^3}{64x\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{}{64\left(bx-\tanh^{-1}(\right.}
\end{aligned}$$

Mathematica [A] time = 0.11106, size = 134, normalized size = 0.8

$$\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{64\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{3/2}}-\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(10b^2x^2\tanh^{-1}(\tanh(a+bx))+8bx\tanh^{-1}(\tanh(a+b\right.}{192x^4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^5,x]
```

```
[Out] (5*b^4*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a +
b*x]]]])/(64*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2)) - (Sqrt[ArcTanh[Tanh[
a + b*x]]]*(15*b^3*x^3 + 10*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 8*b*x*ArcTanh[
Tanh[a + b*x]]^2 - 48*ArcTanh[Tanh[a + b*x]]^3))/(192*x^4*(b*x - ArcTanh[Ta
nh[a + b*x]]))
```

Maple [A] time = 0.118, size = 169, normalized size = 1.

$$2\,b^4\left(\frac{1}{x^4b^4}\left(-\frac{5\left(\operatorname{Artanh}(\tanh(bx+a))\right)^{7/2}}{128\operatorname{Artanh}(\tanh(bx+a))-128\,bx}-\frac{73\left(\operatorname{Artanh}(\tanh(bx+a))\right)^{5/2}}{384}+\left(\frac{55\operatorname{Artanh}(\tanh(bx+a))}{384}-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^5,x)
```

```
[Out] 2*b^4*((-5/128/(arctanh(tanh(b*x+a))-b*x)*arctanh(tanh(b*x+a))^(7/2)-73/384
*arctanh(tanh(b*x+a))^(5/2)+(55/384*arctanh(tanh(b*x+a))-55/384*b*x)*arctan
h(tanh(b*x+a))^(3/2)+(-5/128*a^2-5/64*a*(arctanh(tanh(b*x+a))-b*x-a)-5/128*
(arctanh(tanh(b*x+a))-b*x-a)^2)*arctanh(tanh(b*x+a))^(1/2))/b^4/x^4+5/128/(
arctanh(tanh(b*x+a))-b*x)^(3/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh
(tanh(b*x+a))-b*x)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(\tanh(bx+a))^{\frac{5}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(5/2)/x^5, x)
```

Fricas [A] time = 2.16081, size = 413, normalized size = 2.47

$$\left[\frac{15\sqrt{a}b^4x^4\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(15ab^3x^3+118a^2b^2x^2+136a^3bx+48a^4\right)\sqrt{bx+a}}{384a^2x^4}, -\frac{15\sqrt{-a}b^4x^4\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/384*(15*sqrt(a)*b^4*x^4*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2
*(15*a*b^3*x^3 + 118*a^2*b^2*x^2 + 136*a^3*b*x + 48*a^4)*sqrt(b*x + a))/(a^
2*x^4), -1/192*(15*sqrt(-a)*b^4*x^4*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (15*
a*b^3*x^3 + 118*a^2*b^2*x^2 + 136*a^3*b*x + 48*a^4)*sqrt(b*x + a))/(a^2*x^4
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16573, size = 146, normalized size = 0.87

$$-\frac{\sqrt{2}\left(\frac{15\sqrt{2}b^5\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{\sqrt{2}\left(15(bx+a)^{\frac{7}{2}}b^5+73(bx+a)^{\frac{5}{2}}ab^5-55(bx+a)^{\frac{3}{2}}a^2b^5+15\sqrt{bx+a}a^3b^5\right)}{ab^4x^4}\right)}{384b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/384*sqrt(2)*(15*sqrt(2)*b^5*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a)
+ sqrt(2)*(15*(b*x + a)^(7/2)*b^5 + 73*(b*x + a)^(5/2)*a*b^5 - 55*(b*x + a)
^(3/2)*a^2*b^5 + 15*sqrt(b*x + a)*a^3*b^5)/(a*b^4*x^4))/b
```

3.139 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^6}\,dx$

Optimal. Leaf size=221

$$-\frac{b^3}{64x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{16x^3}+\frac{3b^5}{128\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] (3*b^5*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]
]]])/(128*(b*x - ArcTanh[Tanh[a + b*x]])^(5/2)) + b^4/(128*x*ArcTanh[Tanh[a
 + b*x]]^(3/2)) - b^5/(128*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a +
b*x]]^(3/2)) - b^3/(64*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (3*b^5)/(128*(b*
x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b*x]]]) - (b^2*Sqrt[Arc
Tanh[Tanh[a + b*x]]])/(16*x^3) - (b*ArcTanh[Tanh[a + b*x]]^(3/2))/(8*x^4) -
 ArcTanh[Tanh[a + b*x]]^(5/2)/(5*x^5)
```

Rubi [A] time = 0.174042, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2163, 2161}

$$-\frac{b^3}{64x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{16x^3}+\frac{3b^5}{128\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^6,x]
```

```
[Out] (3*b^5*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]
]]])/(128*(b*x - ArcTanh[Tanh[a + b*x]])^(5/2)) + b^4/(128*x*ArcTanh[Tanh[a
 + b*x]]^(3/2)) - b^5/(128*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a +
b*x]]^(3/2)) - b^3/(64*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (3*b^5)/(128*(b*
x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b*x]]]) - (b^2*Sqrt[Arc
Tanh[Tanh[a + b*x]]])/(16*x^3) - (b*ArcTanh[Tanh[a + b*x]]^(3/2))/(8*x^4) -
 ArcTanh[Tanh[a + b*x]]^(5/2)/(5*x^5)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^6}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{5x^5} + \frac{1}{2}b\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^5}\,dx$$

$$= -\frac{b\tanh^{-1}(\tanh(a+bx))^{3/2}}{8x^4} - \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{5x^5} + \frac{1}{16}\left(3b^2\right)\int \frac{\sqrt{\tanh^{-1}(\tanh}}{x^4}$$

$$= -\frac{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{16x^3} - \frac{b\tanh^{-1}(\tanh(a+bx))^{3/2}}{8x^4} - \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{5x^5}$$

$$= -\frac{b^3}{64x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{16x^3} - \frac{b\tanh^{-1}(\tanh(a+bx)}{8x^4}$$

$$= \frac{b^4}{128x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{b^3}{64x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{b^2\sqrt{\tanh^{-1}(\tanh(a}}{16x^3}$$

$$= \frac{b^4}{128x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{b^5}{128\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+b}$$

$$= \frac{b^4}{128x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{b^5}{128\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+b}$$

$$= \frac{3b^5\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{128\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}} + \frac{b^4}{128x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{}{128\left(bx-\tanh\right.}$$

<u>Mathematica</u> [A] time = 0.12318, size = 150, normalized size = 0.68

$$\frac{1}{640}\left(-\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(10b^3x^3\tanh^{-1}(\tanh(a+bx)) + 8b^2x^2\tanh^{-1}(\tanh(a+bx))^2 - 176bx\tanh^{-1}(\tanh(a+\right.}{x^5\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^6,x]
```

```
[Out] ((-15*b^5*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a
 + b*x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2) - (Sqrt[ArcTanh[Tanh[a
```

```
+ b*x]]]*(15*b^4*x^4 + 10*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 8*b^2*x^2*ArcTan
h[Tanh[a + b*x]]^2 - 176*b*x*ArcTanh[Tanh[a + b*x]]^3 + 128*ArcTanh[Tanh[a
+ b*x]]^4))/(x^5*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2))/640
```

Maple [A] time = 0.122, size = 262, normalized size = 1.2

$$2\,b^5\left(\frac{1}{b^5x^5}\left(\frac{3\,(\mathrm{Artanh}\,(\tanh\,(bx+a)))^{9/2}}{256\,a^2+512\,a\,(\mathrm{Artanh}\,(\tanh\,(bx+a))-bx-a)+256\,(\mathrm{Artanh}\,(\tanh\,(bx+a))-bx-a)^2}-\frac{7\,(\mathrm{Artan}}{128\,\mathrm{Artanh}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^6,x)
```

```
[Out] 2*b^5*((3/256/(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b
*x-a)^2)*arctanh(tanh(b*x+a))^(9/2)-7/128/(arctanh(tanh(b*x+a))-b*x)*arctan
h(tanh(b*x+a))^(7/2)-1/10*arctanh(tanh(b*x+a))^(5/2)+(7/128*arctanh(tanh(b*
x+a))-7/128*b*x)*arctanh(tanh(b*x+a))^(3/2)+(-3/256*a^2-3/128*a*(arctanh(ta
nh(b*x+a))-b*x-a)-3/256*(arctanh(tanh(b*x+a))-b*x-a)^2)*arctanh(tanh(b*x+a)
)^(1/2))/b^5/x^5-3/256/(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(
b*x+a))-b*x-a)^2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x
+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{artanh}\,(\tanh\,(bx+a))^{\frac{5}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(5/2)/x^6, x)
```

Fricas [A] time = 2.28065, size = 462, normalized size = 2.09

$$\left[\frac{15\,\sqrt{a}b^5x^5\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\left(15\,ab^4x^4-10\,a^2b^3x^3-248\,a^3b^2x^2-336\,a^4bx-128\,a^5\right)\sqrt{bx+a}}{1280\,a^3x^5},\ \frac{15\,\sqrt{-a}b^5x^5\,\mathrm{arc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/1280*(15*sqrt(a)*b^5*x^5*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) +
2*(15*a*b^4*x^4 - 10*a^2*b^3*x^3 - 248*a^3*b^2*x^2 - 336*a^4*b*x - 128*a^5)
*sqrt(b*x + a))/(a^3*x^5), 1/640*(15*sqrt(-a)*b^5*x^5*arctan(sqrt(b*x + a)*
sqrt(-a)/a) + (15*a*b^4*x^4 - 10*a^2*b^3*x^3 - 248*a^3*b^2*x^2 - 336*a^4*b*
x - 128*a^5)*sqrt(b*x + a))/(a^3*x^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16527, size = 166, normalized size = 0.75

$$\frac{\sqrt{2}\left(\frac{15\,\sqrt{2}b^6\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{\sqrt{2}\left(15\,(bx+a)^{\frac{9}{2}}b^6-70\,(bx+a)^{\frac{7}{2}}ab^6-128\,(bx+a)^{\frac{5}{2}}a^2b^6+70\,(bx+a)^{\frac{3}{2}}a^3b^6-15\,\sqrt{bx+a}a^4b^6\right)}{a^2b^5x^5}\right)}{1280\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/1280*sqrt(2)*(15*sqrt(2)*b^6*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2
) + sqrt(2)*(15*(b*x + a)^(9/2)*b^6 - 70*(b*x + a)^(7/2)*a*b^6 - 128*(b*x +
 a)^(5/2)*a^2*b^6 + 70*(b*x + a)^(3/2)*a^3*b^6 - 15*sqrt(b*x + a)*a^4*b^6)/
(a^2*b^5*x^5))/b
```

3.140 $\int \frac{x^4}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=99

$$\frac{32x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^3} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2} + \frac{256\tanh^{-1}(\tanh(a+bx))^{9/2}}{315b^5} - \frac{128x\tanh^{-1}(\tanh(a}{35b^4}$$

```
[Out] (2*x^4*Sqrt[ArcTanh[Tanh[a + b*x]]])/b - (16*x^3*ArcTanh[Tanh[a + b*x]]^(3/
2))/(3*b^2) + (32*x^2*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b^3) - (128*x*ArcTan
h[Tanh[a + b*x]]^(7/2))/(35*b^4) + (256*ArcTanh[Tanh[a + b*x]]^(9/2))/(315*
b^5)
```

Rubi [A] time = 0.0653643, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2157, 30}

$$\frac{32x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^3} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2} + \frac{256\tanh^{-1}(\tanh(a+bx))^{9/2}}{315b^5} - \frac{128x\tanh^{-1}(\tanh(a}{35b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```



```
[Out] (2*x^4*Sqrt[ArcTanh[Tanh[a + b*x]]])/b - (16*x^3*ArcTanh[Tanh[a + b*x]]^(3/
2))/(3*b^2) + (32*x^2*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b^3) - (128*x*ArcTan
h[Tanh[a + b*x]]^(7/2))/(35*b^4) + (256*ArcTanh[Tanh[a + b*x]]^(9/2))/(315*
b^5)
```


Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{2x^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{8\int x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$

$$= \frac{2x^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2} + \frac{16\int x^2\tanh^{-1}(\tanh(a}{b^2}$$

$$= \frac{2x^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2} + \frac{32x^2\tanh^{-1}(\tanh(a+}{5b^3}$$

$$= \frac{2x^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2} + \frac{32x^2\tanh^{-1}(\tanh(a+}{5b^3}$$

$$= \frac{2x^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2} + \frac{32x^2\tanh^{-1}(\tanh(a+}{5b^3}$$

$$= \frac{2x^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2} + \frac{32x^2\tanh^{-1}(\tanh(a+}{5b^3}$$

Mathematica [A] time = 0.0427971, size = 83, normalized size = 0.84

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-840b^3x^3\tanh^{-1}(\tanh(a+bx)) + 1008b^2x^2\tanh^{-1}(\tanh(a+bx))^2 - 576bx\tanh^{-1}(\tanh(a\right.}{315b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*Sqrt[ArcTanh[Tanh[a + b*x]]]*(315*b^4*x^4 - 840*b^3*x^3*ArcTanh[Tanh[a +
 b*x]] + 1008*b^2*x^2*ArcTanh[Tanh[a + b*x]]^2 - 576*b*x*ArcTanh[Tanh[a + b
*x]]^3 + 128*ArcTanh[Tanh[a + b*x]]^4))/(315*b^5)
```

Maple [A] time = 0.061, size = 153, normalized size = 1.6

$$2\,\frac{1/9\,\left(\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{9/2}+1/7\,\left(-4\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)+4\,bx\right)\left(\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{7/2}+1/5\,\left(2\,(bx-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 2/b^5*(1/9*arctanh(tanh(b*x+a))^(9/2)+1/7*(-4*arctanh(tanh(b*x+a))+4*b*x)*a
rctanh(tanh(b*x+a))^(7/2)+1/5*(2*(b*x-arctanh(tanh(b*x+a)))^2+(-2*arctanh(t
anh(b*x+a))+2*b*x)^2)*arctanh(tanh(b*x+a))^(5/2)+2/3*(b*x-arctanh(tanh(b*x+
a)))^2*(-2*arctanh(tanh(b*x+a))+2*b*x)*arctanh(tanh(b*x+a))^(3/2)+(b*x-arct
anh(tanh(b*x+a)))^4*arctanh(tanh(b*x+a))^(1/2))
```

Maxima [A] time = 1.79655, size = 86, normalized size = 0.87

$$\frac{2\left(35\,b^5x^5-5\,ab^4x^4+8\,a^2b^3x^3-16\,a^3b^2x^2+64\,a^4bx+128\,a^5\right)}{315\,\sqrt{bx+a}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*b^5*x^5 - 5*a*b^4*x^4 + 8*a^2*b^3*x^3 - 16*a^3*b^2*x^2 + 64*a^4*b
*x + 128*a^5)/(sqrt(b*x + a)*b^5)
```

Fricas [A] time = 2.08191, size = 126, normalized size = 1.27

$$\frac{2\left(35\,b^4x^4-40\,ab^3x^3+48\,a^2b^2x^2-64\,a^3bx+128\,a^4\right)\sqrt{bx+a}}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 - 40*a*b^3*x^3 + 48*a^2*b^2*x^2 - 64*a^3*b*x + 128*a^4)*s
qrt(b*x + a)/b^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(atanh(tanh(a + b*x))), x)
```

Giac [A] time = 1.16333, size = 82, normalized size = 0.83

$$\frac{2\left(35\left(bx+a\right)^{\frac{9}{2}}-180\left(bx+a\right)^{\frac{7}{2}}a+378\left(bx+a\right)^{\frac{5}{2}}a^2-420\left(bx+a\right)^{\frac{3}{2}}a^3+315\sqrt{bx+a}a^4\right)}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2
 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)/b^5
```

3.141 $\int \frac{x^3}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=76

$$-\frac{4x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{b^2}-\frac{32\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^4}+\frac{16x\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^3}+\frac{2x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}$$

```
[Out] (2*x^3*Sqrt[ArcTanh[Tanh[a + b*x]]])/b - (4*x^2*ArcTanh[Tanh[a + b*x]]^(3/2
))/b^2 + (16*x*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b^3) - (32*ArcTanh[Tanh[a +
 b*x]]^(7/2))/(35*b^4)
```

Rubi [A] time = 0.0481367, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2157, 30}

$$-\frac{4x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{b^2}-\frac{32\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^4}+\frac{16x\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^3}+\frac{2x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*x^3*Sqrt[ArcTanh[Tanh[a + b*x]]])/b - (4*x^2*ArcTanh[Tanh[a + b*x]]^(3/2
))/b^2 + (16*x*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b^3) - (32*ArcTanh[Tanh[a +
 b*x]]^(7/2))/(35*b^4)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{2x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{6\int x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$

$$= \frac{2x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{4x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{b^2} + \frac{8\int x\tanh^{-1}(\tanh(a+$$

$$= \frac{2x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{4x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{b^2} + \frac{16x\tanh^{-1}(\tanh(a+b}{5b^3}$$

$$= \frac{2x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{4x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{b^2} + \frac{16x\tanh^{-1}(\tanh(a+b}{5b^3}$$

$$= \frac{2x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{4x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{b^2} + \frac{16x\tanh^{-1}(\tanh(a+b}{5b^3}$$

Mathematica [A] time = 0.0371401, size = 66, normalized size = 0.87

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-70b^2x^2\tanh^{-1}(\tanh(a+bx)) + 56bx\tanh^{-1}(\tanh(a+bx))^2 - 16\tanh^{-1}(\tanh(a+bx))^3 + \right.}{35b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*Sqrt[ArcTanh[Tanh[a + b*x]]]*(35*b^3*x^3 - 70*b^2*x^2*ArcTanh[Tanh[a + b
*x]] + 56*b*x*ArcTanh[Tanh[a + b*x]]^2 - 16*ArcTanh[Tanh[a + b*x]]^3))/(35*
b^4)
```

Maple [A] time = 0.063, size = 123, normalized size = 1.6

$$2\,\frac{1/7\,(\mathrm{Artanh}(\tanh(bx+a)))^{7/2} + 1/5\,(-3\,\mathrm{Artanh}(\tanh(bx+a)) + 3\,bx)\,(\mathrm{Artanh}(\tanh(bx+a)))^{5/2} + 1/3\,\left((bx - \mathrm{Ar}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 2/b^4*(1/7*arctanh(tanh(b*x+a))^(7/2)+1/5*(-3*arctanh(tanh(b*x+a))+3*b*x)*a
rctanh(tanh(b*x+a))^(5/2)+1/3*((b*x-arctanh(tanh(b*x+a)))*(-2*arctanh(tanh(
b*x+a))+2*b*x)+(b*x-arctanh(tanh(b*x+a)))^2)*arctanh(tanh(b*x+a))^(3/2)+(b*
x-arctanh(tanh(b*x+a)))^3*arctanh(tanh(b*x+a))^(1/2))
```

Maxima [A] time = 1.80731, size = 72, normalized size = 0.95

$$\frac{2\left(5\,b^4x^4 - ab^3x^3 + 2\,a^2b^2x^2 - 8\,a^3bx - 16\,a^4\right)}{35\,\sqrt{bx+a}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*b^4*x^4 - a*b^3*x^3 + 2*a^2*b^2*x^2 - 8*a^3*b*x - 16*a^4)/(sqrt(b*x
 + a)*b^4)
```

Fricas [A] time = 2.0453, size = 96, normalized size = 1.26

$$\frac{2\left(5\,b^3x^3 - 6\,ab^2x^2 + 8\,a^2bx - 16\,a^3\right)\sqrt{bx+a}}{35\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^3 - 6*a*b^2*x^2 + 8*a^2*b*x - 16*a^3)*sqrt(b*x + a)/b^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(atanh(tanh(a + b*x))), x)
```

Giac [A] time = 1.16973, size = 66, normalized size = 0.87

$$\frac{2\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)}{35\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2/35*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 3
5*sqrt(b*x + a)*a^3)/b^4
```

3.142 $\int \frac{x^2}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=57

$$\frac{16\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^3} - \frac{8x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2} + \frac{2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}$$

```
[Out] (2*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/b - (8*x*ArcTanh[Tanh[a + b*x]]^(3/2))
/(3*b^2) + (16*ArcTanh[Tanh[a + b*x]]^(5/2))/(15*b^3)
```

Rubi [A] time = 0.0307876, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2157, 30}

$$\frac{16\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^3} - \frac{8x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2} + \frac{2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/b - (8*x*ArcTanh[Tanh[a + b*x]]^(3/2))
/(3*b^2) + (16*ArcTanh[Tanh[a + b*x]]^(5/2))/(15*b^3)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{4\int x\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$

$$= \frac{2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{8x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2} + \frac{8\int \tanh^{-1}(\tanh(a+bx}{3b^2}$$

$$= \frac{2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{8x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2} + \frac{8\,\mathrm{Subst}\left(\int x^{3/2}\,dx, x, \tan\right.}{3b^3}$$

$$= \frac{2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{8x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2} + \frac{16\tanh^{-1}(\tanh(a+bx))}{15b^3}$$

Mathematica [A] time = 0.0337379, size = 49, normalized size = 0.86

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-20bx\tanh^{-1}(\tanh(a+bx)) + 8\tanh^{-1}(\tanh(a+bx))^2 + 15b^2x^2\right)}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*Sqrt[ArcTanh[Tanh[a + b*x]]]*(15*b^2*x^2 - 20*b*x*ArcTanh[Tanh[a + b*x]]
 + 8*ArcTanh[Tanh[a + b*x]]^2))/(15*b^3)
```

Maple [A] time = 0.06, size = 68, normalized size = 1.2

$$2\,\frac{1/5\,(\mathrm{Artanh}\,(\tanh\,(bx+a)))^{5/2} + 1/3\,(-2\,\mathrm{Artanh}\,(\tanh\,(bx+a)) + 2\,bx)\,(\mathrm{Artanh}\,(\tanh\,(bx+a)))^{3/2} + (bx - \mathrm{Artanh}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 2/b^3*(1/5*arctanh(tanh(b*x+a))^(5/2)+1/3*(-2*arctanh(tanh(b*x+a))+2*b*x)*a
rctanh(tanh(b*x+a))^(3/2)+(b*x-arctanh(tanh(b*x+a)))^2*arctanh(tanh(b*x+a))
^(1/2))
```

Maxima [A] time = 1.80668, size = 57, normalized size = 1.

$$\frac{2\left(3\,b^3x^3 - ab^2x^2 + 4\,a^2bx + 8\,a^3\right)}{15\,\sqrt{bx+a}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*b^3*x^3 - a*b^2*x^2 + 4*a^2*b*x + 8*a^3)/(sqrt(b*x + a)*b^3)
```

Fricas [A] time = 2.37504, size = 73, normalized size = 1.28

$$\frac{2\left(3\,b^2x^2 - 4\,abx + 8\,a^2\right)\sqrt{bx+a}}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^2 - 4*a*b*x + 8*a^2)*sqrt(b*x + a)/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(atanh(tanh(a + b*x))), x)
```

Giac [A] time = 1.18468, size = 50, normalized size = 0.88

$$\frac{2\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^{2}\right)}{15\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)/b^3
```

3.143 $\int \frac{x}{\sqrt{\tanh^{-1}(\tanh(a+bx))}} dx$

Optimal. Leaf size=36

$$\frac{2x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}-\frac{4\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2}$$

```
[Out] (2*x*Sqrt[ArcTanh[Tanh[a + b*x]]])/b - (4*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*
b^2)
```

Rubi [A] time = 0.0145474, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2157, 30}

$$\frac{2x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}-\frac{4\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*x*Sqrt[ArcTanh[Tanh[a + b*x]]])/b - (4*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*
b^2)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{2x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{2\int \sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$

$$= \frac{2x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{2\,\mathrm{Subst}\left(\int \sqrt{x}\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b^2}$$

$$= \frac{2x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{4\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2}$$

Mathematica [A] time = 0.0514767, size = 32, normalized size = 0.89

$$\frac{2\left(3bx - 2\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*(3*b*x - 2*ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/(3*b^2)
```

Maple [A] time = 0.062, size = 56, normalized size = 1.6

$$2\,\frac{1/3\,(\mathrm{Artanh}\,(\tanh\,(bx+a)))^{3/2} - a\sqrt{\mathrm{Artanh}\,(\tanh\,(bx+a))} - (\mathrm{Artanh}\,(\tanh\,(bx+a)) - bx - a)\sqrt{\mathrm{Artanh}\,(\tanh\,(bx}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 2/b^2*(1/3*arctanh(tanh(b*x+a))^(3/2)-a*arctanh(tanh(b*x+a))^(1/2)-(arctanh
(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/2))
```

Maxima [A] time = 1.77387, size = 41, normalized size = 1.14

$$\frac{2\left(b^2x^2 - abx - 2\,a^2\right)}{3\,\sqrt{bx+a}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b^2*x^2 - a*b*x - 2*a^2)/(sqrt(b*x + a)*b^2)
```

Fricas [A] time = 2.33754, size = 47, normalized size = 1.31

$$\frac{2\,\sqrt{bx+a}(bx-2\,a)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x + a)*(b*x - 2*a)/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(x/sqrt(atanh(tanh(a + b*x))), x)
```

Giac [A] time = 1.14714, size = 31, normalized size = 0.86

$$\frac{2\left((bx+a)^{\frac{3}{2}} - 3\,\sqrt{bx+a}a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)/b^2
```

3.144 $\int \frac{1}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=16

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}$$

```
[Out] (2*Sqrt[ArcTanh[Tanh[a + b*x]]])/b
```

Rubi [A] time = 0.0044057, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2157, 30}

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*Sqrt[ArcTanh[Tanh[a + b*x]]])/b
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}$$
$$= \frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}$$

Mathematica [A] time = 0.006343, size = 16, normalized size = 1.

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*Sqrt[ArcTanh[Tanh[a + b*x]]])/b
```

Maple [A] time = 0.032, size = 15, normalized size = 0.9

$$2\,\frac{\sqrt{\text{Artanh}\,(\tanh\,(bx+a))}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 2*arctanh(tanh(b*x+a))^(1/2)/b
```

Maxima [A] time = 1.70043, size = 16, normalized size = 1.

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b*x + a)/b
```

Fricas [A] time = 2.05399, size = 26, normalized size = 1.62

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x + a)/b
```

Sympy [A] time = 18.5199, size = 24, normalized size = 1.5

$$\begin{cases} \frac{2\sqrt{\operatorname{atanh}(\tanh(a+bx))}}{b} & \text{for}\ b \neq 0 \\ \frac{x}{\sqrt{\operatorname{atanh}(\tanh(a))}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(atanh(tanh(a + b*x)))/b, Ne(b, 0)), (x/sqrt(atanh(tanh(a)
)), True))
```

Giac [A] time = 1.16846, size = 16, normalized size = 1.

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*x + a)/b
```

3.145 $\int \frac{1}{x\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=49

$$\frac{2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] (2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])
/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]
```

Rubi [A] time = 0.0157421, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2161}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])
/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}$$

Mathematica [A] time = 0.0468943, size = 47, normalized size = 0.96

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x
]]]]])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]
```

Maple [A] time = 0.136, size = 42, normalized size = 0.9

$$-2\,\frac{1}{\sqrt{\mathrm{Artanh}\,(\tanh\,(bx+a))-bx}}\mathrm{Artanh}\left(\frac{\sqrt{\mathrm{Artanh}\,(\tanh\,(bx+a))}}{\sqrt{\mathrm{Artanh}\,(\tanh\,(bx+a))-bx}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] -2/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arc
tanh(tanh(b*x+a))-b*x)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\mathrm{artanh}\,(\tanh\,(bx+a))}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sqrt(arctanh(tanh(b*x + a)))), x)
```

Fricas [A] time = 2.10696, size = 142, normalized size = 2.9

$$\left[\frac{\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)/sqrt(a), 2*sqrt(-a)*arctan(sq
rt(b*x + a)*sqrt(-a)/a)/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(atanh(tanh(a + b*x)))), x)
```

Giac [A] time = 1.12325, size = 28, normalized size = 0.57

$$\frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)
```

3.146 $\int \frac{1}{x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=94

$$\frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{1}{x\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{b\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}$$

```
[Out] (b*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])
/(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) - 1/(x*Sqrt[ArcTanh[Tanh[a + b*x]]])
+ b/((b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rubi [A] time = 0.0528121, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2163, 2161}

$$\frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{1}{x\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{b\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (b*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])
/(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) - 1/(x*Sqrt[ArcTanh[Tanh[a + b*x]]])
+ b/((b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = -\frac{1}{x\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{1}{2}b\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$$

$$= -\frac{1}{x\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx}$$

$$= \frac{b\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}} - \frac{1}{x\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{}{\left(bx-\tanh^{-1}(\tanh\right.}$$

Mathematica [A] time = 0.0559497, size = 78, normalized size = 0.83

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{3/2}} - \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x\left(\tanh^{-1}(\tanh(a+bx))-bx\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (b*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]
]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2) - Sqrt[ArcTanh[Tanh[a + b*x]]]
```

```
/(x*(-(b*x) + ArcTanh[Tanh[a + b*x]]))
```

Maple [A] time = 0.138, size = 95, normalized size = 1.

$$2\,b\left(2\,\frac{\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}}{\left(-4\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)+4\,bx\right)bx}-2\,\frac{1}{\left(-4\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)+4\,bx\right)\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx}}\mathrm{Art}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 2*b*(2*arctanh(tanh(b*x+a))^(1/2)/(-4*arctanh(tanh(b*x+a))+4*b*x)/b/x-2/(-4
*arctanh(tanh(b*x+a))+4*b*x)/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arcta
nh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*sqrt(arctanh(tanh(b*x + a)))), x)
```

Fricas [A] time = 2.15636, size = 232, normalized size = 2.47

$$\left[\frac{\sqrt{a}bx\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)-2\,\sqrt{bx+a}a}{2\,a^2x},-\frac{\sqrt{-a}bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\sqrt{bx+a}a}{a^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*b*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x
 + a)*a)/(a^2*x), -(sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) + sqrt(b*
x + a)*a)/(a^2*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(atanh(tanh(a + b*x)))), x)
```

Giac [A] time = 1.14033, size = 63, normalized size = 0.67

$$-\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{\sqrt{bx+a}b}{ax}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -(b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x + a)*b/(a*x))/
b
```

3.147 $\int \frac{1}{x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=158

$$\frac{3b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}+\frac{3}{4}$$

```
[Out] (3*b^2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]
]]])/(4*(b*x - ArcTanh[Tanh[a + b*x]])^(5/2)) + b/(4*x*ArcTanh[Tanh[a + b*x
]]^(3/2)) - b^2/(4*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3
/2)) - 1/(2*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (3*b^2)/(4*(b*x - ArcTanh[T
anh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rubi [A] time = 0.0962233, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2163, 2161}

$$\frac{3b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}+\frac{3}{4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```


```
[Out] (3*b^2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]
]]])/(4*(b*x - ArcTanh[Tanh[a + b*x]])^(5/2)) + b/(4*x*ArcTanh[Tanh[a + b*x
]]^(3/2)) - b^2/(4*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3
/2)) - 1/(2*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (3*b^2)/(4*(b*x - ArcTanh[T
anh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b*x]]])
```


Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
```

```
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = -\frac{1}{2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{1}{4}b\int \frac{1}{x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$$

$$= \frac{b}{4x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{1}{2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{1}{8}\left(3b^2\right)\int \frac{}{x\tanh^{-1}(}$$

$$= \frac{b}{4x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx}$$

$$= \frac{b}{4x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx}$$

$$= \frac{3b^2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}} + \frac{b}{4x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{}{4\left(bx-\tanh^{-1}(\right.}$$

Mathematica [A] time = 0.0841114, size = 98, normalized size = 0.62

$$\frac{1}{4}\left(\frac{\left(5bx-2\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^2\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] ((-3*b^2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a
+ b*x]]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2) + ((5*b*x - 2*ArcTanh[Ta
nh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/(x^2*(-(b*x) + ArcTanh[Tanh[a +
 b*x]])^2))/4
```

Maple [A] time = 0.147, size = 148, normalized size = 0.9

$$2\,b^2\left(\frac{\sqrt{\mathrm{Artanh}\,(\tanh\,(bx+a))}}{(-4\,\mathrm{Artanh}\,(\tanh\,(bx+a))+4\,bx)\,b^2x^2}+3\,\frac{1}{-4\,\mathrm{Artanh}\,(\tanh\,(bx+a))+4\,bx}\left(2\,\frac{\sqrt{\mathrm{Artanh}\,(\tanh\,(bx+a))}}{(-4\,\mathrm{Artanh}\,(\tanh\,(bx+a))+4\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 2*b^2*(arctanh(tanh(b*x+a))^(1/2)/(-4*arctanh(tanh(b*x+a))+4*b*x)/b^2/x^2+3
/(-4*arctanh(tanh(b*x+a))+4*b*x)*(2*arctanh(tanh(b*x+a))^(1/2)/(-4*arctanh(
tanh(b*x+a))+4*b*x)/b/x-2/(-4*arctanh(tanh(b*x+a))+4*b*x)/(arctanh(tanh(b*x
+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*
x)^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{\mathrm{artanh}\,(\tanh\,(bx+a))}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^3*sqrt(arctanh(tanh(b*x + a)))), x)
```

Fricas [A] time = 2.20169, size = 301, normalized size = 1.91

$$\left[\frac{3\sqrt{a}b^2x^2\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(3abx-2a^2\right)\sqrt{bx+a}}{8a^3x^2}, \frac{3\sqrt{-a}b^2x^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(3abx-2a^2\right)\sqrt{bx+a}}{4a^3x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(3
*a*b*x - 2*a^2)*sqrt(b*x + a))/(a^3*x^2), 1/4*(3*sqrt(-a)*b^2*x^2*arctan(sq
rt(b*x + a)*sqrt(-a)/a) + (3*a*b*x - 2*a^2)*sqrt(b*x + a))/(a^3*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(atanh(tanh(a + b*x)))), x)
```

Giac [A] time = 1.16351, size = 93, normalized size = 0.59

$$\frac{\frac{3b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{3(bx+a)^{\frac{3}{2}}b^3-5\sqrt{bx+a}ab^3}{a^2b^2x^2}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x + a)^(3/
2)*b^3 - 5*sqrt(b*x + a)*a*b^3)/(a^2*b^2*x^2))/b
```

3.148 $\int \frac{1}{x^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=212

$$\frac{5b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{5b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

```
[Out] (5*b^3*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]
]]])/(8*(b*x - ArcTanh[Tanh[a + b*x]])^(7/2)) - b^2/(8*x*ArcTanh[Tanh[a + b
*x]]^(5/2)) + b^3/(8*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^
(5/2)) + b/(12*x^2*ArcTanh[Tanh[a + b*x]]^(3/2)) - (5*b^3)/(24*(b*x - ArcTa
nh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^(3/2)) - 1/(3*x^3*Sqrt[ArcTanh[
Tanh[a + b*x]]]) + (5*b^3)/(8*(b*x - ArcTanh[Tanh[a + b*x]])^3*Sqrt[ArcTanh
[Tanh[a + b*x]]])
```

Rubi [A] time = 0.150657, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2163, 2161}

$$\frac{5b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{5b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```


```
[Out] (5*b^3*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]
]]])/(8*(b*x - ArcTanh[Tanh[a + b*x]])^(7/2)) - b^2/(8*x*ArcTanh[Tanh[a + b
*x]]^(5/2)) + b^3/(8*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^
(5/2)) + b/(12*x^2*ArcTanh[Tanh[a + b*x]]^(3/2)) - (5*b^3)/(24*(b*x - ArcTa
nh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^(3/2)) - 1/(3*x^3*Sqrt[ArcTanh[
Tanh[a + b*x]]]) + (5*b^3)/(8*(b*x - ArcTanh[Tanh[a + b*x]])^3*Sqrt[ArcTanh
[Tanh[a + b*x]]])
```


Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = -\frac{1}{3x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{1}{6}b\int \frac{1}{x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$$

$$= \frac{b}{12x^2\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{1}{3x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{1}{8}b^2\int \frac{}{x^2\tanh^{-1}(\mathrm{t}}$$

$$= -\frac{b^2}{8x\tanh^{-1}(\tanh(a+bx))^{5/2}} + \frac{b}{12x^2\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{1}{3x^3\sqrt{\tanh^{-1}(\tan}}$$

$$= -\frac{b^2}{8x\tanh^{-1}(\tanh(a+bx))^{5/2}} + \frac{b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+b}$$

$$= -\frac{b^2}{8x\tanh^{-1}(\tanh(a+bx))^{5/2}} + \frac{b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+b}$$

$$= -\frac{b^2}{8x\tanh^{-1}(\tanh(a+bx))^{5/2}} + \frac{b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+b}$$

$$= \frac{5b^3\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}} - \frac{b^2}{8x\tanh^{-1}(\tanh(a+bx))^{5/2}} + \frac{}{8\left(bx-\tanh^{-1}(\right.}$$

Mathematica [A] time = 0.107486, size = 117, normalized size = 0.55

$$\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-26bx\tanh^{-1}(\tanh(a+bx))+8\tanh^{-1}(\tanh(a+bx))^2+33b^2x^2\right)}{24x^3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{5b^3\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(}}{\sqrt{\tanh^{-1}(\mathrm{ta}}}\right.}{8\left(\tanh^{-1}(\tanh(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (5*b^3*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a +
b*x]]]])/(8*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(7/2)) + (Sqrt[ArcTanh[Tanh[a
 + b*x]]]*(33*b^2*x^2 - 26*b*x*ArcTanh[Tanh[a + b*x]] + 8*ArcTanh[Tanh[a +
b*x]]^2))/(24*x^3*(b*x - ArcTanh[Tanh[a + b*x]])^3)
```

Maple [A] time = 0.148, size = 200, normalized size = 0.9

$$2\,b^{3}\left(2/3\,\frac{\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}}{\left(-4\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)+4\,bx\right)b^{3}x^{3}}+10/3\,\frac{1}{-4\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)+4\,bx}\left(\frac{\sqrt{\mathrm{Artanh}\left(\tanh\left(bx\right.\right.}}{\left(-4\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right.\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 2*b^3*(2/3*arctanh(tanh(b*x+a))^(1/2)/(-4*arctanh(tanh(b*x+a))+4*b*x)/b^3/x
^3+10/3/(-4*arctanh(tanh(b*x+a))+4*b*x)*(arctanh(tanh(b*x+a))^(1/2)/(-4*arc
tanh(tanh(b*x+a))+4*b*x)/b^2/x^2+3/(-4*arctanh(tanh(b*x+a))+4*b*x)*(2*arcta
nh(tanh(b*x+a))^(1/2)/(-4*arctanh(tanh(b*x+a))+4*b*x)/b/x-2/(-4*arctanh(tan
h(b*x+a))+4*b*x)/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+
a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{4}\sqrt{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4*sqrt(arctanh(tanh(b*x + a)))), x)
```

Fricas [A] time = 2.15512, size = 356, normalized size = 1.68

$$\left[\frac{15\,\sqrt{a}b^{3}x^{3}\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)-2\left(15\,ab^{2}x^{2}-10\,a^{2}bx+8\,a^{3}\right)\sqrt{bx+a}}{48\,a^{4}x^{3}},-\frac{15\,\sqrt{-a}b^{3}x^{3}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(15\,ab^{2}x^{2}-\right.}{24\,a^{4}x^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(a)*b^3*x^3*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*
(15*a*b^2*x^2 - 10*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a^4*x^3), -1/24*(15*sqr
t(-a)*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (15*a*b^2*x^2 - 10*a^2*b*x
 + 8*a^3)*sqrt(b*x + a))/(a^4*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(atanh(tanh(a + b*x)))), x)
```

Giac [A] time = 1.14601, size = 113, normalized size = 0.53

$$-\frac{\frac{15\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3}+\frac{15\,(bx+a)^{\frac{5}{2}}b^4-40\,(bx+a)^{\frac{3}{2}}ab^4+33\,\sqrt{bx+a}a^2b^4}{a^3b^3x^3}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*(15*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) + (15*(b*x + a)
^(5/2)*b^4 - 40*(b*x + a)^(3/2)*a*b^4 + 33*sqrt(b*x + a)*a^2*b^4)/(a^3*b^3*
x^3))/b
```

3.149 $\int \frac{x^4}{\tanh^{-1}(\tanh(a+bx))^{3/2}} \, dx$

Optimal. Leaf size=95

$$\frac{16x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{32x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{b^3} + \frac{128x\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^4} - \frac{256\tanh^{-1}(\tanh(a+}{35b^5}$$

```
[Out] (-2*x^4)/(b*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (16*x^3*Sqrt[ArcTanh[Tanh[a + b
*x]]])/b^2 - (32*x^2*ArcTanh[Tanh[a + b*x]]^(3/2))/b^3 + (128*x*ArcTanh[Tan
h[a + b*x]]^(5/2))/(5*b^4) - (256*ArcTanh[Tanh[a + b*x]]^(7/2))/(35*b^5)
```

Rubi [A] time = 0.0671925, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2157, 30}

$$\frac{16x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{32x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{b^3} + \frac{128x\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^4} - \frac{256\tanh^{-1}(\tanh(a+}{35b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (-2*x^4)/(b*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (16*x^3*Sqrt[ArcTanh[Tanh[a + b
*x]]])/b^2 - (32*x^2*ArcTanh[Tanh[a + b*x]]^(3/2))/b^3 + (128*x*ArcTanh[Tan
h[a + b*x]]^(5/2))/(5*b^4) - (256*ArcTanh[Tanh[a + b*x]]^(7/2))/(35*b^5)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^4}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2x^4}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{8\int \frac{x^3}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{b}$$

$$= -\frac{2x^4}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{16x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{48\int x^2\sqrt{\tanh^{-1}(\tanh}}{b^2}$$

$$= -\frac{2x^4}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{16x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{32x^2\tanh^{-1}(\tanh(a+}{b^3}$$

$$= -\frac{2x^4}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{16x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{32x^2\tanh^{-1}(\tanh(a+}{b^3}$$

$$= -\frac{2x^4}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{16x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{32x^2\tanh^{-1}(\tanh(a+}{b^3}$$

$$= -\frac{2x^4}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{16x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{32x^2\tanh^{-1}(\tanh(a+}{b^3}$$

Mathematica [A] time = 0.0435768, size = 83, normalized size = 0.87

$$-\frac{2\left(-280b^3x^3\tanh^{-1}(\tanh(a+bx)) + 560b^2x^2\tanh^{-1}(\tanh(a+bx))^2 - 448bx\tanh^{-1}(\tanh(a+bx))^3 + 128\tanh^{-1}(t\right.}{35b^5\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (-2*(35*b^4*x^4 - 280*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 560*b^2*x^2*ArcTanh[
Tanh[a + b*x]]^2 - 448*b*x*ArcTanh[Tanh[a + b*x]]^3 + 128*ArcTanh[Tanh[a +
b*x]]^4))/(35*b^5*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Maple [B] time = 0.042, size = 319, normalized size = 3.4

$$2\,\frac{1}{b^5}\left(1/7\,(\mathrm{Artanh}\,(\tanh\,(bx+a)))^{7/2}-4/5\,(\mathrm{Artanh}\,(\tanh\,(bx+a)))^{5/2}\,a-4/5\,(\mathrm{Artanh}\,(\tanh\,(bx+a)))^{5/2}\,(\mathrm{Artanh}\,(\mathrm{t}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 2/b^5*(1/7*arctanh(tanh(b*x+a))^(7/2)-4/5*arctanh(tanh(b*x+a))^(5/2)*a-4/5*
arctanh(tanh(b*x+a))^(5/2)*(arctanh(tanh(b*x+a))-b*x-a)+2*arctanh(tanh(b*x+
a))^(3/2)*a^2+4*arctanh(tanh(b*x+a))^(3/2)*a*(arctanh(tanh(b*x+a))-b*x-a)+2
*arctanh(tanh(b*x+a))^(3/2)*(arctanh(tanh(b*x+a))-b*x-a)^2-4*arctanh(tanh(b
*x+a))^(1/2)*a^3-12*a^2*(arctanh(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(
1/2)-12*a*(arctanh(tanh(b*x+a))-b*x-a)^2*arctanh(tanh(b*x+a))^(1/2)-4*(arct
anh(tanh(b*x+a))-b*x-a)^3*arctanh(tanh(b*x+a))^(1/2)-(a^4+4*a^3*(arctanh(ta
nh(b*x+a))-b*x-a)+6*a^2*(arctanh(tanh(b*x+a))-b*x-a)^2+4*a*(arctanh(tanh(b*
x+a))-b*x-a)^3+(arctanh(tanh(b*x+a))-b*x-a)^4)/arctanh(tanh(b*x+a))^(1/2))
```

Maxima [A] time = 1.78609, size = 86, normalized size = 0.91

$$\frac{2\left(5\,b^5x^5-3\,ab^4x^4+8\,a^2b^3x^3-48\,a^3b^2x^2-192\,a^4bx-128\,a^5\right)}{35\,(bx+a)^{\frac{3}{2}}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*b^5*x^5 - 3*a*b^4*x^4 + 8*a^2*b^3*x^3 - 48*a^3*b^2*x^2 - 192*a^4*b*
x - 128*a^5)/((b*x + a)^(3/2)*b^5)
```

Fricas [A] time = 2.34281, size = 138, normalized size = 1.45

$$\frac{2\left(5\,b^4x^4 - 8\,ab^3x^3 + 16\,a^2b^2x^2 - 64\,a^3bx - 128\,a^4\right)\sqrt{bx + a}}{35\left(b^6x + ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^4*x^4 - 8*a*b^3*x^3 + 16*a^2*b^2*x^2 - 64*a^3*b*x - 128*a^4)*sqrt
(b*x + a)/(b^6*x + a*b^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a + bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(x**4/atanh(tanh(a + b*x))**(3/2), x)
```

Giac [A] time = 1.16118, size = 104, normalized size = 1.09

$$-\frac{2\,a^4}{\sqrt{bx + a}b^5} + \frac{2\left(5\left(bx + a\right)^{\frac{7}{2}}b^{30} - 28\left(bx + a\right)^{\frac{5}{2}}ab^{30} + 70\left(bx + a\right)^{\frac{3}{2}}a^2b^{30} - 140\,\sqrt{bx + a}a^3b^{30}\right)}{35\,b^{35}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -2*a^4/(sqrt(b*x + a)*b^5) + 2/35*(5*(b*x + a)^(7/2)*b^30 - 28*(b*x + a)^(5
/2)*a*b^30 + 70*(b*x + a)^(3/2)*a^2*b^30 - 140*sqrt(b*x + a)*a^3*b^30)/b^35
```

3.150 $\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{12x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2}-\frac{16x\tanh^{-1}(\tanh(a+bx))^{3/2}}{b^3}+\frac{32\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^4}-\frac{2x^3}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] (-2*x^3)/(b*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (12*x^2*Sqrt[ArcTanh[Tanh[a + b
*x]]])/b^2 - (16*x*ArcTanh[Tanh[a + b*x]]^(3/2))/b^3 + (32*ArcTanh[Tanh[a +
 b*x]]^(5/2))/(5*b^4)
```

Rubi [A] time = 0.0486649, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2157, 30}

$$\frac{12x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2}-\frac{16x\tanh^{-1}(\tanh(a+bx))^{3/2}}{b^3}+\frac{32\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^4}-\frac{2x^3}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```


```
[Out] (-2*x^3)/(b*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (12*x^2*Sqrt[ArcTanh[Tanh[a + b
*x]]])/b^2 - (16*x*ArcTanh[Tanh[a + b*x]]^(3/2))/b^3 + (32*ArcTanh[Tanh[a +
 b*x]]^(5/2))/(5*b^4)
```


Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2x^3}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{6\int \frac{x^2}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{b}$$

$$= -\frac{2x^3}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{12x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{24\int x\sqrt{\tanh^{-1}(\tanh(}{b^2}$$

$$= -\frac{2x^3}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{12x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{16x\tanh^{-1}(\tanh(a+}{b^3}$$

$$= -\frac{2x^3}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{12x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{16x\tanh^{-1}(\tanh(a+}{b^3}$$

$$= -\frac{2x^3}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{12x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{16x\tanh^{-1}(\tanh(a+}{b^3}$$

Mathematica [A] time = 0.0396789, size = 66, normalized size = 0.89

$$\frac{2\left(30b^2x^2\tanh^{-1}(\tanh(a+bx)) - 40bx\tanh^{-1}(\tanh(a+bx))^2 + 16\tanh^{-1}(\tanh(a+bx))^3 - 5b^3x^3\right)}{5b^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*(-5*b^3*x^3 + 30*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 40*b*x*ArcTanh[Tanh[a
+ b*x]]^2 + 16*ArcTanh[Tanh[a + b*x]]^3))/(5*b^4*Sqrt[ArcTanh[Tanh[a + b*x]
```

```
]])
```

Maple [B] time = 0.038, size = 201, normalized size = 2.7

$$2\frac{1}{b^4}\left(1/5\,(\mathrm{Artanh}\,(\tanh\,(bx+a)))^{5/2}-(\mathrm{Artanh}\,(\tanh\,(bx+a)))^{3/2}\,a-(\mathrm{Artanh}\,(\tanh\,(bx+a)))^{3/2}\,(\mathrm{Artanh}\,(\tanh\,(bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 2/b^4*(1/5*arctanh(tanh(b*x+a))^(5/2)-arctanh(tanh(b*x+a))^(3/2)*a-arctanh(
tanh(b*x+a))^(3/2)*(arctanh(tanh(b*x+a))-b*x-a)+3*arctanh(tanh(b*x+a))^(1/2
)*a^2+6*a*(arctanh(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/2)+3*(arctan
h(tanh(b*x+a))-b*x-a)^2*arctanh(tanh(b*x+a))^(1/2)-(-a^3-3*a^2*(arctanh(tan
h(b*x+a))-b*x-a)-3*a*(arctanh(tanh(b*x+a))-b*x-a)^2-(arctanh(tanh(b*x+a))-b
*x-a)^3)/arctanh(tanh(b*x+a))^(1/2))
```

Maxima [A] time = 1.79652, size = 70, normalized size = 0.95

$$\frac{2\left(b^4x^4-ab^3x^3+6\,a^2b^2x^2+24\,a^3bx+16\,a^4\right)}{5\,(bx+a)^{\frac{3}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b^4*x^4 - a*b^3*x^3 + 6*a^2*b^2*x^2 + 24*a^3*b*x + 16*a^4)/((b*x + a)^
(3/2)*b^4)
```

Fricas [A] time = 2.38357, size = 108, normalized size = 1.46

$$\frac{2\left(b^3x^3-2\,ab^2x^2+8\,a^2bx+16\,a^3\right)\sqrt{bx+a}}{5\left(b^5x+ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x + 16*a^3)*sqrt(b*x + a)/(b^5*x + a*b
^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(x**3/atanh(tanh(a + b*x))**(3/2), x)
```

Giac [A] time = 1.17376, size = 82, normalized size = 1.11

$$\frac{2\,a^3}{\sqrt{bx+a}b^4} + \frac{2\left((bx+a)^{\frac{5}{2}}b^{16} - 5\,(bx+a)^{\frac{3}{2}}ab^{16} + 15\,\sqrt{bx+a}a^2b^{16}\right)}{5\,b^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2*a^3/(sqrt(b*x + a)*b^4) + 2/5*((b*x + a)^(5/2)*b^16 - 5*(b*x + a)^(3/2)*a
*b^16 + 15*sqrt(b*x + a)*a^2*b^16)/b^20
```

3.151 $\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=55

$$\frac{8x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2}-\frac{16\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^3}-\frac{2x^2}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] (-2*x^2)/(b*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (8*x*Sqrt[ArcTanh[Tanh[a + b*x]
]])/b^2 - (16*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b^3)
```

Rubi [A] time = 0.0300588, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2157, 30}

$$\frac{8x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2}-\frac{16\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^3}-\frac{2x^2}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (-2*x^2)/(b*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (8*x*Sqrt[ArcTanh[Tanh[a + b*x]
]])/b^2 - (16*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b^3)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2x^2}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{4\int \frac{x}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{b}$$

$$= -\frac{2x^2}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{8x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{8\int \sqrt{\tanh^{-1}(\tanh(a+$$

$$= -\frac{2x^2}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{8x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{8\,\text{Subst}\left(\int \sqrt{x}\,dx, x, \tan\right.}{b^3}$$

$$= -\frac{2x^2}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{8x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{16\tanh^{-1}(\tanh(a+bx)}{3b^3}$$

Mathematica [A] time = 0.0320327, size = 49, normalized size = 0.89

$$-\frac{2\left(-12bx\tanh^{-1}(\tanh(a+bx)) + 8\tanh^{-1}(\tanh(a+bx))^2 + 3b^2x^2\right)}{3b^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (-2*(3*b^2*x^2 - 12*b*x*ArcTanh[Tanh[a + b*x]] + 8*ArcTanh[Tanh[a + b*x]]^2
))/(3*b^3*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Maple [B] time = 0.04, size = 106, normalized size = 1.9

$$2\,\frac{1}{b^3}\left(1/3\,\left(\text{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{3/2} - 2\,a\sqrt{\text{Artanh}\left(\tanh\left(bx+a\right)\right)} - 2\,\left(\text{Artanh}\left(\tanh\left(bx+a\right)\right) - bx - a\right)\sqrt{\text{Artanh}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 2/b^3*(1/3*arctanh(tanh(b*x+a))^(3/2)-2*a*arctanh(tanh(b*x+a))^(1/2)-2*(arc
tanh(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/2)-(a^2+2*a*(arctanh(tanh(
b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)/arctanh(tanh(b*x+a))^(1/2))
```

Maxima [A] time = 1.78539, size = 55, normalized size = 1.

$$\frac{2\left(b^3x^3 - 3\,ab^2x^2 - 12\,a^2bx - 8\,a^3\right)}{3\,(bx+a)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b^3*x^3 - 3*a*b^2*x^2 - 12*a^2*b*x - 8*a^3)/((b*x + a)^(3/2)*b^3)
```

Fricas [A] time = 2.26811, size = 85, normalized size = 1.55

$$\frac{2\left(b^2x^2 - 4\,abx - 8\,a^2\right)\sqrt{bx+a}}{3\left(b^4x + ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^2*x^2 - 4*a*b*x - 8*a^2)*sqrt(b*x + a)/(b^4*x + a*b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(x**2/atanh(tanh(a + b*x))**(3/2), x)
```

Giac [A] time = 1.15513, size = 62, normalized size = 1.13

$$-\frac{2\,a^2}{\sqrt{bx+a}b^3}+\frac{2\left((bx+a)^{\frac{3}{2}}b^6-6\,\sqrt{bx+a}ab^6\right)}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -2*a^2/(sqrt(b*x + a)*b^3) + 2/3*((b*x + a)^(3/2)*b^6 - 6*sqrt(b*x + a)*a*b
^6)/b^9
```

3.152 $\int \frac{x}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=34

$$\frac{4\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2}-\frac{2x}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] (-2*x)/(b*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (4*Sqrt[ArcTanh[Tanh[a + b*x]]])/
b^2
```

Rubi [A] time = 0.0148316, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2157, 30}

$$\frac{4\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2}-\frac{2x}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (-2*x)/(b*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (4*Sqrt[ArcTanh[Tanh[a + b*x]]])/
b^2
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2x}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{2\int \frac{1}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{b}$$

$$= -\frac{2x}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b^2}$$

$$= -\frac{2x}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{4\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2}$$

Mathematica [A] time = 0.0572761, size = 29, normalized size = 0.85

$$\frac{4\tanh^{-1}(\tanh(a+bx)) - 2bx}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (-2*b*x + 4*ArcTanh[Tanh[a + b*x]])/(b^2*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Maple [A] time = 0.037, size = 40, normalized size = 1.2

$$2\,\frac{1}{b^2}\left(\sqrt{\text{Artanh}\,(\tanh\,(bx+a))} - \frac{bx - \text{Artanh}\,(\tanh\,(bx+a))}{\sqrt{\text{Artanh}\,(\tanh\,(bx+a))}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 2/b^2*(arctanh(tanh(b*x+a))^(1/2)-(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b
*x+a))^(1/2))
```

Maxima [A] time = 1.78442, size = 41, normalized size = 1.21

$$\frac{2\left(b^2x^2 + 3\,abx + 2\,a^2\right)}{(bx+a)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2*(b^2*x^2 + 3*a*b*x + 2*a^2)/((b*x + a)^(3/2)*b^2)
```

Fricas [A] time = 2.11797, size = 61, normalized size = 1.79

$$\frac{2\,(bx+2\,a)\sqrt{bx+a}}{b^3x+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(b*x + 2*a)*sqrt(b*x + a)/(b^3*x + a*b^2)
```

Sympy [A] time = 35.2198, size = 46, normalized size = 1.35

$$\begin{cases} -\frac{2x}{b\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}} + \frac{4\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}{b^2} & \text{for}\ b \neq 0 \\ \frac{x^2}{2\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a\right)\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Piecewise((-2*x/(b*sqrt(atanh(tanh(a + b*x)))) + 4*sqrt(atanh(tanh(a + b*x)
))/b**2, Ne(b, 0)), (x**2/(2*atanh(tanh(a))**(3/2)), True))
```

Giac [A] time = 1.15173, size = 39, normalized size = 1.15

$$\frac{2\left(\frac{\sqrt{bx+a}}{b}+\frac{a}{\sqrt{bx+ab}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2*(sqrt(b*x + a)/b + a/(sqrt(b*x + a)*b))/b
```

3.153 $\int \frac{1}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=16

$$-\frac{2}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] -2/(b*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rubi [A] time = 0.0048986, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2157, 30}

$$-\frac{2}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(-3/2),x]
```

```
[Out] -2/(b*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^{3/2}}\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}$$
$$= -\frac{2}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Mathematica [A] time = 0.0067302, size = 16, normalized size = 1.

$$-\frac{2}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(-3/2),x]
```

```
[Out] -2/(b*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Maple [A] time = 0.027, size = 15, normalized size = 0.9

$$-2\,\frac{1}{b\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] -2/b/arctanh(tanh(b*x+a))^(1/2)
```

Maxima [A] time = 1.7012, size = 16, normalized size = 1.

$$-\frac{2}{\sqrt{bx+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/(sqrt(b*x + a)*b)
```

Fricas [A] time = 2.00522, size = 43, normalized size = 2.69

$$-\frac{2\sqrt{bx+a}}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*x + a)/(b^2*x + a*b)
```

Sympy [A] time = 35.1407, size = 26, normalized size = 1.62

$$\begin{cases} -\frac{2}{b\sqrt{\operatorname{atanh}(\tanh(a+bx))}} & \text{for } b \neq 0 \\ \frac{x}{\operatorname{atanh}^{\frac{3}{2}}(\tanh(a))} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Piecewise((-2/(b*sqrt(atanh(tanh(a + b*x)))), Ne(b, 0)), (x/atanh(tanh(a))*
*(3/2), True))
```

Giac [A] time = 1.1385, size = 16, normalized size = 1.

$$-\frac{2}{\sqrt{bx+ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -2/(sqrt(b*x + a)*b)
```

3.154 $\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}$$

```
[Out] (-2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]
)/(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) - 2/((b*x - ArcTanh[Tanh[a + b*x]])*
Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rubi [A] time = 0.0357063, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2163, 2161}

$$-\frac{2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (-2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]
)/(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) - 2/((b*x - ArcTanh[Tanh[a + b*x]])*
Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
```

```
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{\int \frac{1}{x\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{-bx+\tanh^{-1}(\tanh(a+b}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}} - \frac{2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a}}$$

Mathematica [A] time = 0.0913641, size = 75, normalized size = 0.96

$$\frac{2}{\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)} - \frac{2\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (-2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x
]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2) + 2/(Sqrt[ArcTanh[Tanh[a + b*
x]]]*(-(b*x) + ArcTanh[Tanh[a + b*x]]))
```

Maple [A] time = 0.115, size = 68, normalized size = 0.9

$$2\,\frac{1}{(\mathrm{Artanh}\,(\tanh\,(bx+a))-bx)\sqrt{\mathrm{Artanh}\,(\tanh\,(bx+a))}} - 2\,\frac{1}{(\mathrm{Artanh}\,(\tanh\,(bx+a))-bx)^{3/2}}\mathrm{Artanh}\left(\frac{\sqrt{\mathrm{Artanh}\,(}}{\sqrt{\mathrm{Artanh}\,(\mathrm{ta}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 2/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^(1/2)-2/(arctanh(tanh(b*x
+a))-b*x)^(3/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*
x)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*arctanh(tanh(b*x + a))^(3/2)), x)
```

Fricas [A] time = 2.07013, size = 266, normalized size = 3.41

$$\left[\frac{(bx+a)\sqrt{a}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\sqrt{bx+a}a}{a^2bx+a^3},\ \frac{2\left((bx+a)\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\sqrt{bx+a}a\right)}{a^2bx+a^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [((b*x + a)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b
*x + a)*a)/(a^2*b*x + a^3), 2*((b*x + a)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt
(-a)/a) + sqrt(b*x + a)*a)/(a^2*b*x + a^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,\mathrm{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(1/(x*atanh(tanh(a + b*x))**(3/2)), x)
```

Giac [A] time = 1.15872, size = 50, normalized size = 0.64

$$\frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{2}{\sqrt{bx+a}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + 2/(sqrt(b*x + a)*a)
```

3.155 $\int \frac{1}{x^2 \tanh^{-1}(\tanh(a+bx))^{3/2}} dx$

Optimal. Leaf size=124

$$-\frac{3b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}-\frac{}{x}$$

```
[Out] (-3*b*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]
]])/(b*x - ArcTanh[Tanh[a + b*x]])^(5/2) - 1/(x*ArcTanh[Tanh[a + b*x]]^(3/2
)) + b/((b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2)) - (3*b
)/((b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rubi [A] time = 0.074482, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2163, 2161}

$$-\frac{3b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}-\frac{}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (-3*b*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]
]])/(b*x - ArcTanh[Tanh[a + b*x]])^(5/2) - 1/(x*ArcTanh[Tanh[a + b*x]]^(3/2
)) + b/((b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2)) - (3*b
)/((b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
```

```
[m, 0] &&  !IntegerQ[n]))
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{1}{x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{1}{2}(3b)\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$$

$$= -\frac{1}{x\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3}}$$

$$= -\frac{1}{x\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3}}$$

$$= -\frac{3b\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}} - \frac{1}{x\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{\left(bx-\tanh^{-1}(\tan\right.}$$

Mathematica [A] time = 0.0832766, size = 91, normalized size = 0.73

$$\frac{3b\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{5/2}} - \frac{\tanh^{-1}(\tanh(a+bx))+2bx}{x\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (3*b*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*
x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2) - (2*b*x + ArcTanh[Tanh[a +
b*x]])/(x*Sqrt[ArcTanh[Tanh[a + b*x]]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

Maple [A] time = 0.104, size = 105, normalized size = 0.9

$$2\,b\left(-\frac{1}{\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}}\left(1/2\,\frac{\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}}{bx}-3/2\,\frac{1}{\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx}}\mathrm{Artanh}\left(\frac{}{\sqrt{}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 2*b*(-1/(arctanh(tanh(b*x+a))-b*x)^2*(1/2*arctanh(tanh(b*x+a))^(1/2)/b/x-3/
2/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arct
anh(tanh(b*x+a))-b*x)^(1/2)))-1/(arctanh(tanh(b*x+a))-b*x)^2/arctanh(tanh(b
*x+a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{2}\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*arctanh(tanh(b*x + a))^(3/2)), x)
```

Fricas [A] time = 2.11259, size = 346, normalized size = 2.79

$$\left[\frac{3\left(b^{2}x^{2}+abx\right)\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(3abx+a^{2}\right)\sqrt{bx+a}}{2\left(a^{3}bx^{2}+a^{4}x\right)},-\frac{3\left(b^{2}x^{2}+abx\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(3abx+a\right.}{a^{3}bx^{2}+a^{4}x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b^2*x^2 + a*b*x)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)
/x) - 2*(3*a*b*x + a^2)*sqrt(b*x + a))/(a^3*b*x^2 + a^4*x), -(3*(b^2*x^2 +
a*b*x)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (3*a*b*x + a^2)*sqrt(b*x
 + a))/(a^3*b*x^2 + a^4*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(1/(x**2*atanh(tanh(a + b*x))**(3/2)), x)
```

Giac [A] time = 1.16568, size = 86, normalized size = 0.69

$$-\frac{3\,b\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}-\frac{3\,(bx+a)b-2\,ab}{\left((bx+a)^{\frac{3}{2}}-\sqrt{bx+a}a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -3*b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) - (3*(b*x + a)*b - 2*a*b
)/(((b*x + a)^(3/2) - sqrt(b*x + a)*a)*a^2)
```

3.156 $\int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))^{3/2}} \, dx$

Optimal. Leaf size=191

$$-\frac{15b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{5b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

```
[Out] (-15*b^2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*
x]]]])/(4*(b*x - ArcTanh[Tanh[a + b*x]])^(7/2)) + (3*b)/(4*x*ArcTanh[Tanh[a
 + b*x]]^(5/2)) - (3*b^2)/(4*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a
+ b*x]]^(5/2)) - 1/(2*x^2*ArcTanh[Tanh[a + b*x]]^(3/2)) + (5*b^2)/(4*(b*x -
 ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^(3/2)) - (15*b^2)/(4*(b*x
 - ArcTanh[Tanh[a + b*x]])^3*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rubi [A] time = 0.127917, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2163, 2161}

$$-\frac{15b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{5b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```


```
[Out] (-15*b^2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*
x]]]])/(4*(b*x - ArcTanh[Tanh[a + b*x]])^(7/2)) + (3*b)/(4*x*ArcTanh[Tanh[a
 + b*x]]^(5/2)) - (3*b^2)/(4*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a
+ b*x]]^(5/2)) - 1/(2*x^2*ArcTanh[Tanh[a + b*x]]^(3/2)) + (5*b^2)/(4*(b*x -
 ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^(3/2)) - (15*b^2)/(4*(b*x
 - ArcTanh[Tanh[a + b*x]])^3*Sqrt[ArcTanh[Tanh[a + b*x]]])
```


Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
```

```
x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))^{3/2}} \, dx = -\frac{1}{2x^2 \tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{1}{4}(3b) \int \frac{1}{x^2 \tanh^{-1}(\tanh(a+bx))^{5/2}} \, dx$$

$$= \frac{3b}{4x \tanh^{-1}(\tanh(a+bx))^{5/2}} - \frac{1}{2x^2 \tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{1}{8}\left(15b^2\right) \int \frac{}{x \tanh^{-1}}$$

$$= \frac{3b}{4x \tanh^{-1}(\tanh(a+bx))^{5/2}} - \frac{3b^2}{4\left(bx - \tanh^{-1}(\tanh(a+bx))\right) \tanh^{-1}(\tanh(a+bx)}$$

$$= \frac{3b}{4x \tanh^{-1}(\tanh(a+bx))^{5/2}} - \frac{3b^2}{4\left(bx - \tanh^{-1}(\tanh(a+bx))\right) \tanh^{-1}(\tanh(a+bx)}$$

$$= \frac{3b}{4x \tanh^{-1}(\tanh(a+bx))^{5/2}} - \frac{3b^2}{4\left(bx - \tanh^{-1}(\tanh(a+bx))\right) \tanh^{-1}(\tanh(a+bx)}$$

$$= -\frac{15b^2 \tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx - \tanh^{-1}(\tanh(a+bx))\right)^{7/2}} + \frac{3b}{4x \tanh^{-1}(\tanh(a+bx))^{5/2}} - \frac{}{4\left(bx - \tanh^{-1}\right.}$$

Mathematica [A] time = 0.116435, size = 115, normalized size = 0.6

$$\frac{1}{4}\left(\frac{9bx\tanh^{-1}(\tanh(a+bx))-2\tanh^{-1}(\tanh(a+bx))^2+8b^2x^2}{x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^3}-\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{7/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] ((-15*b^2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a
 + b*x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(7/2) + (8*b^2*x^2 + 9*b*x*Ar
cTanh[Tanh[a + b*x]] - 2*ArcTanh[Tanh[a + b*x]]^2)/(x^2*Sqrt[ArcTanh[Tanh[a
 + b*x]]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^3))/4
```

Maple [A] time = 0.127, size = 131, normalized size = 0.7

$$2\,b^2\left(\frac{1}{(\mathrm{Artanh}\,(\tanh\,(bx+a))-bx)^3\sqrt{\mathrm{Artanh}\,(\tanh\,(bx+a))}}+\frac{1}{(\mathrm{Artanh}\,(\tanh\,(bx+a))-bx)^3}\left(\frac{1}{b^2x^2}\left(\frac{7\,(\mathrm{Artanh}\,(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 2*b^2*(1/(arctanh(tanh(b*x+a))-b*x)^3/arctanh(tanh(b*x+a))^(1/2)+1/(arctanh
(tanh(b*x+a))-b*x)^3*((7/8*arctanh(tanh(b*x+a))^(3/2)+(-9/8*arctanh(tanh(b*
x+a))+9/8*b*x)*arctanh(tanh(b*x+a))^(1/2))/b^2/x^2-15/8/(arctanh(tanh(b*x+a
))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)
^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\,\mathrm{artanh}\,(\tanh\,(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^3*arctanh(tanh(b*x + a))^(3/2)), x)
```

Fricas [A] time = 2.13596, size = 420, normalized size = 2.2

$$\left[\frac{15\left(b^3x^3+ab^2x^2\right)\sqrt{a}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(15ab^2x^2+5a^2bx-2a^3\right)\sqrt{bx+a}}{8\left(a^4bx^3+a^5x^2\right)}, \frac{15\left(b^3x^3+ab^2x^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}}{a}\right.}{4\left(a^4b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^3*x^3 + a*b^2*x^2)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) +
 2*a)/x) + 2*(15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x + a))/(a^4*b*x^3 +
 a^5*x^2), 1/4*(15*(b^3*x^3 + a*b^2*x^2)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt
(-a)/a) + (15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x + a))/(a^4*b*x^3 + a^
5*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(1/(x**3*atanh(tanh(a + b*x))**(3/2)), x)
```

Giac [A] time = 1.1765, size = 108, normalized size = 0.57

$$\frac{15\,b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{4\sqrt{-a}a^3}+\frac{2\,b^2}{\sqrt{bx+a}a^3}+\frac{7\,(bx+a)^{\frac{3}{2}}b^2-9\sqrt{bx+a}ab^2}{4\,a^3b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 15/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) + 2*b^2/(sqrt(b*x +
a)*a^3) + 1/4*(7*(b*x + a)^(3/2)*b^2 - 9*sqrt(b*x + a)*a*b^2)/(a^3*b^2*x^2)
```

3.157 $\int \frac{1}{x^4 \tanh^{-1}(\tanh(a+bx))^{3/2}} \, dx$

Optimal. Leaf size=245

$$-\frac{35b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{35b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

```
[Out] (-35*b^3*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*
x]]]])/(8*(b*x - ArcTanh[Tanh[a + b*x]])^(9/2)) - (5*b^2)/(8*x*ArcTanh[Tanh
[a + b*x]]^(7/2)) + (5*b^3)/(8*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[
a + b*x]]^(7/2)) + b/(4*x^2*ArcTanh[Tanh[a + b*x]]^(5/2)) - (7*b^3)/(8*(b*x
 - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^(5/2)) - 1/(3*x^3*ArcTa
nh[Tanh[a + b*x]]^(3/2)) + (35*b^3)/(24*(b*x - ArcTanh[Tanh[a + b*x]])^3*Ar
cTanh[Tanh[a + b*x]]^(3/2)) - (35*b^3)/(8*(b*x - ArcTanh[Tanh[a + b*x]])^4*
Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rubi [A] time = 0.182859, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2163, 2161}

$$-\frac{35b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{35b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```


```
[Out] (-35*b^3*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*
x]]]])/(8*(b*x - ArcTanh[Tanh[a + b*x]])^(9/2)) - (5*b^2)/(8*x*ArcTanh[Tanh
[a + b*x]]^(7/2)) + (5*b^3)/(8*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[
a + b*x]]^(7/2)) + b/(4*x^2*ArcTanh[Tanh[a + b*x]]^(5/2)) - (7*b^3)/(8*(b*x
 - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^(5/2)) - 1/(3*x^3*ArcTa
nh[Tanh[a + b*x]]^(3/2)) + (35*b^3)/(24*(b*x - ArcTanh[Tanh[a + b*x]])^3*Ar
cTanh[Tanh[a + b*x]]^(3/2)) - (35*b^3)/(8*(b*x - ArcTanh[Tanh[a + b*x]])^4*
Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{x^4 \tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{1}{3x^3 \tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{1}{2}b \int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$$
$$= \frac{b}{4x^2 \tanh^{-1}(\tanh(a+bx))^{5/2}} - \frac{1}{3x^3 \tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{1}{8}\left(5b^2\right) \int \frac{}{x^2 \tanh^{-}}$$
$$= -\frac{5b^2}{8x \tanh^{-1}(\tanh(a+bx))^{7/2}} + \frac{b}{4x^2 \tanh^{-1}(\tanh(a+bx))^{5/2}} - \frac{1}{3x^3 \tanh^{-1}(\tanh(a}$$
$$= -\frac{5b^2}{8x \tanh^{-1}(\tanh(a+bx))^{7/2}} + \frac{5b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx}$$
$$= -\frac{5b^2}{8x \tanh^{-1}(\tanh(a+bx))^{7/2}} + \frac{5b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx}$$
$$= -\frac{5b^2}{8x \tanh^{-1}(\tanh(a+bx))^{7/2}} + \frac{5b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx}$$
$$= -\frac{5b^2}{8x \tanh^{-1}(\tanh(a+bx))^{7/2}} + \frac{5b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx}$$
$$= -\frac{35b^3 \tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{9/2}} - \frac{5b^2}{8x \tanh^{-1}(\tanh(a+bx))^{7/2}} + \frac{}{8\left(bx-\tanh^{-}\right.}$$

Mathematica [A] time = 0.113197, size = 133, normalized size = 0.54

$$\frac{35b^3 \tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{8\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{9/2}} - \frac{87b^2x^2 \tanh^{-1}(\tanh(a+bx)) - 38bx \tanh^{-1}(\tanh(a+bx))^2 + 8\tanh^{-1}(\tanh(a}{24x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (35*b^3*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a +
 b*x]]]])/(8*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(9/2)) - (48*b^3*x^3 + 87*b^
2*x^2*ArcTanh[Tanh[a + b*x]] - 38*b*x*ArcTanh[Tanh[a + b*x]]^2 + 8*ArcTanh[
Tanh[a + b*x]]^3)/(24*x^3*Sqrt[ArcTanh[Tanh[a + b*x]]]*(-(b*x) + ArcTanh[Ta
```

```
nh[a + b*x]])^4)
```

Maple [A] time = 0.128, size = 186, normalized size = 0.8

$$2\,b^{3}\left(-\frac{1}{\left(\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{4}}\left(\frac{1}{x^{3}b^{3}}\left(\frac{19\,\left(\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{5/2}}{16}+\left(-\frac{17\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)}{6}+\frac{17\,bx}{6}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 2*b^3*(-1/(arctanh(tanh(b*x+a))-b*x)^4*((19/16*arctanh(tanh(b*x+a))^(5/2)+(
-17/6*arctanh(tanh(b*x+a))+17/6*b*x)*arctanh(tanh(b*x+a))^(3/2)+(29/16*a^2+
29/8*a*(arctanh(tanh(b*x+a))-b*x-a)+29/16*(arctanh(tanh(b*x+a))-b*x-a)^2)*a
rctanh(tanh(b*x+a))^(1/2))/b^3/x^3-35/16/(arctanh(tanh(b*x+a))-b*x)^(1/2)*a
rctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))-1/(arc
tanh(tanh(b*x+a))-b*x)^4/arctanh(tanh(b*x+a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{4}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4*arctanh(tanh(b*x + a))^(3/2)), x)
```

Fricas [A] time = 2.12223, size = 478, normalized size = 1.95

$$\left[\frac{105\left(b^{4}x^{4}+ab^{3}x^{3}\right)\sqrt{a}\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)-2\left(105\,ab^{3}x^{3}+35\,a^{2}b^{2}x^{2}-14\,a^{3}bx+8\,a^{4}\right)\sqrt{bx+a}}{48\left(a^{5}bx^{4}+a^{6}x^{3}\right)},-\frac{105\left(b^{4}x^{4}+ab^{3}x^{3}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(105*(b^4*x^4 + a*b^3*x^3)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a)
 + 2*a)/x) - 2*(105*a*b^3*x^3 + 35*a^2*b^2*x^2 - 14*a^3*b*x + 8*a^4)*sqrt(b
*x + a))/(a^5*b*x^4 + a^6*x^3), -1/24*(105*(b^4*x^4 + a*b^3*x^3)*sqrt(-a)*a
rctan(sqrt(b*x + a)*sqrt(-a)/a) + (105*a*b^3*x^3 + 35*a^2*b^2*x^2 - 14*a^3*
b*x + 8*a^4)*sqrt(b*x + a))/(a^5*b*x^4 + a^6*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(1/(x**4*atanh(tanh(a + b*x))**(3/2)), x)
```

Giac [A] time = 1.1462, size = 128, normalized size = 0.52

$$-\frac{35\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{8\sqrt{-a}a^4}-\frac{2\,b^3}{\sqrt{bx+a}a^4}-\frac{57\,(bx+a)^{\frac{5}{2}}b^3-136\,(bx+a)^{\frac{3}{2}}ab^3+87\sqrt{bx+a}a^2b^3}{24\,a^4b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -35/8*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^4) - 2*b^3/(sqrt(b*x +
 a)*a^4) - 1/24*(57*(b*x + a)^(5/2)*b^3 - 136*(b*x + a)^(3/2)*a*b^3 + 87*sq
rt(b*x + a)*a^2*b^3)/(a^4*b^3*x^3)
```

3.158 $\int \frac{x^4}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=99

$$-\frac{16x^3}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{32x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^3}-\frac{128x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^4}+\frac{256\tanh^{-1}(\tanh(a \cdots}{15b^5}$$

```
[Out] (-2*x^4)/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2)) - (16*x^3)/(3*b^2*Sqrt[ArcTanh[
Tanh[a + b*x]]]) + (32*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/b^3 - (128*x*ArcTa
nh[Tanh[a + b*x]]^(3/2))/(3*b^4) + (256*ArcTanh[Tanh[a + b*x]]^(5/2))/(15*b
^5)
```

Rubi [A] time = 0.0660405, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2157, 30}

$$-\frac{16x^3}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{32x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^3}-\frac{128x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^4}+\frac{256\tanh^{-1}(\tanh(a \cdots}{15b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```


```
[Out] (-2*x^4)/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2)) - (16*x^3)/(3*b^2*Sqrt[ArcTanh[
Tanh[a + b*x]]]) + (32*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/b^3 - (128*x*ArcTa
nh[Tanh[a + b*x]]^(3/2))/(3*b^4) + (256*ArcTanh[Tanh[a + b*x]]^(5/2))/(15*b
^5)
```


Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^4}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2x^4}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{8\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx}{3b}$$

$$= -\frac{2x^4}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{16x^3}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{16\int \frac{x^2}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}}{b^2}$$

$$= -\frac{2x^4}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{16x^3}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{32x^2\sqrt{\tanh^{-1}(\tanh(a}}{b^3}$$

$$= -\frac{2x^4}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{16x^3}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{32x^2\sqrt{\tanh^{-1}(\tanh(a}}{b^3}$$

$$= -\frac{2x^4}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{16x^3}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{32x^2\sqrt{\tanh^{-1}(\tanh(a}}{b^3}$$

$$= -\frac{2x^4}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{16x^3}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{32x^2\sqrt{\tanh^{-1}(\tanh(a}}{b^3}$$

Mathematica [A] time = 0.0480728, size = 83, normalized size = 0.84

$$-\frac{2\left(40b^3x^3\tanh^{-1}(\tanh(a+bx)) - 240b^2x^2\tanh^{-1}(\tanh(a+bx))^2 + 320bx\tanh^{-1}(\tanh(a+bx))^3 - 128\tanh^{-1}(\tanh\right.}{15b^5\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (-2*(5*b^4*x^4 + 40*b^3*x^3*ArcTanh[Tanh[a + b*x]] - 240*b^2*x^2*ArcTanh[Ta
nh[a + b*x]]^2 + 320*b*x*ArcTanh[Tanh[a + b*x]]^3 - 128*ArcTanh[Tanh[a + b*
x]]^4))/(15*b^5*ArcTanh[Tanh[a + b*x]]^(3/2))
```

Maple [B] time = 0.045, size = 295, normalized size = 3.

$$2\,\frac{1}{b^5}\left(1/5\,\left(\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{5/2}-4/3\,\left(\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{3/2}a-4/3\,\left(\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{3/2}\left(\operatorname{Artanh}\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] 2/b^5*(1/5*arctanh(tanh(b*x+a))^(5/2)-4/3*arctanh(tanh(b*x+a))^(3/2)*a-4/3*
arctanh(tanh(b*x+a))^(3/2)*(arctanh(tanh(b*x+a))-b*x-a)+6*arctanh(tanh(b*x+
a))^(1/2)*a^2+12*a*(arctanh(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/2)+
6*(arctanh(tanh(b*x+a))-b*x-a)^2*arctanh(tanh(b*x+a))^(1/2)-(-4*a^3-12*a^2*
(arctanh(tanh(b*x+a))-b*x-a)-12*a*(arctanh(tanh(b*x+a))-b*x-a)^2-4*(arctanh
(tanh(b*x+a))-b*x-a)^3)/arctanh(tanh(b*x+a))^(1/2)-1/3*(a^4+4*a^3*(arctanh(
tanh(b*x+a))-b*x-a)+6*a^2*(arctanh(tanh(b*x+a))-b*x-a)^2+4*a*(arctanh(tanh(
b*x+a))-b*x-a)^3+(arctanh(tanh(b*x+a))-b*x-a)^4)/arctanh(tanh(b*x+a))^(3/2)
)
```

Maxima [A] time = 1.78818, size = 86, normalized size = 0.87

$$\frac{2\left(3\,b^5x^5-5\,ab^4x^4+40\,a^2b^3x^3+240\,a^3b^2x^2+320\,a^4bx+128\,a^5\right)}{15\,(bx+a)^{\frac{5}{2}}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*b^5*x^5 - 5*a*b^4*x^4 + 40*a^2*b^3*x^3 + 240*a^3*b^2*x^2 + 320*a^4*
b*x + 128*a^5)/((b*x + a)^(5/2)*b^5)
```

Fricas [A] time = 2.07703, size = 161, normalized size = 1.63

$$\frac{2\left(3\,b^4x^4 - 8\,ab^3x^3 + 48\,a^2b^2x^2 + 192\,a^3bx + 128\,a^4\right)\sqrt{bx+a}}{15\left(b^7x^2 + 2\,ab^6x + a^2b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^4*x^4 - 8*a*b^3*x^3 + 48*a^2*b^2*x^2 + 192*a^3*b*x + 128*a^4)*sqr
t(b*x + a)/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\operatorname{atanh}^{\frac{5}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Integral(x**4/atanh(tanh(a + b*x))**(5/2), x)
```

Giac [A] time = 1.14978, size = 101, normalized size = 1.02

$$\frac{2\left(12\,(bx+a)a^3 - a^4\right)}{3\,(bx+a)^{\frac{3}{2}}b^5} + \frac{2\left(3\,(bx+a)^{\frac{5}{2}}b^{20} - 20\,(bx+a)^{\frac{3}{2}}ab^{20} + 90\,\sqrt{bx+a}a^2b^{20}\right)}{15\,b^{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(12*(b*x + a)*a^3 - a^4)/((b*x + a)^(3/2)*b^5) + 2/15*(3*(b*x + a)^(5/2
)*b^20 - 20*(b*x + a)^(3/2)*a*b^20 + 90*sqrt(b*x + a)*a^2*b^20)/b^25
```

3.159 $\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=76

$$-\frac{4x^2}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{16x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^3}-\frac{32\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^4}-\frac{2x^3}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

```
[Out] (-2*x^3)/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2)) - (4*x^2)/(b^2*Sqrt[ArcTanh[Tan
h[a + b*x]]]) + (16*x*Sqrt[ArcTanh[Tanh[a + b*x]]])/b^3 - (32*ArcTanh[Tanh[
a + b*x]]^(3/2))/(3*b^4)
```

Rubi [A] time = 0.0485976, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2157, 30}

$$-\frac{4x^2}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{16x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^3}-\frac{32\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^4}-\frac{2x^3}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```



```
[Out] (-2*x^3)/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2)) - (4*x^2)/(b^2*Sqrt[ArcTanh[Tan
h[a + b*x]]]) + (16*x*Sqrt[ArcTanh[Tanh[a + b*x]]])/b^3 - (32*ArcTanh[Tanh[
a + b*x]]^(3/2))/(3*b^4)
```


Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2x^3}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{2\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx}{b}$$

$$= -\frac{2x^3}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{4x^2}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{8\int \frac{x}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{b^2}$$

$$= -\frac{2x^3}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{4x^2}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{16x\sqrt{\tanh^{-1}(\tanh(a+}}{b^3}$$

$$= -\frac{2x^3}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{4x^2}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{16x\sqrt{\tanh^{-1}(\tanh(a+}}{b^3}$$

$$= -\frac{2x^3}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{4x^2}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{16x\sqrt{\tanh^{-1}(\tanh(a+}}{b^3}$$

Mathematica [A] time = 0.0388968, size = 65, normalized size = 0.86

$$-\frac{2\left(6b^2x^2\tanh^{-1}(\tanh(a+bx)) - 24bx\tanh^{-1}(\tanh(a+bx))^2 + 16\tanh^{-1}(\tanh(a+bx))^3 + b^3x^3\right)}{3b^4\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (-2*(b^3*x^3 + 6*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 24*b*x*ArcTanh[Tanh[a + b
*x]]^2 + 16*ArcTanh[Tanh[a + b*x]]^3))/(3*b^4*ArcTanh[Tanh[a + b*x]]^(3/2))
```

Maple [B] time = 0.042, size = 186, normalized size = 2.5

$$2\frac{1}{b^4}\left(1/3\,(\mathrm{Artanh}(\tanh(bx+a)))^{3/2}-3\,a\sqrt{\mathrm{Artanh}(\tanh(bx+a))}-3\,(\mathrm{Artanh}(\tanh(bx+a))-bx-a)\sqrt{\mathrm{Artanh}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] 2/b^4*(1/3*arctanh(tanh(b*x+a))^(3/2)-3*a*arctanh(tanh(b*x+a))^(1/2)-3*(arc
tanh(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/2)-(3*a^2+6*a*(arctanh(tan
h(b*x+a))-b*x-a)+3*(arctanh(tanh(b*x+a))-b*x-a)^2)/arctanh(tanh(b*x+a))^(1/
2)-1/3*(-a^3-3*a^2*(arctanh(tanh(b*x+a))-b*x-a)-3*a*(arctanh(tanh(b*x+a))-b
*x-a)^2-(arctanh(tanh(b*x+a))-b*x-a)^3)/arctanh(tanh(b*x+a))^(3/2))
```

Maxima [A] time = 1.7914, size = 70, normalized size = 0.92

$$\frac{2\left(b^4x^4-5\,ab^3x^3-30\,a^2b^2x^2-40\,a^3bx-16\,a^4\right)}{3\,(bx+a)^{\frac{5}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b^4*x^4 - 5*a*b^3*x^3 - 30*a^2*b^2*x^2 - 40*a^3*b*x - 16*a^4)/((b*x +
a)^(5/2)*b^4)
```

Fricas [A] time = 1.93832, size = 131, normalized size = 1.72

$$\frac{2\left(b^3x^3-6\,ab^2x^2-24\,a^2bx-16\,a^3\right)\sqrt{bx+a}}{3\left(b^6x^2+2\,ab^5x+a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^3*x^3 - 6*a*b^2*x^2 - 24*a^2*b*x - 16*a^3)*sqrt(b*x + a)/(b^6*x^2 +
2*a*b^5*x + a^2*b^4)
```

Sympy [A] time = 92.4217, size = 90, normalized size = 1.18

$$\begin{cases} -\frac{2x^3}{3b\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))} - \frac{4x^2}{b^2\sqrt{\operatorname{atanh}(\tanh(a+bx))}} + \frac{16x\sqrt{\operatorname{atanh}(\tanh(a+bx))}}{b^3} - \frac{32\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))}{3b^4} & \text{for } b \neq 0 \\ \frac{x^4}{4\operatorname{atanh}^{\frac{5}{2}}(\tanh(a))} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Piecewise((-2*x**3/(3*b*atanh(tanh(a + b*x))**(3/2)) - 4*x**2/(b**2*sqrt(at
anh(tanh(a + b*x)))) + 16*x*sqrt(atanh(tanh(a + b*x)))/b**3 - 32*atanh(tanh
(a + b*x))**(3/2)/(3*b**4), Ne(b, 0)), (x**4/(4*atanh(tanh(a))**(5/2)), Tru
e))
```

Giac [A] time = 1.16221, size = 80, normalized size = 1.05

$$-\frac{2\left(9(bx+a)a^2-a^3\right)}{3(bx+a)^{\frac{3}{2}}b^4} + \frac{2\left((bx+a)^{\frac{3}{2}}b^8 - 9\sqrt{bx+a}\,ab^8\right)}{3\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(9*(b*x + a)*a^2 - a^3)/((b*x + a)^(3/2)*b^4) + 2/3*((b*x + a)^(3/2)*b
^8 - 9*sqrt(b*x + a)*a*b^8)/b^12
```

3.160 $\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=59

$$-\frac{8x}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{16\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b^3}-\frac{2x^2}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

```
[Out] (-2*x^2)/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2)) - (8*x)/(3*b^2*Sqrt[ArcTanh[Tan
h[a + b*x]]]) + (16*Sqrt[ArcTanh[Tanh[a + b*x]]])/(3*b^3)
```

Rubi [A] time = 0.0296099, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2157, 30}

$$-\frac{8x}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{16\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b^3}-\frac{2x^2}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (-2*x^2)/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2)) - (8*x)/(3*b^2*Sqrt[ArcTanh[Tan
h[a + b*x]]]) + (16*Sqrt[ArcTanh[Tanh[a + b*x]]])/(3*b^3)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2x^2}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{4\int \frac{x}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx}{3b}$$

$$= -\frac{2x^2}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{8x}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{8\int \frac{1}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,d$$

$$= -\frac{2x^2}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{8x}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{8\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, \mathrm{ta}\right.}{3b^3}$$

$$= -\frac{2x^2}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{8x}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{16\sqrt{\tanh^{-1}(\tanh(a+}}{3b^3}$$

Mathematica [A] time = 0.036097, size = 48, normalized size = 0.81

$$-\frac{2\left(4bx\tanh^{-1}(\tanh(a+bx)) - 8\tanh^{-1}(\tanh(a+bx))^2 + b^2x^2\right)}{3b^3\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (-2*(b^2*x^2 + 4*b*x*ArcTanh[Tanh[a + b*x]] - 8*ArcTanh[Tanh[a + b*x]]^2))/
(3*b^3*ArcTanh[Tanh[a + b*x]]^(3/2))
```

Maple [A] time = 0.043, size = 91, normalized size = 1.5

$$2\frac{1}{b^3}\left(\sqrt{\mathrm{Artanh}(\tanh(bx+a))} - \frac{-2\,\mathrm{Artanh}(\tanh(bx+a)) + 2\,bx}{\sqrt{\mathrm{Artanh}(\tanh(bx+a))}} - 1/3\,\frac{a^2 + 2\,a\,(\mathrm{Artanh}(\tanh(bx+a)) - bx - a) +}{(\mathrm{Artanh}(\tanh(bx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] 2/b^3*(arctanh(tanh(b*x+a))^(1/2)-(-2*arctanh(tanh(b*x+a))+2*b*x)/arctanh(t
anh(b*x+a))^(1/2)-1/3*(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b
*x+a))-b*x-a)^2)/arctanh(tanh(b*x+a))^(3/2))
```

Maxima [A] time = 1.79707, size = 57, normalized size = 0.97

$$\frac{2\left(3\,b^3x^3 + 15\,ab^2x^2 + 20\,a^2bx + 8\,a^3\right)}{3\,(bx+a)^{\frac{5}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(3*b^3*x^3 + 15*a*b^2*x^2 + 20*a^2*b*x + 8*a^3)/((b*x + a)^(5/2)*b^3)
```

Fricas [A] time = 1.95248, size = 111, normalized size = 1.88

$$\frac{2\left(3\,b^2x^2 + 12\,abx + 8\,a^2\right)\sqrt{bx+a}}{3\left(b^5x^2 + 2\,ab^4x + a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*b^2*x^2 + 12*a*b*x + 8*a^2)*sqrt(b*x + a)/(b^5*x^2 + 2*a*b^4*x + a^2
*b^3)
```

Sympy [A] time = 92.1434, size = 71, normalized size = 1.2

$$\begin{cases} -\frac{2x^2}{3b\,\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))} - \frac{8x}{3b^2\sqrt{\operatorname{atanh}(\tanh(a+bx))}} + \frac{16\sqrt{\operatorname{atanh}(\tanh(a+bx))}}{3b^3} & \text{for}\ b \neq 0 \\ \frac{x^3}{3\,\operatorname{atanh}^{\frac{5}{2}}(\tanh(a))} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Piecewise((-2*x**2/(3*b*atanh(tanh(a + b*x))**(3/2)) - 8*x/(3*b**2*sqrt(ata
nh(tanh(a + b*x)))) + 16*sqrt(atanh(tanh(a + b*x)))/(3*b**3), Ne(b, 0)), (x
**3/(3*atanh(tanh(a))**(5/2)), True))
```

Giac [A] time = 1.16516, size = 53, normalized size = 0.9

$$\frac{2\sqrt{bx+a}}{b^3}+\frac{2\left(6\,(bx+a)a-a^2\right)}{3\,(bx+a)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*x + a)/b^3 + 2/3*(6*(b*x + a)*a - a^2)/((b*x + a)^(3/2)*b^3)
```

3.161 $\int \frac{x}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{4}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{2x}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

```
[Out] (-2*x)/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2)) - 4/(3*b^2*Sqrt[ArcTanh[Tanh[a +
b*x]]])
```

Rubi [A] time = 0.0140306, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2157, 30}

$$-\frac{4}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{2x}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (-2*x)/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2)) - 4/(3*b^2*Sqrt[ArcTanh[Tanh[a +
b*x]]])
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx &= -\frac{2x}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}+\frac{2\int \frac{1}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx}{3b}\\ &= -\frac{2x}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}+\frac{2\,\mathrm{Subst}\left(\int \frac{1}{x^{3/2}}\,dx,x,\tanh^{-1}(\tanh(a+bx))\right)}{3b^2}\\ &= -\frac{2x}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}-\frac{4}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}\end{aligned}$$

Mathematica [A] time = 0.0552531, size = 31, normalized size = 0.82

$$-\frac{2\left(2\tanh^{-1}(\tanh(a+bx))+bx\right)}{3b^2\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (-2*(b*x + 2*ArcTanh[Tanh[a + b*x]]))/(3*b^2*ArcTanh[Tanh[a + b*x]]^(3/2))
```

Maple [A] time = 0.039, size = 42, normalized size = 1.1

$$2\,\frac{1}{b^2}\left(-\frac{1}{\sqrt{\mathrm{Artanh}\,(\tanh\,(bx+a))}}-1/3\,\frac{bx-\mathrm{Artanh}\,(\tanh\,(bx+a))}{(\mathrm{Artanh}\,(\tanh\,(bx+a)))^{3/2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] 2/b^2*(-1/arctanh(tanh(b*x+a))^(1/2)-1/3*(b*x-arctanh(tanh(b*x+a)))/arctanh
(tanh(b*x+a))^(3/2))
```

Maxima [A] time = 1.76718, size = 42, normalized size = 1.11

$$-\frac{2\left(3\,b^2x^2+5\,abx+2\,a^2\right)}{3\,(bx+a)^{\frac{5}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(3*b^2*x^2 + 5*a*b*x + 2*a^2)/((b*x + a)^(5/2)*b^2)
```

Fricas [A] time = 2.05453, size = 89, normalized size = 2.34

$$-\frac{2\,(3\,bx+2\,a)\sqrt{bx+a}}{3\left(b^4x^2+2\,ab^3x+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*b*x + 2*a)*sqrt(b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Sympy [A] time = 85.0997, size = 51, normalized size = 1.34

$$\begin{cases} -\frac{2x}{3b\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))}-\frac{4}{3b^2\sqrt{\operatorname{atanh}(\tanh(a+bx))}} & \text{for } b\neq 0\\ \frac{x^2}{2\operatorname{atanh}^{\frac{5}{2}}(\tanh(a))} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Piecewise((-2*x/(3*b*atanh(tanh(a + b*x))**(3/2)) - 4/(3*b**2*sqrt(atanh(ta
nh(a + b*x)))), Ne(b, 0)), (x**2/(2*atanh(tanh(a))**(5/2)), True))
```

Giac [A] time = 1.14186, size = 27, normalized size = 0.71

$$-\frac{2\,(3\,bx+2\,a)}{3\,(bx+a)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*b*x + 2*a)/((b*x + a)^(3/2)*b^2)
```

3.162 $\int \frac{1}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=18

$$-\frac{2}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

```
[Out] -2/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2))
```

Rubi [A] time = 0.0045551, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2157, 30}

$$-\frac{2}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(-5/2),x]
```

```
[Out] -2/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^{5/2}}\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= -\frac{2}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.006595, size = 18, normalized size = 1.

$$-\frac{2}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(-5/2),x]
```

```
[Out] -2/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2))
```

Maple [A] time = 0.029, size = 15, normalized size = 0.8

$$-\frac{2}{3\,b}\left(\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] -2/3/b/arctanh(tanh(b*x+a))^(3/2)
```

Maxima [A] time = 1.7228, size = 16, normalized size = 0.89

$$-\frac{2}{3\left(bx+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3/((b*x + a)^(3/2)*b)
```

Fricas [B] time = 2.10169, size = 68, normalized size = 3.78

$$-\frac{2\sqrt{bx+a}}{3\left(b^{3}x^{2}+2\,ab^{2}x+a^{2}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(b*x + a)/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Sympy [A] time = 70.418, size = 27, normalized size = 1.5

$$\begin{cases} -\frac{2}{3b\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))} & \text{for } b \neq 0 \\ \frac{x}{\operatorname{atanh}^{\frac{5}{2}}(\tanh(a))} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Piecewise((-2/(3*b*atanh(tanh(a + b*x))**(3/2)), Ne(b, 0)), (x/atanh(tanh(a
))**(5/2), True))
```

Giac [A] time = 1.14559, size = 16, normalized size = 0.89

$$-\frac{2}{3(bx+a)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3/((b*x + a)^(3/2)*b)
```

3.163 $\int \frac{1}{x \tanh^{-1}(\tanh(a+bx))^{5/2}} \, dx$

Optimal. Leaf size=108

$$\frac{2}{\left(bx - \tanh^{-1}(\tanh(a+bx))\right)^2 \sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{2}{3\left(bx - \tanh^{-1}(\tanh(a+bx))\right) \tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{2\,\mathrm{t}\ldots}{(bx\ldots}$$

```
[Out] (2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])
/(b*x - ArcTanh[Tanh[a + b*x]])^(5/2) - 2/(3*(b*x - ArcTanh[Tanh[a + b*x]])
*ArcTanh[Tanh[a + b*x]]^(3/2)) + 2/((b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[A
rcTanh[Tanh[a + b*x]]])
```

Rubi [A] time = 0.0540267, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2163, 2161}

$$\frac{2}{\left(bx - \tanh^{-1}(\tanh(a+bx))\right)^2 \sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{2}{3\left(bx - \tanh^{-1}(\tanh(a+bx))\right) \tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{2\,\mathrm{t}\ldots}{(bx\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])
/(b*x - ArcTanh[Tanh[a + b*x]])^(5/2) - 2/(3*(b*x - ArcTanh[Tanh[a + b*x]])
*ArcTanh[Tanh[a + b*x]]^(3/2)) + 2/((b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[A
rcTanh[Tanh[a + b*x]]])
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))^{3/2}}}{bx-\tanh^{-1}(\tanh(a+}$$

$$= -\frac{2}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{\left(bx-\tanh^{-1}(\tanh(a\right.}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}} - \frac{2}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+}$$

Mathematica [A] time = 0.143287, size = 91, normalized size = 0.84

$$\frac{2\left(4\tanh^{-1}(\tanh(a+bx))-bx\right)}{3\tanh^{-1}(\tanh(a+bx))^{3/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2} - \frac{2\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (-2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x
]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2) + (2*(-(b*x) + 4*ArcTanh[Tanh
[a + b*x]]))/(3*ArcTanh[Tanh[a + b*x]]^(3/2)*(-(b*x) + ArcTanh[Tanh[a + b*x
]])^2)
```

Maple [A] time = 0.115, size = 93, normalized size = 0.9

$$2\,\frac{1}{(\mathrm{Artanh}\,(\tanh\,(bx+a))-bx)^2\sqrt{\mathrm{Artanh}\,(\tanh\,(bx+a))}} + \frac{2}{3\,\mathrm{Artanh}\,(\tanh\,(bx+a))-3\,bx}(\mathrm{Artanh}\,(\tanh\,(bx+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] 2/(arctanh(tanh(b*x+a))-b*x)^2/arctanh(tanh(b*x+a))^(1/2)+2/3/(arctanh(tanh
(b*x+a))-b*x)/arctanh(tanh(b*x+a))^(3/2)-2/(arctanh(tanh(b*x+a))-b*x)^(5/2)
*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*arctanh(tanh(b*x + a))^(5/2)), x)
```

Fricas [A] time = 2.13391, size = 409, normalized size = 3.79

$$\left[\frac{3\left(b^2x^2+2\,abx+a^2\right)\sqrt{a}\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\left(3\,abx+4\,a^2\right)\sqrt{bx+a}}{3\left(a^3b^2x^2+2\,a^4bx+a^5\right)},\ \frac{2\left(3\left(b^2x^2+2\,abx+a^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-}}{a}\right.\right.}{3\left(a^3b^2x^2+2\,a^4bx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a
) + 2*a)/x) + 2*(3*a*b*x + 4*a^2)*sqrt(b*x + a))/(a^3*b^2*x^2 + 2*a^4*b*x +
 a^5), 2/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(
-a)/a) + (3*a*b*x + 4*a^2)*sqrt(b*x + a))/(a^3*b^2*x^2 + 2*a^4*b*x + a^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \operatorname{atanh}^{\frac{5}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Integral(1/(x*atanh(tanh(a + b*x))**(5/2)), x)
```

Giac [A] time = 1.15142, size = 61, normalized size = 0.56

$$\frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{2\,(3\,bx+4\,a)}{3\,(bx+a)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + 2/3*(3*b*x + 4*a)/((b*x +
 a)^(3/2)*a^2)
```

3.164 $\int \frac{1}{x^2 \tanh^{-1}(\tanh(a+bx))^{5/2}} \, dx$

Optimal. Leaf size=155

$$\frac{5b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3 \sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{5b}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 \tanh^{-1}(\tanh(a+bx))^{3/2}}+\frac{}{(b}$$

```
[Out] (5*b*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]
])/(b*x - ArcTanh[Tanh[a + b*x]])^(7/2) - 1/(x*ArcTanh[Tanh[a + b*x]]^(5/2)
) + b/((b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(5/2)) - (5*b)
/(3*(b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^(3/2)) + (5*b)/
((b*x - ArcTanh[Tanh[a + b*x]])^3*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rubi [A] time = 0.0998613, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2163, 2161}

$$\frac{5b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3 \sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{5b}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 \tanh^{-1}(\tanh(a+bx))^{3/2}}+\frac{}{(b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```


```
[Out] (5*b*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]
])/(b*x - ArcTanh[Tanh[a + b*x]])^(7/2) - 1/(x*ArcTanh[Tanh[a + b*x]]^(5/2)
) + b/((b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(5/2)) - (5*b)
/(3*(b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^(3/2)) + (5*b)/
((b*x - ArcTanh[Tanh[a + b*x]])^3*Sqrt[ArcTanh[Tanh[a + b*x]]])
```


Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
```

```
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{x^2 \tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{1}{x\tanh^{-1}(\tanh(a+bx))^{5/2}} - \frac{1}{2}(5b)\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))^{7/2}}\,dx$$

$$= -\frac{1}{x\tanh^{-1}(\tanh(a+bx))^{5/2}} + \frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx)}$$

$$= -\frac{1}{x\tanh^{-1}(\tanh(a+bx))^{5/2}} + \frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx)}$$

$$= -\frac{1}{x\tanh^{-1}(\tanh(a+bx))^{5/2}} + \frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx)}$$

$$= \frac{5b\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}} - \frac{1}{x\tanh^{-1}(\tanh(a+bx))^{5/2}} + \frac{}{\left(bx-\tanh^{-1}(\tan\right.}$$

Mathematica [A] time = 0.127023, size = 113, normalized size = 0.73

$$\frac{14bx\tanh^{-1}(\tanh(a+bx)) + 3\tanh^{-1}(\tanh(a+bx))^2 - 2b^2x^2}{3x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{5b\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (5*b*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*
x]]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(7/2) + (-2*b^2*x^2 + 14*b*x*ArcTa
nh[Tanh[a + b*x]] + 3*ArcTanh[Tanh[a + b*x]]^2)/(3*x*(b*x - ArcTanh[Tanh[a
+ b*x]])^3*ArcTanh[Tanh[a + b*x]]^(3/2))
```

Maple [A] time = 0.123, size = 130, normalized size = 0.8

$$2b\left(-\frac{1}{(\mathrm{Artanh}(\tanh(bx+a))-bx)^3}\left(1/2\,\frac{\sqrt{\mathrm{Artanh}(\tanh(bx+a))}}{bx}-5/2\,\frac{1}{\sqrt{\mathrm{Artanh}(\tanh(bx+a))-bx}}\mathrm{Artanh}\left(\frac{\sqrt{}}{\sqrt{\mathrm{A}}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] 2*b*(-1/(arctanh(tanh(b*x+a))-b*x)^3*(1/2*arctanh(tanh(b*x+a))^(1/2)/b/x-5/
2/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arct
anh(tanh(b*x+a))-b*x)^(1/2)))-1/3/(arctanh(tanh(b*x+a))-b*x)^2/arctanh(tanh
(b*x+a))^(3/2)-2/(arctanh(tanh(b*x+a))-b*x)^3/arctanh(tanh(b*x+a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\,\mathrm{artanh}(\tanh(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*arctanh(tanh(b*x + a))^(5/2)), x)
```

Fricas [A] time = 2.15793, size = 494, normalized size = 3.19

$$\left[\frac{15\left(b^3x^3+2\,ab^2x^2+a^2bx\right)\sqrt{a}\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)-2\left(15\,ab^2x^2+20\,a^2bx+3\,a^3\right)\sqrt{bx+a}}{6\left(a^4b^2x^3+2\,a^5bx^2+a^6x\right)},-\frac{15\left(b^3x^3+2\,ab^2x^2+a^2b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)*sqrt(a)*log((b*x + 2*sqrt(b*x +
a)*sqrt(a) + 2*a)/x) - 2*(15*a*b^2*x^2 + 20*a^2*b*x + 3*a^3)*sqrt(b*x + a))
/(a^4*b^2*x^3 + 2*a^5*b*x^2 + a^6*x), -1/3*(15*(b^3*x^3 + 2*a*b^2*x^2 + a^2
*b*x)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (15*a*b^2*x^2 + 20*a^2*b*
x + 3*a^3)*sqrt(b*x + a))/(a^4*b^2*x^3 + 2*a^5*b*x^2 + a^6*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15389, size = 88, normalized size = 0.57

$$-\frac{5\,b\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3}-\frac{2\,(6\,(bx+a)b+ab)}{3\,(bx+a)^{\frac{3}{2}}a^3}-\frac{\sqrt{bx+a}}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] -5*b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) - 2/3*(6*(b*x + a)*b + a
*b)/((b*x + a)^(3/2)*a^3) - sqrt(b*x + a)/(a^3*x)
```

3.165 $\int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=224

$$\frac{35b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{35b^2}{12\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^{3/2}}+$$

```
[Out] (35*b^2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x
]]]])/(4*(b*x - ArcTanh[Tanh[a + b*x]])^(9/2)) + (5*b)/(4*x*ArcTanh[Tanh[a
+ b*x]]^(7/2)) - (5*b^2)/(4*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a +
 b*x]]^(7/2)) - 1/(2*x^2*ArcTanh[Tanh[a + b*x]]^(5/2)) + (7*b^2)/(4*(b*x -
ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^(5/2)) - (35*b^2)/(12*(b*x
 - ArcTanh[Tanh[a + b*x]])^3*ArcTanh[Tanh[a + b*x]]^(3/2)) + (35*b^2)/(4*(b
*x - ArcTanh[Tanh[a + b*x]])^4*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rubi [A] time = 0.15198, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2163, 2161}

$$\frac{35b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{35b^2}{12\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```


```
[Out] (35*b^2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x
]]]])/(4*(b*x - ArcTanh[Tanh[a + b*x]])^(9/2)) + (5*b)/(4*x*ArcTanh[Tanh[a
+ b*x]]^(7/2)) - (5*b^2)/(4*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a +
 b*x]]^(7/2)) - 1/(2*x^2*ArcTanh[Tanh[a + b*x]]^(5/2)) + (7*b^2)/(4*(b*x -
ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^(5/2)) - (35*b^2)/(12*(b*x
 - ArcTanh[Tanh[a + b*x]])^3*ArcTanh[Tanh[a + b*x]]^(3/2)) + (35*b^2)/(4*(b
*x - ArcTanh[Tanh[a + b*x]])^4*Sqrt[ArcTanh[Tanh[a + b*x]]])
```


Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{1}{2x^2 \tanh^{-1}(\tanh(a+bx))^{5/2}} - \frac{1}{4}(5b)\int \frac{1}{x^2 \tanh^{-1}(\tanh(a+bx))^{7/2}}\,dx$$
$$= \frac{5b}{4x\tanh^{-1}(\tanh(a+bx))^{7/2}} - \frac{1}{2x^2\tanh^{-1}(\tanh(a+bx))^{5/2}} + \frac{1}{8}\left(35b^2\right)\int \frac{}{x\tanh^{-1}}$$
$$= \frac{5b}{4x\tanh^{-1}(\tanh(a+bx))^{7/2}} - \frac{5b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx)}$$
$$= \frac{5b}{4x\tanh^{-1}(\tanh(a+bx))^{7/2}} - \frac{5b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx)}$$
$$= \frac{5b}{4x\tanh^{-1}(\tanh(a+bx))^{7/2}} - \frac{5b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx)}$$
$$= \frac{5b}{4x\tanh^{-1}(\tanh(a+bx))^{7/2}} - \frac{5b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx)}$$
$$= \frac{35b^2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{9/2}} + \frac{5b}{4x\tanh^{-1}(\tanh(a+bx))^{7/2}} - \frac{}{4\left(bx-\tanh^{-1}\right.}$$

<u>Mathematica</u> [A] time = 0.115992, size = 133, normalized size = 0.59

$$\frac{80b^2x^2\tanh^{-1}(\tanh(a+bx)) + 39bx\tanh^{-1}(\tanh(a+bx))^2 - 6\tanh^{-1}(\tanh(a+bx))^3 - 8b^3x^3}{12x^2\tanh^{-1}(\tanh(a+bx))^{3/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^4} - \frac{35b^2\tanh^{-1}\left(\frac{\sqrt{\tanh}}{\sqrt{\tanh^{-}}}\right.}{4\left(\tanh^{-1}(\tanh(a\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (-35*b^2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a
+ b*x]]]])/(4*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(9/2)) + (-8*b^3*x^3 + 80*b
^2*x^2*ArcTanh[Tanh[a + b*x]] + 39*b*x*ArcTanh[Tanh[a + b*x]]^2 - 6*ArcTanh
[Tanh[a + b*x]]^3)/(12*x^2*ArcTanh[Tanh[a + b*x]]^(3/2)*(-(b*x) + ArcTanh[T
anh[a + b*x]])^4)
```

Maple [A] time = 0.128, size = 157, normalized size = 0.7

$$2b^2\left(3\frac{1}{(\operatorname{Artanh}(\tanh(bx+a))-bx)^4\sqrt{\operatorname{Artanh}(\tanh(bx+a))}}+1/3\frac{1}{(\operatorname{Artanh}(\tanh(bx+a))-bx)^3(\operatorname{Artanh}(\tanh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] 2*b^2*(3/(arctanh(tanh(b*x+a))-b*x)^4/arctanh(tanh(b*x+a))^(1/2)+1/3/(arcta
nh(tanh(b*x+a))-b*x)^3/arctanh(tanh(b*x+a))^(3/2)+1/(arctanh(tanh(b*x+a))-b
*x)^4*((11/8*arctanh(tanh(b*x+a))^(3/2)+(-13/8*arctanh(tanh(b*x+a))+13/8*b*
x)*arctanh(tanh(b*x+a))^(1/2))/b^2/x^2-35/8/(arctanh(tanh(b*x+a))-b*x)^(1/2
)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \operatorname{artanh}(\tanh(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^3*arctanh(tanh(b*x + a))^(5/2)), x)
```

Fricas [A] time = 2.1548, size = 566, normalized size = 2.53

$$\left[\frac{105\left(b^4x^4+2ab^3x^3+a^2b^2x^2\right)\sqrt{a}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(105ab^3x^3+140a^2b^2x^2+21a^3bx-6a^4\right)\sqrt{bx+a}}{24\left(a^5b^2x^4+2a^6bx^3+a^7x^2\right)},\frac{105\left(b^4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(105*(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(a)*log((b*x - 2*sqrt(
b*x + a)*sqrt(a) + 2*a)/x) + 2*(105*a*b^3*x^3 + 140*a^2*b^2*x^2 + 21*a^3*b*
```

```
x - 6*a^4)*sqrt(b*x + a))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2), 1/12*(105*
(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a
)/a) + (105*a*b^3*x^3 + 140*a^2*b^2*x^2 + 21*a^3*b*x - 6*a^4)*sqrt(b*x + a)
)/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15422, size = 126, normalized size = 0.56

$$\frac{35\,b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^4}+\frac{2\left(9\,(bx+a)b^2+ab^2\right)}{3\,(bx+a)^{\frac{3}{2}}a^4}+\frac{11\,(bx+a)^{\frac{3}{2}}b^2-13\,\sqrt{bx+a}ab^2}{4\,a^4b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 35/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^4) + 2/3*(9*(b*x + a)*b
^2 + a*b^2)/((b*x + a)^(3/2)*a^4) + 1/4*(11*(b*x + a)^(3/2)*b^2 - 13*sqrt(b
*x + a)*a*b^2)/(a^4*b^2*x^2)
```

3.166 $\int \frac{1}{x^4 \tanh^{-1}(\tanh(a+bx))^{5/2}} \, dx$

Optimal. Leaf size=278

$$\frac{105b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^5\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{35b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\tanh^{-1}(\tanh(a+bx))^{3/2}}+$$

```
[Out] (105*b^3*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*
x]]]])/(8*(b*x - ArcTanh[Tanh[a + b*x]])^(11/2)) - (35*b^2)/(24*x*ArcTanh[T
anh[a + b*x]]^(9/2)) + (35*b^3)/(24*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[
Tanh[a + b*x]]^(9/2)) + (5*b)/(12*x^2*ArcTanh[Tanh[a + b*x]]^(7/2)) - (15*b
^3)/(8*(b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^(7/2)) - 1/(
3*x^3*ArcTanh[Tanh[a + b*x]]^(5/2)) + (21*b^3)/(8*(b*x - ArcTanh[Tanh[a + b
*x]])^3*ArcTanh[Tanh[a + b*x]]^(5/2)) - (35*b^3)/(8*(b*x - ArcTanh[Tanh[a +
 b*x]])^4*ArcTanh[Tanh[a + b*x]]^(3/2)) + (105*b^3)/(8*(b*x - ArcTanh[Tanh[
a + b*x]])^5*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rubi [A] time = 0.221521, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2163, 2161}

$$\frac{105b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^5\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{35b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\tanh^{-1}(\tanh(a+bx))^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```


```
[Out] (105*b^3*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*
x]]]])/(8*(b*x - ArcTanh[Tanh[a + b*x]])^(11/2)) - (35*b^2)/(24*x*ArcTanh[T
anh[a + b*x]]^(9/2)) + (35*b^3)/(24*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[
Tanh[a + b*x]]^(9/2)) + (5*b)/(12*x^2*ArcTanh[Tanh[a + b*x]]^(7/2)) - (15*b
^3)/(8*(b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^(7/2)) - 1/(
3*x^3*ArcTanh[Tanh[a + b*x]]^(5/2)) + (21*b^3)/(8*(b*x - ArcTanh[Tanh[a + b
*x]])^3*ArcTanh[Tanh[a + b*x]]^(5/2)) - (35*b^3)/(8*(b*x - ArcTanh[Tanh[a +
 b*x]])^4*ArcTanh[Tanh[a + b*x]]^(3/2)) + (105*b^3)/(8*(b*x - ArcTanh[Tanh[
```

```
a + b*x]])^5*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{x^4 \tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{1}{3x^3\tanh^{-1}(\tanh(a+bx))^{5/2}} - \frac{1}{6}(5b)\int \frac{1}{x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}\,dx$$

$$= \frac{5b}{12x^2\tanh^{-1}(\tanh(a+bx))^{7/2}} - \frac{1}{3x^3\tanh^{-1}(\tanh(a+bx))^{5/2}} + \frac{1}{24}\left(35b^2\right)\int \frac{}{x^2 \text{t}}$$

$$= -\frac{35b^2}{24x\tanh^{-1}(\tanh(a+bx))^{9/2}} + \frac{5b}{12x^2\tanh^{-1}(\tanh(a+bx))^{7/2}} - \frac{1}{3x^3\tanh^{-1}(\tan}$$

$$= -\frac{35b^2}{24x\tanh^{-1}(\tanh(a+bx))^{9/2}} + \frac{35b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a}$$

$$= -\frac{35b^2}{24x\tanh^{-1}(\tanh(a+bx))^{9/2}} + \frac{35b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a}$$

$$= -\frac{35b^2}{24x\tanh^{-1}(\tanh(a+bx))^{9/2}} + \frac{35b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a}$$

$$= -\frac{35b^2}{24x\tanh^{-1}(\tanh(a+bx))^{9/2}} + \frac{35b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a}$$

$$= -\frac{35b^2}{24x\tanh^{-1}(\tanh(a+bx))^{9/2}} + \frac{35b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a}$$

$$= \frac{105b^3\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{11/2}} - \frac{35b^2}{24x\tanh^{-1}(\tanh(a+bx))^{9/2}} + \frac{}{24\left(bx-\text{ta}\right.}$$

Mathematica [A] time = 0.195169, size = 150, normalized size = 0.54

$$\frac{1}{24}\left(\frac{208b^3x^3\tanh^{-1}(\tanh(a+bx)) + 165b^2x^2\tanh^{-1}(\tanh(a+bx))^2 - 50bx\tanh^{-1}(\tanh(a+bx))^3 + 8\tanh^{-1}(\tanh}{x^3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^5\tanh^{-1}(\tanh(a+bx))^{3/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] ((315*b^3*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a
 + b*x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(11/2) + (-16*b^4*x^4 + 208*b
^3*x^3*ArcTanh[Tanh[a + b*x]] + 165*b^2*x^2*ArcTanh[Tanh[a + b*x]]^2 - 50*b
*x*ArcTanh[Tanh[a + b*x]]^3 + 8*ArcTanh[Tanh[a + b*x]]^4)/(x^3*(b*x - ArcTa
nh[Tanh[a + b*x]])^5*ArcTanh[Tanh[a + b*x]]^(3/2)))/24
```

Maple [A] time = 0.125, size = 211, normalized size = 0.8

$$2b^3\left(-\frac{1}{(\operatorname{Artanh}(\tanh(bx+a))-bx)^5}\left(\frac{1}{x^3b^3}\left(\frac{41\ (\operatorname{Artanh}(\tanh(bx+a)))^{5/2}}{16}+\left(-\frac{35\operatorname{Artanh}(\tanh(bx+a))}{6}+\frac{35\,bx}{6}\right)\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] 2*b^3*(-1/(arctanh(tanh(b*x+a))-b*x)^5*((41/16*arctanh(tanh(b*x+a))^(5/2)+(
-35/6*arctanh(tanh(b*x+a))+35/6*b*x)*arctanh(tanh(b*x+a))^(3/2)+(55/16*a^2+
55/8*a*(arctanh(tanh(b*x+a))-b*x-a)+55/16*(arctanh(tanh(b*x+a))-b*x-a)^2)*a
rctanh(tanh(b*x+a))^(1/2))/b^3/x^3-105/16/(arctanh(tanh(b*x+a))-b*x)^(1/2)*
arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))-1/3/(
arctanh(tanh(b*x+a))-b*x)^4/arctanh(tanh(b*x+a))^(3/2)-4/(arctanh(tanh(b*x+
a))-b*x)^5/arctanh(tanh(b*x+a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \operatorname{artanh}(\tanh(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4*arctanh(tanh(b*x + a))^(5/2)), x)
```

Fricas [A] time = 2.17269, size = 613, normalized size = 2.21

$$\left[\frac{315\left(b^5x^5+2\,ab^4x^4+a^2b^3x^3\right)\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)-2\left(315\,ab^4x^4+420\,a^2b^3x^3+63\,a^3b^2x^2-18\,a^4bx+8\,a^5\right)\sqrt{bx+}}{48\left(a^6b^2x^5+2\,a^7bx^4+a^8x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(315*(b^5*x^5 + 2*a*b^4*x^4 + a^2*b^3*x^3)*sqrt(a)*log((b*x + 2*sqrt(
b*x + a)*sqrt(a) + 2*a)/x) - 2*(315*a*b^4*x^4 + 420*a^2*b^3*x^3 + 63*a^3*b^
2*x^2 - 18*a^4*b*x + 8*a^5)*sqrt(b*x + a))/(a^6*b^2*x^5 + 2*a^7*b*x^4 + a^8
*x^3), -1/24*(315*(b^5*x^5 + 2*a*b^4*x^4 + a^2*b^3*x^3)*sqrt(-a)*arctan(sqr
t(b*x + a)*sqrt(-a)/a) + (315*a*b^4*x^4 + 420*a^2*b^3*x^3 + 63*a^3*b^2*x^2
- 18*a^4*b*x + 8*a^5)*sqrt(b*x + a))/(a^6*b^2*x^5 + 2*a^7*b*x^4 + a^8*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15118, size = 155, normalized size = 0.56

$$-\frac{105\, b^3 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}a^5}-\frac{315\,(bx+a)^4 b^3-840\,(bx+a)^3 ab^3+693\,(bx+a)^2 a^2b^3-144\,(bx+a)a^3b^3-16\,a^4b^3}{24\left((bx+a)^{\frac{3}{2}}-\sqrt{bx+a}a\right)^3 a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] -105/8*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^5) - 1/24*(315*(b*x +
 a)^4*b^3 - 840*(b*x + a)^3*a*b^3 + 693*(b*x + a)^2*a^2*b^3 - 144*(b*x + a)
*a^3*b^3 - 16*a^4*b^3)/(((b*x + a)^(3/2) - sqrt(b*x + a)*a)^3*a^5)
```

3.167 $\int x^{7/2} \tanh^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=27

$$\frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))-\frac{4}{99}bx^{11/2}$$

```
[Out] (-4*b*x^(11/2))/99 + (2*x^(9/2)*ArcTanh[Tanh[a + b*x]])/9
```

Rubi [A] time = 0.0097488, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$\frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))-\frac{4}{99}bx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (-4*b*x^(11/2))/99 + (2*x^(9/2)*ArcTanh[Tanh[a + b*x]])/9
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{7/2}\tanh^{-1}(\tanh(a+bx))\,dx = \frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx)) - \frac{1}{9}(2b)\int x^{9/2}\,dx$$
$$= -\frac{4}{99}bx^{11/2} + \frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))$$

Mathematica [A] time = 0.0355843, size = 23, normalized size = 0.85

$$\frac{2}{99}x^{9/2}\left(11\tanh^{-1}(\tanh(a+bx)) - 2bx\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (2*x^(9/2)*(-2*b*x + 11*ArcTanh[Tanh[a + b*x]]))/99
```

Maple [A] time = 0.039, size = 20, normalized size = 0.7

$$-\frac{4b}{99}x^{\frac{11}{2}} + \frac{2\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}{9}x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*arctanh(tanh(b*x+a)),x)
```

```
[Out] -4/99*b*x^(11/2)+2/9*x^(9/2)*arctanh(tanh(b*x+a))
```

Maxima [A] time = 0.978504, size = 26, normalized size = 0.96

$$-\frac{4}{99}\,bx^{\frac{11}{2}} + \frac{2}{9}\,x^{\frac{9}{2}}\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -4/99*b*x^(11/2) + 2/9*x^(9/2)*arctanh(tanh(b*x + a))
```

Fricas [A] time = 1.98763, size = 47, normalized size = 1.74

$$\frac{2}{99}\left(9\,bx^5+11\,ax^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 2/99*(9*b*x^5 + 11*a*x^4)*sqrt(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*atanh(tanh(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14606, size = 18, normalized size = 0.67

$$\frac{2}{11}\,bx^{\frac{11}{2}}+\frac{2}{9}\,ax^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] 2/11*b*x^(11/2) + 2/9*a*x^(9/2)
```

3.168 $\int x^{5/2} \tanh^{-1}(\tanh(a + bx)) \, dx$

Optimal. Leaf size=27

$$\frac{2}{7}x^{7/2}\tanh^{-1}(\tanh(a+bx)) - \frac{4}{63}bx^{9/2}$$

```
[Out] (-4*b*x^(9/2))/63 + (2*x^(7/2)*ArcTanh[Tanh[a + b*x]])/7
```

Rubi [A] time = 0.0090049, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$\frac{2}{7}x^{7/2}\tanh^{-1}(\tanh(a+bx)) - \frac{4}{63}bx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (-4*b*x^(9/2))/63 + (2*x^(7/2)*ArcTanh[Tanh[a + b*x]])/7
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{5/2}\tanh^{-1}(\tanh(a+bx))\,dx = \frac{2}{7}x^{7/2}\tanh^{-1}(\tanh(a+bx)) - \frac{1}{7}(2b)\int x^{7/2}\,dx$$
$$= -\frac{4}{63}bx^{9/2} + \frac{2}{7}x^{7/2}\tanh^{-1}(\tanh(a+bx))$$

Mathematica [A] time = 0.0260056, size = 23, normalized size = 0.85

$$\frac{2}{63}x^{7/2}\left(9\tanh^{-1}(\tanh(a+bx)) - 2bx\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (2*x^(7/2)*(-2*b*x + 9*ArcTanh[Tanh[a + b*x]]))/63
```

Maple [A] time = 0.038, size = 20, normalized size = 0.7

$$-\frac{4\,b}{63}x^{\frac{9}{2}} + \frac{2\,\mathrm{Artanh}\,(\tanh\,(bx+a))}{7}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*arctanh(tanh(b*x+a)),x)
```

```
[Out] -4/63*b*x^(9/2)+2/7*x^(7/2)*arctanh(tanh(b*x+a))
```

Maxima [A] time = 0.982933, size = 26, normalized size = 0.96

$$-\frac{4}{63}\,bx^{\frac{9}{2}} + \frac{2}{7}\,x^{\frac{7}{2}}\,\mathrm{artanh}\,(\tanh\,(bx+a))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -4/63*b*x^(9/2) + 2/7*x^(7/2)*arctanh(tanh(b*x + a))
```

Fricas [A] time = 2.04117, size = 46, normalized size = 1.7

$$\frac{2}{63}\left(7\,bx^4 + 9\,ax^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 2/63*(7*b*x^4 + 9*a*x^3)*sqrt(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*atanh(tanh(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12183, size = 18, normalized size = 0.67

$$\frac{2}{9}\,bx^{\frac{9}{2}} + \frac{2}{7}\,ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] 2/9*b*x^(9/2) + 2/7*a*x^(7/2)
```

3.169 $\int x^{3/2} \tanh^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=27

$$\frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx)) - \frac{4}{35}bx^{7/2}$$

```
[Out] (-4*b*x^(7/2))/35 + (2*x^(5/2)*ArcTanh[Tanh[a + b*x]])/5
```

Rubi [A] time = 0.0098923, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$\frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx)) - \frac{4}{35}bx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (-4*b*x^(7/2))/35 + (2*x^(5/2)*ArcTanh[Tanh[a + b*x]])/5
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{3/2} \tanh^{-1}(\tanh(a+bx))\,dx = \frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx)) - \frac{1}{5}(2b)\int x^{5/2}\,dx$$
$$= -\frac{4}{35}bx^{7/2} + \frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx))$$

Mathematica [A] time = 0.0255402, size = 23, normalized size = 0.85

$$\frac{2}{35}x^{5/2}\left(7\tanh^{-1}(\tanh(a+bx)) - 2bx\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (2*x^(5/2)*(-2*b*x + 7*ArcTanh[Tanh[a + b*x]]))/35
```

Maple [A] time = 0.037, size = 20, normalized size = 0.7

$$-\frac{4\,b}{35}x^{\frac{7}{2}} + \frac{2\,\mathrm{Artanh}\,(\tanh\,(bx+a))}{5}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*arctanh(tanh(b*x+a)),x)
```

```
[Out] -4/35*b*x^(7/2)+2/5*x^(5/2)*arctanh(tanh(b*x+a))
```

Maxima [A] time = 0.977613, size = 26, normalized size = 0.96

$$-\frac{4}{35}\,bx^{\frac{7}{2}} + \frac{2}{5}\,x^{\frac{5}{2}}\,\mathrm{artanh}\,(\tanh\,(bx+a))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -4/35*b*x^(7/2) + 2/5*x^(5/2)*arctanh(tanh(b*x + a))
```

Fricas [A] time = 1.99508, size = 46, normalized size = 1.7

$$\frac{2}{35}\left(5\,bx^{3} + 7\,ax^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b*x^3 + 7*a*x^2)*sqrt(x)
```

Sympy [A] time = 26.3986, size = 26, normalized size = 0.96

$$-\frac{4bx^{\frac{7}{2}}}{35} + \frac{2x^{\frac{5}{2}}\operatorname{atanh}\left(\tanh\left(a + bx\right)\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*atanh(tanh(b*x+a)),x)
```

```
[Out] -4*b*x**(7/2)/35 + 2*x**(5/2)*atanh(tanh(a + b*x))/5
```

Giac [A] time = 1.13407, size = 18, normalized size = 0.67

$$\frac{2}{7}\,bx^{\frac{7}{2}} + \frac{2}{5}\,ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] 2/7*b*x^(7/2) + 2/5*a*x^(5/2)
```

3.170 $\int \sqrt{x} \tanh^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=27

$$\frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))-\frac{4}{15}bx^{5/2}$$

```
[Out] (-4*b*x^(5/2))/15 + (2*x^(3/2)*ArcTanh[Tanh[a + b*x]])/3
```

Rubi [A] time = 0.0080778, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$\frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))-\frac{4}{15}bx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (-4*b*x^(5/2))/15 + (2*x^(3/2)*ArcTanh[Tanh[a + b*x]])/3
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sqrt{x} \tanh^{-1}(\tanh(a+bx))\,dx = \frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx)) - \frac{1}{3}(2b)\int x^{3/2}\,dx$$
$$= -\frac{4}{15}bx^{5/2} + \frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))$$

Mathematica [A] time = 0.0259446, size = 23, normalized size = 0.85

$$\frac{2}{15}x^{3/2}\left(5\tanh^{-1}(\tanh(a+bx)) - 2bx\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (2*x^(3/2)*(-2*b*x + 5*ArcTanh[Tanh[a + b*x]]))/15
```

Maple [A] time = 0.036, size = 20, normalized size = 0.7

$$-\frac{4b}{15}x^{\frac{5}{2}} + \frac{2\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))*x^(1/2),x)
```

```
[Out] -4/15*b*x^(5/2)+2/3*x^(3/2)*arctanh(tanh(b*x+a))
```

Maxima [A] time = 0.988791, size = 26, normalized size = 0.96

$$-\frac{4}{15}bx^{\frac{5}{2}} + \frac{2}{3}x^{\frac{3}{2}}\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))*x^(1/2),x, algorithm="maxima")
```

```
[Out] -4/15*b*x^(5/2) + 2/3*x^(3/2)*arctanh(tanh(b*x + a))
```

Fricas [A] time = 1.98817, size = 43, normalized size = 1.59

$$\frac{2}{15}\left(3\,bx^{2}+5\,ax\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b*x^2 + 5*a*x)*sqrt(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x}\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))*x**(1/2),x)
```

```
[Out] Integral(sqrt(x)*atanh(tanh(a + b*x)), x)
```

Giac [A] time = 1.11668, size = 18, normalized size = 0.67

$$\frac{2}{5}\,bx^{\frac{5}{2}}+\frac{2}{3}\,ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*b*x^(5/2) + 2/3*a*x^(3/2)
```

3.171 $\int \frac{\tanh^{-1}(\tanh(a+bx))}{\sqrt{x}}\,dx$

Optimal. Leaf size=25

$$2\sqrt{x}\tanh^{-1}(\tanh(a+bx)) - \frac{4}{3}bx^{3/2}$$

```
[Out] (-4*b*x^(3/2))/3 + 2*Sqrt[x]*ArcTanh[Tanh[a + b*x]]
```

Rubi [A] time = 0.007859, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$2\sqrt{x}\tanh^{-1}(\tanh(a+bx)) - \frac{4}{3}bx^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]/Sqrt[x],x]
```

```
[Out] (-4*b*x^(3/2))/3 + 2*Sqrt[x]*ArcTanh[Tanh[a + b*x]]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))}{\sqrt{x}}\,dx = 2\sqrt{x}\tanh^{-1}(\tanh(a+bx)) - (2b)\int \sqrt{x}\,dx$$
$$= -\frac{4}{3}bx^{3/2} + 2\sqrt{x}\tanh^{-1}(\tanh(a+bx))$$

Mathematica [A] time = 0.0189722, size = 23, normalized size = 0.92

$$\frac{2}{3}\sqrt{x}\left(3\tanh^{-1}(\tanh(a+bx)) - 2bx\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(-2*b*x + 3*ArcTanh[Tanh[a + b*x]]))/3
```

Maple [A] time = 0.04, size = 20, normalized size = 0.8

$$-\frac{4b}{3}x^{\frac{3}{2}} + 2\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))/x^(1/2),x)
```

```
[Out] -4/3*b*x^(3/2)+2*arctanh(tanh(b*x+a))*x^(1/2)
```

Maxima [A] time = 0.989077, size = 26, normalized size = 1.04

$$-\frac{4}{3}\,bx^{\frac{3}{2}} + 2\,\sqrt{x}\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(1/2),x, algorithm="maxima")
```

```
[Out] -4/3*b*x^(3/2) + 2*sqrt(x)*arctanh(tanh(b*x + a))
```

Fricas [A] time = 1.99092, size = 34, normalized size = 1.36

$$\frac{2}{3}(bx + 3a)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*x + 3*a)*sqrt(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}(\tanh(a + bx))}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))/x**(1/2),x)
```

```
[Out] Integral(atanh(tanh(a + b*x))/sqrt(x), x)
```

Giac [A] time = 1.13399, size = 18, normalized size = 0.72

$$\frac{2}{3}bx^{\frac{3}{2}} + 2a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*b*x^(3/2) + 2*a*sqrt(x)
```

3.172 $\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^{3/2}}\,dx$

Optimal. Leaf size=23

$$4b\sqrt{x}-\frac{2\tanh^{-1}(\tanh(a+bx))}{\sqrt{x}}$$

```
[Out] 4*b*Sqrt[x] - (2*ArcTanh[Tanh[a + b*x]])/Sqrt[x]
```

Rubi [A] time = 0.0082348, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$4b\sqrt{x}-\frac{2\tanh^{-1}(\tanh(a+bx))}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]/x^(3/2),x]
```

```
[Out] 4*b*Sqrt[x] - (2*ArcTanh[Tanh[a + b*x]])/Sqrt[x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^{3/2}}\,dx = -\frac{2\tanh^{-1}(\tanh(a+bx))}{\sqrt{x}} + (2b)\int \frac{1}{\sqrt{x}}\,dx$$
$$= 4b\sqrt{x} - \frac{2\tanh^{-1}(\tanh(a+bx))}{\sqrt{x}}$$

Mathematica [A] time = 0.0196906, size = 20, normalized size = 0.87

$$\frac{4bx - 2\tanh^{-1}(\tanh(a+bx))}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]/x^(3/2),x]
```

```
[Out] (4*b*x - 2*ArcTanh[Tanh[a + b*x]])/Sqrt[x]
```

Maple [A] time = 0.034, size = 20, normalized size = 0.9

$$-2\,\frac{\operatorname{Artanh}(\tanh(bx+a))}{\sqrt{x}} + 4\,b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))/x^(3/2),x)
```

```
[Out] -2*arctanh(tanh(b*x+a))/x^(1/2)+4*b*x^(1/2)
```

Maxima [A] time = 0.981791, size = 26, normalized size = 1.13

$$4\,b\sqrt{x} - \frac{2\,\operatorname{artanh}(\tanh(bx+a))}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(3/2),x, algorithm="maxima")
```

```
[Out] 4*b*sqrt(x) - 2*arctanh(tanh(b*x + a))/sqrt(x)
```

Fricas [A] time = 2.0859, size = 28, normalized size = 1.22

$$\frac{2(bx-a)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(b*x - a)/sqrt(x)
```

Sympy [A] time = 1.64159, size = 22, normalized size = 0.96

$$4b\sqrt{x} - \frac{2\,\mathrm{atanh}\,(\tanh(a+bx))}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))/x**(3/2),x)
```

```
[Out] 4*b*sqrt(x) - 2*atanh(tanh(a + b*x))/sqrt(x)
```

Giac [A] time = 1.15908, size = 18, normalized size = 0.78

$$2\,b\sqrt{x} - \frac{2\,a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(3/2),x, algorithm="giac")
```

```
[Out] 2*b*sqrt(x) - 2*a/sqrt(x)
```

3.173 $\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^{5/2}}\,dx$

Optimal. Leaf size=27

$$-\frac{2\tanh^{-1}(\tanh(a+bx))}{3x^{3/2}}-\frac{4b}{3\sqrt{x}}$$

```
[Out] (-4*b)/(3*Sqrt[x]) - (2*ArcTanh[Tanh[a + b*x]])/(3*x^(3/2))
```

Rubi [A] time = 0.0086571, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$-\frac{2\tanh^{-1}(\tanh(a+bx))}{3x^{3/2}}-\frac{4b}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]/x^(5/2),x]
```

```
[Out] (-4*b)/(3*Sqrt[x]) - (2*ArcTanh[Tanh[a + b*x]])/(3*x^(3/2))
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^{5/2}}\,dx = -\frac{2\tanh^{-1}(\tanh(a+bx))}{3x^{3/2}} + \frac{1}{3}(2b)\int \frac{1}{x^{3/2}}\,dx$$
$$= -\frac{4b}{3\sqrt{x}} - \frac{2\tanh^{-1}(\tanh(a+bx))}{3x^{3/2}}$$

Mathematica [A] time = 0.0181129, size = 21, normalized size = 0.78

$$-\frac{2\left(\tanh^{-1}(\tanh(a+bx))+2bx\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]/x^(5/2),x]
```

```
[Out] (-2*(2*b*x + ArcTanh[Tanh[a + b*x]]))/(3*x^(3/2))
```

Maple [A] time = 0.037, size = 20, normalized size = 0.7

$$-\frac{2\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}{3}x^{-\frac{3}{2}} - \frac{4\,b}{3}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))/x^(5/2),x)
```

```
[Out] -2/3*arctanh(tanh(b*x+a))/x^(3/2)-4/3*b/x^(1/2)
```

Maxima [A] time = 0.986474, size = 26, normalized size = 0.96

$$-\frac{4\,b}{3\,\sqrt{x}} - \frac{2\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(5/2),x, algorithm="maxima")
```

```
[Out] -4/3*b/sqrt(x) - 2/3*arctanh(tanh(b*x + a))/x^(3/2)
```

Fricas [A] time = 2.02828, size = 35, normalized size = 1.3

$$-\frac{2\,(3\,bx+a)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*b*x + a)/x^(3/2)
```

Sympy [A] time = 21.2364, size = 27, normalized size = 1.

$$-\frac{4b}{3\sqrt{x}}-\frac{2\,\mathrm{atanh}\,(\tanh\,(a+bx))}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))/x**(5/2),x)
```

```
[Out] -4*b/(3*sqrt(x)) - 2*atanh(tanh(a + b*x))/(3*x**(3/2))
```

Giac [A] time = 1.15382, size = 15, normalized size = 0.56

$$-\frac{2\,(3\,bx+a)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*b*x + a)/x^(3/2)
```

3.174 $\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^{7/2}} dx$

Optimal. Leaf size=27

$$-\frac{2\tanh^{-1}(\tanh(a+bx))}{5x^{5/2}}-\frac{4b}{15x^{3/2}}$$

```
[Out] (-4*b)/(15*x^(3/2)) - (2*ArcTanh[Tanh[a + b*x]])/(5*x^(5/2))
```

Rubi [A] time = 0.0087224, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$-\frac{2\tanh^{-1}(\tanh(a+bx))}{5x^{5/2}}-\frac{4b}{15x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]/x^(7/2),x]
```

```
[Out] (-4*b)/(15*x^(3/2)) - (2*ArcTanh[Tanh[a + b*x]])/(5*x^(5/2))
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^{7/2}}\,dx = -\frac{2\tanh^{-1}(\tanh(a+bx))}{5x^{5/2}} + \frac{1}{5}(2b)\int \frac{1}{x^{5/2}}\,dx$$
$$= -\frac{4b}{15x^{3/2}} - \frac{2\tanh^{-1}(\tanh(a+bx))}{5x^{5/2}}$$

Mathematica [A] time = 0.020748, size = 23, normalized size = 0.85

$$-\frac{2\left(3\tanh^{-1}(\tanh(a+bx))+2bx\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]/x^(7/2),x]
```

```
[Out] (-2*(2*b*x + 3*ArcTanh[Tanh[a + b*x]]))/(15*x^(5/2))
```

Maple [A] time = 0.039, size = 20, normalized size = 0.7

$$-\frac{4\,b}{15}x^{-\frac{3}{2}} - \frac{2\,\mathrm{Artanh}\,(\tanh\,(bx+a))}{5}x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))/x^(7/2),x)
```

```
[Out] -4/15*b/x^(3/2)-2/5*arctanh(tanh(b*x+a))/x^(5/2)
```

Maxima [A] time = 0.979525, size = 26, normalized size = 0.96

$$-\frac{4\,b}{15\,x^{\frac{3}{2}}} - \frac{2\,\mathrm{artanh}\,(\tanh\,(bx+a))}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(7/2),x, algorithm="maxima")
```

```
[Out] -4/15*b/x^(3/2) - 2/5*arctanh(tanh(b*x + a))/x^(5/2)
```

Fricas [A] time = 1.97809, size = 39, normalized size = 1.44

$$-\frac{2\,(5\,bx+3\,a)}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(7/2),x, algorithm="fricas")
```

```
[Out] -2/15*(5*b*x + 3*a)/x^(5/2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))/x**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16366, size = 18, normalized size = 0.67

$$-\frac{2\,(5\,bx+3\,a)}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*(5*b*x + 3*a)/x^(5/2)
```

3.175 $\int x^{7/2}\tanh^{-1}(\tanh(a+bx))^2\,dx$

Optimal. Leaf size=48

$$-\frac{8}{99}bx^{11/2}\tanh^{-1}(\tanh(a+bx))+\frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{16b^2x^{13/2}}{1287}$$

```
[Out] (16*b^2*x^(13/2))/1287 - (8*b*x^(11/2)*ArcTanh[Tanh[a + b*x]])/99 + (2*x^(9
/2)*ArcTanh[Tanh[a + b*x]]^2)/9
```

Rubi [A] time = 0.0231425, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 30}

$$-\frac{8}{99}bx^{11/2}\tanh^{-1}(\tanh(a+bx))+\frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{16b^2x^{13/2}}{1287}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (16*b^2*x^(13/2))/1287 - (8*b*x^(11/2)*ArcTanh[Tanh[a + b*x]])/99 + (2*x^(9
/2)*ArcTanh[Tanh[a + b*x]]^2)/9
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{7/2}\tanh^{-1}(\tanh(a+bx))^2\,dx = \frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))^2 - \frac{1}{9}(4b)\int x^{9/2}\tanh^{-1}(\tanh(a+bx))\,dx$$
$$= -\frac{8}{99}bx^{11/2}\tanh^{-1}(\tanh(a+bx)) + \frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))^2 + \frac{1}{99}\left(8b^2\right)\int x^{11}$$
$$= \frac{16b^2x^{13/2}}{1287} - \frac{8}{99}bx^{11/2}\tanh^{-1}(\tanh(a+bx)) + \frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))^2$$

Mathematica [A] time = 0.060239, size = 40, normalized size = 0.83

$$\frac{2x^{9/2}\left(-52bx\tanh^{-1}(\tanh(a+bx)) + 143\tanh^{-1}(\tanh(a+bx))^2 + 8b^2x^2\right)}{1287}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (2*x^(9/2)*(8*b^2*x^2 - 52*b*x*ArcTanh[Tanh[a + b*x]] + 143*ArcTanh[Tanh[a
+ b*x]]^2))/1287
```

Maple [A] time = 0.042, size = 38, normalized size = 0.8

$$\frac{2\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^2}{9}x^{\frac{9}{2}} - \frac{8b}{9}\left(\frac{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}{11}x^{\frac{11}{2}} - \frac{2b}{143}x^{\frac{13}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*arctanh(tanh(b*x+a))^2,x)
```

```
[Out] 2/9*x^(9/2)*arctanh(tanh(b*x+a))^2-8/9*b*(1/11*x^(11/2)*arctanh(tanh(b*x+a)
)-2/143*x^(13/2)*b)
```

Maxima [A] time = 1.01942, size = 49, normalized size = 1.02

$$\frac{16}{1287}b^2x^{\frac{13}{2}} - \frac{8}{99}bx^{\frac{11}{2}}\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right) + \frac{2}{9}x^{\frac{9}{2}}\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 16/1287*b^2*x^(13/2) - 8/99*b*x^(11/2)*arctanh(tanh(b*x + a)) + 2/9*x^(9/2)
*arctanh(tanh(b*x + a))^2
```

Fricas [A] time = 1.97008, size = 77, normalized size = 1.6

$$\frac{2}{1287}\left(99\,b^2x^6+234\,abx^5+143\,a^2x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 2/1287*(99*b^2*x^6 + 234*a*b*x^5 + 143*a^2*x^4)*sqrt(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*atanh(tanh(b*x+a))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13908, size = 32, normalized size = 0.67

$$\frac{2}{13}\,b^2x^{\frac{13}{2}}+\frac{4}{11}\,abx^{\frac{11}{2}}+\frac{2}{9}\,a^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 2/13*b^2*x^(13/2) + 4/11*a*b*x^(11/2) + 2/9*a^2*x^(9/2)
```

3.176 $\int x^{5/2} \tanh^{-1}(\tanh(a+bx))^2 \, dx$

Optimal. Leaf size=48

$$-\frac{8}{63}bx^{9/2}\tanh^{-1}(\tanh(a+bx))+\frac{2}{7}x^{7/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{16}{693}b^2x^{11/2}$$

```
[Out] (16*b^2*x^(11/2))/693 - (8*b*x^(9/2)*ArcTanh[Tanh[a + b*x]])/63 + (2*x^(7/2
)*ArcTanh[Tanh[a + b*x]]^2)/7
```

Rubi [A] time = 0.0220721, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 30}

$$-\frac{8}{63}bx^{9/2}\tanh^{-1}(\tanh(a+bx))+\frac{2}{7}x^{7/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{16}{693}b^2x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (16*b^2*x^(11/2))/693 - (8*b*x^(9/2)*ArcTanh[Tanh[a + b*x]])/63 + (2*x^(7/2
)*ArcTanh[Tanh[a + b*x]]^2)/7
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{5/2}\tanh^{-1}(\tanh(a+bx))^2\,dx = \frac{2}{7}x^{7/2}\tanh^{-1}(\tanh(a+bx))^2 - \frac{1}{7}(4b)\int x^{7/2}\tanh^{-1}(\tanh(a+bx))\,dx$$
$$= -\frac{8}{63}bx^{9/2}\tanh^{-1}(\tanh(a+bx)) + \frac{2}{7}x^{7/2}\tanh^{-1}(\tanh(a+bx))^2 + \frac{1}{63}\left(8b^2\right)\int x^{9/2}\,dx$$
$$= \frac{16}{693}b^2x^{11/2} - \frac{8}{63}bx^{9/2}\tanh^{-1}(\tanh(a+bx)) + \frac{2}{7}x^{7/2}\tanh^{-1}(\tanh(a+bx))^2$$

Mathematica [A] time = 0.0438811, size = 40, normalized size = 0.83

$$\frac{2}{693}x^{7/2}\left(-44bx\tanh^{-1}(\tanh(a+bx)) + 99\tanh^{-1}(\tanh(a+bx))^2 + 8b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (2*x^(7/2)*(8*b^2*x^2 - 44*b*x*ArcTanh[Tanh[a + b*x]] + 99*ArcTanh[Tanh[a +
 b*x]]^2))/693
```

Maple [A] time = 0.042, size = 38, normalized size = 0.8

$$\frac{2\left(\operatorname{Artanh}(\tanh(bx+a))\right)^2}{7}x^{\frac{7}{2}} - \frac{8b}{7}\left(\frac{\operatorname{Artanh}(\tanh(bx+a))}{9}x^{\frac{9}{2}} - \frac{2b}{99}x^{\frac{11}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*arctanh(tanh(b*x+a))^2,x)
```

```
[Out] 2/7*x^(7/2)*arctanh(tanh(b*x+a))^2-8/7*b*(1/9*x^(9/2)*arctanh(tanh(b*x+a))-
2/99*b*x^(11/2))
```

Maxima [A] time = 1.02176, size = 49, normalized size = 1.02

$$\frac{16}{693}b^2x^{\frac{11}{2}} - \frac{8}{63}bx^{\frac{9}{2}}\operatorname{artanh}(\tanh(bx+a)) + \frac{2}{7}x^{\frac{7}{2}}\operatorname{artanh}(\tanh(bx+a))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 16/693*b^2*x^(11/2) - 8/63*b*x^(9/2)*arctanh(tanh(b*x + a)) + 2/7*x^(7/2)*a
rctanh(tanh(b*x + a))^2
```

Fricas [A] time = 1.96114, size = 74, normalized size = 1.54

$$\frac{2}{693}\left(63\,b^2x^5 + 154\,abx^4 + 99\,a^2x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 2/693*(63*b^2*x^5 + 154*a*b*x^4 + 99*a^2*x^3)*sqrt(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*atanh(tanh(b*x+a))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11666, size = 32, normalized size = 0.67

$$\frac{2}{11}\,b^2x^{\frac{11}{2}} + \frac{4}{9}\,abx^{\frac{9}{2}} + \frac{2}{7}\,a^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 2/11*b^2*x^(11/2) + 4/9*a*b*x^(9/2) + 2/7*a^2*x^(7/2)
```

3.177 $\int x^{3/2} \tanh^{-1}(\tanh(a+bx))^2 \, dx$

Optimal. Leaf size=48

$$-\frac{8}{35}bx^{7/2}\tanh^{-1}(\tanh(a+bx))+\frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{16}{315}b^2x^{9/2}$$

```
[Out] (16*b^2*x^(9/2))/315 - (8*b*x^(7/2)*ArcTanh[Tanh[a + b*x]])/35 + (2*x^(5/2)
*ArcTanh[Tanh[a + b*x]]^2)/5
```

Rubi [A] time = 0.0228219, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 30}

$$-\frac{8}{35}bx^{7/2}\tanh^{-1}(\tanh(a+bx))+\frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{16}{315}b^2x^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (16*b^2*x^(9/2))/315 - (8*b*x^(7/2)*ArcTanh[Tanh[a + b*x]])/35 + (2*x^(5/2)
*ArcTanh[Tanh[a + b*x]]^2)/5
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\tanh^{-1}(\tanh(a+bx))^2\,dx &= \frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx))^2-\frac{1}{5}(4b)\int x^{5/2}\tanh^{-1}(\tanh(a+bx))\,dx\\ &= -\frac{8}{35}bx^{7/2}\tanh^{-1}(\tanh(a+bx))+\frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{1}{35}\left(8b^2\right)\int x^{7/2}\\ &= \frac{16}{315}b^2x^{9/2}-\frac{8}{35}bx^{7/2}\tanh^{-1}(\tanh(a+bx))+\frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx))^2\end{aligned}$$

Mathematica [A] time = 0.0401329, size = 40, normalized size = 0.83

$$\frac{2}{315}x^{5/2}\left(-36bx\tanh^{-1}(\tanh(a+bx))+63\tanh^{-1}(\tanh(a+bx))^2+8b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (2*x^(5/2)*(8*b^2*x^2 - 36*b*x*ArcTanh[Tanh[a + b*x]] + 63*ArcTanh[Tanh[a +
 b*x]]^2))/315
```

Maple [A] time = 0.039, size = 38, normalized size = 0.8

$$\frac{2\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^2}{5}x^{\frac{5}{2}}-\frac{8\,b}{5}\left(\frac{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}{7}x^{\frac{7}{2}}-\frac{2\,b}{63}x^{\frac{9}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*arctanh(tanh(b*x+a))^2,x)
```

```
[Out] 2/5*x^(5/2)*arctanh(tanh(b*x+a))^2-8/5*b*(1/7*x^(7/2)*arctanh(tanh(b*x+a))-
2/63*b*x^(9/2))
```

Maxima [A] time = 1.01492, size = 49, normalized size = 1.02

$$\frac{16}{315}\,b^2x^{\frac{9}{2}}-\frac{8}{35}\,bx^{\frac{7}{2}}\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)+\frac{2}{5}\,x^{\frac{5}{2}}\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 16/315*b^2*x^(9/2) - 8/35*b*x^(7/2)*arctanh(tanh(b*x + a)) + 2/5*x^(5/2)*ar
ctanh(tanh(b*x + a))^2
```

Fricas [A] time = 2.0062, size = 73, normalized size = 1.52

$$\frac{2}{315}\left(35\,b^2x^4 + 90\,abx^3 + 63\,a^2x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^2*x^4 + 90*a*b*x^3 + 63*a^2*x^2)*sqrt(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{\frac{3}{2}}\,\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(x**(3/2)*atanh(tanh(a + b*x))**2, x)
```

Giac [A] time = 1.12977, size = 32, normalized size = 0.67

$$\frac{2}{9}\,b^2x^{\frac{9}{2}} + \frac{4}{7}\,abx^{\frac{7}{2}} + \frac{2}{5}\,a^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 2/9*b^2*x^(9/2) + 4/7*a*b*x^(7/2) + 2/5*a^2*x^(5/2)
```

3.178 $\int \sqrt{x}\tanh^{-1}(\tanh(a+bx))^2\,dx$

Optimal. Leaf size=48

$$-\frac{8}{15}bx^{5/2}\tanh^{-1}(\tanh(a+bx))+\frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{16}{105}b^2x^{7/2}$$

```
[Out] (16*b^2*x^(7/2))/105 - (8*b*x^(5/2)*ArcTanh[Tanh[a + b*x]])/15 + (2*x^(3/2)
*ArcTanh[Tanh[a + b*x]]^2)/3
```

Rubi [A] time = 0.0217704, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 30}

$$-\frac{8}{15}bx^{5/2}\tanh^{-1}(\tanh(a+bx))+\frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{16}{105}b^2x^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (16*b^2*x^(7/2))/105 - (8*b*x^(5/2)*ArcTanh[Tanh[a + b*x]])/15 + (2*x^(3/2)
*ArcTanh[Tanh[a + b*x]]^2)/3
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\tanh^{-1}(\tanh(a+bx))^2\,dx &= \frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))^2 - \frac{1}{3}(4b)\int x^{3/2}\tanh^{-1}(\tanh(a+bx))\,dx \\ &= -\frac{8}{15}bx^{5/2}\tanh^{-1}(\tanh(a+bx)) + \frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))^2 + \frac{1}{15}\left(8b^2\right)\int x^{5/2}\,dx \\ &= \frac{16}{105}b^2x^{7/2} - \frac{8}{15}bx^{5/2}\tanh^{-1}(\tanh(a+bx)) + \frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))^2\end{aligned}$$

Mathematica [A] time = 0.0482864, size = 40, normalized size = 0.83

$$\frac{2}{105}x^{3/2}\left(-28bx\tanh^{-1}(\tanh(a+bx)) + 35\tanh^{-1}(\tanh(a+bx))^2 + 8b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (2*x^(3/2)*(8*b^2*x^2 - 28*b*x*ArcTanh[Tanh[a + b*x]] + 35*ArcTanh[Tanh[a +
 b*x]]^2))/105
```

Maple [A] time = 0.04, size = 38, normalized size = 0.8

$$\frac{2\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^2}{3}x^{\frac{3}{2}} - \frac{8b}{3}\left(\frac{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}{5}x^{\frac{5}{2}} - \frac{2b}{35}x^{\frac{7}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2*x^(1/2),x)
```

```
[Out] 2/3*x^(3/2)*arctanh(tanh(b*x+a))^2-8/3*b*(1/5*x^(5/2)*arctanh(tanh(b*x+a))-
2/35*b*x^(7/2))
```

Maxima [A] time = 1.03271, size = 49, normalized size = 1.02

$$\frac{16}{105}b^2x^{\frac{7}{2}} - \frac{8}{15}bx^{\frac{5}{2}}\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right) + \frac{2}{3}x^{\frac{3}{2}}\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2*x^(1/2),x, algorithm="maxima")
```

```
[Out] 16/105*b^2*x^(7/2) - 8/15*b*x^(5/2)*arctanh(tanh(b*x + a)) + 2/3*x^(3/2)*ar
ctanh(tanh(b*x + a))^2
```

Fricas [A] time = 2.00495, size = 70, normalized size = 1.46

$$\frac{2}{105}\left(15\,b^2x^3+42\,abx^2+35\,a^2x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^2*x^3 + 42*a*b*x^2 + 35*a^2*x)*sqrt(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x}\,\mathrm{atanh}^2\left(\tanh\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2*x**(1/2),x)
```

```
[Out] Integral(sqrt(x)*atanh(tanh(a + b*x))**2, x)
```

Giac [A] time = 1.12525, size = 32, normalized size = 0.67

$$\frac{2}{7}\,b^2x^{\frac{7}{2}}+\frac{4}{5}\,abx^{\frac{5}{2}}+\frac{2}{3}\,a^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/7*b^2*x^(7/2) + 4/5*a*b*x^(5/2) + 2/3*a^2*x^(3/2)
```

3.179 $\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}} dx$

Optimal. Leaf size=46

$$-\frac{8}{3}bx^{3/2}\tanh^{-1}(\tanh(a+bx)) + 2\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2 + \frac{16}{15}b^2x^{5/2}$$

```
[Out] (16*b^2*x^(5/2))/15 - (8*b*x^(3/2)*ArcTanh[Tanh[a + b*x]])/3 + 2*Sqrt[x]*Ar
cTanh[Tanh[a + b*x]]^2
```

Rubi [A] time = 0.0208972, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 30}

$$-\frac{8}{3}bx^{3/2}\tanh^{-1}(\tanh(a+bx)) + 2\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2 + \frac{16}{15}b^2x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^2/Sqrt[x],x]
```

```
[Out] (16*b^2*x^(5/2))/15 - (8*b*x^(3/2)*ArcTanh[Tanh[a + b*x]])/3 + 2*Sqrt[x]*Ar
cTanh[Tanh[a + b*x]]^2
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}}\,dx = 2\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2 - (4b)\int \sqrt{x}\tanh^{-1}(\tanh(a+bx))\,dx$$
$$= -\frac{8}{3}bx^{3/2}\tanh^{-1}(\tanh(a+bx)) + 2\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2 + \frac{1}{3}\left(8b^2\right)\int x^{3/2}\,dx$$
$$= \frac{16}{15}b^2x^{5/2} - \frac{8}{3}bx^{3/2}\tanh^{-1}(\tanh(a+bx)) + 2\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2$$

Mathematica [A] time = 0.0311231, size = 40, normalized size = 0.87

$$\frac{2}{15}\sqrt{x}\left(-20bx\tanh^{-1}(\tanh(a+bx)) + 15\tanh^{-1}(\tanh(a+bx))^2 + 8b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^2/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(8*b^2*x^2 - 20*b*x*ArcTanh[Tanh[a + b*x]] + 15*ArcTanh[Tanh[a +
 b*x]]^2))/15
```

Maple [A] time = 0.04, size = 47, normalized size = 1.

$$\frac{2b^2}{5}x^{\frac{5}{2}} + \frac{(4\,\mathrm{Artanh}(\tanh(bx+a)) - 4\,bx)\,b}{3}x^{\frac{3}{2}} + 2\,(\mathrm{Artanh}(\tanh(bx+a)) - bx)^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2/x^(1/2),x)
```

```
[Out] 2/5*b^2*x^(5/2)+4/3*(arctanh(tanh(b*x+a))-b*x)*b*x^(3/2)+2*(arctanh(tanh(b*
x+a))-b*x)^2*x^(1/2)
```

Maxima [A] time = 1.02751, size = 49, normalized size = 1.07

$$\frac{16}{15}b^2x^{\frac{5}{2}} - \frac{8}{3}bx^{\frac{3}{2}}\,\mathrm{artanh}(\tanh(bx+a)) + 2\sqrt{x}\,\mathrm{artanh}(\tanh(bx+a))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 16/15*b^2*x^(5/2) - 8/3*b*x^(3/2)*arctanh(tanh(b*x + a)) + 2*sqrt(x)*arctan
h(tanh(b*x + a))^2
```

Fricas [A] time = 1.99867, size = 62, normalized size = 1.35

$$\frac{2}{15}\left(3\,b^2x^2+10\,abx+15\,a^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^2 + 10*a*b*x + 15*a^2)*sqrt(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2/x**(1/2),x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**2/sqrt(x), x)
```

Giac [A] time = 1.1664, size = 32, normalized size = 0.7

$$\frac{2}{5}\,b^2x^{\frac{5}{2}}+\frac{4}{3}\,abx^{\frac{3}{2}}+2\,a^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*b^2*x^(5/2) + 4/3*a*b*x^(3/2) + 2*a^2*sqrt(x)
```

3.180 $\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^{3/2}} \, dx$

Optimal. Leaf size=44

$$8b\sqrt{x}\tanh^{-1}(\tanh(a+bx)) - \frac{2\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}} - \frac{16}{3}b^2x^{3/2}$$

```
[Out] (-16*b^2*x^(3/2))/3 + 8*b*Sqrt[x]*ArcTanh[Tanh[a + b*x]] - (2*ArcTanh[Tanh[
a + b*x]]^2)/Sqrt[x]
```

Rubi [A] time = 0.0217644, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 30}

$$8b\sqrt{x}\tanh^{-1}(\tanh(a+bx)) - \frac{2\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}} - \frac{16}{3}b^2x^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^2/x^(3/2),x]
```

```
[Out] (-16*b^2*x^(3/2))/3 + 8*b*Sqrt[x]*ArcTanh[Tanh[a + b*x]] - (2*ArcTanh[Tanh[
a + b*x]]^2)/Sqrt[x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^{3/2}}\,dx = -\frac{2\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}} + (4b)\int \frac{\tanh^{-1}(\tanh(a+bx))}{\sqrt{x}}\,dx$$
$$= 8b\sqrt{x}\tanh^{-1}(\tanh(a+bx)) - \frac{2\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}} - \left(8b^2\right)\int \sqrt{x}\,dx$$
$$= -\frac{16}{3}b^2x^{3/2} + 8b\sqrt{x}\tanh^{-1}(\tanh(a+bx)) - \frac{2\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}}$$

Mathematica [A] time = 0.042224, size = 40, normalized size = 0.91

$$-\frac{2\left(-12bx\tanh^{-1}(\tanh(a+bx)) + 3\tanh^{-1}(\tanh(a+bx))^2 + 8b^2x^2\right)}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^2/x^(3/2),x]
```

```
[Out] (-2*(8*b^2*x^2 - 12*b*x*ArcTanh[Tanh[a + b*x]] + 3*ArcTanh[Tanh[a + b*x]]^2
))/(3*Sqrt[x])
```

Maple [A] time = 0.04, size = 37, normalized size = 0.8

$$-2\,\frac{(\mathrm{Artanh}\,(\tanh\,(bx+a)))^2}{\sqrt{x}} + 8\,b\left(\mathrm{Artanh}\,(\tanh\,(bx+a))\,\sqrt{x} - 2/3\,bx^{3/2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2/x^(3/2),x)
```

```
[Out] -2*arctanh(tanh(b*x+a))^2/x^(1/2)+8*b*(arctanh(tanh(b*x+a))*x^(1/2)-2/3*b*x
^(3/2))
```

Maxima [A] time = 1.01822, size = 49, normalized size = 1.11

$$-\frac{16}{3}\,b^2x^{\frac{3}{2}} + 8\,b\sqrt{x}\,\mathrm{artanh}\,(\tanh\,(bx+a)) - \frac{2\,\mathrm{artanh}\,(\tanh\,(bx+a))^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(3/2),x, algorithm="maxima")
```

```
[Out] -16/3*b^2*x^(3/2) + 8*b*sqrt(x)*arctanh(tanh(b*x + a)) - 2*arctanh(tanh(b*x
 + a))^2/sqrt(x)
```

Fricas [A] time = 2.0064, size = 55, normalized size = 1.25

$$\frac{2\left(b^2x^2+6\,abx-3\,a^2\right)}{3\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^2*x^2 + 6*a*b*x - 3*a^2)/sqrt(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2/x**(3/2),x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**2/x**(3/2), x)
```

Giac [A] time = 1.1522, size = 32, normalized size = 0.73

$$\frac{2}{3}\,b^2x^{\frac{3}{2}}+4\,ab\sqrt{x}-\frac{2\,a^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*b^2*x^(3/2) + 4*a*b*sqrt(x) - 2*a^2/sqrt(x)
```

3.181 $\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^{5/2}}\,dx$

Optimal. Leaf size=48

$$-\frac{2\tanh^{-1}(\tanh(a+bx))^2}{3x^{3/2}}-\frac{8b\tanh^{-1}(\tanh(a+bx))}{3\sqrt{x}}+\frac{16b^2\sqrt{x}}{3}$$

```
[Out] (16*b^2*Sqrt[x])/3 - (8*b*ArcTanh[Tanh[a + b*x]])/(3*Sqrt[x]) - (2*ArcTanh[
Tanh[a + b*x]]^2)/(3*x^(3/2))
```

Rubi [A] time = 0.0230305, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 30}

$$-\frac{2\tanh^{-1}(\tanh(a+bx))^2}{3x^{3/2}}-\frac{8b\tanh^{-1}(\tanh(a+bx))}{3\sqrt{x}}+\frac{16b^2\sqrt{x}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^2/x^(5/2),x]
```

```
[Out] (16*b^2*Sqrt[x])/3 - (8*b*ArcTanh[Tanh[a + b*x]])/(3*Sqrt[x]) - (2*ArcTanh[
Tanh[a + b*x]]^2)/(3*x^(3/2))
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^{5/2}}\,dx = -\frac{2\tanh^{-1}(\tanh(a+bx))^2}{3x^{3/2}} + \frac{1}{3}(4b)\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^{3/2}}\,dx$$
$$= -\frac{8b\tanh^{-1}(\tanh(a+bx))}{3\sqrt{x}} - \frac{2\tanh^{-1}(\tanh(a+bx))^2}{3x^{3/2}} + \frac{1}{3}\left(8b^2\right)\int \frac{1}{\sqrt{x}}\,dx$$
$$= \frac{16b^2\sqrt{x}}{3} - \frac{8b\tanh^{-1}(\tanh(a+bx))}{3\sqrt{x}} - \frac{2\tanh^{-1}(\tanh(a+bx))^2}{3x^{3/2}}$$

Mathematica [A] time = 0.0442962, size = 40, normalized size = 0.83

$$\frac{2\left(-4bx\tanh^{-1}(\tanh(a+bx)) - \tanh^{-1}(\tanh(a+bx))^2 + 8b^2x^2\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^2/x^(5/2),x]
```

```
[Out] (2*(8*b^2*x^2 - 4*b*x*ArcTanh[Tanh[a + b*x]] - ArcTanh[Tanh[a + b*x]]^2))/(
3*x^(3/2))
```

Maple [A] time = 0.041, size = 38, normalized size = 0.8

$$-\frac{2\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^2}{3}x^{-\frac{3}{2}} + \frac{8b}{3}\left(-\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\frac{1}{\sqrt{x}} + 2b\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2/x^(5/2),x)
```

```
[Out] -2/3*arctanh(tanh(b*x+a))^2/x^(3/2)+8/3*b*(-arctanh(tanh(b*x+a))/x^(1/2)+2*
b*x^(1/2))
```

Maxima [A] time = 1.04449, size = 49, normalized size = 1.02

$$\frac{16}{3}b^2\sqrt{x} - \frac{8b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)}{3\sqrt{x}} - \frac{2\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(5/2),x, algorithm="maxima")
```

```
[Out] 16/3*b^2*sqrt(x) - 8/3*b*arctanh(tanh(b*x + a))/sqrt(x) - 2/3*arctanh(tanh(
b*x + a))^2/x^(3/2)
```

Fricas [A] time = 1.9352, size = 55, normalized size = 1.15

$$\frac{2\left(3\,b^2x^2 - 6\,abx - a^2\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*b^2*x^2 - 6*a*b*x - a^2)/x^(3/2)
```

Sympy [A] time = 22.5462, size = 48, normalized size = 1.

$$\frac{16b^2\sqrt{x}}{3} - \frac{8b\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{3\sqrt{x}} - \frac{2\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2/x**(5/2),x)
```

```
[Out] 16*b**2*sqrt(x)/3 - 8*b*atanh(tanh(a + b*x))/(3*sqrt(x)) - 2*atanh(tanh(a +
 b*x))**2/(3*x**(3/2))
```

Giac [A] time = 1.13166, size = 31, normalized size = 0.65

$$2\,b^2\sqrt{x} - \frac{2\left(6\,abx + a^2\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(5/2),x, algorithm="giac")
```

```
[Out] 2*b^2*sqrt(x) - 2/3*(6*a*b*x + a^2)/x^(3/2)
```

3.182 $\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^{7/2}}\,dx$

Optimal. Leaf size=48

$$-\frac{8b\tanh^{-1}(\tanh(a+bx))}{15x^{3/2}}-\frac{2\tanh^{-1}(\tanh(a+bx))^2}{5x^{5/2}}-\frac{16b^2}{15\sqrt{x}}$$

```
[Out] (-16*b^2)/(15*Sqrt[x]) - (8*b*ArcTanh[Tanh[a + b*x]])/(15*x^(3/2)) - (2*Arc
Tanh[Tanh[a + b*x]]^2)/(5*x^(5/2))
```

Rubi [A] time = 0.023184, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 30}

$$-\frac{8b\tanh^{-1}(\tanh(a+bx))}{15x^{3/2}}-\frac{2\tanh^{-1}(\tanh(a+bx))^2}{5x^{5/2}}-\frac{16b^2}{15\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^2/x^(7/2),x]
```

```
[Out] (-16*b^2)/(15*Sqrt[x]) - (8*b*ArcTanh[Tanh[a + b*x]])/(15*x^(3/2)) - (2*Arc
Tanh[Tanh[a + b*x]]^2)/(5*x^(5/2))
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^{7/2}}\,dx = -\frac{2\tanh^{-1}(\tanh(a+bx))^2}{5x^{5/2}} + \frac{1}{5}(4b)\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^{5/2}}\,dx$$
$$= -\frac{8b\tanh^{-1}(\tanh(a+bx))}{15x^{3/2}} - \frac{2\tanh^{-1}(\tanh(a+bx))^2}{5x^{5/2}} + \frac{1}{15}\left(8b^2\right)\int \frac{1}{x^{3/2}}\,dx$$
$$= -\frac{16b^2}{15\sqrt{x}} - \frac{8b\tanh^{-1}(\tanh(a+bx))}{15x^{3/2}} - \frac{2\tanh^{-1}(\tanh(a+bx))^2}{5x^{5/2}}$$

Mathematica [A] time = 0.0428595, size = 40, normalized size = 0.83

$$-\frac{2\left(4bx\tanh^{-1}(\tanh(a+bx)) + 3\tanh^{-1}(\tanh(a+bx))^2 + 8b^2x^2\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^2/x^(7/2),x]
```

```
[Out] (-2*(8*b^2*x^2 + 4*b*x*ArcTanh[Tanh[a + b*x]] + 3*ArcTanh[Tanh[a + b*x]]^2)
)/(15*x^(5/2))
```

Maple [A] time = 0.043, size = 38, normalized size = 0.8

$$-\frac{2\,(\mathrm{Artanh}\,(\tanh\,(bx+a)))^2}{5}x^{-\frac{5}{2}} + \frac{8\,b}{5}\left(-\frac{\mathrm{Artanh}\,(\tanh\,(bx+a))}{3}x^{-\frac{3}{2}} - \frac{2\,b}{3}\frac{1}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2/x^(7/2),x)
```

```
[Out] -2/5*arctanh(tanh(b*x+a))^2/x^(5/2)+8/5*b*(-1/3*arctanh(tanh(b*x+a))/x^(3/2
)-2/3*b/x^(1/2))
```

Maxima [A] time = 1.05763, size = 49, normalized size = 1.02

$$-\frac{16\,b^2}{15\,\sqrt{x}} - \frac{8\,b\,\mathrm{artanh}\,(\tanh\,(bx+a))}{15\,x^{\frac{3}{2}}} - \frac{2\,\mathrm{artanh}\,(\tanh\,(bx+a))^2}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(7/2),x, algorithm="maxima")
```

```
[Out] -16/15*b^2/sqrt(x) - 8/15*b*arctanh(tanh(b*x + a))/x^(3/2) - 2/5*arctanh(ta
nh(b*x + a))^2/x^(5/2)
```

Fricas [A] time = 2.08503, size = 63, normalized size = 1.31

$$-\frac{2\left(15\,b^2x^2+10\,abx+3\,a^2\right)}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(7/2),x, algorithm="fricas")
```

```
[Out] -2/15*(15*b^2*x^2 + 10*a*b*x + 3*a^2)/x^(5/2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2/x**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14518, size = 32, normalized size = 0.67

$$-\frac{2\left(15\,b^2x^2+10\,abx+3\,a^2\right)}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*(15*b^2*x^2 + 10*a*b*x + 3*a^2)/x^(5/2)
```

3.183 $\int x^{7/2} \tanh^{-1}(\tanh(a+bx))^3 \, dx$

Optimal. Leaf size=69

$$\frac{16}{429}b^2x^{13/2}\tanh^{-1}(\tanh(a+bx)) - \frac{4}{33}bx^{11/2}\tanh^{-1}(\tanh(a+bx))^2 + \frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))^3 - \frac{32b^3x^{15/2}}{6435}$$

```
[Out] (-32*b^3*x^(15/2))/6435 + (16*b^2*x^(13/2)*ArcTanh[Tanh[a + b*x]])/429 - (4
*b*x^(11/2)*ArcTanh[Tanh[a + b*x]]^2)/33 + (2*x^(9/2)*ArcTanh[Tanh[a + b*x]
]^3)/9
```

Rubi [A] time = 0.0385126, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 30}

$$\frac{16}{429}b^2x^{13/2}\tanh^{-1}(\tanh(a+bx)) - \frac{4}{33}bx^{11/2}\tanh^{-1}(\tanh(a+bx))^2 + \frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))^3 - \frac{32b^3x^{15/2}}{6435}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (-32*b^3*x^(15/2))/6435 + (16*b^2*x^(13/2)*ArcTanh[Tanh[a + b*x]])/429 - (4
*b*x^(11/2)*ArcTanh[Tanh[a + b*x]]^2)/33 + (2*x^(9/2)*ArcTanh[Tanh[a + b*x]
]^3)/9
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{7/2}\tanh^{-1}(\tanh(a+bx))^3\,dx = \frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))^3 - \frac{1}{3}(2b)\int x^{9/2}\tanh^{-1}(\tanh(a+bx))^2\,dx$$
$$= -\frac{4}{33}bx^{11/2}\tanh^{-1}(\tanh(a+bx))^2 + \frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))^3 + \frac{1}{33}\left(8b^2\right)\int x^{11/}$$
$$= \frac{16}{429}b^2x^{13/2}\tanh^{-1}(\tanh(a+bx)) - \frac{4}{33}bx^{11/2}\tanh^{-1}(\tanh(a+bx))^2 + \frac{2}{9}x^{9/2}\tanh^{-1}$$
$$= -\frac{32b^3x^{15/2}}{6435} + \frac{16}{429}b^2x^{13/2}\tanh^{-1}(\tanh(a+bx)) - \frac{4}{33}bx^{11/2}\tanh^{-1}(\tanh(a+bx))^2 +$$

Mathematica [A] time = 0.0289775, size = 57, normalized size = 0.83

$$-\frac{2x^{9/2}\left(-120b^2x^2\tanh^{-1}(\tanh(a+bx)) + 390bx\tanh^{-1}(\tanh(a+bx))^2 - 715\tanh^{-1}(\tanh(a+bx))^3 + 16b^3x^3\right)}{6435}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (-2*x^(9/2)*(16*b^3*x^3 - 120*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 390*b*x*ArcT
anh[Tanh[a + b*x]]^2 - 715*ArcTanh[Tanh[a + b*x]]^3))/6435
```

Maple [A] time = 0.046, size = 56, normalized size = 0.8

$$\frac{2\,(\mathrm{Artanh}\,(\tanh\,(bx+a)))^3}{9}x^{\frac{9}{2}} - \frac{4\,b}{3}\left(\frac{(\mathrm{Artanh}\,(\tanh\,(bx+a)))^2}{11}x^{\frac{11}{2}} - \frac{4\,b}{11}\left(\frac{\mathrm{Artanh}\,(\tanh\,(bx+a))}{13}x^{\frac{13}{2}} - \frac{2\,b}{195}x^{\frac{15}{2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*arctanh(tanh(b*x+a))^3,x)
```

```
[Out] 2/9*x^(9/2)*arctanh(tanh(b*x+a))^3-4/3*b*(1/11*x^(11/2)*arctanh(tanh(b*x+a)
)^2-4/11*b*(1/13*x^(13/2)*arctanh(tanh(b*x+a))-2/195*x^(15/2)*b))
```

Maxima [A] time = 1.06234, size = 74, normalized size = 1.07

$$-\frac{4}{33}\,bx^{\frac{11}{2}}\,\mathrm{artanh}\,(\tanh\,(bx+a))^2 + \frac{2}{9}\,x^{\frac{9}{2}}\,\mathrm{artanh}\,(\tanh\,(bx+a))^3 - \frac{16}{6435}\left(2\,b^2x^{\frac{15}{2}} - 15\,bx^{\frac{13}{2}}\,\mathrm{artanh}\,(\tanh\,(bx+a))\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -4/33*b*x^(11/2)*arctanh(tanh(b*x + a))^2 + 2/9*x^(9/2)*arctanh(tanh(b*x +
a))^3 - 16/6435*(2*b^2*x^(15/2) - 15*b*x^(13/2)*arctanh(tanh(b*x + a)))*b
```

Fricas [A] time = 1.92607, size = 105, normalized size = 1.52

$$\frac{2}{6435}\left(429\,b^3x^7+1485\,ab^2x^6+1755\,a^2bx^5+715\,a^3x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 2/6435*(429*b^3*x^7 + 1485*a*b^2*x^6 + 1755*a^2*b*x^5 + 715*a^3*x^4)*sqrt(x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*atanh(tanh(b*x+a))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14243, size = 47, normalized size = 0.68

$$\frac{2}{15}\,b^3x^{\frac{15}{2}}+\frac{6}{13}\,ab^2x^{\frac{13}{2}}+\frac{6}{11}\,a^2bx^{\frac{11}{2}}+\frac{2}{9}\,a^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 2/15*b^3*x^(15/2) + 6/13*a*b^2*x^(13/2) + 6/11*a^2*b*x^(11/2) + 2/9*a^3*x^(
9/2)
```

3.184 $\int x^{5/2} \tanh^{-1}(\tanh(a+bx))^3 \, dx$

Optimal. Leaf size=69

$$\frac{16}{231}b^2x^{11/2}\tanh^{-1}(\tanh(a+bx)) - \frac{4}{21}bx^{9/2}\tanh^{-1}(\tanh(a+bx))^2 + \frac{2}{7}x^{7/2}\tanh^{-1}(\tanh(a+bx))^3 - \frac{32b^3x^{13/2}}{3003}$$

```
[Out] (-32*b^3*x^(13/2))/3003 + (16*b^2*x^(11/2)*ArcTanh[Tanh[a + b*x]])/231 - (4
*b*x^(9/2)*ArcTanh[Tanh[a + b*x]]^2)/21 + (2*x^(7/2)*ArcTanh[Tanh[a + b*x]]
^3)/7
```

Rubi [A] time = 0.0377565, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 30}

$$\frac{16}{231}b^2x^{11/2}\tanh^{-1}(\tanh(a+bx)) - \frac{4}{21}bx^{9/2}\tanh^{-1}(\tanh(a+bx))^2 + \frac{2}{7}x^{7/2}\tanh^{-1}(\tanh(a+bx))^3 - \frac{32b^3x^{13/2}}{3003}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (-32*b^3*x^(13/2))/3003 + (16*b^2*x^(11/2)*ArcTanh[Tanh[a + b*x]])/231 - (4
*b*x^(9/2)*ArcTanh[Tanh[a + b*x]]^2)/21 + (2*x^(7/2)*ArcTanh[Tanh[a + b*x]]
^3)/7
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{5/2}\tanh^{-1}(\tanh(a+bx))^3\,dx = \frac{2}{7}x^{7/2}\tanh^{-1}(\tanh(a+bx))^3 - \frac{1}{7}(6b)\int x^{7/2}\tanh^{-1}(\tanh(a+bx))^2\,dx$$
$$= -\frac{4}{21}bx^{9/2}\tanh^{-1}(\tanh(a+bx))^2 + \frac{2}{7}x^{7/2}\tanh^{-1}(\tanh(a+bx))^3 + \frac{1}{21}\left(8b^2\right)\int x^{9/2}\,\mathrm{t}$$
$$= \frac{16}{231}b^2x^{11/2}\tanh^{-1}(\tanh(a+bx)) - \frac{4}{21}bx^{9/2}\tanh^{-1}(\tanh(a+bx))^2 + \frac{2}{7}x^{7/2}\tanh^{-1}($$
$$= -\frac{32b^3x^{13/2}}{3003} + \frac{16}{231}b^2x^{11/2}\tanh^{-1}(\tanh(a+bx)) - \frac{4}{21}bx^{9/2}\tanh^{-1}(\tanh(a+bx))^2 +$$

Mathematica [A] time = 0.0393816, size = 57, normalized size = 0.83

$$\frac{2x^{7/2}\left(104b^2x^2\tanh^{-1}(\tanh(a+bx)) - 286bx\tanh^{-1}(\tanh(a+bx))^2 + 429\tanh^{-1}(\tanh(a+bx))^3 - 16b^3x^3\right)}{3003}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (2*x^(7/2)*(-16*b^3*x^3 + 104*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 286*b*x*ArcT
anh[Tanh[a + b*x]]^2 + 429*ArcTanh[Tanh[a + b*x]]^3))/3003
```

Maple [A] time = 0.044, size = 56, normalized size = 0.8

$$\frac{2\,(\mathrm{Artanh}\,(\tanh\,(bx+a)))^3}{7}x^{\frac{7}{2}} - \frac{12\,b}{7}\left(\frac{(\mathrm{Artanh}\,(\tanh\,(bx+a)))^2}{9}x^{\frac{9}{2}} - \frac{4\,b}{9}\left(\frac{\mathrm{Artanh}\,(\tanh\,(bx+a))}{11}x^{\frac{11}{2}} - \frac{2\,b}{143}x^{\frac{13}{2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*arctanh(tanh(b*x+a))^3,x)
```

```
[Out] 2/7*x^(7/2)*arctanh(tanh(b*x+a))^3-12/7*b*(1/9*x^(9/2)*arctanh(tanh(b*x+a))
^2-4/9*b*(1/11*x^(11/2)*arctanh(tanh(b*x+a))-2/143*x^(13/2)*b))
```

Maxima [A] time = 1.06479, size = 74, normalized size = 1.07

$$-\frac{4}{21}\,bx^{\frac{9}{2}}\,\mathrm{artanh}\,(\tanh\,(bx+a))^2 + \frac{2}{7}\,x^{\frac{7}{2}}\,\mathrm{artanh}\,(\tanh\,(bx+a))^3 - \frac{16}{3003}\left(2\,b^2x^{\frac{13}{2}} - 13\,bx^{\frac{11}{2}}\,\mathrm{artanh}\,(\tanh\,(bx+a))\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -4/21*b*x^(9/2)*arctanh(tanh(b*x + a))^2 + 2/7*x^(7/2)*arctanh(tanh(b*x + a
))^3 - 16/3003*(2*b^2*x^(13/2) - 13*b*x^(11/2)*arctanh(tanh(b*x + a)))*b
```

Fricas [A] time = 2.04076, size = 104, normalized size = 1.51

$$\frac{2}{3003}\left(231\,b^3x^6 + 819\,ab^2x^5 + 1001\,a^2bx^4 + 429\,a^3x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 2/3003*(231*b^3*x^6 + 819*a*b^2*x^5 + 1001*a^2*b*x^4 + 429*a^3*x^3)*sqrt(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*atanh(tanh(b*x+a))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13728, size = 47, normalized size = 0.68

$$\frac{2}{13}\,b^3x^{\frac{13}{2}} + \frac{6}{11}\,ab^2x^{\frac{11}{2}} + \frac{2}{3}\,a^2bx^{\frac{9}{2}} + \frac{2}{7}\,a^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 2/13*b^3*x^(13/2) + 6/11*a*b^2*x^(11/2) + 2/3*a^2*b*x^(9/2) + 2/7*a^3*x^(7/
2)
```

3.185 $\int x^{3/2}\tanh^{-1}(\tanh(a+bx))^3\,dx$

Optimal. Leaf size=69

$$\frac{16}{105}b^2x^{9/2}\tanh^{-1}(\tanh(a+bx))-\frac{12}{35}bx^{7/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx))^3-\frac{32b^3x^{11/2}}{1155}$$

```
[Out] (-32*b^3*x^(11/2))/1155 + (16*b^2*x^(9/2)*ArcTanh[Tanh[a + b*x]])/105 - (12
*b*x^(7/2)*ArcTanh[Tanh[a + b*x]]^2)/35 + (2*x^(5/2)*ArcTanh[Tanh[a + b*x]]
^3)/5
```

Rubi [A] time = 0.0374426, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 30}

$$\frac{16}{105}b^2x^{9/2}\tanh^{-1}(\tanh(a+bx))-\frac{12}{35}bx^{7/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx))^3-\frac{32b^3x^{11/2}}{1155}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (-32*b^3*x^(11/2))/1155 + (16*b^2*x^(9/2)*ArcTanh[Tanh[a + b*x]])/105 - (12
*b*x^(7/2)*ArcTanh[Tanh[a + b*x]]^2)/35 + (2*x^(5/2)*ArcTanh[Tanh[a + b*x]]
^3)/5
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{3/2}\tanh^{-1}(\tanh(a+bx))^3\,dx = \frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx))^3 - \frac{1}{5}(6b)\int x^{5/2}\tanh^{-1}(\tanh(a+bx))^2\,dx$$
$$= -\frac{12}{35}bx^{7/2}\tanh^{-1}(\tanh(a+bx))^2 + \frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx))^3 + \frac{1}{35}\left(24b^2\right)\int x^{7}$$
$$= \frac{16}{105}b^2x^{9/2}\tanh^{-1}(\tanh(a+bx)) - \frac{12}{35}bx^{7/2}\tanh^{-1}(\tanh(a+bx))^2 + \frac{2}{5}x^{5/2}\tanh^{-1}$$
$$= -\frac{32b^3x^{11/2}}{1155} + \frac{16}{105}b^2x^{9/2}\tanh^{-1}(\tanh(a+bx)) - \frac{12}{35}bx^{7/2}\tanh^{-1}(\tanh(a+bx))^2 +$$

Mathematica [A] time = 0.0316131, size = 57, normalized size = 0.83

$$-\frac{2x^{5/2}\left(-88b^2x^2\tanh^{-1}(\tanh(a+bx)) + 198bx\tanh^{-1}(\tanh(a+bx))^2 - 231\tanh^{-1}(\tanh(a+bx))^3 + 16b^3x^3\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (-2*x^(5/2)*(16*b^3*x^3 - 88*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 198*b*x*ArcTa
nh[Tanh[a + b*x]]^2 - 231*ArcTanh[Tanh[a + b*x]]^3))/1155
```

Maple [A] time = 0.046, size = 56, normalized size = 0.8

$$\frac{2\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^3}{5}x^{\frac{5}{2}} - \frac{12\,b}{5}\left(\frac{\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^2}{7}x^{\frac{7}{2}} - \frac{4\,b}{7}\left(\frac{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}{9}x^{\frac{9}{2}} - \frac{2\,b}{99}x^{\frac{11}{2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*arctanh(tanh(b*x+a))^3,x)
```

```
[Out] 2/5*x^(5/2)*arctanh(tanh(b*x+a))^3-12/5*b*(1/7*x^(7/2)*arctanh(tanh(b*x+a))
^2-4/7*b*(1/9*x^(9/2)*arctanh(tanh(b*x+a))-2/99*b*x^(11/2)))
```

Maxima [A] time = 1.06569, size = 74, normalized size = 1.07

$$-\frac{12}{35}\,bx^{\frac{7}{2}}\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2 + \frac{2}{5}\,x^{\frac{5}{2}}\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^3 - \frac{16}{1155}\left(2\,b^2x^{\frac{11}{2}} - 11\,bx^{\frac{9}{2}}\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -12/35*b*x^(7/2)*arctanh(tanh(b*x + a))^2 + 2/5*x^(5/2)*arctanh(tanh(b*x +
a))^3 - 16/1155*(2*b^2*x^(11/2) - 11*b*x^(9/2)*arctanh(tanh(b*x + a)))*b
```

Fricas [A] time = 2.02683, size = 103, normalized size = 1.49

$$\frac{2}{1155}\left(105\,b^3x^5 + 385\,ab^2x^4 + 495\,a^2bx^3 + 231\,a^3x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 2/1155*(105*b^3*x^5 + 385*a*b^2*x^4 + 495*a^2*b*x^3 + 231*a^3*x^2)*sqrt(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{\frac{3}{2}}\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(x**(3/2)*atanh(tanh(a + b*x))**3, x)
```

Giac [A] time = 1.17472, size = 47, normalized size = 0.68

$$\frac{2}{11}\,b^3x^{\frac{11}{2}} + \frac{2}{3}\,ab^2x^{\frac{9}{2}} + \frac{6}{7}\,a^2bx^{\frac{7}{2}} + \frac{2}{5}\,a^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 2/11*b^3*x^(11/2) + 2/3*a*b^2*x^(9/2) + 6/7*a^2*b*x^(7/2) + 2/5*a^3*x^(5/2)
```

3.186 $\int \sqrt{x} \tanh^{-1}(\tanh(a+bx))^3 \, dx$

Optimal. Leaf size=69

$$\frac{16}{35}b^2x^{7/2}\tanh^{-1}(\tanh(a+bx))-\frac{4}{5}bx^{5/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))^3-\frac{32}{315}b^3x^{9/2}$$

```
[Out] (-32*b^3*x^(9/2))/315 + (16*b^2*x^(7/2)*ArcTanh[Tanh[a + b*x]])/35 - (4*b*x
^(5/2)*ArcTanh[Tanh[a + b*x]]^2)/5 + (2*x^(3/2)*ArcTanh[Tanh[a + b*x]]^3)/3
```

Rubi [A] time = 0.0369685, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 30}

$$\frac{16}{35}b^2x^{7/2}\tanh^{-1}(\tanh(a+bx))-\frac{4}{5}bx^{5/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))^3-\frac{32}{315}b^3x^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (-32*b^3*x^(9/2))/315 + (16*b^2*x^(7/2)*ArcTanh[Tanh[a + b*x]])/35 - (4*b*x
^(5/2)*ArcTanh[Tanh[a + b*x]]^2)/5 + (2*x^(3/2)*ArcTanh[Tanh[a + b*x]]^3)/3
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sqrt{x}\tanh^{-1}(\tanh(a+bx))^3\,dx = \frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))^3 - (2b)\int x^{3/2}\tanh^{-1}(\tanh(a+bx))^2\,dx$$
$$= -\frac{4}{5}bx^{5/2}\tanh^{-1}(\tanh(a+bx))^2 + \frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))^3 + \frac{1}{5}\left(8b^2\right)\int x^{5/2}\tan$$
$$= \frac{16}{35}b^2x^{7/2}\tanh^{-1}(\tanh(a+bx)) - \frac{4}{5}bx^{5/2}\tanh^{-1}(\tanh(a+bx))^2 + \frac{2}{3}x^{3/2}\tanh^{-1}(\tan$$
$$= -\frac{32}{315}b^3x^{9/2} + \frac{16}{35}b^2x^{7/2}\tanh^{-1}(\tanh(a+bx)) - \frac{4}{5}bx^{5/2}\tanh^{-1}(\tanh(a+bx))^2 + \frac{2}{3}x^3$$

Mathematica [A] time = 0.0276033, size = 57, normalized size = 0.83

$$-\frac{2}{315}x^{3/2}\left(-72b^2x^2\tanh^{-1}(\tanh(a+bx)) + 126bx\tanh^{-1}(\tanh(a+bx))^2 - 105\tanh^{-1}(\tanh(a+bx))^3 + 16b^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (-2*x^(3/2)*(16*b^3*x^3 - 72*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 126*b*x*ArcTa
nh[Tanh[a + b*x]]^2 - 105*ArcTanh[Tanh[a + b*x]]^3))/315
```

Maple [A] time = 0.046, size = 56, normalized size = 0.8

$$\frac{2\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^3}{3}x^{\frac{3}{2}} - 4b\left(1/5\,x^{5/2}\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^2 - 4/5\,b\left(1/7\,x^{7/2}\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right) - \frac{2\,bx^9}{63}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3*x^(1/2),x)
```

```
[Out] 2/3*x^(3/2)*arctanh(tanh(b*x+a))^3-4*b*(1/5*x^(5/2)*arctanh(tanh(b*x+a))^2-
4/5*b*(1/7*x^(7/2)*arctanh(tanh(b*x+a))-2/63*b*x^(9/2)))
```

Maxima [A] time = 1.05312, size = 74, normalized size = 1.07

$$-\frac{4}{5}\,bx^{\frac{5}{2}}\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2 + \frac{2}{3}\,x^{\frac{3}{2}}\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^3 - \frac{16}{315}\left(2\,b^2x^{\frac{9}{2}} - 9\,bx^{\frac{7}{2}}\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3*x^(1/2),x, algorithm="maxima")
```

```
[Out] -4/5*b*x^(5/2)*arctanh(tanh(b*x + a))^2 + 2/3*x^(3/2)*arctanh(tanh(b*x + a)
)^3 - 16/315*(2*b^2*x^(9/2) - 9*b*x^(7/2)*arctanh(tanh(b*x + a)))*b
```

Fricas [A] time = 2.03469, size = 97, normalized size = 1.41

$$\frac{2}{315}\left(35\,b^3x^4+135\,ab^2x^3+189\,a^2bx^2+105\,a^3x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^3*x^4 + 135*a*b^2*x^3 + 189*a^2*b*x^2 + 105*a^3*x)*sqrt(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x}\,\mathrm{atanh}^3\left(\tanh\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3*x**(1/2),x)
```

```
[Out] Integral(sqrt(x)*atanh(tanh(a + b*x))**3, x)
```

Giac [A] time = 1.16223, size = 47, normalized size = 0.68

$$\frac{2}{9}\,b^3x^{\frac{9}{2}}+\frac{6}{7}\,ab^2x^{\frac{7}{2}}+\frac{6}{5}\,a^2bx^{\frac{5}{2}}+\frac{2}{3}\,a^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*b^3*x^(9/2) + 6/7*a*b^2*x^(7/2) + 6/5*a^2*b*x^(5/2) + 2/3*a^3*x^(3/2)
```

3.187 $\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{\sqrt{x}}\,dx$

Optimal. Leaf size=65

$$\frac{16}{5}b^2x^{5/2}\tanh^{-1}(\tanh(a+bx)) - 4bx^{3/2}\tanh^{-1}(\tanh(a+bx))^2 + 2\sqrt{x}\tanh^{-1}(\tanh(a+bx))^3 - \frac{32}{35}b^3x^{7/2}$$

```
[Out] (-32*b^3*x^(7/2))/35 + (16*b^2*x^(5/2)*ArcTanh[Tanh[a + b*x]])/5 - 4*b*x^(3
/2)*ArcTanh[Tanh[a + b*x]]^2 + 2*Sqrt[x]*ArcTanh[Tanh[a + b*x]]^3
```

Rubi [A] time = 0.0357662, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 30}

$$\frac{16}{5}b^2x^{5/2}\tanh^{-1}(\tanh(a+bx)) - 4bx^{3/2}\tanh^{-1}(\tanh(a+bx))^2 + 2\sqrt{x}\tanh^{-1}(\tanh(a+bx))^3 - \frac{32}{35}b^3x^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3/Sqrt[x],x]
```

```
[Out] (-32*b^3*x^(7/2))/35 + (16*b^2*x^(5/2)*ArcTanh[Tanh[a + b*x]])/5 - 4*b*x^(3
/2)*ArcTanh[Tanh[a + b*x]]^2 + 2*Sqrt[x]*ArcTanh[Tanh[a + b*x]]^3
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{\sqrt{x}}\,dx = 2\sqrt{x}\tanh^{-1}(\tanh(a+bx))^3 - (6b)\int \sqrt{x}\tanh^{-1}(\tanh(a+bx))^2\,dx$$
$$= -4bx^{3/2}\tanh^{-1}(\tanh(a+bx))^2 + 2\sqrt{x}\tanh^{-1}(\tanh(a+bx))^3 + \left(8b^2\right)\int x^{3/2}\tanh^{-1}(\mathrm{t}$$
$$= \frac{16}{5}b^2x^{5/2}\tanh^{-1}(\tanh(a+bx)) - 4bx^{3/2}\tanh^{-1}(\tanh(a+bx))^2 + 2\sqrt{x}\tanh^{-1}(\tanh(a$$
$$= -\frac{32}{35}b^3x^{7/2} + \frac{16}{5}b^2x^{5/2}\tanh^{-1}(\tanh(a+bx)) - 4bx^{3/2}\tanh^{-1}(\tanh(a+bx))^2 + 2\sqrt{x}\,\mathrm{ta}$$

Mathematica [A] time = 0.028104, size = 57, normalized size = 0.88

$$\frac{2}{35}\sqrt{x}\left(56b^2x^2\tanh^{-1}(\tanh(a+bx)) - 70bx\tanh^{-1}(\tanh(a+bx))^2 + 35\tanh^{-1}(\tanh(a+bx))^3 - 16b^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(-16*b^3*x^3 + 56*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 70*b*x*ArcTan
h[Tanh[a + b*x]]^2 + 35*ArcTanh[Tanh[a + b*x]]^3))/35
```

Maple [A] time = 0.039, size = 69, normalized size = 1.1

$$\frac{2\,b^3}{7}x^{\frac{7}{2}} + \frac{(6\,\mathrm{Artanh}\,(\tanh\,(bx+a)) - 6\,bx)\,b^2}{5}x^{\frac{5}{2}} + 2\,(\mathrm{Artanh}\,(\tanh\,(bx+a)) - bx)^2\,bx^{3/2} + 2\,(\mathrm{Artanh}\,(\tanh\,(bx+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3/x^(1/2),x)
```

```
[Out] 2/7*b^3*x^(7/2)+6/5*(arctanh(tanh(b*x+a))-b*x)*b^2*x^(5/2)+2*(arctanh(tanh(
b*x+a))-b*x)^2*b*x^(3/2)+2*(arctanh(tanh(b*x+a))-b*x)^3*x^(1/2)
```

Maxima [A] time = 1.05474, size = 74, normalized size = 1.14

$$-4\,bx^{\frac{3}{2}}\,\mathrm{artanh}\,(\tanh\,(bx+a))^2 + 2\,\sqrt{x}\,\mathrm{artanh}\,(\tanh\,(bx+a))^3 - \frac{16}{35}\left(2\,b^2x^{\frac{7}{2}} - 7\,bx^{\frac{5}{2}}\,\mathrm{artanh}\,(\tanh\,(bx+a))\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] -4*b*x^(3/2)*arctanh(tanh(b*x + a))^2 + 2*sqrt(x)*arctanh(tanh(b*x + a))^3
- 16/35*(2*b^2*x^(7/2) - 7*b*x^(5/2)*arctanh(tanh(b*x + a)))*b
```

Fricas [A] time = 1.9303, size = 85, normalized size = 1.31

$$\frac{2}{35}\left(5\,b^3x^3+21\,ab^2x^2+35\,a^2bx+35\,a^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^3 + 21*a*b^2*x^2 + 35*a^2*b*x + 35*a^3)*sqrt(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3/x**(1/2),x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**3/sqrt(x), x)
```

Giac [A] time = 1.12829, size = 47, normalized size = 0.72

$$\frac{2}{7}\,b^3x^{\frac{7}{2}}+\frac{6}{5}\,ab^2x^{\frac{5}{2}}+2\,a^2bx^{\frac{3}{2}}+2\,a^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/7*b^3*x^(7/2) + 6/5*a*b^2*x^(5/2) + 2*a^2*b*x^(3/2) + 2*a^3*sqrt(x)
```

3.188 $\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^{3/2}} \, dx$

Optimal. Leaf size=63

$$-16b^2x^{3/2}\tanh^{-1}(\tanh(a+bx))+12b\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2-\frac{2\tanh^{-1}(\tanh(a+bx))^3}{\sqrt{x}}+\frac{32}{5}b^3x^{5/2}$$

```
[Out] (32*b^3*x^(5/2))/5 - 16*b^2*x^(3/2)*ArcTanh[Tanh[a + b*x]] + 12*b*Sqrt[x]*A
rcTanh[Tanh[a + b*x]]^2 - (2*ArcTanh[Tanh[a + b*x]]^3)/Sqrt[x]
```

Rubi [A] time = 0.0359327, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 30}

$$-16b^2x^{3/2}\tanh^{-1}(\tanh(a+bx))+12b\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2-\frac{2\tanh^{-1}(\tanh(a+bx))^3}{\sqrt{x}}+\frac{32}{5}b^3x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3/x^(3/2),x]
```

```
[Out] (32*b^3*x^(5/2))/5 - 16*b^2*x^(3/2)*ArcTanh[Tanh[a + b*x]] + 12*b*Sqrt[x]*A
rcTanh[Tanh[a + b*x]]^2 - (2*ArcTanh[Tanh[a + b*x]]^3)/Sqrt[x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^{3/2}}\,dx = -\frac{2\tanh^{-1}(\tanh(a+bx))^3}{\sqrt{x}} + (6b)\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}}\,dx$$
$$= 12b\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2 - \frac{2\tanh^{-1}(\tanh(a+bx))^3}{\sqrt{x}} - \left(24b^2\right)\int \sqrt{x}\tanh^{-1}(\tan$$
$$= -16b^2x^{3/2}\tanh^{-1}(\tanh(a+bx)) + 12b\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2 - \frac{2\tanh^{-1}(\tanh(a}{\sqrt{x}}$$
$$= \frac{32}{5}b^3x^{5/2} - 16b^2x^{3/2}\tanh^{-1}(\tanh(a+bx)) + 12b\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2 - \frac{2\tanh^{-}}{}$$

Mathematica [A] time = 0.0309749, size = 57, normalized size = 0.9

$$\frac{2\left(-40b^2x^2\tanh^{-1}(\tanh(a+bx)) + 30bx\tanh^{-1}(\tanh(a+bx))^2 - 5\tanh^{-1}(\tanh(a+bx))^3 + 16b^3x^3\right)}{5\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3/x^(3/2),x]
```

```
[Out] (2*(16*b^3*x^3 - 40*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 30*b*x*ArcTanh[Tanh[a
+ b*x]]^2 - 5*ArcTanh[Tanh[a + b*x]]^3))/(5*Sqrt[x])
```

Maple [A] time = 0.043, size = 64, normalized size = 1.

$$-2\,\frac{(\mathrm{Artanh}\,(\tanh\,(bx+a)))^3}{\sqrt{x}} + 12\,b\left(1/5\,b^2x^{5/2} + 2/3\,\left(\mathrm{Artanh}\,(\tanh\,(bx+a)) - bx\right)bx^{3/2} + (\mathrm{Artanh}\,(\tanh\,(bx+a)) -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3/x^(3/2),x)
```

```
[Out] -2*arctanh(tanh(b*x+a))^3/x^(1/2)+12*b*(1/5*b^2*x^(5/2)+2/3*(arctanh(tanh(b
*x+a))-b*x)*b*x^(3/2)+(arctanh(tanh(b*x+a))-b*x)^2*x^(1/2))
```

Maxima [A] time = 1.04928, size = 74, normalized size = 1.17

$$12\,b\sqrt{x}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{2}-\frac{2\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{3}}{\sqrt{x}}+\frac{16}{5}\left(2\,b^{2}x^{\frac{5}{2}}-5\,bx^{\frac{3}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(3/2),x, algorithm="maxima")
```

```
[Out] 12*b*sqrt(x)*arctanh(tanh(b*x + a))^2 - 2*arctanh(tanh(b*x + a))^3/sqrt(x)
+ 16/5*(2*b^2*x^(5/2) - 5*b*x^(3/2)*arctanh(tanh(b*x + a)))*b
```

Fricas [A] time = 1.92586, size = 78, normalized size = 1.24

$$\frac{2\left(b^{3}x^{3}+5\,ab^{2}x^{2}+15\,a^{2}bx-5\,a^{3}\right)}{5\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^3*x^3 + 5*a*b^2*x^2 + 15*a^2*b*x - 5*a^3)/sqrt(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atanh}^{3}\left(\tanh\left(a+bx\right)\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3/x**(3/2),x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**3/x**(3/2), x)
```

Giac [A] time = 1.11838, size = 47, normalized size = 0.75

$$\frac{2}{5}\,b^{3}x^{\frac{5}{2}}+2\,ab^{2}x^{\frac{3}{2}}+6\,a^{2}b\sqrt{x}-\frac{2\,a^{3}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/5*b^3*x^(5/2) + 2*a*b^2*x^(3/2) + 6*a^2*b*sqrt(x) - 2*a^3/sqrt(x)
```

3.189 $\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^{5/2}}\,dx$

Optimal. Leaf size=65

$$16b^2\sqrt{x}\tanh^{-1}(\tanh(a+bx)) - \frac{2\tanh^{-1}(\tanh(a+bx))^3}{3x^{3/2}} - \frac{4b\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}} - \frac{32}{3}b^3x^{3/2}$$

```
[Out] (-32*b^3*x^(3/2))/3 + 16*b^2*Sqrt[x]*ArcTanh[Tanh[a + b*x]] - (4*b*ArcTanh[
Tanh[a + b*x]]^2)/Sqrt[x] - (2*ArcTanh[Tanh[a + b*x]]^3)/(3*x^(3/2))
```

Rubi [A] time = 0.0382775, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 30}

$$16b^2\sqrt{x}\tanh^{-1}(\tanh(a+bx)) - \frac{2\tanh^{-1}(\tanh(a+bx))^3}{3x^{3/2}} - \frac{4b\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}} - \frac{32}{3}b^3x^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3/x^(5/2),x]
```

```
[Out] (-32*b^3*x^(3/2))/3 + 16*b^2*Sqrt[x]*ArcTanh[Tanh[a + b*x]] - (4*b*ArcTanh[
Tanh[a + b*x]]^2)/Sqrt[x] - (2*ArcTanh[Tanh[a + b*x]]^3)/(3*x^(3/2))
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^{5/2}}\,dx = -\frac{2\tanh^{-1}(\tanh(a+bx))^3}{3x^{3/2}} + (2b)\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^{3/2}}\,dx$$
$$= -\frac{4b\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}} - \frac{2\tanh^{-1}(\tanh(a+bx))^3}{3x^{3/2}} + \left(8b^2\right)\int \frac{\tanh^{-1}(\tanh(a+}{\sqrt{x}}$$
$$= 16b^2\sqrt{x}\tanh^{-1}(\tanh(a+bx)) - \frac{4b\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}} - \frac{2\tanh^{-1}(\tanh(a+bx))}{3x^{3/2}}$$
$$= -\frac{32}{3}b^3x^{3/2} + 16b^2\sqrt{x}\tanh^{-1}(\tanh(a+bx)) - \frac{4b\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}} - \frac{2\tanh^{-1}(\mathrm{t}}{3}$$

Mathematica [A] time = 0.0294446, size = 55, normalized size = 0.85

$$-\frac{2\left(-24b^2x^2\tanh^{-1}(\tanh(a+bx)) + 6bx\tanh^{-1}(\tanh(a+bx))^2 + \tanh^{-1}(\tanh(a+bx))^3 + 16b^3x^3\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3/x^(5/2),x]
```

```
[Out] (-2*(16*b^3*x^3 - 24*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 6*b*x*ArcTanh[Tanh[a
+ b*x]]^2 + ArcTanh[Tanh[a + b*x]]^3))/(3*x^(3/2))
```

Maple [A] time = 0.043, size = 55, normalized size = 0.9

$$-\frac{2\,(\mathrm{Artanh}\,(\tanh\,(bx+a)))^3}{3}x^{-\frac{3}{2}} + 4\,b\left(-\frac{(\mathrm{Artanh}\,(\tanh\,(bx+a)))^2}{\sqrt{x}} + 4\,b\left(\mathrm{Artanh}\,(\tanh\,(bx+a))\sqrt{x} - 2/3\,bx^{3/2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3/x^(5/2),x)
```

```
[Out] -2/3*arctanh(tanh(b*x+a))^3/x^(3/2)+4*b*(-arctanh(tanh(b*x+a))^2/x^(1/2)+4*
b*(arctanh(tanh(b*x+a))*x^(1/2)-2/3*b*x^(3/2)))
```

Maxima [A] time = 1.06879, size = 74, normalized size = 1.14

$$-\frac{4\,b\,\mathrm{artanh}\,(\tanh\,(bx+a))^2}{\sqrt{x}} - \frac{16}{3}\left(2\,b^2x^{\frac{3}{2}} - 3\,b\sqrt{x}\,\mathrm{artanh}\,(\tanh\,(bx+a))\right)b - \frac{2\,\mathrm{artanh}\,(\tanh\,(bx+a))^3}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(5/2),x, algorithm="maxima")
```

```
[Out] -4*b*arctanh(tanh(b*x + a))^2/sqrt(x) - 16/3*(2*b^2*x^(3/2) - 3*b*sqrt(x)*a
rctanh(tanh(b*x + a)))*b - 2/3*arctanh(tanh(b*x + a))^3/x^(3/2)
```

Fricas [A] time = 1.97198, size = 74, normalized size = 1.14

$$\frac{2\left(b^3x^3+9\,ab^2x^2-9\,a^2bx-a^3\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^3*x^3 + 9*a*b^2*x^2 - 9*a^2*b*x - a^3)/x^(3/2)
```

Sympy [A] time = 26.9452, size = 66, normalized size = 1.02

$$-\frac{32b^3x^{\frac{3}{2}}}{3}+16b^2\sqrt{x}\,\mathrm{atanh}\left(\tanh\left(a+bx\right)\right)-\frac{4b\,\mathrm{atanh}^2\left(\tanh\left(a+bx\right)\right)}{\sqrt{x}}-\frac{2\,\mathrm{atanh}^3\left(\tanh\left(a+bx\right)\right)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3/x**(5/2),x)
```

```
[Out] -32*b**3*x**(3/2)/3 + 16*b**2*sqrt(x)*atanh(tanh(a + b*x)) - 4*b*atanh(tanh
(a + b*x))**2/sqrt(x) - 2*atanh(tanh(a + b*x))**3/(3*x**(3/2))
```

Giac [A] time = 1.16433, size = 46, normalized size = 0.71

$$\frac{2}{3}\,b^3x^{\frac{3}{2}}+6\,ab^2\sqrt{x}-\frac{2\left(9\,a^2bx+a^3\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*b^3*x^(3/2) + 6*a*b^2*sqrt(x) - 2/3*(9*a^2*b*x + a^3)/x^(3/2)
```

3.190 $\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^{7/2}} \, dx$

Optimal. Leaf size=69

$$-\frac{16b^2\tanh^{-1}(\tanh(a+bx))}{5\sqrt{x}}-\frac{4b\tanh^{-1}(\tanh(a+bx))^2}{5x^{3/2}}-\frac{2\tanh^{-1}(\tanh(a+bx))^3}{5x^{5/2}}+\frac{32b^3\sqrt{x}}{5}$$

```
[Out] (32*b^3*Sqrt[x])/5 - (16*b^2*ArcTanh[Tanh[a + b*x]])/(5*Sqrt[x]) - (4*b*Arc
Tanh[Tanh[a + b*x]]^2)/(5*x^(3/2)) - (2*ArcTanh[Tanh[a + b*x]]^3)/(5*x^(5/2
))
```

Rubi [A] time = 0.0376807, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 30}

$$-\frac{16b^2\tanh^{-1}(\tanh(a+bx))}{5\sqrt{x}}-\frac{4b\tanh^{-1}(\tanh(a+bx))^2}{5x^{3/2}}-\frac{2\tanh^{-1}(\tanh(a+bx))^3}{5x^{5/2}}+\frac{32b^3\sqrt{x}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3/x^(7/2),x]
```

```
[Out] (32*b^3*Sqrt[x])/5 - (16*b^2*ArcTanh[Tanh[a + b*x]])/(5*Sqrt[x]) - (4*b*Arc
Tanh[Tanh[a + b*x]]^2)/(5*x^(3/2)) - (2*ArcTanh[Tanh[a + b*x]]^3)/(5*x^(5/2
))
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^{7/2}}\,dx = -\frac{2\tanh^{-1}(\tanh(a+bx))^3}{5x^{5/2}} + \frac{1}{5}(6b)\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^{5/2}}\,dx$$
$$= -\frac{4b\tanh^{-1}(\tanh(a+bx))^2}{5x^{3/2}} - \frac{2\tanh^{-1}(\tanh(a+bx))^3}{5x^{5/2}} + \frac{1}{5}\left(8b^2\right)\int \frac{\tanh^{-1}(\tanh(a}{x^{3/2}}$$
$$= -\frac{16b^2\tanh^{-1}(\tanh(a+bx))}{5\sqrt{x}} - \frac{4b\tanh^{-1}(\tanh(a+bx))^2}{5x^{3/2}} - \frac{2\tanh^{-1}(\tanh(a+bx))^3}{5x^{5/2}}$$
$$= \frac{32b^3\sqrt{x}}{5} - \frac{16b^2\tanh^{-1}(\tanh(a+bx))}{5\sqrt{x}} - \frac{4b\tanh^{-1}(\tanh(a+bx))^2}{5x^{3/2}} - \frac{2\tanh^{-1}(\tanh(}{5x^{5/2}}$$

Mathematica [A] time = 0.0308873, size = 57, normalized size = 0.83

$$\frac{2\left(-8b^2x^2\tanh^{-1}(\tanh(a+bx)) - 2bx\tanh^{-1}(\tanh(a+bx))^2 - \tanh^{-1}(\tanh(a+bx))^3 + 16b^3x^3\right)}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3/x^(7/2),x]
```

```
[Out] (2*(16*b^3*x^3 - 8*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 2*b*x*ArcTanh[Tanh[a +
b*x]]^2 - ArcTanh[Tanh[a + b*x]]^3))/(5*x^(5/2))
```

Maple [A] time = 0.044, size = 56, normalized size = 0.8

$$-\frac{2\,(\mathrm{Artanh}\,(\tanh\,(bx+a)))^3}{5}x^{-\frac{5}{2}} + \frac{12\,b}{5}\left(-\frac{(\mathrm{Artanh}\,(\tanh\,(bx+a)))^2}{3}x^{-\frac{3}{2}} + \frac{4\,b}{3}\left(-\mathrm{Artanh}\,(\tanh\,(bx+a))\,\frac{1}{\sqrt{x}} + 2\,b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3/x^(7/2),x)
```

```
[Out] -2/5*arctanh(tanh(b*x+a))^3/x^(5/2)+12/5*b*(-1/3*arctanh(tanh(b*x+a))^2/x^(
3/2)+4/3*b*(-arctanh(tanh(b*x+a))/x^(1/2)+2*b*x^(1/2)))
```

Maxima [A] time = 1.08763, size = 74, normalized size = 1.07

$$\frac{16}{5}\left(2\,b^2\sqrt{x} - \frac{b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{\sqrt{x}}\right)b - \frac{4\,b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}{5\,x^{\frac{3}{2}}} - \frac{2\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(7/2),x, algorithm="maxima")
```

```
[Out] 16/5*(2*b^2*sqrt(x) - b*arctanh(tanh(b*x + a))/sqrt(x))*b - 4/5*b*arctanh(t
anh(b*x + a))^2/x^(3/2) - 2/5*arctanh(tanh(b*x + a))^3/x^(5/2)
```

Fricas [A] time = 1.96389, size = 78, normalized size = 1.13

$$\frac{2\left(5\,b^3x^3 - 15\,ab^2x^2 - 5\,a^2bx - a^3\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/5*(5*b^3*x^3 - 15*a*b^2*x^2 - 5*a^2*b*x - a^3)/x^(5/2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3/x**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12904, size = 46, normalized size = 0.67

$$2\,b^3\sqrt{x} - \frac{2\left(15\,ab^2x^2 + 5\,a^2bx + a^3\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(7/2),x, algorithm="giac")
```

```
[Out] 2*b^3*sqrt(x) - 2/5*(15*a*b^2*x^2 + 5*a^2*b*x + a^3)/x^(5/2)
```

3.191 $\int \frac{x^{7/2}}{\tanh^{-1}(\tanh(a+bx))} dx$

Optimal. Leaf size=143

$$\frac{2x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{5b^2}+\frac{2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{3b^3}+\frac{2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{b^4}-\frac{2\tan}{}$$

```
[Out] (2*x^(7/2))/(7*b) + (2*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]]))/(5*b^2) + (2
*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2)/(3*b^3) + (2*Sqrt[x]*(b*x - ArcT
anh[Tanh[a + b*x]])^3)/b^4 - (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTan
h[Tanh[a + b*x]]]]*(b*x - ArcTanh[Tanh[a + b*x]])^(7/2))/b^(9/2)
```

Rubi [A] time = 0.127289, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2159, 2162}

$$\frac{2x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{5b^2}+\frac{2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{3b^3}+\frac{2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{b^4}-\frac{2\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (2*x^(7/2))/(7*b) + (2*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]]))/(5*b^2) + (2
*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2)/(3*b^3) + (2*Sqrt[x]*(b*x - ArcT
anh[Tanh[a + b*x]])^3)/b^4 - (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTan
h[Tanh[a + b*x]]]]*(b*x - ArcTanh[Tanh[a + b*x]])^(7/2))/b^(9/2)
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
```

```
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{x^{7/2}}{\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{2x^{7/2}}{7b} - \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$

$$= \frac{2x^{7/2}}{7b} + \frac{2x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{5b^2} + \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2 \int \frac{}{\tanh^{-1}(}}{b^2}$$

$$= \frac{2x^{7/2}}{7b} + \frac{2x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{5b^2} + \frac{2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{3b^3} - \frac{(-}{}$$

$$= \frac{2x^{7/2}}{7b} + \frac{2x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{5b^2} + \frac{2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{3b^3} + \frac{2\sqrt{}}{}$$

$$= \frac{2x^{7/2}}{7b} + \frac{2x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{5b^2} + \frac{2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{3b^3} + \frac{2\sqrt{}}{}$$

Mathematica [A] time = 0.0584617, size = 129, normalized size = 0.9

$$\frac{2\left(-406b^{5/2}x^{5/2}\tanh^{-1}(\tanh(a+bx)) + 350b^{3/2}x^{3/2}\tanh^{-1}(\tanh(a+bx))^2 - 105\sqrt{b}\sqrt{x}\tanh^{-1}(\tanh(a+bx))^3 + 105\left(\right.\right.}{105b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (2*(176*b^(7/2)*x^(7/2) - 406*b^(5/2)*x^(5/2)*ArcTanh[Tanh[a + b*x]] + 350*
b^(3/2)*x^(3/2)*ArcTanh[Tanh[a + b*x]]^2 - 105*Sqrt[b]*Sqrt[x]*ArcTanh[Tanh
[a + b*x]]^3 + 105*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a +
b*x]]]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(7/2)))/(105*b^(9/2))
```

Maple [B] time = 0.132, size = 481, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/arctanh(tanh(b*x+a)),x)
```

```
[Out] 2/7*x^(7/2)/b-2/5/b^2*x^(5/2)*a-2/5/b^2*x^(5/2)*(arctanh(tanh(b*x+a))-b*x-a
)+2/3/b^3*x^(3/2)*a^2+4/3/b^3*x^(3/2)*a*(arctanh(tanh(b*x+a))-b*x-a)+2/3/b^
3*x^(3/2)*(arctanh(tanh(b*x+a))-b*x-a)^2-2/b^4*x^(1/2)*a^3-6/b^4*a^2*(arcta
nh(tanh(b*x+a))-b*x-a)*x^(1/2)-6/b^4*a*(arctanh(tanh(b*x+a))-b*x-a)^2*x^(1/
2)-2/b^4*(arctanh(tanh(b*x+a))-b*x-a)^3*x^(1/2)+2/b^4/((arctanh(tanh(b*x+a)
)-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*a^4+
8/b^4/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(
b*x+a))-b*x)*b)^(1/2))*a^3*(arctanh(tanh(b*x+a))-b*x-a)+12/b^4/((arctanh(ta
nh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1
/2))*a^2*(arctanh(tanh(b*x+a))-b*x-a)^2+8/b^4/((arctanh(tanh(b*x+a))-b*x)*b
)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*a*(arctanh(t
anh(b*x+a))-b*x-a)^3+2/b^4/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^
(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.09325, size = 366, normalized size = 2.56

$$\left[\frac{105\,a^3\sqrt{-\frac{a}{b}}\log\left(\frac{bx+2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\left(15\,b^3x^3-21\,ab^2x^2+35\,a^2bx-105\,a^3\right)\sqrt{x}}{105\,b^4},\ \frac{2\left(105\,a^3\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)+\left(15\,b^3x^3\right.\right.}{105\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] [1/105*(105*a^3*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)
) + 2*(15*b^3*x^3 - 21*a*b^2*x^2 + 35*a^2*b*x - 105*a^3)*sqrt(x))/b^4, 2/10
5*(105*a^3*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (15*b^3*x^3 - 21*a*b^2
*x^2 + 35*a^2*b*x - 105*a^3)*sqrt(x))/b^4]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/atanh(tanh(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12817, size = 95, normalized size = 0.66

$$\frac{2\,a^4\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^4}+\frac{2\left(15\,b^6x^{\frac{7}{2}}-21\,ab^5x^{\frac{5}{2}}+35\,a^2b^4x^{\frac{3}{2}}-105\,a^3b^3\sqrt{x}\right)}{105\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] 2*a^4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) + 2/105*(15*b^6*x^(7/2) -
 21*a*b^5*x^(5/2) + 35*a^2*b^4*x^(3/2) - 105*a^3*b^3*sqrt(x))/b^7
```

3.192 $\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=116

$$\frac{2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{3b^2}+\frac{2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^3}-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh\right.}{b^{7/2}}$$

```
[Out] (2*x^(5/2))/(5*b) + (2*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]]))/(3*b^2) + (2
*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^2)/b^3 - (2*ArcTanh[(Sqrt[b]*Sqrt[x
])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[Tanh[a + b*x]])^(5/2)
)/b^(7/2)
```

Rubi [A] time = 0.0791174, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2159, 2162}

$$\frac{2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{3b^2}+\frac{2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^3}-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh\right.}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/ArcTanh[Tanh[a + b*x]],x]
```


```
[Out] (2*x^(5/2))/(5*b) + (2*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]]))/(3*b^2) + (2
*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^2)/b^3 - (2*ArcTanh[(Sqrt[b]*Sqrt[x
])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[Tanh[a + b*x]])^(5/2)
)/b^(7/2)
```


Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
```

```
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{2x^{5/2}}{5b} - \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$

$$= \frac{2x^{5/2}}{5b} + \frac{2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{3b^2} + \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2 \int \frac{}{\tanh^{-1}(}}{b^2}$$

$$= \frac{2x^{5/2}}{5b} + \frac{2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{3b^2} + \frac{2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^3} - \frac{(-}{}$$

$$= \frac{2x^{5/2}}{5b} + \frac{2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{3b^2} + \frac{2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^3} - \frac{2\,\mathrm{t}}{}$$

Mathematica [A] time = 0.105778, size = 108, normalized size = 0.93

$$\frac{2\left(-35b^{3/2}x^{3/2}\tanh^{-1}(\tanh(a+bx)) + 15\sqrt{b}\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2 - 15\left(\tanh^{-1}(\tanh(a+bx)) - bx\right)^{5/2}\tan^{-1}\left(\frac{}{\sqrt{t}}\right.\right.}{15b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (2*(23*b^(5/2)*x^(5/2) - 35*b^(3/2)*x^(3/2)*ArcTanh[Tanh[a + b*x]] + 15*Sqr
t[b]*Sqrt[x]*ArcTanh[Tanh[a + b*x]]^2 - 15*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(
b*x) + ArcTanh[Tanh[a + b*x]]]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2)))/(
15*b^(7/2))
```

Maple [B] time = 0.126, size = 330, normalized size = 2.8

$$\frac{2}{5\,b}x^{\frac{5}{2}} - \frac{2\,a}{3\,b^2}x^{\frac{3}{2}} - \frac{2\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right) - 2\,bx - 2\,a}{3\,b^2}x^{\frac{3}{2}} + 2\,\frac{\sqrt{x}a^2}{b^3} + 4\,\frac{a\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right) - bx - a\right)\sqrt{x}}{b^3} + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/arctanh(tanh(b*x+a)),x)
```

```
[Out] 2/5*x^(5/2)/b-2/3/b^2*x^(3/2)*a-2/3/b^2*x^(3/2)*(arctanh(tanh(b*x+a))-b*x-a
)+2/b^3*x^(1/2)*a^2+4/b^3*a*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)+2/b^3*(arc
tanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)-2/b^3/((arctanh(tanh(b*x+a))-b*x)*b)^(1/
2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*a^3-6/b^3/((arcta
nh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*
b)^(1/2))*a^2*(arctanh(tanh(b*x+a))-b*x-a)-6/b^3/((arctanh(tanh(b*x+a))-b*x
)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*a*(arctan
h(tanh(b*x+a))-b*x-a)^2-2/b^3/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b
*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)
^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.20097, size = 308, normalized size = 2.66

$$\left[\frac{15\,a^2\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\left(3\,b^2x^2-5\,abx+15\,a^2\right)\sqrt{x}}{15\,b^3},-\frac{2\left(15\,a^2\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-\left(3\,b^2x^2-5\,abx+15\,a^2\right)\sqrt{x}\right)}{15\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] [1/15*(15*a^2*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a))
+ 2*(3*b^2*x^2 - 5*a*b*x + 15*a^2)*sqrt(x))/b^3, -2/15*(15*a^2*sqrt(a/b)*ar
ctan(b*sqrt(x)*sqrt(a/b)/a) - (3*b^2*x^2 - 5*a*b*x + 15*a^2)*sqrt(x))/b^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(x**(5/2)/atanh(tanh(a + b*x)), x)
```

Giac [A] time = 1.13205, size = 80, normalized size = 0.69

$$-\frac{2\,a^{3}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^{3}}+\frac{2\left(3\,b^{4}x^{\frac{5}{2}}-5\,ab^{3}x^{\frac{3}{2}}+15\,a^{2}b^{2}\sqrt{x}\right)}{15\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] -2*a^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2/15*(3*b^4*x^(5/2) -
5*a*b^3*x^(3/2) + 15*a^2*b^2*sqrt(x))/b^5
```

3.193 $\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=89

$$\frac{2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^2}-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}{b^{5/2}}+\frac{2x^{3/2}}{3b}$$

```
[Out] (2*x^(3/2))/(3*b) + (2*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]]))/b^2 - (2*Arc
Tanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]])^(3/2))/b^(5/2)
```

Rubi [A] time = 0.0556897, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2159, 2162}

$$\frac{2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^2}-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}{b^{5/2}}+\frac{2x^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (2*x^(3/2))/(3*b) + (2*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]]))/b^2 - (2*Arc
Tanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]])^(3/2))/b^(5/2)
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
```

```
wiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{2x^{3/2}}{3b} - \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$
$$= \frac{2x^{3/2}}{3b} + \frac{2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^2} + \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2\int \frac{}{\sqrt{x}\tanh}}{b^2}$$
$$= \frac{2x^{3/2}}{3b} + \frac{2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^2} - \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tan\right.}{b^{5/2}}$$

Mathematica [A] time = 0.0892655, size = 86, normalized size = 0.97

$$-\frac{2\sqrt{x}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)}{b^2} + \frac{2\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{b^{5/2}} + \frac{2x^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (2*x^(3/2))/(3*b) - (2*Sqrt[x]*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/b^2 + (2*
ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*(-(b*x) + A
rcTanh[Tanh[a + b*x]])^(3/2))/b^(5/2)
```

Maple [B] time = 0.126, size = 207, normalized size = 2.3

$$\frac{2}{3\,b}x^{\frac{3}{2}} - 2\,\frac{a\sqrt{x}}{b^2} - 2\,\frac{\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx-a\right)\sqrt{x}}{b^2} + 2\,\frac{a^2}{b^2\sqrt{\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)b}}\arctan\left(\frac{}{\sqrt{(\mathrm{Artanh}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/arctanh(tanh(b*x+a)),x)
```

```
[Out] 2/3*x^(3/2)/b-2/b^2*a*x^(1/2)-2/b^2*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)+2/
b^2/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*
```

```
x+a))-b*x)*b)^(1/2))*a^2+4/b^2/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(
b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*a*(arctanh(tanh(b*x+a))-b*x
-a)+2/b^2/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(t
anh(b*x+a))-b*x)*b)^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.10917, size = 244, normalized size = 2.74

$$\left[\frac{3\,a\sqrt{-\frac{a}{b}}\log\left(\frac{bx+2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(bx-3\,a)\sqrt{x}}{3\,b^2},\ \frac{2\left(3\,a\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)+(bx-3\,a)\sqrt{x}\right)}{3\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] [1/3*(3*a*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)) + 2*
(b*x - 3*a)*sqrt(x))/b^2, 2/3*(3*a*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a)
+ (b*x - 3*a)*sqrt(x))/b^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(x**(3/2)/atanh(tanh(a + b*x)), x)
```

Giac [A] time = 1.1186, size = 61, normalized size = 0.69

$$\frac{2\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^2}+\frac{2\left(b^2x^{\frac{3}{2}}-3\,ab\sqrt{x}\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] 2*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/3*(b^2*x^(3/2) - 3*a*
b*sqrt(x))/b^3
```

3.194 $\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=64

$$\frac{2\sqrt{x}}{b}-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}{b^{3/2}}$$

```
[Out] (2*Sqrt[x])/b - (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*
x]]]]*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]])/b^(3/2)
```

Rubi [A] time = 0.03323, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2159, 2162}

$$\frac{2\sqrt{x}}{b}-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (2*Sqrt[x])/b - (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*
x]]]]*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]])/b^(3/2)
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{2\sqrt{x}}{b} - \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$
$$= \frac{2\sqrt{x}}{b} - \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}{b^{3/2}}$$

Mathematica [A] time = 0.0429287, size = 62, normalized size = 0.97

$$\frac{2\sqrt{x}}{b} - \frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (2*Sqrt[x])/b - (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a +
b*x]]]]*Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]])/b^(3/2)
```

Maple [B] time = 0.126, size = 112, normalized size = 1.8

$$2\frac{\sqrt{x}}{b} - 2\frac{a}{b\sqrt{(\mathrm{Artanh}(\tanh(bx+a))-bx)\,b}}\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\mathrm{Artanh}(\tanh(bx+a))-bx)\,b}}\right) - 2\frac{\mathrm{Artanh}(\tanh(bx+a)}{b\sqrt{(\mathrm{Artanh}(\tanh(bx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/arctanh(tanh(b*x+a)),x)
```

```
[Out] 2*x^(1/2)/b-2/b/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arc
tanh(tanh(b*x+a))-b*x)*b)^(1/2))*a-2/b/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)
*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*(arctanh(tanh(b*x+a
))-b*x-a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.08856, size = 189, normalized size = 2.95

$$\left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,\sqrt{x}}{b}, -\frac{2\left(\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-\sqrt{x}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] [(sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)) + 2*sqrt(x))
/b, -2*(sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) - sqrt(x))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(sqrt(x)/atanh(tanh(a + b*x)), x)
```

Giac [A] time = 1.133, size = 42, normalized size = 0.66

$$-\frac{2\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b}+\frac{2\,\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] -2*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) + 2*sqrt(x)/b
```

3.195 $\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=53

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{b}\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(Sqrt[b]
*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]])
```

Rubi [A] time = 0.017706, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2162}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{b}\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(Sqrt[b]
*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]])
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))}\,dx = -\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{b}\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}$$

Mathematica [A] time = 0.026737, size = 51, normalized size = 0.96

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]])/(Sqrt[b
]*Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]])
```

Maple [A] time = 0.126, size = 41, normalized size = 0.8

$$2\frac{1}{\sqrt{(\mathrm{Artanh}(\tanh(bx+a))-bx)\,b}}\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\mathrm{Artanh}(\tanh(bx+a))-bx)\,b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctanh(tanh(b*x+a))/x^(1/2),x)
```

```
[Out] 2/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+
a))-b*x)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.32416, size = 163, normalized size = 3.08

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{bx-a-2\sqrt{-ab}\sqrt{x}}{bx+a}\right)}{ab}, -\frac{2\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqrt(x))/(b*x + a))/(a*b), -2*sqrt
(a*b)*arctan(sqrt(a*b)/(b*sqrt(x)))/(a*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x}\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atanh(tanh(b*x+a))/x**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*atanh(tanh(a + b*x))), x)
```

Giac [A] time = 1.11204, size = 24, normalized size = 0.45

$$\frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(b*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

3.196 $\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=76

$$\frac{2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}$$

```
[Out] (-2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/
(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) + 2/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b
*x]]))
```

Rubi [A] time = 0.035544, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2163, 2162}

$$\frac{2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] (-2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/
(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) + 2/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b
*x]]))
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
```

```
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{b\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))}\,dx}{bx-\tanh^{-1}(\tanh(a+bx))}$$

$$= -\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}} + \frac{2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A] time = 0.0596538, size = 73, normalized size = 0.96

$$-\frac{2}{\sqrt{x}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)} - \frac{2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] (-2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]
)/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2) - 2/(Sqrt[x]*(-(b*x) + ArcTanh[Ta
nh[a + b*x]]))
```

Maple [A] time = 0.13, size = 76, normalized size = 1.

$$-2\,\frac{1}{\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)\sqrt{x}} - 2\,\frac{b}{\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)\sqrt{\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)b}}\arctan\Bigg(\frac{}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/arctanh(tanh(b*x+a)),x)
```

```
[Out] -2/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)-2*b/(arctanh(tanh(b*x+a))-b*x)/((arct
anh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)
```

```
*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.10866, size = 207, normalized size = 2.72

$$\left[\frac{x\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\,\sqrt{x}}{ax}, \frac{2\left(x\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-\sqrt{x}\right)}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] [(x*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x + a)) - 2*sqrt(x
))/(a*x), 2*(x*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - sqrt(x))/(a*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{3}{2}}\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(1/(x**(3/2)*atanh(tanh(a + b*x))), x)
```

Giac [A] time = 1.1469, size = 42, normalized size = 0.55

$$-\frac{2\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a}-\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] -2*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) - 2/(a*sqrt(x))
```

3.197 $\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=101

$$-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}+\frac{2}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{2b}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

```
[Out] (-2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/
(b*x - ArcTanh[Tanh[a + b*x]])^(5/2) + (2*b)/(Sqrt[x]*(b*x - ArcTanh[Tanh[a
 + b*x]])^2) + 2/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rubi [A] time = 0.0572352, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2163, 2162}

$$-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}+\frac{2}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{2b}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] (-2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/
(b*x - ArcTanh[Tanh[a + b*x]])^(5/2) + (2*b)/(Sqrt[x]*(b*x - ArcTanh[Tanh[a
 + b*x]])^2) + 2/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
```

```
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{2}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{b\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))}\,dx}{-bx+\tanh^{-1}(\tanh(a+bx))}$$

$$= \frac{2b}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{}{\left(bx-\text{ta}\right.}$$

$$= -\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}} + \frac{2b}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{}{3x^{3/2}}$$

Mathematica [A] time = 0.172382, size = 89, normalized size = 0.88

$$\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{5/2}} + \frac{2\left(4bx-\tanh^{-1}(\tanh(a+bx))\right)}{3x^{3/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]])
/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2) + (2*(4*b*x - ArcTanh[Tanh[a + b*x
]]))/(3*x^(3/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

Maple [A] time = 0.137, size = 98, normalized size = 1.

$$2\frac{b^2}{(\text{Artanh}(\tanh(bx+a))-bx)^2\sqrt{(\text{Artanh}(\tanh(bx+a))-bx)\,b}}\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\text{Artanh}(\tanh(bx+a))-bx)\,b}}\right) - \frac{}{3\,\text{A}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/arctanh(tanh(b*x+a)),x)
```

```
[Out] 2*b^2/(arctanh(tanh(b*x+a))-b*x)^2/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arc
tan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))-2/3/(arctanh(tanh(b*x+a
))-b*x)/x^(3/2)+2/(arctanh(tanh(b*x+a))-b*x)^2*b/x^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.01692, size = 275, normalized size = 2.72

$$\left[\frac{3\,bx^2\sqrt{-\frac{b}{a}}\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\,(3\,bx-a)\sqrt{x}}{3\,a^2x^2},\,-\frac{2\left(3\,bx^2\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-(3\,bx-a)\sqrt{x}\right)}{3\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] [1/3*(3*b*x^2*sqrt(-b/a)*log((b*x + 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x + a))
+ 2*(3*b*x - a)*sqrt(x))/(a^2*x^2), -2/3*(3*b*x^2*sqrt(b/a)*arctan(a*sqrt(b
/a)/(b*sqrt(x))) - (3*b*x - a)*sqrt(x))/(a^2*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{5}{2}}\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(1/(x**(5/2)*atanh(tanh(a + b*x))), x)
```

Giac [A] time = 1.14017, size = 55, normalized size = 0.54

$$\frac{2\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^2}+\frac{2\,(3\,bx-a)}{3\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] 2*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 2/3*(3*b*x - a)/(a^2*x^
(3/2))
```

3.198 $\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=128

$$-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}}+\frac{2b^2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{2b}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{}{5x^{5/\cdots}}$$

```
[Out] (-2*b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/
(b*x - ArcTanh[Tanh[a + b*x]])^(7/2) + (2*b^2)/(Sqrt[x]*(b*x - ArcTanh[Tanh
[a + b*x]])^3) + (2*b)/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2) + 2/(5*
x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rubi [A] time = 0.0840996, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2163, 2162}

$$-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}}+\frac{2b^2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{2b}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{}{5x^{5/\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] (-2*b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/
(b*x - ArcTanh[Tanh[a + b*x]])^(7/2) + (2*b^2)/(Sqrt[x]*(b*x - ArcTanh[Tanh
[a + b*x]])^3) + (2*b)/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2) + 2/(5*
x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{2}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{b\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))}\,dx}{-bx+\tanh^{-1}(\tanh(a+bx))}$$

$$= \frac{2b}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{b^2\int}{(-bx+}$$

$$= \frac{2b^2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{2b}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{}{5x^{5/2}\left(b\right.}$$

$$= -\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}} + \frac{2b^2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{}{3x^{3/2}}$$

Mathematica [A] time = 0.146365, size = 107, normalized size = 0.84

$$\frac{2\left(-11bx\tanh^{-1}(\tanh(a+bx))+3\tanh^{-1}(\tanh(a+bx))^2+23b^2x^2\right)}{15x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} - \frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] (-2*b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]
)/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(7/2) + (2*(23*b^2*x^2 - 11*b*x*ArcTanh
[Tanh[a + b*x]] + 3*ArcTanh[Tanh[a + b*x]]^2))/(15*x^(5/2)*(b*x - ArcTanh[T
anh[a + b*x]])^3)
```

Maple [A] time = 0.134, size = 120, normalized size = 0.9

$$-\frac{2}{5\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-5\,bx}x^{-\frac{5}{2}}-2\,\frac{b^{2}}{\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{3}\sqrt{x}}+\frac{2\,b}{3\,\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}}x^{-\frac{3}{2}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/arctanh(tanh(b*x+a)),x)
```

```
[Out] -2/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)-2/(arctanh(tanh(b*x+a))-b*x)^3*b^2/
x^(1/2)+2/3/(arctanh(tanh(b*x+a))-b*x)^2*b/x^(3/2)-2*b^3/(arctanh(tanh(b*x+
a))-b*x)^3/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(
tanh(b*x+a))-b*x)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.2064, size = 336, normalized size = 2.62

$$\left[\frac{15\,b^{2}x^{3}\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\left(15\,b^{2}x^{2}-5\,abx+3\,a^{2}\right)\sqrt{x}}{15\,a^{3}x^{3}},\frac{2\left(15\,b^{2}x^{3}\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-\left(15\,b^{2}x^{2}-5\,abx+3\,a^{2}\right)\right.}{15\,a^{3}x^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] [1/15*(15*b^2*x^3*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x +
a)) - 2*(15*b^2*x^2 - 5*a*b*x + 3*a^2)*sqrt(x))/(a^3*x^3), 2/15*(15*b^2*x^3
*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - (15*b^2*x^2 - 5*a*b*x + 3*a^2)
```

```
*sqrt(x))/(a^3*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/atanh(tanh(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15843, size = 70, normalized size = 0.55

$$-\frac{2\,b^{3}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^{3}}-\frac{2\,\left(15\,b^{2}x^{2}-5\,abx+3\,a^{2}\right)}{15\,a^{3}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] -2*b^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/15*(15*b^2*x^2 - 5*a
*b*x + 3*a^2)/(a^3*x^(5/2))
```

3.199 $\int \frac{x^{7/2}}{\tanh^{-1}(\tanh(a+bx))^2}\,dx$

Optimal. Leaf size=135

$$\frac{7x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{3b^3}+\frac{7\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^4}-\frac{7\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh\right.}{b^{9/2}}$$

```
[Out] (7*x^(5/2))/(5*b^2) + (7*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]]))/(3*b^3) +
(7*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^2)/b^4 - (7*ArcTanh[(Sqrt[b]*Sqrt
[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[Tanh[a + b*x]])^(5/
2))/b^(9/2) - x^(7/2)/(b*ArcTanh[Tanh[a + b*x]])
```

Rubi [A] time = 0.103377, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2159, 2162}

$$\frac{7x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{3b^3}+\frac{7\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^4}-\frac{7\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh\right.}{b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/ArcTanh[Tanh[a + b*x]]^2,x]
```



```
[Out] (7*x^(5/2))/(5*b^2) + (7*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]]))/(3*b^3) +
(7*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^2)/b^4 - (7*ArcTanh[(Sqrt[b]*Sqrt
[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[Tanh[a + b*x]])^(5/
2))/b^(9/2) - x^(7/2)/(b*ArcTanh[Tanh[a + b*x]])
```


Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{x^{7/2}}{\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{x^{7/2}}{b\tanh^{-1}(\tanh(a+bx))} + \frac{7\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))}\,dx}{2b}$$

$$= \frac{7x^{5/2}}{5b^2} - \frac{x^{7/2}}{b\tanh^{-1}(\tanh(a+bx))} - \frac{\left(7\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\right)\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+b}$$

$$= \frac{7x^{5/2}}{5b^2} + \frac{7x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{3b^3} - \frac{x^{7/2}}{b\tanh^{-1}(\tanh(a+bx))} + \frac{\left(7\left(-bx+\tan\right.\right.}{}$$

$$= \frac{7x^{5/2}}{5b^2} + \frac{7x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{3b^3} + \frac{7\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^4} - \frac{}{b}$$

$$= \frac{7x^{5/2}}{5b^2} + \frac{7x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{3b^3} + \frac{7\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^4} - \frac{7}{}$$

Mathematica [A] time = 0.186507, size = 144, normalized size = 1.07

$$-\frac{4x^{3/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)}{3b^3} + \frac{\sqrt{x}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^3}{b^4\tanh^{-1}(\tanh(a+bx))} + \frac{6\sqrt{x}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}{b^4} - \frac{7\left(\mathrm{ta}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (2*x^(5/2))/(5*b^2) - (4*x^(3/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/(3*b^3)
 + (6*Sqrt[x]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)/b^4 - (7*ArcTan[(Sqrt[b]
*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*(-(b*x) + ArcTanh[Tanh[a +
 b*x]])^(5/2))/b^(9/2) + (Sqrt[x]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^3)/(b^4
*ArcTanh[Tanh[a + b*x]])
```

Maple [B] time = 0.136, size = 452, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] 2/5*x^(5/2)/b^2-4/3/b^3*x^(3/2)*a-4/3/b^3*x^(3/2)*(arctanh(tanh(b*x+a))-b*x
-a)+6/b^4*x^(1/2)*a^2+12/b^4*a*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)+6/b^4*(
arctanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)+1/b^4*x^(1/2)/arctanh(tanh(b*x+a))*a^
3+3/b^4*x^(1/2)/arctanh(tanh(b*x+a))*a^2*(arctanh(tanh(b*x+a))-b*x-a)+3/b^4
*x^(1/2)/arctanh(tanh(b*x+a))*a*(arctanh(tanh(b*x+a))-b*x-a)^2+1/b^4*x^(1/2
)/arctanh(tanh(b*x+a))*(arctanh(tanh(b*x+a))-b*x-a)^3-7/b^4/((arctanh(tanh(
b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)
)*a^3-21/b^4/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctan
h(tanh(b*x+a))-b*x)*b)^(1/2))*a^2*(arctanh(tanh(b*x+a))-b*x-a)-21/b^4/((arc
tanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x
)*b)^(1/2))*a*(arctanh(tanh(b*x+a))-b*x-a)^2-7/b^4/((arctanh(tanh(b*x+a))-b
*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*(arctan
h(tanh(b*x+a))-b*x-a)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.10656, size = 427, normalized size = 3.16

$$\left[\frac{105\left(a^{2}bx+a^{3}\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\left(6\,b^{3}x^{3}-14\,ab^{2}x^{2}+70\,a^{2}bx+105\,a^{3}\right)\sqrt{x}}{30\left(b^{5}x+ab^{4}\right)},-\frac{105\left(a^{2}bx+a^{3}\right)\sqrt{\frac{a}{b}}\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] [1/30*(105*(a^2*b*x + a^3)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a
)/(b*x + a)) + 2*(6*b^3*x^3 - 14*a*b^2*x^2 + 70*a^2*b*x + 105*a^3)*sqrt(x))
/(b^5*x + a*b^4), -1/15*(105*(a^2*b*x + a^3)*sqrt(a/b)*arctan(b*sqrt(x)*sqr
t(a/b)/a) - (6*b^3*x^3 - 14*a*b^2*x^2 + 70*a^2*b*x + 105*a^3)*sqrt(x))/(b^5
*x + a*b^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1208, size = 103, normalized size = 0.76

$$-\frac{7\,a^{3}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^{4}}+\frac{a^{3}\sqrt{x}}{(bx+a)b^{4}}+\frac{2\left(3\,b^{8}x^{\frac{5}{2}}-10\,ab^{7}x^{\frac{3}{2}}+45\,a^{2}b^{6}\sqrt{x}\right)}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] -7*a^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) + a^3*sqrt(x)/((b*x + a)
*b^4) + 2/15*(3*b^8*x^(5/2) - 10*a*b^7*x^(3/2) + 45*a^2*b^6*sqrt(x))/b^10
```

3.200 $\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))^2}\,dx$

Optimal. Leaf size=108

$$\frac{5\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}{b^{7/2}}-\frac{x^{5/2}}{b\tanh^{-1}(\tanh(}$$

```
[Out] (5*x^(3/2))/(3*b^2) + (5*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]]))/b^3 - (5*A
rcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh
[Tanh[a + b*x]])^(3/2))/b^(7/2) - x^(5/2)/(b*ArcTanh[Tanh[a + b*x]])
```

Rubi [A] time = 0.0764184, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2159, 2162}

$$\frac{5\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}{b^{7/2}}-\frac{x^{5/2}}{b\tanh^{-1}(\tanh(}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (5*x^(3/2))/(3*b^2) + (5*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]]))/b^3 - (5*A
rcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh
[Tanh[a + b*x]])^(3/2))/b^(7/2) - x^(5/2)/(b*ArcTanh[Tanh[a + b*x]])
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{x^{5/2}}{b\tanh^{-1}(\tanh(a+bx))} + \frac{5\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))}\,dx}{2b}$$

$$= \frac{5x^{3/2}}{3b^2} - \frac{x^{5/2}}{b\tanh^{-1}(\tanh(a+bx))} - \frac{\left(5\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\right)\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+b$$

$$= \frac{5x^{3/2}}{3b^2} + \frac{5\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3} - \frac{x^{5/2}}{b\tanh^{-1}(\tanh(a+bx))} + \frac{\left(5\left(-bx+\tan\right.\right.}{}$$

$$= \frac{5x^{3/2}}{3b^2} + \frac{5\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3} - \frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\mathrm{ta}\right.}{b^{7/2}}$$

Mathematica [A] time = 0.134398, size = 119, normalized size = 1.1

$$-\frac{\sqrt{x}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}{b^3\tanh^{-1}(\tanh(a+bx))} - \frac{4\sqrt{x}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)}{b^3} + \frac{5\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{3/2}\tan^{-1}\left(\frac{}{\sqrt{}}\right.}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (2*x^(3/2))/(3*b^2) - (4*Sqrt[x]*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/b^3 + (
5*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*(-(b*x) +
 ArcTanh[Tanh[a + b*x]])^(3/2))/b^(7/2) - (Sqrt[x]*(-(b*x) + ArcTanh[Tanh[a
```

```
+ b*x]])^2)/(b^3*ArcTanh[Tanh[a + b*x]])
```

Maple [B] time = 0.139, size = 294, normalized size = 2.7

$$\frac{2}{3b^2}x^{\frac{3}{2}}-4\frac{a\sqrt{x}}{b^3}-4\frac{(\text{Artanh}(\tanh(bx+a))-bx-a)\sqrt{x}}{b^3}-\frac{a^2}{b^3\text{Artanh}(\tanh(bx+a))}\sqrt{x}-2\frac{a\sqrt{x}(\text{Artanh}(\tanh(bx}{b^3\text{Artanh}(\tanh(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] 2/3*x^(3/2)/b^2-4/b^3*a*x^(1/2)-4/b^3*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)-
1/b^3*x^(1/2)/arctanh(tanh(b*x+a))*a^2-2/b^3*x^(1/2)/arctanh(tanh(b*x+a))*a
*(arctanh(tanh(b*x+a))-b*x-a)-1/b^3*x^(1/2)/arctanh(tanh(b*x+a))*(arctanh(t
anh(b*x+a))-b*x-a)^2+5/b^3/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^
(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*a^2+10/b^3/((arctanh(tanh(b*x+a
))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*a*(
arctanh(tanh(b*x+a))-b*x-a)+5/b^3/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arct
an(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*(arctanh(tanh(b*x+a))-b*
x-a)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.06784, size = 366, normalized size = 3.39

$$\left[\frac{15\left(abx+a^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx+2b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\left(2b^2x^2-10abx-15a^2\right)\sqrt{x}}{6\left(b^4x+ab^3\right)},\frac{15\left(abx+a^2\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)+\left(2b^2x^2-1\right.}{3\left(b^4x+ab^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a*b*x + a^2)*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b) - a)/(b
*x + a)) + 2*(2*b^2*x^2 - 10*a*b*x - 15*a^2)*sqrt(x))/(b^4*x + a*b^3), 1/3*
(15*(a*b*x + a^2)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (2*b^2*x^2 - 10
*a*b*x - 15*a^2)*sqrt(x))/(b^4*x + a*b^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1411, size = 88, normalized size = 0.81

$$\frac{5\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^3}-\frac{a^2\sqrt{x}}{(bx+a)b^3}+\frac{2\left(b^4x^{\frac{3}{2}}-6\,ab^3\sqrt{x}\right)}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 5*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) - a^2*sqrt(x)/((b*x + a)*
b^3) + 2/3*(b^4*x^(3/2) - 6*a*b^3*sqrt(x))/b^6
```

3.201 $\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))^2} \, dx$

Optimal. Leaf size=83

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}{b^{5/2}}-\frac{x^{3/2}}{b\tanh^{-1}(\tanh(a+bx))}+\frac{3\sqrt{x}}{b^2}$$

```
[Out] (3*Sqrt[x])/b^2 - (3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a +
b*x]]]]*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]])/b^(5/2) - x^(3/2)/(b*ArcTanh[Ta
nh[a + b*x]])
```

Rubi [A] time = 0.0536394, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2159, 2162}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}{b^{5/2}}-\frac{x^{3/2}}{b\tanh^{-1}(\tanh(a+bx))}+\frac{3\sqrt{x}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (3*Sqrt[x])/b^2 - (3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a +
b*x]]]]*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]])/b^(5/2) - x^(3/2)/(b*ArcTanh[Ta
nh[a + b*x]])
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{x^{3/2}}{b\tanh^{-1}(\tanh(a+bx))} + \frac{3\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))}\,dx}{2b}$$

$$= \frac{3\sqrt{x}}{b^2} - \frac{x^{3/2}}{b\tanh^{-1}(\tanh(a+bx))} - \frac{\left(3\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\right)\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a}}{2b^2}$$

$$= \frac{3\sqrt{x}}{b^2} - \frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}{b^{5/2}} - \frac{x^{3/2}}{b\tanh^{-1}(\tan}$$

Mathematica [A] time = 0.0769652, size = 81, normalized size = 0.98

$$-\frac{3\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{b^{5/2}} - \frac{x^{3/2}}{b\tanh^{-1}(\tanh(a+bx))} + \frac{3\sqrt{x}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (3*Sqrt[x])/b^2 - x^(3/2)/(b*ArcTanh[Tanh[a + b*x]]) - (3*ArcTan[(Sqrt[b]*S
qrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*Sqrt[-(b*x) + ArcTanh[Tanh[a
 + b*x]]])/b^(5/2)
```

Maple [B] time = 0.135, size = 160, normalized size = 1.9

$$2\,\frac{\sqrt{x}}{b^2}+\frac{a}{b^2\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}\sqrt{x}+\frac{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx-a}{b^2\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}\sqrt{x}-3\,\frac{a}{b^2\sqrt{\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)b}}\,\mathrm{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] 2*x^(1/2)/b^2+1/b^2*x^(1/2)/arctanh(tanh(b*x+a))*a+1/b^2*x^(1/2)/arctanh(ta
nh(b*x+a))*(arctanh(tanh(b*x+a))-b*x-a)-3/b^2/((arctanh(tanh(b*x+a))-b*x)*b
)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*a-3/b^2/((ar
ctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*
x)*b)^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.03279, size = 300, normalized size = 3.61

$$\left[\frac{3\,(bx+a)\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(2\,bx+3\,a)\sqrt{x}}{2\left(b^3x+ab^2\right)},-\frac{3\,(bx+a)\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-(2\,bx+3\,a)\sqrt{x}}{b^3x+ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*x + a)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x +
a)) + 2*(2*b*x + 3*a)*sqrt(x))/(b^3*x + a*b^2), -(3*(b*x + a)*sqrt(a/b)*arc
```

```
tan(b*sqrt(x)*sqrt(a/b)/a) - (2*b*x + 3*a)*sqrt(x))/(b^3*x + a*b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\operatorname{atanh}^2(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(x**(3/2)/atanh(tanh(a + b*x))**2, x)
```

Giac [A] time = 1.13556, size = 62, normalized size = 0.75

$$-\frac{3\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^2}+\frac{a\sqrt{x}}{(bx+a)b^2}+\frac{2\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] -3*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + a*sqrt(x)/((b*x + a)*b^2
) + 2*sqrt(x)/b^2
```

3.202 $\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^2} dx$

Optimal. Leaf size=73

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{b^{3/2}\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{\sqrt{x}}{b\tanh^{-1}(\tanh(a+bx))}$$

```
[Out] -(ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]/(b^(3/2)*Sq
rt[b*x - ArcTanh[Tanh[a + b*x]]])) - Sqrt[x]/(b*ArcTanh[Tanh[a + b*x]])
```

Rubi [A] time = 0.0341577, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 2162}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{b^{3/2}\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{\sqrt{x}}{b\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]/(b^(3/2)*Sq
rt[b*x - ArcTanh[Tanh[a + b*x]]])) - Sqrt[x]/(b*ArcTanh[Tanh[a + b*x]])
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{\sqrt{x}}{b\tanh^{-1}(\tanh(a+bx))} + \frac{\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))}\,dx}{2b}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{b^{3/2}\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}} - \frac{\sqrt{x}}{b\tanh^{-1}(\tanh(a+bx))}$$

Mathematica [A] time = 0.0598, size = 70, normalized size = 0.96

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{b^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}} - \frac{\sqrt{x}}{b\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] -(Sqrt[x]/(b*ArcTanh[Tanh[a + b*x]])) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x
) + ArcTanh[Tanh[a + b*x]]]]/(b^(3/2)*Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]
)
```

Maple [A] time = 0.135, size = 61, normalized size = 0.8

$$-\frac{1}{b\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}\sqrt{x}+\frac{1}{b}\arctan\left(b\sqrt{x}\frac{1}{\sqrt{\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)b}}\right)\frac{1}{\sqrt{\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] -x^(1/2)/b/arctanh(tanh(b*x+a))+1/b/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*ar
ctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.12579, size = 277, normalized size = 3.79

$$\left[-\frac{2\,ab\sqrt{x}+\sqrt{-ab}(bx+a)\log\left(\frac{bx-a-2\,\sqrt{-ab}\sqrt{x}}{bx+a}\right)}{2\left(ab^3x+a^2b^2\right)},-\frac{ab\sqrt{x}+\sqrt{ab}(bx+a)\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{ab^3x+a^2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*b*sqrt(x) + sqrt(-a*b)*(b*x + a)*log((b*x - a - 2*sqrt(-a*b)*sqr
t(x))/(b*x + a)))/(a*b^3*x + a^2*b^2), -(a*b*sqrt(x) + sqrt(a*b)*(b*x + a)*
arctan(sqrt(a*b)/(b*sqrt(x))))/(a*b^3*x + a^2*b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\operatorname{atanh}^{2}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(sqrt(x)/atanh(tanh(a + b*x))**2, x)
```

Giac [A] time = 1.13166, size = 49, normalized size = 0.67

$$\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b} - \frac{\sqrt{x}}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) - sqrt(x)/((b*x + a)*b)
```

3.203 $\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2}\,dx$

Optimal. Leaf size=97

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{b}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}-\frac{1}{b\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{1}{b\sqrt{x}\tanh^{-1}(\tanh(a+bx))}$$

```
[Out] ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]/(Sqrt[b]*(b*x
 - ArcTanh[Tanh[a + b*x]])^(3/2)) - 1/(b*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*
x]])) - 1/(b*Sqrt[x]*ArcTanh[Tanh[a + b*x]])
```

Rubi [A] time = 0.0537793, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2163, 2162}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{b}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}-\frac{1}{b\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{1}{b\sqrt{x}\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]/(Sqrt[b]*(b*x
 - ArcTanh[Tanh[a + b*x]])^(3/2)) - 1/(b*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*
x]])) - 1/(b*Sqrt[x]*ArcTanh[Tanh[a + b*x]])
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{1}{b\sqrt{x}\tanh^{-1}(\tanh(a+bx))} - \frac{\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))}\,dx}{2b}$$

$$= -\frac{1}{b\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{1}{b\sqrt{x}\tanh^{-1}(\tanh(a+bx))} - \frac{\int \frac{}{\sqrt{x}\tanh^{-1}}}{2\left(bx-\tanh^{-}\right.}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{b}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}} - \frac{1}{b\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{}{b\sqrt{x}\,\mathrm{ta}}$$

Mathematica [A] time = 0.0603814, size = 80, normalized size = 0.82

$$\frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))\left(\tanh^{-1}(\tanh(a+bx))-bx\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\sqrt{b}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]/(Sqrt[b]*(-
(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2)) + Sqrt[x]/(ArcTanh[Tanh[a + b*x]]*(-
(b*x) + ArcTanh[Tanh[a + b*x]]))
```

Maple [A] time = 0.132, size = 82, normalized size = 0.9

$$\frac{1}{(\mathrm{Artanh}\,(\tanh\,(bx+a))-bx)\,\mathrm{Artanh}\,(\tanh\,(bx+a))}\sqrt{x}+\frac{1}{\mathrm{Artanh}\,(\tanh\,(bx+a))-bx}\arctan\left(b\sqrt{x}\frac{}{\sqrt{(\mathrm{Artanh}\,(\tan}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctanh(tanh(b*x+a))^2/x^(1/2),x)
```

```
[Out] x^(1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))+1/(arctanh(tanh(b*x
+a))-b*x)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(t
anh(b*x+a))-b*x)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.05718, size = 274, normalized size = 2.82

$$\left[\frac{2\,ab\sqrt{x}-\sqrt{-ab}(bx+a)\log\left(\frac{bx-a-2\,\sqrt{-ab}\sqrt{x}}{bx+a}\right)}{2\left(a^2b^2x+a^3b\right)},\frac{ab\sqrt{x}-\sqrt{ab}(bx+a)\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{a^2b^2x+a^3b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b*sqrt(x) - sqrt(-a*b)*(b*x + a)*log((b*x - a - 2*sqrt(-a*b)*sqrt
(x))/(b*x + a)))/(a^2*b^2*x + a^3*b), (a*b*sqrt(x) - sqrt(a*b)*(b*x + a)*ar
ctan(sqrt(a*b)/(b*sqrt(x))))/(a^2*b^2*x + a^3*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x}\,\text{atanh}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atanh(tanh(b*x+a))**2/x**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*atanh(tanh(a + b*x))**2), x)
```

Giac [A] time = 1.18221, size = 47, normalized size = 0.48

$$\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a}+\frac{\sqrt{x}}{(bx+a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^2/x^(1/2),x, algorithm="giac")
```

```
[Out] arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) + sqrt(x)/((b*x + a)*a)
```

3.204 $\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))^2}\,dx$

Optimal. Leaf size=120

$$-\frac{1}{bx^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{1}{bx^{3/2}\tanh^{-1}(\tanh(a+bx))}+\frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}-\frac{}{\sqrt{x}\left(bx-\mathrm{ta}\right.}$$

```
[Out] (3*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(
b*x - ArcTanh[Tanh[a + b*x]])^(5/2) - 3/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*
x]])^2) - 1/(b*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])) - 1/(b*x^(3/2)*ArcTa
nh[Tanh[a + b*x]])
```

Rubi [A] time = 0.0799972, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2163, 2162}

$$-\frac{1}{bx^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{1}{bx^{3/2}\tanh^{-1}(\tanh(a+bx))}+\frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}-\frac{}{\sqrt{x}\left(bx-\mathrm{ta}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] (3*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(
b*x - ArcTanh[Tanh[a + b*x]])^(5/2) - 3/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*
x]])^2) - 1/(b*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])) - 1/(b*x^(3/2)*ArcTa
nh[Tanh[a + b*x]])
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{1}{bx^{3/2}\tanh^{-1}(\tanh(a+bx))} - \frac{3\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))}\,dx}{2b}$$

$$= -\frac{1}{bx^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{1}{bx^{3/2}\tanh^{-1}(\tanh(a+bx))} + \frac{3\int \frac{1}{x^{3/2}\,\mathrm{ta}}}{2\left(-bx+\mathrm{tar}\right.}$$

$$= -\frac{3}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} - \frac{1}{bx^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{}{bx^{3/2}\,\mathrm{t}}$$

$$= \frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}} - \frac{3}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} - \frac{}{bx^{3/2}\left(\right.}$$

Mathematica [A] time = 0.104345, size = 104, normalized size = 0.87

$$-\frac{b\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2} - \frac{2}{\sqrt{x}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2} - \frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh}}\right.}{\left(\tanh^{-1}(\tanh(a+bx))\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] (-3*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]
)/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2) - 2/(Sqrt[x]*(-(b*x) + ArcTanh[Ta
nh[a + b*x]])^2) - (b*Sqrt[x])/(ArcTanh[Tanh[a + b*x]]*(-(b*x) + ArcTanh[Ta
nh[a + b*x]])^2)
```

Maple [A] time = 0.124, size = 105, normalized size = 0.9

$$-2\,\frac{1}{\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}\sqrt{x}}-\frac{b}{\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}\sqrt{x}-3\,\frac{}{(\mathrm{Artanh}\,(\mathrm{tanh}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] -2/(arctanh(tanh(b*x+a))-b*x)^2/x^(1/2)-1/(arctanh(tanh(b*x+a))-b*x)^2*b*x^
(1/2)/arctanh(tanh(b*x+a))-3/(arctanh(tanh(b*x+a))-b*x)^2*b/((arctanh(tanh(
b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.10007, size = 323, normalized size = 2.69

$$\left[\frac{3\left(bx^{2}+ax\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\,(3\,bx+2\,a)\sqrt{x}}{2\left(a^{2}bx^{2}+a^{3}x\right)},\frac{3\left(bx^{2}+ax\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-(3\,bx+2\,a)\sqrt{x}}{a^{2}bx^{2}+a^{3}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*x^2 + a*x)*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*
x + a)) - 2*(3*b*x + 2*a)*sqrt(x))/(a^2*b*x^2 + a^3*x), (3*(b*x^2 + a*x)*sq
rt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - (3*b*x + 2*a)*sqrt(x))/(a^2*b*x^2
 + a^3*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{3}{2}} \operatorname{atanh}^{2}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(1/(x**(3/2)*atanh(tanh(a + b*x))**2), x)
```

Giac [A] time = 1.15267, size = 66, normalized size = 0.55

$$-\frac{3\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^{2}}-\frac{3\,bx+2\,a}{\left(bx^{\frac{3}{2}}+a\sqrt{x}\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] -3*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) - (3*b*x + 2*a)/((b*x^(3/2
) + a*sqrt(x))*a^2)
```

3.205 $\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))^2}\,dx$

Optimal. Leaf size=145

$$\frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}}-\frac{5}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}-\frac{1}{bx^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{}{bx^{5/2}}$$

```
[Out] (5*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(
b*x - ArcTanh[Tanh[a + b*x]])^(7/2) - (5*b)/(Sqrt[x]*(b*x - ArcTanh[Tanh[a
+ b*x]])^3) - 5/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2) - 1/(b*x^(5/2)
*(b*x - ArcTanh[Tanh[a + b*x]])) - 1/(b*x^(5/2)*ArcTanh[Tanh[a + b*x]])
```

Rubi [A] time = 0.10639, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2163, 2162}

$$\frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}}-\frac{5}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}-\frac{1}{bx^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{}{bx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] (5*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(
b*x - ArcTanh[Tanh[a + b*x]])^(7/2) - (5*b)/(Sqrt[x]*(b*x - ArcTanh[Tanh[a
+ b*x]])^3) - 5/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2) - 1/(b*x^(5/2)
*(b*x - ArcTanh[Tanh[a + b*x]])) - 1/(b*x^(5/2)*ArcTanh[Tanh[a + b*x]])
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{1}{bx^{5/2}\tanh^{-1}(\tanh(a+bx))} - \frac{5\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))}\,dx}{2b}$$

$$= -\frac{1}{bx^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{1}{bx^{5/2}\tanh^{-1}(\tanh(a+bx))} + \frac{5\int \frac{1}{x^{5/2}\tanh^{-1}\ldots}}{2\left(-bx+\tan\ldots\right.}$$

$$= -\frac{5}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} - \frac{1}{bx^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{\ldots}{bx^{5/2}\ldots}$$

$$= -\frac{5b}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} - \frac{5}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} - \frac{\ldots}{bx^{5/2}\ldots}$$

$$= \frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}} - \frac{5b}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} - \frac{\ldots}{3x^{3/2}\left(\ldots\right.}$$

Mathematica [A] time = 0.198996, size = 120, normalized size = 0.83

$$\frac{b^2\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^3} + \frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{7/2}} + \frac{2\left(\tanh^{-1}(\tanh(a+bx\ldots\right.}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] (2*(-7*b*x + ArcTanh[Tanh[a + b*x]]))/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*
x]])^3) + (5*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a
+ b*x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(7/2) + (b^2*Sqrt[x])/(ArcTanh
[Tanh[a + b*x]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^3)
```

Maple [A] time = 0.18, size = 128, normalized size = 0.9

$$-\frac{2}{3\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}}x^{-\frac{3}{2}}+4\frac{b}{\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{3}\sqrt{x}}+\frac{b^{2}}{\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{3}\mathrm{Artar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] -2/3/(arctanh(tanh(b*x+a))-b*x)^2/x^(3/2)+4/(arctanh(tanh(b*x+a))-b*x)^3*b/
x^(1/2)+1/(arctanh(tanh(b*x+a))-b*x)^3*b^2*x^(1/2)/arctanh(tanh(b*x+a))+5/(
arctanh(tanh(b*x+a))-b*x)^3*b^2/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan
(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.08098, size = 402, normalized size = 2.77

$$\left[\frac{15\left(b^{2}x^{3}+abx^{2}\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx+2a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\left(15b^{2}x^{2}+10abx-2a^{2}\right)\sqrt{x}}{6\left(a^{3}bx^{3}+a^{4}x^{2}\right)},-\frac{15\left(b^{2}x^{3}+abx^{2}\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-\left(15b^{2}\right.}{3\left(a^{3}bx^{3}+a^{4}x^{2}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*(b^2*x^3 + a*b*x^2)*sqrt(-b/a)*log((b*x + 2*a*sqrt(x)*sqrt(-b/a) -
 a)/(b*x + a)) + 2*(15*b^2*x^2 + 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 + a^
4*x^2), -1/3*(15*(b^2*x^3 + a*b*x^2)*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x
))) - (15*b^2*x^2 + 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 + a^4*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{5}{2}} \operatorname{atanh}^2(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(1/(x**(5/2)*atanh(tanh(a + b*x))**2), x)
```

Giac [A] time = 1.12245, size = 78, normalized size = 0.54

$$\frac{5\,b^2 \arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} + \frac{b^2\sqrt{x}}{(bx+a)a^3} + \frac{2\,(6\,bx-a)}{3\,a^3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 5*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) + b^2*sqrt(x)/((b*x + a)*
a^3) + 2/3*(6*b*x - a)/(a^3*x^(3/2))
```

3.206 $\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))^2}\,dx$

Optimal. Leaf size=172

$$\frac{7b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{9/2}}-\frac{7b^2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}-\frac{7b}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}-\frac{}{5x^{5/2}}$$

```
[Out] (7*b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(
b*x - ArcTanh[Tanh[a + b*x]])^(9/2) - (7*b^2)/(Sqrt[x]*(b*x - ArcTanh[Tanh[
a + b*x]])^4) - (7*b)/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^3) - 7/(5*x
^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2) - 1/(b*x^(7/2)*(b*x - ArcTanh[Tanh
[a + b*x]])) - 1/(b*x^(7/2)*ArcTanh[Tanh[a + b*x]])
```

Rubi [A] time = 0.136852, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2163, 2162}

$$\frac{7b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{9/2}}-\frac{7b^2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}-\frac{7b}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}-\frac{}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] (7*b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(
b*x - ArcTanh[Tanh[a + b*x]])^(9/2) - (7*b^2)/(Sqrt[x]*(b*x - ArcTanh[Tanh[
a + b*x]])^4) - (7*b)/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^3) - 7/(5*x
^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2) - 1/(b*x^(7/2)*(b*x - ArcTanh[Tanh
[a + b*x]])) - 1/(b*x^(7/2)*ArcTanh[Tanh[a + b*x]])
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
```

```
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{1}{bx^{7/2}\tanh^{-1}(\tanh(a+bx))} - \frac{7\int \frac{1}{x^{9/2}\tanh^{-1}(\tanh(a+bx))}\,dx}{2b}$$

$$= -\frac{1}{bx^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{1}{bx^{7/2}\tanh^{-1}(\tanh(a+bx))} + \frac{7\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))}}{2\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)}$$

$$= -\frac{7}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} - \frac{1}{bx^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{}{bx^{7/2}}$$

$$= -\frac{7b}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} - \frac{7}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} - \frac{}{bx^{7}}$$

$$= -\frac{7b^2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4} - \frac{7b}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} - \frac{}{5x^{5/2}}$$

$$= \frac{7b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{9/2}} - \frac{7b^2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4} - \frac{}{3x^{3/2}}$$

Mathematica [A] time = 0.266968, size = 139, normalized size = 0.81

$$-\frac{2\left(-16bx\tanh^{-1}(\tanh(a+bx))+3\tanh^{-1}(\tanh(a+bx))^2+58b^2x^2\right)}{15x^{5/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^4}-\frac{b^3\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))\left(\tanh^{-1}(\tanh(a+bx))\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] (-7*b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]
)/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(9/2) - (b^3*Sqrt[x])/(ArcTanh[Tanh[a +
 b*x]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4) - (2*(58*b^2*x^2 - 16*b*x*ArcTa
nh[Tanh[a + b*x]] + 3*ArcTanh[Tanh[a + b*x]]^2))/(15*x^(5/2)*(-(b*x) + ArcT
anh[Tanh[a + b*x]])^4)
```

Maple [A] time = 0.178, size = 151, normalized size = 0.9

$$-\frac{b^3}{\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^4\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}\sqrt{x}-7\frac{b^3}{\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^4\sqrt{\left(\mathrm{Artanh}\left(\tanh\left(bx+\right.\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] -1/(arctanh(tanh(b*x+a))-b*x)^4*b^3*x^(1/2)/arctanh(tanh(b*x+a))-7/(arctanh
(tanh(b*x+a))-b*x)^4*b^3/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1
/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))-2/5/(arctanh(tanh(b*x+a))-b*x)^2/
x^(5/2)-6/(arctanh(tanh(b*x+a))-b*x)^4*b^2/x^(1/2)+4/3/(arctanh(tanh(b*x+a)
)-b*x)^3*b/x^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [A] time = 2.19201, size = 460, normalized size = 2.67

$$\left[\frac{105\left(b^3x^4+ab^2x^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\left(105b^3x^3+70ab^2x^2-14a^2bx+6a^3\right)\sqrt{x}}{30\left(a^4bx^4+a^5x^3\right)}, \frac{105\left(b^3x^4+ab^2x^3\right)\sqrt{\frac{b}{a}}\operatorname{arct}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] [1/30*(105*(b^3*x^4 + a*b^2*x^3)*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/
a) - a)/(b*x + a)) - 2*(105*b^3*x^3 + 70*a*b^2*x^2 - 14*a^2*b*x + 6*a^3)*sq
rt(x))/(a^4*b*x^4 + a^5*x^3), 1/15*(105*(b^3*x^4 + a*b^2*x^3)*sqrt(b/a)*arc
tan(a*sqrt(b/a)/(b*sqrt(x))) - (105*b^3*x^3 + 70*a*b^2*x^2 - 14*a^2*b*x + 6
*a^3)*sqrt(x))/(a^4*b*x^4 + a^5*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12369, size = 95, normalized size = 0.55

$$-\frac{7\,b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^4}-\frac{b^3\sqrt{x}}{(bx+a)a^4}-\frac{2\left(45\,b^2x^2-10\,abx+3\,a^2\right)}{15\,a^4x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] -7*b^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4) - b^3*sqrt(x)/((b*x + a)
*a^4) - 2/15*(45*b^2*x^2 - 10*a*b*x + 3*a^2)/(a^4*x^(5/2))
```

3.207 $\int \frac{x^{7/2}}{\tanh^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=135

$$-\frac{7x^{5/2}}{4b^2\tanh^{-1}(\tanh(a+bx))}+\frac{35\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{4b^4}-\frac{35\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(t\right.}{4b^{9/2}}$$

```
[Out] (35*x^(3/2))/(12*b^3) + (35*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]]))/(4*b^4)
 - (35*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*(b*x -
 ArcTanh[Tanh[a + b*x]])^(3/2))/(4*b^(9/2)) - x^(7/2)/(2*b*ArcTanh[Tanh[a +
 b*x]]^2) - (7*x^(5/2))/(4*b^2*ArcTanh[Tanh[a + b*x]])
```

Rubi [A] time = 0.0970262, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2159, 2162}

$$-\frac{7x^{5/2}}{4b^2\tanh^{-1}(\tanh(a+bx))}+\frac{35\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{4b^4}-\frac{35\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(t\right.}{4b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (35*x^(3/2))/(12*b^3) + (35*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]]))/(4*b^4)
 - (35*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*(b*x -
 ArcTanh[Tanh[a + b*x]])^(3/2))/(4*b^(9/2)) - x^(7/2)/(2*b*ArcTanh[Tanh[a +
 b*x]]^2) - (7*x^(5/2))/(4*b^2*ArcTanh[Tanh[a + b*x]])
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{x^{7/2}}{\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{x^{7/2}}{2b\tanh^{-1}(\tanh(a+bx))^2} + \frac{7\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))^2}\,dx}{4b}$$

$$= -\frac{x^{7/2}}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{7x^{5/2}}{4b^2\tanh^{-1}(\tanh(a+bx))} + \frac{35\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))}\,dx}{8b^2}$$

$$= \frac{35x^{3/2}}{12b^3} - \frac{x^{7/2}}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{7x^{5/2}}{4b^2\tanh^{-1}(\tanh(a+bx))} - \frac{\left(35\left(-bx+\tanh^{-1}(\dots\right.\right.}{\dots}$$

$$= \frac{35x^{3/2}}{12b^3} + \frac{35\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{4b^4} - \frac{x^{7/2}}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{\dots}{4b^2\tanh^{-1}\dots}$$

$$= \frac{35x^{3/2}}{12b^3} + \frac{35\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{4b^4} - \frac{35\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx - \dots\right.}{4b^{9/2}}$$

Mathematica [A] time = 0.111725, size = 147, normalized size = 1.09

$$-\frac{21b^{5/2}x^{5/2}\tanh^{-1}(\tanh(a+bx)) - 140b^{3/2}x^{3/2}\tanh^{-1}(\tanh(a+bx))^2 + 105\sqrt{b}\sqrt{x}\tanh^{-1}(\tanh(a+bx))^3 - 105\tanh^{-1}\dots}{12b^{9/2}\tanh^{-1}(\tanh(a+bx))^2 \dots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] -(6*b^(7/2)*x^(7/2) + 21*b^(5/2)*x^(5/2)*ArcTanh[Tanh[a + b*x]] - 140*b^(3/
2)*x^(3/2)*ArcTanh[Tanh[a + b*x]]^2 + 105*Sqrt[b]*Sqrt[x]*ArcTanh[Tanh[a +
b*x]]^3 - 105*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]
]]*ArcTanh[Tanh[a + b*x]]^2*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2))/(12*b^
(9/2)*ArcTanh[Tanh[a + b*x]]^2)
```

Maple [B] time = 0.14, size = 418, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/arctanh(tanh(b*x+a))^3,x)
```

```
[Out] 2/3*x^(3/2)/b^3-6/b^4*a*x^(1/2)-6/b^4*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)-
13/4/b^3/arctanh(tanh(b*x+a))^2*x^(3/2)*a^2-13/2/b^3/arctanh(tanh(b*x+a))^2
*x^(3/2)*a*(arctanh(tanh(b*x+a))-b*x-a)-13/4/b^3/arctanh(tanh(b*x+a))^2*x^(
3/2)*(arctanh(tanh(b*x+a))-b*x-a)^2-11/4/b^4/arctanh(tanh(b*x+a))^2*x^(1/2)
*a^3-33/4/b^4/arctanh(tanh(b*x+a))^2*x^(1/2)*a^2*(arctanh(tanh(b*x+a))-b*x-
a)-33/4/b^4/arctanh(tanh(b*x+a))^2*x^(1/2)*a*(arctanh(tanh(b*x+a))-b*x-a)^2
-11/4/b^4/arctanh(tanh(b*x+a))^2*x^(1/2)*(arctanh(tanh(b*x+a))-b*x-a)^3+35/
4/b^4/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(
b*x+a))-b*x)*b)^(1/2))*a^2+35/2/b^4/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*ar
ctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*a*(arctanh(tanh(b*x+a)
)-b*x-a)+35/4/b^4/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((a
rctanh(tanh(b*x+a))-b*x)*b)^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.09214, size = 509, normalized size = 3.77

$$\left[\frac{105\left(ab^2x^2+2a^2bx+a^3\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx+2b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\left(8b^3x^3-56ab^2x^2-175a^2bx-105a^3\right)\sqrt{x}}{24\left(b^6x^2+2ab^5x+a^2b^4\right)}, \frac{105\left(ab^2x^2+2a^2bx\cdots\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] [1/24*(105*(a*b^2*x^2 + 2*a^2*b*x + a^3)*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*
sqrt(-a/b) - a)/(b*x + a)) + 2*(8*b^3*x^3 - 56*a*b^2*x^2 - 175*a^2*b*x - 10
5*a^3)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4), 1/12*(105*(a*b^2*x^2 + 2*a
^2*b*x + a^3)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (8*b^3*x^3 - 56*a*b
^2*x^2 - 175*a^2*b*x - 105*a^3)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1437, size = 104, normalized size = 0.77

$$\frac{35\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}b^4}-\frac{13\,a^2bx^{\frac{3}{2}}+11\,a^3\sqrt{x}}{4\,(bx+a)^2b^4}+\frac{2\left(b^6x^{\frac{3}{2}}-9\,ab^5\sqrt{x}\right)}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 35/4*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) - 1/4*(13*a^2*b*x^(3/2
) + 11*a^3*sqrt(x))/((b*x + a)^2*b^4) + 2/3*(b^6*x^(3/2) - 9*a*b^5*sqrt(x))
/b^9
```

3.208 $\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))^3} dx$

Optimal. Leaf size=110

$$-\frac{5x^{3/2}}{4b^2\tanh^{-1}(\tanh(a+bx))}-\frac{15\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}{4b^{7/2}}-\frac{x^{5/2}}{2b\tanh^{-1}(\tanh(a+b$$

```
[Out] (15*Sqrt[x])/(4*b^3) - (15*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tan
h[a + b*x]]]]*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]])/(4*b^(7/2)) - x^(5/2)/(2*
b*ArcTanh[Tanh[a + b*x]]^2) - (5*x^(3/2))/(4*b^2*ArcTanh[Tanh[a + b*x]])
```

Rubi [A] time = 0.0711217, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2159, 2162}

$$-\frac{5x^{3/2}}{4b^2\tanh^{-1}(\tanh(a+bx))}-\frac{15\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}{4b^{7/2}}-\frac{x^{5/2}}{2b\tanh^{-1}(\tanh(a+b$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/ArcTanh[Tanh[a + b*x]]^3,x]
```


```
[Out] (15*Sqrt[x])/(4*b^3) - (15*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tan
h[a + b*x]]]]*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]])/(4*b^(7/2)) - x^(5/2)/(2*
b*ArcTanh[Tanh[a + b*x]]^2) - (5*x^(3/2))/(4*b^2*ArcTanh[Tanh[a + b*x]])
```


Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{x^{5/2}}{2b\tanh^{-1}(\tanh(a+bx))^2} + \frac{5\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))^2}\,dx}{4b}$$

$$= -\frac{x^{5/2}}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{5x^{3/2}}{4b^2\tanh^{-1}(\tanh(a+bx))} + \frac{15\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))}\,dx}{8b^2}$$

$$= \frac{15\sqrt{x}}{4b^3} - \frac{x^{5/2}}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{5x^{3/2}}{4b^2\tanh^{-1}(\tanh(a+bx))} - \frac{\left(15\left(-bx+\tanh^{-1}(t\right.\right.}{}$$

$$= \frac{15\sqrt{x}}{4b^3} - \frac{15\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}{4b^{7/2}} - \frac{x}{2b\tanh^{-1}(ta}$$

Mathematica [A] time = 0.0876208, size = 104, normalized size = 0.95

$$\frac{1}{4}\left(-\frac{5x^{3/2}}{b^2\tanh^{-1}(\tanh(a+bx))} - \frac{15\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{b^{7/2}} - \frac{2x^{5/2}}{b\tanh^{-1}(\tanh(a+bx)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] ((15*Sqrt[x])/b^3 - (2*x^(5/2))/(b*ArcTanh[Tanh[a + b*x]]^2) - (5*x^(3/2))/
(b^2*ArcTanh[Tanh[a + b*x]]) - (15*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + A
```

```
rcTanh[Tanh[a + b*x]]]]*Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]])/b^(7/2))/4
```

Maple [B] time = 0.138, size = 249, normalized size = 2.3

$$2\frac{\sqrt{x}}{b^3} + \frac{9a}{4b^2(\mathrm{Artanh}(\tanh(bx+a)))^2}x^{\frac{3}{2}} + \frac{9\,\mathrm{Artanh}(\tanh(bx+a)) - 9bx - 9a}{4b^2(\mathrm{Artanh}(\tanh(bx+a)))^2}x^{\frac{3}{2}} + \frac{7a^2}{4b^3(\mathrm{Artanh}(\tanh(bx+a)))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/arctanh(tanh(b*x+a))^3,x)
```

```
[Out] 2*x^(1/2)/b^3+9/4/b^2/arctanh(tanh(b*x+a))^2*x^(3/2)*a+9/4/b^2/arctanh(tanh
(b*x+a))^2*x^(3/2)*(arctanh(tanh(b*x+a))-b*x-a)+7/4/b^3/arctanh(tanh(b*x+a)
)^2*x^(1/2)*a^2+7/2/b^3/arctanh(tanh(b*x+a))^2*x^(1/2)*a*(arctanh(tanh(b*x+
a))-b*x-a)+7/4/b^3/arctanh(tanh(b*x+a))^2*x^(1/2)*(arctanh(tanh(b*x+a))-b*x
-a)^2-15/4/b^3/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arct
anh(tanh(b*x+a))-b*x)*b)^(1/2))*a-15/4/b^3/((arctanh(tanh(b*x+a))-b*x)*b)^(
1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*(arctanh(tanh(b
*x+a))-b*x-a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.03374, size = 443, normalized size = 4.03

$$\left[\frac{15\left(b^2x^2 + 2abx + a^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx - 2b\sqrt{x}\sqrt{-\frac{a}{b}} - a}{bx+a}\right) + 2\left(8b^2x^2 + 25abx + 15a^2\right)\sqrt{x}}{8\left(b^5x^2 + 2ab^4x + a^2b^3\right)}, -\frac{15\left(b^2x^2 + 2abx + a^2\right)\sqrt{\frac{a}{b}}\arctan\left(\right.}{4\left(b^5x^2 + 2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-
a/b) - a)/(b*x + a)) + 2*(8*b^2*x^2 + 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*x^2
+ 2*a*b^4*x + a^2*b^3), -1/4*(15*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a/b)*arctan
(b*sqrt(x)*sqrt(a/b)/a) - (8*b^2*x^2 + 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*x^2
 + 2*a*b^4*x + a^2*b^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.135, size = 80, normalized size = 0.73

$$-\frac{15\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^3}+\frac{2\,\sqrt{x}}{b^3}+\frac{9\,abx^{\frac{3}{2}}+7\,a^2\sqrt{x}}{4\,(bx+a)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] -15/4*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2*sqrt(x)/b^3 + 1/4*(
9*a*b*x^(3/2) + 7*a^2*sqrt(x))/((b*x + a)^2*b^3)
```

3.209 $\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))^3} \, dx$

Optimal. Leaf size=98

$$-\frac{3\sqrt{x}}{4b^2\tanh^{-1}(\tanh(a+bx))}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4b^{5/2}\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{x^{3/2}}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

```
[Out] (-3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(4*b^(5/
2)*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]) - x^(3/2)/(2*b*ArcTanh[Tanh[a + b*x]
]^2) - (3*Sqrt[x])/(4*b^2*ArcTanh[Tanh[a + b*x]])
```

Rubi [A] time = 0.0528765, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 2162}

$$-\frac{3\sqrt{x}}{4b^2\tanh^{-1}(\tanh(a+bx))}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4b^{5/2}\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{x^{3/2}}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (-3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(4*b^(5/
2)*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]) - x^(3/2)/(2*b*ArcTanh[Tanh[a + b*x]
]^2) - (3*Sqrt[x])/(4*b^2*ArcTanh[Tanh[a + b*x]])
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{x^{3/2}}{2b\tanh^{-1}(\tanh(a+bx))^2} + \frac{3\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^2}\,dx}{4b}$$

$$= -\frac{x^{3/2}}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{3\sqrt{x}}{4b^2\tanh^{-1}(\tanh(a+bx))} + \frac{3\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))}\,dx}{8b^2}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4b^{5/2}\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}} - \frac{x^{3/2}}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{3\sqrt{x}}{4b^2\tanh^{-1}(\tanh(a$$

Mathematica [A] time = 0.0727301, size = 96, normalized size = 0.98

$$-\frac{3\sqrt{x}}{4b^2\tanh^{-1}(\tanh(a+bx))} + \frac{3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{4b^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}} - \frac{x^{3/2}}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] -x^(3/2)/(2*b*ArcTanh[Tanh[a + b*x]]^2) - (3*Sqrt[x])/(4*b^2*ArcTanh[Tanh[a
 + b*x]]) + (3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]
]]])/(4*b^(5/2)*Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]])
```

Maple [A] time = 0.145, size = 85, normalized size = 0.9

$$2\frac{1}{(\mathrm{Artanh}(\tanh(bx+a)))^2}\left(-5/8\frac{x^{3/2}}{b} - 3/8\frac{(\mathrm{Artanh}(\tanh(bx+a))-bx)\sqrt{x}}{b^2}\right) + \frac{3}{4b^2}\arctan\left(b\sqrt{x}\frac{1}{\sqrt{(\mathrm{Artanh}(\tanh($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/arctanh(tanh(b*x+a))^3,x)
```

```
[Out] 2*(-5/8*x^(3/2)/b-3/8*(arctanh(tanh(b*x+a))-b*x)/b^2*x^(1/2))/arctanh(tanh(
b*x+a))^2+3/4/b^2/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((a
rctanh(tanh(b*x+a))-b*x)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.06684, size = 423, normalized size = 4.32

$$\left[-\frac{3\left(b^2x^2+2\,abx+a^2\right)\sqrt{-ab}\log\left(\frac{bx-a-2\sqrt{-ab}\sqrt{x}}{bx+a}\right)+2\left(5\,ab^2x+3\,a^2b\right)\sqrt{x}}{8\left(ab^5x^2+2\,a^2b^4x+a^3b^3\right)},-\frac{3\left(b^2x^2+2\,abx+a^2\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)+(5}{4\left(ab^5x^2+2\,a^2b^4x+a^3b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*s
qrt(x))/(b*x + a)) + 2*(5*a*b^2*x + 3*a^2*b)*sqrt(x))/(a*b^5*x^2 + 2*a^2*b^
4*x + a^3*b^3), -1/4*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b
)/(b*sqrt(x))) + (5*a*b^2*x + 3*a^2*b)*sqrt(x))/(a*b^5*x^2 + 2*a^2*b^4*x +
a^3*b^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\operatorname{atanh}^{3}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(x**(3/2)/atanh(tanh(a + b*x))**3, x)
```

Giac [A] time = 1.11162, size = 63, normalized size = 0.64

$$\frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^2}-\frac{5\,bx^{\frac{3}{2}}+3\,a\sqrt{x}}{4\,(bx+a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 3/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/4*(5*b*x^(3/2) + 3*a*sq
rt(x))/((b*x + a)^2*b^2)
```

3.210 $\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=125

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4b^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}-\frac{1}{4b^2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{1}{4b^2\sqrt{x}\tanh^{-1}(\tanh(a+bx))}-\frac{\cdots}{2b\tanh\cdots}$$

```
[Out] ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]/(4*b^(3/2)*(b
*x - ArcTanh[Tanh[a + b*x]])^(3/2)) - 1/(4*b^2*Sqrt[x]*(b*x - ArcTanh[Tanh[
a + b*x]])) - Sqrt[x]/(2*b*ArcTanh[Tanh[a + b*x]]^2) - 1/(4*b^2*Sqrt[x]*Arc
Tanh[Tanh[a + b*x]])
```

Rubi [A] time = 0.0706845, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2163, 2162}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4b^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}-\frac{1}{4b^2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{1}{4b^2\sqrt{x}\tanh^{-1}(\tanh(a+bx))}-\frac{\cdots}{2b\tanh\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]/(4*b^(3/2)*(b
*x - ArcTanh[Tanh[a + b*x]])^(3/2)) - 1/(4*b^2*Sqrt[x]*(b*x - ArcTanh[Tanh[
a + b*x]])) - Sqrt[x]/(2*b*ArcTanh[Tanh[a + b*x]]^2) - 1/(4*b^2*Sqrt[x]*Arc
Tanh[Tanh[a + b*x]])
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
```

```
[m, 0] &&  !IntegerQ[n]))
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{\sqrt{x}}{2b\tanh^{-1}(\tanh(a+bx))^2} + \frac{\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2}\,dx}{4b}$$

$$= -\frac{\sqrt{x}}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{1}{4b^2\sqrt{x}\tanh^{-1}(\tanh(a+bx))} - \frac{\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))}\,dx}{8b^2}$$

$$= -\frac{1}{4b^2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{\sqrt{x}}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{1}{4b^2\sqrt{x}\tanh^{-1}(\tanh(a+bx))}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4b^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}} - \frac{1}{4b^2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{}{2b\tanh^{-1}}$$

Mathematica [A] time = 0.131997, size = 107, normalized size = 0.86

$$\frac{1}{4}\left(\frac{\sqrt{x}}{b\tanh^{-1}(\tanh(a+bx))^2-b^2x\tanh^{-1}(\tanh(a+bx))} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{b^{3/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{3/2}} - \frac{2\sqrt{x}}{b\tanh^{-1}(\tanh(a+b}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] ((-2*Sqrt[x])/(b*ArcTanh[Tanh[a + b*x]]^2) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[
-(b*x) + ArcTanh[Tanh[a + b*x]]]]/(b^(3/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]]
)^(3/2)) + Sqrt[x]/(-(b^2*x*ArcTanh[Tanh[a + b*x]]) + b*ArcTanh[Tanh[a + b*
x]]^2))/4
```

Maple [A] time = 0.135, size = 98, normalized size = 0.8

$$2\frac{1}{(\mathrm{Artanh}(\tanh(bx+a)))^2}\left(1/8\frac{x^{3/2}}{\mathrm{Artanh}(\tanh(bx+a))-bx}-1/8\frac{\sqrt{x}}{b}\right)+\frac{1}{(4\,\mathrm{Artanh}(\tanh(bx+a))-4\,bx)\,b}\,\mathrm{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/arctanh(tanh(b*x+a))^3,x)
```

```
[Out] 2*(1/8/(arctanh(tanh(b*x+a))-b*x)*x^(3/2)-1/8*x^(1/2)/b)/arctanh(tanh(b*x+a
))^2+1/4/(arctanh(tanh(b*x+a))-b*x)/b/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*
arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.03657, size = 412, normalized size = 3.3

$$\left[-\frac{\left(b^2x^2+2\,abx+a^2\right)\sqrt{-ab}\log\left(\frac{bx-a-2\,\sqrt{-ab}\sqrt{x}}{bx+a}\right)-2\left(ab^2x-a^2b\right)\sqrt{x}}{8\left(a^2b^4x^2+2\,a^3b^3x+a^4b^2\right)},-\frac{\left(b^2x^2+2\,abx+a^2\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)-\left(ab^2x-a\right.}{4\left(a^2b^4x^2+2\,a^3b^3x+a^4b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] [-1/8*((b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqr
t(x))/(b*x + a)) - 2*(a*b^2*x - a^2*b)*sqrt(x))/(a^2*b^4*x^2 + 2*a^3*b^3*x
+ a^4*b^2), -1/4*((b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b)/(b*s
qrt(x))) - (a*b^2*x - a^2*b)*sqrt(x))/(a^2*b^4*x^2 + 2*a^3*b^3*x + a^4*b^2)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\operatorname{atanh}^3(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(sqrt(x)/atanh(tanh(a + b*x))**3, x)
```

Giac [A] time = 1.13072, size = 70, normalized size = 0.56

$$\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}ab} + \frac{bx^{\frac{3}{2}} - a\sqrt{x}}{4(bx+a)^2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 1/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/4*(b*x^(3/2) - a*sqrt(x
))/((b*x + a)^2*a*b)
```

3.211 $\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=152

$$\frac{1}{4b^2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{1}{4b^2x^{3/2}\tanh^{-1}(\tanh(a+bx))}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\sqrt{b}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}+\frac{}{4b\sqrt{x}}$$

```
[Out] (-3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(4*Sqrt[
b]*(b*x - ArcTanh[Tanh[a + b*x]])^(5/2)) + 3/(4*b*Sqrt[x]*(b*x - ArcTanh[Ta
nh[a + b*x]])^2) + 1/(4*b^2*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])) - 1/(2*
b*Sqrt[x]*ArcTanh[Tanh[a + b*x]]^2) + 1/(4*b^2*x^(3/2)*ArcTanh[Tanh[a + b*x
]])
```

Rubi [A] time = 0.0974084, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2163, 2162}

$$\frac{1}{4b^2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{1}{4b^2x^{3/2}\tanh^{-1}(\tanh(a+bx))}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\sqrt{b}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}+\frac{}{4b\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^3),x]
```


```
[Out] (-3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(4*Sqrt[
b]*(b*x - ArcTanh[Tanh[a + b*x]])^(5/2)) + 3/(4*b*Sqrt[x]*(b*x - ArcTanh[Ta
nh[a + b*x]])^2) + 1/(4*b^2*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])) - 1/(2*
b*Sqrt[x]*ArcTanh[Tanh[a + b*x]]^2) + 1/(4*b^2*x^(3/2)*ArcTanh[Tanh[a + b*x
]])
```


Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
```

```
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{1}{2b\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2} - \frac{\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))^2}\,dx}{4b}$$

$$= -\frac{1}{2b\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2} + \frac{1}{4b^2x^{3/2}\tanh^{-1}(\tanh(a+bx))} + \frac{3\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh}}{8b^2}$$

$$= \frac{1}{4b^2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{1}{2b\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{4b^2x^{3/2}\tanh}$$

$$= \frac{3}{4b\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{1}{4b^2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{}{2b\sqrt{x}}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\sqrt{b}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}} + \frac{3}{4b\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{}{4b^2}$$

Mathematica [A] time = 0.0785903, size = 118, normalized size = 0.78

$$\frac{\sqrt{x}}{2\tanh^{-1}(\tanh(a+bx))^2\left(\tanh^{-1}(\tanh(a+bx))-bx\right)} + \frac{3\sqrt{x}}{4\tanh^{-1}(\tanh(a+bx))\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2} + \frac{3}{4\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] (3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]])/(4*Sqrt
[b]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2)) + (3*Sqrt[x])/(4*ArcTanh[Tanh[
a + b*x]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2) + Sqrt[x]/(2*ArcTanh[Tanh[a
+ b*x]]^2*(-(b*x) + ArcTanh[Tanh[a + b*x]]))
```

Maple [A] time = 0.132, size = 112, normalized size = 0.7

$$\frac{1}{\left(2\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-2\,bx\right)\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{2}}\sqrt{x}+\frac{3}{4\,\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}\mathrm{Artanh}\left(\tanh\left(b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctanh(tanh(b*x+a))^3/x^(1/2),x)
```

```
[Out] 1/2*x^(1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^2+3/4/(arctanh(
tanh(b*x+a))-b*x)^2*x^(1/2)/arctanh(tanh(b*x+a))+3/4/(arctanh(tanh(b*x+a))-
b*x)^2/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh
(b*x+a))-b*x)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.1019, size = 423, normalized size = 2.78

$$\left[-\frac{3\left(b^{2}x^{2}+2\,abx+a^{2}\right)\sqrt{-ab}\log\left(\frac{bx-a-2\,\sqrt{-ab}\sqrt{x}}{bx+a}\right)-2\left(3\,ab^{2}x+5\,a^{2}b\right)\sqrt{x}}{8\left(a^{3}b^{3}x^{2}+2\,a^{4}b^{2}x+a^{5}b\right)},-\frac{3\left(b^{2}x^{2}+2\,abx+a^{2}\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)-\left(3\right.}{4\left(a^{3}b^{3}x^{2}+2\,a^{4}b^{2}x+a^{5}b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*s
qrt(x))/(b*x + a)) - 2*(3*a*b^2*x + 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 + 2*a^4*
b^2*x + a^5*b), -1/4*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b
)/(b*sqrt(x))) - (3*a*b^2*x + 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 + 2*a^4*b^2*x
+ a^5*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x}\operatorname{atanh}^3(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atanh(tanh(b*x+a))**3/x**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*atanh(tanh(a + b*x))**3), x)
```

Giac [A] time = 1.16266, size = 63, normalized size = 0.41

$$\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}a^2}+\frac{3\,bx^{\frac{3}{2}}+5\,a\sqrt{x}}{4\,(bx+a)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 3/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/4*(3*b*x^(3/2) + 5*a*sq
rt(x))/((b*x + a)^2*a^2)
```

3.212 $\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=176

$$\frac{3}{4b^2x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{3}{4b^2x^{5/2}\tanh^{-1}(\tanh(a+bx))}+\frac{5}{4bx^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}-\frac{}{2bx^{3/2}}$$

```
[Out] (-15*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])
/(4*(b*x - ArcTanh[Tanh[a + b*x]])^(7/2)) + 15/(4*Sqrt[x]*(b*x - ArcTanh[Ta
nh[a + b*x]])^3) + 5/(4*b*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2) + 3/(4*
b^2*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])) - 1/(2*b*x^(3/2)*ArcTanh[Tanh[a
 + b*x]]^2) + 3/(4*b^2*x^(5/2)*ArcTanh[Tanh[a + b*x]])
```

Rubi [A] time = 0.128015, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2163, 2162}

$$\frac{3}{4b^2x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{3}{4b^2x^{5/2}\tanh^{-1}(\tanh(a+bx))}+\frac{5}{4bx^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}-\frac{}{2bx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^3),x]
```


```
[Out] (-15*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])
/(4*(b*x - ArcTanh[Tanh[a + b*x]])^(7/2)) + 15/(4*Sqrt[x]*(b*x - ArcTanh[Ta
nh[a + b*x]])^3) + 5/(4*b*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2) + 3/(4*
b^2*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])) - 1/(2*b*x^(3/2)*ArcTanh[Tanh[a
 + b*x]]^2) + 3/(4*b^2*x^(5/2)*ArcTanh[Tanh[a + b*x]])
```


Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
```

```
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{1}{2bx^{3/2}\tanh^{-1}(\tanh(a+bx))^2} - \frac{3\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))^2}\,dx}{4b}$$

$$= -\frac{1}{2bx^{3/2}\tanh^{-1}(\tanh(a+bx))^2} + \frac{3}{4b^2x^{5/2}\tanh^{-1}(\tanh(a+bx))} + \frac{15\int \frac{1}{x^{7/2}\tanh^{-1}(\mathrm{ta}}{8b^2}$$

$$= \frac{3}{4b^2x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{1}{2bx^{3/2}\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{4b^2x^{5/2}\tan}$$

$$= \frac{5}{4bx^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{3}{4b^2x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{}{2bx}$$

$$= \frac{15}{4\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{5}{4bx^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{}{4b^2x^5}$$

$$= -\frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}} + \frac{15}{4\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{}{4bx}$$

Mathematica [A] time = 0.166529, size = 141, normalized size = 0.8

$$-\frac{b\sqrt{x}}{2\tanh^{-1}(\tanh(a+bx))^2\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}-\frac{7b\sqrt{x}}{4\tanh^{-1}(\tanh(a+bx))\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^3}--$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] (-15*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]
])/(4*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(7/2)) - 2/(Sqrt[x]*(-(b*x) + ArcTa
nh[Tanh[a + b*x]])^3) - (7*b*Sqrt[x])/(4*ArcTanh[Tanh[a + b*x]]*(-(b*x) + A
rcTanh[Tanh[a + b*x]])^3) - (b*Sqrt[x])/(2*ArcTanh[Tanh[a + b*x]]^2*(-(b*x)
 + ArcTanh[Tanh[a + b*x]])^2)
```

Maple [A] time = 0.142, size = 181, normalized size = 1.

$$-\frac{7b^2}{4\left(\operatorname{Artanh}(\tanh(bx+a))-bx\right)^3\left(\operatorname{Artanh}(\tanh(bx+a))\right)^2}x^{\frac{3}{2}}-\frac{9ab}{4\left(\operatorname{Artanh}(\tanh(bx+a))-bx\right)^3\left(\operatorname{Artanh}(\tanh(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/arctanh(tanh(b*x+a))^3,x)
```

```
[Out] -7/4/(arctanh(tanh(b*x+a))-b*x)^3*b^2/arctanh(tanh(b*x+a))^2*x^(3/2)-9/4/(a
rctanh(tanh(b*x+a))-b*x)^3*b/arctanh(tanh(b*x+a))^2*a*x^(1/2)-9/4/(arctanh(
tanh(b*x+a))-b*x)^3*b/arctanh(tanh(b*x+a))^2*x^(1/2)*(arctanh(tanh(b*x+a))-
b*x-a)-15/4/(arctanh(tanh(b*x+a))-b*x)^3*b/((arctanh(tanh(b*x+a))-b*x)*b)^(
1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))-2/(arctanh(tanh
(b*x+a))-b*x)^3/x^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.13912, size = 466, normalized size = 2.65

$$\left[\frac{15\left(b^2x^3+2\,abx^2+a^2x\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\left(15\,b^2x^2+25\,abx+8\,a^2\right)\sqrt{x}}{8\left(a^3b^2x^3+2\,a^4bx^2+a^5x\right)},\frac{15\left(b^2x^3+2\,abx^2+a^2x\right)\sqrt{\frac{b}{a}}\arctan\right.}{4\left(a^3b^2x^3+2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sq
rt(-b/a) - a)/(b*x + a)) - 2*(15*b^2*x^2 + 25*a*b*x + 8*a^2)*sqrt(x))/(a^3*
b^2*x^3 + 2*a^4*b*x^2 + a^5*x), 1/4*(15*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*sqrt(
b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - (15*b^2*x^2 + 25*a*b*x + 8*a^2)*sqrt
(x))/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{3}{2}}\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(1/(x**(3/2)*atanh(tanh(a + b*x))**3), x)
```

Giac [A] time = 1.10973, size = 80, normalized size = 0.45

$$-\frac{15\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^3}-\frac{2}{a^3\sqrt{x}}-\frac{7\,b^2x^{\frac{3}{2}}+9\,ab\sqrt{x}}{4\,(bx+a)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] -15/4*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/(a^3*sqrt(x)) - 1/4
*(7*b^2*x^(3/2) + 9*a*b*sqrt(x))/((b*x + a)^2*a^3)
```

3.213 $\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=201

$$\frac{5}{4b^2x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{5}{4b^2x^{7/2}\tanh^{-1}(\tanh(a+bx))}-\frac{35b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{9/2}}+\frac{}{12x^{3/2}\,(}$$

```
[Out] (-35*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])
/(4*(b*x - ArcTanh[Tanh[a + b*x]])^(9/2)) + (35*b)/(4*Sqrt[x]*(b*x - ArcTan
h[Tanh[a + b*x]])^4) + 35/(12*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^3) + 7
/(4*b*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2) + 5/(4*b^2*x^(7/2)*(b*x - A
rcTanh[Tanh[a + b*x]])) - 1/(2*b*x^(5/2)*ArcTanh[Tanh[a + b*x]]^2) + 5/(4*b
^2*x^(7/2)*ArcTanh[Tanh[a + b*x]])
```

Rubi [A] time = 0.158671, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2163, 2162}

$$\frac{5}{4b^2x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{5}{4b^2x^{7/2}\tanh^{-1}(\tanh(a+bx))}-\frac{35b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{9/2}}+\frac{}{12x^{3/2}\,(}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^3),x]
```


```
[Out] (-35*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])
/(4*(b*x - ArcTanh[Tanh[a + b*x]])^(9/2)) + (35*b)/(4*Sqrt[x]*(b*x - ArcTan
h[Tanh[a + b*x]])^4) + 35/(12*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^3) + 7
/(4*b*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2) + 5/(4*b^2*x^(7/2)*(b*x - A
rcTanh[Tanh[a + b*x]])) - 1/(2*b*x^(5/2)*ArcTanh[Tanh[a + b*x]]^2) + 5/(4*b
^2*x^(7/2)*ArcTanh[Tanh[a + b*x]])
```


Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
```

```
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{1}{2bx^{5/2}\tanh^{-1}(\tanh(a+bx))^2} - \frac{5\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))^2}\,dx}{4b}$$

$$= -\frac{1}{2bx^{5/2}\tanh^{-1}(\tanh(a+bx))^2} + \frac{5}{4b^2x^{7/2}\tanh^{-1}(\tanh(a+bx))} + \frac{35\int \frac{1}{x^{9/2}\tanh^{-1}(\mathrm{ta}}{8b^2}$$

$$= \frac{5}{4b^2x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{1}{2bx^{5/2}\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{4b^2x^{7/2}\tan}$$

$$= \frac{7}{4bx^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{5}{4b^2x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{}{2bx}$$

$$= \frac{35}{12x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{7}{4bx^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{}{4b^2}$$

$$= \frac{35b}{4\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4} + \frac{35}{12x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{}{4bx^{5/}}$$

$$= -\frac{35b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{9/2}} + \frac{35b}{4\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4} + \frac{}{12x}$$

Mathematica [A] time = 0.255728, size = 156, normalized size = 0.78

$$\frac{1}{12}\left(\frac{6b^2\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^2\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^3} + \frac{33b^2\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^4} + \frac{10}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] ((105*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]
]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(9/2) + (80*b*x - 8*ArcTanh[Tanh[a +
b*x]])/(x^(3/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4) + (33*b^2*Sqrt[x])/(Ar
cTanh[Tanh[a + b*x]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4) + (6*b^2*Sqrt[x])
/(ArcTanh[Tanh[a + b*x]]^2*(-(b*x) + ArcTanh[Tanh[a + b*x]])^3))/12
```

Maple [A] time = 0.142, size = 207, normalized size = 1.

$$\frac{11\,b^3}{4\left(\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^4\left(\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^2}x^{\frac{3}{2}}+\frac{13\,ab^2}{4\left(\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^4\left(\operatorname{Artanh}\left(\tanh\left(b\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/arctanh(tanh(b*x+a))^3,x)
```

```
[Out] 11/4/(arctanh(tanh(b*x+a))-b*x)^4*b^3/arctanh(tanh(b*x+a))^2*x^(3/2)+13/4/(
arctanh(tanh(b*x+a))-b*x)^4*b^2/arctanh(tanh(b*x+a))^2*a*x^(1/2)+13/4/(arct
anh(tanh(b*x+a))-b*x)^4*b^2/arctanh(tanh(b*x+a))^2*x^(1/2)*(arctanh(tanh(b*
x+a))-b*x-a)+35/4/(arctanh(tanh(b*x+a))-b*x)^4*b^2/((arctanh(tanh(b*x+a))-b
*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))-2/3/(ar
ctanh(tanh(b*x+a))-b*x)^3/x^(3/2)+6/(arctanh(tanh(b*x+a))-b*x)^4*b/x^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.12883, size = 545, normalized size = 2.71

$$\left[\frac{105\left(b^3x^4+2\,ab^2x^3+a^2bx^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\left(105\,b^3x^3+175\,ab^2x^2+56\,a^2bx-8\,a^3\right)\sqrt{x}}{24\left(a^4b^2x^4+2\,a^5bx^3+a^6x^2\right)},-\frac{105\left(b^3x^4+2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] [1/24*(105*(b^3*x^4 + 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(-b/a)*log((b*x + 2*a*sq
rt(x)*sqrt(-b/a) - a)/(b*x + a)) + 2*(105*b^3*x^3 + 175*a*b^2*x^2 + 56*a^2*
b*x - 8*a^3)*sqrt(x))/(a^4*b^2*x^4 + 2*a^5*b*x^3 + a^6*x^2), -1/12*(105*(b^
3*x^4 + 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x)))
- (105*b^3*x^3 + 175*a*b^2*x^2 + 56*a^2*b*x - 8*a^3)*sqrt(x))/(a^4*b^2*x^4
+ 2*a^5*b*x^3 + a^6*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15018, size = 96, normalized size = 0.48

$$\frac{35\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^4}+\frac{2\,(9\,bx-a)}{3\,a^4x^{\frac{3}{2}}}+\frac{11\,b^3x^{\frac{3}{2}}+13\,ab^2\sqrt{x}}{4\,(bx+a)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 35/4*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4) + 2/3*(9*b*x - a)/(a^4
*x^(3/2)) + 1/4*(11*b^3*x^(3/2) + 13*a*b^2*sqrt(x))/((b*x + a)^2*a^4)
```

3.214 $\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=228

$$\frac{7}{4b^2x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{7}{4b^2x^{9/2}\tanh^{-1}(\tanh(a+bx))}-\frac{63b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{11/2}}+\frac{\cdots}{4\sqrt{x}\,(\cdots}$$

```
[Out] (-63*b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])
/(4*(b*x - ArcTanh[Tanh[a + b*x]])^(11/2)) + (63*b^2)/(4*Sqrt[x]*(b*x - Arc
Tanh[Tanh[a + b*x]])^5) + (21*b)/(4*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^
4) + 63/(20*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])^3) + 9/(4*b*x^(7/2)*(b*x
 - ArcTanh[Tanh[a + b*x]])^2) + 7/(4*b^2*x^(9/2)*(b*x - ArcTanh[Tanh[a + b*
x]])) - 1/(2*b*x^(7/2)*ArcTanh[Tanh[a + b*x]]^2) + 7/(4*b^2*x^(9/2)*ArcTanh
[Tanh[a + b*x]])
```

Rubi [A] time = 0.20206, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2168, 2163, 2162}

$$\frac{7}{4b^2x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{7}{4b^2x^{9/2}\tanh^{-1}(\tanh(a+bx))}-\frac{63b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{11/2}}+\frac{\cdots}{4\sqrt{x}\,(\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] (-63*b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])
/(4*(b*x - ArcTanh[Tanh[a + b*x]])^(11/2)) + (63*b^2)/(4*Sqrt[x]*(b*x - Arc
Tanh[Tanh[a + b*x]])^5) + (21*b)/(4*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^
4) + 63/(20*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])^3) + 9/(4*b*x^(7/2)*(b*x
 - ArcTanh[Tanh[a + b*x]])^2) + 7/(4*b^2*x^(9/2)*(b*x - ArcTanh[Tanh[a + b*
x]])) - 1/(2*b*x^(7/2)*ArcTanh[Tanh[a + b*x]]^2) + 7/(4*b^2*x^(9/2)*ArcTanh
[Tanh[a + b*x]])
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
```

```
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{1}{2bx^{7/2}\tanh^{-1}(\tanh(a+bx))^2} - \frac{7\int \frac{1}{x^{9/2}\tanh^{-1}(\tanh(a+bx))^2}\,dx}{4b}$$

$$= -\frac{1}{2bx^{7/2}\tanh^{-1}(\tanh(a+bx))^2} + \frac{7}{4b^2x^{9/2}\tanh^{-1}(\tanh(a+bx))} + \frac{63\int \frac{1}{x^{11/2}\tanh^{-1}}}{8b}$$

$$= \frac{7}{4b^2x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{1}{2bx^{7/2}\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{4b^2x^{9/2}\,\mathrm{ta}}$$

$$= \frac{9}{4bx^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{7}{4b^2x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{}{2b}$$

$$= \frac{63}{20x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{9}{4bx^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{}{4b}$$

$$= \frac{21b}{4x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4} + \frac{63}{20x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{}{4bx}$$

$$= \frac{63b^2}{4\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^5} + \frac{21b}{4x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4} + \frac{}{20x^{5}}$$

$$= -\frac{63b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{11/2}} + \frac{63b^2}{4\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^5} + \frac{}{4x}$$

Mathematica [A] time = 0.327386, size = 174, normalized size = 0.76

$$\frac{1}{20}\left(\frac{8\left(-7bx\tanh^{-1}(\tanh(a+bx))+\tanh^{-1}(\tanh(a+bx))^2+36b^2x^2\right)}{x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^5} + \frac{75b^3\sqrt{x}}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^5\tanh^{-1}(\tanh}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] ((75*b^3*Sqrt[x])/((b*x - ArcTanh[Tanh[a + b*x]])^5*ArcTanh[Tanh[a + b*x]])
 - (315*b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x
```

```
]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(11/2) - (10*b^3*Sqrt[x])/(ArcTanh[
Tanh[a + b*x]]^2*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4) + (8*(36*b^2*x^2 - 7*
b*x*ArcTanh[Tanh[a + b*x]] + ArcTanh[Tanh[a + b*x]]^2))/(x^(5/2)*(b*x - Arc
Tanh[Tanh[a + b*x]])^5))/20
```

Maple [A] time = 0.145, size = 229, normalized size = 1.

$$-\frac{2}{5\left(\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{3}}x^{-\frac{5}{2}}-12\,\frac{b^{2}}{\left(\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{5}\sqrt{x}}+2\,\frac{b}{\left(\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{4}x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/arctanh(tanh(b*x+a))^3,x)
```

```
[Out] -2/5/(arctanh(tanh(b*x+a))-b*x)^3/x^(5/2)-12/(arctanh(tanh(b*x+a))-b*x)^5*b
^2/x^(1/2)+2/(arctanh(tanh(b*x+a))-b*x)^4*b/x^(3/2)-15/4/(arctanh(tanh(b*x+
a))-b*x)^5*b^4/arctanh(tanh(b*x+a))^2*x^(3/2)-17/4/(arctanh(tanh(b*x+a))-b*
x)^5*b^3/arctanh(tanh(b*x+a))^2*a*x^(1/2)-17/4/(arctanh(tanh(b*x+a))-b*x)^5
*b^3/arctanh(tanh(b*x+a))^2*x^(1/2)*(arctanh(tanh(b*x+a))-b*x-a)-63/4/(arct
anh(tanh(b*x+a))-b*x)^5*b^3/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x
^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.16307, size = 598, normalized size = 2.62

$$\left[\frac{315\left(b^{4}x^{5}+2\,ab^{3}x^{4}+a^{2}b^{2}x^{3}\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\left(315\,b^{4}x^{4}+525\,ab^{3}x^{3}+168\,a^{2}b^{2}x^{2}-24\,a^{3}bx+8\,a^{4}\right)\sqrt{x}}{40\left(a^{5}b^{2}x^{5}+2\,a^{6}bx^{4}+a^{7}x^{3}\right)},\ 31\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] [1/40*(315*(b^4*x^5 + 2*a*b^3*x^4 + a^2*b^2*x^3)*sqrt(-b/a)*log((b*x - 2*a*
sqrt(x)*sqrt(-b/a) - a)/(b*x + a)) - 2*(315*b^4*x^4 + 525*a*b^3*x^3 + 168*a
^2*b^2*x^2 - 24*a^3*b*x + 8*a^4)*sqrt(x))/(a^5*b^2*x^5 + 2*a^6*b*x^4 + a^7*
x^3), 1/20*(315*(b^4*x^5 + 2*a*b^3*x^4 + a^2*b^2*x^3)*sqrt(b/a)*arctan(a*sq
rt(b/a)/(b*sqrt(x))) - (315*b^4*x^4 + 525*a*b^3*x^3 + 168*a^2*b^2*x^2 - 24*
a^3*b*x + 8*a^4)*sqrt(x))/(a^5*b^2*x^5 + 2*a^6*b*x^4 + a^7*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14484, size = 108, normalized size = 0.47

$$-\frac{63\,b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^5}-\frac{15\,b^4x^{\frac{3}{2}}+17\,ab^3\sqrt{x}}{4\,(bx+a)^2a^5}-\frac{2\left(30\,b^2x^2-5\,abx+a^2\right)}{5\,a^5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] -63/4*b^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^5) - 1/4*(15*b^4*x^(3/2)
 + 17*a*b^3*sqrt(x))/((b*x + a)^2*a^5) - 2/5*(30*b^2*x^2 - 5*a*b*x + a^2)/(
a^5*x^(5/2))
```

3.215 $\int x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=142

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8b^{5/2}}-\frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{8b^2}$$

```
[Out] -(ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[Ta
nh[a + b*x]])^3)/(8*b^(5/2)) + (x^(5/2)*Sqrt[ArcTanh[Tanh[a + b*x]]])/3 - (
x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/(12*b)
 - (Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/
(8*b^2)
```

Rubi [A] time = 0.0775207, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2169, 2165}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8b^{5/2}}-\frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```


```
[Out] -(ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[Ta
nh[a + b*x]])^3)/(8*b^(5/2)) + (x^(5/2)*Sqrt[ArcTanh[Tanh[a + b*x]]])/3 - (
x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/(12*b)
 - (Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/
(8*b^2)
```


Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\int x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{1}{3}x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{1}{6}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\int \frac{x^{3/2}}{\sqrt{\tanh^{-1}(\tan}$$

$$= \frac{1}{3}x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh}}{12b}$$

$$= \frac{1}{3}x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh}}{12b}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8b^{5/2}} + \frac{1}{3}x^{5/2}\sqrt{\tanh^{-1}(\tan}$$

Mathematica [A] time = 0.0855375, size = 104, normalized size = 0.73

$$\frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(8bx\tanh^{-1}(\tanh(a+bx))-3\tanh^{-1}(\tanh(a+bx))^2+3b^2x^2\right)}{24b^2} + \frac{\left(\tanh^{-1}(\tanh(a+bx))\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(3*b^2*x^2 + 8*b*x*ArcTanh[Tanh[a + b
*x]] - 3*ArcTanh[Tanh[a + b*x]]^2))/(24*b^2) + ((-(b*x) + ArcTanh[Tanh[a +
b*x]])^3*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/(8*b^(5/2))
```

Maple [B] time = 0.167, size = 304, normalized size = 2.1

$$\frac{1}{3b}x^{\frac{3}{2}}\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{\frac{3}{2}} - \frac{a}{4b^2}\sqrt{x}\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{\frac{3}{2}} + \frac{a^2}{8b^2}\sqrt{x}\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)} + \frac{a^3}{8}\ln\left(\sqrt{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 1/3*x^(3/2)*arctanh(tanh(b*x+a))^(3/2)/b-1/4/b^2*a*x^(1/2)*arctanh(tanh(b*x
+a))^(3/2)+1/8/b^2*a^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+1/8/b^(5/2)*ln(b^
(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^3+3/8/b^(5/2)*a^2*ln(b^(1/2)*x^
(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)+1/4/b^2*a*(a
rctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+3/8/b^(5/2)*a
*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a
)^2-1/4/b^2*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)
+1/8/b^2*(arctanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+
1/8/b^(5/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*
x+a))-b*x-a)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^{\frac{3}{2}}\sqrt{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)*sqrt(arctanh(tanh(b*x + a))), x)
```

Fricas [A] time = 2.12963, size = 362, normalized size = 2.55

$$\left[\frac{3\,a^3\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(8\,b^3x^2+2\,ab^2x-3\,a^2b\right)\sqrt{bx+a}\sqrt{x}}{48\,b^3},-\frac{3\,a^3\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)-\left(8\,b^3x\right.}{24\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*a^3*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(
8*b^3*x^2 + 2*a*b^2*x - 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^3, -1/24*(3*a^3*s
qrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (8*b^3*x^2 + 2*a*b^2*x
```

```
- 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{\frac{3}{2}}\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(x**(3/2)*sqrt(atanh(tanh(a + b*x))), x)
```

Giac [A] time = 1.16849, size = 81, normalized size = 0.57

$$\frac{1}{24}\sqrt{bx+a}\left(2\left(4x+\frac{a}{b}\right)x-\frac{3a^2}{b^2}\right)\sqrt{x}-\frac{a^3\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{8b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(b*x + a)*(2*(4*x + a/b)*x - 3*a^2/b^2)*sqrt(x) - 1/8*a^3*log(abs(
-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(5/2)
```

3.216 $\int \sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=104

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}-\frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+b}{}$$

```
[Out] -(ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[Ta
nh[a + b*x]])^2)/(4*b^(3/2)) + (x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*x]]])/2 - (
Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/(4*b)
```

Rubi [A] time = 0.0506659, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2169, 2165}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}-\frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[Ta
nh[a + b*x]])^2)/(4*b^(3/2)) + (x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*x]]])/2 - (
Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/(4*b)
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
```

```
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\int \sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{1}{2}x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{1}{4}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\int \frac{\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh}}$$

$$= \frac{1}{2}x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(}}{4b}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4b^{3/2}} + \frac{1}{2}x^{3/2}\sqrt{\tanh^{-1}(\tan}$$

Mathematica [A] time = 0.0688331, size = 84, normalized size = 0.81

$$\frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(\tanh^{-1}(\tanh(a+bx))+bx\right)}{4b} - \frac{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+}\right.}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(b*x + ArcTanh[Tanh[a + b*x]]))/(4*b)
 - ((-(b*x) + ArcTanh[Tanh[a + b*x]])^2*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTan
h[Tanh[a + b*x]]]])/(4*b^(3/2))
```

Maple [B] time = 0.141, size = 174, normalized size = 1.7

$$\frac{1}{2b}\sqrt{x}\left(\text{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{\frac{3}{2}} - \frac{a}{4b}\sqrt{x}\sqrt{\text{Artanh}\left(\tanh\left(bx+a\right)\right)} - \frac{a^2}{4}\ln\left(\sqrt{b}\sqrt{x}+\sqrt{\text{Artanh}\left(\tanh\left(bx+a\right)\right)}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 1/2*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)/b-1/4/b*a*x^(1/2)*arctanh(tanh(b*x+a
))^(1/2)-1/4/b^(3/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^2-1/2
```

```
/b^(3/2)*a*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x
+a))-b*x-a)-1/4/b*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))
^(1/2)-1/4/b^(3/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(
tanh(b*x+a))-b*x-a)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x}\sqrt{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)*sqrt(arctanh(tanh(b*x + a))), x)
```

Fricas [A] time = 2.07159, size = 304, normalized size = 2.92

$$\left[\frac{a^2\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(2\,b^2x+ab\right)\sqrt{bx+a}\sqrt{x}}{8\,b^2},\ \frac{a^2\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+\left(2\,b^2x+ab\right)\sqrt{bx+a}\sqrt{x}}{4\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(a^2*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(2*b
^2*x + a*b)*sqrt(b*x + a)*sqrt(x))/b^2, 1/4*(a^2*sqrt(-b)*arctan(sqrt(b*x +
 a)*sqrt(-b)/(b*sqrt(x))) + (2*b^2*x + a*b)*sqrt(b*x + a)*sqrt(x))/b^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x}\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(x)*sqrt(atanh(tanh(a + b*x))), x)
```

Giac [A] time = 1.15485, size = 65, normalized size = 0.62

$$\frac{1}{4}\sqrt{bx+a}\left(2x+\frac{a}{b}\right)\sqrt{x}+\frac{a^2\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{4b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b*x + a)*(2*x + a/b)*sqrt(x) + 1/4*a^2*log(abs(-sqrt(b)*sqrt(x) +
sqrt(b*x + a)))/b^(3/2)
```

3.217 $\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}\,dx$

Optimal. Leaf size=61

$$\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{\sqrt{b}}$$

```
[Out] -((ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]]))/Sqrt[b]) + Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]
```

Rubi [A] time = 0.0293383, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2169, 2165}

$$\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[x],x]
```

```
[Out] -((ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]]))/Sqrt[b]) + Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
```

```
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}\,dx = \sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{1}{2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\int \frac{1}{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{\sqrt{b}} + \sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+$$

Mathematica [A] time = 0.0393929, size = 62, normalized size = 1.02

$$\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))} + \frac{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}+b\sqrt{x}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]] + ((-(b*x) + ArcTanh[Tanh[a + b*x]])*L
og[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/Sqrt[b]
```

Maple [A] time = 0.069, size = 75, normalized size = 1.2

$$\sqrt{x}\sqrt{\mathrm{Artanh}\,(\tanh\,(bx+a))} + a\ln\left(\sqrt{b}\sqrt{x}+\sqrt{\mathrm{Artanh}\,(\tanh\,(bx+a))}\right)\frac{1}{\sqrt{b}} + (\mathrm{Artanh}\,(\tanh\,(bx+a))-bx-a)\ln\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+1/b^(1/2)*ln(b^(1/2)*x^(1/2)+arctanh(tan
h(b*x+a))^(1/2))*a+1/b^(1/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))
*(arctanh(tanh(b*x+a))-b*x-a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arctanh(tanh(b*x + a)))/sqrt(x), x)
```

Fricas [A] time = 2.15892, size = 251, normalized size = 4.11

$$\left[\frac{a\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,\sqrt{bx+a}b\sqrt{x}}{2\,b}, -\frac{a\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)-\sqrt{bx+a}b\sqrt{x}}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*sqrt(b
*x + a)*b*sqrt(x))/b, -(a*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x)
)) - sqrt(b*x + a)*b*sqrt(x))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2)/x**(1/2),x)
```

```
[Out] Integral(sqrt(atanh(tanh(a + b*x)))/sqrt(x), x)
```

Giac [A] time = 1.14933, size = 49, normalized size = 0.8

$$-\frac{a\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{\sqrt{b}}+\sqrt{bx+a}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] -a*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/sqrt(b) + sqrt(b*x + a)*sqrt(
x)
```

3.218 $\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{3/2}} \, dx$

Optimal. Leaf size=49

$$2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)-\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}$$

```
[Out] 2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]] - (2*Sqrt
[ArcTanh[Tanh[a + b*x]]])/Sqrt[x]
```

Rubi [A] time = 0.0285149, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2168, 2165}

$$2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)-\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(3/2),x]
```

```
[Out] 2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]] - (2*Sqrt
[ArcTanh[Tanh[a + b*x]]])/Sqrt[x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
```

```
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{3/2}}\,dx = -\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}} + b\int \frac{1}{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$$
$$= 2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right) - \frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}$$

Mathematica [A] time = 0.0381369, size = 52, normalized size = 1.06

$$2\sqrt{b}\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))} + b\sqrt{x}\right) - \frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[ArcTanh[Tanh[a + b*x]]])/Sqrt[x] + 2*Sqrt[b]*Log[b*Sqrt[x] + Sqrt[
b]*Sqrt[ArcTanh[Tanh[a + b*x]]]]
```

Maple [B] time = 0.144, size = 149, normalized size = 3.

$$-2\frac{(\mathrm{Artanh}(\tanh(bx+a)))^{3/2}}{(\mathrm{Artanh}(\tanh(bx+a)) - bx)\sqrt{x}} + 2\frac{b\sqrt{x}\sqrt{\mathrm{Artanh}(\tanh(bx+a))}}{\mathrm{Artanh}(\tanh(bx+a)) - bx} + 2\frac{\sqrt{b}\ln\left(\sqrt{b}\sqrt{x} + \sqrt{\mathrm{Artanh}(\tanh(bx+a)}\right)}{\mathrm{Artanh}(\tanh(bx+a)) - bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2)/x^(3/2),x)
```

```
[Out] -2/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+2*b/(arcta
nh(tanh(b*x+a))-b*x)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+2*b^(1/2)/(arctanh(
tanh(b*x+a))-b*x)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a+2*b^(1/2
)/(arctanh(tanh(b*x+a))-b*x)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))
```

```
*(arctanh(tanh(b*x+a))-b*x-a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.06312, size = 243, normalized size = 4.96

$$\left[\frac{\sqrt{b}x\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,\sqrt{bx+a}\sqrt{x}}{x},-\frac{2\left(\sqrt{-b}x\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+a}\sqrt{x}\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*sqrt(b*x +
 a)*sqrt(x))/x, -2*(sqrt(-b)*x*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) +
 sqrt(b*x + a)*sqrt(x))/x]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2)/x**(3/2),x)
```

```
[Out] Integral(sqrt(atanh(tanh(a + b*x)))/x**(3/2), x)
```

Giac [A] time = 1.16806, size = 77, normalized size = 1.57

$$-\sqrt{b}\log\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2\right)+\frac{4\,a\sqrt{b}}{\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(b)*log((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2) + 4*a*sqrt(b)/((sqrt(b)*s
qrt(x) - sqrt(b*x + a))^2 - a)
```

3.219 $\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{5/2}}\,dx$

Optimal. Leaf size=35

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rubi [A] time = 0.0141844, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2167}

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(5/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{5/2}}\,dx=\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A] time = 0.0311113, size = 34, normalized size = 0.97

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{3/2}\left(3bx-3\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(5/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(x^(3/2)*(3*b*x - 3*ArcTanh[Tanh[a + b*x]]
))
```

Maple [A] time = 0.138, size = 29, normalized size = 0.8

$$-\frac{2}{3\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-3\,bx}\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{\frac{3}{2}}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2)/x^(5/2),x)
```

```
[Out] -2/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)*arctanh(tanh(b*x+a))^(3/2)
```

Maxima [A] time = 1.48269, size = 20, normalized size = 0.57

$$-\frac{2\left(bx+a\right)^{\frac{3}{2}}}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(b*x + a)^(3/2)/(a*x^(3/2))
```

Fricas [A] time = 1.99663, size = 46, normalized size = 1.31

$$-\frac{2\left(bx+a\right)^{\frac{3}{2}}}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(b*x + a)^(3/2)/(a*x^(3/2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2)/x**(5/2),x)
```

```
[Out] Integral(sqrt(atanh(tanh(a + b*x)))/x**(5/2), x)
```

Giac [B] time = 1.14085, size = 80, normalized size = 2.29

$$\frac{4\left(3\,b^{\frac{3}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{4}+a^{2}b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{2}-a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] 4/3*(3*b^(3/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^4 + a^2*b^(3/2))/((sqrt(b)
*sqrt(x) - sqrt(b*x + a))^2 - a)^3
```

3.220 $\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{7/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{4b\tanh^{-1}(\tanh(a+bx))^{3/2}}{15x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

```
[Out] (4*b*ArcTanh[Tanh[a + b*x]]^(3/2))/(15*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]
])^2) + (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(5*x^(5/2)*(b*x - ArcTanh[Tanh[a +
 b*x]]))
```

Rubi [A] time = 0.033482, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2171, 2167}

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{4b\tanh^{-1}(\tanh(a+bx))^{3/2}}{15x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(7/2),x]
```

```
[Out] (4*b*ArcTanh[Tanh[a + b*x]]^(3/2))/(15*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]
])^2) + (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(5*x^(5/2)*(b*x - ArcTanh[Tanh[a +
 b*x]]))
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
```

```
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{7/2}}\,dx = \frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{(2b)\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{5/2}}\,dx}{5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$
$$= \frac{4b\tanh^{-1}(\tanh(a+bx))^{3/2}}{15x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A] time = 0.0343497, size = 48, normalized size = 0.67

$$\frac{2\left(5bx-3\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}{15x^{5/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(7/2),x]
```

```
[Out] (2*(5*b*x - 3*ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2))/(15*x^(
5/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

Maple [A] time = 0.154, size = 59, normalized size = 0.8

$$-\frac{2}{5\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-5\,bx}\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{\frac{3}{2}}x^{-\frac{5}{2}}+\frac{4\,b}{15\,\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}}\left(\mathrm{Artanh}\left(\tanh\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2)/x^(7/2),x)
```

```
[Out] -2/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)*arctanh(tanh(b*x+a))^(3/2)+4/15*b/(
arctanh(tanh(b*x+a))-b*x)^2/x^(3/2)*arctanh(tanh(b*x+a))^(3/2)
```

Maxima [A] time = 1.48247, size = 46, normalized size = 0.64

$$\frac{2\left(2\,b^{2}x^{2}-abx-3\,a^{2}\right)\sqrt{bx+a}}{15\,a^{2}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*(2*b^2*x^2 - a*b*x - 3*a^2)*sqrt(b*x + a)/(a^2*x^(5/2))
```

Fricas [A] time = 2.15894, size = 84, normalized size = 1.17

$$\frac{2\left(2\,b^{2}x^{2}-abx-3\,a^{2}\right)\sqrt{bx+a}}{15\,a^{2}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(2*b^2*x^2 - a*b*x - 3*a^2)*sqrt(b*x + a)/(a^2*x^(5/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2)/x**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1564, size = 151, normalized size = 2.1

$$\frac{8\left(15\,b^{\frac{5}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{6}+5\,ab^{\frac{5}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{4}+5\,a^{2}b^{\frac{5}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{2}-a^{3}b^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{2}-a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] 8/15*(15*b^(5/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^6 + 5*a*b^(5/2)*(sqrt(b)
*sqrt(x) - sqrt(b*x + a))^4 + 5*a^2*b^(5/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a
))^2 - a^3*b^(5/2))/((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^5
```

3.221 $\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{9/2}}\,dx$

Optimal. Leaf size=110

$$\frac{16b^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{105x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{8b\tanh^{-1}(\tanh(a+bx))^{3/2}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] (16*b^2*ArcTanh[Tanh[a + b*x]]^(3/2))/(105*x^(3/2)*(b*x - ArcTanh[Tanh[a +
b*x]])^3) + (8*b*ArcTanh[Tanh[a + b*x]]^(3/2))/(35*x^(5/2)*(b*x - ArcTanh[T
anh[a + b*x]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(7*x^(7/2)*(b*x - ArcT
anh[Tanh[a + b*x]]))
```

Rubi [A] time = 0.0543659, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2171, 2167}

$$\frac{16b^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{105x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{8b\tanh^{-1}(\tanh(a+bx))^{3/2}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(9/2),x]
```

```
[Out] (16*b^2*ArcTanh[Tanh[a + b*x]]^(3/2))/(105*x^(3/2)*(b*x - ArcTanh[Tanh[a +
b*x]])^3) + (8*b*ArcTanh[Tanh[a + b*x]]^(3/2))/(35*x^(5/2)*(b*x - ArcTanh[T
anh[a + b*x]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(7*x^(7/2)*(b*x - ArcT
anh[Tanh[a + b*x]]))
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
```

```
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{9/2}}\,dx = \frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(4b)\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{7/2}}\,dx}{7\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

$$= \frac{8b\tanh^{-1}(\tanh(a+bx))^{3/2}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{\left(8b^2\right)\int}{35\left(bx-\right.}$$

$$= \frac{16b^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{105x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{8b\tanh^{-1}(\tanh(a+bx))^{3/2}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2}{7x^{7/2}}$$

Mathematica [A] time = 0.056038, size = 66, normalized size = 0.6

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}\left(-42bx\tanh^{-1}(\tanh(a+bx))+15\tanh^{-1}(\tanh(a+bx))^2+35b^2x^2\right)}{105x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(9/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(3/2)*(35*b^2*x^2 - 42*b*x*ArcTanh[Tanh[a + b*x]]
 + 15*ArcTanh[Tanh[a + b*x]]^2))/(105*x^(7/2)*(b*x - ArcTanh[Tanh[a + b*x]]
)^3)
```

Maple [A] time = 0.143, size = 105, normalized size = 1.

$$-\frac{2}{7\,\mathrm{Artanh}\,(\tanh\,(bx+a))-7\,bx}\,(\mathrm{Artanh}\,(\tanh\,(bx+a)))^{\frac{3}{2}}\,x^{-\frac{7}{2}} - \frac{8\,b}{7\,\mathrm{Artanh}\,(\tanh\,(bx+a))-7\,bx}\left(-\frac{1}{5\,\mathrm{Artanh}\,(\tanh\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2)/x^(9/2),x)
```

```
[Out] -2/7/(arctanh(tanh(b*x+a))-b*x)/x^(7/2)*arctanh(tanh(b*x+a))^(3/2)-8/7*b/(a
rctanh(tanh(b*x+a))-b*x)*(-1/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)*arctanh(t
anh(b*x+a))^(3/2)+2/15*b/(arctanh(tanh(b*x+a))-b*x)^2/x^(3/2)*arctanh(tanh(
b*x+a))^(3/2))
```

Maxima [A] time = 1.4809, size = 61, normalized size = 0.55

$$-\frac{2\left(8\,b^3x^3-4\,ab^2x^2+3\,a^2bx+15\,a^3\right)\sqrt{bx+a}}{105\,a^3x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] -2/105*(8*b^3*x^3 - 4*a*b^2*x^2 + 3*a^2*b*x + 15*a^3)*sqrt(b*x + a)/(a^3*x^
(7/2))
```

Fricas [A] time = 2.25493, size = 112, normalized size = 1.02

$$-\frac{2\left(8\,b^3x^3-4\,ab^2x^2+3\,a^2bx+15\,a^3\right)\sqrt{bx+a}}{105\,a^3x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] -2/105*(8*b^3*x^3 - 4*a*b^2*x^2 + 3*a^2*b*x + 15*a^3)*sqrt(b*x + a)/(a^3*x^
(7/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2)/x**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18521, size = 186, normalized size = 1.69

$$\frac{32\left(70\,b^{\frac{7}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{8}+35\,ab^{\frac{7}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{6}+21\,a^{2}b^{\frac{7}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{4}-7\,a^{3}b^{\frac{7}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{2}+\right.}{105\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{2}-a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] 32/105*(70*b^(7/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^8 + 35*a*b^(7/2)*(sqrt
(b)*sqrt(x) - sqrt(b*x + a))^6 + 21*a^2*b^(7/2)*(sqrt(b)*sqrt(x) - sqrt(b*x
 + a))^4 - 7*a^3*b^(7/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 + a^4*b^(7/2))
/((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^7
```

3.222 $\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{11/2}}\,dx$

Optimal. Leaf size=148

$$\frac{32b^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{315x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{16b^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{105x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{4b\tanh^{-1}(\tanh(a+bx))^{3/2}}{21x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

```
[Out] (32*b^3*ArcTanh[Tanh[a + b*x]]^(3/2))/(315*x^(3/2)*(b*x - ArcTanh[Tanh[a +
b*x]])^4) + (16*b^2*ArcTanh[Tanh[a + b*x]]^(3/2))/(105*x^(5/2)*(b*x - ArcTa
nh[Tanh[a + b*x]])^3) + (4*b*ArcTanh[Tanh[a + b*x]]^(3/2))/(21*x^(7/2)*(b*x
 - ArcTanh[Tanh[a + b*x]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(9*x^(9/2)
*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rubi [A] time = 0.0778354, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2171, 2167}

$$\frac{32b^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{315x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{16b^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{105x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{4b\tanh^{-1}(\tanh(a+bx))^{3/2}}{21x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(11/2),x]
```

```
[Out] (32*b^3*ArcTanh[Tanh[a + b*x]]^(3/2))/(315*x^(3/2)*(b*x - ArcTanh[Tanh[a +
b*x]])^4) + (16*b^2*ArcTanh[Tanh[a + b*x]]^(3/2))/(105*x^(5/2)*(b*x - ArcTa
nh[Tanh[a + b*x]])^3) + (4*b*ArcTanh[Tanh[a + b*x]]^(3/2))/(21*x^(7/2)*(b*x
 - ArcTanh[Tanh[a + b*x]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(9*x^(9/2)
*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{11/2}}\,dx = \frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{9x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(2b)\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{9/2}}\,dx}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

$$= \frac{4b\tanh^{-1}(\tanh(a+bx))^{3/2}}{21x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{9x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{\left(8b^2\right)\int}{21\left(bx-\right.}$$

$$= \frac{16b^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{105x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{4b\tanh^{-1}(\tanh(a+bx))^{3/2}}{21x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2}{9x^{9/2}}$$

$$= \frac{32b^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{315x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4} + \frac{16b^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{105x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{}{21x^7}$$

Mathematica [A] time = 0.0608676, size = 82, normalized size = 0.55

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}\left(-189b^2x^2\tanh^{-1}(\tanh(a+bx)) + 135bx\tanh^{-1}(\tanh(a+bx))^2 - 35\tanh^{-1}(\tanh(a+bx))^3\right.}{315x^{9/2}\left(\tanh^{-1}(\tanh(a+bx)) - bx\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(11/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(3/2)*(105*b^3*x^3 - 189*b^2*x^2*ArcTanh[Tanh[a +
 b*x]] + 135*b*x*ArcTanh[Tanh[a + b*x]]^2 - 35*ArcTanh[Tanh[a + b*x]]^3))/(
315*x^(9/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4)
```

Maple [A] time = 0.158, size = 151, normalized size = 1.

$$-\frac{2}{9\,\mathrm{Artanh}(\tanh(bx+a)) - 9bx}(\mathrm{Artanh}(\tanh(bx+a)))^{\frac{3}{2}}x^{-\frac{9}{2}} - \frac{4b}{3\,\mathrm{Artanh}(\tanh(bx+a)) - 3bx}\left(-\frac{1}{7\,\mathrm{Artanh}(\tanh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2)/x^(11/2),x)
```

```
[Out] -2/9/(arctanh(tanh(b*x+a))-b*x)/x^(9/2)*arctanh(tanh(b*x+a))^(3/2)-4/3*b/(a
rctanh(tanh(b*x+a))-b*x)*(-1/7/(arctanh(tanh(b*x+a))-b*x)/x^(7/2)*arctanh(t
anh(b*x+a))^(3/2)-4/7*b/(arctanh(tanh(b*x+a))-b*x)*(-1/5/(arctanh(tanh(b*x+
a))-b*x)/x^(5/2)*arctanh(tanh(b*x+a))^(3/2)+2/15*b/(arctanh(tanh(b*x+a))-b*
x)^2/x^(3/2)*arctanh(tanh(b*x+a))^(3/2)))
```

Maxima [A] time = 1.4756, size = 76, normalized size = 0.51

$$\frac{2\left(16\,b^4x^4 - 8\,ab^3x^3 + 6\,a^2b^2x^2 - 5\,a^3bx - 35\,a^4\right)\sqrt{bx+a}}{315\,a^4x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] 2/315*(16*b^4*x^4 - 8*a*b^3*x^3 + 6*a^2*b^2*x^2 - 5*a^3*b*x - 35*a^4)*sqrt(
b*x + a)/(a^4*x^(9/2))
```

Fricas [A] time = 2.18912, size = 134, normalized size = 0.91

$$\frac{2\left(16\,b^4x^4 - 8\,ab^3x^3 + 6\,a^2b^2x^2 - 5\,a^3bx - 35\,a^4\right)\sqrt{bx+a}}{315\,a^4x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] 2/315*(16*b^4*x^4 - 8*a*b^3*x^3 + 6*a^2*b^2*x^2 - 5*a^3*b*x - 35*a^4)*sqrt(
b*x + a)/(a^4*x^(9/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2)/x**(11/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19808, size = 224, normalized size = 1.51

$$\frac{64\left(315\,b^{\frac{9}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{10}+189\,ab^{\frac{9}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{8}+84\,a^{2}b^{\frac{9}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{6}-36\,a^{3}b^{\frac{9}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a\right.}\right.}{315\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{2}-a\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(11/2),x, algorithm="giac")
```

```
[Out] 64/315*(315*b^(9/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^10 + 189*a*b^(9/2)*(s
qrt(b)*sqrt(x) - sqrt(b*x + a))^8 + 84*a^2*b^(9/2)*(sqrt(b)*sqrt(x) - sqrt(
b*x + a))^6 - 36*a^3*b^(9/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^4 + 9*a^4*b^
(9/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a^5*b^(9/2))/((sqrt(b)*sqrt(x)
- sqrt(b*x + a))^2 - a)^9
```

3.223 $\int x^{3/2}\tanh^{-1}(\tanh(a+bx))^{3/2}\,dx$

Optimal. Leaf size=177

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}{64b^{5/2}}+\frac{3\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+bx\right.}{64b^2}$$

```
[Out] (3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]])^4)/(64*b^(5/2)) - (x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])*Sq
rt[ArcTanh[Tanh[a + b*x]]])/8 + (x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2*S
qrt[ArcTanh[Tanh[a + b*x]]])/(32*b) + (3*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*
x]])^3*Sqrt[ArcTanh[Tanh[a + b*x]]])/(64*b^2) + (x^(5/2)*ArcTanh[Tanh[a + b
*x]]^(3/2))/4
```

Rubi [A] time = 0.103711, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2169, 2165}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}{64b^{5/2}}+\frac{3\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+bx\right.}{64b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]])^4)/(64*b^(5/2)) - (x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])*Sq
rt[ArcTanh[Tanh[a + b*x]]])/8 + (x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2*S
qrt[ArcTanh[Tanh[a + b*x]]])/(32*b) + (3*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*
x]])^3*Sqrt[ArcTanh[Tanh[a + b*x]]])/(64*b^2) + (x^(5/2)*ArcTanh[Tanh[a + b
*x]]^(3/2))/4
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
```

```
-2]
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\int x^{3/2}\tanh^{-1}(\tanh(a+bx))^{3/2}\,dx = \frac{1}{4}x^{5/2}\tanh^{-1}(\tanh(a+bx))^{3/2} - \frac{1}{8}\left(3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\right)\int x^{3/2}\sqrt{\tanh^{-1}}$$

$$= -\frac{1}{8}x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))} + \frac{1}{4}x^{5/2}\tanh^{-1}(\tanh$$

$$= -\frac{1}{8}x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))} + \frac{x^{3/2}\left(bx-\tanh^{-1}(\right.}{}$$

$$= -\frac{1}{8}x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))} + \frac{x^{3/2}\left(bx-\tanh^{-1}(\right.}{}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}{64b^{5/2}} - \frac{1}{8}x^{5/2}\left(bx-\tanh^{-1}(\right.$$

Mathematica [A] time = 0.0891484, size = 122, normalized size = 0.69

$$\frac{\sqrt{b}\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(11b^2x^2\tanh^{-1}(\tanh(a+bx))+11bx\tanh^{-1}(\tanh(a+bx))^2-3\tanh^{-1}(\tanh(a+bx))^3\right.}{64b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (Sqrt[b]*Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(-3*b^3*x^3 + 11*b^2*x^2*ArcT
anh[Tanh[a + b*x]] + 11*b*x*ArcTanh[Tanh[a + b*x]]^2 - 3*ArcTanh[Tanh[a + b
*x]]^3) + 3*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4*Log[b*Sqrt[x] + Sqrt[b]*Sqr
t[ArcTanh[Tanh[a + b*x]]]])/(64*b^(5/2))
```

Maple [B] time = 0.115, size = 471, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 1/4*x^(3/2)*arctanh(tanh(b*x+a))^(5/2)/b-1/8/b^2*a*x^(1/2)*arctanh(tanh(b*x
+a))^(5/2)+1/32/b^2*a^2*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+3/64/b^2*a^3*x^(
1/2)*arctanh(tanh(b*x+a))^(1/2)+3/64/b^(5/2)*ln(b^(1/2)*x^(1/2)+arctanh(tan
h(b*x+a))^(1/2))*a^4+3/16/b^(5/2)*a^3*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a
))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)+9/64/b^2*a^2*(arctanh(tanh(b*x+a))-b
*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+9/32/b^(5/2)*a^2*ln(b^(1/2)*x^(1/2
)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^2+1/16/b^2*a*(ar
ctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+9/64/b^2*a*(ar
ctanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+3/16/b^(5/2)
*a*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x
-a)^3-1/8/b^2*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(5/
2)+1/32/b^2*(arctanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(tanh(b*x+a))^(3/
2)+3/64/b^2*(arctanh(tanh(b*x+a))-b*x-a)^3*x^(1/2)*arctanh(tanh(b*x+a))^(1/
2)+3/64/b^(5/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tan
h(b*x+a))-b*x-a)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^{\frac{3}{2}}\,\text{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)*arctanh(tanh(b*x + a))^(3/2), x)
```

Fricas [A] time = 2.30879, size = 412, normalized size = 2.33

$$\left[\frac{3\,a^4\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(16\,b^4x^3+24\,ab^3x^2+2\,a^2b^2x-3\,a^3b\right)\sqrt{bx+a}\sqrt{x}}{128\,b^3},-\frac{3\,a^4\sqrt{-b}\arctan\left(\frac{\sqrt{bx}}{b}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/128*(3*a^4*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*
(16*b^4*x^3 + 24*a*b^3*x^2 + 2*a^2*b^2*x - 3*a^3*b)*sqrt(b*x + a)*sqrt(x))/
b^3, -1/64*(3*a^4*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (16
*b^4*x^3 + 24*a*b^3*x^2 + 2*a^2*b^2*x - 3*a^3*b)*sqrt(b*x + a)*sqrt(x))/b^3
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20397, size = 198, normalized size = 1.12

$$\frac{1}{384}\sqrt{2}\left(8\sqrt{2}\left(\sqrt{bx+a}\left(2\left(4x+\frac{a}{b}\right)x-\frac{3a^2}{b^2}\right)\sqrt{x}-\frac{3a^3\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{5}{2}}}\right)a+\sqrt{2}\left(\left(2\left(4\left(6x+\frac{a}{b}\right)x-\frac{5a^2}{b^2}\right)x+\frac{15}{b}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/384*sqrt(2)*(8*sqrt(2)*(sqrt(b*x + a)*(2*(4*x + a/b)*x - 3*a^2/b^2)*sqrt(
x) - 3*a^3*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(5/2))*a + sqrt(2)*
((2*(4*(6*x + a/b)*x - 5*a^2/b^2)*x + 15*a^3/b^3)*sqrt(b*x + a)*sqrt(x) + 1
5*a^4*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(7/2))*b)
```

3.224 $\int \sqrt{x} \tanh^{-1}(\tanh(a+bx))^{3/2} dx$

Optimal. Leaf size=139

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8b^{3/2}}-\frac{1}{4}x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+bx\right.$$

```
[Out] (ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[Tan
h[a + b*x]])^3)/(8*b^(3/2)) - (x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[
ArcTanh[Tanh[a + b*x]]])/4 + (Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt
[ArcTanh[Tanh[a + b*x]]])/(8*b) + (x^(3/2)*ArcTanh[Tanh[a + b*x]]^(3/2))/3
```

Rubi [A] time = 0.070956, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2169, 2165}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8b^{3/2}}-\frac{1}{4}x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+bx\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[Tan
h[a + b*x]])^3)/(8*b^(3/2)) - (x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[
ArcTanh[Tanh[a + b*x]]])/4 + (Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt
[ArcTanh[Tanh[a + b*x]]])/(8*b) + (x^(3/2)*ArcTanh[Tanh[a + b*x]]^(3/2))/3
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\int \sqrt{x}\tanh^{-1}(\tanh(a+bx))^{3/2}\,dx = \frac{1}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))^{3/2} - \frac{1}{2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\int \sqrt{x}\sqrt{\tanh^{-1}(\text{ta}$$

$$= -\frac{1}{4}x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))} + \frac{1}{3}x^{3/2}\tanh^{-1}(\text{tanh}$$

$$= -\frac{1}{4}x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))} + \frac{\sqrt{x}\left(bx-\tanh^{-1}(\text{ta}\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8b^{3/2}} - \frac{1}{4}x^{3/2}\left(bx-\tanh^{-1}(\text{ta}\right.$$

Mathematica [A] time = 0.0706793, size = 105, normalized size = 0.76

$$\frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(8bx\tanh^{-1}(\tanh(a+bx))+3\tanh^{-1}(\tanh(a+bx))^2-3b^2x^2\right)}{24b} + \frac{\left(bx-\tanh^{-1}(\tanh(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(-3*b^2*x^2 + 8*b*x*ArcTanh[Tanh[a +
b*x]] + 3*ArcTanh[Tanh[a + b*x]]^2))/(24*b) + ((b*x - ArcTanh[Tanh[a + b*x]
])^3*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/(8*b^(3/2))
```

Maple [B] time = 0.119, size = 304, normalized size = 2.2

$$\frac{1}{3b}\sqrt{x}\left(\text{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{\frac{5}{2}} - \frac{a}{12b}\sqrt{x}\left(\text{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{\frac{3}{2}} - \frac{a^2}{8b}\sqrt{x}\sqrt{\text{Artanh}\left(\tanh\left(bx+a\right)\right)} - \frac{a^3}{8}\ln\left(\sqrt{b}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 1/3*x^(1/2)*arctanh(tanh(b*x+a))^(5/2)/b-1/12/b*a*x^(1/2)*arctanh(tanh(b*x+
a))^(3/2)-1/8/b*a^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)-1/8/b^(3/2)*ln(b^(1/
2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^3-3/8/b^(3/2)*a^2*ln(b^(1/2)*x^(1/
2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)-1/4/b*a*(arctan
h(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)-3/8/b^(3/2)*a*ln(b
^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^2-1
/12/b*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)-1/8/b
*(arctanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)-1/8/b^(3
/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*
x-a)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x}\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)*arctanh(tanh(b*x + a))^(3/2), x)
```

Fricas [A] time = 2.32663, size = 363, normalized size = 2.61

$$\left[\frac{3\,a^3\sqrt{b}\log\left(2\,bx - 2\,\sqrt{bx+a}\sqrt{b}\sqrt{x} + a\right) + 2\left(8\,b^3x^2 + 14\,ab^2x + 3\,a^2b\right)\sqrt{bx+a}\sqrt{x}}{48\,b^2}, \frac{3\,a^3\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right) + \left(8\,b^3\right.}{24\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*a^3*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(
8*b^3*x^2 + 14*a*b^2*x + 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^2, 1/24*(3*a^3*s
qrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (8*b^3*x^2 + 14*a*b^2*
x + 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x}\,\mathrm{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(sqrt(x)*atanh(tanh(a + b*x))**(3/2), x)
```

Giac [A] time = 1.20014, size = 165, normalized size = 1.19

$$\frac{1}{48}\sqrt{2}\left(6\sqrt{2}\left(\sqrt{bx+a}\left(2x+\frac{a}{b}\right)\sqrt{x}+\frac{a^2\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{3}{2}}}\right)a+\sqrt{2}\left(\sqrt{bx+a}\left(2\left(4x+\frac{a}{b}\right)x-\frac{3a^2}{b^2}\right)\sqrt{x}-\frac{3a^3\log\left(\right|}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(2)*(6*sqrt(2)*(sqrt(b*x + a)*(2*x + a/b)*sqrt(x) + a^2*log(abs(-s
qrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(3/2))*a + sqrt(2)*(sqrt(b*x + a)*(2*(4*
x + a/b)*x - 3*a^2/b^2)*sqrt(x) - 3*a^3*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x
 + a)))/b^(5/2))*b)
```

3.225 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=101

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4\sqrt{b}}-\frac{3}{4}\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+bx\right.$$

```
[Out] (3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]])^2)/(4*Sqrt[b]) - (3*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])*S
qrt[ArcTanh[Tanh[a + b*x]]])/4 + (Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(3/2))/2
```

Rubi [A] time = 0.0476427, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2169, 2165}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4\sqrt{b}}-\frac{3}{4}\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+bx\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/Sqrt[x],x]
```

```
[Out] (3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]])^2)/(4*Sqrt[b]) - (3*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])*S
qrt[ArcTanh[Tanh[a + b*x]]])/4 + (Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(3/2))/2
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{\sqrt{x}}\,dx = \frac{1}{2}\sqrt{x}\tanh^{-1}(\tanh(a+bx))^{3/2} - \frac{1}{4}\left(3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\right)\int \frac{\sqrt{\tanh^{-1}(\tanh}}{\sqrt{x}}$$

$$= -\frac{3}{4}\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))} + \frac{1}{2}\sqrt{x}\tanh^{-1}(\tanh(a+$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4\sqrt{b}} - \frac{3}{4}\sqrt{x}\left(bx-\tanh^{-1}(\tan$$

Mathematica [A] time = 0.0576044, size = 83, normalized size = 0.82

$$\frac{1}{4}\left(\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(5\tanh^{-1}(\tanh(a+bx))-3bx\right)+\frac{3\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tan\right.}{\sqrt{b}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(-3*b*x + 5*ArcTanh[Tanh[a + b*x]]) +
 (3*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTan
h[Tanh[a + b*x]]]])/Sqrt[b])/4
```

Maple [B] time = 0.042, size = 165, normalized size = 1.6

$$\frac{1}{2}\sqrt{x}\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{\frac{3}{2}} + \frac{3a}{4}\sqrt{x}\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)} + \frac{3a^2}{4}\ln\left(\sqrt{b}\sqrt{x}+\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}\right)\frac{1}{\sqrt{b}} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x^(1/2),x)
```

```
[Out] 1/2*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+3/4*a*x^(1/2)*arctanh(tanh(b*x+a))^(
1/2)+3/4/b^(1/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^2+3/2*a/b
^(1/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))
-b*x-a)+3/4*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)
+3/4/b^(1/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b
*x+a))-b*x-a)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(3/2)/sqrt(x), x)
```

Fricas [A] time = 2.27321, size = 311, normalized size = 3.08

$$\left[\frac{3\,a^2\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(2\,b^2x+5\,ab\right)\sqrt{bx+a}\sqrt{x}}{8\,b}, -\frac{3\,a^2\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)-\left(2\,b^2x+5\,ab\right)}{4\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a^2*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(2
*b^2*x + 5*a*b)*sqrt(b*x + a)*sqrt(x))/b, -1/4*(3*a^2*sqrt(-b)*arctan(sqrt(
b*x + a)*sqrt(-b)/(b*sqrt(x))) - (2*b^2*x + 5*a*b)*sqrt(b*x + a)*sqrt(x))/b
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x**(1/2),x)

[Out] Integral(atanh(tanh(a + b*x))**(3/2)/sqrt(x), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(1/2),x, algorithm="giac")

[Out] Timed out
```

3.226 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{3/2}} dx$

Optimal. Leaf size=81

$$-\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{\sqrt{x}}+3b\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}-3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(t\right.$$

```
[Out] -3*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - A
rcTanh[Tanh[a + b*x]]) + 3*b*Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]] - (2*ArcT
anh[Tanh[a + b*x]]^(3/2))/Sqrt[x]
```

Rubi [A] time = 0.056096, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2168, 2169, 2165}

$$-\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{\sqrt{x}}+3b\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}-3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(t\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(3/2),x]
```


```
[Out] -3*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - A
rcTanh[Tanh[a + b*x]]) + 3*b*Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]] - (2*ArcT
anh[Tanh[a + b*x]]^(3/2))/Sqrt[x]
```


Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{3/2}}\,dx = -\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{\sqrt{x}} + (3b)\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}\,dx$$

$$= 3b\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{\sqrt{x}} - \frac{1}{2}\left(3b\left(bx - \tanh^{-1}(\tanh\right.\right.$$

$$= -3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx - \tanh^{-1}(\tanh(a+bx))\right) + 3b\sqrt{x}\sqrt{\tanh}$$

Mathematica [A] time = 0.051877, size = 77, normalized size = 0.95

$$\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(3bx - 2\tanh^{-1}(\tanh(a+bx))\right)}{\sqrt{x}} + 3\sqrt{b}\left(\tanh^{-1}(\tanh(a+bx)) - bx\right)\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(3/2),x]
```

```
[Out] ((3*b*x - 2*ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/Sqrt[x] +
 3*Sqrt[b]*(-(b*x) + ArcTanh[Tanh[a + b*x]])*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[A
rcTanh[Tanh[a + b*x]]]]
```

Maple [B] time = 0.117, size = 280, normalized size = 3.5

$$-2\frac{(\text{Artanh}(\tanh(bx+a)))^{5/2}}{(\text{Artanh}(\tanh(bx+a))-bx)\sqrt{x}}+2\frac{b\sqrt{x}\,(\text{Artanh}(\tanh(bx+a)))^{3/2}}{\text{Artanh}(\tanh(bx+a))-bx}+3\frac{ab\sqrt{x}\sqrt{\text{Artanh}(\tanh(bx+a))}}{\text{Artanh}(\tanh(bx+a))-bx}+3\,\frac{\sqrt{b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x^(3/2),x)
```

```
[Out] -2/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)*arctanh(tanh(b*x+a))^(5/2)+2*b/(arcta
nh(tanh(b*x+a))-b*x)*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+3*b/(arctanh(tanh(b
*x+a))-b*x)*a*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+3*b^(1/2)/(arctanh(tanh(b*
x+a))-b*x)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^2+6*b^(1/2)/(ar
ctanh(tanh(b*x+a))-b*x)*a*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(a
rctanh(tanh(b*x+a))-b*x-a)+3*b/(arctanh(tanh(b*x+a))-b*x)*(arctanh(tanh(b*x
+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+3*b^(1/2)/(arctanh(tanh(b*x+
a))-b*x)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a
))-b*x-a)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{artanh}(\tanh(bx+a))^{\frac{3}{2}}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(3/2)/x^(3/2), x)
```

Fricas [A] time = 2.44476, size = 289, normalized size = 3.57

$$\left[\frac{3\,a\sqrt{b}x\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,\sqrt{bx+a}(bx-2\,a)\sqrt{x}}{2\,x},-\frac{3\,a\sqrt{-b}x\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)-\sqrt{bx+a}(bx-2\,a)\sqrt{}}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*a*sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*sq
rt(b*x + a)*(b*x - 2*a)*sqrt(x))/x, -(3*a*sqrt(-b)*x*arctan(sqrt(b*x + a)*s
qrt(-b)/(b*sqrt(x))) - sqrt(b*x + a)*(b*x - 2*a)*sqrt(x))/x]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x**(3/2),x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**(3/2)/x**(3/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.227 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{5/2}}\,dx$

Optimal. Leaf size=70

$$2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)-\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}}-\frac{2b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}$$

```
[Out] 2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]] - (2*b*Sq
rt[ArcTanh[Tanh[a + b*x]]])/Sqrt[x] - (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*x
^(3/2))
```

Rubi [A] time = 0.0422076, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2168, 2165}

$$2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)-\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}}-\frac{2b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(5/2),x]
```

```
[Out] 2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]] - (2*b*Sq
rt[ArcTanh[Tanh[a + b*x]]])/Sqrt[x] - (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*x
^(3/2))
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{5/2}}\,dx = -\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}} + b\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{3/2}}\,dx$$

$$= -\frac{2b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}} - \frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}} + b^2\int \frac{1}{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

$$= 2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right) - \frac{2b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}} - \frac{2\tanh^{-1}(\tan}{3x^{3}}$$

Mathematica [A] time = 0.0419714, size = 74, normalized size = 1.06

$$-\frac{2\left(-3b^{3/2}x^{3/2}\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}+b\sqrt{x}\right)+3bx\sqrt{\tanh^{-1}(\tanh(a+bx))}+\tanh^{-1}(\tanh(a+bx))^{3/2}\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(5/2),x]
```

```
[Out] (-2*(3*b*x*Sqrt[ArcTanh[Tanh[a + b*x]]] + ArcTanh[Tanh[a + b*x]]^(3/2) - 3*
b^(3/2)*x^(3/2)*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]]))/(3*
x^(3/2))
```

Maple [B] time = 0.135, size = 315, normalized size = 4.5

$$-\frac{2}{3\,\mathrm{Artanh}(\tanh(bx+a))-3\,bx}(\mathrm{Artanh}(\tanh(bx+a)))^{\frac{5}{2}}x^{-\frac{3}{2}} - \frac{4\,b}{3\,(\mathrm{Artanh}(\tanh(bx+a))-bx)^{2}}(\mathrm{Artanh}(\tanh(b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x^(5/2),x)
```

```
[Out] -2/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)*arctanh(tanh(b*x+a))^(5/2)-4/3*b/(a
rctanh(tanh(b*x+a))-b*x)^2/x^(1/2)*arctanh(tanh(b*x+a))^(5/2)+4/3*b^2/(arct
anh(tanh(b*x+a))-b*x)^2*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+2*b^2/(arctanh(t
anh(b*x+a))-b*x)^2*a*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+2*b^(3/2)/(arctanh(
tanh(b*x+a))-b*x)^2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^2+4*b^
(3/2)/(arctanh(tanh(b*x+a))-b*x)^2*a*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a)
)^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)+2*b^2/(arctanh(tanh(b*x+a))-b*x)^2*(a
rctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+2*b^(3/2)/(ar
ctanh(tanh(b*x+a))-b*x)^2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(a
rctanh(tanh(b*x+a))-b*x-a)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(3/2)/x^(5/2), x)
```

Fricas [A] time = 2.31197, size = 302, normalized size = 4.31

$$\left[\frac{3\,b^{\frac{3}{2}}x^2\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,(4\,bx+a)\sqrt{bx+a}\sqrt{x}}{3\,x^2}, -\frac{2\left(3\,\sqrt{-b}bx^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+(4\,bx+a)\sqrt{bx+}\right.}{3\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*b^(3/2)*x^2*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*(4
*b*x + a)*sqrt(b*x + a)*sqrt(x))/x^2, -2/3*(3*sqrt(-b)*b*x^2*arctan(sqrt(b*
x + a)*sqrt(-b)/(b*sqrt(x))) + (4*b*x + a)*sqrt(b*x + a)*sqrt(x))/x^2]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.228 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{7/2}}\,dx$

Optimal. Leaf size=35

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rubi [A] time = 0.0140935, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2167}

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(7/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{7/2}}\,dx=\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A] time = 0.0363926, size = 34, normalized size = 0.97

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{5/2}\left(5bx-5\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(7/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(x^(5/2)*(5*b*x - 5*ArcTanh[Tanh[a + b*x]]
))
```

Maple [A] time = 0.146, size = 29, normalized size = 0.8

$$-\frac{2}{5\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-5\,bx}\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{\frac{5}{2}}x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x^(7/2),x)
```

```
[Out] -2/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)*arctanh(tanh(b*x+a))^(5/2)
```

Maxima [A] time = 1.48283, size = 20, normalized size = 0.57

$$-\frac{2\left(bx+a\right)^{\frac{5}{2}}}{5\,ax^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] -2/5*(b*x + a)^(5/2)/(a*x^(5/2))
```

Fricas [A] time = 2.30816, size = 78, normalized size = 2.23

$$-\frac{2\left(b^2x^2+2\,abx+a^2\right)\sqrt{bx+a}}{5\,ax^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] -2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/(a*x^(5/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33761, size = 45, normalized size = 1.29

$$-\frac{2\,(bx+a)^{\frac{5}{2}}b^{6}}{5\,((bx+a)b-ab)^{\frac{5}{2}}a|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] -2/5*(b*x + a)^(5/2)*b^6/(((b*x + a)*b - a*b)^(5/2)*a*abs(b))
```

3.229 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{9/2}} \, dx$

Optimal. Leaf size=72

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{4b\tanh^{-1}(\tanh(a+bx))^{5/2}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

```
[Out] (4*b*ArcTanh[Tanh[a + b*x]]^(5/2))/(35*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]
])^2) + (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(7*x^(7/2)*(b*x - ArcTanh[Tanh[a +
 b*x]]))
```

Rubi [A] time = 0.0330791, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2171, 2167}

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{4b\tanh^{-1}(\tanh(a+bx))^{5/2}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(9/2),x]
```

```
[Out] (4*b*ArcTanh[Tanh[a + b*x]]^(5/2))/(35*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]
])^2) + (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(7*x^(7/2)*(b*x - ArcTanh[Tanh[a +
 b*x]]))
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
```

```
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{9/2}}\,dx = \frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(2b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{7/2}}\,dx}{7\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$
$$= \frac{4b\tanh^{-1}(\tanh(a+bx))^{5/2}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A] time = 0.040538, size = 48, normalized size = 0.67

$$\frac{2\left(7bx-5\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{5/2}}{35x^{7/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(9/2),x]
```

```
[Out] (2*(7*b*x - 5*ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(5/2))/(35*x^(
7/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

Maple [A] time = 0.129, size = 59, normalized size = 0.8

$$-\frac{2}{7\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-7\,bx}\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{\frac{5}{2}}x^{-\frac{7}{2}}+\frac{4\,b}{35\,\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}}\left(\mathrm{Artanh}\left(\mathrm{tanh}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x^(9/2),x)
```

```
[Out] -2/7/(arctanh(tanh(b*x+a))-b*x)/x^(7/2)*arctanh(tanh(b*x+a))^(5/2)+4/35*b/(
arctanh(tanh(b*x+a))-b*x)^2/x^(5/2)*arctanh(tanh(b*x+a))^(5/2)
```

Maxima [A] time = 1.46936, size = 46, normalized size = 0.64

$$\frac{2\left(2\,b^2x^2 - 3\,abx - 5\,a^2\right)(bx + a)^{\frac{3}{2}}}{35\,a^2x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] 2/35*(2*b^2*x^2 - 3*a*b*x - 5*a^2)*(b*x + a)^(3/2)/(a^2*x^(7/2))
```

Fricas [A] time = 2.18267, size = 105, normalized size = 1.46

$$\frac{2\left(2\,b^3x^3 - ab^2x^2 - 8\,a^2bx - 5\,a^3\right)\sqrt{bx + a}}{35\,a^2x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] 2/35*(2*b^3*x^3 - a*b^2*x^2 - 8*a^2*b*x - 5*a^3)*sqrt(b*x + a)/(a^2*x^(7/2)
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25709, size = 80, normalized size = 1.11

$$\frac{\sqrt{2}\left(\frac{2\,\sqrt{2}(bx+a)b^7}{a^2} - \frac{7\,\sqrt{2}b^7}{a}\right)(bx + a)^{\frac{5}{2}}b}{35\,((bx + a)b - ab)^{\frac{7}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] 1/35*sqrt(2)*(2*sqrt(2)*(b*x + a)*b^7/a^2 - 7*sqrt(2)*b^7/a)*(b*x + a)^(5/2
)*b/(((b*x + a)*b - a*b)^(7/2)*abs(b))
```

3.230 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{11/2}} dx$

Optimal. Leaf size=110

$$\frac{16b^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{315x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{9x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{8b\tanh^{-1}(\tanh(a+bx))^{5/2}}{63x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

```
[Out] (16*b^2*ArcTanh[Tanh[a + b*x]]^(5/2))/(315*x^(5/2)*(b*x - ArcTanh[Tanh[a +
b*x]])^3) + (8*b*ArcTanh[Tanh[a + b*x]]^(5/2))/(63*x^(7/2)*(b*x - ArcTanh[T
anh[a + b*x]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(9*x^(9/2)*(b*x - ArcT
anh[Tanh[a + b*x]]))
```

Rubi [A] time = 0.0522912, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2171, 2167}

$$\frac{16b^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{315x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{9x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{8b\tanh^{-1}(\tanh(a+bx))^{5/2}}{63x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(11/2),x]
```


```
[Out] (16*b^2*ArcTanh[Tanh[a + b*x]]^(5/2))/(315*x^(5/2)*(b*x - ArcTanh[Tanh[a +
b*x]])^3) + (8*b*ArcTanh[Tanh[a + b*x]]^(5/2))/(63*x^(7/2)*(b*x - ArcTanh[T
anh[a + b*x]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(9*x^(9/2)*(b*x - ArcT
anh[Tanh[a + b*x]]))
```


Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
```

```
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{11/2}}\,dx = \frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{9x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(4b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{9/2}}\,dx}{9\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

$$= \frac{8b\tanh^{-1}(\tanh(a+bx))^{5/2}}{63x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{9x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(8b^2 \cdots}{63\left(bx \cdots\right.}$$

$$= \frac{16b^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{315x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{8b\tanh^{-1}(\tanh(a+bx))^{5/2}}{63x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{\cdots}{9x^{9}\cdots}$$

Mathematica [A] time = 0.0447709, size = 66, normalized size = 0.6

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}\left(-90bx\tanh^{-1}(\tanh(a+bx))+35\tanh^{-1}(\tanh(a+bx))^2+63b^2x^2\right)}{315x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(11/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(5/2)*(63*b^2*x^2 - 90*b*x*ArcTanh[Tanh[a + b*x]]
 + 35*ArcTanh[Tanh[a + b*x]]^2))/(315*x^(9/2)*(b*x - ArcTanh[Tanh[a + b*x]]
)^3)
```

Maple [A] time = 0.136, size = 105, normalized size = 1.

$$-\frac{2}{9\,\mathrm{Artanh}\,(\tanh\,(bx+a))-9\,bx}\,(\mathrm{Artanh}\,(\tanh\,(bx+a)))^{\frac{5}{2}}\,x^{-\frac{9}{2}} - \frac{8\,b}{9\,\mathrm{Artanh}\,(\tanh\,(bx+a))-9\,bx}\left(-\frac{\cdots}{7\,\mathrm{Artanh}\,(\tan\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x^(11/2),x)
```

```
[Out] -2/9/(arctanh(tanh(b*x+a))-b*x)/x^(9/2)*arctanh(tanh(b*x+a))^(5/2)-8/9*b/(a
rctanh(tanh(b*x+a))-b*x)*(-1/7/(arctanh(tanh(b*x+a))-b*x)/x^(7/2)*arctanh(t
anh(b*x+a))^(5/2)+2/35*b/(arctanh(tanh(b*x+a))-b*x)^2/x^(5/2)*arctanh(tanh(
b*x+a))^(5/2))
```

Maxima [A] time = 1.48465, size = 61, normalized size = 0.55

$$-\frac{2\left(8\,b^3x^3-12\,ab^2x^2+15\,a^2bx+35\,a^3\right)(bx+a)^{\frac{3}{2}}}{315\,a^3x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] -2/315*(8*b^3*x^3 - 12*a*b^2*x^2 + 15*a^2*b*x + 35*a^3)*(b*x + a)^(3/2)/(a^
3*x^(9/2))
```

Fricas [A] time = 2.02936, size = 135, normalized size = 1.23

$$-\frac{2\left(8\,b^4x^4-4\,ab^3x^3+3\,a^2b^2x^2+50\,a^3bx+35\,a^4\right)\sqrt{bx+a}}{315\,a^3x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] -2/315*(8*b^4*x^4 - 4*a*b^3*x^3 + 3*a^2*b^2*x^2 + 50*a^3*b*x + 35*a^4)*sqrt
(b*x + a)/(a^3*x^(9/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x**(11/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33636, size = 105, normalized size = 0.95

$$-\frac{\sqrt{2}\left(\frac{63\sqrt{2}b^9}{a}+4\left(\frac{2\sqrt{2}(bx+a)b^9}{a^3}-\frac{9\sqrt{2}b^9}{a^2}\right)(bx+a)\right)(bx+a)^{\frac{5}{2}}b}{315\left((bx+a)b-ab\right)^{\frac{9}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(11/2),x, algorithm="giac")
```

```
[Out] -1/315*sqrt(2)*(63*sqrt(2)*b^9/a + 4*(2*sqrt(2)*(b*x + a)*b^9/a^3 - 9*sqrt(
2)*b^9/a^2)*(b*x + a))*(b*x + a)^(5/2)*b/((((b*x + a)*b - a*b)^(9/2)*abs(b))
```

3.231 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{13/2}}\,dx$

Optimal. Leaf size=148

$$\frac{32b^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{1155x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{16b^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{231x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{4b\tanh^{-1}(\tanh(a+bx))^{5/2}}{33x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

```
[Out] (32*b^3*ArcTanh[Tanh[a + b*x]]^(5/2))/(1155*x^(5/2)*(b*x - ArcTanh[Tanh[a +
 b*x]])^4) + (16*b^2*ArcTanh[Tanh[a + b*x]]^(5/2))/(231*x^(7/2)*(b*x - ArcT
anh[Tanh[a + b*x]])^3) + (4*b*ArcTanh[Tanh[a + b*x]]^(5/2))/(33*x^(9/2)*(b*
x - ArcTanh[Tanh[a + b*x]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(11*x^(11
/2)*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rubi [A] time = 0.0755237, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2171, 2167}

$$\frac{32b^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{1155x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{16b^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{231x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{4b\tanh^{-1}(\tanh(a+bx))^{5/2}}{33x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(13/2),x]
```

```
[Out] (32*b^3*ArcTanh[Tanh[a + b*x]]^(5/2))/(1155*x^(5/2)*(b*x - ArcTanh[Tanh[a +
 b*x]])^4) + (16*b^2*ArcTanh[Tanh[a + b*x]]^(5/2))/(231*x^(7/2)*(b*x - ArcT
anh[Tanh[a + b*x]])^3) + (4*b*ArcTanh[Tanh[a + b*x]]^(5/2))/(33*x^(9/2)*(b*
x - ArcTanh[Tanh[a + b*x]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(11*x^(11
/2)*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{13/2}}\,dx = \frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{11x^{11/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(6b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{11/2}}\,dx}{11\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

$$= \frac{4b\tanh^{-1}(\tanh(a+bx))^{5/2}}{33x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{11x^{11/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(8b}{33(b}$$

$$= \frac{16b^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{231x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{4b\tanh^{-1}(\tanh(a+bx))^{5/2}}{33x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{}{11x}$$

$$= \frac{32b^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{1155x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4} + \frac{16b^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{231x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{}{3}$$

Mathematica [A] time = 0.0661309, size = 82, normalized size = 0.55

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}\left(-495b^2x^2\tanh^{-1}(\tanh(a+bx)) + 385bx\tanh^{-1}(\tanh(a+bx))^2 - 105\tanh^{-1}(\tanh(a+b\right.}{1155x^{11/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(13/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(5/2)*(231*b^3*x^3 - 495*b^2*x^2*ArcTanh[Tanh[a +
 b*x]] + 385*b*x*ArcTanh[Tanh[a + b*x]]^2 - 105*ArcTanh[Tanh[a + b*x]]^3))/
(1155*x^(11/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4)
```

Maple [A] time = 0.153, size = 151, normalized size = 1.

$$-\frac{2}{11\,\mathrm{Artanh}(\tanh(bx+a)) - 11\,bx}(\mathrm{Artanh}(\tanh(bx+a)))^{\frac{5}{2}}x^{-\frac{11}{2}} - \frac{12\,b}{11\,\mathrm{Artanh}(\tanh(bx+a)) - 11\,bx}\left(-\frac{}{9\,\mathrm{Artanh}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x^(13/2),x)
```

```
[Out] -2/11/(arctanh(tanh(b*x+a))-b*x)/x^(11/2)*arctanh(tanh(b*x+a))^(5/2)-12/11*
b/(arctanh(tanh(b*x+a))-b*x)*(-1/9/(arctanh(tanh(b*x+a))-b*x)/x^(9/2)*arcta
nh(tanh(b*x+a))^(5/2)-4/9*b/(arctanh(tanh(b*x+a))-b*x)*(-1/7/(arctanh(tanh(
b*x+a))-b*x)/x^(7/2)*arctanh(tanh(b*x+a))^(5/2)+2/35*b/(arctanh(tanh(b*x+a)
)-b*x)^2/x^(5/2)*arctanh(tanh(b*x+a))^(5/2)))
```

Maxima [A] time = 1.50009, size = 76, normalized size = 0.51

$$\frac{2\left(16\,b^4x^4-24\,ab^3x^3+30\,a^2b^2x^2-35\,a^3bx-105\,a^4\right)(bx+a)^{\frac{3}{2}}}{1155\,a^4x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(13/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(16*b^4*x^4 - 24*a*b^3*x^3 + 30*a^2*b^2*x^2 - 35*a^3*b*x - 105*a^4)*
(b*x + a)^(3/2)/(a^4*x^(11/2))
```

Fricas [A] time = 1.95807, size = 162, normalized size = 1.09

$$\frac{2\left(16\,b^5x^5-8\,ab^4x^4+6\,a^2b^3x^3-5\,a^3b^2x^2-140\,a^4bx-105\,a^5\right)\sqrt{bx+a}}{1155\,a^4x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(13/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(16*b^5*x^5 - 8*a*b^4*x^4 + 6*a^2*b^3*x^3 - 5*a^3*b^2*x^2 - 140*a^4*
b*x - 105*a^5)*sqrt(b*x + a)/(a^4*x^(11/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x**(13/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3363, size = 131, normalized size = 0.89

$$-\frac{\sqrt{2}\left(\frac{231\sqrt{2}b^{11}}{a}-2\left(\frac{99\sqrt{2}b^{11}}{a^2}+4\left(\frac{2\sqrt{2}(bx+a)b^{11}}{a^4}-\frac{11\sqrt{2}b^{11}}{a^3}\right)(bx+a)\right)(bx+a)\right)(bx+a)^{\frac{5}{2}}b}{1155\left((bx+a)b-ab\right)^{\frac{11}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(13/2),x, algorithm="giac")
```

```
[Out] -1/1155*sqrt(2)*(231*sqrt(2)*b^11/a - 2*(99*sqrt(2)*b^11/a^2 + 4*(2*sqrt(2)
*(b*x + a)*b^11/a^4 - 11*sqrt(2)*b^11/a^3)*(b*x + a))*(b*x + a))*(b*x + a)^
(5/2)*b/(((b*x + a)*b - a*b)^(11/2)*abs(b))
```

3.232 $\int \sqrt{x}\tanh^{-1}(\tanh(a+bx))^{5/2}\,dx$

Optimal. Leaf size=174

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}{64b^{3/2}}+\frac{5}{32}x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+\right.$$

```
[Out] (-5*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[
Tanh[a + b*x]])^4)/(64*b^(3/2)) + (5*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])
^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/32 - (5*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*
x]])^3*Sqrt[ArcTanh[Tanh[a + b*x]]])/(64*b) - (5*x^(3/2)*(b*x - ArcTanh[Tan
h[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2))/24 + (x^(3/2)*ArcTanh[Tanh[a + b
*x]]^(5/2))/4
```

Rubi [A] time = 0.0975145, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2169, 2165}

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}{64b^{3/2}}+\frac{5}{32}x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (-5*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[
Tanh[a + b*x]])^4)/(64*b^(3/2)) + (5*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])
^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/32 - (5*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*
x]])^3*Sqrt[ArcTanh[Tanh[a + b*x]]])/(64*b) - (5*x^(3/2)*(b*x - ArcTanh[Tan
h[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2))/24 + (x^(3/2)*ArcTanh[Tanh[a + b
*x]]^(5/2))/4
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
```

```
-2]
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\int \sqrt{x}\tanh^{-1}(\tanh(a+bx))^{5/2}\,dx = \frac{1}{4}x^{3/2}\tanh^{-1}(\tanh(a+bx))^{5/2} - \frac{1}{8}\left(5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\right)\int \sqrt{x}\tanh^{-1}$$

$$= -\frac{5}{24}x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2} + \frac{1}{4}x^{3/2}\tanh^{-1}(\mathrm{ta}$$

$$= \frac{5}{32}x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{5}{24}x^{3/2}\left(bx-\mathrm{tan}\right.$$

$$= \frac{5}{32}x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{5\sqrt{x}\left(bx-\tanh^{-1}\right.}{}$$

$$= -\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}{64b^{3/2}} + \frac{5}{32}x^{3/2}\left(bx-\mathrm{tan}\right.$$

Mathematica [A] time = 0.081641, size = 121, normalized size = 0.7

$$\frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-55b^2x^2\tanh^{-1}(\tanh(a+bx))+73bx\tanh^{-1}(\tanh(a+bx))^2+15\tanh^{-1}(\tanh(a+bx))\right.}{192b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(15*b^3*x^3 - 55*b^2*x^2*ArcTanh[Tanh
[a + b*x]] + 73*b*x*ArcTanh[Tanh[a + b*x]]^2 + 15*ArcTanh[Tanh[a + b*x]]^3)
)/(192*b) - (5*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4*Log[b*Sqrt[x] + Sqrt[b]*
Sqrt[ArcTanh[Tanh[a + b*x]]]])/(64*b^(3/2))
```

Maple [B] time = 0.117, size = 471, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] 1/4*x^(1/2)*arctanh(tanh(b*x+a))^(7/2)/b-1/24/b*a*x^(1/2)*arctanh(tanh(b*x+
a))^(5/2)-5/96/b*a^2*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)-5/64/b*a^3*x^(1/2)*
arctanh(tanh(b*x+a))^(1/2)-5/64/b^(3/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x
+a))^(1/2))*a^4-5/16/b^(3/2)*a^3*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1
/2))*(arctanh(tanh(b*x+a))-b*x-a)-15/64/b*a^2*(arctanh(tanh(b*x+a))-b*x-a)*
x^(1/2)*arctanh(tanh(b*x+a))^(1/2)-15/32/b^(3/2)*a^2*ln(b^(1/2)*x^(1/2)+arc
tanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^2-5/48/b*a*(arctanh(t
anh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)-15/64/b*a*(arctanh(ta
nh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)-5/16/b^(3/2)*a*ln(b^
(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^3-1/
24/b*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(5/2)-5/96/b
*(arctanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)-5/64/b*(
arctanh(tanh(b*x+a))-b*x-a)^3*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)-5/64/b^(3/
2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x
-a)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x}\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)*arctanh(tanh(b*x + a))^(5/2), x)
```

Fricas [A] time = 2.21766, size = 425, normalized size = 2.44

$$\left[\frac{15\,a^{4}\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(48\,b^{4}x^{3}+136\,ab^{3}x^{2}+118\,a^{2}b^{2}x+15\,a^{3}b\right)\sqrt{bx+a}\sqrt{x}}{384\,b^{2}},\ \frac{15\,a^{4}\sqrt{-b}\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/384*(15*a^4*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(48*b^4*x^3 + 136*a*b^3*x^2 + 118*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a)*sqrt
(x))/b^2, 1/192*(15*a^4*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x)))
 + (48*b^4*x^3 + 136*a*b^3*x^2 + 118*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a)*sq
rt(x))/b^2]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31174, size = 275, normalized size = 1.58

$$\frac{1}{384}\sqrt{2}\left(48\sqrt{2}\left(\sqrt{bx+a}\left(2x+\frac{a}{b}\right)\sqrt{x}+\frac{a^2\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{3}{2}}}\right)a^2+16\sqrt{2}\left(\sqrt{bx+a}\left(2\left(4x+\frac{a}{b}\right)x-\frac{3a^2}{b^2}\right)\sqrt{x}-\frac{3}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/384*sqrt(2)*(48*sqrt(2)*(sqrt(b*x + a)*(2*x + a/b)*sqrt(x) + a^2*log(abs(
-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(3/2))*a^2 + 16*sqrt(2)*(sqrt(b*x + a)
*(2*(4*x + a/b)*x - 3*a^2/b^2)*sqrt(x) - 3*a^3*log(abs(-sqrt(b)*sqrt(x) + s
qrt(b*x + a)))/b^(5/2))*a*b + sqrt(2)*((2*(4*(6*x + a/b)*x - 5*a^2/b^2)*x +
 15*a^3/b^3)*sqrt(b*x + a)*sqrt(x) + 15*a^4*log(abs(-sqrt(b)*sqrt(x) + sqrt
(b*x + a)))/b^(7/2))*b^2)
```

3.233 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=136

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8\sqrt{b}}+\frac{5}{8}\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+bx\right.$$

```
[Out] (-5*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[
Tanh[a + b*x]])^3)/(8*Sqrt[b]) + (5*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^
2*Sqrt[ArcTanh[Tanh[a + b*x]]])/8 - (5*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]
])*ArcTanh[Tanh[a + b*x]]^(3/2))/12 + (Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(5/2)
)/3
```

Rubi [A] time = 0.069098, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2169, 2165}

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8\sqrt{b}}+\frac{5}{8}\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+bx\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/Sqrt[x],x]
```

```
[Out] (-5*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[
Tanh[a + b*x]])^3)/(8*Sqrt[b]) + (5*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^
2*Sqrt[ArcTanh[Tanh[a + b*x]]])/8 - (5*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]
])*ArcTanh[Tanh[a + b*x]]^(3/2))/12 + (Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(5/2)
)/3
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
```

```
-2]
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{\sqrt{x}}\,dx = \frac{1}{3}\sqrt{x}\tanh^{-1}(\tanh(a+bx))^{5/2} - \frac{1}{6}\left(5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\right)\int \frac{\tanh^{-1}(\tanh}{\sqrt{x}}$$

$$= -\frac{5}{12}\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2} + \frac{1}{3}\sqrt{x}\tanh^{-1}(\tanh($$

$$= \frac{5}{8}\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{5}{12}\sqrt{x}\left(bx-\tanh^{-1}(\tan$$

$$= -\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8\sqrt{b}} + \frac{5}{8}\sqrt{x}\left(bx-\tanh^{-1}(\right.$$

Mathematica [A] time = 0.057602, size = 101, normalized size = 0.74

$$\frac{1}{24}\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-40bx\tanh^{-1}(\tanh(a+bx)) + 33\tanh^{-1}(\tanh(a+bx))^2 + 15b^2x^2\right) + \frac{5\left(\tanh^{-1}(\tanh\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(15*b^2*x^2 - 40*b*x*ArcTanh[Tanh[a +
 b*x]] + 33*ArcTanh[Tanh[a + b*x]]^2))/24 + (5*(-(b*x) + ArcTanh[Tanh[a + b
*x]])^3*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/(8*Sqrt[b])
```

Maple [B] time = 0.046, size = 286, normalized size = 2.1

$$\frac{1}{3}\sqrt{x}\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{\frac{5}{2}} + \frac{5a}{12}\sqrt{x}\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{\frac{3}{2}} + \frac{5a^2}{8}\sqrt{x}\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)} + \frac{5a^3}{8}\ln\left(\sqrt{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^(1/2),x)
```

```
[Out] 1/3*x^(1/2)*arctanh(tanh(b*x+a))^(5/2)+5/12*a*x^(1/2)*arctanh(tanh(b*x+a))^
(3/2)+5/8*a^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+5/8/b^(1/2)*ln(b^(1/2)*x^(
1/2)+arctanh(tanh(b*x+a))^(1/2))*a^3+15/8*a^2/b^(1/2)*ln(b^(1/2)*x^(1/2)+ar
ctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)+5/4*a*(arctanh(tanh(
b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+15/8*a/b^(1/2)*ln(b^(1/2)
*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^2+5/12*(a
rctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+5/8*(arctanh(
tanh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+5/8/b^(1/2)*ln(b^(
1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(5/2)/sqrt(x), x)
```

Fricas [A] time = 2.16983, size = 365, normalized size = 2.68

$$\left[\frac{15\,a^{3}\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(8\,b^{3}x^{2}+26\,ab^{2}x+33\,a^{2}b\right)\sqrt{bx+a}\sqrt{x}}{48\,b}, -\frac{15\,a^{3}\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)-\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*a^3*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*
(8*b^3*x^2 + 26*a*b^2*x + 33*a^2*b)*sqrt(b*x + a)*sqrt(x))/b, -1/24*(15*a^3
*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (8*b^3*x^2 + 26*a*b^
```

```
2*x + 33*a^2*b)*sqrt(b*x + a)*sqrt(x))/b]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.234 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{3/2}} dx$

Optimal. Leaf size=121

$$-\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{\sqrt{x}}+\frac{5}{2}b\sqrt{x}\tanh^{-1}(\tanh(a+bx))^{3/2}-\frac{15}{4}b\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}$$

```
[Out] (15*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x -
ArcTanh[Tanh[a + b*x]])^2)/4 - (15*b*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])
*Sqrt[ArcTanh[Tanh[a + b*x]]])/4 + (5*b*Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(3/2
))/2 - (2*ArcTanh[Tanh[a + b*x]]^(5/2))/Sqrt[x]
```

Rubi [A] time = 0.0660079, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2168, 2169, 2165}

$$-\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{\sqrt{x}}+\frac{5}{2}b\sqrt{x}\tanh^{-1}(\tanh(a+bx))^{3/2}-\frac{15}{4}b\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(3/2),x]
```


```
[Out] (15*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x -
ArcTanh[Tanh[a + b*x]])^2)/4 - (15*b*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])
*Sqrt[ArcTanh[Tanh[a + b*x]]])/4 + (5*b*Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(3/2
))/2 - (2*ArcTanh[Tanh[a + b*x]]^(5/2))/Sqrt[x]
```


Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{3/2}}\,dx = -\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{\sqrt{x}} + (5b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{\sqrt{x}}\,dx$$

$$= \frac{5}{2}b\sqrt{x}\tanh^{-1}(\tanh(a+bx))^{3/2} - \frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{\sqrt{x}} - \frac{1}{4}\Big(15b\Big(bx - \tanh^{-1}(\mathrm{t}$$

$$= -\frac{15}{4}b\sqrt{x}\left(bx - \tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))} + \frac{5}{2}b\sqrt{x}\tanh^{-1}(\mathrm{tanh}$$

$$= \frac{15}{4}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx - \tanh^{-1}(\tanh(a+bx))\right)^2 - \frac{15}{4}b\sqrt{x}\,\Big(bx$$

Mathematica [A] time = 0.0714554, size = 101, normalized size = 0.83

$$\frac{15}{4}\sqrt{b}\left(\tanh^{-1}(\tanh(a+bx)) - bx\right)^2\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))} + b\sqrt{x}\right) - \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-25bx\tanh^{-}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(3/2),x]
```

```
[Out] -(Sqrt[ArcTanh[Tanh[a + b*x]]]*(15*b^2*x^2 - 25*b*x*ArcTanh[Tanh[a + b*x]]
+ 8*ArcTanh[Tanh[a + b*x]]^2))/(4*Sqrt[x]) + (15*Sqrt[b]*(-(b*x) + ArcTanh[
Tanh[a + b*x]])^2*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/4
```

Maple [B] time = 0.116, size = 460, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^(3/2),x)
```

```
[Out] -2/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)*arctanh(tanh(b*x+a))^(7/2)+2*b/(arcta
nh(tanh(b*x+a))-b*x)*x^(1/2)*arctanh(tanh(b*x+a))^(5/2)+5/2*b/(arctanh(tanh
(b*x+a))-b*x)*a*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+15/4*b/(arctanh(tanh(b*x
+a))-b*x)*a^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+15/4*b^(1/2)/(arctanh(tanh
(b*x+a))-b*x)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^3+45/4*b^(1/
2)/(arctanh(tanh(b*x+a))-b*x)*a^2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(
1/2))*(arctanh(tanh(b*x+a))-b*x-a)+15/2*b/(arctanh(tanh(b*x+a))-b*x)*a*(arc
tanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+45/4*b^(1/2)/(a
rctanh(tanh(b*x+a))-b*x)*a*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(
arctanh(tanh(b*x+a))-b*x-a)^2+5/2*b/(arctanh(tanh(b*x+a))-b*x)*(arctanh(tan
h(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+15/4*b/(arctanh(tanh(b*
x+a))-b*x)*(arctanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2
)+15/4*b^(1/2)/(arctanh(tanh(b*x+a))-b*x)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b
*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(5/2)/x^(3/2), x)
```

Fricas [A] time = 2.1141, size = 351, normalized size = 2.9

$$\left[\frac{15\,a^{2}\sqrt{b}x\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(2\,b^{2}x^{2}+9\,abx-8\,a^{2}\right)\sqrt{bx+a}\sqrt{x}}{8\,x}, -\frac{15\,a^{2}\sqrt{-b}x\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)-\left(2\right.}{4\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*a^2*sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(2*b^2*x^2 + 9*a*b*x - 8*a^2)*sqrt(b*x + a)*sqrt(x))/x, -1/4*(15*a^2*sqrt(
-b)*x*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (2*b^2*x^2 + 9*a*b*x - 8
*a^2)*sqrt(b*x + a)*sqrt(x))/x]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.235 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{5/2}} dx$

Optimal. Leaf size=106

$$5b^2\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))} - 5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right) - \frac{2\tanh^{-1}(\tanh}{3x^{3/}}$$

```
[Out] -5*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - A
rcTanh[Tanh[a + b*x]]) + 5*b^2*Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]] - (10*b
*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*Sqrt[x]) - (2*ArcTanh[Tanh[a + b*x]]^(5/2
))/(3*x^(3/2))
```

Rubi [A] time = 0.0632354, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2168, 2169, 2165}

$$5b^2\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))} - 5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right) - \frac{2\tanh^{-1}(\tanh}{3x^{3/}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(5/2),x]
```

```
[Out] -5*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - A
rcTanh[Tanh[a + b*x]]) + 5*b^2*Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]] - (10*b
*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*Sqrt[x]) - (2*ArcTanh[Tanh[a + b*x]]^(5/2
))/(3*x^(3/2))
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{5/2}}\,dx = -\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{3x^{3/2}}+\frac{1}{3}(5b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{3/2}}\,dx$$

$$= -\frac{10b\tanh^{-1}(\tanh(a+bx))^{3/2}}{3\sqrt{x}}-\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{3x^{3/2}}+\left(5b^2\right)\int \frac{\sqrt{\tanh^{-1}(\tan}}{\sqrt{x}}$$

$$= 5b^2\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}-\frac{10b\tanh^{-1}(\tanh(a+bx))^{3/2}}{3\sqrt{x}}-\frac{2\tanh^{-1}(\tanh(a+}{3x^{3/2}}$$

$$= -5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+5b^2\sqrt{x}\sqrt{\tan}$$

Mathematica [A] time = 0.0635914, size = 97, normalized size = 0.92

$$\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-10bx\tanh^{-1}(\tanh(a+bx))-2\tanh^{-1}(\tanh(a+bx))^2+15b^2x^2\right)}{3x^{3/2}}+5b^{3/2}\left(\tanh^{-1}(\tanh(a-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(5/2),x]
```

```
[Out] (Sqrt[ArcTanh[Tanh[a + b*x]]]*(15*b^2*x^2 - 10*b*x*ArcTanh[Tanh[a + b*x]] -
 2*ArcTanh[Tanh[a + b*x]]^2))/(3*x^(3/2)) + 5*b^(3/2)*(-(b*x) + ArcTanh[Tan
h[a + b*x]])*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]]
```

Maple [B] time = 0.119, size = 501, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^(5/2),x)
```

```
[Out] -2/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)*arctanh(tanh(b*x+a))^(7/2)-8/3*b/(a
rctanh(tanh(b*x+a))-b*x)^2/x^(1/2)*arctanh(tanh(b*x+a))^(7/2)+8/3*b^2/(arct
anh(tanh(b*x+a))-b*x)^2*x^(1/2)*arctanh(tanh(b*x+a))^(5/2)+10/3*b^2/(arctan
h(tanh(b*x+a))-b*x)^2*a*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+5*b^2/(arctanh(t
anh(b*x+a))-b*x)^2*a^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+5*b^(3/2)/(arctan
h(tanh(b*x+a))-b*x)^2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^3+15
*b^(3/2)/(arctanh(tanh(b*x+a))-b*x)^2*a^2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b
*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)+10*b^2/(arctanh(tanh(b*x+a))-b*x
)^2*a*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+15*b^
(3/2)/(arctanh(tanh(b*x+a))-b*x)^2*a*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a)
)^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^2+10/3*b^2/(arctanh(tanh(b*x+a))-b*x)
^2*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+5*b^2/(a
rctanh(tanh(b*x+a))-b*x)^2*(arctanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(t
anh(b*x+a))^(1/2)+5*b^(3/2)/(arctanh(tanh(b*x+a))-b*x)^2*ln(b^(1/2)*x^(1/2)
+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(5/2)/x^(5/2), x)
```

Fricas [A] time = 2.15302, size = 362, normalized size = 3.42

$$\left[\frac{15\,ab^{\frac{3}{2}}x^2\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(3\,b^2x^2-14\,abx-2\,a^2\right)\sqrt{bx+a}\sqrt{x}}{6\,x^2},-\frac{15\,a\sqrt{-b}bx^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)-}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*a*b^(3/2)*x^2*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(3*b^2*x^2 - 14*a*b*x - 2*a^2)*sqrt(b*x + a)*sqrt(x))/x^2, -1/3*(15*a*sqrt
(-b)*b*x^2*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (3*b^2*x^2 - 14*a*b
*x - 2*a^2)*sqrt(b*x + a)*sqrt(x))/x^2]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.236 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{7/2}}\,dx$

Optimal. Leaf size=93

$$2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)-\frac{2b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}-\frac{2b\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}}-\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5x^{5/2}}$$

```
[Out] 2*b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]] - (2*b^2*
Sqrt[ArcTanh[Tanh[a + b*x]]])/Sqrt[x] - (2*b*ArcTanh[Tanh[a + b*x]]^(3/2))/
(3*x^(3/2)) - (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*x^(5/2))
```

Rubi [A] time = 0.0587725, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2168, 2165}

$$2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)-\frac{2b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}-\frac{2b\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}}-\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(7/2),x]
```

```
[Out] 2*b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]] - (2*b^2*
Sqrt[ArcTanh[Tanh[a + b*x]]])/Sqrt[x] - (2*b*ArcTanh[Tanh[a + b*x]]^(3/2))/
(3*x^(3/2)) - (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*x^(5/2))
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{7/2}}\,dx = -\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5x^{5/2}} + b\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{5/2}}\,dx$$
$$= -\frac{2b\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}} - \frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5x^{5/2}} + b^2\int \frac{\sqrt{\tanh^{-1}(\tanh(a}}{x^{3/2}}$$
$$= -\frac{2b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}} - \frac{2b\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}} - \frac{2\tanh^{-1}(\tanh(a+bx}{5x^{5/2}}$$
$$= 2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right) - \frac{2b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}} - \frac{2b\tanh^{-1}(}{}$$

Mathematica [A] time = 0.0513485, size = 95, normalized size = 1.02

$$-\frac{2\left(15b^2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))} - 15b^{5/2}x^{5/2}\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))} + b\sqrt{x}\right) + 5bx\tanh^{-1}(\tanh(a+bx))^{3/}\right.}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(7/2),x]
```

```
[Out] (-2*(15*b^2*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]] + 5*b*x*ArcTanh[Tanh[a + b*x]]
^(3/2) + 3*ArcTanh[Tanh[a + b*x]]^(5/2) - 15*b^(5/2)*x^(5/2)*Log[b*Sqrt[x]
+ Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]]))/(15*x^(5/2))
```

Maple [B] time = 0.124, size = 532, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^(7/2),x)
```

```
[Out] -2/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)*arctanh(tanh(b*x+a))^(7/2)-4/15*b/(
arctanh(tanh(b*x+a))-b*x)^2/x^(3/2)*arctanh(tanh(b*x+a))^(7/2)-16/15*b^2/(a
rctanh(tanh(b*x+a))-b*x)^3/x^(1/2)*arctanh(tanh(b*x+a))^(7/2)+16/15*b^3/(ar
ctanh(tanh(b*x+a))-b*x)^3*x^(1/2)*arctanh(tanh(b*x+a))^(5/2)+4/3*b^3/(arcta
nh(tanh(b*x+a))-b*x)^3*a*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+2*b^3/(arctanh(
tanh(b*x+a))-b*x)^3*a^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+2*b^(5/2)/(arcta
nh(tanh(b*x+a))-b*x)^3*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^3+6
*b^(5/2)/(arctanh(tanh(b*x+a))-b*x)^3*a^2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b
*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)+4*b^3/(arctanh(tanh(b*x+a))-b*x)
^3*a*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+6*b^(5
/2)/(arctanh(tanh(b*x+a))-b*x)^3*a*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^
(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^2+4/3*b^3/(arctanh(tanh(b*x+a))-b*x)^3*
(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+2*b^3/(arct
anh(tanh(b*x+a))-b*x)^3*(arctanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(tanh
(b*x+a))^(1/2)+2*b^(5/2)/(arctanh(tanh(b*x+a))-b*x)^3*ln(b^(1/2)*x^(1/2)+ar
ctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(5/2)/x^(7/2), x)
```

Fricas [A] time = 2.08364, size = 365, normalized size = 3.92

$$\left[\frac{15\,b^{\frac{5}{2}}x^{3}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\left(23\,b^{2}x^{2}+11\,abx+3\,a^{2}\right)\sqrt{bx+a}\sqrt{x}}{15\,x^{3}},-\frac{2\left(15\,\sqrt{-b}b^{2}x^{3}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+\right.}{1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] [1/15*(15*b^(5/2)*x^3*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*
(23*b^2*x^2 + 11*a*b*x + 3*a^2)*sqrt(b*x + a)*sqrt(x))/x^3, -2/15*(15*sqrt(
-b)*b^2*x^3*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (23*b^2*x^2 + 11*a
*b*x + 3*a^2)*sqrt(b*x + a)*sqrt(x))/x^3]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.237 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{9/2}}\,dx$

Optimal. Leaf size=35

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*x^(7/2)*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rubi [A] time = 0.0131392, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2167}

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(9/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*x^(7/2)*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{9/2}}\,dx=\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A] time = 0.0422256, size = 34, normalized size = 0.97

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{x^{7/2}\left(7bx-7\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(9/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(7/2))/(x^(7/2)*(7*b*x - 7*ArcTanh[Tanh[a + b*x]]
))
```

Maple [A] time = 0.121, size = 29, normalized size = 0.8

$$-\frac{2}{7\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-7\,bx}\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{\frac{7}{2}}x^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^(9/2),x)
```

```
[Out] -2/7/(arctanh(tanh(b*x+a))-b*x)/x^(7/2)*arctanh(tanh(b*x+a))^(7/2)
```

Maxima [A] time = 1.47513, size = 20, normalized size = 0.57

$$-\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,ax^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] -2/7*(b*x + a)^(7/2)/(a*x^(7/2))
```

Fricas [A] time = 2.10801, size = 100, normalized size = 2.86

$$-\frac{2\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)\sqrt{bx+a}}{7\,ax^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] -2/7*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b*x + a)/(a*x^(7/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24288, size = 45, normalized size = 1.29

$$-\frac{2\,(bx+a)^{\frac{7}{2}}b^8}{7\,((bx+a)b-ab)^{\frac{7}{2}}a|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] -2/7*(b*x + a)^(7/2)*b^8/(((b*x + a)*b - a*b)^(7/2)*a*abs(b))
```

3.238 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{11/2}} \, dx$

Optimal. Leaf size=72

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{9x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{4b\tanh^{-1}(\tanh(a+bx))^{7/2}}{63x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

```
[Out] (4*b*ArcTanh[Tanh[a + b*x]]^(7/2))/(63*x^(7/2)*(b*x - ArcTanh[Tanh[a + b*x]
])^2) + (2*ArcTanh[Tanh[a + b*x]]^(7/2))/(9*x^(9/2)*(b*x - ArcTanh[Tanh[a +
 b*x]]))
```

Rubi [A] time = 0.0327222, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2171, 2167}

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{9x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{4b\tanh^{-1}(\tanh(a+bx))^{7/2}}{63x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(11/2),x]
```

```
[Out] (4*b*ArcTanh[Tanh[a + b*x]]^(7/2))/(63*x^(7/2)*(b*x - ArcTanh[Tanh[a + b*x]
])^2) + (2*ArcTanh[Tanh[a + b*x]]^(7/2))/(9*x^(9/2)*(b*x - ArcTanh[Tanh[a +
 b*x]]))
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
```

```
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{11/2}}\,dx = \frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{9x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(2b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{9/2}}\,dx}{9\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$
$$= \frac{4b\tanh^{-1}(\tanh(a+bx))^{7/2}}{63x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{9x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A] time = 0.0453176, size = 48, normalized size = 0.67

$$\frac{2\left(9bx-7\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{7/2}}{63x^{9/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(11/2),x]
```

```
[Out] (2*(9*b*x - 7*ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(7/2))/(63*x^(
9/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

Maple [A] time = 0.134, size = 59, normalized size = 0.8

$$-\frac{2}{9\,\mathrm{Artanh}\,(\tanh(bx+a))-9\,bx}(\mathrm{Artanh}\,(\tanh(bx+a)))^{\frac{7}{2}}x^{-\frac{9}{2}}+\frac{4\,b}{63\,(\mathrm{Artanh}\,(\tanh(bx+a))-bx)^2}(\mathrm{Artanh}\,(\tanh($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^(11/2),x)
```

```
[Out] -2/9/(arctanh(tanh(b*x+a))-b*x)/x^(9/2)*arctanh(tanh(b*x+a))^(7/2)+4/63*b/(
arctanh(tanh(b*x+a))-b*x)^2/x^(7/2)*arctanh(tanh(b*x+a))^(7/2)
```

Maxima [A] time = 1.48213, size = 46, normalized size = 0.64

$$\frac{2\left(2\,b^{2}x^{2}-5\,abx-7\,a^{2}\right)(bx+a)^{\frac{5}{2}}}{63\,a^{2}x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] 2/63*(2*b^2*x^2 - 5*a*b*x - 7*a^2)*(b*x + a)^(5/2)/(a^2*x^(9/2))
```

Fricas [A] time = 2.06608, size = 130, normalized size = 1.81

$$\frac{2\left(2\,b^{4}x^{4}-ab^{3}x^{3}-15\,a^{2}b^{2}x^{2}-19\,a^{3}bx-7\,a^{4}\right)\sqrt{bx+a}}{63\,a^{2}x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] 2/63*(2*b^4*x^4 - a*b^3*x^3 - 15*a^2*b^2*x^2 - 19*a^3*b*x - 7*a^4)*sqrt(b*x
 + a)/(a^2*x^(9/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**(11/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27105, size = 80, normalized size = 1.11

$$\frac{\sqrt{2}\left(\frac{2\,\sqrt{2}(bx+a)b^{9}}{a^{2}}-\frac{9\,\sqrt{2}b^{9}}{a}\right)(bx+a)^{\frac{7}{2}}b}{63\,((bx+a)b-ab)^{\frac{9}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(11/2),x, algorithm="giac")
```

```
[Out] 1/63*sqrt(2)*(2*sqrt(2)*(b*x + a)*b^9/a^2 - 9*sqrt(2)*b^9/a)*(b*x + a)^(7/2
)*b/(((b*x + a)*b - a*b)^(9/2)*abs(b))
```

3.239 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{13/2}} dx$

Optimal. Leaf size=110

$$\frac{16b^2 \tanh^{-1}(\tanh(a+bx))^{7/2}}{693x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{11x^{11/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{8b\tanh^{-1}(\tanh(a+bx))^{7/2}}{99x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

```
[Out] (16*b^2*ArcTanh[Tanh[a + b*x]]^(7/2))/(693*x^(7/2)*(b*x - ArcTanh[Tanh[a +
b*x]])^3) + (8*b*ArcTanh[Tanh[a + b*x]]^(7/2))/(99*x^(9/2)*(b*x - ArcTanh[T
anh[a + b*x]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(7/2))/(11*x^(11/2)*(b*x - Ar
cTanh[Tanh[a + b*x]]))
```

Rubi [A] time = 0.057208, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2171, 2167}

$$\frac{16b^2 \tanh^{-1}(\tanh(a+bx))^{7/2}}{693x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{11x^{11/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{8b\tanh^{-1}(\tanh(a+bx))^{7/2}}{99x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(13/2),x]
```

```
[Out] (16*b^2*ArcTanh[Tanh[a + b*x]]^(7/2))/(693*x^(7/2)*(b*x - ArcTanh[Tanh[a +
b*x]])^3) + (8*b*ArcTanh[Tanh[a + b*x]]^(7/2))/(99*x^(9/2)*(b*x - ArcTanh[T
anh[a + b*x]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(7/2))/(11*x^(11/2)*(b*x - Ar
cTanh[Tanh[a + b*x]]))
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
```

```
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{13/2}}\,dx = \frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{11x^{11/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(4b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{11/2}}\,dx}{11\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

$$= \frac{8b\tanh^{-1}(\tanh(a+bx))^{7/2}}{99x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{11x^{11/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(8b^2 \ldots}{99\,(bx \ldots}$$

$$= \frac{16b^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{693x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{8b\tanh^{-1}(\tanh(a+bx))^{7/2}}{99x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{\ldots}{11x^{11}\ldots}$$

Mathematica [A] time = 0.0450775, size = 66, normalized size = 0.6

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}\left(-154bx\tanh^{-1}(\tanh(a+bx))+63\tanh^{-1}(\tanh(a+bx))^2+99b^2x^2\right)}{693x^{11/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(13/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(7/2)*(99*b^2*x^2 - 154*b*x*ArcTanh[Tanh[a + b*x]
] + 63*ArcTanh[Tanh[a + b*x]]^2))/(693*x^(11/2)*(b*x - ArcTanh[Tanh[a + b*x
]])^3)
```

Maple [A] time = 0.163, size = 105, normalized size = 1.

$$-\frac{2}{11\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-11\,bx}\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{\frac{7}{2}}x^{-\frac{11}{2}}-\frac{8\,b}{11\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-11\,bx}\Bigg(-\frac{\ldots}{9\,\mathrm{Artanh}\,(\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^(13/2),x)
```

```
[Out] -2/11/(arctanh(tanh(b*x+a))-b*x)/x^(11/2)*arctanh(tanh(b*x+a))^(7/2)-8/11*b
/(arctanh(tanh(b*x+a))-b*x)*(-1/9/(arctanh(tanh(b*x+a))-b*x)/x^(9/2)*arctan
h(tanh(b*x+a))^(7/2)+2/63*b/(arctanh(tanh(b*x+a))-b*x)^2/x^(7/2)*arctanh(ta
nh(b*x+a))^(7/2))
```

Maxima [A] time = 1.47315, size = 61, normalized size = 0.55

$$-\frac{2\left(8\,b^{3}x^{3}-20\,ab^{2}x^{2}+35\,a^{2}bx+63\,a^{3}\right)(bx+a)^{\frac{5}{2}}}{693\,a^{3}x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(13/2),x, algorithm="maxima")
```

```
[Out] -2/693*(8*b^3*x^3 - 20*a*b^2*x^2 + 35*a^2*b*x + 63*a^3)*(b*x + a)^(5/2)/(a^
3*x^(11/2))
```

Fricas [A] time = 2.04776, size = 162, normalized size = 1.47

$$-\frac{2\left(8\,b^{5}x^{5}-4\,ab^{4}x^{4}+3\,a^{2}b^{3}x^{3}+113\,a^{3}b^{2}x^{2}+161\,a^{4}bx+63\,a^{5}\right)\sqrt{bx+a}}{693\,a^{3}x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(13/2),x, algorithm="fricas")
```

```
[Out] -2/693*(8*b^5*x^5 - 4*a*b^4*x^4 + 3*a^2*b^3*x^3 + 113*a^3*b^2*x^2 + 161*a^4
*b*x + 63*a^5)*sqrt(b*x + a)/(a^3*x^(11/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**(13/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19404, size = 105, normalized size = 0.95

$$-\frac{\sqrt{2}\left(\frac{99\sqrt{2}b^{11}}{a}+4\left(\frac{2\sqrt{2}(bx+a)b^{11}}{a^3}-\frac{11\sqrt{2}b^{11}}{a^2}\right)(bx+a)\right)(bx+a)^{\frac{7}{2}}b}{693\left((bx+a)b-ab\right)^{\frac{11}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(13/2),x, algorithm="giac")
```

```
[Out] -1/693*sqrt(2)*(99*sqrt(2)*b^11/a + 4*(2*sqrt(2)*(b*x + a)*b^11/a^3 - 11*sq
rt(2)*b^11/a^2)*(b*x + a))*(b*x + a)^(7/2)*b/(((b*x + a)*b - a*b)^(11/2)*ab
s(b))
```

3.240 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{15/2}} dx$

Optimal. Leaf size=148

$$\frac{16b^2 \tanh^{-1}(\tanh(a+bx))^{7/2}}{429x^{9/2} \left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{32b^3 \tanh^{-1}(\tanh(a+bx))^{7/2}}{3003x^{7/2} \left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{2 \tanh^{-1}(\tanh(a+bx))^{7/2}}{13x^{13/2} \left(bx-\tanh^{-1}(\tanh(a+bx)\right.}$$

```
[Out] (32*b^3*ArcTanh[Tanh[a + b*x]]^(7/2))/(3003*x^(7/2)*(b*x - ArcTanh[Tanh[a +
 b*x]])^4) + (16*b^2*ArcTanh[Tanh[a + b*x]]^(7/2))/(429*x^(9/2)*(b*x - ArcT
anh[Tanh[a + b*x]])^3) + (12*b*ArcTanh[Tanh[a + b*x]]^(7/2))/(143*x^(11/2)*
(b*x - ArcTanh[Tanh[a + b*x]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(7/2))/(13*x^
(13/2)*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rubi [A] time = 0.0786454, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2171, 2167}

$$\frac{16b^2 \tanh^{-1}(\tanh(a+bx))^{7/2}}{429x^{9/2} \left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{32b^3 \tanh^{-1}(\tanh(a+bx))^{7/2}}{3003x^{7/2} \left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{2 \tanh^{-1}(\tanh(a+bx))^{7/2}}{13x^{13/2} \left(bx-\tanh^{-1}(\tanh(a+bx)\right.}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(15/2),x]
```

```
[Out] (32*b^3*ArcTanh[Tanh[a + b*x]]^(7/2))/(3003*x^(7/2)*(b*x - ArcTanh[Tanh[a +
 b*x]])^4) + (16*b^2*ArcTanh[Tanh[a + b*x]]^(7/2))/(429*x^(9/2)*(b*x - ArcT
anh[Tanh[a + b*x]])^3) + (12*b*ArcTanh[Tanh[a + b*x]]^(7/2))/(143*x^(11/2)*
(b*x - ArcTanh[Tanh[a + b*x]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(7/2))/(13*x^
(13/2)*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{15/2}}\,dx = \frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{13x^{13/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(6b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{13/2}}\,dx}{13\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

$$= \frac{12b\tanh^{-1}(\tanh(a+bx))^{7/2}}{143x^{11/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{13x^{13/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(2\cdots}{143\,(\cdots}$$

$$= \frac{16b^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{429x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{12b\tanh^{-1}(\tanh(a+bx))^{7/2}}{143x^{11/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{\cdots}{13\cdots}$$

$$= \frac{32b^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{3003x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4} + \frac{16b^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{429x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{\cdots}{14\cdots}$$

Mathematica [A] time = 0.0713255, size = 82, normalized size = 0.55

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}\left(-1001b^2x^2\tanh^{-1}(\tanh(a+bx))+819bx\tanh^{-1}(\tanh(a+bx))^2-231\tanh^{-1}(\tanh(a+bx\cdots\right.}{3003x^{13/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(15/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(7/2)*(429*b^3*x^3 - 1001*b^2*x^2*ArcTanh[Tanh[a
+ b*x]] + 819*b*x*ArcTanh[Tanh[a + b*x]]^2 - 231*ArcTanh[Tanh[a + b*x]]^3))
/(3003*x^(13/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4)
```

Maple [A] time = 0.241, size = 151, normalized size = 1.

$$-\frac{2}{13\,\mathrm{Artanh}(\tanh(bx+a))-13\,bx}(\mathrm{Artanh}(\tanh(bx+a)))^{\frac{7}{2}}x^{-\frac{13}{2}}-\frac{12\,b}{13\,\mathrm{Artanh}(\tanh(bx+a))-13\,bx}\left(-\frac{\cdots}{11\,\mathrm{Artanh}\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^(15/2),x)
```

```
[Out] -2/13/(arctanh(tanh(b*x+a))-b*x)/x^(13/2)*arctanh(tanh(b*x+a))^(7/2)-12/13*
b/(arctanh(tanh(b*x+a))-b*x)*(-1/11/(arctanh(tanh(b*x+a))-b*x)/x^(11/2)*arc
tanh(tanh(b*x+a))^(7/2)-4/11*b/(arctanh(tanh(b*x+a))-b*x)*(-1/9/(arctanh(ta
nh(b*x+a))-b*x)/x^(9/2)*arctanh(tanh(b*x+a))^(7/2)+2/63*b/(arctanh(tanh(b*x
+a))-b*x)^2/x^(7/2)*arctanh(tanh(b*x+a))^(7/2)))
```

Maxima [A] time = 1.48114, size = 76, normalized size = 0.51

$$\frac{2\left(16\,b^4x^4-40\,ab^3x^3+70\,a^2b^2x^2-105\,a^3bx-231\,a^4\right)(bx+a)^{\frac{5}{2}}}{3003\,a^4x^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(15/2),x, algorithm="maxima")
```

```
[Out] 2/3003*(16*b^4*x^4 - 40*a*b^3*x^3 + 70*a^2*b^2*x^2 - 105*a^3*b*x - 231*a^4)
*(b*x + a)^(5/2)/(a^4*x^(13/2))
```

Fricas [A] time = 2.29772, size = 186, normalized size = 1.26

$$\frac{2\left(16\,b^6x^6-8\,ab^5x^5+6\,a^2b^4x^4-5\,a^3b^3x^3-371\,a^4b^2x^2-567\,a^5bx-231\,a^6\right)\sqrt{bx+a}}{3003\,a^4x^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(15/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(16*b^6*x^6 - 8*a*b^5*x^5 + 6*a^2*b^4*x^4 - 5*a^3*b^3*x^3 - 371*a^4*
b^2*x^2 - 567*a^5*b*x - 231*a^6)*sqrt(b*x + a)/(a^4*x^(13/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**(15/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22904, size = 131, normalized size = 0.89

$$-\frac{\sqrt{2}\left(\frac{429\sqrt{2}b^{13}}{a}-2\left(\frac{143\sqrt{2}b^{13}}{a^2}+4\left(\frac{2\sqrt{2}(bx+a)b^{13}}{a^4}-\frac{13\sqrt{2}b^{13}}{a^3}\right)(bx+a)\right)(bx+a)\right)(bx+a)^{\frac{7}{2}}b}{3003\left((bx+a)b-ab\right)^{\frac{13}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(15/2),x, algorithm="giac")
```

```
[Out] -1/3003*sqrt(2)*(429*sqrt(2)*b^13/a - 2*(143*sqrt(2)*b^13/a^2 + 4*(2*sqrt(2
)*(b*x + a)*b^13/a^4 - 13*sqrt(2)*b^13/a^3)*(b*x + a))*(b*x + a))*(b*x + a)
^(7/2)*b/(((b*x + a)*b - a*b)^(13/2)*abs(b))
```

3.241 $\int \frac{x^{5/2}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}} \, dx$

Optimal. Leaf size=145

$$\frac{5x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{12b^2}+\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx\right.}{8b^{7/2}}$$

```
[Out] (5*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]])^3)/(8*b^(7/2)) + (x^(5/2)*Sqrt[ArcTanh[Tanh[a + b*x]]])/(3*b
) + (5*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])
/(12*b^2) + (5*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a
 + b*x]]])/(8*b^3)
```

Rubi [A] time = 0.0808685, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2169, 2165}

$$\frac{5x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{12b^2}+\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx\right.}{8b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```



```
[Out] (5*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]])^3)/(8*b^(7/2)) + (x^(5/2)*Sqrt[ArcTanh[Tanh[a + b*x]]])/(3*b
) + (5*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])
/(12*b^2) + (5*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a
 + b*x]]])/(8*b^3)
```


Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
```

```
-2]
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{x^{5/2}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b} - \frac{\left(5\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\right)\int \frac{x^{3/2}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}}{6b}$$

$$= \frac{x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b} + \frac{5x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+}}{12b^2}$$

$$= \frac{x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b} + \frac{5x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+}}{12b^2}$$

$$= \frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8b^{7/2}} + \frac{x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+}}{3b}$$

Mathematica [A] time = 0.084714, size = 105, normalized size = 0.72

$$\frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-40bx\tanh^{-1}(\tanh(a+bx))+15\tanh^{-1}(\tanh(a+bx))^2+33b^2x^2\right)}{24b^3} + \frac{5\left(bx-\tanh^{-1}(\tan\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(33*b^2*x^2 - 40*b*x*ArcTanh[Tanh[a +
 b*x]] + 15*ArcTanh[Tanh[a + b*x]]^2))/(24*b^3) + (5*(b*x - ArcTanh[Tanh[a
+ b*x]])^3*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/(8*b^(7/2
))
```

Maple [B] time = 0.171, size = 304, normalized size = 2.1

$$\frac{1}{3b}x^{\frac{5}{2}}\sqrt{\operatorname{Artanh}(\tanh(bx+a))}-\frac{5a}{12b^2}x^{\frac{3}{2}}\sqrt{\operatorname{Artanh}(\tanh(bx+a))}+\frac{5a^2}{8b^3}\sqrt{x}\sqrt{\operatorname{Artanh}(\tanh(bx+a))}-\frac{5a^3}{8}\ln\left(\sqrt{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 1/3*x^(5/2)*arctanh(tanh(b*x+a))^(1/2)/b-5/12/b^2*a*x^(3/2)*arctanh(tanh(b*
x+a))^(1/2)+5/8/b^3*a^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)-5/8/b^(7/2)*ln(b
^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^3-15/8/b^(7/2)*a^2*ln(b^(1/2)*
x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)+5/4/b^3*a*
(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)-15/8/b^(7/2
)*a*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*
x-a)^2-5/12/b^2*(arctanh(tanh(b*x+a))-b*x-a)*x^(3/2)*arctanh(tanh(b*x+a))^(
1/2)+5/8/b^3*(arctanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(tanh(b*x+a))^(1
/2)-5/8/b^(7/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tan
h(b*x+a))-b*x-a)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{\operatorname{artanh}(\tanh(bx+a))}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/sqrt(arctanh(tanh(b*x + a))), x)
```

Fricas [A] time = 2.45734, size = 369, normalized size = 2.54

$$\left[\frac{15a^3\sqrt{b}\log\left(2bx-2\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(8b^3x^2-10ab^2x+15a^2b\right)\sqrt{bx+a}\sqrt{x}}{48b^4},\ \frac{15a^3\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*a^3*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*
(8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^4, 1/24*(15*a^
3*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (8*b^3*x^2 - 10*a*b
^2*x + 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^4]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15882, size = 86, normalized size = 0.59

$$\frac{1}{24}\sqrt{bx+a}\left(2x\left(\frac{4x}{b}-\frac{5a}{b^2}\right)+\frac{15a^2}{b^3}\right)\sqrt{x}+\frac{5a^3\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{8b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(b*x + a)*(2*x*(4*x/b - 5*a/b^2) + 15*a^2/b^3)*sqrt(x) + 5/8*a^3*l
og(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(7/2)
```

3.242 $\int \frac{x^{3/2}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=107

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4b^{5/2}}+\frac{3\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+bx\right.}{4b^2}$$

```
[Out] (3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]])^2)/(4*b^(5/2)) + (x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*x]]])/(2*b
) + (3*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])
/(4*b^2)
```

Rubi [A] time = 0.0515712, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2169, 2165}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4b^{5/2}}+\frac{3\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+bx\right.}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```



```
[Out] (3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]])^2)/(4*b^(5/2)) + (x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*x]]])/(2*b
) + (3*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])
/(4*b^2)
```


Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{x^{3/2}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{2b} - \frac{\left(3\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\right)\int \frac{\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}}{4b}$$

$$= \frac{x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{2b} + \frac{3\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+}}{4b^2}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4b^{5/2}} + \frac{x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+}}{2b}$$

Mathematica [A] time = 0.075713, size = 88, normalized size = 0.82

$$\frac{\sqrt{b}\sqrt{x}\left(5bx-3\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}+3\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tan}\right.}{4b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (Sqrt[b]*Sqrt[x]*(5*b*x - 3*ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b
*x]]] + 3*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[
ArcTanh[Tanh[a + b*x]]]])/(4*b^(5/2))
```

Maple [B] time = 0.167, size = 174, normalized size = 1.6

$$\frac{1}{2b}x^{\frac{3}{2}}\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)} - \frac{3a}{4b^2}\sqrt{x}\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)} + \frac{3a^2}{4}\ln\left(\sqrt{b}\sqrt{x}+\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}\right)b^{-\frac{5}{2}} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 1/2*x^(3/2)*arctanh(tanh(b*x+a))^(1/2)/b-3/4/b^2*a*x^(1/2)*arctanh(tanh(b*x
+a))^(1/2)+3/4/b^(5/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^2+3
/2/b^(5/2)*a*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b
*x+a))-b*x-a)-3/4/b^2*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x
+a))^(1/2)+3/4/b^(5/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arct
anh(tanh(b*x+a))-b*x-a)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/sqrt(arctanh(tanh(b*x + a))), x)
```

Fricas [A] time = 2.29574, size = 316, normalized size = 2.95

$$\left[\frac{3\,a^{2}\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(2\,b^{2}x-3\,ab\right)\sqrt{bx+a}\sqrt{x}}{8\,b^{3}},-\frac{3\,a^{2}\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)-\left(2\,b^{2}x-3\,ab\right)\sqrt{}}{4\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a^2*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(2
*b^2*x - 3*a*b)*sqrt(b*x + a)*sqrt(x))/b^3, -1/4*(3*a^2*sqrt(-b)*arctan(sqr
t(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (2*b^2*x - 3*a*b)*sqrt(b*x + a)*sqrt(x))
/b^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(x**(3/2)/sqrt(atanh(tanh(a + b*x))), x)
```

Giac [A] time = 1.16519, size = 70, normalized size = 0.65

$$\frac{1}{4}\sqrt{bx+a}\sqrt{x}\left(\frac{2x}{b}-\frac{3a}{b^2}\right)-\frac{3a^2\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{4b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b*x + a)*sqrt(x)*(2*x/b - 3*a/b^2) - 3/4*a^2*log(abs(-sqrt(b)*sqrt
(x) + sqrt(b*x + a)))/b^(5/2)
```

3.243 $\int \frac{\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}} \, dx$

Optimal. Leaf size=63

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^{3/2}}+\frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}$$

```
[Out] (ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[Tan
h[a + b*x]]))/b^(3/2) + (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]])/b
```

Rubi [A] time = 0.0283892, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2169, 2165}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^{3/2}}+\frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[Tan
h[a + b*x]]))/b^(3/2) + (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]])/b
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] && !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) && !ILtQ[m + n,
-2]
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
```

```
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\int \frac{1}{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{2b}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^{3/2}} + \frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}$$

Mathematica [A] time = 0.0539003, size = 66, normalized size = 1.05

$$\frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}+b\sqrt{x}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]])/b - ((-(b*x) + ArcTanh[Tanh[a + b*x]
])*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/b^(3/2)
```

Maple [A] time = 0.173, size = 80, normalized size = 1.3

$$\frac{1}{b}\sqrt{x}\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)} - a\ln\left(\sqrt{b}\sqrt{x}+\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}\right)b^{-\frac{3}{2}} - \left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx-a\right)\ln\left(\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b-1/b^(3/2)*ln(b^(1/2)*x^(1/2)+arctanh(t
anh(b*x+a))^(1/2))*a-1/b^(3/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2
))*(arctanh(tanh(b*x+a))-b*x-a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/sqrt(arctanh(tanh(b*x + a))), x)
```

Fricas [A] time = 2.15033, size = 255, normalized size = 4.05

$$\left[\frac{a\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,\sqrt{bx+a}b\sqrt{x}}{2\,b^2},\,\frac{a\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+a}b\sqrt{x}}{b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*sqrt(b
*x + a)*b*sqrt(x))/b^2, (a*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x
))) + sqrt(b*x + a)*b*sqrt(x))/b^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(x)/sqrt(atanh(tanh(a + b*x))), x)
```

Giac [A] time = 1.17339, size = 51, normalized size = 0.81

$$\frac{a \log \left(\left| -\sqrt{b}\sqrt{x} + \sqrt{bx + a} \right| \right)}{b^{\frac{3}{2}}} + \frac{\sqrt{bx + a}\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] a*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(3/2) + sqrt(b*x + a)*sqrt(x
)/b
```

3.244 $\int \frac{1}{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=30

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{b}}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]])/Sqrt[b]
```

Rubi [A] time = 0.0120342, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2165}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]])/Sqrt[b]
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.0304194, size = 33, normalized size = 1.1

$$\frac{2\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}+b\sqrt{x}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (2*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/Sqrt[b]
```

Maple [A] time = 0.095, size = 24, normalized size = 0.8

$$2\,\frac{\ln\left(\sqrt{b}\sqrt{x}+\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))/b^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x}\sqrt{\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x)*sqrt(arctanh(tanh(b*x + a)))), x)
```

Fricas [A] time = 2.07482, size = 162, normalized size = 5.4

$$\left[\frac{\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a)/sqrt(b), -2*sqrt(-b)*arct
an(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x)))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x}\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*sqrt(atanh(tanh(a + b*x)))), x)
```

Giac [A] time = 1.14891, size = 31, normalized size = 1.03

$$-\frac{2\,\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/sqrt(b)
```

3.245 $\int \frac{1}{x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=33

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] (2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rubi [A] time = 0.0133372, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2167}

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx=\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A] time = 0.0392659, size = 32, normalized size = 0.97

$$-\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (-2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(Sqrt[x]*(-(b*x) + ArcTanh[Tanh[a + b*x]]
))
```

Maple [A] time = 0.17, size = 29, normalized size = 0.9

$$-2\,\frac{\sqrt{\mathrm{Artanh}\,(\tanh\,(bx+a))}}{(\mathrm{Artanh}\,(\tanh\,(bx+a))-bx)\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] -2/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)*arctanh(tanh(b*x+a))^(1/2)
```

Maxima [A] time = 1.47852, size = 20, normalized size = 0.61

$$-\frac{2\,\sqrt{bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(b*x + a)/(a*sqrt(x))
```

Fricas [A] time = 2.06163, size = 41, normalized size = 1.24

$$-\frac{2\sqrt{bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*x + a)/(a*sqrt(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{3}{2}}\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/(x**(3/2)*sqrt(atanh(tanh(a + b*x)))), x)
```

Giac [A] time = 1.14597, size = 41, normalized size = 1.24

$$\frac{4\sqrt{b}}{\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 4*sqrt(b)/((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)
```

3.246 $\int \frac{1}{x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{4b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

```
[Out] (4*b*Sqrt[ArcTanh[Tanh[a + b*x]]])/(3*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]]
)^2) + (2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a +
b*x]]))
```

Rubi [A] time = 0.0316608, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2171, 2167}

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{4b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (4*b*Sqrt[ArcTanh[Tanh[a + b*x]]])/(3*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]]
)^2) + (2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a +
b*x]]))
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
```

```
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(2b)\int \frac{1}{x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

$$= \frac{4b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A] time = 0.0405726, size = 46, normalized size = 0.64

$$-\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(\tanh^{-1}(\tanh(a+bx))-3bx\right)}{3x^{3/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (-2*Sqrt[ArcTanh[Tanh[a + b*x]]]*(-3*b*x + ArcTanh[Tanh[a + b*x]]))/(3*x^(3
/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

Maple [A] time = 0.168, size = 59, normalized size = 0.8

$$-\frac{2}{3\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-3\,bx}\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}x^{-\frac{3}{2}}+\frac{4\,b}{3\,\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^2}\sqrt{\mathrm{Artanh}\left(\tanh\left(bx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] -2/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)*arctanh(tanh(b*x+a))^(1/2)+4/3*b/(a
rctanh(tanh(b*x+a))-b*x)^2/x^(1/2)*arctanh(tanh(b*x+a))^(1/2)
```

Maxima [A] time = 1.48287, size = 45, normalized size = 0.62

$$\frac{2\left(2b^2x^2 + abx - a^2\right)}{3\sqrt{bx + a}a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(2*b^2*x^2 + a*b*x - a^2)/(sqrt(b*x + a)*a^2*x^(3/2))
```

Fricas [A] time = 2.12291, size = 61, normalized size = 0.85

$$\frac{2(2bx - a)\sqrt{bx + a}}{3a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*b*x - a)*sqrt(b*x + a)/(a^2*x^(3/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [A] time = 1.15603, size = 74, normalized size = 1.03

$$\frac{8\left(3\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{2}-a\right)b^{\frac{3}{2}}}{3\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{2}-a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*(3*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)*b^(3/2)/((sqrt(b)*sqrt(x) -
 sqrt(b*x + a))^2 - a)^3
```

3.247 $\int \frac{1}{x^{7/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=110

$$\frac{16b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{15\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{8b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{15x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] (16*b^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(15*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b
*x]])^3) + (8*b*Sqrt[ArcTanh[Tanh[a + b*x]]])/(15*x^(3/2)*(b*x - ArcTanh[Ta
nh[a + b*x]])^2) + (2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(5*x^(5/2)*(b*x - ArcTa
nh[Tanh[a + b*x]]))
```

Rubi [A] time = 0.0558492, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2171, 2167}

$$\frac{16b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{15\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{8b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{15x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (16*b^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(15*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b
*x]])^3) + (8*b*Sqrt[ArcTanh[Tanh[a + b*x]]])/(15*x^(3/2)*(b*x - ArcTanh[Ta
nh[a + b*x]])^2) + (2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(5*x^(5/2)*(b*x - ArcTa
nh[Tanh[a + b*x]]))
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(4b)\int \frac{1}{x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

$$= \frac{8b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{15x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(8b^2}{15\,(b}$$

$$= \frac{16b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{15\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{8b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{15x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{}{5x^{5}}$$

Mathematica [A] time = 0.0483617, size = 66, normalized size = 0.6

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-10bx\tanh^{-1}(\tanh(a+bx))+3\tanh^{-1}(\tanh(a+bx))^2+15b^2x^2\right)}{15x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (2*Sqrt[ArcTanh[Tanh[a + b*x]]]*(15*b^2*x^2 - 10*b*x*ArcTanh[Tanh[a + b*x]]
 + 3*ArcTanh[Tanh[a + b*x]]^2))/(15*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])^
3)
```

Maple [A] time = 0.171, size = 105, normalized size = 1.

$$-\frac{2}{5\,\mathrm{Artanh}\,(\tanh\,(bx+a))-5\,bx}\sqrt{\mathrm{Artanh}\,(\tanh\,(bx+a))}x^{-\frac{5}{2}}-\frac{8\,b}{5\,\mathrm{Artanh}\,(\tanh\,(bx+a))-5\,bx}\left(-\frac{1}{3\,\mathrm{Artanh}\,(\tanh\,(b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] -2/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)*arctanh(tanh(b*x+a))^(1/2)-8/5*b/(a
rctanh(tanh(b*x+a))-b*x)*(-1/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)*arctanh(t
anh(b*x+a))^(1/2)+2/3*b/(arctanh(tanh(b*x+a))-b*x)^2/x^(1/2)*arctanh(tanh(b
*x+a))^(1/2))
```

Maxima [A] time = 1.48781, size = 61, normalized size = 0.55

$$-\frac{2\left(8\,b^3x^3+4\,ab^2x^2-a^2bx+3\,a^3\right)}{15\,\sqrt{bx+a}a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/15*(8*b^3*x^3 + 4*a*b^2*x^2 - a^2*b*x + 3*a^3)/(sqrt(b*x + a)*a^3*x^(5/2
))
```

Fricas [A] time = 2.00205, size = 88, normalized size = 0.8

$$-\frac{2\left(8\,b^2x^2-4\,abx+3\,a^2\right)\sqrt{bx+a}}{15\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(8*b^2*x^2 - 4*a*b*x + 3*a^2)*sqrt(b*x + a)/(a^3*x^(5/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17435, size = 104, normalized size = 0.95

$$\frac{32\left(10\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{4}-5\,a\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{2}+a^{2}\right)b^{\frac{5}{2}}}{15\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{2}-a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 32/15*(10*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^4 - 5*a*(sqrt(b)*sqrt(x) - sqrt
(b*x + a))^2 + a^2)*b^(5/2)/((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^5
```

3.248 $\int \frac{1}{x^{9/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=148

$$\frac{16b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{35x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{32b^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{35\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{12b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+$$

```
[Out] (32*b^3*Sqrt[ArcTanh[Tanh[a + b*x]]])/(35*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b
*x]])^4) + (16*b^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(35*x^(3/2)*(b*x - ArcTanh
[Tanh[a + b*x]])^3) + (12*b*Sqrt[ArcTanh[Tanh[a + b*x]]])/(35*x^(5/2)*(b*x
- ArcTanh[Tanh[a + b*x]])^2) + (2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(7*x^(7/2)*
(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rubi [A] time = 0.0830278, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2171, 2167}

$$\frac{16b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{35x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{32b^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{35\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{12b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(9/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```


```
[Out] (32*b^3*Sqrt[ArcTanh[Tanh[a + b*x]]])/(35*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b
*x]])^4) + (16*b^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(35*x^(3/2)*(b*x - ArcTanh
[Tanh[a + b*x]])^3) + (12*b*Sqrt[ArcTanh[Tanh[a + b*x]]])/(35*x^(5/2)*(b*x
- ArcTanh[Tanh[a + b*x]])^2) + (2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(7*x^(7/2)*
(b*x - ArcTanh[Tanh[a + b*x]]))
```


Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{9/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(6b)\int \frac{1}{x^{7/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{7\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

$$= \frac{12b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(24b}{35\,(b}$$

$$= \frac{16b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{35x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{12b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{}{7x}$$

$$= \frac{32b^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{35\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4} + \frac{16b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{35x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{}{35x}$$

Mathematica [A] time = 0.0522516, size = 82, normalized size = 0.55

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-35b^2x^2\tanh^{-1}(\tanh(a+bx))+21bx\tanh^{-1}(\tanh(a+bx))^2-5\tanh^{-1}(\tanh(a+bx))^3+3\right.}{35x^{7/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(9/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (2*Sqrt[ArcTanh[Tanh[a + b*x]]]*(35*b^3*x^3 - 35*b^2*x^2*ArcTanh[Tanh[a + b
*x]] + 21*b*x*ArcTanh[Tanh[a + b*x]]^2 - 5*ArcTanh[Tanh[a + b*x]]^3))/(35*x
^(7/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4)
```

Maple [A] time = 0.169, size = 151, normalized size = 1.

$$-\frac{2}{7\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-7\,bx}\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}x^{-\frac{7}{2}}-\frac{12\,b}{7\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-7\,bx}\left(-\frac{1}{5\,\mathrm{Artanh}\left(\tanh\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(9/2)/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] -2/7/(arctanh(tanh(b*x+a))-b*x)/x^(7/2)*arctanh(tanh(b*x+a))^(1/2)-12/7*b/(
arctanh(tanh(b*x+a))-b*x)*(-1/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)*arctanh(
tanh(b*x+a))^(1/2)-4/5*b/(arctanh(tanh(b*x+a))-b*x)*(-1/3/(arctanh(tanh(b*x
+a))-b*x)/x^(3/2)*arctanh(tanh(b*x+a))^(1/2)+2/3*b/(arctanh(tanh(b*x+a))-b*
x)^2/x^(1/2)*arctanh(tanh(b*x+a))^(1/2)))
```

Maxima [A] time = 1.50141, size = 74, normalized size = 0.5

$$\frac{2\left(16\,b^{4}x^{4}+8\,ab^{3}x^{3}-2\,a^{2}b^{2}x^{2}+a^{3}bx-5\,a^{4}\right)}{35\,\sqrt{bx+a}a^{4}x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(16*b^4*x^4 + 8*a*b^3*x^3 - 2*a^2*b^2*x^2 + a^3*b*x - 5*a^4)/(sqrt(b*x
 + a)*a^4*x^(7/2))
```

Fricas [A] time = 2.09876, size = 109, normalized size = 0.74

$$\frac{2\left(16\,b^{3}x^{3}-8\,ab^{2}x^{2}+6\,a^{2}bx-5\,a^{3}\right)\sqrt{bx+a}}{35\,a^{4}x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*b^3*x^3 - 8*a*b^2*x^2 + 6*a^2*b*x - 5*a^3)*sqrt(b*x + a)/(a^4*x^(7
/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(9/2)/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17013, size = 139, normalized size = 0.94

$$\frac{64\left(35\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{6}-21\,a\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{4}+7\,a^{2}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{2}-a^{3}\right)b^{\frac{7}{2}}}{35\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{2}-a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 64/35*(35*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^6 - 21*a*(sqrt(b)*sqrt(x) - sqr
t(b*x + a))^4 + 7*a^2*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a^3)*b^(7/2)/((
sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^7
```

3.249 $\int \frac{x^{7/2}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=166

$$\frac{7x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b^2}+\frac{35x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}{12b^3}+\frac{35\sqrt{x}\left(bx-\tanh^{-1}(\text{ta}\right.}{}$$

```
[Out] (35*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[
Tanh[a + b*x]])^3)/(8*b^(9/2)) - (2*x^(7/2))/(b*Sqrt[ArcTanh[Tanh[a + b*x]]
]) + (7*x^(5/2)*Sqrt[ArcTanh[Tanh[a + b*x]]])/(3*b^2) + (35*x^(3/2)*(b*x -
ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/(12*b^3) + (35*Sqrt[x
]*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(8*b^4)
```

Rubi [A] time = 0.107418, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2168, 2169, 2165}

$$\frac{7x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b^2}+\frac{35x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}{12b^3}+\frac{35\sqrt{x}\left(bx-\tanh^{-1}(\text{ta}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```


```
[Out] (35*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[
Tanh[a + b*x]])^3)/(8*b^(9/2)) - (2*x^(7/2))/(b*Sqrt[ArcTanh[Tanh[a + b*x]]
]) + (7*x^(5/2)*Sqrt[ArcTanh[Tanh[a + b*x]]])/(3*b^2) + (35*x^(3/2)*(b*x -
ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/(12*b^3) + (35*Sqrt[x
]*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(8*b^4)
```


Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
```

```
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{x^{7/2}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2x^{7/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{7\int \frac{x^{5/2}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{b}$$

$$= -\frac{2x^{7/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{7x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b^2} - \frac{\left(35\left(-bx+\tanh^{-1}(\tan\right.\right.}{}$$

$$= -\frac{2x^{7/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{7x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b^2} + \frac{35x^{3/2}\left(bx-\tanh^{-1}(\tan\right.}{}$$

$$= -\frac{2x^{7/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{7x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b^2} + \frac{35x^{3/2}\left(bx-\tanh^{-1}(\tan\right.}{}$$

$$= \frac{35\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8b^{9/2}} - \frac{2x^{7/2}}{b\sqrt{\tanh^{-1}(\tanh(a+\right.}$$

Mathematica [A] time = 0.102065, size = 122, normalized size = 0.73

$$\frac{\sqrt{x}\left(231b^2x^2\tanh^{-1}(\tanh(a+bx))-280bx\tanh^{-1}(\tanh(a+bx))^2+105\tanh^{-1}(\tanh(a+bx))^3-48b^3x^3\right)}{24b^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{35\left(bx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (Sqrt[x]*(-48*b^3*x^3 + 231*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 280*b*x*ArcTan
h[Tanh[a + b*x]]^2 + 105*ArcTanh[Tanh[a + b*x]]^3))/(24*b^4*Sqrt[ArcTanh[Ta
nh[a + b*x]]]) + (35*(b*x - ArcTanh[Tanh[a + b*x]])^3*Log[b*Sqrt[x] + Sqrt[
b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/(8*b^(9/2))
```

Maple [B] time = 0.124, size = 428, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 1/3*x^(7/2)/b/arctanh(tanh(b*x+a))^(1/2)-7/12/b^2*a*x^(5/2)/arctanh(tanh(b*
x+a))^(1/2)+35/24/b^3*a^2*x^(3/2)/arctanh(tanh(b*x+a))^(1/2)+35/8/b^4*a^3*x
^(1/2)/arctanh(tanh(b*x+a))^(1/2)-35/8/b^(9/2)*a^3*ln(b^(1/2)*x^(1/2)+arcta
nh(tanh(b*x+a))^(1/2))+105/8/b^4*a^2*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)/a
rctanh(tanh(b*x+a))^(1/2)-105/8/b^(9/2)*a^2*(arctanh(tanh(b*x+a))-b*x-a)*ln
(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))+35/12/b^3*a*(arctanh(tanh(b*x+
a))-b*x-a)*x^(3/2)/arctanh(tanh(b*x+a))^(1/2)+105/8/b^4*a*(arctanh(tanh(b*x
+a))-b*x-a)^2*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)-105/8/b^(9/2)*a*(arctanh(t
anh(b*x+a))-b*x-a)^2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))-7/12/b^
2*(arctanh(tanh(b*x+a))-b*x-a)*x^(5/2)/arctanh(tanh(b*x+a))^(1/2)+35/24/b^3
*(arctanh(tanh(b*x+a))-b*x-a)^2*x^(3/2)/arctanh(tanh(b*x+a))^(1/2)+35/8/b^4
*(arctanh(tanh(b*x+a))-b*x-a)^3*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)-35/8/b^(
9/2)*(arctanh(tanh(b*x+a))-b*x-a)^3*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))
^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{2}}}{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(7/2)/arctanh(tanh(b*x + a))^(3/2), x)
```

Fricas [A] time = 2.15873, size = 485, normalized size = 2.92

$$\left[\frac{105\left(a^{3}bx+a^{4}\right)\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(8\,b^{4}x^{3}-14\,ab^{3}x^{2}+35\,a^{2}b^{2}x+105\,a^{3}b\right)\sqrt{bx+a}\sqrt{x}}{48\left(b^{6}x+ab^{5}\right)},\frac{105\left(a^{3}bx\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(105*(a^3*b*x + a^4)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt
(x) + a) + 2*(8*b^4*x^3 - 14*a*b^3*x^2 + 35*a^2*b^2*x + 105*a^3*b)*sqrt(b*x
 + a)*sqrt(x))/(b^6*x + a*b^5), 1/24*(105*(a^3*b*x + a^4)*sqrt(-b)*arctan(s
qrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (8*b^4*x^3 - 14*a*b^3*x^2 + 35*a^2*b^2
*x + 105*a^3*b)*sqrt(b*x + a)*sqrt(x))/(b^6*x + a*b^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1954, size = 101, normalized size = 0.61

$$\frac{\left(\left(2\,x\left(\frac{4\,x}{b}-\frac{7\,a}{b^2}\right)+\frac{35\,a^2}{b^3}\right)x+\frac{105\,a^3}{b^4}\right)\sqrt{x}}{24\,\sqrt{bx+a}}+\frac{35\,a^3\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{8\,b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*((2*x*(4*x/b - 7*a/b^2) + 35*a^2/b^3)*x + 105*a^3/b^4)*sqrt(x)/sqrt(b*
x + a) + 35/8*a^3*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(9/2)
```

3.250 $\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=128

$$\frac{5x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{2b^2}+\frac{15\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}{4b^3}+\frac{15\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(\ldots$$

```
[Out] (15*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[
Tanh[a + b*x]])^2)/(4*b^(7/2)) - (2*x^(5/2))/(b*Sqrt[ArcTanh[Tanh[a + b*x]]
]) + (5*x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*x]]])/(2*b^2) + (15*Sqrt[x]*(b*x -
ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/(4*b^3)
```

Rubi [A] time = 0.076284, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2168, 2169, 2165}

$$\frac{5x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{2b^2}+\frac{15\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}{4b^3}+\frac{15\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(\ldots$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (15*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[
Tanh[a + b*x]])^2)/(4*b^(7/2)) - (2*x^(5/2))/(b*Sqrt[ArcTanh[Tanh[a + b*x]]
]) + (5*x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*x]]])/(2*b^2) + (15*Sqrt[x]*(b*x -
ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/(4*b^3)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
```

```
[m, 0] &&  !IntegerQ[n]))
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2x^{5/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{5\int \frac{x^{3/2}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{b}$$

$$= -\frac{2x^{5/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{5x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{2b^2} - \frac{\left(15\left(-bx+\tanh^{-1}(\mathrm{ta}\right.\right.}{}$$

$$= -\frac{2x^{5/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{5x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{2b^2} + \frac{15\sqrt{x}\left(bx-\tanh^{-1}(\mathrm{ta}\right.}{}$$

$$= \frac{15\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4b^{7/2}} - \frac{2x^{5/2}}{b\sqrt{\tanh^{-1}(\tanh(a+}}$$

Mathematica [A] time = 0.0935498, size = 104, normalized size = 0.81

$$\frac{15\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}+b\sqrt{x}\right)}{4b^{7/2}} - \frac{\sqrt{x}\left(-25bx\tanh^{-1}(\tanh(a+bx))+15\,\mathrm{ta}\right.}{4b^3\sqrt{\tanh^{-1}(\tanh(}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] -(Sqrt[x]*(8*b^2*x^2 - 25*b*x*ArcTanh[Tanh[a + b*x]] + 15*ArcTanh[Tanh[a +
b*x]]^2))/(4*b^3*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (15*(-(b*x) + ArcTanh[Tanh
[a + b*x]])^2*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/(4*b^(
7/2))
```

Maple [B] time = 0.119, size = 261, normalized size = 2.

$$\frac{1}{2b}x^{\frac{5}{2}}\frac{1}{\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}}-\frac{5a}{4b^{2}}x^{\frac{3}{2}}\frac{1}{\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}}-\frac{15a^{2}}{4b^{3}}\sqrt{x}\frac{1}{\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}}+\frac{15a^{2}}{4}\ln\Big(\sqrt{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 1/2*x^(5/2)/b/arctanh(tanh(b*x+a))^(1/2)-5/4/b^2*a*x^(3/2)/arctanh(tanh(b*x
+a))^(1/2)-15/4/b^3*a^2*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)+15/4/b^(7/2)*a^2
*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))-15/2/b^3*a*(arctanh(tanh(b*
x+a))-b*x-a)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)+15/2/b^(7/2)*a*(arctanh(tan
h(b*x+a))-b*x-a)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))-5/4/b^2*(ar
ctanh(tanh(b*x+a))-b*x-a)*x^(3/2)/arctanh(tanh(b*x+a))^(1/2)-15/4/b^3*(arct
anh(tanh(b*x+a))-b*x-a)^2*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)+15/4/b^(7/2)*(
arctanh(tanh(b*x+a))-b*x-a)^2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/arctanh(tanh(b*x + a))^(3/2), x)
```

Fricas [A] time = 2.22357, size = 429, normalized size = 3.35

$$\left[\frac{15\left(a^2bx+a^3\right)\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(2\,b^3x^2-5\,ab^2x-15\,a^2b\right)\sqrt{bx+a}\sqrt{x}}{8\left(b^5x+ab^4\right)},-\frac{15\left(a^2bx+a^3\right)\sqrt{-b}\,\mathrm{arc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*(a^2*b*x + a^3)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x
) + a) + 2*(2*b^3*x^2 - 5*a*b^2*x - 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/(b^5*x
 + a*b^4), -1/4*(15*(a^2*b*x + a^3)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/
(b*sqrt(x))) - (2*b^3*x^2 - 5*a*b^2*x - 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/(b
^5*x + a*b^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20362, size = 85, normalized size = 0.66

$$\frac{\left(x\left(\frac{2\,x}{b}-\frac{5\,a}{b^2}\right)-\frac{15\,a^2}{b^3}\right)\sqrt{x}}{4\,\sqrt{bx+a}}-\frac{15\,a^2\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{4\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(x*(2*x/b - 5*a/b^2) - 15*a^2/b^3)*sqrt(x)/sqrt(b*x + a) - 15/4*a^2*log
(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(7/2)
```

3.251 $\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=86

$$\frac{3\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2}+\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^{5/2}}-\frac{2x^{3/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] (3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]]))/b^(5/2) - (2*x^(3/2))/(b*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (3
*Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]])/b^2
```

Rubi [A] time = 0.0487386, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2168, 2169, 2165}

$$\frac{3\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2}+\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^{5/2}}-\frac{2x^{3/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]]))/b^(5/2) - (2*x^(3/2))/(b*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (3
*Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]])/b^2
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2x^{3/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{3\int \frac{\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{b}$$

$$= -\frac{2x^{3/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{3\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{\left(3\left(-bx+\tanh^{-1}(\tan\right.\right.}{}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^{5/2}} - \frac{2x^{3/2}}{b\sqrt{\tanh^{-1}(\tanh(a+b}}$$

Mathematica [A] time = 0.0784536, size = 81, normalized size = 0.94

$$\frac{\sqrt{x}\left(3\tanh^{-1}(\tanh(a+bx))-2bx\right)}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}+b\sqrt{x}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (Sqrt[x]*(-2*b*x + 3*ArcTanh[Tanh[a + b*x]]))/(b^2*Sqrt[ArcTanh[Tanh[a + b*
x]]]) + (3*(b*x - ArcTanh[Tanh[a + b*x]])*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcT
```

```
anh[Tanh[a + b*x]]]])/b^(5/2)
```

Maple [A] time = 0.121, size = 130, normalized size = 1.5

$$\frac{1}{b}x^{\frac{3}{2}}\frac{1}{\sqrt{\operatorname{Artanh}(\tanh(bx+a))}}+3\frac{a\sqrt{x}}{b^2\sqrt{\operatorname{Artanh}(\tanh(bx+a))}}-3\frac{a\ln\left(\sqrt{b}\sqrt{x}+\sqrt{\operatorname{Artanh}(\tanh(bx+a))}\right)}{b^{5/2}}+3\frac{(\operatorname{Artar}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] x^(3/2)/b/arctanh(tanh(b*x+a))^(1/2)+3/b^2*a*x^(1/2)/arctanh(tanh(b*x+a))^(
1/2)-3/b^(5/2)*a*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))+3/b^2*(arct
anh(tanh(b*x+a))-b*x-a)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)-3/b^(5/2)*(arcta
nh(tanh(b*x+a))-b*x-a)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\operatorname{artanh}(\tanh(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/arctanh(tanh(b*x + a))^(3/2), x)
```

Fricas [A] time = 2.2193, size = 363, normalized size = 4.22

$$\left[\frac{3\left(abx+a^2\right)\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(b^2x+3\,ab\right)\sqrt{bx+a}\sqrt{x}}{2\left(b^4x+ab^3\right)},\frac{3\left(abx+a^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+\left(b^2\right.}{b^4x+ab^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(a*b*x + a^2)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) +
 a) + 2*(b^2*x + 3*a*b)*sqrt(b*x + a)*sqrt(x))/(b^4*x + a*b^3), (3*(a*b*x +
 a^2)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (b^2*x + 3*a*b)
*sqrt(b*x + a)*sqrt(x))/(b^4*x + a*b^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(x**(3/2)/atanh(tanh(a + b*x))**(3/2), x)
```

Giac [A] time = 1.20785, size = 65, normalized size = 0.76

$$\frac{\sqrt{x}\left(\frac{x}{b}+\frac{3a}{b^2}\right)}{\sqrt{bx+a}}+\frac{3a\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(x)*(x/b + 3*a/b^2)/sqrt(b*x + a) + 3*a*log(abs(-sqrt(b)*sqrt(x) + sqrt
(b*x + a)))/b^(5/2)
```

3.252 $\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=52

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]])/b^(3/2) - (2*Sq
rt[x])/(b*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rubi [A] time = 0.0258759, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2168, 2165}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]])/b^(3/2) - (2*Sq
rt[x])/(b*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2\sqrt{x}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{\int \frac{1}{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{b}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)}{b^{3/2}} - \frac{2\sqrt{x}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Mathematica [A] time = 0.0475296, size = 55, normalized size = 1.06

$$\frac{2\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}+b\sqrt{x}\right)}{b^{3/2}} - \frac{2\sqrt{x}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (-2*Sqrt[x])/(b*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (2*Log[b*Sqrt[x] + Sqrt[b]*
Sqrt[ArcTanh[Tanh[a + b*x]]]])/b^(3/2)
```

Maple [A] time = 0.125, size = 42, normalized size = 0.8

$$-2\,\frac{\sqrt{x}}{b\sqrt{\mathrm{Artanh}\,(\tanh\,(bx+a))}} + 2\,\frac{\ln\left(\sqrt{b}\sqrt{x}+\sqrt{\mathrm{Artanh}\,(\tanh\,(bx+a))}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] -2*x^(1/2)/b/arctanh(tanh(b*x+a))^(1/2)+2/b^(3/2)*ln(b^(1/2)*x^(1/2)+arctan
h(tanh(b*x+a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/arctanh(tanh(b*x + a))^(3/2), x)
```

Fricas [A] time = 2.10558, size = 308, normalized size = 5.92

$$\left[\frac{(bx+a)\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,\sqrt{bx+a}b\sqrt{x}}{b^{3}x+ab^{2}},-\frac{2\left((bx+a)\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+a}b\sqrt{x}\right)}{b^{3}x+ab^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [((b*x + a)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*sq
rt(b*x + a)*b*sqrt(x))/(b^3*x + a*b^2), -2*((b*x + a)*sqrt(-b)*arctan(sqrt(
b*x + a)*sqrt(-b)/(b*sqrt(x))) + sqrt(b*x + a)*b*sqrt(x))/(b^3*x + a*b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(sqrt(x)/atanh(tanh(a + b*x))**(3/2), x)
```

Giac [A] time = 1.22081, size = 53, normalized size = 1.02

$$-\frac{2\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{3}{2}}}-\frac{2\sqrt{x}}{\sqrt{bx+a}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(3/2) - 2*sqrt(x)/(sqrt(b*x
 + a)*b)
```

3.253 $\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=33

$$-\frac{2\sqrt{x}}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] (-2*Sqrt[x])/((b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rubi [A] time = 0.0128226, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2167}

$$-\frac{2\sqrt{x}}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (-2*Sqrt[x])/((b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx=-\frac{2\sqrt{x}}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Mathematica [A] time = 0.0313552, size = 32, normalized size = 0.97

$$\frac{2\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (2*Sqrt[x])/(Sqrt[ArcTanh[Tanh[a + b*x]]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])
)
```

Maple [A] time = 0.057, size = 29, normalized size = 0.9

$$2\,{\frac {\sqrt {x}}{ \left({\rm Artanh} \left(\tanh \left(bx+a \right) \right) -bx \right) \sqrt {{\rm Artanh} \left(\tanh \left(bx+a \right) \right) }}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 2*x^(1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x)*arctanh(tanh(b*x + a))^(3/2)), x)
```

Fricas [A] time = 1.98782, size = 53, normalized size = 1.61

$$\frac{2\,\sqrt{bx+a}\sqrt{x}}{abx+a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x + a)*sqrt(x)/(a*b*x + a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x}\,\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(1/(sqrt(x)*atanh(tanh(a + b*x))**(3/2)), x)
```

Giac [A] time = 1.1704, size = 20, normalized size = 0.61

$$\frac{2\sqrt{x}}{\sqrt{bx+a}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x)/(sqrt(b*x + a)*a)
```

3.254 $\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=68

$$\frac{2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{4b\sqrt{x}}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] (-4*b*Sqrt[x])/((b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b*x]
]]) + 2/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]
])
```

Rubi [A] time = 0.032766, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2171, 2167}

$$\frac{2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{4b\sqrt{x}}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (-4*b*Sqrt[x])/((b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b*x]
]]) + 2/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]
])
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
```

```
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = \frac{2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{(2b)\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a$$
$$= -\frac{4b\sqrt{x}}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh\right.}$$

Mathematica [A] time = 0.0467405, size = 43, normalized size = 0.63

$$-\frac{2\left(\tanh^{-1}(\tanh(a+bx))+bx\right)}{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (-2*(b*x + ArcTanh[Tanh[a + b*x]]))/(Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(
-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

Maple [A] time = 0.146, size = 59, normalized size = 0.9

$$-2\,\frac{1}{(\mathrm{Artanh}\,(\tanh\,(bx+a))-bx)\,\sqrt{x}\sqrt{\mathrm{Artanh}\,(\tanh\,(bx+a))}} - 4\,\frac{b\sqrt{x}}{(\mathrm{Artanh}\,(\tanh\,(bx+a))-bx)^2\,\sqrt{\mathrm{Artanh}\,(\tanh\,(bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] -2/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)/arctanh(tanh(b*x+a))^(1/2)-4*b/(arcta
nh(tanh(b*x+a))-b*x)^2*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)
```

Maxima [A] time = 1.49918, size = 43, normalized size = 0.63

$$-\frac{2\left(2\,b^2x^2+3\,abx+a^2\right)}{(bx+a)^{\frac{3}{2}}a^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -2*(2*b^2*x^2 + 3*a*b*x + a^2)/((b*x + a)^(3/2)*a^2*sqrt(x))
```

Fricas [A] time = 2.03001, size = 78, normalized size = 1.15

$$-\frac{2\,(2\,bx+a)\sqrt{bx+a}\sqrt{x}}{a^2bx^2+a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(2*b*x + a)*sqrt(b*x + a)*sqrt(x)/(a^2*b*x^2 + a^3*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17775, size = 68, normalized size = 1.

$$-\frac{2\,b\sqrt{x}}{\sqrt{bx+a}a^2}+\frac{4\,\sqrt{b}}{\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -2*b*sqrt(x)/(sqrt(b*x + a)*a^2) + 4*sqrt(b)/(((sqrt(b)*sqrt(x) - sqrt(b*x
+ a))^2 - a)*a)
```

3.255 $\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=110

$$-\frac{16b^2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{2}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] (-16*b^2*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])^3*Sqrt[ArcTanh[Tanh[a +
 b*x]]]) + (8*b)/(3*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[T
anh[a + b*x]]]) + 2/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[
Tanh[a + b*x]]])
```

Rubi [A] time = 0.0540588, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2171, 2167}

$$-\frac{16b^2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{2}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (-16*b^2*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])^3*Sqrt[ArcTanh[Tanh[a +
 b*x]]]) + (8*b)/(3*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[T
anh[a + b*x]]]) + 2/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[
Tanh[a + b*x]]])
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
```

```
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = \frac{2}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{(4b)\int \frac{1}{x^{3/2}\tanh^{-1}(\tan}}{3\left(bx-\tanh^{-1}(\tan\right.}$$

$$= \frac{8b}{3\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{}{3x^{3/2}\left(bx-\tanh^{-1}(\right.}$$

$$= -\frac{16b^2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{}{3\sqrt{x}\left(bx-\tanh^{-1}(\mathrm{ta}\right.}$$

Mathematica [A] time = 0.0491826, size = 64, normalized size = 0.58

$$\frac{2\left(-6bx\tanh^{-1}(\tanh(a+bx))+\tanh^{-1}(\tanh(a+bx))^2-3b^2x^2\right)}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (2*(-3*b^2*x^2 - 6*b*x*ArcTanh[Tanh[a + b*x]] + ArcTanh[Tanh[a + b*x]]^2))/
(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^3*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Maple [A] time = 0.148, size = 105, normalized size = 1.

$$-\frac{2}{3\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-3\,bx}x^{-\frac{3}{2}}\frac{1}{\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}}-\frac{8\,b}{3\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-3\,bx}\left(-\frac{1}{\mathrm{Artanh}\left(\tanh\left(b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] -2/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)/arctanh(tanh(b*x+a))^(1/2)-8/3*b/(a
rctanh(tanh(b*x+a))-b*x)*(-1/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)/arctanh(tan
h(b*x+a))^(1/2)-2*b/(arctanh(tanh(b*x+a))-b*x)^2*x^(1/2)/arctanh(tanh(b*x+a
))^(1/2))
```

Maxima [A] time = 1.50479, size = 61, normalized size = 0.55

$$\frac{2\left(8\,b^{3}x^{3}+12\,ab^{2}x^{2}+3\,a^{2}bx-a^{3}\right)}{3\,(bx+a)^{\frac{3}{2}}a^{3}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(8*b^3*x^3 + 12*a*b^2*x^2 + 3*a^2*b*x - a^3)/((b*x + a)^(3/2)*a^3*x^(3/
2))
```

Fricas [A] time = 2.05685, size = 104, normalized size = 0.95

$$\frac{2\left(8\,b^{2}x^{2}+4\,abx-a^{2}\right)\sqrt{bx+a}\sqrt{x}}{3\left(a^{3}bx^{3}+a^{4}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(8*b^2*x^2 + 4*a*b*x - a^2)*sqrt(b*x + a)*sqrt(x)/(a^3*b*x^3 + a^4*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/atanh(tanh(b*x+a))**(3/2),x)
```

[Out] Timed out

Giac [A] time = 1.27649, size = 144, normalized size = 1.31

$$\frac{2\,b^{2}\sqrt{x}}{\sqrt{bx+a}a^{3}}-\frac{4\left(3\,b^{\frac{3}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{4}-12\,ab^{\frac{3}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{2}+5\,a^{2}b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{2}-a\right)^{3}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2*b^2*sqrt(x)/(sqrt(b*x + a)*a^3) - 4/3*(3*b^(3/2)*(sqrt(b)*sqrt(x) - sqrt(
b*x + a))^4 - 12*a*b^(3/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 + 5*a^2*b^(3
/2))/(((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^3*a^2)
```

3.256 $\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=148

$$-\frac{32b^3\sqrt{x}}{5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{16b^2}{5\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+b\ldots}}$$

```
[Out] (-32*b^3*Sqrt[x])/(5*(b*x - ArcTanh[Tanh[a + b*x]])^4*Sqrt[ArcTanh[Tanh[a +
 b*x]]]) + (16*b^2)/(5*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^3*Sqrt[ArcTan
h[Tanh[a + b*x]]]) + (4*b)/(5*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt
[ArcTanh[Tanh[a + b*x]]]) + 2/(5*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])*Sqr
t[ArcTanh[Tanh[a + b*x]]])
```

Rubi [A] time = 0.0797879, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2171, 2167}

$$-\frac{32b^3\sqrt{x}}{5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{16b^2}{5\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+b\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (-32*b^3*Sqrt[x])/(5*(b*x - ArcTanh[Tanh[a + b*x]])^4*Sqrt[ArcTanh[Tanh[a +
 b*x]]]) + (16*b^2)/(5*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^3*Sqrt[ArcTan
h[Tanh[a + b*x]]]) + (4*b)/(5*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt
[ArcTanh[Tanh[a + b*x]]]) + 2/(5*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])*Sqr
t[ArcTanh[Tanh[a + b*x]]])
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = \frac{2}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{(6b)\int \frac{1}{x^{5/2}\tanh^{-1}(\tan}}{5\left(bx-\tanh^{-1}(\tan\right.}$$

$$= \frac{4b}{5x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{}{5x^{5/2}\left(bx-\tanh^{-1}\right.}$$

$$= \frac{16b^2}{5\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{}{5x^{3/2}\left(bx-\tanh^{-1}(\right.}$$

$$= -\frac{32b^3\sqrt{x}}{5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{}{5\sqrt{x}\left(bx-\tanh^{-1}(\mathrm{ta}\right.}$$

Mathematica [A] time = 0.0622312, size = 80, normalized size = 0.54

$$-\frac{2\left(15b^2x^2\tanh^{-1}(\tanh(a+bx))-5bx\tanh^{-1}(\tanh(a+bx))^2+\tanh^{-1}(\tanh(a+bx))^3+5b^3x^3\right)}{5x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (-2*(5*b^3*x^3 + 15*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 5*b*x*ArcTanh[Tanh[a +
 b*x]]^2 + ArcTanh[Tanh[a + b*x]]^3))/(5*x^(5/2)*Sqrt[ArcTanh[Tanh[a + b*x]
]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4)
```

Maple [A] time = 0.119, size = 151, normalized size = 1.

$$-\frac{2}{5\,\mathrm{Artanh}\,(\tanh\,(bx+a))-5\,bx}x^{-\frac{5}{2}}\frac{1}{\sqrt{\mathrm{Artanh}\,(\tanh\,(bx+a))}}-\frac{12\,b}{5\,\mathrm{Artanh}\,(\tanh\,(bx+a))-5\,bx}\left(-\frac{1}{3\,\mathrm{Artanh}\,(\tanh\,(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] -2/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)/arctanh(tanh(b*x+a))^(1/2)-12/5*b/(
arctanh(tanh(b*x+a))-b*x)*(-1/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)/arctanh(
tanh(b*x+a))^(1/2)-4/3*b/(arctanh(tanh(b*x+a))-b*x)*(-1/(arctanh(tanh(b*x+a
))-b*x)/x^(1/2)/arctanh(tanh(b*x+a))^(1/2)-2*b/(arctanh(tanh(b*x+a))-b*x)^2
*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)))
```

Maxima [A] time = 1.50686, size = 73, normalized size = 0.49

$$-\frac{2\left(16\,b^4x^4+24\,ab^3x^3+6\,a^2b^2x^2-a^3bx+a^4\right)}{5\,(bx+a)^{\frac{3}{2}}a^4x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/5*(16*b^4*x^4 + 24*a*b^3*x^3 + 6*a^2*b^2*x^2 - a^3*b*x + a^4)/((b*x + a)
^(3/2)*a^4*x^(5/2))
```

Fricas [A] time = 1.99808, size = 128, normalized size = 0.86

$$-\frac{2\left(16\,b^3x^3+8\,ab^2x^2-2\,a^2bx+a^3\right)\sqrt{bx+a}\sqrt{x}}{5\left(a^4bx^4+a^5x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(16*b^3*x^3 + 8*a*b^2*x^2 - 2*a^2*b*x + a^3)*sqrt(b*x + a)*sqrt(x)/(a^
4*b*x^4 + a^5*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40087, size = 217, normalized size = 1.47

$$-\frac{2\,b^{3}\sqrt{x}}{\sqrt{bx+a}a^{4}}+\frac{4\left(5\,b^{\frac{5}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{8}-30\,ab^{\frac{5}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{6}+80\,a^{2}b^{\frac{5}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{4}-50\,a^{3}b^{\frac{5}{2}}\left(\sqrt{b}\sqrt{x}\cdots\right.\right.}{5\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{2}-a\right)^{5}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -2*b^3*sqrt(x)/(sqrt(b*x + a)*a^4) + 4/5*(5*b^(5/2)*(sqrt(b)*sqrt(x) - sqrt
(b*x + a))^8 - 30*a*b^(5/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^6 + 80*a^2*b^
(5/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^4 - 50*a^3*b^(5/2)*(sqrt(b)*sqrt(x)
 - sqrt(b*x + a))^2 + 11*a^4*b^(5/2))/(((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2
 - a)^5*a^3)
```

3.257 $\int \frac{x^{7/2}}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=153

$$-\frac{14x^{5/2}}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{35x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{6b^3}+\frac{35\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}{4b^4}$$

```
[Out] (35*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[
Tanh[a + b*x]])^2)/(4*b^(9/2)) - (2*x^(7/2))/(3*b*ArcTanh[Tanh[a + b*x]]^(3
/2)) - (14*x^(5/2))/(3*b^2*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (35*x^(3/2)*Sqrt
[ArcTanh[Tanh[a + b*x]]])/(6*b^3) + (35*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x
]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/(4*b^4)
```

Rubi [A] time = 0.0959509, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2168, 2169, 2165}

$$-\frac{14x^{5/2}}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{35x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{6b^3}+\frac{35\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```


```
[Out] (35*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[
Tanh[a + b*x]])^2)/(4*b^(9/2)) - (2*x^(7/2))/(3*b*ArcTanh[Tanh[a + b*x]]^(3
/2)) - (14*x^(5/2))/(3*b^2*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (35*x^(3/2)*Sqrt
[ArcTanh[Tanh[a + b*x]]])/(6*b^3) + (35*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x
]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/(4*b^4)
```


Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
```

```
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] && !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) && !ILtQ[m + n,
-2]
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{x^{7/2}}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2x^{7/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}+\frac{7\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx}{3b}$$

$$= -\frac{2x^{7/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}-\frac{14x^{5/2}}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{35\int \frac{x^{3/2}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}}{3b^2}$$

$$= -\frac{2x^{7/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}-\frac{14x^{5/2}}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{35x^{3/2}\sqrt{\tanh^{-1}(\tanh(}}{6b^3}$$

$$= -\frac{2x^{7/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}-\frac{14x^{5/2}}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{35x^{3/2}\sqrt{\tanh^{-1}(\tanh(}}{6b^3}$$

$$= \frac{35\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4b^{9/2}}-\frac{2x^{7/2}}{3b\tanh^{-1}(\tanh(a+b}$$

Mathematica [A] time = 0.105465, size = 121, normalized size = 0.79

$$\frac{35\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}+b\sqrt{x}\right)}{4b^{9/2}}-\frac{\sqrt{x}\left(56b^2x^2\tanh^{-1}(\tanh(a+bx))-175b\right.}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] -(Sqrt[x]*(8*b^3*x^3 + 56*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 175*b*x*ArcTanh[
Tanh[a + b*x]]^2 + 105*ArcTanh[Tanh[a + b*x]]^3))/(12*b^4*ArcTanh[Tanh[a +
b*x]]^(3/2)) + (35*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2*Log[b*Sqrt[x] + Sqrt
[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/(4*b^(9/2))
```

Maple [B] time = 0.125, size = 348, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] 1/2*x^(7/2)/b/arctanh(tanh(b*x+a))^(3/2)-7/4/b^2*a*x^(5/2)/arctanh(tanh(b*x
+a))^(3/2)-35/12/b^3*a^2*x^(3/2)/arctanh(tanh(b*x+a))^(3/2)-35/4/b^4*a^2*x^
(1/2)/arctanh(tanh(b*x+a))^(1/2)+35/4/b^(9/2)*a^2*ln(b^(1/2)*x^(1/2)+arctan
h(tanh(b*x+a))^(1/2))-35/6/b^3*a*(arctanh(tanh(b*x+a))-b*x-a)*x^(3/2)/arcta
nh(tanh(b*x+a))^(3/2)-35/2/b^4*a*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)/arcta
nh(tanh(b*x+a))^(1/2)+35/2/b^(9/2)*a*(arctanh(tanh(b*x+a))-b*x-a)*ln(b^(1/2
)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))-7/4/b^2*(arctanh(tanh(b*x+a))-b*x-a)*
x^(5/2)/arctanh(tanh(b*x+a))^(3/2)-35/12/b^3*(arctanh(tanh(b*x+a))-b*x-a)^2
*x^(3/2)/arctanh(tanh(b*x+a))^(3/2)-35/4/b^4*(arctanh(tanh(b*x+a))-b*x-a)^2
*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)+35/4/b^(9/2)*(arctanh(tanh(b*x+a))-b*x-
a)^2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{2}}}{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(7/2)/arctanh(tanh(b*x + a))^(5/2), x)
```

Fricas [A] time = 2.15042, size = 575, normalized size = 3.76

$$\left[\frac{105\left(a^2b^2x^2+2\,a^3bx+a^4\right)\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(6\,b^4x^3-21\,ab^3x^2-140\,a^2b^2x-105\,a^3b\right)\sqrt{bx+a}\sqrt{x}}{24\left(b^7x^2+2\,ab^6x+a^2b^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(105*(a^2*b^2*x^2 + 2*a^3*b*x + a^4)*sqrt(b)*log(2*b*x + 2*sqrt(b*x +
 a)*sqrt(b)*sqrt(x) + a) + 2*(6*b^4*x^3 - 21*a*b^3*x^2 - 140*a^2*b^2*x - 10
5*a^3*b)*sqrt(b*x + a)*sqrt(x))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5), -1/12*(105
*(a^2*b^2*x^2 + 2*a^3*b*x + a^4)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*
sqrt(x))) - (6*b^4*x^3 - 21*a*b^3*x^2 - 140*a^2*b^2*x - 105*a^3*b)*sqrt(b*x
 + a)*sqrt(x))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20858, size = 101, normalized size = 0.66

$$\frac{\left(\left(3\,x\left(\frac{2\,x}{b}-\frac{7\,a}{b^2}\right)-\frac{140\,a^2}{b^3}\right)x-\frac{105\,a^3}{b^4}\right)\sqrt{x}}{12\,(bx+a)^{\frac{3}{2}}}-\frac{35\,a^2\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{4\,b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/12*((3*x*(2*x/b - 7*a/b^2) - 140*a^2/b^3)*x - 105*a^3/b^4)*sqrt(x)/(b*x +
 a)^(3/2) - 35/4*a^2*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(9/2)
```

3.258 $\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=111

$$-\frac{10x^{3/2}}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{5\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^3}+\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+b$$

```
[Out] (5*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]]))/b^(7/2) - (2*x^(5/2))/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2)) -
(10*x^(3/2))/(3*b^2*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (5*Sqrt[x]*Sqrt[ArcTanh
[Tanh[a + b*x]]])/b^3
```

Rubi [A] time = 0.0659394, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2168, 2169, 2165}

$$-\frac{10x^{3/2}}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{5\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^3}+\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+b$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (5*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]]))/b^(7/2) - (2*x^(5/2))/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2)) -
(10*x^(3/2))/(3*b^2*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (5*Sqrt[x]*Sqrt[ArcTanh
[Tanh[a + b*x]]])/b^3
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
```

```
[m, 0] &&  !IntegerQ[n]))
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx &= -\frac{2x^{5/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}+\frac{5\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx}{3b}\\
&= -\frac{2x^{5/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{5\int \frac{\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}}{b^2}\\
&= -\frac{2x^{5/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{5\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a}}{b^3}\\
&= \frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^{7/2}}-\frac{2x^{5/2}}{3b\tanh^{-1}(\tanh(a+b}\end{aligned}$$

Mathematica [A] time = 0.0903393, size = 101, normalized size = 0.91

$$\frac{5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}+b\sqrt{x}\right)}{b^{7/2}}-\frac{\sqrt{x}\left(10bx\tanh^{-1}(\tanh(a+bx))-15\tanh^{-}\right.}{3b^3\tanh^{-1}(\tanh(a+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] -(Sqrt[x]*(2*b^2*x^2 + 10*b*x*ArcTanh[Tanh[a + b*x]] - 15*ArcTanh[Tanh[a +
b*x]]^2))/(3*b^3*ArcTanh[Tanh[a + b*x]]^(3/2)) + (5*(b*x - ArcTanh[Tanh[a +
 b*x]])*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/b^(7/2)
```

Maple [B] time = 0.127, size = 180, normalized size = 1.6

$$\frac{1}{b}x^{\frac{5}{2}}\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{-\frac{3}{2}}+\frac{5a}{3b^{2}}x^{\frac{3}{2}}\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{-\frac{3}{2}}+5\frac{a\sqrt{x}}{b^{3}\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}}-5\frac{a\ln\left(\sqrt{b}\sqrt{x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] x^(5/2)/b/arctanh(tanh(b*x+a))^(3/2)+5/3/b^2*a*x^(3/2)/arctanh(tanh(b*x+a))
^(3/2)+5/b^3*a*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)-5/b^(7/2)*a*ln(b^(1/2)*x^
(1/2)+arctanh(tanh(b*x+a))^(1/2))+5/3/b^2*(arctanh(tanh(b*x+a))-b*x-a)*x^(3
/2)/arctanh(tanh(b*x+a))^(3/2)+5/b^3*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)/a
rctanh(tanh(b*x+a))^(1/2)-5/b^(7/2)*(arctanh(tanh(b*x+a))-b*x-a)*ln(b^(1/2)
*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/arctanh(tanh(b*x + a))^(5/2), x)
```

Fricas [A] time = 2.09868, size = 512, normalized size = 4.61

$$\left[\frac{15\left(ab^{2}x^{2}+2a^{2}bx+a^{3}\right)\sqrt{b}\log\left(2bx-2\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(3b^{3}x^{2}+20ab^{2}x+15a^{2}b\right)\sqrt{bx+a}\sqrt{x}}{6\left(b^{6}x^{2}+2ab^{5}x+a^{2}b^{4}\right)},\frac{15\left(ab^{2}x^{2}+2a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a*b^2*x^2 + 2*a^2*b*x + a^3)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*
sqrt(b)*sqrt(x) + a) + 2*(3*b^3*x^2 + 20*a*b^2*x + 15*a^2*b)*sqrt(b*x + a)*
sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4), 1/3*(15*(a*b^2*x^2 + 2*a^2*b*x +
a^3)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (3*b^3*x^2 + 20*
a*b^2*x + 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20249, size = 82, normalized size = 0.74

$$\frac{\left(x\left(\frac{3\,x}{b}+\frac{20\,a}{b^2}\right)+\frac{15\,a^2}{b^3}\right)\sqrt{x}}{3\,(bx+a)^{\frac{3}{2}}}+\frac{5\,a\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(x*(3*x/b + 20*a/b^2) + 15*a^2/b^3)*sqrt(x)/(b*x + a)^(3/2) + 5*a*log(a
bs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(7/2)
```

3.259 $\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=75

$$-\frac{2\sqrt{x}}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)}{b^{5/2}}-\frac{2x^{3/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]])/b^(5/2) - (2*x^
(3/2))/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[ArcTanh[T
anh[a + b*x]]])
```

Rubi [A] time = 0.0419205, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2168, 2165}

$$-\frac{2\sqrt{x}}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)}{b^{5/2}}-\frac{2x^{3/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]])/b^(5/2) - (2*x^
(3/2))/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[ArcTanh[T
anh[a + b*x]]])
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2x^{3/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx}{b}$$

$$= -\frac{2x^{3/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{\int \frac{1}{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}}{b^2}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)}{b^{5/2}} - \frac{2x^{3/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{\tanh^{-1}(\tanh(a+}}$$

Mathematica [A] time = 0.0640778, size = 78, normalized size = 1.04

$$-\frac{2\sqrt{x}}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{2\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}+b\sqrt{x}\right)}{b^{5/2}} - \frac{2x^{3/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (-2*x^(3/2))/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[Arc
Tanh[Tanh[a + b*x]]]) + (2*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*
x]]]])/b^(5/2)
```

Maple [A] time = 0.118, size = 59, normalized size = 0.8

$$-\frac{2}{3\,b}x^{\frac{3}{2}}\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{-\frac{3}{2}} - 2\,\frac{\sqrt{x}}{b^2\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}} + 2\,\frac{\ln\left(\sqrt{b}\sqrt{x}+\sqrt{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)}\right)}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] -2/3*x^(3/2)/b/arctanh(tanh(b*x+a))^(3/2)-2*x^(1/2)/b^2/arctanh(tanh(b*x+a)
)^(1/2)+2/b^(5/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/arctanh(tanh(b*x + a))^(5/2), x)
```

Fricas [A] time = 2.12632, size = 451, normalized size = 6.01

$$\left[\frac{3\left(b^2x^2+2\,abx+a^2\right)\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\left(4\,b^2x+3\,ab\right)\sqrt{bx+a}\sqrt{x}}{3\left(b^5x^2+2\,ab^4x+a^2b^3\right)},-\frac{2\left(3\left(b^2x^2+2\,abx+a^2\right)\sqrt{-b}\,a\ldots\right.}{3\left(b\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(
b)*sqrt(x) + a) - 2*(4*b^2*x + 3*a*b)*sqrt(b*x + a)*sqrt(x))/(b^5*x^2 + 2*a
*b^4*x + a^2*b^3), -2/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-b)*arctan(sqrt(b
*x + a)*sqrt(-b)/(b*sqrt(x))) + (4*b^2*x + 3*a*b)*sqrt(b*x + a)*sqrt(x))/(b
^5*x^2 + 2*a*b^4*x + a^2*b^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28144, size = 66, normalized size = 0.88

$$-\frac{2\sqrt{x}\left(\frac{4x}{b}+\frac{3a}{b^2}\right)}{3(bx+a)^{\frac{3}{2}}}-\frac{2\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(x)*(4*x/b + 3*a/b^2)/(b*x + a)^(3/2) - 2*log(abs(-sqrt(b)*sqrt(x)
 + sqrt(b*x + a)))/b^(5/2)
```

3.260 $\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^{5/2}} \, dx$

Optimal. Leaf size=35

$$-\frac{2x^{3/2}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

```
[Out] (-2*x^(3/2))/(3*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2)
)
```

Rubi [A] time = 0.0121499, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2167}

$$-\frac{2x^{3/2}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (-2*x^(3/2))/(3*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2)
)
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^{5/2}} \, dx = -\frac{2x^{3/2}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Mathematica [A] time = 0.0415808, size = 34, normalized size = 0.97

$$\frac{2x^{3/2}}{3\tanh^{-1}(\tanh(a+bx))^{3/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (2*x^(3/2))/(3*ArcTanh[Tanh[a + b*x]]^(3/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]
]))
```

Maple [B] time = 0.118, size = 92, normalized size = 2.6

$$-\frac{1}{b}\sqrt{x}\left(\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{-\frac{3}{2}}+\frac{\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)-bx}{b}\left(\frac{1}{3\operatorname{Artanh}\left(\tanh\left(bx+a\right)\right)-3bx}\sqrt{x}(\operatorname{Artanh}(\tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] -x^(1/2)/b/arctanh(tanh(b*x+a))^(3/2)+(arctanh(tanh(b*x+a))-b*x)/b*(1/3*x^(
1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^(3/2)+2/3/(arctanh(tan
h(b*x+a))-b*x)^2*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/arctanh(tanh(b*x + a))^(5/2), x)
```

Fricas [A] time = 2.09297, size = 77, normalized size = 2.2

$$\frac{2\sqrt{bx+a}x^{\frac{3}{2}}}{3\left(ab^2x^2+2\,a^2bx+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x + a)*x^(3/2)/(a*b^2*x^2 + 2*a^2*b*x + a^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16159, size = 20, normalized size = 0.57

$$\frac{2\,x^{\frac{3}{2}}}{3\,(bx+a)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2)/((b*x + a)^(3/2)*a)
```

3.261 $\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=71

$$\frac{4\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

```
[Out] (-2*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2)
) + (4*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b
*x]]])
```

Rubi [A] time = 0.0315466, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2171, 2167}

$$\frac{4\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (-2*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2)
) + (4*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b
*x]]])
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
```

```
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{2\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+}}{3\left(-bx+\tanh^{-1}(\tanh\right.}$$
$$= -\frac{2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{3\left(bx-\tanh^{-1}(\tanh(\right.}$$

Mathematica [A] time = 0.0353378, size = 47, normalized size = 0.66

$$-\frac{2\sqrt{x}\left(bx-3\tanh^{-1}(\tanh(a+bx))\right)}{3\tanh^{-1}(\tanh(a+bx))^{3/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (-2*Sqrt[x]*(b*x - 3*ArcTanh[Tanh[a + b*x]]))/(3*ArcTanh[Tanh[a + b*x]]^(3/
2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

Maple [A] time = 0.043, size = 58, normalized size = 0.8

$$\frac{2}{3\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-3\,bx}\sqrt{x}\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{-\frac{3}{2}}+\frac{4}{3\,\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}}\sqrt{x}\frac{1}{\sqrt{\mathrm{Artanh}\left(\tanh\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] 2/3*x^(1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^(3/2)+4/3/(arct
anh(tanh(b*x+a))-b*x)^2*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x}\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x)*arctanh(tanh(b*x + a))^(5/2)), x)
```

Fricas [A] time = 2.07893, size = 99, normalized size = 1.39

$$\frac{2\,(2\,bx+3\,a)\sqrt{bx+a}\sqrt{x}}{3\left(a^2b^2x^2+2\,a^3bx+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*b*x + 3*a)*sqrt(b*x + a)*sqrt(x)/(a^2*b^2*x^2 + 2*a^3*b*x + a^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21801, size = 34, normalized size = 0.48

$$\frac{2\,\sqrt{x}\left(\frac{2\,bx}{a^2}+\frac{3}{a}\right)}{3\,(bx+a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(x)*(2*b*x/a^2 + 3/a)/(b*x + a)^(3/2)
```

3.262 $\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=106

$$\frac{16b\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{8b\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^{3/2}}+$$

```
[Out] (-8*b*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^(
3/2)) + 2/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3
/2)) + (16*b*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])^3*Sqrt[ArcTanh[Tanh
[a + b*x]]])
```

Rubi [A] time = 0.0548497, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2171, 2167}

$$\frac{16b\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{8b\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (-8*b*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^(
3/2)) + 2/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3
/2)) + (16*b*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])^3*Sqrt[ArcTanh[Tanh
[a + b*x]]])
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
```

```
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = \frac{2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{(4b)\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a}}{-bx+\tanh^{-1}(\tanh}$$

$$= -\frac{8b\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{\sqrt{x}\left(bx-\tanh^{-1}(\tan\right.}$$

$$= -\frac{8b\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{\sqrt{x}\left(bx-\tanh^{-1}(\tan\right.}$$

Mathematica [A] time = 0.0522689, size = 66, normalized size = 0.62

$$\frac{2\left(6bx\tanh^{-1}(\tanh(a+bx))+3\tanh^{-1}(\tanh(a+bx))^2-b^2x^2\right)}{3\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (2*(-(b^2*x^2) + 6*b*x*ArcTanh[Tanh[a + b*x]] + 3*ArcTanh[Tanh[a + b*x]]^2)
)/(3*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^3*ArcTanh[Tanh[a + b*x]]^(3/2))
```

Maple [A] time = 0.119, size = 104, normalized size = 1.

$$-2\,\frac{1}{(\mathrm{Artanh}\,(\tanh\,(bx+a))-bx)\,\sqrt{x}\,(\mathrm{Artanh}\,(\tanh\,(bx+a)))^{3/2}} - 8\,\frac{b}{\mathrm{Artanh}\,(\tanh\,(bx+a))-bx}\left(1/3\,\frac{}{(\mathrm{Artanh}\,(\tanh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] -2/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)/arctanh(tanh(b*x+a))^(3/2)-8*b/(arcta
nh(tanh(b*x+a))-b*x)*(1/3*x^(1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b
```

```
*x+a))^(3/2)+2/3/(arctanh(tanh(b*x+a))-b*x)^2*x^(1/2)/arctanh(tanh(b*x+a))^
(1/2))
```

Maxima [A] time = 1.51431, size = 61, normalized size = 0.58

$$-\frac{2\left(8\,b^3x^3+20\,ab^2x^2+15\,a^2bx+3\,a^3\right)}{3\,(bx+a)^{\frac{5}{2}}a^3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(8*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 3*a^3)/((b*x + a)^(5/2)*a^3*s
qrt(x))
```

Fricas [A] time = 2.16484, size = 128, normalized size = 1.21

$$-\frac{2\left(8\,b^2x^2+12\,abx+3\,a^2\right)\sqrt{bx+a}\sqrt{x}}{3\left(a^3b^2x^3+2\,a^4bx^2+a^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(8*b^2*x^2 + 12*a*b*x + 3*a^2)*sqrt(b*x + a)*sqrt(x)/(a^3*b^2*x^3 + 2*
a^4*b*x^2 + a^5*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20973, size = 84, normalized size = 0.79

$$-\frac{2\sqrt{x}\left(\frac{5b^2x}{a^3}+\frac{6b}{a^2}\right)}{3(bx+a)^{\frac{3}{2}}}+\frac{4\sqrt{b}}{\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(x)*(5*b^2*x/a^3 + 6*b/a^2)/(b*x + a)^(3/2) + 4*sqrt(b)/(((sqrt(b)
*sqrt(x) - sqrt(b*x + a))^2 - a)*a^2)
```

3.263 $\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=146

$$\frac{32b^2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{16b^2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^{3/2}}+$$

```
[Out] (-16*b^2*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])^3*ArcTanh[Tanh[a + b*x]
]^(3/2)) + (4*b)/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a +
 b*x]]^(3/2)) + 2/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a
+ b*x]]^(3/2)) + (32*b^2*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])^4*Sqrt[
ArcTanh[Tanh[a + b*x]]])
```

Rubi [A] time = 0.0794707, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2171, 2167}

$$\frac{32b^2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{16b^2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```


```
[Out] (-16*b^2*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])^3*ArcTanh[Tanh[a + b*x]
]^(3/2)) + (4*b)/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a +
 b*x]]^(3/2)) + 2/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a
+ b*x]]^(3/2)) + (32*b^2*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])^4*Sqrt[
ArcTanh[Tanh[a + b*x]]])
```


Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = \frac{2}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{(2b)\int \frac{1}{x^{3/2}\tanh^{-1}(\tan}}{bx-\tanh^{-1}(\tan}$$

$$= \frac{4b}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{3x^{3/2}\left(bx-\tanh^{-1}(\mathrm{t}\right.}$$

$$= -\frac{16b^2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{\sqrt{x}\left(bx-\tanh^{-1}(\mathrm{tan}\right.}$$

$$= -\frac{16b^2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{\sqrt{x}\left(bx-\tanh^{-1}(\mathrm{tan}\right.}$$

Mathematica [A] time = 0.058781, size = 79, normalized size = 0.54

$$-\frac{2\left(-9b^2x^2\tanh^{-1}(\tanh(a+bx))-9bx\tanh^{-1}(\tanh(a+bx))^2+\tanh^{-1}(\tanh(a+bx))^3+b^3x^3\right)}{3x^{3/2}\tanh^{-1}(\tanh(a+bx))^{3/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (-2*(b^3*x^3 - 9*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 9*b*x*ArcTanh[Tanh[a + b*
x]]^2 + ArcTanh[Tanh[a + b*x]]^3))/(3*x^(3/2)*ArcTanh[Tanh[a + b*x]]^(3/2)*
(-(b*x) + ArcTanh[Tanh[a + b*x]])^4)
```

Maple [A] time = 0.118, size = 150, normalized size = 1.

$$-\frac{2}{3\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-3\,bx}x^{-\frac{3}{2}}\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{-\frac{3}{2}}-4\,\frac{b}{\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx}\left(-\frac{}{(\mathrm{Artanh}\left(\tanh\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] -2/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)/arctanh(tanh(b*x+a))^(3/2)-4*b/(arc
tanh(tanh(b*x+a))-b*x)*(-1/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)/arctanh(tanh(
b*x+a))^(3/2)-4*b/(arctanh(tanh(b*x+a))-b*x)*(1/3*x^(1/2)/(arctanh(tanh(b*x
+a))-b*x)/arctanh(tanh(b*x+a))^(3/2)+2/3/(arctanh(tanh(b*x+a))-b*x)^2*x^(1/
2)/arctanh(tanh(b*x+a))^(1/2)))
```

Maxima [A] time = 1.51364, size = 76, normalized size = 0.52

$$\frac{2\left(16\,b^4x^4 + 40\,ab^3x^3 + 30\,a^2b^2x^2 + 5\,a^3bx - a^4\right)}{3\,(bx+a)^{\frac{5}{2}}a^4x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(16*b^4*x^4 + 40*a*b^3*x^3 + 30*a^2*b^2*x^2 + 5*a^3*b*x - a^4)/((b*x +
a)^(5/2)*a^4*x^(3/2))
```

Fricas [A] time = 2.0808, size = 150, normalized size = 1.03

$$\frac{2\left(16\,b^3x^3 + 24\,ab^2x^2 + 6\,a^2bx - a^3\right)\sqrt{bx+a}\sqrt{x}}{3\left(a^4b^2x^4 + 2\,a^5bx^3 + a^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(16*b^3*x^3 + 24*a*b^2*x^2 + 6*a^2*b*x - a^3)*sqrt(b*x + a)*sqrt(x)/(a^
4*b^2*x^4 + 2*a^5*b*x^3 + a^6*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32203, size = 161, normalized size = 1.1

$$\frac{2\sqrt{x}\left(\frac{8b^3x}{a^4}+\frac{9b^2}{a^3}\right)}{3(bx+a)^{\frac{3}{2}}}-\frac{8\left(3b^{\frac{3}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^4-9ab^{\frac{3}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2+4a^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(x)*(8*b^3*x/a^4 + 9*b^2/a^3)/(b*x + a)^(3/2) - 8/3*(3*b^(3/2)*(sqr
t(b)*sqrt(x) - sqrt(b*x + a))^4 - 9*a*b^(3/2)*(sqrt(b)*sqrt(x) - sqrt(b*x +
 a))^2 + 4*a^2*b^(3/2))/(((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^3*a^3)
```

3.264 $\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=186

$$\frac{256b^3\sqrt{x}}{15\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^5\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{128b^3\sqrt{x}}{15\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

```
[Out] (-128*b^3*Sqrt[x])/(15*(b*x - ArcTanh[Tanh[a + b*x]])^4*ArcTanh[Tanh[a + b*
x]]^(3/2)) + (32*b^2)/(5*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^3*ArcTanh[T
anh[a + b*x]]^(3/2)) + (16*b)/(15*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2*
ArcTanh[Tanh[a + b*x]]^(3/2)) + 2/(5*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])
*ArcTanh[Tanh[a + b*x]]^(3/2)) + (256*b^3*Sqrt[x])/(15*(b*x - ArcTanh[Tanh[
a + b*x]])^5*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rubi [A] time = 0.104424, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2171, 2167}

$$\frac{256b^3\sqrt{x}}{15\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^5\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{128b^3\sqrt{x}}{15\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```


```
[Out] (-128*b^3*Sqrt[x])/(15*(b*x - ArcTanh[Tanh[a + b*x]])^4*ArcTanh[Tanh[a + b*
x]]^(3/2)) + (32*b^2)/(5*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^3*ArcTanh[T
anh[a + b*x]]^(3/2)) + (16*b)/(15*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2*
ArcTanh[Tanh[a + b*x]]^(3/2)) + 2/(5*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])
*ArcTanh[Tanh[a + b*x]]^(3/2)) + (256*b^3*Sqrt[x])/(15*(b*x - ArcTanh[Tanh[
a + b*x]])^5*Sqrt[ArcTanh[Tanh[a + b*x]]])
```


Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = \frac{2}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{(8b)\int \frac{1}{x^{5/2}\tanh^{-1}(\tan}}{5\left(bx-\tanh^{-1}(\tan\right.}$$

$$= \frac{16b}{15x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{5x^{5/2}\left(bx-\tanh\right.}$$

$$= \frac{32b^2}{5\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{15x^{3/2}\left(bx-\tanh\right.}$$

$$= -\frac{128b^3\sqrt{x}}{15\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{5\sqrt{x}\left(bx-\tanh^{-1}(\right.}$$

$$= -\frac{128b^3\sqrt{x}}{15\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{5\sqrt{x}\left(bx-\tanh^{-1}(\right.}$$

Mathematica [A] time = 0.0647976, size = 100, normalized size = 0.54

$$\frac{2\left(60b^3x^3\tanh^{-1}(\tanh(a+bx))+90b^2x^2\tanh^{-1}(\tanh(a+bx))^2-20bx\tanh^{-1}(\tanh(a+bx))^3+3\tanh^{-1}(\tanh(a+b\right.}{15x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^5\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (2*(-5*b^4*x^4 + 60*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 90*b^2*x^2*ArcTanh[Tan
h[a + b*x]]^2 - 20*b*x*ArcTanh[Tanh[a + b*x]]^3 + 3*ArcTanh[Tanh[a + b*x]]^
4))/(15*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])^5*ArcTanh[Tanh[a + b*x]]^(3/
2))
```

Maple [A] time = 0.12, size = 196, normalized size = 1.1

$$-\frac{2}{5\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-5\,bx}x^{-\frac{5}{2}}\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)\right)^{-\frac{3}{2}}-\frac{16\,b}{5\,\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-5\,bx}\left(-\frac{}{3\,\mathrm{Artanh}\left(\tan\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] -2/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)/arctanh(tanh(b*x+a))^(3/2)-16/5*b/(
arctanh(tanh(b*x+a))-b*x)*(-1/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)/arctanh(
tanh(b*x+a))^(3/2)-2*b/(arctanh(tanh(b*x+a))-b*x)*(-1/(arctanh(tanh(b*x+a))
-b*x)/x^(1/2)/arctanh(tanh(b*x+a))^(3/2)-4*b/(arctanh(tanh(b*x+a))-b*x)*(1/
3*x^(1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^(3/2)+2/3/(arctan
h(tanh(b*x+a))-b*x)^2*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))))
```

Maxima [A] time = 1.51348, size = 90, normalized size = 0.48

$$-\frac{2\left(128\,b^5x^5+320\,ab^4x^4+240\,a^2b^3x^3+40\,a^3b^2x^2-5\,a^4bx+3\,a^5\right)}{15\,(bx+a)^{\frac{5}{2}}a^5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/15*(128*b^5*x^5 + 320*a*b^4*x^4 + 240*a^2*b^3*x^3 + 40*a^3*b^2*x^2 - 5*a
^4*b*x + 3*a^5)/((b*x + a)^(5/2)*a^5*x^(5/2))
```

Fricas [A] time = 2.07344, size = 181, normalized size = 0.97

$$-\frac{2\left(128\,b^4x^4+192\,ab^3x^3+48\,a^2b^2x^2-8\,a^3bx+3\,a^4\right)\sqrt{bx+a}\sqrt{x}}{15\left(a^5b^2x^5+2\,a^6bx^4+a^7x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(128*b^4*x^4 + 192*a*b^3*x^3 + 48*a^2*b^2*x^2 - 8*a^3*b*x + 3*a^4)*sq
rt(b*x + a)*sqrt(x)/(a^5*b^2*x^5 + 2*a^6*b*x^4 + a^7*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.44234, size = 234, normalized size = 1.26

$$-\frac{2\sqrt{x}\left(\frac{11\,b^4x}{a^5}+\frac{12\,b^3}{a^4}\right)}{3\,(bx+a)^{\frac{3}{2}}}+\frac{4\left(45\,b^{\frac{5}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^8-240\,ab^{\frac{5}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^6+490\,a^2b^{\frac{5}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^4-320}{15\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^5a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(x)*(11*b^4*x/a^5 + 12*b^3/a^4)/(b*x + a)^(3/2) + 4/15*(45*b^(5/2)
*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^8 - 240*a*b^(5/2)*(sqrt(b)*sqrt(x) - sqr
t(b*x + a))^6 + 490*a^2*b^(5/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^4 - 320*a
^3*b^(5/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 + 73*a^4*b^(5/2))/(((sqrt(b)
*sqrt(x) - sqrt(b*x + a))^2 - a)^5*a^4)
```

3.265 $\int x^m \tanh^{-1}(\tanh(a+bx))^n \, dx$

Optimal. Leaf size=79

$$\frac{x^m \left(\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)^{-m} \tanh^{-1}(\tanh(a+bx))^{n+1} \text{Hypergeometric2F1}\left(-m, n+1, n+2, -\frac{\tanh^{-1}(\tanh(a+bx))}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{b(n+1)}$$

```
[Out] (x^m*ArcTanh[Tanh[a + b*x]]^(1 + n)*Hypergeometric2F1[-m, 1 + n, 2 + n, -(A
rcTanh[Tanh[a + b*x]]/(b*x - ArcTanh[Tanh[a + b*x]]))])/(b*(1 + n)*((b*x)/(
b*x - ArcTanh[Tanh[a + b*x]]))^m)
```

Rubi [A] time = 0.0432839, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2173}

$$\frac{x^m \left(\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)^{-m} \tanh^{-1}(\tanh(a+bx))^{n+1} {}_2F_1\left(-m, n+1; n+2; -\frac{\tanh^{-1}(\tanh(a+bx))}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] (x^m*ArcTanh[Tanh[a + b*x]]^(1 + n)*Hypergeometric2F1[-m, 1 + n, 2 + n, -(A
rcTanh[Tanh[a + b*x]]/(b*x - ArcTanh[Tanh[a + b*x]]))])/(b*(1 + n)*((b*x)/(
b*x - ArcTanh[Tanh[a + b*x]]))^m)
```

Rule 2173

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(u^m*v^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, -((
a*v)/(b*u - a*v))])/(b*(n + 1)*((b*u)/(b*u - a*v))^m), x] /; NeQ[b*u - a*v,
 0]] /; PiecewiseLinearQ[u, v, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\int x^m \tanh^{-1}(\tanh(a+bx))^n \, dx = \frac{x^m \left(\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)^{-m} \tanh^{-1}(\tanh(a+bx))^{1+n} {}_2F_1\left(-m, 1+n; 2+n; -\frac{\tanh^{-1}(\tanh(a+bx))}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{b(1+n)}$$

Mathematica [A] time = 0.128212, size = 71, normalized size = 0.9

$$\frac{x^{m+1}\tanh^{-1}(\tanh(a+bx))^n\left(\frac{bx}{\tanh^{-1}(\tanh(a+bx))-bx}+1\right)^{-n}\text{Hypergeometric2F1}\left(m+1,-n,m+2,-\frac{bx}{\tanh^{-1}(\tanh(a+bx))-bx}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] (x^(1 + m)*ArcTanh[Tanh[a + b*x]]^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((
b*x)/(-(b*x) + ArcTanh[Tanh[a + b*x]]))])/((1 + m)*(1 + (b*x)/(-(b*x) + Arc
Tanh[Tanh[a + b*x]]))^n)
```

Maple [F] time = 1.392, size = 0, normalized size = 0.

$$\int x^m (\text{Artanh}(\tanh(bx+a)))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctanh(tanh(b*x+a))^n,x)
```

```
[Out] int(x^m*arctanh(tanh(b*x+a))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{artanh}(\tanh(bx+a))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] integrate(x^m*arctanh(tanh(b*x + a))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^m \operatorname{artanh}(\tanh(bx+a))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] integral(x^m*arctanh(tanh(b*x + a))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{atanh}^n(\tanh(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atanh(tanh(b*x+a))**n,x)
```

```
[Out] Integral(x**m*atanh(tanh(a + b*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{artanh}(\tanh(bx+a))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^n,x, algorithm="giac")
```

```
[Out] integrate(x^m*arctanh(tanh(b*x + a))^n, x)
```

3.266 $\int x^4 \tanh^{-1}(\tanh(a+bx))^n \, dx$

Optimal. Leaf size=165

$$-\frac{4x^3 \tanh^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}+\frac{12x^2 \tanh^{-1}(\tanh(a+bx))^{n+3}}{b^3(n+1)(n+2)(n+3)}-\frac{24x \tanh^{-1}(\tanh(a+bx))^{n+4}}{b^4(n+1)(n+2)(n+3)(n+4)}+\frac{24 \tanh^{-1}(\tan}{b^5(n+1)(n+2)(n}$$

```
[Out] (x^4*ArcTanh[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (4*x^3*ArcTanh[Tanh[a +
b*x]]^(2 + n))/(b^2*(1 + n)*(2 + n)) + (12*x^2*ArcTanh[Tanh[a + b*x]]^(3 +
n))/(b^3*(1 + n)*(2 + n)*(3 + n)) - (24*x*ArcTanh[Tanh[a + b*x]]^(4 + n))/(
b^4*(1 + n)*(2 + n)*(3 + n)*(4 + n)) + (24*ArcTanh[Tanh[a + b*x]]^(5 + n))/
(b^5*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n))
```

Rubi [A] time = 0.133738, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2157, 30}

$$-\frac{4x^3 \tanh^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}+\frac{12x^2 \tanh^{-1}(\tanh(a+bx))^{n+3}}{b^3(n+1)(n+2)(n+3)}-\frac{24x \tanh^{-1}(\tanh(a+bx))^{n+4}}{b^4(n+1)(n+2)(n+3)(n+4)}+\frac{24 \tanh^{-1}(\tan}{b^5(n+1)(n+2)(n}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] (x^4*ArcTanh[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (4*x^3*ArcTanh[Tanh[a +
b*x]]^(2 + n))/(b^2*(1 + n)*(2 + n)) + (12*x^2*ArcTanh[Tanh[a + b*x]]^(3 +
n))/(b^3*(1 + n)*(2 + n)*(3 + n)) - (24*x*ArcTanh[Tanh[a + b*x]]^(4 + n))/(
b^4*(1 + n)*(2 + n)*(3 + n)*(4 + n)) + (24*ArcTanh[Tanh[a + b*x]]^(5 + n))/
(b^5*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n))
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^4 \tanh^{-1}(\tanh(a+bx))^n \, dx = \frac{x^4 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4\int x^3 \tanh^{-1}(\tanh(a+bx))^{1+n}\, dx}{b(1+n)}$$
$$= \frac{x^4 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4x^3 \tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12\int x^2 \tanh^{-1}(\tan}{b^2(1+n)(}$$
$$= \frac{x^4 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4x^3 \tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12x^2 \tanh^{-1}(\tanh(a}{b^3(1+n)(2+n)}$$
$$= \frac{x^4 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4x^3 \tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12x^2 \tanh^{-1}(\tanh(a}{b^3(1+n)(2+n)}$$
$$= \frac{x^4 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4x^3 \tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12x^2 \tanh^{-1}(\tanh(a}{b^3(1+n)(2+n)}$$
$$= \frac{x^4 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4x^3 \tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12x^2 \tanh^{-1}(\tanh(a}{b^3(1+n)(2+n)}$$

Mathematica [A] time = 0.10441, size = 146, normalized size = 0.88

$$\frac{\tanh^{-1}(\tanh(a+bx))^{n+1}\left(-4b^3\left(n^3+12n^2+47n+60\right)x^3\tanh^{-1}(\tanh(a+bx))+12b^2\left(n^2+9n+20\right)x^2\tanh^{-1}(\tan}{b^5(n+1)(n+2)(}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] (ArcTanh[Tanh[a + b*x]]^(1 + n)*(b^4*(120 + 154*n + 71*n^2 + 14*n^3 + n^4)*
x^4 - 4*b^3*(60 + 47*n + 12*n^2 + n^3)*x^3*ArcTanh[Tanh[a + b*x]] + 12*b^2*
(20 + 9*n + n^2)*x^2*ArcTanh[Tanh[a + b*x]]^2 - 24*b*(5 + n)*x*ArcTanh[Tanh
[a + b*x]]^3 + 24*ArcTanh[Tanh[a + b*x]]^4))/(b^5*(1 + n)*(2 + n)*(3 + n)*(
4 + n)*(5 + n))
```

Maple [B] time = 0.049, size = 654, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctanh(tanh(b*x+a))^n,x)
```

```
[Out] 1/(5+n)*x^5*exp(n*ln(arctanh(tanh(b*x+a))))+n/b*(arctanh(tanh(b*x+a))-b*x)/
(n^2+9*n+20)*x^4*exp(n*ln(arctanh(tanh(b*x+a))))-4*n*(a^2+2*a*(arctanh(tanh
(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)/b^2/(n^3+12*n^2+47*n+60)*x^
3*exp(n*ln(arctanh(tanh(b*x+a))))+24/b^5/(n^3+12*n^2+47*n+60)/(n^2+3*n+2)*e
xp(n*ln(arctanh(tanh(b*x+a))))*a^5+120/b^5/(n^3+12*n^2+47*n+60)/(n^2+3*n+2)
*exp(n*ln(arctanh(tanh(b*x+a))))*a^4*(arctanh(tanh(b*x+a))-b*x-a)+240/b^5/(
n^3+12*n^2+47*n+60)/(n^2+3*n+2)*exp(n*ln(arctanh(tanh(b*x+a))))*a^3*(arctan
h(tanh(b*x+a))-b*x-a)^2+240/b^5/(n^3+12*n^2+47*n+60)/(n^2+3*n+2)*exp(n*ln(a
rctanh(tanh(b*x+a))))*a^2*(arctanh(tanh(b*x+a))-b*x-a)^3+120/b^5/(n^3+12*n^
2+47*n+60)/(n^2+3*n+2)*exp(n*ln(arctanh(tanh(b*x+a))))*a*(arctanh(tanh(b*x+
a))-b*x-a)^4+24/b^5/(n^3+12*n^2+47*n+60)/(n^2+3*n+2)*exp(n*ln(arctanh(tanh(
b*x+a))))*(arctanh(tanh(b*x+a))-b*x-a)^5-24*(a^2+2*a*(arctanh(tanh(b*x+a))-
b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)^2*n/b^4/(n^3+12*n^2+47*n+60)/(n^2+3*
n+2)*x*exp(n*ln(arctanh(tanh(b*x+a))))+12/b^3*(arctanh(tanh(b*x+a))-b*x)*(a
^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)*n/(2+n)
/(n^3+12*n^2+47*n+60)*x^2*exp(n*ln(arctanh(tanh(b*x+a))))
```

Maxima [A] time = 1.79333, size = 188, normalized size = 1.14

$$\frac{\left(\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^5x^5+\left(n^4+6\,n^3+11\,n^2+6\,n\right)ab^4x^4-4\left(n^3+3\,n^2+2\,n\right)a^2b^3x^3+12\left(n^2+n\right)a^3b^2x^2}{\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] ((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)
*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2 -
 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274
*n + 120)*b^5)
```

Fricas [B] time = 2.18292, size = 788, normalized size = 4.78

$$-\frac{\left(24\,a^4bnx-\left(b^5n^4+10\,b^5n^3+35\,b^5n^2+50\,b^5n+24\,b^5\right)x^5-24\,a^5-\left(ab^4n^4+6\,ab^4n^3+11\,ab^4n^2+6\,ab^4n\right)x^4+4\left(a^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] -((24*a^4*b*n*x - (b^5*n^4 + 10*b^5*n^3 + 35*b^5*n^2 + 50*b^5*n + 24*b^5)*x
^5 - 24*a^5 - (a*b^4*n^4 + 6*a*b^4*n^3 + 11*a*b^4*n^2 + 6*a*b^4*n)*x^4 + 4*
(a^2*b^3*n^3 + 3*a^2*b^3*n^2 + 2*a^2*b^3*n)*x^3 - 12*(a^3*b^2*n^2 + a^3*b^2
*n)*x^2)*cosh(n*log(b*x + a)) + (24*a^4*b*n*x - (b^5*n^4 + 10*b^5*n^3 + 35*
b^5*n^2 + 50*b^5*n + 24*b^5)*x^5 - 24*a^5 - (a*b^4*n^4 + 6*a*b^4*n^3 + 11*a
*b^4*n^2 + 6*a*b^4*n)*x^4 + 4*(a^2*b^3*n^3 + 3*a^2*b^3*n^2 + 2*a^2*b^3*n)*x
^3 - 12*(a^3*b^2*n^2 + a^3*b^2*n)*x^2)*sinh(n*log(b*x + a)))/(b^5*n^5 + 15*
b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atanh(tanh(b*x+a))**n,x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.17808, size = 448, normalized size = 2.72

$$\frac{(bx+a)^n b^5n^4x^5+(bx+a)^n ab^4n^4x^4+10\,(bx+a)^n b^5n^3x^5+6\,(bx+a)^n ab^4n^3x^4+35\,(bx+a)^n b^5n^2x^5-4\,(bx+a)^n a^2b^3n}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^n,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^5*n^4*x^5 + (b*x + a)^n*a*b^4*n^4*x^4 + 10*(b*x + a)^n*b^5*n
^3*x^5 + 6*(b*x + a)^n*a*b^4*n^3*x^4 + 35*(b*x + a)^n*b^5*n^2*x^5 - 4*(b*x
```

```
+ a)^n*a^2*b^3*n^3*x^3 + 11*(b*x + a)^n*a*b^4*n^2*x^4 + 50*(b*x + a)^n*b^5*
n*x^5 - 12*(b*x + a)^n*a^2*b^3*n^2*x^3 + 6*(b*x + a)^n*a*b^4*n*x^4 + 24*(b*
x + a)^n*b^5*x^5 + 12*(b*x + a)^n*a^3*b^2*n^2*x^2 - 8*(b*x + a)^n*a^2*b^3*n
*x^3 + 12*(b*x + a)^n*a^3*b^2*n*x^2 - 24*(b*x + a)^n*a^4*b*n*x + 24*(b*x +
a)^n*a^5)/(b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 12
0*b^5)
```

3.267 $\int x^3 \tanh^{-1}(\tanh(a+bx))^n \, dx$

Optimal. Leaf size=121

$$-\frac{3x^2\tanh^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}+\frac{6x\tanh^{-1}(\tanh(a+bx))^{n+3}}{b^3(n+1)(n+2)(n+3)}-\frac{6\tanh^{-1}(\tanh(a+bx))^{n+4}}{b^4(n+1)(n+2)(n+3)(n+4)}+\frac{x^3\tanh^{-1}(\tanh(a+}{b(n+1)}$$

```
[Out] (x^3*ArcTanh[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (3*x^2*ArcTanh[Tanh[a +
b*x]]^(2 + n))/(b^2*(1 + n)*(2 + n)) + (6*x*ArcTanh[Tanh[a + b*x]]^(3 + n))
/(b^3*(1 + n)*(2 + n)*(3 + n)) - (6*ArcTanh[Tanh[a + b*x]]^(4 + n))/(b^4*(1
 + n)*(2 + n)*(3 + n)*(4 + n))
```

Rubi [A] time = 0.0750075, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2157, 30}

$$-\frac{3x^2\tanh^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}+\frac{6x\tanh^{-1}(\tanh(a+bx))^{n+3}}{b^3(n+1)(n+2)(n+3)}-\frac{6\tanh^{-1}(\tanh(a+bx))^{n+4}}{b^4(n+1)(n+2)(n+3)(n+4)}+\frac{x^3\tanh^{-1}(\tanh(a+}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] (x^3*ArcTanh[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (3*x^2*ArcTanh[Tanh[a +
b*x]]^(2 + n))/(b^2*(1 + n)*(2 + n)) + (6*x*ArcTanh[Tanh[a + b*x]]^(3 + n))
/(b^3*(1 + n)*(2 + n)*(3 + n)) - (6*ArcTanh[Tanh[a + b*x]]^(4 + n))/(b^4*(1
 + n)*(2 + n)*(3 + n)*(4 + n))
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^3 \tanh^{-1}(\tanh(a+bx))^n\,dx = \frac{x^3 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{3\int x^2 \tanh^{-1}(\tanh(a+bx))^{1+n}\,dx}{b(1+n)}$$
$$= \frac{x^3 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{3x^2 \tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{6\int x \tanh^{-1}(\tanh(a}{b^2(1+n)(2+}$$
$$= \frac{x^3 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{3x^2 \tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{6x \tanh^{-1}(\tanh(a+}{b^3(1+n)(2+n)(3}$$
$$= \frac{x^3 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{3x^2 \tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{6x \tanh^{-1}(\tanh(a+}{b^3(1+n)(2+n)(3}$$
$$= \frac{x^3 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{3x^2 \tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{6x \tanh^{-1}(\tanh(a+}{b^3(1+n)(2+n)(3}$$

Mathematica [A] time = 0.0767393, size = 106, normalized size = 0.88

$$\frac{\tanh^{-1}(\tanh(a+bx))^{n+1}\left(-3b^2\left(n^2+7n+12\right)x^2\tanh^{-1}(\tanh(a+bx))+6b(n+4)x\tanh^{-1}(\tanh(a+bx))^2-6\tanh^{-}\right.}{b^4(n+1)(n+2)(n+3)(n+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] (ArcTanh[Tanh[a + b*x]]^(1 + n)*(b^3*(24 + 26*n + 9*n^2 + n^3)*x^3 - 3*b^2*
(12 + 7*n + n^2)*x^2*ArcTanh[Tanh[a + b*x]] + 6*b*(4 + n)*x*ArcTanh[Tanh[a
+ b*x]]^2 - 6*ArcTanh[Tanh[a + b*x]]^3))/(b^4*(1 + n)*(2 + n)*(3 + n)*(4 +
n))
```

Maple [B] time = 0.046, size = 492, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(tanh(b*x+a))^n,x)
```

```
[Out] 1/(4+n)*x^4*exp(n*ln(arctanh(tanh(b*x+a))))+n*(arctanh(tanh(b*x+a))-b*x)/b/
(n^2+7*n+12)*x^3*exp(n*ln(arctanh(tanh(b*x+a))))-6/b^4/(n^4+10*n^3+35*n^2+5
0*n+24)*exp(n*ln(arctanh(tanh(b*x+a))))*a^4-24/b^4/(n^4+10*n^3+35*n^2+50*n+
24)*exp(n*ln(arctanh(tanh(b*x+a))))*a^3*(arctanh(tanh(b*x+a))-b*x-a)-36/b^4
/(n^4+10*n^3+35*n^2+50*n+24)*exp(n*ln(arctanh(tanh(b*x+a))))*a^2*(arctanh(t
anh(b*x+a))-b*x-a)^2-24/b^4/(n^4+10*n^3+35*n^2+50*n+24)*exp(n*ln(arctanh(ta
nh(b*x+a))))*a*(arctanh(tanh(b*x+a))-b*x-a)^3-6/b^4/(n^4+10*n^3+35*n^2+50*n
+24)*exp(n*ln(arctanh(tanh(b*x+a))))*(arctanh(tanh(b*x+a))-b*x-a)^4-3*n/b^2
*(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)/(n^3
+9*n^2+26*n+24)*x^2*exp(n*ln(arctanh(tanh(b*x+a))))+6*n*(a^3+3*a^2*(arctanh
(tanh(b*x+a))-b*x-a)+3*a*(arctanh(tanh(b*x+a))-b*x-a)^2+(arctanh(tanh(b*x+a
))-b*x-a)^3)/b^3/(n^4+10*n^3+35*n^2+50*n+24)*x*exp(n*ln(arctanh(tanh(b*x+a)
)))
```

Maxima [A] time = 1.81333, size = 136, normalized size = 1.12

$$\frac{\left(\left(n^3+6\,n^2+11\,n+6\right)b^4x^4+\left(n^3+3\,n^2+2\,n\right)ab^3x^3-3\left(n^2+n\right)a^2b^2x^2+6\,a^3bnx-6\,a^4\right)(bx+a)^n}{\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
+ n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2
 + 50*n + 24)*b^4)
```

Fricas [B] time = 2.16174, size = 536, normalized size = 4.43

$$\frac{\left(6\,a^3bnx+\left(b^4n^3+6\,b^4n^2+11\,b^4n+6\,b^4\right)x^4-6\,a^4+\left(ab^3n^3+3\,ab^3n^2+2\,ab^3n\right)x^3-3\left(a^2b^2n^2+a^2b^2n\right)x^2\right)\cosh\left(n\log\right.}{b^4n^4+10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] ((6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 6*a^4 + (a*b
^3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2 + a^2*b^2*n)*x^2)*co
sh(n*log(b*x + a)) + (6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4
)*x^4 - 6*a^4 + (a*b^3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2
+ a^2*b^2*n)*x^2)*sinh(n*log(b*x + a)))/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2
+ 50*b^4*n + 24*b^4)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(tanh(b*x+a))**n,x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.21447, size = 305, normalized size = 2.52

$$\frac{(bx+a)^n b^4 n^3 x^4 + (bx+a)^n a b^3 n^3 x^3 + 6\,(bx+a)^n b^4 n^2 x^4 + 3\,(bx+a)^n a b^3 n^2 x^3 + 11\,(bx+a)^n b^4 n x^4 - 3\,(bx+a)^n a^2 b^2 n^2 x^2}{b^4 n^4 + 10\,b^4 n^3 + 35\,b^4 n^2 + 50\,b^4 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^n,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^4*n^3*x^4 + (b*x + a)^n*a*b^3*n^3*x^3 + 6*(b*x + a)^n*b^4*n^
2*x^4 + 3*(b*x + a)^n*a*b^3*n^2*x^3 + 11*(b*x + a)^n*b^4*n*x^4 - 3*(b*x + a
)^n*a^2*b^2*n^2*x^2 + 2*(b*x + a)^n*a*b^3*n*x^3 + 6*(b*x + a)^n*b^4*x^4 - 3
*(b*x + a)^n*a^2*b^2*n*x^2 + 6*(b*x + a)^n*a^3*b*n*x - 6*(b*x + a)^n*a^4)/(
b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

3.268 $\int x^2 \tanh^{-1}(\tanh(a+bx))^n \, dx$

Optimal. Leaf size=82

$$-\frac{2x\tanh^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}+\frac{2\tanh^{-1}(\tanh(a+bx))^{n+3}}{b^3(n+1)(n+2)(n+3)}+\frac{x^2\tanh^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}$$

```
[Out] (x^2*ArcTanh[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (2*x*ArcTanh[Tanh[a + b*
x]]^(2 + n))/(b^2*(1 + n)*(2 + n)) + (2*ArcTanh[Tanh[a + b*x]]^(3 + n))/(b^
3*(1 + n)*(2 + n)*(3 + n))
```

Rubi [A] time = 0.0461799, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2157, 30}

$$-\frac{2x\tanh^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}+\frac{2\tanh^{-1}(\tanh(a+bx))^{n+3}}{b^3(n+1)(n+2)(n+3)}+\frac{x^2\tanh^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] (x^2*ArcTanh[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (2*x*ArcTanh[Tanh[a + b*
x]]^(2 + n))/(b^2*(1 + n)*(2 + n)) + (2*ArcTanh[Tanh[a + b*x]]^(3 + n))/(b^
3*(1 + n)*(2 + n)*(3 + n))
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^2 \tanh^{-1}(\tanh(a+bx))^n\,dx = \frac{x^2 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{2\int x \tanh^{-1}(\tanh(a+bx))^{1+n}\,dx}{b(1+n)}$$
$$= \frac{x^2 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{2x \tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{2\int \tanh^{-1}(\tanh(a+b}{b^2(1+n)(2+}$$
$$= \frac{x^2 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{2x \tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{2\,\text{Subst}\left(\int x^{2+n}\,dx, x,\right.}{b^3(1+n}$$
$$= \frac{x^2 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{2x \tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{2\tanh^{-1}(\tanh(a+bx}{b^3(1+n)(2+n)(3+}$$

Mathematica [A] time = 0.0576271, size = 71, normalized size = 0.87

$$\frac{\tanh^{-1}(\tanh(a+bx))^{n+1}\left(-2b(n+3)x\tanh^{-1}(\tanh(a+bx))+2\tanh^{-1}(\tanh(a+bx))^2+b^2\left(n^2+5n+6\right)x^2\right)}{b^3(n+1)(n+2)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] (ArcTanh[Tanh[a + b*x]]^(1 + n)*(b^2*(6 + 5*n + n^2)*x^2 - 2*b*(3 + n)*x*Ar
cTanh[Tanh[a + b*x]] + 2*ArcTanh[Tanh[a + b*x]]^2))/(b^3*(1 + n)*(2 + n)*(3
 + n))
```

Maple [B] time = 0.044, size = 315, normalized size = 3.8

$$\frac{x^3 e^{n\ln(\text{Artanh}(\tanh(bx+a)))}}{3+n} + \frac{n\left(\text{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)x^2 e^{n\ln(\text{Artanh}(\tanh(bx+a)))}}{b\left(n^2+5n+6\right)} + 2\,\frac{e^{n\ln(\text{Artanh}(\tanh(bx+a)))}a^3}{b^3\left(n^3+6n^2+11n+6\right)} + 6\,\text{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(tanh(b*x+a))^n,x)
```

```
[Out] 1/(3+n)*x^3*exp(n*ln(arctanh(tanh(b*x+a))))+n/b*(arctanh(tanh(b*x+a))-b*x)/
(n^2+5*n+6)*x^2*exp(n*ln(arctanh(tanh(b*x+a))))+2/b^3/(n^3+6*n^2+11*n+6)*ex
p(n*ln(arctanh(tanh(b*x+a))))*a^3+6/b^3/(n^3+6*n^2+11*n+6)*exp(n*ln(arctanh
(tanh(b*x+a))))*a^2*(arctanh(tanh(b*x+a))-b*x-a)+6/b^3/(n^3+6*n^2+11*n+6)*e
xp(n*ln(arctanh(tanh(b*x+a))))*a*(arctanh(tanh(b*x+a))-b*x-a)^2+2/b^3/(n^3+
6*n^2+11*n+6)*exp(n*ln(arctanh(tanh(b*x+a))))*(arctanh(tanh(b*x+a))-b*x-a)^
3-2*n*(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)
/b^2/(n^3+6*n^2+11*n+6)*x*exp(n*ln(arctanh(tanh(b*x+a))))
```

Maxima [A] time = 1.7989, size = 92, normalized size = 1.12

$$\frac{\left(\left(n^2+3\,n+2\right)b^3x^3+\left(n^2+n\right)ab^2x^2-2\,a^2bnx+2\,a^3\right)(bx+a)^n}{\left(n^3+6\,n^2+11\,n+6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3)
```

Fricas [B] time = 2.12992, size = 347, normalized size = 4.23

$$-\frac{\left(2\,a^2bnx-\left(b^3n^2+3\,b^3n+2\,b^3\right)x^3-2\,a^3-\left(ab^2n^2+ab^2n\right)x^2\right)\cosh\left(n\log\left(bx+a\right)\right)+\left(2\,a^2bnx-\left(b^3n^2+3\,b^3n+2\,b^3\right.\right.}{b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] -((2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b
^2*n)*x^2)*cosh(n*log(b*x + a)) + (2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3
)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b^2*n)*x^2)*sinh(n*log(b*x + a)))/(b^3*n^3 +
 6*b^3*n^2 + 11*b^3*n + 6*b^3)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(tanh(b*x+a))**n,x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.17005, size = 189, normalized size = 2.3

$$\frac{(bx+a)^n b^3 n^2 x^3 + (bx+a)^n a b^2 n^2 x^2 + 3\,(bx+a)^n b^3 n x^3 + (bx+a)^n a b^2 n x^2 + 2\,(bx+a)^n b^3 x^3 - 2\,(bx+a)^n a^2 b n x + 2\,(bx+}{b^3 n^3 + 6\,b^3 n^2 + 11\,b^3 n + 6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^n,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^3*n^2*x^3 + (b*x + a)^n*a*b^2*n^2*x^2 + 3*(b*x + a)^n*b^3*n*
x^3 + (b*x + a)^n*a*b^2*n*x^2 + 2*(b*x + a)^n*b^3*x^3 - 2*(b*x + a)^n*a^2*b
*n*x + 2*(b*x + a)^n*a^3)/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3)
```

3.269 $\int x\tanh^{-1}(\tanh(a+bx))^n\,dx$

Optimal. Leaf size=48

$$\frac{x\tanh^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{\tanh^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}$$

```
[Out] (x*ArcTanh[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - ArcTanh[Tanh[a + b*x]]^(2
+ n)/(b^2*(1 + n)*(2 + n))
```

Rubi [A] time = 0.0194814, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2168, 2157, 30}

$$\frac{x\tanh^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{\tanh^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] (x*ArcTanh[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - ArcTanh[Tanh[a + b*x]]^(2
+ n)/(b^2*(1 + n)*(2 + n))
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\tanh^{-1}(\tanh(a+bx))^n\,dx &= \frac{x\tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)}-\frac{\int \tanh^{-1}(\tanh(a+bx))^{1+n}\,dx}{b(1+n)}\\ &= \frac{x\tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)}-\frac{\mathrm{Subst}\left(\int x^{1+n}\,dx,x,\tanh^{-1}(\tanh(a+bx))\right)}{b^2(1+n)}\\ &= \frac{x\tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)}-\frac{\tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)}\end{aligned}$$

Mathematica [A] time = 0.0422084, size = 41, normalized size = 0.85

$$\frac{\left(b(n+2)x-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{n+1}}{b^2(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] ((b*(2 + n)*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(1 + n))/(b^
2*(1 + n)*(2 + n))
```

Maple [B] time = 0.042, size = 175, normalized size = 3.7

$$\frac{x^2e^{n\ln(\mathrm{Artanh}(\tanh(bx+a)))}}{2+n}+\frac{n\left(\mathrm{Artanh}\left(\tanh\left(bx+a\right)\right)-bx\right)xe^{n\ln(\mathrm{Artanh}(\tanh(bx+a)))}}{b\left(n^2+3n+2\right)}-\frac{e^{n\ln(\mathrm{Artanh}(\tanh(bx+a)))}a^2}{b^2\left(n^2+3n+2\right)}-2\,e^{n\ln(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(tanh(b*x+a))^n,x)
```

```
[Out] 1/(2+n)*x^2*exp(n*ln(arctanh(tanh(b*x+a))))+n*(arctanh(tanh(b*x+a))-b*x)/b/
(n^2+3*n+2)*x*exp(n*ln(arctanh(tanh(b*x+a))))-1/b^2/(n^2+3*n+2)*exp(n*ln(ar
ctanh(tanh(b*x+a))))*a^2-2/b^2/(n^2+3*n+2)*exp(n*ln(arctanh(tanh(b*x+a))))*
a*(arctanh(tanh(b*x+a))-b*x-a)-1/b^2/(n^2+3*n+2)*exp(n*ln(arctanh(tanh(b*x+
```

```
a))))*(arctanh(tanh(b*x+a))-b*x-a)^2
```

Maxima [A] time = 1.78231, size = 57, normalized size = 1.19

$$\frac{\left(b^2(n+1)x^2+abnx-a^2\right)(bx+a)^n}{\left(n^2+3\,n+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n/((n^2 + 3*n + 2)*b^2)
```

Fricas [A] time = 2.0834, size = 198, normalized size = 4.12

$$\frac{\left(abnx+\left(b^2n+b^2\right)x^2-a^2\right)\cosh\left(n\log\left(bx+a\right)\right)+\left(abnx+\left(b^2n+b^2\right)x^2-a^2\right)\sinh\left(n\log\left(bx+a\right)\right)}{b^2n^2+3\,b^2n+2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] ((a*b*n*x + (b^2*n + b^2)*x^2 - a^2)*cosh(n*log(b*x + a)) + (a*b*n*x + (b^2
*n + b^2)*x^2 - a^2)*sinh(n*log(b*x + a)))/(b^2*n^2 + 3*b^2*n + 2*b^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(tanh(b*x+a))**n,x)
```

```
[Out] Exception raised: TypeError
```

<u>Giac</u> [A] time = 1.162, size = 103, normalized size = 2.15

$$\frac{(bx+a)^n b^2 n x^2 + (bx+a)^n abnx + (bx+a)^n b^2 x^2 - (bx+a)^n a^2}{b^2 n^2 + 3\,b^2 n + 2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^n,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^2*n*x^2 + (b*x + a)^n*a*b*n*x + (b*x + a)^n*b^2*x^2 - (b*x +
 a)^n*a^2)/(b^2*n^2 + 3*b^2*n + 2*b^2)
```

3.270 $\int \tanh^{-1}(\tanh(a+bx))^n \, dx$

Optimal. Leaf size=20

$$\frac{\tanh^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}$$

```
[Out] ArcTanh[Tanh[a + b*x]]^(1 + n)/(b*(1 + n))
```

Rubi [A] time = 0.0067712, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2157, 30}

$$\frac{\tanh^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^(1 + n)/(b*(1 + n))
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \tanh^{-1}(\tanh(a+bx))^n \, dx &= \frac{\text{Subst}\left(\int x^n \, dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b} \\ &= \frac{\tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} \end{aligned}$$

Mathematica [A] time = 0.0147276, size = 20, normalized size = 1.

$$\frac{\tanh^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^(1 + n)/(b*(1 + n))
```

Maple [A] time = 0.027, size = 21, normalized size = 1.1

$$\frac{(\operatorname{Artanh}(\tanh(bx+a)))^{1+n}}{b(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^n,x)
```

```
[Out] arctanh(tanh(b*x+a))^(1+n)/b/(1+n)
```

Maxima [A] time = 1.69616, size = 28, normalized size = 1.4

$$\frac{(bx+a)(bx+a)^n}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] (b*x + a)*(b*x + a)^n/(b*(n + 1))
```

Fricas [A] time = 2.33867, size = 104, normalized size = 5.2

$$\frac{(bx+a)\cosh(n\log(bx+a))+(bx+a)\sinh(n\log(bx+a))}{bn+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] ((b*x + a)*cosh(n*log(b*x + a)) + (b*x + a)*sinh(n*log(b*x + a)))/(b*n + b)
```

Sympy [A] time = 1.46915, size = 51, normalized size = 2.55

$$\begin{cases} \frac{x}{\operatorname{atanh}(\tanh(a))} & \text{for}\ b = 0 \wedge n = -1 \\ x \operatorname{atanh}^n(\tanh(a)) & \text{for}\ b = 0 \\ \frac{\log(\operatorname{atanh}(\tanh(a+bx)))}{b} & \text{for}\ n = -1 \\ \frac{\operatorname{atanh}(\tanh(a+bx)) \operatorname{atanh}^n(\tanh(a+bx))}{bn+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**n,x)
```

```
[Out] Piecewise((x/atanh(tanh(a)), Eq(b, 0) & Eq(n, -1)), (x*atanh(tanh(a))**n, E
q(b, 0)), (log(atanh(tanh(a + b*x)))/b, Eq(n, -1)), (atanh(tanh(a + b*x))*a
tanh(tanh(a + b*x))**n/(b*n + b), True))
```

Giac [A] time = 1.16034, size = 38, normalized size = 1.9

$$\frac{(bx+a)^n bx + (bx+a)^n a}{bn+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b*x + (b*x + a)^n*a)/(b*n + b)
```

3.271 $\int \frac{\tanh^{-1}(\tanh(a+bx))^n}{x}\,dx$

Optimal. Leaf size=64

$$\frac{\tanh^{-1}(\tanh(a+bx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,-\frac{\tanh^{-1}(\tanh(a+bx))}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{(n+1)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] (ArcTanh[Tanh[a + b*x]]^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, -(ArcTan
h[Tanh[a + b*x]]/(b*x - ArcTanh[Tanh[a + b*x]]))])/((1 + n)*(b*x - ArcTanh[
Tanh[a + b*x]]))
```

Rubi [A] time = 0.021127, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2164}

$$\frac{\tanh^{-1}(\tanh(a+bx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;-\frac{\tanh^{-1}(\tanh(a+bx))}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{(n+1)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^n/x,x]
```

```
[Out] (ArcTanh[Tanh[a + b*x]]^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, -(ArcTan
h[Tanh[a + b*x]]/(b*x - ArcTanh[Tanh[a + b*x]]))])/((1 + n)*(b*x - ArcTanh[
Tanh[a + b*x]]))
```

Rule 2164

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)*Hypergeometric2F1[1, n + 1, n + 2, -((a*v)/(b*u
- a*v))])/((n + 1)*(b*u - a*v)), x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^n}{x}\,dx=\frac{\tanh^{-1}(\tanh(a+bx))^{1+n}\,{}_2F_1\left(1,1+n;2+n;-\frac{\tanh^{-1}(\tanh(a+bx))}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{(1+n)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A] time = 0.0821591, size = 60, normalized size = 0.94

$$\frac{\tanh^{-1}(\tanh(a+bx))^n\left(\frac{\tanh^{-1}(\tanh(a+bx))}{bx}\right)^{-n}\text{Hypergeometric2F1}\left(-n,-n,1-n,1-\frac{\tanh^{-1}(\tanh(a+bx))}{bx}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^n/x,x]
```

```
[Out] (ArcTanh[Tanh[a + b*x]]^n*Hypergeometric2F1[-n, -n, 1 - n, 1 - ArcTanh[Tanh
[a + b*x]]/(b*x)])/(n*(ArcTanh[Tanh[a + b*x]]/(b*x))^n)
```

Maple [F] time = 0.654, size = 0, normalized size = 0.

$$\int \frac{(\text{Artanh}(\tanh(bx+a)))^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^n/x,x)
```

```
[Out] int(arctanh(tanh(b*x+a))^n/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{artanh}(\tanh(bx+a))^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n/x,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^n/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{artanh}(\tanh(bx+a))^n}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n/x,x, algorithm="fricas")
```

```
[Out] integral(arctanh(tanh(b*x + a))^n/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^n(\tanh(a+bx))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**n/x,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**n/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(\tanh(bx+a))^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n/x,x, algorithm="giac")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^n/x, x)
```

3.272 $\int \frac{\tanh^{-1}(\tanh(a+bx))^n}{x^2} dx$

Optimal. Leaf size=71

$$\frac{b\tanh^{-1}(\tanh(a+bx))^n \text{Hypergeometric2F1}\left(1,n,n+1,-\frac{\tanh^{-1}(\tanh(a+bx))}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{bx-\tanh^{-1}(\tanh(a+bx))}-\frac{\tanh^{-1}(\tanh(a+bx))^n}{x}$$

```
[Out] -(ArcTanh[Tanh[a + b*x]]^n/x) + (b*ArcTanh[Tanh[a + b*x]]^n*Hypergeometric2
F1[1, n, 1 + n, -(ArcTanh[Tanh[a + b*x]]/(b*x - ArcTanh[Tanh[a + b*x]]))])/
(b*x - ArcTanh[Tanh[a + b*x]])
```

Rubi [A] time = 0.0415182, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 2164}

$$\frac{b\tanh^{-1}(\tanh(a+bx))^n\, {}_2F_1\left(1,n;n+1;-\frac{\tanh^{-1}(\tanh(a+bx))}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{bx-\tanh^{-1}(\tanh(a+bx))}-\frac{\tanh^{-1}(\tanh(a+bx))^n}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^n/x^2,x]
```

```
[Out] -(ArcTanh[Tanh[a + b*x]]^n/x) + (b*ArcTanh[Tanh[a + b*x]]^n*Hypergeometric2
F1[1, n, 1 + n, -(ArcTanh[Tanh[a + b*x]]/(b*x - ArcTanh[Tanh[a + b*x]]))])/
(b*x - ArcTanh[Tanh[a + b*x]])
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2164

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)*Hypergeometric2F1[1, n + 1, n + 2, -((a*v)/(b*u
```

```
- a*v))])/((n + 1)*(b*u - a*v)), x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^n}{x^2}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^n}{x} + (bn)\int \frac{\tanh^{-1}(\tanh(a+bx))^{-1+n}}{x}\,dx$$

$$= -\frac{\tanh^{-1}(\tanh(a+bx))^n}{x} + \frac{b\tanh^{-1}(\tanh(a+bx))^n\,{}_2F_1\left(1,n;1+n;-\frac{\tanh^{-1}(\tanh(a+bx)}{bx-\tanh^{-1}(\tanh(a+b}\right.}{bx-\tanh^{-1}(\tanh(a+bx))}$$

Mathematica [A] time = 0.0403646, size = 67, normalized size = 0.94

$$\frac{\tanh^{-1}(\tanh(a+bx))^n\left(\frac{\tanh^{-1}(\tanh(a+bx))}{bx}\right)^{-n}\text{Hypergeometric2F1}\left(1-n,-n,2-n,1-\frac{\tanh^{-1}(\tanh(a+bx))}{bx}\right)}{(n-1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^n/x^2,x]
```

```
[Out] (ArcTanh[Tanh[a + b*x]]^n*Hypergeometric2F1[1 - n, -n, 2 - n, 1 - ArcTanh[T
anh[a + b*x]]/(b*x)])/((-1 + n)*x*(ArcTanh[Tanh[a + b*x]]/(b*x))^n)
```

Maple [F] time = 0.836, size = 0, normalized size = 0.

$$\int \frac{(\mathrm{Artanh}(\tanh(bx+a)))^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^n/x^2,x)
```

```
[Out] int(arctanh(tanh(b*x+a))^n/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{artanh}(\tanh(bx+a))^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n/x^2,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^n/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{artanh}\left(\tanh\left(bx+a\right)\right)^{n}}{x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n/x^2,x, algorithm="fricas")
```

```
[Out] integral(arctanh(tanh(b*x + a))^n/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\text{atanh}^{n}\left(\tanh\left(a+bx\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**n/x**2,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**n/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\text{artanh}\left(\tanh\left(bx+a\right)\right)^{n}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n/x^2,x, algorithm="giac")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^n/x^2, x)
```

3.273 $\int \frac{\tanh^{-1}(\tanh(a+bx))^n}{x^3} dx$

Optimal. Leaf size=101

$$\frac{b^2 n \tanh^{-1}(\tanh(a+bx))^{n-1} \text{Hypergeometric2F1}\left(1, n-1, n, -\frac{\tanh^{-1}(\tanh(a+bx))}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{\tanh^{-1}(\tanh(a+bx))^n}{2x^2} - \frac{bn \tanh^{-1}(\tanh(a+bx))^{n-1}}{2x}$$

```
[Out] -(b*n*ArcTanh[Tanh[a + b*x]]^(-1 + n))/(2*x) - ArcTanh[Tanh[a + b*x]]^n/(2*
x^2) + (b^2*n*ArcTanh[Tanh[a + b*x]]^(-1 + n)*Hypergeometric2F1[1, -1 + n,
n, -(ArcTanh[Tanh[a + b*x]]/(b*x - ArcTanh[Tanh[a + b*x]]))])/(2*(b*x - Arc
Tanh[Tanh[a + b*x]]))
```

Rubi [A] time = 0.0668799, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 2164}

$$\frac{b^2 n \tanh^{-1}(\tanh(a+bx))^{n-1} {}_2F_1\left(1, n-1; n; -\frac{\tanh^{-1}(\tanh(a+bx))}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{\tanh^{-1}(\tanh(a+bx))^n}{2x^2} - \frac{bn \tanh^{-1}(\tanh(a+bx))^{n-1}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^n/x^3,x]
```

```
[Out] -(b*n*ArcTanh[Tanh[a + b*x]]^(-1 + n))/(2*x) - ArcTanh[Tanh[a + b*x]]^n/(2*
x^2) + (b^2*n*ArcTanh[Tanh[a + b*x]]^(-1 + n)*Hypergeometric2F1[1, -1 + n,
n, -(ArcTanh[Tanh[a + b*x]]/(b*x - ArcTanh[Tanh[a + b*x]]))])/(2*(b*x - Arc
Tanh[Tanh[a + b*x]]))
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2164

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)*Hypergeometric2F1[1, n + 1, n + 2, -((a*v)/(b*u
- a*v))])/((n + 1)*(b*u - a*v)), x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))^n}{x^3}\,dx &= -\frac{\tanh^{-1}(\tanh(a+bx))^n}{2x^2}+\frac{1}{2}(bn)\int \frac{\tanh^{-1}(\tanh(a+bx))^{-1+n}}{x^2}\,dx\\ &= -\frac{bn\tanh^{-1}(\tanh(a+bx))^{-1+n}}{2x}-\frac{\tanh^{-1}(\tanh(a+bx))^n}{2x^2}-\frac{1}{2}\left(b^2(1-n)n\right)\int \frac{\tanh^{-1}(}{}\\ &= -\frac{bn\tanh^{-1}(\tanh(a+bx))^{-1+n}}{2x}-\frac{\tanh^{-1}(\tanh(a+bx))^n}{2x^2}+\frac{b^2n\tanh^{-1}(\tanh(a+bx)}{2\left(bx\right.}\end{aligned}$$

Mathematica [A] time = 0.0520349, size = 67, normalized size = 0.66

$$\frac{\tanh^{-1}(\tanh(a+bx))^n\left(\frac{\tanh^{-1}(\tanh(a+bx))}{bx}\right)^{-n}\text{Hypergeometric2F1}\left(2-n,-n,3-n,1-\frac{\tanh^{-1}(\tanh(a+bx))}{bx}\right)}{(n-2)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^n/x^3,x]
```

```
[Out] (ArcTanh[Tanh[a + b*x]]^n*Hypergeometric2F1[2 - n, -n, 3 - n, 1 - ArcTanh[T
anh[a + b*x]]/(b*x)])/((-2 + n)*x^2*(ArcTanh[Tanh[a + b*x]]/(b*x))^n)
```

Maple [F] time = 0.913, size = 0, normalized size = 0.

$$\int \frac{(\text{Artanh}(\tanh(bx+a)))^n}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^n/x^3,x)
```

```
[Out] int(arctanh(tanh(b*x+a))^n/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{n}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n/x^3,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^n/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{n}}{x^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n/x^3,x, algorithm="fricas")
```

```
[Out] integral(arctanh(tanh(b*x + a))^n/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^{n}\left(\tanh\left(a+bx\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**n/x**3,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**n/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{n}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n/x^3,x, algorithm="giac")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^n/x^3, x)
```

3.274 $\int x^m \coth^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=37

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))}{m+1}-\frac{bx^{m+2}}{m^2+3m+2}$$

```
[Out] -((b*x^(2 + m))/(2 + 3*m + m^2)) + (x^(1 + m)*ArcCoth[Tanh[a + b*x]])/(1 +
m)
```

Rubi [A] time = 0.0181243, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 30}

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))}{m+1}-\frac{bx^{m+2}}{m^2+3m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] -((b*x^(2 + m))/(2 + 3*m + m^2)) + (x^(1 + m)*ArcCoth[Tanh[a + b*x]])/(1 +
m)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^m \coth^{-1}(\tanh(a+bx))\,dx = \frac{x^{1+m}\coth^{-1}(\tanh(a+bx))}{1+m} - \frac{b\int x^{1+m}\,dx}{1+m}$$
$$= -\frac{bx^{2+m}}{2+3m+m^2} + \frac{x^{1+m}\coth^{-1}(\tanh(a+bx))}{1+m}$$

Mathematica [A] time = 0.0525737, size = 34, normalized size = 0.92

$$x^m\left(\frac{x\left(\coth^{-1}(\tanh(a+bx))-bx\right)}{m+1}+\frac{bx^2}{m+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] x^m*((b*x^2)/(2 + m) + (x*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/(1 + m))
```

Maple [C] time = 0.24, size = 676, normalized size = 18.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arccoth(tanh(b*x+a)),x)
```

```
[Out] 1/(1+m)*x*x^m*ln(exp(b*x+a))-1/4*x*(4*b*x+4*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))
^3-I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)
)^2*m-4*I*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+I*Pi*csgn(I/(exp(2
*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+
1))*m-2*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*
a)+1))^2-I*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+
1))^2*m+2*I*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+2*I*Pi*csgn(I/(e
xp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2
*a)+1))+4*I*Pi-4*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+I*Pi*csgn(I*exp(b*x+a))^
2*csgn(I*exp(2*b*x+2*a))*m+I*Pi*csgn(I*exp(2*b*x+2*a))^3*m-2*I*Pi*csgn(I/(e
xp(2*b*x+2*a)+1))^2*m+2*I*Pi*csgn(I*exp(2*b*x+2*a))^3+2*I*Pi*csgn(I*exp(2*b
*x+2*a)/(exp(2*b*x+2*a)+1))^3-2*I*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*
a))^2*m+2*I*Pi*m+I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3*m-2*I*Pi*
csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*I*Pi*c
```

```
sgn(I/(exp(2*b*x+2*a)+1))^3*m)/(1+m)/(2+m)*x^m
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.14167, size = 72, normalized size = 1.95

$$\frac{\left((bm+b)x^2+(am+2\,a)x\right)x^m}{m^2+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x^2 + (a*m + 2*a)*x)*x^m/(m^2 + 3*m + 2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*acoth(tanh(b*x+a)),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^m*arccoth(tanh(b*x + a)), x)
```

3.275 $\int x^2 \tanh^{-1}(\coth(a+bx))\, dx$

Optimal. Leaf size=23

$$\frac{1}{3}x^3\tanh^{-1}(\coth(a+bx))-\frac{bx^4}{12}$$

```
[Out] -(b*x^4)/12 + (x^3*ArcTanh[Coth[a + b*x]])/3
```

Rubi [A] time = 0.0196044, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 30}

$$\frac{1}{3}x^3\tanh^{-1}(\coth(a+bx))-\frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[Coth[a + b*x]],x]
```

```
[Out] -(b*x^4)/12 + (x^3*ArcTanh[Coth[a + b*x]])/3
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^2 \tanh^{-1}(\coth(a+bx))\,dx = \frac{1}{3}x^3 \tanh^{-1}(\coth(a+bx)) - \frac{1}{3}b\int x^3\,dx$$
$$= -\frac{bx^4}{12} + \frac{1}{3}x^3 \tanh^{-1}(\coth(a+bx))$$

Mathematica [A] time = 0.0237691, size = 20, normalized size = 0.87

$$-\frac{1}{12}x^3\left(bx - 4\tanh^{-1}(\coth(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[Coth[a + b*x]],x]
```

```
[Out] -(x^3*(b*x - 4*ArcTanh[Coth[a + b*x]]))/12
```

Maple [A] time = 0.034, size = 20, normalized size = 0.9

$$-\frac{bx^4}{12} + \frac{x^3 \mathrm{Artanh}\left(\coth\left(bx+a\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(coth(b*x+a)),x)
```

```
[Out] -1/12*b*x^4+1/3*x^3*arctanh(coth(b*x+a))
```

Maxima [A] time = 1.13334, size = 26, normalized size = 1.13

$$-\frac{1}{12}\,bx^4 + \frac{1}{3}\,x^3\,\mathrm{artanh}\left(\coth\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/12*b*x^4 + 1/3*x^3*arctanh(coth(b*x + a))
```

Fricas [A] time = 2.08463, size = 31, normalized size = 1.35

$$\frac{1}{4}\,bx^4 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(coth(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17625, size = 96, normalized size = 4.17

$$-\frac{1}{12}\,bx^4 + \frac{1}{6}\,x^3\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] -1/12*b*x^4 + 1/6*x^3*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1
)/((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))
```

3.276 $\int x \tanh^{-1}(\coth(a+bx))\,dx$

Optimal. Leaf size=23

$$\frac{1}{2}x^2\tanh^{-1}(\coth(a+bx))-\frac{bx^3}{6}$$

```
[Out] -(b*x^3)/6 + (x^2*ArcTanh[Coth[a + b*x]])/2
```

Rubi [A] time = 0.0070311, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6241, 30}

$$\frac{1}{2}x^2\tanh^{-1}(\coth(a+bx))-\frac{bx^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[Coth[a + b*x]],x]
```

```
[Out] -(b*x^3)/6 + (x^2*ArcTanh[Coth[a + b*x]])/2
```

Rule 6241

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Coth[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\tanh^{-1}(\coth(a+bx))\,dx &= \frac{1}{2}x^2\tanh^{-1}(\coth(a+bx))-\frac{1}{2}b\int x^2\,dx\\ &= -\frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(\coth(a+bx))\end{aligned}$$

Mathematica [A] time = 0.0141067, size = 20, normalized size = 0.87

$$-\frac{1}{6}x^2\left(bx-3\tanh^{-1}(\coth(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[Coth[a + b*x]],x]
```

```
[Out] -(x^2*(b*x - 3*ArcTanh[Coth[a + b*x]]))/6
```

Maple [A] time = 0.032, size = 20, normalized size = 0.9

$$-\frac{bx^3}{6}+\frac{x^2\mathrm{Artanh}\left(\coth\left(bx+a\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(coth(b*x+a)),x)
```

```
[Out] -1/6*b*x^3+1/2*x^2*arctanh(coth(b*x+a))
```

Maxima [A] time = 1.14285, size = 26, normalized size = 1.13

$$-\frac{1}{6}\,bx^3+\frac{1}{2}\,x^2\,\mathrm{artanh}\left(\coth\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/6*b*x^3 + 1/2*x^2*arctanh(coth(b*x + a))
```

Fricas [A] time = 2.12269, size = 31, normalized size = 1.35

$$\frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

Sympy [A] time = 124.543, size = 70, normalized size = 3.04

$$\begin{cases} \frac{x^2 \operatorname{atanh}(\coth(a))}{2} & \text{for } b = 0 \\ \left\langle -\frac{\pi}{4}, \frac{\pi}{4} \right\rangle ix^2 & \text{for } a = \log\left(-e^{-bx}\right) \vee a = \log\left(e^{-bx}\right) \\ \frac{x \operatorname{atanh}^2\left(\frac{1}{\tanh(a+bx)}\right)}{2b} - \frac{\operatorname{atanh}^3\left(\frac{1}{\tanh(a+bx)}\right)}{6b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(coth(b*x+a)),x)
```

```
[Out] Piecewise((x**2*atanh(coth(a))/2, Eq(b, 0)), (AccumBounds(-pi/4, pi/4)*I*x*
*2, Eq(a, log(exp(-b*x))) | Eq(a, log(-exp(-b*x)))), (x*atanh(1/tanh(a + b*
x))**2/(2*b) - atanh(1/tanh(a + b*x))**3/(6*b**2), True))
```

Giac [B] time = 1.20196, size = 96, normalized size = 4.17

$$-\frac{1}{6}bx^3 + \frac{1}{4}x^2\log\left(-\frac{\frac{e^{(2bx+2a)}+1}{e^{(2bx+2a)}-1}+1}{\frac{e^{(2bx+2a)}+1}{e^{(2bx+2a)}-1}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] -1/6*b*x^3 + 1/4*x^2*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)
/((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))
```

3.277 $\int \tanh^{-1}(\coth(a+bx))\,dx$

Optimal. Leaf size=16

$$\frac{\tanh^{-1}(\coth(a+bx))^2}{2b}$$

```
[Out] ArcTanh[Coth[a + b*x]]^2/(2*b)
```

Rubi [A] time = 0.0031824, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2157, 30}

$$\frac{\tanh^{-1}(\coth(a+bx))^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Coth[a + b*x]],x]
```

```
[Out] ArcTanh[Coth[a + b*x]]^2/(2*b)
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(\coth(a+bx))\,dx &= \frac{\text{Subst}\left(\int x\,dx, x, \tanh^{-1}(\coth(a+bx))\right)}{b}\\ &= \frac{\tanh^{-1}(\coth(a+bx))^2}{2b}\end{aligned}$$

Mathematica [A] time = 0.0065838, size = 18, normalized size = 1.12

$$x\tanh^{-1}(\coth(a+bx))-\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Coth[a + b*x]],x]
```

```
[Out] -(b*x^2)/2 + x*ArcTanh[Coth[a + b*x]]
```

Maple [A] time = 0.026, size = 15, normalized size = 0.9

$$\frac{(\operatorname{Artanh}(\coth(bx+a)))^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(coth(b*x+a)),x)
```

```
[Out] 1/2*arctanh(coth(b*x+a))^2/b
```

Maxima [A] time = 1.12481, size = 22, normalized size = 1.38

$$-\frac{1}{2}bx^2+x\operatorname{artanh}(\coth(bx+a))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/2*b*x^2 + x*arctanh(coth(b*x + a))
```

Fricas [A] time = 2.10572, size = 23, normalized size = 1.44

$$\frac{1}{2}bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*x
```

Sympy [A] time = 9.43627, size = 46, normalized size = 2.88

$$\begin{cases} x \operatorname{atanh}\left(\coth\left(a\right)\right) & \text{for}\: b = 0 \\ \left\langle -\frac{\pi}{2}, \frac{\pi}{2}\right\rangle i x & \text{for}\: a = \log\left(- e^{- b x}\right) \vee a = \log\left(e^{- b x}\right) \\ \frac{\operatorname{atanh}^{2}\left(\frac{1}{\tanh\left(a + b x\right)}\right)}{2 b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(coth(b*x+a)),x)
```

```
[Out] Piecewise((x*atanh(coth(a)), Eq(b, 0)), (AccumBounds(-pi/2, pi/2)*I*x, Eq(a
, log(exp(-b*x))) | Eq(a, log(-exp(-b*x)))), (atanh(1/tanh(a + b*x))**2/(2*
b), True))
```

Giac [B] time = 1.1999, size = 89, normalized size = 5.56

$$\frac{\log\left(-\frac{\frac{e^{\left(2\,bx+2\,a\right)}+1}{e^{\left(2\,bx+2\,a\right)}-1}+1}{\frac{e^{\left(2\,bx+2\,a\right)}+1}{e^{\left(2\,bx+2\,a\right)}-1}-1}\right)^{2}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] 1/8*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)/((e^(2*b*x + 2*a
) + 1)/(e^(2*b*x + 2*a) - 1) - 1))^2/b
```

3.278 $\int \frac{\tanh^{-1}(\coth(a+bx))}{x} dx$

Optimal. Leaf size=21

$$bx - \log(x)\left(bx - \tanh^{-1}(\coth(a+bx))\right)$$

```
[Out] b*x - (b*x - ArcTanh[Coth[a + b*x]])*Log[x]
```

Rubi [A] time = 0.0303547, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2158, 29}

$$bx - \log(x)\left(bx - \tanh^{-1}(\coth(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Coth[a + b*x]]/x,x]
```

```
[Out] b*x - (b*x - ArcTanh[Coth[a + b*x]])*Log[x]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\coth(a+bx))}{x} dx &= bx - \left(bx - \tanh^{-1}(\coth(a+bx))\right)\int \frac{1}{x} dx\\ &= bx - \left(bx - \tanh^{-1}(\coth(a+bx))\right)\log(x)\end{aligned}$$

Mathematica [A] time = 0.0161014, size = 19, normalized size = 0.9

$$\log(x)\left(\tanh^{-1}(\coth(a+bx)) - bx\right) + bx$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Coth[a + b*x]]/x,x]
```

```
[Out] b*x + (-(b*x) + ArcTanh[Coth[a + b*x]])*Log[x]
```

Maple [A] time = 0.039, size = 21, normalized size = 1.

$$\ln(x)\,\mathrm{Artanh}(\coth(bx+a)) - \ln(x)\,xb + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(coth(b*x+a))/x,x)
```

```
[Out] ln(x)*arctanh(coth(b*x+a))-ln(x)*x*b+b*x
```

Maxima [A] time = 0.95269, size = 46, normalized size = 2.19

$$-b\left(x+\frac{a}{b}\right)\log(x) + b\left(x+\frac{a\log(x)}{b}\right) + \mathrm{artanh}(\coth(bx+a))\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -b*(x + a/b)*log(x) + b*(x + a*log(x)/b) + arctanh(coth(b*x + a))*log(x)
```

Fricas [A] time = 2.13451, size = 22, normalized size = 1.05

$$bx + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] b*x + a*log(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}(\coth(a+bx))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(coth(b*x+a))/x,x)
```

```
[Out] Integral(atanh(coth(a + b*x))/x, x)
```

Giac [C] time = 1.16903, size = 20, normalized size = 0.95

$$bx + \frac{1}{2}(i\pi + 2a)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a))/x,x, algorithm="giac")
```

```
[Out] b*x + 1/2*(I*pi + 2*a)*log(x)
```

3.279 $\int \frac{\tanh^{-1}(\coth(a+bx))}{x^2} dx$

Optimal. Leaf size=17

$$b\log(x) - \frac{\tanh^{-1}(\coth(a+bx))}{x}$$

```
[Out] -(ArcTanh[Coth[a + b*x]]/x) + b*Log[x]
```

Rubi [A] time = 0.0082163, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 29}

$$b\log(x) - \frac{\tanh^{-1}(\coth(a+bx))}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Coth[a + b*x]]/x^2,x]
```

```
[Out] -(ArcTanh[Coth[a + b*x]]/x) + b*Log[x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\coth(a+bx))}{x^2}\,dx = -\frac{\tanh^{-1}(\coth(a+bx))}{x} + b\int \frac{1}{x}\,dx$$
$$= -\frac{\tanh^{-1}(\coth(a+bx))}{x} + b\log(x)$$

Mathematica [A] time = 0.0154534, size = 18, normalized size = 1.06

$$-\frac{\tanh^{-1}(\coth(a+bx))}{x} + b\log(x) + b$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Coth[a + b*x]]/x^2,x]
```

```
[Out] b - ArcTanh[Coth[a + b*x]]/x + b*Log[x]
```

Maple [A] time = 0.033, size = 18, normalized size = 1.1

$$-\frac{\operatorname{Artanh}(\coth(bx+a))}{x} + b\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(coth(b*x+a))/x^2,x)
```

```
[Out] -arctanh(coth(b*x+a))/x+b*ln(x)
```

Maxima [A] time = 1.13869, size = 23, normalized size = 1.35

$$b\log(x) - \frac{\operatorname{artanh}(\coth(bx+a))}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a))/x^2,x, algorithm="maxima")
```

```
[Out] b*log(x) - arctanh(coth(b*x + a))/x
```

Fricas [A] time = 2.08442, size = 27, normalized size = 1.59

$$\frac{bx\log(x) - a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a))/x^2,x, algorithm="fricas")
```

```
[Out] (b*x*log(x) - a)/x
```

Sympy [A] time = 17.0947, size = 42, normalized size = 2.47

$$\begin{cases} \frac{\left\langle -\frac{\pi}{2}, \frac{\pi}{2}\right\rangle i}{x} & \text{for}\: a = \log\left(-e^{-bx}\right) \vee a = \log\left(e^{-bx}\right) \\ b\log(x) - \frac{\operatorname{atanh}\left(\frac{1}{\tanh(a+bx)}\right)}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(coth(b*x+a))/x**2,x)
```

```
[Out] Piecewise((AccumBounds(-pi/2, pi/2)*I/x, Eq(a, log(exp(-b*x))) | Eq(a, log(
-exp(-b*x)))), (b*log(x) - atanh(1/tanh(a + b*x))/x, True))
```

Giac [B] time = 1.18725, size = 95, normalized size = 5.59

$$b\log(|x|) - \frac{\log\left(-\frac{\frac{e^{(2bx+2a)}+1}{e^{(2bx+2a)}-1}+1}{\frac{e^{(2bx+2a)}+1}{e^{(2bx+2a)}-1}-1}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a))/x^2,x, algorithm="giac")
```

```
[Out] b*log(abs(x)) - 1/2*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)/
((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))/x
```

3.280 $\int \frac{\tanh^{-1}(\coth(a+bx))}{x^3} dx$

Optimal. Leaf size=23

$$-\frac{\tanh^{-1}(\coth(a+bx))}{2x^2}-\frac{b}{2x}$$

```
[Out] -b/(2*x) - ArcTanh[Coth[a + b*x]]/(2*x^2)
```

Rubi [A] time = 0.0089536, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 30}

$$-\frac{\tanh^{-1}(\coth(a+bx))}{2x^2}-\frac{b}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Coth[a + b*x]]/x^3,x]
```

```
[Out] -b/(2*x) - ArcTanh[Coth[a + b*x]]/(2*x^2)
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\coth(a+bx))}{x^3}\,dx = -\frac{\tanh^{-1}(\coth(a+bx))}{2x^2} + \frac{1}{2}b\int \frac{1}{x^2}\,dx$$
$$= -\frac{b}{2x} - \frac{\tanh^{-1}(\coth(a+bx))}{2x^2}$$

Mathematica [A] time = 0.0135759, size = 18, normalized size = 0.78

$$-\frac{\tanh^{-1}(\coth(a+bx)) + bx}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Coth[a + b*x]]/x^3,x]
```

```
[Out] -(b*x + ArcTanh[Coth[a + b*x]])/(2*x^2)
```

Maple [A] time = 0.032, size = 20, normalized size = 0.9

$$-\frac{b}{2\,x} - \frac{\mathrm{Artanh}\left(\coth\left(bx+a\right)\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(coth(b*x+a))/x^3,x)
```

```
[Out] -1/2*b/x-1/2*arctanh(coth(b*x+a))/x^2
```

Maxima [A] time = 1.14687, size = 26, normalized size = 1.13

$$-\frac{b}{2\,x} - \frac{\mathrm{artanh}\left(\coth\left(bx+a\right)\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b/x - 1/2*arctanh(coth(b*x + a))/x^2
```

Fricas [A] time = 2.12019, size = 30, normalized size = 1.3

$$-\frac{2\,bx+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a))/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x + a)/x^2
```

Sympy [A] time = 142.164, size = 49, normalized size = 2.13

$$\begin{cases} \frac{\left\langle -\frac{\pi}{4}, \frac{\pi}{4}\right\rangle i}{x^2} & \text{for}\ a=\log\left(-e^{-bx}\right) \vee a=\log\left(e^{-bx}\right) \\ -\frac{b}{2x}-\frac{\operatorname{atanh}\left(\frac{1}{\tanh\left(a+bx\right)}\right)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(coth(b*x+a))/x**3,x)
```

```
[Out] Piecewise((AccumBounds(-pi/4, pi/4)*I/x**2, Eq(a, log(exp(-b*x))) | Eq(a, l
og(-exp(-b*x)))), (-b/(2*x) - atanh(1/tanh(a + b*x))/(2*x**2), True))
```

Giac [B] time = 1.19043, size = 96, normalized size = 4.17

$$-\frac{b}{2\,x}-\frac{\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a))/x^3,x, algorithm="giac")
```

```
[Out] -1/2*b/x - 1/4*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)/((e^(
2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))/x^2
```

3.281 $\int \tanh^{-1}(\cosh(x))\,dx$

Optimal. Leaf size=27

$$-\text{PolyLog}\left(2,-e^x\right)+\text{PolyLog}\left(2,e^x\right)-2x\tanh^{-1}\left(e^x\right)+x\tanh^{-1}(\cosh(x))$$

```
[Out] -2*x*ArcTanh[E^x] + x*ArcTanh[Cosh[x]] - PolyLog[2, -E^x] + PolyLog[2, E^x]
```

Rubi [A] time = 0.0319938, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {6271, 4182, 2279, 2391}

$$-\text{PolyLog}\left(2,-e^x\right)+\text{PolyLog}\left(2,e^x\right)-2x\tanh^{-1}\left(e^x\right)+x\tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Cosh[x]],x]
```

```
[Out] -2*x*ArcTanh[E^x] + x*ArcTanh[Cosh[x]] - PolyLog[2, -E^x] + PolyLog[2, E^x]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(\cosh(x))\,dx &= x\tanh^{-1}(\cosh(x)) + \int x\operatorname{csch}(x)\,dx\\ &= -2x\tanh^{-1}\left(e^x\right) + x\tanh^{-1}(\cosh(x)) - \int \log\left(1-e^x\right)\,dx + \int \log\left(1+e^x\right)\,dx\\ &= -2x\tanh^{-1}\left(e^x\right) + x\tanh^{-1}(\cosh(x)) - \text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^x\right) + \text{Subst}\left(\int \frac{\log(1+x}{x}\right.\\ &= -2x\tanh^{-1}\left(e^x\right) + x\tanh^{-1}(\cosh(x)) - \text{Li}_2\left(-e^x\right) + \text{Li}_2\left(e^x\right)\end{aligned}$$

Mathematica [A] time = 0.0205727, size = 47, normalized size = 1.74

$$\text{PolyLog}\left(2,-e^{-x}\right) - \text{PolyLog}\left(2,e^{-x}\right) + x\left(\log\left(1-e^{-x}\right) - \log\left(e^{-x}+1\right)\right) + x\tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Cosh[x]],x]
```

```
[Out] x*ArcTanh[Cosh[x]] + x*(Log[1 - E^(-x)] - Log[1 + E^(-x)]) + PolyLog[2, -E^
(-x)] - PolyLog[2, E^(-x)]
```

Maple [A] time = 0.04, size = 21, normalized size = 0.8

$$x\text{Artanh}\left(\cosh\left(x\right)\right) + 2\,\text{dilog}\left(e^{-x}\right) - \frac{\text{dilog}\left(e^{-2\,x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(cosh(x)),x)
```

```
[Out] x*arctanh(cosh(x))+2*dilog(exp(-x))-1/2*dilog(exp(-2*x))
```

Maxima [A] time = 1.12202, size = 45, normalized size = 1.67

$$x\,\text{artanh}\left(\cosh\left(x\right)\right) - x\log\left(e^{x}+1\right) + x\log\left(-e^{x}+1\right) - \text{Li}_2\left(-e^{x}\right) + \text{Li}_2\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(cosh(x)),x, algorithm="maxima")
```

```
[Out] x*arctanh(cosh(x)) - x*log(e^x + 1) + x*log(-e^x + 1) - dilog(-e^x) + dilog
(e^x)
```

Fricas [B] time = 2.23709, size = 215, normalized size = 7.96

$$\frac{1}{2}x\log\left(-\frac{\cosh(x)+1}{\cosh(x)-1}\right)-x\log(\cosh(x)+\sinh(x)+1)+x\log(-\cosh(x)-\sinh(x)+1)+\text{Li}_2(\cosh(x)+\sinh(x))-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x*log(-(cosh(x) + 1)/(cosh(x) - 1)) - x*log(cosh(x) + sinh(x) + 1) + x*
log(-cosh(x) - sinh(x) + 1) + dilog(cosh(x) + sinh(x)) - dilog(-cosh(x) - s
inh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{atanh}(\cosh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(cosh(x)),x)
```

```
[Out] Integral(atanh(cosh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{artanh}(\cosh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(cosh(x)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(cosh(x)), x)
```

3.282 $\int x\tanh^{-1}(\cosh(x))\,dx$

Optimal. Leaf size=51

$$-x\text{PolyLog}(2,-e^x)+x\text{PolyLog}(2,e^x)+\text{PolyLog}(3,-e^x)-\text{PolyLog}(3,e^x)+x^2\left(-\tanh^{-1}(e^x)\right)+\frac{1}{2}x^2\tanh^{-1}(\cosh(x$$

```
[Out] -(x^2*ArcTanh[E^x]) + (x^2*ArcTanh[Cosh[x]])/2 - x*PolyLog[2, -E^x] + x*Pol
yLog[2, E^x] + PolyLog[3, -E^x] - PolyLog[3, E^x]
```

Rubi [A] time = 0.0609738, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6273, 4182, 2531, 2282, 6589}

$$-x\text{PolyLog}(2,-e^x)+x\text{PolyLog}(2,e^x)+\text{PolyLog}(3,-e^x)-\text{PolyLog}(3,e^x)+x^2\left(-\tanh^{-1}(e^x)\right)+\frac{1}{2}x^2\tanh^{-1}(\cosh(x$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[Cosh[x]],x]
```

```
[Out] -(x^2*ArcTanh[E^x]) + (x^2*ArcTanh[Cosh[x]])/2 - x*PolyLog[2, -E^x] + x*Pol
yLog[2, E^x] + PolyLog[3, -E^x] - PolyLog[3, E^x]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x \tanh^{-1}(\cosh(x))\,dx &= \frac{1}{2}x^2\tanh^{-1}(\cosh(x)) + \frac{1}{2}\int x^2\mathbf{csch}(x)\,dx \\
&= -x^2\tanh^{-1}(e^x) + \frac{1}{2}x^2\tanh^{-1}(\cosh(x)) - \int x\log(1-e^x)\,dx + \int x\log(1+e^x)\,dx \\
&= -x^2\tanh^{-1}(e^x) + \frac{1}{2}x^2\tanh^{-1}(\cosh(x)) - x\mathbf{Li}_2(-e^x) + x\mathbf{Li}_2(e^x) + \int \mathbf{Li}_2(-e^x)\,dx - \int \mathbf{Li}_2 \\
&= -x^2\tanh^{-1}(e^x) + \frac{1}{2}x^2\tanh^{-1}(\cosh(x)) - x\mathbf{Li}_2(-e^x) + x\mathbf{Li}_2(e^x) + \text{Subst}\left(\int \frac{\mathbf{Li}_2(-x)}{x}\,dx, x, \right. \\
&= -x^2\tanh^{-1}(e^x) + \frac{1}{2}x^2\tanh^{-1}(\cosh(x)) - x\mathbf{Li}_2(-e^x) + x\mathbf{Li}_2(e^x) + \mathbf{Li}_3(-e^x) - \mathbf{Li}_3(e^x)
\end{aligned}$$

Mathematica [A] time = 0.0149095, size = 81, normalized size = 1.59

$$\frac{1}{2}\left(2x\text{PolyLog}(2,-e^{-x}) - 2x\text{PolyLog}(2,e^{-x}) + 2\text{PolyLog}(3,-e^{-x}) - 2\text{PolyLog}(3,e^{-x}) + x^2\log(1-e^{-x}) - x^2\log(e^{-x}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[Cosh[x]],x]
```

```
[Out] (x^2*ArcTanh[Cosh[x]] + x^2*Log[1 - E^(-x)] - x^2*Log[1 + E^(-x)] + 2*x*Pol
yLog[2, -E^(-x)] - 2*x*PolyLog[2, E^(-x)] + 2*PolyLog[3, -E^(-x)] - 2*PolyL
og[3, E^(-x)])/2
```

Maple [C] time = 0.23, size = 479, normalized size = 9.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(cosh(x)),x)
```

```
[Out] x*polylog(2,exp(x))-x*polylog(2,-exp(x))-1/8*I*Pi*csgn(I*(exp(x)-1)^2)*csgn
(I*exp(-x)*(exp(x)-1)^2)^2*x^2-1/8*I*Pi*csgn(I*(exp(x)+1))^2*csgn(I*(exp(x)
+1)^2)*x^2-1/8*I*Pi*csgn(I*exp(-x)*(exp(x)+1)^2)^3*x^2+1/8*I*Pi*csgn(I*exp(
-x))*csgn(I*exp(-x)*(exp(x)+1)^2)^2*x^2-1/8*I*Pi*csgn(I*exp(-x)*(exp(x)-1)^
2)^3*x^2+1/8*I*Pi*csgn(I*(exp(x)-1)^2)^3*x^2+1/4*I*Pi*csgn(I*exp(-x)*(exp(x
)-1)^2)^2*x^2-1/8*I*Pi*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-1)^2)^2*x^2+1
/8*I*Pi*csgn(I*(exp(x)-1)^2)*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-1)^2)*x
^2+1/8*I*Pi*csgn(I*(exp(x)+1)^2)*csgn(I*exp(-x)*(exp(x)+1)^2)^2*x^2-1/8*I*P
i*csgn(I*(exp(x)+1)^2)^3*x^2+1/4*I*Pi*csgn(I*(exp(x)+1))*csgn(I*(exp(x)+1)^
2)^2*x^2-1/4*I*Pi*csgn(I*(exp(x)-1))*csgn(I*(exp(x)-1)^2)^2*x^2+polylog(3,-
exp(x))-polylog(3,exp(x))+1/2*x^2*ln(1-exp(x))-1/2*x^2*ln(exp(x)-1)+1/8*I*P
i*csgn(I*(exp(x)-1))^2*csgn(I*(exp(x)-1)^2)*x^2-1/4*I*Pi*x^2-1/8*I*Pi*csgn(
I*(exp(x)+1)^2)*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)+1)^2)*x^2
```

Maxima [A] time = 1.16409, size = 76, normalized size = 1.49

$$\frac{1}{2}x^2\operatorname{artanh}\left(\cosh\left(x\right)\right)-\frac{1}{2}x^2\log\left(e^x+1\right)+\frac{1}{2}x^2\log\left(-e^x+1\right)-x\operatorname{Li}_2\left(-e^x\right)+x\operatorname{Li}_2\left(e^x\right)+\operatorname{Li}_3(-e^x)-\operatorname{Li}_3(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctanh(cosh(x)) - 1/2*x^2*log(e^x + 1) + 1/2*x^2*log(-e^x + 1) - x
*dilog(-e^x) + x*dilog(e^x) + polylog(3, -e^x) - polylog(3, e^x)
```

Fricas [C] time = 2.14733, size = 327, normalized size = 6.41

$$\frac{1}{4}x^2\log\left(-\frac{\cosh(x)+1}{\cosh(x)-1}\right)-\frac{1}{2}x^2\log(\cosh(x)+\sinh(x)+1)+\frac{1}{2}x^2\log(-\cosh(x)-\sinh(x)+1)+x\text{Li}_2(\cosh(x)+s$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/4*x^2*log(-(cosh(x) + 1)/(cosh(x) - 1)) - 1/2*x^2*log(cosh(x) + sinh(x) +
 1) + 1/2*x^2*log(-cosh(x) - sinh(x) + 1) + x*dilog(cosh(x) + sinh(x)) - x*
dilog(-cosh(x) - sinh(x)) - polylog(3, cosh(x) + sinh(x)) + polylog(3, -cos
h(x) - sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\text{atanh}(\cosh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(cosh(x)),x)
```

```
[Out] Integral(x*atanh(cosh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\text{artanh}(\cosh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(cosh(x)),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(cosh(x)), x)
```

3.283 $\int x^2 \tanh^{-1}(\cosh(x))\,dx$

Optimal. Leaf size=77

$-x^2\text{PolyLog}(2,-e^x)+x^2\text{PolyLog}(2,e^x)+2x\text{PolyLog}(3,-e^x)-2x\text{PolyLog}(3,e^x)-2\text{PolyLog}(4,-e^x)+2\text{PolyLog}($

```
[Out] (-2*x^3*ArcTanh[E^x])/3 + (x^3*ArcTanh[Cosh[x]])/3 - x^2*PolyLog[2, -E^x] +
 x^2*PolyLog[2, E^x] + 2*x*PolyLog[3, -E^x] - 2*x*PolyLog[3, E^x] - 2*PolyL
og[4, -E^x] + 2*PolyLog[4, E^x]
```

Rubi [A] time = 0.0864298, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6273, 4182, 2531, 6609, 2282, 6589}

$-x^2\text{PolyLog}(2,-e^x)+x^2\text{PolyLog}(2,e^x)+2x\text{PolyLog}(3,-e^x)-2x\text{PolyLog}(3,e^x)-2\text{PolyLog}(4,-e^x)+2\text{PolyLog}($

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[Cosh[x]],x]
```


```
[Out] (-2*x^3*ArcTanh[E^x])/3 + (x^3*ArcTanh[Cosh[x]])/3 - x^2*PolyLog[2, -E^x] +
 x^2*PolyLog[2, E^x] + 2*x*PolyLog[3, -E^x] - 2*x*PolyLog[3, E^x] - 2*PolyL
og[4, -E^x] + 2*PolyLog[4, E^x]
```


Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \tanh^{-1}(\cosh(x))\,dx = \frac{1}{3}x^3 \tanh^{-1}(\cosh(x)) + \frac{1}{3}\int x^3\mathbf{csch}(x)\,dx$$
$$= -\frac{2}{3}x^3 \tanh^{-1}(e^x) + \frac{1}{3}x^3 \tanh^{-1}(\cosh(x)) - \int x^2 \log(1-e^x)\,dx + \int x^2 \log(1+e^x)\,dx$$
$$= -\frac{2}{3}x^3 \tanh^{-1}(e^x) + \frac{1}{3}x^3 \tanh^{-1}(\cosh(x)) - x^2\mathrm{Li}_2(-e^x) + x^2\mathrm{Li}_2(e^x) + 2\int x\mathrm{Li}_2(-e^x)\,dx - 2$$
$$= -\frac{2}{3}x^3 \tanh^{-1}(e^x) + \frac{1}{3}x^3 \tanh^{-1}(\cosh(x)) - x^2\mathrm{Li}_2(-e^x) + x^2\mathrm{Li}_2(e^x) + 2x\mathrm{Li}_3(-e^x) - 2x\mathrm{Li}_3(e$$
$$= -\frac{2}{3}x^3 \tanh^{-1}(e^x) + \frac{1}{3}x^3 \tanh^{-1}(\cosh(x)) - x^2\mathrm{Li}_2(-e^x) + x^2\mathrm{Li}_2(e^x) + 2x\mathrm{Li}_3(-e^x) - 2x\mathrm{Li}_3(e$$
$$= -\frac{2}{3}x^3 \tanh^{-1}(e^x) + \frac{1}{3}x^3 \tanh^{-1}(\cosh(x)) - x^2\mathrm{Li}_2(-e^x) + x^2\mathrm{Li}_2(e^x) + 2x\mathrm{Li}_3(-e^x) - 2x\mathrm{Li}_3(e$$

Mathematica [A] time = 0.0314841, size = 109, normalized size = 1.42

$$\frac{1}{24}\big(24x^2\mathrm{PolyLog}(2,-e^{-x}) + 24x^2\mathrm{PolyLog}(2,e^x) + 48x\mathrm{PolyLog}(3,-e^{-x}) - 48x\mathrm{PolyLog}(3,e^x) + 48\mathrm{PolyLog}(4,-e^{-x})$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[Cosh[x]],x]
```

```
[Out] (Pi^4 - 2*x^4 + 8*x^3*ArcTanh[Cosh[x]] - 8*x^3*Log[1 + E^(-x)] + 8*x^3*Log[
1 - E^x] + 24*x^2*PolyLog[2, -E^(-x)] + 24*x^2*PolyLog[2, E^x] + 48*x*PolyL
og[3, -E^(-x)] - 48*x*PolyLog[3, E^x] + 48*PolyLog[4, -E^(-x)] + 48*PolyLog
[4, E^x])/24
```

Maple [C] time = 0.151, size = 501, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(cosh(x)),x)
```

```
[Out] x^2*polylog(2,exp(x))-x^2*polylog(2,-exp(x))-2*polylog(4,-exp(x))+2*polylog
(4,exp(x))+1/12*I*Pi*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)+1)^2)^2*x^3+1/1
2*I*Pi*csgn(I*(exp(x)+1)^2)*csgn(I*exp(-x)*(exp(x)+1)^2)^2*x^3-1/12*I*Pi*cs
gn(I*(exp(x)+1)^2)^3*x^3+1/12*I*Pi*csgn(I*(exp(x)-1)^2)*csgn(I*exp(-x))*csg
```

```
n(I*exp(-x)*(exp(x)-1)^2)*x^3-1/12*I*Pi*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp
(x)-1)^2)^2*x^3-1/12*I*Pi*csgn(I*(exp(x)-1)^2)*csgn(I*exp(-x)*(exp(x)-1)^2)
^2*x^3-1/6*I*Pi*x^3+1/12*I*Pi*csgn(I*(exp(x)-1)^2)^3*x^3+1/3*x^3*ln(1-exp(x
))-1/3*x^3*ln(exp(x)-1)+2*x*polylog(3,-exp(x))-2*x*polylog(3,exp(x))+1/12*I
*Pi*csgn(I*(exp(x)-1))^2*csgn(I*(exp(x)-1)^2)*x^3+1/6*I*Pi*csgn(I*(exp(x)+1
))*csgn(I*(exp(x)+1)^2)^2*x^3+1/6*I*Pi*csgn(I*exp(-x)*(exp(x)-1)^2)^2*x^3-1
/12*I*Pi*csgn(I*exp(-x)*(exp(x)+1)^2)^3*x^3-1/12*I*Pi*csgn(I*(exp(x)+1))^2*
csgn(I*(exp(x)+1)^2)*x^3-1/12*I*Pi*csgn(I*(exp(x)+1)^2)*csgn(I*exp(-x))*csg
n(I*exp(-x)*(exp(x)+1)^2)*x^3-1/6*I*Pi*csgn(I*(exp(x)-1))*csgn(I*(exp(x)-1)
^2)^2*x^3-1/12*I*Pi*csgn(I*exp(-x)*(exp(x)-1)^2)^3*x^3
```

Maxima [A] time = 1.15771, size = 105, normalized size = 1.36

$$\frac{1}{3}x^3\operatorname{artanh}(\cosh(x)) - \frac{1}{3}x^3\log(e^x+1) + \frac{1}{3}x^3\log(-e^x+1) - x^2\mathrm{Li}_2(-e^x) + x^2\mathrm{Li}_2(e^x) + 2x\mathrm{Li}_3(-e^x) - 2x\mathrm{Li}_3(e^x) - 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctanh(cosh(x)) - 1/3*x^3*log(e^x + 1) + 1/3*x^3*log(-e^x + 1) - x
^2*dilog(-e^x) + x^2*dilog(e^x) + 2*x*polylog(3, -e^x) - 2*x*polylog(3, e^x
) - 2*polylog(4, -e^x) + 2*polylog(4, e^x)
```

Fricas [C] time = 2.23189, size = 436, normalized size = 5.66

$$\frac{1}{6}x^3\log\left(-\frac{\cosh(x)+1}{\cosh(x)-1}\right) - \frac{1}{3}x^3\log(\cosh(x)+\sinh(x)+1) + \frac{1}{3}x^3\log(-\cosh(x)-\sinh(x)+1) + x^2\mathrm{Li}_2(\cosh(x)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/6*x^3*log(-(cosh(x) + 1)/(cosh(x) - 1)) - 1/3*x^3*log(cosh(x) + sinh(x) +
 1) + 1/3*x^3*log(-cosh(x) - sinh(x) + 1) + x^2*dilog(cosh(x) + sinh(x)) -
x^2*dilog(-cosh(x) - sinh(x)) - 2*x*polylog(3, cosh(x) + sinh(x)) + 2*x*pol
ylog(3, -cosh(x) - sinh(x)) + 2*polylog(4, cosh(x) + sinh(x)) - 2*polylog(4
, -cosh(x) - sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{atanh}(\cosh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(cosh(x)),x)
```

```
[Out] Integral(x**2*atanh(cosh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{artanh}(\cosh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(cosh(x)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(cosh(x)), x)
```

3.284 $\int x^2 \tanh^{-1}(c + d \tanh(a + bx))\, dx$

Optimal. Leaf size=307

$$-\frac{x\text{PolyLog}\left(3,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2}+\frac{x\text{PolyLog}\left(3,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2}+\frac{\text{PolyLog}\left(4,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^3}-\frac{\text{PolyLog}\left(4,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^3}$$

```
[Out] (x^3*ArcTanh[c + d*Tanh[a + b*x]])/3 + (x^3*Log[1 + ((1 - c - d)*E^(2*a + 2
*b*x))/(1 - c + d)])/6 - (x^3*Log[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c
- d)])/6 + (x^2*PolyLog[2, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))])/(
4*b) - (x^2*PolyLog[2, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))])/(4*b)
 - (x*PolyLog[3, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))])/(4*b^2) + (
x*PolyLog[3, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))])/(4*b^2) + PolyL
og[4, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))]/(8*b^3) - PolyLog[4, -(
((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))]/(8*b^3)
```

Rubi [A] time = 0.468765, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6243, 2190, 2531, 6609, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(3,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2}+\frac{x\text{PolyLog}\left(3,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2}+\frac{\text{PolyLog}\left(4,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^3}-\frac{\text{PolyLog}\left(4,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[c + d*Tanh[a + b*x]],x]
```


```
[Out] (x^3*ArcTanh[c + d*Tanh[a + b*x]])/3 + (x^3*Log[1 + ((1 - c - d)*E^(2*a + 2
*b*x))/(1 - c + d)])/6 - (x^3*Log[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c
- d)])/6 + (x^2*PolyLog[2, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))])/(
4*b) - (x^2*PolyLog[2, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))])/(4*b)
 - (x*PolyLog[3, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))])/(4*b^2) + (
x*PolyLog[3, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))])/(4*b^2) + PolyL
og[4, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))]/(8*b^3) - PolyLog[4, -(
((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))]/(8*b^3)
```


Rule 6243

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Tanh[a + b*x]])/(f*
```

```
(m + 1)), x] + (Dist[(b*(1 - c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^
(2*a + 2*b*x))/(1 - c + d + (1 - c - d)*E^(2*a + 2*b*x)), x], x] - Dist[(b*
(1 + c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*a + 2*b*x))/(1 + c -
d + (1 + c + d)*E^(2*a + 2*b*x)), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &
& IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \tanh^{-1}(c+d\tanh(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(c+d\tanh(a+bx)) + \frac{1}{3}(b(1-c-d))\int \frac{e^{2a+2bx}x^3}{1-c+d+(1-c-d)e^{2a+2}}$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\tanh(a+bx)) + \frac{1}{6}x^3\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{6}x^3\log\Bigg(1$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\tanh(a+bx)) + \frac{1}{6}x^3\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{6}x^3\log\Bigg(1$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\tanh(a+bx)) + \frac{1}{6}x^3\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{6}x^3\log\Bigg(1$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\tanh(a+bx)) + \frac{1}{6}x^3\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{6}x^3\log\Bigg(1$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\tanh(a+bx)) + \frac{1}{6}x^3\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{6}x^3\log\Bigg(1$$

Mathematica [A] time = 11.5652, size = 345, normalized size = 1.12

$$\frac{-6b^2x^2\text{PolyLog}\left(2,\frac{(c-d-1)(\sinh(2(a+bx))-\cosh(2(a+bx)))}{c+d-1}\right)+6b^2x^2\text{PolyLog}\left(2,\frac{(c-d+1)(\sinh(2(a+bx))-\cosh(2(a+bx)))}{c+d+1}\right)-6bx\text{PolyLo}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[c + d*Tanh[a + b*x]])/3 + (4*b^3*x^3*Log[1 + ((-1 + c - d)*(Co
sh[2*(a + b*x)] - Sinh[2*(a + b*x)]))/(-1 + c + d)] - 4*b^3*x^3*Log[1 + ((1
 + c - d)*(Cosh[2*(a + b*x)] - Sinh[2*(a + b*x)]))/(1 + c + d)] - 6*b^2*x^2
*PolyLog[2, ((-1 + c - d)*(-Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)]))/(-1 + c
 + d)] + 6*b^2*x^2*PolyLog[2, ((1 + c - d)*(-Cosh[2*(a + b*x)] + Sinh[2*(a
+ b*x)]))/(1 + c + d)] - 6*b*x*PolyLog[3, ((-1 + c - d)*(-Cosh[2*(a + b*x)]
 + Sinh[2*(a + b*x)]))/(-1 + c + d)] + 6*b*x*PolyLog[3, ((1 + c - d)*(-Cosh
[2*(a + b*x)] + Sinh[2*(a + b*x)]))/(1 + c + d)] - 3*PolyLog[4, ((-1 + c -
d)*(-Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)]))/(-1 + c + d)] + 3*PolyLog[4, (
(1 + c - d)*(-Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)]))/(1 + c + d)])/(24*b^3
)
```

Maple [C] time = 3.866, size = 5366, normalized size = 17.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(c+d*tanh(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [A] time = 2.09237, size = 379, normalized size = 1.23

$$\frac{1}{3}x^3\operatorname{artanh}(d\tanh(bx+a)+c)-\frac{1}{18}bd\left(\frac{4b^3x^3\log\left(\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}+1\right)+6b^2x^2\mathrm{Li}_2\left(-\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right)-6bx\mathrm{Li}_3(-\frac{(c+d+}{c}}{b^4d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctanh(d*tanh(b*x + a) + c) - 1/18*b*d*((4*b^3*x^3*log((c + d + 1)
*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 6*b^2*x^2*dilog(-(c + d + 1)*e^(2*b*x +
 2*a)/(c - d + 1)) - 6*b*x*polylog(3, -(c + d + 1)*e^(2*b*x + 2*a)/(c - d +
 1)) + 3*polylog(4, -(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^4*d) - (4
*b^3*x^3*log((c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1) + 1) + 6*b^2*x^2*dilog
(-(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)) - 6*b*x*polylog(3, -(c + d - 1)*
e^(2*b*x + 2*a)/(c - d - 1)) + 3*polylog(4, -(c + d - 1)*e^(2*b*x + 2*a)/(c
 - d - 1)))/(b^4*d))
```

Fricas [C] time = 2.57471, size = 2588, normalized size = 8.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(b^3*x^3*log(-((c + 1)*cosh(b*x + a) + d*sinh(b*x + a))/((c - 1)*cosh(b
*x + a) + d*sinh(b*x + a))) - 3*b^2*x^2*dilog(sqrt(-(c + d + 1)/(c - d + 1)
)*(cosh(b*x + a) + sinh(b*x + a))) - 3*b^2*x^2*dilog(-sqrt(-(c + d + 1)/(c
- d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(sqrt(-(c + d -
 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(-sqrt(-
(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + a^3*log(2*(c +
d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt(-(c
 + d + 1)/(c - d + 1))) + a^3*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d +
1)*sinh(b*x + a) - 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d + 1))) - a^3*log(
2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1)*s
qrt(-(c + d - 1)/(c - d - 1))) - a^3*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c
 + d - 1)*sinh(b*x + a) - 2*(c - d - 1)*sqrt(-(c + d - 1)/(c - d - 1))) + 6
*b*x*polylog(3, sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x +
a))) + 6*b*x*polylog(3, -sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + si
nh(b*x + a))) - 6*b*x*polylog(3, sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x +
 a) + sinh(b*x + a))) - 6*b*x*polylog(3, -sqrt(-(c + d - 1)/(c - d - 1))*(c
osh(b*x + a) + sinh(b*x + a))) - (b^3*x^3 + a^3)*log(sqrt(-(c + d + 1)/(c -
 d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b^3*x^3 + a^3)*log(-sqrt(-
(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b^3*x^3 +
a^3)*log(sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1
) + (b^3*x^3 + a^3)*log(-sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + si
nh(b*x + a)) + 1) - 6*polylog(4, sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x +
 a) + sinh(b*x + a))) - 6*polylog(4, -sqrt(-(c + d + 1)/(c - d + 1))*(cosh(
b*x + a) + sinh(b*x + a))) + 6*polylog(4, sqrt(-(c + d - 1)/(c - d - 1))*(c
osh(b*x + a) + sinh(b*x + a))) + 6*polylog(4, -sqrt(-(c + d - 1)/(c - d - 1
))*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(c+d*tanh(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{artanh}\left(d \tanh\left(bx+a\right)+c\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(d*tanh(b*x + a) + c), x)
```

3.285 $\int x \tanh^{-1}(c + d \tanh(a + bx))\,dx$

Optimal. Leaf size=231

$$-\frac{\text{PolyLog}\left(3,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^2}+\frac{\text{PolyLog}\left(3,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^2}+\frac{x\text{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{x\text{PolyLog}\left(2,-\frac{(c+d+\cdots}{c-\cdots}\right.}{4b}$$

```
[Out] (x^2*ArcTanh[c + d*Tanh[a + b*x]])/2 + (x^2*Log[1 + ((1 - c - d)*E^(2*a + 2
*b*x))/(1 - c + d)])/4 - (x^2*Log[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c
- d)])/4 + (x*PolyLog[2, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))])/(4*
b) - (x*PolyLog[2, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))])/(4*b) - P
olyLog[3, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))]/(8*b^2) + PolyLog[3
, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))]/(8*b^2)
```

Rubi [A] time = 0.370669, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6243, 2190, 2531, 2282, 6589}

$$-\frac{\text{PolyLog}\left(3,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^2}+\frac{\text{PolyLog}\left(3,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^2}+\frac{x\text{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{x\text{PolyLog}\left(2,-\frac{(c+d+\cdots}{c-\cdots}\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[c + d*Tanh[a + b*x]])/2 + (x^2*Log[1 + ((1 - c - d)*E^(2*a + 2
*b*x))/(1 - c + d)])/4 - (x^2*Log[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c
- d)])/4 + (x*PolyLog[2, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))])/(4*
b) - (x*PolyLog[2, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))])/(4*b) - P
olyLog[3, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))]/(8*b^2) + PolyLog[3
, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))]/(8*b^2)
```

Rule 6243

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Tanh[a + b*x]])/(f*
(m + 1)), x] + (Dist[(b*(1 - c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^
(2*a + 2*b*x))/(1 - c + d + (1 - c - d)*E^(2*a + 2*b*x)), x], x] - Dist[(b*
(1 + c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*a + 2*b*x))/(1 + c -
d + (1 + c + d)*E^(2*a + 2*b*x)), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &
```

```
& IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(c+d\tanh(a+bx))\,dx = \frac{1}{2}x^2\tanh^{-1}(c+d\tanh(a+bx)) + \frac{1}{2}(b(1-c-d))\int \frac{e^{2a+2bx}x^2}{1-c+d+(1-c-d)e^{2a+2b}}$$

$$= \frac{1}{2}x^2\tanh^{-1}(c+d\tanh(a+bx)) + \frac{1}{4}x^2\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{4}x^2\log\Bigg(1$$

$$= \frac{1}{2}x^2\tanh^{-1}(c+d\tanh(a+bx)) + \frac{1}{4}x^2\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{4}x^2\log\Bigg(1$$

$$= \frac{1}{2}x^2\tanh^{-1}(c+d\tanh(a+bx)) + \frac{1}{4}x^2\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{4}x^2\log\Bigg(1$$

$$= \frac{1}{2}x^2\tanh^{-1}(c+d\tanh(a+bx)) + \frac{1}{4}x^2\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{4}x^2\log\Bigg(1$$

Mathematica [A] time = 9.42936, size = 259, normalized size = 1.12

$$\frac{-2bx\text{PolyLog}\left(2,\frac{(c-d-1)(\sinh(2(a+bx))-\cosh(2(a+bx)))}{c+d-1}\right) + 2bx\text{PolyLog}\left(2,\frac{(c-d+1)(\sinh(2(a+bx))-\cosh(2(a+bx)))}{c+d+1}\right) - \text{PolyLog}\Bigg(3,\frac{(c-}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[c + d*Tanh[a + b*x]])/2 + (2*b^2*x^2*Log[1 + ((-1 + c - d)*(Co
sh[2*(a + b*x)] - Sinh[2*(a + b*x)]))/(-1 + c + d)] - 2*b^2*x^2*Log[1 + ((1
 + c - d)*(Cosh[2*(a + b*x)] - Sinh[2*(a + b*x)]))/(1 + c + d)] - 2*b*x*Pol
yLog[2, ((-1 + c - d)*(-Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)]))/(-1 + c + d
)] + 2*b*x*PolyLog[2, ((1 + c - d)*(-Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)])
)/(1 + c + d)] - PolyLog[3, ((-1 + c - d)*(-Cosh[2*(a + b*x)] + Sinh[2*(a +
 b*x)]))/(-1 + c + d)] + PolyLog[3, ((1 + c - d)*(-Cosh[2*(a + b*x)] + Sinh
[2*(a + b*x)]))/(1 + c + d)])/(8*b^2)
```

Maple [C] time = 5.313, size = 5062, normalized size = 21.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(c+d*tanh(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [A] time = 2.1098, size = 290, normalized size = 1.26

$$-\frac{1}{8}\,bd\left(\frac{2\,b^2x^2\log\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}+1\right)+2\,bx\mathrm{Li}_2\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)-\mathrm{Li}_3(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1})}{b^3d}-\frac{2\,b^2x^2\log\left(\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}+1\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*b*d*((2*b^2*x^2*log((c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 2*b
*x*dilog(-(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)) - polylog(3, -(c + d + 1
)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^3*d) - (2*b^2*x^2*log((c + d - 1)*e^(2*b
*x + 2*a)/(c - d - 1) + 1) + 2*b*x*dilog(-(c + d - 1)*e^(2*b*x + 2*a)/(c -
d - 1)) - polylog(3, -(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)))/(b^3*d)) +
1/2*x^2*arctanh(d*tanh(b*x + a) + c)
```

Fricas [C] time = 2.2615, size = 2115, normalized size = 9.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^2*log(-((c + 1)*cosh(b*x + a) + d*sinh(b*x + a))/((c - 1)*cosh(b
*x + a) + d*sinh(b*x + a))) - 2*b*x*dilog(sqrt(-(c + d + 1)/(c - d + 1))*(c
osh(b*x + a) + sinh(b*x + a))) - 2*b*x*dilog(-sqrt(-(c + d + 1)/(c - d + 1)
)*(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(sqrt(-(c + d - 1)/(c - d -
 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(-sqrt(-(c + d - 1)/(c -
 d - 1))*(cosh(b*x + a) + sinh(b*x + a))) - a^2*log(2*(c + d + 1)*cosh(b*x
+ a) + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d
 + 1))) - a^2*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a)
 - 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d + 1))) + a^2*log(2*(c + d - 1)*co
sh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt(-(c + d - 1)
```

```
/(c - d - 1))) + a^2*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b
*x + a) - 2*(c - d - 1)*sqrt(-(c + d - 1)/(c - d - 1))) - (b^2*x^2 - a^2)*l
og(sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b
^2*x^2 - a^2)*log(-sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x
 + a)) + 1) + (b^2*x^2 - a^2)*log(sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x
+ a) + sinh(b*x + a)) + 1) + (b^2*x^2 - a^2)*log(-sqrt(-(c + d - 1)/(c - d
- 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 2*polylog(3, sqrt(-(c + d + 1)
/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*polylog(3, -sqrt(-(c + d
 + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*polylog(3, sqrt(-(c
 + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*polylog(3, -sqr
t(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(c+d*tanh(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{artanh}(d \tanh(bx + a) + c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(d*tanh(b*x + a) + c), x)
```

3.286 $\int \tanh^{-1}(c + d\tanh(a + bx))\,dx$

Optimal. Leaf size=150

$$\frac{\text{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{\text{PolyLog}\left(2,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}+\frac{1}{2}x\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)-\frac{1}{2}x\log\left(\frac{(c+d+1)e^{2a+}}{c-d+1}\right.$$

```
[Out] x*ArcTanh[c + d*Tanh[a + b*x]] + (x*Log[1 + ((1 - c - d)*E^(2*a + 2*b*x))/(
1 - c + d)])/2 - (x*Log[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/2 +
 PolyLog[2, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))]/(4*b) - PolyLog[2
, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))]/(4*b)
```

Rubi [A] time = 0.22185, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6235, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{\text{PolyLog}\left(2,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}+\frac{1}{2}x\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)-\frac{1}{2}x\log\left(\frac{(c+d+1)e^{2a+}}{c-d+1}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[c + d*Tanh[a + b*x]],x]
```

```
[Out] x*ArcTanh[c + d*Tanh[a + b*x]] + (x*Log[1 + ((1 - c - d)*E^(2*a + 2*b*x))/(
1 - c + d)])/2 - (x*Log[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/2 +
 PolyLog[2, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))]/(4*b) - PolyLog[2
, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))]/(4*b)
```

Rule 6235

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Ar
cTanh[c + d*Tanh[a + b*x]], x] + (Dist[b*(1 - c - d), Int[(x*E^(2*a + 2*b*x
))/(1 - c + d + (1 - c - d)*E^(2*a + 2*b*x)), x], x] - Dist[b*(1 + c + d),
Int[(x*E^(2*a + 2*b*x))/(1 + c - d + (1 + c + d)*E^(2*a + 2*b*x)), x], x])
/; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, 1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \tanh^{-1}(c+d\tanh(a+bx))\,dx = x\tanh^{-1}(c+d\tanh(a+bx)) + (b(1-c-d))\int \frac{e^{2a+2bx}x}{1-c+d+(1-c-d)e^{2a+2bx}}\,dx -$$

$$= x\tanh^{-1}(c+d\tanh(a+bx)) + \frac{1}{2}x\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{2}x\log\left(1+\frac{(1+$$

$$= x\tanh^{-1}(c+d\tanh(a+bx)) + \frac{1}{2}x\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{2}x\log\left(1+\frac{(1+$$

$$= x\tanh^{-1}(c+d\tanh(a+bx)) + \frac{1}{2}x\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{2}x\log\left(1+\frac{(1+$$

Mathematica [A] time = 6.08505, size = 131, normalized size = 0.87

$$\frac{\text{PolyLog}\left(2,-\frac{(c+d-1)e^{2(a+bx)}}{c-d-1}\right)-\text{PolyLog}\left(2,-\frac{(c+d+1)e^{2(a+bx)}}{c-d+1}\right)+2bx\left(\log\left(\frac{(c+d-1)e^{2(a+bx)}}{c-d-1}+1\right)-\log\left(\frac{(c+d+1)e^{2(a+bx)}}{c-d+1}+1\right)\right)}{4b}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[c + d*Tanh[a + b*x]],x]
```

```
[Out] x*ArcTanh[c + d*Tanh[a + b*x]] + (2*b*x*(Log[1 + ((-1 + c + d)*E^(2*(a + b*
x)))/(-1 + c - d)] - Log[1 + ((1 + c + d)*E^(2*(a + b*x)))/(1 + c - d)]) +
PolyLog[2, -(((-1 + c + d)*E^(2*(a + b*x)))/(-1 + c - d))] - PolyLog[2, -((
```

```
(1 + c + d)*E^(2*(a + b*x)))/(1 + c - d))])/(4*b)
```

Maple [B] time = 0.158, size = 306, normalized size = 2.

$$-\frac{\operatorname{Artanh}\left(c+d\tanh\left(bx+a\right)\right)\ln\left(d\tanh\left(bx+a\right)-d\right)}{2b}+\frac{\operatorname{Artanh}\left(c+d\tanh\left(bx+a\right)\right)\ln\left(d\tanh\left(bx+a\right)+d\right)}{2b}+\frac{1}{4b}\mathrm{dil}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(c+d*tanh(b*x+a)),x)
```

```
[Out] -1/2/b*arctanh(c+d*tanh(b*x+a))*ln(d*tanh(b*x+a)-d)+1/2/b*arctanh(c+d*tanh(
b*x+a))*ln(d*tanh(b*x+a)+d)+1/4/b*dilog((d*tanh(b*x+a)+c-1)/(c-d-1))+1/4/b*
ln(d*tanh(b*x+a)+d)*ln((d*tanh(b*x+a)+c-1)/(c-d-1))-1/4/b*dilog((d*tanh(b*x
+a)+c+1)/(1+c-d))-1/4/b*ln(d*tanh(b*x+a)+d)*ln((d*tanh(b*x+a)+c+1)/(1+c-d))
-1/4/b*dilog((d*tanh(b*x+a)+c-1)/(c+d-1))-1/4/b*ln(d*tanh(b*x+a)-d)*ln((d*t
anh(b*x+a)+c-1)/(c+d-1))+1/4/b*dilog((d*tanh(b*x+a)+c+1)/(1+c+d))+1/4/b*ln(
d*tanh(b*x+a)-d)*ln((d*tanh(b*x+a)+c+1)/(1+c+d))
```

Maxima [A] time = 2.0972, size = 192, normalized size = 1.28

$$-\frac{1}{4}\,bd\left(\frac{2\,bx\log\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}+1\right)+\mathrm{Li}_2\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)}{b^2d}-\frac{2\,bx\log\left(\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}+1\right)+\mathrm{Li}_2\left(-\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}\right)}{b^2d}\right)+x\,\mathrm{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/4*b*d*((2*b*x*log((c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + dilog(-
(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^2*d) - (2*b*x*log((c + d - 1)*
e^(2*b*x + 2*a)/(c - d - 1) + 1) + dilog(-(c + d - 1)*e^(2*b*x + 2*a)/(c -
d - 1)))/(b^2*d)) + x*arctanh(d*tanh(b*x + a) + c)
```

Fricas [B] time = 2.10806, size = 1602, normalized size = 10.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x*log(-((c + 1)*cosh(b*x + a) + d*sinh(b*x + a))/((c - 1)*cosh(b*x +
 a) + d*sinh(b*x + a))) + a*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)
*sinh(b*x + a) + 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d + 1))) + a*log(2*(c
 + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) - 2*(c - d + 1)*sqrt(
-(c + d + 1)/(c - d + 1))) - a*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d -
 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt(-(c + d - 1)/(c - d - 1))) - a*log(2
*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) - 2*(c - d - 1)*sq
rt(-(c + d - 1)/(c - d - 1))) - (b*x + a)*log(sqrt(-(c + d + 1)/(c - d + 1)
)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-sqrt(-(c + d + 1)/(
c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x + a)*log(sqrt(-(c +
 d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x + a)*log(-
sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - dilog
(sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) - dilog(-s
qrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + dilog(sqrt
(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + dilog(-sqrt(-
(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(c+d*tanh(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{artanh}\left(d \tanh\left(bx + a\right) + c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*tanh(b*x + a) + c), x)
```

3.287 $\int \frac{\tanh^{-1}(c+d\tanh(a+bx))}{x}\,dx$

Optimal. Leaf size=17

$$\text{CannotIntegrate}\left(\frac{\tanh^{-1}(d\tanh(a+bx)+c)}{x},x\right)$$

```
[Out] CannotIntegrate[ArcTanh[c + d*Tanh[a + b*x]]/x, x]
```

Rubi [A] time = 0.140979, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tanh^{-1}(c+d\tanh(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTanh[c + d*Tanh[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTanh[c + d*Tanh[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(c+d\tanh(a+bx))}{x}\,dx = \int \frac{\tanh^{-1}(c+d\tanh(a+bx))}{x}\,dx$$

Mathematica [A] time = 13.6575, size = 0, normalized size = 0.

$$\int \frac{\tanh^{-1}(c+d\tanh(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTanh[c + d*Tanh[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcTanh[c + d*Tanh[a + b*x]]/x, x]
```

Maple [A] time = 0.31, size = 0, normalized size = 0.

$$\int \frac{\operatorname{Artanh}\left(c+d\tanh\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(c+d*tanh(b*x+a))/x,x)
```

```
[Out] int(arctanh(c+d*tanh(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}\left(d\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tanh(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(d*tanh(b*x + a) + c)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{artanh}\left(d\tanh\left(bx+a\right)+c\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tanh(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(arctanh(d*tanh(b*x + a) + c)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}\left(c+d\tanh\left(a+bx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(c+d*tanh(b*x+a))/x,x)
```

```
[Out] Integral(atanh(c + d*tanh(a + b*x))/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}\left(d\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tanh(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*tanh(b*x + a) + c)/x, x)
```

3.288 $\int x^3 \tanh^{-1}(1+d+d\tanh(a+bx))\,dx$

Optimal. Leaf size=155

$$\frac{3x^2\text{PolyLog}\left(3,-(d+1)e^{2a+2bx}\right)}{8b^2}-\frac{3x\text{PolyLog}\left(4,-(d+1)e^{2a+2bx}\right)}{8b^3}+\frac{3\text{PolyLog}\left(5,-(d+1)e^{2a+2bx}\right)}{16b^4}-\frac{x^3\text{PolyLog}\left(2,-\right.}{4b}$$

```
[Out] (b*x^5)/20 + (x^4*ArcTanh[1 + d + d*Tanh[a + b*x]])/4 - (x^4*Log[1 + (1 + d
)*E^(2*a + 2*b*x)])/8 - (x^3*PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))])/(4*b)
+ (3*x^2*PolyLog[3, -((1 + d)*E^(2*a + 2*b*x))])/(8*b^2) - (3*x*PolyLog[4,
-((1 + d)*E^(2*a + 2*b*x))])/(8*b^3) + (3*PolyLog[5, -((1 + d)*E^(2*a + 2*b
*x))])/(16*b^4)
```

Rubi [A] time = 0.295923, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6239, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(3,-(d+1)e^{2a+2bx}\right)}{8b^2}-\frac{3x\text{PolyLog}\left(4,-(d+1)e^{2a+2bx}\right)}{8b^3}+\frac{3\text{PolyLog}\left(5,-(d+1)e^{2a+2bx}\right)}{16b^4}-\frac{x^3\text{PolyLog}\left(2,-\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^5)/20 + (x^4*ArcTanh[1 + d + d*Tanh[a + b*x]])/4 - (x^4*Log[1 + (1 + d
)*E^(2*a + 2*b*x)])/8 - (x^3*PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))])/(4*b)
+ (3*x^2*PolyLog[3, -((1 + d)*E^(2*a + 2*b*x))])/(8*b^2) - (3*x*PolyLog[4,
-((1 + d)*E^(2*a + 2*b*x))])/(8*b^3) + (3*PolyLog[5, -((1 + d)*E^(2*a + 2*b
*x))])/(16*b^4)
```

Rule 6239

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Tanh[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
```

```
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3\tanh^{-1}(1+d+d\tanh(a+bx))\,dx = \frac{1}{4}x^4\tanh^{-1}(1+d+d\tanh(a+bx))+\frac{1}{4}b\int\frac{x^4}{1+(1+d)e^{2a+2bx}}\,dx$$

$$=\frac{bx^5}{20}+\frac{1}{4}x^4\tanh^{-1}(1+d+d\tanh(a+bx))-\frac{1}{4}(b(1+d))\int\frac{e^{2a+2bx}x^4}{1+(1+d)e^{2a+2b}}$$

$$=\frac{bx^5}{20}+\frac{1}{4}x^4\tanh^{-1}(1+d+d\tanh(a+bx))-\frac{1}{8}x^4\log\left(1+(1+d)e^{2a+2bx}\right)+\frac{1}{2}$$

$$=\frac{bx^5}{20}+\frac{1}{4}x^4\tanh^{-1}(1+d+d\tanh(a+bx))-\frac{1}{8}x^4\log\left(1+(1+d)e^{2a+2bx}\right)-\frac{x}{}$$

$$=\frac{bx^5}{20}+\frac{1}{4}x^4\tanh^{-1}(1+d+d\tanh(a+bx))-\frac{1}{8}x^4\log\left(1+(1+d)e^{2a+2bx}\right)-\frac{x}{}$$

$$=\frac{bx^5}{20}+\frac{1}{4}x^4\tanh^{-1}(1+d+d\tanh(a+bx))-\frac{1}{8}x^4\log\left(1+(1+d)e^{2a+2bx}\right)-\frac{x}{}$$

$$=\frac{bx^5}{20}+\frac{1}{4}x^4\tanh^{-1}(1+d+d\tanh(a+bx))-\frac{1}{8}x^4\log\left(1+(1+d)e^{2a+2bx}\right)-\frac{x}{}$$

$$=\frac{bx^5}{20}+\frac{1}{4}x^4\tanh^{-1}(1+d+d\tanh(a+bx))-\frac{1}{8}x^4\log\left(1+(1+d)e^{2a+2bx}\right)-\frac{x}{}$$

Mathematica [A] time = 4.36327, size = 144, normalized size = 0.93

$$\frac{1}{16}\left(\frac{6x^2\text{PolyLog}\left(3,-\frac{e^{-2(a+bx)}}{d+1}\right)}{b^2}+\frac{6x\text{PolyLog}\left(4,-\frac{e^{-2(a+bx)}}{d+1}\right)}{b^3}+\frac{3\text{PolyLog}\left(5,-\frac{e^{-2(a+bx)}}{d+1}\right)}{b^4}+\frac{4x^3\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{d+1}\right)}{b}-2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (4*x^4*ArcTanh[1 + d + d*Tanh[a + b*x]] - 2*x^4*Log[1 + 1/((1 + d)*E^(2*(a
+ b*x)))] + (4*x^3*PolyLog[2, -(1/((1 + d)*E^(2*(a + b*x))))])/b + (6*x^2*P
olyLog[3, -(1/((1 + d)*E^(2*(a + b*x))))])/b^2 + (6*x*PolyLog[4, -(1/((1 +
d)*E^(2*(a + b*x))))])/b^3 + (3*PolyLog[5, -(1/((1 + d)*E^(2*(a + b*x))))])
/b^4)/16
```

Maple [C] time = 11.484, size = 1769, normalized size = 11.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*arctanh(1+d+d*tanh(b*x+a)),x)

[Out] 1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/2/b^4*d*a^3/(1+d)*dilog(1+exp(b*x+a)*(-d-1)^(1/2))+1/2/b^4*d*a^3/(1+d)*dilog(1-exp(b*x+a)*(-d-1)^(1/2))-3/8/b^4*d*a^4/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))-1/4/b^4*d*a^3/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))+1/2/b^4*d*a^4/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))+1/2/b^4*d*a^4/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))+1/2/b^3*a^3/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))*x-1/4/b*d/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*x^3+3/8/b^2*d/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2*a))*x^2+1/2/b^3*a^3/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))*x-1/2/b^3*a^3/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x-3/8/b^3*d/(1+d)*polylog(4,-(1+d)*exp(2*b*x+2*a))*x+1/8*x^4*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1)-1/16*I*x^4*Pi*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^3-1/8/b^4*d*a^4/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1)+1/16*I*x^4*Pi*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))^2+1/16*I*x^4*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/8*I*x^4*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/2/b^3*d*a^3/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x+1/2/b^3*d*a^3/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))*x+1/2/b^3*d*a^3/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))*x+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a))^3+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))^3+1/8*I*x^4*Pi*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2+1/20*b*x^5-1/16*I*x^4*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/8/b^4*a^4/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1)+1/16*I*x^4*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))-1/8*I*x^4*Pi-1/8*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x^4+1/2/b^4*a^4/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))-3/8/b^4*a^4/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))-1/4/b/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*x^3-1/4/b^4/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*a^3+3/8/b^2/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2*a))*x^2-3/8/b^3/(1+d)*polylog(4,-(1+d)*exp(2*b*x+2*a))*x+1/2/b^4*a^4/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))+3/16/b^4*d/(1+d)*polylog(5,-(1+d)*exp(2*b*x+2*a))+1/2/b^4*a^3/(1+d)*dilog(1+exp(b*x+a)*(-d-1)^(1/2))+1/2/b^4*a^3/(1+d)*dilog(1-exp(b*x+a)*(-d-1)^(1/2))-1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2+1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))^2-1/8/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x^4+3/16/b^4/(1+d)*polylog(5,-(1+d)*exp(2*b*x+2*a))-1/4*x^4*ln(exp(b*x+a))-1/8*x^4*ln(d)

Maxima [A] time = 3.27983, size = 201, normalized size = 1.3

$$\frac{1}{4}x^4\,\text{artanh}\,(d\tanh\,(bx+a)+d+1)+\frac{1}{40}\left(\frac{2\,x^5}{d}-\frac{5\left(2\,b^4x^4\log\left((d+1)e^{(2\,bx+2\,a)}+1\right)+4\,b^3x^3\text{Li}_2\left(-(d+1)e^{(2\,bx+2\,a)}\right)-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arctanh(d*tanh(b*x + a) + d + 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*log
((d + 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog(-(d + 1)*e^(2*b*x + 2*a)) -
 6*b^2*x^2*polylog(3, -(d + 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, -(d + 1)
*e^(2*b*x + 2*a)) - 3*polylog(5, -(d + 1)*e^(2*b*x + 2*a)))/(b^5*d))*b*d
```

Fricas [C] time = 2.00258, size = 1378, normalized size = 8.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/40*(2*b^5*x^5 + 5*b^4*x^4*log(-((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/
(d*cosh(b*x + a) + d*sinh(b*x + a))) - 20*b^3*x^3*dilog(1/2*sqrt(-4*d - 4)*
(cosh(b*x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-1/2*sqrt(-4*d - 4)*(co
sh(b*x + a) + sinh(b*x + a))) - 5*a^4*log(2*(d + 1)*cosh(b*x + a) + 2*(d +
1)*sinh(b*x + a) + sqrt(-4*d - 4)) - 5*a^4*log(2*(d + 1)*cosh(b*x + a) + 2*
(d + 1)*sinh(b*x + a) - sqrt(-4*d - 4)) + 60*b^2*x^2*polylog(3, 1/2*sqrt(-4
*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -1/2*sqrt(
-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, 1/2*sqrt(-4
*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, -1/2*sqrt(-4*
d - 4)*(cosh(b*x + a) + sinh(b*x + a))) - 5*(b^4*x^4 - a^4)*log(1/2*sqrt(-4
*d - 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 5*(b^4*x^4 - a^4)*log(-1/2*s
qrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 120*polylog(5, 1/2*sqr
t(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) + 120*polylog(5, -1/2*sqrt(-4*
d - 4)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(1+d+d*tanh(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{artanh}(d\tanh(bx+a)+d+1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^3*arctanh(d*tanh(b*x + a) + d + 1), x)
```

3.289 $\int x^2 \tanh^{-1}(1+d+d\tanh(a+bx))\,dx$

Optimal. Leaf size=128

$$\frac{x\text{PolyLog}\left(3,-(d+1)e^{2a+2bx}\right)}{4b^2}-\frac{\text{PolyLog}\left(4,-(d+1)e^{2a+2bx}\right)}{8b^3}-\frac{x^2\text{PolyLog}\left(2,-(d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{6}x^3\log\left((d+1)e^{2a+2}\right.$$

```
[Out] (b*x^4)/12 + (x^3*ArcTanh[1 + d + d*Tanh[a + b*x]])/3 - (x^3*Log[1 + (1 + d
)*E^(2*a + 2*b*x)])/6 - (x^2*PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))])/(4*b)
+ (x*PolyLog[3, -((1 + d)*E^(2*a + 2*b*x))])/(4*b^2) - PolyLog[4, -((1 + d)
*E^(2*a + 2*b*x))]/(8*b^3)
```

Rubi [A] time = 0.254388, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6239, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{x\text{PolyLog}\left(3,-(d+1)e^{2a+2bx}\right)}{4b^2}-\frac{\text{PolyLog}\left(4,-(d+1)e^{2a+2bx}\right)}{8b^3}-\frac{x^2\text{PolyLog}\left(2,-(d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{6}x^3\log\left((d+1)e^{2a+2}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```


```
[Out] (b*x^4)/12 + (x^3*ArcTanh[1 + d + d*Tanh[a + b*x]])/3 - (x^3*Log[1 + (1 + d
)*E^(2*a + 2*b*x)])/6 - (x^2*PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))])/(4*b)
+ (x*PolyLog[3, -((1 + d)*E^(2*a + 2*b*x))])/(4*b^2) - PolyLog[4, -((1 + d)
*E^(2*a + 2*b*x))]/(8*b^3)
```


Rule 6239

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Tanh[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
```

```
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \tanh^{-1}(1+d+d\tanh(a+bx))\,dx &= \frac{1}{3}x^3\tanh^{-1}(1+d+d\tanh(a+bx))+\frac{1}{3}b\int \frac{x^3}{1+(1+d)e^{2a+2bx}}\,dx \\
&= \frac{bx^4}{12}+\frac{1}{3}x^3\tanh^{-1}(1+d+d\tanh(a+bx))-\frac{1}{3}(b(1+d))\int \frac{e^{2a+2bx}x^3}{1+(1+d)e^{2a+2b}} \\
&= \frac{bx^4}{12}+\frac{1}{3}x^3\tanh^{-1}(1+d+d\tanh(a+bx))-\frac{1}{6}x^3\log\left(1+(1+d)e^{2a+2bx}\right)+\frac{1}{2} \\
&= \frac{bx^4}{12}+\frac{1}{3}x^3\tanh^{-1}(1+d+d\tanh(a+bx))-\frac{1}{6}x^3\log\left(1+(1+d)e^{2a+2bx}\right)-\frac{x}{-} \\
&= \frac{bx^4}{12}+\frac{1}{3}x^3\tanh^{-1}(1+d+d\tanh(a+bx))-\frac{1}{6}x^3\log\left(1+(1+d)e^{2a+2bx}\right)-\frac{x}{-} \\
&= \frac{bx^4}{12}+\frac{1}{3}x^3\tanh^{-1}(1+d+d\tanh(a+bx))-\frac{1}{6}x^3\log\left(1+(1+d)e^{2a+2bx}\right)-\frac{x}{-} \\
&= \frac{bx^4}{12}+\frac{1}{3}x^3\tanh^{-1}(1+d+d\tanh(a+bx))-\frac{1}{6}x^3\log\left(1+(1+d)e^{2a+2bx}\right)-\frac{x}{-}
\end{aligned}
$$

Mathematica [A] time = 4.87942, size = 118, normalized size = 0.92

$$
\frac{1}{24}\left(\frac{6x\text{PolyLog}\left(3,-\frac{e^{-2(a+bx)}}{d+1}\right)}{b^2}+\frac{3\text{PolyLog}\left(4,-\frac{e^{-2(a+bx)}}{d+1}\right)}{b^3}+\frac{6x^2\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{d+1}\right)}{b}-4x^3\log\left(\frac{e^{-2(a+bx)}}{d+1}+1\right)+8x^3\tanh\right.
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (8*x^3*ArcTanh[1 + d + d*Tanh[a + b*x]] - 4*x^3*Log[1 + 1/((1 + d)*E^(2*(a
+ b*x)))] + (6*x^2*PolyLog[2, -(1/((1 + d)*E^(2*(a + b*x))))])/b + (6*x*Pol
yLog[3, -(1/((1 + d)*E^(2*(a + b*x))))])/b^2 + (3*PolyLog[4, -(1/((1 + d)*E
^(2*(a + b*x))))])/b^3)/24
```

Maple [C] time = 14.46, size = 1710, normalized size = 13.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(1+d+d*tanh(b*x+a)),x)
```

```
[Out] -1/3*x^3*ln(exp(b*x+a))-1/6*x^3*ln(d)-1/4/b*d/(1+d)*polylog(2,-(1+d)*exp(2*
b*x+2*a))*x^2-1/6*I*x^3*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/12
*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+
1))^2-1/2/b^3*a^3/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))-1/8/b^3*d/(1+d)*polyl
og(4,-(1+d)*exp(2*b*x+2*a))-1/6*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x^3-1/2/
b^3*a^2/(1+d)*dilog(1+exp(b*x+a)*(-d-1)^(1/2))-1/2/b^3*a^2/(1+d)*dilog(1-ex
p(b*x+a)*(-d-1)^(1/2))+1/3/b^3/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*a^3-1/4/b/(
1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*x^2+1/4/b^3/(1+d)*polylog(2,-(1+d)*ex
p(2*b*x+2*a))*a^2+1/4/b^2/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2*a))*x-1/2/b^3*
a^3/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))+1/4/b^3*d/(1+d)*polylog(2,-(1+d)*ex
p(2*b*x+2*a))*a^2+1/4/b^2*d/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2*a))*x-1/2/b^
2*a^2/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))*x-1/2/b^2*a^2/(1+d)*ln(1-exp(b*x+
a)*(-d-1)^(1/2))*x-1/2/b^3*d*a^3/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))-1/2/b^
3*d*a^3/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))-1/2/b^3*d*a^2/(1+d)*dilog(1+exp
(b*x+a)*(-d-1)^(1/2))-1/2/b^3*d*a^2/(1+d)*dilog(1-exp(b*x+a)*(-d-1)^(1/2))+
1/2/b^2/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x*a^2+1/3/b^3*d/(1+d)*ln(1+(1+d)*e
xp(2*b*x+2*a))*a^3+1/12*b*x^4-1/6*I*Pi*x^3+1/6/b^3*a^3/(1+d)*ln(exp(2*b*x+2
*a)*d+exp(2*b*x+2*a)+1)+1/6/b^3*d*a^3/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2
*a)+1)-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d/(ex
p(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))*csgn
(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a
)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))^2-1/2/
b^2*d*a^2/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))*x+1/2/b^2*d/(1+d)*ln(1+(1+d)*
exp(2*b*x+2*a))*x*a^2-1/2/b^2*d*a^2/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))*x-1
/8/b^3/(1+d)*polylog(4,-(1+d)*exp(2*b*x+2*a))-1/6/(1+d)*ln(1+(1+d)*exp(2*b*
x+2*a))*x^3+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))^3+1/6*I*x^3*Pi*csgn(I*d/(e
xp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2-1/12*I*x^3*Pi*csgn(I*d/(exp(2*b*x+2*a)+1
)*exp(2*b*x+2*a))^3+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))
^3-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)
+1))^3+1/12*I*x^3*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+1/12*I*x^3
*Pi*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(
exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))^2-1/12*I*x^3*Pi*csgn(I*d)*csgn(I*d/(exp
(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2+1/6*x^3*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)
+1)-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)*d+exp(2
*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1)
)+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/12*I*x^3*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2
*a)/(exp(2*b*x+2*a)+1))*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))
```

Maxima [A] time = 3.27233, size = 169, normalized size = 1.32

$$\frac{1}{3}x^3\operatorname{artanh}\left(d\tanh\left(bx+a\right)+d+1\right)+\frac{1}{36}\left(\frac{3x^4}{d}-\frac{2\left(4b^3x^3\log\left((d+1)e^{(2bx+2a)}+1\right)+6b^2x^2\mathrm{Li}_2\left(-(d+1)e^{(2bx+2a)}\right)-\right.}{b^4d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctanh(d*tanh(b*x + a) + d + 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*log
((d + 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-(d + 1)*e^(2*b*x + 2*a)) -
 6*b*x*polylog(3, -(d + 1)*e^(2*b*x + 2*a)) + 3*polylog(4, -(d + 1)*e^(2*b*
x + 2*a)))/(b^4*d))*b*d
```

Fricas [C] time = 1.98671, size = 1161, normalized size = 9.07

$$b^4x^4 + 2\,b^3x^3\log\left(-\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 6\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4\,d-4}(\cosh(bx+a)+\sinh(bx+a))\right) - 6\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(b^4*x^4 + 2*b^3*x^3*log(-((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/(d
*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b^2*x^2*dilog(1/2*sqrt(-4*d - 4)*(co
sh(b*x + a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-1/2*sqrt(-4*d - 4)*(cosh(b
*x + a) + sinh(b*x + a))) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*s
inh(b*x + a) + sqrt(-4*d - 4)) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d +
 1)*sinh(b*x + a) - sqrt(-4*d - 4)) + 12*b*x*polylog(3, 1/2*sqrt(-4*d - 4)*
(cosh(b*x + a) + sinh(b*x + a))) + 12*b*x*polylog(3, -1/2*sqrt(-4*d - 4)*(c
osh(b*x + a) + sinh(b*x + a))) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(-4*d - 4)*(
cosh(b*x + a) + sinh(b*x + a)) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sqrt(-4*d
- 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 12*polylog(4, 1/2*sqrt(-4*d - 4
)*(cosh(b*x + a) + sinh(b*x + a))) - 12*polylog(4, -1/2*sqrt(-4*d - 4)*(cos
h(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(1+d+d*tanh(b*x+a)),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{artanh}(d \tanh(bx+a)+d+1)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(d*tanh(b*x + a) + d + 1), x)
```

3.290 $\int x \tanh^{-1}(1+d+d\tanh(a+bx))\,dx$

Optimal. Leaf size=101

$$\frac{\text{PolyLog}\left(3,-(d+1)e^{2a+2bx}\right)}{8b^2}-\frac{x\text{PolyLog}\left(2,-(d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{4}x^2\log\left((d+1)e^{2a+2bx}+1\right)+\frac{1}{2}x^2\tanh^{-1}(d\tanh(a+b$$

```
[Out] (b*x^3)/6 + (x^2*ArcTanh[1 + d + d*Tanh[a + b*x]])/2 - (x^2*Log[1 + (1 + d)
*E^(2*a + 2*b*x)])/4 - (x*PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))])/(4*b) + P
olyLog[3, -((1 + d)*E^(2*a + 2*b*x))]/(8*b^2)
```

Rubi [A] time = 0.223576, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6239, 2184, 2190, 2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3,-(d+1)e^{2a+2bx}\right)}{8b^2}-\frac{x\text{PolyLog}\left(2,-(d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{4}x^2\log\left((d+1)e^{2a+2bx}+1\right)+\frac{1}{2}x^2\tanh^{-1}(d\tanh(a+b$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcTanh[1 + d + d*Tanh[a + b*x]])/2 - (x^2*Log[1 + (1 + d)
*E^(2*a + 2*b*x)])/4 - (x*PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))])/(4*b) + P
olyLog[3, -((1 + d)*E^(2*a + 2*b*x))]/(8*b^2)
```

Rule 6239

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Tanh[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(1+d+d\tanh(a+bx))\,dx = \frac{1}{2}x^2\tanh^{-1}(1+d+d\tanh(a+bx)) + \frac{1}{2}b\int\frac{x^2}{1+(1+d)e^{2a+2bx}}\,dx$$

$$= \frac{bx^3}{6} + \frac{1}{2}x^2\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{2}(b(1+d))\int\frac{e^{2a+2bx}x^2}{1+(1+d)e^{2a+2bx}}$$

$$= \frac{bx^3}{6} + \frac{1}{2}x^2\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{4}x^2\log\left(1+(1+d)e^{2a+2bx}\right) + \frac{1}{2}$$

$$= \frac{bx^3}{6} + \frac{1}{2}x^2\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{4}x^2\log\left(1+(1+d)e^{2a+2bx}\right) - \frac{xL}{}$$

$$= \frac{bx^3}{6} + \frac{1}{2}x^2\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{4}x^2\log\left(1+(1+d)e^{2a+2bx}\right) - \frac{xL}{}$$

$$= \frac{bx^3}{6} + \frac{1}{2}x^2\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{4}x^2\log\left(1+(1+d)e^{2a+2bx}\right) - \frac{xL}{}$$

Mathematica [A] time = 4.92203, size = 91, normalized size = 0.9

$$\frac{2bx\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{d+1}\right)+\text{PolyLog}\left(3,-\frac{e^{-2(a+bx)}}{d+1}\right)+2b^2x^2\left(2\tanh^{-1}(d\tanh(a+bx)+d+1)-\log\left(\frac{e^{-2(a+bx)}}{d+1}+1\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (2*b^2*x^2*(2*ArcTanh[1 + d + d*Tanh[a + b*x]] - Log[1 + 1/((1 + d)*E^(2*(a
 + b*x)))]) + 2*b*x*PolyLog[2, -(1/((1 + d)*E^(2*(a + b*x))))] + PolyLog[3,
 -(1/((1 + d)*E^(2*(a + b*x))))])/(8*b^2)
```

Maple [C] time = 3.849, size = 1627, normalized size = 16.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(1+d+d*tanh(b*x+a)),x)
```

```
[Out] -1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d/(exp(2*b*x
+2*a)+1)*exp(2*b*x+2*a))^2+1/6*b*x^3-1/4/b^2*d*a^2/(1+d)*ln(exp(2*b*x+2*a)*
```

```
d+exp(2*b*x+2*a)+1)+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^
3-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1
))^3+1/4*I*x^2*Pi*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2+1/4*x^2*ln(
exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1)-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1))*
csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2
*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/8*I*x^2*Pi*csgn(I/(exp(2
*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1)
)^2+1/8*I*x^2*Pi*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))*csgn(I/(exp(2*
b*x+2*a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))^2-1/8*I*x^2*Pi*csgn(I*d)*c
sgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2+1/2/b^2*a/(1+d)*dilog(1-exp(b*
x+a)*(-d-1)^(1/2))-1/4/b^2/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*a^2-1/4/b/(1+d)
*polylog(2,-(1+d)*exp(2*b*x+2*a))*x-1/4/b^2/(1+d)*polylog(2,-(1+d)*exp(2*b*
x+2*a))*a+1/2/b^2*a^2/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))+1/2/b^2*a^2/(1+d)
*ln(1-exp(b*x+a)*(-d-1)^(1/2))+1/8/b^2*d/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2
*a))+1/2/b^2*a/(1+d)*dilog(1+exp(b*x+a)*(-d-1)^(1/2))-1/4*d/(1+d)*ln(1+(1+d
)*exp(2*b*x+2*a))*x^2+1/8*I*x^2*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*
a))-1/4*I*x^2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/2*x^2*ln(exp
(b*x+a))-1/4*x^2*ln(d)-1/8*I*x^2*Pi*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2
*a))^3-1/4*I*Pi*x^2-1/4/b^2*a^2/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1)
+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a))^3-1/4/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))
*x^2+1/8/b^2/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2*a))+1/2/b^2*d*a/(1+d)*dilog
(1+exp(b*x+a)*(-d-1)^(1/2))+1/2/b^2*d*a/(1+d)*dilog(1-exp(b*x+a)*(-d-1)^(1/
2))-1/2/b/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x*a-1/4/b^2*d/(1+d)*ln(1+(1+d)*e
xp(2*b*x+2*a))*a^2-1/4/b*d/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*x-1/4/b^2
*d/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*a+1/2/b*a/(1+d)*ln(1+exp(b*x+a)*(
-d-1)^(1/2))*x+1/2/b*a/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))*x+1/2/b^2*d*a^2/
(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))+1/2/b^2*d*a^2/(1+d)*ln(1-exp(b*x+a)*(-d
-1)^(1/2))+1/8*I*x^2*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))
*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))+1/8*I*x^2*Pi*csgn(I/(exp(2*b*x
+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-
1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+
2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))-1/2
/b*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x*a+1/2/b*d*a/(1+d)*ln(1+exp(b*x+a)*(
-d-1)^(1/2))*x+1/2/b*d*a/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))*x
```

Maxima [A] time = 3.25204, size = 136, normalized size = 1.35

$$\frac{1}{24}\left(\frac{4\,x^3}{d}-\frac{3\left(2\,b^2x^2\log\left((d+1)e^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(-(d+1)e^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3(-(d+1)e^{(2\,bx+2\,a)})\right)}{b^3d}\right)bd+\frac{1}{2}\,x^2\,\mathrm{artanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/24*(4*x^3/d - 3*(2*b^2*x^2*log((d + 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dilog
(-(d + 1)*e^(2*b*x + 2*a)) - polylog(3, -(d + 1)*e^(2*b*x + 2*a)))/(b^3*d))
*b*d + 1/2*x^2*arctanh(d*tanh(b*x + a) + d + 1)
```

Fricas [C] time = 2.0149, size = 954, normalized size = 9.45

$$\frac{2\,b^3x^3 + 3\,b^2x^2\log\left(-\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 6\,bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4\,d-4}(\cosh(bx+a)+\sinh(bx+a))\right) - 6\,bx\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{\cdot}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 + 3*b^2*x^2*log(-((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/
(d*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b*x*dilog(1/2*sqrt(-4*d - 4)*(cosh
(b*x + a) + sinh(b*x + a))) - 6*b*x*dilog(-1/2*sqrt(-4*d - 4)*(cosh(b*x + a
) + sinh(b*x + a))) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*
x + a) + sqrt(-4*d - 4)) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*si
nh(b*x + a) - sqrt(-4*d - 4)) - 3*(b^2*x^2 - a^2)*log(1/2*sqrt(-4*d - 4)*(c
osh(b*x + a) + sinh(b*x + a)) + 1) - 3*(b^2*x^2 - a^2)*log(-1/2*sqrt(-4*d -
 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 6*polylog(3, 1/2*sqrt(-4*d - 4)*
(cosh(b*x + a) + sinh(b*x + a))) + 6*polylog(3, -1/2*sqrt(-4*d - 4)*(cosh(b
*x + a) + sinh(b*x + a))))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\mathrm{atanh}\,(d\tanh\,(a+bx)+d+1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(1+d+d*tanh(b*x+a)),x)
```

```
[Out] Integral(x*atanh(d*tanh(a + b*x) + d + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{artanh}\left(d \tanh\left(bx+a\right)+d+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(d*tanh(b*x + a) + d + 1), x)
```

3.291 $\int \tanh^{-1}(1+d+d\tanh(a+bx))\,dx$

Optimal. Leaf size=69

$$-\frac{\text{PolyLog}\left(2,-(d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{2}x\log\left((d+1)e^{2a+2bx}+1\right)+x\tanh^{-1}(d\tanh(a+bx)+d+1)+\frac{bx^2}{2}$$

```
[Out] (b*x^2)/2 + x*ArcTanh[1 + d + d*Tanh[a + b*x]] - (x*Log[1 + (1 + d)*E^(2*a
+ 2*b*x)])/2 - PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))]/(4*b)
```

Rubi [A] time = 0.136717, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6231, 2184, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,-(d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{2}x\log\left((d+1)e^{2a+2bx}+1\right)+x\tanh^{-1}(d\tanh(a+bx)+d+1)+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcTanh[1 + d + d*Tanh[a + b*x]] - (x*Log[1 + (1 + d)*E^(2*a
+ 2*b*x)])/2 - PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))]/(4*b)
```

Rule 6231

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Ar
cTanh[c + d*Tanh[a + b*x]], x] + Dist[b, Int[x/(c - d + c*E^(2*a + 2*b*x)),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \tanh^{-1}(1+d+d\tanh(a+bx))\,dx = x\tanh^{-1}(1+d+d\tanh(a+bx)) + b\int \frac{x}{1+(1+d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^2}{2} + x\tanh^{-1}(1+d+d\tanh(a+bx)) - (b(1+d))\int \frac{e^{2a+2bx}x}{1+(1+d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^2}{2} + x\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{2}x\log\left(1+(1+d)e^{2a+2bx}\right) + \frac{1}{2}\int \text{lo}$$
$$= \frac{bx^2}{2} + x\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{2}x\log\left(1+(1+d)e^{2a+2bx}\right) + \frac{\text{Subst}}{}$$
$$= \frac{bx^2}{2} + x\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{2}x\log\left(1+(1+d)e^{2a+2bx}\right) - \frac{\text{Li}_2\left(-\right.}{}$$

Mathematica [B] time = 4.3156, size = 201, normalized size = 2.91

$$\frac{-2\text{PolyLog}\left(2,-\sqrt{-d-1}e^{a+bx}\right) - 2\text{PolyLog}\left(2,\sqrt{-d-1}e^{a+bx}\right) - 2\log\left(e^{a+bx}\right)\log\left(1-\sqrt{-d-1}e^{a+bx}\right) - 2\log\left(e^{a+bx}\right)\log}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] x*ArcTanh[1 + d + d*Tanh[a + b*x]] + (b^2*x^2 + Log[E^(a + b*x)]^2 - 2*Log[
E^(a + b*x)]*Log[1 - Sqrt[-1 - d]*E^(a + b*x)] - 2*Log[E^(a + b*x)]*Log[1 +
 Sqrt[-1 - d]*E^(a + b*x)] + 2*Log[E^(a + b*x)]*Log[E^(-a - b*x) + (1 + d)*
E^(a + b*x)] - 2*b*x*Log[(2 + d)*Cosh[a + b*x] + d*Sinh[a + b*x]] - 2*PolyL
```

```
og[2, -(Sqrt[-1 - d]*E^(a + b*x))] - 2*PolyLog[2, Sqrt[-1 - d]*E^(a + b*x)]
)/(4*b)
```

Maple [B] time = 0.116, size = 247, normalized size = 3.6

$$\frac{\operatorname{Artanh}(1+d+d\tanh(bx+a))\ln(d\tanh(bx+a)+d)}{2b}-\frac{\operatorname{Artanh}(1+d+d\tanh(bx+a))\ln(d\tanh(bx+a)-d)}{2b}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(1+d+d*tanh(b*x+a)),x)
```

```
[Out] 1/2/b*arctanh(1+d+d*tanh(b*x+a))*ln(d*tanh(b*x+a)+d)-1/2/b*arctanh(1+d+d*ta
nh(b*x+a))*ln(d*tanh(b*x+a)-d)+1/8/b*ln(d*tanh(b*x+a)+d)^2-1/4/b*dilog(1+1/
2*d*tanh(b*x+a)+1/2*d)-1/4/b*ln(d*tanh(b*x+a)+d)*ln(1+1/2*d*tanh(b*x+a)+1/2
*d)-1/4/b*dilog(1/2*(d*tanh(b*x+a)+d)/d)-1/4/b*ln(d*tanh(b*x+a)-d)*ln(1/2*(
d*tanh(b*x+a)+d)/d)+1/4/b*dilog((d*tanh(b*x+a)+d+2)/(2*d+2))+1/4/b*ln(d*tan
h(b*x+a)-d)*ln((d*tanh(b*x+a)+d+2)/(2*d+2))
```

Maxima [A] time = 3.26336, size = 97, normalized size = 1.41

$$\frac{1}{4}bd\left(\frac{2x^2}{d}-\frac{2bx\log\left((d+1)e^{(2bx+2a)}+1\right)+\operatorname{Li}_2\left(-(d+1)e^{(2bx+2a)}\right)}{b^2d}\right)+x\operatorname{artanh}(d\tanh(bx+a)+d+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*b*d*(2*x^2/d - (2*b*x*log((d + 1)*e^(2*b*x + 2*a) + 1) + dilog(-(d + 1)
*e^(2*b*x + 2*a)))/(b^2*d)) + x*arctanh(d*tanh(b*x + a) + d + 1)
```

Fricas [B] time = 2.05621, size = 709, normalized size = 10.28

$$\frac{b^2x^2+bx\log\left(-\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right)+a\log\left(2(d+1)\cosh(bx+a)+2(d+1)\sinh(bx+a)+\sqrt{-4d-4}\right)+a\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + b*x*log(-((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/(d*cosh(b
*x + a) + d*sinh(b*x + a))) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sin
h(b*x + a) + sqrt(-4*d - 4)) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*si
nh(b*x + a) - sqrt(-4*d - 4)) - (b*x + a)*log(1/2*sqrt(-4*d - 4)*(cosh(b*x
+ a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-1/2*sqrt(-4*d - 4)*(cosh(b*x +
a) + sinh(b*x + a)) + 1) - dilog(1/2*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b
*x + a))) - dilog(-1/2*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{atanh}\left(d\tanh\left(a+bx\right)+d+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(1+d+d*tanh(b*x+a)),x)
```

```
[Out] Integral(atanh(d*tanh(a + b*x) + d + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{artanh}\left(d\tanh\left(bx+a\right)+d+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*tanh(b*x + a) + d + 1), x)
```

3.292 $\int \frac{\tanh^{-1}(1+d+d\tanh(a+bx))}{x}\,dx$

Optimal. Leaf size=18

$$\text{CannotIntegrate}\left(\frac{\tanh^{-1}(d\tanh(a+bx)+d+1)}{x},x\right)$$

```
[Out] CannotIntegrate[ArcTanh[1 + d + d*Tanh[a + b*x]]/x, x]
```

Rubi [A] time = 0.0790353, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\tanh^{-1}(1+d+d\tanh(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTanh[1 + d + d*Tanh[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTanh[1 + d + d*Tanh[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(1+d+d\tanh(a+bx))}{x}\,dx = \int \frac{\tanh^{-1}(1+d+d\tanh(a+bx))}{x}\,dx$$

Mathematica [A] time = 4.23546, size = 0, normalized size = 0.

$$\int \frac{\tanh^{-1}(1+d+d\tanh(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTanh[1 + d + d*Tanh[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcTanh[1 + d + d*Tanh[a + b*x]]/x, x]
```

Maple [A] time = 0.31, size = 0, normalized size = 0.

$$\int \frac{\operatorname{Artanh}\left(1+d+d\tanh\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(1+d+d*tanh(b*x+a))/x,x)
```

```
[Out] int(arctanh(1+d+d*tanh(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}\left(d\tanh\left(bx+a\right)+d+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*tanh(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(d*tanh(b*x + a) + d + 1)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{artanh}\left(d\tanh\left(bx+a\right)+d+1\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*tanh(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(arctanh(d*tanh(b*x + a) + d + 1)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}\left(d\tanh\left(a+bx\right)+d+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(1+d+d*tanh(b*x+a))/x,x)
```

```
[Out] Integral(atanh(d*tanh(a + b*x) + d + 1)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(d\tanh(bx+a)+d+1)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*tanh(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*tanh(b*x + a) + d + 1)/x, x)
```

3.293 $\int x^3 \tanh^{-1}(1-d-d\tanh(a+bx))\,dx$

Optimal. Leaf size=168

$$\frac{3x^2\text{PolyLog}\left(3,-(1-d)e^{2a+2bx}\right)}{8b^2}-\frac{3x\text{PolyLog}\left(4,-(1-d)e^{2a+2bx}\right)}{8b^3}+\frac{3\text{PolyLog}\left(5,-(1-d)e^{2a+2bx}\right)}{16b^4}-\frac{x^3\text{PolyLog}\left(2,\right.}{4}$$

```
[Out] (b*x^5)/20 + (x^4*ArcTanh[1 - d - d*Tanh[a + b*x]])/4 - (x^4*Log[1 + (1 - d
)*E^(2*a + 2*b*x)])/8 - (x^3*PolyLog[2, -((1 - d)*E^(2*a + 2*b*x))])/(4*b)
+ (3*x^2*PolyLog[3, -((1 - d)*E^(2*a + 2*b*x))])/(8*b^2) - (3*x*PolyLog[4,
-((1 - d)*E^(2*a + 2*b*x))])/(8*b^3) + (3*PolyLog[5, -((1 - d)*E^(2*a + 2*b
*x))])/(16*b^4)
```

Rubi [A] time = 0.296893, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6239, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(3,-(1-d)e^{2a+2bx}\right)}{8b^2}-\frac{3x\text{PolyLog}\left(4,-(1-d)e^{2a+2bx}\right)}{8b^3}+\frac{3\text{PolyLog}\left(5,-(1-d)e^{2a+2bx}\right)}{16b^4}-\frac{x^3\text{PolyLog}\left(2,\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^5)/20 + (x^4*ArcTanh[1 - d - d*Tanh[a + b*x]])/4 - (x^4*Log[1 + (1 - d
)*E^(2*a + 2*b*x)])/8 - (x^3*PolyLog[2, -((1 - d)*E^(2*a + 2*b*x))])/(4*b)
+ (3*x^2*PolyLog[3, -((1 - d)*E^(2*a + 2*b*x))])/(8*b^2) - (3*x*PolyLog[4,
-((1 - d)*E^(2*a + 2*b*x))])/(8*b^3) + (3*PolyLog[5, -((1 - d)*E^(2*a + 2*b
*x))])/(16*b^4)
```

Rule 6239

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Tanh[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
```

```
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3 \tanh^{-1}(1-d-d\tanh(a+bx))\,dx = \frac{1}{4}x^4\tanh^{-1}(1-d-d\tanh(a+bx)) + \frac{1}{4}b\int \frac{x^4}{1+(1-d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{4}(b(1-d))\int \frac{e^{2a+2bx}x^4}{1+(1-d)e^{2a+2b}$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1-d)e^{2a+2bx}\right) +$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1-d)e^{2a+2bx}\right) -$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1-d)e^{2a+2bx}\right) -$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1-d)e^{2a+2bx}\right) -$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1-d)e^{2a+2bx}\right) -$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1-d)e^{2a+2bx}\right) -$$

Mathematica [A] time = 4.37523, size = 144, normalized size = 0.86

$$\frac{1}{16}\left(\frac{6x^2\text{PolyLog}\left(3,\frac{e^{-2(a+bx)}}{d-1}\right)}{b^2} + \frac{6x\text{PolyLog}\left(4,\frac{e^{-2(a+bx)}}{d-1}\right)}{b^3} + \frac{3\text{PolyLog}\left(5,\frac{e^{-2(a+bx)}}{d-1}\right)}{b^4} + \frac{4x^3\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{d-1}\right)}{b} - 2x^4\text{lo}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (4*x^4*ArcTanh[1 - d - d*Tanh[a + b*x]] - 2*x^4*Log[1 - 1/((-1 + d)*E^(2*(a
 + b*x)))] + (4*x^3*PolyLog[2, 1/((-1 + d)*E^(2*(a + b*x)))])/b + (6*x^2*Po
lyLog[3, 1/((-1 + d)*E^(2*(a + b*x)))])/b^2 + (6*x*PolyLog[4, 1/((-1 + d)*E
^(2*(a + b*x)))])/b^3 + (3*PolyLog[5, 1/((-1 + d)*E^(2*(a + b*x)))])/b^4)/1
6
```

Maple [C] time = 15.613, size = 1773, normalized size = 10.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3*arctanh(-1+d+d*tanh(b*x+a)),x)
```

```
[Out] -1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*
x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))+1
/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2
*b*x+2*a)/(exp(2*b*x+2*a)+1))+3/16/b^4*d/(d-1)*polylog(5,(d-1)*exp(2*b*x+2*
a))-1/2/b^4*a^3/(d-1)*dilog(1-exp(b*x+a)*(d-1)^(1/2))+1/8*I*x^4*Pi+1/2/b^4*
d*a^3/(d-1)*dilog(1-exp(b*x+a)*(d-1)^(1/2))-3/16/b^4/(d-1)*polylog(5,(d-1)*
exp(2*b*x+2*a))+1/8/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x^4+3/8/b^4/(d-1)*ln(1
-(d-1)*exp(2*b*x+2*a))*a^4+1/4/b/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*x^3+
1/4/b^4/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*a^3-1/2/b^4*a^4/(d-1)*ln(1-ex
p(b*x+a)*(d-1)^(1/2))+1/2/b^4*d*a^3/(d-1)*dilog(1+exp(b*x+a)*(d-1)^(1/2))-1
/2/b^3*a^3/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))*x-1/2/b^3*a^3/(d-1)*ln(1+exp(
b*x+a)*(d-1)^(1/2))*x-3/8/b^4*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*a^4-1/4/b*
d/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*x^3-1/4/b^4*d/(d-1)*polylog(2,(d-1)
*exp(2*b*x+2*a))*a^3+3/8/b^2*d/(d-1)*polylog(3,(d-1)*exp(2*b*x+2*a))*x^2-3/
8/b^3*d/(d-1)*polylog(4,(d-1)*exp(2*b*x+2*a))*x+1/2/b^4*d*a^4/(d-1)*ln(1-ex
p(b*x+a)*(d-1)^(1/2))+1/2/b^4*d*a^4/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))+1/2/
b^3/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x*a^3+1/16*I*x^4*Pi*csgn(I*exp(b*x+a))
^2*csgn(I*exp(2*b*x+2*a))-1/8*I*x^4*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+
2*a))^2+1/16*I*x^4*Pi*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^3+1/16*I*
x^4*Pi*csgn(I*exp(2*b*x+2*a))^3+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*
b*x+2*a)+1))^3+1/20*b*x^5-1/16*I*x^4*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)+
1)*exp(2*b*x+2*a))^2+1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I/(exp(2
*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))^2+1/16*I*x^4*Pi*csgn(I*(e
xp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a
)*d-exp(2*b*x+2*a)-1))^2-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a))*csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*
a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))^3-1/2/b^4*a^3/(d-1)*dilog(1+exp(
b*x+a)*(d-1)^(1/2))+1/8*x^4*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1)-1/2/b^4*a
^4/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))-1/8*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a)
)*x^4+1/16*I*x^4*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csg
n(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))-1/8*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a
)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))^2-1/8/b^4*d*a^4/(d-1)*ln(exp(2*b*
x+2*a)*d-exp(2*b*x+2*a)-1)-1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2
*a)+1))*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2+1/8/b^4*a^4/(d-1)*ln(
exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1)-1/4*x^4*ln(exp(b*x+a))-1/8*x^4*ln(d)-3/8
/b^2/(d-1)*polylog(3,(d-1)*exp(2*b*x+2*a))*x^2+3/8/b^3/(d-1)*polylog(4,(d-1
)*exp(2*b*x+2*a))*x-1/2/b^3*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x*a^3+1/2/b^
3*d*a^3/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))*x+1/2/b^3*d*a^3/(d-1)*ln(1+exp(b
*x+a)*(d-1)^(1/2))*x
```

Maxima [A] time = 3.27785, size = 197, normalized size = 1.17

$$-\frac{1}{4}x^4\operatorname{artanh}(d\tanh(bx+a)+d-1)+\frac{1}{40}\left(\frac{2x^5}{d}-\frac{5\left(2b^4x^4\log\left(-(d-1)e^{(2bx+2a)}+1\right)+4b^3x^3\mathrm{Li}_2\left((d-1)e^{(2bx+2a)}\right)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/4*x^4*arctanh(d*tanh(b*x + a) + d - 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*lo
g(-(d - 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog((d - 1)*e^(2*b*x + 2*a))
- 6*b^2*x^2*polylog(3, (d - 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, (d - 1)*
e^(2*b*x + 2*a)) - 3*polylog(5, (d - 1)*e^(2*b*x + 2*a)))/(b^5*d))*b*d
```

Fricas [C] time = 2.00922, size = 1281, normalized size = 7.62

$$2b^5x^5-5b^4x^4\log\left(-\frac{d\cosh(bx+a)+d\sinh(bx+a)}{(d-2)\cosh(bx+a)+d\sinh(bx+a)}\right)-20b^3x^3\mathrm{Li}_2\left(\sqrt{d-1}(\cosh(bx+a)+\sinh(bx+a))\right)-20b^3x^3\mathrm{Li}_2\left(-\sqrt{\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/40*(2*b^5*x^5 - 5*b^4*x^4*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/((d -
2)*cosh(b*x + a) + d*sinh(b*x + a))) - 20*b^3*x^3*dilog(sqrt(d - 1)*(cosh(b
*x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-sqrt(d - 1)*(cosh(b*x + a) +
sinh(b*x + a))) - 5*a^4*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x +
a) + 2*sqrt(d - 1)) - 5*a^4*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*
x + a) - 2*sqrt(d - 1)) + 60*b^2*x^2*polylog(3, sqrt(d - 1)*(cosh(b*x + a)
+ sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -sqrt(d - 1)*(cosh(b*x + a) + sin
h(b*x + a))) - 120*b*x*polylog(4, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a
))) - 120*b*x*polylog(4, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) - 5*
(b^4*x^4 - a^4)*log(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 5*(b
^4*x^4 - a^4)*log(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 120*p
olylog(5, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) + 120*polylog(5, -sq
rt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**3*atanh(-1+d+d*tanh(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -x^3 \operatorname{artanh}(d \tanh(bx+a) + d - 1) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x^3*arctanh(d*tanh(b*x + a) + d - 1), x)
```

3.294 $\int x^2 \tanh^{-1}(1 - d - d\tanh(a + bx))\,dx$

Optimal. Leaf size=139

$$\frac{x\text{PolyLog}\left(3,-(1-d)e^{2a+2bx}\right)}{4b^2}-\frac{\text{PolyLog}\left(4,-(1-d)e^{2a+2bx}\right)}{8b^3}-\frac{x^2\text{PolyLog}\left(2,-(1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{6}x^3\log\left((1-d)e^{2a+}\right.$$

```
[Out] (b*x^4)/12 + (x^3*ArcTanh[1 - d - d*Tanh[a + b*x]])/3 - (x^3*Log[1 + (1 - d
)*E^(2*a + 2*b*x)])/6 - (x^2*PolyLog[2, -((1 - d)*E^(2*a + 2*b*x))])/(4*b)
+ (x*PolyLog[3, -((1 - d)*E^(2*a + 2*b*x))])/(4*b^2) - PolyLog[4, -((1 - d)
*E^(2*a + 2*b*x))]/(8*b^3)
```

Rubi [A] time = 0.258435, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6239, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{x\text{PolyLog}\left(3,-(1-d)e^{2a+2bx}\right)}{4b^2}-\frac{\text{PolyLog}\left(4,-(1-d)e^{2a+2bx}\right)}{8b^3}-\frac{x^2\text{PolyLog}\left(2,-(1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{6}x^3\log\left((1-d)e^{2a+}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^4)/12 + (x^3*ArcTanh[1 - d - d*Tanh[a + b*x]])/3 - (x^3*Log[1 + (1 - d
)*E^(2*a + 2*b*x)])/6 - (x^2*PolyLog[2, -((1 - d)*E^(2*a + 2*b*x))])/(4*b)
+ (x*PolyLog[3, -((1 - d)*E^(2*a + 2*b*x))])/(4*b^2) - PolyLog[4, -((1 - d)
*E^(2*a + 2*b*x))]/(8*b^3)
```

Rule 6239

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Tanh[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
```

```
x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \tanh^{-1}(1-d-d\tanh(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(1-d-d\tanh(a+bx)) + \frac{1}{3}b\int \frac{x^3}{1+(1-d)e^{2a+2bx}}\,dx$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{3}(b(1-d))\int \frac{e^{2a+2bx}x^3}{1+(1-d)e^{2a+2b}}$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-d)e^{2a+2bx}\right) +$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-d)e^{2a+2bx}\right) -$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-d)e^{2a+2bx}\right) -$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-d)e^{2a+2bx}\right) -$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-d)e^{2a+2bx}\right) -$$

Mathematica [A] time = 5.03482, size = 119, normalized size = 0.86

$$\frac{1}{24}\left(\frac{6x\text{PolyLog}\left(3,\frac{e^{-2(a+bx)}}{d-1}\right)}{b^2} + \frac{3\text{PolyLog}\left(4,\frac{e^{-2(a+bx)}}{d-1}\right)}{b^3} + \frac{6x^2\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{d-1}\right)}{b} - 4x^3\log\left(1-\frac{e^{-2(a+bx)}}{d-1}\right) + 8x^3\tanh\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (8*x^3*ArcTanh[1 - d - d*Tanh[a + b*x]] - 4*x^3*Log[1 - 1/((-1 + d)*E^(2*(a
 + b*x)))] + (6*x^2*PolyLog[2, 1/((-1 + d)*E^(2*(a + b*x)))])/b + (6*x*Poly
Log[3, 1/((-1 + d)*E^(2*(a + b*x)))])/b^2 + (3*PolyLog[4, 1/((-1 + d)*E^(2*
(a + b*x)))])/b^3)/24
```

Maple [C] time = 13.838, size = 1716, normalized size = 12.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*arctanh(-1+d+d*tanh(b*x+a)),x)
```

```
[Out] -1/2/b^3*d*a^2/(d-1)*dilog(1-exp(b*x+a)*(d-1)^(1/2))-1/2/b^3*d*a^2/(d-1)*di
log(1+exp(b*x+a)*(d-1)^(1/2))+1/3/b^3*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*a^
3-1/4/b*d/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*x^2+1/4/b^3*d/(d-1)*polylog
(2,(d-1)*exp(2*b*x+2*a))*a^2+1/4/b^2*d/(d-1)*polylog(3,(d-1)*exp(2*b*x+2*a)
)*x-1/2/b^2/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x*a^2+1/2/b^2*a^2/(d-1)*ln(1-e
xp(b*x+a)*(d-1)^(1/2))*x-1/2/b^3*d*a^3/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))-1
/2/b^3*d*a^3/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))+1/12*I*x^3*Pi*csgn(I/(exp(2
*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1)
)^2+1/12*I*x^3*Pi*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))*csgn(I/(exp(2
*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))^2-1/3*x^3*ln(exp(b*x+a))-
1/6*x^3*ln(d)-1/6*I*x^3*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/12
*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+
1))^2+1/12*b*x^4+1/8/b^3/(d-1)*polylog(4,(d-1)*exp(2*b*x+2*a))+1/6/(d-1)*ln
(1-(d-1)*exp(2*b*x+2*a))*x^3+1/6*x^3*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1)-
1/6/b^3*a^3/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1)+1/12*I*x^3*Pi*csgn(
I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))^3+1/6*I*x^3*Pi-1/
12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d/(exp(2*b*x+2
*a)+1)*exp(2*b*x+2*a))^2+1/6/b^3*d*a^3/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2*b*x+
2*a)-1)+1/2/b^2*a^2/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))*x-1/12*I*x^3*Pi*csgn
(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/6*I*x^3*Pi
*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))^2+1/12*I*x^
3*Pi*csgn(I*exp(2*b*x+2*a))^3+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*
x+2*a)+1))^3-1/6*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x^3-1/3/b^3/(d-1)*ln(1-
(d-1)*exp(2*b*x+2*a))*a^3+1/4/b/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*x^2-1
/4/b^3/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*a^2-1/4/b^2/(d-1)*polylog(3,(d
-1)*exp(2*b*x+2*a))*x+1/2/b^3*a^3/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))+1/2/b^
3*a^3/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))-1/8/b^3*d/(d-1)*polylog(4,(d-1)*ex
p(2*b*x+2*a))+1/2/b^3*a^2/(d-1)*dilog(1-exp(b*x+a)*(d-1)^(1/2))+1/2/b^3*a^2
/(d-1)*dilog(1+exp(b*x+a)*(d-1)^(1/2))+1/12*I*x^3*Pi*csgn(I*exp(b*x+a))^2*c
sgn(I*exp(2*b*x+2*a))-1/12*I*x^3*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)+1)*e
xp(2*b*x+2*a))^2+1/12*I*x^3*Pi*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^
3+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/12*I*x^3*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2
*a)/(exp(2*b*x+2*a)+1))*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))-1/12*I*
x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1
))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))+1/2/b^2*d
/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x*a^2-1/2/b^2*d*a^2/(d-1)*ln(1-exp(b*x+a)
*(d-1)^(1/2))*x-1/2/b^2*d*a^2/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))*x
```

Maxima [A] time = 3.27003, size = 166, normalized size = 1.19

$$-\frac{1}{3}\,x^{3}\,\operatorname{artanh}\left(d\tanh\left(bx+a\right)+d-1\right)+\frac{1}{36}\left(\frac{3\,x^{4}}{d}-\frac{2\left(4\,b^{3}x^{3}\log\left(-(d-1)e^{(2\,bx+2\,a)}+1\right)+6\,b^{2}x^{2}\mathrm{Li}_{2}\left((d-1)e^{(2\,bx+2\,a)}\right)-\right.}{b^{4}d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/3*x^3*arctanh(d*tanh(b*x + a) + d - 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*lo
g(-(d - 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog((d - 1)*e^(2*b*x + 2*a))
- 6*b*x*polylog(3, (d - 1)*e^(2*b*x + 2*a)) + 3*polylog(4, (d - 1)*e^(2*b*x
 + 2*a)))/(b^4*d))*b*d
```

Fricas [C] time = 2.02436, size = 1083, normalized size = 7.79

$$b^4x^4 - 2\,b^3x^3\log\left(-\frac{d\cosh(bx+a)+d\sinh(bx+a)}{(d-2)\cosh(bx+a)+d\sinh(bx+a)}\right) - 6\,b^2x^2\mathrm{Li}_2\left(\sqrt{d-1}(\cosh(bx+a)+\sinh(bx+a))\right) - 6\,b^2x^2\mathrm{Li}_2\left(-\sqrt{d-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(b^4*x^4 - 2*b^3*x^3*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/((d - 2)
*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b^2*x^2*dilog(sqrt(d - 1)*(cosh(b*x
+ a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-sqrt(d - 1)*(cosh(b*x + a) + sinh
(b*x + a))) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a) +
 2*sqrt(d - 1)) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x +
a) - 2*sqrt(d - 1)) + 12*b*x*polylog(3, sqrt(d - 1)*(cosh(b*x + a) + sinh(b
*x + a))) + 12*b*x*polylog(3, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)))
 - 2*(b^3*x^3 + a^3)*log(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) -
 2*(b^3*x^3 + a^3)*log(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) -
12*polylog(4, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) - 12*polylog(4,
-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**2*atanh(-1+d+d*tanh(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -x^2 \operatorname{artanh}(d\tanh(bx+a)+d-1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctanh(d*tanh(b*x + a) + d - 1), x)
```

3.295 $\int x \tanh^{-1}(1-d-d\tanh(a+bx))\,dx$

Optimal. Leaf size=110

$$\frac{\text{PolyLog}\left(3,-(1-d)e^{2a+2bx}\right)}{8b^2}-\frac{x\text{PolyLog}\left(2,-(1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{4}x^2\log\left((1-d)e^{2a+2bx}+1\right)+\frac{1}{2}x^2\tanh^{-1}(d(-\tanh(a$$

```
[Out] (b*x^3)/6 + (x^2*ArcTanh[1 - d - d*Tanh[a + b*x]])/2 - (x^2*Log[1 + (1 - d)
*E^(2*a + 2*b*x)])/4 - (x*PolyLog[2, -((1 - d)*E^(2*a + 2*b*x))])/(4*b) + P
olyLog[3, -((1 - d)*E^(2*a + 2*b*x))]/(8*b^2)
```

Rubi [A] time = 0.223967, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6239, 2184, 2190, 2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3,-(1-d)e^{2a+2bx}\right)}{8b^2}-\frac{x\text{PolyLog}\left(2,-(1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{4}x^2\log\left((1-d)e^{2a+2bx}+1\right)+\frac{1}{2}x^2\tanh^{-1}(d(-\tanh(a$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcTanh[1 - d - d*Tanh[a + b*x]])/2 - (x^2*Log[1 + (1 - d)
*E^(2*a + 2*b*x)])/4 - (x*PolyLog[2, -((1 - d)*E^(2*a + 2*b*x))])/(4*b) + P
olyLog[3, -((1 - d)*E^(2*a + 2*b*x))]/(8*b^2)
```

Rule 6239

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Tanh[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(1-d-d\tanh(a+bx))\,dx = \frac{1}{2}x^2\tanh^{-1}(1-d-d\tanh(a+bx)) + \frac{1}{2}b\int \frac{x^2}{1+(1-d)e^{2a+2bx}}\,dx$$

$$= \frac{bx^3}{6} + \frac{1}{2}x^2\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{2}(b(1-d))\int \frac{e^{2a+2bx}x^2}{1+(1-d)e^{2a+2bx}}$$

$$= \frac{bx^3}{6} + \frac{1}{2}x^2\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{4}x^2\log\left(1+(1-d)e^{2a+2bx}\right) + \frac{1}{2}$$

$$= \frac{bx^3}{6} + \frac{1}{2}x^2\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{4}x^2\log\left(1+(1-d)e^{2a+2bx}\right) - \frac{x}{}$$

$$= \frac{bx^3}{6} + \frac{1}{2}x^2\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{4}x^2\log\left(1+(1-d)e^{2a+2bx}\right) - \frac{x}{}$$

$$= \frac{bx^3}{6} + \frac{1}{2}x^2\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{4}x^2\log\left(1+(1-d)e^{2a+2bx}\right) - \frac{x}{}$$

Mathematica [A] time = 5.02232, size = 93, normalized size = 0.85

$$\frac{2bx\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{d-1}\right)+\text{PolyLog}\left(3,\frac{e^{-2(a+bx)}}{d-1}\right)+2b^2x^2\left(2\tanh^{-1}(d(-\tanh(a+bx))-d+1)-\log\left(1-\frac{e^{-2(a+bx)}}{d-1}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (2*b^2*x^2*(2*ArcTanh[1 - d - d*Tanh[a + b*x]] - Log[1 - 1/((-1 + d)*E^(2*(
a + b*x)))]) + 2*b*x*PolyLog[2, 1/((-1 + d)*E^(2*(a + b*x)))] + PolyLog[3,
1/((-1 + d)*E^(2*(a + b*x)))])/(8*b^2)
```

Maple [C] time = 6.896, size = 1635, normalized size = 14.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*arctanh(-1+d+d*tanh(b*x+a)),x)
```

```
[Out] 1/4*I*x^2*Pi+1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2
*b*x+2*a)-1))^3+1/2/b^2*d*a^2/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))+1/2/b^2*d*
```

```
a/(d-1)*dilog(1-exp(b*x+a)*(d-1)^(1/2))+1/2/b^2*d*a/(d-1)*dilog(1+exp(b*x+a
)*(d-1)^(1/2))-1/4/b^2*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*a^2-1/4/b*d/(d-1)
*polylog(2,(d-1)*exp(2*b*x+2*a))*x+1/8*I*x^2*Pi*csgn(I*d/(exp(2*b*x+2*a)+1)
*exp(2*b*x+2*a))^3-1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*c
sgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2+1/6*b*x^3+1/8*I*x^2*Pi*csgn(I*
(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2
*a)*d-exp(2*b*x+2*a)-1))^2+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*
a)+1))^3-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp
(2*b*x+2*a)+1))^2-1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)
/(exp(2*b*x+2*a)+1))^2-1/8*I*x^2*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)+1)*e
xp(2*b*x+2*a))^2+1/4/b^2/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*a^2+1/4/b/(d-1)*p
olylog(2,(d-1)*exp(2*b*x+2*a))*x+1/4/b^2/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*
a))*a-1/2/b^2*a^2/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))-1/2/b^2*a^2/(d-1)*ln(1
+exp(b*x+a)*(d-1)^(1/2))+1/8/b^2*d/(d-1)*polylog(3,(d-1)*exp(2*b*x+2*a))-1/
2/b^2*a/(d-1)*dilog(1-exp(b*x+a)*(d-1)^(1/2))-1/2/b^2*a/(d-1)*dilog(1+exp(b
*x+a)*(d-1)^(1/2))-1/4*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x^2-1/4*I*x^2*Pi*
csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))^2+1/8*I*x^2*
Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/4*I*x^2*Pi*csgn(I*exp(b*x+
a))*csgn(I*exp(2*b*x+2*a))^2+1/4/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x^2-1/8/b
^2/(d-1)*polylog(3,(d-1)*exp(2*b*x+2*a))-1/4/b^2*d*a^2/(d-1)*ln(exp(2*b*x+2
*a)*d-exp(2*b*x+2*a)-1)-1/2*x^2*ln(exp(b*x+a))-1/4*x^2*ln(d)-1/4/b^2*d/(d-1
)*polylog(2,(d-1)*exp(2*b*x+2*a))*a+1/2/b/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*
x*a-1/2/b*a/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))*x-1/2/b*a/(d-1)*ln(1+exp(b*x
+a)*(d-1)^(1/2))*x+1/2/b^2*d*a^2/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))+1/4/b^2
*a^2/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1)+1/8*I*x^2*Pi*csgn(I/(exp(2
*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1)
)^2+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a))^3+1/4*x^2*ln(exp(2*b*x+2*a)*d-exp(2
*b*x+2*a)-1)-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)
*d-exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x
+2*a)-1))+1/8*I*x^2*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*
csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))+1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+
2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1
/2/b*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x*a+1/2/b*d*a/(d-1)*ln(1-exp(b*x+a)
*(d-1)^(1/2))*x+1/2/b*d*a/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))*x
```

Maxima [A] time = 3.25065, size = 135, normalized size = 1.23

$$\frac{1}{24}\left(\frac{4\,x^3}{d}-\frac{3\left(2\,b^2x^2\log\left(-(d-1)e^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left((d-1)e^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3\left((d-1)e^{(2\,bx+2\,a)}\right)\right)}{b^3d}\right)bd-\frac{1}{2}\,x^2\operatorname{artanh}(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/24*(4*x^3/d - 3*(2*b^2*x^2*log(-(d - 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dilo
g((d - 1)*e^(2*b*x + 2*a)) - polylog(3, (d - 1)*e^(2*b*x + 2*a)))/(b^3*d))*
b*d - 1/2*x^2*arctanh(d*tanh(b*x + a) + d - 1)
```

Fricas [C] time = 1.92609, size = 895, normalized size = 8.14

$$\frac{2\,b^3x^3 - 3\,b^2x^2\log\left(-\frac{d\cosh(bx+a)+d\sinh(bx+a)}{(d-2)\cosh(bx+a)+d\sinh(bx+a)}\right) - 6\,bx\mathrm{Li}_2\left(\sqrt{d-1}(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))\right) - 6\,bx\mathrm{Li}_2\left(-\sqrt{d-1}(\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 - 3*b^2*x^2*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/((d -
2)*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b*x*dilog(sqrt(d - 1)*(cosh(b*x +
a) + sinh(b*x + a))) - 6*b*x*dilog(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x +
 a))) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a) + 2*sqr
t(d - 1)) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a) - 2
*sqrt(d - 1)) - 3*(b^2*x^2 - a^2)*log(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x
 + a)) + 1) - 3*(b^2*x^2 - a^2)*log(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x
+ a)) + 1) + 6*polylog(3, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) + 6*
polylog(3, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int x\,\mathrm{atanh}\left(d\tanh\left(a+bx\right)+d-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*atanh(-1+d+d*tanh(b*x+a)),x)
```

```
[Out] -Integral(x*atanh(d*tanh(a + b*x) + d - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -x\,\mathrm{artanh}\left(d\tanh\left(bx+a\right)+d-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x*arctanh(d*tanh(b*x + a) + d - 1), x)
```

3.296 $\int \tanh^{-1}(1-d-d\tanh(a+bx))\,dx$

Optimal. Leaf size=76

$$-\frac{\text{PolyLog}\left(2,-(1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{2}x\log\left((1-d)e^{2a+2bx}+1\right)+x\tanh^{-1}(d(-\tanh(a+bx))-d+1)+\frac{bx^2}{2}$$

```
[Out] (b*x^2)/2 + x*ArcTanh[1 - d - d*Tanh[a + b*x]] - (x*Log[1 + (1 - d)*E^(2*a
+ 2*b*x)])/2 - PolyLog[2, -((1 - d)*E^(2*a + 2*b*x))]/(4*b)
```

Rubi [A] time = 0.138354, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6231, 2184, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,-(1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{2}x\log\left((1-d)e^{2a+2bx}+1\right)+x\tanh^{-1}(d(-\tanh(a+bx))-d+1)+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```


```
[Out] (b*x^2)/2 + x*ArcTanh[1 - d - d*Tanh[a + b*x]] - (x*Log[1 + (1 - d)*E^(2*a
+ 2*b*x)])/2 - PolyLog[2, -((1 - d)*E^(2*a + 2*b*x))]/(4*b)
```


Rule 6231

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Ar
cTanh[c + d*Tanh[a + b*x]], x] + Dist[b, Int[x/(c - d + c*E^(2*a + 2*b*x)),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(1-d-d\tanh(a+bx))\,dx &= x\tanh^{-1}(1-d-d\tanh(a+bx))+b\int \frac{x}{1+(1-d)e^{2a+2bx}}\,dx\\ &= \frac{bx^2}{2}+x\tanh^{-1}(1-d-d\tanh(a+bx))-(b(1-d))\int \frac{e^{2a+2bx}x}{1+(1-d)e^{2a+2bx}}\,dx\\ &= \frac{bx^2}{2}+x\tanh^{-1}(1-d-d\tanh(a+bx))-\frac{1}{2}x\log\left(1+(1-d)e^{2a+2bx}\right)+\frac{1}{2}\int \log\\ &= \frac{bx^2}{2}+x\tanh^{-1}(1-d-d\tanh(a+bx))-\frac{1}{2}x\log\left(1+(1-d)e^{2a+2bx}\right)+\frac{\text{Subst}\big(}{}\\ &= \frac{bx^2}{2}+x\tanh^{-1}(1-d-d\tanh(a+bx))-\frac{1}{2}x\log\left(1+(1-d)e^{2a+2bx}\right)-\frac{\text{Li}_2\big(-(1}{}\end{aligned}$$

Mathematica [B] time = 4.27504, size = 200, normalized size = 2.63

$$\frac{-2\text{PolyLog}\left(2,-\sqrt{d-1}e^{a+bx}\right)-2\text{PolyLog}\left(2,\sqrt{d-1}e^{a+bx}\right)-2\log\left(e^{a+bx}\right)\log\left(1-\sqrt{d-1}e^{a+bx}\right)-2\log\left(e^{a+bx}\right)\log\left(\sqrt{d}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] x*ArcTanh[1 - d - d*Tanh[a + b*x]] + (b^2*x^2 + Log[E^(a + b*x)]^2 - 2*Log[
E^(a + b*x)]*Log[1 - Sqrt[-1 + d]*E^(a + b*x)] - 2*Log[E^(a + b*x)]*Log[1 +
 Sqrt[-1 + d]*E^(a + b*x)] + 2*Log[E^(a + b*x)]*Log[E^(-a - b*x)*(-1 + (-1
+ d)*E^(2*(a + b*x)))] - 2*b*x*Log[(-2 + d)*Cosh[a + b*x] + d*Sinh[a + b*x]
```

```
] - 2*PolyLog[2, -(Sqrt[-1 + d]*E^(a + b*x))] - 2*PolyLog[2, Sqrt[-1 + d]*E
^(a + b*x)])/(4*b)
```

Maple [B] time = 0.147, size = 280, normalized size = 3.7

$$\frac{\operatorname{Artanh}\left(-1+d+d\tanh\left(bx+a\right)\right)\ln\left(d\tanh\left(bx+a\right)-d\right)}{2\,b}-\frac{\operatorname{Artanh}\left(-1+d+d\tanh\left(bx+a\right)\right)\ln\left(d\tanh\left(bx+a\right)+\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctanh(-1+d+d*tanh(b*x+a)),x)
```

```
[Out] 1/2/b*arctanh(-1+d+d*tanh(b*x+a))*ln(d*tanh(b*x+a)-d)-1/2/b*arctanh(-1+d+d*
tanh(b*x+a))*ln(d*tanh(b*x+a)+d)+1/4/b*ln(-1/2*d*tanh(b*x+a)-1/2*d+1)*ln(1/
2*d*tanh(b*x+a)+1/2*d)-1/4/b*ln(-1/2*d*tanh(b*x+a)-1/2*d+1)*ln(d*tanh(b*x+a
)+d)+1/4/b*dilog(1/2*d*tanh(b*x+a)+1/2*d)+1/8/b*ln(d*tanh(b*x+a)+d)^2-1/4/b
*dilog(1/2*(d*tanh(b*x+a)+d)/d)-1/4/b*ln(d*tanh(b*x+a)-d)*ln(1/2*(d*tanh(b*
x+a)+d)/d)+1/4/b*dilog((d*tanh(b*x+a)+d-2)/(2*d-2))+1/4/b*ln(d*tanh(b*x+a)-
d)*ln((d*tanh(b*x+a)+d-2)/(2*d-2))
```

Maxima [A] time = 3.28686, size = 99, normalized size = 1.3

$$\frac{1}{4}\,bd\left(\frac{2\,x^2}{d}-\frac{2\,bx\log\left(-(d-1)e^{(2\,bx+2\,a)}+1\right)+\operatorname{Li}_2\left((d-1)e^{(2\,bx+2\,a)}\right)}{b^2d}\right)-x\operatorname{artanh}\left(d\tanh\left(bx+a\right)+d-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*b*d*(2*x^2/d - (2*b*x*log(-(d - 1)*e^(2*b*x + 2*a) + 1) + dilog((d - 1)
*e^(2*b*x + 2*a)))/(b^2*d)) - x*arctanh(d*tanh(b*x + a) + d - 1)
```

Fricas [B] time = 1.96903, size = 668, normalized size = 8.79

$$b^2x^2-bx\log\left(-\frac{d\cosh(bx+a)+d\sinh(bx+a)}{(d-2)\cosh(bx+a)+d\sinh(bx+a)}\right)+a\log\left(2\,(d-1)\cosh\left(bx+a\right)+2\,(d-1)\sinh\left(bx+a\right)+2\,\sqrt{d-1}\right)+a\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - b*x*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/((d - 2)*cosh(b
*x + a) + d*sinh(b*x + a))) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sin
h(b*x + a) + 2*sqrt(d - 1)) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sin
h(b*x + a) - 2*sqrt(d - 1)) - (b*x + a)*log(sqrt(d - 1)*(cosh(b*x + a) + si
nh(b*x + a)) + 1) - (b*x + a)*log(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x +
a)) + 1) - dilog(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) - dilog(-sqrt
(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \operatorname{atanh}\left(d\tanh\left(a+bx\right)+d-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atanh(-1+d+d*tanh(b*x+a)),x)
```

```
[Out] -Integral(atanh(d*tanh(a + b*x) + d - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\operatorname{artanh}\left(d\tanh\left(bx+a\right)+d-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(d*tanh(b*x + a) + d - 1), x)
```

3.297 $\int \frac{\tanh^{-1}(1-d-d\tanh(a+bx))}{x}\,dx$

Optimal. Leaf size=21

$$\text{CannotIntegrate}\left(\frac{\tanh^{-1}(d(-\tanh(a+bx))-d+1)}{x},x\right)$$

```
[Out] CannotIntegrate[ArcTanh[1 - d - d*Tanh[a + b*x]]/x, x]
```

Rubi [A] time = 0.0667474, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tanh^{-1}(1-d-d\tanh(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTanh[1 - d - d*Tanh[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTanh[1 - d - d*Tanh[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(1-d-d\tanh(a+bx))}{x}\,dx=\int \frac{\tanh^{-1}(1-d-d\tanh(a+bx))}{x}\,dx$$

Mathematica [A] time = 5.20367, size = 0, normalized size = 0.

$$\int \frac{\tanh^{-1}(1-d-d\tanh(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTanh[1 - d - d*Tanh[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcTanh[1 - d - d*Tanh[a + b*x]]/x, x]
```

Maple [A] time = 0.329, size = 0, normalized size = 0.

$$\int -\frac{\operatorname{Artanh}\left(-1+d+d\tanh\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctanh(-1+d+d*tanh(b*x+a))/x,x)
```

```
[Out] int(-arctanh(-1+d+d*tanh(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\operatorname{artanh}\left(d\tanh\left(bx+a\right)+d-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*tanh(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -integrate(arctanh(d*tanh(b*x + a) + d - 1)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\operatorname{artanh}\left(d\tanh\left(bx+a\right)+d-1\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*tanh(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(-arctanh(d*tanh(b*x + a) + d - 1)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\operatorname{atanh}\left(d\tanh\left(a+bx\right)+d-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atanh(-1+d+d*tanh(b*x+a))/x,x)
```

```
[Out] -Integral(atanh(d*tanh(a + b*x) + d - 1)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\operatorname{artanh}\left(d\tanh\left(bx+a\right)+d-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*tanh(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(-arctanh(d*tanh(b*x + a) + d - 1)/x, x)
```

3.298 $\int x^2 \tanh^{-1}(c + d\coth(a + bx))\,dx$

Optimal. Leaf size=303

$$-\frac{x\text{PolyLog}\left(3,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2}+\frac{x\text{PolyLog}\left(3,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2}+\frac{\text{PolyLog}\left(4,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^3}-\frac{\text{PolyLog}\left(4,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^3}$$

```
[Out] (x^3*ArcTanh[c + d*Coth[a + b*x]])/3 + (x^3*Log[1 - ((1 - c - d)*E^(2*a + 2
*b*x))/(1 - c + d)])/6 - (x^3*Log[1 - ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c
- d)])/6 + (x^2*PolyLog[2, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(4*b
) - (x^2*PolyLog[2, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(4*b) - (x*
PolyLog[3, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(4*b^2) + (x*PolyLog
[3, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(4*b^2) + PolyLog[4, ((1 -
c - d)*E^(2*a + 2*b*x))/(1 - c + d)]/(8*b^3) - PolyLog[4, ((1 + c + d)*E^(2
*a + 2*b*x))/(1 + c - d)]/(8*b^3)
```

Rubi [A] time = 0.456686, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6245, 2190, 2531, 6609, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(3,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2}+\frac{x\text{PolyLog}\left(3,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2}+\frac{\text{PolyLog}\left(4,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^3}-\frac{\text{PolyLog}\left(4,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[c + d*Coth[a + b*x]],x]
```


```
[Out] (x^3*ArcTanh[c + d*Coth[a + b*x]])/3 + (x^3*Log[1 - ((1 - c - d)*E^(2*a + 2
*b*x))/(1 - c + d)])/6 - (x^3*Log[1 - ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c
- d)])/6 + (x^2*PolyLog[2, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(4*b
) - (x^2*PolyLog[2, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(4*b) - (x*
PolyLog[3, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(4*b^2) + (x*PolyLog
[3, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(4*b^2) + PolyLog[4, ((1 -
c - d)*E^(2*a + 2*b*x))/(1 - c + d)]/(8*b^3) - PolyLog[4, ((1 + c + d)*E^(2
*a + 2*b*x))/(1 + c - d)]/(8*b^3)
```


Rule 6245

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Coth[a + b*x]])/(f*
```

```
(m + 1)), x] + (-Dist[(b*(1 - c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E
^(2*a + 2*b*x))/(1 - c + d - (1 - c - d)*E^(2*a + 2*b*x)), x], x] + Dist[(b
*(1 + c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*a + 2*b*x))/(1 + c -
 d - (1 + c + d)*E^(2*a + 2*b*x)), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \tanh^{-1}(c+d\coth(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(c+d\coth(a+bx)) - \frac{1}{3}(b(1-c-d))\int \frac{e^{2a+2bx}x^3}{1-c+d+(-1+c+d)e^{2a+2b}}$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{6}x^3\log\left(1-\right.$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{6}x^3\log\left(1-\right.$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{6}x^3\log\left(1-\right.$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{6}x^3\log\left(1-\right.$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{6}x^3\log\left(1-\right.$$

Mathematica [A] time = 10.0535, size = 353, normalized size = 1.17

$$-6b^2x^2\text{PolyLog}\left(2,\frac{(c-d-1)(\cosh(2(a+bx))-\sinh(2(a+bx)))}{c+d-1}\right)+6b^2x^2\text{PolyLog}\left(2,\frac{(c-d+1)(\cosh(2(a+bx))-\sinh(2(a+bx)))}{c+d+1}\right)-6bx\text{PolyLog}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*ArcTanh[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[c + d*Coth[a + b*x]])/3 + (4*b^3*x^3*Log[(2*(Cosh[a + b*x] - S
inh[a + b*x])*(d*Cosh[a + b*x] + (-1 + c)*Sinh[a + b*x]))/(-1 + c + d)] - 4
*b^3*x^3*Log[1 + ((1 + c - d)*(-Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)]))/(1
+ c + d)] - 6*b^2*x^2*PolyLog[2, ((-1 + c - d)*(Cosh[2*(a + b*x)] - Sinh[2*
(a + b*x)]))/(-1 + c + d)] + 6*b^2*x^2*PolyLog[2, ((1 + c - d)*(Cosh[2*(a +
 b*x)] - Sinh[2*(a + b*x)]))/(1 + c + d)] - 6*b*x*PolyLog[3, ((-1 + c - d)*
(Cosh[2*(a + b*x)] - Sinh[2*(a + b*x)]))/(-1 + c + d)] + 6*b*x*PolyLog[3, (
(1 + c - d)*(Cosh[2*(a + b*x)] - Sinh[2*(a + b*x)]))/(1 + c + d)] - 3*PolyL
og[4, ((-1 + c - d)*(Cosh[2*(a + b*x)] - Sinh[2*(a + b*x)]))/(-1 + c + d)]
+ 3*PolyLog[4, ((1 + c - d)*(Cosh[2*(a + b*x)] - Sinh[2*(a + b*x)]))/(1 + c
 + d)])/(24*b^3)
```

Maple [C] time = 3.53, size = 5294, normalized size = 17.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(c+d*coth(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [A] time = 2.12903, size = 374, normalized size = 1.23

$$\frac{1}{3}x^3 \operatorname{artanh}\left(d\coth\left(bx+a\right)+c\right)-\frac{1}{18}bd\left(\frac{4b^3x^3\log\left(-\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}+1\right)+6b^2x^2\mathrm{Li}_2\left(\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right)-6bx\mathrm{Li}_3\left(\frac{(c+d+1}{c-}\right.}{b^4d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctanh(d*coth(b*x + a) + c) - 1/18*b*d*((4*b^3*x^3*log(-(c + d + 1
)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 6*b^2*x^2*dilog((c + d + 1)*e^(2*b*x +
 2*a)/(c - d + 1)) - 6*b*x*polylog(3, (c + d + 1)*e^(2*b*x + 2*a)/(c - d +
1)) + 3*polylog(4, (c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^4*d) - (4*b
^3*x^3*log(-(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1) + 1) + 6*b^2*x^2*dilog(
(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)) - 6*b*x*polylog(3, (c + d - 1)*e^(
2*b*x + 2*a)/(c - d - 1)) + 3*polylog(4, (c + d - 1)*e^(2*b*x + 2*a)/(c - d
 - 1)))/(b^4*d))
```

Fricas [C] time = 2.57225, size = 2561, normalized size = 8.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(b^3*x^3*log(-(d*cosh(b*x + a) + (c + 1)*sinh(b*x + a))/(d*cosh(b*x + a
) + (c - 1)*sinh(b*x + a))) - 3*b^2*x^2*dilog(sqrt((c + d + 1)/(c - d + 1))
*(cosh(b*x + a) + sinh(b*x + a))) - 3*b^2*x^2*dilog(-sqrt((c + d + 1)/(c -
d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(sqrt((c + d - 1)
/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(-sqrt((c +
 d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + a^3*log(2*(c + d +
1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt((c + d
+ 1)/(c - d + 1))) + a^3*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*si
nh(b*x + a) - 2*(c - d + 1)*sqrt((c + d + 1)/(c - d + 1))) - a^3*log(2*(c +
 d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt((c
 + d - 1)/(c - d - 1))) - a^3*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d -
1)*sinh(b*x + a) - 2*(c - d - 1)*sqrt((c + d - 1)/(c - d - 1))) + 6*b*x*pol
ylog(3, sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 6*
b*x*polylog(3, -sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a
))) - 6*b*x*polylog(3, sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(
b*x + a))) - 6*b*x*polylog(3, -sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a)
 + sinh(b*x + a))) - (b^3*x^3 + a^3)*log(sqrt((c + d + 1)/(c - d + 1))*(cos
h(b*x + a) + sinh(b*x + a)) + 1) - (b^3*x^3 + a^3)*log(-sqrt((c + d + 1)/(c
 - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b^3*x^3 + a^3)*log(sqrt(
(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b^3*x^3 +
a^3)*log(-sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1
) - 6*polylog(4, sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x +
a))) - 6*polylog(4, -sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*
x + a))) + 6*polylog(4, sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh
(b*x + a))) + 6*polylog(4, -sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) +
sinh(b*x + a))))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(c+d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{artanh}(d \coth(bx + a) + c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(d*coth(b*x + a) + c), x)
```

3.299 $\int x \tanh^{-1}(c + d\coth(a + bx))\,dx$

Optimal. Leaf size=229

$$-\frac{\text{PolyLog}\left(3,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^2}+\frac{\text{PolyLog}\left(3,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^2}+\frac{x\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{x\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}$$

```
[Out] (x^2*ArcTanh[c + d*Coth[a + b*x]])/2 + (x^2*Log[1 - ((1 - c - d)*E^(2*a + 2
*b*x))/(1 - c + d)])/4 - (x^2*Log[1 - ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c
- d)])/4 + (x*PolyLog[2, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(4*b)
- (x*PolyLog[2, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(4*b) - PolyLog
[3, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)]/(8*b^2) + PolyLog[3, ((1 + c
 + d)*E^(2*a + 2*b*x))/(1 + c - d)]/(8*b^2)
```

Rubi [A] time = 0.371601, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6245, 2190, 2531, 2282, 6589}

$$-\frac{\text{PolyLog}\left(3,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^2}+\frac{\text{PolyLog}\left(3,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^2}+\frac{x\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{x\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[c + d*Coth[a + b*x]])/2 + (x^2*Log[1 - ((1 - c - d)*E^(2*a + 2
*b*x))/(1 - c + d)])/4 - (x^2*Log[1 - ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c
- d)])/4 + (x*PolyLog[2, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(4*b)
- (x*PolyLog[2, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(4*b) - PolyLog
[3, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)]/(8*b^2) + PolyLog[3, ((1 + c
 + d)*E^(2*a + 2*b*x))/(1 + c - d)]/(8*b^2)
```

Rule 6245

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Coth[a + b*x]])/(f*
(m + 1)), x] + (-Dist[(b*(1 - c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E
^(2*a + 2*b*x))/(1 - c + d - (1 - c - d)*E^(2*a + 2*b*x)), x], x] + Dist[(b
*(1 + c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*a + 2*b*x))/(1 + c -
 d - (1 + c + d)*E^(2*a + 2*b*x)), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
```

```
&& IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(c+d\coth(a+bx))\,dx = \frac{1}{2}x^2\tanh^{-1}(c+d\coth(a+bx)) - \frac{1}{2}(b(1-c-d))\int \frac{e^{2a+2bx}x^2}{1-c+d+(-1+c+d)e^{2a+2bx}}$$

$$= \frac{1}{2}x^2\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{4}x^2\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{4}x^2\log\left(1-\right.$$

$$= \frac{1}{2}x^2\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{4}x^2\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{4}x^2\log\left(1-\right.$$

$$= \frac{1}{2}x^2\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{4}x^2\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{4}x^2\log\left(1-\right.$$

$$= \frac{1}{2}x^2\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{4}x^2\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{4}x^2\log\left(1-\right.$$

Mathematica [A] time = 8.6012, size = 267, normalized size = 1.17

$$\frac{-2bx\text{PolyLog}\left(2,\frac{(c-d-1)(\cosh(2(a+bx))-\sinh(2(a+bx)))}{c+d-1}\right)+2bx\text{PolyLog}\left(2,\frac{(c-d+1)(\cosh(2(a+bx))-\sinh(2(a+bx)))}{c+d+1}\right)-\text{PolyLog}\left(3,\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*ArcTanh[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[c + d*Coth[a + b*x]])/2 + (2*b^2*x^2*Log[(2*(Cosh[a + b*x] - S
inh[a + b*x])*(d*Cosh[a + b*x] + (-1 + c)*Sinh[a + b*x]))/(-1 + c + d)] - 2
*b^2*x^2*Log[1 + ((1 + c - d)*(-Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)]))/(1
+ c + d)] - 2*b*x*PolyLog[2, ((-1 + c - d)*(Cosh[2*(a + b*x)] - Sinh[2*(a +
 b*x)]))/(-1 + c + d)] + 2*b*x*PolyLog[2, ((1 + c - d)*(Cosh[2*(a + b*x)] -
 Sinh[2*(a + b*x)]))/(1 + c + d)] - PolyLog[3, ((-1 + c - d)*(Cosh[2*(a + b
*x)] - Sinh[2*(a + b*x)]))/(-1 + c + d)] + PolyLog[3, ((1 + c - d)*(Cosh[2*
(a + b*x)] - Sinh[2*(a + b*x)]))/(1 + c + d)])/(8*b^2)
```

Maple [C] time = 5.571, size = 4990, normalized size = 21.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(c+d*coth(b*x+a)),x)
```

```
[Out] 1/2/b^2*a^2*c/(1+c+d)*ln((c*exp(b*x+a)+exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)
+exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))+1/2/b^2*a^2*d/(1+c+d)*ln((-c*exp(b*x+
a)-exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2)
)+1/2/b^2*a^2*d/(1+c+d)*ln((c*exp(b*x+a)+exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/
2)+exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))+1/2/b*a/(1+c+d)*ln((-c*exp(b*x+a)-e
xp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))*x-
1/8*I*Pi*x^2*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d-exp(2*b*x+2*
a)+1))*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d-exp(2*b*x+2*a)+1)/
(exp(2*b*x+2*a)-1))^2+1/8*I*Pi*x^2*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+
2*a)+1)*d+exp(2*b*x+2*a)-1))*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1
)*d+exp(2*b*x+2*a)-1)/(exp(2*b*x+2*a)-1))^2-1/4/b^2*a^2/(c+d-1)*ln(exp(2*b*
x+2*a)*c+exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-c+d+1)-1/2/b*d/(1+c+d)*ln(1-(1+c+d
)*exp(2*b*x+2*a)/(1+c-d))*x*a+1/2/b^2*a^2/(c+d-1)*ln((c*exp(b*x+a)+exp(b*x+
a)*d+((c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))+1/4*c/(c+
d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*x^2+1/4*d/(c+d-1)*ln(1-(c+d-1)*ex
p(2*b*x+2*a)/(c-d-1))*x^2-1/8/b^2*c/(c+d-1)*polylog(3,(c+d-1)*exp(2*b*x+2*a
)/(c-d-1))-1/8/b^2*d/(c+d-1)*polylog(3,(c+d-1)*exp(2*b*x+2*a)/(c-d-1))+1/2/
b^2*a/(c+d-1)*dilog((-c*exp(b*x+a)-exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)+exp
(b*x+a))/((c-d-1)*(c+d-1))^(1/2))+1/2/b^2*a/(c+d-1)*dilog((c*exp(b*x+a)+exp
(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))-1/4/
b^2/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*a^2-1/4/b/(c+d-1)*polylog(
2,(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*x-1/4/b^2/(c+d-1)*polylog(2,(c+d-1)*exp(2
*b*x+2*a)/(c-d-1))*a+1/2/b^2*a^2/(c+d-1)*ln((-c*exp(b*x+a)-exp(b*x+a)*d+((c
-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))+1/4*x^2*ln((exp(2
*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d+exp(2*b*x+2*a)-1)-1/4/(c+d-1)*ln(1-(c+d
-1)*exp(2*b*x+2*a)/(c-d-1))*x^2+1/8/b^2/(c+d-1)*polylog(3,(c+d-1)*exp(2*b*x
+2*a)/(c-d-1))+1/8*I*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*((exp(2*b*x+2
*a)-1)*c+(exp(2*b*x+2*a)+1)*d-exp(2*b*x+2*a)+1))*csgn(I*((exp(2*b*x+2*a)-1)
*c+(exp(2*b*x+2*a)+1)*d-exp(2*b*x+2*a)+1)/(exp(2*b*x+2*a)-1))-1/8*I*Pi*x^2*
csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*
d+exp(2*b*x+2*a)-1))*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d+exp(
2*b*x+2*a)-1)/(exp(2*b*x+2*a)-1))-1/4/b^2*a^2/(1+c+d)*ln(exp(2*b*x+2*a)*c+e
xp(2*b*x+2*a)*d+exp(2*b*x+2*a)-c+d-1)+1/2/b*a/(1+c+d)*ln((c*exp(b*x+a)+exp(
b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))*x-1/4
/b*d/(1+c+d)*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*x-1/4/b*c/(1+c+d)*po
lylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*x-1/2/b/(1+c+d)*ln(1-(1+c+d)*exp(2*
b*x+2*a)/(1+c-d))*x*a+1/2/b^2*a*c/(1+c+d)*dilog((-c*exp(b*x+a)-exp(b*x+a)*d
+((1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))+1/2/b^2*a*c/(
1+c+d)*dilog((c*exp(b*x+a)+exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))
/((1+c-d)*(1+c+d))^(1/2))+1/2/b^2*a*d/(1+c+d)*dilog((-c*exp(b*x+a)-exp(b*x+
a)*d+((1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))+1/2/b^2*a
*d/(1+c+d)*dilog((c*exp(b*x+a)+exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)+exp(b*x
```

+a))/((1+c-d)*(1+c+d))^(1/2))-1/4/b^2*d/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)
/(1+c-d))*a^2-1/4/b^2*d/(1+c+d)*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*a
-1/4/b^2*c/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*a^2-1/4/b^2*c/(1+c+
d)*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*a+1/2/b^2*a^2*c/(1+c+d)*ln((-c
*exp(b*x+a)-exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1+c-d)*(1+c+
d))^(1/2))-1/4*d/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*x^2-1/4*c/(1+
c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*x^2-1/4/b/(1+c+d)*polylog(2,(1+c+
d)*exp(2*b*x+2*a)/(1+c-d))*x+1/2/b^2*a^2/(1+c+d)*ln((-c*exp(b*x+a)-exp(b*x+
a)*d+((1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))+1/2/b^2*a
^2/(1+c+d)*ln((c*exp(b*x+a)+exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)+exp(b*x+a)
)/((1+c-d)*(1+c+d))^(1/2))-1/4/b^2/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c
-d))*a^2-1/4/b^2/(1+c+d)*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*a+1/2/b^
2*a/(1+c+d)*dilog((c*exp(b*x+a)+exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)+exp(b*
x+a))/((1+c-d)*(1+c+d))^(1/2))+1/8/b^2*c/(1+c+d)*polylog(3,(1+c+d)*exp(2*b*
x+2*a)/(1+c-d))+1/8/b^2*d/(1+c+d)*polylog(3,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))
+1/2/b^2*a/(1+c+d)*dilog((-c*exp(b*x+a)-exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2
)-exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))-1/4*I*Pi*x^2-1/8*I*Pi*x^2*csgn(I/(ex
p(2*b*x+2*a)-1))*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d-exp(2*b*
x+2*a)+1)/(exp(2*b*x+2*a)-1))^2-1/8*I*Pi*x^2*csgn(I*((exp(2*b*x+2*a)-1)*c+(
exp(2*b*x+2*a)+1)*d-exp(2*b*x+2*a)+1)/(exp(2*b*x+2*a)-1))^3-1/8*I*Pi*x^2*cs
gn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d+exp(2*b*x+2*a)-1)/(exp(2*b*
x+2*a)-1))^3+1/4*I*Pi*x^2*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d
-exp(2*b*x+2*a)+1)/(exp(2*b*x+2*a)-1))^2-1/2/b*c/(1+c+d)*ln(1-(1+c+d)*exp(2
*b*x+2*a)/(1+c-d))*x*a+1/2/b*a*c/(1+c+d)*ln((-c*exp(b*x+a)-exp(b*x+a)*d+((1
+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))*x+1/2/b*a*c/(1+c+
d)*ln((c*exp(b*x+a)+exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/((1+c-
d)*(1+c+d))^(1/2))*x+1/2/b*a*d/(1+c+d)*ln((-c*exp(b*x+a)-exp(b*x+a)*d+((1+c
-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))*x+1/2/b*a*d/(1+c+d)
*ln((c*exp(b*x+a)+exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/((1+c-d)
*(1+c+d))^(1/2))*x+1/2/b*c/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*x*a
+1/2/b*d/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*x*a-1/2/b*c*a/(c+d-1)
*ln((-c*exp(b*x+a)-exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/((c-d-1
)*(c+d-1))^(1/2))*x-1/2/b*c*a/(c+d-1)*ln((c*exp(b*x+a)+exp(b*x+a)*d+((c-d-1
)*(c+d-1))^(1/2)-exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))*x-1/2/b*d*a/(c+d-1)*l
n((-c*exp(b*x+a)-exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/((c-d-1)*
(c+d-1))^(1/2))*x-1/2/b*d*a/(c+d-1)*ln((c*exp(b*x+a)+exp(b*x+a)*d+((c-d-1)*
(c+d-1))^(1/2)-exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))*x-1/4/b^2*a^2*c/(1+c+d)
*ln(exp(2*b*x+2*a)*c+exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-c+d-1)-1/4/b^2*a^2*d/(
1+c+d)*ln(exp(2*b*x+2*a)*c+exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-c+d-1)-1/4*x^2*l
n((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d-exp(2*b*x+2*a)+1)-1/4/(1+c+d)*l
n(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*x^2+1/8/b^2/(1+c+d)*polylog(3,(1+c+d)*e
xp(2*b*x+2*a)/(1+c-d))+1/4/b^2*a^2*c/(c+d-1)*ln(exp(2*b*x+2*a)*c+exp(2*b*x+
2*a)*d-exp(2*b*x+2*a)-c+d+1)+1/4/b^2*d*a^2/(c+d-1)*ln(exp(2*b*x+2*a)*c+exp(
2*b*x+2*a)*d-exp(2*b*x+2*a)-c+d+1)+1/8*I*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*
csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d+exp(2*b*x+2*a)-1)/(exp(2*

```
b*x+2*a)-1))^2-1/2/b^2*c*a^2/(c+d-1)*ln((-c*exp(b*x+a)-exp(b*x+a)*d+((c-d-1
)*(c+d-1))^(1/2)+exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))-1/2/b^2*c*a^2/(c+d-1)
*ln((c*exp(b*x+a)+exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/((c-d-1)
*(c+d-1))^(1/2))-1/2/b^2*d*a^2/(c+d-1)*ln((-c*exp(b*x+a)-exp(b*x+a)*d+((c-d
-1)*(c+d-1))^(1/2)+exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))-1/2/b^2*d*a^2/(c+d-
1)*ln((c*exp(b*x+a)+exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/((c-d-
1)*(c+d-1))^(1/2))-1/2/b^2*c*a/(c+d-1)*dilog((-c*exp(b*x+a)-exp(b*x+a)*d+((
c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))-1/2/b^2*c*a/(c+d
-1)*dilog((c*exp(b*x+a)+exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/((
c-d-1)*(c+d-1))^(1/2))-1/2/b^2*d*a/(c+d-1)*dilog((-c*exp(b*x+a)-exp(b*x+a)*
d+((c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))-1/2/b^2*d*a/
(c+d-1)*dilog((c*exp(b*x+a)+exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)-exp(b*x+a)
)/((c-d-1)*(c+d-1))^(1/2))+1/4/b^2*c/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(c
-d-1))*a^2+1/4/b*c/(c+d-1)*polylog(2,(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*x+1/4/
b^2*c/(c+d-1)*polylog(2,(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*a+1/4/b^2*d/(c+d-1)
*ln(1-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*a^2+1/4/b*d/(c+d-1)*polylog(2,(c+d-1)
*exp(2*b*x+2*a)/(c-d-1))*x+1/4/b^2*d/(c+d-1)*polylog(2,(c+d-1)*exp(2*b*x+2*
a)/(c-d-1))*a-1/2/b/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*x*a+1/2/b*
a/(c+d-1)*ln((-c*exp(b*x+a)-exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)+exp(b*x+a)
)/((c-d-1)*(c+d-1))^(1/2))*x+1/2/b*a/(c+d-1)*ln((c*exp(b*x+a)+exp(b*x+a)*d+
((c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))*x
```

Maxima [A] time = 2.13306, size = 288, normalized size = 1.26

$$-\frac{1}{8}\,bd\left(\frac{2\,b^2x^2\log\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}+1\right)+2\,bx\mathrm{Li}_2\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)-\mathrm{Li}_3(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1})}{b^3d}-\frac{2\,b^2x^2\log\left(-\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}+1\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*b*d*((2*b^2*x^2*log(-(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 2*
b*x*dilog((c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)) - polylog(3, (c + d + 1)
*e^(2*b*x + 2*a)/(c - d + 1)))/(b^3*d) - (2*b^2*x^2*log(-(c + d - 1)*e^(2*b
*x + 2*a)/(c - d - 1) + 1) + 2*b*x*dilog((c + d - 1)*e^(2*b*x + 2*a)/(c - d
 - 1)) - polylog(3, (c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)))/(b^3*d)) + 1/
2*x^2*arctanh(d*coth(b*x + a) + c)
```

Fricas [C] time = 2.48163, size = 2094, normalized size = 9.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^2*log(-(d*cosh(b*x + a) + (c + 1)*sinh(b*x + a))/(d*cosh(b*x + a
) + (c - 1)*sinh(b*x + a))) - 2*b*x*dilog(sqrt((c + d + 1)/(c - d + 1))*(co
sh(b*x + a) + sinh(b*x + a))) - 2*b*x*dilog(-sqrt((c + d + 1)/(c - d + 1))*
(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(sqrt((c + d - 1)/(c - d - 1)
)*(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(-sqrt((c + d - 1)/(c - d -
 1))*(cosh(b*x + a) + sinh(b*x + a))) - a^2*log(2*(c + d + 1)*cosh(b*x + a)
 + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt((c + d + 1)/(c - d + 1)
)) - a^2*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) - 2*
(c - d + 1)*sqrt((c + d + 1)/(c - d + 1))) + a^2*log(2*(c + d - 1)*cosh(b*x
 + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt((c + d - 1)/(c - d
 - 1))) + a^2*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a)
 - 2*(c - d - 1)*sqrt((c + d - 1)/(c - d - 1))) - (b^2*x^2 - a^2)*log(sqrt(
(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b^2*x^2 -
a^2)*log(-sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1
) + (b^2*x^2 - a^2)*log(sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh
(b*x + a)) + 1) + (b^2*x^2 - a^2)*log(-sqrt((c + d - 1)/(c - d - 1))*(cosh(
b*x + a) + sinh(b*x + a)) + 1) + 2*polylog(3, sqrt((c + d + 1)/(c - d + 1))
*(cosh(b*x + a) + sinh(b*x + a))) + 2*polylog(3, -sqrt((c + d + 1)/(c - d +
 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*polylog(3, sqrt((c + d - 1)/(c -
d - 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*polylog(3, -sqrt((c + d - 1)/(
c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(c+d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{artanh}\left(d\coth\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(d*coth(b*x + a) + c), x)
```

3.300 $\int \tanh^{-1}(c + d\coth(a + bx))\,dx$

Optimal. Leaf size=150

$$\frac{\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}+\frac{1}{2}x\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)-\frac{1}{2}x\log\left(1-\frac{(c+d+1)e^{2a}}{c-d+1}\right.$$

```
[Out] x*ArcTanh[c + d*Coth[a + b*x]] + (x*Log[1 - ((1 - c - d)*E^(2*a + 2*b*x))/(
1 - c + d)])/2 - (x*Log[1 - ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/2 +
 PolyLog[2, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)]/(4*b) - PolyLog[2, (
(1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)]/(4*b)
```

Rubi [A] time = 0.227116, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6237, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}+\frac{1}{2}x\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)-\frac{1}{2}x\log\left(1-\frac{(c+d+1)e^{2a}}{c-d+1}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[c + d*Coth[a + b*x]],x]
```

```
[Out] x*ArcTanh[c + d*Coth[a + b*x]] + (x*Log[1 - ((1 - c - d)*E^(2*a + 2*b*x))/(
1 - c + d)])/2 - (x*Log[1 - ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/2 +
 PolyLog[2, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)]/(4*b) - PolyLog[2, (
(1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)]/(4*b)
```

Rule 6237

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Ar
cTanh[c + d*Coth[a + b*x]], x] + (-Dist[b*(1 - c - d), Int[(x*E^(2*a + 2*b*
x))/(1 - c + d - (1 - c - d)*E^(2*a + 2*b*x)), x], x] + Dist[b*(1 + c + d),
 Int[(x*E^(2*a + 2*b*x))/(1 + c - d - (1 + c + d)*E^(2*a + 2*b*x)), x], x])
 /; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, 1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \tanh^{-1}(c+d\coth(a+bx))\,dx = x\tanh^{-1}(c+d\coth(a+bx)) - (b(1-c-d))\int \frac{e^{2a+2bx}x}{1-c+d+(-1+c+d)e^{2a+2bx}}\,dx$$

$$= x\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{2}x\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{2}x\log\left(1-\frac{(1+c}{1}\right.$$

$$= x\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{2}x\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{2}x\log\left(1-\frac{(1+c}{1}\right.$$

$$= x\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{2}x\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{2}x\log\left(1-\frac{(1+c}{1}\right.$$

Mathematica [A] time = 4.78232, size = 131, normalized size = 0.87

$$x\tanh^{-1}(d\coth(a+bx)+c) - \frac{-\text{PolyLog}\left(2,\frac{(c+d-1)e^{2(a+bx)}}{c-d-1}\right)+\text{PolyLog}\left(2,\frac{(c+d+1)e^{2(a+bx)}}{c-d+1}\right)-2bx\left(\log\left(1-\frac{(c+d-1)e^{2(a+bx)}}{c-d-1}\right)\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[c + d*Coth[a + b*x]],x]
```

```
[Out] x*ArcTanh[c + d*Coth[a + b*x]] - (-2*b*x*(Log[1 - ((-1 + c + d)*E^(2*(a + b
*x)))/(-1 + c - d)] - Log[1 - ((1 + c + d)*E^(2*(a + b*x)))/(1 + c - d)]) -
 PolyLog[2, ((-1 + c + d)*E^(2*(a + b*x)))/(-1 + c - d)] + PolyLog[2, ((1 +
```

```
c + d)*E^(2*(a + b*x)))/(1 + c - d)])/(4*b)
```

Maple [B] time = 0.137, size = 306, normalized size = 2.

$$\frac{\operatorname{Artanh}\left(c+d\coth\left(bx+a\right)\right)\ln\left(d\coth\left(bx+a\right)+d\right)}{2b}-\frac{\operatorname{Artanh}\left(c+d\coth\left(bx+a\right)\right)\ln\left(d\coth\left(bx+a\right)-d\right)}{2b}+\frac{1}{4b}\operatorname{dilog}\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(c+d*coth(b*x+a)),x)
```

```
[Out] 1/2/b*arctanh(c+d*coth(b*x+a))*ln(d*coth(b*x+a)+d)-1/2/b*arctanh(c+d*coth(b
*x+a))*ln(d*coth(b*x+a)-d)+1/4/b*dilog((d*coth(b*x+a)+c-1)/(c-d-1))+1/4/b*l
n(d*coth(b*x+a)+d)*ln((d*coth(b*x+a)+c-1)/(c-d-1))-1/4/b*dilog((d*coth(b*x+
a)+c+1)/(1+c-d))-1/4/b*ln(d*coth(b*x+a)+d)*ln((d*coth(b*x+a)+c+1)/(1+c-d))-
1/4/b*dilog((d*coth(b*x+a)+c-1)/(c+d-1))-1/4/b*ln(d*coth(b*x+a)-d)*ln((d*co
th(b*x+a)+c-1)/(c+d-1))+1/4/b*dilog((d*coth(b*x+a)+c+1)/(1+c+d))+1/4/b*ln(d
*coth(b*x+a)-d)*ln((d*coth(b*x+a)+c+1)/(1+c+d))
```

Maxima [A] time = 2.09648, size = 192, normalized size = 1.28

$$-\frac{1}{4}\,bd\left(\frac{2\,bx\log\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}+1\right)+\operatorname{Li}_2\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)}{b^2d}-\frac{2\,bx\log\left(-\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}+1\right)+\operatorname{Li}_2\left(\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}\right)}{b^2d}\right)+x\,\mathrm{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/4*b*d*((2*b*x*log(-(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + dilog(
(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^2*d) - (2*b*x*log(-(c + d - 1)
*e^(2*b*x + 2*a)/(c - d - 1) + 1) + dilog((c + d - 1)*e^(2*b*x + 2*a)/(c -
d - 1)))/(b^2*d)) + x*arctanh(d*coth(b*x + a) + c)
```

Fricas [B] time = 2.30651, size = 1586, normalized size = 10.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x*log(-(d*cosh(b*x + a) + (c + 1)*sinh(b*x + a))/(d*cosh(b*x + a) +
(c - 1)*sinh(b*x + a))) + a*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)
*sinh(b*x + a) + 2*(c - d + 1)*sqrt((c + d + 1)/(c - d + 1))) + a*log(2*(c
+ d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) - 2*(c - d + 1)*sqrt((
c + d + 1)/(c - d + 1))) - a*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1
)*sinh(b*x + a) + 2*(c - d - 1)*sqrt((c + d - 1)/(c - d - 1))) - a*log(2*(c
 + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) - 2*(c - d - 1)*sqrt(
(c + d - 1)/(c - d - 1))) - (b*x + a)*log(sqrt((c + d + 1)/(c - d + 1))*(co
sh(b*x + a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-sqrt((c + d + 1)/(c - d
+ 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x + a)*log(sqrt((c + d - 1)
/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x + a)*log(-sqrt((c
 + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - dilog(sqrt((c
 + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) - dilog(-sqrt((c +
d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + dilog(sqrt((c + d -
1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + dilog(-sqrt((c + d - 1)/
(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(c+d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{artanh}\left(d \coth\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*coth(b*x + a) + c), x)
```

3.301 $\int \frac{\tanh^{-1}(c+d\coth(a+bx))}{x}\,dx$

Optimal. Leaf size=17

$$\text{CannotIntegrate}\left(\frac{\tanh^{-1}(d\coth(a+bx)+c)}{x},x\right)$$

```
[Out] CannotIntegrate[ArcTanh[c + d*Coth[a + b*x]]/x, x]
```

Rubi [A] time = 0.14102, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tanh^{-1}(c+d\coth(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTanh[c + d*Coth[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTanh[c + d*Coth[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(c+d\coth(a+bx))}{x}\,dx = \int \frac{\tanh^{-1}(c+d\coth(a+bx))}{x}\,dx$$

Mathematica [A] time = 13.6226, size = 0, normalized size = 0.

$$\int \frac{\tanh^{-1}(c+d\coth(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTanh[c + d*Coth[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcTanh[c + d*Coth[a + b*x]]/x, x]
```

Maple [A] time = 0.329, size = 0, normalized size = 0.

$$\int \frac{\operatorname{Artanh}\left(c+d\coth\left(bx+a\right)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(c+d*coth(b*x+a))/x,x)
```

```
[Out] int(arctanh(c+d*coth(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}\left(d\coth\left(bx+a\right)+c\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*coth(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(d*coth(b*x + a) + c)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{artanh}\left(d\coth\left(bx+a\right)+c\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*coth(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(arctanh(d*coth(b*x + a) + c)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(c+d*coth(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}\left(d\coth\left(bx+a\right)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*coth(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*coth(b*x + a) + c)/x, x)
```

3.302 $\int x^3 \tanh^{-1}(1+d+d\coth(a+bx))\,dx$

Optimal. Leaf size=152

$$\frac{3x^2\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{8b^2}-\frac{3x\text{PolyLog}\left(4,(d+1)e^{2a+2bx}\right)}{8b^3}+\frac{3\text{PolyLog}\left(5,(d+1)e^{2a+2bx}\right)}{16b^4}-\frac{x^3\text{PolyLog}\left(2,(d+1\right.}{4b}$$

```
[Out] (b*x^5)/20 + (x^4*ArcTanh[1 + d + d*Coth[a + b*x]])/4 - (x^4*Log[1 - (1 + d
)*E^(2*a + 2*b*x)])/8 - (x^3*PolyLog[2, (1 + d)*E^(2*a + 2*b*x)])/(4*b) + (
3*x^2*PolyLog[3, (1 + d)*E^(2*a + 2*b*x)])/(8*b^2) - (3*x*PolyLog[4, (1 + d
)*E^(2*a + 2*b*x)])/(8*b^3) + (3*PolyLog[5, (1 + d)*E^(2*a + 2*b*x)])/(16*b
^4)
```

Rubi [A] time = 0.303812, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6241, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{8b^2}-\frac{3x\text{PolyLog}\left(4,(d+1)e^{2a+2bx}\right)}{8b^3}+\frac{3\text{PolyLog}\left(5,(d+1)e^{2a+2bx}\right)}{16b^4}-\frac{x^3\text{PolyLog}\left(2,(d+1\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] (b*x^5)/20 + (x^4*ArcTanh[1 + d + d*Coth[a + b*x]])/4 - (x^4*Log[1 - (1 + d
)*E^(2*a + 2*b*x)])/8 - (x^3*PolyLog[2, (1 + d)*E^(2*a + 2*b*x)])/(4*b) + (
3*x^2*PolyLog[3, (1 + d)*E^(2*a + 2*b*x)])/(8*b^2) - (3*x*PolyLog[4, (1 + d
)*E^(2*a + 2*b*x)])/(8*b^3) + (3*PolyLog[5, (1 + d)*E^(2*a + 2*b*x)])/(16*b
^4)
```

Rule 6241

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Coth[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
```

```
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3 \tanh^{-1}(1+d+d\coth(a+bx))\,dx = \frac{1}{4}x^4\tanh^{-1}(1+d+d\coth(a+bx)) + \frac{1}{4}b\int \frac{x^4}{1+(-1-d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\coth(a+bx)) + \frac{1}{4}(b(1+d))\int \frac{e^{2a+2bx}x^4}{1+(-1-d)e^{2a+2l}}$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1+d)e^{2a+2bx}\right) + \frac{1}{2}$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1+d)e^{2a+2bx}\right) - \frac{x^3}{}$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1+d)e^{2a+2bx}\right) - \frac{x^3}{}$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1+d)e^{2a+2bx}\right) - \frac{x^3}{}$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1+d)e^{2a+2bx}\right) - \frac{x^3}{}$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1+d)e^{2a+2bx}\right) - \frac{x^3}{}$$

Mathematica [A] time = 4.29145, size = 141, normalized size = 0.93

$$\frac{1}{16}\left(\frac{6x^2\text{PolyLog}\left(3,\frac{e^{-2(a+bx)}}{d+1}\right)}{b^2} + \frac{6x\text{PolyLog}\left(4,\frac{e^{-2(a+bx)}}{d+1}\right)}{b^3} + \frac{3\text{PolyLog}\left(5,\frac{e^{-2(a+bx)}}{d+1}\right)}{b^4} + \frac{4x^3\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{d+1}\right)}{b} - 2x^4\log\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] (4*x^4*ArcTanh[1 + d + d*Coth[a + b*x]] - 2*x^4*Log[1 - 1/((1 + d)*E^(2*(a
+ b*x)))] + (4*x^3*PolyLog[2, 1/((1 + d)*E^(2*(a + b*x)))])/b + (6*x^2*Poly
Log[3, 1/((1 + d)*E^(2*(a + b*x)))])/b^2 + (6*x*PolyLog[4, 1/((1 + d)*E^(2*
(a + b*x)))])/b^3 + (3*PolyLog[5, 1/((1 + d)*E^(2*(a + b*x)))])/b^4)/16
```

Maple [C] time = 13.915, size = 1741, normalized size = 11.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*arctanh(1+d+d*coth(b*x+a)),x)

[Out] 1/2/b^3*a^3/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))*x+1/2/b^3*a^3/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))*x-1/2/b^3*a^3/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x-1/4/b*d/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*x^3+3/8/b^2*d/(1+d)*polylog(3,(1+d)*exp(2*b*x+2*a))*x^2-3/8/b^3*d/(1+d)*polylog(4,(1+d)*exp(2*b*x+2*a))*x+1/2/b^4*d*a^3/(1+d)*dilog(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b^4*d*a^3/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))+1/2/b^4*d*a^4/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))-3/8/b^4*d*a^4/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))-1/4/b^4*d*a^3/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))+1/8*x^4*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1)-1/8/b^4*a^4/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1)-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^3+1/8*I*x^4*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/8/b^4*d*a^4/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1)+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))-1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2+1/2/b^3*d*a^3/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))*x-1/2/b^3*d*a^3/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x+1/2/b^3*d*a^3/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))*x+1/2/b^4*d*a^4/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))+1/16*I*x^4*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/8*I*x^4*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a))^3+1/20*b*x^5+1/16*I*x^4*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))+3/16/b^4/(1+d)*polylog(5,(1+d)*exp(2*b*x+2*a))-1/8/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x^4-1/8*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x^4-1/4/b^4*a^3/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))-1/4/b/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*x^3+3/8/b^2/(1+d)*polylog(3,(1+d)*exp(2*b*x+2*a))*x^2-3/8/b^3/(1+d)*polylog(4,(1+d)*exp(2*b*x+2*a))*x+1/2/b^4*a^3/(1+d)*dilog(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b^4*a^3/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))+3/16/b^4*d/(1+d)*polylog(5,(1+d)*exp(2*b*x+2*a))+1/2/b^4*a^4/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b^4*a^4/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))-3/8/b^4*a^4/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^3-1/16*I*x^4*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^3-1/8*I*x^4*Pi+1/16*I*x^4*Pi*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^2-1/16*I*x^4*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2+1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^2-1/4*x^4*ln(exp(b*x+a))-1/8*x^4*ln(d)

Maxima [A] time = 3.2687, size = 197, normalized size = 1.3

$$\frac{1}{4}\,x^4\,\mathrm{artanh}\,(d\coth\,(bx+a)+d+1)+\frac{1}{40}\left(\frac{2\,x^5}{d}-\frac{5\left(2\,b^4x^4\log\left(-(d+1)e^{(2\,bx+2\,a)}+1\right)+4\,b^3x^3\mathrm{Li}_2\left((d+1)e^{(2\,bx+2\,a)}\right)-6\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arctanh(d*coth(b*x + a) + d + 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*log
(-(d + 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog((d + 1)*e^(2*b*x + 2*a)) -
 6*b^2*x^2*polylog(3, (d + 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, (d + 1)*e
^(2*b*x + 2*a)) - 3*polylog(5, (d + 1)*e^(2*b*x + 2*a)))/(b^5*d))*b*d
```

Fricas [C] time = 2.09132, size = 1281, normalized size = 8.43

$$\frac{2\,b^5x^5+5\,b^4x^4\log\left(-\frac{d\cosh(bx+a)+(d+2)\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right)-20\,b^3x^3\mathrm{Li}_2\left(\sqrt{d+1}(\cosh\,(bx+a)+\sinh\,(bx+a))\right)-20\,b^3x^3\mathrm{Li}_2\left(-\sqrt{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/40*(2*b^5*x^5 + 5*b^4*x^4*log(-(d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/
(d*cosh(b*x + a) + d*sinh(b*x + a))) - 20*b^3*x^3*dilog(sqrt(d + 1)*(cosh(b
*x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-sqrt(d + 1)*(cosh(b*x + a) +
sinh(b*x + a))) - 5*a^4*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x +
a) + 2*sqrt(d + 1)) - 5*a^4*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*
x + a) - 2*sqrt(d + 1)) + 60*b^2*x^2*polylog(3, sqrt(d + 1)*(cosh(b*x + a)
+ sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -sqrt(d + 1)*(cosh(b*x + a) + sin
h(b*x + a))) - 120*b*x*polylog(4, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a
))) - 120*b*x*polylog(4, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) - 5*
(b^4*x^4 - a^4)*log(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 5*(b
^4*x^4 - a^4)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 120*p
olylog(5, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) + 120*polylog(5, -sq
rt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(1+d+d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{artanh}\left(d \coth\left(bx + a\right) + d + 1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(1+d+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^3*arctanh(d*coth(b*x + a) + d + 1), x)
```

3.303 $\int x^2 \tanh^{-1}(1 + d + d \coth(a + bx))\, dx$

Optimal. Leaf size=126

$$\frac{x\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{4b^2}-\frac{\text{PolyLog}\left(4,(d+1)e^{2a+2bx}\right)}{8b^3}-\frac{x^2\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{6}x^3\log\left(1-(d+1)e^{2a+2b}\right.$$

```
[Out] (b*x^4)/12 + (x^3*ArcTanh[1 + d + d*Coth[a + b*x]])/3 - (x^3*Log[1 - (1 + d
)*E^(2*a + 2*b*x)])/6 - (x^2*PolyLog[2, (1 + d)*E^(2*a + 2*b*x)])/(4*b) + (
x*PolyLog[3, (1 + d)*E^(2*a + 2*b*x)])/(4*b^2) - PolyLog[4, (1 + d)*E^(2*a
+ 2*b*x)]/(8*b^3)
```

Rubi [A] time = 0.258996, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6241, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{x\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{4b^2}-\frac{\text{PolyLog}\left(4,(d+1)e^{2a+2bx}\right)}{8b^3}-\frac{x^2\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{6}x^3\log\left(1-(d+1)e^{2a+2b}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[1 + d + d*Coth[a + b*x]],x]
```


```
[Out] (b*x^4)/12 + (x^3*ArcTanh[1 + d + d*Coth[a + b*x]])/3 - (x^3*Log[1 - (1 + d
)*E^(2*a + 2*b*x)])/6 - (x^2*PolyLog[2, (1 + d)*E^(2*a + 2*b*x)])/(4*b) + (
x*PolyLog[3, (1 + d)*E^(2*a + 2*b*x)])/(4*b^2) - PolyLog[4, (1 + d)*E^(2*a
+ 2*b*x)]/(8*b^3)
```


Rule 6241

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Coth[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
```

```
x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \tanh^{-1}(1+d+d\coth(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(1+d+d\coth(a+bx)) + \frac{1}{3}b\int \frac{x^3}{1+(-1-d)e^{2a+2bx}}\,dx$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1+d+d\coth(a+bx)) + \frac{1}{3}(b(1+d))\int \frac{e^{2a+2bx}x^3}{1+(-1-d)e^{2a+2b}$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+d)e^{2a+2bx}\right) + \frac{1}{2}$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+d)e^{2a+2bx}\right) - \frac{x^2}{}$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+d)e^{2a+2bx}\right) - \frac{x^2}{}$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+d)e^{2a+2bx}\right) - \frac{x^2}{}$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+d)e^{2a+2bx}\right) - \frac{x^2}{}$$

Mathematica [A] time = 4.84449, size = 116, normalized size = 0.92

$$\frac{1}{24}\left(\frac{6x\text{PolyLog}\left(3,\frac{e^{-2(a+bx)}}{d+1}\right)}{b^2} + \frac{3\text{PolyLog}\left(4,\frac{e^{-2(a+bx)}}{d+1}\right)}{b^3} + \frac{6x^2\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{d+1}\right)}{b} - 4x^3\log\left(1-\frac{e^{-2(a+bx)}}{d+1}\right) + 8x^3\tanh^{-}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] (8*x^3*ArcTanh[1 + d + d*Coth[a + b*x]] - 4*x^3*Log[1 - 1/((1 + d)*E^(2*(a
+ b*x)))] + (6*x^2*PolyLog[2, 1/((1 + d)*E^(2*(a + b*x)))])/b + (6*x*PolyLo
g[3, 1/((1 + d)*E^(2*(a + b*x)))])/b^2 + (3*PolyLog[4, 1/((1 + d)*E^(2*(a +
 b*x)))])/b^3)/24
```

Maple [C] time = 16.177, size = 1684, normalized size = 13.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(1+d+d*coth(b*x+a)),x)
```

```
[Out] -1/4/b*d/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*x^2-1/12*I*x^3*Pi*csgn(I/(ex
p(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^3+1/12*I*x^3*Pi*csgn(I
*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^3-1/3*x^3*ln(exp(b*x+a))-1/6*x^3*ln(d)-
1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2
*a)-1))^2-1/12*I*x^3*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a
))^2-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d/(exp(
2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/6*I*x^3*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(
2*b*x+2*a))^2+1/12*b*x^4-1/6*I*Pi*x^3-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))*
csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x
+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^2+
1/6/b^3*d*a^3/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1)+1/6/b^3*a^3/(1+d)
*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1)+1/6*I*x^3*Pi*csgn(I*d/(exp(2*b*x+2*a
)-1)*exp(2*b*x+2*a))^2+1/12*I*x^3*Pi*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a
)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^2+1/6*
x^3*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1)-1/2/b^3*d*a^2/(1+d)*dilog(1+exp(b
*x+a)*(1+d)^(1/2))+1/2/b^2/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x*a^2+1/3/b^3*d
/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a^3-1/2/b^3*d*a^3/(1+d)*ln(1-exp(b*x+a)*(
1+d)^(1/2))-1/2/b^3*d*a^3/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))+1/4/b^3*d/(1+d
)*polylog(2,(1+d)*exp(2*b*x+2*a))*a^2+1/4/b^2*d/(1+d)*polylog(3,(1+d)*exp(2
*b*x+2*a))*x-1/2/b^2*a^2/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))*x-1/2/b^2*a^2/(
1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))*x-1/2/b^3*d*a^2/(1+d)*dilog(1-exp(b*x+a)*
(1+d)^(1/2))+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))^3-1/12*I*x^3*Pi*csgn(I*d/
(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^3+1/4/b^2/(1+d)*polylog(3,(1+d)*exp(2*b*
x+2*a))*x-1/2/b^3*a^3/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))-1/2/b^3*a^3/(1+d)*
ln(1+exp(b*x+a)*(1+d)^(1/2))-1/6*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x^3-1/8
/b^3*d/(1+d)*polylog(4,(1+d)*exp(2*b*x+2*a))-1/2/b^3*a^2/(1+d)*dilog(1-exp(
b*x+a)*(1+d)^(1/2))-1/2/b^3*a^2/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))+1/3/b
^3/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a^3-1/4/b/(1+d)*polylog(2,(1+d)*exp(2*b
*x+2*a))*x^2+1/4/b^3/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*a^2+1/12*I*x^3*P
i*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/6/(1+d)*ln(1-(1+d)*exp(2*b*
x+2*a))*x^3-1/8/b^3/(1+d)*polylog(4,(1+d)*exp(2*b*x+2*a))+1/12*I*x^3*Pi*csg
n(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1
)*exp(2*b*x+2*a))-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*
x+2*a)*d+exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp
(2*b*x+2*a)-1))+1/2/b^2*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x*a^2-1/2/b^2*d*
a^2/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))*x-1/2/b^2*d*a^2/(1+d)*ln(1+exp(b*x+a
)*(1+d)^(1/2))*x+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a
)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))
```

Maxima [A] time = 3.31507, size = 166, normalized size = 1.32

$$\frac{1}{3}x^3\,\mathrm{artanh}\left(d\coth\left(bx+a\right)+d+1\right)+\frac{1}{36}\left(\frac{3\,x^4}{d}-\frac{2\left(4\,b^3x^3\log\left(-(d+1)e^{(2\,bx+2\,a)}+1\right)+6\,b^2x^2\mathrm{Li}_2\left((d+1)e^{(2\,bx+2\,a)}\right)-\right.}{b^4d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctanh(d*coth(b*x + a) + d + 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*log
(-(d + 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog((d + 1)*e^(2*b*x + 2*a)) -
 6*b*x*polylog(3, (d + 1)*e^(2*b*x + 2*a)) + 3*polylog(4, (d + 1)*e^(2*b*x
+ 2*a)))/(b^4*d))*b*d
```

Fricas [C] time = 1.96704, size = 1083, normalized size = 8.6

$$\frac{b^4x^4 + 2\,b^3x^3\log\left(-\frac{d\cosh(bx+a)+(d+2)\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 6\,b^2x^2\mathrm{Li}_2\left(\sqrt{d+1}(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))\right) - 6\,b^2x^2\mathrm{Li}_2\left(-\sqrt{d+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(b^4*x^4 + 2*b^3*x^3*log(-(d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/(d
*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b^2*x^2*dilog(sqrt(d + 1)*(cosh(b*x
+ a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-sqrt(d + 1)*(cosh(b*x + a) + sinh
(b*x + a))) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a) +
 2*sqrt(d + 1)) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x +
a) - 2*sqrt(d + 1)) + 12*b*x*polylog(3, sqrt(d + 1)*(cosh(b*x + a) + sinh(b
*x + a))) + 12*b*x*polylog(3, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)))
 - 2*(b^3*x^3 + a^3)*log(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) -
 2*(b^3*x^3 + a^3)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) -
12*polylog(4, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) - 12*polylog(4,
-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(1+d+d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{artanh}(d\coth(bx+a)+d+1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1+d+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(d*coth(b*x + a) + d + 1), x)
```

3.304 $\int x \tanh^{-1}(1+d+d\coth(a+bx))\,dx$

Optimal. Leaf size=100

$$\frac{\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{8b^2}-\frac{x\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(d+1)e^{2a+2bx}\right)+\frac{1}{2}x^2\tanh^{-1}(d\coth(a+bx)$$

```
[Out] (b*x^3)/6 + (x^2*ArcTanh[1 + d + d*Coth[a + b*x]])/2 - (x^2*Log[1 - (1 + d)
*E^(2*a + 2*b*x)])/4 - (x*PolyLog[2, (1 + d)*E^(2*a + 2*b*x)])/(4*b) + Poly
Log[3, (1 + d)*E^(2*a + 2*b*x)]/(8*b^2)
```

Rubi [A] time = 0.229627, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6241, 2184, 2190, 2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{8b^2}-\frac{x\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(d+1)e^{2a+2bx}\right)+\frac{1}{2}x^2\tanh^{-1}(d\coth(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcTanh[1 + d + d*Coth[a + b*x]])/2 - (x^2*Log[1 - (1 + d)
*E^(2*a + 2*b*x)])/4 - (x*PolyLog[2, (1 + d)*E^(2*a + 2*b*x)])/(4*b) + Poly
Log[3, (1 + d)*E^(2*a + 2*b*x)]/(8*b^2)
```

Rule 6241

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Coth[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(1+d+d\coth(a+bx))\,dx = \frac{1}{2}x^2\tanh^{-1}(1+d+d\coth(a+bx)) + \frac{1}{2}b\int \frac{x^2}{1+(-1-d)e^{2a+2bx}}\,dx$$

$$= \frac{bx^3}{6} + \frac{1}{2}x^2\tanh^{-1}(1+d+d\coth(a+bx)) + \frac{1}{2}(b(1+d))\int \frac{e^{2a+2bx}x^2}{1+(-1-d)e^{2a+2bx}}$$

$$= \frac{bx^3}{6} + \frac{1}{2}x^2\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{4}x^2\log\left(1-(1+d)e^{2a+2bx}\right) + \frac{1}{2}$$

$$= \frac{bx^3}{6} + \frac{1}{2}x^2\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{4}x^2\log\left(1-(1+d)e^{2a+2bx}\right) - \frac{x\mathrm{Li}}{}$$

$$= \frac{bx^3}{6} + \frac{1}{2}x^2\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{4}x^2\log\left(1-(1+d)e^{2a+2bx}\right) - \frac{x\mathrm{Li}}{}$$

$$= \frac{bx^3}{6} + \frac{1}{2}x^2\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{4}x^2\log\left(1-(1+d)e^{2a+2bx}\right) - \frac{x\mathrm{Li}}{}$$

Mathematica [A] time = 4.87623, size = 90, normalized size = 0.9

$$\frac{2bx\,\mathrm{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{d+1}\right) + \mathrm{PolyLog}\left(3,\frac{e^{-2(a+bx)}}{d+1}\right) + 2b^2x^2\left(2\tanh^{-1}(d\coth(a+bx)+d+1) - \log\left(1-\frac{e^{-2(a+bx)}}{d+1}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] (2*b^2*x^2*(2*ArcTanh[1 + d + d*Coth[a + b*x]] - Log[1 - 1/((1 + d)*E^(2*(a
 + b*x)))]) + 2*b*x*PolyLog[2, 1/((1 + d)*E^(2*(a + b*x)))] + PolyLog[3, 1/
((1 + d)*E^(2*(a + b*x)))])/(8*b^2)
```

Maple [C] time = 7.336, size = 1603, normalized size = 16.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(1+d+d*coth(b*x+a)),x)
```

```
[Out] -1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x
+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))+1/
```

```
8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b
*x+2*a)/(exp(2*b*x+2*a)-1))-1/8*I*x^2*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*
b*x+2*a))^3+1/6*b*x^3-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*
b*x+2*a)/(exp(2*b*x+2*a)-1))^2-1/8*I*x^2*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2
*a)-1)*exp(2*b*x+2*a))^2-1/4*I*x^2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2
*a))^2-1/4/b^2*d*a^2/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1)+1/8*I*x^2*
Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+1/8/b^2/(1+d)*polylog(3,(1+d
)*exp(2*b*x+2*a))-1/4/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x^2-1/8*I*x^2*Pi*csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*
x+2*a))^2+1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)
*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^2-1/2*x^2*ln(exp(b*x+a))-1/4*x^2*ln(d
)-1/4/b^2*a^2/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1)-1/4*I*Pi*x^2+1/8*
I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^3-1/8*I*x^2*Pi*csgn(I/(e
xp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^3+1/4*I*x^2*Pi*csgn(I
*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a))
^3-1/2/b/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x*a-1/4*d/(1+d)*ln(1-(1+d)*exp(2*
b*x+2*a))*x^2+1/8/b^2*d/(1+d)*polylog(3,(1+d)*exp(2*b*x+2*a))+1/2/b^2*a/(1+
d)*dilog(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b^2*a/(1+d)*dilog(1+exp(b*x+a)*(1+d)
^(1/2))-1/4/b^2/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a^2-1/4/b/(1+d)*polylog(2,
(1+d)*exp(2*b*x+2*a))*x-1/4/b^2/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*a+1/2
/b^2*a^2/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b^2*a^2/(1+d)*ln(1+exp(b*x+
a)*(1+d)^(1/2))+1/8*I*x^2*Pi*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))*cs
gn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^2-1/4/b^2*d/(1
+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a^2-1/4/b*d/(1+d)*polylog(2,(1+d)*exp(2*b*x+
2*a))*x-1/4/b^2*d/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*a+1/2/b*a/(1+d)*ln(
1-exp(b*x+a)*(1+d)^(1/2))*x+1/2/b*a/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))*x+1/
2/b^2*d*a^2/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b^2*d*a^2/(1+d)*ln(1+exp
(b*x+a)*(1+d)^(1/2))+1/2/b^2*d*a/(1+d)*dilog(1-exp(b*x+a)*(1+d)^(1/2))+1/2/
b^2*d*a/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))-1/8*I*x^2*Pi*csgn(I*exp(2*b*x
+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2+1/4*x^2*ln(exp(2*b*x+2*a
)*d+exp(2*b*x+2*a)-1)+1/8*I*x^2*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b
*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))-1/2/b*d/(1+d)*ln(1-
(1+d)*exp(2*b*x+2*a))*x*a+1/2/b*d*a/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))*x+1/
2/b*d*a/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))*x
```

Maxima [A] time = 3.294, size = 135, normalized size = 1.35

$$\frac{1}{24}\left(\frac{4\,x^3}{d} - \frac{3\left(2\,b^2x^2\log\left(-(d+1)e^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left((d+1)e^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3\left((d+1)e^{(2\,bx+2\,a)}\right)\right)}{b^3d}\right)bd + \frac{1}{2}\,x^2\,\mathrm{artanh}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/24*(4*x^3/d - 3*(2*b^2*x^2*log(-(d + 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dilo
g((d + 1)*e^(2*b*x + 2*a)) - polylog(3, (d + 1)*e^(2*b*x + 2*a)))/(b^3*d))*
b*d + 1/2*x^2*arctanh(d*coth(b*x + a) + d + 1)
```

Fricas [C] time = 2.05746, size = 895, normalized size = 8.95

$$\frac{2\,b^3x^3 + 3\,b^2x^2\log\left(-\frac{d\cosh(bx+a)+(d+2)\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 6\,bx\mathrm{Li}_2\left(\sqrt{d+1}(\cosh(bx+a)+\sinh(bx+a))\right) - 6\,bx\mathrm{Li}_2\left(-\sqrt{d+1}(\mathrm{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 + 3*b^2*x^2*log(-(d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/
(d*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b*x*dilog(sqrt(d + 1)*(cosh(b*x +
a) + sinh(b*x + a))) - 6*b*x*dilog(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x +
 a))) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a) + 2*sqr
t(d + 1)) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a) - 2
*sqrt(d + 1)) - 3*(b^2*x^2 - a^2)*log(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x
 + a)) + 1) - 3*(b^2*x^2 - a^2)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x
+ a)) + 1) + 6*polylog(3, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) + 6*
polylog(3, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\mathrm{atanh}\,(d\coth(a+bx)+d+1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(1+d+d*coth(b*x+a)),x)
```

```
[Out] Integral(x*atanh(d*coth(a + b*x) + d + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\mathrm{artanh}\,(d\coth(bx+a)+d+1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1+d+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(d*coth(b*x + a) + d + 1), x)
```

3.305 $\int \tanh^{-1}(1+d+d\coth(a+bx))\,dx$

Optimal. Leaf size=69

$$-\frac{\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{2}x\log\left(1-(d+1)e^{2a+2bx}\right)+x\tanh^{-1}(d\coth(a+bx)+d+1)+\frac{bx^2}{2}$$

```
[Out] (b*x^2)/2 + x*ArcTanh[1 + d + d*Coth[a + b*x]] - (x*Log[1 - (1 + d)*E^(2*a
+ 2*b*x)])/2 - PolyLog[2, (1 + d)*E^(2*a + 2*b*x)]/(4*b)
```

Rubi [A] time = 0.142, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6233, 2184, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{2}x\log\left(1-(d+1)e^{2a+2bx}\right)+x\tanh^{-1}(d\coth(a+bx)+d+1)+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[1 + d + d*Coth[a + b*x]],x]
```


```
[Out] (b*x^2)/2 + x*ArcTanh[1 + d + d*Coth[a + b*x]] - (x*Log[1 - (1 + d)*E^(2*a
+ 2*b*x)])/2 - PolyLog[2, (1 + d)*E^(2*a + 2*b*x)]/(4*b)
```


Rule 6233

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Ar
cTanh[c + d*Coth[a + b*x]], x] + Dist[b, Int[x/(c - d - c*E^(2*a + 2*b*x)),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \tanh^{-1}(1+d+d\coth(a+bx))\,dx = x\tanh^{-1}(1+d+d\coth(a+bx)) + b\int \frac{x}{1+(-1-d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^2}{2} + x\tanh^{-1}(1+d+d\coth(a+bx)) + (b(1+d))\int \frac{e^{2a+2bx}x}{1+(-1-d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^2}{2} + x\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{2}x\log\left(1-(1+d)e^{2a+2bx}\right) + \frac{1}{2}\int \mathrm{lo}$$
$$= \frac{bx^2}{2} + x\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{2}x\log\left(1-(1+d)e^{2a+2bx}\right) + \frac{\mathrm{Subst}}{}$$
$$= \frac{bx^2}{2} + x\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{2}x\log\left(1-(1+d)e^{2a+2bx}\right) - \frac{\mathrm{Li}_2\left((1\right.}{}$$

Mathematica [B] time = 3.45542, size = 197, normalized size = 2.86

$$\frac{-2\mathrm{PolyLog}\left(2,-\sqrt{d+1}e^{a+bx}\right) - 2\mathrm{PolyLog}\left(2,\sqrt{d+1}e^{a+bx}\right) - 2\log\left(e^{a+bx}\right)\log\left(1-\sqrt{d+1}e^{a+bx}\right) - 2\log\left(e^{a+bx}\right)\log\left(\sqrt{\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] x*ArcTanh[1 + d + d*Coth[a + b*x]] + (b^2*x^2 + Log[E^(a + b*x)]^2 - 2*Log[
E^(a + b*x)]*Log[1 - Sqrt[1 + d]*E^(a + b*x)] - 2*Log[E^(a + b*x)]*Log[1 +
Sqrt[1 + d]*E^(a + b*x)] + 2*Log[E^(a + b*x)]*Log[E^(-a - b*x)*(-1 + (1 + d
)*E^(2*(a + b*x)))] - 2*b*x*Log[d*Cosh[a + b*x] + (2 + d)*Sinh[a + b*x]] -
```

```
2*PolyLog[2, -(Sqrt[1 + d]*E^(a + b*x))] - 2*PolyLog[2, Sqrt[1 + d]*E^(a +
b*x)])/(4*b)
```

Maple [B] time = 0.108, size = 247, normalized size = 3.6

$$\frac{\operatorname{Artanh}(1+d+d\coth(bx+a))\ln(d\coth(bx+a)+d)}{2b}-\frac{\operatorname{Artanh}(1+d+d\coth(bx+a))\ln(d\coth(bx+a)-d)}{2b}+\frac{(\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(1+d+d*coth(b*x+a)),x)
```

```
[Out] 1/2/b*arctanh(1+d+d*coth(b*x+a))*ln(d*coth(b*x+a)+d)-1/2/b*arctanh(1+d+d*co
th(b*x+a))*ln(d*coth(b*x+a)-d)+1/8/b*ln(d*coth(b*x+a)+d)^2-1/4/b*dilog(1+1/
2*d*coth(b*x+a)+1/2*d)-1/4/b*ln(d*coth(b*x+a)+d)*ln(1+1/2*d*coth(b*x+a)+1/2
*d)-1/4/b*dilog(1/2*(d*coth(b*x+a)+d)/d)-1/4/b*ln(d*coth(b*x+a)-d)*ln(1/2*(
d*coth(b*x+a)+d)/d)+1/4/b*dilog((d*coth(b*x+a)+d+2)/(2*d+2))+1/4/b*ln(d*cot
h(b*x+a)-d)*ln((d*coth(b*x+a)+d+2)/(2*d+2))
```

Maxima [A] time = 3.25139, size = 97, normalized size = 1.41

$$\frac{1}{4}\,bd\left(\frac{2\,x^2}{d}-\frac{2\,bx\log\left(-(d+1)e^{(2\,bx+2\,a)}+1\right)+\operatorname{Li}_2\left((d+1)e^{(2\,bx+2\,a)}\right)}{b^2d}\right)+x\operatorname{artanh}\left(d\coth\left(bx+a\right)+d+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*b*d*(2*x^2/d - (2*b*x*log(-(d + 1)*e^(2*b*x + 2*a) + 1) + dilog((d + 1)
*e^(2*b*x + 2*a)))/(b^2*d)) + x*arctanh(d*coth(b*x + a) + d + 1)
```

Fricas [B] time = 1.9311, size = 668, normalized size = 9.68

$$\frac{b^2x^2+bx\log\left(-\frac{d\cosh(bx+a)+(d+2)\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right)+a\log\left(2\left(d+1\right)\cosh\left(bx+a\right)+2\left(d+1\right)\sinh\left(bx+a\right)+2\sqrt{d+1}\right)+a\log\left(2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + b*x*log(-(d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/(d*cosh(b
*x + a) + d*sinh(b*x + a))) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sin
h(b*x + a) + 2*sqrt(d + 1)) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sin
h(b*x + a) - 2*sqrt(d + 1)) - (b*x + a)*log(sqrt(d + 1)*(cosh(b*x + a) + si
nh(b*x + a)) + 1) - (b*x + a)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x +
a)) + 1) - dilog(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) - dilog(-sqrt
(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{atanh}\left(d\coth\left(a+bx\right)+d+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(1+d+d*coth(b*x+a)),x)
```

```
[Out] Integral(atanh(d*coth(a + b*x) + d + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{artanh}\left(d\coth\left(bx+a\right)+d+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*coth(b*x + a) + d + 1), x)
```

3.306 $\int \frac{\tanh^{-1}(1+d+d\coth(a+bx))}{x}\,dx$

Optimal. Leaf size=18

$$\text{CannotIntegrate}\left(\frac{\tanh^{-1}(d\coth(a+bx)+d+1)}{x},x\right)$$

```
[Out] CannotIntegrate[ArcTanh[1 + d + d*Coth[a + b*x]]/x, x]
```

Rubi [A] time = 0.0776305, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tanh^{-1}(1+d+d\coth(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTanh[1 + d + d*Coth[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTanh[1 + d + d*Coth[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(1+d+d\coth(a+bx))}{x}\,dx = \int \frac{\tanh^{-1}(1+d+d\coth(a+bx))}{x}\,dx$$

Mathematica [A] time = 4.18748, size = 0, normalized size = 0.

$$\int \frac{\tanh^{-1}(1+d+d\coth(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTanh[1 + d + d*Coth[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcTanh[1 + d + d*Coth[a + b*x]]/x, x]
```

Maple [A] time = 0.349, size = 0, normalized size = 0.

$$\int \frac{\operatorname{Artanh}\left(1+d+d\coth\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(1+d+d*coth(b*x+a))/x,x)
```

```
[Out] int(arctanh(1+d+d*coth(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}\left(d\coth\left(bx+a\right)+d+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*coth(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(d*coth(b*x + a) + d + 1)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{artanh}\left(d\coth\left(bx+a\right)+d+1\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*coth(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(arctanh(d*coth(b*x + a) + d + 1)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}\left(d\coth\left(a+bx\right)+d+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(1+d+d*coth(b*x+a))/x,x)
```

```
[Out] Integral(atanh(d*coth(a + b*x) + d + 1)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(d\coth(bx+a)+d+1)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*coth(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*coth(b*x + a) + d + 1)/x, x)
```

3.307 $\int x^3 \tanh^{-1}(1-d-d\coth(a+bx))\,dx$

Optimal. Leaf size=165

$$\frac{3x^2\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{8b^2}-\frac{3x\text{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)}{8b^3}+\frac{3\text{PolyLog}\left(5,(1-d)e^{2a+2bx}\right)}{16b^4}-\frac{x^3\text{PolyLog}\left(2,(1-\right.}{4b}$$

```
[Out] (b*x^5)/20 + (x^4*ArcTanh[1 - d - d*Coth[a + b*x]])/4 - (x^4*Log[1 - (1 - d
)*E^(2*a + 2*b*x)])/8 - (x^3*PolyLog[2, (1 - d)*E^(2*a + 2*b*x)])/(4*b) + (
3*x^2*PolyLog[3, (1 - d)*E^(2*a + 2*b*x)])/(8*b^2) - (3*x*PolyLog[4, (1 - d
)*E^(2*a + 2*b*x)])/(8*b^3) + (3*PolyLog[5, (1 - d)*E^(2*a + 2*b*x)])/(16*b
^4)
```

Rubi [A] time = 0.299263, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6241, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{8b^2}-\frac{3x\text{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)}{8b^3}+\frac{3\text{PolyLog}\left(5,(1-d)e^{2a+2bx}\right)}{16b^4}-\frac{x^3\text{PolyLog}\left(2,(1-\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[1 - d - d*Coth[a + b*x]],x]
```


```
[Out] (b*x^5)/20 + (x^4*ArcTanh[1 - d - d*Coth[a + b*x]])/4 - (x^4*Log[1 - (1 - d
)*E^(2*a + 2*b*x)])/8 - (x^3*PolyLog[2, (1 - d)*E^(2*a + 2*b*x)])/(4*b) + (
3*x^2*PolyLog[3, (1 - d)*E^(2*a + 2*b*x)])/(8*b^2) - (3*x*PolyLog[4, (1 - d
)*E^(2*a + 2*b*x)])/(8*b^3) + (3*PolyLog[5, (1 - d)*E^(2*a + 2*b*x)])/(16*b
^4)
```


Rule 6241

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Coth[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
```

```
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3 \tanh^{-1}(1-d-d\coth(a+bx))\,dx = \frac{1}{4}x^4\tanh^{-1}(1-d-d\coth(a+bx)) + \frac{1}{4}b\int \frac{x^4}{1+(-1+d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\coth(a+bx)) + \frac{1}{4}(b(1-d))\int \frac{e^{2a+2bx}x^4}{1+(-1+d)e^{2a+2}}$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1-d)e^{2a+2bx}\right) + \frac{1}{2}$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1-d)e^{2a+2bx}\right) - \frac{x}{}$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1-d)e^{2a+2bx}\right) - \frac{x}{}$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1-d)e^{2a+2bx}\right) - \frac{x}{}$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1-d)e^{2a+2bx}\right) - \frac{x}{}$$
$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1-d)e^{2a+2bx}\right) - \frac{x}{}$$

Mathematica [A] time = 4.28041, size = 147, normalized size = 0.89

$$\frac{1}{16}\left(\frac{6x^2\text{PolyLog}\left(3,-\frac{e^{-2(a+bx)}}{d-1}\right)}{b^2} + \frac{6x\text{PolyLog}\left(4,-\frac{e^{-2(a+bx)}}{d-1}\right)}{b^3} + \frac{3\text{PolyLog}\left(5,-\frac{e^{-2(a+bx)}}{d-1}\right)}{b^4} + \frac{4x^3\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{d-1}\right)}{b} - \right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (4*x^4*ArcTanh[1 - d - d*Coth[a + b*x]] - 2*x^4*Log[1 + 1/((-1 + d)*E^(2*(a
 + b*x)))] + (4*x^3*PolyLog[2, -(1/((-1 + d)*E^(2*(a + b*x))))])/b + (6*x^2
*PolyLog[3, -(1/((-1 + d)*E^(2*(a + b*x))))])/b^2 + (6*x*PolyLog[4, -(1/((-
1 + d)*E^(2*(a + b*x))))])/b^3 + (3*PolyLog[5, -(1/((-1 + d)*E^(2*(a + b*x)
))))])/b^4)/16
```

Maple [C] time = 14.384, size = 1801, normalized size = 10.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3*arctanh(-1+d+d*coth(b*x+a)),x)
```

```
[Out] 1/2/b^4*d*a^3/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))-3/8/b^4*d/(d-1)*ln(1+(d
-1)*exp(2*b*x+2*a))*a^4-1/4/b*d/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*x^3-
1/4/b^4*d/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*a^3+3/8/b^2*d/(d-1)*polylo
g(3,-(d-1)*exp(2*b*x+2*a))*x^2-3/8/b^3*d/(d-1)*polylog(4,-(d-1)*exp(2*b*x+2
*a))*x+1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(
exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))^2-1/8/b^4*d*a^4/(d-1)*ln(exp(2*b*x+2*a)
*d-exp(2*b*x+2*a)+1)-1/8*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)
*d-exp(2*b*x+2*a)+1))^2+1/8*x^4*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1)+1/8*I
*x^4*Pi+1/16*I*x^4*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^3+1/16*I*
x^4*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2
*a)/(exp(2*b*x+2*a)-1))-1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b
*x+2*a)/(exp(2*b*x+2*a)-1))^2+1/16*I*x^4*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp
(2*b*x+2*a))-1/8*I*x^4*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/16*
I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))^3+1
/8/b^4*a^4/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1)+1/16*I*x^4*Pi*csgn(I
*exp(2*b*x+2*a))^3+1/20*b*x^5+1/16*I*x^4*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2*a)
/(exp(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))+1/16*I*x^4
*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^3-1/2/b^4*a^3/(d-1)*dilog(1+e
xp(b*x+a)*(1-d)^(1/2))-1/2/b^4*a^3/(d-1)*dilog(1-exp(b*x+a)*(1-d)^(1/2))+3/
16/b^4*d/(d-1)*polylog(5,-(d-1)*exp(2*b*x+2*a))-1/2/b^4*a^4/(d-1)*ln(1-exp(
b*x+a)*(1-d)^(1/2))+3/8/b^4*a^4/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))+1/4/b^4*a^
3/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))-1/2/b^4*a^4/(d-1)*ln(1+exp(b*x+a)*
(1-d)^(1/2))+1/4/b/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*x^3-3/8/b^2/(d-1)
*polylog(3,-(d-1)*exp(2*b*x+2*a))*x^2+3/8/b^3/(d-1)*polylog(4,-(d-1)*exp(2*
b*x+2*a))*x-1/8*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^4+1/8/(d-1)*ln(1+(d-1)
*exp(2*b*x+2*a))*x^4-3/16/b^4/(d-1)*polylog(5,-(d-1)*exp(2*b*x+2*a))+1/16*I
*x^4*Pi*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)-
1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))^2+1/2/b^4*d*a^3/(d-1)*dilog(1-exp(b
*x+a)*(1-d)^(1/2))-1/2/b^3*a^3/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))*x+1/2/b^4
*d*a^4/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^4*d*a^4/(d-1)*ln(1-exp(b*x+
a)*(1-d)^(1/2))+1/2/b^3*a^3/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x-1/2/b^3*a^3/
(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))*x-1/16*I*x^4*Pi*csgn(I*d)*csgn(I*d/(exp(
2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*
b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/16*I*x^4*Pi*cs
gn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2-1/4*x^
4*ln(exp(b*x+a))-1/8*x^4*ln(d)+1/2/b^3*d*a^3/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1
/2))*x+1/2/b^3*d*a^3/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))*x-1/2/b^3*d/(d-1)*l
n(1+(d-1)*exp(2*b*x+2*a))*x*a^3-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-1))*cs
gn(I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*
b*x+2*a)*d-exp(2*b*x+2*a)+1))
```

Maxima [A] time = 3.31231, size = 201, normalized size = 1.22

$$-\frac{1}{4}x^4\,\mathrm{artanh}\left(d\coth\left(bx+a\right)+d-1\right)+\frac{1}{40}\left(\frac{2x^5}{d}-\frac{5\left(2b^4x^4\log\left((d-1)e^{(2bx+2a)}+1\right)+4b^3x^3\mathrm{Li}_2\left(-(d-1)e^{(2bx+2a)}\right)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/4*x^4*arctanh(d*coth(b*x + a) + d - 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*lo
g((d - 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog(-(d - 1)*e^(2*b*x + 2*a))
- 6*b^2*x^2*polylog(3, -(d - 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, -(d - 1
)*e^(2*b*x + 2*a)) - 3*polylog(5, -(d - 1)*e^(2*b*x + 2*a)))/(b^5*d))*b*d
```

Fricas [C] time = 1.98223, size = 1378, normalized size = 8.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/40*(2*b^5*x^5 - 5*b^4*x^4*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/(d*cos
h(b*x + a) + (d - 2)*sinh(b*x + a))) - 20*b^3*x^3*dilog(1/2*sqrt(-4*d + 4)*
(cosh(b*x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-1/2*sqrt(-4*d + 4)*(co
sh(b*x + a) + sinh(b*x + a))) - 5*a^4*log(2*(d - 1)*cosh(b*x + a) + 2*(d -
1)*sinh(b*x + a) + sqrt(-4*d + 4)) - 5*a^4*log(2*(d - 1)*cosh(b*x + a) + 2*
(d - 1)*sinh(b*x + a) - sqrt(-4*d + 4)) + 60*b^2*x^2*polylog(3, 1/2*sqrt(-4
*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -1/2*sqrt(
-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, 1/2*sqrt(-4
*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, -1/2*sqrt(-4*
d + 4)*(cosh(b*x + a) + sinh(b*x + a))) - 5*(b^4*x^4 - a^4)*log(1/2*sqrt(-4
*d + 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 5*(b^4*x^4 - a^4)*log(-1/2*s
qrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 120*polylog(5, 1/2*sqr
t(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) + 120*polylog(5, -1/2*sqrt(-4*
d + 4)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**3*atanh(-1+d+d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -x^3 \operatorname{artanh}\left(d \coth\left(bx + a\right) + d - 1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x^3*arctanh(d*coth(b*x + a) + d - 1), x)
```

3.308 $\int x^2 \tanh^{-1}(1-d-d\coth(a+bx))\,dx$

Optimal. Leaf size=137

$$\frac{x\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{4b^2}-\frac{\text{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)}{8b^3}-\frac{x^2\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{6}x^3\log\left(1-(1-d)e^{2a+2\ldots}\right.$$

```
[Out] (b*x^4)/12 + (x^3*ArcTanh[1 - d - d*Coth[a + b*x]])/3 - (x^3*Log[1 - (1 - d
)*E^(2*a + 2*b*x)])/6 - (x^2*PolyLog[2, (1 - d)*E^(2*a + 2*b*x)])/(4*b) + (
x*PolyLog[3, (1 - d)*E^(2*a + 2*b*x)])/(4*b^2) - PolyLog[4, (1 - d)*E^(2*a
+ 2*b*x)]/(8*b^3)
```

Rubi [A] time = 0.263704, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6241, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{x\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{4b^2}-\frac{\text{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)}{8b^3}-\frac{x^2\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{6}x^3\log\left(1-(1-d)e^{2a+2\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (b*x^4)/12 + (x^3*ArcTanh[1 - d - d*Coth[a + b*x]])/3 - (x^3*Log[1 - (1 - d
)*E^(2*a + 2*b*x)])/6 - (x^2*PolyLog[2, (1 - d)*E^(2*a + 2*b*x)])/(4*b) + (
x*PolyLog[3, (1 - d)*E^(2*a + 2*b*x)])/(4*b^2) - PolyLog[4, (1 - d)*E^(2*a
+ 2*b*x)]/(8*b^3)
```

Rule 6241

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Coth[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
```

```
x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \tanh^{-1}(1-d-d\coth(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(1-d-d\coth(a+bx)) + \frac{1}{3}b\int \frac{x^3}{1+(-1+d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1-d-d\coth(a+bx)) + \frac{1}{3}(b(1-d))\int \frac{e^{2a+2bx}x^3}{1+(-1+d)e^{2a+2}}$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-d)e^{2a+2bx}\right) + \frac{1}{2}$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-d)e^{2a+2bx}\right) - \frac{x}{}$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-d)e^{2a+2bx}\right) - \frac{x}{}$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-d)e^{2a+2bx}\right) - \frac{x}{}$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-d)e^{2a+2bx}\right) - \frac{x}{}$$

Mathematica [A] time = 4.9474, size = 121, normalized size = 0.88

$$\frac{1}{24}\left(\frac{6x\text{PolyLog}\left(3,-\frac{e^{-2(a+bx)}}{d-1}\right)}{b^2} + \frac{3\text{PolyLog}\left(4,-\frac{e^{-2(a+bx)}}{d-1}\right)}{b^3} + \frac{6x^2\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{d-1}\right)}{b} - 4x^3\log\left(\frac{e^{-2(a+bx)}}{d-1}+1\right) + 8x^3\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (8*x^3*ArcTanh[1 - d - d*Coth[a + b*x]] - 4*x^3*Log[1 + 1/((-1 + d)*E^(2*(a
 + b*x)))] + (6*x^2*PolyLog[2, -(1/((-1 + d)*E^(2*(a + b*x))))])/b + (6*x*P
olyLog[3, -(1/((-1 + d)*E^(2*(a + b*x))))])/b^2 + (3*PolyLog[4, -(1/((-1 +
d)*E^(2*(a + b*x))))])/b^3)/24
```

Maple [C] time = 15.92, size = 1742, normalized size = 12.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*arctanh(-1+d+d*coth(b*x+a)),x)
```

```
[Out] 1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^3-1/3*x^3*ln(exp(b*
x+a))-1/6*x^3*ln(d)-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b
*x+2*a)/(exp(2*b*x+2*a)-1))^2-1/12*I*x^3*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2
*a)-1)*exp(2*b*x+2*a))^2-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a
)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/6*I*x^3*Pi*csgn(I/(ex
p(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))^2-1/6*I*x^3*Pi*csgn(I*
exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/12*I*x^3*Pi*csgn(I*(exp(2*b*x+2*a)*d
-exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2
*a)+1))^2+1/12*b*x^4+1/6/b^3*d*a^3/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)
+1)-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2
*a)-1))^2+1/6*I*x^3*Pi-1/2/b^3*d*a^2/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))-
1/2/b^3*d*a^2/(d-1)*dilog(1-exp(b*x+a)*(1-d)^(1/2))-1/2/b^3*d*a^3/(d-1)*ln(
1+exp(b*x+a)*(1-d)^(1/2))+1/12*I*x^3*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b
*x+2*a))^3+1/6*x^3*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1)+1/12*I*x^3*Pi*csgn
(I/(exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*
x+2*a)+1))^2-1/2/b^3*d*a^3/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))+1/3/b^3*d/(d-
1)*ln(1+(d-1)*exp(2*b*x+2*a))*a^3-1/4/b*d/(d-1)*polylog(2,-(d-1)*exp(2*b*x+
2*a))*x^2+1/4/b^3*d/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*a^2+1/4/b^2*d/(d
-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))*x-1/2/b^2/(d-1)*ln(1+(d-1)*exp(2*b*x+2
*a))*x*a^2+1/2/b^2*a^2/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))*x+1/2/b^2*a^2/(d-
1)*ln(1-exp(b*x+a)*(1-d)^(1/2))*x+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(
exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))^3-1/6*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a)
)*x^3-1/8/b^3*d/(d-1)*polylog(4,-(d-1)*exp(2*b*x+2*a))+1/2/b^3*a^2/(d-1)*di
log(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^3*a^2/(d-1)*dilog(1-exp(b*x+a)*(1-d)^(1
/2))-1/3/b^3/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a^3+1/4/b/(d-1)*polylog(2,-(d
-1)*exp(2*b*x+2*a))*x^2-1/4/b^3/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*a^2-
1/4/b^2/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))*x+1/2/b^3*a^3/(d-1)*ln(1+exp
(b*x+a)*(1-d)^(1/2))+1/2/b^3*a^3/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))+1/12*I*
x^3*Pi*csgn(I*exp(2*b*x+2*a))^3-1/6/b^3*a^3/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2
*b*x+2*a)+1)+1/12*I*x^3*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+1/6/
(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^3+1/8/b^3/(d-1)*polylog(4,-(d-1)*exp(2*b
*x+2*a))+1/12*I*x^3*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*
csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+
2*a))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))+
1/2/b^2*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x*a^2-1/2/b^2*d*a^2/(d-1)*ln(1+e
xp(b*x+a)*(1-d)^(1/2))*x-1/2/b^2*d*a^2/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))*x
-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*
x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))
```

Maxima [A] time = 3.29164, size = 169, normalized size = 1.23

$$-\frac{1}{3}\,x^3\,\mathrm{artanh}\left(d\coth\left(bx+a\right)+d-1\right)+\frac{1}{36}\left(\frac{3\,x^4}{d}-\frac{2\left(4\,b^3x^3\log\left((d-1)e^{(2\,bx+2\,a)}+1\right)+6\,b^2x^2\mathrm{Li}_2\left(-(d-1)e^{(2\,bx+2\,a)}\right)-\right.}{b^4d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/3*x^3*arctanh(d*coth(b*x + a) + d - 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*lo
g((d - 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-(d - 1)*e^(2*b*x + 2*a))
- 6*b*x*polylog(3, -(d - 1)*e^(2*b*x + 2*a)) + 3*polylog(4, -(d - 1)*e^(2*b
*x + 2*a)))/(b^4*d))*b*d
```

Fricas [C] time = 2.07972, size = 1161, normalized size = 8.47

$$b^4x^4 - 2\,b^3x^3\log\left(-\frac{d\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+(d-2)\sinh(bx+a)}\right) - 6\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4\,d+4}(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))\right) - 6\,b^2x^2\mathrm{Li}_2\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(b^4*x^4 - 2*b^3*x^3*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/(d*cosh(
b*x + a) + (d - 2)*sinh(b*x + a))) - 6*b^2*x^2*dilog(1/2*sqrt(-4*d + 4)*(co
sh(b*x + a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-1/2*sqrt(-4*d + 4)*(cosh(b
*x + a) + sinh(b*x + a))) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*s
inh(b*x + a) + sqrt(-4*d + 4)) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d -
 1)*sinh(b*x + a) - sqrt(-4*d + 4)) + 12*b*x*polylog(3, 1/2*sqrt(-4*d + 4)*
(cosh(b*x + a) + sinh(b*x + a))) + 12*b*x*polylog(3, -1/2*sqrt(-4*d + 4)*(c
osh(b*x + a) + sinh(b*x + a))) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(-4*d + 4)*(
cosh(b*x + a) + sinh(b*x + a)) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sqrt(-4*d
+ 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 12*polylog(4, 1/2*sqrt(-4*d + 4
)*(cosh(b*x + a) + sinh(b*x + a))) - 12*polylog(4, -1/2*sqrt(-4*d + 4)*(cos
h(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**2*atanh(-1+d+d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -x^2 \operatorname{artanh}\left(d \coth\left(bx + a\right) + d - 1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctanh(d*coth(b*x + a) + d - 1), x)
```

3.309 $\int x \tanh^{-1}(1-d-d\coth(a+bx))\,dx$

Optimal. Leaf size=109

$$\frac{\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{8b^2}-\frac{x\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(1-d)e^{2a+2bx}\right)+\frac{1}{2}x^2\tanh^{-1}(d(-\coth(a+b$$

```
[Out] (b*x^3)/6 + (x^2*ArcTanh[1 - d - d*Coth[a + b*x]])/2 - (x^2*Log[1 - (1 - d)
*E^(2*a + 2*b*x)])/4 - (x*PolyLog[2, (1 - d)*E^(2*a + 2*b*x)])/(4*b) + Poly
Log[3, (1 - d)*E^(2*a + 2*b*x)]/(8*b^2)
```

Rubi [A] time = 0.231253, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6241, 2184, 2190, 2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{8b^2}-\frac{x\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(1-d)e^{2a+2bx}\right)+\frac{1}{2}x^2\tanh^{-1}(d(-\coth(a+b$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcTanh[1 - d - d*Coth[a + b*x]])/2 - (x^2*Log[1 - (1 - d)
*E^(2*a + 2*b*x)])/4 - (x*PolyLog[2, (1 - d)*E^(2*a + 2*b*x)])/(4*b) + Poly
Log[3, (1 - d)*E^(2*a + 2*b*x)]/(8*b^2)
```

Rule 6241

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Coth[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(1-d-d\coth(a+bx))\,dx = \frac{1}{2}x^2\tanh^{-1}(1-d-d\coth(a+bx)) + \frac{1}{2}b\int \frac{x^2}{1+(-1+d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^3}{6} + \frac{1}{2}x^2\tanh^{-1}(1-d-d\coth(a+bx)) + \frac{1}{2}(b(1-d))\int \frac{e^{2a+2bx}x^2}{1+(-1+d)e^{2a+2b}$$
$$= \frac{bx^3}{6} + \frac{1}{2}x^2\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{4}x^2\log\left(1-(1-d)e^{2a+2bx}\right) + \frac{1}{2}$$
$$= \frac{bx^3}{6} + \frac{1}{2}x^2\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{4}x^2\log\left(1-(1-d)e^{2a+2bx}\right) - \frac{xI}{}$$
$$= \frac{bx^3}{6} + \frac{1}{2}x^2\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{4}x^2\log\left(1-(1-d)e^{2a+2bx}\right) - \frac{xI}{}$$
$$= \frac{bx^3}{6} + \frac{1}{2}x^2\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{4}x^2\log\left(1-(1-d)e^{2a+2bx}\right) - \frac{xI}{}$$

Mathematica [A] time = 4.79792, size = 94, normalized size = 0.86

$$\frac{2bx\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{d-1}\right)+\text{PolyLog}\left(3,-\frac{e^{-2(a+bx)}}{d-1}\right)+2b^2x^2\left(2\tanh^{-1}(d(-\coth(a+bx))-d+1)-\log\left(\frac{e^{-2(a+bx)}}{d-1}+1\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (2*b^2*x^2*(2*ArcTanh[1 - d - d*Coth[a + b*x]] - Log[1 + 1/((-1 + d)*E^(2*(
a + b*x)))]) + 2*b*x*PolyLog[2, -(1/((-1 + d)*E^(2*(a + b*x))))] + PolyLog[
3, -(1/((-1 + d)*E^(2*(a + b*x))))])/(8*b^2)
```

Maple [C] time = 9.189, size = 1659, normalized size = 15.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*arctanh(-1+d+d*coth(b*x+a)),x)
```

```
[Out] -1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*x
+2*a)+1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))+1/
```

```
8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b
*x+2*a)/(exp(2*b*x+2*a)-1))+1/4*I*x^2*Pi-1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)
)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2+1/4*x^2*ln(exp(2*b*x+2*a)*d-e
xp(2*b*x+2*a)+1)+1/6*b*x^3+1/8*I*x^2*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b
*x+2*a))-1/4/b^2*d*a^2/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1)+1/8*I*x^
2*Pi*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)-1)*
(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))^2-1/8*I*x^2*Pi*csgn(I*d)*csgn(I*d/(exp
(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*
b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/2*x^2*ln(exp(b
*x+a))-1/4*x^2*ln(d)+1/8*I*x^2*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a
))^3+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a))^3-1/2/b^2*a/(d-1)*dilog(1+exp(b*x+
a)*(1-d)^(1/2))-1/4*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^2-1/2/b^2*a/(d-1)*
dilog(1-exp(b*x+a)*(1-d)^(1/2))+1/4/b^2/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a^
2+1/4/b/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*x+1/4/b^2/(d-1)*polylog(2,-(
d-1)*exp(2*b*x+2*a))*a-1/2/b^2*a^2/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))-1/2/b
^2*a^2/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))+1/8/b^2*d/(d-1)*polylog(3,-(d-1)*
exp(2*b*x+2*a))+1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d-ex
p(2*b*x+2*a)+1))^3+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^3
-1/4*I*x^2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/4/(d-1)*ln(1+(d
-1)*exp(2*b*x+2*a))*x^2-1/8/b^2/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))+1/8*
I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*
a)*d-exp(2*b*x+2*a)+1))^2-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2-1/4*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(
exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))^2+1/4/b^2*a^2/(d-1)*ln(exp(2*b*x+2*a)*d
-exp(2*b*x+2*a)+1)+1/2/b^2*d*a/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^
2*d*a/(d-1)*dilog(1-exp(b*x+a)*(1-d)^(1/2))-1/4/b^2*d/(d-1)*ln(1+(d-1)*exp(
2*b*x+2*a))*a^2-1/4/b*d/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*x-1/4/b^2*d/
(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*a+1/2/b/(d-1)*ln(1+(d-1)*exp(2*b*x+2
*a))*x*a-1/2/b*a/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))*x-1/2/b*a/(d-1)*ln(1-ex
p(b*x+a)*(1-d)^(1/2))*x+1/2/b^2*d*a^2/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))+1/
2/b^2*d*a^2/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))-1/2/b*d/(d-1)*ln(1+(d-1)*exp
(2*b*x+2*a))*x*a+1/2/b*d*a/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))*x+1/2/b*d*a/(
d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))*x+1/8*I*x^2*Pi*csgn(I*d)*csgn(I*exp(2*b*x
+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))
```

Maxima [A] time = 3.27199, size = 136, normalized size = 1.25

$$\frac{1}{24}\left(\frac{4\,x^3}{d}-\frac{3\left(2\,b^2x^2\log\left((d-1)e^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(-(d-1)e^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3(-(d-1)e^{(2\,bx+2\,a)})\right)}{b^3d}\right)bd-\frac{1}{2}\,x^2\,\mathrm{artanh}\,($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

[Out] 1/24*(4*x^3/d - 3*(2*b^2*x^2*log((d - 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dilog
(-(d - 1)*e^(2*b*x + 2*a)) - polylog(3, -(d - 1)*e^(2*b*x + 2*a)))/(b^3*d))
*b*d - 1/2*x^2*arctanh(d*coth(b*x + a) + d - 1)

Fricas [C] time = 1.91705, size = 954, normalized size = 8.75

$$2\,b^3x^3 - 3\,b^2x^2\log\left(-\frac{d\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+(d-2)\sinh(bx+a)}\right) - 6\,bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4\,d+4}(\cosh(bx+a)+\sinh(bx+a))\right) - 6\,bx\mathrm{Li}_2\left(-\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-x*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="fricas")

[Out] 1/12*(2*b^3*x^3 - 3*b^2*x^2*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/(d*cos
h(b*x + a) + (d - 2)*sinh(b*x + a))) - 6*b*x*dilog(1/2*sqrt(-4*d + 4)*(cosh
(b*x + a) + sinh(b*x + a))) - 6*b*x*dilog(-1/2*sqrt(-4*d + 4)*(cosh(b*x + a
) + sinh(b*x + a))) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*
x + a) + sqrt(-4*d + 4)) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*si
nh(b*x + a) - sqrt(-4*d + 4)) - 3*(b^2*x^2 - a^2)*log(1/2*sqrt(-4*d + 4)*(c
osh(b*x + a) + sinh(b*x + a)) + 1) - 3*(b^2*x^2 - a^2)*log(-1/2*sqrt(-4*d +
 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 6*polylog(3, 1/2*sqrt(-4*d + 4)*
(cosh(b*x + a) + sinh(b*x + a))) + 6*polylog(3, -1/2*sqrt(-4*d + 4)*(cosh(b
*x + a) + sinh(b*x + a))))/b^2

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-x*atanh(-1+d+d*coth(b*x+a)),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -x \operatorname{artanh}\left(d \coth\left(bx + a\right) + d - 1\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x*arctanh(d*coth(b*x + a) + d - 1), x)
```

3.310 $\int \tanh^{-1}(1 - d - d\coth(a + bx))\,dx$

Optimal. Leaf size=76

$$-\frac{\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b} - \frac{1}{2}x\log\left(1-(1-d)e^{2a+2bx}\right) + x\tanh^{-1}(d(-\coth(a+bx)) - d + 1) + \frac{bx^2}{2}$$

```
[Out] (b*x^2)/2 + x*ArcTanh[1 - d - d*Coth[a + b*x]] - (x*Log[1 - (1 - d)*E^(2*a
+ 2*b*x)])/2 - PolyLog[2, (1 - d)*E^(2*a + 2*b*x)]/(4*b)
```

Rubi [A] time = 0.137386, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6233, 2184, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b} - \frac{1}{2}x\log\left(1-(1-d)e^{2a+2bx}\right) + x\tanh^{-1}(d(-\coth(a+bx)) - d + 1) + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcTanh[1 - d - d*Coth[a + b*x]] - (x*Log[1 - (1 - d)*E^(2*a
+ 2*b*x)])/2 - PolyLog[2, (1 - d)*E^(2*a + 2*b*x)]/(4*b)
```

Rule 6233

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Ar
cTanh[c + d*Coth[a + b*x]], x] + Dist[b, Int[x/(c - d - c*E^(2*a + 2*b*x)),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \tanh^{-1}(1-d-d\coth(a+bx))\,dx = x\tanh^{-1}(1-d-d\coth(a+bx)) + b\int \frac{x}{1+(-1+d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^2}{2} + x\tanh^{-1}(1-d-d\coth(a+bx)) + (b(1-d))\int \frac{e^{2a+2bx}x}{1+(-1+d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^2}{2} + x\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{2}x\log\left(1-(1-d)e^{2a+2bx}\right) + \frac{1}{2}\int \log($$
$$= \frac{bx^2}{2} + x\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{2}x\log\left(1-(1-d)e^{2a+2bx}\right) + \frac{\text{Subst}\left(\int\right.}{}$$
$$= \frac{bx^2}{2} + x\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{2}x\log\left(1-(1-d)e^{2a+2bx}\right) - \frac{\text{Li}_2\left((1-\right.}{4}$$

Mathematica [B] time = 4.05503, size = 208, normalized size = 2.74

$$\frac{-2\text{PolyLog}\left(2,-\sqrt{1-d}e^{a+bx}\right) - 2\text{PolyLog}\left(2,\sqrt{1-d}e^{a+bx}\right) - 2\log\left(e^{a+bx}\right)\log\left(1-\sqrt{1-d}e^{a+bx}\right) - 2\log\left(e^{a+bx}\right)\log\left(\sqrt{1-\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] x*ArcTanh[1 - d - d*Coth[a + b*x]] + (b^2*x^2 + Log[E^(a + b*x)]^2 - 2*Log[
E^(a + b*x)]*Log[1 - Sqrt[1 - d]*E^(a + b*x)] - 2*Log[E^(a + b*x)]*Log[1 +
Sqrt[1 - d]*E^(a + b*x)] + 2*Log[E^(a + b*x)]*Log[E^(-a - b*x)*(1 + (-1 + d
)*E^(2*(a + b*x)))] - 2*b*x*Log[d*Cosh[a + b*x] + (-2 + d)*Sinh[a + b*x]] -
```

```
2*PolyLog[2, -(Sqrt[1 - d]*E^(a + b*x))] - 2*PolyLog[2, Sqrt[1 - d]*E^(a +
b*x)])/(4*b)
```

Maple [B] time = 0.107, size = 280, normalized size = 3.7

$$-\frac{\operatorname{Artanh}\left(-1+d+d\coth\left(bx+a\right)\right)\ln\left(d\coth\left(bx+a\right)+d\right)}{2b}+\frac{\operatorname{Artanh}\left(-1+d+d\coth\left(bx+a\right)\right)\ln\left(d\coth\left(bx+a\right)-d\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctanh(-1+d+d*coth(b*x+a)),x)
```

```
[Out] -1/2/b*arctanh(-1+d+d*coth(b*x+a))*ln(d*coth(b*x+a)+d)+1/2/b*arctanh(-1+d+d
*coth(b*x+a))*ln(d*coth(b*x+a)-d)+1/4/b*ln(-1/2*d*coth(b*x+a)-1/2*d+1)*ln(1
/2*d*coth(b*x+a)+1/2*d)-1/4/b*ln(-1/2*d*coth(b*x+a)-1/2*d+1)*ln(d*coth(b*x+
a)+d)+1/4/b*dilog(1/2*d*coth(b*x+a)+1/2*d)+1/8/b*ln(d*coth(b*x+a)+d)^2-1/4/
b*dilog(1/2*(d*coth(b*x+a)+d)/d)-1/4/b*ln(d*coth(b*x+a)-d)*ln(1/2*(d*coth(b
*x+a)+d)/d)+1/4/b*dilog((d*coth(b*x+a)+d-2)/(2*d-2))+1/4/b*ln(d*coth(b*x+a)
-d)*ln((d*coth(b*x+a)+d-2)/(2*d-2))
```

Maxima [A] time = 3.27488, size = 99, normalized size = 1.3

$$\frac{1}{4}\,bd\left(\frac{2\,x^2}{d}-\frac{2\,bx\log\left((d-1)e^{(2\,bx+2\,a)}+1\right)+\operatorname{Li}_2\left(-(d-1)e^{(2\,bx+2\,a)}\right)}{b^2d}\right)-x\operatorname{artanh}\left(d\coth\left(bx+a\right)+d-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*b*d*(2*x^2/d - (2*b*x*log((d - 1)*e^(2*b*x + 2*a) + 1) + dilog(-(d - 1)
*e^(2*b*x + 2*a)))/(b^2*d)) - x*arctanh(d*coth(b*x + a) + d - 1)
```

Fricas [B] time = 2.00741, size = 709, normalized size = 9.33

$$b^2x^2-bx\log\left(-\frac{d\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+(d-2)\sinh(bx+a)}\right)+a\log\left(2\,(d-1)\cosh\left(bx+a\right)+2\,(d-1)\sinh\left(bx+a\right)+\sqrt{-4\,d+4}\right)+a\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - b*x*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/(d*cosh(b*x + a
) + (d - 2)*sinh(b*x + a))) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sin
h(b*x + a) + sqrt(-4*d + 4)) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*si
nh(b*x + a) - sqrt(-4*d + 4)) - (b*x + a)*log(1/2*sqrt(-4*d + 4)*(cosh(b*x
+ a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-1/2*sqrt(-4*d + 4)*(cosh(b*x +
a) + sinh(b*x + a)) + 1) - dilog(1/2*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b
*x + a))) - dilog(-1/2*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \operatorname{atanh}\left(d\coth\left(a+bx\right)+d-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atanh(-1+d+d*coth(b*x+a)),x)
```

```
[Out] -Integral(atanh(d*coth(a + b*x) + d - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\operatorname{artanh}\left(d\coth\left(bx+a\right)+d-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(d*coth(b*x + a) + d - 1), x)
```

3.311 $\int \frac{\tanh^{-1}(1-d-d\coth(a+bx))}{x}\,dx$

Optimal. Leaf size=21

$$\text{CannotIntegrate}\left(\frac{\tanh^{-1}(d(-\coth(a+bx))-d+1)}{x},x\right)$$

```
[Out] CannotIntegrate[ArcTanh[1 - d - d*Coth[a + b*x]]/x, x]
```

Rubi [A] time = 0.0888822, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tanh^{-1}(1-d-d\coth(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTanh[1 - d - d*Coth[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTanh[1 - d - d*Coth[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(1-d-d\coth(a+bx))}{x}\,dx=\int \frac{\tanh^{-1}(1-d-d\coth(a+bx))}{x}\,dx$$

Mathematica [A] time = 4.30794, size = 0, normalized size = 0.

$$\int \frac{\tanh^{-1}(1-d-d\coth(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTanh[1 - d - d*Coth[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcTanh[1 - d - d*Coth[a + b*x]]/x, x]
```

Maple [A] time = 0.322, size = 0, normalized size = 0.

$$\int -\frac{\operatorname{Artanh}\left(-1+d+d\coth\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctanh(-1+d+d*coth(b*x+a))/x,x)
```

```
[Out] int(-arctanh(-1+d+d*coth(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\operatorname{artanh}\left(d\coth\left(bx+a\right)+d-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*coth(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -integrate(arctanh(d*coth(b*x + a) + d - 1)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\operatorname{artanh}\left(d\coth\left(bx+a\right)+d-1\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*coth(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(-arctanh(d*coth(b*x + a) + d - 1)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\operatorname{atanh}\left(d\coth\left(a+bx\right)+d-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atanh(-1+d+d*coth(b*x+a))/x,x)
```

```
[Out] -Integral(atanh(d*coth(a + b*x) + d - 1)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\operatorname{artanh}\left(d\coth\left(bx+a\right)+d-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*coth(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(-arctanh(d*coth(b*x + a) + d - 1)/x, x)
```

3.312 $\int (e+fx)^3 \tanh^{-1}(\tan(a+bx))\,dx$

Optimal. Leaf size=302

$$\frac{3if^2(e+fx)\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3}-\frac{3if^2(e+fx)\text{PolyLog}\left(4,ie^{2i(a+bx)}\right)}{8b^3}+\frac{3f(e+fx)^2\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{8b^2}-\frac{3f(e$$

```
[Out] ((I/4)*(e + f*x)^4*ArcTan[E^((2*I)*(a + b*x))])/f + ((e + f*x)^4*ArcTanh[Ta
n[a + b*x]])/(4*f) - ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (3*f*(e +
f*x)^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (3*f*(e + f*x)^2*Pol
yLog[3, I*E^((2*I)*(a + b*x))])/(8*b^2) + (((3*I)/8)*f^2*(e + f*x)*PolyLog[
4, (-I)*E^((2*I)*(a + b*x))])/b^3 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E
^((2*I)*(a + b*x))])/b^3 - (3*f^3*PolyLog[5, (-I)*E^((2*I)*(a + b*x))])/(16
*b^4) + (3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```

Rubi [A] time = 0.231503, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6251, 4181, 2531, 6609, 2282, 6589}

$$\frac{3if^2(e+fx)\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3}-\frac{3if^2(e+fx)\text{PolyLog}\left(4,ie^{2i(a+bx)}\right)}{8b^3}+\frac{3f(e+fx)^2\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{8b^2}-\frac{3f(e$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*ArcTanh[Tan[a + b*x]],x]
```


```
[Out] ((I/4)*(e + f*x)^4*ArcTan[E^((2*I)*(a + b*x))])/f + ((e + f*x)^4*ArcTanh[Ta
n[a + b*x]])/(4*f) - ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (3*f*(e +
f*x)^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (3*f*(e + f*x)^2*Pol
yLog[3, I*E^((2*I)*(a + b*x))])/(8*b^2) + (((3*I)/8)*f^2*(e + f*x)*PolyLog[
4, (-I)*E^((2*I)*(a + b*x))])/b^3 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E
^((2*I)*(a + b*x))])/b^3 - (3*f^3*PolyLog[5, (-I)*E^((2*I)*(a + b*x))])/(16
*b^4) + (3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```


Rule 6251

```
Int[ArcTanh[Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcTanh[Tan[a + b*x]])/(f*(m + 1)), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
```

```
 e, f}, x] && IGtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (e+fx)^3 \tanh^{-1}(\tan(a+bx))\,dx = \frac{(e+fx)^4 \tanh^{-1}(\tan(a+bx))}{4f} - \frac{b\int (e+fx)^4 \sec(2a+2bx)\,dx}{4f}$$
$$= \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4 \tanh^{-1}(\tan(a+bx))}{4f} + \frac{1}{2}\int (e+fx)^3 \log\left(\right.$$
$$= \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4 \tanh^{-1}(\tan(a+bx))}{4f} - \frac{i(e+fx)^3\text{Li}_2\left(-ie^{2i\right.}}{4b}$$
$$= \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4 \tanh^{-1}(\tan(a+bx))}{4f} - \frac{i(e+fx)^3\text{Li}_2\left(-ie^{2i\right.}}{4b}$$
$$= \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4 \tanh^{-1}(\tan(a+bx))}{4f} - \frac{i(e+fx)^3\text{Li}_2\left(-ie^{2i\right.}}{4b}$$
$$= \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4 \tanh^{-1}(\tan(a+bx))}{4f} - \frac{i(e+fx)^3\text{Li}_2\left(-ie^{2i\right.}}{4b}$$
$$= \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4 \tanh^{-1}(\tan(a+bx))}{4f} - \frac{i(e+fx)^3\text{Li}_2\left(-ie^{2i\right.}}{4b}$$

<u>Mathematica</u> [B] time = 1.19349, size = 654, normalized size = 2.17

$$\frac{1}{4}x\left(6e^2fx+4e^3+4ef^2x^2+f^3x^3\right)\tanh^{-1}(\tan(a+bx)) + \frac{6b^2e^2f\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right) - 6b^2e^2f\text{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*ArcTanh[Tan[a + b*x]],x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcTanh[Tan[a + b*x]])/4 + (
-8*b^4*e^3*x*Log[1 - I*E^((2*I)*(a + b*x))] - 12*b^4*e^2*f*x^2*Log[1 - I*E^
((2*I)*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] - 2*b^4
*f^3*x^4*Log[1 - I*E^((2*I)*(a + b*x))] + 8*b^4*e^3*x*Log[1 + I*E^((2*I)*(a
 + b*x))] + 12*b^4*e^2*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 8*b^4*e*f^2*x
^3*Log[1 + I*E^((2*I)*(a + b*x))] + 2*b^4*f^3*x^4*Log[1 + I*E^((2*I)*(a + b
*x))] - (4*I)*b^3*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))] + (4*I)*
b^3*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))] + 6*b^2*e^2*f*PolyLog[3,
(-I)*E^((2*I)*(a + b*x))] + 12*b^2*e*f^2*x*PolyLog[3, (-I)*E^((2*I)*(a + b*
x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] - 6*b^2*e^2*f*Pol
yLog[3, I*E^((2*I)*(a + b*x))] - 12*b^2*e*f^2*x*PolyLog[3, I*E^((2*I)*(a +
b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^((2*I)*(a + b*x))] + (6*I)*b*e*f^2*Po
lyLog[4, (-I)*E^((2*I)*(a + b*x))] + (6*I)*b*f^3*x*PolyLog[4, (-I)*E^((2*I)
*(a + b*x))] - (6*I)*b*e*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))] - (6*I)*b*f^
```

```
3*x*PolyLog[4, I*E^((2*I)*(a + b*x))] - 3*f^3*PolyLog[5, (-I)*E^((2*I)*(a +
 b*x))] + 3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```

Maple [C] time = 4.766, size = 7429, normalized size = 24.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*arctanh(tan(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{16}\left(f^3x^4+4\,ef^2x^3+6\,e^2fx^2+4\,e^3x\right)\log\left(2\,\cos\left(2\,bx+2\,a\right)^2+2\,\sin\left(2\,bx+2\,a\right)^2+4\,\sin\left(2\,bx+2\,a\right)+2\right)-\frac{1}{16}\left(f^3x^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctanh(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/16*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*log(2*cos(2*b*x + 2*a)
^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/16*(f^3*x^4 + 4*e*f
^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*
a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(1/2*((b*f^3*x^4 + 4*b*e*f^2*x^3
+ 6*b*e^2*f*x^2 + 4*b*e^3*x)*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f^3*x^4
 + 4*b*e*f^2*x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*sin(4*b*x + 4*a)*sin(2*b*x +
2*a) + (b*f^3*x^4 + 4*b*e*f^2*x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*cos(2*b*x +
2*a))/(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x
)
```

Fricas [C] time = 2.88607, size = 4516, normalized size = 14.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctanh(tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/32*(3*f^3*polylog(5, (I*tan(b*x + a)^2 + 2*tan(b*x + a) - I)/(tan(b*x +
a)^2 + 1)) - 3*f^3*polylog(5, (I*tan(b*x + a)^2 - 2*tan(b*x + a) - I)/(tan(
b*x + a)^2 + 1)) + 3*f^3*polylog(5, (-I*tan(b*x + a)^2 + 2*tan(b*x + a) + I
)/(tan(b*x + a)^2 + 1)) - 3*f^3*polylog(5, (-I*tan(b*x + a)^2 - 2*tan(b*x +
 a) + I)/(tan(b*x + a)^2 + 1)) - (4*I*b^3*f^3*x^3 + 12*I*b^3*e*f^2*x^2 + 12
*I*b^3*e^2*f*x + 4*I*b^3*e^3)*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x +
a) - I + 1)/(tan(b*x + a)^2 + 1) + 1) - (4*I*b^3*f^3*x^3 + 12*I*b^3*e*f^2*x
^2 + 12*I*b^3*e^2*f*x + 4*I*b^3*e^3)*dilog(-((I + 1)*tan(b*x + a)^2 - 2*tan
(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1) + 1) - (-4*I*b^3*f^3*x^3 - 12*I*b^3
*e*f^2*x^2 - 12*I*b^3*e^2*f*x - 4*I*b^3*e^3)*dilog(-(-(I - 1)*tan(b*x + a)^
2 + 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) - (-4*I*b^3*f^3*x^3 -
 12*I*b^3*e*f^2*x^2 - 12*I*b^3*e^2*f*x - 4*I*b^3*e^3)*dilog(-(-(I - 1)*tan(
b*x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) + 2*(b^4*f^3
*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^
2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(((I + 1)*tan(b*x + a)^2 + 2*tan(
b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) - 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f
+ 4*a^3*b*e*f^2 - a^4*f^3)*log(((I + 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) +
 I - 1)/(tan(b*x + a)^2 + 1)) + 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*
e*f^2 - a^4*f^3)*log(((I + 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I - 1)/(t
an(b*x + a)^2 + 1)) - 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 +
4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(
((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) + 2
*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e
^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log((-(I - 1)*tan(b*x + a)^
2 + 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) - 2*(b^4*f^3*x^4 + 4*b^4*
e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f +
 4*a^3*b*e*f^2 - a^4*f^3)*log((-(I - 1)*tan(b*x + a)^2 - 2*tan(b*x + a) + I
 + 1)/(tan(b*x + a)^2 + 1)) - 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*
f^2 - a^4*f^3)*log(((I - 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) + I + 1)/(tan
(b*x + a)^2 + 1)) + 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*
f^3)*log(((I - 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I + 1)/(tan(b*x + a)^
2 + 1)) - 4*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x)
*log(-(tan(b*x + a) + 1)/(tan(b*x + a) - 1)) - (6*I*b*f^3*x + 6*I*b*e*f^2)*
polylog(4, (I*tan(b*x + a)^2 + 2*tan(b*x + a) - I)/(tan(b*x + a)^2 + 1)) -
(6*I*b*f^3*x + 6*I*b*e*f^2)*polylog(4, (I*tan(b*x + a)^2 - 2*tan(b*x + a) -
 I)/(tan(b*x + a)^2 + 1)) - (-6*I*b*f^3*x - 6*I*b*e*f^2)*polylog(4, (-I*tan
(b*x + a)^2 + 2*tan(b*x + a) + I)/(tan(b*x + a)^2 + 1)) - (-6*I*b*f^3*x - 6
*I*b*e*f^2)*polylog(4, (-I*tan(b*x + a)^2 - 2*tan(b*x + a) + I)/(tan(b*x +
a)^2 + 1)) - 6*(b^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*polylog(3, (I*tan(
b*x + a)^2 + 2*tan(b*x + a) - I)/(tan(b*x + a)^2 + 1)) + 6*(b^2*f^3*x^2 + 2
*b^2*e*f^2*x + b^2*e^2*f)*polylog(3, (I*tan(b*x + a)^2 - 2*tan(b*x + a) - I
)/(tan(b*x + a)^2 + 1)) - 6*(b^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*polyl
```

```
og(3, (-I*tan(b*x + a)^2 + 2*tan(b*x + a) + I)/(tan(b*x + a)^2 + 1)) + 6*(b
^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*polylog(3, (-I*tan(b*x + a)^2 - 2*t
an(b*x + a) + I)/(tan(b*x + a)^2 + 1)))/b^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (e + fx)^3 \operatorname{atanh}(\tan(a + bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*atanh(tan(b*x+a)),x)
```

```
[Out] Integral((e + f*x)**3*atanh(tan(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx + e)^3 \operatorname{artanh}(\tan(bx + a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctanh(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*arctanh(tan(b*x + a)), x)
```

3.313 $\int (e+fx)^2 \tanh^{-1}(\tan(a+bx))\,dx$

Optimal. Leaf size=234

$$\frac{f(e+fx)\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{4b^2}-\frac{f(e+fx)\text{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{4b^2}+\frac{if^2\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3}-\frac{if^2\text{PolyLog}\left(4,ie^{2i\right.}}{8b^3}$$

```
[Out] ((I/3)*(e + f*x)^3*ArcTan[E^((2*I)*(a + b*x))])/f + ((e + f*x)^3*ArcTanh[Ta
n[a + b*x]])/(3*f) - ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*(e + f*
x)*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(4*b^2) - (f*(e + f*x)*PolyLog[3,
I*E^((2*I)*(a + b*x))])/(4*b^2) + ((I/8)*f^2*PolyLog[4, (-I)*E^((2*I)*(a +
b*x))])/b^3 - ((I/8)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])/b^3
```

Rubi [A] time = 0.165241, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6251, 4181, 2531, 6609, 2282, 6589}

$$\frac{f(e+fx)\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{4b^2}-\frac{f(e+fx)\text{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{4b^2}+\frac{if^2\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3}-\frac{if^2\text{PolyLog}\left(4,ie^{2i\right.}}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*ArcTanh[Tan[a + b*x]],x]
```

```
[Out] ((I/3)*(e + f*x)^3*ArcTan[E^((2*I)*(a + b*x))])/f + ((e + f*x)^3*ArcTanh[Ta
n[a + b*x]])/(3*f) - ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*(e + f*
x)*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(4*b^2) - (f*(e + f*x)*PolyLog[3,
I*E^((2*I)*(a + b*x))])/(4*b^2) + ((I/8)*f^2*PolyLog[4, (-I)*E^((2*I)*(a +
b*x))])/b^3 - ((I/8)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])/b^3
```

Rule 6251

```
Int[ArcTanh[Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcTanh[Tan[a + b*x]])/(f*(m + 1)), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
 e, f}, x] && IGtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (e+fx)^2 \tanh^{-1}(\tan(a+bx))\,dx = \frac{(e+fx)^3 \tanh^{-1}(\tan(a+bx))}{3f} - \frac{b\int (e+fx)^3 \sec(2a+2bx)\,dx}{3f}$$

$$= \frac{i(e+fx)^3 \tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{(e+fx)^3 \tanh^{-1}(\tan(a+bx))}{3f} + \frac{1}{2}\int (e+fx)^2 \log\left(\right.$$

$$= \frac{i(e+fx)^3 \tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{(e+fx)^3 \tanh^{-1}(\tan(a+bx))}{3f} - \frac{i(e+fx)^2 \mathrm{Li}_2\left(-ie^{2i\right.}}{4b}$$

$$= \frac{i(e+fx)^3 \tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{(e+fx)^3 \tanh^{-1}(\tan(a+bx))}{3f} - \frac{i(e+fx)^2 \mathrm{Li}_2\left(-ie^{2i\right.}}{4b}$$

$$= \frac{i(e+fx)^3 \tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{(e+fx)^3 \tanh^{-1}(\tan(a+bx))}{3f} - \frac{i(e+fx)^2 \mathrm{Li}_2\left(-ie^{2i\right.}}{4b}$$

$$= \frac{i(e+fx)^3 \tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{(e+fx)^3 \tanh^{-1}(\tan(a+bx))}{3f} - \frac{i(e+fx)^2 \mathrm{Li}_2\left(-ie^{2i\right.}}{4b}$$

Mathematica [A] time = 0.64761, size = 409, normalized size = 1.75

$$\frac{1}{3}x\left(3e^2+3efx+f^2x^2\right)\tanh^{-1}(\tan(a+bx)) + \frac{-6ib^2(e+fx)^2\mathrm{PolyLog}\left(2,-ie^{2i(a+bx)}\right)+6ib^2(e+fx)^2\mathrm{PolyLog}\left(2,ie^{2i(a+\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*ArcTanh[Tan[a + b*x]],x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcTanh[Tan[a + b*x]])/3 + (-12*b^3*e^2*x*Lo
g[1 - I*E^((2*I)*(a + b*x))] - 12*b^3*e*f*x^2*Log[1 - I*E^((2*I)*(a + b*x))
] - 4*b^3*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] + 12*b^3*e^2*x*Log[1 + I*E
^((2*I)*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 4*b^3
*f^2*x^3*Log[1 + I*E^((2*I)*(a + b*x))] - (6*I)*b^2*(e + f*x)^2*PolyLog[2,
(-I)*E^((2*I)*(a + b*x))] + (6*I)*b^2*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a
+ b*x))] + 6*b*e*f*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] + 6*b*f^2*x*PolyLog
[3, (-I)*E^((2*I)*(a + b*x))] - 6*b*e*f*PolyLog[3, I*E^((2*I)*(a + b*x))] -
 6*b*f^2*x*PolyLog[3, I*E^((2*I)*(a + b*x))] + (3*I)*f^2*PolyLog[4, (-I)*E^
((2*I)*(a + b*x))] - (3*I)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])/(24*b^3)
```

Maple [C] time = 8.575, size = 5543, normalized size = 23.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*arctanh(tan(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{12}\left(f^2x^3+3efx^2+3e^2x\right)\log\left(2\cos\left(2bx+2a\right)^2+2\sin\left(2bx+2a\right)^2+4\sin\left(2bx+2a\right)+2\right)-\frac{1}{12}\left(f^2x^3+3efx^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctanh(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x
 + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*
log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) -
 integrate(2/3*((b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(4*b*x + 4*a)*cos(
2*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*sin(4*b*x + 4*a)*sin(2
*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(2*b*x + 2*a))/(cos(
4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

Fricas [C] time = 2.4866, size = 3359, normalized size = 14.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctanh(tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/48*(3*I*f^2*polylog(4, (I*tan(b*x + a)^2 + 2*tan(b*x + a) - I)/(tan(b*x +
 a)^2 + 1)) + 3*I*f^2*polylog(4, (I*tan(b*x + a)^2 - 2*tan(b*x + a) - I)/(t
an(b*x + a)^2 + 1)) - 3*I*f^2*polylog(4, (-I*tan(b*x + a)^2 + 2*tan(b*x + a
) + I)/(tan(b*x + a)^2 + 1)) - 3*I*f^2*polylog(4, (-I*tan(b*x + a)^2 - 2*ta
n(b*x + a) + I)/(tan(b*x + a)^2 + 1)) + (6*I*b^2*f^2*x^2 + 12*I*b^2*e*f*x +
 6*I*b^2*e^2)*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(tan
(b*x + a)^2 + 1) + 1) + (6*I*b^2*f^2*x^2 + 12*I*b^2*e*f*x + 6*I*b^2*e^2)*di
log(-((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)
```

```
 + 1) + (-6*I*b^2*f^2*x^2 - 12*I*b^2*e*f*x - 6*I*b^2*e^2)*dilog(-(-(I - 1)*
tan(b*x + a)^2 + 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) + (-6*I*
b^2*f^2*x^2 - 12*I*b^2*e*f*x - 6*I*b^2*e^2)*dilog(-(-(I - 1)*tan(b*x + a)^2
 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) - 4*(b^3*f^2*x^3 + 3*b
^3*e*f*x^2 + 3*b^3*e^2*x + 3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(((I + 1
)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) + 4*(3*a*b
^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(((I + 1)*tan(b*x + a)^2 + 2*I*tan(b*x +
 a) + I - 1)/(tan(b*x + a)^2 + 1)) - 4*(3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2
)*log(((I + 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I - 1)/(tan(b*x + a)^2 +
 1)) + 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x + 3*a*b^2*e^2 - 3*a^2*b
*e*f + a^3*f^2)*log(((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1)/(tan(
b*x + a)^2 + 1)) - 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x + 3*a*b^2*e
^2 - 3*a^2*b*e*f + a^3*f^2)*log((-(I - 1)*tan(b*x + a)^2 + 2*tan(b*x + a) +
 I + 1)/(tan(b*x + a)^2 + 1)) + 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*
x + 3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log((-(I - 1)*tan(b*x + a)^2 - 2*t
an(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + 4*(3*a*b^2*e^2 - 3*a^2*b*e*f +
 a^3*f^2)*log(((I - 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) + I + 1)/(tan(b*x
+ a)^2 + 1)) - 4*(3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(((I - 1)*tan(b*x
 + a)^2 - 2*I*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + 8*(b^3*f^2*x^3
+ 3*b^3*e*f*x^2 + 3*b^3*e^2*x)*log(-(tan(b*x + a) + 1)/(tan(b*x + a) - 1))
+ 6*(b*f^2*x + b*e*f)*polylog(3, (I*tan(b*x + a)^2 + 2*tan(b*x + a) - I)/(t
an(b*x + a)^2 + 1)) - 6*(b*f^2*x + b*e*f)*polylog(3, (I*tan(b*x + a)^2 - 2*
tan(b*x + a) - I)/(tan(b*x + a)^2 + 1)) + 6*(b*f^2*x + b*e*f)*polylog(3, (-
I*tan(b*x + a)^2 + 2*tan(b*x + a) + I)/(tan(b*x + a)^2 + 1)) - 6*(b*f^2*x +
 b*e*f)*polylog(3, (-I*tan(b*x + a)^2 - 2*tan(b*x + a) + I)/(tan(b*x + a)^2
 + 1)))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e+fx\right)^2 \operatorname{atanh}\left(\tan\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*atanh(tan(b*x+a)),x)
```

```
[Out] Integral((e + f*x)**2*atanh(tan(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx+e\right)^2 \operatorname{artanh}\left(\tan\left(bx+a\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctanh(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*arctanh(tan(b*x + a)), x)
```

3.314 $\int (e+fx)\tanh^{-1}(\tan(a+bx))\,dx$

Optimal. Leaf size=162

$$\frac{f\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{8b^2}-\frac{f\text{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{8b^2}-\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx)\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b}$$

```
[Out] ((I/2)*(e + f*x)^2*ArcTan[E^((2*I)*(a + b*x))])/f + ((e + f*x)^2*ArcTanh[Ta
n[a + b*x]])/(2*f) - ((I/4)*(e + f*x)*PolyLog[2, (-I)*E^((2*I)*(a + b*x))])
/b + ((I/4)*(e + f*x)*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*PolyLog[3,
(-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (f*PolyLog[3, I*E^((2*I)*(a + b*x))])/
(8*b^2)
```

Rubi [A] time = 0.108099, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6251, 4181, 2531, 2282, 6589}

$$\frac{f\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{8b^2}-\frac{f\text{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{8b^2}-\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx)\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*ArcTanh[Tan[a + b*x]],x]
```


```
[Out] ((I/2)*(e + f*x)^2*ArcTan[E^((2*I)*(a + b*x))])/f + ((e + f*x)^2*ArcTanh[Ta
n[a + b*x]])/(2*f) - ((I/4)*(e + f*x)*PolyLog[2, (-I)*E^((2*I)*(a + b*x))])
/b + ((I/4)*(e + f*x)*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*PolyLog[3,
(-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (f*PolyLog[3, I*E^((2*I)*(a + b*x))])/
(8*b^2)
```


Rule 6251

```
Int[ArcTanh[Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcTanh[Tan[a + b*x]])/(f*(m + 1)), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
 e, f}, x] && IGtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
```

```
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (e+fx)\tanh^{-1}(\tan(a+bx))\,dx = \frac{(e+fx)^2\tanh^{-1}(\tan(a+bx))}{2f} - \frac{b\int (e+fx)^2\sec(2a+2bx)\,dx}{2f}$$

$$= \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} + \frac{(e+fx)^2\tanh^{-1}(\tan(a+bx))}{2f} + \frac{1}{2}\int (e+fx)\log\left(1\right.$$

$$= \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} + \frac{(e+fx)^2\tanh^{-1}(\tan(a+bx))}{2f} - \frac{i(e+fx)\text{Li}_2\left(-ie^{2i\cdots}\right.}{4b}$$

$$= \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} + \frac{(e+fx)^2\tanh^{-1}(\tan(a+bx))}{2f} - \frac{i(e+fx)\text{Li}_2\left(-ie^{2i\cdots}\right.}{4b}$$

$$= \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} + \frac{(e+fx)^2\tanh^{-1}(\tan(a+bx))}{2f} - \frac{i(e+fx)\text{Li}_2\left(-ie^{2i\cdots}\right.}{4b}$$

Mathematica [A] time = 0.291724, size = 263, normalized size = 1.62

$$-be\left(\frac{i\text{PolyLog}\left(2,-ie^{i(2a+2bx)}\right)}{4b^2}-\frac{i\text{PolyLog}\left(2,ie^{i(2a+2bx)}\right)}{4b^2}-\frac{ix\tan^{-1}\left(e^{2ia+2ibx}\right)}{b}\right)+\frac{f\left(2ibx\text{PolyLog}(2,-\sin(2(a+bx))+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)*ArcTanh[Tan[a + b*x]],x]
```

```
[Out] e*x*ArcTanh[Tan[a + b*x]] + (f*x^2*ArcTanh[Tan[a + b*x]])/2 - b*e*(((-I)*x*
ArcTan[E^((2*I)*a + (2*I)*b*x)])/b + ((I/4)*PolyLog[2, (-I)*E^(I*(2*a + 2*b
*x))])/b^2 - ((I/4)*PolyLog[2, I*E^(I*(2*a + 2*b*x))])/b^2) + (f*((4*I)*b^2
*x^2*ArcTan[Cos[2*(a + b*x)] + I*Sin[2*(a + b*x)]] + (2*I)*b*x*PolyLog[2, I
*Cos[2*(a + b*x)] - Sin[2*(a + b*x)]] - (2*I)*b*x*PolyLog[2, (-I)*Cos[2*(a
+ b*x)] + Sin[2*(a + b*x)]] - PolyLog[3, I*Cos[2*(a + b*x)] - Sin[2*(a + b*
x)]] + PolyLog[3, (-I)*Cos[2*(a + b*x)] + Sin[2*(a + b*x)]]))/(8*b^2)
```

Maple [C] time = 4.416, size = 2543, normalized size = 15.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*arctanh(tan(b*x+a)),x)
```

```
[Out] -1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^3+1/4*I*Pi*
x*e*csgn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2+1/2*(1/2*f*x^2+
e*x)*ln(exp(2*I*(b*x+a))+I)-1/4*I*Pi*x*e*csgn((1-I)*(exp(2*I*(b*x+a))-I)/(e
xp(2*I*(b*x+a))+1))^3+1/4/b^2*f*a^2*ln(-exp(2*I*(b*x+a))+I)-1/2/b*e*a*ln(-e
xp(2*I*(b*x+a))+I)+1/2/b*a*e*ln(exp(2*I*(b*x+a))+I)-1/4/b^2*f*a^2*ln(exp(2*
I*(b*x+a))+I)+1/4/b^2*f*(I*b*x+I*a)^2*ln(1-I*exp(2*I*(b*x+a)))+1/8*f*polylo
g(3,-I*exp(2*I*(b*x+a)))/b^2-1/8*f*polylog(3,I*exp(2*I*(b*x+a)))/b^2-1/4*ln
(exp(2*I*(b*x+a))-I)*x^2*f-1/2*ln(exp(2*I*(b*x+a))-I)*x*e-1/8*I*Pi*f*csgn((
1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3*x^2+1/2*I/b*e*(I*b*x+I*a)
*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))-1/2*I/b*e*(I*b*x+I*a)*ln(1+exp(
I*(b*x+a))*(-1)^(3/4))-1/2*I/b*e*(I*b*x+I*a)*ln(1-exp(I*(b*x+a))*(-1)^(3/4)
)+1/2*I/b^2*f*a*dilog(1+exp(I*(b*x+a))*(-1)^(3/4))+1/4/b^2*f*(I*b*x+I*a)*po
lylog(2,I*exp(2*I*(b*x+a)))-1/4/b^2*f*(I*b*x+I*a)^2*ln(1+I*exp(2*I*(b*x+a))
)-1/4/b^2*f*(I*b*x+I*a)*polylog(2,-I*exp(2*I*(b*x+a)))+1/8*I*Pi*f*csgn((1+I
)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2*x^2+1/2*I/b^2*f*a*dilog(1-ex
p(I*(b*x+a))*(-1)^(3/4))-1/2*I/b^2*f*a*dilog(((-I)^(1/2)-exp(I*(b*x+a)))/(-
```

I)^(1/2))-1/2*I/b^2*f*a*dilog(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))+1/2*I /b*e*(I*b*x+I*a)*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))-1/4*I*Pi*x*e-1/ 8*I*Pi*f*x^2+1/4*I*Pi*x*e*csgn((1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a)) +1))^2-1/2*I/b*e*dilog(1+exp(I*(b*x+a))*(-1)^(3/4))-1/2*I/b*e*dilog(1-exp(I *(b*x+a))*(-1)^(3/4))+1/2*I/b*e*dilog(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))+1/2*I/b*e*dilog(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))-1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^3*x^2+1/8*I*Pi*f*csgn((1-I)*(e xp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2*x^2+1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))*csgn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2*x^2-1/2*I/b^2*f*a*(I*b*x+I*a)*ln(((-I)^(1/2)-exp(I*(b*x+a)))/ (-I)^(1/2))-1/2*I/b^2*f*a*(I*b*x+I*a)*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))-1/8*I*Pi*f*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2*x^2+1/8*I*Pi*f*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2*x^2+1/8*I*Pi*f*csgn(I*(exp(2*I* (b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2*x^2+1/4*I* Pi*x*e*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b *x+a))+1))^2+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x +a))+I)/(exp(2*I*(b*x+a))+1))^2+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I)/(e xp(2*I*(b*x+a))+1))^3-1/4*I*Pi*x*e*csgn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I *(b*x+a))+1))^3-1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))*csgn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))*x^2+1/8*I*Pi*f*csgn (I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*csgn((1-I)*(exp(2*I*(b*x+a))- I)/(exp(2*I*(b*x+a))+1))*x^2-1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))*csgn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))+1/4* I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*csgn((1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))-1/4*I*Pi*x*e*csgn(I/(exp(2*I*(b*x+a)) +1))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))*csgn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2-1/8*I*Pi*f*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(e xp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))*x^2-1 /4*I*Pi*x*e*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))-1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2-1/4*I*Pi*x* e*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*csgn((1-I)*(exp(2*I*(b* x+a))-I)/(exp(2*I*(b*x+a))+1))^2-1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))-I)/(ex p(2*I*(b*x+a))+1))*csgn((1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2* x^2-1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))-I)/(ex p(2*I*(b*x+a))+1))^2*x^2+1/2*I/b^2*f*a*(I*b*x+I*a)*ln(1+exp(I*(b*x+a))*(-1) ^(3/4))+1/2*I/b^2*f*a*(I*b*x+I*a)*ln(1-exp(I*(b*x+a))*(-1)^(3/4))+1/8*I*Pi* f*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3*x^2-1/8*I*Pi*f*csgn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^3*x^2+1/8*I*Pi*f*csgn(I/(ex p(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))-I) /(exp(2*I*(b*x+a))+1))*x^2+1/4*I*Pi*x*e*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I *(exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{8}\left(fx^2+2\,ex\right)\log\left(2\,\cos\left(2\,bx+2\,a\right)^2+2\,\sin\left(2\,bx+2\,a\right)^2+4\,\sin\left(2\,bx+2\,a\right)+2\right)-\frac{1}{8}\left(fx^2+2\,ex\right)\log\left(2\,\cos\left(2\,bx+2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctanh(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/8*(f*x^2 + 2*e*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin
(2*b*x + 2*a) + 2) - 1/8*(f*x^2 + 2*e*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2
*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(((b*f*x^2 + 2*b*e*x)*co
s(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f*x^2 + 2*b*e*x)*sin(4*b*x + 4*a)*sin(
2*b*x + 2*a) + (b*f*x^2 + 2*b*e*x)*cos(2*b*x + 2*a))/(cos(4*b*x + 4*a)^2 +
sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

Fricas [C] time = 2.16103, size = 2314, normalized size = 14.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctanh(tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/16*((2*I*b*f*x + 2*I*b*e)*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a)
 - I + 1)/(tan(b*x + a)^2 + 1) + 1) + (2*I*b*f*x + 2*I*b*e)*dilog(-((I + 1)
*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1) + 1) + (-2*I
*b*f*x - 2*I*b*e)*dilog(-(-(I - 1)*tan(b*x + a)^2 + 2*tan(b*x + a) + I + 1)
/(tan(b*x + a)^2 + 1) + 1) + (-2*I*b*f*x - 2*I*b*e)*dilog(-(-(I - 1)*tan(b*
x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) - 2*(b^2*f*x^2
 + 2*b^2*e*x + 2*a*b*e - a^2*f)*log(((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a
) - I + 1)/(tan(b*x + a)^2 + 1)) + 2*(2*a*b*e - a^2*f)*log(((I + 1)*tan(b*x
 + a)^2 + 2*I*tan(b*x + a) + I - 1)/(tan(b*x + a)^2 + 1)) - 2*(2*a*b*e - a^
2*f)*log(((I + 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I - 1)/(tan(b*x + a)^
2 + 1)) + 2*(b^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a^2*f)*log(((I + 1)*tan(b*x
+ a)^2 - 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) - 2*(b^2*f*x^2 + 2*b
^2*e*x + 2*a*b*e - a^2*f)*log((-(I - 1)*tan(b*x + a)^2 + 2*tan(b*x + a) + I
 + 1)/(tan(b*x + a)^2 + 1)) + 2*(b^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a^2*f)*l
og((-(I - 1)*tan(b*x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1))
 + 2*(2*a*b*e - a^2*f)*log(((I - 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) + I +
```

```
 1)/(tan(b*x + a)^2 + 1)) - 2*(2*a*b*e - a^2*f)*log(((I - 1)*tan(b*x + a)^2
 - 2*I*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + 4*(b^2*f*x^2 + 2*b^2*e
*x)*log(-(tan(b*x + a) + 1)/(tan(b*x + a) - 1)) + f*polylog(3, (I*tan(b*x +
 a)^2 + 2*tan(b*x + a) - I)/(tan(b*x + a)^2 + 1)) - f*polylog(3, (I*tan(b*x
 + a)^2 - 2*tan(b*x + a) - I)/(tan(b*x + a)^2 + 1)) + f*polylog(3, (-I*tan(
b*x + a)^2 + 2*tan(b*x + a) + I)/(tan(b*x + a)^2 + 1)) - f*polylog(3, (-I*t
an(b*x + a)^2 - 2*tan(b*x + a) + I)/(tan(b*x + a)^2 + 1)))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e+fx\right)\operatorname{atanh}\left(\tan\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*atanh(tan(b*x+a)),x)
```

```
[Out] Integral((e + f*x)*atanh(tan(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx+e\right)\operatorname{artanh}\left(\tan\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctanh(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*arctanh(tan(b*x + a)), x)
```

3.315 $\int \tanh^{-1}(\tan(a+bx))\,dx$

Optimal. Leaf size=79

$$-\frac{i\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b}+ix\tan^{-1}\left(e^{2i(a+bx)}\right)+x\tanh^{-1}(\tan(a+bx))$$

```
[Out] I*x*ArcTan[E^((2*I)*(a + b*x))] + x*ArcTanh[Tan[a + b*x]] - ((I/4)*PolyLog[
2, (-I)*E^((2*I)*(a + b*x))])/b + ((I/4)*PolyLog[2, I*E^((2*I)*(a + b*x))])
/b
```

Rubi [A] time = 0.04757, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6247, 4181, 2279, 2391}

$$-\frac{i\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b}+ix\tan^{-1}\left(e^{2i(a+bx)}\right)+x\tanh^{-1}(\tan(a+bx))$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tan[a + b*x]],x]
```

```
[Out] I*x*ArcTan[E^((2*I)*(a + b*x))] + x*ArcTanh[Tan[a + b*x]] - ((I/4)*PolyLog[
2, (-I)*E^((2*I)*(a + b*x))])/b + ((I/4)*PolyLog[2, I*E^((2*I)*(a + b*x))])
/b
```

Rule 6247

```
Int[ArcTanh[Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcTanh[Tan[a + b
*x]], x] - Dist[b, Int[x*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \tanh^{-1}(\tan(a+bx))\,dx = x\tanh^{-1}(\tan(a+bx)) - b\int x\sec(2a+2bx)\,dx$$

$$= ix\tan^{-1}\left(e^{2i(a+bx)}\right) + x\tanh^{-1}(\tan(a+bx)) + \frac{1}{2}\int \log\left(1-ie^{i(2a+2bx)}\right)dx - \frac{1}{2}\int \log\left(1+\right.$$

$$= ix\tan^{-1}\left(e^{2i(a+bx)}\right) + x\tanh^{-1}(\tan(a+bx)) - \frac{i\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{i(2a+2bx)}\right)}{4b} + \frac{i\,\text{Sub}}{}$$

$$= ix\tan^{-1}\left(e^{2i(a+bx)}\right) + x\tanh^{-1}(\tan(a+bx)) - \frac{i\text{Li}_2\left(-ie^{2i(a+bx)}\right)}{4b} + \frac{i\text{Li}_2\left(ie^{2i(a+bx)}\right)}{4b}$$

Mathematica [A] time = 0.0267131, size = 74, normalized size = 0.94

$$\frac{i\left(-\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)+\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)+4bx\left(\tan^{-1}\left(e^{2i(a+bx)}\right)-i\tanh^{-1}(\tan(a+bx))\right)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tan[a + b*x]],x]
```

```
[Out] ((I/4)*(4*b*x*(ArcTan[E^((2*I)*(a + b*x))] - I*ArcTanh[Tan[a + b*x]]) - Pol
yLog[2, (-I)*E^((2*I)*(a + b*x))] + PolyLog[2, I*E^((2*I)*(a + b*x))]))/b
```

Maple [B] time = 0.138, size = 178, normalized size = 2.3

$$\frac{\arctan(\tan(bx+a))\,\text{Artanh}(\tan(bx+a))}{b} + \frac{\arctan(\tan(bx+a))}{2b}\ln\left(1+\frac{i(1+i\tan(bx+a))^2}{1+(\tan(bx+a))^2}\right) - \frac{\frac{i}{4}}{b}\text{polylog}\left(2,\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tan(b*x+a)),x)
```

```
[Out] 1/b*arctan(tan(b*x+a))*arctanh(tan(b*x+a))+1/2/b*arctan(tan(b*x+a))*ln(1+I*
(1+I*tan(b*x+a))^2/(1+tan(b*x+a)^2))-1/4*I/b*polylog(2,-I*(1+I*tan(b*x+a))^
2/(1+tan(b*x+a)^2))-1/2/b*arctan(tan(b*x+a))*ln(1-I*(1+I*tan(b*x+a))^2/(1+t
an(b*x+a)^2))+1/4*I/b*polylog(2,I*(1+I*tan(b*x+a))^2/(1+tan(b*x+a)^2))
```

Maxima [B] time = 1.59488, size = 246, normalized size = 3.11

$$\frac{4\,(bx+a)\,\text{artanh}\,(\tan\,(bx+a)) + \left(\arctan\left(\frac{1}{2}\,\tan\,(bx+a)+\frac{1}{2},\frac{1}{2}\,\tan\,(bx+a)+\frac{1}{2}\right) - \arctan\left(\frac{1}{2}\,\tan\,(bx+a)-\frac{1}{2},-\frac{1}{2}\,\text{ta}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(b*x + a)*arctanh(tan(b*x + a)) + (arctan2(1/2*tan(b*x + a) + 1/2, 1
/2*tan(b*x + a) + 1/2) - arctan2(1/2*tan(b*x + a) - 1/2, -1/2*tan(b*x + a)
+ 1/2))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log(1/2*tan(b*x + a)^2 + tan(b*
x + a) + 1/2) + (b*x + a)*log(1/2*tan(b*x + a)^2 - tan(b*x + a) + 1/2) - I*
dilog((1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2) + I*dilog(-(1/2*I - 1/2)*ta
n(b*x + a) + 1/2*I + 1/2) + I*dilog((1/2*I - 1/2)*tan(b*x + a) + 1/2*I + 1/
2) - I*dilog(-(1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2))/b
```

Fricas [B] time = 2.29526, size = 1494, normalized size = 18.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*log(-(tan(b*x + a) + 1)/(tan(b*x + a) - 1)) - 2*(b*x + a)*log(((
I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) + 2*a
*log(((I + 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) + I - 1)/(tan(b*x + a)^2 +
1)) - 2*a*log(((I + 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I - 1)/(tan(b*x
+ a)^2 + 1)) + 2*(b*x + a)*log(((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I
 + 1)/(tan(b*x + a)^2 + 1)) - 2*(b*x + a)*log((-(I - 1)*tan(b*x + a)^2 + 2*
tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + 2*(b*x + a)*log((-(I - 1)*tan
```

```
(b*x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + 2*a*log(((I -
 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) - 2*a*
log(((I - 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1
)) + I*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(tan(b*x +
a)^2 + 1) + 1) + I*dilog(-((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1)
/(tan(b*x + a)^2 + 1) + 1) - I*dilog(-(-(I - 1)*tan(b*x + a)^2 + 2*tan(b*x
+ a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) - I*dilog(-(-(I - 1)*tan(b*x + a)^2
 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{atanh}(\tan(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tan(b*x+a)),x)
```

```
[Out] Integral(atanh(tan(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{artanh}(\tan(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(tan(b*x + a)), x)
```

3.316 $\int \frac{\tanh^{-1}(\tan(a+bx))}{e+fx}\,dx$

Optimal. Leaf size=17

$$\text{CannotIntegrate}\left(\frac{\tanh^{-1}(\tan(a+bx))}{e+fx},x\right)$$

```
[Out] CannotIntegrate[ArcTanh[Tan[a + b*x]]/(e + f*x), x]
```

Rubi [A] time = 0.0400403, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tanh^{-1}(\tan(a+bx))}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTanh[Tan[a + b*x]]/(e + f*x),x]

[Out] Defer[Int][ArcTanh[Tan[a + b*x]]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tan(a+bx))}{e+fx}\,dx = \int \frac{\tanh^{-1}(\tan(a+bx))}{e+fx}\,dx$$

Mathematica [A] time = 5.06333, size = 0, normalized size = 0.

$$\int \frac{\tanh^{-1}(\tan(a+bx))}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTanh[Tan[a + b*x]]/(e + f*x),x]
```

```
[Out] Integrate[ArcTanh[Tan[a + b*x]]/(e + f*x), x]
```

Maple [A] time = 0.602, size = 0, normalized size = 0.

$$\int \frac{\text{Artanh}\left(\tan\left(bx+a\right)\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tan(b*x+a))/(f*x+e),x)
```

```
[Out] int(arctanh(tan(b*x+a))/(f*x+e),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{artanh}\left(\tan\left(bx+a\right)\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tan(b*x+a))/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tan(b*x + a))/(f*x + e), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{artanh}\left(\tan\left(bx+a\right)\right)}{fx+e},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tan(b*x+a))/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral(arctanh(tan(b*x + a))/(f*x + e), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}(\tan(a+bx))}{e+fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tan(b*x+a))/(f*x+e),x)
```

```
[Out] Integral(atanh(tan(a + b*x))/(e + f*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(\tan(bx+a))}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tan(b*x+a))/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(arctanh(tan(b*x + a))/(f*x + e), x)
```

3.317 $\int x^2 \tanh^{-1}(c + d\tan(a + bx))\,dx$

Optimal. Leaf size=395

$$\frac{x\text{PolyLog}\left(3,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b^2}-\frac{x\text{PolyLog}\left(3,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b^2}+\frac{i\text{PolyLog}\left(4,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{8b^3}-\frac{i\text{PolyLog}\left(4,-\frac{(c}{}\right.}{8b^3}$$

```
[Out] (x^3*ArcTanh[c + d*Tan[a + b*x]])/3 + (x^3*Log[1 + ((1 - c + I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 - c - I*d)])/6 - (x^3*Log[1 + ((1 + c - I*d)*E^((2*I)*a
+ (2*I)*b*x))/(1 + c + I*d)])/6 - ((I/4)*x^2*PolyLog[2, -(((1 - c + I*d)*E^
((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/b + ((I/4)*x^2*PolyLog[2, -(((1 + c
 - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/b + (x*PolyLog[3, -(((1 -
 c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/(4*b^2) - (x*PolyLog[3,
 -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/(4*b^2) + ((I/8
)*PolyLog[4, -(((1 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/b^3
 - ((I/8)*PolyLog[4, -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d
))])/b^3
```

Rubi [A] time = 0.49861, antiderivative size = 395, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6267, 2190, 2531, 6609, 2282, 6589}

$$\frac{x\text{PolyLog}\left(3,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b^2}-\frac{x\text{PolyLog}\left(3,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b^2}+\frac{i\text{PolyLog}\left(4,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{8b^3}-\frac{i\text{PolyLog}\left(4,-\frac{(c}{}\right.}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[c + d*Tan[a + b*x]])/3 + (x^3*Log[1 + ((1 - c + I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 - c - I*d)])/6 - (x^3*Log[1 + ((1 + c - I*d)*E^((2*I)*a
+ (2*I)*b*x))/(1 + c + I*d)])/6 - ((I/4)*x^2*PolyLog[2, -(((1 - c + I*d)*E^
((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/b + ((I/4)*x^2*PolyLog[2, -(((1 + c
 - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/b + (x*PolyLog[3, -(((1 -
 c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/(4*b^2) - (x*PolyLog[3,
 -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/(4*b^2) + ((I/8
)*PolyLog[4, -(((1 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/b^3
 - ((I/8)*PolyLog[4, -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d
))])/b^3
```

Rule 6267

```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Tan[a + b*x]])/(f*(m
 + 1)), x] + (-Dist[(I*b*(1 + c - I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)
*E^(2*I*a + 2*I*b*x))/(1 + c + I*d + (1 + c - I*d)*E^(2*I*a + 2*I*b*x)), x]
, x] + Dist[(I*b*(1 - c + I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*I*
a + 2*I*b*x))/(1 - c - I*d + (1 - c + I*d)*E^(2*I*a + 2*I*b*x)), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, 1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
```

```
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\tanh^{-1}(c+d\tan(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(c+d\tan(a+bx))+\frac{1}{3}(b(i(1-c)-d))\int\frac{e^{2ia+2ibx}x^3}{1-c-id+(1-c+id)e^{2ia+}}$$

$$=\frac{1}{3}x^3\tanh^{-1}(c+d\tan(a+bx))+\frac{1}{6}x^3\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)-\frac{1}{6}x^3\log\Big(1-$$

$$=\frac{1}{3}x^3\tanh^{-1}(c+d\tan(a+bx))+\frac{1}{6}x^3\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)-\frac{1}{6}x^3\log\Big(1-$$

$$=\frac{1}{3}x^3\tanh^{-1}(c+d\tan(a+bx))+\frac{1}{6}x^3\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)-\frac{1}{6}x^3\log\Big(1-$$

$$=\frac{1}{3}x^3\tanh^{-1}(c+d\tan(a+bx))+\frac{1}{6}x^3\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)-\frac{1}{6}x^3\log\Big(1-$$

$$=\frac{1}{3}x^3\tanh^{-1}(c+d\tan(a+bx))+\frac{1}{6}x^3\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)-\frac{1}{6}x^3\log\Big(1-$$

Mathematica [A] time = 0.853248, size = 346, normalized size = 0.88

$$\frac{1}{3}x^3\tanh^{-1}(d\tan(a+bx)+c)+\frac{-6ib^2x^2\text{PolyLog}\left(2,\frac{(-c+id+1)e^{2i(a+bx)}}{c+id-1}\right)+6ib^2x^2\text{PolyLog}\left(2,-\frac{(c-id+1)e^{2i(a+bx)}}{c+id+1}\right)+6bx\text{Poly}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[c + d*Tan[a + b*x]])/3 + (4*b^3*x^3*Log[1 + ((-1 + c - I*d)*E^
((2*I)*(a + b*x)))/(-1 + c + I*d)] - 4*b^3*x^3*Log[1 + ((1 + c - I*d)*E^((2
*I)*(a + b*x)))/(1 + c + I*d)] - (6*I)*b^2*x^2*PolyLog[2, ((1 - c + I*d)*E^
((2*I)*(a + b*x)))/(-1 + c + I*d)] + (6*I)*b^2*x^2*PolyLog[2, -(((1 + c - I
*d)*E^((2*I)*(a + b*x)))/(1 + c + I*d))] + 6*b*x*PolyLog[3, ((1 - c + I*d)*
E^((2*I)*(a + b*x)))/(-1 + c + I*d)] - 6*b*x*PolyLog[3, -(((1 + c - I*d)*E^
((2*I)*(a + b*x)))/(1 + c + I*d))] + (3*I)*PolyLog[4, ((1 - c + I*d)*E^((2*
I)*(a + b*x)))/(-1 + c + I*d)] - (3*I)*PolyLog[4, -(((1 + c - I*d)*E^((2*I)
```

```
*(a + b*x)))/(1 + c + I*d))])/(24*b^3)
```

Maple [C] time = 4.986, size = 6967, normalized size = 17.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(c+d*tan(b*x+a)),x)

[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*tan(b*x+a)),x, algorithm="maxima")

[Out] 1/12*x^3*log((c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*b
*x + 2*a) + (c^2 + d^2 + 2*c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 + 2*(c^2 -
 d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 2*c + 1) - 1/12*x^3*log((c^2 + d^2 - 2*c
 + 1)*cos(2*b*x + 2*a)^2 + 4*(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 - 2*c
+ 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 + 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*x + 2
*a) - 2*c + 1) - 4*b*d*integrate(-1/3*(2*(c^2 + d^2 - 1)*x^3*cos(2*b*x + 2*
a)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^3*sin(2*b*x + 2*a)^
2 + (c^2 - d^2 - 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2*b*x + 2*a) - (c
^2 - d^2 - 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^3*cos(2*b*x
 + 2*a) + (c^2 - d^2 - 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 + d^
4 + 2*(c^2 + 1)*d^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*cos(4*b*x +
 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x + 2*a)^2 +
(c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 + d^4
 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 + 2*(c^4 + d^4 -
 2*(3*c^2 - 1)*d^2 - 2*c^2 + 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4
*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*(c^4 - d^
4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3 - c)*d - 2*(c*d^3 + (
c^3 - c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*sin(2*b*x + 2*a))*si
n(4*b*x + 4*a) + 8*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1), x)
```

Fricas [C] time = 2.9781, size = 5873, normalized size = 14.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/48*(8*b^3*x^3*log(-(d*tan(b*x + a) + c + 1)/(d*tan(b*x + a) + c - 1)) + 6
*I*b^2*x^2*dilog(-((2*I*(c + 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 - 2*I*(c
+ 1)*d + (2*I*c^2 + 4*(c + 1)*d - 2*I*d^2 + 4*I*c + 2*I)*tan(b*x + a) + 4*c
 + 2)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) - 6
*I*b^2*x^2*dilog(-((-2*I*(c + 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 + 2*I*(c
 + 1)*d + (-2*I*c^2 + 4*(c + 1)*d + 2*I*d^2 - 4*I*c - 2*I)*tan(b*x + a) + 4
*c + 2)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) -
 6*I*b^2*x^2*dilog(-((2*I*(c - 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 - 2*I*(
c - 1)*d + (2*I*c^2 + 4*(c - 1)*d - 2*I*d^2 - 4*I*c + 2*I)*tan(b*x + a) - 4
*c + 2)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1) + 1) +
 6*I*b^2*x^2*dilog(-((-2*I*(c - 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 + 2*I*
(c - 1)*d + (-2*I*c^2 + 4*(c - 1)*d + 2*I*d^2 + 4*I*c - 2*I)*tan(b*x + a) -
 4*c + 2)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1) + 1)
 + 4*a^3*log(((I*(c + 1)*d + d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)*d + (I*c
^2 + I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/(tan(b*x + a)^2 + 1)) + 4*a
^3*log(((I*(c + 1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*(c + 1)*d + (I*c^2 + I
*d^2 + 2*I*c + I)*tan(b*x + a) + 2*c + 1)/(tan(b*x + a)^2 + 1)) - 4*a^3*log
(((I*(c - 1)*d + d^2)*tan(b*x + a)^2 - c^2 + I*(c - 1)*d + (I*c^2 + I*d^2 -
 2*I*c + I)*tan(b*x + a) + 2*c - 1)/(tan(b*x + a)^2 + 1)) - 4*a^3*log(((I*(
c - 1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*(c - 1)*d + (I*c^2 + I*d^2 - 2*I*c
 + I)*tan(b*x + a) - 2*c + 1)/(tan(b*x + a)^2 + 1)) - 6*b*x*polylog(3, ((c^
2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*tan(b*x + a)^2 - c^2 - 2*I*(c + 1)*d + d
^2 + (2*I*c^2 - 4*(c + 1)*d - 2*I*d^2 + 4*I*c + 2*I)*tan(b*x + a) - 2*c - 1
)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) - 6*b*x*pol
ylog(3, ((c^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*tan(b*x + a)^2 - c^2 + 2*I*(
c + 1)*d + d^2 + (-2*I*c^2 - 4*(c + 1)*d + 2*I*d^2 - 4*I*c - 2*I)*tan(b*x +
 a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)
) + 6*b*x*polylog(3, ((c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*tan(b*x + a)^2
- c^2 - 2*I*(c - 1)*d + d^2 + (2*I*c^2 - 4*(c - 1)*d - 2*I*d^2 - 4*I*c + 2*
I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^
2 - 2*c + 1)) + 6*b*x*polylog(3, ((c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*tan
(b*x + a)^2 - c^2 + 2*I*(c - 1)*d + d^2 + (-2*I*c^2 - 4*(c - 1)*d + 2*I*d^2
 + 4*I*c - 2*I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)
^2 + c^2 + d^2 - 2*c + 1)) - 4*(b^3*x^3 + a^3)*log(((2*I*(c + 1)*d + 2*d^2)
*tan(b*x + a)^2 + 2*c^2 - 2*I*(c + 1)*d + (2*I*c^2 + 4*(c + 1)*d - 2*I*d^2
```

```
+ 4*I*c + 2*I)*tan(b*x + a) + 4*c + 2)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^
2 + c^2 + d^2 + 2*c + 1)) - 4*(b^3*x^3 + a^3)*log(((-2*I*(c + 1)*d + 2*d^2)
*tan(b*x + a)^2 + 2*c^2 + 2*I*(c + 1)*d + (-2*I*c^2 + 4*(c + 1)*d + 2*I*d^2
 - 4*I*c - 2*I)*tan(b*x + a) + 4*c + 2)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)
^2 + c^2 + d^2 + 2*c + 1)) + 4*(b^3*x^3 + a^3)*log(((2*I*(c - 1)*d + 2*d^2)
*tan(b*x + a)^2 + 2*c^2 - 2*I*(c - 1)*d + (2*I*c^2 + 4*(c - 1)*d - 2*I*d^2
- 4*I*c + 2*I)*tan(b*x + a) - 4*c + 2)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^
2 + c^2 + d^2 - 2*c + 1)) + 4*(b^3*x^3 + a^3)*log(((-2*I*(c - 1)*d + 2*d^2)
*tan(b*x + a)^2 + 2*c^2 + 2*I*(c - 1)*d + (-2*I*c^2 + 4*(c - 1)*d + 2*I*d^2
 + 4*I*c - 2*I)*tan(b*x + a) - 4*c + 2)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)
^2 + c^2 + d^2 - 2*c + 1)) + 3*I*polylog(4, ((c^2 + 2*I*(c + 1)*d - d^2 + 2
*c + 1)*tan(b*x + a)^2 - c^2 - 2*I*(c + 1)*d + d^2 + (2*I*c^2 - 4*(c + 1)*d
 - 2*I*d^2 + 4*I*c + 2*I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*ta
n(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) - 3*I*polylog(4, ((c^2 - 2*I*(c + 1)*d
 - d^2 + 2*c + 1)*tan(b*x + a)^2 - c^2 + 2*I*(c + 1)*d + d^2 + (-2*I*c^2 -
4*(c + 1)*d + 2*I*d^2 - 4*I*c - 2*I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 +
2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) - 3*I*polylog(4, ((c^2 + 2*
I*(c - 1)*d - d^2 - 2*c + 1)*tan(b*x + a)^2 - c^2 - 2*I*(c - 1)*d + d^2 + (
2*I*c^2 - 4*(c - 1)*d - 2*I*d^2 - 4*I*c + 2*I)*tan(b*x + a) + 2*c - 1)/((c^
2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1)) + 3*I*polylog(4,
((c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*tan(b*x + a)^2 - c^2 + 2*I*(c - 1)*d
 + d^2 + (-2*I*c^2 - 4*(c - 1)*d + 2*I*d^2 + 4*I*c - 2*I)*tan(b*x + a) + 2*
c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1)))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(c+d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{artanh}(d\tan(bx+a)+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*tan(b*x+a)),x, algorithm="giac")

[Out] integrate(x^2*arctanh(d*tan(b*x + a) + c), x)
```

3.318 $\int x \tanh^{-1}(c + d\tan(a + bx))\,dx$

Optimal. Leaf size=295

$$\frac{\text{PolyLog}\left(3,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{8b^2}-\frac{\text{PolyLog}\left(3,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{8b^2}-\frac{ix\text{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b}+\frac{ix\text{PolyLog}\left(2,-\frac{(c-}{}\right.}{4b}$$

```
[Out] (x^2*ArcTanh[c + d*Tan[a + b*x]])/2 + (x^2*Log[1 + ((1 - c + I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 - c - I*d)])/4 - (x^2*Log[1 + ((1 + c - I*d)*E^((2*I)*a
+ (2*I)*b*x))/(1 + c + I*d)])/4 - ((I/4)*x*PolyLog[2, -(((1 - c + I*d)*E^((
2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/b + ((I/4)*x*PolyLog[2, -(((1 + c - I
*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/b + PolyLog[3, -(((1 - c + I*
d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))]/(8*b^2) - PolyLog[3, -(((1 + c
- I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))]/(8*b^2)
```

Rubi [A] time = 0.403314, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6267, 2190, 2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{8b^2}-\frac{\text{PolyLog}\left(3,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{8b^2}-\frac{ix\text{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b}+\frac{ix\text{PolyLog}\left(2,-\frac{(c-}{}\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[c + d*Tan[a + b*x]])/2 + (x^2*Log[1 + ((1 - c + I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 - c - I*d)])/4 - (x^2*Log[1 + ((1 + c - I*d)*E^((2*I)*a
+ (2*I)*b*x))/(1 + c + I*d)])/4 - ((I/4)*x*PolyLog[2, -(((1 - c + I*d)*E^((
2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/b + ((I/4)*x*PolyLog[2, -(((1 + c - I
*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/b + PolyLog[3, -(((1 - c + I*
d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))]/(8*b^2) - PolyLog[3, -(((1 + c
- I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))]/(8*b^2)
```

Rule 6267

```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Tan[a + b*x]])/(f*(m
 + 1)), x] + (-Dist[(I*b*(1 + c - I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)
*E^(2*I*a + 2*I*b*x))/(1 + c + I*d + (1 + c - I*d)*E^(2*I*a + 2*I*b*x)), x]
```

```
, x] + Dist[(I*b*(1 - c + I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*I*
a + 2*I*b*x))/(1 - c - I*d + (1 - c + I*d)*E^(2*I*a + 2*I*b*x)), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, 1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(c+d\tan(a+bx))\,dx = \frac{1}{2}x^2\tanh^{-1}(c+d\tan(a+bx)) + \frac{1}{2}(b(i(1-c)-d))\int \frac{e^{2ia+2ibx}x^2}{1-c-id+(1-c+id)e^{2ia+2ib}}$$

$$= \frac{1}{2}x^2\tanh^{-1}(c+d\tan(a+bx)) + \frac{1}{4}x^2\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) - \frac{1}{4}x^2\log\left(1+\right.$$

$$= \frac{1}{2}x^2\tanh^{-1}(c+d\tan(a+bx)) + \frac{1}{4}x^2\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) - \frac{1}{4}x^2\log\left(1+\right.$$

$$= \frac{1}{2}x^2\tanh^{-1}(c+d\tan(a+bx)) + \frac{1}{4}x^2\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) - \frac{1}{4}x^2\log\left(1+\right.$$

$$= \frac{1}{2}x^2\tanh^{-1}(c+d\tan(a+bx)) + \frac{1}{4}x^2\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) - \frac{1}{4}x^2\log\left(1+\right.$$

Mathematica [A] time = 0.582998, size = 257, normalized size = 0.87

$$\frac{1}{2}x^2\tanh^{-1}(d\tan(a+bx)+c) + \frac{-2ibx\text{PolyLog}\left(2,\frac{(-c+id+1)e^{2i(a+bx)}}{c+id-1}\right)+2ibx\text{PolyLog}\left(2,-\frac{(c-id+1)e^{2i(a+bx)}}{c+id+1}\right)+\text{PolyLog}\left(3,\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[c + d*Tan[a + b*x]])/2 + (2*b^2*x^2*Log[1 + ((-1 + c - I*d)*E^
((2*I)*(a + b*x)))/(-1 + c + I*d)] - 2*b^2*x^2*Log[1 + ((1 + c - I*d)*E^((2
*I)*(a + b*x)))/(1 + c + I*d)] - (2*I)*b*x*PolyLog[2, ((1 - c + I*d)*E^((2*
I)*(a + b*x)))/(-1 + c + I*d)] + (2*I)*b*x*PolyLog[2, -(((1 + c - I*d)*E^((
2*I)*(a + b*x)))/(1 + c + I*d))] + PolyLog[3, ((1 - c + I*d)*E^((2*I)*(a +
b*x)))/(-1 + c + I*d)] - PolyLog[3, -(((1 + c - I*d)*E^((2*I)*(a + b*x)))/(
1 + c + I*d))])/(8*b^2)
```

Maple [C] time = 9.453, size = 6593, normalized size = 22.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(c+d*tan(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -2*b*d*integrate(-(2*(c^2 + d^2 - 1)*x^2*cos(2*b*x + 2*a)^2 + 2*c*d*x^2*sin
(2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^2*sin(2*b*x + 2*a)^2 + (c^2 - d^2 - 1)*
x^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x + 2*a) - (c^2 - d^2 - 1)*x^2*co
s(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^2*cos(2*b*x + 2*a) + (c^2 - d^2
 - 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 + d^4 + 2*(c^2 + 1)*d^2
+ (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*cos(4*b*x + 4*a)^2 + 4*(c^4 + d
^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x + 2*a)^2 + (c^4 + d^4 + 2*(c^2
+ 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 -
 2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 + 2*(c^4 + d^4 - 2*(3*c^2 - 1)*d^2 -
 2*c^2 + 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(c*d^3 + (c^3 - c)*
d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*(c^4 - d^4 - 2*c^2 + 1)*cos(2
*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3 - c)*d - 2*(c*d^3 + (c^3 - c)*d)*cos(2*b*
x + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*sin(2*b*x + 2*a))*sin(4*b*x + 4*a) + 8*(
c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1), x) + 1/8*x^2*log((c^2 + d^2 + 2
*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 + 2*
c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 + 2*(c^2 - d^2 + 2*c + 1)*cos(2*b*x +
 2*a) + 2*c + 1) - 1/8*x^2*log((c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a)^2 + 4
*(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 - 2*c + 1)*sin(2*b*x + 2*a)^2 + c^
2 + d^2 + 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*x + 2*a) - 2*c + 1)
```

Fricas [C] time = 2.73826, size = 4547, normalized size = 15.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/16*(4*b^2*x^2*log(-(d*tan(b*x + a) + c + 1)/(d*tan(b*x + a) + c - 1)) + 2
*I*b*x*dilog(-((2*I*(c + 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 - 2*I*(c + 1)
```

```
*d + (2*I*c^2 + 4*(c + 1)*d - 2*I*d^2 + 4*I*c + 2*I)*tan(b*x + a) + 4*c + 2
)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) - 2*I*b
*x*dilog(-((-2*I*(c + 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 + 2*I*(c + 1)*d
+ (-2*I*c^2 + 4*(c + 1)*d + 2*I*d^2 - 4*I*c - 2*I)*tan(b*x + a) + 4*c + 2)/
((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) - 2*I*b*x
*dilog(-((2*I*(c - 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 - 2*I*(c - 1)*d + (
2*I*c^2 + 4*(c - 1)*d - 2*I*d^2 - 4*I*c + 2*I)*tan(b*x + a) - 4*c + 2)/((c^
2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1) + 1) + 2*I*b*x*dil
og(-((-2*I*(c - 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 + 2*I*(c - 1)*d + (-2*
I*c^2 + 4*(c - 1)*d + 2*I*d^2 + 4*I*c - 2*I)*tan(b*x + a) - 4*c + 2)/((c^2
+ d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1) + 1) - 2*a^2*log(((I
*(c + 1)*d + d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)*d + (I*c^2 + I*d^2 + 2*I
*c + I)*tan(b*x + a) - 2*c - 1)/(tan(b*x + a)^2 + 1)) - 2*a^2*log(((I*(c +
1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*(c + 1)*d + (I*c^2 + I*d^2 + 2*I*c + I
)*tan(b*x + a) + 2*c + 1)/(tan(b*x + a)^2 + 1)) + 2*a^2*log(((I*(c - 1)*d +
 d^2)*tan(b*x + a)^2 - c^2 + I*(c - 1)*d + (I*c^2 + I*d^2 - 2*I*c + I)*tan(
b*x + a) + 2*c - 1)/(tan(b*x + a)^2 + 1)) + 2*a^2*log(((I*(c - 1)*d - d^2)*
tan(b*x + a)^2 + c^2 + I*(c - 1)*d + (I*c^2 + I*d^2 - 2*I*c + I)*tan(b*x +
a) - 2*c + 1)/(tan(b*x + a)^2 + 1)) - 2*(b^2*x^2 - a^2)*log(((2*I*(c + 1)*d
 + 2*d^2)*tan(b*x + a)^2 + 2*c^2 - 2*I*(c + 1)*d + (2*I*c^2 + 4*(c + 1)*d -
 2*I*d^2 + 4*I*c + 2*I)*tan(b*x + a) + 4*c + 2)/((c^2 + d^2 + 2*c + 1)*tan(
b*x + a)^2 + c^2 + d^2 + 2*c + 1)) - 2*(b^2*x^2 - a^2)*log(((-2*I*(c + 1)*d
 + 2*d^2)*tan(b*x + a)^2 + 2*c^2 + 2*I*(c + 1)*d + (-2*I*c^2 + 4*(c + 1)*d
+ 2*I*d^2 - 4*I*c - 2*I)*tan(b*x + a) + 4*c + 2)/((c^2 + d^2 + 2*c + 1)*tan
(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) + 2*(b^2*x^2 - a^2)*log(((2*I*(c - 1)*d
 + 2*d^2)*tan(b*x + a)^2 + 2*c^2 - 2*I*(c - 1)*d + (2*I*c^2 + 4*(c - 1)*d -
 2*I*d^2 - 4*I*c + 2*I)*tan(b*x + a) - 4*c + 2)/((c^2 + d^2 - 2*c + 1)*tan(
b*x + a)^2 + c^2 + d^2 - 2*c + 1)) + 2*(b^2*x^2 - a^2)*log(((-2*I*(c - 1)*d
 + 2*d^2)*tan(b*x + a)^2 + 2*c^2 + 2*I*(c - 1)*d + (-2*I*c^2 + 4*(c - 1)*d
+ 2*I*d^2 + 4*I*c - 2*I)*tan(b*x + a) - 4*c + 2)/((c^2 + d^2 - 2*c + 1)*tan
(b*x + a)^2 + c^2 + d^2 - 2*c + 1)) - polylog(3, ((c^2 + 2*I*(c + 1)*d - d^
2 + 2*c + 1)*tan(b*x + a)^2 - c^2 - 2*I*(c + 1)*d + d^2 + (2*I*c^2 - 4*(c +
 1)*d - 2*I*d^2 + 4*I*c + 2*I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c +
1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) - polylog(3, ((c^2 - 2*I*(c + 1)*
d - d^2 + 2*c + 1)*tan(b*x + a)^2 - c^2 + 2*I*(c + 1)*d + d^2 + (-2*I*c^2 -
 4*(c + 1)*d + 2*I*d^2 - 4*I*c - 2*I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 +
 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) + polylog(3, ((c^2 + 2*I*(
c - 1)*d - d^2 - 2*c + 1)*tan(b*x + a)^2 - c^2 - 2*I*(c - 1)*d + d^2 + (2*I
*c^2 - 4*(c - 1)*d - 2*I*d^2 - 4*I*c + 2*I)*tan(b*x + a) + 2*c - 1)/((c^2 +
 d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1)) + polylog(3, ((c^2 -
 2*I*(c - 1)*d - d^2 - 2*c + 1)*tan(b*x + a)^2 - c^2 + 2*I*(c - 1)*d + d^2
+ (-2*I*c^2 - 4*(c - 1)*d + 2*I*d^2 + 4*I*c - 2*I)*tan(b*x + a) + 2*c - 1)/
((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1)))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(c+d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{artanh}(d \tan(bx + a) + c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(d*tan(b*x + a) + c), x)
```

3.319 $\int \tanh^{-1}(c + d\tan(a + bx))\,dx$

Optimal. Leaf size=194

$$-\frac{i\text{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b}+\frac{i\text{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b}+\frac{1}{2}x\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)-\frac{1}{2}x\log\left(1+\frac{(c\cdots}{\cdots}\right.$$

```
[Out] x*ArcTanh[c + d*Tan[a + b*x]] + (x*Log[1 + ((1 - c + I*d)*E^((2*I)*a + (2*I
)*b*x))/(1 - c - I*d)])/2 - (x*Log[1 + ((1 + c - I*d)*E^((2*I)*a + (2*I)*b*
x))/(1 + c + I*d)])/2 - ((I/4)*PolyLog[2, -(((1 - c + I*d)*E^((2*I)*a + (2*
I)*b*x))/(1 - c - I*d))])/b + ((I/4)*PolyLog[2, -(((1 + c - I*d)*E^((2*I)*a
 + (2*I)*b*x))/(1 + c + I*d))])/b
```

Rubi [A] time = 0.239275, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6259, 2190, 2279, 2391}

$$-\frac{i\text{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b}+\frac{i\text{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b}+\frac{1}{2}x\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)-\frac{1}{2}x\log\left(1+\frac{(c\cdots}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[c + d*Tan[a + b*x]],x]
```

```
[Out] x*ArcTanh[c + d*Tan[a + b*x]] + (x*Log[1 + ((1 - c + I*d)*E^((2*I)*a + (2*I
)*b*x))/(1 - c - I*d)])/2 - (x*Log[1 + ((1 + c - I*d)*E^((2*I)*a + (2*I)*b*
x))/(1 + c + I*d)])/2 - ((I/4)*PolyLog[2, -(((1 - c + I*d)*E^((2*I)*a + (2*
I)*b*x))/(1 - c - I*d))])/b + ((I/4)*PolyLog[2, -(((1 + c - I*d)*E^((2*I)*a
 + (2*I)*b*x))/(1 + c + I*d))])/b
```

Rule 6259

```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Tanh[c + d*Tan[a + b*x]], x] + (-Dist[I*b*(1 + c - I*d), Int[(x*E^(2*I*a +
2*I*b*x))/(1 + c + I*d + (1 + c - I*d)*E^(2*I*a + 2*I*b*x)), x], x] + Dist[
I*b*(1 - c + I*d), Int[(x*E^(2*I*a + 2*I*b*x))/(1 - c - I*d + (1 - c + I*d)
*E^(2*I*a + 2*I*b*x)), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c + I*d)^2
, 1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \tanh^{-1}(c+d\tan(a+bx))\,dx = x\tanh^{-1}(c+d\tan(a+bx)) + (b(i(1-c)-d))\int \frac{e^{2ia+2ibx}x}{1-c-id+(1-c+id)e^{2ia+2ibx}}\,dx$$

$$= x\tanh^{-1}(c+d\tan(a+bx)) + \frac{1}{2}x\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) - \frac{1}{2}x\log\left(1+\frac{(1+c}{1}\right.$$

$$= x\tanh^{-1}(c+d\tan(a+bx)) + \frac{1}{2}x\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) - \frac{1}{2}x\log\left(1+\frac{(1+c}{1}\right.$$

$$= x\tanh^{-1}(c+d\tan(a+bx)) + \frac{1}{2}x\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) - \frac{1}{2}x\log\left(1+\frac{(1+c}{1}\right.$$

Mathematica [B] time = 32.4898, size = 4654, normalized size = 23.99

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[c + d*Tan[a + b*x]],x]
```

```
[Out] x*ArcTanh[c + d*Tan[a + b*x]] + (d*(-(a*Log[-(Sec[(a + b*x)/2]^2*((-1 + c)*
Cos[a + b*x] + d*Sin[a + b*x]))]) + a*Log[Sec[(a + b*x)/2]^2*(Cos[a + b*x]
```

```
+ c*Cos[a + b*x] + d*Sin[a + b*x])] + (a + b*x)*Log[(-d + Sqrt[1 - 2*c + c^
2 + d^2])/(-1 + c) + Tan[(a + b*x)/2]] + I*Log[((-1 + c)*(1 + I*Tan[(a + b*
x)/2]))/(-1 + c + I*d - I*Sqrt[1 - 2*c + c^2 + d^2])]*Log[(-d + Sqrt[1 - 2*
c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/2]] - I*Log[-(((-1 + c)*(I + Tan[(
a + b*x)/2]))/(I - I*c - d + Sqrt[1 - 2*c + c^2 + d^2]))]*Log[(-d + Sqrt[1
- 2*c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/2]] + (a + b*x)*Log[(d + Sqrt[
1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]] + I*Log[((-1 + c)*(-I + T
an[(a + b*x)/2]))/(I - I*c + d + Sqrt[1 - 2*c + c^2 + d^2])]*Log[(d + Sqrt[
1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]] - I*Log[((-1 + c)*(I + Ta
n[(a + b*x)/2]))/(-I + I*c + d + Sqrt[1 - 2*c + c^2 + d^2])]*Log[(d + Sqrt[
1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]] - (a + b*x)*Log[-((d + Sq
rt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2]] - I*Log[((1 + c)*(-I
+ Tan[(a + b*x)/2]))/(-I - I*c + d + Sqrt[1 + 2*c + c^2 + d^2])]*Log[-((d +
 Sqrt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2]] + I*Log[((1 + c)*(
I + Tan[(a + b*x)/2]))/(I + I*c + d + Sqrt[1 + 2*c + c^2 + d^2])]*Log[-((d
+ Sqrt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2]] - (a + b*x)*Log[(
-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(1 + c)] + I*Log
[((1 + c)*(1 - I*Tan[(a + b*x)/2]))/(1 + c - I*d + I*Sqrt[1 + 2*c + c^2 + d
^2])]*Log[(-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(1 +
c)] - I*Log[((1 + c)*(1 + I*Tan[(a + b*x)/2]))/(1 + c + I*d - I*Sqrt[1 + 2*
c + c^2 + d^2])]*Log[(-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x
)/2])/(1 + c)] + I*PolyLog[2, (d + Sqrt[1 - 2*c + c^2 + d^2] - (-1 + c)*Tan
[(a + b*x)/2])/(I - I*c + d + Sqrt[1 - 2*c + c^2 + d^2])] - I*PolyLog[2, (d
 + Sqrt[1 - 2*c + c^2 + d^2] - (-1 + c)*Tan[(a + b*x)/2])/(-I + I*c + d + S
qrt[1 - 2*c + c^2 + d^2])] - I*PolyLog[2, (-d + Sqrt[1 - 2*c + c^2 + d^2] +
 (-1 + c)*Tan[(a + b*x)/2])/(I - I*c - d + Sqrt[1 - 2*c + c^2 + d^2])] + I*
PolyLog[2, (-d + Sqrt[1 - 2*c + c^2 + d^2] + (-1 + c)*Tan[(a + b*x)/2])/(-I
 + I*c - d + Sqrt[1 - 2*c + c^2 + d^2])] - I*PolyLog[2, (d + Sqrt[1 + 2*c +
 c^2 + d^2] - (1 + c)*Tan[(a + b*x)/2])/(-I - I*c + d + Sqrt[1 + 2*c + c^2
+ d^2])] + I*PolyLog[2, (d + Sqrt[1 + 2*c + c^2 + d^2] - (1 + c)*Tan[(a + b
*x)/2])/(I + I*c + d + Sqrt[1 + 2*c + c^2 + d^2])] + I*PolyLog[2, (-d + Sqr
t[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(-I - I*c - d + Sqrt[1 +
 2*c + c^2 + d^2])] - I*PolyLog[2, (-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c
)*Tan[(a + b*x)/2])/(I + I*c - d + Sqrt[1 + 2*c + c^2 + d^2])])*((-2*a)/(b*
(-1 + c^2 + d^2 - Cos[2*(a + b*x)] + c^2*Cos[2*(a + b*x)] - d^2*Cos[2*(a +
b*x)] + 2*c*d*Sin[2*(a + b*x)])) + (2*(a + b*x))/(b*(-1 + c^2 + d^2 - Cos[2
*(a + b*x)] + c^2*Cos[2*(a + b*x)] - d^2*Cos[2*(a + b*x)] + 2*c*d*Sin[2*(a
+ b*x)]))))/(Log[(-d + Sqrt[1 - 2*c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/
2]] + Log[(d + Sqrt[1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]] - Log
[-((d + Sqrt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2]] - Log[(-d +
 Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(1 + c)] + (Log[(-d
+ Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(1 + c)]*Sec[(a + b
*x)/2]^2)/(2*(1 - I*Tan[(a + b*x)/2])) - (Log[(-d + Sqrt[1 - 2*c + c^2 + d^
2])/(-1 + c) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(2*(1 + I*Tan[(a + b*x
)/2])) + (Log[(-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(
```

1 + c)]*Sec[(a + b*x)/2]^2)/(2*(1 + I*Tan[(a + b*x)/2])) + ((I/2)*Log[(d + Sqrt[1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(-I + Tan[(a + b*x)/2]) - ((I/2)*Log[-((d + Sqrt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(-I + Tan[(a + b*x)/2]) - ((I/2)*Log[(-d + Sqrt[1 - 2*c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(I + Tan[(a + b*x)/2]) - ((I/2)*Log[(d + Sqrt[1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(I + Tan[(a + b*x)/2]) + ((I/2)*Log[-((d + Sqrt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(I + Tan[(a + b*x)/2]) + ((a + b*x)*Sec[(a + b*x)/2]^2)/(2*((-d + Sqrt[1 - 2*c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/2])) + ((I/2)*Log[((-1 + c)*(1 + I*Tan[(a + b*x)/2]))/(-1 + c + I*d - I*Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/((-d + Sqrt[1 - 2*c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/2]) - ((I/2)*Log[-(((-1 + c)*(I + Tan[(a + b*x)/2]))/(I - I*c - d + Sqrt[1 - 2*c + c^2 + d^2]))]*Sec[(a + b*x)/2]^2)/((-d + Sqrt[1 - 2*c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/2]) + ((a + b*x)*Sec[(a + b*x)/2]^2)/(2*((d + Sqrt[1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2])) + ((I/2)*Log[((-1 + c)*(-I + Tan[(a + b*x)/2]))/(I - I*c + d + Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/((d + Sqrt[1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]) - ((I/2)*Log[((-1 + c)*(I + Tan[(a + b*x)/2]))/(-I + I*c + d + Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/((d + Sqrt[1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]) - ((a + b*x)*Sec[(a + b*x)/2]^2)/(2*(-((d + Sqrt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2])) - ((I/2)*Log[((1 + c)*(-I + Tan[(a + b*x)/2]))/(-I - I*c + d + Sqrt[1 + 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-((d + Sqrt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2]) + ((I/2)*Log[((1 + c)*(I + Tan[(a + b*x)/2]))/(I + I*c + d + Sqrt[1 + 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-((d + Sqrt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2]) + ((I/2)*(-1 + c)*Log[1 - (d + Sqrt[1 - 2*c + c^2 + d^2] - (-1 + c)*Tan[(a + b*x)/2])/(I - I*c + d + Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(d + Sqrt[1 - 2*c + c^2 + d^2] - (-1 + c)*Tan[(a + b*x)/2]) - ((I/2)*(-1 + c)*Log[1 - (d + Sqrt[1 - 2*c + c^2 + d^2] - (-1 + c)*Tan[(a + b*x)/2])/(-I + I*c + d + Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(d + Sqrt[1 - 2*c + c^2 + d^2] - (-1 + c)*Tan[(a + b*x)/2]) + ((I/2)*(-1 + c)*Log[1 - (-d + Sqrt[1 - 2*c + c^2 + d^2] + (-1 + c)*Tan[(a + b*x)/2])/(I - I*c - d + Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-d + Sqrt[1 - 2*c + c^2 + d^2] + (-1 + c)*Tan[(a + b*x)/2]) - ((I/2)*(-1 + c)*Log[1 - (-d + Sqrt[1 - 2*c + c^2 + d^2] + (-1 + c)*Tan[(a + b*x)/2])/(-I + I*c - d + Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-d + Sqrt[1 - 2*c + c^2 + d^2] + (-1 + c)*Tan[(a + b*x)/2]) - ((I/2)*(1 + c)*Log[1 - (d + Sqrt[1 + 2*c + c^2 + d^2] - (1 + c)*Tan[(a + b*x)/2])/(-I - I*c + d + Sqrt[1 + 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(d + Sqrt[1 + 2*c + c^2 + d^2] - (1 + c)*Tan[(a + b*x)/2]) + ((I/2)*(1 + c)*Log[1 - (d + Sqrt[1 + 2*c + c^2 + d^2] - (1 + c)*Tan[(a + b*x)/2])/(I + I*c + d + Sqrt[1 + 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(d + Sqrt[1 + 2*c + c^2 + d^2] - (1 + c)*Tan[(a + b*x)/2]) - ((1 + c)*(a + b*x)*Sec[(a + b*x)/2]^2)/(2*(-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])) + ((I/2)*(1 + c)*Log[((

```
1 + c)*(1 - I*Tan[(a + b*x)/2]))/(1 + c - I*d + I*Sqrt[1 + 2*c + c^2 + d^2]
)]*Sec[(a + b*x)/2]^2)/(-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b
*x)/2]) - ((I/2)*(1 + c)*Log[((1 + c)*(1 + I*Tan[(a + b*x)/2]))/(1 + c + I*
d - I*Sqrt[1 + 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-d + Sqrt[1 + 2*c +
c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2]) - ((I/2)*(1 + c)*Log[1 - (-d + Sqrt[
1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(-I - I*c - d + Sqrt[1 + 2
*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-d + Sqrt[1 + 2*c + c^2 + d^2] + (1
+ c)*Tan[(a + b*x)/2]) + ((I/2)*(1 + c)*Log[1 - (-d + Sqrt[1 + 2*c + c^2 +
d^2] + (1 + c)*Tan[(a + b*x)/2])/(I + I*c - d + Sqrt[1 + 2*c + c^2 + d^2])]
*Sec[(a + b*x)/2]^2)/(-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x
)/2]) + (a*Cos[(a + b*x)/2]^2*(-(Sec[(a + b*x)/2]^2*(d*Cos[a + b*x] - (-1 +
 c)*Sin[a + b*x])) - Sec[(a + b*x)/2]^2*((-1 + c)*Cos[a + b*x] + d*Sin[a +
b*x])*Tan[(a + b*x)/2]))/((-1 + c)*Cos[a + b*x] + d*Sin[a + b*x]) + (a*Cos[
(a + b*x)/2]^2*(Sec[(a + b*x)/2]^2*(d*Cos[a + b*x] - Sin[a + b*x] - c*Sin[a
 + b*x]) + Sec[(a + b*x)/2]^2*(Cos[a + b*x] + c*Cos[a + b*x] + d*Sin[a + b*
x])*Tan[(a + b*x)/2]))/(Cos[a + b*x] + c*Cos[a + b*x] + d*Sin[a + b*x]))
```

Maple [B] time = 0.09, size = 612, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(c+d*tan(b*x+a)),x)
```

```
[Out] 1/b*arctan(tan(b*x+a))*arctanh(c+d*tan(b*x+a))-1/2/b*arctan((c+d*tan(b*x+a)
)/d-c/d)*ln(d*((c+d*tan(b*x+a))/d-c/d)+c+1)+1/2/b*arctan((c+d*tan(b*x+a))/d
-c/d)*ln(d*((c+d*tan(b*x+a))/d-c/d)+c-1)-1/4*I/b*ln(d*((c+d*tan(b*x+a))/d-c
/d)+c+1)*ln((I*d-d*((c+d*tan(b*x+a))/d-c/d))/(1+c+I*d))+1/4*I/b*ln(d*((c+d*
tan(b*x+a))/d-c/d)+c+1)*ln((I*d+d*((c+d*tan(b*x+a))/d-c/d))/(I*d-c-1))-1/4*
I/b*dilog((I*d-d*((c+d*tan(b*x+a))/d-c/d))/(1+c+I*d))+1/4*I/b*dilog((I*d+d*
((c+d*tan(b*x+a))/d-c/d))/(I*d-c-1))+1/4*I/b*ln(d*((c+d*tan(b*x+a))/d-c/d)+
c-1)*ln((I*d-d*((c+d*tan(b*x+a))/d-c/d))/(I*d+c-1))-1/4*I/b*ln(d*((c+d*tan(
b*x+a))/d-c/d)+c-1)*ln((I*d+d*((c+d*tan(b*x+a))/d-c/d))/(1-c+I*d))+1/4*I/b*
dilog((I*d-d*((c+d*tan(b*x+a))/d-c/d))/(I*d+c-1))-1/4*I/b*dilog((I*d+d*((c+
d*tan(b*x+a))/d-c/d))/(1-c+I*d))
```

Maxima [B] time = 1.8042, size = 502, normalized size = 2.59

$$4\,(bx+a)\,\mathrm{artanh}\,(d\tan(bx+a)+c) + \left(\arctan\left(\frac{d^2\tan(bx+a)+(c+1)d}{c^2+d^2+2\,c+1}, \frac{(c+1)d\tan(bx+a)+c^2+2\,c+1}{c^2+d^2+2\,c+1}\right) - \arctan\left(\frac{d^2\tan(bx+a)+(c-1)d}{c^2+d^2-2\,c+1}, \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(b*x + a)*arctanh(d*tan(b*x + a) + c) + (arctan2((d^2*tan(b*x + a) +
 (c + 1)*d)/(c^2 + d^2 + 2*c + 1), ((c + 1)*d*tan(b*x + a) + c^2 + 2*c + 1)
/(c^2 + d^2 + 2*c + 1)) - arctan2((d^2*tan(b*x + a) + (c - 1)*d)/(c^2 + d^2
 - 2*c + 1), ((c - 1)*d*tan(b*x + a) + c^2 - 2*c + 1)/(c^2 + d^2 - 2*c + 1)
))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log((d^2*tan(b*x + a)^2 + 2*(c + 1)*
d*tan(b*x + a) + c^2 + 2*c + 1)/(c^2 + d^2 + 2*c + 1)) + (b*x + a)*log((d^2
*tan(b*x + a)^2 + 2*(c - 1)*d*tan(b*x + a) + c^2 - 2*c + 1)/(c^2 + d^2 - 2*
c + 1)) - I*dilog(-(I*d*tan(b*x + a) - d)/(I*c + d + I)) + I*dilog(-(I*d*ta
n(b*x + a) - d)/(I*c + d - I)) - I*dilog((I*d*tan(b*x + a) + d)/(-I*c + d +
 I)) + I*dilog((I*d*tan(b*x + a) + d)/(-I*c + d - I)))/b
```

Fricas [B] time = 2.77094, size = 3212, normalized size = 16.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*log(-(d*tan(b*x + a) + c + 1)/(d*tan(b*x + a) + c - 1)) - 2*(b*x
 + a)*log(((2*I*(c + 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 - 2*I*(c + 1)*d +
 (2*I*c^2 + 4*(c + 1)*d - 2*I*d^2 + 4*I*c + 2*I)*tan(b*x + a) + 4*c + 2)/((
c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) - 2*(b*x + a)*l
og(((-2*I*(c + 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 + 2*I*(c + 1)*d + (-2*I
*c^2 + 4*(c + 1)*d + 2*I*d^2 - 4*I*c - 2*I)*tan(b*x + a) + 4*c + 2)/((c^2 +
 d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) + 2*(b*x + a)*log(((
2*I*(c - 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 - 2*I*(c - 1)*d + (2*I*c^2 +
4*(c - 1)*d - 2*I*d^2 - 4*I*c + 2*I)*tan(b*x + a) - 4*c + 2)/((c^2 + d^2 -
2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1)) + 2*(b*x + a)*log(((-2*I*(c
 - 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 + 2*I*(c - 1)*d + (-2*I*c^2 + 4*(c
- 1)*d + 2*I*d^2 + 4*I*c - 2*I)*tan(b*x + a) - 4*c + 2)/((c^2 + d^2 - 2*c +
 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1)) + 2*a*log(((I*(c + 1)*d + d^2)*t
an(b*x + a)^2 - c^2 + I*(c + 1)*d + (I*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a
) - 2*c - 1)/(tan(b*x + a)^2 + 1)) + 2*a*log(((I*(c + 1)*d - d^2)*tan(b*x +
 a)^2 + c^2 + I*(c + 1)*d + (I*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a) + 2*c
+ 1)/(tan(b*x + a)^2 + 1)) - 2*a*log(((I*(c - 1)*d + d^2)*tan(b*x + a)^2 -
c^2 + I*(c - 1)*d + (I*c^2 + I*d^2 - 2*I*c + I)*tan(b*x + a) + 2*c - 1)/(ta
n(b*x + a)^2 + 1)) - 2*a*log(((I*(c - 1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*
```

```
(c - 1)*d + (I*c^2 + I*d^2 - 2*I*c + I)*tan(b*x + a) - 2*c + 1)/(tan(b*x +
a)^2 + 1)) + I*dilog(-((2*I*(c + 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 - 2*I
*(c + 1)*d + (2*I*c^2 + 4*(c + 1)*d - 2*I*d^2 + 4*I*c + 2*I)*tan(b*x + a) +
 4*c + 2)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1)
 - I*dilog(-((-2*I*(c + 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 + 2*I*(c + 1)*
d + (-2*I*c^2 + 4*(c + 1)*d + 2*I*d^2 - 4*I*c - 2*I)*tan(b*x + a) + 4*c + 2
)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) - I*dil
og(-((2*I*(c - 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 - 2*I*(c - 1)*d + (2*I*
c^2 + 4*(c - 1)*d - 2*I*d^2 - 4*I*c + 2*I)*tan(b*x + a) - 4*c + 2)/((c^2 +
d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1) + 1) + I*dilog(-((-2*I
*(c - 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 + 2*I*(c - 1)*d + (-2*I*c^2 + 4*
(c - 1)*d + 2*I*d^2 + 4*I*c - 2*I)*tan(b*x + a) - 4*c + 2)/((c^2 + d^2 - 2*
c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1) + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(c+d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{artanh}\left(d\tan\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*tan(b*x + a) + c), x)
```

3.320 $\int \frac{\tanh^{-1}(c+d\tan(a+bx))}{x}\,dx$

Optimal. Leaf size=17

$$\text{CannotIntegrate}\left(\frac{\tanh^{-1}(d\tan(a+bx)+c)}{x},x\right)$$

```
[Out] CannotIntegrate[ArcTanh[c + d*Tan[a + b*x]]/x, x]
```

Rubi [A] time = 0.150291, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tanh^{-1}(c+d\tan(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTanh[c + d*Tan[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTanh[c + d*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(c+d\tan(a+bx))}{x}\,dx=\int \frac{\tanh^{-1}(c+d\tan(a+bx))}{x}\,dx$$

Mathematica [A] time = 4.53994, size = 0, normalized size = 0.

$$\int \frac{\tanh^{-1}(c+d\tan(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTanh[c + d*Tan[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcTanh[c + d*Tan[a + b*x]]/x, x]
```

Maple [A] time = 0.375, size = 0, normalized size = 0.

$$\int \frac{\operatorname{Artanh}\left(c+d\tan\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(c+d*tan(b*x+a))/x,x)
```

```
[Out] int(arctanh(c+d*tan(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}\left(d\tan\left(bx+a\right)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tan(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(d*tan(b*x + a) + c)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\operatorname{artanh}\left(d\tan\left(bx+a\right)+c\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tan(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(arctanh(d*tan(b*x + a) + c)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(c+d*tan(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}\left(d\tan\left(bx+a\right)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tan(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*tan(b*x + a) + c)/x, x)
```

3.321 $\int x^2 \tanh^{-1}(1 - id + d\tan(a + bx))\,dx$

Optimal. Leaf size=170

$$-\frac{x\text{PolyLog}\left(3,-(1-id)e^{2ia+2ibx}\right)}{4b^2}-\frac{i\text{PolyLog}\left(4,-(1-id)e^{2ia+2ibx}\right)}{8b^3}+\frac{ix^2\text{PolyLog}\left(2,-(1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{6}x^3\log\left(1+\right.$$

```
[Out] (I/12)*b*x^4 + (x^3*ArcTanh[1 - I*d + d*Tan[a + b*x]])/3 - (x^3*Log[1 + (1
- I*d)*E^((2*I)*a + (2*I)*b*x)])/6 + ((I/4)*x^2*PolyLog[2, -((1 - I*d)*E^((
2*I)*a + (2*I)*b*x))])/b - (x*PolyLog[3, -((1 - I*d)*E^((2*I)*a + (2*I)*b*x
))])/(4*b^2) - ((I/8)*PolyLog[4, -((1 - I*d)*E^((2*I)*a + (2*I)*b*x))])/b^3
```

Rubi [A] time = 0.291613, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {6263, 2184, 2190, 2531, 6609, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(3,-(1-id)e^{2ia+2ibx}\right)}{4b^2}-\frac{i\text{PolyLog}\left(4,-(1-id)e^{2ia+2ibx}\right)}{8b^3}+\frac{ix^2\text{PolyLog}\left(2,-(1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{6}x^3\log\left(1+\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[1 - I*d + d*Tan[a + b*x]],x]
```

```
[Out] (I/12)*b*x^4 + (x^3*ArcTanh[1 - I*d + d*Tan[a + b*x]])/3 - (x^3*Log[1 + (1
- I*d)*E^((2*I)*a + (2*I)*b*x)])/6 + ((I/4)*x^2*PolyLog[2, -((1 - I*d)*E^((
2*I)*a + (2*I)*b*x))])/b - (x*PolyLog[3, -((1 - I*d)*E^((2*I)*a + (2*I)*b*x
))])/(4*b^2) - ((I/8)*PolyLog[4, -((1 - I*d)*E^((2*I)*a + (2*I)*b*x))])/b^3
```

Rule 6263

```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Tan[a + b*x]])/(f*(m
 + 1)), x] + Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c + I*d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
```

```
x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \tanh^{-1}(1-id+d\tan(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(1-id+d\tan(a+bx)) + \frac{1}{3}(ib)\int \frac{x^3}{1+(1-id)e^{2ia+2ibx}}\,dx$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id+d\tan(a+bx)) - \frac{1}{3}(b(i+d))\int \frac{e^{2ia+2ibx}x^3}{1+(1-id)e^{2ia+2}}$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id+d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-id)e^{2ia+2ibx}\right) +$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id+d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-id)e^{2ia+2ibx}\right) +$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id+d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-id)e^{2ia+2ibx}\right) +$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id+d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-id)e^{2ia+2ibx}\right) +$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id+d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-id)e^{2ia+2ibx}\right) +$$

Mathematica [A] time = 0.410717, size = 155, normalized size = 0.91

$$\frac{1}{3}x^3\tanh^{-1}(d\tan(a+bx)-id+1) - \frac{6ib^2x^2\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{d+i}\right)+6bx\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{d+i}\right)-3i\text{PolyLog}\left(4,-\frac{ie^{-2i}}{d}\right.}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[1 - I*d + d*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[1 - I*d + d*Tan[a + b*x]])/3 - (4*b^3*x^3*Log[1 + I/((I + d)*E
^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, (-I)/((I + d)*E^((2*I)*(a +
 b*x)))] + 6*b*x*PolyLog[3, (-I)/((I + d)*E^((2*I)*(a + b*x)))] - (3*I)*Pol
yLog[4, (-I)/((I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

Maple [C] time = 14.966, size = 2346, normalized size = 13.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(1-I*d+d*tan(b*x+a)),x)
```

```
[Out] 1/12*I*x^3*Pi*csgn(I*d)*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(
I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b
*x+a))+1))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I))*csgn(I*(I*exp(
2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))-1/2*I/b^3*a^3/(I+d
)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))+1/3*I/b^3/(I+d)*ln(1-I*(I+d)*exp(
2*I*(b*x+a)))*a^3-1/4*I/b^2/(I+d)*polylog(3,I*(I+d)*exp(2*I*(b*x+a)))*x-1/2
*I/b^3*a^3/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/8*I/b^3*d/(I+d)*
polylog(4,I*(I+d)*exp(2*I*(b*x+a)))+1/6*I/b^3*a^3/(I+d)*ln(I*exp(2*I*(b*x+a
))+exp(2*I*(b*x+a))*d+I)-1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(
b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a)
)*d+I)/(exp(2*I*(b*x+a))+1))+1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2
*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*
x+a))*d+I)/(exp(2*I*(b*x+a))+1))^2-1/4*I/b^3*d/(I+d)*polylog(2,I*(I+d)*exp(
2*I*(b*x+a)))*a^2-1/2*I/b^2*a^2/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2
))*x-1/6*I/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*x^3-1/2/b^3*a^2/(I+d)*dilog
(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/2/b^3*a^2/(I+d)*dilog(1+I*exp(I*(b*
x+a))*(-I*(I+d))^(1/2))-1/4/b/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*x^2
-1/6*x^3*ln(d)+1/4/b^3/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*a^2-1/6*d/
(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*x^3+1/12*I*b*x^4-1/4/b^2*d/(I+d)*polyl
og(3,I*(I+d)*exp(2*I*(b*x+a)))*x-1/2/b^3*a^3*d/(I+d)*ln(1-I*exp(I*(b*x+a))*
(-I*(I+d))^(1/2))-1/2/b^3*a^3*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2
))+1/6/b^3*a^3*d/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)+1/3/b^3*
d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a^3-1/6*I*Pi*x^3-1/2*I/b^2*a^2/(I+d)
*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x+1/8/b^3/(I+d)*polylog(4,I*(I+d)*
exp(2*I*(b*x+a)))+1/2*I/b^3*a^2*d/(I+d)*dilog(1-I*exp(I*(b*x+a))*(-I*(I+d))
^(1/2))+1/2*I/b^3*a^2*d/(I+d)*dilog(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))+1/
2*I/b^2/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*x*a^2+1/4*I/b*d/(I+d)*polylog(
2,I*(I+d)*exp(2*I*(b*x+a)))*x^2-1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+ex
p(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))^3-1/2/b^2*a^2*d/(I+d)*ln(1+I*exp(
I*(b*x+a))*(-I*(I+d))^(1/2))*x+1/2/b^2*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)
))*x*a^2-1/2/b^2*a^2*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x+1/12
*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2-1/12*I*x^3*Pi*csg
n(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^3-1/12*I*x^3*Pi*csgn((I*exp(2*I*
(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))^3+1/12*I*x^3*Pi*csgn(I
*exp(2*I*(b*x+a)))^3+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a)
)+1))^3+1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^3+1/1
2*I*x^3*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))-1/6*I*x^3*Pi*c
sgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2-1/12*I*x^3*Pi*csgn(I*d)*cs
gn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2-1/12*I*x^3*Pi*csgn(I*exp(2*
I*(b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x
+a)))^2+1/12*I*x^3*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2
*I*(b*x+a))+1))^2+1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))
*d+I))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1
))^2-1/3*x^3*ln(exp(I*(b*x+a)))+1/6*x^3*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+
a))*d+I)+1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))
```

```
)*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))-1/12*I*x^3*Pi*csgn(I*d/(exp
(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x
+a)))^2+1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(
d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x
+a))+1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^2-1/12*I*x^3*Pi*csgn
(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^2+1/12*I
*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*
x+a))*d+I)/(exp(2*I*(b*x+a))+1))^2
```

Maxima [B] time = 1.16957, size = 460, normalized size = 2.71

$$\frac{\frac{12\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\operatorname{artanh}(d\tan(bx+a)-i\,d+1)}{b^2}-\frac{-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2+\left(8i\,(bx+a)^3-18i\,(bx+a)^2a+18i\,(bx+a)a^2\right)\arctan}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctanh(d*tan(b*
x + a) - I*d + 1)/b^2 - (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
+ a)^2*a^2 + (8*I*(b*x + a)^3 - 18*I*(b*x + a)^2*a + 18*I*(b*x + a)*a^2)*ar
ctan2(-d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) + cos(2*b*
x + 2*a) + 1) + (-12*I*(b*x + a)^2 + 18*I*(b*x + a)*a - 9*I*a^2)*dilog((I*d
 - 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(b*x + a)
*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^2 + 2*d
*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, (I*d
 - 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (I*d - 1)*e^(2*I*b*x + 2*I*a)))
/b^2)/b
```

Fricas [C] time = 2.08637, size = 981, normalized size = 5.77

$$\frac{i\,b^4x^4+2\,b^3x^3\log\left(-\frac{\left((d+i)e^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{d}\right)+6i\,b^2x^2\operatorname{Li}_2\left(\frac{1}{2}\,\sqrt{4i\,d-4}e^{(i\,bx+i\,a)}\right)+6i\,b^2x^2\operatorname{Li}_2\left(-\frac{1}{2}\,\sqrt{4i\,d-4}e^{(i\,bx+i\,a)}\right)-i}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(I*b^4*x^4 + 2*b^3*x^3*log(-((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*
b*x - 2*I*a)/d) + 6*I*b^2*x^2*dilog(1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) +
6*I*b^2*x^2*dilog(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - I*a^4 + 2*a^3*log
(((2*d + 2*I)*e^(I*b*x + I*a) + I*sqrt(4*I*d - 4))/(2*d + 2*I)) + 2*a^3*log
(((2*d + 2*I)*e^(I*b*x + I*a) - I*sqrt(4*I*d - 4))/(2*d + 2*I)) - 12*b*x*po
lylog(3, 1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - 12*b*x*polylog(3, -1/2*sqrt
(4*I*d - 4)*e^(I*b*x + I*a)) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(4*I*d - 4)*e^
(I*b*x + I*a) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sqrt(4*I*d - 4)*e^(I*b*x +
I*a) + 1) - 12*I*polylog(4, 1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - 12*I*pol
ylog(4, -1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(1-I*d+d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{artanh}\left(d \tan\left(bx + a\right) - i\,d + 1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(d*tan(b*x + a) - I*d + 1), x)
```

3.322 $\int x \tanh^{-1}(1 - id + d\tan(a + bx))\,dx$

Optimal. Leaf size=133

$$-\frac{\text{PolyLog}\left(3,-(1-id)e^{2ia+2ibx}\right)}{8b^2}+\frac{ix\text{PolyLog}\left(2,-(1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{4}x^2\log\left(1+(1-id)e^{2ia+2ibx}\right)+\frac{1}{2}x^2\tanh^{-1}(d\tan$$

```
[Out] (I/6)*b*x^3 + (x^2*ArcTanh[1 - I*d + d*Tan[a + b*x]])/2 - (x^2*Log[1 + (1 -
 I*d)*E^((2*I)*a + (2*I)*b*x)])/4 + ((I/4)*x*PolyLog[2, -((1 - I*d)*E^((2*I
)*a + (2*I)*b*x))])/b - PolyLog[3, -((1 - I*d)*E^((2*I)*a + (2*I)*b*x))]/(8
*b^2)
```

Rubi [A] time = 0.246391, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6263, 2184, 2190, 2531, 2282, 6589}

$$-\frac{\text{PolyLog}\left(3,-(1-id)e^{2ia+2ibx}\right)}{8b^2}+\frac{ix\text{PolyLog}\left(2,-(1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{4}x^2\log\left(1+(1-id)e^{2ia+2ibx}\right)+\frac{1}{2}x^2\tanh^{-1}(d\tan$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[1 - I*d + d*Tan[a + b*x]],x]
```

```
[Out] (I/6)*b*x^3 + (x^2*ArcTanh[1 - I*d + d*Tan[a + b*x]])/2 - (x^2*Log[1 + (1 -
 I*d)*E^((2*I)*a + (2*I)*b*x)])/4 + ((I/4)*x*PolyLog[2, -((1 - I*d)*E^((2*I
)*a + (2*I)*b*x))])/b - PolyLog[3, -((1 - I*d)*E^((2*I)*a + (2*I)*b*x))]/(8
*b^2)
```

Rule 6263

```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Tan[a + b*x]])/(f*(m
 + 1)), x] + Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c + I*d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
```

```
x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(1-id+d\tan(a+bx))\,dx = \frac{1}{2}x^2\tanh^{-1}(1-id+d\tan(a+bx))+\frac{1}{2}(ib)\int\frac{x^2}{1+(1-id)e^{2ia+2ibx}}\,dx$$
$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id+d\tan(a+bx))-\frac{1}{2}(b(i+d))\int\frac{e^{2ia+2ibx}x^2}{1+(1-id)e^{2ia+2ib}}$$
$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id+d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1-id)e^{2ia+2ibx}\right)+$$
$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id+d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1-id)e^{2ia+2ibx}\right)+$$
$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id+d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1-id)e^{2ia+2ibx}\right)+$$
$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id+d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1-id)e^{2ia+2ibx}\right)+$$

Mathematica [A] time = 0.290247, size = 119, normalized size = 0.89

$$\frac{1}{2}x^2\tanh^{-1}(d\tan(a+bx)-id+1)-\frac{2ibx\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{d+i}\right)+\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{d+i}\right)+2b^2x^2\log\left(1+\frac{ie^{-2i(a+bx)}}{d+i}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[1 - I*d + d*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[1 - I*d + d*Tan[a + b*x]])/2 - (2*b^2*x^2*Log[1 + I/((I + d)*E
^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/((I + d)*E^((2*I)*(a + b*x
)))] + PolyLog[3, (-I)/((I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

Maple [C] time = 7.477, size = 2256, normalized size = 17.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(1-I*d+d*tan(b*x+a)),x)
```

```
[Out] 1/2*I/b*a/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x-1/8*I*x^2*Pi*csgn
(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^3+1/4*x^2*ln(I*exp(2*I*(b*x+a))+e
```

```
xp(2*I*(b*x+a))*d+I)-1/8*I*x^2*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))
*d+I)/(exp(2*I*(b*x+a))+1))^3+1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*ex
p(2*I*(b*x+a)))^3-1/4*I/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*x^2-1/4/b^2*a^
2*d/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)-1/8*I*x^2*Pi*csgn(I/(
exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^2-1/8*I/
b^2/(I+d)*polylog(3,I*(I+d)*exp(2*I*(b*x+a)))-1/4*d/(I+d)*ln(1-I*(I+d)*exp(
2*I*(b*x+a)))*x^2+1/2*I/b^2*a^2/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2
))+1/6*I*b*x^3+1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))
*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))-1/4*I*x^2*Pi*csgn(I*exp(I*(b
*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2+1/8*I*x^2*Pi*csgn(I*d)*csgn(I*exp(2*I*(b
*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a))
)+1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I))*csgn(I*(I*
exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))^2-1/8*I*x^2*Pi
*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))+1
)*exp(2*I*(b*x+a)))^2+1/2*I/b^2*a^2/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^
(1/2))-1/4*x^2*ln(d)+1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a
))*d+I)/(exp(2*I*(b*x+a))+1))*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I
)/(exp(2*I*(b*x+a))+1))^2+1/8*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(
2*I*(b*x+a)))-1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*cs
gn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2+1/8*I*x^2*Pi*csgn(I/(exp(2*
I*(b*x+a))+1))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b
*x+a))+1))^2-1/4*I*Pi*x^2+1/2/b^2*a/(I+d)*dilog(1-I*exp(I*(b*x+a))*(-I*(I+d
))^(1/2))+1/2/b^2*a/(I+d)*dilog(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/4/b/
(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*x-1/4/b^2/(I+d)*polylog(2,I*(I+d)
*exp(2*I*(b*x+a)))*a-1/8/b^2*d/(I+d)*polylog(3,I*(I+d)*exp(2*I*(b*x+a)))-1/
8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a
))+1))^3-1/2*I/b^2*a*d/(I+d)*dilog(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/2
*I/b^2*a*d/(I+d)*dilog(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/8*I*x^2*Pi*cs
gn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^2-1/8*
I*x^2*Pi*csgn(I*d)*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2-1/8*I*
x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1
))*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))-1/2
/b*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*x*a+1/2/b*a*d/(I+d)*ln(1-I*exp(I*
(b*x+a))*(-I*(I+d))^(1/2))*x+1/2/b*a*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d
))^(1/2))*x-1/2*I/b/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*x*a+1/4*I/b*d/(I+d
)*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*x+1/4*I/b^2*d/(I+d)*polylog(2,I*(I+d)
*exp(2*I*(b*x+a)))*a+1/2*I/b*a/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2)
)*x-1/4/b^2*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a^2+1/2/b^2*a^2*d/(I+d)*
ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))+1/2/b^2*a^2*d/(I+d)*ln(1+I*exp(I*(b
*x+a))*(-I*(I+d))^(1/2))-1/4*I/b^2*a^2/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*
(b*x+a))*d+I)-1/4*I/b^2/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a^2+1/8*I*x^2*
Pi*csgn(I*exp(2*I*(b*x+a)))^3+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I
*(b*x+a))+1))^3+1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^
2+1/8*I*x^2*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x
+a))+1))^2-1/2*x^2*ln(exp(I*(b*x+a)))-1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))
```

```
+1))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I))*csgn(I*(I*exp(2*I*(b
*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))+1/8*I*x^2*Pi*csgn(I/(exp
(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I
*(b*x+a))+1))
```

Maxima [B] time = 1.08567, size = 333, normalized size = 2.5

$$\frac{12\left((bx+a)^2-2\,(bx+a)a\right)\operatorname{artanh}(d\tan(bx+a)-i\,d+1)}{b}-\frac{-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\mathrm{Li}_2\left((i\,d-1)e^{(2i\,bx+2i\,a)}\right)+\left(6i\,(bx+a)^2-12i\,(bx+a)a\right)\arctan(-d\cos(2\,bx+2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arctanh(d*tan(b*x + a) - I*d + 1)/b
- (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((I*d - 1)*e^(2*I*b
*x + 2*I*a)) + (6*I*(b*x + a)^2 - 12*I*(b*x + a)*a)*arctan2(-d*cos(2*b*x +
2*a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) + cos(2*b*x + 2*a) + 1) + 3*((b
*x + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin
(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) + 3*polylo
g(3, (I*d - 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

Fricas [C] time = 1.94992, size = 813, normalized size = 6.11

$$\frac{2i\,b^3x^3+3\,b^2x^2\log\left(-\frac{\left((d+i)e^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{d}\right)+2i\,a^3+6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i\,d-4}e^{(i\,bx+i\,a)}\right)+6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4i\,d-4}e^{(i\,bx+i\,a)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*I*b^3*x^3 + 3*b^2*x^2*log(-((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*
I*b*x - 2*I*a)/d) + 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(4*I*d - 4)*e^(I*b*x +
I*a)) + 6*I*b*x*dilog(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - 3*a^2*log(((2
*d + 2*I)*e^(I*b*x + I*a) + I*sqrt(4*I*d - 4))/(2*d + 2*I)) - 3*a^2*log(((2
*d + 2*I)*e^(I*b*x + I*a) - I*sqrt(4*I*d - 4))/(2*d + 2*I)) - 3*(b^2*x^2 -
a^2)*log(1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) - 3*(b^2*x^2 - a^2)*log(-
1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) - 6*polylog(3, 1/2*sqrt(4*I*d - 4)
```

```
*e^(I*b*x + I*a)) - 6*polylog(3, -1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(1-I*d+d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{artanh}\left(d \tan\left(bx + a\right) - i\, d + 1\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(d*tan(b*x + a) - I*d + 1), x)
```

3.323 $\int \tanh^{-1}(1 - id + d\tan(a + bx))\,dx$

Optimal. Leaf size=93

$$\frac{i\text{PolyLog}\left(2,-(1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{2}x\log\left(1+(1-id)e^{2ia+2ibx}\right)+x\tanh^{-1}(d\tan(a+bx)-id+1)+\frac{1}{2}ibx^2$$

```
[Out] (I/2)*b*x^2 + x*ArcTanh[1 - I*d + d*Tan[a + b*x]] - (x*Log[1 + (1 - I*d)*E^
((2*I)*a + (2*I)*b*x)])/2 + ((I/4)*PolyLog[2, -((1 - I*d)*E^((2*I)*a + (2*I
)*b*x))])/b
```

Rubi [A] time = 0.151419, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6255, 2184, 2190, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2,-(1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{2}x\log\left(1+(1-id)e^{2ia+2ibx}\right)+x\tanh^{-1}(d\tan(a+bx)-id+1)+\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[1 - I*d + d*Tan[a + b*x]],x]
```

```
[Out] (I/2)*b*x^2 + x*ArcTanh[1 - I*d + d*Tan[a + b*x]] - (x*Log[1 + (1 - I*d)*E^
((2*I)*a + (2*I)*b*x)])/2 + ((I/4)*PolyLog[2, -((1 - I*d)*E^((2*I)*a + (2*I
)*b*x))])/b
```

Rule 6255

```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Tanh[c + d*Tan[a + b*x]], x] + Dist[I*b, Int[x/(c + I*d + c*E^(2*I*a + 2*I*
b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \tanh^{-1}(1-id+d\tan(a+bx))\,dx = x\tanh^{-1}(1-id+d\tan(a+bx))+(ib)\int \frac{x}{1+(1-id)e^{2ia+2ibx}}\,dx$$
$$=\frac{1}{2}ibx^2+x\tanh^{-1}(1-id+d\tan(a+bx))-(b(i+d))\int \frac{e^{2ia+2ibx}x}{1+(1-id)e^{2ia+2ibx}}\,dx$$
$$=\frac{1}{2}ibx^2+x\tanh^{-1}(1-id+d\tan(a+bx))-\frac{1}{2}x\log\left(1+(1-id)e^{2ia+2ibx}\right)+\frac{1}{2}\int$$
$$=\frac{1}{2}ibx^2+x\tanh^{-1}(1-id+d\tan(a+bx))-\frac{1}{2}x\log\left(1+(1-id)e^{2ia+2ibx}\right)-\frac{i\,\mathrm{Sul}}{}$$
$$=\frac{1}{2}ibx^2+x\tanh^{-1}(1-id+d\tan(a+bx))-\frac{1}{2}x\log\left(1+(1-id)e^{2ia+2ibx}\right)+\frac{i\mathrm{Li}_2}{}$$

Mathematica [B] time = 12.9263, size = 766, normalized size = 8.24

$$\frac{x\sec^2(a+bx)(\cos(bx)+i\sin(bx))(\sin(bx)+i\cos(bx))(d\sin(a+bx)+(2-id)\cos(a+bx))\left(\mathrm{PolyLog}\left(2,-\frac{1}{2}(\cos(a)+\right.\right.}{(\tan(a+bx)-i)(d\tan(a+bx)-id+2)(id\sin(a+bx)+(d+2i)\cos(a+bx))\left(\frac{\sec^2(bx)\log\left(\frac{\sec(bx)(d\,s}{}\right.}{\tan}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[1 - I*d + d*Tan[a + b*x]],x]
```

```
[Out] x*ArcTanh[1 - I*d + d*Tan[a + b*x]] + (x*((2*I)*b*x*Log[2*Cos[b*x]*(Cos[b*x
] - I*Sin[b*x])] - Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((2*I + d)*Cos[a + b*x
] + I*d*Sin[a + b*x]))/(2*(I + d))]*Log[1 - I*Tan[b*x]] + Log[(Sec[b*x]*((2
 - I*d)*Cos[a + b*x] + d*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*Log[1 +
I*Tan[b*x]] - PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] - PolyLog[2, (Sec[b*x]
*(d*Cos[a] + I*(2*I + d)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*(I + d
))] + PolyLog[2, -((Cos[a] + I*Sin[a])*(d*Cos[a] + I*(2*I + d)*Sin[a])*(-I
+ Tan[b*x]))/2])*Sec[a + b*x]^2*(Cos[b*x] + I*Sin[b*x])*(I*Cos[b*x] + Sin[b
*x])*((2 - I*d)*Cos[a + b*x] + d*Sin[a + b*x]))/(((2*I + d)*Cos[a + b*x] +
I*d*Sin[a + b*x])*((I*Log[1 + I*Tan[b*x]]*Sec[b*x]*(d*Cos[a] + I*(2*I + d)*
Sin[a]))/((2*I + d)*Cos[a + b*x] + I*d*Sin[a + b*x]) + (Log[1 - I*Tan[b*x]]
*Sec[b*x]*((-I)*d*Cos[a] + (2*I + d)*Sin[a]))/((2*I + d)*Cos[a + b*x] + I*d
*Sin[a + b*x]) + 2*b*x*(1 - I*Tan[b*x]) + (Log[(Sec[b*x]*((2 - I*d)*Cos[a +
 b*x] + d*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*Sec[b*x]^2)/(-I + Tan[b
*x]) - (Log[1 + ((Cos[a] + I*Sin[a])*(d*Cos[a] + I*(2*I + d)*Sin[a])*(-I +
Tan[b*x]))/2]*Sec[b*x]^2)/(-I + Tan[b*x]) + Log[(Sec[b*x]*(Cos[a] - I*Sin[a
])*((2*I + d)*Cos[a + b*x] + I*d*Sin[a + b*x]))/(2*(I + d))]*(-I + Tan[b*x]
) - (Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((2*I + d)*Cos[a + b*x] + I*d*Sin[a
+ b*x]))/(2*(I + d))]*Sec[b*x]^2)/(I + Tan[b*x]))*(-I + Tan[a + b*x])*(2 -
I*d + d*Tan[a + b*x]))
```

Maple [B] time = 0.134, size = 292, normalized size = 3.1

$$\frac{\frac{i}{2}\operatorname{Artanh}\left(1-id+d\tan\left(bx+a\right)\right)\ln\left(id+d\tan\left(bx+a\right)\right)}{b}-\frac{\frac{i}{2}\operatorname{Artanh}\left(1-id+d\tan\left(bx+a\right)\right)\ln\left(-id+d\tan\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(1-I*d+d*tan(b*x+a)),x)
```

```
[Out] 1/2*I/b*arctanh(1-I*d+d*tan(b*x+a))*ln(I*d+d*tan(b*x+a))-1/2*I/b*arctanh(1-
I*d+d*tan(b*x+a))*ln(-I*d+d*tan(b*x+a))-1/4*I/b*dilog((2-I*d+d*tan(b*x+a))/
(-2*I*d+2))-1/4*I/b*ln(I*d+d*tan(b*x+a))*ln((2-I*d+d*tan(b*x+a))/(-2*I*d+2)
)+1/4*I/b*dilog(1/2*I*(-I*d+d*tan(b*x+a))/d)+1/4*I/b*ln(I*d+d*tan(b*x+a))*l
n(1/2*I*(-I*d+d*tan(b*x+a))/d)-1/8*I/b*ln(-I*d+d*tan(b*x+a))^2+1/4*I/b*dilo
g(1-1/2*I*d+1/2*d*tan(b*x+a))+1/4*I/b*ln(-I*d+d*tan(b*x+a))*ln(1-1/2*I*d+1/
2*d*tan(b*x+a))
```

Maxima [B] time = 1.48609, size = 355, normalized size = 3.82

$$-\frac{4\,(bx+a)d\left(\frac{\log(d\tan(bx+a)-i\,d+2)}{d}-\frac{\log(\tan(bx+a)-i)}{d}\right)+d\left(-\frac{2i\left(\log(d\tan(bx+a)-i\,d+2)\log\left(-\frac{i\,d\tan(bx+a)+d+2i}{2\,d+2i}+1\right)+\mathrm{Li}_2\left(\frac{i\,d\tan(bx+a)+d+2i}{2\,d+2i}\right)\right)}{d}+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*(b*x + a)*d*(log(d*tan(b*x + a) - I*d + 2)/d - log(tan(b*x + a) - I
)/d) + d*(-2*I*(log(d*tan(b*x + a) - I*d + 2)*log(-(I*d*tan(b*x + a) + d +
2*I)/(2*d + 2*I) + 1) + dilog((I*d*tan(b*x + a) + d + 2*I)/(2*d + 2*I)))/d
+ (2*I*log(d*tan(b*x + a) - I*d + 2)*log(tan(b*x + a) - I) - I*log(tan(b*x
+ a) - I)^2)/d - 2*I*(log(1/2*d*tan(b*x + a) - 1/2*I*d + 1)*log(tan(b*x + a
) - I) + dilog(-1/2*d*tan(b*x + a) + 1/2*I*d))/d + 2*I*(log(tan(b*x + a) -
I)*log(-1/2*I*tan(b*x + a) + 1/2) + dilog(1/2*I*tan(b*x + a) + 1/2))/d) - 8
*(b*x + a)*arctanh(d*tan(b*x + a) - I*d + 1))/b
```

Fricas [B] time = 1.99135, size = 605, normalized size = 6.51

$$\frac{i\,b^2x^2+bx\log\left(-\frac{\left((d+i)e^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{d}\right)-i\,a^2-(bx+a)\log\left(\frac{1}{2}\sqrt{4i\,d-4}e^{(i\,bx+i\,a)}+1\right)-(bx+a)\log\left(-\frac{1}{2}\sqrt{4i\,d-4}e^{(i\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*b^2*x^2 + b*x*log(-((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2
*I*a)/d) - I*a^2 - (b*x + a)*log(1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) -
 (b*x + a)*log(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) + a*log(((2*d + 2*
I)*e^(I*b*x + I*a) + I*sqrt(4*I*d - 4))/(2*d + 2*I)) + a*log(((2*d + 2*I)*e
^(I*b*x + I*a) - I*sqrt(4*I*d - 4))/(2*d + 2*I)) + I*dilog(1/2*sqrt(4*I*d -
 4)*e^(I*b*x + I*a)) + I*dilog(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(1-I*d+d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{artanh}(d\tan(bx+a) - i\,d + 1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*tan(b*x + a) - I*d + 1), x)
```

3.324 $\int \frac{\tanh^{-1}(1-id+d\tan(a+bx))}{x}\,dx$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(\frac{\tanh^{-1}(d\tan(a+bx)-id+1)}{x},x\right)$$

```
[Out] CannotIntegrate[ArcTanh[1 - I*d + d*Tan[a + b*x]]/x, x]
```

Rubi [A] time = 0.0964059, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tanh^{-1}(1-id+d\tan(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTanh[1 - I*d + d*Tan[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTanh[1 - I*d + d*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(1-id+d\tan(a+bx))}{x}\,dx=\int \frac{\tanh^{-1}(1-id+d\tan(a+bx))}{x}\,dx$$

Mathematica [A] time = 1.04263, size = 0, normalized size = 0.

$$\int \frac{\tanh^{-1}(1-id+d\tan(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTanh[1 - I*d + d*Tan[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcTanh[1 - I*d + d*Tan[a + b*x]]/x, x]
```

Maple [A] time = 0.388, size = 0, normalized size = 0.

$$\int \frac{\operatorname{Artanh}(1 - id + d\tan(bx + a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(1-I*d+d*tan(b*x+a))/x,x)
```

```
[Out] int(arctanh(1-I*d+d*tan(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-i\,bx + \frac{1}{4}\,(i\,\pi - 4i\,a - 2\,\log(d))\log(x) - \frac{1}{2}i\int \frac{\arctan(-d\cos(2\,bx + 2\,a) + \sin(2\,bx + 2\,a), -d\sin(2\,bx + 2\,a) - \cos(2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1-I*d+d*tan(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -I*b*x + 1/4*(I*pi - 4*I*a - 2*log(d))*log(x) - 1/2*I*integrate(arctan2(-d*
cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)
 - 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*si
n(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\log\left(-\frac{\left((d+i)e^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{d}\right)}{2\,x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1-I*d+d*tan(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*log(-((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)/d)
/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(1-I*d+d*tan(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}\left(d\tan\left(bx+a\right)-i\,d+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1-I*d+d*tan(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*tan(b*x + a) - I*d + 1)/x, x)
```

3.325 $\int x^2 \tanh^{-1}(1 + id - d\tan(a + bx))\, dx$

Optimal. Leaf size=171

$$-\frac{x\text{PolyLog}\left(3,-(1+id)e^{2ia+2ibx}\right)}{4b^2}-\frac{i\text{PolyLog}\left(4,-(1+id)e^{2ia+2ibx}\right)}{8b^3}+\frac{ix^2\text{PolyLog}\left(2,-(1+id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{6}x^3\log\left(1+\right.$$

```
[Out] (I/12)*b*x^4 + (x^3*ArcTanh[1 + I*d - d*Tan[a + b*x]])/3 - (x^3*Log[1 + (1
+ I*d)*E^((2*I)*a + (2*I)*b*x)])/6 + ((I/4)*x^2*PolyLog[2, -((1 + I*d)*E^((
2*I)*a + (2*I)*b*x))])/b - (x*PolyLog[3, -((1 + I*d)*E^((2*I)*a + (2*I)*b*x
))])/(4*b^2) - ((I/8)*PolyLog[4, -((1 + I*d)*E^((2*I)*a + (2*I)*b*x))])/b^3
```

Rubi [A] time = 0.289115, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6263, 2184, 2190, 2531, 6609, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(3,-(1+id)e^{2ia+2ibx}\right)}{4b^2}-\frac{i\text{PolyLog}\left(4,-(1+id)e^{2ia+2ibx}\right)}{8b^3}+\frac{ix^2\text{PolyLog}\left(2,-(1+id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{6}x^3\log\left(1+\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[1 + I*d - d*Tan[a + b*x]],x]
```

```
[Out] (I/12)*b*x^4 + (x^3*ArcTanh[1 + I*d - d*Tan[a + b*x]])/3 - (x^3*Log[1 + (1
+ I*d)*E^((2*I)*a + (2*I)*b*x)])/6 + ((I/4)*x^2*PolyLog[2, -((1 + I*d)*E^((
2*I)*a + (2*I)*b*x))])/b - (x*PolyLog[3, -((1 + I*d)*E^((2*I)*a + (2*I)*b*x
))])/(4*b^2) - ((I/8)*PolyLog[4, -((1 + I*d)*E^((2*I)*a + (2*I)*b*x))])/b^3
```

Rule 6263

```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Tan[a + b*x]])/(f*(m
 + 1)), x] + Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c + I*d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
```

```
x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \tanh^{-1}(1+id-d\tan(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(1+id-d\tan(a+bx)) + \frac{1}{3}(ib)\int \frac{x^3}{1+(1+id)e^{2ia+2ibx}}\,dx$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1+id-d\tan(a+bx)) - \frac{1}{3}(b(i-d))\int \frac{e^{2ia+2ibx}x^3}{1+(1+id)e^{2ia+2}}$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1+id-d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+id)e^{2ia+2ibx}\right) -$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1+id-d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+id)e^{2ia+2ibx}\right) -$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1+id-d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+id)e^{2ia+2ibx}\right) -$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1+id-d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+id)e^{2ia+2ibx}\right) -$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1+id-d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+id)e^{2ia+2ibx}\right) -$$

Mathematica [A] time = 0.391566, size = 156, normalized size = 0.91

$$\frac{1}{3}x^3\tanh^{-1}(d(-\tan(a+bx))+id+1) - \frac{6ib^2x^2\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{d-i}\right) + 6bx\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{d-i}\right) - 3i\text{PolyLog}\left(4,\frac{ie^{-2i(}}{d-}\right.}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[1 + I*d - d*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[1 + I*d - d*Tan[a + b*x]])/3 - (4*b^3*x^3*Log[1 - I/((-I + d)*
E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, I/((-I + d)*E^((2*I)*(a +
b*x)))] + 6*b*x*PolyLog[3, I/((-I + d)*E^((2*I)*(a + b*x)))] - (3*I)*PolyLo
g[4, I/((-I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

Maple [C] time = 14.246, size = 2456, normalized size = 14.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*arctanh(-1-I*d+d*tan(b*x+a)),x)
```

```
[Out] 1/12*I*x^3*Pi*csgn(I*d)*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(
I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))-1/2*I/b^3*a^3/(-d+I)*ln(1-I*exp(
I*(b*x+a))*(-I*(-d+I))^(1/2))+1/6*I/b^3*a^3/(-d+I)*ln(I*exp(2*I*(b*x+a))-ex
p(2*I*(b*x+a))*d+I)+1/8*I/b^3*d/(-d+I)*polylog(4,I*(-d+I)*exp(2*I*(b*x+a)))
+1/3*I/b^3/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b*x+a)))*a^3+1/12*I*x^3*Pi*csgn(d/
(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^3+1/12*I*x^3*Pi*csgn((exp(2*I*(b*x+a
))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3-1/12*I*x^3*Pi*csgn(d/(ex
p(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2+1/6*d/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b
*x+a)))*x^3-1/6*x^3*ln(d)+1/2*I/b^2/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b*x+a)))*
x*a^2-1/2*I/b^2*a^2/(-d+I)*ln(1+I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))*x+1/4*I
/b^3*d/(-d+I)*polylog(2,I*(-d+I)*exp(2*I*(b*x+a)))*a^2+1/12*I*b*x^4+1/12*I*
x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b
*x+a))-I)/(exp(2*I*(b*x+a))+1))^2+1/6*I*x^3*Pi-1/2/b^3*a^2/(-d+I)*dilog(1+I
*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))-1/2/b^3*a^2/(-d+I)*dilog(1-I*exp(I*(b*x+
a))*(-I*(-d+I))^(1/2))-1/4/b/(-d+I)*polylog(2,I*(-d+I)*exp(2*I*(b*x+a)))*x^
2+1/4/b^3/(-d+I)*polylog(2,I*(-d+I)*exp(2*I*(b*x+a)))*a^2-1/12*I*x^3*Pi*csg
n(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3-1/6*I
/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b*x+a)))*x^3+1/2/b^2*a^2*d/(-d+I)*ln(1+I*exp
(I*(b*x+a))*(-I*(-d+I))^(1/2))*x+1/2/b^2*a^2*d/(-d+I)*ln(1-I*exp(I*(b*x+a))
*(-I*(-d+I))^(1/2))*x+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))^3+1/12*I*x^3*P
i*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^3+1/12*I*x^3*Pi*csgn(I*d/(e
xp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^3+1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a
))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*d-I
*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2-1/12*I*x^3*Pi*csgn(I*(exp(2*I*
(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a
))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))+1/12*I*x^3*Pi*csgn(I*exp(I
*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))-1/12*I*x^3*Pi*csgn((exp(2*I*(b*x+a))*
d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2-1/6/b^3*a^3*d/(-d+I)*ln(I*e
xp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d+I)-1/3/b^3*d/(-d+I)*ln(1-I*(-d+I)*exp(2*
I*(b*x+a)))*a^3-1/6*I*x^3*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))
)^2+1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I))*csgn(I*
(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2-1/12*I*x^
3*Pi*csgn(I*d)*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2-1/12*I*x^3
*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I*(b*x+a)
)+1)*exp(2*I*(b*x+a)))^2+1/2/b^3*a^3*d/(-d+I)*ln(1+I*exp(I*(b*x+a))*(-I*(-d
+I))^(1/2))+1/2/b^3*a^3*d/(-d+I)*ln(1-I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))+1
/4/b^2*d/(-d+I)*polylog(3,I*(-d+I)*exp(2*I*(b*x+a)))*x-1/4*I/b^2/(-d+I)*pol
ylog(3,I*(-d+I)*exp(2*I*(b*x+a)))*x-1/2*I/b^3*a^3/(-d+I)*ln(1+I*exp(I*(b*x+
a))*(-I*(-d+I))^(1/2))-1/3*x^3*ln(exp(I*(b*x+a)))+1/12*I*x^3*Pi*csgn(I/(exp
(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I
*(b*x+a))+1))-1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))
*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2+1/12*I*x^3*Pi*csgn(I*d/(ex
p(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*
x+a)))-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))/(
exp(2*I*(b*x+a))+1))^2-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*
```

```
I*(b*x+a))/(exp(2*I*(b*x+a))+1))^2-1/2*I/b^2*a^2/(-d+I)*ln(1-I*exp(I*(b*x+a
))*(-I*(-d+I))^(1/2))*x-1/2*I/b^3*a^2*d/(-d+I)*dilog(1-I*exp(I*(b*x+a))*(-I
*(-d+I))^(1/2))+1/8/b^3/(-d+I)*polylog(4,I*(-d+I)*exp(2*I*(b*x+a)))-1/12*I*
x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b
*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a
))+1))-1/2*I/b^3*a^2*d/(-d+I)*dilog(1+I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))-1
/4*I/b*d/(-d+I)*polylog(2,I*(-d+I)*exp(2*I*(b*x+a)))*x^2-1/2/b^2*d/(-d+I)*l
n(1-I*(-d+I)*exp(2*I*(b*x+a)))*x*a^2+1/6*x^3*ln(exp(2*I*(b*x+a))*d-I*exp(2*
I*(b*x+a))-I)
```

Maxima [B] time = 1.18215, size = 459, normalized size = 2.68

$$-\frac{\frac{12\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\operatorname{artanh}(d\tan(bx+a)-i\,d-1)}{b^2}+\frac{-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2+\left(8i\,(bx+a)^3-18i\,(bx+a)^2a+18i\,(bx+a)a^2\right)\operatorname{arcta}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctanh(d*tan(b
*x + a) - I*d - 1)/b^2 + (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
 + a)^2*a^2 + (8*I*(b*x + a)^3 - 18*I*(b*x + a)^2*a + 18*I*(b*x + a)*a^2)*a
rctan2(d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + cos(2*b
*x + 2*a) + 1) + (-12*I*(b*x + a)^2 + 18*I*(b*x + a)*a - 9*I*a^2)*dilog((-I
*d - 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(b*x +
a)*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^2 - 2
*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, (-
I*d - 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (-I*d - 1)*e^(2*I*b*x + 2*I*
a)))/b^2)/b
```

Fricas [C] time = 2.08202, size = 994, normalized size = 5.81

$$\frac{i\,b^4x^4-2\,b^3x^3\log\left(-\frac{de^{(2i\,bx+2i\,a)}}{(d-i)e^{(2i\,bx+2i\,a)}-i}\right)+6i\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{-4i\,d-4}e^{(i\,bx+i\,a)}\right)+6i\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{-4i\,d-4}e^{(i\,bx+i\,a)}\right)-i\,a^4+2\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(I*b^4*x^4 - 2*b^3*x^3*log(-d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*x
+ 2*I*a) - I)) + 6*I*b^2*x^2*dilog(1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) +
6*I*b^2*x^2*dilog(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - I*a^4 + 2*a^3*lo
g(((2*d - 2*I)*e^(I*b*x + I*a) + I*sqrt(-4*I*d - 4))/(2*d - 2*I)) + 2*a^3*l
og(((2*d - 2*I)*e^(I*b*x + I*a) - I*sqrt(-4*I*d - 4))/(2*d - 2*I)) - 12*b*x
*polylog(3, 1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - 12*b*x*polylog(3, -1/2*
sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(-4*I*d -
 4)*e^(I*b*x + I*a) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sqrt(-4*I*d - 4)*e^(I
*b*x + I*a) + 1) - 12*I*polylog(4, 1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) -
12*I*polylog(4, -1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**2*atanh(-1-I*d+d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -x^2 \operatorname{artanh}\left(d \tan\left(bx+a\right) - i\,d - 1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctanh(d*tan(b*x + a) - I*d - 1), x)
```

3.326 $\int x \tanh^{-1}(1 + id - d\tan(a + bx))\, dx$

Optimal. Leaf size=134

$$-\frac{\text{PolyLog}\left(3,-(1+id)e^{2ia+2ibx}\right)}{8b^2}+\frac{ix\text{PolyLog}\left(2,-(1+id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{4}x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)+\frac{1}{2}x^2\tanh^{-1}(d(-\text{t}$$

```
[Out] (I/6)*b*x^3 + (x^2*ArcTanh[1 + I*d - d*Tan[a + b*x]])/2 - (x^2*Log[1 + (1 +
 I*d)*E^((2*I)*a + (2*I)*b*x)])/4 + ((I/4)*x*PolyLog[2, -((1 + I*d)*E^((2*I
)*a + (2*I)*b*x))])/b - PolyLog[3, -((1 + I*d)*E^((2*I)*a + (2*I)*b*x))]/(8
*b^2)
```

Rubi [A] time = 0.244279, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6263, 2184, 2190, 2531, 2282, 6589}

$$-\frac{\text{PolyLog}\left(3,-(1+id)e^{2ia+2ibx}\right)}{8b^2}+\frac{ix\text{PolyLog}\left(2,-(1+id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{4}x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)+\frac{1}{2}x^2\tanh^{-1}(d(-\text{t}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[1 + I*d - d*Tan[a + b*x]],x]
```


```
[Out] (I/6)*b*x^3 + (x^2*ArcTanh[1 + I*d - d*Tan[a + b*x]])/2 - (x^2*Log[1 + (1 +
 I*d)*E^((2*I)*a + (2*I)*b*x)])/4 + ((I/4)*x*PolyLog[2, -((1 + I*d)*E^((2*I
)*a + (2*I)*b*x))])/b - PolyLog[3, -((1 + I*d)*E^((2*I)*a + (2*I)*b*x))]/(8
*b^2)
```


Rule 6263

```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Tan[a + b*x]])/(f*(m
 + 1)), x] + Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c + I*d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
```

```
x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(1+id-d\tan(a+bx))\,dx=\frac{1}{2}x^2\tanh^{-1}(1+id-d\tan(a+bx))+\frac{1}{2}(ib)\int\frac{x^2}{1+(1+id)e^{2ia+2ibx}}\,dx$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id-d\tan(a+bx))-\frac{1}{2}(b(i-d))\int\frac{e^{2ia+2ibx}x^2}{1+(1+id)e^{2ia+2ib}}$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id-d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)+$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id-d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)+$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id-d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)+$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id-d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)+$$

Mathematica [A] time = 0.298871, size = 120, normalized size = 0.9

$$\frac{1}{2}x^2\tanh^{-1}(d(-\tan(a+bx))+id+1)-\frac{2ibx\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{d-i}\right)+\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{d-i}\right)+2b^2x^2\log\left(1-\frac{ie^{-2i(a+bx)}}{d-i}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[1 + I*d - d*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[1 + I*d - d*Tan[a + b*x]])/2 - (2*b^2*x^2*Log[1 - I/((-I + d)*
E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/((-I + d)*E^((2*I)*(a + b*x)
))] + PolyLog[3, I/((-I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

Maple [C] time = 3.953, size = 2358, normalized size = 17.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*arctanh(-1-I*d+d*tan(b*x+a)),x)
```

```
[Out] 1/4*I*x^2*Pi+1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^3
-1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I
```

```
*(b*x+a))+1))^2-1/2*I/b/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b*x+a)))*x*a+1/2*I/b*
a/(-d+I)*ln(1+I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))*x+1/2*I/b^2*a*d/(-d+I)*di
log(1-I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))+1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x
+a))+1))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))
+1))^2+1/8*I*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I))*csgn(
I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2-1/4*I/(
-d+I)*ln(1-I*(-d+I)*exp(2*I*(b*x+a)))*x^2-1/8*I*x^2*Pi*csgn((exp(2*I*(b*x+a
))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2+1/6*I*b*x^3-1/4*I/b^2*a^
2/(-d+I)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d+I)-1/4*I/b^2/(-d+I)*ln(1-
I*(-d+I)*exp(2*I*(b*x+a)))*a^2+1/2*I/b^2*a^2/(-d+I)*ln(1+I*exp(I*(b*x+a))*(
-I*(-d+I))^(1/2))+1/2*I/b^2*a^2/(-d+I)*ln(1-I*exp(I*(b*x+a))*(-I*(-d+I))^(1
/2))+1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(e
xp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))-1/4*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))*cs
gn(I*exp(2*I*(b*x+a)))^2+1/8*I*x^2*Pi*csgn(I*d)*csgn(I*exp(2*I*(b*x+a))/(ex
p(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))-1/8*I*x^
2*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a
))+1)*exp(2*I*(b*x+a)))^2+1/4*x^2*ln(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-
I)-1/4*x^2*ln(d)+1/8*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+
a)))-1/8*I*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I
*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+
a))+1))-1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I*d
/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2+1/4/b^2*d/(-d+I)*ln(1-I*(-d+I)*ex
p(2*I*(b*x+a)))*a^2-1/2/b^2*a^2*d/(-d+I)*ln(1+I*exp(I*(b*x+a))*(-I*(-d+I))^
(1/2))-1/2/b^2*a^2*d/(-d+I)*ln(1-I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))-1/8*I*
x^2*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2-1/8*I*x^2*Pi*csgn(I*
exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^2-1/8*I*x^2
*Pi*csgn(I*d)*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2+1/2*I/b*a/(
-d+I)*ln(1-I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))*x-1/2/b*a*d/(-d+I)*ln(1+I*ex
p(I*(b*x+a))*(-I*(-d+I))^(1/2))*x-1/8*I*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I
*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3+1/8*I*x^2*Pi*csgn(d/(exp(2*I*(
b*x+a))+1)*exp(2*I*(b*x+a)))^3-1/8*I/b^2/(-d+I)*polylog(3,I*(-d+I)*exp(2*I*
(b*x+a)))+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))^3+1/8*I*x^2*Pi*csgn(I*exp(2
*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^3+1/8*I*x^2*Pi*csgn((exp(2*I*(b*x+a))*d-I
*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3+1/8/b^2*d/(-d+I)*polylog(3,I*(
-d+I)*exp(2*I*(b*x+a)))+1/2/b^2*a/(-d+I)*dilog(1+I*exp(I*(b*x+a))*(-I*(-d+I
))^(1/2))+1/2/b^2*a/(-d+I)*dilog(1-I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))-1/4/
b/(-d+I)*polylog(2,I*(-d+I)*exp(2*I*(b*x+a)))*x-1/4/b^2/(-d+I)*polylog(2,I*
(-d+I)*exp(2*I*(b*x+a)))*a+1/4*d/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b*x+a)))*x^2
+1/8*I*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*
x+a))+1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+
1))^2+1/4/b^2*a^2*d/(-d+I)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d+I)-1/2*
x^2*ln(exp(I*(b*x+a)))+1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*exp
(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))-1/8*I*x^2*Pi*c
sgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I
))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))+1
```

```
/2*I/b^2*a*d/(-d+I)*dilog(1+I*exp(I*(b*x+a))*(-I*(-d+I))^(1/2))-1/4*I/b*d/(
-d+I)*polylog(2,I*(-d+I)*exp(2*I*(b*x+a)))*x-1/4*I/b^2*d/(-d+I)*polylog(2,I
*(-d+I)*exp(2*I*(b*x+a)))*a-1/2/b*a*d/(-d+I)*ln(1-I*exp(I*(b*x+a))*(-I*(-d+
I))^(1/2))*x+1/2/b*d/(-d+I)*ln(1-I*(-d+I)*exp(2*I*(b*x+a)))*x*a
```

Maxima [B] time = 1.09568, size = 332, normalized size = 2.48

$$-\frac{\frac{12\left((bx+a)^2-2(bx+a)a\right)\operatorname{artanh}(d\tan(bx+a)-id-1)}{b}+\frac{-4i(bx+a)^3+12i(bx+a)^2a-6ibx\operatorname{Li}_2\left((-id-1)e^{(2ibx+2ia)}\right)+\left(6i(bx+a)^2-12i(bx+a)a\right)\arctan(d\cos(2bx+\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arctanh(d*tan(b*x + a) - I*d - 1)/b
 + (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((-I*d - 1)*e^(2*I
*b*x + 2*I*a)) + (6*I*(b*x + a)^2 - 12*I*(b*x + a)*a)*arctan2(d*cos(2*b*x +
 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + cos(2*b*x + 2*a) + 1) + 3*(
(b*x + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*s
in(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) + 3*poly
log(3, (-I*d - 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

Fricas [C] time = 2.00718, size = 822, normalized size = 6.13

$$\frac{2ib^3x^3-3b^2x^2\log\left(-\frac{de^{(2ibx+2ia)}}{(d-i)e^{(2ibx+2ia)}-i}\right)+2ia^3+6ibx\operatorname{Li}_2\left(\frac{1}{2}\sqrt{-4id-4}e^{(ibx+ia)}\right)+6ibx\operatorname{Li}_2\left(-\frac{1}{2}\sqrt{-4id-4}e^{(ibx+ia)}\right)-3a^2\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*I*b^3*x^3 - 3*b^2*x^2*log(-d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*
x + 2*I*a) - I)) + 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(-4*I*d - 4)*e^(I*b*x +
I*a)) + 6*I*b*x*dilog(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - 3*a^2*log(((
2*d - 2*I)*e^(I*b*x + I*a) + I*sqrt(-4*I*d - 4))/(2*d - 2*I)) - 3*a^2*log((
(2*d - 2*I)*e^(I*b*x + I*a) - I*sqrt(-4*I*d - 4))/(2*d - 2*I)) - 3*(b^2*x^2
 - a^2)*log(1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1) - 3*(b^2*x^2 - a^2)*l
og(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1) - 6*polylog(3, 1/2*sqrt(-4*I*
d - 4)*e^(I*b*x + I*a)) - 6*polylog(3, -1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a
```

```
)))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*atanh(-1-I*d+d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -x \operatorname{artanh}\left(d \tan\left(bx+a\right) - i d - 1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x*arctanh(d*tan(b*x + a) - I*d - 1), x)
```

3.327 $\int \tanh^{-1}(1 + id - d\tan(a + bx))\,dx$

Optimal. Leaf size=94

$$\frac{i\text{PolyLog}\left(2,-(1+id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{2}x\log\left(1+(1+id)e^{2ia+2ibx}\right)+x\tanh^{-1}(d(-\tan(a+bx))+id+1)+\frac{1}{2}ibx^2$$

```
[Out] (I/2)*b*x^2 + x*ArcTanh[1 + I*d - d*Tan[a + b*x]] - (x*Log[1 + (1 + I*d)*E^
((2*I)*a + (2*I)*b*x)])/2 + ((I/4)*PolyLog[2, -((1 + I*d)*E^((2*I)*a + (2*I
)*b*x))])/b
```

Rubi [A] time = 0.147865, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6255, 2184, 2190, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2,-(1+id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{2}x\log\left(1+(1+id)e^{2ia+2ibx}\right)+x\tanh^{-1}(d(-\tan(a+bx))+id+1)+\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[1 + I*d - d*Tan[a + b*x]],x]
```

```
[Out] (I/2)*b*x^2 + x*ArcTanh[1 + I*d - d*Tan[a + b*x]] - (x*Log[1 + (1 + I*d)*E^
((2*I)*a + (2*I)*b*x)])/2 + ((I/4)*PolyLog[2, -((1 + I*d)*E^((2*I)*a + (2*I
)*b*x))])/b
```

Rule 6255

```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Tanh[c + d*Tan[a + b*x]], x] + Dist[I*b, Int[x/(c + I*d + c*E^(2*I*a + 2*I*
b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \tanh^{-1}(1+id-d\tan(a+bx))\,dx = x\tanh^{-1}(1+id-d\tan(a+bx)) + (ib)\int \frac{x}{1+(1+id)e^{2ia+2ibx}}\,dx$$
$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1+id-d\tan(a+bx)) - (b(i-d))\int \frac{e^{2ia+2ibx}x}{1+(1+id)e^{2ia+2ibx}}\,dx$$
$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1+id-d\tan(a+bx)) - \frac{1}{2}x\log\left(1+(1+id)e^{2ia+2ibx}\right) + \frac{1}{2}\int$$
$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1+id-d\tan(a+bx)) - \frac{1}{2}x\log\left(1+(1+id)e^{2ia+2ibx}\right) - \frac{i\,\mathrm{Sul}}{}$$
$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1+id-d\tan(a+bx)) - \frac{1}{2}x\log\left(1+(1+id)e^{2ia+2ibx}\right) + \frac{i\mathrm{Li}_2}{}$$

Mathematica [B] time = 15.1089, size = 723, normalized size = 7.69

$$x\tanh^{-1}(d(-\tan(a+bx))+id+1) - \frac{x\sec(a+bx)(\cos(bx)+i\sin(bx))(\sin(bx)+i\cos(bx))\left(-\mathrm{PolyLog}\left(2,\frac{1}{2}(\cos(a)+\right.\right.}{(\tan(a+bx)-i)(id\sin(a+bx)+(d-2i)\cos(a+bx))\left(-\frac{\sec^2(bx)\log}{}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[1 + I*d - d*Tan[a + b*x]],x]
```

```
[Out] x*ArcTanh[1 + I*d - d*Tan[a + b*x]] - (x*((-2*I)*b*x*Log[2*Cos[b*x]*(Cos[b*
x] - I*Sin[b*x])] + Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-2*I + d)*Cos[a + b
*x] + I*d*Sin[a + b*x]))/(2*(-I + d))]*Log[1 - I*Tan[b*x]] - Log[(Sec[b*x]*
((2 + I*d)*Cos[a + b*x] - d*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*Log[1
 + I*Tan[b*x]] + PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] + PolyLog[2, (Sec[b
*x]*(d*Cos[a] + (2 + I*d)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*(-I +
 d))] - PolyLog[2, ((Cos[a] + I*Sin[a])*(d*Cos[a] + (2 + I*d)*Sin[a])*(-I +
 Tan[b*x]))/2])*Sec[a + b*x]*(Cos[b*x] + I*Sin[b*x])*(I*Cos[b*x] + Sin[b*x]
))/(((-2*I + d)*Cos[a + b*x] + I*d*Sin[a + b*x])*((I*Log[1 - I*Tan[b*x]]*Se
c[b*x]*(d*Cos[a] + (2 + I*d)*Sin[a]))/((-2*I + d)*Cos[a + b*x] + I*d*Sin[a
+ b*x]) + (Log[1 + I*Tan[b*x]]*Sec[b*x]*((-I)*d*Cos[a] + (-2*I + d)*Sin[a])
)/((-2*I + d)*Cos[a + b*x] + I*d*Sin[a + b*x]) - (Log[(Sec[b*x]*((2 + I*d)*
Cos[a + b*x] - d*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*Sec[b*x]^2)/(-I
+ Tan[b*x]) + (Log[1 - ((Cos[a] + I*Sin[a])*(d*Cos[a] + (2 + I*d)*Sin[a])*(
-I + Tan[b*x]))/2]*Sec[b*x]^2)/(-I + Tan[b*x]) - Log[1 - (Sec[b*x]*(d*Cos[a
] + (2 + I*d)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*(-I + d))]*(-I +
Tan[b*x]) + (Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-2*I + d)*Cos[a + b*x] + I
*d*Sin[a + b*x]))/(2*(-I + d))]*Sec[b*x]^2)/(I + Tan[b*x]) + (2*I)*b*x*(I +
 Tan[b*x]))*(-I + Tan[a + b*x]))
```

Maple [B] time = 0.136, size = 328, normalized size = 3.5

$$\frac{\frac{i}{2}\mathrm{Artanh}\left(-1-id+d\tan\left(bx+a\right)\right)\ln\left(-id+d\tan\left(bx+a\right)\right)}{b}-\frac{\frac{i}{2}\mathrm{Artanh}\left(-1-id+d\tan\left(bx+a\right)\right)\ln\left(id+d\tan\left(bx+a\right.\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctanh(-1-I*d+d*tan(b*x+a)),x)
```

```
[Out] 1/2*I/b*arctanh(-1-I*d+d*tan(b*x+a))*ln(-I*d+d*tan(b*x+a))-1/2*I/b*arctanh(
-1-I*d+d*tan(b*x+a))*ln(I*d+d*tan(b*x+a))-1/4*I/b*dilog((-2-I*d+d*tan(b*x+a
))/(-2*I*d-2))-1/4*I/b*ln(I*d+d*tan(b*x+a))*ln((-2-I*d+d*tan(b*x+a))/(-2*I*
d-2))+1/4*I/b*dilog(1/2*I*(-I*d+d*tan(b*x+a))/d)+1/4*I/b*ln(I*d+d*tan(b*x+a
))*ln(1/2*I*(-I*d+d*tan(b*x+a))/d)-1/4*I/b*ln(1+1/2*I*d-1/2*d*tan(b*x+a))*l
n(-1/2*I*d+1/2*d*tan(b*x+a))+1/4*I/b*ln(1+1/2*I*d-1/2*d*tan(b*x+a))*ln(-I*d
+d*tan(b*x+a))-1/4*I/b*dilog(-1/2*I*d+1/2*d*tan(b*x+a))-1/8*I/b*ln(-I*d+d*t
an(b*x+a))^2
```

Maxima [B] time = 1.50011, size = 358, normalized size = 3.81

$$-\frac{4\,(bx+a)d\left(\frac{\log(d\tan(bx+a)-i\,d-2)}{d}-\frac{\log(\tan(bx+a)-i)}{d}\right)-d\left(\frac{2i\left(\log(d\tan(bx+a)-i\,d-2)\log\left(-\frac{i\,d\tan(bx+a)+d-2i}{2\,d-2i}+1\right)+\mathrm{Li}_2\left(\frac{i\,d\tan(bx+a)+d-2i}{2\,d-2i}\right)\right)}{d}-2i\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*(b*x + a)*d*(log(d*tan(b*x + a) - I*d - 2)/d - log(tan(b*x + a) - I
)/d) - d*(2*I*(log(d*tan(b*x + a) - I*d - 2)*log(-(I*d*tan(b*x + a) + d - 2
*I)/(2*d - 2*I) + 1) + dilog((I*d*tan(b*x + a) + d - 2*I)/(2*d - 2*I)))/d -
 (2*I*log(d*tan(b*x + a) - I*d - 2)*log(tan(b*x + a) - I) - I*log(tan(b*x +
 a) - I)^2)/d + 2*I*(log(-1/2*d*tan(b*x + a) + 1/2*I*d + 1)*log(tan(b*x + a
) - I) + dilog(1/2*d*tan(b*x + a) - 1/2*I*d))/d - 2*I*(log(tan(b*x + a) - I
)*log(-1/2*I*tan(b*x + a) + 1/2) + dilog(1/2*I*tan(b*x + a) + 1/2))/d) + 8*
(b*x + a)*arctanh(d*tan(b*x + a) - I*d - 1))/b
```

Fricas [B] time = 1.94248, size = 612, normalized size = 6.51

$$\frac{i\,b^2x^2-bx\log\left(-\frac{de^{(2i\,bx+2i\,a)}}{(d-i)e^{(2i\,bx+2i\,a)}-i}\right)-i\,a^2-(bx+a)\log\left(\frac{1}{2}\sqrt{-4i\,d-4}e^{(i\,bx+i\,a)}+1\right)-(bx+a)\log\left(-\frac{1}{2}\sqrt{-4i\,d-4}e^{(i\,bx+i\,a)}+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*b^2*x^2 - b*x*log(-d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*x + 2*I*a
) - I)) - I*a^2 - (b*x + a)*log(1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1) -
 (b*x + a)*log(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1) + a*log(((2*d - 2
*I)*e^(I*b*x + I*a) + I*sqrt(-4*I*d - 4))/(2*d - 2*I)) + a*log(((2*d - 2*I)
*e^(I*b*x + I*a) - I*sqrt(-4*I*d - 4))/(2*d - 2*I)) + I*dilog(1/2*sqrt(-4*I
*d - 4)*e^(I*b*x + I*a)) + I*dilog(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)))/
b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atanh(-1-I*d+d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\operatorname{artanh}\left(d\tan\left(bx+a\right)-i\,d-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(d*tan(b*x + a) - I*d - 1), x)
```

3.328 $\int \frac{\tanh^{-1}(1+id-d\tan(a+bx))}{x} dx$

Optimal. Leaf size=23

$$\text{CannotIntegrate}\left(\frac{\tanh^{-1}(d(-\tan(a+bx))+id+1)}{x},x\right)$$

[Out] CannotIntegrate[ArcTanh[1 + I*d - d*Tan[a + b*x]]/x, x]

Rubi [A] time = 0.0912821, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tanh^{-1}(1+id-d\tan(a+bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[ArcTanh[1 + I*d - d*Tan[a + b*x]]/x,x]

[Out] Defer[Int][ArcTanh[1 + I*d - d*Tan[a + b*x]]/x, x]

Rubi steps

$$\int \frac{\tanh^{-1}(1+id-d\tan(a+bx))}{x} dx = \int \frac{\tanh^{-1}(1+id-d\tan(a+bx))}{x} dx$$

Mathematica [A] time = 0.998412, size = 0, normalized size = 0.

$$\int \frac{\tanh^{-1}(1+id-d\tan(a+bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcTanh[1 + I*d - d*Tan[a + b*x]]/x,x]

[Out] Integrate[ArcTanh[1 + I*d - d*Tan[a + b*x]]/x, x]

Maple [A] time = 0.381, size = 0, normalized size = 0.

$$\int -\frac{\operatorname{Artanh}\left(-1-id+d\tan\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctanh(-1-I*d+d*tan(b*x+a))/x,x)
```

```
[Out] int(-arctanh(-1-I*d+d*tan(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-i\,bx+\frac{1}{4}\left(-i\,\pi-4i\,a-2\,\log\left(-d\right)\right)\log\left(x\right)+\frac{1}{2}i\int\frac{\arctan\left(d\cos\left(2\,bx+2\,a\right)+\sin\left(2\,bx+2\,a\right),-d\sin\left(2\,bx+2\,a\right)+\cos\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1-I*d+d*tan(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -I*b*x + 1/4*(-I*pi - 4*I*a - 2*log(-d))*log(x) + 1/2*I*integrate(arctan2(d
*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + cos(2*b*x + 2*a
) + 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*s
in(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\log\left(-\frac{de^{\left(2i\,bx+2i\,a\right)}}{(d-i)e^{\left(2i\,bx+2i\,a\right)}-i}\right)}{2\,x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1-I*d+d*tan(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(-1/2*log(-d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*x + 2*I*a) - I))
/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atanh(-1-I*d+d*tan(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\operatorname{artanh}\left(d\tan\left(bx+a\right)-i\,d-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1-I*d+d*tan(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(-arctanh(d*tan(b*x + a) - I*d - 1)/x, x)
```

3.329 $\int (e+fx)^3 \tanh^{-1}(\cot(a+bx))\,dx$

Optimal. Leaf size=302

$$\frac{3if^2(e+fx)\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3}-\frac{3if^2(e+fx)\text{PolyLog}\left(4,ie^{2i(a+bx)}\right)}{8b^3}+\frac{3f(e+fx)^2\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{8b^2}-\frac{3f(e$$

```
[Out] ((I/4)*(e + f*x)^4*ArcTan[E^((2*I)*(a + b*x))])/f + ((e + f*x)^4*ArcTanh[Co
t[a + b*x]])/(4*f) - ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (3*f*(e +
f*x)^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (3*f*(e + f*x)^2*Pol
yLog[3, I*E^((2*I)*(a + b*x))])/(8*b^2) + (((3*I)/8)*f^2*(e + f*x)*PolyLog[
4, (-I)*E^((2*I)*(a + b*x))])/b^3 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E
^((2*I)*(a + b*x))])/b^3 - (3*f^3*PolyLog[5, (-I)*E^((2*I)*(a + b*x))])/(16
*b^4) + (3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```

Rubi [A] time = 0.233002, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6253, 4181, 2531, 6609, 2282, 6589}

$$\frac{3if^2(e+fx)\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3}-\frac{3if^2(e+fx)\text{PolyLog}\left(4,ie^{2i(a+bx)}\right)}{8b^3}+\frac{3f(e+fx)^2\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{8b^2}-\frac{3f(e$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*ArcTanh[Cot[a + b*x]],x]
```


```
[Out] ((I/4)*(e + f*x)^4*ArcTan[E^((2*I)*(a + b*x))])/f + ((e + f*x)^4*ArcTanh[Co
t[a + b*x]])/(4*f) - ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (3*f*(e +
f*x)^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (3*f*(e + f*x)^2*Pol
yLog[3, I*E^((2*I)*(a + b*x))])/(8*b^2) + (((3*I)/8)*f^2*(e + f*x)*PolyLog[
4, (-I)*E^((2*I)*(a + b*x))])/b^3 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E
^((2*I)*(a + b*x))])/b^3 - (3*f^3*PolyLog[5, (-I)*E^((2*I)*(a + b*x))])/(16
*b^4) + (3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```


Rule 6253

```
Int[ArcTanh[Cot[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcTanh[Cot[a + b*x]])/(f*(m + 1)), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
```

```
e, f}, x] && IGtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (e+fx)^3 \tanh^{-1}(\cot(a+bx))\,dx = \frac{(e+fx)^4 \tanh^{-1}(\cot(a+bx))}{4f} - \frac{b\int (e+fx)^4 \sec(2a+2bx)\,dx}{4f}$$

$$= \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4 \tanh^{-1}(\cot(a+bx))}{4f} + \frac{1}{2}\int (e+fx)^3 \log\left(1\right.$$

$$= \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4 \tanh^{-1}(\cot(a+bx))}{4f} - \frac{i(e+fx)^3 \mathrm{Li}_2\left(-ie^{2i(\right.}{4b}$$

$$= \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4 \tanh^{-1}(\cot(a+bx))}{4f} - \frac{i(e+fx)^3 \mathrm{Li}_2\left(-ie^{2i(\right.}{4b}$$

$$= \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4 \tanh^{-1}(\cot(a+bx))}{4f} - \frac{i(e+fx)^3 \mathrm{Li}_2\left(-ie^{2i(\right.}{4b}$$

$$= \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4 \tanh^{-1}(\cot(a+bx))}{4f} - \frac{i(e+fx)^3 \mathrm{Li}_2\left(-ie^{2i(\right.}{4b}$$

$$= \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4 \tanh^{-1}(\cot(a+bx))}{4f} - \frac{i(e+fx)^3 \mathrm{Li}_2\left(-ie^{2i(\right.}{4b}$$

Mathematica [B] time = 0.301206, size = 654, normalized size = 2.17

$$\frac{1}{4}x\left(6e^2fx + 4e^3 + 4ef^2x^2 + f^3x^3\right)\tanh^{-1}(\cot(a+bx)) + \frac{6b^2e^2f\mathrm{PolyLog}\left(3, -ie^{2i(a+bx)}\right) - 6b^2e^2f\mathrm{PolyLog}\left(3, ie^{2i(a+bx)}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*ArcTanh[Cot[a + b*x]],x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcTanh[Cot[a + b*x]])/4 + (
-8*b^4*e^3*x*Log[1 - I*E^((2*I)*(a + b*x))] - 12*b^4*e^2*f*x^2*Log[1 - I*E^
((2*I)*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] - 2*b^4
*f^3*x^4*Log[1 - I*E^((2*I)*(a + b*x))] + 8*b^4*e^3*x*Log[1 + I*E^((2*I)*(a
 + b*x))] + 12*b^4*e^2*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 8*b^4*e*f^2*x
^3*Log[1 + I*E^((2*I)*(a + b*x))] + 2*b^4*f^3*x^4*Log[1 + I*E^((2*I)*(a + b
*x))] - (4*I)*b^3*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))] + (4*I)*
b^3*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))] + 6*b^2*e^2*f*PolyLog[3,
(-I)*E^((2*I)*(a + b*x))] + 12*b^2*e*f^2*x*PolyLog[3, (-I)*E^((2*I)*(a + b*
x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] - 6*b^2*e^2*f*Pol
yLog[3, I*E^((2*I)*(a + b*x))] - 12*b^2*e*f^2*x*PolyLog[3, I*E^((2*I)*(a +
b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^((2*I)*(a + b*x))] + (6*I)*b*e*f^2*Po
lyLog[4, (-I)*E^((2*I)*(a + b*x))] + (6*I)*b*f^3*x*PolyLog[4, (-I)*E^((2*I)
*(a + b*x))] - (6*I)*b*e*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))] - (6*I)*b*f^
```

```
3*x*PolyLog[4, I*E^((2*I)*(a + b*x))] - 3*f^3*PolyLog[5, (-I)*E^((2*I)*(a +
 b*x))] + 3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```

Maple [C] time = 4.522, size = 7429, normalized size = 24.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*arctanh(cot(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{16}\left(f^{3}x^{4}+4\,ef^{2}x^{3}+6\,e^{2}fx^{2}+4\,e^{3}x\right)\log\left(2\,\cos\left(2\,bx+2\,a\right)^{2}+2\,\sin\left(2\,bx+2\,a\right)^{2}+4\,\sin\left(2\,bx+2\,a\right)+2\right)-\frac{1}{16}\left(f^{3}x^{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctanh(cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/16*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*log(2*cos(2*b*x + 2*a)
^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/16*(f^3*x^4 + 4*e*f
^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*
a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(1/2*((b*f^3*x^4 + 4*b*e*f^2*x^3
+ 6*b*e^2*f*x^2 + 4*b*e^3*x)*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f^3*x^4
 + 4*b*e*f^2*x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*sin(4*b*x + 4*a)*sin(2*b*x +
2*a) + (b*f^3*x^4 + 4*b*e*f^2*x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*cos(2*b*x +
2*a))/(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x
)
```

Fricas [C] time = 2.89513, size = 3679, normalized size = 12.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctanh(cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/32*(3*f^3*polylog(5, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - 3*f^3*poly
log(5, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + 3*f^3*polylog(5, -I*cos(2*b
*x + 2*a) + sin(2*b*x + 2*a)) - 3*f^3*polylog(5, -I*cos(2*b*x + 2*a) - sin(
2*b*x + 2*a)) - (4*I*b^3*f^3*x^3 + 12*I*b^3*e*f^2*x^2 + 12*I*b^3*e^2*f*x +
4*I*b^3*e^3)*dilog(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - (4*I*b^3*f^3*x^
3 + 12*I*b^3*e*f^2*x^2 + 12*I*b^3*e^2*f*x + 4*I*b^3*e^3)*dilog(I*cos(2*b*x
+ 2*a) - sin(2*b*x + 2*a)) - (-4*I*b^3*f^3*x^3 - 12*I*b^3*e*f^2*x^2 - 12*I*
b^3*e^2*f*x - 4*I*b^3*e^3)*dilog(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) -
(-4*I*b^3*f^3*x^3 - 12*I*b^3*e*f^2*x^2 - 12*I*b^3*e^2*f*x - 4*I*b^3*e^3)*di
log(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - 4*(b^4*f^3*x^4 + 4*b^4*e*f^2*
x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x)*log(-(cos(2*b*x + 2*a) + sin(2*b*x + 2
*a) + 1)/(cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1)) - 2*(4*a*b^3*e^3 - 6*a^
2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(cos(2*b*x + 2*a) + I*sin(2*b*x +
 2*a) + I) + 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*lo
g(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I) + 2*(b^4*f^3*x^4 + 4*b^4*e*f^2
*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^
3*b*e*f^2 - a^4*f^3)*log(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1) - 2*(b^
4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 -
 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(I*cos(2*b*x + 2*a) - sin(2*
b*x + 2*a) + 1) + 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^
4*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(-I*c
os(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1) - 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3
+ 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e
*f^2 - a^4*f^3)*log(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1) - 2*(4*a*b^
3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(-cos(2*b*x + 2*a) +
I*sin(2*b*x + 2*a) + I) + 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2
- a^4*f^3)*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I) - (-6*I*b*f^3*x
- 6*I*b*e*f^2)*polylog(4, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - (-6*I*b*
f^3*x - 6*I*b*e*f^2)*polylog(4, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - (6
*I*b*f^3*x + 6*I*b*e*f^2)*polylog(4, -I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)
) - (6*I*b*f^3*x + 6*I*b*e*f^2)*polylog(4, -I*cos(2*b*x + 2*a) - sin(2*b*x
+ 2*a)) - 6*(b^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*polylog(3, I*cos(2*b*
x + 2*a) + sin(2*b*x + 2*a)) + 6*(b^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*
polylog(3, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - 6*(b^2*f^3*x^2 + 2*b^2*
e*f^2*x + b^2*e^2*f)*polylog(3, -I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + 6
*(b^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*polylog(3, -I*cos(2*b*x + 2*a) -
 sin(2*b*x + 2*a)))/b^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e + fx\right)^{3} \operatorname{atanh}\left(\cot\left(a + bx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*atanh(cot(b*x+a)),x)
```

```
[Out] Integral((e + f*x)**3*atanh(cot(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx + e\right)^{3} \operatorname{artanh}\left(\cot\left(bx + a\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctanh(cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*arctanh(cot(b*x + a)), x)
```

3.330 $\int (e+fx)^2 \tanh^{-1}(\cot(a+bx))\,dx$

Optimal. Leaf size=234

$$\frac{f(e+fx)\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{4b^2}-\frac{f(e+fx)\text{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{4b^2}+\frac{if^2\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3}-\frac{if^2\text{PolyLog}\left(4,ie^{2i\right.}}{8b^3}$$

```
[Out] ((I/3)*(e + f*x)^3*ArcTan[E^((2*I)*(a + b*x))])/f + ((e + f*x)^3*ArcTanh[Co
t[a + b*x]])/(3*f) - ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*(e + f*
x)*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(4*b^2) - (f*(e + f*x)*PolyLog[3,
I*E^((2*I)*(a + b*x))])/(4*b^2) + ((I/8)*f^2*PolyLog[4, (-I)*E^((2*I)*(a +
b*x))])/b^3 - ((I/8)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])/b^3
```

Rubi [A] time = 0.169751, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6253, 4181, 2531, 6609, 2282, 6589}

$$\frac{f(e+fx)\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{4b^2}-\frac{f(e+fx)\text{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{4b^2}+\frac{if^2\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3}-\frac{if^2\text{PolyLog}\left(4,ie^{2i\right.}}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*ArcTanh[Cot[a + b*x]],x]
```


```
[Out] ((I/3)*(e + f*x)^3*ArcTan[E^((2*I)*(a + b*x))])/f + ((e + f*x)^3*ArcTanh[Co
t[a + b*x]])/(3*f) - ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*(e + f*
x)*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(4*b^2) - (f*(e + f*x)*PolyLog[3,
I*E^((2*I)*(a + b*x))])/(4*b^2) + ((I/8)*f^2*PolyLog[4, (-I)*E^((2*I)*(a +
b*x))])/b^3 - ((I/8)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])/b^3
```


Rule 6253

```
Int[ArcTanh[Cot[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcTanh[Cot[a + b*x]])/(f*(m + 1)), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
 e, f}, x] && IGtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (e+fx)^2 \tanh^{-1}(\cot(a+bx))\,dx = \frac{(e+fx)^3 \tanh^{-1}(\cot(a+bx))}{3f} - \frac{b\int (e+fx)^3 \sec(2a+2bx)\,dx}{3f}$$

$$= \frac{i(e+fx)^3 \tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{(e+fx)^3 \tanh^{-1}(\cot(a+bx))}{3f} + \frac{1}{2}\int (e+fx)^2 \log\left(1\right.$$

$$= \frac{i(e+fx)^3 \tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{(e+fx)^3 \tanh^{-1}(\cot(a+bx))}{3f} - \frac{i(e+fx)^2 \text{Li}_2\left(-ie^{2i(\right.}}{4b}$$

$$= \frac{i(e+fx)^3 \tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{(e+fx)^3 \tanh^{-1}(\cot(a+bx))}{3f} - \frac{i(e+fx)^2 \text{Li}_2\left(-ie^{2i(\right.}}{4b}$$

$$= \frac{i(e+fx)^3 \tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{(e+fx)^3 \tanh^{-1}(\cot(a+bx))}{3f} - \frac{i(e+fx)^2 \text{Li}_2\left(-ie^{2i(\right.}}{4b}$$

$$= \frac{i(e+fx)^3 \tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{(e+fx)^3 \tanh^{-1}(\cot(a+bx))}{3f} - \frac{i(e+fx)^2 \text{Li}_2\left(-ie^{2i(\right.}}{4b}$$

Mathematica [A] time = 0.188489, size = 409, normalized size = 1.75

$$\frac{1}{3}x\left(3e^2+3efx+f^2x^2\right)\tanh^{-1}(\cot(a+bx)) + \frac{-6ib^2(e+fx)^2\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)+6ib^2(e+fx)^2\text{PolyLog}\left(2,ie^{2i(a+\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*ArcTanh[Cot[a + b*x]],x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcTanh[Cot[a + b*x]])/3 + (-12*b^3*e^2*x*Lo
g[1 - I*E^((2*I)*(a + b*x))] - 12*b^3*e*f*x^2*Log[1 - I*E^((2*I)*(a + b*x))
] - 4*b^3*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] + 12*b^3*e^2*x*Log[1 + I*E
^((2*I)*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 4*b^3
*f^2*x^3*Log[1 + I*E^((2*I)*(a + b*x))] - (6*I)*b^2*(e + f*x)^2*PolyLog[2,
(-I)*E^((2*I)*(a + b*x))] + (6*I)*b^2*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a
+ b*x))] + 6*b*e*f*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] + 6*b*f^2*x*PolyLog
[3, (-I)*E^((2*I)*(a + b*x))] - 6*b*e*f*PolyLog[3, I*E^((2*I)*(a + b*x))] -
 6*b*f^2*x*PolyLog[3, I*E^((2*I)*(a + b*x))] + (3*I)*f^2*PolyLog[4, (-I)*E^
((2*I)*(a + b*x))] - (3*I)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])/(24*b^3)
```

Maple [C] time = 8.488, size = 5543, normalized size = 23.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*arctanh(cot(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{12}\left(f^2x^3+3\,efx^2+3\,e^2x\right)\log\left(2\,\cos\left(2\,bx+2\,a\right)^2+2\,\sin\left(2\,bx+2\,a\right)^2+4\,\sin\left(2\,bx+2\,a\right)+2\right)-\frac{1}{12}\left(f^2x^3+3\,efx^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctanh(cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x
 + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*
log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) -
 integrate(2/3*((b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(4*b*x + 4*a)*cos(
2*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*sin(4*b*x + 4*a)*sin(2
*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(2*b*x + 2*a))/(cos(
4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

Fricas [C] time = 2.66629, size = 2647, normalized size = 11.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctanh(cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/48*(-3*I*f^2*polylog(4, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - 3*I*f^2*
polylog(4, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + 3*I*f^2*polylog(4, -I*c
os(2*b*x + 2*a) + sin(2*b*x + 2*a)) + 3*I*f^2*polylog(4, -I*cos(2*b*x + 2*a
) - sin(2*b*x + 2*a)) + (6*I*b^2*f^2*x^2 + 12*I*b^2*e*f*x + 6*I*b^2*e^2)*di
log(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + (6*I*b^2*f^2*x^2 + 12*I*b^2*e*
f*x + 6*I*b^2*e^2)*dilog(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + (-6*I*b^2
*f^2*x^2 - 12*I*b^2*e*f*x - 6*I*b^2*e^2)*dilog(-I*cos(2*b*x + 2*a) + sin(2*
b*x + 2*a)) + (-6*I*b^2*f^2*x^2 - 12*I*b^2*e*f*x - 6*I*b^2*e^2)*dilog(-I*co
```

```
s(2*b*x + 2*a) - sin(2*b*x + 2*a)) + 8*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3
*e^2*x)*log(-(cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1)/(cos(2*b*x + 2*a) -
sin(2*b*x + 2*a) + 1)) + 4*(3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(cos(2*
b*x + 2*a) + I*sin(2*b*x + 2*a) + I) - 4*(3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f
^2)*log(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I) - 4*(b^3*f^2*x^3 + 3*b^3
*e*f*x^2 + 3*b^3*e^2*x + 3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(I*cos(2*b
*x + 2*a) + sin(2*b*x + 2*a) + 1) + 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*
e^2*x + 3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(I*cos(2*b*x + 2*a) - sin(2
*b*x + 2*a) + 1) - 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x + 3*a*b^2*e
^2 - 3*a^2*b*e*f + a^3*f^2)*log(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1)
 + 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x + 3*a*b^2*e^2 - 3*a^2*b*e*f
 + a^3*f^2)*log(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1) + 4*(3*a*b^2*e^
2 - 3*a^2*b*e*f + a^3*f^2)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + I)
- 4*(3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(-cos(2*b*x + 2*a) - I*sin(2*b
*x + 2*a) + I) + 6*(b*f^2*x + b*e*f)*polylog(3, I*cos(2*b*x + 2*a) + sin(2*
b*x + 2*a)) - 6*(b*f^2*x + b*e*f)*polylog(3, I*cos(2*b*x + 2*a) - sin(2*b*x
 + 2*a)) + 6*(b*f^2*x + b*e*f)*polylog(3, -I*cos(2*b*x + 2*a) + sin(2*b*x +
 2*a)) - 6*(b*f^2*x + b*e*f)*polylog(3, -I*cos(2*b*x + 2*a) - sin(2*b*x + 2
*a)))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e + fx\right)^2 \operatorname{atanh}\left(\cot\left(a + bx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*atanh(cot(b*x+a)),x)
```

```
[Out] Integral((e + f*x)**2*atanh(cot(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx + e\right)^2 \operatorname{artanh}\left(\cot\left(bx + a\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctanh(cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*arctanh(cot(b*x + a)), x)
```

3.331 $\int (e+fx)\tanh^{-1}(\cot(a+bx))\,dx$

Optimal. Leaf size=162

$$\frac{f\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{8b^2}-\frac{f\text{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{8b^2}-\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx)\text{PolyLog}\left(2,ie^{2i(a+b}\right.}{4b}$$

```
[Out] ((I/2)*(e + f*x)^2*ArcTan[E^((2*I)*(a + b*x))])/f + ((e + f*x)^2*ArcTanh[Co
t[a + b*x]])/(2*f) - ((I/4)*(e + f*x)*PolyLog[2, (-I)*E^((2*I)*(a + b*x))])
/b + ((I/4)*(e + f*x)*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*PolyLog[3,
(-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (f*PolyLog[3, I*E^((2*I)*(a + b*x))])/
(8*b^2)
```

Rubi [A] time = 0.109397, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6253, 4181, 2531, 2282, 6589}

$$\frac{f\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{8b^2}-\frac{f\text{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{8b^2}-\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx)\text{PolyLog}\left(2,ie^{2i(a+b}\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*ArcTanh[Cot[a + b*x]],x]
```


```
[Out] ((I/2)*(e + f*x)^2*ArcTan[E^((2*I)*(a + b*x))])/f + ((e + f*x)^2*ArcTanh[Co
t[a + b*x]])/(2*f) - ((I/4)*(e + f*x)*PolyLog[2, (-I)*E^((2*I)*(a + b*x))])
/b + ((I/4)*(e + f*x)*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*PolyLog[3,
(-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (f*PolyLog[3, I*E^((2*I)*(a + b*x))])/
(8*b^2)
```


Rule 6253

```
Int[ArcTanh[Cot[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcTanh[Cot[a + b*x]])/(f*(m + 1)), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
 e, f}, x] && IGtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
```

```
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (e+fx)\tanh^{-1}(\cot(a+bx))\,dx = \frac{(e+fx)^2\tanh^{-1}(\cot(a+bx))}{2f} - \frac{b\int (e+fx)^2\sec(2a+2bx)\,dx}{2f}$$

$$= \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} + \frac{(e+fx)^2\tanh^{-1}(\cot(a+bx))}{2f} + \frac{1}{2}\int (e+fx)\log\left(1 -\right.$$

$$= \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} + \frac{(e+fx)^2\tanh^{-1}(\cot(a+bx))}{2f} - \frac{i(e+fx)\mathrm{Li}_2\left(-ie^{2i(a+\right.}}{4b}$$

$$= \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} + \frac{(e+fx)^2\tanh^{-1}(\cot(a+bx))}{2f} - \frac{i(e+fx)\mathrm{Li}_2\left(-ie^{2i(a+\right.}}{4b}$$

$$= \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} + \frac{(e+fx)^2\tanh^{-1}(\cot(a+bx))}{2f} - \frac{i(e+fx)\mathrm{Li}_2\left(-ie^{2i(a+\right.}}{4b}$$

Mathematica [A] time = 0.122783, size = 263, normalized size = 1.62

$$-be\left(\frac{i\text{PolyLog}\left(2,-ie^{i(2a+2bx)}\right)}{4b^2}-\frac{i\text{PolyLog}\left(2,ie^{i(2a+2bx)}\right)}{4b^2}-\frac{ix\tan^{-1}\left(e^{2ia+2ibx}\right)}{b}\right)+\frac{f\left(2ibx\text{PolyLog}(2,-\sin(2(a+bx))\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)*ArcTanh[Cot[a + b*x]],x]
```

```
[Out] e*x*ArcTanh[Cot[a + b*x]] + (f*x^2*ArcTanh[Cot[a + b*x]])/2 - b*e*(((-I)*x*
ArcTan[E^((2*I)*a + (2*I)*b*x)])/b + ((I/4)*PolyLog[2, (-I)*E^(I*(2*a + 2*b
*x))])/b^2 - ((I/4)*PolyLog[2, I*E^(I*(2*a + 2*b*x))])/b^2) + (f*((4*I)*b^2
*x^2*ArcTan[Cos[2*(a + b*x)] + I*Sin[2*(a + b*x)]] + (2*I)*b*x*PolyLog[2, I
*Cos[2*(a + b*x)] - Sin[2*(a + b*x)]] - (2*I)*b*x*PolyLog[2, (-I)*Cos[2*(a
+ b*x)] + Sin[2*(a + b*x)]] - PolyLog[3, I*Cos[2*(a + b*x)] - Sin[2*(a + b*
x)]] + PolyLog[3, (-I)*Cos[2*(a + b*x)] + Sin[2*(a + b*x)]]))/(8*b^2)
```

Maple [C] time = 5.203, size = 2544, normalized size = 15.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*arctanh(cot(b*x+a)),x)
```

```
[Out] -1/2*I/b*e*dilog(1+exp(I*(b*x+a))*(-1)^(3/4))-1/2*I/b*e*dilog(1-exp(I*(b*x+
a))*(-1)^(3/4))+1/2*I/b*e*dilog(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))+1/2
*I/b*e*dilog(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))+1/8*I*Pi*f*csgn(I*(exp
(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2*x^2-1
/2*(-1/2*f*x^2-e*x)*ln(exp(2*I*(b*x+a))+I)-1/2*I/b^2*f*a*dilog(((-I)^(1/2)-
exp(I*(b*x+a)))/(-I)^(1/2))-1/2*I/b^2*f*a*dilog(((-I)^(1/2)+exp(I*(b*x+a)))
/(-I)^(1/2))+1/8*f*polylog(3,-I*exp(2*I*(b*x+a)))/b^2-1/8*f*polylog(3,I*exp
(2*I*(b*x+a)))/b^2-1/8*I*Pi*f*csgn((1+I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x
+a))-1))^2*x^2+1/4/b^2*f*a^2*ln(-exp(2*I*(b*x+a))+I)-1/2/b*e*a*ln(-exp(2*I*
(b*x+a))+I)+1/2/b*a*e*ln(exp(2*I*(b*x+a))+I)-1/4/b^2*f*a^2*ln(exp(2*I*(b*x+
a))+I)+1/4/b^2*f*(I*b*x+I*a)^2*ln(1-I*exp(2*I*(b*x+a)))-1/8*I*Pi*f*csgn((1-
I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2*x^2+1/2*I/b^2*f*a*dilog(1+e
xp(I*(b*x+a))*(-1)^(3/4))+1/2*I/b^2*f*a*dilog(1-exp(I*(b*x+a))*(-1)^(3/4))-
1/4*I*Pi*x*e*csgn((1+I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2-1/4*I*
Pi*x*e*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2-1/8*I*Pi*f*c
sgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1
```

))^2*x^2-1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^3+1
/4*I*Pi*x*e*csgn((1+I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^3+1/4*I*P
i*x*e+1/8*I*Pi*f*x^2+1/2*I/b*e*(I*b*x+I*a)*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(
-I)^(1/2))+1/2*I/b*e*(I*b*x+I*a)*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))
-1/2*I/b*e*(I*b*x+I*a)*ln(1+exp(I*(b*x+a))*(-1)^(3/4))-1/2*I/b*e*(I*b*x+I*a
)*ln(1-exp(I*(b*x+a))*(-1)^(3/4))+1/4/b^2*f*(I*b*x+I*a)*polylog(2,I*exp(2*I
*(b*x+a)))-1/4/b^2*f*(I*b*x+I*a)^2*ln(1+I*exp(2*I*(b*x+a)))-1/4/b^2*f*(I*b*
x+I*a)*polylog(2,-I*exp(2*I*(b*x+a)))-1/4*ln(exp(2*I*(b*x+a))-I)*x^2*f-1/2*
ln(exp(2*I*(b*x+a))-I)*x*e+1/8*I*Pi*f*csgn((1+I)*(exp(2*I*(b*x+a))-I)/(exp(
2*I*(b*x+a))-1))^3*x^2-1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x
+a))-1))^3*x^2-1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))
csgn((1+I)(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2*x^2+1/4*I*Pi*x*e*c
sgn(I*(exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1
))^2+1/8*I*Pi*f*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))+I)/(e
xp(2*I*(b*x+a))-1))^2*x^2+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I
*(b*x+a))-1))^3+1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1)
)^3*x^2+1/2*I/b^2*f*a*(I*b*x+I*a)*ln(1+exp(I*(b*x+a))*(-1)^(3/4))+1/2*I/b^2
*f*a*(I*b*x+I*a)*ln(1-exp(I*(b*x+a))*(-1)^(3/4))-1/4*I*Pi*x*e*csgn(I/(exp(2
I(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2-1/2*I/b
^2*f*a*(I*b*x+I*a)*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))-1/2*I/b^2*f*a
*(I*b*x+I*a)*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))-1/8*I*Pi*f*csgn(I*(
exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(
exp(2*I*(b*x+a))-1))*x^2-1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*
(b*x+a))-1))*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))+1/4*I*Pi
*x*e*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((1-I)*(exp(2*I*
(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2-1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I
))*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a
))-1))-1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))+I))*csgn(I/(exp(2*I*(b*x+a))-1))
csgn(I(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*x^2+1/8*I*Pi*f*csgn(I*(e
xp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(e
xp(2*I*(b*x+a))-1))^2*x^2+1/4*I*Pi*x*e*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*
(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2-1/4*I*Pi*x*e*csgn(I*(exp(2*I*(
b*x+a))-I)/(exp(2*I*(b*x+a))-1))*csgn((1+I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(
b*x+a))-1))^2+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I))*csgn(I/(exp(2*I*(b*
x+a))-1))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))+1/8*I*Pi*f*csgn
(I*(exp(2*I*(b*x+a))-I))*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+
a))-I)/(exp(2*I*(b*x+a))-1))*x^2+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I)/(
exp(2*I*(b*x+a))-1))*csgn((1+I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))-
1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(
2*I*(b*x+a))-1))^2-1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*
(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2*x^2+1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))
-I)/(exp(2*I*(b*x+a))-1))*csgn((1+I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))
-1))*x^2+1/4*I*Pi*x*e*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))
^3+1/8*I*Pi*f*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^3*x^2

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{8}\left(fx^2+2\,ex\right)\log\left(2\,\cos\left(2\,bx+2\,a\right)^2+2\,\sin\left(2\,bx+2\,a\right)^2+4\,\sin\left(2\,bx+2\,a\right)+2\right)-\frac{1}{8}\left(fx^2+2\,ex\right)\log\left(2\,\cos\left(2\,bx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctanh(cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/8*(f*x^2 + 2*e*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin
(2*b*x + 2*a) + 2) - 1/8*(f*x^2 + 2*e*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2
*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(((b*f*x^2 + 2*b*e*x)*co
s(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f*x^2 + 2*b*e*x)*sin(4*b*x + 4*a)*sin(
2*b*x + 2*a) + (b*f*x^2 + 2*b*e*x)*cos(2*b*x + 2*a))/(cos(4*b*x + 4*a)^2 +
sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

Fricas [C] time = 2.36915, size = 1725, normalized size = 10.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctanh(cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/16*((2*I*b*f*x + 2*I*b*e)*dilog(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) +
(2*I*b*f*x + 2*I*b*e)*dilog(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + (-2*I*
b*f*x - 2*I*b*e)*dilog(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + (-2*I*b*f*
x - 2*I*b*e)*dilog(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + 4*(b^2*f*x^2 +
 2*b^2*e*x)*log(-(cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1)/(cos(2*b*x + 2*a
) - sin(2*b*x + 2*a) + 1)) + 2*(2*a*b*e - a^2*f)*log(cos(2*b*x + 2*a) + I*s
in(2*b*x + 2*a) + I) - 2*(2*a*b*e - a^2*f)*log(cos(2*b*x + 2*a) - I*sin(2*b
*x + 2*a) + I) - 2*(b^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a^2*f)*log(I*cos(2*b*
x + 2*a) + sin(2*b*x + 2*a) + 1) + 2*(b^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a^2
*f)*log(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1) - 2*(b^2*f*x^2 + 2*b^2*e
*x + 2*a*b*e - a^2*f)*log(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1) + 2*(
b^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a^2*f)*log(-I*cos(2*b*x + 2*a) - sin(2*b*
x + 2*a) + 1) + 2*(2*a*b*e - a^2*f)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2
*a) + I) - 2*(2*a*b*e - a^2*f)*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) +
 I) + f*polylog(3, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - f*polylog(3, I*
cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + f*polylog(3, -I*cos(2*b*x + 2*a) + s
```

```
in(2*b*x + 2*a)) - f*polylog(3, -I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)))/b^
2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (e+fx)\operatorname{atanh}(\cot(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*atanh(cot(b*x+a)),x)
```

```
[Out] Integral((e + f*x)*atanh(cot(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx+e)\operatorname{artanh}(\cot(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctanh(cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*arctanh(cot(b*x + a)), x)
```

3.332 $\int \tanh^{-1}(\cot(a+bx))\,dx$

Optimal. Leaf size=79

$$-\frac{i\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b}+ix\tan^{-1}\left(e^{2i(a+bx)}\right)+x\tanh^{-1}(\cot(a+bx))$$

```
[Out] I*x*ArcTan[E^((2*I)*(a + b*x))] + x*ArcTanh[Cot[a + b*x]] - ((I/4)*PolyLog[
2, (-I)*E^((2*I)*(a + b*x))])/b + ((I/4)*PolyLog[2, I*E^((2*I)*(a + b*x))])
/b
```

Rubi [A] time = 0.0462235, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6249, 4181, 2279, 2391}

$$-\frac{i\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b}+ix\tan^{-1}\left(e^{2i(a+bx)}\right)+x\tanh^{-1}(\cot(a+bx))$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Cot[a + b*x]],x]
```

```
[Out] I*x*ArcTan[E^((2*I)*(a + b*x))] + x*ArcTanh[Cot[a + b*x]] - ((I/4)*PolyLog[
2, (-I)*E^((2*I)*(a + b*x))])/b + ((I/4)*PolyLog[2, I*E^((2*I)*(a + b*x))])
/b
```

Rule 6249

```
Int[ArcTanh[Cot[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcTanh[Cot[a + b
*x]], x] - Dist[b, Int[x*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \tanh^{-1}(\cot(a+bx))\,dx = x\tanh^{-1}(\cot(a+bx)) - b\int x\sec(2a+2bx)\,dx$$
$$= ix\tan^{-1}\left(e^{2i(a+bx)}\right) + x\tanh^{-1}(\cot(a+bx)) + \frac{1}{2}\int \log\left(1-ie^{i(2a+2bx)}\right)dx - \frac{1}{2}\int \log\left(1+ie^{i\ldots}\right.$$
$$= ix\tan^{-1}\left(e^{2i(a+bx)}\right) + x\tanh^{-1}(\cot(a+bx)) - \frac{i\,\mathrm{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{i(2a+2bx)}\right)}{4b} + \frac{i\,\mathrm{Subst}\ldots}{}$$
$$= ix\tan^{-1}\left(e^{2i(a+bx)}\right) + x\tanh^{-1}(\cot(a+bx)) - \frac{i\mathrm{Li}_2\left(-ie^{2i(a+bx)}\right)}{4b} + \frac{i\mathrm{Li}_2\left(ie^{2i(a+bx)}\right)}{4b}$$

Mathematica [A] time = 0.0368159, size = 74, normalized size = 0.94

$$\frac{i\left(-\mathrm{PolyLog}\left(2,-ie^{2i(a+bx)}\right)+\mathrm{PolyLog}\left(2,ie^{2i(a+bx)}\right)+4bx\left(\tan^{-1}\left(e^{2i(a+bx)}\right)-i\tanh^{-1}(\cot(a+bx))\right)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Cot[a + b*x]],x]
```

```
[Out] ((I/4)*(4*b*x*(ArcTan[E^((2*I)*(a + b*x))] - I*ArcTanh[Cot[a + b*x]]) - Pol
yLog[2, (-I)*E^((2*I)*(a + b*x))] + PolyLog[2, I*E^((2*I)*(a + b*x))]))/b
```

Maple [B] time = 0.109, size = 422, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(cot(b*x+a)),x)
```

```
[Out] -1/4*I/b*dilog(-I*cosh(2*arctanh(cot(b*x+a)))-I*sinh(2*arctanh(cot(b*x+a)))
)-1/4*I/b*ln((1-I)/(-cot(b*x+a)^2+1)^(1/2)+(1+I)*cot(b*x+a)/(-cot(b*x+a)^2+
1)^(1/2))*ln(-I*cosh(2*arctanh(cot(b*x+a)))-I*sinh(2*arctanh(cot(b*x+a))))+
1/2*I/b*ln((1-I)/(-cot(b*x+a)^2+1)^(1/2)+(1+I)*cot(b*x+a)/(-cot(b*x+a)^2+1)
^(1/2))*arctanh(cot(b*x+a))+1/4*I/b*dilog(I*cosh(2*arctanh(cot(b*x+a)))+I*s
inh(2*arctanh(cot(b*x+a))))+1/4*I/b*ln((1+I)/(-cot(b*x+a)^2+1)^(1/2)+(1-I)*
cot(b*x+a)/(-cot(b*x+a)^2+1)^(1/2))*ln(I*cosh(2*arctanh(cot(b*x+a)))+I*sinh
(2*arctanh(cot(b*x+a))))-1/2*I/b*ln((1+I)/(-cot(b*x+a)^2+1)^(1/2)+(1-I)*cot
(b*x+a)/(-cot(b*x+a)^2+1)^(1/2))*arctanh(cot(b*x+a))-1/4*I/b*arctanh(cot(b*
x+a))*ln(-I*cosh(2*arctanh(cot(b*x+a)))-I*sinh(2*arctanh(cot(b*x+a))))+1/4*
I/b*arctanh(cot(b*x+a))*ln(I*cosh(2*arctanh(cot(b*x+a)))+I*sinh(2*arctanh(c
ot(b*x+a))))
```

Maxima [B] time = 1.62672, size = 248, normalized size = 3.14

$$\frac{4\,(bx+a)\,\mathrm{artanh}\left(\frac{1}{\tan(bx+a)}\right)+\left(\arctan\left(\frac{1}{2}\,\tan(bx+a)+\frac{1}{2},\frac{1}{2}\,\tan(bx+a)+\frac{1}{2}\right)-\arctan\left(\frac{1}{2}\,\tan(bx+a)-\frac{1}{2},-\frac{1}{2}\,\tan\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(b*x + a)*arctanh(1/tan(b*x + a)) + (arctan2(1/2*tan(b*x + a) + 1/2,
 1/2*tan(b*x + a) + 1/2) - arctan2(1/2*tan(b*x + a) - 1/2, -1/2*tan(b*x + a
) + 1/2))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log(1/2*tan(b*x + a)^2 + tan(
b*x + a) + 1/2) + (b*x + a)*log(1/2*tan(b*x + a)^2 - tan(b*x + a) + 1/2) -
I*dilog((1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2) + I*dilog(-(1/2*I - 1/2)*
tan(b*x + a) + 1/2*I + 1/2) + I*dilog((1/2*I - 1/2)*tan(b*x + a) + 1/2*I +
1/2) - I*dilog(-(1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2))/b
```

Fricas [B] time = 2.26752, size = 1030, normalized size = 13.04

$$\frac{4\,bx\log\left(-\frac{\cos(2\,bx+2\,a)+\sin(2\,bx+2\,a)+1}{\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a)+1}\right)+2\,a\log\left(\cos\left(2\,bx+2\,a\right)+i\,\sin\left(2\,bx+2\,a\right)+i\right)-2\,a\log\left(\cos\left(2\,bx+2\,a\right)-i\,\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*log(-(cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1)/(cos(2*b*x + 2*a)
 - sin(2*b*x + 2*a) + 1)) + 2*a*log(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) +
 I) - 2*a*log(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I) - 2*(b*x + a)*log(
I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1) + 2*(b*x + a)*log(I*cos(2*b*x +
2*a) - sin(2*b*x + 2*a) + 1) - 2*(b*x + a)*log(-I*cos(2*b*x + 2*a) + sin(2*
b*x + 2*a) + 1) + 2*(b*x + a)*log(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a) +
1) + 2*a*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + I) - 2*a*log(-cos(2*b
*x + 2*a) - I*sin(2*b*x + 2*a) + I) + I*dilog(I*cos(2*b*x + 2*a) + sin(2*b*
x + 2*a)) + I*dilog(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - I*dilog(-I*cos
(2*b*x + 2*a) + sin(2*b*x + 2*a)) - I*dilog(-I*cos(2*b*x + 2*a) - sin(2*b*x
 + 2*a)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{atanh}\left(\cot\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(cot(b*x+a)),x)
```

```
[Out] Integral(atanh(cot(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{artanh}\left(\cot\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(cot(b*x + a)), x)
```

3.333 $\int \frac{\tanh^{-1}(\cot(a+bx))}{e+fx} \, dx$

Optimal. Leaf size=17

$$\text{CannotIntegrate}\left(\frac{\tanh^{-1}(\cot(a+bx))}{e+fx},x\right)$$

```
[Out] CannotIntegrate[ArcTanh[Cot[a + b*x]]/(e + f*x), x]
```

Rubi [A] time = 0.0418305, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tanh^{-1}(\cot(a+bx))}{e+fx} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTanh[Cot[a + b*x]]/(e + f*x),x]
```

```
[Out] Defer[Int][ArcTanh[Cot[a + b*x]]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\cot(a+bx))}{e+fx} \, dx = \int \frac{\tanh^{-1}(\cot(a+bx))}{e+fx} \, dx$$

Mathematica [A] time = 0.888917, size = 0, normalized size = 0.

$$\int \frac{\tanh^{-1}(\cot(a+bx))}{e+fx} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTanh[Cot[a + b*x]]/(e + f*x),x]
```

```
[Out] Integrate[ArcTanh[Cot[a + b*x]]/(e + f*x), x]
```

Maple [A] time = 1.075, size = 0, normalized size = 0.

$$\int \frac{\operatorname{Artanh}(\cot(bx+a))}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(cot(b*x+a))/(f*x+e),x)
```

```
[Out] int(arctanh(cot(b*x+a))/(f*x+e),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(\cot(bx+a))}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(cot(b*x+a))/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(cot(b*x + a))/(f*x + e), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{artanh}(\cot(bx+a))}{fx+e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(cot(b*x+a))/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral(arctanh(cot(b*x + a))/(f*x + e), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}\left(\cot\left(a+bx\right)\right)}{e+fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(cot(b*x+a))/(f*x+e),x)
```

```
[Out] Integral(atanh(cot(a + b*x))/(e + f*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}\left(\cot\left(bx+a\right)\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(cot(b*x+a))/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(arctanh(cot(b*x + a))/(f*x + e), x)
```

3.334 $\int x^2 \tanh^{-1}(c + d\cot(a + bx))\,dx$

Optimal. Leaf size=391

$$\frac{x\text{PolyLog}\left(3,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b^2}-\frac{x\text{PolyLog}\left(3,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b^2}+\frac{i\text{PolyLog}\left(4,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{8b^3}-\frac{i\text{PolyLog}\left(4,\frac{(c+id+1)e^{2i}}{c-id+1}\right)}{8b^3}$$

```
[Out] (x^3*ArcTanh[c + d*Cot[a + b*x]])/3 + (x^3*Log[1 - ((1 - c - I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 - c + I*d)])/6 - (x^3*Log[1 - ((1 + c + I*d)*E^((2*I)*a
+ (2*I)*b*x))/(1 + c - I*d)])/6 - ((I/4)*x^2*PolyLog[2, ((1 - c - I*d)*E^((
2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/b + ((I/4)*x^2*PolyLog[2, ((1 + c + I*
d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/b + (x*PolyLog[3, ((1 - c - I*d
)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/(4*b^2) - (x*PolyLog[3, ((1 + c
+ I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/(4*b^2) + ((I/8)*PolyLog[4,
 ((1 - c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/b^3 - ((I/8)*PolyL
og[4, ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/b^3
```

Rubi [A] time = 0.486339, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6269, 2190, 2531, 6609, 2282, 6589}

$$\frac{x\text{PolyLog}\left(3,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b^2}-\frac{x\text{PolyLog}\left(3,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b^2}+\frac{i\text{PolyLog}\left(4,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{8b^3}-\frac{i\text{PolyLog}\left(4,\frac{(c+id+1)e^{2i}}{c-id+1}\right)}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[c + d*Cot[a + b*x]])/3 + (x^3*Log[1 - ((1 - c - I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 - c + I*d)])/6 - (x^3*Log[1 - ((1 + c + I*d)*E^((2*I)*a
+ (2*I)*b*x))/(1 + c - I*d)])/6 - ((I/4)*x^2*PolyLog[2, ((1 - c - I*d)*E^((
2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/b + ((I/4)*x^2*PolyLog[2, ((1 + c + I*
d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/b + (x*PolyLog[3, ((1 - c - I*d
)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/(4*b^2) - (x*PolyLog[3, ((1 + c
+ I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/(4*b^2) + ((I/8)*PolyLog[4,
 ((1 - c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/b^3 - ((I/8)*PolyL
og[4, ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/b^3
```

Rule 6269

```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Cot[a + b*x]])/(f*(m
 + 1)), x] + (-Dist[(I*b*(1 - c - I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)
*E^(2*I*a + 2*I*b*x))/(1 - c + I*d - (1 - c - I*d)*E^(2*I*a + 2*I*b*x)), x]
, x] + Dist[(I*b*(1 + c + I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*I*
a + 2*I*b*x))/(1 + c - I*d - (1 + c + I*d)*E^(2*I*a + 2*I*b*x)), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, 1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \tanh^{-1}(c+d\cot(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(c+d\cot(a+bx)) + \frac{1}{3}(b(i(1+c)-d))\int \frac{e^{2ia+2ibx}x^3}{1+c-id+(-1-c-id)e^{2ia+}$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\cot(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) - \frac{1}{6}x^3\log\left(1-\right.$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\cot(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) - \frac{1}{6}x^3\log\left(1-\right.$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\cot(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) - \frac{1}{6}x^3\log\left(1-\right.$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\cot(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) - \frac{1}{6}x^3\log\left(1-\right.$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\cot(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) - \frac{1}{6}x^3\log\left(1-\right.$$

Mathematica [A] time = 0.902668, size = 339, normalized size = 0.87

$$\frac{1}{3}x^3\tanh^{-1}(d\cot(a+bx)+c) + \frac{-6ib^2x^2\text{PolyLog}\left(2,\frac{(c+id-1)e^{2i(a+bx)}}{c-id-1}\right)+6ib^2x^2\text{PolyLog}\left(2,\frac{(c+id+1)e^{2i(a+bx)}}{c-id+1}\right)+6bx\text{PolyLog}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[c + d*Cot[a + b*x]])/3 + (4*b^3*x^3*Log[1 - ((-1 + c + I*d)*E^
((2*I)*(a + b*x)))/(-1 + c - I*d)] - 4*b^3*x^3*Log[1 - ((1 + c + I*d)*E^((2
*I)*(a + b*x)))/(1 + c - I*d)] - (6*I)*b^2*x^2*PolyLog[2, ((-1 + c + I*d)*E
^((2*I)*(a + b*x)))/(-1 + c - I*d)] + (6*I)*b^2*x^2*PolyLog[2, ((1 + c + I*
d)*E^((2*I)*(a + b*x)))/(1 + c - I*d)] + 6*b*x*PolyLog[3, ((-1 + c + I*d)*E
^((2*I)*(a + b*x)))/(-1 + c - I*d)] - 6*b*x*PolyLog[3, ((1 + c + I*d)*E^((2
*I)*(a + b*x)))/(1 + c - I*d)] + (3*I)*PolyLog[4, ((-1 + c + I*d)*E^((2*I)*
(a + b*x)))/(-1 + c - I*d)] - (3*I)*PolyLog[4, ((1 + c + I*d)*E^((2*I)*(a +
 b*x)))/(1 + c - I*d)])/(24*b^3)
```

Maple [C] time = 5.454, size = 6775, normalized size = 17.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(c+d*cot(b*x+a)),x)

[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*cot(b*x+a)),x, algorithm="maxima")

[Out] 1/12*x^3*log((c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*b
*x + 2*a) + (c^2 + d^2 + 2*c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 - 2*(c^2 -
 d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 2*c + 1) - 1/12*x^3*log((c^2 + d^2 - 2*c
 + 1)*cos(2*b*x + 2*a)^2 + 4*(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 - 2*c
+ 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 - 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*x + 2
*a) - 2*c + 1) - 4*b*d*integrate(1/3*(2*(c^2 + d^2 - 1)*x^3*cos(2*b*x + 2*a
)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^3*sin(2*b*x + 2*a)^2
 - (c^2 - d^2 - 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2*b*x + 2*a) + (c^
2 - d^2 - 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^3*cos(2*b*x
+ 2*a) - (c^2 - d^2 - 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 + d^4
 + 2*(c^2 + 1)*d^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*cos(4*b*x +
4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x + 2*a)^2 + (
c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 + d^4
+ 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 + 2*(c^4 + d^4 -
2*(3*c^2 - 1)*d^2 - 2*c^2 - 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*
(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) - 4*(c^4 - d^4
 - 2*c^2 + 1)*cos(2*b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 - c)*d + 2*(c*d^3 + (c
^3 - c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*sin(2*b*x + 2*a))*sin
(4*b*x + 4*a) + 8*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1), x)
```

Fricas [C] time = 3.29963, size = 4749, normalized size = 12.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/48*(8*b^3*x^3*log(-(d*cos(2*b*x + 2*a) + (c + 1)*sin(2*b*x + 2*a) + d)/(d
*cos(2*b*x + 2*a) + (c - 1)*sin(2*b*x + 2*a) + d)) + 6*I*b^2*x^2*dilog(-(c^
2 + d^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2
+ 2*(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 +
 2*c + 1) + 1) - 6*I*b^2*x^2*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2
 + 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*si
n(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1) + 1) - 6*I*b^2*x^2*dilog(-(
c^2 + d^2 - (c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^
2 + 2*(c - 1)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2
 - 2*c + 1) + 1) + 6*I*b^2*x^2*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d
^2 - 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*
sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1) + 1) + 4*a^3*log(1/2*c^2
+ I*(c + 1)*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*
(I*c^2 + I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) + c + 1/2) - 4*a^3*log(1/2*c^2
 + I*(c - 1)*d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2
*(I*c^2 + I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) - c + 1/2) + 4*a^3*log(-1/2*c
^2 + I*(c + 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1
/2*(I*c^2 + I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) - c - 1/2) - 4*a^3*log(-1/2
*c^2 + I*(c - 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) +
 1/2*(I*c^2 + I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) + c - 1/2) - 6*b*x*polylo
g(3, ((c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*(
c + 1)*d - I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*c + 1)) - 6*
b*x*polylog(3, ((c^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-
I*c^2 - 2*(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*c
 + 1)) + 6*b*x*polylog(3, ((c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x
+ 2*a) + (I*c^2 - 2*(c - 1)*d - I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a))/(c^2 +
 d^2 - 2*c + 1)) + 6*b*x*polylog(3, ((c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*
cos(2*b*x + 2*a) + (-I*c^2 - 2*(c - 1)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2
*a))/(c^2 + d^2 - 2*c + 1)) - 4*(b^3*x^3 + a^3)*log((c^2 + d^2 - (c^2 + 2*I
*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c + 1)*d + I*d^
2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1)) - 4*(b^3*
x^3 + a^3)*log((c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x
 + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) + 2*c
+ 1)/(c^2 + d^2 + 2*c + 1)) + 4*(b^3*x^3 + a^3)*log((c^2 + d^2 - (c^2 + 2*I
*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c - 1)*d + I*d^
2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1)) + 4*(b^3*
```

```
x^3 + a^3)*log((c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x
 + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) - 2*c
+ 1)/(c^2 + d^2 - 2*c + 1)) - 3*I*polylog(4, ((c^2 + 2*I*(c + 1)*d - d^2 +
2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin(2
*b*x + 2*a))/(c^2 + d^2 + 2*c + 1)) + 3*I*polylog(4, ((c^2 - 2*I*(c + 1)*d
- d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*(c + 1)*d + I*d^2 - 2*I*c -
 I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*c + 1)) + 3*I*polylog(4, ((c^2 + 2*I*(
c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*(c - 1)*d - I*d^2 -
 2*I*c + I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*c + 1)) - 3*I*polylog(4, ((c^2
 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*(c - 1)*d
+ I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*c + 1)))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(c+d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{artanh}\left(d\cot\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(d*cot(b*x + a) + c), x)
```

3.335 $\int x \tanh^{-1}(c + d\cot(a + bx))\,dx$

Optimal. Leaf size=293

$$\frac{\text{PolyLog}\left(3,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{8b^2}-\frac{\text{PolyLog}\left(3,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{8b^2}-\frac{ix\text{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b}+\frac{ix\text{PolyLog}\left(2,\frac{(c+id+1)e^{2i}}{c-id+1}\right)}{4b}$$

```
[Out] (x^2*ArcTanh[c + d*Cot[a + b*x]])/2 + (x^2*Log[1 - ((1 - c - I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 - c + I*d)])/4 - (x^2*Log[1 - ((1 + c + I*d)*E^((2*I)*a
+ (2*I)*b*x))/(1 + c - I*d)])/4 - ((I/4)*x*PolyLog[2, ((1 - c - I*d)*E^((2*
I)*a + (2*I)*b*x))/(1 - c + I*d)])/b + ((I/4)*x*PolyLog[2, ((1 + c + I*d)*E
^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/b + PolyLog[3, ((1 - c - I*d)*E^((2
*I)*a + (2*I)*b*x))/(1 - c + I*d)]/(8*b^2) - PolyLog[3, ((1 + c + I*d)*E^((
2*I)*a + (2*I)*b*x))/(1 + c - I*d)]/(8*b^2)
```

Rubi [A] time = 0.400496, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6269, 2190, 2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{8b^2}-\frac{\text{PolyLog}\left(3,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{8b^2}-\frac{ix\text{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b}+\frac{ix\text{PolyLog}\left(2,\frac{(c+id+1)e^{2i}}{c-id+1}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[c + d*Cot[a + b*x]])/2 + (x^2*Log[1 - ((1 - c - I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 - c + I*d)])/4 - (x^2*Log[1 - ((1 + c + I*d)*E^((2*I)*a
+ (2*I)*b*x))/(1 + c - I*d)])/4 - ((I/4)*x*PolyLog[2, ((1 - c - I*d)*E^((2*
I)*a + (2*I)*b*x))/(1 - c + I*d)])/b + ((I/4)*x*PolyLog[2, ((1 + c + I*d)*E
^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/b + PolyLog[3, ((1 - c - I*d)*E^((2
*I)*a + (2*I)*b*x))/(1 - c + I*d)]/(8*b^2) - PolyLog[3, ((1 + c + I*d)*E^((
2*I)*a + (2*I)*b*x))/(1 + c - I*d)]/(8*b^2)
```

Rule 6269

```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Cot[a + b*x]])/(f*(m
 + 1)), x] + (-Dist[(I*b*(1 - c - I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)
*E^(2*I*a + 2*I*b*x))/(1 - c + I*d - (1 - c - I*d)*E^(2*I*a + 2*I*b*x)), x]
```

```
, x] + Dist[(I*b*(1 + c + I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*I*
a + 2*I*b*x))/(1 + c - I*d - (1 + c + I*d)*E^(2*I*a + 2*I*b*x)), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, 1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(c+d\cot(a+bx))\,dx = \frac{1}{2}x^2\tanh^{-1}(c+d\cot(a+bx)) + \frac{1}{2}(b(i(1+c)-d))\int \frac{e^{2ia+2ibx}x^2}{1+c-id+(-1-c-id)e^{2ia+2}}$$

$$= \frac{1}{2}x^2\tanh^{-1}(c+d\cot(a+bx)) + \frac{1}{4}x^2\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) - \frac{1}{4}x^2\log\left(1-\frac{(1}{}\right.$$

$$= \frac{1}{2}x^2\tanh^{-1}(c+d\cot(a+bx)) + \frac{1}{4}x^2\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) - \frac{1}{4}x^2\log\left(1-\frac{(1}{}\right.$$

$$= \frac{1}{2}x^2\tanh^{-1}(c+d\cot(a+bx)) + \frac{1}{4}x^2\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) - \frac{1}{4}x^2\log\left(1-\frac{(1}{}\right.$$

$$= \frac{1}{2}x^2\tanh^{-1}(c+d\cot(a+bx)) + \frac{1}{4}x^2\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) - \frac{1}{4}x^2\log\left(1-\frac{(1}{}\right.$$

Mathematica [A] time = 0.541223, size = 253, normalized size = 0.86

$$\frac{1}{2}x^2\tanh^{-1}(d\cot(a+bx)+c) + \frac{-2ibx\text{PolyLog}\left(2,\frac{(c+id-1)e^{2i(a+bx)}}{c-id-1}\right)+2ibx\text{PolyLog}\left(2,\frac{(c+id+1)e^{2i(a+bx)}}{c-id+1}\right)+\text{PolyLog}\left(3,\frac{(c+i}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[c + d*Cot[a + b*x]])/2 + (2*b^2*x^2*Log[1 - ((-1 + c + I*d)*E^
((2*I)*(a + b*x)))/(-1 + c - I*d)] - 2*b^2*x^2*Log[1 - ((1 + c + I*d)*E^((2
*I)*(a + b*x)))/(1 + c - I*d)] - (2*I)*b*x*PolyLog[2, ((-1 + c + I*d)*E^((2
*I)*(a + b*x)))/(-1 + c - I*d)] + (2*I)*b*x*PolyLog[2, ((1 + c + I*d)*E^((2
*I)*(a + b*x)))/(1 + c - I*d)] + PolyLog[3, ((-1 + c + I*d)*E^((2*I)*(a + b
*x)))/(-1 + c - I*d)] - PolyLog[3, ((1 + c + I*d)*E^((2*I)*(a + b*x)))/(1 +
 c - I*d)])/(8*b^2)
```

Maple [C] time = 9.795, size = 6425, normalized size = 21.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(c+d*cot(b*x+a)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -2*b*d*integrate((2*(c^2 + d^2 - 1)*x^2*cos(2*b*x + 2*a)^2 + 2*c*d*x^2*sin(
2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^2*sin(2*b*x + 2*a)^2 - (c^2 - d^2 - 1)*x
^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x + 2*a) + (c^2 - d^2 - 1)*x^2*cos
(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^2*cos(2*b*x + 2*a) - (c^2 - d^2
- 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 + d^4 + 2*(c^2 + 1)*d^2 +
 (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*cos(4*b*x + 4*a)^2 + 4*(c^4 + d^
4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x + 2*a)^2 + (c^4 + d^4 + 2*(c^2 +
 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 -
2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 + 2*(c^4 + d^4 - 2*(3*c^2 - 1)*d^2 -
2*c^2 - 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(c*d^3 + (c^3 - c)*d
)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) - 4*(c^4 - d^4 - 2*c^2 + 1)*cos(2*
b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 - c)*d + 2*(c*d^3 + (c^3 - c)*d)*cos(2*b*x
 + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*sin(2*b*x + 2*a))*sin(4*b*x + 4*a) + 8*(c
*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1), x) + 1/8*x^2*log((c^2 + d^2 + 2*
c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 + 2*c
 + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 - 2*(c^2 - d^2 + 2*c + 1)*cos(2*b*x +
2*a) + 2*c + 1) - 1/8*x^2*log((c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a)^2 + 4*
(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 - 2*c + 1)*sin(2*b*x + 2*a)^2 + c^2
 + d^2 - 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*x + 2*a) - 2*c + 1)
```

Fricas [C] time = 3.22085, size = 3839, normalized size = 13.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/16*(4*b^2*x^2*log(-(d*cos(2*b*x + 2*a) + (c + 1)*sin(2*b*x + 2*a) + d)/(d
*cos(2*b*x + 2*a) + (c - 1)*sin(2*b*x + 2*a) + d)) + 2*I*b*x*dilog(-(c^2 +
```

```
d^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*
(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c
 + 1) + 1) - 2*I*b*x*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2 + 2*c +
 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin(2*b*x
+ 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1) + 1) - 2*I*b*x*dilog(-(c^2 + d^2 -
(c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c - 1
)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1)
+ 1) + 2*I*b*x*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*co
s(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a)
 - 2*c + 1)/(c^2 + d^2 - 2*c + 1) + 1) - 2*a^2*log(1/2*c^2 + I*(c + 1)*d -
1/2*d^2 - 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 +
 2*I*c + I)*sin(2*b*x + 2*a) + c + 1/2) + 2*a^2*log(1/2*c^2 + I*(c - 1)*d -
 1/2*d^2 - 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2
- 2*I*c + I)*sin(2*b*x + 2*a) - c + 1/2) - 2*a^2*log(-1/2*c^2 + I*(c + 1)*d
 + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^
2 + 2*I*c + I)*sin(2*b*x + 2*a) - c - 1/2) + 2*a^2*log(-1/2*c^2 + I*(c - 1)
*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*
d^2 - 2*I*c + I)*sin(2*b*x + 2*a) + c - 1/2) - 2*(b^2*x^2 - a^2)*log((c^2 +
 d^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2
*(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*
c + 1)) - 2*(b^2*x^2 - a^2)*log((c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2 + 2
*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin(2*
b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1)) + 2*(b^2*x^2 - a^2)*log((c^2 +
 d^2 - (c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2
*(c - 1)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*
c + 1)) + 2*(b^2*x^2 - a^2)*log((c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^2 - 2
*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*sin(2*
b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1)) - polylog(3, ((c^2 + 2*I*(c +
1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*(c + 1)*d - I*d^2 + 2*I
*c + I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*c + 1)) - polylog(3, ((c^2 - 2*I*(
c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*(c + 1)*d + I*d^2
- 2*I*c - I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*c + 1)) + polylog(3, ((c^2 +
2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*(c - 1)*d - I*
d^2 - 2*I*c + I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*c + 1)) + polylog(3, ((c^
2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*(c - 1)*d
 + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*c + 1)))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(c+d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{artanh}\left(d\cot\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(d*cot(b*x + a) + c), x)
```

3.336 $\int \tanh^{-1}(c + d\cot(a + bx))\,dx$

Optimal. Leaf size=194

$$-\frac{i\text{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b}+\frac{i\text{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b}+\frac{1}{2}x\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)-\frac{1}{2}x\log\left(1-\frac{(c+ia}{c}\right.$$

```
[Out] x*ArcTanh[c + d*Cot[a + b*x]] + (x*Log[1 - ((1 - c - I*d)*E^((2*I)*a + (2*I
)*b*x))/(1 - c + I*d)])/2 - (x*Log[1 - ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*
x))/(1 + c - I*d)])/2 - ((I/4)*PolyLog[2, ((1 - c - I*d)*E^((2*I)*a + (2*I)
*b*x))/(1 - c + I*d)])/b + ((I/4)*PolyLog[2, ((1 + c + I*d)*E^((2*I)*a + (2
*I)*b*x))/(1 + c - I*d)])/b
```

Rubi [A] time = 0.241336, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6261, 2190, 2279, 2391}

$$-\frac{i\text{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b}+\frac{i\text{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b}+\frac{1}{2}x\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)-\frac{1}{2}x\log\left(1-\frac{(c+ia}{c}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[c + d*Cot[a + b*x]],x]
```

```
[Out] x*ArcTanh[c + d*Cot[a + b*x]] + (x*Log[1 - ((1 - c - I*d)*E^((2*I)*a + (2*I
)*b*x))/(1 - c + I*d)])/2 - (x*Log[1 - ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*
x))/(1 + c - I*d)])/2 - ((I/4)*PolyLog[2, ((1 - c - I*d)*E^((2*I)*a + (2*I)
*b*x))/(1 - c + I*d)])/b + ((I/4)*PolyLog[2, ((1 + c + I*d)*E^((2*I)*a + (2
*I)*b*x))/(1 + c - I*d)])/b
```

Rule 6261

```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Tanh[c + d*Cot[a + b*x]], x] + (-Dist[I*b*(1 - c - I*d), Int[(x*E^(2*I*a +
2*I*b*x))/(1 - c + I*d - (1 - c - I*d)*E^(2*I*a + 2*I*b*x)), x], x] + Dist[
I*b*(1 + c + I*d), Int[(x*E^(2*I*a + 2*I*b*x))/(1 + c - I*d - (1 + c + I*d)
*E^(2*I*a + 2*I*b*x)), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - I*d)^2
, 1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \tanh^{-1}(c+d\cot(a+bx))\,dx = x\tanh^{-1}(c+d\cot(a+bx)) + (b(i(1+c)-d))\int \frac{e^{2ia+2ibx}x}{1+c-id+(-1-c-id)e^{2ia+2ibx}}\,dx$$

$$= x\tanh^{-1}(c+d\cot(a+bx)) + \frac{1}{2}x\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) - \frac{1}{2}x\log\left(1-\frac{(1+c\ldots}{1\ldots}\right)$$

$$= x\tanh^{-1}(c+d\cot(a+bx)) + \frac{1}{2}x\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) - \frac{1}{2}x\log\left(1-\frac{(1+c\ldots}{1\ldots}\right)$$

$$= x\tanh^{-1}(c+d\cot(a+bx)) + \frac{1}{2}x\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) - \frac{1}{2}x\log\left(1-\frac{(1+c\ldots}{1\ldots}\right)$$

Mathematica [B] time = 31.4373, size = 4463, normalized size = 23.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[c + d*Cot[a + b*x]],x]
```

```
[Out] x*ArcTanh[c + d*Cot[a + b*x]] - (d*(a*Log[-(Sec[(a + b*x)/2]^2*(d*Cos[a + b
*x] + (-1 + c)*Sin[a + b*x]))] - a*Log[-(Sec[(a + b*x)/2]^2*(d*Cos[a + b*x]
```

+ Sin[a + b*x] + c*Sin[a + b*x]))] - (a + b*x)*Log[-((-1 + c + Sqrt[1 - 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]] - I*Log[(d*(-I + Tan[(a + b*x)/2]))/(-1 + c - I*d + Sqrt[1 - 2*c + c^2 + d^2])]*Log[-((-1 + c + Sqrt[1 - 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]] + I*Log[(d*(I + Tan[(a + b*x)/2]))/(-1 + c + I*d + Sqrt[1 - 2*c + c^2 + d^2])]*Log[-((-1 + c + Sqrt[1 - 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]] + (a + b*x)*Log[-((1 + c + Sqrt[1 + 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]] + I*Log[(d*(-I + Tan[(a + b*x)/2]))/(1 + c - I*d + Sqrt[1 + 2*c + c^2 + d^2])]*Log[-((1 + c + Sqrt[1 + 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]] - I*Log[(d*(I + Tan[(a + b*x)/2]))/(1 + c + I*d + Sqrt[1 + 2*c + c^2 + d^2])]*Log[-((1 + c + Sqrt[1 + 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]] - (a + b*x)*Log[(1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/d] - I*Log[-((d*(-I + Tan[(a + b*x)/2]))/(1 - c + I*d + Sqrt[1 - 2*c + c^2 + d^2]))]*Log[(1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/d] + I*Log[-((d*(I + Tan[(a + b*x)/2]))/(1 - c - I*d + Sqrt[1 - 2*c + c^2 + d^2]))]*Log[(1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/d] + (a + b*x)*Log[(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/d] + I*Log[-((d*(-I + Tan[(a + b*x)/2]))/(-1 - c + I*d + Sqrt[1 + 2*c + c^2 + d^2]))]*Log[(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/d] - I*Log[-((d*(I + Tan[(a + b*x)/2]))/(-1 - c - I*d + Sqrt[1 + 2*c + c^2 + d^2]))]*Log[(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/d] - I*PolyLog[2, (-1 + c + Sqrt[1 - 2*c + c^2 + d^2] - d*Tan[(a + b*x)/2])/(-1 + c - I*d + Sqrt[1 - 2*c + c^2 + d^2])] + I*PolyLog[2, (-1 + c + Sqrt[1 - 2*c + c^2 + d^2] - d*Tan[(a + b*x)/2])/(-1 + c + I*d + Sqrt[1 - 2*c + c^2 + d^2])] - I*PolyLog[2, (1 + c - Sqrt[1 + 2*c + c^2 + d^2] - d*Tan[(a + b*x)/2])/(1 + c + I*d - Sqrt[1 + 2*c + c^2 + d^2])] + I*PolyLog[2, (1 + c + Sqrt[1 + 2*c + c^2 + d^2] - d*Tan[(a + b*x)/2])/(1 + c - I*d + Sqrt[1 + 2*c + c^2 + d^2])] - I*PolyLog[2, (1 + c + Sqrt[1 + 2*c + c^2 + d^2] - d*Tan[(a + b*x)/2])/(1 + c + I*d + Sqrt[1 + 2*c + c^2 + d^2])] + I*PolyLog[2, (1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/(1 - c - I*d + Sqrt[1 - 2*c + c^2 + d^2])] - I*PolyLog[2, (1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/(1 - c + I*d + Sqrt[1 - 2*c + c^2 + d^2])] + I*PolyLog[2, (-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/(-1 - c + I*d + Sqrt[1 + 2*c + c^2 + d^2])])*((2*a)/(b*(1 - c^2 - d^2 - Cos[2*(a + b*x)] + c^2*Cos[2*(a + b*x)] - d^2*Cos[2*(a + b*x)] - 2*c*d*Sin[2*(a + b*x)])) - (2*(a + b*x))/(b*(1 - c^2 - d^2 - Cos[2*(a + b*x)] + c^2*Cos[2*(a + b*x)] - d^2*Cos[2*(a + b*x)] - 2*c*d*Sin[2*(a + b*x)]))))/(-Log[-((-1 + c + Sqrt[1 - 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]] + Log[-((1 + c + Sqrt[1 + 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]] - Log[(1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/d] + Log[(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/d] - ((I/2)*Log[-((-1 + c + Sqrt[1 - 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(-I + Tan[(a + b*x)/2]) + ((I/2)*Log[-((1 + c + Sqrt[1 + 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(-I + Tan[(a + b*x)/2]) - ((I/2)*Log[(1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/d]*Sec[(a + b*x)/2]^2)/(-I + Tan[(a + b*x)/2]) + ((I/2)*Log[(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/d]

```
*Sec[(a + b*x)/2]^2)/(-I + Tan[(a + b*x)/2]) + ((I/2)*Log[-((-1 + c + Sqrt[
1 - 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(I + Tan[(
a + b*x)/2]) - ((I/2)*Log[-((1 + c + Sqrt[1 + 2*c + c^2 + d^2])/d) + Tan[(a
 + b*x)/2]]*Sec[(a + b*x)/2]^2)/(I + Tan[(a + b*x)/2]) + ((I/2)*Log[(1 - c
+ Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/d]*Sec[(a + b*x)/2]^2)/(I
 + Tan[(a + b*x)/2]) - ((I/2)*Log[(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*T
an[(a + b*x)/2])/d]*Sec[(a + b*x)/2]^2)/(I + Tan[(a + b*x)/2]) - ((a + b*x)
*Sec[(a + b*x)/2]^2)/(2*(-((-1 + c + Sqrt[1 - 2*c + c^2 + d^2])/d) + Tan[(a
 + b*x)/2])) - ((I/2)*Log[(d*(-I + Tan[(a + b*x)/2]))/(-1 + c - I*d + Sqrt[
1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-((-1 + c + Sqrt[1 - 2*c + c^2
+ d^2])/d) + Tan[(a + b*x)/2]) + ((I/2)*Log[(d*(I + Tan[(a + b*x)/2]))/(-1
+ c + I*d + Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-((-1 + c + Sq
rt[1 - 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]) + ((a + b*x)*Sec[(a + b*x)/
2]^2)/(2*(-((1 + c + Sqrt[1 + 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2])) + (
(I/2)*Log[(d*(-I + Tan[(a + b*x)/2]))/(1 + c - I*d + Sqrt[1 + 2*c + c^2 + d
^2])]*Sec[(a + b*x)/2]^2)/(-((1 + c + Sqrt[1 + 2*c + c^2 + d^2])/d) + Tan[(
a + b*x)/2]) - ((I/2)*Log[(d*(I + Tan[(a + b*x)/2]))/(1 + c + I*d + Sqrt[1
+ 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-((1 + c + Sqrt[1 + 2*c + c^2 + d
^2])/d) + Tan[(a + b*x)/2]) - ((I/2)*d*Log[1 - (-1 + c + Sqrt[1 - 2*c + c^2
 + d^2] - d*Tan[(a + b*x)/2])/(-1 + c - I*d + Sqrt[1 - 2*c + c^2 + d^2])]*S
ec[(a + b*x)/2]^2)/(-1 + c + Sqrt[1 - 2*c + c^2 + d^2] - d*Tan[(a + b*x)/2]
) + ((I/2)*d*Log[1 - (-1 + c + Sqrt[1 - 2*c + c^2 + d^2] - d*Tan[(a + b*x)/
2])/(-1 + c + I*d + Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-1 + c
 + Sqrt[1 - 2*c + c^2 + d^2] - d*Tan[(a + b*x)/2]) - ((I/2)*d*Log[1 - (1 +
c - Sqrt[1 + 2*c + c^2 + d^2] - d*Tan[(a + b*x)/2])/(1 + c + I*d - Sqrt[1 +
 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(1 + c - Sqrt[1 + 2*c + c^2 + d^2]
- d*Tan[(a + b*x)/2]) + ((I/2)*d*Log[1 - (1 + c + Sqrt[1 + 2*c + c^2 + d^2]
 - d*Tan[(a + b*x)/2])/(1 + c - I*d + Sqrt[1 + 2*c + c^2 + d^2])]*Sec[(a +
b*x)/2]^2)/(1 + c + Sqrt[1 + 2*c + c^2 + d^2] - d*Tan[(a + b*x)/2]) - ((I/2
)*d*Log[1 - (1 + c + Sqrt[1 + 2*c + c^2 + d^2] - d*Tan[(a + b*x)/2])/(1 + c
 + I*d + Sqrt[1 + 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(1 + c + Sqrt[1 +
2*c + c^2 + d^2] - d*Tan[(a + b*x)/2]) - (d*(a + b*x)*Sec[(a + b*x)/2]^2)/(
2*(1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])) - ((I/2)*d*Log[
-((d*(-I + Tan[(a + b*x)/2]))/(1 - c + I*d + Sqrt[1 - 2*c + c^2 + d^2]))]*S
ec[(a + b*x)/2]^2)/(1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])
 + ((I/2)*d*Log[-((d*(I + Tan[(a + b*x)/2]))/(1 - c - I*d + Sqrt[1 - 2*c +
c^2 + d^2]))]*Sec[(a + b*x)/2]^2)/(1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Ta
n[(a + b*x)/2]) - ((I/2)*d*Log[1 - (1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*T
an[(a + b*x)/2])/(1 - c - I*d + Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2
]^2)/(1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2]) + ((I/2)*d*Lo
g[1 - (1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/(1 - c + I*d
 + Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(1 - c + Sqrt[1 - 2*c +
c^2 + d^2] + d*Tan[(a + b*x)/2]) + (d*(a + b*x)*Sec[(a + b*x)/2]^2)/(2*(-1
- c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])) + ((I/2)*d*Log[-((d*
(-I + Tan[(a + b*x)/2]))/(-1 - c + I*d + Sqrt[1 + 2*c + c^2 + d^2]))]*Sec[(
```

```
a + b*x)/2]^2)/(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2]) -
((I/2)*d*Log[-((d*(I + Tan[(a + b*x)/2]))/(-1 - c - I*d + Sqrt[1 + 2*c + c^
2 + d^2]))]*Sec[(a + b*x)/2]^2)/(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan
[(a + b*x)/2]) - ((I/2)*d*Log[1 - (-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*T
an[(a + b*x)/2])/(-1 - c + I*d + Sqrt[1 + 2*c + c^2 + d^2])]*Sec[(a + b*x)/
2]^2)/(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2]) - (a*Cos[(a
 + b*x)/2]^2*(-(Sec[(a + b*x)/2]^2*((-1 + c)*Cos[a + b*x] - d*Sin[a + b*x])
) - Sec[(a + b*x)/2]^2*(d*Cos[a + b*x] + (-1 + c)*Sin[a + b*x])*Tan[(a + b*
x)/2]))/(d*Cos[a + b*x] + (-1 + c)*Sin[a + b*x]) + (a*Cos[(a + b*x)/2]^2*(-
(Sec[(a + b*x)/2]^2*(Cos[a + b*x] + c*Cos[a + b*x] - d*Sin[a + b*x])) - Sec
[(a + b*x)/2]^2*(d*Cos[a + b*x] + Sin[a + b*x] + c*Sin[a + b*x])*Tan[(a + b
*x)/2]))/(d*Cos[a + b*x] + Sin[a + b*x] + c*Sin[a + b*x]))
```

Maple [B] time = 0.088, size = 629, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(c+d*cot(b*x+a)),x)
```

```
[Out] -1/2/b*arctanh(c+d*cot(b*x+a))*Pi+1/b*arctanh(c+d*cot(b*x+a))*arccot(cot(b*
x+a))+1/2/b*arctan((c+d*cot(b*x+a))/d-c/d)*ln(d*((c+d*cot(b*x+a))/d-c/d)+c+
1)-1/2/b*arctan((c+d*cot(b*x+a))/d-c/d)*ln(d*((c+d*cot(b*x+a))/d-c/d)+c-1)+
1/4*I/b*ln(d*((c+d*cot(b*x+a))/d-c/d)+c+1)*ln((I*d-d*((c+d*cot(b*x+a))/d-c/
d))/(1+c+I*d))-1/4*I/b*ln(d*((c+d*cot(b*x+a))/d-c/d)+c+1)*ln((I*d+d*((c+d*c
ot(b*x+a))/d-c/d))/(I*d-c-1))+1/4*I/b*dilog((I*d-d*((c+d*cot(b*x+a))/d-c/d)
)/(1+c+I*d))-1/4*I/b*dilog((I*d+d*((c+d*cot(b*x+a))/d-c/d))/(I*d-c-1))-1/4*
I/b*ln(d*((c+d*cot(b*x+a))/d-c/d)+c-1)*ln((I*d-d*((c+d*cot(b*x+a))/d-c/d))/
(I*d+c-1))+1/4*I/b*ln(d*((c+d*cot(b*x+a))/d-c/d)+c-1)*ln((I*d+d*((c+d*cot(b
*x+a))/d-c/d))/(1-c+I*d))-1/4*I/b*dilog((I*d-d*((c+d*cot(b*x+a))/d-c/d))/(I
*d+c-1))+1/4*I/b*dilog((I*d+d*((c+d*cot(b*x+a))/d-c/d))/(1-c+I*d))
```

Maxima [B] time = 1.88544, size = 529, normalized size = 2.73

$$\frac{4\,(bx+a)\,\operatorname{artanh}\left(c+\frac{d}{\tan(bx+a)}\right)+\left(\arctan\left(\frac{(c+1)d+\left(c^2+2\,c+1\right)\tan(bx+a)}{c^2+d^2+2\,c+1},\frac{(c+1)d\tan(bx+a)+d^2}{c^2+d^2+2\,c+1}\right)-\arctan\left(\frac{(c-1)d+\left(c^2-2\,c+1\right)\tan(bx+a)}{c^2+d^2-2\,c+1}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(b*x + a)*arctanh(c + d/tan(b*x + a)) + (arctan2(((c + 1)*d + (c^2 +
 2*c + 1)*tan(b*x + a))/(c^2 + d^2 + 2*c + 1), ((c + 1)*d*tan(b*x + a) + d^
2)/(c^2 + d^2 + 2*c + 1)) - arctan2(((c - 1)*d + (c^2 - 2*c + 1)*tan(b*x +
a))/(c^2 + d^2 - 2*c + 1), ((c - 1)*d*tan(b*x + a) + d^2)/(c^2 + d^2 - 2*c
+ 1)))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log((2*(c + 1)*d*tan(b*x + a) +
(c^2 + 2*c + 1)*tan(b*x + a)^2 + d^2)/(c^2 + d^2 + 2*c + 1)) + (b*x + a)*lo
g((2*(c - 1)*d*tan(b*x + a) + (c^2 - 2*c + 1)*tan(b*x + a)^2 + d^2)/(c^2 +
d^2 - 2*c + 1)) + I*dilog(-((c + 1)*tan(b*x + a) - I*c - I)/(I*c + d + I))
- I*dilog(-((c - 1)*tan(b*x + a) - I*c + I)/(I*c + d - I)) + I*dilog(-((c -
 1)*tan(b*x + a) + I*c - I)/(-I*c + d + I)) - I*dilog(-((c + 1)*tan(b*x + a
) + I*c + I)/(-I*c + d - I)))/b
```

Fricas [B] time = 2.97562, size = 2920, normalized size = 15.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*log(-(d*cos(2*b*x + 2*a) + (c + 1)*sin(2*b*x + 2*a) + d)/(d*cos(
2*b*x + 2*a) + (c - 1)*sin(2*b*x + 2*a) + d)) + 2*a*log(1/2*c^2 + I*(c + 1)
*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*
d^2 + 2*I*c + I)*sin(2*b*x + 2*a) + c + 1/2) - 2*a*log(1/2*c^2 + I*(c - 1)*
d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d
^2 - 2*I*c + I)*sin(2*b*x + 2*a) - c + 1/2) + 2*a*log(-1/2*c^2 + I*(c + 1)*
d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d
^2 + 2*I*c + I)*sin(2*b*x + 2*a) - c - 1/2) - 2*a*log(-1/2*c^2 + I*(c - 1)*
d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d
^2 - 2*I*c + I)*sin(2*b*x + 2*a) + c - 1/2) - 2*(b*x + a)*log((c^2 + d^2 -
(c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c + 1
)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1))
 - 2*(b*x + a)*log((c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2
*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) +
2*c + 1)/(c^2 + d^2 + 2*c + 1)) + 2*(b*x + a)*log((c^2 + d^2 - (c^2 + 2*I*(
c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c - 1)*d + I*d^2
+ 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1)) + 2*(b*x +
a)*log((c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a)
+ (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^
2 + d^2 - 2*c + 1)) + I*dilog(-(c^2 + d^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*
```

```
c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*
b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1) + 1) - I*dilog(-(c^2 + d^2 - (c
^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d
 - I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1) + 1
) - I*dilog(-(c^2 + d^2 - (c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x +
 2*a) + (-I*c^2 + 2*(c - 1)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c +
 1)/(c^2 + d^2 - 2*c + 1) + 1) + I*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c - 1)*d
 - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c +
 I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1) + 1))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(c+d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{artanh}\left(d\cot\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*cot(b*x + a) + c), x)
```

3.337 $\int \frac{\tanh^{-1}(c+d\cot(a+bx))}{x}\,dx$

Optimal. Leaf size=17

$$\text{CannotIntegrate}\left(\frac{\tanh^{-1}(d\cot(a+bx)+c)}{x},x\right)$$

```
[Out] CannotIntegrate[ArcTanh[c + d*Cot[a + b*x]]/x, x]
```

Rubi [A] time = 0.142022, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tanh^{-1}(c+d\cot(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTanh[c + d*Cot[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTanh[c + d*Cot[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(c+d\cot(a+bx))}{x}\,dx=\int \frac{\tanh^{-1}(c+d\cot(a+bx))}{x}\,dx$$

Mathematica [A] time = 4.90867, size = 0, normalized size = 0.

$$\int \frac{\tanh^{-1}(c+d\cot(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTanh[c + d*Cot[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcTanh[c + d*Cot[a + b*x]]/x, x]
```

Maple [A] time = 0.371, size = 0, normalized size = 0.

$$\int \frac{\text{Artanh}\left(c+d\cot\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(c+d*cot(b*x+a))/x,x)
```

```
[Out] int(arctanh(c+d*cot(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{artanh}\left(d\cot\left(bx+a\right)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*cot(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(d*cot(b*x + a) + c)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{artanh}\left(d\cot\left(bx+a\right)+c\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*cot(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(arctanh(d*cot(b*x + a) + c)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(c+d*cot(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}\left(d\cot\left(bx+a\right)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*cot(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*cot(b*x + a) + c)/x, x)
```

3.338 $\int x^2 \tanh^{-1}(1 + id + d\cot(a + bx))\,dx$

Optimal. Leaf size=168

$$-\frac{x\text{PolyLog}\left(3,(1+id)e^{2ia+2ibx}\right)}{4b^2}-\frac{i\text{PolyLog}\left(4,(1+id)e^{2ia+2ibx}\right)}{8b^3}+\frac{ix^2\text{PolyLog}\left(2,(1+id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{6}x^3\log\left(1-(1+\right.$$

```
[Out] (I/12)*b*x^4 + (x^3*ArcTanh[1 + I*d + d*Cot[a + b*x]])/3 - (x^3*Log[1 - (1
+ I*d)*E^((2*I)*a + (2*I)*b*x)])/6 + ((I/4)*x^2*PolyLog[2, (1 + I*d)*E^((2*
I)*a + (2*I)*b*x)])/b - (x*PolyLog[3, (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/(
4*b^2) - ((I/8)*PolyLog[4, (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/b^3
```

Rubi [A] time = 0.296257, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {6265, 2184, 2190, 2531, 6609, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(3,(1+id)e^{2ia+2ibx}\right)}{4b^2}-\frac{i\text{PolyLog}\left(4,(1+id)e^{2ia+2ibx}\right)}{8b^3}+\frac{ix^2\text{PolyLog}\left(2,(1+id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{6}x^3\log\left(1-(1+\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[1 + I*d + d*Cot[a + b*x]],x]
```

```
[Out] (I/12)*b*x^4 + (x^3*ArcTanh[1 + I*d + d*Cot[a + b*x]])/3 - (x^3*Log[1 - (1
+ I*d)*E^((2*I)*a + (2*I)*b*x)])/6 + ((I/4)*x^2*PolyLog[2, (1 + I*d)*E^((2*
I)*a + (2*I)*b*x)])/b - (x*PolyLog[3, (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/(
4*b^2) - ((I/8)*PolyLog[4, (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/b^3
```

Rule 6265

```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Cot[a + b*x]])/(f*(m
 + 1)), x] + Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c - I*d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
```

```
x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\tanh^{-1}(1+id+d\cot(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(1+id+d\cot(a+bx))+\frac{1}{3}(ib)\int\frac{x^3}{1+(-1-id)e^{2ia+2ibx}}\,dx$$
$$=\frac{1}{12}ibx^4+\frac{1}{3}x^3\tanh^{-1}(1+id+d\cot(a+bx))+\frac{1}{3}(b(i-d))\int\frac{e^{2ia+2ibx}x^3}{1+(-1-id)e^{2ia\cdots}}$$
$$=\frac{1}{12}ibx^4+\frac{1}{3}x^3\tanh^{-1}(1+id+d\cot(a+bx))-\frac{1}{6}x^3\log\left(1-(1+id)e^{2ia+2ibx}\right)+\cdots$$
$$=\frac{1}{12}ibx^4+\frac{1}{3}x^3\tanh^{-1}(1+id+d\cot(a+bx))-\frac{1}{6}x^3\log\left(1-(1+id)e^{2ia+2ibx}\right)+\cdots$$
$$=\frac{1}{12}ibx^4+\frac{1}{3}x^3\tanh^{-1}(1+id+d\cot(a+bx))-\frac{1}{6}x^3\log\left(1-(1+id)e^{2ia+2ibx}\right)+\cdots$$
$$=\frac{1}{12}ibx^4+\frac{1}{3}x^3\tanh^{-1}(1+id+d\cot(a+bx))-\frac{1}{6}x^3\log\left(1-(1+id)e^{2ia+2ibx}\right)+\cdots$$
$$=\frac{1}{12}ibx^4+\frac{1}{3}x^3\tanh^{-1}(1+id+d\cot(a+bx))-\frac{1}{6}x^3\log\left(1-(1+id)e^{2ia+2ibx}\right)+\cdots$$

Mathematica [A] time = 0.416638, size = 155, normalized size = 0.92

$$\frac{1}{3}x^3\tanh^{-1}(d\cot(a+bx)+id+1)-\frac{6ib^2x^2\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{d-i}\right)+6bx\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{d-i}\right)-3i\text{PolyLog}\left(4,-\frac{ie^{-2i\cdots}}{\cdots}\right)\cdots}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[1 + I*d + d*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[1 + I*d + d*Cot[a + b*x]])/3 - (4*b^3*x^3*Log[1 + I/((-I + d)*
E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, (-I)/((-I + d)*E^((2*I)*(a
 + b*x)))] + 6*b*x*PolyLog[3, (-I)/((-I + d)*E^((2*I)*(a + b*x)))] - (3*I)*
PolyLog[4, (-I)/((-I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

Maple [C] time = 16.132, size = 2456, normalized size = 14.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(1+I*d+d*cot(b*x+a)),x)
```

```
[Out] -1/2/b^2*d/(-d+I)*ln(1+I*(-d+I)*exp(2*I*(b*x+a)))*x*a^2+1/2/b^2*a^2*d/(-d+I
)*ln(1-I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))*x+1/2/b^2*a^2*d/(-d+I)*ln(1+I*exp
(I*(b*x+a))*(I*(-d+I))^(1/2))*x-1/4/b/(-d+I)*polylog(2,-I*(-d+I)*exp(2*I*(b
*x+a)))*x^2+1/4/b^3/(-d+I)*polylog(2,-I*(-d+I)*exp(2*I*(b*x+a)))*a^2+1/6*d/
(-d+I)*ln(1+I*(-d+I)*exp(2*I*(b*x+a)))*x^3+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*
x+a))/(exp(2*I*(b*x+a))-1))^3+1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*e
xp(2*I*(b*x+a)))^3+1/6*I/b^3*a^3/(-d+I)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+
a))*d-I)-1/6*x^3*ln(d)+1/12*I*b*x^4-1/12*I*x^3*Pi*csgn(I*d)*csgn(I*d/(exp(2
*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a))/(e
xp(2*I*(b*x+a))-1))*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-1/3/b
^3*d/(-d+I)*ln(1+I*(-d+I)*exp(2*I*(b*x+a)))*a^3+1/6*I*x^3*Pi-1/6*I/(-d+I)*l
n(1+I*(-d+I)*exp(2*I*(b*x+a)))*x^3-1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))
-1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+1/12*
I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*
(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*
csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^2+1/12*I*x^3*Pi*csgn(I*d/(exp
(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x
+a)))+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))^3-1/4*I/b^2/(-d+I)*polylog(3,-
I*(-d+I)*exp(2*I*(b*x+a)))*x-1/2*I/b^3*a^3/(-d+I)*ln(1-I*exp(I*(b*x+a))*(I*
(-d+I))^(1/2))-1/2*I/b^3*a^3/(-d+I)*ln(1+I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))
-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*
I*(b*x+a))-1))^2+1/12*I*x^3*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x
+a)))+1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I))*csgn(
I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2-1/6*I*x
^3*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2+1/2/b^3*a^3*d/(-d+I
)*ln(1-I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))+1/2/b^3*a^3*d/(-d+I)*ln(1+I*exp(I
*(b*x+a))*(I*(-d+I))^(1/2))-1/6/b^3*a^3*d/(-d+I)*ln(I*exp(2*I*(b*x+a))-exp(
2*I*(b*x+a))*d-I)-1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)
))^2+1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*
I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x
+a))-1))^2-1/3*x^3*ln(exp(I*(b*x+a)))-1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a)
)*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*d-I*
exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))-1/2*I/b^3*a^2*d/(-d+I)*dilog(1+I*
exp(I*(b*x+a))*(I*(-d+I))^(1/2))-1/4*I/b*d/(-d+I)*polylog(2,-I*(-d+I)*exp(2
*I*(b*x+a)))*x^2+1/6*x^3*ln(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)+1/8*I/
b^3*d/(-d+I)*polylog(4,-I*(-d+I)*exp(2*I*(b*x+a)))+1/3*I/b^3/(-d+I)*ln(1+I*
(-d+I)*exp(2*I*(b*x+a)))*a^3+1/4/b^2*d/(-d+I)*polylog(3,-I*(-d+I)*exp(2*I*(
b*x+a)))*x-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+
a))*d-I*exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+
I)/(exp(2*I*(b*x+a))-1))+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp
(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))-1/12*I*x^3*
Pi*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2-1
/2/b^3*a^2/(-d+I)*dilog(1-I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))-1/2/b^3*a^2/(-
d+I)*dilog(1+I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))+1/12*I*x^3*Pi*csgn(d/(exp(2
*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^3-1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d
```

```
-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^3+1/12*I*x^3*Pi*csgn((exp(2*I*
(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^3+1/4*I/b^3*d/(-d+I)
*polylog(2,-I*(-d+I)*exp(2*I*(b*x+a)))*a^2+1/2*I/b^2/(-d+I)*ln(1+I*(-d+I)*e
xp(2*I*(b*x+a)))*x*a^2-1/2*I/b^2*a^2/(-d+I)*ln(1-I*exp(I*(b*x+a))*(I*(-d+I)
)^(1/2))*x+1/8/b^3/(-d+I)*polylog(4,-I*(-d+I)*exp(2*I*(b*x+a)))+1/12*I*x^3*
Pi*csgn(I*d)*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))*csgn(I*d/(exp(2*
I*(b*x+a))-1)*exp(2*I*(b*x+a)))-1/2*I/b^2*a^2/(-d+I)*ln(1+I*exp(I*(b*x+a))*
(I*(-d+I))^(1/2))*x-1/2*I/b^3*a^2*d/(-d+I)*dilog(1-I*exp(I*(b*x+a))*(I*(-d+
I))^(1/2))
```

Maxima [B] time = 1.18607, size = 462, normalized size = 2.75

$$\frac{\frac{12\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\operatorname{artanh}(d\cot(bx+a)+i\,d+1)}{b^2}-\frac{-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2+\left(-8i\,(bx+a)^3+18i\,(bx+a)^2a-18i\,(bx+a)a^2\right)\operatorname{arctan}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctanh(d*cot(b*
x + a) + I*d + 1)/b^2 - (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
+ a)^2*a^2 + (-8*I*(b*x + a)^3 + 18*I*(b*x + a)^2*a - 18*I*(b*x + a)*a^2)*a
rctan2(d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) - cos(2*b*
x + 2*a) + 1) + (-12*I*(b*x + a)^2 + 18*I*(b*x + a)*a - 9*I*a^2)*dilog((I*d
 + 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(b*x + a)
*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^2 + 2*d
*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, (I*d
 + 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (I*d + 1)*e^(2*I*b*x + 2*I*a)))
/b^2)/b
```

Fricas [C] time = 2.00875, size = 506, normalized size = 3.01

$$\frac{2i\,b^4x^4+4\,b^3x^3\log\left(-\frac{\left((d-i)e^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{d}\right)+6i\,b^2x^2\operatorname{Li}_2\left(-(-i\,d-1)e^{(2i\,bx+2i\,a)}\right)-2i\,a^4+4\,a^3\log\left(\frac{(d-i)e^{(2i\,bx+2i\,a)}+i}{d-i}\right)-6}{24\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/24*(2*I*b^4*x^4 + 4*b^3*x^3*log(-((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*
I*b*x - 2*I*a)/d) + 6*I*b^2*x^2*dilog(-(-I*d - 1)*e^(2*I*b*x + 2*I*a)) - 2*
I*a^4 + 4*a^3*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)/(d - I)) - 6*b*x*polylo
g(3, (I*d + 1)*e^(2*I*b*x + 2*I*a)) - 4*(b^3*x^3 + a^3)*log((-I*d - 1)*e^(2
*I*b*x + 2*I*a) + 1) - 3*I*polylog(4, (I*d + 1)*e^(2*I*b*x + 2*I*a)))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(1+I*d+d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{artanh}(d \cot(bx + a) + i d + 1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(d*cot(b*x + a) + I*d + 1), x)
```

3.339 $\int x \tanh^{-1}(1 + id + d\cot(a + bx))\,dx$

Optimal. Leaf size=132

$$-\frac{\text{PolyLog}\left(3,(1+id)e^{2ia+2ibx}\right)}{8b^2}+\frac{ix\text{PolyLog}\left(2,(1+id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(1+id)e^{2ia+2ibx}\right)+\frac{1}{2}x^2\tanh^{-1}(d\cot(a$$

```
[Out] (I/6)*b*x^3 + (x^2*ArcTanh[1 + I*d + d*Cot[a + b*x]])/2 - (x^2*Log[1 - (1 +
 I*d)*E^((2*I)*a + (2*I)*b*x)])/4 + ((I/4)*x*PolyLog[2, (1 + I*d)*E^((2*I)*
a + (2*I)*b*x)])/b - PolyLog[3, (1 + I*d)*E^((2*I)*a + (2*I)*b*x)]/(8*b^2)
```

Rubi [A] time = 0.246078, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6265, 2184, 2190, 2531, 2282, 6589}

$$-\frac{\text{PolyLog}\left(3,(1+id)e^{2ia+2ibx}\right)}{8b^2}+\frac{ix\text{PolyLog}\left(2,(1+id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(1+id)e^{2ia+2ibx}\right)+\frac{1}{2}x^2\tanh^{-1}(d\cot(a$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[1 + I*d + d*Cot[a + b*x]],x]
```


```
[Out] (I/6)*b*x^3 + (x^2*ArcTanh[1 + I*d + d*Cot[a + b*x]])/2 - (x^2*Log[1 - (1 +
 I*d)*E^((2*I)*a + (2*I)*b*x)])/4 + ((I/4)*x*PolyLog[2, (1 + I*d)*E^((2*I)*
a + (2*I)*b*x)])/b - PolyLog[3, (1 + I*d)*E^((2*I)*a + (2*I)*b*x)]/(8*b^2)
```


Rule 6265

```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Cot[a + b*x]])/(f*(m
 + 1)), x] + Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c - I*d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(1+id+d\cot(a+bx))\,dx = \frac{1}{2}x^2\tanh^{-1}(1+id+d\cot(a+bx))+\frac{1}{2}(ib)\int\frac{x^2}{1+(-1-id)e^{2ia+2ibx}}\,dx$$
$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id+d\cot(a+bx))+\frac{1}{2}(b(i-d))\int\frac{e^{2ia+2ibx}x^2}{1+(-1-id)e^{2ia+2i}}$$
$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id+d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1+id)e^{2ia+2ibx}\right)+\frac{1}{2}$$
$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id+d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1+id)e^{2ia+2ibx}\right)+$$
$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id+d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1+id)e^{2ia+2ibx}\right)+$$
$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id+d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1+id)e^{2ia+2ibx}\right)+$$

Mathematica [A] time = 0.297686, size = 119, normalized size = 0.9

$$\frac{1}{2}x^2\tanh^{-1}(d\cot(a+bx)+id+1)-\frac{2ibx\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{d-i}\right)+\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{d-i}\right)+2b^2x^2\log\left(1+\frac{ie^{-2i(a+bx)}}{d-i}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[1 + I*d + d*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[1 + I*d + d*Cot[a + b*x]])/2 - (2*b^2*x^2*Log[1 + I/((-I + d)*
E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/((-I + d)*E^((2*I)*(a + b
*x)))] + PolyLog[3, (-I)/((-I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

Maple [C] time = 9.899, size = 2358, normalized size = 17.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(1+I*d+d*cot(b*x+a)),x)
```

```
[Out] 1/8/b^2*d/(-d+I)*polylog(3,-I*(-d+I)*exp(2*I*(b*x+a)))+1/4*I*x^2*Pi+1/8*I*x
^2*Pi*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^3+1/8*I*x^2*Pi*csgn(I*e
```

```
xp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^3+1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x
+a))-1)*exp(2*I*(b*x+a)))^3-1/4*I/b^2/(-d+I)*ln(1+I*(-d+I)*exp(2*I*(b*x+a))
)*a^2+1/2*I/b^2*a^2/(-d+I)*ln(1-I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))+1/2*I/b^
2*a^2/(-d+I)*ln(1+I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))-1/8*I*x^2*Pi*csgn(I*(e
xp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I
*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))+1/4*x^2*ln(exp(2*I*
(b*x+a))*d-I*exp(2*I*(b*x+a))+I)+1/6*I*b*x^3+1/8*I*x^2*Pi*csgn(I/(exp(2*I*(
b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+
a))-1))^2+1/8*I*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I))*cs
gn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2-1/4*
I*x^2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2-1/8*I*x^2*Pi*csg
n(I*d)*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-1/8*I*x^2*Pi*csgn(
I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(
2*I*(b*x+a)))^2-1/4*x^2*ln(d)+1/8*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*
exp(2*I*(b*x+a)))-1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x
+a))/(exp(2*I*(b*x+a))-1))^2-1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn
(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^2-1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*
(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))
^2+1/8*I*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(
b*x+a))-1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a)
)-1))^2+1/4/b^2*a^2*d/(-d+I)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d-I)-1/
2/b^2*a^2*d/(-d+I)*ln(1-I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))-1/2/b^2*a^2*d/(-
d+I)*ln(1+I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))+1/4/b^2*d/(-d+I)*ln(1+I*(-d+I)
*exp(2*I*(b*x+a)))*a^2-1/4*I/b^2*a^2/(-d+I)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(
b*x+a))*d-I)+1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*c
sgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b
*x+a)))^3-1/2*x^2*ln(exp(I*(b*x+a)))+1/2*I/b*a/(-d+I)*ln(1+I*exp(I*(b*x+a))
*(I*(-d+I))^(1/2))*x+1/4*d/(-d+I)*ln(1+I*(-d+I)*exp(2*I*(b*x+a)))*x^2+1/2/b
^2*a/(-d+I)*dilog(1-I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))+1/2/b^2*a/(-d+I)*dil
og(1+I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))-1/4/b/(-d+I)*polylog(2,-I*(-d+I)*ex
p(2*I*(b*x+a)))*x-1/4/b^2/(-d+I)*polylog(2,-I*(-d+I)*exp(2*I*(b*x+a)))*a-1/
2/b*a*d/(-d+I)*ln(1-I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))*x-1/2/b*a*d/(-d+I)*l
n(1+I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))*x+1/2/b*d/(-d+I)*ln(1+I*(-d+I)*exp(2
*I*(b*x+a)))*x*a+1/8*I*x^2*Pi*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I
)/(exp(2*I*(b*x+a))-1))^3-1/8*I*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I
*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^3-1/8*I/b^2/(-d+I)*polylog(3,-I*(-d+I)*e
xp(2*I*(b*x+a)))-1/4*I/(-d+I)*ln(1+I*(-d+I)*exp(2*I*(b*x+a)))*x^2-1/8*I*x^2
*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+
a))+I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-
1))+1/8*I*x^2*Pi*csgn(I*d)*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))*cs
gn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))+1/2*I/b^2*a*d/(-d+I)*dilog(1+
I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))-1/4*I/b*d/(-d+I)*polylog(2,-I*(-d+I)*exp
(2*I*(b*x+a)))*x-1/4*I/b^2*d/(-d+I)*polylog(2,-I*(-d+I)*exp(2*I*(b*x+a)))*a
-1/2*I/b/(-d+I)*ln(1+I*(-d+I)*exp(2*I*(b*x+a)))*x*a+1/2*I/b^2*a*d/(-d+I)*di
log(1-I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))-1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+
```

```
a))-1)*exp(2*I*(b*x+a)))^2-1/8*I*x^2*Pi*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*
(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*c
sgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))+1
/2*I/b*a/(-d+I)*ln(1-I*exp(I*(b*x+a))*(I*(-d+I))^(1/2))*x
```

Maxima [B] time = 1.10757, size = 335, normalized size = 2.54

$$\frac{\frac{12\left((bx+a)^2-2\,(bx+a)a\right)\operatorname{artanh}(d\cot(bx+a)+i\,d+1)}{b}-\frac{-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\operatorname{Li}_2\left((i\,d+1)e^{(2i\,bx+2i\,a)}\right)+\left(-6i\,(bx+a)^2+12i\,(bx+a)a\right)\arctan(d\cos(2\,bx+2}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arctanh(d*cot(b*x + a) + I*d + 1)/b
- (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((I*d + 1)*e^(2*I*b
*x + 2*I*a)) + (-6*I*(b*x + a)^2 + 12*I*(b*x + a)*a)*arctan2(d*cos(2*b*x +
2*a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1) + 3*((b
*x + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin
(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) + 3*polylo
g(3, (I*d + 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

Fricas [C] time = 1.81524, size = 431, normalized size = 3.27

$$\frac{4i\,b^3x^3+6\,b^2x^2\log\left(-\frac{\left((d-i)e^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{d}\right)+4i\,a^3+6i\,bx\operatorname{Li}_2\left(-(-i\,d-1)e^{(2i\,bx+2i\,a)}\right)-6\,a^2\log\left(\frac{(d-i)e^{(2i\,bx+2i\,a)}+i}{d-i}\right)-6\,(b}{24\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/24*(4*I*b^3*x^3 + 6*b^2*x^2*log(-((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*
I*b*x - 2*I*a)/d) + 4*I*a^3 + 6*I*b*x*dilog(-(-I*d - 1)*e^(2*I*b*x + 2*I*a)
) - 6*a^2*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)/(d - I)) - 6*(b^2*x^2 - a^2
)*log((-I*d - 1)*e^(2*I*b*x + 2*I*a) + 1) - 3*polylog(3, (I*d + 1)*e^(2*I*b
*x + 2*I*a)))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(1+I*d+d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{artanh}\left(d\cot\left(bx+a\right)+i\,d+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(d*cot(b*x + a) + I*d + 1), x)
```

3.340 $\int \tanh^{-1}(1+id+d\cot(a+bx))\,dx$

Optimal. Leaf size=93

$$\frac{i\text{PolyLog}\left(2,(1+id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{2}x\log\left(1-(1+id)e^{2ia+2ibx}\right)+x\tanh^{-1}(d\cot(a+bx)+id+1)+\frac{1}{2}ibx^2$$

```
[Out] (I/2)*b*x^2 + x*ArcTanh[1 + I*d + d*Cot[a + b*x]] - (x*Log[1 - (1 + I*d)*E^
((2*I)*a + (2*I)*b*x)])/2 + ((I/4)*PolyLog[2, (1 + I*d)*E^((2*I)*a + (2*I)*
b*x)])/b
```

Rubi [A] time = 0.149287, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6257, 2184, 2190, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2,(1+id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{2}x\log\left(1-(1+id)e^{2ia+2ibx}\right)+x\tanh^{-1}(d\cot(a+bx)+id+1)+\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[1 + I*d + d*Cot[a + b*x]],x]
```

```
[Out] (I/2)*b*x^2 + x*ArcTanh[1 + I*d + d*Cot[a + b*x]] - (x*Log[1 - (1 + I*d)*E^
((2*I)*a + (2*I)*b*x)])/2 + ((I/4)*PolyLog[2, (1 + I*d)*E^((2*I)*a + (2*I)*
b*x)])/b
```

Rule 6257

```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Tanh[c + d*Cot[a + b*x]], x] + Dist[I*b, Int[x/(c - I*d - c*E^(2*I*a + 2*I*
b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \tanh^{-1}(1+id+d\cot(a+bx))\,dx = x\tanh^{-1}(1+id+d\cot(a+bx)) + (ib)\int \frac{x}{1+(-1-id)e^{2ia+2ibx}}\,dx$$
$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1+id+d\cot(a+bx)) + (b(i-d))\int \frac{e^{2ia+2ibx}x}{1+(-1-id)e^{2ia+2ibx}}\,dx$$
$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1+id+d\cot(a+bx)) - \frac{1}{2}x\log\left(1-(1+id)e^{2ia+2ibx}\right) + \frac{1}{2}\int$$
$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1+id+d\cot(a+bx)) - \frac{1}{2}x\log\left(1-(1+id)e^{2ia+2ibx}\right) - \frac{i\,\mathrm{Sub}}{}$$
$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1+id+d\cot(a+bx)) - \frac{1}{2}x\log\left(1-(1+id)e^{2ia+2ibx}\right) + \frac{i\mathrm{Li}_2}{}$$

Mathematica [B] time = 36.436, size = 709, normalized size = 7.62

$$\frac{x\csc^2(a+bx)(\cos(bx)-i\sin(bx))(\cos(bx)+i\sin(bx))\left(i\mathrm{PolyLog}\left(2,\frac{(\cos(a)-}{}\right.\right.}{(\cot(a+bx)+i)(d\cot(a+bx)+id+2)\left(\frac{(d-2i)\cos(a+bx)(\log(1-i\tan(bx))-\log(1+i\tan(bx)))}{d\cos(a+bx)+(2+id)\sin(a+bx)}+\frac{d\sin(a+bx)(\log(1-i\tan(bx))-\log(1+i\tan(b}{(d-2i)\sin(a+bx)-id\cos(a+bx)}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[1 + I*d + d*Cot[a + b*x]],x]
```

```
[Out] x*ArcTanh[1 + I*d + d*Cot[a + b*x]] + (x*Csc[a + b*x]^2*(2*b*x*Log[2*Cos[b*
x]*(Cos[b*x] - I*Sin[b*x])] + I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*(d*Cos[a
+ b*x] + (2 + I*d)*Sin[a + b*x]))/(2*(-I + d))]*Log[1 - I*Tan[b*x]] - I*Log
[(Sec[b*x]*((-I)*Cos[a] + Sin[a])*(d*Cos[a + b*x] + (2 + I*d)*Sin[a + b*x])
)/2]*Log[1 + I*Tan[b*x]] + I*PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] - I*Pol
yLog[2, (Sec[b*x]*((2 + I*d)*Cos[a] - d*Sin[a])*(Cos[a + b*x] + I*Sin[a + b
*x]))/2] + I*PolyLog[2, ((Cos[a] - I*Sin[a])*((-2 - I*d)*Cos[a] + d*Sin[a])
*(I + Tan[b*x]))/(2*(-I + d))])*(Cos[b*x] - I*Sin[b*x])*(Cos[b*x] + I*Sin[b
*x]))/((I + Cot[a + b*x])*(2 + I*d + d*Cot[a + b*x])*((2*I)*b*x + Log[1 + (
Sec[b*x]*((-2 - I*d)*Cos[a] + d*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2]
 - Log[(Sec[b*x]*((-I)*Cos[a] + Sin[a])*(d*Cos[a + b*x] + (2 + I*d)*Sin[a +
 b*x]))/2] + ((-2*I + d)*Cos[a + b*x]*(Log[1 - I*Tan[b*x]] - Log[1 + I*Tan[
b*x]]))/(d*Cos[a + b*x] + (2 + I*d)*Sin[a + b*x]) + (d*(Log[1 - I*Tan[b*x]]
 - Log[1 + I*Tan[b*x]])*Sin[a + b*x])/((-I)*d*Cos[a + b*x] + (-2*I + d)*Sin
[a + b*x]) + 2*b*x*Tan[b*x] - I*Log[1 + (Sec[b*x]*((-2 - I*d)*Cos[a] + d*Si
n[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2]*Tan[b*x] + I*Log[(Sec[b*x]*((-I)*
Cos[a] + Sin[a])*(d*Cos[a + b*x] + (2 + I*d)*Sin[a + b*x]))/2]*Tan[b*x] - I
*Log[1 - I*Tan[b*x]]*Tan[b*x] + I*Log[1 + I*Tan[b*x]]*Tan[b*x]))
```

Maple [B] time = 0.124, size = 299, normalized size = 3.2

$$\frac{-\frac{i}{2}\mathrm{Artanh}\left(1+id+d\cot\left(bx+a\right)\right)\ln\left(id+d\cot\left(bx+a\right)\right)}{b}+\frac{\frac{i}{2}\mathrm{Artanh}\left(1+id+d\cot\left(bx+a\right)\right)\ln\left(id-d\cot\left(bx+a\right)\right)}{b}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(1+I*d+d*cot(b*x+a)),x)
```

```
[Out] -1/2*I/b*arctanh(1+I*d+d*cot(b*x+a))*ln(I*d+d*cot(b*x+a))+1/2*I/b*arctanh(1
+I*d+d*cot(b*x+a))*ln(I*d-d*cot(b*x+a))+1/4*I/b*dilog(1/2*I*(-I*d-d*cot(b*x
+a))/d)+1/4*I/b*ln(I*d-d*cot(b*x+a))*ln(1/2*I*(-I*d-d*cot(b*x+a))/d)-1/4*I/
b*dilog((-2-I*d-d*cot(b*x+a))/(-2*I*d-2))-1/4*I/b*ln(I*d-d*cot(b*x+a))*ln((
-2-I*d-d*cot(b*x+a))/(-2*I*d-2))-1/8*I/b*ln(I*d+d*cot(b*x+a))^2+1/4*I/b*dil
og(1+1/2*I*d+1/2*d*cot(b*x+a))+1/4*I/b*ln(I*d+d*cot(b*x+a))*ln(1+1/2*I*d+1/
2*d*cot(b*x+a))
```

Maxima [B] time = 1.53743, size = 389, normalized size = 4.18

$$-\frac{4\,(bx+a)d\left(\frac{\log((i\,d+2)\tan(bx+a)+d)}{d}-\frac{\log(i\,\tan(bx+a)+1)}{d}\right)-d\left(\frac{2i\left(\log((i\,d+2)\tan(bx+a)+d)\log\left(\frac{(d-2i)\tan(bx+a)-i\,d}{2i\,d+2}+1\right)+\mathrm{Li}_2\left(-\frac{(d-2i)\tan(bx+a)-i\,d}{2i\,d+2}\right.\right.}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*(b*x + a)*d*(log((I*d + 2)*tan(b*x + a) + d)/d - log(I*tan(b*x + a)
 + 1)/d) - d*(2*I*(log((I*d + 2)*tan(b*x + a) + d)*log(((d - 2*I)*tan(b*x +
 a) - I*d)/(2*I*d + 2) + 1) + dilog(-((d - 2*I)*tan(b*x + a) - I*d)/(2*I*d
+ 2)))/d + 2*I*(log(1/2*(d - 2*I)*tan(b*x + a) - 1/2*I*d)*log(I*tan(b*x + a
) + 1) + dilog(-1/2*(d - 2*I)*tan(b*x + a) + 1/2*I*d + 1))/d - (2*I*log((I*
d + 2)*tan(b*x + a) + d)*log(I*tan(b*x + a) + 1) - I*log(I*tan(b*x + a) + 1
)^2)/d - 2*I*(log(I*tan(b*x + a) + 1)*log(-1/2*I*tan(b*x + a) + 1/2) + dilo
g(1/2*I*tan(b*x + a) + 1/2))/d) - 8*(b*x + a)*arctanh(I*d + d/tan(b*x + a)
+ 1))/b
```

Fricas [A] time = 1.82227, size = 340, normalized size = 3.66

$$\frac{2i\,b^2x^2+2\,bx\log\left(-\frac{\left((d-i)e^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{d}\right)-2i\,a^2-2\,(bx+a)\log\left((-i\,d-1)e^{(2i\,bx+2i\,a)}+1\right)+2\,a\log\left(\frac{(d-i)e^{(2i\,bx+2i\,a)}+i}{d-i}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*I*b^2*x^2 + 2*b*x*log(-((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x
 - 2*I*a)/d) - 2*I*a^2 - 2*(b*x + a)*log((-I*d - 1)*e^(2*I*b*x + 2*I*a) + 1
) + 2*a*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)/(d - I)) + I*dilog(-(-I*d - 1
)*e^(2*I*b*x + 2*I*a)))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(1+I*d+d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{artanh}\left(d\cot\left(bx+a\right)+i\,d+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*cot(b*x + a) + I*d + 1), x)
```

3.341 $\int \frac{\tanh^{-1}(1+id+d\cot(a+bx))}{x}\,dx$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(\frac{\tanh^{-1}(d\cot(a+bx)+id+1)}{x},x\right)$$

```
[Out] CannotIntegrate[ArcTanh[1 + I*d + d*Cot[a + b*x]]/x, x]
```

Rubi [A] time = 0.0858041, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\tanh^{-1}(1+id+d\cot(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTanh[1 + I*d + d*Cot[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTanh[1 + I*d + d*Cot[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(1+id+d\cot(a+bx))}{x}\,dx = \int \frac{\tanh^{-1}(1+id+d\cot(a+bx))}{x}\,dx$$

Mathematica [A] time = 0.954385, size = 0, normalized size = 0.

$$\int \frac{\tanh^{-1}(1+id+d\cot(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTanh[1 + I*d + d*Cot[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcTanh[1 + I*d + d*Cot[a + b*x]]/x, x]
```

Maple [A] time = 0.395, size = 0, normalized size = 0.

$$\int \frac{\operatorname{Artanh}\left(1+id+d\cot\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(1+I*d+d*cot(b*x+a))/x,x)
```

```
[Out] int(arctanh(1+I*d+d*cot(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-i\,bx+\frac{1}{4}\left(-i\,\pi-4i\,a-2\,\log\left(-d\right)\right)\log\left(x\right)+\frac{1}{2}i\int\frac{\arctan\left(d\cos\left(2\,bx+2\,a\right)+\sin\left(2\,bx+2\,a\right),-d\sin\left(2\,bx+2\,a\right)+\cos\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+I*d+d*cot(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -I*b*x + 1/4*(-I*pi - 4*I*a - 2*log(-d))*log(x) + 1/2*I*integrate(arctan2(d
*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + cos(2*b*x + 2*a
) - 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*s
in(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\log\left(-\frac{\left((d-i)e^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{d}\right)}{2\,x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+I*d+d*cot(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*log(-((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)/d)
/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(1+I*d+d*cot(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}\left(d\cot\left(bx+a\right)+i\,d+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+I*d+d*cot(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*cot(b*x + a) + I*d + 1)/x, x)
```

3.342 $\int x^2 \tanh^{-1}(1 - id - d\cot(a + bx))\,dx$

Optimal. Leaf size=169

$$-\frac{x\text{PolyLog}\left(3,(1-id)e^{2ia+2ibx}\right)}{4b^2}-\frac{i\text{PolyLog}\left(4,(1-id)e^{2ia+2ibx}\right)}{8b^3}+\frac{ix^2\text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{6}x^3\log\left(1-(1-\right.$$

```
[Out] (I/12)*b*x^4 + (x^3*ArcTanh[1 - I*d - d*Cot[a + b*x]])/3 - (x^3*Log[1 - (1
- I*d)*E^((2*I)*a + (2*I)*b*x)])/6 + ((I/4)*x^2*PolyLog[2, (1 - I*d)*E^((2*
I)*a + (2*I)*b*x)])/b - (x*PolyLog[3, (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/(
4*b^2) - ((I/8)*PolyLog[4, (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/b^3
```

Rubi [A] time = 0.295491, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6265, 2184, 2190, 2531, 6609, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(3,(1-id)e^{2ia+2ibx}\right)}{4b^2}-\frac{i\text{PolyLog}\left(4,(1-id)e^{2ia+2ibx}\right)}{8b^3}+\frac{ix^2\text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{6}x^3\log\left(1-(1-\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[1 - I*d - d*Cot[a + b*x]],x]
```

```
[Out] (I/12)*b*x^4 + (x^3*ArcTanh[1 - I*d - d*Cot[a + b*x]])/3 - (x^3*Log[1 - (1
- I*d)*E^((2*I)*a + (2*I)*b*x)])/6 + ((I/4)*x^2*PolyLog[2, (1 - I*d)*E^((2*
I)*a + (2*I)*b*x)])/b - (x*PolyLog[3, (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/(
4*b^2) - ((I/8)*PolyLog[4, (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/b^3
```

Rule 6265

```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Cot[a + b*x]])/(f*(m
 + 1)), x] + Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c - I*d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
```

```
x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \tanh^{-1}(1-id-d\cot(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(1-id-d\cot(a+bx)) + \frac{1}{3}(ib)\int \frac{x^3}{1+(-1+id)e^{2ia+2ibx}}\,dx$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id-d\cot(a+bx)) + \frac{1}{3}(b(i+d))\int \frac{e^{2ia+2ibx}x^3}{1+(-1+id)e^{2ia+}}$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id-d\cot(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-id)e^{2ia+2ibx}\right) +$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id-d\cot(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-id)e^{2ia+2ibx}\right) +$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id-d\cot(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-id)e^{2ia+2ibx}\right) +$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id-d\cot(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-id)e^{2ia+2ibx}\right) +$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id-d\cot(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-id)e^{2ia+2ibx}\right) +$$

Mathematica [A] time = 0.404761, size = 155, normalized size = 0.92

$$\frac{1}{3}x^3\tanh^{-1}(d(-\cot(a+bx))-id+1) - \frac{6ib^2x^2\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{d+i}\right) + 6bx\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{d+i}\right) - 3i\text{PolyLog}\left(4,\frac{ie^{-2i(a}}{d+}\right.}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[1 - I*d - d*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[1 - I*d - d*Cot[a + b*x]])/3 - (4*b^3*x^3*Log[1 + 1/((-1 + I*d
)*E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, I/((I + d)*E^((2*I)*(a +
 b*x)))] + 6*b*x*PolyLog[3, I/((I + d)*E^((2*I)*(a + b*x)))] - (3*I)*PolyLo
g[4, I/((I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

Maple [C] time = 18.589, size = 2346, normalized size = 13.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*arctanh(-1+I*d+d*cot(b*x+a)),x)
```

```
[Out] 1/2*I/b^3*a^2*d/(I+d)*dilog(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))+1/12*I*x^3*
Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^3+1/12*I*x^3*Pi*csgn(I*d/(
exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^3+1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+
a))-1)*exp(2*I*(b*x+a)))^2-1/2/b^3*a^3*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I+
d))^(1/2))+1/6/b^3*a^3*d/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)+
1/3/b^3*d/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*a^3-1/4/b^2*d/(I+d)*polylog(
3,-I*(I+d)*exp(2*I*(b*x+a)))*x-1/6*x^3*ln(d)+1/12*I*b*x^4-1/12*I*x^3*Pi*csg
n(I*d)*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-1/12*I*x^3*Pi*csgn
(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp
(2*I*(b*x+a)))^2-1/6*I*Pi*x^3+1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*cs
gn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^2-1/12
*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(
b*x+a))-1)*exp(2*I*(b*x+a)))^2-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(
I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^2+1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*
(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))
+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))^3-1/2*I/b^3*a^3/(I+d)*ln(1-I*exp(I*
(b*x+a))*(I*(I+d))^(1/2))-1/8*I/b^3*d/(I+d)*polylog(4,-I*(I+d)*exp(2*I*(b*x
+a)))-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))/(e
xp(2*I*(b*x+a))-1))^2+1/12*I*x^3*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I
*(b*x+a)))-1/6*I*x^3*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2+1
/6*I/b^3*a^3/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)+1/3*I/b^3/(I
+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*a^3-1/4*I/b^2/(I+d)*polylog(3,-I*(I+d)*e
xp(2*I*(b*x+a)))*x-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(I*exp
(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*
x+a))*d-I)/(exp(2*I*(b*x+a))-1))-1/3*x^3*ln(exp(I*(b*x+a)))-1/6*I/(I+d)*ln(
1+I*(I+d)*exp(2*I*(b*x+a)))*x^3-1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+ex
p(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^3+1/6*x^3*ln(I*exp(2*I*(b*x+a))+e
xp(2*I*(b*x+a))*d-I)+1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+
a))*d-I))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a)
)-1))^2-1/2*I/b^3*a^3/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/12*I*x
^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1)
)*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))+1/12
*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a)
)-1))*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^
2-1/2/b^3*a^3*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/2*I/b^2*a^2/
(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x+1/12*I*x^3*Pi*csgn(I*exp(2*I
*(b*x+a)))*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b
*x+a))-1))-1/2*I/b^2*a^2/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x+1/8
/b^3/(I+d)*polylog(4,-I*(I+d)*exp(2*I*(b*x+a)))-1/12*I*x^3*Pi*csgn((I*exp(2
*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^3-1/2/b^3*a^2/(I+d)
*dilog(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/2/b^3*a^2/(I+d)*dilog(1-I*exp(
I*(b*x+a))*(I*(I+d))^(1/2))-1/4/b/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+a))
)*x^2+1/4/b^3/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*a^2-1/6*d/(I+d)*ln
(1+I*(I+d)*exp(2*I*(b*x+a)))*x^3+1/2*I/b^2/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+
a)))*x*a^2+1/4*I/b*d/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*x^2-1/4*I/b
```

```
^3*d/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*a^2+1/12*I*x^3*Pi*csgn(I*d)
*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))*csgn(I*d/(exp(2*I*(b*x+a))-1
)*exp(2*I*(b*x+a)))-1/2/b^2*a^2*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/
2))*x-1/2/b^2*a^2*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x+1/2/b^2*
d/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*x*a^2+1/2*I/b^3*a^2*d/(I+d)*dilog(1-
I*exp(I*(b*x+a))*(I*(I+d))^(1/2))+1/12*I*x^3*Pi*csgn((I*exp(2*I*(b*x+a))+ex
p(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^2-1/12*I*x^3*Pi*csgn(d/(exp(2*I*(
b*x+a))-1)*exp(2*I*(b*x+a)))^3
```

Maxima [B] time = 1.18004, size = 463, normalized size = 2.74

$$-\frac{\frac{12\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\operatorname{artanh}(d\cot(bx+a)+i\,d-1)}{b^2}+\frac{-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2+\left(-8i\,(bx+a)^3+18i\,(bx+a)^2a-18i\,(bx+a)a^2\right)\operatorname{arct}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctanh(d*cot(b
*x + a) + I*d - 1)/b^2 + (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
 + a)^2*a^2 + (-8*I*(b*x + a)^3 + 18*I*(b*x + a)^2*a - 18*I*(b*x + a)*a^2)*
arctan2(-d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - cos(2
*b*x + 2*a) + 1) + (-12*I*(b*x + a)^2 + 18*I*(b*x + a)*a - 9*I*a^2)*dilog((
-I*d + 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(b*x
+ a)*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^2 -
 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3,
(-I*d + 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (-I*d + 1)*e^(2*I*b*x + 2*
I*a)))/b^2)/b
```

Fricas [C] time = 1.93778, size = 505, normalized size = 2.99

$$\frac{2i\,b^4x^4-4\,b^3x^3\log\left(-\frac{de^{(2i\,bx+2i\,a)}}{(d+i)e^{(2i\,bx+2i\,a)}-i}\right)+6i\,b^2x^2\operatorname{Li}_2\left(-(i\,d-1)e^{(2i\,bx+2i\,a)}\right)-2i\,a^4+4\,a^3\log\left(\frac{(d+i)e^{(2i\,bx+2i\,a)}-i}{d+i}\right)-6\,bx\operatorname{polylog}}{24\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/24*(2*I*b^4*x^4 - 4*b^3*x^3*log(-d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b*
x + 2*I*a) - I)) + 6*I*b^2*x^2*dilog(-(I*d - 1)*e^(2*I*b*x + 2*I*a)) - 2*I*
```

```
a^4 + 4*a^3*log(((d + I)*e^(2*I*b*x + 2*I*a) - I)/(d + I)) - 6*b*x*polylog(
3, (-I*d + 1)*e^(2*I*b*x + 2*I*a)) - 4*(b^3*x^3 + a^3)*log((I*d - 1)*e^(2*I
*b*x + 2*I*a) + 1) - 3*I*polylog(4, (-I*d + 1)*e^(2*I*b*x + 2*I*a)))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**2*atanh(-1+I*d+d*cot(b*x+a)),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -x^2 \operatorname{artanh}(d \cot(bx + a) + i d - 1) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="giac")

[Out] integrate(-x^2*arctanh(d*cot(b*x + a) + I*d - 1), x)
```

3.343 $\int x \tanh^{-1}(1 - id - d \cot(a + bx)) \, dx$

Optimal. Leaf size=133

$$-\frac{\text{PolyLog}\left(3,(1-id)e^{2ia+2ibx}\right)}{8b^2}+\frac{ix\text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(1-id)e^{2ia+2ibx}\right)+\frac{1}{2}x^2\tanh^{-1}(d(-\cot($$

```
[Out] (I/6)*b*x^3 + (x^2*ArcTanh[1 - I*d - d*Cot[a + b*x]])/2 - (x^2*Log[1 - (1 -
 I*d)*E^((2*I)*a + (2*I)*b*x)])/4 + ((I/4)*x*PolyLog[2, (1 - I*d)*E^((2*I)*
a + (2*I)*b*x)])/b - PolyLog[3, (1 - I*d)*E^((2*I)*a + (2*I)*b*x)]/(8*b^2)
```

Rubi [A] time = 0.249908, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6265, 2184, 2190, 2531, 2282, 6589}

$$-\frac{\text{PolyLog}\left(3,(1-id)e^{2ia+2ibx}\right)}{8b^2}+\frac{ix\text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(1-id)e^{2ia+2ibx}\right)+\frac{1}{2}x^2\tanh^{-1}(d(-\cot($$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[1 - I*d - d*Cot[a + b*x]],x]
```

```
[Out] (I/6)*b*x^3 + (x^2*ArcTanh[1 - I*d - d*Cot[a + b*x]])/2 - (x^2*Log[1 - (1 -
 I*d)*E^((2*I)*a + (2*I)*b*x)])/4 + ((I/4)*x*PolyLog[2, (1 - I*d)*E^((2*I)*
a + (2*I)*b*x)])/b - PolyLog[3, (1 - I*d)*E^((2*I)*a + (2*I)*b*x)]/(8*b^2)
```

Rule 6265

```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Cot[a + b*x]])/(f*(m
 + 1)), x] + Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c - I*d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(1-id-d\cot(a+bx))\,dx = \frac{1}{2}x^2\tanh^{-1}(1-id-d\cot(a+bx))+\frac{1}{2}(ib)\int\frac{x^2}{1+(-1+id)e^{2ia+2ibx}}\,dx$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id-d\cot(a+bx))+\frac{1}{2}(b(i+d))\int\frac{e^{2ia+2ibx}x^2}{1+(-1+id)e^{2ia+2i}}$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id-d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1-id)e^{2ia+2ibx}\right)+\frac{1}{2}$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id-d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1-id)e^{2ia+2ibx}\right)+\frac{ix}{}$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id-d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1-id)e^{2ia+2ibx}\right)+\frac{ix}{}$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id-d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1-id)e^{2ia+2ibx}\right)+\frac{ix}{}$$

Mathematica [A] time = 0.29709, size = 119, normalized size = 0.89

$$\frac{1}{2}x^2\tanh^{-1}(d(-\cot(a+bx))-id+1)-\frac{2ibx\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{d+i}\right)+\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{d+i}\right)+2b^2x^2\log\left(1+\frac{e^{-2i(a+bx)}}{-1+id}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[1 - I*d - d*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[1 - I*d - d*Cot[a + b*x]])/2 - (2*b^2*x^2*Log[1 + 1/((-1 + I*d
)*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/((I + d)*E^((2*I)*(a + b*x
)))] + PolyLog[3, I/((I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

Maple [C] time = 10.754, size = 2256, normalized size = 17.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*arctanh(-1+I*d+d*cot(b*x+a)),x)
```

```
[Out] 1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^3+1/8*I*x^2*Pi*c
sgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^3+1/2*I/b^2*a^2/(I+d)*ln(1+I
```

exp(I(b*x+a))*(I*(I+d))^(1/2))+1/2*I/b^2*a^2/(I+d)*ln(1-I*exp(I*(b*x+a))*
(I*(I+d))^(1/2))-1/4*I/b^2*a^2/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))
*d-I)-1/8/b^2*d/(I+d)*polylog(3,-I*(I+d)*exp(2*I*(b*x+a)))-1/4/b^2/(I+d)*po
lylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*a+1/2/b^2*a/(I+d)*dilog(1+I*exp(I*(b*x+a
))*(I*(I+d))^(1/2))-1/4*d/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*x^2+1/2/b^2*
a/(I+d)*dilog(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))+1/4*x^2*ln(I*exp(2*I*(b*x
+a))+exp(2*I*(b*x+a))*d-I)+1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*
(b*x+a))*d-I))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b
*x+a))-1))^2+1/6*I*b*x^3+1/2/b^2*a^2*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I+d)
)^(1/2))-1/4*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2-1/8
*I*x^2*Pi*csgn(I*d)*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-1/8*I
*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))*csgn(I*d/(exp(2*I*(b*
x+a))-1)*exp(2*I*(b*x+a)))^2-1/4/b^2*d/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))
*a^2+1/2/b^2*a^2*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/8*I*x^2*P
i*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^3-1/4*x^2*ln(d)+1/8*I*x^2*P
i*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))-1/4*I*Pi*x^2+1/8*I*x^2*
Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))*c
sgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^2+1/8*I
*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*
x+a))*d-I)/(exp(2*I*(b*x+a))-1))^2-1/8*I/b^2/(I+d)*polylog(3,-I*(I+d)*exp(2
I(b*x+a)))-1/4*I/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*x^2-1/8*I*x^2*Pi*cs
gn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^2-1/8*
I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x
+a))-1))^2-1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csg
n(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+1/8*I*x^2*Pi*csgn(I*d/(exp(2*I
*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a))
)-1/4/b^2*a^2*d/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)+1/8*I*x^2
*Pi*csgn(I*exp(2*I*(b*x+a)))^3-1/4/b/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+
a)))*x-1/8*I*x^2*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I
*(b*x+a))-1))^3-1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-
I)/(exp(2*I*(b*x+a))-1))^3-1/2*x^2*ln(exp(I*(b*x+a)))-1/2*I/b^2*a*d/(I+d)*d
ilog(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/4*I/b^2/(I+d)*ln(1+I*(I+d)*exp(2
I(b*x+a)))*a^2-1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d
-I)/(exp(2*I*(b*x+a))-1))*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(e
xp(2*I*(b*x+a))-1))+1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a
)))^2+1/8*I*x^2*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*
(b*x+a))-1))^2+1/8*I*x^2*Pi*csgn(I*d)*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x
+a))-1))*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))-1/2*I/b/(I+d)*ln(1
+I*(I+d)*exp(2*I*(b*x+a)))*x*a+1/4*I/b*d/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(
b*x+a)))*x+1/4*I/b^2*d/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*a-1/2/b*d
/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*x*a+1/2/b*a*d/(I+d)*ln(1+I*exp(I*(b*x
+a))*(I*(I+d))^(1/2))*x+1/2/b*a*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/
2))*x-1/2*I/b^2*a*d/(I+d)*dilog(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))+1/2*I/b
*a/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x+1/2*I/b*a/(I+d)*ln(1-I*ex
p(I*(b*x+a))*(I*(I+d))^(1/2))*x-1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*c

```
sgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I))*csgn(I*(I*exp(2*I*(b*x+a))
+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b
*x+a)))*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+
a))-1))
```

Maxima [B] time = 1.10368, size = 336, normalized size = 2.53

$$-\frac{\frac{12\left((bx+a)^2-2(bx+a)a\right)\operatorname{artanh}(d\cot(bx+a)+id-1)}{b}+\frac{-4i(bx+a)^3+12i(bx+a)^2a-6ibx\operatorname{Li}_2\left((-id+1)e^{(2ibx+2ia)}\right)+\left(-6i(bx+a)^2+12i(bx+a)a\right)\arctan(-d\cos(2b}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arctanh(d*cot(b*x + a) + I*d - 1)/b
 + (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((-I*d + 1)*e^(2*I
*b*x + 2*I*a)) + (-6*I*(b*x + a)^2 + 12*I*(b*x + a)*a)*arctan2(-d*cos(2*b*x
 + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1) + 3
*((b*x + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)
*sin(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) + 3*po
lylog(3, (-I*d + 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

Fricas [C] time = 2.14886, size = 428, normalized size = 3.22

$$\frac{4ib^3x^3-6b^2x^2\log\left(-\frac{de^{(2ibx+2ia)}}{(d+i)e^{(2ibx+2ia)}-i}\right)+4ia^3+6ibx\operatorname{Li}_2\left(-(id-1)e^{(2ibx+2ia)}\right)-6a^2\log\left(\frac{(d+i)e^{(2ibx+2ia)}-i}{d+i}\right)-6\left(b^2x^2-a^2\right)\mathrm{lo}}{24b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/24*(4*I*b^3*x^3 - 6*b^2*x^2*log(-d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b*
x + 2*I*a) - I)) + 4*I*a^3 + 6*I*b*x*dilog(-(I*d - 1)*e^(2*I*b*x + 2*I*a))
- 6*a^2*log(((d + I)*e^(2*I*b*x + 2*I*a) - I)/(d + I)) - 6*(b^2*x^2 - a^2)*
log((I*d - 1)*e^(2*I*b*x + 2*I*a) + 1) - 3*polylog(3, (-I*d + 1)*e^(2*I*b*x
 + 2*I*a)))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*atanh(-1+I*d+d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -x \operatorname{artanh}\left(d\cot\left(bx+a\right)+i\,d-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x*arctanh(d*cot(b*x + a) + I*d - 1), x)
```

3.344 $\int \tanh^{-1}(1 - id - d\cot(a + bx))\,dx$

Optimal. Leaf size=94

$$\frac{i\text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{2}x\log\left(1-(1-id)e^{2ia+2ibx}\right)+x\tanh^{-1}(d(-\cot(a+bx))-id+1)+\frac{1}{2}ibx^2$$

```
[Out] (I/2)*b*x^2 + x*ArcTanh[1 - I*d - d*Cot[a + b*x]] - (x*Log[1 - (1 - I*d)*E^
((2*I)*a + (2*I)*b*x)])/2 + ((I/4)*PolyLog[2, (1 - I*d)*E^((2*I)*a + (2*I)*
b*x)])/b
```

Rubi [A] time = 0.1492, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6257, 2184, 2190, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{2}x\log\left(1-(1-id)e^{2ia+2ibx}\right)+x\tanh^{-1}(d(-\cot(a+bx))-id+1)+\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[1 - I*d - d*Cot[a + b*x]],x]
```

```
[Out] (I/2)*b*x^2 + x*ArcTanh[1 - I*d - d*Cot[a + b*x]] - (x*Log[1 - (1 - I*d)*E^
((2*I)*a + (2*I)*b*x)])/2 + ((I/4)*PolyLog[2, (1 - I*d)*E^((2*I)*a + (2*I)*
b*x)])/b
```

Rule 6257

```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Tanh[c + d*Cot[a + b*x]], x] + Dist[I*b, Int[x/(c - I*d - c*E^(2*I*a + 2*I*
b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, 1]
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \tanh^{-1}(1-id-d\cot(a+bx))\,dx = x\tanh^{-1}(1-id-d\cot(a+bx)) + (ib)\int \frac{x}{1+(-1+id)e^{2ia+2ibx}}\,dx$$
$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1-id-d\cot(a+bx)) + (b(i+d))\int \frac{e^{2ia+2ibx}x}{1+(-1+id)e^{2ia+2ibx}}\,dx$$
$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1-id-d\cot(a+bx)) - \frac{1}{2}x\log\left(1-(1-id)e^{2ia+2ibx}\right) + \frac{1}{2}\int \mathrm{l}$$
$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1-id-d\cot(a+bx)) - \frac{1}{2}x\log\left(1-(1-id)e^{2ia+2ibx}\right) - \frac{i\,\mathrm{Sub}}{}$$
$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1-id-d\cot(a+bx)) - \frac{1}{2}x\log\left(1-(1-id)e^{2ia+2ibx}\right) + \frac{i\mathrm{Li}_2\big(}{}$$

Mathematica [B] time = 27.1782, size = 605, normalized size = 6.44

$$\frac{x\csc^2(a+bx)(\cos(bx)-i\sin(bx))(\cos(bx)+i\sin(bx))\left(i\mathrm{PolyLog}\left(2,\frac{(\cos(a)-i\sin(a))(\tan(bx)+i)(d\sin(a)+(2-id)\cos(a))}{2(d+i)}\right)-i\mathrm{Poly}\right.}{(\cot(a+bx)+i)(d\cot(a+bx)+id-2)\left(\frac{\sec^2(bx)\log\big(}{}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[1 - I*d - d*Cot[a + b*x]],x]
```

```
[Out] x*ArcTanh[1 - I*d - d*Cot[a + b*x]] + (x*Csc[a + b*x]^2*(2*b*x*Log[2*Cos[b*
x]*(Cos[b*x] - I*Sin[b*x])] + I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*(d*Cos[a
+ b*x] + I*(2*I + d)*Sin[a + b*x]))/(2*(I + d))]*Log[1 - I*Tan[b*x]] - I*Lo
g[(I*Sec[b*x]*(d*Cos[a + b*x] + I*(2*I + d)*Sin[a + b*x]))/(2*Cos[a] - (2*I
)*Sin[a])]*Log[1 + I*Tan[b*x]] + I*PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] -
 I*PolyLog[2, (Sec[b*x]*((2 - I*d)*Cos[a] + d*Sin[a])*(Cos[a + b*x] + I*Sin
[a + b*x]))/2] + I*PolyLog[2, ((Cos[a] - I*Sin[a])*((2 - I*d)*Cos[a] + d*Si
n[a])*(I + Tan[b*x]))/(2*(I + d))])*(Cos[b*x] - I*Sin[b*x])*(Cos[b*x] + I*S
in[b*x]))/((I + Cot[a + b*x])*(-2 + I*d + d*Cot[a + b*x])*(-((Log[1 - I*Tan
[b*x]]*Sec[b*x]*((2*I + d)*Cos[a] + I*d*Sin[a]))/(d*Cos[a + b*x] + I*(2*I +
 d)*Sin[a + b*x])) + (Log[1 + I*Tan[b*x]]*Sec[b*x]*((2*I + d)*Cos[a] + I*d*
Sin[a]))/(d*Cos[a + b*x] + I*(2*I + d)*Sin[a + b*x]) + (Log[(I*Sec[b*x]*(d*
Cos[a + b*x] + I*(2*I + d)*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*Sec[b*
x]^2)/(1 + I*Tan[b*x]) - 2*b*x*(I + Tan[b*x]) + I*Log[1 - (Sec[b*x]*((2 - I
*d)*Cos[a] + d*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2]*(I + Tan[b*x])))
```

Maple [B] time = 0.122, size = 335, normalized size = 3.6

$$\frac{\frac{i}{2}\mathrm{Artanh}\left(-1+id+d\cot(bx+a)\right)\ln\left(id+d\cot(bx+a)\right)}{b}-\frac{\frac{i}{2}\mathrm{Artanh}\left(-1+id+d\cot(bx+a)\right)\ln\left(id-d\cot(bx+a)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctanh(-1+I*d+d*cot(b*x+a)),x)
```

```
[Out] 1/2*I/b*arctanh(-1+I*d+d*cot(b*x+a))*ln(I*d+d*cot(b*x+a))-1/2*I/b*arctanh(-
1+I*d+d*cot(b*x+a))*ln(I*d-d*cot(b*x+a))+1/4*I/b*dilog(1/2*I*(-I*d-d*cot(b*
x+a))/d)+1/4*I/b*ln(I*d-d*cot(b*x+a))*ln(1/2*I*(-I*d-d*cot(b*x+a))/d)-1/4*I
/b*dilog((2-I*d-d*cot(b*x+a))/(-2*I*d+2))-1/4*I/b*ln(I*d-d*cot(b*x+a))*ln((
2-I*d-d*cot(b*x+a))/(-2*I*d+2))+1/4*I/b*ln(1-1/2*I*d-1/2*d*cot(b*x+a))*ln(I
*d+d*cot(b*x+a))-1/4*I/b*ln(1-1/2*I*d-1/2*d*cot(b*x+a))*ln(1/2*I*d+1/2*d*co
t(b*x+a))-1/4*I/b*dilog(1/2*I*d+1/2*d*cot(b*x+a))-1/8*I/b*ln(I*d+d*cot(b*x+
a))^2
```

Maxima [B] time = 1.51454, size = 386, normalized size = 4.11

$$-\frac{4(bx+a)d\left(\frac{\log((id-2)\tan(bx+a)+d)}{d}-\frac{\log(i\tan(bx+a)+1)}{d}\right)+d\left(-\frac{2i\left(\log((id-2)\tan(bx+a)+d)\log\left(\frac{(d+2i)\tan(bx+a)-id}{2id-2}+1\right)+\mathrm{Li}_2\left(-\frac{(d+2i)\tan(bx+a)-id}{2id-2}\right)\right)}{d}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*(b*x + a)*d*(log((I*d - 2)*tan(b*x + a) + d)/d - log(I*tan(b*x + a)
 + 1)/d) + d*(-2*I*(log((I*d - 2)*tan(b*x + a) + d)*log(((d + 2*I)*tan(b*x
+ a) - I*d)/(2*I*d - 2) + 1) + dilog(-((d + 2*I)*tan(b*x + a) - I*d)/(2*I*d
 - 2)))/d - 2*I*(log(-1/2*(d + 2*I)*tan(b*x + a) + 1/2*I*d)*log(I*tan(b*x +
 a) + 1) + dilog(1/2*(d + 2*I)*tan(b*x + a) - 1/2*I*d + 1))/d + (2*I*log((I
*d - 2)*tan(b*x + a) + d)*log(I*tan(b*x + a) + 1) - I*log(I*tan(b*x + a) +
1)^2)/d + 2*I*(log(I*tan(b*x + a) + 1)*log(-1/2*I*tan(b*x + a) + 1/2) + dil
og(1/2*I*tan(b*x + a) + 1/2))/d) + 8*(b*x + a)*arctanh(I*d + d/tan(b*x + a)
 - 1))/b
```

Fricas [A] time = 2.29961, size = 336, normalized size = 3.57

$$\frac{2ib^2x^2 - 2bx\log\left(-\frac{de^{(2ibx+2ia)}}{(d+i)e^{(2ibx+2ia)}-i}\right) - 2ia^2 - 2(bx+a)\log\left((id-1)e^{(2ibx+2ia)}+1\right) + 2a\log\left(\frac{(d+i)e^{(2ibx+2ia)}-i}{d+i}\right) + i\,\mathrm{Li}_2\left(-(i\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*I*b^2*x^2 - 2*b*x*log(-d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b*x + 2
*I*a) - I)) - 2*I*a^2 - 2*(b*x + a)*log((I*d - 1)*e^(2*I*b*x + 2*I*a) + 1)
+ 2*a*log(((d + I)*e^(2*I*b*x + 2*I*a) - I)/(d + I)) + I*dilog(-(I*d - 1)*e
^(2*I*b*x + 2*I*a)))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atanh(-1+I*d+d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\operatorname{artanh}\left(d\cot\left(bx+a\right)+i\,d-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(d*cot(b*x + a) + I*d - 1), x)
```

3.345 $\int \frac{\tanh^{-1}(1-id-d\cot(a+bx))}{x}\,dx$

Optimal. Leaf size=23

$$\text{CannotIntegrate}\left(\frac{\tanh^{-1}(d(-\cot(a+bx))-id+1)}{x},x\right)$$

```
[Out] CannotIntegrate[ArcTanh[1 - I*d - d*Cot[a + b*x]]/x, x]
```

Rubi [A] time = 0.0960844, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\tanh^{-1}(1-id-d\cot(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTanh[1 - I*d - d*Cot[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTanh[1 - I*d - d*Cot[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(1-id-d\cot(a+bx))}{x}\,dx=\int \frac{\tanh^{-1}(1-id-d\cot(a+bx))}{x}\,dx$$

Mathematica [A] time = 1.03994, size = 0, normalized size = 0.

$$\int \frac{\tanh^{-1}(1-id-d\cot(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTanh[1 - I*d - d*Cot[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcTanh[1 - I*d - d*Cot[a + b*x]]/x, x]
```

Maple [A] time = 0.387, size = 0, normalized size = 0.

$$\int -\frac{\operatorname{Artanh}\left(-1+id+d\cot\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctanh(-1+I*d+d*cot(b*x+a))/x,x)
```

```
[Out] int(-arctanh(-1+I*d+d*cot(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-i\,bx+\frac{1}{4}\left(i\,\pi-4i\,a-2\,\log\left(d\right)\right)\log\left(x\right)-\frac{1}{2}i\int\frac{\arctan\left(-d\cos\left(2\,bx+2\,a\right)+\sin\left(2\,bx+2\,a\right),-d\sin\left(2\,bx+2\,a\right)-\cos\left(2\right.\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+I*d+d*cot(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -I*b*x + 1/4*(I*pi - 4*I*a - 2*log(d))*log(x) - 1/2*I*integrate(arctan2(-d*
cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)
 + 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*si
n(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\log\left(-\frac{de^{\left(2i\,bx+2i\,a\right)}}{\left(d+i\right)e^{\left(2i\,bx+2i\,a\right)}-i}\right)}{2\,x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+I*d+d*cot(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(-1/2*log(-d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b*x + 2*I*a) - I))
/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atanh(-1+I*d+d*cot(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\operatorname{artanh}\left(d\cot\left(bx+a\right)+i\,d-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+I*d+d*cot(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(-arctanh(d*cot(b*x + a) + I*d - 1)/x, x)
```

3.346 $\int \tanh^{-1}(e^x)\,dx$

Optimal. Leaf size=21

$$\frac{1}{2}\text{PolyLog}(2,e^x)-\frac{1}{2}\text{PolyLog}(2,-e^x)$$

```
[Out] -PolyLog[2, -E^x]/2 + PolyLog[2, E^x]/2
```

Rubi [A] time = 0.0115109, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2282, 5912}

$$\frac{1}{2}\text{PolyLog}(2,e^x)-\frac{1}{2}\text{PolyLog}(2,-e^x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[E^x],x]
```

```
[Out] -PolyLog[2, -E^x]/2 + PolyLog[2, E^x]/2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \tanh^{-1}\left(e^x\right)\,dx = \text{Subst}\left(\int \frac{\tanh^{-1}(x)}{x}\,dx, x, e^x\right)$$
$$= -\frac{\text{Li}_2\left(-e^x\right)}{2} + \frac{\text{Li}_2\left(e^x\right)}{2}$$

Mathematica [B] time = 0.0349408, size = 51, normalized size = 2.43

$$-\frac{1}{2}\text{PolyLog}\left(2,-e^x\right)+\frac{1}{2}\text{PolyLog}\left(2,e^x\right)+\frac{1}{2}x\log\left(1-e^x\right)-\frac{1}{2}x\log\left(e^x+1\right)+x\tanh^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[E^x],x]
```

```
[Out] x*ArcTanh[E^x] + (x*Log[1 - E^x])/2 - (x*Log[1 + E^x])/2 - PolyLog[2, -E^x]
/2 + PolyLog[2, E^x]/2
```

Maple [A] time = 0.044, size = 31, normalized size = 1.5

$$\ln\left(e^x\right)\text{Artanh}\left(e^x\right)-\frac{\text{dilog}\left(e^x\right)}{2}-\frac{\text{dilog}\left(e^x+1\right)}{2}-\frac{\ln\left(e^x\right)\ln\left(e^x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(exp(x)),x)
```

```
[Out] ln(exp(x))*arctanh(exp(x))-1/2*dilog(exp(x))-1/2*dilog(exp(x)+1)-1/2*ln(exp
(x))*ln(exp(x)+1)
```

Maxima [B] time = 0.964469, size = 78, normalized size = 3.71

$$-\frac{1}{2}x(\log\left(e^x+1\right)-\log\left(e^x-1\right))+x\,\text{artanh}\left(e^x\right)+\frac{1}{2}\log\left(-e^x\right)\log\left(e^x+1\right)-\frac{1}{2}x\log\left(e^x-1\right)+\frac{1}{2}\text{Li}_2\left(e^x+1\right)-\frac{1}{2}\text{Li}_2\left(-e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(exp(x)),x, algorithm="maxima")
```

```
[Out] -1/2*x*(log(e^x + 1) - log(e^x - 1)) + x*arctanh(e^x) + 1/2*log(-e^x)*log(e
^x + 1) - 1/2*x*log(e^x - 1) + 1/2*dilog(e^x + 1) - 1/2*dilog(-e^x + 1)
```

Fricas [B] time = 1.58103, size = 263, normalized size = 12.52

$$\frac{1}{2}x\log\left(-\frac{\cosh(x)+\sinh(x)+1}{\cosh(x)+\sinh(x)-1}\right)-\frac{1}{2}x\log(\cosh(x)+\sinh(x)+1)+\frac{1}{2}x\log(-\cosh(x)-\sinh(x)+1)+\frac{1}{2}\operatorname{Li}_2(\cosh$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(exp(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x*log(-(cosh(x) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) - 1/2*x*log(cos
h(x) + sinh(x) + 1) + 1/2*x*log(-cosh(x) - sinh(x) + 1) + 1/2*dilog(cosh(x)
 + sinh(x)) - 1/2*dilog(-cosh(x) - sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{atanh}(e^x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(exp(x)),x)
```

```
[Out] Integral(atanh(exp(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{artanh}(e^x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(exp(x)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(e^x), x)
```

3.347 $\int x \tanh^{-1}(e^x)\, dx$

Optimal. Leaf size=43

$$-\frac{1}{2}x\text{PolyLog}(2,-e^x)+\frac{1}{2}x\text{PolyLog}(2,e^x)+\frac{1}{2}\text{PolyLog}(3,-e^x)-\frac{1}{2}\text{PolyLog}(3,e^x)$$

```
[Out] -(x*PolyLog[2, -E^x])/2 + (x*PolyLog[2, E^x])/2 + PolyLog[3, -E^x]/2 - Poly
Log[3, E^x]/2
```

Rubi [A] time = 0.0429293, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6213, 2531, 2282, 6589}

$$-\frac{1}{2}x\text{PolyLog}(2,-e^x)+\frac{1}{2}x\text{PolyLog}(2,e^x)+\frac{1}{2}\text{PolyLog}(3,-e^x)-\frac{1}{2}\text{PolyLog}(3,e^x)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[E^x],x]
```

```
[Out] -(x*PolyLog[2, -E^x])/2 + (x*PolyLog[2, E^x])/2 + PolyLog[3, -E^x]/2 - Poly
Log[3, E^x]/2
```

Rule 6213

```
Int[ArcTanh[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + a + b*f^(c + d*x)], x], x] - Dist[1/2, Int[x^m
*Log[1 - a - b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IGtQ[m,
 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x\tanh^{-1}(e^x)\,dx &= -\left(\frac{1}{2}\int x\log(1-e^x)\,dx\right)+\frac{1}{2}\int x\log(1+e^x)\,dx\\ &= -\frac{1}{2}x\text{Li}_2(-e^x)+\frac{x\text{Li}_2(e^x)}{2}+\frac{1}{2}\int \text{Li}_2(-e^x)\,dx-\frac{1}{2}\int \text{Li}_2(e^x)\,dx\\ &= -\frac{1}{2}x\text{Li}_2(-e^x)+\frac{x\text{Li}_2(e^x)}{2}+\frac{1}{2}\text{Subst}\left(\int\frac{\text{Li}_2(-x)}{x}\,dx,x,e^x\right)-\frac{1}{2}\text{Subst}\left(\int\frac{\text{Li}_2(x)}{x}\,dx,x,e^x\right)\\ &= -\frac{1}{2}x\text{Li}_2(-e^x)+\frac{x\text{Li}_2(e^x)}{2}+\frac{\text{Li}_3(-e^x)}{2}-\frac{\text{Li}_3(e^x)}{2}\end{aligned}$$

Mathematica [A] time = 0.0263989, size = 71, normalized size = 1.65

$$\frac{1}{4}\left(-2x\text{PolyLog}(2,-e^x)+2x\text{PolyLog}(2,e^x)+2\text{PolyLog}(3,-e^x)-2\text{PolyLog}(3,e^x)+x^2\log(1-e^x)-x^2\log(e^x+1)+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[E^x],x]
```

```
[Out] (2*x^2*ArcTanh[E^x] + x^2*Log[1 - E^x] - x^2*Log[1 + E^x] - 2*x*PolyLog[2,
-E^x] + 2*x*PolyLog[2, E^x] + 2*PolyLog[3, -E^x] - 2*PolyLog[3, E^x])/4
```

Maple [A] time = 0.036, size = 62, normalized size = 1.4

$$\frac{x^2\text{Artanh}(e^x)}{2}-\frac{x^2\ln(e^x+1)}{4}-\frac{x\text{polylog}(2,-e^x)}{2}+\frac{\text{polylog}(3,-e^x)}{2}+\frac{x^2\ln(1-e^x)}{4}+\frac{x\text{polylog}(2,e^x)}{2}-\frac{\text{polylog}(3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(exp(x)),x)
```

```
[Out] 1/2*x^2*arctanh(exp(x))-1/4*x^2*ln(exp(x)+1)-1/2*x*polylog(2,-exp(x))+1/2*p
olylog(3,-exp(x))+1/4*x^2*ln(1-exp(x))+1/2*x*polylog(2,exp(x))-1/2*polylog(
3,exp(x))
```

Maxima [B] time = 0.962817, size = 80, normalized size = 1.86

$$\frac{1}{2}x^2\operatorname{artanh}(e^x) - \frac{1}{4}x^2\log(e^x+1) + \frac{1}{4}x^2\log(-e^x+1) - \frac{1}{2}x\text{Li}_2(-e^x) + \frac{1}{2}x\text{Li}_2(e^x) + \frac{1}{2}\text{Li}_3(-e^x) - \frac{1}{2}\text{Li}_3(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(exp(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctanh(e^x) - 1/4*x^2*log(e^x + 1) + 1/4*x^2*log(-e^x + 1) - 1/2*x
*dilog(-e^x) + 1/2*x*dilog(e^x) + 1/2*polylog(3, -e^x) - 1/2*polylog(3, e^x
)
```

Fricas [C] time = 1.60316, size = 375, normalized size = 8.72

$$\frac{1}{4}x^2\log\left(-\frac{\cosh(x)+\sinh(x)+1}{\cosh(x)+\sinh(x)-1}\right) - \frac{1}{4}x^2\log(\cosh(x)+\sinh(x)+1) + \frac{1}{4}x^2\log(-\cosh(x)-\sinh(x)+1) + \frac{1}{2}x\text{Li}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(exp(x)),x, algorithm="fricas")
```

```
[Out] 1/4*x^2*log(-(cosh(x) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) - 1/4*x^2*log
(cosh(x) + sinh(x) + 1) + 1/4*x^2*log(-cosh(x) - sinh(x) + 1) + 1/2*x*dilog
(cosh(x) + sinh(x)) - 1/2*x*dilog(-cosh(x) - sinh(x)) - 1/2*polylog(3, cosh
(x) + sinh(x)) + 1/2*polylog(3, -cosh(x) - sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\operatorname{atanh}(e^x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(exp(x)),x)
```

```
[Out] Integral(x*atanh(exp(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{artanh}\left(e^{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(exp(x)),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(e^x), x)
```

3.348 $\int x^2 \tanh^{-1}(e^x) \, dx$

Optimal. Leaf size=58

$$-\frac{1}{2}x^2\text{PolyLog}(2,-e^x)+\frac{1}{2}x^2\text{PolyLog}(2,e^x)+x\text{PolyLog}(3,-e^x)-x\text{PolyLog}(3,e^x)-\text{PolyLog}(4,-e^x)+\text{PolyLog}($$

```
[Out] -(x^2*PolyLog[2, -E^x])/2 + (x^2*PolyLog[2, E^x])/2 + x*PolyLog[3, -E^x] -
x*PolyLog[3, E^x] - PolyLog[4, -E^x] + PolyLog[4, E^x]
```

Rubi [A] time = 0.0670158, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6213, 2531, 6609, 2282, 6589}

$$-\frac{1}{2}x^2\text{PolyLog}(2,-e^x)+\frac{1}{2}x^2\text{PolyLog}(2,e^x)+x\text{PolyLog}(3,-e^x)-x\text{PolyLog}(3,e^x)-\text{PolyLog}(4,-e^x)+\text{PolyLog}($$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[E^x],x]
```

```
[Out] -(x^2*PolyLog[2, -E^x])/2 + (x^2*PolyLog[2, E^x])/2 + x*PolyLog[3, -E^x] -
x*PolyLog[3, E^x] - PolyLog[4, -E^x] + PolyLog[4, E^x]
```

Rule 6213

```
Int[ArcTanh[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + a + b*f^(c + d*x)], x], x] - Dist[1/2, Int[x^m
*Log[1 - a - b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IGtQ[m,
 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
```

```
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x^2 \tanh^{-1}(e^x)\,dx &= -\left(\frac{1}{2}\int x^2\log(1-e^x)\,dx\right)+\frac{1}{2}\int x^2\log(1+e^x)\,dx\\
&= -\frac{1}{2}x^2\text{Li}_2(-e^x)+\frac{1}{2}x^2\text{Li}_2(e^x)+\int x\text{Li}_2(-e^x)\,dx-\int x\text{Li}_2(e^x)\,dx\\
&= -\frac{1}{2}x^2\text{Li}_2(-e^x)+\frac{1}{2}x^2\text{Li}_2(e^x)+x\text{Li}_3(-e^x)-x\text{Li}_3(e^x)-\int \text{Li}_3(-e^x)\,dx+\int \text{Li}_3(e^x)\,dx\\
&= -\frac{1}{2}x^2\text{Li}_2(-e^x)+\frac{1}{2}x^2\text{Li}_2(e^x)+x\text{Li}_3(-e^x)-x\text{Li}_3(e^x)-\text{Subst}\left(\int\frac{\text{Li}_3(-x)}{x}\,dx,x,e^x\right)+\text{Subst}\left(\int\right.\\
&= -\frac{1}{2}x^2\text{Li}_2(-e^x)+\frac{1}{2}x^2\text{Li}_2(e^x)+x\text{Li}_3(-e^x)-x\text{Li}_3(e^x)-\text{Li}_4(-e^x)+\text{Li}_4(e^x)
\end{aligned}$$

Mathematica [A] time = 0.0279609, size = 93, normalized size = 1.6

$$\frac{1}{6}\left(-3x^2\text{PolyLog}(2,-e^x)+3x^2\text{PolyLog}(2,e^x)+6x\text{PolyLog}(3,-e^x)-6x\text{PolyLog}(3,e^x)-6\text{PolyLog}(4,-e^x)+6\text{PolyI}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[E^x],x]
```

```
[Out] (2*x^3*ArcTanh[E^x] + x^3*Log[1 - E^x] - x^3*Log[1 + E^x] - 3*x^2*PolyLog[2
, -E^x] + 3*x^2*PolyLog[2, E^x] + 6*x*PolyLog[3, -E^x] - 6*x*PolyLog[3, E^x
```

```
] - 6*PolyLog[4, -E^x] + 6*PolyLog[4, E^x])/6
```

Maple [A] time = 0.036, size = 79, normalized size = 1.4

$$\frac{x^3\operatorname{Artanh}(\mathrm{e}^x)}{3}-\frac{x^3\ln(\mathrm{e}^x+1)}{6}-\frac{x^2\operatorname{polylog}(2,-\mathrm{e}^x)}{2}+x\operatorname{polylog}(3,-\mathrm{e}^x)-\operatorname{polylog}(4,-\mathrm{e}^x)+\frac{x^3\ln(1-\mathrm{e}^x)}{6}+\frac{x^2\operatorname{polyl}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(exp(x)),x)
```

```
[Out] 1/3*x^3*arctanh(exp(x))-1/6*x^3*ln(exp(x)+1)-1/2*x^2*polylog(2,-exp(x))+x*p
olylog(3,-exp(x))-polylog(4,-exp(x))+1/6*x^3*ln(1-exp(x))+1/2*x^2*polylog(2
,exp(x))-x*polylog(3,exp(x))+polylog(4,exp(x))
```

Maxima [A] time = 0.961828, size = 103, normalized size = 1.78

$$\frac{1}{3}x^3\operatorname{artanh}(e^x)-\frac{1}{6}x^3\log(e^x+1)+\frac{1}{6}x^3\log(-e^x+1)-\frac{1}{2}x^2\mathrm{Li}_2(-e^x)+\frac{1}{2}x^2\mathrm{Li}_2(e^x)+x\mathrm{Li}_3(-e^x)-x\mathrm{Li}_3(e^x)-\mathrm{Li}_4(-e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(exp(x)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctanh(e^x) - 1/6*x^3*log(e^x + 1) + 1/6*x^3*log(-e^x + 1) - 1/2*x
^2*dilog(-e^x) + 1/2*x^2*dilog(e^x) + x*polylog(3, -e^x) - x*polylog(3, e^x
) - polylog(4, -e^x) + polylog(4, e^x)
```

Fricas [C] time = 1.68578, size = 463, normalized size = 7.98

$$\frac{1}{6}x^3\log\left(-\frac{\cosh(x)+\sinh(x)+1}{\cosh(x)+\sinh(x)-1}\right)-\frac{1}{6}x^3\log(\cosh(x)+\sinh(x)+1)+\frac{1}{6}x^3\log(-\cosh(x)-\sinh(x)+1)+\frac{1}{2}x^2\mathrm{Li}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(exp(x)),x, algorithm="fricas")
```

```
[Out] 1/6*x^3*log(-(cosh(x) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) - 1/6*x^3*log
(cosh(x) + sinh(x) + 1) + 1/6*x^3*log(-cosh(x) - sinh(x) + 1) + 1/2*x^2*dil
og(cosh(x) + sinh(x)) - 1/2*x^2*dilog(-cosh(x) - sinh(x)) - x*polylog(3, co
sh(x) + sinh(x)) + x*polylog(3, -cosh(x) - sinh(x)) + polylog(4, cosh(x) +
sinh(x)) - polylog(4, -cosh(x) - sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{atanh}(e^x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(exp(x)),x)
```

```
[Out] Integral(x**2*atanh(exp(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{artanh}(e^x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(exp(x)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(e^x), x)
```

3.349 $\int \tanh^{-1}\left(e^{a+bx}\right) dx$

Optimal. Leaf size=35

$$\frac{\text{PolyLog}\left(2,e^{a+bx}\right)}{2b}-\frac{\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b}$$

```
[Out] -PolyLog[2, -E^(a + b*x)]/(2*b) + PolyLog[2, E^(a + b*x)]/(2*b)
```

Rubi [A] time = 0.0139707, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2282, 5912}

$$\frac{\text{PolyLog}\left(2,e^{a+bx}\right)}{2b}-\frac{\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[E^(a + b*x)],x]
```

```
[Out] -PolyLog[2, -E^(a + b*x)]/(2*b) + PolyLog[2, E^(a + b*x)]/(2*b)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \tanh^{-1}\left(e^{a+bx}\right) dx = \frac{\text{Subst}\left(\int \frac{\tanh^{-1}(x)}{x} dx, x, e^{a+bx}\right)}{b}$$
$$= -\frac{\text{Li}_2\left(-e^{a+bx}\right)}{2b} + \frac{\text{Li}_2\left(e^{a+bx}\right)}{2b}$$

Mathematica [A] time = 0.0773172, size = 68, normalized size = 1.94

$$\frac{-\text{PolyLog}\left(2,-e^{a+bx}\right)+\text{PolyLog}\left(2,e^{a+bx}\right)+bx\left(\log\left(1-e^{a+bx}\right)-\log\left(e^{a+bx}+1\right)+2\tanh^{-1}\left(e^{a+bx}\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[E^(a + b*x)],x]
```

```
[Out] (b*x*(2*ArcTanh[E^(a + b*x)] + Log[1 - E^(a + b*x)] - Log[1 + E^(a + b*x)])
 - PolyLog[2, -E^(a + b*x)] + PolyLog[2, E^(a + b*x)])/(2*b)
```

Maple [B] time = 0.042, size = 67, normalized size = 1.9

$$\frac{\ln\left(e^{bx+a}\right)\text{Artanh}\left(e^{bx+a}\right)}{b}-\frac{\text{dilog}\left(e^{bx+a}\right)}{2b}-\frac{\text{dilog}\left(e^{bx+a}+1\right)}{2b}-\frac{\ln\left(e^{bx+a}\right)\ln\left(e^{bx+a}+1\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(exp(b*x+a)),x)
```

```
[Out] 1/b*ln(exp(b*x+a))*arctanh(exp(b*x+a))-1/2/b*dilog(exp(b*x+a))-1/2/b*dilog(
exp(b*x+a)+1)-1/2/b*ln(exp(b*x+a))*ln(exp(b*x+a)+1)
```

Maxima [B] time = 0.961612, size = 144, normalized size = 4.11

$$\frac{(bx+a)\,\text{artanh}\left(e^{(bx+a)}\right)}{b}-\frac{(bx+a)\left(\log\left(e^{(bx+a)}+1\right)-\log\left(e^{(bx+a)}-1\right)\right)-\log\left(-e^{(bx+a)}\right)\log\left(e^{(bx+a)}+1\right)+(bx+a)\log\left(e\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] (b*x + a)*arctanh(e^(b*x + a))/b - 1/2*((b*x + a)*(log(e^(b*x + a) + 1) - l
og(e^(b*x + a) - 1)) - log(-e^(b*x + a))*log(e^(b*x + a) + 1) + (b*x + a)*l
og(e^(b*x + a) - 1) - dilog(e^(b*x + a) + 1) + dilog(-e^(b*x + a) + 1))/b
```

Fricas [B] time = 1.54787, size = 419, normalized size = 11.97

$$bx\log\left(-\frac{\cosh(bx+a)+\sinh(bx+a)+1}{\cosh(bx+a)+\sinh(bx+a)-1}\right) - bx\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right) - a\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x*log(-(cosh(b*x + a) + sinh(b*x + a) + 1)/(cosh(b*x + a) + sinh(b*x
 + a) - 1)) - b*x*log(cosh(b*x + a) + sinh(b*x + a) + 1) - a*log(cosh(b*x +
 a) + sinh(b*x + a) - 1) + (b*x + a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1
) + dilog(cosh(b*x + a) + sinh(b*x + a)) - dilog(-cosh(b*x + a) - sinh(b*x
+ a)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{atanh}\left(e^{a+bx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(exp(b*x+a)),x)
```

```
[Out] Integral(atanh(exp(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{artanh}\left(e^{(bx+a)}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(e^(b*x + a)), x)
```

3.350 $\int x \tanh^{-1}\left(e^{a+bx}\right) dx$

Optimal. Leaf size=71

$$\frac{\text{PolyLog}\left(3,-e^{a+bx}\right)}{2b^2}-\frac{\text{PolyLog}\left(3,e^{a+bx}\right)}{2b^2}-\frac{x\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b}+\frac{x\text{PolyLog}\left(2,e^{a+bx}\right)}{2b}$$

```
[Out] -(x*PolyLog[2, -E^(a + b*x)])/(2*b) + (x*PolyLog[2, E^(a + b*x)])/(2*b) + P
olyLog[3, -E^(a + b*x)]/(2*b^2) - PolyLog[3, E^(a + b*x)]/(2*b^2)
```

Rubi [A] time = 0.057715, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6213, 2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3,-e^{a+bx}\right)}{2b^2}-\frac{\text{PolyLog}\left(3,e^{a+bx}\right)}{2b^2}-\frac{x\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b}+\frac{x\text{PolyLog}\left(2,e^{a+bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[E^(a + b*x)],x]
```

```
[Out] -(x*PolyLog[2, -E^(a + b*x)])/(2*b) + (x*PolyLog[2, E^(a + b*x)])/(2*b) + P
olyLog[3, -E^(a + b*x)]/(2*b^2) - PolyLog[3, E^(a + b*x)]/(2*b^2)
```

Rule 6213

```
Int[ArcTanh[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + a + b*f^(c + d*x)], x], x] - Dist[1/2, Int[x^m
*Log[1 - a - b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IGtQ[m,
 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x\tanh^{-1}\left(e^{a+bx}\right)dx &= -\left(\frac{1}{2}\int x\log\left(1-e^{a+bx}\right)dx\right)+\frac{1}{2}\int x\log\left(1+e^{a+bx}\right)dx\\ &= -\frac{x\text{Li}_2\left(-e^{a+bx}\right)}{2b}+\frac{x\text{Li}_2\left(e^{a+bx}\right)}{2b}+\frac{\int \text{Li}_2\left(-e^{a+bx}\right)dx}{2b}-\frac{\int \text{Li}_2\left(e^{a+bx}\right)dx}{2b}\\ &= -\frac{x\text{Li}_2\left(-e^{a+bx}\right)}{2b}+\frac{x\text{Li}_2\left(e^{a+bx}\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\text{Li}_2(-x)}{x}dx,x,e^{a+bx}\right)}{2b^2}-\frac{\text{Subst}\left(\int \frac{\text{Li}_2(x)}{x}dx,x,e^{a+bx}\right)}{2b^2}\\ &= -\frac{x\text{Li}_2\left(-e^{a+bx}\right)}{2b}+\frac{x\text{Li}_2\left(e^{a+bx}\right)}{2b}+\frac{\text{Li}_3\left(-e^{a+bx}\right)}{2b^2}-\frac{\text{Li}_3\left(e^{a+bx}\right)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.0517436, size = 113, normalized size = 1.59

$$\frac{-2bx\text{PolyLog}\left(2,-e^{a+bx}\right)+2bx\text{PolyLog}\left(2,e^{a+bx}\right)+2\text{PolyLog}\left(3,-e^{a+bx}\right)-2\text{PolyLog}\left(3,e^{a+bx}\right)+b^2x^2\log\left(1-e^{a+bx}\right)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[E^(a + b*x)],x]
```

```
[Out] (2*b^2*x^2*ArcTanh[E^(a + b*x)] + b^2*x^2*Log[1 - E^(a + b*x)] - b^2*x^2*Lo
g[1 + E^(a + b*x)] - 2*b*x*PolyLog[2, -E^(a + b*x)] + 2*b*x*PolyLog[2, E^(a
 + b*x)] + 2*PolyLog[3, -E^(a + b*x)] - 2*PolyLog[3, E^(a + b*x)])/(4*b^2)
```

Maple [B] time = 0.042, size = 153, normalized size = 2.2

$$\frac{x^2\text{Artanh}\left(e^{bx+a}\right)}{2}-\frac{\ln\left(e^{bx+a}+1\right)x^2}{4}+\frac{\ln\left(e^{bx+a}+1\right)a^2}{4b^2}-\frac{x\text{polylog}\left(2,-e^{bx+a}\right)}{2b}+\frac{\text{polylog}\left(3,-e^{bx+a}\right)}{2b^2}+\frac{\ln\left(1-e^{bx+a}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(exp(b*x+a)),x)
```

```
[Out] 1/2*x^2*arctanh(exp(b*x+a))-1/4*ln(exp(b*x+a)+1)*x^2+1/4/b^2*ln(exp(b*x+a)+
1)*a^2-1/2*x*polylog(2,-exp(b*x+a))/b+1/2*polylog(3,-exp(b*x+a))/b^2+1/4*ln
(1-exp(b*x+a))*x^2-1/4/b^2*ln(1-exp(b*x+a))*a^2+1/2*x*polylog(2,exp(b*x+a))
/b-1/2*polylog(3,exp(b*x+a))/b^2-1/2/b^2*a^2*arctanh(exp(b*x+a))
```

Maxima [A] time = 1.00834, size = 146, normalized size = 2.06

$$\frac{1}{2}x^2 \operatorname{artanh}\left(e^{(bx+a)}\right) - \frac{1}{4}b\left(\frac{b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right)-2\,\mathrm{Li}_3(-e^{(bx+a)})}{b^3} - \frac{b^2x^2\log\left(-e^{(bx+a)}+1\right)+2\,bx\mathrm{L}}{b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctanh(e^(b*x + a)) - 1/4*b*((b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x
*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3 - (b^2*x^2*log(-e^(b
*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3)
```

Fricas [C] time = 1.6175, size = 585, normalized size = 8.24

$$\frac{b^2x^2\log\left(-\frac{\cosh(bx+a)+\sinh(bx+a)+1}{\cosh(bx+a)+\sinh(bx+a)-1}\right) - b^2x^2\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)+2\,bx\mathrm{Li}_2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^2*log(-(cosh(b*x + a) + sinh(b*x + a) + 1)/(cosh(b*x + a) + sinh
(b*x + a) - 1)) - b^2*x^2*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 2*b*x*di
log(cosh(b*x + a) + sinh(b*x + a)) - 2*b*x*dilog(-cosh(b*x + a) - sinh(b*x
+ a)) + a^2*log(cosh(b*x + a) + sinh(b*x + a) - 1) + (b^2*x^2 - a^2)*log(-c
osh(b*x + a) - sinh(b*x + a) + 1) - 2*polylog(3, cosh(b*x + a) + sinh(b*x +
 a)) + 2*polylog(3, -cosh(b*x + a) - sinh(b*x + a)))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{atanh}\left(e^{a}e^{bx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(exp(b*x+a)),x)
```

```
[Out] Integral(x*atanh(exp(a)*exp(b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{artanh}\left(e^{(bx+a)}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(e^(b*x + a)), x)
```

3.351 $\int x^2 \tanh^{-1}\left(e^{a+bx}\right) dx$

Optimal. Leaf size=101

$$\frac{x\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^2}-\frac{x\text{PolyLog}\left(3,e^{a+bx}\right)}{b^2}-\frac{\text{PolyLog}\left(4,-e^{a+bx}\right)}{b^3}+\frac{\text{PolyLog}\left(4,e^{a+bx}\right)}{b^3}-\frac{x^2\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b}+$$

```
[Out] -(x^2*PolyLog[2, -E^(a + b*x)])/(2*b) + (x^2*PolyLog[2, E^(a + b*x)])/(2*b)
 + (x*PolyLog[3, -E^(a + b*x)])/b^2 - (x*PolyLog[3, E^(a + b*x)])/b^2 - Pol
yLog[4, -E^(a + b*x)]/b^3 + PolyLog[4, E^(a + b*x)]/b^3
```

Rubi [A] time = 0.092033, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6213, 2531, 6609, 2282, 6589}

$$\frac{x\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^2}-\frac{x\text{PolyLog}\left(3,e^{a+bx}\right)}{b^2}-\frac{\text{PolyLog}\left(4,-e^{a+bx}\right)}{b^3}+\frac{\text{PolyLog}\left(4,e^{a+bx}\right)}{b^3}-\frac{x^2\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[E^(a + b*x)],x]
```

```
[Out] -(x^2*PolyLog[2, -E^(a + b*x)])/(2*b) + (x^2*PolyLog[2, E^(a + b*x)])/(2*b)
 + (x*PolyLog[3, -E^(a + b*x)])/b^2 - (x*PolyLog[3, E^(a + b*x)])/b^2 - Pol
yLog[4, -E^(a + b*x)]/b^3 + PolyLog[4, E^(a + b*x)]/b^3
```

Rule 6213

```
Int[ArcTanh[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + a + b*f^(c + d*x)], x], x] - Dist[1/2, Int[x^m
*Log[1 - a - b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IGtQ[m,
 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \tanh^{-1}\left(e^{a+bx}\right) dx &= -\left(\frac{1}{2}\int x^2 \log\left(1-e^{a+bx}\right) dx\right)+\frac{1}{2}\int x^2 \log\left(1+e^{a+bx}\right) dx \\
&= -\frac{x^2\text{Li}_2\left(-e^{a+bx}\right)}{2b}+\frac{x^2\text{Li}_2\left(e^{a+bx}\right)}{2b}+\frac{\int x\text{Li}_2\left(-e^{a+bx}\right) dx}{b}-\frac{\int x\text{Li}_2\left(e^{a+bx}\right) dx}{b} \\
&= -\frac{x^2\text{Li}_2\left(-e^{a+bx}\right)}{2b}+\frac{x^2\text{Li}_2\left(e^{a+bx}\right)}{2b}+\frac{x\text{Li}_3\left(-e^{a+bx}\right)}{b^2}-\frac{x\text{Li}_3\left(e^{a+bx}\right)}{b^2}-\frac{\int \text{Li}_3\left(-e^{a+bx}\right) dx}{b^2}+\frac{\int \text{Li}_3 \\
&= -\frac{x^2\text{Li}_2\left(-e^{a+bx}\right)}{2b}+\frac{x^2\text{Li}_2\left(e^{a+bx}\right)}{2b}+\frac{x\text{Li}_3\left(-e^{a+bx}\right)}{b^2}-\frac{x\text{Li}_3\left(e^{a+bx}\right)}{b^2}-\frac{\text{Subst}\left(\int \frac{\text{Li}_3(-x)}{x} dx, x, e^{a+b}}{b^3} \\
&= -\frac{x^2\text{Li}_2\left(-e^{a+bx}\right)}{2b}+\frac{x^2\text{Li}_2\left(e^{a+bx}\right)}{2b}+\frac{x\text{Li}_3\left(-e^{a+bx}\right)}{b^2}-\frac{x\text{Li}_3\left(e^{a+bx}\right)}{b^2}-\frac{\text{Li}_4\left(-e^{a+bx}\right)}{b^3}+\frac{\text{Li}_4\left(e^{a+bx}\right)}{b^3}
\end{aligned}
$$

Mathematica [A] time = 0.0493919, size = 149, normalized size = 1.48

$$
\frac{-3b^2x^2\text{PolyLog}\left(2,-e^{a+bx}\right)+3b^2x^2\text{PolyLog}\left(2,e^{a+bx}\right)+6bx\text{PolyLog}\left(3,-e^{a+bx}\right)-6bx\text{PolyLog}\left(3,e^{a+bx}\right)-6\text{PolyLog}}{6b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[E^(a + b*x)],x]
```

```
[Out] (2*b^3*x^3*ArcTanh[E^(a + b*x)] + b^3*x^3*Log[1 - E^(a + b*x)] - b^3*x^3*Lo
g[1 + E^(a + b*x)] - 3*b^2*x^2*PolyLog[2, -E^(a + b*x)] + 3*b^2*x^2*PolyLog
[2, E^(a + b*x)] + 6*b*x*PolyLog[3, -E^(a + b*x)] - 6*b*x*PolyLog[3, E^(a +
 b*x)] - 6*PolyLog[4, -E^(a + b*x)] + 6*PolyLog[4, E^(a + b*x)])/(6*b^3)
```

Maple [B] time = 0.048, size = 185, normalized size = 1.8

$$\frac{x^3\operatorname{Artanh}\left(e^{bx+a}\right)}{3}+\frac{\operatorname{polylog}\left(4,e^{bx+a}\right)}{b^3}-\frac{\operatorname{polylog}\left(4,-e^{bx+a}\right)}{b^3}+\frac{a^3\operatorname{Artanh}\left(e^{bx+a}\right)}{3\,b^3}-\frac{\ln\left(e^{bx+a}+1\right)a^3}{6\,b^3}+\frac{\ln\left(1-e^{bx+a}\right)}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(exp(b*x+a)),x)
```

```
[Out] 1/3*x^3*arctanh(exp(b*x+a))+polylog(4,exp(b*x+a))/b^3-polylog(4,-exp(b*x+a)
)/b^3+1/3/b^3*a^3*arctanh(exp(b*x+a))-1/6/b^3*ln(exp(b*x+a)+1)*a^3+1/6/b^3*
ln(1-exp(b*x+a))*a^3-1/6*ln(exp(b*x+a)+1)*x^3-1/2*x^2*polylog(2,-exp(b*x+a)
)/b+x*polylog(3,-exp(b*x+a))/b^2+1/6*ln(1-exp(b*x+a))*x^3+1/2*x^2*polylog(2
,exp(b*x+a))/b-x*polylog(3,exp(b*x+a))/b^2
```

Maxima [A] time = 1.03676, size = 192, normalized size = 1.9

$$\frac{1}{3}\,x^3\operatorname{artanh}\left(e^{(bx+a)}\right)-\frac{1}{6}\,b\left(\frac{b^3x^3\log\left(e^{(bx+a)}+1\right)+3\,b^2x^2\mathrm{Li}_2\left(-e^{(bx+a)}\right)-6\,bx\mathrm{Li}_3(-e^{(bx+a)})+6\,\mathrm{Li}_4(-e^{(bx+a)})}{b^4}-\frac{b^3x^3\log}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctanh(e^(b*x + a)) - 1/6*b*((b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2
*x^2*dilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4, -e
^(b*x + a)))/b^4 - (b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x
+ a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(b*x + a)))/b^4)
```

Fricas [C] time = 1.62964, size = 732, normalized size = 7.25

$$\frac{b^3x^3\log\left(-\frac{\cosh(bx+a)+\sinh(bx+a)+1}{\cosh(bx+a)+\sinh(bx+a)-1}\right)-b^3x^3\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)+3\,b^2x^2\text{Li}_2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(b^3*x^3*log(-(cosh(b*x + a) + sinh(b*x + a) + 1)/(cosh(b*x + a) + sinh
(b*x + a) - 1)) - b^3*x^3*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 3*b^2*x^
2*dilog(cosh(b*x + a) + sinh(b*x + a)) - 3*b^2*x^2*dilog(-cosh(b*x + a) - s
inh(b*x + a)) - a^3*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 6*b*x*polylog(
3, cosh(b*x + a) + sinh(b*x + a)) + 6*b*x*polylog(3, -cosh(b*x + a) - sinh(
b*x + a)) + (b^3*x^3 + a^3)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 6*pol
ylog(4, cosh(b*x + a) + sinh(b*x + a)) - 6*polylog(4, -cosh(b*x + a) - sinh
(b*x + a)))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(exp(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{artanh}\left(e^{(bx+a)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(e^(b*x + a)), x)
```

3.352 $\int \tanh^{-1}\left(a+bf^{c+dx}\right) dx$

Optimal. Leaf size=168

$$\frac{\text{PolyLog}\left(2,1-\frac{2}{a+bf^{c+dx}+1}\right)}{2d\log(f)}-\frac{\text{PolyLog}\left(2,1-\frac{2bf^{c+dx}}{(1-a)\left(a+bf^{c+dx}+1\right)}\right)}{2d\log(f)}-\frac{\log\left(\frac{2}{a+bf^{c+dx}+1}\right)\tanh^{-1}\left(a+bf^{c+dx}\right)}{d\log(f)}+\frac{\log\left(\frac{2bf}{(1-a)(a+}\right.}{}$$

```
[Out] -((ArcTanh[a + b*f^(c + d*x)]*Log[2/(1 + a + b*f^(c + d*x))])/(d*Log[f])) +
 (ArcTanh[a + b*f^(c + d*x)]*Log[(2*b*f^(c + d*x))/((1 - a)*(1 + a + b*f^(c
 + d*x)))])/(d*Log[f]) + PolyLog[2, 1 - 2/(1 + a + b*f^(c + d*x))]/(2*d*Log
[f]) - PolyLog[2, 1 - (2*b*f^(c + d*x))/((1 - a)*(1 + a + b*f^(c + d*x)))]/
(2*d*Log[f])
```

Rubi [A] time = 0.129825, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2282, 6111, 5920, 2402, 2315, 2447}

$$\frac{\text{PolyLog}\left(2,1-\frac{2}{a+bf^{c+dx}+1}\right)}{2d\log(f)}-\frac{\text{PolyLog}\left(2,1-\frac{2bf^{c+dx}}{(1-a)\left(a+bf^{c+dx}+1\right)}\right)}{2d\log(f)}-\frac{\log\left(\frac{2}{a+bf^{c+dx}+1}\right)\tanh^{-1}\left(a+bf^{c+dx}\right)}{d\log(f)}+\frac{\log\left(\frac{2bf}{(1-a)(a+}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a + b*f^(c + d*x)],x]
```

```
[Out] -((ArcTanh[a + b*f^(c + d*x)]*Log[2/(1 + a + b*f^(c + d*x))])/(d*Log[f])) +
 (ArcTanh[a + b*f^(c + d*x)]*Log[(2*b*f^(c + d*x))/((1 - a)*(1 + a + b*f^(c
 + d*x)))])/(d*Log[f]) + PolyLog[2, 1 - 2/(1 + a + b*f^(c + d*x))]/(2*d*Log
[f]) - PolyLog[2, 1 - (2*b*f^(c + d*x))/((1 - a)*(1 + a + b*f^(c + d*x)))]/
(2*d*Log[f])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6111

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \tanh^{-1}\left(a+bf^{c+dx}\right)dx = \frac{\text{Subst}\left(\int \frac{\tanh^{-1}(a+bx)}{x}\,dx, x, f^{c+dx}\right)}{d\log(f)}$$
$$= \frac{\text{Subst}\left(\int \frac{\tanh^{-1}(x)}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bf^{c+dx}\right)}{bd\log(f)}$$

$$= -\frac{\tanh^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1+a+bf^{c+dx}}\right)}{d\log(f)} + \frac{\tanh^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(1-a)\left(1+a+bf^{c+dx}\right)}\right)}{d\log(f)} + \text{Su}$$
$$= -\frac{\tanh^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1+a+bf^{c+dx}}\right)}{d\log(f)} + \frac{\tanh^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(1-a)\left(1+a+bf^{c+dx}\right)}\right)}{d\log(f)} - \text{Li}_2$$
$$= -\frac{\tanh^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1+a+bf^{c+dx}}\right)}{d\log(f)} + \frac{\tanh^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(1-a)\left(1+a+bf^{c+dx}\right)}\right)}{d\log(f)} + \text{Li}_2$$

Mathematica [A] time = 0.0770035, size = 108, normalized size = 0.64

$$\frac{\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a-1}\right)-\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+1}\right)+dx\log(f)\left(\log\left(\frac{a+bf^{c+dx}-1}{a-1}\right)-\log\left(\frac{a+bf^{c+dx}+1}{a+1}\right)+2\tanh^{-1}\left(a+bf^{c+dx}\right)\right)}{2d\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a + b*f^(c + d*x)],x]
```

```
[Out] (d*x*Log[f]*(2*ArcTanh[a + b*f^(c + d*x)] + Log[(-1 + a + b*f^(c + d*x))/(-
1 + a)] - Log[(1 + a + b*f^(c + d*x))/(1 + a)]) + PolyLog[2, -((b*f^(c + d*
x))/(-1 + a))] - PolyLog[2, -((b*f^(c + d*x))/(1 + a))])/(2*d*Log[f])
```

Maple [A] time = 0.076, size = 164, normalized size = 1.

$$\frac{\ln\left(bf^{dx+c}\right)\text{Artanh}\left(a+bf^{dx+c}\right)}{d\ln\left(f\right)} + \frac{1}{2d\ln\left(f\right)}\text{dilog}\left(\frac{bf^{dx+c}+a-1}{a-1}\right) + \frac{\ln\left(bf^{dx+c}\right)}{2d\ln\left(f\right)}\ln\left(\frac{bf^{dx+c}+a-1}{a-1}\right) - \frac{1}{2d\ln\left(f\right)}\text{dilo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a+b*f^(d*x+c)),x)
```

```
[Out] 1/d/ln(f)*ln(b*f^(d*x+c))*arctanh(a+b*f^(d*x+c))+1/2/d/ln(f)*dilog((b*f^(d*
x+c)+a-1)/(a-1))+1/2/d/ln(f)*ln(b*f^(d*x+c))*ln((b*f^(d*x+c)+a-1)/(a-1))-1/
2/d/ln(f)*dilog((1+a+b*f^(d*x+c))/(1+a))-1/2/d/ln(f)*ln(b*f^(d*x+c))*ln((1+
a+b*f^(d*x+c))/(1+a))
```

Maxima [A] time = 0.995855, size = 284, normalized size = 1.69

$$\frac{\operatorname{artanh}\left(bf^{dx+c}+a\right)\log\left(f^{dx+c}\right)}{d\log\left(f\right)}-\frac{b\left(\frac{\log\left(bf^{dx+c}+a+1\right)}{b}-\frac{\log\left(bf^{dx+c}+a-1\right)}{b}\right)\log\left(f^{dx+c}\right)-b\left(\frac{\log\left(bf^{dx+c}+a+1\right)\log\left(-\frac{bf^{dx+c}+a+1}{a+1}+1\right)+\mathrm{Li}_2\left(\frac{b\cdots}{\cdots}\right)}{b}\cdots\right.}{2\,d\log\left(f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a+b*f^(d*x+c)),x, algorithm="maxima")
```

```
[Out] arctanh(b*f^(d*x + c) + a)*log(f^(d*x + c))/(d*log(f)) - 1/2*(b*(log(b*f^(d
*x + c) + a + 1)/b - log(b*f^(d*x + c) + a - 1)/b)*log(f^(d*x + c)) - b*((l
og(b*f^(d*x + c) + a + 1)*log(-(b*f^(d*x + c) + a + 1)/(a + 1) + 1) + dilog
((b*f^(d*x + c) + a + 1)/(a + 1)))/b - (log(b*f^(d*x + c) + a - 1)*log(-(b*
f^(d*x + c) + a - 1)/(a - 1) + 1) + dilog((b*f^(d*x + c) + a - 1)/(a - 1)))
/b))/(d*log(f))
```

Fricas [A] time = 1.80078, size = 900, normalized size = 5.36

$$\frac{dx\log\left(f\right)\log\left(-\frac{b\cosh\left((dx+c)\log\left(f\right)\right)+b\sinh\left((dx+c)\log\left(f\right)\right)+a+1}{b\cosh\left((dx+c)\log\left(f\right)\right)+b\sinh\left((dx+c)\log\left(f\right)\right)+a-1}\right)+c\log\left(b\cosh\left((dx+c)\log\left(f\right)\right)+b\sinh\left((dx+c)\log\left(f\right)\right)+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a+b*f^(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(d*x*log(f)*log(-(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) +
 a + 1)/(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)) + c*
log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a + 1)*log(f) - c
```

```
*log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)*log(f) -
(d*x + c)*log(f)*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) +
 a + 1)/(a + 1)) + (d*x + c)*log(f)*log((b*cosh((d*x + c)*log(f)) + b*sinh(
(d*x + c)*log(f)) + a - 1)/(a - 1)) - dilog(-(b*cosh((d*x + c)*log(f)) + b*
sinh((d*x + c)*log(f)) + a + 1)/(a + 1) + 1) + dilog(-(b*cosh((d*x + c)*log
(f)) + b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1) + 1))/(d*log(f))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a+b*f**(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{artanh}\left(bf^{dx+c} + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a+b*f^(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(b*f^(d*x + c) + a), x)
```

3.353 $\int x \tanh^{-1}\left(a+bf^{c+dx}\right) dx$

Optimal. Leaf size=211

$$-\frac{\text{PolyLog}\left(3,\frac{bf^{c+dx}}{1-a}\right)}{2d^2\log^2(f)}+\frac{\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+1}\right)}{2d^2\log^2(f)}+\frac{x\text{PolyLog}\left(2,\frac{bf^{c+dx}}{1-a}\right)}{2d\log(f)}-\frac{x\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+1}\right)}{2d\log(f)}-\frac{1}{4}x^2\log\left(-a-bf^{c+dx}\right.$$

```
[Out] -(x^2*Log[1 - a - b*f^(c + d*x)])/4 + (x^2*Log[1 + a + b*f^(c + d*x)])/4 +
(x^2*Log[1 - (b*f^(c + d*x))/(1 - a)])/4 - (x^2*Log[1 + (b*f^(c + d*x))/(1
+ a)])/4 + (x*PolyLog[2, (b*f^(c + d*x))/(1 - a)])/(2*d*Log[f]) - (x*PolyLo
g[2, -((b*f^(c + d*x))/(1 + a))])/(2*d*Log[f]) - PolyLog[3, (b*f^(c + d*x))
/(1 - a)]/(2*d^2*Log[f]^2) + PolyLog[3, -((b*f^(c + d*x))/(1 + a))]/(2*d^2*
Log[f]^2)
```

Rubi [A] time = 0.150448, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6213, 2532, 2531, 2282, 6589}

$$-\frac{\text{PolyLog}\left(3,\frac{bf^{c+dx}}{1-a}\right)}{2d^2\log^2(f)}+\frac{\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+1}\right)}{2d^2\log^2(f)}+\frac{x\text{PolyLog}\left(2,\frac{bf^{c+dx}}{1-a}\right)}{2d\log(f)}-\frac{x\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+1}\right)}{2d\log(f)}-\frac{1}{4}x^2\log\left(-a-bf^{c+dx}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[a + b*f^(c + d*x)],x]
```



```
[Out] -(x^2*Log[1 - a - b*f^(c + d*x)])/4 + (x^2*Log[1 + a + b*f^(c + d*x)])/4 +
(x^2*Log[1 - (b*f^(c + d*x))/(1 - a)])/4 - (x^2*Log[1 + (b*f^(c + d*x))/(1
+ a)])/4 + (x*PolyLog[2, (b*f^(c + d*x))/(1 - a)])/(2*d*Log[f]) - (x*PolyLo
g[2, -((b*f^(c + d*x))/(1 + a))])/(2*d*Log[f]) - PolyLog[3, (b*f^(c + d*x))
/(1 - a)]/(2*d^2*Log[f]^2) + PolyLog[3, -((b*f^(c + d*x))/(1 + a))]/(2*d^2*
Log[f]^2)
```


Rule 6213

```
Int[ArcTanh[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + a + b*f^(c + d*x)], x], x] - Dist[1/2, Int[x^m
*Log[1 - a - b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IGtQ[m,
 0]
```

Rule 2532

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[((f + g*x)^(m + 1)*Log[d + e*(F^(c*(a +
b*x)))^n])/(g*(m + 1)), x] + (Int[(f + g*x)^m*Log[1 + (e*(F^(c*(a + b*x)))^
n)/d], x] - Simp[((f + g*x)^(m + 1)*Log[1 + (e*(F^(c*(a + b*x)))^n)/d])/(g*
(m + 1)), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}\left(a+bf^{c+dx}\right)dx=-\left(\frac{1}{2}\int x\log\left(1-a-bf^{c+dx}\right)dx\right)+\frac{1}{2}\int x\log\left(1+a+bf^{c+dx}\right)dx$$
$$=-\frac{1}{4}x^2\log\left(1-a-bf^{c+dx}\right)+\frac{1}{4}x^2\log\left(1+a+bf^{c+dx}\right)+\frac{1}{4}x^2\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{4}x^2\log\Big(1$$
$$=-\frac{1}{4}x^2\log\left(1-a-bf^{c+dx}\right)+\frac{1}{4}x^2\log\left(1+a+bf^{c+dx}\right)+\frac{1}{4}x^2\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{4}x^2\log\Big(1$$
$$=-\frac{1}{4}x^2\log\left(1-a-bf^{c+dx}\right)+\frac{1}{4}x^2\log\left(1+a+bf^{c+dx}\right)+\frac{1}{4}x^2\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{4}x^2\log\Big(1$$
$$=-\frac{1}{4}x^2\log\left(1-a-bf^{c+dx}\right)+\frac{1}{4}x^2\log\left(1+a+bf^{c+dx}\right)+\frac{1}{4}x^2\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{4}x^2\log\Big(1$$

Mathematica [A] time = 0.118409, size = 177, normalized size = 0.84

$$\frac{-2\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a-1}\right)+2\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+1}\right)+2dx\log(f)\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a-1}\right)-2dx\log(f)\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+1}\right)+}{4d^2\log^2(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[a + b*f^(c + d*x)],x]
```

```
[Out] (2*d^2*x^2*ArcTanh[a + b*f^(c + d*x)]*Log[f]^2 + d^2*x^2*Log[f]^2*Log[1 + (
b*f^(c + d*x))/(-1 + a)] - d^2*x^2*Log[f]^2*Log[1 + (b*f^(c + d*x))/(1 + a)
] + 2*d*x*Log[f]*PolyLog[2, -((b*f^(c + d*x))/(-1 + a))] - 2*d*x*Log[f]*Pol
yLog[2, -((b*f^(c + d*x))/(1 + a))] - 2*PolyLog[3, -((b*f^(c + d*x))/(-1 +
a))] + 2*PolyLog[3, -((b*f^(c + d*x))/(1 + a))])/(4*d^2*Log[f]^2)
```

Maple [B] time = 0.131, size = 596, normalized size = 2.8

$$\frac{x^2\ln\left(1+a+bf^{dx+c}\right)}{4}-\frac{x^2\ln\left(1-a-bf^{dx+c}\right)}{4}+\frac{x^2}{4}\ln\left(1-\frac{bf^{dx}f^c}{1-a}\right)+\frac{cx}{2d}\ln\left(1-\frac{bf^{dx}f^c}{1-a}\right)+\frac{c^2}{4d^2}\ln\left(1-\frac{bf^{dx}f^c}{1-a}\right)+\frac{}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a+b*f^(d*x+c)),x)
```

```
[Out] 1/4*x^2*ln(1+a+b*f^(d*x+c))-1/4*x^2*ln(1-a-b*f^(d*x+c))+1/4*ln(1-b*f^(d*x)*
f^c/(1-a))*x^2+1/2/d*ln(1-b*f^(d*x)*f^c/(1-a))*x*c+1/4/d^2*ln(1-b*f^(d*x)*f
^c/(1-a))*c^2+1/2/ln(f)/d*polylog(2,b*f^(d*x)*f^c/(1-a))*x+1/2/ln(f)/d^2*po
lylog(2,b*f^(d*x)*f^c/(1-a))*c-1/2/ln(f)^2/d^2*polylog(3,b*f^(d*x)*f^c/(1-a
))+1/4/d^2*c^2*ln(1-a-b*f^(d*x)*f^c)-1/2/ln(f)/d^2*c*dilog((b*f^(d*x)*f^c+a
-1)/(a-1))-1/2/d*c*ln((b*f^(d*x)*f^c+a-1)/(a-1))*x-1/2/d^2*c^2*ln((b*f^(d*x
)*f^c+a-1)/(a-1))-1/4*ln(1-b*f^(d*x)*f^c/(-1-a))*x^2-1/2/d*ln(1-b*f^(d*x)*f
^c/(-1-a))*x*c-1/4/d^2*ln(1-b*f^(d*x)*f^c/(-1-a))*c^2-1/2/ln(f)/d*polylog(2
,b*f^(d*x)*f^c/(-1-a))*x-1/2/ln(f)/d^2*polylog(2,b*f^(d*x)*f^c/(-1-a))*c+1/
2/ln(f)^2/d^2*polylog(3,b*f^(d*x)*f^c/(-1-a))-1/4/d^2*c^2*ln(1+a+b*f^(d*x)*
f^c)+1/2/ln(f)/d^2*c*dilog((1+a+b*f^(d*x)*f^c)/(1+a))+1/2/d*c*ln((1+a+b*f^(
d*x)*f^c)/(1+a))*x+1/2/d^2*c^2*ln((1+a+b*f^(d*x)*f^c)/(1+a))
```

Maxima [A] time = 1.07484, size = 262, normalized size = 1.24

$$-\frac{1}{4}\,bd\left(\frac{\log\left(\frac{bf^{dx}f^{c}}{a+1}+1\right)\log\left(f^{dx}\right)^{2}+2\,\mathrm{Li}_{2}\left(-\frac{bf^{dx}f^{c}}{a+1}\right)\log\left(f^{dx}\right)-2\,\mathrm{Li}_{3}(-\frac{bf^{dx}f^{c}}{a+1})}{bd^{3}\log\left(f\right)^{3}}-\frac{\log\left(\frac{bf^{dx}f^{c}}{a-1}+1\right)\log\left(f^{dx}\right)^{2}+2\,\mathrm{Li}_{2}\left(-\frac{bf}{a}\right.}{bd^{3}\log\left(f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a+b*f^(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*b*d*((log(b*f^(d*x)*f^c/(a + 1) + 1)*log(f^(d*x))^2 + 2*dilog(-b*f^(d*
x)*f^c/(a + 1))*log(f^(d*x)) - 2*polylog(3, -b*f^(d*x)*f^c/(a + 1)))/(b*d^3
*log(f)^3) - (log(b*f^(d*x)*f^c/(a - 1) + 1)*log(f^(d*x))^2 + 2*dilog(-b*f^
(d*x)*f^c/(a - 1))*log(f^(d*x)) - 2*polylog(3, -b*f^(d*x)*f^c/(a - 1)))/(b*
d^3*log(f)^3))*log(f) + 1/2*x^2*arctanh(b*f^(d*x + c) + a)
```

Fricas [C] time = 1.74833, size = 1195, normalized size = 5.66

$$\frac{d^{2}x^{2}\log\left(f\right)^{2}\log\left(-\frac{b\cosh\left((dx+c)\log\left(f\right)\right)+b\sinh\left((dx+c)\log\left(f\right)\right)+a+1}{b\cosh\left((dx+c)\log\left(f\right)\right)+b\sinh\left((dx+c)\log\left(f\right)\right)+a-1}\right)-c^{2}\log\left(b\cosh\left((dx+c)\log\left(f\right)\right)+b\sinh\left((dx+c)\log\left(f\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a+b*f^(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(d^2*x^2*log(f)^2*log(-(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log
(f)) + a + 1)/(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)
) - c^2*log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a + 1)*lo
g(f)^2 + c^2*log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a -
1)*log(f)^2 - 2*d*x*dilog(-(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log
(f)) + a + 1)/(a + 1) + 1)*log(f) + 2*d*x*dilog(-(b*cosh((d*x + c)*log(f))
+ b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1) + 1)*log(f) - (d^2*x^2 - c^2)*l
og(f)^2*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a + 1)/(
a + 1)) + (d^2*x^2 - c^2)*log(f)^2*log((b*cosh((d*x + c)*log(f)) + b*sinh((
d*x + c)*log(f)) + a - 1)/(a - 1)) + 2*polylog(3, -(b*cosh((d*x + c)*log(f)
) + b*sinh((d*x + c)*log(f)))/(a + 1)) - 2*polylog(3, -(b*cosh((d*x + c)*lo
g(f)) + b*sinh((d*x + c)*log(f)))/(a - 1)))/(d^2*log(f)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a+b*f**(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{artanh}\left(bf^{dx+c} + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a+b*f^(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(b*f^(d*x + c) + a), x)
```

3.354 $\int x^2 \tanh^{-1}\left(a + bf^{c+dx}\right) dx$

Optimal. Leaf size=264

$$-\frac{x\text{PolyLog}\left(3,\frac{bf^{c+dx}}{1-a}\right)}{d^2\log^2(f)}+\frac{x\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+1}\right)}{d^2\log^2(f)}+\frac{\text{PolyLog}\left(4,\frac{bf^{c+dx}}{1-a}\right)}{d^3\log^3(f)}-\frac{\text{PolyLog}\left(4,-\frac{bf^{c+dx}}{a+1}\right)}{d^3\log^3(f)}+\frac{x^2\text{PolyLog}\left(2,\frac{bf^{c+d}}{1-a}\right.}{2d\log(f)}$$

```
[Out] -(x^3*Log[1 - a - b*f^(c + d*x)])/6 + (x^3*Log[1 + a + b*f^(c + d*x)])/6 +
(x^3*Log[1 - (b*f^(c + d*x))/(1 - a)])/6 - (x^3*Log[1 + (b*f^(c + d*x))/(1
+ a)])/6 + (x^2*PolyLog[2, (b*f^(c + d*x))/(1 - a)])/(2*d*Log[f]) - (x^2*Po
lyLog[2, -((b*f^(c + d*x))/(1 + a))])/(2*d*Log[f]) - (x*PolyLog[3, (b*f^(c
+ d*x))/(1 - a)])/(d^2*Log[f]^2) + (x*PolyLog[3, -((b*f^(c + d*x))/(1 + a))
])/(d^2*Log[f]^2) + PolyLog[4, (b*f^(c + d*x))/(1 - a)]/(d^3*Log[f]^3) - Po
lyLog[4, -((b*f^(c + d*x))/(1 + a))]/(d^3*Log[f]^3)
```

Rubi [A] time = 0.196633, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6213, 2532, 2531, 6609, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(3,\frac{bf^{c+dx}}{1-a}\right)}{d^2\log^2(f)}+\frac{x\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+1}\right)}{d^2\log^2(f)}+\frac{\text{PolyLog}\left(4,\frac{bf^{c+dx}}{1-a}\right)}{d^3\log^3(f)}-\frac{\text{PolyLog}\left(4,-\frac{bf^{c+dx}}{a+1}\right)}{d^3\log^3(f)}+\frac{x^2\text{PolyLog}\left(2,\frac{bf^{c+d}}{1-a}\right.}{2d\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[a + b*f^(c + d*x)],x]
```

```
[Out] -(x^3*Log[1 - a - b*f^(c + d*x)])/6 + (x^3*Log[1 + a + b*f^(c + d*x)])/6 +
(x^3*Log[1 - (b*f^(c + d*x))/(1 - a)])/6 - (x^3*Log[1 + (b*f^(c + d*x))/(1
+ a)])/6 + (x^2*PolyLog[2, (b*f^(c + d*x))/(1 - a)])/(2*d*Log[f]) - (x^2*Po
lyLog[2, -((b*f^(c + d*x))/(1 + a))])/(2*d*Log[f]) - (x*PolyLog[3, (b*f^(c
+ d*x))/(1 - a)])/(d^2*Log[f]^2) + (x*PolyLog[3, -((b*f^(c + d*x))/(1 + a))
])/(d^2*Log[f]^2) + PolyLog[4, (b*f^(c + d*x))/(1 - a)]/(d^3*Log[f]^3) - Po
lyLog[4, -((b*f^(c + d*x))/(1 + a))]/(d^3*Log[f]^3)
```

Rule 6213

```
Int[ArcTanh[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + a + b*f^(c + d*x)], x], x] - Dist[1/2, Int[x^m
*Log[1 - a - b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IGtQ[m,
```

```
0]
```

Rule 2532

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[((f + g*x)^(m + 1)*Log[d + e*(F^(c*(a +
b*x)))^n])/(g*(m + 1)), x] + (Int[(f + g*x)^m*Log[1 + (e*(F^(c*(a + b*x)))^
n)/d], x] - Simp[((f + g*x)^(m + 1)*Log[1 + (e*(F^(c*(a + b*x)))^n)/d])/(g*
(m + 1)), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \tanh^{-1}\left(a+bf^{c+dx}\right)dx = -\left(\frac{1}{2}\int x^2\log\left(1-a-bf^{c+dx}\right)dx\right)+\frac{1}{2}\int x^2\log\left(1+a+bf^{c+dx}\right)dx$$
$$= -\frac{1}{6}x^3\log\left(1-a-bf^{c+dx}\right)+\frac{1}{6}x^3\log\left(1+a+bf^{c+dx}\right)+\frac{1}{6}x^3\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{6}x^3\log$$
$$= -\frac{1}{6}x^3\log\left(1-a-bf^{c+dx}\right)+\frac{1}{6}x^3\log\left(1+a+bf^{c+dx}\right)+\frac{1}{6}x^3\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{6}x^3\log$$
$$= -\frac{1}{6}x^3\log\left(1-a-bf^{c+dx}\right)+\frac{1}{6}x^3\log\left(1+a+bf^{c+dx}\right)+\frac{1}{6}x^3\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{6}x^3\log$$
$$= -\frac{1}{6}x^3\log\left(1-a-bf^{c+dx}\right)+\frac{1}{6}x^3\log\left(1+a+bf^{c+dx}\right)+\frac{1}{6}x^3\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{6}x^3\log$$
$$= -\frac{1}{6}x^3\log\left(1-a-bf^{c+dx}\right)+\frac{1}{6}x^3\log\left(1+a+bf^{c+dx}\right)+\frac{1}{6}x^3\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{6}x^3\log$$

Mathematica [A] time = 0.0879626, size = 235, normalized size = 0.89

$$\frac{3d^2x^2\log^2(f)\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a-1}\right)-3d^2x^2\log^2(f)\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+1}\right)+6\text{PolyLog}\left(4,-\frac{bf^{c+dx}}{a-1}\right)-6\text{PolyLog}\left(4,-\frac{bf^{c}}{a+}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[a + b*f^(c + d*x)],x]
```

```
[Out] (2*d^3*x^3*ArcTanh[a + b*f^(c + d*x)]*Log[f]^3 + d^3*x^3*Log[f]^3*Log[1 + (
b*f^(c + d*x))/(-1 + a)] - d^3*x^3*Log[f]^3*Log[1 + (b*f^(c + d*x))/(1 + a)
] + 3*d^2*x^2*Log[f]^2*PolyLog[2, -((b*f^(c + d*x))/(-1 + a))] - 3*d^2*x^2*
Log[f]^2*PolyLog[2, -((b*f^(c + d*x))/(1 + a))] - 6*d*x*Log[f]*PolyLog[3, -
((b*f^(c + d*x))/(-1 + a))] + 6*d*x*Log[f]*PolyLog[3, -((b*f^(c + d*x))/(1
+ a))] + 6*PolyLog[4, -((b*f^(c + d*x))/(-1 + a))] - 6*PolyLog[4, -((b*f^(c
 + d*x))/(1 + a))])/(6*d^3*Log[f]^3)
```

Maple [B] time = 0.126, size = 672, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a+b*f^(d*x+c)),x)
```

```
[Out] 1/6*x^3*ln(1+a+b*f^(d*x+c))-1/6*x^3*ln(1-a-b*f^(d*x+c))+1/6*ln(1-b*f^(d*x)*
f^c/(1-a))*x^3-1/2/d^2*ln(1-b*f^(d*x)*f^c/(1-a))*x*c^2-1/3/d^3*ln(1-b*f^(d*
x)*f^c/(1-a))*c^3+1/2/ln(f)/d*polylog(2,b*f^(d*x)*f^c/(1-a))*x^2-1/2/ln(f)/
d^3*polylog(2,b*f^(d*x)*f^c/(1-a))*c^2-1/ln(f)^2/d^2*polylog(3,b*f^(d*x)*f^
c/(1-a))*x+1/ln(f)^3/d^3*polylog(4,b*f^(d*x)*f^c/(1-a))-1/6/d^3*c^3*ln(1-a-
b*f^(d*x)*f^c)+1/2/ln(f)/d^3*c^2*dilog((b*f^(d*x)*f^c+a-1)/(a-1))+1/2/d^2*c
^2*ln((b*f^(d*x)*f^c+a-1)/(a-1))*x+1/2/d^3*c^3*ln((b*f^(d*x)*f^c+a-1)/(a-1)
)-1/6*ln(1-b*f^(d*x)*f^c/(-1-a))*x^3+1/2/d^2*ln(1-b*f^(d*x)*f^c/(-1-a))*x*c
^2+1/3/d^3*ln(1-b*f^(d*x)*f^c/(-1-a))*c^3-1/2/ln(f)/d*polylog(2,b*f^(d*x)*f
^c/(-1-a))*x^2+1/2/ln(f)/d^3*polylog(2,b*f^(d*x)*f^c/(-1-a))*c^2+1/ln(f)^2/
d^2*polylog(3,b*f^(d*x)*f^c/(-1-a))*x-1/ln(f)^3/d^3*polylog(4,b*f^(d*x)*f^c
/(-1-a))+1/6/d^3*c^3*ln(1+a+b*f^(d*x)*f^c)-1/2/ln(f)/d^3*c^2*dilog((1+a+b*f
^(d*x)*f^c)/(1+a))-1/2/d^2*c^2*ln((1+a+b*f^(d*x)*f^c)/(1+a))*x-1/2/d^3*c^3*
ln((1+a+b*f^(d*x)*f^c)/(1+a))
```

Maxima [A] time = 1.06354, size = 338, normalized size = 1.28

$$\frac{1}{3}x^3\operatorname{artanh}\left(bf^{dx+c}+a\right)-\frac{1}{6}bd\left(\frac{\log\left(\frac{bf^{dx}f^c}{a+1}+1\right)\log\left(f^{dx}\right)^3+3\operatorname{Li}_2\left(-\frac{bf^{dx}f^c}{a+1}\right)\log\left(f^{dx}\right)^2-6\log\left(f^{dx}\right)\operatorname{Li}_3\left(-\frac{bf^{dx}f^c}{a+1}\right)+6\operatorname{Li}}{bd^4\log\left(f\right)^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a+b*f^(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctanh(b*f^(d*x + c) + a) - 1/6*b*d*((log(b*f^(d*x)*f^c/(a + 1) +
1)*log(f^(d*x))^3 + 3*dilog(-b*f^(d*x)*f^c/(a + 1))*log(f^(d*x))^2 - 6*log(
f^(d*x))*polylog(3, -b*f^(d*x)*f^c/(a + 1)) + 6*polylog(4, -b*f^(d*x)*f^c/(
a + 1)))/(b*d^4*log(f)^4) - (log(b*f^(d*x)*f^c/(a - 1) + 1)*log(f^(d*x))^3
+ 3*dilog(-b*f^(d*x)*f^c/(a - 1))*log(f^(d*x))^2 - 6*log(f^(d*x))*polylog(3
, -b*f^(d*x)*f^c/(a - 1)) + 6*polylog(4, -b*f^(d*x)*f^c/(a - 1)))/(b*d^4*lo
g(f)^4))*log(f)
```

Fricas [C] time = 1.60706, size = 1454, normalized size = 5.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a+b*f^(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(d^3*x^3*log(f)^3*log(-(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log
(f)) + a + 1)/(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)
) - 3*d^2*x^2*dilog(-(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) +
 a + 1)/(a + 1) + 1)*log(f)^2 + 3*d^2*x^2*dilog(-(b*cosh((d*x + c)*log(f))
+ b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1) + 1)*log(f)^2 + c^3*log(b*cosh(
(d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a + 1)*log(f)^3 - c^3*log(b*
cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)*log(f)^3 - (d^3*
x^3 + c^3)*log(f)^3*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)
) + a + 1)/(a + 1)) + (d^3*x^3 + c^3)*log(f)^3*log((b*cosh((d*x + c)*log(f)
) + b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1)) + 6*d*x*log(f)*polylog(3, -(
b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)))/(a + 1)) - 6*d*x*log(f
)*polylog(3, -(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)))/(a - 1)
) - 6*polylog(4, -(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)))/(a
+ 1)) + 6*polylog(4, -(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)))
/(a - 1)))/(d^3*log(f)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a+b*f**(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{artanh}\left(bf^{dx+c} + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a+b*f^(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(b*f^(d*x + c) + a), x)
```

3.355 $\int e^{c(a+bx)} \tanh^{-1}(\sinh(ac+bcx))\,dx$

Optimal. Leaf size=107

$$\frac{(1-\sqrt{2})\log\left(-e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc}+\frac{(1+\sqrt{2})\log\left(-e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc}+\frac{e^{ac+bcx}\tanh^{-1}(\sinh(c(a+bx)))}{bc}$$

```
[Out] (E^(a*c + b*c*x)*ArcTanh[Sinh[c*(a + b*x)]])/(b*c) + ((1 - Sqrt[2])*Log[3 -
 2*Sqrt[2] - E^(2*c*(a + b*x))])/(2*b*c) + ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2]
 - E^(2*c*(a + b*x))])/(2*b*c)
```

Rubi [A] time = 0.170877, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {2194, 6275, 2282, 12, 1247, 632, 31}

$$\frac{(1-\sqrt{2})\log\left(-e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc}+\frac{(1+\sqrt{2})\log\left(-e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc}+\frac{e^{ac+bcx}\tanh^{-1}(\sinh(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTanh[Sinh[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcTanh[Sinh[c*(a + b*x)]])/(b*c) + ((1 - Sqrt[2])*Log[3 -
 2*Sqrt[2] - E^(2*c*(a + b*x))])/(2*b*c) + ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2]
 - E^(2*c*(a + b*x))])/(2*b*c)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6275

```
Int[((a_.) + ArcTanh[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Dist[a + b*ArcTanh[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/
(1 - u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTanh[u]), x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int e^{c(a+bx)}\tanh^{-1}(\sinh(ac+bcx))\,dx = \frac{\text{Subst}\left(\int e^x\tanh^{-1}(\sinh(x))\,dx,x,ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tanh^{-1}(\sinh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{e^x\cosh(x)}{1-\sinh^2(x)}\,dx,x,ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tanh^{-1}(\sinh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{2x(-1-x^2)}{1-6x^2+x^4}\,dx,x,e^{ac+bcx}\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tanh^{-1}(\sinh(c(a+bx)))}{bc} - \frac{2\,\text{Subst}\left(\int \frac{x(-1-x^2)}{1-6x^2+x^4}\,dx,x,e^{ac+bcx}\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tanh^{-1}(\sinh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{-1-x}{1-6x+x^2}\,dx,x,e^{2ac+2bcx}\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tanh^{-1}(\sinh(c(a+bx)))}{bc} + \frac{\left(1-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-3+2\sqrt{2}+x}\,dx,x,e^{2ac+2bcx}\right)}{2bc} +$$
$$= \frac{e^{ac+bcx}\tanh^{-1}(\sinh(c(a+bx)))}{bc} + \frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}-e^{2ac+2bcx}\right)}{2bc} + \frac{\left(1+\sqrt{2}\right)}{}$$

Mathematica [A] time = 0.174728, size = 153, normalized size = 1.43

$$\frac{\log\left(-2e^{c(a+bx)}-e^{2c(a+bx)}+1\right)+\log\left(2e^{c(a+bx)}-e^{2c(a+bx)}+1\right)-2e^{c(a+bx)}\tanh^{-1}\left(\frac{1}{2}e^{-c(a+bx)}-\frac{1}{2}e^{c(a+bx)}\right)-2\sqrt{2}\tanh^{-1}\left(\frac{e^{c(}}{}\right.}{2bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcTanh[Sinh[a*c + b*c*x]],x]
```

```
[Out] (-2*E^(c*(a + b*x))*ArcTanh[1/(2*E^(c*(a + b*x))) - E^(c*(a + b*x))/2] - 2*
Sqrt[2]*ArcTanh[(-1 + E^(c*(a + b*x)))/Sqrt[2]] + 2*Sqrt[2]*ArcTanh[(1 + E^
(c*(a + b*x)))/Sqrt[2]] + Log[1 - 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x))] +
Log[1 + 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x))])/(2*b*c)
```

Maple [C] time = 0.473, size = 868, normalized size = 8.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctanh(sinh(b*c*x+a*c)),x)
```

```
[Out] 1/2/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1)+1/4*I/b/c*Pi
*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+2*exp(c*(
b*x+a))-1))^2*exp(c*(b*x+a))-1/2*I/b/c*exp(c*(b*x+a))*Pi-1/4*I/b/c*Pi*csgn(
I*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp
(-c*(b*x+a))*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*exp(c*(b*x+a))+1/4*I/b/
c*Pi*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))^3*exp(c
*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))*csgn(
I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))^2*exp(c*(b*x+a))-
1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))^
3*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))
*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))^2*exp(c*(b*x
+a))+1/4*I/b/c*Pi*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))*csgn(I*exp
(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1)
)*exp(c*(b*x+a))+1/2*I/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))+2*e
xp(c*(b*x+a))+1))^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csg
n(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))^2*exp(c*(b*x+a)
)-1/2/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))-2*exp(c*(b*x+a))-1)+1/2/b/c*ln
(exp(2*c*(b*x+a))-(1+2^(1/2))^2)*2^(1/2)-1/2/b/c*ln(exp(2*c*(b*x+a))-(2^(1/
2)-1)^2)*2^(1/2)-2*a/b+1/2/b/c*ln(exp(2*c*(b*x+a))-(1+2^(1/2))^2)+1/2/b/c*l
n(exp(2*c*(b*x+a))-(2^(1/2)-1)^2)
```

<u>Maxima</u> [B] time = 1.48345, size = 248, normalized size = 2.32

$$\frac{\operatorname{artanh}\left(\sinh\left(bcx+ac\right)\right)e^{\left(\left(bx+a\right)c\right)}}{bc}+\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{\left(bcx+ac\right)}+1}{\sqrt{2}+e^{\left(bcx+ac\right)}-1}\right)}{2\,bc}-\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{\left(bcx+ac\right)}-1}{\sqrt{2}+e^{\left(bcx+ac\right)}+1}\right)}{2\,bc}+\frac{\log\left(e^{\left(2\,bcx+2\,ac\right)}+2\,e^{\left(bcx+ac\right)}-\right.}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(sinh(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctanh(sinh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + 1/2*sqrt(2)*log(-(sqrt(2
) - e^(b*c*x + a*c) + 1)/(sqrt(2) + e^(b*c*x + a*c) - 1))/(b*c) - 1/2*sqrt(
2)*log(-(sqrt(2) - e^(b*c*x + a*c) - 1)/(sqrt(2) + e^(b*c*x + a*c) + 1))/(b
*c) + 1/2*log(e^(2*b*c*x + 2*a*c) + 2*e^(b*c*x + a*c) - 1)/(b*c) + 1/2*log(
e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x + a*c) - 1)/(b*c)
```

Fricas [B] time = 1.76314, size = 625, normalized size = 5.84

$$\frac{(\cosh(bcx+ac)+\sinh(bcx+ac))\log\left(-\frac{\sinh(bcx+ac)+1}{\sinh(bcx+ac)-1}\right)+\sqrt{2}\log\left(\frac{3\left(2\sqrt{2}+3\right)\cosh(bcx+ac)^2-4\left(3\sqrt{2}+4\right)\cosh(bcx+ac)\sinh(bcx+ac)+3\left(2\right.}{\cosh(bcx+ac)^2+\sinh(bcx+ac)^2-3}\right.}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(sinh(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log(-(sinh(b*c*x + a*c) + 1)/(
sinh(b*c*x + a*c) - 1)) + sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(b*c*x + a*c)^
2 - 4*(3*sqrt(2) + 4)*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2*sqrt(2) +
3)*sinh(b*c*x + a*c)^2 - 2*sqrt(2) - 3)/(cosh(b*c*x + a*c)^2 + sinh(b*c*x +
 a*c)^2 - 3)) + log(2*(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 - 3)/(cosh
(b*c*x + a*c)^2 - 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b*c*x + a*c)
^2)))/(b*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atanh(sinh(b*c*x+a*c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27287, size = 220, normalized size = 2.06

$$\frac{e^{((bx+a)c)}\log\left(-\frac{e^{(bcx+ac)}-e^{(-bcx-ac)}+2}{e^{(bcx+ac)}-e^{(-bcx-ac)}-2}\right)}{2\,bc}+\frac{\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2\,e^{(2\,bcx+2\,ac)}-6\right|}{\left|4\sqrt{2}+2\,e^{(2\,bcx+2\,ac)}-6\right|}\right)}{2\,bc}+\frac{\log\left(\left|e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1\right|\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(sinh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] 1/2*e^((b*x + a)*c)*log(-(e^(b*c*x + a*c) - e^(-b*c*x - a*c) + 2)/(e^(b*c*x
 + a*c) - e^(-b*c*x - a*c) - 2))/(b*c) + 1/2*sqrt(2)*log(abs(-4*sqrt(2) + 2
```

```
*e^(2*b*c*x + 2*a*c) - 6)/abs(4*sqrt(2) + 2*e^(2*b*c*x + 2*a*c) - 6))/(b*c)
 + 1/2*log(abs(e^(4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1))/(b*c)
```

3.356 $\int e^{c(a+bx)} \tanh^{-1}(\cosh(ac+bcx))\,dx$

Optimal. Leaf size=49

$$\frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}+\frac{e^{ac+bcx}\tanh^{-1}(\cosh(c(a+bx)))}{bc}$$

```
[Out] (E^(a*c + b*c*x)*ArcTanh[Cosh[c*(a + b*x)]])/(b*c) + Log[1 - E^(2*c*(a + b*
x))]/(b*c)
```

Rubi [A] time = 0.0746668, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2194, 6275, 2282, 12, 260}

$$\frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}+\frac{e^{ac+bcx}\tanh^{-1}(\cosh(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTanh[Cosh[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcTanh[Cosh[c*(a + b*x)]])/(b*c) + Log[1 - E^(2*c*(a + b*
x))]/(b*c)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6275

```
Int[((a_.) + ArcTanh[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Dist[a + b*ArcTanh[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/
(1 - u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTanh[u]), x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)}\tanh^{-1}(\cosh(ac+bcx))\,dx &= \frac{\text{Subst}\left(\int e^x\tanh^{-1}(\cosh(x))\,dx,x,ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\tanh^{-1}(\cosh(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int e^x\,\mathbf{csch}(x)\,dx,x,ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\tanh^{-1}(\cosh(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int \frac{2x}{-1+x^2}\,dx,x,e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\tanh^{-1}(\cosh(c(a+bx)))}{bc}+\frac{2\,\text{Subst}\left(\int \frac{x}{-1+x^2}\,dx,x,e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\tanh^{-1}(\cosh(c(a+bx)))}{bc}+\frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}
\end{aligned}$$

Mathematica [A] time = 0.0879195, size = 60, normalized size = 1.22

$$\frac{\log\left(1-e^{2c(a+bx)}\right)+e^{c(a+bx)}\tanh^{-1}\left(\frac{1}{2}e^{-c(a+bx)}\left(e^{2c(a+bx)}+1\right)\right)}{bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcTanh[Cosh[a*c + b*c*x]],x]
```

```
[Out] (E^(c*(a + b*x))*ArcTanh[(1 + E^(2*c*(a + b*x)))/(2*E^(c*(a + b*x)))] + Log
[1 - E^(2*c*(a + b*x))])/(b*c)
```

Maple [C] time = 0.375, size = 887, normalized size = 18.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctanh(cosh(b*c*x+a*c)),x)
```

```
[Out] 1/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))+1)-1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a
))*(exp(c*(b*x+a))+1)^2)^3*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a
))-1)^2)^3*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*(exp(
c*(b*x+a))+1)^2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+1)^2)*exp(c*(b*x+a)
)+1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1)^2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(
b*x+a))+1)^2)^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(
b*x+a))-1)^2)^3*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1))^2*cs
gn(I*(exp(c*(b*x+a))-1)^2)*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a
))+1)^2)^3*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-
c*(b*x+a))*(exp(c*(b*x+a))+1)^2)^2*exp(c*(b*x+a))+1/2*I/b/c*Pi*csgn(I*exp(-
c*(b*x+a))*(exp(c*(b*x+a))-1)^2)^2*exp(c*(b*x+a))-1/2*I/b/c*exp(c*(b*x+a))*
Pi-1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+
a))-1)^2)^2*exp(c*(b*x+a))+1/2*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1))*csgn(I*(
exp(c*(b*x+a))+1)^2)^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1
))^2*csgn(I*(exp(c*(b*x+a))+1)^2)*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*exp(-c
*(b*x+a)))*csgn(I*(exp(c*(b*x+a))-1)^2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+
a))-1)^2)*exp(c*(b*x+a))-1/2*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1))*csgn(I*(ex
p(c*(b*x+a))-1)^2)^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1)^
2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-1)^2)^2*exp(c*(b*x+a))-1/b/c*exp(
c*(b*x+a))*ln(exp(c*(b*x+a))-1)-2*a/b+1/b/c*ln(exp(2*c*(b*x+a))-1)
```

Maxima [A] time = 1.00298, size = 86, normalized size = 1.76

$$\frac{\operatorname{artanh}\left(\cosh\left(bcx+ac\right)\right)e^{\left(\left(bx+a\right)c\right)}}{bc}+\frac{\log\left(e^{\left(bcx+ac\right)}+1\right)}{bc}+\frac{\log\left(e^{\left(bcx+ac\right)}-1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(cosh(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctanh(cosh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + log(e^(b*c*x + a*c) + 1)
/(b*c) + log(e^(b*c*x + a*c) - 1)/(b*c)
```

Fricas [A] time = 1.76039, size = 238, normalized size = 4.86

$$\frac{(\cosh(bcx+ac)+\sinh(bcx+ac))\log\left(-\frac{\cosh(bcx+ac)+1}{\cosh(bcx+ac)-1}\right)+2\log\left(\frac{2\sinh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(cosh(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log(-(cosh(b*c*x + a*c) + 1)/(
cosh(b*c*x + a*c) - 1)) + 2*log(2*sinh(b*c*x + a*c)/(cosh(b*c*x + a*c) - si
nh(b*c*x + a*c))))/(b*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atanh(cosh(b*c*x+a*c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23512, size = 198, normalized size = 4.04

$$\frac{\left(e^{(bcx)}\log\left(-\frac{e^{(2bcx+2ac)}}{e^{(2bcx+2ac)}-2e^{(bcx+ac)}+1}-\frac{2e^{(bcx+ac)}}{e^{(2bcx+2ac)}-2e^{(bcx+ac)}+1}-\frac{1}{e^{(2bcx+2ac)}-2e^{(bcx+ac)}+1}\right)+2e^{(-ac)}\log\left(\left|e^{(2bcx+2ac)}-1\right|\right)\right)e^{(ac)}}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(cosh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] 1/2*(e^(b*c*x)*log(-e^(2*b*c*x + 2*a*c)/(e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x +
 a*c) + 1) - 2*e^(b*c*x + a*c)/(e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x + a*c) + 1
) - 1/(e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x + a*c) + 1)) + 2*e^(-a*c)*log(abs(e
^(2*b*c*x + 2*a*c) - 1)))*e^(a*c)/(b*c)
```

3.357 $\int e^{c(a+bx)}\tanh^{-1}(\tanh(ac+bcx))\,dx$

Optimal. Leaf size=45

$$\frac{e^{ac+bcx}\tanh^{-1}(\tanh(c(a+bx)))}{bc}-\frac{e^{ac+bcx}}{bc}$$

```
[Out] -(E^(a*c + b*c*x)/(b*c)) + (E^(a*c + b*c*x)*ArcTanh[Tanh[c*(a + b*x)]])/(b*
c)
```

Rubi [A] time = 0.0543779, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2194, 6275}

$$\frac{e^{ac+bcx}\tanh^{-1}(\tanh(c(a+bx)))}{bc}-\frac{e^{ac+bcx}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTanh[Tanh[a*c + b*c*x]],x]
```

```
[Out] -(E^(a*c + b*c*x)/(b*c)) + (E^(a*c + b*c*x)*ArcTanh[Tanh[c*(a + b*x)]])/(b*
c)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6275

```
Int[((a_.) + ArcTanh[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Dist[a + b*ArcTanh[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/
(1 - u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTanh[u]), x]]
```

Rubi steps

$$\int e^{c(a+bx)} \tanh^{-1}(\tanh(ac+bcx))\,dx = \frac{\text{Subst}\left(\int e^x \tanh^{-1}(\tanh(x))\,dx, x, ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tanh^{-1}(\tanh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int e^x\,dx, x, ac+bcx\right)}{bc}$$
$$= -\frac{e^{ac+bcx}}{bc} + \frac{e^{ac+bcx}\tanh^{-1}(\tanh(c(a+bx)))}{bc}$$

Mathematica [A] time = 0.0816431, size = 46, normalized size = 1.02

$$\frac{e^{c(a+bx)}\left(\tanh^{-1}\left(\frac{e^{2c(a+bx)}-1}{e^{2c(a+bx)}+1}\right)-1\right)}{bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcTanh[Tanh[a*c + b*c*x]],x]
```

```
[Out] (E^(c*(a + b*x))*(-1 + ArcTanh[(-1 + E^(2*c*(a + b*x)))/(1 + E^(2*c*(a + b*
x)))]))/(b*c)
```

Maple [A] time = 0.038, size = 68, normalized size = 1.5

$$\frac{(xbc+ac)\,e^{xbc+ac} - e^{xbc+ac} + e^{xbc+ac}\left(\text{Artanh}\left(\tanh\left(xbc+ac\right)\right) - xbc - ac\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctanh(tanh(b*c*x+a*c)),x)
```

```
[Out] 1/b/c*((b*c*x+a*c)*exp(b*c*x+a*c)-exp(b*c*x+a*c)+exp(b*c*x+a*c)*(arctanh(ta
nh(b*c*x+a*c))-x*b*c-a*c))
```

Maxima [A] time = 0.994307, size = 58, normalized size = 1.29

$$\frac{\text{artanh}\left(\tanh\left(bcx+ac\right)\right)e^{((bx+a)c)}}{bc} - \frac{e^{(bcx+ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(tanh(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctanh(tanh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - e^(b*c*x + a*c)/(b*c)
```

Fricas [A] time = 1.73324, size = 112, normalized size = 2.49

$$\frac{(bcx + ac - 1)\cosh(bcx + ac) + (bcx + ac - 1)\sinh(bcx + ac)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(tanh(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] ((b*c*x + a*c - 1)*cosh(b*c*x + a*c) + (b*c*x + a*c - 1)*sinh(b*c*x + a*c))
/(b*c)
```

Sympy [A] time = 11.3783, size = 58, normalized size = 1.29

$$\begin{cases} 0 & \text{for } c = 0 \wedge (b = 0 \vee c = 0) \\ xe^{ac}\operatorname{atanh}(\tanh(ac)) & \text{for } b = 0 \\ \frac{e^{ac}e^{bcx}\operatorname{atanh}(\tanh(ac+bcx))}{bc} - \frac{e^{ac}e^{bcx}}{bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atanh(tanh(b*c*x+a*c)),x)
```

```
[Out] Piecewise((0, Eq(c, 0) & (Eq(b, 0) | Eq(c, 0))), (x*exp(a*c)*atanh(tanh(a*c
)), Eq(b, 0)), (exp(a*c)*exp(b*c*x)*atanh(tanh(a*c + b*c*x))/(b*c) - exp(a*
c)*exp(b*c*x)/(b*c), True))
```

Giac [A] time = 1.10506, size = 47, normalized size = 1.04

$$\frac{\left(b^2c^2x + abc^2 - bc\right)e^{(bcx+ac)}}{b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(tanh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] (b^2*c^2*x + a*b*c^2 - b*c)*e^(b*c*x + a*c)/(b^2*c^2)
```

3.358 $\int e^{c(a+bx)} \tanh^{-1}(\coth(ac + bcx))\, dx$

Optimal. Leaf size=45

$$\frac{e^{ac+bcx}\tanh^{-1}(\coth(c(a+bx)))}{bc} - \frac{e^{ac+bcx}}{bc}$$

```
[Out] -(E^(a*c + b*c*x)/(b*c)) + (E^(a*c + b*c*x)*ArcTanh[Coth[c*(a + b*x)]])/(b*
c)
```

Rubi [A] time = 0.0543121, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2194, 6275}

$$\frac{e^{ac+bcx}\tanh^{-1}(\coth(c(a+bx)))}{bc} - \frac{e^{ac+bcx}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTanh[Coth[a*c + b*c*x]],x]
```

```
[Out] -(E^(a*c + b*c*x)/(b*c)) + (E^(a*c + b*c*x)*ArcTanh[Coth[c*(a + b*x)]])/(b*
c)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6275

```
Int[((a_.) + ArcTanh[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Dist[a + b*ArcTanh[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/
(1 - u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTanh[u]), x]]
```

Rubi steps

$$\int e^{c(a+bx)}\tanh^{-1}(\coth(ac+bcx))\,dx = \frac{\text{Subst}\left(\int e^x\tanh^{-1}(\coth(x))\,dx,x,ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tanh^{-1}(\coth(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int e^x\,dx,x,ac+bcx\right)}{bc}$$
$$= -\frac{e^{ac+bcx}}{bc} + \frac{e^{ac+bcx}\tanh^{-1}(\coth(c(a+bx)))}{bc}$$

Mathematica [A] time = 0.0832905, size = 46, normalized size = 1.02

$$\frac{e^{c(a+bx)}\left(\tanh^{-1}\left(\frac{e^{2c(a+bx)}+1}{e^{2c(a+bx)}-1}\right)-1\right)}{bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcTanh[Coth[a*c + b*c*x]],x]
```

```
[Out] (E^(c*(a + b*x))*(-1 + ArcTanh[(1 + E^(2*c*(a + b*x)))/(-1 + E^(2*c*(a + b*
x)))]))/(b*c)
```

Maple [C] time = 0.257, size = 351, normalized size = 7.8

$$\frac{e^{c(bx+a)}\ln\left(e^{c(bx+a)}\right)}{bc} + \frac{\frac{i}{4}e^{c(bx+a)}}{bc}\left(2\pi\left(\text{csgn}\left(\frac{i}{e^{2c(bx+a)}-1}\right)\right)^2 - 2\pi\left(\text{csgn}\left(\frac{i}{e^{2c(bx+a)}-1}\right)\right)^3 - \pi\,\text{csgn}\left(\frac{i}{e^{2c(bx+a)}-1}\right)\text{csg}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctanh(coth(b*c*x+a*c)),x)
```

```
[Out] 1/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a)))+1/4*I*(2*Pi*csgn(I/(exp(2*c*(b*x+a)
)-1))^2-2*Pi*csgn(I/(exp(2*c*(b*x+a))-1))^3-Pi*csgn(I/(exp(2*c*(b*x+a))-1))
*csgn(I*exp(2*c*(b*x+a)))*csgn(I*exp(2*c*(b*x+a))/(exp(2*c*(b*x+a))-1))+Pi*
csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*exp(2*c*(b*x+a))/(exp(2*c*(b*x+a))-1))^
2-Pi*csgn(I*exp(c*(b*x+a)))^2*csgn(I*exp(2*c*(b*x+a)))+2*Pi*csgn(I*exp(c*(b
*x+a)))*csgn(I*exp(2*c*(b*x+a)))^2-Pi*csgn(I*exp(2*c*(b*x+a)))^3+Pi*csgn(I*
exp(2*c*(b*x+a)))*csgn(I*exp(2*c*(b*x+a))/(exp(2*c*(b*x+a))-1))^2-Pi*csgn(I
*exp(2*c*(b*x+a))/(exp(2*c*(b*x+a))-1))^3+4*I-2*Pi)/b/c*exp(c*(b*x+a))
```

Maxima [A] time = 0.986189, size = 58, normalized size = 1.29

$$\frac{\operatorname{artanh}\left(\coth\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc}-\frac{e^{(bcx+ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(coth(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctanh(coth(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - e^(b*c*x + a*c)/(b*c)
```

Fricas [A] time = 1.93226, size = 55, normalized size = 1.22

$$\frac{(bcx+ac-1)e^{(bcx+ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(coth(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] (b*c*x + a*c - 1)*e^(b*c*x + a*c)/(b*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atanh(coth(b*c*x+a*c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19302, size = 54, normalized size = 1.2

$$\frac{\left(e^{(bcx)}\log\left(-e^{(2\,bcx+2\,ac)}\right)-2\,e^{(bcx)}\right)e^{(ac)}}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(coth(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] 1/2*(e^(b*c*x)*log(-e^(2*b*c*x + 2*a*c)) - 2*e^(b*c*x))*e^(a*c)/(b*c)
```

3.359 $\int e^{c(a+bx)} \tanh^{-1}(\text{sech}(ac + bcx))\, dx$

Optimal. Leaf size=49

$$\frac{\log\left(1 - e^{2c(a+bx)}\right)}{bc} + \frac{e^{ac+bcx}\tanh^{-1}(\text{sech}(c(a+bx)))}{bc}$$

```
[Out] (E^(a*c + b*c*x)*ArcTanh[Sech[c*(a + b*x)]])/(b*c) + Log[1 - E^(2*c*(a + b*
x))]/(b*c)
```

Rubi [A] time = 0.0678644, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2194, 6275, 2282, 12, 260}

$$\frac{\log\left(1 - e^{2c(a+bx)}\right)}{bc} + \frac{e^{ac+bcx}\tanh^{-1}(\text{sech}(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTanh[Sech[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcTanh[Sech[c*(a + b*x)]])/(b*c) + Log[1 - E^(2*c*(a + b*
x))]/(b*c)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6275

```
Int[((a_.) + ArcTanh[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Dist[a + b*ArcTanh[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/
(1 - u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTanh[u]), x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int e^{c(a+bx)} \tanh^{-1}(\mathbf{sech}(ac+bcx))\,dx &= \frac{\text{Subst}\left(\int e^x \tanh^{-1}(\mathbf{sech}(x))\,dx, x, ac+bcx\right)}{bc} \\
&= \frac{e^{ac+bcx}\tanh^{-1}(\mathbf{sech}(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int e^x \mathbf{csch}(x)\,dx, x, ac+bcx\right)}{bc} \\
&= \frac{e^{ac+bcx}\tanh^{-1}(\mathbf{sech}(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{2x}{-1+x^2}\,dx, x, e^{ac+bcx}\right)}{bc} \\
&= \frac{e^{ac+bcx}\tanh^{-1}(\mathbf{sech}(c(a+bx)))}{bc} + \frac{2\,\text{Subst}\left(\int \frac{x}{-1+x^2}\,dx, x, e^{ac+bcx}\right)}{bc} \\
&= \frac{e^{ac+bcx}\tanh^{-1}(\mathbf{sech}(c(a+bx)))}{bc} + \frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}
\end{aligned}
$$

Mathematica [A] time = 0.0825592, size = 59, normalized size = 1.2

$$
\frac{\log\left(1-e^{2c(a+bx)}\right)+e^{c(a+bx)}\tanh^{-1}\left(\frac{2e^{c(a+bx)}}{e^{2c(a+bx)}+1}\right)}{bc}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcTanh[Sech[a*c + b*c*x]],x]
```

```
[Out] (E^(c*(a + b*x))*ArcTanh[(2*E^(c*(a + b*x)))/(1 + E^(2*c*(a + b*x)))] + Log
[1 - E^(2*c*(a + b*x))])/(b*c)
```

Maple [C] time = 0.335, size = 872, normalized size = 17.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctanh(sech(b*c*x+a*c)),x)
```

```
[Out] 1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1))^2*csgn(I*(exp(c*(b*x+a))-1)^2)*exp(
c*(b*x+a))+1/2*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1))*csgn(I*(exp(c*(b*x+a))+1
)^2)^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))+1))*csgn(I*(exp
(c*(b*x+a))-1)^2/(exp(2*c*(b*x+a))+1))^2*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I
/(exp(2*c*(b*x+a))+1))*csgn(I*(exp(c*(b*x+a))+1)^2/(exp(2*c*(b*x+a))+1))^2*
exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1))^2*csgn(I*(exp(c*(b*x
+a))+1)^2)*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1)^2/(exp(2*c
*(b*x+a))+1))^3*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1)^2)^3*
exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1)^2)*csgn(I*(exp(c*(b*x
+a))-1)^2/(exp(2*c*(b*x+a))+1))^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp(c
*(b*x+a))+1)^2)*csgn(I/(exp(2*c*(b*x+a))+1))*csgn(I*(exp(c*(b*x+a))+1)^2/(e
xp(2*c*(b*x+a))+1))*exp(c*(b*x+a))-1/2*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1))*
csgn(I*(exp(c*(b*x+a))-1)^2)^2*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(c*(b
*x+a))-1)^2/(exp(2*c*(b*x+a))+1))^3*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp
(c*(b*x+a))+1)^2)^3*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1)^2
)*csgn(I*(exp(c*(b*x+a))+1)^2/(exp(2*c*(b*x+a))+1))^2*exp(c*(b*x+a))+1/4*I/
b/c*Pi*csgn(I*(exp(c*(b*x+a))-1)^2)*csgn(I/(exp(2*c*(b*x+a))+1))*csgn(I*(ex
p(c*(b*x+a))-1)^2/(exp(2*c*(b*x+a))+1))*exp(c*(b*x+a))-1/b/c*exp(c*(b*x+a))
*ln(exp(c*(b*x+a))-1)+1/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))+1)-2*a/b+1/b/c
*ln(exp(2*c*(b*x+a))-1)
```

Maxima [A] time = 0.999006, size = 86, normalized size = 1.76

$$\frac{\operatorname{artanh}\left(\operatorname{sech}\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc}+\frac{\log\left(e^{(bcx+ac)}+1\right)}{bc}+\frac{\log\left(e^{(bcx+ac)}-1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(sech(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctanh(sech(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + log(e^(b*c*x + a*c) + 1)
/(b*c) + log(e^(b*c*x + a*c) - 1)/(b*c)
```

Fricas [A] time = 1.94412, size = 236, normalized size = 4.82

$$\frac{\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\log\left(\frac{\cosh(bcx+ac)+1}{\cosh(bcx+ac)-1}\right)+2\,\log\left(\frac{2\,\sinh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(sech(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log((cosh(b*c*x + a*c) + 1)/(c
osh(b*c*x + a*c) - 1)) + 2*log(2*sinh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sin
h(b*c*x + a*c))))/(b*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atanh(sech(b*c*x+a*c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18145, size = 132, normalized size = 2.69

$$\frac{e^{((bx+a)c)}\log\left(-\frac{\frac{2}{e^{(bcx+ac)}+e^{(-bcx-ac)}}+1}{\frac{2}{e^{(bcx+ac)}+e^{(-bcx-ac)}}-1}\right)}{2\,bc}+\frac{\log\left(\left|e^{(2\,bcx+2\,ac)}-1\right|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(sech(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] 1/2*e^((b*x + a)*c)*log(-(2/(e^(b*c*x + a*c) + e^(-b*c*x - a*c)) + 1)/(2/(e
^(b*c*x + a*c) + e^(-b*c*x - a*c)) - 1))/(b*c) + log(abs(e^(2*b*c*x + 2*a*c
) - 1))/(b*c)
```

3.360 $\int e^{c(a+bx)}\tanh^{-1}(\mathbf{csch}(ac+bcx))\,dx$

Optimal. Leaf size=107

$$\frac{\left(1-\sqrt{2}\right)\log\left(-e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc}+\frac{\left(1+\sqrt{2}\right)\log\left(-e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc}+\frac{e^{ac+bcx}\tanh^{-1}(\text{csch}(c(a+bx)))}{bc}$$

```
[Out] (E^(a*c + b*c*x)*ArcTanh[Csch[c*(a + b*x)]])/(b*c) + ((1 - Sqrt[2])*Log[3 -
 2*Sqrt[2] - E^(2*c*(a + b*x))])/(2*b*c) + ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2]
 - E^(2*c*(a + b*x))])/(2*b*c)
```

Rubi [A] time = 0.170098, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {2194, 6275, 2282, 12, 1247, 632, 31}

$$\frac{\left(1-\sqrt{2}\right)\log\left(-e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc}+\frac{\left(1+\sqrt{2}\right)\log\left(-e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc}+\frac{e^{ac+bcx}\tanh^{-1}(\text{csch}(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTanh[Csch[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcTanh[Csch[c*(a + b*x)]])/(b*c) + ((1 - Sqrt[2])*Log[3 -
 2*Sqrt[2] - E^(2*c*(a + b*x))])/(2*b*c) + ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2]
 - E^(2*c*(a + b*x))])/(2*b*c)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6275

```
Int[((a_.) + ArcTanh[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Dist[a + b*ArcTanh[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/
(1 - u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTanh[u]), x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int e^{c(a+bx)}\tanh^{-1}(\operatorname{csch}(ac+bcx))\,dx = \frac{\text{Subst}\left(\int e^x \tanh^{-1}(\operatorname{csch}(x))\,dx, x, ac+bcx\right)}{bc}$$

$$= \frac{e^{ac+bcx}\tanh^{-1}(\operatorname{csch}(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{e^x \coth(x)\operatorname{csch}(x)}{1-\operatorname{csch}^2(x)}\,dx, x, ac+bcx\right)}{bc}$$

$$= \frac{e^{ac+bcx}\tanh^{-1}(\operatorname{csch}(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{2x(1+x^2)}{1-6x^2+x^4}\,dx, x, e^{ac+bcx}\right)}{bc}$$

$$= \frac{e^{ac+bcx}\tanh^{-1}(\operatorname{csch}(c(a+bx)))}{bc} + \frac{2\,\text{Subst}\left(\int \frac{x(1+x^2)}{1-6x^2+x^4}\,dx, x, e^{ac+bcx}\right)}{bc}$$

$$= \frac{e^{ac+bcx}\tanh^{-1}(\operatorname{csch}(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{1+x}{1-6x+x^2}\,dx, x, e^{2ac+2bcx}\right)}{bc}$$

$$= \frac{e^{ac+bcx}\tanh^{-1}(\operatorname{csch}(c(a+bx)))}{bc} + \frac{\left(1-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-3+2\sqrt{2}+x}\,dx, x, e^{2ac+2bcx}\right)}{2bc} +$$

$$= \frac{e^{ac+bcx}\tanh^{-1}(\operatorname{csch}(c(a+bx)))}{bc} + \frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}-e^{2ac+2bcx}\right)}{2bc} + \frac{\left(1+\sqrt{2}\right.}{}$$

Mathematica [A] time = 0.172125, size = 150, normalized size = 1.4

$$\frac{\log\left(-2e^{c(a+bx)}-e^{2c(a+bx)}+1\right)+\log\left(2e^{c(a+bx)}-e^{2c(a+bx)}+1\right)-2\sqrt{2}\tanh^{-1}\left(\frac{e^{c(a+bx)}-1}{\sqrt{2}}\right)+2\sqrt{2}\tanh^{-1}\left(\frac{e^{c(a+bx)}+1}{\sqrt{2}}\right)+2e^{c(a+b}}{2bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcTanh[Csch[a*c + b*c*x]],x]
```

```
[Out] (-2*Sqrt[2]*ArcTanh[(-1 + E^(c*(a + b*x)))/Sqrt[2]] + 2*Sqrt[2]*ArcTanh[(1
+ E^(c*(a + b*x)))/Sqrt[2]] + 2*E^(c*(a + b*x))*ArcTanh[(2*E^(c*(a + b*x)))
/(-1 + E^(2*c*(a + b*x)))] + Log[1 - 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x))]
 + Log[1 + 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x))])/(2*b*c)
```

Maple [C] time = 0.385, size = 842, normalized size = 7.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctanh(csch(b*c*x+a*c)),x)
```

```
[Out] 1/2/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1)+1/4*I/b/c*Pi
*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))*csgn(I/(exp(2*c*(b*x+a))-1)
)*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1)/(exp(2*c*(b*x+a))-1))*exp(c
*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))*csgn(
I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x
+a))-1/4*I/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(-exp(2*c*(b*x+a))+2*
exp(c*(b*x+a))+1)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(
I*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(
I/(exp(2*c*(b*x+a))-1)*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*exp(c*(b*x+a)
)+1/4*I/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I/(exp(2*c*(b*x+a))-1)*(ex
p(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(-
exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1)/(exp(2*c*(b*x+a))-1))^3*exp(c*(b*x+a))
+1/4*I/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*csgn(I/(exp(2*c
*(b*x+a))-1)*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4*I/
b/c*Pi*csgn(I/(exp(2*c*(b*x+a))-1)*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))^3
*exp(c*(b*x+a))-1/2/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))-2*exp(c*(b*x+a))
-1)+1/2/b/c*ln(exp(2*c*(b*x+a))-(1+2^(1/2))^2)*2^(1/2)-1/2/b/c*ln(exp(2*c*(
b*x+a))-(2^(1/2)-1)^2)*2^(1/2)-2*a/b+1/2/b/c*ln(exp(2*c*(b*x+a))-(1+2^(1/2)
)^2)+1/2/b/c*ln(exp(2*c*(b*x+a))-(2^(1/2)-1)^2)
```

<u>Maxima</u> [B] time = 1.47965, size = 248, normalized size = 2.32

$$\frac{\operatorname{artanh}\left(\operatorname{csch}\left(bcx+ac\right)\right)e^{\left(\left(bx+a\right)c\right)}}{bc}+\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{\left(bcx+ac\right)}+1}{\sqrt{2}+e^{\left(bcx+ac\right)}-1}\right)}{2\,bc}-\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{\left(bcx+ac\right)}-1}{\sqrt{2}+e^{\left(bcx+ac\right)}+1}\right)}{2\,bc}+\frac{\log\left(e^{\left(2\,bcx+2\,ac\right)}+2\,e^{\left(bcx+ac\right)}-\right.}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(csch(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctanh(csch(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + 1/2*sqrt(2)*log(-(sqrt(2
) - e^(b*c*x + a*c) + 1)/(sqrt(2) + e^(b*c*x + a*c) - 1))/(b*c) - 1/2*sqrt(
2)*log(-(sqrt(2) - e^(b*c*x + a*c) - 1)/(sqrt(2) + e^(b*c*x + a*c) + 1))/(b
*c) + 1/2*log(e^(2*b*c*x + 2*a*c) + 2*e^(b*c*x + a*c) - 1)/(b*c) + 1/2*log(
e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x + a*c) - 1)/(b*c)
```

Fricas [B] time = 1.76078, size = 624, normalized size = 5.83

$$\frac{(\cosh(bcx+ac)+\sinh(bcx+ac))\log\left(\frac{\sinh(bcx+ac)+1}{\sinh(bcx+ac)-1}\right)+\sqrt{2}\log\left(\frac{3\,(2\,\sqrt{2}+3)\cosh(bcx+ac)^2-4\,(3\,\sqrt{2}+4)\cosh(bcx+ac)\sinh(bcx+ac)+3\,(2\,\cdots}{\cosh(bcx+ac)^2+\sinh(bcx+ac)^2-3}\right.}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(csch(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log((sinh(b*c*x + a*c) + 1)/(s
inh(b*c*x + a*c) - 1)) + sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(b*c*x + a*c)^2
 - 4*(3*sqrt(2) + 4)*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2*sqrt(2) + 3
)*sinh(b*c*x + a*c)^2 - 2*sqrt(2) - 3)/(cosh(b*c*x + a*c)^2 + sinh(b*c*x +
a*c)^2 - 3)) + log(2*(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 - 3)/(cosh(
b*c*x + a*c)^2 - 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b*c*x + a*c)^
2)))/(b*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{ac}\int e^{bcx}\operatorname{atanh}(\operatorname{csch}(ac+bcx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atanh(csch(b*c*x+a*c)),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*atanh(csch(a*c + b*c*x)), x)
```

Giac [A] time = 1.25607, size = 234, normalized size = 2.19

$$\frac{e^{((bx+a)c)}\log\left(-\frac{\frac{2}{e^{(bcx+ac)}-e^{(-bcx-ac)}}+1}{\frac{2}{e^{(bcx+ac)}-e^{(-bcx-ac)}}-1}\right)}{2\,bc}+\frac{\sqrt{2}\log\left(\frac{\left|-4\,\sqrt{2}+2\,e^{(2\,bcx+2\,ac)}-6\right|}{\left|4\,\sqrt{2}+2\,e^{(2\,bcx+2\,ac)}-6\right|}\right)}{2\,bc}+\frac{\log\left(\left|e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1\right|\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(csch(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] 1/2*e^((b*x + a)*c)*log(-(2/(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + 1)/(2/(e
^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 1))/(b*c) + 1/2*sqrt(2)*log(abs(-4*sqr
t(2) + 2*e^(2*b*c*x + 2*a*c) - 6)/abs(4*sqrt(2) + 2*e^(2*b*c*x + 2*a*c) - 6
))/(b*c) + 1/2*log(abs(e^(4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1))/(b
*c)
```

3.361 $\int \frac{\left(a+b\tanh^{-1}(cx^n)\right)\left(d+e\log\left(fx^m\right)\right)}{x}\,dx$

Optimal. Leaf size=136

$$-\frac{bd\text{PolyLog}(2,-cx^n)}{2n}+\frac{bd\text{PolyLog}(2,cx^n)}{2n}-\frac{be\log\left(fx^m\right)\text{PolyLog}(2,-cx^n)}{2n}+\frac{be\log\left(fx^m\right)\text{PolyLog}(2,cx^n)}{2n}+\frac{bem}{}$$

```
[Out] a*d*Log[x] + (a*e*Log[f*x^m]^2)/(2*m) - (b*d*PolyLog[2, -(c*x^n)])/(2*n) -
(b*e*Log[f*x^m]*PolyLog[2, -(c*x^n)])/(2*n) + (b*d*PolyLog[2, c*x^n])/(2*n)
 + (b*e*Log[f*x^m]*PolyLog[2, c*x^n])/(2*n) + (b*e*m*PolyLog[3, -(c*x^n)])/
(2*n^2) - (b*e*m*PolyLog[3, c*x^n])/(2*n^2)
```

Rubi [A] time = 0.537622, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2301, 6742, 6095, 5912, 6071, 6069, 2374, 6589}

$$-\frac{bd\text{PolyLog}(2,-cx^n)}{2n}+\frac{bd\text{PolyLog}(2,cx^n)}{2n}-\frac{be\log\left(fx^m\right)\text{PolyLog}(2,-cx^n)}{2n}+\frac{be\log\left(fx^m\right)\text{PolyLog}(2,cx^n)}{2n}+\frac{bem}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTanh[c*x^n])*(d + e*Log[f*x^m]))/x,x]
```

```
[Out] a*d*Log[x] + (a*e*Log[f*x^m]^2)/(2*m) - (b*d*PolyLog[2, -(c*x^n)])/(2*n) -
(b*e*Log[f*x^m]*PolyLog[2, -(c*x^n)])/(2*n) + (b*d*PolyLog[2, c*x^n])/(2*n)
 + (b*e*Log[f*x^m]*PolyLog[2, c*x^n])/(2*n) + (b*e*m*PolyLog[3, -(c*x^n)])/
(2*n^2) - (b*e*m*PolyLog[3, c*x^n])/(2*n^2)
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6071

```
Int[(Log[(d_.)*(x_)^(m_.)]*(ArcTanh[(c_.)*(x_)^(n_.)]*(b_.) + (a_)))/(x_),
x_Symbol] :> Dist[a, Int[Log[d*x^m]/x, x], x] + Dist[b, Int[(Log[d*x^m]*Arc
Tanh[c*x^n])/x, x], x] /; FreeQ[{a, b, c, d, m, n}, x]
```

Rule 6069

```
Int[(ArcTanh[(c_.)*(x_)^(n_.)]*Log[(d_.)*(x_)^(m_.)])/(x_), x_Symbol] :> Di
st[1/2, Int[(Log[d*x^m]*Log[1 + c*x^n])/x, x], x] - Dist[1/2, Int[(Log[d*x^
m]*Log[1 - c*x^n])/x, x], x] /; FreeQ[{c, d, m, n}, x]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx^n)\right)\left(d+e\log\left(fx^m\right)\right)}{x}\,dx = \int \left(\frac{d\left(a+b\tanh^{-1}(cx^n)\right)}{x}+\frac{e\left(a+b\tanh^{-1}(cx^n)\right)\log\left(fx^m\right)}{x}\right)dx$$

$$= d\int \frac{a+b\tanh^{-1}(cx^n)}{x}\,dx + e\int \frac{\left(a+b\tanh^{-1}(cx^n)\right)\log\left(fx^m\right)}{x}\,dx$$

$$= (ae)\int \frac{\log\left(fx^m\right)}{x}\,dx + (be)\int \frac{\tanh^{-1}(cx^n)\log\left(fx^m\right)}{x}\,dx + \frac{d\,\mathrm{Subst}\left(\int \frac{a\cdots}{}\right.}{}$$

$$= ad\log(x) + \frac{ae\log^2\left(fx^m\right)}{2m} - \frac{bd\mathrm{Li}_2(-cx^n)}{2n} + \frac{bd\mathrm{Li}_2(cx^n)}{2n} - \frac{1}{2}(be)\int \frac{\log\left(\cdots\right.}{}$$

$$= ad\log(x) + \frac{ae\log^2\left(fx^m\right)}{2m} - \frac{bd\mathrm{Li}_2(-cx^n)}{2n} - \frac{be\log\left(fx^m\right)\mathrm{Li}_2(-cx^n)}{2n} + \frac{bd\cdots}{}$$

$$= ad\log(x) + \frac{ae\log^2\left(fx^m\right)}{2m} - \frac{bd\mathrm{Li}_2(-cx^n)}{2n} - \frac{be\log\left(fx^m\right)\mathrm{Li}_2(-cx^n)}{2n} + \frac{bd\cdots}{}$$

Mathematica [C] time = 0.283437, size = 114, normalized size = 0.84

$$\frac{bcx^n\left(d+e\log\left(fx^m\right)\right)\text{HypergeometricPFQ}\left(\left\{\frac{1}{2},\frac{1}{2},1\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},c^2x^{2n}\right)}{n} - \frac{bcemx^n\text{HypergeometricPFQ}\left(\left\{\frac{1}{2},\frac{1}{2},\frac{1}{2},1\right\},\left\{\cdots\right.\right.}{n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcTanh[c*x^n])*(d + e*Log[f*x^m]))/x,x]
```

```
[Out] -((b*c*e*m*x^n*HypergeometricPFQ[{1/2, 1/2, 1/2, 1}, {3/2, 3/2, 3/2}, c^2*x
^(2*n)])/n^2) + (b*c*x^n*HypergeometricPFQ[{1/2, 1/2, 1}, {3/2, 3/2}, c^2*x
^(2*n)]*(d + e*Log[f*x^m]))/n + (a*Log[x]*(2*d - e*m*Log[x] + 2*e*Log[f*x^m
]))/2
```

Maple [C] time = 0.285, size = 668, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^n))*(d+e*ln(f*x^m))/x,x)
```

```
[Out] 1/4*I/n*dilog(1-c*x^n)*Pi*b*e*csgn(I*f)*csgn(I*f*x^m)^2+1/4*I/n*Pi*dilog(c*
x^n+1)*b*e*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/2*I/n*Pi*ln(x^n)*a*e*csgn(
I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/2*I/n*Pi*ln(x^n)*a*e*csgn(I*x^m)*csgn(I*f*
x^m)^2+1/4*I/n*dilog(1-c*x^n)*Pi*b*e*csgn(I*x^m)*csgn(I*f*x^m)^2-1/4*I/n*Pi
*dilog(c*x^n+1)*b*e*csgn(I*x^m)*csgn(I*f*x^m)^2+1/2*I/n*Pi*ln(x^n)*a*e*csgn
(I*f)*csgn(I*f*x^m)^2-1/2*I/n*Pi*ln(x^n)*a*e*csgn(I*f*x^m)^3+1/4*I/n*Pi*dil
og(c*x^n+1)*b*e*csgn(I*f*x^m)^3+1/2*e*b/n*ln(1-c*x^n)*ln(c*x^n)*m*ln(x)+1/2
*e*b/n*m*ln(x)*polylog(2,c*x^n)-1/2*e*b/n*ln(1-c*x^n)*ln(c*x^n)*ln(x^m)+1/2
*e*b/n*dilog(c*x^n)*m*ln(x)+1/2*b*e*m*polylog(3,-c*x^n)/n^2-1/2*b*e*m*polyl
og(3,c*x^n)/n^2-1/4*I/n*Pi*dilog(c*x^n+1)*b*e*csgn(I*f)*csgn(I*f*x^m)^2-1/2
*e*b/n*dilog(c*x^n)*ln(x^m)+1/2/n*dilog(1-c*x^n)*ln(f)*b*e-1/2/n*ln(f)*dilo
g(c*x^n+1)*b*e+1/n*ln(f)*ln(x^n)*a*e-1/2*e*b/n*dilog(c*x^n+1)*ln(x^m)-1/4*I
/n*dilog(1-c*x^n)*Pi*b*e*csgn(I*f*x^m)^3-1/2*e*b/n*m*ln(x)*polylog(2,-c*x^n
)+1/2*e*b/n*dilog(c*x^n+1)*m*ln(x)-1/2/n*dilog(c*x^n+1)*b*d+1/n*ln(x^n)*a*d
+1/2*e*a/m*ln(x^m)^2-1/4*I/n*dilog(1-c*x^n)*Pi*b*e*csgn(I*f)*csgn(I*x^m)*cs
gn(I*f*x^m)+1/2/n*dilog(1-c*x^n)*b*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{ae\log\left(fx^m\right)^2}{2\,m} + ad\log(x) - \frac{1}{4}\left(bem\log(x)^2 - 2\,be\log(x)\log(x^m) - 2\left(e\log\left(f\right) + d\right)b\log(x)\right)\log(cx^n + 1) + \frac{1}{4}\left(bem\,\mathrm{lo}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*e*log(f*x^m)^2/m + a*d*log(x) - 1/4*(b*e*m*log(x)^2 - 2*b*e*log(x)*lo
g(x^m) - 2*(e*log(f) + d)*b*log(x))*log(c*x^n + 1) + 1/4*(b*e*m*log(x)^2 -
2*b*e*log(x)*log(x^m) - 2*(e*log(f) + d)*b*log(x))*log(-c*x^n + 1) + integr
ate(1/2*(2*b*c*e*n*x^n*log(x)*log(x^m) - (b*c*e*m*n*log(x)^2 - 2*(e*n*log(f
) + d*n)*b*c*log(x))*x^n)/(c^2*x*x^(2*n) - x), x)
```

Fricas [C] time = 1.75005, size = 980, normalized size = 7.21

$$\frac{2\,aemn^2\log(x)^2 - 2\,bem\mathrm{polylog}\left(3, c\cosh\left(n\log(x)\right) + c\sinh\left(n\log(x)\right)\right) + 2\,bem\mathrm{polylog}\left(3, -c\cosh\left(n\log(x)\right) - c\,\mathrm{sin}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*e*m*n^2*log(x)^2 - 2*b*e*m*polylog(3, c*cosh(n*log(x)) + c*sinh(n*
log(x))) + 2*b*e*m*polylog(3, -c*cosh(n*log(x)) - c*sinh(n*log(x))) + 2*(b*
e*m*n*log(x) + b*e*n*log(f) + b*d*n)*dilog(c*cosh(n*log(x)) + c*sinh(n*log(
x))) - 2*(b*e*m*n*log(x) + b*e*n*log(f) + b*d*n)*dilog(-c*cosh(n*log(x)) -
c*sinh(n*log(x))) - (b*e*m*n^2*log(x)^2 + 2*(b*e*n^2*log(f) + b*d*n^2)*log(
x))*log(c*cosh(n*log(x)) + c*sinh(n*log(x)) + 1) + (b*e*m*n^2*log(x)^2 + 2*
(b*e*n^2*log(f) + b*d*n^2)*log(x))*log(-c*cosh(n*log(x)) - c*sinh(n*log(x))
 + 1) + 4*(a*e*n^2*log(f) + a*d*n^2)*log(x) + (b*e*m*n^2*log(x)^2 + 2*(b*e*
n^2*log(f) + b*d*n^2)*log(x))*log(-(c*cosh(n*log(x)) + c*sinh(n*log(x)) + 1
)/(c*cosh(n*log(x)) + c*sinh(n*log(x)) - 1)))/n^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**n))*(d+e*ln(f*x**m))/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{artanh}\,(cx^n)+a)\left(e\log\left(fx^m\right)+d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)*(e*log(f*x^m) + d)/x, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
    Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: # isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```